# MECHANICS-PROBLEMS

## FOR ENGINEERING STUDENTS

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Gift of
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# PREFACE

THIS book contains many problems similar to those that are found in text-books; but besides many have been developed from actual engineering conditions. My object has been to correlate more closely these every-day practical examples with the important subjects in Mechanics. As an aid to this end, fifteen illustrations and forty-seven line cuts have been introduced. Although the book is not intended to take the place of text-books or lecture notes, it is hoped that it will in many instances supersede the hektograph or cyclostile sheets which many instructors now find it necessary to prepare and issue.

From my own experience as student, engineer in practice, and college instructor, I am convinced that the main object of a course in Mechanics should be to prepare the students to solve its problems. Therefore I have endeavored to make these problems — five hundred in number — fulfill the requirements for thorough and interesting instruction. They have been arranged in the following order: Work, Force, Motion. This order I believe to be on the whole the most satisfactory; but some instructors prefer to teach first the subject of Force; others begin with Motion. To meet these varying demands and other general uses an alphabetical classification of problems has been prepared, which makes it possible to find any single problem, or a collection of problems that pertain to a given subject.

At the beginning of each important section of the book one problem is solved so as to explain the method of solving similar problems, and to serve as a guide for solutions to be put in note-books. I believe that students learn the subject best and analyze most carefully the steps taken by thus keeping this set of college notes unabbreviated and in plain, concise form. In my own classes I require with considerable firmness that these notes shall be well-kept.

Besides the problems taken from engineering practice valuable ones have been selected from the following examination papers: Science and Art (England), Woolwich (England), Annapolis (United States), West Point (United States), and Harvard (United States); and some taken in whole or in part from text-books on Mechanics by Thornton, Lodge, Perry, Goodman, Goodeve (England), Wright, Hoskins, Johnson, Bowser (United States).

Photographs or electroplates have been furnished for certain of the illustrations as follows:

Page 17 by Otto Gas Engine Works; pages 20 and 32, Pelton Water Wheel Company; page 24, Wellington-Wild Coal Company; page 25, Harrisburg Foundry and Machine Company; page 29, Fall River Iron Works Company; page 35, Associated Factory Mutual Fire Insurance Companies; page 63, Maryland Steel Company; page 64, Bucyrus Company; page 112, A. J. Lloyd & Co.

FRANK B. SANBORN.

TUFTS COLLEGE, MASS.,
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WORK. Up grade on the Pennsylvania Railroad at Tyrone, Penn.

# MECHANICS–PROBLEMS

## I. WORK

### FOOT-POUNDS

**1.** A train weighing 100 tons moves 30 miles an hour along a horizontal road; the resistances are 8 pounds per ton. Find the quantity of work expended each hour.

$$\begin{aligned}
\text{Work} &= \text{force} \times \text{distance} \\
\text{Force} &= 8 \times 100 \\
&= 800 \text{ pounds} \\
\text{Distance} &= 30 \times 5\,280 \\
&= 158\,400 \text{ feet} \\
\therefore \text{Work} &= 800 \text{ pounds} \times 158\,400 \text{ feet} \\
&= 126\,720\,000 \text{ foot-pounds each hour.}
\end{aligned}$$

**2.** Find the work done by an engine in drawing a train one mile along a level railway, when the constant resistances of friction, air, and so on, are one ton.

**3.** A hole is punched through a plate of wrought-iron one-half inch in thickness, and the pressure operating the punch is estimated at 36 tons. Assuming that the resistance to the punch is uniform, find the number of foot-pounds of work done.

**4.** Find what work is being done per minute — that is, find the activity or the power of a pumping engine which is raising 2 000 gallons of water an hour from a mine 300 feet deep.

**5.** If a weight of 1 130 pounds be lifted up 20 feet by 20 men twice in a minute, how much work does each man do per hour?

**6.** A number of men can each do, on the average, 495 000 foot-pounds of work per day of 8 hours. How many such men are required to work at the rate of 10 horse-power — 33 000 × 10 foot-pounds per minute?

**7.** It is said that a horse can do about 13 200 000 foot-pounds of work in a day of 8 hours, walking at the rate of $2\frac{1}{2}$ miles per hour. What pull in pounds could such a horse exert continuously during the working-day?

**8.** The surface of the water in a well is at a depth of 20 feet from the surface of the ground, and when 500 gallons have been pumped out the surface is lowered to 26 feet. Find the number of units of work done in the operation.

**9.** One of the largest chimneys in America is that of the Clark Thread Co. at Newark, N.J. Its height is 335 feet, interior diameter 11 feet, outside diameter at base $28\frac{1}{2}$ feet, at top 14 feet. Find the work done in raising the material from the ground to its place in the chimney.

**10.** A chain hanging vertically 520 feet long, weighing 20 pounds per foot, is wound up. What work is done?

**11.** A chain of weight 300 pounds and length 150 feet, with a weight of 500 pounds at the end of it, is to be wound up by a capstan. What work will be done?

**12.** A stream of water is 20 feet wide, its average depth is 3 feet, and the average velocity in the cross-section is 3 miles per hour. If there is an available fall of 200 feet, how much potential energy is possessed by the quantity flowing each minute? (Weight of water may be taken as 62.5 pounds per cubic foot.)

**13.** A horse draws 150 pounds of earth out of a well, by means of a rope going over a fixed pulley, which moves at the rate of $2\frac{1}{2}$ miles an hour. Neglecting friction, how many units of work does this horse perform a minute?

**14.** A cylindrical shaft 14 feet in diameter must be sunk to a depth of 10 fathoms through chalk, the weight of which is 144 pounds per cubic foot. Find the work done.

**15.** A well is to be dug 20 feet deep and 4 feet in diameter. Find the work in raising the material, supposing that a cubic foot of it weighs 140 pounds.

**16.** A horse draws earth from a trench by means of a rope going over a pulley. He pulls up, twice every 5 minutes, a man weighing 130 pounds, and a barrowful of earth weighing 260 pounds. Each time the horse goes forward 40 feet. Find the useful work done per hour.

**17.** A body weighing 50 pounds slides a distance of 8 feet down a plane inclined 20° to the horizontal, against a constant retarding force of 4 pounds. Compute the total work done upon the body by (gravity) its weight and the friction.

**18.** What electrical current expressed in ampères will be used by a 250-volt electric hoist when raising 2 500 pounds of coal per minute from a ship's hold 150 feet below dump cars on trestle work, the efficiency of the whole arrangement being 50 per cent?

$$\left\{ \begin{array}{ll} 1 \text{ horse-power} & = 746 \text{ watts} \\ \text{Watts} & = \text{volts} \times \text{ampères} \end{array} \right\}$$

**19.** The hammer of a pile-driver, weighing 500 pounds, is raised to a height of 20 feet and then allowed to fall upon the head of a pile, which is driven into the ground 1 inch by the blow. Find the average force which the hammer exerts upon the head of the pile.

$$\begin{aligned} \text{Work} &= \text{force} \times \text{distance} \\ &= 500 \times 20 \\ &= 10\,000 \text{ foot-pounds} \\ \text{Distance} &= \tfrac{1}{12} \text{ foot} \\ \therefore 10\,000 \text{ foot-pounds} &= \text{force} \times \tfrac{1}{12} \text{ foot} \\ \therefore \text{force} &= 10\,000 \times 12 \\ &= 120\,000 \text{ pounds} \end{aligned}$$

**20.** A hammer weighing 1 ton falls from a height of 24 feet on the end of a vertical pile, and drives it half an inch deeper into the ground. Assume the driving force of the hammer on the pile to be constant while it lasts, and find its amount expressed in tons weight.

**21.** What energy is stored in a cross-bow whose cord has been pulled 15 inches with a maximum force of 224 pounds?

**22.** A train of 150 tons is running at 60 miles an hour. What force is required to stop it in a quarter of a mile?

**23.** A railway car of 4 tons, moving at the rate of 5 miles an hour, strikes a pair of buffers which yield to the extent of 6 inches. Find the average force exerted upon them.

**24.** If 25 cubic feet of water are pumped every 5 minutes from a mine 140 fathoms deep, what amount of work is expended per minute?

**25.** In pumping 1 000 gallons from a water-cistern with vertical sides the surface of the water is lowered 5 feet. Find the work done, the discharge being 10 feet above the original surface.

**26.** A pumping-engine is partly worked by a weight of 2 tons, which at each stroke of the pump falls through 4 feet; the pump makes 10 strokes a minute. How many gallons of water (one gallon weighing $8\frac{1}{3}$ pounds) are lifted per minute by the weight from a depth of 200 feet?

$$\begin{aligned}
\text{Work} &= \text{force} \times \text{distance} \\
\text{Force} &= 2 \times 2\,000 \times 10 \\
&= 40{,}000 \text{ pounds} \\
\text{Distance} &= 4 \text{ feet} \\
\text{Work} &= 40\,000 \text{ pounds} \times 4 \text{ feet} \\
&= 160\,000 \text{ foot-pounds}
\end{aligned}$$

To find the number of gallons of water that can be lifted by this amount of work:

$$\text{Work} = \text{force} \times \text{distance}$$
$$160\,000 \text{ foot-pounds} = \text{force} \times 200 \text{ feet}$$
$$\text{force} = \frac{160\,000}{200}$$
$$= 800 \text{ pounds}$$
$$= 96 \text{ gallons}$$

**27**. A uniform beam weighs 1 000 pounds, and is 20 feet long; it hangs by one end, round which it can turn freely. How many foot-pounds of work must be done to raise it from its lowest to its highest position?

**28**. A weight of 200 pounds is to be raised to a height of 40 feet by a cord passing over a fixed smooth pulley; it is found that a constant force P, pulling the cord at its other end for three-fourths of the ascent, communicates sufficient velocity to the weight to enable it to reach the required height. Find P.

**29**. A horse drawing a cart along a level road at the rate of 2 miles per hour performs 29 216 foot-pounds of work in 3 minutes. What pull in pounds does the horse exert in drawing the cart?

**30**. A body weighing 10 pounds slides down an inclined plane whose height is 25 feet; it reaches the foot of the plane with a velocity of 30 feet per second. During the motion how many foot-pounds of energy have been expended on friction and other resistances?

(The velocity of a body falling unimpeded is $v = \sqrt{2gh}$.)

**31.** If a horse walking once round a circle 10 yards across raises a ton weight 18 inches, what force does he exert over and above that necessary to overcome friction?

**32.** If, neglecting frictions, a power of 10 pounds, acting on an arm 2 feet long, produces in a screw-press a pressure of half a ton, what would be the pitch of the screw?

**33.** What is the ratio of the weight to the power, in a screw-press working without friction, when the screw makes 4 turns in the inch, and the arm to which the power is applied is 2 feet long?

**34.** What force applied at the end of an arm 18 inches long will produce a pressure of 1 000 pounds upon the head of a smooth screw when 11 turns cause the head to advance two-thirds of an inch?

**35.** Find the mechanical advantage in a differential screw, if the length of the power arm is 2 feet, and there are 4 threads to the inch in the large screw, and 5 threads to the inch in the small screw.

**36.** In a differential pulley, if the radii of the pulleys in the fixed block are as 3 to 2; and if the weight of the lower block is $1\frac{1}{2}$ pounds, what weight can be raised by a force of 5 pounds?

**37.** In a wheel and axle the diameter of the wheel is 7 feet, of the axle 7 inches. What weight can be raised by a force of 10 pounds acting at the circumference of the wheel?

**38.** A weight of 448 pounds is raised by a cord which passes round a drum 3 feet in diameter, having on its shaft a toothed wheel also 3 feet in diameter; a pinion 8 inches in diameter, and driven by a winch handle 16 inches long, gears with the wheel. Find the power to be applied to the winch handle in order to raise the weight.

Fig. 1.

**39.** A tackle is formed of two blocks, each weighing 15 pounds, the lower one being a single movable pulley, and the upper or fixed block having two sheaves; the parts of the cord are vertical, and the standing end is fixed to the movable block. What pull on the cord will support 200 pounds hung from the movable block? and what will then be the pull on the staple at the upper block?

**40.** A weight of 400 pounds is being raised by a pair of pulley blocks, each having two sheaves; the standing part of the rope is fixed to the upper block, and the parts of the rope, whose weight may be disregarded, are considered to be vertical; each block weighs 10 pounds. What is the pressure on the point from which the upper block hangs?

**41.** Two equal weights, each 112 pounds, are joined by a rope which runs over two pulleys A and B 12 feet apart and in the same horizontal line. If a weight of 10 pounds is attached to the rope half-way between A and B, find the distance in inches to which the rope is deflected below the level of A B.

**42.** A weight of 500 pounds, by falling through 36 feet, lifts, by means of a machine, a weight of 60 pounds to a height of 200 feet. How many units of work have been expended on friction, and what proportion does the expenditure bear to the whole amount of work done?

**43.** The pull on a tram-car was registered when the car was at the following distances along the track: 0, 200 pounds; 10 feet, 150 pounds; 25 feet, 160 pounds; 32 feet, 156 pounds; 41 feet, 163 pounds; 56 feet, 170 pounds; 60 feet, 165 pounds; 73 feet, 160 pounds. What is the average (space) pull on the car, and what is the effective work done in pulling the car through the distance of 73 feet?

**44.** In lifting an anchor of $1\frac{1}{2}$ tons from a depth of 15 fathoms in 6 minutes, what is the useful man-power, if a man-power is defined as 3 500 foot-pounds per minute?

**45.** Four hundred weight of material are drawn from a depth of 80 fathoms by a rope weighing 1.15 pounds per linear foot. How much work is done altogether, and how much per cent is done in lifting the rope? How many units of 33 000 foot-pounds per minute would be required to raise the material in $4\frac{1}{2}$ minutes?

## HORSE-POWER

**46.** A weight of 3 tons is raised through 50 feet in a quarter of a minute. What horse-power must be used?

Work done in $\frac{1}{4}$ minute $= 3 \times 2\,000 \times 50$ foot-pounds.
Work done in 1 minute $= 4 \times 3 \times 2\,000 \times 50$ foot-pounds.
Now, 1 horse-power $= 33\,000$ foot-pounds per minute.

$$\therefore \text{ required horse-power} = \frac{4 \times 3 \times 2\,000 \times 50}{33\,000}$$
$$= 36\frac{4}{11}$$

**47.** A man weighing 155 pounds carries a weight of 65 pounds to a vertical height of 20 feet. How many foot-pounds of work has he done? If he make 20 such journeys in an hour, at what rate in horse-power does he work?

**48.** A windmill raises by means of a pump 22 tons of water per hour to a height of 60 feet. Supposing it to work uniformly, calculate its horse-power.

**49.** The travel of the table of a planing-machine which cuts both ways is 9 feet. If the resistance while cutting be taken at 400 pounds, and the number of revolutions or double strokes per hour be 80, find the horse-power absorbed in cutting.

**50.** A forge hammer weighing 300 pounds makes 100 lifts a minute; the perpendicular height of each lift is 2 feet. What is the horse-power of the engine that operates 20 such hammers?

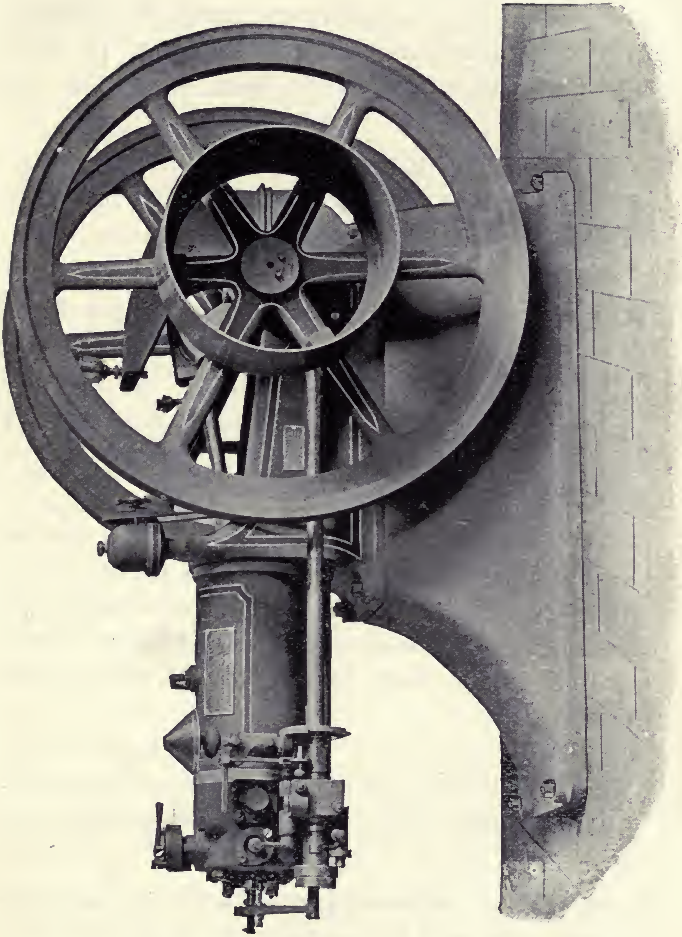


Fig. 2.

**51.** What would be the indicated horse-power of a gas engine which has a piston 12 inches in diameter and a crank 8 inches long? The engine working at

150 revolutions a minute, there being an explosion every 2 revolutions and the mean effective pressure in the cylinder during a cycle being 62 pounds per square inch.

**52.** How many horse-power would it take to raise 3 hundred weight of coal a minute from a pit whose depth is 660 feet ?

**53.** Find the horse-power of an engine which is to raise 30 cubic feet of water per minute from a depth of 440 feet.

**54.** Find the horse-power required to draw a train of 100 tons, at the rate of 30 miles an hour, along a level railroad, the resistance from friction being 16 pounds per ton.

**55.** Each of the two cylinders in a locomotive engine is 16 inches in diameter and the length of crank is 1 foot. If the driving-wheels make 105 revolutions per minute, and the mean effective steam-pressure is 85 pounds per square inch, what is the horse-power ?

**56.** The weight of a train is 95.5 tons, and the drawbar pull is 6 pounds per ton. Find the horse-power required to keep the train running at 25 miles per hour.

**57.** A train, whose weight including the engine is 100 tons, is drawn by an engine of 150 horse-power ; friction is 14 pounds per ton — all other resistances neglected. Find the maximum speed which the engine is capable of maintaining on a level track.

**58.** A dynamo is driven by an engine that develops 230 horse-power. If the efficiency of dynamo is 0.81 what "activity" in kilowatts is represented by the current generated?

(1 kilowatt = 1.340 horse-power.)

**59.** Electric lamps giving 1 candle-power for 4 watts (*a*) how many 10- and (*b*) how many 16-candle lamps may be worked per electric horse-power? The combined efficiency of engine, dynamo, and gearing being 70 per cent, what is the candle-power available for every indicated horse-power?

**60.** The section of a stream is 12 square feet, the average velocity of the water is 2 feet per second; there is an available fall of 25 feet; what is the horse-power available? A turbine here drives a dynamo machine which sends electric power to a motor at a distance. The efficiency of the turbine is 70 per cent; of the dynamo, 87 per cent; 10 per cent of the energy from the dynamo is wasted in transmission and the efficiency of the motor is 72 per cent. How much power is given out by the motor? The voltage of the dynamo is 102. What is the current in ampères?

**61.** A 500-volt electric motor imparts velocity to an 8-ton car so that at the end of 20 seconds it is moving on a level track at the rate of 10 miles an hour; the total efficiency of the motor and car is 60 per cent. What ampères are necessary?

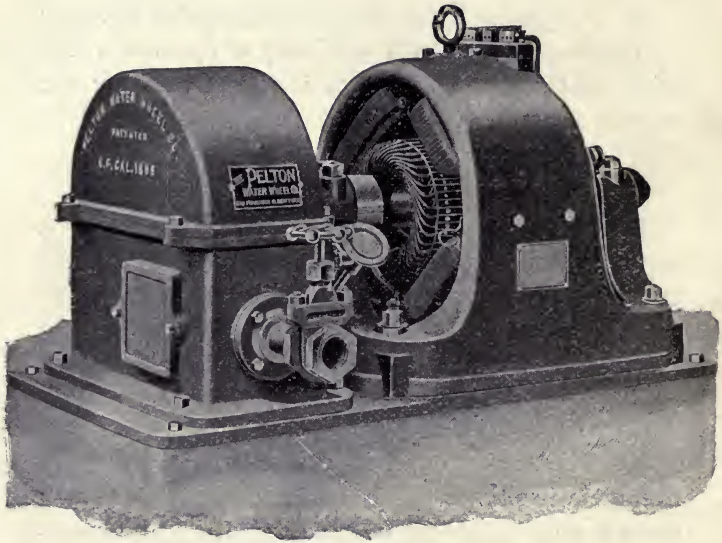



Fig. 3.

**62**. A water-motor is driven by two jets 1 inch in diameter, flowing with velocity of 80 feet per second. Theoretic horse-power would be 9.9; and if efficiency of wheel is 85 per cent, and the generator which the wheel drives also 85 per cent, what power in kilowatts is represented by the current that is produced?

**63**. What is the necessary difference of tensions in a driving-belt 30 inches wide, which is running 4 200 feet a minute and transmitting 300 horse-power?

**64**. Find the speed of a driving-pulley 3.5 feet in diameter to transmit 6 horse-power, the driving-force of the belt being 150 pounds.

**65.** A belt can stand a pull of 100 pounds only. Find the least speed at which it can be driven to transmit 20 horse-power.

**66.** A pulley 3 feet 6 inches in diameter, and making 150 revolutions a minute, drives by means of a belt, a machine which absorbs 7 horse-power. What must be the width of the belt so that its greatest tension may be 70 pounds per inch of width, it being assumed that the tension in the driving-side is twice that on the slack side?

**67.** An endless cord stretched and running over grooved pulleys with a linear velocity of 3 000 feet per minute, transmits five horse-power. Find the tension of the cord in pounds.

**68.** In the transmission of power by a rope the wheel carrying the rope is 14 feet in diameter, and makes 30 revolutions per minute, the tension of the rope being 100 pounds. Find the amount of power transmitted as estimated in horse-power.

**69.** A locomotive engine, which can work up to 100 horse-power, is attached to a train, whose mass (including the locomotive itself) is 100 tons. Assuming the total resistance to be constant and equivalent to 10 pounds weight per ton, find the greatest speed of the train in miles per hour.

When traveling at this speed the steam is shut off. Find the distance and the time in which the train would be reduced to rest by the resistance alone.

**70.** A train weighing 100 tons runs at 42 miles an hour on a level track, the resistance being 8 pounds per ton. Find its speed up a 1 per cent grade (1 foot rise in 100 feet horizontal) if the engine-power is kept constant.

**71.** In 1895 a passenger engine on the Lake Shore Railroad made a run of 86 miles at the rate of 73 miles an hour. Weight of train, 250 tons; resistance on level track, 15 pounds per ton. The engine was a 10-wheeler, having drivers 5 feet 8 inches in diameter and cylinders 17 × 24 inches. Show that to develop 730 horse-power the average effective cylinder-pressure must have been about 37 pounds per square inch.

**72.** What must be the effective horse-power of a locomotive which moves at the steady speed of 35 miles an hour on level rails, the weight of engine and train being 120 tons, and the resistance 16 pounds per ton? What additional horse-power would be necessary if the rails were laid along a gradient of 1 in 142?

**73.** In example 72 find in each case how far the train would move after steam was shut off, assuming the above constant resistance and neglecting rotary motions. Find also the speed of the train after the latter had moved over a distance of 1 000 feet from the point where steam was shut off.

**74.** Find the total horse-power of two engines which are taking a train of 250 tons down a grade of

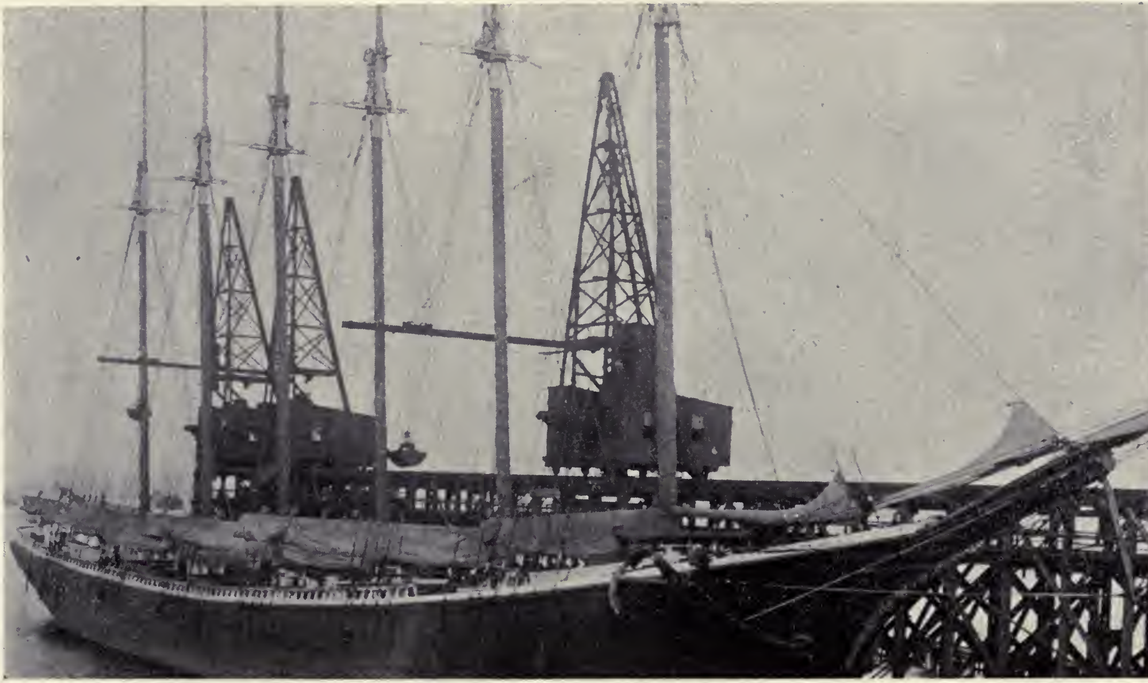
1 in 200 at 60 miles an hour, supposing the resistance on the level at this speed to be 35 pounds a ton.

**75.** A train of 50 tons moves up a rough incline of 1 in 10, the resistance caused by friction being 16 pounds per ton. What horse-power must the engine exert in order to maintain a uniform speed of 3 miles an hour?

**76.** Find the horse-power of a locomotive which is to move at the rate of 20 miles an hour up an incline which rises 1 foot in 100, the weight of the locomotive and load being 60 tons, and the resistance from friction 12 pounds per ton.

**77.** A steam-crane, working at 3 horse-power, is able to raise a weight of 10 tons to a height of 50 feet in 20 minutes. What part of the work is done against friction? If the crane is kept at similar work for 8 hours, how many foot-pounds of work are wasted on friction?

**78.** The six-master shown on the next page carries 5 500 tons of coal. It is unloaded by small engines which take up 1 ton at each hoist; average lift from hold of ship to top of chutes which lead to cars, 35 feet; weight of bucket, 1 ton; 2 trips are made per minute, and 25 per cent of power of engine is lost in friction and transmission. Find the horse-power of each engine required when two towers are working.



Six-masted Schooner Unloading Coal.

The illustration of six-master on opposite page accompanies Problem 78.

**79.** An average size coal barge will carry 1 600 tons. If it is unloaded by two simple direct engines, the coal being hoisted 65 feet to an elevated hopper on the wharf, weight of bucket 1 ton, and carrying 1 ton of coal, what horse-power of engines would be required to unload the 1 600 tons in 20 hours?

**80.** The coal from the hopper is run into a car which carries 2 tons, and goes down a grade 25 feet long in 25 seconds; it strikes a cross-bar, or "stopper," which, acting through a distance of 30 feet, brings the car to rest. What is the average force that acts?

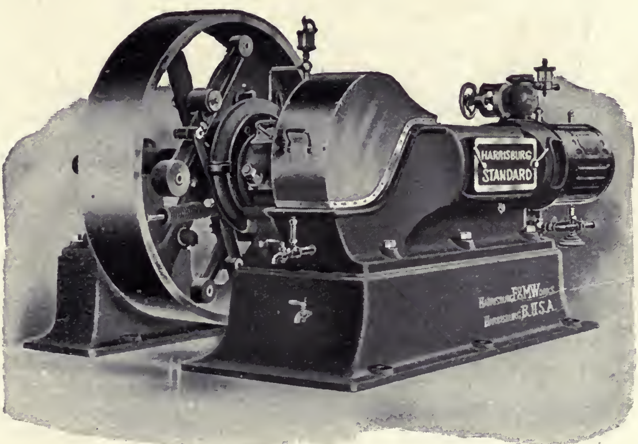



Fig. 4.

**81.** The engine shown in Fig. 4 has steam cylinder 15 inches in diameter; length of stroke, 15 inches;

revolutions per minute, 275; mean effective pressure, 76 pounds per square inch. Find the horse-power.

**82.** The indicator cards illustrated herewith were taken from an engine of the type shown in problem 81, diameter of steam cylinder being 14 inches, length of stroke 12 inches, revolutions per minute 300. Scale on cut the mean ordinates, which were produced by indicator springs of stiffness 40 pounds to an inch, and compute the indicated horse-power of the engine.

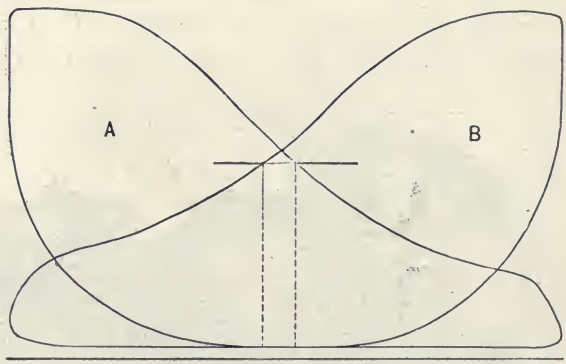



Fig. 5. Full Load Indication.

**83.** The indicator cards shown below were taken from one of the triple-expansion pumping-engines at the East Boston Station of the Metropolitan Sewerage. The cards were from two ends of a high-pressure cylinder. Refer to the cards and compute the indicated horse-power. (A twenty-four hours' duty trial of this pumping-engine was made January 17–18, 1901, by engineering students of Tufts College.)

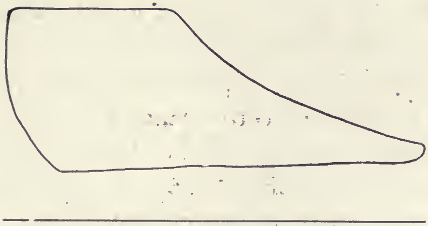


Fig. 6. Head end. Card shown, one-half size; area of original, 4.69 square inches; stiffness of spring, 50 pounds per square inch; length of stroke, 30 inches; revolutions per minute, 84.

Fig. 7. Crank end. Card shown, one-half size; area of original, 4.62 square inches; stiffness of spring, 50 pounds per square inch; length of stroke, 30 inches; revolutions per minute, 84.

**84.** The average breadth of an indicator diagram for one end of a piston is 1.58 inches, and for the other end it is 1.42 inches, and 1 inch represents 32 pounds per square inch. Piston, 12 inches diameter; crank, 1 foot long; revolutions per minute, 110. What is the indicated horse-power?

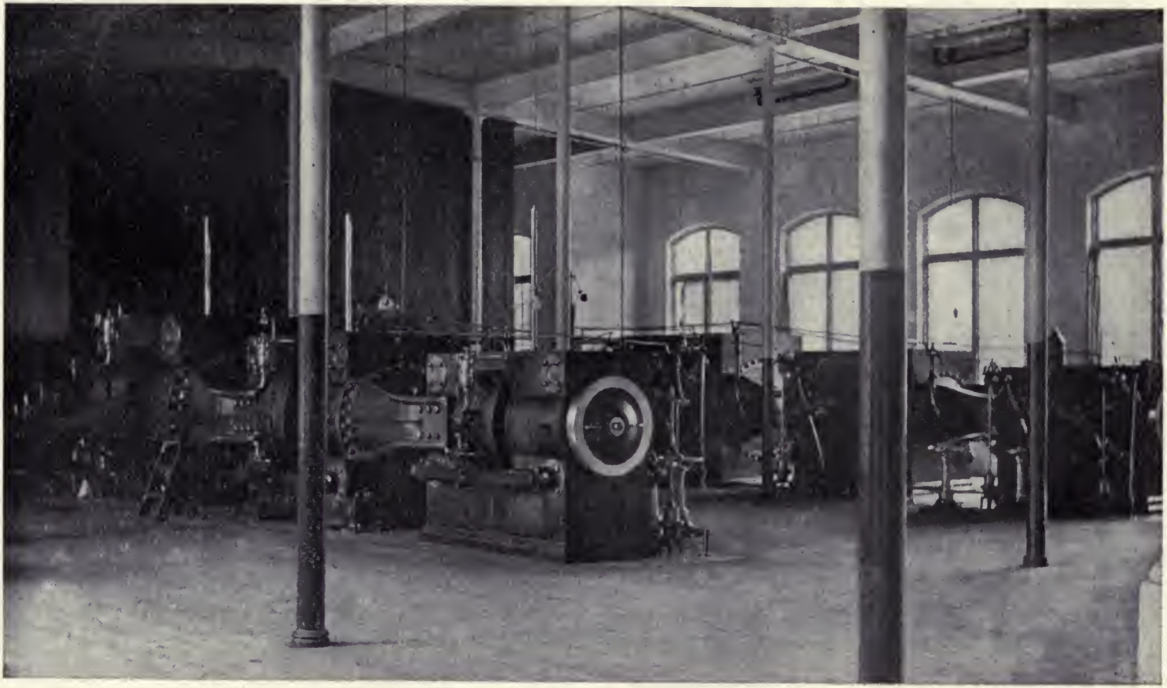
**85.** The cylinder of a steam-engine has an internal diameter of 3 feet; length of stroke, 6 feet; and it makes 10 strokes per minute. Under what effective pressure per square inch would it have to work in order that the piston may develop 125 horse-power?

The illustration of triple-expansion engines on opposite page accompanies Problem 86.

**86.** Four pairs of triple-expansion steam-engines are used to drive the cotton machinery of the largest Fall River corporation. One of these engines shown in illustration has cylinders $26\frac{1}{2}$ inches diameter, $36\frac{1}{2}$, and 54. The steam pressures are: In main pipe, 150 pounds per square inch; in receiver between high and intermediate cylinders, 40 pounds; in receiver between intermediate and low, 5 pounds. Vacuum is 27 inches. The mean effective pressures in the cylinders are respectively 54 pounds per square inch, $23\frac{1}{2}$ and $12\frac{1}{2}$. Length of stroke is 5 feet; piston speed, 660 feet per minute. Calculate the horse-power.

**87.** An engine is required to drive an overhead traveling crane for lifting a load of 30 tons at 4 feet per minute. The power is transmitted by means of $2\frac{1}{4}$-inch shafting, making 160 revolutions per minute. The length of the shafting is 250 feet; the power is transmitted from the shaft through two pairs of bevel gears (efficiency 90% each, including bearings), and one worm and wheel (efficiency 85%, including bearings). Taking the mechanical efficiency of the steam-engine at 80%, calculate the required horse-power of the engine.

**88.** An engine working at 50 horse-power is driven by steam at 75 pounds pressure acting on pistons in two cylinders. If the area of each piston is 72 square inches, and the length of stroke 2 feet, how many revolutions does the fly-wheel make per minute?



Triple-Expansion Engines in a Cotton Mill.

**89**. The steam-engine in use at the Worsted Weaving Mill of the Pacific Mills at Lawrence, Mass., is a Corliss type cross-compound with steam cylinders 19 and 36 inches diameter; stroke, 42 inches; revolutions, 100 per minute; mean effective pressures, 60 pounds and 13 pounds. Find how many looms weaving worsted dress-goods said engine will drive, each loom requiring $\frac{1}{4}$ horse-power.

**90**. A ship laden with coal must be unloaded at the rate of 22 tons of coal in 10 minutes. If the height of lift is 150 feet, what horse-power of engines will be required?

**91**. The fuel used in running a steam-engine is coal of such composition that the combustion of 1 pound produces heat sufficient to raise the temperature of 12 000 pounds of water 1° Fahr. It is found that $3\frac{1}{2}$ pounds of fuel are consumed per horse-power per hour. What is the efficiency of the entire apparatus?

**92**. A steam-engine uses coal of such composition that the combustion of 1 pound generates 10 000 British thermal units. If 40 pounds of coal are used per hour, and if the efficiency is 0.08, what horse-power is realized?

**93**. The cylinder of a Corliss-type steam-engine is 30 inches in diameter, stroke 48 inches, and it makes 85 revolutions per minute. The steam pressure being 90 pounds per square inch, what is the horse-power of the engine?

**94.** The piston of a steam-engine is 15 inches in diameter; its stroke is $2\frac{1}{2}$ feet, and it makes 20 revolutions per minute; the mean pressure of the steam on it is 15 pounds per square inch. How many foot-pounds of work are done by the steam per minute, and what is the horse-power of the engine?

**95.** An engine has a 6-foot stroke, the shaft makes 30 revolutions per minute, the average steam pressure is 25 pounds per square inch. Required the horse-power when the area of the piston is 1 800 square inches, the modulus of the engine being $\frac{11}{12}$.

**96.** The diameter of a steam-engine cylinder is 9 inches; the length of crank, 9 inches; the number of revolutions per minute, 110; the mean effective pressure of the steam 35 pounds per square inch. Find the indicated horse-power.

**97.** Find the horse-power of an engine which is drawing 120 tons up an incline of 1 in 300 at 30 miles an hour against wind and frictional resistances of 20 pounds a ton.

**98.** The area of a cross-section of the Charles River at Riverside, Massachusetts, is 408 square feet. The velocity of current as found by rod floats and current meter, April 17 and 22, 1902, was 1.12 feet per second. What would be the theoretic horse-power of this quantity of water at the Waltham dam, which gives a fall of 12.58 feet?

The illustration on opposite page of canal and mills at Manchester, N.H. accompanies Problem 103.

Fig. 8.—Water-Power.

**99.** Find the useful horse-power of a water-wheel, supposing the stream to be 100 feet wide and 5 feet deep, and to flow with a velocity of $\frac{1}{2}$ foot per second; the height of the fall is 24 feet, and the efficiency of the wheel 70 per cent.

**100.** A small stream has mean velocity of 35 feet per minute, fall of 13 feet and a mean section of 5 feet by 2. On this stream is erected a water-wheel whose modulus is 0.65. Find the horse-power of the wheel.

**101.** Given the stream in example 100, what must be the height of the fall to grind 10 bushels of corn per hour, if the modulus of the wheel is 0.4?

**102.** How many cubic feet of water must be descending the fall per minute in example 100, in order that the wheel may grind at the rate of 28 bushels per hour?

Water-Power Canal and Mills, Amoskeag, N. H.

**103**. The illustration on the preceding page shows the canal at Manchester, N.H., as it passes the mills of the Amoskeag Manufacturing Company, and the Manchester Mills Company. Width is 51 feet, depth of water 8.9 feet, velocity of flow 1.13 feet per second. What quantity of water is flowing? The height of fall for the turbines being 27.3 feet, what is the theoretic horse-power?

**104**. The mean section of the Merrimac Canal just before it enters the mills of the Merrimac Manufacturing Company at Lowell, Mass., is 48.2 feet by 10.6 feet; mean velocity on Nov. 23, 1901, was 2.37 feet per second; the water-wheels had a net fall of 35.67 feet, and gave an efficiency of about 77 per cent. Find the number of broad looms weaving cotton sheetings that may be driven $2\frac{1}{2}$ looms requiring one horse-power.

**105**. The estimated discharge of the nine turbines at Niagara Falls in 1898 was 430 cubic feet per second for each turbine. The average pressure head on the wheels was that due to a fall of about 136 feet. Compute the actual horse-power available from all turbines, allowing an efficiency of 82 per cent.

**106**. The average flow over Niagara Falls is 270 000 cubic feet per second. The height of fall is 161 feet. In round numbers what horse-power is developed?

**107**. Calculate the horse-power that can be obtained for one minute from an accumulator which makes one stroke in a minute and has a ram of 20 inches diameter, 23 feet stroke, loaded to a pressure of 750 pounds per square inch.

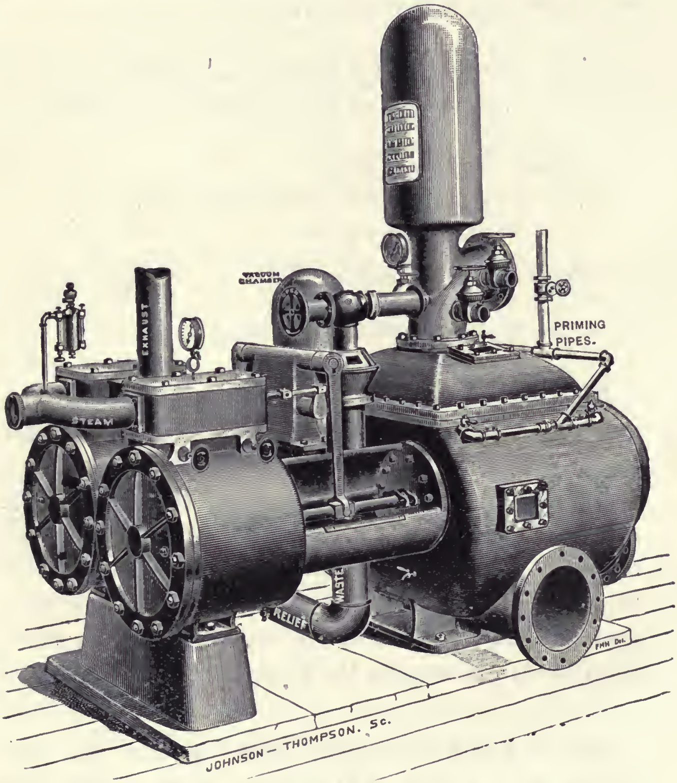



Fig. 9. An Underwriter Fire-Pump with Standard Fittings.

**108.** A fire-pump for protection of a 50 000-spindle cotton-mill will deliver 1 000 gallons of water per minute at 100 pounds pressure. Large boiler capacity is required for such a fire-pump and for the above size 150 horse-power would be used. What portion of this boiler capacity would be required in actual work of delivering water?

1 000 gallons = 8 355 pounds per minute
100 pounds per square inch pressure = 230.4 feet elevation

$$\text{Work} = 8\,355 \times 230.4$$
$$= 1\,924\,992 \text{ foot-pounds per minute}$$
$$= 58.3 \text{ horse-power}$$

$$\text{Portion of boiler used} = \frac{58.3}{150}$$
$$= .39 \text{ (About one-third.)}$$

**109.** An Underwriter fire-pump to protect a medium-sized factory will deliver four streams of water through $1\frac{1}{8}$-inch smooth nozzles with pressure at base of play pipes of 50 pounds per square inch. This would correspond to a discharge of 1 060 gallons per minute. Loss of pressure through nozzle can be neglected; and loss in quantity of discharge by slippage, short strokage, and so on will be about 10 per cent. Find the work done by the pump.

**110.** A pump of medium size used for fire protection of a factory will deliver three $1\frac{1}{8}$-inch fire streams at 80 pounds pressure. To give ample boiler capacity 70 per cent should be provided as surplus capacity. What should be the total boiler capacity?

**111.** A fire-engine pump is provided with a nozzle, the sectional area of which is 1 square inch, and the water is projected through the nozzle with an average normal velocity of 130 feet per second. Find (1) the number of cubic feet discharged per second; (2) the weight of water discharged per minute; (3) the kinetic energy of each pound of water as it leaves the nozzle; (4) the horse-power of the engine required to drive the pump, assuming the efficiency to be 70 per cent.

**112.** What must be the horse-power of a pumping-engine working 12 hours per day, to supply 50 000 persons with 110 gallons of water each per day, supposing the water to be raised to the mean height of 190 feet, the efficiency of engine and pump being 70 per cent?

**113.** If 3.8 pounds of soft coal produces one horse-power, how many pounds will be required by the above pumping-engine for a year's supply?

**114.** Find the horse-power necessary to pump out the Saint Mary's Falls Canal Lock, Sault Ste Marie, in 24 hours, the length of the lock being 500 feet, width 80 feet, and depth of water 18 feet, the water being delivered at a height of 42 feet above the bottom of the lock.

**115.** There were 6 000 cubic feet of water in a mine whose depth was 60 fathoms, when an engine of 50 horse-power began to operate a pump; the engine continued to work 5 hours before the mine was cleared of water. Supposing one-fourth of the work of the engine to have been wasted, find the number of cubic feet of water which ran into the mine during the 5 hours.

**116**. A nozzle discharges a stream 1 inch in diameter with a velocity of 80 feet per second. (*a*) How much kinetic energy is possessed by the amount of water which flows out in 1 minute? (*b*) If this energy could all be utilized by a water-wheel, what would be its power?

**117**. Suppose the nozzle referred to in example 116 to drive a water-wheel connected with a pump which lifts water 20 feet. If the efficiency of the whole apparatus is 0.48, how much water is lifted per minute?

**118**. The mean section of the branch of the First Level Canal at the headgates of No. 1 Mill, Whiting Paper Co., Holyoke, Mass., is 78 feet wide by 14 deep; from this canal to the Second Level there is a fall of 20 feet, but about 2 feet is lost in penstock and tail-race. The turbines that are driven have an efficiency of 77 %. Find how many 96-inch Fourdrinier Paper Machines can be driven, each machine requiring 100 horse-power.

**119**. What is the horse-power of a water-fall of 18 feet when the stream above the fall passes through a section of 6 square feet at the rate of $2\frac{1}{2}$ miles an hour.

**120.** What horse-power is involved in lowering by 2 feet the level of the surface of a lake 2 square miles in area in 300 hours, the water being lifted to an average height of 5 feet?

**121.** Taking the average power of a man as $\frac{1}{10}$th of a horse-power, and the efficiency of the pump used as 0.4, in what time will 10 men empty a tank of 50 feet $\times$ 30 feet $\times$ 6 feet filled with water, the lift being an average height of 30 feet?

**122.** A shaft 560 feet deep and 5 feet in diameter is full of water. How many foot-pounds of work are required to empty it, and how long would it take an engine of $3\frac{1}{2}$ horse-power to do the work?

**123.** Required the number of horse-power to raise 2 200 cubic feet of water an hour from a mine whose depth is 63 fathoms.

**124.** What weight of coal will an engine of 4 horse-power raise in one hour from a pit whose depth is 200 feet?

**125.** A cut is being made on a 4-inch wrought-iron shaft revolving at 10 revolutions per minute; the traverse feed is 0.3 inch per revolution; the pressure on the tool is found to be 435 pounds. What is the horse-power expended at the tool? How much metal is removed per hour per horse-power when the depth of cut is .06 inch, the breadth .06 inch (triangular section)?

**126.** A man rides a bicycle up a hill whose slope is 1 in 20 at the rate of 4 miles an hour. The weight of man and machine is $187\frac{1}{2}$ pounds. What work per minute is he doing?

**127.** At the top of the hill the bicyclist referred to in example 126 is met by a strong head-wind, and he finds that he has to work twice as hard to keep the same rate of 4 miles an hour on the level. What force is the wind exerting against him?

**128.** A bicyclist works at the rate of one-tenth of a horse-power, and goes 12 miles an hour on the level. Prove that the constant resistance of the road is 3.125 pounds.

Prove that up an incline of 1 vertical to 50 horizontal the speed will be reduced to about 5.8 miles per hour, supposing that the man and machine together weigh 168 pounds.

**129.** A man rows *a* miles per hour uniformly. If R pounds be the resistance of the water, and P foot-pounds of useful work are done at each stroke, find the number of strokes made per minute.

**130.** A train runs from rest down an incline of 1 in 100, for a distance of 1 mile (no engine attached); it then runs up an equal grade with its acquired velocity for a distance of 500 yards before stopping. Assuming the principle of work, find the total resistance, frictional or other, in pounds per ton, which has been opposing its motion.

**131.** In the Westinghouse brake tests (Jan., 1887), at Weehawken, a passenger-train moving 22 miles an hour on a down grade of 1% was stopped in 91 feet. There was 94% of the train braked. Taking the frictional resistance as 8 pounds per ton, find the net brake resistance per ton and the grade to which this is equivalent.

**132.** The rise and fall of the tide at Boston, Mass., is about 9 feet. If the in-coming water for one square mile of ocean surface could be stored, and its potential energy used during the out-going tide with an average fall of $4\frac{1}{2}$ feet, what horse-power would be utilized?

**133.** A six-inch rapid-fire gun discharges 5 projectiles per minute, each of weight 100 pounds, with a velocity of 2 800 feet per second. What is the horse-power expended?

**134.** A 500-volt motor drives a 10-ton car up a 5 per cent grade at a speed of 12 miles per hour: 75 per cent of the work of the motor is usefully expended. What electric current, expressed in ampères, will be required?

**135.** The resistance offered by still water to the passage of a certain steamer at 10 knots an hour is 15 000 pounds. A portion of the power of the engines equal to 12% of the total is absorbed in the "slip" (i.e., in pushing aside and backward the water acted on by the screw or paddle) and 8% of the total is absorbed in friction of machinery. What must be the total horse-power of the engines?

**136**. The United States warship Columbia has a speed of 23 knots, with an indicated horse-power of 22 000. Find the resistance offered to her passage.

**137**. A freight-car weighing 20 000 pounds requires a net pull of 10 pounds per ton to overcome frictional resistance. If "kicked" to a level side track with velocity of 10 miles per hour, how far will it run before stopping?

**138**. An express train of weight 250 tons cover 40 miles in 40 minutes. Taking the train resistances on a level track to be 20 pounds per ton at this speed, find the horse-power that engine must develop.

**139**. The speed of the "Exposition Flyer" on the Lake Shore and Michigan Southern Railroad, when running at its maximum, is 100 miles per hour. At that speed what pull by the engine would represent one horse-power? What pull when running at 50 miles an hour?

**140**. "Up to the highest usual speeds of commercial ships we may assume without great error that, for vessels not dissimilar in form and character going at the usual speeds, the indicated horse-power is $H = D^{\frac{2}{3}}V^3 \div c$ where D is the displacement in tons and V is the speed in knots and $c$ is a constant, which for many classes of vessel may be taken as not far different from 240." — PERRY'S *Applied Mechanics*. What is the indicated horse-power of a vessel of 1 330 tons, moving at a speed of 12 knots, if it obeys the above rule?

## ENERGY

**141.** The weight of a ram is 600 pounds, and at the end of a blow it has a velocity of 40 feet per second. What work is done in raising it?

**142.** A hoisting-engine lifts an elevator weighing 1 ton through 50 feet when it attains a velocity of 4 feet per second. If the steam is shut off, how much higher will it rise?

**143.** Find the horse-power of a man who strikes 25 blows per minute on an anvil with a hammer of weight 14 pounds, the velocity of the hammer on striking being 32 feet per second.

The height from which hammer would have to fall to acquire the same velocity would be found from the fundamental formula,

$$v = \sqrt{2\,gh}$$

$$32 = \sqrt{64 \times h}$$

$$\therefore\ h = 16 \text{ feet}$$

Now $$\text{work} = \text{force} \times \text{distance}.$$

Or work may be found directly from the formula,

$$\text{work} = \tfrac{1}{2}\,mv^2,$$

which formula is easily deduced from the above.

**144.** Show that to give a velocity of 20 miles an hour to a train requires the same energy as to lift it vertically through a height of 13.4 feet.

**145.** What is the kinetic energy of a cable car moving at 6 miles per hour, loaded with 36 passengers, each of average weight 154 pounds? Weight of car, $2\frac{1}{2}$ tons. What is its momentum? If stopped in 2 seconds, what is the average force? (The space average force would be equivalent to a constant force sufficient to stop the car.)

**146.** What is the kinetic energy of an electric car weighing $2\frac{1}{2}$ tons, moving at 10 miles an hour, and loaded with 50 passengers, each of average weight 150 pounds?

**147.** A ball weighing 5 ounces, and moving at 1 000 feet per second, pierces a shield, and moves on with a velocity of 400 feet per second. What energy is lost in piercing the shield?

**148.** A shot of 1 000 pounds moving at the rate of 1 600 feet per second strikes a fixed target. How far will the shot penetrate the target, exerting upon it an average pressure equal to a weight of 12 000 tons?

**149.** A bullet weighing 1 ounce leaves the mouth of a rifle with a velocity of 1 500 feet per second. If the barrel be 4 feet long, calculate the mean pressure of the powder, neglecting all friction.

Let P = mean pressure of powder in pounds

Kinetic energy of bullet $= \frac{1}{2}\, m\, v^2$

$$= \frac{1}{2} \times (\tfrac{1}{16} \div 32) \times (1\,500)^2$$

Work done on bullet $= P \times 4$

$$\therefore\ P \times 4 = \left(\frac{1}{32 \times 32}\right) \times (1\,500)^2$$

$$P = \frac{1\,500^2}{4 \times 32 \times 32}$$

$$= 549\tfrac{81}{256} \text{ pounds weight.}$$

**150**. The bullet referred to in example 149 penetrates a sand bank to the depth of 3 feet. What is the mean pressure exerted by the sand, and how long does the motion continue in the sand?

**151**. A bullet leaves the barrel of a gun with the velocity of 1 000 feet per second; supposing it to weigh 2 ounces, find the work stored up in the bullet and the height from which it must fall to acquire that velocity.

**152**. What is the kinetic energy of a 5-hundred weight projectile moving with a velocity of 2 000 feet per second?

**153**. A half-ton shot is discharged from an 81-ton gun with a velocity of 1 620 feet per second. With what velocity will the gun recoil, neglecting the mass of the powder? Will the gun or the shot be able to do more work before coming to rest, and in what proportion?

**154**. A cannon when fired recoils with a velocity of 10 feet per second, and runs up a platform having an incline of 1 in 4. Find the distance (measured horizontally) that it goes before coming to rest.

**155**. A nail 2 inches long was driven into a block by successive blows from a hammer weighing 5.01 pounds; after one blow it was found that the head of the nail projected 0.8 inches above the surface of the block; the hammer was then raised to a height of 1.5 feet and allowed to fall upon the head of the nail, which, after the blow, was found to be 0.46 inches above the surface. Find the force which the hammer exerted upon the head of the nail at this blow.

**156.** A hammer weighing 1 pound has a velocity of 20 feet per second at the instant it strikes the head of a nail. Find the force which the hammer exerts on the nail if it is driven into the wood $\frac{1}{10}$th of an inch.

**157.** What is the energy of a pendulum bob weighing half a ton and swinging past its equilibrium position at the rate of 1 foot a second?

**158.** A ball weighing 8 pounds, tied to one end of a fine thread 6 feet long, swings backward and forward in the arc of a semicircle. What is the kinetic energy, and what is its velocity as it passes through the lowest point of the semicircle?

**159.** The weight of a fly-wheel is 8 000 pounds and the diameter, 20 feet; diameter of axle, 14 inches; coefficient of friction, 0.2. If the wheel is disconnected from the engine when making 27 revolutions per minute, find how many revolutions it will make before it stops ($g$ taken $= 32.2$).

Work stored in wheel $= \frac{1}{2}\,\frac{8\,000}{32.2}\left(\frac{20 \times 27 \times 3\frac{1}{7}}{60}\right)^2$

Work done by friction in $x$ revolutions

$$= 0.2 \times 8\,000\,\frac{3\frac{1}{7} \times 14}{12} \times x$$

$$\frac{1}{2}\,\frac{8\,000}{32.2}\,\frac{(20 \times 27 \times 3\frac{1}{7})^2}{60} = 0.2 \times 8\,000 \times \frac{3\frac{1}{7} \times 14}{12} \times x$$

$$x = \frac{5 \times 81 \times 3\frac{1}{7} \times 3}{32.2 \times 7}$$

$$= 16.9 \text{ revolutions.}$$

**160.** A small heavy body weighing 20 pounds slides down a rough circular arc 10 feet in radius whose plane is vertical. It begins to move from one end of a horizontal diameter, and is found to reach the lowest point with a velocity of 12 feet a second. How many foot-pounds of work have been done against friction during the motion? And if the same proportionate loss of energy occurred in the next portion of the same circle, how high would it ascend?

**161.** Find the useful work done each second by a fire-engine which discharges water at the rate of 500 gallons per minute with a velocity of 72 feet per second.

**162.** The ordinary fire-engine when in full operation burns soft coal, and will consume in an hour about 60 pounds per fire-stream of 250 gallons per minute. Therefore at the Thanksgiving fire in Boston, in 1893, a 500-gallon engine that was running 48 hours required how many pounds of coal?

**163.** A massive slow-moving fly-wheel, mounted on a horizontal axis 1 foot in diameter, possesses 1 500 foot-pounds of kinetic energy, which is used to raise a weight of 25 pounds by means of a rope coiled round the axis. Assuming that a weight of 5 pounds is able to overcome the friction, how many times will the wheel revolve before it comes to rest? How many revolutions in the opposite direction must be made before the original energy is restored to the wheel?

**164.** A fly-wheel weighs 10 000 pounds, and is of such a size that the matter composing it may be treated as if concentrated on the circumference of a circle 12 feet in radius. What is its kinetic energy when moving at the rate of 15 revolutions a minute? How many turns would it make before coming to rest, if the steam were cut off, and it moved against a friction of 400 pounds exerted on the circumference of an axle 1 foot in diameter? ($g=32$.)

**165.** A blacksmith's helper using a 16-pound sledge strikes 20 times a minute and with a velocity of 30 feet per second. Find his rate of work.

**166.** A body weighing 200 pounds is moved from a state of rest, and is found subsequently to be moving at the rate of 12 feet a second. How many foot-pounds of work must have been expended on it by the forces exerting motion over and above those expended on the resistances?

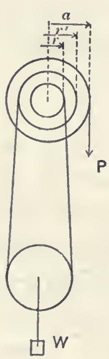


Fig. 10.

**167.** In a jack-screw the pitch of the screw is 1 inch, the lever is 2 feet long, and the force applied at the end of the lever is 25 pounds. Find the weight that can be lifted, friction being neglected.

**168.** Determine by the principle of work (neglecting friction), the relation between the effort P and the load W in case of the differential wheel-and-axle of Fig. 10.

One revolution,

$$\text{Work of P} = P \times 2\,a\pi$$

$$\text{Work of weight} = \tfrac{1}{2}W \times 2\,r' \times \pi - \tfrac{1}{2}W \times 2\,r\pi$$

$$P \times 2\,\pi a = \tfrac{1}{2}W\pi \times 2\,(r' - r)$$

$$P \times 2\,a = W\,(r' - r)$$

$$\frac{P}{W} = \frac{r' - r}{2\,a}.$$

**169**. A body is suspended by an elastic string whose unstretched length is 4 feet. Under a pull of 10 pounds the string stretches to a length of 5 feet. Required the work done on the body by the tension of the string while its length changes from 6 feet to 4 feet.

**170**. A body falls down the whole length of an inclined plane on which the coefficient of friction is 0.2. The height of the plane is 10 feet and the base 30 feet. On reaching the bottom it rolls horizontally on a plane, having the same coefficient of friction. Find how far it will roll.

**171**. Two bodies A and B, weighing 50 pounds and 10 pounds respectively, are connected by a thread; B is placed on a smooth table, and A hangs over the edge. When A has fallen 10 feet, what is the kinetic energy (or accumulated work) of the bodies jointly, and what of them severally?

**172.** A bead whose weight is W is free to slide on a smooth circular wire in a vertical plane. A string attached to the bead passes over a smooth peg at the highest point of the circle and sustains a weight P. Determine by the principle of virtual work the position of equilibrium.

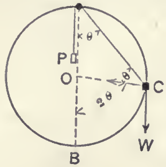


Fig. 11.

# II. FORCE

## FORCES ACTING AT A POINT

**173.** At what angle must the forces 6 pounds and 8 pounds be inclined, if their resultant is 10 pounds?

Draw the parallelogram of forces, one side being 6 units, one 8, and the diagonal between them 10 units. Required the angle between the 6- and 8-pound forces. Solve by trigonometry or geometry. Required angle $= 90°$.

**174.** What is the resultant of forces 60 pounds and 80 pounds acting at right angles to each other?

**175.** Two men pull a body horizontally by means of ropes. One exerts a force of 28 pounds directly north, the other a force of 42 pounds in direction N. 42° E. What single force would be equivalent to the two?

**176.** Three cords are knotted together; one of these is pulled to the north with a force of 6 pounds, another to the east with a force of 8 pounds. With what force must the third be pulled to keep the whole at rest?

**177.** Two persons lifting a body exert forces of 44 pounds and 60 pounds on opposite sides of the ver-

Force illustrated by two fire streams being delivered by the pump service of the large cotton mills of B.B. & R. Knight at Natick, Rhode Island. One stream is being held by men in correct position, the other by men who have been crowded into an awkward and dangerous position. Pressure shown on the gauge at the hydrant was 75 pounds per square inch.

tical, but each with an inclination of 28°. What single force would produce the same effect?

**178.** A force of 50 units acts along a line inclined at an angle of 30° to the horizon. Find, by construction or otherwise, its horizontal and vertical components.

**179.** Explain the boatman's meaning when he says that greater force is developed when a mule hauls a canal boat with a long rope than with a short one. Is the same true of a steam-tug when towing a four-master?

**180.** Two strings, one of which is horizontal, and the other inclined to the vertical at an angle of 30°, support a weight of 10 pounds. Find the tension in each string.

**181.** Two forces of 20 pounds, and one of 21 act at a point. The angle between the first and second is 120°, and between the second and third, 30°. Find the resultant.

**182.** Forces of 9 pounds, 12, 13, and 26, act at a point so that the angles between the successive forces are equal. Find their resultant.

**183.** A weightless rod, 3 feet long, is supported horizontally, one end being hinged to a vertical wall, and the other attached by a string to a point 4 feet above the hinge; a weight of 120 pounds is hung from the end supported by the string. Calculate the tension in the string, and the pressure along the rod.

**184.** A weight of 100 pounds is fixed to the top of a weightless rod or strut 5 feet long whose lower end rests in a corner between a floor and a vertical wall, while its upper end is attached to the wall by a horizontal wire 4 feet long. Calculate the tension in the wire, and the thrust in the rod.

**185.** A rod AB is hinged at A and supported in a horizontal position by a string BC making an angle of 45° with the rod; the rod has a weight of 10 pounds suspended from B. Find the tension in the string and the force at the hinge. (The weight of the rod can be neglected.)

**186.** A simple triangular truss of 30 feet span and 5 feet depth supports a load of 4 tons at the apex. Find the forces acting on rafters and tie rod.

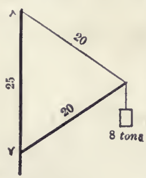



Fig. 12.

**187.** A derrick is set as shown in sketch, the load being 8 tons. Find the stress in the boom and the tackle.

**188.** A stiff-leg steel derrick, with mast 55 feet high, boom 85 feet long, set with tackle 40 feet long, as shown in cut, is raising two boilers of 50 tons weight. Find stresses in boom and tackle. (See illustration on page 55.)

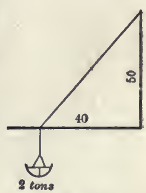



Fig. 13.

**189.** Find the stress in tackle and compression in boom of towers for six-master shown on page 24 when bucket, weighing with its load 2 tons, is set in position shown by Fig. 13.

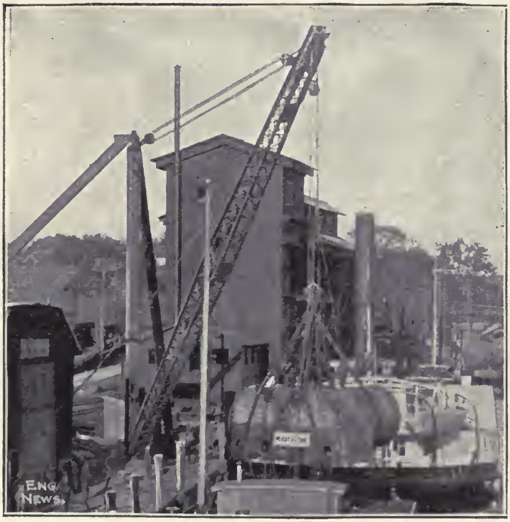



Fig. 14.

**190.** A balloon capable of raising a weight of 360 pounds is held to the ground by a rope which makes an angle of 60° with the horizon. Determine the tension of the rope and the horizontal pressure of the wind on the balloon.

**191.** A uniform beam 10 feet long, weighing 80 pounds, is suspended from a horizontal ceiling by two strings attached at its ends, and at points 16 feet apart in the ceiling. Find the tension in each string.

**192.** A boat is towed along a canal 50 feet wide, by mules on both banks; the length of each rope from its point of attachment to the bank is 72 feet:

the boat moves straight down the middle of the canal. Find the total effective pull in that direction, when the pull on each rope is 800 pounds.

**193**. A boat is being towed by a rope making an angle of 30° with the boat's length; the resultant pressure of the water and rudder is inclined at 60° to the length of the boat, and the tension in the rope is equal to the weight of half a ton. Find the resultant force in the direction of the boat's length.

**194.** In a direct-acting steam-engine the piston-pressure is 22 500 pounds; the connecting-rod makes a maximum angle of 15° with the line of action of the piston. Find the pressure on the guides.

**195**. A man weighing 160 pounds sits in a loop at the end of a rope 10 feet 3 inches long, the other end being fastened to a point above. What horizontal force will pull him 2 feet 3 inches from the vertical, and what will then be the pull on the rope?

**196**. A man weighing 160 pounds sits in a hammock suspended by ropes which are inclined at 30° and 45° to vertical posts. Find the pull in each rope.

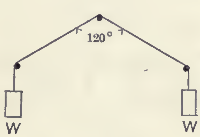


Fig. 15.

**197**. Two equal weights, W, are attached to the extremities of a flexible string which passes over three tacks arranged in the form of an isosceles triangle with the base horizontal, the vertical angle at the upper tack being 120°. Find the pressure on each tack.

**198.** A rod AB 5 feet long, without weight, is hung from a point C by two strings, which are attached to its ends and to the point; the string AC is 3 feet long, and the string BC 2 feet; a weight of 2 pounds is hung from A and a weight of 3 pounds from B. Find the tension of the strings and the condition that these may be in equilibrium.

**199.** A weight of 10 pounds is suspended by two strings, 7 and 24 inches long, the other ends of which are fastened to the extremities of a rod 25 inches in length. Find the tension of the strings when the weight hangs immediately below the middle point of the rod.

**200.** AB is a wall, and C a fixed point at a given perpendicular distance from it; a uniform rod of given length is placed on C with one end against AB. If all the surfaces are smooth, find the position in which the rod is in equilibrium.

**201.** AB is a uniform beam weighing 300 pounds. The end A rests against a smooth vertical wall, the end B is attached to a rope CB. Point C is vertically above A, length of beam is 4 feet, rope 7 feet. Represent the forces acting, and find the pressure against the wall and the tension in the rope.

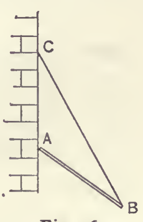


Fig. 16.

**202.** A wagon weighing 2 200 pounds rests on a slope of inclination 30°. What are the equivalent forces parallel and perpendicular to the plane?

**203.** AB is a rod that can turn freely round one end A; the other end B rests against a smooth inclined plane. In what direction does the plane react upon the rod? Illustrate your answer by a diagram showing the rod, the plane, and the reaction.

**204.** A wagon weighing 2 tons is to be drawn up a smooth road which rises 4 feet vertically in a distance of 32 feet horizontally by a rope parallel to the road. What must the pull of the rope exceed in order that it may move the wagon?

**205.** What weight can be drawn up a smooth plane rising 1 in 5 by a pull of 200 pounds (*a*) when the pull is parallel with the plane? (*b*) when it is horizontal?

**206.** A horse is attached to a dump-car by a chain, which is inclined at an angle of 45° to the rails; the force exerted by the horse is 672 pounds. What is the effective force along the rails?

**207.** The angle of inclination of a smooth inclined plane is 45°: a force of 3 pounds acts horizontally, and a force of 4 pounds acts parallel to the plane. Find the weight which they will be just able to support.

**208.** A body rests on a plane of height 3 feet, length 5 feet. If the body weighs 14 pounds, what force acting along the plane could support it, and what would be the pressure on the plane?

**209.** A number of loaded trucks each containing one ton, standing on a given part of a smooth tramway, where the inclination is 30°, support an equal number of empty trucks on another part, where the inclination is 45°. Find the weight of a truck.

**210.** Two planks of lengths 7 yards and 6 yards rest with one end of each on a horizontal plane, the other ends in contact above that plane; two weights are supported one on each plank, and are connected by a string passing over a pulley at the junction of the planks; the weight on the first plank is 21 pounds. What is the weight on the other, friction not being considered?

**211.** The weight of a wheel with its load is 2 tons, diameter of wheel 5 feet. Find the least horizontal force necessary to pull it over a stone 4 inches high. (When the wheel begins to rise three forces are acting: P, W, and R the reaction. It is required to find P.)

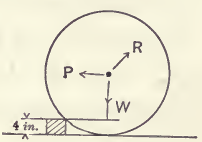


Fig. 17.

**212.** A rectangular box, containing a 200-pound ball, stands on a horizontal table, and is tilted about one of its lower edges through an angle of 30.° Find the pressure between the ball and the box.

**213.** An iron sphere weighing 50 pounds is resting against a smooth vertical wall and a smooth plane which is inclined 60° to the horizon. Find the pressure on the wall and plane.

**214**. A beam weighing 400 pounds rests with its ends on two inclined planes whose angles of inclination to the horizontal are 20° and 30°. Find the pressures on the planes.

**215**. A thread 14 feet long is fastened to two points A and B which are in the same horizontal line and 10 feet apart; a weight of 25 pounds is tied to the thread at a point P so chosen that AP is 6 feet — therefore BP is 8 feet long. The weight being thus suspended, find by means of construction or otherwise, what are the tensions of the parts AP and BP of the thread.

**216**. AC and BC are two threads 4 feet and 5 feet long, respectively, fastened to fixed points A and B, which are in the same horizontal line 6 feet apart; a weight of 50 pounds is fastened to C. Find, by means of a line construction drawn to scale, the pull it causes at the points A and B. Each of the threads AC and BC is, of course, in a state of tension. What are the forces producing the tension?

**217**. A boiler weighing 3 000 pounds is supported by tackles from the fore and main yards. If the tackles make angles of 25° and 35° respectively with the vertical, what is the tension of each?

**218**. A piece of wire 26 inches long, and strong enough to support directly a load of 100 pounds, is attached to two points 24 inches apart in the same horizontal line. Find the maximum load that can be suspended at the middle of the piece of wire without breaking it.

**219.** A picture of 50 pounds weight hanging vertically against a smooth wall is supported by a string passing over a smooth hook; the ends of the string are fastened to two points in the upper rim of the frame, which are equidistant from the center of the rim, and the angle at the peg is 60°. Find the tension in the string.

**220.** A weight W attached by two connecting cords of lengths $a$ and $b$ to two fixed points A and B, and separated by a horizontal interval $c$, are in equilibrium under the action of gravity. Required the stresses P and Q in the cords.

**221.** Two equal rods AB and BC are loosely jointed together at B. C and A rest on two fixed supports in the same horizontal line, and are connected by a cord equal in length to AB. If a weight of 12 pounds be suspended from B, what is the pressure produced along AB and BC, and the tension in the cord?

**222.** A new device for unloading cars of coal is illustrated and described in the *Journal of the Association of Engineering Societies* for October, 1901. The loaded car is taken in a cradle, and the coal dumped from the car into a bin from which it is distributed. Total weight of car, coal and cradle, is about 70 tons. This weight is supported by two sets of Y

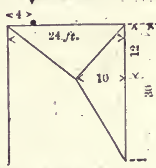


Fig. 18.

braces and posts about as shown in sketch. Calculate the stresses acting in the members.

**223**. A tripod whose vertex is A, and whose legs are AB, AC, AD, of lengths 8 feet, 8.5, and 9 respectively, sustains a load of 2 tons. The ends B, C, D, form a triangle whose sides are BC 7 feet, CD 6 feet, BD 8 feet. Find by graphical construction the compressive forces in each leg.

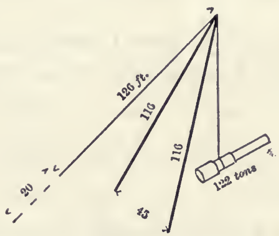


Fig. 19.

**224**. Figs. 19–20 show a pair of shears erected at Sparrow's Point, Md., for the Maryland Steel Company. The two front legs are hollow steel tubes 116 feet long. They are 45 feet apart at the bottom. The back leg is 126 feet long, and is connected to hydraulic machines for operating the shears. How much are the forces acting in these legs when a Krupp gun weighing 122 tons is being lifted?

**225**. Each leg of a pair of shears is 50 feet long. They are spread 20 feet at the foot. The back stay is 75 feet long. Find the forces acting on each member when lifting a load of 20 tons at a distance of 20 feet from the foot of the shear legs, neglecting the weight of structure.

**226**. Shear legs each 50 feet long, 30 feet apart on horizontal ground, meet at point C, which is 45 feet vertically above the ground; stay from C is inclined

Fig. 20.

at 40° to the horizon; a load of 10 tons hangs from C. Find the force in each leg and stay.

**227.** A vertical crane post is 10 feet high, jib 30 feet long, stay 24 feet long, meeting at a point C. There are two back stays making angles of 45° with the horizontal; they are in planes due north and due


Dipper Dredge "Pan American."

west from the post. A weight of 5 tons hangs from C. Find the forces in the jib and stays — 1st, when C is southeast of the post; 2d, when C is due east; 3d, when C is due south.

**228**. The view on opposite page shows one of the largest dipper dredges ever built, the "Pan American," constructed at Buffalo in 1899 for use on the Great Lakes. An A-frame, the legs of which are 57 feet long and 40 feet apart at the bottom, is held at the apex by four cables which are 100 feet long. The boom is 53 feet long and weighs 30 tons. The handle, which weighs about 4 tons, is 60 feet long, and carries on its end a dipper weighing 16 tons, which will dredge up $8\frac{1}{4}$ cubic yards, or about 12 tons, of material at one load.

The dipper is operated by a wire rope passing over a pulley on the outward end of the boom. In the position represented by the outline sketch, the boom is inclined to the water surface at an angle of 30°, the dipper is carrying the full load, and the handle is in a horizontal position

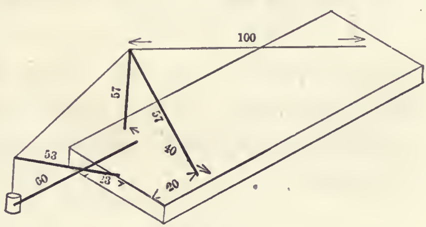



Fig. 21.

with its middle point supported at a point on the boom 23 feet from the foot of the boom. The apex of the A-frame is vertically above the foot of the boom. Compute the forces acting in the 100-foot back-stays, (considering them to be one rope only, in the position as per sketch) in the legs of the A-frame, in the boom, and in the wire rope which raises the dipper.

**229.** Draw a triangle ABC with its base AB horizontal, and its vertex C downwards; let AC and BC represent threads fastened to fixed points at A and B, and at C to a third thread, which carries a given weight W. Given the angles of the triangle ABC, find the tensions of the threads.

**230.** ABC is a rigid equilateral triangle, weight not considered; the vertex B is fastened by a hinge to a wall, while the vertex C rests against the wall under B. If a given weight is hung from A, find the reactions at B and C. What are the magnitudes and directions of the forces exerted by the weight on the wall at B and C?

**231.** ABCD is a square; forces of 1 pound, 6, and 9 act in directions AB and AD, respectively. Find the magnitude of their resultant correct to two places of decimals.

**232.** Draw a square ABCD and its diagonal AC; two forces of 10 units act from A to B, and from C to D respectively forming a couple; a third force of 15 units acts from C to A. Find their resultant, and show in a diagram how it acts.

**233.** A, B, C, D, are the angular points of a square taken in order; three forces act on a particle at A, viz. one of 7 units from A to B, a second of 10 units from D to A, and a third of $5\sqrt{2}$ units along the diagonal from A to C. Find, by construction or otherwise, the resultant of these three forces.

**234.** Forces P, 2P, 3P, and 4P act along the sides of a square A, B, C, D, taken in order. Find the magnitude, direction, and line of action of the resultant.

**235.** A sinker is attached to a fishing-line which is then thrown into running water. Show by means of a diagram the forces which act on the sinker so as to maintain equilibrium.

**236.** A uniform rod 6 feet long, weighing 10 pounds, is supported by a smooth pin and by a string 6 feet long which is attached to the rod 1 foot from one end and to a nail vertically above the pin, 4 feet distant. Show by construction the position in which the rod will come to rest.

**237.** A light rod AB can turn freely round a hinge at A; it rests in an inclined position against a smooth peg near the end B; a weight is hung from the middle of the rod. Show in a diagram the forces which keep the rod at rest, and name them.

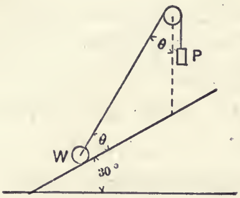



Fig. 22.

**238.** A weight W on a plane inclined 30° to the horizontal is supported as shown in cut. The angles $\theta$ being equal. Find the ratio of the power to the weight.

**239.** Discuss the action of the wind in propelling a sailing-vessel.

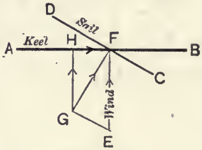



Fig. 23.

Let AB be the keel, CD the sail. Let the force of the wind be represented in magnitude and direction by EF. The component GF of EF, perpendicular to the sail, is the effective component in propelling the ship; the other component EG, parallel to the sail, is useless; but GF drives the ship forward and sidewise. The component GH of GF, perpendicular to AB, produces side motion, or leeway; and the other component HF, along the keel, produces forward motion, or headway.

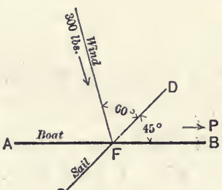



Fig. 24.

**240.** A sailing-boat is being driven forward by a force of 300 pounds as shown in Fig. 24. What force is P acting in direction of motion of the boat?

**241.** Discuss the action of the rudder of a vessel in counteracting leeway. Show that one effect of the action of the rudder is to diminish the vessel's motion.

**242.** A thread of length $l$ has its ends fastened to two points in a line of length $c$, and inclined to the vertical with angle $\theta$; a weight W hangs on the thread by means of a smooth hook. Find the position in which the weight comes to rest and the tension in the thread.

**243**. A smooth ring weighing 40 pounds slides along a cord that is attached to two fixed points in a horizontal line. The distance between the points being one-half length of cord, find position in which weight will come to rest and the tension in the string near the points of attachment.

**244**. A small heavy ring A, which can slide upon a smooth vertical hoop, is kept in a given position by a string AB, B being the highest point of the hoop. Show that the pressure between the ring and the hoop is equal to the weight of the ring.

**245**. Draw a figure showing the mechanical conditions of equilibrium when a uniform beam rests with one extremity against a smooth vertical wall, and the other inside a smooth hemispherical bowl.

**246**. A ball 8 inches in diameter, weighing 100 pounds, rests on a plane inclined 30° to the horizon, and is held in equilibrium by a string 4 inches long attached to a sphere and to an inclined plane. Represent the forces acting, and find their values.

**247**. A uniform sphere rests on a smooth inclined plane, and is held by a horizontal string. To what point on the surface of the sphere must the string be attached? Draw a figure showing the forces in action, assuming that the weight acts through the center of the sphere.

**248.** A particle of weight W is supported within a smooth hemispherical bowl by a string of given length having one end attached to a point in the rim. State clearly the forces which keep the particle in equilibrium; find their magnitudes if the length of the string and the radius of the bowl be given and the rim be in a horizontal plane.

**249.** A uniform bar of weight 20 pounds, length 12 feet, rests with one end inside a smooth hemispherical bowl, and is supported by the edge of the bowl at a point distant 10 feet from the above end, 2 feet of the bar being outside the bowl. Draw the forces producing equilibrium, and find their values.

**250.** A rod of weight 10 pounds rests in a smooth hemispherical bowl, which is fixed with its rim horizontal; the rod is 12 feet long, and 2 feet of it are outside the bowl. The inclination of the rod to the horizon is 30°. Draw a figure representing the reactions of the bowl, and calculate these reactions.

**251.** The platform of a suspension foot-bridge 100 feet span, 10 feet width, supports a load, including its own weight, of 150 pounds per square foot. The two suspension cables have a dip of 20 feet. Find the force acting on each cable close to the tower, and in the middle, assuming the cable to hang in a parabolic curve.

**252.** The slopes of a simple triangular roof-truss are 30° and 45°, and the span is 50 feet. The trusses are spaced 10 feet apart, and the weight of the roof covering and snow is 50 pounds per square foot. Find the tension in the tie-rod.

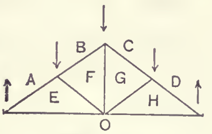


Fig. 25.

**253.** In a roof of 32 feet span and height 12 feet the trusses are 10 feet apart, and the members EF, GH, come to the middle points of the rafters. If the weight of the roof-covering is 25 pounds per square foot, draw the stress diagram and scale off the stressess.

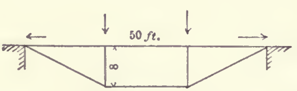


Fig. 26.

**254.** A highway bridge of span 50 feet, breadth 40 feet, has two queen-post trusses of depth 8 feet; and each truss is divided by two posts into three equal parts. The bridge is designed to carry a load of 100 pounds per square foot of floor surface. Find the stresses developed.

**255.** Find the stresses in a king-post truss represented in the figure. Distance between trusses is 12 feet, height of truss 10 feet, length of span 40 feet, uniform load of 200 pounds per linear foot, and a load of 1 000 pounds at the foot of the post.

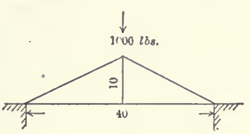


Fig. 27.

## MOMENTS

**256.** The length of a bar is 12 inches; weights 1 pound and 2 pounds are attached to its ends. At what point must the bar be supported to effect a balance?

Draw a figure. Let distance from fulcrum to the 1-pound weight be $x$. Suppose that weights could cause one complete revolution around fulcrum; then

Work done by 1-pound weight = Work done by 2-pound weight
Now, work done by 1-pound weight = force × distance

$$= 1 \times 2\pi x$$

Work done by 2-pound weight $= 2 \times 2\pi(12 - x)$

$$\therefore\ 1 \times 2\pi x = 2 \times 2\pi(12 - x)$$

and

$$1 \times x = 2 \times (12 - x)$$

Or in words: The weight producing motion × its perpendicular lever arm = the weight moved × its perpendicular lever arm. Thus observe the direct relation existing between the principles of Work and those of Moments.

DEFINITION. — The Moment of a force about a point is the product of the force times the perpendicular distance from the point to the line of action of the force; or briefly, Moment is force × perpendicular. Clockwise motion is usually taken positive; opposite, negative.

From the above equation, $x$ would be found equal to 8 inches.

**257.** Two weights of 126 pounds and 220 pounds respectively, are suspended from the extremities of a straight bar 26 inches in length. Find the point at which their resultant will intersect the bar.

**258.** A rigid rod 7 feet long, without weight rests on a fixed point 2 feet 6 inches from one end; to this end a weight of 18 pounds is attached. What weight must be hung from the other end so that the rod may be horizontal?

**259**. A light rod of length 3 yards has weights of 15 pounds and 3 pounds suspended at the middle and extremity respectively; it balances on a fulcrum. Find the position of the fulcrum, and the pressure on it.

**260**. A stiff pole 12 feet long sticks out horizontally from a vertical wall. It would break if a weight of 28 pounds were hung at the end. How far out along the pole may a boy of weight 112 pounds venture with safety?

**261**. The length of an oar is 8 feet, of which 2 feet are inside the rowlock; a man exerts a pressure on the extremity of the handle of 100 pounds. What is the pressure on the rowlock, and resultant pressure causing the boat to move?

**262**. Find the propelling force on an eight-oared shell, if each man pulls his oar with a force of 56 pounds, and the length of the oar outside the rowlock is three times the length inside the rowlock.

**263**. A light bar, 5 feet long, has weights of 9 pounds and 5 pounds respectively suspended from its extremities, and 10 pounds from its middle point. Find the point on which it will balance.

**264.** A lever AB of the first order, 8 feet long, has the fulcrum 2 feet from B, a weight of 5 pounds is hung from A, and one of 17 pounds from B. Putting the weight of the lever itself out of the question, from what point must a weight of 2.5 pounds be hung to keep the lever horizontal?

**265.** A weight of 100 pounds is supported by a rope which passes over a fixed pulley and is attached to a lever at a point 2 feet from the fulcrum which is at the end; length of lever is 12 feet. What weight must be suspended at the other end to keep the lever horizontal?

**266.** Eight sailors raise an anchor, of weight 2 688 pounds, by pushing on the spokes of a capstan which has a radius of 14 inches. If they all push at equal distances from the center exerting a force of 55 pounds each, what is the distance?

Let $x$ be the distance from the axis.
Take moments about the axis,

$$8 \times 56 \times x = 2\,688 \times 14.$$
$$\therefore\ x = 84 \text{ inches}$$
$$= 7 \text{ feet.}$$

**267.** Is there any reason why a man should put his shoulder to the spoke of the wheel rather than to the body of the wagon in helping it up hill?

**268.** A rod AB, of length 15 feet, is supported by props at A and B; a weight of 200 pounds is suspended from the rod at a point 7 feet from A. Find the pressure on the prop at A.

**269.** A light bar, 9 feet long, to which is attached a weight of 150 pounds, at a point 3 feet from one end, is borne by two men. Find what portion of the weight is borne by each man, when the bar is horizontal.

**270.** A light rod, 16 inches long, rests on two pegs 9 inches apart, with its center midway between them. The greatest weights, which can be suspended separately from the two ends of the rod without disturbing the equilibrium, are 4 pounds and 5 pounds respectively. There is another weight fixed to the rod. Find that weight and its position.

**271.** A light rod AB, 20 inches long, rests upon two pegs whose distance apart is equal to half the length of the rod. How must it be placed so that the pressure on the pegs may be equal when weights 2W, 3W, are suspended from A, B, respectively?

**272.** The horizontal roadway of a bridge is 30 feet long and its weight, 6 tons, may be supposed to act at its middle point, and it rests on similar supports at its ends. What pressure is borne by each of the supports when a carriage weighing 2 tons is one-third of the way across the bridge?

**273.** "We have a set of hay-scales, and sometimes we have to weigh wagons that are too long to go on them. Can we get the correct weight by weighing one end at a time and then adding the two weights?"

**274.** A rod, 18 inches long, can turn about one of its ends, and a weight of 5 pounds is fixed to a point 6 inches from the fixed end. Find the force which must be applied at the other end to preserve equilibrium.

**275.** A straight uniform lever weighing 10 pounds rests on a fulcrum one-third of its length from one end; it is loaded with a weight of 4 pounds at that end. Find what vertical force must act at the other end to keep the lever at rest.

**276.** A weight of 56 pounds is attached to one end of a uniform bar which is ten feet long, and weighs 20 pounds; the fulcrum is 2 feet from the end to which the weight is attached. What weight must be applied at the other end to balance?

**277.** AB is a horizontal uniform bar $1\frac{1}{2}$ feet long, and F a point in it 10 inches from A. Suppose that AB is a lever turning on a fulcrum under F, and carrying a weight of 40 pounds at B; weight of lever, 4 pounds. If it is kept horizontal by a fixed pin above the rod, 7 inches from F and 3 inches from A, find the pressure on the fulcrum and on the fixed pin.

**278.** A lever, 16 feet long, balances about a point 4 feet from one end; if a weight of 120 pounds be attached to the other end, it balances about a point 6 feet from that end. Find the weight of the lever.

**279.** A uniform lever is 18 inches long, and each inch in length weighs 1 ounce. Find the place of the fulcrum when a weight of 27 ounces at one end of the lever balances a weight of 9 ounces at the other end.

**280.** A rod, of weight 4 pounds, and of length 16 feet, balances about a point 4 feet from one end. If a weight of 10 pounds be hung 2 feet from this end, find the weight that must be hung from the other extremity that the rod may then balance about its middle point.

**281.** A piece of shafting 10 feet long, and weighing 100 pounds, rests horizontally on two horses placed at its ends. A pulley weighing 75 pounds is keyed to the shaft at a point distant $3\frac{1}{3}$ feet from one end. How many pounds applied at the other end by a man lifting vertically will just raise it?

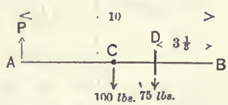


**Fig. 28.**

100 pounds the weight of shaft acts downward at the middle point; 75 pounds the weight of pulley acts downward at D, $3\frac{1}{2}$ feet from B. Find the required force acting upward.

Take moments about B,

$$+P \times AB - 100 \times CB - 75 \times BD = 0.$$

$$\therefore\ P \times 10 = 100 \times 5 + 75 \times 3\tfrac{1}{3}.$$

$$P = 75 \text{ pounds.}$$

**282.** Six men are to carry an iron rail 60 feet long and weighing 90 pounds per yard; each man sustains one-sixth of the weight. Two men are to lift from one end and the other four by means of a cross-bar. Where must the cross-bar be placed?

**283.** A block of stone weighing 300 pounds is to be removed by two men using a light plank 6 feet long. How must the stone be placed so that one man will carry two-thirds of the weight and the other one-third?

**284.** A rod 2 feet long, with a weight of 7 pounds at its middle point, is placed upon two nails, A and B. AB is horizontal and 7 inches long. Find the distances to which the ends of the rod must extend beyond the nails, if the difference of the pressures on the nails be 5 pounds.

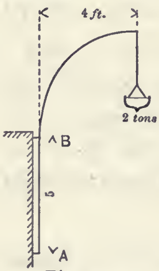


Fig. 29.

**285.** A davit is supported by a foot-step A and a collar B, placed 5 feet apart. A boat weighing 2 tons is about to be lowered, and is hanging 4 feet horizontally from vertical through the foot-step and collar. Determine the forces which must be acting at A and B.

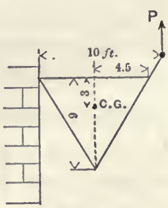


Fig. 30.

**286.** The resistance of 208 pounds found in example 80 pulls out of vertical a triangular mass of rocks by acting at P as represented in sketch. The thickness of the triangular mass being $1\frac{1}{2}$ feet, weight of stone 150 pounds per cubic foot, and voids taking one-third of space, find total weight of rocks, and height that center of gravity will be raised if P acts through a distance of 30 feet.

**287.** Like parallel forces of 10 and 20 units act perpendicularly to AB at A and B; a force of 15 units acts from A to B. Find the resultant of the three forces, and show in a diagram how it acts.

**288.** A rod is acted on at one end by a force of 3 downwards, and at a distance of two feet from this end by a force of 5 upwards. Where must a force of 2 be applied to keep the rod at rest?

**289.** Three parallel forces of 1 pound each act on a horizontal bar. The right hand one acts vertically upwards, the two others vertically downwards, at distances 2 feet and 3 feet respectively, from the first. Draw their resultant, and state exactly its magnitude and position.

**290.** A rod is suspended horizontally on two points, A and B, 12 feet apart; between A and B points C and D are taken, such that AC = BD = 3 feet; a weight of 120 pounds is hung at C, and a weight of 240 pounds at D; the weight of the rod is neglected. Take a point O, midway between A and B, and find with respect to O the algebraical sum of the moments of the forces acting on the rod on one side of O.

**291.** A horizontal rod without weight, 6 feet long, rests on two supports at its extremities; a weight of

672 pounds is suspended from the rod at a distance of $2\frac{1}{2}$ feet from one end. Find the reaction at each point of support. If one support could bear a pressure of only 112 pounds, what is the greatest distance from the other support at which the weight could be suspended?

**292.** Three equal parallel forces act at the corners of an equilateral triangle. Find the point of application of their resultant.

**293.** Find the center of the three parallel forces 4 pounds, 6, and 8, which act respectively at the corners of an equilateral triangle.

**294.** P, Q, R, are parallel forces acting in the same direction at the angular points respectively of an equilateral triangle ABC. If $P = 2Q = 3R$, find the position of their center; also find its position if the direction of the force Q is reversed.

**295.** Show that if two forces be represented in magnitude and direction by two sides of a triangle, taken in order, the sum of their moments about every point in the base is the same.

**296.** Draw a square whose angular points in order are A, B, C, D, and suppose equal forces (P) to act from D to A, A to B, and B to C respectively, and a fourth force (2P) to act from C to D. Find a point such that, if the moments of the forces are taken with respect to it, the algebraic sum is zero.

**297**. ABCD is a square, the length of each side being 4 feet, and four forces act as follows: 2 pounds from D to A, 3 pounds from B to A, 4 pounds from C to B, and 5 pounds from D to B. Find the algebraical sum of the moments of the forces about C.

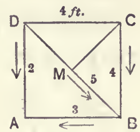


Fig. 31.

The forces act as in the figure.
Draw CM perpendicular to DB.
Then, $$CM = DM.$$
$$\therefore CD^2 = CM^2 + MD^2 = 2CM^2.$$
$$\therefore CM = \frac{CD}{\sqrt{2}}.$$
$$\therefore CM = \frac{4}{\sqrt{2}} = 2.83 \text{ nearly.}$$

$\therefore$ Algebraical sum of the moments about C
$$= -2 \times DC + 3 \times CB + 4 \times 0 - 5 \times CM$$
$$= -2 \times 4 + 3 \times 4 + 0 + 5\ (2.83)$$
$$= -8 + 12 \times 14.15$$
$$= -10.15 \text{ units.}$$

**298**. ABCD is a square, and AC is a diagonal: forces P, Q, R, act along parallel lines at B, C, D, respectively, Q acts in the direction A to C, P in same direction, and R in opposite direction. Find, and show in a diagram, the position of the center when Q = 5P and R = 7P.

**299**. Draw a rectangle, ABCD, such that the side AB is three-fourths of the side BC; forces of 3, 9, and 5 units act from B to A, B to C, and D to A respectively. Find their resultant by construction or otherwise, and show in your diagram exactly how it acts.

**300.** Prove that, if parallel forces 1, 2, 3, 4, 5, 6, are situated at the angles of a regular hexagon, the distance of their center from the center of the circumscribing circle is two-sevenths of the radius of that circle.

**301.** Six forces, represented by the sides of a regular hexagon taken in order, act along the sides to turn the hexagon round an axis perpendicular to its plane. Show that the moment of the forces is the same through whatever point within the hexagon the axis passes.

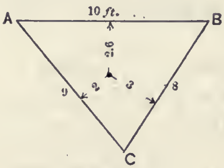


Fig. 32.

**302.** A triangular table, sides 8 feet, 9 feet, and 10 feet, is supported by legs at each corner, and 350 pounds is placed on it 3 feet from the 8-foot side, 2 feet from the 9-foot side, and 2.6 feet from the 10-foot side. What are the pressures on the legs?

**303.** A triangular shaped platform right-angled at A, with side AB 10 feet long, side AC 40 feet long, is loaded with freight at 50 pounds per square foot surface. Find the load carried by each of the three corner-posts.

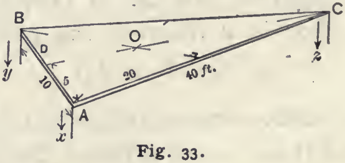


Fig. 33.

O is center of gravity, $\frac{1}{3}$ distance from base to vertex

$$\text{Area} = 40 \times \tfrac{1}{2} \times 10$$
$$= 200 \text{ square feet}$$
$$\text{Load} = 200 \times 50 = 10\,000.$$

Moments about axis AB,

$+ z \times CA - 10\,000 \times$ perp. from C.G. to axis $AB = 0$

$$z \times 40 - 10\,000 \times \tfrac{1}{3} \times 40 = 0$$
$$z \times 40 = 10\,000 \times \tfrac{1}{3} \times 40$$
$$z = \tfrac{10\,000}{3} = 3\,333\tfrac{1}{3}.$$

Moments about CA,

$$y \times 10 - 10\,000 \times \tfrac{1}{3} \times 10 = 0$$
$$y = 3\,333\tfrac{1}{3}.$$

Moments about CB,

$$x \times \text{perp. to BC} - 10\,000 \times \tfrac{1}{3} \text{ perp. to BC} = 0$$
$$x = 3\,333\tfrac{1}{3}.$$

**304.** A floor $20 \times 30$ feet is supported mainly by four posts, one at each corner. There is a load of 20 pounds per square foot uniformly distributed, and at point O, 5 feet from 30-foot side and 7 feet from 20-foot side there is a metal planer weighing 5 tons. Find the load on each post.

**305.** Weights 5, 6, 9, and 7 respectively, are hung from the corners of a horizontal square, 27 inches in a side. Find, by taking moments about two adjacent edges of the square, the point where a single force must be applied to balance the effect of the forces at the corners.

**306.** Four vertical forces, 5, 7, 10 and 12 pounds, act at the corners of a square of 20-inch sides. Find resultant and its point of application.

Let ABCD be the square,

Resultant $= 5 + 7 + 10 + 12$

$= 34$ pounds.

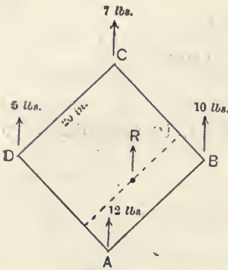


Fig. 34.

To find its point of application:

Resultant of 7 and 10 will be a force of 17 pounds acting from point in line CB distant $\frac{3}{10}$ of 20 inches from B. The resultant of 5 and 12 will be 17 pounds acting at a point in line AD distant $\frac{5}{12}$ of 20 inches from A. The resultant of these two resultants will be a force of 17 + 17 pounds, 34 pounds, acting at a point half way between them and at a perpendicular distance from AB of

$\frac{1}{2}$ of $[\frac{3}{10} \times 20 + \frac{5}{12} \times 20] = 7\frac{1}{6}$ inches.

**307.** A uniform beam, weighing 400 pounds, is suspended by means of two chains fastened one at each end of the beam. When the beam is at rest it is found that the chains make angles of 100° and 115° with the beam. Find the tension in the chains.

**308.** What is the resultant of a couple of moment 15, and a force 3?

**309.** A brakeman sets up a brake on a freight car by pulling 50 pounds with one hand and pushing 50 pounds with the other; his forces act tangentially to the brake wheel, the diameter of which is $1\frac{1}{2}$ feet. Another time he produces the same brake resistance by using a lever in handwheel and pulling 25 pounds. How far from handwheel must his hands be placed?

**310.** When are couples said to be like and when unlike? When will two unlike couples balance each other? (1) If a system of forces is represented in magnitude and position by the sides of a plane polygon taken in order, show that the system must be equivalent to a couple. (2) If the sides of a parallelogram taken in order represent a system of forces acting upon a body, express the moment of the couple to which the system of forces is equivalent.

**311**. Show that a force and a couple in one plane may be reduced to a single force. Given in position a force of 10 pounds, and a couple consisting of two forces of 4 pounds each, at a distance of 2 inches, acting with the hands of a clock, draw the equivalent single force.

**312**. The length of the side of a square ABCD is 12 inches. Along the sides AB and CD forces of 10 pounds act, and along AD, CB forces of 20 pounds. Find the moment of the equivalent couple.

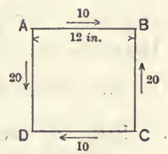


Fig. 35.

Moments about D,

$$-12 \times 10 + 12 \times 20 = \text{moment of equivalent-couple}$$
$$12 \times 10 = \text{moment of equivalent-couple}$$

**313.** Forces P and Q act at A, and are completely represented by AB and AC, sides of a triangle ABC. Find a third force R such that the three forces together may be equivalent to a couple whose moment is represented by half the area of the triangle.

**314**. A tradesman has a balance with arms of unequal length, but tries to be fair by weighing his material first from one scale pan, then from the other. Show that he will defraud himself.

**315**. A tradesman uses a balance with arms in ratio of 5 to 6; he weighs out from alternate pans what appears to be 30 pounds. How much does he gain or lose?

**316**. The beam of a balance is 6 feet long, and it appears correct when empty; a certain body placed in one scale weighs 120 pounds, when placed in the other, 121 pounds. Show that the fulcrum must be distant about $\frac{1}{13}$ of an inch from the center of the beam.

**317**. The weight of a steelyard is 12 pounds, its movable weight is 3 pounds. Find the distance between successive pound graduations, if the length of the short arm is 3 inches.

**318**. A weight of 247 pounds is attached to one end of a horizontal straight lever, which is 22 inches long, and may be regarded as having no weight; the force is applied at the other end, and makes an angle of 27° with the lever; the fulcrum is 3 inches from the weight. Find the magnitude of the force when it just balances the weight.

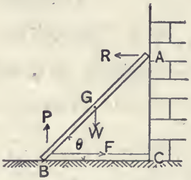


Fig. 36.

**319**. A uniform beam rests at a given inclination, $\theta$, with one end against a smooth vertical wall, and the other end on smooth horizontal ground: it is held from slipping by a string extending horizontally from the foot of the beam to the foot of the wall. Find the tension in the string and the pressures at the ground and wall.

AB is the beam, AC the wall, BC the string, W the weight of the beam acting at its middle point G.

There are three forces supporting the beam: vertical reaction P, horizontal reaction R, and tension in the string F.

Take moments about B, the point of intersection of two of the forces — their lever arms would be zero.

$$R \times AC = W \times \frac{BC}{2}.$$

Substitute for AC its value $BC \times \tan\theta$, then

$$(1)\ R = \frac{W}{2\tan\theta}$$

but R must equal F, both being horizontal resisting forces that maintain equilibrium; likewise P and W must be equal.

$$\therefore (2)\ F = \frac{W}{2\tan\theta} \text{ and}$$

$$(3)\ P = W$$

**320.** A uniform beam rests with a smooth end against the junction of the horizontal ground and a vertical wall; it is supported by a string fastened to the other end of the beam and to a staple in the vertical wall. Find the tension of the string, and show that it will be half the weight of the beam if the length of the string be equal to the height of the staple above the ground.

**321.** A uniform rod 8 feet long, weighing 18 pounds, is fastened at one end to a vertical wall by a smooth hinge, and is free to move in a vertical plane perpendicular to the wall. It is kept horizontal by a string 10 feet long, attached to its free end and to a point in the wall. Find the tension in the string, and the pressure on the hinge.

**322.** A uniform beam, 12 feet in length, rests with one end against the base of a wall which is 20 feet high. If the beam be held by a rope 13 feet long, attached to the top of the beam and to the summit of the wall, find the tension of the rope, neglecting its weight, and assuming the weight of the beam to be 100 pounds.

**323.** A foot-bridge with 18 feet span, 6 feet breadth has two king-post trusses, one on each side, 3 feet deep. The bridge is loaded with people which makes 100 pounds per square foot of floor surface. Find the stress in the post.

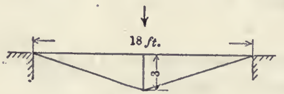


Fig. 37.

**324.** A beam AB rests on the smooth ground at A and on a smooth inclined plane at B; a string is fastened at B and, passing over a smooth peg at the top of the plane, supports a weight P. If W is the weight of the beam, and $\alpha$ the inclination of the plane, find P and the reactions on the rod.

Draw the figure.

The weight W acts at the middle point C. The reaction of the ground at A is R, upwards.

The reaction of the plane at B is $R_1$, perpendicular to the plane.

Let the angle BAD $= \theta$.

The tension of the string at B = tension of the string throughout = P.

There are four forces acting on the beam, W, R, $R_1$, P.

Resolve vertically and horizontally.

**325.** A pole 12 feet long, weighing 25 pounds, rests with one end against the foot of a wall, and from a point 2 feet from the other end a cord runs horizontally to a point in the wall 8 feet from the ground. Find the tension of the cord and the pressure of the lower end of the pole.

**326.** A light smooth stick 3 feet long is loaded at one end with 8 ounces of lead; the other end rests against a smooth vertical wall, and across a nail which is 1 foot from the wall. Find the position of equilibrium and the pressure on the nail and on the wall.

**327.** A trapezoidal wall has a vertical back and a sloping front face; width of base, 10 feet; width of top, 7 feet; height, 30 feet. What horizontal force must be applied at a point 20 feet from the top in order to overturn it? Thickness of wall, 1 foot; weight of masonry in wall, 130 pounds per cubic foot.

**328.** Six men using a rope 50 feet long were just able to pull over a chimney 75 feet high. How far up from the bottom of the chimney was it advisable to attach the rope?

**329.** If 150 000 pounds is the thrust along the connecting rod of the engine, in example 86, $2\frac{1}{2}$ feet the crank radius, and the connecting-rod is inclined to the crank axis at 150°, show that the moment of the thrust about the crank-pin is one-half the greatest possible moment.

**330.** A trap-door of uniform thickness, 5 feet long and 3 feet wide, and weighing 5 hundred weight, is held open at angle of 35° with the horizontal by means of a chain. One end of chain is hooked at middle of top edge of door, and the other is fastened at wall 4 feet above hinges. Find the force in the chain and the force at each hinge.

**331.** The sketch represents a coal wagon weighing with its load $4\frac{1}{2}$ tons. How many pounds applied at P by usual methods of hand power will just lift the wagon when in the position shown in the sketch?

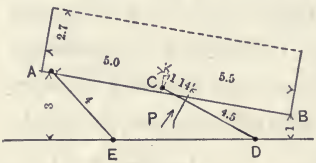


Fig. 38.

AE is a rod in tension. CD is a connecting-bar. Divide the problem into three parts:

(*a*) Draw the forces acting.

(*b*) Find horizontal distance from C to the vertical through the center of gravity.

(*e*) Find force to apply at C parallel to P; then find P.

## CENTER OF GRAVITY

**332.** A rod of uniform section and density, weighing 3 pounds, rests on two points, one under each end; a movable weight of 4 pounds is placed on the rod. Where must it be placed so that one of the points may sustain a pressure of 3 pounds, and the other a pressure of 4 pounds?

**333**. Two rods of uniform density weighing 2 pounds and 3 pounds respectively are put together so that the 3-pound one stands on the middle of the other. Find the center of gravity of the whole.

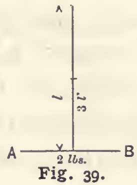


Fig. 39.

Take moments about AB,

$$+3 \times \tfrac{1}{2}l - 5 \times x = 0$$
$$x = \tfrac{3}{10} \text{ of } l.$$

**334**. A curtain rod 5 feet long, weighing 4 pounds, has four rings on it each weighing 1 pound. The two end rings are 4 feet apart, the two middle rings 2 feet apart, and one ring is distant 6 inches from the middle of the rod. Find their center of gravity.

**335**. Three particles of masses, 3 pounds, 7 pounds, and 10 pounds, are respectively 5 feet above, 6 feet above, and 12 feet below a horizontal line. What is the position of their center of gravity with reference to the horizontal line?

**336**. A rod ABC, 16 inches long, rests in a horizontal position upon two supports at A and B one foot apart, and it is found that the least upward or downward forces applied at C which would move the rod are 4 ounces and 5 ounces respectively. Find the weight of the rod and the position of its center of gravity.

**337**. A straight line AB represents a rod 10 feet long, supported horizontally by two points, one under each end; C is a point in AB, 3 feet from A. What pressure is produced on the points A and B by a weight of 30 pounds hung at C? What additional pressure is exerted on the points of support if the rod is of uniform density and weighs 20 pounds?

**338.** A thin plate of metal is in the shape of a square and equilateral triangle, having one side common; the side of the square is 12 inches long. Find the center of gravity of the plate.

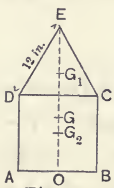


Fig. 40.

Let $G_1$ be the center of gravity of the triangle, $G_2$ of the square, G of the whole plate.

From symmetry $EG_1\,GG_2O$ will be a straight line bisecting the plate, and

$$OG_2 = 6 \text{ inches}$$
$$OG_1 = 15.5 \text{ inches}$$

Let $w$ = weight of metal per square inch

$$\text{Area of triangle} = \tfrac{1}{2} \times 12 \times \sqrt{12^2 - 6^2}$$
$$= 62.4 \text{ square inches}$$
$$\text{Weight} = 62.4 \text{ pounds} \times w \text{ pounds}$$
$$\text{Area of square} = 144 \text{ square inches}$$
$$\text{Weight} = 144 \times w \text{ pounds}$$

Take moments about the axis AB,

$$\text{Weight of triangle} \times OG_1 + \text{weight of square} \times OG_2 - \text{total weight} \times OG = 0$$

$$62.4w \times 15.5 + 144w \times 6 - (62.4w + 144w) \times OG = 0$$
$$\therefore\ OG = 8.86 \text{ inches.}$$

**339.** ABC is a triangle with a right angle at A. AB = 3 inches; AC = 4 inches; weights of 2 ounces, 3 and 4, are placed at A, B, and C. Find the position of their center of gravity.

**340**. A uniform triangle ABC of weight W, and lying on a horizontal table, is just raised by a vertical force applied at A. Find the magnitude of this force, and that of the resultant pressure between the base BC and the table.

**341**. A uniform circular disk has a circular hole punched out of it, extending from the circumference half way to the center. Find the center of gravity of the remainder.

**342**. A box, including its cover, is made of six equal square boards; where is its center of gravity when its lid is turned back through an angle of 180°?

**343**. ABCD is a thin rectangular plate weighing 50 pounds, AB is 10 feet, BC 2 feet; the plate is suspended by the middle point of its upper edge AB, and then, of course, AB is horizontal, but if a weight of 5 pounds is placed at A, AB will become inclined to the horizon. Show how to find the angle of inclination either by calculation or by construction.

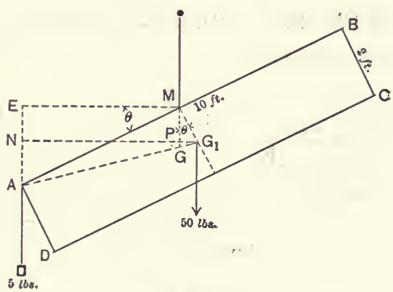



Fig. 41.

Moments about P,

$$PG_1 \times 50 = PN \times 5$$
$$PG_1 = MG_1 \times \sin\theta$$
$$= 1 \times \sin\theta.$$

From the above equations,

$$PN = EM = 10 \times \sin\theta$$
$$\tan\theta = \frac{EA}{EM}$$
$$EA = MA \times \sin\theta$$
$$= 5 \times \sin\theta$$
$$\therefore\ \tan\theta = \frac{5 \times \sin\theta}{10 \times \sin\theta}$$
$$= \tfrac{1}{2}$$
$$\theta = \tan^{-1}\tfrac{1}{2}.$$

**344**. A circular disk, 8 inches in diameter, has a hole 2 inches in diameter punched out of it, the center of the hole being 3 inches from the circumference of the disk. Find the center of gravity of the remaining portion.

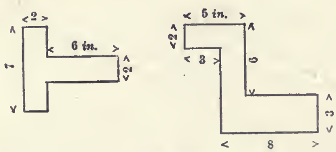



Fig. 42.

**345**. Find the centers of area of the above sections of uniform plate metals.

**346**. Into a hollow cylindrical vessel 11 inches high and weighing 10 pounds, the center of gravity of which is 5 inches from the base, a uniform solid cylinder 6 inches long and weighing 20 pounds is just fitted. Find the common center of gravity.

$G_1$ center of gravity of hollow cylinder
$G_2$ center of gravity of solid cylinder.
Moments about AB,

$$+10 \times 5 + 20 \times 3 - 30 \times x = 0$$
$$+50 + 60 - 30x = 0$$
$$30x = 110$$
$$x = 3\tfrac{2}{3} \text{ inches.}$$

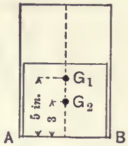


Fig. 43.

**347.** Give examples of stable and unstable equilibrium. A cone and a hemisphere of the same material are cemented together at the common circular base. If they are on a horizontal plane, and the hemisphere in contact with the plane, find the height of the cone in order that the equilibrium may be neutral. (The center of gravity of a hemisphere divides a radius in the ratio of 3 to 5.)

**348.** A thread 9 feet long has its ends fastened to the ends of a rod 6 feet long; the rod is supported in such a manner as to be capable of turning freely round a point 2 feet from one end; a weight is placed on the thread, like a bead on a string. Find the position in which the rod will come to rest, it being supposed that the rod is without weight, and that there is no friction between the weight and the thread.

**349.** A circular disk weighs 9 ounces; a thin straight wire as long as the radius of the circle weighs 7 ounces; if the wire is placed on the disk so as to be a chord of the circle, the center of gravity of the whole will be at a distance from the center of the circle equal to some fractional part of the radius. Find that fraction by construction or calculation.

**350.** A cone and a hemisphere are on the same base. What height must the cone be in order that the center of gravity of the whole solid shall be at the center of the common base?

$r$ = radius common base.
$h$ = height of cone.

## FRICTION

The coefficients of friction for various pairs of substances have been found experimentally by Morin; these results however can be used only for approximate computation; actual trial should be made for specific cases.

| | |
|---|---|
| Oak on oak, fibers parallel to direction of motion | 0.48 |
| perpendicular | 0.34 |
| endwise | 0.19 |
| Metals on oak dry, fibers parallel | 0.5 to 0.6 |
| Metals on metals, dry | 0.15 to 0.2 |
| Smooth surfaces with unguents well greased | 0.05 |

**351.** What push would be required to move a stone of weight 3 pounds along ice, if the coefficient of friction is 0.1?

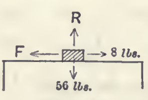


Fig. 44.

**352.** A weight of 56 pounds is moved along a horizontal table by a force of 8 pounds. How much is the coefficient of friction?

The pull of 8 pounds is required to overcome friction, and is equal to the friction.

Friction = coefficient × Reaction (perpendicular to plane of table).

$$\begin{aligned} F &= \mu \times R \\ &= \mu \times 56 \text{ pounds} \\ 8 &= \mu \times 56 \\ u &= \tfrac{8}{56} \\ &= \tfrac{1}{7}. \end{aligned}$$

**353**. A block of wood weighing 1 pound is just dragged along a horizontal table by a force of 1 pound. What is the direction of the resultant reaction?

**354**. What is the angle of friction or limiting angle of resistance? When a body urged against a rough fixed plane by certain forces is at rest, to what extent is the direction of the reaction of the plane against it known?

**355**. A block of stone is dragged along the ground by a horse exerting a force of 224 pounds. If $\mu = 0.6$, what is the weight of the block?

**356**. A weight of 500 pounds is placed on a table, and can hardly be slid by a horizontal pull of 155 pounds. Find the coefficient of friction, and the number of degrees in the angle of friction by measuring from a drawing made to a scale.

**357**. A stone just slides down a hill of inclination 30°. What is the coefficient of friction?

**358**. A block rests on a plane which is tilted till the block commences to slide. The inclination is found to be 8.4 inches at starting, and afterwards 6.3 inches on a horizontal length of 2 feet. Find the coefficient of friction when the block starts to slide, and after it has started.

**359**. A horse draws a load weighing 2 000 pounds up a grade of 1 in 20; the resistance on the level is 100 pounds per ton. Find the pull on the traces when they are parallel with the incline.

**360**. How much work has a man, weighing 224 pounds, done in walking twenty miles up a slope of 1 vertical to 40 horizontal? What force could drag a dead load of the same weight up the same hill (*a*) if friction be negligible, (*b*) if friction be $\frac{1}{4}$ of the weight?

**361**. Three artillerymen drag a gun weighing 1 700 pounds up a hill rising 2 vertically in 17 horizontally. Suppose the resistance to the wheels going up the hill be 16 pounds per hundred weight, what pull parallel to the hill must each exert to move it?

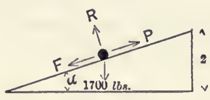


Fig. 45.

When the gun is about to move forward the pull P will be acting up the plane, and parallel to it; the friction F down the plane, holding back; the force R perpendicular to inclined plane, partly supporting the gun, and W the weight of the gun acting vertically downward. Weight of gun is given — 1 700 pounds. Resolve into components perpendicular and parallel to the plane. The perpendicular component will be the supporting force of the plane — its reaction R; the parallel component will be the part of the pull P required by weight of the gun.

**362**. Find the force which, acting in a given direction, will just support a body of given weight on a rough inclined plane. The height is to the base of the plane as 3 to 4, and it is found that the body is just supported on it by a horizontal force equal to half the weight of the body. Find the coefficient of friction between the body and the plane.

**363.** Two equal weights are attached to a string that is laid over the top of two inclined planes having the same altitude and placed back to back, the angles of inclination being 30° and 60° respectively. Show that the weights will be on the point of moving if the coefficient of friction between each plane and weight be $\frac{1}{2+\sqrt{3}}$.

**364.** The roughness of a plane, of inclination 30° is such that a body of weight 500 pounds can rest on it. What is the least force required to draw the body up the plane? ($\alpha$ in sketch will equal the angle of friction.)

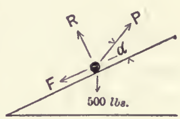


Fig. 46.

**365.** Find the least force that will drag a body weighing 100 pounds along a rough horizontal plane, the coefficient of friction being $\frac{1}{\sqrt{3}}$. Find also the resistant reaction of the plane.

**366.** A weight of 5 pounds can just be supported on a rough inclined plane by a weight of 2 pounds, or can just support a weight of 4 pounds suspended by a string passing over a smooth pulley at the vertex.

Find the coefficient of friction, and the inclination of the plane.

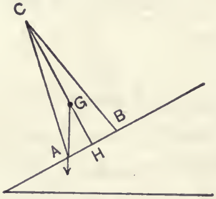



Fig. 47.

**367.** A heavy cone is placed on a rough inclined plane, the inclination of which is gradually increased. Find whether the cone will begin by sliding down the plane or toppling over.

Assume first that the equilibrium is broken by the body toppling over; if this does not require too great a value of the coefficient or angle of friction, then the equilibrium will be broken in that way.

Let ABC represent the vertical section of the cone, CH its axis, G its center of gravity; then $HG = \frac{HC}{4}$.

Let the inclination of the plane be such that the vertical through G passes through A, the lowest point of the base. If the cone can rest here without sliding, then the slightest increase of inclination will cause it to topple over.

Let $\phi = ACH = \frac{1}{2}$ vertical angle of the cone.

The forces on the cone are the weight along GA; and the reaction of the plane.

$\therefore$ the reaction of the plane is along AG.

This can only be so if AG makes with the normal to the plane an angle less than $\alpha$, the angle of friction.

$$\therefore AGH \text{ must be } < \alpha$$

or

$$\tan AGH \text{ must be } < \mu$$

But

$$\frac{\tan AGH}{\tan ACH} = \frac{\frac{AH}{HG}}{\frac{AH}{HC}} = \frac{HC}{HG} = \frac{4}{1}.$$

$$\therefore \tan AGH = 4 \tan ACH = 4 \tan \phi$$

If $\mu > 4 \tan \phi$, the cone will topple over.

If $\mu < 4 \tan \phi$, the cone will begin to slide, and its motion will start as soon as the inclination of the plane is $\alpha$, or $\tan^{-1} \mu$.

**368.** A rectangular block ABCD whose height is double its base, stands with its base AD on a rough floor, coefficient of friction $\frac{1}{6}$. If it be pulled by a horizontal force at C till motion ensues, determine whether it will slip on the floor, or begin to turn over round D.

**369.** A cubical block rests on a rough plank with its edges parallel to the edges of the plank. If, as the plank is gradually raised, the block turns over on it before slipping, how much at least must be the coefficient of friction?

**370.** The poles supporting a lawn-tennis net are kept in a vertical position by guy-ropes, one to each pole, which pass round pegs 2 feet distant from the poles. If the coefficient of friction between the ropes and pegs be $\frac{4}{3}$, show that the inclination of the latter to the vertical must be less than $\tan^{-1}\frac{2}{11}$, the height of the poles being 4 feet.

**371.** The table of a small planing-machine which weighs 112 pounds, makes six double strokes of $4\frac{1}{2}$ feet each per minute. The coefficient of friction between the sliding surfaces is .07. What is the work in foot-pounds per minute performed in moving the table?

**372.** A rough wedge has been inserted into a block, and is only acted on by the reactions. If it is on the point of slipping out, and the coefficient of friction is $\frac{1}{\sqrt{3}}$, what is the angle of the wedge?

**373.** A cotter, or wedge, having a taper of 1 in 8, is driven into a cottered joint with an estimated pressure of 600 pounds. Taking the coefficient of friction between the two surfaces as 0.2, find the force which the cotter, or wedge, exerts at the joint perpendicular to the pressure of 600 pounds; also find the pull necessary to withdraw the cotter.

**374.** A steel wedge 12 inches long, tapered from 2 inches thick down to 0, is used to wedge up a pump plunger weighing 3 000 pounds by means of a maul weighing 5 pounds. The coefficient of friction is 0.15 and the striking velocity of the maul is 25 feet per second. How far will each blow drive the wedge?

**375.** The locomotive of the Empire State Express has four drivers and a total weight of 124 000 pounds; the weight on the drivers is 84 000 pounds; the coefficient of friction between wheels and rails is 0.18. Find the greatest pull which the engine can exert in pulling itself and a train. What is the total weight of itself and train which it can draw up a grade of 1 in 100, if the resistance to motion is 12 pounds per ton on the level?

**376.** A wheel of weight W rests between two planes, each inclined to the vertical at angle $\alpha$; the plane of the wheel is perpendicular to the line of intersection of the two planes, which is itself horizontal. If $\mu$ be the coefficient of friction, find the least couple necessary to turn the wheel.

**377.** A uniform ladder of weight W rests on rough ground and against a rough wall, the coefficients of friction being respectively $\mu$ and $\mu'$. What is the least inclination it can make with the horizon?

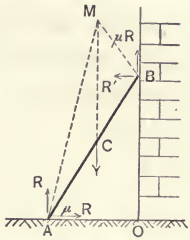



Fig. 48.

Let AB be the ladder, AO the ground, BO the wall. The ladder is about to slip down; therefore the limiting friction acts at A toward the wall, and at B upwards,

Let the normal reaction at A = R.

$$\therefore \text{ Friction at } A = \mu R.$$

Similarly we have R′ and $\mu'$R′ at B.

The three forces acting are the weight of the ladder and the resultant reactions at A and B.

These three forces must meet in a point M vertically above C, W the weight of the ladder acting at the middle point C.

Resolve the forces horizontally and vertically.

$$W = R + \mu' R'$$
$$R' = \mu R$$
$$W = R + \mu\mu' R$$

Take moments about B,

$$R \times AO = W\frac{AO}{2} + \mu R \times BO$$
$$\therefore R = \frac{W}{2} + \mu R \times \frac{BO}{AO}$$
$$= \frac{W}{2} + \mu R \tan\theta$$
$$\therefore W = 2\, R\, [1 - \mu \tan\theta].$$

Placing this equal to value of W found above,

$$2\, R\, [1 - \mu \tan\theta] = R + \mu\mu' R$$
$$\therefore 1 - \mu\mu' = 2\,\mu \tan\theta$$
$$\therefore \tan\theta = \frac{1 - \mu\mu'}{2\,\mu}$$

**378.** A ladder inclined at an angle of 60° to the horizon, rests with one end on rough pavement, and the other end against a smooth vertical wall; the ladder begins to slide down when a weight is put at its middle point. Show that the coefficient of friction is $\frac{\sqrt{3}}{6}$.

**379.** A uniform ladder weighing 100 pounds and 52 feet long is inclined at an angle of 45° with a rough vertical wall and a rough horizontal plane. If the coefficient of friction is at each end $\frac{2}{3}$, how far up the ladder can a man weighing 200 pounds ascend before the ladder begins to slip?

**380.** A uniform ladder 70 feet long is equally inclined to a vertical wall and the horizontal ground, both rough; a man with a hod — weight 224 pounds — ascends the ladder which weighs 448 pounds. How far up the ladder can the man ascend before it slips, the tangent of the angle of resistance for the wall being $\frac{1}{3}$ and for the ground $\frac{1}{2}$?

**381.** A uniform beam rests with one end on a rough horizontal plane, and the other against a rough vertical wall, and when inclined to the horizon at an angle of 30°, is on the point of slipping down: suppose the surfaces equally rough, find $\mu$.

**382.** The mean diameter of the threads of a $\frac{1}{2}$ inch bolt is .45 inches, the slope of the thread .07 and the coefficient of friction 0.16. Find the tension in a bolt when tightened up by a force of 20 pounds on the end of a wrench 12 inches long.

**383.** Experiment shows that a weight can lift only three-quarters of its own weight by means of a rope over a single pulley, this being the consequence of the stiffness of the rope and the friction of the axis. Hence show that the mechanical advantage of four such pulleys arranged in two blocks is about 2.05.

**384** A weight of 5 tons is to be raised from the hold of a steamer by means of a rope which takes $3\frac{1}{2}$ turns around the drum of a steam-windlass. If $\mu = 0.234$, what force must a man exert on the other end of the rope?

$$T_1 = T_2 e^{2\pi\mu n}$$
$$\log_{10} T_1 = \log_{10} T_2 + 2.7288\, n\mu.$$

**385.** A man by taking $2\frac{1}{2}$ turns around a post with a rope, and holding back with a force of 200 pounds, just keeps the rope from surging. Supposing $\mu = 0.168$, find the tension at the other end of the rope.

**386.** A weight of 2 000 pounds is to be lowered into the hold of a ship by means of a rope which passes over and around a spar lashed across the hatch-coamings so as to have an arc of contact of $1\frac{1}{4}$ circumferences. If $\mu = \frac{7}{22}$, what force must a man exert at the end of the rope to control the weight?

**387.** A hawser is subjected to a stress of 10 000 pounds. How many turns must be taken around the bitts, in order that a man who cannot pull more than 250 pounds may keep it from surging, supposing $\mu = 0.168$?

**88.** A rope drive carrying 20 ropes, has a pulley 16 feet in diameter, and transmits 600 horse-power when running at 90 revolutions per minute. Taking $\mu = 0.7$ and the angle of contact $180°$, find the tensions on the tight and slack sides of the rope.

$$T_1 = T_2 e^{\mu\theta}$$
$$T_1 - T_2 = 218.8$$
$$T_2 = T_1 - 218.8$$
$$T_1 = (T_1 - 218.8)e^{\mu\theta}$$
$$T_1 (e^{\mu\theta} - 1) = 218.8\, e^{\mu\theta}$$
$$T_1 = \frac{.218.8\, e^{\mu\theta}}{e^{\mu\theta} - 1}$$
$$= \frac{218.8 \times 2.72^{.7\times\pi}}{2.72^{.7\times\pi} - 1}.$$

**389.** A single fixed pulley, 6 inches in diameter, turns on an axle 2 inches in diameter; coefficient of friction 0.2. A weight of 500 pounds is lifted by means of this pulley. Find the force P that is required.

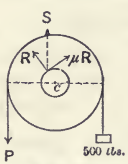


Fig. 49.

Taking moments about C, the center

$$-P \times 3 + 500 \times 3 + \mu R \times 1 = 0$$
$$\mu = 0.2$$

Since $$S = R^2 + \mu^2 R^2$$

and $$S = P + W$$

$$R = \frac{P + W}{\sqrt{1 + \mu^2}}$$
$$= \frac{P + 500}{1.02}$$

$$\therefore P \times 3 = 500 \times 3 + 0.2 \times \frac{P + 500}{1.02} \times 1$$
$$P = 570 \text{ pounds.}$$

**390.** A single fixed pulley, 2 feet in radius, turns on an axle 1 inch in radius; the weight of the pulley is 80 pounds. A weight of 500 pounds is lifted by means of this pulley; What force P is required? The coefficient of friction between axle and bearing is 0.1; the rope is supposed to be flexible, and without weight, and P to act vertically.

**391.** Let P and W be inclined to each other at an angle of 90°; radius of pulley is 6 inches; radius of axle $\frac{3}{4}$ inch; coefficient of friction, 0.2. Determine the relation of P and W in case of incipient motion.

**392.** A leather belt will stand a pull of 200 pounds. It passes around one-half the circumference of a pulley 4 feet in diameter making 150 revolutions per minute. What power will it transmit if the coefficient of friction between the belt and pulley is 0.1?

**393.** Find the width of a belt necessary to transmit 10 horse-power to a pulley 12 inches in diameter, so that the greatest tension may not exceed 40 pounds per inch of width when the pulley makes 1 500 revolutions per minute, the weight of the belt per square foot being 1.5 pounds, and the coefficient of friction 0.25.

**394.** A belt laps 150° around a 3-foot pulley, making 130 revolutions per minute; the coefficient of friction is 0.35. What is the maximum pull on the belt when 20 horse-power is being transmitted and the belt is just on the point of slipping?

**395.** The power of an engine is tested by putting a belt over the fly-wheel, which is 5 feet in diameter, and on one end of the belt hanging a weight of 300 pounds and to the other attaching a spring balance. The fly-wheel is observed to make 150 revolutions per minute and the spring balance reads 180 pounds. What is the brake horse-power?

**396.** In problem 395, if the belt laps more than 180° the dynamometer pulls on belt being 300 pounds and 180 pounds, coefficient of friction 0.159, what part of the circumference is encircled?

**397.** A 12-inch Pelton water-motor of 3 horse-power is tested by a friction brake encircling three-fourths of the 4-inch pulley of the motor and having a lever arm extending 22 inches from center of pulley to scales. The scales read 5 pounds when motor is making 1150 revolutions per minute. What horse-power is being developed?

**398.** Find the horse-power necessary to turn a shaft 9 inches in diameter making 75 revolutions per minute, if the total load on it is 12 tons and $\mu = .015$.

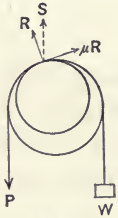


Fig. 50.

**399.** A shaft makes 50 revolutions per minute. If the load on the bearing be 8 tons and the diameter of the bearing be 7 inches, at what rate is heat being generated, the average coefficient of friction being 0.05?

$$
\begin{aligned}
S &= 8 \text{ tons} \\
&= P + W \\
R &= \frac{P + W}{\sqrt{1 + \mu^2}} \\
&= \frac{8 \times 2\,000}{\sqrt{1.0\,025}} \\
&= \frac{16\,000}{1.001} \\
&= 15\,980 \\
\mu R &= 799 \text{ pounds,}
\end{aligned}
$$

7-inch diameter = 22-inch circumference.

$$\tfrac{22}{12} \times 50 = \tfrac{1100}{12} = 92 \text{ feet per minute}$$

Foot-pounds generated by heat

$$
\begin{aligned}
&= 799 \times 92 \\
&= 73\,500 \text{ foot-pounds per minute.}
\end{aligned}
$$

**400.** A horizontal axle 10 inches in diameter has a vertical load upon it of 20 tons, and a horizontal pull of 4 tons. The coefficient of friction is 0.02. Find the heat generated per minute, and the horse-power wasted in friction, when making 50 revolutions per minute.

**401.** A horizontal axle 10 inches in diameter has upon it a vertical load of 20 tons, and a horizontal pull of 4 tons. The coefficient of friction is 0.02. Find by the rough method given below the heat generated per minute, and the horse-power wasted in friction when making 50 revolutions per minute, taking the resistance as 2 pounds per square inch.

("A rough and ready estimate of the work absorbed by a bearing is to assume that the frictional resistance of the surface of the bearing is 3 pounds per square inch for ordinary lubrications, 2 pounds for pad, 1 pound for bath, the surface being reckoned on the nominal area." — GOODMAN.)

**402.** The shaft of a 1 000-kilowatt dynamo is 25 inches in diameter, makes 100 revolutions per minute, and carries a total load of 45 000 pounds. The coefficient of friction being 0.05, find the horse-power lost in heat that is generated by friction.

**403.** Calculate the horse-power absorbed by a foot-step bearing with flat end 8 inches in diameter when supporting a load of 4 000 pounds, and making 100 revolutions per minute, coefficient of friction 0.03.

**404.** Find the horse-power absorbed in overcoming the friction of a foot-step bearing 4 inches in diameter, the total load being $1\frac{1}{2}$ tons, the number of revolutions 100 per minute, and the average coefficient of friction 0.07.

$$\text{Moment of friction} = \tfrac{2}{3}\,\mu WR \quad \text{(See text-books.)}$$

$$\text{Work per minute} = \tfrac{2}{3}\,\mu W \times \frac{\frac{1}{2}D}{12} \times 2\pi \times N$$

(D being in inches.)

$$= \frac{\mu WDN}{\frac{18}{\pi}}$$

$$= \frac{\mu WDN}{5.73}.$$

## III. MOTION

**405**. A body moving with a velocity of 5 feet per second is acted on by a force which produces a constant acceleration of 3 feet per second. What is the velocity at the end of 20 seconds?

Velocity gained = acceleration per second × number of seconds.

$$\begin{aligned} v &= f \times t \\ &= 3 \times 20 \\ &= 60 \text{ feet per second} \\ \text{Final velocity} &= 60 + 5 \\ &= 65 \text{ feet per second.} \end{aligned}$$

**406**. The initial velocity of a stone is 12 feet per second; this velocity decreases uniformly at the rate of 2 feet per second. How far will the stone have traveled in 5 seconds?

**407**. Two trains A and B moving towards each other on parallel rails uniformly at the rate of 30 miles and 45 miles an hour, respectively, are 5 miles apart at a given instant. How far apart will they be at the end of 6 minutes from that instant, and at what distances are they from the first position of A?

**408**. The velocity of a train is known to have been increasing uniformly; at one o'clock its velocity was 12 miles per hour, at 10 minutes past one its velocity was 36 miles an hour. What was its velocity at $7\frac{1}{2}$ minutes past one?


MOTION. Impact of Mallet and Polo Ball. Find Distance, Velocity, Time.

**409.** A train moving at the rate of 30 miles an hour is brought to rest in 2 minutes; the retardation is uniform. How far did it travel?

**410.** A body acted on by a constant force begins to move from a state of rest; it is observed to move through 55 feet in a certain 2 seconds, and through 77 feet in the next 2 seconds. What distance did it describe in the first 6 seconds of its motion?

**411.** A stone skimming on ice passes a certain point with a velocity of 20 feet per second and suffers a retardation of one unit. Find the space passed over in the next 10 seconds, and the whole space traversed when the stone had come to rest.

**412.** An ice boat weighing 1 000 pounds is driven for 30 seconds from rest by a wind force of 100 pounds. Find the velocity acquired and the distance passed over.

**413.** Two bodies are let fall from the same point at an interval of 2 seconds. Find the distance between them after the first has fallen for 6 seconds.

For 1st body, $s = \frac{1}{2} gt^2$
$= \frac{1}{2} \times 32 \times 6^2$
$= 576$ feet.

For 2d body, $s = \frac{1}{2} gt^2$
$= \frac{1}{2} \times 32 \times 4^2$
$= 256$ feet.

$\therefore$ distance apart $= 576 - 256$
$= 320$ feet.

**414.** A stone is projected vertically upwards with a velocity of 80 feet per second from the summit of a tower 96 feet high. In what time will it reach the ground, and with what velocity?

**415.** A stone is dropped into a well, and the sound of its striking is heard $2\frac{7}{12}$ seconds after it is let fall; the velocity of sound in air is 1 200 feet per second. What is the depth of the well?

Let $\qquad s =$ depth of well.

$$\therefore \text{ time for sound to come up} = \frac{s}{1\,200} \text{ seconds.}$$

Time for stone to fall is found from formula

$$s = \tfrac{1}{2} g t^2.$$

$$\therefore\ t^2 = \frac{2\,s}{g} = \frac{s}{16}$$

$$\therefore\ t = \frac{\sqrt{s}}{4}$$

Time for stone to fall + time for sound to come up $= 2\frac{7}{12}$.

$$\therefore\ \frac{s}{1\,200} + \frac{\sqrt{s}}{4} = \frac{31}{12}$$

$$\therefore\ s + 300\sqrt{s} = 3\,100$$

$$\therefore\ s \pm 150^2 + 300\sqrt{s} = 3\,100 \pm 150^2$$

$$\therefore\ \sqrt{s} = -\,310 \text{ an inadmissible value,}$$

or $\qquad \sqrt{s} = +\,10$

$$\therefore\ s = 100 \text{ feet, depth of well.}$$

**416.** A stone is let fall from a tower of height *a* feet; another is projected upwards vertically from the foot of the tower; the two start at the same moment.

What is the initial velocity of the second if they meet halfway up the tower?

**417**. A stone is dropped into a well, and the sound of the splash is heard 7.7 seconds afterwards. Find the depth of the well, supposing the velocity of sound to be 1 120 feet per second.

**418**. A bucket is dropped into a well and in 4 seconds the sound of its striking the water is heard. How deep is the well?

**419**. A balloon has been ascending vertically at a uniform rate for $4\frac{1}{2}$ seconds, and a stone let fall from it reaches the ground in 7 seconds. Find the velocity of the balloon and the height from which the stone is let fall.

**420**. From a balloon which is ascending with a velocity of 32 feet per second, a ball is let fall and reaches the ground in 7 seconds. How high was the balloon when the stone was dropped?

**421**. A ball is let fall to the ground from a certain height, and at the same time another ball is thrown upwards with just sufficient velocity to carry it to the point from which the first one fell. Find when and where they will meet.

**422**. A cake of ice slides down a smooth chute, at an angle of 30° to the horizon. Through how many feet vertically will it fall in the fourth second of its motion?

Space = average velocity × total time

Average velocity = $\frac{1}{2}$ final velocity (for constant acceleration)

Final velocity = gain per second (the acceleration) × number of seconds

$$= ft$$

Average velocity = $\frac{1}{2} ft$

$$\therefore \text{ space} = \frac{1}{2} ft \times t$$
$$= \frac{1}{2} ft^2$$
$$= \frac{1}{2} gt^2, \text{ for a falling body.}$$
$$s = \frac{1}{2} \times 32 \times 4^2, \text{ in four seconds}$$
$$= 256 \text{ feet.}$$
$$s = \frac{1}{2} \times 32 \times 3^2, \text{ in three seconds}$$
$$= 144 \text{ feet.}$$

During fourth second,

$$s = 256 - 144$$
$$= 112 \text{ feet on incline.}$$

If the chute has inclination of 30° the vertical component of distance will be

$$s = 112 \times \sin 30^\circ$$
$$= 112 \times \frac{1}{2}$$
$$= 56 \text{ feet.}$$

**423**. A cable car "runs wild" down a smooth track of inclination 20° to the horizontal. How far does it go during the first 8 seconds after starting from rest?

**424**. A velocity of $6\sqrt{2}$ along the diagonal of a square is resolved into two rectangular components along the sides of the square. How much is each component?

**425.** A body is sliding with velocity $u$ down an inclined plane whose inclination to the horizon is 30°. Find the horizontal and vertical components of this velocity.

**426.** A deer is running at the rate of 20 miles an hour, and a sportsman fires at him when he is at the nearest point, 200 yards distant. How many feet in advance of him should aim be taken if the velocity of the bullet be 1 000 feet per second?

**427.** A boat is rowed at the rate of 5 miles an hour on a river that runs 4 miles an hour. In what direction must the boat be pointed to cross the river perpendicularly? With what velocity does it move?

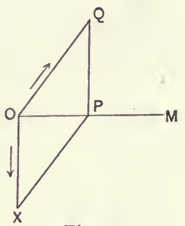


Fig. 51.

Let OX be 4 units in length to represent the velocity of the stream.

Draw OM perpendicular to OX. The resultant velocity is to be in the direction OM.

With center X and radius of 5 units describe an arc cutting OM in P.

Join XP, and complete the parallelogram of velocities OXPQ.

OQ is the required direction.

The angle $QOP = \sin^{-1} \frac{4}{5}$.

Therefore the boat must not be rowed straight across, but up stream at an angle of 53° 10′.

To find the resultant velocity.

$$\begin{aligned} OP^2 &= OQ^2 - QP^2 \\ &= 5^2 - 4^2 \\ &= 25 - 16 \\ &= 9 \\ \therefore OP &= 3 \end{aligned}$$

∴ the boat crosses the river at the rate of 3 miles an hour.

**428.** A river flows at the rate of 2 miles per hour. A boat is rowed in such a way that in still water its velocity would be 5 feet per second in a straight line. The river is 3 000 feet wide; the boat, starting from one shore, is headed 60° up-stream. Where will it strike the opposite shore?

**429.** A bullet moving upwards with velocity of 1 000 feet per second, hits a balloon rising with velocity 100 feet per second. Find the relative velocity.

**430.** A train at 45 miles an hour, passes a carriage moving 10 yards a second in the same direction along a parallel road. Find the relative velocity.

**431.** To a passenger in a train, raindrops seem to be falling at an angle of 30° to the vertical; they are really falling vertically, with velocity 80 feet per second. What is the speed of the train?

**432.** Two roads cross at right angles; along one a man walks northward at 4 miles per hour, along the other a carriage goes at 8 miles per hour. What is the velocity of the man relative to the carriage?

**433.** A steamer is proceeding E. with a velocity of 6 miles per hour; the wind appears to blow from the N.; the steamer increases its velocity to 12 miles per hour, and the wind now appears to blow from the N. E. What is the true direction of the wind and its velocity?

**434.** A ship is sailing north-east with a velocity of 10 miles per hour, and to a passenger on board the wind appears to blow from the north with a velocity of 10 $\sqrt{2}$ miles per hour. Find the true velocity of the wind.

**435**. Two trains, whose lengths are respectively 130 feet and 110 feet, moving in opposite directions on parallel rails, are observed to be 4 seconds in completely passing each other, the velocity of the longest train being double that of the other. Find at what rate per hour each train is moving.

**436**. A fly-wheel revolves 12 times a second. What is the angular velocity about the center of a point on its rim?

**437**. A train weighing 60 tons has a velocity of 40 miles an hour when the steam is shut off. If the resistance to motion is 10 pounds per ton, and no brakes are applied, how far will it have traveled when the velocity has reduced to 10 miles per hour?

**438**. A freight train of 100 tons weight is moving at the rate of 30 miles per hour when the steam is shut off and the brakes applied to the locomotive. Supposing the only friction is that at the locomotive, the weight of which is 20 tons, what is the coefficient of friction if the train stops after moving 2 miles?

**439**. A steamer approaching a dock with engines reversed so as to produce a uniform retardation is observed to make 500 feet during the first 30 seconds of the retarded motion and 200 feet during the next

30 seconds. In how many more seconds will the headway be completely stopped?

**440**. A ball is thrown along a rough floor, coefficient of friction $\frac{1}{2}$. What will be its velocity after 3 seconds, if the original velocity is 50 feet per second?

**441**. A body is projected up an inclined plane, of angle 30°, with a velocity of 80 feet per second. Find —

(1) How long it will be before coming to rest?
(2) How far it will go up the plane?
(3) How long it will be in returning to its starting-point?
(4) With what velocity will it return to its starting-point?

**442**. A cannon when fired recoils with a velocity of 10 feet per second and runs up a platform having an incline of 1 in 4. Find the horizontal distance it goes before coming to rest.

**443**. A locomotive weighing 100 tons is observed to be increasing its speed at the rate of 100 feet a minute. What is the effective force acting on it?

**444**. What force must be exerted by an engine to move a train of mass 100 tons with 10 units of acceleration, if frictional resistances are 5 pounds per ton?

**445**. A train of 100 tons, excluding the engine, runs up a 1% grade with an acceleration of 1 foot per second. If the friction is 10 pounds per tons, find the pull on the drawbar between engine and train.

**446**. A force of 5 pounds is made to move a body weighing 50 pounds. What is the acceleration produced?

$$\text{Power producing motion} : \text{whole mass moved} = f : g$$
$$5 : 50 = f : 32$$
$$50f = 5 \times 32$$
$$f = 3\tfrac{1}{5} \text{ feet per second.}$$

**447**. A body moving along a straight line is known to be acted on by a constant force; at a certain instant it is moving at the rate of 12 feet a second, and in the next 10 seconds it describes a distance of 470 feet. What velocity does it gain in each second of its motion?

$$s = Vt + \tfrac{1}{2} ft^2$$
$$470 = 12 \times 10 + \tfrac{1}{2} f\, 10^2$$
$$350 = \tfrac{1}{2} f \times 100$$
$$f = \tfrac{350}{50}$$
$$= 7 \text{ feet per second.}$$

**448**. A body whose mass is 108 pounds is placed on a smooth horizontal plane, and under the action of a certain force describes from rest a distance $11\frac{1}{9}$ feet in 5 seconds. What is the force acting?

**449**. Masses of 5 pounds and 11 are connected by a weightless thread; the 11-pound weight is placed on a smooth horizontal table, while the other hangs over the edge. If both are then allowed to move under the action of gravity, what is the tension of the thread?

**450**. A 10-pound weight hangs over the edge of a table and pulls a 45-pound box along; the coefficient of friction between the table and the box is 0.05. Find the acceleration and the tension in the string.

**451**. The table of a box-machine weighs 50 pounds and is pulled back to its starting position, a distance of 6 feet, by a falling weight of 20 pounds. What time, neglecting friction, will thus be used in return motion?

**452**. A weight of 10 pounds rests 6 feet from the edge of a smooth horizontal table that is 3 feet high. A string 7 feet long passes over a smooth pulley at the edge of the table and connects with a 10-pound weight. If this second weight is allowed to fall, in what time will it cause the first weight to reach the edge of the table?

**453**. A balloon is moving upward with a speed which is increasing at the rate of 4 feet per second in each second. Find how much the weight of a body of 10 pounds as tested by a spring balance on it, would differ from its weight under ordinary circumstances.

**454**. A man who is just strong enough to lift 150 pounds can lift a barrel of flour of 200 pounds weight when going down on an elevator. How fast is the velocity of elevator increasing per second?

**455.** An elevator, starting from rest, has a downward acceleration of $\frac{1}{2}\,g$ for 1 second, then moves uniformly for 2 seconds, then has an upward acceleration of $\frac{1}{3}\,g$ until it comes to rest. (*a*) How far does it descend? (*b*) A person whose weight is 140 pounds experiences what pressure from the elevator during each of the three periods of its motion?

**456.** If a train ascends by its own momentum a grade of 1 in 40 for a distance of 1 mile, the resistance from friction, etc., being 10 pounds per ton, find its initial velocity.

$$\begin{aligned} \text{Work} = 2\,000 \times 132 &= 264\,000 \text{ foot-pounds} \\ 10 \times 5\,280 &= \frac{52\,800}{316\,800} \text{ foot-pounds} \\ \tfrac{1}{2}\,mv^2 &= 316\,800 \\ \tfrac{1}{2} \times \tfrac{2000}{32} \times v^2 &= 316\,800 \\ v^2 &= 10\,100 \\ v &= 100 +, \text{ feet per second,} \end{aligned}$$

or the 316 800 foot-pounds could be accomplished by the 1 ton starting with a velocity acquired by falling from a height, *h*.

$$\begin{aligned} 2\,000 \times h &= 316\,800 \\ h &= 158.4. \end{aligned}$$

If a body fell from this height its velocity would be

$$\begin{aligned} v &= 8\sqrt{158.4} \\ &= 100 +, \text{ feet per second as above.} \end{aligned}$$

**457.** A particle whose mass is 10 pounds moves along a horizontal plane against a friction of one-fifth of its weight for a distance of 20 feet before coming torest. What must have been its velocity at the beginning of the 20 feet?

**458.** Some railroad cars start from rest down an incline a mile long with a gradient of 1 in 100. Find how many yards they will travel on the level, after leaving the incline, before they come to rest, the frictional resistance to their motion being 10 pounds per ton. Find also, in miles per hour, the greatest velocity that they will acquire.

**459.** Two weights of 120 and 100 pounds are suspended by a fine thread passing over a fixed pulley without friction. What space will either of them pass over in the third second of their motion from rest?

**460.** A cord passing over a smooth pulley carries 10 pounds at one end and 54 pounds at the other. What will be the velocity of the weight 5 seconds from rest, and what will be the tension in the cord?

**461.** An engine draws a three-ton cage up a coal-pit shaft at a speed uniformly increasing at the rate of 5 feet per second. What is the tension in the rope?

$$
\begin{aligned}
P &= \frac{W}{g} f \\
&= \tfrac{6000}{32} \times 5 \\
&= 937.5 \text{ pounds} \\
\text{Pull} &= 937.5 + 6\,000 \\
&= 6\,937.5 \text{ pounds.}
\end{aligned}
$$

**462**. Two strings pass over a smooth pulley; on one side both strings are attached to a weight of 5 pounds, on the other side one string is attached to a weight of 3 pounds, the other to one of 4 pounds. Find the tensions during motion.

**463**. A body is projected with a velocity of 50 feet per second in a direction inclined 40° upward from the horizontal. Determine the magnitude and direction of the velocity at the end of 2 seconds ($g$ being taken equal to 32.15).

Time to reach highest point,

$$t = \frac{u \sin a}{g}$$

$$= \frac{50 \times .643}{32.15}$$

$$= 1 \text{ second.}$$

Therefore in another second the body will fall and be in position similar to its initial.

**464**. A body is projected with a velocity of 20 feet per second down a plane whose inclination is 25°; the coefficient of friction being 0.4. Determine the space traversed in 2 seconds.

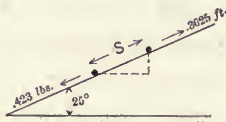


Fig. 52.

Power producing motion : total mass moved $= f : g$

$$.423 - .3625 : 1 = f : 32$$

$$f = 1.93 \text{ feet per second}$$

$$s = Vt + \tfrac{1}{2} ft^2$$

$$= 20 \times 2 + \tfrac{1}{2} \times 1.93 \times 2^2$$

$$= 43.9 \text{ feet.}$$

**465.** A body slides down a rough inclined plane 100 feet long, the sine of whose angle of inclination is 0.6 and coefficient of friction is $\frac{1}{2}$. Find the velocity at the bottom. If projected up the plane with a velocity which just carries it to the top, find that velocity and the height it would reach if thrown vertically upwards with the same velocity.

**466.** A bullet is fired with a velocity of 1 000 feet per second. What must be the angle of inclination, in order that it may strike a point in the same horizontal plane, at a distance of 15 625 feet?

**467.** From the top of a tower a stone is thrown up at an angle of 30°, with a velocity of 288 feet per second; the height of the tower is 160 feet. Find the time required for the stone to reach the ground, and how far it will have gone from the foot of the tower.

**468.** From a train moving at 60 miles per hour a stone is dropped; the stone starts at a height of 8 feet above the ground. What is the horizontal distance through which the stone has gone while falling?

**469.** From a quarry blast a stone has a velocity of 200 feet per second, in a direction inclined at an angle of 60° to the horizontal plane. To what height will it rise, and how far away will it strike the ground?

**470.** A bullet is fired with a velocity of which the horizontal and vertical components are 80 and 120 feet per second respectively. Find the range and greatest height.

$$\text{Time of flight} = \frac{2\,\mu \sin a}{g}$$
$$= \frac{2 \times 120}{32}$$
$$= \tfrac{30}{4} = 7\tfrac{1}{2} \text{ seconds.}$$
$$\text{Range} = 80 \times 7\tfrac{1}{2}$$
$$= 600 \text{ feet}$$
$$\text{Greatest height} = \frac{120^2}{64}$$
$$= 225 \text{ feet.}$$

**471.** A ball is discharged with the initial velocity of 1 100 feet. How many miles is the greatest possible range?

**472.** A cannon ball is fired horizontally from a hill that is on the coast and 900 feet high: neglecting the resistance of the atmosphere, find the time which elapses before it strikes the sea.

**473.** A projectile is fired horizontally from the top of a hill 300 feet high to a ship at sea. Its initial velocity is 2 000 feet per second and its weight 500 pounds. What will be its range, and what will be the energy of the blow which it strikes? Neglect resistance.

**474.** The top of a fortification wall is 50 feet above the level of a city. From a man-of-war in the bay 250 feet below the top of the wall and distant horizontally 3 000 feet, a projectile is fired with velocity of 1 000 feet per second. The projectile just clears the wall. Where will it land inside the city?

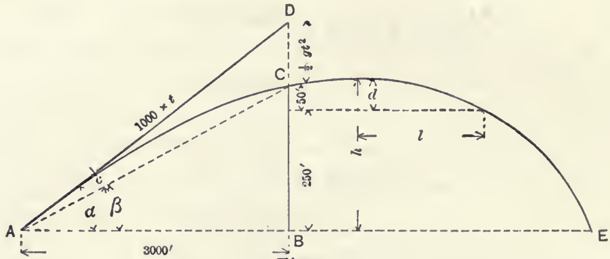



Fig. 53.

Find the distance AC and the angle B. Then consider flight from A to C.

$$\text{Horizontal range} = \frac{u^2 \sin 2c}{g}$$

$$= \sqrt{3\,000^2 + 300^2} = \frac{1\,000^2 \times \sin 2c}{32}$$

from which C may be found and

$$a = \beta + c$$

Compute $h$, greatest height; subtract 250, to find $d$.

$$d = \tfrac{1}{2}gt^2.$$

Find $t$; then find the horizontal, $l$. This added to one-half the range will give the position where projectile will land.

(This problem may well be done by methods of calculus.)

**475**. A rifle projects its shot horizontally with a velocity of 1 000 feet per second; the shot strikes the ground at a distance of 1 000 yards. What is the height of the rifle above the ground?

**476.** What is the pressure exerted horizontally on the rails by an engine of 20 tons weight going round a curve of 600 yards radius at 30 miles an hour?

$$\text{Velocity} = 44 \text{ feet per second.}$$

$$\therefore \text{ pressure} = \frac{mv^2}{r}$$

$$= \frac{20 \times 2\,000 \times 44^2}{1\,800 \times 32} \text{ pounds weight}$$

$$= 1\,344 \text{ pounds.}$$

**477.** A train of 60 tons weight is rounding a curve of radius one mile, with a velocity of 20 miles an hour. What is the horizontal pressure on the rails?

**478.** An engine of mass 24 tons is moving round a curve of 400 yards radius, and the horizontal pressure exerted on the rails is 4.84 tons weight. What is the velocity of the engine?

**479.** The mass of the bob of a conical pendulum is 2 pounds, the length of the string is 3 feet, the angle of inclination to vertical is 45°. What is the tension?

**480.** The mass of the bob is 20 pounds, the length of the string is 2 feet, the tension of the string is $500\pi^2$ pounds weight. How many revolutions per second is the pendulum making?

**481.** If a conical pendulum be 10 feet long, the half angle of the cone 30°, and the mass of the bob 12 pounds, find the tension of the thread and the time of one revolution.

**482.** A weight of 10 pounds is fastened by a string which passes through a hole in a smooth horizontal table to a weight of 1 pound, which hangs vertically; the first weight is revolving on the table about the hole as a center. How many revolutions are there per minute if the horizontal portion of the string is 15 inches long?

**483.** A ball is hung by a string in a passenger car which is rounding a curve of 1 000 feet radius, with a velocity of 30 miles an hour. Find the inclination of the string to the vertical.

**484.** A pendulum of length 156.556 inches oscillates in two seconds at London. What is the value of $g$?

**485.** Given $l$ the length of a simple pendulum, $\pi\sqrt{\frac{l}{g}}$ the time of an oscillation: show how to find approximately the height of a mountain when a seconds pendulum, by being taken from sea level to its summit, loses $n$ beats in 24 hours. If $n=15$, what is the height of the mountain, the radius of the earth being 4 000 miles?

**486.** Water is flowing in a service pipe at the rate of 24 feet per second; the pipe is 50 feet long. If the water be uniformly shut off by a stop valve in one-tenth of a second, show that the water pressure in the pipe near the valve is increased by 162.5 pounds per square inch.

**487.** If in the above pipe the pressure of the "water hammer" had been 400 pounds per square inch, with what velocity would water have been flowing at the beginning?

**488.** A cricket ball of mass 6 ounces is struck so that its velocity is changed from 10 feet per second in one direction to 20 feet per second in the opposite. What was the impulse?

**489.** A hammer of 10 tons weight falling from a height of 4 feet drives a wooden pile and comes to rest in $\frac{1}{32}$ second. How far does it drive the pile? And, assuming the force is uniform, find it and the impulse.

**490.** An 8-inch projectile weight 250 pounds, strikes a sand butt with velocity of 2 000 feet per second and is stopped in 25 feet. If the resistance is uniform, what is its value in pounds, and how long did it take to stop the projectile?

**491.** A shot of mass 20 pounds is fired from a gun of mass 2 000 pounds, and length 10 feet; the gun rests at the foot of an inclined plane, rising 1 in 15. If the muzzle velocity of the shot be 1 200 feet per second, how far up the plane will the gun recoil?

**492.** An 8-hundred weight shot leaves a 40-ton gun with velocity of 2 000 feet per second: the length of the gun is 20 feet. What is the average force of the powder?

**493.** A man weighing 160 pounds jumps with a velocity of $16\frac{1}{4}$ feet per second into a boat weighing

100 pounds. With what velocity will the boat move away?

**494.** An 800-pound shot is fired from an 81-ton gun, with a muzzle velocity of 1 400 per second: a steady resistance of 9 tons begins to act immediately after the explosion. How far will the gun move?

**495.** A one-ounce bullet fired out of a 20-pound rifle pressed against a mass of 180 pounds, kicks the latter back with an initial velocity of 6 inches per second. Find the initial velocity of the bullet.

$$\text{Momentum forward} = \text{momentum backward}$$
$$MV = mv$$
$$\tfrac{1}{16} \times V = (180 + 20) \times \tfrac{1}{2}$$
$$V = 1\,600 \text{ feet per second.}$$

**496.** A ball of mass 4 pounds and velocity 4 feet per second meets directly a ball of mass 5 pounds with opposite velocity of 2 feet per second; $e = \frac{1}{2}$. Find the velocities after impact.

**497.** A freight train, weighing 200 tons, and traveling 20 miles per hour, runs into a passenger train of 50 tons standing on the same track. Find the velocity at which the broken cars of the passenger train will be forced along the track, supposing $e = \frac{1}{5}$.

**498.** A shell bursts into two fragments, whose weights are 12 and 20 pounds. The former travels onward with a velocity of 700 feet per second, and the latter with a velocity of 380 feet per second.

What was the momentum of the shell when the explosion occurred?

**499**. There are two bodies whose masses are 15 pounds and 20 pounds respectively; the former, moving at the rate of 12 feet a second overtakes and impinges directly on the latter moving at the rate of 6 feet a second. Find their common velocity at the end of compression and their joint energies just before impact and at the end of compression.

**500**. A body A weighing 10 pounds, and moving at the rate of 15 feet a second, strikes another body B weighing 20 pounds, and moving at the rate of 10 feet a second, in the direction at right angles to that of A's motion. The bodies are to be treated as points, and the impact is supposed to take place in the direction of A's motion. Find the velocities and directions of the motions of the bodies after impact, the restitution being perfect (coefficient of elasticity $= 1$).

# A FEW IMPORTANT UNIT VALUES TO BE USED IN SOLVING THESE PROBLEMS

| | |
|---|---|
| 1 hundred weight | = 100 pounds |
| 1 ton | = 2 000 pounds |
| 1 fathom | = 6 feet |
| 1 knot | = 6 080 feet |
| 1 cubic foot of water | = $62\frac{1}{2}$ pounds |
| | = $7\frac{1}{2}$ gallons |
| 1 gallon of water | = $8\frac{1}{3}$ pounds |
| 1 British thermal unit | = 778 foot-pounds of energy |
| *g*, accelleration of gravity | = 32 feet per second, unless otherwise specified |
| 1 horse-power | = 746 watts |
| 1 kilowatt | = 1.34 horse-power |

Watts = volts × ampères

## TRIGONOMETRIC FUNCTIONS

| | 0° | 30° | 45° | 60° | 90° | 120° |
|---|---|---|---|---|---|---|
| Sine | 0 | $\frac{1}{2}$ .500 | $\frac{1}{\sqrt{2}}$ .707 | $\frac{\sqrt{3}}{2}$ .866 | 1 | $\frac{\sqrt{3}}{2}$ .866 |
| Cosine | 1 | $\frac{\sqrt{3}}{2}$ .866 | $\frac{1}{\sqrt{2}}$ .707 | $\frac{1}{2}$ .500 | 0 | $-\frac{1}{2}$ −.500 |
| Tangent | 0 | $\frac{1}{\sqrt{3}}$ .577 | 1 | $\sqrt{3}$ 1.732 | Infinite | $-\sqrt{3}$ −1.732 |

$\text{Sin} = \frac{\text{Perp}}{\text{Hypot}}$ $\quad$ $\text{Cos} = \frac{\text{Base}}{\text{Hypot}}$ $\quad$ $\text{Tan} = \frac{\text{Perp}}{\text{Base}}$

$\text{Tan} = \frac{\text{Sin}}{\text{Cos}}$ $\quad$ $\text{Cot} = \frac{1}{\text{Tan}}$ $\quad$ $\text{Sec} = \frac{1}{\text{Cos}}$

$\text{Cosec} = \frac{1}{\text{Sin}}$ $\quad$ $\text{Vers} = 1 - \cos$ $\quad$ $a : b = \text{Sin}\, A : \text{Sin}\, B$

$\text{Sin}\,(A + B) = \text{Sin}\, A\ \text{Cos}\, B + \text{Cos}\, A\ \text{Sin}\, B$ $\quad$ $c = \sqrt{a^2 + b^2 - 2ab\ \text{Cos}\, C}$

# ANSWERS TO PROBLEMS

**3.** 3 360 foot-pounds.
**7.** 125 pounds.
**10.** 2 704 000 foot-pounds.
**11.** 97 500 foot-pounds.
**12.** 198 000 000 foot-pounds.
**17.** 104 8 foot-pounds.
**22.** About $13\frac{3}{4}$ tons weight.
**27.** 20 000 foot-pounds.
**28.** $266\frac{2}{3}$ pounds.
**30.** 110 foot-pounds.
**32.** $1\frac{17}{49}$ inches.
**33.** $603\frac{3}{7} : 1$.
**34.** .54 pounds.
**35.** $3\,017\frac{1}{7}$.
**36.** $28\frac{1}{2}$ pounds.
**37.** 120 pounds.
**38.** 112 pounds.
**39.** $71\frac{2}{3}$ pounds; $301\frac{1}{3}$.
**40.** $522\frac{1}{2}$ pounds.
**42.** 6 000 foot-pounds; 3 : 2.
**43.** 161 pounds; 11 800 foot-pounds.
**44.** 14.4 man-power.
**45.** 324 500 foot-pounds; 41 per cent; 226 units.
**47.** 4 400 foot-pounds; $\frac{2}{45}$ horse-power.
**48.** $1\frac{1}{3}$ horse-power.
**50.** $36\frac{4}{11}$ horse-power.
**51.** 21 horse-power.
**57.** $40\frac{5}{28}$ miles per hour.
**60.** 68 horse power; 303 ampères.
**63.** About 80 pounds per inch of width.
**66.** 4 inches.
**69.** $37\frac{1}{2}$ miles per hour; $2\frac{1}{192}$ miles; 6 minutes, 25 seconds.
**70.** 12 miles an hour.
**72.** 179.2 horse-power; 224 horse-power.
**73.** 5 728 feet; 2 545.8 feet; 31.8 miles per hour; 27.3 miles per hour.
**74.** 1 000 horse-power.
**75.** 87 horse-power.
**80.** 208 pounds.
**81.** 140 horse power.
**82.** 107 horse-power.
**87.** 5.7.
**89.** 2 540 looms.
**91.** 0.061 nearly.
**98.** 650 horse-power.
**100.** 5.6 horse-power.
**111.** (1) .9 cubic feet; (2) 1.69 tons; (3) 262.4 foot-pounds; (4) 39.5 horse-power.
**117.** 62.5 cubic feet.
**125.** 0.14 horse-power; 97 cubic inches.
**127.** $37\frac{1}{2}$ pounds.
**129.** $\frac{88\,a\,R}{P}$ strokes per minute.
**130.** 11.2 pounds per ton.
**131.** 367 pounds; 19.5 per cent.
**135.** 576 horse-power.
**140.** 871 horse-power.
**141.** 15 000 foot-pounds.
**142.** 3 inches.
**143.** 0.17 horse power.
**145.** 13 400 foot-pounds; 3 045.6 momentum; 1 522.8 pounds; 88 feet.
**150.** $\frac{1}{125}$ second.
**151.** 15 528 feet.

**153.** 10 feet per second; shot does more work as 162 : 1.
**154.** 6 feet 3 inches.
**155.** 265 pounds.
**158.** 48 foot-pounds; $8\sqrt{6}$ feet per second.
**160.** 155 foot-pounds; energy at lowest point 45 foot-pounds.
**163.** $171\frac{1}{2}$ revolutions; 236 revolutions.
**164.** 55 700 foot-pounds; 44.2 turns.
**169.** 20 foot-pounds.
**170.** 20 feet.
**171.** (both) 500 foot-pounds.
(A) $\frac{1}{6}$ (500) foot-pounds.
(D) $\frac{5}{6}$ (500) foot-pounds.
**172.** $\cos\theta = \frac{P}{2W}$.
**173.** 90 degrees.
**174.** 100 pounds.
**176.** 10 pounds.
**177.** A force of 92.1 pounds at angle 32°40′ with force of 44 pounds.
**178.** $25\sqrt{3}$; 25.
**181.** 29 pounds.
**183.** 150 pounds, 90.
**184.** $133\frac{1}{3}$ pounds, $166\frac{2}{3}$.
**185.** $10\sqrt{2}$ pounds; 10.
**186.** 6 tons on the tie; 6.32 tons on the rafters.
**187.** $6\frac{2}{5}$ tons.
**190.** 416 pounds, 208.
**191.** 50 pounds.
**193.** 580 pounds.
**194.** 6 030 pounds.
**195.** 36 pounds, 164.
**196.** 117.1 pounds, 82.8.
**198.** $\sqrt{5}$ pounds, $2\sqrt{5}$.
**199.** $2\frac{4}{5}$ pounds, $9\frac{3}{5}$.
**200.** $\cos^3\theta = \frac{b}{a}$; $b$ being distance from C to AB, $2a$ the length of rod.
**203.** Perpendicular to plane.
**204.** 500 pounds.
**205.** 1 020 pounds, 1 000.
**206.** 475 pounds.
**207.** 8.657 pounds.
**208.** $8\frac{3}{5}$ pounds, $11\frac{1}{5}$.
**212.** 100 pounds, $100\sqrt{3}$.
**213.** 86.6 pounds, 100.
**215.** 20 pounds, 15.
**216.** 36.4 pounds, 27.3.
**218.** 77 pounds.
**220.** D being the area of the triangle, $P = \frac{Wb}{4cD}(a^2 + c^2 - b^2)$; $Q = \frac{Wa}{4cD}(b^2 + c^2 - a^2)$
**223.** 1.16 tons, 0.55, 0.53.
**225.** Legs 17.2 tons; back stay 18.8.
**226.** 7.8 tons, 6.5.
**229.** $CB = W\frac{\cos A}{\sin C}$; $CA = W\frac{\cos B}{\sin C}$.
**230.** At C force is horizontal and $= \frac{W\sqrt{3}}{2}$; at B $\tan^{-1}\frac{\sqrt{3}}{2}$ to vertical and $= \frac{W\sqrt{7}}{2}$.
**231.** 14.24 pounds.
**232.** Force 15 parallel to CA.
**233.** 13 units at $\tan^{-1}\frac{5}{12}$ with AB.
**234.** $2\sqrt{2}\,P$ parallel to CA at distance from AC equal to $\frac{3\sqrt{2}}{2}$ AB.
**238.** $1 : \sqrt{3}$.
**242.** Tension $= \frac{Wl}{2\sqrt{l^2 - c^2\sin^2\theta}}$.
**246.** Reaction 115.5 pounds; tension 57.7.
**251.** 60 000 pounds close to the tower; 47 000 in the middle.

**256.** 4 inches from end.
**257.** 9.47 inches from end.
**258.** 10 pounds.
**259.** 9 inches from middle; 18 pounds.
**260.** 3 feet.
**262.** $148\frac{2}{3}$ pounds.
**263.** 5 inches from middle.
**264.** $3\frac{3}{5}$ feet from B.
**265.** $8\frac{1}{3}$ pounds.
**268.** $106\frac{2}{3}$ pounds.
**269.** 100 pounds, 50.
**270.** 3 pounds; $\frac{1}{2}$ inch from middle.
**271.** Its ends 7 inches and 3 from the pegs.
**272.** $4\frac{1}{3}$ tons, $3\frac{2}{3}$.
**274.** $1\frac{2}{3}$ pounds.
**276.** $6\frac{1}{2}$ pounds.
**277.** 89.143 pounds; 45.143.
**278.** 120 pounds.
**279.** 6 inches from end.
**280.** 9.5 pounds.
**283.** 4 feet from one man.
**284.** 11 inches, 6.
**287.** $15\sqrt{5}$ pounds.
**288.** 5 feet from end.
**289.** 1 pound at distance 5 feet.
**290.** 540.
**291.** 280 pounds, 392; 1 foot.
**292.** On line bisecting vertical angle, $\frac{2}{3}$ from vertex.
**293.** $\frac{2\sqrt{3}}{9}a$, $\frac{\sqrt{3}}{3}a$, $4\sqrt{3}\,a$ from the sides, if each side $=2a$.
**294.** $\frac{6\sqrt{3}}{11}a$, $\frac{3\sqrt{3}}{11}a$, $\frac{2\sqrt{3}}{11}a$ from sides; outside the triangle at distance $\frac{6\sqrt{3}}{5}a$, $\frac{3\sqrt{3}}{5}a$, $\frac{2\sqrt{3}}{5}a$.
**296.** Any point of line parallel to CD passing through X which is in BC produced so that $CX=2\,BC$.
**298.** At X in BD produced so that $2\,DX=BD$.
**299.** 5 units acting parallel to BD, cutting BC produced in X so that $4\,CX=BC$.
**305.** At point 15 and 16 inches from adjacent sides.
**307.** 200 pounds, 220.
**308.** A parallel force, distant 5 units.
**309.** $2\frac{1}{4}$ feet from rim.
**313.** If D be the middle point of BC, R is represented in magnitude by 2 AD, and acts through X parallel to DA. X being in DC or DB so that $DX=\frac{BC}{8}$.
**315.** He loses 1 pound.
**317.** 1 inch.
**318.** 85.9 pounds.
**321.** Each equal 15 pounds.
**322.** $32\frac{1}{2}$ pounds.
**323.** 2 700 pounds.
**325.** 11.25 pounds, 31.25.
**326.** Length of stick from nail to wall $=\sqrt[3]{3}$ feet. Pressure $=8\sqrt[3]{3}$ and $8\sqrt{\sqrt[3]{9}-1}$ ounces.
**327.** 18 900 pounds.
**328.** 35.3 feet.
**331.** 15 000 pounds.
**332.** $\frac{3}{8}$ of length from end where pressure is 4 pounds.
**335.** $3\frac{3}{20}$ feet below.
**336.** 7 ounces; C. of G. $9\frac{1}{7}$ inches from A.
**337.** 21 pounds at A, 9 pounds at B; additional pressure 10 pounds.
**339.** 1 inch from AC, $1\frac{7}{9}$ inches from AB.
**340.** $\frac{1}{3}$ W; $\frac{2}{3}$ W.
**348.** $2\cos\theta=3\cos\frac{\pi-\theta}{3}$.

**349.** $\frac{9\sqrt{3}}{32}$.

**350.** $h = r\sqrt{3}$.

**351.** $\frac{3}{10}$ pound.

**353.** 45 degrees to the vertical.

**354.** It makes angle $\lambda$ with normal.

**355.** 373 pounds.

**357.** $\frac{1}{\sqrt{3}}$.

**358.** 0.35; 0.26.

**359.** 200 pounds.

**362.** $\frac{2}{11}$.

**365.** 50 pounds, $50\sqrt{3}$.

**366.** $\mu = \frac{1}{4}$; Inclination $= \tan^{-1}\frac{3}{4}$.

**371.** 423.3 foot-pounds.

**372.** 60 degrees.

**375.** 7.56 tons, 396.

**376.** $\frac{\mu W r}{(1+\mu^2)\sin\alpha}$;
$r$ = radius.
W = weight of wheel.

**379.** 47 feet.

**380.** 50 feet.

**381.** $\frac{1}{\sqrt{3}}$.

**382.** 1 920 pounds.

**384.** 65 pounds.

**385.** 2 800 pounds.

**386.** 164 pounds.

**387.** $3\frac{1}{2}$.

**388.** 246 pounds, 27.2.

**390.** 504.12 pounds.

**391.** $\frac{W}{P} = 0.952$ or 1.05.

**393.** 8 inches.

**394.** 970 pounds.

**395.** $8\frac{4}{7}$.

**396.** $\frac{222}{360}$.

**398.** 1.92 horse-power.

**400.** 155 thermal units.

**401.** 4.13 horse-power.

**403.** 0.51 horse-power.

**404.** 0.44.

**406.** 35 feet.

**407.** $2\frac{1}{2}$ miles; from A 3 miles, B $\frac{1}{2}$ mile.

**408.** 30 miles per hour.

**409.** 2 640 feet.

**410.** 99 feet.

**411.** 150 feet, 200.

**412.** 65.5 miles per hour; 0.27 miles.

**414.** 6 seconds; 112 feet per second.

**416.** $\sqrt{ag}$ feet per second.

**417.** 784 feet.

**418.** 231 feet.

**419.** 306.05 feet; 68.01 feet per second.

**420.** 4 080 feet.

**421.** $\frac{3h}{4}$ from ground; in time $\sqrt{\frac{h}{2g}}$.

**423.** 352 feet.

**424.** 6 and 6.

**425.** $\frac{u\sqrt{3}}{2}$; $\frac{u}{2}$.

**426.** 17.6 feet.

**431.** $\frac{80}{\sqrt{3}}$ feet per second.

**432.** 8.94 miles per hour.

**433.** N. W. $6\sqrt{2}$ miles per hour.

**434.** 10 miles per hour.

**435.** $13\frac{7}{11}$ miles per hour, $27\frac{3}{11}$.

**436.** $24\pi$.

**437.** 2.14 miles.

**438.** $\frac{11}{24}$.

**439.** 5 seconds.

**440.** 2 feet per second.

**441.** (1) 5 seconds; (2) 200 feet; (3) 5 seconds; (4) 80 feet per second.

**442.** 6 feet 3 inches.

**444.** $31\frac{53}{112}$ tons weight.

**445.** $4\frac{5}{8}$ tons.

**449.** $3\frac{7}{16}$ pounds.

**452.** 1 second.

**454.** 8 feet per second.

**455.** (*a*) $\frac{13}{8}g$ feet; (*b*) 70 pounds, 140, $186\frac{2}{3}$.

**457.** 16 feet per second.

**458.** 6 547.2 feet; 29.5 miles per hour.
**459.** 7.25 feet.
**462.** $2\frac{1}{2}$ pounds, $3\frac{1}{3}$.
**463.** 50 feet per second; 40 degrees downward from horizontal.
**465.** $16\sqrt{5}$ feet per second; 100 feet.
**466.** 15 degrees or 75 degrees.
**467.** 10 seconds; 1 440 $\sqrt{3}$ feet.
**468.** $42\sqrt{2}$ feet.
**469.** 470 feet.
**471.** $7\frac{31}{192}$ miles.
**475.** 144 feet
**477.** 0.3 ton weight.
**478.** 60 miles per hour.
**479.** 2.83 pounds.
**480.** 10.
**482.** $15\frac{3}{11}$ revolutions.
**483.** $\tan^{-1}\frac{121}{2000}$.
**484.** 32.19.
**485.** 3 $666\frac{2}{3}$ feet.
**487.** 60 feet per second.
**488.** $11\frac{1}{4}$ units.
**489.** 3 inches; 160 tons plus weight of mass; 10 000.
**491.** $33\frac{3}{4}$ feet.
**492.** 1 250 tons.
**493.** 10 feet per second.
**494.** 6.9 feet.
**496.** $-1$ feet per second, 2.
**497.** 13.1 miles per hour.
**498.** 496.8 units.
**499.** $8\frac{4}{7}$ feet per second.
**500.** A returns at 5 feet per second. B moves at 45 degrees with its course and velocity of $10\sqrt{2}$ feet per second.

# ALPHABETICAL CLASSIFICATION OF PROBLEMS.