"Researches in Geometry"

BY

DR. P. S. G. DUBASH, D. SC.,
DR. CHROM., DR. SAN. SC.,
F.S.P., F.B.E A., ETC.

To the Late
Mr. JALBHOY BHARDA, B.A.,
the Principal of the New High School, Bombay
this trifling pamphlet is dedicated.

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"RESEARCHES IN GEOMETRY"

BY

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Dr. Chrom., Phys. M., Phil. B.**

Patron of the International College of Chromatics ;

Founder's Chairman of the Workers' Welfare League of India (London.)

Agrégé de la Société Internationale de Philologie, Science et Beaux-Arts ;

Fratulo de la Brita Esperanto Asocio ;

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Diplomed in the first-class final of "Electro-metallurgy and Allied Sciences" of the City and Guilds of London Institute.

Diplomed first in "Sanitary Science as applied to Public Works" of the Royal Sanitary Institute.

Author of

(1) "Dreaming ;" (2) "Rationalistic and other Poems ;" (3) "Continuity from Electrons to Infinity ;" (4) "Colour and the Child ;" (5) "Romance of Souls" and (6) "Hygenie of Town-Planning and Vegetation."

Discoverer of

(1) Fish-culture systems of sewage and refuse disposal ;

(2) The actual hearing of colours by the blind, (as distinct from colour-hearing) ;

(3) The National Museums of the Great Dead ; (having statuettes made from the ashes of cremation of the great dead men of different nations.)

(4) The Arithmetic Phenomena of Sum-digital Recurrance ;

(5) The Law of Complimentary Colours ;

(6) Researches in Colour-vision and Colour-Psychology ;

(7) The definition of man—a smile-able animal.

CAJORI

INTRODUCTION.

In order to prevent any very excitable critic from committing suicide, I write this introduction in the first place, and secondly because I have some interesting facts to give to my readers.

This booklet has only a few geometric constructions for inscribing regular polygons in given circles-without the use of a protractor. I was never taught to do this either in my school or colleges, and so, when I succeeded in getting these methods I was very much satisfied but to make sure about the credit of being the first discoverer I wrote to Mr. Jalbhai Bharda the principal of the New High School, Bombay and asked him if any one had already done this. As Mr. Bharda delayed in answering I wrote to Mr. Sohrabsha J. Bulsara, M.A., who informed me that Burchet had obtained some methods for doing this, and so, I was a little disappointed and had decided to drop the idea of giving publicity to these. In the mean time my beloved principal Mr. Bharda died. A few nights after his passing away I dreamt that he told me that though it was true that Burçhet had descovered some methods, mine were some what different and that there was something good about some of my geometric constructions. So I sent my manuscripts to Mr. Bulsara and requested him to compare my methods with those of Burchet. He informed me that mine were different from Burchet's. So I have once again emboldened myself to give publicity to these in spite of the fright of the critics saying that I am claiming too much.

Construction No. 1 and Construction No. 2 had appeared in the "Indian Engineering" of the 30th October, 1920 under the title "Squaring the Circle and Trisecting Angles." Critics having found nothing new or interesting with the rest of the article I have only selected these two from it. It was through the generosity of Mr. Milne of the "Indian Engineering" that I got my article well criticised by the public. Research of any kind is more or less at a discount in India, and mathematical research so much so, that it is not easy to give publicity to this if the author is not a professional mathematician and commits slight technical errors as I do. So in spite of the grumbling against some Anglo-Indians I am glad to say that Mr. Milne gave me a very fair hearing. My attempt was purely to get a practical solution of the great problem of squaring a circle, and though my construction gives correct results to two places after the decimal point, and can be varied to give correct results to four places, I maintain mine is a simpler construction than others and its correctness can be proved, which is more than what can be said for some others. Still this is just the point on which some critics find fault with my construction. To please these I have put forward another construction for squaring a circle on the lines suggested in Encyclopædia Britannica. I may add a few words here about the construction for trisecting angles. The construction for trisecting a sixty-degrees angle as discovered by me was discovered before as Mr. Bharda told me personally. However very few others have said this. In this brochure I have published another construction for trisecting any angle upto 54°.

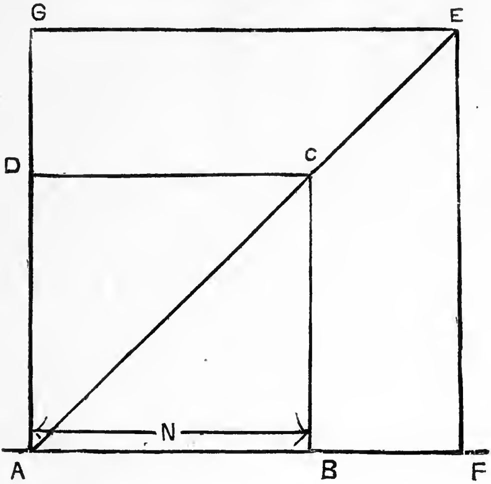
I have already thanked Mr. Milne of the "Indian Engineering" but I have still to thank Mr. Wilson who encouraged me still more by publishing my article "The Lost Theorems of Euclid" which appeared in the "Indian Engineering" of the 30th April, 1921. While an European editor gave this article a fair trial, an Indian mathematician gave it unfair criticisms, by giving the reader of the criticism "An Exhausted Mine" which appeared in the "Bombay Chronicle" an entirely wrong impression. The critic repeated most of the things I had said myself but gave them an appearance of rediculing me. The title itself is indicative of the fact that I granted that Euclid might have known these theorems. Yet the critic harped on this point much. Again he forgot that Newtons Laws of Motion were more simple and commonplace than my propositions yet they were called after him because he systematised them.

Mr. J. Connolly of the "Indian Engineering" who followed Mr. Wilson has been the most encouraging, so far, not only by publishing my arithmetical research "Sum-digital Recurrance" but even by giving me the blocks of the diagrams for which I have to thank him very much indeed.

P. S. G. DUBASH.

CONSTRUCTION No. I.

To square a square or to construct a square equal in area to the square of (n^2) on a given dimension n.



If a square, n^2, is constructed on a given dimension n then to construct a square equal in area to the square of the first square, *i.e.* $(n^2)^2$.

On line AB of N units in length construct a square ABCD. Join A and C and prolong it to a convenient length. Now mark off AE by taking the diagonal as many times as the number N of any definite units it represents.

If N equals		1·5	then make AE	1·5 AC	
"	"	2·0	"	" 2 0 AC	
"	"	2·25	"	., 2·15 AC	and so on.

Then from E draw a line perpendicular to line AD prolonged and cutting it at G and from E draw another perpendicular to AB prolonged cutting it at F. Now this square AFEG is equal to $(N^2)^2$.

Proof AB equals N and as ABC is a right-angle triangle

$$AC^2 = AB^2 + BC^2 = N^2 + N^2, \text{ i.e., } 2N^2$$

therefore $AC = \sqrt{2N^2} = N\sqrt{2}$

$AE^2 = AF^2 + FE^2$, but as AE is the locus of all points equidistant from AB and

AD ∴ EF=EG, but as EG and EF are perpendiculars to AG and AF the figure AFEG must be a square.

$$\therefore AE = N\ (N\sqrt{2})$$

$$\therefore AE^2 = \left\{ N\ (N\sqrt{2}) \right\}^2 = AF^2 + FE^2 = 2\ AF^2$$

$$= N^4 \times 2 = 2AF^2$$

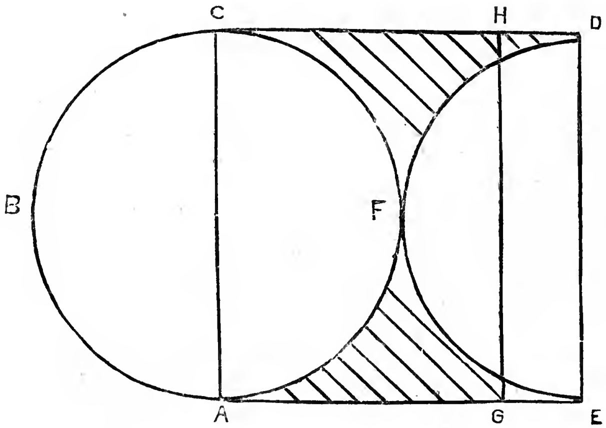
$$\therefore N^4 = AF^2$$

$$\therefore AF^2 = (N^2)^2$$

Q. E. D.

CONSTRUCTION No. II.

To construct a square equal in area to the area of a given circle.



This is applicable in those cases where the dimensions are very large such as say 100 feet.

Let ABCF be any circle. On the diameter AC construct a square ACDE. Divide the line AE into 200 parts. Cut off GE equal to 43 such parts and draw a line parallel to DE and call it GH. Now by the usual geometrical method construct a square equal in area to the area of this newly derived rectangle AGHC. This square will be equal in area to the area of the given circle.

PROOF

If on DE as diameter we describe a semi-circle DFE then we see that the area of the square on the diameter is larger than the area of the two semi-circles AFC and DFE which together are equal in area to the circle ABCF. The square AEDC is greater than the circle ABCF by the sum of the areas CFD and AFE. This difference is

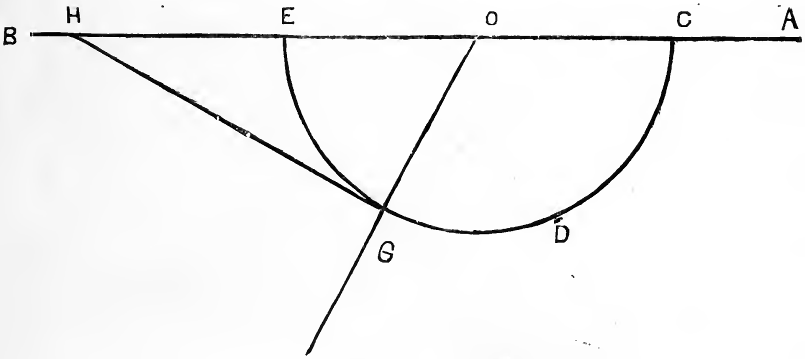
$$\left(D^2 - \frac{\pi D^2}{4}\right) = D^2\,(0{\cdot}215) = \frac{43}{200}\,D^2.$$

So the area of the rectangle is equal to the area of the circle because we have deducted from the square an area equal to the area of the sum of the areas CFD and AFE.

This would give an exact solution to the unsolved problem of ages of constructing a square equal in area of a given circle provided the diameter of the circle is large enough to enable one to divide it into 200 equal parts.

CONSTRUCTION No. III.

TO SQUARE A CIRCLE.



In the Cyclopædia Britannica on the subject of quadrature (or squaring a circle) it is said that the line of final solution of this problem of thousands of years is to obtain in some way a line equal to the circumferance of the given circle to be squared and on that line as base to construct a triangle with an altitude equal to the radius and then get a square equal in area to the area of this triangle.

Construction.—Draw any line AB of convenient length. On it describe a semi-circle CDE of the given circle. From O make an angle EOG of 62 degrees making OG cut the circumferance at G. From G draw a tangent to the circle

CDE and let it produced cut AB at H. Now CH will be equal to the semi-circumferance CDE.

∴ By taking CH twice we get the circumferance of the given circle.

∴ Taking 2 (C H) as base and r as altitude.

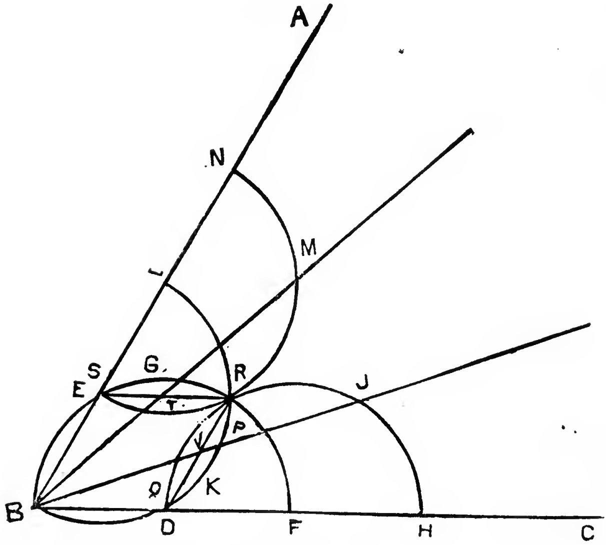
The area of the triangle $= 2\text{ CH} \times \frac{r}{2}$ but $2\text{ CH} = 2\pi r$

$$2\pi r \times \frac{r}{2} = \pi r^2$$

= area of the given circle.

Q. E. F.

CONSTRUCTION No. IV.



To trisect an angle of 60 degrees.

Let ABC be any angle of 60 degrees.

With any convenient radius cut BC at D and BA at E.

With the same radius and D and E as centres describe semi-circles FGB and LKB.

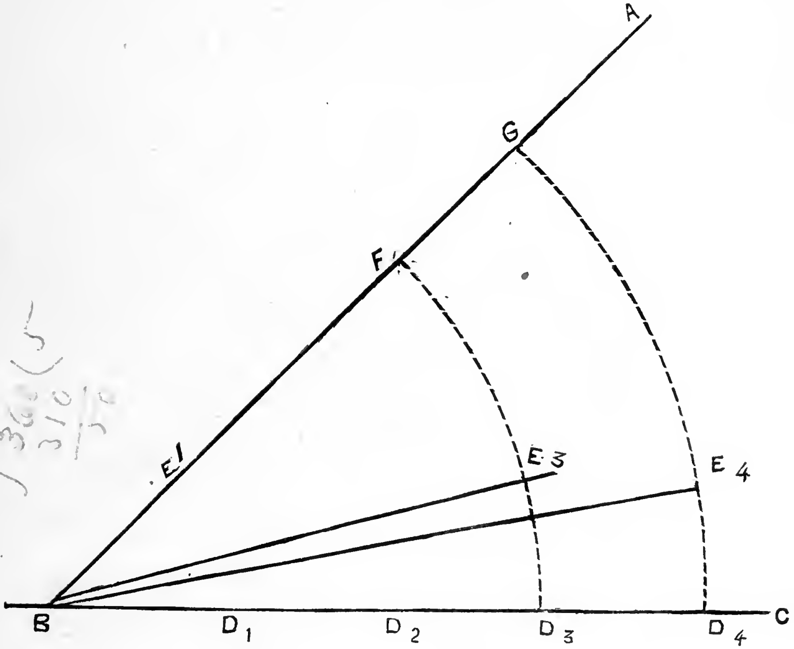
With same radius and F and L as centres describe HJD and NME semi-circles.

These four semi-circles will intersect so that the second of one line and the first of the other line and the second of the other line and the first of one line will create almond shaped areas with two extreme pairs of points PQ and SR. Join PQ and SR and bisect them. The points of bisections V and T joined to the point B will trisect the angle.

Q. E. F.

N.B.—In this figure D and Q, E and S and R and P are very close together and even coincide.

CONSTRUCTION No. V.



"Trisecting and Quartsecting Angles upto 54°"

Construction.—Let ABC be any angle upto 54°.

With any convenient radius cut BC at D_1, and BA at E_1. With the same radius mark off D_2, D_3 and D_4 along B.C.

With BD_3 as radius describe an are D_3F.

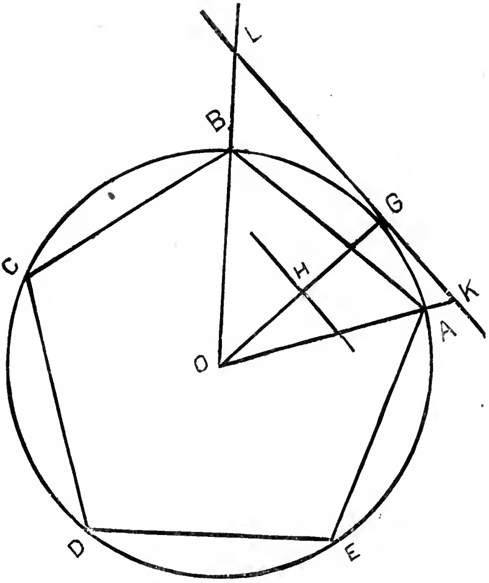
With the pair of compasses opened to radius equal to D_1E_1 mark off E_3 on D_3 F cutting it at E_3. Join E_3 and B. Now the angle E_3BC is one third ABC.

In the similar way by marking off along the arc D_4G, E_4 so that D_4E_4 is equal to D_1E_1 and by joining E_4 to B we get E_4BC equal to one fourth the angle ABC. In this case the angle ABC is 45° and the angle E_3BC is fifteen degrees and the angle E_4BC is 11°$\frac{1}{4}$.

N.B.—With the same construction we can divide an angle of thirty degrees into five equal parts also.

CONSTRUCTION No. VI.

TO INSCRIBE A REGULAR PENTAGON IN A CIRCLE.

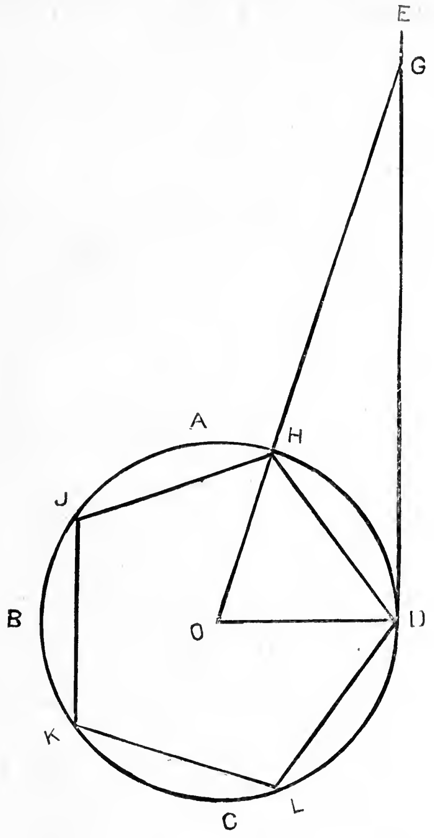


Construction.—Let ABCDE be any given circle. From G draw any radius OG. Bisect it at H. From G draw a line perpendicular to OG. On it mark off GK=GH and GL=OG. Join L and O and K and O. Now the angle LOK will be 72°. Let B be the point where LO cuts the circle and let A be the point KO cuts it. With a chord equal to AB mark off the points C, D and E. Join these points and ABCDE will be the inscribed regular pentagon.

N.B.—This construction is quite good enough for all cases. It is easy to see that angle LOG is 45°. Now as GK$=\frac{1}{2}$ GO tan GOK$=0.5$. Now tangent of $\angle 27° = 0.5095$. So the error in ratio is 0.0095 which is negligible for small figures.

CONSTRUCTION No. VII.

TO INSCRIBE AN AP-REGULAR PENTAGON WITHOUT A PROTECTOR.



(For Architects and Engineers.)

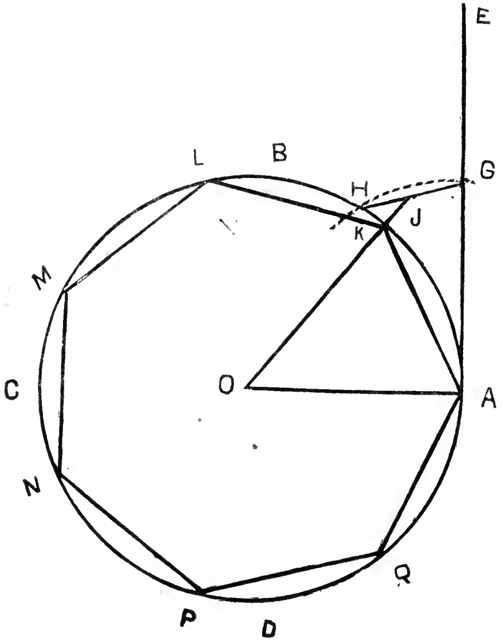
In the circle ABCD to inscribe a regular pentagon HJKLD.

Construction.—From D draw a radius DO and a perpendicular DE to DO. Mark off DG equal to three times the length of the radius DO. Join GO cutting the circumferance of the circle ABCD at H. Openning a pair of compasses to DH mark off other points J, K and L. Join DH, HJ, JK, KL, and LD and DHJKL is an inscribed ap-regular pentagon.

N.B.—In ninety per cent of cases the angle GOD comes exactly 72° but sometimes it is less. So, for practical purposes it is quite good but the theorectical mathematicians will have difficulty that it should ever be 72° considering that we have made tan GOD=3 while tan 72° is 3.0777. Still it is good enough judged by the practical results.

CONSTRUCTION No. VIII.

TO CONSTRUCT A REGULAR HEPTAGON.



In the circle ABCD to inscribe a regular heptagon AKLMNPQ.

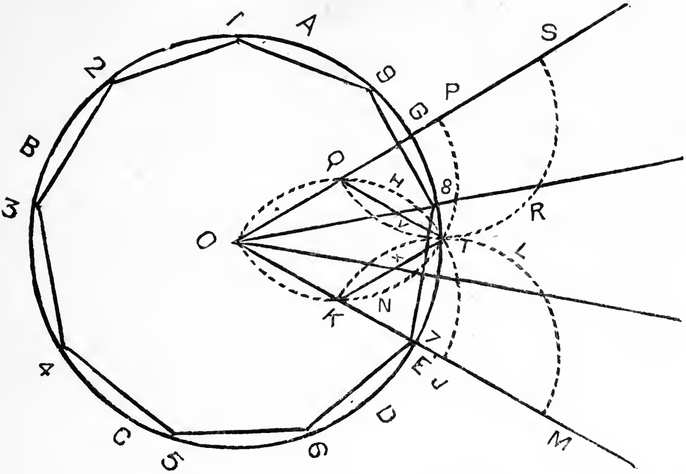
Construction.—Draw any radius OA. From A erect a perpendicular AE. With radius OA and centre A describe an arc HG cutting the circle ABCD at H and the perpendicular AE at G. Join HG. Bisect HG at J. Join J and O cutting the circle ABCD at K. With arc KA mark off points L, M, N, P and Q. Join AK, KL, LM, MN, NP, PQ, and QA. Then AKLMNPQ is a regular heptagon.

N.B.—Though I am not offering any absolute proof I can show that this construction is correct.

The point H is a point of a regular hexagon being marked by radius OA described. The point G if connected with O will give a line which will cut the circle at a point which will be of a regular Octagon. If we bisect the arc between them the result is approximate. But by bisecting HG and joining JO we get a point which is correct.

CONSTRUCTION No. IX.

"TO CONSTRUCT A NONAGON AND A DECOCTAGON WITHOUT A PROTRACTOR"



In any circle ABCD to construct a nonagon (1,-2,-3-4-5-6-7-8-9)

Construction.—Draw any radius OE and cut off the arc GE=OE and join OG, ∴ GOE being an equilateral triangle the <GOE is 60°.

With any convenient radius and any centre K along OE describe a semi-circle OHJ, and with J as centre and same radius describe another semi-circle KLM on OE produced. On line OG produced describe similarly two semi-circles ONP and QRS. Now these four semi-circles intersect one another creating two almond shape figures QHT and KNT. Join Q & T and K & T and bisect QT at V and KT at X. Join O & V and O & X and these lines will trisect the angle GOE. Let OV produced cut the circle ABCD at 8. Then the angle "8OE" is 40°.

With the arc 8E mark off point 1,2,3,4,5,6,7, 8 and 9 and joining them get the nonagon (1-2-3-4-5-6-7-8-9.)

If we marks off eighteen points with an arc equal to "8G" we can get a decoctagon as the angle "QO8" is 20°.

N.B.—As I have used up the letters of the alphabet I have called the nonagon (1-2-3-4-5-6-7-8-9.)

Press Opinions on Dr. P. S. G. Dubash's

"Hygiene of Town-Planning and Vegetation."

"The Times" (London). "A concise treatise for students and municipal bodies dealing primarily with town-planning as it affects the health and well-being of the workers".

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"Oxford Chronicle" (Oxford). "The wide spread interest in housing conditions renders particularly opportune a book on "Hygiene of Town-Planning and Vegetation" by Dr. P. S. G. Dubash who has made a careful study of the subject in this country, on the continent and in India".

"Popular Science Siftings" (London). "The book shows that the ramifications of the subject are amazing in their variety. We gravely doubt if those who have hitherto gone out light-heartedly to plain cities have known their subject. Here they may investigate it to the very foundations and to the ultimate heights. It aims at being a text-book of well-established fact all cut and dried and put in a nut-shell; and it succeeds in its aims. So much so that we commend it not only to the garden city planners but to every person who intends to build a house or rent one. It contains facts about air, land, construction and hygiene which will prove a revelation and inspiration to those home planners and others who have never pondered on these things. The world owes a great debt to Dr. Dubash for this painstaking hand-book".

"The Journal of State Medicine". (Royal Institute of Public Health). "In this little book, the author emphasizes the necessity of sanitary considerations, he does not forget the aesthetic side and devotes many pages to the advantages of tree planting. The writer gives much detailed information on house-planning, as also on the laying out and construction of streets, gardens and open spaces. The author has accumulated a considerable amount of interesting information in a small space. His book is written in a concise manner and will prove helpful to students of sanitation and hygiene and also to municipal corporations. Only the best opinions on various aspects of the subjects treated are given so that the desired information may be considered reliable".

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BY

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