THE

RESISTANCE AND PROPULSION

OF

SHIPS.

BY

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PREFACE.

DURING the last twenty or thirty years the literature relating to the Resistance and Propulsion of ships has received many valuable and important additions. Of the few books in English published in this period, those of the highest value have been restricted either in scope or in mode of treatment. For the most part, however, the important additions to the subject have been published only in the transactions of engineering and scientific societies, or in the technical press. Such papers and special articles are far from providing a connected account of the trend of modern thought and practice, and the present work has been undertaken in the hope that there might be a field of usefulness for a connected and fairly comprehensive exposition of the subject from the modern scientific and engineering standpoint.

Such a work must depend in large measure on the extant literature of the subject, and the author would here express his general acknowledgments to those who have preceded him in this field. In many places special obligations are due, and it has been the intention to give special references at such points to the original papers or sources. With the material drawn from the general literature of the subject there has been combined a considerable amount of original matter which it is hoped will contribute somewhat to whatever of interest or value the work as a whole may possess.

A free use has been made of calculus and mechanics in the development of the subject, the nature of the treatment requiring the use of these powerful auxiliaries. At the same time most of the important results and considerations bearing on them are discussed in general terms and from the descriptive standpoint, and all operations involved in the actual solution of problems are reduced to simple expression in terms of elementary mathematical processes. It will thus be possible, by the omission of the parts involving higher mathematics, to still obtain a fairly connected idea of most of the subject from the descriptive standpoint, and to apply all methods developed for purposes of design.

In the development of such methods the purpose has been to supply plain paths along which the student may proceed step by step from the initial conditions to the desired results, and to arrange the method in such a way as shall conduce to the most intelligent application of engineering judgment and experience. In the design of screw-propellers especially, where the number of controlling conditions is necessarily large, the purpose has been to present a mode of solution in which the most important controlling conditions are represented in the formulæ, and in which the determination of the numerical values of these representatives by auxiliary computation, estimate, or assumption is forced upon the attention of the student in such a way as to call at each step for a definite act of engineering judgment.

In so far as the method may differ from others, it is intended to present the series of operations in such way as to favor the recognition of a considerable number of relatively simple steps, and the full and free application at each step of such judgment and precedent as may be at hand. Such methods are perhaps better adapted to the discipline of the student than to the uses of the trained designer, and it is obviously for the former rather than for the latter that such a work should be prepared. At the same time it seems not unlikely that even for the trained designer the splitting up of concrete judgments into their separate factors is often an operation of value, and one which will the more readily enable him to adapt his methods to rapidly changing precedents and conditions of design.

The natural limitations of size and the need of homogeneity have rendered the work in many respects less complete than the Author might have wished, and many important developments especially in pure theory have received but scanty notice. As presented, the work represents substantially the lectures on Resistance and Propulsion given by the Author to students of Cornell University in the School of Marine Construction, and many features both in subject-matter and mode of treatment have been introduced as a result of the experience thus obtained in dealing with these subjects.

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INTRODUCTORY NOTE RELATING TO UNITS OF MEASUREMENT.

THE units of weight, and of force in general, used in naval architecture are the *pound* and the *ton*, the latter usually of 2240 lbs. It will always be so understood in the present volume.

The units of velocity are the *foot per second*, the *foot per minute*, and the *knot*. The latter, while often used in the sense of a distance, is really a speed or velocity. As adopted by the U. S. Navy Department it is a speed of 6080.27 ft. per hour. The British Admiralty knot is a speed of 6080 ft. per hour. The distance 6080.27 ft. is the length of a minute of arc on a sphere whose area equals that of the earth. For all purposes with which we are concerned in the present volume the U. S. and British Admiralty knots may be considered the same.

On the inland waters of the United States the statute mile of 5280 feet is frequently employed in the measurement of speed instead of the knot, and the short or legal ton of 2000 lbs. for the measurement of displacement and weight in general.

Revolutions of engines are usually referred to the minute as unit.

RESISTANCE AND PROPULSION OF SHIPS.

CHAPTER I.

RESISTANCE.

1. General Ideas.

In the science of hydrostatics it is shown, for a body wholly or partially immersed in a liquid and at rest relative to such liquid, that the horizontal resultant of all forces between the liquid and the body is zero, and that the vertical resultant equals the weight of the body. If, however, there is relative motion between the liquid and the body, the hydrostatic conditions of equilibrium no longer hold, and we find in general a force acting between the liquid and the body in such direction as to oppose the movement, and thus tend to reduce the relative velocity to zero. This force which we now consider is therefore one called into existence by the motion. It will in general have both a horizontal and a vertical component. The latter, while generally omitted from consideration, may in special cases reach an amount requiring recognition. The existence of such a resisting force, while due ultimately to the relative motion, may be considered more immediately as arising from a change in the amount and distribution of the surface forces acting on the body. When there is no relative motion between body and liquid, the surface forces are wholly normal pressures and their distribution is such that, as above noted, the horizontal resultant is zero and the vertical resultant equals the weight. The instant such motion arises, however, the surface forces undergo marked changes in both amount and character. The normal pressures are more or less changed in amount and distribution, and, in addition, tangential forces which were entirely absent when the body was at rest are now called into existence. In consequence the horizontal resultant is no longer zero, but a certain amount R, the equal of which must be constantly applied in the direction of motion if uniform movement under the new conditions is to be maintained. The entire vertical resultant must, of course, still equal the weight. This resultant, however, may be considered as made up of two parts, one due to the motion, and the other to the statical buoyancy of the portion immersed. The sum of these two will equal the weight. Hence when in motion the statical buoyancy will not in general be the same either in amount or distribution as for the condition of rest.

The amount and distribution of the surface forces must depend ultimately on the following conditions:

(1) The body.... { (*a*) Geometrical form and dimensions of immersed portion. (*b*) Character of wetted surface.

(2) The liquid... { (*a*) Density. (*b*) Viscosity.

(3) The relative motion.

Since it is the relative velocity between the body and liquid with which we are concerned, it is evident that we may approach the problem from two standpoints according as we consider the liquid at rest and the body moving through it, or the body at rest and the liquid flowing past it. The former is known as Euler's method, and the latter as Lagrange's. Each method has certain advantages, and it will be found useful to view the phenomena in part, at least, from both standpoints.

We may also view the constitution of the resisting force in two ways. The first, as a summation of varying forces acting between the water and the elements of the surface as mentioned above. On the other hand considering the water, we find as the result of this disturbance in the distribution and amount of the liquid forces a certain series of phenomena varying somewhat with the location of the body relative to the surface of the liquid. These we will briefly examine.

As the first and simplest case we will suppose the body wholly immersed and so far below the surface that the disturbances in hydrostatic pressure become inappreciable near the surface. It thus results that there are no surface effects or changes of level.

Let *ABK*, Fig. 1, represent the body in question, considered at rest with the liquid flowing by. Now considering the motion of the particles of water, it is found by experience that two well-marked types of motion may be distinguished.

The particles at a considerable distance from the body will move past in paths indistinguishable from straight lines. As we approach the body we shall find that the paths become gently curved outward around the body as shown at *LMN*. Such paths are in general space rather than plane curves. The nearer we approach to the body the more pronounced the curvature, but in all these paths the distinguishing feature is the smooth, easy-flowing form and the absence of anything approaching doubling or looping. Such curves are known as stream-lines. Passing in still nearer the body, however, we shall find, if its form is blunt or rounded, a series of large eddies or vortices seemingly formed at or near the stern. These float away and involve much of the water extending from the body to some little distance sternward. The water between the eddies will also be found to be moving in a more

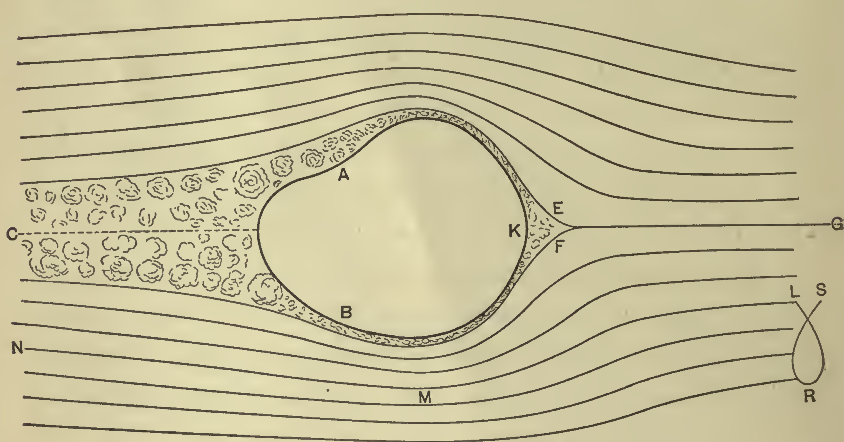

FIG. 1.

or less confused and irregular manner. These eddies with the water about and between them constitute the so-called "wake." At high speeds also, and with a blunt or broad forward end, there may be found in addition just forward of the body a small mass of water in which the motion is not well defined and smooth. Again, at the sides and between the smooth-flowing stream-lines and the body we shall find a belt of confused eddying water in which the loops and spiral paths are very small, the whole constituting a relatively thin layer of water thrown into the most violent confusion as regards the paths of its particles. The extent of the disturbance decreases gradually from the body outward to the water involved in the smooth stream-line motion. The characteristic feature of the motion of the water involved in these vortices, eddies, etc., is therefore its irregularity and complexity as distinguished from the smoothness and simplicity of that first considered. The volumes *KEF* and *ABC* are sometimes known as the liquid prow and stern. The former, however, is usually inappreciable in comparison with the latter. The term "dead-water" is also sometimes applied to the mass of water thus involved.

We may also consider the path of the particles by Euler's method of investigation. This will be simply the motion of the particle relative to the surrounding body of water considered as at rest. For the outlying particles, such as those forming the curves *LMN*, we shall have now simply a movement out and back as the body moves by. The paths out and back are not usually the same, and the particle does not necessarily or usually return to its starting-point. If we now suppose the body moving toward *G*, the path of a particle originally at *L* will be some such curve as *LRS*. This is a single definite path, the particle being moved out from *L* and finally brought to *S* without final velocity, where it therefore remains. For particles involved in the eddies and whirls however, the paths are entirely indefinite, and consist of confused interlacing spirals and loops. The particle is not definitely taken hold of at one point and as definitely left at another, but instead may be carried with the body some distance and then left with a certain velocity and energy, the latter gradually and ultimately becoming dissipated as heat.

Let us now remove the special restriction relating to the location of the body, and let us suppose the motion to take place at or near the surface of the liquid. The normal distribution of hydrostatic pressure near the surface will now be disturbed, and, as an additional result, changes of elevation will occur constituting certain series of waves. As we shall see later, the energy involved in these waves is partly propagated on and retained within the system, and partly propagated away and lost.

Now returning to the general consideration of resistance, it is evident that we may approach its estimation from two standpoints. (1) We may seek to study the amount and nature of the disturbance in the distributed liquid forces acting on the immersed surface. (2) We may seek to know the resistance or force between the body and the liquid, through its effects on the latter as manifested in the various ways above described. The latter is the point of view usually taken. From this standpoint it seems natural to charge a portion of the resistance to each manifestation, and thus to look for a part in the production of the curved stream-lines, a part in the production of the eddies at the bow and stern, a part in the eddying belt due to tangential or frictional forces, and a part in the waves.

These various manifestations we shall proceed to take up in order.

Stream-lines and stream-line motion have played so prominent a part in the various views which have been held on resistance, and serve so well to show certain features of the general problem, that we shall find it profitable to first examine them in some detail.

2. STREAM-LINES.

For the definition and general description of what is meant by a stream-line we may refer to the preceding section. We have now to define a stream-tube or tube of flow. Let *PQRS*, Fig. 2, be any closed curve in a plane perpendicular

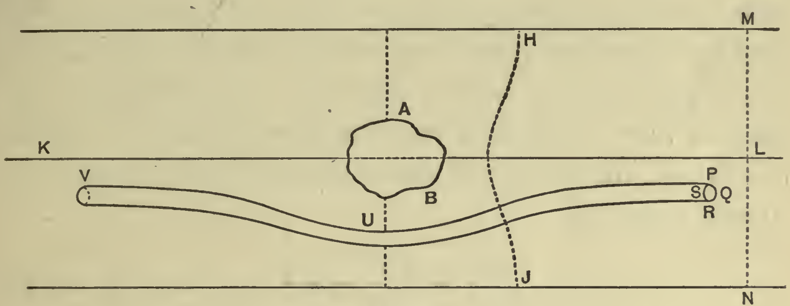

FIG. 2.

to the line of motion *LK*. Let this curve be located at a point so far from *AB* that the stream-lines *PUV*, etc., are at *P* sensibly parallel to *KL*. The particles comprised in the contour *PQRS* at any instant will, in their passage past *AB*, trace out a series of paths which by their summation will form a closed tube or pipe. Such is called a tube of flow. From these definitions, a particle of water which is within the tube at *PR* will always remain within it, and no others will enter. We shall therefore have a tube of varying section in which the water will flow as though its walls were of a frictionless rigid material, instead of the geometrical boundary specified. Starting at *PR* with a certain amount of water filling the cross-section of the tube, we find the motion parallel to *KL*. As we approach and pass *AB* the direction and velocity of flow will change, but the latter always in such way as to maintain the tube constantly full. After passing beyond *AB* the direction of motion will approach KL, and finally at a sufficient distance will again become parallel to it. Supposing the fundamental conditions to remain unchanged, the entire configuration of stream-lines will remain constant, and at any one point there will be no variation from one moment to the next. It follows that the conditions for steady motion as defined in hydrodynamics are fulfilled, and hence that the equations of such motion are directly applicable to the liquid moving within the tube. Assuming for the present that the liquid is hydrodynamically perfect, that is, that there are no forces due to viscosity, the equation for steady motion is

$$\frac{p}{\sigma} + z + \frac{v^2}{2g} = h,^* \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where p = pressure per unit area;
σ = density;
z = elevation above a fixed datum;
v = velocity;
h = a constant for each stream-line, called the total head.

$\frac{p}{\sigma}$ is called the presure-head;

z is the actual head;

$\frac{v^2}{2g}$ is called the velocity-head.

The total head, being constant for each line or indefinitely small tube, but variable from one to another, may be consid-

* We shall not here develop the fundamental equations of hydrodynamics, but shall assume the student familiar with them or within reach of a text-book on the subject.

ered as a distinguishing characteristic of the tube or line of flow. If we take the value of such characteristic far from *AB*, as at *PR*, we shall have for $p \div \sigma$ simply the statical pressure-head measured by the distance from *PR* to the surface of the liquid. For z we have the vertical distance from *PR* to the origin *O* at any fixed depth. The sum of these two equals the distance from *O* to the surface. Hence, as is perfectly permissible, if we take *O* at the surface, $p \div \sigma + z = 0$, and $h = v_0^2 \div 2g$, where v_0 is the known velocity with which the liquid as a whole is moving past *AB*. Hence with *O* at the surface the equation to a line becomes

$$\frac{p}{\sigma} + z + \frac{v^2}{2g} = \frac{v_0^2}{2g}. \quad \ldots \ldots \quad (2)$$

It is evident that as we move along the tube p and v will depend on z and on the cross-sectional area of the tube. If we take a case where the tube is sensibly contained in a horizontal plane, z will be constant and p and v will vary in opposite directions. The cross-sectional area will be found to increase somewhat just before *AB* is reached. Hence in this neighborhood v will decrease and p will increase. As we pass on, the area will decrease, becoming a minimum at *U*. Here p will be a minimum and v a maximum. The same changes are then repeated in reverse order as we pass on from *U* to *V*. The reasons for the variation in cross-sectional area as above stated may be seen as follows:

Consider a tube of flow of large diameter to entirely inclose *AB* and the liquid about it. We consider the sides of the tube so far away that the stream-lines constituting its boundary are sensibly parallel to *KL*. All phenomena may therefore be considered as taking place within the tube. The cross-sectional area available for the flow of the liquid will evidently be the area taken normal to the stream-lines. At the entering end and far from AB this will be a right section, as MN. Just forward of AB, however, where the stream-lines curve outward, the orthogonal section will be curved or dished as indicated by HJ. The area of this will be greater than that of the right section at MN, and hence the average area of the tubes of flow must be greater. This effect will also be more pronounced near the body, where the change in curvature is greater. As we pass on to U the stream-lines are again parallel to the sides of the large tube, and hence the net section available for flow will be the right section of the tube minus the section of AB. Hence the average cross-section of the tubes of flow must be less here than at the entering end. As above, the difference will be more pronounced near the body than far removed from it. Similarly just behind AB the orthogonal section will be curved and the average area will be increased, as at HJ.

We have now to show that in any tube of flow in which the two ends are equal in area, opposite in direction, and in the same line, the total effect of the internal pressures is 0; that is, that the pressures developed have no tendency to transport the tube in any direction whatever.

Fig. 3.

Let us first consider any closed tube or pipe, as in Fig. 3, no matter what the contour or the variation of sectional area. Suppose it filled with a perfect or non-viscous liquid moving with no tangential force between itself and the walls of the pipe. The conditions are therefore similar to those for a closed tube of flow in a perfect liquid. Suppose the liquid within this tube to have acquired a motion of flow around the tube. There being no forces to give rise to a dissipation of energy, the motion will continue indefinitely. We know from mechanics that the forces in such case form a balanced system and hence that there is no resultant force tending to move the pipe in any direction. That such is the case may also be seen by considering that if there were a resultant external force we might obtain work by allowing such resultant to overcome a resistance. Such performance of work could not react on the velocity of the liquid, and hence we should obtain work without the expenditure of energy.

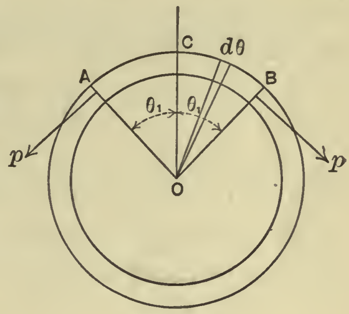

FIG. 4.

Suppose next the pipe to be circular in contour and of uniform section, as in Fig. 4.

Let a = area of section of pipe;
r = mean radius of contour;
σ = density of liquid = 62.5 for fresh and 64 for salt water;
f = centrifugal force due to liquid.

Then the volume of any small element is $ard\theta$, and the corresponding centrifugal force is

$$df = \frac{\sigma ard\theta v^2}{gr} = \frac{\sigma av^2}{g}d\theta.$$

Let the total angle AOB be $2\theta_1$. Let θ be the angle between OC and any element. Then the component of df along OC is

$$df \cos\theta = \frac{\sigma a v^2 \cos\theta d\theta}{g}.$$

Integrating, we have for the total resultant along OC

$$Q = \frac{2\sigma a v^2}{g} \sin\theta_1.$$

This total force outward from O toward C may be equilibrated by a pair of tangential tensions at A and B. Let each of these be denoted by p. Then we have

$$2p \sin\theta_1 = Q = \frac{2\sigma a v^2}{g} \sin\theta_1,$$

or $$p = \frac{\sigma a v^2}{g}.$$

That is, the forces due to the movement of the liquid in any arc give rise to two tangential tensions at the ends of the arc, and these tensions are independent of its length and depend simply on a and v. This is also evidently true no matter what the remainder of the contour, or whether it is closed or not. Hence in any pipe, closed in contour or not, containing a portion of uniform area and curved in the arc of a circle, the forces due to the flow of the liquid through this portion will be such as would be balanced by two tangential tensions at the ends of the arc, each equal to $\sigma a v^2 \div g$.

Let us now consider a pipe of any irregular contour and sectional area, as in Fig. 5. Let the ends A and D be of the same area, but turned in any direction relative to each other.

Suppose liquid as above to flow through the pipe. Required to determine the resultant of the internal pressures.

Suppose the circuit completed by means of a pipe *AFED* of uniform section and made up of circular arcs *AF* and *DE* and a straight length *FE*. Then if a circulation is set up in this closed contour such that the velocities at *A* and *D* are the same as before, the conditions in *ABCD* will remain unaffected. We know as above that the entire resultant is 0, and that the effects due to the liquid in *AF* and *DE* are represented by tensions at the extremities of those arcs each

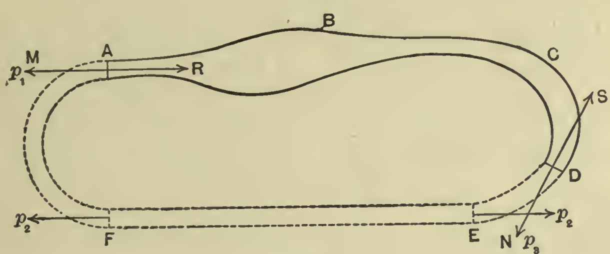

FIG. 5.

of value $\sigma a v^2 \div g$. The effect due to the straight part *FE* must be 0, and the two tensions at *F* and *E* will balance each other. Hence for the entire resultant of the forces in the dotted part of the contour we shall have the equal tangential forces $p_1 = AM$ and $p_3 = DN$ at *A* and *D*. Now since the system of forces for the entire contour is balanced, it follows that the forces due to the liquid in *ABCD* must be represented by two forces equal and opposite to *AM* and *DN*. Hence *AR* and *DS* must represent in direction, point of application, and amount the resultant of the forces due to the water in *ABCD*.

As a special case let *ABCDE*, Fig. 6, be a curved pipe of any varying sectional area, but with its ends *A* and *E* equal, opposite in direction, and in the same line. Applying the above principle, it follows that the resultant will be that of

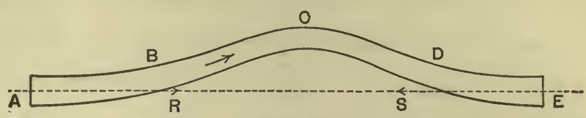

FIG. 6.

the two forces AR and ES, equal and opposite in direction and in the same line. Hence in such case the resultant is 0 and there is no tendency to move the pipe in any direction, and in particular no force tending to move it in the direction AE. This serves to establish the initial proposition relative to the forces in such a tube of flow. In the most general case the straight portions at the two ends, while parallel, are not in the same straight line. The preceding treatment still holds, however, and we shall have for such case the resultant represented by two equal and opposite forces not in the same line and therefore forming a couple.

Let now AB, Fig. 7, be a body about which the liquid flows without tangential forces in stream-line paths. Let

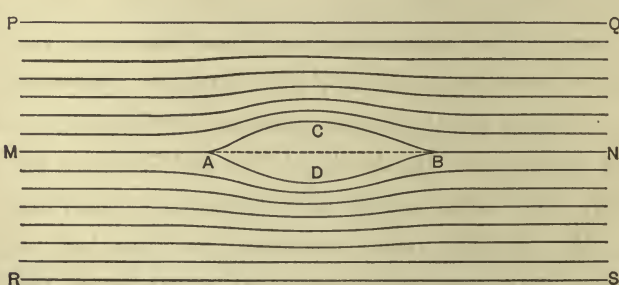

FIG. 7.

this body be immersed at an indefinite depth so that surface effects are insensible. We wish to show that the total force communicated to the body by the stream-lines is 0. This may be seen by considering *PQRS* a tube of flow entirely surrounding *ACBD*, with its straight ends equal in area, opposite in direction, and in the same line, and of variable sectional area, *ACBD* forming an internal boundary. The general proposition above then applies, and the resultant of all forces acting on the internal boundary *ACBD* will be 0. Otherwise the tube *PQRS* may be considered as made up of an indefinite number of small tubes for each of which the resultant may be a couple. It is easily seen, however, that the forces constituting these couples will, for the whole tube, constitute a system uniformly distributed over the ends *PR* and *QS*, and hence the entire resultant will be 0.

This important conclusion may also be reached by considering that if a resultant force were communicated from the liquid to *AB*, we might obtain work by allowing *AB* to move and thus overcome some resistance. By the supposition of the absence of all tangential force between *AB* and the liquid, this would take place without affecting the velocity of the latter. In fact without tangential force the velocity of the liquid can in no wise be affected. Hence we should obtain work without the expenditure of energy, and the suppositions leading to this result are therefore inadmissible.

We may at this point note a further result of the relationship expressed by the general equation of hydrodynamics above.

Suppose z to remain substantially constant and v to increase continually. The pressure will correspondingly decrease, and at the limit for a certain velocity we shall have $p = 0$. An indefinitely small increase of velocity would lead to a tendency to set up a negative pressure or tension in the liquid. This could not be actually realized, and the result would be, instead, a breaking up of the stream into minute turbulent whirls and eddies. The existence of such turbulence may therefore imply a previous condition in which there existed a tendency toward an indefinite decrease in the pressure, and such may be considered as its hydrodynamical significance. In the actual case the stream becomes unsteady and breaks up before the pressure becomes actually reduced to zero, although it is always much reduced as compared with the pressure for steady flow. A further result of the great reduction in pressure is usually found in the giving up by the water of the air which it holds in solution. The air thus liberated appears in the form of small bubbles mingled with the water, causing the foamy or yeasty appearance which usually accompanies this phenomenon.

3. Geometry of Stream-lines.

Among the first to note the relation of stream-lines to the problem of ship resistance was Prof. Rankine. His investigations covered the examination of their general properties and the derivation of forms for ships which, under the special conditions assumed, would give for the relative motion of ship and water smooth stream-line paths. We will now show methods of constructing stream-lines for various special cases as developed by Rankine and other investigators since his time.

The stream-lines which surround a body are in general space-curves. The properties of such lines of double curvature are so complex that their investigation is a work of much difficulty and labor. Many helpful and instructive suggestions, however, may be derived from a study of plane water-lines to which the present notice will be restricted.

In order to imagine a physical condition which would give such lines, suppose a body partly immersed in a liquid on the surface of which is a rigid frictionless plane. Let a similar parallel plane touch the under side of the body. Then let the liquid between the planes move past the body, or *vice versa* let the body move relative to the liquid. The relative motion of liquid and body may then be considered as taking place in water-lines contained in horizontal planes. At the limit we may suppose the planes approached very near, the body being then simply that portion contained between them. The liquid will then move in a thin horizontal sheet in plane water-lines.

It is shown in hydromechanics that for such water-lines there exists a function ψ such that

$$u = \text{velocity along } x = \frac{\delta\psi}{\delta y}$$

and $$v = \text{velocity along } y = -\frac{\delta\psi}{\delta x}.$$

That is, that the velocity in any direction is the rate of change of this function in a direction perpendicular to the given one. It is also shown that this function is constant for any one water-line, and that it may therefore be considered as a characteristic or equation to such line. It is likewise shown that its value for any one line is proportional to the total amount of flow between such line and some one line taken as a standard or line of reference.

For the simplest case let us assume two planes as above, indefinite in extent and very near. At a given point let liquid be introduced at a uniform rate. Such a point of introduction is called a source. In such case the liquid will move in straight lines radiating from the source. Similarly we might have assumed liquid abstracted at a uniform rate. Such a point of abstraction is called a sink. In such case likewise, the liquid will move in straight lines converging toward the sink.

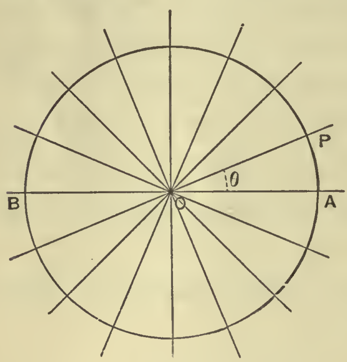

FIG. 8.

In Fig. 8 let O be the point and OA the line of reference from which the total flow is measured. Let a be the amount introduced per second or other unit of time, and θ the angle between OA and any line OP. Then the flow between OA and OP is that corresponding to the angle AOP. Hence we have

$$\psi = \frac{a\theta}{2\pi}.$$

The quantity $a \div 2\pi$ is called the strength of the source or sink.

It is also shown in hydromechanics that the function ψ for a complex system of sinks and sources is simply the algebraic sum of the separate functions, considering ψ for a source as $+$ and for a sink as $-$. Hence for such a system in general we have

$$\bar{\psi} = \Sigma\psi.$$

For two sources of equal strength we have

$$\psi_1 = \frac{a\theta_1}{2\pi} \quad \text{and} \quad \psi_2 = \frac{a\theta_2}{2\pi}.$$

Hence $$\bar{\psi} = \frac{a}{2\pi}(\theta_1 + \theta_2).$$

For a sink and source of equal strength we have

$$\bar{\psi} = \frac{a}{2\pi}(\theta_1 - \theta_2).$$

In all such cases of simple combination the actual stream-lines are most readily found by a construction due to Maxwell and illustrated in Fig. 9.

From O_1 and O_2 let radiating lines be drawn at angular intervals proportional to $a \div 2\pi$. Then the number of lines

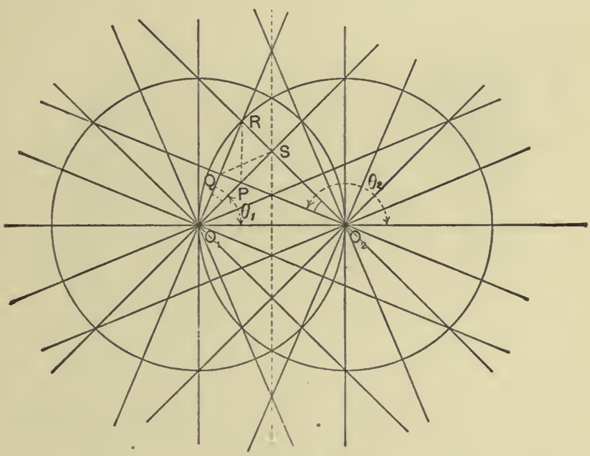

Fig. 9.

will also be proportional to $a \div 2\pi$. Take any point P. Then it is readily seen that by going across the quadrilateral $PQRS$ to R we find another point for which θ_1 is increased by

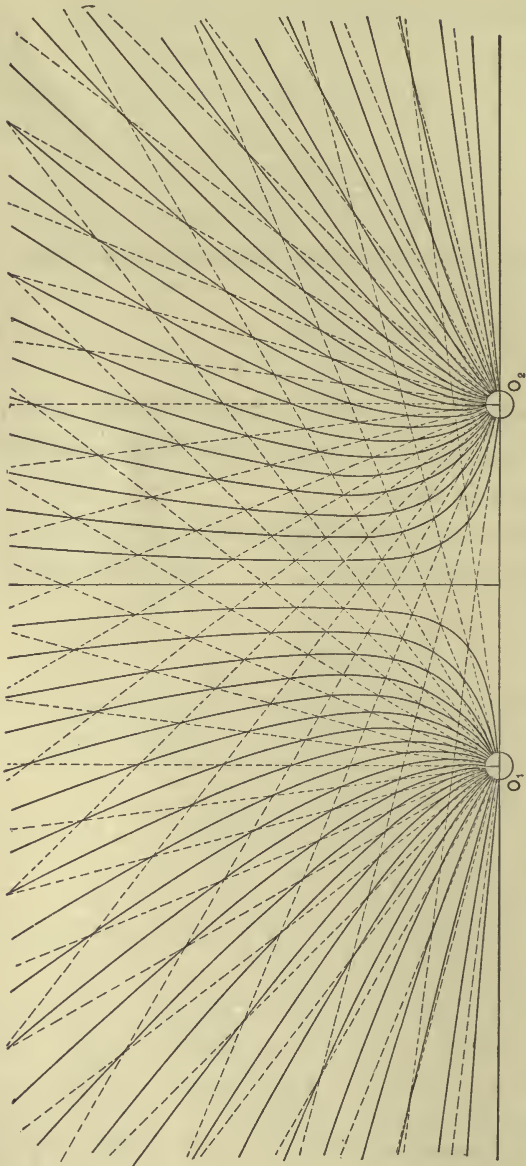

FIG. 10.

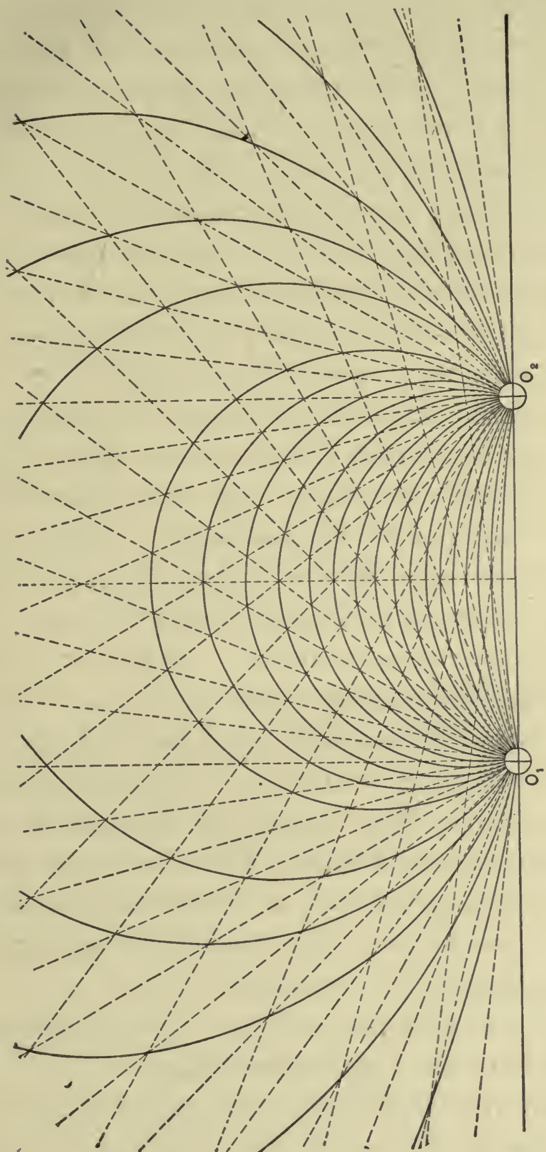

FIG. 11.

the angle RO_1S and θ_2 decreased by the equal angle QO_2S. Hence at R we shall have $(\theta_1 + \theta_2)$ of the same value as at P. Hence P and R will be points on one of the stream-lines belonging to the two sources O_1 and O_2. It follows that the series of stream-lines for these sources will be obtained by drawing continuous curved lines diagonally across the series of quadrilaterals formed by the intersections of the radiating lines from O_1 and O_2. Similarly it may be seen that for Q and S the values of $(\theta_1 - \theta_2)$ are the same, and hence that these two points are on one of the stream-lines for a source O_1 and a sink O_2. In like manner, then, the series of stream-lines for such a doublet, as it is termed, will be found by drawing continuous curvilinear diagonals in the other direction across the same set of quadrilaterals. These constructions are illustrated in Figs. 10 and 11, and it is readily seen in the latter case that the curves are arcs of circles.

Maxwell showed in general that for all cases of combination of two systems of sinks and sources or their equivalents, the resulting system of stream-lines could be found geometrically by laying down those for the two component systems, and then taking curved lines continuously diagonal to the series of quadrilaterals thus formed. In this way the result of a combination of any number of sinks and sources may be found. In practice the labor involved is very great when they exceed three or four in number.

In case the sinks and sources are not all of the same strength, exactly the same method holds.

We have now to show as a special case the result of the combination of a source and a continuous stream with straight stream-lines. The latter may be considered as a part of the system due to a source of infinite strength situated at an infinite distance away. The geometrical method for the determination of the resulting stream-line system is exactly similar to the preceding, and comes under the general rule of procedure as stated above. It is illustrated in Fig. 12. The straight lines radiating from O are replaced each by a curved line as shown. Outside of this system of curved lines springing from O are the deflected lines of the uniformly flowing stream. It is readily seen that the line ABC separates one of these systems from the other; also that there is no flow whatever across this line, and hence that it separates the liquid introduced by the source O from that originally in the stream. It is also evident that if, instead of the source O and its system of stream-lines, we should substitute a frictionless surface ABC extending away indefinitely, the lines of the uniform stream would have exactly the same form which they now have as a result of the combination of the source and stream.

Taking similarly a uniform stream in combination with a source and sink, we have the result of Fig. 13. In this case it is seen that the liquid introduced at O_1 and withdrawn at O_2 does not spread away indefinitely, but is confined to the portion bounded by the line ABC, and that, as before, this line separates the liquid thus introduced and withdrawn, from that originally in the stream. Hence also if, instead of the doublet O_1O_2, we should introduce a thin flat solid with frictionless contour ABC . . . , the resulting system of stream-lines in the uniform stream would be the same as that here resulting from the stream and the doublet.

It is thus seen that we may in this way arrive at the system of stream-lines due to a frictionless solid of certain form plunged in a uniformly flowing stream.

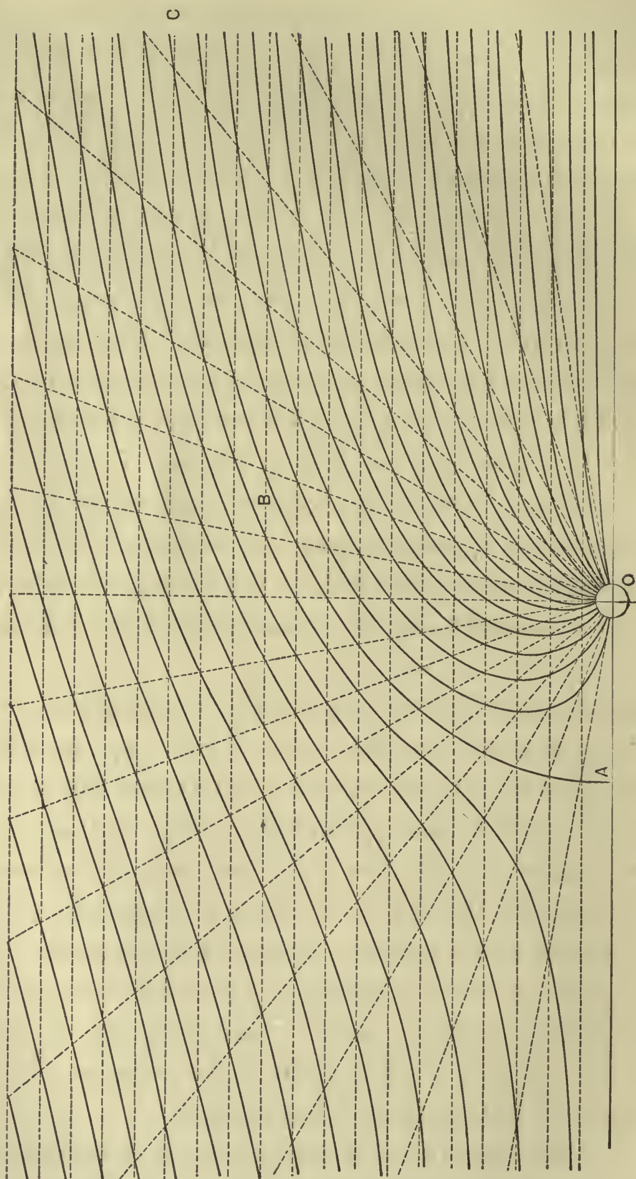

FIG. 12.

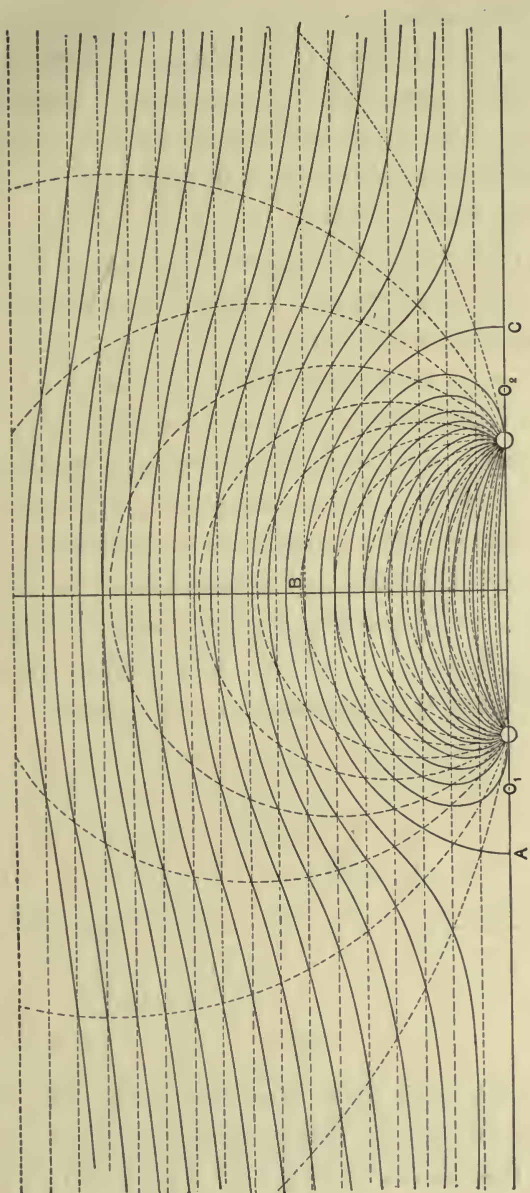

FIG. 13.

The form *ABC* . . . is not, however, suitable for the water-line of a ship, and it is only by an extension of the method that stream-lines for ship-shaped forms may be determined. This extension involves the appropriate choice and location of sources and sinks such that the resulting line of separation shall as closely as may be desired resemble a ship's water-line. The most general method of effecting this extension is that due to Mr. D. W. Taylor,* which we may briefly summarize as follows:

Instead of a system of separate sources and sinks, Mr. Taylor imagines a source-and-sink line or narrow slot through a part of which liquid is introduced, and through the remainder of which it is withdrawn. The variation of strength along this line corresponds to a difference in width of slot. The distribution of strength may be represented graphically by a line *ABC*, Fig. 14, where *OB* represents the source portion and *BP* the sink portion, the strength at any point being proportional to the ordinate of *ABC*.

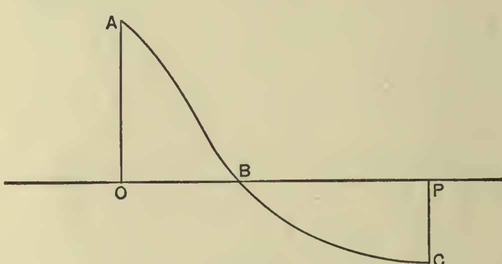

FIG. 14.

The areas *ABO* and *BPC* must be equal, else the total amounts introduced and withdrawn would not be the same. The author then shows by graphical representation and inte-

* Transactions Institute of Naval Architects, vol. xxxv. p. 385.

gration how to find the current function or equation for the stream-lines corresponding to such a sink-and-source line, and how by varying the character of *ABC*, Fig. 14, to vary the character of the resulting contour *ABC* . . . , Fig. 13, so as to bring it to any desired degree of similarity to a given water-line.

The distribution of stream-lines being known, it is a simple matter to find, by the help of the equation for steady motion, § 2 (1), the corresponding distribution of pressure. This is illustrated in Fig. 15. *ABC* represents one quarter of the horizontal section of a body producing a system of stream-lines as shown. Then at points along a line OY_1 the excess in pressure over the normal may be represented by the ordinate to the curve *DE* relative to OY_1 as axis. Similarly the defect in speed at the same points may be represented by a curve such as *FG*. In like manner at points along the line BY_2 the excess of speed is represented by the curve *HI* and the defect of pressure by *JK*. At a considerable distance from the body these values all approach indefinitely near to their normal values and the variations therefrom become indefinitely small, as indicated by the diagram.

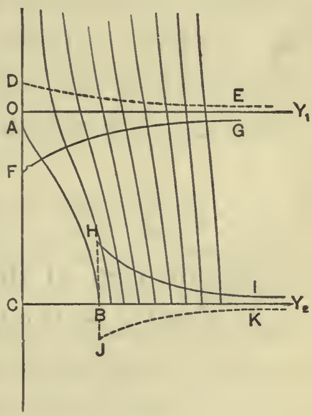

FIG. 15.

Since, under the special conditions assumed in the treatment of stream-lines, the total resultant force between the body and the liquid is 0, the resistance to the motion of the solid through the liquid will also be 0. The investigation of stream-lines is not therefore of importance in the determination of the actual resistance in any given case, but rather as showing, under the assumptions of plane stream-lines and no tangential force between the surface and the liquid, what forms would give rise to smooth and easy stream-lines, and hence presumably in an actual liquid would experience the minimum of resistance.

In the actual case the liquid is quite free instead of being confined between two rigid planes. This introduces into the form of the stream-lines, especially near the surface, profound changes, so that the indications received from plane stream-lines must be used with caution, and only so far as comparison with the results of experimental investigation may seem to justify. See further § 14, on wave-resistance.

4. Résumé of General Considerations.

In considering the application of the general principles developed in the present chapter to the resistance of ships, it must be borne in mind that for the present we take no account of any influence due to the presence of a propelling agent. The resistance which we here consider is the actual or tow-rope resistance—the resistance which the ship would oppose to being moved as by towing through the water at the given speed. The modifications due to the presence of a propeller or paddle-wheel will be considered in § 45.

We have before us, therefore, the general problem of the resistance of a partially immersed body in an actual liquid, as water. For convenience we shall examine the subject under the following heads:

(1) *Stream-line Resistance.*—The liquid being no longer without viscosity or internal shearing stresses, the stream-line conditions of a perfect liquid will not be quite realized, and

the maintenance of these lines in the actual liquid will involve a slight drain of energy from the body. This involves the transmission of a resultant force between the liquid and the body opposed to the relative motion of the two, or, in the present case, to the motion of the latter. This gives the first item of the total resistance as above.

(2) *Eddy Resistance.*—We come next to the water involved in the liquid prow and stern. As already noted, this does not follow open stream-line paths. It is thrown into irregular involved motion, the energy of which in the actual liquid is continually degrading into heat. The water involved in this motion does not, moreover, remain the same, but gradually changes by the drawing in of new particles and the expulsion of others.

The maintenance of this liquid prow and stern will give rise, therefore, to a constant drain of energy from the moving body, the effect of which is manifested as a resistance to the movement. This gives the second item as above.

(3) *Surface, Skin, or Frictional Resistance.*—We take next the belt of eddying water produced by the action of the tangential forces between the surface and the liquid. These eddies stand in the same relation to the motion of the body as those giving rise to the eddy resistance so called. Their maintenance constantly drains energy from the body, and this gives rise to a resistance to the movement. This gives the third item as above.

(4) *Wave Resistance.*—Finally we have the wave systems which always accompany the motion of a body near the surface of the water. As remarked in § 1 and as we shall later show, the energy involved in these systems is not retained intact, but a portion is constantly being propagated away and

lost. The maintenance of the wave systems requires, therefore, a constant supply of energy from the moving body, and this, as in the previous cases, gives rise to a resistance to the motion. This gives the fourth item as above.

It may be noted that in the most general view (1) and (4) are not independent. Waves may be considered as the result of modified stream-lines near the surface, and hence in its most general significance (1) might be considered as including (1) and (4). As here classified, however, we mean by (1) the resistance due to the maintenance of the system of stream-lines which would be formed were the body moved with the given speed at an indefinite depth below the surface. The modification due to wave-formation is then taken care of by (4). In any case (1) is very small, since, with the velocities occurring in practice, water behaves nearly as a non-viscous liquid. Similarly (2) and (3) might with propriety be termed the eddy-resistance, since both items arise from the maintenance of systems of eddies. Since their immediate causes are distinct, however, it is convenient to consider them separately.

The sum of (1) and (2) is frequently termed the head-resistance, though it is sometimes considered as of two parts, called *head* and *tail* resistances, the former being due to the excess of pressure in the liquid prow, and the latter to the defect in the eddying liquid stern. Inasmuch as they cannot be separated, however, we shall use the one term for both.

With ships of ordinary form this is so small as to be relatively negligible. In other cases, as in the motion of the paddle-wheel, propeller-blade, bilge-keel in oscillation, etc., this is one of the chief items of the resistance to the motion of the given body.

In ships of usual form we are therefore concerned chiefly with surface and wave resistance.

The sum of (1) and (4) or of (1), (2), and (4) is often known as the *residual* resistance. This term has reference to the usual view of ship-resistance which recognizes but two chief subdivisions:

(*a*) Surface or skin resistance.

(*b*) Residual resistance, including (4), above, and all other parts not properly classified under (*a*).

It must not be forgotten that this classification of resistance according to the various effects produced on the water is somewhat arbitrary. A more logical mode of subdivision might be made by starting from the body and considering that each element of immersed surface is subjected to a certain force. Such force may be resolved into two components; one tangential, the other normal. This is entirely general and includes all effects no matter from what cause arising. Comparing with the other method of subdivision it is evident that the longitudinal components of the tangential forces will give by their summation the surface or frictional resistance. Hence the longitudinal components of the normal forces must give by their summation the remaining or residual resistance, corresponding to (1), (2), and (4), or with ship-shaped forms almost entirely to (4). This subdivision is entirely independent of any supposition with regard to the effects on the water, and is the most direct or immediate aspect of resistance itself. The two parts may be termed *tangential* and *normal* resistances.

No matter what view we may take of the subdivision of resistance, we are obliged to go to experimental investigation for all satisfactory information as to its amount. We proceed, therefore, to the description of the various experimental investigations, and to the discussion of the results which may be derived therefrom.

5. Resistance of Deeply Immersed Planes Moving Normal to Themselves.

If the body in Fig. 1 is supposed to be reduced to a plane normal to the line of motion, the result will be a system of stream-lines and eddies somewhat as in Fig. 16, though the

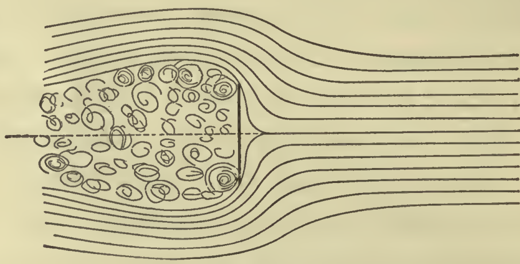

Fig. 16.

line of demarcation between the two is not fixed nor so definite as must be suggested by a rough diagrammatic representation such as the figure gives. The tangential resistance will be 0 because all tangential force is at right angles to the direction of motion. This does not mean that the actual resistance is independent of the character of the surface, but simply that the actual tangential forces have no longitudinal component. The nature of the surface will presumably slightly affect the stream-line formation and the amount and nature of the liquid prow and stern, so that ultimately it will have its effect on the resistance. Viewed immediately, however, the resistance is due wholly to the eddies and stream-lines in an imperfect liquid. Experiment shows that the resistance in such case may be approximately expressed by an equation of the form

$$R = f\frac{\sigma}{2g}Av^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where R = resistance in pounds;

f = a coefficient;

σ = density of liquid = 62.5 for fresh and 64 for salt water;

g = acceleration due to gravity = 32.2 ft.;

A = area in square feet;

v = velocity in feet per second.

For salt water $\sigma \div 2g$ is very commonly put equal to 1. In such case the equation becomes

$$R = fAv^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The values of the coefficient f which have been determined experimentally have varied widely. This is doubtless due to the fact that such experiments have been carried on by different observers under different conditions, notably as to amount of surface, depth of immersion, and actual velocities employed. The subject has not been examined with sufficient completeness to show satisfactorily the general relation of the resistance to the three conditions, *area*, *immersion*, and *velocity*. In fact there is considerable doubt as to whether R should increase strictly as the area, we are not sure that the index of v should for all velocities be 2, and the relation of f to varying immersion is not satisfactorily known. The attempt, therefore, to express the results of experiments under widely varying conditions by the simple formula above, throwing on f the consequences of all differences between the true and the assumed law, may with justice lead to widely varying values for this coefficient.

Among the earliest experiments were those by Col. Beaufoy made at the Greenland Docks in London between 1793 and 1798. Taking A in square feet, v in feet per second, and R in pounds, the mean value of f deduced from these experiments is about 1.1.

Mr. Wm. Froude's experiments on planes about 3 ft. wide gave with similar units a mean value of about 1.7.

Dubuat at a single speed of about 3.28 ft. per second found for a plane maintained stationary in a moving stream a value of $f = 1.86$. For a stationary liquid and moving plane, the speed being the same, he found $f = 1.43$. If in the first case the liquid moved steadily and without eddies or internal irregularities, and in the second case the liquid was strictly at rest, the relative speed being the same in each case, these two results should be the same. The discrepancy was doubtless due to the difficulty of making satisfactory observations on the mean velocity of a flowing stream, and also to the now well-known difference in effect between a stream moving without and with turbulence.

More recently Joessel's experiments in the river Loire, made on a plane of sheet iron .98 ft. high by 1.31 ft. long and immersed so as to have .66 ft. of water over the upper edge, gave a mean value of $f = 1.6$. The maximum velocity was not above about 4.25 ft. per second.

Still more recently Mr. R. E. Froude has determined this coefficient under conditions which insure the highest degree of experimental accuracy. The resulting value agrees very closely with the average of Beaufoy's values mentioned above, and was about 1.1.

An unexplained divergence from all the values above given is indicated by the results of experiments on bilge-keels. An attempt to fit this formula of resistance to these results leads to a coefficient varying from 5 or 6 to 15 or 16. Its excessive variability, as well as its great divergence from the other values, indicates that we have here involved some element or condition not represented by the formula, and whose importance has been hitherto unsuspected. It is understood that experiments under the direction of R. E. Froude are in progress which, it is expected, will throw light on these points.

It may be well here to note that if the immersion is sufficiently great to reduce the surface disturbance to a negligible amount, the resistance will be practically independent of any further increase of depth. This is readily seen from the fact that the resistance arises, not from the pressure, which of course increases with the depth, but rather from a difference in the distribution of such pressure, and that this is practically independent of depth so long as surface disturbance is absent. Still otherwise we may view the resistance as due to the energy absorbed by the stream-line and eddy systems; and since below the minimum depth above mentioned the configuration of these will be constant, the energy absorbed and the resulting resistance will likewise remain the same. These conclusions are borne out by Beaufoy's experiments.

It will be observed that, according to our definition, head-resistance is the resistance due to the eddy and stream-line formation supposing the immersion of the body so great that surface effects are negligible. It is not the resistance due to the actual stream-line and eddy formation if near the surface, for this would include the wave-resistance as well. In the present case, therefore, tangential resistance being absent, we must consider the change in the total resistance as the plane approaches the surface as due to the resistance arising from the wave-formation. In any experiment, therefore, if the plane is near the surface, the resistance actually measured will consist of two parts: head-resistance proper and wave-making resistance.

As is well known, a very slight wave may involve considerable energy, especially if its length is great. Hence apparently slight surface effects may count for much in the actual total resistance.

It seems likely that the explanation of much of the variation in the value of the coefficient f noted above may be found in these considerations. The actual amount measured is of course total resistance. If the immersion is not sufficient to eliminate surface disturbances, then this total resistance will include a portion due to the wave-formation. This must depend on depth, a condition not represented in the formula above. As we shall see later also, wave-resistance varies with velocity according to a much higher index than 2. The formula above is therefore wholly unsuited to represent the wave-resistance portion, and any attempt to make it represent a total resistance of which the wave-making portion may be relatively important will naturally lead to widely divergent values of f. In the case of bilge-keels the surface disturbance is of very considerable amount, and it seems likely that much of the resistance is due to the wave-making effects produced. The attempt to fit the above formula to the total results may therefore naturally lead to a value of f widely differing from its value for head-resistance alone.

6. Resistance of Deeply Immersed Planes Moving Obliquely to their Normal.

If the plane, instead of standing at right angles to the direction of motion, is inclined at an angle θ, we have a result somewhat as indicated in Fig. 17. In such case the force in

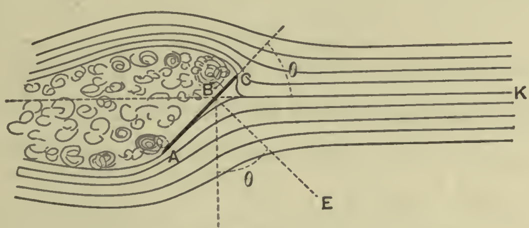

FIG. 17.

the line of the normal *EB* as determined by Joessel's experiments is given by the formula

$$P_\theta = 1.622 \frac{\sin\theta}{.39 + .61\sin\theta}\frac{\sigma}{2g}Av^2.* \quad . \quad . \quad . \quad (1)$$

The longitudinal component, or the resistance in the line of motion, is therefore

$$R_\theta = 1.622 \frac{\sin^2\theta}{.39 + .61\sin\theta}\frac{\sigma}{2g}Av^2. \quad . \quad . \quad . \quad (2)$$

Hence $$\frac{P_\theta}{P_{90}} = \frac{\sin\theta}{.39 + .61\sin\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and $$\frac{R_\theta}{R_{90}} = \frac{\sin^2\theta}{.39 + .61\sin\theta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

---

* Where not otherwise explained, the nomenclature of this section is the same as that of the preceding.

According to Beaufoy's results we may put approximately

$$\frac{P_\theta}{P_{90}} = \sin\theta, \quad \ldots \ldots \quad (5)$$

and

$$\frac{R_\theta}{R_{90}} = \sin^2\theta. \quad \ldots \ldots \quad (6)$$

The experiments of Wm. Froude also agreed in giving

$$\frac{P_\theta}{P_{90}} = \sin\theta. \quad \ldots \ldots \quad (7)$$

It must be admitted that the condition of our information on this subject is far from satisfactory, and further experiments are much needed.

Joessel's experiments were also directed toward the determination of the location of the center of the system of distributed pressures on an oblique plane. The results were as follows:

Let l be the length of plane in the direction of motion, and x the distance to the center of pressure from the leading edge. Then

$$x = (.195 + .305 \sin\theta)l. \quad \ldots \quad (8)$$

In the narrow range of speeds, 0 to 4.25 ft. per second, this law seemed to be independent of the velocity.

Referring again to Fig. 17, it is known that the stream flowing toward AC will divide and pass around, a part by C and a part by A. Let BK be the plane which separates one of these two streams from the other. Then Lord Rayleigh has shown that B is determined by the following proportion:

$$\frac{AB}{AC} = \frac{2 + 4\cos\theta - 2\cos^2\theta + (\pi - \theta)\sin\theta}{4 + \pi\sin\theta}.$$

The tangential force is evidently less than if both streams flowed in the same direction, as when the plane is moved parallel to itself. Let k be the ratio between the tangential forces in the two cases. Then Cotterill * has shown that

$$k = \frac{4 \cos \theta - (\pi - 2\theta) \sin \theta}{4 + \pi \sin \theta}.$$

The graphical representation of these relations is shown in Fig. 18.

The importance of reducing eddy resistance by an appropriate choice of form for a ship-shaped body, and the avoid-

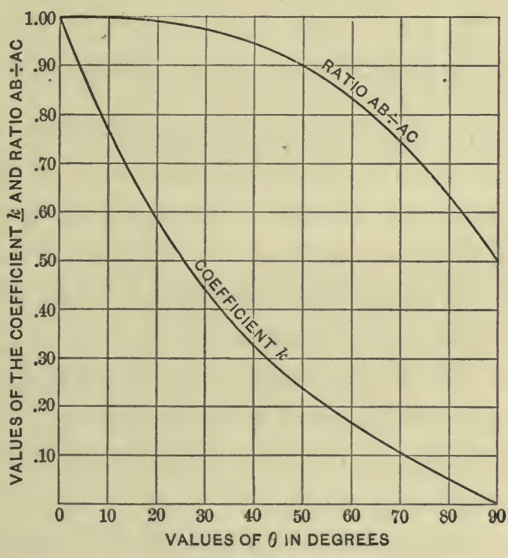

Fig. 18.

ance where possible of all surfaces normal to the direction of motion and of all sudden changes of direction in the surface, is readily seen. It thus appears as shown by Taylor † for a

* Transactions of the Institute of Naval Architects, vol. xx. p. 152.
† "Resistance and Screw Propulsion," p. 27.

ship 300 feet long at 20 knots, using the results of § 7 for tangential resistance, that the resistance of one square foot moving normal to its own direction is about 770 times as great as that of the same area exposed to tangential or skin-resistance only.

We may also add in this connection that if the lines of a ship are too full aft for her speed, large, unstable eddies are liable to appear about the stern, suddenly shifting about from one quarter to the other and causing sudden and unsymmetrical changes in the total resistance and in the action of the rudder, thus rendering such ships very difficult to steer.

7. Tangential or Skin Resistance.

The origin of this form of resistance has been referred to in § 1. Experiments for the determination of its amount have been made by Beaufoy, Tidman, and Wm. Froude. Inasmuch as those of the latter are the most recent and extensive, we shall be content with a brief résumé of the results.

In these experiments boards $\frac{3}{16}$ inch thick, 19 inches deep, and of different lengths from 4 to 50 feet were covered with various substances and towed lengthwise in a tank of fresh water, their speed and resistance being carefully measured by appropriate apparatus.

For the discussion of the results of the experiments the tangential resistance is supposed to vary according to the formula

$$R = fAv^n,$$

where R = resistance in pounds;

f = a coefficient;

A = area in square feet;

v = velocity in feet per second or knots, according to the value of f used;

n = an exponent.

A summary of the experimental results is shown in the following table:

TABLE I. SHOWING RESULTS OF EXPERIMENTS ON SKIN-RESISTANCE.

Nature of Surface.	Length of Surface or Distance from Cutwater.											
	2 feet.			8 feet.			20 feet.			50 feet.		
	A	B	C	A	B	C	A	B	C	A	B	C
Varnish......	2.00	.41	.390	1.85	.325	.264	1.85	.278	.240	1.83	.250	.226
Paraffin......	1.95	.38	.370	1.94	.314	.260	1.93	.271	.237			
Tinfoil......	2.16	.30	.295	1.99	.278	.263	1.90	.262	.244	1.83	.246	.232
Calico	1.93	.87	.725	1.92	.626	.504	1.89	.531	.447	1.87	.474	.423
Fine sand....	2.00	.81	.690	2.00	.583	.450	2.00	.480	.384	2.06	.405	.337
Medium sand.	2.00	.90	.730	2.00	.625	.488	2.00	.534	.465	2.00	.488	.456
Coarse sand..	2.00	1.10	.880	2.00	.714	.520	2.00	.588	.490			

Column A gives the value of n for the particular length and character of surface.

Column B gives for the whole surface the mean resistance in pounds per square foot at a speed of 600 feet per minute or 10 feet per second.

Column C gives for the same speed the resistance per square foot at a distance from the forward end equal to that stated in the heading.

It is thus seen that the resistance per unit area decreases as we go from forward aft. Thus for varnish the resistance per square foot at distances of 2, 8, 20, and 50 feet from the bow are respectively .39, .264, .24, .226. It necessarily results, as is shown by the table, that the mean resistance per square foot decreases with increased length. The explanation of this as given by Mr. Froude is as follows:

The board or surface under test moves surrounded by a skin of eddying water, the maintenance of the energy of which gives rise to the resistance under consideration. The forward part of the board is constantly entering water at rest, while the after portions come in contact with water already acted on by the forward end, and possessing, therefore, to a greater or less extent, this eddying motion. In these eddies the movement of the water-particles next the board is forward, more or less of the water being involved, and with a greater or less velocity according to the length of time they have been acted on, and to the other circumstances of the surface and motion. It follows that the eddying motion will be more and more pronounced as we go from the bow aft. Hence a part of the board near the forward end advancing into still water will naturally meet a much greater resistance than the same area located near the after end, and entering water which already possesses in large measure the eddying motion.

We will now consider in detail the character of the equation for tangential resistance as given above, especially as to the relation between the quantities involved and the circumstances of the experiment.

The quantity A is taken as the area without modification or change. It is seen as above that the length factor of area is the one of importance, so that we may consider, so far as its effect on the quantities of the formula is concerned, A to be represented by length. Suppose now that we have given a series of values of R for a range of values of quality, length, and speed. Each such value will give a single equation involving the two unknowns f and n. These may be both variable from point to point, and not knowing on what they may depend we cannot consider any two equations as simultaneous, and thus use them for the determination of f and n. In other words, without other conditions the relations of f and n to the circumstances of the experiment are indeterminate. Some arbitrary assumption must therefore be made in regard to one, and the other determined in accordance therewith. Taking any set of values of R for fixed quality, and length at varying speeds, it seems most natural to assume that f will be constant for all such values of R, and to then determine n in accordance with this assumption. This is equivalent to assuming that f is dependent on quality and length, and independent of speed. We may also consider this coefficient as providing for effects due to changes of temperature and density of liquid. It may also be well to note that this coefficient, and in fact the entire equation, is considered as independent of the depth of immersion.

While thus taking f as independent of speed variation, it is not independent of the amount taken as the unit of speed. This appears as follows: In the formula, give v a value 1. Then $R = fA$, or $f = R \div A$. Hence $f =$ the mean value of R per square foot *at the unit speed.*

The value of f being thus known, it may be substituted in the various equations representing the values of R at varying speeds, and the values of n thus determined. It is seen that the values of n may vary with the speed, and they will likewise depend on the particular value of f used.

It thus appears that f will depend on length, quality of surface, density, and temperature, while n will depend on speed and f, and hence on speed and the other various conditions above.

Applying these principles to the results of Froude's experiments* we find in general for a given quality, that f decreases with increasing length at a decreasing rate: also that f varies with quality in the way we should naturally expect.

We turn now to n and consider its relation to the three principal conditions, quality, length, and speed. We first note that we can form no scale of quality, although there is plainly a dependence, n being in general larger for rough than for smooth surfaces. For length we find, by comparing the results for constant quality and speed, that the relation of n to length is somewhat variable with the quality. The results are shown graphically in Fig. 19. In (a) and (b) for varnish and calico respectively there is a falling off, at first marked and then more gradual as the length of 50 feet is approached. In (c) for fine sand the value is practically constant, and very nearly so in (d) for medium sand. Curiously, also, the values for (c) are greater than those for (d).

The relation of n to speed is much less simple or regular. In Fig. 20 are shown graphically the variations of n with speed for four qualities as above. For convenience in dealing with the data available, 500 ft. per minute was chosen as the unit speed. Special comment on the curves is unnecessary. It may be stated that they were derived by methods described in § 78 from the curves published in the British Association Reports above referred to.

The use of these data for large high-speed ships involves the following considerations:

Given values for f and n for any proposed quality of surface, for planes from 2 to 50 ft. in length, and moving at

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* See complete report in British Association Reports for 1874, of which the table on page 41 is a partial résumé.

speeds from 200 to 600 ft. per minute. Required appropriate values for ships perhaps 600 ft. in length and moving at

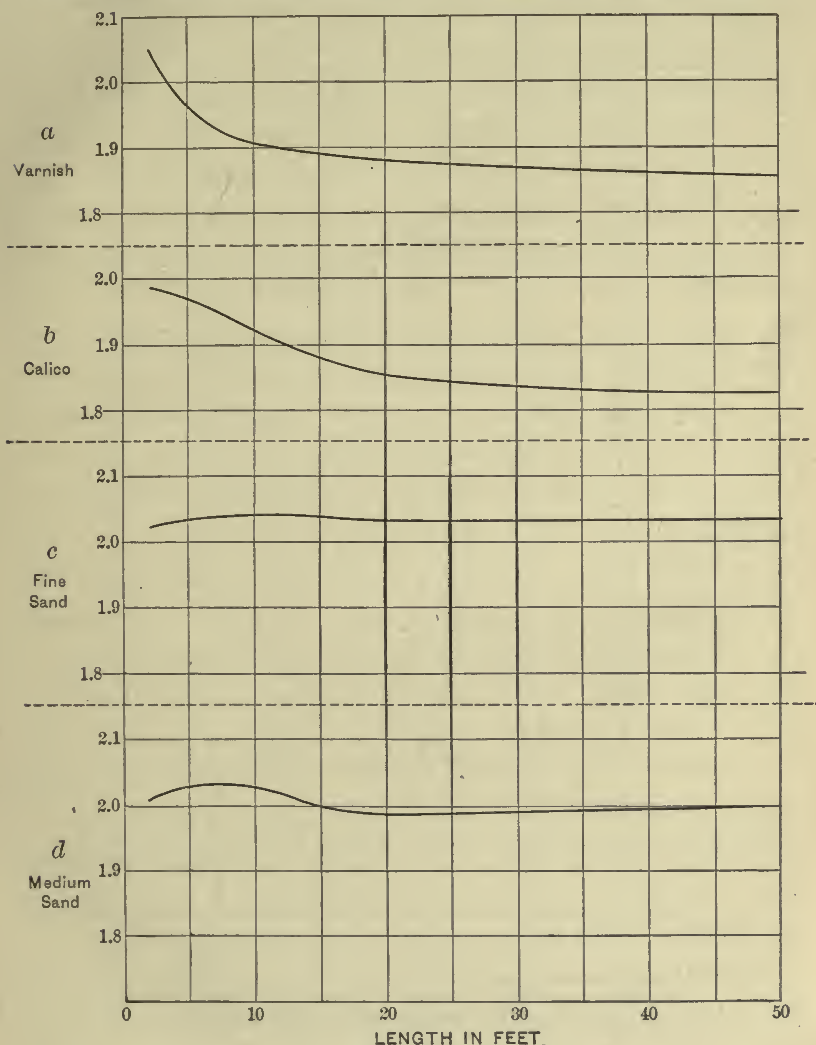

FIG. 19.

speeds of 20 to 25 knots per hour or 2000 to 2500 ft. per minute.

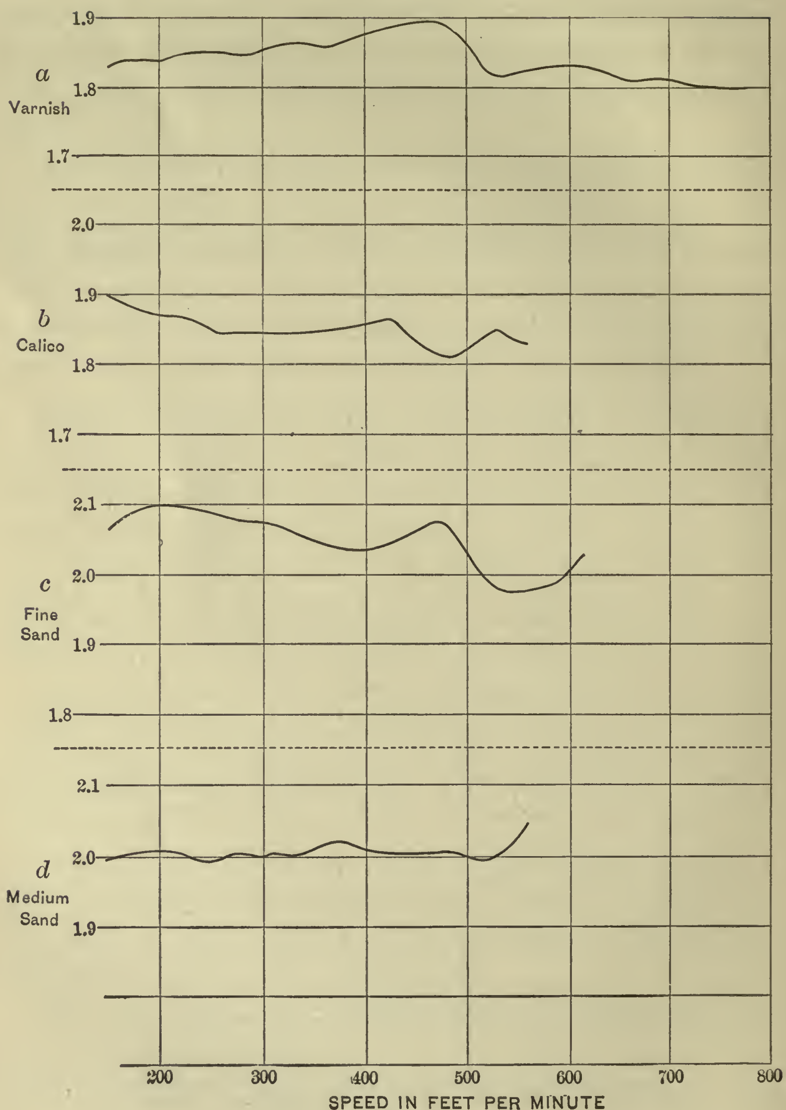
1.9
a
Varnish
1.8
1.7
2.0
1.9
b
Calico
1.8
1.7
2.1
c
Fine
Sand
2.0
1.9
1.8
2.1
d
Medium
Sand
2.0
1.9
200
300
400
500
600
700
800
SPEED IN FEET PER MINUTE

FIG. 20.

The method actually in use is either to assume f and n as practically unchanged beyond the maximum length and speed involved in the experimental data, or to attempt an extrapolation to the given limits, of the laws governing their variation within the experimental range. The extreme uncertainty of either method, especially as regards the variation of n, is seen by considering the curves of Fig. 19 extended by judgment to lengths ten times as great as the maximum there given, or those of Fig. 20 to speeds three or four times as great as the highest there given. The use of such methods of estimation is only justified by the lack of more extended information. For actual computation definite values must, of course, be selected. It must be remembered, however, that their accuracy for large ships at high speeds is a matter of very great uncertainty. Thus in the tables for f given below it is doubtful if the figures in the third significant place have any real significance whatever. Similarly for n, values 1.83, 1.825, or 1.829 as used may perhaps answer as well as any other, but it must not be forgotten that no significance can attach to figures in the third place of decimals, and but slight importance to those in the second place.

On the other hand we may remark that a difference of 1 in the hundredths of n will cause for usual speeds a difference of only about 1 per cent in the value of v^n, so that such an amount of uncertainty is not greater than that to which we are accustomed in most engineering work. This discussion shows, however, to what an extent extrapolation is necessary in any attempt to compute skin-resistance, and how great is the need of more extended experimental investigation.

In this connection the ideas of M. Risbec as given in a recent paper * will be of interest. This author considers that it is more rational to assume that fundamentally, skin-resistance, involving as it does the energy of eddying water, will be proportional to the square of the speed, and to write the formula

$$R = fAv^2.$$

The coefficient f is then to be determined in accordance with the experimental data, and is considered as including a certain function of "perturbation," the value of which offsets the varying values of n in the more usual mode of consideration. The possibility of such a treatment of the original data will be obvious from the preceding discussion of the relation of the quantities in the equation to the conditions of the experiment. We have simply to find for the various values of quality, length, and speed the quotient

$$\frac{R}{Av^2} = f.$$

M. Risbec then represents the quotient f as a function of four parameters for each quality of surface, of the speed v, and of the length L. The complexity of the resulting equation and the uncertainty of the exact nature of the parameters, especially as affected by their extension to lengths and speeds far beyond those contained in the fundamental data, make it questionable whether any advantage is to be derived from this mode of consideration. It is, however, of interest as showing a possible mode of considering the relation of the formula to the characteristics of the experiment.

* Bulletin de l'Association Technique Maritime, vol. v. p. 45.

8. Skin-resistance of Ship-formed Bodies.

While in the preceding section we have referred in general to the application of the data therein discussed to the resistance of ship-formed bodies, it must be remembered that fundamentally the data relate simply to thin boards, sensibly equivalent to planes.

Now at any point of a ship-formed body let the horizontal component of the tangential force be denoted by p. Let ds be an element of area, and θ the inclination to the longitudinal of the water-lines at this point. Then $p \cos \theta ds$ is the longitudinal component of the tangential force or the tangential resistance for this element. For the whole ship we shall have, therefore, for the skin-resistance

$$R = \int p \cos \theta ds;$$

or if $\bar{p}$ denote a mean value of p, we have

$$R = \bar{p} \int ds \cos \theta.$$

The value of the integral is known as the reduced wetted surface,* and it thus appears that strictly speaking it is with this rather than with the actual surface that we are concerned in dealing with skin-resistance. The difference between the two, however, is usually less than 1 per cent, and the uncertainties surrounding the whole question as already discussed show that no significant error will be made by using either

* As shown in the theory of statical naval architecture the value of the reduced wetted surface is more simply expressed by

$$\text{Reduced surface} = \int ds \cos \theta = \int G dx,$$

where G is the variable girth or length of section, and dx is the element of length.

the one or the other. The experiments of Wm. Froude on the greyhound led him to use for A the actual wetted surface, and the coefficients derived from his results and given in Table II, p. 53, are intended to correspond to this value of the area. In continental Europe the reduced surface is frequently used in this connection.

The value of the wetted surface may be derived by approximate integration as shown in the theory of statical naval architecture, or by the use of one of the following empirical and approximate formulæ:

Let S = wetted surface in square feet;
D = displacement in tons;
L = length in feet;
H = draft in feet;
B = beam in feet;
b = block coefficient of fineness.

Then we may have

$$S = 15.6 \sqrt{DL}.*$$

$$S = L[1.7H + bB].†$$

In addition to the above notation let us put

k = coefficient taken from table;
c = " " " " ;
G_1 = girth of midship section;
p = prismatic coefficient of fineness.

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* Transactions Society of Naval Architects and Marine Engineers, vol. I. p. 226.

† Transactions Institute of Naval Architects, vol. XXXVI. p. 72.

TABLE I.

Coefficient K.

$\sqrt{pb}$	$B \div H$: 2	2.5	3	4	5
.45	....	1.869	1.792	1.729	1.706
.46	....	43	72	21	00
.47	....	17	59	13	1.696
.48	....	1.792	46	06	92
.49	....	73	34	00	88
.50	1.806	57	27	1.696	86
.51	1.780	43	17	92	84
.52	63	32	11	88	82
.53	49	23	02	84	80
.54	36	15	1.698	82	78
.55	27	07	94	80	76
.56	17	00	88	78	76
.57	11	1.696	86	76	76
.58	05	92	82	76	76
.59	1.699	88	80	76	78
.60	94	84	78	76	80
.61	90	80	76	76	82
.62	86	78	76	78	84
.63	84	76	76	80	86
.64	82	76	76	82	90
.65	80	76	76	84	94
.66	78	76	78	86	98
.67	78	76	82	90	1.702
.68	78	76	84	96	09
.69	78	78	86	1.700	15
.70	78	80	90	06	21
.71	82	82	94	13	29
.72	84	86	1.700	19	37
.73	86	90	05	27	47
.74	90	94	11	35	57
.75	94	1.700	17	45	70
.76	99	07	27	56	84
.77	1.705	13	36	71	1.800
.78	11	19	46	86	21
.79	17	29	61	1.806	49
.80	27	39	76	35	75
.81	36	49	97	65	1.904
.82	49	65	1.825	92	31
.83	61	80	50	1.917	59
.84	80	1.803	81	47	85
.85	1.802	37	1.909	73	2.010

TABLE II.

Coefficient c.

$\frac{B}{H}$	
2.0	.933
2.1	.829
2.2	.824
2.3	.820
2.4	.816
2.5	.812
2.6	.809
2.7	.805
2.8	.802
2.9	.899
3.0	.896
3.1	.893
3.2	.890
3.3	.888
3.4	.885
3.5	.882
3.6	.880
3.7	.878
3.8	.875
3.9	.873
4.0	.871
4.1	.869
4.2	.867
4.3	.865
4.4	.863
4.5	.860
4.6	.859
4.7	.857
4.8	.855
4.9	.853
5.0	.851
5.1	.850
5.2	.848
5.3	.846
5.4	.845
5.5	.843
5.6	.842
5.7	.840
5.8	.839
5.9	.837
6.0	.836
6.5	.829
7.0	.823

Intermediate values of K and c may be obtained by interpolation.

Then to a somewhat greater degree of accuracy we have

$$S = kcLG_1p(1 - .5pb).*$$

The first two formulæ have the advantage of simplicity, while the latter is more accurate, its errors being usually less than one per cent.

For the reduced surface we have, from the foot-note on p. 49,

$$S_r = \int G dx.$$

With the same notation as before we have the following empirical relation between S and S_r:

$$S = S_r\left(1 + (p - .2)\left(\frac{B}{L}\right)^2\right).\dagger$$

9. Actual Values of the Quantities f and n for Skin-resistance.

We will now give a résumé of the practical values which have been proposed by various authorities for the coefficient f and the exponent n.

The general formula is

$$R = fAv^n,$$

where R = resistance in pounds;
f = coefficient;
A = area of wetted surface in square feet;
v = speed in knots per hour;
n = index.

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* Transactions Society of Naval Architects and Marine Engineers, vol. II. p. 297.

† *Ibid.*, vol. III. p. 138.

TABLE I, SHOWING FROUDE'S EXPERIMENTAL VALUES.

Nature of Surface.	Length of Surface.							
	2 feet.		8 feet.		20 feet.		50 feet.	
	n	*f*	*n*	*f*	*n*	*f*	*n*	*f*
Varnish..........	2.00	.0117	1.85	.0121	1.85	.0104	1.83	.0097
Paraffin............	1.95	.0119	1.94	.0100	1.93	.0088		
Tin-foil............	2.16	.0064	1.99	.0081	1.90	.0089	1.83	.0095
Calico..............	1.93	.0281	1.92	.0206	1.89	.0184	1.87	.0170
Fine sand..........	2.00	.0231	2.00	.0166	2.00	.0137	2.06	.0104
Medium sand.......	2.00	.0257	2.00	.0178	2.00	.0152	2.00	.0139
Coarse sand........	2.00	.0314	2.00	.0204	2.00	.0168		

TABLE II.

VALUES FOR SHIPS BASED ON FROUDE'S EXPERIMENTS.

(SALT WATER.)

Length in Feet.	Coefficient of Skin-resistance.	Index for the Variation of Resistance with Speed.	Length in Feet.	Coefficient of Skin-resistance.	Index for the Variation of Resistance with Speed.
	f	*n*		*f*	*n*
8	.01197	1.825	80	.00933	1.825
9	.01177	"	90	.00928	"
10	.01161	"	100	.00923	"
12	.01131	"	120	.00916	"
14	.01106	"	140	.00911	"
16	.01086	"	160	.00907	"
18	.01069	"	180	.00904	"
20	.01055	"	200	.00902	"
25	.01029	"	250	.00897	"
30	.01010	"	300	.00892	"
35	.00993	"	350	.00889	"
40	.00981	"	400	.00886	"
45	.00971	"	450	.00883	"
50	.00963	"	500	.00880	"
60	.00950	"	550	.00877	"
70	.00940	"	600	.00874	"

TABLE III.

VALUES FOR SHIPS BASED ON TIDMAN'S EXPERIMENTS.

(SALT WATER.)

Length in Feet.	Iron Bottom Clean and Well Painted.		Copper or Zinc Sheathing.			
			Smooth.		Rough.	
	f	*n*	*f*	*n*	*f*	*n*
10	.01124	1.853	.0100	1.9175	.01400	1.870
20	.01075	1.849	.00990	1.900	.01350	1.861
30	.01018	1.844	.00903	1.865	.01310	1.853
40	.00998	1.8397	.00978	1.840	.01275	1.847
50	.00991	1.8357	.00976	1.830	.01250	1.843
100	.00970	1.829	.00966	1.827	.01200	1.843
150	.00957	1 829	.00953	1.827	.01183	1.843
200	.00944	1.829	.00943	1.827	.01170	1.843
250	.00933	1.829	.00936	1.827	.01160	1.843
300	.00923	1.829	.00930	1.827	.01152	1.843
350	.00916	1.829	.00927	1.827	.01145	1.843
400	.00910	1.829	.00926	1.827	.01140	1.843
450	.00906	1,829	.00926	1.827	.01137	1 843
500	.00904	1.829	.00926	1.827	.01136	1.843

TABLE IV.

VALUES FOR PARAFFIN MODELS BASED ON TIDMAN'S EXPERIMENTS.

(FRESH WATER.)

Length in Feet.	Coefficient of Skin resistance.	Index for the Variation of Resistance with Speed.	Length in Feet.	Coefficient of Skin-resistance.	Index for the Variation of Resistance with Speed.
	f	*n*		*f*	*n*
2	.01176	1.94	12	.00908	1.94
3	.01123	"	12.5	.00901	"
4	.01083	"	13	.00895	"
5	.01050	"	13.5	.00889	"
6	.01022	"	14	.00883	"
7	.00997	"	14.5	.00878	"
8	.00973	"	15	.00873	"
9	.00953	"	16	.00864	"
10	.00937	"	17	.00855	"
10.5	.00928	"	18	.00847	"
11	.00920	"	19	.00840	"
11.5	.00914	"	20	.00834	"

The various values for f given in the above tables are based on the experiments of Froude and Tidman. As other special values we may mention the following:

Colthurst * gives the value $f = .0167$ for rough-sawn wood and $f = .01$ for smooth-planed wood. The same authority states that for fine "grass" or marine growth f may rise to a value from .048 to .062. For clean copper the value .0073 is given, and for fresh-painted iron the value .01. This last value is also that used by Prof. Rankine. These values all relate to ships of usual length, or at least over 50 feet.

In this connection the results given in a recent paper by Chief Naval Constructor Hichborn † are of interest. From the data given in this paper the following tabular presentation is derived: Column a shows the percentage increase of total resistance with foul bottom over that with clean, reckoned on the latter as base.

Ship.	Speed.	a.
Charleston	8.8	70
Yorktown	9	72
Philadelphia	11	52
San Francisco	9	33
Bennington	7	200
Baltimore	11	20

It thus appears, as we should expect, that the term "foul" is entirely indefinite, and may mean almost any condition in which the resistance is sensibly increased. In the extreme case of the table the total resistance of the Benning-

* Cited by Pollard and Dudebout, Théorie du Navire, vol. III. p. 376.

† Transactions Society of Naval Architects and Marine Engineers, vol. II. p. 159.

ton when foul is three times that when clean. At the low speeds at which these runs were made, most of the resistance will be due to the skin, so that while the above figures refer to total resistance, the ratios for the skin-resistance will not be far different, though slightly greater.

These possibilities impress in the strongest manner the extreme importance of a clean bottom especially for economical cruising at moderate speeds, and for the possible attainment of the highest speeds.

10. Waves.

As introductory to the subject of wave-resistance we shall give in the present section a brief account of the commonly accepted theories of wave-motion in liquids, with a statement of the more important results, but without the details of the mathematical development.*

In order to examine in a satisfactory manner the influence of waves upon a ship, some working theory as to their dynamical constitution is required. It must be understood that such a theory is not to be viewed as final truth in itself, but rather as a convenient method of correlating as nearly as possible the known facts, and of deducing by appropriate

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* Among others, the following works may be consulted by those interested:

Rankine *et al.*: "Shipbuilding, Theoretical and Practical." London, 1866.

J. Scott Russell: "System of Modern Naval Architecture," vol. I. London, 1865.

Encyclopedia Britannica, article Wave.

Pollard et Dudebout: Théorie du Navire, vol. III. Paris, 1892.

Gatewood: Journal U. S. Naval Institute 1883, p. 223. Annapolis.

Stokes: Cambridge Transactions 1847.

Encyclopedia Metropolitana, article Tides and Waves.

mathematical means the probable nature and extent of the influences which are to be examined. The most simple of the theories proposed to this end is that known as the trochoidal. The theory receives its name from the trochoid, which is the form of the contour of waves resulting from this mode of viewing their constitution. The trochoid may be defined as the locus of a point P, Fig. 21, which revolves uniformly

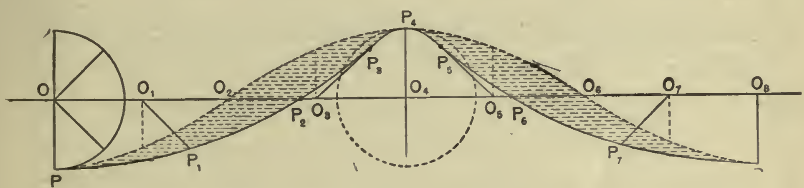


FIG. 21.

about a center O, which center itself moves uniformly along a straight line OO_8. In this case if O moves uniformly along OO_8, and OP revolves uniformly counter-clockwise, the curve $P_0P_1 \ldots P_8$ will result. For comparison there is shown in dotted lines a sinusoid of equal length and height, and the difference between the two is indicated by the shaded area.

A trochoid may also be defined as the locus of a point in the radius of a circle, which circle rolls uniformly along a straight line. Thus in Fig. 22 let a circle of radius OQ roll along on the under side of AB. Let P be the tracing-point. Then the locus of P will be a curve $CPDE$, and the identity in character between the two curves in Figs. 21 and 22 is readily seen. If the tracing-point P is on the circumference of the circle the locus becomes the common cycloid as a special case of the trochoid. If the point passes beyond Q, as to R, the locus is still called a trochoid. The specific names prolate-trochoid and curtate-trochoid are given to these two classes of curves. It is only with the former as shown in the figures that we are here concerned.

Waves whose contours are very nearly trochoidal are but rarely met with, though a moderately heavy swell in calm weather at a considerable time after the subsidence of the storm to which it owes its existence, quite closely conforms

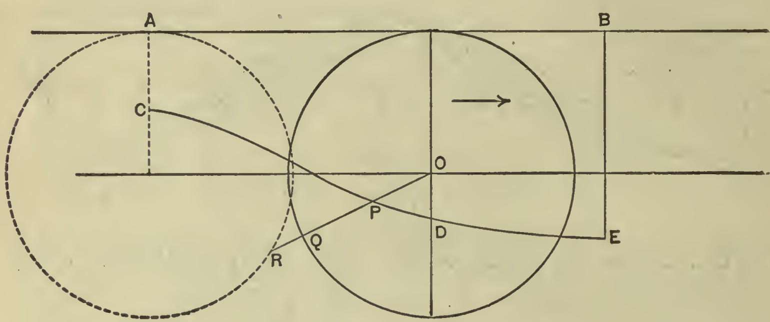


Fig. 22.

to this type. The trochoidal wave is, moreover, the type most readily examined by mathematical means, and is therefore naturally taken for purposes of theoretical investigation.

The ideal constitution of a trochoidal wave involves the following suppositions:

(1) That for any given wave *all* particles revolve in fixed circular orbits, in vertical planes parallel to the direction of propagation and normal to the lines of crests and hollows, with the same constant angular velocity, and in direction with the propagation when in a crest.

(2) That for any layer of water originally horizontal the radii are equal and the centers are on the same horizontal line.

(3) That the radii decrease with increase of depth according to a law to be stated later.

(4) That all particles originally in the same vertical are always in the same phase.

It follows that the surface layer of particles and all successive layers originally horizontal will take the form of trochoidal surfaces of continually decreasing altitudes according to the law of decreasing radii. It further results that all corresponding crests and hollows for successive layers will lie in the same vertical line, and hence that all wave-lengths for the successive trochoidal subsurfaces will be equal, and that the velocity of propagation for all must be the same.

This ideal constitution of a wave may be illustrated in Fig. 23. The diagram represents a section of a certain por-

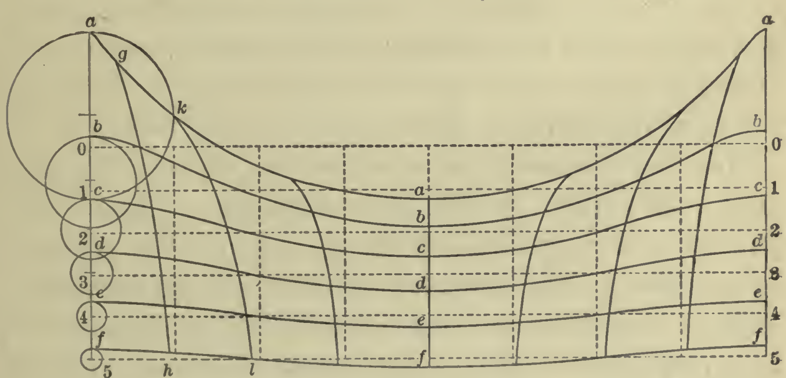


FIG. 23.

tion of still water divided by horizontal and vertical planes into rectangular blocks as shown in dotted lines. The full lines represent the same blocks of water when thrown into wave-motion. The horizontal planes 0, 1, 2, etc., for still water are, in the wave, thrown into the trochoidal surfaces *a*, *b*, *c*, etc. The corresponding circular orbits are shown on the left, and their centers are seen to be elevated above the corresponding still-water planes by successively decreasing amounts as we go from the surface downward. To avoid confusion in the diagram the circular orbits are elsewhere omitted. The motion involved in the figure may be viewed from several standpoints. Thus we may consider the trochoidal layers and note, as the wave progresses, the change in thickness and form at any one point. Or again we may consider the vertical columns of water between the distorted verticals *af*, *gh*, *kl*, etc., and note that as the wave progresses these columns shorten and broaden and then lengthen and narrow, at the same time waving to and fro with the period of the wave.

Again, each rectangular block of still water is represented in the wave by a distorted block, and the successive forms into which one of these blocks is thrown may be seen by following along the series of distorted quadrilaterals between the pair of corresponding trochoidal surfaces for the wave.

In order that such a wave may actually exist in a liquid it must fulfil certain hydrodynamic conditions. These are: (1) That at the upper surface the pressure must be constant and normal to the surface. (2) That the liquid cannot undergo expansion or compression during the process of wave-propagation. These conditions may be made to furnish a relation btween the length and the velocity of propagation, and a law for the decreasing radii of the circular orbits. These are as follows:

Let $L =$ length in feet;

$V =$ linear velocity in feet per second;

$R =$ radius of rolling circle;

$\omega =$ angular velocity;

$T =$ periodic time;

$g =$ value of gravity $= 32.2$.

$$\left.\begin{array}{ll} \text{Then} & V = R\omega = R\sqrt{\frac{g}{R}} = \sqrt{gR} = \sqrt{\frac{gL}{2\pi}}, \\ \text{or} & V = 2.265\sqrt{L} \text{ ft. per sec.,} \\ \text{and} & V = 1.341\sqrt{L} \text{ knots per hour,} \end{array}\right\} \quad . \quad . \quad (1)$$

$$\text{also} \qquad T = \frac{L}{V} = .4415\sqrt{L}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Let $r_0 =$ radius of orbit at surface;

$r =$ radius of orbit at depth of center z below center of surface orbits;

$z =$ any given depth of orbit center below center of surface orbits;

$e =$ base of Naperian logarithms.

$$\left.\begin{array}{ll} \text{Then} & \log_e \frac{r}{r_0} = -\frac{z}{R}, \\ \text{or} & r = r_0 e^{-\frac{z}{R}} = r_0 e^{-\frac{2\pi z}{L}}. \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If these various conditions are fulfilled, therefore, it appears that the geometrical motion here supposed, if once inaugurated, is hydrodynamically possible. This does not prove that in any actual wave the particles move as herein assumed. Observation shows, however, that these suppositions are near the truth, and the actual motion and characteristics of waves, so far as admitting measurement, are for the most part in fairly satisfactory agreement with those resulting from the type of wave-motion assumed. It seems, therefore, reasonable to accept the theory as a working basis for the investigation of such characteristics of wave-motion as we are here concerned with.

We may now state a further series of results following upon the conditions assumed.

The point of maximum slope of any trochoid corresponds to a value of θ given by

$$\cos\theta = \frac{r}{R}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

or

$$\sin\theta = \frac{\sqrt{R^2 - r^2}}{R}.$$

If ϕ is this maximum slope, then

$$\tan\phi = \frac{r}{\sqrt{R^2 - r^2}},$$

and

$$\sin\phi = \frac{r}{R} = \frac{\pi h}{L}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where h = height of wave = $2r$.

The normal at any point intersects the vertical through the center of the rolling circle at the constant distance R above it, and therefore always at the point of contact of the rolling circle with the line on which it rolls.

The pressure on any trochoidal subsurface is constant and equal to that due to the depth of the corresponding plane surface in still water.

Let z = depth of the line of centers;
r_0 = radius of surface orbit;
r = " " orbit with center at depth z;
R = " " rolling circle.

Then the pressure in the crest of a wave is less than that for an equal depth in still water in the ratio

$$\frac{z - \left(\frac{r_0^2 - r^2}{2R}\right)}{z + r_0 - r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Similarly the pressure in the hollow of a wave is greater than that for an equal depth in still water in the ratio

$$\frac{z - \left(\frac{r_0^2 - r^2}{2R}\right)}{z - r_0 + r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

At any point the resultant force due to gravity and to centrifugal force acts along the normal and hence through the instantaneous center of motion of the rolling circle as specified above. In Fig. 24 let $OA = R$. Then if the force due to gravity is represented by OA or R, that due to centrifugal force will be represented by the radius OP, and the resultant by the line AP.

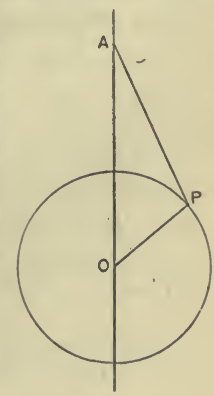


FIG. 24.

Let m = mass of particle;

g = value of gravity = 32.2;

ω = angular velocity = $\sqrt{\frac{2\pi g}{L}}$;

f = resultant force represented by AP.

Then we have

$$f = m\sqrt{g^2 + r^2\omega^4 - 2gr\omega^2 \cos\theta}. \quad . \quad . \quad . \quad (8)$$

The line of orbit centers for any trochoidal surface or sheet is raised above the original still-water level of the corresponding plane by the distance

$$\frac{\pi r^2}{L} = \frac{r^2}{2R} = \frac{r_0^2 \epsilon^{-\frac{2z}{R}}}{2R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Let H denote the still-water depth of a layer corresponding to the trochoidal sheet with line of centers at a depth z below the center line of surface orbits. Then

$$H = z - \frac{r_0^2 - r^2}{2R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The total energy of a wave is always half kinetic and half potential.

Let E denote total energy in foot-pounds;
- σ " density;
- L " length in feet;
- r_0 " radius of orbit at surface;
- h " height of wave $= 2r_0$;
- R " radius of rolling circle.

Then

$$E = \frac{\sigma L r_0^2}{2}\left(1 - \frac{r_0^2}{2R^2}\right),$$

or

$$E = \frac{\sigma L h^2}{8}\left(1 - 4.935\left(\frac{h}{L}\right)^2\right).$$

If we put $\sigma = 64$, we have

$$E = 8Lh^2\left(1 - 4.935\left(\frac{h}{L}\right)^2\right). \quad . \quad . \quad . \quad . \quad (11)$$

The total power involved in a series of waves per foot of breadth is

$$\text{Power} = .0329\sqrt{L}h^2\left(1 - 4.935\left(\frac{h}{L}\right)^2\right). \quad . \quad . \quad . \quad (12)$$

Combinations of Wave Systems.—Given two trochoidal systems in general with parallel crests, moving in the same direction. The result of such a combination geometrically, as is well known, is to produce a series of groups of waves, each group reaching a maximum of height where two crests correspond, and a minimum where crest and hollow combine. Each group therefore culminates in a wave of maximum altitude with comparatively small disturbance betweeen the successive groups. It may be shown that such geometrical combination does not fulfil the necessary hydrodynamical conditions, and therefore cannot exist permanently as a combination of actual trochoidal systems. The actual result of the combination of two wave systems is to produce a more or less mixed resultant system, with characteristics relative to which those given by the geometrical combination of trochoidal systems are intended simply as approximations. We do find, however, actual combinations of wave systems agreeing in all their general characteristics with those given by the geometrical combination referred to, and it therefore seems fair to use the method as a working basis for the examination of such points as we are interested in.

As a special case let us assume that the two series are of

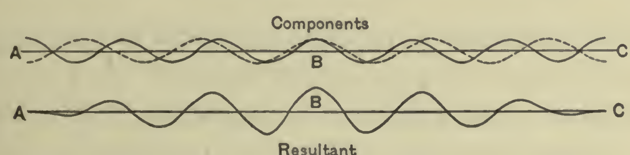


FIG. 25.

equal altitude, h, but of different lengths, L_1 and L_2. Then the geometrical combination will give a result as indicated in Fig. 25. At A and C the phases are opposite, and the values of h being equal the resultant disturbance is 0, and we have for a little space practically undisturbed water. At a point B midway between, the phases are the same and the two effects are added, giving a wave of height $2h$, the maximum possible in the entire series. The group from A to C is then indefinitely repeated to the right and left as far as the given conditions hold.

For such a group the length AC is given by the equation

$$AC = \frac{L_1 L_2}{2(L_2 - L_1)}. \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Let v = velocity of propagation of the resultant group as a whole;

v_1 = velocity of propagation of the component of length L_1;

v_2 = velocity of propagation of the component of length L_2.

$$\left. \begin{aligned} \text{Then} \qquad v &= \frac{v_1 v_2}{v_1 + v_2}, \\ \text{or} \qquad \frac{1}{v} &= \frac{1}{v_1} + \frac{1}{v_2}. \end{aligned} \right\} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

As L_1 approaches L_2 in value the length of the group AC will increase and the velocity will approach more and more nearly to one half that of the components. At the limit where $L_1 = L_2$ the length would become indefinite and the velocity would become one half that of the components. Since any given group is the expression of a certain distribution of energy, it is considered that the velocity of propagation of the energy is the same as that of the group.

We will now take the special case of two trochoidal systems of different altitudes, but of equal lengths and therefore of equal velocities.

In Fig. 26 let O denote the orbit center for a given surface particle as affected by either component. Strictly speaking the two centers are not at the same level, but the difference is very small and is usually neglected. At any given instant of time let DOA denote the phase-angle θ_1 of the first component, and DOB the phase-angle θ_2 of the second component. Then AOB will represent the constant difference of phase between the two components. Also let $OA = r_1$ and $OB = r_2$. Then the combination

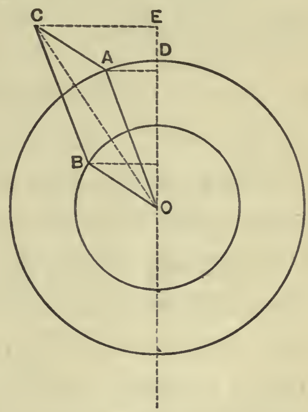


FIG. 26.

of the two components will result in a trochoidal system of wave-length equal to that of its components, and of surface orbit radius $r = OC$, and with phase relationships with its components as given by the angles COA and COB. In other words, if the parallelogram $OACB$ is made to revolve uniformly about O while the latter moves uniformly along x, the two speeds being appropriate to the wave-length taken, then the point A will trace the contour of the first component, B that of the second, and C that of the resultant.

Let a denote the linear difference between similar points on the two components, ω the angular velocity, V the linear velocity, and R the radius of the rolling circle. Then the angle denoting the phase difference AOB is given by

$$AOB = \frac{\omega a}{V} = \frac{a}{R} = \frac{2\pi a}{L}, \text{ in angular measure,}$$

or $$AOB = \frac{360a}{L} \text{ in degrees.}$$

From the triangle OAC we derive for the altitude of the resultant system

$$h = h_1^2 + h_2^2 + 2h_1h_2 \cos \frac{360a}{L}. \quad . \quad . \quad . \quad (15)$$

As to the degree of fulfilment which the ideal theory finds in actual waves it may be said in general, and without entering into details, that, so far as observations can be made, most of the actual characteristics are in fair agreement with the results derived from theory. The agreement is by no means complete, and in fact differs in certain particulars in such way as to lead to the belief that a truly trochoidal wave is but rarely if ever met with. On the other hand the agreement is sufficiently close to seemingly warrant the use of the theory for all purposes of the naval architect, at least in so far as he is concerned with waves in deep water.

In regard to the limiting sizes of waves the following results may be given:

From reliable observation it may be fairly concluded that waves longer than 1200 to 1500 feet are very rarely met with, though there seems ground for accepting reports of exceptional waves of a length approaching 3000 feet. The corresponding periods would be 15 to 17 seconds for the first named, and about 24 seconds for the last.

With regard to height the evidence is much more conflicting. It appears, however, that from 40 to 45 feet is about the maximum value which can be accepted as reliable, and values approaching these figures are met with but rarely. From 25 to 30 feet may be taken as the ordinary limit. It should be noted, however, that these heights relate to a series of regular waves as contemplated by the theory, and not to exceptional results arising from the interference and combination of different systems.

With regard to ratio of height to length, it is found usually to decrease with increase of length. The highest values of this ratio appear to be about 1 : 6, such being found only with comparatively short waves. More commonly the ratio is from 1 : 12 to 1 : 25. The corresponding values of the maximum inclination are respectively about 32°, 15°, and 7°.

Shallow-water Waves.—When we come to waves in shallow water we find, as nearly as can be determined, the paths of the particles to be elliptical with the longer axis horizontal. In order to meet this departure from the trochoidal system, the so-called elliptical trochoidal or shallow-water wave system is used. This ideal system may be generated as follows: Given an ellipse, Fig. 27, with its major axis horizontal, moving uniformly along X without angular change. Draw any line OB at an angle θ with the vertical. Through B draw a vertical and through A a horizontal. They will intersect on the ellipse at P. Let OB revolve uniformly to the left. Then the locus of the point P

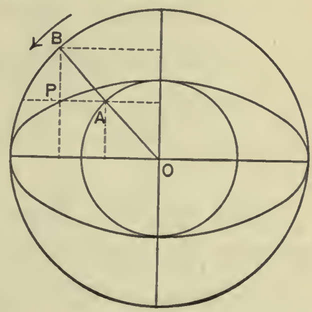
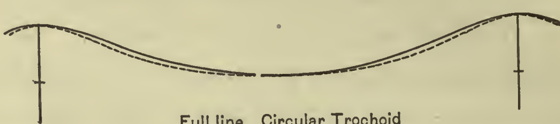


FIG. 27.

thus determined by the movement of the ellipse along OX and of OB about O will be an elliptical trochoid. Comparing this contour with that resulting from the circle as a base, as in Fig. 28, the former is seen to lie slightly within the latter, thus making a flatter hollow and steeper crest, these differences becoming more pronounced as the difference between the two axes of the ellipse is greater



Full line, Circular Trochoid
Dotted line, Elliptical Trochoid

FIG, 28.

In the present connection the feature of especial interest is the velocity of propagation. For this the following expression may be derived:

Let V = velocity in feet per second;
r_0 = semi-major axis at surface;
u_0 = " minor " " "
L = wave length;
g = value of gravity = 32.2.

Then $$V = \sqrt{\frac{gL}{2\pi}}\sqrt{\frac{u_0}{r_0}}. \qquad (16)$$

This equation shows by comparison with (1), that the velocity of a shallow-water wave is less than that of a deep-water wave of equal length, a conclusion borne out by experience.

Let d = depth of the water. Then when $2\pi d \div L$ is small it may be shown that

$$V = \sqrt{gd}. \qquad (17)$$

It should be noted that the ideal wave system thus derived, does not, and seemingly cannot, be made to fulfil the hydrodynamic conditions necessary to its actual existence as assumed. This does not prevent, however, such form of motion from being, possibly, a close approximation to what actually takes place, nor does it seriously interfere with its usefulness as an approximate working theory of shallow-water waves.

Waves of Translation.—The waves thus far considered occur naturally in indefinite series, and represent the result of a widely distributed disturbance. The typical wave of translation is individual in character, and represents the result of a local disturbance. In Fig. 29 let AB be a trough or canal

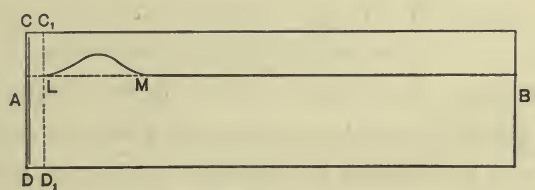


FIG. 29.

of a length AB, large relative to its breadth and depth. Let CD be a false end or partition movable within the canal along the direction AB. If now CD be moved rapidly from CD to C_1D_1, the water will become at first banked up in front of it; but as CD is retarded and finally brought to rest at C_1D_1, the banked-up water will leave C_1D_1 and travel on as a crest or hump elevated entirely above the general level of the tank, somewhat as indicated in the diagram. Such a wave is called positive.

Let us again suppose that the partition was at C_1D_1 and the water at rest. Let it then be moved from C_1D_1 to CD.

As a result a hollow or depression will be formed existing wholly below the general level of the surface. This hollow will then be propagated on in a manner similar to the crest previously described. Such a wave is called negative.

A positive wave may also be formed by plunging into the water at one end of the canal a block which will occupy the volume CDD_1C_1, or by the sudden introduction of a certain additional amount of liquid into the end of the trough. Similarly a negative wave may be formed by withdrawing a block, or by withdrawing, as by a lifting-pump, a certain amount of water.

Positive waves if properly formed are quite permanent, only slowly losing their energy and form by external friction and internal viscous forces. If not properly formed the wave will soon break up into an irregular series, the members of which are propagated with different velocities, so that the energy is soon dissipated and the form disappears. The negative wave is much less permanent than the positive, and can be made to preserve its integrity for a short time only.

The horizontal movement of any sheet of particles situated in a transverse plane seems to be the same. The vertical motion varies according to its distance from the bottom of the canal, and seems to be very nearly proportional to such distance. The path of any particle seems to be very nearly a semi-ellipse, as shown at ACO in Fig. 30. It therefore appears that, due to the passage of the wave, a particle which was formerly at A has been transported and left at O; furthermore, that all particles in the transverse plane of A will undergo the same longitudinal translation, and will be left in a transverse plane containing O. Care must be here taken to distinguish between the motion of the particle and the motion of propagation. While the particle moves through its path as ACO, the wave-form will move forward its wave-length OF or LM, Fig. 29.

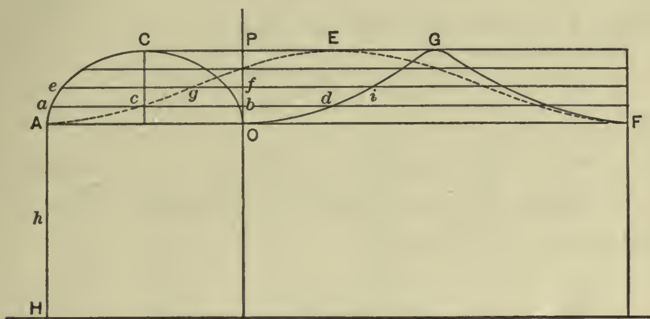


FIG. 30.

The approximate contour of a wave of translation may be determined by the following construction:

Let ACO be the ellipse representing the path of a surface particle, and let AH be the depth of the canal. Then lay off $AF = 2\pi h$ and construct the sinusoid AEF. Next lay off to the right from points on this curve, as A, c, g, etc., distances AO, ab, ef, etc., or the intercepts between the ellipse and the vertical OP. The points O, d, i, etc., thus determined will give the desired contour of the ideal wave of translation corresponding to the conditions assumed.

Let v = velocity of propagation;
h = depth of water when at rest;
k = height of wave above still-water level;
L = wave-length = OF, Fig. 30;
α = range of translation = AO. Fig. 30;
V = volume of water constituting wave:

$b =$ breadth of canal;
$A =$ area of profile of wave.

$$\text{Then} \qquad v = \sqrt{g(h+k)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

When k is relatively small we have

$$v = \sqrt{gh}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

It is found that k cannot exceed h. On reaching this value, or before, the wave breaks at the crest. Hence as a limiting value of v we have

$$v = \sqrt{2gh}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

For the wave-length

$$L = 2\pi h - \alpha, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

or where α is relatively small

$$L = 2\pi h. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

For the value of A we have

$$A = \alpha h. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

As with oscillatory waves, the energy is one-half kinetic and one-half potential, the entire amount being

$$\text{Energy} = 2\sigma V\bar{z}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

where $\sigma =$ density;
$V =$ volume;
$\bar{z} =$ elevation of center of gravity of wave above the still-water level.

TABLE I.

SHOWING RELATION BETWEEN L, T, AND V FOR TROCHOIDAL WAVES.

L	T	V in Feet per Sec.	V in Knots.	L	T	V in Feet per Sec.	V in Knots.
10	1.40	7.2	4.2	350	8.30	42.4	25.1
20	2.00	10.1	6.0	400	8.80	45.3	26.8
30	2.42	12.4	7.3	450	9.40	48.0	28.4
40	2.80	14.3	8.5	500	9.90	50.6	30.0
50	3.10	16.0	9.5	550	10.40	53.1	31.4
60	3.40	17.6	10.4	600	10.80	55.5	32.8
70	3.70	19.0	11.2	650	11.20	57.8	34.2
80	4.00	20.2	12.0	700	11.70	59.9	35.5
90	4.20	21.5	12.7	750	12.10	62.0	36.7
100	4.40	22.7	13.4	800	12.50	64.1	37.9
150	5.40	27.7	16.4	850	12.90	66.0	39.1
200	6.20	32.0	19.0	900	13.20	68.0	40.2
250	7.00	35.8	21.2	950	13.60	69.8	41.3
300	7.60	39.2	23.2	1000	14.00	71.6	42.4

TABLE II.

SHOWING VALUES OF $\frac{r}{r_0}$ FOR VARYING VALUES OF $\frac{z}{L}$.

$\frac{z}{L}$	$\frac{r}{r_0}$	$\frac{z}{L}$	$\frac{r}{r_0}$	$\frac{z}{L}$	$\frac{r}{r_0}$	$\frac{z}{L}$	$\frac{r}{r_0}$
.01	.9391	.07	.6442	.25	.2079	.60	.0231
.02	.8820	.08	.6049	.30	.1518	.70	.0123
.03	.8283	.09	.5681	.35	.1109	.80	.0066
.04	.7778	.10	.5336	.40	.0810	.90	.0035
.05	.7304	.15	.3897	.45	.0592	1.00	.0019
.06	.6859	.20	.2846	.50	.0432		

TABLE III.

SHOWING THE VARIATION OF PRESSURE VERTICALLY DOWNWARD IN THE CREST AND HOLLOW OF A WAVE.

$\frac{z}{L}$	Crest.		Hollow.	
	Actual Depth / L.	Value of Ratio [10] (6).	Actual Depth / L.	Value of Ratio [10] (7).
.05	.0635	.730	.0365	1.269
.10	.1233	.766	.0767	1.231
.15	.1805	.794	.0836	1.200
.20	.2358	.818	.1642	1.174
.25	.2896	.837	.2105	1.152
.30	.3424	.854	.2576	1.135
.35	.3945	.868	.3055	1.120
.40	.4460	.879	.3540	1.108
.45	.4975	.889	.4025	1.098
.50	.5478	.898	.4521	1.088
.60	.6488	.913	.5512	1.075
.70	.7493	.924	.6507	1.064
.80	.8497	.932	.7502	1.056
.90	.9498	.939	.8502	1.049
1.00	1.0499	.946	.9501	1.045
1.50	1.6000	.963	1.4500	1.029
2.00	2.0500	.972	1.9500	1.022

TABLE IV.

SHOWING POWER OF OCEAN WAVES IN HORSE-POWER UNITS.

$\frac{L}{h}$	Length of Wave in Feet.							
	25	50	75	100	150	200	300	400
50	.04	.23	.64	1.31	3.62	7.43	20.46	42
40	.06	.36	1.00	2.05	5.65	11.59	31.95	66
30	.12	.64	1.77	3.64	10.02	20.57	56.70	116
20	.25	1.44	3.96	8.13	21.79	45.98	126.70	260
15	.42	2.83	6.97	14.13	39.43	80.94	223.06	457
10	.98	5.53	15.24	31.29	86.22	177.00	487.75	1001

Table V.

COMPARISON BETWEEN WAVES IN SHALLOW AND DEEP WATER.

Depth of Water as a Fraction of the Wave-length.	Ratio between Quantities for Shallow Water and Corresponding Quantities for Deep Water.		
	Ratio of Axes of Surface Orbits.	Length for Given Velocity.	Velocity for Given Length.
.01	.063	15.90	.251
.02	.124	8.08	.352
.03	.186	5.376	.431
.04	.246	4.065	.496
.05	.304	3.289	.552
.075	.439	2.277	.663
.10	.557	1.796	.746
.15	.736	1.358	.858
.20	.847	1.180	.920
.25	.917	1.091	.958
.30	.955	1.047	.977
.35	.975	1.026	.987
.40	.987	1.013	.993
.45	.993	1.007	.996
.50	.996	1.004	.998
.60	.999	1.001	.999
.75	.9999	1.0001	.9999
1.00	.99999	1.00001	.999999

11. Wave-formation due to the Motion of a Ship-formed Body through the Water.

In § 2 it is shown that if the surface of the water were covered by a rigid plane through which the ship could pass without resistance, and which closed after it, no waves could be formed, but a disturbance in the distribution of pressure would result, such that about the bow and stern there would be an excess and about the middle portions a defect. If the hypothetical rigid plane is removed, we shall have instead of such a modification of pressure, and as the natural manifestation of the tendency to produce it, an elevation of the surface about the bow and stern and a depression about the middle portions. The result of these initial or fundamental tendencies is, however, much modified by propagation and the interference or combination of their elementary results.

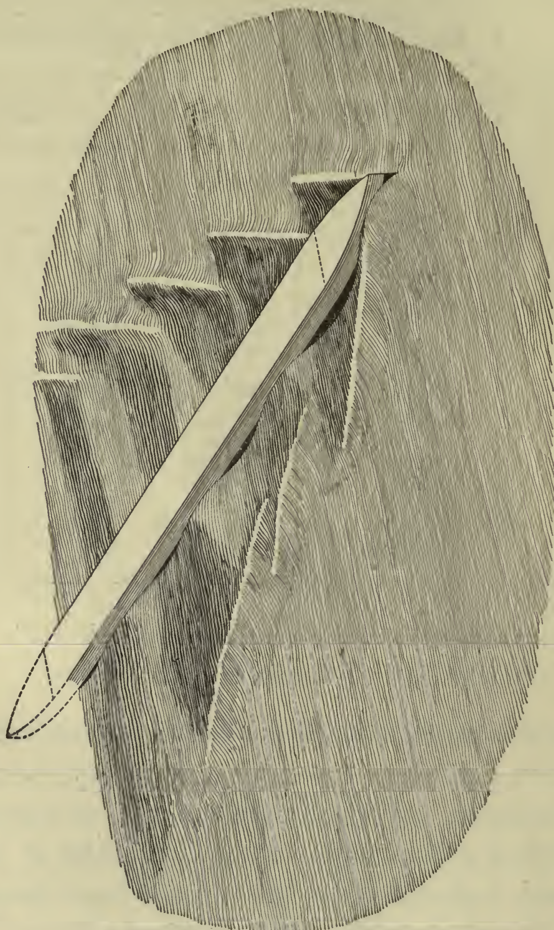


Fig. 31.

In any event, however, the distribution of normal pressure over the surface of the ship will be much changed from that corresponding to hydrostatic conditions, such change resulting in a force directed from forward aft. This resultant will be of such amount that the increased expenditure of energy necessary to overcome it will just equal the energy necessary to the maintenance of the system of waves and stream-lines.

Having thus referred to the modified stream-line system

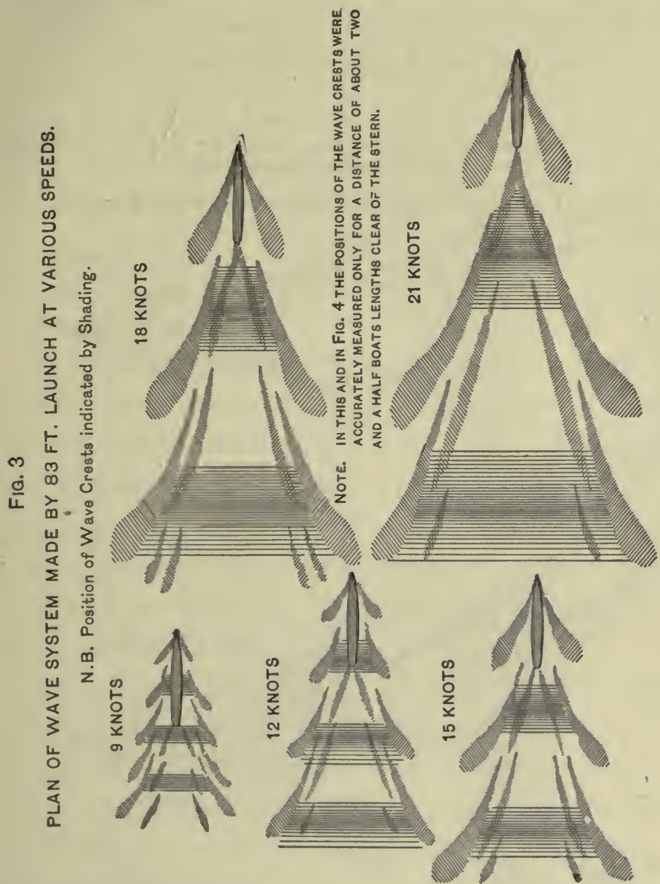


Fig. 32.

and its attendant phenomena as a sufficient initial cause for the formation of waves, we may examine their actual character and distribution in more detail.

In Figs. 31, 32, and 33 are shown wave patterns as determined by Mr. R. E. Froude for the various cases mentioned. We have here indicated two quite distinct varieties of wave form and distribution:

(*a*) Waves with crests perpendicular to the line of motion, constituting the transverse series;

(*b*) Waves with crests oblique to the line of motion, constituting the oblique or divergent series.

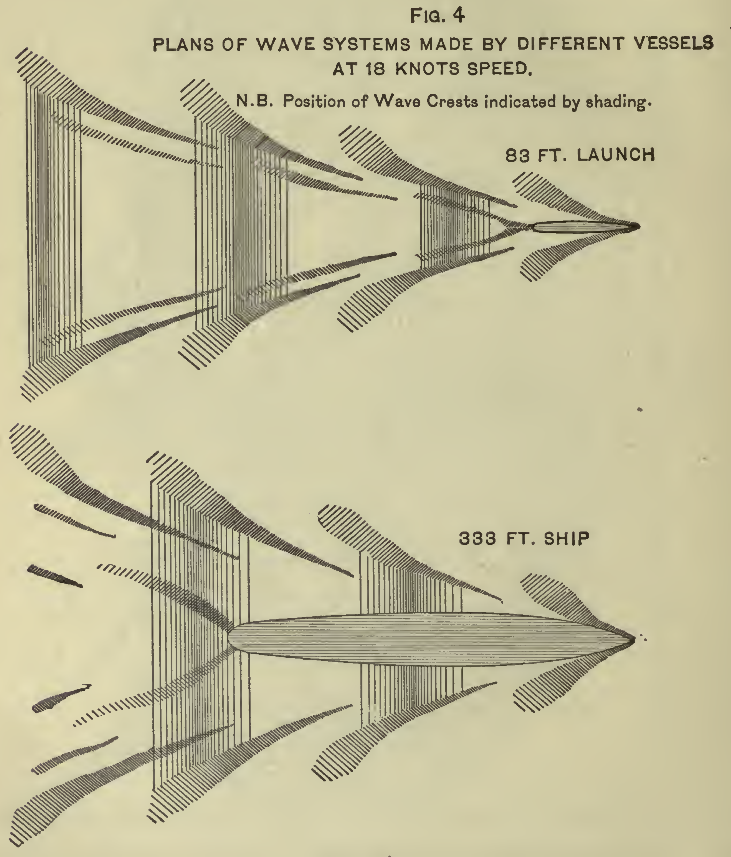


FIG. 33.

The series of transverse waves are distributed along the line of motion of the ship, and may be considered as a group of trochoidal waves as described in § 10. The speed of the individual waves is the same as that of the ship, and hence their length from crest to crest corresponds closely to that of a deep-water trochoidal wave traveling at the same speed as the ship. A crest is always found at or near the bow, and it is this crest which may be considered as the initial result of the wave-forming tendency in this region. To avoid confusion this crest is omitted from the diagram, and only the more pronounced local elevations constituting the divergent crests are shown. As we shall explain later, however, the energy belonging to this wave is drained away sternward relative to the ship, and gives rise to a series of secondary waves, or echoes as they are termed, the primary and its echoes thus forming the series as a whole.

In a long parallel-sided ship the crests and hollows show for some distance from the bow, gradually decreasing in altitude as they spread transversely, and thus involve more and more water with a gradually decreasing energy. If the parallel body is so long that the waves have by this process of dissipation become inappreciable, then it will be found that a similar principal crest will be generated near the stern, the energy of which, by the same process of sternward propagation as at the bow, will give rise to a stern series of echoes. The diminution of pressure amidships will be so slight and will be distributed over so great a distance that the primary hollow will usually be scarcely noticeable. If instead of the long parallel body we have the usual form, we shall have the primary crest at the bow and stern, and a more pronounced tendency to form a primary hollow amidships. The energy of the waves thus formed will be propagated sternward as before, thus giving rise to echoes and to three more or less plainly marked initial systems. These, however, will coalesce, and the whole will form a complete system with crests and hollows of size and distribution depending on the character of the various components. As the variations in velocity due to the stream-line motion are only slight, the altitude of such waves will be small, and at ordinary speeds they are scarcely noticeable. As the increase in velocity amidships is, moreover, quite gradual, the corresponding initial hollow will usually be slight, and a correspondingly unimportant element in the combined series. In many cases, however, at high speeds and when the elements of combination enter properly, the combined series may show a very pronounced hollow amidships.

Turning to the divergent series, we have the configuration shown in the diagrams involving in each case an initial divergent crest formed on either side of the bow. These may be considered as the primary result of certain tendencies to which we shall directly revert. The same drain of energy and propagation sternward obtains here as with the transverse series, and, as we shall explain, the result is the series of echoes as shown, arranged in skew or imbricated order; i.e., with their crest-lines overlapping like the shingles of a house.

We have now to explain the formation of the primary divergent wave at the bow. We have already explained the initial formation of the transverse crest at the bow. This crest in itself is of small altitude, and extends over a considerable area according to the speed and to the rate of variation of the pressure in the stream-lines. Referring to Fig. 34, we have in plan a suggestion of the contour or level lines for the variation of level which would result from the initial impulse corresponding to the conditions from forward aft. The line *LBHDGFK* is supposed to lie on the surface of the smooth water. *AB* and *EF* are humps or crests, and *CD* is a depression or hollow. Now the tendencies which lead to the general elevation of the water throughout *AB* become very much accentuated near *A*, the forward end of the surface of discontinuity between the ship and the liquid. As a result there will be raised against the surface of the bow at *A* a

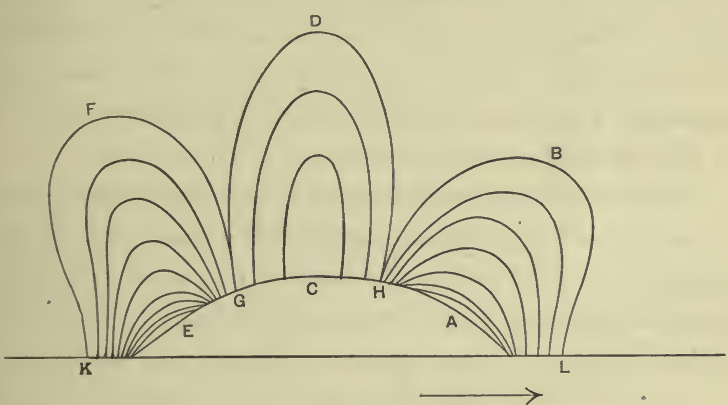


FIG. 34.

more or less pronounced wall of water. This may be considered as a locally exaggerated manifestation of the general tendency which would give rise to the hump as a whole. Otherwise we readily see in this wall the simple result of driving the oblique face *A* rapidly through the water, or, *vice versa*, the natural result of placing an oblique face *A* in a rapidly flowing stream.

While therefore this special elevation of water is due fundamentally to the same general cause as the remainder of the hump, and while it must be considered as a part of this general elevation *AB*, yet it is elevated so definitely above the surface of the remainder of the hump, that immediately we find this local elevation undergoing propagation somewhat like a positive wave of translation. More exactly or more generally, we may view this propagation as a result of the unequal distribution of pressure in the water in this neighborhood. The pressure is greatest quite near the ship, and decreases rapidly as we go slightly outward. The water thus affected will naturally yield in all directions in which the pressure is less. Hence it will yield upward and outward, and thus give rise to an initial elevation of water and to a propagation outward in the direction of the most rapid decrease of pressure. This at first will be nearly normal to the water-line or surface of the ship.

Let us now attach ourselves to the ship and consider the stream as flowing by. Relative to the ship the initial propagating impulse will be in the direction *OP*, sensibly perpendicular to *OQ*, Fig. 35. Let v be the velocity. Then if at

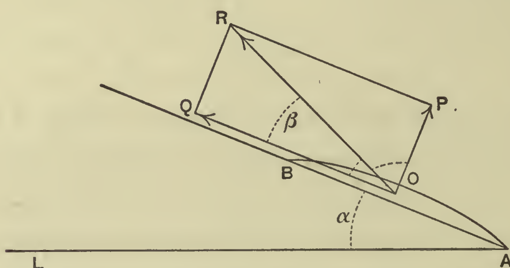


FIG. 35.

a given instant the stream could be suddenly arrested, the water at that instant forming the elevation *AB* would be propagated along *OP* with velocity v. In the actual case, however, the propagation takes place not into still water, but into water moving sternward with its natural stream-line velocity, approximately parallel to AQ. The elevation of water thus propagated will have two velocities—one, v, along OP, and another, w, along OQ. Hence the actual propagation will be the resultant of these two, and its direction OR will make an angle β with OQ such that $\tan \beta = v \div w$. Also the direction OR will make an angle γ with the direction of motion AL such that

$$\tan \gamma = \tan (\alpha + \beta) = \frac{\tan \alpha + \frac{v}{w}}{1 - \frac{v}{w} \tan \alpha}.$$

The velocity v may be considered as depending on the rate of decrease of pressure from AQ outward, or on the pressure very near the surface of the ship in this neighborhood. We might consider a portion of the velocity of propagation as due to the elevation AB considered as a positive wave of translation. Inasmuch, however, as the distribution of pressure seems to be the important factor in this propagation, it seems preferable to refer the whole phenomenon, generation and propagation, to this general cause. It is therefore evident that v will increase and decrease with the velocity of the ship according to a law too complex for general expression.

The velocity w is the velocity along the stream-lines. It will be at this point slightly less than u, the velocity of the ship, or the velocity of the flow as a whole. For fine ships or small values of α its value cannot differ widely from $u \cos \alpha$. For large values of α this would give too small a value.

Turning to the value of $\tan \gamma$ above, it is seen that the only quantity depending on speed is the ratio $v \div w$. Both of these, as we have seen, depend on the speed of the ship, and it seems not unnatural to suppose that v like w may very nearly vary directly with u, the speed of the ship. In such case $v \div w$, and with it γ, will remain very nearly the same at varying speeds. This has been found experimentally to be the case, more especially for fine ships at moderate and high speeds. For very full ships the value of γ seems to increase slightly with the speed, indicating that for such forms v increases more rapidly than w. Before leaving this value of tan γ it may be well to show the method of finding the mean value of tan α for the bow. In Fig. 36 let the origin be at B and x reckoned plus aft. Then if α

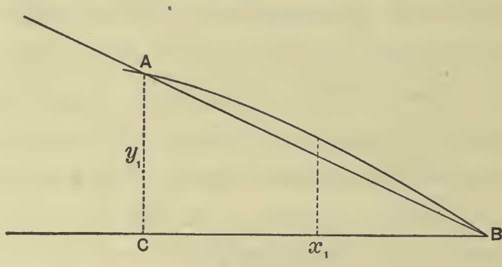


FIG. 36.

denotes the inclination of the curve AB we have

$$\tan \alpha = \frac{dy}{dx},$$

and $$\text{mean of } \tan \alpha = \frac{1}{x_1}\int_0^{x_1} \tan \alpha dx = \frac{1}{x_1}\int_0^{x_1} dy = \frac{y_1}{x_1}.$$

Integrating similarly for the mean vertically, we have

$$\text{mean of } \frac{y_1}{x_1} = \frac{1}{z_1 x_1}\int y_1 dz = \text{area of half-section } AC \div z_1 x_1.$$

Hence

$$\text{mean of } \tan \alpha \text{ for bow} = \frac{\text{area of half-section } AC}{z_1 x_1}.$$

In usual forms BC may be taken at 10 to 15 per cent of the length.

At very high velocities of the ship the water in the elevation AB, Fig. 35, instead of forming a simple sharply-marked mound will curl and break. The general result will be the same, however, as the breaking is simply equivalent to pouring a mass of water along AB, and the distribution and rate of variation of the pressure will be such as to give the same general outward propagation as above described.

Having thus discussed the propagation of a single instantaneous elevation of water AB and the considerations on which its direction depends, we next observe that as the propagation removes the water forming the elevation at any one instant, it is immediately succeeded by another like elevation, so that as a result there is a continual elevation propagated relative to the boat along the direction OR. As the elevation recedes from AQ, however, the crest tends to broaden and thus loses altitude and finally becomes insensible. Toward the outer end its direction may change somewhat, due to the superposition of the tendencies at that point on the initial impulse along OP. As the side of the ship is left, the most rapid decrease of pressure will be directed more nearly transversely, or even aft, toward the hollow amidships. In consequence of this, together with the changed value of w, the outer end may curve around somewhat toward the stern. Due, however, to the decreasing altitude, this change is usually unnoticeable.

We have thus described the development of the primary member of the divergent system of waves at the bow. A similar primary is formed at the stern through the action of the same general causes. In this case the initial wall or special elevation of water is formed under the stern and is propagated outward into the outlying water, which, due to the frictional wake and to the stream-line motion in this neighborhood, has a somewhat lower velocity than at the bow. While thus the initial direction of propagation of the elevation would usually be somewhat aft of the direction for the similar elevation at the stern, yet the other component being less than at the bow the resulting direction is not far different at the two ends. At high speeds instead of the special elevation being formed against the side of the ship it is formed immediately astern. In this case the two streams flowing one on either side of the ship combined with that flowing under the ship may be considered as meeting and giving rise to an initial elevation along the line *AB*, Fig. 37.

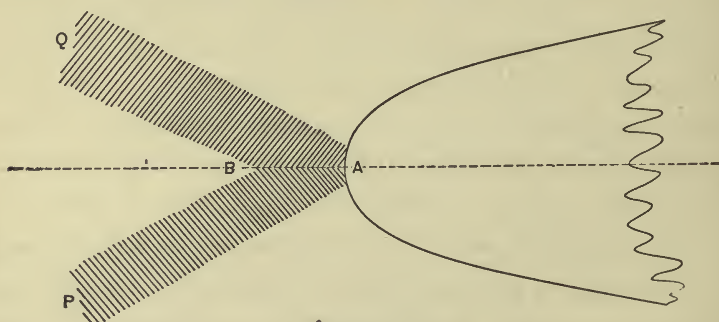


FIG 37.

This being propagated transversely outward on each side into the outlying water gives rise as before to the two divergent waves *ABP* and *ABQ*. At certain speeds and with certain forms of boat this wave is very pronounced, taking the form of a flat-topped hill of water sloping away in a fan-shaped figure and gradually decreasing in altitude. At still higher speeds this wave recedes still farther astern and usually becomes somewhat less pronounced. Especially is this the case with that form of after-body which approximates to a wedge with flat side down and edge near the water surface at the stern, as found on most modern torpedo-boats and many other high-speed craft.

Having thus discussed the formation of the primary members of the various series of waves involved in the problem of resistance, it is time to take up the question of the propagation of energy already referred to.

12. THE PROPAGATION OF A TRAIN OF WAVES.

This subject has been briefly considered in § 10. We must now examine in more detail the consequences of the statements there made. In Fig. 38 let the full line represent

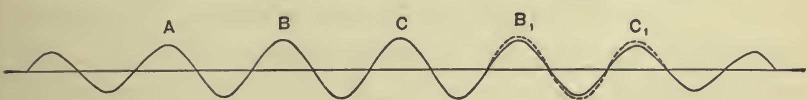


FIG. 38.

a group of waves of which C marks the center and principal member, the successive crests on either side gradually decreasing in altitude until they become negligible. Then observation in agreement with theory shows that such a configuration or group is propagated with only about one half the velocity of the individual waves A, B, C, etc. Now the group as a whole may be considered as the expression of a certain distribution of energy, and hence the velocity of the group may be considered as the velocity with which the energy is propagated, as distinguished from the velocity of the individual waves. It must be considered, therefore, that in waves of trochoidal character the energy is propagated at only one half the velocity of the waves themselves. Viewin the matter from this standpoint, let us examine the propagation of the group of waves in Fig. 38. The wave C representing the principal member is propagated forward with a certain velocity. As it moves forward, however, it leaves behind a part of its energy, and hence gradually decreases in altitude, and is no longer the maximum member of the group. The energy thus left behind is absorbed by B, which therefore increases and becomes after a certain time the maximum member B_1, and hence the new center of the group. The wave C in the meantime has gone from C to C_1, while the principal member of the group, as such, has gone from C to B_1. The latter evidently fixes the velocity of the group. There is thus a continual propagation of the individual waves forward, while relative to the waves the individual characteristics are propagated backward. The group velocity is therefore the difference between these two. It results that any individual wave as A travels along through the entire group, taking on successively the characteristics of the various members, and finally dying out at the forward edge while a new member rises at the after edge; and thus the process continues. Or, again, we may consider that the leading waves are continually dwindling and disappearing for lack of the energy which they leave behind, while the following waves similarly grow by the access of energy which they as continually receive. Thus if v is the velocity of the waves forward, then the velocity of the group and of the energy is $v/2$ forward. Relative to the group the velocity of the waves is $v/2$ forward, and of the energy 0. Relative to the waves the velocity of the group and of the energy is $v/2$ backward.

In thus considering the group velocity of a train of waves we must note that the exact proof supposes trochoidal motion, and hence circular paths for the motion of the particles. This cannot be the case where the waves vary in altitude, and under such circumstances the group cannot be indefinitely permanent, even neglecting viscous degradation. This is borne out by observation. Again, in shallow water the orbits being oval or approximately elliptical, the group-velocity is greater than one half the wave-velocity, and hence in such case the energy is transmitted at a velocity greater than one half that of the individual waves. In the extreme case we have the wave of translation in which the group becomes reduced to the individual, the velocity of one being necessarily that of the other. In such case, then, the energy is transmitted at a velocity equal to that of the individual wave.

13. Formation of the Echoes in the Transverse and Divergent Systems of Waves.

Taking first the primary bow transverse wave, we see that as an individual it must necessarily keep pace with the ship. Considering, however, that it is constituted approximately as a trochoidal wave, its energy will be continually draining to the rear and giving rise to a successive series of echoes. These from their gradual spreading sideways soon lose all significant altitude and become negligible.

Turning now to the divergent system, we find in the draining backward of the energy relative to the ship combined with the natural internal propagation peculiar to this wave, the explanation of the overlapping or skew arrangement of the crests. We may consider that these divergent waves constitute virtually a special series superimposed, as it were, on the flatter transverse series. In Fig. 39 let AB be the primary member. Now this crest, considered simply as a configuration, is by virtue of its fixed relation to the ship carried forward unchanged in the direction OP with a velocity v equal to that of the ship. A part of the energy, however, will naturally lag behind, the relative amount approximating more or less closely to one half, according as the characteristics of the wave approach more or less closely

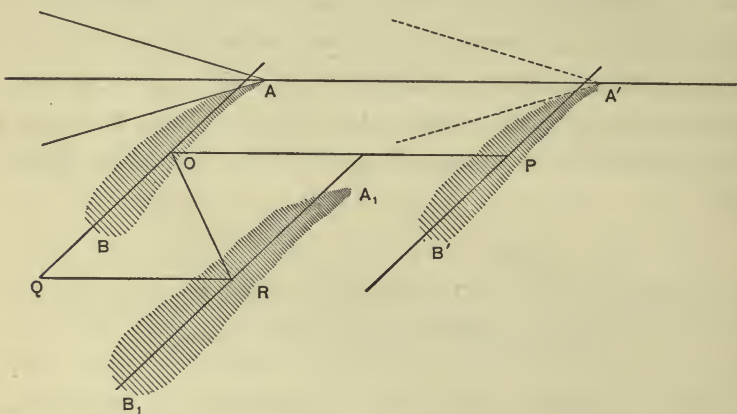


FIG. 39.

to those of trochoidal form. Relative to still water, therefore, there will be a propagation of the energy forward at a velocity less than v, and approaching $v/2$ as the wave approaches trochoidal form. In addition to this propagation of energy along OP, there will be likewise, due to the peculiar constitution of this elevation of water, a propagation or transmission of energy outward along OQ. The resultant direction of propagation is therefore along some line OR. The result is that the energy in AB may be considered as located at a later instant in A_1B_1, while the primary crest at

$A'B'$ exists by virtue of energy drawn from the ship between A and A'. Or otherwise, relative to the ship and the constantly renewed primary crest AB, energy is constantly draining to the rear and outward along OQ, the result being a transmission obliquely outward in such manner as to give rise to the series of overlapping crest-lines as shown.

In a similar way a second echo is formed, and so on until by lateral extension and viscous degradation the altitude becomes negligible. In like manner the stern series of divergent waves may give rise to a similar series of echoes.

We have already referred to Figs. 32 and 33 in illustration of the wave-pattern whose formation we have attempted to explain. We may now call attention to certain further details.

We first note the location of the primary bow and stern waves. At low speeds the bow divergent waves are formed close to the bow, while as the speed increases their mean location draws somewhat farther aft. At low speeds the stern divergent waves are formed on the quarter and diverge independently as shown in Fig. 32. As the speed increases they draw aft and together, and finally coalesce into an elongated mound of water spreading away in V shape, as already noted.

The bow and stern transverse primary waves have the same general location longitudinally as the divergent primaries. Indeed, as we have already pointed out, the latter are merely local exaggerations of the general primary elevation at these points.

The length between the primary bow and stern waves increases somewhat with the speed. At moderate or high speeds it is usually considered as slightly greater than the length of the ship. The available data is not sufficient to determine satisfactorily the amount of excess, but it is usually taken at from 5 to 10 per cent of the length.

Numerous measurements of the wave-lengths for the transverse system are found to be in close accord with the length for the natural trochoidal wave of the same speed as the ship, and as given by formula, § 10 (1). It may therefore be considered as shown by experience that the wave-length of the transverse series is in satisfactory agreement with what it should be, considering them as a series of trochoidal waves having the speed of the ship. In consequence it seems allowable to assume for the waves of this system such further properties of trochoidal waves as may be convenient for their investigation.

With the divergent system the case is somewhat different. These waves, so far as their configuration is concerned, travel forward with the velocity of the ship. If γ is the angle of their crest with the longitudinal, however, the component of their velocity normal to the crest-line will be $u \sin \gamma$. Now measurement indicates that the wave-length in this direction agrees fairly well with that of a natural trochoidal wave having a velocity $u \sin \gamma$. It follows that in a sense we may consider this system as made up of a series of bits of trochoidal waves with a velocity $u \sin \gamma$ and a length corresponding. For this length we shall have

$$L = \frac{2\pi}{g} u^2 \sin^2 \gamma.$$

Hence for the length from crest to crest on a longitudinal line we should have

$$L_1 = \frac{2\pi}{g} u^2 \sin \gamma.$$

For the transverse system we have likewise

$$L_2 = \frac{2\pi}{g} u^2.$$

Hence the longitudinal length of the transverse series should be somewhat greater than for the divergent series. This is borne out by the diagrams, reference to which shows that the crests of the former fall farther and farther astern relative to those of the latter. While the difference may not be so great as is indicated by the formulæ above, the tendency is in this direction, and the errors of the formulæ may naturally be referred to the imperfect manner in which such waves fulfil the trochoidal characteristics.

14. Wave-making Resistance.

Having thus considered the formation of the various wave systems attending the motion of a ship through the water, we may next examine their relation to resistance.

Taking first the bow transverse system, we have seen that the energy of the waves is, relative to the ship, constantly passing sternward. At the same time the energy of the system as a whole must be maintained constant. Hence this drain of energy sternward must be made up by energy derived from the ship, and it is simply the transmission of this energy which gives rise to this part of the wave resistance. The exact fraction of energy which falls astern relative to the ship will depend on the geometrical character of the wave; and it must not be considered that the ratio 1/2 is anything more than an approximation, which, however, may serve for illustrative purposes. Hence let us assume that one half the energy is naturally propagated with the form, and that one half must be made up from the ship. It follows that for every two wave-lengths run the ship must supply the energy necessary to the formation of one wave.

Let E be the energy of the wave, L the wave-length, and R the resistance due to its maintenance. Then since resistance equals the work done or energy transmitted divided by the distance, we have

$$R = \frac{E}{2L} \quad \text{or} \quad R \sim \frac{E}{L}.$$

A similar expression would hold for the element of resistance due to each of the divergent waves at the bow, E being the energy and L the longitudinal wave-length for this system.

It thus appears that the work which is done by the ship at the bow on the water in maintaining these systems of waves is equivalent to adding one new wave to each system for every two wave-lengths traveled. Similar considerations hold for the resistance due to the stern wave systems.

We have thus far considered the bow and stern wave systems in their individual aspect. We have now to consider the possible influence of the former upon the latter.

We first note that, so far as the divergent system is concerned, no direct influence is possible, since these waves travel off obliquely and come in contact with the ship at the bow only. Again, if the ship is very long for her speed, especially if she has a long middle body, the bow transverse system will become negligible before reaching the stern, so that each system will produce its individual effect. In the general case, however, the bow transverse system will not have entirely disappeared by the time it reaches the stern, and there will be formed at this point a combination system resulting from the remainder of the bow system and the natural stern system.

The total energy of the entire wave systems may be considered as that of the bow transverse and divergent systems plus that of the stern divergent system, possibly modified by the bow transverse system, plus that of the stern combination system minus that part of the latter which is received from the bow system. For the transverse systems alone the above value is equal to the sum of the energy of the stern combination system plus the loss of energy in the bow system when it reaches the stern.

Let L denote the distance between the natural bow and stern primary crests as discussed in § 13, and let λ be the wave-length, which we may take as that of a trochoidal wave of the velocity of the ship. Let $L = n\lambda + a$. Then a is the distance from the stern primary crest forward to the nearest crest of the bow system. Hence $a \div \lambda$ is the phase difference ratio and $2\pi a \div \lambda$ is the phase difference angle. Let h_1 be the altitude of the bow wave, kh_1 the remaining altitude of the bow system when it reaches the stern, and h_2 the natural altitude of the stern wave. Then referring to § 10 (15) it is seen that the combination system will be trochoidal and of altitude

$$h^2 = k^2h_1^2 + h_2^2 + 2kh_1h_2 \cos \frac{2\pi a}{\lambda}.$$

Referring now to § 10 (11), it is seen that the energy of a wave per unit breadth measured along the crest is proportional to the product of the wave-length by the square of the altitude. Let B_1 be the breadth transversely at the bow. This is of course indefinite, since h_1 gradually decreases as we go from the bow outward. We may, however, consider h_1^2 as the mean of the squares of the altitudes for a breadth B_1 considered as comprising all of the wave whose elevation is sensible. The energy of the bow primary wave will then be proportional to $h_1^2 B_1 \lambda$. Let B and h denote similar quantities for the combination system at the stern. Then the energy of the stern combination wave will be proportional to $h^2 B \lambda$. A portion of the latter, however, proportional to $k^2 h_1^2 B_1 \lambda$, is derived from the bow system. Hence the energy necessary for the maintenance of the whole system, and to be supplied by the ship in running a distance 2λ, is proportional to $h_1^2 B_1 \lambda + h^2 B \lambda - k^2 h_1^2 B_1 \lambda$. Now assuming that B_1 and B are sensibly the same, substituting for the value of h above and putting the result proportional to the work done by the ship in overcoming a resistance R_w through a distance 2λ, we have

$$2R_w \lambda \sim \left(h_1^2 + h_2^2 + 2kh_1h_2 \cos \frac{2\pi a}{\lambda}\right) B\lambda.$$

Whence $$R_w \sim \left(h_1^2 + h_2^2 + 2kh_1h_2 \cos \frac{2\pi a}{\lambda}\right) B. \quad . \quad . \quad (1)$$

Let $s = mL$ = wave-making length of ship, in which, as already noted, m is usually 1.05 to 1.10. Then s will consist of a certain number of multiples of λ with a remainder a. Hence $\cos (2\pi s \div \lambda) = \cos (2\pi a \div \lambda)$. Also, $\lambda = 2\pi v^2 \div g$, § 10 (1). Hence $2\pi s \div \lambda = gs \div v^2 = gmL \div v^2$, and hence

$$R_w \sim \left(h_1^2 + h_2^2 + 2kh_1h_2 \cos \frac{gmL}{v^2}\right) B. \quad . \quad . \quad (2)$$

Now in general we may put

$$h \sim \lambda,$$

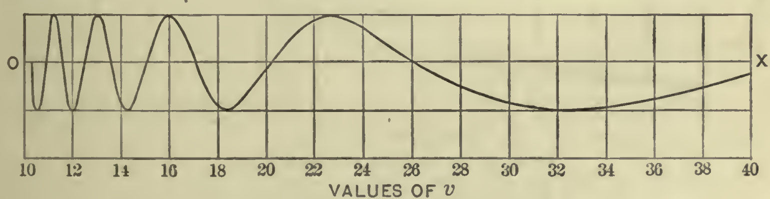
and $$h \sim v^2,$$

and hence $$h_1^2 \sim h_2^2 \sim v^4.$$

Hence we may represent the resistance due to the natural bow system by a term of the form H^2v^4, and that due to the natural stern system by a similar term, J^2v^4. Then from the derivation of (2) it follows that we should likewise represent the total resistance of the combined series by an expression of the form

$$R_w = \left(H^2 + J^2 + 2HJk \cos \frac{gmL}{v^2}\right)Bv^4. \quad . \quad . \quad (3)$$

This expression is periodic in character, the mean value being $(H^2 + J^2)Bv^4$. To illustrate the variation from this



Values of $2HJk \cos \frac{gmL}{v^2}$ laid off from OX as axis.

FIG. 40.

value due to the third term we may refer to Fig. 40. We have here illustrated the values of the expression

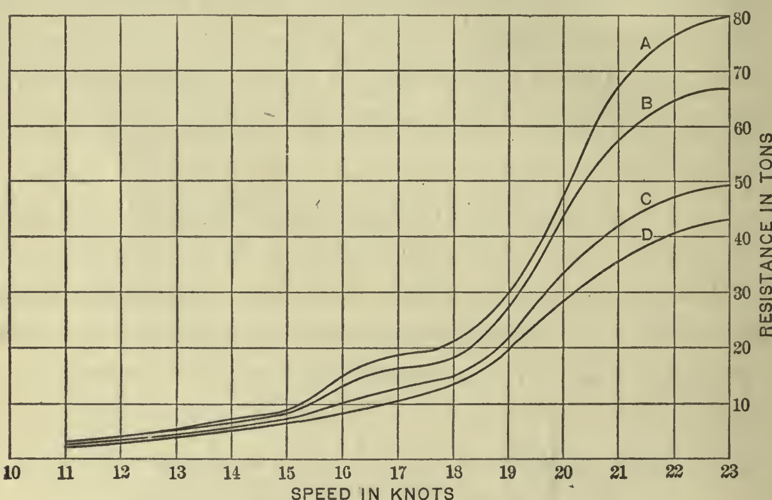
$$2HJk \cos \frac{gmL}{v^2},$$

using the following numerical values of the terms

$$2HJk = 1,$$

$$mL = 100.$$

At low and moderate speeds the amount of wave-resistance is so small that these fluctuations are entirely imperceptible. As the speed is increased, however, a point is reached where the wave-resistance forms an important part of the total resistance, and where evidences of such a periodic fluctuation in its amount are plainly indicated. This is illustrated in Fig. 41,* showing the residual resistance-



CURVES OF RESIDUAL RESISTANCE

Ship	Length	Beam	Draft†	Disp't
A	400′	38.2	20.7	5930
B	"	"	19.1	5390
C	"	"	16.5	4480
D	"	"	15.4	4090

FIG. 41.

curves of a ship at several drafts as determined by Mr. R. E. Froude.

For the waves of the divergent series we may take likewise the energy as proportional to the length, breadth, and

* Transactions Institute of Naval Architects, vol. XXII. p. 220.
† Inclusive of 9 inches of keel.

square of the altitude. Hence h, B, and λ denoting in general the same characteristics as above, we should have for each train

$$\text{Energy} \sim h^2 B\lambda,$$

$$R \sim h^2 B.$$

As further illustrating the above principles, we may refer to the following experiments of Mr. Froude on the influence of length of parallel middle body on wave-making resistance.

For the purposes of the investigation, an initial model was taken as representing a ship 160 feet long. Successive lengths of parallel midd'e body representing 20 feet in length were then added until the total length represented 500 feet. These models were tried at various speeds and the resistance measured. The frictional resistance being then computed and subtracted, the residual resistance was considered as sensibly due to the maintenance of the wave systems. In order to show the results graphically, the information was disposed as in Fig. 42.*

Any given curve above AA represents the residual resistance at a constant speed, as marked, for varying values of L as shown on the base AA. Similarly the corresponding line below AA shows the frictional resistance at the same speed for the same series of ships, the ordinates in this case being measured downwards.

We may note the following points indicated by this diagram:

(1) The curves of residual resistance show plainly a periodic variation, the length of each period or distance from

* Transactions Institute of Naval Architects, vol. XVIII. p. 77.

maximum to maximum being approximately constant at any one speed.

(2) The length of the period or spacing between maxima

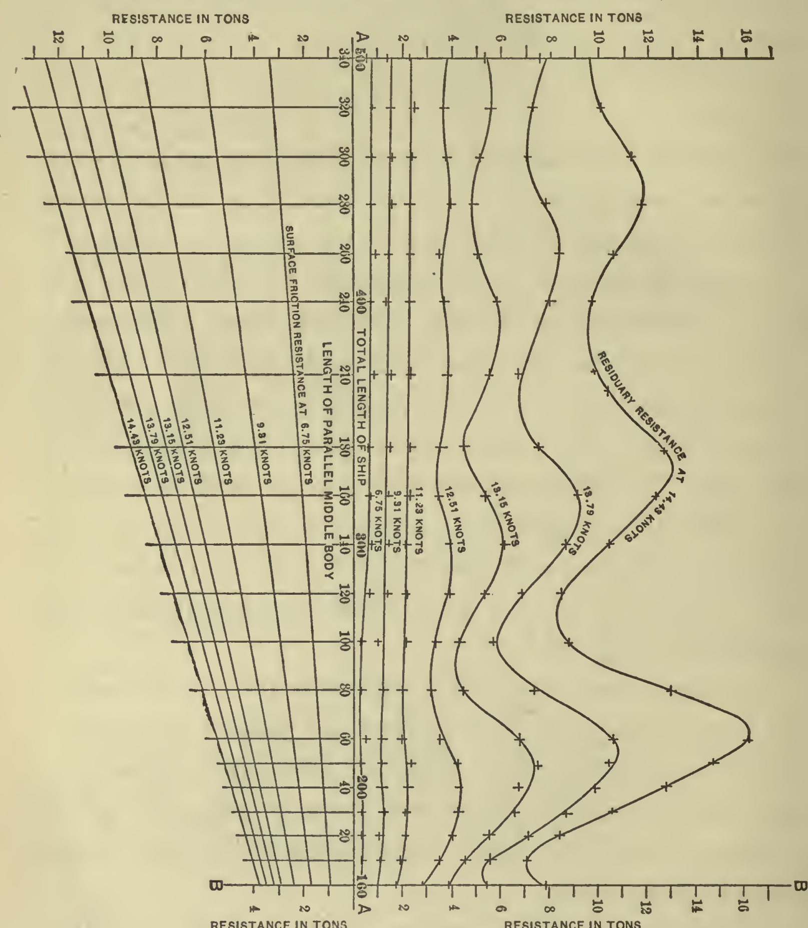


FIG. 42.

is greater as the speed increases, the value being found to vary nearly as the square of the speed.

(3) The amplitude of the variation increases as the speed increases.

(4) The amplitude of the variation decreases as the length increases.

The increase of (3) is due to the rapidly increasing importance of wave-making resistance in general, after passing a certain speed, and also more directly to the value of k, which usually increases with the speed. The decrease of (4) is due to the decrease in k as L increases. If L were sufficiently great there would be no interference, and hence no fluctuation or periodic variation in the resistance.

Turning back now to formula (2) we may further examine the relation of h_1, h_2, and B to the speed.

In perfect stream-line motion in horizontal planes and without elevation of the surface we have, denoting the velocity by u,

$$p - p_0 = \frac{\sigma}{2g}(u_0^2 - u^2). \quad \text{See § 2 (1)}$$

Hence along any one line for a slight change in velocity

$$dp = -\frac{\sigma}{g}udu.$$

If now we put $du = eu$, where e represents a certain fraction, we have

$$dp = -\frac{\sigma}{g}eu^2.$$

Hence with varying initial velocity, the pressure increments or decrements at any given point will vary as the square of the velocity. In actual wave-line motion at moderate speeds the elevation will correspond nearly to the increment of pressure, and hence we should find the values of h_1 and h_2 nearly proportional to u^2. At higher speeds the stream-line motion is less perfect, and it seems probable that the variation of pressure for a given percentage of speed fluctuation will increase somewhat more slowly than as the square of the speed. Assuming, however, in general this relationship, we may as before represent the resistance due to the natural bow and stern waves by H^2u^4 and J^2u^4. We shall then have again as in (3)

$$R_w = u^4\left(H^2 + J^2 + 2kHJ\cos\frac{gmL}{u^2}\right)B.$$

Next as to B. It has been usually assumed that B is very nearly constant at varying speeds, and that it may be taken as approximately proportional to the linear dimensions of the ship. In such case we should have

$$R_w \sim lu^4.$$

Some have thought that it should be considered rather as varying with h, and hence with u^2. In such case we should have

$$R_w \sim u^6,$$

and hence independent directly of the dimensions of the ship. Risbec * suggests a combination of these two terms in the form

$$R_w \sim Alu^4 + Bu^6,$$

and believes that as u increases B will become more and more predominant in the value of R_w. It may be noted that Risbec's suggestion is equivalent to

$$R_w \sim Ju^n,$$

in which J may depend on the dimensions of the ship and n may vary from 4 to 6.

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* Bulletin de l'Association Technique Maritime, vol. v. p. 52.

Similar considerations hold with regard to the resistance due to the divergent system, each member of which may be considered of the form

$$R = Cu^4 \quad \text{or} \quad C_1u^n,$$

or as a summation of such terms, according as the law is assumed to be constituted. Hence we may consider that in reference to speed the total wave-making resistance will probably vary with some index between 4 and 6, or as a sum of terms with these or intermediate exponents. The probably decreasing values of P and Q with increasing speed would, if they were taken as constant, decrease the exponents of u from the values they would otherwise have, and thus perhaps partly compensate for the increase in exponent due to the variation of the term B.

The actual relation, however, of the total wave-making resistance to the speed and to the dimensions of the ship is not known, nor can it be satisfactorily determined from the data available. There seems to be ground, however, for the belief that in many cases it shows a higher rate of variation than that given by the exponent 4. See also § 26. For the present, therefore, we are thrown back on more empirical formulæ, intended simply to express as nearly as may be the results of experience. Such formulæ will be found in § 29. With the accumulation of data from model experiments we may perhaps hope ultimately for a satisfactory determination of the constants in (1) and (2) or in other like formulæ shown by the data to be more suitable to the purpose in view.

15. Relation of Resistance in General to the Density of the Liquid.

For the head-resistance and skin-resistance per unit area we have seen that the density of the liquid enters as a factor. Similarly here the density must be a factor of the energy of a wave, and thus all parts of the resistance, and hence resistance as a whole, will vary with the density. The variation in the resistance of a given ship with change of density of the liquid is, however, in large measure offset by the oppositely varying amount of immersed body of ship. As between fresh and salt water the difference is slight, and is usually neglected except in the case of model experiments or very careful estimates. The densities of fresh and salt water are usually taken in the ratio of 1.0 : 1.032, or less accurately in the ratio 35 : 36.

16. Froude's Experiments on Four Models.

In 1876 Wm. Froude published a paper* showing the results of experiments on the comparative resistance of models of four ships of the same displacement but of certain differences of form. These models were called *A*, *B*, *C*, *D*; *A* being that of the Merkara. The dimensions corresponding to the four models were as follows:

	Displacement.	Length of Fore Body.	Length of Middle Body.	Length of After Body.	Total Length.	Beam.	Draft.	Wetted Surface.
A	3980	144	72	144	360	37.2	16.25	18660
B	3980	179.5	..	179.5	359	45.88	18	19130
C	3980	154.5	..	154.5	309	49.4	19.32	17810
D	3980	95	95	95	285	45.56	17.89	16950

* Transactions Institute of Naval Architects, vol. XVII. p. 181.

The forms of the fore and after bodies of *B*, *C*, and *D* were derived from those of *A* by expansion longitudinally, transversely, and vertically as required. The fore and after bodies themselves have therefore the same ratios of fullness and other characteristics as those of *A*, and the resulting differences in resistance may therefore be considered as due to the effect of the presence or absence of the parallel middle body.

The results may be summarized by saying that at all speeds *B* and *C* had less resistance than *A* or *D*. As between *B* and *C*, the latter having the less wetted surface had the less resistance. As between *A* and *D*, the latter, though having less wetted surface, had at the speeds tried (from 9 knots upward) greater resistance, though at the lowest speeds the directions of the two resistance-curves were such as to indicate an intersection, and lower values for *A* for very low speeds. At high speeds *B* and *C* changed places relatively, the smaller wave-resistance due to *B*'s greater ratio of *L* to *B* being more influential at such speeds than its greater wetted surface. At all speeds *A* had less resistance than *D*. For the former the resistance began to increase rapidly at about 17 knots and for the latter at about 14 knots. For *C* this relatively rapid increase in resistance began to appear at about 19 knots, while for no speed up to 20 knots was such tendency noticeable with *B*.

It appears therefore in general that resistance will be decreased if, instead of a parallel middle body, the fore and after bodies are expanded so as to obtain a ship of the same displacement as with parallel middle body, even if the ratio *B* to *L* is thereby somewhat increased.

17. Modification of Resistance due to Irregular Movement.

It is well known experimentally in all cases of bodies moving in a liquid, that a certain amount of water is momentarily bound to the body and must be considered as practically taking part in its motion, especially in any rapid changes involving accelerations and retardations. This amount, it also appears, may be from 15 to 20 per cent of the displacement of the ship. It follows that the resistance to an acceleration will not be that due simply to the mass of the ship, but rather to the combined mass of ship and water. At the same time it must be remembered that during retardation a corresponding effort is given out. It is, however, a general principle that more energy is required to maintain a system in irregular movement, the velocity oscillating about a mean value u, than at the same uniform value of the velocity. This is because in general R varies with u according to a higher index than 1, and because in such case the mean of a series of such powers of u is greater than the mean value of u raised to such power. This is readily verified for an index 2 or 3.

The mean resistance of a ship in irregular motion but at a mean speed u will therefore in general be greater than if the velocity could be made uniform at the same amount.

The actual modes of propulsion employed usually involve slight irregularities in motion, and these may be more or less increased by irregularities of wind, current, tide, depth of water, etc. Under usually favorable conditions it is not likely that the increase of R due to this cause is important, although its existence and explanation are of interest.

18. Variation of Resistance due to Rough Water.

The resistance of a ship in a seaway is known actually to be very considerably greater than that in smooth water for the same average speed. The causes of this may be given under three heads:

(1) The direct action of the waves and the rolling and pitching tend to increase the irregularities of motion, thereby affecting the resistance as described in the last section.

(2) The wave-disturbance of the water tends to confuse and disturb the regular stream-line motion, thereby entailing a greater drain of energy from the ship than in smooth water at equal speed.

(3) The pitching and rolling, notably the former, place the ship continually in less favorable positions relative to propulsion, so that on the whole the mean resistance is increased.

These causes also react unfavorably on the propelling apparatus, lowering its efficiency and increasing the irregularities of motion. While this last result is secondary, the final effect on the propulsion is the same as though the resistance of the ship were virtually increased.

No rules can, of course, be given for the estimate of such irregular modifications. It is a matter of experience, however, that the following qualities are favorable to uninterrupted speed in rough water:

(1) Considerable length, good free-board, and steadiness, especially absence from pitching.

(2) Large size and consequently great weight.

The reasons for the good effects of the former are apparent. The good effects of the latter are due to the increase of inertia and the consequent decrease in the irregularities of movement.

19. Increase of Resistance due to Shallow Water or to the Influence of Banks and Shoals.

It is a well-known fact that the resistance of a ship or boat at any given speed is very much increased by shallow water or by the proximity of banks, as in the navigation of canals and narrow rivers. The primary cause of this excess of resistance is readily found in the disturbed stream-line motion. As a result of the decreased cross-section within which the stream-lines must be contained, there arises an entire change in their form and in the wave configuration about the boat. The latter in general is very much exaggerated relative to its condition in open deep water, and the net consequence is a very considerable increase in the amount of energy necessary to the maintenance of the configuration of the water, and hence a corresponding increase in the resistance.

We may next inquire as to the limits within which these results become of notable significance.

Taking first the question of shallowness of water, the other dimensions being unlimited, we have a large number of instances of trial-trips in which the increase of resistance due to shallow water is more or less clearly shown.

White * gives the following instance:

The Blenheim in water averaging 9 fathoms made 20 knots with 15750 I.H.P. The wave phenomena were most striking and unusual. In water from 22 to 36 fathoms with the same power the speed was 21.5 knots.

In the trial of the U. S. S. New York both ways over a

* Manual of Naval Architecture, 1894, p. 467.

40-mile course it was found that at the same point each way where the depth of water was charted as 37 fathoms the revolutions fell off slightly, while the steam-pressure became increased. The water over the remainder of the course was from 15 to 20 fathoms deeper than at this point. The mean speed was 21 knots.

The characteristics of wave-motion in shallow water have been given in § 10. It is there shown that the paths of the particles become elliptical instead of circular, the ellipse becoming flatter as the water becomes shallower. Now it has been suggested that so long as the depth is sufficient to allow the wave corresponding to the speed of the ship to assume sensibly its trochoidal form and constitution no sensible increase in resistance will result. The propriety of considering this as more than a roughly approximate relationship may be questioned, since we do not know that the natural wave formed by the ship is more than roughly trochoidal in form.

By reference to § 10, Table V, it appears that if the depth equals one half the wave-length the difference between the two kinds of waves would be quite negligible. This leads by the aid of § 10 (1) to the following table:

Speed.	Minimum Depth in Feet.
10	28
12	40
14	55
16	71
18	90
20	111
22	135
24	160
26	188
28	218
30	250

From considerations drawn from a study of the distribution of stream-lines under certain ideal circumstances, D. W. Taylor* concludes that a depth of water equal to ten times the draft is an outside limit, and that no sensible increase in resistance is likely to be met with so long as the depth is not less than six times the draft.

The result in the case of the New York, if reliable, indicates that the influence extends to a considerably greater depth than six times the draft, or than the amount given by the table above. The point is one that cannot be settled except by experiment, satisfactorily extended data of which are not yet available.

Turning now to the conditions existing in canals, we have not only the restriction due to the bottom, but also that due to the sides as well. Proceeding at once to a representative case, we may note briefly the phenomena attending propulsion in a canal of which the cross-sectional area is only some four or five times that of the greatest section of the ship.

In such cases at moderate speeds the bow wave is quite marked, rising more or less transversely on each side or a little ahead of the bow, and followed amidships or astern by an equally notable hollow. Following astern is a train of waves, each member of which moves with the velocity of the boat, while the train as a whole moves at a lesser velocity, as explained in § 12. The slower the boat moves the more are the phenomena like those noted in connection with motion on the open sea. As the speed increases the more definitely does the boat place herself on the rear edge of a wave as in Fig. 43, and the more pronounced does the series of following waves become. As the speed of the boat approaches

* Transactions Institute of Naval Architects, vol. XXXVI. p. 234.

that corresponding to the propagation of a permanent wave of translation, $u = \sqrt{gh}$, the waves begin to take on more and more the characteristics of waves of translation. They transmit a greater and greater proportion of energy and the train becomes shorter and shorter. The resistance, however, continually increases due to the rapidly growing size of the waves and their consequently increased demand for energy. As the boat reaches the critical velocity or passes slightly

FIG. 43.

beyond it, the train of waves contracts to a single member, which takes its place with crest amidships or with the boat slightly on its forward slope. The resistance then decreases, and the boat may be towed along at this speed with a much less expenditure of power than just before. The cause of this sudden change in the law of resistance is found in the relation of the boat to the wave. The necessary energy to form the permanent solitary wave having been expended, and the boat having been placed in the most favorable position relative to such wave, the maintenance of the wave and the velocity of the boat may be obtained at a comparatively small further expenditure. The resistance for the most part is that due to the maintenance of the wave in an imperfect liquid, with the increased tendencies toward degradation due to irregularities in the sides and bottom.

Following is a table showing results of experiments made by J. Scott Russell, the pioneer of experimental investigation in this direction.

Weight of Boat in Pounds.	Speed in Miles per Hour.	Resistance in Pounds.
10239	4.72	112
	5.92	201
	6.19	275
	9.04	250
	10.48	268.5
12579	6.19	250
	7.57	500
	8.52	400
	9.04	280

Speeds at which these conditions obtain cannot, however, be practically maintained in canals, due chiefly to the wear and tear on the banks from the wash of the wave, and the comparatively high speeds necessary for depths of water exceeding 10 or 12 feet.

We are therefore rather concerned as to the effect of the banks and bottom on the resistance at moderate speeds, and as to the necessary ratio between the section of the canal and that of the ship in order that the increase in resistance may be negligible.

Satisfactory data for a general discussion of these points are not available. We have, however, certain partial results as follows:

Elnathan Sweet* gives the following formula as representing the results of a series of experiments made by him on the Erie Canal:

$$R = \frac{.10303 s v^2}{r - .597}.$$

* Transactions American Society of Civil Engineers, vol. IX. p. 99.

R = resistance in pounds;
s = wetted surface;
v = speed in feet per second;
r = ratio of section of canal to that of boat.

The speed varied from 2 to 3 feet per second, or from 1.27 to 2.09 miles per hour. s varied from 2870 to 3090; r from 4.28 to 5; depth of water from 7 to 8 feet; draft of boat from 6 to 7 feet; displacement from 260 to 310 tons.

The formula is a special case of the general formula proposed by J. Scott Russell and Du Buat for resistance in a restricted channel, viz.,

$$R = \frac{Av^2s}{r + B},$$

where A and B are constants to be determined by experiment. The ranges of values of r, v, and s are manifestly too small to warrant the application of these results to conditions widely different from those covered by the experiments.

Conder * states that in one instance where $r = 3$ a speed of 7 miles in the open was reduced with the same power to 5; while in the discussion of the same paper Gordon states that in the Rangoon, where $r = 10$, a speed of ten miles was similarly reduced to 7.

M. Caméré gives the following results cited by Pollard and Dudebout:†

Let resistance be represented by a formula

$$R = fsv^2,$$

* Minutes of Proceedings of Institution of Civil Engineers, London vol. LXXVI. p. 660.

† Théorie du Navire, vol. III. p. 458.

where the letters have the same significance as above and f is a coefficient.

r	Values of f at Speeds per Second of		
	3'.28	4'.92	6'.56
16.4	.00464	.00513	.00564
9.0	.00836	.00891	.00946
3.45	.0280		

More recently M. de Mas has conducted extensive experiments in France, the results of which have been made public by Derôme.* Among the many interesting features of this investigation the following may be mentioned:

Three boats were tested by towing on the Seine. The ratio of the section of the stream to the immersed section of boat was about 55, for which the results should be, presumably, not far from those for a ratio indefinitely large. The results are given in Table I, from which it appears that the

TABLE I.

Boat.	Length.	Displacement.	Resistance in Pounds at Speeds per Minute of				
			98'.4	196'.8	295'.2	393'.6	492'
Alma..........	124.6	286	119	357	783	1464	2467
René..........	99.4	...	112	353	783	1466	2469
Adrien........	67.4	148	112	353	783	1466	2469

resistance was independent of the length within the limits of speed and length employed. The same three boats were afterward tested in the Bourgogne Canal at speeds

* Proceedings Sixth International Congress on Inland Navigation, 1894. Quoted also by Leslie Robinson, Proceedings Institute Mechanical Engineers, 1897.

increasing by increments of 49.2 feet per minute, from 49.2 to 246 feet per minute, with the same result as regards the substantial independence of resistance on length. The general truth of this proposition must by no means be assumed from these experiments, though there seems reason for believing that, due to the peculiar form of canal-boats, a change of length has relatively small influence on the resistance.

A number of boats with particulars as in Table II were tested in both river and canal, with results as given in Table III.

TABLE II.

PARTICULARS OF BOATS.

Name.	Length.	Average Breadth at Midship Section.	Block Coefficient of Fineness.
Peniche	125.3	16.4	.99
Flûte	122.8	16.5	.94
Toue	118.4	16.5	.97
Prussian	111.9	16.1	.93

TABLE III.

Draft.	Boat.	Cross-section of Canal. A	Area of Midship Section.	Ratio of A to Area of Midship Section.	Speed 98.4 Feet per Min. Resistance. Canal R	River r	$\frac{R}{r}$	Speed 196.8 Feet per Min. Resistance. Canal R	River r	$\frac{R}{r}$
5'.25	Peniche	317.9	86.66	3.67	379	225	1.69	1896	664	2.86
	Flûte	"	86.44	3.68	247	119	2.07	1060	357	2.97
	Toue	"	86.44	3.68	240	97	2.48	1021	278	3.67
4'.27	Flûte	"	70.30	4.52	154	97	1.59	626	315	1.99
	Prussian	"	68.68	4 62	119	49	2.45	474	176	2.69
	Margotat	"	69.98	4.54	117	46	2.52	434	148	2.94
3'.28	Flûte	"	54.04	5.88	106	86	1.23	421	284	1.48

An interesting test was also made in various waterways of the boat Jeanne, 99 feet long, 16.4 feet wide, and of the Flûte class. The results are given in Table IV.

Table IV.

TEST OF JEANNE IN DIFFERENT WATERWAYS.

Draft.	Boat.	Ratio of A to Area of Midship Section.	49′.2 per Min.			98′.4 per Min.			147′.6 per Min.			196′.8 per Min.			246′ per Min.		
			R	r	$\frac{R}{r}$	R	r	$\frac{R}{r}$	R	r	$\frac{R}{r}$	R	r	$\frac{R}{r}$	R	r	$\frac{R}{r}$
3′.28	Seine	116.	26.5	26.5	1.00	28.7	28.7	1.00	146	146	1.00	245	245	1.00	384	384	1.00
	A	8.31	26.5	"	1.00	28.7	"	1.00	150	"	1.03	271	"	1.11	459	"	1.20
	B	5.89	35.3	"	1.33	103.6	"	1.42	227	"	1.56	408	"	1.67	692	"	1.83
	C	4.64	37.5	"	1.41	108.2	"	1.48	238	"	1.63	452	"	1.85	796	"	2.07
	D	3.82	39.7	"	1.50	123.5	"	1.70	284	"	1.95	558	"	2.28	1019	"	2.66
4′.27	Seine	89.3	26.5	"	1.00	83.8	83.8	1.00	172	172	1.00	295	295	1.00	474	474	1.00
	A	6.39	30.9	"	1.17	105.8	"	1.26	238	"	1.38	441	"	1.49	756	"	1.60
	B	4.54	48.5	"	1.74	154.3	"	1.84	342	"	1.98	622	"	2.10	1074	"	2.26
	C	3.57	50.7	"	1.92	178.6	"	2.13	410	"	2.38	814	"	2.75	1501	"	3.17
	D	2.94	70.5	"	2.67	255.7	"	3.05	657	"	3.82	1389	"	4.70	2646	"	5.58
5′.25	Seine	72.5	28.7	28.7	1.00	90.4	90.4	1.00	187	187	1.00	320	320	1.00	511	511	1.00
	A	5.19	39.7	"	1.42	141.1	"	1.56	326	"	1.74	620	"	1.94	1124	"	2.20
	B	3.68	70.5	"	2.46	244.7	"	2.71	562	"	3.00	1047	"	3.28	1839	"	3.59
	C	2.90	72.8	"	2.56	264.6	"	2.93	688	"	3.67	1532	"	4.79	2965	"	5.80

While the information relating to many points of this problem is still quite meager, it is sufficiently well established that where the ratio of section of canal to section of ship is not more than six or eight, a very considerable increase in the resistance will result, even at moderate speeds. Again, mere sectional area of canal is not sufficient, for we might have a very wide and shallow canal in which the ratio of sections would be very great, but in which we should have the shallow-water resistance as already discussed. In order to reduce this increase of resistance to a small or negligible quantity, it seems likely that the transverse dimensions of the channel should be everywhere from six to ten times the corresponding dimensions of the greatest section of the boat.

In this connection it is interesting to note that the influence of fine extremities is less and less useful to reduce resistance as the ratio of section of canal to boat is less. It is readily seen that a point may be reached where the resistance may be less for a somewhat narrow and full form than for an equal displacement and equal length, but with fine ends and a larger midship section, and hence with a greater constriction of available cross-section for stream-line motion.

20. Increase of Resistance due to Slope of Currents.

In ascending rivers where the current is noticeable there must be a corresponding up grade and a resulting vertical component to the movement. The total energy expended in any given time will therefore include, in addition to that necessary to overcome the water-resistance, an amount Db, where D equals displacement and b is the total change in elevation effected during the interval in question. The corresponding resistance is naturally $D \sin \alpha = D\alpha$, where α is the angle of grade and always small. Naturally the amount of this part of the total resistance in such a case is usually small, though it may reach such an amount as to prevent the ascent of a river where the speed of the current is considerably less than the possible speed of the ship in smooth water.

In special cases α may reach a value of from .001 to .0015. At moderate speeds the smooth-water resistance may not be more than $.01D$ or even less, so that it is readily seen that this part of the resistance may reach an amount considerable in comparison with that for such speeds under usual conditions.

21. Influence on Resistance due to Changes of Trim.

In treating of resistance we have thus far considered only the horizontal component. It is quite evident, however, if the ship were to be maintained in her statical trim and draft and towed through the water at any given speed, that the resultant of all the changes of pressure would in general have a vertical as well as a horizontal component. In many cases, especially at high speeds, it becomes important to take cognizance of the existence of this vertical force. Considered as a single vertical resultant, it usually acts through a point forward of the statical center of buoyancy. In the actual case, the ship being free to yield to this force, she will change trim, the bow rising somewhat more than the stern sinks, thus decreasing the displacement by a small amount. At certain high speeds these modifications, especially the change of trim, become very noticeable. At still higher speeds, especially with the cut-up form of after body mentioned in § 11, it would appear that the trim may begin to go back toward its normal value, though the displacement will probably continue to decrease with the increase of speed.

From another point of view we may consider that the motion of the boat, especially at high speeds, gives rise to a wave system, and that the boat naturally accommodates itself in trim to the characteristics of this system. In general at high speeds such system will involve a hollow under or near the stern and a crest near the bow, thus placing the boat on the back slope of the wave and naturally giving rise to the change of trim. An instance is given by White* of measure-

* Manual of Naval Architecture, p. 470.

ments taken by Yarrow on a torpedo-boat about 80 feet long at a speed of 18.5 knots. The change of trim was about one-half inch per foot of length or forty inches between bow and stern. Relative to the water surface forward the bow raised rather more than one foot, while relative to the surface aft it settled less than six inches. The remainder of the change in trim is accounted for by the change in the level of the water itself due to the wave formation.

Like the other phenomena attending the motion of a solid in a liquid, this change of trim and of displacement cannot at present be treated by abstract mathematics. The investigation of the performance of models and such comparative methods are alone of use in aiding to form an estimate of the amount of such change in any given case.

The influence of the change of trim on resistance is probably slight. Directly it is a result and not a cause of resistance. Indirectly it may be a cause of a modification of resistance on account of the difference which it may cause in the form of the immersed body. With usual forms the influence of such slight changes of this character as usually occur is not likely to be important. It is probable that the principal loss arising from a change of trim is due to decreased efficiency in the propulsive apparatus rather than to actual increase in the resistance.

In connection with these considerations it may be pointed out that there is reason to believe that for extremely high speeds relative to length, such as are attained by some torpedo-boats and fast launches, and notably by recent torpedo-boats and torpedo-boat destroyers, the coefficients in the formulæ of resistance which hold for larger ships and for more moderate speeds undergo considerable change. A direct extension of the results for lower speeds to such extreme values is not therefore allowable, and the various constants and characteristics of empirical formulæ for resistance and power must be determined by direct experiment under such conditions, or under others comparable with them. See § 26.

In the boats mentioned speeds of 28 to 31 knots have been attained on lengths of from 180 to 200 feet. The resistance at such speeds is shown to vary with a lower index than for more moderate speeds, as, e.g., 18 to 24 knots, and the performance as a whole seems to indicate a general modification in the relation of the resistance to the attendant conditions.

As causes of this change in relationship we may look to the change in displacement, and consequently in wetted surface, and to the changed location of the wave system relative to the boat.

Photographs of boats at very high speeds show that under such conditions the decrease in wetted surface may be quite considerable, though data are lacking to furnish a satisfactory basis for estimate in any given case.

The change in the resistance of a boat in a canal when it approaches a speed such that it can be forced over on to the top or forward slope of the primary wave has already been mentioned.

It is not to be expected that these conditions can be paralleled in open water; but we know that as the speed increases the location of the bow primary wave falls farther and farther aft, and photographs taken under these conditions as well as the decreased change of trim seem to show that at a speed sufficiently high the boat is forced at least partly over the primary bow wave, so that such wave, once formed, might require a less expenditure for the maintenance of its energy and the speed of the boat than would be indicated by an extension of the law for lower speeds to this extreme point.

It is not likely that this favorable region in the law of resistance can at present be taken advantage of by other than boats of the types mentioned. For larger ships, e.g., of $L = 400$ feet, such speeds would exceed 40 knots, and the power required would be so great that with present engineering resources its development on the weight available and its economical application would be impracticable.

22. Influence of Bilge-keels on Resistance.

Bilge-keels are usually located near the turn of the bilge, and stand normal to the surface rather than vertical. They usually run for two thirds or three quarters the length of the ship, and are supposed to follow as nearly as may be the natural stream line path in order to interpose the minimum resistance. Experiments by Wm. Froude and others have shown that the additional resistance due to bilge-keels is slight. Some of the model experiments by Mr. Froude indicated that the additional resistance due to the bilge-keels was less than that due to their surface friction as usually computed (§ 8). It has been suggested that this may be due to the fact that the keels are in a skin of water moving forward more or less with the ship, and that in consequence the resistance per unit area is less than that normally due to the relative velocity of ship and water as a whole. A more just view is perhaps to consider that with the bilge-keels the average skin-resistance coefficient for the ship as a whole is somewhat less than for the ship without the keels; so that while with the keels the total area is greater, the coefficient is less, and the product is but slightly increased. The reason for the decrease in the average coefficient may be readily seen by comparing two bodies of cylindrical form moving axially through the water. Let one be smooth and the other deeply corrugated or channelled longitudinally, both of the same outside diameter. Both will tend to set in motion a cylinder of water of about the same diameter, and with the channelled cylinder this tendency will be so effective that at and near the bottom of the channels the relative velocity of the surface and the water will be very slight, and hence the resistance small. The average coefficient of surface resistance for the channelled cylinder will therefore be less than for the plain, and while its area is much greater, the total surface resistance will not be increased in a proportional degree. Recent Italian experiments on the model of the Sardegna * showed for speeds of the ship from 16 to 21 knots an increase of resistance from 1 to 5 per cent increasing with speed, or with given power a decrease in speed from practically nothing to a little over 1 per cent. These experiments seemed to indicate a somewhat rapid increase in the resistance due to the bilge-keels from speeds of about 18 knots upward. It may be suggested that this was due in large measure to a change of trim at these speeds and to a disturbance of the stream-line flow, in virtue of which the keels became placed oblique to the line of flow and thus offered a certain amount of head-resistance.

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* Rivista Marittima, Oct. 1895.

23. Air-resistance.

All above-water portions of a ship in motion are, of course, exposed to air-resistance. This from the irregularity of the motion of the air and the great irregularity of the surface exposed is very difficult of computation. Except in rigged ships with a high velocity of the wind, however, this effect is not considered of great importance, and usually no attempt is made to estimate its amount. From data obtained from planes moving in air it appears that the resistance may be expressed by the formula

$$R = fAv^2.$$

In this equation A is the projected area of the surface exposed, the projection being made on a plane perpendicular to the direction of motion. For an unrigged ship this is readily found, being simply the area of an end view of the part of the ship above water. Where the ship is rigged the amount to be taken is very indefinite, and no estimate can be made except by comparative methods, for which, unfortunately, the amount of data available is insufficient. Wm. Froude was led to believe that in the case of the Greyhound the influence of the rigging was about equal to that of the hull. This would depend very much, however, on the nature and amount of rigging, and doubtless on the speed of the wind. The velocity v is to be taken as the longitudinal component of the relative velocity of the wind and ship. Thus in calm v will be the speed of the ship, in a following wind it will be less, and in a head wind more.

The value of the coefficient f rests on experimental determination. The experiments of Wm. Froude and others indicate that with pounds, feet, and seconds as units its value may be taken as about .0017, and with pounds, feet, and knots per hour as about .0048. The experimental values are derived from planes of small or moderate size, and there is considerable uncertainty as to the values suitable for use on large areas. More data on this point are much needed.

In the case of the Greyhound unrigged, with a relative speed $v = 15$ knots the effect produced was measured by 330 pounds. For $v = 10$ knots this would correspond to about 150 pounds. This is about 1.5 per cent of the water-resistance at the same speed. With rigging it was assumed that this might be doubled, so that in a calm the ship would experience an air-resistance of perhaps 3 per cent of her water-resistance. For moderate speeds, or so long as the water-resistance varies sensibly as the square of the speed, we might expect the relation between the air and water resistance to remain sensibly constant. For higher speeds the water-resistance will increase more rapidly than the square of the speed, so that the ratio of air to water resistance will decrease at these speeds.

If instead of calm air we suppose a speed of 10 knots through the water against a head wind of velocity say 30 knots, we should have $v = 40$, and the air-resistance would be some 16 times as much as before, or unrigged about 25 per cent of the water-resistance and rigged about 50 per cent. While these figures may not be exact, they are sufficiently so for illustrative purposes, and show plainly the importance which wind-resistance may assume in extreme cases.

On one hand the constant decrease of sails and rigging on all types of steam-vessels tends to decrease the importance of air-resistance. On the other, the constant effort to increase speed on ocean-liners, war-ships, torpedo-boats, etc., calls increased attention to all causes which in any manner may affect the resistance. Additional experiments in this direction are greatly needed. Such experiments might be carried out by allowing a ship to drift in a fairly smooth sea under the influence of the wind alone, and measuring the velocity of the ship relative to the water and that of the wind relative to the ship. Then from model experiments or other known data relative to the ship her resistance at the speed attained may be known, and this must equal that due to the air moving past the ship with the relative velocity observed. Otherwise a ship might be moored and subjected to the wind, the strain being measured by suitable dynamometric apparatus.

24. Influence of Foul Bottom on Resistance.

There seems no reason for believing that the condition of the bottom of a ship will essentially affect anything but the skin-resistance.

The effect of foul bottom on skin-resistance has been discussed in § 9, and it is again mentioned here simply to make complete the special causes which may affect the total resistance.

25. Speed at which Resistance begins to Rapidly Increase.

The analysis of curves of resistance usually shows that for very low speeds the total resistance varies nearly as the square of the speed, such relation remaining nearly constant until a speed is approached at which the resistance begins rather suddenly to increase more rapidly, the index becoming 3 or even 4 for a time, while at still higher speeds the index may fall again to 2 or sometimes apparently to even less. Such characteristics in the curve of residual resistance are shown in Fig. 41. The speed at which the first rapid increase is located depends chiefly, no doubt, on the wave-making features of the ship; and while it is not expressible with any great accuracy by any simple formula, yet it is sometimes considered that for general purposes it may be taken as proportional to the speed of a wave of length equal to that of the ship. Hence we should have

$$u \sim \sqrt{\frac{gL}{2\pi}} = b\sqrt{L},$$

where b is some constant. With knots and feet as units, b is frequently taken as approximately 1, in which case we have

$$u = \sqrt{L}.$$

This must be considered as giving simply an indication of the locality of a region where the resistance will begin to rapidly increase, rather than the actual location of a definite speed. It must also be remembered that generally the location of this region will be higher as the ratio of length to beam is greater, and as the prismatic coefficient is less.

26. The Law of Comparison or of Kinematic Similitude.

It is the purpose of the law of comparison to furnish for two ships of similar geometrical form but of different size, and moving at different speeds, a relation between the residual resistances in the two cases. This involves, as we know, principally the wave-making or modified stream-line resistance.

This being the purpose, care should be had in noting the fundamental assumptions on which the statement of the law depends.

Consider the stream-lines in an indefinite liquid moving past a body as in § 1. Let the motion throughout these stream-lines be steady. That is, let there be no discontinuity as in the breaking of waves at the surface or the formation of eddies within the body of the liquid. The usual equation for steady motion will then apply, and denoting the total head by k we shall have for any stream-line,

$$z + \frac{v^2}{2g} + \frac{p}{\sigma} = k. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Now suppose that we have a second body similar geometrically to the first but λ times larger in all directions. The second body may then be considered as a magnification of the first in the linear ratio λ. Let it be placed, like the first, in a liquid of indefinite extent moving past it with some velocity nv, such that the stream-line distributions in the two cases are also geometrically similar and in the linear ratio λ. It follows that the entire systems in the two cases are geometrically similar, and may be considered as derived the one from the other by linear expansion or diminution in the ratio λ. For a stream-line in the second system situated similarly to that taken in the first we shall have

λz for z,

nv " v,

and we may put p_1 " p,

and k_1 " k.

Then we have

$$\lambda z + \frac{n^2 v^2}{2g} + \frac{p_1}{\sigma} = k_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Now we may most readily conceive of a stream-line *AC*, Fig. 44, as due to a head from some indefinitely large reser-

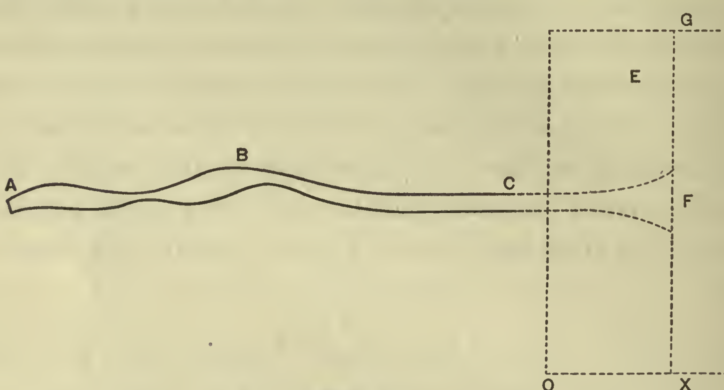

FIG. 44.

voir *E* situated at an indefinite distance away. This, evidently, will in no way change the nature of things at *ABC*. Now suppose the stream-line continued back to a place *F* where the velocity is inappreciable. If the stream-line is indefinitely small *F* will be of small finite area, and we may so consider it as small as we choose, and hence as very small compared with the dimensions of the reservoir. Then if for the present *ABC* represent a stream-line of the first system, equation (1) must hold throughout its length, and we shall have at *F*

$$z + \frac{p}{\sigma} = k. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In this equation let us take z as measured from a base-line *OX*. The pressure p will be simply the statical pressure due to the height *FG*, and $p \div \sigma$ will in feet be equal to this height. Hence $z + p \div \sigma$ is here equal to XG, the total depth of the reservoir to the base-line OX. From (3) this is the value of k. Hence throughout the entire course of the stream-line represented by (1), the constant k represents this depth GX.

Now the second system is a magnification of the first in the linear ratio λ. There will be therefore a similar point F similarly situated in a similar indefinite reservoir whose depth to a similar base-line will be λk, and this will be the value of k_1, the constant for the stream-line in the second system, whose equation is given in (2).

We have thus far simply considered the velocity-ratio n such that the stream-lines in the two systems will be geometrically similar. We must now determine what the value of this ratio will be in terms of the linear ratio λ.

Since the stream-lines are similar throughout, and since the flow is steady and the condition of continuity is fulfilled, we must have the velocities along similar lines exactly proportional at similar points. That is, if v_0, v_1, and V_0, V_1 denote velocities in the first and second systems at pairs of similar points, we must have

$$\frac{V_0}{V_1} = \frac{v_0}{v_1}, \quad \text{or} \quad \frac{V_1}{v_1} = \frac{V_0}{v_0}.$$

This shows that n is a constant, and that if we can determine the velocity-ratio for any pair of similar points the constant value of n will thus be known. The velocity-ratio at a pair of corresponding points is most readily found by aid of Fig. 44, by considering A an open end in each system. We shall then have $p = 0$, and for the first system

$$\frac{v^2}{2g} = k - z,$$

and for the second

$$\frac{n^2v^2}{2g} = \lambda k - \lambda z = \lambda(k - z).$$

Hence, dividing, we have

$$n^2 = \lambda \quad \text{and} \quad n = \sqrt{\lambda}.$$

It follows that in order to fulfil the conditions of similarity throughout the two systems we must have a constant velocity-ratio equal to $\sqrt{\lambda}$. Putting this for n in (2) we have

$$\lambda z + \frac{\lambda v^2}{2g} + \frac{p_1}{\sigma} = \lambda k, \quad . \quad . \quad . \quad . \quad . \quad (4)$$

whence, comparing (1) and (4), $p_1 = \lambda p$.

The same relation will hold for every other stream-line, and it follows that throughout the entire second system the pressure will be λ times that at corresponding points in the first system. In other words, the magnification in pressure will exactly correspond to that in linear dimension. This relation will therefore hold between the pressures at corresponding points on the surfaces of the two bodies, because by supposition the surfaces throughout are in contact with the stream-lines. In the stream-lines of Fig. 1 the body is supposed to be indefinitely immersed, so that no surface changes take place. All of the preceding equations and conclusions hold, however, for any stream-line in a perfect liquid, and therefore for a stream-line system about a body partially immersed, so long as the conditions of continuity of flow are fulfilled. Now it is due to the distribution of pressure throughout such a stream-line system that the stream-line resistance arises. Hence this resistance will be the value of the integral of the longitudinal component of the pressure acting at the inner surfaces of the stream-line system. Denote an element of the surface by ds and the angle between its normal and the longitudinal by θ. Then we have for our first system

$$R_w = \int p \cos \theta \, ds.$$

In the second system the entire distribution of pressure is multiplied by λ, and the linear dimensions are increased in the same ratio. Hence the ratio between corresponding elements of surface will be λ^2, and in the second system we shall have

$$R'_w = \int \lambda p \cos \theta \, \lambda^2 ds = \lambda^3 \int p \cos \theta \, ds = \lambda^3 R_w.$$

That is, with two systems under the conditions we have assumed, the ratio between the resistances due to stream-line pressure is λ^3. We may also note that this is likewise the relation between the volumes of the two immersed bodies, and hence between their displacements.

The law of comparison thus derived involves three considerations, to which careful attention should be given:

(1) The supposition of geometrical similarity;

(2) The definition of corresponding speeds·

(3) The statement of the law.

Geometrical similarity for the two systems being assumed, corresponding speeds are described as those which will produce similar stream-line or similar wave configurations, and are defined as speeds in the ratio of the square root of the linear-dimension ratio λ. Based on these conditions, we then have the following statement of the law:

At corresponding speeds the resistances of similar ships are in the ratio of the cubes of like linear dimensions; or as the cube of the linear-dimension ratio; or as the volume-ratio.

The truth of the law thus derived, it must be remembered, rests on the application of the formula for steady motion to stream-line flow. It is therefore only applicable to that part of the total resistance which is due to the stream-line motion, and hence, so far as the above derivation is concerned, is not applicable to skin or eddy resistance. It also presupposes the following as necessary conditions for its exact truth:

(1) A liquid without viscosity;

(2) The absence of discontinuity of flow such as would be caused by breaking waves or eddies.

So far as the first condition is concerned, the viscosity of water relative to the velocities with which we are concerned is so slight that this departure from the exact conditions will make no essential difference in the application of the law. With regard to the second condition we must distinguish between the effect of the eddies, and that due to breaking waves. Eddies will be formed by possible irregularities in the surface, and as an expression of the general tangential action or skin-resistance. Such eddies may, however, be considered as virtually a part of the ship so far as the transmission of pressure between it and the stream-line system is concerned. The actual stream-line system involved is therefore that which envelops the eddies. If the two ships are geometrically similar, we may evidently assume with safety that the existence of these eddies will not essentially disturb the geometrical similarity of the enveloping systems of stream-lines. Hence while we are not here authorized to apply the law of comparison to eddy and skin resistance, it appears that the existence of the eddies as the expression of these two forms of resistance should not interfere essentially with the application of the law to the systems of stream-lines actually formed. The breaking of a wave involves a discontinuity of stream-line flow, and in such case we are not strictly entitled to extend the law to the stream-line or wave resistance. Within the limits prescribed by this condition, however, we may consider the law as satisfactorily established. We may also consider it as highly probable that even with breaking waves the general configurations in the two systems will, at corresponding speeds, still be essentially similar, and that the resistances involved will still essentially follow the same law of comparison. It must be remembered, however, that while such may be the case, its verification will depend on experiment rather than on mathematical reasoning.

It is possible to view the question of the law of comparison from an entirely different standpoint, and thus to extend the scope of its possible application. It will be noted that the derivation above given establishes the law under certain limited conditions. It does not follow, however, that the same law may not be applicable under wider conditions. To this question we now turn our attention.

Assume a series of ships all of similar geometrical form, but diverse in dimension. Assume that the resistances of such ships as a family at varying speeds may be represented by a general equation consisting of a series of terms all of the form

$$Al^{\left(3-\frac{n}{2}\right)}v^n.$$

We shall then have

$$R = \Sigma Al^{\left(3-\frac{n}{2}\right)}v^n. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In this equation l is some typical dimension, necessarily the same throughout the series, v is the speed, and A is one of a series of constants depending on form characteristics, and hence constant throughout the series. Such a general equation might therefore consist of a series of terms, as for example

$$R = Al^{5/2}v + Bl^2v^2 + Cl^{3/2}v^3 + Dlv^4, \quad . \quad . \quad (6)$$

in which n was given successively the values 1, 2, 3, 4. It is by no means necessary that n should be integral, so that terms of the form $El^{1.4}v^{3.2}$, $Fl^{1.6}v^{2.8}$, etc., may occur. In fact, so far as we are at present concerned, n may have any value whole or fractional, and the number of terms may be indefinite.

In regard to the limits for the values of these exponents we note that if $n = 0$,

$$3 - \frac{n}{2} = 3;$$

if $n = 6$,

$$3 - \frac{n}{2} = 0.$$

The first would give a term independent of speed and dependent only on size, and varying directly as volume or weight. This seems hardly likely, and we may consider $n = 0$ as the lower limit of values of n. The upper limit $n = 6$ gives a term independent of dimension and depending wholly on speed. As indicated elsewhere, it has been thought that such a term is probable, or at least possible. In any event, however, it is evidently the upper limit for n unless we admit the possibility of a term involving a decrease of resistance with increase of size, which seems quite unlikely. Hence n may presumably have any value between 0 and 6, whole or fractional.

Taking therefore (6) simply as an illustration of the kind of terms to be found in such an equation, we note again that A, B, C, etc., are form constants, and remain unchanged throughout the particular family of ships. We assume then that it may be possible to express the resistance of any ship of this family at any speed by substituting in the appropriate equation using the form constants A, B, C, etc., and the given values of l and v. For any other type of form there will naturally be another series of form constants with their series of values of exponents for l and v, not necessarily the same as in the equation for the first type.

We see also that for the same ship at varying speeds this equation assumes the possibility of representing the resistance as a summation of terms involving the variable v with various exponents, whole or fractional; while for different ships at the same speed we should have the resistance expressed as a sum of terms involving the variable l with another set of exponents, whole or fractional.

A little thought will show that these assumptions are more elastic than those involved in the derivation of the law from the equations of hydrodynamics. They admit a wide range of variation, and an indefinite number of terms in the expression for R, binding the exponents of l and v only to the one condition that the exponent of l plus one half that of v shall equal 3. Taking (5) therefore as the symbolic equation let us compare by its means the resistances of two ships similar in form with a linear-dimension ratio λ, and at speeds in the ratio $\sqrt{\lambda}$. We have then, using subscripts 1 and 2,

$$l_2 = \lambda l_1,$$

$$v_2 = \sqrt{\lambda} v_1,$$

$$R_1 = \Sigma A L_1^{3-\frac{n}{2}} v_1^n,$$

$$R_2 = \Sigma A (\lambda L_1)^{3-\frac{n}{2}} (\sqrt{\lambda} v_1)^n = \lambda \Sigma^3 A L_1^{3-\frac{n}{2}} v_1^n,$$

or $$R_2 = \lambda^3 R_1.$$

Hence for any resistance which follows the law as expressed in (5) the law of comparison as above stated will hold. Furthermore, the elasticity of equation (5) makes it quite possible or even likely that it may properly represent a larger portion of the total resistance than that due directly to the stream-line or wave-formation, and hence that the law of comparison may be applicable to a somewhat greater part of the total resistance than that to which the first mode of derivation properly makes it.

Eddy-resistance, for example, is usually considered as varying with some area, and as the square of the speed, § 5, or as represented by a term Bl^2v^2. This is seen to fall into place as one of the general series of terms in (5).

Next as to skin-resistance. This is represented by a term of the form fAv^n, § 7, where f is a coefficient which may decrease with length, A is area, and n is an exponent usually less than 2. This term is seen to violate the conditions for members of (5), and the amount by which n is less than 2 and the amount of decrease of f with increase of l and hence of A, furnish together a general index of the amount by which skin-resistance is not amenable to the law of comparison. With smooth bottom and the values derived from

Froude's experiments, the difference between the values of the resistance as computed and as derived by comparison may be considerable. To take an illustrative case, suppose $l_2 \div l_1 = 4$, and hence $v_2 \div v_1 = 2$. Also, let the values of f be .01 and .0093, and the exponent $n = 1.85$. Then the ratio between the resistance by comparison will be 64, while by computation it will be

$$\frac{A_2 f_2}{A_1 f_1}\left(\frac{v_2}{v_1}\right)^{1.85} = 16 \times .93 \times 2^{1.85} = 53.64.$$

The ratio of these is .84, showing that by formula the value is 16 per cent less than by comparison. If the skin-resistance were about one half the total, this would involve a difference of about 8 per cent of the total resistance.

In many cases, however, especially with rough bottom, the value of the exponent is nearly or quite 2, and the decrease in f is very small. In such cases this term would likewise fall practically under the general law with the others.

The question of the form of the terms expressing wave-resistance has been discussed in § 14. If we take the two terms Alv^4 and Bv^6, we find that each comes into place as one of the terms of (5), and hence both fulfil the law of comparison.

We will now give a series of equivalent expressions for the velocity and resistance ratios for two similar ships at corresponding speeds. We have

$$\frac{v_2}{v_1} = \lambda^{\frac{1}{2}} = \left(\frac{l_2}{l_1}\right)^{\frac{1}{2}} = \left(\frac{A_2}{A_1}\right)^{\frac{1}{4}} = \left(\frac{v_2}{v_1}\right)^{\frac{1}{6}} = \left(\frac{D_2}{D_1}\right)^{\frac{1}{6}}, \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$\frac{R_2}{R_1} = \lambda^3 = \left(\frac{l_2}{l_1}\right)^3 = \left(\frac{A_2}{A_1}\right)^{\frac{3}{2}} = \frac{V_2}{V_1} = \frac{D_2}{D_1} = \left(\frac{l_2 v_2}{l_1 v_1}\right)^2 = \frac{l_2}{l_1}\left(\frac{v_2}{v_1}\right)^4$$

$$= \frac{A_2}{A_1}\left(\frac{v_2}{v_1}\right)^2 = \left(\frac{l_2}{l_1}\right)^{\frac{3}{2}}\left(\frac{v_2}{v_1}\right)^3 = \left(\frac{D_2}{D_1}\right)^{\frac{1}{2}}\left(\frac{v_2}{v_1}\right)^3 = \left(\frac{v_2}{v_1}\right)^6. \quad . \quad . \quad . \quad (8)$$

Any and all of these, and others which may be written similarly, are equivalent forms of the ratios involved in the law of comparison for resistance.

The actual confidence felt in the application of this law, either to the total resistance or to the residual resistance, must be considered as resting rather on the result of experiment than on any abstract proof necessarily involving assumptions not perfectly fulfilled in the actual case. This agreement may also be considered as indicating the degree of closeness with which we may consider either the residual or total resistance as capable of expression by the sum of a series of terms as symbolized in (5).

As direct experimental investigation relating to the law of comparison, we may mention the experiments referred to in greater detail in § 30, in which in two separate cases the resistances of a model and of its corresponding ship when compared according to this law were found to be in very satisfactory agreement, the greatest errors being within 3 per cent.

In this connection we may also note the experimental determinations made by Wm. and R. E. Froude on the wave-configurations made by similar forms moving at corresponding speeds.

The wave systems due to a model of the Greyhound and to the boat herself at corresponding speeds were compared by Wm. Froude. Their similarity was found striking even to details of the bow wave system. It was also found for the model that up to a speed of about 1.6 feet per second the resistance-curve followed almost exactly the curve of skin-resistance as computed by formula. It was a matter of observation that up to this speed the model moved without raising sensible waves. At higher speeds the total resistance increased somewhat rapidly over that due to skin-resistance alone, and seemed to follow the plainly visible augmentation in the size of the wave system. Mr. R. E. Froude also mapped with care the wave-configuration about two models, one four times the size of the other, the larger travelling at twice the speed of the smaller. The smaller represented a launch 83 feet long at 9 knots per hour, and the larger a ship 333 feet long at 18 knots per hour. The configurations thus resulting have been already referred to and are shown in Figs. 32 and 33, and their great similarity in position and relation to the ship is strikingly seen. In Fig. 33 is also shown the system for the smaller model at the same speed as the larger, so that we have here the comparison of two wave systems made by different similar ships at the same speed. The general similarity of distribution relative to the water is quite apparent.

The first comparison indicates therefore that similar ships at corresponding speeds will produce similar wave-configurations relative each to the ship; while the second indicates that similar ships at the *same* speed will produce approximately similar wave-configurations relative to the water.

This latter consideration may lead to the suggestion that the wave pattern for a given type of ship is in large measure independent of the size of the ship, and simply dependent on character of form and on speed. Hence the energy involved in the waves would be likewise dependent only on the same functions, and likewise the resistance. It is these considerations pushed to their full limit which gives Bv^6 as the form of the term for wave-resistance in § 14. In one important feature, however, the wave pattern cannot be independent of the size of the ship. This feature is the composite stern system which in general depends on the relative positions of the components due to the primary bow and stern waves, and this in turn depends on the relation between the wave-making length and the speed. Hence in part, at least, wave-making resistance must depend on the dimensions, and its proper expression may perhaps involve terms in both v^4 and v^6, as already suggested in § 14.

27. Application of the Law of Comparison.

Case I. Denote two similar ships by S_1 and S_2. If the law of comparison is assumed to apply to the entire resistance, we have to form simple proportions as expressed by any of the various forms in § 26 (7) and (8). The given information must necessarily involve three terms:

(1) A given ship S_1 at a given speed;

(2) The resistance R_1 at this speed;

(3) A similar ship S_2 at a corresponding speed.

The fourth term, R_2, is then readily found from the proportions in § 26 (8).

Let Fig. 45 be a graphical representation of the resistance of a ship at varying speeds. We now wish to show that the same curve may also be considered as a diagram of resistance for all similar ships, the speed and resistance scales being suitably modified.

To this end we see that any ordinate as AB gives the resistance R_1 for the speed $v = OA$. Suppose now we have a ship λ times as large in linear dimension. Then at a speed $\sqrt{\lambda}v$ the resistance will be $\lambda^3 R_1$. Hence if the scales are so changed that OA to the new scale denotes $\sqrt{\lambda}v$, and AB denotes $\lambda^3 R_1$, the one will properly correspond to the other; and the same relation holding for other points, we shall have to the new scales a diagram of the resistance of the new ship

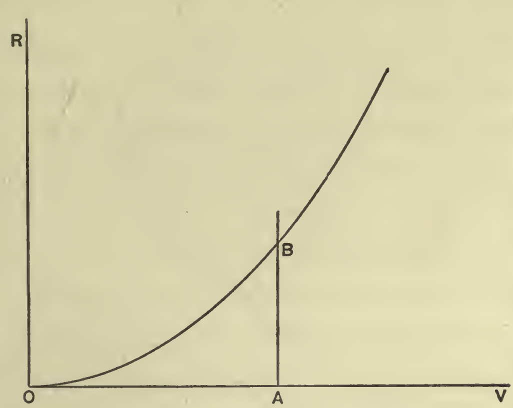

FIG. 45.

at varying speeds. We note that to effect this change the linear amount representing unit speed must be divided by $\lambda^{\frac{1}{2}}$, and the linear amount representing unit of resistance must be divided by λ^3. A curve of this character, therefore, by the proper treatment of the scales will represent graphically for this type or family, the resistance of any ship at any speed, within the limits determined by the original data and by the modification of the scales.

CASE II. Let OP_1, Fig. 46, denote the curve of total resistance for a given ship S_1 at varying speeds. Let the skin-resistance be computed according to the methods of § 8. and set off from OP_1 downward at the various speeds as CB at A. This will give a curve OQ as the graphical representation of the residual resistance, while the intercept between OQ and OP_1 will show the value of the skin-resistance. The residual resistance as given by OQ is then treated directly by the law of comparison, either by proportion, or by a change of scales, as just explained. On the latter supposition OQ may also be considered as representing to appropriate scales

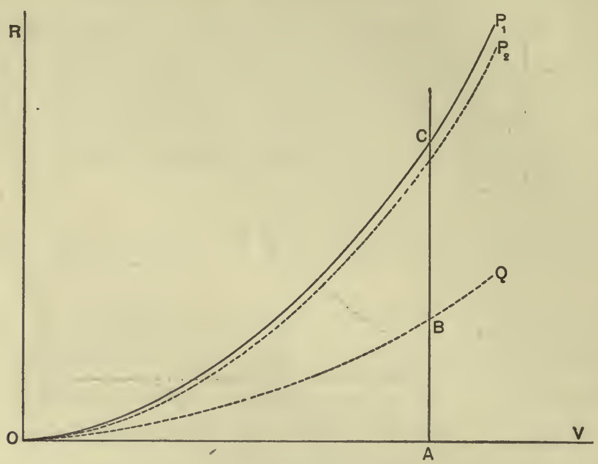

FIG. 46.

the residual resistance of the second ship S_2 at varying speeds. The residual resistance of S_2 being thus found, the frictional resistance is computed as in § 8 by an appropriate selection of the constants. The two being added the entire resistance is thus determined. For graphically representing the entire resistance of S_2 we may proceed as follows: The values of the skin-resistance of S_2 having been computed, find ordinates for its representation to a scale λ^3 times as large as that used for S_1. Lay these off from OQ, using a speed scale $\sqrt{\lambda}$ times as large as that for S_1. The result will be a curve OP_2, representing to these changed scales the total resistance of S_2. If instead of the ship S_1 we have a model, the principles and operations are exactly similar, and we may thus consider the above as a general explanation of the method of connecting the results of a model experiment with the ship which it represents.

28. General Remarks on the Theories of Resistance.

From the preceding sections treating on the resistance of ships, it is quite evident that pure theory is quite unable to furnish the indications necessary to the solution of any given problem. The true function of theory as at present existing must be considered as that of providing a means to a systematic study of the phenomena attending the motion of a ship-formed body through the water, and of establishing the basis for an intelligent comparison of the results of one vessel with those of another, in order that experimental data may be made available for purposes of prediction and design.

It may be here remarked that the so-called "form of least resistance" in its abstract sense has for the designer only an indirect interest. It is of course true for any one condition of displacement, character of surface, state of the liquid, and speed, that there must be some form of least resistance, or at least some form than which none can offer less resistance. Such form, however, will doubtless change with every change in the four fundamental characteristics above, and with the question of resistance must always be associated those of safety, strength, and carrying capacity, and often adaptation to special conditions. The purpose is not therefore in any given case to produce a form of least resistance as such, but rather a form which shall the most economically combine the several qualifications in the particular proportion called for by the circumstances of the problem.

29. Actual Formulæ for Resistance.

Of the many formulæ which have been proposed for the direct computation of resistance we shall mention but few. In practical application it is usually power rather than resistance which is computed, though with the necessary assumptions, of course, the one may be derived from the other. In Chapter V the question of power will be discussed, and various additional formulæ and methods will be given for its determination, from which, by inverse processes, the resistance might be determined if desired.

We have first

$$R = \frac{aD^{2/3}v^2}{7k}, \quad \ldots \ldots \quad (1)$$

where D is displacement in tons;

v is speed in knots;

R is resistance in tons;

a and k are two variable coefficients—the coefficient of propulsion and the admiralty coefficient.

These coefficients are explained in § 46 and § 62, and, as is readily seen, the formula is simply derived from the so-called admiralty formula for power, and except where used with the law of comparison, as explained in § 62, its results are only to be considered as roughly approximate.

Middendorf * has deduced from numerous experiments on screw-steamers the following formula, of which the first term expresses the residual and the second term the skin resistance:

$$R = \frac{.428BMv^{2.5}}{\sqrt{B^2 + eL^2}} + .0096Sv^2. \quad \ldots \quad (2)$$

Where the surface is quite smooth, the exponent of v in the second term may be made 1.85 instead of 2.

* Büsley, Die Schiffsmaschine, 1886, vol. II. p. 579.

The units are pounds, feet, and knots. B is beam; L, length; M, area of midship section; S, wetted surface; while e is a coefficient to be taken from the following table, using the prismatic coefficient as argument:

p	e	p	e
.70 and under..	2	.81	1.50
.71	1.99	.82	1.42
.72	1.98	.83	1.32
.73	1.96	.84	1.18
.74	1.93	.85	1.06
.75	1.89	.86	.90
.76	1.85	.87	.74
.77	1.81	.88	.55
.78	1.75	.89	.31
.79	1.69	.90	.02
.80	1.62		

Büsley considers that this formula may be expected to give good results except for very long, fine ships, in which case the values are somewhat too large.

Taylor* suggests for speeds up to that for which v^2 in knots divided by the wave-making length is not more than about 1.2, the following:

$$R = fsv^{1.83} + \frac{12.5bDv^4}{L^2}, \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where the first term gives the skin resistance and the second the residual or wave-making. The units are pounds for R, tons for D, and feet and knots for the other quantities. D is

* Transactions Society of Naval Architects and Marine Engineers, vol. II. p. 143.

displacemnt; L, length; s, surface; f, the coefficient of skin-resistance as discussed in §§ 8, 9; and b the block coefficient.

In this connection we may also mention Calvert's* formula for the resistance of sections of propeller-blades or bodies of similar form moving at a slight obliquity to the face.

Let v = speed in feet per second;
B = breadth in feet;
D = depth in feet;
θ = angle of inclination;
m = an exponent depending on θ, as indicated by the following table:

θ	m	θ	m
5°	.20	50°	.77
10°	.35	60°	.84
20°	.50	70°	.90
30°	.60	80°	.96
40°	.69	90°	1.00

We then have

$$R = 6V^{1.85}B^{m}D \sin \theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

30. Experimental Methods of Determining the Resistance of Ship-formed Bodies.

Experiments on resistance have been made in a few cases on actual vessels. For the most part, however, such experiments have been limited to models usually from 8 to 15 feet in length.

Where the actual vessels have been used, they have been

* Transactions Institute of Naval Architects, vol. XXVIII. p. 303.

towed either from a boom or off the quarter of the towing ship, in order to bring them clear of the wave system formed by the latter. The tow-rope strain was then measured by a suitable dynamometer and the speed by specially constructed and carefully rated propeller logs. In this way tests were made by Wm. Froude * on the Greyhound at a time when tests of models were generally considered of little value. The comparison of the data thus derived with that given by a model of the same ship reduced in the ratio 1 : 16 showed a very satisfactory agreement throughout, and did much to establish confidence in model experiments, which from that time have been regularly carried on in England and on the Continent.

A later comparison of the resistance of an actual vessel with that of the model was made by Yarrow in 1883 † between a torpedo-boat and its model. The range of speeds covered by the comparison was up to 15 knots. Here also there was virtual agreement, the actual values being some 3 per cent greater than those determined by means of the model.

These and other results, as well as the general agreement (when special disturbing causes are eliminated) found between predictions based on model experiments and actual results, have all led to a high degree of confidence in the value and reliability of model experiments for the determination of the resistance to be expected in full-sized ships.

The models are usually of paraffine, shaped on a special machine in which the block cast roughly to shape moves under rapidly-revolving cutters which are moved by a panta-

* Naval Science, vol. III. p. 240.

† Transactions Institute of Naval Architects, vol. XXIV. p. 111.

graphic connection with a tracing-point, the latter being carried by the operator along the water-lines of the vessel whose form is to be reproduced. In this way successive water-lines are traced out at the proper successive heights on the surface of the future model, the cross-sections at the end of the operation resembling series of steps, the inner corners of which are on the surface desired. The outer corners are then worked off smoothly by hand-tools until the inner angle of the path cut by the tool barely remains. The surface is then smoothed and polished, and is ready for the trials.

The necessary equipment consists of a tank or canal some 20 to 25 feet wide, 10 or 12 feet deep, and from 300 to 400 feet long. A truck running on rails spans this canal and carries the necessary apparatus for holding the model and measuring the pull at the various speeds attained. The latter are measured by electrical contacts at known distances on the track, and also through the revolutions of the truck-wheels. All the elements of the entire determination, including seconds of time, dynamometer pulls, changes of trim and draft, are automatically registered on a drum driven by connection with the truck-wheels. The models are taken hold of in such way that they are free to assume whatever draft and trim may suit the conditions from moment to moment. If desired also, special measurements by photography and otherwise may be made of the wave systems produced.

For investigating the effect of the propeller on resistance a special truck is provided carrying the propeller connected with driving-power in such way that it may be driven at any desired number of revolutions per minute. The propeller thus driven and carried entirely independent of the model is then brought up into its proper position astern of the model, and the revolutions adjusted until the total thrust developed is equal to the modified resistance of the model. The conditions are then similar to those which would exist were the propeller by means of this thrust driving the model at the same speed. A comparison of the resistance thus found with its value without the propeller shows the increase due to the presence of the latter.

With the same apparatus the performance of model propellers may be investigated, and thus most valuable information bearing on the design of full-sized propellers may be determined. To these matters we shall refer later in § 49.

31. Oblique Resistance.

Euler's Theory.—Let the resistance for motion longitudinally ahead be denoted by $R_1 = A_1v^2$, and that for motion transversely by $R_2 = A_2v^2$. Then if the actual motion of a vessel is with a velocity u in a direction making an angle θ with the longitudinal, the components of the velocity longitudinally and transversely will be $u \cos \theta$ and $u \sin \theta$. Euler then assumed that the corresponding components of the total resistance will be $A_1u^2 \cos^2 \theta$ and $A_2u^2 \sin^2 \theta$, the resulting total resistance being

$$R = u^2 \sqrt{A_1^2 \cos^4 \theta + A_2^2 \sin^4 \theta}, \quad . \quad . \quad . \quad (1)$$

and the tangent of its direction with the longitudinal

$$\tan \alpha = \frac{A_2 \sin^2 \theta}{A_1 \cos^2 \theta} = \frac{A_2}{A_1} \tan^2 \theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

These equations can only be considered as giving a rough approximation to the actual values. They serve, however, to introduce the questions of principal interest which are the direction and point of application of this total force R.

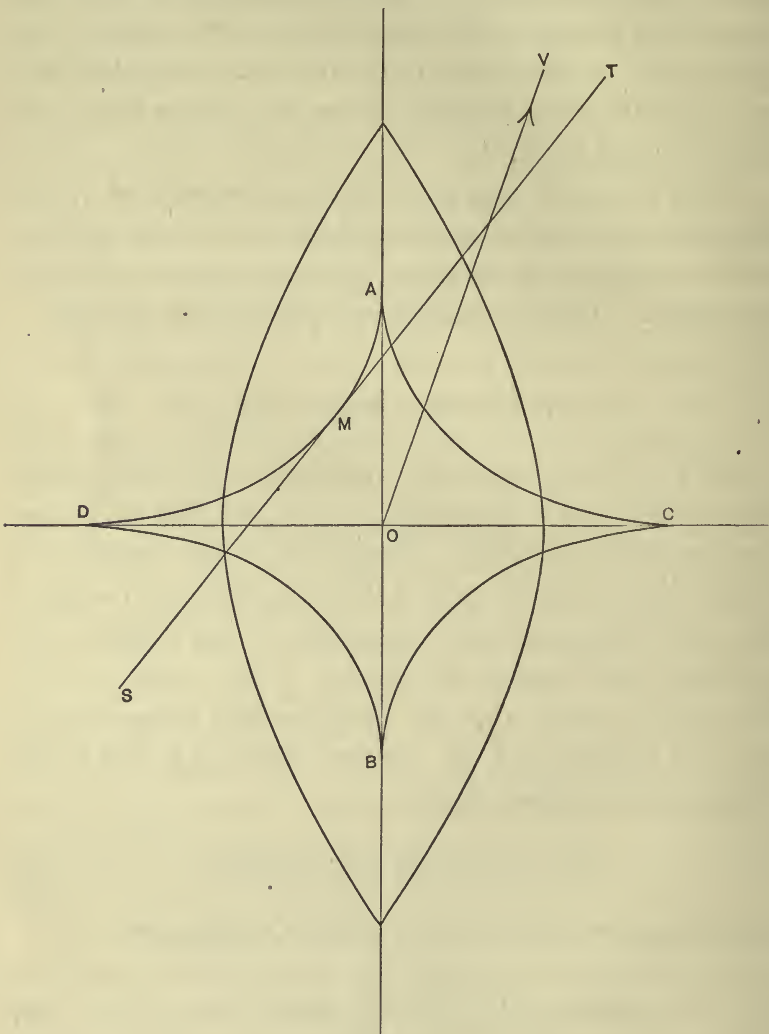

FIG. 47.

These have relation to stability of route when the vessel is moved obliquely as by a tow-line. In Fig. 47, for a certain direction OV and speed u, let TS be the direction of R, and hence ST the direction of the applied towing force. Let M be the location of the point of application of the resistance. The tow-line must be attached somewhere in the line ST. If the point of application is toward T from M, it is readily seen that the couple developed by any slight yaw or deviation from the given direction OV would give rise to a moment tending to return the ship to this direction, and to the fulfilment of the conditions originally assumed. Under these circumstances, then, there will be stability of route. If, on the other hand, the tow-line were attached beyond M toward S, the moment due to a yaw would tend to still further increase the divergence, and there would hence be instability of route, with a tendency on the part of the vessel to swing about and go with the port side forward. For each value of the direction OV there will evidently be a point M, and the collection of these for all possible directions will give a locus, as shown in the figure. There is very little experimental data relating to the form and characteristics of this locus. Certain general conclusions, however, may be reached as follows:

It is evident that the line ST must touch the locus at M. Also, since physically there will be but one such point for any given position of OV, it follows that ST can touch the locus but once, and hence will be tangent to it at M. Hence the locus will be the envelope of the series of lines ST. From the results of Jöessel relative to the location of the centre of resistance for planes, § 6, it seems reasonable to assume that the center for longitudinal motion will be at some point A, and that as the angle is varied it will somewhat rapidly fall aft on either side, thus giving a cusp at this point. Similarly there would be like cusps at B, the center for motion astern, and at C and D, the centers for motion nearly transversely, the exact angle depending on the amount of difference between the fore and after bodies of the ship. We then assume the remainder of the locus to be as indicated in the diagram. If therefore any point of attachment on the ship be taken for the tow-rope, the angle between the latter and the keel for stability of route will be determined by drawing a tangent from this point to the envelope. There will be two or four such tangents in general, according as the point is within or without the envelope. Observation will show, however, which one is applicable to any given case. This determines α, the angle between the keel and the tow-rope. In regard to θ, the inclination of the direction of motion, there seems physical reason to believe that the latter will always lie between the keel and the direction of the tow-rope.

This does not agree with the indications of (2) for very small values of θ, but we may well doubt the exactness of this equation at the limit where one of the component velocities is very small. We may expect, therefore, that the relation between α and θ will be somewhat as indicated in Fig. 48, where values of $(\alpha - \theta)$ and α are plotted in rectangular coördinates. The value of $(\alpha - \theta)$ is $(+)$ between O and A, or for α approximately between 0 and 90°, and $(-)$ for α between A and B. For $\alpha = 0$, 180° and some angle near 90°, α and θ will coincide and $(\alpha - \theta)$ will be 0, as indicated. If, therefore, the information expressed by these two diagrams is at hand, any problem involving the relation between the point of attachment of a tow-rope, its direction for stability of route, and the direction of the motion is readily solved. Unfortunately we have no such accurate knowledge for representative cases, and in lieu the problem must practically be solved by trial and error. Guyou has

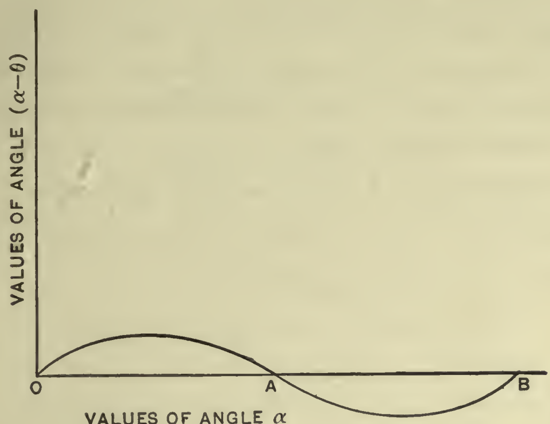

FIG. 48.

determined by experiment for a launch that the point A lay about 11 per cent of the length forward of the center.

The character of the two diagrams above will depend on the ship fundamentally, and in the second place on the following conditions:

(1) *Trim.*—The greater the trim by the stern the greater the difference between the forward and after bodies, and the greater the difference between the branches AC and CB of Fig. 47. The farther aft as a whole, relative to the ship, also, will the envelope be located.

(2) *Speed.*—It is found by experiment that when v increases, α increases slightly. This indicates that the transverse component of resistance increases more rapidly than the longitudinal. The value of α is not therefore independent of speed, as in Euler's equations.

(3) *Helm.*—The position of the rudder is capable of

changing the whole character of the envelope of Fig. 47 by introducing an additional oblique force at the stern and by modifying the stream-line motion. It follows that within certain limits the relative and absolute values of θ and α may be varied at pleasure by the appropriate use of the helm.

The manœuvring of ships when moored by single cable or in a bridle, or when towed by single line or bridle, are familiar illustrations of the practical application of these principles.

CHAPTER II.

PROPULSION.

32. General Statement of the Problem.

The fundamental problem of propulsion is to find a thrust whereby we may overcome the resistance which the ship meets when moving through the water. Leaving aside propulsion by sails, the water about the ship presents itself as the only medium by means of which this necessary thrust can be developed.

Now water is a yielding medium, and the conditions are entirely different from those encountered in pushing a boat in shallow water by means of a pole, where a firm and unyielding hold is obtained on the bottom. With water or any such fluid medium the development of a thrust must depend fundamentally on the utilization of its inertia and viscid forces. We may view the production of a thrust from two standpoints:

(*a*) We know that the production of a change of momentum requires the action of a force and the expenditure of energy, and that conversely the matter acted on will react on the agent producing the change of momentum, such reaction being in fact the resistance opposed to the change of momentum. If therefore we provide an agent attached to the ship which shall produce a change of momentum in matter of any kind, such change of momentum being directed astern, or at least having a sternward component, there will result on the agent a reaction having a forward component, such reaction being then available as a propulsive thrust. The fundamental conditions necessary are, therefore, that a change of momentum must be produced in matter by an agent attached to the ship, and that such change must have a sternward component. In the usual case water is the matter acted on, and that in which the change of momentum is produced. A propulsive thrust would, however, be given by a gun firing projectiles over the stern or by a boy throwing apples or stones in the same direction, as well as by the means found more useful for actual propulsion.

We now turn to the second method of viewing a thrust.

(*b*) We have seen in Chapter I that no body can be moved through a liquid without experiencing a resistance. Let us then consider the pair of bodies made of the ship and a propeller of some kind, and let us attempt to produce relative motion by so moving the latter relative to the former that its motion shall have a sternward component. Such motion will meet with a resistance which, as we know, is simply the resultant of the distributed system of pressures and tangential forces acting on the surface of the moving body. This resistance will react along the line of relative motion, and will therefore have a forward component, and may hence be made to yield a propulsive thrust. It results that the resistance of the ship and that of the propeller, both taken along the line of motion of the former, or what is the same thing, along the line of motion of the system as a whole, must be equal. We shall next examine geometrically the conditions attendant on this second mode of view.

33. Action of a Propulsive Element.

Let *SS*, Fig. 49, represent the ship and *C* the element, the latter being simply any body so connected with the ship that its relative motion is along a line *AB* with a velocity v. The line *AB* is not necessarily horizontal, but may have any

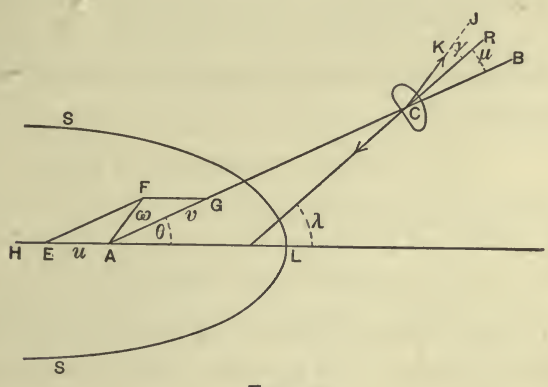

Fig. 49.

direction in space, and is simply defined as making an angle θ with the longitudinal *HL*. As a result of this motion of *C* let the ship move with a velocity u along *LH*. In the usual way we determine $AF = w$ as the direction and amount of motion of *C* relative to the surrounding body of still water. This we lay off at *CK*, remembering that its direction in space will be determined by *LH* and *AB*. While the actual motion of *C* is thus along *CJ*, let its form be such that the resultant of the system of distributed forces acting on its surface is along some line in space *CR*, making an angle γ with *CJ*. Denote the inclination of *CR* to *AB* by μ and to the longitudinal by λ, and denote the value of this resultant by *R*. For simplicity, Fig. 49 is drawn as though all the lines were in the horizontal plane. As noted above, such is not necessarily the case, but with the angles λ, μ, γ, and θ defined as above, the statement of the configuration is entirely general.

The resistance to the relative motion of C along AB will evidently be $R \cos \mu$, and for the total work performed by the propelling power we shall have

$$W = R \cos \mu \,.\, v = Rv \cos \mu. \quad . \quad . \quad . \quad . \quad (1)$$

For the available thrust we shall have the longitudinal component of R,

$$T = R \cos \lambda = \text{resistance of ship}; \quad . \quad . \quad (2)$$

and for the work done on the ship,

$$W_1 = R \cos \lambda \,.\, u = Ru \cos \lambda. \quad . \quad . \quad . \quad . \quad (3)$$

The resistance along the line of motion of C relative to still water is $R \cos \gamma$, and for the work thus expended we have

$$W_2 = R \cos \gamma \,.\, w = Rw \cos \gamma. \quad . \quad . \quad . \quad . \quad (4)$$

We must also have

$$W = W_1 + W_2,$$

or

$$v \cos \mu = u \cos \lambda + w \cos \gamma. \quad . \quad . \quad . \quad (5)$$

This last result may also be derived directly from the triangle AEF.

Definition of Efficiency.—In the remaining chapters of this work we shall have frequent occasion to use the term *efficiency.* In its general sense it is simply the ratio of returns to total investment; or of the useful result received, to the total expense necessary to its attainment. The fundamental definition must be carefully kept in mind, for in some cases it is not easy to say what is the useful return, or what the total expense.

In the present case there is no ambiguity, and we have for the efficiency

$$e = \frac{W_1}{W} = \frac{u \cos \lambda}{v \cos \mu} = \frac{u \cos \lambda}{u \cos \lambda + w \cos \gamma}. \quad . \quad . \quad (6)$$

We may note that the three velocties $v \cos \mu$, $u \cos \lambda$, and $w \cos \gamma$ are the projections of the velocities v, u, and w on the direction of total resistance CR, and that the efficiency is simply the ratio of the two projections of u and v along this line.

As a particular case, let the foregoing equations relate to an element of the surface of C instead of the whole body, or otherwise we may consider the element as a small plane. This is represented in Fig. 50. In this diagram, as before, we

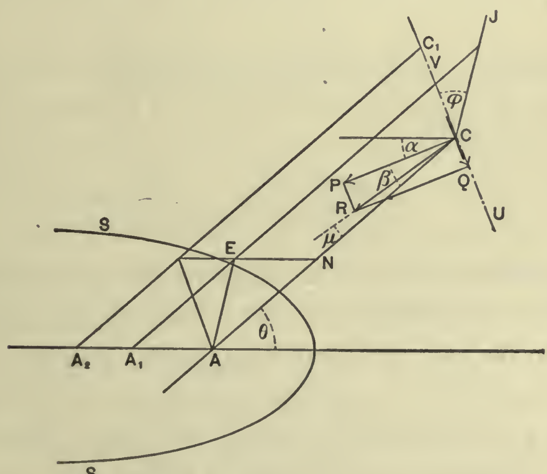

FIG. 50.

assume that the lines are not necessarily in one plane. The total force R may here be represented by its two components

P and Q normal and tangential to the plane. Denote the inclination of P to the longitudinal by α and to the line AC by β. Then the inclinations of Q to the same lines are $90 + \alpha$ and $90 - \beta$, respectively. We have then for the thrust

$$T = P \cos \alpha - Q \sin \alpha, \quad . \quad . \quad . \quad . \qquad (7)$$

and for the component of R along AC

$$R \cos \mu = P \cos \beta + Q \sin \beta. \quad . \quad . \quad . \quad . \qquad (8)$$

Hence for the useful work

$$W_1 = (P \cos \alpha - Q \sin \alpha)u, \quad . \quad . \quad . \quad . \qquad (9)$$

and for the total work

$$W = (P \cos \beta + Q \sin \beta)v; \quad . \quad . \quad . \quad . \qquad (10)$$

whence
$$e = \frac{(P \cos \alpha - Q \sin \alpha)u}{(P \cos \beta + Q \sin \beta)v}. \quad . \quad . \quad . \qquad (11)$$

If $Q = 0$, we have

$$e = \frac{u \cos \alpha}{v \cos \beta} = \frac{u}{v \cos \beta \sec \alpha}. \quad . \quad . \quad . \qquad (12)$$

Suppose now the element supported on a smooth unyielding surface as indicated by UV. Then when it has moved relative to the ship the same distance AN as before, the ship will have moved to A_2 and the element to C_1. In such case the only work expended is that on the ship. Hence e must equal 1. Hence in (12) v, α, and β remain the same, while u must equal $v \cos \beta \sec \alpha$. Or in other words, $v \cos \beta \sec \alpha$ is the speed which the ship would have if C were supported on the smooth unyielding surface. This is readily seen geometrically if the lines of Fig. 50 are considered in one plane; otherwise an application of spherical trigonometry is necessary to a geometrical proof.

The difference between the velocity $v \cos \beta \sec \alpha$ and the actual velocity u is called the slip. This we denote by S and the ratio $S \div v \cos \beta \sec \alpha$ by s. Hence we have

$$u = v \cos \beta \sec \alpha - S,$$

and
$$e = \frac{u}{v \cos \beta \sec \alpha} = 1 - s. \quad . \quad . \quad . \quad . \quad (13)$$

This is therefore the limiting value of the efficiency when the tangential forces Q are 0.

It will be noted that the results thus far developed are entirely independent of any supposition relating to P or Q or the laws according to which they vary; and further, that the limiting value of e in (11) or (12) is purely a geometrical function.

Application to Typical Cases.—(*a*) Suppose the element a small plane *normal* to the direction of motion and moved *in* the direction of motion, and hence normal to itself. In this case all tangential forces on the element balance, and R is simply the difference of the two normal forces, one on the front and the other on the back. Referring to Fig. 49, we have, therefore, 0 as the value of the angles λ, μ, γ, and θ.

Hence
$$T = R,$$

and
$$W = Rv = Tv;$$

$$W_1 = Ru = Tu;$$

$$w = v - u = S;$$

$$e = \frac{u}{v} = 1 - s. \quad . \quad . \quad . \quad . \quad . \quad (14)$$

(*b*) Let the element be any solid symmetrical body, or at least of such form that the total resistance R is in the direction of motion, and let the latter be longitudinal. Then in Fig. 49 we have 0 for the value of λ, μ, γ, and θ. Hence the same equations apply as for (*a*).

(*c*) Let the element be a plane revolving about a transverse axis attached to the boat as in a radial paddle-wheel. If we suppose the boat propelled by a wheel consisting of a large number of such elements, we know that in order to obtain the necessary reaction the elements must move backward somewhat relative to the water, and that if u be the speed of the boat and v the peripheral speed of the paddles, v will be greater than u. If v were equal to u, it is readily seen that the element relative to the water would describe a cycloid, the motion being in fact exactly as though the boat were carried on wheels of a radius equal to that of the element, such wheels rolling without slip on a rigid plane horizontal foundation. In the actual case the motion will be as though the wheels were reduced in diameter in the ratio of u to v, and likewise rolled without slipping on such reduced diameter. As a result the element will now travel in a curtate trochoid, as illustrated in Fig. 51. In this figure the paddle is denoted by the heavy line, and the lower portion of the path of a point P is shown by the curve $PQRS$. The line of centers is at AB, and the slip being taken at 20 per cent, the effective radius OO is 80 per cent of OR. The rolling circle will then touch the line CD, which therefore will contain the instantaneous centers of the path of the point P. The location of the center of the wheel for successive angular positions of the element being as shown by the numbers along AB, the corresponding instantaneous centers

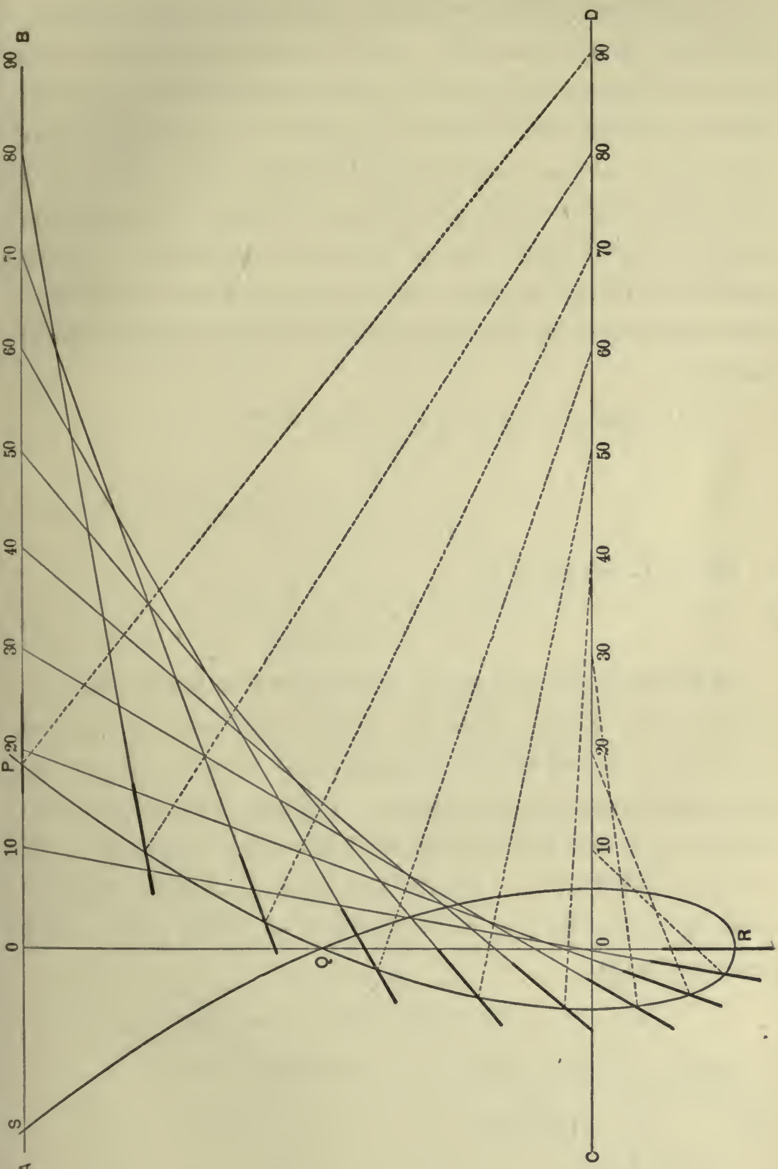

FIG. 51.

are located at points vertically underneath and similarly numbered on the line CD. The location of these centers serves to show in any position the direction of motion of the point P relative to still water, and hence approximately the like motion of the entire blade. Taking the blade as a whole, the various elements lying between the inner and outer edges will have varying values of r and hence of v, and hence will travel in different trochoidal curves, all determined, however, by the conditions and in the manner above described.

For this case in Fig. 50 we should have

$$\alpha = \theta;$$

$$\beta = 0.$$

Hence in (7) we have

$$T = P \cos \theta - Q \sin \theta. \quad . \quad . \quad . \quad . \quad (15)$$

The sign of $\sin \theta$ changes as the element passes through the vertical. Since, however, before reaching the vertical the element is inclined on one side, and after passing it the inclination is on the other side, it follows that Q itself also changes sign, and hence that the horizontal component of Q is always subtractive. Hence the value of T will always be correctly given by (15), considering θ as always plus. We have also in (8)

$$R \cos \mu = P.$$

Hence $$W_1 = (P \cos \theta - Q \sin \theta)u;$$

$$W = Pv;$$

$$e = \frac{(P \cos \theta - Q \sin \theta)u}{Pv}.$$

(d) Let the element be a plane moving as on the end of an oar, but suppose the plane of the oar to remain always vertical. This case is readily seen to be similar to the radial paddle-wheel, and so far as the general equations go, those given above in (c) apply here as well.

(e) Let the element be a plane always vertical and revolving about an axis fixed to the boat, as in certain forms of feathering paddle-wheels (§ 38). The path of the element relative to the water will be the same as before, but we shall have

$$\alpha = 0;$$

$$\beta = \theta.$$

Hence in equations (7)–(11)

$$T = P;$$

$$R \cos \mu = P \cos \theta + Q \sin \theta;$$

$$W_1 = Pu;$$

$$W = (P \cos \theta + Q \sin \theta)v;$$

$$e = \frac{Pu}{(P \cos \theta + Q \sin \theta)v}.$$

(f) Let the element be as in (c) and (e), but so connected that its plane shall always pass through the upper point of the circle which it traverses relative to the boat, as in certain forms of feathering paddle-wheel. (See Fig. 52.) In this case we have

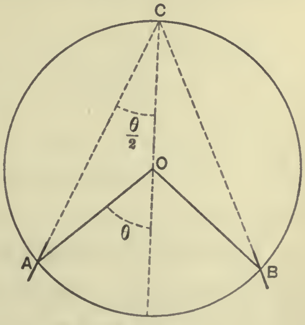

FIG. 52.

$$\alpha = \frac{\theta}{2};$$

$$\beta = \frac{\theta}{2}.$$

Hence in equations (7)–(11)

$$T = P \cos \frac{\theta}{2} - Q \sin \frac{\theta}{2};$$

$$R \cos \mu = P \cos \frac{\theta}{2} + Q \sin \frac{\theta}{2};$$

$$W_1 = Tu, \quad W = Rv \cos \mu;$$

and

$$e = \frac{\left(P \cos \frac{\theta}{2} - Q \sin \frac{\theta}{2}\right) u}{\left(P \cos \frac{\theta}{2} + Q \sin \frac{\theta}{2}\right) v}.$$

Let us now compare the limiting efficiencies of these three styles of paddle-wheel elements, assuming that u and v are the same in each case. The latter condition is necessary in order to make a comparison possible, and is perfectly allowable and natural, since it merely requires an adjustment of the amount of surface to the size or resistance of the ship.

Remembering that our equations apply simply at a given instant, we have,

for (c), $$e = \frac{u \cos \theta}{v};$$

for (e), $$e = \frac{u}{v \cos \theta};$$

for (f), $$e = \frac{u}{v}.$$

The order of excellence, so far as efficiency goes, is seen to be (e), (f), (c). For (f) the efficiency is constant or independent of θ, and therefore this is the value for all parts of the revolution during which the element is acting, and therefore the value for this element as a whole. For other like elements the value will vary inversely as v, and hence inversely as their radial distances from the axis of revolution. The resultant limiting efficiency is, of course, a mean of these elementary values.

For (c) and (e), however, the case is different. The efficiency for an entire stroke will be the useful work divided by the gross work for the same period. Hence we should have,

$$\text{for } (c), \qquad e = \frac{\Sigma Pu \cos \theta}{\Sigma Pv};$$

$$\text{for } (e), \qquad e = \frac{\Sigma Pu}{\Sigma Pv \cos \theta}.$$

Now P being really a function of θ and the velocities, it is evident that no simpler general expression can be obtained except by making special assumptions as to the form of P. It seems reasonable to believe, however, that under similar conditions the order of excellence with regard to both limiting and actual efficiency might be expected to remain the same as above.

34. Definitions relating to Screw Propellers.

If one line revolves about another perpendicular to it as an axis, and at the same time move its point of contact along this axis, then points in this revolving line will describe *helical curves*, and the line as a whole or any part of it will generate a *helical surface*.

A *screw propeller* may for our present purpose be defined as consisting of one or more blades having on the rear or driving side an approximately helical surface, such blades being joined to a common boss or central portion through which they receive their motion of rotation in a transverse plane relative to the ship.

Fig. 53 shows a four-bladed right-hand propeller.

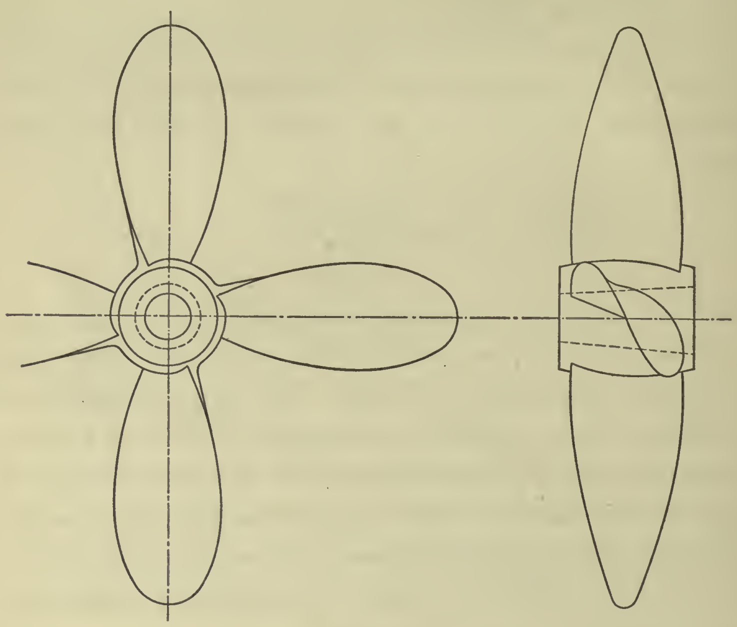

FIG. 53.

A propeller is said to be *right-hand* or *left-hand* according as it turns with or against the hands of a watch when viewed from aft and driving the ship ahead.

The *face* or *driving-face* of a blade is the rear face. It is that face which acts on the water, and which in return receives the excess of pressure which gives the driving thrust.

The *back* of a blade is on the forward side. Care must be taken to avoid confusion in the use of these terms.

The *leading* and *following edges* of a blade are respectively the forward and after edges.

The *diameter* of a propeller is the diameter of the circle swept by the tips of the blades.

The *pitch* is the axial distance between two successive convolutions of the helical surface. In an actual propeller the term should be understood as strictly referring to a small element of the driving-face only. With this understanding the *pitch* may be defined as the longitudinal distance which the ship would be driven were such element to work on a smooth unyielding surface, as for example the corresponding helical surface of a fixed nut. Pitch is thus seen to be purely a geometrical function of the propeller. Its value may vary from point to point over the entire driving-face, or it may be constant. In the latter case the propeller is said to be of uniform pitch. If it increases as we go from the hub toward the outer circumference, the pitch is said to increase radially. If it is greater on the following than on the leading edge, the pitch is said to increase axially. The latter mode of variation is usually implied by the simple term *increasing* or *expanding pitch*.

The *area*, *developed area*, or *helicoidal area* of a blade is the actual surface of the driving-face. For the propeller it is, of course, the sum of the areas of the blades.

The *projected area* is, correspondingly, the area of the projection on a transverse plane of one blade or of all the blades.

The *disk area* is the area of the circle swept by the tips of the blades.

The *boss* or *hub* is the central body to which the blades are all united, and which in turn is attached to the shaft.

35. Propulsive Action of the Element of a Screw Propeller.

It follows from the definition in the preceding section that a helicoidal path must lie in the surface of a cylinder, and that the pitch is equal to the distance between two successive convolutions of the helix measured along the same element of the cylinder. It is shown in geometry that if the surface of such a cylinder is developed, the helical path becomes a straight line, the diagonal of the rectangle representing the portion of the cylinder containing one convolution. Thus in Fig. 54, if the cylindrical surface be cut along the line *AB*, and the lower part rolled out and flattened, the helix *AHB* will become the diagonal *AC* of a rectangle *ABCD*, Fig. 55. In this rectangle *AB* denotes the pitch and *AD* the circumference of the circle of diameter *AD*, Fig. 54. The angle *CAD*, Fig. 55, is called the *pitch-angle*, and the ratio *AB* ÷ *AD*, Fig. 54, is called the *pitch-ratio*.

A radius *AL* moving so as to always rest on *LL* and *AHB*, Fig. 54, at the same time keeping perpendicular to

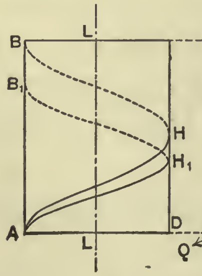

Fig. 54.

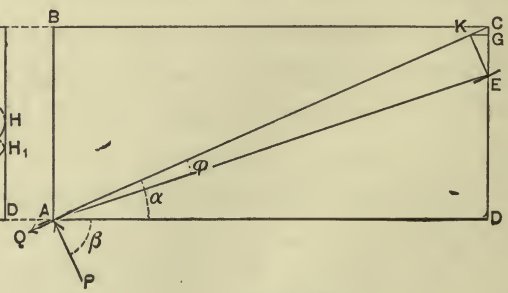

Fig. 55.

LL, will then describe a helical surface such as we are now concerned with. Let us consider a small element of this surface at the radius LA. If this element acted without slip, as already defined, then it is evident that for one revolution the ship would be driven ahead a distance AB and the element would travel in the helical path AHB. In the actual case due to the yielding of the water the longitudinal component of the motion will be AB_1 instead of AB, while the circular component will remain the same. The element will hence actually traverse a helical path AH_1B_1 instead of AHB. Let this be represented in the developed diagram, Fig. 55, by AE. Then $DE = AB_1$, the amount of longitudinal motion, and $CE = BB_1$, the amount of slip. The angle CAE is often termed the *slip angle*.

Let diameter AD, Fig. 54, $= d$;
radius AL " " $= r$;
pitch AB " " $= p$;
pitch-ratio $= c$;
revolutions $= N$;
area of element $= dA$;
angle CAE, Fig. 55, $= \phi$.

Then, using the nomenclature of Fig. 50, we have

$$\theta = 90^\circ;$$

$$CAD = \alpha;$$

$$PAD = \beta.$$

Hence $$\alpha + \beta = 90^\circ;$$

and $$\tan \alpha = \frac{p}{2\pi r};$$

$$p = 2\pi r \tan \alpha;$$

$$c = \frac{p}{d} = \frac{p}{2r} = \pi \tan \alpha.$$

We will now apply the general equations of § 33 to the element under consideration.

We have for the thrust

$$T = P \cos \alpha - Q \sin \alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Also, $R \cos \mu = P \sin \alpha + Q \cos \alpha$; . . . . (2)

$$W_1 = Tu = (P \cos \alpha - Q \sin \alpha)u; \quad . \quad . \quad . \quad . \quad (3)$$

$$W = Rv \cos \mu = (P \sin \alpha + Q \cos \alpha)v; \quad . \quad . \quad (4)$$

$$e = \frac{(P \cos \alpha - Q \sin \alpha)u}{(P \sin \alpha + Q \cos \alpha)v}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

For the limiting value when $Q = 0$, as in previous cases,

$$e = \frac{u \cos \alpha}{v \sin \alpha} = \frac{u}{v \tan \alpha} = \frac{DE}{DC} = (1 - s).$$

We have next to derive expressions for the forces P and Q considered as determined by the motion of the element.

We may first note that these forces will vary directly with the density of the liquid. Inasmuch, however, as this factor enters equally into the resistance of the propeller and that of the ship, it is evident that so far as the relations of the two are concerned, the design of the propeller will be independent of the density. That is, the same propeller should drive the ship with equal efficiency in either fresh or salt water. In any event the density factor will be provided for by the constants used to express the values of these forces.

The actual direction of motion of the element relative to still water is determined by the angles α and ϕ.

Now let Fig. 55 represent to an appropriate scale the distances moved per minute instead of per revolution.

Then

$$AD = 2\pi rN = v;$$

$$DC = pN = 2\pi rN \tan \alpha;$$

$$DE = pN(1 - s) = 2\pi rN \tan \alpha (1 - s) = u;$$

$$AE = 2\pi rN \sqrt{1 + (1 - s)^2 \tan^2 \alpha};$$

$$AC = 2\pi rN \sqrt{1 + \tan^2 \alpha}.$$

Taking § 5 (1) and § 6 (5) as the general expression of P, we should have

$$P = a\, dA\, \overline{AE}^2 \sin \phi.$$

But $\sin \phi = \dfrac{EK}{AE}$ and $EK = CE \cos \alpha = spN \cos \alpha$.

Hence $P = a\, dA\, \overline{AE}\, \overline{EK}$.

Whence, substituting and reducing, we find

$$P = 4\pi^2 r^2 N^2 s \sin \alpha \sqrt{1 + (1 - s)^2 \tan^2 \alpha}\; a\, dA.$$

Likewise from § 7, taking the relative gliding velocity of water and element as AC, and assuming a variation as the square of the speed, we have

$$Q = f\, \overline{AC}^2 dA = 4\pi^2 r^2 N^2 (1 + \tan^2 \alpha) f\, dA.$$

The coefficient f will depend on the length of the element, character of surface, and angle ϕ.

Now let dT, dU, and dW denote for this element the thrust, the useful, and the total work, instead of T, W_1, and W.

Then from (1), (3), (4), we have

$$dT = 4\pi^2 r^2 N^2 dA(as \sin\alpha \cos\alpha \sqrt{1 + (1 - s)^2 \tan^2\alpha} - f \sin\alpha (1 + \tan^2\alpha));$$

$$dU = 8\pi^3 r^3 N^3 (1 - s) dA(as \sin^2\alpha \sqrt{1 + (1 - s)^2 \tan^2\alpha} - f \sin\alpha \tan\alpha (1 + \tan^2\alpha));$$

$$dW = 8\pi^3 r^3 N^3 dA(as \sin^2\alpha \sqrt{1 + (1 - s)^2 \tan^2\alpha} + f \cos\alpha (1 + \tan^2\alpha)).$$

Put $\sin^2\alpha \sqrt{1 + (1 - s)^2 \tan^2\alpha} = B;$

$$\sin\alpha \tan\alpha (1 + \tan^2\alpha) = C;$$

$$\cos\alpha (1 + \tan^2\alpha) = E.$$

Whence $C = E \tan^2\alpha$.

Then the above equations become

$$dT = 4\pi^2 r^2 N^2 dA(asB \cot\alpha - fC \cot\alpha); \quad . \quad . \quad . \quad (6)$$

$$dU = 8\pi^3 r^3 N^3 (1 - s) dA(asB - fC); \quad . \quad . \quad . \quad . \quad (7)$$

$$dW = 8\pi^3 r^3 N^3 dA(asB + fE). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Whence

$$e = (1 - s)\frac{asB - fC}{asB + fE} = (1 - s)\frac{\frac{a}{f}sB - C}{\frac{a}{f}sB + E}. \quad . \quad . \quad (9)$$

Now put $2r = yp$ where y is a variable denoting the location of the element in terms of the pitch. It is called the *diameter ratio*, and is evidently the reciprocal of the pitch-ratio c for the element in question. For the propeller as a whole the values of c and y will be, of course, those of the outer element. Where we wish to especially distinguish these limiting or outer values we shall denote them by c_1 and y_1.

We then have

$$dT = \pi^2 p^2 N^2 y^2 f dA\left(\frac{a}{f} sB \cot \alpha - C \cot \alpha\right); \quad . \quad . \quad (10)$$

$$dU = \pi^3 p^3 N^3 y^3 (1 - s) f dA\left(\frac{a}{f} sB - C\right); \quad . \quad . \quad . \quad . \quad (11)$$

$$dW = \pi^3 p^3 N^3 y^3 f dA\left(\frac{a}{f} sB + E\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

36. Propulsive Action of the Entire Propeller.

If the expressions in the preceding section for dT, dU, and dW could be integrated over the surface of the blade, we should have values for the entire T, U, and W. Assuming p constant, these may be represented by

$$T = \pi^2 p^2 N^2 \int y^2 (asB \cot \alpha - fC \cot \alpha) dA; \quad . \quad . \quad (1)$$

$$U = \pi^3 p^3 N^3 (1 - s) \int y^3 (asB - fC) dA; \quad . \quad . \quad . \quad . \quad (2)$$

$$W = \pi^3 p^3 N^3 \int y^3 (asB + fE) dA. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Also, dividing (2) by (3), we have, similar to § 35, (9),

$$e = (1 - s) \frac{\int y^3 \left(\frac{a}{f} sB - C\right) dA}{\int y^3 \left(\frac{a}{f} sB + E\right) dA}. \quad . \quad . \quad . \quad (4)$$

Of the variables involved in this integration we may remark that y, s, B, C, E, and dA are determined by the geometry of the propeller and by the slip. They are therefore readily known. It is not the same, however, with a and f. While these are called constants, there is no reason for assuming that their values will be the same for different elements of the surface, or for different aggregations of such elements. In fact such indications as we have point toward a decided variation in value with different values of the total area, pitch-ratio, slip, thickness of blade, condition of surface, and location of element.

In regard to the coefficient f it must be remembered that it here represents not merely a surface effect, but rather the resultant force in the direction of the plane of the driving-face. Due to the increase of thickness from the tips inward, we should expect a corresponding increase in the value of this coefficient.

The coefficient for direct resistance is taken in the form $a \sin \phi$, and the supposition of constancy in the value of a throughout the integration virtually assumes that the normal resistance varies directly with the surface. This, however, can only be even approximately true up to a certain point. We have specified in § 32 the two general methods of considering the propulsive action of an element. Thus far we have made use of the second one only. In considering the variation in value of this coefficient a, we shall find it useful to here briefly consider the propulsive action of the element of the screw propeller viewed from the first standpoint. In § 37 and § 38 a more general discussion of the application of this method will be found.

From this point of view the action of the element is to accelerate in a sternward direction the water acted on. This, for the element, is a cylindrical shell of water of radius equal to that of the location of the element, and of thickness determined by its height. For the propeller it would be a cylindrical body of water of diameter sensibly equal to that of the propeller, and with a hollow core of diameter sensibly equal to that of the hub. As we shall show later, the maximum amount of acceleration depends on the slip and on the geometrical configuration of the element or of the propeller. If, now, we consider the element very narrow, as for example a part of a narrow lath-like blade, the amount of acceleration produced either by the element or by the blade as a whole will be very much less than this geometrical maximum above referred to. The thrust obtained will therefore be correspondingly less. As we increase the width of such a blade the factor a will at first be nearly constant for each element of increase, so that for a time the thrust will increase directly with the area. At length, however, as we approach a condition where the maximum acceleration and maximum thrust are nearly attained, the increase in thrust for each additional element of area will be less and less, or in other words, the thrust will increase at a slower and slower rate relative to the increase of area. We shall therefore finally reach a width of blade such that no additional increase will add sensibly to the thrust. It is evident that as we thus increase the area from our very narrow initial blade the average value of a or the thrust received per unit area will steadily decrease, at first slowly and then more rapidly. It even seems possible that, under certain conditions, after having passed a certain point the increase of area might be attended by an actual decrease in total thrust due to the increasing confusion of the streams of water acted on. These points will be again referred to in § 49. We wish now simply to indicate that this coefficient will probably vary with the total area in the general manner above described.

Again, the actual values of ϕ due to the influence of the thickness of the leading edge may perhaps be quite different from the values determined geometrically, and the law of variation with the sine of the angle is, again, not quite exact.

It is therefore evident that the coefficient a instead of being constant will probably vary for different locations and configurations of the element, for different values of the slip, and for different amounts of total area of blade. If therefore we wished to make direct use of these expressions for T, U, and W as they stand, we should need a series of experimental values of these coefficients appropriate to the circumstances of the case in hand. There is, however, no such set of values available, so that, even if it were desirable, no direct use can at present be made of these expressions in the form given above.

It is not without interest, however, to note in passing the results of a special examination into the question of the distribution of the values of a and f radially along the blade. While no direct experimental determination of the values for an element are available, we have in Froude's experimental investigation on propellers* the integrated results corresponding to T, U, W, and e. As we shall refer more especially to these experiments at a later point, it is sufficient at present to state that the data thus obtained give the values of T, U, and e for a series of propellers of constant

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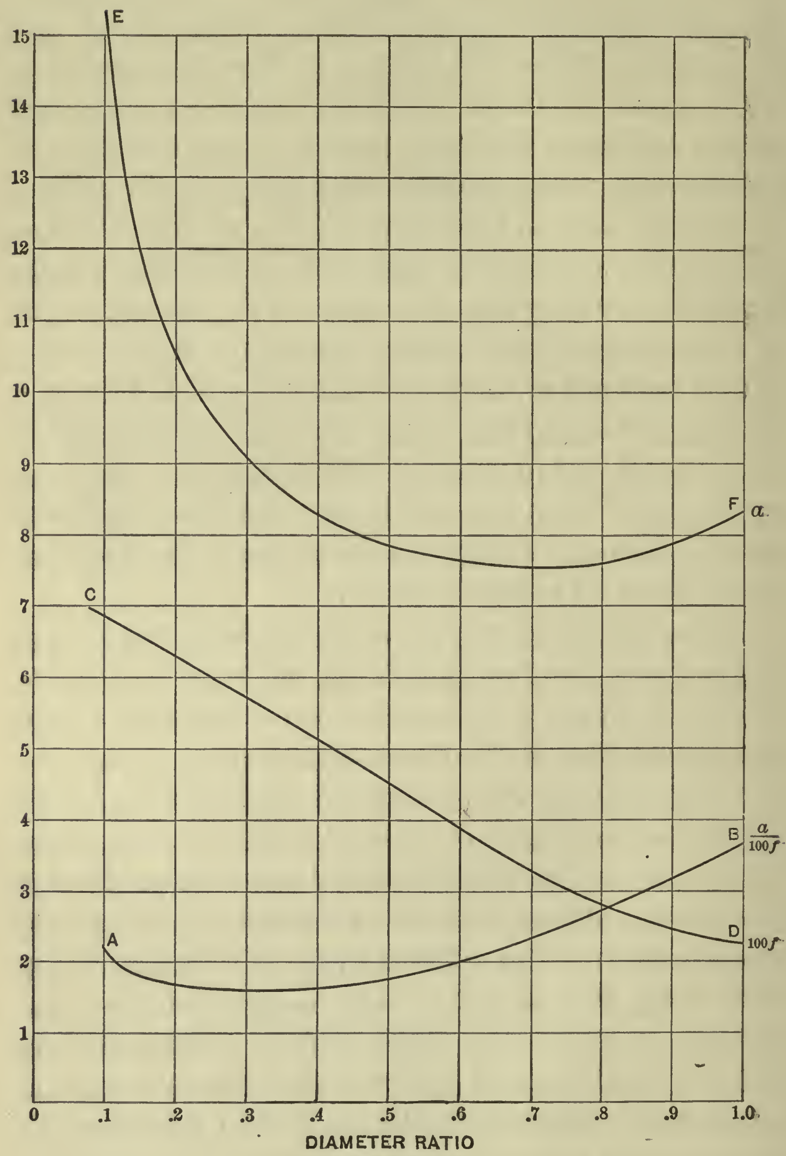
* Transactions Institute of Naval Architects, vol. XXVII. p. 250.

diameter and variable pitch (and hence of variable value y_1) at a series of slips. Now it is evident, taking any one slip, that some distribution of the values of a and f substituted in the fundamental formulæ (1), (2), (4) will give the observed results, and hence that by a process of trial and error an approximation to the probable distribution may be reached.

Referring also to (4), it is seen that e depends solely on $a \div f$, and hence that the distribution of this ratio may be determined inversely from the values of e for various external or outer values of y. Likewise from (1) it is seen that T may be expressed as a function of f and $a \div f$, and hence, if the latter is first determined, the distribution of the values of f may be inferred from that of the values of T. The values of $a \div f$ and f being thus found, the values of a immediately follow. Without entering into the details of the operation we may state the results as follows:

Taking the slip as 20 per cent, it was found that a distribution of $a \div f$ as given by AB, Fig. 56, would give by substitution in (4) the observed values of the efficiency. Next, that a distribution of f as shown in CD would, in connection with the values of $a \div f$, give by substitution in (1) the observed values of T. The resulting values of a are shown by EF. It was also clearly indicated that no other distribution widely differing from this in character could fulfil the given conditions. The values of f vary, as we should expect; while those of a present a more complex variation, due probably to the varying width of blade at different radial distances on the same propeller, and at points having the same value of the diameter ratio on different propellers, and to other causes which do not appear from the data at hand.

For most purposes, however, we are not so much con-


E
F
a
C
B
$\frac{a}{100f}$
A
D
100f
15
14
13
12
11
10
9
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6
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4
3
2
1
0
.1
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.4
.5
.6
.7
.8
.9
1.0
DIAMETER RATIO

FIG. 56.

cerned with the distribution of the values of a and f as with mean values, or rather with values of the integrals of (1), (2), (3), (4). These integrals depend on the values of y_1 and s, and on the shape, proportions, and area of the blade. Supposing for the present the latter elements constant, it is evident that if we can find the values of these integrals for a sufficient number of values of y_1 and s, we shall be able to obtain by interpolation the thrust, useful work, and efficiency corresponding to any given set of conditions.

We will now briefly refer to the experiments most available for this purpose. In the paper above referred to Mr. Froude gives the results of a series of experiments which he had carried on at the Admiralty Experimental Tank for the purpose of investigating the relation of thrust and efficiency to pitch-ratio and slip. The propellers were of 2, 3, and 4 blades, and were all of 8.16 inches diameter, and of pitch-ratios 1.225, 1.4, 1.8, 2.2. The blades were elliptical in developed form, as shown in Fig. 57. The hub took the place of the inner part of the ellipse, and the form was slightly changed at this point to give a somewhat wider line of attachment between hub and blade. The maximum width of the blade was .4 the radius. These propellers were tested at various values of the slip. It was found that within the limits of the experiment the results might be expressed quite closely by the following laws:

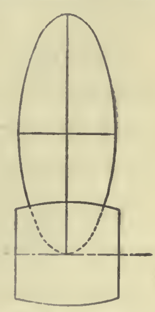

FIG. 57.

(a) For constant slip and varying values of y_1 the thrust may be expressed by an equation of the form

$$T = A(y_1 - .17).$$

(*b*) For constant efficiency and varying values of y_1 the thrust may be expressed in the form

$$T = By_1^{.8}.$$

These equations were made use of to reduce the observations to order and to eliminate minor instrumental and observational errors, and also to interpolate for intermediate values of y_1 and s. Where the nature of the law seemed to authorize it, the values were extended slightly beyond the limits of the experimental determination.

The data thus determined were then put into graphical form in sets of curves, one of which gave T as a function of the slip for varying values of the pitch-ratio c_1. These curves were so plotted as to be independent of absolute dimension, so that by the necessary assumptions as discussed in § 49 and by the introduction of the proper functions of any proposed dimensions they become applicable to any given case.

In regard to the curves of efficiency, it was found that by expressing e as a function of s for varying values of y_1 or c_1 a series of curves were obtained which were essentially the same curve plotted to varying horizontal scales. It therefore became possible, by an appropriate choice of horizontal scale for each pitch-ratio, to bring all of these efficiency curves into essential identity, and hence to use one for all. This fusion of the various efficiency curves was thus obtained by substituting for the abscissa s a quantity called abscissa value such that

$$\text{Abscissa value} = sf(c_1).$$

Any values of s and c_1 being then given, and the corresponding form of $f(c_1)$ being known, the abscissa value follows, and thence from the curve the efficiency is known.

As the subject is to be treated somewhat differently here, we shall refer the reader to the original paper for further details, especially as to the methods given for applying these results to the problem of design.

Mention may also be made of a series of experiments on screw propellers by Thornycroft and Barnaby.* The general results of these experiments also closely correspond with those by Mr. Froude, and as the latter are the more extensive we shall use them as the source for our experimental values.

In order to place the equations (1), (2), (3), (4) in the most convenient form for application to these experiments we proceed as follows:

Let b denote the varying width of blade, and b_1 the maximum width; also r, as before, the radius of any element and r_1 the outer radius. Then

$$\frac{r}{r_1} = \frac{2r}{2r_1} = \frac{py}{d_1} = c_1 y,$$

or $$r = c_1 r_1 y.$$

The propellers had four blades. Hence we have

$$dA = 4bdr = 4bc_1 r_1 dy = 2bdc_1 dy.$$

For an elliptical blade as described

$$b = \frac{2b_1}{r_1}\sqrt{r_1 r - r^2} = 2b_1\sqrt{c_1 y - c_1^2 y^2}. \quad . \quad . \quad (5)$$

Hence $$dA = 4b_1 dc_1\sqrt{c_1 y - c_1^2 y^2}\, dy. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

* Transactions Institute Civil Engineers, vol. CII, p. 74.

This relates the area of the elliptical blades to the diameter of the propeller and to the maximum width b_1. We wish next to substitute for this some form of relationship in which the geometrical character of the contour of the blade will not be explicitly involved. To this end we relate the area of the blades directly to $\pi d^2 \div 4$, or to the disk area of the propeller. To this end put

$$A = h\left(\frac{\pi d^2}{4}\right) \quad \text{or} \quad h = \frac{4A}{\pi d^2}. \quad . \quad . \quad . \quad . \quad (7)$$

We shall call h the *area-ratio*.

It is found that the area of a blade such as represented in Fig. 57 is about .891 the area of the ellipse. Hence for one such blade we have

$$\text{Area} = .891\pi\frac{d}{4}\frac{b_1}{2} = .35db_1.$$

Hence for the entire propeller in this case

$$A = 1.4db_1.$$

$$\text{or} \qquad db_1 = .714A = .714\frac{\pi hd^2}{4} = .561hd^2. \quad . \quad . \quad (8)$$

Substituting this in (6) we have, as the more general relation desired,

$$dA = 2.24hd^2c_1\sqrt{c_1y - c_1^2y^2}\,dy. \quad . \quad . \quad . \quad . \quad (9)$$

In the present case we remember that $b_1 = .4r_1 = .2d$. Hence from (8) we have

$$hd^2 = 1.78b_1d = .356d^2 \quad \text{or} \quad h = .356.$$

We must bear this in mind as the standard value of h for the experimental propellers.

Substituting for dA in (1), (2), (3), we have

$$T=p^2N^2d^2\left[\pi^2 2.24h\int y^2(asB\cot\alpha - fC\cot\alpha)c_1\sqrt{c_1y - c_1^2y^2}dy\right];$$

$$U=p^3N^3d^2\left[\pi^3(1-s)2.24h\int y^3(asB - fC)c_1\sqrt{c_1y - c_1^2y^2}\,dy\right];$$

$$W=p^3N^3d^2\left[\pi^3 2.24h\int y^3(asB + fE)c_1\sqrt{c_1y - c_1^2y^2}\,dy\right].$$

The quantities within the brackets are seen to be functions of the proportions of the propeller and of the slip, or explicitly of y and s. Denote for any given case their values by H, K, L, respectively. We must here remember that we are not at present applying these equations to propellers in general, but simply to the propellers of this series of experiments, in which the size, proportion, and shape of the blades remained constant, and y_1 and s were the only variables. Hence we may properly consider that the values of H, K, and L are functions of these variables only. We have therefore

$$T=(pN)^2d^2H; \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$U=(pN)^3d^2K; \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$W=(pN)^3d^2L; \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$e=\frac{K}{L}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

So far as any ultimate use is to be made of these equations, we may simply consider U and e. W, if desired, is readily found by dividing U by e, and T by dividing U by the speed u.

We are therefore to derive actual values of K by substituting in equation (11) the values found from Froude's experiments. To avoid the presence of large numbers in our results we first substitute for p, d, and N, as follows:

$$p' = \frac{\text{pitch in feet}}{10} = \frac{p}{10};$$

$$d' = \frac{\text{diameter in feet}}{10} = \frac{d}{10};$$

$$N' = \frac{\text{revolutions per min.}}{10} = \frac{N}{10}.$$

Then let us denote the resulting value of K by the two factors kl, where k is a function of the slip s and is intended to represent the influence of this variable, and l is a function of pitch or diameter ratio, c or y_1, and is intended to represent the influence of this variable.

We have then

$$U = (p'N')^3 d'^2 kl. \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Now putting into (14) an actual value of U as given by multiplying Froude's curves of thrust by the speed u, and reducing to horse-power units, we are able to derive the value of the combined factor kl corresponding to the particular values of y_1 and s assumed. If this be done for a large number of values of y_1 and s, we shall have a corresponding series of values of kl. These we may naturally seek to express, each as a function of the variable on which it depends.

Keeping in mind proposition (a) above, it is evident that we may put

$$l = (y_1 - .17). \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

An examination of the series of values of kl then showed that the values of k could be very nearly expressed by the quadratic formula

$$k = (.034 + .805s - .68s^2). \quad . \quad . \quad . \quad (16)$$

Tabular values of k and l will be found in § 50. In the form of an equation the final result becomes

$$U = (p'N')^2 d'^2 (.034 + 805s - .68s^2)(y_1 - .17). \quad (17)$$

In this equation U is in horse-power units; p', N', and d are as defined above; while s and y_1 are to be expressed as decimals.

The resulting values of U will then be found to closely represent all values falling within the limits of s and y_1 covered by the experiments.

The formula given in (17) is therefore simply empirical, expressing the results of the experiments noted above, and therefore fundamentally applicable simply to the propellers there employed. In Chapter IV we shall consider the extension of this equation to propellers in general.

We now turn to the question of efficiency. From (4) it appears that in general e is a function of s, a, f, B, C, E, or of a, f, s, y_1. If we now assume that the influence of the general variation of a and f on e is unimportant, it would follow that we might express in general e as a function of y_1 or c, and s,—the former as fixing the character of the propeller, the latter the nature of its use. This assumption is undoubtedly inadmissible when the character of the blade as regards shape and area-ratio h widely varies, and the influence of dimension alone is perhaps not unimportant. As a working basis, however, let us first assume the blades all similar and of the standard form and proportions as used in Froude's experiments, and let us neglect the influence due to actual dimension. Then, knowing the efficiency of the model propellers, that of any other similar propeller will be the same as that for the model of the same pitch-ratio and working at the same slip. If next we assume the area-ratio h and the shape of blade to vary, the results determined will be correspondingly less reliable. Still from such indications as we have it seems likely that for slight variations in area-ratio or shape the influence on the efficiency would be small. In default of more exact information, therefore, we shall take the results of these experiments as our standard of efficiency, remembering that as we depart from the shape and proportions of the blades there used the results will correspondingly decrease in probable accuracy.

As above mentioned, the efficiencies of the model propellers were expressed by Froude as a function of c and s by means of the "abscissa value," through the agency of which the efficiency curves for all values of c were brought into coincidence. For our present purposes, however, we prefer to put the information in a somewhat different form, as follows:

Determining by Froude's diagrams the values of c and s for a series of constant values of e, it was found on plotting the results that each locus of c and s for constant e was very nearly a straight line, and that the bundle of lines radiate very nearly from a single point. The actual character of a locus of c and s thus determined is fixed by propositions (a) and (b), which of course are approximate, and not physically exact. Starting from these propositions, it may be shown that the actual locus is not a straight line, and that its equation is quite complex. Between the limits of the experiments, however, the equation represents a locus which is very nearly rectilinear. The character thus found by geometrical determination is therefore justified by the nature of the propositions on which the fundamental diagrams are based. The same result is also implicitly involved in Froude's determination of the fact that the efficiency curves for varying pitch-ratios may by means of the *abscissa value*, as referred to above, be reduced all to the same form, and hence one made to serve for all.

The diagram deduced as above explained is shown in Fig. 58. The approximate focus of the lines is at about $c = -6.8$, $s = -.21$, so that the approximate equation to the set of lines is $s + .21 = n(c + 6.8)$. The actual lines of the diagram are those which most closely represent the points determined from the original data, and do not therefore exactly coincide with a bundle emanating from the focus mentioned. The difference is very slight, however, and is presumably much less than the necessary difference between the experimental and true values in any given case, and is therefore quite insignificant. This diagram may be carefully examined with profit. Geometrically it may be regarded as a series of level lines cutting the surface which would represent e as a function of c and s. A plane perpendicular to the axis of s as indicated by AB would cut out a curve of efficiency for constant slip as a function of pitch-ratio, while a plane perpendicular to the axis of c would cut out a curve of efficiency for constant pitch-ratio as a function of slip. Such curves are shown in Figs. 59 and 60. It is the latter which is made use of by Mr. Froude. Looking again at the diagram in general, it is seen that it readily indicates the relation to e of any propeller of standard form, given its pitch-ratio and slip; and, *vice versa*, it shows the range of values of

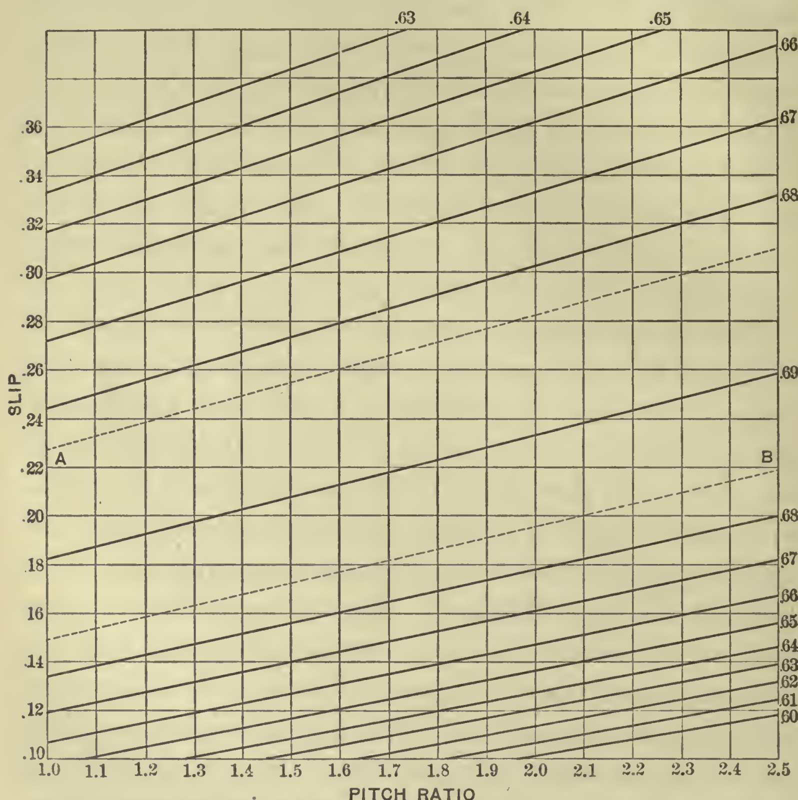

FIG. 58.—DIAGRAM OF EFFICIENCY AS A FUNCTION OF SLIP AND PITCH RATIO.

these variables within which any particular efficiency may be expected. Thus the line for 69 per cent shows the relative values of c and s along which the best efficiency was found, while the area between the two lines for 68 per cent shows a region or range of values of c and s within which the efficiency was not below 68 per cent. Again, it shows plainly the more rapid falling off as we approach small slip rather than large, and in consequence that it is better to work in a region of slip greater than that for maximum efficiency rather than less. Further uses of the diagram will appear with application to problems of design, but the above will

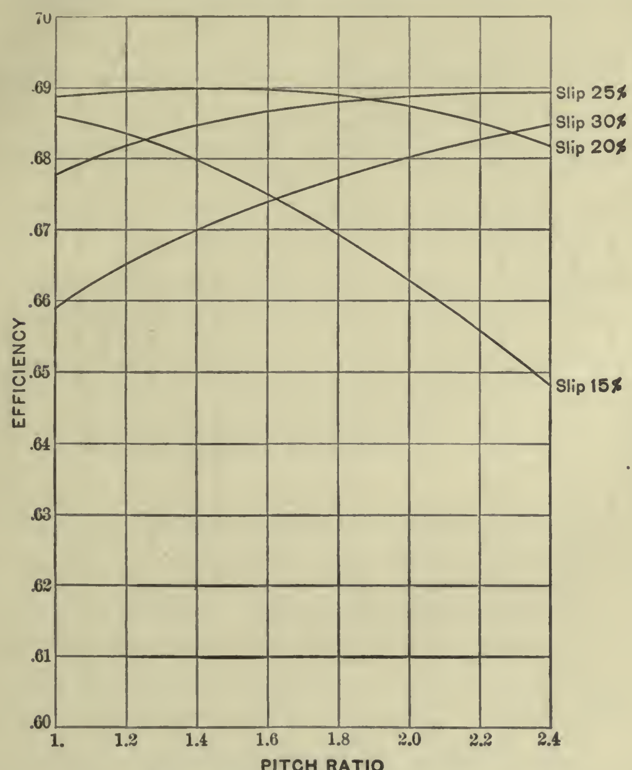

FIG. 59.—CURVES OF EFFICIENCY.

suffice to indicate the main lines along which its study may be directed.

The reader should perhaps at this point be cautioned that the absolute values of the efficiency here given are probably less accurate than the relative values. Unavoidable errors enter into the measurement of many of the quantities involved in efficiency, and in addition some modifications were made by Mr. Froude in order to reduce the values to agree-

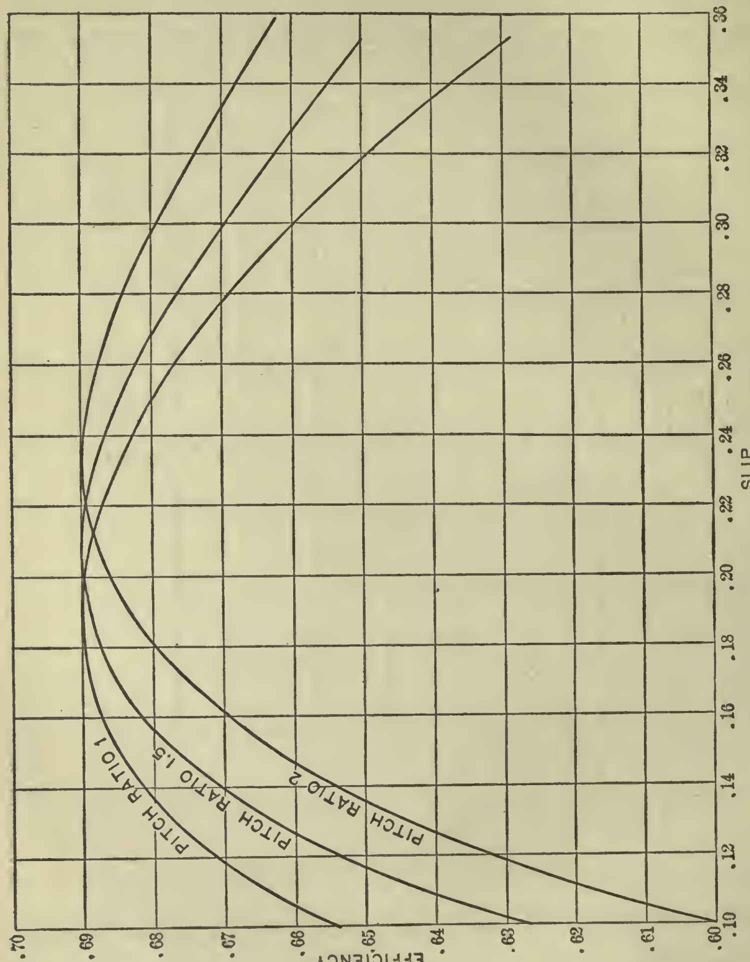

FIG. 60.—CURVES OF EFFICIENCY.

ment with proposition (*b*) above. The numbers here given should therefore be considered not as absolute values, but as close approximations; while the indications as to change of value for change of condition may be accepted as presumably somewhat more accurate in character.

37. Action of a Screw Propeller viewed from the Standpoint of the Water acted on and the Acceleration imparted.

In the treatment of the screw propeller in §§ 35, 36, we have preferred the method there followed to that which involves a consideration of the amount of water acted on and its acceleration, because the former seems more fruitful practically, and to furnish a more valuable means of relating theory to practice than the latter. The latter method, however, has chiefly attracted the attention of mathematicians and theoretical investigators, and it may be well to consider briefly some of the leading features of the theory from this point of view, especially as they are of general interest as well.

The water acted on must necessarily travel in helical paths. That is, its velocity will have a longitudinal and a rotary component. The longitudinal component is that which directly yields the reaction from which comes the thrust. The rotary component as well as the longitudinal absorbs work from the propelling agent. Its existence also gives rise to centrifugal force, due to which the pressure is less in the interior of the column than at its outer boundaries. The water in coming aft into the zone of influence of the propeller gradually has its velocity increased, and a certain amount of acceleration is therefore received before the water reaches the propeller itself. Mr. R. E. Froude * has shown

* Transactions Institute Naval Architects, vol. xxx. p. 390.

in the limiting case where no rotation is produced, and where the propeller blades are so narrow that little or no sensible acceleration can be received in the time taken to pass through the propeller itself, that one half the acceleration will be received forward of the propeller and one half in the race aft of the propeller.

With rotation, however, we shall have in and aft of the propeller the defect of pressure in the interior of the column due to centrifugal force. In the actual case, therefore, somewhat more than one half of the acceleration will be produced forward of the propeller, a certain amount while passing through, and the remainder aft of the propeller.

It is also evident that the higher the pitch-ratio the more the rotation, and *vice versa*. Hence with propellers of high pitch-ratio we may expect that most of the acceleration will be received forward of the propeller, while with low values of pitch-ratio the proportion there received may be but little more than one half the total. The column as a whole must, for continuity of flow, decrease in cross-section as it passes through the propeller, and to such distance aft as the velocity increases.

If the amount of water acted on could be known and the total acceleration produced, the thrust could be immediately determined, and knowing the speed, the useful work would follow. The total work would be the sum of the useful, plus the work represented by the kinetic energy imparted to the water. The latter representing the waste is seen to depend on the amount of water acted on and on the initial and final velocities. It is also clear that we may divide the energy spent in the wake into three parts, as follows: (1) that due to longitudinal change of velocity; (2) that due to circular change of velocity or rotation; (3) that due to turbulence and eddies. The portion (1) is of course necessary to the production of a useful thrust, while (2) and (3), though not fundamentally necessary, cannot be entirely avoided. The energy involved in the race rotation has been made the subject of examination by R. E. Froude,* who shows by certain ideal limiting cases that its influence on efficiency will be practically negligible except with large values of the slip and high pitch-ratios.

If in a simple ideal case M denotes the amount of water acted on and v the longitudinal change of velocity, § 42, (4), then the thrust will be proportional to Mv and the energy involved to Mv^2. Now for a given value of Mv it is readily seen that Mv^2 will be a minimum when M is a maximum and v a minimum. That is, for a given thrust the energy due to longitudinal acceleration is least when the mass acted on is large and the acceleration is small. For high efficiency, therefore, this consideration points to the use of large propellers acting at small slip rather than the reverse. This corresponds in § 35 to the result consequent on the assumption of the decrease or absence of tangential forces. Actually the existence of race rotation, eddies and turbulence due to skin-resistance, etc., as well as structural considerations, limit the size and slip for the best efficiency as already shown in § 36. Compare also § 49.

It is thus possible to discuss the action of the screw propeller from the standpoint of the present section. In the actual case, however, none of the terms here involved can be exactly related to known quantities, so that, as in the other mode of treatment, we are thrown upon experiment for the

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* Transactions Institute of Naval Architects, vol. XXXIII. p. 265.

ultimate control of whatever formulæ may be proposed. We give, however, in § 38, as an illustration, the treatment of the paddle-wheel from this point of view.*

38. The Paddle-wheel Treated from the Standpoint of § 37.

The paddle-wheel may be investigated in the same general way as the screw propeller in §§ 35, 36. The lesser relative importance of this instrument of propulsion does not, however, seem to justify the necessary work, and furthermore we have no such experimental data for the final control of our equations as with the screw propeller. We will therefore consider it briefly from the standpoint mentioned in § 32 and referred to more especially in § 37.

The thrust is measured by the sternward momentum of the water imparted by means of the paddle.

Let v_0 be the initial mean velocity of the water in f. s.;

v_1 be the final mean velocity of the water in f. s.;

A be the cross-sectional area of the stream affected;

u be the velocity of the ship in f. s.;

D be the diameter of the circle described by the center of pressure of the paddles (this point may

* For the more important details of the various theories which have been developed along the lines here referred to, reference may be made to the following:

"The Mechanical Principles of the Action of Propellers." By Prof. Rankine. Transactions Institute of Naval Architects, vol. VI. p. 13.

"The Minimum Area of Blade in a Screw Propeller necessary to Form a Complete Column." By Prof. J. H. Cotterill. *Ibid.*, vol. XX. p. 152.

"The Part played in the Operation of Propulsion by Differences in Fluid Pressure." By R. E. Froude. *Ibid.*, vol. XXX. p. 390.

"The Theoretical Effect of the Race Rotation on Screw-Propeller Efficiency." By R. E. Froude. *Ibid.*, vol. XXXIII. p. 265.

"Marine Propellers." By S. W. Barnaby. London and New York.

be taken as sensibly at the center of the paddle radially);

N be the number of revolutions per minute.

Then πDNA is the volume acted on per minute and for sea-water: $64.4\pi DNA \div 60$ is the mass in pounds acted on per second.

$v_1 - v_0$ is the change in velocity per second. Hence

$$\text{thrust} = T = \frac{64.4\pi DNA(v_1 - v_0)}{60 \times 32.2} = \frac{\pi DNA}{30}(v_1 - v_0).$$

The thrust equals the resistance, and hence, solving for A, we have

$$A = \frac{30R}{\pi DN(v_1 - v_0)}.$$

In this expression the value of $(v_1 - v_0)$ is uncertain. Most writers, following Rankine, have considered that

$$v_1 - v_0 = \text{slip of propeller or paddle-wheel} = s\frac{\pi DN}{60}.$$

This is known to be quite inexact, but we may use it for the present illustrative purpose. We have, therefore,

$$A = \frac{60 \times 30R}{(\pi DN)^2 s}.$$

Or we may put more generally and more safely

$$A \sim \frac{R}{(DN)^2 s}. \quad \cdots\cdots \quad (1)$$

And likewise, area of one paddle $\sim A$. Hence if we denote the area of one paddle by a, we may put

$$a \sim \frac{R}{(DN)^2 s}. \quad \cdots\cdots \quad (2)$$

For purposes of design we may preferably put the equation in another form, as follows:

We have $$R \sim a(DN)^2 s; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$u = \frac{\pi DN(1 - s)}{60}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$\therefore$ Useful work or $Ru \sim a(DN)^3 s(1 - s)$. . . (5)

Also $Ru \sim$ total work or I.H.P.

Denoting this by I, we have

$$I \sim a(DN)^3 s(1 - s), \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

or $$a \sim \frac{I}{(DN)^3 s(1 - s)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

By comparison with paddle-wheels which have performed well, constants may be found thus relating the area of blade to known and to previously assumed quantities.

For radial paddles the slip s may be taken from 20 to 30 per cent, and for feathering paddles from 15 to 20 per cent.

We have therefore

$$a = K \frac{I}{(D'N')^3 s(1 - s)}. \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The value of K will depend on the units chosen for D', N', and I. For numerical convenience we may take these as follows:

$I =$ I.H.P. absorbed by *one* wheel;

$D' =$ (diam. of wheel in feet) $\div$ 10 $= D \div 10$;

$N' =$ (revolutions per minute) $\div$ 10 $= N \div 10$.

We may then take K from 2.0 to 2.5.

This fixes the area of one float. The number of floats may be made about $.8D$ for radial wheels, and from $.5D$ to $.7D$ for feathering wheels. The proportions of floats are usually

Length = 3 to 4 times breadth.

The greatest immersion of the upper edge at mean draft should be from .3 to .8 the breadth, according as the boat is to navigate smooth or rough water and to vary little or much in draft.

If u is in knots per hour, instead of (4) we shall have

$$u = \frac{\pi DN(1 - s)}{6080 \div 60} = \frac{\pi DN(1 - s)}{101.3}. \quad . \quad . \quad . \quad (9)$$

Hence $$DN = \frac{101.3u}{\pi(1 - s)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

To illustrate these formulæ take the following data:

$I = 1000$ on both wheels and 500 on one wheel;
$u = 18$ knots;
$s = .20$ per cent, assumed.

Then $$DN = \frac{101.3 \times 18}{\pi \times .8} = 725.5,$$

and $$D'N' = 7.255.$$

Hence, taking $K = 2.4$,

$$a = \frac{2.4 \times 500}{(7.255)^3 \times .16} = 19.6 \text{ sq. ft., or say 20 sq. ft.}$$

The floats might therefore be $8' \times 2'.5$.

We may now divide DN in any proportion suitable to the circumstances of the case. Thus if we take $N = 40$, we have $D = 18'.1$; or if we put $D = 15'$, we have $N = 48.4$.

Kinematic Arrangement of Feathering Paddles—In Figs. 61 and 62 are shown the usual kinematic arrangements for

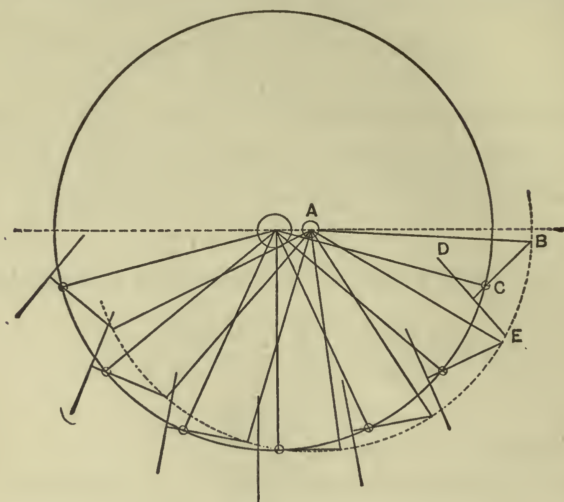

FIG. 61.

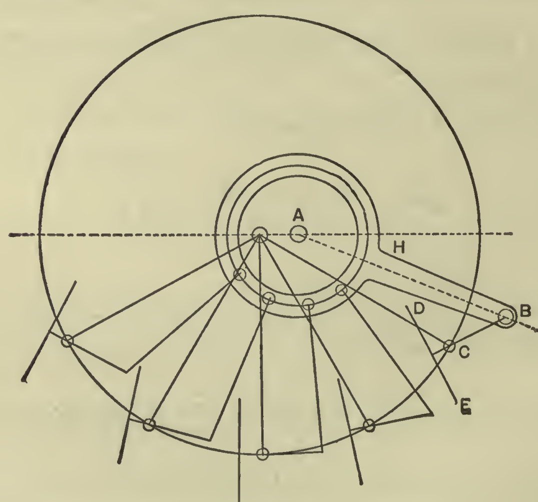

FIG. 62.

feathering paddles. In Fig. 61 the links similar to AB are all pivoted at A. This link actuates the lever BC and blade

DE as shown, and similarly for the others. In Fig. 62 *HB* is a drive-link actuated by an eccentric with center at *A*. The remainder of the links are pivoted to the eccentric strap as shown, and are connected to blades similar to *DE*, as in Fig. 61. In both diagrams the plane of the blade may be beyond the pivot *C* as shown, or it may contain *C*, or it may lie between *B* and *C*. The arrangement of Fig. 62 is more commonly used, that of Fig. 61 being occasionally employed for small wheels.

39. Hydraulic Propulsion.

In this mode of propulsion the water is drawn into pumps or other similarly acting propelling agents situated within the boat. It is then acted on by the pump-vanes and delivered with an increased velocity toward the stern. The necessary reaction is thus obtained and the boat is moved. This mode of propulsion from its slight use does not merit any extended consideration. We may, however, instructively glance briefly at certain features.

Let M be the water acted on per second in pounds. For simplicity we may assume the water when first brought under the influence of the propelling agent to be at rest relative to the surrounding still water. Let v_1 be its velocity sternward relative to the same datum when delivered by the propelling apparatus. The change of velocity per second is then v_1. The change of momentum which will equal the thrust will be therefore

$$T = \frac{Mv_1}{g}.$$

The useful work is therefore

$$W_1 = Tu = \frac{M}{g} v_1 u.$$

The total work will be Tu plus that involved in the acceleration and disturbance of the water, and possibly in raising it against gravity. If for simplicity we neglect the loss due to eddies and disturbance and suppose the water discharged under the same average head as that under which it enters, we shall have for the total work

$$W = Tu + \frac{Mv_1^2}{2g}.$$

Hence $$e = \frac{u}{u + \frac{v_1}{2}}.$$

This is sometimes called Rankine's ideal efficiency, since it is the limiting value of the efficiency under the suppositions made. In the actual case the additional losses due to eddies, etc., will raise the waste work to $\frac{Mv_1^2}{g}$ or more, and decrease the efficiency to

$$e = \frac{u}{u + v_1} \text{ or less.}$$

Now for anything approaching moderate or high speeds the necessary thrust will require correspondingly large values of M and v_1, or of both. The first means heavy machinery and the second low efficiency. In any practical case the value of v_1 is so large as to reduce the efficiency below that of a properly installed screw propeller or paddle-wheel. Ideal conditions for e would require small v_1, and hence large

M; but these are impossible to realize in practice, and the efficiency is therefore necessarily small.

For special purposes where moderate speed is sufficient, where simplicity is an important element, or where an ordinary propeller might be liable to race violently or become fouled, or where great manœuvring power is required as in a steam life-boat, this mode of propulsion may have advantages which will justify its use. The manœuvring power is usually obtained by multiple-discharge nozzles so situated and so adjustable that the reaction may be directed in any line, and the boat propelled in either direction or transversely, or turned in either direction as on a pivot.

40. Screw Turbines or Screw Propellers with Guide-blades.

Thornycroft's screw turbine may be taken as an illustrative example of this type of propelling agent. The propeller proper consists of a hub *ABC*, Fig. 63, with blades *ADEB*

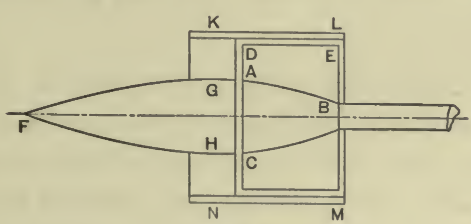

Fig. 63.

very much longer axially and shorter radially than the common propeller-blade. In consequence the inclination of *DE* to the axis is slight. Aft of this is a projection *FGHKLMN* fixed to the ship. This consists of a prolongation of the boss by the spindle-formed body *FGH*, a cylindrical shell or casing *KLMN*, and a series of blades connecting this casing to the spindle-formed body as indicated at *KG* and *HN*. These blades have a slight inclination in the opposite direction to that of the propeller-blades *ADEB*. The pitch of the latter blades is variable. The product of the revolutions by the pitch at *E* is intended to be equal to the supposed velocity of feed, or the velocity of the stern of the boat through the wake unaffected by the action of the propeller. The product of the revolutions by the pitch at *D* is likewise intended to be equal to the speed of delivery, and the increase of size in *ABC* and consequent reduction in area of stream are intended to be such as to give for this variable velocity a steady flow from *BE* to *DA*. The water on leaving the blades of the propeller has a considerable rotary component, due to which it impinges on the fixed blades *KG*. By their guidance it is brought gradually to an axial direction, and thus finally delivered. The long projection *FGH* is for the purpose of providing for something approaching steady flow after leaving the guide-blades, and to insure the gradual union of the various streams with each other and with the outside water.

From the character of the actual wake as explained in § 41, it is quite certain that while in a general way the intention of the design as to the distribution of pitch and sectional area of stream to suit the velocities of feed and discharge may be partly realized, yet such result must be far from exact, and it is doubtful if such variation of pitch can have much influence on the actual result.

Remembering the relation of efficiency to the characteristics of a propeller, it is evident that there is no reason for expecting any marked difference in this feature between this form and the common propeller. If, then, there is any increase in ultimate efficiency it will probably be found in the relation of the propelling agent to the ship. Either the augment of resistance due to the propeller must be less or the wake factor greater (§ 44), or both. There seems to be no reason for assuming any essential difference in the wake factor, but it may be fairly expected that due to the pressure on the blades KG, the normal direction of which will have a component forward, there may be some return for the increase in hull-resistance due to the action of the propeller. On the other hand the additional attachment furnishes a considerable increase of resistance, and hence it is doubtful if the net resistance is much less than with a propeller of the usual form. We should therefore expect about the same ultimate efficiency. Such conclusions are borne out by experience.

The presence of the surrounding casing does, however, prevent the indraught of air and the radial escape of the water, and thus increases the resistance of the blades to revolution, and hence the thrust. It results that the requisite thrust may be obtained from a propeller of smaller diameter than when of the usual form, especially if the draft is small and the tips of the blades of a common propeller would come near the surface of the water. In such cases there seems to be a field of usefulness for propellers of this character, and many of the Thornycroft type have been installed with very satisfactory results.

CHAPTER III.

REACTION BETWEEN SHIP AND PROPELLER.

41. The Constitution of the Wake.

In our treatment of propulsion we have to this point assumed the propelling agent to act in undisturbed water. In the actual case this is far from correct, and we have now to examine the effect of irregularly disturbed water on the preceding results.

We will first note the cause and nature of the disturbance. Taking the screw propeller as the typical propelling agent, its place of action is at the stern in what is termed the wake. The water in this immediate neighborhood is subject to the following disturbing causes:

(1) *Stream-line Motion.*—In § 2 we have seen that the relative motion between the water and the ship is less at the stern than between the ship and the outlying undisturbed water, so that relative to the latter the stream-line action will give a motion forward. Due to the same general cause, the direction of the relative motion of the water and ship is aft, with an upward and inward component as it follows the contour of the vessel. This brings its direction of flow oblique to the plane of the propeller Due to the difference in their location, this is more strongly marked with twin-screws than with a single screw. The influence of this obliquity of flow will be discussed at a later point. This part

of the wake is frequently referred to the wave-motion due to the system of waves generated by the motion of the ship. We prefer, however, to refer it to stream-line motion, as a more fundamental aspect of the cause. This naturally includes all features of the wake contained in the freely-flowing water, or in that not involved in frictional and dead-water eddies.

(2) *Skin resistance or Frictional Wake.*—The nature of the action between the skin of the ship and the water has been discussed in § 7, from which it appears that the whole surface is accompanied by a layer of turbulent water partaking more or less of the forward movement. This motion is greater as we approach the after end, and at the stern we shall have a considerable mass of water moving forward relative to the surrounding body of still water.

As to the distribution of the resulting complex wake, we have the following considerations:

Near the surface the form of the ship is relatively full and the convergence of the stream-lines is correspondingly more marked. The disturbance of the normal horizontal distribution of the water by the resulting wave-motion is also relatively more marked near the surface. As to the velocity of the frictional wake, there seems little reason to expect any marked variation with depth for points near the ship's surface. The velocity will, however, rapidly decrease at points successively farther and farther from the surface.

As to the actual distribution of wake-velocity, an interesting series of experiments has been carried out by G. A. Calvert.* Across the stern of a model 28 feet long and representing a full-bodied cargo steamer was fitted a frame

* Institution of Naval Architects, vol. XXXIV. p. 61.

upon which were stretched several vertical wires extending from the deck to some distance below the keel. These wires carried Pitot tubes fitted with vanes and free to swing to the direction of flow. An outrigger extending into undisturbed water carried a similar tube. The heights recorded in these tubes were then translated into velocities by the usual formula, $v = \sqrt{2gh}$.

This furnished the means of determining the velocity of the ship relative to any point in the wake and relative to still water, and hence the velocity of the water at this point of the wake relative to still water. The wake-velocities thus found were expressed as percentages of the velocity relative to still water. In the section of the wake including the screw aperture, the boss alone of the screw being fitted, the percentages in a vertical line near the stern-post varied from 64 to 17 per cent from the surface downward; in a horizontal line near the surface, from 64 to 7 per cent from the stern-post outward; and in a horizontal line near the keel, from 17 to 10 per cent from the stern-post outward. The average amount at this section was about 19 per cent.

An attempt was next made to determine the portion of this due to the frictional wake. To this end a thin plank of the same length as the model was towed at the same speeds. Similar measuring appliances being fitted, the wake speeds at points near the surface of the plank at distances of 1, 7, 14, 21, and 28 feet from the forward end were found respectively 16, 37, 45, 48, and 50 per cent of the speed of the plank. It was also shown that approximately the wake-velocities decreased in a geometrical progression as the distances from the surface increased in arithmetical progression.

From various considerations, for the details of which the original paper may be consulted, the author concludes that of the 19 per cent average wake about 5 per cent was due to frictional wake, 9 per cent to wave-motion, or stream-line motion as we prefer to term it, and the remaining 5 per cent unaccounted for otherwise is charged to the influence of "dead water" or eddying water about the stern due to the comparative fullness of form.

It thus appears that the wake-velocity in general is exceedingly variable throughout the cross-section of the wake stream. Also from this and other experimental determinations to be referred to at a later point, it appears that including simply the water immediately astern of the ship, and hence that likely to be influenced by propellers, its average value may vary from 6 or 8 per cent to 20 or 25 per cent of the speed of the vessel. In general it is found that the wake value is greater as the ship is longer and fuller, and less as it is shorter and finer. For empirical equations relating its value to the characteristics of the ship see § 50, (9) and (10).

42. Definitions of Different Kinds of Slip, of Mean Slip, and of Mean Pitch.

It will be remembered that, unless otherwise stated, all velocities are referred to the surrounding body of still water as datum. We will first suppose the wake to be uniform.

Let u = speed of ship;

v_0 = speed of wake if propeller were not acting and the ship were towed at the speed u;

v_1 = actual speed of water at or just forward of the propeller;

v_2 = final speed of water due to influence of the propeller;

$v =$ pitch $\times$ revolutions $= pN =$ speed of advance of propeller through wake if there were no slip.

Then $u - v_0 =$ speed of advance of propeller relative to the wake;

$u - v_1 =$ speed of advance of propeller relative to the water immediately about it.

$$v - u = \textit{apparent slip of propeller} = S_1; \quad . \quad . \quad . \quad (1)$$

$$v - (u - v_0) = (v - u) + v_0 = S_1 + v_0 = \textit{true slip of propeller} = S_2;$$

or $\quad S_1 + v_0 = \textit{true slip of propeller} = S_2; \quad . \quad . \quad . \quad . \quad . \quad (2)$

$$v_0 - v_2 = \textit{total slip of water} = S_3. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Usually v_0 is directed forward and v_2 aft, and hence S_3 will be numerically the sum of v_0 and v_2.

It should be especially noted that these three kinds of slip are separate and distinct, though of course not independent. In particular it should be noted that S_2 and S_3 are not the same.

In the simple ideal case of Fig. 55 we may readily relate the value of S_3 to S_2 or s_2 as follows:

It is shown in mechanics that the action of a surface in deflecting a stream—omitting skin-resistance and the formation of eddies—involves simply a change in direction without change in velocity, and the total force interaction between the stream and the surface is measured in direction and amount by the resultant change in momentum. In that diagram the water is considered as approaching the blade with velocity represented in amount and direction by EA, and as leaving with the same velocity parallel to the blade. This is represented in amount and direction by making $KA = EA$. Then by the usual composition of motions the resultant change is represented in direction and amount by EK. The longitudinal component of this is EG, and this is therefore the acceleration which is directly useful in providing thrust.

Now if the slip angle CAE is small we may put

$$EG = EC \cos^2 \alpha = s_2 pN \cos^2 \alpha = S_2 \cos^2 \alpha. \quad . \quad (4)$$

It thus appears that the velocity S_3 is variable for a given value of S_2, increasing as α decreases, and therefore increasing from the hub outward, at all points, however, being less than S_2.

Effect on the Preceding due to the Irregularity of the Wake.—From (2) we have

$$S_2 = S_1 + v_0;$$

whence
$$s_2 = s_1 + \frac{v_0}{v}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Now remembering the actual wake as described in § 41 it appears that $v_0 \div v$ may vary from perhaps 0 to 50 per cent or more. The value of s_1 is usually found between 10 and 20 per cent. Hence s_2 may vary from say 5 or 10 per cent to 70 per cent or perhaps even more. It thus appears that the value of the true slip is variable over the surface of the blade and from point to point in the revolution between the very wide limits noted above, and these may not perhaps be the widest extremes. It is indeed quite possible that for certain elements the slip might be 0 or even negative, while for others its value might rise possibly to nearly 100 per cent. It is thus seen that the idea of slip for the propeller as a whole, acting in the wake, has lost entirely the simplicity of meaning which we were able to give to it when dealing with a single element in undisturbed water.

In spite, however, of the wide range of extreme values which it seems possible that the slip may have, the conditions for most of the blade and for most of the revolution are such as to give rise to a much narrower range of fluctuation, and most of the work is undoubtedly done between a comparatively narrow range of variation. We are thus led to the definition of *mean* slip. This admits of definition in various ways.

(*a*) It might be taken as the arithmetical mean for an entire revolution, of the various slips of the elements composing the driving-face.

(*b*) Since these elementary values are quite unequal in relative importance according to their location and the value of the slip, it might seem more fair to give to each element a weight corresponding to the gross work absorbed by it. Thus if w_1, w_2, w_3, etc., denote the amount of work absorbed by each element, and s_1, s_2, s_3, etc., the corresponding values of the slip, then

$$\text{mean slip} = \frac{w_1 s_1 + w_2 s_2 + \ldots}{w_1 + w_2 + \ldots} = \frac{\Sigma ws}{W}.$$

(*c*) Instead of giving to each elementary slip a weight proportional to its gross work, we may take its thrust or useful work. Denoting the elementary thrusts by t_1, t_2, etc., we should then have

$$\text{mean slip} = \frac{t_1 s_1 + t_2 s_2}{t_1 + t_2} = \frac{\Sigma ts}{T}.$$

An approximate method of applying the weights thus indicated in (*b*) and (*c*) will be given below under the discussion of mean pitch.

(*d*) In § 35, (12), the total work of an element is expressed as a function of the geometrical form of the propeller and the slip. For the entire propeller with variable slip the total work will be represented by the summation of such elements. Suppose now that the slip instead of variable is constant at a value $\bar{s}$, all other conditions remaining as before, and that the corresponding summation for total work W is the same as before. Then evidently $\bar{s}$ may be considered relative to the variable distribution of slip as a mean or equivalent value. This is the same as defining *mean slip* as the slip at which the same propeller in a uniform stream at the same number of revolutions would absorb the same total work as in the actual case.

(*e*) Instead of total work as in (*d*), the definition may be similarly founded on useful work or thrust. This is the same as defining *mean slip* as the slip at which the same propeller in a uniform stream at the same number of revolutions would give the same useful work or the same thrust as in the actual case.

In all of our references thus far to an entire propeller we have assumed it to be of uniform pitch on the driving-face. On this assumption we have discussed the variability of slip over the surface and throughout the revolution, and have given various definitions of mean slip as above. Propellers are frequently made, however, with pitch variable over the driving-face, and thus is introduced another element of variation into the distribution of slip, and also the need for some definition of the terms *mean pitch* and *mean slip* as applied to such propellers.

For the former we may take a geometrical basis and define *mean pitch* as the mean of the distributed values taken over the driving-face. Where the pitch varies simply from the leading to the following edge, the mean of the two values at these points is frequently taken as the mean pitch instead of the more distributed mean as above.

Instead of taking the simple mean of the distributed values, we might perhaps more properly give to the pitch of each element a weight proportional to the work which it absorbs or to the thrust which it develops. By reference to § 35, (8) and (6), it appears that this would be very closely realized by giving to the pitch of each element a weight represented in the first case by the product of its area by the *cube* of its velocity or by the *cube* of its radius, and in the second case by the product of the area by the *square* of the velocity or radius. The following actual case of a four-bladed model propeller may be given as an illustration of the determination of the reduced mean pitch in the second manner. The mean pitch across the blades was taken at four radial distances nearly equal to .3, .5, .7, and .9 the radius, and the area was taken as proportional to the breadth. The work for one blade was then arranged as follows:

Radius.	r^2	Breadth.	Pitch.	r^2b	r^2bp
1.77″	3.13	3.30″	16.13″	10.33	166.6
2.97	8.82	3.60	15.90	31.75	504.8
4.07	16.56	3.30	15.96	54.65	872.2
5.26	27.67	2.16	15.78	59.76	943.2
				156.49	2486.8
					15.89

The quotient of the sum of the column r^2b into the sum r^2bp gives for this blade the reduced mean pitch equal to 15.89″. For the other blades the values similarly found were 15.45″, 15.80″, and 15.54″. This gives as the final mean for the entire propeller the value 15.67″.

A value of the mean pitch being thus found from either a simple or weighted mean, a definition of mean slip follows thus:

Assume a propeller similar to the one given in all respects except as to pitch, which shall be uniform and of the mean value as above defined, and let this propeller working in a uniform stream at the same number of revolutions absorb the same amount of total work as the given propeller. Then the slip at which such conditions would be fulfilled would be a mean or equivalent slip for the given propeller. This is evidently similar to (d) above for the case of uniform pitch. We may also as in (e) take the useful work or thrust as the basis for a similar definition of mean slip.

Instead of defining mean pitch on a purely geometrical basis, we may give it a dynamical definition as follows:

Let the given propeller work in undisturbed water with given revolutions and speed. Let there be a propeller of uniform pitch with the same diameter, area, and shape of blades, and let it work in undisturbed water at the same revolutions and speed. Then the pitch at which the latter propeller would have the same turning moment, or at which it would absorb the same work, as the first, may be considered as the equivalent mean pitch of the former. Similarly the definition may be based on equivalent thrusts instead of equivalent turning moments.

It may be remarked that the pitch thus determined would doubtless vary with the revolutions, and with the relation between revolutions and speed, so that it could not be considered as a fixed dimension of the propeller.

A dynamical definition of mean pitch having been thus taken, the definition of mean slip would most naturally follow according to (*d*) or (*e*) above.

In all such cases where the mean pitch or slip of a propeller is based on its performance in comparison with that of a propeller of uniform pitch, the influence of the thickness or rounded back is virtually involved. To this we may briefly allude.

Given a body of the cross-section of a propeller blade as in Fig. 64, moved relative to the water in the direction of the

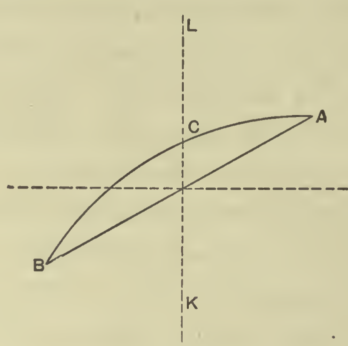

FIG. 64.

face BA. Then the distribution of the stream-lines is such that relative to the pressure on AB there is an excess from A to C and a more or less pronounced defect between B and C. The result is in general represented by a pressure on the face AB having its center much nearer B than A, and hence tending to turn the blade about in the clockwise direction as here viewed. If the direction KL is axial, there will result in such case a positive or forward thrust. That is, if the propeller be run in undisturbed water at such revolutions relative to the speed that the slip of the driving-face is 0, there will in general result a slight positive or forward thrust due to this distribution of the stream-line motion, and it is not until the slip of the driving-face is still further decreased and made slightly negative that the fore and aft components of the total surface forces exactly balance and give a zero thrust. It may also be noted that this zero thrust is actually the resultant of a component aft due to the tangential forces or those due to skin-friction and edge-resistance, and of a component forward due to the normal forces or those due to the stream-line pressures. If, therefore, the former could be reduced or eliminated the propeller would, under the conditions just assumed, still show a positive thrust, and it would require a still further increase of negative slip to reduce the longitudinal component of the normal pressures to zero. These results have been frequently noted experimentally, and have been made the subject of quantitative measurement in a series of experiments carried on by the author.*

This is what is frequently referred to by the statement that the addition of thickness results in a virtual increase of pitch, because if the equivalent pitch be taken as (longitudinal speed for zero thrust) ÷ (revolutions), the result will be greater than that derived by the measurement of the driving-face, and the excess would be still greater could the tangential forces be eliminated.

We have thus discussed various possible definitions of mean pitch and mean slip in order to show the variety of meaning which may be given to these terms, and the consequent inexactness of significance attending their use without some agreement as to the basis of definition. Our use of the equations of § 36 will virtually assume the definition of mean slip as based on (*e*). That is, we shall assume, as will be

* Transactions Society Naval Architects and Marine Engineers, vol. v.

explained later, that for the actual variable slip may be substituted some constant equivalent value for use in the equations giving the value of the total useful work.

In regard to propellers of variable pitch it may be noted that such design is usually intended to provide for some variable distribution of slip over the surface, assuming the propeller to work in a uniform stream. When, however, we remember the great variability of the stream or wake, it is quite evident that any attempt to secure any specified distribution would be entirely futile, and that in any given case the actual distribution will be quite different from that intended. Hence any effects resulting from a variable pitch will be quite accidental, and it is very doubtful if, in the present state of our knowledge, there is anything to be gained by introducing such a feature into our designs. These conclusions seem to be borne out by experience, for propellers of uniform pitch have shown themselves in practice to be equal in efficiency to those in which the pitch is variable according to various laws. We shall therefore pay no further attention to variable pitch as a feature of screw propellers, but shall in all cases assume them to be of uniform pitch over the entire driving-face.

43. Influence of Obliquity of Stream and of Shaft on the Action of a Screw Propeller.

In general the line of the shaft, the direction of the stream, and the direction of advance are all different. This is illustrated in Fig. 65, where AC represents the direction and speed of advance of the ship relative to still water, BC the direction and speed of the stream relative to the same, OF the direction of the shaft, and OA the speed and direction of the element relative to the ship. Then by the composition of relative motions it follows that BA represents the direction and speed of the stream relative to the ship, and BO that of the stream relative to the blade or element. Let v_0 denote the velocity of the stream relative to still water as represented

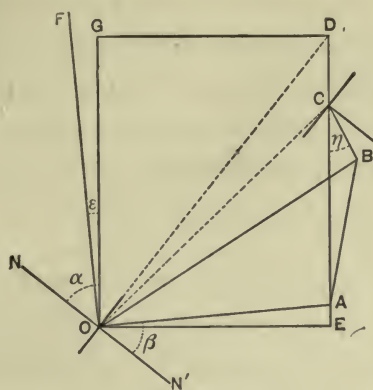

FIG. 65.

by BC, and denote the angle BCA by η. Then, as in § 33, we denote the angle between OG and the normal ON to the element by α. The inclination of v_0 to the normal is then $(\alpha - \eta)$, and $v_0 \cos(\alpha - \eta)$ is the component of v_0 in the direction of the normal, and hence the velocity in this direction which would be impressed on the element by the stream of velocity v_0, if there were no slip. The result of this in the direction of advance would be a velocity $v_0 \cos(\alpha - \eta) \sec \alpha$.

From § 33 the longitudinal velocity without slip in still water would be $v \cos \beta \sec \alpha$. Hence denoting the total speed of advance if there were no slip by u', we shall have

$$u' = v \cos \beta \sec \alpha + v_0 \cos(\alpha - \eta) \sec \alpha$$
$$= (v \cos \beta + v_0 \cos(\alpha - \eta)) \sec \alpha.$$

In the present case, with the shaft at an angle ϵ with the line of advance, the angle β for a given element is constant, while α varies. The exact values of α depend on the solution of a spherical triangle, as is readily seen, and these values will vary through a range 2ϵ. In consequence, the values of the speed of advance without slip, u', will vary through a corresponding range. Denote the mean value of u' by u_0' and the range of its variation by $2\Delta u'$. Also, let θ denote the angular location of the element reckoned from the position for which u' is maximum. Then it is found, if ϵ and v_0 are relatively small, that we have approximately

$$u' = u_0' + \Delta u' \cos \theta.$$

The value of u' thus given is seen to vary from $u_0' + \Delta u'$ through u_0', $u_0' - \Delta u'$, u_0 to $u_0' + \Delta u'$ for the successive quadrants of an entire revolution. The actual speed of advance being u, it follows that the difference $u' - u$ or the actual slip S, and the percentage slip s may likewise be expressed in the same approximate form. Hence we shall have

$$S = S_0 + \Delta S \cos \theta;$$

$$s = s_0 + \Delta s \cos \theta.$$

Denote the mean value of α by α_0. This is seen to be the angle between the normal and the direction of the shaft. Then

$$u_0' = (v \cos \beta + v_0 \cos (\alpha_0 - \eta)) \sec \alpha_0;$$

and $$S_0 = u_0' - u.$$

We must now find the value of $\Delta u' = \Delta S$. We have for the maximum value of u' approximately

$$u'_{\text{max.}} = (v \cos \beta + v_0 \cos (\alpha_0 + \epsilon - \eta)) \sec (\alpha_0 + \epsilon).$$

Then $$u'_{\text{max.}} - u_0' = \Delta u' = \Delta S.$$

Considering ϵ small, this difference is readily put in the following approximate form:

$$\Delta u' = \Delta S = u_0' \frac{\epsilon}{\cot \alpha_0 - \epsilon}.$$

It thus appears that the values of ΔS and hence of Δs vary over the entire surface of the blade, and hence the entire distribution of S and s will vary from point to point over the surface and from one position to another in the revolution. The value of the range of slip $2\Delta S$ is seen to increase with ϵ and with α_0. Hence the value of ΔS will continuously decrease for locations of the element from the hub outward. The position from which θ must be counted is seen from the following considerations. The origin for θ is the prositon for maximum value of u'; and since v_0 is small, u' will depend chiefly on v, and will reach its maximum very near the position for which sec α is maximum or α maximum or when $\alpha = \alpha_0 + \epsilon$. Hence the 0 position of θ is readily seen to be near that in which the normal is parallel to the plane determined by the shaft and direction of advance, and in which the normal and direction of advance lie on opposite sides of the shaft.

Thus with twin screws, the starboard turning to the right and the port to the left, let the shafts incline outward from the engine aft, as usually fitted. Then it follows from the above that for each blade of each propeller the slip is maximum near its highest position and minimum near its lowest. Similarly, with a right-hand propeller and the shaft inclined downward the slip of any blade is maximum when it is horizontal and directed to the right, and minimum when horizontal and directed to the left. With right- and left-hand twin screws, as above noted, the effect of the obliquity of the shafts is still further increased by the natural obliquity of the inflowing streams as they follow the contour of the ship, thus giving a direction of stream relative to still water somewhat like BC of Fig. 65. Inclination of the shafts in the opposite direction would tend to correct the variability of the slip due to these streams, but from structural reasons this is rarely permissible.

We have discussed in the preceding section the influence of the variability of the wake on slip, and in the present section the influence of obliquity, assuming thus far a uniform stream or wake. Combining these influences we shall evidently have in any given case variations of slip over the surface and throughout the revolution ranging through limits, variable themselves, but probably as wide as from 0 or a negative value to 50 or 75 per cent or more. We shall have therefore the variability noted in §§ 41, 42, with an added amount due to obliquity. As noted in § 41, however, most of the work will be done between narrower ranges, but due to obliquity even these may be considerable. It becomes therefore a matter of importance to inquire to what extent the formulæ of § 36 may be invalidated by the existence of this variability of slip.

44. Effect of the Wake and its Variability on the Equations of § 36.

We will first take the influence of the variability of the wake on the efficiency.

Let e_1, e_2, etc., denote the efficiencies of the various elements. Then using the previous nomenclature we shall have for the entire propeller

$$e = \frac{U}{W} = \frac{w_1e_1 + w_2e_2 + \ldots}{w_1 + w_2 + \ldots} = \frac{\Sigma we}{W}. \quad . \quad . \quad (1)$$

If now the value of e varies by a linear law with slip for the range of variation, we shall have

$$e_1 = e_0 + as_1;$$
$$e_2 = e_0 + as_2.$$
$$e_3 = e_0 + as_3.$$
etc.

Hence we find

$$\frac{U}{W} = e_0 + a\frac{w_1s_1 + w_2s_2 + \ldots}{w_1 + w_2 + \ldots}. \quad . \quad . \quad . \quad (2)$$

We know that efficiency does not actually vary by a linear law with slip, but if most of the work is done between a relatively narrow range of variation of slip, the variation of efficiency may be approximately expressed by such a law. The resulting efficiency in (2) is then seen to be that corresponding to the mean slip as defined in § 42 (*b*). The actual efficiency will be somewhat less than that given by (2), and we may consider that this equation simply indicates the general conditions under which no great loss of efficiency shall result from variable slip, viz., that most of the work must be done within a comparatively narrow range of slip variation.

This conclusion is borne out by the experiments of R. E. Froude, and we may consider that the latter constitute really the basis for our assumptions relative to the wake. These are:

That the actual turbulent variable wake may be considered as sensibly equivalent to a single uniform wake, and that the performance of the propeller may be considered as equivalent to that which would result from the presence of such a wake.

The true mean slip S_2 is then greater than the apparent slip S_1 by the amount of this uniform substituted wake, which we may denote by v_0. *Vice versa* it results that the amount of the uniform substituted wake is equal to the difference between the true mean slip based on the definition of § 42, (*e*), and the apparent slip S_1.

Stating again the relation somewhat differently, we note that the true slip is greater by variable amounts than the apparent slip, and the actual thrust developed is greater than that which would result in undisturbed water by the screw working at the apparent slip. At some greater slip in undisturbed water, however, the thrust developed would be the same as that actually obtained. This greater slip may then, according to § 42 (*e*) be considered as the equivalent or mean true slip in the actual case, and the difference between the true and apparent slips thus defined will represent the amount of uniform wake considered as equivalent to the actual variable wake.

This definition of equivalent wake is based on an equivalence of thrusts, as stated in § 42 (*e*). We might also base a definition on an equivalence of turning moments or total works, as stated in definition (*d*). If now the wake itself were uniform these two definitions or modes of determination would evidently lead to the same value of the wake. The difference in the two values thus determined indicates therefore the effect due to the turbulence or irregularity of the wake. Experiments on this point are not sufficiently extended to furnish very complete evidence as to the exact influence which turbulence may play, but its value is believed to be small, and in any event in default of sufficiently extended information we are compelled to assume its influence as negligible. The actual point where the influence of turbulence or irregularity of wake touches our methods of design is in its influence on efficiency, to which reference has been made in the early part of the present section.

We therefore virtually assume in all cases, for the actual turbulent wake, the substitution without change in efficiency, of a uniform wake of velocity v_0, the amount of this velocity being based on an equivalence of thrusts as previously explained. Let

$$\frac{v_0}{u - v_0} = w.$$

This relates v_0 to $(u - v_0)$, the speed of advance of the propeller through the water about the stern. Whence

$$v_0 = \frac{wu}{1 + w}.$$

From § 42, (5), we have

$$s_2 = s_1 + \frac{v_0}{v} = s_1 + \frac{w}{1 + w}\frac{u}{v} = s_1 + \frac{w}{1 + w}(1 - s_1);$$

whence $$s_2 = \frac{s_1 + w}{1 + w};$$

$$s_1 = s_2(1 + w) - w;$$

$$w = \frac{s_2 - s_1}{1 - s_2}; \qquad (3)$$

$$1 + w = \frac{1 - s_1}{1 - s_2}. \qquad (4)$$

Let us next consider the influence of the uniform wake on the expressions for useful work and efficiency in § 36, (2) and (4).

It is readily seen that the useful work must be obtained by multiplying the thrust by the speed of the ship relative to still water, and hence by $(1 - s_1)pN$. In all other places, however, where slip enters into these expressions it will be the true slip s_2. Hence we shall have in such case

$$U = \pi^3 p^3 N^3 (1 - s_1) \int y^3 (asB - fC) dA;$$

$$W = \pi^3 p^3 N^3 \int y^3 (asB + fE) dA.$$

The ratio of these two will give an apparent efficiency. The true efficiency must, of course, be that for the propeller working at a slip of s_2. The presence of the wake cannot change the efficiency of the propeller itself, while it may increase the amount of useful result U. The increase in U must be credited to the wake rather than to the propeller. These points will be again referred to in § 46. For these reasons we call the ratio of U to W an apparent rather than a real propeller efficiency. This we may denote by e_1, while the true efficiency, which we will denote by e_2, will have the value as given in § 36, (4). Hence we have

$$e_1 = (1 - s_1) \frac{\int y^3 \left(\frac{a}{f} sB - C\right) dA}{\int y^3 \left(\frac{a}{f} sB + E\right) dA}, \quad . \quad . \quad . \quad (5)$$

and $$e_2 = e_1 \frac{1 - s_2}{1 - s_1} = \frac{e_1}{1 + w}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

45. Augmentation of Resistance due to Action of Propeller.

We have thus far considered the influence of the ship, through the wake which it produces, on the action of the propeller. We now turn to the influence of the propeller on the resistance of the ship.

We have already in § 37 considered the action of the propeller in producing in front of itself a defect of pressure and thus imparting a portion of the total acceleration produced, before the water acted on reaches the propeller itself. This defect of pressure is for the most part due to the centrifugal force consequent upon the rotation of the race by the propeller. The rotation will evidently be greater as the blades stand more nearly fore and aft, and hence as the pitch-ratio is higher. This defect of pressure will interfere seriously with the natural stream-line motion. The water instead of being able to close around the stern and form the natural stern wave will be more or less disturbed and drawn away to the propeller. The result of this is a diminution of the pressure which would naturally exist about the stern if the ship were towed at the same speed. The result of this is, of course, an increase in the amount of resistance to be overcome by the propelling agent. This increase when viewed from the standpoint of the resistance may be termed the augment or augmentation of resistance.

Where paddle-wheels are used as the propelling agent, a like augmentation is experienced though it arises from somewhat different causes. The action of the paddle-wheels at the sides and nearly amidships gives rise to a race of water moving faster relative to the ship than would be the case if the wheels were not there. The result is an increase in the skin-resistance. The wheels also undoubtedly exercise a disturbing influence on the natural stream-line motion, and they may thus introduce an additional element into the augmentation of resistance. The amount of augmentation due to paddle-wheels has not been as extensively determined experimentally as for screw propellers, but various experiments by Dr. Tidman, R. E. Froude, and Messrs. Denny indicate that it does not much differ in amount from that for a screw propeller under like conditions of speed.

46. Analysis of the Power necessary for Propulsion.

Let $W =$ the indicated horse-power;

$W_f =$ the power absorbed by the friction of the engine and shafting, and by any attached pumps.

Then $W - W_f = W_p =$ power delivered to propeller.

Let $T_0 =$ the necessary thrust, supposing the ship towed at the given speed u;

$T =$ the actual thrust $= T_0 +$ the amount of augmentation.

Then $T_0 u$ is called the effective horse-power; and Tu is called the thrust horse-power. The ratio $\frac{T_0 u}{W}$ is called the propulsive coefficient.

The ratio $\frac{T_0 u}{W_p}$, which is much more definitely related to propulsive efficiency, has unfortunately received no special name. We shall here distinguish it as the coefficient h.

As defined in § 33, the apparent propeller efficiency is

$$e_1 = \frac{Tu}{W_p};$$

while as in § 44 the true propeller efficiency is

$$e_2 = \frac{T(u - v_0)}{W_p} = e_1 \frac{u - v_0}{u} = \frac{e_1}{1 + w}.$$

The efficiency e_2 is the ultimate test of the performance of the propeller considered simply as a propeller. It is given a certain amount of work W_p. Working in undisturbed water at the same true slip, $S_2 = (v - u + v_0)$, it would develop the thrust T, and deliver an amount of work $T(v - S_2) = T(u - v_0)$ as in the numerator of e_2 above.

If, however, we consider the propeller simply as a means of getting the ship through the water, the useful work will be $T_0 u$, since this is the amount which would be required to tow the ship at the speed u. The ratio $T_0 u \div W_p$ is therefore the final test of the value of the propeller as a means of actually propelling the ship. If the ship produced no wake and the propeller no augmentation of resistance, this ratio $T_0 u \div W_p$ and the ratio $e_2 = T(u - v_0) \div W_p$ would be the same. The relation between them, therefore, involves the mutual interaction of ship and propeller. The ratio expressing this relation is hence termed the hull efficiency. We have therefore

$$\text{Hull efficiency} = \frac{\text{coefficient } h}{\text{true propeller efficiency}};$$

or coefficient h = hull efficiency × true propeller efficiency. Hence

$$\text{Hull efficiency} = \frac{T_0 u}{W_p} \div \frac{T(u - v_0)}{W_p} = \frac{T_0}{T} \cdot \frac{u}{u - v_0} = \frac{T_0}{T}(1 + w),$$

and

$$\text{Coefficient } h = \frac{T_0}{T} \frac{u}{u - v_0} e_2 = \frac{T_0}{T}(1 + w) e_2.$$

The hull efficiency is thus seen to be the product of two factors $T_0 \div T$ and $u \div (u - v_0)$. The first is the ratio of the true to the augmented resistance, or the reciprocal of the coefficient of augmentation as above defined. This factor has been termed by R. E. Froude the "Thrust-deduction factor." We shall, however, prefer to consider it as the reciprocal of the coefficient of augmentation, $T \div T_0$. The effect of this factor, as seen, is to decrease the hull efficiency in the ratio $T_0 \div T$. It therefore implies a loss in efficiency due to the augmentation of resistance.

The other factor $u \div (u - v_0) = (1 + w)$, as already defined in § 44. The effect of this factor, as seen, is to increase the hull efficiency in the ratio $(1 + w)$. It therefore implies a gain in efficiency due to the location of the propeller in a forward wake instead of in undisturbed water. This gain in efficiency is seen to be due simply to the gain in the speed of advance. That is, if the propeller were working in undisturbed water at the same revolutions and true slip S_2 and consequent thrust T, the speed of advance would be $(v - S_2)$. In the actual case, due to the fact that the wake water is borne forward, so to speak, to meet the propeller, the same true slip S_2 is obtained with a speed of advance u greater than $(v - S_2)$ by the speed of the wake v_0. Hence the two speeds of advance are u and $u - v_0$, and their ratio is $(1 + w)$, as defined.

Of the two factors thus constituting hull efficiency, one is seen to be greater and the other less than 1. Hence they tend mutually to offset each other, and the product or the hull efficiency will usually not vary widely from 1. In fact many determinations by R. E. Froude indicate that the hull efficiency may usually be taken as 1 without sensible error, and that the causes which seem to increase one factor will correspondingly decrease the other, and *vice versa*. Hence the coefficient h may usually be taken as sensibly equal to the true propeller efficiency e_2. It should not, however, be forgotten that the hull efficiency is by no means necessarily 1, and that circumstances might arise in which such an assumption would involve a sensible error.

This analysis of the total power and the various relationships involved may be illustrated by the diagram of Fig. 66.

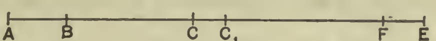

FIG. 66.

The total power, revolutions, and thrust are supposed to remain constant. The subdivision is then shown both with and without wake. This is as follows:

AE = I.H.P. = total power;

AB = power absorbed by friction of engine and attached pumps;

BE = power delivered to propeller = W_p;

BC = power absorbed in the eddies, rotation, and sternward acceleration communicated to the water acted on, assuming the existence of a wake;

BC_1 = power absorbed under the preceding head, assuming that there is no wake and that the propeller works in undisturbed water;

CE = thrust horse-power = Tu, assuming the existence of a wake;

C_1E = thrust horse-power without wake = $T(u - v_0)$ = actual thrust T multiplied by the reduced speed $(u - v_0)$ which would correspond to the assumed power, revolutions, and slip with no wake;

FE = power absorbed by the augmentation of resistance. This for convenience is assumed as the same with or without wake;

CF = power absorbed by the true or towed resistance in the case with wake = effective horse-power;

C_1F = power absorbed by the true or towed resistance in the case without wake.

Therefore with fixed power, revolutions, thrust, and resistance the wake would increase the speed from $(u - v_0)$ to u or in the ratio $(1 + w)$, and hence the useful effect in the same ratio.

The various ratios are also illustrated as follows:

$$\frac{CF}{AE} = \text{propulsive coefficient};$$

$$\frac{CF}{BE} = \text{coefficient } h;$$

$$\frac{CE}{BE} = \text{apparent propeller efficiency};$$

$$\frac{C_1E}{BE} = \text{true} \quad \text{"} \quad \text{"}$$

$$\frac{CE}{CF} = \text{coefficient of augmentation};$$

$$\frac{CE}{C_1E} = \frac{u}{u_1} = \frac{u}{u - v_0} = (1 + w) = \text{wake-return factor};$$

$$\frac{CF}{C_1E} = \text{hull efficiency}.$$

It will be noted that the diagram of Fig. 66 is not applicable to the *same ship* with and without wake, for the power, revolutions, thrust, and resistance are supposed to remain constant, while the speed changes from $(u - v_0)$ to u. The diagram is applicable, however, to the performance of the *propeller* with and without wake, under the conditions that revolutions, thrust, and resistance shall remain constant, and hence that the propeller at the two speeds $(u - v_0)$ and u shall be opposed to the same resistance. This is usually the most useful mode of analysis, as it serves to connect the propeller in its actual surroundings with the same propeller in undisturbed water, developing at the same revolutions and true slip the same thrust.

We may, however, as in Fig. 67, illustrate the effect of the wake on the same ship at constant speed, resistance, and

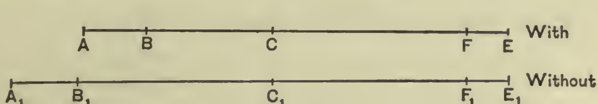

FIG. 67.

thrust of the propeller, but varying revolutions and power. In this diagram $ABCFE$ denote for the ship with wake the same points as in Fig. 66, while $A_1B_1C_1F_1E_1$ denote similar points for the same ship at the same speed and resistance, but without wake. The power represented by CE is of course equal to that represented by C_1E_1. The propeller power BE and the total power AE are, however, less than the corresponding amounts B_1E_1 and A_1E_1. This saving of power comes about as follows: As shown in § 36, (1), thrust varies with revolutions and with slip. Now if there were no wake and the slip were S_1, the necessary thrust would be developed by a certain number of revolutions. With the wake v_0 and the same speed u the slip becomes increased to $S_1 + v_0 = S_2$, and the revolutions necessary to develop the fixed thrust are correspondingly decreased. Now with constant thrust the turning moment or torque, and hence the mean effective pressure in the cylinders, will all remain very nearly constant. Hence the propeller horse-power W^p and the indicated horse-power W will vary very nearly as the revolutions. Hence while the work CE will equal C_1E_1, the work BE and the total work AE will be less respectively than B_1E_1 and A_1E_1 very nearly in the same ratio as that in which the revolutions are reduced. In this way, then, it is seen that the presence of the wake makes it possible to obtain the necessary thrust and propulsion of the ship with a lower number of revolutions and with a correspondingly decreased engine-power than in undisturbed water.

The wake is due to the motion of the ship, and its kinetic energy is simply the energy which has been put into it by the ship in its movement through the water. The energy of the wake comes therefore from the engine, or more ultimately from the coal or fuel, as the source of energy. The reduction of the engine-power and fuel-consumption, as above explained, may therefore be considered as a return from the wake, or as a reutilization of a small part of the energy which has been expended in its formation.

The amount of this reduction will depend on the relation of thrust, revolutions, and slip, and cannot be expressed in simple terms. From § 36, (14) and (16), however, we have

$$T = \frac{U}{pN(1 - s)} = \text{approximately } BN^2(.034 + .85s),$$

where B includes all terms relating to the geometry of the propeller. Hence

$$N^2 = \frac{T}{B(.034 + .85s)}. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The effect of v_0 on s is readily determined, and for constant T the influence of this on N^2 is shown approximately by this equation. Hence follows the effect on N and on power. As an illustration, suppose $s_1 = .15$ and $w = .133$. Then from § 44, (4), $s_2 = .25$, and from (1) above

$$\frac{N_1^2}{N_2^2} = \frac{.2465}{.1615} = 1.526 \quad \text{and} \quad \frac{N_1}{N_2} = 1.24.$$

Hence with these values the propulsion without the wake would require the development of about 24 per cent more power than in the actual case.

We will now give a numerical illustration of the analysis illustrated in Fig. 66.

Let $$s_1 = .16;$$

$$s_2 = .27.$$

This would mean that the propeller in undisturbed water with 27 per cent slip would give the same thrust as obtained in the actual case at 16 per cent apparent slip, the revolutions being the same in each case.

Then from § 44, $1 + w = 1.151$;

$$w = .151.$$

Let AE, the I.H.P., be denoted by.......	1
Let AB be............................	.14
Then $BE = W_p$.............. ... ...=	.86
Let $CE = Tu$........................=	.67
Then e_1 = apparent propeller efficiency..=	.779
And e_2 = true propeller efficiency.......=	.677
Also $C_1E = CE \div 1 + w$.............=	.582
Hence CC_1.........................=	.088

Let coefficient of augmentation be......= 1.165
Then CF = E.H.P....................= .575
And FE..............................= .095
Then coefficient h.................= .669
Propulsive coefficient.................= .575
Hull efficiency........................= .988

The following table shows the results of certain experiments carried out on a Dutch tugboat.* During the trials the thrusts were measured at the thrust-block by a hydraulic dynamometer, while the values of the E.H.P. were determined from model experiments.

I.H.P.	T.H.P.	E.H.P.	Rev.	Speed.	Principal Dimensions.
31.03	19.76	15.80	94	6.97	
50.56	33.16	27.42	111	8.07	Length = 72′
80.24	53.22	42.74	127.5	9.02	Beam = 14′ 9″
132.35	89.43	70.69	148	10.07	Draft forward = 3′ 10″
170.83	118.85	87.75	160.5	10.47	Draft aft = 7′ 4½″
230.58	161.40	108.46	175	10.84	Displacement = 69 tons
260.32	180.29	120.22	180.5	11.01	

The ratio of T.H.P. to I.H.P. varies with increase of speed from .64 to .69, and that of E.H.P. to I.H.P. from .54 to .46. The pitch of the propeller was 7.63 feet, and, as may be determined, the slip varied from about 1.5 to 19 per cent. The increase of propulsive coefficient at low values of the speed and slip is somewhat abnormal, but whether due to errors in the data or to an increased value of the ship efficiency at these points does not appear. The series of experiments is a valuable one, and it may be hoped that the near future will give us many more of the same character.

* See *Steamship* for October 1897.

47. INDICATED THRUST.

This term, which frequently occurs in engineering literature, is defined as follows:

$$\text{Indicated thrust in tons} = \frac{33000 \text{ I.H.P.}}{pN2240}. \quad . \quad (1)$$

This is a thrust which would correspond to an absence of slip and a total efficiency of 1. This is a wholly impossible set of conditions, but as it is considered convenient for the expression of certain relationships, we will show its connection with a much better known quantity. Let C be an engine constant defined by the equation

$$C = \frac{\text{L.P. cylinder area} \times 2 \times \text{stroke in feet}}{33000} = \frac{2LA_3}{33000}. \quad (2)$$

Then the mean effective pressure reduced to the L.P. cylinder is the pressure defined by the equation

$$\overline{(\text{m.e.p.})} = \frac{\text{I.H.P.}}{CN} = \frac{33000 \text{ I.H.P.}}{2LA_3N}. \quad . \quad . \quad . \quad (3)$$

Now comparing indicated thrust and reduced mean effective pressure, it is readily seen that there is a constant ratio between them, and hence that the one is in constant proportion to the other. Hence we may remember that the indicated thrust is simply a quantity proportional to the reduced mean effective pressure.

We will also show here another expression for the reduced mean effective pressure. Let A_1, A_2, A_3 be the areas of the successive cylinders of a triple-expansion engine, A_3 being the value for the low-pressure cylinder. Let p_1, p_2, and p_3 be the successive actual mean effective pressures in the same cylinders. Then evidently

$$(\overline{\text{m.e.p.}}) = \frac{p_1 A_1}{A_3} + \frac{p_2 A_2}{A_3} + p_3. \quad . \quad . \quad . \quad . \quad (4)$$

The values in (3) and (4) are evidently the same. For multiple-expansion engines of any number of stages the same general method applies. For triple-expansion engines with initial pressures from 160 to 180 pounds absolute, or 145 to 165 by gauge, the reduced mean effective pressure is usually from 30 to 40 pounds per square inch.

48. Negative Apparent Slip.

If there were no wake, the apparent slip s_1 and the true slip s_2 would be the same. With the development of the wake, however, the true slip remaining the same, the apparent slip decreases until some point is reached where an equilibrium of conditions is maintained.

From § 44 we have

$$s_1 = s_2(1 + w) - w = s_2 - (1 - s_2)w.$$

Now if, as stated above, s_2 remains constant and w increases, s_1 will continuously decrease. If w reaches a sufficient value, s_1 will become 0 and then negative. The condition that $s_1 = 0$ is

$$s_2 = (1 - s_2)w \quad \text{or} \quad s_2 = \frac{w}{1 + w}, \quad . \quad . \quad . \quad (1)$$

or

$$w = \frac{s_2}{1 - s_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The question of the possibility or otherwise of a 0 or negative value of s_1 is simply a question of the possibility or otherwise of a wake value equal to or greater than this value $s_2 \div (1 - s_2)$. There is no inherent reason why such a wake should not exist, or why it should not be formed by the motion of the ship through the water.

Such a condition is not, however, desirable, or indicative of a good propeller efficiency. It indicates the existence of either one or both of the following:

(1) A low true slip, and consequently a low efficiency as shown in all efficiency curves, as in § 36.

(2) An excessive ship-resistance corresponding to the formation of a wake of this velocity.

While, therefore, the apparent propeller efficiency will be high, the true efficiency may nevertheless be quite low, and the actual resistance per ton of displacement will naturally be excessive.

As a slightly different way of expressing the condition of 0 or negative slip, we may take § 42, (2). Thus

$$S_1 = S_2 - v_0.$$

Hence if $$S_1 \overline{\gtrless} 0, \quad S_2 \overline{\gtrless} v_0.$$

This may be considered as implying simply that the propeller is capable of furnishing the necessary thrust with a true slip S_2 equal to or less than the wake-velocity v_0. This is evidently equivalent to the condition expressed in (1)

$$s_2 \overline{\gtrless} \frac{w}{1 + w}.$$

Now in Chapter IV practical methods will be given for designing a propeller to fulfil any given program of conditions. To design a propeller which would probably show 0 or negative slip we have therefore simply to fix as one of the conditions a value of s_2 equal to or less than that given by (1), using such value of w as shall be deemed appropriate. Thus for illustration, the value of w is very commonly about .1. Hence $w \div (1 + w) = .09$, and a propeller designed to fulfil the given conditions with a true slip of from 7 to 9 per cent would probably show a 0 or slightly negative apparent slip. Such a propeller would be very large and wasteful, and, as above stated, quite undesirable. These results in actual practice are avoided by the assumption of a much larger value of the true slip. Their mention is simply introduced here in order to show the entire possibility and significance of a 0 or negative value of the apparent slip. While, therefore, the possibility of these results is unquestionable, it is, however, presumable that many reported cases of negative slip have arisen from errors of measurement, especially in the pitch. In particular might this be the case with propellers of variable pitch, in which the definition of mean pitch is necessarily arbitrary in character, according as we make it purely geometrical, or give it a dynamical basis by giving to the pitch of each element a weight proportional to the thrust which it develops or the work which it absorbs. Due to this necessarily arbitrary character of the definition of mean pitch with such propellers, the expressions *mean pitch* and *apparent slip* lose much of the significance which we are able to give to them with propellers of uniform pitch. See also § 42.

The distribution of forces on a screw-propeller blade, referred to in §§ 42 and 52, Figs. 64 and 85, indicates furthermore the possibility of a small positive thrust with a small negative true slip measured with reference to the face. The slip of the water (§ 42), however, must always be positive.

CHAPTER IV.

PROPELLER DESIGN.

49. Connection of Model Experiments with Actual Propellers.

We now take up the considerations relating to the application of experimental data derived from models to the design of full-sized propellers.

We must first remember that the actual data given by Froude's experiments, as described in § 36, relate to certain propellers .68 ft. in diameter, with certain pitch-ratios, at certain revolutions and slips, and with blades of an elliptical shape, of a certain area, material, and thickness, and with hubs of a certain diameter relative to that of the propeller itself. The data thus found was so regular in character and agreed so well with the fundamental propositions (*a*), (*b*), § 36, that it seemed fair to accept these propositions as empirically true for the range of values covered, and hence to accept as reliable the interpolations thereby effected. It therefore follows that the results may be considered as applicable to all propellers of this diameter and character of blade within the given limits of pitch-ratio, and working between the given limits of slip. We will now consider the justice of extending these results to full-sized propellers.

The derivation of § 36, (11), (12), shows that the whole question depends on the supposition that the forces P and Q

vary as the area and as the square of the speed, or that the values of K, L, and $K \div L = e$ are independent of actual dimensions, and are simply functions of pitch-ratio and slip.

In regard to efficiency, a satisfactory general correspondence between the values for model experiments and actual propellers has been testified to by numerous comparisons. In particular it has been found that the general shape of the efficiency-curve, as shown in Fig. 60, is characteristic of the performance of full-sized propellers. According to Mr. Froude's comparisons, we may depend safely on the general character of the efficiency data and on the values themselves considered relatively, while absolutely they may involve a slight error. That is, we may safely depend on the experimental data to show us correctly the general conditions for the best efficiency and how the efficiency will vary for given changes in pitch-ratio and slip, while there may be slight errors in the actual values of the efficiency itself. In any case the amount of error is presumably small, and experience seems to indicate that we shall be quite safe in using these data, at least as a general guide to the desirable range of values of pitch-ratio and slip, within which to work in any given case of design.

The changes which may be involved in passing from the model to the full-sized propeller are of three kinds:

(1) Change in diameter, and hence in all other dimensions in proportion;

(2) Change in area-ratio, $h = A \div \frac{\pi d^2}{4}$;

(3) Change in shape of blade.

In any actual case any or all of these might be involved. The above remarks relating to efficiency imply only change

(1). That is, geometrical similarity between the model and the propeller is supposed to be maintained. Hence only under this condition are the results of the model experiments strictly applicable, and as variations (2) and (3) enter in to a greater and greater degree, the use of these results must be attended with a continually decreasing degree of accuracy and confidence. It would be, however, a great convenience if we might feel the liberty of introducing within moderate limits changes (2) and (3), or more especially the former. Now mathematical investigation, into the details of which we will not enter here, indicates that the variation of efficiency with variations (2) and (3) is very slow, and hence that we may presumably introduce such changes in moderate degree without sensibly affecting the efficiency.

Turning now to the value of K in § 36, (11), we note that it involves geometrical ratios and the two coefficients a and f. Let us now consider the effect of changes (1), (2), and (3) on these coefficients.

From the close analogy between f and the skin-resistance coefficient of § 7, it would seem probable that it would be more especially affected by change (1), and hence that it should receive some correction similar to that for length, as discussed in that section. The data for such correction do not exist, but such indications as are available indicate that it is not large in amount. Of still less importance are the effects due to (2) and (3).

For the coefficient a the case is by no means the same. Considering, as we fairly may, that so long as proportion and form remain the same the total normal resistance varies as the area and as the square of the speed, it follows that a may be considered independent of change (1). Its variation with change (3) within the limits usually involved will also be slight. We come, therefore, to the variation of a with area-ratio h, all other characteristics remaining the same.

Suppose we start with very narrow, thin, elliptical blades of maximum width $.011r$, or area-ratio $h = .01$. These blades with given revolutions and slip will develop a certain thrust T and useful work U corresponding to certain average and distributed values of the coefficients a and f. Now if the width and hence the area of these blades be doubled, the area-ratio being now .02, we shall undoubtedly find that the value of a will remain sensibly unchanged, and hence the total thrust will be doubled. If we continue thus to increase the width, and hence the area, we shall undoubtedly for a time find the same rate of increase in the thrust, indicating a sensibly constant value of a. If the path of the blade were rectilinear, this would undoubtedly hold true up to some increase of area beyond actual experience. With the screw propeller, however, the path is helicoidal; and we readily see that as the width and area are increased the thrust cannot increase indefinitely, but must rather tend toward a limit. In other words, as the area is increased the thrust is increased at a slower and slower rate. This implies a gradual and continual decrease in the coefficient a, while presumably the value of f remains nearly the same, or at least falls off much less rapidly than a. This decrease in a is due to mutual interference in the streams acted on by the different blades. While therefore a decreases, f remains nearly the same, and the thrust per unit area falls off accordingly. In this way we shall finally reach a point where increase of area gives no increase of thrust. The value thus developed is therefore a maximum, and cannot be exceeded, no matter what the area. Indeed, certain indications seem to point toward a possible falling off of thrust with excessive increase of area.

The ideal maximum thrust obtainable for given diameter, pitch, revolutions, and slip is that corresponding to the formation of what is termed a *complete column*; that is, a column of water in which each part has received the full acceleration which, with geometrically perfect action, the propeller is capable of imparting. The amount of this maximum thrust may be investigated as follows:

Referring to Fig. 55, it is shown that the value of the longitudinal acceleration is given by

$$EG = EC \cos^2 \alpha = S_2 \cos^2 \alpha = s_2 pN \cos^2 \alpha.$$

Now for an element of the total column consisting of a shell of water of thickness dr, moving with this acceleration, the thrust will be

$$dT = \text{mass of water per second} \times \text{acceleration } EG.$$

Hence

$$dT = \frac{\sigma}{g}\pi d \,.\, dr \,.\, \text{velocity of feed} \times EG.$$

We consider the velocity of feed as represented by DG, or as the speed of advance DE + the acceleration EG. The value of EG above and found in § 42 assumes the angle CAE small. On the same assumption we may also put

$$CG = EC \sin^2 \alpha = S_2 \sin^2 \alpha = s_2 pN \sin^2 \alpha.$$

Hence

$$DG = pN(1 - s_2 \sin^2 \alpha);$$

and putting in general s for s_2, we have

$$dT = \frac{\sigma}{g}\pi d \,.\, drsp^2N^2 \cos^2 \alpha(1 - s \sin^2 \alpha).$$

But $\tan \alpha = p \div \pi d$, and $d = 2r = py$. Making these substitutions and reducing, we find

$$dT = \frac{\sigma}{g}\pi^3 p^4 N^2 s y^3 \frac{(\pi^2 y^2 + 1 - s)}{(\pi^2 y^2 + 1)^2} dy. \quad . \quad . \quad (1)$$

Hence for the entire propeller the maximum ideal thrust will be given by the integration of this expression between the proper limits for y. This is seen to be entirely independent of the breadth of blade or area of its surface. The thrust thus found would be the maximum possible on the suppositions made, and would correspond to the perfect action of each element, and to the absence of skin-resistance. It is therefore greater than could be obtained in any actual case, no matter what the amount of surface.

On the assumption that the value of a is constant, Cotterill* has used the above expression for dT to find the breadth of blade necessary to form a complete column at various values of y. This was done by equating the thrust above to that in a form similar to § 35, (6), omitting f and with an assumed value 1.7 for a, and solving for dA. Since, however, this value is not accurate for a, and since, moreover, it cannot remain constant but must fall off in marked degree with large increase of area, the results thus found can only be considered as rough approximations to the lower limits, beyond which we must go in order to approach the "complete column" condition. So far as these indications went, however, they showed that propellers of ordinary proportions do not form complete columns, and hence do not give the maximum thrust corresponding to their diameter,

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* Transactions Institute of Naval Architects, vol. XX. p. 152.

pitch, revolutions, and slip. This instead of being a fault is a decided advantage, for it is very sure that a propeller working near the upper limit of the thrust would be less efficient than if the surface and thrust were less, all other conditions remaining the same. This arises from the fact that the last increments of thrust must be obtained by the addition of a disproportionate amount of surface with its accompanying skin-resistance. This will result in an increase of f relative to a, and in a corresponding loss in efficiency (§ 36, (4)). Hence while such a propeller may give a large thrust for its diameter, pitch, slip, and revolutions, the proportion of useful to total work will be comparatively poor.

Let us now return to the expression for the thrust in (1). By integrating the function of y between various values of the outer limit we obtain the values of the maximum ideal thrust for the entire propellers of corresponding pitch-ratios. A comparison of these with the values actually obtained by experiment under similar conditions of diameter, pitch, revolutions, and slips will be of interest.

The integrations were effected by approximate methods, and the results are shown by CD, Fig. 68. These are plotted to represent the maximum ideal thrust in tons for propellers of 10 feet diameter at 100 revolutions and 20 per cent slip, and of various values of the pitch-ratio as given on the axis of abscissæ. The curve AB in the same diagram gives similarly the actual values of the thrust as derived for the same conditions from Froude's experimental models, assuming geometrical similarity. This curve gives therefore the actual thrusts for propellers the same as above, and of area-ratio $h = .36$. With other values of the slip the results were entirely similar, thus indicating still more clearly that the propeller of usual proportions is far from developing the maximum ideal thrust. The ratio of these two thrusts is shown by the curve EF.

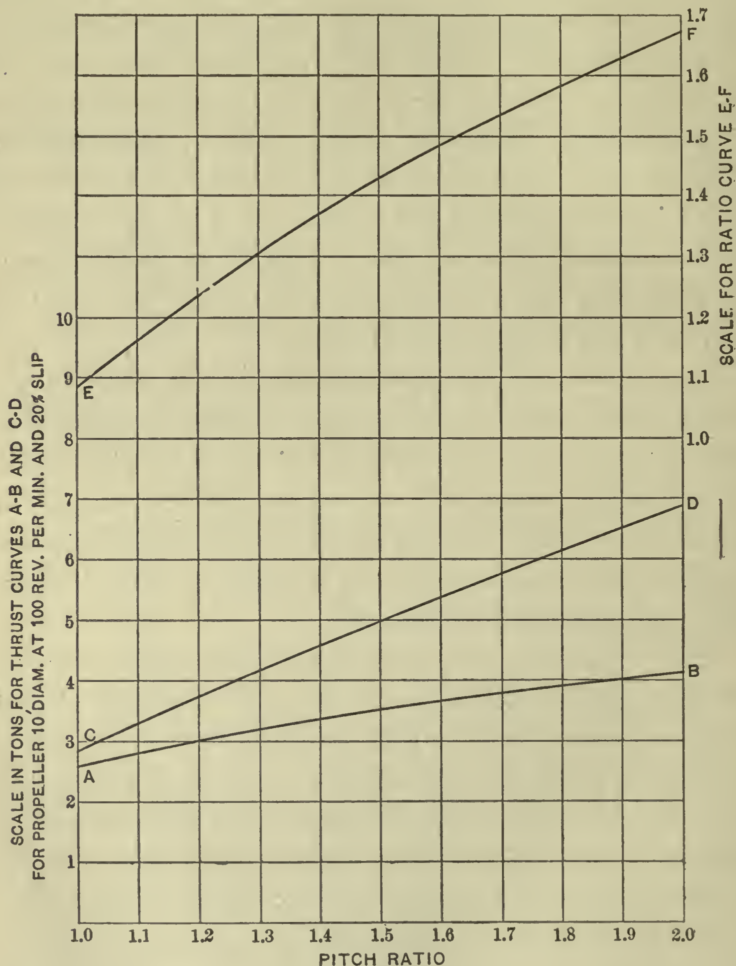

FIG. 68.

It would be more especially useful to know the law connecting increase of thrust with h. Unfortunately the information on this point is very scanty. R. E. Froude gives as the relative thrusts of 4, 3, and 2 blades, each of the same form and area as in Fig. 57, the numbers

$$1 : .865 : .65,$$

while the areas are in the ratios

$$1 : .75 : 50.$$

This shows a gain in the thrust per unit area as the area is decreased. It would seem fair to assume this gain to be due fundamentally to the decrease in area. This is also in accord with Isherwood's experiments,* which indicated that for a given blade area the thrust was practically independent of the number of blades. We will assume therefore that the above relative thrusts and areas correspond, the area 1, however, corresponding to an area-ratio of .36 and the others respectively to .27 and .18. The relative values of the thrusts will certainly depend on the pitch-ratio and on the slip, so that those given above cannot be true generally. They are believed to apply more particularly to a value of the pitch-ratio about 1.3, and to average values of the slip. For propellers of about this pitch-ratio we have therefore three values as above, a zero-point for $h = 0$, and an ideal maximum as in Fig. 68. Let us now take arbitrarily the thrust of the four-bladed propeller—or rather the thrust for area-ratio .36—as 1, and indicate our results graphically. This is shown in Fig. 69, where the abscissæ give the values of h and the ordinates the values of the thrust in terms of that for $h = .36$, as unity. P, Q, and R are the points for 2, 3, and 4 blades, or for $h = .18$, .27, .36, the corresponding

* *Engineering*, vol. xx. pp. 369, 370.

ordinates being as plotted, .65, .865, 1. In Fig. 68 the relative value of the ideal maximum for pitch-ratio 1.3 is seen to be 1.3. This value is laid off as the line *CD*. We have, therefore, *O*, *P*, *Q*, and *R* as experimental points, with *CD* as an ideal limit, the actual limit being slightly less. The conclusion seems clear that the variation of thrust with h must follow some such line as *OPQRS*.

For any other pitch-ratio a similar curve must pass through *R*, since we take the thrust for $h = .36$ as relatively 1. Now putting in the upper limits for pitch-ratio = 2 and 1, we have *AB* and *EF*. While the data on which we are working is very meagre, it seems to be a fair conclusion that the curves for these values of the pitch-ratio will be similar to *OHV* and *OGT*, with intermediate curves for intermediate values of the pitch-ratio.*

Mention may also be made here of the experiments of Mr. A. Blechynden,† the results of which indicate a general confirmation of the conclusions we have just drawn.

From the diagrams of Fig. 69 it would follow that with a propeller of high pitch-ratio the thrust increases nearly as the area to a considerably greater value of h than with low pitch-ratio. Also remembering that with present practice h is usually found between .35 and .45, it is seen that a relative increase of area is much more admissible on a propeller of high pitch-ratio than on one of low, and that in the latter

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* An experimental investigation of the influence of the amount of surface on the performance of screw propellers is now being carried on by the author, the results of which, so far as determined, justify these general conclusions, drawn independently of the experimental work. For a preliminary statement of the work, reference may be made to the Transactions of the Society of Naval Architects and Marine Engineers, vol. v.

† Transactions N. E. Coast Institute of Engineers and Ship-builders, vol. III. p. 179.

case we are near the point where increase of area will give very slight return in increase of thrust. These conclusions also are borne out by the indications of actual experience, and should be borne in mind in connection with the proper value of h.

It must be remembered that there is very little experi-

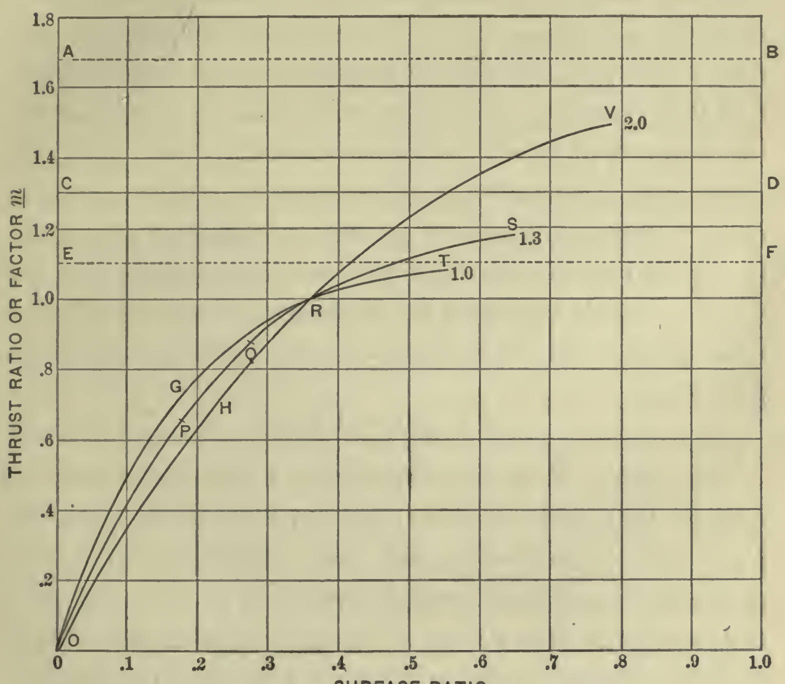


FIG. 69.

mental data bearing on the relations shown in Fig. 69, and the diagram is simply an attempt to express the available data, extended by reference to an ideal maximum which may be computed as a function of the conditions of operation of the propeller.

We have thus shown the general nature of the variation of thrust, useful work, and hence of the coefficient a, with variation in h.

Now of the three kinds of change with which we may be involved in passing from the model to the full-sized propeller, let us first consider only (1) and (2). From the preceding discussion it will be seen that the change of a with h and its resultant effect on thrust and useful work is presumably the only one of importance, and the only one for which we shall need to provide. Referring to § 36, (10) and (11), it is seen that this variation is equivalent to a change in the value of the integrals H or K, the values of which should gradually decrease as h increases, thus corresponding to the decreasing rate of increase of thrust with area. Instead of attempting to express this variation in K as a function of pitch-ratio and h, let us simply represent its ratio to the standard value, or value for $h = .36$, by a factor m. We have then, instead of § 36, (14),

$$U = (p'N')^3 d'^2 klm. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The value of m will depend on h and pitch-ratio. If $h = .36$ then m is always 1, and (2) becomes the same as § 36, (14), corresponding thus to the general condition of geometrical similarity between the model and the propeller. If h is not .36, then a value for m must be selected having in view the general results indicated in Fig. 69. It is evident, in fact, that the values of m are represented by the ordinates to the curves OGT, OPS, etc. Hence by inspection a value may be selected which shall approximately correspond to the given conditions of pitch-ratio and h.

It should be especially noted that as here treated the question of the number of blades does not directly enter. The value of m is determined by the area and not by the number of blades. Some attention must, of course, be paid to the usual proportions between length and breadth of blade, but area is made the fundamentally controlling feature. In general we shall find the following a safe guide:

With area-ratio from .15 to .25, 2 blades may be used;
" " " .25 to .40, 3 " " " "
" " " .35 to .50, 4 " " " "

We may now consider the effect due to change (3)—a change in the form of the blade. The effect arises here not from a change in a or f, but from a change in the form of dA as a function of y.

The thrust for any given element varies sensibly as the square of the speed, and hence, for given revolutions, as the square of the radius. Hence the entire thrust will vary sensibly as the integration of each element of area multiplied by the square of its radius, or as the moment of inertia of the blade area about the axis. Denote this moment for an elliptical blade by I_0, and for another blade of equal area but different form by I. Then the form of blade being the only variable element between the two propellers, we should expect that the values of the thrust would be sensibly in the ratio of I_0 to I, or, since the areas are equal, in the ratio of the squares of the radii of gyration. It should be remembered that this ratio is intended simply to provide for the results of a change in *form*, the coefficient m being intended to provide for the results of a change in *area*. A change of form, however, implies a change of area in certain parts of the blade, and hence a probable change in the value of the coefficient a, and hence a slight departure in the value of the thrust from proportionality to the moment of inertia or square of the radius of gyration.

A special examination of this point seems to indicate, however, that the error should not be sensible so long as the blade is in general oblong or oval in form, while if distinctly widest near the tips the actual gain in thrust will be somewhat less than that indicated by the ratio of the moments of inertia. We should be hardly justified, moreover, in extending our efficiency data to blades of this form; so that it may be understood as a general limitation that the blades to which these equations and methods are to be applied should be of a generally oblong or oval form, and that if they are markedly trapezoidal, with the broad end at the tip, the use of these methods will be attended with more uncertainty. At the same time if an estimate relative to such blades has to be made, and no means of more direct comparison are available, the use of the relations indicated will probably furnish the best estimate obtainable under the circumstances, the true values of the forces and work involved being probably somewhat smaller than those given by the estimate. In a large number of applications of these formulæ, made under the author's direction, for the purpose of analyzing the performance of propellers, they were applied to all varieties of form, including many of extreme proportion. The general closeness of correspondence and the consistency of the results were much greater than had been anticipated, and, so far as this investigation indicated, these formulæ and methods would seem likely to furnish a satisfactory degree of accuracy for most designing purposes.

In order to introduce this influence due to variation in form into our equations, let

$$\frac{I}{I_0} = \frac{\rho^2 A}{\rho_0^2 A} = \frac{\rho^2}{\rho_0^2} = i,$$

ρ being the radius of gyration of the blade area about the axis. The value of i may be determined by an approximate integration as follows. The value of I is evidently represented by the integral

$$I = \int br^2 dr,$$

in which b is the breadth at radius r. The approximate value of this for any given blade may be obtained in the manner indicated in the following form, referring to the propeller of Fig. 75:

r	r^2	b	br^2
.2	.04	19	.76
.3	.09	22	1.98
.4	.16	23.5	3.76
.5	.25	24	6.00
.6	.36	24	8.64
.7	.49	23.3	11.42
.8	.64	21.2	13.57
.9	.81	17.0	13.77
1.0	1 00	5.0	5.00
		179.0	64.90
		12.0	2.88
		167	62.02
			.3714

The column headed r gives the fractions of the entire radius at which the breadths b are measured. The next column gives the squares of these values. The third column gives the breadths of the blade at the corresponding radial distances. The fourth column gives the products br^2, which are integrated here by the trapezoidal rule as sufficiently exact for the purpose in view. The column is summed, and from the result the half sum of the end values is subtracted, giving the value 62.02. This quantity is proportional to I, the omitted factor being, as is readily seen, $r^3 \div 10$, whence

$$I = 6.202r^3.$$

The area of the blade is determined from column 3 by a similar integration, the omitted factor here being $r \div 10$. Hence

$$A = 16.7r.$$

It is found in practice more convenient to use the ratio $\rho^2 \div \rho_0^2$ rather than $I \div I_0$. In this case, therefore, we have

$$\rho^2 = \frac{I}{A} = \frac{6.202r^3}{16.7r} = .3714r^2,$$

and $$I = .3714r^2A.$$

By a similar proceeding we should find for any elliptical blade with the same relative size of hub

$$\rho_0^2 = .357r^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and $$I_0 = .357r^2A. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Hence $$i = \frac{\rho^2}{\rho_0^2} = \frac{.3714r^2}{.357r^2} = 1.04.$$

The values given in (3) and (4) are the same for any elliptical blade, no matter what the surface-ratio, provided the diameter of hub is .2 that of the propeller. Denote the ratio of hub to propeller by x. Then for other values of x the following empirical formula will give very closely the values of i for all elliptical blades in terms of that for $x = .2$ as unity:

$$i = 1 + 1.1(x - .20). \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

For blades of other form ρ^2 may be found by an integration similar to that shown above, and i thus determined. In Figs. 70 to 79 are shown a series of blades of the same area, and for $x = .2$, but of varying form, with their values of the ratio i. These illustrate the relation of i to form for constant value of x, and will aid in making an estimate of its value independent of the detailed computation.

Introducing, therefore, the factor i into (2), we have

$$U = (p'N')^3 d'^2 (iklm). \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Influence due to Thickness of Blades.—On this subject but little information is available. We know in general that an increase of thickness will increase f, and hence decrease thrust and efficiency. Data, however, are lacking for a quantitative estimate of the amount of such influence. So long as the thickness is not unusual the results should not show any marked variation due to this cause, and we shall not further refer to this influence. We should remember, however, that a saving in thickness so long as rigidity and the necessary strength are not sacrificed is always desirable on the score of efficiency.

GENERAL REMARKS.

We therefore consider (6), together with the equations, diagrams, and tables relating to m, k, and l, and the efficiency information as derived in § 36 and shown in Figs. 58 or 71, as our fundamental data for propeller design.

In discussing the application of these data to actual cases involving the various changes designated under the headings (1), (2), (3), and especially those whose effects are represented by the coefficients i and m, the assumptions used have been necessarily to some extent hypothetical, and the experimental basis for a determination of the values of these coefficients is unfortunately very meager. If, however, the admissible changes be limited to (1), that is, to changes in dimension only, with constant shape of blade and area-ratio, then m and i are both 1, and we simply reproduce Froude's experimental results as applied to such a series of propellers. The general agreement between the results given by intelligent use of the method thus restricted, with successful experience, indicates that it may be employed with a high degree of confidence in its reliability, and that in any event it will be much better than uncertain comparison between the results of experiment where the accuracy of the data may be called in question, or than any method which involves a less detailed analysis and representation of the different variables involved in the problem. The introduction of the coefficients m and i into the method is for the purpose of providing a greater elasticity in the choice of form and area-ratio, without a serious loss of general reliability. While undoubtedly the method may be used with more confidence when form and area-ratio are constant and the same as in the model propellers, yet for changes of moderate amount in these features the effects must be very closely represented by the coefficients i and m as given, and no serious departure from reliable results will be thereby introduced. In any event where reliable data from more direct comparison in such cases is not available, this method will probably furnish the most reliable results to be found under the circumstances.

Again, as will appear more fully by illustrative examples, this general method does not take the place of judgment and experience. So far as the method itself is concerned, there may result an indefinite number of propellers, each more or less completely fulfilling the conditions imposed, and in the discrimination between these full scope may be found for the use of judgment and experience. It is intended rather as an aid to the intelligent use of experience, and to the systematic analysis of the results of trial data.

50. Problems of Propeller Design.

Formula (6), § 49, relates simply to the useful work of a propeller operating in undisturbed water. In the actual case the propeller operates in the forward wake. In Chapter III we have discussed this wake, and shown that we may substitute for the actual turbulent wake a uniform stream of velocity v_0 giving a true slip $S_2 = S_1 + v_0$, and a speed of advance of the propeller relative to the water in which it works, of $PN - S_2 = PN - S_1 - v_0 = u - v_0$. Now in order to connect the propeller in undisturbed water with the actual case, we assume that for a given propeller at given revolutions working in the wake of velocity v_0 and developing a thrust of T and a speed relative to the wake of $(u - v_0)$, the useful work and efficiency will be the same as if it were working in undisturbed water at the same revolutions and true slip, and hence with an equal speed of advance $(u - v_0)$. That is, referring to Fig. 70, we must consider the useful work of our formula as based on the true propeller efficiency and not the apparent, or on the speed of advance of the propeller through the water in which it works, and not relative to still water. Hence the useful work thus defined will be represented by $C_1E = T(u - v_0)$, and not by $CE = Tu$.

It is therefore the work C_1E which must be substituted in the formula, § 49, (6).

The following will therefore be the most natural order of procedure:

The I.H.P. being given or AE, we assume AB. This gives us *BE*. We then select or assume the true efficiency at which we propose to work, or else fix the true slip and pitch-ratio which together determine the efficiency. Then

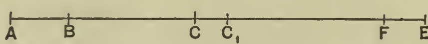


FIG. 70.

BE multiplied by this efficiency gives C_1E, the useful work to be substituted in our formula. Again, it may arise that instead of the I.H.P. we have the E.H.P., or assume the propulsive coefficient, and thus pass from *AE* to *CF* direct. Then *CF* multiplied by hull efficiency $= C_1E$, the useful work, as before. As already noted, unless special information is available, the hull efficiency is usually taken as unity.

We must next determine the value of pN.

Let $u =$ speed in knots. Then

$101.\dot{3}u =$ speed in feet per minute.

And $$pN = \frac{101.\dot{3}u}{(1 - s_1)}.$$

But from § 44, (4),

$$(1 - s_1) = (1 + w)(1 - s_2).$$

Hence $$pN = \frac{101.\dot{3}u}{(1 + w)(1 - s_2)}.$$

This determination calls for an estimate of the wake factor w. Unfortunately the information available as a basis for such estimate is very meager. Froude in connection with the paper on propellers previously referred to gives a series of values for ships of various characteristics, and beyond this next to no information is available in the general literature of the subject. Adding to these a few other known values, Prof. McDermott * has connected them with the characteristics of the ships by empirical formulæ, as follows:

Let w denote wake factor;

p " prismatic or cylindrical coefficient;

m " midship-section coefficient;

L " length in feet.

Then

$$w = .16\left(\frac{p}{m}L^{\frac{1}{3}} - .6\right) \text{ for single-screw ships;}$$

$$w = .13\left(\frac{p}{m}L^{\frac{1}{3}} - 1.1\right) \text{ for twin-screw ships.}$$

These equations represent simply the available data, and as this is relatively meager, care must be exercised in their use. Where no special information is otherwise obtainable, however, they will probably serve to give a fair approximation to the value desired. Further suggestions on this point as well as on several others relating to the choice of values for the various quantities involved will be given in the following section. We will now, without more delay, illustrate the application of our formulæ, diagrams, and tables to actual problems.

We first repeat in collected form, for convenience, the principal equations which we may have occasion to use:

$$U = (p'N')^3 d'^2 (iklm) = e_2 W_p; \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$p'N' = \frac{pN}{100}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$d' = \frac{d}{10}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

* Transactions Society of Naval Architects and Marine Engineers, vol. IV. p. 164.

$$pN = \frac{101.3u}{1 - s_1}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$(1 - s_1) = (1 + w)(1 - s_2); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$(1 + w) = \frac{1 - s_1}{1 - s_2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$k = (.034 + .805s - .68s^2); \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$l = (y_1 - .17); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$w = .16\left(\frac{p}{m}L^{\frac{1}{6}} - .6\right) \text{ for single screw-ships;} \quad (9)$$

$$w = .13\left(\frac{p}{m}L^{\frac{1}{6}} - 1.1\right) \text{ for twin-screw ships;} \quad . \quad (10)$$

m is estimated by aid of Fig. 69, p. 253;

i is determined by computation or by judgment, guided by Figs. 72 to 81, and § 49 (5).

The values of k, l, and of $101.3u$ may also be taken from Tables I, II, III, for the values of slip, pitch-ratio, and speed given, and by interpolation for intermediate values.

TABLE I. VALUES FOR FACTOR k.

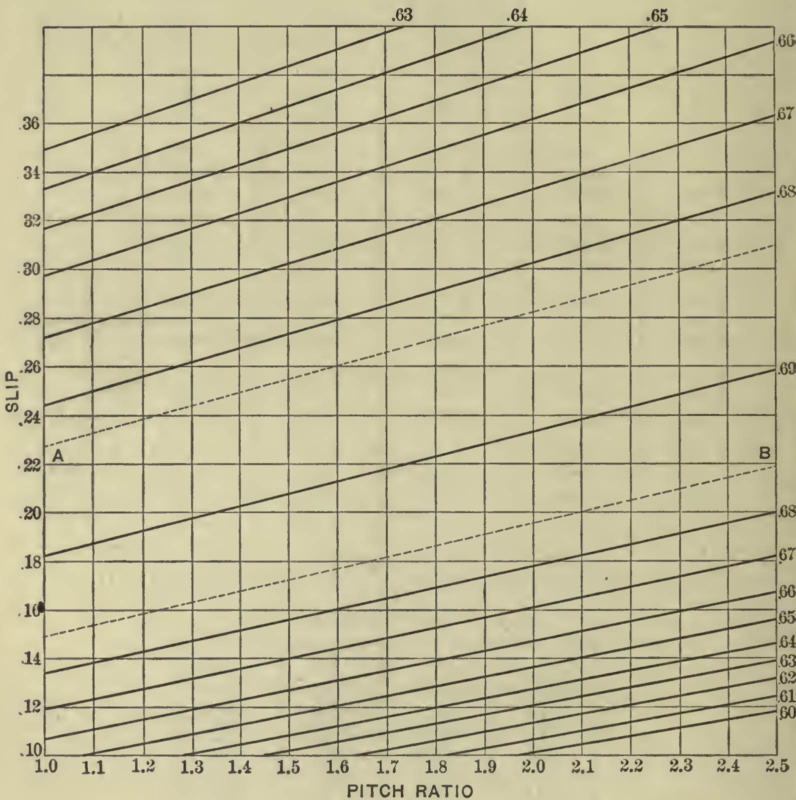
Slip.	k	Slip.	k
.10	.108	.25	.193
.11	.114	.26	.197
.12	.121	.27	.202
.13	.127	.28	.206
.14	.133	.29	.210
.15	.140	.30	.214
.16	.145	.31	.218
.17	.151	.32	.222
.18	.157	.33	.226
.19	.162	.34	.229
.20	.168	.35	.233
.21	.173	.36	.236
.22	.178	.37	.239
.23	.183	.38	.242
.24	.188	.39	.245

TABLE II. VALUES FOR FACTOR l.

Pitch-ratio.	l	Pitch-ratio.	l	Pitch-ratio.	l
.90	.941	1.40	.544	1.90	.356
.92	.917	1.42	.534	1.92	.351
.94	.894	1.44	.524	1.94	.345
.96	.872	1.46	.515	1.96	.340
.98	.850	1.48	.506	1.98	.335
1.00	.830	1.50	.497	2.00	.330
1.02	.810	1.52	.488	2.02	.325
1.04	.792	1.54	.479	2.04	.320
1.06	.773	1.56	.471	2.06	.315
1.08	.756	1.58	.463	2.08	.310
1.10	.739	1.60	.455	2.10	.306
1.12	.723	1.62	.447	2.12	.302
1.14	.707	1.64	.440	2.14	.297
1.16	.692	1.66	.432	2.16	.293
1.18	.677	1.68	.425	2.18	.289
1.20	.663	1.70	.418	2.20	.285
1.22	.650	1.72	.411	2.22	.280
1.24	.636	1.74	.405	2.24	.276
1.26	.624	1.76	.398	2.26	.272
1.28	.611	1.78	.392	2.28	.269
1.30	.599	1.80	.386	2.30	.265
1.32	.588	1.82	.379	2.32	.261
1.34	.576	1.84	.373	2.34	.257
1.36	.565	1.86	.368	2.36	.254
1.38	.555	1.88	.362	2.38	.250

TABLE III. GIVING FEET PER MIN. CORRESPONDING TO KNOTS, OR VALUES OF $101.\dot{3}u$.

u	$101.\dot{3}u$.	u	$101.\dot{3}u$.
10	1013	25	2533
11	1115	26	2635
12	1216	27	2736
13	1317	28	2837
14	1419	29	2939
15	1520	30	3040
16	1621	31	3141
17	1723	32	3243
18	1824	33	3344
19	1925	34	3445
20	2027	35	3547
21	2128	36	3648
22	2229	37	3749
23	2331	38	3851
24	2432	39	3952


.63
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SLIP
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.32
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.28
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.24
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.20
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.16
.14
.12
.10
A
B
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1.1
1.2
1.3
1.4
1.5
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2.4
2.5
PITCH RATIO

FIG. 71.

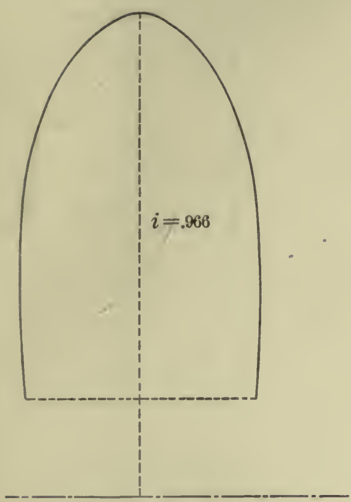

FIG. 72.

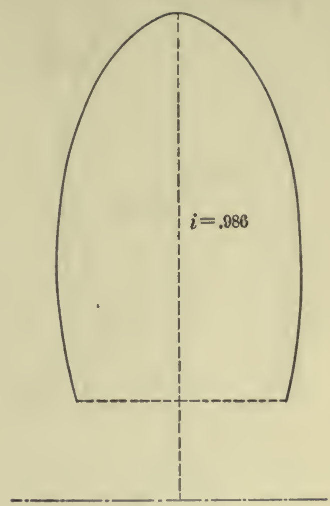

FIG. 73.

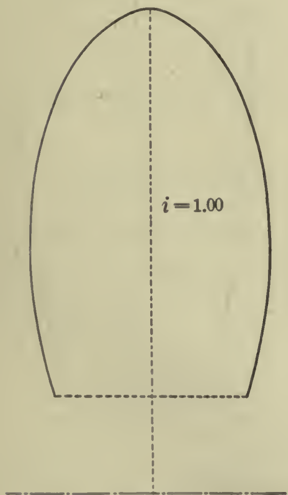

FIG. 74.

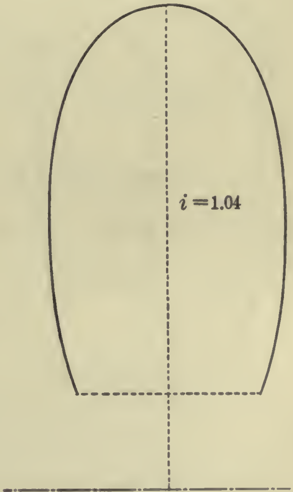

FIG. 75.

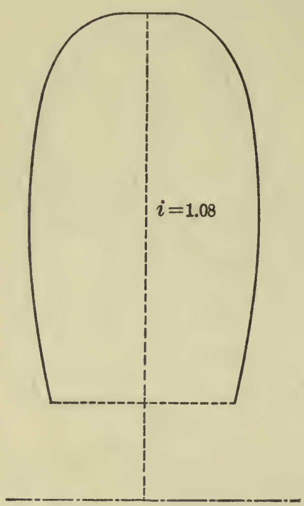

FIG. 76.

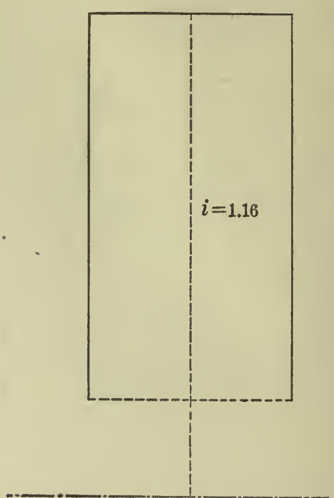

FIG. 77.

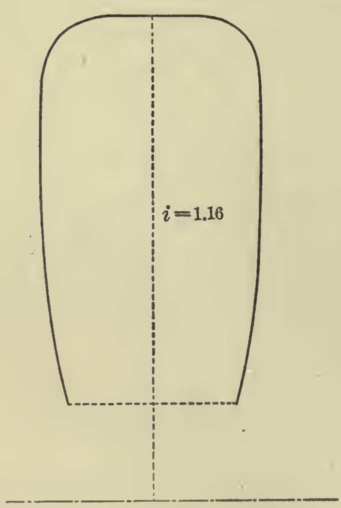

FIG. 78.

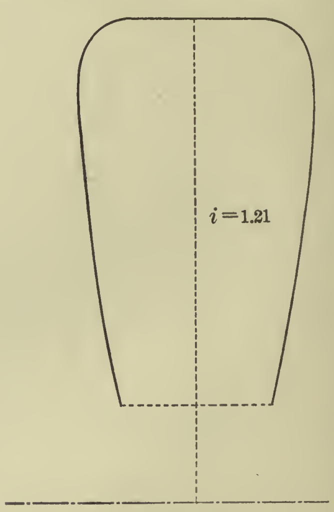

FIG. 79.

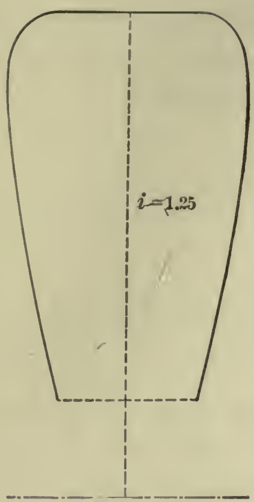

FIG. 80.

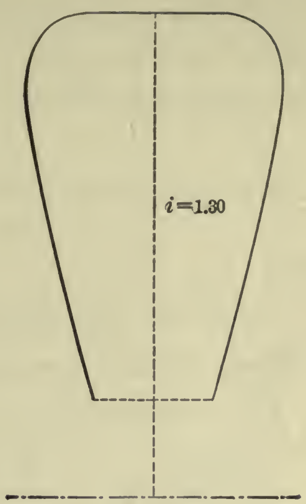

FIG. 81.

EXAMPLES.

(*a*) As the first example let us take

$$\text{I.H.P.} = 4600;$$
$$\text{speed} = u = 16 \text{ knots};$$

friction power = 13 per cent = 598 H.P. (*AB*, Fig. 70). Hence

$$\text{propeller power} = 4002 \text{ H.P. } (BE, \text{ Fig. } 70).$$

Next from the data given suppose w found or estimated as .14. We will also select .26 as the proposed value of the true slip or s_2. We then find from (6)

$$1 - s_1 = .844 \quad \text{and} \quad s_1 = .156.$$

Hence $$pN = \frac{16 \times 101.3}{.844} = 1922;$$

and $$p'N' = 19.22.$$

Now supposing neither p nor N fixed, we may proceed as follows: We first select a pitch-ratio of, let us say, $c = 1.26$.

We will also assume in this problem that the blades are elliptical, with area-ratio .36, so that both i and m are 1. From Tables I and II and Fig. 71 we then find

$$k = .197;$$
$$l = .624;$$
$$e_2 = .68.$$

Hence

useful work $U = .68 \times 4002 = 2721$ (C_1E, Fig. 70).

Then substituting we have

$$2721 = (19.22)^3 d'^2 \times .197 \times .624;$$

whence $d'^2 = 3.141;$

and $d = 17.72;$

$$p = 22.33;$$
$$N = 86.1.$$

(*b*) Let the conditions remain the same as above, except as regards the blades. Let the area-ratio be .48 and the shape such that i is taken $= 1.03$. From Fig. 69 it appears that we may take m about 1.1. Hence we have

$$2721 = (19.22)^3 d'^2 \times 1.03 \times .197 \times .624 \times 1.1;$$

whence $d'^2 = 2.766;$

and $d = 16.6;$

$$p = 20.9;$$
$$N = 92.$$

(*c*) Suppose with the same data as in (*a*) we fix both pitch and diameter and propose to find the necessary shape of blade or amount of area. Thus let $p = 24$ feet and $d = 18$ feet. Then with the same value of pN we find $N = 80.1$. Also $c = 1.33$, and we have

$$k = .197, \text{ as before};$$
$$l = .582;$$
$$e_2 = .681;$$
and $$U = 2725.$$

Substituting and solving for the combined factor (*im*), we then find

$$(im) = \frac{2725}{7100 \times 3.24 \times .197 \times .582} = 1.03.$$

This may be realized with an elliptical blade of area-ratio about .40 or with an area-ratio of .36 and shape similar to Fig. 73 or by any desired combination of the two features area and shape such that the product *im* shall equal 1.03.

(*d*) Given the same data as in (*a*), let us fix N at 110. Then with the same value of pN we have

$$p = 1922 \div 110 = 17.5.$$

Let us now assume for trial a diameter of 16 feet. Then

$$c = 17.5 \div 16 = 1.1;$$

and we have

$$k = .197, \text{ as before};$$
$$l = .739;$$
$$e_2 = .677;$$
and $$U = 2710.$$

Substituting for (*im*) as in (*c*), we have

$$(im) = \frac{2710}{7100 \times 2.56 \times .197 \times .739} = 1.024.$$

A slight increase of area-ratio or a slight filling out of the form at the outer end of the blade will therefore fulfil the indications here given.

The propellers thus far considered have been implicitly supposed to have four blades. To illustrate the application of the equations to propellers of two or three blades we may take the following:

(*e*) Let

$$\text{I.H.P.} = 5000;$$
$$u = 20;$$
$$W_p = .87 \times 5000 = 4350;$$
$$w = .08;$$
$$s_2 = .25.$$

Then

$$(1 - s_1) = .81,$$

and

$$pN = \frac{2027}{.81} = 2503,$$

whence

$$(p'N') = 25.03.$$

Take, also,

$$\text{pitch-ratio} = 1.4;$$
$$\text{number of blades} = 3;$$
$$\text{area-ratio} = .30;$$
$$i = 1.00.$$

Then we find

$$k = .193;$$
$$l = .544;$$
$$m = .92;$$
$$e_2 = .68;$$
$$U = 2958.$$

Then, substituting, we have

$$d'^2 = \frac{2958}{15681 \times .193 \times .544 \times .92} = 1.952.$$

Whence we find

$$d = 14;$$
$$p = 19.6;$$
$$N = 128.$$

As an example of a two-bladed propeller we may take the following:

(*f*) I.H.P. = 8;

$$u = 7;$$
$$W_p = .85 \times 8 = 6.8;$$
$$w = .10;$$
$$s_2 = .30.$$

Then $(1 - s_1) = .77,$

and $pN = 921;$

whence $(p'N') = 9.21.$

Take, also, pitch-ratio = 1.1;

number of blades = 2,

area-ratio = .20;

$$i = 1.16.$$

Then we find

$$k = .214;$$
$$l = .739;$$
$$m = .72;$$
$$e_2 = .66;$$
$$U = 4.49.$$

Then, substituting, we have

$$d^5 = \frac{4.49}{781 \times 1.16 \times .214 \times .739 \times .72} = .0435.$$

Whence we find

$$d = 2.09;$$
$$p = 2.3;$$
$$N = 400.$$

(g) As a further illustration of special design let us take the case of a tugboat. If the design is based on the conditions of use we must naturally take the speed low and the slip high.

Let
$$\text{I.H.P.} = 300;$$
$$u = 7;$$
$$W_p = .83 \times 300 = 249;$$
$$w = .12;$$
$$s_2 = .42.$$

Then
$$(1 - s_1) = .65;$$
$$(pN) = 1091;$$
and
$$(p'N') = 10.91.$$

Take also
$$\text{pitch-ratio} = 1.2;$$
$$\text{number of blades} = 4;$$
$$\text{area-ratio} = .54;$$
$$i = 1.20.$$

Then we find
$$k = .254 \text{ (from (7))};$$
$$l = .663;$$
$$m = 1.15;$$
$$e_2 = .60 \text{ (by extension of Fig. 71)};$$
$$U = 149.4.$$

Then substituting we have
$$d'^2 = \frac{149.4}{1299 \times 1.15 \times 1.2 \times .254 \times .663} = .4950.$$

Whence we find
$$d = 7.04;$$
$$p = 8.45;$$
$$N = 129.$$

The usual operations involved may be somewhat generalized as follows:

Given I.H.P., speed u, friction, wake factor w, and true slip s_2.

Whence W_p or BE, Fig. 70, s_1, pN, and k.

Then (1) Given neither p nor N.

Assume pitch-ratio c, i, and m.

Thence find e_2, U, and l.

Thence by (1) find d.

Thence p and N.

(2) Given either p or N.

Thence the other N or p.

Assume d.

Thence find c, e_2, U, and l.

Thence by (1) find (im) and divide between the two according to choice.

Thence follow shape and area-ratio.

(3) Given d.

Assume c or p.

Thence find p or c and N, also e_2, U, and l.

Thence as in (2).

By an inversion of the above processes these equations may be used for the analysis of a given set of trial data, and thus for the determination of the values of s_2 and w in the given case. The use of the equations for this purpose assumes them as applicable to the given propeller, and hence we should expect the results to be generally more reliable the more closely the actual propeller approaches to the standard form and area. The operation is as follows:

(h) Given I.H.P., p, N, d, i, m, l, and s_1, and assume engine-friction.

Thence find W_p and pitch-ratio.

Thence from (1) the value of $\frac{k}{e_2}$.

In Fig. 82 we have plotted the values of $\frac{k}{e_2}$ on pitch-ratio as abscissa, each curve for constant value of s_2 as shown. We have therefore simply to take on this diagram

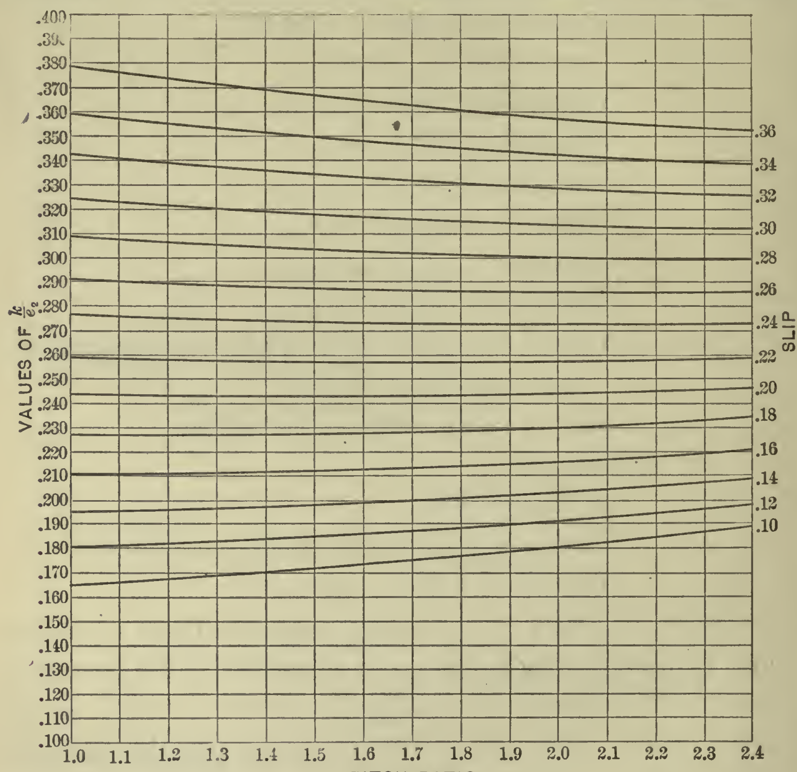

Fig. 82

the point corresponding to the given value of pitch-ratio and $k \div e_2$. The location of this point relative to the curves for constant slip will then show the corresponding value of s_2.

Thus, for example, taking the data of (*a*) except s_2 and w, we find

$$\frac{k}{e_2} = \frac{4002}{7100 \times 3.141 \times .624} = .288.$$

Entering the diagram on pitch-ratio 1.26 we find .26 as the nearest value of the slip, corresponding to the value originally taken in (*a*). By substitution in (6) we then find $w = .14$, while from Fig. 71 $e_2 = .68$.

As a further example we may take the following data from the U. S. cruiser Detroit:

We have given as mean results for the two propellers:

$$D = 11;$$
$$p = 13;$$
$$\text{pitch-ratio} = 1.18;$$
$$\text{I.H.P.} = 2577;$$
$$N = 170.1;$$
$$u = 18.71;$$
$$s_1 = .142;$$
$$\text{area-ratio} = .305;$$
$$m = .90;$$
$$i = 1.09;$$
$$l = .677.$$

Engine-friction is taken at 14 per cent.

Hence $W_p = 2216$.

Then we have

$$\frac{k}{e_2} = \frac{2216}{1.21 \times 10808 \times 1.09 \times .677 \times .9} = .255.$$

From Fig. 82 we thence find $s_2 = .215$.

Hence $$1 + w = \frac{.858}{.785} = 1.093,$$

and $$w = .093;$$

while from Fig. 71 $e_2 = .688$.

The use of the equations in this manner in a large number of cases carried out under the direction of the author showed that it is somewhat difficult to satisfactorily determine the value of w from such analysis alone. This is not so much due to inaccuracy in the equations as to the fact that a slight change or error in the data will make a relatively much larger error in the value of w. This arises from the fact that the value of w is determined from the quantity $(1 + w)$. Now an error of moderate amount in the data or in the suitability of the equations to the data might give, for example, an error of 2 or 3 per cent in the value of $(1 + w)$, making, for example, its value 1.12 when it should be 1.10. This would make a difference of .02 in w, or an error of 20 per cent in excess of the true value. So far as ordinary applications are concerned, however, and especially so far as the problem of design is involved, it is the factor $(1 + w)$ and not w which is used, and hence changes of relatively large percentage amount in w will have comparatively small influence on the solution of such problems. It may be also mentioned that a comparison of the values of w determined in this manner with those given by equations (9) and (10) and by general estimate showed in nearly all cases that a difference of a few per cent in the friction of the engine, or more especially of a very small amount in the pitch of the propeller, would be sufficient to account for the difference in the values of w. The uncertainty as to the real value of the pitch in cases of variable pitch has been considered in § 42; and bearing these facts in mind as well as the possibilities of slight errors of measurement in all cases, and the further uncertainty regarding engine-friction, it would appear that the degree of fulfilment is closer than might naturally have been expected, and that for problems of design the equations and methods may be used with a high degree of confidence in the results.

The influence of an incorrect estimate of w in a problem of design may be illustrated by the following example:

(i) In problem (a) above, suppose .14 to be the correct value of w, but let its estimated value be taken as .09. Then with the same assumed value of s_2 we shall have

$$1 - s_1 = .807;$$

$$pN = 2009.$$

Following the work through as in (a), we should find the following results:

$$d = 16.6,$$

$$p = 20.9,$$

$$N = 96.1,$$

instead of those found in (a).

Now suppose this design accepted and the propeller fitted to the ship. We may then ask two questions: (1) What will be the change in efficiency and what the power necessary at the desired speed of 16 knots? or, (2) What will be the resultant speed if the power 4002 is delivered to the propeller as in (a)?

We first assume the speed of 16 knots attained with a value of $C_1E = 2721$, as in (a). We then have, by substitution in (1),

$$2721 = (p'N')^2(1.66)^2k \,.\, 624.$$

In this equation ($p'N'$) and k are both unknown, but both dependent on s_1. Hence solving, we have

$$(p'N')^2k = 1582.$$

We must now find by a process of trial a value of s_2 such that the resulting values of k and $p'N'$ (using of course the true value .14 for w) will give for $(p'N')^3k$ the value 1582, or a sufficiently close approximation to such value. For the value $s_2 = .26$ assumed in (a) we have $k = .197$ and $(p'N')^3 = 7100$. Whence $(p'N')^3k = 1399$. The value 1582 required, indicates a value of s_2 larger than .26, and one or two trials are sufficient to show that the desired result is very near to .28, for which $(p'N')^3k = 1587$. We may take, therefore, as the result

$$s_2 = .28;$$

$$pN = 1975;$$

$$N = 94.5;$$

$$e_2 = .673, \text{ from Fig. } 71,$$

$$W_p = 2721 \div 67.3 = 4043.$$

According to these results, therefore, the revolutions would be slightly less than the 96.1 expected, the efficiency would be decreased by .007, and there would be required 41 additional H.P.

If we next suppose but 4002 H.P. delivered to this propeller and inquire as to the resulting speed, we have a problem requiring for its solution a resistance-curve for the ship. For all practical purposes, however, we may assume the same loss of efficiency .007, and that power varies as the cube of the speed. This would give for the ratio of the power in the two cases the value

$$.68 \div .673 = 1.01,$$

and therefore

$$\text{Ratio of speeds} = \sqrt[3]{1.01} = 1.003.$$

Hence with 4002 H.P. at the propeller in the case assumed we should have a speed of

$$u = 16 \div 1.003 = 15.95,$$

or a resulting loss of .05 knot.

In a similar fashion if we suppose w overestimated by .05, or taken at .19 instead of .14, we may carry the work through, finding for the propeller the following:

$$pN = 1840;$$
$$d = 18.9;$$
$$p = 23.8;$$
$$N = 77.3.$$

Then, as before, we should find that such a propeller applied to the ship in (*a*) with $w = .14$ instead of .19 would give rise to the following values:

$$s_2 = .24;$$
$$pN = 1872;$$
$$N = 78.7;$$
$$e_2 = 68.5, \text{ from Fig. 71};$$
$$W_p = 2721 \div 685 = 3972.$$

In the same manner as above we should also find that with 4002 H.P. delivered to this propeller the speed would be increased by about .04 knot or to 16.04 knots.

These illustrations of the influence of an error in the estimation of w may perhaps be taken as extreme cases, for we should under all usual conditions be able to estimate w within .05, and it thus appears that the errors resulting from an incorrect estimate of w are in themselues not likely to be serious, especially if the propeller is designed to work at or near its maximum efficiency. Some further discussion of these points will be found in the following section.

Additional Methods of Propeller Design.—The formulæ and methods of design whose applications are shown in the present section are such as naturally result from the general method of treatment developed in this work. The same original data (that of Froude's experiments) may be used in a variety of other ways as illustrated by Froude,* Barnaby,† Caird,‡ and McDermott.§ There have been long in use also briefer and less accurate methods of screw-propeller design than those involving the detailed considerations discussed in the present work. A brief reference to such methods may here be made.

The value of pN is found from the given speed u and an assumed value of the apparent slip s_1 (see (4)). The values of p and N are then selected by judgment.

The value of d may then be found from the formula

$$d = K_1\sqrt{\frac{\text{I.H.P.}}{(PN)^3}},$$

in which K_1 is usually taken between 18000 and 25000.

It is readily seen that this is equivalent to taking the value of the product i, k, l, m, (1) as constant, or assuming its value by judgment between limits corresponding to those specified for K_1.

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* Transactions Institute of Naval Architects, vol. XXVII. p. 250.

† Transactions Institution of Civil Engineers, vol. CII. p. 74.

‡ Transactions Institute of Engineers and Shipbuilders in Scotland, 1895–96, p. 21.

§ Transactions Society of Naval Architects and Marine Engineers, vol. IV. p. 159.

The helicoidal area A is sometimes determined by the formula

$$A = K_2\sqrt{\frac{\text{I.H.P.}}{N}},$$

in which K_2 may vary from 8 to 15.

The helicoidal area may also be determined by the formula

$$A = \frac{\text{Indicated thrust}}{K_3} = \frac{\text{I.H.P.} \times 33000}{K_3(PN)} \text{ (see § 47)},$$

in which K_3 may vary from 1000 to 1500.

Having thus determined A, the resulting values of the disk area and of the diameter d may readily be found by the use of standard proportions.

Since helicoidal area should vary nearly as d^2, it is evident that the three equations above are mutually inconsistent in form, though when used with judgment and with abundance of data for comparison they may be made to yield good results. The third, relating helicoidal area to indicated thrust, is probably the most satisfactory of the three.

The disk area may also be related to the wetted surface of the ship by the formula

$$\text{Disk area} = \frac{\text{Wetted surface}}{K_4},$$

in which K_4 is usually found between 70 and 90, while for specially high-powered craft it may fall to from 50 to 70.

The disk area may also be related to the area of midship section by the formula

$$\text{Disk area} = \frac{\text{Area of midship section}}{K_5},$$

in which K_5 is usually found between 2 and 2.5, while for specially high-powered craft it may fall to from 1.2 to 2.

The disk area may also be related to the displacement D by the formula

$$\text{Disk area} = \frac{D^{\frac{2}{3}}}{K_6},$$

in which K_6 is usually found between .8 and 1.1, while for specially high-powered craft it may fall to from .6 to .8.

In formulæ relating the disk area to the ship the result found is considered as the area for all the propellers, one, two, or three, as fitted. The value may then be divided as required, and the diameter immediately found.

The methods and formulæ briefly referred to in these equations involve necessarily a large element of judgment, with but slight opportunity for its orderly exercise. They have, furthermore, no provision for estimating in detail the probable results due to variations in the different characteristics of the propeller. With the more detailed methods presented and illustrated in the present chapter, the range of judgment is somewhat narrowed, its application is simpler, and full provision is made for the orderly estimate of the probable results due to changes in the more important characteristics.

51. General Suggestions relating to the Choice of Values in the Equations for Propeller Design, and to the Question of Number and Location of Propellers.

In the method of the preceding section the I.H.P. is taken as a part of the fundamental data. This implicitly involves values of the augmentation of resistance due to the propeller and of the wake factor. While, therefore, the augmentation of resistance is not explicitly involved in our equations, it is really represented by the assumed or determined I.H.P. The wake factor w is not only involved in the I.H.P., but we are also called on to assign to it a definite value.

The principal source of information relative to both the wake factor and the factor of augmentation (or its reciprocal "thrust deduction," as termed by Froude) is in the experiments on models. These points are discussed by Mr. R. E. Froude in the paper* to which previous reference has been made, and to which the reader may be referred for details. The conclusions were that for both primary uses of the data, viz., the determination of the standing of a propeller on the scale of efficiency and the relation of its thrust to revolutions, such sources of error as may properly seem to exist apparently tend to neutralize each other, so that we have to deal not with a single error, nor with several tending in one direction, but with a balance of errors in which, though the individual values may be sensible, it is quite likely that their combination will not form an error of serious amount. Comparing the numbers of revolutions for the models with those for the full-sized ships, Mr. Froude states that the model data on the average were found to overestimate the revolutions of twin-screws by about 2 or 3 per cent, and to underestimate those of single-screws by about the same amount.

As shown by example (i), § 50, the result of an underestimate of w when the design is carried through for a given

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* Transactions Institute of Naval Architects, vol. XXVII. p. 250.

pitch-ratio, as in (*a*), will be to obtain a propeller too small and with a larger true slip than that assumed, while an over-rated w will give rise *vice versa* to a propeller too large and with a true slip less than that assumed. If the assumed conditions correspond nearly to the maximum efficiency as indicated by the diagram of Fig. 71, then it is probably better that w should be underrated rather than the reverse. This is because the efficiency falls off from the maximum much more slowly for an increasing than for a decreasing true slip. If, on the other hand, the conditions fixing the efficiency are such as to place the propeller well over beyond the maximum in the direction of increasing slip, then it is better to overrate the value of w rather than the reverse. This is because the consequences will be a decreased true slip and a better efficiency than that assumed. Bearing these points in mind we may usually, by the aid of the diagram in Fig. 71, take the values of w and s_2 in such way that the probable error will either increase the efficiency or insure the minimum amount of decrease. These remarks relate simply, of course, to considerations of efficiency. It will very commonly be found that the final results adopted are largely dependent on structural and other considerations, aside from efficiency.

According to Blechynden, whose experiments on propellers have been mentioned previously, the best results will generally be obtained from Froude's data by using values which would place the propeller on the efficiency surface of Fig. 71, somewhat beyond the crest in the direction of increasing slip. According to the same authority propellers of low pitch-ratio, as for example 1.1 or 1.15, are not efficient when used for the propulsion of ships of full form, such as cargo-vessels of block coefficient from .7 to .8. On this point, however, there is much conflicting testimony, there being considerable data indicating in a general way the suitability of moderate pitch-ratios for ships of full form, and of higher pitch-ratios for ships of moderate and fine forms. The question seems to turn principally on the effect due to the augmentation, this being more pronounced on a full than on a fine after body. Now the augmentation increases in general with the diameter and with the pitch-ratio. Hence, so far as this effect is concerned, we should expect the best results from moderate pitch-ratio and small diameter, and these two conditions, by equation (1), § 50, are seen to be mutually consistent. On the other hand, with very full ships there will be at the stern a certain amount of dead or eddying water. If the propeller is so small and so near the stern as to work for the most part in this eddy, its supply of water will be incomplete and irregular, and the efficiency will be correspondingly decreased. Judged, therefore, from this standpoint we should expect the best results from a propeller placed as far aft as the structural arrangements will allow, and of a diameter sufficient to reach out through the eddying water into that possessing regular stream-line motion. Such a value of the diameter would naturally go with a relatively large pitch-ratio. We have therefore in this case, as in nearly all others with which we have to deal, opposing considerations, one indicating a small diameter and the other a large. The actual best value in any given case will necessarily depend on the balance of these considerations, and for this, in the present state of our information, no general rule can be framed.

In connection with the influence of a very full stern, reference may be made to an extreme case, instances of which have been said to occur. In cases where the inflow of water is more or less hindered, the action of the propeller may approach more and more nearly to that of a centrifugal pump, delivering the water with a large radial or transverse velocity and with slight longitudinal acceleration, and thus absorbing energy with but slight gain of thrust. The defect of pressure forward of the propeller will, however, produce in full measure its effect on the hull, and in the extreme case the thrust gained may be less than the sternward resultant due to this defect, and thus the final resultant force on the ship may be aft rather than forward. In such case the ship would move aft no matter in what direction the propeller turned. Such cases have been reported, although, of course, they are not possible with usual forms.

The Number of Propellers.—This question is one which depends chiefly on the desired subdivision of the power, the available draft, and the desired number of revolutions of the engines.

In many cases the total power is more than it seems desirable to transmit with one shaft, and thus two or more shafts and propellers are fitted. This was the case with the U. S. triple-screw cruisers Columbia and Minneapolis, in which it seemed desirable at the time of their design to subdivide the total power into three units for transmission to the propellers. This was only one of several reasons for adopting this arrangement; but as such it occupied a prominent place in the discussion of the general problem. Similar considerations hold for modern ocean-liners developing from 20,000 to 30,000 I.H.P. In these cases it seems desirable to transmit the power to the propellers in two units, and if there were no other reasons for subdivision, it is doubtful if under present conditions engineers would wish to transmit the entire amount by one shaft. Then, aside from reasons of this character, we have the more important considerations of subdivision for safety and for manœuvring power. The duplication of propelling machinery gives vastly greater security against total breakdown or disablement, and this, as well as the added manœuvring power for war-ships, furnishes considerations of sufficient importance to justify or rather demand the fitting of twin-screws in all such cases.

Again, the average draft may be so limited or the power so great that a single screw would be of too large diameter for the necessary immersion of its blades. This would be the case with most of the fast liners and war-ships, and with all moderately fast vessels of light draft. In extreme cases the draft may be so small that three or more propellers might be needed in order to absorb the necessary power with the limited diameter permissible. There seems to be no reason, so far as efficiency of operation is concerned, why such subdivision, if needed, should not be made. For such cases, however, attention may be called to the peculiar advantages of the turbine propeller as described in § 40.

With regard to the question of revolutions, it is obvious that the smaller the propeller, pitch-ratio and speed being the same, the higher will be the revolutions. It may arise, therefore, that the desired number of revolutions, if high, can only be attained with twin- or multiple-screws.

In the question of the applicability of triple-screws to cruising war-ships, one of the chief considerations not already mentioned related to the resultant possibility of cruising at low speeds with one or two engines working at nearly full power, rather than with all engines working at a small fraction of full power. In such cases the thermodynamic and mechanical efficiencies should be improved. The screws not in operation, however, are dragged, and this increases the resistance. The slip also usually increases, especially with one screw, and this may result in a loss of propulsive efficiency. In practice, therefore, it has been found, both in this country and in Europe, that the use of one or two screws at low speeds does not result in increased efficiency. In a number of cases the coal required for a given speed with one, two, and three screws showed but slight variation, and the trials seemed to indicate an approximate equivalence in general efficiency. Notwithstanding this, triple-screws have been extensively introduced into the practice of the German naval authorities as well as elsewhere, the chief reasons being the possibility of smaller and lower engines, smaller castings and easier construction, better stowage of the engines in the lean after body of fast ships, and advantages arising in time of action from a triple subdivision of the power and engine-room personnel.

In the general question of single- or multiple-screws there is probably no necessary difference in *propeller* efficiency alone, except such as may arise from a better possible selection of characteristics in one case or another. With the proper estimate of wake and proper design throughout there is reason to believe that no especial advantage in *propeller* efficiency can be expected on either hand, and that the choice must be made rather upon the considerations discussed above.

Location of Propellers. — The location of a propeller depends on the question of augmentation of resistance, on its relation to the wake, and on various structural considerations. As already seen, the augmentation of resistance is more pronounced with a full than with a fine form, and with the former especially will vary in marked degree with the distance of the propeller from the stern-post. In such cases it is believed that the propeller, if single, should be at a distance from the stern-post not less than one half or two thirds its diameter. As the distance is decreased from about this amount the flow of water to the propeller becomes more and more indirect and incomplete, the thrust developed becomes less than it should be, and the augmentation effect becomes more and more pronounced. With vessels of moderate and pronounced fineness these effects are less notable in character, and the propeller if single is usually placed directly aft of the stern-post, at such a distance as the necessary structural arrangements make convenient. In torpedo-boats, launches, and yachts the propeller is frequently dropped below the keel line and is sometimes placed aft of the rudder. This favors a full and free flow of water to the propeller, and tends to decrease augmentation of resistance.

Twin-screws are usually placed in the same plane and slightly forward of the stern-post. In such case the average distance from the disk of the screw forward to the ship's surface is much greater than with a single-screw, and the augmentation of resistance is correspondingly less. In some cases an aperture has been cut through the dead-wood opposite the propellers in order to give an outlet at this point, and thus relieve the stern from the periodic shock to which it might be subject with the blades passing very near the surface. In other cases the screws have been located in different transverse planes, each with apertures, and with their disks overlapping. The advantages claimed are decreased distance between the shafts with a given diameter of propeller, or greater diameter with given distance between the shafts. It seems doubtful, however, whether the final results are such as to recommend this arrangement for general adoption.

With three propellers the center one is usually more deeply immersed than those at the sides, the three shaft-centers projected on a transverse plane forming a triangle with apex downward. The center propeller is also naturally farther aft than those at the side, and with this distribution there seems to be little interference between the propellers, and no apparent loss of efficiency in action.

In general it should be borne in mind that the farther a propeller is located from the stern of the ship the less in general will be the resultant augmentation of resistance, and the less also the valuable return from the wake. It is evident, therefore, that so long as the propeller is at a sufficient distance from the stern to obtain a good flow of water and to avoid an abnormal degree of augmentation there will be in general no advantage in its farther removal, for the loss in wake return will offset any further gain by way of a decrease of augmentation.

52. Special Conditions affecting the Operation of Screw Propellers.

Indraught of Air.—The application of the equations of the preceding sections presupposes that the propeller has available a full supply of what is sometimes termed "solid water"; that is, water without sensible admixture of air. It is found as a result of the defect of pressure which exists in the interior of the stream flowing to and through the propeller, that if the blades in the upper part of their path approach near the surface of the water, and more especially if they cut or break the surface, that air is drawn down and mixed with the water. The result is a more or less frothy or foamy mixture, the density of which is much less than that of water. In consequence the thrust for given revolutions and slip is decreased, or for given thrust the revolutions and slip must be increased. In either case the efficiency is generally decreased. If the tips of the blades come very near the surface or actually cut it, the loss in thrust and efficiency may become serious, especially with large pitch-ratio and high peripheral velocity.

It should be clearly understood that a loss of efficiency is by no means necessarily attendant on the working of a propeller in foam, or on the mere fact that the tips of the blades come near the surface or actually cut it. The whole question, so far as the *operation* of the propeller is concerned, is determined by the resultant true slip at which it works. It would be perfectly possible to design a propeller with blades cutting the surface but with sufficient diameter so that even with the indraught and admixture of air the true slip would be within the proper range for good efficiency.

In order, however, to keep down the size of the propeller and to insure its working in water practically free from air, its diameter should be so chosen with reference to the draft of water at the stern, as modified by the possible change of trim and stern-wave, that the tips of the blades will always be well covered. Experience seems to indicate that the desirable immersion of the tips of the blades when nearest the surface should not be less than about .2 or .3 the diameter of the propeller. In addition to the indraught of air from the surface there is always under normal conditions a certain amount of air held in absorption by the water. This air is given off more or less completely about the edges and backs of the blades in consequence of the partial vacua which exist at these points. This will give rise unavoidably to a small amount of foam in the wake, the consequences of which, however, are believed to be insensibly small.

Very Full Form at the Stern.—Mention has been made of this feature in the preceding section.

Racing.—When a ship is in a seaway the blades are continually varying their distances from the surface or actually emerging from it, and the revolutions undergo correspondingly sudden and perhaps extreme variations, unless by some form of governing device the supply of steam to the engine is suitably controlled. Even at the best the revolutions, and hence the slip, will frequently vary through very wide ranges. Such variations may also be due partly to the effect of the internal motion of the water composing a wave, the movement being forward in the crest and aft in the hollow. Such variations in the speed of the propeller are favorable to the production of eddies, and thus to a waste of energy and loss of efficiency. There is in addition a further loss in efficiency due to the wide range of slip through which the propeller works. We know, as indicated by Fig. 71, that for the best results the propeller should work at or close about a single value of the slip. But in violent racing the slip for the propeller as a whole may vary from perhaps a negative value to 50 or 75 per cent or even more. Having reference to Fig. 71 it is readily seen that such wide variation in the value of the slip must inevitably cause a serious falling off in the average efficiency. Other things being equal, these effects will be more pronounced the more readily the propeller-blades are partially uncovered, and hence the nearer they are to the surface of the water in normal trim. We have here, therefore, an additional reason for good immersion of the propellers. It is here that twin-screws gain one of their advantages, their diameter being less and possible immersion greater than for a single-screw to absorb the same total power.

Cavitation or the Effect due to Very High Propeller Velocities.—If a blade such as AB, Fig. 83, is drawn through the

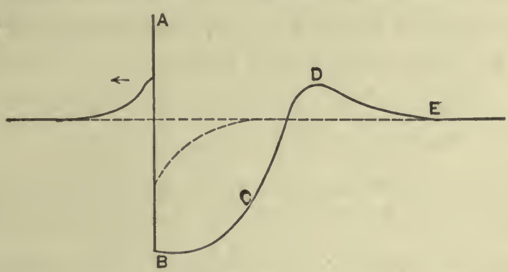

FIG. 83.

water as indicated, the space immediately in its rear is ren dered more or less empty on account of the time required for the water to close in behind the moving blade. At slow speeds this open space will only extend slightly below the surface of the water, while if the speed is sufficiently great it may reach to the bottom of the blade, being shaped in contour somewhat as shown by BCD. The velocity in feet per second with which water tends to flow into an open space is given by the general formula $v = \sqrt{2gh}$. Where the space is open to the air, as in Fig. 83, $h =$ simply the head of water. Thus, for example, at the bottom of a board immersed to a depth of 4 feet $v = 16$, while if the depth is 1 foot $v = 8$. Where the space is shut off from the air and is a perfect vacuum, $h =$ the head due to the atmospheric pressure plus that due to the water, or $h = 34$ feet plus depth of water. In any actual case the vacuum cannot be perfect, but will contain water-vapor and some air given off from the water. In consequence the values for such case will be less than those resulting from the head given above, which must be considered rather as an upper or limiting value.

Now if the velocity of the blades of a propeller is sufficiently high it is evident that such spaces will be formed at the back—in this case of course entirely under water. When this phenomenon is present in any marked degree the water acted on by the propeller is found to become turbulent instead of measurably continuous, and with further increase of revolutions the gain in thrust is very slight.

This phenomenon has been investigated by M. Normand * for a torpedo-boat fixed in location instead of in free route. The diameter of propeller was about 6.5 feet and mean pitch 7.7 feet. The center of the shaft was maintained at four immersions varying from 4.8 to 3.85 feet. The curves connecting thrust with revolutions are as shown in Fig. 84, in which the ordinate is the square of the revolutions N, and the abscissa is the resultant thrust. The regular increase of thrust with N^2 for the deepest immersion is shown by the approximately straight line OD. For the lightest immersion and moderate revolutions the thrust was slightly greater than for the deepest immersion, but the difference was not large, and the increase with the square of the revolutions was regular. After reaching about 135 revolutions, however, the rate of increase of thrust with revolutions began quickly to decrease; and from this point on there was but slight increase of thrust for increase of revolutions up to about 230, the

* Bulletin de l'Association Technique Maritime, vol. IV. p. 68.

maximum number obtained. Very near the end, however, as shown by the curve, the rate of increase was slightly greater than at intermediate points. M. Normand suggests that the curve between B and C corresponds to a period of

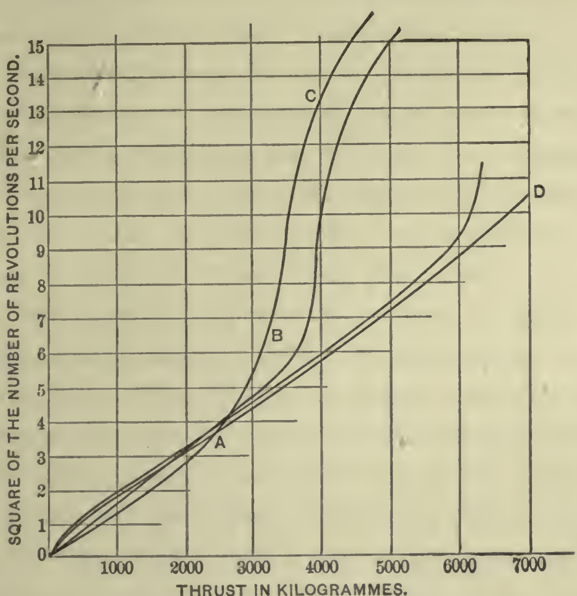

FIG. 84.

instability of the open spaces, during which they are being formed and again filled with more or less irregularity, thus increasing the turbulence of the water, while beyond C the condition is more permanent, and the increase of thrust with N^2 is somewhat more rapid. The remaining curves give similarly the thrusts for intermediate immersions.

The direct application of this to the case of vessels in free route is somewhat uncertain, because the conditions are quite different. The slip is much less, the amount of water handled much more, and the actual velocity of the blades through the water greater for the same revolutions. It seems quite certain that in free route the revolutions might be considerably greater without rupture of the column than for a boat fixed in location.

Still more recently these phenomena have been independently examined by Parsons* and Barnaby† in England. According to the latter, the limiting conditions when cavitation is about to set in are more readily expressed in terms of the average thrust developed per unit of projected blade area than in terms of velocity, and both investigations furnished for this limiting thrust substantially the same value of 11¼ pounds per square inch, the immersion of the tips of the blades being 11 inches. At the point where this average thrust per square inch of projected blade area was reached signs of cavitation began to appear, and on increasing the revolutions the phenomenon became more or less pronounced, and but little additional thrust was gained. Barnaby states also that for every additional foot of immersion the thrust per square inch may be increased about ⅜ pound. The value should also vary slightly with pitch-ratio, being less if the latter is large; but the influence due to this feature is so small as to be practically negligible.

It follows, therefore, according to these investigations, that from 11 to 12 pounds per square inch of projected blade area may be considered as the maximum thrust which can be efficiently developed, and if more than this is required it can only be attained with difficulty and at a loss of efficiency. The phenomenon of cavitation has presented itself more or less prominently in connection with the modern torpedo-

* Transactions Institute of Naval Architects, vol. XXXVIII.
† *Ibid.*

boat destroyer and boats of similar type or condition. In such cases 2500 to 3500 I.H.P. provided for each of two propellers 6 to 7 feet in diameter at 400 revolutions and upward develop a speed of 30 knots and upward. It seems probable that this phenomenon may cause trouble with further increase of speed, especially in boats of this character; and the all-important question in such cases is therefore as to the best disposition of proportions and dimensions for the development of the maximum thrust per unit area without undue sacrifice of efficiency due to this cause. The obvious way to avoid the trouble, where possible, is to provide a sufficient projected area of blade to reduce the average thrust below the limiting value, and the use of nine propellers on the Turbinia is an illustration of this treatment of the difficulty.

Springing of Propeller-blades.—From the laws regulating the distribution of pressure over a plane moving through the water as explained in § 6, it follows that the outer portions of propeller-blades, or such portions as may be considered substantially equivalent to thin flat surfaces, will tend to place themselves normal to the stream or line of flow. This would result in a tendency to increase the pitch and slip. The natural tendency of the inner portions or those with a sensibly rounded back will be quite different. It is readily shown by experiment that when placed slightly oblique the tendency of such sections is indeed to place themselves at right angles to the stream, but, as illustrated in Fig. 85, the incipient movement is clockwise instead of the reverse, the final position toward which the

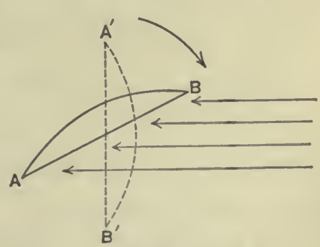

FIG. 85.

section tends being as shown in dotted lines, with convex side toward the stream. This will result in a tendency to decrease the pitch and slip. (Compare also § 42, Fig. 64.)

In addition to the tendency to spring due to the irregularity of the distribution of pressure, the blade as a whole will bend more or less under the influence of the thrust. This bending will be accompanied by a slight untwisting of the blade, thus tending toward an increase of pitch and slip. This influence is relatively more important than the others mentioned above, and hence in the complex effect the result is usually a slight untwisting of the blade with increase of pitch and slip. With blades of the usual stiffness it does not seem likely that this effect will be sufficient to produce any sensible influence on the performance of the propeller. In extreme cases, however, where the blades have been thinned beyond the proper limit, the effective pitch and slip might be so increased as to sensibly affect the efficiency—usually for the worse.

53. The Direct Application of the Law of Comparison to Propeller Design.

Instead of the method of propeller design indicated in the preceding sections, it is possible to apply directly the laws of comparison as explained in § 26. As required by the nature of the law, we assume:

(1) Similarity in geometrical form and situation with a linear ratio λ, the same as that which exists between the ships themselves.

(2) Corresponding speeds as defined for ships, or linear speeds in the ratio of $\lambda^{\frac{1}{2}}$. This condition will evidently apply

only to corresponding points on the two propellers. Let r_1 and r_2 be the radii of such points on the two propellers, and N_1, N_2 the revolutions. Then $2\pi r_1 N_1$ and $2\pi r_2 N_2$ will be the actual or linear speeds of these points. Hence we shall have for corresponding speeds

$$\frac{2\pi r_2 N_2}{2\pi r_1 N_1} = \lambda^{\frac{1}{2}}.$$

But $$\frac{r_2}{r_1} = \lambda.$$

Hence $$\frac{N_2}{N_1} = \frac{1}{\lambda^{\frac{1}{2}}}.$$

Hence at corresponding speeds the revolutions will be inversely as the linear speeds, or inversely as the square root of the linear dimension-ratio.

It must be remembered, further, that these assumptions implicitly involve equal percentages of slip, equal values of the wake factor, equal values of the factor of augmentation due to the propeller, and equal mechanical efficiencies for the engines.

It then results, on the assumptions of § 26, that the ratio between the powers necessary to drive the two ships is $\lambda^{\frac{7}{2}}$. With the propellers thus assumed, it is seen that they fulfil in all points the conditions of the law of comparison, considering it applicable to the whole resistance. Hence the ratio of the resistances to transverse motion, the ratio of the thrusts, and in fact the ratio of all similar components of the forces acting on them, will be λ^3. The speed-ratio is $\lambda^{\frac{1}{2}}$. Hence the ratio between the amounts of useful work will be $\lambda^{\frac{7}{2}}$, the same as that for the total amounts; or, in other words, the two propellers thus situated will give similar results with equal efficiencies.

We have therefore

$$\frac{d_2}{d_1} = \frac{p_2}{p_1} = \lambda;$$

$$\frac{A_2}{A_1} = \qquad \lambda^2;$$

$$\frac{N_2}{N_1} = \frac{u_1}{u_2} = \frac{1}{\lambda^{\frac{1}{2}}}.$$

For example, let $\lambda = 1.44$, $u_1 = 16$ knots, $N_1 = 100$. Then $\lambda^{\frac{1}{2}} = 1.2$. $\lambda^3 =$ displacement-ratio $= 2.986$, and $\lambda^{\frac{7}{2}} =$ power-ratio $= 3.58$. Then $u_2 = 19.2$ and $N_2 = 83.3$. It follows, therefore, that the larger ship would require a propeller 1.2 times the size for the smaller, and with an expenditure of 3.58 times the power at 83.3 revolutions would drive it at a speed of 19.2 knots, with the same efficiency as for the smaller.

This method by comparison is really a special case of that discussed in the preceding section. The method by comparison is a restricted mode of applying the results of a given propeller to the design of one of similar form under similar conditions and at corresponding speeds, while the method of § 50 assumes such laws and relations as will provide for the application of the results of a given propeller to the design of one of similar form under similar conditions but at any speeds. The relation between § 50, (1), and the law of comparison is readily seen by applying the former to the design of a propeller using the data of a similar case at corresponding speed. Taking first the latter, we have

$$U_1 = (p'N')_1^3 d_1'^2 (iklm)_1 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

We suppose in this equation U, d, p, and N as well as the speed and all characteristics of the propeller to be the given data. Then the resulting combined factor $(iklm)$ is readily found. We next propose, by means of this general equation, to use this data for the design of a propeller for a similar ship at corresponding speed. Assuming the same slip, pitch-ratio, and other proportions for the propeller, the factor $(iklm)$ will be the same in both cases. U_2 from the law of comparison for power will be $\lambda^{\frac{7}{2}}U_1$, and since the speeds are corresponding, $(p'N')_2$ will equal $\lambda^{\frac{1}{2}}(p'N')_1$. Hence for the second case we shall have

$$\lambda^{\frac{7}{2}}U_1 = \lambda^{\frac{3}{2}}(p'N')_1^3 d_2'^2 (iklm)_1 \quad . \quad . \quad . \quad . \qquad (2)$$

Comparing this with (1), it is evident that

$$d_2'^2 = \lambda^2 d_1'^2$$

or
$$d_2 = \lambda d_1;$$

whence
$$p_2 = \lambda p_1,$$

and
$$N_2 = \frac{1}{\lambda^{\frac{1}{2}}} N_1.$$

Hence the propeller will have the same factor of similitude as the ship, and will have its revolutions in the inverse ratio of the square root of this factor, the same as if determined by the law of comparison discussed above. In other words, the application of § 50, (1), to such a case in the manner above shown would give exactly the same results as would be given by the direct application of the law of comparison itself.

54. The Strength of Propeller-blades.

The blade of a propeller may be considered as a beam supported at one end, the root, and loaded in a variable and complex manner according to the distribution of pressure over its surface. As in § 35, the amount of this pressure may be approximated to, and by the proper treatment as explained in mechanics, and by approximate integration, the amount of the resultant bending moment at the root may be determined.*

In such case, however, we must not forget that the assumptions are necessarily somewhat inexact, that the actual pressure will undergo wide variations due to irregularities in the wake, and that as in all structural design a considerable factor of safety must be introduced in order to allow for the unknown and uncertain elements necessarily involved in the operation of the propeller. We propose, therefore, to substitute for the actual blade a rectangular plane area set at an angle α to the transverse. The value of this angle will be considered at a later point.

Let n be the number of blades, and T the actual thrust. Then $T \div n =$ thrust for one blade, and $(T \div n) \sec \alpha$ will be the normal pressure on the blade. The center of this pressure will correspond to the center of a system of forces varying as the square of their distance from the axis. Hence denoting the outer radius and that of the hub by r_1 and r_2 we have, as the distance from the axis to this center of pressure,

$$\bar{r} = \frac{3}{4} \cdot \frac{r_1^4 - r_2^4}{r_1^3 - r_2^3}.$$

If we take approximately $r_2 = .2r_1$, we find

$$\bar{r} = .756r_1, \quad \text{or sensibly} \quad .75r_1.$$

We shall also omit all account of bending moment due to the tangential forces, as they will in any case be small in

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* See Taylor, "Resistance of Ships and Screw Propulsion," p. 208.

amount, and will be sufficiently provided for by the factor of safety, and by the reference of our final equation to actual results for the determination of the value of the empirical factor which is to be introduced. The bending moment at the root reckoned normal to the face is therefore

$$M = (.75r_1 - r_2)\frac{T}{n}\sec\alpha. \quad . \quad . \quad . \quad . \quad (1)$$

Now let b denote the length of the section where the blade joins the hub, and t the maximum thickness. We may safely consider that the geometrical characteristics of this section will not widely vary. In actual form it closely resembles a circular or parabolic segment. The general equation for the beam gives us

$$M = \frac{KI}{y_0},$$

where K = stress at outer fiber;

y_0 = distance from neutral axis of section to such fiber;

I = moment of inertia of section about neutral axis.

With the suppositions above made regarding the nature of the section we should have

$$\frac{I}{y_0} \text{ proportional to } bt^2.$$

Hence we may put

$$\frac{I}{y_0} = Qbt^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and
$$(.75r_1 - r_2)\frac{T}{n}\sec\alpha = KQbt^2. \quad . \quad . \quad . \quad (3)$$

Now, turning to Fig. 66, we readily see that

$$T = \frac{a(\text{I.H.P.}) \times 33000}{(1 - s_1)pN}, \quad . \quad . \quad . \quad . \quad (4)$$

where a represents the ratio (Thrust H.P.) ÷ (I.H.P.).

For α we shall take the inclination at the assumed center of pressure, or rather at $.7r_1$ in order to allow somewhat for the rounding off of the blades at the outer end. We have, then,

$$\tan \alpha = \frac{p}{2\pi r_1} = \frac{p}{.7\pi d}.$$

Then if $p \div d = c$ as heretofore, we have

$$\tan \alpha = \frac{c}{2.2} \quad \text{and} \quad \sec \alpha = \sqrt{1 + \frac{c^2}{4.84}}.$$

Collecting and substituting in (3) we have, finally,

$$bt^2 = \frac{33000(.75r_1 - r_2)a(\text{I.H.P.}) \sqrt{1 + c^2 \div 4.84}}{(1 - s_1)pNKQn}. \quad (5)$$

This may be further simplified by making the following assumptions, which are quite admissible, considering the uncertainties necessarily involved:

$r_2 = .2r_1$, as above,

$(1 - s_1)$ and a sensibly constant.

Then uniting all constants in one, which we denote by A^2, we have

$$bt^2 = \frac{A^2 d(\text{I.H.P.}) \sqrt{1 + c^2 \div 4.84}}{pNn} \quad . \quad . \quad . \quad . \quad (6)$$

Also, let us put $\frac{\sqrt{1 + c^2 \div 4.84}}{c} = e$ and I.H.P. $= H$.

Then

$$bt^2 = \frac{A^2He}{Nn}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

or

$$t = A\sqrt{\frac{He}{bNn}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

In this it must be remembered that A includes the strength of the material, and hence its value must be taken accordingly. This result, it must be remembered, gives the thickness at the actual root of the blade. The fillet by which it is connected to the hub is of course extra, and is not here considered. The values of e may be taken from the following table:

$c = p \div d$	e	$c = p \div d$	e
1	1.10	1.7	.74
1.1	1.02	1.8	.72
1.2	.95	1.9	.70
1.3	.89	2.0	.68
1.4	.85	2.1	.66
1.5	.81	2.2	.64
1.6	.77	2.3	.63

Now comparing this equation with a large number of propellers which have shown sufficient strength, it is found that for bronze or steel A may be taken at from 9 to 12, while for cast iron its value should be increased to from 14 to 17.

Collecting for convenience these various items, we have therefore

$$t = A\sqrt{\frac{He}{bNn}},$$

where t = thickness in inches at root of blade;

H = I.H.P.;

e = factor taken from the table above;

b = length in inches of section at root of blade;

N = revolutions per min.;

n = number of blades;

$$A = \begin{cases} 9 \text{ to } 12 \text{ for bronze or steel;} \\ 14 \text{ to } 17 \text{ for cast iron.} \end{cases}$$

MATERIALS SUITABLE FOR SCREW-PROPELLERS.

Though the fundamental purpose in the present volume is the design of the dimensions and form of propellers, yet some brief notice of the materials suitable may not be out of place. Cast iron, cast steel, brass, gun-metal, and the various bronzes are used. Cast iron is the cheapest, but being relatively weak and brittle the blades are necessarily thicker and less efficient than with steel or bronze. The strength available is usually from 20 000 to 25 000 lbs. per square inch of section. Cast-iron blades may be broken short off by striking logs or obstacles, and if such collisions are likely to occur it might be better to have cast-iron blades than steel or bronze, as the rupture of a blade might spare more serious results arising from strains on the engine and stern of the ship. Usually, however, we wish the blades to hold and not to break, and for this purpose cast iron is at a relative disadvantage.

Cast steel is stronger than cast iron, its ultimate strength in castings suitable for propeller-blades ranging from 50 000 to 60 000 lbs. per square inch of section. The sections may therefore be made thinner, and a better efficiency obtained in so far as dependent on this feature. The surface is naturally not as smooth as that of cast iron, but with improved methods of production the difference in this feature is insignificant.

Bronzes have naturally a smoother surface, and seem, furthermore, to have a lower coefficient of skin-resistance. This added to their strength and good casting qualities makes possible a relatively smooth thin blade with sharp edges, all of which are features favorable to good efficiency. The strength available with the best bronzes varies from 40 000 to 60 000 lbs. per square inch of section. With ordinary gun-metal from 25 000 to 35 000 lbs. per square inch of section may be allowed, while with common brass not more than 20 000 to 25 000 lbs. should be depended on. Of these various alloys, manganese-bronze is probably more used than any other, due to its better combination of desirable qualities such as strength and stiffness, good casting qualities, resistance to corrosion, etc. Care is needed in the manipulation of the bronzes in melting, pouring and cooling in order to obtain the full benefit, but with such care the product is homogeneous and reliable. Its greater relative cost restricts its use, however, to war-ships, yachts, and launches, ocean-liners, and other cases where the importance of a saving in propulsive efficiency is considered worth obtaining at a slight increase in first cost.

The durability of propeller-blades is in the order: bronze, cast iron, cast steel. The two latter usually deteriorate by general corrosion and local pitting, the average life being usually from five to ten years. The life of bronze blades is practically indefinite, or at least as great as that of the ship itself.

55. Geometry of the Screw Propeller.

Some general notions relating to the geometrical form of the screw propeller have been given in § 34. We have now to give a more detailed account of the principal forms which may be produced.

We may define the surface of a screw propeller in general as a surface generated by a line l moving on two other lines as guides, of which one, a, is straight, forming the axis, and the other, b, lies in the surface of a cylinder of which a is the axis. This surface being developed, we have the actual form of b. For the common uniform-pitch propeller with blades at right angles to the axis it is readily seen that l is straight and at right angles to a, and that the developed b is straight and at an angle α with the transverse plane such that we shall have

$$\text{pitch} = 2\pi r \tan \alpha, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where r is the radius of the cylinder in which b is supposed to lie.

The line l is the generatrix;
" " a " " axis;
" " b " " guide, or guide-iron.

We will denote b when developed by $\bar{b}$.

The variations usually found in l are as follows:

(1) Straight and at right angles to a;
(2) " " inclined to a;
(3) Bent or curved in an axial plane;
(4) " " " " a transverse plane.

The axis a is invariable in character.

The guide $\bar{b}$ may vary as follows:

(1) Straight;

(2) Curved.

The curvature under (2) is usually such that the angle α, and hence the pitch, increases from the forward to the after edge of the blade.

In the actual generation of the surface we may also have variations according as to whether l changes its angular position relative to a or not. If the longitudinal component of the motion of the point of contact of l with b is greater or less than that of the point of contact of l with a, the pitch will vary radially, or from the hub outward. In such a case practically two guides would be required—one near the hub to suit the pitch at that radius, and the other as usual at the outer end and suited to the pitch at that point.

Let c denote the ratio of the velocities of the points of contact of l with a and with b. Then we may have three cases:

(1) c equal to 1;

(2) c greater than 1;

(3) c less than 1.

All common varieties of helicoidal surfaces may be generated by giving to l, $\bar{b}$, and c the different variations as above enumerated. We may have, however, other forms of helicoidal surface which cannot be generated by a rigid line in any of the ways above mentioned. We may most readily conceive of such a surface as made up of the summation of an indefinite number of guides b, and thus as generated by a variable guide moving radially, and perhaps angularly and longitudinally as well, and taking at the same time the successive shapes and inclinations as determined by the nature of the pitch at the successive radial locations. In this way any form of blade and any distribution of pitch, no matter how complex, may be geometrically determined, and by the use of a reasonable number of guides actually produced in the form of a pattern, or moulded direct in the foundry.

Usually, however, the surfaces of propellers are such as can be generated by a rigid line, and of the great variety which may thus be formed but few are commonly found. These are as follows:

$l_1b_1c_1$ gives the common propeller of uniform pitch;

$l_2b_1c_1$ gives the propeller of uniform pitch with straight elements bent back from the plane of revolution;

$l_3b_1c_1$ gives the propeller of uniform pitch as in the last, but with the elements curved away from the plane of revolution;

$l_4b_1c_1$ gives the propeller of uniform pitch with elements bent or curved in the plane of revolution;

$l_1b_2c_1$ gives the common expanding-pitch propeller, the pitch increasing from forward to after edge.

We may also form these various combinations with c_2 or c_3, and thus introduce radial variation of pitch in any way desired.

It is also possible to greatly vary the appearance of a propeller-blade by varying the location of the part actually taken for the blade. Thus from a helicoidal surface of uniform pitch, by appropriately locating and shaping the contour, we may produce a blade which has the appearance of being bent back from the plane of revolution, and which on casual examination may seem to be the same as that given by $l_2b_1c_1$ or $l_3b_1c_1$, though such is by no means the case.

While thus an indefinite variety of propellers are possible as regards shape of blade and distribution of pitch, it must be clearly understood that so far as experimental or actual results are concerned we are not in a position to select among them any one set of characteristics as the best, and especially any one distribution of pitch, or indeed to say that any distribution of pitch will produce a propeller superior to that of uniform pitch. It is possible that some distribution may be superior to the uniform, but we are not at present in a position to either confirm or disprove this by experimental data. We find therefore a very general tendency to consider the propeller of uniform pitch as the standard in type. The bent-back blades as given by $l_2\bar{b}_1c_1$ has been much used on torpedo-boats, yachts, and small craft, the presumable reasons being a possibly better hold on the water, and a decrease in the so-called short-circuiting around the tips of the blades.

In regard to the influence of any fanciful variation of pitch, we have but to remember the nature of the wake as discussed in § 41. It is there shown that relative to a uniform pitch any propeller-blade must necessarily work with an excessively varying distribution of slip, both over its surface and during a revolution, and hence relative to a uniform slip it must act as though the pitch were correspondingly variable. A propeller of pitch variable according to some definite law will therefore not give a corresponding distributed variation of slip, and its actual variation of pitch is so small as to be entirely swallowed up in the greater wake variation of slip, so that its whole relation to the wake will be but slightly different from that of a uniform-pitch propeller, and in any event entirely different from that in a uniform stream.

Any benefit supposed to arise from some particular distribution of pitch and slip will therefore be wholly fanciful or accidental, since in the actual case the variation of slip corresponding to that of pitch will not be even approximated to. See also §§ 42 and 44.

The Laying Down of a Screw Propeller.—We take as the simple and standard case a propeller of uniform pitch as given by $l_1\bar{b}_1c_1$. We must have the following details given or assumed:

Diameter.
Pitch.
Area.
Diameter and Shape of Hub.
Shape of Blade.

We may also assume for simplicity that the blades are to be cast with the hub, and that they are to be symmetrical about a radial center line.

Let XX, Fig. 86, be the axis and OY a perpendicular, on which the center line OB is taken equal to the radius of the propeller. The hub is then laid off as in the figure. The blade area being known, we readily find the mean half-width, and using this as a general guide sketch in a trial half-contour AGB. This area may then be checked by planimeter or otherwise, and adjusted until it is correct in amount and the contour is satisfactory. The length of the root-section is usually taken about .9 that of the maximum width.

The distance CB is then divided into any convenient number of parts, measuring from either B, C, or O, as may be most convenient.

From (1) the value of tan α for any point F is

$$\tan \alpha = \frac{p}{2\pi r} = \frac{p}{2\pi} \div r = \frac{p}{2\pi} \div OF.$$

If, therefore, we take a point E such that $OE = p \div 2\pi$, the angle EFO will give the corresponding value of α. A similar relation holds for all other points, so that if from E a

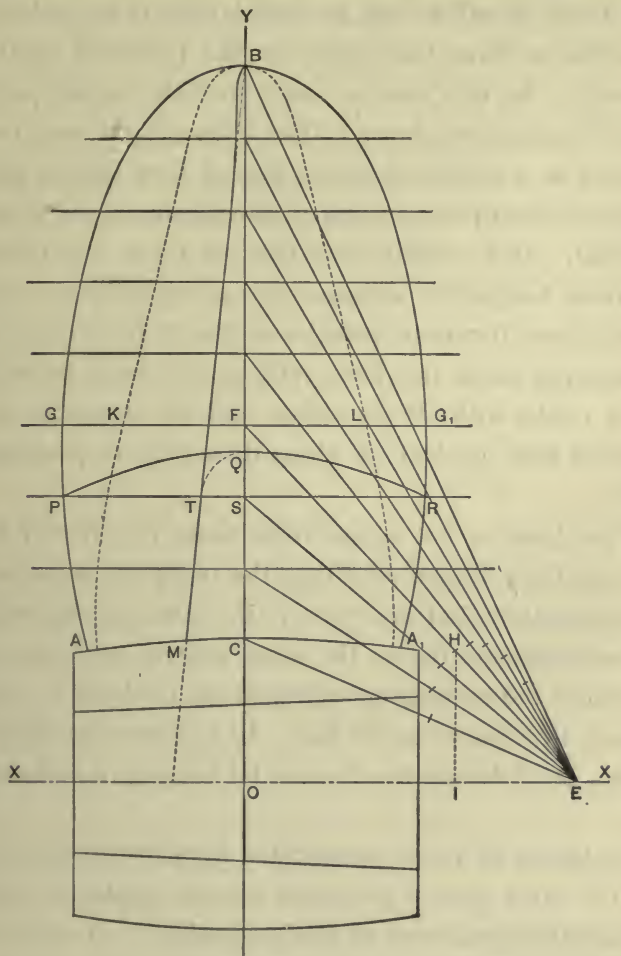

FIG. 86.

bundle of lines be drawn to the several points on the radius, the inclinations of these lines to OB will give the corresponding values of α.

Hence if EH is taken equal to FG, it is evident that EI is the projection of FG on a longitudinal plane, and HI its projection on a transverse plane. If, therefore, FK is made equal to EI, K will be the projected view of G, and making $FL = FK$, we have two points on the projected contour of the blade. In this way we may find the entire projected view of the blade as shown. The other projections are readily found in a similar manner; but as they involve only an exercise in descriptive geometry, the details need not be here explained. It is readily seen that, so far as the contour is concerned, this whole construction is approximate, and not accurate; and it may be mentioned that in the projection on a transverse plane the lines such as GG_1 may be taken as arcs of circles with OF as radius, and the distances such as HI would then be laid off along these arcs, or practically as chords.

If the blade is not symmetrical about OB, it may be laid off in any form desired, and then the two parts must be projected separately, but otherwise in the same general manner.

For detachable blades the same general method is used, the form at the root being appropriately modified to suit the means of attachment to the hub. In any case the blade joins the hub, or, if detachable, its circular boss, by a well-rounded fillet.

For blades of more complicated form and distribution of pitch the same general principles may be applied to find the approximate appearance of the projections. It may be observed, however, that at best these are but approximate, and moreover are of no practical value to the pattern-maker or moulder. The information of which he makes direct use is much more restricted in amount, as will be explained at a later point.

Returning to Fig. 86, the next step is to lay down the thickness *CM* at the root of the blade. A straight line, *MB*, is then drawn to the tip. This will give the middle-line thickness of the various sections. At the tip such line must be run into a curve as shown, in order to insure good casting. For bronze propellers such thickness may be taken at about $\frac{1}{32}$ inch per foot of diameter. For cast iron it must be slightly thicker. The varying sections or thickness strips may now be put in. Thus *SQ* is made equal to *ST*, and a circular or parabolic arc is run through *P*, *Q*, and *R*. When the outer sections are reached, as in Fig. 87, such an arc would give too

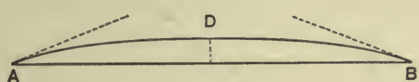

FIG. 87.

thin an edge for reliable casting. We may, therefore, draw lines at the ends as shown, making the edge of the requisite angle, and then run in a smooth curve tangent to these lines at the ends, and parallel to *AB* at the centre *D*. This minimum angle may vary from as little as 5° for small bronze propellers, such as are used on torpedo-boats and fast yachts, to 15° or 20° for cast-iron propellers of large size, such as are used in ordinary mercantile practice. The smaller angles are to be recommended.

The thickness is, of course, placed on the back of the blade; that is, forward of the helicoidal surface which has been laid out as the driving-face. For the sections near the root, however, certain modifications are sometimes made, based on the following considerations: *AB*, Fig. 88, denoting the face, we have by the usual construction $EC =$ pitch and $DC =$ slip. Then, considering the water as undisturbed and flowing axially to the propeller, we have DO as the direction of relative motion. Draw HJ parallel to DO. Then, with the usual form of section, the back of the blade from B to J will be met by this stream, and the resultant pressure will have a component from forward aft, thus reducing the

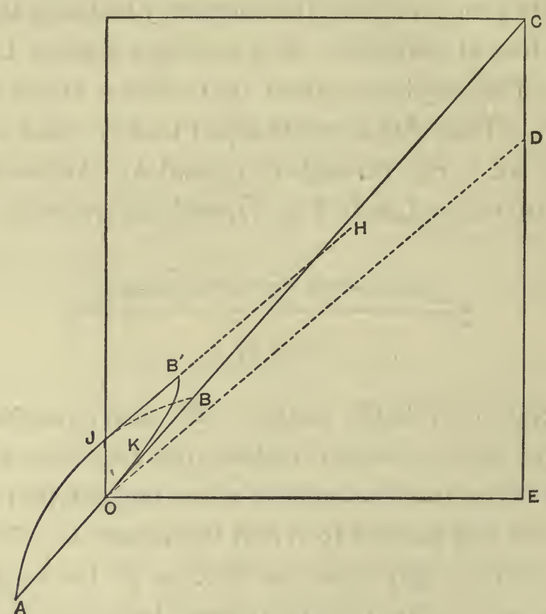

FIG. 88.

effective thrust. If, however, the section is made as shown by the full line, we have the back merely exposed to frictional resistance, while the direct flow meets the face only, and may thus give a slightly increased thrust. This change in section amounts to a great increase of pitch on the driving-face from B' to K. The simplicity of this construction, however, is doubtless far from representing the path of the water in the actual case, and while it is possible that some such modification of the section at the root may be advantageous, we are not in a position to base such an opinion upon experimental data.

We have above referred to the fact that the information actually needed by the pattern-maker or moulder does not include the various geometrical projections there referred to. The information strictly necessary is simply the following: The principal dimensions and characteristics of the propeller and hub; the expanded form or contour of the blade, as in $AGBG_1A_1$, Fig. 86; the shape and angular position of the generatrix, or else the form of a series of guides at successive radial distances; the distribution of the thickness. The necessary information may therefore be classified under these four heads:

(1) The chief dimensions;
(2) The contour of the blade;
(3) The law of the pitch;
(4) The distribution of the thickness;

Thus for a uniform pitch propeller, the information given in Fig. 86 aside from the projection *AKB* is all that is needed to prepare a pattern in wood or sweep up the form in loam.

Instead of the above method some designers prefer to lay down first the projection on a transverse plane, making the projected area a certain fraction of the disk area. See, for example, Fig. 53. The expanded form and the other projection may then be found by the inverse of the methods given above.

In Plate *A* is given a copy of the working drawing for the propellers of the U. S. battleships Indiana and Massachusetts, showing the information provided in these cases of naval war-ship design.

Measuring the Pitch of a Screw Propeller.—From (1) we have $p = 2\pi r \tan \alpha$. If therefore at a given value of r the

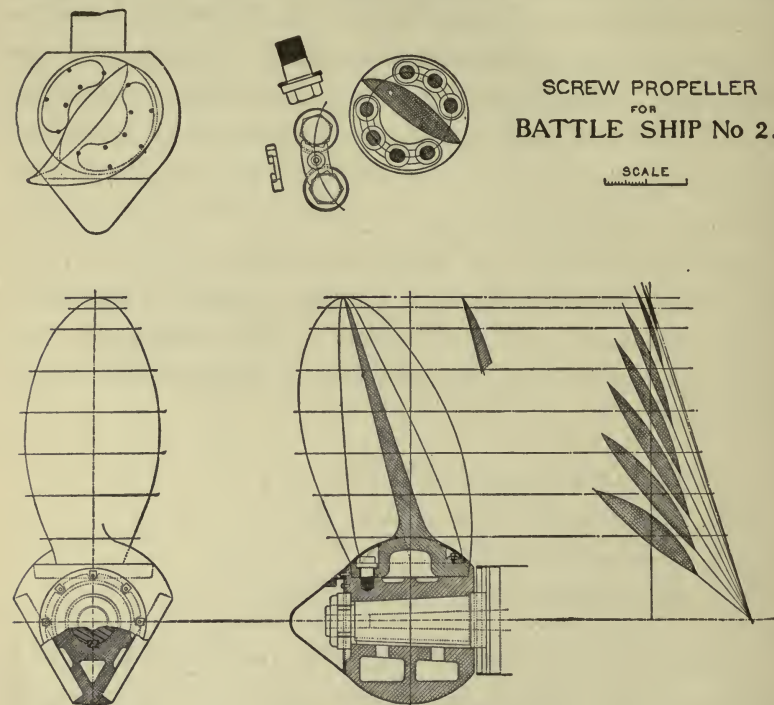

PLATE A.

value of tan α can be determined, the pitch is known. Such measurement requires necessarily the establishment of a transverse plane, or plane perpendicular to the axis of the propeller. If the propeller is lying on a floor, the latter may be used, care being taken that the propeller is so blocked up that its axis shall be perpendicular to the floor. Otherwise a plug carrying perpendicular to the axis a swinging arm may be fitted to the shaft hole, thus giving a movable line of reference. If the propeller is in place on the ship, the arm may be attached to the hub or to the shaft nut, or so arranged in any way as to give a line of reference movable or adjustable in the transverse plane.

We will first note an approximate method which consists in determining at what value of r we have $\alpha = 45^\circ$, tan $\alpha = 1$, and hence $p = 2\pi r$. The transverse plane being established as above, a bevel or triangle with angle 45° or even an improvised sheet of metal may be applied, and the corresponding location of r approximately found.

For more accurate measures we have to determine at a selected value of r two sides of a triangle, as in Fig. 89. AB

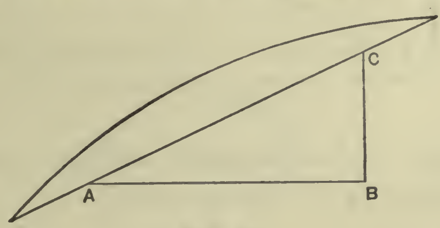

FIG. 89.

lies in a transverse plane and on the surface of a cylinder of radius r. AC lies on the face of the blade and on the same cylinder, being in fact their intersection. BC is parallel to the axis. The triangle ABC developed is evidently similar to ADC of Fig. 55 for the same radius and pitch, the relation being as shown in Fig. 90. If, therefore, we have an arm swinging about the axis, we may by running a line perpendicular to this arm at a fixed value of r determine any desired number of points on the line AC. The length of AC may then be measured by a flexible rule or batten, or, if it is short, it will not differ sensibly from the straight line joining its extremities. BC is then determined as the difference between the axial distances from the points A and C to the transverse plane or swinging arm. We have then

$$\sin \alpha = \frac{BC}{AC} \quad \text{and} \quad \tan \alpha = \frac{BC}{\sqrt{AC^2 - BC^2}},$$

$$\text{and} \qquad p = \frac{2\pi r BC}{\sqrt{AC^2 - BC^2}}.$$

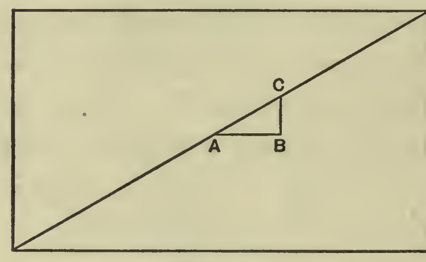

FIG. 90.

Still otherwise, we may measure the angle β subtended at the axis by the arc AB. We then have

$$AB = r\beta, \quad \tan \alpha = \frac{BC}{AB} = \frac{BC}{r\beta},$$

$$\text{and} \qquad p = \frac{2\pi BC}{\beta}.$$

If the distance AC is small relative to the dimensions of the propeller it gives the pitch simply for that particular small part of the surface, and if the pitch is thus determined in various parts of the surface it will usually be found to vary very considerably even in a propeller intended to be of uniform pitch. This arises from minor departures from exactness in the pattern, mould, and casting. In rough work such variation may be as great as 10 per cent on either side of the intended value, though with proper care the limit of variation should be much less.

The details of the operation of measuring the pitch will frequently vary with the means at hand and the particular circumstances of the case; but if the fundamental geometrical problem to be solved is kept clearly in mind, such necessary variations will present no difficulty.

Table for connecting the Slip Angle ϕ with the Slip Ratio s.—This is deduced as follows: Referring to Fig. 55, we have

$$\tan \alpha = \frac{p}{2\pi r} \quad \text{and} \quad \tan(\alpha - \phi) = \frac{(1 - s)p}{2\pi r},$$

whence, with the nomenclature there used, we readily find

$$\tan \phi = \frac{\pi y s}{\pi^2 y^2 + (1 - s)}.$$

Table I gives the values of ϕ for varying values of y and s.

Table for Determining the Modification of Pitch due to the Twisting of a Blade.—In propellers with detachable blades it is customary to provide for twisting the blade slightly on the hub, and thus modifying the pitch. The amount of modification at any given radial distance or value of y, is found as follows:

Referring to Fig. 55, we have in general

$$p = 2\pi r \tan \alpha = \pi d \tan \alpha,$$

whence $$\tan \alpha = \frac{p}{\pi d} = \frac{1}{\pi y}.$$

Let β be the amount of twist. Then, evidently,

$$p_1 = \pi d \tan(\alpha \pm \beta),$$

and

$$\frac{p_1}{p_0} = \frac{\tan(\alpha \pm \beta)}{\tan \alpha}.$$

Since β is always quite small, this may be put in the form

$$\frac{p_1}{p_0} = \frac{1 \pm \beta \cot \alpha}{1 \mp \beta \tan \alpha} = \frac{1 \pm \beta \pi y}{1 \mp \dfrac{\beta}{\pi y}}.$$

Table II gives values of $p_1 \div p_0$ for varying values of β and y.

It is thus seen that the amount of change varies considerably for different portions of the blade. Hence if the pitch were uniform before twisting, it cannot be so afterward. The twist will have, in fact, transformed the surface from one of uniform pitch, to one with a more or less irregular distribution from root to tip of blade. This, of course, is by no means necessarily prejudicial, and so long as the angle of twist is small we have no reason experimentally to expect any particular effect either good or bad due merely to the *irregularity* of the pitch. The twisting thus provided for is for the purpose of changing the effective or mean pitch of the propeller as a whole, and the above considerations indicate that so long as the amount of twist is small, we have no experimental basis for objecting to this mode of obtaining such modification.

As a final question we may ask what will be the mean pitch of the blade thus modified. This will depend on the definition of mean pitch as discussed in § 42, to which reference may be made.

TABLE I.

VALUE OF THE SLIP ANGLE ϕ.

Slip.	.05		.10		.15		.20		.25		.30		.35		.40	
Diameter Ratio.	°	′	°	′	°	′	°	′	°	′	°	′	°	′	°	′
.1	0	51	1	48	2	51	4	00	5	17	6	44	8	21	10	12
.2	1	21	2	47	4	20	6	00	7	49	9	46	11	53	14	11
.3	1	28	3	01	4	39	6	22	8	11	10	06	12	06	14	13
.4	1	25	2	54	4	26	6	02	7	41	9	24	11	10	12	59
.5	1	19	2	40	4	04	5	30	6	58	8	28	10	00	11	35
.6	1	12	2	25	3	41	4	57	6	15	7	34	8	56	10	18
.7	1	5	2	12	3	19	4	28	5	38	6	48	7	59	9	12
.8	1	00	2	00	3	01	4	03	5	05	6	08	7	12	8	16
.9	0	54	1	49	2	45	3	41	4	37	5	34	6	32	7	30
1.0	0	50	1	40	2	31	3	22	4	14	5	06	5	58	6	51

TABLE II.*

FACTOR FOR MODIFICATION OF PITCH DUE TO TWISTING OF BLADE.

Twist of Blade.	1°		2°		3°		4°		5°		6°	
	Factor for		Factor for		Factor for		Factor for		Factor for		Factor for	
Diameter Ratio.	Increase.	Decrease.	Increase.	Decrease.	Increase.	Decrease.	Increase.	Decrease.	Increase.	Decrease.	Increase.	Decrease.
.1	1.065	.943	1.138	.891	1.221	.843	1.315	.800	1.425	.761	1.553	.725
.2	1.040	.963	1.083	.927	1.127	.893	1.175	.861	1.226	.830	1.281	.800
.3	1.036	.966	1.073	.933	1.111	.901	1.152	.870	1.194	.840	1.237	.811
.4	1.037	.965	1.074	.930	1.112	.897	1.152	.864	1.193	.832	1.236	.801
.5	1.039	.962	1.079	.925	1.120	.886	1.162	.852	1.205	.817	1.249	.783
.6	1.043	.958	1.086	.918	1.130	.877	1.176	.837	1.222	.798	1.269	.760
.7	1.047	.954	1.094	.909	1.142	.864	1.192	.820	1.242	.777	1.293	.734
.8	1.051	.950	1.103	.900	1.156	.851	1.210	.802	1.264	.754	1.320	.707
.9	1.056	.945	1.113	.891	1.170	.837	1.229	.783	1.288	.730	1.348	.678
1.0	1.060	.940	1.122	.880	1.184	.821	1.247	.763	1.311	.705	1.376	.648

* Taylor's "Resistance of Ships and Screw-propulsion," Table XV.

CHAPTER V.

POWERING SHIPS.

56. INTRODUCTORY.

THE various constituents of the power which it is necessary to develop in the cylinders of a marine engine in order to propel a ship at a given speed have been already discussed in § 46. We have now to consider the various suppositions which will enable us to make an estimate of the total power thus required in any given case. These are of two general classes and lead to two general methods for the estimate of power.

(1) An assumption of the necessary constants which will enable us to compute the resistance and the power required to overcome it, together with that required for the various other constituents going to make up the I.H.P. This also may be done in either of two ways:

(*a*) We may make the various assumptions necessary to compute the different constituents individually. This involves the following elements: the power for the resistance proper or the E.H.P.; the amount involved in the augmentation of resistance; the wake factor; the propeller efficiency; and the amount absorbed by friction. By a proper combination of these as indicated by Fig. 66 we readily work back to the total I.H.P.

(*b*) We may make the assumptions necessary to compute the E.H.P. as above, and then assume the propulsive coefficient or ratio E.H.P. ÷ I.H.P., whence the latter follows directly from the former, and without special assumptions as to its other individual elements.

(2) The second general method involves certain assumptions which justify the extension of the law of comparison from resistance to power, and thus enable us to compute the I.H.P. in any given case from that actually observed for similar ships at corresponding speeds.

We shall discuss these methods in order.

57. The Computation of W_p, or the Work absorbed by the Propeller.

The necessary information and assumptions requisite for the computation of R have been discussed in Chapter I and in § 45. The amount of augmentation of resistance must be taken by judgment guided by such information relating to similar cases as may be available. Using the E.H.P. as a base, the additional amount of power thus required will usually be found between 10 and 25 per cent. Definite information with regard to the amount of this constituent of I.H.P. is only generally obtainable by model experiments as referred to in § 30. With such appliances, the true and augmented resistances are readily found.

Unless there is available some definite information based on model experiments, relative to the probable amount of augmentation, there is little use in attempting its independent estimate.

The determination of the wake factor w has been already referred to in §§ 44 and 49. With the wake factor and augmentation known or assumed, we readily pass, as explained in § 46, from *CF* to C_1E, Fig. 66.

If, instead of estimating augmentation and wake factor separately we should consider the ship efficiency (§ 46) as 1, then we have, in Fig. 66, $CF = C_1E$ and propeller power or $BE = CF$ divided by propeller efficiency.

The considerations relating to propeller efficiency have been discussed in §§ 49 to 51. Guided by such information as is available and as seems applicable to the case in hand, its value is assumed. This, as we have seen, may in usual cases be expected to lie between 65 and 69 per cent.

58. Engine Friction.

We come next to the amount of power absorbed by friction between the propeller and the cylinders.

We may remark at the beginning that there is very little exact or satisfactory information on these points. Model experiments are here of no avail, at least for purposes of actual measurement. Such measurement would require at the various speeds the simultaneous determination of the power in the cylinders and of the power received at the propeller. If, for example, just forward of the propeller there could be inserted a transmission dynamometer which would measure the turning moment, then the product of this by 2π times the number of revolutions per minute would give the power transmitted to the propeller. Such determinations are obviously difficult and costly, and hence more or less indirect methods are resorted to.

In the first place it is assumed that the total power absorbed by friction may be expressed by an equation of the form

$$W_f = hN + lW, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where h is a constant for any given engine, N is the number of revolutions, and hN is the amount due to what is termed the initial friction, while W is the I.H.P. and l is likewise a constant for the given engine.

The supposition that the total frictional power may be thus expressed is quite arbitrary, but seems to answer the purpose required as closely as present data can determine. It assumes simply that there will exist at the various joints and rubbing-surfaces a certain tangential resistance of which a part proportional to h is sensibly independent of revolutions or load, and represents what is termed the initial friction. The existence of this factor is chiefly due to the pressure at the various stuffing-boxes, piston-rings, etc., and to the weight of the shafting and other members supported in bearings. This part of the total friction is also sometimes known as that due to the *dead* load. The work necessary to overcome such a constant resistance will vary directly as the revolutions, and hence will be represented by a term of the form hN. It is further assumed that the remainder of the tangential resistance at the rubbing-surfaces will vary directly as the load on the engine, or as the mean effective pressure, and hence the work necessary to overcome this part will be proportional to the product of mean effective pressure and revolutions, or to total power developed. Hence this part of the frictional work will be represented by a term of the form lW.

Both of these terms may be expected to vary with the type and condition of the engine, and with the presence or absence of attached pumps. In general the friction of horizontal engines is greater than that of vertical. The power absorbed in friction also increases in general with the number of cylinders and with the multiplication of rubbing-joints. The friction of new engines is also naturally greater than when they have become somewhat worn. Again, incorrect adjustment or alignment either at the beginning or due to wear or working of the ship may give rise to excessive frictional loss. In such cases the condition is likely to manifest itself by hot bearings and other signs that something is wrong at the points thus affected. The presence of attached air-pumps occasions an absorption of power not only to overcome the friction of the pump, but for its regular running, which amount must be subtracted from the I.H.P. in order to properly obtain that sent to the propeller. This item is not to be viewed as a loss, but simply in such cases as a constituent of the total power which must be eliminated in order to determine the net amount delivered to the propeller. With independent pumps this item, of course, does not exist as a constituent of the power of the main engines.

The power required to operate pumps has been determined by various special and individual trials. The power absorbed by the initial friction has been determined by slowly running the unloaded main engine by the turning engine, or by derivation from curves of revolution and power as described under the next sub-head. The power absorbed by the load friction has been estimated from various coefficients derived from stationary engines in which the different quantities are susceptible of measurement, and from analyses of the total I.H.P. in which the various efficiencies and ratios are derived by comparison from model experiments.

Determination of Initial Friction from Speed-trial Data.—In § 47 we have defined the reduced mean effective pressure, and we may evidently consider this as balanced against the mean total resistance to the movement of the engine. This total resistance may be considered as the sum of two parts, one due to the useful load and the other due to the friction load.

We have already referred to the frictional or tangential resistance at the joints and rubbing-surfaces under two heads: that due to initial conditions and that due to the load, corresponding to initial and load friction. Evidently the frictional work will require an additional pressure on the pistons or an additional amount of reduced mean effective pressure as above defined, and such additional amount may naturally be considered in two parts, one due to initial friction and the other to the load friction. We may thus consider the entire reduced mean effective pressure as made up of three parts, one corresponding to the net or delivered power and two due to friction as above. Denoting these respectively by p, p_1, p_2, we have

$$\bar{p} = p_1 + p_2 + p. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From the nature of initial friction as above defined it follows that p_1 is constant and is in fact proportional to the factor h there used, while p_2 and also p will vary with the load.

If now we have a series of corresponding values of H and N, we may derive by § 47 (3) the values of $\bar{p}$. Plotting these on N as an abscissa, we shall have a curve as in Fig. 91. Now when $N = 0$, p_2 and p must necessarily be 0, and hence $\bar{p}$ will be equal to p_1. If therefore the curve be extended back by judgment, it will cut the axis of Y at a point A which will give an approximation OA to the value of p_1. Then for the

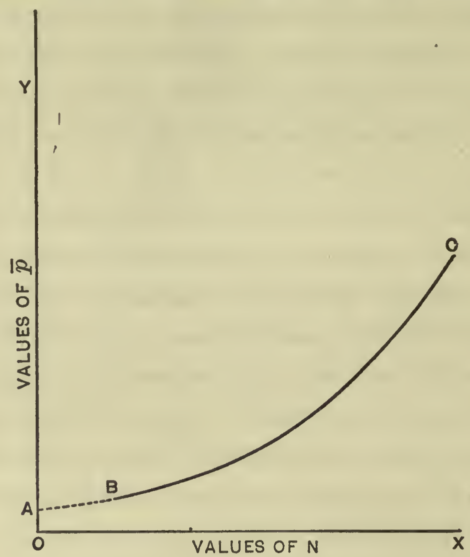

FIG. 91.

general value of the initial friction power at any number of revolutions N we have

$$\frac{2p_1LA_3N}{33000} = hN, \text{ as above.}$$

Hence we see that $$h = \frac{2p_1LA_3}{33000}.$$

In order that such an approximation may be of value, special care should be taken with the determinations relating to the lower points on the curve, and the revolutions should be reduced to the lowest number at which the engine can be kept in continuous movement.

We may also for the same result use the same data somewhat differently as follows:

From the common horse-power formula we have

$$\frac{\partial H}{\partial N} = \frac{2\bar{p}LA}{33000}.$$

Hence $$\left.\frac{\partial H}{\partial N}\right]_0 = \frac{2p_1LA_2}{33000}.$$

Hence if H be plotted on N as an abscissa we shall have a curve as in Fig. 92. If this be extended back through O by

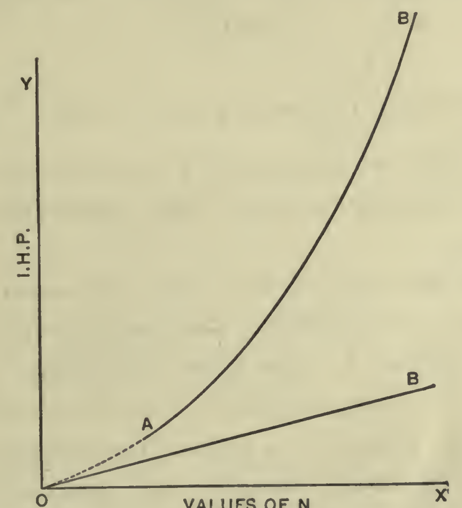

FIG. 92.

judgment, it will have at O a certain inclination to OX, and the tangent of this angle or of BOX will therefore be an approximation to the value of $\left.\frac{\partial H}{\partial N}\right]_0$; and from this the corresponding value of p_1 may be found. See also § 78 for another method of finding the initial friction.

The existing data relating to initial friction gathered in this and other ways indicate that in amount it may be expected to lie between 5 and 8 per cent of the full power of the engine, or that p_1 will be from 5 to 8 per cent of the $\bar{p}$ for full load. This would correspond to a value of from 2 to 3 pounds per square inch on the low-pressure piston for triple-expansion engines under usual conditions as above noted. Let this percentage ratio be denoted by i and denote full-power conditions by accents. Then we should have

Work of initial friction at full load $= hN' = iH'$,

$$\text{or} \qquad h = i\frac{H'}{N'};$$

and work for initial friction in general $= hN = \dfrac{iH'N}{N'}$, where, as above stated, we may expect i to be found between .05 and .08. These figures relate more especially to the main engines alone.

Value of the Load Friction.—We have above stated that this is taken as proportional to the whole power H. It might seem more correct to take it as proportional to the total power minus the initial friction. The difference involved is, however, quite negligible in view of the general uncertainty surrounding the whole question, and we naturally prefer, as the simpler of the two, the method as stated. We have therefore

$$\text{Load friction} = lH.$$

The existing data relating to load friction indicate that l may be expected to vary between nearly the same limits as i, or from .05 to .08.

General Equation for Friction.—Combining the two parts, we have

$$\text{Total friction} = \frac{iH'N}{N'} + lH.$$

If we plot the total power and the power absorbed by these two components of friction on revolutions as an abscissa, we shall have a diagram as in Fig. 93, where the ordinates

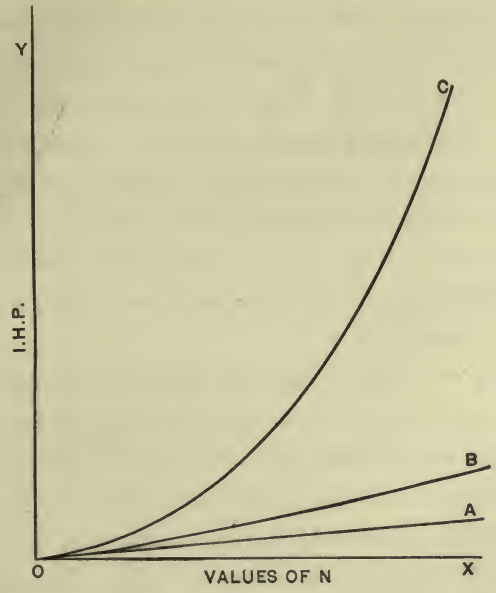

FIG. 93.

between X and OA give the values of the initial friction, those between OA and OB the values of the load friction, and those between OB and OC the values of the net or delivered power. This equation and diagram serve to illustrate the increasing relative importance of friction at reduced speeds. Thus suppose at full power the friction is taken at 14 per cent, equally divided between load and initial friction. Denoting full power by 100, we should then have for one-half speed approximately one-eighth power or 12.5. The load friction would be reduced in the same proportion, and would therefore become $7 \div 8 = .875$. The initial friction would be reduced in the ratio of the revolutions or approximately to one half its full-power value, or to 3.5. Hence the total friction will be 4.375, and the percentage 4.375 ÷ 12.5 = 35 per cent. This illustrates forcibly the wastefulness of running at reduced power, and the impossibility of obtaining good propulsive efficiencies under these conditions.

At full power, therefore, we may expect that the friction of a marine engine will absorb an amount lying not far from, say, 15 per cent as a mean value. With the best modern design and construction this figure may fall to 12 per cent or even possibly to 10 per cent, while with older engines or poorer construction or lack of adjustment it may rise to 18 or 20 per cent or even more.

It should be understood that the division of the power absorbed by the friction as indicated in the present section is not altogether satisfactory, and the only excuse for its use is our ignorance of a more correct analysis. It is not to be expected that p_1 will be absolutely constant at all revolutions or that p_2 will vary exactly with the load. Further experimental investigation is much needed on these points.

59. Power required for Auxiliaries.

In the present work we are fundamentally concerned, of course, with the power needed for propulsion. Inasmuch, however, as we not infrequently meet with air-pumps attached to the main engine, as referred to in the last section, and also with estimates of power or power data, including that required for auxiliaries, it will be well to note briefly the power which the usual auxiliaries may be expected to absorb.

There has been a great reduction in this amount within the past few years. Auxiliary machinery has been so improved in design and construction that its cost in power required for operation is much less than it was from ten to twenty years ago. At the earlier period from 5 to 8 per cent of the I.H.P. was believed to be absorbed by the resistance of the various pumps, most of which were attached to the main engine. Blechynden,* writing for a period about ten years since, gives the following estimate, based on data derived mostly from merchant steamers with triple-expansion engines:

Dead load and air-pump...	7.8	Per cent of I.H.P.
Circulating-pump.........	1.5	
Feed-pump	.6	
Bilge-pump....	.5	

If we take an estimate for the air-pump of about 1 per cent, which would not be far in error for the cases involved, we should have 3.6 per cent required for all auxiliaries. These figures agree generally with those obtained in this country about ten years ago, especially in the early vessels of the modern steel navy. Taking more recently the results of the latest practice, especially in war-ship design, we have the following results:

Air-pump	.1 to .2	Per cent of I.H.P.
Circulating-pump....	.2 to .5	
Feed-pump.........	.5 to .6	
Blowers	.5 to 1.5	

Omitting the item for blowers, we have for the remaining three items from .8 to 1.3 per cent, or slightly over 1 per

* Transactions N. E. Coast Inst. Engineers and Shipbuilders, vol. VII. p. 194.

cent as a mean value. The chief improvement has been in air- and circulating-pumps, which together a few years ago absorbed from 2 to 3 per cent, while with the latest practice this figure has been reduced to about one quarter of this amount. The item for bilge-pumps in earlier estimates is not so often met with now, having decreased to almost a negligible amount—probably less than .1 per cent. The amount absorbed by blowers is naturally variable between wide limits, being dependent on the degree of forced draught and other circumstances.

It may be doubted whether the power of an attached air-pump can be brought to as small a figure as for an independent pump. This is due chiefly to the fact that the attached pump usually runs faster than the independent, and faster than is needed to maintain a vacuum. In consequence the power is necessarily greater, and for such cases probably not less than .5 per cent will be required.

60. Illustrative Example.

We will now illustrate the application of the preceding sections to the computation of the I.H.P. in a given case, assuming where necessary the various values involved.

Referring to Fig. 66, let true resistance $= R$.

Then E.H.P. $= CF = Ru$.

Let the factor of augmentation be 1.16.

Then actual resistance $= 1.16R$.

And thrust horse-power $= CE = 1.16Ru$.

Let true propeller efficiency $= .67$.

Let wake factor $w = .15$.

Then $(1 + w) = 1.15$.

And horse-power $C_1E = 1.16Ru \div 1.15 = 1.01Ru$.

Then propeller horse-power $= BE = 1.01Ru \div .67 = 1.51Ru$.

Next, for the frictional loss let i and l each $= .07$.

Then I.H.P. $= AE = 1.51Ru \div .86 = 1.756Ru$.

$$\text{Propulsive coefficient} = \frac{Ru}{1.756Ru} = .57.$$

These suppositions correspond to the following subdivision of the total I.H.P. in percentages.

Initial friction	= .07
Load friction	= .07
Thrust horse-power $= \frac{1.16}{1.756}$	= .661
Propeller loss decreased by wake gain.	= .199
Total	= 1.000

If, as in § 56 (1) (*b*), the propulsive coefficient were to be assumed directly, we should in the preceding example divide Ru by .57, thus giving $1.756Ru$ at once.

Unless special data are available for the estimate of the coefficients and ratios involved in the process above, it will be found usually quite as satisfactory to assume the propulsive coefficient directly. This again can only be done intelligently when experimental data are at hand from somewhat similar ships under generally like conditions. The range of values, it will be remembered, is usually from .50 to .60.

61. Powering by the Law of Comparison.

The assumptions involved are as follows:

(1) It is assumed that the entire resistance is subject to the law of comparison. The propriety of this assumption has been considered in § 26, to which reference may be made. As there shown, the error is quite small, and may be considered as well within the limit of general uncertainty surrounding the entire problem. Further, with a slightly roughened bottom, as actually exists for most of the time, the index of the speed in the term for skin-resistance will approach the value 2.00, and the error will correspondingly decrease.

(2) It is assumed that the term *similar* is extended to include *ship*, *propeller*, and *machinery*. This involves the general geometrical similarity between the ships and between the propellers, the ratio being, of course, the same for each. Geometrical similarity is not necessary for the machinery, but it should be of the same general type in each case, or at least there should be good ground for assuming an equal engine friction in each case.

(3) It is assumed that the term *corresponding speeds* is extended to cover the speeds of the propellers relative to their ships and to each other, as well as the speeds of the ships relative to the water.

Hence, as in § 53, we shall have similar points on the two propellers describing similar paths with velocities in the ratio $\lambda^{\frac{1}{2}}$, and as there shown we shall have also

$$\frac{N_2}{N_1} = \left(\frac{L_1}{L_2}\right)^{1/2} = \frac{1}{\lambda^{1/2}}.$$

(4) It is assumed if (2) and (3) are fulfilled that the thrusts will bear the same relation to the resistance in each

case, that the slips true and apparent will be the same per cent in the two cases, that the efficiencies will be the same, and likewise the augmentation and wake factors. It is assumed, in short, that the entire relation of the propeller to the ship will be the same in the two cases.

The propriety of these assumptions evidently depends on the same general basis as the application of the law of comparison to the resistance of the ship itself.

As a result of the preceding assumptions it follows that similar ships with similar propellers at corresponding speeds will have equal propulsive coefficients.

Or otherwise we may less safely consider that independent of the fulfillment of (2) and (3), the propulsive efficiencies and relations are the same in the two cases.

Let the subscripts 1 and 2 denote the two cases. Then we have

R_2 and R_1 are the two resistances;

u_2 and u_1 are the two speeds;

l_2 and l_1 are any two similar dimensions;

$\lambda = l_2 \div l_1$ is the linear ratio;

$a =$ the propulsive coefficient;

H_2 and H are the two powers.

Then

$$H_1 = \frac{R_1 u_1}{a},$$

$$H_2 = \frac{R_2 u_2}{u},$$

and
$$\frac{H_2}{H_1} = \frac{R_2 u_2}{R_1 u_1}.$$

Now we have in § 26 a series of expressions for $R_2 \div R_1$ in terms of various functions of the ship and of the speeds. Substituting these, and remembering that $u_2 \div u_1 = \sqrt{\lambda} =$

$\sqrt{l_2 \div l_1}$, we readily find the following series of expressions for $H_2 \div H_1$, the ratio between the powers necessary to propel similar ships at corresponding speeds:

$$\frac{H_2}{H_1} = \left(\frac{l_2}{l_1}\right)^{\frac{7}{2}} = \left(\frac{A_2}{A_1}\right)^{\frac{7}{4}} = \left(\frac{V_2}{V_1}\right)^{\frac{7}{6}} = \left(\frac{D_2}{D_1}\right)^{\frac{7}{6}} = \left(\frac{D_2}{D_1}\right)\left(\frac{l_2}{l_1}\right)^{\frac{1}{2}} = \left(\frac{D_2}{D_1}\right)^{\frac{1}{2}}\left(\frac{l_2}{l_1}\right)^{2}$$

$$= \frac{D_2 u_2}{D_1 u_1} = \left(\frac{D_2}{D_1}\right)^{\frac{5}{6}}\left(\frac{u_2}{u_1}\right)^{2} = \left(\frac{D_2}{D_1}\right)^{\frac{2}{3}}\left(\frac{u_2}{u_1}\right)^{3} = \left(\frac{l_2}{l_1}\right)^{2}\left(\frac{u_2}{u_1}\right)^{3} = \left(\frac{A_2}{A_1}\right)\left(\frac{u_2}{u_1}\right)^{3}$$

$$= \left(\frac{D_2}{D_1}\right)^{\frac{1}{2}}\left(\frac{u_2}{u_1}\right)^{4} = \left(\frac{D_2}{D_1}\right)^{\frac{1}{3}}\left(\frac{u_2}{u_1}\right)^{5} = \left(\frac{l_2}{l_1}\right)\left(\frac{u_2}{u_1}\right)^{5} = \left(\frac{D_2}{D_1}\right)^{\frac{1}{6}}\left(\frac{u_2}{u_1}\right)^{6} = \left(\frac{u_2}{u_1}\right)^{7}. \quad (1)$$

It is obvious that the list of expressions in which the law of comparison may be thus represented is by no means exhausted, and that all are equivalent and equally correct, and will give identical results, provided the fundamental conditions are fulfilled; i.e., if the ships are similar and the speeds corresponding.

If therefore we know the data in any one case it is a simple matter to derive the value for a similar ship at a corresponding speed.

As an illustration, let

$$D_1 = 2000;$$
$$H_1 = 3000;$$
$$u_1 = 16;$$
$$D_2 = 3200.$$

In § 26 various expressions have been given for corresponding speeds. Here, having given only D_1 and D_2, we take naturally $u_2 = u_1\left(\frac{3200}{2000}\right)^{\frac{1}{6}} = 1.085 u_1 = 17.4.$

Then $H_2 = H_1 \frac{D_2 u_2}{D_1 u_1} = 3000 \times \frac{3200}{2000} \times 1.085 = 5230.$

In a similar way, taking other expressions for $\frac{H_2}{H_1}$, the same value would be found. Hence the similar ship of displacement 3200 at 17.4 knots would require 5230 I.H.P.

Assuming that the similarity between the two ships is perfect, the two chief sources of error in the use of this method are in the extension of the law of comparison to skin-resistance, and in the possibility of a difference in the propulsive coefficients in the two cases. The nature of the error due to the first of these has been discussed above and in § 26. In regard to the latter it may be seen that if the speed of the first ship, for example, is low or far from the normal speed the propulsive coefficient will probably be poor, and less than might be properly expected for the second ship if near her normal speed The use of the power of the first ship in such a case would therefore, other things being equal, result in an overestimate of the power for the second.

In all such cases, therefore, judgment must be used as to what extent the necessary conditions are fulfilled, and as to the amount of reliance to be placed on the values resulting from any given comparison.

62. The Admiralty Displacement Coefficient.

Among the various formulæ for powering which have been in use for many years, none has obtained so wide and general acceptance as that involving the use of the so-called *Admiralty displacement constant or coefficient*. This formula dates from a period long anterior to the introduction of the law of comparison, and we shall find it of interest to examine it in the light of that law.

The formula is expressed by the equation

$$H = \frac{D^{\frac{2}{3}} u^3}{K}.$$

In practice it was found that there was a general tendency to constancy in the values of K, ranging as they usually did from 200 to 250. The use of the formula required simply the estimate of the constant suited to the ship and to the speed. This, of course, was a matter of considerable uncertainty except as experience in similar cases furnished data as a basis. The values of K naturally vary for the same ship at different speeds, or for different ships at the same speed. These values are also seen to vary directly as the efficiency of propulsion. That is, in any given case the greater the value of K the less the value of H, and hence the greater the propulsive efficiency. In numerous cases in which progressive speed trials have been made the results for the value of K are similar to those shown in Fig. 94. At low speeds the

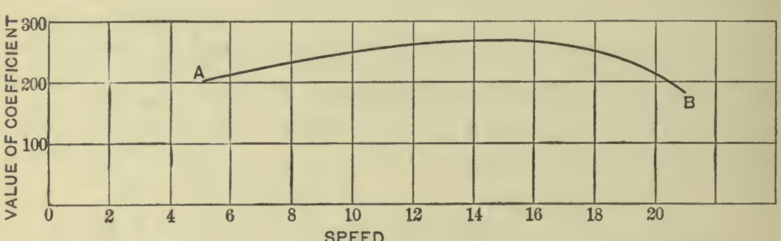

FIG. 94.

value is low, due to the general mechanical inefficiency of both engine and propeller at powers and speeds far below those for which they were designed. As the speed increases the value of K increases, indicating that due to an increasing efficiency the total power is related to the speed by an index of n less than 3. This continues to some speed at which, on this basis, the propeller is the most efficient and K has its maximum value. Then as the speed is increased the resistance begins to increase excessively, and perhaps the propulsive efficiency begins to decrease. The result is that H becomes related to u with an index greater than 3, and we have a corresponding fall in the value of K.

So long as speeds were moderate and abundant information was available relating to nearly equal ships at nearly the same speeds, the use of this coefficient was reasonably satisfactory. Its assumption, however, was almost entirely a matter of judgment, and there were few guiding principles by means of which any definite value might be determined as suitable in any given case.

The use of the formula in this way has therefore in recent years fallen into disfavor, and it is well understood that thus employed it can only be expected to give a rough approximation to the power suitable in any proposed case.

Let us now examine the relation of this formula to the law of comparison. We have in one case

$$H_1 = \frac{D_1^{\frac{2}{3}} u_1^3}{K_1};$$

and in another:

$$H_2 = \frac{D_2^{\frac{2}{3}} u_2^3}{K_2}.$$

Whence
$$\frac{H_2}{H_1} = \frac{K_1}{K_2} \cdot \frac{D_2^{\frac{2}{3}} u_2^3}{D_1^{\frac{2}{3}} u_1^3}. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Now suppose that D_1 and D_2 are similar ships and u_1 and u_2 are corresponding speeds. Then from § 61 (1) we have, by the law of comparison,

$$\frac{H_2}{H_1} = \frac{D_2^{\frac{2}{3}} u_2^3}{D_1^{\frac{2}{3}} u_1^3}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence, comparing (1) with (2), it follows that $K_1 \div K_2 = 1$ or $K_1 = K_2$. We therefore derive the important result that—

Similar ships at corresponding speeds have the same Admiralty coefficient.

This may be taken as a statement of the law of extended comparison, and as such it will be equally correct with any of the other statements implied in the general equation of § 61, (1). This statement supplies therefore the principle necessary to guide in the choice of Admiralty coefficients, and once such principle is recognized and followed, this formula is instantly raised from its former condition of relative unreliability and placed on the same level, or rather made identical with the law of extended comparison, as already so numerously expressed above.

As an illustrative example let us take the following:

$$D_2 = 8000;$$
$$u_2 = 18.$$

Among the available data is a speed-power curve for a similar ship of displacement 6000. We have now to find the speed at which the latter ship will correspond with the former. Without involving any other data we may take the ratio as $(D_2 \div D_1)^{\frac{1}{6}} = 1.049$. Hence $u_1 = 18 \div 1.049 = 17.16$. Turning now to the given curve at a speed of 17.16, suppose we find $H = 6320$. We have then $D_1 = 6000$, $u_1 = 17.16$, and $H_1 = 6320$. Substituting this in the Admiralty formula we find $K = 264$. Now by the law of comparison as above shown, this coefficient is suitable for the similar ship of displacement 8000 at the corresponding speed 18. Hence, making the substitution in the formula, we find for the proposed case

$$H = \frac{400 \times 5832}{264} = 8836.$$

63. The Admiralty Midship-section Coefficient.

In addition to the Admiralty displacement formula, the equation

$$H = \frac{(\text{area of midship section})u^3}{K_1}$$

has been used to a considerable extent, especially in continental Europe. Where the value of K_1 is selected by judgment alone, it stands on the same basis as the $D^{\frac{2}{3}}$ formula, though it is hardly as satisfactory, since the length of the ship is entirely unrepresented in the formula, except as its value is taken into account in the judgment by which the value of K_1 is selected. Comparing it, however, with § 61 (1), it is seen to be identical with the $D^{\frac{2}{3}}$ formula in its relation to the law of extended comparison. If, therefore, it is used in this way for similar ships at corresponding speeds, it becomes simply another mode of application of this extended law. We may thus say, as before:

Similar ships at corresponding speeds have the same midship-section power coefficient.

64. Formulæ involving Wetted Surface.

Formulæ have been proposed and used quite extensively of the general form

$$H = \frac{Su^3}{K_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

As proposed, these formulæ were to be used in a manner similar to that in which the two Admiralty formulæ have usually been used. That is, the values of K_2 were to be selected by judgment. They are, when thus used, open to the same objections and are under the same lack of confidence as the two above described when used in the same way.

In the method known as "Kirk's Analysis" an approximation to the wetted surface was found by computing that of a substituted block model with prismatic middle body and wedge-shaped fore and after bodies. The appropriate value of K_2 was then assumed from the data available in similar cases. This method was more especially intended for the powering of cargo steamers of moderate size and speed, with an abundance of data available to serve as a guide in the selection of the values of the constant. Under these circumstances the method gave fairly satisfactory results, as indeed would any of the other methods here discussed when used in the same way.

In Rankine's "augmented surface" method the wetted surface, or more commonly the reduced surface, § 8, was multiplied by a factor called the coefficient of augmentation. This coefficient is a function of the form of the ship at the bow, or more specifically of the angles of maximum inclination of the various water-lines at the bow. The area thus obtained was called the "augmented surface." Its value was used for S in (1) above, and appropriate values of K_2 were assumed exactly as in other cases.

The derivation of this formula proceeded on the assumption that the only resistance necessary to consider was the frictional, or at least that the resistance might be considered as varying with the square of the speed and hence the power as the cube of the speed; and further, that the influence of the form on the velocity of gliding or the relative velocity of skin and water could be represented by this function of the angles of entrance. The method is interesting as containing an attempt to represent the influence of the form at the bow. It is entirely inadequate, however, to properly provide for either variation of form in general, or for varying speeds. It is in fact subject to exactly the same limitations and errors as the other formulæ here discussed, and requires the same kind of judgment for the proper selection of the coefficient involved.

The relation of such formulæ to the law of extended comparison is seen to be identical with that of the two previously discussed. In general it is seen that they are all special cases of the general term in which the ship is represented by an area or second-degree function, and the speed has the index 3. If therefore they are used as expressions of the laws of extended comparison they will have the same authority and will give the same results as any of the other expressions previously given. All such formulæ may therefore be made expressions of the law of extended comparison by the simple rule:

Similar ships at corresponding speeds will have the same coefficient in the formula for power.

65. Other Special Constants.

The various forms for the law of extended comparison in § 61 show that a large variety of "constants" or coefficients might be found which would be the same for similar ships at corresponding speeds. The constants considered in §§ 62, 63, and 64 are all derived from the function l^2u^3. Of the various constants which might be found in a similar manner we will only refer to those derived from the functions Du and

$D^{\frac{1}{2}}u^4$. Denoting these by K_3 and K_4, we have for the corresponding formulæ for power

$$H = \frac{Du}{K_3},$$

and $$H = \frac{D^{\frac{1}{2}}u^4}{K^4};$$

whence $$K_3 = \frac{Du}{H},$$

and $$K_4 = \frac{D^{\frac{1}{2}}u^4}{H}.$$

If for similar ships at corresponding speeds the same values of the constant K_3 be taken, or the same values of the constant K_4, the law of comparison will be fulfilled and the formulæ may thus be used for the determination of power by means of this law in the same manner as with those discussed in §§ 62, 63, and 64.

We will now briefly discuss the use of these various constants in cases where the geometrical similarity is not complete.

66. The Law of Comparison where the Ships are Not Exactly Similar.

Strictly speaking, the law of comparison has no significance where the ships are not similar. Where they are, any and all of the various expressions for corresponding speeds, for resistance, and for power will give exactly the same results. When the similarity is not perfect, however, the various expressions for corresponding speeds, for resistance and for power will not give the same results; and while strictly none are applicable, it is quite evident that they may all be considered as approximations, more or less near according to the nature of the similarity and to the particular expression used. It is evident therefore that in such cases, and hence in most cases likely to arise in actual practice, it is not altogether a matter of indifference what expressions are used for the relations between the speeds and between the powers in the two cases. We must first note clearly the two questions involved: (*a*) The relation between the powers; and (*b*) the relation between the speeds, or the definition of "corresponding speeds." With regard to the latter, we are free to take any of the various expressions of § 26, (7) and (8). Corresponding speeds are intended to be those for which the liquid surroundings of the two similar ships shall also be similar. These depend principally on the wave-formation, and this on the length of the ship more than on any other one feature. It therefore seems preferable to fix the corresponding speeds by the ratio of the lengths, and we take therefore

$$\frac{u_2}{u_1} = \lambda^{\frac{1}{2}} = \left(\frac{l_2}{l_1}\right)^{\frac{1}{2}} = \left(\frac{L_2}{L_1}\right)^{\frac{1}{2}}.$$

For the power-ratio we will discuss the three functions Du, $D^{\frac{2}{3}}u^3$, $D^{\frac{1}{2}}u^4$, or the functions which give rise to the three coefficients K_3, K, K_4, as above derived.

Taking the first, its use is equivalent to the assumption that for similar ships at corresponding speeds the power varies as the function Du, or in symbols

$$H \sim Du.$$

Denoting the chief dimensions of the ship by L, B, h, the block coefficient by b, and remembering that $u \sim L^{\frac{1}{2}}$ we have for similar ships at corresponding speeds

$$R \sim D \sim BhLb \sim \frac{BhLb}{u^2}u^2 \sim Bhbu^2.$$

If therefore we assume more broadly that resistance in general can be expressed as the product of some function of the ship by the square of the speed, the use of the constant K_3 is equivalent to the assumption that such function is Bhb.

Similarly we have

$$R \sim D \sim BhLb \sim \frac{BhLb}{u^4}u^4 \sim \frac{Bhb}{L}u^4.$$

Hence, likewise, if we assume that resistance in general can be expressed as the product of some function of the ship by the fourth power of the speed, the use of the constant K_3 is equivalent to the assumption that such function is $\frac{Bhb}{L}$.

Now taking a general equation of resistance involving the second and fourth powers of the speed, let us introduce those functions into the coefficients. We have then

$$R = P(Bhb)u^2 + Q\left(\frac{Bhb}{L}\right)u^4 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The equation of resistance in this form is thus seen to correspond to the use of the coefficient K_3 as above.

In a similar manner we find for the form of the general equation which corresponds to the use of the coefficient K the following:

$$R = P(D^{\frac{2}{3}})u^2 + Q\left(\frac{D^{\frac{2}{3}}}{L}\right)u^4 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Likewise for that which corresponds to the use of K_4,

$$R = P(DL)^{\frac{1}{2}}u^2 + Q(Bhb)^{\frac{1}{3}}u^4 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

But in § 8 we have seen that approximately $(DL)^{\frac{1}{2}} \sim$ wetted surface or $(DL)^{\frac{1}{2}} \sim S$. Whence we may write (3) in the form

$$R = P(S)u^2 + Q(Bhb)^{\frac{1}{3}}u^4 \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

We may also show the relation of the three functions to the dimensions of the ship and the block coefficient as follows:

For comparison the function Du is equivalent to $L^{\frac{3}{2}}Bhb$;
" " " " $D^{\frac{2}{3}}u^3$ " " " $L^{\frac{13}{6}}(Bhb)^{\frac{2}{3}}$;
" " " " $D^{\frac{1}{2}}u^4$ " " " $L^{\frac{5}{2}}(Bhb)^{\frac{1}{2}}$.

From these last expressions it is readily seen how the different values would vary in cases where the similarity is not exact. Thus suppose that the second ship is relatively longer than the first. Then the length-ratio will be greater than the others, and the power as derived from the above expressions will increase in amount from the first to the last. *Vice versa*, if the second ship were broader, deeper, or fuller, the first of the above functions would give the largest value, and the others successively less.

Looked at from the standpoint of the formula of resistance, it would appear as though (3), and hence the function $D^{\frac{1}{2}}u^4$, or the constant K_4 should be the most nearly correct. Experience seems to indicate, however, that there is no one form equally applicable to all modes of variation from exact similarity, and sufficient data are lacking for the definite relation of the appropriate form of function to the nature of the departure from similarity.

Mention may also be made of the function $D^{\frac{7}{6}}$, which is sometimes used, especially in connection with a speed-ratio $(D_2 \div D_1)^{\frac{1}{6}}$.

Of these various functions, $D^{\frac{2}{3}}u^3$ and $D^{\frac{1}{2}}u^4$ or the constants K and K_4 may be recommended in preference to the others. The latter is perhaps likely to be the more correct, though the former, from the fact that it is the well-known Admiralty coefficient, has been much used; and such use is likely to continue so long as no more definite information is available regarding the relative values of the various functions and coefficients available.

We may illustrate the use of these expressions by an example as follows. Given the following data:

	L	B	h	D	u	H
(1)........	300	40	20	3700	15	3129
(2)........	400	48	24	7640		

Then $$\left(\frac{L_2}{L_1}\right)^{\frac{1}{2}} = (1.33)^{\frac{1}{2}} = 1.155.$$

Hence $$u_2 = 15 \times 1.155 = 17.32.$$

We will now find the value of H_2, using the following ratios, the results being as shown:

$\frac{H_2}{H_1}$		H_2
$\left(\frac{D_2}{D_1}\right)^{\frac{7}{6}}$	$= 2.32$	7260
$\left(\frac{D_2}{D_1}\right)^{\frac{2}{3}}\left(\frac{u_2}{u_1}\right)^3$	$= 2.497$	7814
$\frac{D_2 u_2}{D_1 u_1}$	$= 2.385$	7464
$\left(\frac{D_2}{D_1}\right)^{\frac{1}{2}}\left(\frac{u_2}{u_1}\right)^4 = \left(\frac{D_2}{D_1}\right)^{\frac{1}{2}}\left(\frac{L_2}{L_1}\right)^2$	$= 2.563$	8020

Where the variation from similarity is marked, the method of comparison, of course, fails entirely, and it must be remembered that its reliability rapidly decreases as departures from similarity increase. The discussion of the present section is therefore not intended to show a means of extending the method of comparison to widely dissimilar forms, but simply to show what forms of expression are most likely to take rational account of such slight variations as experience indicates may be admitted without sensibly affecting the application of the law as stated.

67. English's Mode of Comparison.*

Given two similar ships of displacement D_1 and D_2 at speeds V_1 and V_2, not in general "corresponding." Then it is always possible to make two models of different dimension such that at the same speed one shall correspond to D_1 and the other to D_2. From the principles of § 26 we should then have the following:

Residual Resistance.	Skin-resistance.	Displacement.	Speed.
W_1	S_1	D_1	V_1
W_2	S_2	D_2	V_2
w_1	s_1	d_1	$V_1\left(\frac{d_1}{D_1}\right)^{\frac{1}{6}}$
w_2	s_2	$d_1\frac{D_2}{D_1}\left(\frac{V_1}{V_2}\right)^6 = d_2$	$V_1\left(\frac{d_1}{D_1}\right)^{\frac{1}{6}}$

Let n be the ratio of the total resistances of the models at the speed $V_1\left(\frac{d_1}{D_1}\right)^{\frac{1}{6}}$. Then

$$w_1 = W_1\frac{d_1}{D_1},$$

and $$(w_2 + s_2) = n(w_1 + s_1).$$

Also $$W_2 = w_2\frac{D_2}{d_2}.$$

* Proceedings Institution of Mechanical Engineers, 1896, p. 79.

Substituting for these values as above, we find

$$W_2 = \left(\frac{V_2}{V_1}\right)^6 \left(nW_1 + \frac{D_1}{d_1}(ns_1 - s_2)\right).$$

Then

$$\text{Total resistance of } D_2 = S_2 + W_2.$$

The values of S_1, S_2, s_1, s_2, are to be computed in the usual way, using appropriate values for the skin-resistance coefficient. Assuming also for the ship D_1 the ratio between thrust horse-power and I.H.P. ($CE \div AE$, Fig. 66), we can find $S_1 + W_1$, and hence W_1. The value of n is found by towing the models d_1 and d_2 simultaneously from the ends of a bar supported at an intermediate point. The point of support is then adjusted until the bar remains at right angles to the direction of motion. In this condition the resistances are evidently in the inverse ratio of the lever-arms, and hence a measurement of the latter will give the value of n as desired. In this way it becomes possible to determine the value of W_2 above, and hence of the total resistance $S_2 + W_2$.

As an extension of the above method of treatment, let us assume the law of comparison to cover the total resistance, instead of the residual only. Let the experiment be made as before and the value of n similarly determined. Then denoting total resistances by R_1, R_2, r_1, r_2, we have

$$r_2 = nr_1;$$

$$R_2 = r_2\frac{D_2}{d_2} = nr_1\frac{D_2}{d_2};$$

$$r_1 = R_1\frac{d_1}{D_1}.$$

Hence $$R_2 = nR_1 \frac{d_1}{D_1} \frac{D_2}{d_2} = nR_1 \left(\frac{V_2}{V_1}\right)^6,$$

or $$\frac{R_2}{R_1} = n\left(\frac{V_2}{V_1}\right)^6;$$

and therefore $$\frac{H_2}{H_1} = \text{ratio of powers} = n\left(\frac{V_2}{V_1}\right)^7.$$

This method being wholly comparative, no dynamometric apparatus is necessary, and the experimental determinations are comparatively simple. Since, however, the results are comparative and not absolute, its field of usefulness will be much restricted in comparison with the usual method as described in § 26.

68. The Various Sections of a Displacement Speed Power Surface.

Let us consider a general series of ships, all of the same type of form. The law of extended comparison furnishes the connecting law between all the ships of such a series, no matter what the displacement or speed. It is therefore evident that any speed-power curve for a given displacement may be considered as the speed-power curve for the whole series by means of an appropriate modification of horizontal and vertical scales. Suppose, for example, that we have a curve for displacement D_1 plotted with scales as follows: Abscissa, 1 linear unit $= p$ knots. Ordinate, 1 linear unit $=$ q I.H.P. Then the same curve will also represent the speed-power relation for D_2 with the following scales:

$$\text{Abscissa, 1 linear unit} = p\left(\frac{D_2}{D_1}\right)^{\frac{1}{6}} \text{knots;}$$

$$\text{Ordinate, 1 linear unit} = q\left(\frac{D_2}{D_1}\right)^{\frac{7}{6}} \text{I.H.P.}$$

With such scale modifications, therefore, one curve would represent the whole series of values of power for the various values of displacement and speed.

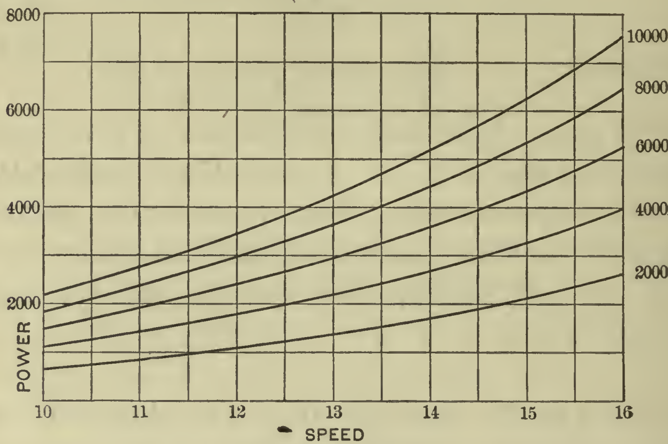

FIG. 95.—Relation between Speed and Power. Displacement constant for each curve, as shown on the right.

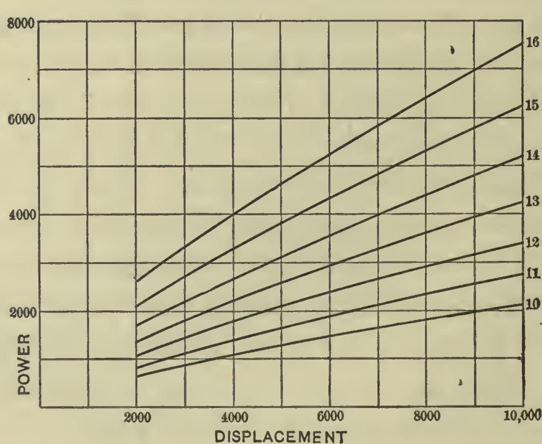

FIG. 96.—Relation between Displacement and Power. Speed constant for each curve, as shown on the right.

With a uniform scale such a series of values would require for their complete representation a surface of which the ordinate is the value of H, located by abscissæ corresponding to the given values of D and u. The various sections of this surface by planes parallel to the axes of H and u would represent each a speed-power curve for a given value of D.

The general values may therefore be represented by a single curve with varying scales, or with uniform scales and varying curves.

The surface above referred to may also be cut by planes parallel to the axes of H and D or D and u, thus cutting out

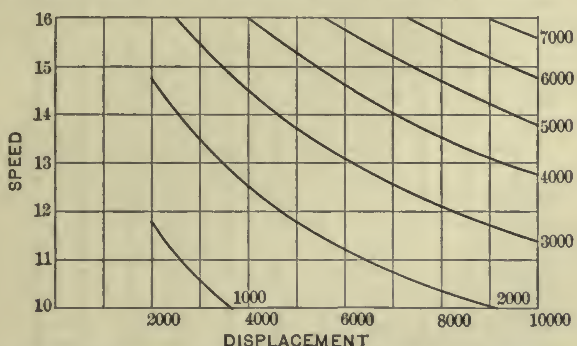

FIG. 97.—Relation between Displacement and Speed. Power constant for each curve, as shown on the right.

curves, one set showing the relation between H and D for a fixed value of u, and the other the relation between D and u for a fixed value of H. The various curves of section are shown in Figs. 95, 96, and 97, a study of the general characteristics of which may be recommended.

69. Application of the Law of Comparison to show the General Relation between Size and Carrying Capacity for a Given Speed.

In Fig. 98, let AB denote the curve of E.H.P. for a given ship of 5000 tons displacement at speeds from 10 to 20 knots. Now suppose this model taken as the type for a series of vessels of displacement from 5000 to 20 000 tons. Then by the appropriate transformation the E.H.P. for each of these vessels may be determined from the given curve, as explained in § 68. Again, assuming a constant propulsive coefficient of

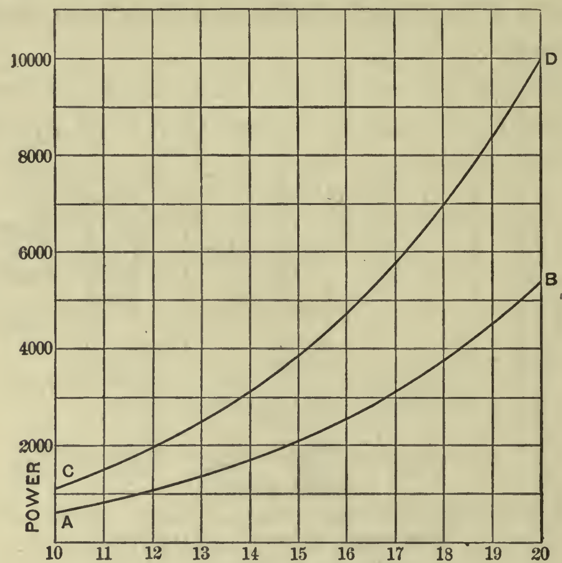

FIG. 98.

say .54, the values of the I.H.P. may be similarly obtained from a curve *CD*, the ordinates of which are equal to those of *AB* ÷ .54. If a constant propulsive coefficient could be assumed for the 5000-type ship, *CD* would be its curve of I.H.P. Actually, however, with the same machinery at varying speeds, the propulsive coefficient will vary so that *CD* would not in such case correspond to an experimental curve derived from any one ship, and we may more properly consider it simply as a curve proportional to E.H.P. Let us now find the I.H.P. for our series of similar ships, all at 20 knots speed. The speed at which the 5000-ton ship will correspond is

$$u = 20\left(\frac{5000}{D}\right)^{\frac{1}{6}}.$$

Taking the ordinate of *CD* at this speed, let it be denoted by *y*. Then the I.H.P. for the similar ship at 20 knots will be

$$H = y\left(\frac{D}{5000}\right)^{\frac{7}{6}}.$$

In this way we derive *AB*, Fig. 99, which shows the I.H.P. at this speed for the supposed series of ships. The

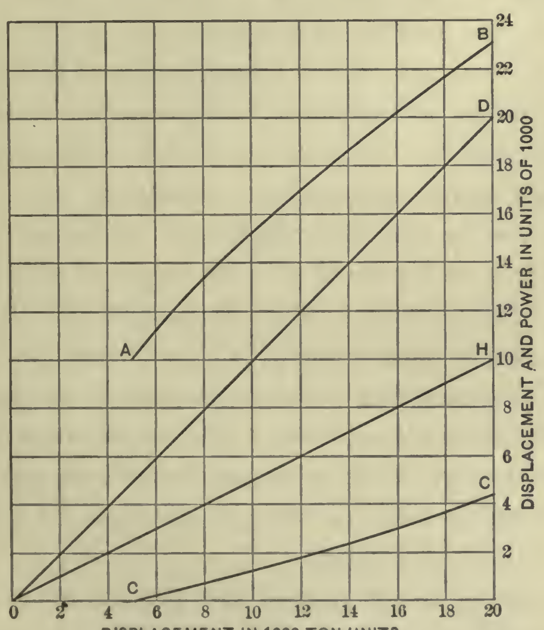

FIG. 99.

form of the curve shows plainly that *H* increases at a much slower rate than *D*; in fact in this particular case the rate is nearly as the square root of the displacement. Next suppose the weight of the hull and fittings to be .50 of the total displacement for the entire series. This ratio will vary considerably according to the type of structure, and would not, moreover, remain the same in ships of so great difference in size. In any case, however, the variation will not be sufficient to affect the general nature of the relations which we wish to establish, and some such assumption is necessary in order to introduce the necessary simplification into the relations involved. The ordinates to the line OD, Fig. 99, will then equal the displacement, and those between OH and OD the weight of the hull, etc. Taking the total weight of machinery and boilers as one-ninth ton per I.H.P. or 9 I.H.P. per ton, and coal at 1.8 pounds per I.H.P. per hour for all purposes, and assuming enough coal for a seven-days' run and neglecting the variation in D due to the consumption of coal and stores, or, otherwise, considering that the mean displacement for the trip corresponds to the values of the diagram, it is readily found that the weight of machinery and coal will be represented by the ordinate between OH and CC. The remainder lying between CC and the axis of X is evidently cargo-carrying or earning capacity. It appears that in the case chosen the 5000-ton ship could barely make the trip without cargo, while as D increases the earning capacity is seen to rapidly increase, rising to 3000 tons for $D = 16\,000$ and to 4200 for $D = 20\,000$.

Again, if we take it roughly that the operating expenses will vary with the power, the time being the same, the diagram shows how rapidly, relative to expenses, the earning capacity increases as D is increased. Thus for $D = 9000$ the expenses are represented by 14 400 and the earning capacity by about 1000 tons, while by an increase of D to 16 000 or by 77 per cent the expenses are increased by about 45 per cent and the earning capacity by 200 per cent.

Again, in Fig. 100, let us note the variation of carrying capacity with speed, the length of voyage being constant.

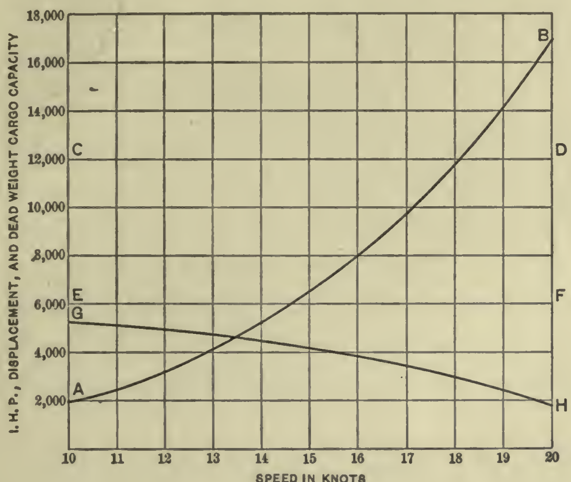

FIG. 100.

We will take for illustration the 12 000-ton ship. AB is the curve of I.H.P. The constant ordinate between CD and EF denotes the constant weight of hull and fittings. The ordinates between EF and GH denote the weight of machinery and coal, the latter decreasing with the decrease in power, but increasing with the increase of time. The ordinates between CD and GH give, then, the total weights on the same suppositions as before. The remaining ordinates between GH and X show the variation of carrying capacity with speed, and how dearly the latter is bought at the expense of the former.

The particular values for these various relationships in any given case will depend on many considerations here omitted. The general nature of the results, however, is sufficiently indicated by the diagrams, which are chiefly intended as suggestive applications of the law of comparison to the study of special problems of this character.

CHAPTER VI.

TRIAL TRIPS.

70. Introductory.

The general purpose of a trial trip is to determine the speed and power which may be maintained continuously for a certain distance or time. In addition to this fundamental purpose it is always desirable to observe such data as bear in any way on the general problem of resistance and propulsion, while in particular cases various characteristics may be observed relating to different points on which information is desired.

For the direct determination of speed it is sufficient to obtain simultaneous observations of distance and time. We shall not here consider in detail the measurement of power, assuming it to be determined by indicators in the usual manner.

The details of the various measurements necessary for the determination of speed may vary somewhat with the length of the course. We may have (1) a long course, as for example from 20 to 100 miles or more, over which but one run, or at most but one run in each direction, is to be made; or (2) a short course, as of one or two miles, usually the former, over which as many runs may be made as desired.

We may also distinguish between the purpose of the trials as follows: (*a*) the determination of the speed or power, or both, under a single set of conditions, such as full power or half-power for example; or (*b*) the determination of the speed and power under a series of varying conditions so that the continuous relation between speed and power for widely varying values of either may be determined.

We may next briefly mention the chief points relating to the ship which may influence speed, and the conditions to be fulfilled for maximum speed.

(*a*) *Displacement.*—This should be as light as possible, though this consideration is not independent of (*b*) and (*c*).

(*b*) *Propeller.*—The surfaces should be smooth, blades thin, edges sharp, and it must be well immersed.

(*c*) *Trim.*—The resistance will not presumably vary sensibly for slight changes of trim, but it may result that a very considerable change, such for example as might be necessary to immerse a propeller when the displacement is very light, would materially increase the resistance for the given displacement. In such case there is also the added loss due to the obliquity of the thrust.

(*d*) *Bottom.*—This should be smooth and freshly painted.

(*e*) *Wind and Sea.*—The wind should be light, and preferably off the beam or a little ahead, in order to give the benefit of increased natural draft for the boilers. The sea should, of course, be smooth.

For the attainment of the fundamental and secondary purposes of trial trips the following data are requisite or desirable:

Power.
Distance.
Time.
Revolutions.

All conditions affecting the relation of the ship to resistance and propulsion, such as—

Mean draft.

Trim at rest.

Condition of bottom if known.

Wave profile alongside of vessel.

Change of trim when under way.

Depth of water.

State and direction of wind and waves.

In addition to these, which are observed at the time of the trial, the characteristics of the ship and of the propeller are supposed to be known.

71. Detailed Consideration of the Observations to be Made.

Distance.—Omitting special reference to *power*, we first note that the distance with which we are concerned is that which the propeller has driven the ship through the water, and not the distance over the ground or between fixed points on shore. The influence of tides and currents must, therefore, be eliminated before the distance actually traversed through the water can be known.

For the marking of distance itself we have two general methods—buoys or ships at anchor, and range-marks. The former are more suitable for a long course, where the change in location caused by swinging to the tide will be of no relative importance. For a short course such errors would be inadmissible, and the limits of the course must be marked with all possible accuracy. In such case range-marks on shore are made use of. Thus in Fig. 101 *AB* and *CD* are two pairs of poles or range-marks determining the two parallel lines *AB* and *CD*. The course is then some line *EF* at right angles to these ranges, and at such a distance from the shore that the depth of water shall be sufficient to avoid sensible retardation (§ 19). In addition to the ranges, two requisites are therefore necessary in order to definitely fix the course—a direction and a location. The direction is usually known by compass course, and it is also desirable to mark it by ranges and buoys. The latter means is also used to indicate its approxi-

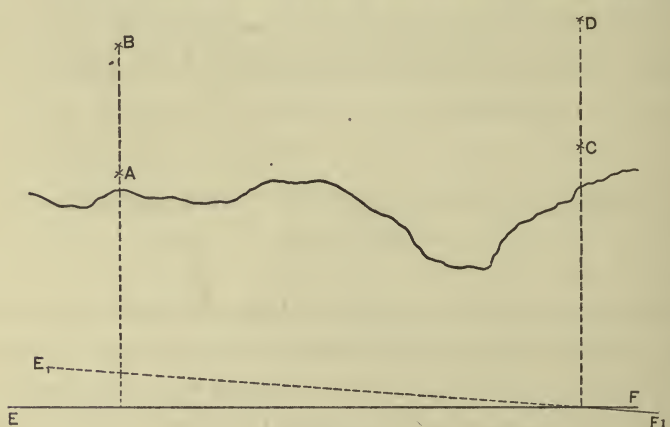

FIG. 101.

mate location with reference to the proper depth of water. At each end of the course there should be plenty of room for making turns and for gaining headway before entering the run proper. The considerations developed in § 77 show that the length of this preliminary run should be preferably not less than about one mile. This seems to be in agreement with experience, as Mr. Archibald Denny suggests * that the ship should have a straight run of at least one-half mile, and better still one mile, before entering the course proper.

* Transactions International Congress of Marine Engineers and Naval Architects in Chicago, 1893. Paper XXXI.

The necessity of a course at right angles to AB and CD rather than on some oblique line as E_1F_1 is evident without special discussion.

Tidal Observations.—As will be shown later, the tidal influence for a short-course trial may be approximately eliminated by appropriate treatment of the other data observed. With a long-course trial in one direction only such elimination is not possible, and with a single run in each direction the data cannot be satisfactorily used without special treatment for the elimination of tidal influence. It is therefore generally desirable to have made special tidal observations. For a long course, boats or ships may be anchored at intervals of five or ten miles along the course, and should make observations continually as often as once in fifteen minutes throughout the trial. Such observations should preferably be made at about the half mean draft of the vessel undergoing trial. The observation may give simply the component of the tide in the direction of the course, or perhaps preferably the whole velocity and its direction. Such observations are usually made with a so-called patent or taffrail log. This consists essentially of a small screw propeller, which in the case of vessels in motion is towed astern, the revolutions being communicated through the towing cord or by other means, to a counter on the ship. In the case of a vessel at anchor near the line of the course, such a log is buoyed a short distance from the ship at the appropriate depth, and observations made as usual. If the component of the tide in the direction of the course only is desired, the log must be maintained in this direction. If the entire velocity and the direction are both to be observed, the log must have a vane attached and must be free to turn to the tide. As shown by Froude's experiments, such instruments, if made with all the care appertaining to the production of a piece of truly scientific apparatus, are quite reliable. As usually furnished, however, they are liable to a correction, and should be rated in water known to be still at a speed near that at which they are used. A slight error in the log is, however, of less importance in tidal observations than when such instruments are used to determine directly the velocity of the ship itself. In addition to the tidal observations, the time of passage of the ship at each station is also noted.

We may then assume that we have thus at our disposal a mass of data giving at from four to eight points along the course a series of tidal observations at intervals of about fifteen minutes for the whole trial.

For a short course, if tidal observations are made at all, the number of locations will naturally be much less, according to circumstances and the degree of accuracy with which the measure of this disturbance is desired.

Time.—For the measure of time careful observation with accurate watches is sufficient for long-course trials. For short-course trials the probable error of reading with the usual form of seconds-hand watch may become sensible. For satisfactory accuracy stop-watches may be used, or preferably, for scientific purposes, some form of chronograph by which an electric signal is recorded the instant contact is made by the pressure of a key.

Revolutions.—For the counting of revolutions on a long-course trial, the ordinary engine-counter is quite sufficient. For a short-course trial the same means may be used, though, as with time, more satisfactory accuracy is attained by the use of some chronographic arrangement. In Weaver's speed and revolution counter a paper tape is fed at a uniform speed under a series of electrically controlled pens. One of these under control of a clock makes a mark every second. Another is in circuit with a contact maker, and is used to note the instant of entering and leaving the course. Other pens are electrically connected with each main shaft and thus register every revolution. Such an instrument is, of course, only used in short-course trials.

Special Conditions.—Most of these are self-explanatory.

The wave profile may be determined by measuring down from the rail by batten. The use of photography from a neighboring vessel may also be suggested. The change of trim from the condition at rest and when under way may be determined by the use of a pendulum adjusted to swing in a longitudinal plane.

72. Elimination of Tidal Influence.

We first suppose the observations made on a short-course trial. These may be considered as furnishing, for any given run, the average tidal velocity, and the correction consists simply in subtracting or adding, as the case may require.

With the long course, where several hours may be required to traverse its length, the assumption of a sensibly uniform tide is not admissible, and the more detailed observations already referred to become necessary.

An approximate method of applying the correction is as follows:

Let T_1, T_2, T_3, etc., denote the lengths in minutes of the successive time intervals between passing the posts of observation. Let v_1, v_2, v_3, etc., be the tidal velocities per minute along the course at each successive station at the instant of passage of the ship, + being considered as denoting a velocity in the same direction as the ship. Then $\frac{1}{2}(v_1 + v_2)$, $\frac{1}{2}(v_2 + v_3)$, $\frac{1}{2}(v_3 + v_4)$, etc., are taken as the mean tidal velocities for the successive intervals. The corresponding tidal influences will be:

For the first, $T_1\left(\frac{v_1 + v_2}{2}\right) = \frac{T_1}{2}(v_1 + v_2)$;

" " second, $\frac{T_2}{2}(v_2 + v_3)$;

" " third, $\frac{T_3}{2}(v_3 + v_4)$;

etc. etc.

The entire tidal correction will be then simply the algebraic sum of these partial corrections.

As a method somewhat more accurate in principle, the following may be suggested:

The tidal characteristics, as is well known, are expressible in terms of circular functions of time and distance from a given origin, and the tidal velocity at the ship at any instant must necessarily vary approximately as a sinusoidal function of space and time. In Fig. 102 let the ordinates at O, P,

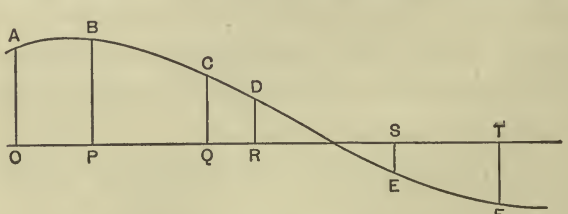

FIG. 102.

Q, etc., denote the tidal velocities at the successive points of observation laid off on a time abscissa. Then OP, PQ, etc., are the successive time intervals between the points of observation. Now without attempting any refined analysis of the tidal components, we may quite closely obtain a continuous value of the tidal velocity at the ship at each successive instant by passing through the points A, B, C, etc., a smooth curve, keeping in mind the characteristics of a sinusoidal form. If now the area of this curve be found by a planimeter or by approximate integration, the result will give the total distance or set due to the tide, + or — according to the circumstances of the case. The total correction found in either of these ways is then applied to the total length of the course, thus giving the corrected length, from which in conjunction with the time the speed is directly found.

We will next consider the elimination of tidal influence for short-course trials without special observations.

For a measured mile the time required to make a double run with and against the tide will be so small that without large error the tidal velocity may be considered constant throughout. In such case the simple mean of the two runs will evidently give the true mean speed; thus:

$$\frac{(u_1 + v) + (u_2 - v)}{2} = \frac{u_1 + u_2}{2}.$$

An extension of this, known as the method of "continued averages," has also been much used. This is as follows: Suppose four runs made, two in each direction, giving apparent speeds u_1, u_2, u_3, u_4. These are treated as shown by the form on the next page

The third average is then taken as the correct speed, thus deduced from the speeds observed. This is equivalent to giving the second and third runs three times the weight of the first and fourth, and the final result may be more readily found by this formula than by the actual formation of the successive columns of averages.

Speeds.	1st Averages.	2d Averages.	3d and Final Average.
u_1			
	$\frac{u_1 + u_2}{2}$		
u_2		$\frac{u_1 + 2u_2 + u_3}{4}$	
	$\frac{u_2 + u_3}{2}$		$\frac{u_1 + 3u_2 + 3u_3 + u_4}{8}$
u_3		$\frac{u_2 + 2u_3 + u_4}{4}$	
	$\frac{u_3 + u_4}{2}$		
u_4			

The correctness of this method depends on certain assumptions which we may briefly note.

Let the true speed remain constant at u. Let the tidal velocity be expressed as a function of the time as follows:

$$v = a + bt + ct^2.$$

Let the runs be made at equal time intervals which may be denoted by t. Then for the apparent speeds we shall have

$$u_1 = u + a;$$
$$u_2 = u - a - b - c;$$
$$u_3 = u + a + 2b + 4c;$$
$$u_4 = u - a - 3b - 9c.$$

By substituting in the formula for continued averages above, we readily find

$$\frac{u_1 + 3u_2 + 3u_3 + u_4}{8} = u.$$

It thus appears that if the true speed remains constant, and if the tidal influence varies as a quadratic function of the time, and if the time intervals are equal, this method will give the true speed. In the actual case, however, not one of these conditions will in general be fulfilled. It is therefore doubtful if the result of four runs thus reduced is any more accurate than the simple average, or than the average of two runs made with as small a time interval as possible.

It may also be noted that when the tide is just on the turn either way the water is frequently affected by eddies and counter-currents, which though small in velocity are so confused in distribution as to defy any attempt at elimination. When, however, the tide is at about half-ebb or half-flow, the velocity is greater but quite regular. This results naturally from the sinusoidal character of tidal motion, as a result of which the change of velocity is least when the velocity is greatest at about half tide, and the change is greatest when the velocity is 0 at full tide or slack water. It follows that half tide with a clearly defined regular tidal velocity uniformly distributed over the course is in general preferable to slack or high water with the accompanying eddies and irregularities.

Further methods of dealing with tidal influences will be found in the next section.

73. Speed Trials for the Purpose of obtaining a Continuous Relation between Speed, Revolutions, and Power.

We have thus far been concerned with the actual true speed attained, without reference to the corresponding number of revolutions or the I.H.P. necessary. For the investigation of the propulsive performance, however, it is quite desirable to obtain sufficient data to furnish a continuous relation between these three quantities. For this purpose the short-course trial alone is suitable. The actual conduct of the trial is in no wise different from that already described, but it is continued through a decreasing series of speeds with their appropriate revolutions and powers, to a number of revolutions as low as can be maintained uniformly by the engines. The problem of the proper disposition of the data thus found is more complicated than when speed alone is desired. We now desire to determine not only the true speed, but also the revolutions and I.H.P. which *correspond* to it.

Now in practice it is found difficult, even for any one run, to maintain the revolutions and power constant. With care the variation may be slight, but at the best the question may always arise as to the proper interpretation of the data observed.

Let H_1, N_1, u_1 be the power, revolutions, and true speed for the first run, and H_2, N_2, u_2 those for the second, v being the tidal influence taken as constant for the two runs. Then the actual observations furnish us with

$$H_1, \quad N_1, \quad (u_1 + v);$$
$$H_2, \quad N_2, \quad (u_2 - v).$$

For two runs intended to be at the same speed, the variation will be so slight that we may properly assume the slip of the propeller to be the same for both. As a result the true speed will vary directly with the revolutions, and the true mean speed will correspond to the mean number of revolutions. That is,

$$\frac{u_1 + u_2}{2} \text{ corresponds to } \frac{N_1 + N_2}{2}.$$

Now with power we have in general

$$H = Bu^n. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where B and n depend alike on the ship and on the speed. Where the variation in speed is not large, we may safely take the variation in B and n as negligible. In the present case, therefore, we should have

$$H_1 = Bu_1^n;$$

$$H_2 = Bu_2^n.$$

Whence $$u_1 = \left(\frac{H_1}{B}\right)^{\frac{1}{n}};$$

$$u_2 = \left(\frac{H_2}{B}\right)^{\frac{1}{n}};$$

and $$\frac{u_1 + u_2}{2} = \frac{H_1^{\frac{1}{n}} + H_2^{\frac{1}{n}}}{2B^{\frac{1}{n}}}.$$

and $$B\left(\frac{u_1 + u_2}{2}\right)^n = \left(\frac{H_1^{\frac{1}{n}} + H_2^{\frac{1}{n}}}{2}\right)^n. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The left-hand member of this equation is in the same general form as (1) above, and it must therefore give the value of the power which corresponds to the true mean speed. The relation of this to the observed powers H_1 and H_2 is then shown by the right-hand side of (2) above. The exponent n will usually be found between 3 and 4. As an illustration let

$$n = 3;$$
$$H_1 = 3200;$$
$$H_2 = 3600.$$

Then by substitution we find

$$\left(\frac{H_1^{\frac{1}{n}} + H_2^{\frac{1}{n}}}{2}\right)^n = 3396.$$

The value by simple average would be 3400, and as the uncertainties in the measurement of power are relatively far greater than the difference thus resulting, and as these values of H_1 and H_2 represent a variation greater than would be likely to occur under the conditions assumed, it seems fair to conclude that in such case we may properly take the mean

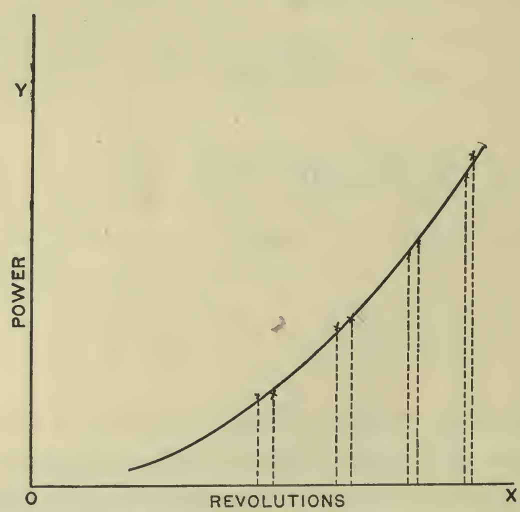

FIG. 103.

power as corresponding to the mean speed. For two runs thus made we may take, therefore, the mean revolutions, mean speed, and mean power, as all corresponding.

The next pair of runs being made at a lower power, another set of means is found, and so for the entire series.

Still otherwise, we may effect the general averaging by graphical process as follows:

Let power be plotted on revolutions as in Fig. 103, each pair of spots corresponding to a double run intended to be made at the same power. Then a fair curve passing through and between the spots may be taken as the best approximation to the continuous relation between revolutions and power. We may then plot the mean of the speeds as an ordinate, on the mean of the revolutions as an abscissa, as in *OB*, Fig. 104, thus giving a continuous relation between speed and revolutions. A straight line *OA* shows similarly

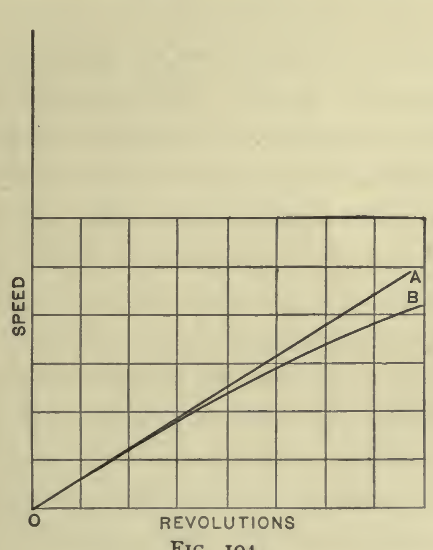

FIG. 104.

the speed if the apparent slip were 0. Hence the vertical intercept between *OA* and *OB* shows the corresponding loss in speed at each point. From these diagrams we may then obtain and plot if desired a curve showing the continuous relation between power and speed.

We may also derive the relation between revolutions and speed graphically by plotting each separate speed on revolutions as an abscissa, thus obtaining two curves, one for speeds with and the other for speeds against the tide, with pairs of points at nearly the same revolutions on each. A mean curve will then give the desired relation between revolutions and speed.

A slightly different mode of graphically determining the relation between revolutions and true speed is that due to Mr. D. W. Taylor,* which is briefly as follows:

The course is steamed over back and forth, going over each time the same route off as well as on the course, so that the time intervals between the middle points of the runs may vary inversely as the speeds. For each successive run the revolutions are decreased by as nearly as possible an equal decrement. This gives a series of runs at certain revolutions with the tide, and another series with other revolutions against the tide. Each of these is then plotted, giving curves of speeds plotted on revolutions, with and against the tide. A mean curve is then taken as the curve of true speed. The author then shows by means of an illustrative example that with a fulfilment of the above conditions well within practicable limits a variable tidal influence will be eliminated with quite sufficient accuracy for all practical purposes.

The advantage claimed for this method is that a given range of speed can be satisfactorily covered with a smaller number of runs than when they are made in pairs as previously described.

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* Journal Am. Soc. of Naval Engineers, vol. IV. p. 587.

74. Relation between Speed Power and Revolutions for a Long-distance Trial.

While the intention may be to carry on such a trial at uniform power, in any actual case there will naturally arise considerable variation in the various quantities involved. The mean speed and the mean revolutions are readily found, and with any probable amount of variation we may presumably consider them as corresponding the one to the other. With regard to the power which corresponds to such mean speed or revolutions, however, there is more room for uncertainty.

The data available will be a series of indicator-cards taken at intervals throughout the trial, and giving presumably the mean effective pressures in the cylinders at the instant in question. The revolutions are also taken at the same time, and thus each observation gives the data for determining a value of the power at that instant. The mean of these revolutions, however, will not be the true mean, due to individual errors of observation, and to the fact that they are merely a series of detached values of N, while the true mean is obtained from the continuous counter giving the total number for the whole run. On this account, therefore, and for finding the mean power, it seems preferable to use the mean effective pressures from the cards, and the true mean revolutions from the total counter. The method for doing this we may develop as follows: In any given propeller it is readily seen that the relation between the mean effective pressure in the cylinders, or more definitely between the mean effective pressure reduced to the l. p. cylinder (§ 47), and the thrust of the propeller or resistance of the ship, must be very nearly constant except for very widely varying conditions. We may therefore properly assume that the reduced mean effective pressure will vary very nearly as the resistance. Denoting these pressures by p_1, p_2, etc., we may, with all significant accuracy, assume them expressible in the form $p_1 = bN_1^2$, etc. Hence, extending the method outlined in § 73 for mean power, we should here use for the final mean pressure $\bar{p}$ the square of the mean square roots of the various values p_1, p_2, etc. This pressure used with the true mean revolutions would then give an approximation to the corresponding power having all significant accuracy, at least so far as obtainable from the data at hand.

A still greater refinement, but usually without practical value, would be the substitution for 2 of an index to be derived from a special examination of the relation between speed or revolutions and power.

If the variation in the power is not unusually large, the square of the mean square root will not sensibly differ from the simple arithmetical mean, and in such case the latter may properly be used.

As an illustration of the amount of the variation, let three values of p be denoted by 64, 81, and 100. Then the square of the mean square root is 81, and the arithmetical mean is 81.66, or somewhat less than one per cent in excess.

We therefore conclude that in all usual cases we may, for the power corresponding to the mean revolutions or speed, use the simple arithmetical mean of the pressures, as above, with the mean revolutions. At the same time the substitution of the square of the mean square root for the arithmetical mean will undoubtedly give a more correct result, and if the difference is significant, the latter method should be used.

75. Long-course Trial with Standardized Screw.

We have thus far in all cases assumed the measurement of distance to be effected by means independent of the ship's propeller. A method of long-course trial has been used to some extent, however, by means of which the propeller itself, having been standardized, is made the instrument for the measurement of the distance. This is known as the standardized screw method, and has been used in certain government trials at the suggestion of the Bureau of Steam Engineering of the Navy Department.

The ship is first taken on the measured mile and the continuous relation between speed and revolutions as discussed in § 73 is carefully determined. The ship being in the same condition as regards draft, trim, condition of bottom, etc., then puts to sea in smooth water and calm weather, and runs such a distance or time as may be desired. For the best accuracy readings of the counter should be taken every ten or fifteen minutes. In any event the total time and total revolutions are known. Indicator-cards are, of course, taken at appropriate intervals throughout the run.

Now if the revolutions have been uniform, or if they have varied somewhat, but not out of a range within which the apparent slip may be taken as sensibly constant, then, as in § 73, the true mean speed and the mean revolutions will correspond, and the former may be determined directly from the latter by the aid of the curve of standardization. If in an extreme case the revolutions are widely variable and the slip is widely variable with revolutions, this correspondence will not be accurate. The cause of the error and a method for its elimination may be seen as follows:

Let *OX*, Fig. 105, be a time abscissa on which are laid off as ordinates the successive numbers of revolutions per minute at the instants at which they are taken. A smooth curve *AB* drawn through the points thus found gives a continuous relation between time and revolutions. Then from the curve of standardization the speed corresponding to each number of revolutions may be found. These being plotted at the same points will give a curve *CD*, showing the continuous relation between time and speed. The area *OCDX* being found by planimeter or by approximate integration, the result will be

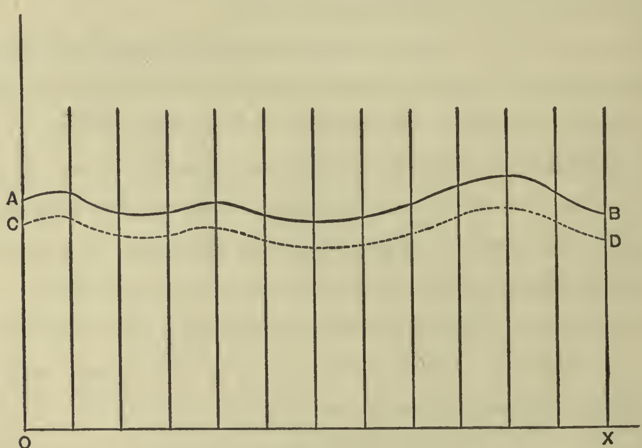


FIG. 105.

proportional to the entire distance, and the time being known, the true mean speed is thus determined. If now the apparent slip were constant, or *OB*, Fig. 104, a straight line, the curves *AB* and *CD* would be similar and the integration of *AB* would serve the same purpose as that of *CD*. But the integration of *AB* is furnished by the revolution counter, and hence the mean revolutions thus determined would with constant slip give the true distance or true speed. Again, if the revolutions are constant, it is evident that both AB and CD become straight lines parallel to OX, and their constant ratio would be that between speed and revolutions given by Fig. 104, for the particular value of the latter.

If these conditions are not fulfilled, that is, if revolutions vary with time, and slip with revolutions, then the curves AB and CD would no longer be similar, and the use of the former, or, in other words, of the mean revolutions, would no longer be theoretically exact.

With the usual amount of variation in these quantities, however, the error will be negligible and the mean revolutions may be used.

The treatment of the power has already been considered in § 73.

As some of the advantages of this method mention may be made of the following:

(*a*) There are no tidal corrections to consider. This arises from the fact that by means of the standardization the propeller is made the instrument for measuring distance.

(*b*) The ship under trial is independent of the presence of other vessels for course-markers and for tidal observations.

(*c*) The results are not invalidated by departure from a straight course. Any course desired may be followed so long as it is not a curve of small enough radius to give rise to a sensible retardation.

(*d*) The results attained are constantly known during the trial, so that the exact status of the ship relative to the expected performance is known from beginning to end.

(*e*) Any derangement to the machinery which might render a trial over a definite course not only inconclusive, but also devoid of any return of valuable data, will have no effect on the result, since the trial may be considered as ended at the expiration of any complete ten-minute interval, and whether the results are considered satisfactory or not, at least they are known, and valuable data are obtained.

On the other hand the limitation of this mode of trial so far as relates to similarity of conditions between the standardization trial and regular trial must be carefully noted. The apparent slip will vary with the condition of the bottom, and with the draft, trim, and condition of the wind and sea; and since calm weather and a smooth sea are the only conditions which can be definitely described, it is necessary that both trials should be made substantially under such conditions.

76. The Influence of Acceleration and Retardation on Trial-trip Data.

If the revolutions and power or mean effective pressure are subject to rapid fluctuations, they will no longer have the same relation to speed as for uniform conditions. If the pressure rapidly increases, a portion of the resultant increase of thrust and work done goes toward accelerating the motion of the ship, and the mean pressure, revolutions, and power will all be greater than for the existing speed under uniform conditions. Similarly with a rapidly decreasing pressure the mean pressure, revolutions, and power will be less than for the existing speed under uniform conditions.

The variation of the pressure is usually not sufficiently rapid to make this item of practical importance, though the possibility of its existence as a disturbing feature may be borne in mind.

It could only be eliminated by a knowledge of the varying acceleration, an experimental determination of which would be a matter of no small difficulty. A further problem involving somewhat similar considerations is that concerned with the time and distance necessary to effect a change of speed either of increase or decrease. This is of such general interest and importance in connection with the subject-matter of the present chapter that we may properly turn to its consideration at this point.

77. The Time and Distance required to Effect a Change of Speed.

Suppose the mean effective pressure in the cylinders to be dependent alone on the points of cut-off. With any fixed condition of these, therefore, and neglecting variations due to friction, the engine-turning moment will be constant, and hence its equivalent, the moment resisting the transverse motion of the propeller. Again, assuming that the distribution of pressure over the surface of the propeller is sensibly the same for constant turning moment no matter what the speed of the ship, it follows that the relation between the actual thrust and the mean effective pressure is geometrical, and hence if the former remains constant, so will the latter. While these suppositions may not be quite exact, they must in any case be very near the truth. Suppose, therefore, the ship being at rest, that steam of a given pressure is turned on the engine with the cut-offs in such position that their joint result would give a mean effective pressure sufficient to give a thrust which would maintain the ship at a speed of u_1 knots. In consequence the ship will gradually gather headway and after a time attain sensibly the speed u_1, during which a distance s will have been traversed. In thus considering thrust and resistance it is evident that actual thrust and actual or augmented and not true or towed resistance must be taken.

Let R_1 denote the resistance at speed u_1 and in general R that at speed u. We shall have then a constant thrust equal to R_1 acting against a variable hydraulic resistance R. The difference $R_1 - R$ will be a net force available for the acceleration of the ship. We must also remember that a certain amount of water is more or less closely associated with the motion of the ship, and partakes of its acceleration and retardation. We may therefore use μD instead of D. We have then

$$\frac{du}{dt} = \frac{g(R_1 - R)}{\mu D} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In the fundamental formulæ of dynamics R is measured in pounds force and D in pounds mass. As their ratio is unchanged by a change in the unit, they may both be measured in tons. We have therefore, as the units in this equation, feet, seconds, and tons. Now represent R in the general form

$$R = cD^{\frac{2}{3}}u^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

in which c will be later defined so that R and D may be in tons and u in feet per second or knots per hour. In this equation c is not necessarily a constant. This will provide therefore for any actual law of augmented resistance. We have then, from (1),

$$\frac{du}{dt} = \frac{gD^{\frac{2}{3}}c}{\mu D}(u_1^2 - u^2) = \frac{gc}{\mu D^{\frac{1}{3}}}(u_1^2 - u^2). \quad . \quad . \quad (3)$$

Dividing by $\frac{ds}{dt} = u$, we have

$$\frac{du}{ds} = \frac{gc(u_1^2 - u^2)}{\mu D^{\frac{1}{3}} u} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Hence from (3) and (4)

$$dt = \frac{\mu D^{\frac{1}{3}}}{gc} \frac{du}{u_1^2 - u^2} . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$ds = \frac{\mu D^{\frac{1}{3}}}{gc} \frac{udu}{u_1^2 - u^2} . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

If now to simplify integration we assume c a constant, we have, reckoning t and s from starting,

$$t = \frac{\mu D^{\frac{1}{3}}}{2gcu_1} \log_e \frac{u_1 + u}{u_1 - u} ; \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$s = \frac{\mu D^{\frac{1}{3}}}{2gc} \log_e \frac{u_1^2}{(u_1^2 - u^2)} . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

From (7) and (8) it appears that when $u = u_1$, both t and s become ∞. This is simply a result of the form of the equations, and indicates that under the conditions assumed, the velocity would go on indefinitely increasing, but would never actually attain the amount u_1. This is the natural and necessary result of the decreasing value of $\frac{du}{dt}$ as given in (3) when u approaches u_1, and while it may be a suprising result, it is in exact accord with the physical conditions assumed. In the actual case, the mean effective pressure is never exactly constant, and hence the corresponding velocity u_1 is constantly variable about a mean value. The important point practically is not how long or how far it will take to acquire the velocity u_1, but rather a velocity sensibly equal to it, as for example $.99u_1$, or $.999u_1$. These relations are illustrated

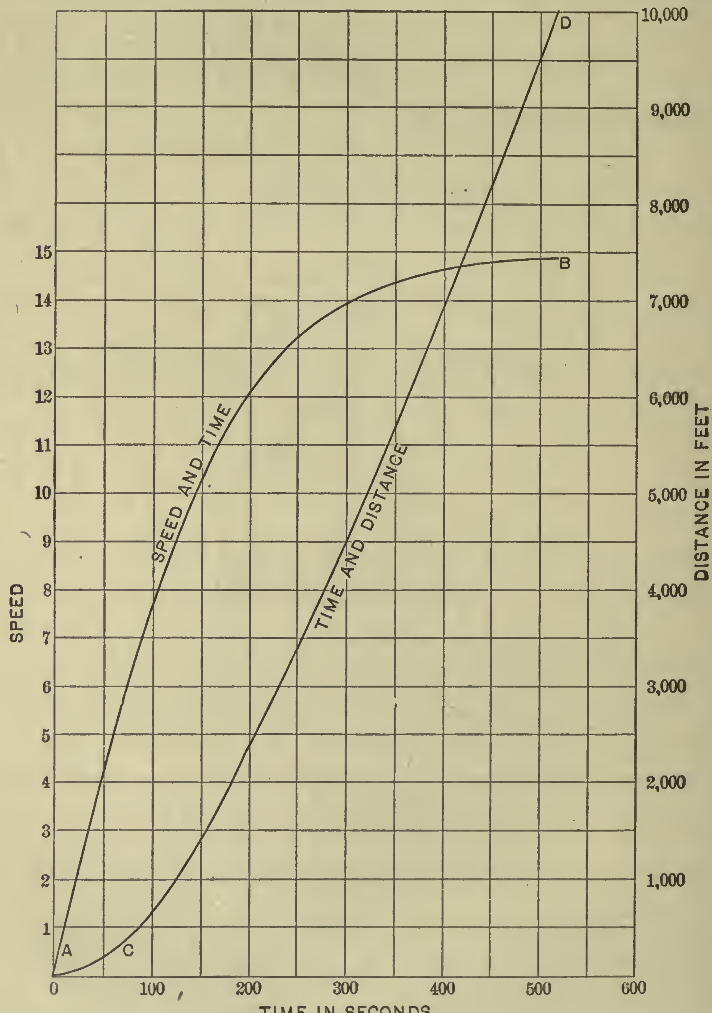


FIG. 106.—Time and Distance for gaining Speed.

in Fig. 106, where values of (7) and (8) are plotted for successive values of u. We will now explain the reduction of the equations to a form suitable for computation.

Let K be the Admiralty displacement coefficient as defined in § 62, and e the ratio between the thrust horse-power T.H.P. and the I.H.P. Then we have

$$\text{I.H.P.} = \frac{D^{\frac{2}{3}}u^3}{K} \quad \text{and} \quad \text{T.H.P.} = \frac{eD^{\frac{2}{3}}u^3}{K},$$

where the units are the *horse-power*, *ton*, and *knot*.

To reduce to the units, foot-ton per minute, ton, and foot per second, we have

$$\text{T.H.P.} = \frac{eD^{\frac{2}{3}}u^3 \times 33000}{K \times 2240 \times (1.689)^3},$$

and $$R = \frac{eD^{\frac{2}{3}}u^3 \times 33000}{K \times 2240 \times (1.689)^3 \times 60u} = \left(\frac{.051e}{K}\right) D^{\frac{2}{3}}u^2.$$

Comparing this with the general form for R in (2), it is evident that $c = .051e \div K$. This may be taken as the defining equation for c such that when substituted in (2) it will give R in tons when D is in tons and u in feet per second. Substituting this value of c in (7) and (8) we should then have the values of t in seconds and s in feet, expressed in terms of D in tons and u in feet per second. While retaining the second and foot as units for t and s, it will be more convenient to introduce on the right-hand side of the equations the necessary factors, so that common logarithms may be used instead of hyperbolic, and speed may be measured in knots instead of feet per second. Making these substitutions in (7) and (8), and putting $\mu = 1.1$, we find

$$t \text{ (sec.)} = .457\frac{D^{\frac{1}{3}}K}{u_1 e} \log \frac{u_1 + u}{u_1 - u}; \quad . \quad . \quad . \quad (9)$$

$$s \text{ (feet)} = .772\frac{D^{\frac{1}{3}}K}{e} \log \frac{u_1^2}{(u_1^2 - u^2)}. \quad . \quad . \quad . \quad (10)$$

For illustration take $K = 240$ and $K \div e = 400$, $D = 5000$, $u_1 = 15$. Then we have

$$t = 208.4 \log \frac{15 + u}{15 - u}, \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and

$$s = 5280 \log \frac{225}{225 - u^2}. \quad . \quad . \quad . \quad . \quad (12)$$

The numerical values of (11) and (12) are plotted in Fig. 106, as above referred to. The curve *AB* giving the time is asymptotic to the 15-knot line, thus showing geometrically the indefinite approach of the speed to the 15 knots as a limit. The curve *CD* shows likewise the relation between time and distance, approaching more and more nearly a straight line in form as the 15-knot speed is gradually approximated to.

We will now suppose the vessel going ahead under uniform conditions at speed u_1, and that the engine is suddenly reversed and driven with full mean effective pressure astern. In such case we shall have a different geometrical relation between the longitudinal force due to the screw and the mean effective pressure, than for motion ahead. In general, however, we shall have in such case

$$\frac{du}{dt} = -\frac{g(P + R)}{\mu D}, \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where $P =$ backward pull due to engine, and $R =$ resistance to velocity of ship forward.

We will now assume, similar to (1),

$$R = c_1 D^{\frac{2}{3}} u^2,$$

and

$$P = c_2 D^{\frac{2}{3}} u_1^2,$$

where for P, u_1 is the speed at which the given mean pressure would drive the ship ahead. We have then for (13)

$$\frac{du}{dt} = -\frac{g}{\mu D^{\frac{2}{3}}}(c_2u_1^2 + c_1u^2),$$

whence

$$dt = -\frac{\mu D^{\frac{2}{3}}}{g}\,\frac{du}{c_2u_1^2 + c_1u^2}. \quad . \quad . \quad . \quad . \quad (14)$$

$$ds = -\frac{\mu D^{\frac{2}{3}}}{g}\,\frac{udu}{c_2u_1^2 + c_1u^2}. \quad . \quad . \quad . \quad . \quad (15)$$

Integrating in the usual manner, we have

$$t = \frac{\mu D^{\frac{2}{3}}}{\sqrt{c_1c_2}gu_1}\left[\tan^{-1}\sqrt{\frac{c_1}{c_2}} - \tan^{-1}\left(\sqrt{\frac{c_1}{c_2}}\frac{u}{u_1}\right)\right]; \quad . \quad (16)$$

$$s = \frac{\mu D^{\frac{2}{3}}}{2gc_1}\log_e\left(\frac{1 + \frac{c_1}{c_2}}{1 + \frac{c_1u^2}{c_2u_1^2}}\right). \quad . \quad . \quad . \quad . \quad (17)$$

In selecting values of c_1 and c_2 we may bear in mind the following considerations: The coefficient c_2 is less than the c for the go-ahead condition due to the rounded backs of the blades, and the consequent decreased longitudinal component of the distributed surface pressures. For a given mean pressure, therefore, the pull would be less than the thrust. As to the ratio between the two, definite information is lacking; and in default of such definite data we will take the pull as .8 the thrust. Again, the value of c_1 corresponds to the actual resistance of the ship to motion ahead when the propeller is backing. In this condition the latter is sending a stream of water forward against the stern, thereby producing an excess of pressure at this point instead of a defect as when working ahead. The actual resistance to motion ahead is therefore decreased instead of increased, or the augmentation may be considered as negative. The ratio between c_1 and the c for motion ahead at the same speed would be equal therefore to that between the true resistance minus this decrement, and the true resistance plus the usual augmentation. Taking the augmentation and decrement about the same, this ratio would be not far from .8, and we will therefore take c_2 as equal to c_1. Comparing also with the value of c, we have

$$c_1 = c_2 = .8c = \frac{.041K}{e}.$$

Taking $\mu = 1.1$, substituting these various values, and reducing speed to knots and hyperbolic to common logarithms we have

$$t = .493\frac{D^{\frac{1}{3}}K}{u_1 e}\left(.785 - \tan^{-1}\frac{u}{u_1}\right); \quad . \quad . \quad . \quad (18)$$

$$s = .959\frac{D^{\frac{1}{3}}K}{e}\left[.30103 - \log\left(1 + \left(\frac{u}{u_1}\right)^2\right)\right]. \quad . \quad (19)$$

If $u = 0$, we have simply

$$t = .387\frac{D^{\frac{1}{3}}K}{u_1 e}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

$$s = .289\frac{D^{\frac{1}{3}}K}{e} = .747u_1t. \quad . \quad . \quad . \quad . \quad (21)$$

With the same numerical values as before, we have

$$t = 225\left(.785 - \tan^{-1}\frac{u}{15}\right); \quad . \quad . \quad . \quad . \quad . \quad (22)$$

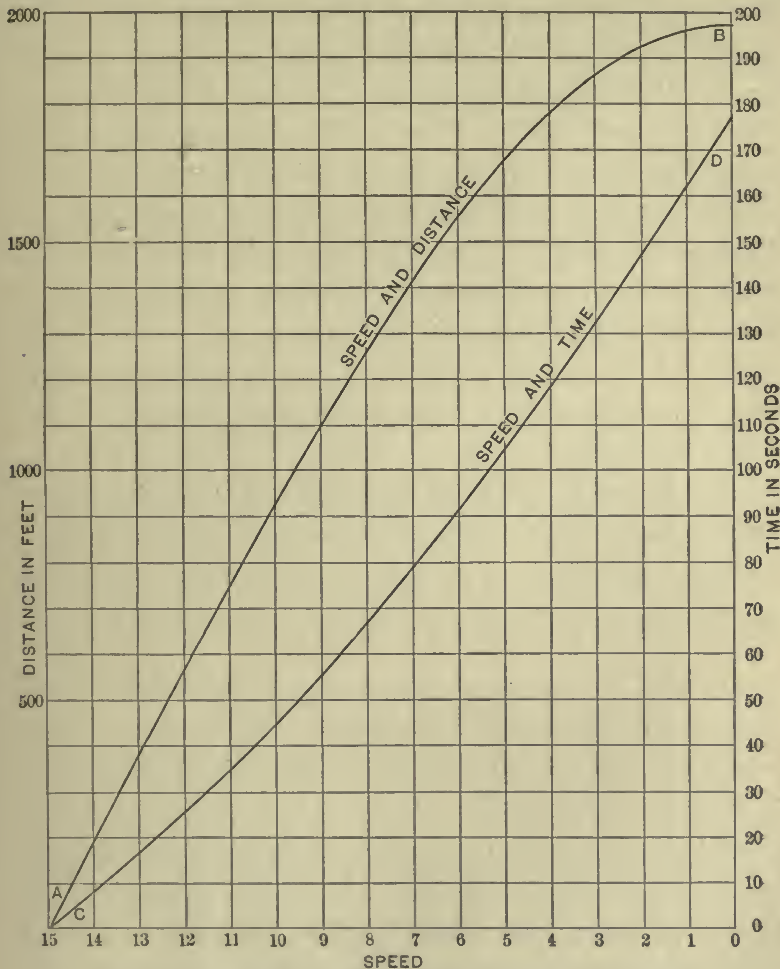
$$s = 6560\left[.30103 - \log\left(1 + \left(\frac{u}{15}\right)^2\right)\right]; \quad . \quad (23)$$

and if $u = 0$,

$$t = 177,$$

$$s = 1975.$$

The varying values of t and s for any speed u are given in Fig. 107, which is thus a diagram of the extinction of forward



TIME AND DISTANCE FOR STOPPING FROM FULL SPEED

FIG. 107.

velocity. AB gives the relation between speed and distance, and CD that between speed and time.

For simplicity of operation we have assumed that the augmented resistance varies throughout as the square of the speed. This, of course, is far from being actually the case. The ratio $K \div e$ may be expected to vary considerably in the case of a vessel gaining speed from rest or stopping from full speed. If the law of variation of $K \div e$ were known—that is, the law of the variation of the actual resistance with speed, both with engines turning ahead and backing—the fundamental equations could readily be solved by approximate or graphical integration. The difference in the result, however, would be slight, and would not change the general nature of the results obtained by the substitution of the constant value of $K \div e$. It may be remarked, however, that in the selection of a value of K a mean rather than maximum value should be taken.

An interesting conclusion from (21) is that the distance in which a vessel may be brought to rest from motion ahead is sensibly independent of the speed, and depends only on the size and the ratio $\frac{K}{e}$, or if e is taken as practically constant, on D and K. The reason why there must actually be a tendency toward some such uniformity in the distance run may readily be seen. In the example taken this distance would be from five to six lengths. In the case of very full or poorly formed vessels, or with foul bottom, we might have an average value of K much less than 240. As an illustration, let $K =$ 180 and $K \div e = 300$. Then the distance traversed in such case would be only three fourths of that under previous conditions, and we should have for the same size of ship

$$t = 133,$$

$$s = 1481.$$

These figures agree with the general results of the experiments reported to the British Association,* by which it was found that ships could usually be stopped in from 4 to 6 lengths, and that this distance seemed practically independent of the power, or in other words, of the speed at which the experiment was made.

We have assumed in all the foregoing that the engine is instantly reversed, and that the engine instantly gathers its motion either ahead or back, as the case may be. This is not actually possible, so that due allowance should be made in comparing experimental results with those given by the foregoing approximate theory.

In any case the general nature of these results is correct, and it is readily seen that they have an important bearing on the manœuvring of vessels and on the proper conditions for trial trips. In measured-mile trials especially the importance is clearly shown of a long start in order that the ship may attain sensibly her maximum speed before entering upon the course.

78. The Geometrical Analysis of Trial Data.

Considering the three items of data, power, speed, and revolutions, we may plot the former on either of the latter as abscissa giving curves as in Figs. 108, 109, or we may plot speed on revolutions giving a curve as in Fig. 110. The various uses of these curves need no especial explanation. In using the power-speed curve for purposes of comparison, § 61, however, it must be remembered that the given curve strictly applies only to the ship in the given condition of dis-

* Reports for 1878, p. 421.

placement, trim, condition of bottom, and of weather and water, and that it cannot be applied to the same ship for all

FIG. 108.

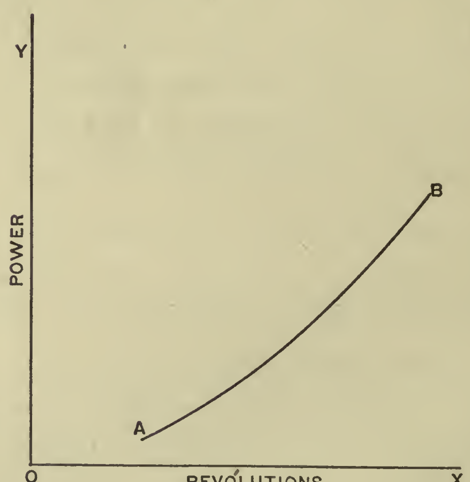


FIG. 109.

circumstances without due allowance having been made for such differences in condition as may exist.

It is frequently of interest to know the index of the speed with which the power varies at any given speed, and similarly for the variation of resistance with speed, or either with revolutions, or in fact of any quantity which may be expressed in the general form

$$y = ax^n. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For example, we may determine that at 10 knots the power varies as the cube of the speed and the resistance as

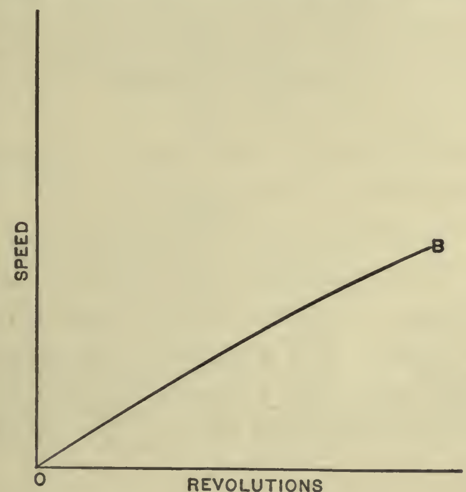


FIG. 110.

the square, while at 15 knots the power may vary as the index 3.7 and the resistance therefore as the index 2.7.

We will now develop methods of deriving the value of this index.

Let x_1 be the given value of the abscissa. Then the usual method has been to assume the index sensibly constant for a short range on either side of this value, say between the values x_0 and x_2, equidistant above and below x_1. Then from the general equation we have

$$y_0 = ax_0^n,$$

$$y_2 = ax_2^n,$$

whence $$\frac{y_2}{y_0} = \left(\frac{x_2}{x_0}\right)^n,$$

and $$n = \log \frac{y_2}{y_0} \div \log \frac{x_2}{x_0} = \frac{\log y_2 - \log y_0}{\log x_2 - \log x_0}. \quad . \quad . \quad . \quad (2)$$

When, however, the curve is at hand showing the relation between y and x as is usually the case, the following geometrical method will be found much more rapid and satisfactory, besides being exact in principle for a continually varying value of n.

We must first obtain a clear idea of the nature of the index with which we are here concerned.

Let it be a question of the variation of any function y with x. Let the value at x_0 be y_0. Then let x be increased by a small increment such that its new value divided by its old is $(1 + e)$, where e is a quantity very small compared to 1. This will result in a change in the value of y in a ratio which we may denote by $(1 + f)$. Now obviously we may put

$$(1 + e)^m = (1 + f),$$

or $$m = \frac{\log (1 + f)}{\log (1 + e)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This we take as defining the nature of the exponent m with which we are here concerned. It would therefore follow that if $m = 3$ and the speed were increased 1 per cent it would result in an increase of the power in the ratio $(1.01)^3$.

So long as the amount of increase is indefinite, the character of our exponent will lack mathematical distinctness, and we are therefore naturally led to define m as the ratio of log $(1 + f)$ to log $(1 + e)$ when e is indefinitely decreased.

Denoting the increments by dx and dy, we have then by definition

$$m = \frac{\log\left(1 + \frac{dy}{y}\right)}{\log\left(1 + \frac{dx}{x}\right)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

By Maclaurin's theorem of expansion this becomes

$$m = \frac{\frac{dy}{y}}{\frac{dx}{x}} = \frac{dy}{dx} \div \frac{y}{x}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

We have also

$$m = \frac{\frac{dy}{y}}{\frac{dx}{x}} = \frac{d(\log y)}{d(\log x)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

We must now show that the exponent m thus defined is not in general the same as the n of the general equation

$$y = ax^n.$$

We have $\log y = \log a + n \log x$.

Now remembering that in the cases with which we have to deal n is not a constant, we differentiate, considering both n and x as variables. We thus have

$$\frac{dy}{dx} = anx^{n-1} + ax^n \log x \frac{dn}{dx}.$$

Also $$\frac{y}{x} = ax^{n-1};$$

whence $$\frac{dy}{dx} \div \frac{y}{x} = n + x \log x \frac{dn}{dx},$$

or $$m = n + x \log x \frac{dn}{dx}. \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Hence it follows that unless n is constant, m and n will not be the same. It also follows that in the general case the exponent found by the expression in (2), being an approximation to the value of m, will not satisfy the fundamental equation (1) from which it is found, except for a particular value of a; and that a series of values thus found will not in general satisfy the fundamental equation (1) for any constant value of a. This still further shows the fact of the difference in character between m and n as here defined.

Taking then the value of m as thus defined, we proceed to show a method for its geometrical determination.

In Fig. 111 let OP be the graphical representation of the

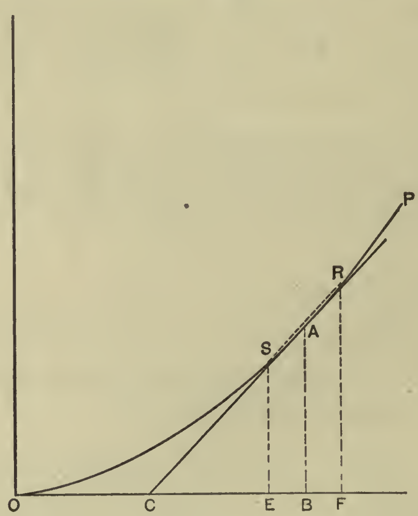


FIG. 111.

law in question. Let A be any given point corresponding to abscissa OB, and let CA be a tangent to OP at A. Then

$$\frac{dy}{dx} = \frac{AB}{BC} \quad \text{and} \quad \frac{y}{x} = \frac{AB}{OB}.$$

Hence $$m = \frac{dy}{dx} \div \frac{y}{x} = \frac{OB}{BC}, \quad \quad (8)$$

or $m =$ the quotient of the abscissa by the subtangent.

In this determination the only uncertainty is that involved in drawing a tangent to a curve. This may, however, be done quite accurately as follows: We may evidently assume without sensible error that the curve in the vicinity of A may be considered as parabolic of the second degree. We then take two points E and F equidistant from B and note the corresponding points S and R on the curve. Then by a well-known property of such curves a line drawn through A parallel to RS will be tangent to the curve at A.

The exponent n, which is essentially different in character from m, may be defined simply as the exponent which will satisfy the given fundamental equation $y = ax^n$. Its value is then most readily found by taking logarithms, whence

$$n = \frac{\log y - \log a}{\log x}. \quad \quad (9)$$

It may be readily seen that the value of n will depend on the particular unit used for x, but once this unit defined, the value of n becomes definite. Let $x = 1$, or the unit. Then we have

$$a = y,$$

or the value of the constant $a =$ the ordinate for $x = 1$.

We may therefore put more generally

$$y = y_1 x^n.$$

The exponent m may be seen to relate simply to the nature of the law at the point in question. We may therefore term it the index of instantaneous variation. It must necessarily be the exponent involved when we say that at any point y varies as a certain power of x. On the other hand, the exponent n depends on the entire law of variation from the point where $x = 1$ to the point in question. It therefore represents the resultant effect, between these points, of the

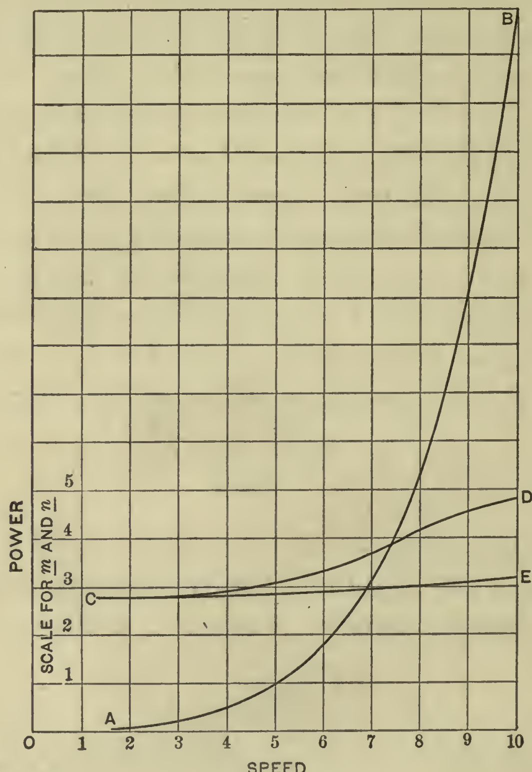


FIG. 112.

entire series of values of m. If the law is such that m is constant, m and n become identical as in the common algebraic curves. The distinction between m and n for such curves as we are here concerned with must not be forgotten.

The application of the preceding to the variation of power or resistance with speed is obvious. In Fig. 112 are given examples in which *AB* is the given fundamental curve, *CD* is the locus of the exponent *m*, and *CE* is that of the exponent *n*, where one mile is the unit. These diagrams are self-explanatory and will repay careful examination. Evidently the same investigation may be made with reference to the power and revolution curve. The speed and revolution curve gives the apparent slip, which may also, if desired, be plotted separately as *AB*, Fig. 113. If *OA*, Fig. 104, were a straight

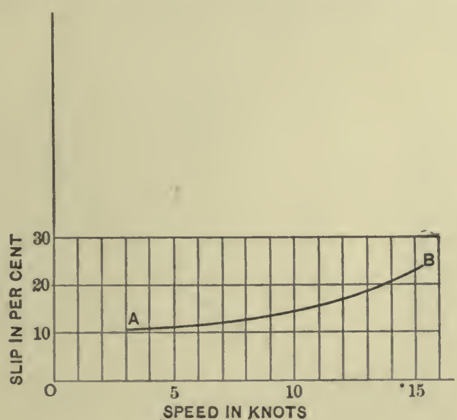


FIG. 113.

line the apparent slip would be constant, *AB* would be horizontal, and the indices relating power to revolutions and speed would be the same. Practically this is never found, though frequently the slip will vary but slightly through a considerable range.

We have in § 58 given a method of finding the mean effective pressure corresponding to the initial friction of the engine. We will now give another due to Wm. Froude.

In Fig. 114 let *AB* be a curve of mean pressure, proportional, as we have seen in § 47, to the curve of indicated thrust. Now if we assume that this pressure varies as the resistance, and the latter for speeds below u_1 as speed with the constant index n, we have only to continue BA by a curve tangent to it at A and with reference to some at present unknown axis EX, fulfilling the condition that $m = n$ at all points. Froude took $m = 1.87$, and the construction is readily seen to be an immediate result of the general method established above for the determination of m. The construction is as follows: At A draw a tangent to the curve.

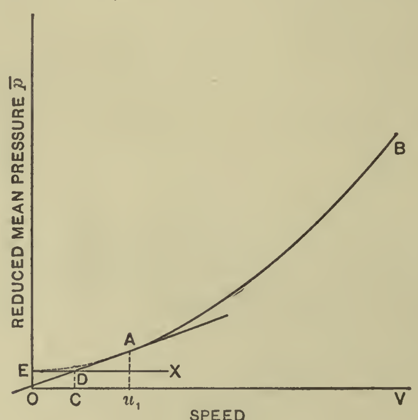


FIG. 114.

Lay off $u_1C = u_1O \div m$, erect a perpendicular CD, and draw EX. Then a curve EA run in tangent at E to EX and at A to AD will approximately fulfil the conditions required for the continuation of the pressure curve downward, and OE will therefore be the value of p_0, or the value for the initial friction. Due to the uncertainties of drawing a tangent at the end of a curve and the uncertainty that the pressure varies as the index 1.87, or indeed with any other constant index of the speed, the final result necessarily partakes of the uncertainty inherent in all attempts to extend a graphical law beyond the range covered by the observations.

79. APPLICATION OF LOGARITHMIC CROSS-SECTION PAPER.

In logarithmic cross-section paper, the divisions are similar to those on a slide-rule, or in other words, the distances from

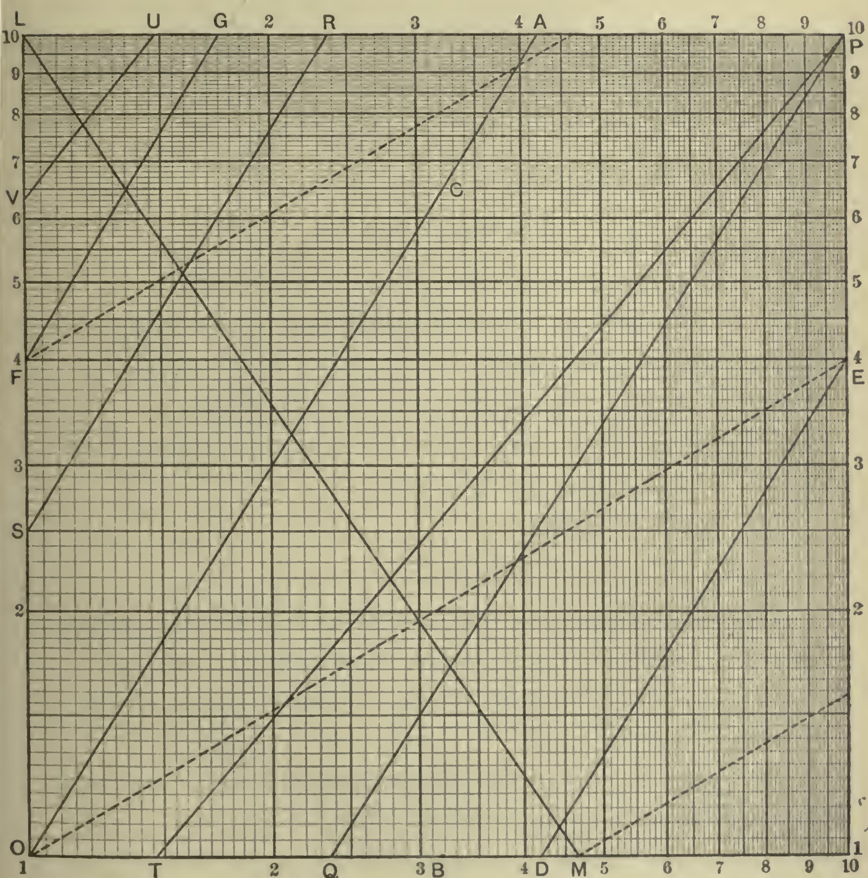


FIG. 115.

the origin are proportional to the logarithms of the numbers, as in Fig. 115. Cross-section paper ruled in this way has so many special applications to various problems in naval architecture, that a brief description of the more important may be of interest.

Given an equation $y = ax^n$.

Whence $\log y = \log a + n \log x$.

Now remembering that on this paper the abscissa and ordinate are not x and y, but $\log x$ and $\log y$, the above equation is really

$$\text{ordinate} = \text{constant} + n \text{ times abscissa.}$$

This for usual plotting is in the form

$$y = Ax + B,$$

and hence represents a straight line.

It follows that any such equation will on this paper be represented by a straight line inclined to X at an angle whose tangent is n, and cutting Y at a point which gives a. This fundamental property has a number of important applications, of which we may note the following:

(a) If $a = 1$, we have

$$y = x^n.$$

Hence a straight line on this paper will give a locus of roots or powers, of index whole or fractional, to determine which it is simply necessary to draw a line or a series of lines at an angle $\tan^{-1} n$ to X.

As commonly made, this paper has but a single logarithmic scale from 1 to 10, subdivided according to the size of the unit. By drawing on this, however, a number of lines dependent on the nature of the index, a single sheet may be made to give the nth power or root of any number from 0 to ∞. For illustration we will take the case of a table of $\frac{2}{3}$

powers. That is, we wish to express graphically the locus of the equation $y = x^{\frac{2}{3}}$. Let Fig. 116 denote a square cross-sectioned with logarithmic scales as described. Suppose that

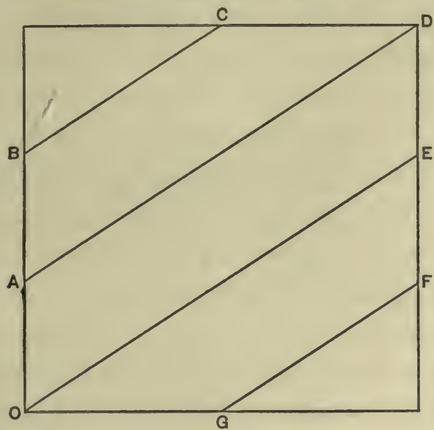


FIG. 116.

there were joined to it and to each other on the right and above, an indefinite series of such squares similarly divided. Then considering in passing from one square to an adjacent one to the right or above, that the unit becomes of the next higher order, it is evident that such a series of squares would, with the proper variation of the unit, represent all values of either x or y between 0 and ∞.

Suppose next the original square divided on the horizontal edge into 2 parts and on the vertical into 3, the points of division being at A, B, C, E, F, G. Then lines joining these points as shown in the diagram will be at an inclination to the horizontal whose tangent is $\frac{2}{3}$. Now beginning at O, OE will give the values of $x^{\frac{2}{3}}$ for values of x from 1 to 10. For greater values of x the line would run into the next adjacent square, and the location of this line, if continued, may be seen to be exactly similar to that of BC in the square before us.

It follows that, considering the units as of the next higher order, the line BC will give values of $x^{\frac{2}{3}}$ for x between O and G or 10 and 31.6 +. For larger values of x we should run into the adjacent square above with change of unit for y, but without change for x. We should here traverse a line similar to GF. Therefore by proper choice of units we may use GF for values of $x^{\frac{2}{3}}$ where x lies between 31.6 + and 100. We should then run into the next square on the right requiring the unit for x to be of the next higher order, and traverse a line similar to AD, which takes us finally to the opposite corner and completes the cycle, the last line giving us values of $x^{\frac{2}{3}}$ for x between 100 and 1000. Following this, the same series of lines would result for numbers of succeeding orders.

A little consideration of the subject will show that the value of $x^{\frac{2}{3}}$ for any value of x between 1 and ∞ may thus be read approximately from one or another of these lines; and if for any value between 1 and ∞, then likewise for any value between 0 and 1. The location of the decimal point is readily found by a little attention to the numbers involved. A rule for its location might be derived, but is of little additional value in practice. The limiting values of x for any given line may be marked on it, thus enabling a proper choice to be readily made.

The principles involved in this case may be readily extended to any other, and it will be found in general that if the exponent be represented by $\frac{m}{n}$, the complete set of lines may be drawn by dividing one side of the square into m and the other into n parts, and joining the points of division as in Fig. 116. In all there will be $(m + n - 1)$ lines, and opposite to any point on X there will be n lines corresponding to the n different beginnings of the nth root of the mth power, while opposite to any point on Y will be m lines corresponding to the m different beginnings of the mth root of the nth power. Where the complete number of lines would be quite large, it is usually unnecessary to draw them all, and the number may be limited to those necessary to cover the needed range in the values of x.

If, instead of the equation $y = x^n$, we have a constant term as a multiplier, giving an equation in the more general form $y = Bx^n$, or $Bx^{\frac{m}{n}}$, there will be the same number of lines and at the same inclination, but all shifted vertically through a distance equal to log B. If, therefore, we start on the axis of Y at the point B, we may draw in the same series of lines and in a similar manner.

It will be noted, of course, that the index $m \div n$ may be used either way for the same set of lines. That is, the same lines will give the $\frac{2}{3}$ or $\frac{3}{2}$ power of numbers, or the second or $\frac{1}{2}$ power, etc.

(b) If in the general equation $y = ax^n$ the exponent n is variable, the locus plotted on logarithmic paper will be curved instead of straight, and the index of instantaneous variation m, at any given point (§ 78), will be the tangent of the inclination of the tangent line at such point. This is readily seen from § 78, (6). Also, n will be the tangent of the inclination of a line drawn from the point where the locus cuts Y (or where $x = 1$) to the given point. This readily follows from § 78, (9). This serves also to clearly illustrate the difference in character between m and n.

(c) Proportions of the form

$$y_2 : y_1 :: x_2^n : x_1^n$$

are frequently met with. As an equation this is

$$y_2 = y_1\left(\frac{x_2}{x_1}\right)^n,$$

$$\text{or} \quad \log y_2 = \log y_1 + n(\log x_2 - \log x_1).$$

Now in Fig. 117 let A and B denote the points x_1 and x_2, and C and D be at the heights denoted by y_1 and y_2. Then, remembering that the actual distances involved are the logarithms of the corresponding quantities, we readily see that $BD = AC + nAB$ or $AC + nCE$. Therefore, a line from C to D will be at an inclination to X whose tangent is n. If,

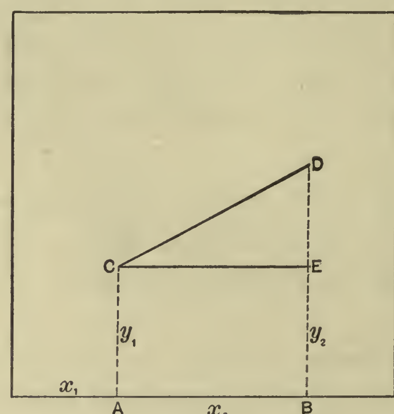


FIG. 117.

therefore, we have any ready means of passing from C, a point determined by the coordinates $x_1 y_1$, along an oblique line inclined to the axis of X at an angle whose tangent is n, it is evident that we shall pass along a continuous series of points $x_2 y_2$ so related to $x_1 y_1$ that the proportion above mentioned will be fulfilled. To solve any such proportion, therefore, we have simply to start at the given point, $x_1 y_1$, and pass along the oblique line until we reach a point whose abscissa is x_2. The corresponding ordinate will be the desired value of y_2. Or, *vice versa*, if we stop at any given value of the ordinate y_2, the corresponding abscissa will be x_2, so related to the other quantities that the proportion is fulfilled. To provide the necessary means for moving in the right direction from any point whatever as $x_1 y_1$, a series of equidistant lines at the proper angle may be ruled. With this aid a very close approximation to the proper values of $x_2 y_2$ may be made. As an instance of the use of this proportion, suppose that we wish to find the corresponding speeds for two vessels from the sixth roots of the displacements.

We have
$$\frac{u_2}{u_1} = \left(\frac{D_2}{D_1}\right)^{\frac{1}{6}}.$$

Given the u_1 and D_1 we find the point, considering one axis as that of speed and the other as that of displacement. Then by passing along a line inclined at an angle $\tan^{-1} \frac{1}{6}$ to the axis of D, we shall pass through a continuous series of points which will give corresponding displacements and speeds. Stopping at the displacement D_2 we read off directly the desired speed u_2; or, *vice versa*, stopping at a speed u_2 we read off the corresponding displacement D_2.

Having thus found the corresponding speeds, we may by another application of the same principle find the I.H.P. by using, for example, the equation

$$\frac{H_2}{H_1} = \left(\frac{D_2}{D_1}\right)^{\frac{7}{6}}.$$

As a further illustration we may take the determination of Admiralty coefficients from the equation

$$K = \frac{D^{\frac{2}{3}} u^3}{H}.$$

All of the operations here involved may be carried out on logarithmic paper and the value of the coefficients rapidly determined.

STEAMSHIP AND PROPELLER DATA.

In Tables A and B will be found a collection of data relating to ship and propeller performance. A few of the cases given in Table B are not represented in A due to the difficulty of obtaining satisfactory values of some of the data required. In Table B it should be noted that the value of the I.H.P. is that required for one propeller, and not that required for the ship as a whole in the case of twin or triple screws.

The data presented in these tables have been gathered from various sources, as near as possible in all cases to the original publication. All ratios and derivative quantities have, furthermore, been independently computed from the fundamental data, and while it can hardly be hoped that among so many figures all errors have been avoided, it is hoped that their number is small and their character not significant. The values of the form coefficient i for the propeller have been computed from the developed forms of the blades, and show how great a variation may exist in this factor, even among propellers which might by general estimation be considered the same in form.

In Table B, column headed *Screw*, the letters have the following significance:

C.................... center
S...... .... starboard
P...... .... port
M.................... mean

TABLE A.

Name.	Displacement. Tons.	Length. Feet.	Beam. Feet.	Draft. Feet.	Area Midship Section.	Block Coefficient.	Midship Section Coefficient.	Prismatic Coefficient.
Annapolis	951	168.0	36.00	11.43	355	.504	.902	.559
Atlanta	3025	270.0	42.00	17.00	635	.549	.889	.618
Augusta Victoria	9486	459.2	56.10	23.07	1153	.559	.890	.628
Baltimore (1)	4392	315.0	48.50	19.52	817	.516	.863	.598
" (2)	4500	315.0	48.50	19 87	833	.519	.864	.600
Bancroft	832	187.5	32.00	11.44	277	.424	.757	.560
Bayern (German Navy)	7370	321.5	60.00	19.62	1038	.682	.882	.773
Bayern (N. Ger. Lloyd)	7070	390.3	45.00	24.10	975	.605	.930	.650
Bennington	1706	228.0	36.00	14.00	429	.520	.850	.612
Boston	3235	270.0	42.00	17.67	670	.565	.903	.626
Brooklyn	8150	400.5	64.68	21.87	1225	.504	.866	.582
Castine	1068	190.0	32.00	11.78	325	.522	.861	.606
Charleston	3557	300.0	46.16	17.85	716	.504	.869	.580
Chicago	4543	315.0	48.25	19.00	796	.551	.868	.635
City of Lowell	2445	319.9	48.00	12.83	467	.434	.758	.573
Clara	38.4	91.0	11.75	4.23	25	.389	.658	.591
Columbia (U. S. Navy)	7350	411.6	58.40	22.40	1120	.478	.856	.558
Columbia (H.A.P.A.G.)	9482	463.0	56.00	22.90	1121	.559	.874	.640
Concord	1723	228.0	36.00	14.10	432	.521	.850	.613
Cushing	105.3	138.7	14.25	4.87	47.7	.383	.687	.557
Dania	8372	373.0	44.48	26.00	1065	.701	.950	.737
Detroit	2068	257.0	37.00	14.46	468	.526	.875	.601
Dolphin	1413	240.0	32.00	13.79	338	.467	.766	.609
Elbe	6350	420.0	45.00	20.00	791	.612	.915	.669
Fearless	1560	220.0	34.00	14.00	438	.521	.920	.566
Foote	143.9	160.0	16.06	5.07	58.5	.387	.718	.539

TABLE A—(*continued.*)

Name.	Displacement. Tons.	Length. Feet.	Beam. Feet.	Draft. Feet.	Area Midship Section.	Block Coefficient.	Midship Section Coefficient.	Prismatic Coefficient.
Forth	3584	300.0	46.00	17.62	744	.516	.918	.562
Fürst Bismarck	10490	502.0	57.50	23.30	1202	.546	.897	.609
Gresham	820	188.0	32.00	9.92	252	.481	.793	.606
Guardian	222	104.5	20.00	8.40	116	.480	.748	.642
Hammonia III	5910	373.0	45.00	20.25	845	.609	.927	.657
Helena	1340	250.0	39.60	8.63	327	.549	.957	.574
Hohenzollern (German Navy)	4180	382.5	45.92	18.20	722	.458	.863	.530
Imperieuse	7645	315.0	61.00	25.00	1287	.557	.844	.660
Indiana	10225	348.0	69.25	23.87	1540	.622	.931	.668
Iowa	11363	360.0	72.23	24.04	1640	.636	.945	.673
Iris	3290	300.0	46.08	18.08	700	.488	.889	.549
Iwana	198	92.5	20.95	8.16	128	.438	.750	.584
Kaiser Wilhelm II	8575	449.6	51.00	23.00	1001	.589	.884	.666
Katahdin (1)	2125	250.2	41.67	14.85	461	.480	.745	.644
" (2)	2139	250.2	41.67	14.90	463	.482	.746	.646
Lahn	7685	448.4	49.00	21.88	927	.581	.897	.648
Launch No. 4	23.3	54.8	11.88	3.88	25	.399	.670	.596
Lookout	42.9	96.0	13.60	3.70	25.3	.463	.750	.617
Machias	1068	190.0	32.00	11.78	325	.522	.861	.606
Manning	950	188.0	32.83	12.00	293	.481	.797	.604
McCulloch	1060	200.0	33.33	12.33	308	.482	.801	.602
Marblehead	2054	257.0	37.00	14.40	464	.525	.871	.603
Marietta	991	174.1	34.00	11.96	349	.512	.896	.571
Massachusetts	10265	348.0	69.25	24.08	1545	.619	.927	.668
Minneapolis	7387	412.0	58.00	22.50	1125	.481	.862	.558
Monterey	4000	256.0	59.04	14.43	772	.642	.906	.709
Montgomery	2091	257.0	37.00	14.00	471	.550	.909	.605

TABLE A—(*continued.*)

Name.	Displacement. Tons.	Length. Feet.	Beam. Feet.	Draft. Feet.	Area Midship Section.	Block Coefficient.	Midship Section Coefficient.	Prismatic Coefficient.
Narkeeta	190	92.5	20.95	7.92	123	.433	.742	.583
Nashville	1364	221.4	38.00	10.95	370	.518	.889	.583
Newark	3980	310.8	49.17	18.27	775	.499	.863	.578
Newport	972	168.0	36 00	11.71	344	.501	.852	.588
New York (U. S. Navy)	8480	380.0	64.25	23.89	1350	.509	.880	.578
Normannia	10590	498.7	57.40	22.25	1169	.582	.915	.636
Olympia	5586	340.0	53.00	20.73	970	.523	.883	.592
Oregon	10250	348.0	69.25	24.00	1534	.620	.923	.672
Petrel	855	175.0	31.00	11.21	286	.492	.823	.598
Philadelphia	4325	315.0	48.57	19.21	815	.515	.874	.589
Porter	164	175.0	17.00	4.79	61.9	.403	.760	.530
Powerful	14200	500.0	71.00	27.00	....	.518	......	....
Rodney	9690	325.0	68.00	26.70	1560	.575	.859	.669
Rugia	6750	352 0	43.00	22.20	904	.703	.947	.742
San Francisco	4088	310.0	49.15	18.75	770	.501	.836	.599
Spree	8895	463.0	51.83	21.92	1015	.592	.893	.663
Surprise	1544	250.0	32.50	13.83	392	.481	.872	.552
Terrible	14200	500.0	71.00	27.00	....	.518	....	....
Venetia	4025	320.0	39.83	17.50	625	.662	.940	.704
Vesuvius	771	246.2	26.42	9.51	192	.436	.764	.571
Vicksburg	964	168.0	36.00	11.65	342	.500	.852	.587
Virginia	4070	320.0	40.00	17.60	623	.662	.927	.714
Wahneta	176.5	92.5	20.95	7.60	116.4	.419	.731	.573
Wheeling	1000	174.1	34.00	12.03	352	.512	.896	.571
Wilmington	1342	250.0	39.60	8.65	327	.549	.957	.574
Worth	9878	354.3	64.00	24.37	1435	.626	.920	.680
Yorktown	1680	228.0	36.00	13.84	432	.518	.867	.597

TABLE B.

Name.	Propeller.								Trial Data.			
	Screw.	Number of Blades.	Diam.	Pitch.	Pitch-ratio.	Area.	Area-ratio.	Shape Factor. *i*	I H.P.	Rev.	Speed.	Apparent Slip in Per Cent.
Annapolis	C	4	9.50	11.00	1.16	30.0	.423	.98	1208	146.8	13.17	17.4
Ascania	C	4	15.00	14.50	.97	56.1	.318	1.03	1100	78.0	10.00	10.4
Atlanta	C	4	17.00	24.28	1.43	96.2	.424	.99	2749	68.8	13.45	18.4
"	C	4	17.00	24.28	1.43	96.2	.424	.99	2145	63.8	12.77	16.5
"	C	4	17.00	24.28	1.43	96.2	.424	.99	904	47.8	9.89	13.7
Augusta Victoria	M	4	16.57	31.33	1.89	104.8	.486	1.17	6000	75.0	18.00	22.4
Baltimore (1)	S	3	14.50	20.00	1.38	57.2	.346	1.07	4361	118.1	19.57	16.0
" (1)	P	3	14.50	20.00	1.38	57.2	.346	1.07	4317	118.0	19.57	16.0
" (2)	S	3	14.50	21.50	1.48	57.2	.346	1.07	4846	116.4	20.00	19.0
" (2)	P	3	14.50	21.50	1.48	57.2	.346	1.07	4985	116.1	20.00	18.8
Bancroft	M	3	7.00	7.75	1.11	15.0	.390	1.06	585	225.0	14.52	15.7
Bayern (German Navy)	M	4	17.69	18.67	1.06	90.5	.368	1.02	2061	76.9	13.69	3.4
" " "	M	2	17.69	18.67	1.06	45.2	.184	1.02	2217	84.9	13.80	11.7
" " "	M	4	17.69	18.67	1.06	90.5	.368	1.02	853	56.7	10.44	00.1
" " "	M	2	17.69	18.67	1.06	45.2	.184	1.02	908	64.1	10.84	8.2
" " "	M	4	16.40	18.67	1.14	85.0	.402	1.08	2402	84.0	14.04	9.3
" " "	M	3	16.40	18.67	1.14	63.7	.302	1.08	2355	87.8	13.96	13.7
" " "	M	2	16.40	18.67	1.14	42.5	.201	1.08	2567	96.0	14.29	19.2
" " "	M	4	16.40	17.81	1.09	85.0	.402	1.08	2744	91.6	14.29	11.2
" " "	M	3	16.40	17.81	1.09	63.7	.302	1.08	2452	92.3	14.06	13.3
" " "	M	4	16.40	18.67	1.14	85.0	.402	1.08	898	64.4	10.65	10.2
Bayern (N. German Lloyd)	C	4	18.05	24.61	1.36	86.3	.337	1.16	3500	66.0	14.00	12.6
Bennington	S	3	10.50	13.71	1.31	24.3	.281	1.04	1642	150.8	17.50	14.2
"	P	3	10.50	13.69	1.30	24.3	.281	1.04	1680	151.1	17.50	14.3
Blitz	M	3	11.48	11.48	1.00	26.7	.258	.95	1382	147.3	16.04	3.9

TABLE B—(*continued.*)

Name.	Propeller.								Trial Data.			
	Screw.	Number of Blades.	Diam.	Pitch.	Pitch-ratio.	Area.	Area-ratio.	Shape Factor. i	I.H.P.	Rev.	Speed.	Apparent Slip in Per Cent.
Blitz	M	3	11.48	12.70	1.11	26.7	.258	.95	1311	136.0	16.08	5.1
"	M	3	11.48	13.62	1.19	26.7	.258	.95	1257	128.5	16.03	7.2
"	M	3	10.50	13.62	1.30	25.5	.294	1.06	1269	138.2	16.28	12.3
Boston	C	4	17.00	23.67	1.39	95.7	.422	.99	3936	72.2	15.58	7.6
"	C	4	17.00	23.67	1.39	95.7	.422	.99	2423	64.8	14.35	5.2
Brooklyn	S	3	16.50	19.98	1.21	75.7	.354	1.10	9229	136.2	21.91	18.4
"	P	3	16.50	20.01	1.21	75.7	.354	1.10	9020	136.9	21.91	19.0
Castine	S	3	7.66	8.74	1.14	15.4	.334	1.07	1076	217.0	16.03	14.4
"	P	3	7.66	8.74	1.14	15.4	.334	1.07	1052	220.6	16.03	15.8
Charleston	S	3	14.00	17.60	1.26	54.8	.356	1.03	3160	115.4	18.20	9.2
"	P	3	14.00	17.60	1.26	54.8	.356	1.03	3156	113.9	18.20	8.0
Chicago	S	4	15.50	24.59	1.59	77.9	.413	.93	2464	70.0	15.33	9.7
"	P	4	15.50	24.59	1.59	77.9	.413	.93	2142	70.8	15.33	10.7
"	S	4	15.50	24.59	1.59	77.9	.413	.93	1462	58.8	13.27	7.0
"	P	4	15.50	24.59	1.59	77.9	.413	.93	1331	59.8	13.27	8.5
"	S	4	15.50	24.59	1.59	77.9	.413	.93	826	47.3	10.47	8.8
"	P	4	15.50	24.59	1.59	77.9	.413	.93	615	46.3	10.47	6.9
City of Lowell	M	4	11.08	16.63	1.50	46.9	.486	1.08	2174	125.9	19.27	6.8
" "	M	4	11.08	16.63	1.50	46.9	.486	1.08	1364	106.7	16.20	7.5
Clara	C	4	3.75	6.25	1.67	5.45	.494	1.05	147	280.0	12.83	25.7
"	C	4	3.88	6.50	1.68	5.03	.425	.99	178	270.0	13.40	22.6
"	C	4	3.88	6.50	1.68	5.03	.425	.99	142	252.0	13.00	19.6
Columbia (U. S. Navy)	S	3	15.00	21.50	1.43	53.7	.304	1.06	6606	134.0	22.80	19.8
" " "	P	3	15.00	21.50	1.43	53.7	.304	1.06	5560	132.9	22.80	19.1
" " "	C	3	14.00	21.50	1.54	53.3	.346	1.07	5826	127.7	22.80	15.9

TABLE B—(*continued.*)

Name.	Propeller.								Trial Data.			
	Screw.	Number of Blades	Diam.	Pitch.	Pitch-ratio.	Area.	Area-ratio.	Shape Factor. i	I.H.P.	Rev.	Speed.	Apparent Slip in Per Cent.
Columbia (H.A.P.A.G.)	M	3	18.00	31.00	1.72	92.9	.365	1.05	6225	75.0	18.70	18.5
Concord	S	3	10.50	13.20	1.26	26.5	.306	1.04	1678	151.5	17.00	13.9
"	P	3	10.50	13.20	1.26	26.5	.306	1.04	1636	153.3	17.00	14.9
Cushing	M	4	4.25	8.42	1.98	9.3	.656	1.15	877	372.1	22.48	27.3
Dania	C	4	19.69	21.65	1.10	107.0	.351	1.09	3400	67.0	13.00	9.2
Detroit	S	3	11.00	13.00	1.18	29 0	.305	1.09	2600	170.1	18.71	14.2
"	P	3	11.00	13.00	1.18	29.0	.305	1.09	2555	170.1	18.71	14.2
Dolphin	C	4	13.75	24.08	1.75	70.7	.476	1.00	2144	74.2	15.50	12.1
Elbe	C	4	21.75	29.00	1.33	135.6	.365	1.08	6130	66.5	16.25	14.6
Fearless	M	3	10.50	12.62	1.20	24.0	.278	1.08	1557	150.4	16.91	9.7
Foote	S	3	5.20	8.00	1.54	7.38	.347	1.04	....	398.1	24.53	22.0
"	P	3	5.20	8.00	1.54	7.38	.347	1.04	....	393.3	24.53	21.0
Forth	M	3	13.00	17.50	1.35	47.0	.354	1.10	3080	122.6	18.18	14.2
Fürst Bismarck	M	3	19.03	27.89	1.47	86.1	.303	1.01	7972	90.8	20.70	17.1
Gresham	C	4	10.00	12 50	1.25	40 0	.509	....	2347	165.3	17.32	15.1
Guardian	M	4	7.33	11.00	1.50	23.9	.566	1.19	530	138.6	12.33	18.0
"	M	4	7.33	11.00	1.50	23.9	.566	1.19	402	128.7	11.84	15.3
"	M	4	7.33	11.00	1.50	23.9	.566	1.19	187	102.8	9.94	11.0
Hammonia III	C	4	18.01	27.98	1.55	110.8	.435	1.18	4350	64.9	15.24	15.0
" "	C	4	18.01	27.98	1.55	110.8	.435	1.18	3360	61.2	14.58	13.7
" "	C	4	18.01	27.98	1.55	110.8	.435	1.18	1850	50.5	12.10	13.2
" "	C	4	18.01	27.98	1.55	110.8	.435	1.18	581	30.7	7.70	9.2
Hay	C	4	6.24	6.73	1.08	8.34	.273	1.03	154	127.2	7.98	5.6
"	C	4	6.24	5.74	.92	8.34	.273	1.03	145	140.3	7.77	2.3
"	C	4	5.58	8 37	1.50	7.64	.312	1.06	133	135.0	7.82	29.9

TABLE B—(*Continued.*)

Name.	Propeller.								Trial Data.			
	Screw.	Number of Blades.	Diam.	Pitch.	Pitch-ratio.	Area.	Area-ratio.	Shape Factor. i	I.H.P.	Rev.	Speed.	Apparent Slip in Per Cent.
Hay	C	4	5.58	7.71	1.38	7.64	.312	1.06	140	144.1	8.19	25.3
"	C	4	5.58	7.05	1.26	7.64	.312	1.06	156	156.2	8.54	21.4
"	C	4	5.58	6.40	1.15	7.64	.312	1.06	202	177.5	9.22	17.8
"	C	4	5.58	6.40	1.15	7.64	.312	1.06	83	133.0	7.32	12.9
"	C	4	5.58	6.96	1.25	7.64	.312	1.06	202	187.5	9.30	27.8
"	C	4	5.58	6.96	1.25	7.64	.312	1.06	82	139.0	7.30	23.5
Helena	S	3	7.00	7.10	1.01	18.7	.486	1.23	958	280.0	15.50	21.0
"	P	3	7.00	7.10	1.01	18.7	.486	1.23	987	279.9	15.50	21.0
Hohenzollern (German Navy)	M	4	14.76	22.64	1.53	52.0	.304	1.06	4817	107.0	21.53	10.0
Hohenzollern (N. Ger. Lloyd)	C	4	17.06	21.00	1.23	97.2	.425	1.15	2540	70.0	13.23	8.8
Imperieuse	M	4	18.16	22.06	1.22	87.0	.346	1.05	5092	88.0	17.21	10.1
Indiana	S	3	15.50	16.00	1.03	53.9	.285	1.04	4826	130.8	15.55	24.7
"	P	3	15.50	16.00	1.03	53.9	.285	1.04	4672	131.3	15.55	25.0
Iowa	S	3	16.50	19.97	1.21	75.7	.354	1.10	5949	108.6	17.09	20.1
"	P	3	16.50	20.00	1.21	75.7	.354	1.10	5885	110.5	17.09	21.6
Iris	M	4	18.54	18.17	.98	194.4	.720	1.07	3752	91.0	16.58	−1.6
"	M	4	18.54	18.17	.98	194.4	.720	1.07	2626	82.2	15.12	−2.5
"	M	4	18.54	18.17	.98	194.4	.720	1.07	1280	65.1	12.06	−3.3
"	M	2	18.54	18.17	.98	97.2	.360	1.07	2184	88.9	15.73	1.3
"	M	2	18.54	18.17	.98	97.2	.360	1.07	1653	81.2	14.52	.0
"	M	2	18.54	18.17	.98	97.2	.360	1.07	819	65.1	11.58	.8
"	M	4	16.29	19.96	1.23	144.0	.691	1.03	3857	97.2	18.57	3.0
"	M	4	16.29	19.96	1.23	144.0	.691	1.03	2554	85.4	16.56	1.6
"	M	4	16.29	19.96	1.23	144.0	.691	1.03	917	61.3	12.38	−2.6
"	M	2	18.13	21.27	1.17	112.0	.434	1.01	3778	93.3	18.59	5.1

TABLE B—(*Continued.*)

Name.	Propeller.								Trial Data.			
	Screw.	Number of Blades.	Diam.	Pitch.	Pitch-ratio.	Area.	Area-ratio.	Shape Factor i.	I.H.P.	Rev.	Speed.	Apparent Slip in Per Cent.
Iris	M	2	18.13	21.27	1.17	112.0	.434	1.01	1979	76.9	15.75	2.4
"	M	2	18.13	21.27	1.17	112.0	.434	1.01	883	59.4	12.48	.0
Iwana	C	4	7.50	12.50	1.67	22.5	.509	.96	349	115.5	11.58	18.8
Kaiser Wilhelm II	C	4	21.82	25.26	1.16	142.4	.381	1.04	6650	70.5	16.40	6.7
Katahdin (1)	S	3	11.83	14.00	1.18	35.8	.326	1.08	2691	147.7	16.11	21.0
" (1)	P	3	11.83	14.00	1.18	35.8	.326	1.08	2219	143.7	16.11	18.8
" (2)	S	3	10.88	14.33	1.32	38.4	.413	1.16	2636	153.5	15.88	26.8
" (2)	P	3	10.88	14.33	1.32	38.4	.413	1.16	2230	146.7	15.88	23.5
" (2)	S	3	10.88	14.33	1.32	38.4	.413	1.16	2252	147.6	15.70	24.8
" (2)	P	3	10.88	14.33	1.32	38.4	.413	1.16	2159	142.9	15.70	22.3
" (2)	S	3	10.88	14.33	1.32	38.4	.413	1.16	1185	125.3	14.42	18.6
" (2)	P	3	10.88	14.33	1.32	38.4	.413	1.16	1534	127.4	14.42	19.9
" (2)	S	3	12.00	14.00	1.17	36.0	.318	1.06	2515	137.1	16.07	15.1
" (2)	P	3	12.00	14.00	1.17	36.0	.318	1.06	2389	141.9	16.07	18.0
" (2)	S	3	12.00	14.00	1.17	36.0	.318	1.06	2355	135.2	15.84	15.2
" (2)	P	3	12.00	14.00	1.17	36.0	.318	1.06	2263	140.2	15.84	18.2
" (2)	S	3	12.00	14.00	1.17	36.0	.318	1.06	1597	125.8	14.87	14.4
" (2)	P	3	12.00	14.00	1.17	36.0	.318	1.06	1443	126.7	14.87	15.1
Lahn	C	4	22.25	30.00	1.35	160.0	.411	1.15	8929	70.0	19.00	8.3
Launch No. 4	C	2	4.33	5.14	1.19	6.13	.416	1.26	39.3	196.5	8.50	14.6
" "	C	2	4.33	5.14	1.19	4.80	.325	1.26	39.3	200.2	8.50	16.2
" "	C	2	4.33	5.14	1.19	3.06	.207	1.26	40.6	208.4	8.50	19.4
" "	C	2	4.33	5.14	1.19	1.74	.118	1.26	42.6	221.0	8.50	24.2
" "	C	3	4.33	7.00	1.62	6.85	.464	1.24	41.5	151.1	8.50	18.5
" "	C	3	4.33	7.00	1.62	5.29	.359	1.32	42.6	157.9	8.50	22.0

TABLE B—(*Continued.*)

Name.	Propeller.								Trial Data.			
	Screw.	Number of Blades.	Diam.	Pitch.	Pitch-ratio.	Area.	Area-ratio.	Shape Factor i.	I.H.P.	Rev.	Speed.	Apparent Slip in Per Cent.
Lookout	C	4	4.65	8.40	1.81	7.94	.468	1.12	84.5	152.3	9.53	24.4
"	C	4	5.00	9.00	1.80	9.34	.476	1.14	95	140.0	10.19	18.0
Machias	S	3	7.66	8.87	1.16	15.4	.335	1.07	868	218.6	15.46	19.2
"	P	3	7.66	8.87	1.16	15.4	.335	1.07	927	214.3	15.46	17.6
Manning	C	4	11.00	12.33	1.12	40.0	.421		2405	156.2	16.68	12.3
McCulloch	C	4	11.00	12.50	1.14	43.0	.452	1.15	2239	151.8	17.23	8.0
Marblehead	S	3	11.00	12.00	1.09	33.3	.351	1.11	2656	179.0	18.44	13.0
"	P	3	11.00	12.00	1.09	33.3	.351	1.11	2647	173.4	18.44	10.2
Marietta	S	3	6.75	7.25	1.07	17.0	.475	1.03	505	231.1	13.02	21.3
"	P	3	6.75	7.25	1.07	17.0	.475	1.03	518	232.5	13.02	21.8
Massachusetts	S	3	15.50	16.00	1.03	53.9	.285	1.04	5050	132.3	16.21	22.4
"	P	3	15.50	16.00	1.03	53.9	.285	1.04	5078	133.1	16.21	22.9
Minneapolis	S	3	15.00	22.00	1.47	53.7	.304	1.06	6587	131.9	23.07	19.4
"	P	3	15.00	22.00	1.47	53.7	.304	1.06	6561	133.1	23.07	20.2
"	C	3	14.50	21.50	1.48	53.3	.323	1.07	7219	132.2	23.07	17.7
Monterey	S	3	10.16	11.66	1.15	38.9	.480	1.17	2504	162.9	13.60	27.5
"	P	3	10.16	11.66	1.15	38.9	.480	1.17	2483	161.2	13.60	26.7
Montgomery	S	3	11.00	12.75	1.16	29.0	.305	1.09	2763	179.7	19.06	15.7
"	P	3	11.00	12.75	1.16	29.0	.305	1.09	2721	180.9	19.06	16.3
Narkeeta	C	4	7.50	12.50	1.67	22.5	.509	.96	356	111.8	11.22	18.6
Nashville	S	3	6.67	7.00	1.05	13.0	.373	1.12	1242	308.5	16.30	23.5
"	P	3	6.67	7.00	1.05	13.0	.373	1.12	1247	308.3	16.30	23.4
Newark	S	3	14.50	18.97	1.31	52.8	.320	1.08	4468	127.3	19.00	20.3
"	P	3	14.50	18.97	1.31	52.8	.320	1.08	4114	126.6	19.00	19.9
Newport	C	3	10.00	10.45	1.05	23.3	.297	1.05	976	142.7	12.29	16.5

TABLE B—(*continued.*)

Name.	Propeller.								Trial Data.			
	Screw.	Number of Blades.	Diam.	Pitch.	Pitch-ratio.	Area.	Area-ratio.	Shape Factor. i	I.H.P.	Rev.	Speed.	Apparent Slip in per Cent.
New York	S	3	16.00	21.00	1.31	69.1	.344	1.10	8636	134.6	21.00	24.7
" "	P	3	16.00	21.00	1.31	69.1	.344	1.10	8312	135.0	21.00	24.9
Normannia	M	3	18.12	26.74	1.48	87.6	.313	1.07	8122	92 5	20.75	15.0
"	M	3	18.12	26.74	1.48	87.6	.313	1.07	4808	80.0	18.63	11.7
"	M	3	18.12	26.74	1.48	87.6	.313	1.07	2155	60.0	14.53	8.2
"	M	3	18.12	26.74	1.48	87.6	.313	1.07	785	40.0	10.12	4.2
Olympia	S	3	14.75	19.00	1.29	68.0	.398	1.16	8298	140.0	21.69	17.4
"	P	3	14.75	19.00	1.29	68.0	.398	1.16	8552	138.5	21.69	16.5
Oregon	S	3	15.00	15.60	1.04	66.0	.373	1.10	5254	128.3	16.79	15.0
"	P	3	15.00	15.60	1.04	66.0	.373	1.10	5637	128.2	16.79	15.0
Petrel	C	3	9.75	12.25	1.26	23.6	.316	.90	1051	126.7	11.79	23.0
Philadelphia	S	3	14.50	20.39	1.41	57.2	.346	1.07	4247	119 6	19.68	18.2
"	P	3	14.50	20.39	1.41	57.2	.346	1.07	4286	119.5	19.68	18.2
Porter	S	3	5.40	8.75	1.62	10.7	.467	....	....	390.	28.63	15.0
"	P	3	5.40	8.75	1.62	10.7	.467	....	....	389 2	28.63	14.8
Powerful	M	3	19.50	23.71	1.22	78.0	.261	....	12950	114.5	21.80	18.6
"	M	3	19 50	23.71	1.22	78.0	.261	....	9230	102.8	20.98	12.4
Rodney	M	4	15.50	19.42	1.25	72.0	.382	1.10	5805	107.2	16.92	17.6
Rugia	C	4	17.50	21.33	1.22	73.0	.304	1.05	2600	68.6	13.00	10.0
San Francisco	S	3	13.50	18.75	1.39	57.6	.402	1.09	4788	125.8	19.52	16.2
" "	P	3	13.50	18.75	1.39	57.6	.402	1.09	4793	123.8	19.52	14.8
Spree	C	4	22.47	31.17	1.39	160.0	.403	1.15	12327	69.6	19.55	8.7
Sumatra	C	4	7.88	9.51	1.21	22.0	.450	1.13	340	118.0	10.00	9.7
Surprise	M	3	11.00	14.75	1.34	24.0	.253	1.08	1523	132.1	17.00	11.4
Terrible	M	3	19.5	24.00	1.23	92.0	.308	....	12824	112.3	22.41	15.7

TABLE B—(*continued.*)

Name.	Propeller.								Trial Data.			
	Screw.	Number of Blades.	Diam.	Pitch.	Pitch-ratio.	Area.	Area-ratio.	Shape Factor. *i*	I.H.P.	Rev.	Speed.	Apparent Slip in per Cent.
Terrible	M	3	19.5	24.00	1.23	92.0	.308	....	9250	102.7	20.81	14.4
Venetia	C	4	16.00	18.50	1.16	70.0	.348	1.03	1678	73.0	11.81	11.4
Vesuvius	S	3	7.75	9.37	1.21	15.9	.337	.97	1895	267.8	21.42	13.5
"	P	3	7.75	9.37	1.21	15.9	.337	.97	1817	270.0	21.42	14.2
Vicksburg	C	3	10.00	10.63	1.06	23.3	.297	1.05	1084	146.6	12.71	17.3
Virginia	C	4	16.00	16.25	1.02	62.2	.309	1.06	1836	82.0	11.55	12.2
Wahneta	C	4	7.50	12.50	1.67	22.5	.509	.96	378	114.6	11.63	17.7
Wheeling	S	3	6.75	7.25	1.07	17.0	.475	1.03	520	230.4	12.88	21.9
"	P	3	6.75	7.25	1.07	17.0	.475	1.03	530	232.4	12.88	22.6
Wilmington	S	3	7.00	7.13	1.02	18.7	.486	1.23	968	272.8	15.08	21.4
"	P	3	7.00	7.13	1.02	18.7	.486	1.23	890	273.3	15.08	21.6
Worth	M	3	16.13	17.72	1.10	58.0	.284	.95	5114	109.2	16.60	13.1
Yorktown	S	3	10.50	12.50	1.19	25.4	.294	1.04	1804	160.7	16.65	16.0
"	P	3	10.50	12.50	1.19	25.4	.294	1.04	1775	160.8	16.65	16.1
"	S	3	10.50	12.50	1.19	25.4	.294	1.04	1135	140.8	14.81	14.7
"	P	3	10.50	12.50	1.19	25.4	.294	1.04	1184	140.4	14.81	14.5
"	S	3	10.50	12.50	1.19	25.4	.294	1.04	342	94.2	10.61	8.7
"	P	3	10.50	12.50	1.19	25.4	.294	1.04	386	97.7	10.61	12.0
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"	M	3	10.00	12.8	1.28	20.1	.256	.98	890	151.3	15.15	20.7
"	M	2	10.00	12.8	1.28	13.4	.171	.98	911	166.8	15.42	26.8
"	M	4	10.00	12.8	1.28	26.8	.341	.98	198	84.7	9.02	15.7
"	M	3	10.00	12.8	1.28	20.1	.256	.98	207	94.3	9.86	17.2
"	M	2	10.00	12.8	1.28	13.4	.171	.98	282	112.7	10.90	23.4
"	M	3	9.02	12.8	1.42	18.7	.294	1.06	976	168.8	15.71	26.3
"	M	2	9.02	12.8	1.42	12.5	.196	1.06	868	170.8	15.43	28.5

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