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RESIDENTIAL SUCCESSION AND LAND-USE DYNAMICS
IN A VINTAGE MODEL OF URBAN HOUSING

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Summary:

This paper adapts the vintage model of urban housing developed in Brueckner
(1979) to a two-class city. Computer simulation of the model highlights
the differences between static and dynamic urban areas. The contour of
building heights in irregular in the dynamic city and spatial mixing of the
two income groups occurs.

Residential Succession and Land-Use Dynamics
in a Vintage Model of Urban Housing

by

Jan K. Brueckner

I.

In the static urban spatial model, different income groups live
segregated in doughnut-shaped annular areas, with the poorer groups liv-
ing closer to the center of the city. This result follows from the in-
verse relationship between the steepness of a group's bid-rent curve for
housing and its income level. The reasoning is that the only way each
bid-rent curve can be maximal in some part of the city, so that all in-
come groups are represented in the urban population, is for the steepest
curve to be maximal in the central part of the city, the next steepest
to be maximal in an adjacent annulus, and so on. For a rigorous discus-
sion of static multi-class cities, see Hartwick et. al. (1976).

Although the standard model generates complete segregation by income,
real world location patterns are far less neat. While many studies show
positive correlations between income and distance to the central business
district (CBD), casual observation at the neighborhood level often pro-
vides evidence of substantial spatial mixing of different income groups,
contrary to the predictions of the model.

Partly in order to generate location patterns which conform better
to reality, this paper adapts the vintage model of urban housing developed
in Brueckner (1979) to a two-class city. One of the principal insights
afforded by computer simulation of the model is that spatial mixing of
different income groups is a likely occurrence when the durability of

housing is recognized. Furthermore, the simulation results illustrate
that residential succession, where dwellings are bid away from their
original inhabitants, can lead to spatial patterns of land-use intensity
which are strikingly different from those predicted by the static model.
Erratic patterns of building heights which recapitulate actual cities
replace the smoothly declining contour of building heights predicted
by the static model.

The vintage model used in this paper constitutes an improvement over
previous models of durable housing (see Muth (1973), Anas (1978), and
Fisch (1977), for example) because of its simplicity and its explicitly
spatial character. The tractability of the model is largely due to the
assumptions of housing producer myopia (current prices are expected to
persist forever) and the exogeneity of utility (the city is open and
costless migration equates the urban utility level to the exogenous ex-
ternal level). The next section of the paper reviews the structure of
the vintage model for a one-class city, while Section III generalizes
the model to include two income groups. Section IV contains the simu-
lation results, and Section V presents conclusions.

II.

Precisely stated, the fundamental assumptions of the model are that
the utility level of urban residents is given by an exogenous function
of time, $u(t)$, and that housing producers expect the price per unit of
housing services in their structures at all future times to equal the
current price. It is also assumed that structural modification of build-
ings and their dwellings is possible only through demolition and rede-
velopment. Quality deterioration, however, causes the amount of services

provided by a dwelling to decline at a constant percentage rate α during
its life.

The open city assumption means that $v(x,q) = u(t)$, where v is the
quasi-concave utility function for the identical urban residents, x is
the consumption of the numeraire non-housing good, and q is the consump-
tion of housing services. Inverting this equation yields $x = x(q,u(t))$,
which, for a given q, gives the consumption level of the numeraire which
generates utility equal to $u(t)$. Substituting for x in the consumer bud-
get constraint, the price per unit of housing services which allows the
consumer to reach utility level $u(t)$ in a dwelling with service level q
located k miles from the CBD is given by

$$p = \frac{y(t) - c(t,k) - x(q,u(t))}{q} \equiv G(t,k,q), \qquad (1)$$

where $y(t)$ is exogenous income at time t and $c(t,k)$ is commuting cost
from k to the CBD at t. Let $p(t,k) \equiv \max_{\{q\}} G(t,k,q)$ be the maximum achiev-
able price per unit of housing services at t and k, and let $q(t,k)$ be the
dwelling size which calls forth this price. It is easily shown that the
budget line $x + p(t,k)q + c(t,k) = y(t)$ is tangent to the indifference
curve with utility level $u(t)$ at the point where $q = q(t,k)$.

With myopia, the housing producer expects the price per unit of hous-
ing services in new structures to persist indefinitely. Therefore, the
producer divides a structure built k miles from the CBD at time t into
dwellings of size $q(t,k)$, which yield the maximum price per unit of ser-
vices at t. To construct the building, the producer purchases inputs
of land ℓ and non-land capital N, which yield housing services accord-
ing to the constant returns function $H(N,\ell)$. Now since the price per unit

of services in the new building is expected to persist forever, the expected present value of revenue for the building is higher the longer the anticipated operating period. Thus the producer assumes the building will have an infinite life, and accordingly arranges to finance his construction cost over an infinite period. With constant returns, the scale of output is indeterminate, but housing producers choose S, the N/ℓ ratio in structures, by maximizing the expected present value of profit (EPVP) per acre. At time t, EPVP per acre is given by

$$\frac{p(t,k)h(S)}{\alpha+r} - n(t)S - R(t,k),\qquad (2)$$

where r is the discount rate, $n(t)$ is the exogenous unit price of N at t, $R(t,k)$ is the endogenous land price per acre at t and k, and $h(S) \equiv H(S,1)$. Equation (2) comes from integrating $p(t,k)h(S)e^{-\alpha(\omega-t)} - rn(t)S - rR(t,k)$, expected profits per acre at time ω, weighted by the discount factor $e^{-r(\omega-t)}$, from $\omega=t$ to $\omega=\infty$. Euler's theorem makes the maximized value of (2) equal to zero and allows $R(t,k)$ to be solved for in terms of $p(t,k)$, $n(t)$, α, and r.

To implement the model, it is assumed that production and utility functions are Cobb-Douglas, $H(N,\ell) \equiv N^\beta \ell^{1-\beta}$ and $v(x,q) \equiv x^\theta q^{1-\theta}$, and that utility income, and commuting costs are given by $u(t) \equiv u_0 e^{ut}$, $y(t) \equiv y_0 e^{yt}$, and $c(t,k) \equiv c_0 k e^{yt}$. Note that the last two assumptions give $y(t) - c(t,k) \equiv (y_0 - c_0 k)e^{yt}$. Using (1), the assumptions yield

$$p(t,k) = (1-\theta)\theta^{\frac{\theta}{1-\theta}} \left(\frac{y_0 - c_0 k}{u_0}\right)^{\frac{1}{1-\theta}} e^{\frac{y-u}{1-\theta}t} \qquad (3)$$

and yield a similar solution for $q(t,k)$. The price per unit of services in new dwellings decreases with k and increases (decreases) over time as

the rate of increase of income is greater than (less than) the rate of in-
crease of utility. The further assumption $n(t) \equiv n_0 e^{nt}$ yields solutions
for $R(t,k)$ and S_{tk}, the optimal value of S in (2).

A central feature of the model is that the myopia of housing pro-
ducers is never validated. The price per unit of services beyond a con-
struction date τ does not remain equal to $p(\tau,k)$ in general. The actual
price at t in dwellings constructed at time τ at distance k is denoted
$f(t,k;\tau)$ and is given by

$$f(t,k;\tau) = G(t,k,q(\tau,k)e^{-\alpha(t-\tau)}) \qquad (4)$$

Note that $f(\tau,k;\tau) = p(\tau,k)$.

Although producer myopia means that the anticipated lifespan of a
new structure is infinite, producers may demolish their buildings and re-
develop at a finite time. If the EPVP per acre from continuing to operate
a building falls short of the EPVP per acre from clearing the original land
and constructing a new building, then the producer redevelops. Note that
if redevelopment occurs, the producer must continue to amortize the cost
of the non-land capital in the original structure, which is destroyed with
demolition. When demolition costs are zero, the above condition for the
desirability of redevelopment translates into the compact condition

$$R(t,k) \geq \frac{f(t,k;\tau)\ h(S_{\tau k})}{\alpha+r}\ e^{-\alpha(t-\tau)}, \qquad (5)$$

which says that redevelopment of structures of vintage τ located at k
is desirable at t if land's resale price per acre equals or exceeds the
expected present value of revenue (EPVR) per acre from continuing to

operate the structures. Note that $h(S_{\tau k})e^{-\alpha(t-\tau)}$ is the output of housing services per acre at t from structures of vintage τ located at k and that the producer expects the price per unit of housing services to equal $f(t,k;\tau)$ in all future periods. Substituting in (5) for $R(t,k)$, $f(t,k;\tau)$, and $h(S_{\tau k})$ using the above assumptions on functional forms yields the following condition for determining the demolition age z of structures:

$$\frac{(1-\theta)(1-\beta)\exp[(C+B)z]}{1-\theta \exp[-\frac{1-\theta}{\theta} Bz]} = 1, \tag{6}$$

where $C = (\beta(y-u-n(1-\theta))/(1-\beta)(1-\theta)) + \alpha$ and $B = ((\theta y-u)/(1-\theta)) - \alpha$. Note that if a demolition age exists, it is independent of a structure's construction date τ and location k. A unique positive solution to (6) exists as long as $C > 0$. If $C \leq 0$, certain values of B exist for which (6) has no positive solutions; buildings deteriorate indefinitely in these cases.

It is interesting to note that relaxing the assumption of zero demolition costs introduces the possibility of building abandonment. Abandonment occurs when no consumer will live in a building at a positive price, or $f(t,k;\tau) \leq 0$. To see that abandonment can never occur when demolition costs are zero, note that the positivity of $R(t,k)$ in (5) means that (5) will be first satisfied while $f(t,k;\tau)$ is still positive; redevelopment will occur before abandonment. If demolition costs per acre, $D(t)$, are positive, however, the appropriate LHS for (5) is $R(t,k) - D(t)$, and it is possible that $f(t,k;\tau)$ turns negative before (5) is satisfied, implying that abandonment may occur before redevelopment.

In Brueckner (1978), the model sketched in this section was used to explore the spatial growth of an urban area over time. In the next section, the model is adapted to a city with two income classes.

III.

In order to avoid consideration of the spatial growth of the city in the following analysis, the outer urban boundary is assumed to be fixed over time. This follows from assuming that the price of agricultural land changes at the same rate as the price of urban land at the outer boundary.

In the two-class model, both income groups have the utility function $x^\theta q^{1-\theta}$, and utility, income, and commuting costs for the groups are $u_0^i e^{u_i^i t}$, $y_0^i e^{y_i t}$, $c_0^i k e^{y_i t}$, $i=1,2$, yielding $y_i(t) - c_i(t,k) \equiv (y_0^i - c_0^i k)e^{y_i t}$, $i=1,2$.

The location of the areas in which new construction for the groups occurs is considered first. Suppose an entire open city is constructed at $t=0$. At each distance, housing will be built for the group willing to pay the highest price per unit of housing services for new units. Noting (3), housing is constructed for group 1 at k if

$$\frac{y_{01} - c_{01}k}{u_{01}} > \frac{y_{02} - c_{02}k}{u_{02}} \tag{7}$$

and for group 2 if the reverse inequality holds. Now both sides of (7) are downward-sloping straight lines with at most one intersection. If both groups are to live in the city, the value of k at the intersection must be positive and must yield positive values for both sides of (7). Under these conditions, it is easily shown that initially, group 1 lives in an outer annulus and group 2 lives in an inner annulus only if

$$\frac{y_{01}}{c_{01}} > \frac{y_{02}}{c_{02}} \tag{8}$$

while the opposite is true if the reverse inequality holds. Thus the group with the highest value of y_0/c_0 lives in an outer annulus at time zero.[1]

Although all construction is simultaneous in the original city, it will become clear below that subsequent redevelopment of buildings need not occur simultaneously, even in the case of buildings constructed at the same date for a particular group. Recall that in the one-class model, buildings constructed at the same date were always redeveloped simultaneously. Nonetheless, if any new construction for group i occurs at time t, the new buildings will be located at distances where $p_i(t,\cdot)$ is maximal. Accordingly, the "new-construction area" for group i at t is defined to be the set of values of k for which $p_i(t,\cdot)$ is maximal. Now $p_1(t,k) \gtrless p_2(t,k)$ as $u_{02}(y_{01}-c_{01}k)\exp[(y_1-u_1)t] \gtrless u_{01}(y_{02}-c_{02}k)\exp[(y_2-u_2)t]$, using (3), and it may be shown that $p_1(t,\cdot)$ and $p_2(t,\cdot)$ are maximal in an outer and an inner annulus respectively when (8) holds, and vice versa when the reverse inequality holds. The boundary between the annuli at t, $\hat{k}(t)$, is found by equating the two sides of the above inequality. Note that if $y_1 - u_1 = y_2 - u_2$, $\hat{k}'(t) = 0$. This discussion establishes that when (8) holds, the new-construction area for group 1 (2) is an outer (inner) annulus; any new building constructed at t for group 1 (2) will be located beyond (inside of) $\hat{k}(t)$. Finally, it is important to realize that there will be many periods in the city's history where no new construction takes place.

While housing is originally constructed for the group which bids the highest price per unit of services for new dwellings, changes in income and utility levels, commuting costs, and dwelling service levels can generate residential succession, where one group out bids the other group for dwellings originally constructed for it. In the two-class model, the function G from (1) will be indexed by i. Letting $w_i(t,k;\tau) \equiv q_i(\tau,k)e^{-\alpha(t-\tau)}$ denote the service level at t in a dwelling originally constructed at τ and k for group i, the maximum price per unit of services that a member of group j will pay to live in the dwelling is given by $G_j(t,k,w_i(t,k;\tau))$. The dwelling will be bid away from group i when this expression exceeds $f_i(t,k;\tau) \equiv G_i(t,k,w_i(t,k;\tau))$. Using (1), this requires

$$y_j(t) - c_j(t,k) - x(w_i(t,k;\tau), u_j(t)) >$$

$$y_i(t) - c_i(t,k) - x(w_i(t,k;\tau), u_i(t)) \qquad (9)$$

Setting j=1 and i=2 and using the solution for $q_i(\tau,k)$ and the maintained assumption on functional forms, (9) reduces to

$$\frac{y_{01} - c_{01}k}{y_{02} - c_{02}k} > (1 - F(t,\tau)e^{(y_2-y_1)t}) \qquad (10)$$

where F is a complicated function which does not involve k.[2] Similarly, setting j=2 and i=1, (9) reduces to

$$\frac{y_{02} - c_{02}k}{y_{01} - c_{01}k} > (1 + M(t,\tau))e^{(y_1-y_2)t} \qquad (11)$$

where M is another complicated function.[3] Inequalities (10) and (11) imply particular spatial patterns for residential succession. If

$y_{01}/c_{01} > y_{02}/c_{02}$, then the LHS of (10) is increasing in k, and thus, if (10) is satisfied for some k' in the city, it also holds for all k > k'. This result implies that if members of group 1 are able at time t to bid away any group 2 units of vintage τ, these units will be the outermost units of that vintage. Similarly, since the LHS of (11) is decreasing in k when $y_{01}/c_{01} > y_{02}/c_{02}$, if (11) holds for some k', it holds for all k < k'. This implies that if group 2 is able at time t to bid away any group 1 units of vintage τ, these units will be the innermost units of that vintage. Since $y_{01}/c_{01} > y_{02}/c_{02}$ implies that the new-construction area for group 1 (2) is an outer (inner) annulus, the above results imply that if a group is able to bid away any of the other group's units of a particular vintage, these units will be the ones closest to the given group's new-construction area. The same conclusion follows if $y_{01}/c_{01} < y_{02}/c_{02}$. In this case group 1's (2's) new-construction area is an inner (outer) annulus, while units of the other group of a particular vintage which are bid away by group 1 (2) will be the innermost (outermost) units of that vintage. These facts are confirmed by the simulation results reported in Section IV..

Using a non-spatial model, Muth (1973) reached conclusions which are consistent with the above results. He showed that dwellings bid away from the high-income group by the poor will be those which provide low levels of housing services. Our model implies that if the poor live in the inner annulus, dwellings of a given vintage which they bid away from the high-income group will be those providing the lowest level of services for that vintage. This follows because the service level of a group's dwellings of a particular vintage increases with distance to the CBD and because the

poor bid away only the innermost high-income dwellings of any vintage. Empirical support for these conclusions is provided in Brueckner (1978), where residential succession is shown to proceed most vigorously in areas with "small" dwellings.

At this point it will be useful to sketch some possible residential histories for the city. Suppose first that $\hat{k}'(t) = 0$, so that the new construction areas remain fixed over time. Then suppose that each group is never able to bid away any of the other group's dwellings. In this case, each annulus behaves as if it were described by the one-class model; all the buildings in an annulus are redeveloped simultaneously at intervals given by the solution to (6) based on the appropriate parameter values. The length of these intervals will be different in general in the two annuli. Residential succession, however, might occur while $k'(t) = 0$. Although dwellings will be bid away from their original occupants, the fact that new-construction areas are fixed means that the original occupant type always returns when buildings are redeveloped. It will be shown below, however, that buildings which undergo succession age beyond their normal demolition date. Finally, the new-construction areas may change over time, and this may or may not be accompanied by residential succession. Since the former case clearly involves the most complicated dynamic process, the simulations below were chosen to illustrate it.

To conclude the analysis of the model, it is shown that a building which undergoes succession and is redeveloped for its original occupants is redeveloped later than a building constructed at the same date which does not undergo succession. If a building is built at τ and k

for group i and redeveloped for that group, redevelopment occurs when $R_i(\cdot,k)$ equals the EPVR per acre for the structure. If the building has been bid away by group j, the EPVR per acre at t is given by

$$\frac{G_j(t,k,w_i(t,k;\tau))\ h(S^i_{\tau k})}{\alpha+r}\ e^{-\alpha(t-\tau)}, \tag{12}$$

noting (5). If succession has not occurred, the EPVR per acre at t equals $f_i(t,k;\tau)h(S^i_{\tau k})e^{-\alpha(t-\tau)}/(\alpha+r)$. Now if the building is occupied by group j at t, then $G_j(t,k,w_i(t,k;\tau)) > f_i(t,k;\tau)$. This means that (12) exceeds $R_i(\cdot,k)$ at the no-succession redevelopment date, where the second EPVR expression above equals $R_i(\cdot,k)$. Thus the building is not ready for redevelopment at the no-succession redevelopment date if residential succession has occurred. While this argument holds for fixed k, it is also true when two buildings in different locations are compared. Since buildings of the same vintage in different locations are redeveloped at the same date when they do not undergo succession, it follows that a building which undergoes succession is redeveloped later than another building of the same vintage in a different location which does not. This fact is amply illustrated in the simulations.

IV.

This section reports results for two selected simulations out of the large number that were computed. In all the simulations, $c_{01} = c_{02} = 1$, $y_{01} = 150$, $y_{02} = 87.5$, $u_{01} = 2$, and $\hat{u}_{01} = 1$, which imply $\hat{k}(0) = 25$ and that new construction for the initially richer group always occurs in the outer annulus. As in Brueckner (1978), α, the rate of deterioration of structures, equals .01; β, the exponent of N in the housing production

function equals .75; θ, the exponent of the non-housing good in the utility function, equals .6; and n, the rate of increase of the price of non-land capital, equals zero. The city is divided into discrete rings one "block" wide, with k at all points in each ring set equal to the outer radius of the ring. The outer boundary of the city is fixed at k=65. It is also assumed that new construction and changes of location by consumers occur only at integer times.

The residential history of each ring was computed as follows. At t=0, new structures were built for the appropriate group. At each subsequent time, the group with the highest bid for dwellings in the ring was assigned to them, and the EPVR per acre was compared to the appropriate land price (given by the identity of the new construction area in which the ring is located). If redevelopment was warranted, new structures were built for the appropriate group.

The tables below report, for selected times, four pieces of information: OCC, the identity of the occupants of the buildings in each ring; OROCC, the identity of the group for whom the buildings were originally constructed; AGE, the age of the buildings; and S*, the log of a normalized S value in each ring, which represents building height. S* was computed by dividing S by its value at t=0 and k=0, multiplying by 100, and taking the log. The reason for the normalization was to avoid having to choose an arbitrary level for the price of N, n_0, which sets the level of S. For a structure of vintage τ constructed for group i at distance k, the buidling height measure is given by

$$s_{\tau k}^{i*} = 4.605 + \frac{1}{(1-\theta)(1-\beta)} \log[\frac{u_{02}(y_{0i}-c_{0i}k)}{y_{02} u_{0i}}] + [\frac{(y_i-u_i)}{(1-\theta)(1-\beta)} \tau] \quad (13)$$

Note that building heights change only when redevelopment occurs. Group bid prices for the units in each ring and other potentially interesting quantities are not reported to save space.

Our goal was to generate urban histories where residential succession and changes in the new-construction areas of the groups occur together. In the first simulation, $y_1 = .08$, $y_2 = .07$, $u_1 = .079$, and $u_2 = .066$. These assumptions imply $\hat{k}'(t) > 0$, which says that the new-construction areas for groups 1 and 2 shrink and grow respectively over time, and they also generate dramatic residential succession in the group 1 new-construction area. Note that $y_1 > y_2$ and $y_{01} > y_{02}$ imply $y_1(t) > y_2(t)$ for all t, and that since $y_i-u_i > 0$, i=1,2, building heights from (13) increase over time. The length of the first simulation is 50 years.

The second simulation illustrates residential succession in the group 2 new-construction area. To generate this outcome under the requirements $y_i-u_i > 0$, i=1,2, and $y_1(t) > y_2(t)$ for all t during the simulation, it was necessary to choose negative y_i and u_i.[4] The values are $y_1 = -.045$, $y_2 = -.03$, $u_1 = -.066$, $u_2 = -.05$, which imply $\hat{k}'(t) < 0$. The second simulation runs for 40 years.

In the first simulation, the demolition age of buildings which never undergo succession and are redeveloped for their original occupants is 9 years regardless of which group inhabits them. While more realistic 45-50 year lifespans can be generated by other parameter choices, short building lives permitted relatively short simulations. Blocks 0-22 and

57-65 never change hands in the first simulation, so all but blocks 0 and 65 from this group are omitted in Table 1, which contains selected results from the first simulation. While the initial boundary between the groups is k=25, group 2 has bid away group 1 dwellings out to block 34 by year 6. Table 1a shows the situation in year 8, where dwellings out to block 44 have undergone succession. As predicted by the analysis of Section II, only the innermost group 1 dwellings have been bid away. In year 9, buildings which did not undergo succession are redeveloped, while, as predicted by the analysis in Section II, redevelopment is delayed for those structures which were bid away from group 1. Table 1b shows that in year 10, buildings in blocks 47 and 48 are redeveloped for the original group 1 inhabitants, while dwellings in blocks 26-46 continue to age beyond their normal redevelopment time and are still occupied by group 2. The implication of this phenomenon for the pattern of land-use may be seen in the S* column. The smooth decline of building heights in the central part of the city is interrupted by a sharp decrease at block 26, which is followed by a range of smoothly decreasing heights that ends with an abrupt increase at block 46. This irregular building height pattern, which is graphed in Figure 1, could never be generated by the static model.[5]

Redevelopment of buildings in blocks 26-46, which proceeds in years 11-16, is complete by year 16, shown in Table 1d. Table 1c shows a peculiar pocket of low intensity land use in blocks 30-32 in year 15, where 15-year-old buildings still stand.

In year 18, succession begins again as group 2 bids away 9-year-old buildings in blocks 49 and 50, preventing their redevelopment. By year

19, shown in Table 1e, those buildings have been redeveloped for group 1, while group 2 has bid away the 8- and 9-year-old dwellings in blocks 44-48 and the 7-year-old units in block 42. These results bear out the predictions of the above analysis because only the innermost units of vintages which undergo succession are bid away. The striking feature of Table 13, however, is the presence of members of group 2 deep within the group 1 new-construction area, separated by many blocks from the rest of their group. The static urban model, of course, fails to generate this realistic spatial mixing of income groups.

In years 20 and 21, shown in Tables 1f and 1g, group 2 consolidates its position in the group 1 new-construction area by filling in gaps left by the residential leapfrogging of years 18 and 19. By year 28, redevelopment for group 1 has proceeded back to block 39, but members of group 2 have again penetrated deeply into the group 1 area at blocks 50-51. The process of filling the gap starts again and is complete by year 37 (see Tables 1i-1k), at which time a new group 2 beachhead is established at block 51. By year 45, two widely separated group 2 neighborhoods exist deep within the group 1 new-construction area (see Tables 1ℓ and 1m). Finally, note the striking areas of low intensity land-use in years 28 and 40, where neighborhoods of old buildings are surrounded by much younger structures. Figure 1 shows the graph of S^* in year 28.

In the second simulation, the demolition ages of buildings which never undergo succession and are redeveloped for the original group are 7 and 8 years for groups 1 and 2 respectively. Since the original group 1 new-construction area is controlled by group 1 throughout the

simulation, blocks 31-64 are omitted from Table 2. By year 7, shown in
Table 2a, group 1 has bid away group 2 dwellings in to block 16, and in
year 8, shown in Table 2b, redevelopment leaves a familiar cluster of
old buildings surrounded by new structures. The simultaneous redevelop-
ment of buildings in blocks 0-15 and blocks 16-18, which were bid away
by group 2, appears to contradict the above analysis, but is in fact due
to the use of discrete time units. Table 2c shows the situation in year
14, where buildings which were redeveloped for group 2 are again bid
away by group 1. Note that while block 19 was jumped, it has been bid
away by year 15, shown in Table 2d. By year 17, shown in Table 2e,
buildings out to block 18 have been redeveloped for group 1, while a
group of old buildings in blocks 19-23 breaks the smooth contour of
building heights, as shown in Figure 2. The by now familiar cycle of
succession followed by redevelopment for group 1 followed by further
succession is evident in years 22, 26, 30, 36, and 37, shown in Tables
2f-2j. Note the leapfrogging by group 1 in years 22, 30, and 37, and
note the presence of a lone block of 8-year-old buildings surrounded
by new structures in year 35. The building height contour for this
year is shown in Figure 2.

V.

The simulations highlight the differences between static and dy-
namic two-class cities. First, both simulations show that the blurring
of the boundary between the two income groups is a natural outcome when
structures are durable. This result corresponds better to location
patterns in real cities than does the perfect segregation implied by

the static model. Second, postponement of demolition for buildings which undergo residential succession generates pockets of low intensity land-use which interrupt the smooth decline of the building height contour. This implication is welcome in view of the complex patterns of building heights found in real cities, which are often inexplicable under the static model. In sum, the spatial vintage model developed in this paper strengthens the link between reality and urban economic theory.

An important task for future research is the adaptation of the model to a closed city, where utility is endogenous. Apparently, little can be said in general about the dynamic properties of a closed urban area, even in the one-class case; insights will have to be drawn principally from simulation results. It appears, however, that the computational problems involved in closed city simulations will be substantial. Less ambitious research goals might include use of the open city model to analyse the effects of housing subsidy programs, changes in zoning laws, or discontinuous changes in transportation costs. Such studies would no doubt provide further insight into the dynamics of an urban economy with durable structures.

Block	1a: t=8 OCC	OROCC	AGE	S*	1b: t=10 OCC	OROCC	AGE	S*	1c: t=15 OCC	OROCC	AGE	S*	1d: t=16 OCC	OROCC	AGE	S*
0	2	2	8	4.605	2	2	1	5.775	2	2	6	5.775	2	2	7	5.775
22	2	2	8	1.709	2	2	1	2.879	2	2	6	2.879	2	2	7	2.879
23	2	2	8	1.555	2	2	1	2.725	2	2	6	2.725	2	2	7	2.725
24	2	2	8	1.399	2	2	1	2.569	2	2	6	2.569	2	2	7	2.569
25	2	2	8	1.240	2	2	1	2.410	2	2	6	2.410	2	2	7	2.410
26	2	2	8	1.160	2	2	2	2.410	2	2	6	2.529	2	2	7	2.899
27	2	1	8	1.079	2	2	1	1.160	2	2	6	2.699	2	2	7	2.735
28	2	1	8	0.998	2	2	10	1.079	2	2	6	2.735	2	2	1	2.699
29	2	1	8	0.915	2	2	10	0.998	2	2	1	2.899	2	2	1	2.529
30	2	1	8	0.832	2	1	10	0.915	2	1	0	2.529	2	1	2	2.487
31	2	1	8	0.749	2	1	10	0.832	2	1	0	2.699	2	1	2	2.349
32	2	1	8	0.664	2	1	10	0.749	2	1	15	0.664	2	1	3	2.264
33	2	1	8	0.579	2	1	10	0.664	2	1	15	0.749	2	1	3	2.079
34	2	1	8	0.493	2	1	10	0.579	2	1	15	0.832	2	1	3	1.993
35	2	1	8	0.407	2	1	10	0.493	2	1	0	1.907	2	1	2	1.907
36	2	1	8	0.319	2	1	10	0.407	2	1	0	1.993	2	1	2	1.719
37	2	1	8	0.231	2	1	10	0.319	2	1	1	2.079	2	1	1	1.631
38	2	1	8	0.142	2	1	10	0.231	2	1	1	1.542	2	1	1	1.542
39	2	1	8	0.053	2	1	10	0.142	2	1	2	1.631	2	1	0	1.353
40	2	1	8	-0.038	2	1	10	0.053	2	1	2	1.719	2	1	0	1.262
41	2	1	8	-0.129	2	1	10	-0.038	2	1	2	1.171	2	1	1	1.171
42	2	1	8	-0.221	2	1	10	-0.129	2	1	3	1.262	2	1	1	0.979
43	2	1	8	-0.314	2	1	10	-0.221	2	1	3	1.353	2	1	2	0.886
44	2	1	8	-0.408	2	1	10	-0.314	1	1	4	0.886	1	1	2	0.692
45	2	1	8	-0.503	2	1	10	-0.408	1	1	4	0.979	1	1	3	0.597
46	2	1	8	-0.599	2	1	10	-0.503	1	1	5	0.692	1	1	3	0.501
47	2	1	8	-0.695	1	1	10	-0.599	1	1	5	0.597	1	1	4	0.305
48	2	1	8	-0.793	1	1	0	0.305	1	1	5	0.501	1	1	4	0.207
49	2	1	8	-0.891	1	1	0	0.207	1	1	5	0.305	1	1	5	0.009
50	1	1	8	-0.991	1	1	10	0.009	1	1	6	0.207	1	1	5	-0.091
51	1	1	8	-1.091	1	1	10	-0.091	1	1	6	0.009	1	1	6	-0.191
52	1	1	8	-1.193	1	1	10	-0.191	1	1	6	-0.091	1	1	6	-0.293
53	1	1	8	-1.296	1	1	10	-0.293	1	1	6	-0.191	1	1	7	-0.396
54	1	1	8	-1.399	1	1	10	-0.396	1	1	6	-0.293	1	1	7	-0.499
55	1	1	8	-1.504	1	1	10	-0.499	1	1	6	-0.396	1	1	7	-0.604
56	1	1	8	-1.610	1	1	10	-0.604	1	1	6	-0.499	1	1	7	-0.710
57	1	1	8	-1.717	1	1	10	-0.710	1	1	6	-0.604	1	1	7	-0.817
...																
65	1	1	8	-2.616	1	1	1	-0.817	1	1	6	-0.817	1	1	7	-0.817

Table 1 (cont.)

Block	1i: t=31 OCC	OROCC	AGE	S*	1j: t=35 OCC	OROCC	AGE	S*	1k: t=37 OCC	OROCC	AGE	S*	1l: t=40 OCC	OROCC	AGE	S*
0	2	2	4	8.115	2	2	8	8.115	2	2	1	9.285	2	2	4	9.285
⋯																
22	2	2	4	5.219	2	2	8	5.219	2	2	1	6.389	2	2	4	6.389
23	2	2	4	5.065	2	2	8	5.065	2	2	1	6.235	2	2	4	6.235
24	2	2	4	4.909	2	2	8	4.909	2	2	1	6.079	2	2	4	6.079
25	2	2	4	4.750	2	2	8	4.750	2	2	1	5.920	2	2	4	5.920
26	2	2	8	4.069	2	2	3	5.239	2	2	5	5.239	2	2	8	5.239
27	2	2	8	3.905	2	2	3	5.075	2	2	5	5.075	2	2	8	5.075
28	2	2	7	3.869	2	2	2	5.039	2	2	4	5.039	2	2	7	5.039
29	2	2	7	3.699	2	2	2	4.869	2	2	4	4.869	2	2	7	4.869
30	2	2	6	3.657	2	2	1	4.827	2	2	3	4.827	2	2	6	4.827
31	2	2	1	4.131	2	2	5	4.131	2	2	7	4.131	2	2	1	5.301
32	2	2	1	3.953	2	2	5	3.953	2	2	7	3.953	2	2	1	5.123
33	2	2	1	3.771	2	2	5	3.771	2	2	7	3.771	2	2	1	4.941
34	2	2	0	3.716	2	2	4	3.716	2	2	6	3.716	2	2	0	4.886
35	2	2	0	3.527	2	2	4	3.527	2	2	6	3.527	2	2	0	4.697
36	2	1	1	3.319	2	1	5	3.319	2	1	7	3.319	2	1	10	3.319
37	1	1	1	3.231	1	1	5	3.231	1	1	7	3.231	1	1	10	3.231
38	1	1	2	3.042	1	1	6	3.042	1	1	8	3.042	1	1	11	3.042
39	1	1	3	2.853	2	1	7	2.853	2	1	9	2.853	2	1	12	2.853
40	1	1	4	2.662	2	1	8	2.662	2	1	10	2.662	2	1	13	2.662
41	1	1	4	2.571	2	1	8	2.571	2	1	10	2.571	2	1	13	2.571
42	1	1	5	2.379	2	1	9	2.379	2	1	11	2.379	2	1	14	2.379
43	1	1	6	2.186	2	1	10	2.186	2	1	12	2.186	2	1	0	3.686
44	1	1	7	1.992	2	1	11	1.992	2	1	13	1.992	2	1	2	3.392
45	2	1	8	1.797	2	1	12	1.797	2	1	1	3.097	2	1	4	3.097
46	2	1	8	1.701	2	1	12	1.701	2	1	1	3.001	2	1	4	3.001
47	2	1	10	1.405	2	1	2	2.605	1	1	4	2.605	1	1	7	2.605
48	2	1	10	1.307	2	1	2	2.507	1	1	4	2.507	1	1	7	2.507
49	1	1	1	2.109	1	1	5	2.109	1	1	7	2.109	1	1	10	2.109
50	1	1	1	2.009	1	1	5	2.009	1	1	7	2.009	1	1	10	2.009
51	1	1	3	1.709	1	1	7	1.709	2	1	9	1.709	1	1	1	2.809
52	1	1	1	1.507	1	1	8	1.507	1	1	0	2.507	1	1	3	2.507
53	1	1	1	1.404	1	1	8	1.404	1	1	0	2.404	1	1	3	2.404
54	1	1	1	1.301	1	1	8	1.301	1	1	1	2.201	1	1	4	2.201
55	1	1	1	1.196	1	1	8	1.196	1	1	1	2.096	1	1	4	2.096
56	1	1	1	1.090	1	1	8	1.090	1	1	1	1.990	1	1	4	1.990
57	1	1	1	0.983	1	1	8	0.983	1	1	1	1.883	1	1	4	1.883
⋯																
65	1	1	4	0.084	1	1	8	0.084	1	1	1	0.984	1	1	4	0.984

Table 1 (cont.)

Block	le OCC	le OROCC	le t=19 AGE	le S*	1f OCC	1f OROCC	1f t=20 AGE	1f S*	1g OCC	1g OROCC	1g t=21 AGE	1g S*	1h OCC	1h OROCC	1h t=28 AGE	1h S*
0	2	2	1	6.945	2	2	2	6.945	2	2	3	6.945	2	2	1	8.115
...																
22	2	2	1	4.049	2	2	2	4.049	2	2	3	4.049	2	2	1	5.219
23	2	2	1	3.895	2	2	2	3.894	2	2	3	3.895	2	2	1	5.065
24	2	2	1	3.739	2	2	2	3.739	2	2	3	3.739	2	2	1	4.909
25	2	2	1	3.580	2	2	2	3.580	2	2	3	3.580	2	2	1	4.750
26	2	2	5	2.899	2	2	6	2.899	2	2	7	2.899	2	2	5	4.069
27	2	2	5	2.735	2	2	6	2.735	2	2	7	2.735	2	2	5	3.905
28	2	2	4	2.699	2	2	5	2.699	2	2	6	2.699	2	2	4	3.869
29	2	2	4	2.529	2	2	5	2.529	2	2	6	2.529	2	2	4	3.699
30	2	2	3	2.487	2	2	4	2.487	2	2	5	2.487	2	2	3	3.657
31	1	1	3	2.349	1	1	4	2.349	2	1	5	2.349	2	1	12	2.349
32	1	1	3	2.264	1	1	4	2.264	2	1	5	2.264	2	1	12	2.264
33	1	1	4	2.079	1	1	5	2.079	2	1	6	2.079	2	1	13	2.079
34	1	1	4	1.993	1	1	5	1.993	2	1	6	1.993	2	1	13	1.993
35	1	1	4	1.907	1	1	5	1.907	2	1	6	1.907	2	1	13	1.907
36	1	1	5	1.719	2	1	6	1.719	2	1	7	1.719	2	1	14	1.719
37	1	1	5	1.631	2	1	6	1.631	2	1	7	1.631	2	1	14	1.631
38	1	1	5	1.542	1	1	6	1.542	2	1	7	1.542	2	1	14	1.542
39	1	1	6	1.353	2	1	7	1.353	2	1	8	1.353	1	1	0	2.853
40	2	2	6	1.262	2	1	7	1.262	2	1	8	1.262	1	1	1	2.662
41	1	1	6	1.171	2	1	7	1.171	2	1	8	1.171	1	1	2	2.571
42	2	2	7	0.979	2	1	7	0.979	2	1	9	0.979	1	1	3	2.379
43	2	2	7	0.886	2	1	8	0.886	2	1	9	0.886	1	1	4	2.186
44	2	2	8	0.692	2	1	8	0.692	2	1	10	0.692	1	1	5	1.992
45	2	2	8	0.597	2	1	9	0.597	2	1	10	0.597	1	1	5	1.797
46	2	2	8	0.501	2	1	9	0.501	2	1	10	0.501	1	1	7	1.701
47	1	1	9	0.305	2	1	9	0.305	1	1	0	1.405	1	1	7	1.405
48	1	1	9	0.207	2	1	10	0.207	1	1	0	1.307	1	1	9	1.307
49	1	1	0	1.009	1	1	10	1.009	1	1	2	1.009	2	1	9	1.009
50	1	1	0	0.909	1	1	1	0.909	1	1	2	0.909	2	1	0	0.909
51	1	1	1	0.709	1	1	1	0.709	1	1	3	0.709	1	1	1	1.709
52	1	1	1	0.607	1	1	2	0.607	1	1	3	0.607	1	1	1	1.507
53	1	1	1	0.504	1	1	2	0.504	1	1	3	0.504	1	1	1	1.404
54	1	1	1	0.401	1	1	2	0.401	1	1	3	0.401	1	1	1	1.301
55	1	1	1	0.296	1	1	2	0.296	1	1	3	0.296	1	1	1	1.196
56	1	1	1	0.190	1	1	2	0.190	1	1	3	0.190	1	1	1	1.090
57	1	1	1	0.083	1	1	2	0.083	1	1	3	0.083	1	1	1	0.983
...																
65	1	1	1	-0.816	1	1	2	-0.816	1	1	3	-0.816	1	1	1	0.084

Table 2

Block	2a: t=7 OCC	OROCC	AGE	S*	2b: t=8 OCC	OROCC	AGE	S*	2c: t=14 OCC	OROCC	AGE	S*	2d: t=15 OCC	OROCC	AGE	S*
0	2	2	7	4.605	2	2	0	6.205	2	2	6	6.205	2	2	7	6.205
1	2	2	7	4.490	2	2	0	6.090	2	2	6	6.090	2	2	7	6.090
2	2	2	7	4.374	2	2	0	5.974	2	2	6	5.974	2	2	7	5.974
3	2	2	7	4.256	2	2	0	5.856	2	2	6	5.856	2	2	7	5.856
4	2	2	7	4.137	2	2	0	5.737	2	2	6	5.737	2	2	7	5.737
5	2	2	7	4.017	2	2	0	5.617	2	2	6	5.617	2	2	7	5.617
6	2	2	7	3.895	2	2	0	5.495	2	2	6	5.495	2	2	7	5.495
7	2	2	7	3.771	2	2	0	5.371	2	2	6	5.371	2	2	7	5.371
8	2	2	7	3.646	2	2	0	5.246	2	2	6	5.246	2	2	7	5.246
9	2	2	7	3.520	2	2	0	5.120	2	2	6	5.120	2	2	7	5.120
10	2	2	7	3.392	2	2	0	4.992	2	2	6	4.992	2	2	7	4.992
11	2	2	7	3.262	2	2	0	4.862	2	2	6	4.862	2	2	7	4.862
12	2	2	7	3.130	2	2	0	4.730	2	2	6	4.730	2	2	7	4.730
13	2	2	7	2.997	2	2	0	4.597	2	2	6	4.597	2	2	7	4.597
14	2	2	7	2.862	2	2	0	4.462	2	2	6	4.462	2	2	7	4.462
15	2	2	7	2.725	2	2	0	4.325	2	2	6	4.325	2	2	7	4.325
16	2	2	7	2.586	2	2	0	4.186	2	2	6	4.186	2	2	7	4.186
17	2	2	7	2.445	2	2	0	4.045	2	2	6	4.045	2	2	7	4.045
18	2	2	7	2.302	2	2	0	3.902	2	2	6	3.902	2	2	7	3.902
19	2	2	7	2.157	2	2	8	2.157	1	2	5	3.957	1	2	6	3.957
20	2	2	7	2.010	2	2	8	2.010	1	2	5	3.810	1	2	6	3.810
21	2	2	7	1.861	2	2	8	1.861	1	2	5	3.661	1	2	6	3.661
22	2	2	7	1.709	2	2	8	1.709	1	2	5	3.509	1	2	6	3.509
23	2	2	7	1.555	2	2	8	1.555	1	2	5	3.355	1	2	6	3.355
24	2	2	7	1.399	2	2	8	1.399	1	2	5	3.210	1	2	6	3.210
25	2	2	7	1.240	2	2	8	1.240	1	2	5	3.130	1	2	6	3.130
26	2	1	0	2.630	2	1	1	2.630	1	1	0	4.100	1	1	1	4.100
27	2	1	0	2.549	2	1	1	2.549	1	1	0	4.019	1	1	1	4.019
28	2	1	0	2.468	2	1	1	2.468	1	1	0	3.938	1	1	1	3.938
29	2	1	0	2.385	2	1	1	2.385	1	1	0	3.855	1	1	1	3.855
30	2	1	0	2.302	2	1	1	2.302	1	1	0	3.772	1	1	1	3.772
65 ...	1	1	0	-1.146	1	1	1	-1.146	1	1	0	0.324	1	1	1	0.324

Block	1m: t=45 OCC	OROCC	AGE	S*
0	2	2	0	10.455
...				
22	2	2	0	7.559
23	2	2	0	7.405
24	2	2	0	7.249
25	2	2	0	7.090
26	2	2	4	6.409
27	2	2	4	6.245
28	2	2	3	6.209
29	2	2	3	6.039
30	2	2	2	5.997
31	2	2	6	5.301
32	2	2	6	5.123
33	2	2	6	4.941
34	2	2	5	4.886
35	2	2	5	4.697
36	2	2	1	5.025
37	2	2	0	4.959
38	2	2	0	4.759
39	1	2	1	4.453
40	1	1	2	4.262
41	1	1	2	4.171
42	1	1	4	3.879
43	1	1	5	3.686
44	2	1	7	3.392
45	2	1	9	3.097
46	2	1	9	3.001
47	2	1	12	2.605
48	2	1	12	2.507
49	1	1	3	3.309
50	1	1	3	3.209
51	1	1	6	2.809
52	1	1	8	2.507
53	1	1	8	2.404
54	2	1	9	2.201
55	2	1	9	2.096
56	1	1	0	2.890
57	1	1	0	2.783
...				
65	1	1	0	1.884

Table (cont.)

Block	2i: t=35				2j: t=37			
	OCC	OROCC	AGE	S*	OCC	OROCC	AGE	S*
0	2	2	3	11.005	2	5	5	11.005
1	2	2	3	10.890	2	5	5	10.890
2	2	2	3	10.774	2	5	5	10.774
3	2	2	3	10.656	2	5	5	10.656
4	2	2	3	10.537	2	5	5	10.537
5	2	2	3	10.417	2	5	5	10.417
6	2	2	3	10.295	2	5	5	10.295
7	2	2	3	10.171	2	5	5	10.171
8	2	2	3	10.046	2	5	5	10.046
9	2	2	3	9.920	2	5	5	9.920
10	2	2	3	9.792	2	5	5	9.792
11	2	2	3	9.662	2	5	5	9.662
12	2	2	3	9.530	2	5	5	9.530
13	2	2	3	9.397	2	5	5	9.397
14	2	2	3	9.262	2	5	5	9.262
15	2	2	3	9.125	2	5	5	9.125
16	2	2	3	8.986	2	5	5	8.986
17	2	2	3	8.845	2	5	5	8.845
18	2	1	1	9.102	2	2	3	9.102
19	2	1	0	9.157	2	2	2	9.157
20	2	1	8	7.410	2	2	1	9.210
21	1	1	0	8.905	1	2	2	8.905
22	1	1	1	8.618	1	2	3	8.618
23	1	1	3	8.119	1	2	5	8.119
24	1	1	5	7.620	1	2	0	9.090
25	1	1	5	7.540	1	2	0	9.010
26	1	1	0	8.510	1	2	2	8.510
27	1	1	0	8.429	1	2	2	8.429
28	1	1	0	8.348	1	2	2	8.348
29	1	1	0	8.265	1	2	2	8.265
30	1	1	0	8.182	1	2	2	8.182
...								
65	1	1	0	4.734	1	1	2	4.734

Table 2 (cont.)

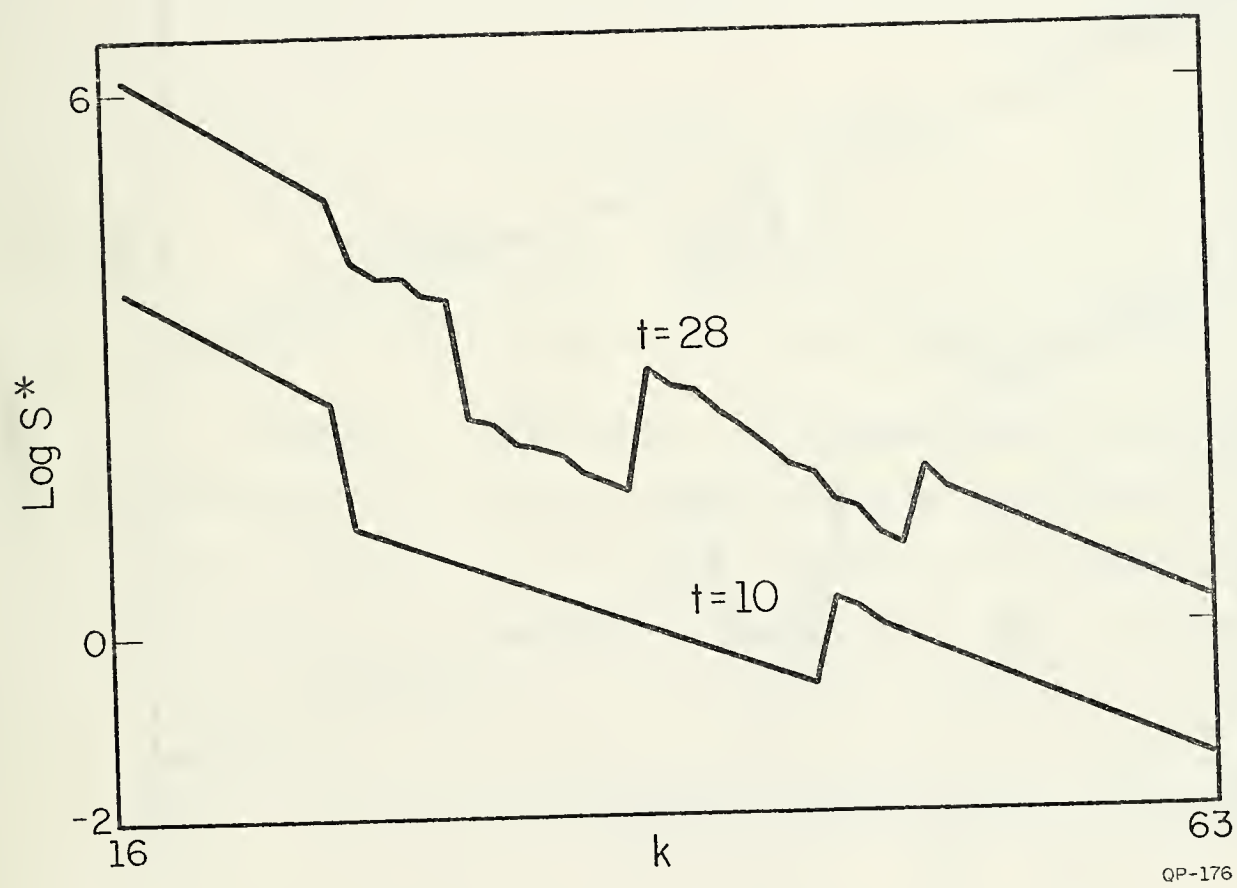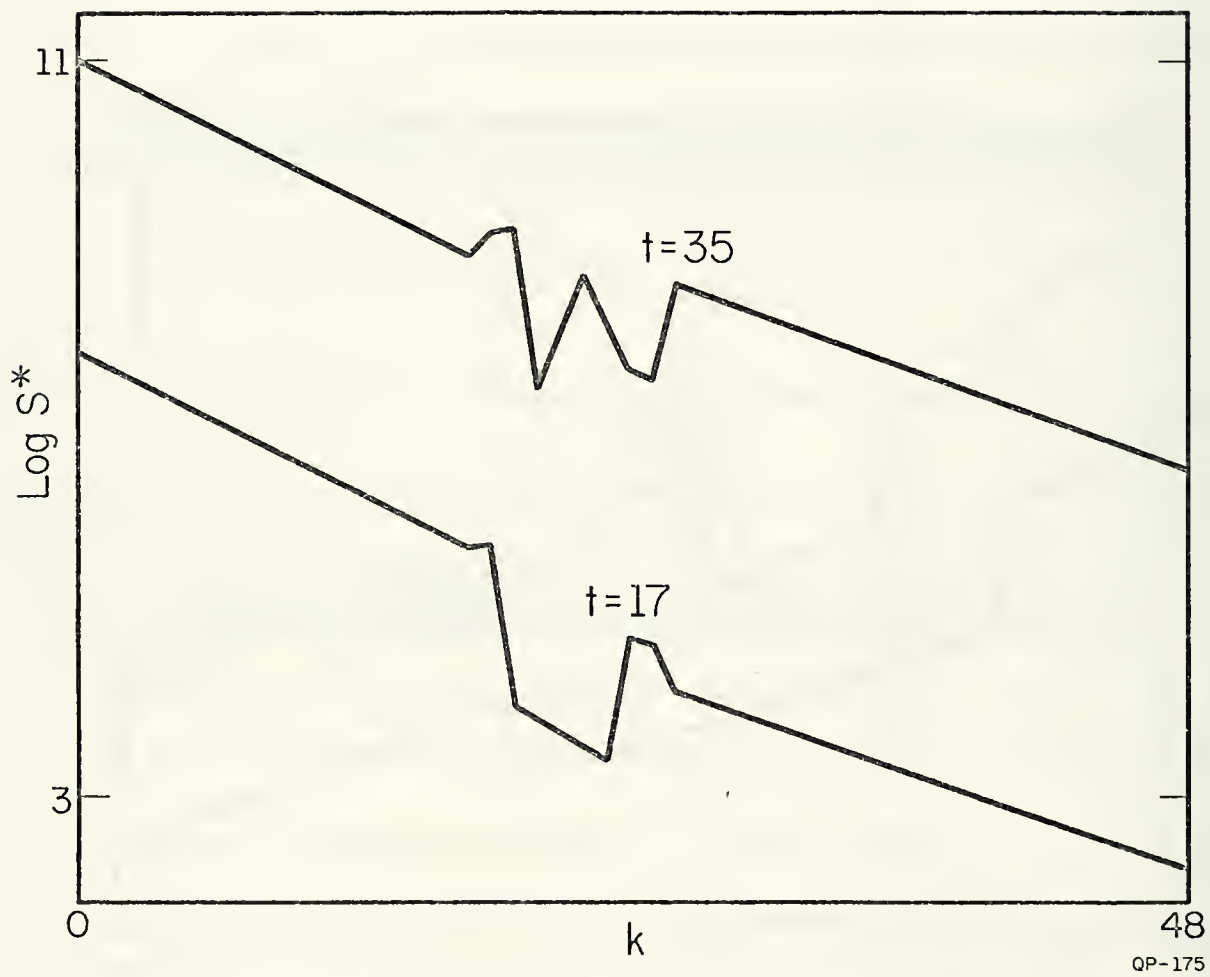| Block	2e: t=17				2f: t=22				2g: t=26				2h: t=30			
	OCC	OROCC	AGE	S*	OCC	OROCC	AGE	S*	OCC	OROCC	AGE	S*	OCC	OROCC	AGE	S*
0	2	2	1	7.805	2	2	6	7.805	2	2	2	9.405	2	2	6	9.405
1	2	2	1	7.690	2	2	6	7.690	2	2	2	9.290	2	2	6	9.290
2	2	2	1	7.574	2	2	6	7.574	2	2	2	9.174	2	2	6	9.174
3	2	2	1	7.456	2	2	6	7.456	2	2	2	9.056	2	2	6	9.056
4	2	2	1	7.337	2	2	6	7.337	2	2	2	8.937	2	2	6	8.937
5	2	2	1	7.217	2	2	6	7.217	2	2	2	8.817	2	2	6	8.817
6	2	2	1	7.095	2	2	6	7.095	2	2	2	8.695	2	2	6	8.695
7	2	2	1	6.971	2	2	6	6.971	2	2	2	8.571	2	2	6	8.571
8	2	2	1	6.846	2	2	6	6.846	2	2	2	8.446	2	2	6	8.446
9	2	2	1	6.720	2	2	6	6.720	2	2	2	8.320	2	2	6	8.320
10	2	2	1	6.592	2	2	6	6.592	2	2	2	8.192	2	2	6	8.192
11	2	2	1	6.462	2	2	6	6.462	2	2	2	8.062	2	2	6	8.062
12	2	2	1	6.330	2	2	6	6.330	2	2	2	7.930	2	2	6	7.930
13	2	2	1	6.197	2	2	6	6.197	2	2	2	7.797	2	2	6	7.797
14	2	2	1	6.062	2	2	6	6.062	2	2	2	7.662	2	2	6	7.662
15	2	2	1	5.925	2	2	6	5.925	2	2	2	7.525	2	2	6	7.525
16	2	2	1	5.786	2	2	6	5.786	2	2	2	7.386	2	2	6	7.386
17	2	2	1	5.645	2	2	6	5.645	2	2	2	7.245	2	2	6	7.245
18	2	2	0	5.702	2	2	6	5.702	2	2	2	7.386	2	2	6	7.386
19	2	2	8	3.957	2	2	5	5.757	2	2	0	7.502	2	2	6	7.502
20	2	2	8	3.509	2	2	4	5.610	2	2	8	5.757	2	2	6	7.557
21	2	2	8	3.661	2	2	4	5.461	2	2	8	5.610	2	2	6	7.410
22	2	2	8	3.509	2	2	4	5.309	2	2	8	5.461	2	2	4	7.261
23	2	2	8	3.355	2	2	4	5.179	2	2	8	5.309	2	2	3	7.148
24	2	2	1	4.680	2	2	6	4.680	2	2	3	6.070	2	2	3	7.540
25	2	2	1	4.600	2	2	7	4.600	2	2	3	6.150	2	2	5	7.620
26	2	2	3	4.100	2	2	1	5.570	2	2	5	5.570	2	2	0	7.040
27	2	2	3	4.019	2	2	1	5.489	2	2	5	5.489	2	2	0	6.969
28	2	2	3	3.938	2	2	1	5.408	2	2	5	5.408	2	2	3	6.878
29	2	2	3	3.855	2	2	1	5.325	2	2	5	5.325	2	2	3	6.795
30	2	2	3	3.772	2	2	1	5.242	2	2	5	5.242	2	2	3	6.712
...																
65	1	1	3	0.324	1	1	1	1.794	1	1	5	1.794	1	1	2	3.264

FIGURE 1.

FIGURE 2.

Footnotes

[1]The inverse relation between income and distance to the CBD referred to in the introduction requires that marginal commuting costs do not rise rapidly with income. The result that the group with the higher y_0/c_0 ratio lives in the outer annulus at $t=0$ is a special case of this general result.

[2]$F(t,\tau) = \theta(\exp[u_2(t-\tau)/\theta] - \exp[u_1(t-\tau)/\theta])$.

$$\exp[(\alpha(1-\theta)/\theta - y_2)(t-\tau)]$$

[3]$M(t,\tau) = F(t,\tau)(u_{02}/u_{01})^{1/\theta} \cdot \exp[(u_2-u_1)\tau/\theta] \cdot \exp[(y_2-y_1)(t-\tau)]$

[4]A peculiarity of the model is that while the identity of the high income group may change as time progresses because of differences in rates of income growth, the location of new-construction areas is determined only by initial income levels when $c_{01} = c_{02}$. This means that the current income of the group whose new-construction area is an outer annulus may be lower than that of the other group over parts of the city's history, a result which is impossible in the static urban spatial model. To avoid this unappealing outcome, we chose y_1 and y_2 so that $y_1(t) > y_2(t)$ over the time span of the simulations. Under this requirement and the constraint $y_i - u_i > 0$, $i=1,2$, it was necessary to choose negative y_i and u_i to generate residential succession in the group 2 area.

[5]Although the unbroken parts of the curves in the figures should be slightly concave, they are drawn as straight lines since the difference is almost imperceptible.

References

A. Anas, Dynamics of urban residential growth, J. Urban Econ., 5, 66-87 (1978).

J. Brueckner, The determinants of residential succession, J. Urban Econ., 4, 45-59 (1977).

J. Brueckner, A vintage model of urban growth, in "Urban Transportation, Planning, and the Dynamics of Land Use" (Moses, ed.), Academic Press, New York (1979).

O. Fisch, Dynamics of the housing market, J. Urban Econ., 4, 428-447 (1977).

J. Hartwick et al., Comparative statics of a residential economy with several classes, J. Econ. Theory, 13, 396-413 (1976).

R. Muth, A vintage model of the housing market, Pap. Proc. Regional Sci. Assoc., 30, 141-156 (1973).