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**Reinsurance Decision Making and
Expected Utility**

*Danny Samson
Howard Thomas*

College of Commerce and Business Administration
Bureau of Economic and Business Research
University of Illinois, Urbana-Champaign

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Reinsurance Decision Making and Expected Utility

Danny Samson, Assistant Professor
Department of Business Administration

Howard Thomas, Professor
Department of Business Administration

Abstract

Utility theory is developed and applied in this article as a choice criterion for decisions concerning which types and extents of reinsurances are most appropriate for an insurer. Using a unidimensional utility function, reinsurance options are evaluated by calculating an upper bound premium (i.e., the maximum that the insurer should consider paying for a particular reinsurance agreement), which can be compared with market rates. Comparisons between reinsurance options can thus be accurately made as a function of the probability density function of the original loss, the modifications made by various ceding agreements, and the risk attitude of the insurer.

Reinsurance Decision Making and Expected Utility

Introduction

The decision analysis approach involves the stages of decision structuring, assessment, and the use of a choice criterion for deciding between alternatives.

From a series of discussions held with insurer managers (mainly in Australia and the U.K.) their primary concerns with respect to reinsurance were found to be related to the choice of an appropriate reinsurance type and the extent of coverage to be purchased. In particular, management expressed a need for an evaluation process that would be capable of aiding their judgement about how much to pay for reinsurance.

The structuring phase of the reinsurance decision analysis is an important part of the process [12] and involves not only the consideration of the elements of the reinsurance decision, but also the relationship of the reinsurance decision to the other risk related decisions of the organization. The other risk related decisions include the choices of reserving policies, risk pooling, and investment policies (see [12] for a comprehensive discussion) and since all of these decisions affect the total risk picture of the firm, they should not be made independently.

Choice Between Alternatives

A number of possible decision rules have been or could be used for reinsurance decision-making. Berliner [3] has examined the probability of ruin criterion and developed some implications for using this rule

for the determination of reinsurance loadings. The 'probability of ruin' approach, which features strongly in risk theory, suffers from two shortcomings:

1.) The arbitrary nature of such a criterion makes it difficult to implement. The definition of a particular probability of ruin (e.g., 0.01, 0.001, 0.0001) has little rational basis.

2.) Even if a meaningful probability of ruin could be defined, the making of decisions such that a particular probability of ruin is achieved considers only one possible state (i.e., ruin) and its (low) probability, which is a narrow and limited way of making decisions.

Benktander [2] has suggested that optimal reinsurance positions can be determined using variance as a measure of risk. However, mean-variance models may not be adequate representations of the decision-maker's preference in situations where the probability distributions of strategic variables are not approximately symmetric. In reinsurance, distributions of underwriting profit may be significantly skewed and may also be discontinuous or truncated where non-proportional reinsurance is considered. A strong aversion may exist to profits being below a certain point in the distribution (e.g., zero) and mean-variance models normally can not represent such preferences. Similar types of models that consider higher moments such as a three parameter model can overcome the difficulty of skewness but generally not that of discontinuity.

The Development of the Expected Utility Rule in the Reinsurance Decision Context

Borch [4] has developed the expected utility criterion in insurance/reinsurance contexts, and Friefelder [9] has applied utility theory to property-liability insurance rate calculations. Friefelder [9, p. 518]

suggests that "the inherent weakness of the mean-variance and the safety-first techniques are obvious" and further that "the best method of determining property-liability insurance rates is through the use of utility theory" [9 p. 519]. Friefelder's arguments can be applied equally strongly to the reinsurance decision, whereby the expected utility decision criterion can be usefully applied to the problem of the evaluation of reinsurance alternatives.

Cummins and Friefelder [7] have developed and illustrated a decision rule based on a maximum probable yearly aggregate loss estimate, but suggest that "more attention should be devoted to developing practical methods for applying consistent decision-making rules such as expected utility maximization to risk management problems" [7 p. 51].

In developing reinsurance decision criteria based on expected utility, it is assumed herein that utility can be expressed as a unidimensional function of net asset states and that all payoffs can be related to net assets and hence valued according to their effect on net assets. The analytical procedure used is to consider the expected utility of the firm both with and without reinsurance and to solve the indifference equation (which equates these expected utilities) for the assumed unknown reinsurance premium. This premium then represents the maximum premium that should be paid by the cedant for the specified reinsurance, since paying the maximum premium and receiving the indemnity leaves the cedant with the same expected utility as in the case of no reinsurance (i.e., a condition of indifference). The process of risk transfer from the policyowner (I) to the direct underwriter (D) and then to the reinsurer (R) can be expressed as two separate

actions. The primary insurance will only take place if both of the following conditions are satisfied.

$$E[U_D(A_D + P_D - X)] \geq E[U_D(A_D)] \quad (1)$$

$$E[U_I(A_I - P_I)] \geq E[U_I(A_I - X)] \quad (2)$$

where:

U_D and U_I are the utility functions of the direct underwriter and the policyowner respectively

A_D and A_I are random variables representing the net assets of the direct underwriter and the policyowner respectively

P_D is the premium paid to the underwriter

P_I is the premium paid by the policyowner

X is the random variable representing the loss

E is the expectation operator.

Obviously an agreement will only be reached and a contract made if the potential policyowner is willing to pay a premium at least as much as that demanded by the underwriter, i.e.,

$$P_I \geq P_D \quad (3)$$

An illustration can be made of the case where both parties have linear utility functions as follows:

$$U_D = a_D + b_D (A_D) \quad (4)$$

$$U_I = a_I + b_I (A_I) \quad (5)$$

where a_D , b_D , a_I , and b_I are constants.

Then equations (1) and (2) become:

$$E[a_D + b_D(A_D + P_D - X)] \geq E[a_D + b_D(A_D)] \quad (6)$$

$$E[a_I + b_I(A_I - P_I)] \geq E[a_I + b_I(A_I - X)] \quad (7)$$

Assuming that X is independent of both A_D and A_I , equations (6) and (7) yield:

$$P_D \geq E(X)$$

$$P_I \leq E(X)$$

and when coupled to the condition represented by equation (3), the result is a unique solution which is intuitively obvious, i.e.,

$$P_D = P_I = E(X) \quad (8)$$

For the case where both parties are risk neutral neither party benefits in terms of increased expected utility from a risk transfer where the premium is equal to the expected value of the risk. This case is one where the parties are indifferent to the insurance and where in real cases no risk transfer would occur because of the administrative costs involved.

A more usual situation is one where the potential policyowner is averse to the risk borne and would be prepared to pay more than the expected value of the risk to get rid of it.

Consider the case of a risk bearer being risk averse and having an exponential utility function defined by:

$$U_I(A_I) = -e^{-k_I A_I} \quad (9)$$

Equation (2) becomes:

$$E(-e^{-k_I(A_I - P_I)}) \geq E(-e^{-k_I(A_I - X)}) \quad (10)$$

and assuming A_I and X to be independently generated yields:

$$e^{k_I P_I} \leq E \left[e^{k_I X} \right] \quad (11)$$

If the underwriter had an exponential utility function:

$$U_D(A_D) = -e^{-k_D A_D} \quad (12)$$

the condition under which the risk would be accepted would be:

$$e^{k_D P_D} \geq E \left[e^{k_D X} \right] \quad (13)$$

As an illustrative example, if X is uniformly distributed between the lower limit of A and an upper limit of B , the condition shown as equation 11 becomes:

$$e^{k_I P_I} \leq \int_A^B \frac{1}{B-A} e^{k_I X} dx$$

$$P_I \leq \frac{1}{k_I} \ln \frac{1}{k_I(B-A)} (e^{k_I B} - e^{k_I A}) \quad (14)$$

and similarly equation (13) becomes:

$$P_D \geq \frac{1}{k_D} \ln \frac{1}{k_D(B-A)} (e^{k_D B} - e^{k_D A}) \quad (15)$$

Keeney and Raiffa [10] have defined the constant k to be a measure of risk aversion in the function:

$$U(A) = -e^{-kA}$$

and it would therefore be expected that an insurance contract would only be struck (i.e., $P_I \geq P_D$) if $k_I \geq k_D$, i.e., if the insured party was more risk averse than the underwriter.

In the present case it is assumed that the reinsurance decision is made independently of the underwriting decision although this must not necessarily be so in practice. In considering reinsurance under these conditions, the ceding of a portion of a risk can be considered in terms of how it modifies the loss distribution faced by the ceding insurer.

The expected utility condition for a reinsurance decision for the direct underwriter is:

$$E[U_D(A+P_i-P_R-Y)] \geq E[U_D(A+P_i-X)]$$

where A represents the insurers assets, P_i the primary insurance premium, P_R is the reinsurance premium and Y is the random variable representing the modified loss (retained risk). Letting $A_D = A+P_i$,

$$E[U_D(A_D-P_R-Y)] \geq E[U_D(A_D-X)] \quad (16)$$

The expected utility condition for the reinsurer would be similar in form to equation (1) since the reinsurer is simply underwriting a risk for a given premium.

Equation (16) can be considered as the basic condition for the evaluation by the ceding insurer of a reinsurance proposal. Rather than check to see whether the inequality holds for various alternatives, the analysis can be performed by assuming the equality condition and a unique solution for P_R can be sought (known as P_{RMAX}).

If in other circumstances there was information available about the premiums for various reinsurance options, an alternative procedure

could be used to find the best option, i.e., the option with the highest value of $E [U(A_D - P_R - Y)]$. In developing and calculating the expected utilities the assumption is made that A_D and X (and hence A_D and Y) have zero covariance. A_D represents the net assets of the firm including all factors except claims from the account or groups of accounts being considered for reinsurance. In cases where A_D and X (and hence A_D and Y) are known to be dependent, this can be taken into account in finding the expected utilities. In some cases the stronger assumption will be made that A_D is a known constant, and whilst this is likely to be unreasonable in some practical circumstances (since A_D represents the end of period net assets and would include forecasts of uncertain quantities such as investment returns), it does not weaken the power of the model nor of the illustrations. The alternative option to assuming a known constant A_D makes for a more complex calculation procedure, but presents no conceptual or analytical difficulties.

Illustrations of the Expected Utility Decision Criterion

The expected utility decision criterion can be used to calculate the value of reinsurance in terms of a certainty equivalent. The following illustrations are based on equation (16).

The reinsurance forms considered will be the proportional (quota share) form and the non-proportional form. The method is general regarding the risk basis used, for the assumption relates to the loss distribution for a reinsurance proposal, and would lead to the same utility evaluation whether based on a single risk, account, or group of accounts. The expected utility models could be used to compare the options of reinsurance using various risk bases by considering them as separate cases and examining the resultant expected utilities of the models.

The method can be applied (i.e., equation (16) can be solved) for any utility function and claims distribution. Friefelder [9] and Cozzolino [6] have suggested the use of an exponential utility function.

Case 1: Exponential Utility, Non-Proportional Reinsurance, Lognormal Claims Distribution

Shpilberg [13] has fitted the lognormal distribution to fire loss severity and Van der Laan and Boermans [14] have used this distribution to describe claims from motor insurance policies.

Consider X as being lognormally distributed with the following probability density function:

$$f(X) = \frac{1}{X\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln X - \mu}{\sigma}\right)^2}$$

where μ and σ are parameters of the distribution.

Equation (16) becomes:

$$e^{k_D P_{RMAX}} = \frac{\int_0^{\infty} e^{k_D X} \cdot \frac{1}{X\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln X - \mu}{\sigma}\right)^2} dX}{\int_0^C e^{k_D X} \cdot \frac{1}{X\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln X - \mu}{\sigma}\right)^2} dX + (1 - P_1) e^{k_D C}} \quad (17)$$

where C is the retention level and $P_1 = \text{pr}(X < C)$.

Equation (17) can be numerically integrated.

For the lognormal distribution with parameters μ and σ the mean and variance are given by Aitchison and Brown [1, p. 8] as:

$$\text{Mean: } \alpha = e^{\mu + \frac{1}{2}\sigma^2}$$

$$\text{Variance: } \beta^2 = e^{2\mu + \sigma^2}(e^{\sigma^2} - 1)$$

For illustrative purposes, the parameters found by Van der Laan and Boermans [14] applied to motor insurance claims are used as a guide:

$$\text{i.e. } \mu = 5$$

$$\sigma^2 = 1.5$$

$$\text{hence } \alpha = 314.19$$

$$\beta^2 = 4083780$$

$$\beta = 2020.8$$

Table 1 and Figure 1 show the solution of equation (17) for P_{RMAX} as a function of various values of k_D^1 and C.

It should be noted that for cases 1 and 2 where numerical integration procedures are used, the numerical solutions may be inexact due to the rounding errors associated with such procedures.

¹Units of k_D are $\$^{-1}$ and units of α and β are $\$$.

Table 1
 Values of P_{RMAX} for Case 1

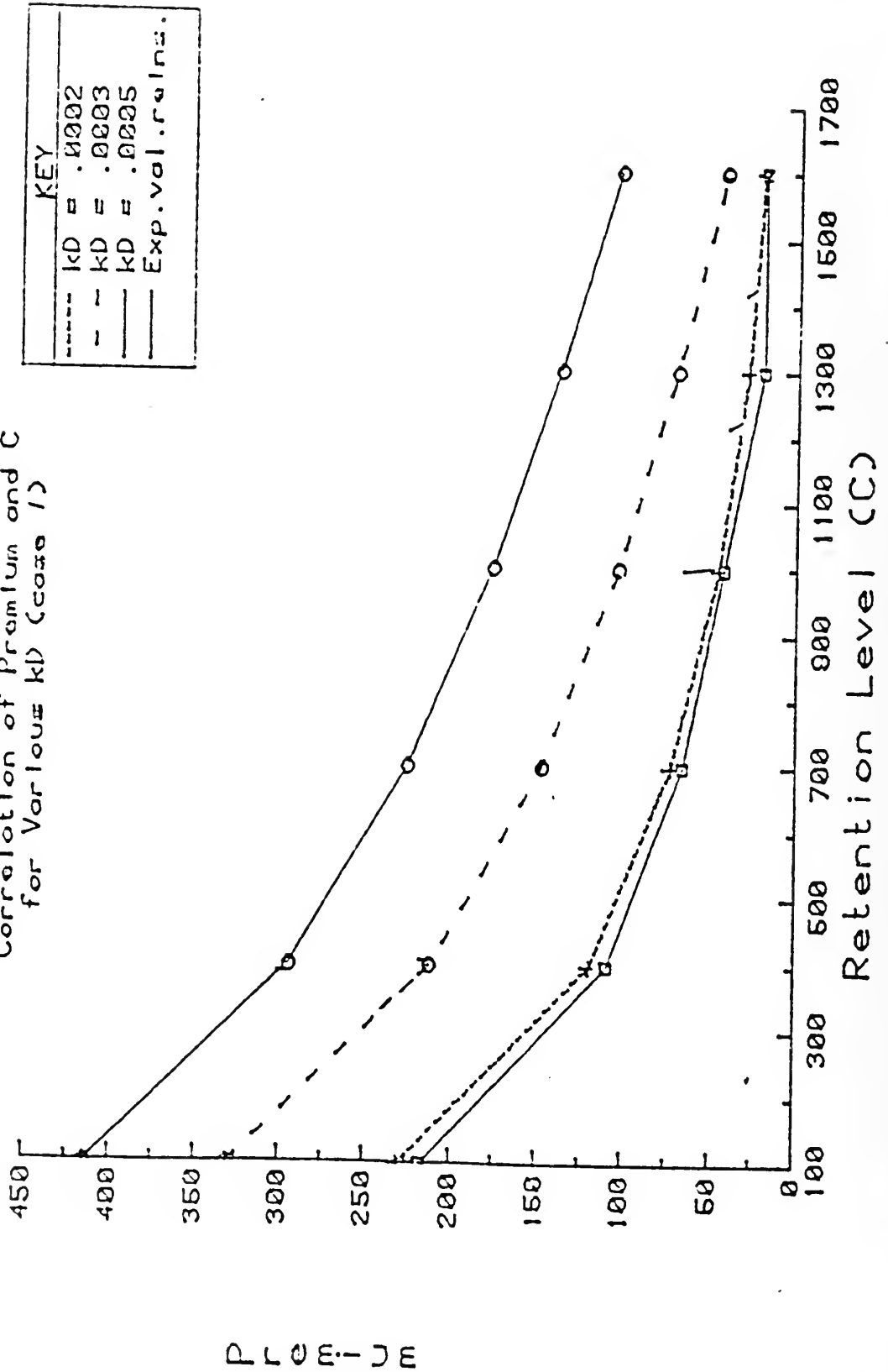
C	$k_D = .0002$.0003	.0005	Expected value of reinsurance
100	.239.5	330.9	450.0	216.9
400	128.2	212.38	294.3	109.2
700	81.27	148.2	226.16	65.79
1000	53.82	104.4	177.3	42.87
1300	36.76	71.07	138.35	29.14
1600	24.81	44.15	105.15	20.29

The expected value of reinsurance as a function of C (also shown in Table 1) is given by:

$$\int_C^{\infty} \frac{(x-C)}{X\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln X-u}{\sigma}\right)^2} dx$$

The risk premium (the amount paid over and above the expected loss or burning cost) is the difference between P_{RMAX} and the expected value of reinsurance. This premium is a monotonically increasing function of k_D (the risk aversion measure), and a monotonically decreasing function of C (the retention limit).

Figure 1
Correlation of Premium and C
for Various kd (Case 1)



Case 2: Exponential Utility, Proportional Reinsurance
Lognormal Claims Distribution

Aitchison and Brown [1, p. 11] give the theorem:

"If X is $\Lambda(\mu, \sigma^2)$ [lognormally distributed with parameters μ and σ^2] and b and c are constants, where $c > 0$ (say $c=e^a$) then cX^b is $\Lambda(a+b\mu, b^2\sigma^2)$."

In the case of proportional reinsurance of lognormally distributed claims, X is the original claims amount and Y is the retained claims (after reinsurance). Hence Y is $\Lambda(\ln F + \mu, \sigma^2)$.

Equation (16) becomes:

$$e^{k_D P_{RMAX}} = \frac{\int_0^{\infty} e^{k_D X} \cdot \frac{1}{X\sqrt{2\pi\sigma}} \cdot e^{-\frac{1}{2}\left(\frac{\ln X - \mu}{\sigma}\right)^2} dX}{\int_0^{\infty} e^{k_D Y} \frac{1}{Y\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{\ln Y - (\ln F + \mu)}{\sigma}\right)^2} dY} \quad (18)$$

where $Y = FX$. (F is the fraction retained).

Equation (18) can be evaluated numerically. Table 2 and Figure 2 show values of P_{RMAX} for various k_D and F values. In this illustration, as in case 1, $\mu = 5$, and $\sigma^2 = 1.5$.

Table 2
 Values of P_{RMAX} for Case 6

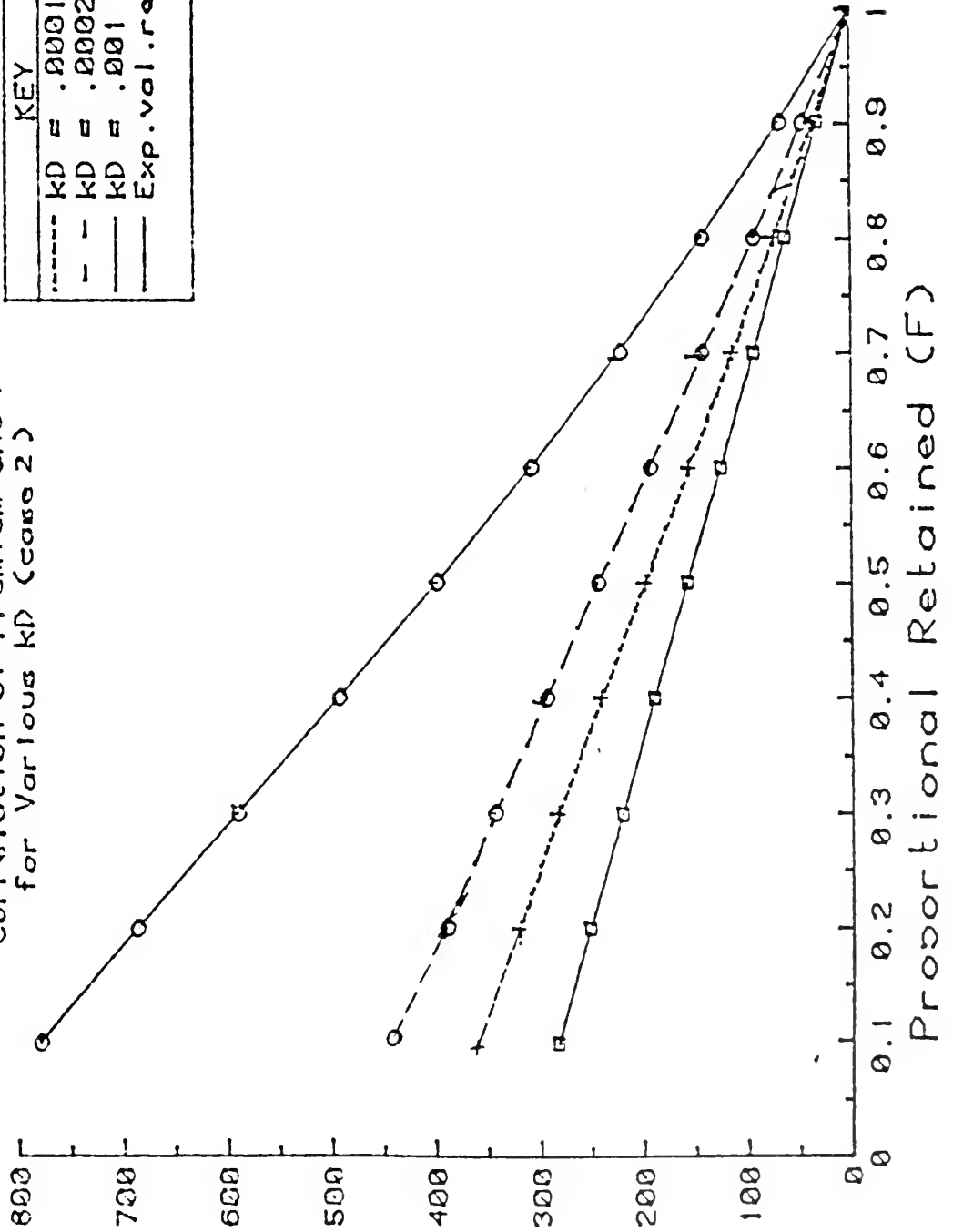
F	$k_D = .0001$.0002	.001	Expected Value of reinsurance
.1	362.9	448.7	779.3	282.8
.2	319.7	389.4	686.5	251.4
.3	282.7	342.8	589.6	219.9
.4	240.5	293.2	491.7	188.5
.5	197.9	242.7	396.7	157.1
.6	155.8	192.5	306.5	125.6
.7	114.9	142.9	221.6	94.26
.8	75.27	94.29	142.4	62.83
.9	36.94	46.18	68.67	31.41
1	0	0	0	0

The expected value of reinsurance is given by:

$$\int_0^{\infty} (1-F)X \cdot \frac{1}{X\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{\ln X - \mu}{\sigma}\right)^2} dX = (1-F) E(X)$$

Figure 2
Correlation of Premium and F
for Various KD (Case 2)

KEY	
.....	KD = .0001
- - - -	KD = .0002
— — — —	KD = .001
————	Exp. val. rains.



Case 3: Exponential Utility, Non-Proportional Reinsurance, Exponential Claims Distribution

Consider an insurer facing a loss distribution that is exponentially distributed. Such an insurer may wish to evaluate non proportional reinsurance possibilities on a per event basis (i.e., "occurrence basis reinsurance": See [5, p. 70]).

For this situation P_{RMAX} can be found from the equation

$$\int_0^C -e^{-k_D(A_D - P_{RMAX} - Y)} \cdot \lambda e^{-\lambda Y} dY + \int_C^\infty -e^{-k_D(A_D - C - P_{RMAX})} \lambda e^{-\lambda Y} dY = \int_0^\infty -e^{-k_D(A_D - X)} \cdot \lambda e^{-\lambda X} dX \quad (19)$$

The solution is

$$P_{RMAX} = \frac{1}{k_D} \ln \frac{1}{1 - \frac{k_D}{\lambda} e^{-(k_D - \lambda)C}} \quad (20)$$

(which holds for $\lambda > k$)

Table 3 and Figure 3 show values of P_{RMAX} for case 3 as a function of k_D and C for $\lambda = \frac{1}{\$2,380.95}$ (this value of λ is one used by Friefelder [9, p. 526]). Also shown in Table 3 is the expected value of the re-insurances, which is given by

$$\int_C^\infty (X-C) \lambda e^{-\lambda X} dX = \frac{e^{-\lambda C}}{\lambda}$$

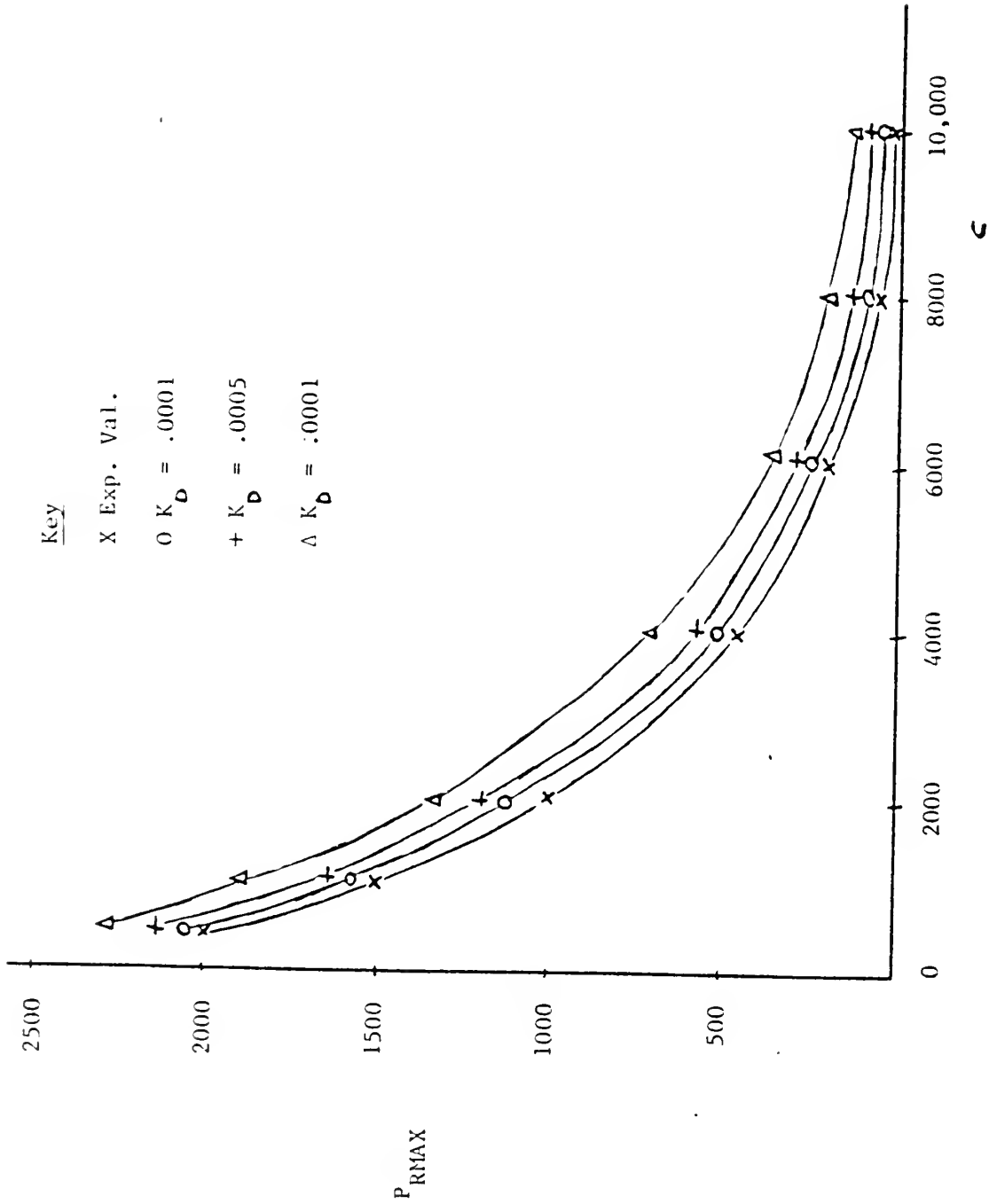
Table 3

Values of P_{RMAX} for Case 3

$$\text{Assuming } \lambda = \frac{1}{\$2,380.95}$$

C	'k = .00001	.00005	.0001	Expected values of reinsurance
200	2218	2343	2527	2189
500	1959	2084	2268	1929
1000	1593	1716	1898	1564
1500	1296	1416	1594	1268
2000	1054	1170	1342	1028
3000	698	800	956	675
4000	463	549	685	443
5000	307	378	493	291
6000	204	260	355	191
7000	135	179	257	126
8000	90	124	186	83.7
9000	59	86	135	54.3
10000	39	59	98	36.7

Figure 3
Correlation of Premium and C for
various K_D (case 3)



Case 4: Exponential Utility Function, Proportional Reinsurance, Exponential Claims Distribution

Equation (16) becomes:

$$\int_0^{\infty} -e^{-k_D(A_D - P_{RMAX} - Y)} \frac{\lambda}{F} e^{-\frac{\lambda}{F}Y} dY = \int_0^{\infty} -e^{-k_D(A_D - X)} \lambda e^{-\lambda X} dX \quad (21)$$

Since $Y = FX$ (where F is the fraction retained).

The solution to this equation is

$$P_{RMAX} = \frac{1}{k_D} \ln \left[\frac{\lambda - Fk_D}{\lambda - k_D} \right] \quad (22)$$

for $\lambda > k_D$

The expected value of reinsurance for this case is

$$\int_0^{\infty} (1-F)X \cdot \lambda e^{-\lambda X} dX = \frac{1-F}{\lambda}$$

Table 4 and Figure 4 show values of P_{RMAX} for case 4 with

$\lambda = \frac{1}{\$2,380.95}$ for various values of k_D and F .

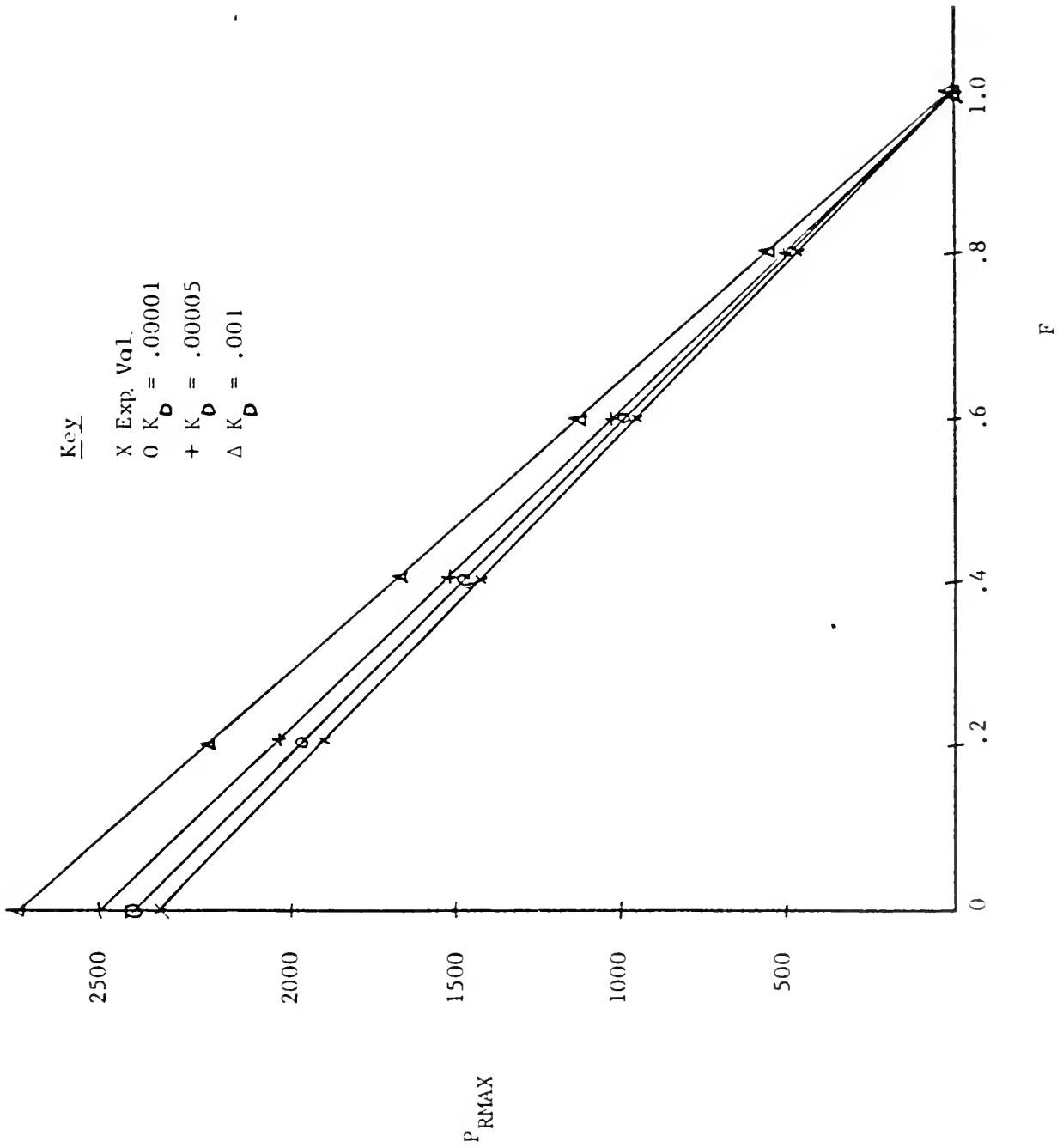
Table 4

Values of P_{RMAX} for Case 4

Assuming $\lambda = \frac{1}{\$2,380.95}$

F	k = .00001	.00005	.0001	Exp Val
0	2409	2535	2760	2380
.1	2171	2295	2519	2142
.2	1932	2053	2271	1904
.3	1692	1807	2016	1666
.4	1452	1559	1754	1428
.5	1212	1307	1484	1190
.6	970	1052	1206	952
.7	729	794	919	714
.8	486	533	623	476
.9	243	286	316	238
1	0	0	0	0

Figure 4
Correlation of Premium and F for
various K_D (case 4)



Case 5: Logarithmic Utility Function, Non-Proportional Reinsurance
Lognormal Claims Distribution

Consider a utility function having a logarithmic utility function as shown by $U(A_D) = \ln(A_D+k)$ where k is a constant. Equation (16) becomes:

$$\int_0^C \ln(A_D - P_{RMAX} + k - Y) \cdot \frac{1}{Y\sqrt{2\pi\sigma}} e^{\frac{-1}{2\sigma^2}(\ln Y - \mu)^2} dY + (1 - P_1) [\ln(A_D - P_{RMAX} + k - C)]$$

$$= \int_0^\infty \ln(A_D - X + k) \frac{1}{X\sqrt{2\pi\sigma}} e^{\frac{-1}{2\sigma^2}(\ln X - \mu)^2} dX \quad (23)$$

where C = retention level and $P_1 = \text{pr}(X < C)$.

This equation can be solved using numerical methods for values of P_{RMAX} as a function of A_D , k , X , C , μ and σ .

Since the logarithmic utility function does not possess the property of constant risk aversion, A_D does not cancel out in equation (23) as it did in equations (16) through (22). Hence estimated values of A_D (the end of period assets) must be input to the decision.

If there is little uncertainty associated with A_D , it could be treated as a known constant (by using a point estimate). However, it may be necessary to treat A_D as a random variable as a result of other sources of uncertainty (e.g., uncertain investment returns such as stock prices and dividends or real estate values) that affect assets and are unresolved at the time the decision is taken.

Discussion of the Expected Utility Criterion

As pointed out by Friefelder [9] the expected utility approach has many conceptual advantages over other approaches. In reinsurance decision making, where a process of negotiation often occurs between the ceding party and reinsurer, a knowledge of the upper bound reinsurance premium (i.e., P_{RMAX}) would be a most useful information base for comparing alternatives and trading off between variables. This knowledge of an upper bound premium may be particularly useful when market conditions are such that reinsurance premiums are relatively high. When excess capacity exists and reinsurance prices are low it may be useful to calculate and compare the utilities of a number of competing reinsurance alternatives, each of which may be attractive in an absolute sense (i.e., each of which have higher utility than the "no reinsurance" alternative).

The choice of the utility function is an important part of the process. The exponential utility function has properties that make its application simpler than many other functions, and Friefelder [9] has suggested that it is appropriate (principally on the basis of equity considerations) in establishing ratemaking policies. These reasons alone do not make its use compulsory, as the most important criterion for the choice of a utility function form is its accuracy as a representation of the preferences (and risk attitude) of the organization or its representative over the domain of net asset states. The exponential and logarithmic utility functions were used in the preceding section as examples of functions that have the property of risk aversion. Risk aversion is a consistent property of studies of risk-taking in business situations. Indeed, Libby and Fishburn [11] find risk aversion demonstrated in business decision-making in the following manner:

...risk is combined with return in a hybrid model that combines compensatory and non-compensatory decision rules. A model in which risk first plays a role as a ruin constraint and then interacts with the mean as a trade-off parameter defined as target semi-variance is most supportable [11, p. 289]

Other simple functions may be applicable for some organizations, or in some cases more complex functions (which may involve combinations of simple functions) may be appropriate. Keeney and Raiffa [10] and Farquhar [8] give detailed discussions and bibliographies of this subject. It is important to note that equation (16) can be applied to any reinsurance decision, regardless of both the form of the firm's utility function and the nature of the claims distribution.

Given that an appropriate utility function can be found, the application to reinsurance decision making of this choice criterion is a most useful one, as all of the important elements of the decision (claims, type of insurance, retention level, premium, organizational risk attitude) can be considered in aggregate and a unidimensional value can be obtained (i.e., given the input information, the model aids in making the tradeoffs).

As an example of how this type of analysis can help the manager to evaluate reinsurance options, consider an insurer with an exponential utility function ($k = .00005$) facing a loss distribution as shown in cases 3 and 4. The firm is able to screen and evaluate the insurance proposals using the data in Tables 3 and 4. For example, non-proportional reinsurance with a retention limit of \$8,000 is attractive only if the premium is less than \$124. Similarly from Table 3 a 50 percent proportional reinsurance would be worth a maximum of \$1,307. This figure represents a risk premium of \$117 since the expected

value of this reinsurance is \$1,190. Since proportional reinsurance premiums are typically calculated as being a proportion of the original insurance premium less an allowance for commissions and expenses, the insurer can use the expected utility criterion as a guide towards finding its optimal reinsurance position. This position can be found by comparing various reinsurance premiums obtained from the market with appropriate values of P_{RMAX} (using a similar method to that of the cases developed herein, i.e., based on equation (16)).

Conclusions

The decision analysis paradigm, which has previously been successfully developed for insurance ratemaking processes, can be used to effectively aid in the reinsurance decision. The expected utility decision criterion can be used to give the ceding party a guide for which reinsurance alternatives are preferable and at what prices.

The expected utility method is general, and can be used as an evaluation tool for any appropriate utility function, loss function, and reinsurance type and extent.

References

1. Aitchison, J. and Brown, J. A. C., The Lognormal Distribution, Cambridge University Press, Cambridge, 1957.
2. Benktander, G., "A Note on Optimal Reinsurance," Astin Bulletin, VIII, 1975, p. 154.
3. Berliner, B., "Some Thoughts on Reinsurance Loadings Under a Ruin Criterion," Scandinavian Actuarial Journal, 1974, p. 76.
4. Borch, K., The Mathematical Theory of Insurance, Lexington Books, Lexington, Mass., 1974.
5. Carter, R., Reinsurance, Kluwer, London, 1979.
6. Cozzolino, J. M., "A Method For the Evaluation of Retained Risk," Journal of Risk and Insurance, 45, 3, 1978.
7. Cummins, J. D. and Friefelder, L., "A Comparative Analysis of Alternative Maximum Probable Yearly Aggregate Loss Estimates," Journal of Risk and Insurance, 45, 2, p. 27, March 1978.
8. Farquhar, P. H., Utility Assessment Methods, Working Paper 81-5, Graduate School of Admin., UC Davis, California, 1982.
9. Friefelder, L., "Exponential Utility Theory Ratemaking: An Alternative Ratemaking Approach," Journal of Risk and Insurance, 46, 3, September 1979.
10. Keeney, R. L. and Raiffa, H., Decisions with Multiple Objectives, John Wiley, NY, 1976.
11. Libby, R. and Fishburn, R. C., "Behavioral Models in Risk Taking in Business Decisions: A Survey and Evaluation," Journal of Accounting Research 1977, 15, p. 272-292.
12. Samson, D., The Role of Decision Analysis in Reinsurance Decision Making, Unpublished Ph.D. Thesis, University of New South Wales, Australia, 1982.
13. Shpilberg, D., "The Probability Distribution of Fire Loss Amount", Journal of Risk and Insurance, 44, 1, p. 104, March 1977.
14. Van der Laan, B. S. and Boermans, F., Some Probability Distributions in the Field of Motor Car Damages, Report 7001, Netherlands School of Economics, 1970.

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