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RELATION OF SOLIDS IN MILK TO FAT AND SPECIFIC GRAVITY OF THE MILK

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The use of formulas for computing the percentage of total solids or of solids-not-fat in milk has been common both in the United States and in Europe for many years. These formulas are based upon the specific gravity and the percentage of fat in the milk.

The idea that a relation exists between specific gravity, fat, and solids in milk seems to have occurred first to Behrend and Morgen.[1] Clausnizer and Mayer,[2] and Hehner[3] published formulas in the attempt to show this relation. These formulas were based on inaccurate data and have been abandoned.

Fleischmann and Morgen[4] published a formula which was later corrected by Fleischmann.[5] This formula is $T = .2665 \frac{100\ S - 100}{S} + 1.2\ F$, in which T = percentage of total solids, S = specific gravity of milk at 15° C., and F = percentage of fat in milk.

Richmond[6] has developed the formula $T = .262 \frac{G}{D} + 1.2\ F$, in which G = Quevenne lactometer reading, D = specific gravity, and F = percentage of fat, and has found that the simpler formula $T = \frac{G}{4} + \frac{6}{5} F + .14$ gives results which correspond very closely with it if the specific gravity is between 1.020 and 1.036. This formula is commonly used in England.

Babcock[7] published the formula total solids $= \frac{L + .7\ f}{3.8} + f$, in which L = Quevenne lactometer reading and f = percentage of fat. Babcock[8] later stated this relation as total solids $= \frac{L}{4} + 1.2\ F$. With the addition of .14 this is the same as Richmond's formula.

This formula of Babcock is most commonly used in the United States and for that reason was selected for making all computations of total solids which are recorded in this bulletin. Babcock's formula for solids-not-fat as used at the present time is S.N.F. $= \frac{L}{4} + .2\ F + .14$.

The purpose of this investigation was to show, by means of a statistical analysis, the relation existing between the percentages of total solids as determined by weight (A.O.A.C. method) and the corresponding percentages of total solids as computed by the formula $T.S. = \frac{L}{4} + 1.2\ F$ when applied to a large number of milk samples; also the effect upon the results of the size of the lot of milk sampled. The relation between the percentages of solids-not-fat as determined by difference and the corresponding percentages of solids-not-fat as computed by the formula $S.N.F. = \frac{L}{4} + .2\ F$ also was studied.

PLAN OF INVESTIGATION

Source of Milk Samples

This investigation involved a statistical study of the percentages of total solids and of solids-not-fat by weight, and the corresponding percentages of total solids and of solids-not-fat by formula, as determined from three different groups of milk samples, namely:

(1) *1158 Samples from Individual Cows.*—Most of these samples were composites of the milk produced during three-day periods selected at regular intervals thruout the lactations of the cows. A few of these samples represent only one milking. The cows used were all in the dairy herds at the University of Illinois and represent the Ayrshire, Guernsey, Holstein and Jersey breeds, and Guernsey-Holstein crosses.

(2) *134 Random Samples of Mixed Milk.*—These samples were taken from cans of milk delivered at milk plants, from weigh-tanks, and from storage and pasteurizing vats. No record was kept of the sizes of the lots of milk from which the samples were taken; they varied, however, from less than 10 gallons to 150 gallons or more.

(3) *40 Samples Taken from Large Lots of Milk.*—These samples were all taken from storage or pasteurizing vats in plants which handle milk in large quantities. Records were kept of the sizes of these lots, which varied from 135 gallons to 3,000 gallons.

In every case care was taken to make certain that the sample obtained was representative of the lot of milk from which it was secured.

Chemical Analysis

All samples were put into glass jars and securely sealed to prevent evaporation of the water from the samples. The samples were also preserved with formaldehyde in approximately the quantity recommended by Palmer.[9]

Determinations were made of specific gravity, percentage of fat, and percentage of total solids.

The specific gravity was obtained at 15.5° C. with a chainomatic specific-gravity balance. At least two adjustments and readings of the vernier scale on the balance were made in determining the specific gravity of each sample.

The percentage of fat was determined by the Roese-Gottlieb method, about 5 grams of milk being weighed into a Röhrig tube. The volumes of reagents used were reduced from those given in the methods of analysis[10] of the A.O.A.C. to correspond to the weight of milk used. Duplicate determinations were made in each case and the average of the duplicates reported as the percentage determined.

The percentage of total solids was determined by weighing 2 to 3 grams of the sample into a weighed flat bottom lead dish and heating to constant weight at the temperature of boiling water. Duplicate determinations were made and the average of the duplicates reported as the percentage determined.

The percentage of solids-not-fat was determined by subtracting the percentage of fat by weight from the percentage of total solids by weight. To avoid confusion, this percentage of solids-not-fat will be spoken of as percentage of solids-not-fat by weight.

Statistical Analysis

The percentage of total solids was computed from the specific gravity and percentage fat content of each sample according to the formula $T.S. = \frac{L}{4} + 1.2\ F$. The percentage of total solids by weight for each sample was subtracted algebraically from its corresponding percentage of total solids by formula. In this way 1,158 differences were determined for the first group of samples, 134 differences for the second group, and 40 differences for the third group.[a]

The mean and the standard deviation of the differences for each group of samples were determined. A comparison was then made between the means of the differences of the respective groups of samples and likewise between the standard deviations.

The mean of the differences of each group of samples ± 2.14[b] times the corresponding standard deviation of the differences gave limits such that the chances are 30:1 that any single difference determined by the above methods for that group of samples will fall within them.

RESULTS

The results from the chemical and statistical analysis of the three groups of samples include the percentage of fat, the percentage of total

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[a]The percentage of solids-not-fat was computed from the specific gravity and the percentage of fat of each sample according to the formula $S.N.F. = \frac{L}{4} + .2\ F$. The percentage of solids-not-fat by weight for each sample was subtracted algebraically from its corresponding percentage of solids-not-fat by formula. These differences are identical with the differences between the total solids by weight and by formula for the same samples.

[b]Determined from the equation given on page xviii of Karl Pearson's Tables for Statisticians.

solids by weight, the percentage of solids-not-fat by weight, the percentage of total solids by formula, the percentage of solids-not-fat by formula, the specific gravity at 15.5° C., and also the algebraic differences determined by subtracting the percentages of total solids by weight from their corresponding percentages of total solids by formula. The differences for the solids-not-fat are identical with the corresponding differences for the total solids. These differences are shown graphically

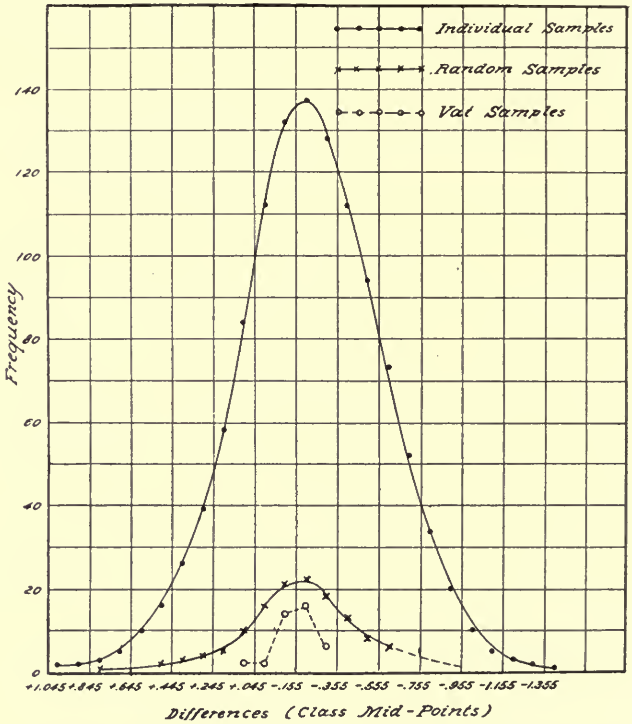

FIG. 1.—FREQUENCY DISTRIBUTION OF THE DIFFERENCES BETWEEN TOTAL SOLIDS AND SOLIDS-NOT-FAT FOR THREE GROUPS OF SAMPLES: INDIVIDUAL, RANDOM, AND VAT

in Fig. 1. Table 1 includes the mean, the standard deviation, and the limits at odds of 30:1 of the differences determined for each group of samples.

The mean of the differences[a] for the first group of samples (samples of milk from individual cows) is — .173 percent. In other words, the percent-

[a]Differences between the percentages of total solids by weight and the corresponding percentages of total solids by the formula $T.S. = \frac{L}{4} + 1.2\,F.$

ages of total solids determined by weight for samples of milk from individual cows are on the average .173 percent greater than the percentages of total solids computed for the same samples by the formula $T.S. = \frac{L}{4} + 1.2\ F$. The mean of the differences for the second group of samples (random samples from milk of more than one cow) is — .105 percent. The mean of the differences for the third group of samples (samples from large vats of milk) is also — .105 percent. Hence, the percentages of total solids determined by weight in milk from two or more cows and in milk from many cows, are on the average .105 percent greater than their corresponding percentages of total solids computed by the formula

$$T.S. = \frac{L}{4} + 1.2\ F.$$

TABLE 1.—MEANS AND STANDARD DEVIATIONS OF DIFFERENCES BETWEEN SOLIDS BY WEIGHT AND SOLIDS BY FORMULA

Samples	Number of differences	Mean percentage	Standard deviation, percent	Limits at odds of 30:1 percent
Individual......	1158	—.173±.0067	.340±.0048	—.173±.727 or +.554 and —.900
Random........	134	—.105±.0014	.242±.0010	—.105±.519 or +.414 and —.624
Vat............	40	—.105±.0011	.100±.0008	—.105±.214 or +.109 and —.319

Altho the means of the differences for the three types of samples differ very little from each other, it does not follow that the formula is equally as accurate in computing the percentages of total solids within them, as will be shown later on.

The standard deviation of the differences for the three groups of samples are .340, .242, and .100 percent respectively. *The standard deviation is a measure of variability; hence, as the number of cows contributing to the milk samples is increased, the variability in the differences between the percentages of total solids by weight and their corresponding percentages of total solids by formula, is markedly decreased.* This relation is illustrated very clearly in Fig. 1, wherein the differences for each group of samples are shown graphically. In Fig. 1 it will be found that the differences for the first group of samples range from +1.045 to —1.355 percent, for the second group of samples from +.845 to —.555 percent, and for the third group of samples from +.145 to —.255 percent.

Putting this variability into practical terms we have in Table 1 limits such that the odds, or chances, are 30:1 that any single difference determined by the above methods for each group of samples will fall within them. The limits for the first group of samples are +.554 and —.900 percent; i. e., for samples from individual cows the chances are 30:1 that the percentages of total solids by the formula ($T.S. = \frac{L}{4} + 1.2\ F$) will lie within +.554 and —.900 percent of their corresponding percentages of total solids by weight.

The limits at odds of 30:1 for the second group of samples are +.414 and —.624 percent, and for the third group of samples +.109 and —.319 percent. Hence, as the number of cows contributing to the samples of milk are increased the limits are decreased within which the chances are 30:1 that the percentage of total solids computed by the above formula will deviate from its corresponding percentage of total solids by weight.

If the formula ($T.S. = \frac{L}{4} + 1.2\ F$) be corrected for samples of milk from individual cows by adding the mean of the above differences for these samples, it will read $T.S. = \frac{L}{4} + 1.2\ F + .173$. In accordance with this formula the limits at odds of 30:1 will also be corrected to ±.727 percent. The limits of ±.727 percent were determined by adding .173 percent to the above limits of +.554 and —.900 percent. In like manner the formula $T.S. = \frac{L}{4} + 1.2\ F$ will be corrected for the second and third groups of samples to read $T.S. = \frac{L}{4} + 1.2\ F + .105$. Altho the corrected formulas for the second and third groups of samples are the same, the limits at odds of 30:1 are much different and will be ±.519 percent and ±.214 percent respectively. Hence, it can readily be seen that the standard deviation or variability within the above differences determines the accuracy of the formula $T.S. = \frac{L}{4} + 1.2\ F$ in computing the percentages of total solids in milk from the respective sources. The mean of the above differences has a bearing on the accuracy of the formula when combined with the standard deviation, but without the latter it has very little meaning.

The probable errors of the means and the standard deviations of the differences as reported in Table 1 are all many times less than their constants. Hence, it may safely be assumed that the samples from which these constants were derived are representative of the general populations of samples of the respective types.

As the differences between the percentages of solids-not-fat by weight and of solids-not-fat by formula are identical with the corresponding differences for the total solids, the means, standard deviations, and limits at odds of 30:1 are the same.

The formula for solids-not-fat (S.N.F. $= \frac{L}{4} + .2$ F) corrected for samples of milk from individual cows will read S.N.F. $= \frac{L}{4} + .2$ F $+ .173$; for the second and third groups it will read S.N.F. $= \frac{L}{4} + .2$ F $+ .105$.

SUMMARY

The formula T.S. $= \frac{L}{4} + 1.2$ F $+ .173$ in computing the percentage of T.S. in milk from individual cows is accurate in so far as the chances are 30:1 that the percentages of total solids computed by it will lie within $\pm .727$ percent of their corresponding percentages of total solids by weight. In other words, it may be expected that, on the average, the percentages of total solids by the above formula for 30 samples out of every 31 samples will lie within $\pm .727$ percent of their corresponding percentages of total solids by weight. Likewise, the percentage of total solids by the above formula for one sample out of every 31 samples will lie without the range of $\pm .727$ percent of its corresponding percentage of total solids by weight.

The formula T.S. $= \frac{L}{4} + 1.2$ F $+ .105$ in computing the percentage of total solids in random samples of milk from two or more cows is accurate in so far as the chances are 30:1 that the percentages of total solids computed by it will lie within $\pm .519$ percent of their corresponding percentages of total solids by weight. Hence, it may be expected that on the average the percentages of total solids computed by the above formula for 30 samples out of every 31 samples will lie within the range of $\pm .519$ percent of their corresponding percentages of total solids by weight. Likewise, the percentage of total solids by the above formula for one sample out of every 31 samples will lie without the range of $\pm .519$ percent of its corresponding percentage of total solids by weight.

The formula T.S. $= \frac{L}{4} + 1.2$ F $+ .105$ may also be used in computing the percentage of total solids in milk from large vats and storage tanks, i. e., milk from many cows. In using this formula to compute the percentage of total solids in such milk, it may be expected that on the average the percentages of total solids computed by it in 30 samples out of every 31 samples will lie within $\pm .214$ percent of their corresponding percentages of total solids by weight. In like manner the percentage of total solids by the formula for one sample out of every 31 samples will lie without the range of $\pm .214$ percent.

The formula S.N.F. $= \frac{L}{4} + .2$ F $+ .173$ in computing the percentage of solids-not-fat in the milk from individual cows is accurate in so far as the chances are 30:1 that the percentages of solids-not-fat computed by it will lie within $\pm .727$ percent of their corresponding percentages of solids-not-fat by weight.

The formula $S.N.F. = \frac{L}{4} + .2\,F + .105$ may be used to compute the percentage of solids-not-fat in the mixed milk from two or more cows. For such samples, taken at random, the chances are 30:1 that the percentages of solids-not-fat computed by it will lie within $\pm .514$ percent of their corresponding percentages of solids-not-fat by weight. For samples taken from large vats and storage tanks, the chances are 30:1 that the computed percentages of solids-not-fat will lie within $\pm .214$ percent of their corresponding percentages of solids-not-fat by weight.

CONCLUSIONS

1. The accuracy of the formula $T.S. = \frac{L}{4} + 1.2\,F$ in computing the percentage of total solids in milk increases as the number of cows contributing to the milk is increased.

2. The formula $T.S. = \frac{L}{4} + 1.2\,F + .173$ may be used in computing the percentage of total solids in milk from individual cows, but the variability within the results is too great to make it of any practical use.

3. The formula $T.S. = \frac{L}{4} + 1.2\,F + .105$ when used to compute the percentage of total solids in milk from many cows gives results which very closely approximate those determined by direct chemical analysis, and in plants handling large quantities of milk may be used with relative satisfaction.

4. The accuracy of the formula $S.N.F. = \frac{L}{4} + .2\,F$ in computing the percentage of solids-not-fat in milk increases as the number of cows contributing to the milk is increased.

5. The formula $S.N.F. = \frac{L}{4} + .2\,F + .173$ may be used in computing the percentage of solids-not-fat in milk from individual cows, but the variability within the results is too great to make it of any practical use.

6. The formula $S.N.F. = \frac{L}{4} + .2\,F + .105$ may be used in computing the percentage of solids-not-fat in large lots of milk. The results obtained approximate closely enough to those obtained by chemical analysis to be of practical value.

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