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The Regulation of Dual Trading:
Winners, Losers and Market Impact - Updated & Revised

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Winners, Losers and Market Impact***

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**The Regulation of Dual Trading:
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Abstract

Dual trading reduces the net order flow and market depth, but has no effect on the expected total trading volume and price efficiency. Trading volume and gross (of commission fees) profits of the informed traders are lower with dual trading while trading volume and gross losses of the uninformed traders are unaffected. The effect of dual trading on the uninformed traders is the same irrespective of whether they act as noise traders or as rational, risk-averse hedgers.

When the broker's commission income is independent of her customer's trading volume, the competitive commission fee is lower with dual trading. The utility of the uninformed traders (net of commission fees) increases with dual trading while the net profits of the informed traders decrease. If the commission income depends on the trading volume, however, the competitive commission fee may increase with dual trading. This is because the decrease in informed trading volume may reduce the broker's expected commission income more than the amount of her trading profits.

Dual trading refers to the practice of brokers trading for their own accounts in addition to bringing their customers' orders to the market. The practice has, over the years, been controversial with proponents of dual trading vouching for its salutary effects on market liquidity and price efficiency and opponents emphasizing the potential conflicts of interest between dual trading brokers and their customers. On the regulatory front, the anti-dual-trading camp currently holds sway. In March 1993, the Commodities Futures Trading Commission (CFTC) proposed limiting dual trading in the largest futures markets. The Chicago Mercantile Exchange (CME) banned dual trading in all active contracts effective May 20, 1991. But, the issue is by no means settled, as the Exchange still faces great resentment over the ban.¹

I develop a model to study how a ban on dual trading will affect aggregate market characteristics (market liquidity and price efficiency). I also study its distributional effects by looking at the impact on brokers' commission fees as well as the trading volumes and profits (both gross and net of commission costs) of the informed and the uninformed traders. The main contribution of this paper lies in highlighting the effect of dual trading on trading volume. In particular, I show that the trading volume of the informed traders is reduced because the broker mimics or piggybacks on the informed trades.² Because their orders are now executed at a higher (in absolute value) price,³ the broker's piggybacking makes it costly for the informed traders to buy and sell the amounts they would have chosen to trade in the absence of dual trading.

An important consequence of this result is that the competitive commission fee charged by the broker could be higher with dual trading.

This is because the broker loses commission income due to the reduction in informed trading volume and this loss may more than offset her trading profits. This possibility has not been pointed out in previous studies of dual trading (for example, Fishman and Longstaff (1992)).

The effect of dual trading on market liquidity is mixed. Liquidity is hurt because dual trading reduces the market depth. The reason is that the broker, in addition to mimicking informed trades, also offsets a portion of the uninformed order flow. The total effect is to decrease the net order flow and increase the adverse selection problem for the marketmaker (who is assumed to observe the net order flow only). Yet, the uninformed trading volume is a higher proportion of the total expected trading volume with dual trading. In fact, the total expected trading volume is not affected by dual trading as the loss in informed trading volume is just made up by the broker's own trading activity. Further, price informativeness does not change with dual trading because market depth and the net order flow decrease in the same proportion.⁴

Initially, commission fees are ignored. The model is an extension of Kyle (1985). Uninformed noise traders and a group of informed traders submit market orders to a broker who places them (along with her own orders) with a marketmaker for execution. The broker's motive for dual trading comes from her private observations of the size of her customers' orders.⁵ In equilibrium, she is able to infer all of her informed customers' information from her observations and profit through mimicking or piggybacking on the informed trades. The marketmaker batches the total order flow and executes them at a single price. The

price is determined by assuming that the marketmaker makes zero profits conditional on observing the total order flow.

For the model with no commission fees, the main result is that the profits of the informed traders are reduced with dual trading while uninformed traders neither gain nor lose. This result does not change when the model is extended to allow for rational behavior by uninformed traders. Following Spiegel and Subrahmanyam (1992), uninformed traders (who are risk-averse) trade in order to hedge their endowments of shares of the risky asset. The hedgers' trading volume and expected utility do not change with dual trading. Although the broker still offsets a portion of the hedgers' order flow, any impact of this on the hedgers' utility is exactly made up by the reduction in market depth.⁶

Next, I analyze trading behavior when traders have to pay commission fees to the broker. As in Fishman and Longstaff (1992), the broker incurs a fixed and a variable cost of brokerage. Further, the brokerage business is competitive so that the broker's total income (trading profits plus expected commission income) is zero. Two cases are analyzed. In the first, the commission fees are assumed fixed (i.e., independent of the order size) and are paid irrespective of whether traders trade or not. This implies that traders never forego a trade. Since the broker's commission income is independent of her customers' trading volume, the competitive commission fee is always lower with dual trading. The net profits of the informed traders decrease and the net utility of the uninformed traders increase with dual trading.⁷

The second case is an extended example based on Krishnan's (1992) binomial version of Kyle (1985). In this example, the commission fee is proportional to the amount traded. It illustrates that, when the broker's commission income depends upon the trading volume, the competitive commission fee may be higher with dual trading.

In related literature, Roell (1990) has a model of dual trading in which a broker observes the trades of some uninformed traders. Her model does not include the effect of commission fees. Informed traders have higher profits when dual trading is banned. Uninformed traders whose trades are observed by the broker have higher profits with dual trading. Those whose trades are not observed by the broker are hurt by dual trading.

In Fishman and Longstaff (1992), the broker has private information about whether her customer is informed or uninformed. Before commissions, all customers lose with dual trading. Including commissions, dual trading benefits the uninformed traders and hurts the informed traders. In contrast to this paper, these authors assume that trading volume is fixed at one unit. As a result, the informed trader in their model fails to take into account the broker's mimicking behavior when formulating her optimal trading strategy. A further implication of this assumption is that the broker's commission fees are always lower with dual trading. They also do not model the behavior of the customer when she is uninformed. On the other hand, they allow the customer and the dual trader to trade at different prices and they also model the effect of frontrunning by the broker.

The remainder of this paper is organized as follows. Section I develops the basic dual trading model with noise traders, ignoring commission fees. In Section II, this is contrasted with a model where dual trading is completely banned. Results on the market impact of dual trading are obtained. Section III extends the model to allow for rational behavior by uninformed traders. Section IV studies the effect of dual trading on commission fees charged by the broker. Section V concludes. All proofs are contained in the appendix.

I. THE DUAL TRADING MODEL AND SOLUTION

A. The Dual Trading Model

I consider a market in which a single risky asset with unknown liquidation value v is traded. There is a group of m informed traders each of whom receive, prior to trading, signals s^i about the unknown value v . The signals are of the form $s^i = v + e^i$, $i=1, \dots, m$ where the error terms e^i are independent of each other. In addition, there is a group of uninformed noise traders who trade for liquidity reasons. Initially, the uninformed traders' motives for trading are not modelled. Later, the basic model is extended to allow for rational behavior by the uninformed traders.

Each informed trader $i = 1, \dots, m$ submits a market order x_d^i to a broker. The noise traders also collectively submit market orders worth u to the same broker. The latter then places the total of the submitted orders $(x_d + u)$, where $x_d = \sum_{i=1}^m x_d^i$, to a marketmaker for execution.⁸ In the dual trading model, the broker may also trade an amount d for her own account. She may want to do so because, by observing the market orders x_d^i of the informed, she is able to infer some or all of their

information s^i . In addition, it may be profitable for the broker to take the opposite position of the group of uninformed traders.⁹ At this stage, I ignore commission fees (which are introduced in Section IV).

It is assumed that, when the broker places her customers' order with the marketmaker, she simultaneously sends along her own order d as well. The marketmaker then fixes a single price p_d at which she will execute the total order flow $y_d = x_d + d + u$. Following Kyle (1985), the marketmaker is assumed to be risk-neutral and competitive. Conditional on observing y_d , she earns zero expected profits.

The random variables in the model are v , u and e^i , $i=1, \dots, m$. All these variables are normally distributed with zero mean and finite variances Σ_v , Σ_u and Σ_e , respectively. Thus the m error terms are drawn from an identical distribution.¹⁰ In addition, all investors follow linear trading rules $x_d^i = A_d s^i$, $i=1, \dots, m$ (for the informed) and $d = B_1 x_d + B_2 u$ (for the broker). This implies that the marketmaker's pricing rule is also linear: $p_d(y_d) = \Gamma_d y_d$, where $1/\Gamma_d$ is the now-familiar market depth parameter.

There are three distinct stages to this trading game:

(1) Informed traders receive their information and decide how much they want to trade. In making this decision, each informed trader is aware that, first, she is in competition with the other informed traders and, second, that the broker will "piggyback" on the information conveyed by her trading decision. The informed traders care about the broker's piggybacking because they receive a less favorable price for their trades as a consequence. Noise traders simply submit u .

(2) The broker observes u and x_d^i and infers that each informed trader has some information s^{i*} , $i=1, \dots, m$. Based on her inferences and u , she decides to trade an amount d .

(3) The marketmaker fixes a price $p_d = \Gamma_d(x_d + u + d)$, where $p_d = E(v | y_d)$ and so $\Gamma_d = \text{Cov}(v, y_d) / \text{Var}(y_d)$.

This suggests the following solution method. Fix Γ_d and suppose that each informed trader $i=1, \dots, m$ has decided to trade some amount x_d^i and uninformed traders have submitted demand u . From each x_d^i the broker infers information s^{i*} . She then chooses d to maximize her expected profits, where the expectation is taken with respect to the vector $(s^{1*}, \dots, s^{m*}, u)$. Each informed trader i then chooses x_d^i as a best response to $d(s^{1*}, \dots, s^{m*}, u)$, the rival informed traders' decisions x_d^j , $j \neq i$ and uninformed trades u . Finally, Γ_d is obtained from the optimal trading rules and the marketmaker's zero profit assumption.

Depending upon what the equilibrium beliefs of the broker are, there can be potentially many equilibria to the signalling game between the informed traders and the broker. Fortunately, in this model, the signalling game affords a unique solution: there is a single fully separating equilibrium. In other words, the informed traders' information is fully revealed to the broker and so in equilibrium $s^{i*} = s^i$, $i=1, \dots, m$.

B. The Dual Trading Solution

First, I solve the signalling game between the informed traders and the broker. Given her observations of x_d^i and u , the broker chooses d to maximize her conditional expected profits given by

$E(\pi | s^{1*}, \dots, s^{m*}, u)$, where $\pi = (v - \Gamma_d Y_d)d$. From the first-order condition, the optimal $d = [E(v | s^{1*}, \dots, s^{m*}) - \Gamma_d(x_d + u)]/2\Gamma_d$. The second-order condition is satisfied by $\Gamma_d > 0$. Define $t = \Sigma_v / (\Sigma_v + \Sigma_e)$ and note that $0 \leq t \leq 1$. t is a measure of the unconditional precision of s^i , $i=1, \dots, m$. For example, if $t=1$ then s^i is a perfect signal. Then $E(v | s^{1*}, \dots, s^{m*}) = ts^*/Q$ where $Q = [1+t(m-1)]$ and $s^* = \sum_{i=1}^m s^{i*}$. Therefore, the broker's optimal trade is:

$$d = \frac{ts^*}{2Q\Gamma_d} - \frac{x_d + u}{2}. \quad (1)$$

In a separating equilibrium $s^{i*} = s^i = x_d^i/A_d$ for each $i=1, \dots, m$. So, the presence of the dual trader is seen to have two opposite effects on an informed trader's incentive to trade. Suppose $x_d > 0$ (a buy order). If x_d is increased, the broker infers that the informed traders' information is improved and so s^* is higher as well. The broker trades more, d is higher and so is the resulting price. Thus, this signalling effect tends to inhibit informed traders from trading aggressively.

On the other hand, a higher x_d also reduces d from the second term in (1). This is a "second-mover disadvantage" for the broker as she has to accommodate market orders of any size by the informed and tends to encourage informed trades. For finite m , however, the signalling effect always dominates the second-mover effect, so that $B_1 > 0$ in equilibrium (x_d and d always have the same sign). The broker optimally mimics the trading decisions of the informed. Also, the broker optimally takes the opposite position of the aggregate uninformed trades.

Given (1), each informed trader i chooses x_d^i to maximize her conditional expected profits $E(I_d^1 | s^i)$, where

$$I_d^1 = \left(v - \Gamma_d d - \Gamma_d x_d^i - \Gamma_d \sum_{j \neq i} x_d^j - \Gamma_d u \right) x_d^i.$$

After incorporating the optimal value of d from equation (1) into I_d^1 , the first-order condition for x_d^i , $i \neq j$ is:

$$\frac{t(1+Q)s^i}{2} = \Gamma_d \left[x_d^i + 0.5(m-1)E(x_d^j | s^i) \right] + \frac{ts^{i*}}{Q}. \quad (2)$$

Equation (2) says that the marginal value of an additional trade for the i -th informed trader is equal to its marginal cost. This cost has two components: the change in the price due to her own and her rivals' expected trades plus the change in the broker's inference as to her information. After using the facts that (i) $s^{i*} = s^i = x_d^i/A_d$ in equilibrium and (ii) $E(s^j | s^i) = ts^i$ for $j \neq i$, A_d is obtained as the coefficient of s^i in (2):

$$A_d = \frac{t^2(m-1)}{\Gamma_d Q(Q+1)}. \quad (3)$$

From (3), $A_d = 0$ when $m = 1$. But $A_d = 0$ cannot be a separating equilibrium since the functions $x_d^i = A_d s^i$, $i=1, \dots, m$ are then no longer invertible.

Lemma 1: When $m = 1$, there is no solution to the dual trading model.

The result can be interpreted as follows. The inhibiting effect of the broker's piggybacking or mimicking behavior on any individual

informed trader is inversely related to m , the number of informed customers the broker has. For $m = 1$, this inhibiting effect exactly offsets the marginal value of an extra trade for the individual informed trader as the first-order condition (2) reduces to:

$$t(s^i - s^{i*}) = \Gamma_d x_d^i. \quad (4)$$

So, for any $x_d^i > 0$, the marginal cost of an additional trade for the informed always exceeds its marginal benefits.¹¹ Substituting (3) into (1), the optimal dual trading function is given by:

$$d = \frac{x_d}{t(m-1)} - \frac{u}{2}. \quad (5)$$

Finally, by using (3) and (5) in conjunction with the marketmaker's zero profit assumption the optimal value of market depth is derived as:

$$\Gamma_d = 2 \frac{\sqrt{mt\Sigma_v}}{(1+Q)\sqrt{\Sigma_u}}. \quad (6)$$

Proposition 1 fully characterizes the dual trading equilibrium.

Proposition 1: If $m > 1$ and $t > 0$, there exists an unique solution to the dual trading model in which $x_d^i = A_d s^i$, $i=1, \dots, m$, $d = B_1 x_d - \frac{u}{2}$ and $p_d = \Gamma_d y_d$ where A_d is given by (3), B_1 by the coefficient of x_d in (5) and Γ_d by (6).

II. THE MARKET IMPACT OF DUAL TRADING

In weighing the costs and benefits of dual trading, a regulator might be interested in its effect on aggregate market characteristics (total trading volume and profits, market depth and price efficiency) as well as its distributional effect on individual groups of market participants. These groups include the informed and uninformed traders and the broker. The distributional impact of dual trading may be discerned by considering its effects on the trading volumes of the informed and the uninformed,¹² the broker's commission fees and traders' expected profits net of commission costs. The impact of dual trading on aggregate market characteristics is studied in this section and the distributional question is analyzed in the following two sections.

A. The Nondual Trading Model

I will compare the dual trading solution obtained in Section I with the solution obtained when dual trading is completely banned. The broker is then a pure intermediary, bringing her customers' orders to the market.¹³ The resulting trading game is a Cournot-Nash game in trading quantities. Each informed trader places an order x_n^i with the broker based on her information s^i . The broker submits the total order flow $y_n = x_n + u$ (where x_n is total informed trades in the nondual trading market) to the marketmaker for execution. The price determined is $p_n = \Gamma_n y_n$. Lemma 2 describes the nondual trading equilibrium.

Lemma 2: If there is no dual trading, a solution always exists provided $t > 0$. The informed traders trade $x_n^i = A_n s^i$ and the price is $p_n = \Gamma_n y_n$, where:

$$A_n = \frac{t}{\Gamma_n(1+Q)}, \quad \Gamma_n = \frac{\sqrt{mt\Sigma_v}}{(1+Q)\sqrt{\Sigma_u}}. \quad (7)$$

Proof: See the proof of Lemma 1 in Admati and Pfleiderer (1988).

B. Trading Volume and Gross Profits, Market Depth and Price Efficiency

Due to piggybacking by the broker, it is reasonable to expect that $A_d < A_n$. The difference in informed trading intensities A_i , $i = d, n$ depends upon the market depths Γ_d and Γ_n . By inspection of (6) and (7), $\Gamma_d = 2\Gamma_n$: relative to the market without dual trading, market depth is half its value with dual trading. This reflects the fact that the dual trading broker offsets half of the uninformed order flow. Given this result, the difference in informed trading intensities:

$$A_n - A_d = \frac{t}{\Gamma_d Q} \quad (8)$$

which is positive for $t > 0$. t/Q represents what the broker learns about the unknown v from observing the m -vector of informed trades. The more informative is this observation, the greater is the relative shrinkage of the informed order flow in the dual trading market. The difference $(A_n - A_d)$ is also positively related to market depth, since a deeper market allows the broker to trade larger amounts with less concern about the price impact.

However, the broker herself provides an additional source of trading activity in the dual trading market. Considering the dual trading and informed trading activities together:

$$x_d + d = \frac{1}{\Gamma_d} \frac{ts}{1+Q} - \frac{u}{2} \quad (9)$$

$$x_n = \frac{1}{\Gamma_n} \frac{ts}{1+Q}. \quad (10)$$

From (9) and (10), any difference in the net order flows between the two markets occur because the broker offsets part of the noise trading. As a result, total informed order flow (by the informed traders and the broker) is halved due to the halving of market depth and, further, the net noise trading order flow (of the broker plus the noise traders) is halved. Hence, $y_n = 2y_d$: the net order flow is twice as high without dual trading.

The decrease in the net order flow occurs because the broker takes the opposite position of the uninformed trades. Ignoring the effect on informed trading activity, the trading volume is actually higher with dual trading. Define the expected uninformed trading volume as $\sqrt{\Sigma_u} / \sqrt{2\pi}$. Then, the increase in the uninformed trading volume due to dual trading is $\sqrt{\Sigma_u} / 2\sqrt{2\pi}$. Define the expected net informed trading volume in market $i = d, n$ as $A_i \sqrt{m\Sigma_s} / \sqrt{2\pi}$. Since $A_d < A_n$, informed trading volume is lower with dual trading. In fact, it can be shown that the two effects are exactly offsetting so that the expected total trading volume is unaffected by dual trading.

So what can we conclude about the effect of dual trading on market liquidity? The decrease in the depth parameter indicates an increase in the adverse selection problem for the marketmaker. Yet, uninformed trading volume is actually higher relative to the informed trading volume when dual trading occurs. This is not reflected in the depth parameter since the marketmaker is assumed to observe the order flow after all trades are netted out. On the other hand, the result that trading volume is constant implies that dual trading does not affect liquidity.

Define price efficiency PI_i , $i = d, n$ as $\Sigma_v - \text{var}(v|p_i)$. It can be shown that $PI_i = (\Gamma_i)^2 \Sigma_{y_i}$, where Σ_{y_i} is the variance of the net order flow in market $i = d, n$. PI_i depends positively on the volatility of the net order flow (since this depends positively on the informed trading intensities A_i) and inversely on market depth (since prices are less sensitive to volume, and hence to information, in deep markets). It follows that $PI_d = PI_n$ because, relative to the nondual trading market, the reduction in the variance of the net order flow is exactly offset by the reduction in market depth in the dual trading regime.

Let I_i denote the combined unconditional expected profits of the informed group (before observing any signals or paying any commissions) in the i -th market, $i = d, n$. Since the price level $p_i = \Gamma_i y_i$, it is the same in the two markets. It follows that $I_d < I_n$: gross profits of the informed are strictly lower with dual trading (since the price level is the same but the informed order flow is lower). Total trading profits in the dual trading market is $I_d + W$, where W is the broker's unconditional expected trading profits. Total informed trading profits

in the dual trading market (I_d plus that part of the broker's profits obtained from mimicking informed trades) is half of I_n (since the price level is the same but total informed order flow is half of x_n). But, the broker also profits from her observance of noise trading and the amount of this profit equals $\Gamma_d \Sigma_U / 4$ or $\Gamma_n \Sigma_U / 2$, which is exactly half of I_n . Thus $I_d + W = I_n$.

In equilibrium, the uninformed traders as a group suffer losses and the amount of their losses mirrors the total trading profits of the informed traders and the broker. Denote $L_g = I_d - I_n + W$ as the difference in the gross losses of the uninformed traders between the dual and nondual trading regimes. Therefore, $L_g = 0$.

Proposition 2: (1) $y_n = 2y_d$ and $\Gamma_d = 2\Gamma_n$. The net order flow and market depth in the dual trading market are half their values in the nondual trading market. (2) The expected total trading volume, the gross trading profits and price efficiency are the same with or without dual trading. (3) $I_d < I_n$ and $L_g = 0$. Gross profits of the informed are lower with dual trading. Gross losses of the uninformed are unchanged.

In the following section, the basic model is extended to allow for rational behavior by the uninformed traders.

III. HEDGING BY UNINFORMED TRADERS

There are h risk-averse uninformed traders ("hedgers") who trade for purely risk-sharing reasons. The development of the model here follows Spiegel and Subrahmanyam (1992). Each hedger j has random endowment w^j , which is assumed to be normally distributed with mean zero and variance Σ_w . w^j , $j=1, \dots, h$ are independent of each other and all

other random variables in the model. All hedgers have negative exponential utility functions with risk-aversion parameter R .

Suppose that in market i all hedgers $j=1, \dots, h$ submit market orders u_i^j to the broker and follow linear trading rules of the form $u_i^j = D_i w^j$. Let the total uninformed trading volume in market i be $u_i = \sum_{j=1}^h u_i^j$. If π_i^j is the profit of the j -th hedger in the i -th market, then u_i^j is chosen to maximize her utility or certainty-equivalent profits $G_i^j = E(\pi_i^j | w^j) - \frac{R}{2} \text{Var}(\pi_i^j | w^j)$. Let G_i , $i = d, n$ be the sum of the utilities of all h hedgers in the i -th market. The informed traders and the broker's maximization problem remains the same as before, since each w^j is independent of v .¹⁴ Market depth is now positively related to the magnitude of the "hedge factor" D_i (since this increases the variance of the total order flow) and to the risk aversion parameter R . Further, the equilibrium $D_i < 0$ since the marginal utility of the hedgers from a purchase (sale) is negative if endowments are positive (negative). Lemma 3 describes the equilibrium for the nondual trading market. The appendix describes the dual trading equilibrium.

Lemma 3: An equilibrium to the hedger model exists if R satisfies:

$$R\Sigma_v(2-t) > 2\sqrt{mt\Sigma_v} / \sqrt{h\Sigma_w}. \quad (11)$$

In equilibrium, each hedger $j=1, \dots, h$ trades $u_n^j = D_n w^j$, where $D_n < 0$, market depth is $1/\Gamma_n$, and:

$$-D_n \Gamma_n = \frac{\sqrt{mt\Sigma_v}}{(1+Q)\sqrt{h\Sigma_w}} \quad (12)$$

where

$$D_n = \frac{2\sqrt{mt\Sigma_v/h\Sigma_w} - R\Sigma_v(2-t)}{R\Sigma_v(2-t) - \frac{R\Sigma_v mt}{h(1+Q)}}. \quad (13)$$

Proof: See the proof of Proposition 1 in Spiegel and Subrahmanyam (1992).

(11) states that equilibrium exists if the amount of risk-aversion and noise in the market exceeds the amount of information available. From (12) and (A13) in the appendix, $\Gamma_d ab(D_d) = 2\Gamma_n ab(D_n)$, where $ab(D_i)$ denotes the absolute value of D_i . How does dual trading affect the choice of D_i ? The price impact of liquidity order flows is given by $\Gamma_i u_i$. Conjecture that $\Gamma_d = 2\Gamma_n$. Then, since the net liquidity order flow are halved with dual trading, the price impact of liquidity trades (including those by the broker) is identical in both markets and so $D_d = D_n$. This in turn means that $u_d = u_n$ and the initial conjecture regarding the market depths must be correct as well.¹⁵ As before, the

price levels are identical in the two markets and, so by implication, $G_d = G_n$. Hence, the results about the market impact of dual trading reached in the previous section is robust to the introduction of rational uninformed traders.

Proposition 3: $D_d = D_n$, $G_d = G_n$ and $\Gamma_d = 2\Gamma_n$. The hedgers' trading volumes and gross utilities are the same with or without dual trading. The depth of the market is halved when dual trading is permitted.

IV. TRADING WITH COMMISSION FEES

Suppose that, in market i , the broker charges a commission fee of $\$c_i$ to cover her costs of brokerage. Agents make their decisions in the following sequence. At stage zero, the broker determines the commission fee c_i . At stage one, the informed traders observe their signal realizations and c_i . Uninformed traders observe their endowment realizations and c_i . Then, both groups of traders decide how much to trade. At stage two, if dual trading is allowed, the broker's trade size is also contingent on c_d and her observance of informed and uninformed trades (if any). Finally, a price is set in the manner specified earlier.

The broker faces a fixed cost k_0 and a variable cost k_1 of conducting business, both costs being non-negative. Following Fishman and Longstaff (1992), I assume that the brokerage business is competitive and so the commission fee c_i is chosen so that the broker's expected trading profits plus expected commission income equals zero.

In Section A, I will assume that there is a fixed commission fee per trade and it has to be paid irrespective of whether traders make a

trade or not. Here, the commission may be interpreted as a market access fee. The variable cost k_1 is also incurred on a per trade basis. In this case, the commission does not affect traders' decisions regarding how much and whether to trade. Thus, the number of trades that occur in equilibrium is the same as the total number of traders in the market. Since the broker's commission income does not change, the competitive commission fee is always lower with dual trading.

In Section B, I construct an example illustrating that when the broker's commission income depends upon the trading volume, the competitive commission fee may be higher with dual trading because of the negative effect on the informed trading volume. In this example, the commission fee is proportional to the order size and has to be paid only when a trader buys or sells a share of the asset. With dual trading, informed traders reduce their order size due to the broker's mimicking behavior. This reduction in the broker's expected commission income more than offsets her expected trading profits.

A. The Effect of Dual Trading on the Commission When It Has to Be Paid Irrespective of Whether Traders Trade or Not

When the j -th informed trader trades, her net profits in market i are $N_i^j = I_i^j - c_i$. If she does not trade, her losses are $-c_i$. So, she will always trade. Also, her equilibrium order size and gross profits will not change since the first-order conditions for her trading decision are not affected. Similar remarks apply to the uninformed traders. Thus, the total number of traders ($m+h$) is also the number of trades occurring in equilibrium. The commission fees are determined by the following equations:

$$c_d = k_1 + \frac{k_0 - W}{m+h} \quad (14)$$

$$c_n = k_1 + \frac{k_0}{m+h} \quad (15)$$

(14) and (15) equate the profit margin per trade ($c_i - k_1$) in market i to the fixed cost of brokerage per trade, net of the broker's profits. Since the broker's dual trading profits are positive, $c_d < c_n$. Further, the difference in commission fees ($c_n - c_d$) is equal to the dual trading profits per trade. For the j -th uninformed trader, her net profits are $\pi_d^j - c_d$ in the dual trading market and $\pi_n^j - c_n$ in the nondual market. From Proposition 3, her gross profits (and utility) are unchanged with dual trading. Since $c_d < c_n$, her net profits (and utility) increase with dual trading. For the informed traders, Proposition 2 says that their gross profits decrease by W , the amount of dual trading profits. The value of the reduction in commission fees to them is $\frac{m}{m+h}W$. Thus, the net profits of informed traders decrease with dual trading.

Proposition 4: Suppose that in market i , traders pay a fixed commission fee of c_i irrespective of whether they trade. Then (1) $c_d < c_n$ and (2) net of c_i , the utility of the uninformed traders are higher and the profits of the informed traders are lower with dual trading.

B. An Example to Show That the Competitive Commission Fee May Be Higher With Dual Trading

This section develops a binomial version of the Kyle model based on Krishnan (1992). The value of the risky asset v can take on one of

two values 0 or 1, each with probability one-half. Uninformed traders are modeled as noise traders, as in the basic model. There is a single informed trader with perfect information about the asset value. Her trading strategy is $X(v)$. Denoting x as a realization of $X(v)$, I assume that x is an element of the set $S = \{-1, -.5, +.5, +1\}$. The net order flow y_n is the sum of x and the noise trades and also takes on values in the set S . There is a broker who charges a proportional commission fee of $\$c_i$ per share in market i .

Consider the nondual trading market first. The marketmaker's pricing rule is $P(y_n)$. She sets a price p that earns her zero expected profits after observing y_n . Thus, if y_n takes on the value z in the set S , then the price p is set according to the following rule:

$$\tilde{p} = E(\tilde{v} | \tilde{y}_n = z) = \frac{P_x(y_n = z | v = 1)}{P_x(y_n = z | v = 1) + P_x(y_n = z | v = 0)} \quad (16)$$

where $P_r(A)$ stands for the probability of event A occurring. Because of noise trading, x and y_n need not be the same. The marketmaker uses her observation of y_n to make an inference about x and, therefore, v . This inference process is assumed to take the following form:

Assumption A1: $P_r(y_n = z | X(v) = z) = r_1$ and $P_r(y_n \neq z | X(v) = z) = (1 - r_1)/3 = q_1$, where $r_1 > q_1$.

Assumption A1 states that the informed order flow and the net order flow are positively correlated.¹⁶ In equilibrium, informed traders maximize profits and the marketmaker makes zero profits conditional on the order flow realized. Two observations are in order.

First, in this example, the equilibrium prices belong to the open unit interval $(0, 1)$. This implies that if $v = 0(1)$ the informed trader will optimally sell (buy). Second, given Assumption A1, $P(1) = P(0.5)$ and so the profits of the informed trader are strictly proportional to the order size.¹⁷ Thus, informed traders find it profitable to buy (sell) one share rather than half a share if $v = 1(0)$.

For trading to be profitable, trading revenues must exceed the commission costs. This requires the following condition to be satisfied:

$$\frac{2r_1q_1}{r_1+q_1} + q_1 > c_n. \quad (17)$$

Lemma 4: Assume that A1 holds. If (17) is satisfied, trading occurs in equilibrium and the optimal trading strategy of the informed trader is to buy 1 share if $v = 1$ and to sell 1 share if $v = 0$. The equilibrium prices are $P(1) = r_1/(r_1+q_1)$ and $P(-1) = q_1/(r_1+q_1)$.

Next, consider the dual trading market. I assume that the informed trader's market orders belong to the set S , as before, but the broker's trades are restricted to the set $\{-0.25, +0.25\}$.¹⁸ The combined order flows of the informed trader and the dual trader then belong to the set T , where $T = \{-1.25, -0.75, -0.25, +0.25, +0.75, +1.25\}$. The net order flow y_d (including noise trading) is also a realization from the set T . The marketmaker's inference process is captured by the following assumption:

Assumption A2: $P_r(y_d=z | X(v)=z)=r_1$ if $z = 0.25$ or 0.75 and $P_r(y_d=1.25 | X(v)=1.25)=r_2$, where $r_2 > r_1$. $P_r(y_d \neq z | X(v)=z)=(1-r_1)/5=(0.6)q_1$ if $z = 0.25$ or 0.75 ; $P_r(y_d \neq 1.25 | X(v)=1.25)=q_2$, where $r_2 > q_2$.

The idea behind Assumption A2 is that the largest sized order is more likely to come from an informed trader than a small or medium-sized order. The practical effect of the assumption is that, in the proposed equilibrium, $P(1.25) > P(0.75) = P(0.25)$. Suppose the dual trader buys 0.25 shares whenever she observes a positive informed order flow. Then the informed trader may prefer to buy half a share (in which case $y_d = 0.75$ with high probability) instead of one share (when $y_d = 1.25$ with high probability) if the additional cost of buying an extra half share exceeds the marginal revenue. This is precisely the piggybacking effect discussed in the more general model of Sections I and II. The following lemma states the conditions under which the dual trading equilibrium exhibits the piggybacking effect.

Lemma 5: In the dual trading equilibrium, trading occurs if:

$$\frac{1.2r_1q_1}{(0.6)q_1+r_1} + 1.2q_1 > c_d. \quad (18)$$

If the informed trader buys (sells) the dual trader believes that $v = 1(0)$ and she buys (sells) 1/4 of a share. Suppose the following condition also holds:

$$2q_2 - (0.6)q_1 + \frac{2q_2r_2}{q_2+r_2} - \frac{r_1(0.6)q_1}{(0.6)q_1+r_1} < \frac{c_d}{2}. \quad (19)$$

Then, the informed trader buys (sells) $1/2$ of a share if $v = 1(0)$. The equilibrium prices are $P(0.75)$ and $P(-0.75)$.

The commission fees are set competitively. The variable cost k_1 is computed on a per share basis. In deriving the competitive fees, I assume that (17), (18) and (19) hold and so trading occurs in equilibrium at the levels described in Lemmas 4 and 5. Of course, these three conditions themselves depend upon the fees. Later, I provide a numerical example where the fees are set as described below and (17), (18) and (19) are also satisfied.

In the nondual trading market, c_n is determined by the following equation:

$$\left(r_1 + \frac{q_1}{2} \right) (c_n - k_1) = k_0. \quad (20)$$

$r_1 + q_1/2$ is the expected trading volume in the nondual trading market. This expression is derived in the following way. With probability one-half, $v = 1$ and the informed trader buys one share. With probability r_1 , the order flow is 1 and with probability q_1 , the order flow is $1/2$. A similar calculation holds good on the sell side.

For the dual trading market, I calculate the broker's customer trading volume by computing the expected total trading volume in the way described above and then subtracting off $1/4$ of a share, which is the broker's expected trading volume. Then, c_d is given by the following expression:

$$0.5(1.5r_1 + 1.8q_1 - 0.5)(c_d - k_1) = k_0 - W \quad (21)$$

where W is given by:

$$W = \frac{0.6q_1}{r_1 + 0.6q_1} (r_1 + 0.3q_1). \quad (22)$$

From Assumption A2, $q_1 < 0.15$ and the broker's expected customer trading volume is lower with dual trading. This creates the possibility that the loss in the broker's expected commission income will more than offset her expected trading profits. Proposition 5 relates when this happens.

Proposition 5: Suppose (17), (18) and (19) are satisfied. $c_d > c_n$ if:

$$k_0 \left(\frac{1 + r_1 - 1.6q_1}{4r_1 + 2q_1} \right) > W. \quad (23)$$

Comparing (22) and (23), higher values of r_1 and k_0 make $c_d > c_n$ more likely. Higher values of r_1 is analogous to increase the trading intensity of the informed traders in the more general model, enhancing the "bite" of the piggybacking effect. A numerical example satisfying Proposition 5 is provided below.

A Numerical Example of Proposition 5. Suppose $r_1 = 0.81$, $q_1 = 0.063$, $r_2 = 0.86$, $q_2 = 0.028$, $k_0 = 0.08$ and $k_1 = 0.043$ (4.3 cents per share). The competitive commission fees are $c_n = 13.81$ cents per share and $c_d = 14.64$ cents per share. In the nondual trading market, the expected

gross profits of the informed traders are 18 cents per share in equilibrium; in the dual trading market it is 14.86 cents per share. Thus, trading always occurs in equilibrium. The piggybacking effect exists because, with dual trading, if the informed trader is trading $1/2$ of a share, there is a loss of 3.7 cents per share from trading another $1/2$ of a share. The net profits of the informed traders fall from 4.28 cents per share to 0.22 cents per share with dual trading. The losses of the uninformed traders also decrease by 3.4 per share with dual trading.

V. CONCLUSION

The paper considers the effects of allowing brokers to trade on their own account (to dual trade) in addition to their usual intermediary function. In the model without commission fees, dual trading leads to a reduction in the net order flow and the market depth, but expected trading volume and price efficiency is not affected. Informed traders have lower profits with dual trading because they are forced to be less aggressive in anticipation of the broker mimicking their trades and piggybacking on their information. The broker also takes the opposite position of uninformed trades and offsets a portion of the uninformed order flow. Since the net uninformed order flow and the market depth both decrease by the same proportion, however, the price impact of an uninformed trade and the variance of uninformed profits are unchanged. So, uninformed traders neither gain nor lose with dual trading.

The effect of dual trading on the commission fee charged by the broker depends upon whether the broker's commission income is

independent of the trading volume. Two cases are analyzed. In the first, commission fees are fixed and have to be paid irrespective of whether traders choose to trade or not. Here, the competitive commission is lower and, net of the commission, informed traders have lower profits and uninformed traders have higher utility with dual trading.

The second case involves a binomial version of Kyle (1985) where the commission fee is proportional to the amount traded. The example illustrates that the competitive commission fee may be higher with dual trading if the loss in the broker's expected commission income due to the reduction in the informed trading volume is greater than her trading profits.

FOOTNOTES

¹See The Chicago Tribune, February 4, 1992 and The Wall Street Journal, February 7, 1992 for details.

²Chang and Locke (1992) find no positive correlation between the order imbalances of dual traders and customers in the current futures market. They conclude that piggybacking behavior by dual traders is absent. However, since no distinction is made between informed and uninformed customer trades, the lack of a positive correlation is not conclusive.

³This effect is partially due to the fact that traders transact in a batch market where the price is increasing in the net order flow, as in Kyle (1985). This characteristic of the model maximizes the negative impact of piggybacking on informed trading. But the effect would remain in a multiperiod setting where the orders of the customers and the broker are executed and priced separately, so long as some subset of the informed customers make repeat purchases or sales via the same broker. From Kyle (1985), the optimal dynamic trading strategy of an informed trader is to dribble her trades over time.

⁴Empirical studies of the effect of dual trading on market depth and the bid-ask spread have produced mixed results. Many of the studies (Chang and Locke (1992), Park and Sarkar (1992) and CFTC (1990)) find that, in the markets studied so far, dual trading does not significantly affect liquidity as measured either by the bid-ask spread or the depth. Smith and Whaley (1990), however, show a significant increase in the

effective bid-ask spread as a result of a restriction on dual trading. Walsh and Dinehart (1991) also find some evidence that dual trading is associated with narrower bid-ask spreads.

⁵The broker is assumed to have no private information of her own. For a model with a privately informed broker, see Sarkar (1991).

⁶Specifically, the price impact of the hedgers' trades is given by the product of the net uninformed order flow and the inverse of the market depth. Both the net uninformed order flow and the market depth are halved. Hence the result.

⁷The results when the broker's commission income is independent of the trading volume is consistent with Fishman and Longstaff (1992) where the trading volume is fixed at one share.

⁸I will adopt the convention of labelling the decision variables of individual agents with a superscript and market variables with a subscript. The subscript d will refer to the solution in the dual trading model and the subscript n to the nondual trading solution.

⁹The broker is assumed to have no independent information regarding v . In a previous version of this paper (Sarkar (1991)), the broker has her own information but does not observe uninformed trades u . Also $m = 1$. This makes for some interesting interactions between the information of the single insider and that of the broker. For example, for low precision of the broker's information, the insider's trades is actually decreasing in the precision of her own information!

¹⁰For $m = 2$, I have checked that the results are unchanged if the informed traders have information of different precisions. I conjecture that this is true for general m .

¹¹In Sarkar (1991), an equilibrium exists even with $m = 1$ so long as the precision of the broker's information is positive. The reason is that, if the broker has an independent source of information about v , she attaches relatively less weight to her observation of the informed trade. The change in the broker's inference, when the informed trader buys or sells an extra share, no longer fully offsets the marginal value of that extra share traded.

¹²Since, in the model so far, the trading volume of the uninformed is not a choice variable, the impact of dual trading on uninformed trading volume will not be considered until Section III (when I do model the uninformed trading decision).

¹³Some brokers can commit not to dual trade (as occurs in reality). Those customers who value the superior trading skills of dual trading brokers (as suggested in Grossman (1989)) will continue to patronize them. Others, perhaps more concerned with frontrunning and other potential abuses, may choose the brokers committed not to dual trade. Thus, my model should be seen as a reduced form of this more general situation where traders and brokers are matched according to their varying needs.

¹⁴Of course, the actual informed and dual trading volumes will be different since market depth will be different, in general.

¹⁵Note that this is precisely the situation considered in the previous section, where u was exogenously fixed at the same level in the two markets. The appendix proves this result formally.

¹⁶The assumption is stricter than it needs to be. For example,

when y and $X(v)$ are not equal, the probabilities could vary with the realized value of y .

¹⁷See the proof of Lemma 4 in the appendix.

¹⁸This assumption can also be relaxed. It is motivated by the desire to make the point of this example as simply as possible.

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APPENDIX

Proof of Lemma 1

Let $E(v|s^{i*}, \dots, s^{m*}) = as^*$, where $s^* = \sum_{i=1}^m s^{i*}$. Applying Bayes' rule, $a = \frac{1/\Sigma_e}{1/\Sigma_v + m/\Sigma_e} = \frac{t}{1+(m-1)t}$, where $t = \Sigma_v/(\Sigma_v + \Sigma_e)$. This gives the optimal dual trade $d(s^*, x_d)$ as given in (1).

Incorporating (1), each informed trader i 's profits I_d^i can be written as:

$$I_d^i = \left(v - \frac{t}{1+(m-1)t} \frac{s^*}{2} - \frac{\Gamma_d}{2} x_d - \frac{\Gamma_d u}{2} \right) x_d^i \quad (A1)$$

Substituting $s^{i*} = s^i = \frac{x_d^i}{A_d}$ for each $i = 1, \dots, m$ into (A1):

$$E(I_d^i | s^i) = \left[t s^i - \frac{x_d^i}{2} \left(\Gamma_d + \frac{t}{1+(m-1)t} \frac{1}{A_d} \right) - \sum_{j \neq i} \frac{E(x_d^j | s^i)}{2} \left(\Gamma_d + \frac{t}{1+(m-1)t} \frac{1}{A_d} \right) \right] x_d^i \quad (A2)$$

Substituting $E(x_d^j | s^i) = A_d t s^i$ for each $j \neq i$ into (A2) and then differentiating with respect to x_d^i gives (2). When $m = 1$, (2) has the form:

$$x_d^i \left(\frac{t}{A_d} + \Gamma_d \right) = t s^i \quad (A3)$$

It is easily checked that there is no $A_d > 0$ such that $x_d^i = A_d s^i$ has a solution.

Proof of Proposition 1

From (3) and (5), $y_d = x_d + d + u = \frac{x_d[1+t(m-1)]}{t(m-1)} + \frac{u}{2}$, or:

$$y_d = \frac{ts}{[2+t(m-1)]} \cdot \frac{1}{\Gamma_d} + \frac{u}{2}, \text{ where } s = \sum_{i=1}^m s^i \quad (\text{A4})$$

(6) follows from solving $\Gamma_d = \text{covariance}(v, y_d) / \text{variance}(y_d)$.

Proof of Proposition 2

Price informativeness $PI_i, i = d, n$ is defined as:

$$PI_i = \Sigma_v - \text{Variance}(v|p_i) = \Gamma_i^2 \Sigma_{y_i}. \quad (\text{A5})$$

Since $\Gamma_d = 2\Gamma_n$ but $y_d = \frac{y_n}{2}$, $PI_d = PI_n$.

$I_i = E([v-p_i]x_i)$ is the total unconditional expected profits of the informed traders in market $i = d, n$. $I_d < I_n$ since $p_d = p_n$ and $x_d < x_n$.

$$\begin{aligned} I_d + W &= E([v-p_d][x_d+d]) \\ &= E\left\{[v-p_n]\left[\frac{x_n}{2} - \frac{u}{2}\right]\right\} = \frac{I_n}{2} + \frac{\Gamma_n \Sigma_u}{2}. \end{aligned} \quad (\text{A6})$$

The result follows because $\Gamma_n \Sigma_u = \frac{\sqrt{mt\Sigma_v\Sigma_u}}{1+Q} = I_n$.

The expected trading volume in the nondual trading market is

$\frac{A_n \sqrt{m\Sigma_s}}{\sqrt{2\pi}} + \frac{\sqrt{\Sigma_u}}{\sqrt{2\pi}}$. The expected trading volume in the dual trading market is $\frac{Q}{Q-1} \frac{A_d \sqrt{m\Sigma_s}}{\sqrt{2\pi}} + \frac{3}{2} \frac{\sqrt{\Sigma_u}}{\sqrt{2\pi}}$. Substituting in for the values of A_d and A_n , we get the result that the expected trading volumes are the same.

Proof of Proposition 3

In the dual trading market, the profits of the j -th hedger is:

$$\pi_d^j = v(u_d^j + w^j) - \Gamma_d u_d^j \left(u_d^j + D_d \sum_{m \neq j} w^m + X_d + d \right). \quad (\text{A7})$$

Since $X_d + d = \frac{ts}{\Gamma_d(1+Q)} - \frac{u_d}{2}$, this expression becomes:

$$\pi_d^j = v(u_d^j + w^j) - \frac{\Gamma_d u_d^j}{2} \left(u_d^j + D_d \sum_{m \neq j} w^m \right) - \frac{ts}{1+Q} u_d^j \quad (\text{A8})$$

$$E(\pi_d^j | w^j) = -\frac{\Gamma_d}{2} (u_d^j)^2 \quad (\text{A9})$$

$$\begin{aligned} \text{Var}(\pi_d^j | w^j) &= \Sigma_v (w^j)^2 + (u_d^j)^2 \left[\Sigma_v \left(1 - \frac{mt(2+Q)}{(1+Q)^2} \right) \right. \\ &\quad \left. + \frac{(\Gamma_d D_d)^2}{4} (h-1) \Sigma_w \right] + 2 \Sigma_v w^j u_d^j \frac{(2-t)}{1+Q}. \end{aligned} \quad (\text{A10})$$

The certainty-equivalent profits $G_d^j = E(\pi_d^j | w^j) - \frac{R}{2} \text{Var}(\pi_d^j | w^j)$.

Differentiating G_d^j with respect to u_d^j :

$$-\Gamma_d u_d^j - R u_d^j \left[\Sigma_v \left(1 - \frac{mt(2+Q)}{(1+Q)^2} \right) + \frac{(\Gamma_d D_d)^2}{4} (h-1) \Sigma_w \right] - R \Sigma_v w^j \frac{(2-t)}{1+Q} = 0. \quad (\text{A11})$$

Equating D_d to the coefficient of w^j in (A11) and solving:

$$R D_d \frac{(\Gamma_d D_d)^2}{4} (h-1) \Sigma_w + D_d \left[\Gamma_d + R \Sigma_v \left(1 - \frac{mt(Q+2)}{(1+Q)^2} \right) \right] + \frac{R \Sigma_v (2-t)}{1+Q} = 0. \quad (\text{A12})$$

Solving for Γ_d :

$$\Gamma_d D_d = \frac{-2\sqrt{mt\Sigma_v}}{(1+Q)\sqrt{h\Sigma_v}}. \quad (\text{A13})$$

From (12) and (A13), $\Gamma_d D_d = 2\Gamma_n D_n$. Solving for D_d shows $D_d = D_n$. It follows that $G_n^j = G_d^j$ for each j . Thus $G_d = G_n$. Finally, $D_d = D_n$ implies that $\Gamma_d = 2\Gamma_n$.

Proof of Proposition 4

From (14) and (15), $c_n - c_d = \frac{W}{m+h}$. The total benefit to all of the broker's customers is $(m+h)(c_n - c_d) = W$. The share of the m informed traders in the pie is $\frac{m}{m+h}W < W$, which is their loss from dual trading.

Proof of Lemma 4

Suppose the equilibrium is as given in Lemma 4. Then, $X(1) = 1$ and $X(0) = -1$. The informed trader's expected profits from buying one share is:

$$N_n(1) = r_1[1 - P(1)] + q_1[3 - P(1/2) - P(-1/2) - P(-1)] - c_n \quad (\text{A14})$$

where

$$P(1) = \frac{P_r(y_n=1|v=1)}{P_r(y_n=1|v=1) + P_r(y_n=1|v=0)} = \frac{r_1}{r_1 + q_1} \quad (\text{A15})$$

$$P(-1) = \frac{P_r(y_n=-1|v=1)}{P_r(y_n=-1|v=1) + P_r(y_n=-1|v=0)} = \frac{q_1}{q_1 + r_1} \quad (\text{A16})$$

and

$$P(1/2) = P(-1/2) = 1/2.$$

Rewriting (A14) after substituting in for the equilibrium prices:

$$N_n(1) = q_1 + \frac{2r_1 q_1}{r_1 + q_1} - c_n. \quad (\text{A17})$$

Suppose instead the informed trader's strategy is $X(1) = 1/2$ and $X(0) = -1/2$. (We confine ourselves to symmetric strategies of the form $X(1) = +z$ and $X(0) = -z$.) Then, it is easily checked that $N_n(1) = 2N_n(1/2)$. Note also that the informed trader will never sell when $v = 1$ because her net profits are always negative. A similar proof obtains on the sell side and so is not repeated here.

Proof of Lemma 5

Suppose the dual trader observes $x > 0$. She infers that $v = 1$. Given that $p \in (0,1)$, it is optimal for her to buy $1/4$ of a share. Similarly, if $x < 0$ it is optimal for her to sell $1/4$ of a share.

Next, consider the informed trader's problem. Suppose $X(1) = 1/2$ and $X(0) = -1/2$. She knows that the broker will buy and sell $1/4$ of a share. If $v = 1$ and she buys $1/2$ of a share, $x+d = 3/4$ and her expected profits are:

$$N_d(1/2) = \frac{r_1}{2} [1 - P(3/4)] + \frac{0.6q_1}{2} [5 - P(-3/4) - P(5/4) - P(1/4) - P(-1/4) - P(-5/4)] - \frac{c_d}{2} \quad (\text{A18})$$

$$\text{where } P(3/4) = \frac{r_1}{r_1 + 0.6q_1} \quad (\text{A19})$$

$$P(-3/4) = \frac{0.6q_1}{0.6q_1 + r_1} \quad (\text{A20})$$

and $P(5/4) = P(-5/4) = P(1/4) = P(-1/4) = 1/2$.

Substituting in for the equilibrium prices:

$$N_d(1/2) = \frac{1}{2} \left\{ 1.2q_1 + \frac{1.2r_1q_1}{r_1 + 0.6q_1} - c_d \right\}. \quad (\text{A21})$$

Suppose that the informed trader decided to buy and sell 1 share instead. Proceeding the same way as before, her expected profits are:

$$N_d(1) = 2q_2 + \frac{2r_2q_2}{r_2 + q_2} - c_d. \quad (\text{A22})$$

It is optimal for her to buy 1/2 of a share instead of one share if $[N_d(1) - N_d(1/2)] < 0$, which is (19) in the text. Trading occurs in equilibrium if $N_d(1/2) > 0$, which is (18) in the text. Again, we can repeat this exercise on the sell side.

Proof of Proposition 5

The expected trading volume in the nondual trading market is $E(|y_n|) = 2 \cdot \frac{1}{2} \cdot \left(r_1 + \frac{q_1}{2} \right) = r_1 + \frac{q_1}{2}$. In the dual trading market, the expected total trading volume is $E(|y_d|) = \left[r_1 \frac{3}{4} + 0.6q_1 \left(\frac{1}{4} + \frac{5}{4} \right) \right]$. The expected customer trading volume for the broker is $E(|y_d|) - E(|d|) = 0.75r_1 + 0.9q_1 - 0.25$. The competitive commission fee c_i in market $i = d, n$ solves the equation:

$$c_i [E(|y_i|) - E(|d|)] + W = k_1 [E(|y_i|) - E(|d|)] + k_0. \quad (\text{A23})$$

Of course, in the nondual market $d = 0$ and $W = 0$. (20) and (21) in the text follow from (A23). The broker's expected trading profits are:

$$\begin{aligned}
W &= \frac{1}{2} [W(1/4) + W(-1/4)] \\
&= \frac{1}{2} \left[\frac{r_1}{4} (1 - P(3/4) + P(-3/4)) + \frac{0.6q_1}{4} (5 - P(-3/4) + P(3/4)) \right] \\
&= W(1/4)
\end{aligned}$$

where $W(1/4)$ is the expression derived in (22) of the text.

From (20) and (21):

$$c_d - c_n = \frac{k_0 - W}{0.75r_1 + 0.9q_1 - 0.25} - \frac{k_0}{r_1 + 0.5q_1}. \quad (\text{A24})$$

Simplifying, we obtain (23) in the text.

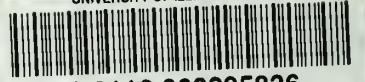
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