...suration.

ELEMENTS

OF

MENSURATION.

CONTAINING

RULES FOR THE SOLUTION OF THE PRINCIPAL PROBLEMS EXPRESSED IN THE MOST CONCISE MANNER, ACCOMPANIED BY EXPLANATIONS ADAPTED TO THE UNDERSTANDING OF PUPILS WHO HAVE NOT PREVIOUSLY STUDIED GEOMETRY

TOGETHER WITH

Numerous Examples Illustrating the Various Rules.

BY

M. H. RODGERS,

TEACHER OF MATHEMATICS IN THE GIRLS' HIGH AND NORMAL SCHOOL OF PHILADELPHIA.

PHILADELPHIA:
PUBLISHED BY E. H. BUTLER & CO.
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Fr. Cajori

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Rodgers' Mensuration.

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OFFICE OF THE CONTROLLERS OF PUBLIC SCHOOLS,
FIRST SCHOOL DISTRICT OF PENNSYLVANIA.

Philadelphia, February 12th, 1862.

At a meeting of the Controllers of Public Schools, First District of Pennsylvania, held at the Controller's Chamber, on Tuesday, February 11th, 1862, the following Resolution was adopted:—

Resolved, That Rodgers' Mensuration be introduced to be used in the Public Schools of this District.

ROBERT J. HEMPHILL,
Secretary.

PREFACE.

PERHAPS in no branch has the method of instruction, to the young student, been so greatly improved during the last ten years as in Mathematics.

Pupils are no longer satisfied with committing mechanically rules expressing to them no meaning, because not understood. They no longer receive the words of their text-books, nor the assertions of their teachers, as *sufficient proof of the truth of the information* imparted to them. Having been taught to inquire the *reason why*, they are unwilling to receive a rule without investigating whence it came, and being convinced *why* it will solve the required problem.

In the pursuance of such investigations, the mind, refusing to become a mere automaton, must reason, comprehend, and decide for itself.

The dislike evinced by so many of the young for the study of Mathematics, may be traced to the fact that the text-books placed in their hands are frequently too abstruse for their comprehension.

The *want of simple explanations* in treatises on Mensuration has been greatly felt, more especially by those pupils who have not previously studied Geometry. To such, the solutions of the problems are in a great degree unintelligible, and the retention of the rules rendered doubly difficult, because solely dependent upon the memory.

It has been the aim of the Author, in preparing the present work, to *supply* this *deficiency*.

The wording and explanations of the rules have been made as clear and brief as possible, and explanations have been given in all cases where they came within the comprehension of pupils unacquainted with Geometry and higher Algebra.

Particular attention has been paid to the *correctness* and the *arrangement* of DEFINITIONS.

The EXAMPLES, which are original in construction, have been made very *numerous*, and illustrative of every variety of application of the different problems.

Great pains have been taken in the SOLUTIONS given of examples to place the *dimensions* in the *figures*, an omission common to most authors.

The same care has been employed to avoid the *improper use of abstract numbers*, a fault found in most treatises on this subject, the *solutions of examples* being performed with *abstract numbers*, yet producing *concrete numbers* for *results*.

Hoping this work will accomplish the object for which it was written, the Author commends it to the favorable attention of teachers.

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MENSURATION.

Mensuration is practical geometry, or that part of geometry which teaches how to find the lengths of lines, the areas of surfaces, and the volumes of solids.

DEFINITIONS.

A *mathematical point* is that which indicates *position* only, and has no *magnitude.*

The point of a fine needle, or the representation of a point on the black board, is a *physical* point, perceptible to the senses, and as such possesses magnitude; but a mathematical point, having no magnitude, cannot, strictly speaking, be represented.

The beginning and termination of a line are points, but these form no part of the line itself.

A *line* is that which has *length* only.

All lines are either *straight, curved*, or combinations of these two.

A *straight line* is one that does not change its direction between its extremities, and is the *shortest distance* between two points.

A *curved line* is one that changes its direction at every point.

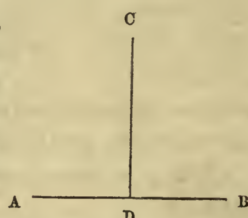
A *broken line* is one that is made up of a number of limited straight lines. When the word *line* only is used, a *straight line* is meant.

Lines, according to the manner in which they are drawn, are termed *parallel*, *oblique*, *perpendicular*, *vertical*, and *horizontal*.

Parallel lines are those, which being situated in the same plane, if produced to any extent, both ways, never meet.

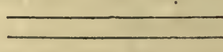
Oblique lines are those which do not maintain the same distance apart, but meet if sufficiently produced.

A *perpendicular line* is one that meets another, making the angles on each side equal. The angles thus formed are called *right angles*. Thus the line C D is perpendicular to the line A B, and the line A B is perpendicular to C D.



Vertical lines are those which are perpendicular to the horizon or water level. Vertical lines at different points on the earth's surface are not parallel, but converge towards the centre.

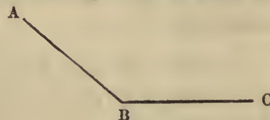
Horizontal lines are those which are parallel to the horizon, or water level.



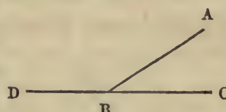
A *line* is *intersected* when crossed or cut by another. Thus the line A B intersects D C. The line is said to be *bisected* when it is divided into two *equal* parts. Thus the line E D is bisected.

ANGLES.

A plane rectilineal angle is the opening or inclination of two straight lines meeting in a point. The two lines which form it are called its *sides*, and the point where they meet its *vertex*. Thus A B C is an angle.



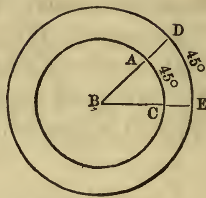
An *angle* is *read* by naming the letters or figures which stand at the *vertex*, and at the *ends* of the lines which form it, always placing the one at the *vertex* in the *middle*. Thus the angle A B C or the angle A B D. If there be but one angle at the vertex, it may be indicated by reading the letter at that point only, as the angle A.



Angles are *measured* by *arcs* of *circles*, and their size is expressed in degrees, minutes, and seconds.

To *measure* an angle, make the *vertex* of the angle the *centre* of the *circle*, then with the whole or part of one of its sides for a radius, describe a circle around its centre. Divide the circumference into 360 degrees, and the *number of degrees* contained in the *arc* (or part of the circumference) between its sides constitutes the *measure* of the angle.

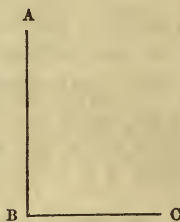
The *size of an angle* is not altered by having its sides produced. The lengthening of its sides increases the circumference of the circle in the same proportion, therefore it will take the same number of degrees to measure it as before. The *size of the degrees* is altered, but not their *number*.



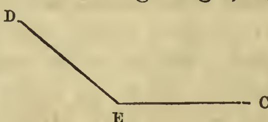
A B C is an angle of 45°, as is also D B E.

Angles are divided into *right*, *acute*, and *obtuse angles.*

A *right angle* is formed by a line meeting another *perpendicularly;* as, the angle A B C. It always contains 90 degrees.

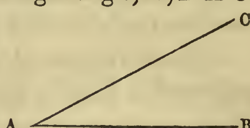


An *obtuse angle* is one *greater* than a right angle; as, D E C. It may contain any number of degrees between 90 and 180. It is called obtuse because its vertex is less pointed than that of a right angle.



An *acute angle* is one *less* than a right angle; as, B A C. It is called acute because its vertex is more pointed than that of a right angle.

All angles, except right angles, are called *oblique angles.*



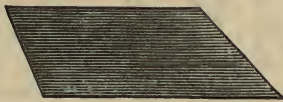
SURFACES.

A *surface* is that which has *length* and *breadth* only. Its boundaries are *lines.*

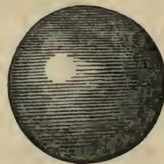
Surfaces are the *boundaries*, *faces*, or *limits* of solids;

they must be considered as making no part of the solids themselves, and therefore can have no *thickness.*

A *plane surface* is one with which a *straight line*, if laid in any direction, will exactly *coincide.*



A *curved surface* is one that, like a curved line, changes its direction at *every point.*

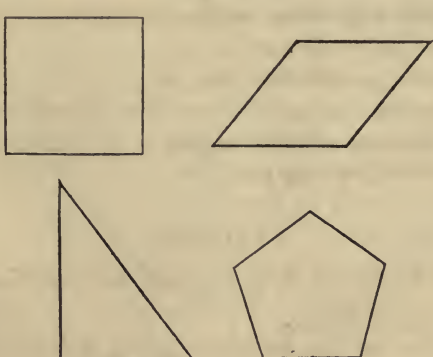


A *concave surface* is one which is rounded out on the inside into a spherical form, like the inner surface of a hollow globe.

A *convex surface* is one which swells on the outside into a round or spherical form, like the outside of a globe.

POLYGONS.

A *polygon* is any *plane figure* bounded by *straight lines;* as the following figures:



The *perimeter* of a polygon is the *sum* of the sides which bound it.

A *regular polygon* is one whose sides are all equal.

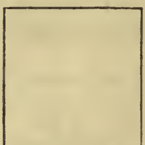
A *polygon* takes its *name* from the *number* of its *sides*.

A polygon of	three	sides	is called a	trigon or triangle.
"	four	"	"	tetragon or quadrilateral.
"	five	"	"	pentagon.
"	six	"	"	hexagon.
"	seven	"	"	heptagon.
"	eight	"	"	octagon.
"	nine	"	"	nonagon.
"	ten	"	"	decagon.
"	eleven	"	"	undecagon.
"	twelve	"	"	dodecagon.

There are *six tetragons* or *quadrilaterals*, *four* of which are called *parallelograms*.

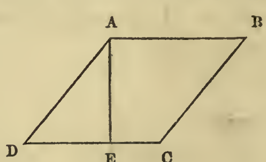
A *parallelogram* is a *quadrilateral*, whose *opposite sides* are *parallel*.

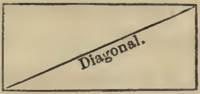
A *square* is a parallelogram whose *sides are equal*, and whose *angles are right angles*.



A *rhombus* is a parallelogram whose *sides are equal*, but whose *angles are not right angles*.

The *altitude* of any parallelogram is measured by a straight line drawn from the top of the figure perpendicular to the base; as, A E. The *base* is the line D C on which it stands.





A *rectangle* is a parallelogram whose *opposite sides only* are equal, and whose *angles are right angles.*

A *rhomboid* is a parallelogram whose *opposite sides only* are equal, but whose *angles are not right angles.*

A *diagonal* of a polygon is a straight line joining the vertices of two opposite angles.

The *area* or *superficial contents* of any figure is the amount of surface which it contains.

Similar polygons are those which have the same number of angles, which are equal each to each, and the sides about these angles, taken in the same order, proportional.

In similar polygons, the corresponding sides, angles, diagonals, &c., in each, are termed *homologous.*

Similar polygons are to each other as the squares of their homologous sides, or as the squares of their like dimensions.

A *Problem* is a question that requires a solution.

An *Axiom* is a self-evident truth ; as, "Things which are equal to the same thing, are equal to each other."

"Things which are doubles of equal things, are equal to each other."

"A straight line is the shortest distance between two points," &c.

TABLES.

LINEAR OR LONG MEASURE.

Linear or Long Measure is used to measure distances.

TABLE.

12 inches (*in.*)	make	1 foot,	*ft.*
3 feet	"	1 yard,	*yd.*
5½ yards, or 16½ feet	"	1 rod,	*rd.*
4 rods, 22 yards, or 66 feet	"	1 chain,	*ch.*
10 chains, or 40 rods	"	1 furlong,	*fur.*
8 furlongs	"	1 mile,	*m.*
3 miles	"	1 league,	*lea.*
69½ miles	"	1 degree, *deg.* or °	
360 degrees	"	The circumference of the earth.	

SURFACE OR SQUARE MEASURE.

Square Measure is used in measuring the areas of surfaces.

TABLE.

144 square inches (*sq. in.*)	make	1 square foot,	*sq. ft.*
9 square feet	"	1 square yard,	*sq. yd.*
30¼ square yards, or 272¼ square feet	"	1 square rod,	*sq. rd.*
16 square rods	"	1 square chain,	*sq. ch.*
2½ square chains, or 40 square rods	"	1 rood,	*R.*
4 roods, 160 square rods, or 10 square chains	"	1 acre,	*A.*
640 acres	"	1 square mile,	*sq. m.*

MENSURATION OF SURFACES.

PARALLELOGRAMS.

PROBLEM I.

To find the area of any parallelogram when the base and altitude are given.

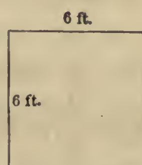
Rule.

Multiply the base by the altitude.

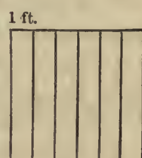
Note.—When the base and altitude are the same, this will be equivalent to squaring the side.

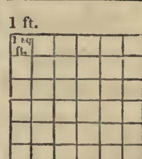
EXPLANATION.

1. What is the area of a square whose side is 6 feet?



If this square is 6 feet on each side, when the figure is divided into 6 equal parts, through its sides, each *strip* or *division* will be 1 *ft.* wide. If one of these strips be again divided, in the *opposite direction*, into 6 equal parts, these *subdivisions* will be each 1 *ft. long* and 1 *ft. wide*, or will contain 1 *sq. ft.* Hence in this strip or section there are 6 sq. ft., and, as there are 6 strips in all, there will be 6 times 6 sq. ft. in the figure, or 36 sq. ft.

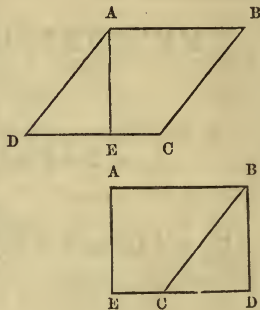




Res. 36 sq. ft.

A rhombus and rhomboid may be changed to a rectangle, thus:

Let A B C D be a rhombus, then through the altitude A E cut off the part A E D, and apply it to the other side of the figure, it will then become the rectangle A B D E. Since the base and altitude of both figures are alike, if their product gives the area of the rectangle, it will also give the area of the rhombus.



EXAMPLES.

2. What is the area of a square whose side is 12 feet? 144

3. How many acres in a garden lot 737 feet 6 inches square? 12.48A +

4. How many acres in a square piece of land whose sides are each 40 rods? 10 A.

5. How many square miles in a right-angled field whose sides are each 200 chains? 6 1/4 miles

6. What is the area of a rectangle whose length is 5 feet, and breadth 7 feet? 35 ft.

7. What is the area of a rectangle whose length is 13 chains, and breadth 9 chains? 117 chains

8. What is the area of a rhomboid whose length is 2 feet 5 inches, and its altitude 2 feet? 4 5/6 ft

9. What is the area of a rhombus whose base is 13 feet, and altitude 5 feet? 65 ft.

10. What is the area of a rhombus whose base is 15 feet, and altitude 13 feet? 195 ft.

11. What is the difference in area between a field 35 rods square, and one of half the dimensions?

12. How many small squares, each containing 4 square inches, are contained in a large one which is 3 feet square?

13. How many squares will it take to equal in area one of 40 rods square, if the smaller ones are each half the dimensions of the larger?

14. What is the area of a square field whose perimeter is 160 rods?

15. What is the area of a square whose perimeter is 140 chains?

16. If the sum of the base and altitude of a rhomboid is 64 feet, and the base is to the altitude as 3 to 5, what is the area?

17. If the sum of the base and altitude of a rhomboid is 128 feet, and the base is to the altitude as 7 to 9, what is the area?

18. The altitude of a rectangle equals 10 times the square root of 256, and the base is to the altitude as 1 to 2. What is the area?

19. How much will it cost to plaster a room, if the length is 25 feet, the width 16 feet, and height 17 feet 5 inches, at 15 cents a square yard?

20. How many yards in length of carpet, which is 3 quarters wide, will it take to cover a floor 22 feet wide, and 30 feet 3 inches long?

21. How many bricks, 8 inches long, and 4 inches wide, will it take to pave a yard which is 16 feet by 25 feet?

22. How many square yards of paper will it take to cover the walls of a room, which are 16 feet wide, 27 feet long, and 17 feet high, deducting $\frac{1}{16}$ of the surface for the doors

and windows? How much will it cost to lay it on, at a cent a square yard? 1522 yds cost $15.22

NOTE.—If the *area* of any parallelogram equals the *product* of its *base* and *altitude, the area divided by either dimension gives the other;* for if the product of two numbers, and one of them be given to find the other, we divide the product by the given factor.

As the *area* of a *square* is the product of two *equal factors,* the *square root* of its *area* equals the *side.*

23. The area of a square field is 663.0625 sq. rods. What is the length of its side?

SOLUTION. $\sqrt{663.0625 \text{ sq. rods}} = 25.75$ rods. Res.

24. The area of a square is 1600 sq. chains. What is the length of its side? 40 chains

25. What is the side of a square field whose area is 4 acres? 25.2+ rods

26. What is the side of a square lot whose area is 3 acres, 3 roods, and 25 sq. rods? 25 rods

27. The difference in area between two squares is 1600 sq. rods, and the area of the smaller one is 900 sq. rods. What is the side of the larger? 50 rods

28. If it take 64 blocks of stone, whose sides are each 20 inches, to lay a square pavement, how long and wide is the surface paved? 160 in

29. If the area of a rectangle is 149.875 sq. chains, and its length 27.25 chains, what is its breadth? 5.5 chains

30. The area of a rectangle is 2400 sq. rods, and its breadth 40 rods. What is its length? 60 rods

31. A lot 150 feet wide cost $210000, at the rate of $8 a square foot. What was its length? 175 ft.

32. I wish to saw a square yard from a plank 15 inches wide. How far from the end of the plank shall I commence to cut it? 56 2/5 in.

33. A rectangular field is 50 rods long, and contains 10 acres. How long must another field be, which has the same width, to contain 5 acres?

34. What is the difference between the area of a lot 30 feet square, and that of 2 others, each 15 feet square?

35. A man having a field 70 rods square, sold to D 100 square rods, and to E 5 acres. What fractional part of the field remained unsold?

36. A colonel forming his regiment into a square, finds he has 15 men over, but to increase the square, so that it shall contain one more in rank and file, he requires 48 more men. How many men had he?

37. How much ground will be required to enlarge a square lot, 5 feet on every side, whose area is 225 square feet?

NOTE.—The following examples are given to show that *the same perimeter does not always enclose the same area.*

That is, if a string 20 inches long be laid on a table in the form of a square, it will enclose an area of 25 sq. inches; but if it be placed in the form of a rectangle, whose sides are 8 and 2 inches, it will contain only 16 sq. inches, &c.

38. What is the difference in area between a rectangle which is 80 feet by 20 feet, and a square which has the same perimeter?

39. What is the perimeter of a square having the same area as a rectangle, whose length is 20 feet, and breadth 5 feet?

40. What must be the perimeter of a square, in order that it may contain the same area as a rectangle of 48 by 12 feet?

41. What is the difference in area between 2 rectangles, the first being 4 feet by 5 feet, and the second 1 foot by 8 feet?

42. The perimeters of 2 squares are to each other as 1 to 2. What relation do their areas hold to each other?

PROBLEM II.

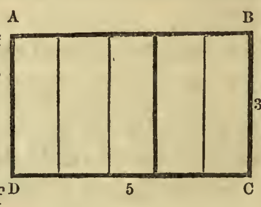
The area of a rectangle and the proportion of its sides being given, to find the length of those sides.

RULE.

To find the less side, multiply the area by the less number of the proportion, divide the product by the greater, and extract the square root of the quotient. Then find the greater side by simple proportion.

EXPLANATION.

Let A B C D be a rectangle whose sides are to each other as 3 is to 5.



If we divide it through its *longest* side into 5 *equal* strips, each strip will be 3 times as long as it is wide, and will equal $\frac{1}{5}$ of the rectangle.

If one of these divisions is 3 times as long as it is wide, 3 of them, that is, $\frac{3}{5}$ of the rectangle, will equal a square whose side is the less side of the rectangle, and the square root of these $\frac{3}{5}$ will be the side.

If the sides are to each other as 3 to 5, having the less side, we find the greater by simple proportion.

EXAMPLES.

1. The area of a rectangle is 60 sq. inches, and its length is to its breadth as 5 to 3. What are its sides?

SOLUTION. $\frac{3}{5}$ of 60 sq. in. = 36 sq. in.

$\sqrt{36 \text{ sq. in.}}$ = 6 inches, the less side.

3 : 5 : : 6 in. : 10 in., the greater side.

Res. Length 10 inches, breadth 6 inches.

2. A rectangle containing 192 square feet is 3 times as long as it is wide. How wide is it? 8 ft wide 24 ft lo

3. What is the breadth of a rectangle containing 150 sq. inches, whose length is to its breadth as 3 is to 2? 15 to 10

4. A rectangular lot contains 2400 sq. feet, and its length is 1½ times the breadth. What is the length? 60 ft 4

5. A rectangle contains 180 square feet, and the length is to the breadth as 5 to 4. What are the sides? 15 ft 12 f

6. In 2 square fields there are 8586 square rods, and their sides are to each other as 5 to 9. What is the length of a side of each? 45 rods 81 rods

NOTE.—If the sides are to each other as 5 to 9, their areas, being produced by the squares of their sides, will be to each other as 25 : 81. We therefore divide the sum of their areas in the proportion of 25 to 81, and extract the square root for the sides.

7. The contents of 2 square fields are 257125 square rods, and their sides are to each other as 6 to 7. What are their sides? 330 rods 385 rods

8. The area of 2 squares is 8352 sq. feet, and their sides are to each other as 3 to 7. What are their sides? 36 ft. 84

9. What are the areas of 2 square lots whose sides are to each other as 7 to 8, the contents of both lots being 1017 sq. feet? 441 ft 575 ft.

10. What relation do the sides of 2 squares hold to each other whose areas are 81, and 729 sq. feet? 1 to 3

PROBLEM III.

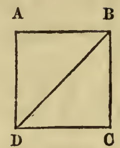
The side of a square given to find the diagonal.

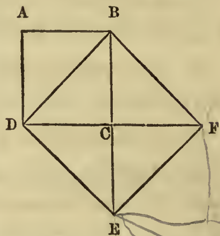
RULE.

Extract the square root of double the area of the square.

EXPLANATION.

Let A B C D be a square, and B D the diagonal. If we extend the line B C until C E equals it, and the line D C until C F equals it, and connect the points B and F, F and E, E and D, by straight lines, we will have described a square on the diagonal. Since this square contains four parts, each equal to half the original square, it must be twice as large.





Hence the square described on the diagonal equals twice the square in which it is found.

When we have given the side to find the diagonal, the square of the side gives the *area* of the square in which the diagonal is found, and double this, equals the square on the diagonal; then we have given the area of a square to find one side, therefore extract the square root.

EXAMPLES.

1. The side of a square is 25 feet. What is its diagonal?

SOLUTION. 25 ft.$^2 = 625$ sq. feet, the area of the square. Now double this gives the square of the diagonal, and the square root of the square of the diagonal equals the diagonal itself. Hence, $\sqrt{625 \text{ sq. feet} \times 2} = 35.35$ feet.

Res. 35.35 feet.

2. The side of a square is 37 chains. What is its diagonal?

3. The side of a square is 5 feet 7 inches. What is its diagonal?

4. The area of a square field is 6 acres. What is its diagonal?

5. What is the diagonal of a square whose side is 5 yards 2 feet 3 inches?

6. What is the diagonal of a square whose area is equal to that of a rhombus whose base is 19 feet, and altitude 16 feet?

7. What is the diagonal of a square equal in area to a rectangle, 50 feet by 75 feet?

8. A cubical room is 16 feet long. What is the diagonal of the floor?

9. A cubical room is 20 feet high. What is the diagonal of the floor? 28. 282 ft

10. The superficial contents of a cubical room are 1350 sq. feet. What is the diagonal of its sides?

NOTE.—The sides of the room being equal, the contents divided by 6 gives the area of one side, or the area of a square of which we are to find the diagonal.

11. The superficial contents of a cubical room are 3750 sq. feet. What is the diagonal of the ceiling? 35. 35 ft

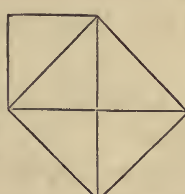
PROBLEM IV.

The diagonal of a square given to find the side.

RULE.

Extract the square root of half the square of the diagonal.

EXPLANATION.

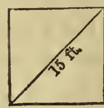


The square of the diagonal gives the area of the square described upon it, which is twice the square in which it is found. Therefore the square root of half of this square will be the side.

EXAMPLES.

1. What is the side of a square whose diagonal is 15 feet?

SOLUTION. $15 \text{ ft.}^2 = 225$ sq. feet, the square on the diagonal. 225 sq. feet $\div 2 = 112.5$ sq. feet, the area of the square.



$$\sqrt{112.5} \text{ sq. feet} = 10.606 \text{ feet.}$$

Res. 10.606 feet.

2. The diagonal of a square is 40 rods. What is the length of one side? 28.284 rods

3. The diagonal of a square is 55 chains. What is the side? 38.89 Chs.

4. The diagonal of a square field is 75 rods. What is the side? 53.033 rods

5. The diagonal of a square is 13 feet 5 inches. What is the length of one side? 9.486 ft

6. The diagonal of a square is 31 chains. What is the area? 480.5 chains

7. How many acres in a square lot whose diagonal is 16 chains 3 rods and 11 feet?

8. In going from the north-east to the south-west corner of a square field containing 4 acres, how much farther will it be to go along its sides than diagonally across? 14.84

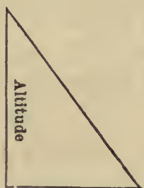
DEFINITIONS.

TRIANGLES.

A *triangle* is a plane figure bounded by *three straight* lines. A B C is a triangle.

The *base* of a triangle is generally the *side* on which it *stands;* as, A C.

The *altitude* is measured by a straight line drawn from the vertex opposite the base, perpendicular to the base. In *right-angled* triangles it equals one of the *sides*, in *acute-angled* triangles it is drawn *within*, and in *obtuse-angled* triangles sometimes *without* the figure.



Triangles are divided, according to their *sides*, into *equilateral*, *isosceles*, and *scalene* triangles.

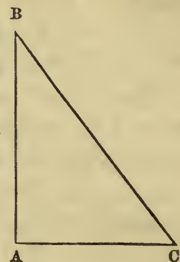
An *equilateral* triangle is one whose *sides* are *all equal.*

An *isosceles* triangle is one *two* of whose sides are *equal.*

A *scalene* triangle is one whose sides are all *unequal.*

Triangles are divided according to their *angles* into *right, acute,* and *obtuse-angled triangles.*

A *right-angled* triangle is one that has *one right angle.* The *hypothenuse* of a right-angled triangle is the *longest* side, or the side which is opposite to the right angle; as, the side B C. The side A C is called the *base,* and A B the *perpendicular.*



An *acute-angled* triangle is one whose angles are *all acute.*

An *obtuse-angled* triangle is one that has *one obtuse* angle.

When the *angles* of a triangle are *all equal,* it is termed *equiangular.*

Similar triangles are those which have all the angles of the one equal to all the angles of the other, each to each, and the sides about the equal angles proportional.

Similar triangles are to each other as the *squares* described on their *homologous sides.*

Similar triangles have their *like dimensions proportional.*

Triangles having the same *base* are to each other as their *altitudes.*

Triangles having the same *altitude* are to each other as their *bases*.

When two numbers are multiplied by the *same* number, their products are said to be *equimultiples* of those numbers.

Equimultiples of quantities have the same ratio as the quantities themselves.

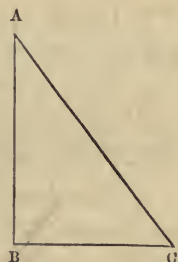
TRIANGLES.

PROBLEM V.

To find the area of a triangle when the base and altitude are known.

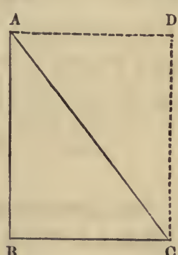
RULE.

Multiply the base by the altitude, and divide the product by 2.



EXPLANATION.

Let A B C be a triangle having B C for its base, and A B for its altitude. Add to this, in an inverted position, a triangle having the same base and altitude. Since the opposite sides of this figure are equal, it must be a *parallelogram*, and each triangle equals the *half* of it; as the *base* and *altitude* are *equal* in *both figures*, if the area of the parallelogram equals the product of the base and altitude, the *area of the triangle will equal half that product.*



EXAMPLES.

1. What is the area of a triangle whose base is 14 inches, and altitude 7 inches?

SOLUTION. 14 in. $\times$ 7 in. $=$ 98 sq. in. 98 sq. in. $\div$ 2 $=$ 49 sq. in. Res. 49 sq. in.

2. What is the area of a triangle whose base is 60 feet, and altitude 40 feet?

3. What is the area of a triangle whose base is 5 feet 5 inches, and perpendicular 6 feet 3 inches?

4. How many acres in a triangle whose sides, containing the right angle, are 60 rods and 57.38 rods?

5. What is the area of a triangle whose base is 3 rods 5 feet, and altitude 6 yards 2 inches?

6. How many acres in a triangle whose base is 36.25 chains, and altitude 27.59 chains?

7. How many acres in a triangle whose base is 60 rods, and its altitude $\frac{5}{7}$ of its base?

8. What is the area of a triangle which has the same base and altitude as a rectangle, whose sides are 50 feet, and 27 feet 6 inches?

9. How much was paid an acre for a triangular lot, which cost \$375, and whose base is 40, and altitude 60 rods?

10. What will be the cost of paving a triangular yard with bricks, at 24 cents per square yard, its base being 21 yards, and altitude 15 yards?

PROBLEM VI.

The area of a triangle and base given to find the altitude. Or, the area and either dimension, to find the other.

RULE.

Double the area, and divide by the given dimension.

EXPLANATION.

The *area* is *half* the product of the *base* and *altitude*, hence if we *double* the area we have the *product* of the *base* and *altitude;* then we have given the *product* of the base and altitude, and *one* of them to find the *other*. When we have the product of two factors, and one of them to find the other, we divide the product by the given factor.

EXAMPLES.

1. What is the altitude of a triangle whose area is 15 sq. inches, and base 5 inches?

SOLUTION. 15 sq. inches $\times$ 2 = 30 sq. inches. 30 sq. inches $\div$ 5 inches = 6 inches, Result.

15 sq. inches = area.

2. What is the altitude of a triangle whose area is 768 sq. feet, and whose base is 24 feet? 64 ft

3. What is the base of a triangle whose area is 5 sq. rods, 6 sq. yards, 3 sq. inches, and whose altitude is 3 rods 2 inches? 3.56

4. How many chains in the base of a triangle whose area is 56 sq. chains, and altitude 100 rods? 17.92+ rods

5. What is the altitude of a triangle whose area is 4 acres, and its base 350 rods? 3.65+ rods

6. What is the perpendicular of a triangular lot which cost $500, at the rate of $5 per square rod, if the base is 8 rods? 25 rods

7. A triangular field contains 600 sq. yards, and its base is to its altitude as 1 to 3. What are its base and altitude? 60 al[illegible] and 20 base

8. What is the base of a lot, which is laid out in the form of a right-angled triangle, and rents for \$62.50, at the rate of 25 cents per square rod, the altitude being 10 rods? 50

9. The base and altitude of a triangle, containing 121½ sq. chains, are to each other in the proportion of 1 to 3. What are the base and altitude? 25 ch. 9 chains

10. The area of a triangle is 283½ sq. feet, and its base and altitude are to each other as 7 to 9. What are its base and altitude? 27 ft 21 ft

PROBLEM VII.

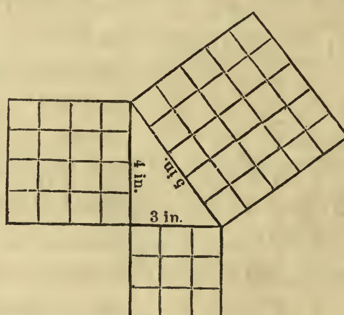
The base and perpendicular of a right-angled triangle being given, to find the hypothenuse.

RULE.

Extract the square root of the sum of their squares.

EXPLANATION.

Let us take for an example a triangle whose base is 3 inches, whose perpendicular is 4 inches, and hypothenuse 5 inches. Here the square of the hypothenuse will be 25 sq. in., the square of the base 9 sq. in., and that of the perpendicular 16 sq. in. Adding the last two, their sum gives 25 sq. in., which equals the square of the hypothenuse; hence *the square of the hypothenuse equals the sum of the squares of the other two sides.* Then when we have the base and perpendicular to find the hypothenuse, the sum of their squares gives the square of the



hypothenuse, and the square root of this square is the hypothenuse.

NOTE.—The solution of this problem, known as the 47th of Euclid, is said to have been discovered by Pythagoras, who was so rejoiced thereat that he sacrificed a hundred oxen to the gods, in thankfulness, for enabling him to solve it. The Persians are said to call it *The Bride*, because it has such a large family or number of propositions dependent upon it.

EXAMPLES.

1. What is the hypothenuse of a right-angled triangle whose perpendicular is 24 feet, and base 35 feet?

SOLUTION. 35 ft.^{2} = 1225 sq. ft. 24 ft.^{2} = 576 sq. ft. 576 sq. ft. + 1225 sq. ft. = 1801 sq. ft. $\sqrt{1801}$ sq. ft. = 42.43 ft. Res. 42.43 ft.

2. What is the hypothenuse of a triangle whose base is 57.05 rods, and perpendicular 60 rods?

3. What is the hypothenuse of a triangle whose base is 5 yards 2 feet 6 inches, and perpendicular 7 yards 1 foot 2 inches?

4. What is the diagonal of a field whose length is 25 chains, and breadth 23 chains? 33.95

5. What is the diagonal of the floor of a cubical room whose height is 16 feet? 22.627 ft

6. What is the diagonal of a cubical room whose length is 18 feet?* 31.17 ft

7. What is the diagonal of a room 35 feet long, 25 feet wide, and 20 feet high? 47 ft 23 ft

8 The area of a rectangular field is 12 acres, and its length is to its breadth as 3 to 1. What is its diagonal? 73.99

9. What is the longest straight line that can be drawn in a rectangle whose sides are 30 and 40 inches?

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* See Key.

10. In the centre of a field 40 rods square there is planted a pole 75 feet long. How long will a line be that will reach from the top of the pole to either corner of the garden?

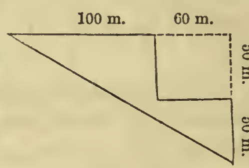
11. What is the length of a ladder one end of which rests against a tree 20 feet from the ground, and the other on the ground at a distance of 16 feet 5 inches from its trunk?

12. What is the length of a ladder one end of which is placed 25 feet from a building, and the other end against the house 30 feet from the ground?

13. What is the length of one of the equal sides in an isosceles triangle whose base is 42 feet, and altitude 30 feet?

NOTE.—*The perpendicular of an isosceles or equilateral triangle divides the base into two equal parts.*

14. A ship sailed from port north 50 miles, then west 60 miles, when she stopped to unload her cargo, after which she sailed still farther north 50 miles and west 100 miles. How far was she then in a direct line from the port whence she started?



15. A vessel sailed first south 25 miles, then east 75 miles, again south 80 miles, and east 70 miles. How far was she then from the port whence she started?

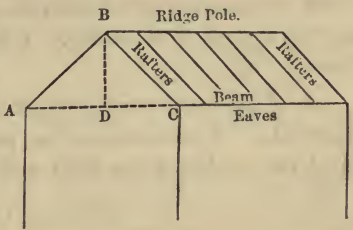
16. What is the perimeter of a right-angled triangle whose area is 121.5 sq. chains, and whose base is 3 times the altitude?

17. What is the perimeter of a triangle whose base of 45 feet is divided by its perpendicular into two parts, which are to each other as 2 to 7, the perpendicular being 50 feet?

18. What is the perimeter of a right-angled triangle whose area is 2437.5 sq. chains, and whose base is to its altitude as 13 to 15?

19. What is the longest side of a right-angled triangle whose area is 700 square feet, and whose base is to its altitude as 7 to 8?

Gable end—the triangular end of a house or other building from the eaves to the top, as A B C. B D is the *altitude* of the gable end.



Ridge pole—the upper horizontal timber in a roof, against which the rafters pitch.

Rafter—a piece of timber that extends from the plate of a building towards the ridge, and serves to support the covering of the roof.

Eaves—the edges of the roof of a building, which usually project beyond the face of the walls so as to throw off the water.

Beam or plate—the largest or principal piece of timber in a building, that lies across the walls, and serves to support the principal rafters.

20. The distance between the lower ends of two equal rafters is 30 feet, and the perpendicular distance of the ridge pole above the foot of the rafters is 10 feet. How long are the rafters?

21. The gable ends of a house are 24 feet wide, and the ridge is 15 feet above the eaves. How much will it cost to tin the roof, at 8 cents per sq. foot, if it is 30 feet long?

22. How wide is the gable end of a house, in which the rafters form a right angle at the top, and are 16 feet long on one side and 12 on the other, the eaves being at the same height?

23. A house, whose gable ends are 24 feet wide, is 38 feet long, and the height of the ridge above the beam is 10

feet. The roof projects 1 foot over the ends and eaves in all directions. How many shingles will be required to roof it, supposing each shingle to be 4 inches wide, and each course 6 inches?

24. The area of an isosceles triangle is 588 square chains, and its altitude is 42 chains. What is the length of one of its equal sides?

25. A castle 120 feet high is surrounded by a ditch 40 feet wide. What must be the length of a rope to reach from the outside of the ditch to the top of the castle?

PROBLEM VIII.

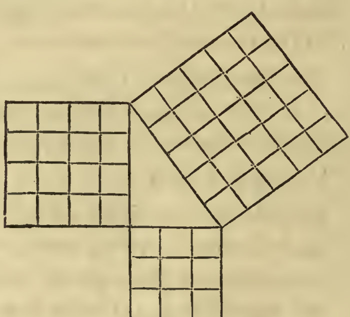
The hypothenuse and either side of a right-angled triangle being given, to find the other side.

RULE.

Extract the square root of the difference of their squares.

EXPLANATION.

If the square of the hypothenuse equals the sum of the squares of the other two sides, *the square of the hypothenuse minus the square of the given side will equal the square of the required side.* When we have the square to find the side, extract the square root.



EXAMPLES.

1. The hypothenuse of a triangle is 5 inches, and the base is 3 inches. What is the perpendicular?

SOLUTION. $5 \text{ in.}^2 = 25$ sq. in. $3 \text{ in.}^2 = 9$ sq. in. 25 sq. in. — 9 sq. in. = 16 sq. in. $\sqrt{16 \text{ sq. in.}} = 4$ in.

Result. 4 in.

2. The hypothenuse of a right-angled triangle is 13 feet, and the altitude 11 feet. What is the base?

3. What is the perpendicular of a right-angled triangle whose hypothenuse is 29 feet, and base 17 feet?

4. The hypothenuse of a triangle is 35 feet, and the perdicular 20 feet. What is the base?

5. The hypothenuse of a triangle is 17 feet and 5 inches, and the base 11 feet and 7 inches. What is the perpendicular?

6. One end of a ladder, which is 7 yards long, rests on the ground 13 feet from the trunk of a tree, the other end leans against the tree 5 feet from its top. How high is the tree?

7. A ladder 40 feet long is so placed in the street that it reaches a window 30 feet from the ground, and, when turned to the opposite side, without changing the position of the foot, reaches another window 25 feet from the ground. How wide is the street?

8. The diagonal of a rectangular field is 50 rods, and its length 40 rods. What is its breadth?

9. Two boys flying a kite wished to ascertain its height; the one held the string close to the ground, and the other placed himself directly under the kite; they found the distance between them was 60 feet, and that the length of line out was 95 feet. How high was the kite?

10. The top of a flag-staff being broken off in a storm, the broken part rested upon the upright, and the top on the ground 30 feet from its foot. The broken part measured 45 feet. How high was the staff?

11. What is the altitude of an equilateral triangle whose sides are each 40 feet?

12. The perimeter of a field in the form of an equilateral triangle is 30 chains. How many acres does the field contain?

13. How wide is the gable of a house, in which the rafters are 25 feet long, and the height of the ridge pole above the eaves 12 feet?

14. The gable end of a house is 24 feet wide, and the rafters are 16 feet on each side of the roof. What is the perpendicular distance of the ridge pole above the eaves?

15. How many square feet of boards will it take to close up the gables of a barn whose rafters are 20 feet long, and the distance of the ridge above the foot of the rafters 10 feet?

16. A triangular lot whose hypothenuse is 95 feet, and perpendicular 76 feet, rents for $300 a year. How much is that a square foot?

17. A ladder 50 feet long is placed against a house of the same height, so that the end which rests on the ground is 18 feet from the house. How far from the top of the house is it placed?

18. Two men started from the same place, and travelled, one south at the rate of 4 miles an hour, and the other south-east at the rate of 5 miles an hour. After travelling 11 hours, they turned and travelled directly towards each other, at the same rate as before, until they met. How far did each one travel?

PROBLEM IX.

The base and the sum of the other two sides of a right-angled triangle being given, to find those sides. Or,

The sum of two numbers and the difference between their squares being given, to find those numbers.

Since the square of the base equals the difference between the squares of the other two sides, we have the following

RULE.

Divide the square of the base, or the difference between the squares of two numbers, by their sum, and the quotient will be the difference of those numbers.

Add this sum and difference together, and divide the result by 2 for the larger number or hypothenuse.

Subtract the hypothenuse from the sum for the perpendicular.

EXPLANATION.

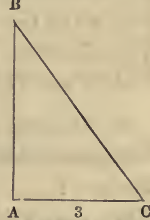
The sum of 5 and 4 = 9.
The difference of 5 and 4 = 1.
The product of this sum and difference = 9.
The difference between the squares of these two numbers, or $5^2 - 4^2 = 9$; hence,

The difference between the squares of two numbers equals the product of two factors, viz., the sum and difference of those numbers; therefore,

The difference between the squares of two numbers divided by their sum gives their difference.

When the sum and difference of two numbers are given to find the larger, we add them together and divide by 2.

Let A B C be a triangle whose base is 3, and the sum of the other two sides 9. Then the square of the base, or 9, equals the difference between the squares of the other two sides; this, divided by 9, their sum, gives 1, their difference, whence



A B + B C = 9

$(9 + 1) \div 2 = 5$, the larger side, or hypothenuse.

$9 - 5 = 4$, the smaller side, or perpendicular.

EXAMPLES.

1. The base of a triangle is 18 inches, and the sum of the hypothenuse and perpendicular is 54 inches. What is the length of the hypothenuse?

2. The base of a triangle is 20 feet, and the sum of the hypothenuse and perpendicular is 40 feet. What is the perpendicular?

3. The base of a triangle is 210 chains, and the sum of the hypothenuse and perpendicular is 630 chains. What are the hypothenuse and perpendicular?

4. At what distance above the ground did a tree, which was 90 feet high, break off, when the broken part rests on the upright, and the top on the ground 30 feet from the foot of the tree?

5. A flag-staff 125 feet high was broken by the wind, the top struck the ground 40 feet from the foot of the staff, and the broken end rested on the upright part. What was the length of the broken piece?

6. The base of a right-angled triangle is 28 feet, and the *difference* of the other two sides is 14 feet. What are those sides?

7. A pole standing in a field was broken by a storm, and fell so that one end rested on the ground 33 feet from the foot, while the other remained attached to the upright part. The difference between the parts of the pole after it was broken was 11 feet. How high was the pole at first?

8. The sum of the hypothenuse and perpendicular of a triangle is 104 feet, and their difference is 26 feet. What is the base?

9. A field enclosed in the form of a right-angled triangle has 136 rods of fence in its perpendicular and hypothenuse;

the difference of fence in the same two sides is 34 rods. What is the area of the field?

10. A started from Philadelphia and travelled south 20 miles. B started from the same city, and after travelling west for a certain distance, turned and journeyed in a straight line until he reached the point at which A had stopped. The distance A travelled was $\frac{1}{3}$ of the distance they both travelled. How far was B from A when he changed his course?

PROBLEM X.

The three sides of a triangle being given, to find a perpendicular which will divide it into two right-angled triangles.

NOTE.—If we consider the *longest* side to be the *base* of the triangle, and draw from the *vertex opposite* the *base* a line *perpendicular* to *it*, this line will be the *perpendicular* of

the triangle, and will divide the base into two parts. These *parts* form the *bases* of two right-angled triangles (right-angled because the line is perpendicular to the base), whose *hypothenuses* are the *sides*, and whose *perpendiculars* are the *perpendicular* of the *triangle*.

To find the *smaller part*, or the *base* of the *smaller right-angled triangle*, we have the following

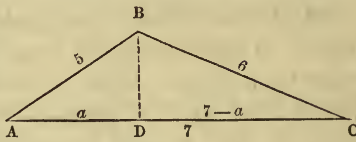
RULE.

From the square of the base subtract the difference of the squares of the other two sides, and divide the remainder by twice the base.

We then have the base and hypothenuse of the smaller right-angled triangle to find the perpendicular.

EXPLANATION.

Let A B C be a triangle having 5, 6, and 7 for its sides; what is the length of a perpendicular which will divide it into two right-angled triangles?



Consider A C the base, and draw the line B D perpendicular to it. Since the two smaller triangles thus formed are right-angled, we first find A D, in order to obtain B D.

Let A D be represented by the letter a, then D C will equal $7 - a$.

The square of the perpendicular of a right-angled triangle equals the square of the hypothenuse minus the square of the base; hence the square of B D equals $25 - a^2$, and for the same reason the square of B D equals $36 - (7 - a)^2$; therefore these differences must be equal. For things which equal the same thing, are equal to each other; and we have

$25 - a^2 = 36 - (7 - a)^2$; completing the square,

$25 - a^2 = 36 - 49 + 14a - a^2$; cancel and transpose,

$$49 + 25 - 36 = 14a.$$

$$38 = 14a.$$

$$a = \tfrac{38}{14}, \text{ or } 2\tfrac{5}{7}.$$

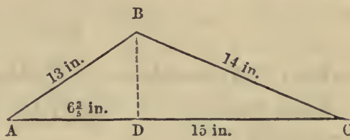
Hence the rule, from the square of the base, or 49, subtract the difference of the squares of the other two sides, or 11, and we have 38, which, divided by twice the base, or 14, gives $2\frac{5}{7}$, which is the value of a, or the length of the line A D.

We then have A D the base, and A B the hypothenuse, to find the perpendicular.

EXAMPLES.

1. The sides of a triangle are 13, 14, and 15 inches.

What is the length of a perpendicular that will divide it into two right-angled triangles, or what is its altitude?



SOLUTION. $14 \text{ in.}^2 - 13 \text{ in.}^2 = 27$ sq. inches.

$15 \text{ in.}^2 - 27$ sq. in. $=$ 225 sq. in. $-$ 27 sq. in. $=$ 198 sq. in.

198 sq. in. $\div$ (15 in. $\times$ 2) $= 6\frac{3}{5}$ in., the length of A D.

$\sqrt{13 \text{ in.}^2 - 6\frac{3}{5} \text{ in.}^2} = \sqrt{\frac{3136}{25}}$ sq. in. $= 11\frac{1}{5}$ in., the altitude. Res. $11\frac{1}{5}$ inches.

2. What is the perpendicular of a triangle whose sides are 3, 4, and 6 inches?

3. What is the altitude of a triangle whose sides are 15, 18, and 25 inches?

4. What is the perpendicular of a triangle whose sides are 22, 27, and 40 feet?

5. What is the perpendicular of a triangle whose sides are 60, 70, and 90 chains?

6. What is the altitude of a scalene triangle whose base is 240 chains, and the other sides 100 and 200 chains?

7. What is the altitude of a triangle whose base is 68 rods, and its other sides 50 and 25 rods?

8. What is the perpendicular of a triangle whose sides are 350, 150, and 220 chains?

9. What is the height of the gable end of a house whose width is 28 feet, the rafters being 18 feet long on one side and 25 feet on the other?

10. What is the altitude of a triangle whose base is 15 feet, and its other sides 11 and 12 feet?

PROBLEM XI.

The three sides of a triangle being given, to find the area.

RULE.

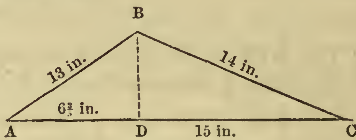
First find the altitude by Problem X.,

Then multiply the base by the altitude, and divide the product by 2.

NOTE.—The rules used in the solution of this problem have been explained.

EXAMPLES.

1. What is the area of a triangle whose sides are 13, 14, and 15 inches?



SOLUTION. $14 \text{ in.}^2 - 13 \text{ in.}^2 = 27$ sq. inches.

$15 \text{ in.}^2 - 27$ sq. in. $= 225$ sq. in. $- 27$ sq. in., or 198 sq. in.

198 sq. in. $\div$ (15 in. $\times$ 2) $= 6\frac{3}{5}$ in., the length of A D.

$\sqrt{13 \text{ in.}^2 - 6\frac{3}{5} \text{ in.}^2} = \sqrt{\frac{3136}{25} \text{ sq. in.}} = \frac{56}{5} \text{ in.} = 11\frac{1}{5}$ inches, the length of B D.

$11\frac{1}{5}$ in. $\times \frac{15}{2}$ in. $= 84$ sq. inches.

Res. 84 square inches.

2. What is the area of a triangle whose sides are 5, 6, and 7 inches?

3. What is the area of a triangle whose sides are 3, 4, and 6 inches?

4. What is the area of a triangle whose sides are 15, 18, and 25 inches?

5. What is the area of a triangle whose sides are 25, 30, and 46 inches?

6. How many acres in a field whose sides are 25, 30, and 50 rods?

7. How many square yards in a lot whose sides are 76, $43\frac{1}{2}$, and 35 yards?

8. What is the rent of a field whose sides are 80, 90, and 125 rods, at the rate of $2.00 per acre?

9. How many square feet of boards will it take to board up the gable ends of a house, the rafters being 18 feet long on one side and 20 feet on the other, and the gables 30 feet wide?

10. What ratio does a triangle whose sides are 65, 70, and 75 feet, hold to another whose sides are 13, 14, and 15 feet?

11. What is the area of an equilateral triangle whose sides are each 20 inches?

12. What is the area of an isosceles triangle whose base is 26 inches, and each of the equal sides 38 inches?

PROBLEM XII.

The area of a triangle and the proportion of its sides being given, to find their length.

RULE.

Find the area of a triangle whose sides equal the numbers expressing the given proportion.

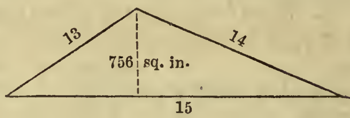
Then as this area is to the given area, so is the square of either side to the square of the required side.

EXPLANATION.

Since these triangles have the *proportion* of their *sides* the *same*, they are *similar*, and will therefore be to each other as the squares of their like dimensions. Having found one of the required sides, the rest can be ascertained by simple proportion.

EXAMPLES.

1. The area of a triangle is 756 square inches, and its sides are to each other as 13, 14, and 15. What is the length of each side?



SOLUTION. 15 in.^{2} — (14 in.^{2} — 13 in.^{2}) = 198 sq. in.
198 sq. in. ÷ 30 in. = $6\frac{3}{5}$ in., the smaller base.
13 in.^{2} — $6\frac{3}{5}$ in.^{2} = $\frac{3136}{25}$ sq. in.
$\sqrt{\frac{3136}{25}}$ sq. in. = $\frac{56}{5}$ in., the altitude.
$\frac{56}{5}$ in. × $\frac{15}{2}$ in. = 84 sq. in., the area of a triangle whose sides are to each other as 13, 14, and 15.
84 sq. in. : 756 sq. in. : : 13 in.^{2} : square of the similar side, or 1521 sq. in.
$\sqrt{1521 \text{ sq. in.}}$ = 13 in., the side.

13 : 14 : : 39 in. : 42 in.
13 : 15 : : 39 in. : 45 in.

39, 42, and 45 in. Res.

2. The area of a triangle is 756 square feet, and its sides are to each other as 13, 14, and 15 feet. What is the length of each side?

3. The area of a triangle is 1.781 acres, and its sides are to each other in the proportion of 5, 6, and 10 rods. What are those sides?

4. What are the sides of a triangle containing 486 sq. chains, if they are in the proportion of 3, 4, and 5 chains?

5. The sides of a triangular plot of ground are in the proportion of 2, 3, and 4 feet, and it contains 418.282 sq. feet. What is the length of each side?

6. If a triangular piece of ground containing 27 acres measures 60 rods on one side, what would be the corresponding side of a similar triangle containing 3 acres?

7. The area of a triangle is 6 acres, and its base is 10 chains. What is the area of a similar triangle whose base is 30 chains?

8. What relation does a triangle whose base and altitude are 5 and 7 feet, hold to one whose area is 105 square feet?

9. If the perpendicular and base of a right-angled triangle are 18 and 24 chains, what will be the sides of a triangle containing 54 sq. chains, which I cut off from it parallel to its base?

10. I wish to enclose 336 square rods in the form of a triangle whose sides shall be to each other as 13, 14, and 15. What must be the length of each side?

PROBLEM XIII.

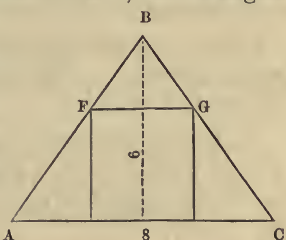
The base and perpendicular of a triangle being given, to find the side of the inscribed square.

RULE.

Divide their product by their sum.

EXPLANATION.

Since the line F G is parallel to A C, the triangles F B G and A B C are similar, and therefore have their like sides proportional, that is, 8 the base of the larger : 6 its altitude : : the side of the square, or base of the smaller : 6 — that side, or its altitude.



In every proportion the products of the extremes and means are equal, therefore 48 — 8 times the side of the square equals 6 times its side; hence 48 equals 14 times the side of the square. and the side will be $\frac{48}{14}$, or $3\frac{3}{7}$, whence the rule,

Divide the product of the base and perpendicular, or 48, by their sum, or 14.

EXAMPLES.

1. The base of a right-angled triangle is 4 feet, and the perpendicular 9 feet. What is the side of the inscribed square?

2. The base of an isosceles triangle is 20 chains, and the perpendicular 30 chains. What is the side of the inscribed square?

3. The base of a right-angled triangle is 75 feet, and its perpendicular 50 feet. What is the area of the inscribed square?

4. A triangle contains 10 acres, and its base is 50 rods. What is the side of the inscribed square?

5. The equal sides of an isosceles triangle are eacn 20 feet, and its base is 30 feet. What is the area of the greatest square that can be drawn within it?

6. The hypothenuse of a triangle is 45 feet, and its base is 27 feet. What is the side of the inscribed square?

7. Having inscribed a square in a triangle whose base is 16 feet, and altitude 26 feet, I find I have also formed 3 smaller triangles. What is the area of the one similar to the original triangle?

8. Having inscribed a square in a triangle whose base is 50, and altitude 70 feet, I wish to know the area of the 2 small right-angled triangles whose altitudes form sides of the square?

9. The sides of a scalene triangle are 13, 14, and 15 feet.

What is the side of the greatest square that can be drawn within it?

10. A triangular farm, whose base was 400 and altitude 320 rods, was divided among 3 children as follows: the eldest received the greatest square that could be drawn within it, the youngest the 2 right-angled triangles left after the eldest had received his portion, and the second the remainder. How many acres did each one's share contain?

PROBLEM XIV.

Two triangles which are alike in one dimension, having their areas and the unequal dimension of the one given, to find the corresponding dimension of the other.

RULE.

As the area of the one whose dimension is given, is to the one whose dimension is required, so is the given to the required dimension.

EXPLANATION.

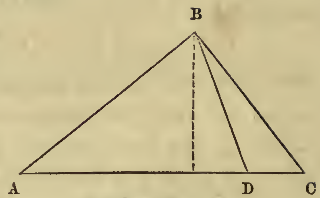
Triangles which are alike in one dimension are to each other as their unequal dimensions, that is, when their bases are equal they are to each other as their altitudes, and when their altitudes are equal they are to each other as their bases.

The area of a triangle is the product of two factors, the base and half the altitude, hence the *areas* of two triangles having the *same altitude* are *equimultiples* of their *bases*, but equimultiples of quantities have the same ratio as the quantities themselves, whence the rule.

The same reasoning applies when their *bases* are *equal.*

EXAMPLES.

1. A triangle whose base is 10 feet contains 120 square feet. What is the base of a triangle having the same altitude, whose area is 6 square feet?



A B C = 120 sq. ft.
A B D = 6 sq. ft.
A D C = 10 ft.

SOLUTION.—120 sq. ft. : 6 sq. ft. : : 10 ft. : required base or ½ foot. Res. ½ of a foot or 6 inches.

2. A triangle whose base is 13 feet contains 65 square feet. What is the base of a triangle having the same altitude, whose area is 25 square feet?

3. The areas of two triangles, having equal bases, are 5 and 37 acres, and the altitude of the smaller is 40 rods. What is the altitude of the larger?

4. Two triangles having the same altitude contain 50 and 75 square rods. What is the base of the larger if that of the smaller is 5 rods?

5. What relation do the areas of 2 triangles hold to each other, whose altitudes are 15 and 25 feet, their bases being the same?

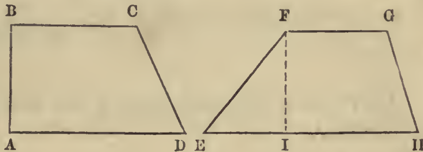
6. Given the area of the triangle A B C equals 2 acres, the triangle A B D equals 15 sq. chains, and the line A C equals 10 chains, to find D C?

7. I have a triangular piece of land whose area is 19 acres and 2 sq. chains, the base of which is 24 chains. I desire to divide it into 3 triangles holding the relation to each other of 1, 2, and 3. What must be the base of each, if they all retain the same altitude as the original figure?

DEFINITIONS.

THE TRAPEZOID, TRAPEZIUM, AND REGULAR POLYGONS HAVING MORE THAN FOUR SIDES.

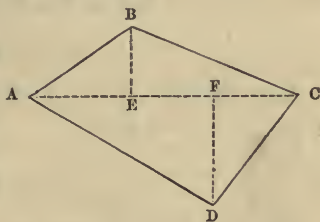
A *trapezoid* is a quadrilateral, *two* of whose sides only are *parallel;* as, A B C D, or E F G H.



The *altitude* of a trapezoid is the *perpendicular distance* between its *parallel sides;* as, B A or F I.

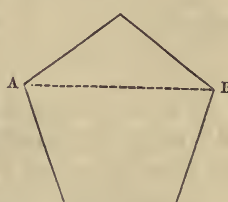
A *trapezium* is a quadrilateral *none* of whose sides are *parallel;* as, A B C D.

The *diagonal* A C divides it into two triangles, of which B E and F D form the perpendiculars.



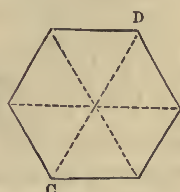
A *pentagon* is a polygon of *five sides.*

A *diagonal* of a regular pentagon, as A B, divides it into a *trapezoid* and *triangle.*



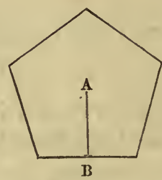
A *hexagon* is a polygon of *six sides.*

A *diagonal* passing through the *centre* of a regular hexagon, as C D, divides it into *two equal trapezoids.*



The *side* of a regular hexagon inscribed in a circle equals the *radius* of the circle.

The *apothem* of a regular polygon is a straight line drawn from the centre, perpendicular to one of its sides; as, A B.



It is frequently termed the *perpendicular.*

All *regular polygons* having the *same number* of *sides* are *similar.*

THE TRAPEZOID.

PROBLEM XV.

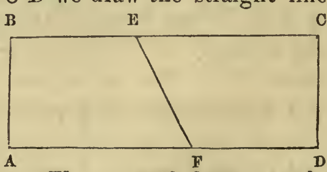
The parallel sides and altitude of a trapezoid being given, to find the area.

RULE.

Multiply half the sum of the parallel sides by the altitude.

EXPLANATION.

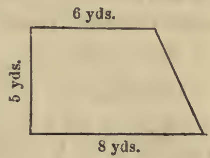
If in the rectangle A B C D we draw the straight line E F, making B E equal F D, we have two equal *trapezoids.* The *altitude* of *either* equals the *altitude* of the *rectangle,* and the *parallel sides* of *either* equal its *base.* The area of the rectangle equals the base multiplied by the altitude, or, which is the same thing, the sum of the parallel sides of either trapezoid multipled by the altitude. Since the *trapezoids* are *halves*



of the rectangle, *each* will equal *half* the *same product*, or half the sum of the parallel sides multiplied by the altitude.

EXAMPLES.

1. What is the area of a trapezoid whose parallel sides are 6 and 8 yards, and whose perpendicular is 5 yards?



SOLUTION. (6 yards + 8 yards) ÷ 2 = 7 yards, or half the sum of its parallel sides.

7 yards × 5 yards = 35 sq. yards.

Res. 35 sq. yards.

2. What is the area of a trapezoid whose parallel sides are 30 and 20 chains, and whose altitude is 26 chains?

3. One side of a quadrilateral, having 2 right angles, is 20 feet, and the side parallel to it is 24 feet. What is its area, if the perpendicular is 9 yards?

4. What is the area of a trapezoid, having 2 right angles, if the sides containing these angles are 4, 6, and 10 feet; the longest side being the perpendicular?

5. How many square feet are contained in a plank which is 16 inches wide at one end, and 12 inches at the other; the length being 28 inches?

6. A farmer has a field in the form of a trapezoid, which he wishes to sell for ¼ as many dollars per acre as there are acres in it. What is its value if its parallel sides are 600 and 424 rods, and its perpendicular 100 rods?

NOTE.—Since the area of a trapezoid equals half the product of its parallel sides by its altitude, double the area divided by the altitude will give its parallel sides, &c.

7. The parallel sides of a trapezoid are 60 and 40 chains, and its area is 150 acres. What is its altitude?

8. The area of a trapezoid is 150 acres, and its altitude is 30 chains. What is the sum of its parallel sides?

9. I have a plank 16 inches long, which contains 540 square inches. Two of its ends are parallel, and hold the same relation to each other as 2 to 3. What is the length of each end?

10. From one side of a rectangular field 80 rods long, I wish to cut off a trapezoid of 13 acres, whose parallel sides shall be to each other as 5 to 8. What will be the length of each side?

THE TRAPEZIUM.

PROBLEM XVI.

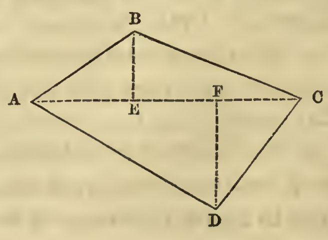
The diagonal and perpendiculars of a trapezium being given, to find the area.

RULE.

Multiply the diagonal by half the sum of the perpendiculars.

EXPLANATION.

The *diagonal* of a *trapezium* divides it into two *triangles*, and at the same time forms their *bases*. The *perpendiculars* of the trapezium are the *altitudes* of the triangles. Hence we find the area of the trapezium by finding the areas of the two triangles whose bases and

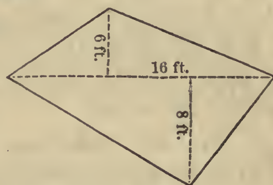


altitudes are known, or, which is the same thing, multiply the diagonal by half the sum of the two perpendiculars.

NOTE.—If we multiply a number by two different multipliers and add the results, we will have the same as if the number were multiplied by the sum of the multipliers.

EXAMPLES.

1. What is the area of a trapezium whose diagonal is 16 feet, and whose perpendiculars to this diagonal are 6 and 8 feet?



SOLUTION. $(16 \text{ ft.} \times 6 \text{ ft.}) \div 2 = 48$ sq. ft.

$(16 \text{ ft.} \times 8 \text{ ft.}) \div 2 = 64$ sq. ft.

112 sq. ft.

Or, $\left(\frac{6 \text{ ft.} + 8 \text{ ft.}}{2}\right) \times 16 \text{ ft.} = 112$ sq. ft.

Res. 112 sq. feet.

2. What is the area of a trapezium whose diagonal is 80 feet, and whose perpendiculars to this diagonal are 24 and 20 feet?

3. What is the number of square chains in a trapezium whose diagonal is 40 rods, and whose perpendiculars are 15 and 20 rods?

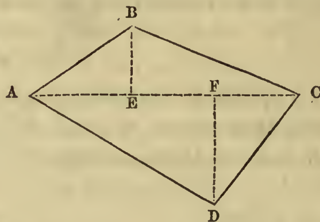
4. How many square yards of paving are there in a quadrilateral whose diagonal is 60 feet, and perpendiculars 20 and 30½ feet?

5. How many acres in a quadrangular field whose diagonal is 50 chains, and perpendiculars 21 chains 3 rods, and 13 chains?

6. The diagonal of a trapezium, separating the shorter from the longer sides, is 80 rods, and the sides are 40, 50, 60, and 70 rods. How many acres does it contain?

7. The diagonal of a trapezium, separating the shorter from the longer sides, is 80 rods, and the sides are 40, 50, 60, and 70 rods. What are the lengths of the two perpendiculars to this diagonal?

8. What is the area of the trapezium A B C D, in which A C is 160 chains, A B is 70 chains, D C is 80 chains, A E 50 chains, and F C 60 chains?



9. In the trapezium A B C D the perpendiculars B E and F D are 40 and 80 rods, and the sides A B and B C are 60 and 100 rods. What is its area?

10. In the trapezium A B C D the diagonal A C is 100 feet, the sides A D and D C are 80 and 50 feet, and the perpendicular B E is 30 feet. What is the area?

NOTE.—In performing the last three examples, draw the figure and place the dimensions, the solution will then become evident.

REGULAR POLYGONS OF MORE THAN FOUR SIDES.

PROBLEM XVII.

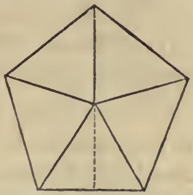
The perimeter and apothem of a regular polygon being given, to find the area.

RULE.

Multiply half the perimeter by the apothem.

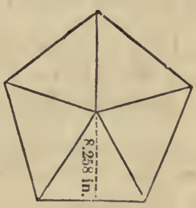
EXPLANATION.

If straight lines be drawn from the centre of a polygon to the vertices of all its angles, we will have as many *equal triangles* as the *polygon* has *sides*. Since these triangles have the *perimeter*, or sum of the sides, of *the polygon* for their *bases*, and its *apothem* for their *altitude*, the area of any one of them will equal half of one of the sides multiplied by the apothem, and all of them, or the polygon, will equal half the perimeter multiplied by the apothem.



EXAMPLES.

1. What is the area of a regular pentagon whose perimeter is 60 inches, and apothem, or perpendicular to one of its sides, 8.258 inches?



SOLUTION. (60 in. ÷ 2) = 30 in., or half the perimeter.

30 in. × 8.258 in. = 247.74 sq. in.

Res. 247.74 sq. in.

2. What is the area of a regular pentagon whose perimeter is 40 inches, and apothem 5.505 inches?

3. What is the area of a regular hexagon whose side is 7 feet, and whose perpendicular from the centre to one of its sides is 6.062 feet?

4. How many acres in a regular octagon whose side is 8 chains, and apothem 9.656 chains?

5. What is the area of a regular nonagon whose side is 5 feet, and apothem 6.868 feet?

6. What is the perimeter of a regular dodecagon whose

area is 279.9 sq. inches, and whose apothem is 9.33 inches?

7. What is the apothem of a regular undecagon whose area is 149.842 square feet, and whose sides are each 4 feet?

8. What is the apothem of a regular heptagon whose area is 232.568 square feet, and whose perimeter is 56 feet?

9. What is the area of a hexagon whose side is 6 feet, and whose diagonal passing through the centre is 12 feet?

10. What is the area of a regular octagon whose diagonals passing through the centre are 20.904 inches, and whose sides are 8 inches?

PROBLEM XVIII.

One side of a regular polygon being given, to find the area.

RULE.

First ascertain, by the table, the area of a similar polygon whose side is 1.

As the square of its side is to the square of the given side, so is its area to the required area.

NOTE.—Since the first term of this proportion is always the 1^2, dividing by it will not affect the product of the second and third terms.

EXPLANATION.

All *similar polygons* are to each other as the *squares* of their *like dimensions*

The following table consists of the areas of regular polygons whose sides are 1.

Number of Sides.	Names of Polygons.	Areas.
3	Trigon or triangle	0.433013
4	Tetragon or quadrilateral	1.000000
5	Pentagon	1.720477
6	Hexagon	2.598076
7	Heptagon	3.633912
8	Octagon	4.828427
9	Nonagon	6.181824
10	Decagon	7.694209
11	Undecagon	9.365640
12	Dodecagon	11.196152

EXAMPLES.

1. What is the area of a regular pentagon whose side is 5 inches?

SOLUTION. 1 in.^{2} : 5 in.^{2} : : 1.720477 sq. in. : the required area, or 43.011925 sq. in.

Res. 43.011925 sq. in.

2. How many acres in a regular nonagon whose side is 17 rods?

3. How many triangles, each containing .433013 square inches, can be formed from a regular decagon whose side is 5 inches?

4. How many squares of 9 square inches each are contained in a regular hexagon whose side is 3 inches?

5. What is the apothem of a regular octagon whose side is 8 chains?

6. What is the perimeter of a regular trigon whose area is 3.897117 square feet?

7. What is the diagonal passing through the centre of a regular hexagon whose side is 6 feet, and apothem 5.196 feet?

8. What is the diagonal passing through the centre of a regular octagon whose perimeter is 50 feet, and apothem 7.543 feet?

IRREGULAR POLYGONS OF MORE THAN FOUR SIDES.

Irregular polygons are those whose sides are unequal.

PROBLEM XIX.

To find the area of an irregular polygon having more than four sides.

RULE.

Draw straight lines dividing the polygon into trapezoids, trapeziums, and triangles, as may be most convenient.

Then find the areas of the figures thus formed, and add them together.

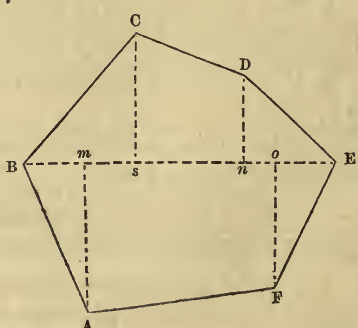
EXPLANATION.

If the polygon can be divided into a number of figures, whose area can be readily found, it is evident it will equal the sum of their areas.

EXAMPLES.

1. What is the area of the polygon A B C D E, whose dimensions are as follows:

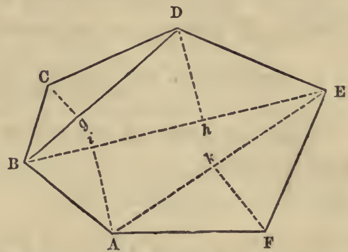
B A = 14 feet,
B C = 13 feet,
E F = 10 feet,
E D = 9 feet,
B m = 3 feet,
B s = 6 feet,
B n = 14 feet,
B o = 16 feet,
B E = 20 feet?



2. What is the area of a polygon whose dimensions are one-half of those in the first example?

3. How many acres in an irregular tract of land having the following dimensions:

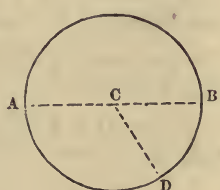
B E = 95 chains,
B D = 75 chains,
A E = 80 chains,
A i = 28 chains,
D h = 30 chains,
C g = 15 chains,
F k = 20 chains?



4. What is the area of a tract of land whose dimensions are one-half of those in the third example?

DEFINITIONS.

CIRCLES.



A circle is a plane figure bounded by a curved line which is everywhere equidistant from a point within called the centre.

The *circumference or periphery* of a circle is the curved line which bounds it.

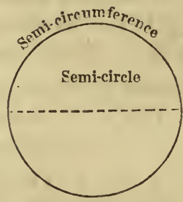
The *diameter* of a circle is a *straight line* passing through the centre, and terminating at both ends in the circumference; as, A B.

The *radius* of a circle is a *straight line* drawn from the centre to the circumference; as, C D.

Since the circumference is everywhere equidistant from the centre, all the *radii* of a *circle* are *equal.*

Since the *diameters* measure the distance from the centre to the circumference *twice,* they are also *equal,* and *each* is *double* a *radius.*

A diameter is the longest straight line that can be drawn in a circle, and divides the circle and circumference into two equal parts—called *semi-circle* and *semi-circumference.*



Circles are divided into 360 equal parts called degrees, each degree is divided into 60 minutes, and each minute into 60 seconds.

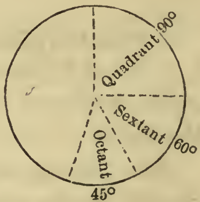
Circles whether *great* or *small* contain the *same number* of *degrees.* The difference consists in the *size* of the degrees, not in their *number.*

Quadrants, sextants, and octants receive their names from the number of degrees they contain being aliquot parts of 360°.

A quadrant is ¼ of a circle, and contains 90°.

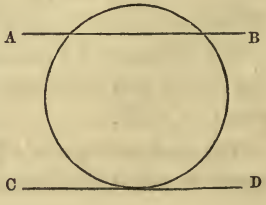
A sextant is ⅙ of a circle, and contains 60°.

An octant is ⅛ of a circle, and contains 45°.



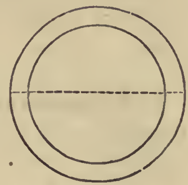
A tangent is a straight line which touches the circumference at *one* point, without cutting it; as, C D.

The point where it touches is called the point of *contact.*

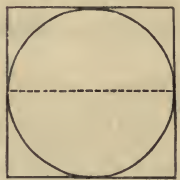


A secant is a straight line which intersects the circumference in *two* points, and lies partly within, and partly without the circle; as, A B.

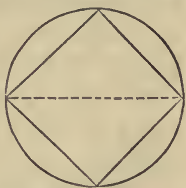
Concentric circles are those which have the same centre but unequal radii. Their circumferences form parallel curves.



A polygon is *described upon a circle* when each of its sides is tangent to the circumference



A polygon is *inscribed in a circle* when the vertices of all its angles are in the circumference. The *circle* is then said to be *circumscribed* about the *polygon*.



A circle includes a *greater area* than any *polygon* having the *same perimeter.*

All *circles* are *similar*, because their circumferences always hold the same relation to their radii.

Circles therefore are to each other as the *squares* of their like dimensions.

The *circumferences* of two circles are to each other as their *radii.*

A *square described upon* a circle is *double* the *inscribed square*, for *its side* equals the *diagonal* of the *inscribed* square, and the square of the diagonal equals twice the square in which it is found.

CIRCLES.

PROBLEM I.

The diameter of a circle being given, to find the circumference.

RULE.

Multiply the diameter by 3.1416.

EXPLANATION.

Mathematicians have never been able to find the *exact* ratio of the diameter of a circle to the circumference, nor have they accomplished the squaring of the circle, that is, they have never succeeded in drawing a square or other polygon having the *same* area as any given circle.

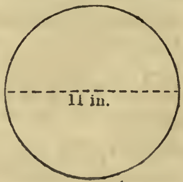
The ratio of the diameter to the circumference was shown by Metius to be as 113 to 355, which is sufficiently accurate for practical use.

$355 \div 113 = 3.141592+$, but for convenience we extend the decimal only to 4 places, calling the last 6.

Hence the rule, multiply the diameter by 3.1416.

EXAMPLES.

1. What is the circumference of a circle whose diameter is 11 inches?



SOLUTION. 11 in. $\times$ 3.1416 = 34.5576 in. 34.5576 in. = 2 feet, 10.5576 inches.

NOTE.—If the circumference is the product of two factors, viz., the diameter and 3.1416, the circumference divided by 3.1416 gives the diameter.

2. What is the circumference of a cart-wheel whose diameter is 6 feet?

3. What length of tire will it take to band a carriage-wheel 5 feet 7 inches in diameter?

4. What is the diameter of a circle whose circumference is 2 feet 10.5576 inches?

5. What is the thickness of a round tree whose girt is 15 feet?

6. The dial plate of a clock is 2 feet in circumference. What is the length of the minute hand?

7. At what rate per hour does the city of Quito move from west to east if the equatorial diameter of the earth is 7926 miles, and it turns once on its axis in 24 hours?

8. What is the circumference of a circle whose diameter equals the diagonal of a square containing 3 acres?

9. If the minute hand of a watch is 1 inch long, over how much of the circumference of the face does it pass in 20 minutes?

10. What is the diameter of a circle, having the same perimeter as a right-angled triangle,whose base and perpendicular are 18 and 24 feet?

11. The base of a right-angled triangle is 28 chains, and the sum of the other two sides is 56 chains. What is the diameter of a circle having the same perimeter?

12. What is the circumference of the greatest circle which I can draw upon a blackboard with a string of 7 inches?

13. How many times will a wheel, 5 feet in diameter,

turn round in going 5 miles, 2 furlongs, 3 chains, 2 rods, 3 yards, 1 foot, and 5 inches?

14. A horse is fastened in a meadow, by a rope 30 feet long, to the top of a post 6 feet high. What is the circumference of the greatest circle over which he can graze?

15. What is the diameter of a wheel which makes 336 revolutions in a minute, when the cars are going 30 miles an hour?

PROBLEM II.

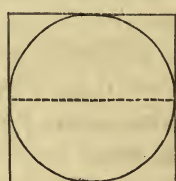
To find the area of a circle, the diameter or circumference being given.

RULE.

Multiply the square of the diameter by .7854, or the square of the circumference by .07958.

EXPLANATION.

The square of the diameter gives the area of the *square described upon the circle*, which holds the same relation to the circle as 1 to .7854; therefore, as

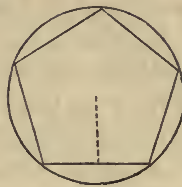


1 : .7854 : : *the area of the square described upon the circle* : *the circle.*

Since the first term of the proportion is 1, we simply multiply the area of the square, or the square of the diameter, by .7854.

The circumference is 3.1416 times the diameter, therefore its square is $(3.1416)^2$, or 9.86965056 times the square of the diameter, and must be multiplied by $\frac{1}{9.86965056}$ of .7854, or .07958.

NOTE.—The area of a circle also equals ½ the product of the circumference by the radius; for if in the circle we inscribe a regular polygon, and draw its apothem, the polygon equals ½ the product of the perimeter by the apothem. If the *number* of the *sides* of the polygon be indefinitely *increased*, until they are mere *points*, the perimeter will become the circumference, the apothem the radius, and the polygon the circle; therefore the circle will equal ½ the product of the circumference by the radius, or ¼ the product of the circumference by the diameter, which is



(diam. $\times$ diam. $\times$ 3.1416) $\div$ 4.

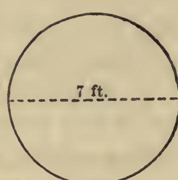
Since dividing *one* of several factors, *before multiplying them together*, is the *same* as to divide their *product*, we divide 3.1416 by 4, and have the work reduced to

(diam. $\times$ diam. $\times$ .7854).

Now the square of the diameter = the square described on the circle, and since the square of the diameter $\times$ .7854 = the circle, the square described upon the circle : circle : : 1 : .7854, as stated in the explanation to the rule.

EXAMPLES.

1. What is the area of a circle whose diameter is 7 feet?



SOLUTION. 7 ft.^{2} $\times$ .7854 = 38.4846 sq. feet.

Res. 38.4846 sq. feet.

2. What is the area of a circle whose diameter is 11 inches?

3. What is the area of a circle whose circumference is 15 feet 2 inches?

4. What is the area of a circle described with a string of 11 inches?

5. How many square yards are there in a circle whose diameter is 27 feet 5 inches?

6. What is the value of a circular piece of ground whose circumference is 200 chains, at $50 an acre?

7. What is the difference in area between a circle whose circumference is 60 feet, and an equilateral triangle having the same perimeter?

8. What is the area of a circle whose diameter corresponds to the diagonal of a rectangle 24 feet long, and 18 feet wide?

9. What is the area of a circular race course, which a horse, at the rate of a mile in 3 minutes, can trot around in 5 minutes?

10. What is the difference in area between a circle 5 chains in diameter and a square described upon it?

11. How many acres in a semi-circular lot whose radius is 20 rods?

12. A horse is fastened in a meadow, by a rope 30 feet long, to the top of a post 6 feet high. What is the area of the greatest circle over which he can graze?

13. What is the difference in area between a circle 60 rods in circumference and a rectangle, having the same perimeter, whose length is twice its breadth?

14. What relation will the quantity of water that can be forced into a basin in 1 hour, through a pipe 3 feet in diameter, hold to that which can be forced through 3 pipes, each 1 foot in diameter, in the same length of time?

15. What is the area of a circle which contains as many square feet as its circumference numbers long feet?

PROBLEM III.

To find the diameter or circumference when the area is given.

RULE.

To find the diameter—divide the area by .7854 and extract the square root of the quotient.

To find the circumference—divide the area by .07958 and extract the square root of the quotient.

EXPLANATION.

This rule is simply the reverse of the preceding one.

EXAMPLES.

1. What is the diameter of a circle containing 490.875 square feet?

SOLUTION. 490.875 sq. feet $\div$ .7854 $=$ 625 sq. feet.

$\sqrt{625 \text{ sq. feet}} = 25$ feet.

Res. 25 feet.

2. What is the diameter of a circle containing 78.54 square feet?

3. What is the circumference of a circle containing 1.9895 square feet?

4. What is the diameter of a circular acre?

5. What is the circumference of a wheel which turns 200 times in running around a circular bowling-green, whose area is 795.8 square rods?

6. What is the diameter of a circular fish-pond having the same area as a square one whose side is 20 rods?

7. What is the circumference of a circle having the same area as a triangle whose sides are 13, 14, and 15 chains?

8. What is the diameter of a circle having the same area as a rectangle whose diagonal is 10 rods, and length 6 rods?

9. The area of a circular park is 4 square miles. How long will it take to drive around it, at the rate of 5 miles per hour?

10. There is a circular garden containing 3216.9984 square feet, in the centre of which stands a pole 64 feet high. The pole being broken by the wind, one end of the broken part rested on the upright, and the other on the ground at the extremity of the garden. What was the length of the broken part?

NOTE.—Similar surfaces are to each other as the squares of their like dimensions.

If the areas of circles are produced by multiplying the squares of the diameters by .7854, or the squares of the circumferences by .07958—these areas must be equimultiples of the squares of the diameters and circumferences, and as equimultiples of quantities have the same ratio as the quantities themselves, the areas are to each other as the squares of their diameters or circumferences.

11. If the diameter of a wheel is 18 inches, what is the circumference of one 3 times as large?

12. If a rope 4 inches in circumference is composed of 200 threads, how many threads will be required to make one 9 inches in circumference?

13. If the wheels of a car which are $2\frac{1}{2}$ feet in diameter make 7 revolutions per second, how many revolutions would a wheel 5 feet in diameter make, if it passes over the same distance in the same length of time?

14. If a pipe 1 foot in diameter will fill a cistern in 6 hours, how large a pipe will it take to fill a cistern 3 times as large in the same time?

PROBLEM IV.

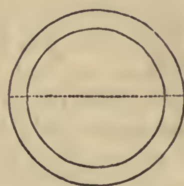
To find the areas of circular rings formed by concentric circles.

RULE.

Find the difference between the areas of the circles forming the rings.

EXPLANATION.

If two circles, of unequal radii, be drawn around a common centre, the excess of the larger over the smaller forms a circular ring, therefore the area of this ring equals the difference between the areas of the two circles.



EXAMPLES.

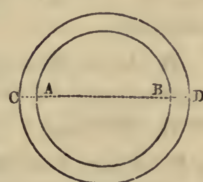
1. What is the area of a circular ring formed by 2 concentric circles whose diameters are 4 and 2 feet?

SOLUTION. $4\text{ ft.}^2 \times .7854 = 12.5664$ sq. ft., the area of the circle C D.

$2\text{ ft.}^2 \times .7854 = 3.1416$ sq. ft., the area of the circle A B.

12.5664 sq. ft. — 3.1416 sq. ft. = 9.4248 sq. ft., the area of the circular ring.

Res. 9.4248 sq. ft.



A B = 2 ft.
C D = 4 ft.

2. What is the area of a circular ring formed by 2 concentric circles whose diameters are 4 and 8 feet?

3. What is the area of a circular ring formed by 2 concentric circles whose circumferences are 25.1328 feet and 18.8496 feet?

4. I have a circle 4 inches in diameter. What is the

surface of the ring formed by putting it in the centre of a circle twice as large?

5. The radii of 3 concentric circles are 3, 4, and 5 feet. What are the areas of the 2 circular rings thus formed?

6. A bricklayer is to pave a walk, 3 feet in width, around a circular grass-plat 15 feet in diameter. How many bricks will it take if they are 8 inches long and 4 wide, making no allowance for waste?

7. Within a circular acre is a pond 5 rods in diameter. What fractional part of the acre is not covered by the pond?

8. A mason is to curb a cylindrical well, at 1 shilling per square foot; the breadth of the curb is to be 1 foot 6 inches. How much will it cost, if the diameter within the curb is 3 feet?

9. If the minute hand of a clock is 6 inches, and the hour hand 5 inches long, what is the difference of the surfaces over which they travel from sunrise to sunset at the time of the vernal equinox, when the days and nights are equal?

10. A circular garden, containing 1 acre, is bordered by a gravel walk of uniform width which takes up $\frac{1}{4}$ of its area. What is the width of the walk?

11. Three men bought a grindstone 3 feet in diameter, for which they paid equally. What part of the diameter must each grind down for his share?

12. Four men purchased a grindstone 70 inches in diameter, towards which the first contributed $2.50, the second $2.00, the third $1.50, and the fourth $1.00. What part of the diameter must each grind down for his share, if the one who contributed $2.50 grinds first, and the rest follow according to the amounts they paid?

PROBLEM V.

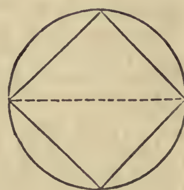
The diameter of a circle being given, to find the side of the inscribed square.

RULE.

Extract the square root of half the square of the diameter.

EXPLANATION.

The *diameter of the circle* forms the *diagonal of the inscribed square.*

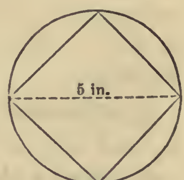


Hence we have the diagonal of a square given to find its side, as in Problem IV. of Polygons.

When the *circumference* is given, *first* find the *diameter.*

EXAMPLES.

1. What is the side of a square inscribed in a circle whose diameter is 5 inches?



SOLUTION. $5 \text{ in.}^2 = 25$ sq. in.

$\sqrt{25 \text{ sq. in.} \div 2} = 3.535$ in.

Res. 3.535 inches.

2. What is the side of a square inscribed in a circle whose diameter is 2 feet 3 inches?

3. What is the side of a square inscribed in a circle whose radius is 2 inches?

4. What is the diagonal of a square inscribed in a circle whose circumference is 20.4204 feet?

5. What is the area of a circle circumscribed about a square whose side is 5 chains?

6. How much more land in a circle 30 rods in diameter than in its inscribed square?

7. An eccentric father bequeathed a circular portion of his estate, containing 800 acres, to his wife, son, and four daughters, in the following manner: the wife and son were to have the two largest isosceles triangles that could be inscribed in the circle on its diameter, and each daughter ¼ of the remainder. How many acres did each receive?

8. How many squares inscribed in a circle equal one described upon it? Explain why this is so.

9. Why does multiplying the diameter by .7071, or the circumference by .2251, produce the side of the inscribed square?

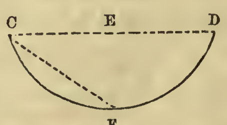
10. Why does multiplying the diameter of a circle by .8862, or the circumference by .2821, give the side of a square having the same area as the circle?

DEFINITIONS.

ARCS, SECTORS, AND SEGMENTS OF CIRCLES.

An arc of a circle is any part of its circumference; as, A B.

The chord or subtense of an arc is a straight line joining its extremities; as, C D.

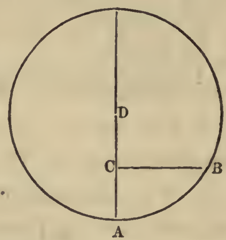


Half the chord of the arc is the half of the line C D; as, C E or E D.

The chord of half the arc is a straight line joining the extremities of half the arc; as, F C.

The chord of half the arc is the *hypothenuse* of a right-angled triangle, whose perpendicular is half the chord of the whole arc, as may be seen by joining the points E and F.

The sine of an arc is a straight line drawn from one of its extremities perpendicular to a diameter passing through the other extremity; thus, C B is the sine of the arc A B.

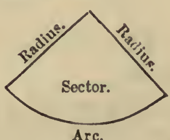


The versed sine of an arc is that part of the diameter which is intercepted between the arc and its sine; thus, C A is the versed sine of the arc A B.

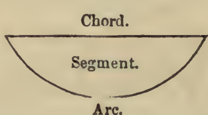
The cosine of an arc is that part of the diameter which is intercepted between the centre of the circle and the sine of the arc; as, D C.

It always equals the *radius minus the versed sine.*

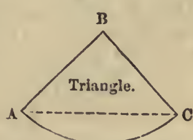
A sector of a circle is that part included between an arc and two radii drawn to its extremities.



A segment of a circle is that part included between an arc and its chord.



The *difference* between a *sector* and *segment*, having the *same arc*, is a triangle whose base is the chord of the arc, and altitude the cosine of half the arc; as, A B C.



Two sectors, segments, or arcs of circles are similar when they correspond to equal angles at the centre.

CIRCULAR AND ANGULAR MEASURE.

This measure is applied to the measurement of circles and angles.

60 seconds (″)	make 1 minute,	′
60 minutes	" 1 degree,	°
30 degrees	" 1 sign,	*S.*
12 signs, or 360 degrees . .	" 1 circle,	*C.*

ARCS OF CIRCLES.

PROBLEM VI.

The number of degrees in a circular arc and the radius of the circle being given, to find the length of the arc.

RULE.

First find the circumference of the circle whose radius is given.

Then as 360 degrees is to the number of degrees in the arc, so is the circumference to the required arc.

EXPLANATION.

The *circumference* is the length of 360 degrees, and we wish to find the length of the number of degrees in the *arc*, therefore as an arc is less than a circumference, we have by simple proportion as 360° : number of ° in the arc : : circumference : arc.

EXAMPLES.

1. What is the length of an arc of 95° whose diameter is 5 inches?

SOLUTION. 5 in. $\times$ 3.1416 = 15.708 in., the circumference.

360° : 95° : : 15.708 in. : 4.145 in., the arc.

Res. 4.145 inches.

2. What is the length of an arc of 30°, the radius of the circle being 7 feet?

3. What is the length of an arc of 30° 5′ 7″, the radius of the circle being 20 feet?

4. What is the length of a degree of the earth's circumference, if its equatorial diameter is 7926 miles?

5. How far is an inhabitant of any place on the equator carried in 5 hours, the diameter of the earth being 7926 miles?

PROBLEM VII.

The chord of the arc and the chord of half the arc being given, to find the length of the arc.

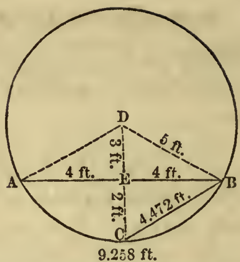
RULE.

From 8 times the chord of half the arc, subtract the chord of the whole arc, and divide the remainder by 3.

NOTE.—When the chord of the arc and the chord of half the arc are not given, but other terms from which to find them, they can generally be obtained from those terms, by a knowledge of the properties of a right-angled triangle, as will be seen by drawing the figure and placing the given dimensions.

EXAMPLES.

1. What is the length of the arc, if the versed sine of half the arc is 2 feet, and the radius of the circle is 5 feet?



SOLUTION. If the radius is 5 feet and the versed sine of half the arc is 2 feet,

5 feet — 2 feet = 3 feet, the cosine of the same.

Placing these dimensions in the figure, we have the hypothenuse and perpendicular of the triangle D B E, therefore its base, or half the chord of the whole arc, equals the $\sqrt{5\text{ ft.}^2 - 3\text{ ft.}^2} = 4$ ft., the length of E B, also of E A. Therefore A B, the chord of the arc = 8 feet. We now have the perpendicular and base of the right-angled triangle B C E to get the hypothenuse B C, or chord of half the arc. Hence $\sqrt{2\text{ ft.}^2 + 4\text{ ft.}^2} = 4.472$ ft. or C B.

From 8 times the chord of half the arc, subtract, &c.

$$4.472\text{ ft.} \times 8 = 35.776\text{ ft.}$$

$$\frac{35.776\text{ ft.} - 8\text{ ft.}}{3} = 9.258\text{ ft.}$$

Res. 9.258 feet.

2. What is the length of the arc, if the versed sine of half the arc is 6 feet, and the radius of the circle is 10 feet?

3. What is the length of the arc, if the versed sine of half the arc is 2 feet, and the radius of the circle is 6 feet?

4. What is the arc, if the versed sine of half the arc is 2 feet, and the cosine of the same is 8 feet?

5. What is the length of an arc whose chord is 26 yards, and the diameter of the circle 100 yards?

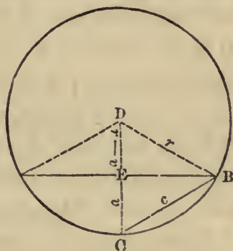
6. What is the length of an arc whose chord is 30 chains, and the versed sine of half the arc is 8 chains?

7. What is the length of the arc, when the diameter of the circle is 36 feet, and the chord of half the arc is 12 feet?

NOTE.—The *chord of half the arc* always equals the square root of the product of two *factors*, viz., the *diameter* and *versed sine* of half the arc, therefore the *square* of the chord of half the arc equals their product, and being *divided* by *either gives* the *other;* that is,

The sq. of the chord of half the arc ÷ by the diam. = the versed sine of half the arc.

The sq. of the cord of half the arc ÷ by the versed sine of half the arc = the diameter.



Let the versed sine $= v$.

Let the radius $= r$.

Let the chord of half the arc $= c$.

Let the cosine of half the arc $= r - v$.

Then from the 2 right-angled triangles C B E and D B E we have

$c^2 - v^2 =$ the square of the line E B.

Also $r^2 - (r - v)^2 =$ the square of the line E B.

Hence $c^2 - v^2 = r^2 - (r - v)^2$, for things which equal the same thing are equal to each other.

Completing the multiplication in the equation,

$c^2 - v^2 = r^2 - r^2 + 2rv - v^2$, cancelling,

$c^2 = 2rv$, or, since $2r =$ the diameter,

$c^2 = dv$, or the square of the chord of half the arc = the product of the diameter and versed sine, and

$$c^2 \div d = v.$$

$$c^2 \div v = d.$$

SOLUTION OF THE 7TH EXAMPLE.

12 ft.^{2} ÷ 36 ft. = 4 ft., the versed sine of half the arc.

$\sqrt{12\text{ ft.}^2 - 4\text{ ft.}^2} = 11.313$ ft., half of the whole chord.

11.313 ft. × 2 = 22.626 ft., the chord.

$$\frac{12\text{ ft.} \times 8 - 22.626\text{ ft.}}{3} = 24.458\text{ ft., the arc.}$$

Res. 24.458 feet.

8. What is the length of the arc when the diameter of the circle is 50 yards, and the chord of half the arc is 10 yards?

9. What is the arc, if its chord is 8 feet, and the versed sine of half the arc is 3 feet?

10. What is the arc, if the chord of half the arc is 16 feet, and the diameter of the circle is 32 feet?

SECTORS OF CIRCLES.

PROBLEM VIII.

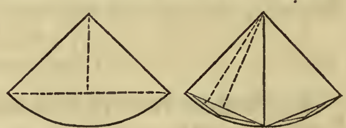
To find the area of a sector.

RULE.

Multiply the arc by the radius, and divide the product by 2.

EXPLANATION.

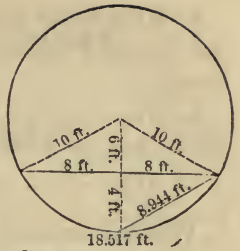
Draw a triangle in the sector having the radii and chord for its sides. The area of the triangle equals ½ the product of its base and altitude, but if the *number* of *triangles*,



drawn in the sector, be indefinitely increased, their *bases* will equal the *arc* of the sector, their *altitudes* its *radius*, and the *triangles* the *sector*. Hence the rule, multiply the arc by the radius and divide the product by 2.

EXAMPLES.

1. What is the area of a sector, if the chord of its arc is 16 feet, and the radius of the circle 10 feet?



SOLUTION. $\sqrt{10 \text{ ft.}^2 - 8 \text{ ft.}^2} = 6$ ft., the cosine of ½ the arc.

10 ft. — 6 ft. = 4 ft., the versed sine of ½ the arc.

$\sqrt{8 \text{ ft.}^2 + 4 \text{ ft.}^2} = 8.944$ ft., chord of ½ the arc.

$\frac{8.944 \text{ ft.} \times 8 - 16 \text{ ft.}}{3} = 18.517$ ft., the arc

$\frac{18.517 \text{ ft.} \times 10 \text{ ft.}}{2} = 92.585$ sq. ft., the sector.

Res. 92.585 sq. ft.

2. What is the area of a sector whose arc is 75 feet, and the radius of the circle 30 feet?

3. What is the area of a sector, the chord of whose arc is 18 feet, and the diameter of the circle 30 feet?

4. What is the area of a sector, if the chord of half the arc is 5 feet, and its versed sine is 3 feet?

5. What is the area of a sector, if the diameter of the circle is 20 feet, and the versed sine of half the arc 2 feet?

6. What is the area of a sector whose arc is 90°, if the diameter of the circle is 2 feet 3 inches?

7. What is the area of a sextant, the diameter of the circle being 20 yards?

8. What is the area of an octant, the circumference of the circle being 25.1328 feet?

9. What is the area of a sector whose arc is a quadrant, the diameter of the circle being 5 feet?

10. If a sector of a circle, whose diameter is 6 feet, contain 3.5343 sq. feet, what will be the area of a similar sector, in a circle whose diameter is 10 feet?

SEGMENTS OF CIRCLES.

PROBLEM IX.

To find the area of a segment.

RULE.

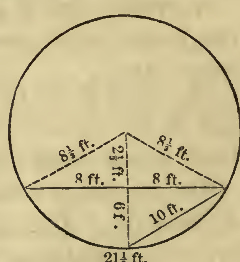
First find the area of a sector having the same arc. From this subtract the area of the triangle whose base is the chord of the arc, and whose altitude is the cosine of half the arc.

EXPLANATION.

Since a sector *exceeds* a segment having the same arc, by the area of a triangle whose base is the chord of the arc, and altitude the cosine of half the arc, the *difference* between that *sector* and *triangle* will give the *segment.*

EXAMPLES.

1. What is the area of a segment the chord of whose arc is 16 feet, and the chord of half the arc 10 feet?



SOLUTION. $\sqrt{10 \text{ ft.}^2 - 8 \text{ ft.}^2} =$ 6 ft., the versed side of $\frac{1}{2}$ the arc.

100 sq. ft. $\div$ 6 ft. $= 16\frac{2}{3}$ ft., the diameter.

$16\frac{2}{3}$ ft. $\div 2 = 8\frac{1}{3}$ ft., the radius.

$8\frac{1}{3}$ ft. $-$ 6 ft. $= 2\frac{1}{3}$ ft., the cosine of $\frac{1}{2}$ the arc.

$$\frac{10 \text{ ft.} \times 8 - 16 \text{ ft.}}{3} = 21\tfrac{1}{3} \text{ ft., the arc.}$$

$$\frac{21\frac{1}{3} \text{ ft.} \times 8\frac{1}{3} \text{ ft.}}{2} = 88\tfrac{8}{9} \text{ sq. ft., the sector.}$$

$$\frac{16 \text{ ft.} \times 2\frac{1}{3} \text{ ft.}}{2} = 18\frac{2}{3} \text{ sq. ft., the triangle.}$$

$88\frac{8}{9}$ sq. ft. — $18\frac{2}{3}$ sq. ft. $= 70\frac{2}{9}$ sq. ft., the segment.
Res. $70\frac{2}{9}$ sq. feet.

2. What is the area of a segment, the chord of whose arc is 24 yards, and the chord of half the arc 15 yards?

3. What is the area of a segment, the versed sine of half the arc being 1 foot, and the radius of the circle 5 feet?

4. What is the area of a segment, the chord of whose arc is 8 feet, and the cosine of half the arc 3 feet?

5. Compute, by the rules for segments, the area of 1 of the 4 segments, formed by inscribing a square in a circle whose diameter is 40 rods?

DEFINITIONS.

ZONES.

A circular zone is a part of a circle included between two parallel chords, and their intercepted arcs.

The breadth of the zone is that part of the diameter contained between the two parallel chords.

The *chords* may lie on the *same* or *different* sides of the diameter, and *one* of them may form the *diameter*.

They may be *equal* or *unequal*, but if *equal* the *diameter* passes through the *middle* of the *zone*.

ZONES.

PROBLEM X.

To find the area of a circular zone.

RULE.

First draw the chords of the intercepted arcs.

Then to twice the area of one of the segments thus formed, add the area of the trapezoid or rectangle formed at the same time.

When one of the chords is the diameter of the circle, take from the semi-circle the segment formed by the smaller chord of the zone and its arc.

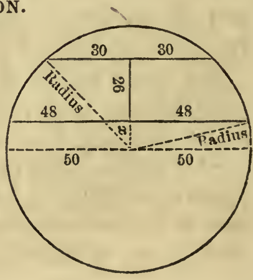
NOTE.—The rules for trapezoids, rectangles, segments, &c., have already been explained.

To find the diameter of the circle,

Divide the *difference* between the square of ½ the larger chord, and the sum of the square of the breadth plus the square of ½ the smaller chord by twice the breadth, and the quotient will be the *base* of a right-angled triangle whose *perpendicular* is ½ the larger chord, and whose *hypothenuse* is the *radius* of the circle.

Double the radius for the diameter.

EXPLANATION.



If two parallel chords of a zone are 96 and 60, and its breadth 26, what is the diameter?

Let $26 + x =$ distance from centre to shortest chord

Let $x =$ distance from centre to longest chord.

Then from the properties of a right-angled triangle,

$$30^2 + (26 + x)^2 = \text{radius}^2;\ \text{also},$$

$$48^2 + x^2 = \text{radius}^2;\ \text{hence},$$

$$30^2 + (26 + x)^2 = 48^2 + x^2,\ \text{or}$$

$$900 + 676 + 52x + x^2 = 2304 + x^2,\ \text{cancel and collect},$$

$$1576 + 52x = 2304.$$

$$52x = 2304 - 1576.$$

$$52x = 728.$$

$x = 14$, the base, of which 48 is the perpendicular, and the radius the hypothenuse. Then

$$\sqrt{14^2 + 48^2} = 50,\ \text{the radius}.$$

$$50 \times 2 = 100,\ \text{the diameter}.$$

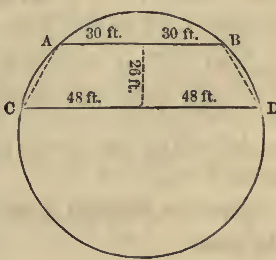
Since from the equation $52x = 2304 - 1576$, we obtain x, we have the rule,

Divide the difference between the square of ½ the larger chord, and the sum of the square of the breadth plus the square of ½ the smaller chord, by twice the breadth, to find the base of a right-angled triangle, whose perpendicular is ½ the larger chord, and whose hypothenuse is the radius.

NOTE.—If the square of ½ the larger chord *exceed* the sum of the square of the breadth plus the square of ½ the smaller chord, the *chords* are on the *same side* of the diameter; but if it be *less* than the sum, *they* are on *different sides*.

EXAMPLES.

1. What is the area of a circular zone, whose parallel chords are 96 and 60 feet, and whose breadth is 26 feet?

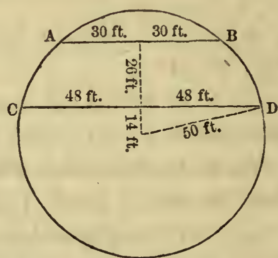


SOLUTION. To perform this example we will first find the area of the trapezoid, then the area of one of the segments, the double of which added to the trapezoid will equal the zone.

$$\frac{(60 \text{ ft.} + 96 \text{ ft.}) \times 26 \text{ ft.}}{2} = 2028 \text{ sq. ft., the trapezoid.}$$

To find the segment we can obtain the chord of its arc and also the diameter of the circle, which will, by the rules for segments, be sufficient.

To find the diameter, divide the difference between the square of ½ the larger chord, and the sum of the square of the breadth plus the square of ½ the smaller chord by twice the breadth, and we will have the base of a right-angled triangle, whose perpendicular is ½ the larger chord, and whose hypothenuse is the radius.



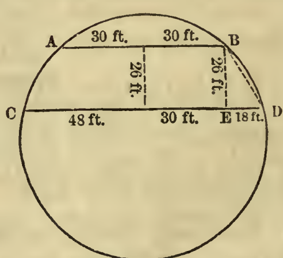
$48 \text{ ft.}^2 - (30 \text{ ft.}^2 + 26 \text{ ft.}^2) = 728$ sq. feet.

$728 \text{ sq. ft.} \div 52 \text{ ft.} = 14$ ft.

$\sqrt{14 \text{ ft.}^2 + 48 \text{ ft.}^2} = 50$ ft., the radius.

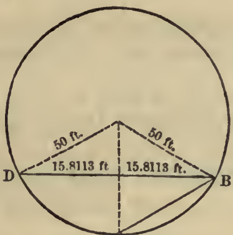
$50 \text{ ft.} \times 2 = 100$ ft., the diameter.

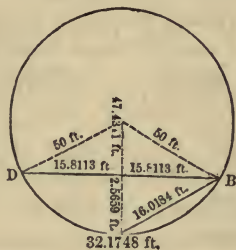
To find the chord of the arc of the segment, if from ½ of the larger chord of the zone we take ½ of the smaller, the remainder will be the base of the right-angled triangle B E D, whose perpendicular is 26 ft., the breadth of the zone, and whose hypothenuse is the chord of the arc of the segment, hence



$\sqrt{18 \text{ ft.}^2 + 26 \text{ ft.}^2} = 31.6228$ ft., the chord B D of the segment.

The dimensions of this segment can be more readily placed by drawing a segment to represent it in another circle. If we retain the same dimensions the area of the segment will be the same.





NOTE.—Always place the dimensions as soon as they are found.

$\sqrt{50\text{ ft.}^2 - 15.8113\text{ ft.}^2} = 47.4341$ ft., the cosine of half the arc.

50 ft. — 47.4341 ft. = 2.5659 ft., the versed sine of half the arc.

Since the square root of the (diameter × versed sine of half the arc) = chord of half the arc,

$\sqrt{100\text{ ft.} \times 2.5659\text{ ft.}} = 16.0184$ ft., the chord of half the arc.

From 8 times the chord of half the arc subtract the chord of the arc, and ÷ the remainder by 3.

$$\frac{16.0184\text{ ft.} \times 8 - 31.6227\text{ ft.}}{3} = 32.1748\text{ ft., the length}$$

of the arc.

Half the product of radius and arc = sector.

$$\frac{32.1748\text{ ft.} \times 50\text{ ft.}}{2} = 804.37\text{ sq. ft., the area of sector.}$$

Half the product of the cosine of half the arc and the chord of the arc = the triangle.

$$\frac{31.6227 \text{ ft.} \times 47.4341 \text{ ft.}}{2} = 749.9971 \text{ sq. ft. the triangle.}$$

Sector 804.37 sq. ft. — 749.9971 sq. ft., triangle = 54.3729 sq. ft., segment.

54.3729 sq. ft. × 2 = 108.7458 sq. ft., the two segments.

Trapezoid.		Segments.		Zone.
2028 sq. ft.	+	108.7458 sq. ft.	=	2136.7458 sq. ft.

Res. 2136.7458 sq. ft.

2. What is the area of a circular zone whose parallel chords are 18 and 24 inches, and whose breadth is 3 inches?

3. What is the area of a circular zone whose breadth is 7 feet, and whose parallel chords are 42 and 56 feet?

4. What is the area of a circular zone whose parallel chords are 18 and 24 yards, and whose breadth is 21 yards?

5. What is the area of a circular zone whose parallel chords are 24 and 32 rods, and whose breadth is 28 rods?

6. What is the area of a circular zone whose breadth is 48 feet, and whose parallel chords are each 36 feet?

7. What is the area of a circular zone whose smaller chord is 10 inches, and whose larger chord is 20 inches, being the diameter of the circle?

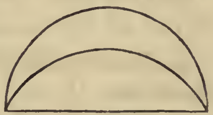
8. What is the area of a circular zone whose parallel chords are each 12 feet, and whose breadth is 16 feet?

9. What is the area of a circular zone whose larger chord being the diameter of the circle is 10 feet, and whose breadth is 4 feet?

DEFINITIONS.

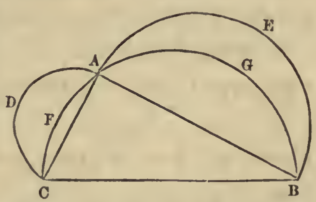
THE LUNE.

A lune is the space included between the intersecting arcs of two eccentric circles.



These arcs with their *chord* form *two segments*, whose *difference* constitutes the *lune.*

The first curvilinear figure whose surface was exactly calculated was the lune of Hippocrates. This lune is formed by drawing semi-circles on the sides of a right-angled triangle; thus, if we describe semi-circles on the sides C B, C A, and A B, we have the lunes C D A and A E B.



These lunes equal the triangle C A B, for, the largest semi-circle $= \frac{1}{2}$ the C B $^2 \times .7854$, and the smaller ones $= \frac{1}{2}$ the (C A $^2 +$ A B ^{2}) $\times .7854$.

But from the properties of a right-angled triangle we have C A $^2 +$ A B $^2 =$ C B ^{2}; therefore, the larger semi-circle equals the two smaller ones.

If from these equals we subtract the segments C F A and A G B, we have left the triangle C A B equal to the two lunes. For if equals be taken from equals, the remainders will be equal.

If the perpendicular and base are equal, the lunes will be equal, and each will equal $\frac{1}{4}$ of a square inscribed in a circle whose diameter is the hypothenûse.

LUNES.

PROBLEM XI.

To find the area of a lune.

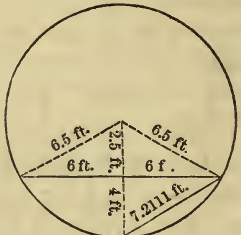
RULE.

Find the difference between the two segments formed by the arcs of the lune, and their chord.

NOTE.—The reason for this rule is too obvious to require any explanation.

EXAMPLES.

1. The chord of the segments forming a lune is 12 feet, and the heights of the segments are 4 and 3 feet. What is the area of the lune?



SOLUTION. $6\text{ ft.}^2 + 4\text{ ft.}^2 = 52$ sq. ft.

$\sqrt{52\text{ sq. ft.}} = 7.2111$ ft., the chord of ½ the arc.

52 sq. ft. ÷ 4 ft. = 13 ft., the diameter.

13 ft. ÷ 2 = 6.5 ft., radius.

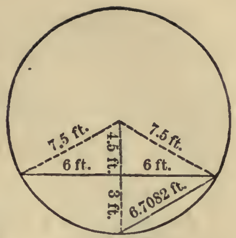
6.5 ft. — 4 ft. = 2.5 ft., the cosine of ½ the arc.

$$\frac{7.2111\text{ ft.} \times 8 - 12\text{ ft.}}{3} = 15.2296\text{ ft., the arc.}$$

$$\frac{15.2296\text{ ft.} \times 6.5\text{ ft.}}{2} = 49.4962\text{ sq. ft., the sector.}$$

$$\frac{12\text{ ft.} \times 2.5\text{ ft.}}{2} = 15\text{ sq. ft., the triangle.}$$

49.4962 sq. ft. — 15 sq. ft. = 34.4962 sq. ft., the larger segment.



$6 \text{ ft.}^2 + 3 \text{ ft.}^2 = 45$ sq. ft.

$\sqrt{45 \text{ sq. ft.}} = 6.7082$ ft., the chord of $\frac{1}{2}$ the arc.

$45 \text{ sq. ft.} \div 3 \text{ ft} = 15$ ft., the diameter.

$15 \text{ ft.} \div 2 = 7.5$ ft., the radius.

$7.5 \text{ ft.} - 3 \text{ ft.} = 4.5$ ft., the cosine of $\frac{1}{2}$ the arc.

$$\frac{6.7082 \text{ ft.} \times 8 - 12 \text{ ft.}}{3} = 13.8885 \text{ ft., the arc.}$$

$$\frac{13.8885 \text{ ft.} \times 7.5 \text{ ft.}}{2} = 52.0818 \text{ sq. ft., the sector.}$$

$$\frac{12 \text{ ft.} \times 4.5 \text{ ft.}}{2} = 27 \text{ sq. ft., the triangle.}$$

$52.0818 \text{ sq. ft.} - 27 \text{ sq. ft.} = 25.0818$ sq. ft., the smaller segment.

$34.4962 \text{ sq. ft.} - 25.0818 \text{ sq. ft.} = 9.4144$ sq. ft., the lune.

Res. 9.4144 sq. ft.

2. The chord of the segments forming a lune is 1 foot 4 inches, and the heights of the segments are 7 and 5 inches. What is the area of the lune?

3. The chord of the segments is 32 inches, and the heights of the segments are 12 and 6 inches. What is the area of the lune?

4. Two eccentric circles 24 and 18 inches in diameter intersect each other so as to form a lune. What is the area of the lune if the chord of the intersecting arcs is 8 in.?

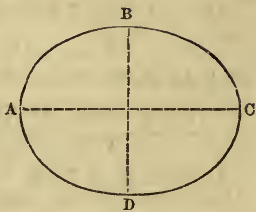
5. What is the area of the lunes formed by describing semi-circles on the three sides of a right-angled triangle, if those sides are 6, 8, and 10 inches?

6. What is the area of a lune formed by describing a semi-circle on the side of a square inscribed in a circle, whose diameter is 10 yards?

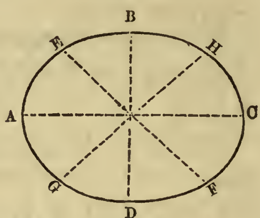
DEFINITIONS.

THE ELLIPSE.

An ellipse is a section of a cone, generated by a plane being passed through its slant sides obliquely to the base; as, the curve A B C D.

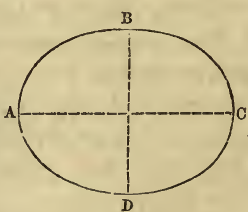


A diameter of an ellipsis is any straight line passing through its centre, and terminating at both ends in the circumference; as, the lines A C, B D, E F, and G H.



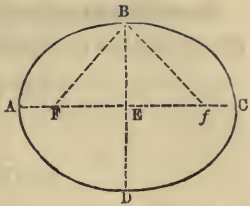
The transverse axis of an ellipse is its *longest diameter;* as, A C. It is also called the *major axis.*

The conjugate axis of an ellipse is its *shortest diameter;* as, B D. This axis is perpendicular to the transverse axis, and is sometimes called the *minor axis.*



The vertices of a diameter are the points in which the diameter meets the circumference: thus, A and C are the vertices of the transverse axis A C; B and D are the vertices of the conjugate axis B D.

The foci of an ellipse are two points in the *longest diameter*, from which if two straight lines be drawn meeting each other in the circumference, their sum will equal that diameter; as, the points F and *f*.



To find the foci of an ellipse, take a straight line equal to half the longest diameter, and, having placed *one end* of it on either vertex of the shortest diameter, let the *other end* fall where it will on the longest diameter. The *points* where it *meets* that diameter on *each side* of the *centre* of the ellipse are the *foci.*

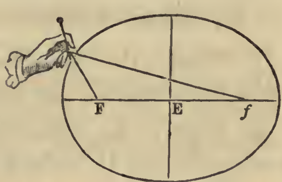
If the lines B F and B *f* each equal half the diameter A C, the points F and *f* are the foci; for the straight lines B F and B *f*, drawn from them, meet each other in the circumference, and equal the longest diameter.

The centre of an ellipse is the middle point of the straight line which joins the foci; as, E.

All the diameters *bisect* each other at the *centre.*

The eccentricity of an ellipse is the distance from the centre to either focus; as, E F, or E *f*.

The simplest method of constructing an ellipse when its major and minor axis are given, is to place them so that they bisect each other, and then find the foci.



Then take a string the length of the major axis and fasten its ends at the foci. The curve which a pencil will then describe, on both sides of the major axis, by being pressed against the string stretched to its greatest extent, will be the circumference of the ellipse.

Two ellipses are *similar* when their axes are respectively proportional to each other.

THE ELLIPSE.

PROBLEM XII.

The transverse and conjugate axes of an ellipse being given, to find the circumference.

RULE.

Multiply the square root of half the sum of the squares of the transverse and conjugate axes by 3.1416.

EXAMPLES.

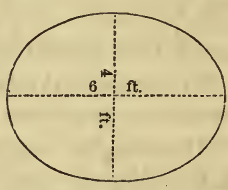
1. What is the circumference of an ellipse whose transverse and conjugate axes are 4 and 6 feet?

SOLUTION.

$$\sqrt{\frac{4\text{ ft.}^2 + 6\text{ ft.}^2}{2}} = 5.099\text{ ft.}$$

$$5.099\text{ ft.} \times 3.1416 = 16.0190184\text{ ft.}$$

Res. 16.0190184 feet.



2. What is the circumference of an ellipse whose transverse and conjugate axes are 70 and 50 feet?

3. What is the circumference of an ellipse whose semi-axes are 10 and 15 inches?

4. What is the circumference of an elliptical field whose major and minor axes are 24 and 20 chains?

5. What is the circumference of an ellipse, if the rectangle which is described upon it is twice as long as it is wide, and contains 18 square yards?

6. What is the eccentricity of an ellipse whose transverse and conjugate axes are 10 and 6 inches?

7. How far are the foci of an ellipse from the vertices of the transverse diameter, if the transverse and conjugate diameters are 20 and 12 feet?

PROBLEM XIII.

The transverse and conjugate axes of an ellipse being given, to find the area.

RULE.

Multiply the square of a mean proportional between the two axes by .7854.

EXPLANATION.

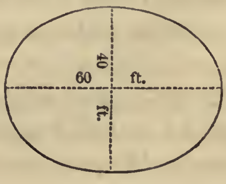
It can be proved by geometrical analysis that a mean proportional between the axes of an ellipse gives the diameter of an equivalent circle.

Therefore to get the area of an ellipse we first find a mean proportional between the axes by extracting the square root of their product.

We then have the diameter of a circle equal in area to the ellipse; and since the square of the diameter $\times$ by .7854 equals the circle, the square of the mean proportional $\times$ by .7854 will equal the ellipse.

EXAMPLES.

1. What is the area of an ellipse whose transverse and conjugate axes are 60 and 40 feet?



SOLUTION. $\sqrt{60 \text{ ft.} \times 40 \text{ ft.}} =$ 48.989 ft.

$48.989 \text{ ft.}^2 \times .7854 = 1884.96$ sq. ft.

Res. 1884.96 sq. feet.

2. How many acres are there in an elliptical field whose longest and shortest diameters are 80 and 60 chains?

3. What is the area of an elliptical park whose major and minor axes are 92 and 78 rods?

4. The area of an elliptical fish-pond is 19.635 sq. rods. What is the diameter of a circular one of equal area?

5. The area of an ellipse is 6.2832 sq. feet, and its conjugate axis is 2 feet. What is the transverse axis?

6. The axes of an ellipse containing 808.1766 sq. feet are to each other as 3 to 7. What are the axes?

7. In finding the area of an ellipse, why is it the same to multiply the square of a mean proportional between the two axes by .7854 as to multiply the product of the two axes by .7854?

MENSURATION OF SOLIDS.

DEFINITIONS.

POLYHEDRONS.

A *volume, solid*, or *body*, is a quantity of space having three dimensions, viz., *length*, *breadth*, and *thickness.*

The terms *solid* and *body*, as generally applied, would infer the existence of matter, whereas the reasonings of geometry carefully exclude every such idea. For this reason the term *volume* is preferable, as it denotes a quantity of space limited in every direction, irrespective of what that space may be filled with, or whether it be entirely void. An *empty barrel* is just as much a *solid*, mathematically considered, as *one filled with lead;* a *body capable* of containing, as *that which* contains.

A *polyhedron* is a solid or volume bounded by polygons; these polygons are called the *faces*, and the straight lines in which the faces meet, the *sides* or *edges* of the polyhedron. The solid angles formed by three or more of these faces meeting at a common point are termed *polyhedral angles.*

Polyhedrons, from the *number* of their faces, are denominated *tetrahedrons, pentahedrons, hexahedrons, heptahedrons,* &c.

A *regular polyhedron* is one whose faces are equal regular polygons, and whose polyhedral angles are all equal.

The *diagonal* of a polyhedron is a straight line joining the vertices of any two polyhedral angles not adjacent.

Similar polyhedrons are those which are bounded by an equal number of mutually similar faces, similarly situated.

All *regular polyhedrons* of the same name are *similar.*

The corresponding parts of similar solids are termed *homologous.*

Similar solids or volumes (that is solids or volumes whose dimensions vary proportionally) are to each other as the *cubes* of their *like dimensions.*

Solids of the same name, having *two* dimensions *alike*, are to each other as their *third* dimensions.

Solids of the same name, having *one* dimension *alike* in *each*, are to each other as the *products* of the other *two.*

Solids, generally, are to each other as the *products* of their *bases* and *altitudes.*

The *principal irregular* polyhedrons are *prisms* and *pyramids.*

A *prism* is a polyhedron whose ends are two equal parallel polygons, and whose sides are right-angled parallelograms.

The two equal parallel polygons form the *upper* and *lower bases* of the prism, and the right-angled parallelograms its *convex surface.*

The convex surface plus the areas of the bases form the *entire surface.*

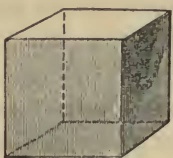
The *altitude* of a prism is the perpendicular distance between its bases.

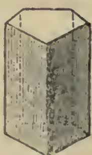
A *regular prism* is one whose edges are perpendicular to its bases. Hence, in a regular prism the edges and altitude are *equal.*

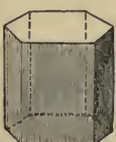
Other prisms are termed *oblique*, and in such prisms the edges are *greater* than the altitude.

Prisms, from the *shapes* of their *bases*, are classified into *triangular* or *trigonal*, *quadrangular* or *tetragonal*, *pentagonal*, *hexagonal*, *heptagonal*, *octagonal prisms*, &c.


Triangular Prism.


Quadrangular Prism.

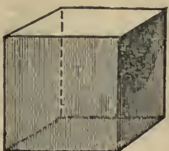

Pentagonal Prism.


Hexagonal Prism.


Octagonal Prism.

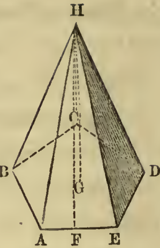
A *parallelopipedon* is a prism whose faces are all right-angled parallelograms.

When these are all *equal* the solid is termed a *cube*, otherwise a *rectangular parallelopipedon*.


Cube.


Rectangular Parallelopipedon.

A *pyramid* is a polyhedron whose sides are triangles, uniting in a point at the *apex*, and terminating in the edges of a *polygon*.



The polygon A B C D E is the *base* of the pyramid, and the point H its *apex* or *vertex*.

The triangles H B A, H A E, H E D, H D C, and H C B, form its *convex surface.*

The *altitude* of a pyramid is the perpendicular distance from its apex to the plane of its base; as, H G.

The *slant height* of a pyramid is the altitude of the triangles forming its convex surface; as, H F.

A *regular pyramid* is one whose base is a regular polygon, and in which the *perpendicular* let fall from the apex upon the base, passes through the *centre* of the base.

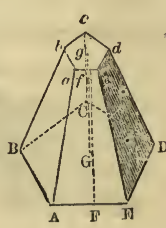
This perpendicular is termed the *axis* of the pyramid.

In *irregular* pyramids the perpendicular, measuring the altitude, sometimes falls without the pyramid, and is perpendicular to the plane of the base produced.

Pyramids are classified from the *shapes* of their *bases*, in the *same manner* as *prisms.*

Every pyramid is *the third* of a prism having an equal base and altitude.

The *frustum* of a pyramid is that part which remains after the top has been cut off, by passing a plane through the sides of the pyramid parallel to its base.



The part cut off by the plane is a *smaller pyramid*, *similar* to the *larger*, and is called the *segment* of a pyramid.

The section *a b c d e*, made by the plane parallel to the base, is a polygon *similar* to the base, and forms the *upper base* of the frustum.

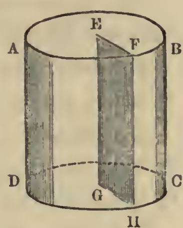
The *altitude* of a frustum is the perpendicular distance between its upper and lower bases; as, *g* G.

The *convex surface* of a frustum is composed of as many trapezoids, as there are sides in the polygons forming its bases.

The *slant height* of a frustum is the altitude of the trapezoids forming the convex surface; as, *f* F.

A *frustum of a pyramid* is equal to the sum of three *pyramids*, whose *altitudes* are the *altitude* of the *frustum*, and whose *bases* are its *lower base*, its *upper base*, and a *mean proportional between the two bases*

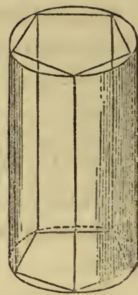
CYLINDERS AND CONES.



A *cylinder* is a solid or volume generated by the revolution of a right-angled parallelogram about one of its sides, which remains fixed; thus, the volume A B C D, is formed by revolving the rectangle E F H G about the side E G, which remains fixed.

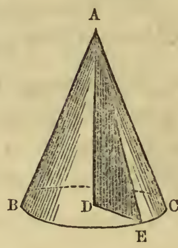
The fixed side E G is the *axis* of the cylinder.

The side F H describes the *convex surface* of the cylinder, and the sides E F and G H its *bases*. Hence, the *diameter* of the *base* of a *cylinder* equals *twice* the *breadth* of the *revolving parallelogram*, and its *altitude*, the *length* of the *parallelogram*.



A *prism is inscribed in a cylinder* when the polygons forming its bases are inscribed in the bases of the cylinder, for its edges will then be contained in the convex surface of the cylinder.

Similar cylinders are those generated by the revolution of similar rectangles about their homologous sides, or those whose axes are proportional to the radii of their bases.



A *cone* is a solid or volume generated by the revolution of a right-angled triangle about one of its sides, containing the right angle, which remains fixed; thus, the volume A B C is formed by the revolution of the right-angled triangle A D E about the fixed side A D.

The fixed side A D is the *axis* of the cone.

The *hypothenuse* A E describes the *convex surface* of the cone, and the *base* D E the *circle* forming the *base* of the cone.

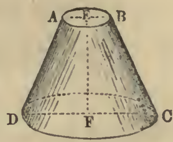
The *vertex* A of the generating triangle is the *apex* or *vertex* of the cone.

The *altitude* of a cone is the perpendicular distance from its apex to the plane of its base, or it is the *perpendicular* of the generating triangle; as, A D.

The *slant height* of a cone is measured by a straight line drawn from the apex to the circumference of the base, or it is the *hypothenuse* of the generating triangle; as, A E.

Every *cone* is the *third* of a *cylinder* having an equal base and altitude.

Similar cones are those generated by the revolution of similar triangles about their homologous sides, or those whose *axes* are *proportional* to the *radii* of their *bases*.



The *frustum of a cone* is that part which remains, after the top has been cut off, by passing a plane through the sides of the cone parallel to its base.

The part cut off by the plane is a *smaller cone, similar* to the *larger*, and is called the *segment* of a cone.

The section A B, made by the plane parallel to the base, is a circle, and forms the *upper base* of the frustum.

The *altitude* of the frustum is the perpendicular distance between its upper and lower bases; as, E F.

The *slant height* of the frustum is measured by a straight line drawn from the circumference of the upper to that of the lower base; as, A D.

CUBIC OR SOLID MEASURE.

This measure is used in finding the volumes of solids.

TABLE.

1728	cubic inches (*cu. in.*)	make	1 cubic foot,	*cu. ft.*
27	cubic feet	"	1 cubic yard,	*cu. yd.*
16	cubic feet .	"	1 cord foot,	*c. ft.*
8	cord feet, or 128 cubic feet, .	"	1 cord of wood,	*C.*
$4492\frac{1}{2}$	cubic feet .	"	1 cubic rod,	*cu. rd.*
32768000	cubic rods .	"	1 cubic mile,	*cu. m.*

ALSO,

231	cubic inches	make	1 wine gallon,	*W. gal.*
282	cubic inches	"	1 ale gallon,	*A. gal.*
$268\frac{4}{5}$	cubic inches	"	1 dry gallon,	*D. gal.*
$2150\frac{42}{100}$	cubic inches	"	1 bushel,	*bu.*
$24\frac{3}{4}$	cubic feet	"	1 perch of stone,	*pch.*

NOTE.—A perch of stone is $16\frac{1}{2}$ feet long, $1\frac{1}{2}$ feet wide, and 1 foot high.

9*

PRISMS AND CYLINDERS.

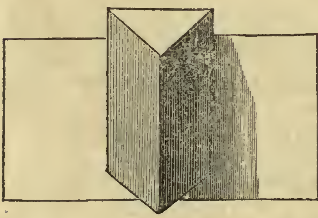
PROBLEM I.

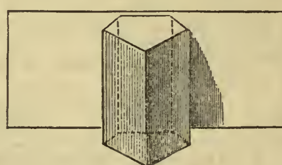
To find the convex surface of any solid having two equal ends, and a uniform distance around it, such as cylinders and all classes of prisms.

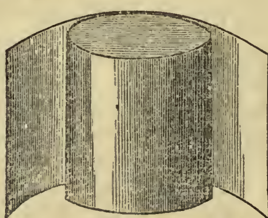
RULE.

Multiply the distance around the base by the altitude. Add the areas of both ends to the convex surface for the entire surface.

EXPLANATION.







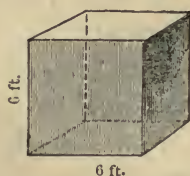
If we cover the convex surface of any of these solids with a rectangular plane surface, such as a sheet of paper, making the *width* of the paper equal the *height* of the figure, and its *length* just the *distance around* its *base*, the surface of the paper must equal the convex surface of the solid. But the paper when removed is a *rectangle* whose length is the distance around the base of the solid, and whose width is its altitude. Therefore we get the *convex surface* of the *solid* by finding the *area* of the *rectangle*, or by multiplying the distance around the base by the altitude.

The entire surface exceeds the convex simply by the areas of the ends.

EXAMPLES.

1. What are the superficial contents of a cube whose side is 6 feet?

SOLUTION. The distance around the base of this cube is 24 feet. Therefore 24 ft. × 6 ft., the altitude, gives 144 sq. ft. or the convex surface. 6 ft.^{2} = 36 sq. feet, the area of one end. 36 sq. ft. × 2 = 72 sq. feet or both ends. Hence, 144 sq. feet + 72 sq. ft. = 216 square feet.

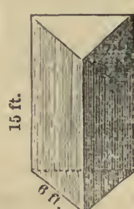


Res. 216 sq. feet.

2. What are the superficial contents of a triangular prism, whose altitude is 15 feet, and each side of the triangle forming the base 6 feet?

SOLUTION. 6 ft. × 3 = 18 ft., or the perimeter of the base. 18 ft. × 15 ft = 270 sq. ft., the convex surface.

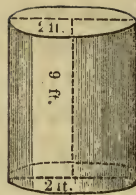
The area of the equilateral triangle forming the ends is 15.588 sq. ft. 15.588 sq. ft. × 2 = 31.176 sq. ft., area of both ends.



31.176 sq. ft. + 270 sq. ft. = 301.176 sq. ft.

Res. 301.176 sq. ft.

3. What is the entire surface of a cylinder whose altitude is 9 feet, and the diameter of the base 2 feet?



SOLUTION. 2 ft. × 3.1416 = 6.2832 ft., circumference of the base.

6.2832 ft. × 9 ft. = 56.5488 sq. ft., the convex surface.

2 ft.^{2} × .7854 × 2 = 6.2832 sq. ft., the area of both ends.

56.5488 sq. ft. + 6.2832 sq. ft. = 62.832 sq. ft. Res. 62.832 sq. ft.

4. What are the superficial contents of a cube whose side is 5 feet 5 inches?

5. What is the surface of a cube whose side is 7 feet 5 inches?

6. What are the superficial contents of a parallelopipedon which is 35 feet long, 13 feet wide, and 7 feet high?

7. What are the superficial contents of a brick 8 inches long, 4 inches wide, and 2 inches thick?

8. What is the surface of a tetragonal prism whose length is 9 feet 2 inches, whose width is 7 feet 3 inches, and thickness 5 feet 6 inches?

9. What is the convex surface of a triangular prism whose altitude is 10 feet, and each side of its base 8 feet?

10. What is the entire surface of a triangular prism whose altitude is 12 feet, and the perimeter, or distance around the base, 21 feet, the base being an equilateral triangle?

11. What is the convex surface of a pentagonal prism whose altitude is 15 inches, and each side of the polygon forming the base 3 inches?

12. What is the entire surface of a trigonal prism whose altitude is 8 inches, and each side of its equilateral ends 3 inches?

13. What is the entire surface of a pentagonal prism whose altitude is 1 foot 6 inches, and each side of the base 5 inches?

14. What is the entire surface of an octagonal prism whose altitude is 3 feet, and each side of the base 7 inches?

15. What is the entire surface of a hexagonal prism whose altitude is 7 feet 5 inches, and each side of the base 1 foot 3 inches?

16. What is the convex surface of a cylinder whose altitude is $5\frac{1}{2}$ feet, and the diameter of the base $1\frac{1}{2}$ feet?

17. What is the entire surface of a cylinder whose altitude is 7 feet, and the diameter of the base 2 feet 6 inches?

18. What is the convex surface of a cylinder whose revolving rectangle is 7 feet long, and 2 feet wide?

19. What is the entire surface of a cylinder described by a gate, revolving upon a pivot in its centre, which is 7 feet high and 6 wide?

20. What are the superficial contents of a room 15 feet wide, 20 feet long, and 16 feet high?

21. What is the circumference of the base of a cylinder whose convex surface is 64.7955 square feet, and altitude 2 yards 1 foot 6 inches?

NOTE.—The convex surface being the product of two factors, viz. the distance around the base and the altitude, if divided by either gives the other.

The entire surface minus the area of the ends equals the convex surface.

22. What is the altitude of a cylinder whose entire surface is 103.6728 square feet, and the diameter of the base 3 feet?

23. What is the side of a cubic block of marble whose superficial contents are 20 sq. yards, 1 sq. ft., 72 sq. in.?

24. What is the altitude of a pentagonal prism, if its entire surface is 3.722 sq. ft., and each side of its base 5 in.?

25. What is the distance around an octagonal room 16 feet high, if it requires $246\frac{14}{27}$ yards of paper, $\frac{3}{4}$ of a yard wide, to cover its walls?

PROBLEM II.

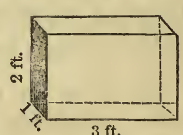
To find the solidity of any solid figure having two equal ends, and a uniform distance around it, such as cylinders and all classes of prisms.

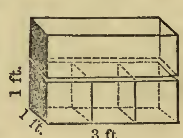
RULE.

Multiply the area of the base by the altitude.

EXPLANATION.

Let the accompanying solid be a prism whose length is 3 feet, width 1 foot, and height 2 feet. Then if it be divided through its altitude into as many equal parts as there are feet in the altitude, by passing planes through it parallel to the base, we will have *two smaller prisms*, each 3 feet long, 1 foot wide, and 1 foot high. Take one of these and divide it through its length into 3 equal parts; each subdivision will be 1 foot long, wide, and thick, or contain 1 *cubic foot*. This smaller prism therefore will contain 3 cubic feet, just as many cubic feet as there are *square feet* in its *base;* as there is the same number of smaller prisms in the original solid as there are feet in the altitude, or *two*, the *contents* of the *first prism* will be twice 3 cubic feet, or *the number of square feet in the base multiplied by the number of feet in the altitude.*





NOTE.—When the prism is a *cube*, multiplying the area of the base by the altitude is equivalent to *cubing its side.*

NOTE.—The *rules* for finding the *solidities* of *prisms* and *cylinders* are *alike;* for if we inscribe a prism in a cylinder, the solidity of the prism will equal the product of its base and altitude; but by increasing the *number* of its sides indefinitely, the perimeter of the prism will become the circumference of the cylinder, and, their altitudes being the same, the prism will become the cylinder; hence, the solidity of a cylinder equals the product of its base and altitude.

EXAMPLES.

1. What is the solidity of a triangular prism whose altitude is 12 feet, and each side of its base 6 feet?

SOLUTION. The area of the equilateral triangle forming the base is 15.588 sq. ft.

15.588 sq. ft. × 12 ft. = 187.056 cu. ft.

Res. 187.056 cu. ft.

2. What is the solidity of a cube whose side is 3 inches?

3. What are the solid contents of a block of marble which measures 5 feet 3 inches on each side?

4. What is the solidity of a brick which is 8 inches long, 4 inches wide, and 2 inches thick?

5. What are the solid contents of a rectangular parallelopipedon 25 feet long, 13 feet wide, and 7 feet thick

6. What are the solid contents of a block of red sandstone which is 9 feet 2 inches long, 7 feet 3 inches wide, and 5 feet 6 inches thick?

7. What is the solidity of a triangular prism whose altitude is 15 feet, and each side of the base 6 feet?

8. A man has a bin 8 feet square and 3 feet high. How many bushels will it contain?

9. What will be the cost of a block of granite 7 feet 2 inches long, 5 feet wide, and 3 feet 7 inches thick, at 25 cents a cubic foot?

10. What will be the capacity of a room which is 16 feet high, 30 feet long, and 22 feet wide?

11. What is the solidity of a trigonal prism whose altitude is 16 feet, and each side of the trigonal base $1\frac{1}{2}$ feet?

12. Snow having fallen into an open cellar 30 feet long and 20 feet wide, to the depth of $\frac{7}{9}$ of a foot, it became necessary to remove it. How much was paid for having it removed at the rate of 4 cents per dozen bushels?

13. What is the weight of a block of granite which is 3 feet 2 inches long, 3 feet 1 inch wide, and 2 feet 4 inches thick, its specific gravity being 2650, that is, a cubic foot weighs 2650 oz. Av.

14. What is the solidity of a pentagonal prism whose length is 15 feet, and each side of the base 2 feet.

15. What is the solidity of an octagonal prism whose altitude is 45 feet, and each side of the base 5 feet 3 inches?

16. What is the solidity of a trigonal prism, each side of the base being 6.5 feet, and the altitude 29 feet?

17. How many gallons of water will a section of water-pipe contain whose diameter is 2 feet, and length 6 feet?

18. A lot is 175 feet long and 150 feet wide. How deep must a ditch $4\frac{1}{2}$ feet wide be dug around the outside, that the dirt thrown out may raise the surface of the lot 1 foot?

19. How many bricks 8 inches long, 4 wide, and 2 thick, would be required to build a rectangular cistern whose outside dimensions are as follows: 15 feet long, 8 feet wide, and 10 feet deep, the wall to be $\frac{1}{2}$ a foot thick?

20. What is the length of a wall 3 feet thick and 7 feet high, which cost $500, at the rate of $1 per cubic foot?

21. What must be the depth of a cistern, the length and width being each $\frac{1}{3}$ of the depth, so as to contain 30 hhds.?

22. A stone pillar 80 feet long is cut into 4 such pieces

that $\frac{1}{2}$ the first, $\frac{1}{3}$ the second, $\frac{1}{4}$ the third, and $\frac{1}{5}$ the fourth are equal. What are the lengths of the pieces?

23. A log of wood is 15 ft. long, and 36 in. in diameter. How many cubic feet will it contain when hewn square?

24. Some boys having thrown a quantity of stone into a cylindrical tank $3\frac{1}{2}$ feet in diameter, and partly filled with water, the water rose 9 inches. How many perches of stone were thrown in?

25. What is the area of the top of a cubical box which contains 125 cubic inches?

26. How many bushels of oats will a bin contain which is 6 ft. long, 4 ft. 6 in. wide, and 3 ft. 3 in. deep?

27. A cistern is to be built in cylindrical form to hold 13 hogsheads of water (wine measure). What will be its altitude if the diameter of the base is 6 feet?

28. How many bricks 8 inches long, 4 inches wide, and 2 inches thick, are contained in the walls of a building which is 30 feet long, 20 feet wide, and 50 feet high to the eaves, the gables being 8 feet above the eaves, and the walls 1 foot thick?

NOTE.—*Similar solids* are to each other as the *cubes* of their *like dimensions.*

Solids of the same name, having *two* dimensions *alike*, are to each other as their *third* dimensions.

Solids of the same name, having *one* dimension *alike* in each, are to each other as the *products* of the *other two.*

Solids, generally, are to each other as the *products* of their *bases* by their *altitudes.*

29. If a ship's cable contain 534 threads when it is 2 inches in diameter, how many threads will it contain when it is 4 inches in diameter?

30. If a section of a cylindrical pillar, $1\frac{1}{2}$ feet in diameter, weigh 500 lbs., what is the diameter of a section of equal length of another pillar whose weight is 350 lbs.

31. An iron pillar, 5 inches in diameter contains 5 cubic

feet. What must be the diameter of a pillar of equal length to contain 8 cubic feet?

32. A stone is $1\frac{1}{3}$ times as wide as it is thick, and $1\frac{1}{2}$ times as long as it is thick, and contains 432 cubic feet. What are its dimensions?

33. The contents of a box are 1620 cubic feet, and its sides are to each other as 3, 4, and 5. What are its length, width, and breadth?

34. How much wood is there in a pile 6 ft. 3 in. long, 3 ft. 5 in. wide, and 5 ft. 7 in. high? If it is worth $5, how much will a pile of twice these dimensions cost?

35. How many cubic blocks, whose sides are 2 inches, can be cut from one whose side is 1 foot?

36. How many more bushels of grain will a bin measuring 6 ft. each way hold, than 2 others measuring 3 ft. each way?

37. What relation will the quantity of water that can be forced through a pipe 4 feet in diameter in 1 hour, hold to that which can be forced in 2 hours, through 2 pipes each 3 feet in diameter?

PROBLEM III.

The inner diameter and thickness of a cylindrical ring being given, to find the surface.

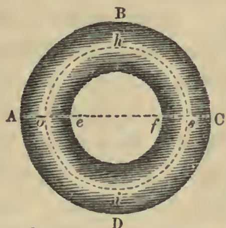
RULE.

Since the *ring* can be changed to a *cylinder*, having the *dotted line* of the ring for its *altitude*, and its *thickness* for its *diameter*, we apply the rule used for cylinders.

EXPLANATION.

Let A B C D be a *cylindrical* ring, *e f* the *inner* diameter, A *e* the *thickness*, and *o h s i* the dotted line running through the centre.

If the ring be cut through, perpendicular to the diameter, and then stretched out, the body becomes *cylindrical* in form. As the *inner* circumference of the ring was *less* than the *outer*, one side of this cylinder will be longer than the other, but if this excess be cut off the lower end and applied inversely to the upper, the figure becomes a perfect cylinder.



The *diameter*, or *thickness* of the ring, A *e*, is the *diameter* of the cylinder.

The dotted circle *o h s i* is the altitude of the cylinder. The surface of the ring equals the convex surface of the cylinder, and the solidity in both cases is the same. Hence, the rules are the same as those for cylinders.

NOTE.—If half the thickness be added to each end of the inner diameter, the sum will be the diameter of the dotted circle.

EXAMPLES.

1. What is the surface of a cylindrical ring whose inner diameter is 10 inches, and its thickness 2 inches?

SOLUTION.

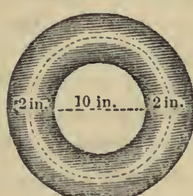
10 in. + 2 in. = 12 in., the diameter of the dotted circle.

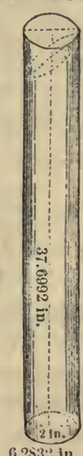
12 in. × 3.1416 = 37.6992 in., the dotted circle, or the altitude of the cylinder.

2 in. × 3.1416 = 6.2832 in., the circumference of the thickness of the ring, or the circumference of the cylinder.

37.6992 in. × 6.2832 in. = 236.871 sq. inches.

Res. 236.871 sq. inches.





2. What is the surface of a cylindrical ring whose inner diameter is 9.456 feet, and its thickness 4 inches?

3. The circumference of the thickness of a cylindrical ring is 3.1416 ft., and the circumference of a circle running through the middle of the ring is 240 ft. What is its surface?

4. The entire diameter of a cylindrical ring is 20 feet, and its inner diameter 15 feet. What is its surface?

5. The thickness of a cylindrical ring is 3 inches, and the outer circumference 63 inches. What is its surface?

NOTE.—The solidity of a cylindrical ring is found in the same manner as the solidity of a cylinder.

6. What is the solidity of a cylindrical ring whose inner diameter is 6 inches, and its thickness 2 inches?

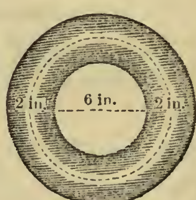
SOLUTION.

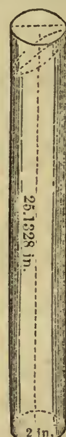
6 in. + 2 in. = 8 in., the diameter of the dotted circle.

8 in. × 3.1416 = 25.1328 in., the circumference of the dotted circle, or the altitude of the cylinder.

2 in.^{2} × .7854 = 3.1416 sq. in., the area of the base of the cylinder.

25.1328 in. × 3.1416 sq. in. = 78.9572 cu. in., the solidity of ring.





Res. 78.9572 cu. in.

7. What is the solidity of a cylindrical ring whose thickness is 4 inches, and the inner diameter 24 inches?

8. What is the solidity of an anchor ring whose inner diameter is 7 inches, and its thickness 2 inches?

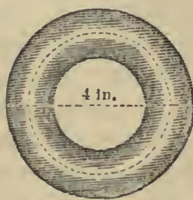
9. What is the solidity of a cylindrical ring attached to an ox-yoke, for the purpose of supporting the tongue of the cart, whose inner diameter is 4 inches, and the thickness $\frac{7}{8}$ of an inch?

10. What is the solidity of a cylindrical ring fastened to

a hitching-post, whose inner diameter is $2\frac{1}{2}$ inches, and thickness $\frac{1}{4}$ of an inch?

NOTE.—The following examples are simply reversions of the preceding ones.

11. What is the thickness of a cylindrical ring whose outer circumference is 18.8496 inches, and inner diameter 4 inches.



SOLUTION. 18.8496 in. ÷ 3.1416 = 6 in., the entire diameter of the ring.

6 in. — 4 in. = 2 in., the difference between the two diameters, or twice the thickness.

2 in. ÷ 2 = 1 in., the thickness. Res. 1 inch.

12. What is the thickness of a cylindrical ring if the outer circumference is 80, and the inner diameter 20 feet?

13. The solid contents of a cylindrical ring are 28 cubic feet, and a circle passing through the middle of it is 6 feet in circumference. What is the thickness?

14. The solid contents of a cylindric ring are 360 cubic feet, and the thickness 2 feet. What is its inner diameter?

15. What is the inner diameter of a cylindrical ring whose solidity is 35 cubic feet, and thickness $1\frac{1}{2}$ feet?

PYRAMIDS AND CONES.

PROBLEM IV.

To find the convex surface of a right pyramid or cone.

RULE.

Multiply the distance around the base by half the slant height.

Add the area of the base to the convex surface for the entire surface.

EXPLANATION.

The convex surface of a pyramid is composed of *triangles* whose *bases* form *the distance around the base of the pyramid*, and whose *altitudes* are its *slant height*. *Each of these triangles* equals the product of its base by one-half the slant height of the pyramid; hence, *the areas of all the triangles* will equal the product of the sum of their bases by one-half the slant height, or the convex surface of the pyramid equals the product of the distance around its base by one-half the slant height.

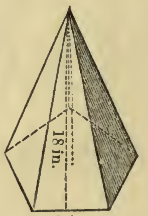
NOTE.—If we inscribe a right pyramid in a cone, its convex surface equals the product of the distance around its base by one-half its slant height; but if the *number* of the sides of the pyramid be indefinitely increased, the perimeter of its base will equal the circumference of the base of the cone, its slant height will equal the slant height of the cone, and the pyramid will become the cone. Hence, the rules for pyramids and cones are the same.

EXAMPLES.

1. What is the entire surface of a pentagonal pyramid, each side of whose base is 5 inches, and whose slant height is 18 inches?

SOLUTION. 5 in. $\times$ 5 = 25 in., the distance around the base.

25 in. $\times$ (18 in. $\div$ 2) = 225 sq. in., the convex surface.



Find the area of the base by Problem XVIII., page 56.

1 in.^{2} : 5 in.^{2} : : 1.720477 sq. in. : the base, or 43.011925 sq. in.

225 sq. in. + 43.011925 sq. in. = 268.011925 sq. in., the entire surface. Res. 268.011925 sq. inches.

2. What is the convex surface of a triangular pyramid, each side of its base being 3 in., and its slant height 16 in.?

3. What is the entire surface of an octagonal pyramid whose slant height is 17.5 ft., and each side of the base 3 ft.?

4. What is the convex surface of a cone whose slant height is 15 ft., and the diameter of the base 2 ft. 6 in.?

5. What is the entire surface of a cone whose slant height is 3 ft., and the diameter of the base 8 ft. 3 in.?

6. How much will it cost to paint the convex surface of a square pyramid, at 10 cents per square yard, if each side of the base is 5 feet, and the slant height 30 feet?

7. What is the entire surface of a solid, the hypothenuse of whose generating triangle is 10 feet, and the perpendicular about which the revolution is performed 8 feet?

8. What is the convex surface of the largest Egyptian pyramid, if its base is a square measuring 693 feet on a side, and its altitude is 500 feet?

9. If the circumference of the base of a solid, generated by the revolution of a right-angled triangle about its perpendicular, is 18.8496 feet, and its altitude 4 feet, what is the hypothenuse of the generating triangle?

10. What is the diagonal of the base of a square pyramid whose convex surface contains 40 square yards, and whose slant height is 5 yards?

PROBLEM V.

To find the solidity of a right pyramid or cone.

RULE.

Multiply the area of the base by one-third the altitude.

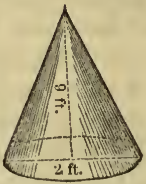
EXPLANATION.

It can be proved, by geometrical analysis, that a *pyramid* or *cone* is *one-third of a prism* or *cylinder* having an *equal base* and *altitude*.

If the solidity of a prism or cylinder equals the product of its base and altitude, the product of the base and altitude of a pyramid or cone will equal a solid three times as great as itself; hence, one-third of this product, or, which is the same thing, the product of the base by one-third the altitude, equals the pyramid or cone.

EXAMPLES.

1. What is the solidity of a cone the diameter of whose base is 2 feet, and altitude 9 feet?



SOLUTION. $2 \text{ ft.}^2 \times .7854 = 3.1416$ sq. ft., the area of the base.

$3.1416 \text{ sq. ft.} \times (9 \text{ ft.} \div 3) = 9.4248$ cu. ft., the cone.

Res. 9.4248 cu. feet.

2. What is the solidity of a cone whose altitude is 8 feet, and the diameter of whose base is 3 feet?

3. What is the solidity of a triangular pyramid, each side of its base being 3 feet, and its altitude 5 feet?

4. What are the solid contents of a hexagonal pyramid whose altitude is 27 in., and each side of its base 6 in.?

5. What was the solidity of the marble monument which Queen Semiramis is said to have erected in Babylon at the tomb of her husband Ninus, its shape being that of a square pyramid, which measured 150 feet in altitude. and 20 feet on each side of the base?

6. What are the solid contents of a cone generated by the revolution of a triangle, whose hypothenuse is 15 feet, about its perpendicular, the base being 9 feet?

7. What would be the solidity of a cone generated by a triangle similar to that in the sixth example, but whose perpendicular is 8 feet?

NOTE.—If the solidity of a pyramid or cone equals the product of the base by one-third the altitude, three times the solidity will give the product of the base and altitude, and this product divided by either gives the other.

8. What is the altitude of a triangular pyramid, com-

posed of marble, which cost $23.382, at the rate of $2.00 a cubic foot, each side of the base being 3 feet?

9. I wish to divide a sugar loaf, conical in form, and 20 inches high, into four equal parts, by sections parallel to the base. What must be the height of each part?

10. If the altitude of a triangular pyramid is 10 ft., what is the altitude of its segment cut off by a plane parallel to the base, which contains one-fourth the solidity of the pyramid?

11. What is the diameter of the base of a cone which cost $1272.348, at $5 a cubic foot, the diameter of the base being to the altitude as 3 to 4?

12. If a cone 6 feet in altitude weighs 270 pounds, what will be the altitude of a similar cone weighing 640 pounds?

FRUSTUMS OF PYRAMIDS AND CONES.

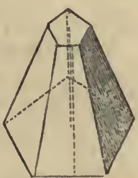
PROBLEM VI.

To find the convex surface of the frustum of a right pyramid or cone.

RULE.

Multiply the sum of the distances around the upper and lower bases of the frustum by one-half its slant height. Add the areas of the two bases to the convex surface for the entire surface.

EXPLANATION.



The convex surface of the frustum of a pyramid is composed of *trapezoids*, whose *parallel sides* form the *distances around the upper and lower bases of the frustum*, and whose *altitudes* are its *slant height.*

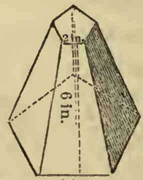
Each of these trapezoids equals the product of the sum of its parallel sides by one-half the slant height of the frustum; hence, the *areas of all the trapezoids* will equal the product of the sum of their parallel sides by one-half the slant height, or the convex surface of the frustum equals the product of the sum of the distances around the upper and lower bases by one-half its slant height.

NOTE.—If we inscribe a frustum of a right pyramid in the frustum of a right cone, its convex surface equals the sum of the distances around its upper and lower bases multiplied by one-half its slant height; but if the *number* of the sides of the inscribed frustum be indefinitely increased, the perimeters of its bases will equal the circumferences of the bases of the cone, its slant height will equal the slant height of the cone, and the frustum of the pyramid will become the frustum of the cone. Hence, the rules for the frustums of pyramids and cones are the same.

NOTE.—The frustum of a cone is generated by the revolution of a *trapezoid* about its *perpendicular*, which remains fixed.

EXAMPLES.

1. What is the surface of the frustum of a pentagonal pyramid, the slant height being 6 inches, each side of the upper base 2 inches, and each side of the lower base 4 inches?



SOLUTION. 2 in. $\times$ 5 = 10 in., the distance around the upper base.

4 in. $\times$ 5 = 20 in., the distance around the lower base.

10 in. + 20 in. = 30 in., the sum of these distances.

30 in. $\times$ (6 in. $\div$ 2) = 90 sq. in., the convex surface.

Find the areas of the bases by Prob. XVIII., page 56.

1 in.^{2} : 2 in.^{2} : : 1.720477 sq. in. : the upper base, or 6.881908 sq. in.

1 in.^{2} : 4 in.^{2} : : 1.720477 sq. in. : the lower base, or 27.527632 sq. in.

90 sq. in. + 6.881908 sq. in. + 27.527632 sq. in. = 124.40954 sq. in., the surface of the frustum.

Res. 124.40954 sq. inches.

2. What is the convex surface of the frustum of a hexagonal pyramid, the slant height being 8 inches, each side of the upper end 3 inches, and each side of the lower end 6 inches?

3. How many square yards are contained in the surface of the frustum of an octagonal pyramid whose slant height is 7 feet, each side of its upper base 3 feet, and of its lower base 5 feet?

4. What is the convex surface of a frustum of a cone whose slant height is 8 feet, and the diameters of whose bases are 4 and 6 feet?

5. What is the surface of the frustum of a cone described by a trapezoid whose parallel sides are 3 and 6 inches, and whose altitude is 4 inches?

6. How much will it cost to line a cistern with cement at 10 cents a square foot, if it is 8 feet square at the top, 4 feet at the bottom, and 10 feet deep?

PROBLEM VII.

To find the solidity of the frustum of a right pyramid or cone.

RULE.

Add together the areas of the two bases and a mean proportional between them, and multiply this sum by one-third the altitude.

NOTE.—To get a mean proportional between two numbers, extract the square root of their product.

NOTE.—The sum of the two bases and the mean proportional between them can be obtained with much less work by multiplying the *squares of the diameters plus their product by* .7854 when the frustum is of a *cone*, or *the squares of the sides plus their product by the number found in the tabular area* when the frustum is of a *pyramid.*

For reason, see Key.

EXPLANATION.

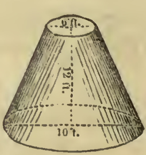
It can be proved by geometrical analysis that the frustum of a regular pyramid is equal in solidity to the sum of three *pyramids*, having for their *altitudes* the *altitude of the frustum*, and for their *bases* the *lower base of the frustum*, the *upper base of the frustum*, and *a mean proportional between them.*

Every triangular pyramid equals the product of its base by one-third its altitude, therefore the *three pyramids* will equal the sum of their bases, multiplied by one-third their altitude, or the sum of the upper and lower bases of the frustum plus a mean proportional between them, multiplied by one-third the altitude of the frustum. Since they equal in solidity the frustum, the solidity of the frustum will also equal the sum of its upper and lower bases plus a mean proportional between them, multiplied by one-third its altitude.

NOTE.—It has been shown, in the note under Problem VI., that the rules for the frustums of pyramids and cones are similar.

EXAMPLES.

1. What is the solidity of the frustum of a cone, the diameters of its upper and lower bases being 6 and 10 feet, and its altitude 12 feet?



SOLUTION. $(6 \text{ ft.}^2 + 10 \text{ ft.}^2 + (6 \text{ ft.} \times 10 \text{ ft.})) \times .7854 = 153.9384$ sq. ft., the sum of the bases and a mean proportional between them.

$153.9384 \text{ sq. ft.} \times (12 \text{ ft.} \div 3) = 615.7536$ cu. ft., the solidity of the frustum. Res. 615.7536 cu. ft.

2. What are the solid contents of the frustum of a cone the diameters of whose ends are 3 and 7 inches, and whose altitude is 9 inches?

3. How many perches of stone are contained in the frustum of a pyramid composed of granite, whose altitude

is 16 feet and whose bases are squares, the upper one measuring 4 feet on each side and the lower one 5 feet?

4. What will be the cost of a stick of hewn timber which is 2 feet 6 inches square at one end, 1 foot 6 inches square at the other, and 15 feet long, at the rate of 5 cents per cubic foot?

5. How many hogsheads of rain water will a cistern contain, which is 12 feet in diameter at the bottom, 8 feet at the top, and 9 feet deep?

6. A crucible which is 4 inches in diameter at the top, 2 inches at the bottom, and $3\frac{6261}{9163}$ inches in depth is filled with melted silver. How many cubes, each containing 1 solid inch, can be made from the metal?

7. How many feet high is the frustum of a tetragonal pyramid which contains 2 cubic yards and 13 cubic feet, if the upper end measures 3 feet on each side and the lower end 7 feet?

8. What is the altitude of the trapezoid describing the frustum of a cone, if the parallel sides of the trapezoid are 4 and 5 inches, and the frustum contains 1149.8256 cubic inches?

NOTE.—If the frustums of cones or pyramids have the same altitude, and their ends proportional, their solidities will be to each other as the squares of their diameters or like sides.

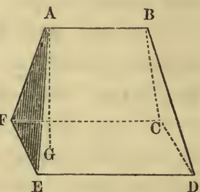
9. A marble monument shaped in the form of a frustum of a hexagonal pyramid measures 3 feet at each side of the base, 1 foot at the top, and 15 feet high. What are the sides of a monument containing 675.49976 cubic feet, whose altitude is the same as the former, and whose ends hold the same relation to each other?

10. If the diameters of the top and bottom of a basket are 10 and 8 inches, and the depth 9 inches, what must be the dimensions of a similarly-shaped basket to contain 8 times as much?

DEFINITIONS.

THE WEDGE.

The *wedge* is a solid bounded by five polygons; viz., a *rectangle* forming its back or base, two *trapezoids* forming its faces, and two *triangles* its ends.



In the wedge A B C D E F, the rectangle C D E F is the base, the trapezoids A B C F and A B D E are the faces, and the triangles A E F and B D C the ends of the wedge.

The *edge* of a wedge is the straight line in which the trapezoids forming its faces meet; as, A B.

The *altitude* of a wedge is the perpendicular distance from the edge to the base; as, A G.

THE WEDGE.

PROBLEM VIII.

To find the solidity of a wedge.

RULE.

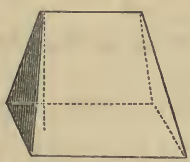
When the base exceeds the edge in length, at each end of the edge pass a plane through the wedge perpendicular to its base. The portion cut off by each plane will be a *quadrangular pyramid*, having half the difference in length between the base and edge, and the breadth of the base for its base, and the altitude of the wedge for its altitude.

The middle portion will equal a *triangular prism*, whose altitude is the edge of the wedge, and whose ends are triangles, having for their bases the breadth of the base of the wedge, and for their altitudes the altitude of the wedge.

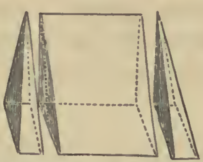
Hence, the *sum* of the *solidities* of the *prism* and *two pyramids* will equal that of the *wedge.*

When the edge exceeds the base in length, add to each end of the wedge a quadrangular pyramid, having half this excess, and the breadth of the base for its base, and the altitude of the wedge for its altitude. The solid thus formed will be a *triangular prism*, whose altitude is the edge of the wedge, and whose ends are triangles, having for their bases the breadth of the base of the wedge, and for their altitudes the altitude of the wedge.

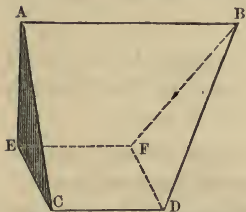
Hence, the *difference* between the *solidities* of the *prism* and *two added pyramids* will equal that of the *wedge.*



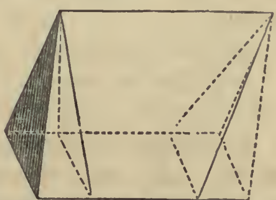
A wedge whose *base exceeds* its *edge* in *length.*



The same wedge divided into two quadrangular pyramids, and a triangular prism.



A wedge whose *edge exceeds* its *base* in *length.*



The same wedge with the two quadrangular pyramids added, making it a triangular prism.

NOTE.—The rule needs no further explanation.

EXAMPLES.

1. What are the solid contents of a wedge whose base is 30 inches long and 10 broad, the length of the edge being 20 inches, and the altitude 18 inches?

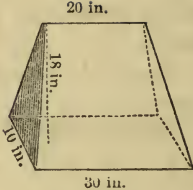
SOLUTION. 30 in. — 20 in. = 10 in., the difference in length of the base and edge.

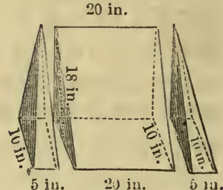
10 in. ÷ 2 = 5 in., one side of the base of each quadrangular pyramid.

10 in. × 5 in. × (18 in. ÷ 3) = 300 cu. in., the solidity of each pyramid.

300 cu. in. × 2 = 600 cu. in., the solidity of both pyramids.

10 in. × (18 in. ÷ 2) = 90 sq. in., the area of the triangle forming the base of the prism.





90 sq. in. × 20 in. = 1800 cu. in., the solidity of the triangular prism.

600 cu. in., the pyramids, + 1800 cu. in., the prism, = 2400 cu. in., the solidity of the wedge.

Res. 2400 cu. inches.

2. What are the solid contents of a wedge whose base is 3 feet 4 inches long, 1 foot 3 inches broad, the length of the edge being 2 feet 3 inches, and the altitude 2 feet 9 inches?

3. What are the solid contents of a wedge whose base is 1 yard 3 inches long and 26 inches broad, the length of the edge being 31 inches, and the altitude 24 inches?

4. What is the solidity of a stone, in the form of a wedge, which is 10 feet 2 inches long at the base and 3 feet 3 inches broad, the altitude of the stone being 6 feet, and the length of its edge one-half the length of its base?

5. What is the difference in volume between two wedges, each being 60 inches in altitude, and 25 inches broad at the base, but the base of one being 70 inches long and its edge 50 inches, while the base of the other is 50 inches long and its edge 70 inches?

DEFINITIONS.

THE PRISMOID.

A *rectangular prismoid* is a polyhedron whose ends are two unequal but parallel rectangles, and whose sides are trapezoids. Or, it is a *frustum of a wedge* formed by passing a plane through it parallel to its base.

The *altitude* of a prismoid is the perpendicular distance between its ends; as, A B.

THE PRISMOID.

PROBLEM IX.

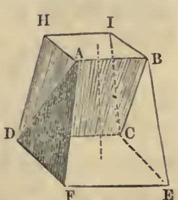
To find the solidity of a rectangular prismoid.

RULE.

Add the solidity of a wedge whose altitude and base are the altitude and lower base of the prismoid, and whose edge is the length of the upper base, to that of a wedge having the same altitude, but whose base is the upper base of the prismoid, and whose edge is the length of its lower base.

EXPLANATION.

A prismoid may be considered as composed of two *wedges* having for their *altitudes* the *altitude* of the *prismoid*, for their *bases* the *bases* of the *prismoid*, and for their *edges* the *lengths* of its *upper* and *lower bases*.



For if through the lines A B and C D we pass the plane A B C D, it will divide the prismoid into the two wedges

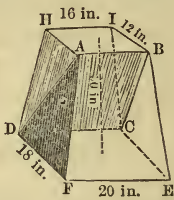
A B C D F E and C D H I B A, having for their altitudes the altitude of the prismoid, for their bases the upper and lower bases of the prismoid, and for their edges the lengths of its upper and lower bases.

Hence, the *solidities* of the *two wedges* will equal that of the *prismoid*.

EXAMPLES.

1. How many cubic inches are there in a rectangular prismoid, the greater end being 20 by 18 inches, the less 16 by 12 inches, and the altitude 30 inches?

Solution. Divide the prismoid into the wedges A B C D F E and C D H I B A.



$\frac{20 \text{ in.} - 16 \text{ in.}}{2} = 2$ in. The wedge A B C D F E may be divided into two quadrangular pyramids—whose bases are 18 in. by 2 in., whose altitudes are 30 in.—and a trianglar prism whose altitude is 16 in., and whose ends are triangles having 18 and 30 in. for their bases and altitudes.

18 in. $\times$ 2 in. $\times$ (30 in. $\div$ 3) $=$ 360 cu. in., the solidity of each pyramid.

360 cu. in. $\times$ 2 $=$ 720 cu. in., the solidity of both pyramids.

18 in. $\times$ (30 in. $\div$ 2) $=$ 270 sq. in., the base of the triangular prism.

270 sq. in. $\times$ 16 in. $=$ 4320 cu. in., the solidity of the prism.

720 cu. in. $+$ 4320 cu. in. $=$ 5040 cu. in., the wedge A B C D F E.

The wedge C D H I B A has its edge C D 20 in., and its base H I B A 16 in. long.

$\frac{20 \text{ in.} - 16 \text{ in.}}{2} = 2$ in.

Hence, the quadrangular pyramids necessary to be added to the wedge, in order to change it to a triangular prism having the same altitude as the wedge, have their bases 12 in. by 2 in., and their altitudes 30 in. The wedge, with the addition of these pyramids, equals a prism whose altitude is 20 in., and whose ends are triangles having 12 and 30 in. for their bases and altitudes.

12 in. × (30 in. ÷ 2) = 180 sq. in., the base of the triangular prism.

180 sq. in. × 20 in. = 3600 cu. in., the solidity of the prism.

2 in. × 12 in. × (30 in. ÷ 3) = 240 cu. in., the solidity of each pyramid.

240 cu. in. × 2 = 480 cu. in., the solidity of both pyramids.

The prism. The pyramids.
3600 cu. in. — 480 cu. in. = 3120 cu. in., the wedge C D H I B A.

5040 cu. in + 3120 cu. in. = 8160 cu. in., the prismoid.

Res. 8160 cu inches.

2. How many cubic feet are there in a rectangular prismoid, if the smaller end is 3 feet by 2 feet 6 inches, the greater 4 feet by 3 feet 4 inches, and the altitude 5 feet?

3. What is the solidity of a block of granite, cut in the shape of a frustum of a wedge, the upper base being 1 yard 4 inches by 2 feet 6 inches, the lower 1 yard 2 feet by 1 yard 1 foot 2 inches, and the altitude 1 yard 1 foot and 10 inches?

4. How many bushels will a box contain, shaped in the form of a rectangular prismoid, the top being 2 by 2½ feet, the bottom 3 by 3½ feet, and the depth 1⅓ feet?

5. What will be the cost of the base of a marble monument, at $1.50 per cubic foot, if it is 12 by 10 feet at the bottom, 8 by 6 feet at the top, and 6 feet high?

DEFINITIONS.

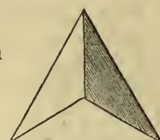
THE REGULAR POLYHEDRONS.

A REGULAR POLYHEDRON is one whose faces are equal regular polygons, and whose polyhedral angles are all equal.

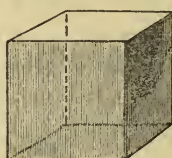
There are *five* regular polyhedrons, which derive their names from the *number* of their *sides*.

They are the *tetrahedron*, the *hexahedron* or *cube*, the *octahedron*, *dodecahedron*, and *icosahedron*.

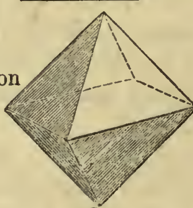
The *tetrahedron* is a regular polyhedron bounded by four triangles.



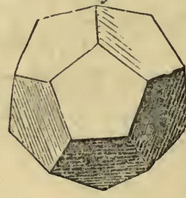
The *hexahedron* or cube is a regular polyhedron bounded by six squares.

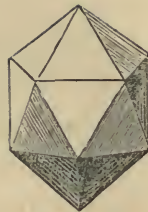


The *octahedron* is a regular polyhedron bounded by eight triangles.



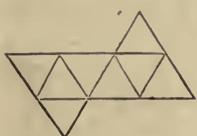
The *dodecahedron* is a regular polyhedron bounded by twelve pentagons.

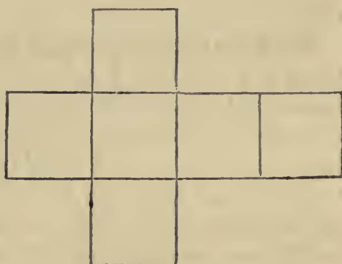


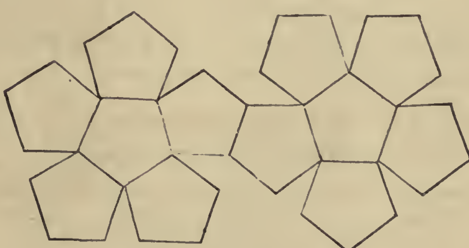


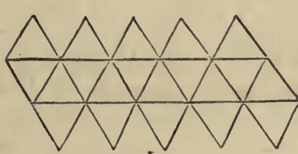
The *icosahedron* is a regular polyhedron bounded by twenty triangles.

If the following figures be cut out of pasteboard, and the lines cut partly through, so that the parts may be turned up and glued together, the solids thus formed will represent the five regular polyhedrons.









REGULAR POLYHEDRONS.

PROBLEM X.

To find the surface of a regular polyhedron.

RULE.

Multiply the square of the given edge by the surface of a similar polyhedron whose edges are 1.

EXPLANATION.

Since the *faces* of a regular polyhedron are *equal, regular polygons*, the *edges* must be *all equal.*

The faces of polyhedrons of the same name are similar polygons; but similar polygons are to each other as the squares of their like dimensions, or, conversely, the squares of their like dimensions are to each other as their surfaces. Hence, as the 1^2 : given edge^2 : : surface of a similar polyhedron whose edges are 1 : the surface of the required polyhedron.

Since the first term equals 1, dividing by it will not alter the product of the second and third terms. Hence the rule, Multiply the square of the given edge by the surface of a similar polyhedron whose edges are 1.

A TABLE OF THE SURFACES AND SOLIDITIES OF REGULAR POLYHEDRONS WHOSE EDGES ARE 1.

Names.	No. of Faces	Surfaces.	Solidities.
Tetrahedron,	4	1.7320508	0.1178513
Hexahedron or Cube,	6	6.0000000	1.0000000
Octahedron,	8	3.4641016	0.4714045
Dodecahedron,	12	20.6457288	7.6631189
Icosahedron,	20	8.6602540	2.1816950

EXAMPLES.

1. What is the surface of an octahedron measuring 5 in. on each side?

SOLUTION.

1 in.^{2} : 5 in.^{2} : : 3.4641016 sq. in. : Res. or 86.60254 sq. in

Res. 86.60254 sq. inches.

2. What is the surface of a tetrahedron whose sides are each 1 foot 5 inches?

3. What is the surface of an icosahedron whose sides are 3 times as long as those of a hexahedron, whose surface contains 6 square feet?

4. How much will it cost to paint the surface of a dodecahedron whose sides are each 3 feet, at the rate of 10 cents per square yard?

5. What is the side of an octahedron whose surface contains 31.1769144 square yards?

PROBLEM XI.

To find the solidity of a regular polyhedron.

RULE.

Multiply the cube of the given edge by the solidity of a similar polyhedron whose edges are 1.

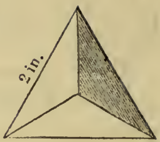
EXPLANATION.

Similar solids are to each other as the cubes of their like dimensions, or, conversely, the cubes of their like dimensions are to each other as their solidities. Hence, as the 1^3 : given edge^3 : : solidity of a similar polyhedron whose edges are 1 : to the solidity of the required polyhedron.

Since the 1^3 is 1, dividing by it will not affect the product of the second and third terms. Hence the rule, Multiply the cube of the given edge by the solidity of a similar polyhedron whose edges are 1.

EXAMPLES.

1. What are the solid contents of a tetrahedron whose side is 2 inches?



SOLUTION. 1 in.^{3} : 2 in.^{3} : : .1178513 cu. in. : Res. or .9428104 cu. in.

Res. .9428104 cu. inches.

2. What is the solidity of a tetrahedron whose side is 4 in.?

3. How many gallons of wine would a hollow dodecahedron contain whose sides are each 2 feet?

4. How many hexahedrons, measuring 1 inch on each side, would equal in volume an icosahedron whose sides are twice as long?

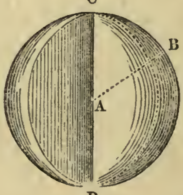
5. How many inches long is the side of an octahedron whose solidity is 12.7279215 cu. feet?

DEFINITIONS.

THE SPHERE.

A *sphere* is a solid or volume bounded by a curved surface, every part of which is equidistant from a point within, called the centre.

Or it is a volume generated by the revolution of a semi-circle about its diameter, as an axis, which remains fixed.



A *radius* of a sphere is any straight line drawn from its centre to its surface; as, A B.

Hence, from the definition of a sphere, all the *radii* of a *sphere* are *equal*.

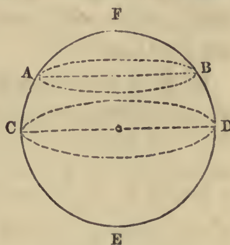
A *diameter* of a sphere is any straight line passing through its centre, and terminating at both ends in its circumference; as, C D.

Since a diameter measures twice the distance from the centre to the surface, the diameters are all equal, and each is double a radius.

Every section of a sphere made by a plane is a *circle*.

A *plane* is *tangent* to a sphere when it has but one point in common with the surface.

A *great circle* of a sphere is one the centre of whose plane is the centre of the sphere; as, C D.



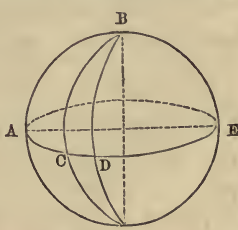
Or it is one which divides a sphere into *two equal parts*.

A *small circle* of a sphere is one the centre of whose plane is not the centre of the sphere; as, A B.

Or it is one which divides a sphere into *two unequal parts*.

The *poles* of a circle of a sphere are those two points in the surface of the sphere, every way equidistant from the circumference of that circle. The points F and E are the poles of the circles A B and C D.

A *spherical angle* is an angle included between the arcs of two great circles intersecting each other on the surface of the sphere; as, the angles B A C, B C D, and B D E.

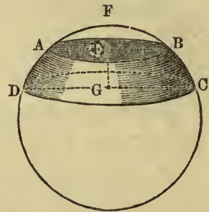


The *vertices* of the angles are the points at which the arcs intersect.

A *spherical polygon* is any portion of the surface of the sphere bounded by arcs of great circles of that sphere; as, the surface B E D.

Spherical polygons receive their *names* from the *number* of their *bounding arcs*, as plane ones from the number of straight lines which bound them.

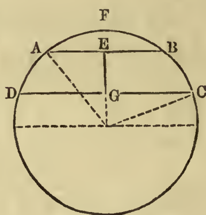
A *spherical zone* is a portion of the surface of a sphere included between two parallel planes; as, A B C D.



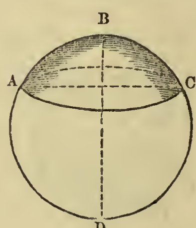
If either plane is tangent to the sphere, the zone will have but one base; as, A F B.

The perpendicular distance between these two planes forms the *altitude* of the zone; as, E G.

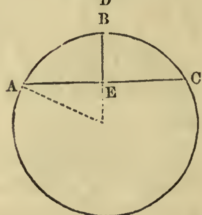
If we bisect the sphere A F B C D by a plane perpendicular to the bases of the zone, the section thus formed will be the *circle* A F B C D. The chords A B and D C will form the diameters of the upper and lower bases, and E G the altitude of the zone.



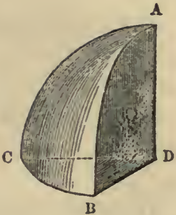
A *spherical segment* is a portion of a sphere included between two parallel planes. These planes form the bases of the segment; but if either plane is tangent to the sphere, the segment will have but one base; as, A B C.



If we bisect the sphere A B C D by a plane perpendicular to the base of the segment, the section thus formed will be the *circle* A B C D, of which the chord A C forms the diameter of the base of the segment, and B E the altitude.



A *spherical pyramid* is a portion of a sphere included between the planes of a solid angle whose vertex is at the centre of the sphere; as, A D B C.

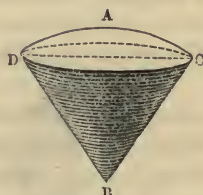


The spherical polygon A B C (or surface of the sphere), intercepted by these planes, forms the *base* of the pyramid.

A *spherical sector* is a solid or volume generated by the revolution of a circular sector about a straight line, drawn through the vertex of the sector as an axis.

Let the circular sector A B C be revolved about A B as an axis, then will it describe the spherical sector D A C B.

The *zone* described by A C equals the *base* of the spherical sector.



The *cone* C D B has the *same base* as the *spherical segment* C A D, and forms the *difference* in solidity between the spherical sector and segment.

All *spheres* are *similar*, because the proportion between their dimensions is always the same.

The *surfaces of spheres* are to each other as the squares of their radii or diameters.

The *solidities of spheres* are to each other as the cubes of their radii or diameters.

SPHERES.

PROBLEM XII.

To find the surface of a sphere.

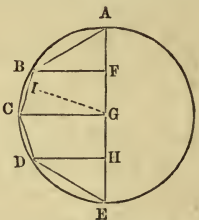
RULE.

Multiply its diameter by the circumference of a great circle of the sphere.

NOTE.—The convex surface of a zone or segment of a sphere equals its height multiplied by the circumference of a great circle of that sphere.

EXPLANATION.

A sphere is formed by the revolution of a semi-circle about its diameter as an axis.



Inscribe in the semi-circle A C E the regular semi-polygon A B C D E, and from the vertices of its angles draw straight lines perpendicular to the diameter A E.

Now it can be proved, by geometrical analysis, that if the semi-polygon be revolved about the diameter A E, the surface described by either of its sides, as B C, equals the product of its height measured on the axis or diameter A E, multiplied by the circumference of the inscribed circle, or the circumference of G I. The same is true of the surface described by each side of the semi-polygon; therefore, the surface described by the semi-polygon equals the product of the sum of these heights, or of the diameter, multiplied by the circumference of the inscribed circle.

But if the *number* of the sides of the semi-polygon be indefinitely increased, its perimeter will eventually equal the circumference of the semi-circle, G I, the radius of the inscribed circle, will equal the radius G C of the sphere, and the surface described by the semi-polygon will become the surface described by the semi-circle. Hence, the surface of a sphere equals the product of its diameter multiplied by the circumference of a great circle of that sphere.

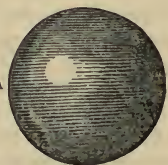
EXAMPLES.

1. What is the surface of a sphere or globe whose diameter is 5 inches?

SOLUTION. 5 in. × 3.1416 = 15.708 in., the circumference of a great circle of the sphere.*

15.708 in. × 5 in. = 78.54 sq. in., the surface of the sphere.

Res. 78.54 sq. inches.



A B = 5 inches.

2. What is the surface of a sphere whose diameter is 5 feet 7 inches?

3. What is the surface of a bombshell which is 12 inches in diameter?

4. What is the surface of the moon, whose diameter is 2160 miles, supposing her to be a perfect sphere?

5. What is the diameter of a ball which cost 141*l.* 7.44*s.* to gild it, at the rate of 36*s.* per square foot?

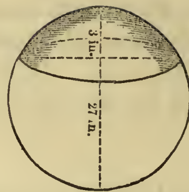
6. What is the surface of a ball, which, when put into a cylindrical vessel 4 inches in depth, just reaches from the top to the bottom of the vessel?

7. What is the surface of the greatest globe that can be cut from a cube of 9 inches?

8. Why is the surface of a sphere equal to the area of four of its great circles?

9. If the diameter of a sphere is 27 inches, what is the surface of a spherical segment belonging to it whose height is 3 inches?

* The circumference of a great circle of a sphere equals the diameter × by 3.1416.



SOLUTION. 27 in. × 3.1416 = 84.8232 in., the circumference of a great circle.

84.8232 in. × 3 in. = 254.4696 sq. in., surface of the spherical segment.

Res. 254.4696 sq. inches.

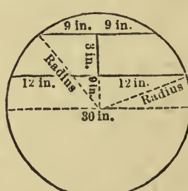
10. What is the surface of a spherical segment whose height is 5 ft., if the diameter of the sphere of which it is a part is 20 ft.?

11. What is the surface of a spherical segment the radius of whose *base* is 9 inches, and whose height is 3 inches?

NOTE.—To find the *radius* of the sphere when the zone or segment has but *one base*, divide the *sum* of the squares of the height and radius of the base by twice the height.*

To find the *radius* when the zone or segment has *two bases*, divide the *difference* between the square of ½ the diameter of the larger base, and the sum of the squares of ½ the diameter of the smaller base and height, by twice the height. The quotient will be the base of a right-angled triangle whose perpendicular is ½ the diameter of the larger base, and whose hypothenuse is the required *radius.*†

12. What is the surface of a spherical zone, the diameters of whose bases are 24 and 18 in., and whose height is 3 in.?



SOLUTION.

12 in.² — (9 in.² + 3 in.²) = 54 sq. in.

54 sq. in. ÷ (3 in. × 2) = 9 in.

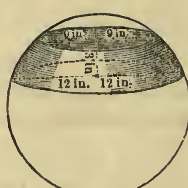
Then from the properties of a right-angled triangle,

$\sqrt{12 \text{ in.}^2 + 9 \text{ in.}^2} = 15$ in., the radius of the sphere.

15 in. × 2 = 30 in., the diameter.

30 in. × 3.1416 = 94.248 in., the circumference of a great circle.

94.248 in. × 3 in. = 282.744 sq. in., the surface of the zone.



Res. 282.744 sq. inches.

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* For explanation, see Key. † For explanation, see note, p. 82.

13. What is the surface of the torrid zone, if its height is 3150.68 miles, and the diameter of the earth is 7912 miles?

PROBLEM XIII.

To find the solidity of a sphere.

RULE.

Multiply its surface by one-third of its radius.

NOTE.—The solidity of a spherical pyramid or sector also equals the product of the surface of the polygon or zone forming the base, by one-third of the radius of the sphere to which it belongs.

EXPLANATION.

If we inscribe a *regular polyhedron* in a sphere, we may consider the polyhedron to be composed of *pyramids*, each having for its *vertex* the *centre of the sphere*, and for its *base* one of the *faces* of the *polyhedron*. The solidity of each of these pyramids equals the product of its base by one-third of its altitude, and the solidity of all the pyramids, or of the polyhedron, equals the product of the sum of their bases, or the surface of the polyhedron, by one-third of their common altitude.

Now if we increase the *number* of the faces of the polyhedron, its surface, or the bases of the pyramids, will equal the surface of the sphere, the altitude of the pyramids will equal the radius of the sphere, and the polyhedron will become the sphere. Hence, the solidity of a sphere equals the product of its surface by one-third of its radius.

NOTE.—The solidity of a *spherical pyramid* or *sector* is obtained in the same manner as that of a sphere, for either may be considered as composed of an indefinite number of *pyramids* whose *bases* form the *base of the pyramid* or *sector*, and whose *vertices*, like the vertex of the pyramid and sector, are at the *centre of the sphere.*

EXAMPLES.

1. What is the solidity of a sphere whose diameter is 7 ft.?

A B = 7 ft.

SOLUTION.

7 ft. × 3.1416 = 21.9912 ft., the circumference of a great circle.

21.9912 ft. × 7 ft. = 153.9384 sq. ft., the surface of the sphere.

7 ft. ÷ 2 = $3\frac{1}{2}$ ft., radius.

153.9384 sq. ft. × ($3\frac{1}{2}$ ft. ÷ 3) = 179.5948 cu. ft., the solidity of the sphere. Res. 179.5948 cu. ft.

2. What is the solidity of a sphere whose diameter is 5 ft.?

3. What is the solidity of a globe whose radius is 10 rods?

4. How many cubic miles does the earth contain if its diameter is 7912 miles, supposing it to be a perfect sphere?

5. The diameter of a small circle of a certain globe is 8 ft., and the distance from the centre of its plane to the centre of the sphere is 3 ft. What are the solid contents of the sphere?

6. What is the diameter of a globe whose solidity is 179.5948 cu. feet?

7. What is the diameter of a globe containing as many cubic feet as a cone 2 feet in diameter, and 3 feet high?

8. What is the radius of that sphere whose solid contents equal as many cubic feet, as its surface contains square feet?

9. Why does cubing the diameter of a sphere, and multiplying it by .5236 give its solidity?

10. What is the solidity of the greatest cube that can be cut from a globe 6 inches in diameter?

11. If a globe 3 feet in diameter weighs 260 lbs., what will be the weight of another whose diameter is 6 feet?

12. What is the diameter of a sphere which contains 8 times as much as one 5 feet in diameter?

13. What is the solidity of a spherical pyramid the area of whose base is 100 sq. feet, and the diameter of the sphere 20 feet?

14. What is the solidity of a spherical sector whose base is a zone 5 feet in altitude, in a sphere 24 feet in diameter?

15. What is the solidity of a spherical sector of the earth, whose base is the north frigid zone, the height of which is 327.19 miles, the diameter of the earth being 7912 miles?

PROBLEM XIV.

To find the solidity of a spherical segment with one base.

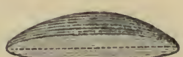
RULE.

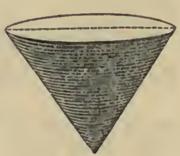
When the spherical segment is less than a hemisphere, from the solidity of a spherical sector having the zone of the segment for its base, subtract the solidity of the cone which forms the difference between the segment and sector.

When the spherical segment is greater than a hemisphere, it equals the sum of this sector and cone.

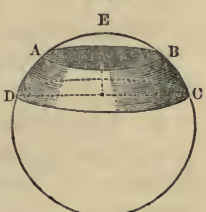
NOTE.—The solidity of a spherical segment having two bases equals the difference between the solidities of two segments, the one extending from the top of the sphere to its upper base, the other from the top of the sphere to its lower base.

EXPLANATION.





If a spherical sector be divided into two parts by passing a plane through the circumference of its base, the section will be a circle, and the solid will be divided into a cone and a spherical segment; therefore, if we take the cone from the spherical sector the remainder will be the spherical segment.

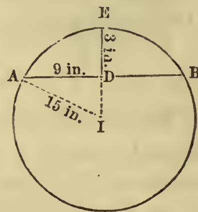


When the spherical segment has two bases, as A B C D, it is evident that the solidity of a segment extending from E to the base C D will exceed the given segment, by a segment extending from E to the base A B. Hence, the difference between the solidities of these two will be the solidity of the required segment.

EXAMPLES.

1. What is the solidity of a spherical segment the radius of whose base is 9 inches, and whose height is 3 inches?

SOLUTION.

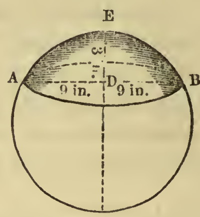


9 in.² + 3 in.² = 90 sq. in.

90 sq. in. ÷ 6 in. = 15 in., the radius of the sphere.

15 in. × 2 = 30 in., the diameter.

30 in. × 3.1416 = 94.248 in., the circumference of a great circle.



94.248 in. × 3 in. = 282.744 sq. in., the convex surface of the spherical segment.

282.744 sq. in. × (15 in. ÷ 3) = 1413.72 cu. in., the solidity of a spherical sector having the same base as the segment.

18 in.² × .7854 = 254.4696 sq. in., the base of the cone forming the difference between the sector and segment.

254.4696 sq. in. × (12 in. ÷ 3) = 1017.8784 cu. in., the solidity of the cone.

1413.72 cu. in. — 1017.878 cu. in. = 395.8416 cu. in., the solidity of the spherical segment.

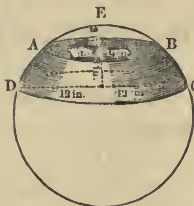
Res. 395.8416 cu. inches.

2. What is the solidity of a spherical segment the radius of whose base is 21 inches, and whose height is 7 inches?

3. What is the solidity of a spherical segment, the height of the zone forming the base being 2 feet, and the diameter of the sphere being 20 feet?

4. What is the solidity of a spherical segment whose height is 18 inches, the diameter of the sphere being 30 inches?

5. What is the solidity of a spherical segment the diameters of whose bases are 24 and 18 inches, and whose height is 3 inches?



SOLUTION. $12 \text{ in.}^2 - (9 \text{ in.}^2 + 3 \text{ in.}^2) = 54$ sq. in.

54 sq. in. $\div (3 \text{ in.} \times 2) = 9$ in., the base of a right-angled triangle whose hypothenuse is the radius.

$\sqrt{12 \text{ in.}^2 + 9 \text{ in.}^2} = 15$ in., the radius of the sphere.

$15 \text{ in.} \times 2 = 30$ in., the diameter.

Then from the solidity of the spherical segment D E C take the segment A E B.

$30 \text{ in.} \times 3.1416 = 94.248$ in., the circumference of a great circle.

$94.248 \text{ in.} \times 6 \text{ in.} = 565.488$ sq. in., the convex surface of the spherical segment D E C.

565.488 sq. in. $\times (15 \text{ in.} \div 3) = 2827.44$ cu. in., the solidity of the spherical sector, having for its base the base of the spherical segment, and for its vertex the centre of the sphere.

$24 \text{ in.}^2 \times .7854 = 452.3904$ sq. in., the base of the cone forming the difference between this spherical sector and segment.

452.3904 sq. in. $\times (9 \text{ in.} \div 3) = 1357.1712$ cu. in., the solidity of the cone.

2827.44 cu. in. $- 1357.1712$ cu. in. $= 1470.2688$ cu. in., the solidity of the spherical segment D E C.

$94.248 \text{ in.} \times 3 \text{ in.} = 282.744$ sq. in., the convex surface of the spherical segment A E B.

$282.744 \text{ in.} \times (15 \text{ in.} \div 3) = 1413.72$ cu. in., the solidity of the spherical sector, having for its base the base of the spherical segment A E B, and for its vertex the centre of the sphere.

18 in.^{2} $\times$ .7854 = 254.4696 sq. in., the base of the cone forming the difference between this spherical sector and segment.

254.4696 sq. in. $\times$ (12 in. $\div$ 3) = 1017.8784 cu. in., the solidity of the cone.

1413.72 cu. in. — 1017.8784 cu. in. = 395.8416 cu. in., the solidity of the spherical segment A E B.

1470.2688 cu. in. — 395.8416 cu. in. = 1074.4272 cu. in., the solidity of the required spherical segment A B C D.

Res. 1074.4272 cu. inches.

6. What is the solidity of a spherical segment the diameters of whose bases are 12 and 16 feet, and whose height is 2 feet?

7. What is the solidity of a spherical segment the diameters of whose bases are 6 and 10 feet, and whose height is 4 feet?

8. What is the solidity of a spherical segment the diameters of whose bases are 24 and 32 feet, and whose height is 28 feet?

THE END.

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