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ABSTRACT

The pricing of stock index futures is examined by combining a multiperiod asset pricing model with a familiar futures pricing relationship in the literature. By adding a stationary stochastic process for changes in stock prices and a marginal utility of wealth variable, we derive several empirically testable results in addition to showing that the changes in the corresponding futures prices are nearly white noise. Specifically, we find that the futures price changes should have means and variances that depend on the time to maturity. Using price changes on three popular stock index futures, we find evidence of risk premia and variances that change as the contracts approach maturity.

RISK PREMIA AND THE VARIATION OF STOCK INDEX FUTURES

In this paper, we examine the risk premia and the variability of prices on the three major stock index futures which began trading in 1982. There are several models in the literature which have been used to study futures prices. The most familiar model is the martingale model described in Samuelson (1965). In this model, the futures price is the market's expectation of the spot price at maturity and the futures price itself is a martingale. Recent papers by Cornell and French (1983) and Modest and Sundaresan (1983) have examined stock index futures by assuming that futures contracts are identical to forward contracts and then applying an arbitrage relationship in which the arbitrage occurs by an investor simultaneously buying (or selling short) the portfolio of stocks in an index and selling (buying) a futures contract on the corresponding index. This approach is essentially the cost of carry model. In this paper, we use a recent arbitrage-based model in which the arbitrage occurs by an investor simultaneously taking positions in the futures contract and borrowing or lending at risk free interest rates. This model explicitly accounts for the daily settlement feature of futures contracts and permits a difference between futures prices and corresponding forward prices. In addition this model does not require that investors have access to an asset whose value follows the spot price. The cost of carry model has this requirement and it can be applied to futures on the Standard & Poors Index of 500 Common Stocks and the New York Stock Exchange

Composite, but it cannot be applied directly to the Value Line index futures because of the difficulties and expenses of creating a portfolio that follows an index which is a geometric average. The model used in this paper is not based on arbitrage that involves buying or selling the spot index or a portfolio of stocks, and it can be applied to all three indices.

Much of the recent literature on pricing futures and forward contracts has been based on models which incorporate stochastic interest rates; these models naturally incorporate some notion of risk aversion and a stochastic investment opportunity set. In these models of futures prices, we no longer have a martingale result, even though the martingale model seems to be a good empirical approximation for some markets. Here, we examine how stock index futures might deviate from the martingale model and what kind of risk premia might be imbedded in the prices. We also examine the issue of how the price variation might change as the contracts approach maturity.[1] In the theoretical section of the paper, we get a result which is close to the martingale model, but contains some subtle differences. We then apply the model to stock index futures and empirically examine the deviations from the martingale model. We find evidence of time-varying risk premia and variances in our samples of weekly changes in the logarithm of prices on stock index futures. The empirical results are presented in Section II. Some additional results on the risk premia imbedded in the S&P 500 are presented in Section III.

I. A Model of Futures Prices

In this section, the behavior of prices on stock index futures is examined within the context of a model with risk aversion and stochastic interest rates. A discrete-time intertemporal asset pricing model is combined with Proposition 2 in Cox, Ingersol, and Ross (1981), hereafter CIR, to develop an equilibrium relationship for futures prices which is then applied to stock index futures. This futures pricing equation is similar to several that have been derived in the literature, and we extend the model to derive some empirically testable implications for stock index futures. Essentially, we use the asset pricing model to value the cashflow in the arbitrage relation of Proposition 2. Using an arbitrage argument, CIR show that a futures price, at time t for a contract that matures at t+s, is equal to the value of the following cashflow at maturity:

$$P_{t+s}\left[\prod_{j=0}^{s-1}(1+R_{t+j})\right], \qquad (1)$$

where P_{t+s} is the price at maturity of the good or asset on which the contract is written and R_t is the interest rate from t to t+1. The one-period interest rate enters because the arbitrage argument uses borrowing and lending at the one-period (one-day) rate to handle the cashflows that arise because of daily settlement.

The next step is to value this cashflow. CIR, in their Section 4, examine futures prices and forward prices in a continuous-time, continuous-state model, and they suggest in their equation (47) the approach that is followed here. In a model in which agents solve a multiperiod optimization problem, Lucas (1978) has derived the following relatioship for asset prices:

$$p_{it} = E_t\left[\frac{\beta\, J'(w_{t+1})}{J'(w_t)} (p_{i,t+1} + x_{i,t+1})\right], \tag{2}$$

where $J'(\cdot)$ is the marginal utility of real wealth, p_{it} is the real price of asset i at the end of period t, and x_{it} is the real cashflow or dividend received for holding asset i during t. E_t is the conditional expectation operator and the conditioning set is information available at period t.[2] Equation (2) can be solved recursively to produce the following relationship:

$$p_{it} = \sum_{j=1}^{\infty} \beta^j E_t\left[\frac{J'(w_{t+j})}{J'(w_t)} x_{i,t+j}\right]. \tag{3}$$

One can easily verify that (3) is a solution to the difference equation in (2). This valuation model simply states that the value of an asset is equal to the expected value of its future cashflows weighted by the corresponding marginal utility of wealth. This relationship can be applied to any asset and can be applied to value a single cashflow as in (1).

In this model, all the relevant variables are denominated in consumption units because individuals are optimizing the utility of real consumption; hence all variables are real quantities as opposed to nominal or dollar quantities. To convert to a valuation model in nominal terms, we first define the nominal cashflows and prices as follows:

$$x_{it} \equiv D_t X_{it}$$

$$p_{it} \equiv D_t P_{it},$$

where X_{it} and P_{it} are the nominal dividends and asset prices, respectively, and D_t is the consumption price deflator (or the reciprocal of the consumption price index). We then define a new variable, $\lambda_t \equiv D_t J'(w_t)$, which is the product of the marginal utility of real wealth and the consumption price deflator, and substitute these expressions into equation (3) to get an asset pricing model in nominal terms:[3]

$$P_{it} = \sum_{j=1}^{\infty} \beta^j E_t[\frac{\lambda_{t+j}}{\lambda_t} X_{i,t+j}]. \tag{4}$$

The model in (4) can now be used to value nominal cashflows, and we use it to value the single cashflow in (1) to get

$$H_t(s) = E_t[\frac{\beta^s \lambda_{t+s}}{\lambda_t} P_{t+s}(\prod_{j=0}^{s-1} (1+R_{t+j}))], \tag{5}$$

where $H_t(s)$ is the futures price at time t for a contract that matures at (t+s). Equation (5) is not new; CIR, Richard and Sundaresan (1981), and French (1983) have derived similar pricing relationships. If P_t represents the value for a portfolio of stocks or a stock index without dividends, then $H_t(s)$ is the price for the corresponding stock index futures. We can also use the asset pricing model in nominal terms to derive an equilibrium relationship for nominal risk-free interest rates including the one-period rates R_t. Let $B_t(t+k)$ be the price of a default-free discount bond that matures at time (t+k) paying \$1, then

$$B_t(t+k) = E_t\left(\frac{\beta^k \lambda_{t+k}}{\lambda_t}\right).$$

For one-period nominal interest rates, we have

$$B_t(t+1) = \frac{1}{1+R_t} = E_t\left(\frac{\beta \lambda_{t+1}}{\lambda_t}\right),$$

and this relationship is used in the analysis of (5).

This equilibrium pricing relationship for stock index futures is not very useful in its present form. From equation (5), one can explore the conditions for the futures price to be above or below the expected value of the future level of the index, but empirically testable implications are difficult to derive. To derive some testable implications, we add the assumption that log stock price changes, $\Delta \ln P_t$, and changes in the marginal utility of wealth variable, $\Delta \ln \lambda_t$, are part of a stationary multiple time series representation with normally distributed innovations:[4]

$$\Delta \ln P_t = \overline{P} + \sum_{j=0}^{\infty} \underline{b}_j' \, \underline{\varepsilon}_{t-j}$$

$$\Delta \ln \lambda_t = \overline{\lambda} + \sum_{j=0}^{\infty} \underline{a}_j' \, \underline{\varepsilon}_{t-j}.$$

The innovations have mean zero and a covariance matrix $\Omega = E(\underline{\varepsilon}_t \underline{\varepsilon}_t')$. From equation (5), we evaluate the following moment generating function:

$$\frac{H_t(s)}{P_t} = E_t[\exp\{(\ln P_{t+s} - \ln P_t) + \sum_{j=0}^{s-1} \ln(1+R_{t+j}) + s \ln \beta + (\ln \lambda_{t+s} - \ln \lambda_t)\}]. \qquad (6)$$

Noting that the one-period interest rates are related to the conditional expectations of changes in the marginal utility of wealth variable, we make the following substitutions:

$$\sum_{j=0}^{s-1} \ln(1+R_{t+j}) = -\sum_{j=0}^{s-1} \ln\{E_{t+j}[\frac{\beta\lambda_{t+j+1}}{\lambda_{t+j}}]\}$$

$$= -\sum_{j=0}^{s-1} \ln\{E_{t+j}[\exp\{\ln\beta + \Delta\ln\lambda_{t+j+1}\}]\}$$

$$= -[s\ln\beta + s\bar{\lambda} + \frac{s}{2}\,\underline{a}_0'\Omega\underline{a}_0 + \sum_{j=1}^{s}\sum_{k=1}^{\infty}\underline{a}_k'\underline{\varepsilon}_{t+j-k}],$$

and

$$\ln\lambda_{t+s} - \ln\lambda_t = \sum_{j=1}^{s}\Delta\ln\lambda_{t+j} = s\bar{\lambda} + \sum_{j=1}^{s}\sum_{k=0}^{\infty}\underline{a}_k'\underline{\varepsilon}_{t+j-k}$$

Combining these two expressions, we get

$$s\ln\beta + (\ln\lambda_{t+s} - \ln\lambda_t) + \sum_{j=0}^{s-1}\ln(1+R_{t+j}) = -\frac{s}{2}\,\underline{a}_0'\Omega\underline{a}_0 + \sum_{j=1}^{s}\underline{a}_0'\underline{\varepsilon}_{t+j}.$$

This expression is substituted into (6) and we get

$$\ln(\frac{H_t(s)}{P_t}) = E_t(\ln P_{t+s} - \ln P_t) + \frac{1}{2}\,\mathrm{Var}_t(\ln P_{t+s} - \ln P_t)$$
$$+ \mathrm{Cov}_t[(\ln P_{t+s} - \ln P_t),\ \sum_{j=1}^{s}\underline{a}_0'\underline{\varepsilon}_{t+j}], \qquad (7)$$

where Var_t and Cov_t are the conditional variances and covariances, respectively. The covariance term in (7) can be written as a function of the parameters of the multiple time series for $\Delta\ln P_t$ and $\Delta\ln\lambda_t$:

$$\mathrm{Cov}_t[(\ln P_{t+s} - \ln P_t),\ \sum_{j=1}^{s}\underline{a}_0'\underline{\varepsilon}_{t+j}] = (\sum_{j=0}^{s-1}(s-j)\underline{b}_j)'\Omega\underline{a}_0.$$

In words, this term is the conditional covariance between the change in the log price and the sum of the one-step ahead forecast errors for the marginal utility of wealth variable. This covariance term determines the risk premium in the futures price. If the covariance is negative, then we define the risk premium to be positive, and the futures price is less than the market's expectation of the spot price at maturity (backwardation). If the covariance is positive, the risk premium is negative and the futures price is greater than the expected spot price (contango).[5] If $\underline{a}_0 = \underline{0}$, this conditional covariance is zero and we get the martingale result for futures prices.

To study the variation of futures prices, we examine the change in the log of the futures price:

$$\begin{aligned}\Delta \ln H_t(s) \equiv \ln H_t(s) - \ln H_{t-1}(s+1) &= E_t(\ln P_{t+s}) - E_{t-1}(\ln P_{t+s}) \\ &+ \frac{1}{2}\,\mathrm{Var}_t(\ln P_{t+s} - \ln P_t) - \frac{1}{2}\,\mathrm{Var}_t(\ln P_{t+s} - \ln P_{t-1}) \\ &+ \mathrm{Cov}[(\ln P_{t+s} - \ln P_t),\ \sum_{j=1}^{s} \underline{a}_0'\underline{\varepsilon}_{t+j}] \\ &- \mathrm{Cov}[(\ln P_{t+s} - \ln P_{t-1}),\ \sum_{j=1}^{s+1} \underline{a}_0'\underline{\varepsilon}_{t-1+j}].\end{aligned}$$

The expression for $E_t(\ln P_{t+s}) - E_{t-1}(\ln P_{t+s})$ is evaluated by applying the rules for revising forecasts for a fixed future period found in Nerlove, Grether, and Carvalho (1979, p. 88). The conditional variances and covariances are separately evaluated and we arrive at the following equation:

$$\Delta \ln H_t(s) = (\sum_{j=0}^{s} \underline{b}_j)'\underline{\varepsilon}_t - \frac{1}{2}(\sum_{j=0}^{s} \underline{b}_j)'\Omega(\sum_{j=0}^{s} \underline{b}_j) - (\sum_{j=0}^{s} \underline{b}_j)'\Omega\underline{a}_0. \quad (8)$$

The first term in equation (8) is a linear combination of the innovations for the current period, hence this term is a random variable which is independent of the past. The last two terms are not random as they are functions of the parameters of the multiple time series, but these terms can change as we approach maturity (as s decreases). The series $\Delta \ell n \dot{H}_t(s)$ will resemble a serially uncorrelated process if the changes in the last two terms are small relative to the variation in the first term. This is precisely the case one would anticipate for stock index futures. Because stock prices experience much variation and resemble random walks, it is reasonable to conjecture that the coefficients $\underline{b}_1$, $\underline{b}_2$, ... are small in absolute value relative to the coefficients in $\underline{b}_0$. But s in the summations decreases as we approach maturity and over time we can have variations in the last two terms of (8) and changes in the variance of the random term. The possible changes in the last two terms, however, would be relatively small, and the variation in $\Delta \ell n H_t$ would be dominated by the random variation of the first term; hence, the price changes should be close to white noise. LeRoy (1982) has noted that in models with risk aversion the martingale property does not generally hold for futures prices, but in this model with risk aversion and stochastic interest rates, the prices on stock index futures are near martingales. This observation suggests that an empirical researcher may not be able to detect any serial correlation in the price changes, even though the futures prices do not exactly satisfy the martingale property. The last term in equation (8) represents the change in the risk premium and we refer to it as the mean parameter in the futures price change. If $\underline{a}_0 = \underline{0}$, the futures price is a martingale and the mean parameter is zero for all maturities. If there are risk premia in the futures price, then the mean parameter will be nonzero and may even change as we approach maturity.

Samuelson (1965) has argued that the variation of futures prices will change as the contract approaches maturity; in fact, he argued that the volatility should increase as the contract approaches maturity, which at first seems counterintuitive. Rutledge (1976), however, has shown that the variance of futures prices will remain constant if the spot price follows a random walk; Samuelson's result applies when the process on the spot price is stationary. In our model, the stock price is a non-stationary process and the variance of the futures price and the mean parameter are constant if stock prices follow a random walk. If stock prices are not a random walk, then the variance will change as we approach maturity, but we cannot predict the direction of the change without further information.

From the model in equation (8), we have several hypotheses that can be examined empirically by using actual log price changes on stock index futures: (1) there may be risk premia and nonzero mean parameters which may vary, (2) the volatility or variance of price changes may vary as we approach maturity, and (3) a time series, $\Delta \ell n H_t$, constructed from prices on near contracts may have some periodicity due to the dependence of the mean parameter and the variance on time to maturity. In the next section, we present empirical evidence on these hypotheses by studying the behavior of prices on the Standard & Poor's 500 futures, the New York Stock Exchange Composite futures, and the Kansas City Value Line futures, hereafter, the S&P 500, the NYSC, and the KCVL, respectively. These three index futures are studied because they are the most actively traded stock index futures, and they have a history of prices dating back to 1982.

II. Empirical Analysis

Now we turn to the empirical implications of the model for stock index futures. Specifically, we explore whether there are nonzero mean parameters and whether the mean parameters and the variances change as we approach maturity. The standard model for analyzing futures prices is one in which price changes are independent of past price changes and the variance is constant. In many studies, the possibility of a changing variance is ignored, but this issue has been examined by Rutledge and others for commodity futures.[6] First, we simplify the model for $\Delta \ell n H_t(s)$ because we examine data on stock index futures only and we do not attempt the difficult task of formulating a multiple time series model. Equation (8) in Section I implies a model of the following form for univariate time series analysis:

$$\Delta \ell n H_t(s) = \sigma(s)\, e_t - \frac{1}{2} \sigma^2_{(s)} + \mu(s),$$

where e_t is a standard normal random variable and is serially independent. $\mu(s)$ and $\sigma^2_{(s)}$ are the mean and variance parameters which depend on time to maturity. We examine four hypotheses: (1) changing means and changing variances, (2) a zero mean and changing variances, (3) a constant mean and a constant variance, and (4) a zero mean and a constant variance. One implication of the model is that the mean parameter must be constant if the variance is constant.

The model is applied to weekly changes in the log of prices for the S&P 500, the NYSC, and the KCVL futures. Thursday settlement prices are used to measure prices.[7] Weekly price changes, instead of daily price changes, are studied so that we can avoid the weekend and day-of-the-week effects which have been found in stock returns. Because most of the activity (open interest and trading volume) has been in the near contracts, we focus the analysis on the near contracts only. For each of the three futures, we have constructed a time series of log price changes on the near contract. We start with the log price change on the nearest contract and follow it to one week before maturity, then for the subsequent week we pick up the log price change on the next contract which has thirteen weeks to maturity during the sample period. The series run from the beginning of trading in 1982 up to the first week of March 1985. The series for the S&P 500 and the NYSC have 147 and 148 observations, respectively, and the series for the KCVL has 158 observations. The series contain 13 different times to maturity so that hypothesis (1) has 13 means and 13 variances, or 26 parameters to estimate. The three remaining hypotheses impose restrictions on the means and variances and are therefore testable.

The parameters are estimated by the method of maximum likelihood and the likelihood ratio statistic, $-2\ell n\theta$, is used to test the various restrictions. Using the assumption that the innovations are normally distributed, we can write the log-likelihood function for the most general model (hypothesis 1) as follows:

$$\ell nL = - \sum_{s=1}^{13} \{\frac{T_s}{2} \ell n\sigma^2_{(s)} + \frac{1}{2\sigma^2_{(s)}} \sum_{t-1}^{T_s} (y_{st} - \mu(s) + \frac{1}{2}\sigma^2_{(s)})^2\},$$

where we have omitted the proportionality constant $-\frac{T}{2}\ell n(2\pi)$. T_s represents the number of observations for a given time to maturity, y_{st} represents the observations on $\Delta\ell nH_t(s)$, and T is the summation of

T_s, s=1,...,13. The estimates under hypothesis 1 are presented in Table I. Only a few of the mean parameters are statistically significant, and it is difficult to detect any patterns in the mean or variance estimates. If we exclude the mean parameters for one week to maturity on the NYSC and the KCVL contracts, it appears that the means are smaller in absolute value for the last six weeks before maturity. For all three contracts, the variance estimates suggest that there is first an increase in the variance and then a drop as we approach maturity. If we divide the times to maturity into three groups (1-4 weeks, 5-9 weeks, 10-13 weeks) and estimate variances for each group, we find that the variance estimates are highest for the middle group of 5 to 9 weeks to maturity and F tests of the variance ratios are significant for the NYSE and KCVL futures at the 5% level.

The more interesting results, however, involve the tests of the restrictions, and these results are shown in Tables II, III, and IV. For each series, we conduct five tests using the likelihood ratio statistic. For the S&P 500 and KCVL futures, we reject the hypothesis of a zero mean parameter and a constant variance at standard significance levels.[8] For every test of a restrictive hypothesis (2, 3, or 4) against hypothesis 1, we reject the restrictive hypothesis at standard significance levels for the S&P 500 and the KCVL. These results indicate evidence of risk premia and variances that change with time to maturity. The results for the NYSE are not as strong. At the 10 percent significance level, there is evidence that the variances change with time to maturity, but the tests do not indicate evidence of non-zero mean parameters. The results in Table III for the NYSE favor the martingale model, but there is some weak evidence that the variance changes with time to maturity. It is interesting to note that if we test for a nonzero mean parameter in a model with a constant variance, we do not reject the null hypothesis of a zero mean. Hence a myopic researcher would find support for the martingale model. If we test the hypothesis of a constant variance in a model with no risk premium, we find only marginal evidence against the constant variance model. Both of these restrictions, however, are rejected for the S&P 500 and the KCVL when they are tested against the model of Section I. Finally, we have computed several test statistics to check for serial correlation in the time series, but the detailed report is omitted here. None of these tests indicate evidence of serial correlation, either before or after a correction for the periodic components.

III. Additional Evidence on the Risk Premia

In this section, we present some casual empirical evidence on the risk premium in stock index futures. If the martingale model is a good approximation for stock index futures, then we have a measure for the market's expected level of the index to prevail at maturity of the contract. Given an estimate of the expected dividends on the stocks in the index, we can then compute the expected return for the index. Expected returns on major stock market indices are frequently used as proxies for expected returns on the market portfolio. The issue is whether futures prices for the major stock indices produce realistic numbers for the expected returns on common stock portfolios.

The cost of carry model mentioned in the introduction produces a futures price which is less than the expected spot price if expected returns on the stock indexes exceed risk free interest rates. Given the universal agreement on this latter point, the cost of carry model implies a downward bias in the futures price relative to the expected spot price (or a positive risk premium). By invoking the assumption of risk aversion, we can use the model in equation (7) to argue that the risk premium in stock index futures should be positive. In equation (7), this argument relies on the properties of risk aversion to determine the predicted sign of the conditional covariance between stock price changes and the marginal utility of wealth variable. A more intuitive interpretation proceeds as follows. The portfolio which matches a major stock index represents a significant portion of wealth or at least one of the most volatile components of total wealth. With risk aversion, marginal utility of wealth is a decreasing function of wealth and the covariance between the portfolio return and marginal utility of wealth should be negative. If we are holding this portfolio and we take a short position in an equivalent number of contracts in the stock index futures, we reduce substantially the risk of our portfolio and the magnitude of the negative covariance of our portfolio return with our marginal utility of wealth.[9] For this opportunity, we are willing to pay a premium in the form of accepting a lower expected return on the hedged portfolio. The lower expected return requires the futures price to be less than the expected index level.

We examine this issue by using prices and dividends for the S&P 500 over quarterly intervals which coincide with the maturity of the S&P 500 futures. Standard & Poor's publishes a quarterly dividend series for its index of 500 common stocks, adjusted to the level of the index.[10]

The dividend series for 1980:I to 1984:III is reproduced in Table V. Note the stability of the series. Clearly, dividends over a one-quarter time horizon are quite predictable. Although, the market may not be able to predict perfectly dividends over the next quarter, the forecasts of market participants must be quite accurate, and the range for expected dividends must certainly fall within a narrow band around the actual levels. We have divided the period June 17, 1982, to September 20, 1984, into nine separate thirteen-week periods, each terminating with the expiration of a futures contract on the S&P 500. We use the futures price at the beginning of each period as our measure of the expected level of the index for the end of the period. We then calculate two estimates for the expected return: one using the actual dividends during the quarter as a measure of expected dividends, and a second estimate using the actual dividend plus ten percent. During this period 1980-84, the largest quarterly increase in dividends was only 6.8 percent. The numbers are then compared to the risk-free return measured by the rate of return on holding a thirteen-week Treasury bill over the same period. The numbers are summarized in Table VI.

During the first four periods, estimates for expected returns are less than the corresponding risk-free returns. There were times when the futures prices were less than the current spot prices. During the last five periods, the estimated expected returns were higher than the risk-free rates, but nowhere near the levels that we would consider appropriate for expected returns on the S&P 500. Most estimates of the "market risk premium" in the literature indicate that the expected return on a large portfolio of stocks, on an annual basis, is 8 to 9 percentage points higher than the return on risk-free securities.[11] On a quarterly basis, this difference would be around 2 percentage points. More recent estimates indicate a market risk premium in the range of 5 1/2 to 6 percent annually. Using the 5 1/2 percent number and quarterly compounding, we have computed implied expected returns for the S&P 500 which are shown in column four. All of the estimates using futures prices in columns two and three are substantially less than the estimates in column four. We interpret this casual empirical evidence as implying that there is a positive risk premium in stock index futures and that futures prices must be significantly less than the corresponding expected spot prices for the S&P 500. Otherwise, one must argue that the frequently cited estimates for the market risk premium are much too high.

IV. Summary and Conclusions

We have examined the behavior of price changes on stock index futures within a model with risk aversion and stochastic interest rates, and we find that the log price changes should closely resemble a white noise process, but with subtle deviations resulting from time-varying risk premia and variances. We then test these results by using weekly price changes on stock index futures. The empirical results indicate strong support for the implications of the model and reject the restrictions implied by the martingale model and by models with constant variances. In addition, we present some casual empirical evidence which indicates that there are substantial risk premia in the prices for the S&P 500 futures.

TABLE I

Maximum Likelihood Estimates

S&P 500:

s	$\mu(s)$	$\sigma^2(s)$
13	-.003504	.0006493
12	.018453*	.0002867
11	-.012982*	.0001714
10	.013130	.0008258
9	.015183*	.0003242
8	.004347	.0003102
7	-.004755	.0008848
6	.004037	.0010075
5	-.005395	.0004744
4	-.002884	.0006705
3	.001154	.0007713
2	.007979	.0002264
1	-.006897	.0002078

Log-Likelihood = 494.0411 T = 147

NYSC:

s	$\mu(s)$	$\sigma^2(s)$
13	-.008775	.0001970
12	.015274	.0008868
11	.006679	.0004539
10	.002480	.0002963
9	-.005410	.0008054
8	.000363	.0009666
7	.006138	.0008159
6	-.001611	.0006504
5	.002456	.0009072
4	.005151	.0001930
3	-.004464	.0002119
2	-.003565	.0004817
1	.010946*	.0002636

Log-Likelihood = 493.5052 T = 148

KCVL:

s	$\mu(s)$	$\sigma^2(s)$
13	-.005837	.0002012
12	.009837	.0010459
11	.012220	.0003868
10	.006391	.0003934
9	-.013015	.0004642
8	.009617	.0009541
7	.005706	.0010283
6	-.001546	.0006288
5	-.003309	.0010296
4	.004505	.0002821
3	-.004847	.0002370
2	-.003961	.0005317
1	.015056*	.0002580

Log-Likelihood = 524.1906 T = 158

*Significant at the 5% level

TABLE II

Hypothesis Tests, S&P 500

	Number of Estimated Parameters	Log-Likelihood
H_1: Different means, different variances	26	494.0411
H_2: Zero means premium, different variances	13	478.0932
H_3: One mean, one variance	2	471.4512
H_4: Zero means, one variance	1	470.9099

Test of H_2 vs. H_1

$$-2\ell n\theta = \chi^2(13) = 31.90^*$$

Test of H_3 vs. H_1

$$-2\ell n\theta = \chi^2(24) = 45.18^*$$

Test of H_4 vs. H_1

$$-2\ell n\theta = \chi^2(25) = 46.26^*$$

Test of H_4 vs. H_2

$$-2\ell n\theta = \chi^2(12) = 14.37$$

Test of H_4 vs. H_3

$$-2\ell n\theta = \chi^2(1) = 1.08$$

NOTES: *Significant at 1% level
**Significant at 2.5% level
***Significant at 5% level

TABLE III

Hypothesis Tests, NYSC

	Number of Estimated Parameters	Log-Likelihood
H_1: Different means, different variances	26	493.5052
H_2: Zero means, different variances	13	485.8286
H_3: One mean, one variance	2	475.9087
H_4: Zero means, one variance	1	475.3334

Test of H_2 vs. H_1

$$-2\ln\theta = \chi^2(13) = 15.35$$

Test of H_3 vs. H_1

$$-2\ln\theta = \chi^2(24) = 35.19$$

Test of H_4 vs. H_1

$$-2\ln\theta = \chi^2(25) = 36.34$$

Test of H_4 vs. H_2

$$-2\ln\theta = \chi^2(12) = 20.99$$

Test of H_4 vs. H_3

$$-2\ln\theta = \chi^2(1) = 1.15$$

TABLE IV

Hypothesis Tests, KCVL

		Number of Estimated Parameters	Log-Likelihood
H_1:	Different means, different variances	26	524.1906
H_2:	Zero means, different variances	13	512.4029
H_3:	One mean, one variance	2	502.9200
H_4:	Zero means, one variance	1	502.2197

Test of H_2 vs. H_1

$$-2\ln\theta = \chi^2(13) = 23.58^{***}$$

Test of H_3 vs. H_1

$$-2\ln\theta = \chi^2(24) = 42.54^{**}$$

Test of H_4 vs. H_1

$$-2\ln\theta = \chi^2(25) = 43.94^{**}$$

Test of H_4 vs. H_2

$$-2\ln\theta = \chi^2(12) = 20.37$$

Test of H_4 vs. H_3

$$-2\ln\theta = \chi^2(1) = 1.40$$

TABLE V

Quarterly Dividends, S&P 500

Quarter	Dividends
1980:I	1.46
II	1.56
III	1.56
IV	1.58
1981:I	1.58
II	1.67
III	1.69
IV	1.69
1982:I	1.67
II	1.76
III	1.73
IV	1.71
1983:I	1.71
II	1.79
III	1.79
IV	1.80
1984:I	1.80
II	1.92
III	1.86

SOURCE: Standard & Poor's, Statistical Service, Current Statistics, November 1984.

TABLE VI

Period	Risk-Free Return	Expected Return, S&P 500: Perfect Foresight on Dividends	Expected Return, S&P 500: A High Estimate for Expected Dividends	Expected Return With 5 1/2% Market Risk Premium
6/17/82 - 9/16/82	3.271%	-0.669%	-.508%	4.663%
9/16/82 - 12/16/82	2.064	1.204	1.342	3.439
12/16/82 - 3/17/83	2.011	1.752	1.878	3.486
3/17/83 - 6/16/83	2.101	1.036	1.156	3.477
6/16/83 - 9/15/83	2.230	2.247	2.353	3.608
9/15/83 - 12/15/83	2.349	2.415	2.525	3.728
12/15/83 - 3/15/84	2.370	2.895	3.006	3.750
3/15/84 - 6/21/84	2.425	3.310	3.432	3.805
6/21/84 - 9/20/84	2.543	2.557	2.677	3.925

Notes: The index levels and the bid-ask rates for Treasury Bills are taken from the Wall Street Journal. The returns on 13-week T-bills are computed from the average of the bid and ask; the discount basis rates have been converted to holding period returns. The expected returns with the 5 1/2% market risk premium are computed as follows: $(1 + ER) = (1 + RF) \times (1.055)^{.25}$.

FOOTNOTES

[1]For a discussion of these issues and others, see Kamara (1982) and LeRoy (1982). The models of futures prices and forward prices are contained in Cox, Ingersoll, and Ross (1981), Richard and Sundaresan (1981), and French (1983).

[2]If the intertemporal utility function is time-additive, then the marginal utility of real wealth equals the marginal utility of consumption in real terms and the model collapses to a consumption-based CAPM. This asset pricing relation has been used in studies by Grossman and Shiller (1981), Hansen and Singleton (1982), Ferson (1983), and Dunn and Singleton (1983).

[3]λ_t is similar to $\tilde{\lambda}(\tau)$ in equation (8) of French (1983). For the case where utility of consumption is time-additive and $U(c_t) = \ln c_t$, the λ_t variable is the reciprocal of consumption in nominal terms.

[4]Here we are focusing on stock prices without dividends. By specifying a multiple time series representation, we can derive our results in a relatively general setting. This specification for stock price changes includes the random walk model as a special case, and it includes a factor model similar to the factor model for returns in the Arbitrage Pricing Theory of Ross (1976, 1977). It differs from the usual factor model in the APT by specifically allowing for random variation in expected price changes. The normality assumption is required so that we can evaluate the expectation in (5), which becomes a moment generating function. We are implicitly assuming that there is some dividend process so that the asset pricing model produces stock prices which are, at least to some approximation, lognormally distributed.

[5]Here we define the risk premium to be $E_t(P_{t+s}) - H_t(s)$.

[6]See the survey by Kamara, particularly p. 280.

[7]The futures prices for all three indices are taken from two sources: the Wall Street Journal and a data tape from MJK Associates, Computerized Commodity Data Sources. The S&P 500 matures on the third Thursday of the contract month. The last trading day on the NYSC is the next to last trading day of the contract month, and the last trading day on the KCVL is the last trading day of the contract month. In most cases, the last trading days for the NYSC and KCVL fell on Wednesdays, Thursdays, or Fridays.

[8]Note that rejection of a zero mean parameter indicates evidence of a risk premium.

[9]We would be willing to pay more for an asset that has a positive covariance with marginal utility of wealth than an otherwise equivalent asset that has a negative covariance. The asset with a negative covariance has the following characteristics: when its price is higher than expected, marginal utility of wealth tends to be lower

than expected, and when its price is lower, marginal utility of wealth tends to be higher. Positive surprises tend to come when they will be worth less. The asset with a positive covariance with marginal utility of wealth has the opposite set of characteristics. Positive surprises tend to come when they will be worth more.

[10]The numbers are reported as four-quarter moving totals, but one can use the numbers in the technical appendix to Leroy and Porter (1981) to recover the quarterly dividend series.

[11]See the studies by Ibbotson and Sinquefield (1977) and Merton (1980).

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