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ABSTRACT

We present a model in which intertemporal interdependencies and divergence of time preferences alter the form of optimal contracts between a risk-neutral agent and risk-neutral principals. The agent wishes to maximize the discounted sum of benefits over his or her lifetime; the principals each want to maximize only current rewards. We demonstrate the existence of conditions under which all principals (except the last period's) can be made strictly better off by allowing the agent to capture some surplus each period (via an incentive contract), instead of forcing the agent down to the reservation wage with a fixed-payment contract. The incentive contracts serve as a means of aligning the time preferences of the agent and the principals.

I. INTRODUCTION

Much of the principal-agent literature in economics, finance and accounting deals with the structure of contracts under a variety of assumptions about risk preferences and the availability of information. These papers usually abstain from institutional constraints. For example, if both the principal and the agent are risk-neutral and contract for one period, and one period only, it is well-known that the Pareto optimal contract has the principal "selling" the output to the agent for a fixed fee; in this way, the principal is able to recapture all of the rents the agent could have earned because of asymmetries of information. (See, for example, Shavell [1979].) Why is it that we don't see these property rights being sold more often in the real world, when we can safely assume risk-neutrality of the agent? Sappington [1983] suggests that the agent's limited liability prevents such contracts. Here we provide an alternative explanation. Our explanation is that the agent serves a series of interdependent principals over his or her professional career. In our framework, the time horizons of the principals and the agent are not indentical, adding a new source of "preference divergence." These interdependencies and non-alignment of time preferences change the form of the contracts that we would expect between the agent and the principals.

If the agent and the principals are myopic (current reward) maximizers and risk-neutral, then the optimal contract between the agent and each principal would have the principal selling the outcome to the agent for a fixed payment; in this way, the principal would capture all of the surplus from the agent. If, however, the agent is interested in maximizing the discounted sum of payments over a finite working lifetime, and the principal in each period wants to maximize only current reward, then are the optimal contracts still fixed-payment contracts? Intuition would tell us that if the outcomes are independent, in that the actions taken by the agent on behalf of one principal have no effect on subsequent principals' payoffs, then myopic maximization is still optimal. When the payoffs are not independent, however, myopic contracting may not be optimal from either the agent's or from the principals' points of view, or both.

The way in which we introduce interdependencies between principals' payoffs, and thus intertemporal links, is by assuming that rewards in future periods depend upon current states and future states. In particular, we assume that one of two possible states can occur each period. Label these "win" or "lose". Associated with each state is a reward function which depends upon past states, via a summary variable of past history. We call this variable "reputation". The principal in each period holds "title" to the reward in that period. Each agent is endowed with an initial amount of reputation when the process starts. We also assume that rewards are increasing in reputation.

It should be noted that the reputation variable we define is not reputation in the sense of, for example, Kreps and Wilson [1982]. In those models, the agent's type, which remains the same over time, is not known to the principal and reputation serves as a signal about the agent's unknown type.[1] In our model, reputation can be thought of as the agent's type and we assume that this information is known with certainty by the principal. The distinguishing feature of our model is that the agent's type changes stochastically over time as a function of states that occurred in past periods. The agent can control this stochastic process by providing effort that effects the probability of the states. Effort is assumed to increase (decrease) the probability of winning (losing) in the current period.

Intertemporal interdependencies create a new source of divergence of preferences if the time horizons of the principals and the agent are not the same. We assume that the agent wishes to maximize a discounted stream of rewards over his or her lifetime. The principals, however, wish to maximize only their own current reward, since their relationship with the agent lasts for only one period. The structure of the model is such that if the win state occurs in the current period, the agent is strictly better off next period, ceteris paribus, than if the lose state occurs in the current period. There are, thus, future as well as current rents that can accrue to the agent from increased effort. The contracts that the principals offer to the agent should be designed to capture as much of these total rents as possible.

The total rents in any period depend on past history through the reputation variable. In any period, a principal chooses a contract to offer the agent. The agent then chooses a level of effort that maximizes expected discounted rewards from the current period to the end of the planning horizon. The model we consider in this paper is one in which there are two periods. Our results would, however, hold for any finite number of periods. Suppose that at the last period the risk-neutral principal offers the agent a contract that captures all the rents of that last period for the principal and forces the agent down to his or her reservation utility. In such a contract the agent pays the principal a fixed amount equal to the maximum expected total surplus. If this type of contract is used in the last period, then the agent can not capture any expected rents from winning in the first period. That is, the agent's expected reward in the final period is independent of the state that occurs in the first period. Thus, in terms of last period rewards, the agent is indifferent between winning and losing in the first period. The optimal contract in the first period is then (again) a fixed price contract in which the agent pays the principal the maximum expected total surplus in the first period.

In contrast, if the principal in the last period offers the agent an incentive contract that allows the agent to keep some expected surplus, and more importantly such that the amount of surplus that the agent keeps depends upon what happened in the first period, then the agent is not indifferent between winning and losing in the first period in terms of the effect on final period utility. Clearly, the agent is strictly better off ex ante with contracts each period which guarantee an expected payment strictly more than the reservation utility each period, rather than contracts each period that guarantee exactly the reservation wage in each period. Can the principals be made better off as well?

Our results show that (1) the last period principal is always worse off; (2) conditions exist such that the principals in all but the last period are strictly better off; and (3) conditions exist such that the total expected discounted surplus from the beginning of the planning horizon is strictly greater with incentive contracts.

The implications of (1) and (2) above is that there is an implicit intertemporal transfer from the last period principal to earlier principals. The difficulty with using (3) to claim Pareto superiority for incentive contracts is that we must be able to transfer surplus from one period to another and from one principal to another. However, since the principals are different each period, such a transfer becomes problematic.

The intuition behind our results is that the incentive contract may be better able to align the time preferences of the principals (in all but the last period) and the agent. In the final period, by definition the asymmetry in time preferences no longer exists between the principal and the agent; in that case the fixed-payment contract is always better for the principal. In contrast (in all but the final period) with an incentive contract the reward for winning is the current reward plus the discounted net increase in the surplus in the future; with a fixed-payment contract, the only reward for winning is the current reward. Thus, with the incentive contract the agent provides more effort (which is costly to the agent) and this increased effort increases the probability of winning; the effect of the latter on expected rewards more than compensates for the increase in costs to the agent.

Consider our model in the context of the relationship between a lawyer (the agent) and the many clients (principals) that the lawyer serves orver his or her professinal career. We can use our results to explain, at least in part, the absence of contracts in which the lawyer "buys" a case from the client.

If we think of lawyers as a group with bargaining power, we can conclude that they may have the ability to control the types of contracts the members of the group may ethically and legallly accept. Clearly, incentive contracts benefit the lawyers at the expense of some of the clients. Why then would legislators be willing to pass laws prohibiting lawyers from buying cases from clients? Our results show that all the clients, except the final one, may be made strictly better off with incentive contracts. If the legislators themselves have a short time perspective, the final client's welfare will be greatly discounted in any computation of benefits versus costs. In addition, the legislators can also claim, as a result of (3) above, that since the total surplus can be greater with incentive contracts, society as a whole is somehow "better off" - ignoring, of course, the fact that some members of society are strictly worse off as individuals.

We further demonstrate that as the net future benefit of winning increases, the agent's share of current reward decreases and the principal's share of current reward increases. This result may explain, for example, why lawyers are willing to do pro bono work in potentially landmark cases: lawyers take "payment" in the future from increased reputation.

II. THE MODEL

In this section we develop the formal model we use to obtain our results. For ease of exposition, we will restrict our attention to the two period case; however, as stated in the introduction, our results hold for any finite number of periods.

At the beginning of each period, the agent is endowed with some level of reputation, which we denote by r and r_1, where r denotes reputation at the beginning of the planning horizon when there are two periods to go and r_1 denotes the reputation at the beginning of the last period in the planning horizon. We describe below the precise relationship between r and r_1. The current level of reputation is known by both the agent and the current period's principal.

The model we consider is a two-state model. For ease of exposition, we call these states "win" and "lose". At the beginning of the planning horizon, the agent's reputation is given. If the win state occurs, then $r_1 = r_W(r)$; if the lose state occurs, then $r_1 = r_L(r)$, where $r_W' > 0$ and $r_L' > 0$. The relationship between these functions is as follows: for all r,

$$r_W(r) > r; \quad r_W(r) > r_L(r). \qquad (2.1)$$

Winning improves reputation over the previous period, and winning is better than losing. However, losing may <u>or may not improve</u> reputation.

If the lose state occurs in the current period, the current reward is defined (without loss of generality) to be equal to zero.

If the win state occurs, the reward in the current period is a function of the beginning of the period reputation. We denote this reward function by $X(\cdot)$, where $X' > 0$ and $X'' \leq 0$. Reward in the first (last) period is then given by $X(r)$ ($X(r_1)$). Since r_1 depends on whether the win or lose state occurs in the first period and on r, $X(\cdot)$ in the final period depends upon initial reputation and the outcome in the first period. Given the relationship described in (2.1) above, we have

$$X(r_W(r)) > X(r); \quad X(r_W(r)) > X(r_L(r)). \tag{2.2}$$

Current reward is increasing in both initial reputation r and the outcome in the previous period.

The $X(\cdot)$ function can be interpreted in the following way: in the the lawyer's case, $X(\cdot)$ would represent the size of the cash award the lawyer can get for the client (if the case is won). If the lawyer has a high level of reputation, perhaps from winning several past cases, the lawyer will be in a better negotiating position with opposing lawyers with inferior track records, thus increasing the potential jury or out of court settlement award to the client.

The probability that the win state occurs in any period is denoted by $p(a)$ and the probability of the lose state by $(1 - p(a))$, where a is the level of "effort" taken by the agent in the current period. These probabilities depend only on the current level of effort, and are independent of the agent's reputation as well.[2] Feasible effort levels are contained in the interval $[0,\alpha]$, where α is not necessarily finite. The p(a) function satisfies the following assumptions:

$$p(0) = 0;$$

$$p'(a) > 0\ ,\ p''(a) \leq 0,\ \text{and}\ p'''(a) \geq 0\ \text{for all}\ a \in [0,\alpha]; \quad (2.3)$$

$$\lim_{a \to \alpha} p(a) \leq 1.$$

The last assumption in (2.3) implies that even if the agent were to take the maximum level of effort that is feasible, there still may be some residual uncertainty remaining; the agent may not have the ability to make the win state a sure thing. The assumption on the third derivative of p(a) is necessary to obtain our results. This assumption states that p(a) is a very smooth increasing, concave function of a.

To demonstrate the existence of sufficent conditions for incentive contracts that allow the agent to keep some of the rents each period to be superior to fixed-payment contracts (for all but the final principal) when both the agent and the principal are risk-neutral, we restrict attention to linear sharing rules.[3] If sufficient conditions exist under which linear sharing rules are superior, then conditions must exist for <u>general</u> optimal incentive contracts to dominate fixed-payment contracts. Clearly, this restriction simplifies the analysis considerably. Define K as the fraction of current reward kept by the agent; (1 - K) is the fraction that goes to the principal.

We assume that the agent's disutility from effort has a monetary equivalent expressed by the function g(a), which satisfies the following:

$$g(0) = 0;$$

$$g'(a) > 0,\ g''(a) \geq,\ \text{and}\ g'''(a) \geq 0\ \text{for all}\ a \in [0,\alpha];\ \text{and}$$

$$\lim_{a \to \alpha} g(a) = \infty.$$

These conditions require g(a) to be a very smooth, increasing, convex function of a; in addition, disutility of effort goes to infinity as a goes to its upper limit. The requirements on the p(a) and g(a) functions assure that the problem we model has a non-trivial solution: that is, $a = \alpha$ is not a feasible level of effort. To assure that $a = 0$ is not a feasible solution we require $p'(0)X(r) - g'(0) > 0$; that is, there is a positive net marginal benefit of increasing effort above zero.

In the tradition of dynamic programming, we use backward induction arguments to demonstrate our results. At the beginning of the last period, the agent's reputation, r_1, is given and known to the agent and the current principal. For a given contract K, the agent will solve the following maximization problem:

$$\max_{a}\ p(a)KX(r_1) - g(a). \tag{2.5}$$

Without loss of generality, we assume that the agent's reservation wage is zero. Given our assumptions on $p(\cdot)$ and $g(\cdot)$, we know that the following first order condition is both necessary and sufficient for the solution to the problem in (2.5):

$$p'(a)KX(r_1) - g'(a) = 0. \tag{2.6}$$

The principal wishes to choose a share K and an effort level a to solve:[4]

$$\begin{aligned} &\max_{K,a} \; p(a)(1 - K)X(r_1) \\ &\text{s.t. } p'(a)KX(r_1) - g'(a) = 0; \; \text{(incentive compatibility)} \\ &\qquad p(a)KX(r_1) - g(a) \geq 0. \quad \text{(individual rationality)} \end{aligned} \tag{2.7}$$

It is immediate that $K \leq 0$ is not a feasible solution; since $g'(a) > 0$ and $p'(a) > 0$, there would not be any a satisfying the first constraint. The incentive compatibility constraint is satisfied with equality since the principal would never choose a K such that the agent would be induced to choose $a = 0$; the principal would be strictly better off with a "small" K that induces a "small" amount of effort $a > 0$. Such a (K,a) pair exists because $p'(0)X(r) - g'(0) > 0$.

To simplify the analysis, we give a sufficient condition such that the individual rationality constraint is satisfied by any (K,a) pair that satisfies the incentive compatibility constraint:

$$g(a)/p(a) \text{ is non-decreasing in a for all } a \in [0,\alpha]. \tag{2.8}$$

Since g(a) is convex increasing and p(a) is concave increasing, this assumption can be interpreted as g(a) increases at least as fast as p(a). We can state:

PROPOSITION 2.1

If $g(a)/p(a)$ is non-decreasing in a for all $a \in [0,\alpha]$, then the individual rationality constraint in (2.7) is redundant.

(All proofs are contained in the Appendix).

From the agent's first order condition we can express K as a function of a. We can thus rewrite the principal's problem as the selection of a (and implicitly K) to solve the unconstrained maximization problem[5]

$$\max_{a} \; p(a)X(r_1) - p(a)\frac{g'(a)}{p'(a)}. \tag{2.9}$$

Let $a_1(r_1)$ solve (2.9). Then the principal's and the agent's expected rewards can be written, respectively, as

$$U_1(r_1) \equiv p(a_1(r_1))X(r_1) - p(a_1(r_1))\frac{g'(a_1(r_1))}{p'(a_1(r_1))}.$$

and (2.10)

$$V_1(r_1) \equiv p(a_1(r_1))\frac{g'(a_1(r_1))}{p'(a_1(r_1))} - g(a_1(r_1)).$$

Note that

$$U_1(r_1) + V_1(r_1) = p(a_1(r_1))X(r_1) - g(a_1(r_1)), \tag{2.11}$$

which is the total surplus to be divided between the principal and the agent in the final period.

When the contract between the agent and the principal is a fixed-payment contract, the agent buys the reward from the principal for a payment $P(r_1)$, where

$$P(r_1) \equiv \max_{a} p(a)X(r_1) - g(a).$$

The principal's expected utility is then

$$U_1^*(r_1) = P(r_1), \tag{2.12}$$

and the agent's expected utility is the reservation wage (assumed here to be zero). Since $U_1^*(r_1) + V_1^*(r_1) \geq U_1(r_1) + V_1(r_1)$ and $V_1(r_1) \geq V_1^*(r_1)$, we have $U_1^*(r_1) \geq U_1(r_1)$, the expected result that the principal is no worse off with the fixed-payment contract and the agent is no worse off with the incentive contract.

The principals in our model are concerned only with the current period's reward. The agent, however, wishes to maximize the discounted sum of rewards over his or her "lifetime". The agent's reward in the final period depends on the agent's reputation at the beginning of the final period, which in turn depends on the agent's initial level of reputation and the state that occurs in the first period. For ease of notation, we make the following definitions:

$$V_{1W}(r) \equiv V_1(r_{1W}(r)) \quad \text{and} \quad V_{1L}(r) \equiv V_1(r_{1L}(r)). \tag{2.13}$$

At the beginning of the planning horizon, the agent faces the first principal. For a given contract K, the agent wishes to choose the level of effort a to mazimize the discounted sum of rewards over the two periods, where $\beta \leq 1$ is the discount rate. Thus, the agent wishes to solve:

$$\max_{a} \; p(a)KX(r) - g(a) + \beta[p(a)V_{1W}(r) + (1-p(a))\, V_{1L}(r)]. \qquad (2.14)$$

Under our assumptions on p(a) and g(a), the following first order condition is again both necessary and sufficient:

$$p'(a)KX(r) - g'(a) + \beta p'(a)[V_{1W}(r) - V_{1L}(r)] = 0. \qquad (2.15)$$

The principal wishes to choose a and K to solve:

$$\max_{a} \; p(a)(1 - K)X(r)$$

$$\text{s.t. } p'(a)[KX(r) + \beta(V_{1W}(r) - V_{1L}(r))] - g'(a) = 0; \qquad (2.16)$$

$$p(a)KX(r) - g(a) + \beta[p(a)V_{1W}(r) + (1-p(a))V_{1L}(r)] \geq 0.$$

Under the assumption in (2.8), we have the following Proposition, which is the first period analog of Proposition 2.1:

PROPOSITION 2.2

If g(a)/p(a) is non-decreasing in a for all $a \in [0,\alpha]$, then the individual rationality constraint in (2.16) is redundant.

The principal's problem can again be reduced to finding a solution to an unconstrained maximization problem:

$$\underset{a}{\text{MAX}}\ p(a)X(r) - p(a)\frac{g'(a)}{p'(a)} + \beta p(a)[V_{1W}(r) - V_{1L}(r)]. \tag{2.18}$$

To assure that the solution to this problem is unique, we make the following additional assumption on the relationship between g(a) and p(a):

$$\frac{g'(a)}{p'(a)} \quad \text{is a strictly convex function for all } a \in [0,\alpha]. \tag{2.19}$$

It should be noted that $g'(a)/p'(a)$ is increasing in a by our original assumptions on p(a) and g(a). Let $a_2(r)$ solve the problem in (2.18). The current principal's expected reward in then

$$U_2(r) \equiv p(a_2(r))[X(r) - \frac{g'(a_2(r))}{p'(a_2(r))} + \beta(V_{1W}(r) - V_{1L}(r))]. \tag{2.20}$$

The agent's sum of discounted rewards over the two periods can then be written as:

$$V_2(r) \equiv p(a_2(r))\frac{g'(a_2(r))}{p'(a_2(r))} - g(a_2(r)) + \beta V_{1L}(r). \tag{2.21}$$

Note that

$$U_2(r) + V_2(r) = p(a_2(r))X(r) - g(a_2(r)) + \beta[p(a_2(r))V_{1W}(r) + (1-p(a_2(r))V_{1L}(r)]. \qquad (2.22)$$

If a fixed-payment contract is used in the first period, the agent is not able to capture any rents from increased reputation in the last period, since the agent receives the reservation wage (in expectation) in the last period independent of the first period outcome. The principal's expected reward is:

$$U_2^*(r) = P_2(r) \equiv \max_a p(a)X(r) - g(a) \qquad (2.23)$$

Clearly, $V_2(r) \geq 0$ for all r. Can we find conditions such that the principal in the first period is strictly better off with the incentive contract; i.e., such that $U_2(r) > U_2^*(r)$?

In the next section we provide conditions for such an outcome. We also give conditions under which the ex ante expected total discounted surplus is strictly greater with incentive contracts than with fixed-payment contracts.

III. DOMINANCE OF INCENTIVE CONTRACTS

In this section we analyze the properties of the model developed in the previous section and provide sufficient conditions for an incentive contract to be superior to a fixed-payment contract for the principal in the first period.

To show the existence of such conditions, we first provide some comparative statics results. These results are used in the proof of the sufficiency of the conditions we provide for the superiority of incentive contracts. In addition, the comparative statics results provide some intuition into the properties of the optimal linear sharing rules between the principals and the agent.

The rewards for the agent and the principal in the final period depend on the initial level of reputation r, and the state that occurred in the first period. Define the agent's expected reward with the incentive contract, the principal's expected reward with the incentive contract, and the total expected surplus with the fixed-payment contract for the states W (win) and L (lose) as:

$$V_1(r_{1s}(r)),\ U_1(r_{1s}(r)),\ \text{and}\ U_1^*(r_{1s}(r)) \quad \text{for } s = W, L. \qquad (3.1)$$

Since $r_{1W}(r)$ and $r_{1L}(r)$ have been defined as strictly increasing functions of r, to show that the functions in (3.1) are increasing in r we need only show that they are increasing in their direct arguments, r_1.

All of our comparative statics results on the last period are contained in the following proposition:

PROPOSITION 3.1

Under the assumptions of our model:

(i) $a_1(r_1)$;

(ii) $K_1(r_1)X(r_1)$;

(iii) $(1 - K_1(r_1))X(r_1)$;

(iv) $U_1(r_1)$; and

(v) $V_1(r_1)$ are all increasing functions of r_1, and

(iv) $V_1'(r_1) < p(a_1(r_1))X'(r_1)$.

The proposition demonstrates that a higher level of current reputation increases the agent's level of effort, the total reward to both the agent and the principal in the win state, and the expected reward to both the principal and the agent. In addition, any increase in current reward as a result of increased reputation accrues to both the agent and the the current principal since $V_1'(r_1) < p(a_1(r_1))X'(r_1)$.

Recall (from Section II) that we can write, for any initial level of reputation r, the principal's expected rewards with an incentive contract and with a fixed-payment contract respectively as:

$$U_2(r) = \max_a p(a)\left[X(r) - \frac{g'(a)}{p'(a)} + \beta(V_{1W}(r) - V_{1L}(r))\right], \qquad (3.2)$$

and

$$U_2^*(r) = \max_a p(a)X(r) - g(a). \qquad (3.3)$$

For ease of exposition, we let $A(r;W,L) \equiv V_{1W}(r) - V_{1L}(r)$; $A(\cdot)$

represents the net effect of winning over losing in the first period on the last period's rewards for the agent. From Proposition 3.1 and the definition of $V_{1W}(r)$ and $V_{1L}(r)$ we have

$$A(r;W,L) > 0. \tag{3.4}$$

In the following Theorem we give sufficient conditions under which the first period principal is strictly better off with an incentive contract than with a fixed-payment contract.

THEOREM 3.1

If (i) $X(r)$ is unbounded from above; and

(ii) $r_{1s}(r) = r_1(r,s)$ for $s=W,L$

where $r_1(\cdot,\cdot)$ is strictly increasing and unbounded from above in both of its arguments, then there exists a $\underline{W}(r)$ for every r such that for all $W > \underline{W}(r)$, $U_2(r) > U_2^*(r)$.

The interpretation of Theorem 3.1 is straightforward: if the differential effect next period between winning and losing in the current period is "large enough", the first period principal is strictly better off allowing the agent to capture some of the next period's rents from increased reputation. This large incremental benefit of winning over losing causes the agent to work harder (provide more effort) in the current period, and therefore increases the probability of winning in the current period; this extra effort benefits the principal as well.

We now give some comparative statics results on rewards in the first period:

PROPOSITION 3.2

Under the assumptions of the model,

(i) $V_2(r)$ is an increasing function of r;

When $X'(r) + \beta[V'_{1W}(r) - V'_{1L}(r)] > (<) 0$,

(ii) $U_2(r)$ is increasing (decreasing) in r;

(iii) $a_2(r)$ is increasing (decreasing) in r; and

(iv) $(1 - K_2(r))X(r)$ is increasing (decreasing) in r.

(v) $K_2(r)X(r)$ is (increasing) in r.

The agent's discounted sum of expected rewards has been shown to increase as initial reputation increases. Therefore, the agent is strictly better off with a higher level of reputation at the start of the planning horizon. Whether the principal in the first period is at least as well off with an agent with a high level of initial reputation as with an agent with a low level of initial reputation depends, in equilibrium, on the rate of change of the principal's expected utility, $U'_2(r) = p(a_2(r))(X'(r) + \beta(V'_{1W}(r) - V'_{1L}(r))$.

Thus, the first period principal's expected reward is increasing or decreasing in reputation as $X'(r) + \beta(V'_{1W}(r) - V'_{1L}(r))$ is increasing or decreasing in r. This expression represents the total discounted rents from winning in the first period. The first term, $X'(r)$, represents the change in the current periods total reward with increased reputation; by assumption, this change is positive. The second term, $\beta(V'_{1W}(r) - V'_{1L}(r))$, represents the discounted future net benefit of winning, to the agent, in the current period; i.e., the future rents that result from winning rather than losing in the current period. When this term is positive, the sum of the two terms is positive, and we have $U_2(r)$ increasing in r. When the latter term is negative, the change in $U_2(r)$ will depend on the relative magnitude of the two terms. Only when then marginal benefit of increased reputation is much larger if the agent loses in the current period than if the agent wins, will $U_2(r)$ be a decreasing function of r.

However, in general we cannot sign $(V'_{1W}(r) - V'_{1L}(r))$. When β is small, the $X'(r)$ term dominates, and we know that $U'_2 > 0$ since the future is heavily discounted by the agent. Since β is bounded from above by 1, we cannot necessarily make β large enough so that $X'(r) + \beta(V'_{1W}(r) - V'_{1L}(r)) > 0$ when the term in brackets is negative. Also when β is small, $X(r) + \beta(V_{1W}(r) - V_{1L}(r))$ gets closer to $X(r)$. As a result, for sufficiently small β, $U^*_2(r)$ will be strictly greater than $U_2(r)$. In this case, the first period principal will be strictly better off with a fixed price contract than with an incentive contract.

Just as the sign of $U'_2(r)$ depends upon the sign of $X'(r) + \beta(V'_{1W}(r) - V'_{1L}(r))$, so too do the signs of $a'_2(r)$, $\partial(1-K_2(r))X(r)/\partial r$, and $\partial K_2(r)X(r)/\partial r$. Only when $X'(r) + \beta[V'_{1W}(r) - V'_{1L}(r)] < 0$ is the relationship between reputation and (i) effort, (i) the agent's share, and (iii) the principal's share unambiguous. In this case the marginal benefit of increased reputation (to the agent) is much larger next period if the agent loses in the current period than if the agent wins in the current period. Increasing reputation increases the agent's effort and the principal's share and decreases the agent's share. The agent is trading off probability of winning for reward.

In the following proposition we give sufficient conditions for $X'(r) + \beta[V_{1W}'(r) - V_{1L}'(r)] \geq 0$, and thus sufficient conditions for $U_2(r)$, $a_2(r)$, and $(1-K_2(r))X(r)$ to be non-decreasing in r.

PROPOSITION 3.3

Under the assumptions of the model, and

if $X'(r) \geq r_{1L}'(r) \cdot X'(r_{1L})$,

then $X'(r) + \beta[V_{1W}'(r) - V_{1L}'(r)] \geq 0$.

When $X'(r) \geq r_{1L}'(r) \cdot X'(r_{1L})$ we have an impounding process and a reward structure in which an increase in current reputation increases current reward if the win state occurs this period more than it increases reward in the future if the lose state occurs this period. For example, if $r_{1L}(r) = \theta_L r + L$, where $\theta_L \leq 1$, we have a decay process. Since X(r) is assumed to be a concave function of r, our conditions are met.

The implication of this proposition is that the first period principal, if given a choice between agents with different levels of current reputation, should choose the agent with the highest level of current reputation.

Suppose that the initial level of reputation is fixed at some level r, but we allow W (or L) to vary so that $(V_{1W}(r) - V_{1L}(r)) = A(r;W,L)$ is increasing; for ease of exposition in what follows, denote this difference by A_r. This difference represents the net future benefit of winning over losing in the current period. We can describe what happens to the agent's and principal's current reward in the win state as A_r increases or decreases. This proposition can be used as an explanation of pro bono work in potentially landmark cases: the lawyer is willing to forego current payment because the net benefit of winning over losing is so large that the lawyer is willing to take payment in the "future". Another interpretation is that the client needs to supply less current incentive to get the lawyer to work; the lawyer is motivated by future rewards as well.

PROPOSITION 3.4

The agent's (principal's) share of current reward in the first period, $K_2(r)X(r)$ ($(1 - K_2(r))X(r)$) is a decreasing (increasing) function of A_r.

To summarize, in this section we have demonstrated that in the last period both the agent's and principal's expected rewards are increasing functions of initial reputation, as are the agent's level of effort and the share of current reward going to the principal. In the first period, the agent's sum of discounted expected reward is increasing in initial reputation; however, the first period principal's expected reward is increasing in initial reputation only if the total discounted rents from winning in the first period are increasing in initial reputation. Finally, and most importantly, we have demonstrated the existence of conditions under which a risk-neutral first period principal is strictly better off with an incentive contract than with a fixed-payment contract, even though the agent is also risk-neutral. These results come about because in effect, an implicit intertemporal transfer is made between the last and first period principals.

IV. SURPLUS MAXIMIZING INCENTIVE CONTRACTS: AN EXAMPLE

In this section we provide an example in which the total ex ante expected surplus is strictly greater with incentive contracts that with fixed-payment contracts. We assume that $p(a)$, $g(a)$, $r_{1s}(r)$, and $X(r)$ have the following functional forms:

(i) $p(a) = (1 - e^{-a}) \qquad a \geq 0.$

(ii) $g(a) = a \qquad a \geq 0.$

(iii) $r_{1s}(r) = \theta r + s \quad s = W, L;\ \theta \leq 1$ and $W > L.$

(iv) $X(r) = \ln(r + 1) + 1.$

It is straightforward (though tedious) to verify that the following hold:

(i) $V_1(r_{1s}(r)) = \sqrt{r_{1s}(r)} - 1 - \ln\sqrt{r_{1s}(r)}$ and

$A(r) \equiv V_1(r_{1W}(r)) - V_1(r_{1L}(r));$

(ii) $U_1(r_{1s}(r)) = (\sqrt{r_{1s}(r)} - 1)^2;$

(iii) $U_1^*(r_{1s}(r)) = r_{1s}(r) - 1 - \ln r_{1s}(r);$

(iv) $V_2(r) = \sqrt{r+\beta A(r)} - 1 - \ln(\sqrt{r+\beta A(r)}) + \beta V_1(r_{1L}(r));$

(v) $U_2(r) = (\sqrt{r+\beta A(r)} - 1)^2;$ and

(vi) $U_2^*(r) = r - 1 - \ln r.$

When $\beta = .9$ and $\theta = .9$, for example, there are several sets of parameter values for r, W and L such that the total expected surplus is larger with the incentive contracts than with the fixed-payment contracts. In particular, if $r = .2$, $L = 0$, and $W = 30$, we get first period reward of $X(r) = 1.1832$, and last period rewards of $X(r_{1w}) = 4.46687$ and $X(r_{1L}) = 1.16551$. The **ex ante** total expected surplus with the fixed-payment contract is $U_2^*(r) + \beta E[U_1^*(r_{1s}(r)] = .31950$, where expectation is taken with respect to the probabilities of winning and losing in the first period with the fixed-payment contract. In contrast, the ex ante total expected surplus with incentive contracts is $V_2(r) + U_2(r) + \beta E[U_1(r_{1s}(r))] = .32043$, where the expectation is taken with respect to the probabilities of winning and losing in the first period with the incentive contracts. The superiority of incentive contracts over fixed price contracts (for this example) increases monotonically as W is increased.

The above result depends directly on the functional forms and parameter values chosen. However, we have chosen reasonable functional forms and parameter values that give us non-pathological results (all effort levels are positive and all shares are positive). Thus, we have demonstrated the existence of interesting situations in which the total surplus can be made larger by allowing the agent's to capture rents each period instead of forcing the agent's down to their reservation wages.

V. CONCLUSIONS

We have developed a model in which intertemporal interdependcies and divergence of time preferences alter the form of optimal contracts between a risk-neutral agent and risk-neutral principals. The agent wishes to maximize the discounted sum of benefits over his or her lifetime; the principals each want to maximize only current rewards.

Conditions exist under which all principals (except the final period's principal) can be made strictly better off by allowing the agent to capture some surplus each period, instead of forcing the agent down to the reservation wage with a fixed-payment contract. The incentive contracts serve as a means of aligning the time preferences of the agent and the principals.

In addition, we have a non-pathological example in which the total ex ante suplus generated by the incentive contracts is strictly larger than that generated by the fixed-payment contracts. Our results can be used as a partial explanation of why we don't see risk-neutral agents buying output from risk-neutral principals, as well as why there are implicit or explicit prohibitions against such contracts in some professions (such as the legal profession).

APPENDIX

Proof of Proposition 2.1:

We wish to show that the following constrained maximization problems are equivalent; that is, $(K(r_1), a(r_1))$ solves

$$\max_{K,a} p(a)(1 - K)X(r_1)$$
$$\text{s.t. } p'(a)KX(r_1) - g'(a) = 0;$$
$$p(a)KX(r_1) - g(a) \geq 0$$

iff $(K(r_1), a(r_1))$ solves

$$\max_{K,a} p(a)(1 - K)X(r_1)$$
$$\text{s.t } p'(a)KX(r_1) - g'(a) = 0.$$

We demonstrate this result by showing that the feasible sets for both of these problems are the same. Let

$$F(r_1) \equiv \{(K,a) | p'(a)KX(r_1) - g'(a) = 0,\ p(a)KX(r_1) - g(a) \geq 0\};$$
$$G(r_1) \equiv \{(K,a) | p'(a)KX(r_1) - g'(a) = 0\}.$$

Clearly, $F(r_1)$ is contained in $G(r_1)$. We must now show that $G(r_1)$ is contained in $F(r_1)$. Let $(\hat{K}, \hat{a})$ be such that $p'(\hat{a})\hat{K}X(r_1) - g'(\hat{a}) = 0$ and assume that $p(\hat{a})\hat{K}X(r_1) - g(\hat{a}) < 0$. Since $g(a)/p(a)$ is non-decreasing in a, we know that $g'(\hat{a})p(\hat{a}) - p'(\hat{a})g(\hat{a}) \geq 0$. Solving for

$\hat{K}$, we get

$$\hat{K} = \frac{g'(\hat{a})}{p'(\hat{a})X(r_1)}$$

Substituting this into $p(\hat{a})\hat{K}X(r_1) - g(\hat{a}) < 0$, gives us

$$p(\hat{a}) \frac{g'(\hat{a})}{p'(\hat{a})X(r_1)} X(r_1) - g(\hat{a}) =$$

$$\frac{p(\hat{a})g'(\hat{a}) - p'(\hat{a})g(\hat{a})}{p'(\hat{a})} < 0.$$

Since $p'(\hat{a}) > 0$, this contradicts our assumption that $g(a)/p(a)$ is non-decreasing in a. Therefore, $p(\hat{a})\hat{K}X(r_1) - g(\hat{a}) \geq 0$, and we have $G(r_1)$ is contained in $F(r_1)$. QED

Proof of Proposition 2.2:

We wish to demonstrate that the following constrained maximization problems are equivalent: that is, that $(K(r),a(r))$ solves

$$\max_{a} \; p(a)(1 - K)X(r)$$

$$\text{s.t. } p'(a)[KX(r) + \beta(V_{1W}(r) - V_{1L}(r))] - g'(a) = 0;$$

$$p(a)KX(r) - g(a) + \beta[p(a)V_{1W}(r) + (1-p(a))V_{1L}(r)] \geq 0.$$

iff $(K(r),a(r))$ solves

$$\max_a \; p(a)(1 - K)X(r)$$

$$\text{s.t. } p'(a)[KX(r) + \beta(V_{1W}(r) - V_{1L}(r))] - g'(a) = 0.$$

We proceed exactly as the proof of Proposition 2.1. Let

$$F(r) \equiv \{(K,a) \mid p'(a)[KX(r) + \beta(V_{1W}(r) - V_{1L}(r))] - g'(a) = 0$$
$$\text{and } p(a)KX(r) - g(a) + \beta(p(a)V_{1W}(r) + (1-p(a))V_{1L}(r)) \geq 0\}$$

and

$$G(r) \equiv \{(K,a) \mid p'(a)[KX(r) + \beta(V_{1W}(r) - V_{1L}(r))] - g'(a) = 0.\}$$

Clearly, $F(r)$ is contained in $G(r)$. We again must demonstrate that $G(r)$ is contained in $F(r)$. Let $(\hat{K},\hat{a}) \in G(r)$. Then

$$\hat{K} = \frac{g'(\hat{a})}{p'(\hat{a})X(r)} - \frac{\beta(V_{1W}(r) - V_{1L}(r))}{X(r)}$$

and

$$p(\hat{a})\frac{g'(\hat{a})}{p'(\hat{a})} - \beta p(\hat{a})[V_{1W}(r) - V_{1L}(r)] - g(\hat{a})$$
$$+ \beta[p(a)V_{1W}(r) + (1-p'(\hat{a})V_{1L}(r)] =$$

$$p(\hat{a})\frac{g'(\hat{a})}{p'(\hat{a})} - g(\hat{a}) + \beta V_{1L}(r) \geq 0,$$

since $p(\hat{a})g'(\hat{a}) - g(\hat{a})p'(\hat{a}) \geq 0$ by assumption and $V_{1L}(r) \geq$ from Proposition 2.1. Therefore, $G(r)$ is contained in $F(r)$.

Proof of Proposition 3.1:

Note that we do not proceed in order from (i) through (vi).

(i) By definition, $a_1(r_1)$ is chosen to solve

$$\max_{a} \; p(a)X(r_1) - p(a)\frac{g'(a)}{p'(a)}$$

The first order conditions for this problem are then

$$p'(a)X(r_1) - g'(a) - p(a)\left[\frac{g''(a)p'(a) - p''(a)g'(a)}{\{p'(a)\}^2}\right] = 0. \qquad (A.1)$$

These conditions are necessary and sufficient by concavity of p(a), convexity of g(a) and strict convexity of g'(a)/p'(a). Let the left hand side of (A.1) be denoted by $F_1(a,r_1)$ for ease of exposition. Totally differentiating this function with respect to r_1 gives

$$\frac{\partial F_1}{\partial a} a_1' + \frac{\partial F_1}{\partial r_1} = 0.$$

From the second order conditions we have F_1 non-increasing in a. The sign of a_1' is thus the same as the sign of

$$\frac{\partial F_1}{\partial r_1} = p(a_1(r_1))X'(r_1) > 0.$$

Therefore, we have our desired result that a_1 is increasing in r_1.

(ii) To show that $K_1(r_1)X(r_1)$ is increasing in r_1, we note that

$$K_1(r_1)X(r_1) = \frac{g'(a_1(r_1))}{p'(a_1(r_1))}$$

Therefore

$$\frac{\partial K_1(r_1)X(r_1)}{\partial r_1} = \frac{(g''p' - p''g')}{\{p'\}^2} a_1' > 0.$$

(iv) By definition

$$U_1(r_1) = p(a_1(r_1))X(r_1) - p(a_1(r_1))\frac{g'(a_1(r_1))}{p'(a_1(r_1))}.$$

Therefore

$$U_1'(r_1) = p'a_1'X + pX' - p'a_1'\frac{g'}{p'} - p\frac{(g''p' - p''g')}{\{p'\}^2}a_1'$$

$$= [p'X - g' - p\frac{(g''p' - p''g')}{\{p'\}^2}]a_1' + pX'$$

$$= pX'$$

in equilibrium. Since $X' > 0$, we have our desired result.

(v) By definition

$$V_1(r_1) = p(a_1(r_1))\frac{g'(a_1(r_1))}{p'(a_1(r_1))} - g(a_1(r_1)).$$

Therefore,

$$V_1'(r_1) = p'a_1'\frac{g'}{p'} + p\{ \frac{g''p' - p''g'}{(p')^2} \}a_1' - g'a_1'$$

$$= p\left\{\frac{g''p' - p''g'}{(p')^2}\right\}a_1' > 0, \tag{A.2}$$

from part (i) above.

(vi) From (A.2) we have

$$V_1'(r_1) = p\left\{\frac{g''p' - p''g'}{(p')^2}\right\}a_1' \tag{A.3}$$

Totally differentiating (A.1) with respect to r_1 gives us

$$a_1' = -\frac{-p'X'}{p''X - g'' - \frac{\{g''p'-p''g'\}}{p'} - p\frac{\partial^2}{\partial a^2}\frac{g'}{p'}} \tag{A.4}$$

Substituting (A.4) into (A.3) gives

$$V_1'(r_1) = \frac{p\{g''p' - p''g'\}p'X'}{(p')^2\left(-p''X + g'' + \frac{\{g''p'-p''g'\}}{p'} + p\frac{\partial^2}{\partial a^2}\frac{g'}{p'}\right)}. \tag{A.5}$$

From (A.1) we have that in equilibrium,

$$X = \frac{p}{(p')^2}\{g''p'-p''g'\} + \frac{g'}{p'}$$

so that

$$p''X - g' = \frac{p''p}{(p')^3}\{g''p'-p''g'\} + p''\frac{g'}{p'} - g'.$$

$$= \{g''p'-p''g'\}\left[\frac{p''p}{(p')^3} - \frac{1}{p'}\right].$$

Therefore, the denominator in (A.5) can be written as

$$-\{g''p'-p''g'\}\left[\frac{p''p}{(p')^3} - \frac{1}{p'}\right] + \{g''p'-p''g'\}\left[\frac{1}{p'}\right] =$$

$$\{g''p'-p''g'\}\frac{(2(p')^2 - p''p)}{(p')^3} + p\frac{\partial^2}{\partial a^2}\frac{g'}{p'} > \qquad \text{(A.6)}$$

$$\{g''p'-p''g'\}\frac{(2(p')^2 - p''p)}{(p')^3}$$

since g'/p' is assumed strictly convex in a. Substituting (A.6) into (A.5) gives us

$$V_1'(r_1) < \frac{p\{g''p'-p''g\}X'}{\{g''p'-p''g'\}\frac{(2(p')^2-p''p)}{(p')^2}} =$$

$$\frac{p(p')^2}{2(p')^2-p''p}X' <$$

$$pX',$$

since $p'>0$, $p''<0$ and $p\leq1$ implies that $(p')^2/(2(p')^2-p''p) < 1$; we thus have our desired result.

(iii) To demonstrate $(1 - K_1(r_1))X(r_1)$ is increasing in r_1 we use (A.4) to solve for X' in terms of a_1', and

$$\frac{\partial(1 - K_1(r_1))X(r_1)}{\partial r_1} = X'(r_1) - \frac{\{g''p' - p''g'\}}{(p')^2}a_1'$$

$$= \left\{ \frac{-p''}{p'} X + \frac{g''}{p'} + \left[\frac{\partial}{\partial a} \frac{g'}{p'}\right] + p\left[\frac{\partial^2}{\partial a^2} \frac{g'}{p'}\right] - \left[\frac{\partial}{\partial a} \frac{g'}{p'}\right] \right\} a_1'$$

$$= \left\{ \frac{-p''}{p'} X + \frac{g''}{p'} + \left[\frac{\partial^2}{\partial a^2} \frac{g'}{p'}\right] \right\} a_1' > 0$$

by assumptions of p, g, and g'/p', and the result that $a_1' > 0$.

QED

Proof of Theorem 3.1:

(i) We first prove that $A(r;W,L)$ is strictly increasing and unbounded from above in W. From Proposition 3.1, $V_1(r_1)$ is strictly increasing in r_1. Since $V_{1W}(r) \equiv V_1(r_{1W}(r))$ and $r_{1W}(r) \equiv r_1(r_1,W)$ is assumed strictly increasing in W, we have $V_{1W}(\cdot)$ strictly increasing in W. Since $V_{1L}(r)$ is independent of W, we have $A(r;W,L)$ strictly increasing in W. $A(r;W,L)$ unbounded from above is immediate from $X(r)$ unbounded from above, so that $V_1(r)$ is unbounded from above, and $r_1(\cdot,\cdot)$ unbounded from above.

(ii) We define the following function for ease of exposition:

$$M(r,A) \equiv \text{MAX}\ p(a)\left[\ X(r) - \frac{g'(a)}{p'(a)} + A\ \right]. \tag{A.7}$$

Note that the right hand side of (A.7) is just (3.2) with $\beta(V_{1W}(r) - V_{1L}(r))$ replaced by A. Let

$$A(r) \equiv \underset{a}{\text{MIN}} \left\{ \frac{U_2(r)}{p(a)} + \frac{g'(a)}{p'(a)} - X(r) \right\}. \tag{A.8}$$

To demonstrate that the MIN is obtained in (A.8), we note that $1/p(a)$ is decreasing and convex in a, and that $g'(a)/p'(a)$ is increasing and strictly convex in a. Thus, the MIN will be obtained at some $\underline{a}(r)$. Then

$$p(\underline{a}(r))[X(r) - \frac{g'(\underline{a}(r))}{p'(\underline{a}(r))} + A(r)] = U_2(r).$$

By definition,

$$M(r,A(r)) \geq p(\underline{a}(r))[X(r) - \frac{g'(\underline{a}(r))}{p'(\underline{a}(r))} + A(r) = U_2(r).$$

Also from the definition of $M(r,A)$ we have

$$M(r,A) \geq M(r,A(r)) \qquad \text{for all } A \geq A(r).$$

Clearly, $A(r) > 0$ for all r. We now must show that we can find a $W(r)$, for fixed L, such that

$$\beta(V_1(r_1(r,w)) - V_1(r_1(r,l))) \geq A(r). \tag{A.9}$$

It is sufficient for (A.9) that

(i) at $W = L$, $\beta(V_1(r_1(r,W)) - V_1(r_1(r,L))) = 0$, and

(ii) $V_1(r_1(r,W)) - V_1(r_1(r,L))$ is an increasing, unbounded function of W.

Let

$$W(r) \equiv \text{MIN}\{\, W \mid \beta(V_1(r_1(r,W)) - V_1(r_1(r,L))) \geq A(r)\}.$$

Then, for all $W \geq W(r)$,

$$\max_a p(a)\left[X(r) - \frac{g'(a)}{p'(a)} + \beta(V_1(r_1(r,W)) - V_1(r_1(r,L))\right) \geq$$

$$\max_a p(a)X(r) - g(a),$$

which is our desired result.

QED

Proof of Proposition 3.2:

(i) It is straightforward to verify that in equilibrium,

$$V_2(r) = p(a_2(r)) \frac{g'(a_2(r))}{p'(a_2(r))} - g(a_2(r)) + \beta V_1(r_{1L}(r))$$

and (A.10)

$$V_2'(r) = p\left(\frac{g''p' - p''g'}{(p')^2} \right)a_2' + \beta V_{1L}'.$$

We thus need an explicit expression for a_2' in order to sign V_2'. We note that the principal chooses $a_2(r)$ to solve

$$p'(a)[X(r) + \beta(V_{1W}(r) - V_{1L}(r))] - g'(a)$$

$$- p(a)\left[\frac{g''(a)p'(a) - p''(a)g'(a)}{(p'(a))^2}\right] = 0. \qquad \text{(A.11)}$$

Denoting the left hand side of (A.10) by $F_2(a,r)$, we can totally differentiate the left hand side of (A.10) with respect to r and rearrange terms to get:

$$a_2'(r) = \frac{p'[X'(r) + \beta(V_{1W}' - V_{1L}')]}{-\partial F_2/\partial a} \tag{A.12}$$

where $V_{1i} = V_1'(r_{1i})\cdot r_{1i}'(r)$ for i=W,L. Substituting (A.12) into (A.10) gives us

$$V_2'(r) = \frac{p}{(p')^2}\{g''p' - p''g'\}\frac{p'(X' + \beta(V_{1W}' - V_{1L}'))}{-F_{21}} + \beta V_{1L}'$$

$$= \frac{1}{-F_{21}}\ [\frac{p}{p'}\{g''p' - p''g'\}\{X' + \beta(V_{1W}' - V_{1L}')\} - \beta V_{1L}'F_{21}]$$

We must thus sign the term in the square brackets. From (A.11) we have

$$F_{21} = p''[X + \beta(V_{1W} - V_{1L})] - g'' - \frac{\{g''p' - p''g'\}}{p'} - p\frac{\partial^2}{\partial a^2}\frac{g'}{p'}. \tag{A.13}$$

Solving for $X + \beta(V_{1W} - V_{1L})$ from (A.11) gives us

$$X + \beta(V_{1W} - V_{1L}) = \frac{g'}{p'} + \frac{p}{(p')^3}\{g''p' - p''g'\}. \tag{A.14}$$

Using (A.13) in (A.14) results in

$$F_{21} = p''[\frac{g'}{p'} + \frac{p}{(p')^3}\{g''p' - p''g'\}] - g'' - \frac{\{g''p' - p''g'\}}{p'}$$

$$= \frac{\{p''g' - g''p'\}}{p'} + \frac{p'p''}{(p')^3}\{g''p' - p''g'\} - \frac{\{g''p' - p''g'\}}{p'}$$

$$= \{\frac{p'p'' - 2(p')^2}{(p')^3}\}\{g''p' - p''g'\} - p\frac{\partial^2}{\partial a^2}\frac{g'}{p'}. \tag{A.15}$$

Substituting the right hand side of (A.15) into the term in square brackets in (A.13) and collecting terms results in

$$\operatorname{sgn} V_2'(r) = \operatorname{sgn} \{ \frac{p}{p'}(g''p' - p''g')(X' + \beta V_{1W}')$$

$$- \beta V_{1L}'(g''p' - p''g')(\frac{p}{p'} + \frac{pp'' - 2(p')^2}{(p')^3})$$

$$+ \beta V_{1L}' \frac{\partial^2}{\partial a^2} \frac{g'}{p'} \}.$$

Since $V_{1i}' > 0$ for i=W,L and and g'/p' is strictly convex, a sufficient condition for $V_2' > 0$ is therefore

$$\frac{p}{p'} + \frac{pp'' - 2(p')^2}{(p')^3} < 0.$$

which follows immediately from assumptions on p.

(ii) This result is immediate from the fact that in equilibrium,

$$U_2'(r) = p'(a_2(r))\{X'(r) + \beta(V_{1W}'(r) - V_{1L}'(r))\}$$

(iii) From (A.12) we have

$$a_2'(r) = \frac{p'[X'(r) + \beta(V_{1W}' - V_{1L}')]}{-\partial F_2/\partial a}$$

Since $-\partial F_2/\partial a = F_{21} > 0$ by the assumptions on $p(\cdot)$, $g(\cdot)$ and $g'(\cdot)/p'(\cdot)$, and $V_{1W} > V_{1L}$ from Proposition 3.1, we have the sign of a_2' the same as the sign of $X' + \beta(V_{1W}' - V_{1L}')$.

(iv) Using the individual rationality constraint to solve for $K_2(r)X(r)$; and (A.12) to solve for $X + \beta\Delta 1_{\delta} \div \theta \simeq \equiv \Delta 1_{\omega} \div \theta \simeq\simeq$ $\iota\nu$ $\tau\epsilon\theta\mu\sigma$ $\rho\partial$ $\partial[g'/p']/\partial a$ in equilibrium, we have

$$\frac{\partial(1 - K_2(r))X(r)}{\partial r_2} = X' - \frac{\partial}{\partial a}\left[\frac{g'}{p'}\right]a_2' - \beta(V_{1L}' - V_{1W}')$$

$$= \left\{\frac{-p''}{p'}(X + \beta(V_{1W}-V_{1L})) + \frac{g''}{p'} + \frac{p}{p'}\,\frac{\partial^2}{\partial a^2}\left[\frac{g'}{p'}\right]\right\}a_2'.$$

Since the term in curly brackets is strictly positive, the derivative of $(1 - K_2(r))X(r)$ with respect to r has the same sign as a_2', which in turn has the same sign as $(X' + \beta(V_{1W}' - V_{1L}'))$, which is our desired result.

QED

Proof of Proposition 3.3:

This result is immediate from

(i) $V_1'(r_1) < p(a_1(r_1))X'(r_1) < X'(r_1)$ for all r_1,

(ii) $V_{1L}'(r) = V_1(r_{1L}(r))\cdot r_{1L}'(r)\cdot > 0$, and

(iii) $\beta \leq 1$.

QED

Proof of Proposition 3.4:

(Note that primes now denote derivatives with respect to A_r). Totally differentiating the principal's first order condition with respect to A_r gives us:

$$F_{21} \cdot a_2' + \beta p'(a_2) = 0. \tag{A.16}$$

Since $X(r)$ does not depend on A_r, we know that $\partial(1 - K_2(r))X(r)/\partial A_r = -\partial K_2(r)X(r)/\partial A_r$. In equilbrium,

$$\frac{\partial K_2(r)X(r)}{\partial A_r} = \frac{\partial}{\partial a}\left[\frac{g'}{p'}\right]a_2' - \beta \tag{A.17}$$

Using (A.16) to solve for a_2' and substituting gives:

$$\frac{\partial K_2(r)X(r)}{\partial A_r} = \frac{-\beta}{F_{21}}\{p''(X + A_r) - g'' - p\frac{\partial^2}{\partial a^2}[\frac{g'}{p'}]\} < 0.$$

Thus, we have our desired result that the agent's share, $K_2(r)X(r)$, is decreasing in A_r and the principal's share, $(1 - K_2(r))X(r)$, is increasing in A_r.

QED

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FOOTNOTES

1. In a recent paper Laffont and Tirole [1988] allow the principal only to offer short term (one-period) contracts to an agent in a two-period model. However, the agent's ability is unknown to the principal and the principal's objective is the maximization of welfare over the two periods.

2. That is, the effect of effort and reputation are "separable". Effort effects the probability of winning but not the reward size. Reputation, on the other hand, effects the reward size, but not the probability of winning.

3. Note that we have implicitly excluded linear sharing rules that give the agent KX - k if the agent wins and -k if the agent loses. With this kind of sharing rule, the principal can force the agent down to the reservation wage each period by the appropriate choice of k; this kind of contract would provide the same reputational incentives as selling X to the agent, and the principal would be strictly worse off.

4. The assumptions on p(a) and g(a) assure that the "first order approach" is valid, since the first order conditions are both necessary and sufficient. Thus, the first order conditions define a unique level of effort a.

5. Problem (2.9) is obtained by solving the incentive compatibility constraint for K and substituting into the objective function in (2.7).