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Rudiments of Insurance Purchasing: A Graphical State-Claims Analysis

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Rudiments of Insurance Purchasing:
A Graphical State-Claims Analysis

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State-Claims Analysis

Abstract

The purpose of this note is mainly pedagogical. It shows how some of the main results existing in the insurance literature can be easily and effectively illustrated using a state-claims approach instead of the traditional utility analysis, which is more popular in insurance research and teaching. This approach is used to show how the level of insurance purchased changes as a function of a fixed and proportional loading factor, and under varying degrees of risk aversion, and alternative state-dependent utility environment.

Keywords: coinsurance, loading, optimal insurance, risk aversion, state-claims approach, state-dependent utility, utility.

Insurance Abstracts and Reviews Codes: M50, E30, B70.

I. INTRODUCTION

Graphical analysis is often used as an aid for understanding insurance purchasing decisions. A standard graphical tool employed in the analysis of insurance purchasing decisions is based on the representation of preferences for wealth in the wealth-utility plane; i.e. a graph with final wealth given by the abscissa and the utility of wealth given as the ordinate of points on the graph. Unfortunately, this graph is cumbersome to use in illustrating partial insurance coverage (see Schlesinger [1983]).

An alternative graphical presentation is based on a diagram, in which the axes depict the levels of final wealth realized in the respective states of the world.¹ This state-claims approach was used, for example by Ehrlich and Becker [1977], Rothchild and Stiglitz [1976] and Stiglitz [1977] in their classic analyses of insurance equilibrium. Despite its simplicity, this approach appears to be less popular in research and teaching than the traditional utility analysis.

The purpose of this note is mainly pedagogical and intended to show how several insurance concepts are easily illustrated in a state-claims framework, where insurance is being viewed as a contingent claim. The results are neither new nor exhaustive in examining insurance theorems that can be illustrated using the state-claims approach; but they are, hopefully, presented in a manner that provides sufficient new insights into consumer choice in the insurance market.

Both the standard utility approach, and the state-claims model often focus on a simple two-state world in which either a total loss occurs or no loss at all. This simplification allows easy illustration of some of the

main results existing in the insurance literature. Being confined to a two-state world, however, sterilizes some of the real life aspects - e.g., the distinction between coinsurance and deductible-type policies disappears in such an environment.

The paper sets up the simple state-claims framework and uses it to examine how the level of insurance purchased changes when the price of insurance changes. We first consider the case of no fixed loading fees as in Ehrlich and Becker [1972], while using a slightly different framework. We then extend the analysis to the case where the premium includes a fixed as well as a proportional loading. Next it is shown how increased risk aversion leads to the purchase of more coverage. The last application examines the purchase of insurance when individuals have state-dependent preferences.

II. THE STATE-CLAIMS FRAMEWORK

Consider a risk averse individual with initial wealth W that is subject to a possible loss L , ($0 < L \leq W$), with probability of loss $p > 0$. The individual's preferences are represented by the increasing, strictly concave, twice-differentiable von Neumann-Morgenstern utility of final wealth, $U(Y)$, where Y denotes the final random wealth. We restrict our analysis to allow for only two possible outcomes. In particular, let

$$Y = \begin{cases} y_1 = W & \text{with probability } 1-p \\ y_2 = W-L & \text{with probability } p. \end{cases} \quad (1)$$

The wealth levels in the definitional equation (1) represent final wealth in the absence of any insurance.

We assume that insurance is available, and the consumer selects the amount of insurance by choosing a coinsurance level, α . The premium, $\Pi(\alpha)$, is based on a common formula, which has fixed and proportional loading

elements:

$$\Pi(\alpha) = (1+\lambda)\alpha pL + \gamma, \quad (2)$$

where:

α = coinsurance factor: the insurer pays αL in the event of a loss and the insured retains $(1-\alpha)L$ of the loss,

λ = a loading factor ($\lambda \geq 0$),

and

γ = fixed cost per contract ($\gamma \geq 0$).

Following a purchase of such an insurance policy the individual's final wealth is:

$$Y = \begin{cases} y_1 = W - \Pi(\alpha) & \text{with probability } 1-p \\ y_2 = W - \Pi(\alpha) - (1-\alpha)L & \text{with probability } p. \end{cases} \quad (3)$$

III. THE LEVEL OF INSURANCE PURCHASED

The above framework can be used to analyze the optimal level of insurance, α . To illustrate the use of state-claims techniques, first consider the question of whether or not full coverage is desirable, which is probably the most common use of state-claims analysis. We shall distinguish between two pricing situations:

a. No Fixed loading fee

Assume that there is no fixed loading charge for insurance, i.e., $\gamma = 0$. Point E in Figure 1, depicts the levels of final wealth in the two states of the world where no insurance is bought (the situation described in equation 1). At this point the consumer has a wealth of W with a probability $1-p$ (state 1), and a wealth of $W-L$ with a probability p (state 2). A situation of certainty is represented by the case where the consumer has the same wealth in both states, i.e., a point on the 45° line (the "certainty line").

The consumer may change his initial position by purchasing insurance. Buying full coverage would bring him to a point G on the certainty line. In order to reach this situation he pays the premium $(1+\lambda)pL$ (equation 2) in state 1, and his net gain in state 2 is the insurance benefit minus the premium (i.e., $L - (1+\lambda)pL$). Consequently, the slope of line \overline{EF} which passes through G equals $- [1 - (1+\lambda)p]/(1+\lambda)p$. Points along EF lying between E and G represent partial coverage where the coinsurance factor, α , increases monotonically as we go from E to G. Indeed, for any point on EF, such as H, the level of coverage is represented by $\alpha = d(E,H)/d(E,G)$, where $d(\cdot, \cdot)$ represents the distance between two points (i.e., the standard Euclidean metric. A proof appears in the Appendix.)

INSERT FIGURE 1 ABOUT HERE

Let us consider whether or not G is optimal by considering what would happen if we were at G and then decided to reduce the level of coverage by a small amount. In general, the individual's marginal rate of substitution between one dollar in state 1 and one dollar in state 2 is

$(1-p)U'(y_1)/pU'(y_2)$.² Since $y_1 = y_2$ at G, this implies a marginal rate of substitution of $(1-p)/p$. This is not only true at G, but at all contingent claims along the certainty line. This mathematical property is referred to as "ray homotheticity" along the certainty line.

If coverage is reduced from G by a small amount, the insurance market allows trading of state 1 wealth for state 2 wealth at a rate of $[1-(1+\lambda)p]/(1+\lambda)p$. This rate equals the subjective marginal rate of substitution $(1-p)/p$, only when the insurers do not charge any loading on top of the expected loss (i.e., $\lambda=0$, in addition to our assumption that $\gamma=0$).

Thus, in the case of no fixed loading a full coverage is desirable only when insurance is priced according to the policy's actuarial value, ($\lambda=0$).

The curve U_1U_1 in Figure 1 illustrates the so-called indifference curve that includes G. This curve represents the locus of contingent claims that yield the individual the same expected utility as G. If the individual is strictly risk averse, this curve will be strictly convex to the origin (see Appendix.) It is easy to show that the claims lying southwest of this curve represent lower expected utility. Thus, as drawn, G is preferred to E. That is, full coverage is preferred to no coverage. This need not necessarily be true, and it is generally possible for G to yield a lower expected utility than E.

In the case where there is a positive proportional loading, ($\lambda > 0$), the subjective substitution rate at point G is larger than the objective (market) rate of substitution. Equality of the objective and subjective rate may only happen at a point Southeast of G. In the case illustrated, the expected-utility-maximizing level of insurance would leave the individual with claim H where the consumer's marginal rate of substitution of wealth between the no-loss and loss states is the same as the ratio of market prices for the two states. In other words, partial coverage is optimal. This well-known result is usually attributed to Mossin [1968] and Smith [1968].

The optimal level of coverage is

$$\alpha^* = d(E,H)/d(E,G) \tag{4}$$

The change from the initial position at E is characterized as follows:

$$d(E,J) = \text{premium paid for coverage} = (1+\lambda)p\alpha^*L$$

$$\begin{aligned} d(H,J) &= \text{net indemnity in the loss state (net of premium)} \\ &= \alpha^*L - (1+\lambda)p\alpha^*L \end{aligned}$$

$$d(E,J) + d(H,J) = \text{total insurance indemnity} = \alpha^*L.$$

Due to the convexity of the indifference curve passing through H , $U_0 U_0$, it must lie everywhere above \overline{EG} . In particular, it must lie above both points E and G so that partial coverage (point H) is better than no coverage as well as full coverage and hence is a global optimum. For a higher loading factor, λ , the slope of EG would change, and the optimum H would tend to be closer to point E , as we will show shortly. Note that if λ is high enough, "no insurance" may become the constrained optimum. If we allow $\alpha < 0$, then such an α can be optimal for high enough λ so that the insured "goes short" in insurance. This peculiar solution is not like insurance in the usual sense, but is not unsimilar to writing an option rather than buying one.²

b. Premiums include a fixed loading fee ($\gamma > 0$).

Now supposed that the premium includes a fixed loading fee ($\gamma > 0$), where the premium is given by (2). For the sake of comparison, assume that $\lambda > 0$ and is the same as in Figure 1. If any insurance is to be purchased, a fixed fee of γ must be paid. The opportunity of post-insurance wealth levels is given by the line $\overline{E'F'}$ in Figure 2, which is parallel to \overline{EF} (with the same slope $-[1-(1+\lambda)p]/(1+\lambda)p$). E' is derived by deducting γ from E in both states of the world. Of course E' itself is not a legitimate final outcome, since the consumer who prefers to purchase no insurance will not agree to pay the loading γ . However, along the opportunity locus, $\overline{E',F'}$, we can, once again, find the optimal level of coverage, at point H' . In general, H' can entail either a higher, lower, or the same level of coverage, α , as does H . Any difference in coverage is due to what is known in economic terms as the "income effect" (subtracting γ from wealth in all states).

INSERT FIGURE 2 ABOUT HERE

H' may offer a lower level of expected utility than E (see Figure 2).

I.e., although H' is the optimal claim (conditional on the payment of γ), the individual may be better off by not purchasing any insurance and "going bare", so to speak, thus, remaining at E. Whether or not "going bare" is the optimal decision depends on the magnitude of the fixed loading γ , and on the individual's preferences.

If $\lambda = 0$, but $\gamma > 0$, then the marginal price of insurance equals its actuarial value so that full coverage is optimal if any coverage is purchased at all. In this case, γ is the expected profit of the insurer. Whether or not full coverage is preferred to no coverage depends on whether or not γ is "too large". In this case, "too large" means that γ exceeds the Arrow-Pratt risk premium for E (See Stiglitz [1977], Schlesinger [1983].)

c. Changing the price of insurance

If the loading parameters γ and λ , are changed, it is likely that the optimal level of coverage will also change. Small changes in γ can result in quite drastic changes in coverage levels as the consumer goes from no coverage to some positive level, or vice versa. Small changes in λ can have the same effect when $\gamma > 0$. However, when $\gamma = 0$, the level of coverage is a continuous function of λ and small changes in λ do not cause drastic jumps in the level of coverage. Note, that there is no guarantee that α must be monotonically decreasing with λ : it is possible to have an increase in coverage as λ increases, although this is not particularly likely. (See Hoy and Robson [1981] and Borch [1985]).

An example of such an occurrence is illustrated in Figure 3. As λ increases the individual is offered contracts along $\overline{EG''}$ rather than \overline{EG} . It is easy to show that the policy whose contingent claim representation lies

closest to the 45° certainty line (where "closest" is measured via perpendicular distance) has the higher level of coverage.³ Drawing a 45° line through H, it can be seen that the optimal claim H^{*} along $\overline{EG^*}$ contains a higher level of coverage than does H. Consequently, more insurance is purchased along $\overline{EG^*}$, even though the price for every level of coverage is proportionately higher. However, since $\alpha = 1$ when $\lambda = 0$ and since $\alpha = 0$ for high enough λ , the coverage level is a monotonic decreasing function of λ in some neighborhood of $\lambda = 0$, and for high enough λ .

INSERT FIGURE 3 ABOUT HERE

IV. CHANGES IN RISK AVERSION

For a higher level of risk aversion, the level of coverage will increase (unless coverage is already full, $\alpha = 1$)⁴. Also, for a fixed λ , the smallest γ that will induce "no coverage" as an optimum will increase with the individual's level of risk aversion.⁵ Both of these results become obvious in the state-claims framework, once the effects of increased risk aversion on individual preferences are understood.⁶

A utility function $V(Y)$ is said to be more risk averse than $U(Y)$ if V is more concave than U ; i.e., if there exists a twice differentiable function g , such that

$$V(Y) = g[U(Y)] \text{ , } g' > 0, g'' < 0.$$

Now consider the marginal rate of substitution between state 1 claims and state 2 claims for utilities U and V . For $y_1 > y_2$,

$$\begin{aligned} MRS_u &= \frac{(1-p)U'(Y_1)}{pU'(Y_2)} \\ &\geq \frac{g'[U(Y_1)]}{g'[U(Y_2)]} \cdot \frac{(1-p)U'(Y_1)}{pU'(Y_2)} = MRS_v \end{aligned} \tag{5}$$

since g is concave. If $Y_1 = Y_2$, $MRS_U = MRS_V$; and if $Y_1 < Y_2$, $MRS_U < MRS_V$.

The implications of (5) and the corresponding conditions for $Y_1 \leq Y_2$ are that the indifference curve for V is tangent to that for U at any certainty claim, such as A in Figure 4. Furthermore, the indifference curve for V is more convex, lying everywhere above that for U except at the point of tangency. The indifference curve for V passing through point B in Figure 4, for example, would have a flatter slope at B than does the curve U . Restricting ourselves to $Y_1 \geq Y_2$ it is easy to use (5) to prove the two claims made at the beginning of this section.

INSERT FIGURE 4 ABOUT HERE

V. STATE-DEPENDENT PREFERENCES

As a final illustration of state-claims techniques for simple insurance modelling, we consider the case where preferences themselves differ in the two states of the world. Insurance purchasing when preferences are state dependent can be readily viewed using the state-claims approach.⁸ To this end, we express preferences via the function $U(Y, \theta)$ where $\theta = 1$ or 2 , indicating which of the two states occurs. Although it is not necessary, we assume that the individual has a distaste for state 2, apart from wealth considerations, (i.e., that $U(Y, 1) > U(Y, 2)$ for all Y). For example, it might be that state 2 involves the loss of an "irreplaceable commodity" with a sentimental value in addition to its market value.⁹ We let $U'(Y, \theta)$ denote $\partial U(Y, \theta) / \partial Y$ and assume that $U' > 0$ and $U'' < 0$ for both $\theta = 1$ and $\theta = 2$, and for all values of Y .

For simplicity, consider only the case of insurance sold at its

actuarial value, (i.e., the insurance premium is given by equation (2) with $\lambda = \gamma = 0$). Extensions to the cases of positive loadings are straightforward. Consider the available insurance contracts, which are represented by the segment \overline{EG} in Figure 5, having a slope $-(1-p)/p$. When preferences were state independent, full coverage (point G) was optimal. In the current case, the individual's marginal rate of substitution of state 1 claims for state 2 claims at full coverage G, is

$$\text{MRS} = \frac{(1-p)U'(Y_1,1)}{pU'(Y_2,2)} \quad (6)$$

In general $U'(Y,1)$ can be greater, less than, or equal to $U'(Y,2)$ for any particular value of Y . If, for example, marginal utility is lower for each wealth level in the loss state (state 2), then the subjective marginal rate of substitution at G is greater than the market prices ratio ($\text{MRS}_G > (1-p)/p$) and consequently less than full coverage should be purchased. Indeed, it follows that the optimal level of coverage must entail contingent claims where $U'(Y_1,1) = U'(Y_2,2)$, such as at point H in Figure 5. The locus of all contingent claims satisfying this condition is called the "reference set" of U , (denoted by $RS(U)$), an example of which appears in Figure 5.¹⁰ The reference set represents all contingent claims with equal marginal utilities in both states of nature, whereas the certainty line has described the situations with equal wealth in both states.

INSERT FIGURE 5 ABOUT HERE

In general, $RS(U)$ must not lie entirely below the certainty line as drawn in Figure 5. If $U'(Y,1) < U'(Y,2)$ for all Y , then $RS(U)$ will lie entirely above the certainty line. In such a case, more than 100 percent coverage would be optimal, if such coverage is not prohibited.

An interesting oddity occurs if we consider an initial position

somewhere between G and H on \overline{EG} , rather than at E, and assume that $U'(Y,1) > U'(Y,2)$ once again. In such a case, the optimal contingent claims contract would still be H, but the optimal level of α would be negative! That is, the individual would be willing to pay an "indemnity" following a loss in order to receive a "premium" that can be used when no loss occurs. It is not difficult to explain situations in which such contracts might be desirable. For example, consider a bachelor with no wife or children and no favorite charities. Rather than be interested in a life insurance policy that pays if he dies, this individual might be more interested in a contract that pays him when he lives, and takes some of his assets at his time of death -- an idea that is not all that different from a life annuity contract.

VI. CONCLUDING REMARKS

In analyzing the workings of markets, it is always useful to have as many tools as possible at one's disposal. In this paper, we have shown how a simple state-claims analysis can often lead to clearer insights and understandings of certain market principles for the purchase of insurance. Hopefully, such insights can lead to new hypotheses about how insurance markets work.

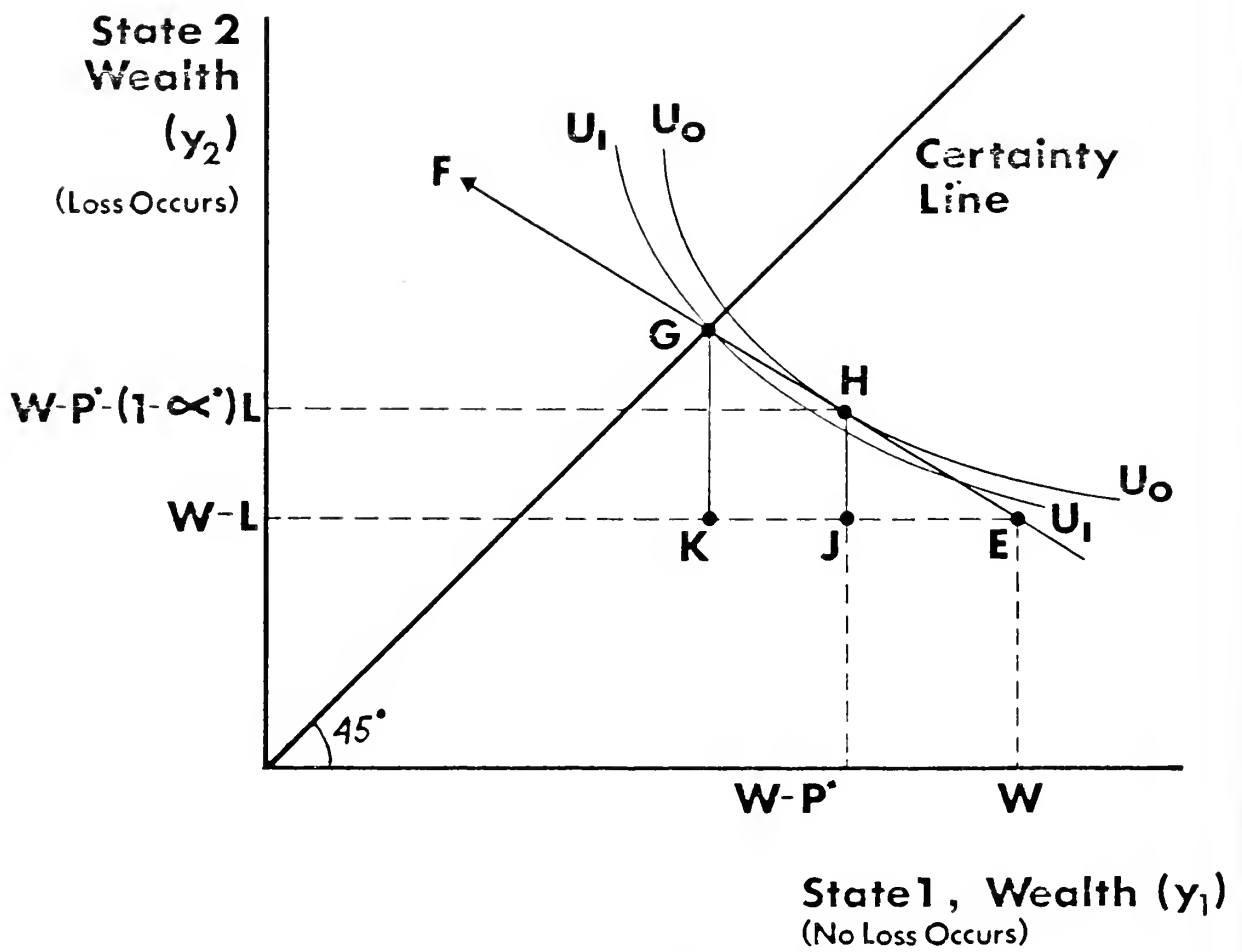


Figure 1

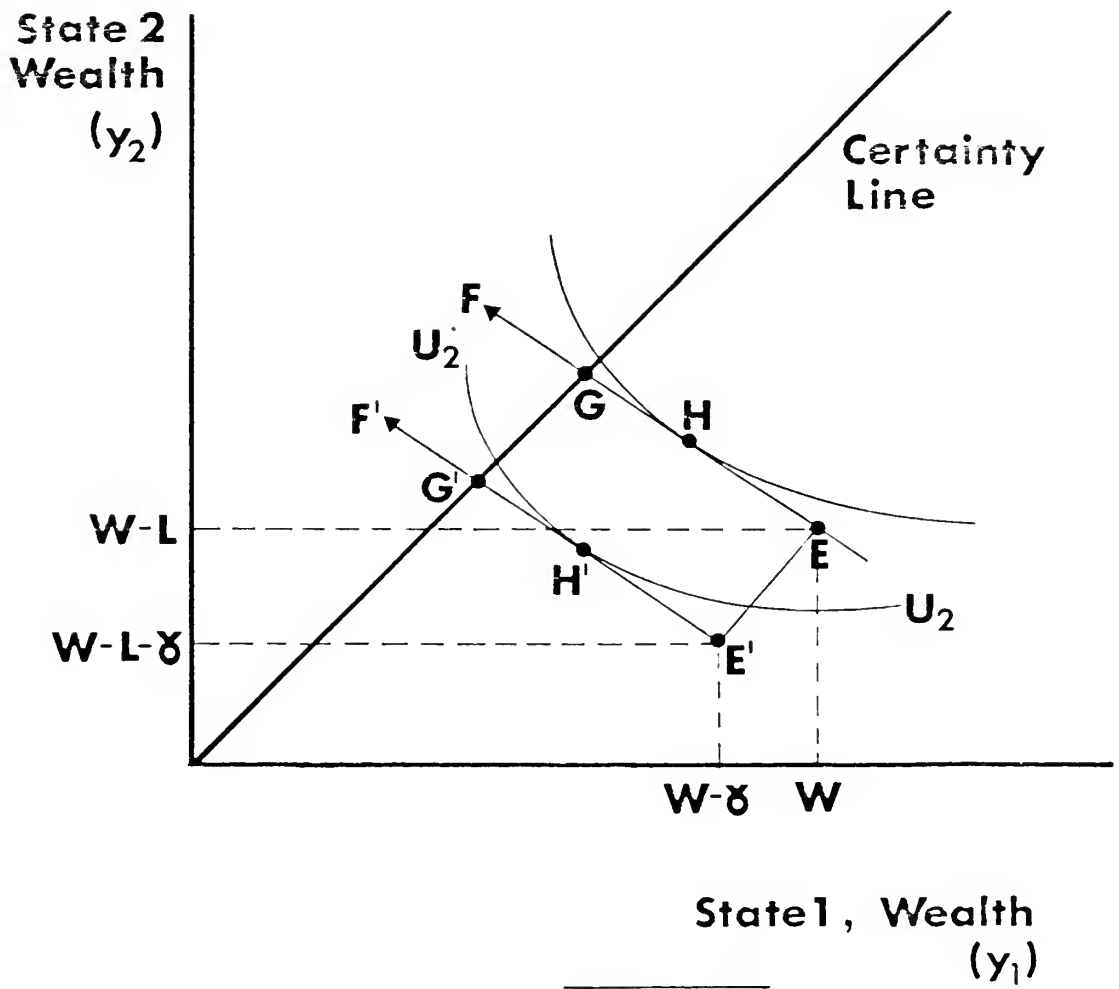


Figure 2

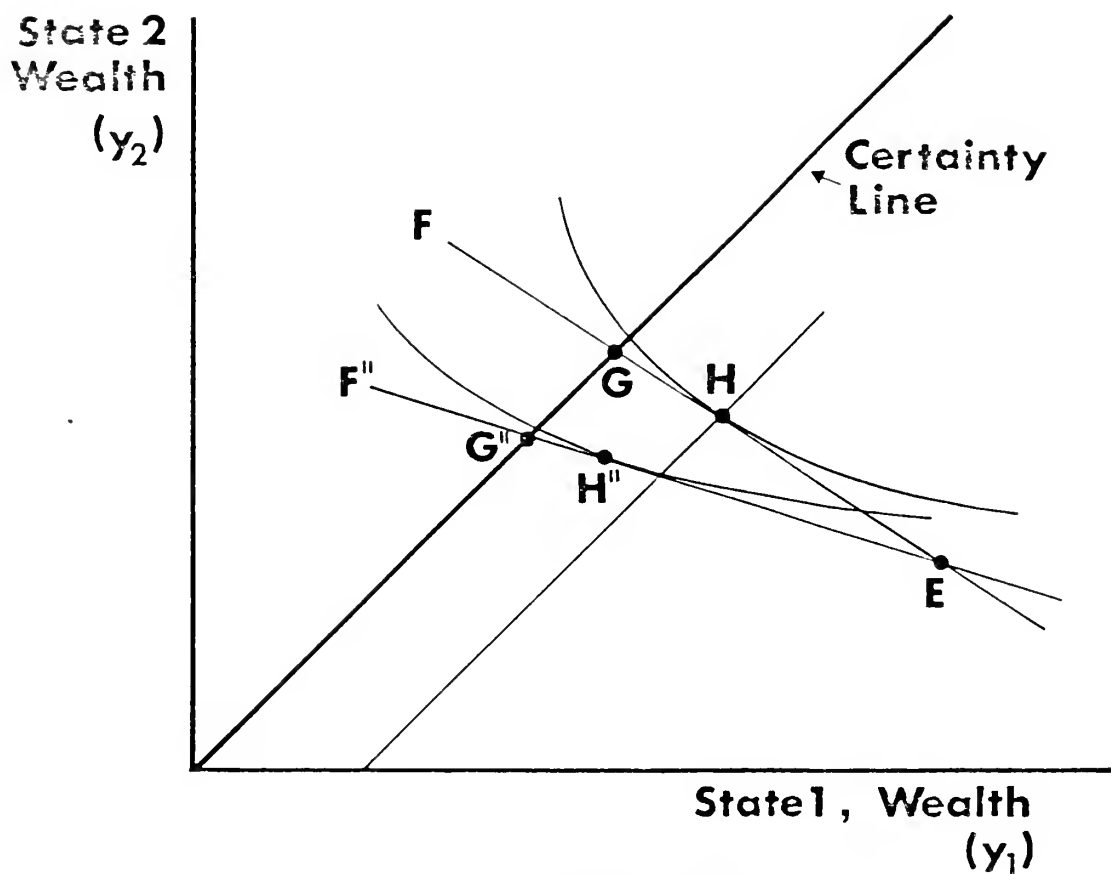


Figure 3

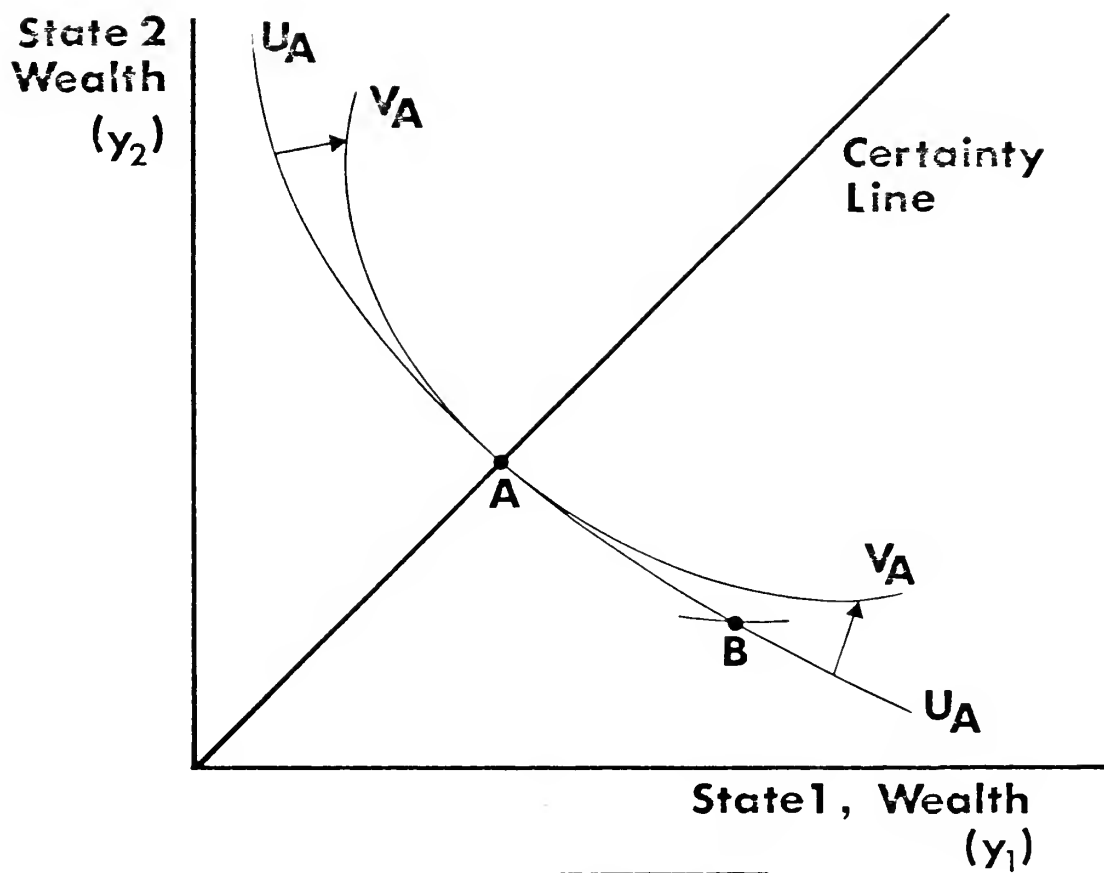


Figure 4

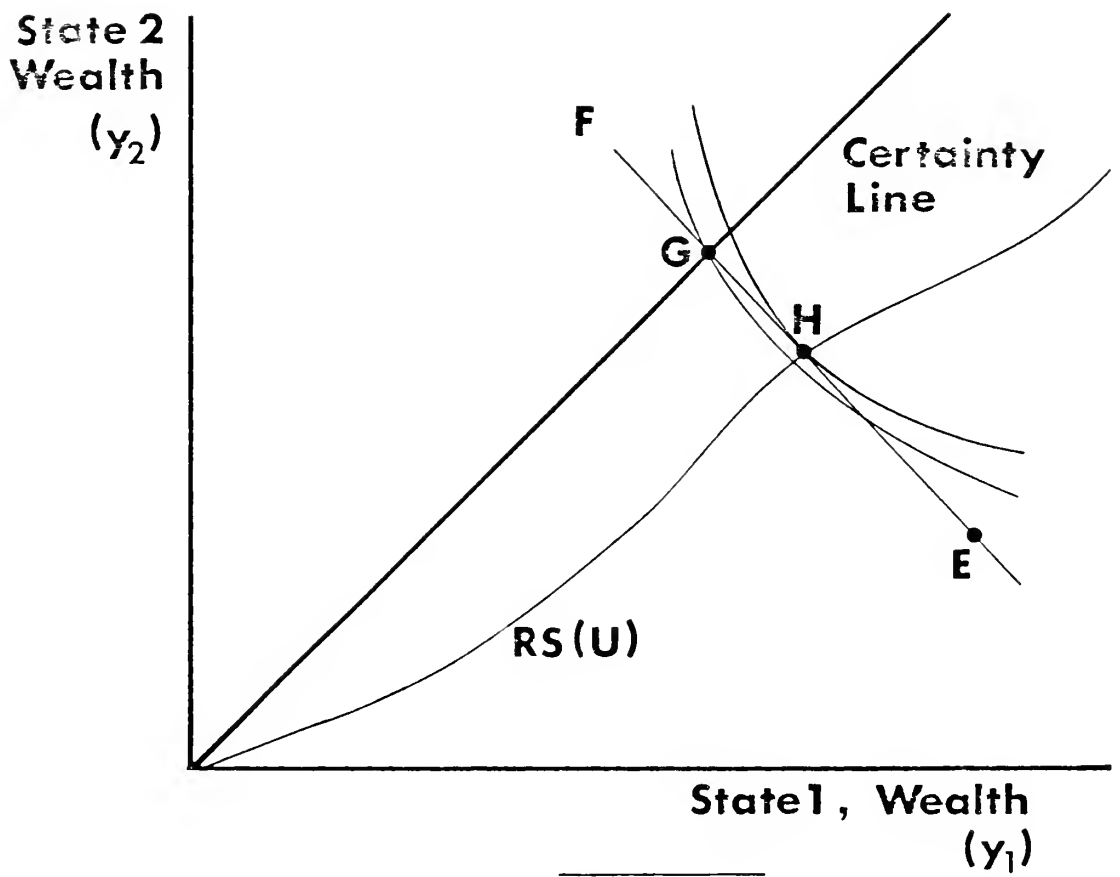


Figure 5

FOOTNOTES

1. See, for example, Hirshleifer [1966] and Hirshleifer and Riley [1979].
2. This rate of substitution is easily obtained by setting the expected utility equal to a constant.
3. A negative α entails the consumer making a net payment contingent on his or her own loss in return for a payment in the no loss state. This occurs only when λ is high enough to compensate for the extra risk involved in such a contract.
4. This was shown by Schlesinger [1981]
5. This result, as far as we know, is not published anywhere, although it is too straightforward to be considered particularly significant.
6. The details of the proofs are left to the reader.
7. The measure of risk aversion at a particular wealth level is $r = -U''(Y)/U'(Y)$. See Pratt [1964], and Kihlstrom and Mirman [1974].
8. There are many papers on state-dependent preferences. The state-claims approach with state dependent utilities is basically discussed in Cook and Graham [1977]. The model used here is similar to that of Karni [1983], although Karni does not model insurance purchasing per se.
9. See Cook and Graham [1977], Schlesinger [1984] and Shioshansi [1983].
10. This notion is due to Karni [1983].

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APPENDIX

A. Proof that $\alpha = d(E,H,)/d(E,G)$.

Let H denote the contingent claim following the purchase of insurance. If H=G or H=E, the proof is trivial. So assume that H lies strictly between E and G as in figure 1. Now, as labeled in figure 1, triangles EJH and EKG are similar triangles. Also, we argued in the text that $\alpha L = d(E,J) + d(J,H)$. In a similar manner, it is trivially seen that $L = d(E,K) + d(K,G)$. But the ratio $\alpha = \frac{d(E,J) + d(J,H)}{L}$ is then equivalent to the sum of the legs of right triangles EJH divided by those of EKG. Since we have similar triangles, the ratio of hypoteneuses must have the same ratio, which proves the claim.

B. Proof that risk averse preferences imply convex indifference curves.

This follows if we can show that expected utility is a quasiconcave function of (Y_1, Y_2) . To this end, suppose $(Y_1, Y_2) = (X_1, X_2) + (1-p)(Z_1, Z_2)$, where (X_1, X_2) and (Z_1, Z_2) yield the same expected utility and where $0 < p < 1$. We need only to show that (Y_1, Y_2) yields a higher expected utility. Now,

$$\begin{aligned} EU(Y_1, Y_2) &= (1-p)U(X_1 + (1-p)Z_1) + pU(X_2 + (1-p)Z_2) \\ &= (1-p)[U(X_1) + (1-p)U(Z_1)] + p[U(X_2) + (1-p)U(Z_2)] \\ &= EU(X_1, X_2) + (1-p)EU(Z_1, Z_2), \end{aligned}$$

where the inequality follows from the strict concavity of U.

Since $EU(X_1, X_2) = EU(Z_1, Z_2)$, we are done.

C. Proof that the state claim lying closer to the certainty line entails a higher level of insurance coverage.

We show this only for the case where $\alpha = 0$. The case where $\alpha > 0$ is easy and is left to the reader. First, suppose that for two different values, we obtain claims H and H', equidistant from the certainty line. This is drawn in Figure A. Since triangles EHH' and EGG' are similar triangles,

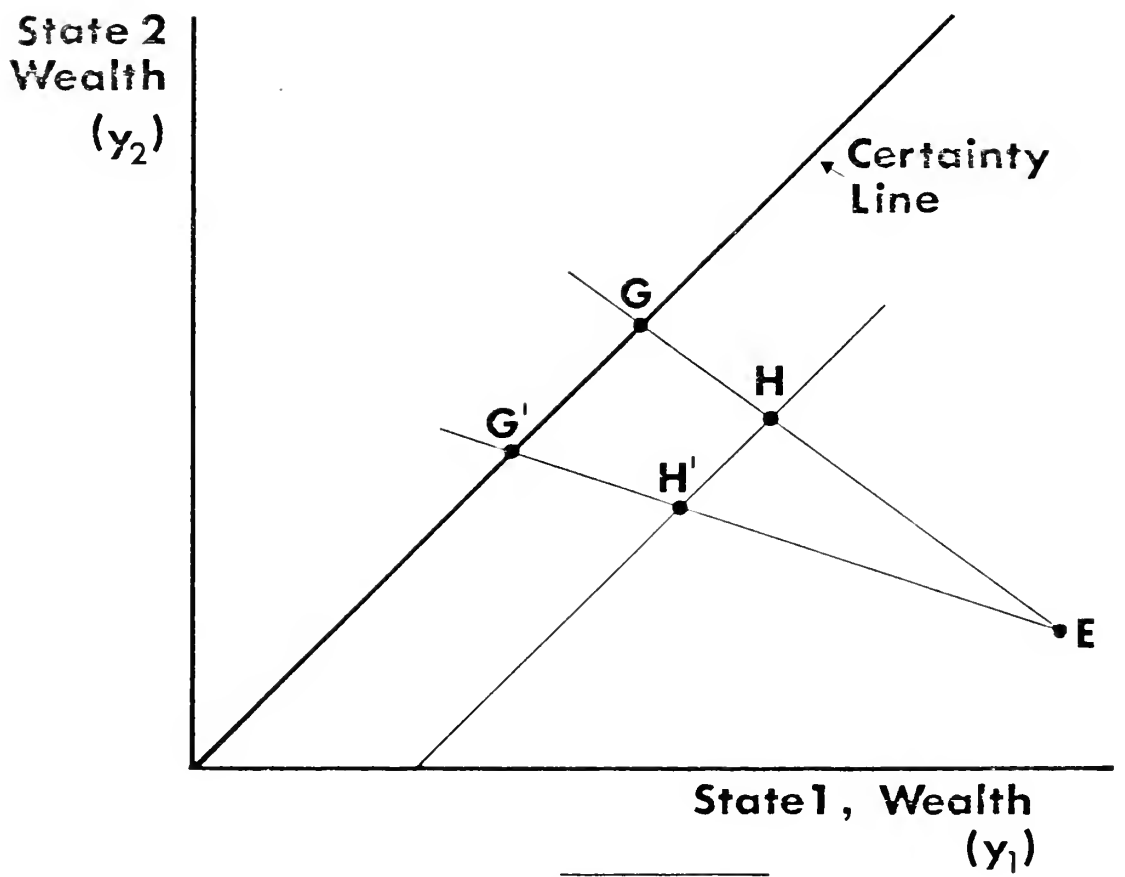
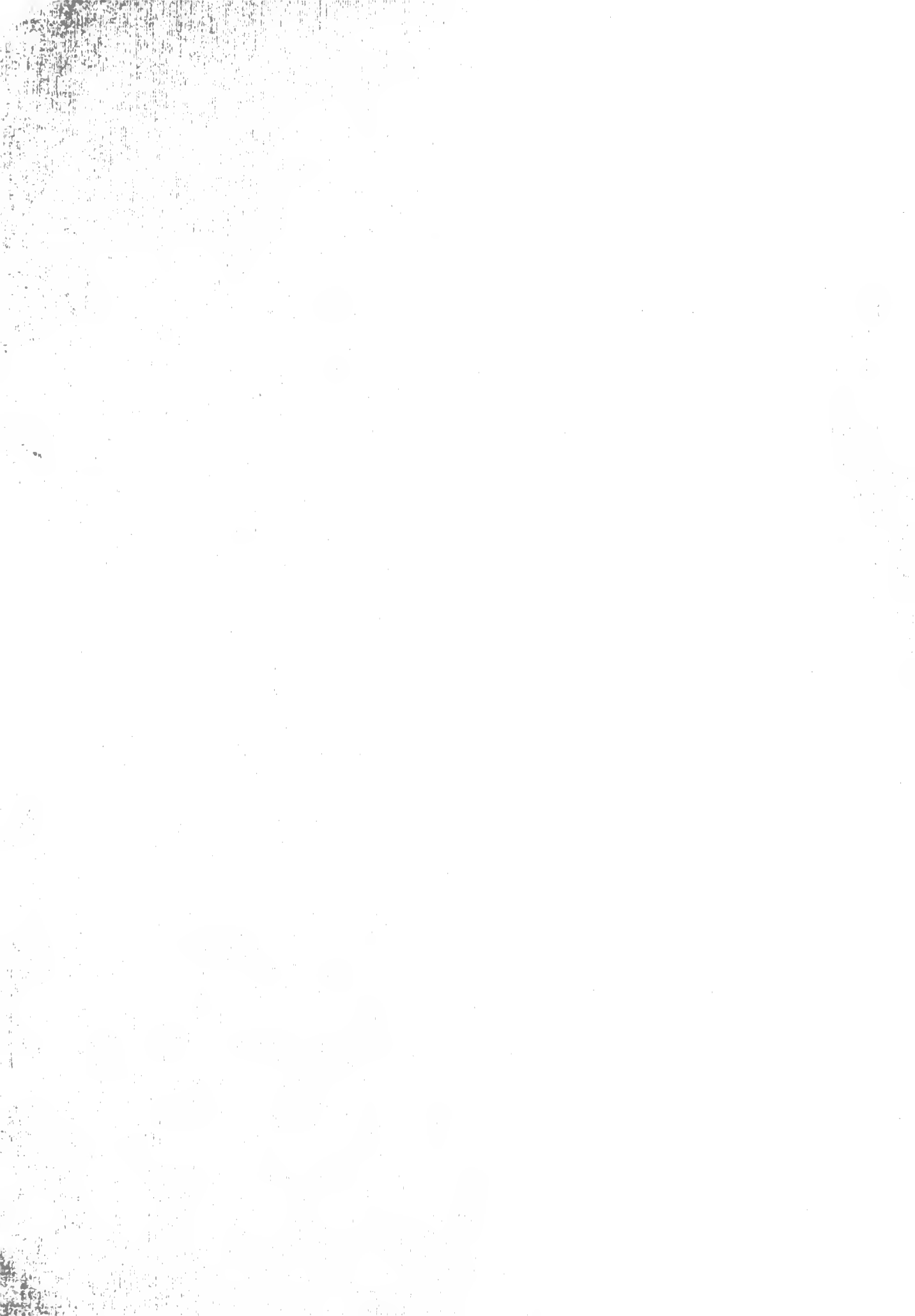


Figure A

it follows from part A of this appendix that H and H' represent claims with identical levels of coverage. The claim now follows trivially from part A of the appendix.

INSERT FIGURE A ABOUT HERE



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