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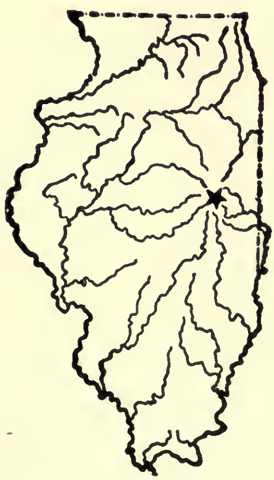
RELATION BETWEEN PERCENTAGE FAT CONTENT AND YIELD OF MILK

Correction of Milk Yield for Fat Content

By W. L. GAINES and F. A. DAVIDSON



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Contents of Bulletin No. 245

RELATION BETWEEN PERCENTAGE FAT CONTENT AND YIELD OF MILK

Correction of Milk Yield for Fat Content

By W. L. GAINES, Chief in Milk Production, and
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INTRODUCTION

Among dairymen, it is a matter of common observation that, in general, the milk yields of cows tend to vary inversely with the percentage fat content of the milk. Various statistical investigations, by the method of correlation, support this observation. Such investigations have shown the existence of a significant negative coefficient of correlation between the two variables, percentage fat content of the milk and yield of milk. The present study is a further analysis of this relation based on 23,302 records of the milk and fat production of cows. It purposes to show the nature of the relation between percentage fat content of the milk and yield of milk; and to formulate a method of correcting milk yield to equate for the influence of fat content.

The dairy cow occupies her position in our agriculture primarily as a producer of milk. Cows are highly variable in milk yield; and this variability is a large factor in the immediate economy of milk production, and of great import in the possible future dairy development of the cow. But variability in milk yield is affected by many factors, and if the milk yield of a cow is to be used as an indication of her position on the scale of merit with reference to immediate economy and future development, it is desirable to distinguish the factors affecting milk secretion, and to have a measure of the effect of each. The advanced registry* records of the dairy breeds usually list three such factors, each of which is very potent in its influence on milk yield; viz., time (length of record), percentage of fat, and age of cow.**

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*The term *advanced registry* is used frequently, as here, in a general sense to apply to any of the breeds.

**Among other factors affecting milk yield the list following may be suggestive: food supply (amount, character); body food reserves (body fat, mineral store); growth of cow; size of cow; pregnancy preceding lactation (sex of fetus, sire of fetus, normal term, premature or delayed delivery, birth weight); pregnancy during time of lactation; ovariotomy; frequency of milking; character of milking; previous development (exercise and training of the mammary function at preceding lactations); physical exercise; comfort (temperature, flies, etc.). Undoubtedly some of these are of considerable importance, while others may have little or no influence. Of the many important factors it is remarkable that we have had an adequate measure of the influence of only one, that of age. It is hoped the data following will supply a measure for one other, that of percentage fat content of the milk.

The age of a cow has long been recognized as a factor affecting the milk yield. The advanced registry system when first established in 1885 took account of this fact. Various data have been published from time to time showing the absolute or relative milk yields by cows at varying ages. Gowen[1] has shown that the relation between these two variables—age and yield—may be closely expressed by a logarithmic curve, and has given the equations for the curves for the Holstein, Jersey, and Guernsey breeds. These equations have been found valuable in this laboratory in equating the milk yields of cows so as to make the yields directly comparable in so far as the age factor is concerned.*

SOURCE OF DATA

The data used in this study have been taken from the records of cow testing associations in Illinois (unpublished**) and the published records of the Holstein-Friesian Association, the American Jersey Cattle Club, the American Guernsey Cattle Club, the Ayrshire Breeders' Association, and the Brown Swiss Breeders' Association.

The records of the cow testing associations include only two breeds—Holstein and Jersey—in numbers large enough to be of value for present purposes. The Holstein records are of both grades and pure-breds, located in commercial dairy herds in the whole-milk districts of Illinois. The records used are from those associations only that are known to have had competent and reliable testers in charge, and whose members sold whole milk at about the same price (price being a factor in the amount of feed given the cows, and consequently a factor in milk yield). Very little advanced registry testing was practiced.

The Jersey records are of both grades and pure-breds, obtained in one association over a period of five years. The number of cows involved is consequently less than the number of records used. Whole milk was sold, the market paying, however, in exact proportion to the fat content of the milk. The quality of the cows and the conditions under which the records were made were similar to those of the Holsteins. No advanced registry testing was practiced.

*The use of corrections is common in the physical sciences. When the chemist determines the volume of a gas, he corrects his measurement to certain standard conditions; he makes a correction for temperature, a second correction for barometric pressure, and a third correction for the tension of aqueous vapor. Biological corrections of the kind under consideration here are just as much needed and just as useful as those used in the physical sciences. In many cases it is impossible to standardize the cause of variation, and in such cases the only recourse is to standardize the effect thru the use of a correction factor. The determination of biological corrections is complicated by the multiplicity of reactions occurring simultaneously in the living organism, and this condition may subject the determination to error and to the necessity of revision as additional evidence accumulates.

**The writers acknowledge the courtesy of Professor C. S. Rhode of the Dairy Department, University of Illinois, in supplying part of these records.

The records of the breed associations are the well-known advanced registry records. They need no explanation here, but it may be in point to recall that they are made under a wide range of conditions, and that in some cases no expense in feed, care, and manipulation is spared in order to secure a maximum recorded production. The goal of advanced registry testing (except possibly the Ayrshire) is based on the fat record rather than on the milk record. Consequently, there is a stimulus toward any manipulation that increases the real or apparent fat percentage as well as the yield of milk.

It is very seldom that all the cows of a herd are included in the advanced registry system, whereas it is very rarely that they are not all included in the cow testing association system. As compared with the cow testing association records, the advanced registry records represent a higher capacity portion of the total population, producing under conditions nearer the optimum for maximum production.

HYPOTHESIS

Preliminary study suggested that the relation between the percentage fat content of the milk and the yield of milk is simple and logical; namely, (1) the solids-not-fat, as well as the fat itself, are concerned in the relation; (2) the relation depends on the *energy value* of the fat and the solids-not-fat, rather than directly on the amount of solids present; (3) the energy value of the total solids of the milk is *constant,* if all factors which affect milk yield, other than the solids content, are compensated.

If the above is in fact the case, the physiological relation between the two variables fat percentage and milk yield is revealed, and a base is established from which to correct milk yield for the influence of fat content. If the percentage fat content of the milk is a factor affecting milk yield according to a definite physiological relation, and this relation can be expressed mathematically, the use of such mathematical expression in the correction of milk yield for fat content is justified from a physiological standpoint. Indeed, correction by such a method is preferable to the use of an expression describing the relation found in the advanced registry data because advanced registry selection and practices may to some extent distort the true relation. For the purpose of the present study, the following hypothesis is therefore adopted:

The milk yield of cows with varying fat percentages is such that the total energy value of the milk is constant if the effects of all factors other than composition are equalized.

That is, by way of further explanation, there are many things which influence the amount of milk that cows produce; for example, the fat percentage of the milk, length of record, the individuality of the cow, age, feed, and so forth. The influence of fat percentage (or rather, composition as measured by fat percentage) is the particular factor now under study; and the proposition of the hypothesis is that if the effects of each of the other factors are made equal for each cow, the energy value of the total milk produced by each cow will be the same—a constant. The influence of fat percentage on yield is, according to the hypothesis, a function of the energy value of the fat plus the energy value of the solids-not-fat present in the milk; and the influence is measured directly by the energy value of the solids. The milk yield must, by the hypothesis, be inversely proportional to the energy value of the solids per unit of milk.

Now, in order to subject the hypothesis to test it is necessary to meet the condition that all factors except composition be equalized. It is impossible to do this directly for all factors. Indirect methods, based on statistical principles must be used.

If we take a large number of cows, representative of the same breed, working under similar conditions, and separate them into classes on the basis of the percentage fat content of their milk, and determine the *average* milk yield of each class, we may assume that, as between the *averages* so obtained, all factors in milk yield are equalized, except the one on which the classification is based. On statistical principles, which need not be elaborated here, this will be true, within a certain probability of error, except as to factors which are also correlated with fat percentage. The factor solids-not-fat is such an exception, and that is why it is treated together with the fat. *What we really have to consider is the influence of composition on milk yield.*

There are undoubtedly some other factors, such as the size of the cow, which are correlated with fat percentage, but the *net* effect of *all* such factors is regarded as being so small as to be negligible.

The hypothesis is concerned with the energy value of the milk solids, but the records used give only the amount of milk and the fat content. It is necessary, therefore, to estimate the energy value, and the method and justification for this will appear shortly.

It is on the principles outlined above that the hypothesis was suggested by study of the data. On the same principles the validity of the hypothesis is put to test in the following pages, representative data from *all* the records available being used.

PRESENTATION

The records used give the fat percentage to the closest second decimal, and the milk yield in pounds and tenths. Each group of records is arranged in a correlation table (see pages 599 to 621), class intervals of 0.1 for fat percentage and 1,000 pounds for milk yield (20 or 50 pounds for the seven-day records) being used. The coefficients of correlation and other constants have been derived by standard methods and are given in Table 21. The mean milk yields of the several fat percentage classes have been computed from the correlation tables and are given in tabular form (pages 600 to 618) and in the accompanying graphs. Included with these latter data, in both tables and graphs, are two other sets of data: first, the corresponding milk yields calculated from a fitted curve of constant energy, and the deviations of the observed from the calculated values; second, the corresponding milk yields calculated from a fitted logarithmic curve, and the deviations of the observed from the calculated values.

The Constant Energy Curve.—It is well known that the solids-not-fat content of normal milk varies with the fat content in a very definite ratio. Gowen[2] finds, in Holstein cows, a correlation of $+.8991 \pm .0071$ between these two constituents; indicating that the solids-not-fat content of milk may be determined with reasonable accuracy from the fat content. The energy value of milk fat and of solids-not-fat is also definite and well established. Stocking and Brew[3] working with extensive data compiled from various sources have prepared a table which shows these several relations.* From their table we derive:**

*Overman[4] also gives data which bear on this relation. He has compiled several thousand complete analyses of milk of stated known purity and studied them from the standpoint of the food value of milk of varying percentage fat content. His data cover a range of fat percentage from 3 to 7, and show a linear relation between fat percentage and food (energy) value per quart of milk. The curve is in excellent agreement with that of Stocking and Brew as to direction but somewhat lower (about 7 percent) in absolute values. The difference in absolute values is accounted for in large part by a variance in the energy values used by the two authorities for fat and solids-not-fat.

**Symbols are used as follows:

D = deviation of observed from calculated milk yield, in pounds.
E = energy of milk solids, in large calories.
ECM = milk yield corrected for energy value to 4 percent fat.
f = frequency.
F = milk fat, in pounds.
FCM = milk yield corrected for fat to 4 percent fat.
F-SCM = milk yield corrected for fat and solids to 4 percent fat.
M = milk, in pounds.
ME = mean error, $\frac{\Sigma(+D) - \Sigma(-D)}{n}$
M_e = milk yield, in pounds, calculated from constant energy curve.
M_f = milk yield, in pounds, calculated from constant fat curve.
M_l = milk yield, in pounds, calculated from logarithmic curve.
M_o = mean milk yield, in pounds, observed.
n = number of values.
r = coefficient of correlation.
RE = root mean square error, $\sqrt{\frac{\Sigma D^2}{n}}$
S-N-F = solids-not-fat, in pounds.
s-n-f = percentage solids-not-fat content of milk.
t = percentage fat content of milk.
Σ = summation.

$$\begin{aligned} \text{s-n-f} &= 7.1 + 0.4\,t \\ E &= 132.06\,M + 4964\,F \quad \text{(and, since } F = .01\,Mt) \\ &= 132.06\,M + 49.64\,Mt \\ &= 49.64\,M\,(2.66 + t) \end{aligned}$$

By hypothesis, E is constant, say 49.64a (a being a constant the value of which is to be determined). Then, for the amount (pounds) of milk, M_e, necessary to satisfy this value of E, we have:

$$49.64\,M_e\,(2.66 + t) = 49.64a$$

and,

$$M_e = \frac{a}{2.66 + t}$$

This curve is arbitrarily so fitted to the observations that at values of t corresponding to those of the observations,

$$\Sigma\,M_e = \Sigma\,M_o$$

Hence,

$$\Sigma\,\frac{a}{2.66 + t} = \Sigma\,M_o$$

or,

$$a\left(\Sigma\,\frac{1}{2.66 + t}\right) = \Sigma\,M_o$$

and,

$$a = \frac{\Sigma\,M_o}{\Sigma\,\frac{1}{2.66 + t}}$$

The method of fitting causes the sum of the *plus* deviations and the sum of the *minus* deviations to be equal in value. It does not necessarily reduce either the mean error, $\frac{\Sigma\,(+D) - \Sigma\,(-D)}{n}$, or the root mean-square error, $\sqrt{\frac{\Sigma\,D^2}{n}}$, to a minimum. However, the method answers for present purposes. The constants are given in Table 22.

The reader should bear in mind that *the energy curve is an expression of the hypothesis.* While it is "fitted" to the observations, this "fitting" only adjusts the mean level of the curve to the mean level of the observations of milk yield. Its shape and general direction are fixed and inflexible. If it conforms to the observations, that conformity is evidence that fat percentage affects milk yield and that the effect of fat percentage on milk yield is measured directly by the energy value of the milk solids. If the energy curve conforms to the observations, it is evidence in support of the validity of the hypothesis.

The Logarithmic Curve.—In general, the data suggest that a curve of the type $y = a + bx + c \log_{10} (x + \alpha)$ should be adapted to fit the observations. Further, this type of curve has been found applicable to the expression of many biological relations. Consequently, it has been used here, and has been fitted to each set of data by the method of moments, using Miner's[5] equations and tables. The constants for the several equations are given in Table 22. The logarithmic curve is used purely for purposes of comparison.

Comparisons.—The graphs (Figs. 1 to 10) give a visual impression of how well the observations support the hypothesis: first, by comparing the energy curve, which represents the hypothesis, with the observations themselves; second, by comparing the energy curve with the fitted logarithmic curve. A further comparison with the logarithmic curve is afforded by the tabular presentation (see pages 599 to 621). Here a numerical expression of the fit of the energy curve is attempted by giving, for both curves, the mean error, and the root mean-square error. These errors are given also in the graphs. If the error of the energy curve is not greater than that of the logarithmic curve, then, so far as the logarithmic curve is a guide, the observations support the hypothesis. Likewise, an error for the energy curve greatly in excess of that for the logarithmic curve, shows a lack of support. The errors for the several sets of data are brought together in Table 23.

The Constant Fat Curve.—Since a fat standard is used as the basis of admission to the advanced registry, and since fat yield is quite generally used as a measure of a cow's production, it has seemed desirable to consider the yield of milk required for a constant yield of fat. The equation for the curve of constant fat is $M_f = \frac{a}{t}$, and this curve has been fitted by determining a after the same manner as in the energy curve. The data are given only in summary form (Tables 22 and 23), except that for the purpose of illustration the curve is drawn into one of the graphs (Fig. 1).

Age Correction.—It has been found unnecessary to use an age-correction factor for the milk yields, except in two cases where a comparatively small number of records is used. For a limited number of records it serves to smooth the data materially, and would probably be useful in smoothing the values for the end and near-end frequencies in other cases. The two cases corrected are the Brown Swiss and the early Holstein seven-day records. The Brown Swiss records, as published, give the age only by groups. Yields are corrected to age of maximum yield by using Gowen's[1] equation for the Holstein breed. The Holstein seven-day records have been corrected to the age of 8 years 9 months by using data given by Miner.[5]

DISCUSSION

The Coefficient of Correlation.—Table 21 shows the correlation between fat percentage and milk yield to be negative in every case. The coefficient is not very high in any case but is significant in every case. The correlation for the Holstein and Jersey cow testing associations ($r = -.198 \pm .012$ and $-.212 \pm .021$, respectively) have the most meaning from the standpoint of the normal relation between percentage fat content and yield of milk because the populations they represent are the least selected of any of the groups.

Advanced registry selection (except the Ayrshire), by reason of the entrance requirements, tends to increase the negative correlation. This appears prominently in the case of the Jersey seven-day records, where $r = -.506 \pm .026$. The entrance requirement in this class is twelve pounds of fat regardless of age. Inspection of the correlated distributions (Table 7) shows that a considerable part of the total population is cut off in the upper left portion (low fat percentage and low milk yield). The effect of this is to give a higher negative value to r than would be obtained from a distribution representative of the whole Jersey population (see also Fig. A). Exactly the same principle operates, in lesser degree, in the other advanced registry groups, except in the Ayrshire. The Ayrshire standard is peculiar in that there is a minimum milk requirement besides the usual fat requirement. The effect of the additional milk requirement is nil at values of t below 3.57–4.29 (the value varying with age), but above that point the milk requirement tends to give a positive correlation. There are also other complications (see Roberts,[6] page 73).

The low value of r is caused in part by the great variability in milk yield, due to the inherent quantitative differences in the function of milk secretion and to the extreme susceptibility of this function to environmental factors. Everyone knows, of course, that a knowledge of the percentage fat content of a cow's milk does not justify an estimate of her milk yield. But the fact of a significant correlation shows that there exists some definite relation between the fat percentage and the mean milk yield of a number of cows. The nature of this relation is brought out more clearly by the graphs and tables for each group of records.

Holstein Cow Testing Association Records.—The data for these records are found in Tables 1 and 2, and Fig. 1. Survey of the graph shows that the energy curve is very nearly coincident with the logarithmic curve. Its mean error is one pound greater, and its root mean-square error six pounds less than for the logarithmic curve. If there is any choice between the two it would seem to be in favor of the energy curve, either on the basis of the magnitude of the errors or the general impression formed by study of the graph.

The curve of constant fat is added in Fig. 1 for the sake of illustration. It is quite obvious that fat yield is not an equitable measure of production within the classes represented. The fat curve has a mean error of 555 pounds and a root mean-square error of 775 pounds (Table 23), or nearly double that of the energy curve. It is not given in the data for the remaining records since it bears a similar relation to the energy curve in all cases.

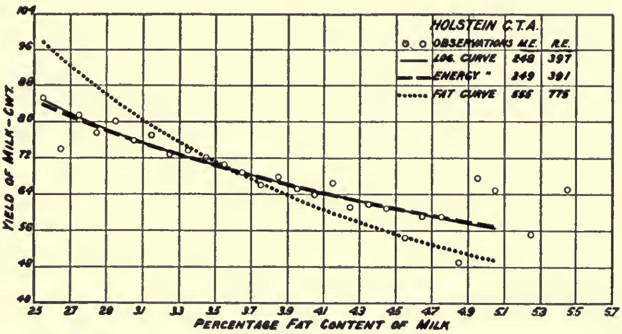

FIG. 1.—RELATION BETWEEN FAT PERCENTAGE AND MILK YIELD: HOLSTEIN-FRIESIAN COW TESTING ASSOCIATION YEARLY RECORDS
2,773 records. Data from Table 2, page 600

The 2,773 records concerned here should be thoroly representative of the Holstein breed, under good conditions of management, in herds dependent primarily upon the sale of milk for their income. The majority of the cows were high grades.

It will be noted that while the general tendency of the observations is plainly in the direction of the energy curve, there are many rather wide deviations from it. Toward either end, where the frequencies are small, wide fluctuations are natural. But even with larger frequencies there are some apparently wide deviations. For example, the class at $t = 3.345$ shows a deviation of 132 pounds. Considering the 337 records of this class by themselves, the mean, 7,384, has a probable error of 79.6. From this we might expect a deviation of 132 about once in five. For the other classes, having smaller frequencies, the chance of error in the mean would be still greater. Consequently, some irregularity in the observations is to be expected and is no reason for discrediting the data.

Jersey Cow Testing Association Records.—The data for these records appear in Tables 3 and 4, and Fig. 2. The observations are less regular in distribution than those in Fig. 1. The number of records is much smaller, 970, and the number of cows represented still smaller.

Again, the energy curve is practically coincident with the logarithmic curve. Its errors are greater by 6 pounds for the mean and

7 pounds for the root. This is less than 2 percent, and considering the nature of the data is very close.

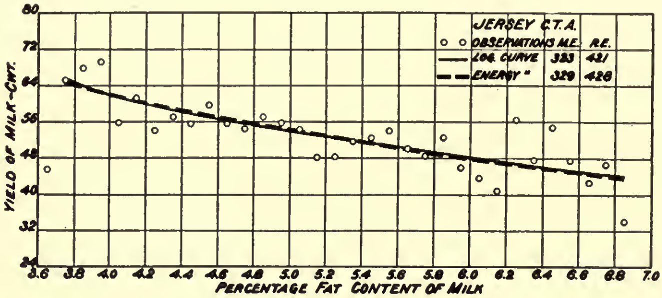

FIG. 2.—RELATION BETWEEN FAT PERCENTAGE AND MILK YIELD: JERSEY COW TESTING ASSOCIATION YEARLY RECORDS
970 records. Data from Table 4, page 602

. The records of the Holstein and Jersey cow testing associations are regarded as supporting the hypothesis remarkably well. No similar records for other breeds were available for study. We have to consider next the advanced registry records, but before doing so it is necessary first to discuss the nature of the selection effected by the requirements for admission to the advanced registry.

Nature of Advanced Registry Selection.—Fig. A is a diagram designed to illustrate the nature of advanced registry selection. It is intended to represent a correlation surface for fat percentage and milk yield for a very large number of cows under official test conditions. All cows above the line AB would be excluded by an entrance requirement of 360 pounds of fat, and all cows below the line could qualify. If the broken line represents the periphery of the population, and the population increases in density with some uniformity toward its center, then it is clear from the diagram that an increasing proportion of cows is eliminated as we go from higher to lower fat percentage. Since it is the poorest grades of cows that are eliminated, the qualitative effect must be to improve the mean grade of those left. And improvement would increase as we go from higher to lower fat percentage, because of the increasing proportion of the population eliminated.

The proportion of a total population that would fail to qualify for the advanced registry is uncertain. Roberts[6] refers to 98 Ayrshires which were tested and failed to qualify (with an entrance requirement of 214.3 to 322 pounds of fat, according to age) presumably comparable with 1,091 that did qualify. Since the poorest cows are probably not tested at all, it would seem that advanced registry requirements would exclude at least 10 percent of the total population if all were tested.

FIG. A.—ILLUSTRATING THE NATURE OF THE SELECTION EFFECTED BY A CONSTANT FAT PRODUCTION REQUIREMENT

The broken line is intended to represent the periphery of a very large cow population. The line AB is drawn thru the points corresponding to 360 pounds of fat. Note that selection is more severe at low fat percentages than at higher fat percentages. The figure is purely diagrammatic.

On the basis of energy yield, the nature of advanced registry selection is shown clearly by Table A. The table shows that selection becomes increasingly more severe in going from higher to lower fat percentages. It is therefore to be expected that the mean energy yield shown by advanced registry records will be greater at lower values of t than at higher values of t. Since the energy curve is adjusted to the mean of the observations, there will be a tendency toward *plus* deviations at the left end of the graphs, and a less marked tendency toward *minus* deviations at the right, assuming the energy

TABLE A.—ILLUSTRATING THE NATURE OF THE SELECTION EFFECTED IN ADVANCED REGISTRY BY A FIXED FAT ENTRANCE REQUIREMENT

(Note increasing increment in E in going from higher to lower fat percentages)

t	F	M	E	Increment in E
			000 omitted	
8.0	360	4 500	2 381	
7.0	360	5 143	2 466	85
6.0	360	6 000	2 579	113
5.0	360	7 200	2 738	159
4.0	360	9 000	2 976	238
3.0	360	12 000	3 372	396
2.0	360	18 000	4 164	792

curve to represent the true relation for the unselected population. Bearing this in mind, we may now consider the advanced registry data in relation to the hypothesis.

Jersey Register of Merit Long-Time Records.—The data for this group are given in Tables 5 and 6, and Fig. 3. Considering the graph, it will be seen that the energy curve does not conform closely to the logarithmic curve. That the energy curve does not go thru the observations quite so well as the logarithmic curve is shown both by inspection and by the errors. The logarithmic curve, of course, is

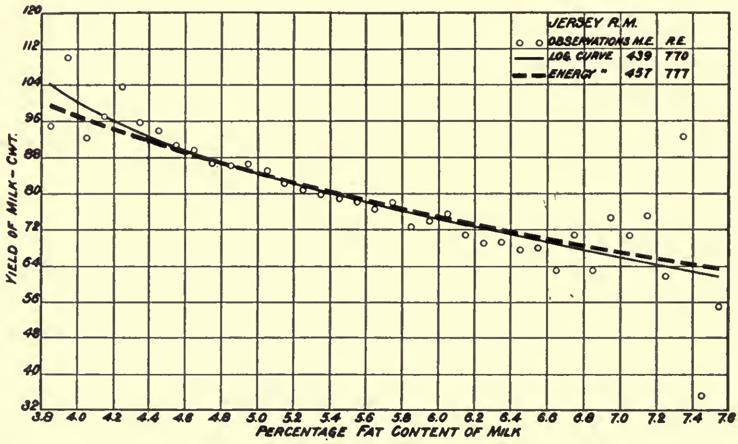

FIG. 3.—RELATION BETWEEN FAT PERCENTAGE AND MILK YIELD: JERSEY REGISTER OF MERIT LONG-TIME RECORDS

8,038 records. Data from Table 6, page 604. (One observation at $t = 8.145$. $M_0 = 4500$ is omitted in the graph.)

determined solely by the observations on the selected advanced registry population; whereas the energy curve can be expected to conform to observations only on a random sample of the population. In how far is the selection effected by the entrance requirements an explanation of the difference between the two curves (accepting the logarithmic curve as representing the observations)? Without attempting to answer quantitatively, it is apparent that the effect of selection would be to produce a difference similar to that actually found. Think of the energy curve as placed slightly lower on the graph so that the two curves coincide at the right-hand end. Compare, now, the curved wedge-shaped surface between the two, with the curved wedge-shaped surface of the population excluded by the entrance requirements as illustrated in Fig. A. It would seem quite possible that the differential selection of the entrance requirements is entirely responsible for the deviations of the logarithmic curve, or observations, from the energy curve.

Jersey Register of Merit Seven-Day Records.—The data for this group are given in Tables 7 and 8, and Fig. 4. Judged by the errors, the energy curve fits nearly as well as the logarithmic curve. Visual impression from the graph, however, is favorable to the logarithmic curve. It will be noted that the difference between the two is similar to that found in the case of the long-time records.

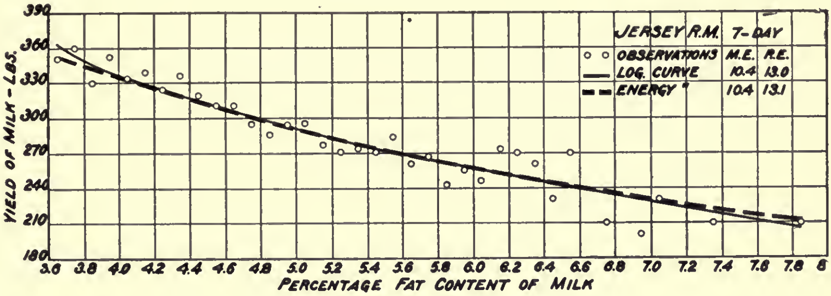

FIG. 4.—RELATION BETWEEN FAT PERCENTAGE AND MILK YIELD: JERSEY REGISTER OF MERIT SEVEN-DAY RECORDS

367 records. Data from Table 8, page 606

Guernsey Advanced Register Records.—The data for this group are given in Table 9 and Fig. 5. The relation between the two curves, as shown in the graph, is very similar to that noted and discussed for the Jersey long-time records.

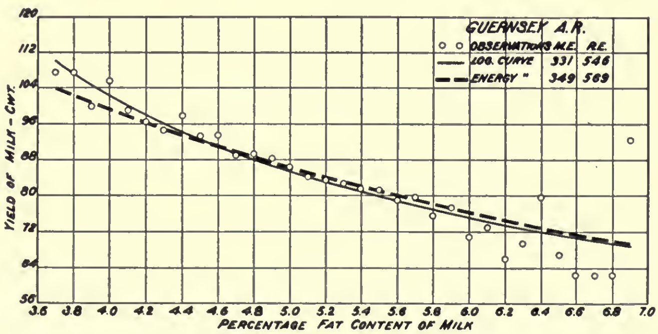

FIG. 5.—RELATION BETWEEN FAT PERCENTAGE AND MILK YIELD: GUERNSEY ADVANCED REGISTER RECORDS

3,564 records. Data from Table 9, page 607

Ayrshire Advanced Registry Records.—The data for this group are given in Table 10 and Fig. 6. The observations show, from left to right, first a descending tendency and then an ascending tendency.

The logarithmic curve is not fitted to the whole data because the type used is not adapted. As previously pointed out, the Ayrshire entrance requirements are peculiar. At values of t above 3.57–4.29, selection becomes more severe, and consequently the mean energy yield of the right-hand end groups is increased. Making allowance for this, the Ayrshire data differ from the energy curve in a manner similar to that for the Jersey and Guernsey, and in accordance with expectation.

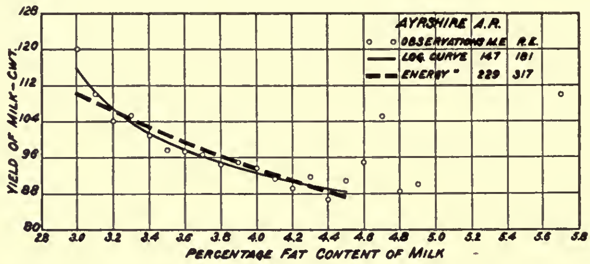

FIG. 6.—RELATION BETWEEN FAT PERCENTAGE AND MILK YIELD: AYRSHIRE ADVANCED REGISTRY RECORDS

1,091 records. Data from Table 10, page 608

Brown Swiss Register of Production Records (Age-Corrected).—The data for this group are given in Tables 11, 12 , and 13 and Fig. 7. The age-correction factor applied here is that for the Holstein breed (for lack of better data) and may be subject to some error. The graph shows a great deal of irregularity in the observations, which is possibly due to the small number of records, 311. The general trend of the observations is in conformity with the energy curve, but the data are hardly satisfactory for the purpose of fitting a curve.

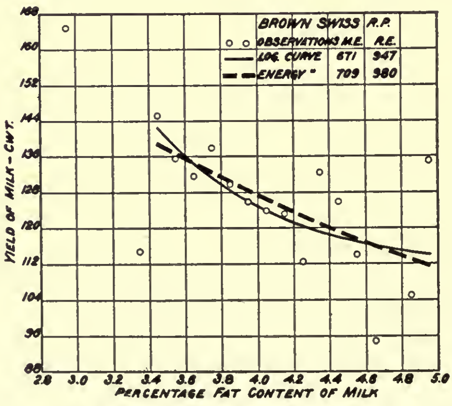

FIG. 7.—RELATION BETWEEN FAT PERCENTAGE AND MILK YIELD: BROWN SWISS REGISTER OF PRODUCTION RECORDS

311 records, age-corrected. Data from Table 13, page 611

Holstein-Friesian Advanced Register Long-Time Records.—The data for this group are given in Tables 14, 15, 16, and 17, and in Figs. 8 and 9. Considering Fig. 8, which is based on Vols. 24 to 30, the records are seen to be exceptional. The center of the group shows some tendency to conform to the energy curve. But the right-hand portion is very remarkable. As the data stand, they do not support the hypothesis.* That the conditions of official testing were responsi-

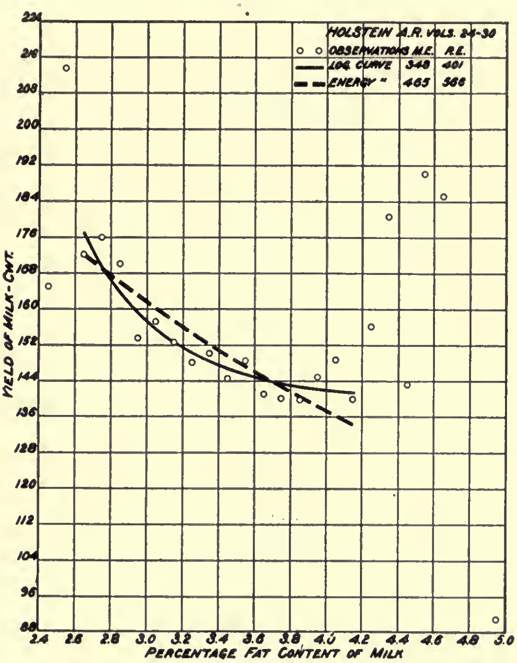

FIG. 8.—RELATION BETWEEN FAT PERCENTAGE AND MILK YIELD: HOLSTEIN-FRIESIAN ADVANCED REGISTER LONG-TIME RECORDS
5,266 records, 1912-1919. Data from Table 15, page 613 (cf. Figs. 1, 9, 10).

ble for the exceptional results shown is indicated by the fact that Holstein cows, under the conditions of the cow testing association (Fig. 1), showed an entirely consistent behavior in their records. Eckles[7] has shown experimentally that the condition of the cow at freshening materially affects milk secretion, qualitatively, the fat percentage being increased by a fat condition of the cow. It is commonly believed that the Holstein cow is especially susceptible to this influence. It may be offered in explanation that a part of the advanced registry

*It may be noted that the energy basis is, nevertheless, a more equitable basis of comparison than the fat basis, as shown by the errors, Table 23.

Holstein cows, having normally a somewhat low value of t and a high value of M, were in high condition at freshening, and thereby the value of t was greatly increased while the value of M was not decreased. Such a condition might distort the data to produce the effect observed.

Because of the exceptional nature of the above records, which may be called ''modern,'' the earlier records of Vols. 18 to 24 are considered. They are the first 1,003 long-time official records of the breed. The data (Fig. 9) show a somewhat similar tendency, but in lesser degree. It will be seen that judged by the logarithmic curve, the energy curve does not fit well; but judged by the observations themselves, it fits fairly well. This view is supported by the errors, which are not much greater for the energy curve than for the logarithmic curve.

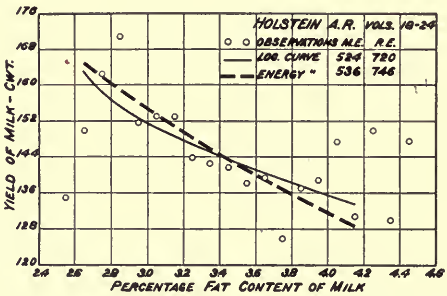

FIG. 9.—RELATION BETWEEN FAT PERCENTAGE AND MILK YIELD: HOLSTEIN-FRIESIAN ADVANCED REGISTER LONG-TIME RECORDS

1,033 records, 1906-1913. Data from Table 17, page 615. These are the first 1,003 long-time official records of the breed. Note the partial disappearance of the discordantly high milk yields shown at the higher fat percentages in Fig. 8.

Holstein-Friesian A.R.O. Seven-Day Records (Age-Corrected).— The data for this group are given in Tables 18, 19, and 20, and Fig. 10. The records represent the first 277 cows admitted (1894-1898) to the Holstein-Friesian advanced registry under the system of official tests. The graph shows a slight tendency of the data in the direction noted for the long-time records. While there is not the closest agreement between the energy curve and the logarithmic curve, yet it is evident that the energy curve goes thru the observations strikingly well. Its mean error is less than that of the logarithmic curve, but its root error is somewhat greater.*

*It should be noted that the ''modern'' seven-day records do not support the hypothesis at all. They show, in fact, a tendency to constant milk yield. Reference is had to those seven-day records made shortly after calving. For records made some time after calving, the energy relation may hold. The seven-day records are being studied further from this standpoint.

While the "modern" Holstein advanced registry records do not support the hypothesis, the early records, both long-time and seven-day, are regarded as supporting it very satisfactorily.

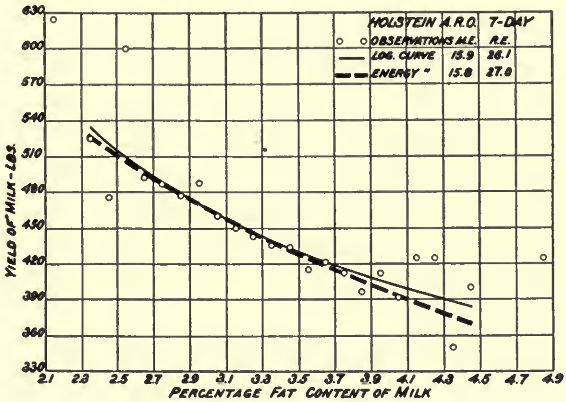

FIG. 10.—RELATION BETWEEN FAT PERCENTAGE AND MILK YIELD: HOLSTEIN-FRIESIAN ADVANCED REGISTER SEVEN-DAY RECORDS

277 records, 1894-1898, age-corrected. Data from Table 20, page 618. These are the records of the first 277 cows of the breed admitted to the advanced register under the present system of official supervision. Note the practically complete disappearance of the discordantly high milk yields at the higher fat percentages shown in Fig. 8.

Summary of Evidence.—The coefficient of correlation (Table 21) indicates the presence of a definite relation between percentage fat content and yield of milk. As brought out further by the mean milk yields and by the fitted curves and their errors (as shown in the graphs and tables) this relation is in excellent conformity with that expressed in the hypothesis for all the records except the "modern" Holstein advanced registry records.

It is therefore held that the hypothesis is verified by the evidence at hand.*

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*The hypothesis of this paper has a bearing on the problems of inheritance of the characters fat percentage and milk yield. The low value of r as found in correlating the two variables has given rise to the notion that the genetic factors responsible for the two characters are independently transmitted and capable of combination in any way. Such may be the fact. But we must remember that to secure the simultaneous development of high fat percentage and high milk yield would involve an extraordinary expenditure of energy. The hypothesis suggests that a high energy yield is no more certain of attainment with a high fat percentage than with a low fat percentage; and that there is no more likelihood of securing a super-dairy breed by crossing a high fat percentage breed with a high milk yielding breed than there is within either breed itself, so far as the direct influence of the genetic factors determining the two characters in question is concerned.

An alternative explanation of the relation between fat percentage and milk yield deserves consideration. If the mean fat percentage of the various milk

CORRECTION OF MILK YIELD FOR FAT CONTENT

Derivation of Formula.—The principal product of the dairy cow is her mammary product—milk,—variable in quantity and chemical composition. Composition itself, as measured by fat percentage, is a factor having a certain definite influence on quantity. It is desirable to express the mammary product in terms of milk of some certain standard composition, and this is readily possible because of the nature of the influence of composition on yield. The choice of the standard composition to be used is not predetermined, except that it be the normal composition of milk of some particular fat percentage. For the Holstein breed the choice might be the mean of the breed, say milk of 3.4 percent fat; for the Jersey breed, likewise, 5.4 percent fat. For general convenience and utility, it is better to have a single standard for all cows, and normal milk of 4.0 percent fat has been chosen as being near a mean and most convenient of use.

The problem is now to equate the milk yield at varying fat percentages to the standard of a milk having a fat content of 4.0 percent. The equation takes the form,

$$\text{Fat corrected yield of milk (pounds), FCM} = \frac{\text{Energy yield}}{\text{Energy of 1 pound of 4.0\% milk}}$$

$$= \frac{132.06\,M + 4964\,F}{330.62}$$

$$= .4\,M + 15\,F$$

Application of Formula.—It will be recalled that the "modern" Holstein advanced registry records do not support the proposition on which this formula is based. There may be some doubt as to whether the formula may equitably be applied to this class of records. The

yield classes be determined, it is found that there is a decrease in fat percentage from lower to higher milk yields. On the basis of this, Gowen[2] (p. 95) has offered, in explanation of the relation between fat percentage and milk yield, the proposition that a higher milk yield requires a greater expenditure of energy (in total) than a lower milk yield, and that the fat or fat precursors of the milk are drawn on to meet this energy requirement, thus reducing, to some extent, the proportion of fat in the milk at the higher milk yields. According to Gowen's view, milk yield is cause and fat percentage is effect (milk yield, however, affecting fat percentage only to a minor degree); whereas, according to the view of the present paper, fat percentage (together with the correlated solids-not-fat percentage) is cause and milk yield is effect (fat percentage being, however, only one of many factors affecting milk yield). Both views have in common the recognition of an energy requirement in explanation of the relation between the two variables. To the writers, it does not seem reasonable to suppose that the fat or fat precursors of the milk should be the sole source of the energy required in milk secretion: whereas, it does seem reasonable to suppose that the energy requirement should be a determining factor in the amount of milk secreted, and that the energy requirement should be in proportion to the energy content of the solids of the milk secreted.

only apparent reason that it might not be so applied would be that the composition of the milk of advanced registry Holstein cows (under the particular conditions surrounding the production of their records) is different from that of other cows (or Holstein cows under ordinary conditions), where the fat percentage is the same. There is no evidence that it is different, but on the other hand there is some evidence* that it is not different. In the judgment of the writers, the discordant results noted are due to subjecting part of the population to unusual conditions and the discord would disappear if the whole population were subjected to the same condition. The formula is therefore regarded as applicable to the class of records in question.

The proposition on which the formula is based is supported by the use of mean milk yields of groups. The question arises, is it applicable to individuals? The relation $E = 132.06\,M + 4964\,F$ is naturally subject to some variation, and to that extent there is the probability of error in applying the formula to the individual. That such error would not be great is indicated by two facts: first, the fat itself represents more than half the energy of the milk (except when $t < 3.8$); second, the solids-not-fat, which are responsible for the remainder of the energy, are closely correlated with the fat ($r = +.9$). Hence, the formula may be applied to the individual with the probability of only slight injustice.**

The recommendation is therefore made that for comparative purposes in considering the milk production of cows, the yield of milk be corrected by the formula $.4\,M + 15\,F$; where M = milk yield, in pounds, and F = fat yield, in pounds.***

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*Unpublished data, Illinois Agricultural Experiment Station.

**It would be more accurate to determine the energy value calorimetrically. The greater accuracy is not regarded as a sufficient offset to the difficulties involved in the calorimetric determination to warrant its use, ordinarily. If the energy value is determined directly the equation would take the form:

$$E\,C\,M = \frac{E}{330.62} = .3025\,E$$

Where both the solids and fat are determined, the equation might take the form:

$$F\text{-}S\,C\,M = \frac{4220\,F + 1860\,S\text{-}N\text{-}F}{330.62} = 12.764\,F + 5.626\,S\text{-}N\text{-}F$$

***As to the equity of this correction, further evidence, of a different sort, is to be had from the feed required for the production of milk of different fat percentages. On this, a great deal of experimental work is summarized and generalized in the feeding standards for milk production. Table B analyzes several standards on the basis of the energy value of the milk solids. It will be noted from the table that the feed required per unit energy of milk is practically a constant for the varying fat percentages (except with the Eckles standard). In point of feed required, the evidence of the feeding standards supports the equity of the correction formula.

The results of experimental work in milk production are generally stated in terms of milk and of fat. There are often economic conditions that make it desirable to lay stress on one or the other of these terms. In other cases, where a physiological comparison is desired, it may be desirable to have a single expression to cover both terms, and for such purpose the above formula should be of value. To illustrate specifically, take the results of grading up from scrub cows

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TABLE B.—RELATION BETWEEN PERCENTAGE FAT CONTENT OF MILK AND FEED REQUIRED PER UNIT ENERGY OF THE MILK SOLIDS

(The table shows the relative values of the feed required by various feeding standards as given in Larson and Putney,[8] at the several fat percentages indicated. Four-percent milk is taken as 100 for each standard. The energy of the milk solids is estimated by the formula given in the text.)

Feeding standard	Fat content of milk									
	2.5%	3.0%	3.5%	4.0%	4.5%	5.0%	5.5%	6.0%	6.5%	7.0%
Haecker......		98	99	100	101	101	101	101	102	
Savage.......	95	97	99	100	101	101	101	101	101	100
Henry and Morrison....	96	97	99	100	101	101	101	101	101	101
Eckles........		102	101	100	102	104	109	115	121	
Armsby......	91	92	96	100	100	103	103	103	105	105

We may now develop a point of some practical interest, namely, the relative feed cost of producing milk as affected by the percentage fat content. Table B shows the nutrients required for milk production (exclusive of maintenance) at different fat percentages to be in proportion to the energy value of the milk. We have seen above that the energy yield of cows is constant, so far as it is affected by the fat percentage of their milk. Therefore, the nutrients required for maintenance, per unit energy of the milk produced, are a constant so far as they are affected by percentage fat content (disregarding any correlation between fat percentage and size of cow). It follows, then, that the relative feed cost of producing milks of different fat percentages is substantially in accordance with the equation:

$$\text{Feed cost per cwt. milk} = X(.4 + .15t),$$

where X is the feed cost per cwt. of 4.0-percent milk. To illustrate, if the feed cost of 4.0-percent milk is \$2.00 per cwt., then the corresponding cost of 3.0-percent milk is \$2.00 $[.4 + (.15)(3)] = \$1.70$; and, of 5.0-percent milk, \$2.00 $[.4 + (.15)(5)] = \$2.30$; and so forth. Or, to put it perhaps more simply, a difference of 1 in the percentage fat content of the milk corresponds to a difference in feed cost which is equal to 15 percent of the feed cost of 4.0-percent milk.

It is plain, at this point, that the argument of this paper is essentially that the energy value of the milk solids is an equitable basis of comparison of the production of cows. That the energy value should be expressed in terms of average milk of 4.0-percent fat content is purely a matter of convenience and desire to retain the term "milk." It was stated near the outset that the relation between fat percentage and milk yield is "simple and logical." The reason for the statement is seen now, since the laws of energetics may be expected to be involved in the secretion of milk, as they are in other life activities. Apparently, the water of the milk represents no expenditure of energy on the part of the mammary gland. The osmotic pressure of the milk and the blood are the same, so that there is no balance of osmotic energy with which to reckon. Consequently, the energy relation goes back entirely to the solids of the milk.

by the use of dairy bred bulls as reported in Bulletin 188 of the Iowa Agricultural Experiment Station. The daughters of the Holstein bull used showed a milk production equal to 190 percent of that of the dams, and a fat production equal to 159 percent. The average production of the dams was 3,894 pounds FCM and of the daughters, 6,602 pounds FCM. The production of the daughters, on this basis, is 170 percent of that of the dams; and we may say that the dairy-bred bull has increased the dairy capacity of the first generation by 70 percent when compared with the stock with which he was mated (age is not taken into account here).

The formula should be of especial value in comparing the production of cows having a considerable difference in the percentage fat content of their milk. Table C has been prepared from the published records of five dairy breeds in order to show the relation between the highest milk and highest fat records in each breed. Decimals are omitted from the milk and fat records.* Suppose it is desired to compare the records of the first two cows in the table. Cow B.P. has a recorded fat production 100 pounds greater than cow S.P.P.; but the latter has more milk by 10,364 pounds. Which is the better record? From the physiological standpoint of work performed, and reduced to terms of 4.0-percent milk, S.P.P. has the better record by 2,645 pounds. In like manner, comparison may be made between the breeds, if desired.

TABLE C.—HIGHEST MILK AND HIGHEST FAT RECORDS OF FIVE BREEDS (August, 1922)

Name and Number	Milk *lbs.*	Fat *lbs.*	Fat %	FCM* *lbs.*	FCM* *relative values*
Segis Pietertje Prospect HFHB 221846..	37 381	1 159	3.10	32 337	100
Bella Pontiac, CHB 46321..............	27 017	1 259	4.66	29 692	92
Murne Cowen, AGCC 195977..........	24 008	1 098	4.57	26 073	81
Countess Prue, AGCC 43785...........	18 627	1 103	5.92	23 996	74
Garclaugh May Mischief, ABA 27944...	25 329	895	3.53	23 557	73
Lily of Willowmore, ABA 22269.........	22 596	956	4.23	23 378	72
Fauvic's Star, AJCC 313018............	20 616	1 006	4.88	23 336	72
Lad's Iota, AJCC 350672...............	18 632	1 048	5.63	23 173	72
Hawthorn Dairy Maid, BSBA 6753.....	22 623	927	4.10	22 954	71

*Milk yield corrected for fat to 4.0-percent fat.

SUMMARY

The relation between percentage fat content and yield of milk is shown by analysis of ten groups of cow records, comprizing 23,302 records in all. Accordant results are shown by nine groups:

*The writers feel that the extensive printing of meaningless decimals in data of this nature as practiced by Agricultural Experiment Stations and Breed Associations, is without justification.

970 Jersey Cow Testing Association yearly records
8,038 Jersey Register of Merit long-time records
367 Jersey Register of Merit seven-day records
3,564 Guernsey Advanced Register long-time records
1,091 Ayrshire Advanced Registry long-time records
311 Brown-Swiss Register of Production long-time records
2,773 Holstein Cow Testing Association yearly records
277 Holstein-Friesian Advanced Register seven-day records (Vols. 1-9)
1,003 Holstein-Friesian Advanced Register long-time records (Vols. 18-24)

Discordant results are shown by one group:

5,266 Holstein-Friesian Advanced Register long-time records (Vols. 24-30)

The relation supported by the majority of the data is made the basis of a correction formula for milk yield designed to equate for the influence of fat percentage on yield.

CONCLUSIONS

The percentage fat content of the milk is a factor affecting milk yield. So far as affected by fat percentage, the milk yield is inversely proportional to the energy value of the milk solids per unit of milk; that is, the energy value of the milk solids, in the total milk yield, is a constant. For a group of comparable cows, the relation between fat percentage and milk yield is expressed by the equation $M = \frac{a}{2.66 + t}$; where M is the average milk yield (in pounds), t is fat percentage, and a is a constant determined in value by the productive level of the particular group. As corollaries: $F = .01a - \frac{.0266a}{2.66 + t}$; $S\text{-}N\text{-}F = .004a + \frac{.06036a}{2.66 + t}$; and, $S = .014a + \frac{.03376a}{2.66 + t}$; where F is fat, S-N-F is solids-not-fat, S is solids (all, in pounds) and a is the same constant as above.

The milk yields of cows may be corrected for the influence of fat content to the physiological equivalent of 4.0-percent (fat) milk by the equation, $F\,C\,M = .4M + 15F$; where F C M is "fat corrected milk," M is the actual milk yield, and F is the actual fat yield (all, in pounds).

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TABLE 1.—CORRELATION OF THE VARIABLES PERCENTAGE FAT CONTENT AND YIELD OF MILK

Data from Holstein (Grade and Pure-Bred) Cow Testing Association Yearly Records

Yield of milk (cwt.); class mid-points	Percentage fat content of milk; class mid-points (add .045)																														
	2.5	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0	5.1	5.2	5.3	5.4	Total
25	...	...	...	...	1	...	...	...	2	1	1	1	1	2	1	2	...	...	...	...	1	...	...	...	...	...	...	...	...	...	13
35	...	2	1	...	...	3	2	5	9	11	7	7	8	7	8	6	2	5	6	1	2	2	1	1	...	...	...	...	...	...	96
45	...	...	1	3	3	15	14	22	30	29	35	24	27	20	24	14	10	7	5	4	4	4	1	3	...	...	...	...	...	...	299
55	...	1	1	4	11	10	28	38	56	50	47	49	54	28	25	25	12	8	9	5	4	1	3	...	...	...	...	1	...	1	471
65	1	1	4	5	12	30	38	49	60	66	51	61	42	39	30	21	18	12	5	4	1	3	...	...	3	1	...	...	...	...	557
75	...	...	2	9	18	29	33	48	56	56	56	57	43	29	20	11	10	12	4	4	3	2	...	1	1	...	...	...	...	1	505
85	...	3	2	4	14	23	37	32	50	46	44	30	25	23	20	9	8	2	2	2	1	1	2	...	...	...	...	...	...	...	380
95	2	2	...	5	10	8	20	23	36	39	24	15	10	14	7	7	1	1	3	...	...	1	...	...	...	...	...	...	...	...	228
105	...	1	2	6	13	8	18	11	22	15	10	9	4	4	3	2	2	...	1	...	...	...	...	...	...	...	...	...	...	...	131
115	...	...	4	...	1	7	6	3	8	4	5	2	3	2	...	2	1	...	1	...	...	...	...	...	...	...	...	...	...	...	49
125	...	...	...	...	2	5	3	7	5	3	2	...	1	1	2	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	32
135	...	...	...	...	1	1	3	...	...	2	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	8
145	...	...	...	...	...	...	...	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1
155	...	...	...	...	...	...	...	...	1	...	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	2
165	...	...	...	...	...	...	...	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1
Total	3	10	17	36	86	139	202	238	337	322	283	256	218	169	140	99	65	47	36	20	16	14	7	5	4	1	0	1	0	2	2773

TABLE 2.—COMPARISON OF MEAN MILK YIELDS AS OBSERVED, AND AS CALCULATED FROM FITTED LOGARITHMIC AND CONSTANT ENERGY CURVES

Grade and Pure-Bred Holstein Cow Testing Association Records
(See Table 1)

t	f	M_o	M_1	D	M_e	D
2.545	3	8 500	8 483	+ 17	8 366	+ 134
2.645	10	7 400	8 276	− 876	8 209	− 809
2.745	17	8 147	8 089	+ 58	8 057	+ 90
2.845	36	7 778	7 919	− 141	7 910	− 132
2.945	86	8 023	7 761	+ 262	7 769	+ 254
3.045	139	7 608	7 615	− 7	7 633	− 25
3.145	202	7 718	7 478	+ 240	7 502	+ 216
3.245	238	7 277	7 348	− 71	7 375	− 98
3.345	337	7 384	7 225	+ 159	7 252	+ 132
3.445	322	7 220	7 107	+ 113	7 133	+ 87
3.545	283	7 058	6 994	+ 64	7 018	+ 40
3.645	256	6 871	6 886	− 15	6 907	− 36
3.745	218	6 610	6 781	− 171	6 799	− 189
3.845	169	6 808	6 680	+ 128	6 695	+ 113
3.945	140	6 529	6 582	− 53	6 593	− 64
4.045	99	6 389	6 487	− 98	6 495	− 106
4.145	65	6 654	6 394	+ 260	6 399	+ 255
4.245	47	6 117	6 304	− 187	6 307	− 190
4.345	36	6 194	6 216	− 22	6 217	− 23
4.445	20	6 100	6 130	− 30	6 129	− 29
4.545	16	5 438	6 045	− 607	6 044	− 606
4.645	14	5 929	5 963	− 34	5 961	− 32
4.745	7	5 929	5 882	+ 47	5 881	+ 48
4.845	5	4 900	5 802	− 902	5 802	− 902
4.945	4	6 750	5 724	+1026	5 726	+1024
5.045	1	6 500	5 647	+ 853	5 652	+ 848
5.245	1	5 500*				
5.445	2	6 500*				
Mean error			248		249	
Root mean-square error			397		391	

*Excluded in fitting curves and computing errors.

TABLE 3.—CORRELATION OF THE VARIABLES PERCENTAGE FAT CONTENT AND YIELD OF MILK

Data from Jersey (Grade and Pure-Bred) Cow Testing Association Yearly Records

Percentage fat content of milk; class mid-points (add .045)

Yield of milk (cwt.); class mid-points	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0	5.1	5.2
25	....	....	....	....	....	....	....	....	....	....	....	....	....	....	1	1	1
35	....	....	....	....	1	1	4	4	2	2	7	7	7	7	12	13	11
45	2	....	....	....	3	3	3	6	9	9	12	13	13	13	15	16	17
55	....	....	2	1	4	5	8	10	23	13	20	20	20	29	17	17	14
65	....	1	3	3	2	4	2	9	8	12	12	7	13	11	17	6	9
75	....	....	1	2	2	....	2	6	4	8	6	4	11	7	7	1	3
85	....	....	....	1	....	1	1	....	....	1	2	3	2	1	2	....	....
95	....	....	1	....	....	2	....	....	....	1	....	....	....	1	....	....	....
Total	2	1	7	7	12	16	20	35	46	46	59	54	66	69	71	54	55

Continued below

Yield of milk (cwt.); class mid-points	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0	6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	Total
25	....	....	2	3	2	1	1	1	....	....	....	....	....	....	....	....	13
35	12	14	3	6	6	8	7	4	2	....	....	....	1	....	....	....	141
45	14	14	17	8	8	9	3	9	1	1	3	1	....	2	1	1	226
55	18	24	18	16	6	10	9	2	....	3	1	....	....	....	1	....	311
65	7	14	8	5	7	11	1	1	....	2	....	1	1	....	....	....	177
75	3	2	3	2	....	....	....	....	....	....	....	....	....	....	....	....	74
85	2	2	2	....	....	1	....	....	....	....	....	....	....	....	....	....	21
95	....	....	1	....	....	1	....	....	....	....	....	....	....	....	....	....	7
Total	56	70	54	40	29	41	21	17	3	6	4	2	2	2	2	1	970

TABLE 4.—COMPARISON OF MEAN MILK YIELDS AS OBSERVED, AND AS CALCULATED FROM FITTED LOGARITHMIC AND CONSTANT ENERGY CURVES

Grade and Pure-Bred Jersey Cow Testing Association Records
(See Table 3)

t	f	M_o	M_l	D	M_e	D
3.645	2	4 541*				
3.745	1	6 515	6 554	− 39	6 469	+ 46
3.845	7	6 785	6 385	+ 400	6 369	+ 416
3.945	7	6 928	6 249	+ 679	6 273	+ 655
4.045	12	5 583	6 133	− 550	6 179	− 596
4.145	16	6 124	6 030	+ 94	6 088	+ 36
4.245	20	5 400	5 937	− 537	6 000	− 600
4.345	35	5 700	5 851	− 151	5 915	− 215
4.445	46	5 565	5 770	− 205	5 832	− 267
4.545	46	5 980	5 694	+ 286	5 750	+ 230
4.645	59	5 568	5 621	− 53	5 672	− 104
4.745	54	5 444	5 552	− 108	5 595	− 151
4.845	66	5 712	5 485	+ 227	5 520	+ 192
4.945	69	5 572	5 420	+ 152	5 448	+ 124
5.045	71	5 430	5 356	+ 74	5 377	+ 53
5.145	54	4 815	5 295	− 480	5 308	− 493
5.245	55	4 827	5 235	− 408	5 241	− 414
5.345	56	5 160	5 176	− 16	5 176	− 16
5.445	70	5 243	5 119	+ 124	5 112	+ 131
5.545	54	5 407	5 062	+ 345	5 050	+ 357
5.645	40	5 000	5 007	− 7	4 989	+ 11
5.745	29	4 845	4 952	− 107	4 930	− 85
5.845	41	5 256	4 898	+ 358	4 872	+ 384
5.945	21	4 595	4 845	− 250	4 815	− 220
6.045	17	4 382	4 792	− 410	4 760	− 378
6.145	3	4 088	4 740	− 652	4 705	− 617
6.245	6	5 666	4 689	+ 977	4 653	+1013
6.345	4	4 779	4 638	+ 141	4 601	+ 178
6.445	2	5 495	4 588	+ 907	4 550	+ 945
6.545	2	4 735	4 538	+ 197	4 501	+ 234
6.645	2	4 281	4 488	− 207	4 453	− 172
6.745	2	4 670	4 439	+ 231	4 405	+ 265
6.845	1	3 417	4 390	− 973	4 359	− 942
Mean error			323		329	
Root mean-square error			421		428	

*Excluded in fitting curves and computing errors.

TABLE 5.—CORRELATION OF THE VARIABLES PERCENTAGE FAT CONTENT AND YIELD OF MILK

Data from Jersey Register of Merit, Vols. 1916, 1917, 1918, 1919
All Long-Time Records, Including Reentries

Percentage fat content of milk; class mid-points (add .045)

Yield of milk (cwt.); class mid-points	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9
35	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
45	...	...	...	...	...	...	...	...	...	...	...	...	1	2	5	7	12	14	20	22	20	24
55	...	...	...	...	...	1	1	8	8	16	25	24	42	61	68	79	69	62	80	62	63	74
65	...	...	2	2	4	2	18	19	37	40	65	61	70	80	92	97	130	141	121	119	125	94
75	...	...	1	2	1	15	13	33	38	61	107	102	112	153	159	134	156	109	108	81	80	73
85	...	...	2	8	7	20	32	38	52	70	95	74	92	110	97	100	96	92	84	68	46	41
95	1	...	3	7	9	22	20	29	33	44	59	51	69	55	48	56	56	54	47	43	41	33
105	...	1	1	5	2	14	12	22	20	31	40	43	45	32	38	31	30	33	30	25	28	20
115	...	1	1	3	7	10	11	13	20	14	22	19	21	25	21	22	24	15	13	28	9	6
125	...	...	...	2	...	5	11	5	13	4	13	16	12	17	11	13	12	15	6	5	5	7
135	...	...	1	...	1	1	2	6	7	4	11	9	10	8	8	5	4	2	5	7	2	2
145	...	...	...	...	...	1	3	2	3	3	5	5	6	5	2	4	4	3	3	2	1	2
155	...	...	...	...	1	...	2	3	...	1	2	3	1	1	2	3	...	1	2	1	1	2
165	...	...	...	1	1	...	...	...	2	2	...	...	...	1	3	...	2	...	...	1	...	...
175	...	...	...	...	1	...	...	...	...	1	1	...	2	...	1	...	1	...	...	...	...	...
185	...	...	...	...	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
195	...	...	...	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Total	1	2	11	30	35	92	125	178	233	291	445	407	483	550	555	551	596	541	519	464	421	378

Continued below

Yield of milk (cwt.); class mid-points	6.0	6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7.0	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8.0	8.1	Total
35	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1	...	...	...	...	...	...	...	1
45	18	27	10	15	14	14	11	4	4	3	...	1	1	...	...	1	...	...	...	...	...	1	251
55	54	42	49	26	28	14	12	5	4	2	6	2	1	...	...	...	...	...	...	...	...	...	988
65	66	55	37	30	27	20	16	10	5	5	1	1	...	...	...	1	...	...	...	...	...	...	1593
75	66	45	24	18	14	13	4	9	...	3	...	1	...	...	...	...	...	...	...	...	...	...	1735
85	41	27	20	19	7	3	...	2	...	4	...	...	1	...	...	...	...	...	...	...	...	...	1348
95	22	15	15	4	5	6	...	...	3	3	...	...	...	...	...	...	...	...	...	...	...	...	853
105	17	10	3	5	2	...	1	...	...	...	...	1	...	...	...	...	...	...	...	...	...	...	542
115	8	3	1	4	2	2	2	1	...	...	1	1	...	...	...	...	...	...	...	...	...	...	330
125	6	2	...	...	2	3	...	1	...	...	1	...	...	...	...	...	...	...	...	...	...	...	187
135	3	1	...	...	...	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	100
145	1	...	1	...	1	...	...	1	...	1	...	...	...	...	...	...	...	...	...	...	...	...	59
155	2	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	28
165	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	14
175	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	7
185	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1
195	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1
Total	304	228	160	121	102	75	47	33	16	21	9	7	3	0	1	2	0	0	0	0	0	1	8038

TABLE 6.—COMPARISON OF MEAN MILK YIELDS AS OBSERVED, AND AS CALCULATED FROM LOGARITHMIC AND CONSTANT ENERGY CURVES

Jersey Register of Merit, Vols. 1916, 1917, 1918, 1919
All Long-Time Records, Including Reentries

(See Table 5)

t	f	M_o	M_l	D	M_e	D
3.845	1	9 500	10 415	− 915	9 940	− 440
3.945	2	11 000	10 146	+ 854	9 789	+1211
4.045	11	9 227	9 913	− 686	9 643	− 416
4.145	30	9 700	9 707	− 7	9 502	+ 198
4.245	35	10 357	9 520	+ 837	9 364	+ 993
4.345	92	9 572	9 349	+ 223	9 231	+ 341
4.445	125	9 384	9 190	+ 194	9 101	+ 283
4.545	178	9 061	9 041	+ 20	8 974	+ 87
4.645	233	8 954	8 900	+ 54	8 851	+ 103
4.745	291	8 654	8 766	− 112	8 732	− 78
4.845	445	8 606	8 639	− 33	8 615	− 9
4.945	407	8 643	8 516	+ 127	8 502	+ 141
5.045	483	8 497	8 399	+ 98	8 392	+ 105
5.145	550	8 214	8 285	− 71	8 284	− 70
5.245	555	8 073	8 175	− 102	8 179	− 106
5.345	551	7 979	8 068	− 89	8 077	− 98
5.445	596	7 885	7 964	− 79	7 978	− 93
5.545	541	7 810	7 862	− 52	7 881	− 71
5.645	519	7 646	7 763	− 117	7 786	− 140
5.745	464	7 801	7 666	+ 135	7 693	+ 108
5.845	421	7 255	7 572	− 317	7 603	− 348
5.945	378	7 378	7 478	− 100	7 514	− 136
6.045	304	7 535	7 387	+ 148	7 428	+ 107
6.145	228	7 078	7 297	− 219	7 343	− 265
6.245	160	6 907	7 209	− 302	7 261	− 354
6.345	121	6 929	7 122	− 193	7 180	− 251
6.445	102	6 746	7 036	− 290	7 102	− 356
6.545	75	6 807	6 951	− 144	7 025	− 218
6.645	47	6 308	6 869	− 561	6 949	− 641
6.745	33	7 075	6 786	+ 289	6 875	+ 200
6.845	16	6 306	6 704	− 398	6 803	− 497
6.945	21	7 452	6 624	+ 828	6 732	+ 720
7.045	9	7 056	6 544	+ 512	6 662	+ 394
7.145	7	7 500	6 466	+1034	6 595	+ 905
7.245	3	6 166	6 387	− 221	6 528	− 362
7.345	1*	9 250	6 310	+2940	6 463	+2787
7.445	1	3 500	6 234	−2734	6 399	−2899
7.545	2	5 500	6 158	− 658	6 336	− 836
8.145	1	4 500**	.....	......	.....	......
Mean error			439		457	
Root mean-square error			770		777	

*Taken from Roberts,[6] Table V.
**Excluded in fitting curves and computing errors.

TABLE 7.—CORRELATION OF THE VARIABLES PERCENTAGE FAT CONTENT AND YIELD OF MILK

Data from Jersey Register of Merit, Vols. 1911, 1913, 1915, 1916, 1917, 1918, 1919
All Seven-Day Records of Cows Four Years Old and Over

Percentage fat content of milk; class mid-points (add .045)

Yield of milk (lbs.); class mid-points	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0	5.1	5.2	5.3	5.4	5.5	5.6	5.7	Continued below
190	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
210	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1	3	1	
230	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	3	8	6	6	1	2	3	
250	...	...	...	...	...	...	...	...	...	...	...	3	4	3	6	6	4	4	3	...	2	1	
270	...	...	...	...	...	...	...	...	1	3	6	14	10	3	5	4	4	5	2	2	3	2	
290	...	...	...	...	...	...	5	3	7	2	11	3	7	5	2	4	4	3	2	1	1	...	
310	...	...	...	3	1	3	2	...	2	9	...	3	1	1	4	2	...	3	...	...	...	...	
330	...	1	1	1	4	...	2	...	...	4	3	1	...	3	3	1	3	...	2	...	...	...	
350	1	2	...	...	...	...	2	1	3	1	3	3	1	...	3	1	...	...	2	1	...	1	
370	...	...	...	...	1	1	1	...	...	1	3	1	1	...	...	1	...	1	...	1	...	1	
390	...	...	...	...	...	1	...	1	...	...	...	2	1	...	1	...	...	1	...	...	...	...	
410	...	1	...	1	...	...	...	1	2	...	...	...	...	1	...	...	...	...	...	...	...	...	
430	...	...	...	...	...	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	
450	...	...	...	1	...	...	...	...	...	...	1	...	...	...	...	...	1	...	...	...	1	...	
Total	1	4	1	6	6	5	13	6	15	20	27	30	25	16	24	22	24	23	17	7	12	9	

Yield of milk (lbs.); class mid-points	5.8	5.9	6.0	6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7.0	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	Total
190	...	...	...	...	...	...	1	...	...	...	...	1	...	...	...	...	...	...	...	...	...	2
210	3	6	1	1	1	...	1	...	...	1	...	1	...	...	...	1	...	...	...	...	1	22
230	4	4	3	...	1	...	1	...	...	...	...	...	1	...	...	...	...	...	...	...	...	43
250	...	1	...	2	...	1	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	41
270	1	1	...	1	1	1	1	1	...	...	...	...	...	...	...	...	...	...	...	...	...	71
290	...	...	2	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	62
310	...	...	...	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	35
330	...	...	...	2	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	32
350	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	25
370	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	14
390	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	7
410	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	6
430	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	2
450	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	5
Total	9	14	6	6	5	2	5	1	0	1	0	2	1	0	0	1	0	0	0	0	1	367

TABLE 8.—COMPARISON OF MEAN MILK YIELDS AS OBSERVED, AND AS CALCULATED FROM LOGARITHMIC AND CONSTANT ENERGY CURVES

Jersey Register of Merit, Vols. 1911, 1913, 1915, 1916, 1917, 1918, 1919
All Seven-Day Records of Cows Four Years and More of Age

(See Table 7)

t	f	M_o	M_l	D_o	M_e	D
3.645	1	350.0	362.7	−12.7	353.2	− 3.2
3.745	4	360.0	354.4	+ 5.6	347.7	+12.3
3.845	1	330.0	347.1	−17.1	342.3	−12.3
3.945	6	353.3	340.5	+12.8	337.2	+16.1
4.045	6	333.3	334 5	− 1.2	332.1	+ 1.2
4.145	5	338.0	328.9	+ 9.1	327.2	+10.8
4.245	13	325.4	323.6	+ 1.8	322.5	+ 2.9
4.345	6	336.7	318.7	+18.0	317.9	+18.8
4.445	15	319.3	313.9	+ 5.4	313.4	+ 5.9
4.545	20	311.0	309.4	+ 1.6	309.1	+ 1.9
4.645	27	311.5	305.1	+ 6.4	304.8	+ 6.7
4.745	30	295.3	301.0	− 5.7	300.7	− 5.4
4.845	25	286.0	297.0	−11.0	296.7	−10.7
4.945	16	295.0	293.1	+ 1.9	292.8	+ 2.2
5.045	24	295.8	289.3	+ 6.5	289.0	+ 6.8
5.145	22	277.3	285.7	− 8.4	285.3	− 8.0
5.245	24	271.7	282.1	−10.4	281.7	−10.0
5.345	23	273.5	278.6	− 5.1	278.2	− 4.7
5.445	17	271.2	275.2	− 4.0	274.8	− 3.6
5.545	7	284.3	271.9	+12.4	271.4	+12.9
5.645	12	261.7	268.6	− 6.9	268.1	− 6.4
5.745	9	267.8	265.4	+ 2.4	265.0	+ 2.8
5.845	9	243.3	262.2	−18.9	261.8	−18.5
5.945	14	255.7	259.1	− 3.4	258.8	− 3.1
6.045	6	246.7	256.1	− 9.4	255.8	− 9.1
6.145	6	273.3	253.1	+20.2	252.9	+20.4
6.245	5	270.0	250.1	+19.9	250.1	+19.9
6.345	2	260.0	247.2	+12.8	247.3	+12.7
6.445	5	230.0	244.3	−14.3	244.6	−14.6
6.545	1	270.0	241.4	+28.6	241.9	+28.1
6.745	1	210.0	235.9	−25.9	236.8	−26.8
6.945	2	200.0	230.3	−30.3	231.8	−31.8
7.045	1	230.0	227.6	+ 2.4	229.5	+ .5
7.345	1	210.0	219.6	− 9.6	222.6	−12.6
7.845	1	210.0	206.7	+ 3.3	212.0	− 2.0
Mean error.......................			10.4		10.4	
Root mean-square error............			13.0		13.1	

TABLE 9.—COMPARISON OF MEAN MILK YIELDS AS OBSERVED, AND AS CALCULATED FROM LOGARITHMIC AND CONSTANT ENERGY CURVES

Guernsey Advanced Register*

All Entries and Reentries to and Including Vol. XXIX

t	f	M_o	M_l	D	M_e	D
3.7	4	10 750	10 992	− 242	10 393	+ 357
3.8	4	10 749	10 718	+ 31	10 232	+ 517
3.9	8	10 000	10 456	− 456	10 076	− 76
4.0	16	10 562	10 216	+ 346	9 925	+ 637
4.1	41	9 908	9 995	− 87	9 778	+ 130
4.2	68	9 661	9 790	− 129	9 635	+ 26
4.3	111	9 484	9 599	− 115	9 497	− 13
4.4	122	9 782	9 421	+ 361	9 362	+ 420
4.5	187	9 346	9 253	+ 93	9 232	+ 114
4.6	211	9 359	9 095	+ 264	9 105	+ 254
4.7	246	8 912	8 945	− 33	8 981	− 69
4.8	275	8 949	8 804	+ 145	8 861	+ 88
4.9	275	8 824	8 670	+ 154	8 744	+ 80
5.0	294	8 644	8 542	+ 102	8 629	+ 15
5.1	305	8 436	8 420	+ 16	8 518	− 82
5.2	273	8 367	8 303	+ 64	8 410	− 43
5.3	241	8 279	8 191	+ 88	8 304	− 25
5.4	216	8 166	8 083	+ 83	8 201	− 35
5.5	204	8 151	7 980	+ 171	8 101	+ 50
5.6	135	7 909	7 881	+ 28	8 003	− 94
5.7	87	7 951	7 785	+ 166	7 907	+ 44
5.8	76	7 552	7 693	− 141	7 813	− 261
5.9	52	7 750	7 604	+ 146	7 722	+ 28
6.0	42	7 083	7 518	− 435	7 633	− 550
6.1	21	7 297	7 435	− 138	7 546	− 249
6.2	20	6 599	7 354	− 755	7 461	− 862
6.3	10	6 950	7 276	− 326	7 377	− 427
6.4	7	7 964	7 199	+ 765	7 296	+ 668
6.5	7	6 678	7 111	− 433	7 216	− 538
6.6	3	6 249	7 055	− 806	7 138	− 889
6.7	1	6 250	6 985	− 735	7 062	− 812
6.8	1	6 249	6 917	− 668	6 987	− 738
6.9	1	9 250	6 852	+2398	6 914	+2336
Mean error			331		349	
Root mean-square error			546		569	

*From Roberts,[6] Table X.

TABLE 10.—COMPARISON OF MEAN MILK YIELDS AS OBSERVED, AND AS CALCULATED FROM LOGARITHMIC AND CONSTANT ENERGY CURVES

Ayrshire Advanced Registry (Ayrshire Breeders' Association Year Book), 1907, 1911, 1913, 1914*

t	f	M_o	M_l	D	M_e	D
3.0	1	12 000	11 581	+ 419	11 012	+ 988
3.1	2	11 000	11 063	− 63	10 820	+ 180
3.2	5	10 400	10 683	− 283	10 636	− 236
3.3	14	10 536	10 385	+ 151	10 457	+ 79
3.4	49	10 082	10 142	− 60	10 285	− 203
3.5	56	9 768	9 938	− 170	10 118	− 350
3.6	75	9 720	9 763	− 43	9 956	− 236
3.7	106	9 651	9 610	+ 41	9 799	− 148
3.8	149	9 440	9 476	− 36	9 648	− 208
3.9	121	9 488	9 356	+ 132	9 501	− 13
4.0	132	9 360	9 249	+ 111	9 358	+ 2
4.1	117	9 120	9 152	− 32	9 220	− 100
4.2	96	8 911	9 064	− 153	9 085	− 174
4.3	65	9 154	8 984	+ 170	8 955	+ 199
4.4	46	8 663	8 910	− 247	8 828	− 165
4.5	28	9 089	8 843	+ 246	8 704	+ 385
4.6	10	9 500**	.....	......	.....	......
4.7	10	10 500**	.....	......	.....	......
4.8	6	8 833**	.....	......	.....	......
4.9	2	9 000**	.....	......	.....	......
5.7	1	11 000**	.....	......	.....	......
Mean error			147		229	
Root mean-square error			181		317	

*From Roberts,[6] Table XX.
**Excluded in fitting curves and computing errors.

TABLE 11.—CORRELATION OF THE VARIABLES PERCENTAGE FAT CONTENT AND YIELD OF MILK

Data from Brown Swiss Register of Production (Brown Book of Brown Swiss Cattle Breeders' Association), Jan. 1, 1922
All Records Except Ten-Month Records

Percentage fat content of milk; class mid-points (add .045)

Yield of milk (cwt.); class mid-points	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	Total
55	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	1	.....	1	.....	.....	1	.....	.....	.....	.....	3
65	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	2	4	3	.....	.....	.....	2	.....	.....	.....	11
75	.....	.....	.....	.....	.....	.....	.....	.....	1	3	3	2	1	2	1	.....	3	1	.....	2	.....	19
85	.....	.....	.....	.....	.....	.....	.....	.....	.....	4	6	9	6	7	.....	1	.....	.....	.....	.....	.....	33
95	.....	.....	.....	.....	.....	.....	3	6	3	5	8	9	5	7	3	2	1	.....	.....	1	.....	53
105	.....	.....	.....	.....	.....	1	3	1	5	3	5	5	2	3	3	.....	1	1	.....	1	1	35
115	1	.....	.....	.....	1	2	3	4	5	4	4	3	3	6	4	3	1	.....	.....	.....	.....	44
125	.....	.....	.....	.....	.....	1	3	1	1	4	3	5	4	1	1	2	1	.....	.....	.....	.....	27
135	.....	.....	.....	.....	.....	.....	1	4	6	2	8	3	4	0	2	1	.....	.....	.....	.....	.....	31
145	.....	.....	.....	.....	.....	.....	1	2	5	2	3	6	1	3	.....	.....	.....	.....	.....	.....	.....	23
155	.....	.....	.....	.....	.....	1	1	.....	4	2	.....	1	1	.....	.....	.....	1	.....	.....	.....	.....	11
165	.....	.....	.....	.....	.....	.....	1	1	.....	1	3	2	.....	1	1	.....	.....	.....	.....	.....	.....	10
175	1	.....	.....	.....	.....	.....	.....	.....	2	1	.....	1	2	.....	.....	.....	.....	.....	.....	.....	.....	7
185	.....	.....	.....	.....	.....	1	1	.....	.....	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	.....	.....	3
195	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	.....	.....	1
Total	2	0	0	0	1	6	17	19	32	31	43	49	35	34	15	9	9	4	0	4	1	311

TABLE 12.—CORRELATION OF THE VARIABLES PERCENTAGE FAT CONTENT AND YIELD OF MILK

Data from Brown Swiss Register of Production
Same as Table 11, Age-Corrected*

Percentage fat content of milk; class mid-points (add .045)

Yield of milk (cwt.); class mid-points	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	Total
75	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	1	1	1	.....	.....	1	.....	.....	1	.....	5
85	.....	.....	.....	.....	.....	.....	.....	.....	.....	1	1	2	5	4	1	.....	1	2	.....	.....	.....	17
95	.....	.....	.....	.....	.....	.....	1	4	2	4	7	9	2	10	.....	1	2	.....	.....	1	.....	43
105	.....	.....	.....	.....	.....	.....	2	.....	3	3	5	8	6	4	2	2	1	2	.....	.....	.....	38
115	.....	.....	.....	.....	1	1	2	3	5	6	7	7	5	4	2	1	1	.....	.....	.....	.....	45
125	.....	.....	.....	.....	.....	2	3	.....	3	2	5	6	1	3	3	2	1	.....	.....	2	.....	33
135	.....	.....	.....	.....	.....	.....	2	4	5	2	8	1	6	1	2	.....	1	.....	.....	.....	1	33
145	.....	.....	.....	.....	.....	1	4	3	5	5	3	4	3	3	1	1	.....	.....	.....	.....	.....	33
155	1	.....	.....	.....	.....	.....	.....	3	.....	3	.....	3	2	2	1	1	.....	.....	.....	.....	.....	16
165	.....	.....	.....	.....	.....	1	1	1	4	3	5	3	.....	2	1	1	.....	.....	.....	.....	.....	22
175	1	.....	.....	.....	.....	.....	.....	1	3	2	1	3	1	.....	2	.....	.....	.....	.....	.....	.....	14
185	.....	.....	.....	.....	.....	.....	1	.....	2	.....	.....	1	2	.....	.....	.....	.....	.....	.....	.....	.....	6
195	.....	.....	.....	.....	.....	1	1	.....	.....	.....	1	1	1	.....	.....	.....	1	.....	.....	.....	.....	6
Total	2	0	0	0	1	6	17	19	32	31	43	49	35	34	15	9	9	4	0	4	1	311

*Yield of milk equated to age of maximum production by Gowen's equation for Holstein-Friesian cows, as follows:

Two-year-old class........× 1.36517
Three-year-old class......× 1.21749
Four-year-old class.......× 1.12500
Five-year-old class........× 1.06457
Mature class.............× 1.00000

TABLE 13.—COMPARISON OF MEAN MILK YIELDS AS OBSERVED, AND AS CALCULATED FROM LOGARITHMIC AND CONSTANT ENERGY CURVES

Brown Swiss Register of Production, Jan. 1, 1922
All Long-Time Records, Age-Corrected

(See Table 12)

t	f	M_o	M_l	D	M_e	D
2.945	2	16 500*	.....	......	.....	......
3.345	1	11 500*	.....	......	.....	......
3.445	6	14 500	14 261	+ 239	13 886	+ 614
3.545	17	13 559	13 835	− 276	13 663	− 104
3.645	19	13 184	13 465	− 281	13 446	− 262
3.745	32	13 781	13 143	+ 638	13 236	+ 545
3.845	31	12 984	12 862	+ 122	13 033	− 49
3.945	43	12 593	12 616	− 23	12 835	− 242
4.045	49	12 398	12 402	− 4	12 643	− 245
4.145	35	12 329	12 214	+ 115	12 458	− 129
4.245	34	11 265	12 051	− 786	12 277	−1012
4.345	15	13 233	11 910	+1323	12 103	+1130
4.445	9	12 611	11 787	+ 824	11 932	+ 679
4.545	9	11 389	11 683	− 294	11 766	− 377
4.645	4	9 500	11 595	−2095	11 605	−2105
4.845	4	10 500	11 461	− 961	11 296	− 796
4.945	1	13 500	11 414	+2086	11 147	+2353
Mean error.......................			671		709	
Root mean-square error............			947		980	

*Excluded in fitting curves and computing errors.

TABLE 14.—CORRELATION OF THE VARIABLES PERCENTAGE FAT CONTENT AND YIELD OF MILK

Data from Holstein-Friesian Advanced Register, Vols. 24 to 30 Inclusive
All Long-Time Records, Including Reentries

Percentage fat content of milk; class mid-points (add .045)

Yield of milk (cwt.); class mid-points	2.4	2.5	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	Total
65	....	....	....	....	....	....	....	....	....	....	....	3	2	1	2	3	2	....	....	....	....	....	....	....	....	....	13
75	....	....	....	....	....	1	2	3	1	4	8	7	12	13	3	5	....	2	....	....	1	....	....	....	....	....	62
85	....	....	....	....	....	2	4	10	23	24	28	18	33	14	13	3	....	4	1	....	....	....	....	....	....	....	177
95	....	....	....	....	1	5	3	22	25	27	37	34	40	14	14	11	5	7	2	....	....	1	....	....	....	1	249
105	....	....	1	3	2	4	21	33	48	64	56	38	36	25	24	8	5	3	1	....	....	....	....	....	....	....	372
115	....	....	....	2	8	24	30	44	56	71	79	58	42	34	23	14	4	10	3	2	1	....	....	....	....	....	505
125	....	....	1	3	10	23	33	52	73	61	71	60	50	37	19	8	10	4	1	2	3	....	....	....	....	....	521
135	....	1	2	3	13	27	28	47	71	91	75	66	56	45	25	8	7	7	4	1	2	....	....	....	....	....	579
145	....	....	1	5	14	18	28	54	79	67	60	63	38	37	19	10	10	3	4	1	....	1	....	....	....	....	512
155	1	....	5	6	11	27	36	49	65	70	58	41	29	20	28	9	7	5	4	1	....	....	....	....	....	....	472
165	....	1	....	5	6	17	33	51	52	66	50	37	32	16	9	7	2	4	3	....	1	....	....	....	....	....	392
175	1	....	2	11	7	16	24	43	38	46	34	30	31	26	9	7	5	4	2	5	1	....	....	....	....	....	342
185	....	1	3	5	4	14	20	24	28	44	27	31	28	13	7	8	2	4	3	....	....	....	1	....	....	....	267
195	....	....	2	6	3	7	11	27	23	32	27	30	14	7	6	1	1	3	1	1	1	....	....	....	....	....	203
205	....	....	....	4	10	8	9	8	21	23	16	21	9	6	8	3	3	1	1	2	1	1	....	....	....	....	155
215	....	....	1	2	4	4	8	9	19	13	16	16	8	6	5	3	3	....	1	....	....	1	....	....	....	....	119
225	....	....	2	3	8	2	7	8	11	15	9	10	5	4	2	7	2	....	1	1	....	....	....	....	....	....	97
235	....	....	1	1	2	3	6	7	3	12	6	10	8	2	1	1	....	....	1	2	....	1	....	....	....	....	67
245	....	....	....	....	3	....	4	6	6	12	6	4	5	1	2	2	1	1	1	1	....	1	....	....	....	....	56
255	....	....	....	1	3	2	2	3	2	5	3	4	3	2	1	2	1	1	....	....	....	....	....	....	....	....	35
265	....	1	....	1	1	1	1	1	3	3	3	3	2	2	1	....	1	....	....	....	....	....	....	....	....	....	24
275	....	....	....	1	....	....	4	4	2	1	1	2	1	1	....	....	....	....	....	1	....	....	....	....	....	....	18
285	....	....	....	....	1	1	3	1	1	1	3	....	....	....	1	....	....	1	....	....	....	....	....	....	....	....	13
295	....	....	....	....	....	1	1	3	....	....	2	2	....	....	....	....	....	....	....	....	....	....	....	....	....	....	9
305	....	....	....	1	1	....	....	....	....	1	....	....	1	....	....	....	....	....	....	....	....	....	....	....	....	....	4
315	....	1	....	....	....	....	....	1	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	2
325	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	0
335	....	....	....	....	....	....	....	1	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	1
Total	2	5	21	63	112	207	318	511	650	753	675	588	485	326	222	120	71	64	34	20	11	6	1	0	0	1	5266

TABLE 15.—COMPARISON OF MEAN MILK YIELDS AS OBSERVED, AND AS CALCULATED FROM LOGARITHMIC AND CONSTANT ENERGY CURVES

Holstein-Friesian Advanced Register, Vols. 24-30
All Long-Time Records, Including Reentries

(See Table 14)

t	f	M_o	M_1	D	M_e	D
2.445	2	16 500*	......	......	......	......
2.545	5	21 360*	......	......	......	......
2.645	21	17 214	17 667	− 453	17 212	+ 2
2.745	63	17 595	16 924	+ 671	16 893	+ 702
2.845	112	17 000	16 365	+ 635	16 586	+ 414
2.945	207	15 355	15 927	− 572	16 290	− 935
3.045	318	15 729	15 576	+ 153	16 005	− 276
3.145	511	15 273	15 289	− 16	15 730	− 457
3.245	650	14 800	15 053	− 253	15 463	− 663
3.345	753	15 028	14 857	+ 171	15 206	− 178
3.445	675	14 446	14 694	− 248	14 956	− 510
3.545	588	14 840	14 558	+ 282	14 715	+ 125
3.645	485	14 091	14 446	− 355	14 482	− 391
3.745	326	14 009	14 353	− 344	14 256	− 247
3.845	222	13 973	14 278	− 305	14 037	− 64
3.945	120	14 475	14 217	+ 258	13 824	+ 651
4.045	71	14 880	14 170	+ 710	13 618	+1262
4.145	64	13 984	14 135	− 151	13 418	+ 566
4.245	34	15 617*	......	......	......	......
4.345	20	18 050*	......	......	......	......
4.445	11	14 318*	......	......	......	......
4.545	6	19 000*	......	......	......	......
4.645	1	18 500*	......	......	......	......
4.945	1	9 100*	......	......	......	......
Mean error			348		465	
Root mean-square error			401		566	

*Excluded in fitting curves and computing errors.

TABLE 16.—CORRELATION OF THE VARIABLES PERCENTAGE FAT CONTENT AND YIELD OF MILK

Data from Holstein-Friesian Advanced Register, Vols. 18 to 24 Inclusive
All Long-Time Records, Including Reentries

Percentage fat content of milk; class mid-points (add .045)

Yield of milk (cwt.); class mid-points	2.5	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	Total
65	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	1	.....	1	.....	.....	.....	.....	2
75	.....	.....	.....	.....	.....	.....	.....	.....	.....	1	4	.....	4	.....	1	1	1	.....	.....	1	13
85	.....	.....	.....	.....	.....	.....	2	5	6	7	8	6	10	3	3	.....	.....	.....	.....	.....	50
95	.....	.....	.....	.....	2	4	4	4	14	5	4	6	5	2	6	1	.....	.....	.....	.....	57
105	.....	.....	1	.....	.....	1	4	11	13	8	12	11	6	5	2	.....	2	.....	.....	.....	76
115	.....	.....	.....	.....	3	5	7	13	11	17	16	7	11	6	1	.....	2	.....	.....	.....	99
125	.....	.....	.....	.....	2	10	12	15	15	23	14	13	8	3	2	2	1	1	1	.....	122
135	1	1	.....	1	4	6	11	6	18	12	15	3	6	4	4	.....	3	2	1	1	99
145	.....	2	1	1	2	11	12	10	10	10	12	12	5	4	4	2	.....	1	.....	.....	99
155	.....	.....	1	.....	2	4	5	16	12	8	14	8	4	4	3	1	1	.....	.....	.....	83
165	.....	.....	.....	.....	2	10	12	6	16	12	9	6	4	6	2	1	.....	1	.....	1	88
175	.....	1	.....	.....	4	6	11	10	9	7	6	5	3	4	2	.....	1	.....	.....	.....	69
185	.....	.....	.....	1	.....	2	2	5	10	8	8	4	3	.....	2	.....	.....	.....	.....	.....	45
195	.....	.....	.....	2	.....	.....	3	5	6	6	5	2	1	.....	.....	.....	1	1	.....	.....	32
205	.....	.....	.....	.....	.....	3	1	4	3	2	5	2	.....	.....	1	1	.....	.....	.....	1	23
215	.....	.....	.....	.....	1	2	2	1	1	4	1	1	.....	.....	2	1	.....	.....	.....	.....	16
225	.....	.....	.....	.....	1	.....	2	1	2	1	.....	1	2	1	.....	.....	.....	.....	.....	.....	11
235	.....	.....	.....	.....	.....	.....	2	.....	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	.....	.....	3
245	.....	.....	.....	.....	1	.....	.....	.....	1	1	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	3
255	.....	.....	.....	.....	.....	1	1	.....	.....	.....	.....	.....	.....	.....	1	1	.....	.....	.....	.....	4
265	.....	.....	1	.....	.....	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	2
275	.....	.....	.....	.....	.....	1	.....	.....	.....	.....	.....	1	.....	1	.....	.....	.....	.....	.....	.....	3
285	.....	.....	.....	.....	.....	1	1	1	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	3
295	.....	.....	.....	.....	.....	.....	1	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	1
Total	1	4	4	5	24	67	95	113	148	132	133	89	72	44	36	12	12	6	2	4	1003

TABLE 17.—COMPARISON OF MEAN MILK YIELDS AS OBSERVED, AND AS CALCULATED FROM LOGARITHMIC AND CONSTANT ENERGY CURVES

Holstein-Friesian Advanced Register, Vols. 18-24
All Long-Time Records, Including Reentries

(See Table 16)

t	f	M_o	M_l	D	M_e	D
2.545	1	13 500*	.....	......	.....	......
2.645	4	15 000	16 286	−1286	16 505	−1505
2.745	4	16 250	15 856	+ 394	16 199	+ 51
2.845	5	17 100	15 542	+1558	15 905	+1195
2.945	24	15 167	15 286	− 119	15 621	− 454
3.045	67	15 321	15 063	+ 258	15 347	− 26
3.145	95	15 290	14 863	+ 427	15 084	+ 206
3.245	113	14 385	14 679	− 294	14 828	− 443
3.345	148	14 264	14 507	− 243	14 581	− 317
3.445	132	14 182	14 344	− 162	14 342	− 160
3.545	133	13 816	14 188	− 372	14 111	− 295
3.645	89	13 938	14 039	− 101	13 887	+ 51
3.745	72	12 583	13 895	−1312	13 671	−1088
3.845	44	13 704	13 755	− 51	13 460	+ 244
3.945	36	13 890	13 618	+ 272	13 256	+ 634
4.045	12	14 750	13 485	+1265	13 059	+1691
4.145	12	13 083	13 355	− 272	12 867	+ 216
4.245	6	15 000*	......	......	......	......
4.345	2	13 000*	......	......	......	......
4.445	4	14 750*	......	......	......	......
Mean error......................			524		536	
Root mean-square error............			720		746	

*Excluded in fitting curves and computing errors.

TABLE 18.—CORRELATION OF THE VARIABLES PERCENTAGE FAT CONTENT AND YIELD OF MILK

Data from Holstein-Friesian Advanced Register, Vols. 1 to 9 Inclusive
Seven-Day Records, First Entries Only

Yield of milk (lbs.); class mid-points	Percentage fat content of milk; class mid-points (add .045)																												
	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	Total
225	....	....	....	....	....	....	....	1	....	....	2	3	2	1	3	3	2	....	1	1	....	....	....	....	....	....	....	....	19
275	....	....	....	....	....	1	3	4	5	9	9	5	3	9	3	2	....	....	3	2	....	....	....	1	....	....	....	....	59
325	....	....	1	1	....	3	5	4	5	5	4	5	5	3	6	3	1	3	....	1	....	3	2	....	....	....	....	....	60
375	1	....	....	....	....	1	1	1	5	3	1	2	5	1	3	5	1	1	2	2	....	....	....	1	....	....	....	....	36
425	....	....	....	....	1	....	2	7	5	7	10	9	2	3	1	....	4	3	1	....	2	1	....	....	....	....	....	1	59
475	....	....	....	....	....	....	1	3	....	1	5	3	5	2	3	1	....	....	....	....	....	....	....	....	....	....	....	....	24
525	....	....	....	....	....	....	....	1	2	3	1	1	....	2	....	....	....	....	1	....	....	....	....	....	....	....	....	....	11
575	....	....	....	....	....	1	....	....	1	1	....	....	....	1	1	....	....	....	....	....	....	....	....	....	....	....	....	....	5
625	....	....	....	....	....	....	....	1	....	1	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	2
675	....	....	....	....	....	....	....	....	1	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	1
725	....	....	....	....	1	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	1
Total	1	0	1	1	2	6	12	22	24	30	32	28	22	22	20	14	8	7	8	6	2	4	2	2	0	0	0	1	277

TABLE 19.—CORRELATION OF THE VARIABLES PERCENTAGE FAT CONTENT AND YIELD OF MILK

Data from Holstein-Friesian Advanced Register, Same as Table 18
Seven-Day Records, Age-Corrected

Percentage fat content of milk; class mid-points (add .045)

Yield of milk; (lbs.); class mid-points	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	Total
275																	1												1
325								1		1	1				2	1		1		1			1						9
375							1	2	1	7	5	7	7	9	9	7	1	3	4	3		1	1	1					69
425						2	3	4	7	5	10	7	5	5	4	1	4	2	3	1	2	2		1				1	69
475				1	1	2	3	8	8	10	10	11	8	5	2	3	2	1		1		1							77
525			1			1	2	4	4	3	5	3	2	1	2	1			1										30
575							3	2	3	2	1			2	1	1													15
625	1					1				2																			4
675								1																					1
725					1				1																				2
Total	1	0	1	1	2	6	12	22	24	30	32	28	22	22	20	14	8	7	8	6	2	4	2	2	0	0	0	1	277

TABLE 20.—COMPARISON OF MEAN MILK YIELDS AS OBSERVED, AND AS CALCULATED FROM LOGARITHMIC AND CONSTANT ENERGY CURVES

Holstein-Friesian A. R. O. Seven-Day Records
First 277 Original Entries, Age-Corrected

(See Table 19)

	f	M_o	M_l	D	M_e	D
2.145	1	625.0*	.....	......	.....	......
2.345	1	525.0	534.4	− 9.4	526.5	− 1.5
2.445	1	475.0	521.3	−46.3	516.2	−41.2
2.545	2	600.0	509.4	+90.6	506.2	+93.8
2.645	6	491.7	498.5	− 6.8	496.7	− 5.0
2.745	12	487.5	488.4	− 0.9	487.5	0.0
2.845	22	477.3	478.9	− 1.6	478.6	− 1.3
2.945	24	487.5	470.1	+17.4	470.1	+17.4
3.045	30	460.0	461.8	− 1.8	461.9	− 1.9
3.145	32	450.0	454.1	− 4.1	453.9	− 3.9
3.245	28	442.9	446.9	− 4.0	446.2	− 3.3
3.345	22	436.4	440.0	− 3.6	438.8	− 2.4
3.445	22	434.1	433.5	+ 0.6	431.6	+ 2.5
3.545	20	415.0	427.4	−12.4	424.6	− 9.6
3.645	14	421.4	421.6	− 0.2	417.9	+ 3.5
3.745	8	412.5	416.1	− 3.6	411.4	+ 1.1
3.845	7	396.4	410.8	−14.4	405.1	− 8.7
3.945	8	412.5	405.8	+ 6.7	398.9	+13.6
4.045	6	391.7	401.1	− 9.4	393.0	− 1.3
4.145	2	425.0	396.5	+28.5	387.2	+37.8
4.245	4	425.0	392.2	+32.8	381.6	+43.4
4.345	2	350.0	388.1	−38.1	376.2	−26.2
4.445	2	400.0	384.1	+15.9	370.9	+29.1
4.845	1	425.0*	.....	......	.....	......
Mean error			15.9		15.8	
Root mean-square error			26.1		27.0	

*Excluded in fitting curves and computing errors.

TABLE 21.—MATHEMATICAL CONSTANTS OF THE CORRELATIONS BETWEEN PERCENTAGE FAT CONTENT AND YIELD OF MILK

Table No.	Records	Length of records	Number of records	Milk Yield: Mean	Milk Yield: Standard deviation	Milk Yield: Coefficient of variation	Fat Percentage: Mean	Fat Percentage: Standard deviation	Fat Percentage: Coefficient of variation	Coefficient of correlation
				pounds	*pounds*	*percent*	*pounds*	*pounds*	*percent*	
1	Holstein C.T.A.	Yearly...	2773	7218.2 ± 29.33	2290.1 ± 20.74	31.73 ± .313	3.538 ± .0049	.3810 ± .0035	10.76 ± .0975	−.1979 ± .0123
14	Holstein A.R. Vols. 24-30...	Long time	5266	14886.7 ± 38.32	4123.3 ± 27.10	27.69 ± .196	3.413 ± .0029	.3095 ± .0020	9.08 ± .0596	−.1138 ± .0092
16	Holstein A.R. Vols. 18-24...	Long time	1003	14229.3 ± 24.60	3741.3 ± 17.39	26.29 ± .438	3.445 ± .0062	.2919 ± .0044	8.46 ± .1270	−.1358 ± .0209
18	Holstein A.R.O.	7-day....	277	365.6 ± 3.71	91.5 ± 2.62	25.05 ± .756	3.280 ± .0166	.4099 ± .0117	12.49 ± .3630	−.1326 ± .0398
19	HolsteinA.R.O., age-corrected..	7-day....	277	447.6 ± 2.86	70.50 ± 2.02	15.75 ± .463	3.280 ± .0166	.4099 ± .0117	12.49 ± .3630	−.4086 ± .0338
3	Jersey C.T.A...	Yearly...	970	5364.2 ± 28.40	1311.0 ± 20.09	24.42 ± .396	5.139 ± .0118	.5480 ± .0084	10.66 ± .1630	−.2120 ± .0207
5	Jersey R.M....	Long time	8038	8005.2 ± 15.86	2108.6 ± 11.22	26.34 ± .149	5.413 ± .0041	.5383 ± .0029	9.95 ± .0529	−.2949 ± .0069
7	Jersey R.M....	7-day....	367	288.6 ± 1.82	51.6 ± 1.29	17.88 ± .459	5.086 ± .0227	.6460 ± .0161	12.69 ± .3210	−.5059 ± .0262
*	Guernsey A.R..	Long time	3564	8644.4 ± 23.70	2095.4 ± 16.70	24.24 ± .200	5.033 ± .0050	.4710 ± .0040	9.35 ± .0800	−.2960 ± .0270
**	Ayrshire A.R...	Long time	1091	9417.1 ± 41.70	2044.4 ± 29.50	21.71 ± .330	3.933 ± .0060	.3180 ± .0050	8.08 ± .1200	−.1380 ± .0200
11	Brown Swiss R.P..........	Long time	311	11287.3 ± 107.64	2815.0 ± 76.13	24.94 ± .715	4.002 ± .0115	.2995 ± .0081	7.48 ± .2020	−.2910 ± .0350
12	Brown Swiss R.P., age-corrected........	Long time	311	12628.1 ± 108.46	2836.5 ± 76.71	22.47 ± .638	4.002 ± .0115	.2995 ± .0081	7.48 ± .2020	−.2516 ± .0358

*From Roberts,[6] Table X. **From Roberts,[6] Table XX.

TABLE 22.—EQUATIONS TO FITTED CURVES EXPRESSING RELATION BETWEEN PERCENTAGE FAT CONTENT AND YIELD OF MILK

(Y=yield of milk, in pounds; t=percentage fat content of milk)

Reference to—			Logarithmic $Y = a + bx + c \log_{10} (x + \alpha)$ $(x = 10t - \beta)$					Constant energy $Y=\frac{a}{2.66+t}$	Constant fat $Y=\frac{a}{t}$
Fig. No.	Table No.	Records	a	b	c	α	β	a**	a
1	2	Holstein C.T.A. (yearly).........	10 732.0	− 38.95	− 2 718.2	5.5	24.45	43 548	24 909
8	15	Holstein-Friesian A.R. (Vols. 24-30) (yearly).........	22 026.3	+165.50	− 8 317.2	2.5	25.45	91 309	50 532
9	17	Holstein-Friesian A.R. (Vols. 18-24) (yearly).........	16 645.7	− 88.50	− 1 539.9	.5	25.45	87 559	48 457
10	20	Holstein-Friesian A.R.O. (7-day), age-corrected....	1 017.5	+ 1.87	− 442.3	11.5	22.45	2 635	1 460
2	4	Jersey C.T.A. (yearly).........	6 952.4	− 36.80	− 907.9	1.5	36.45	41 432	27 087
3	6	Jersey R.M. (yearly).........	12 117.5	− 49.10	− 2 530.2	3.5	37.45	64 659	43 142
4	8	Jersey R.M. (7-day)*.........	430.9	− 1.71	− 89.9	4.5	35.45	2 227	1 457
5	9	Guernsey A.R. (yearly).........	18 869.2	+ 12.70	− 7 482.2	9.5	36.00	66 100	43 177
6	10	Ayrshire A.R. (yearly).........	13 039.9	+ 28.00	− 3 736.8	1.5	29.00	62 325	36 091
7	13	Brown Swiss R.P. (yearly), age-corrected........	31 457.9	+249.10	−17 084.4	9.5	33.45	84 776	51 317

*The records of cows under four years of age were excluded in making up this correlation table because the entrance requirement (12 pounds of fat, regardless of age) is relatively high for young cows, and it was thought this might tend unduly to exclude cows of lower fat percentage under the age of four years.

**This constant should afford an equitable physiological basis of comparison for the production of the groups and breeds. The data have not been treated with this object in mind, however, and there are three things which are not always comparable, viz.: length of record, age, and period when the records were made. Judging by the cow testing association records, the values of a indicate that the Jersey is 95 percent (41432/43548) as high a producer as the Holstein. By the advanced registry records, she is only 71 percent (64659/91309) as high. Looked at in another way, the Jersey advanced registry records are more representative of what the breed does under commercial conditions than is the case with the Holstein. Comparing all the breeds, the rank in descending order of production is: Holstein, Brown Swiss, Guernsey, Jersey, and Ayrshire. This suggests that size, rather than efficiency of the mammary apparatus, may be the cause of that rank.

TABLE 23.—ERRORS OF FITTED CURVES

(The mean error is given first; the root mean-square error, second)

Reference to—		Type of Curve		
Table No.	Records	Logarithmic	Constant energy	Constant fat
2	Holstein C•T.A.	248 397	249 391	555 775
15	Holstein-Friesian A.R. (V. 24-30)	348 401	465 566	1001 1203
17	Holstein-Friesian A.R. (V. 18-24)	524 720	536 746	981 1355
20	Holstein-Friesian A.R. O. (7-day)	15.9 26.1	15.8 27.0	37.4 49.6
4	Jersey C. T. A.	323 421	329 428	426 551
6	Jersey R.M. (yearly)	439 770	457 777	498 867
8	Jersey R.M. (7-day)	10.4 13.0	10.4 13.1	16.1 21.3
9	Guernsey A.R.	331 546	349 569	386 673
10	Ayrshire A.R.	147 181	229 317	437 521
13	Brown Swiss R.P.	671 947	709 980	772 1101

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(*The headings in capitals are subjects of entire bulletins*)