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# The Regulation of Dual Trading: Winners, Losers and Market Impact

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WINNERS, LOSERS AND MARKET IMPACT\*

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**THE REGULATION OF DUAL TRADING:  
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**ABSTRACT**

I show that dual trading has no aggregate market impact because total trading volume, market depth and price efficiency are all unaffected by a ban on dual trading. However, trading volume and gross (of commission fees) profits of the informed traders are higher with dual trading while trading volume and gross losses of the uninformed traders are unaffected. This effect of the ban on the uninformed is the same irrespective of whether they act as noise traders or as rational, risk-averse hedgers.

Commission rates charged by the broker may decrease when dual trading is banned if informed trading volume as a proportion of the broker's total customer trading volume, the broker's fixed costs and the amount of information in the market are high. Net profits of the informed and net losses of the uninformed both increase when dual trading is banned.



Dual trading refers to the practice of brokers trading for their own accounts in addition to bringing their customers' orders to the market. The practice has, over the years, generated intense controversy with proponents of dual trading vouching for its salutary effects on market liquidity and price efficiency and opponents emphasizing the potential conflicts of interest between dual trading brokers and their customers. On the regulatory front, the anti-dual-trading camp currently holds sway with the Chicago Mercantile Exchange (CME) banning dual trading in all active contracts effective May 20, 1991. The issue is by no means settled, as the Exchange still faces great resentment over the ban.<sup>1</sup>

I develop a model to study how a ban on dual trading will affect aggregate market characteristics (total trading volume, market depth and price efficiency). I also study its distributional effects by looking at the impact on brokers' commission rates as well as the trading volumes and profits (both gross and net of commission costs) of the informed and the uninformed traders. The main conclusion is that the market impact of dual trading is minimal, but that uninformed traders tend to benefit at the expense of the informed traders.

The microstructure of the basic model follows Kyle (1985). Uninformed noise traders<sup>2</sup> and a group of  $m$  informed traders submit market orders to a broker who places them (along with her own orders) with a marketmaker for execution. The marketmaker batches the total order flow and executes them at a single price. The price is determined by assuming that the marketmaker makes zero profits conditional on observing the total order flow.

The broker's motive for dual trading comes from her private observations of the size of her customers' orders.<sup>3</sup> In equilibrium, she is able to infer all of her informed customers' information from her observations and profit through mimicking or piggybacking on the informed trades. Because their orders are now executed at a higher (in absolute value) price, the broker's piggybacking hurts the informed traders, who react by restricting their order sizes.<sup>4</sup> Thus, informed trading volume is higher when dual trading is banned.

However, total trading volume is unaffected by a ban on dual trading. This is because, with dual trading, the broker's own trading activity exactly makes up for the slack in informed trading volume. As a result, market depth, price informativeness and the price level are also the same with or without dual trading. It follows that the profits of the informed (ignoring commission costs) are lower with dual trading. Finally, total trading (i.e., informed plus dual trading) profits are identical across markets and, therefore, so are the losses of the noise traders.

These results are unchanged when the basic model is extended to allow for rational behavior by uninformed traders. Following Spiegel and Subrahmanyam (1992), uninformed traders (who are risk-averse) trade in order to "hedge" their endowments of shares of the risky asset. Since total informed trading volume (of the informed customers and the broker<sup>5</sup>) are equal with or without dual trading, then so are the riskiness of the market and the trading volume of the "hedgers."

Next, I analyze trading behavior when investors have to pay commission rates. Suppose that commission rates are proportional to the

order size. Informed traders exploit their information signals less because of commission costs, reducing the informativeness of the order flow to the marketmaker. Traders conjecture, therefore, that market depth has two components: a "direct" adverse selection component as in Kyle (1985) plus an "indirect" adverse selection effect equal to the commission rate.

With this formulation, the equilibrium market depth is exactly the same as in the model without commission rates. This is because the marketmaker must make zero expected profits in either case and the amount of information used by informed traders do not change. So, the marketmaker must reduce the "direct" adverse selection component to exactly offset the effect of the commission rate. Informed trading volume and the informativeness of the order flow are restored to their pre-commission-rate values.

As in Fishman and Longstaff (1992), it is assumed the broker incurs fixed and variable costs of brokerage. Further, the brokerage business is competitive so that the broker's total income (trading profits plus commission income) is zero. Then, a ban on dual trading may actually reduce commission rates because the broker can more than offset her loss of trading profits through higher commission income (since informed trading volume is higher).

Commission fees paid by the informed (as well as the combined amount paid by all of the broker's customers) are always lower with dual trading. But, this is not enough to offset the reduction in informed gross profits and so net profits of the informed traders are always lower with dual trading. Similarly, although uninformed traders may pay

more commission fees (if commission rates are higher with dual trading), their losses net of commission costs are always lower than with dual trading.

Roell (1990) has a model of dual trading in which a broker observes the trade of some uninformed traders. Uninformed traders benefit because, through separation from informed traders, they are able to obtain a better price. In Fishman and Longstaff (1992), the broker has private information about whether her customer is informed or uninformed. Their results on the effect of dual trading on the gross and net trading profits of the informed and the uninformed are identical to the ones here. However, they assume that trading volume is fixed at one unit. As a result, the informed trader in their model fails to take into account the broker's mimicking behavior when formulating her optimal trading strategy. A further implication of this assumption is that the broker's commission rates are always lower with dual trading. They also do not model the behavior of the customer when she is uninformed. On the other hand, they allow the customer and the dual trader to trade at different prices and they also model the effect of frontrunning by the broker.

My results on the market impact of dual trading are broadly consistent with the empirical work of Park and Sarkar (1992), who find that market depth is unaffected by a restriction on dual trading in the S&P 500 futures market and that total trading volume decreased, but only by about 4.59%. Also, a study by the Commodity Futures Trading Commission (1989) find that customers of dual trading brokers do not obtain significantly different bid-ask spreads relative to customers of



exclusive brokers. However, the results are inconsistent with the work of Smith and Whaley (1990), who find an average increase of 33% in the effective bid-ask spread when studying the same set of restrictions on dual trading analyzed by Park and Sarkar (1992). In addition, Walsh and Dinehart (1991) find some evidence that dual trading is associated with narrower bid-ask spreads.

Section I develops the basic dual trading model with noise traders, ignoring commission costs. In section II, this is contrasted with a model where dual trading is completely banned. Results on the market impact of dual trading are obtained. Section III extends the basic model to introduce the effect of proportional commission rates on the informed traders' optimal trading strategy. Then, the effect of dual trading on commission rates and traders' net profits is explored. Section IV further extends the model to allow for rational behavior by uninformed traders. Proportional commission rates are re-introduced into this extended model and their effect analyzed. The study concludes in section V. All proofs are contained in the appendix.

## I. THE DUAL TRADING MODEL AND SOLUTION

### A. *The Dual Trading Model*

I consider a market in which a single risky asset with unknown liquidation value  $v$  is traded. There is a group of  $m$  informed traders each of whom receive, prior to trading, signals  $s^i$  about the unknown value  $v$ . The signals are of the form  $s^i = v + e^i, i = 1, \dots, m$  where the error terms  $e^i$  are independent of each other. In addition, there is a group of uninformed noise traders who trade for liquidity reasons. Initially, the uninformed traders' motives for trading are not modelled.

Later, the basic model is extended to allow for rational behavior by the uninformed traders.

Each informed trader  $i = 1, \dots, m$  submits a market order  $x^i$  to a broker. The noise traders also collectively submit market orders worth  $u$  to the same broker. The latter then places the total of the submitted orders  $(x_d + u)$ , where  $x_d = \sum_{i=1}^m x^i$ , to a marketmaker for execution.<sup>6</sup> In the dual trading model, the broker may also trade an amount  $d$  for her own account. She may want to do so because, by observing the market orders  $x^i$  of the informed, she is able to infer some or all of their information  $s^i$ . The act of dual trading makes the broker a de facto informed trader.<sup>7</sup>

It is assumed that, when the broker places her customers' orders with the marketmaker, she simultaneously sends along her own order  $d$  as well. The marketmaker then fixes a single price  $p_d$  at which she will execute the total order flow  $y_d = x_d + d + u$ . Following Kyle (1985), the marketmaker is assumed to be risk-neutral and competitive. Conditional on observing  $y_d$ , she earns zero expected profits.

The random variables in the model are  $v$ ,  $u$  and  $e^i$ ,  $i = 1, \dots, m$ . All these variables are normally distributed with zero mean and finite variances  $\Sigma_v$ ,  $\Sigma_u$  and  $\Sigma_e$ , respectively. Thus the  $m$  error terms are drawn from an identical distribution.<sup>8</sup> In addition, all investors follow linear trading rules  $x^i = A_d s^i$ ,  $i = 1, \dots, m$  (for the informed) and  $d = B \sum_{i=1}^m x^i$  (for the broker). This implies that the marketmaker's pricing rule is also linear:  $p(y_d) = \Gamma_d y_d$ , where  $1/\Gamma_d$  is the now-familiar market depth parameter.

There are three distinct stages to this trading game:

- (1) Informed traders receive their information and decide how much they want to trade. In making this decision, each informed trader is aware that, first, she is in competition with the other informed traders and, second, that the broker will "piggyback" on the information conveyed by her trading decision. The informed traders care about the broker's piggybacking because they receive a less favorable price for their trades as a consequence. Noise traders simply submit  $u$ .
- (2) The broker observes  $u$  and  $x^i$  and infers that each informed trader has some information  $s^{i*}, i = 1, \dots, m$ . Based on her inferences, she decides to trade an amount  $d$ .
- (3) The marketmaker fixes a price  $p_d = \Gamma_d(x_d + u + d)$ , where  $p_d = E(v | y_d)$  and so  $\Gamma_d = \text{Cov}(v, y_d) / \text{Var}(y_d)$ .

This suggests the following solution method. Fix  $\Gamma_d$  and suppose that each informed trader  $i = 1, \dots, m$  has decided to trade some amount  $x^i$ . From each  $x^i$  the broker infers information  $s^{i*}$ . She then chooses  $d$  to maximize her expected profits, where the expectation is taken with respect to the vector  $(s^{1*}, \dots, s^{m*})$ . Each informed trader  $i$  then chooses  $x^i$  as a best response to  $d(s^{1*}, \dots, s^{m*})$  and the rival informed traders' decisions  $x^j, j \neq i$ . Finally,  $\Gamma_d$  is obtained from the optimal trading rules and the marketmaker's zero profit assumption.

Depending upon what the equilibrium beliefs of the broker are, there can be potentially many equilibria to the signalling game between the informed traders and the broker. Fortunately, in this model, the signalling game affords a unique solution: there is a single fully

separating equilibrium. In other words, the informed traders' information is fully revealed to the broker and so  $s^{i*} = s^i, i = 1, \dots, m$  in equilibrium.

### B. The Dual Trading Solution

First, I solve the signalling game between the informed traders and the broker.<sup>9</sup> Given her observations of  $x_d$  and  $u$ , the broker chooses  $d$  to maximize her conditional expected profits  $E(\pi | s^{1*}, \dots, s^{m*}, u)$ , where  $\pi = (v - \Gamma_d y_d)d$ . From the first-order condition, the optimal  $d = [E(v | s^{1*}, \dots, s^{m*}) - \Gamma_d x_d] / 2\Gamma_d$ . The second order condition is satisfied by  $\Gamma_d > 0$ . Define  $t = \Sigma_v / (\Sigma_v + \Sigma_e)$  and note that  $0 \leq t \leq 1$ .  $t$  is a measure of the unconditional precision of  $s^i, i = 1, \dots, m$ . For example, if  $t = 1$  then  $s^i$  is a perfect signal. Then,  $E(v | s^{1*}, \dots, s^{m*}) = ts^* / Q$  where  $Q = [1 + t(m-1)]$  and  $s^* = \sum_{i=1}^m s^{i*}$ . Therefore, the broker's optimal trade is:

$$d = \frac{ts^*}{2Q\Gamma_d} - \frac{x_d}{2} \quad (1)$$

In a separating equilibrium  $s^{i*} = s^i = x^i / A_d$  for each  $i = 1, \dots, m$ . So, in equilibrium, the presence of the dual trader will have two opposite effects on the informed traders' incentive to trade. Suppose  $x^i > 0$  (a buy order). If  $x^i$  is increased, the broker infers that the informed trader's information is improved and so  $s^{i*}$  is higher as well. The broker trades more,  $d$  is higher and so is the resulting price. Thus, this signalling effect tends to inhibit informed traders from trading aggressively.

On the other hand, a higher  $x^i$  also reduces  $d$  from the second term in (1). This is a "second-mover disadvantage" for the broker as she has to accommodate market orders of any size by the informed and tends to encourage informed trades. For finite  $m$ , however, the signalling effect always dominates the second mover effect, so that  $B > 0$  in equilibrium ( $x_d$  and  $d$  always have the same sign). The broker optimally mimics the "consensus" trading decision of the informed group.<sup>10</sup>

Given (1), each informed trader  $i$  chooses  $x^i$  to maximize her conditional expected profits  $E(I_d^i | s^i)$ , where  $I_d^i = (v - \Gamma_d d - \Gamma_d x^i - \Gamma_d \sum_{j \neq i} x^j - \Gamma_d u) x^i$ . After incorporating the optimal value of  $d$  from equation (1) into  $I_d^i$ , the first-order condition for  $x^i$ ,  $i \neq j$  is:

$$\frac{t(1+Q)s^i}{2} = \Gamma_d [x^i + 0.5(m-1)E(x^j | s^i)] + \frac{ts^{i*}}{Q} \quad (2)$$

Equation (2) says that the marginal value of an additional trade for the  $i$ -th informed trader is equal to its marginal cost. This cost has two components: the change in the price due to her own and her rivals' expected trades plus the change in the broker's inference as to her information.

After using the facts that (i)  $s^{i*} = s^i = x^i/A_d$  in equilibrium and (ii)  $E(s^j | s^i) = ts^i$  for  $j \neq i$ ,  $A_d$  is obtained as the coefficient of  $s^i$  in (2):

$$A_d = \frac{t^2(m-1)}{\Gamma_d Q(Q+1)} \quad (3)$$

From (3),  $A_d = 0$  when  $m = 1$ . But  $A_d = 0$  cannot be a separating equilibrium since the functions  $x^i = A_d s^i, i = 1, \dots, m$  are then no longer invertible.<sup>11</sup>

**Lemma 1: When  $m = 1$ , there is no solution to the dual trading model.**

The result can be interpreted as follows. The inhibiting effect of the broker's piggybacking or mimicking behavior on any individual informed trader is inversely related to  $m$ , the number of informed customers the broker has. For  $m = 1$ , this inhibiting effect exactly offsets the marginal value of an extra trade for the individual informed trader. To see this, notice that the first-order condition (2) for  $m = 1$  simplifies to:

$$t(s^i - s^{i*}) = \Gamma_d x^i \quad (4)$$

So, for any  $x^i > 0$ , the marginal cost of an additional trade for the informed always exceeds its marginal benefits.<sup>12</sup>

Substituting (3) into (1), the optimal dual trading function is given by:

$$d = \frac{x_d}{t(m-1)} \quad (5)$$

Finally, by using (3) and (5) in conjunction with the marketmaker's zero profit assumption the optimal value of market depth is derived as:

$$\Gamma_d = \frac{\sqrt{mt} \sum_v}{(1+Q) \sqrt{\sum_u}} \quad (6)$$

Proposition 1 fully characterizes the dual trading equilibrium.

**Proposition 1:** If  $m > 1$  and  $t > 0$ , there exists a unique solution to the dual trading model in which  $x^i = A_d s^i, i = 1, \dots, m$ ,  $d = Bx_d$  and  $p_d = \Gamma_d y_d$  where  $A_d$  is given by (3),  $B$  by the coefficient of  $x_d$  in (5) and  $\Gamma_d$  by (6).

What determines the extent of dual trading in the market? First, consider the effect of increasing the number of informed traders  $m$  on dual trading  $d$ . As  $m$  increases, the broker's observation of the trade of any individual informed trader is less valuable. But, at the same time, she observes more informed trades. The net effect of increasing  $m$  is to weaken the signalling effect and so reduce  $d$ .<sup>13</sup>

The effect of increasing the information precision  $t$  is to make informed trades more sensitive to the information signals and so make the broker's observations more informative. This tends to increase  $d$ . But, a higher  $t$  also increases informed trading volume  $x_d$  and this tends to reduce  $d$  via the second-mover effect. Thus,  $d$  is increasing in  $t$  only if  $t(m-1) < 1$ --i.e., if the total amount of information in the market is sufficiently low. In fact, when  $t(m-1) < 1$ , it follows from (5) that  $d > x_d$ . Since, informationally speaking, the broker is equal to  $m$  informed traders, it may be said that dual trading dominates the market if  $d > x_d$ .

Corollary 1: (1)  $d$  is decreasing in  $m$ .  $\text{Sign} [\delta d / \delta t] = \text{sign} [1 - t(m-1)]$ .  
 (2)  $d > x_d$  if  $t(m-1) < 1$ .

## II. THE MARKET IMPACT OF DUAL TRADING

In weighing the costs and benefits of dual trading, a regulator might be interested in its effect on aggregate market characteristics (total trading volume and profits, market depth and price efficiency) as well as its distributional effect on individual groups of market participants. These groups include the informed and uninformed traders and the broker. The distributional impact of dual trading may be discerned by considering its effects on the trading volumes of the informed and the uninformed,<sup>14</sup> the broker's commission rates and traders' expected profits net of commission costs. The impact of dual trading on aggregate market characteristics is studied in this section and the distributional question is analyzed in the next.

### A. *The Nondual Trading Model*

I will compare the dual trading solution obtained in section I with the solution obtained when dual trading is completely banned. The broker is then a pure intermediary, bringing her customers' orders to the market.<sup>15</sup> The resulting trading game is a Cournot-Nash game in trading quantities. Each informed trader places an order  $x^i$  with the broker based on her information  $s^i$ . The broker submits the total order flow  $y_n = x_n + u$  (where  $x_n$  is total informed trading volume in the nondual trading market) to the marketmaker for execution. The price determined is  $p_n = \Gamma_n y_n$ . Lemma 2 describes the nondual trading equilibrium.



**Lemma 2:** If there is no dual trading, a solution always exists provided  $t > 0$ . The informed traders trade  $x^i = A_n s^i$  and the price is  $p_n = \Gamma_n y_n$ , where:

$$A_n = \frac{t}{\Gamma_n(1+Q)}, \quad \Gamma_n = \frac{\sqrt{mt \sum_v}}{(1+Q) \sqrt{\sum_u}} \quad (7)$$

### ***B. Trading Volume and the Gross Profits, Market Depth and Price***

#### ***Efficiency***

Due to "piggybacking" by the broker, it is reasonable to expect that  $x_d < x_n$ . The difference in informed trading volume depends upon the trading intensities  $A_n$  and  $A_d$ , as well as the market depth parameters  $\Gamma_d$  and  $\Gamma_n$ . But, by inspection of (6) and (7), the market depth parameters have the same value. So:

$$x_n - x_d = \frac{ts}{Q} \frac{1}{\Gamma(1+Q)}, \quad \text{where } \Gamma = \Gamma_d = \Gamma_n. \quad (8)$$

which is positive for  $t > 0$  and  $s > 0$ .  $ts/Q$  represents what the broker learns about the unknown  $v$  from observing the  $m$ -vector of informed trades. The more informative is this observation, the greater is the relative shrinkage of informed trades in the dual trading market. The difference in informed trading volume is also positively related to market depth, since a deeper market allows the broker to place larger orders with less concern about its impact on the price.

However, the broker herself provides an additional source of trading activity in the dual trading market. Now, in general, the

difference in total trading volume between the two markets ( $y_d - y_n$ ) depends upon the trading intensities  $A_d$ ,  $A_n$  and  $B$  and the market depth parameters  $\Gamma_d$  and  $\Gamma_n$ . However:

$$y_d - y_n = \frac{ts}{1+Q}[\Gamma_d - \Gamma_n] \quad (9)$$

and so the difference depends upon the market depth parameters only. Since the market depth parameters have the same value, total trading volumes are exactly the same in both markets--the broker's trading activity precisely offsets the slack in informed trading volume.

In fact, the equality of market depth in the two markets implies that the informativeness of the total order flow is unchanged. Informed traders use their information less when there is dual trading, but the broker also exploits the same information vector so that the total information usage remains the same. This suggests that price efficiency  $PI_i$ ,  $i = d, n$  (where  $PI_i$  is defined as  $\Sigma_v - \text{var}(v|p_i)$ ) is also identical in both markets. Thus, the aggregate market impact of dual trading is nil (although distributional effects are present since  $x_d < x_n$ ).

Let  $I_i$  denote the combined unconditional expected profits of the informed group (before observing any signals or paying any commissions) in the  $i$ -th market,  $i = d, n$ . Since both market depth and total trading volumes are identical in the two markets, then so is the price level. Hence, total trading profits must be the same in the two markets, i.e.,  $I_d + \pi = I_n$ . From (5),  $d > 0$  in equilibrium whenever  $t > 0$  and  $m > 1$  and so the broker's trading profits  $\pi$  are strictly positive. It follows

that  $I_d < I_n$ : gross profits of the informed are strictly lower with dual trading.

As the marketmaker makes zero expected profits, uninformed traders lose money in equilibrium and the amount of their loss mirrors the total trading profits of the informed and the broker. Denote  $L_g = I_d - I_n + \pi$  as the difference in the gross losses of the uninformed between the dual and nondual trading regimes. Therefore,  $L_g = 0$ .

**Proposition 2:** (1)  $Y_d = Y_n$ ,  $I_d + \pi = I_n$ ,  $\Gamma_d = \Gamma_n$  and  $PI_d = PI_n$ . Total trading volume and gross profits, market depth and price efficiency are the same with or without dual trading. (2)  $I_d < I_n$  and  $L_g = 0$ . Gross profits of the informed are lower with dual trading. Gross losses of the uninformed are unchanged.

### III. COMMISSION RATES AND NET TRADING PROFITS

#### A. A Model of Trading With Commission Rates

Suppose that, in market  $i = d, n$ , the broker charges a per trade commission fee of  $\$c_i$  to cover her costs of brokerage. Agents make their decisions in the following sequence. At stage zero, the broker determines the commission rate  $c_i$ . At stage one, the informed traders observe their signals and  $c_i$ . Then, they decide how much to trade. Noise traders trade  $u$ . The rest of the model proceeds as before. Stage zero is analyzed in section IIIB. The subsequent stages are analyzed here.

I will assume that the commission rate is quadratic in the trades. In other words, if trader  $i$  buys or sells  $z^i$  shares the commission fee paid is  $\$c_i(z^i)^2$ . This is the most tractable way of ensuring positive

commission fees on all trades, whether a buy or sell. Of course, the optimal trading decisions must now be based on traders' profits, net of her commission costs. Let  $z^i$  denote the solution to the  $i$ -th informed trader's problem in this case, with  $z_i$  indicating the aggregate informed trading volume in the  $i$ -th market,  $i = d, n$ . As a short-hand, I will refer to the model with brokerage costs as the  $z$ -model to contrast with the earlier  $x$ -model (based on traders' gross profits).

In defining an equilibrium concept for the  $z$ -model, there are two important modelling issues to contend with. First, the distribution of  $z^i$  is potentially non-normal because, for realizations of  $s^i$  close to zero, net profits may be negative if the commission rate is high. Thus, there could be a no-trade interval around zero.<sup>16</sup> Second, there are many different ways in which the commission rate  $c_i$  could interact with the informed traders' information signals.<sup>17</sup>

Let  $N_i^j$  and  $N_i$  be the  $j$ -th informed trader's net profits and the combined net profits of the informed group in the  $i$ -th market,  $i = d, n$ . Still writing  $y_i$  for the total order flow (with no presumption that its equilibrium value is unchanged), suppose that traders conjecture the marketmaker's pricing function to be  $p_i = \tau_i y_i$ ,  $i = d, n$ . In the nondual trading market, the expected net profits of the  $j$ -th informed trader can be expressed as:

$$E(N_n^j | s^j) = [t s^j - \tau_n z^j - c_n z^j - \tau_n E(z^{-j} | s^j)] z^j \quad (10)$$

where  $z^{-j} = \sum_{i \neq j} z^i$  is the total trading volume of the rival informed traders.

Now, consider an "indirect" formulation of this problem. The individual informed trader no longer directly takes into account the amount of commission fees she has to pay. Instead, she conjectures that  $p_i = (\lambda_i + c_i)y_i$ . The logic behind this formulation is as follows. Market depth is still determined by the marketmaker's adverse selection considerations only. However, it now has two components: a direct adverse selection effect given by  $\lambda_i$  and an indirect effect caused by the transactions cost element represented by  $c_i$ . An increase in  $c_i$  reduces the intensity with which information is exploited, decreasing the variance of the order flow and so reduces market depth. Thus, for a given amount of the order flow  $y_i$ , the price level and the commission rate are positively correlated.<sup>18</sup>

In the "indirect" problem, expected net profits of the  $j$ -th informed trader in the nondual trading case is:

$$E(N_n^j | S^j) = [ts^j - \lambda_n z^j - c_n z^j - (\lambda_n + c_n)E(z^{-j} | S^j)]z^j \quad (11)$$

Equations (10) and (11) look very similar, except that (11) contains an extra term which represents the effect on market depth of the expected commission fees paid by rival informed traders. The "indirect" analysis for the dual trading case is similar. The broker does not pay any commission fees. But, since her trades are derived from the informed trades, it is rational for her to make the same conjecture as the informed traders do. In equilibrium, such a conjecture is self-fulfilling.

The "indirect" representation has several advantages over the "direct" formulation. First, for the informed traders, it can be shown that equilibrium net profits are always positive and so the equilibrium trading volume is normally distributed.<sup>19</sup> Second, it affords a closed-form solution which the "direct" model does not.<sup>20</sup> Therefore, it is the "indirect" formulation that will be followed here. Lemma 3 describes the resulting equilibria.

**Lemma 3:** The model with proportional commission rates has the same equilibrium solution as the model without commission rates with  $\lambda_i + c_i = \Gamma_i$ ,  $z_i = x_i$  and  $N_i = I_i$  for  $i = d, n$ .

Lemma 3 says that first, market depth is the same with or without commission rates. Since the marketmaker is restricted to making zero profits irrespective of whether commissions are charged or not, the equilibrium level of depth must be the same if the informativeness of the order flow is the same. The normality of the equilibrium trading volume in the z-model ensures that every trade which was feasible without commission rates is also feasible with commission rates. Hence, the order flow is equally informative in both cases. This argument also implies that the equilibrium informed and dual trading volume will be invariant with respect to the commission rate. Further, equilibrium net profits in the z-model must equal the equilibrium gross profits in the x-model.

What is different is the composition of the market depth parameter in the two cases. The direct adverse selection component of market depth  $\lambda_i$  is smaller in the model with commission costs because, after

adjusting for the transactions cost effect, the residual adverse selection problem is less severe for the marketmaker.

**B. The Effect of Dual Trading on Commission Rates**

Suppose that the broker faces a fixed cost  $k_0$  and a variable cost  $k_1$  of conducting business, both costs being non-negative. To be consistent with the representation of the commission rate, it will be assumed that the variable cost of a trade  $z^i$  is  $k_1(z^i)^2$ . Following Fishman and Longstaff (1992), I assume that the brokerage business is competitive and so the commission rate  $c_i$  is chosen such that the broker's expected trading profits plus expected commission income equals zero. To avoid introducing further notation, I will write  $z_i$ ,  $i = d, n$  and  $u$  to mean  $E\{(z_i)^2\}$  and  $E(u^2)$ , respectively. Then the broker's commission rates with ( $c_d$ ) and without ( $c_n$ ) dual trading must satisfy:

$$c_d = k_1 + \frac{k_0 - \pi}{z_d + u} \quad (12)$$

$$c_n = k_1 + \frac{k_0}{z_n + u} \quad (13)$$

Note that, if expected customer trading volume is the same in both markets (as in Fishman and Longstaff (1992)), then  $z_d = z_n$  and so  $c_d < c_n$  of necessity. But if  $z_d < z_n$  (as here), then  $c_d > c_n$  is possible. The broker can offset the loss of her trading profits when dual trading is banned through higher commission income generated by the greater volume of customer trades and thus maintain lower commission rates  $c_n$ . Proposition 3 fully characterizes the relationship between  $c_d$  and  $c_n$ .

**Proposition 3:**  $\text{Sign}(c_d - c_n) = \text{sign}(k_0 - \sqrt{mt \sum_v \sum_u})$ .  $c_d < c_n$  is likely for higher values of uninformed trading  $\Sigma_u$  and lower values of the fixed brokerage cost  $k_0$  and total information  $mt$ .

To explain proposition 3, it would be helpful to express the difference in commission rates in terms of their basic parameters, thus:

$$c_d - c_n = k_0 \left[ \frac{1}{z_d + u} - \frac{1}{z_n + u} \right] - \frac{\pi}{z_d + u} \quad (14)$$

According to (14), fixed costs per unit of customer trading volume are higher with dual trading since  $z_d < z_n$ . This tends to make  $c_d > c_n$ . If fixed costs are small and uninformed trading volume  $u$  is large relative to total customer trading volume, this factor becomes relatively small in magnitude. In addition, the broker's trading profits allow her to reduce commission rates with dual trading. From corollary 1, if the amount of information  $mt$  is small then dual trading is extensive and, by implication, so are dual trading profits. This tends to make  $c_d < c_n$ .

The difference in commission fees paid by the informed can be expressed as:

$$c_d z_d - c_n z_n = k_0 \left[ \frac{z_d}{z_d + u} - \frac{z_n}{z_n + u} \right] - \pi \frac{z_d}{z_d + u} - k_1 (z_n - z_d) \quad (15)$$

The proportion of fixed costs paid for by the informed (relative to that by the uninformed) is lower with dual trading and so is the burden of the broker's variable costs borne by them. In addition, a



proportion of dual trading profits also serves to reduce the commission costs of the informed. Hence all three terms in the right hand side (RHS) of (15) are negative; the informed always pay lower commission fees with dual trading.

For the uninformed traders, however, since their trading volume is fixed at  $u$ , the difference in their commission fees is simply equal to the increase in the proportion of the fixed costs paid for by them--adjusted by the share of dual trading profits used to reduce their commission costs:

$$u(c_d - c_n) = k_0 \left( \frac{u}{z_d + u} - \frac{u}{z_n + u} \right) - \pi \frac{u}{z_d + u} \quad (16)$$

Considering (15) and (16) together, the difference in commission costs paid by all customers between the dual and the nondual trading markets is:

$$c_d(z_d + u) - c_n(z_n + u) = -\pi - k_1(z_n - z_d) < 0 \quad (17)$$

Since the fixed costs must be fully paid for in both markets, it does not enter into (17). Total commission costs for customers are lower in the dual trading market because the broker's trading profits cushion part of the costs and because the broker's variable costs are lower (due to the reduced customer trading volume).

Let  $I_{iz}$  be the gross profits of the informed traders in the  $z$ -model for market  $i = d, n$ . Then, the difference in the gross losses of the uninformed between the dual and the nondual trading markets is given

by  $L_g = I_{dz} - I_{nz} + \pi$ . Where previously (in the model without commission costs)  $L_g = 0$ , it can be shown that now  $L_g = c_d z_d - c_n z_n < 0$ --i.e.,  $L_g$  reflects the difference in commission costs paid by the informed traders. The reason is that, in equilibrium, informed traders are fully compensated by the marketmaker for their commission costs through the adjustment in market depth. Thus, it is the uninformed traders who (indirectly) pay for the informed traders' commission costs.

The difference in uninformed net losses are  $L_n = L_g + u(c_d - c_n)$ . From the discussion above,  $L_n$  can be expressed as the difference between total commission costs paid by customers in the two markets:

$$L_n = c_d(z_d + u) - c_n(z_n + u) \quad (18)$$

which, from (17), is negative--uninformed net losses decrease with dual trading.

The net profits of the informed traders are  $N_i = I_{iz} - c_i z_i$ . Informed traders pay lower commission costs with dual trading, but this is not sufficient to fully offset a reduction in their gross profits. To see this, note that the difference in informed net profits between the dual and the nondual trading markets simply mirrors the difference in uninformed net losses (after adjusting for changes in the informed trading volumes):

$$N_d - N_n = L_n + k_1(z_n - z_d) \quad (19)$$

From (17), (18) and (19),  $N_d - N_n = -\pi < 0$ . The result is intuitive since, from lemma 3, informed net profits in the z-model have the same equilibrium values as informed gross profits in the x-model.

Proposition 4 summarizes the results on commission rates and customers' net profits.

**Proposition 4:** (1)  $c_d(x_d+u) < c_n(x_n+u)$  and  $c_d x_d < c_n x_n$ . Commission costs of all customers, as well as that of the informed separately, are lower with dual trading. (2)  $N_d < N_n$ . Informed net profits are lower with dual trading. (3)  $L_n < 0$ . Net losses of the uninformed are also lower with dual trading.

So far, the trading motivations of uninformed traders have not been modelled. In the next section, the basic model is extended to allow for rational behavior by uninformed traders.

#### IV. HEDGING BY UNINFORMED TRADERS

Initially, suppose that there are no commission costs. Later, I will indicate how the results generalize with the introduction of proportional commission costs.

##### A. *The Model With Uninformed Traders as Rational Hedgers*

There are  $h$  risk-averse uninformed traders ("hedgers") who trade for purely risk-sharing reasons. The development of the model here follows Spiegel and Subrahmanyam (1992). Each hedger  $j$  has random endowment  $w^j$ , which is assumed to be normally distributed with mean zero and variance  $\Sigma_w$ .  $w^j, j = 1, \dots, h$  are independent of each other and all

other random variables in the model. All hedgers have negative exponential utility functions with risk-aversion parameter  $R$ .

Suppose that all hedgers submit market orders  $u^j$  to the broker and follow linear trading rules of the form  $u^j = Dw^j, j = 1, \dots, h$ . Let the total uninformed trading volume be  $u = \sum_{j=1}^h u^j$ . If  $\pi_i^j$  is the profit of the  $j$ -th hedger in market  $i = d, n$ , then  $u^j$  is chosen to maximize her utility or certainty-equivalent profits  $V_i^j = \{E(\pi_i^j | w_j) - \frac{R}{2} \text{Var}(\pi_i^j | w_j)\}$ . Let  $V_i, i = d, n$  be the sum of the utilities of all  $h$  hedgers in the  $i$ -th market. The informed traders and the broker's maximization problem remains the same as before, since each  $w^j$  is independent of  $v$ .<sup>21</sup> Market depth is now positively related to the magnitude of the "hedge factor"  $D^2$  (since this increases the variance of the total order flow) and to the risk aversion parameter  $R$ . Further, the equilibrium  $D < 0$  since the marginal utility of the hedgers from a purchase (sale) is negative if endowments are positive (negative). Lemma 4 describes the equilibrium.

**Lemma 4:** An equilibrium to the hedger model exists if  $R$  satisfies equation (A20) in the appendix. In equilibrium, each hedger  $j = 1, \dots, h$  trades  $u^j = D_i w^j$ , where  $D_i < 0$ , market depth is  $1/\theta_i, i = d, n$  and:

$$-D_i \theta_i \sqrt{h \sum_w} = \Gamma \sqrt{\sum_u} \quad (20)$$

$\Gamma = \Gamma_d = \Gamma_n$  is defined in (6) or (7) and  $D_i$  in equation (A19) of the appendix.

As in Spiegel and Subrahmanyam (1992), equilibrium exists if the amount of risk-aversion and noise in the market exceeds the amount of

information available. Notice that the RHS of (20) is independent of  $i$ . Further, since total informed trading volume is the same *function* of  $\theta_i$  in both markets,  $\theta_d(x_d+d) = \theta_n x_n$ . In other words, equilibrium informed trades have the same impact on the price level in both markets and so the variance of uninformed profits must also be the same across the two markets. Thus, the amount of risk and so the number of shares hedged by the uninformed traders are equal between the dual and the nondual trading markets:  $D_d = D_n$ . From (20),  $\theta_d = \theta_n$  and by implication  $V_d = V_n$ . Hence, the conclusion that dual trading has no aggregate market impact remains unchanged when uninformed traders behave rationally.

**Proposition 5:**  $D_d = D_n$ ,  $V_d = V_n$  and  $\theta_d = \theta_n$ . The hedgers' trading volume and gross utility and the depth of the market are the same with or without dual trading.

#### ***B. The Effect of Proportional Commission Rates***

From the discussion in section IIIA, the commission rate affects trading volume through the market depth parameter only. Since the latter is determined solely on the marketmaker's adverse selection considerations,  $\theta_i$ ,  $i = d, n$  does not change with the introduction of proportional commission rates. So, the equilibrium value of  $D$  does not change either. Further, since the hedgers' trading volume is equal across the dual and the nondual trading markets, the comparison between the dual and nondual trading markets put forth in section IIIB is still valid here. In particular, proposition 4 holds precisely as stated.

A remaining issue of interest is whether there are parameter values for which commission rates are higher with dual trading in the noise trader solution, but are lower in the current model? In comparing the noise trader model to the hedger model, the pivotal variable is the expected volume of uninformed trading. If this were greater in the hedger model,<sup>23</sup> then (since neither the total information nor the brokerage costs have changed) it follows from proposition 3 that  $c_d < c_n$  would be more likely as well. Corollary 2 states conditions under which  $c_d < c_n$  in the hedger model and, simultaneously,  $c_d > c_n$  in the noise trader model.

**Corollary 2:** If  $|D|\sqrt{h\sum_w} > k_0 > \sqrt{\sum_u}$ , then  $c_d > c_n$  in the noise trader model but  $c_d < c_n$  in the hedger model.

The following example illustrates this corollary. Suppose that  $\Sigma_u = \Sigma_w = \Sigma_v = 1$ . Let  $k_0 = 1.2$ ,  $t = 0.5$  and  $m = 2$ . Then  $\sqrt{mt\sum_v\sum_u} = 1 < k_0$  and so from proposition 3,  $c_d > c_n$  in the noise trader model. Now, let  $R = 2$  and  $h = 4$ . Then, from equations (A19) and (A21) in the appendix,  $|D| = 0.71$  and  $|D|\sqrt{h\sum_w} = 1.42 > k_0$  and so  $c_d < c_n$  in the hedger model. High values of the risk-aversion parameter and the number of hedgers make this result likely.

For a larger set of parameter values, when  $c_d > c_n$  in the noise trader model,  $c_d < c_n$  is possible (but not guaranteed) in the hedger model. If the amount of noise is pegged at the same level in the two models, only the weaker condition that the uninformed traders "overhedge" is required. No restrictions are needed on the fixed cost  $k_0$ . This is stated in Corollary 3.

Corollary 3: Suppose  $\Sigma_U = h\Sigma_W$ . Then, if  $|D| > 1$ , there will be some parameter values for which  $c_d > c_n$  in the noise trader model and  $c_d < c_n$  in the hedger model.  $|D| > 1$  if  $A\sqrt{\sum_v \sum_w mt} > 2(1+Q)\sqrt{h}$ , which is satisfied for high values of  $A$  and  $\Sigma_W$  and for  $h < mt$ .

## V. CONCLUSION

Dual trading has no aggregate market impact since total trading volume, market depth and price efficiency are all invariant with respect to a ban on dual trading. However, the dual trading activity comes at the expense of other informed traders who are the broker's customers. These informed traders restrict their trading volume in anticipation of the broker mimicking their trades and "piggybacking" on their information.

The reduction in the business of informed customers leads to a loss in commission income for the broker if dual trading is permitted. If this loss is greater than the broker's trading profits, then commission rates may increase with dual trading. This is likely if the broker's fixed brokerage costs, the total amount of information in the market and informed trading as a proportion of total customer trades are high. Informed profits and uninformed losses, both gross and net of commission costs, are lower with dual trading.

The basic model is extended to model uninformed traders as risk-averse hedgers. Since total informed trading (of informed traders plus the broker) is identical with or without dual trading, the hedgers also trade the same amount in both markets. Although the comparison between the two regulatory regimes remains unchanged, parameter values are derived under which commission rates will be lower with dual trading

in the hedger model but will be higher with dual trading in the noise trader model.



## FOOTNOTES

<sup>1</sup>See The Chicago Tribune, February 4, 1992 and The Wall Street Journal, February 7, 1992 for details.

<sup>2</sup>Later, the basic model is extended to allow for rational behavior by uninformed traders.

<sup>3</sup>The broker is assumed to have no private information of her own. For a model with a privately informed broker, see Sarkar (1991).

<sup>4</sup>The assumption of a batch market maximizes the negative impact of piggybackers on informed trading. But the effect would remain in a setting where the orders of the customers and the broker are executed (and priced) separately, so long as some subset of the informed customers make repeat purchases or sales via the same broker. From Kyle (1985), the optimal dynamic trading strategy of an informed trader is in fact to dribble her trades over time.

<sup>5</sup>By virtue of being able to infer the information of her informed customers, the broker can be considered to be a de facto informed trader.

<sup>6</sup>I will adopt the convention of labelling the decision variables of individual agents with a superscript and market variables with a subscript. The subscript  $d$  will refer to the solution in the dual trading model and the superscript  $n$  to the nondual trading solution.

<sup>7</sup>The broker is assumed to have no independent information regarding  $v$ . In a previous version of this paper (Sarkar (1991)), the broker had her own information and  $m = 1$ . This made for some interesting

interactions between the information of the single insider and that of the broker. For example, for low precision of the broker's information, the insider's trades is actually decreasing in the precision of her own information!

<sup>8</sup>For  $m = 2$ , I have checked that the results are unchanged if the informed traders have information of different precisions. I conjecture that this is true for general  $m$ .

<sup>9</sup>The exposition in this section follows Mailath (1987).

<sup>10</sup>Depending upon the realization of their information signals, some informed traders may place buy orders and others sell orders. If  $x_d > 0$ , then the buy signals are stronger and  $d > 0$  also. Notice that  $x_d = 0$  is not ruled out.

<sup>11</sup>This result is similar in spirit to a corollary in Gould and Verrecchia (1985), where a privately informed specialist sets a price which is then observed by a single trader. They show that equilibrium does not exist if the trader has no private information of her own.

<sup>12</sup>In Sarkar (1991), an equilibrium exists even with  $m = 1$  so long as the precision of the broker's information is positive. The reason is that, if the broker has an independent source of information about  $v$ , she attaches relatively less weight to her observation of the informed trade. The change in the broker's inference, when the informed trader buys or sells an extra share, no longer fully offsets the marginal value of that extra share traded.

<sup>13</sup>The reason is that, although the signal realizations are distinct, they all pertain to a single asset. Thus, much of the

additional information obtained by the broker is redundant. Formally, there are  $m(m-1)$  covariance terms for  $m$  distinct signals.

<sup>14</sup>Since in the model so far, the trading volume of the uninformed is not a choice variable, this particular question is deferred until section IV (when I do model the uninformed trading decision).

<sup>15</sup>This interpretation of a nondual trading market as one where the broker does not trade at all appears to be consistent with market realities (for example, the S&P 500 index futures market where such a ban is currently effective). In the previous version of this paper (Sarkar, 1991) the broker was assumed to be independently informed and this raised troubling issues as to what happens to the broker's information when dual trading is banned. A related issue concerns the choice of brokers. If some brokers can commit not to dual trade (as occurs in reality), why should all customer trades not be redirected to them? Since customers do in fact continue to knowingly patronize dual trading brokers (in futures markets, the broker has to announce her intention to dual trade at the beginning of the trading day), it must be the case that these brokers provide customers benefits not available from nondual trading brokers. For example, Grossman (1989) has conjectured that the opportunity to engage in dual trading rewards brokers with superior trading skills. Thus, my model should be seen as a reduced form of this more general situation where traders and brokers are matched according to their varying needs. I thank the Editor, Douglas W. Diamond, for bringing these points to my attention.

<sup>16</sup>I wish to thank the anonymous referee for pointing this out to me.

<sup>17</sup>Here are two examples.  $z^i = F_i s^i$  with  $A_i > F_i$ . Here the commission rate lowers the trading intensity. Or,  $z^i = x^i - T_i c_i$ . Here, the trading intensity  $A_i$  is unchanged. But  $T_i$ , the marginal effect of  $c$  on the trading volume, could be affected by  $A_i$ .

<sup>18</sup>Thus, a fixed per trade commission would not affect market depth by this argument.

<sup>19</sup>This is because, in equilibrium, net profits in the  $z$ -model are equal to the equilibrium gross profits in the  $x$ -model.

<sup>20</sup>The solution for the "direct" representation involves a cubic equation in the market depth parameter.

<sup>21</sup>Of course, the actual informed and dual trading volumes will be different since market depth will be different, in general.

<sup>22</sup>The "hedge factor" is different from the hedge ratio familiar in the futures/options literature since, here, the asset being used for hedging purposes is the same one that is being hedged. I am grateful to Bjorn Flesaker for explaining this to me.

<sup>23</sup>Of course, a comparison of the actual uninformed trading volume is not meaningful since it is not a choice variable in the noise trader model. However, the expected noise trading volume and the equilibrium expected hedging volume can be compared.

**APPENDIX**

**Proof of Lemma 1**

Let  $E(v|s^{i^*}, \dots, s^{m^*}) = as^*$ , where  $s^* = \sum_{i=1}^m s^{i^*}$ . Applying Bayes' rule,  $a = \frac{1/\sum_e}{1/\sum_v + m/\sum_e} = \frac{t}{1+(m-1)t}$ , where  $t = \Sigma_v/(\Sigma_v + \Sigma_e)$ . This gives the optimal dual trade  $d(s^*, x_d)$  as given in (1).

Incorporating (1), each informed trader  $i$ 's profits  $I_d^i$  can be written as:

$$I_d^i = \left( v - \frac{t}{1+(m-1)t} \frac{s^*}{2} - \frac{\Gamma_d}{2} x_d - \Gamma_d u \right) x^i \quad (A1)$$

Substituting  $s^{i^*} = s^i = \frac{x^i}{A_d}$  for each  $i = 1, \dots, m$  into (A1):

$$\begin{aligned} E(I_d^i | s^i) &= \left[ t s^i - \frac{x^i}{2} \left( \Gamma_d + \frac{t}{1+(m-1)t} \frac{1}{A_d} \right) \right. \\ &\quad \left. - \sum_{j \neq i} \frac{E(x^j | s^i)}{2} \left( \Gamma_d + \frac{t}{1+(m-1)t} \frac{1}{A_d} \right) \right] x^i \end{aligned} \quad (A2)$$

Substituting  $E(x^j | s^i) = A_d t s^i$  for each  $j \neq i$  into (A2) and then differentiating with respect to  $x^i$  gives (2). When  $m = 1$ , (2) has the form:

$$x^i \left( \frac{t}{A_d} + \Gamma_d \right) = t s^i \quad (A3)$$

It is easily checked that there is no  $A_d > 0$  such that  $x^i = A_d s^i$  has a solution.

Proof of Proposition 1

From (5),  $y_d = x_d + d + u = \frac{x_d[1+t(m-1)]}{t(m-1)} + u$ , or:

$$y_d = \frac{ts}{[2+t(m-1)]} \cdot \frac{1}{\Gamma_d} + u, \text{ where } s = \sum_{i=1}^m s^i \quad (\text{A4})$$

(6) follows from solving  $\Gamma_d = \text{covariance}(v, y_d) / \text{variance}(y_d)$ .

Proof of Corollary 1

From (3), (5) and (6),  $d = \frac{\sqrt{t}}{Q\sqrt{m}} \frac{\sqrt{\sum u}}{\sqrt{\sum v}} s$ . Ignoring terms unrelated to  $m$  and  $t$ ,  $\frac{\delta d}{\delta t} = \frac{1-t(m-1)}{2Q^2\sqrt{m}t} > 0$  for  $t(m-1) < 1$ .

$$\frac{\delta d}{\delta m} = \frac{-\sqrt{t}}{Q\sqrt{m}} \left( \frac{1}{2m} + \frac{t}{Q} \right) < 0.$$

Proof of Lemma 2

Each informed trader "i" chooses  $x^i$  to maximize  $E(I_n^i | s^i)$ , where:

$$I_n^i = \left( v - \Gamma_n x^i - \Gamma_n \sum_{j \neq i} x^j \right) x^i \quad (\text{A5})$$

The first-order condition for  $x^i$  yields:

$$x^i = \frac{ts^i}{2\Gamma_n} [1 - (m-1)\Gamma_n A_n] \quad (\text{A6})$$

Solving (A6) yields the equilibrium value for  $A_n$ . Solving for  $\Gamma_n$  in the usual way, (7) is obtained. From the proof of proposition 1:

$$\Gamma_d(x_d+d) = \frac{ts}{[2+t(m-1)]} = \Gamma_n x_n. \quad (\text{A7})$$

Proof of Proposition 2

(1) From (A7), it follows that  $y_d = y_n$  since  $\Gamma_d = \Gamma_n$ . Since  $y_d = x_d + d + u$  and  $y_n = x_n + u$ , it follows that  $x_d < x_n$  since  $d > 0$  for finite  $m$ . Price informativeness  $PI_i, i = d, n$  is defined as:

$$PI_i = \sum_v - \text{Variance}(v|p_i) = \Gamma_i^2 \sum_{y_i}. \quad (\text{A8})$$

Since  $\Gamma_d = \Gamma_n$  and  $y_d = y_n$ ,  $PI_d = PI_n$ .

(2)  $I_i = E([v-p_i]x_i)$  is the total unconditional expected profits of the informed traders in market  $i = d, n$ .  $I_d < I_n$  since  $p_d = p_n$  and  $x_d < x_n$ .

$$\begin{aligned} I_d + \pi &= E([v-p_d][x_d+d]) \\ &= E([v-p_n]x_n) = I_n. \end{aligned}$$

Proof of Lemma 3

Differentiating (11) with respect to  $z^j$ :

$$z^j = \frac{ts^j}{2(\lambda_n+c_n)} \cdot [1-(m-1)A_n(\lambda_n+c_n)] \quad (\text{A9})$$

Solving for  $A_n$  from (A9),  $A_n = \frac{t}{(\lambda_n+c_n)(1+Q)}$ . Solving for  $(\lambda_n+c)$ ,  $(\lambda_n+c) = \frac{\sqrt{mt} \sum_v}{(1+Q) \sqrt{\sum_u}} = \Gamma_n$ .

The proof for the dual trading case is similar.

Proof of Proposition 3

From (12) and (13) in the text:

$$c_d - c_n = \frac{1}{x_d + u} \cdot \left[ k_0 \frac{(x_n - x_d)}{y_n} - \pi \right] \quad (\text{A10})$$

Since  $x_n$ ,  $x_d$  and  $u$  are  $E(x_n)^2$ ,  $E(x_d)^2$  and  $E(u^2)$ , we have:

$$x_n - x_d = \frac{mt \sum_v}{Q\Gamma^2(1+Q)^2} = \frac{\sum_u}{Q} \quad (\text{A11})$$

$$y_n = \sum_u (1+Q) \quad (\text{A12})$$

$$\pi = \frac{\sqrt{mt \sum_v \sum_u}}{Q(1+Q)} \quad (\text{A13})$$

Substituting (A11)-(A13) into (A10), the result follows.

Proof of Proposition 4

By definition,  $N_i = I_{iz} + c_i z_i$  for  $i = d, n$ . From lemma 3,  $N_i = I_i$  and so:

$$\begin{aligned} L_g &= I_{dz} - I_{nz} + \pi \\ &= I_d - I_n + \pi + c_d z_d - c_n z_n \\ &= c_d z_d - c_n z_n \text{ from part 1 of proposition 2.} \end{aligned}$$

$$\begin{aligned} L_n &= L_g + u(c_d - c_n) \\ &= c_d(z_d + u) - c_n(z_n + u). \end{aligned}$$



Proof of Lemma 4

$$\pi_i^j = v(u^j + w^j) - \theta_i u^j \left( u^j + D_i \sum_{m \neq j} w^m + x_i \right) \quad (\text{A14})$$

where  $x_d$  is total informed (including dual) trades. From the maximization problem of the informed traders,  $\theta_i x_i = \frac{ts}{1+Q}$  for  $i = d, n$ .

$$E(\pi_i^j | w^j) = -\theta_i (u^j)^2 \quad (\text{A15})$$

$$\begin{aligned} E(\pi_i^j | w^j) &= -\theta_i (u^j)^2 \\ \text{Var}(\pi_i^j | w^j) &= E \left[ v w^j + u^j \left( v - \frac{ts}{1+Q} - \theta_i D_i \sum_{m \neq j} w^m \right) \right]^2 \\ &= \sum_v (w^j)^2 + (u^j)^2 \left[ \sum_v \left( 1 - \frac{mt(Q+2)}{(Q+1)^2} \right) + (\theta_i D_i)^2 (h-1) \sum_w \right] \\ &\quad + 2 \sum_v u^j w^j \frac{(2-t)}{1+Q} \end{aligned} \quad (\text{A16})$$

Differentiating  $V_i^j$  with respect to  $u^j$  and then equating  $D_i$  with the coefficient of  $w^j$  in the resulting first-order condition yields:

$$\begin{aligned} RD_i^3 \theta_i^2 (h-1) \sum_x + D_i \left[ 2\theta_i + R \sum_v \left( 1 - \frac{mt(Q+2)}{(Q+1)^2} \right) \right] \\ + R \sum_v \frac{(2-t)}{Q+1} = 0 \end{aligned} \quad (\text{A17})$$

Solving for  $\theta_i$ :

$$\theta_i D_i = \frac{-\sqrt{mt \sum_v}}{\sqrt{h \sum_w}} \frac{1}{Q+1} \quad (\text{A18})$$

It follows from (A18) that since  $\theta_i > 0$  to satisfy the second-order condition for the informed traders,  $D_i < 0$  in equilibrium. Substituting for  $\theta_i D_i$  in (A17) and solving for  $D_i$ :

$$D_i = \frac{2\sqrt{mt\sum_v / (h\sum_w)} - R\sum_v (2-t)}{R\sum_v (2-t) - \frac{R\sum_v mt}{h(Q+1)}} \quad (\text{A19})$$

Since  $D_i < 0$ , equilibrium exists if:

$$R\sum_v (2-t) > \frac{2\sqrt{mt\sum_v}}{\sqrt{h\sum_w}} \quad (\text{A20})$$

The denominator of  $D_i$  in (A19) is always positive, so (A20) is sufficient. To show this, rewrite the denominator as:

$$\begin{aligned} \text{Denom} &= R\sum_v \left[ 1 - \frac{mt(Q+2)}{(Q+1)^2} \right] + \frac{(h-1)mt}{h(Q+1)^2} \\ &= \frac{R\sum_v}{(Q+1)^2} [(1-t)(4+mt) + t^2] + \frac{(h-1)mt}{h(Q+1)^2} \\ &> 0. \end{aligned}$$

In the second step, the definition  $Q = 1 + t(m-1)$  has been used.

#### Proof of Proposition 5

Since the RHS of (A19) is independent of  $i$ ,  $D_d = D_n$ . Since the RHS of (A18) is independent of  $i$ ,  $\theta_d = \theta_n$  given that  $D_d = D_n$ . It follows from inspection of (A15) and (A16) that  $V_d = V_n$ .

Proof of Corollary 2

From proposition 3,  $c_d < c_n$  if  $k_0 < \sqrt{mt \sum_v \sum_u}$ . Since  $E(u^2) = hD^2 \Sigma_w$  in the hedger model, the appropriate condition now is:

$$k_0 < |D| \sqrt{mt \sum_v} \sqrt{h \sum_w} \quad (\text{A21})$$

where  $|D|$  is the absolute value of  $D$ . The result follows immediately from a comparison of proposition 3 and (A21).

Proof of Corollary 3

If  $\sum_u = h \sum_w$ ,  $k_0 < \sqrt{mt \sum_v \sum_u} < |D| \sqrt{mt \sum_v} \sqrt{h \sum_w}$  when  $|D| > 1$ .

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