RELATIVITY FOR ALL

HERBERT DINGLE

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WITH TWO DIAGRAMS

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PREFACE

AN apology is needed for the production of a popular work on Relativity after Einstein himself has undertaken such a task. As a matter of fact, our purpose here is somewhat different from Einstein's. This little book is written, not so much for those who wish to understand a physical theory from the point of view of a physicist, as for the large body of intelligent men and women who look on physics as one of many avenues into the secret of the Universe, and wish to know its windings in their relation to the larger field of human inquiry.

The dominant aim throughout the book has been to make the ideas definite and intelligible to the ordinary mind. All other considerations—strict philosophical phraseology, literary graces, conventional forms of presentation; everything, in fact, but truth—have been subordinated to this end. Between the Charybdis of inaccuracy and the Scylla of abstruseness, the course is narrow and the sea is rough. The vessel will hardly escape buffetings from either side. It is hoped, nevertheless, that in the present voyage a passage will be made without fatal mishap.

Those who wish to pursue the subject more deeply, from either the philosophical or the scientific standpoint, are recommended to the works of Professor A. N. Whitehead, F.R.S. The author is glad to acknowledge his deep indebtedness to Professor Whitehead for invaluable help and unwearying kindness in unveiling the mysteries of a difficult subject.

H. D.

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PART I

THE FOUNDATIONS OF SCIENCE

CHAPTER I

HOW THE THEORY AROSE

"Space is thought's, and the wonders thereof, and the secret of space;
Is thought not more than the thunders and lightnings? shall thought give place?
Time, father of life, and more great than the life it begat and began,
Earth's keeper and heaven's and their fate, lives, thinks and hath substance in man."

WHEN Swinburne wrote these words, he was thinking what a wonderful being he was. That they would ever come to be a poetical expression of cold, scientific ideas about matter, time, and space, was probably the thought farthest from his inaccessible mind. Yet so it is. The new doctrine of Relativity entails a complete uprooting of the conceptions that have formerly been held to lie inviolable at the foundations of thought and experience. The theory is not merely a metaphysical speculation. It has arisen in order to explain certain facts of observation, which seem to point to it as the most probable statement of the nature of the Universe which we perceive.

Let us think for a moment of the way in which we are accustomed—tacitly, almost subconsciously—to regard the physical world. We think of it as a number of pieces of matter. These pieces of matter exist in space. We do not generally take the trouble to define to ourselves exactly what we mean by "space," but we understand one another quite well when we refer to it in conversation. It is a sort of receptacle, without limit in any direction, in which the material of the world exists and moves about. When we say a certain object is "there," we have a clear idea of what we mean, and we feel confident that, provided the object does not move, it will always be "there," no matter what we do ourselves. If we could take the wings of the morning, and dwell in the uttermost parts of the sea, the object would still be "there." Then we have also an idea of time. We do not define this either, but we know what it means. When something has happened, it belongs to the past. Nothing can ever bring the same happening into the present or the future again. It happened at some definite time, and every person in the Universe who observed it would agree with every other person as to what that time was, supposing every one had an accurate clock.

These three "things"—matter, space, and time—are the three independent, immovable foundation-stones of the World, as we are accustomed to regard it, and Science has hitherto adopted them as the cnly possible data in terms of which to express its discoveries. For instance, the law of gravitation expresses the way in which matter will move near other matter, *i.e.* it describes how the position of *matter* in *space* changes as *time* advances. All other physical laws have been essentially of the same kind.

But recently, scientists have had reason to question whether space, time, and matter are really the absolute and fundamental things we have supposed. The doubt arises in the following way. As the result of a considerable accumulation of experience, it has been impressed upon physicists that space is not empty, but is filled, in every nook and cranny of its infinite extent, with a kind of invisible, intangible super-matter, which has been called the "ether." The first—and still one of the most convincing—of the indications of this substance came from the study of the propagation of light through space. Certain remarkable laboratory experiments seemed to assert that light could travel from a luminous body to the eye in no other form than that of a train of waves, like the ripples spreading out in a pool of water when a stone is thrown into it. The facts pointed unanimously in this direction, and their combined force was almost irresistible. But waves are unthinkable without some medium in which they exist. There must obviously be something through which light waves travel: what is that something? It is not the air, because light reaches us from stars millions of miles away, and there is, to the best of our knowledge, no air or matter of any kind reaching all the way from the stars to us. It must be something of whose existence we have not previously been aware; something that fills all space, for light comes to us from all directions and from unimaginable distances. It must, moreover, penetrate the pores and secret places of matter itself, for does not light pass through some bodies, which are said to be "transparent"? Scientists, then, were led to the idea of a space filled with this infinite, all-permeating ether.

Now there is nothing in all this to challenge our common sense. The conception of an omnipresent ether offers no difficulties to the imagination. Space might as well be full as empty, so far as mere possibility is concerned. Nevertheless, we should feel more satisfied on the matter if we had some direct sign of the ether's existence. An experiment giving immediate evidence of it would be more convincing than its appearance as the last link in a chain of reasoning. This was recognized by physicists, and many attempts were made to betray the ether into a declaration of its reality. One of the most promising of these was the search for the velocity of the earth as it travels through the ether. Since the ether filled all space, it had to be regarded as being at rest as a whole. It could not move bodily because it was infinite and there was nothing for it to move into. Consequently, the velocity of the earth through the ether could be looked upon as its "absolute" velocity—something more fundamental than its velocity of revolution round the Sun, which ignores any possible motion of the Sun itself.

To understand the most famous of all experiments made to measure this absolute velocity, we must picture the earth swimming through the ether at some speed which we are to find out. Suppose that, on the earth's surface, and travelling with it, there are two objects—a lamp and a mirror (A), represented in Fig. 1. Suppose also that these objects lie in the line of absolute motion of the earth—whatever that may be—the mirror in advance of the lamp. Let the lamp be uncovered for an instant, so that it sends a beam towards the mirror (A). Now light travels with a definite velocity (186,000 miles a second). It moves at this speed through the ether towards the mirror (A).

But the mirror (A) is running away from the beam, since it is fixed to the earth, and the earth is moving through the ether. Consequently, the light should take longer to reach the mirror (A) than it would if the earth were not moving. On reaching the mirror (A), the light is reflected back to the lamp. But now the lamp is moving to meet it, so that

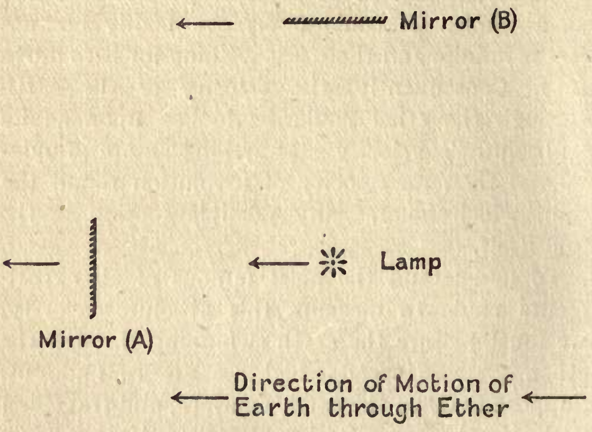

FIG. 1.

the light will make the return journey more quickly than it would have done if the earth had been at rest. It is a very simple matter to calculate what the time should be for the total journey to and fro, in terms of the unknown absolute velocity of the earth. But now suppose that, at the same time as the beam of light left the lamp, another beam left the same point at right angles to the first, towards the mirror (B). This beam would move across the line of motion of the earth, and the time it would take to perform its complete journey to the mirror (B) and back, can be calculated also. On making the calculations, we find that the second beam should return to the lamp before the first, and we can tell exactly how much sooner it should arrive—in terms, of course, of the unknown velocity. The interval should vary throughout the year, owing to the change in the direction and rate of motion of the earth.

Now an experiment on these lines—the famous Michelson-Morley experiment—was actually performed in 1887. The apparatus used was so delicate that it was capable of detecting a far smaller quantity than that which it was expected to measure. Every one awaited the result with confidence. But when the experiment was made, it was found that the two beams arrived back *at the same time.* The apparatus was turned round, so that the mirrors were in different positions relative to the lamp; the experiment was repeated at different times of the year; but always the result was the same—the two beams took precisely the same time for their respective journeys.

Now it must be recognized at once that this was a most extraordinary thing. Here was an experiment, performed with every care and apparently with full understanding of what was being done, which completely failed to give the result that common sense would have thought inevitable. For what the experiment seems to imply is this. We know that, if a bird flies from one end of a train to the other, he will complete the journey sooner if the train is moving towards him than he would do if it were at rest. The experiment suggests that, if he only moves as quickly as light, he will appear to the engine-driver to reach the end in the same time, no matter whether the train is at rest, or moving towards him, or moving away from him. It seems impossible, but experience shows it to be true.

If any explanation is to be given, therefore, it must necessarily involve something revolutionary. Various suggestions were offered, but, in the light of future investigations at any rate, none of them was so satisfactory or far-reaching as the most revolutionary of all—the principle of relativity. Let us ask ourselves why common sense says that the bird cannot reach the end of the train in the same time, when the train is moving, as he does when it is at rest. We reply that he has to travel different distances in space in the two cases. If, during his flight, the train has moved, its far end, when he reaches it, will be at a point in space different from that which it occupied at the beginning. The times taken by the two journeys will therefore be different. But, in saying this, we are assuming that "space" and "time" mean the same things for the engine-driver at rest as they do for the engine-driver in motion. What if they are different ? In that case, of course, we shall not know what to expect. If what one man calls an hour, another calls a minute, and what the first pronounces a yard, the second asserts to be a mile—*and if there is no possible criterion for testing their statements, so that both are equally right or equally wrong*—then it will not be surprising if results are obtained which would otherwise be considered impossible. This is, in essence, just what the principle of relativity says. It declares that the conceptions of space and time —and, as will subsequently appear, of matter also—are not absolute and independent, but are relative to the observer. What do we mean by this ? We will try to explain it in the next chapter.

CHAPTER II

SPACE, TIME, AND MATTER

SUPPOSE a being, endowed with full human intelligence, but without any experience or knowledge of the world, were suddenly created and placed, say, on Hampstead Heath: what would he perceive? The answer we should naturally give to this question is contained in the first chapter—he would perceive material things in space and time. The answer of the relativist, however, is different. According to him, the man would perceive a number of happenings, occurrences, events. Their interpretation as material objects in space and time would come later, and would be the result of his intelligent ordering of the events among themselves.

Let us take an example. Suppose our visitor sees a wasp alight on a flower. That is an event. Next, suppose the wasp alights on his hand. That is another event. We have here, then, two events, and to the man they would at first be merely two events and nothing more. But now, suppose he begins to use his intelligence, and tries to impose some order or arrangement on the circumstances in which he finds himself. He notices that there is something common to the two events, and also to a number of intermediate events, with which we need not concern ourselves. He has an impression of an "object" with black and yellow bands, which characterizes the whole series of events from the wasp on the flower to the wasp on the hand. This "character" of the events he calls "matter," and the particular example with which we are dealing, a "wasp." He has now the first of the three entities which we supposed were his original perceptions—matter.

But that is not enough. If he confines himself to what is common to the two events, he will not be able to distinguish them, one from the other. He must construct some other relation between them. He does this by saying that they are "in different places": the flower is in one "place," and the hand in another. In this way he forms an idea of place, and by extending the same relation to other events which he perceives, he becomes conscious of "infinite space." Matter, space—two types of relation between events—have arisen as conceptions derived from a common source—the events themselves.

Are these conceptions sufficient to enable our observer to think clearly and to comprehend the world around him? Not quite: matter and space will not relate all the events which he perceives. Consider a third event: suppose he feels a stinging sensation in his hand. How can he relate this to the second of the events we have already considered—the arrival of the wasp on his hand? The space relations are the same—if we make the legitimate assumption that there is no movement between the two events. The material relations are the same—the wasp and the hand. He must find a third type of relation. He therefore says that one of the events occurs "before" the other. By generalizing this relation, he forms the conception of "time."

Matter, space, and time, then, according to the relativist, are types of relation between events. Together they appear to be capable of relating the whole of inanimate Nature in a consistent and orderly way. Our visitor employs them for purposes of thought; he hands them down to his successors, generation after generation, until, ultimately, they come to be regarded as the fundamental perceptions of the human mind, and the poor event, the legitimate father of them all, sinks to the rank of a dependent.

This idea of the derivative character of matter, space, and time lies at the heart of the modern principle of relativity. It deserves particular emphasis, for, if it is once firmly grasped, the greater part of the difficulty of the subject disappears. It is the event that is the immediate entity of perception; Nature is the sum-total of events, and every instrument of thought that our minds employ can be traced back to its ultimate origin in events. Two observers of Nature see, not necessarily the same matter, but the same events, because events finally constitute the external physical world. What about the spatial, temporal, and material relations the observers impose on the events: will they be the same? Evidently it is not necessary that they should be. We have no right to say, without experimental test, that a man on the Earth and a man on Mars, say, who are moving relatively to one another, will both declare Regent Street to be half a mile long. What they may both be immediately aware of are the two events which are the existence of the two ends of the street during their perception of them. If one man relates them spatially by saying that they are half a mile apart, there is no fundamental necessity, so far as we know, for the other man to do the same. It is essentially a matter for experiment.

Let us illustrate this point, which is of basic importance, by an example—which, however, must not be pressed too close. Consider two events: first, a young man sees a young maiden; second, he shows signs of agitation. Consider, further, two observers of these events—the young man himself and another young maiden. Each of them relates the events in a certain way. The young man calls his relation, "love"; the second young maiden—supposing she is honest with her-self—calls her relation "jealousy." Here, then, we have a type of relation between events which we definitely recognize as relative. Why is this so? We do not know: it is a complete mystery. But we certainly do know that the events are related differently by different persons. May not, then, the space and time relations also be relative? "Ah!" you exclaim, "but the two observers in your example are in different circumstances. They have different predispositions, different histories, different emotional states. Consequently, their emotional relations between events will inevitably be different. But space and time are independent of our predispositions, our histories, our emotional states. They are on an entirely different footing." That is quite true: space and time do not change with our emotions. But it does not follow that they do not change with anything. Emotions are relative because they depend on our emotional state. Might not space and time depend on our spatio-temporal state? Might they not be modified by motion, for example, *i.e.*, by a change of our position in space as our position in time advances? It seems to be possible, at any rate. If I am moving relatively to you, it does not seem to be imperative that my spatial and my temporal relations, between events that are observed by both of us, shall be the same as yours. "But," you reply, "it is idle to talk of what seems to be possible.

Is it not a fact that there is no such difference between us? If my watch does not keep mean solar time when I am on an express train, have I not legitimate ground of complaint against my watchmaker? If the road becomes longer or shorter when I am travelling along it on a bus, shall not my habits justly be open to suspicion? You may be right with your possibilities, but experience shows that they are not actual." But then, after all, experience has its limitations. Perhaps, in a journey from London to Manchester, your watch keeps "perfect" time, so far as you can judge: it may yet have varied by an amount too small for you to detect. If you could travel at the same rate for 100,000 years, or if you could move at a speed of 100,000 miles a second, might not your extended experience show that the former conclusion was too hasty? Experience is certainly the final judge in the case before us, but it gives a verdict strictly according to the facts in its possession. If we wish to get at the truth, we must elicit all the facts.

The question, then, comes down to this: Can we make an experiment that will decide definitely whether space and time are different for observers in relative motion? Such an experiment is not inconceivable, but it would be one of colossal difficulty. At present, all we can hope to do is to see if there are any facts which receive a simple explanation if we assume the relativity of time and space, but which can be interpreted only with difficulty, or not at all, if those relations are absolute. Now facts of this kind are presented to us in the Michelson-Morley experiment, and in other attempts to observe the drift of matter through ether. They are to be found also in certain astronomical observations, to which we shall refer later. The theory of relativity, in fact, has passed with honours every test it has so far been found possible to apply. It is only the absence of direct experimental confirmation that prevents it from being recognized as a proven law of the Universe.

We shall consider the precise nature of the relativity in the next chapter, but it will be useful to state at once the kind of effect the relativist requires. The table opposite shows the loss of a watch during one day, and the shortening of a 1-foot rule in the direction of motion, as they would appear to an observer moving relatively to them at different speeds, who tests them by standard instruments moving with him.[1]

[1] As we have said, the principle from which these results are obtained will be stated in the next chapter. The reader, however, may, at this point, be somewhat curious as to its nature. We will, therefore, say in advance that the theory of relativity assumes that it is impossible for any observer ever to obtain experimental evidence of relative motion between matter and ether. This means (cf. the Michelson-Morley experiment) that all observers, whatever their state of relative motion, will

TABLE I.

Speed.	Example in Nature.	Loss of Watch per Day.	Shortening of 1-foot Rule.
60 miles per hour . .	Express train.	$\frac{1}{3,000,000,000}$ second.	$\frac{1}{20,000,000,000,000}$ inch.
67,000 miles per hour .	Earth's velocity round Sun.	$\frac{1}{2300}$ second.	$\frac{1}{17,000,000}$ inch.
700,000 miles per hour .	Velocity of star Arcturus relative to Earth.	$\frac{1}{20}$ second.	$\frac{1}{144,000}$ inch.
93,000 miles per second .	Half velocity of light.	3 hours 10 min.	$1\frac{3}{5}$ inch.
161,000 miles per second	—	12 hours.	6 inches.
186,000 miles per second	Velocity of light.	24 hours.	12 inches.

It is clear now why the relativity of space and time, if it is true, did not declare itself long ago. The effect is so small for speeds which we ordinarily use that it is quite impossible to detect it. High velocities, or observations extending over long periods, are necessary before its importance begins to appear.

We have still to consider the nature of matter. Is that relative also ? If two of the primary relations between events vary with the observer, it seems probable that the third will do so as well. As a matter of fact, the principle of relativity requires that there shall be a change in matter arising from motion. The quantity of matter in a body (not its "size," which is an attribute of space, but the actual amount of matter in it; what we call its "mass," and usually measure by weighing the body) should be different for different observers moving relatively to one another. Or,

obtain the same measure of the velocity of light. Such a result can only occur if the spaces and times used by the observers are related in one particular way, which, granted the principle, is readily determined by mathematical calculation. In this manner the foregoing table has been constructed. Thus, to a hypothetical observer on Arcturus, a beam of light would travel $\frac{1}{144000}$ inch per foot less than the same beam to an observer on the Earth. But the former observer would find that it did so in a period of time less than that measured by the Earth man by $\frac{1}{20}$th second per day. These numbers are such that both observers would obtain the same value for the velocity of the beam, and are the only ones that are consistent with the application of the same principle to all possible relative velocities.

in other words, if a body moved with gradually increasing velocity relative to us, we should find that its mass, supposing we could measure it, would grow continuously.[1]

Now this, again, seems at first to be contrary to experience. From the time of Lavoisier, in the eighteenth century, at least, the invariability of mass has been one of the cardinal doctrines of chemistry and physics, and experience has tended consistently to confirm it. Can it conceivably be an illusion ? Here, as before, we must understand clearly what experience shows. All that we can fairly deduce from the experiments which support the law of invariability of mass is that, under the particular conditions of those experiments, the mass of a body does not vary by any amount that we can measure. The case might be totally different if we had instruments of much greater

[1] The mass of a body, as we shall see in Chapter VI, is the resistance the body offers to change of velocity. If this resistance remained constant, then any force capable of increasing the velocity at all would, if it continued acting, go on increasing it indefinitely. From the point of view of relativity, however, this is impossible, because, as in the Michelson-Morley experiment, space and time adjust themselves so as to make the velocity of light the highest relative velocity possible between two bodies. (See also Chapter IV.) Consequently, resistance to change of velocity (*i.e.* mass) must increase as speed increases, in some way which will allow of its becoming infinite when the velocity of light is reached. The calculation of the necessary change is a simple piece of mathematics, and gives the results embodied in Table 2.

precision, or if we could compare the mass of a body at rest with its mass when moving at an extremely high speed. As with time and space, the demands of relativity are so humble at ordinary velocities that they might be granted in full without giving us the slightest suspicion that they are made. As the body moves faster and faster, the theory gathers confidence, and asks for more and more. Table 2 shows, for the same velocities as before, the increase in mass of a body containing 1 lb. of matter when it is at rest with respect to the observer.

TABLE 2.

Speed.	Increase of Mass.
60 miles per hour	$\frac{1}{250,000,000,000,000}$ lb.
67,000 miles per hour . . .	$\frac{1}{200,000,000}$ lb.
700,000 miles per hour . . .	$\frac{1}{1,730,000}$ lb.
93,000 miles per second . . .	$\frac{1}{6}$ lb.
161,000 miles per second . . .	1 lb.
186,000 miles per second . . .	Infinitely great.

Can we test this by experiment ? It is exceedingly difficult. We must rely again on indirect evidence. Nature has provided us with extremely small electrified particles which move with enormous speed. It has been found possible to measure their mass at different rates of motion. The result is that they actually do show a change of mass with speed, of just the amount required by the theory. Moreover, theoretical results deduced from the assumption of such a change when these bodies move inside material atoms, have been verified with almost incredible accuracy. The theory again has been successful in every test to which it has been subjected. But we must remark that these are highly special cases. The particles are electrically charged—they are believed, in fact, to be particles of electricity itself—and the effects observed can be explained equally well on the ordinary electro-magnetic theory, without reference to relativity. We cannot, therefore, put them forward as unequivocal evidence for relativity. But we can say that, so far as experiment has yet gone, there is nothing that has put the theory in the slightest difficulty, or necessitated any modification of its fundamental principles.

Let us see, then, just where we stand. We have said that, since the only element in Nature is the event, space, time, and matter may be relative to the observer. We have considered experiments which seem to make it very probable that they are relative. It remains for us now to investigate the principles governing the magnitude of their variation with change of speed. It is to this question that we turn in the next chapter.

CHAPTER III

THE "FOUR-DIMENSIONAL CONTINUUM"

IT has already been pointed out—and it is so important that it will bear repetition—that the principle of relativity is a deduction from facts of observation. It is emphatically not an arm-chair doctrine, proceeding from the inner recesses of the brain without reference to the results of experience. When the modern relativist says that space, time, and matter are different ideas for different observers, he does so because he believes that the interpretation of experimental facts which thereby becomes possible is the simplest and, on the whole, the most plausible that he can devise. Consequently, he is not content with the mere statement that these relations change with one's state of motion. He must say exactly by how much they change. As we implied in the first chapter, if a yard and an hour to one observer become a quite indiscriminate length and time to another, anything might happen. But, actually, "anything" does not happen: particular things happen—for example, the Michelson-Morley experiment. The magnitude of the yard and the hour to the second observer must be such as to explain those particular things, and cannot be anything else.

The numerical side of the theory of relativity is derived from the failure of all attempts to detect the relative motion of matter and ether. The relativist assumes that, in the nature of things, it is impossible to observe such a motion; in other words, that space and time change with motion in such a way as always to make the measure of the velocity of matter through ether equal to nothing. If this is granted, the calculation of the necessary change becomes a simple piece of mathematics.

We are endeavouring in this book to avoid the use of general mathematical formulæ. We shall, therefore, not give the theoretical expressions for the dependence of the time, space, and mass units on velocity. The reader who is interested in this side of the question may find them in any of the more technical expositions of the principle. We call attention rather to Tables 1 and 2 in Chapter II, from which most of the essential facts may be understood quite as well as from the general expressions. It will be observed at once that, until the relative velocity reaches a very high value, the change is almost infinitesimally small. As the velocity grows, however, the change increases at a more rapid rate, until, at the velocity of light, the state of affairs would appear to be inconceivable.

We shall return to this point in the next chapter. For the moment we will direct our attention to a very simple relation between the space and time used by any one observer—a relation which summarizes the quantitative requirements of the principle. It is not brought out in the tables, and, as it is of very great importance, we will deal with it at length.

All who have tried to get at the meaning of relativity have, at one time or another, come across the blessed phrase, "the four-dimensional continuum." What does it mean? Let us be quite clear, first of all, as to the meaning of the word "dimension." Suppose we have a room, enclosed by four square, vertical walls, and a square floor and ceiling. Suppose an electric lamp globe is suspended somewhere in its interior. How can we describe—to an architect, say—the exact position of the globe in the room? (We are not concerning ourselves now with the question of what we mean by "position," which occupied us in the last chapter. We are using the word in its ordinary, everyday sense, just as we might have done if we had had no reason to doubt the absolute nature of space.) We should tell him something about it if we said the globe was 7 feet above the ground. But that would not be enough. If that were all, the globe might be anywhere in a movable floor placed at that height. Suppose, then, we say, further, that it is 6 feet from the wall containing the door. That is better, but still he is not satisfied. There may be a whole line of objects on our imaginary movable floor which satisfy this condition, and the globe might be at any point in it. But let us now say that it is 5 feet from the adjacent wall containing the window. We have then determined its position completely. There is only one point in the line that is 5 feet from the wall containing the window: or, in other words, there is only one point in the room that is 7 feet above the ground, 6 feet from one specified wall, and 5 feet from another. We have definitely fixed the position of the globe by these measurements.

Now there are other ways in which we could have done this, but, in all of them, three independent measurements are necessary and sufficient. This is expressed, in technical language, by saying that space has three dimensions. Another aspect of the same property of space is embodied in the statement that three independent measurements are necessary to calculate the spatial volume occupied by a body—*e.g.* its length, breadth, and height.

Three independent measurements, we say, will define the position of a body in relation to a given structure (*e.g.* a room), and these three measurements may be made in different ways.

We must now point out that, in whatever way they are made, there is a certain relation between them that is always satisfied. Let the continuous lines in Fig. 2 indicate the floor and the two walls of the room with which we are dealing, and let the globe be suspended in its defined position, A. It

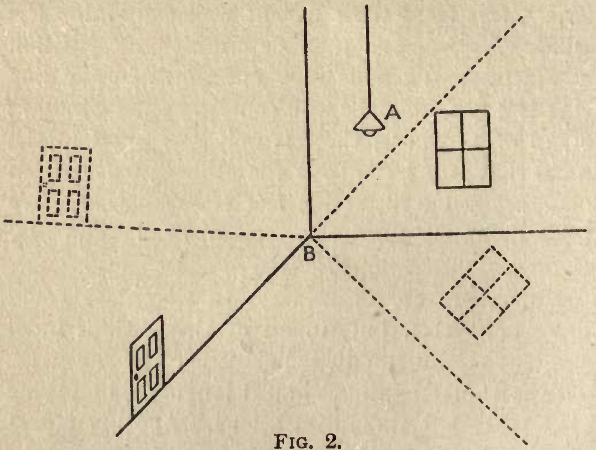

FIG. 2.

will be 7, 6, and 5 feet from the respective planes of reference. It is easily proved that the distance from the globe, A, to the corner, B, in which these three planes meet, is obtained by adding together the three quantities, 7^2, 6^2, 5^2, and finding the square root of the sum. Thus, $7^2+6^2+5^2=110$, and the square root of 110 is nearly $10\frac{1}{2}$, so that the globe is nearly $10\frac{1}{2}$ feet from the corner, B.

Now suppose—as the result of an earthquake, say—the room is twisted into the position shown by the dotted lines in the figure, in such a way that the points A and B remain in exactly the same positions as before. The distance AB will then be unaltered—about 10½ feet. But the distances of the lamp from the walls and floor will now be quite different. Yet—and this is the point we are trying to illustrate—provided the walls and floor still remain at right angles to one another, those distances must be such that the sum of their squares is equal to 110. Thus, if the lamp is 9 feet and 5 feet from the two walls, it must be 2 feet from the floor, for $9^2+5^2+2^2=110$. It cannot possibly be at any other distance, however the room is twisted within the restrictions we have mentioned.

Now we need not have had the earthquake to alter the position of the room. We could have imagined the walls and floor to be anywhere we liked, and defined the position of the globe relative to the point B by reference to the imaginary room. Or, without supposing the room twisted at all, we could have fixed the position by three other measurements of a different kind. The essential point is that, however we do it, there is a definite way of combining the measurements so as to give the number 110, and the measurements are bound to be related among themselves so that the combination will give this number.

To summarize, then, the statement that space has three dimensions implies two things: first, three independent measurements are necessary to fix one point relative to another; second, whatever three measurements we select for this purpose, there is a certain combination of them that is the same for all selections.

Bearing this in mind, we are now in a position to understand what a four-dimensional continuum is. We repeat that, in all that we have said about the three dimensions of space, we have been speaking in terms of absolute space. We have supposed that the distances 7, 6, 5 feet and the number 110 have the same value for all observers. From the point of view of relativity, as we know, this is only true as long as we are at rest in the room. If we begin to move, the distances change, and our measurement of time changes also. In view of the constancy of the number 110, whatever alterations took place in the separate measurements in our supposed absolute space, it is perhaps natural to inquire whether there is any way of combining our new space and new time measurements, whatever they may be, in such a way as to obtain the same result as that given by the corresponding combination of our old space and old time measurements. As a matter of fact, there is such a combination.

To illustrate this, we must choose two events instead of two points, A and B, for, to deal with time, we must introduce something containing the temporal quality. Actually, the perceived existence of the corner and the lamp are events, for perception itself takes time. It will simplify matters, however, if our data are more readily recognized as events. Let us consider the total interval (which we usually analyse into space interval and time interval) between the lighting of the lamp, A, and the arrival of a spider at the corner, B. Suppose, first, that we are at rest in the room. Then the square of the spatial distance between these two events is, as we have seen, $7^2+6^2+5^2$, *i.e.* 110. Suppose the time interval is 10 units;[1] so that its square is 100. The difference between the squares of the space and time intervals (which, we shall soon see, is an important quantity) will then be $7^2+6^2+5^2-10^2=110-100=10$. Now consider how the events would appear to us if we were moving relatively to the room. As we know, both the space interval and the time interval would be different. Suppose our speed to be such as to give us a space interval of 9 feet between the events. What would the time interval be? It is found that it must be just large enough for its square to differ from the square of the space interval by exactly the same amount (namely, 10) as in the former case. It must therefore be nearly $8\frac{1}{2}$ units,

[1] The magnitude of a time unit for this purpose is one thousand-millionth of a second.

for $9^2 - (8\frac{1}{2})^2 = 10$, very nearly. And, however we move, this condition must always be satisfied.[1]

And that is all that the four-dimensional continuum means. There is nothing essentially mysterious about it. The mental panic that it sometimes creates seems to arise from the complete illusion that there is actually a fourth dimension in space—a sort of additional direction of spatial extension, of which we could obtain experience if we possessed an additional sense. In reality, the fourth dimension does not exist, except as an academic expression of our familiar experience of time. Nothing exists fundamentally but events. It is not true to say that they take place in a four-dimensional continuum. Strictly speaking, they do not take place at all : they simply exist in themselves. We do not say that Nature "takes place," and Nature is simply the aggregate of events. The four-dimensional continuum is merely the mathematician's shorthand way of saying, first, that four measurements are necessary to define the complete interval between two events, and second,

[1] Strictly speaking, this particular combination—(space interval)^2 – (time interval)^2—is invariant only in free space. In the neighbourhood of heavy bodies—or, to use scientific language, in strong gravitational fields—a slightly different combination must be taken. This will be dealt with in a later chapter. In all circumstances, the general statement—that there is a particular combination of the four measurements that is constant for all observers—is strictly true.

that a certain combination of these four measurements is constant for all observers, whatever their spaces and times might be. It has no other physical meaning. It derives its name simply by analogy to the familiar three dimensions of space.

We have dwelt in detail on this point, because a great deal of the terror inspired by the idea of relativity is due to preliminary misconceptions. The theory is approached as if it were something quite outside the scope of ordinary intelligence and everyday experience. It cannot be said too often that this is a complete mistake. The only reason why the practical effects of the principle (supposing it to be true) were not recognized long ago is that they are exceedingly minute—not at all that they require new organs of sense and intelligence. If this is once thoroughly understood, the subject will lose its esoteric appearance and begin to be instructive.

CHAPTER IV

THE VELOCITY OF LIGHT

WE have already called attention to the remarkable properties which seem to be possessed by the velocity of light. Suppose we have two observers, A and B, moving relatively to one another with this velocity. Tables 1 and 2 show us what to expect. It would appear that events separated by a finite time interval to A would be simultaneous to B. For since, under these conditions, B's watch would lose twenty-four hours in one of A's days (see Table 1), time would appear to stand still for B, and the whole of A's world—past, present, and future—would be concentrated for B's perception in a moment of time. The relations of A and B are, of course, reciprocal, for it is just as true to say that A is moving relatively to B as to say that B is moving relatively to A. Hence, B's world is presented instantaneously to A also. Our observers, then, would appear each to inhabit a double world of events: one world is perceived instantaneously, and the other stretches through time.

It is very interesting to think that the whole panorama of the world's history—from the dawn of created things to the last sunset of time—might be conjured up for our inspection, if only we could move past it quickly enough. Like every good thing, however, the realization of such a prospect entails some compensating conditions. We say nothing for the moment about the difficulty of attaining the requisite relative speed, or of perceiving terrestrial events clearly when it is attained. But it should be pointed out that the perception, supposing it were possible, would occupy but an instant of our time—a duration so minute that our sluggish intelligences would be powerless to apprehend it. Moreover, the picture would present a very different aspect from that to which we are accustomed. According to Table 1, the dimension of every material object in the direction of relative motion would be nothing. The instantaneous world would consist of a collection of plane objects, having no thickness at all.

The historian, then, has no great incentive to study the production of high velocities. But the economist appears to be in better case. It seems from Table 2 that if matter moves past him with sufficient speed, it can increase its content to any desired amount. The widow no longer needs an Elijah to conserve her supplies; if she agrees to the conditions, Maskelyne can do it. But here again, the juggling fiends of

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relativity are not to be believed. They may keep the word of promise to our ear; they will certainly break it to our hope. The "mass" that grows with velocity is not "size"; the size of the matter, in fact, actually diminishes, owing to the shortening in the direction of motion. What mass really means (see Chapter VI) is inertia, or resistance of matter to change of motion. So that, all that Table 2 implies is that the faster a body moves past an observer, the more difficult does it become to increase the relative speed: the body itself gets smaller and smaller all the time.

From the practical point of view, then, any anticipations of increased wealth or novel orders of experience that the theory might have aroused, are likely to meet with disappointment. It might, however, be some consolation to the intellect to know that it is not called upon to conceive the possibility of such anticipations. But we have yet to indicate another property of this remarkable velocity of light: it is the highest velocity that one body can have relative to another—a natural maximum of speed, to exceed which is for ever impossible. Let us look once more at Table 2. At the velocity of light we see that the mass of a body is "infinitely great." Remembering that "mass" means resistance to change of motion, it follows that there is an infinitely great resistance to the change of speed of a body moving with this critical velocity. Not only can a body never exceed this speed. If it once reaches it, it can never begin to move more slowly, for the resistance to *change* of motion is as great in one direction as in the other. The relative velocity of the body is fixed.

Now there appears to be an obvious objection to this. Suppose a body is moving past me with the velocity of light. If I begin to move in the same direction, do I not decrease our relative velocity ? Or, if I move in the opposite direction, do I not increase it ? The answer in each case is, No. When I begin to move, my space and time alter, and my new measure of the relative velocity, with my new space and time, is exactly the same as the old measure, with the old space and time. At whatever speed I move, the result is the same. This is, in fact, the essence of the Michelson-Morley experiment, in which light moved relatively to the observer with exactly the same speed, in whatever direction it was travelling, or however the earth was moving.

All this follows, as we have said more than once, from the experimental failure to detect any change at all in the measured velocity of light, arising from motion of the observer, and the assumption that no such change exists. It is grounded in actual physical experience. Einstein has gone a step further, and has attempted to deduce from it the meaning of the term "simultaneity." He points out that when we say that two things happen "at the same time," we may have a general idea of what we mean, but we should be in difficulties if we were asked to give a rigid explanation of the phrase. In the light of the theory of relativity, he suggests a definition on the following lines. Suppose we have two events, and an observer situated midway in his spatial distance between them. Suppose light signals reach the observer from the two events at the same time. Then, according to Einstein, the two events themselves were simultaneous. To another observer, in motion relative to the first, the events would not necessarily be simultaneous, because the same light signals might reach him, at the mid-point of his spatial distance, at different times. This, it should be noted, is not a thing to be proved. It is a definition, not a proposition. If we were to try to test it, we should require some means of determining whether the two events actually were simultaneous, and we could not do this without knowing what the simultaneity of events means, *i.e.*, without falling back on the statement itself. Also, it is not a complete philosophical definition of simultaneity. It can only be applied to events separated in space from the observer, and it assumes, moreover, that the observer knows what he means when he says the light signals reach him "at the same time."

This book is concerned solely with experimental

Science and its lessons. We shall, therefore, not pursue the present line of thought, which enters the borderland of metaphysics. Nevertheless, for the sake of completeness, it should perhaps be stated that the choice of light signals for the purpose of the definition goes slightly beyond experimental justification. The choice is made, of course, in order that the application of the definition shall give the peculiar properties of the velocity of light that the theory of relativity requires. But we cannot be quite sure whether those properties belong to the actual velocity of light or to another velocity whose magnitude is too close to it to be distinguished by the experiments on which the theory is founded. The latter possibility is favoured by Whitehead. His reasons are, in the main, philosophical, and therefore do not fall within our scope. For general description, it is sufficient to indicate that there is a peculiar velocity, very close to that of light, which cannot possibly be exceeded by any body relative to any other.

PART II

THE LAWS OF NATURE

CHAPTER V

WHAT IS A NATURAL LAW?

AT the very beginning of scientific inquiry lies the assumption that Nature works in an orderly way. The aim of the scientist is to express, in as simple a statement as possible, the principles underlying the order and arrangement of phenomena. To do this, he has to observe what Nature does. He can provoke her in various ways, and from her response he can draw certain conclusions as to her character. He states those conclusions in terms which he believes will be understood by all—in terms, hitherto, of matter, space, and time.

There was a period in scientific history when it was thought that the whole world of possible experience could be described without going beyond these three fundamental ideas. It was hoped that one all-embracing law, expressing the relation of matter to time and space (or, in other words, the movements of matter), would be the complete and ultimate reward of the physicist—a law from which the whole of physical history, throughout the infinity of space, would issue and run its inevitable course from age to age.

That hope passed away. There were actions of Nature that would not be pressed within the limits of such a law. An ether seemed inevitable: light, electricity—these were not to be made subordinate to matter and motion; they demanded a status of their own. Apart altogether from spiritual facts (with which we are not concerned in any part of this book), a strict materialism was found to be untenable. As a matter of fact, there never was a time when it could be said to have triumphed: its hope, even when brightest, was for the future. The Newtonian law of gravitation, from which it sprang, demanded something other than matter, space, and time; namely, gravitation. It presumed a force, which modified the movements of matter. It is true that the universal scope of this force made it appear very likely that it belonged, in some way, to the essential qualities of matter itself. But, until this was proved, it could not be said that matter and motion had established their sway over the whole physical universe.

It is not without regret that one sees the failure of a simple explanation of things. To the mind that aims at the unification of phenomena, the discovery of a new element in Nature brings disappointment as well as elation. But facts are invincible. The progress of physics demanded the admission that there were other physical existences besides matter, time, and space. Nevertheless, there was no need to modify ideas as to what was a law of Nature. The new entities could manifest themselves only by their effect on matter in space and time. Electricity, for example, was, in itself, merely a hypothesis—though, apparently, a necessary one. All that was observed was a peculiar kind of material movement. A natural law was still a statement of the way in which matter moved in space and time, though, to make the statement simple, it was necessary to introduce other conceptions.

The reader will be prepared, by the first part of this book, for a new conception of natural law. Relativity gives the death-blow to whatever might remain of the old form of materialism. Not only does it make it impossible to reduce Nature to matter and motion: it makes the description of the course of Nature in these terms an incomplete, and therefore a false one. What has hitherto been called a law of Nature becomes a law of our particular aspect of Nature, which is only one of an infinite number of aspects. Space, time, and matter are seen to be an inadequate alphabet for a universal language. They may, or may not, be capable of forming all the words we need, but there are tongues whose sounds they certainly will not fit, and these tongues, as well as ours, belong to Nature. We must go back to the primitive sounds for the expression of natural laws. We can afterwards spell them out in our own characters for our own special use, but the laws themselves must be universal.

In other words, the only possible terms for the statement of a law of Nature are events. Any materia-spatio-temporal statement that is peculiar to a particular observer, and has not its exact equivalent in the relations of other observers, is not a natural law.

Relativity demands, therefore, a review of existing laws. Only those which can be generalized in the way we have suggested can survive; the others must be restated. Since it is the inevitable custom of physicists to express their conclusions in mathematical form, the test must be a mathematical one. We therefore pass over its details, which are of interest only to the specialist. The general idea, however, we hope has been made clear.

The new point of view is of especial interest because it suggests the possibility of a more complete unification of Nature than any previously imagined. With one hand relativity destroys the throne of matter and motion: with the other it erects an altar to the event. Matter, space, and time, even if they could have explained everything in Nature, were, after all, three independent things: the event is one thing. But, as we have seen, the three things failed: can the one thing succeed? It has a better chance, for the following reason. Matter, space, and time, when they were thought to be fundamental, were, on that very account, incapable of modification to meet new discoveries. If they did the unexpected, it was not they, but something else, that was responsible. They were absolute, wrapped in immutability as in an impenetrable garment. Force had to be invented; electricity, magnetism were postulated—all because matter, space, and time were held to be above caprice. But if we start with the event, there is only one deity on our Olympus. Matter, space, and time are his broken lights, and there is no sacrilege in supposing them liable to change. Consequently, what we formerly attributed to force might perhaps be derived from a modification of space or time or matter—in which case, force can be dispensed with. Possibly, also, the other extraneous entities that we have called into being might be treated in the same way.

It should particularly be noted that it is not sufficient merely to conceive an idea of this kind. It must be something more than a philosophical possibility before it can apply for recognition as a scientific hypothesis. A particular modification, that will tally exactly with experiment, must be found, and the new laws, expressed in terms of the modified relations, must be capable of generalization so as to include the experience of all observers, in the way we have already pointed out. If these conditions cannot be fulfilled, the particular unification suggested must be abandoned.

We may say at once that it has been found possible to describe the phenomena of gravitation by a certain modification of space. Electromagnetism is being treated in the same way, with every prospect of success. We choose the case of gravitation to illustrate the argument we have been trying to develop in this chapter. General statements are sometimes difficult to follow, and are liable to be misunderstood. The next chapter, then, is devoted to the work of Newton on the movements of bodies, while the succeeding one attempts to explain the attitude of the relativist to the same facts.

CHAPTER VI

THE WORK OF NEWTON

WHEN Newton began his work, the time was ripe for a great generalization. Galileo had studied the manner in which bodies on the earth's surface fell to the ground, and had shown how the distances fallen through increased with the time of flight. Kepler had succeeded, after many failures, in expressing, in three famous laws, how the planets moved round the Sun. The manner in which different bodies moved was known with great exactitude. What was lacking was a co-ordination between the fall of a body to the Earth and the journey of a planet round the Sun.

It was not obvious to all that such a co-ordination was necessary. One could regard the elliptic motion of the Earth and the linear motion of a falling stone as distinct phenomena. It was natural, said some, for the Earth to move in an ellipse, and it was natural for the stone to fall in a straight line. To attempt to get behind these facts was absurd. They were ultimate facts of Nature.

Newton, however, regarded them in a different light. Matter was matter, and if it moved in a certain way in one set of circumstances, and in a different way in another set, the reason must be sought in the circumstances, and not placed to the account of the moving bodies. We must remember that Newton thought in terms of absolute space, time, and matter.

From this point of view he was faced with a double problem. He had, first of all, to determine what was the natural tendency of matter in no circumstances at all, *i.e.*, when it was entirely uninfluenced by anything outside itself; secondly, he had to consider the effect on this tendency of the various circumstances occurring in Nature. For the second task he was guided by observed facts: the effect must be such as to produce the phenomena we actually find. But for the first he had to fall back on his own inspired imagination. There was no experience to guide him, because he could never be quite sure that any movements he observed were free from external influence.

In making his fundamental assumption, Newton took as his starting-point the essential deadness, or "inertia," of matter. He put forward the hypothesis that matter by itself could do nothing to change its state of motion. If it was at rest, it would remain at rest until something moved it. If it was moving, it would continue to move, in exactly the same direction, and with exactly the same speed, until it was disturbed by outside agencies. This he declared to be the natural condition of matter, and any departure of a body from either of these states—of rest or of uniform motion in a straight line—was evidence that something was interfering with it. To this something, whatever change it produced, he gave the name "force."

We should understand quite clearly that "force," in the Newtonian sense, is not a thing observed: it is a hypothesis. What we observe is a change of motion. According to Newton's assumption, this change implies a cause, and force is created to act the part. Newton did not discover force; he invented it. He was thus at liberty to deal with it as he liked. He could define its magnitude in whatever fashion best suited him—so long as its calculated effects were consistent with the facts. He set himself, then, to define force in such a way that it would be possible to find one particular force capable of explaining, at the same time, the movements of bodies on the Earth and in the Heavens.

Now since force is the supposed cause of change of motion of a body, it must be measured in some way by the rate at which it produces the change. Newton, still with his one object in view, tried, first of all, the simplest possible definition. He assumed force to be actually equal to the rate at which it changes motion. But he recognized that he would probably not achieve much success with a definition of this kind, unless he understood by motion something more than mere velocity. The idea that the agency producing a given change of velocity was of the same magnitude, no matter what was the bulk of the moving body, did not appear very promising as the basis of a universal law. He therefore defined the quantity of motion of a body as the product of its "mass" and its velocity. It followed that the change of velocity produced in a body by a given force was greater, the smaller the mass of the body, the change of *motion* being the same in both cases.

It was in this way that the idea of mass first became definite in physics. From its derivation, its meaning is simply inertia, or resistance to change of velocity. Since inertia is taken to be the fundamental property of matter, the mass of a body can also be interpreted as the quantity of matter in it. The motion of the body, then, according to Newton, arises from the quantity of matter it contains and the velocity with which it is moving. A change in either of these things will alter the motion and reveal the existence of a force.

Later experiments showed that, to the degree of accuracy attainable, the mass of a body never varied. If disintegration took place, the sum of the masses of the various parts was always exactly equal to the mass of the original body, whatever treatment the body or its parts received. In this way the idea of matter became absolute. Change of motion was due entirely to change of velocity, the mass remaining constant all the time. Newton's law of force, therefore, amounted to a statement that the force acting on a body was equal to the product of the mass and the change of velocity (or, the "acceleration") which it produced.

Newton had now provided himself with general laws expressing the possible motions of matter. He had next to apply them to the facts of Nature, and see if it were possible to devise a single force that would give rise to the varied motions actually observed. The problem was no easy one; it required a Newton to solve it. A stone fell in a constant direction, but with varying speed; the Earth revolved with almost uniform speed, but with continually changing direction. Moreover, a heavy body fell from a given height to the Earth in the same time as a light one. All these experimental facts had to be the inevitable result of one simple hypothesis.

As every one knows, Newton was almost completely successful. He assumed that, between every two pieces of matter in the Universe—or, at any rate, in our own Solar System—there existed a force of attraction (gravitation) which was proportional to the product of the masses of the two bodies divided by the square of the distance between them. This force acted on each of the bodies, pulling them towards one another, with accelerations which, of course, depended on their masses. The force pulling the stone to the Earth also pulled the Earth to the stone, but it produced a far greater acceleration in the stone than it did in the Earth, because of the great difference between the masses of the two bodies. All bodies on the Earth's surface, however, would fall with the same acceleration. For, suppose one body had twice the mass of another. The force pulling it towards the Earth would then be twice the force pulling the second body towards the Earth. But the resistance to the force (*i.e.* the mass) would also be twice as great for the first body as for the second. Consequently, the same acceleration would be produced in both bodies. To explain why the planets did not fall into the Sun if they were attracted by it, it was necessary only to suppose that they had some motion of their own, independent of that produced by gravitation. The attractive force would then fulfil its function by constantly changing the direction of motion and, to a slight extent, the speed.

Newton's laws of motion and gravitation have been the basis of physics for more than two hundred years. Their success in explaining and predicting new phenomena has been almost complete. It is true that not every material movement can be said to come within the scope of gravitation. Electricity, magnetism, radiation—all have had to be recognized as origins of force. Nevertheless, observations have been consistent with the idea that the forces they produce are in accordance with Newton's definition. Almost every observed change of motion in Nature can be explained as the result of a Newtonian force, arising from particular physical conditions. But there are one or two that have defied such explanation. They are exceedingly small—so small, in fact, that one might be inclined at first to neglect them. But astronomical observations are very exact, and they leave no doubt at all that there are motions in the Solar System that so far it has not been possible to bring under the sway of Newton's laws. One of the most important of these is exhibited by the planet Mercury—the nearest to the Sun of all the planets yet discovered. Mercury, like all the Sun's satellites, revolves round its primary in an ellipse. There is one point in its orbit (its "perihelion") which is nearer to the Sun than any other. Now Mercury is, of course, attracted by the other planets as well as by the Sun, and calculation shows that, as a result, its perihelion should gradually change its position in space (the absolute space of the Newtonian system). Mercury has been under observation for many years, and it is found that the position of the perihelion does change, but not by quite the same amount as the calculations require. The difference in one century amounts only to the apparent length of a 1-foot rule placed one mile away, but this is much greater than the possible errors of observation. It must have some cause not yet revealed, or else the Newtonian laws are not quite exact. Until the advent of the theory of relativity, it can be said that there was no explanation of this phenomenon.

CHAPTER VII

RELATIVITY AND THE MOVEMENTS OF BODIES

THE attitude of the relativist to Newton's laws of motion and gravitation is not exactly that of criticism. The more he studies them, the more are their wonderful, almost magical, beauty and simplicity brought home to him. He looks on them as on Prospero's fairy visions—perfect in their kind, but springing from such stuff as dreams are made on. It is the tacit assumptions underlying the laws that are the objects of his attack.

Newton, as we have said, presupposed absolute space, time, and matter. If these things are relative, the laws become, not false, but meaningless. A body left to itself moves in a straight line. But what is a straight line? A line which is straight in A's space may be curved in B's. Again, force is measured by the rate of change of motion of a body. But what is the "rate" of change? In whose time system must it be measured? How, indeed, are we to know whether force exists or not? A, with his space and time, finds a change of motion: B, with his space and time, finds none. A asserts a force; B denies it: which is right? Once more, two bodies attract one another with a force proportional to the product of their masses divided by the square of the distance between them. But what are their masses? Is A to measure them, or B?

Clearly, we must start afresh if the relativist is right. We must go behind the motions of bodies, which we observe from our own particular standpoint, and think in terms of events, which are common to all. Let us take an example. On 8th February 1921, the Moon was between the Earth and the Sun. On 22nd February 1921, it was in the opposite direction from that of the Sun, as seen from the Earth. Newton gave laws to account for the elliptic motion of the Moon from the first of these positions to the second, as we observe it. The relativist looks for the connection between the series of events which we speak of as the successive positions of the Moon between the dates mentioned, but which another observer might regard differently. He takes a standpoint beyond the Moon, the Sun, the dates, and studies the events from which they spring. He asks why those particular events are what they are, and not something else. Afterwards, when he has found the answer to his question, he descends to Mother Earth again, and translates it into our language of space, time, and matter.

Now, in dealing with events, we must make use of the only relation among them known to us so far; namely, that their complete separation —what we have called, for convenience, the interval between them in the four-dimensional continuum—is constant for all observers. This separation, as we have seen, is obtained by subtracting the square of the time interval from the square of the space interval as measured by the same observer, and finding the square root of the difference.

The complete separation, we say, is constant for all observers. But that does not tell us anything about the course of Nature. It does not tell us why (speaking in our own terms, for brevity) the Moon should travel from its position on the 8th February to its position on the 22nd February in an ellipse, as seen from the Earth, and not in a straight line or a circle. If it moved in either of these paths, the complete separation between the two events which we describe as its positions on the dates given, would still have the same value for all observers, though a different value from that which it actually has. Clearly, to obtain a law of Nature we must make some hypothesis as to the actual value of the interval between events.

Einstein assumed Nature to be such that the total four-dimensional interval between any two events, when computed from event to event along the actual succession, has a maximum value. That is to say—to refer to our example again—if the Moon moved in any path slightly different from that which it chooses, then the total interval between the two events which we call its positions on the 8th February and the 22nd February would be smaller than it is. This will probably appear very surprising at first, because, accustomed as we are to intervals in space, it seems inconceivable that there can be a path which has not a slightly greater neighbouring one. But we must note that the interval with which we are dealing is to be calculated in the hypothetical four-dimensional continuum, and not in space. Referring to Chapter III, we see that it depends on the difference between the squares of the space and time intervals. This difference can be increased by diminishing the time interval as well as by augmenting the space interval, so that we shall be getting nearer to the idea of Einstein if we think of the actual path of the Moon as being that in which it can cover the greatest spatial distance in the shortest time.

It must be recognized that Einstein's hypothesis was a guess, though an inspired one. It depended for its justification on its ability to explain the facts of observation. It is very like Newton's guess about the movement of matter in the free state. Newton assumed that free matter would move in a straight line, *i.e.* that it would take the minimum spatial distance between any two points in its path. Einstein assumed that an actual event would be separated from its neighbour by the maximum four-dimensional distance. There is this important difference, however, between the two assumptions. Newton was thinking of an ideal case, which hardly occurs in Nature, for matter is never quite free. To account for actual motions he had to introduce force. But Einstein dealt with actual events. If his assumption was successful, he would therefore have no need of force or any agency at all outside the events themselves.

Now this assumption of Einstein's can be tested, for, from our knowledge of the Moon's path, for instance, we can calculate the four-dimensional interval between any two events in it, and compare the result with the interval between the same two events, supposing the Moon had travelled in a slightly different path; *i.e.* supposing the series of events we call the Moon's successive positions in space had been slightly different from what they are. This has been done, using the geometry of Euclid in the calculation. The result shows that the actual path does not give the maximum four-dimensional interval.

We are faced, then, with two possibilities. Either Einstein's assumption is contrary to Nature, or else the definitions and axioms of Euclid are not relevant to the space in which the members of the Solar System travel. Or—in other words—Einstein's assumption must be either abandoned altogether, or modified by the employment of a different type of combination of the four measurements (see Chapter III) for the purpose of defining the four-dimensional interval in the neighbourhood of material bodies. Which of these alternatives can we adopt ? The natural impulse is, of course, towards the former. It seems to be impossible to cast doubt on Euclid. But, once more, the matter is not to be decided by prejudice: it must submit to experiment. If we assume Einstein to be on the right lines, then space must be non-Euclidean, and geometrical measurements in it should show results contrary to Euclid's assertions. On the other hand, if experiment shows space to be completely Euclidean, then Einstein's assumption falls to the ground, and some other hypothesis becomes necessary. Experiment, as always, is the final court of appeal.

Let us pause for a moment to see what we mean by space being "non-Euclidean." There is nothing occult about it ; it is essentially a statement about actual physical fact—to be tested by ordinary experience. It simply means that the assumptions that Euclid made about space are not applicable to the actual space which we use as a relation between events. The non-Euclidean character of space, if it is actual, would lead us to expect some, at least, of the propositions of Euclid to be falsified by exact measurements in space. For example, the sum of the three angles of a triangle might not be exactly equal to two right angles.

The test seems an easy one, but, as before, we are baffled by the extreme minuteness of the crucial effect. The differences between practicable measurements in the space of Euclid and in the space which must be assumed in order to justify the hypothesis of Einstein, are beyond the power of existing instruments to detect. Our only hope—at present, at least—is to assume Einstein's hypothesis, and see if the consequences agree with fact better than those of any other assumption.

The result of investigations of this kind has been all in favour of Einstein. There are two points on which the Einstein and the Newtonian theories give definitely conflicting results. The first is connected with the orbit of the planet Mercury. The Newtonian laws, as we have seen, leave a small movement of the perihelion of this planet unexplained. Einstein's modified assumption gives a motion equal to that observed, within the limits of experimental error. It does not need any additional modification for this explanation, nor was the hypothesis constructed for the purpose of explaining the motion. The result follows directly from the assumption that Mercury moves in the maximum four-dimensional path, and the consequent supposition of non-Euclidean space. This is the only explanation of the phenomenon that has not been devised *ad hoc* and found inapplicable to, or conflicting with, other observations.

The second point at issue between the Einstein and Newtonian theories involves observations that had not been made previous to the formulation of the principle of relativity. A ray of light, passing close to a heavy body, should, on Einstein's assumption, suffer a slight change of direction, as if it were pulled towards the body. According to Newton's principles, there seems to be no reason why the light should be bent at all. It is possible, however, that light possesses the equivalent of weight in a material body, and, if so, the gravitational force should cause a bending similar to that predicted by the theory of relativity, but of only about half the amount. The two theories are therefore definitely at variance in their predictions, and an experimental test becomes possible. This test was made at the total solar eclipse of 29th May 1919. The heavy body chosen was the Sun, and the light examined was that emitted by stars which were almost directly behind the Sun as seen from the Earth. During the eclipse the sunlight was extinguished, and the stars became visible, apparently very close to the obscured Sun. Now these stars necessarily appeared to be in the directions of their own light, by which they were seen. If, therefore, that light was bent, they would appear to be displaced from their normal positions in the sky, which were known with great exactitude. The amount of the displacement would be a measure of the bending of the light. The result was that bending of the light did occur, of just the amount (within the limits of experimental error) required by the Einstein hypothesis. Once more, experiment justified the assumption that the space of experience is non-Euclidean.

There is a third possible consequence of the theory that is not predicted by Newton's laws; namely, that the colour of the light emitted by a glowing substance in a very massive body, such as the Sun, should be slightly different from that of light from the same kind of material on the Earth. It is not quite certain, however, that this conclusion necessarily follows from the theory. Einstein himself considers that it does, but there are distinguished mathematicians who hold the opposite view. The question is one of great difficulty. Experimental tests have been made, but the colour of the light may be influenced in so many ways, and the results are so complicated, that no certain conclusions can yet be drawn from them.

We are left, then, with these facts. The theory of relativity, requiring that space does not conform to the definitions and axioms of Euclid, explains all the movements of bodies that are accounted for by the Newtonian law of gravitation. In addition, it explains a movement of the planet Mercury that stands outside the Newtonian law, and it has predicted the true path of a ray of light, which no other theory seems able to do. It requires the supposition of no imaginary existences, such as force, but proceeds entirely and completely from a single hypothesis as to the association of events. There is no known phenomenon with which it is at variance.

CHAPTER VIII

SOME PROBLEMS OF RELATIVITY

A GREAT idea invariably creates as many problems as it solves: that is a sign of its greatness. The thoughtful student will not be baffled by its novelty or lose himself in its details. He will patiently probe it to the core, and lay bare to his mind its inner meaning and its relation to the world of experience. And in so doing he will meet with difficulties—not, perhaps, the difficulties that are dealt with in books, for they were the authors', and may not be his own. The attainment of a new point of view is hindered, not so much by the roughness of the road as by the tendency to return to the old standpoint. It is our prepossessions that hold us back, drawing us, like a magnet, to themselves. Each of us has his own standpoint and prejudices; each will have his own difficulties.

It may be said of the principle of relativity that, for the most part, it offers the same problems to all plain men. Its point of view is so remote from that to which most of us are accustomed that we are relatively together, and approach it along parallel roads. This, our concluding chapter, is devoted to a few general comments on some of the more prominent difficulties that are our common lot. It makes no claim to be exhaustive or final; its sole purpose is to help.

It is important that we should recognize that the principle of relativity is not a complex, fantastic theory—a sort of last hope, called in to save the human mind from defeat by the manœuvres of phenomena. It is, on the contrary, a straightforward attack on the problems of Nature, an attempt to see them as they are. It is a quest after the simple. It is inevitable that it should pursue its ends at the expense of plausibility. We are accustomed to the complex. We think in terms of three things—matter, space, and time—and we are so much at home with them that we do not, perhaps, give our race full credit for the consistency and success with which it has applied them to the interpretation of the Universe. Juggling with three balls is not an easy matter, and we have done it almost to perfection. It is not surprising that, when we are left with one, we are at a loss to know how to perform our tricks. But let us once clearly realize the conditions; let us suitably arrange our mirrors to make up for the lost balls, and we shall find our repertoire augmented and our own effort simplified. That is, in essence, the central meaning of relativity. It takes us to the view-point of the Gods, from which we see things as they are, unmodified by reflection in matter, space, and time. It is a step towards truth, and truth is simple to the simple-minded.

The question of the existence of the ether in the light of relativity has aroused much discussion. Apparently there are still differences of opinion on this point. It would be unwise, therefore, to make any definite pronouncement. We shall merely offer a few suggestions from our own point of view. There seems to be nothing in the theory of relativity that is incompatible with the ether. On the other hand, the theory has no need of the ether's existence. There has probably been a little misunderstanding in some quarters as to what relativity really implies. The theory is based on the assumption that it is inconceivable that we shall ever detect relative motion between matter and ether. But that does not necessarily mean that there is no ether. The relative motion is hidden because the velocity with which the ether (if it exists) transmits waves is almost, if not quite, identical with the peculiar velocity that takes part in the relativity formulæ for change of the time, space, and matter units with motion. If the changes of these so-called fundamental units are granted, then experiment appears to decide neither for nor against the ether's existence. There is still the possibility that the ether possesses some physical property, other than the power to transmit waves, for which no compensation is made by the relativity transformations. The essence of relativity is the universal character of the event, and the subordination of space, time, and matter. A subordinate ether in addition would appear not to be inconsistent with this. The ether may, at any rate, have an existence as real as that of matter, and that is all that is demanded of it by the physical facts which called it into existence.

We have already dealt more than once with the idea that relativity is not a physical theory, but a metaphysical one. This book has failed in its purpose if it has not made it clear that the entire theory is built up with the one object of accounting for actual physical facts, and stands or falls at the dictate of experiment. The theory of relativity is no more metaphysical than the wave theory of light, or Newton's laws of motion. It partakes of their nature, and is vulnerable to the same weapons. It appears, perhaps, at first blush to be metaphysical, because it deals with the nature of matter, space, and time, which are part of the playground of metaphysics. But it is with these things as objective entities that relativity is concerned. Its assertions about them are susceptible to actual physical tests with clocks, scales, and balances. They may be considered metaphysically, it is true, but relativity does not so consider them. Love is fair game for the metaphysician, but we do not fall in love metaphysically. It is essentially a matter of experience.

But perhaps the greatest difficulty of relativity is presented to the imagination. The consequences of the theory are so extraordinary that we cannot picture them. It seems impossible, for instance, that the order of events in time can be different for different persons. We must remember, however, that relativity does not entail anything that is *contrary* to experience. It would have no right to exist if that were so. Its predictions, that appear to us so strange, are related to matters as yet *outside* our experience, about which we can only form conjectures. Everything that we come across in our everyday life is left untouched. Space, time, and matter have an absolute meaning for observers relatively at rest. For them there is a real and definite meaning in the statements that a man is 6 feet high, that is it nine o'clock, and that sugar is 8d. a pound. With all terrestrial movements, even, the statements are, for practical purposes, exact. They are quite as true as the statement that, in sunlight, grass is green. It is only when we get into conditions that are far beyond our common experience that the reasoned effects belie our anticipations. With a Sun composed of sodium, grass would no longer be green, and space, time, and matter, in appropriate circumstances, suffer a corresponding change. It is not true to say that relativity is revolting to our experience. All we can say is that we did not expect it. But, after all, Nature has nothing to do with our expectations.

It is well with Science when reason and imagination go hand in hand. One assists the other, and the mind is satisfied. But the history of Science shows that it is not in this way that the greatest advances have been made. The imagination of Faraday saw the electric field threaded by "lines of force," to which bodies were harnessed and in obedience to which they moved in their courses. But the mind was baffled: reason could find no lines of force until Clerk-Maxwell brought them within its light. In the words of Helmholtz: "Now that the mathematical interpretation of Faraday's conceptions regarding the nature of electric and magnetic forces has been given by Clerk-Maxwell, we see how great a degree of exactness and precision was really hidden behind the words, which to Faraday's contemporaries appeared either vague or obscure. . . . I confess that many times I have myself sat hopelessly looking upon some paragraph of Faraday's descriptions of lines of force, or of the galvanic current being an axis of power."

Would it not have been unwise to restrain the imagination of Faraday because reason could not follow it?

Or—to take an example more closely allied to the present discussion—think of the dawn of the idea that the Earth was round and rotated on an axis. Reason demanded it; there was no other explanation of facts. But where was the imagination? Was it conceivable that we were whirling through space with breathless speed, and yet were insensible of the motion? Could one really believe that there were people on the Earth who were upside down and yet did not fall off? It was impossible; common sense scorned the suggestion. Yet, would progress have been possible if we had listened to common sense and silenced the voice of reason?

We are in much the same position to-day. Can we not take heart from the experience of the past? To-day the dullest schoolboy knows our place in the Solar System, and finds it not hard to accept. Surely it is not impossible that the paradox of relativity will one day become a part of our common knowledge and fashion our view of Nature. Reason calls for it: if it is true, the imagination will not be left far behind.

Meanwhile, reason will march along and explore fresh country, and we can at least meditate on its conquests. Whatever our attitude towards them may be, we must recognize that the world is a far more wonderful thing than we have ever imagined. Conceptions which we thought were universal appear as merely one set of an infinite number of possible conceptions. Our idea of Nature—consistent though it has been, and, therefore, to some extent, true—is yet not the whole truth. We have "swayed about upon a rocking-horse, and thought it Pegasus." The theory of relativity does not countenance the bombast of Swinburne, with which we opened: it should make us very humble.

Finally, we must not imagine that the theory of relativity is complete and self-sufficient. Rather is it the beginning of a new chapter in Science. Some pages of that chapter have already been written, and the work is even now in progress. Electro-magnetism is being examined; the nature of space, and its extent, are being subjected to searching inquiry. We have tried, in this book, to indicate the view-point; the view we must leave till our eyes are adjusted to the new distances. But the view is there, and the reward of our labours will be limited only by the quality of our vision.

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