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ROBUST MEASUREMENT OF BETA RISK

Abstract

Many empirical studies find that the distribution of stock returns departs from normality. In such cases, it is desirable to employ a statistical estimation procedure which may be more efficient than ordinary least squares. This paper describes various robust methods which have attracted increasing attention in the statistical literature, in the context of estimating beta risk. The empirical analysis documents tne potential efficiency gains from using robust methods as an alternative to ordinary least squares, based on both simulated and actual returns data.

Much interest in financial economics centers on efficient estimation of the parameters of the return generating process. The estimated beta from a market model, for example, is widely used to generate a security's expected rate of return for discounting cash flows, and to compute risk-adjusted returns for performance evaluation. In practice, however, there are various problems in estimating betas. It is well known that the ordinary least squares (OLS) estimator of beta is particularly sensitive to the presence of outliers and, more generally, to departures from normality (see, e.g., Chow (1983, p. 88), Judge et al. (1988, p. 890), and Ruppert and Carroll (1980)). Early evidence (Mandelbrot (1963) and Fama (1965)) suggested that the distribution of stock returns might be "fat-tailed" relative to a normal distribution, resulting in outliers. More recent evidence (Kon (1984) and Roll (1988)) considers a mixture of normal distributions, possibly reflecting "good" observations, interspersed with unusual news-related observations. Accordingly, alternative methods of estimating beta may be desirable in the presence of extreme return observations.

In this paper, we describe an alternative to OLS for estimating beta or the parameters of other financial models, that is robust to departures from normality and which has attracted increasing attention in the statistical literature. Koenker (1982) defines robustness as,

> In ordinary parlance "robust" means sturdy--capable of withstanding the "slings and arrows of outrageous fortune." In statistics and more loosely in economics it has come to signify a certain resilience of conclusions to deviations from assumptions of hypothetical models. In effect robustness is a continuity requirement. An inference θ is robust to hypothesis H if a small wiggle in H induces a small wiggle in θ.

Our interest in a robust alternative to OLS stems from recent research questions that suggest that the problem of fat-tailed stock return distributions may be substantively more severe than earlier realized. Specifically, earlier studies on the properties of return distributions focused on a random sample from the largest firms such as the Dow Jones 30. However, empirical research has come to focus increasingly on firms whose returns are expected to exhibit fatter tails relative to randomly selected large firms. For example, papers studying recommendations by financial analysts found that in many cases analysts focus on, analyze and subsequently recommend companies whose returns previously exhibited extreme movements (see Copeland and Mayers (1982) and Bjering, Lakonishok and Vermaelen (1983)). Fat tailed distributions are also expected in returns of companies that filed under Chapter 11, companies that cut dividends and smaller companies in general. In studies in the area of corporate control there are good reasons to expect returns to have fat tailed distributions. According to Lakonishok and Vermaelen (1990), small companies involved in stock repurchase tender offers had substantial negative abnormal returns prior to the announcement of the offer. This is consistent with the common belief that these stocks are underpriced. The free cash flow argument advanced by Jensen (1986) also suggests that companies involved in takeovers exhibit extreme performance prior to announcements related to corporate control changes. Roll (1988) finds that the biggest improvement in explanatory power of the "market model" and multi-factor model when news dates are excluded is associated with firms involved in takeover situations. For firms involved in takeovers, returns around many of the news dates are probably outlier observations.

The estimation of beta risk is of major concern in recent studies of the "overreaction" hypothesis. The methodology in general involves identifying "losers" and "winners" on the basis of returns realized over some past period; their price performance in subsequent periods is then examined. However, firms ranked as extreme losers and winners have experienced dramatic firm-specific news, so that realized returns contain many outlier observations. In such cases, OLS estimates of beta are inefficient, and appropriate methods of measuring beta risk are required (see DeBondt and Thaler (1987), Chan (1988), and Ball and Kothari (1989)).

Similar issues arise with risk estimation using returns on a cross-sectional sample of firms. Ibbotson (1975), in his study of the performance of new issues, proposed a cross-sectional beta estimate for securities separated in time. Clarkson and Thompson (1990) utilize this approach to examine the effects of differential information on risk assessment, based on the arguments of Klein and Bawa (1976, 1977), and Barry and Brown (1984, 1985). Specifically, the differential information hypothesis implies that the beta of an initial public offering (IPO) should decline as time passes and information on the issuer increases. Clarkson and Thompson (1990), as well as Barry, Muscarella, Peavy and Vetsuypens (1988) find extremely high values for beta on the first trading day, with an abrupt decline thereafter. However, numerous extreme observations on returns are observed during the first few trading days for IPOs. For example, Barry et al. (1988) find that returns range from -62 percent to 117 percent for the first day of trading. As a result, OLS cross-sectional beta estimates will not be very reliable. The issue of risk is also essential in understanding the well-known underpricing of IPOs, as well as their subsequent price behavior (Ritter (1991)).

It should be pointed out that the issue does not concern biases in the OLS estimator. It is commonly believed, for example, that contemporaneously estimated betas for winners (losers) are biased upwards (downwards) (considerations of leverage aside). On the other hand, Appendix 1 isolates the consequences of using OLS to estimate the betas of winners or losers and demonstrates the lack of bias. Nonetheless, an alternative, unbiased, robust estimation method may still offer efficiency gains (relative to OLS) over a wide class of thick-tailed distributions.

This paper documents the magnitude of these efficiency gains in situations which are likely to arise in many applications of interest. In such situations, interest centers on a model for returns absent some treatment (e.g., in event studies), where it would be useful to employ a procedure which is robust to data errors or non-normality in the returns distribution. Specifically, the empirical analysis confronts OLS and various robust methods for estimating beta risk. The results are based on simulated data (providing a known benchmark), and also on two applications to actual data. The first application concerns beta estimation for a sample of losers, winners and randomly selected firms; the second application considers beta estimation for IPOs, based on cross-sectional regressions. The results confirm that substantial efficiency gains can indeed be achieved by the use of robust methods instead of OLS. While our empirical results deal with the estimation of beta risk, the methods are applicable in other contexts, such as the prediction of returns using financial variables (Chan, Hamao and Lakonishok (1990)).

The remainder of the paper is organized as follows: the first section provides the motivation for our study and literature review, the second section includes a review of the various robust methods of estimating linear models; Section III presents our empirical results; Section IV contains a summary and conclusions.

I. Motivation and Literature Review

In estimating the parameters of a linear model, such as beta risk, the assumption about the distribution of the error is crucial. If the error term has a Gaussian distribution, the OLS estimator of the parameters has minimum variance of the <u>entire</u> class of unbiased estimators (see Rao (1973)). Moreover, using Jensen's inequality the optimality of the OLS procedure under Gaussian conditions can be established for any convex loss function (see Rao (1973), section 5a). When normality of the error term cannot be assumed, OLS will provide the best unbiased estimator of the parameters of the linear model only if attention is restricted to those estimators which are linear functions of the dependent variable. In many situations, however, this set may be unnecessarily restrictive. Moreover, outliers can have a potent effect, completely altering least squares estimates (Ruppert and Carroll (1980), Koenker (1982)).

Statistically, a fat-tailed distribution may be modelled as arising from a mixture of normal distributions. For example, the underlying data may come from a standard normal distribution, but are contaminated by aberrant observations from another normal distribution with higher variance. Such a distribution will have heavier tails than a normal distribution.

In the finance literature, earlier research suggests that the distribution of daily stock returns exhibits "fatter tails" than a normal distribution. For example, Fama (1965) fits a stable Paretian distribution to daily returns and finds a characteristic exponent less than two; Praetz (1972) and Blattberg and Gonedes (1974) provide evidence in favor of the student (t) distribution; Kon (1984) finds that returns on the 30 Dow Jones stocks can be described as a mixture of between 2 to 4 normal distributions. In addition, Blume (1968) shows that the residuals from estimating betas using OLS have approximately the same distribution as the underlying stock returns. Taken together, the empirical evidence suggests that the distribution of residuals departs from normality and is likely to be characterized by fat tails.

Roll, in his presidential address (1988), suggests an economic model which is consistent with stock returns being generated by a mixture of distributions. He basically assumes that stock returns are interspersed with extreme values, which are related to news events, thereby substantially increasing the kurtosis of the return distribution. Damodaran (1985, 1987) also argues that the kurtosis of a firm's return process reflects the frequency of information released about the firm.

Robust statistical methods provide an alternative to least squares, and have recently attracted growing attention although not in finance. Such estimators give less weight to "outlier" observations, for example by minimizing the sum of absolute deviations (the method of minimum absolute deviations, MAD) instead of the sum of squared deviations. Sharpe (1971) and Cornell and Dietrich (1978) applied the MAD method to estimate betas. Their samples included the largest firms and a sample of mutual funds. The results revealed that the difference between the two methods is small, so that the MAD method (for reasons that will be explained in the next section) did not prove itself a clearly superior method.

II. Robust Methods of Estimating Linear Models

This section of the paper provides an informal motivation for the statistical methods used in this paper. Although the discussion focuses on estimating the parameters of the "market model" applied to excess security returns, the methods can be generalized straightforwardly. More detailed accounts may be found in Koenker and Bassett (1978), Bassett and Koenker (1982) and Koenker and Portnoy (1988). Just as the ordinary least squares estimator can be obtained from minimizing the sum of squared residuals, the estimators we consider are based on minimizing the criterion function,

$$\sum_{t=1}^{T} \rho_\theta(u_t)$$

for $\rho_\theta(u_t) = \theta|u_t|$ if $u_t \geq 0$, or $(1-\theta)|u_t|$ if $u_t < 0$. Here $0 < \theta < 1$, $u_t = r_t - \alpha - \beta r_{mt}$, $t = 1,\ldots,T$, and $|\cdot|$ denotes absolute value.

Since the minimand is the sum of the absolute values of the residuals, deviant observations are given less importance than under a squared error criterion. For example, the case of $\theta = 1/2$ corresponds to the minimum absolute deviations (MAD) estimator of the regression parameters. More generally, large (small) values of the "weight" θ attach a heavy penalty to observations with large positive (negative) residuals. Each fitted regression line (corresponding to a different value of θ) passes through at least two data points, with at most $T\theta$ sample observations lying below the fitted line, and at least $(T-2)\theta$ observations lying above the line. For example, when $\theta = 1/2$, the median fitted residual is zero: half of the data points lie above the line, while half lie below. Varying θ between 0 and 1 yields a set of "regression quantile" estimates $\hat{\beta}(\theta)$, analogous to the quantiles of any sample of data, that is, the set of order statistics.[1]

The characterization above suggests, at least on an intuitive level, the following features of these regression quantiles. Specifically, the effect of large positive or negative outlying observations will tend to be concentrated in the regression quantiles corresponding to extreme (high or low) values of θ. Note, however, that no observations are discarded in the course of computing these statistics. Moreover, the behavior of returns in the sample determines the variation in the regression quantiles as θ changes. From this perspective, choosing an estimate of β corresponding to one value of θ, such as the MAD estimate, ignores potentially useful information in the sample. Accordingly, the performance of the MAD estimator may be improved upon by an estimator which incorporates several regression quantiles.

In the statistical literature, considerable attention has been devoted to the problem of obtaining robust estimates of the population mean via linear combinations of sample quantiles (e.g., trimmed means). In the same spirit, regression quantiles serve as the basis for the robust estimators of regression parameters that we consider. The general form of such trimmed regression quantile (TRQ) estimators is

$$\hat{\beta}_\alpha = \frac{1}{(1-2\alpha)} \int_\alpha^{1-\alpha} \hat{\beta}(\theta)d\theta$$

where $0 < \alpha < 1/2$. This estimator is a weighted average of the regression quantile statistics and hence belongs to the class of L-estimators.[2] Each regression quantile is weighted by its (data dependent) "relative frequency" of occurrence, given by its corresponding interval of θ-values. The form of the estimator suggests that it is analogous to a trimmed mean, with trimming proportion α: the "extreme" quantiles, where the influence of outlying observations should be most heavily concentrated, are deleted. As the sample size goes to infinity, another intuitively natural interpretation of $\hat{\beta}_\alpha$ is possible: consider fitting the α-th, and $(1-\alpha)$-th, regression quantile lines through the data. Then exclude all observations lying on or below the α-th regression quantile line (corresponding to large negative outliers), as well as all observations lying on or above the $(1-\alpha)$-th quantile line (corresponding to large positive outliers). The remaining observations are then used to calculate the ordinary least squares estimator; in large samples, the resulting "trimmed least squares" estimator is equivalent to $\hat{\beta}_\alpha$.

Although the discussion has concentrated on estimation, statistical inference concerning the trimmed regression quantile estimator $\hat{\beta}_\alpha$ is also possible. In large samples, $\hat{\beta}_\alpha$ is consistent and normally distributed with variance-covariance matrix $\sigma_\alpha^2(X'X)^{-1}$, where X is the matrix of regressors (see Koenker and Portnoy (1988)). A consistent estimator of σ_α^2 is

$$s_\alpha^2 = \frac{1}{(1-2\alpha)^2} \{\frac{SSE_\alpha}{(T-2)} + \alpha[\overline{x}'(\hat{\beta}(\alpha)-\hat{\beta}_\alpha)]^2 + (1-\alpha)[\overline{x}'(\hat{\beta}(1-\alpha)-\hat{\beta}_\alpha)]^2\}.$$

SSE_α is the sum of squared residuals from the trimmed least squares estimator, based on a sample of T observations. $\overline{x}$ is a column vector containing the sample means of the regressors, while $\hat{\beta}(\theta)$ is the vector of parameter estimates for the θ-th regression quantile. Simulation evidence in Koenker and Portnoy (1988) suggests that the asymptotic approximation is not unreasonable, even in samples of 25 to 50 observations.

In addition, we also consider estimators that are finite linear combinations of regression quantiles:

$$\beta_\omega = \sum_{i=1}^{N} \omega_i \hat{\beta}(\theta_i)$$

with weights $0 < \omega_i < 1$, $i = 1,\ldots,N$, and $\sum_{i=1}^{N} \omega_i = 1$.

Two specific cases of such weighted averages are Tukey's trimean, a weighted average of the regression quartiles:

$$\hat{\beta}_{TRM} = 0.25\hat{\beta}(1/4) + 0.5\hat{\beta}(1/2) + 0.25\hat{\beta}(3/4);$$

and the Gastwirth estimator, given by

$$\hat{\beta}_{GAS} = 0.3\hat{\beta}(1/3) + 0.4\hat{\beta}(1/2) + 0.3\hat{\beta}(2/3).$$

These estimators are computationally simpler than the estimator $\hat{\beta}_\alpha$, while still exploiting the behavior of several regression quantile statistics.[3]

The properties of the TRQ estimator $\hat{\beta}_\alpha$ may usefully be compared to other robust estimation methods. An alternative class of estimators, M-estimators,[4] are obtained as solutions to the problem of minimizing a function of scaled regression residuals (Huber (1981)). The computation of such M-estimators, however, requires a (robust) estimate of the scale parameter of the distribution of errors; such information is not required for the TRQ method. Along similar lines, Ruppert and Carroll (1980) investigate the properties of the least squares estimator, after trimming some proportion of the residuals obtained from a preliminary fit. However, the properties of this estimator are very sensitive to the preliminary estimate. More recently, an improved, one-step trimmed least squares procedure has been developed by Welsh (1987). Simulation evidence in Koenker (1987) suggests that the TRQ estimator and Welsh's estimator have roughly similar performance. Iteratively re-weighted least squares estimators have also been proposed by Krasker and Welsch (1982) in the form of "bounded influence" estimation. This method mitigates the effects not only of heavy-tailed error terms, but also of aberrant observations on explanatory variables. Specifically, in calculating the ordinary least squares estimator, each observation receives a (data-determined) weight, which limits the influence of outlying residuals or of outlying observations in the explanatory variables. Although we do not apply all these alternative methods, this is not to preclude their potential usefulness in the present context, or in other contexts.[5] A final, not unimportant, consideration is that software for implementing the TRQ method is readily available (see Koenker and D'Orey (1987), White (1987)).

III. Empirical Results

The empirical results are based on the following return generating process:

$$r_{it} - r_{ft} = \alpha_i + \beta_i(r_{mt}-r_{ft}) + \varepsilon_{it}, \qquad \begin{array}{l} i = 1,\ldots,N, \\ t = 1,\ldots,T, \end{array} \tag{1}$$

where r_{it}, r_{mt} and r_{ft} are the rates of return on security i, on the market index and on the risk-free asset, respectively, for period t. Our analysis rests on both simulated returns data and actual returns data. The main advantages of a simulation are that the true values of the underlying parameters are known, and that the extent of departures from normality can be controlled. Beginning with a baseline simulation using data generated from a normal distribution, we simulate the effects of various forms of departures from normality. Additional experiments use actual market returns data, and the empirical distribution of market model residuals.

In each simulation, the following procedure is replicated 25,000 times. Thirty-six paired observations on the excess market return and the residual are drawn from the hypothesized distributions. A simulated series for the stock rate of return is generated from the model (1), with the true parameters α and β set to zero and one, respectively. The generated rate of return series is then regressed on the market premium using seven methods: OLS, minimum absolute deviations (MAD), the trimmed regression quantile estimator (with trimming proportion α set to 0.10, 0.20 or 0.25), and the trimean and Gastwirth estimators. The methods are compared, using summary statistics on the cross-sectional distribution (across the 25,000 replications) of the estimated market model parameters. The statistics are: the mean estimated intercept, $\hat{\alpha}$, and mean estimated slope, $\hat{\beta}$, together with their cross-sectional standard deviations; the mean absolute deviation (ABSD) of the estimated betas away from one; the cross-sectional variance of beta estimated with OLS, relative to the cross-sectional variance from a given robust procedure (V_{OLS}/V_M). In comparing the different methods we focus on the relative variance statistics, which is the most common measure of relative efficiency in the statistics literature.[6]

The first set of results, reported in Table 1 (columns (2) to (5)), focuses on the "ideal" case: the excess market rate of return and the market model residual are drawn from a bivariate normal distribution. The excess market return is specified to have a population mean of 6.10 percent and a standard deviation of 5.79 percent (taken from the historical summary statistics in French, Schwert and Stambaugh (1987), Table 3). The residual has a mean of zero, and a standard deviation of 11.66 percent, yielding a typical R^2

of 20 percent for the OLS market model regression (see Roll (1988)). By construction, the market excess return and the residual are mutually uncorrelated. In this setting, the OLS procedure performs best, as expected; the loss in efficiency for the trimmed regression quantile methods (with moderate values for α) is only about 10 percent.

Further results are obtained from simulations where the distribution of the error term diverges from normality. The distribution of the market excess return is as specified before. We first consider a case where the error terms are drawn from a Student t-distribution with 3 degrees of freedom.[7] These results are presented in the last four columns of Table 1. The results for the robust methods reveal a substantial improvement over OLS. In terms of the relative variance, the improvement is lowest for MAD (at 47 percent) and is as high as 77 percent for TRQ with large trimming proportion. As in the "ideal" case, all procedures provide unbiased estimates of the slope and intercept, and do not differ substantially in the precision of estimating the intercept (except for MAD, which again has the worst performance).

The next set of simulations is based on a closer approximation to the actual distribution of market model residuals, and of excess market returns. We randomly selected 50 firms from the NYSE with monthly data over the period 1983-1985. For each of these securities we estimate the market model (1) using OLS and the return on the equally-weighted NYSE index. Hence, we obtain 1800 (36 x 50) realizations for the residual term. The bootstrapping technique is used to obtain artificial series of realizations for the residuals and for the equity premiums. For the residuals, 36 observations are drawn randomly from the 1800 observations. Similarly, 36 observations are randomly drawn from the 720 monthly observations on the Ibbotson-Sinquefield equity risk premium series from 1926 to 1985. A simulated series for the stock rate of return is generated from the model (1), with the true parameters α and β again set to zero and one, respectively. The market model is then fit to these artificial data, using the seven methods outlined earlier. As in the previous tables, this experiment is repeated 25,000 times.

Many applications in finance utilize daily data on returns. The performance of the different robust methods, relative to OLS, as a function of the sampling interval, is an open empirical issue. On the one hand, Koenker's (1982) results indicate that the efficiency of the robust estimators could be low for highly skewed distributions of the residual. To the extent that daily returns are more skewed than monthly returns (Brown and Warner (1985)) we would not expect the robust methods to perform well. On the other hand, the distributions of the daily and monthly data may also differ with respect to higher moments. For example, if the distribution of daily returns exhibits thicker tails than monthly returns, the robust methods may yield better performance.

The methodology for the daily data is similar in spirit to the earlier procedure using monthly data. In particular, we analyze daily returns for the same sample of 50 random firms, over the same three-year period. Studies utilizing daily returns data for estimating betas typically employ a one-year sample period and use raw returns. Accordingly, we regress the raw return on the level of the market return for each of the three years, thus obtaining 758 residuals for each firm in our sample.[8] From these, we randomly draw 250 observations and pair them with observations on the equally-weighted index from the CRSP database. The artificially generated return series are then subjected to the same procedures employed earlier with the monthly data. For the monthly returns data, the coefficients of sample skewness and kurtosis are 0.17 and 3.60, respectively. In comparison, the daily returns data exhibit higher skewness (0.36) and substantially higher kurtosis (7.56).

The results for the random group of firms are reported in Table 2. Columns (2) through (5) present the results when the sampling interval is monthly. All the robust methods except MAD outperform OLS, with an improvement of around 20 percent for the better cases. In columns (6) through (9) the results are based on a daily sampling interval. Here all the robust methods, including MAD, substantially outperform OLS, with the improvement in excess of 50 percent for many of the estimators. In accordance with the sample statistics for kurtosis reported above, the fatter tails of the distribution of returns accounts for the superior performance of the robust methods when applied to daily data, compared to monthly data.

Table 2 documents the advantages afforded by robust methods when the data are sampled at daily, instead of monthly, intervals. Table 3 further evaluates the performance of the different methods, based on monthly data, when they are applied to a sample of "winners" and "losers." Specifically, we selected from all NYSE firms the 50 firms with the highest ("winners"), and the 50 firms with the lowest ("losers") compound return over 1983-1985. The same bootstrapping procedure employed earlier is replicated with the sample of winners and also with the sample of losers.

In Table 3, results are presented using the bootstrap method for losers (columns (2)-(5)) and winners (columns (6)-(9)). The improvement in the better robust methods over OLS is about 20 percent for "losers" and 40 percent for "winners." A possible explanation for the mild improvement is that our experiment utilizes the residuals fitted from a prior OLS regression. The least squares method is sensitive to extreme observations and thus tends to accommodate these observations. Therefore, the distribution of the fitted residuals departs less from normality than the true residuals. Accordingly, our results tend to understate the potential improvement.[9]

Next, we present some evidence based on the actual returns to our sample of losers, winners and randomly selected firms. For the sake of brevity, we only compare in Table 4 the OLS method with the TRQ method, with $\alpha = 0.10$. The table reports the average (across firms in the sample) of the absolute difference between beta estimated with the robust method and beta estimated with OLS, as well as the ratio of the cross-sectional variances of estimated betas from the two methods. Unlike the simulation experiments, however, the true beta for each security is unknown and, as a result, the evidence in Table 4 should be interpreted with some caution. Nonetheless, it is reassuring to note that, for the random sample, the average absolute difference in estimated betas is small--roughly 0.1. For winners and losers, however, the differences are more substantial, and they are as high as 0.2 for losers. For all three groups of firms, there is a major reduction in the cross-sectional dispersion of betas estimated with TRQ, relative to betas fitted with OLS. The efficiency gains are 29 and 36 percent for losers and winners, respectively, while the gain is smaller for the random sample, as expected.[10]

Finally, results are presented in Table 5 using Ibbotson's (1975) cross-sectional regression methodology. The data are returns on a sample of 661 initial public offerings (IPOs) over the period 1978-1985. A cross-sectional regression, relating the return on each IPO to the contemporaneous return on the NASDAQ index, is estimated for each of the 10 days following the listing. The OLS method produces a very high beta, 4.26, for the first trading day, and 1.62 on the following day, a one-day drop of 2.64. It is hard to imagine that increases in information over 24 hours can explain such a huge drop in beta. Thus it is reassuring to find that the TRQ method produces a less extreme beta on the first day (3.20), and a smaller drop on the second trading day (to 0.92, for a decline of 2.28). As another basis of comparison, the OLS method produces larger day-to-day changes in betas than the TRQ method. The average daily absolute change in betas (from day 1 to day 10) is 0.58 and 0.48 for the OLS and TRQ procedures, respectively.[11]

Infrequent trading might introduce substantial biases in the cross-sectional betas for both methods. To reduce the bias, two day returns were utilized in the regression. The first cross-sectional regression now produces much lower betas; 3.39 for the OLS method and 2.52 for the TRQ method. The drop in beta (from the first to the second regression) is once more less abrupt for the TRQ method, 1.44, versus 2.12 for the OLS method.[12] In summary, the OLS and robust methods produce substantially different betas especially for the first few trading days (when extreme observations are more common) and, the robust betas seem to be more in line with our priors.

IV. Summary and Conclusions

This paper describes and applies several robust methods for the estimation of parameters in a linear regression model when the distribution of the residuals displays thicker tails than a Gaussian distribution. These methods are applicable when observations on the dependent variable take on extreme outlying values, not accounted for by movements in the explanatory variables. An economic model for such behavior rests in terms of "good" observations, mixed with news-driven "bad" observations (see, e.g., Roll (1988)). As such, robust estimation methods hold promise for studies of firms involved in takeover activity, bankruptcy proceedings, stock repurchase offers, dividend and earnings announcements and initial public offerings, for example.

Our results with simulated and actual data support the potential efficiency gains from robust methods, relative to least squares. In the case where the distribution of the residuals is Gaussian, there is only a minor efficiency loss of about 10 to 20 percent. When the residuals follow a

Student-t distribution with 3 degrees of freedom, efficiency gains of about 80 percent are possible. Substantial improvements from using the robust methods are also observed when the simulations are based on the actual distribution of residuals and excess market returns using both monthly and daily data.

With respect to actual data, in the context of winner and loser stocks as well as randomly selected stocks, the robust methods outperform substantially the OLS method. In estimating cross-sectional betas for a sample of initial public offerings, the results indicate that the robust methods should be considered as a serious alternative to OLS.

In comparing the different methods, the performance of the minimum absolute deviations estimator is disappointing, confirming previous results. However, we go further by providing improved alternatives, particularly in the form of the trimmed regression quantile estimator. These alternative methods are straightforward to implement.

We find the overall results from our analysis to be encouraging. Further research might focus on a more extensive analysis of the extent of leptokurtosis observed in returns data, as well as how such kurtosis is correlated with firm-specific news events, economic characteristics such as size, and the time-interval over which returns are measured. Further, the present methods may be useful in other empirical applications.

Footnotes

[1]For a continuous random variable Z with distribution function F, its θ-th quantile, ζ_θ, is such that $F(\zeta_\theta) = \theta$.

[2]L-estimators are obtained as linear combinations of order statistics. Examples include the median and trimmed means. A trimmed mean is simply the sample mean, after some proportion α of the observations at each extreme of the sample are deleted.

[3]Koenker and Bassett (1978) develop the asymptotic distribution theory for the trimean and Gastwirth estimators.

[4]In our context, an M-estimator is obtained by minimizing

$$\sum_{t=1}^{T} \rho\left(\frac{r_t-\alpha-\beta r_{mt}}{\sigma}\right)$$

for some function ρ, where σ is a scale parameter. An example for ρ is

$$\rho\left(\frac{u_t}{\sigma}\right) = \begin{cases} 1/2\ u_t^2 & \text{if } |u_t| < k\sigma \\ k|u_t| - 1/2\ k^2 & \text{if } |u_t| \geq k\sigma. \end{cases}$$

Setting $k = \infty$ yields the OLS estimator, while a finite, positive value for k places less weight on extreme residuals.

[5]Carroll and Welsh (1988) study the effects of an asymmetric distribution for the error term on robust regression procedures. They stress that estimates of slope parameters under most robust methods (including the regression quantile method) are unaffected by asymmetric errors. However, the Krasker and Welsch (1982) bounded influence estimator is inconsistent when the errors are asymmetrically distributed. This result helps to justify restricting attention to the regression quantile approach.

[6]The ratio of variances also has an appealing intuitive interpretation. If, for example, the variance ratio is 1.5, then the researcher using OLS will need a sample that is 50 percent larger in order to achieve the same efficiency as the alternative procedure.

[7]According to Blattberg and Gonedes (1974) and Kon (1984), the degrees of freedom parameter for the Student model should be between 2 and 10, in order to explain the observed leptokurtosis in daily returns data. In the case of smaller companies during turbulent periods, a Student-t distribution with 3 degrees of freedom might be a fair representation even with weekly or monthly data.

[8]The results are essentially unaltered if betas are fitted using all three years of data, or if only the most recent year's worth of data are used.

[9]Consistent with the proof in the appendix, the betas for the winner and loser stocks are essentially one. The intercept estimated with OLS is zero by design. On the other hand, the intercepts estimated under the robust methods are about -0.1, -0.2 and -0.7 percent for the loser, random and winner stocks and are statistically significant. The reason for these departures is that the distribution of the bootstrapped residuals is not symmetric, but skewed to the right, and more so for winners than for losers. Since the robust methods attach less weight to extreme observations, the estimated intercept is below zero and smallest for the group of winner stocks.

[10]The cross-sectional variability in estimated betas reflects both the variability in the underlying true betas as well as sampling error. For a given sample, however, the variation across firms in true betas is the same across methods. Accordingly, the variance ratio statistics in Table 4 understate the reduction in measurement error for betas afforded by the robust methods.

[11]Additional confirmatory evidence on the adverse influence of extreme observations is provided by repeating the OLS regression, but deleting the 10 most extreme return observations on each day. With this crude adjustment, the first and second day betas are 3.58 and 1.27, respectively (a one-day decline of 2.31, almost identical to the decline with the TRQ method); the average daily absolute change in betas is 0.53. Note that deleting observations, however, is an ad hoc modification, and unnecessarily discards sample information.

[12]Following the first regression the betas from the TRQ regression are close to one. They are higher than the one day return betas because the impact of infrequent trading is reduced.

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Appendix

This appendix considers the nature of any biases arising from using ordinary least squares (OLS) to estimate the market model from a sample of "winners" or "losers." The model to be estimated is

$$r_{it} = \alpha_i + \beta_i r_{mt} + \varepsilon_{it}, \qquad \begin{array}{l} t = 1,\ldots,T, \\ i = 1,\ldots,N. \end{array} \tag{1}$$

r_{it} is the excess return (over the risk-free rate) on security i. r_{mt} is the excess return on the market index. α_i, β_i are unknown parameters. ε_{it} is a serially uncorrelated residual, with $E(\varepsilon_{it}) = 0$, $var(\varepsilon_{it}) = \sigma^2_{\varepsilon i}$. The notion of a "winner" or "loser" security in a given sample corresponds most closely to a security whose non-market, residual component, ε_{it}, has a positive, or negative, sample average. That is, $\bar{\varepsilon}_i > 0$ for "winners," while $\bar{\varepsilon}_i < 0$ for "losers," where the overbar denotes the sample arithmetic mean. This characterization appears to match what DeBondt and Thaler (1987), Beaver and Landsman (1981) have in mind.

The ordinary least squares estimator of β_i, $\hat{\beta}_i$, is given by

$$\hat{\beta}_i = \sum_{t=1}^{T} \frac{(r_{mt}-\bar{r}_m)r_{it}}{\sum_{t=1}^{T} (r_{mt}-\bar{r}_m)^2} .$$

Substituting for r_{it} from (1) yields

$$\hat{\beta}_i = \beta_i + \sum_{t=1}^{T} \frac{(r_{mt}-\bar{r}_m)\varepsilon_{it}}{\Sigma(r_{mt}-\bar{r}_m)^2} . \tag{2}$$

Since the residuals have an expected value of zero, $E(\hat{\beta}_i) = \beta_i$. That is, on average (in repeated samples), the ordinary least squares method yields a correct estimate of the true underlying risk coefficient β. In any given sample, of course, the estimated $\hat{\beta}_i$ may differ from the true β_i. From (2), however, the sign of the difference $(\hat{\beta}_i-\beta_i)$ in a given sample depends only on the sample covariance between the excess market returns and the residuals (which covariance should equal zero in the population if the market model is correctly specified). In particular, the sign of $(\hat{\beta}_i - \beta_i)$ does not depend on the sign of the sample mean $\bar{\varepsilon}_i$, and hence on whether the security is a "winner" or "loser."

On the other hand, the sign of the sample mean residual does affect the least squares estimate of α_i, the intercept term, in a given sample:

$$\hat{\alpha}_i - \alpha_i = \bar{r}_i - \hat{\beta}_i \bar{r}_m - \bar{r}_i + \beta_i \bar{r}_m + \bar{\varepsilon}_i$$

$$= -(\hat{\beta}_i - \beta_i)\bar{r}_m + \bar{\varepsilon}_i \ . \qquad (3)$$

Under the null hypothesis $\alpha_i = 0$, the estimated intercept (correctly) measures the sample average excess return not attributable to market movements, up to measurement error in β.

Estimates of the standard errors of $\hat{\alpha}$ and $\hat{\beta}$ also require an estimate of the standard deviation of the regression residuals (adjusted for degrees of freedom):

$$\hat{\sigma}^2_{\varepsilon i} = \frac{1}{(T-2)} \sum_{t=1}^{T} (r_{it} - \hat{\alpha}_i - \hat{\beta}_i r_{mt})^2$$

$$= \frac{1}{(T-2)} \sum_{t=1}^{T} (\alpha_i - \hat{\alpha}_i + (\beta_i - \hat{\beta}_i) r_{mt} + \varepsilon_{it})^2$$

Substituting from (2) and (3),

$$\hat{\sigma}^2_{\varepsilon i} = (\hat{\beta}_i - \beta_i)^2 \sum_{t=1}^{T} \frac{(r_{mt} - \bar{r}_{mt})^2}{(T-2)} + \sum_{t=1}^{T} \frac{(\varepsilon_{it} - \bar{\varepsilon}_i)^2}{(T-2)} \ .$$

This expression for $\hat{\sigma}^2_{\varepsilon i}$ also does not depend on the sign of the realized mean residual, $\bar{\varepsilon}_i$. Accordingly, limiting the sample to "losers" or "winners" creates no bias in the OLS estimator. This is, of course, not to exclude the possibility that improved (more efficient) estimators may exist.

Table 1

Summary statistics for the sampling distribution of estimated intercept ($\hat{\alpha}$) and slope ($\hat{\beta}$) from market model regression

$$r_t - r_{ft} = \alpha + \beta(r_{mt} - r_{ft}) + \varepsilon_t$$

Sampling distribution based on 25,000 replications of simulated data with 36 observations per replication. Simulated data based on: $\alpha = 0$; $\beta = 1$; $(r_{mt} - r_{ft})$ normally distributed with mean 0.1%, standard deviation 5.79%. In columns (2)-(5), ε_t is normally distributed with mean 0 and standard deviation 11.66%; in columns (6)-(9), ε_t follows a student 't' distribution with 3 degrees of freedom. Columns (2) and (6) present the sample mean of $\hat{\alpha}$ (standard deviation in parentheses); columns (3) and (7) present the sample mean of $\hat{\beta}$ (standard deviation in parentheses). Columns (4) and (8) report the average absolute deviation of estimated beta away from one. Columns (5) and (9) report the cross-sectional variance of beta estimated with OLS, divided by the cross-sectional variance of beta estimated with each robust method.

Method[a]	$\hat{\alpha}$	$\hat{\beta}$	ABSD	V_{OLS}/V_M	$\hat{\alpha}$	$\hat{\beta}$	ABSD	V_{OLS}/V_M
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
OLS	0.0002	0.9994	0.28	1.00	0.0003	0.9956	0.26	1.00
	(0.0199)	(0.3499)			(0.0199)	(0.3464)		
MAD	0.0004	0.9967	0.44	0.64	0.0002	0.9981	0.22	1.47
	(0.0245)	(0.4373)			(0.0158)	(0.2860)		
TRQ (α=0.10)	0.0002	0.9992	0.29	0.93	0.0002	0.9989	0.21	1.67
	(0.0204)	(0.3631)			(0.0150)	(0.2680)		
TRQ (α=0.20)	0.0003	0.9988	0.30	0.87	0.0002	0.9987	0.20	1.77
	(0.0211)	(0.3748)			(0.0145)	(0.2603)		
TRQ (α=0.25)	0.0003	0.9982	0.30	0.84	0.0002	0.9984	0.20	1.77
	(0.0215)	(0.3818)			(0.0144)	(0.2601)		
Trimean	0.0003	0.9994	0.31	0.82	0.0002	0.9988	0.21	1.67
	(0.0215)	(0.3853)			(0.0148)	(0.2680)		
Gastwirth	0.0003	0.9994	0.31	0.80	0.0002	0.9984	0.21	1.70
	(0.0220)	(0.3922)			(0.0147)	(0.2653)		

[a] Estimation methods are Ordinary Least Squares (OLS); Minimum Absolute Deviations (MAD); Trimmed Regression Quantile (TRQ) with trimming proportion α: 0.10, 0.20 or 0.25; Tukey's trimean; Gastwirth estimator.

Table 2

Summary statistics for the sampling distribution of estimated intercept ($\hat{\alpha}$) and slope ($\hat{\beta}$) from market model regression. For monthly data (columns (2)-(5)) the model is

$$(r_t - r_{ft}) = \alpha + \beta(r_{mt} - r_{ft}) + \varepsilon_t$$

Sampling distribution based on 25,000 replications of simulated data with 36 observations per replication. Simulated data based on: $\alpha = 0$; $\beta = 1$; $(r_{mt} - r_{ft})$ drawn from empirical distribution of Ibbotson-Sinquefield risk premium (1926-1985); ε_t drawn from empirical distribution of market model residuals fitted from 50 randomly selected NYSE stocks (1983-1985). For daily data (columns (6)-(9)) the model is

$$r_t = \alpha + \beta r_{mt} + \varepsilon_t$$

The sampling distribution is based on 25,000 replications of simulated data with 250 observations per replication. Simulated data are based on: $\alpha = 0$; $\beta = 1$; r_{mt} drawn from the empirical distribution of the CRSP equally-weighted index (1962-1985); ε_t drawn from the empirical distribution of daily residuals fitted from 50 randomly selected NYSE stocks (1983-1985). Columns (2) and (6) present the sample mean of $\hat{\alpha}$ (standard deviation in parentheses); columns (3) and (7) present the sample mean of $\hat{\beta}$ (standard deviation in parentheses). Columns (4) and (8) report the average absolute deviation of estimated beta away from one. Columns (5) and (9) report the cross-sectional variance of beta estimated with OLS, divided by the cross-sectional variance of beta estimated with each method.

Method[a]	$\hat{\alpha}$	$\hat{\beta}$	ABSD	V_{OLS}/V_M	$\hat{\alpha}$	$\hat{\beta}$	ABSD	V_{OLS}/V_M
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
OLS	0.0000 (0.0200)	1.0024 (0.3977)	0.30	1.00	0.0001 (0.0013)	0.9912 (0.2127)	0.16	1.00
MAD	-0.0026 (0.0205)	1.0059 (0.4119)	0.32	0.93	-0.0004 (0.0012)	0.9887 (0.1806)	0.14	1.39
TRQ (α=0.10)	-0.0019 (0.0183)	1.0016 (0.3658)	0.28	1.18	-0.0004 (0.0012)	0.9913 (0.1764)	0.13	1.45
TRQ (α=0.20)	-0.0021 (0.0183)	1.0027 (0.3663)	0.28	1.18	-0.0005 (0.0012)	0.9910 (0.1715)	0.13	1.54
TRQ (α=0.25)	-0.0021 (0.0185)	1.0030 (0.3695)	0.28	1.16	-0.0005 (0.0012)	0.9904 (0.1712)	0.13	1.54
Trimean	-0.0022 (0.0186)	1.0032 (0.3758)	0.29	1.12	-0.0004 (0.0012)	0.9911 (0.1730)	0.13	1.51
Gastwirth	-0.0021 (0.0189)	1.0032 (0.3765)	0.29	1.12	-0.0005 (0.0012)	0.9894 (0.1754)	0.13	1.47

[a]Estimation methods are: Ordinary Least Squares (OLS); Minimum Absolute Deviations (MAD); Trimmed Regression Quantiles (TRQ) with trimming proportion α = 0.10, 0.20 or 0.25; Tukey's trimean; Gastwirth estimator.

Table 3

Summary statistics for the sampling distribution of estimated intercept ($\hat{\alpha}$) and slope ($\hat{\beta}$) from market model regression

$$(r_t - r_{ft}) = \alpha + \beta(r_{mt} - r_{ft}) + \varepsilon_t$$

Sampling distribution based on 25,000 replications of simulated data with 36 observations per replication. Simulated data based on: $\alpha = 0$; $\beta = 1$; $(r_{mt} - r_{ft})$ drawn from empirical distribution of Ibbotson-Sinquefield risk premium (1926-1985). In columns (2)-(5), ε_t is drawn from empirical distribution of market model residuals fitted from 50 NYSE stocks with lowest compound return (1983-1985); in columns (6)-(9), ε_t is drawn from empirical distribution of market model residuals fitted from 50 NYSE stocks with highest compound return (1983-1985).

Method[a]	$\hat{\alpha}$	$\hat{\beta}$	ABSD	V_{OLS}/V_M	$\hat{\alpha}$	$\hat{\beta}$	ABSD	V_{OLS}/V_M
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
OLS	0.0001	0.9977	0.30	1.00	0.0001	1.0007	0.28	1.00
	(0.0198)	(0.4019)			(0.2008)	(0.3887)		
MAD	-0.0012	0.9965	0.31	1.00	-0.0083	1.0025	0.28	1.14
	(0.0193)	(0.4010)			(0.0176)	(0.3645)		
TRQ (α=0.10)	-0.0008	0.9962	0.28	1.22	-0.0056	1.0006	0.25	1.41
	(0.0177)	(0.3644)			(0.0161)	(0.3278)		
TRQ (α=0.20)	-0.0013	0.9960	0.28	1.23	-0.0071	1.0017	0.25	1.42
	(0.0176)	(0.3628)			(0.0159)	(0.3264)		
TRQ (α=0.25)	-0.0013	0.9963	0.28	1.21	-0.0075	1.0022	0.25	1.40
	(0.0177)	(0.3648)			(0.0160)	(0.3288)		
Trimean	-0.0012	0.9956	0.29	1.17	-0.0069	1.0001	0.26	1.35
	(0.0180)	(0.3714)			(0.0163)	(0.3346)		
Gastwirth	-0.0014	0.9969	0.29	1.17	-0.0076	1.0018	0.26	1.34
	(0.0181)	(0.3718)			(0.0163)	(0.3353)		

[a]Estimation methods are: Ordinary Least Squares (OLS); Minimum Absolute Deviations (MAD); Trimmed Regression Quantiles (TRQ) with trimming proportion α = 0.10, 0.20 or 0.25; Tukey's trimean; Gastwirth estimator.

Table 4

Comparison between betas estimated with Ordinary Least Squares (OLS) and Trimmed Regression Quantile (TRQ) with trimming proportion $\alpha = 0.10$. Data are monthly returns (Jan. 1983 - Dec. 1985) on 50 randomly selected NYSE firms (Random); on 50 NYSE firms with lowest compound return over the period (Losers) on 50 NYSE firms with highest compound return over the period (Winners). The first row reports the average absolute difference between beta estimated with TRQ and beta estimated with OLS; the second row reports the cross-sectional variance of beta estimated with OLS, divided by cross-sectional variance of beta estimated with TRQ.

	Random	Losers	Winners
Mean $\lvert\beta_{TRQ} - \beta_{OLS}\rvert$	0.0819	0.2259	0.1429
V_{OLS}/V_M	1.1322	1.2913	1.3601

Table 5

Cross-sectional beta estimates from Ordinary Least Squares (OLS) and Trimmed Regression Quantiles (TRQ) with trimming proportion $\alpha = 0.10$, for 661 initial public offerings by trading day relative to the day of listing.

Trading Day	One Day Returns		Two Day Returns	
	OLS	TRQ	OLS	TRQ
1	4.26	3.20	3.39	2.52
2	1.62	0.92		
3	1.24	0.71	1.27	1.08
4	1.74	1.25		
5	1.42	0.90	1.33	1.19
6	1.19	0.72		
7	1.36	0.85	1.15	0.97
8	1.02	0.63		
9	1.46	0.87	1.27	1.08
10	1.27	0.74		