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# Revised Wave Runup Curves for Smooth Slopes

by

Philip N. Stoa

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$d_s/H_o'$  was extended to  $d_s/H_o' = 8$ ; relative depth ( $d_s/H_o'$ ) is important even for  $d_s/H_o' > 3$  for waves which do not break on the structure slope.

A flow chart is given to assist in choosing the proper figure and in interpreting the results when applied to untested bottom slopes (i.e., bottom slopes flatter than 1 on 10).

Also given are example problems and a curve for scale-effect corrections.

## PREFACE

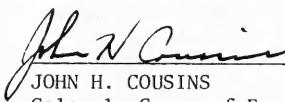
This report describes a means of determining wave runup on coastal structures having uniformly sloping, smooth surfaces. The report is based principally on small-scale test results and analyses of Saville (1956) and Savage (1959) as reanalyzed by Stoa (1978). The work was conducted under the coastal engineering research program of the U.S. Army Coastal Engineering Research Center (CERC).

The technical guidelines presented in this report supersede the design runup curves for smooth slopes given in the Shore Protection Manual (SPM) (U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1977). The revised runup curves given here include a wider range of relative depth,  $d_s/H_o^1$ . These results are based on experiments using regular waves. Ahrens (1977a, 1977b) presented methods for estimating runup and overtopping, respectively, from irregular waves based on results of regular wave testing.

The report was prepared by Philip N. Stoa, Oceanographer, under the general supervision of Robert A. Jachowski, Chief, Coastal Design Criteria Branch.

Comments on this publication are invited.

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## CONTENTS

	Page
CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI) . . . . .	6
SYMBOLS AND DEFINITIONS . . . . .	7
I INTRODUCTION. . . . .	9
II RUNUP CURVES. . . . .	11
1. Smooth Structure Fronted by Horizontal Bottom . . . . .	11
2. Smooth Structure Fronted by 1 on 10 Bottom Slope and Zero Toe Depth ( $d_s = 0$ ) . . . . .	11
3. Smooth Structure Fronted by 1 on 10 Bottom Slope and Toe Depth Greater than Zero ( $d_s > 0$ ) . . . . .	11
III MAXIMUM RUNUP . . . . .	22
IV SMOOTH-SLOPE SCALE-EFFECT CORRECTION. . . . .	24
V EXAMPLE PROBLEMS. . . . .	24
LITERATURE CITED. . . . .	35

## TABLES

1 Example runup for $T = 7$ seconds, constant depth, and $(H'_o)_{max} = 16$ feet. . . . .	31
2 Example runup for $T = 13$ seconds, constant depth, and $(H'_o)_{max} = 16$ feet. . . . .	31
3 Example runup for constant wave steepness, $H'_o/gT^2 = 0.0101$ . . . . .	33
4 Summary of maximum runup for different conditions . . . . .	33

## FIGURES

1 Definition sketch of variables applicable to wave runup . . . . .	10
2 Relative runup for smooth slope on horizontal bottom; $d_s/H'_o = 3$ . . . . .	12
3 Relative runup for smooth slope on horizontal bottom; $d_s/H'_o = 5$ . . . . .	13
4 Relative runup for smooth slope on horizontal bottom; $d_s/H'_o = 8$ . . . . .	14
5 Relative runup for smooth slopes on 1 on 10 bottom; $d_s = 0$ ; $d/H'_o = 3$ . . . . .	15

## CONTENTS

## FIGURES--Continued

	Page
6 Relative runup for smooth slopes on 1 on 10 bottom; $d_s = 0; d/H_O^! = 5$ . . . . .	16
7 Relative runup for smooth slopes on 1 on 10 bottom; $d_s = 0; d/H_O^! = 8$ . . . . .	17
8 Relative runup for smooth slopes on 1 on 10 bottom; $\ell/L \geq 0.5; d_s/H_O^! = 0.6$ . . . . .	18
9 Relative runup for smooth slopes on 1 on 10 bottom; $\ell/L \geq 0.5; d_s/H_O^! = 1.0$ . . . . .	19
10 Relative runup for smooth slopes on 1 on 10 bottom; $\ell/L \geq 0.5; d_s/H_O^! = 1.5$ . . . . .	20
11 Relative runup for smooth slopes on 1 on 10 bottom; $\ell/L \geq 0.5; d_s/H_O^! = 3.0$ . . . . .	21
12 Flow chart for the evaluation of wave runup . . . . .	23
13 Runup scale-effect correction factor, $k$ . . . . .	25

**CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI)  
UNITS OF MEASUREMENT**

U.S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	by	To obtain
inches	25.4	millimeters
	2.54	centimeters
square inches	6.452	square centimeters
cubic inches	16.39	cubic centimeters
feet	30.48	centimeters
	0.3048	meters
square feet	0.0929	square meters
cubic feet	0.0283	cubic meters
yards	0.9144	meters
square yards	0.836	square meters
cubic yards	0.7646	cubic meters
miles	1.6093	kilometers
square miles	259.0	hectares
knots	1.852	kilometers per hour
acres	0.4047	hectares
foot-pounds	1.3558	newton meters
millibars	$1.0197 \times 10^{-3}$	kilograms per square centimeter
ounces	28.35	grams
pounds	453.6 0.4536	grams kilograms
ton, long	1.0160	metric tons
ton, short	0.9072	metric tons
degrees (angle)	0.01745	radians
Fahrenheit degrees	5/9	Celsius degrees or Kelvins <sup>1</sup>

<sup>1</sup>To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use formula:  $C = (5/9)(F - 32)$ .

To obtain Kelvin (K) readings, use formula:  $K = (5/9)(F - 32) + 273.15$ .

## SYMBOLS AND DEFINITIONS

d	water depth
$d_s$	water depth at toe of structure
g	acceleration of gravity (32.2 feet per second squared or 9.81 meters per second squared)
H	wave height
$H'_o$	the deepwater wave height, neglecting refraction, equivalent to the wave height, H, measured in a given water depth
$K_s$	shoaling coefficient, $H/H'_o$
k	runup scale-effect correction factor
L	wavelength
$L_o$	deepwater wavelength; wavelength in water depth, d, such that $d/L \geq 0.5$
$\ell$	horizontal length of slope fronting toe of structure
R	runup; the vertical rise of water on structure face resulting from wave action
T	wave period
$\beta$	bottom slope; used for the slope fronting a structure and is different from the structure slope
$\theta$	structure slope; may be beach slope if runup on the beach face is being investigated



## REVISED WAVE RUNUP CURVES FOR SMOOTH SLOPES

by

Philip N. Stoa

### I. INTRODUCTION

Wave runup is the vertical distance above stillwater level (SWL) reached by a wave incident to a structure or beach. Prediction of wave runup on coastal structures is necessary to determine an adequate crest elevation to prevent overtopping or to help determine the extent of overtopping. Wave runup curves for structures with either smooth or rough slopes have previously been presented in the Shore Protection Manual (SPM) (U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1977). Runup data of Saville (1956) and Savage (1959), together with data from other reports, have been reanalyzed (Stoa, 1978). This report presents revised smooth-slope runup curves which vary in certain regions from those presented in the SPM. A scale-effect correction curve is also given for application to smooth-slope runup.

Wave runup is primarily a function of characteristics of the structure and incident wave; wave characteristics are also a function of water depth and bottom slope. The variables are shown in Figure 1 and are defined as:  $R$ , runup;  $\theta$ , angle of structure slope;  $d$ , water depth;  $d_g$ , water depth at toe of structure;  $\beta$ , angle of bottom slope at the structure toe; and  $\ell$ , horizontal length of the bottom slope seaward of the structure toe.  $L$  and  $H$  are the wavelength and wave height, respectively, as measured in a water depth,  $d$ . The same wave may be described by an equivalent deepwater wave ( $d/L \geq 0.5$ ) for which the dimensions would be  $L_o$  and  $H_o'$ .  $L_o$  is the deepwater wavelength and  $H_o'$  is the equivalent unrefracted deepwater wave height.  $L_o$  may be determined if the wave period,  $T$ , is known ( $L_o = gT^2/2\pi$ ); this report uses  $gT^2$  as the principal measure of deepwater wavelength.  $H_o'$  is used because it avoids the problem of defining the wave height in varying depths over a sloping bottom where the wave may already have broken. The wave height in deep water is related to wave height in a shallower depth by the shoaling coefficient,  $H/H_o'$  or  $K_s$ . The shoaling coefficient and wavelength,  $L$ , may be determined from Tables C-1 or C-2 in the SPM when  $L_o$  and the required depth are known.

The runup curves are given for three different cases: (a) horizontal bottom at the structure toe; (b) 1 on 10 sloping bottom at the structure toe, with a zero toe depth ( $d_g = 0$ ); and (c) 1 on 10 sloping bottom at the structure toe, with toe depths greater than zero ( $d_g > 0$ ). Case (c) has, generally, the potential for the largest waves attacking the structure. A bottom slope of 1 on 10 is relatively steep for ocean coastlines, and its occurrence would be restricted to beach faces with coarse sediments (see Fig. 4-33 in the SPM), backshore areas subject to flooding, or some nearshore areas. However, most bottom slopes would be flatter than 1 on 10. Experimental data for runup on structures fronted by flatter slopes

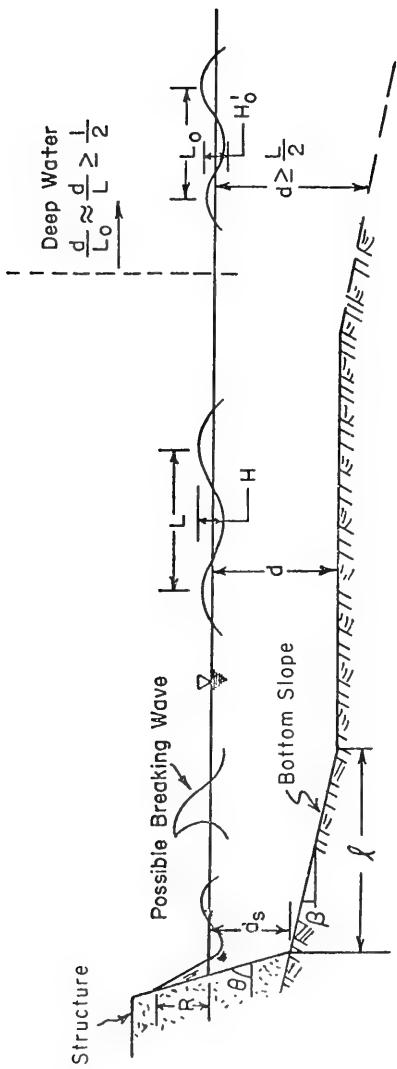


Figure 1. Definition sketch of variables applicable to wave runup (Stora, 1978).

are very limited; brief qualitative comments regarding runup in such circumstances are given in later sections.

The incident wave characteristics seaward of the toe of the bottom slope are partly determined by the corresponding water depth and are important in determination of runup. The methods presented in Sections II,2 and II,3 are designed to account for the incident wave characteristics at the toe of the bottom slope as determined in model experiments. Natural underwater slopes are rarely so well defined; straight-line approximations of irregular slopes should be determined by the designer. Intersections of the straight lines will define the location of a change in slope.

## II. RUNUP CURVES

### 1. Smooth Structure Fronted by Horizontal Bottom.

Relative runup,  $R/H_O'$ , for a smooth structure fronted by a horizontal bottom is given in Figures 2, 3, and 4 for specific values of relative depth,  $d_S/H_O'$ . As shown by comparing the figures, relative runup on the flatter slopes is not a function of  $d_S/H_O'$ . However, relative runup on the steep slopes is sensitive to depth effects; relative runup for a given wave steepness,  $H_O'/gT^2$ , is largest at the lowest  $d_S/H_O'$  value. Thus, proper consideration of depth effects must be included in design.

Relative depth values of  $2 < d_S/H_O' < 3$  may occur for structures on horizontal bottoms, but experimental data are limited. Figure 2 ( $d_S/H_O' = 3$ ) is recommended for cases in which  $d_S/H_O' < 3$ . Large  $d_S/H_O'$  values may occur, for example, in reservoirs; runup determinations for  $d_S/H_O' > 8$  should be based on Figure 4 ( $d_S/H_O' = 8$ ).

### 2. Smooth Structure Fronted by 1 on 10 Bottom Slope and Zero Toe Depth ( $d_S = 0$ ).

When  $d_S = 0$ , wave conditions are determined using the depth,  $d$ , at the toe of the 1 on 10 bottom slope. Figures 5, 6, and 7 show the results for  $d/H_O'$  (not  $d_S/H_O'$ ) values of 3, 5, and 8 with a 1 on 10 bottom slope.

Runup on a structure fronted by a beach slope flatter than 1 on 10 would be expected to be less than indicated in Figures 5, 6, and 7 for comparable wave conditions. However, these figures are recommended for use when a flatter bottom slope is present and  $d_S = 0$ .

### 3. Smooth Structure Fronted by 1 on 10 Bottom Slope and Toe Depth Greater than Zero ( $d_S > 0$ ).

Design curves for runup on a smooth structure with  $d_S > 0$ , fronted by a 1 on 10 bottom slope, are given in Figures 8 to 11. The curves apply to cases where the relative bottom-slope length is  $\ell/L \geq 0.5$ . For values of  $\ell/L < 0.5$  but for high  $d_S/H_O'$  values (e.g.,  $d_S/H_O' \geq 3$ ), the runup values from figures for structures on horizontal bottoms (Figs. 2, 3, and

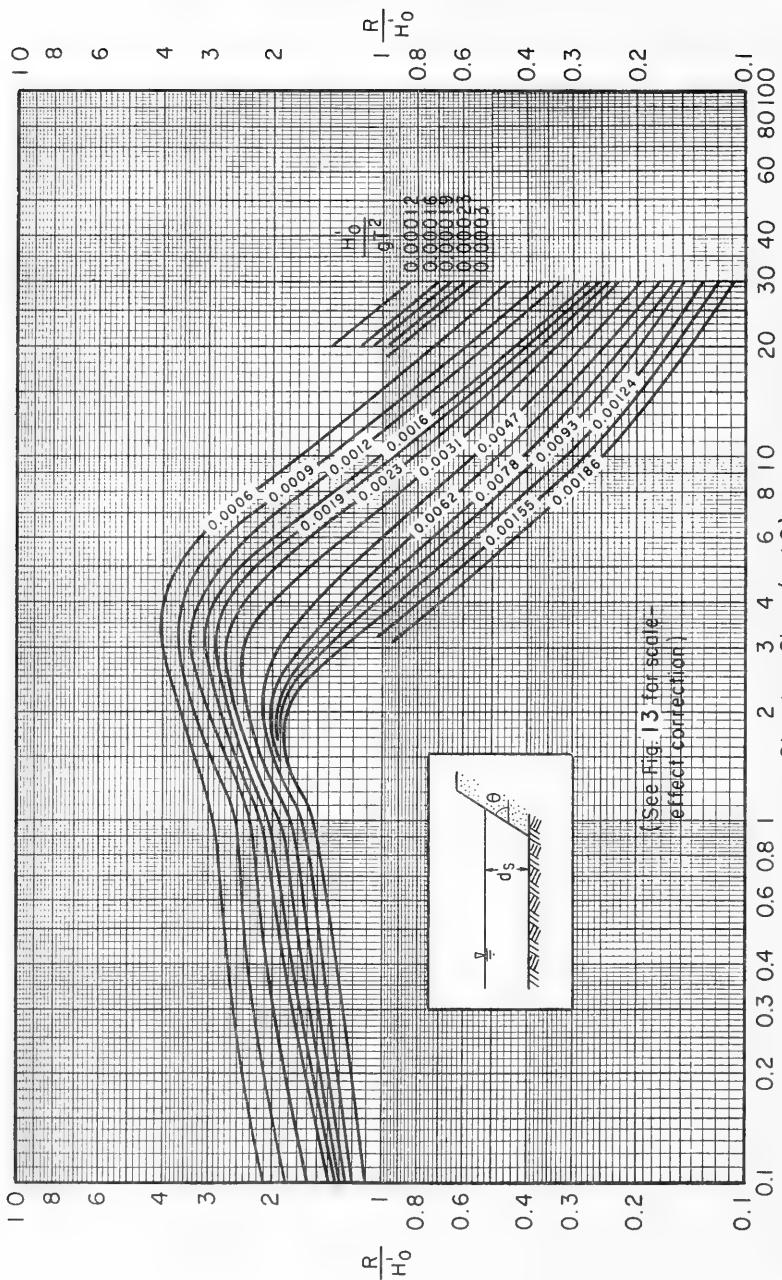


Figure 2. Relative runup for smooth slope on horizontal bottom;  
 $d_g/H_0 = 3$  (Stoia, 1978).

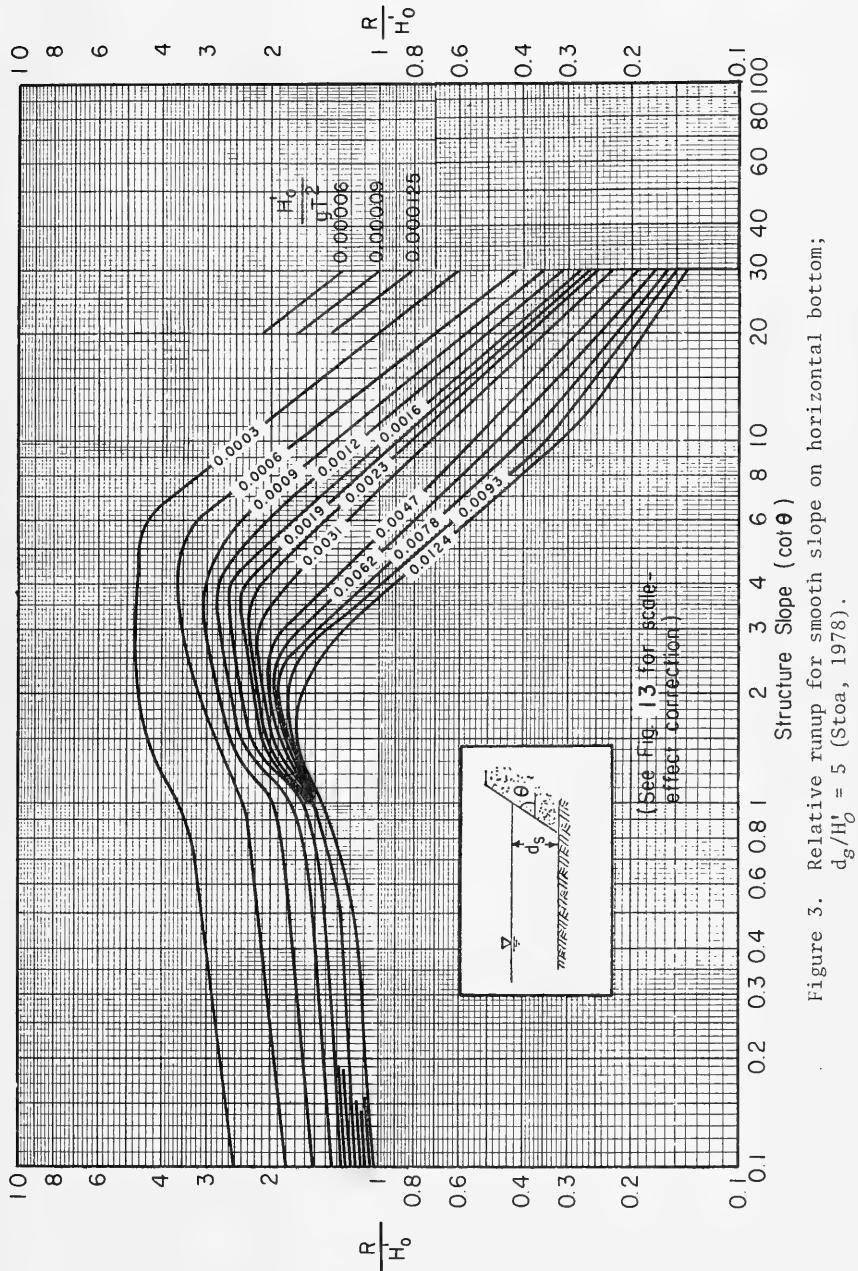


Figure 3. Relative runup for smooth slope on horizontal bottom;  
 $d_s/H_0 = 5$  (Stoe, 1978).

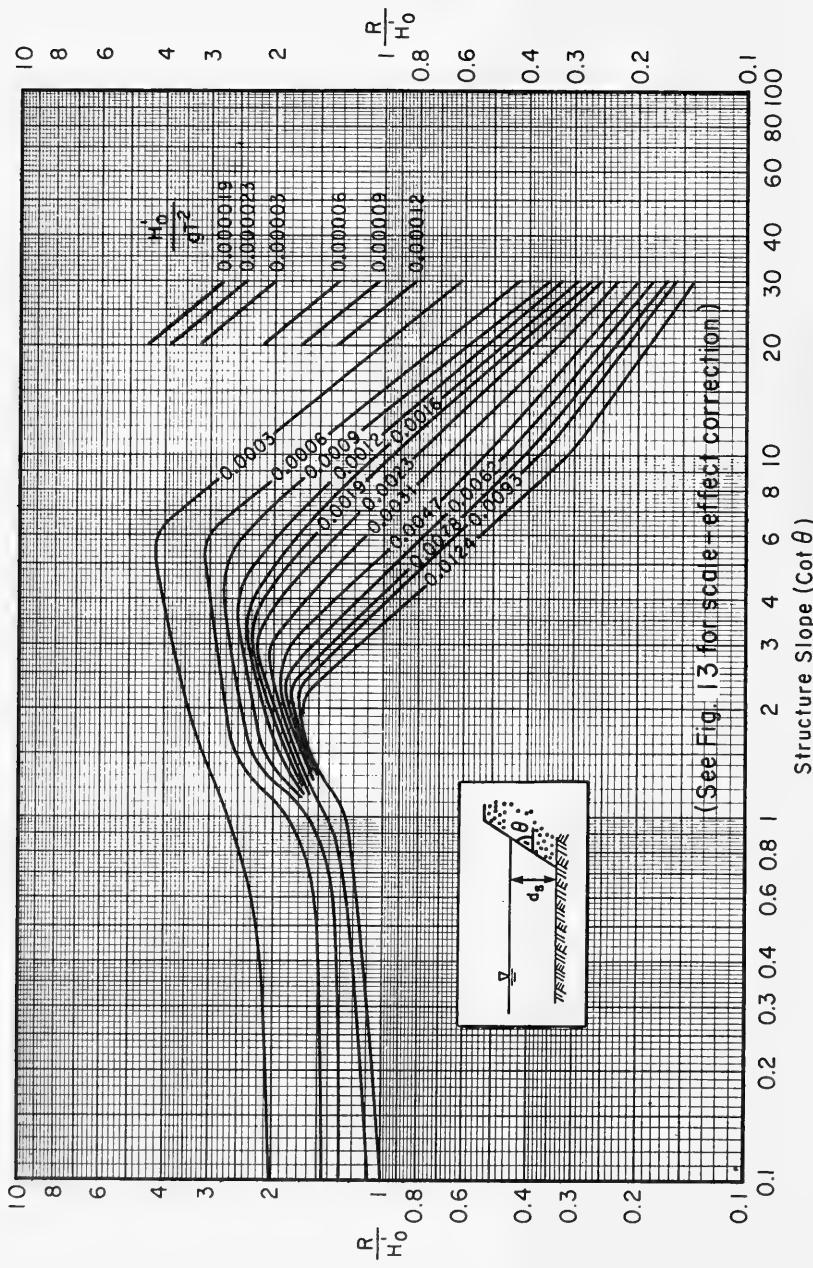


Figure 4. Relative runup for smooth slope on horizontal bottom;  
 $d_s/H_0' = 8$  (Stoia, 1978).

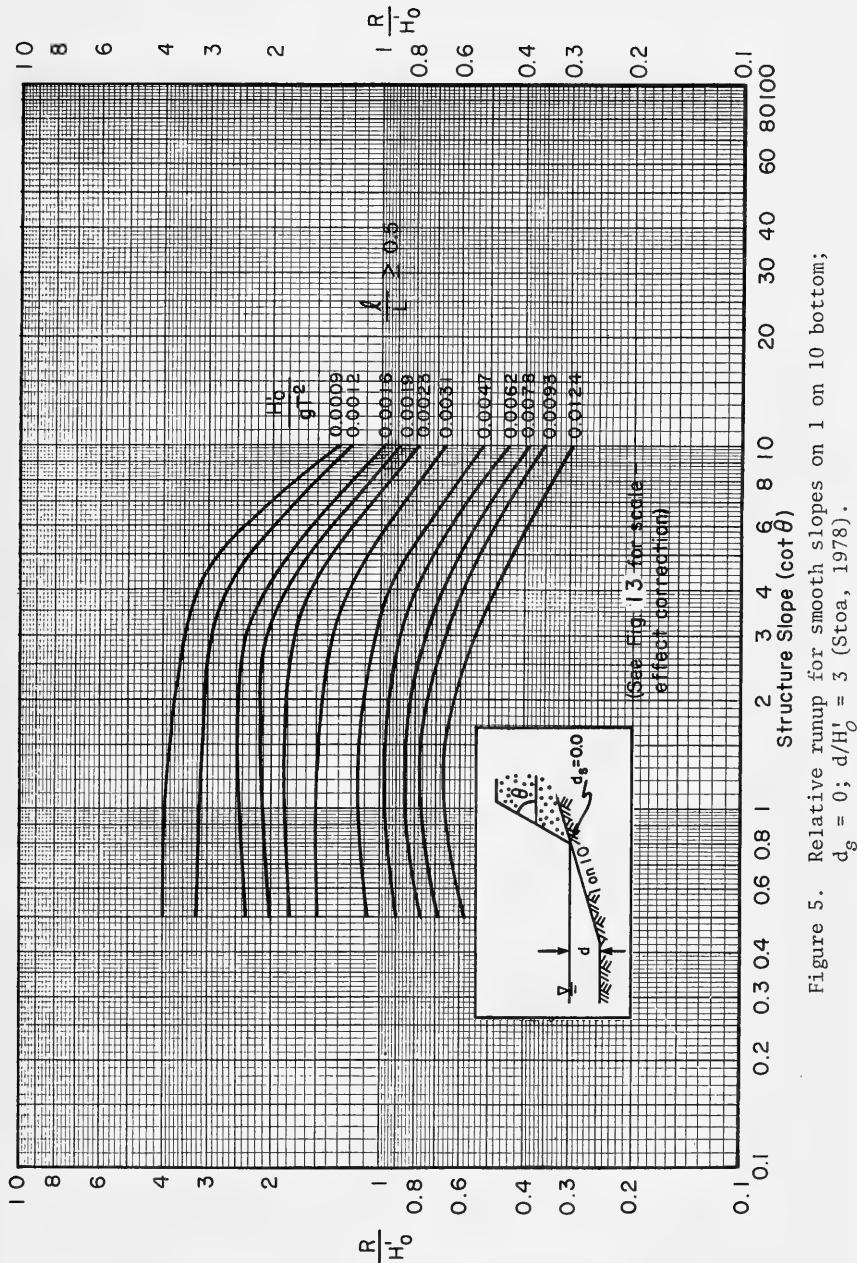


Figure 5. Relative runup for smooth slopes on 1 on 10 bottom;  
 $d_s = 0$ ;  $d/H_0 = 3$  (Stoa, 1978).

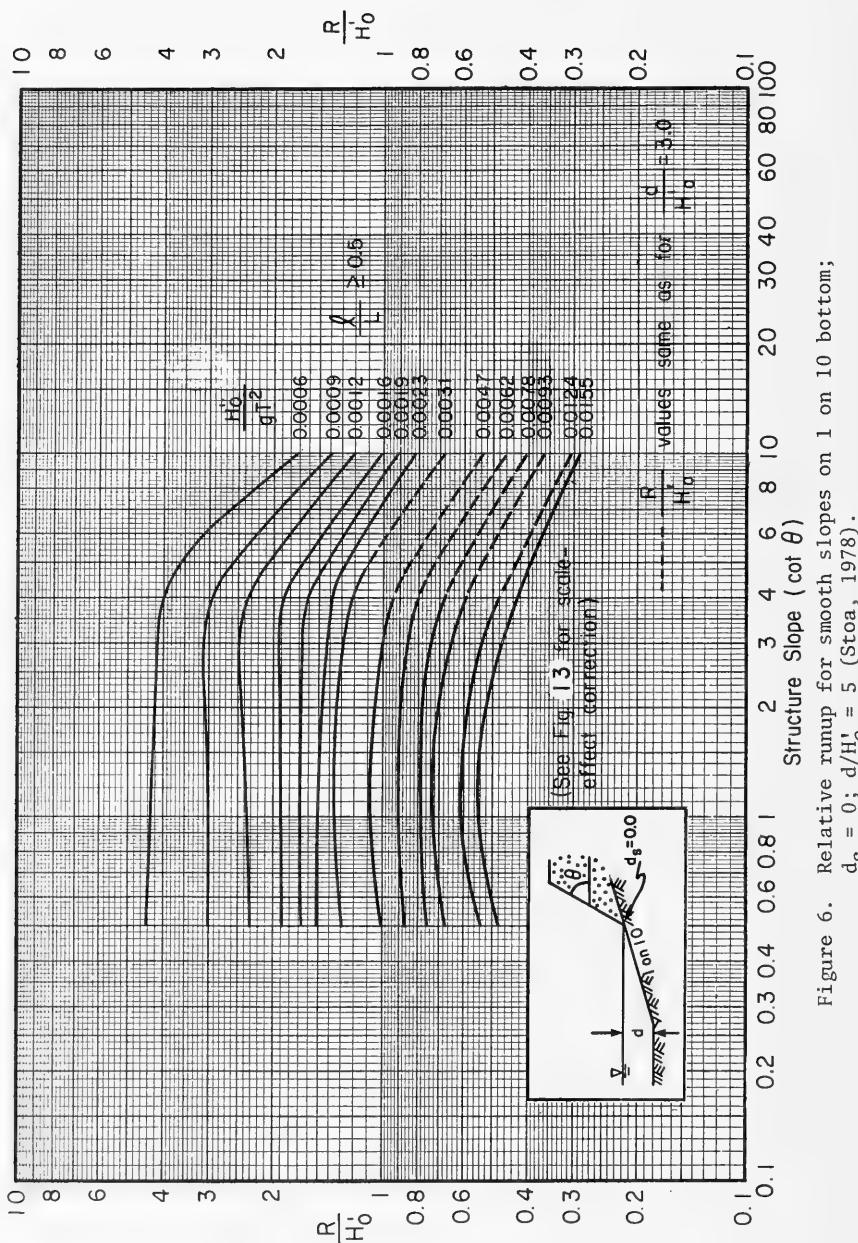


Figure 6. Relative runup for smooth slopes on 1 on 10 bottom;  
 $d_s = 0$ ;  $d/H'_0 = 5$  (Stora, 1978).

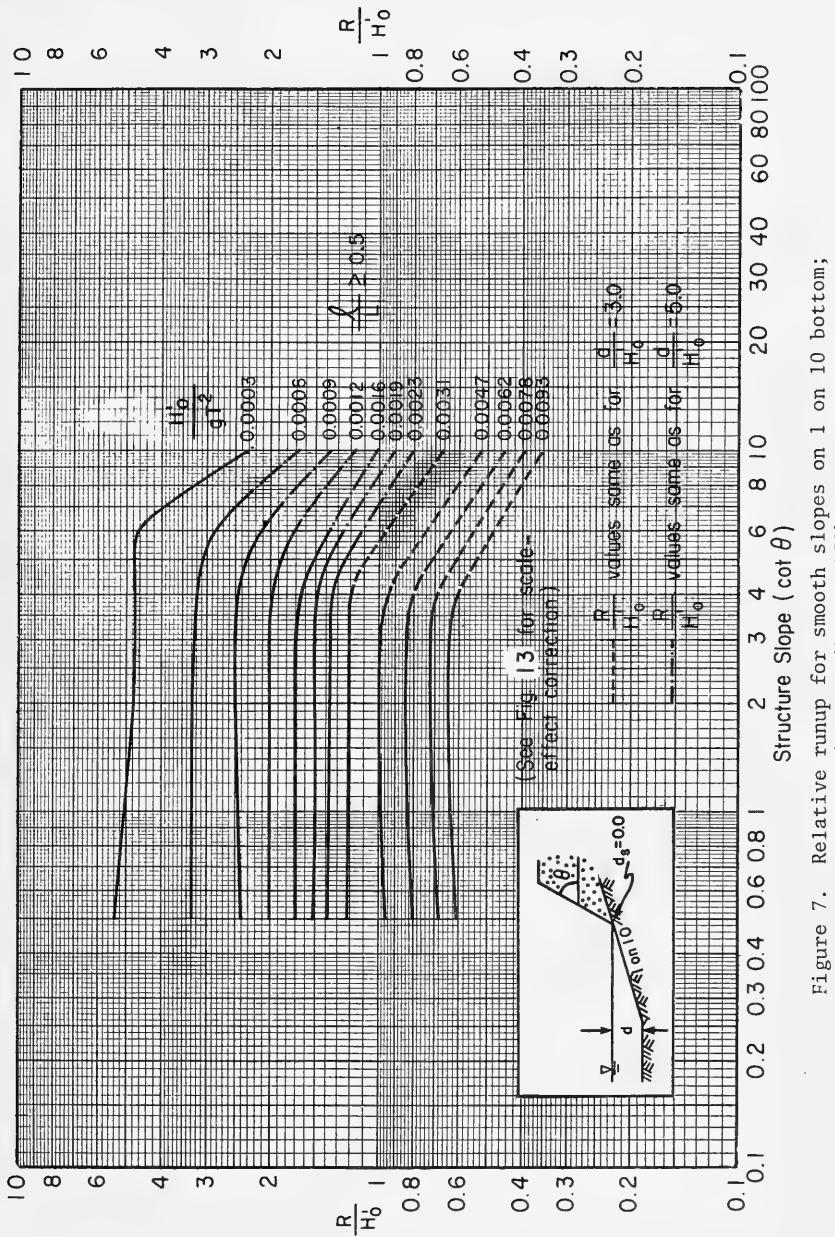


Figure 7. Relative runup for smooth slopes on 1 on 10 bottom;  
 $d_s = 0$ ;  $d/H'_0 = 8$  (Stoia, 1978).

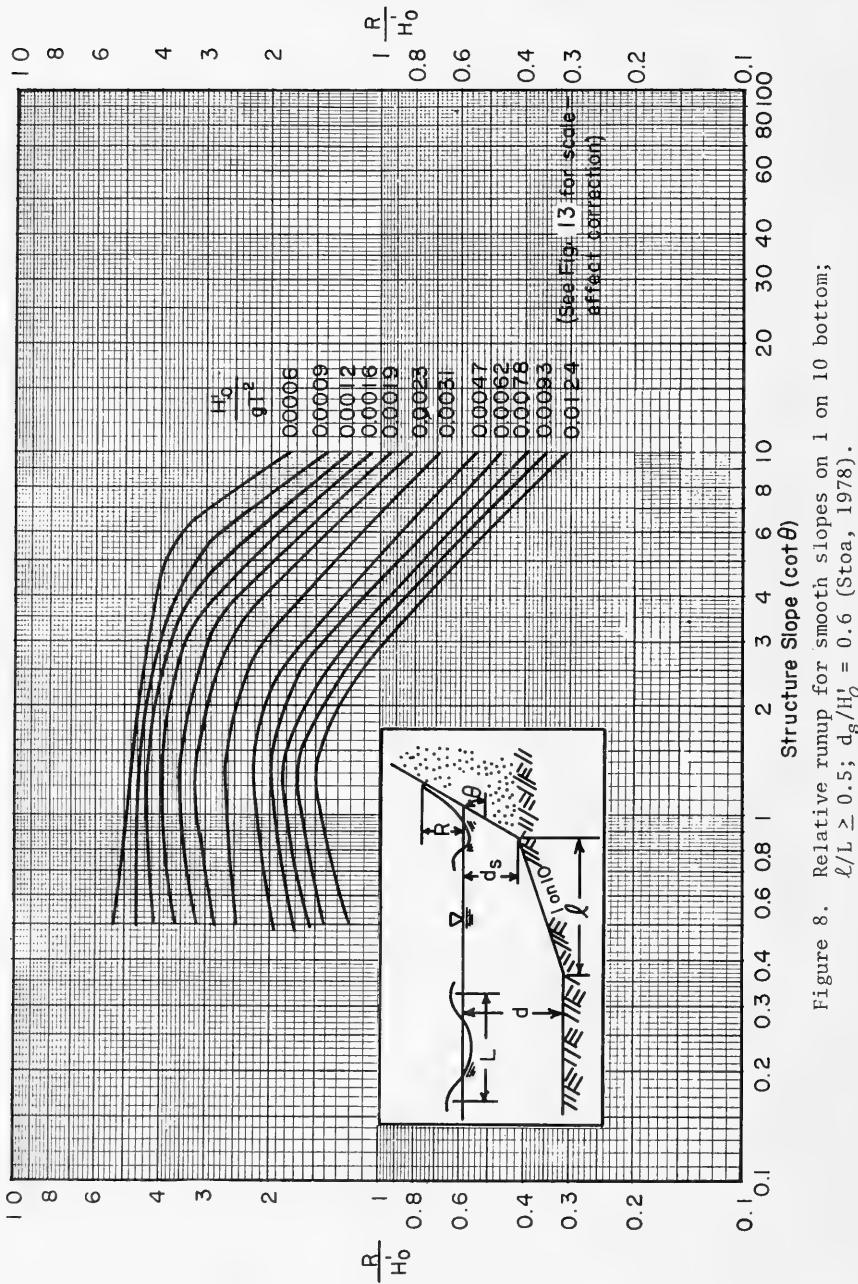


Figure 8. Relative runup for smooth slopes on 1 on 10 bottom;  
 $\ell/L \geq 0.5$ ;  $d_g/H'_0 = 0.6$  (Stoia, 1978).

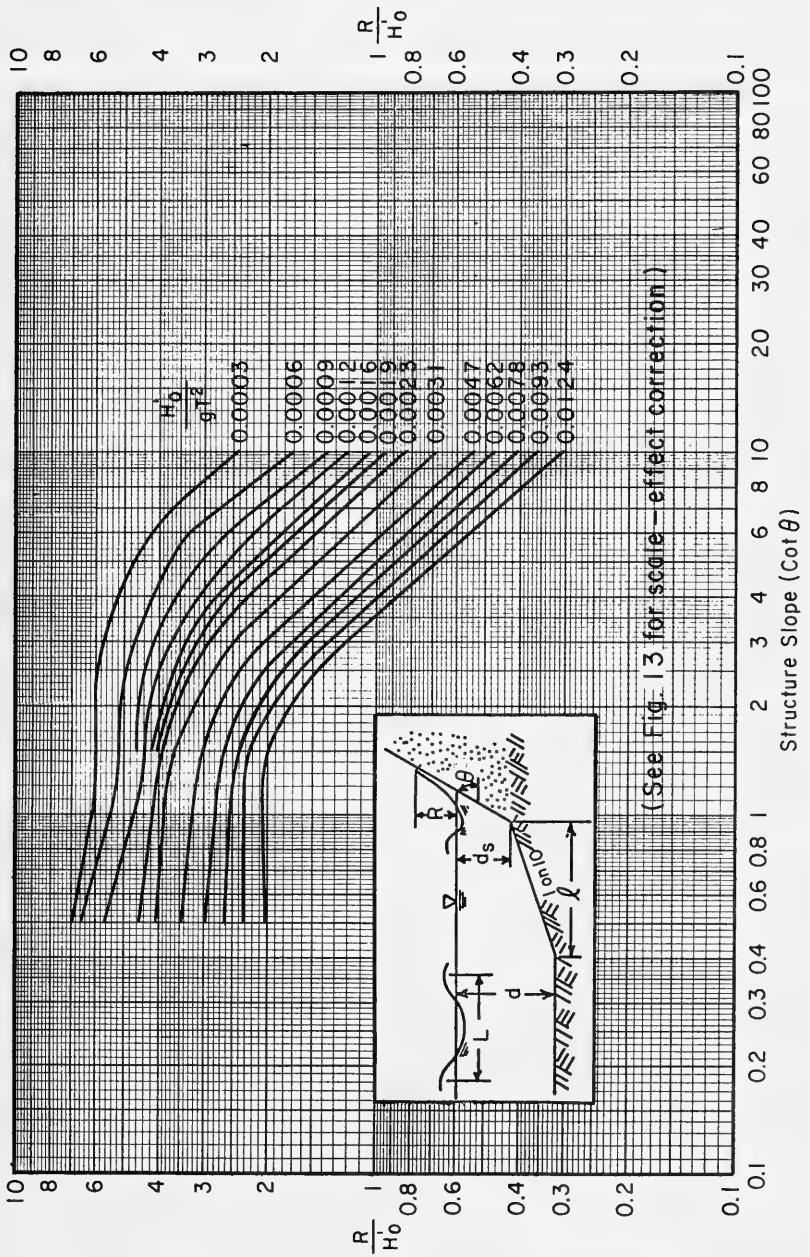
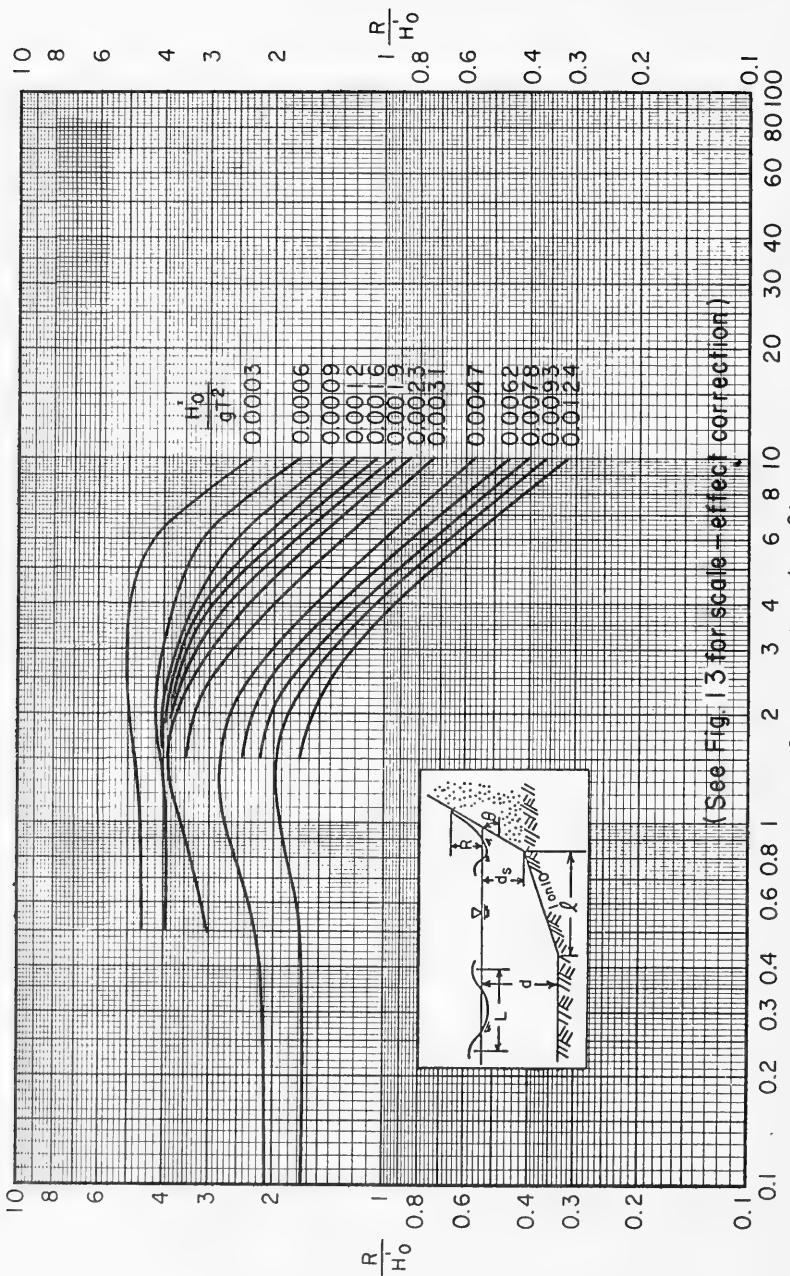
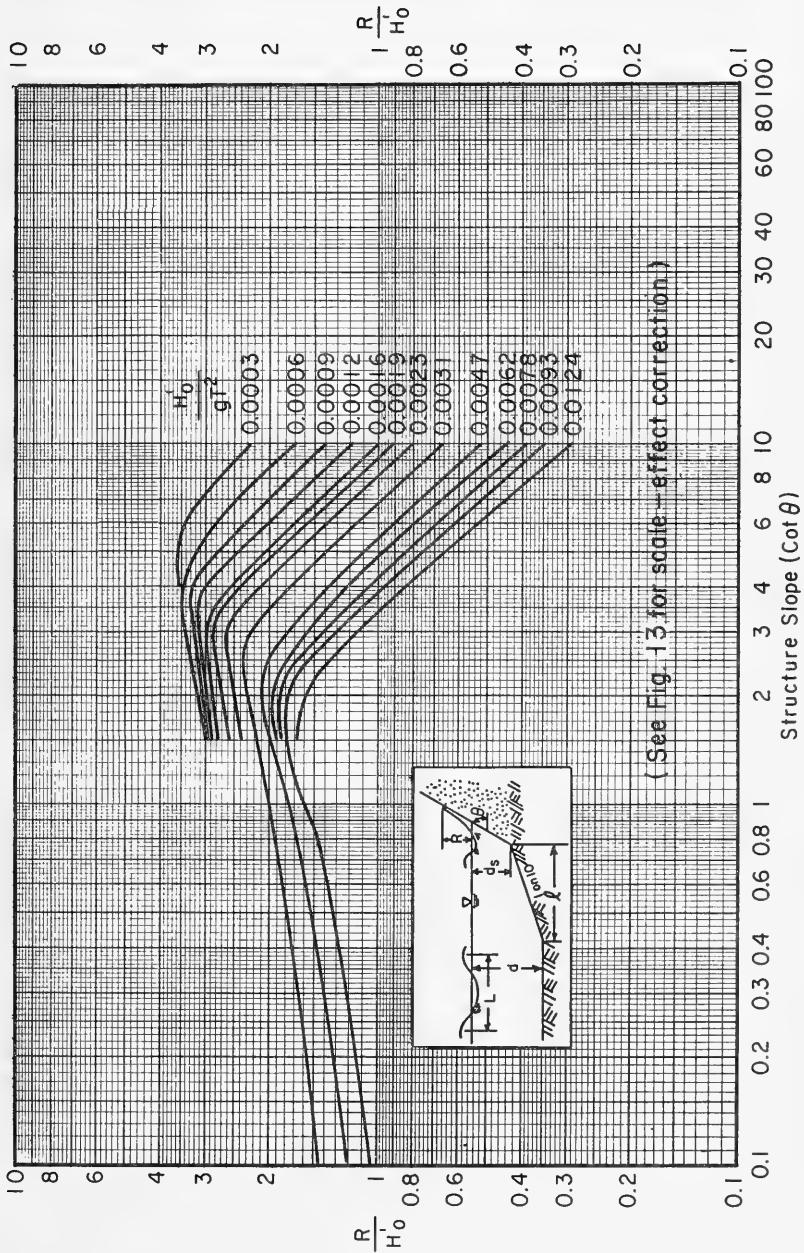


Figure 9. Relative runup for smooth slopes on 1 on 10 bottom;  $\ell/L \geq 0.5$ ;  $d_g/H_B' = 1.0$  (Stora, 1978).



(See Fig. 13 for scale-effect correction)

Figure 10. Relative runup for smooth slopes on 1 on 10 bottom;  
 $\ell/L \geq 0.5$ ;  $d_g/H_0^1 = 1.5$  (Stoa, 1978).



{ See Fig. 13 for scale effect correction }

Figure 11. Relative runup for smooth slopes on 1 on 10 bottom;  
 $\ell/L \geq 0.5$ ;  $d_s/H_0^1 = 3.0$  (Stora, 1978).

4) should be used as upper bounds of relative runup on structures fronted by a 1 on 10 slope with the same  $d_s/H_O'$  value. In the case of  $\ell/L < 0.5$  with low values of  $d_s/H_O'$  (e.g., 0.6, 1, etc.), it should be expected that relative runup will be somewhat higher than predicted from the curves (Figs. 8 to 11), and probably not exceeding 15 to 20 percent higher. However, the effect of the length of a 1 on 10 bottom slope diminishes as the structure slope decreases, and effectively ceases to be significant for  $\cot \theta \geq 4$ . These comments are incorporated in a flow chart (Fig. 12) for determining which figure to use to find the runup on a structure fronted by a sloping bottom.

Because there are insufficient data available for cases where bottom slopes are flatter than 1 on 10, it is recommended that the curves given in this report, applicable to structures fronted by 1 on 10 bottom slopes, be used; in most cases, results are expected to give higher estimates of  $R$  (see Fig. 12). For the larger  $d_s/H_O'$  values (e.g.,  $d_s/H_O' > 2.5$ ), relative runup on structures fronted by gentle bottom slopes will be equal to or less than that given in Figures 2, 3, and 4 (horizontal bottom) for the appropriate  $d_s/H_O'$  value. Relative runup on structure slopes flatter than 1 on 4 is largely unaffected by changes in bottom slope. Relative runup on steep structures fronted by a gentle bottom slope will be equal to or less than values given in Figures 8 and 9 but may be slightly higher than those given in Figure 10 ( $d_s/H_O' = 1.5$ ).

### III. MAXIMUM RUNUP

This section discusses the maximum runup from regular waves when a range of conditions is possible. Maximum runup from irregular waves is not discussed, but an approach to estimation of maximum runup from irregular waves is given by Ahrens (1977a). In his method, runup resulting from a significant wave is determined from design curves such as given here, and then runup for the irregular waves is assumed to follow a Rayleigh distribution.

Maximum runup,  $R$ , for a range of regular wave conditions, is not necessarily associated with the maximum relative runup,  $R/H_O'$ . For structures sited on horizontal bottoms, and for a given wave steepness,  $H_o/gT^2$ , both the maximum relative runup and the maximum dimensional runup occur at the minimum value of  $d_s/H_O'$ .

For structures sited on a 1 on 10 sloping bottom, maximum dimensional runup,  $R$ , may or may not be coincident with the maximum relative runup determined for a range of wave conditions. If depth,  $d_s$ , and wave steepness are assumed constant, then maximum relative runup occurs when  $1.0 \leq d_s/H_O' \leq 1.5$ , but maximum dimensional runup,  $R$ , is found when  $d_s/H_O'$  is a minimum (in this report, when  $d_s > 0$ , then  $(d_s/H_O')_{min} = 0.6$ ). In cases where a bottom slope flatter than 1 on 10 is present, for a given wave steepness, the maximum relative runup will occur for somewhat higher  $d_s/H_O'$  values ( $1.5 \leq d_s/H_O' \leq 2.0$ ). However, if wave height,  $H_O'$ , and wave steepness are held constant, the maximum dimensional runup,  $R$ , will be coincident with maximum relative runup as  $d_s/H_O'$  varies (i.e., as

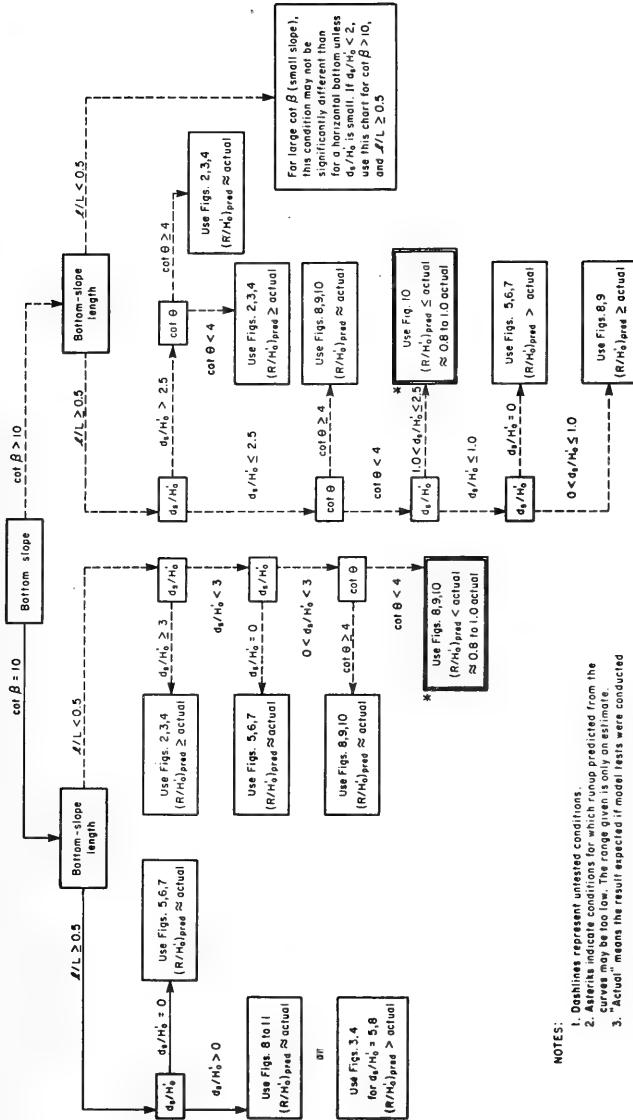


Figure 12. Flow chart for the evaluation of wave runup.

$d_s$  changes). The maximums ( $R/H_0^t$  and  $R$ ) may occur at any value of  $d_s/H_0^t$  (including  $d_s/H_0^t = 0$ ) depending on the wave steepness being considered. Runup maximums would occur at intermediate values of  $d_s/H_0^t$  ( $1.0 \leq d_s/H_0^t \leq 1.5$ ) for high values of  $H_0^t/gT^2$ , but at low values of  $d_s/H_0^t$  for low values of  $H_0^t/gT^2$ .

For a given wave period and constant depth,  $d_s$  (with wave steepness varying as  $d_s/H_0^l$  varies), maximum dimensional runup is generally not coincident with maximum relative runup; furthermore, the maximum dimensional runup may occur at other than the minimum  $d_s/H_0^l$  value.

The designer of a structure subject to runup will usually have a range of wave conditions for which maximum runup must be determined. The preceding discussion emphasizes the need to determine the maximum actual runup by finding the runup for each of several wave conditions. Example problem 3 (Sec. V) highlights some of the relationships discussed here and shows the maximum runup values for different sets of initial wave conditions.

#### IV. SMOOTH-SLOPE SCALE-EFFECT CORRECTION

The smooth-slope runup curves plotted in Figures 2 to 11 are based on small-scale wave-flume tests. A limited number of large-scale tests (Saville, 1958) indicated scale effects were present in the runup results. Figure 13 presents values of the correction factor,  $k$ , as a function of structure slope; the curve is modified from that given in the SPM, and is extended over steeper slopes.

Selection of a particular structure slope may be dependent on evaluation of runup on different slopes. The trends in runup on different structure slopes are presumed correct as given by the design curves (Figs. 2 to 11). Comparisons of runup for different structure slopes should be based on the design curves, with the scale-effect correction applied only to the final selected runup value.

## V. EXAMPLE PROBLEMS

The following example problem solutions use Tables C-1 or C-2 in the SPM and the applicable curves in this report.

GIVEN: An impermeable structure has a smooth slope of 1 on 3 and is subjected to a design wave,  $H = 8$  feet (2.4 meters), measured at a gage located in a depth,  $d = 30$  feet (9.1 meters). Design wave period is  $T = 8$  seconds. The structure is fronted by a 1 on 90 bottom slope extending seaward of the point of wave measurement. Design depth at structure toe is  $d_s = 25$  feet (7.6 meters). (Assume no wave refraction between the wave gage and structure.)

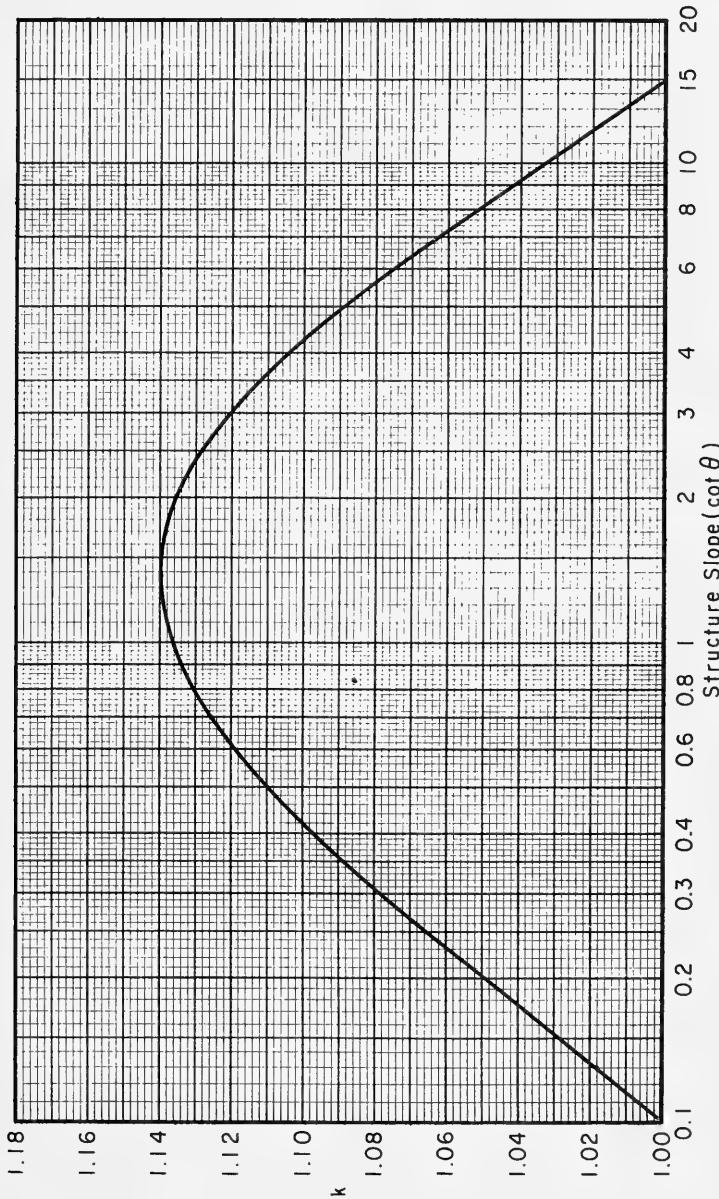


Figure 13. Runup scale-effect correction factor,  $k$  (Stoa, 1978).

FIND: The height above SWL to which the structure must be built to prevent overtopping by the design wave.

SOLUTION: The wave height must be converted to a deepwater value. Using the depth where wave height was measured, calculate

$$\frac{d}{L_o} = \frac{d}{gT^2/2\pi} = \frac{d}{(32.2/2\pi)T^2} = \frac{30}{5.12(8)^2}$$

$$\frac{d}{L_o} = 0.09155 .$$

To determine the shoaling coefficient,  $H/H'_o$ , Table C-1 in the SPM is used, and

$$\frac{H}{H'_o} \approx 0.9406 .$$

Therefore,

$$H'_o = \frac{H}{0.9406} = \frac{8}{0.9406}$$

$$H'_o = 8.5 \text{ feet } (2.6 \text{ meters}) .$$

Calculate, also,

$$\frac{H'_o}{gT^2} = \frac{8.5}{32.2(8)^2} = 0.00412$$

and, for  $d_s = 25$  feet

$$\frac{d_s}{H'_o} = \frac{25}{8.5} = 2.94 .$$

The bottom slope is very gentle (1 on 90). Assuming that the slope approximates a horizontal bottom, the appropriate set of curves for  $d_s/H'_o = 2.9$  is Figure 2 (for  $d_s/H'_o = 3$ ). For a 1 on

### 3 slope and

$$\frac{H_O^!}{gT^2} = 0.00412, \quad \frac{R}{H_O^!} = 2.1 .$$

The runup, uncorrected for scale effects, is

$$R = 17.9 \text{ feet (5.5 meters)}$$

The scale-correction factor,  $k$ , can be determined from Figure 13, and, for  $\cot \theta = 3$ , the correction factor is  $k = 1.12$ .

Thus, the corrected runup is

$$R = (1.12) (17.9) = 20.0 \text{ feet (6.1 meters)}$$

\* \* \* \* \* \* \* \* \* \* \* \* \* \* \* \* \* EXAMPLE PROBLEM 2 \* \* \* \* \* \* \* \* \* \* \* \* \* \* \* \* \*

GIVEN: An impermeable, smooth, 1 on 2 structure is fronted by a 1 on 10 bottom slope. Toe depth for the structure is  $d_s = 10$  feet (3 meters), but the bottom slope extends seaward to a depth of 50 feet (15.2 meters), beyond which the slope is approximately 1 on 100. The design wave approaches normal to the structure and has a height of  $H = 9$  feet (2.7 meters) and period of  $T = 9$  seconds, measured at a depth of 55 feet (16.8 meters).

FIND: The height of wave runup using the appropriate set of curves.

SOLUTION: The wave height given is not the deepwater wave height; it is measured, however, above the gentle 1 on 100 bottom slope which approximates a horizontal surface. To determine the shoaling coefficient,  $K_s$ , for the location of measurement, calculate

$$\frac{d}{L_O} = \frac{d}{gT^2/2\pi}$$

$$= \frac{55}{(5.12)(9)^2}$$

$$\frac{d}{L_0} = 0.1326 .$$

From Table C-1 in the SPM,

$$K_S = \frac{H}{H_O^t} \approx 0.9162 .$$

Therefore,

$$H_O^t = \frac{H}{K_S} = \frac{9}{0.9162} = 9.82 \text{ feet (3.0 meters)}$$

$$\frac{d_S}{H_O^t} = \frac{10}{9.82} = 1.018 \approx 1.0 ,$$

and

$$\frac{H_O^t}{gT^2} = \frac{9.82}{(32.2)(9)^2} = 0.00377 .$$

Because there is a steeply sloping bottom fronting the structure, the value of  $\ell/L$  must be determined:

$$\ell = (50 - 10)(10) = 400 \text{ feet (122 meters)} .$$

Next determine the wavelength in water depth of 50 feet (the depth at the toe of the 1 on 10 slope). For

$$\frac{d}{L_O} = \frac{50}{(32.2/2\pi)(9)^2} = 0.12045 ,$$

and from Table C-1,

$$\frac{d}{L} \approx 0.1585 .$$

Therefore,

$$L = \frac{d}{d/L} = \frac{50}{0.1585} = 315.46 \text{ feet (96.1 meters)} .$$

Then,

$$\frac{\ell}{L} = \frac{400}{315.46} = 1.27 ;$$

thus,

$$\frac{\ell}{L} > 0.5 .$$

The appropriate set of design curves is then determined; the flow chart in Figure 12 shows that Figure 9 has the appropriate curves, and that the results are presumed correct at model scales. From Figure 9, for  $H'_o/gT^2 = 0.0038$ ,

$$\frac{R}{H'_o} \approx 3.0 .$$

The runup is

$$R = \left( \frac{R}{H'_o} \right) (H'_o) = (3.0)(9.82)$$

$$R = 29.5 \text{ feet (9.0 meters)} .$$

For  $\cot \theta = 2$ , the scale-correction factor, from Figure 13, is

$$k = 1.136 .$$

Thus, the corrected runup is

$$R = (1.136)(29.5) = 33.5 \text{ feet (10.2 meters)} .$$

\* \* \* \* \* \* \* \* \* \* \* \* \* \* \* \* EXAMPLE PROBLEM 3 \* \* \* \* \* \* \* \* \* \* \*

GIVEN: A design geometrically similar to that in example problem 2, where an impermeable, smooth, 1 on 2 structure is fronted by a 1 on 10 bottom slope. Toe depth for the structure is  $d_s = 10$  feet, but the bottom slope extends seaward to a depth of 50 feet beyond which

the slope is approximately 1 on 100. However, a range of wave periods and deepwater wave heights are known;  $H_o' \leq 16$  feet (4.9 meters).

FIND: Maximum runup for three different wave conditions:  $T_{max} = 7$  seconds;  $T_{max} = 13$  seconds; and constant wave steepness,  $H_o'/gT^2 = 0.0101$ , with  $T_{max} = 7$  seconds.

SOLUTION: For any given  $d_s/H_o'$  value, the design curves show that relative runup is highest for the longest wave period (or the lowest wave steepness,  $H_o'/gT^2$ ). However, for constant toe depth,  $d_s$ , and for constant wave steepness, the largest wave height (or lowest  $d_s/H_o'$  value) usually results in the largest absolute runup,  $R$ . When a sloping bottom is present, and wave period and toe depth ( $d_s$ ) are held constant, the maximum runup may occur at other than the minimum  $d_s/H_o'$  value. Thus, runup for a range of  $d_s/H_o'$  values should be investigated.

In the following development, *preliminary* determinations of runup are not corrected for scale effect. Only the final runup, as determined for selected wave conditions and structure slope, is corrected.

(a) The maximum wave height given is  $H_o' = 16$  feet; for this location, the resultant  $d_s/H_o'$  value is

$$\frac{d_s}{H_o'} = \frac{10}{16} = 0.63 ,$$

which is approximately the lowest value used in Figures 8 to 11. The maximum runup may be determined by constructing a table for varying conditions. Because the maximum wave period is less here than in example problem 2,  $L$  is also less; thus,  $\ell/L > 0.5$  and Figures 8 to 11 may be used. Furthermore, Figure 12 indicates that the results in Figures 8 to 11 are approximately correct, to model scale. For  $d_s = 10$  feet,  $T = 7$  seconds, and  $gT^2 = 1,577.8$  feet. Table 1 may be constructed with  $T$  held constant at 7 seconds because the maximum wave period results in the highest relative runup for each value of  $d_s/H_o'$ . The maximum runup of 23.5 feet (7.2 meters) in Table 1 does not occur for the largest wave height because the largest waves break seaward of the structure for the given wave period.

Table 1. Example runup for  $T = 7$  seconds,  
constant depth, and  $(H_O^!)_{max} = 16$  feet.

Fig.	$d_s/H_O^!{^1}$	$H_O^!$ (ft)	$H_O^!/gT^2$	$R/H_O^!$	R (ft)
8	$\approx 0.6$	16.00	0.01014	1.38	22.10 <sup>2</sup>
9	1.0	10.00	0.00634	2.35	23.50
10	1.5	6.67	0.00423	$\approx 2.80$	18.70
11	3.0	3.33	0.00211	2.60	8.66

<sup>1</sup> $d_s/H_O^!$  values selected to correspond with values in figures;  $d_s = 10$  feet.

<sup>2</sup> $R_{max} = 23.5$  feet.

(b) For the second condition where  $T_{max} = 13$  seconds, the maximum runup would occur for the lowest  $d_s/H_O^!$  value. To check  $\ell/L$ , for  $d = 50$  feet:

$$\frac{d}{L_O} = \frac{50}{(32.2/2\pi)(13)^2} = 0.0577$$

$$\frac{d}{L} = 0.1020$$

$$L = 490.2 \text{ feet} ;$$

$$\frac{\ell}{L} = \frac{400}{490.2} = 0.82 > 0.5 .$$

Table 2 may be constructed for  $d_s = 10$  feet,  $T = 13$  seconds,  $gT^2 = 5,441.8$  feet, and using Figures 8 to 11. Table 2 shows that, in this case, not only is runup higher for the longer wave period than in Table 1, but the maximum runup occurs at a lower  $d_s/H_O^!$  value for the maximum deepwater wave.

Table 2. Example runup for  $T = 13$  seconds,  
constant depth, and  $(H_O^!)_{max} = 16$  feet.

Fig.	$d_s/H_O^!{^1}$	$H_O^!$ (ft)	$H_O^!/gT^2$	$R/H_O^!$	R (ft)
8	$\approx 0.6$	16.00	0.002940	2.60	41.6 <sup>2</sup>
9	1.0	10.00	0.001840	3.80	38.0
10	1.5	6.67	0.001230	3.90	26.0
11	3.0	3.33	0.000612	3.15	10.5

<sup>1</sup> $d_s = 10$  feet.

<sup>2</sup> $R_{max} = 41.6$  feet.

(c) For the third condition, suppose that wave steepness is expected to be most important, and that the structure is being designed for a constant wave steepness of  $H_o'/gT^2 = 0.0101$  and a maximum period of 7 seconds.

Table 3 shows the characteristic relationship that the largest runup,  $R$ , occurs for the lowest  $d_s/H_o'$  value when  $H_o'/gT^2$  and  $d_s$  are constant; the largest relative runup has lower dimensional runup. However, Table 3 does not indicate the maximum runup to be expected on this structure for the given conditions; Table 1 shows the maximum (uncorrected for scale effects) to be 23.5 feet when a maximum period of 7 seconds is given. Thus, care should be exercised in determining runup for a particular structure. The results of the three parts of this problem are summarized in Table 4, and the calculated values are corrected for scale effect based on Figure 13.

\* \* \* \* \* \* \* \* \* \* \* \* \* \* \* EXAMPLE PROBLEM 4 \* \* \* \* \* \* \* \* \* \* \*

GIVEN: An impermeable structure has a smooth slope of 1 on 1.5 and is subjected to a design wave,  $H_o' = 5$  feet (1.5 meters). Design wave period is  $T = 6$  seconds. The design water depth at the toe of the structure is  $d_s = 0.0$  foot. The bottom has a 1 on 10 slope from the structure toe to a depth,  $d = 15$  feet (4.6 meters), at which point the bottom slope changes to 1 on 200.

FIND: Determine runup on the structure caused by a wave train approaching normally.

SOLUTION: The toe depth is zero, and the bottom slope is 1 on 10; assuming that the more seaward 1 on 200 bottom slope approximates a horizontal bottom, Figures 5, 6, and 7 are applicable, subject to the value of  $d/H_o'$ .

$$\frac{d}{H_o'} = \frac{15}{5} = 3$$

Therefore, Figure 5 is applicable;

$$\frac{H_o'}{gT^2} = \frac{5}{(32.2)(6)^2} = 0.0043$$

The relative runup for a 1 on 1.5 structure slope is determined by interpolation to be

Table 3. Example runup for constant wave steepness,  $H_o'/gT^2 = 0.0101$ .

Fig.	$H_o'/gT^2$	$d_s/H_o'$	$H_o'$ (ft)	$T^2$ (s)	$R/H_o'$	$R$ (ft)
8	0.0101	$\approx 0.6$	16.00	7.0	1.38	22.1 <sup>4</sup>
9	0.0101	1.0	10.00	5.5	1.88	18.8
10	0.0101	1.5	6.67	4.5	1.75	11.7
11	0.0101	3.0	3.33	3.2	1.73	5.8

<sup>1</sup> $d_s = 10$  feet.

<sup>2</sup> $T_{max} = 7$  seconds.

<sup>3</sup> $\cot \theta = 2$ .

<sup>4</sup> $R_{max} = 22.1$  feet.

Table 4. Summary of maximum runup for different conditions.

Table	Wave condition	Maximum <sup>1</sup>	Scale-effect correction	Maximum
		R (ft)	k	R (ft)
1	Constant period; $T = 7$ seconds	23.5	1.136	26.7
2	Constant period; $T = 13$ seconds	41.6	1.136	47.3
3	Constant steepness: $H_o'/gT^2 = 0.0101$ ; $T_{max} = 7$ seconds	22.1	1.136	25.1

<sup>1</sup>Uncorrected for scale effect.

$$\frac{R}{H_O^t} \approx 1.23 .$$

Therefore,

$$R = (1.23)(5)$$

$$R = 6.15 \text{ feet } (1.87 \text{ meters})$$

The scale-correction factor,  $k$ , from Figure 13, is

$$k = 1.14 .$$

The corrected runup is

$$R = (1.14)(6.15) = 7.0 \text{ feet } (2.1 \text{ meters}) .$$

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627



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