MATHEMATICAL EXAMINATION PAPERS

FOR ADMISSION INTO

THE ROYAL MILITARY COLLEGE, SANDHURST

SANDHURST

MATHEMATICAL PAPERS

FOR ADMISSION INTO

The Royal Military College

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Royal Military College, Sandhurst.

July 1881.

I.—ALGEBRA AND MENSURATION.

1. Simplify the following expressions:—

(a) $\left(\frac{a}{b}+\frac{b}{a}\right)^4-2\left(\frac{a^2}{b^2}-\frac{b^2}{a^2}\right)^2+\left(\frac{a}{b}-\frac{b}{a}\right)^4$;

(b) $\dfrac{x^6-y^6}{x^3+2x^2y+2xy^2+y^3}$;

(c) $\sqrt{a^2+\frac{b^2}{4}+\frac{c^2}{9}+ab+\frac{2ac}{3}+\frac{bc}{3}}$.

2. Prove that $(a^m)^n = a^{mn}$, whether (m) and (n) be positive or negative, and reduce to their simplest forms

$$\sqrt[6]{(a^3b\sqrt[5]{a^3bc})^5} \quad\text{and}\quad \frac{\dfrac{\sqrt{1+x}}{\sqrt{1-x}}-\dfrac{\sqrt{1-x}}{\sqrt{1+x}}}{\dfrac{\sqrt{1+x}}{\sqrt{1-x}}+\dfrac{\sqrt{1-x}}{\sqrt{1+x}}}.$$

3. How may the least common multiple of three algebraical expressions be found? Find that of

$$2x^2 - x - 1, \quad 2x^2 + 3x + 1, \quad x^2 - 1.$$

4. Prove the rule for extracting the cube root of an algebraical quantity, and extract the cube root of

$$8a^6 - 12a^5b + 42a^4b^2 - 37a^3b^3 + 63a^2b^4 - 27ab^5 + 27b^6.$$

5. The sides of 3 cubes have equal differences, and their sum is 15 inches: the solid content of the 3 is 495 cubic inches: find their dimensions and volume.

6. If α and β be the roots of the equation $ax^2 + bx + c = 0$, prove that $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$. What forms will the equation assume, if the roots be equal, and (1) of the same sign, (2) of opposite signs?

7. Solve the equations

(1) $(a + x)(b + x) - a(b + c) = \frac{a^2c}{b} + x^2$;

(2) $x^2 + x = 6y, \quad x^3 + 1 = 9y$;

(3) $5y - 9z = 18, \quad 4x - 3z = 14, \quad 7x - 6y = 14.$

8. Prove that the value of the fraction $\frac{a + b + c}{p + q + r}$ lies between those of the greatest and least of the fractions $\frac{a}{p}, \frac{b}{q}, \frac{c}{r}$: and show in what cases $\sqrt[m]{-a} \times \sqrt{-b}$ has a real and positive, a real and negative, or no real value.

9. Two men leave two places, A and B, distant (d) miles from each other, and travel (a) and (b) miles a day respectively in the same straight line AB: what is their distance apart at the end of (t) days, and after what time will they come together?

Explain the results (1) when $a = b$, (2) when $a = b$ and $d = 0$.

10. If three equal circles, whose radii are each 7 inches, touch each other, find the area enclosed between them to three places of decimals ($\pi = \frac{22}{7}$).

11. Two thin vessels, without lids, each contain a cubic foot: the one is a rectangular parallelopiped on a square base whose height is half its length: the other a right circular cylinder whose height is equal to the radius of the base: compare the amounts of material which it would require to make them, the thickness being the same for both ($\pi = 3\cdot1416$).

12. A solid consisting of a right cone standing on a hemisphere is placed in a right cylinder full of water, and touches the bottom. Find the weight of water displaced; having given that the radius of the cylinder is 3 feet, and its height 4, the radius of the hemisphere 2, and the height of the cone 4; and that a cubic foot of water weighs 1,000 oz.

13. Prove that $\log_b a \times \log_a b = 1$, and find the value of $\sqrt[6]{9\sqrt{3\sqrt{2}}}$.

Given $\log 2 = \cdot30103$ $\qquad$ $\log 3 = \cdot47712$
$\log 1626 = 3\cdot21112$ $\qquad$ $\log 1627 = 3\cdot21139$.

14. The number of births in a town is 25 in every thousand of the population annually, and the deaths 20 in every thousand; in how many years will the population double itself?

Given $\log 67 = 1\cdot82607$.

15. Find the number of digits in 2^{64}, and if

$$a^1 \,.\, a^2 \,.\, a^3 \,\ldots\, a^n = p,$$

find (in terms of a and p) an expression for the number of factors a^1, a^2, a^3, &c.

II.—EUCLID AND TRIGONOMETRY.

1. Show that if a straight line be divided into any two parts, the rectangle contained by the whole and one of the parts, is equal to the rectangle contained by the two parts, together with the square on the aforesaid part.

To what algebraical formula does this proposition correspond?

2. Describe a square that shall be equal to a given rectilineal figure.

A given square is divided into four equal squares by two straight lines drawn through its centre. If one of these squares be supposed to be removed, show how to construct a square equal to the remaining gnomon.

3. Prove that the angle at the centre of a circle is double of the angle at the circumference on the same base, that is, on the same part of the circumference.

Deduce from this proposition the fact that the angle in a semicircle is a right angle.

4. From a given circle cut off a segment containing an angle equal to a given rectilineal angle.

5. Describe a circle about a given triangle.

6. Define the terms "alternando" and "convertendo"; "*similar* figures" and "*reciprocal* figures."

If four magnitudes be proportionals, they shall also be proportionals, when taken inversely.

7. If a straight line be drawn parallel to one of the sides of a triangle, it shall cut the other sides, or those sides produced, proportionally.

In a triangle ABC, BC is bisected in D, AD is bisected in E, and BE produced to meet AC in F: show that $AF = \frac{1}{3}AC$, and $EF = \frac{1}{4}BF$.

8. Equal triangles which have one angle of the one equal to one angle of the other, have their sides about the equal angles reciprocally proportional.

9. Name the trigonometrical ratios, and define them.

Prove that $\sec^2 A + \operatorname{cosec}^2 A = \sec^2 A \operatorname{cosec}^2 A$.

10. Show that

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}; \text{ and that}$$

$$\frac{\cos 2A - \cos 4A}{\sin 4A - \sin 2A} - \frac{\cos A - \cos 3A}{\sin 3A - \sin A} = \frac{\sin A}{\cos 2A \cos 3A}.$$

11. Find the value of $\sin 15°$, and prove that

$$\sin 3(A - 15°) = 4 \cos(A - 45°) \cos(A + 15°) \sin(A - 15°).$$

12. Show that the sines of the angles of a triangle are proportional to the opposite sides. If α, β, γ are the perpendiculars from the angular points of a triangle upon the opposite sides, prove that

$$a \sin A + b \sin B + c \sin C = 2(\alpha \cos A + \beta \cos B + \gamma \cos C).$$

13. The sides BC, CA, AB of a triangle are as 4 : 5 : 6. Find the angle B.

$L \cos 27° 53' = 9{\cdot}9464040$ $\log 2 = {\cdot}3010300$.
$L \cos 27° 54' = 9{\cdot}9463371$

14. In a triangle of given perimeter prove that the radii of the escribed circles are proportional to the tangents of the semi-angles opposite to the sides to which they are escribed. Show that the area of the triangle formed by joining the centres of the escribed circles of the original triangle $= \dfrac{s.a}{\sin A}$, when s is the semiperimeter of the original triangle.

15. The angular elevation of a steeple at a place due south of it is 45°, and at another place due west of the former station the elevation is 15°: show that the height of the steeple is $\frac{a}{2}(3^{\frac{1}{4}} - 3^{-\frac{1}{4}})$, a being the distance between the places.

December 1881.

I.—ALGEBRA AND MENSURATION.

1. Simplify the following expressions

(1) $\left(1 + \dfrac{x^2 + y^2 - z^2}{2xy}\right) \div \left(1 + \dfrac{y^2 + z^2 - x^2}{2yz}\right)$,

(2) $\dfrac{1}{\sqrt{3} - \sqrt{2}} + \dfrac{1}{\sqrt{5} - \sqrt{4}}$,

(3) $\left[a^{-1}b\left\{a^{-4}b^3(a^3b\sqrt{ab})^2\right\}^{\frac{1}{3}}\right]^{-1}$.

2. Multiply $x^{\frac{2}{3}} - 2ax^{\frac{1}{3}} + a^2$ by $x^{\frac{1}{3}} + a$ and divide $x^3 + 3\sqrt[3]{2} \,.\, x + 1$ by $x + \sqrt[3]{2} - 1$.

3. Assuming that $a^m \times a^n = a^{m+n}$, for all values of m and n, show what meaning must be assigned to the following expressions :—

$$(1)\ a^0, \qquad (2)\ a^{-1}, \qquad (3)\ a^{\frac{2}{3}},$$

and reduce to its simplest form

$$(a + b\sqrt{-1})^4 + (a - b\sqrt{-1})^4.$$

4. Find the Least Common Multiple of

$$3x^2 - 11x + 6,\ 2x^2 - 7x + 3, \text{ and } 6x^2 - 7x + 2.$$

5. Extract the square root of

$$a^2 - \frac{3a\sqrt{a}}{2} - \frac{3\sqrt{a}}{2} + \frac{41a}{16} + 1,$$

and the cube root of

$$x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1.$$

6. Solve the equations

$$(1)\quad \frac{3x+2}{3(x-3)} + \frac{3x-2}{3(x+3)} = \frac{3x^2 - x - 2}{x^2 - 9}.$$

$$(2)\quad \begin{aligned} 3x + 2y + 5z &= 1, \\ 5x + 3y - 2z &= 2, \\ 2x - 5y - 3z &= 7. \end{aligned}$$

$$(3)\quad \sqrt{3x+7} + \sqrt{2x+3} = 7.$$

7. At what time between 6 and 7 o'clock are the hands of a clock at right angles?

8. A circular plot is surrounded by a ring of gravel b feet wide; if the radius of the circle, including the ring, be a feet, find the relation between a and b, so that the areas of grass and gravel may be equal.

9. A man divides his property between his wife, his three sons, and his three brothers, in such a manner that after the legacy duty on each share is paid, the widow's share is one-third of that of each of the sons, and equal to the whole received by the brothers ; find the proportion which each receives, the legacy duty for a son being one per cent., for a brother three per cent., and for a widow nothing, and the brothers' shares being equal.

10. If α and β be the roots of the equation

$$x^2 - (1 + a)x + \tfrac{1}{2}(1 + a + a^2) = 0,$$

prove that $\alpha^2 + \beta^2 = a$, and form the equation whose roots are α^2 and β^2.

11. A sphere whose diameter is one foot, is cut out of a cubic foot of lead, and the remainder is melted down into the form of another sphere; find its diameter ($\pi = 3\cdot1416$).

12. The diameter of the earth being 7900 miles and that of the moon 2160, compare the areas of their surfaces, and find the radius of a sphere, whose surface is equal to their sum.

13. Define the terms logarithm, mantissa, and characteristic, and prove

(1) $\log_b a \,.\, \log_a b = 1$;

(2) $\log_a (x^m y^n) = m \log_a x + n \log_a y$.

14. Find log 98 and log $(\frac{4}{343})$, given log 2 = ·30103, log 7 = ·845098, and solve the equation

$$(\tfrac{1}{2})^{x+4} = (25)^{3x+2}, \text{ given } \log_{10} 5 = \cdot6989700.$$

II.—EUCLID AND TRIGONOMETRY.

1. Show that if a straight line be divided into any two parts, the square on the whole line is equal to the squares on the two parts, together with twice the rectangle contained by the parts.

If the two complements are together equal to the squares on the two parts, show, either by geometry or algebra, that the straight line is bisected.

2. In every triangle, the square on the side subtending either of the acute angles is less than the squares on the sides containing that angle by twice the rectangle contained by either of those sides, and the straight line intercepted between the acute angle and the perpendicular let fall on it from the opposite angle.

Prove that in every triangle the squares on the two sides are together double of the squares on half the base and on the straight line joining its point of bisection to the vertex.

3. Show that the opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles.

Also show that the sum of one pair of opposite sides of any quadrilateral described about a circle is equal to the sum of the other pair.

4. If from a point without a circle two straight lines be drawn, one of which cuts the circle and the other touches it, show that the rectangle contained by the whole line which cuts the circle and the part of it without the circle is equal to the square on the line which touches it.

Find the locus of points from which the tangents drawn to two intersecting circles are equal.

5. Describe an isosceles triangle, having each of the angles at the base double of the third angle. Also show that this problem supplies a geometrical construction for determining sin 18°.

6. Inscribe an equilateral and equiangular hexagon in a given circle, and compare the area of this hexagon with that of a similar one described about the circle.

7. If two triangles have one angle of the one equal to one angle of the other, and the sides about those equal angles proportionals, the triangles shall be equiangular, and shall have those angles equal which are opposite to the homologous sides.

If two chords AB, AC drawn from any point A in the circumference of the circle ABC, be produced to meet the tangent at the other extremity of the diameter through A in D, E; prove that the triangle AED is similar to ABC.

8. In right-angled triangles the rectilineal figure described upon the side opposite to the right angle is equal to the similar and similarly described figures upon the sides containing the right angle.

9. What is meant by the unit of circular measure?

Prove the formula $\theta = \dfrac{\text{arc}}{\text{radius}}$.

Find the length of that part of a circular railway curve which subtends an angle of $22\frac{1}{2}^{\circ}$ to a radius of a mile ($\pi = 3{\cdot}1416$).

10. Prove from a figure that

$$\cos(A - B) = \cos A \cos B + \sin A \sin B,$$

when A lies between 315° and 360°, and $A - B$ between 180° and 225°.

11. Prove that $2 \sin \frac{A}{2} = \pm \sqrt{1 + \sin A} \pm \sqrt{1 - \sin A}$.

Show, *à priori*, the reason for the four different values of $\sin \frac{A}{2}$ formed from $\sin A$.

12. Prove that

(1) $\sec^4\theta + \tan^4\theta = 1 + 2 \sec^2\theta \tan^2\theta$,

(2) $\sin^{-1}\left(\frac{x - a + b}{2b}\right)^{\frac{1}{2}} = \frac{1}{2} \cos^{-1} \frac{a - x}{b}$,

(3) $\tan 7° 30' = (\cot 30° - \operatorname{cosec} 45°)(\sec 45° - 1)$.

13. Show that in any triangle ABC

$$(b + c) \cos A + (c + a) \cos B + (a + b) \cos C = a + b + c;$$

and if $r, R, r_a\, r_b\, r_c$ are the radii of the circles inscribed in, circumscribed about, and escribed to the triangle ABC,

$$\frac{1}{r_a - r} + \frac{1}{r_b + r_c} = \frac{4R}{a^2}.$$

14. If $\tan \theta = \dfrac{2\sqrt{ab} \sin \frac{C}{2}}{a - b}$, find θ from the following data :—

$$a = 5,\ b = 2,\ C = 120°,\ \log 3 = \cdot477121,$$

$$L \tan 61° 17 = 10\cdot261329$$
$$L \tan 61° 18 = 10\cdot261629.$$

15. An observer is situated in a boat vertically beneath the centre of the roadway of a suspension bridge. He finds that its length subtends at his eye an angle α. At a measured distance q down stream, at a point immediately opposite the centre of the roadway, he finds it subtends an angle β. Supposing the surface of the river horizontal, find an expression for the length of the roadway and its height above the surface of the stream.

July 1882.

I.—ALGEBRA AND MENSURATION.

1. Explain the law of indices; and show that

$$a^0 = 1,\ a^{\frac{1}{n}} = \sqrt[n]{a},\ a^m + a^n = a^{m+n}.$$

Also reduce to its most simple form

$$\frac{(\frac{3}{2})^{\frac{1}{2}} - (\frac{2}{3})^{\frac{3}{2}}}{6^{\frac{1}{2}} + (\frac{2}{3})^{\frac{1}{2}}}.$$

2. Find the value of $(x^2 + y^2)^{\frac{1}{3}}$, when $x = 1 + 2\sqrt{-1}$, $y = 2 + \sqrt{-1}$.

Eliminate x and y from the equations

$$x^2(x - a^2) = y(a^3 + 1),$$
$$x + y = a^3(x - y),$$
$$2(a + 1)(x^2 + 1) + 2a^2 = (a^3 + 1)(x + y).$$

3. Simplify the fraction $\dfrac{(38 + 17\sqrt{5})^{\frac{1}{3}}}{(9 + 4\sqrt{5})^{\frac{1}{2}}}$,

and express with a rational denominator the sum of

$$\frac{1}{1+\sqrt{2}+\sqrt{3}} + \frac{1}{1+\sqrt{2}-\sqrt{3}} + \frac{1}{1-\sqrt{2}+\sqrt{3}} + \frac{1}{-1+\sqrt{2}+\sqrt{3}}.$$

4. Show that in the process of finding the greatest common measure of two expressions, the introduction of a new factor does not affect the result.

Find the G.C.M. of $a^2x^3 + a^5 - 2abx^3 + b^2x^3 + a^3b^2 - 2a^4b$ and $2a^2x^4 - 5a^4x^2 + 3a^6 - 2b^2x^4 + 5a^2b^2x^2 - 3a^4b^2$.

5. Prove that, when $n + 1$ digits of a square root have been obtained by the ordinary rule, n more may be obtained by division only. Apply this to find the square root of 3 to six places of decimals.

Find the cube root of $8a(3b^2 - a^2) + 8b^2\sqrt{8b^2 - 3a^2}$.

6. Solve the equation $x^2 - ax + \dfrac{a^2 - b^2}{4} = 0$; and find the value of b when the two roots are equal.

7. Solve the equations

(1) $\frac{x}{x-7} - \frac{x+7}{x} = \frac{49}{x}.$

(2) $\frac{2}{3}\sqrt{25 - x^2} + \sqrt{5-x} = \sqrt{5+x}.$

(3) $\frac{x}{3} - \frac{3}{x} = 4\frac{4}{5}.$

(4) $\begin{cases} x^2 + y^2 = \dfrac{820}{xy} \\ x^2 - y^2 = 9 \end{cases}.$

8. *A*, *B*, and *C* run a mile race at uniform speed; *A* wins by 160 yards, *B* comes in second, beating *C* by $76\frac{12}{23}$ yards in distance and by $\frac{1}{4}$ minute in time. What is the pace of each?

9. Prove from first principles that

$$\log\left(\frac{a^x b^y}{c^z}\right) = x \log a + y \log b - z \log c.$$

Find to three places of decimals the values of x from the equation

$$(4)^{2x} - 8(4)^x + 12 = 0,$$

having given log 2 = ·3010300, log 3 = ·4771213.

10. Explain what is meant by the characteristic of a logarithm.

Given $\log 1\frac{1}{4} = \cdot 0969100$ and $\log \cdot\dot{1} = \bar{1}\cdot 0457575$, and the logs given in Q. 9, find the logarithms of $2\frac{1}{2}$, $2\frac{1}{4}$, and $\cdot\dot{2}$.

Also having given that log 7 = ·8450980 and log 9·824394 = ·9923057, find the 540th root of ·00007.

11. A right pyramid whose base is a square of 7 inches side and whose perpendicular height is 8 inches is cut into two parts by a plane parallel to the base and 6 inches from it. Find the volumes of the two parts and their total surfaces.

(In the following questions consider $\pi = \frac{22}{7}$.)

12. A piece of paper in the form of a circular sector, of which the radius is 7 inches and the curved side 11 inches, is formed into a conical cup. Find the area of the conical surface, and also of the base of the cone.

13. What is the weight of a cylinder formed of sheet iron half an inch thick, with an outer circumference of 10 ft. $7\frac{2}{7}$ ins. and a width of 3 ft. 6 ins.?

240 cub. ins. iron weigh 1,000 oz. avoirdupois.

II.—EUCLID AND TRIGONOMETRY.

1. Divide a given straight line into two parts, so that the rectangle contained by the whole, and one of the parts, shall be equal to the square of the other part.

Solve this problem also algebraically.

If the rectangle be double of the square, how will the line be divided?

2. If from any point without a circle two straight lines be drawn, one of which cuts the circle and the other touches it: the rectangle contained by the whole line, which cuts the circle, and the part of it without the circle, shall be equal to the square of the line which touches it.

If from a point without a circle there be drawn two straight lines, one of which is perpendicular to a diameter, and the other cuts the circle: the square of the perpendicular is equal to the rectangle contained by the whole cutting line, and the part without the circle, together with the rectangle contained by the segments of the diameter.

3. Inscribe a circle in a given triangle. If the triangle be right-angled, show that the hypotenuse, together with the diameter, is equal to the sum of the other sides.

4. Inscribe an equilateral and equiangular pentagon in a given circle.

Show that the straight line, which joins two opposite angles of the pentagon, is parallel to one of its sides.

5. The rectangle contained by the diagonals of a quadrilateral figure inscribed in a circle is equal to both the rectangles contained by its opposite sides.

If the diagonals cut each other at right angles, show that the rectangles contained by the opposite sides are together double of the quadrilateral figure.

6. Distinguish between the circular measure of an angle and its measure in degrees; and prove that to turn circular measure into seconds we must multiply by 206265, and to turn seconds into circular measure we must multiply by ·000004848.

7. Compare the Trigonometrical Ratios of any angle and its supplement; and determine the Trigonometrical Ratios of 495°.

Find all the angles between 0° and 500° which satisfy the equation $\sin^2\theta = \frac{3}{4}$.

8. Find geometrically an expression for the cosine of the difference of two angles in terms of the trigonometrical ratios of those angles.

Find all the values of x, which satisfy the equation

$$\cos x \cos 3x = \cos 2x \cos 6x.$$

Solve the equation

$$\tan^{-1}\tfrac{1}{3} + \tan^{-1}\tfrac{1}{4} + \tan^{-1}\tfrac{1}{5} + \tan^{-1}x = \frac{\pi}{4}.$$

9. If r, r_1, r_2, r_3 be the radii of the inscribed and escribed circles of a triangle, and x, y, z the perpendiculars from the angles on the opposite sides, prove that

$$(1)\quad xyz = \frac{(a+b+c)^3}{abc}r^3,$$

$$(2)\quad \sin\frac{A}{2} = \frac{\sqrt{(s-b)(s-c)}}{bc} = \frac{r}{\sqrt{(r_2-r)(r_3-r)}}.$$

10. In a triangle prove that

$$\tan\left(\frac{A}{2} + B\right) = \frac{c+b}{c-b}\tan\frac{A}{2},$$

and if $3c = 7b$, and $A = 6^\circ\ 37'\ 24''$, find the other angles.

Given $\log 2 = ·30103$

$L \tan 3^\circ\ 18'\ 42'' = 8·7624069$

$L \tan 9^\circ\ 13'\ 50'' = 9·1603083$ diff for $10'' = 1486$.

December 1882.

I.—ALGEBRA AND MENSURATION.

Great importance will be attached to accuracy in results.

1. Reduce to its simplest form

$$\left\{\frac{a^4 - y^4}{a^2 - 2ay + y^2} \div \frac{a^2 + ay}{a - y}\right\} \times \left\{\frac{a^5 - a^3y^2}{a^3 + y^3} \div \frac{a^4 - 2a^3y + a^2y^2}{a^2 - ay + y^2}\right\},$$

and find the value of $\dfrac{\sqrt{3}}{2 - \sqrt{3}} + \dfrac{2 - \sqrt{2}}{2 + \sqrt{2}}$ to 3 places of decimals.

2. Divide

$x^{12} - x^{-12} + 6(x^8 - x^{-8}) + 9(x^4 - x^{-4})$ by $x^6 - x^{-6} + 3(x^2 - x^{-2})$,

and reduce $\dfrac{x^4 - x^3 - x + 1}{x^4 - 2x^3 - x^2 - 2x + 1}$ to its lowest terms.

3. If $x + \sqrt{y} = a + \sqrt{v}$, prove that $x = a$ and $\sqrt{y} = \sqrt{v}$.

Extract the square root of $2 - x - 2\sqrt{1 - x}$.

4. Prove the rule for converting a mixed circulating decimal into a vulgar fraction, and state under what circumstances fractions are convertible into finite or infinite decimals.

Multiply $7{\cdot}0\dot{9}\dot{1}$ by $70{\cdot}\dot{9}\dot{1}$.

5. Find the cube of $(a + 2\sqrt{-b^2})^{\frac{1}{3}} - (a - 2\sqrt{-b^2})^{\frac{1}{3}}$.

6. Solve the equations

$$(1) \quad \sqrt{x} + \sqrt{a + x} = \frac{2a}{\sqrt{a + x}}.$$

$$(2) \quad x^2 + y^2 + (1 + xy)^2 = 14; \quad xy(x + y) = 6.$$

$$(3) \quad x - y^{\frac{1}{2}} + z^{\frac{1}{3}} = 2; \quad 4x + 6y^{\frac{1}{2}} + 5z^{\frac{1}{3}} = 31; \quad 5x - 11y^{\frac{1}{2}} + 13z^{\frac{1}{3}} = 22.$$

7. A merchant at the end of the first year had doubled his original stock, the second year he gained £80 more than the square root of his increased stock, the third year he cleared half the square of all that he had at the end of the second, and found himself with £18,240. How much had he at first?

8. Show that if unity be added to the product of any four consecutive numbers, the sum is a perfect square.

9. Solve the equation $ax^2 - bx + c = 0$, and find the value of the sum of the cubes of the roots; also form an equation whose roots are the sum and difference of the roots of the original equation.

10. Define a logarithm, and find the values of $\log_{10} \cdot 01$, $\log_7 343$.

Solve the equation $5^{x+2} = 8^{2x-1}$, finding the value of x to four places of decimals.

[Given $\log 2 = \cdot 30103$.]

11. Prove that $\log_a b \cdot \log_b c \cdot \log_c a = 1$.

Also find $\log_{10} \cdot 005$, $\log_{10} \left(\frac{64}{35}\right)^{\frac{5}{8}}$, $\log_{1000} \frac{343}{8}$.

[Given $\log_{10} 7 = \cdot 845098$.]

12. The base of a pyramid covers a square of $13\frac{2177}{5445}$ acres, and its height is 480 feet: find a side of the square, and the volume of the pyramid.

13. A spherical shell of iron, whose diameter is 1 foot, is filled with lead: find the thickness of the iron, when the weights of iron and lead are equal. A cubic inch of iron weighs 4·2 oz., and a cubic inch of lead weighs 6·6 oz.

II.—EUCLID AND TRIGONOMETRY.

1. In every triangle, the square on the side subtending an acute angle is less than the squares on the sides containing that angle by twice the rectangle contained by either of these sides, and the straight line intercepted between the perpendicular let fall on it from the opposite angle and the acute angle.

The squares on the straight lines drawn from the right angle to the two points of trisection of the hypotenuse of a right-angled triangle are together equal to five times the square on the line between the points of trisection.

2. On a given straight line describe a segment of a circle containing an angle equal to a given rectilineal angle.

Given the vertical angle and the segments of the base of a triangle made by the inscribed circle; construct the triangle.

3. Inscribe a regular hexagon in a circle; and compare its area with that of the circumscribed equilateral triangle.

4. If a straight line be drawn parallel to one of the sides of a triangle, it shall cut the other sides, or those sides produced, proportionally; and if the sides, or the sides produced, be cut proportionally, the straight line which joins the points of section shall be parallel to the remaining side of the triangle.

A', B', C' are the middle points of BC, CA, AB, sides of a triangle ABC, show that a triangle can be constructed whose sides are parallel and equal to AA', BB', CC'.

5. If the vertical angle of a triangle be bisected by a straight line which cuts the base, show that the segments of the base are to one another in the same ratio as the sides of the triangle.

If the points are given in which the external and internal bisectors of the vertical angle meet the base, find the locus of the vertex of the triangle.

6. Name and define the trigonometrical ratios. Prove that $\sec^2 A + \operatorname{cosec}^2 A = \sec^2 A \operatorname{cosec}^2 A$.

If the cosecant of an angle between 90° and 180° is $\frac{2}{\sqrt{3}}$, what is the secant? And if the cosine of an angle between 540° and 630° is $-\frac{1}{2}$, what is the cosecant?

7. Prove the following identities

(1) $(\sin 2A)^2 = 2\cos^2 A(1 - \cos 2A)$.

(2) $2 \operatorname{cosec} 4A + 2 \cot 4A = \cot A - \tan A$.

(3) $2\tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{7} + 2\tan^{-1}\frac{1}{5} = \frac{\pi}{4}$.

8. In a plane triangle ABC prove that

(1) $\tan A \tan B \tan C = \tan A + \tan B + \tan C$,

(2) $a \sin A + b \sin B + c \sin C = 2(\alpha \cos A + \beta \cos B + \gamma \cos C)$,

where abc are the sides and $\alpha\beta\gamma$ the perpendiculars let fall on them from the opposite sides respectively.

9. Prove that the area of a triangle $= \frac{a^2}{4} \sin 2B + \frac{b^2}{4} \sin 2A$; and if R, r are the radii of the circumscribing and inscribed circles

$$Rr = \frac{abc}{4(a+b+c)}.$$

10. Given $\log 1\frac{1}{5} = \cdot 0791812$ and $\log 2\frac{2}{5} = \cdot 3802112$, find the value of $\sqrt[5]{(3\cdot 6)^3} \times \sqrt[4]{\frac{1}{25}} \div \sqrt[3]{8\frac{4}{72}}$, the mantissæ for 46929 and 46930 being 6714413 and 6714506.

In a triangle ABC, $b = 14$, $c = 11$, $A = 60°$; find the other angles, having given $L \tan 11° \, 44' \, 29'' = 9\cdot 31774$.

11. A measured line is drawn from a point on a horizontal plane in a direction at right angles to the line joining that point to the base of a tower standing on the plane. The angles of elevation of the tower from the two ends of the measured line are 30° and 18°. Find the height of the tower in terms of l, the length of the measured line.

July 1883.

I.—ALGEBRA AND MENSURATION.

1. Reduce to its simplest form

$$\left\{\frac{2x}{x+y} - \frac{x^2}{x^2-y^2} + \frac{2y}{x-y}\right\} \times \left(\frac{1}{x} + \frac{1}{y}\right) \div \left(\frac{3}{x-y} - \frac{2}{x} + \frac{1}{y}\right).$$

2. Divide $(ax + by)^3 + (ax - by)^3 + (bx - ay)^3 + (bx + ay)^3$ by $(a+b)^2x^2 - 3ab(x^2 - y^2)$.

Find the Greatest Common Measure of

$10x^3 - 54x^2 + 87x - 45$ and $5x^4 - 36x^3 + 87x^2 - 90x + 54$.

3. Prove that

$$\frac{(a^2+x^2)^{\frac{1}{2}}+(a^2-x^2)^{\frac{1}{2}}}{(a^2+x^2)^{\frac{1}{2}}-(a^2-x^2)^{\frac{1}{2}}}-\left\{\frac{a^4}{x^4}-1\right\}^{\frac{1}{2}}=\frac{a^2}{x^2}.$$

Find the square root of

$$2ab-ax-bx-2\sqrt{ab(ab-ax-bx+x^2)}$$

4. Prove that the cube of a number which has n digits cannot have less than $3n-2$ nor more than $3n$ digits.

Find the cube root of 40·353607; and of $1-3x$ to four terms in ascending powers of x.

5. Evaluate $x^2+\frac{1}{x^2}$, $x^3+\frac{1}{x^3}$, and $x^4+\frac{1}{x^4}$,

(1) when $x+\frac{1}{x}=a$, (2) when $x=a+\sqrt{a^2-1}$;

also $2^{-2}3^{\frac{1}{2}}x^{\frac{5}{6}}+2^{-3}3^{-\frac{1}{2}}x^{\frac{2}{3}}-10(27x)^{-\frac{1}{3}}$ when $x=64$.

6. Solve the equations

(1) $$\frac{x^2-4x+4}{x-1}+\frac{x^2-3x-1}{x-2}-2\frac{x^2-5x+5}{x-3}=0;$$

(2) $$\frac{1}{x}+\frac{1}{y}=\frac{x+y}{6}=\frac{5}{x+y+1};$$

(3) $x^2+yz=44$, $xy+xz=36$, $x+y+z=15$.

7. *A* walks m miles in n hours; *B* walks $6n$ miles in $\frac{1}{3}m$ hours; the difference of their rates of walking is $\frac{1}{2}$ mile per hour. Find the rate at which each walks.

8. A bar of metal, of a certain weight, made of gold and silver mixed, is worth £111. 3*s*. 0*d*.; if it were all gold, it would be worth £162; and if the proportion of the metals were reversed, it would be worth £60. 6*s*. 0*d*. An ounce of gold is worth 27*s*. more than a pound (troy) of silver. Find the prices of gold and silver per ounce, and the quantity of each metal in the bar.

9. Prove that, if α β are the roots of the equation

$$ax^2+(a+b)x+b=0,$$

$$\text{then } \frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{a}{b}+\frac{b}{a}.$$

10. Prove that the logarithm of a quotient is equal to the logarithm of the dividend diminished by the logarithm of the divisor.

Given $\log_e 2 = \cdot 69314718$, $\log_e 3 = 1\cdot 09861229$; find the logarithm to the base e of

(1) $\frac{8}{\sqrt{27}}$; (2) $\frac{3e^2}{512}$; (3) $\sqrt{\frac{2}{3}} \times \sqrt[3]{\frac{9}{16}} \times \sqrt[4]{\frac{64}{27}}$.

11. Find the logarithm of $3\cdot 375$ to the base $2\cdot 25$.

What is the characteristic of $\log_7 5463$?

Given $\log_{10} 4 = \cdot 60206$, and $8^x \, 125^{2-x} = 2^{4x+3} \, 5^x$, find x to three places of decimals.

12. A rectangular building, 256 feet long and 48 feet wide, has a roof running lengthwise, of which the height of the centre ridge above the top of the walls is 18 feet and the slope at each end the same as at the sides; find the area of the roof, supposing that there are no eaves.

If a flat roof were substituted for the above, to what additional height must the walls be raised in order to have the same volume of air in the building?

13. An iron boiler is in the form of a circular cylinder 9 ft. long, with hemispherical ends. Its extreme diameter is 3 feet, the metal is one inch thick, and the weight is $3623\frac{18}{20}$ pounds. What is the weight of a cubic inch of iron? [$\pi = \frac{22}{7}$.]

II.—EUCLID AND TRIGONOMETRY.

1. Prove that the parallelograms about the diameter of a square are likewise squares.

2. Divide a given straight line into two parts, so that the rectangle contained by the whole and one of the parts may be equal to the square on the other part.

If AB be so divided in C, and D be the middle point of the longer part AC, prove that a triangle, whose sides are equal to AD, AC, BD respectively, will be right-angled.

3. Draw a straight line from a given point, either without or in the circumference, which shall touch a given circle.

Describe a triangle, having given the vertical angle, one of the sides containing it, and the length of the perpendicular let fall from the vertex upon the base.

4. A straight line passing through the centre of a circle cuts, but not at right angles, another straight line, which does not pass through the centre. Show that the rectangle contained by the segments of the one straight line is equal to the rectangle contained by the segments of the other.

5. Describe a circle about a given square.

Prove that, if tangents be drawn to the circle at the angular points of the square, another square will be formed, whose area is double that of the given square.

6. Find a mean proportional between two given straight lines.

In the triangle ABC the two sides AB, AC are equal, and the circle having B as centre and BC as radius cuts AC in D; prove that BC is a mean proportional between AC, CD.

7. In equal circles, angles, whether at the centres or at the circumferences, have the same ratio which the circumferences on which they stand have to one another.

8. Explain how angles are measured (1) by degrees, minutes, and seconds, (2) by circular measure, and show how to connect the circular measure of any angle with its measure in degrees, &c.

Find the distance in miles between two places on the Equator, which differ in longitude by $6^\circ\ 18'$, assuming the earth's equatorial diameter to be 7925·6 miles. ($\pi = \frac{22}{7}$.)

9. Define the *sine* and *tangent* of an angle.

Show that $\sin A = \pm \dfrac{\tan A}{\sqrt{1 + \tan^2 A}}$, and explain the meaning of the double sign.

Which sign would you use when A is between 180° and 270°?

10. Prove that

$$\cos(A - B) = \cos A \cos B + \sin A \sin B.$$

If $\sin A + \sin B = 1$, and $\cos A + \cos B = \sqrt{2}$, prove that $A - B = n \cdot 360^\circ \pm 60^\circ$.

11. Show that

$$(1) \quad \sin \frac{A}{2} + \cos \frac{A}{2} = \pm \sqrt{1 + \sin A}.$$

(2) $\tan(60^\circ + A) - \tan(60^\circ - A) = \dfrac{8 \cot A}{\cot^2 A - 3}$.

(3) $\sec^{-1} 3 = 2 \cot^{-1} \sqrt{2}$.

12. Prove that the sides of any plane triangle are proportional to the sines of the opposite angles.

Prove also that in any plane triangle

$$a \sin \frac{B - C}{2} = (b - c) \cos \frac{A}{2}.$$

13. Given $a = 3$, $b = 2{\cdot}75$, $c = 1{\cdot}75$, find B.

$\log 2 = {\cdot}30103$ $\quad L \tan 32^\circ 19' = 9{\cdot}8008365$.
diff. $1' = 2796$.

December 1883.

I.—ALGEBRA AND MENSURATION.

1. Find the value of the expression

$$a - b\sqrt{a+b} + (a-b)\sqrt[3]{\left\{\sqrt{2ab} + \frac{a}{b^3}(a^2 - b^2)^{\frac{3}{2}}\right\}},$$

when $a = 2$, $b = 6$.

2. Simplify

(1) $\dfrac{a^2 + bx}{(a+b)(a-x)} + \dfrac{b^2 + ax}{(b+a)(b-x)} + \dfrac{x^2 + ab}{(x-a)(x-b)}$,

(2) $\dfrac{x^4 + x^2y^2 + y^4}{x^2 + y^2} \times \dfrac{x^2 + y(2x + y)}{x^3 - y^3} \div \dfrac{x^3 + y^3}{x^2 - y(2x - y)}$,

3. Prove the rule for finding the Greatest Common Measure of two Algebraic expressions. Find the Greatest Common Measure of

$$x^3 + (5m - 3)x^2 + 3m(2m - 5)x - 18m^2,$$
and $x^3 + (m - 3)x^2 - m(2m + 3)x + 6m^2$.

Also find the Least Common Multiple of

$x^3 - ax^2 + a^2x - a^3$, $x^3 + ax^2 + a^2x + a^3$, and $x^3 + ax^2 - a^2x - a^3$.

4. Find the value of

$$\frac{x^3 + x^2 - 5x + 3}{x^4 - 2x^3 - x^2 + 4x - 2} \text{ when } x = 1,$$

and of $\dfrac{(x^2 - a^2)^{\frac{3}{2}} + x - a}{(1 + x - a)^3 - 1}$ when $x = a$.

5. Resolve $\dfrac{x-5}{x^2-x-2}$ into partial fractions, and find a value for x that will make $x^4 - 6x^3 + 13x^2 - 8x + 16$ a perfect square.

6. Simplify $(5 - 3\sqrt{3})^{\frac{1}{3}} \div (2 - \sqrt{3})$,
and find a cube root of

$$8x(3y^2 - x^2) + 8y^2\sqrt{8y^2 - 3x^2}.$$

7. Solve the equations

(1) $\sqrt{x} + \sqrt{4+x} = \dfrac{4}{\sqrt{x}}$.

(2) $\begin{cases} y + z = x + 4a; \\ z + x = y + 2a; \\ x + y = z. \end{cases}$

(3) $x^2 + y^2 = \sqrt{a^2b^2 + x^2y^2}$
$x + y = \sqrt{b^2 + xy}$.

8. A and B together can do a piece of work in a certain time. If they each did one-half of the work separately, A would have to work one day less, B two days more than before. Find the time in which A and B together can do the work.

9. If x_1, x_2 are the roots of $ax^2 + bx + c = 0$, find in terms of a, b, c the values of

$(b + ax_1)^{-2} + (b + ax_2)^{-2}$, and $(c - ax_1^2)^{-1} + (c - ax_2^2)^{-1}$.

10. Prove that $\log ab = \log a + \log b$, and adapt to logarithmic computation

$$(a + \sqrt{a^2 - 1})^{\frac{1}{2}} + (a - \sqrt{a^2 - 1})^{\frac{1}{2}}.$$

Given that $\log \frac{1}{2} = \bar{1}\cdot6989700$ and $\log \frac{1}{3} = \bar{1}\cdot5228787$,
find the logarithms of

$$\sqrt[5]{6},\quad \tfrac{2}{3}\sqrt[3]{14\cdot4},\quad \tfrac{72}{125}\sqrt[3]{270} \times \tfrac{3}{16}\sqrt[3]{625}.$$

11. Six iron tubes, the cross section of any one of them being a square are joined together, so as to form a hexagonal ring, which can be filled with water. If the area of the cross-section be sixteen square inches, and the distance between any two internal opposite corners of the hexagon be one yard, find the whole surface of the tubing and its contents, neglecting the thickness of the sheet iron.

12. A cylindrical tower 24 feet diameter and 30 feet high is capped with a hemispherical dome. The top of the dome is cut off, and over the orifice formed is built a cylindrical lantern 8 feet diameter and 10 feet high, closed at the top by a plane surface. Find the whole exterior surface of the building.

II.—EUCLID AND TRIGONOMETRY.

1. In every triangle the square of the side subtending either of the acute angles is less than the squares of the sides containing that angle by twice the rectangle contained by either of these sides, and the straight line intercepted between the perpendicular let fall upon it from the opposite side and the acute angle.

Prove this in the case where the perpendicular falls without the triangle.

2. Draw a straight line from a given point, either without or in the circumference, which shall touch a given circle.

From any point in the circumference of the larger of two concentric circles straight lines are drawn touching the inner circle. A third tangent is drawn touching that part of the inner circumference which is convex to the aforesaid point. Show that the perimeter of the triangle thus formed is invariable.

3. The angles in the same segment of a circle are equal to one another.

4. On a given base describe a triangle having a given vertical angle and one of its sides double of the other.

5. Divide a right angle into five equal parts, having first proved the proposition in Euclid on which your method depends.

6. Similar triangles are to one another in the duplicate ratio of their homologous sides.

Explain the following terms: homologous sides, ex æquali, alternando, duplicate ratio.

7. Two equal triangles are drawn upon the same base, and a straight line is drawn through them parallel to the base; show that the parts of it intercepted between the sides of each triangle are equal to one another.

8. A, B, and C are the angles of a triangle; the number of grades in A and C is four times the number of those in B, and the number of degrees in B and C is double that in A. Find the angles in degrees, grades, and circular measure.

9. Define the cosecant of an angle, and trace the changes in its sign and magnitude as the angle increases from $\frac{\pi}{2}$ to $\frac{3\pi}{2}$.

Prove geometrically that $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$.

10. Write down all the values of θ which satisfy the equations

(1) $\operatorname{cosec}^2\theta = \dfrac{2\sec\theta}{\tan^2\theta}$.

(2) $\sin^4\theta - \cos^4\theta = 1$.

(3) $\sin^2 2\theta - \sin^2\theta = \frac{1}{4}$.

11. If θ be less than $\frac{\pi}{2}$, prove that $\sin\theta < \theta$, and $> \theta - \frac{\theta^3}{4}$: and explain the use of the formula

$$\sin A + \sin(72^\circ + A) - \sin(72^\circ - A) = \sin(36^\circ + A) - \sin(36^\circ - A).$$

12. In any triangle, prove that

(1) $\dfrac{a^2 + b^2 - ab\cos C}{a\sin A + b\sin B + c\sin C} = \dfrac{a}{2\sin A}$.

(2) $\tan\dfrac{B - C}{2} = \dfrac{b - c}{b + c}\cot\dfrac{A}{2}$.

If in a triangle $A = 60^\circ$, $b = 9$, $c = 6$, find the other angles, having given that

$\log 2 = \cdot 30103$	$L \tan 19^\circ 6' = 9\cdot 53943$
$\log 3 = \cdot 47712$	$L \tan 19^\circ 7' = 9\cdot 53984$

13. Find the radii of the inscribed and escribed circles of a triangle.

Prove that the area of the triangle formed by joining the points of contact of the inscribed circle equals $2r^2 \cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$.

14. The shadow of a tower is observed to be half the known height of the tower, and some time after to be equal to the full height; how much will the sun have gone down in the interval?

$$\log 2 = \cdot 3010300 \qquad L \tan 63^\circ 26' = 10\cdot 3009994.$$
$$\text{Diff. for } 1' = \cdot 0003159.$$

July 1884.

I.—ALGEBRA AND MENSURATION.

[*Great importance will be attached to accuracy.*]

1. Resolve into their elementary factors

(1) $x^5 - y^5 - (x - y)^5$, (2) $a^3 + b^3 + c^3 - 3abc$,

and find the condition that $x^3 + (p + q)x + a$ may be divisible by $x + p + q$.

2. Reduce the following expression to its simplest terms

$$\frac{x-y}{x+y} \times \frac{x^2 - xy + y^2}{x^2 + xy + y^2} \times \frac{x^3 - y^3}{x^3 + y^3} \div \frac{(x-y)^4}{x^4 - y^4},$$

and prove that if

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f},$$

$$\frac{ma + nc + pe}{mb + nd + pf} = \frac{ax + cy + ez}{bx + dy + fz}.$$

3. Cube

$$(x + \sqrt{-y^2})^{\frac{1}{3}} + (x - \sqrt{-y^2})^{\frac{1}{3}},$$

and extract the cube root of

$$8a^3 - 18a^2b + 4a^2c - 6abc + \frac{27ab^2}{2} + \frac{2ac^2}{3} + \frac{9b^2c}{4}$$
$$- \frac{27b^3}{8} - \frac{bc^2}{2} + \frac{c^3}{27}.$$

4. Find the Greatest Common Measure of

$$6x^4 + 5ax^3 + 3a^2x^2 + 13a^3x - 3a^4, \text{ and}$$
$$3x^3 + 16ax^2 + 15a^2x - 4a^3.$$

5. Show that the product of any four consecutive numbers increased by unity is a perfect square, and extract the square root of

$$25a - 10b\sqrt{a} + b^2 - 10\sqrt{a} + 2b + 1.$$

6. Solve the equations

$$(1)\quad \frac{2x-1}{2x+1} + \frac{2x+1}{2x-1} = 3.$$

$$(2)\quad \sqrt[3]{a+x} + \sqrt[3]{a-x} = m.$$

$$(3)\quad \begin{aligned} xy + x + y &= 7; \\ xz + x + z &= 8; \\ yz + y + z &= 17. \end{aligned}$$

7. If α, β be the roots of the equation $x^2 + 2px + q = 0$, express $\alpha^4 + \beta^4$ in terms of p and q, and form the equation whose roots are $\frac{1}{2}(4 \pm \sqrt{7})$.

8. There are two cubical piles of wood, one of which stands on 16 square yards more than the other: each costs as many shillings per cubic yard as there are yards in an edge of the other, and the total cost of the two is twelve guineas. Find approximately the area on which each stands.

9. Determine the meaning of a^{-x}, and multiply

$$x^{-\frac{1}{3}} - \frac{y^{\frac{1}{3}}}{\sqrt[6]{x}} + y^{\frac{2}{3}} \text{ by } \frac{1}{x^{\frac{1}{6}}} + \sqrt[3]{y}.$$

Also simplify $(16^{\frac{3}{2}})^{-\frac{1}{6}}$, and extract the square root of $3 \pm \sqrt{5}$.

10. Find the relation between the logarithms of the same number to different bases, and if the logarithm of a given number to the base 4 is ·35184, find its logarithm to base 8.

11. Given $\log 2 = {}$·30103, $\log 7 = {}$·845098, find $\log_{10}$ ·005 and $\log_{10}\left(\frac{64}{35}\right)^{\frac{5}{6}}$, and solve the equations

$$\begin{aligned} 18y^x - y^{2x} &= 81; \\ 3^x &= y^2. \end{aligned}$$

12. In a given circle a regular hexagon is inscribed, and in the hexagon another circle, then hexagons and circles alternately *ad infinitum.* Find a series to express the spaces included between each circle and hexagon, and show that it approximates to the area of the original circle.

13. A ball of lead, 4 inches in diameter, is covered with gold. Find the thickness of the gold, in order that (1) the volumes of gold and lead may be equal, (2) the surface of the gold may be twice that of the lead.

14. If a right cone on a circular base be divided into three portions by two sections parallel to the base at equal distances from the base and vertex, compare the three volumes into which it is divided.

II.—EUCLID AND TRIGONOMETRY.

[*Great importance will be attached to accuracy.*]

1. Describe a square which shall be equal to a given rectilineal figure.

Given a square and one side of a rectangle which is equal to the square, find the other side.

2. Describe a circle which shall touch a given circle, and touch a given straight line at a given point.

How many solutions are there; and what is the nature of the contact, when the given straight line (1) falls without, (2) cuts the given circle?

3. The angle at the centre of a circle is double of the angle at the circumference on the same base, *i.e.* on the same part of the circumference.

If two straight lines AEB, CED in a circle intersect in E, the angles subtended by AC and BD at the centre are together double of the angle AEC.

4. Describe an isosceles triangle, having each of the angles at the base double of the third angle.

If A be the vertex and BD the base of the constructed triangle, and C the point of division in the side AB, show that AE is parallel to CD, D and E being the points of intersection of the two circles employed in the construction.

5. Show how to describe a circle about a given triangle.

6. If two triangles have one angle of the one equal to one angle of the other, and the sides about the equal angles proportionals, the triangles shall be equiangular to one another, and shall have those angles equal which are opposite to the homologous sides.

If from a point without a circle two straight lines be drawn, one of which touches, and the other cuts the circle ; a line drawn from the same point in any direction equal to the tangent will be parallel to the chord of the arc intersected by two lines drawn from its other extremity to the intersections of the circle with the cutting line.

7. In any right-angled triangle, any rectilineal figure described on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle.

8. The angles of a quadrilateral inscribed in a circle, taken in order, when multiplied by 1, 4, 6, 9 respectively are in arithmetical progression ; find their values.

9. What is meant by the circular measure of an angle? Prove the formula $\theta = \frac{\text{arc}}{\text{radius}}$. What would this formula become if the unit of circular measure was 36°?

10. Prove from a figure that

$$\cos (A - B) = \cos A \cos B + \sin A \sin B,$$

when A lies between 315° and 360°, and $A - B$ between 180° and 225°.

11. Solve the following equations, giving the general expressions for the angles :

(1) $\cos \theta = 0$; (2) $\sin \theta + \cos \theta = 1$;

(3) $\tan \theta + \tan 4\theta + \tan 7\theta = \tan \theta \, . \tan 4\theta \, . \tan 7\theta$.

12. Prove that

$$\operatorname{cosec} 2\theta + \cot 2\theta = \cot \theta,$$

and find the value of

$$\cot^{-1}\frac{ab + 1}{a - b} + \cot^{-1}\frac{bc + 1}{b - c} + \cot^{-1}\frac{ca + 1}{c - a}.$$

13. Show that in any triangle

$$a \sin A + b \sin B + c \sin C =$$
$$\sqrt{a^2 + b^2 + c^2} \cdot \sqrt{\sin^2 A + \sin^2 B + \sin^2 C}.$$

If D, E, F are the feet of the perpendiculars from A, B, C on a, b, c, and R_a, R_b, R_c the radii of the circles described about the triangles AFE, BDF, CED respectively, then

$$R_a : R_b : R_c :: \cos A : \cos B : \cos C.$$

14. A person standing due south of a lighthouse observes that his shadow cast by the light at the top is 24 feet long: on walking 100 yards due east he finds his shadow to be 30 feet. Supposing him to be 6 feet high, find the height of the light from the ground.

15. In a triangle ABC, if BC and the angles ABC and ACB are given, show how to find the remaining sides: thus $BC = 1652$, angle $ABC = 26° 30'$, and angle $ACB = 47° 15'$; find AB and AC.

$L \sin 73° 45' = 9{\cdot}9822938.$
$L \sin 47° 15' = 9{\cdot}8658868.$
$L \sin 26° 30' = 9{\cdot}6495274.$
$\log 1652 = 3{\cdot}2180100.$
$\log 7678 = 3{\cdot}8852481.$ D = 57.
$\log 12636 = 4{\cdot}1016096.$ D = 344.

December 1884.

I.—ALGEBRA AND MENSURATION.

[*Great importance will be attached to accuracy.*]

1. Multiply

$$a + b^{\frac{2}{3}} + c^{\frac{1}{2}} - b^{\frac{1}{3}}c^{\frac{1}{4}} - c^{\frac{1}{4}}a^{\frac{1}{2}} - a^{\frac{1}{2}}b^{\frac{1}{3}} \text{ by } a^{\frac{1}{2}} + b^{\frac{1}{3}} + \ ^{\frac{1}{4}}$$

and divide

$$\frac{x^4}{3} - \frac{11x^3}{12} + \frac{41x^2}{8} - \frac{23x}{4} + 6 \text{ by } \frac{2x^2}{3} - \frac{5x}{6} + 1.$$

2. Simplify

$$\frac{1}{x-1} - \frac{1}{2x+2} - \frac{x+3}{2x^2+2};$$

and show that if

$$x = \frac{4ab}{a+b}, \text{ then } \frac{x+2a}{x-2a} + \frac{x+2b}{x-2b} = 2.$$

3. Prove that the Least Common Multiple of any two numbers is equal to the product of the numbers divided by their Greatest Common Measure.

Find the Least Common Multiple of

$$x^5 + x^4 + x^3 + x^2 + x + 1, \text{ and } x^5 - x^4 + x^3 - x^2 + x - 1.$$

4. Show that when $n + 1$ digits of a square root have been obtained by the common rule, n more may be obtained by division only. Apply this to find the square root of 3 to six places of decimals.

Find the square root of

$$(x^2 + y^2)(x^2 + z^2) + 2x(x^2 + yz)(y + z) + 4x^2yz.$$

5. Solve the equations

$$(1) \quad \left.\begin{aligned} x - y + 2z &= 11 \\ 2x - y + z &= 9 \\ x - 2y + z &= 0 \end{aligned}\right\}.$$

$$(2) \quad 2\sqrt{x} - \sqrt{4x - \sqrt{7x+2}} = 1.$$

$$(2) \quad \left.\begin{aligned} x + \sqrt[3]{xy^2} &= \frac{10}{x} \\ y + \sqrt[3]{x^2y} &= \frac{810}{y} \end{aligned}\right\}.$$

6. Find the value of $(x^2 + y^2)^{\frac{1}{2}}$, when $x = 1 + 2\sqrt{-1}$, and $y = 2 + \sqrt{-1}$. Express $\sqrt{2n\sqrt{-1}}$ in the form of a binomial surd.

7. There is a number consisting of two digits which, when it is multiplied by the digit on the left hand, gives 280 as the product; if the sum of the digits is multiplied by the same digit the product is 55. Required the number.

8. If α and β are the roots of the equation $x^2 + px + q = 0$, form the equation whose roots are $(\alpha - \beta)^2$ and $(\alpha + \beta)^2$.

Show that $(x^2 - bc)(2x - b - c)^{-1}$ has no real value between b and c.

9. Define a logarithm and the base of a system of logarithms.

Prove that

$$\log xy = \log x + \log y\,;\ \log x^n = n \log x.$$

Given $\log 7 = \cdot 8450980$, $\log 58751 = 4\cdot 7690153$, $\log 58752 = 4\cdot 7690227$,

find $\sqrt[5]{\cdot 07}$ to seven significant figures.

10. Solve the equation $a^x(a^x - 1) = 1$; and find the value of

$$10 \log \tfrac{3}{2} + 7 \log \tfrac{5}{18} + 4 \log \tfrac{48}{25}.$$

11. A circular disk of cardboard one foot in diameter is divided into six equal sectors by pencil lines drawn through the centre. In each sector there is described a circle touching the two bounding radii of the sector and also the arc joining their ends at its middle point. If the circles are cut out from the six sectors, find the area of cardboard remaining.

12. A hollow paper cone, whose vertical angle is 60°, is held with its vertex downwards, and in it there is placed a sphere of radius two inches. The portion of the cone remote from the apex is now cut away along the line where the paper touches the sphere. Find the exterior surface of the body thus found.

13. A hollow right prism stands upon a base which is an equilateral triangle. The vertical faces of the prism are squares, the side of a square being one foot. The prism is filled with water, and the largest possible sphere is then submerged in it. Find the amount of water remaining in the prism.

$$[\pi = \tfrac{22}{7},\ \sqrt{3} = 1\cdot 73.]$$

II.—EUCLID AND TRIGONOMETRY.

[*In the first seven questions ordinary abbreviations may be employed, but the method of proof must be geometrical. Great importance will be attached to accuracy.*]

1. If a straight line be bisected, and produced to any point, the rectangle contained by the whole line thus produced, and the part

of it produced, together with the square on half the line bisected, is equal to the square on the straight line which is made up of the half and the part produced.

Produce a given straight line so that the rectangle contained by the whole line thus produced, and the part produced, may be equal to the square on half the line.

2. If any point be taken in the diameter of a circle which is not the centre, of all the straight lines which can be drawn from this point to the circumference, the greatest is that in which the centre is, and the other part of the diameter is the least.

3. The opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles.

If the sides *AD*, *BC* of a quadrilateral *ABCD* inscribed in a circle are produced to meet in *E*, and a circle described on *CD* as a chord meets *DE* and *CE* produced in *F* and *G*; *FG* will be parallel to *AB*.

4. On a given straight line describe a segment of a circle containing an angle equal to a given rectilineal angle less than a right angle.

5. Describe a circle touching one side of a triangle and the other two sides produced.

Prove that if the diameter of the circle is equal to the perimeter of the triangle, the triangle is right-angled.

6. The sides about the equal angles of triangles which are equiangular to one another are proportionals; and those which are opposite to the equal angles are homologous sides.

If *ABC* be a semi-circle of which *O* is the centre, and *OB* perpendicular to *AC*, and *ADE* a chord cutting *OB* in *D*, prove that the circle described about *BDE* will be touched by *AB*.

7. Describe a rectilineal figure which shall be similar to one given rectilineal figure, and equal to another given rectilineal figure.

8. Explain fully what is meant by circular measure, and if a right angle is reckoned as 100, find the value on the same scale of the angle whose circular measure is $\frac{1}{2}$. ($\pi = 3{\cdot}1416$.)

9. Show that $\cos(90^\circ + A) = -\sin A$.

If $\tan^2\theta + 3\cot^2\theta = 4$, find $\sin\theta$.

10. Prove that $\sin(A - B) = \sin A\cos B - \cos A\sin B$, and, by giving suitable values to A, deduce the corresponding formulæ for $\cos(A \pm B)$.

If $\sec^2\alpha = \frac{4}{3}$, find a general expression for all the angles which have the same tangent as α.

11. Show that

(1) $\cos^2 A - \cos^2 B = \sin(B + A)\sin(B - A)$.

(2) $\tan A - \tan\frac{A}{2} = \tan\frac{A}{2}\sec A$.

(3) $3\tan^{-1}a = \tan^{-1}\frac{3a - a^3}{1 - 3a^2}$.

12. Prove that in any plane triangle

(1) $c^2 = a^2 + b^2 - 2ab\cos C$.

(2) $a\cos A\cos 2B + b\cos B\cos 2A + c\cos C = 0$.

13. Given $b = 8{\cdot}4$ inches, $c = 12$ inches, $B = 37^\circ\,36'$, find A.

$L\sin 37^\circ\,36' = 9{\cdot}7854332$ $\log 7 = {\cdot}845098$

$L\sin 60^\circ\,39' = 9{\cdot}9403381$

Diff. $1' = 711$.

July 1885.

I.—ALGEBRA AND MENSURATION.

1. Prove that

$$(y + z)(z + x)(x + y) = (yz + zx + xy)(x + y + z) - xyz.$$

Divide

$$a + b + c + 3(b^{\frac{1}{3}} + c^{\frac{1}{3}})(c^{\frac{1}{3}} + a^{\frac{1}{3}})(a^{\frac{1}{3}} + b^{\frac{1}{3}}) \text{ by } a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}}.$$

2. Prove the rule for finding the G.C.M. of two quantities; find that of $a^{x+4} - a^x b^4 + a^4 b^y - b^{y+4}$ and $a^{x+2} - a^x b^2 + a^2 b^y - b^{y+2}$; and find the L.C.M. of $x^3 - 3x - 2$ and $x^3 + 2x^2 - x - 2$.

D

3. Simplify

$$a + b - \frac{1}{a + \frac{1}{b}} - \frac{1}{b + \frac{1}{a}}$$

and

$$\left\{ \frac{\sqrt{x + a}}{\sqrt{x - a}} - \frac{\sqrt{x - a}}{\sqrt{x + a}} \right\} \frac{\sqrt{x^3 - a^3}}{\sqrt{(x + a)^2 - ax}}.$$

4. Find the square root of $x^4 - 4x^3 + 10x^2 - 12x + 9$, and write down the expansion of $\left(x + \frac{2}{x}\right)^5$.

5. Find the square root of $18 + 8\sqrt{5}$, and simplify

$$\sqrt{3\sqrt{5} - \sqrt{2} + \sqrt{7 + 2\sqrt{10}}}.$$

6. Solve the equations

$$(1) \quad \tfrac{3}{7}(6x - 7) + \frac{1 - 7x}{6} = x.$$

$$(2) \quad 4x^{\frac{3}{2}} - 27x^{\frac{3}{4}} = 40.$$

$$(3) \quad \left.\begin{aligned} x^3 + y^3 &= 35 \\ x + y &= 5 \end{aligned}\right\}.$$

7. Prove that the roots of $x^2 + px + q = 0$ are equal when $p^2 - 4q = 0$; also that one is half the other, if $9q = 2p^2$.

8. A certain sum of money had to be divided equally among 15 persons; if £2 more had been available for division, each person would have received 5 per cent. more. What was the sum?

9. Given $\log 4{\cdot}2 = {\cdot}6232493$, $\log {\cdot}012 = \bar{2}{\cdot}0791813$, and $\log {\cdot}0441 = \bar{2}{\cdot}6444386$, find the logarithms of the nine digits.

10. From the equation $105^x = 100$ find, by aid of the logarithms in the last question, the value of x to four places of decimals. Given $\log 2000{\cdot}1 = 3{\cdot}3010517$ construct a table of proportional parts, and find $\log {\cdot}2000088$.

11. The length of a hall is three times the breadth: the cost of whitewashing the ceiling at $5\frac{1}{3}d.$ per square yard is £4. 12*s.* $7\cdot\dot{1}d.$, and the cost of papering the four walls at 1*s.* 9*d.* per square yard is £35: find the height of the hall.

12. A cube of marble, of which an edge is 1 foot, has all its corners evenly ground down so as to leave facets in the shape of equilateral triangles, while the faces of the original cube again become squares; find, approximately, the total area of the body so formed.

13. A hemispherical bowl, whose internal radius is 1 foot, is filled with water and placed on a horizontal table. In the water there is placed, with its vertex touching the centre of the bottom of the bowl and its axis vertical, a cone whose vertical angle is 90°; find the amount of water left in the bowl after the intrusion of the cone.

II.—EUCLID AND TRIGONOMETRY.

[*In the first eight questions ordinary abbreviations may be employed, but the method of proof must be geometrical.*]

1. If a straight line be divided into two equal parts and also into two unequal parts, the rectangle contained by the unequal parts, together with the square on the line between the points of section, is equal to the square on half the line.

2. In obtuse-angled triangles, if a perpendicular be drawn from either of the acute angles to the opposite side produced, the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle, by twice the rectangle contained by the side on which, when produced, the perpendicular falls, and the straight line intercepted without the triangle, between the perpendicular and the obtuse angle.

The sum of the squares on the distances of the middle point of either of the diagonals of any quadrilateral from the angles of the quadrilateral is equal to half the sum of the squares on the four sides.

3. If two circles touch one another internally, the straight line which joins their centres, being produced, shall pass through the point of contact.

4. The angle at the centre of a circle is double of the angle at the circumference upon the same base, *i.e.*, upon the same part of the circumference.

From the point A the lines AB and AC are drawn touching the circle $BDEC$, whose centre is O, in the points B and C. If DOE be any diameter, and the lines BE and CD intersect in P, prove that the angle BPC is equal to the angle between either BA or CA and OA produced.

5. If from any point without a circle there be drawn two straight lines, one of which cuts the circle, and the other meets it, and if the rectangle contained by the whole line which cuts the circle, and the part of it without the circle, be equal to the square on the line which meets the circle, the line which meets the circle shall touch it.

6. Describe a circle about a given triangle.

The isosceles triangles ABC and ADE have a common vertex A, and the sides BA and AD in the same straight line. If the bases BC and DE produced intersect in F, and the circumscribing circles intersect in A and P, prove that the angles ACP and ADP are together equal to the angle CFD.

7. The sides about the equal angles of triangles which are equiangular to one another are proportionals; and those which are opposite to the equal angles are homologous sides, that is, are the antecedents or the consequents of the ratios.

Prove that the locus of the point, whose distances from two given straight lines are to each other in a constant ratio, is a straight line passing through the point of intersection of the given lines, and give a construction for drawing that line when the ratio is known.

8. Similar triangles are to one another in the duplicate ratio of their homologous sides.

9. Define the circular measure of an angle, and find from your definition the circular measure of a right angle.

Express the angle 44° as a fraction of the angle 1·5 circular measure, assuming π to be $\frac{22}{7}$.

10. Prove that $\cos(A + B) = \cos A \cos B - \sin A \sin B$, and deduce the values of $\cos 2A$ and $\cos 3A$ in terms of $\cos A$.

Write down the values of $\sin 3A$ and $\tan 3A$.

11. Prove that

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2};$$

and that

$$\sin A - \sin B = 2 \sin \frac{A-B}{2} \cos \frac{A+B}{2}.$$

If $\sin A$ be the geometric mean between $\sin B$ and $\cos B$, prove that $\cos 2A = 2 \sin(45^\circ - B) \cos(45^\circ + B)$.

Prove also that

$$2 \sin^{-1} \tfrac{1}{2} = \cos^{-1} \tfrac{1}{2}.$$

12. In the triangle ABC, if d be the diameter of the circumscribing circle, prove that

(1) $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = d.$$

(2) $$\frac{b \cos C + c \cos B}{a} = \frac{c \cos A + a \cos C}{b} = \frac{a \cos B + b \cos A}{c} = 1.$$

If D, E, F be the feet of the perpendiculars from A, B, and C upon the opposite sides, prove that the diameters of the circles circumscribing the triangles AEF, BDF, and CDE are respectively

$$a \cot A,\ b \cot B, \text{ and } c \cot C.$$

13. In the triangle ABC,

$$A = 40^\circ,\ a = 140·5,\ b = 170·6$$

find B and C.

Given that

$$L \sin 40^\circ = 9·8080675.$$
$$\log 1405 = 3·1476763.$$
$$\log 1706 = 3·2319790.$$
$$L \sin 51^\circ 18' = 9·8923342.$$
$$L \sin 51^\circ 19' = 9·8924354.$$

December 1885.

I.—ALGEBRA AND MENSURATION.

1. Show that $(ax + by + cz)^3 + (cx - by + az)^3$ is divisible by $(a + c)(x + z)$.

Find the H.C.D. of $x^5 + 11x - 12$ and $x^5 + 11x^3 + 54$.

2. Find an expression containing no higher power of x than the first which added to

$$x^4 + 6x^3 + 13x^2 + 6x + 1$$

will make it a complete square.

3. Solve the equations

(1) $\dfrac{x-3}{2} + \dfrac{x-2}{3} = \dfrac{2}{x-3} + \dfrac{3}{x-2}$.

(2) $\dfrac{x^2+x+1}{x+1} + \dfrac{x^2+3x+1}{x+3} = \dfrac{8x^2+16x-1}{4x+8}$.

(3) $x^2 + y^2 - xy = 61$.
$x + y - \sqrt{xy} = 7$.

4. If α and β are the roots of the equation $x^2 + px + q = 0$, express $\alpha^2 + \beta^2$ and $\alpha^3 + \beta^3$ in terms of p and q.

5. Prove that if $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f} = k$ and m, n, p are any numbers

$$k^4 = \frac{m^2a^3c + n^2a^6d^2 + p^2e^4}{m^2b^3d + n^2b^6c^3 + p^2f^4}.$$

6. Three numbers are as 1, 2, 3; the sum of their squares is sixty-three times the sum of the numbers. Find them.

7. If the first term of an A.P. is a and the common difference d, find an expression for the sum of n terms.

If the common difference is $-d$, and the sum of n terms $\dfrac{(2a+d)^2}{9d}$ find n.

8. Write down the general term (the r^{th}) in the expansion of $(1 - x)^{-n}$, and find the condition that the sum of the coefficients of the first three terms may equal that of the fourth. Generalise this result.

9. Prove that if m, n, and a are any numbers,

$$\frac{\log_a n}{\log_{ma} n} = 1 + \log_a m.$$

Given that $\log_{10} 2 = \cdot 30103$, find $\log_{10} 500$ and $\log_{10} \cdot 0008$.

10. Assuming the expansion for $\log_e (1 + x)$, prove that

$$\log_e \frac{n+1}{n} = 2\left\{\frac{1}{2n+1} + \tfrac{1}{3}\left(\frac{1}{2n+1}\right)^3 + \ldots\right\}.$$

What is the practical use of this series?

11. Find the area of the triangular field *ABC* from the following measurements on the ordnance survey of 25 inches to the mile. *AC* 4·1 inches, perpendicular from *B* on *AC* 1·59 inches. Calculate the same area from the three sides, *AB* measuring 3·3 inches and *BC* 2 inches. Express the mean of the two in acres.

12. A well 5 feet in diameter and 30 feet deep is to have a lining of bricks fitting close together without mortar, 9 inches thick. Required approximately in tons the weight of the bricks, supposing a brick $9 \times 4\frac{1}{2} \times 3$ inches to weigh 5 lbs.

13. A hollow shell 12 inches in diameter is placed in a conical vessel whose vertical angle is 60°, and water poured into it until it just covers the shell and fills the cavity in it. When the shell emptied of the water in it is removed, and a solid ball of the same diameter substituted for it, the water stands $\frac{1}{2}$ inch above it; find approximately the thickness of the shell.

II.—EUCLID AND TRIGONOMETRY.

1. Divide a given straight line into two parts, so that the rectangle contained by the whole line and one of the parts shall be equal to the square on the other part.

2. Describe a square that shall be equal to a given rectilineal figure.

Having given one side of a rectangle, whose area is equal to that of a given square, find the other side.

3. If a straight line drawn through the centre of a circle bisects a straight line in it which does not pass through the centre, it shall cut it at right angles; and conversely, if it cut it at right angles, it shall bisect it. Prove this.

4. From a given circle cut off a segment, which shall contain an angle equal to a given angle. The chord of a given segment of a circle is produced to a fixed point; on this line so produced describe a segment of a circle similar to the given one.

5. If two straight lines cut one another within a circle, prove that the rectangle contained by the segments of one of them shall be equal to the rectangle contained by the segments of the other.

6. Inscribe an equilateral and equiangular pentagon in a given circle.

Construct a regular pentagon which shall have the line joining two alternate vertices of a given length.

7. Prove that equal parallelograms, which have one angle of the one equal to one angle of the other, have their sides about the equal angles reciprocally proportional.

8. In a right-angled triangle, show that if a perpendicular be drawn from the right angle to the base, the triangles on each side of it are similar to the whole triangle and to each other.

ABC is a triangle; AD is drawn perpendicular to BC, meeting it in D between B and C; if BA is a mean proportional between BD and BC, prove that BAC is a right angle.

9. Define the two common units of angular measure. Find the circular measure of 42°, and find the angle whose circular measure is $\frac{5}{6}$.

10. Assuming that

$$\left.\begin{array}{l}\sin(A+B) = \sin A \cos B + \cos A \sin B \\ \cos(A+B) = \cos A \cos B - \sin A \sin B\end{array}\right\}$$

find in terms of the ratios of A the values of $\sin 2A$, $\cos 2A$, $\tan 2A$, $\sin \frac{A}{2}$, $\cos \frac{A}{2}$, and $\tan \frac{A}{2}$.

11. Prove that

$$1 - \tan^2 A \tan^2 B = \frac{\cos^2 B - \sin^2 A}{\cos^2 A \cos^2 B};$$

and from the equation $2 \tan^2\theta = \sec^2\theta$, find a general expression for θ.

12. Prove that in any triangle

$$\cos A + \cos B = \frac{2(a + b)}{c} \sin^2 \tfrac{1}{2} C.$$

A lighthouse appears to a man in a boat 300 yards from its foot to subtend an angle of 6° 20′ 24·7″. Find in feet the height of the lighthouse, having given

$L \tan 6° 20' = 9·0452836$ $\log 3 = ·4771213.$
Difference for $1' = ·0011507$

13. Prove that in any triangle

$$\tan 2A + \tan 2B + \tan 2C = \tan 2A \tan 2B \tan 2C.$$

Hence show that if x, y, z are three numbers such that $x + y + z = xyz$

$$x(1 - y^2)(1 - z^2) + y(1 - z^2)(1 - x^2) + z(1 - x^2)(1 - y^2) = 4xyz.$$

HIGHER MATHEMATICS.

III.—CONIC SECTIONS AND DIFFERENTIAL CALCULUS.

[*Full marks may be gained by doing about* two-thirds *of this paper.*]

1. Define a *parabola*, and prove that the ordinate of any point on a parabola is a mean proportional between the abscissa of the point and the latus rectum.

Two of the angular points of an equilateral triangle are on the axis of a parabola, one of them being the vertex. If the curve pass through the third angular point, show that each side of the triangle will be two-thirds of the latus rectum.

2. The locus of the foot of the perpendicular from either focus upon the tangent at any point of an ellipse is the circle described on the axis major as diameter.

Prove also that the straight line joining the foot of the perpendicular with the centre of the ellipse bisects the focal distance.

3. Explain what is meant by an *asymptote* of a curve.

If any chord PP' of an hyperbola be produced to meet the two asymptotes in Q, Q', prove that $PQ = P'Q'$.

4. Express the area of a triangle in terms of the rectangular coordinates of its angular points.

Hence determine the area of the triangle formed by joining the points, whose coordinates are

$$(a, b + c), \quad (a, b - c), \quad (-a, c),$$

and verify your result geometrically.

5. Investigate the equation of a straight line in the form

$$\frac{x}{a} + \frac{y}{b} = 1.$$

If this straight line cut the axes at the points A, B respectively, write down the equations of (1) the straight line drawn from the middle point of AB to the origin, (2) the straight line bisecting AB at right angles, and show that

$$x - y = \frac{a - b}{2}$$

is the equation of a straight line bisecting the angle between these two straight lines.

6. Find the radius and coordinates of the centre of each of the circles

$$x^2 + y^2 - 2ax - 4ay = 4a^2,$$
$$x^2 + y^2 - 3ax + 4ay = 0.$$

Also find the equation of their common chord, and show that $4a$ is the length of each of their common tangents.

7. Define the *excentric angle* at any point of an ellipse, and find the equation of a chord of an ellipse in terms of the excentric angles of its extremities.

Prove that if the chord be parallel to either diagonal of the circumscribing rectangle, the sum of the excentric angles of its extremities will be an odd multiple of $\frac{\pi}{2}$.

8. Prove that $y = mx + c$ will touch the hyperbola

$$b^2x^2 - a^2y^2 = a^2b^2, \text{ if } b^2 + c^2 = a^2m^2.$$

If from any point on the circle $x^2 + y^2 = a^2$, a pair of tangents be drawn to the parabola $y^2 = 2ax$, prove that the chord of contact will always touch the rectangular hyperbola $x^2 - y^2 = a^2$.

9. Prove that the polar equation of any conic section, with the focus as pole, may be expressed in the form

$$\frac{a}{r} = 1 + e \cos \theta.$$

Hence, or otherwise, show that the semi-latus rectum of a conic section is the harmonic mean between the segments of any focal chord.

10. Find the differential coefficients of

$$\sin x, \qquad a^x, \qquad \tan^{-1}\left(\frac{1}{\sqrt{x^2 - 1}}\right).$$

11. If $x = \epsilon^\theta \sin (\sqrt{3} . \theta)$, prove that

$$\frac{d^3x}{d\theta^3} + 8x = 0.$$

12. Investigate a method of finding the limiting value of a function of an independent variable, when it assumes the indeterminate form $\frac{0}{0}$.

Find the value, when $x = 1$, of

$$\frac{(x - 1)^2}{\epsilon^{2(x-1)} - x^2}.$$

13. State clearly the conditions which must be satisfied in order that a function of an independent variable may have a *maximum* or *minimum* value.

Find all the maxima and minima values of

$$\frac{x^3}{3x^2 - a^2}.$$

14. Find all the asymptotes of the curve

$$x^2(y + x) = a^2y,$$

and trace the curve.

15. Prove that the square of the radius of curvature at any point of a parabola varies as the cube of the focal distance of the point.

IV.—STATICS AND DYNAMICS.

[*Full marks may be obtained for about* three-quarters *of this paper. Great importance will be attached to accuracy.*]

1. Assuming the parallelogram of forces for the direction, prove it for the magnitude of the resultant.

The sides AB and AC of the triangle ABC are bisected in D and E: prove that the resultant of the forces BE and DC is parallel and proportional to BC.

2. Define the moment of a force about a point.

If the moments of two forces in one plane about a point in that plane are equal and opposite, prove that their resultant passes through the point.

3. Show how to find the centre of gravity of any number of weights rigidly connected, and prove that the centre of gravity of a triangle coincides with that of three equal weights at the angles.

The sides AB and AD of the quadrilateral $ABCD$ are bisected in E and F, and EF is divided at H, so that $EH : FH ::$ area of triangle ADC : area of triangle ABC. Prove that the centre of gravity of the quadrilateral is in CH at two-thirds of CH from C.

4. Find the mechanical advantage of the system of pulleys of two blocks, one fixed and the other movable, the same string passing round all of them.

If a basket be suspended from the lower block, and a man in the basket support himself and the basket by pulling at the free end of the string, find the tension he exerts, neglecting the inclinations of the strings to the vertical, the weight of man and basket together being W.

5. State the laws of friction.

A circular hoop of radius one foot hangs upon a horizontal bar, and a man hangs by one hand from the hoop. If the coefficient of friction between the hoop and bar be $\frac{1}{\sqrt{3}}$, find the shortest possible distance from the bar to the man's hand, the weight of the hoop being neglected.

6. A pair of steps consists of a ladder, a support equal in length to the ladder and hinged to it at the top, and two parallel equal ropes connecting the ladder and the support. If it stands on a smooth horizontal plane, and a man walks slowly up the ladder, prove that the tension in either string at any time varies as the man's height above the ground, the weight of the steps being neglected.

7. Explain what is meant by relative velocity, and state how the relative velocity of two moving particles is determined.

A treadwheel with axis horizontal and diameter two feet makes ten revolutions a minute. At what rate per hour does a man upon it walk over its surface, on the supposition that he always keeps at the same height above the ground? $\left(\pi = \frac{22}{7}\right)$.

8. Obtain the formula $S = Vt \pm \frac{ft^2}{2}$.

A falling particle in the last second of its fall passes through 224 feet. Find the height from which it fell, the acceleration of gravity being 32.

9. The path of a projectile is a parabola. Prove this, pointing out the laws of motion assumed in the proof.

If PQ, QR, and RS be arcs described in equal times, and V_1, V_2, V_3, V_4 be the velocities at P, Q, R, and S, prove that $V_2^2 - V_3^2$ is an arithmetical mean between $V_1^2 - V_2^2$ and $V_3^2 - V_4^2$.

10. State the third law of motion, and find the acceleration of a particle on a smooth inclined plane. Find also the vertical and horizontal accelerations of the particle.

Two bodies start from rest at the same point, one down a smooth plane at the angle 45° to the horizon, and the other falling freely.

Prove that either body as seen by a person moving with the other appears to be moving from the observer in a straight line perpendicular to the plane with uniform acceleration.

11. Find the velocities of two elastic spheres after direct impact. State generally how to do this for oblique impact.

12. What is the time of oscillation of a simple pendulum of given length?

A clock gains five seconds a day: show how it may be made to keep right time.

13. Two weights hang from the extremities of a string passing over a smooth fixed pulley. Find the acceleration. In what time will a weight of 37 lbs. draw another of 24 lbs. through 32 feet? and what velocity will each particle have at the end of that time?

July 1886.

I.—ALGEBRA AND MENSURATION.

1. Divide

$$a^3(b^2 - c^2) + b^3(c^2 - a^2) + c^3(a^2 - b^2) \text{ by } ab + bc + ca.$$

2. Find the square root of

$$a^2(x^6 + 2x^3 + 1) + 2ab(x^5 + x^4 + x^2 + x) + b^2(x^4 + 2x^3 + x^2).$$

3. Solve the equations

$$(1)\quad \frac{x + 11}{x - 7} - \frac{x - 7}{x + 11} = 5\frac{7}{22}.$$

$$(2)\quad \left.\begin{aligned} (x + y)^2 + 3(x - y) &= 30 \\ xy + 3(x - y) &= 11 \end{aligned}\right\}.$$

4. Prove that a surd cannot be equal to the sum of a rational quantity and a surd.

Show that if $\sqrt[3]{x^3 + \sqrt{x^6 - 8y^6}} + \sqrt[3]{x^3 - \sqrt{x^6 - 8y^6}} = a$, then $2x^3 + 6ay^2 - a^3 = 0$.

5. A person invests a sum of money, and obtains an income of £100 a year; if the rate of interest were 1 per cent. less, he would have to invest £500 more to obtain the same income. Find the sum invested and the rate of interest.

6. If $a : b :: c : d$ and $p : q :: r : s$, prove that

$$ap + cr : bq + ds :: \sqrt{acpr} : \sqrt{bdqs}.$$

7. Investigate formulæ for finding the sum of a geometrical series, to n terms and to infinity.

The sum of a certain geometrical series to 6 terms is $13\frac{25}{32}$, and the sum of the same series to infinity is 14; find the first term and the common ratio.

8. Prove that each term in the expansion of $(a - b)^{-n}$ is greater than the preceding term if b is greater than a, and n is greater than $\frac{a}{b}$.

Find the coefficient of x^r in the expansion of $(1 - x)^2 (1 + x)^n$.

Expand $(1 - \frac{2}{3}x)^{\frac{3}{2}}$ to five terms.

9. Prove that $\log_{a^m} b^m \times \log_{b^n} a^n = 1$.

Given $\log_{10} 8 = \cdot 9030900$, $\log 28 = 1\cdot 447158$; find the logarithms of 50 and 350 to the base 10, and the logarithm of 10 to the base 12·5.

10. Expand $\log_e (1 + x)$ in a series of ascending powers of x; and prove that

$$\log_e 3 = 1 + \frac{1}{3 \cdot 2^2} + \frac{1}{5 \cdot 2^4} + \frac{1}{7 \cdot 2^6}, \text{ \&c., to infinity.}$$

11. The area of a rectangular field is $\frac{3}{5}$ of an acre; and its length is double its breadth; determine the lengths of its sides approximately.

If a pony is tethered to the middle point of one of the longer sides, find the length of the tether in yards, correct to two places of decimals, in order that he may graze over half the field. ($\pi = 3\cdot 1416$.)

12. An obelisk $68\frac{1}{2}$ feet high has a square section throughout; it is $7\frac{1}{2}$ feet wide at the base, and gradually tapers to a width of 5 feet, the summit being in the form of a pyramid $7\frac{1}{2}$ feet high; it is made of granite, of which a cubic foot weighs 156 lbs.; find the weight of the obelisk.

13. A solid cone 15 inches high is placed on its base in a cylindrical vessel the inner diameter of which is the same as that of the base of the cone; water is then poured into the cylinder until it is 9 inches deep; the cone being removed, another cone, with the same sized base, but of different altitude, is substituted for it; and the surface of the water just reaches the top of this cone. Find the height of the second cone.

II.—EUCLID AND TRIGONOMETRY.

[*In answering the questions on geometry ordinary abbreviations may be employed, but the methods of proof must be geometrical. Great importance will be attached to accuracy.*]

1. If a straight line be divided into two equal and also into two unequal parts, prove that the rectangle contained by the two unequal parts, together with the square on the line between the points of section, is equal to the square on half the line.

2. In obtuse-angled triangles if a perpendicular be drawn from either of the acute angles to the opposite side produced, prove that the square on the side subtending the obtuse angle is greater than the squares on the sides containing the obtuse angle, by twice the rectangle contained by the side upon which, when produced, the perpendicular falls, and by the straight line intercepted without the triangle between the perpendicular and the obtuse angle.

If an angle of a triangle is one-third of four right angles, show that the square on the side subtending that angle is equal to the squares on the sides containing it, together with the rectangle contained by those sides.

3. A segment of a circle being given, show how to describe the circle of which it is the segment.

4. Describe a circle which shall pass through a given point and touch each of two given straight lines which are not parallel.

5. Prove that the sides about the equal angles of equiangular triangles are proportionals, those sides which are opposite to the equal angles being homologous.

Apply this proposition to prove that the rectangle contained by the segments of any chord passing through a given point within a circle is constant.

6. If from the vertical angle of a triangle a straight line is drawn perpendicular to the base, prove that the rectangle contained by the sides of the triangle is equal to the rectangle contained by the perpendicular and the diameter of the circle described about the triangle.

7. Two sides of a triangle, whose perimeter is constant, are given in position; show that the third side always touches a certain circle.

8. Express the values of the tangent, secant, and cosecant in terms of the sine of the angle, and also in terms of the cosine of twice the angle. Find the values of the tangent, secant, and cosecant of 22° 30′.

9. Prove the following:—

$$(\sin 2A)^2 = 2\cos^2 A(1 - \cos 2A)$$

$$\tan 6A = \frac{\cos 5A - \cos 7A}{\sin 7A - \sin 5A},$$

and if $\frac{a}{b} = \frac{\cos A}{\cos B}$, prove that

$$a \tan A + b \tan B = (a + b) \tan \frac{A + B}{2}.$$

10. Show that in any triangle

$$(b + c - a)\tan\frac{A}{2} = (c + a - b)\tan\frac{B}{2} = (a + b - c)\tan\frac{C}{2}$$

$$= \left\{\frac{(b + c - a)(c + a - b)(a + b - c)}{a + b + c}\right\}^{\frac{1}{2}}.$$

If R, r_a, r_b, and r_c are the radii respectively of the circumscribed and three escribed circles of a triangle, show that

$$Rr_a(s-a) = Rr_b(s-b) = Rr_c(s-c) = \frac{abc}{4}$$

where s is the semi-perimeter of the triangle.

11. Find the area of a triangle in terms of (1) two sides and the angle between them; (2) two angles and the side between them.

12. Define the characteristic and the mantissa of a logarithm. Find the logarithm of 5 when the base is 3; and the logarithm of $\frac{1}{3}$ when the base is 5. $\text{Log}_{10} 3 = \cdot 4771213$; $\log_{10} 2 = \cdot 3010300$.

13. Two sides of a triangle are 9 and 7, and the included angle is $38° 56' 32''\cdot 8$. Find the base and remaining angles.

$$\left.\begin{array}{l} L \tan 19° 29' = 9\cdot 5487471 \\ L \tan 19° 28' = 9\cdot 5483452 \end{array}\right\}.$$

HIGHER MATHEMATICS.

III.—CONIC SECTIONS AND DIFFERENTIAL CALCULUS.

[*Full marks may be obtained for about* three-quarters *of this paper. Great importance will be attached to accuracy.*]

1. Define a conic section.

The endless string PAQ passes through a ring A fixed to a table, and two beads P and Q, strung upon it, move on the table so that it is always stretched with PQ perpendicular to and bisected by a fixed straight line through A. Find the locus of either P or Q.

2. Prove that the tangents at the extremities of a focal chord of a conic section intersect in the directrix, and that they are at right angles to one another when the curve is a parabola.

Two circles pass through the focus of a parabola and touch the parabola, one at each extremity of a focal chord. Prove that they intersect at right angles.

3. In the ellipse prove that

$$PN^2 : AN \,.\, NA' :: BC^2 : AC^2,$$

the letters having their ordinary meaning.

Deduce the corresponding property of the parabola by considering that curve as the limiting case of the ellipse.

4. Indicate by a figure the positions of the points determined by the equations

(1) $y = (x - 1)e^x + (x + 1)e^{-x}$ when $x = 1$ and -1 respectively.

(2) $r = a\dfrac{\theta - 2}{\theta + 2}$ when $\theta = 1$ and -1 respectively.

5. Find the area of the triangle contained between the lines

$$x - y + 1 = 0, \quad x + y - 1 = 0, \quad 2x + 3y + 4 = 0.$$

6. Prove that the circles

$$x^2 + y^2 - 2x - 4y - 4 = 0$$
$$x^2 + y^2 - 8x - 12y + 36 = 0$$

intersect at right angles, and find the length of their common chord.

7. Find the equation of the tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

If two points be taken on the minor axis of an ellipse, each at a distance (ae) from the centre, prove that the sum of the squares of the perpendiculars from these points upon any tangent to the ellipse is equal to $2a^2$.

8. Prove that the equation of the hyperbola referred to the asymptotes is of the form

$$xy = m^2.$$

Prove also that the polar equation (between p and r) of the rectangular hyperbola is

$$pr = 2m^2.$$

9. Find from definition the differential coefficients of e^x and $\tan x$.

10. Find the limiting value of

$$\frac{\tan ax - \tan ma}{\sin^2 ax - \sin^2 ma} \text{ when } x = m.$$

11. Find the conditions that $f(x)$ should be a maximum or minimum.

A line AB of given length always passes through a fixed point and has one end A on the axis of x; find its inclination to that axis when the distance of its middle point from the axis of y is a maximum.

12. Find the radius of curvature ρ at any point of a polar curve.

If $r \propto \cos^2 \frac{\theta}{2}$, prove that $\rho \propto \sqrt{r}$.

13. Trace the curve

$$(x - a)^2 y^2 = x(x + a)^3.$$

IV.—STATICS AND DYNAMICS.

[*Full marks may be obtained for about* three-quarters *of this paper. Great importance will be attached to accuracy.*]

1. Prove the parallelogram of forces for the direction of the resultant.

The sides AB, BC, CD, and DA of the quadrilateral $ABCD$ are bisected in E, F, G, and H respectively. Prove that if two forces parallel and equal to AB and DC act at any point, their resultant will be parallel to HF and equal to $2HF$.

2. If the moments of two forces in one plane about any point in the plane be equal and opposite, prove that their resultant passes through that point.

3. Define, and show how to find, the centre of gravity of given weights rigidly connected. Find the centres of gravity of a triangle and a pyramid respectively.

If a quadrilateral when suspended by one of its angles rests with the diagonal through that angle vertical, prove that this diagonal bisects the area of the quadrilateral.

4. State the conditions of equilibrium of any number of forces in one plane. The uniform beam DE of weight W rests at the point E on the smooth inclined plane AB, and is movable about a hinge at D, which is parallel to the line of intersection of the horizontal and inclined planes.

If θ and a be the inclinations of DE and AB to the horizon, find the pressure at E and the strain on the hinge.

5. State the laws of friction.

If, in the last question, the inclined plane AB, remaining smooth, be the face of a prism of the same weight as the beam resting upon the horizontal plane in which D is situated, find the value of θ in the limiting position of equilibrium, assuming that the horizontal plane is rough and that the coefficient of friction between it and the base of the prism is μ. Find also the greatest value of μ for which slipping is possible.

6. Find the relation of the power to the weight in a system of heavy pulleys in which each hangs by a separate string.

If there be four pulleys with weights W, $2W$, $3W$, and $4W$, beginning with the lowest, find the power necessary to support the weight $15W$.

7. Explain the term "relative velocity," and state how to find it in the case of two particles whose actual velocities are known.

Two persons are describing concentric circles of radii 50 and 100 yards respectively in the same time, viz., 4 minutes, and in the same direction, each walking uniformly. Find their relative velocity when they subtend an angle of 60° at the common centre of the circles.

8. Prove the formula $s = vt + \frac{ft^2}{2}$ in the case of uniformly accelerated motion. By what laws of motion can it be made applicable to the case of a projectile?

9. A particle is projected from a point in a plane inclined at the angle a to the horizon with the velocity v at the angle θ to the plane. The direction of projection is perpendicular to the line of intersection of the inclined and horizontal planes. Find the range on the plane.

If the plane be smooth and inelastic determine the motion of the projectile after it strikes the plane, and prove that the time between its striking the plane and returning to the original point of projection is

$$\frac{4v \cos(\theta + \alpha)}{g \sin 2\alpha}.$$

10. Find the acceleration of a weight on a smooth inclined plane.

Two particles P and Q start at the same instant from A down two smooth inclined lines meeting at A, prove that the straight line PQ is always parallel to itself.

11. Find the acceleration when two unequal weights are connected by a string over a smooth pulley.

If the weights be originally equal, and one be uniformly increased, and the other uniformly diminished, each at the rate of $\frac{1}{n}$th part of its original value per second, find the velocity at the end of t seconds.

12. Find the velocities of two imperfectly elastic balls of given masses after direct impact.

Prove that momentum is conserved but that vis viva is lost.

December 1886.

I.—ALGEBRA AND MENSURATION.

1. Simplify

$$\frac{a^2 + bc + ca + ab}{a^2 + 2bc + 2ca + ab} \times \frac{a^3 + 8c^3}{a^4 + a^2c^2 + 6ac^3 + 4c^4}.$$

2. Find the square root of

$$x^4 + y^4 - xy(x^2 + y^2)\sqrt{2} + \tfrac{5}{2}x^2y^2.$$

3. Solve the equations

(1) $\sqrt{(x-2)(x-3)} + 5\sqrt{\frac{x-2}{x-3}} = \sqrt{x^2 + 6x + 8}$;

(2) $\begin{cases} x^3 + 1 = 81(y^2 + y) \\ x^2 + x = 9(y^3 + 1). \end{cases}$

4. Find the value when $x = \frac{1}{4}\sqrt{3}$, of

$$\frac{1 + 2x}{1 + \sqrt{1 + 2x}} + \frac{1 - 2x}{1 - \sqrt{1 - 2x}}.$$

5. A and B start from the same point, B five days after A. A travels 1 mile the first day, 2 miles the second, 3 miles the third, and so on; B travels 12 miles a day. When will they be together?

Explain the double answer.

6. If A vary as B when C is invariable, and A vary as C when B is invariable, then will A vary as the product of B and C when both B and C are variable.

If P and Q vary respectively as $y^{\frac{1}{2}}$ and $y^{\frac{1}{3}}$ when z is constant, and as $z^{\frac{1}{3}}$ and $z^{\frac{1}{2}}$ respectively when y is constant, and if $x = P + Q$, find the equation connecting x, y, and z; it being known that

$$\text{when } y = z = 64, \quad x = 12;$$
$$\text{and when } y = 4z = 16, \quad x = 2.$$

7. Find the number of permutations of n things taken r at a time.

Out of 21 consonants and 5 vowels, how many different arrangements can be made with 4 consonants and 2 vowels in each arrangement?

8. (1) Find the numerically greatest term in the expansion of $(1 + x)^n$, where x is less than unity, and n is positive.

(2) Find by the Binomial Theorem the cube root of 128 to seven places of decimals.

(3) Find the coefficient of x^r in the expansion of $(6-7x+2x^2)^{-1}$ in ascending powers of x.

9. If a, b, c are in Geometrical Progression, then $\log_a x$, $\log_b x$, $\log_c x$ are in Harmonical Progression.

Given

$$\left.\begin{aligned} \log_{10} 126765 &= 5{\cdot}10300, \\ \log_{10} 200000 &= 5{\cdot}30103, \\ \log_{10} 300000 &= 5{\cdot}47712; \end{aligned}\right\}$$

find the values of $\frac{2^{70}}{5^{30}}$; $\log_{10} 313$; $\log_6 10$ to three places of decimals.

10. Expand a^x in ascending powers of x; and prove that

$$\log_e(x+1)=2\log_e x-\log_e(x-1)-2\left\{\frac{1}{2x^2-1}+\frac{1}{3}\frac{1}{(2x^2-1)^3}+\&c.\right\}$$

11. Sketch plan and calculate the area of a field *ABEGFDC* from the following notes:—

	Yards to *G*	
	204	
to *F* 94	198	
	122	10 to *E*
to *D* 64	117	
to *C* 14	88	
	63	70 to *B*
	From *A*	

N.B.—The middle column gives the distances from *A* of the feet of the perpendiculars drawn from the angular points *F*, *D*, *C*, *E*, *B*, upon the straight line *GA*; the columns to left and right give the length of these perpendiculars measured to the left and right respectively.

12. A frustrum of a pyramid has rectangular ends, the sides of the base being 25 and 36 feet. If the height of the frustrum be 60 feet, and its volume 50480 cubic feet; find the area of the top.

Find, to the nearest foot, the radius of the sphere whose volume is equal to the volume of the frustrum. ($\pi = 3\frac{1}{7}$.)

13. Find the cost of the canvas, 2 feet wide, at 3*s.* 6*d.* a yard, required to make a conical tent, 12 feet in diameter and 8 feet high. ($\pi = 3\frac{1}{7}$.)

II.—EUCLID AND TRIGONOMETRY.

[*In answering the questions on geometry ordinary abbreviations may be employed, but the method of proof must be geometrical. Great importance will be attached to accuracy.*]

1. Divide a given straight line into two parts, so that the rectangle contained by the whole line and one of the parts may be equal to the square on the other part.

If the straight line PQ is divided in R, so that $PQ \cdot QR = PR^2$, and PR is divided in S, so that $PR \cdot RS = PS^2$, prove that $PS = RQ$.

2. Describe a square that shall be equal to a given rectilineal figure.

3. Show that on the same straight line and on the same side of it, there cannot be two similar segments of circles, not coinciding with one another.

4. Prove that the angle at the centre of a circle is double of the angle at the circumference on the same base, that is on the same part of the circumference.

If a chord be drawn through one of the points of intersection of two circles to cut the two circles, its whole length will subtend a constant angle at the other point of intersection.

5. About a given circle describe a triangle equiangular to a given triangle. Given the vertical angle and the segments of the base of a triangle made by the inscribed circle, construct it.

6. Equal triangles which have one angle of the one equal to one angle of the other, have their sides about the equal angles reciprocally proportional.

Construct an isosceles triangle equal in area to a given scalene triangle, and having a common vertical angle.

7. If BN and CN are drawn perpendicular respectively to the sides AB, AC of a triangle ABC, and meet in N, show that the rectangle $BC \, . \, AN$ is equal to the sum of the rectangles $AC \, . \, BN$ and $AB \, . \, CN$.

8. Establish the formulæ

(1) $\cos A(1 - \tan 2A \tan A) = \cos 3A(1 + \tan 2A \tan A)$.

(2) $2 \operatorname{cosec} 4A + 2 \cot 4A = \cot A - \tan A$.

9. Show that the sines of the angles of a triangle are proportional to the opposite sides, and hence deduce from the result in question 7 that

$$\sin (B + C) = \sin B \cos C + \cos B \sin C.$$

10. If α, β, and γ are the perpendiculars from the angles of a triangle ABC on the opposite sides, show that

$$a \sin A + b \sin B + c \sin C = 2(\alpha \cos A + \beta \cos B + \gamma \cos C).$$

Also prove that

$$\frac{\sin A}{a} + \frac{\sin B}{b} + \frac{\sin C}{c} = \frac{12 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}}{a + b + c}.$$

11. In any triangle find *two* symmetrical expressions for R (the radius of the circle circumscribing the triangle) in terms of a, b, c, A, B, C.

Prove also that $abc \cdot r = 4R(s - a)(s - b)(s - c)$, where R and r are the radii of the circumscribed and inscribed circles respectively, and $s = \frac{1}{2}(a + b + c)$.

12. Prove that $\log \frac{a}{b} = \log a - \log b$; and show how, by the use of a subsidiary angle, the logarithm of $a^2 + b^2$ may be found.

Two sides of a triangle are 11 and 9 feet respectively, and the included angle is 35° 5′ 49″. Find the base and the remaining angles.

$$\left.\begin{aligned} L \tan 17^\circ\, 33' &= 9{\cdot}50004 \\ L \tan 17^\circ\, 32' &= 9{\cdot}49960 \end{aligned}\right\}.$$

13. A line $\sqrt{3 + \sqrt{5}}$ units in length is drawn from any point in a horizontal plane in a direction (in the plane) at right angles to the line joining that point to the base of a flagstaff standing in the plane. The angles of elevation of the flagstaff at the two ends of the first line are 30° and 18°. Find the height of the flagstaff.

HIGHER MATHEMATICS.

III.—CONIC SECTIONS AND DIFFERENTIAL CALCULUS.

[*Full marks may be obtained for about* three-quarters *of this paper. Great importance will be attached to accuracy.*]

1. Prove that the tangent at any point of a parabola bisects the angle between the focal distance and the perpendicular upon the directrix.

If P be a point on the parabola, N the foot of the ordinate, and G the intersection of the normal at P with the axis: prove that the chord cut off from SP by the circle described round NPG is of the same length for all positions of P.

2. Prove that the feet of the perpendiculars from the foci on the tangent to an ellipse lie on a fixed circle.

Given the focus and three tangents to an ellipse, construct the ellipse.

3. Define the hyperbola, and prove from the definition that the difference of the distances of any point on the hyperbola from the foci is constant.

A circle of variable radius touches each of two given circles externally: prove that its centre lies upon one branch of an hyperbola, and find the eccentricity. How must the enunciation be altered so that both branches may be described?

4. Find the equation of a straight line.

Find the lengths of the sides of the parallelogram bounded by the four lines

$$\sqrt{3}.x + y = 0, \quad \sqrt{3}.y + x = 0, \quad \sqrt{3}.x + y = 1,$$
$$\sqrt{3}.y + x = 1.$$

5. Find the equation of the parabola.

Two parabolas have parallel axes: prove that the locus of the middle points of all lines intercepted between them parallel to the axes is another parabola.

6. Find the equation of the tangent to the parabola $y^2 = 4ax$ at the point x', y'.

If this tangent meet another parabola, $y^2 = 4bx$, in the points whose ordinates are y_1 and y_2, prove that y' is the harmonic mean between y_1 and y_2.

7. Find the equation of the diameter of a given system of parallel chords in an ellipse.

Prove that the locus of the middle points of all the chords of an ellipse which pass through a given point on the major axis is a similar ellipse passing through the given point.

8. Prove that the lines $y = \pm \frac{b}{a}x$ are asymptotes to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

If $a = b$, prove that that portion of every normal which is intercepted between the axes is bisected by the hyperbola.

9. Prove that the limit, when $x = 0$ of $\frac{e^x - 1}{x}$ is 1, and deduce the differential coefficient of e^x.

Find also the differential coefficient of $\sec x$.

10. Differentiate

(1) $y = \frac{1 - x}{\sqrt{1 + x^2}}$. (2) $y = x^c \log_a x$.

11. Find the value of $x^2 \operatorname{cosec} x \cot x$, when $x = 0$.

12. Investigate the condition that $f(x)$ may be a maximum or minimum.

Prove that the greatest value of $\frac{2x\sqrt{9 + 3x^2}}{9 + 7x^2}$ is $\frac{1}{2}$.

13. Prove the formulæ $\frac{1}{p^2} = \left(x\frac{dy}{ds} - y\frac{dx}{ds}\right)^{-2} = u^2 + \left(\frac{du}{d\theta}\right)^2$ in plane curves, where $u = \frac{1}{r}$.

In the curve $r = \frac{a}{\theta}$ prove that the polar subtangent is of constant length.

14. Find the asymptote of, and trace the curve $ay^2 - x^2y - x^3 = 0$.

IV.—STATICS AND DYNAMICS.

[*Full marks may be obtained for about* three-quarters *of this paper. Great importance will be attached to accuracy.*]

1. State the parallelogram of forces. Assuming it for direction, prove it for magnitude.

The sides BC and AC of the triangle ABC are bisected in D and E respectively. Prove that four forces represented in direction, magnitude, and lines of action by AD, BE, CE, and CD are in equilibrium.

2. Define the moment of a force about a point, and prove that its magnitude is proportional to the area of the triangle with the point as vertex and the line representing the forces as base. How is the sign determined ?

Three forces act at a point O within the triangle ABC represented by OA, OB, OC. Prove that if L, M, and N be the algebraical sum of their moments in any one direction round A, B, and C respectively, then $L = -(M + N)$.

3. Show how to find the centre of gravity of any given heavy particles rigidly connected.

Weights 3, 3, 4, 5, 1, 6 are placed in the foregoing order at the successive angles of a regular hexagon; find their centre of gravity.

4. Determine the condition of equilibrium when a heavy body is placed upon a horizontal plane.

A pile of four penny pieces stands upon a table; find the greatest possible horizontal distance between the centres of the highest and lowest.

5. What are the conditions of equilibrium of any number of forces in one plane?

Two heavy rods AOB and COD equal to each other in all respects are movable about a pivot at O where AO and CO are one-third of AB and CD respectively; the system stands upright with B and D upon a smooth floor, and A and C connected by a weightless string of length AO or CO. Find the tension of the string in terms of the weight of either rod.

6. State the laws of friction.

If in the last case the plane be rough, and the string AC be removed, find the least value of the coefficient of friction in order that the same position may be retained.

7. Find the mechanical advantage of the screw.

8. Show how to find the relative velocity of two moving bodies.

Find the relative velocity of the extremities of the hands of a clock of lengths one foot and 9 inches respectively, at three o'clock, assuming π to be $\frac{22}{7}$.

9. Find the path of a projectile in a vacuum.

After how long a time will the direction of motion be perpendicular to that of projection, V being the velocity of projection, a the angle of projection?

10. A stone thrown by a person from the stern of a steamer with relative velocity V at an angle α would just reach the prow if the vessel were moving uniformly.

Where would it reach the deck if with the same circumstances of projection the vessel were moving with uniform acceleration $\frac{g}{n}$?

11. By what principles are the velocities of spherical balls, after direct impact, calculated (1) when the spheres are inelastic; (2) when they are elastic?

An 80-ton gun on a smooth horizontal plane projects a bolt of 5 cwt. horizontally with a velocity of 1,200 ft per second. What is the velocity of recoil?

12. Find the acceleration and tension of the string when two unequal weights connected by a string over a smooth pulley are in free motion under gravity.

If ACB be the string, C the pulley, and a weight of 5 lbs. be attached at A, a weight of 3 lbs. at B, and another of 3 lbs. between B and C, and if B be originally 11 ft. above the ground, find the distance above B of the second weight in order that the latter may just reach the ground. Find also the time of motion. [$g = 32$.]

July 1887.

I.—ALGEBRA AND MENSURATION.

(Algebra, up to and including the Binomial Theorem; the Theory and Use of Logarithms; and Mensuration.)

[*Great importance will be attached to accuracy.*]

1. Simplify the expressions:—

(1) $\left\{\frac{a+b}{2(a-b)} - \frac{a-b}{2(a+b)} + \frac{2b^2}{a^2-b^2}\right\}\frac{a-b}{b}$.

(2) $[(ab)^{\frac{1}{2}} + (ac)^{\frac{1}{2}} + (bd)^{\frac{1}{2}} + (cd)^{\frac{1}{2}}][(ab)^{\frac{1}{2}} - (ac)^{\frac{1}{2}} - (bd)^{\frac{1}{2}} + (cd)^{\frac{1}{2}}]$.

2. Multiply together $x^{\frac{1}{n}} - x^{-\frac{1}{n}}$ and $x^{\frac{2}{n}} + 1 + x^{-\frac{2}{n}}$; and divide $x^{12} + \frac{1}{x^{12}} + 6\left(x^8 + \frac{1}{x^8}\right) + 15\left(x^4 + \frac{1}{x^4}\right) + 20$ by $x^6 + \frac{1}{x^6} + 3\left(x^2 + \frac{1}{x^2}\right)$.

3. Find the value of

$$(9 + 4\sqrt{5})^{\frac{1}{3}} + (9 - 4\sqrt{5})^{\frac{1}{3}},$$

and extract the square root of

$$x^2 - bx - \frac{ab}{3} + \frac{2ax}{3} + \frac{b^2}{4} + \frac{a^2}{9}.$$

4. Solve the equations

(1) $\sqrt{4x^2 + 2x + 7} = 12x^2 + 6x - 119.$

(2) $\begin{cases} x^2 - 2xy + y^2 + 2x + 2y - 3 = 0. \\ y(x - y + 1) + x(x - y - 1) = 0. \end{cases}$

5. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, prove that

$$\frac{a}{b} = \cdots = \frac{(3c^2 + 4e^2)^{\frac{1}{2}}}{(3d^2 + 4f^2)^{\frac{1}{2}}}.$$

6. If of n things p are alike and the remainder unlike, show how to find the number of combinations of them taken r together.

In how many ways can a guard of 10 soldiers be selected from a company of 100, and in how many of these would two particular individuals be included?

7. Prove the Binomial Theorem for positive integral indices.

If $(1 + x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + \ldots$, show that

$$2C_0 + 2^2\frac{C_1}{2} + 2^3\frac{C_2}{3} + 2^4\frac{C_3}{4} + \ldots = \frac{3^{n+1} - 1}{n + 1}.$$

8. On a division in the House of Commons, if the number of members for the motion had been increased by 50 from the other side, the motion would have been carried by 5 to 3; but if those against the motion had received 60 from the other party the motion would have been lost by 4 to 3. Did the motion succeed, and how many members voted on the question?

9. Prove that $e^x = 1 + x + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \frac{x^4}{\lfloor 4} + \ldots$

If $x = y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \ldots$, find y in ascending powers of x.

10. Find the number whose logarithm is $-$ 1·6805452, having given

$$\log 20{\cdot}867 = 1{\cdot}3194600,$$
$$\log 20{\cdot}866 = 1{\cdot}3194392.$$

11. A public garden occupies two acres, and is in the form of a square. If a pathway goes completely round its inner edge, and occupies one-eighth of an acre, what is its width?

12. A cubical box, 5 feet deep, is filled with layers of spherical balls, whose diameters, where they touch, are in vertical and horizontal lines.

Find what portion of the space in the box would be left vacant if the diameter of a ball is half an inch.

13. A cathedral has two spires and a dome; each of the former consists, in the upper part, of a pyramid 60 feet high, standing on a square base, of which a side is 20 feet. The dome is a hemisphere of 40 feet radius.

Find the cost of covering the three with lead at $7\frac{1}{2}d.$ per square foot.

[Take $\pi = \frac{22}{7}$.]

II.—EUCLID AND TRIGONOMETRY.

[*Ordinary abbreviations may be used in answers to the first six questions, but the method of proof must be geometrical. Great importance will be attached to accuracy.*]

1. If a straight line be bisected, and produced to any point, prove that the rectangle contained by the whole line thus produced, and the part of it produced, together with the square on half the line bisected, is equal to the square on the straight line which is made up of the half and the part produced.

If ABC be an isosceles triangle, and D be a point on the base BC produced; show that the difference of the squares on AD and AC is equal to the rectangle contained by BD and CD.

2. Draw a straight line from a given point, either without or in the circumference, to touch a given circle.

3. If ABC be an acute-angled triangle inscribed in a circle (BAC being the greatest angle of the triangle), and if AD, AE chords of the circle, be drawn cutting BC in F and G respectively, so that the angles AFG and AGF are each equal to BAC; find the angles of the pentagon $ABDEC$ in terms of the angles of the triangle ABC.

4. Inscribe a circle in a given triangle.

Show that the radius of the circle inscribed in an equilateral triangle is one-third of that of any one of the escribed circles.

5. If two triangles have one angle of the one equal to one angle of the other, and the sides about two other angles proportionals; then, if each of the remaining angles be either less, or not less, than a right angle, or if one of them be a right angle, the triangles shall be equiangular to one another, and shall have those angles equal about which the sides are proportionals.

6. Prove that similar triangles are to one another in the duplicate ratio of their homologous sides.

If ABC be an obtuse-angled triangle, having the obtuse angle BAC; and if AD, AE, be drawn to meet BC in D and E, so that the angles ADB, AEC, are each equal to the angle BAC; show that the three triangles ABD, AEC, ABC, are similar to one another; and are to one another as the squares on the sides of the triangle ABC.

7. Given a circle whose radius is unity, and an acute angle at the centre of the circle; find lines to represent the circular measure of the angle and its trigonometrical ratios.

Trace the changes in the value of the cosine as the angle increases to $360°$.

8. Find the values of $\sin 15°$ and $\cos 15°$.

Show that

$$\sin 10° + \sin 20° + \sin 40° + \sin 50° = \sin 70° + \sin 80°.$$

9. Prove that

(1) $\cos 4\theta = \dfrac{\cot^2\theta - 6 + \tan^2\theta}{\cot^2\theta + 2 + \tan^2\theta}$;

(2) $\tan(A + B + C) =$

$$\frac{(\tan A + \tan B + \tan C) - \tan A \tan B \tan C}{1 - (\tan B \tan C + \tan C \tan A + \tan A \tan B)}.$$

10. Prove that, in a plane triangle,

(1) $a^2 = b^2 + c^2 - 2bc \cos A$;

(2) $\sin \dfrac{A}{2} = \sqrt{\dfrac{(s - b)(s - c)}{bc}}$.

11. In a plane triangle, given

$$b = 2 \text{ ft. } 6 \text{ in.}, \quad c = 2 \text{ ft.}, \quad A = 22^\circ 20';$$

find the other angles; and then show that the side a is very approximately 1 foot; given

$$\begin{aligned} \log 2 &= \cdot 30103, \\ \log 3 &= \cdot 47712, \\ L \sin 22^\circ 20' &= 9\cdot 57977, \\ \log \cot 11^\circ 10' &= \cdot 70465, \\ L \tan 29^\circ 22' 20'' &= 9\cdot 75038, \\ L \tan 29^\circ 22' 30'' &= 9\cdot 75043, \\ L \sin 49^\circ 27' 34'' &= 9\cdot 88079. \end{aligned}$$

12. Show that, if R be the radius of the circle circumscribed to a triangle,

$$R = \frac{a}{2 \sin A}.$$

HIGHER MATHEMATICS.

III.—CONIC SECTIONS AND DIFFERENTIAL CALCULUS.

[*Full marks may be obtained for about* four-fifths *of this paper.*]

1. In a conic section the portion of any tangent between the point of contact and the directrix subtends a right angle at the focus.

2. In a parabola with vertex A the perpendicular SY upon the tangent at P from the focus S is a mean proportional between AS and SP.

If the tangents at the extremities of any focal chord Pp meet at T, prove that ST is a mean proportional between AS and Pp.

3. In the ellipse the normal at any point bisects the angle between the focal distances of that point.

A number of ellipses have their major axes of equal lengths and one focus fixed; if they all touch a given straight line, find the curve upon which the remaining focus of each is situated.

4. Prove that the triangles between every tangent to an hyperbola and the asymptotes are equal in area.

5. Every plane section of a right cone parallel to one of the generating lines is a parabola.

Give a construction for finding the focus.

6. Find the equation of a straight line in terms of the intercepts upon the axes.

Two parallel straight lines AA' and BB' cut the axis of x in A, A', and the axis of y in B, B', their distances from the origin O being as $1 : m$. Prove that the coordinates of the point of intersection of AB' and $A'B$ are $\frac{OA'}{1+m}$ and $\frac{OB'}{1+m}$ respectively.

7. Find the equation of the tangent to the parabola $y^2 = 4ax$ in terms of the tangent of its inclination to the axis.

Hence show that two tangents can be drawn to a parabola from a point (hk) *without* the parabola.

8. Prove that the locus of a point, the sum of the squares of whose distances from two intersecting straight lines is constant, is an ellipse.

Find the directions of its principal axes.

9. Find the differential coefficients of a^x and $\tan x$ from first principles, and differentiate the functions

(1) $x\sqrt{a^2 - x^2}$,

(2) $\frac{\sin^{-1} x}{\sqrt{1 - x^2}}$.

10. If $y^2 = a - bx^2$, prove that $\frac{d^2y}{dx^2} + \frac{1}{y}\left(\frac{dy}{dx}\right)^2 = \frac{1}{x} \cdot \frac{dy}{dx}$.

11. State the different indeterminate forms which functions may assume for given values of the independent variable.

Find the value of $\frac{\log(1 - x^2)}{\log \cos x}$ in the limiting case when $x = 0$.

12. Investigate the conditions that $f(x)$ should be a maximum or minimum.

A rough common is bordered by a road ABC turning through a right angle at B, the portions AB and BC being straight lines one mile and half a mile respectively. Find the shortest time in which a man could go from A to C, his average speeds on the road and common being 150 and 120 yards per minute respectively.

13. Prove that the length of the polar subtangent to the curve $r = f(\theta)$ is $r^2\frac{d\theta}{dr}$.

Apply this to find the polar subtangent to the curve $r = a \sec^2 \frac{\theta}{2}$.

14. Prove that the radius of curvature at a point in a curve referred to polar ordinates is $r\frac{dr}{dp}$.

If a small portion of a curve is given to you, having a radius of curvature greater than the dimensions of the paper on which it is drawn, what construction would you make (with the aid of compasses and a ruler) to determine the radius of curvature *by this method?*

IV.—STATICS AND DYNAMICS.

[*Full marks may be obtained for about* four-fifths *of this paper. Great importance will be attached to accuracy.*]

1. If A, B, C, D, be the angular points taken in order of a quadrilateral, prove that the four forces represented in magnitude, direction, and line of action by AB, AD, CB, and CD cannot be in equilibrium unless the quadrilateral is a parallelogram.

2. Define the moment of a force about a point as to magnitude and direction, and prove that the moment of the resultant of two forces about a point is equal to the sum of the moments of the components.

Show that the sum of the moments of any three forces represented in magnitude and position by the sides of a triangle taken in order is the same about every point in the plane of the triangle.

3. Find the centre of gravity of a number of weights rigidly connected. Find the centre of gravity of a pyramid on any polygonal base.

4. State the laws of friction.

The triangular lamina ABC, right-angled at B, stands with BC upon a rough horizontal plane. If the plane be gradually tilted round an axis in its own plane perpendicular to BC, with the angle B downwards, prove that it will begin to slide or topple over according as the coefficient of friction is less or greater than tan A.

5. State the conditions of equilibrium when any number of forces act upon a rigid body in one plane.

A pair of compasses, each of whose legs is a uniform bar of weight W, is supported hinge downwards by two smooth pegs at the middle points of the legs in the same horizontal line, the legs being kept apart at an angle A with one another by a weightless rod joining their extremities; find the thrust in this rod.

6. Find the relation of the power to the weight in a system of (n) weightless movable pulleys in which each hangs by a separate string.

A man of 12 stone weight is suspended by the lowest of such a system of four pulleys, and supports himself by pulling at the end of the string which passes over the fixed pulley. Find the amount of his pull on this string, neglecting the inclinations of the strings to the vertical.

7. Prove the formulæ

$$s = ut + \frac{ft^2}{2}, \qquad v^2 = u^2 + 2fs,$$

in uniformly accelerated motion.

Two particles start at the same instant from rest, with velocities u and v respectively; the motion of the first is uniformly retarded, while that of the second is uniform. Prove that by the time the first comes to rest the distances traversed by them are to one another as $u : 2v$.

8. Find the range of a projectile on a horizontal plane.

A room has a square floor and height equal to half its length. How must a ball be projected from the middle point of the floor perpendicularly to one wall, so as just to graze the middle point of the roof on the rebound? The ball and the walls are supposed to be perfectly elastic.

9. Find the acceleration of a particle down an inclined plane, (1) smooth, (2) rough.

Two inclined planes of equal length, the one rough, the other smooth, are tilted at angles a, a', respectively, so that a particle takes the same time in sliding down either. Find the coefficient of friction of the rough plane.

10. State the three laws of motion.

A heavy body is placed upon a smooth table and connected by a string laid on the table, and passing over its edge, with a weight of 1 lb. hanging vertically. Find the weight of the body that it may acquire a velocity of 1 foot per second in the first second, gravity being 32.

11. How is impact measured?

A body, whose mass is m lbs., is attached to a weightless and inextensible string passing over a smooth pulley and fastened at the other end to another body of m' lbs. lying on a table, the line joining the position of m' and the pulley being inclined a to the vertical. If the string be at first slack and become stretched after m has fallen through 1 foot, find the impulsive tension of the string and the velocity then imparted to m'.

12. Prove that in the last case the acceleration of m, directly after the string becomes stretched, is $\frac{m - m' \cos a}{m + m'} g$.

Find also the acceleration of m'.

13. Find the radial acceleration of a particle describing a given circle with given velocity.

A weightless inextensible string of length $(2l)$ is fastened at its extremities to two points A and B in the same horizontal line, distance (l) apart, and supports the weight W tied to its middle point. If W be projected perpendicular to the plane AWB with double the velocity requisite for it to describe a complete circle, find the greatest and least tension of the string.

If one portion of the string be cut when W is halfway between its highest and lowest points, determine the subsequent motion.

December 1887.

I.—ALGEBRA AND MENSURATION.

(Algebra, up to and including the Binomial Theorem; the Theory and Use of Logarithms; and Mensuration.)

[*Great importance will be attached to accuracy.*]

1. Reduce to its lowest terms

$$\frac{a^2 + b^2 + c^2 + 2ab + 2bc + 2ca}{a^2 - b^2 - c^2 - 2bc},$$

and simplify

$$\frac{\dfrac{a}{a+b} + \dfrac{b}{a-b}}{\dfrac{a}{a-b} - \dfrac{b}{a+b}}.$$

2. Find the Greatest Common Measure of

$$6x^5 - 9x^4 + 19x^3 - 12x^2 + 19x - 15 \text{ and}$$
$$4x^4 - 2x^3 + 10x^2 + x + 15;$$

and the Least Common Multiple of

$$x^2 - 3xy - 10y^2,\ x^2 + 2xy - 35y^2,\ x^2 - 8xy + 15y^2.$$

3. Simplify

$$\frac{1}{2} \cdot \frac{\sqrt{x^2 - 1}}{x + \sqrt{x^2 - 1} - 1} \cdot \frac{1 + \sqrt{\dfrac{x - 1}{x + 1}}}{1 - \dfrac{x - 1}{x + 1}}$$

$$+ \frac{1}{4} \cdot \frac{\sqrt{x + 1} - \sqrt{x - 1}}{x - \sqrt{x^2 - 1}} \cdot \frac{\sqrt{x - 1}}{\sqrt{\dfrac{x + 1}{x - 1}} + 1}.$$

and find the square root of $\frac{9}{4} + 6x - 17x^2 - 28x^3 + 49x^4$.

4. Solve the equations

(1) $(7 + x)(8 - x) - \frac{7x}{3} = 17x + 1 - x^2$.

(2) $ax + y = x + by = \frac{1}{2}(x + y) + 1$.

5. Solve the quadratic equations

(1) $\frac{3x - 6}{5 - x} + \frac{11 - 2x}{10 - 4x} = 3\frac{1}{2}$.

(2) $\left.\begin{array}{l} \dfrac{x^3}{y} = 108 - x^4 \\ \dfrac{y^3}{x} = \dfrac{4}{3} - y^4 \end{array}\right\}.$

6. A market-woman bought apples at three for a penny, and as many more at four for a penny; and thinking to make her money again, she sold them at seven for *2d.* She lost, however, *3d.* by the business. How much did she sell them for?

7. Find two numbers in the ratio of $1\frac{1}{2} : 2\frac{2}{3}$, such that, when increased each by 15, they shall be in the ratio of $1\frac{2}{3} : 2\frac{1}{2}$.

8. Find the total number of combinations of n things.

If C_r denote the number of combinations of n things taken r together, prove that

$$\frac{1}{n} C_1 + \frac{1}{n - 1} C_2 + \frac{1}{n - 2} C_3 + \ldots + \frac{1}{1} C_n = \frac{2}{n + 1}(2^n - 1).$$

9. Prove the truth of the Binomial Theorem when the index is fractional.

Give the fifth term of $(a^{-\frac{1}{3}} + 2x^{-\frac{1}{3}})^{-\frac{1}{6}}$.

10. Prove that $e^x = 1 + x + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \ldots$

Given $\log \frac{1}{2} = \bar{1}\cdot69897$, find x from the equation $20^x = 100$.

Also find the logarithm of 256 to the base $2\sqrt{2}$.

11. A halfpenny piece is one inch in diameter. Six halfpennies are placed so that each coin touches two others, their centres being all on the circumference of a circle. Find the area which they enclose.

12. A circular disk of lead, 3 inches in thickness and 12 inches diameter, is wholly converted into shot of the same density, and of ·05 inch radius each. How many shot does it make?

13. The interior of a building is in the form of a cylinder of 15 feet radius and 12 feet altitude, surmounted by a cone whose vertical angle is a right angle. Find the area of surface and the cubical content of the building.

II.—EUCLID AND TRIGONOMETRY.

[*Ordinary abbreviations may be used in answers to the first six questions, but the method of proof must be geometrical. Great importance will be attached to accuracy.*]

1. If a side of a triangle be produced, the exterior angle is equal to the two interior and opposite angles: and the three interior angles of every triangle are together equal to two right angles.

Prove the corollary with regard to the interior angles of any rectilineal figure; and show from it that four angles of a regular pentagon are equal to three angles of a regular decagon.

2. Describe a square equal to a given rectilineal figure.

3. Equal straight lines in a circle are equally distant from the centre; and those which are equally distant from the centre are equal to one another.

4. Draw a straight line from a given point, either without or in the circumference, which shall touch a given circle.

A straight line AB is divided into three equal parts at C and D; and CPD is an equilateral triangle. Prove that AP is a tangent to the circle BPC.

5. Inscribe an equilateral and equiangular hexagon in a given circle.

6. ABC is a triangle inscribed in a circle ABC. AD, perpendicular to BC, meets it in D, and is produced to meet the circle in E; and AF, parallel to CE, meets CB, or CB produced, in F.

Show (1) that the rectangle $BA \cdot AC$ is equal to that contained by AD and the diameter of the circle; (2) that the rectangle $BA \cdot AC$ is equal to the rectangle $AF \cdot EB$.

7. Show that

(1) $\sin 3A + \sin 2A - \sin A = 4 \sin A \cos \frac{A}{2} \cos \frac{3A}{2}$.

(2) $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{2}{9} = 45°$.

8. Find an expression for all the angles which have a given tangent.

If $4 \cot 2\theta = \cot^2\theta - \tan^2\theta$, prove that all possible values of θ are given by

$$\theta = n\pi \pm \frac{\pi}{4}.$$

9. Prove the formula

$$\sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}.$$

Hence show that

$$\frac{\sin^2\frac{A}{2}}{s-c} + \frac{\sin^2\frac{B}{2}}{b} + \frac{\sin^2\frac{C}{2}}{c} = \frac{1}{b}.$$

10. Explain carefully, and illustrate by figures, the case in which the solution of a plane triangle is ambiguous.

If $a = 9$, $b = 12$, $A = 30°$, find the values of c, having given

log 12 = 1·07918,	Log sin 30° = 9·69897,
log 9 = ·95424,	Log sin 11° 48′ 39″ = 9·31108,
log 171 = 2·23301,	Log sin 41° 48′ 39″ = 9·82391,
log 368 = 2·56635,	Log sin 108° 11′ 21″ = 9·97774.

11. Find the area of a triangle in terms of its sides.

If each of three circles, of radius r, touch the other two, show that the area included between the circles is nearly equal to the square upon one-fifth part of one of their diameters. (Take $\pi = 3·1416$.)

12. A flagstaff DE stands on a horizontal plane at D. From a point A in the plane, 30 feet from D, a road AC rises, making an angle of 30° with DA produced. If AC be measured $180\sqrt{3}$ feet up the road, it is found that the line of sight from C to E, the top of the flagstaff, will just pass through a point B in AD produced, such that DB equals 100 feet. Find the height of the flagstaff.

HIGHER MATHEMATICS.

III.—CONIC SECTIONS AND DIFFERENTIAL CALCULUS.

[*Full marks may be obtained for about* four-fifths *of this paper.*]

1. Define a parabola, and prove that, if A is the vertex, S the focus, and PN the ordinate of a point P of the curve,

$$PN^2 = 4AS \cdot AN.$$

B is any point in the axis, and the circle on AB as diameter intersects the parabola in two points P and Q; if PQ meets the axis in N, prove that the normals at P and Q bisect NB.

2. Prove that in any conic section the semi-latus rectum is the harmonic mean between the two segments of any focal chord.

3. Define conjugate diameters of an ellipse, and prove that, if the diameter DCd is conjugate to PCp, then PCp is conjugate to DCd.

In an ellipse SR, $S'R$, drawn from the foci perpendicular respectively to a pair of conjugate diameters, intersect in R; prove that the locus of R is an ellipse, having SS' for one of its axes.

4. The axes being rectangular, find the tangent of the angle between the two straight lines

$$y = mx + c, \qquad y = m'x + c'.$$

Find the equations of the straight lines through the point (4, 5) which are inclined at the angle 45° to the straight line

$$2y = x + 2.$$

5. Prove that the equation of the tangent at the point (x, y) of the circle, $x^2 + y^2 = c^2$, is

$$Xx + Yy = c^2,$$

and that, if α and β are the intercepts of the axes by this tangent,

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{1}{c^2}$$

6. Interpret the equations

(1) $r \cos \theta = a$, (2) $r = a \cos \theta$,

(3) $r \sin^2\theta = 4a \cos \theta$, (4) $2a = r(1 + \cos \theta)$,

$$(5) \quad \frac{1}{r^2} = \frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}.$$

7. Find the equation of the ellipse which has its centre at the origin, its axes coincident with the coordinate axes, and which passes through the two points whose coordinates are (2, 2) and (3, 1).

8. Trace the curve represented by the equation

$$xy = c^2,$$

and prove that the locus of the feet of the perpendiculars from the origin on the tangents to this curve is the curve

$$(x^2 + y^2)^2 = 4c^2xy.$$

9. Obtain from the definition the differential coefficients, with regard to x, of $\sin x$ and $\sin^{-1} x$. Differentiate with regard to x the expression

$$(1)\ \frac{x^2 + x + 1}{x^2 - x + 1}, \qquad (2)\ e^{\sin 2x}, \qquad (3)\ (\sin 2x)^{\sin x}.$$

10. Show how to determine, and to distinguish between, the maxima and minima values of a function of one variable.

If
$$\frac{dy}{dx} = \frac{(x-1)(x-2)^2}{(x-3)^3(x-4)^5},$$
find the values of x for which y has a maximum or minimum value.

11. If $y = e^x \log x$, prove that

$$x\frac{d^2y}{dx^2} + (1 - 2x)\frac{dy}{dx} + (x-1)y = 0.$$

Applying Leibnitz's theorem, prove that, when $x = 0$,

$$(n+2)\frac{d^{n+2}y}{dx^{n+2}} - (2n+3)\frac{d^{n+1}y}{dx^{n+1}} + (n+1)\frac{d^ny}{dx^n} = 0.$$

12. Prove that, at a point of inflection in a curve, $\frac{d^2y}{dx^2}$ changes sign.

Find the two points of inflection of the curve

$$\frac{y}{a} = \frac{5x^2}{9a^2} + \left(\frac{x-a}{a}\right)^{\frac{5}{3}}.$$

13. Find the asymptotes of, and trace, the curves

$$(1)\ (y - x)(y^2 + x^2) = a^2x, \qquad (2)\ r(3\theta - \pi) = a\theta.$$

IV.—STATICS AND DYNAMICS.

[*Full marks may be obtained for about* four-fifths *of this paper. Great importance will be attached to accuracy.*]

1. If three forces, acting in one plane, equilibrate, prove that either they are parallel, or their lines of action meet in one point.

A picture is to be hung against a smooth vertical wall, at a given inclination to the wall, and with its lower edge touching the wall at a given height, and is to be supported by a single string fastened to a given point of the wall; find by a graphical construction the point on the back of the picture to which the string must be attached.

2. Find the position of the centre of gravity of the area of a triangle.

If ABC be a triangle, and E, F, the middle points of the sides AC, AB, prove that the centre of gravity of the area $CEFB$ divides the distance between EF and BC in the ratio of five to four.

3. If a rod without weight forms part of a system in equilibrium, and if the rod is connected with the system at its two ends only, prove that the stress at each end is in the direction of the rod.

Three rods AB, BC, CA, lying on a horizontal table, are jointed together at A, B, and C; and a tightened string connects two given points in AB and AC; find the directions of the stresses at A, B, and C.

4. If a heavy body is suspended from a fixed point about which it can move freely, prove that, when it is at rest, its centre of gravity is in the vertical line through the point of suspension.

A heavy triangular lamina is at rest inside a smooth hemispherical bowl; prove that the pressures at the three angular points are equal.

5. Enunciate the principle of Virtual Work.

Four equal heavy rods, jointed together, and forming a square $ABCD$, are suspended from the joint A, and the square form is maintained by a string connecting A and C; find the tension of the string, prove that the stresses at B and D are horizontal, and find these stresses.

6. A heavy body rests on a rough plane, inclined at the angle a to the horizontal, and is supported by a string in the direction of the line of greatest slope on the plane; if the coefficient of friction is less than tan a, find the greatest and least tensions of the string which are consistent with equilibrium.

7. Describe the Common Balance, and find its position of equilibrium when unequal weights are placed in the scale-pans.

8. Explain what is meant by the acceleration of a point moving in a straight line, and how it is measured.

Taking 72 as the measure of an acceleration when four feet and three seconds are units of length and time, find its measure when eight feet and twelve seconds are the units.

9. An elastic spherical ball moving on a smooth horizontal plane impinges directly on another ball of the same size at rest; find the ratio of the masses of the two balls in order that the first ball may be reduced to rest.

If the second ball, after a direct rebound from a vertical wall, again come into direct collision with the first ball, determine the subsequent motions of the two balls, it being given that the coefficient of elasticity between the wall and the second ball is the same as between the two balls.

10. Prove that the time of descent of a heavy particle down any chord of a sphere from its highest point is the same.

Find the position of the straight line from a given point outside a sphere to the surface of the sphere, down which the time of descent is the shortest.

11. Explain what is meant by the statement that action and reaction are equal and opposite.

A gun is mounted on a gun-carriage movable on a smooth horizontal plane, and the gun is elevated at the angle α to the horizontal. A shot is fired off, and leaves the gun in the direction inclined at the angle θ to the horizontal; if the mass of the gun and its carriage be n times that of the shot, prove that

$$\tan\theta = \left(1 + \frac{1}{n}\right)\tan\alpha.$$

12. Enunciate and explain the principle of the Conservation of Energy.

A shot is fired from a gun, which is fixed, with a certain charge of powder; if the quantity of powder is quadrupled, in what ratio will the velocity of the shot be increased?

July 1888.

I.—ALGEBRA AND MENSURATION.

(Algebra, up to and including the Binomial Theorem ; the Theory and Use of Logarithms ; and Mensuration.)

[*Great importance will be attached to accuracy.*]

1. Simplify

(1) $\left(\dfrac{a+b}{a-b}+\dfrac{a-b}{a+b}\right)\div\left(\dfrac{a^{\frac{1}{2}}+b^{\frac{1}{2}}}{a^{\frac{1}{2}}-b^{\frac{1}{2}}}-\dfrac{a^{\frac{1}{2}}-b^{\frac{1}{2}}}{a^{\frac{1}{2}}+b^{\frac{1}{2}}}\right)$,

(2) $\dfrac{(a+b\sqrt{-1})(a-b\sqrt{-1})(a^4-b^4)}{\left\{(a+b\sqrt{-1})^2+(a-b\sqrt{-1})^2\right\}\left\{a^3+b^3+ab(a+b)\right\}}$.

2. Show that

$$a^4+b^4+c^4+d^4-2a^2(b^2+c^2+d^2)-2b^2(c^2+d^2)-2c^2d^2 + 8abcd = \left\{(a+b)^2-(c+d)^2\right\}\left\{(a-b)^2-(c-d)^2\right\}.$$

3. Prove the rule for finding the Greatest Common Measure of two algebraical quantities, and find two algebraical quantities of the fourth degree in x having $a + 2x$ as their Greatest Common Measure.

4. Extract the square root of

$$4a^2+9b^2-11ab+4a^{\frac{3}{2}}b^{\frac{1}{2}}-6a^{\frac{1}{2}}b^{\frac{3}{2}},$$

and the cube root of

$$\frac{125}{8}x^3+\frac{225}{8}x^2y+\frac{135}{8}xy^2+\frac{27}{8}y^3+\frac{75x^2z}{4}+\frac{90xyz}{4}+\frac{27y^2z}{4} + \frac{15xz^2}{2}+\frac{9yz^2}{2}+z^3.$$

5. Solve the equations

(1) $2x^2-2x+2\sqrt{2x^2-7x+6}=5x-6.$

(2) $5x-3y=x+y+20.$

$x^3+y^2=11x-5y.$

6. A number consists of 3 digits in Geometrical Progression. The sum of the right-hand and left-hand digits exceeds twice the middle digit by unity; and the sum of the left-hand and middle digits is two-thirds of the sum of the middle and right-hand digits. Find the number.

7. Define Harmonical Progression, and insert 4 Harmonic Means between 2 and 12.

8. Prove that the number of combinations of n things taken r at a time is the same as the number of them taken $n - r$ at a time.

How many different sets at lawn-tennis might be formed out of a party of 10 ladies and 6 gentlemen, each set containing 2 ladies and 2 gentlemen?

9. Find the greatest term in the expansion of $(x + a)^n$; and write down the 5th term in the expansion of $(a + \sqrt{-b})^{\frac{1}{2}}$.

10. Assuming that $\log_e(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \&c.$, find a series for $\log_e \frac{n+1}{n}$, and explain the use of it.

11. Given log 2 = ·301030, log 3 = ·477121, and log 7 = ·845098, write down the logarithms of ·005, 6·3, and of $(\frac{49}{216})^{\frac{1}{3}}$.

Find the value of x from the equation $18^{8-4x} = (54\sqrt{2})^{3x-2}$ (using base $3\sqrt{2}$).

12. The Great Pyramid of Egypt stands on a base approximately 750 feet square, and its height is 450 feet. Find (1) its weight, if 1 cubic yard weighs $2\frac{1}{4}$ tons; (2) the length of a wall, 5 feet high and $1\frac{1}{2}$ feet thick, which might be built of the materials.

13. In a sphere of 6-inch radius a conical hole is bored so that the vertex is at the centre. If 50 square inches of surface be removed, what is the volume of the sphere which is left?

II.—ALGEBRA AND MENSURATION.

EXTRA PAPER.

(Algebra, up to and including the Binomial Theorem ; the Theory and Use of Logarithms ; and Mensuration.)

[*Great importance will be attached to accuracy.*]

1. Simplify

(1) $$\left\{\frac{y^2 - yz + z^2}{x} + \frac{x^2}{y+z} - \frac{3}{\frac{1}{y} + \frac{1}{z}}\right\} \cdot \frac{\frac{2}{y} + \frac{2}{z}}{\frac{1}{yz} + \frac{1}{zx} + \frac{1}{xy}} + (x+y+z)^2 ;$$

(2) $$\frac{2\sqrt{a-x}}{3\sqrt{a+x} - 2\sqrt{a-x}} + \frac{3\sqrt{a+x}}{3\sqrt{a+x} + 2\sqrt{a-x}}.$$

2. Resolve the expression $2(a^6 + b^6) - ab(a^2 + b^2)(2ab - 3a^2 + 3b^2)$ into 5 simple factors.

3. Prove that the sum or difference of any multiples of A and B is divisible by all the common divisors of A and B.

What numerical value of y will make the expressions

$$2(y^3 + y^2)x^3 + (11y^2 - 2y)x^2 + (y^2 + 5y)x + 5y - 1$$
$$\text{and } 2(y^2 + y)x^2 + (11y - 2)x + 4$$

have a common measure other than unity?

4. Extract the square roots of

(1) $$1 + \frac{41}{16}a - \frac{3 + 3a}{2}a^{\frac{1}{2}} + a^2 ;$$

(2) $$3(x-1) + 2\sqrt{2x^2 - 7x - 4}.$$

5. Solve the equations

(1) $$7\sqrt{x-8} - \sqrt{21x + 12} = 2\sqrt{3} ;$$

(2) $$x^2 + 3xy = 12 - xy = 16y^2 - xy - x^2.$$

6. A wine merchant bought a cask of sherry for £9, and after losing 3 gallons by leakage, sold the rest of the cask at 6*s*. per gallon above cost price, thereby realising a profit of $33\frac{1}{3}$ per cent. on his whole outlay. How many gallons did the cask contain?

7. There are $2m + 1$ terms in Arithmetic Progression. The first term is a, and the last is b; what is the middle term?

A series whose 1st, 2nd, and 3rd terms are respectively

$$\frac{1}{\sqrt{2}}, \quad \frac{1}{1 + \sqrt{2}}, \quad \frac{1}{4 + 3\sqrt{2}}$$

is either Arithmetic or Geometric. Determine which it is, and write down the 4th term.

8. Determine what value of r will make the number of combinations of $2n$ things taken r together greatest.

Prove that

$$^{2n+4}C_{n+2} + {}^{2n+4}C_{n+3} = {}^{2n+5}C_{n+3},$$

the symbol ^nC_r being used to denote the number of combinations of n things taken r together.

9. Expand $(1 - x)^{-4}$ to 5 terms by the Binomial Theorem, and write down the $\overline{r+4}$th term in its simplest form.

Apply the Binomial Theorem to prove that the sum of the series

$$1 + \frac{3}{4} + \frac{3 \cdot 5}{4 \cdot 8} + \frac{3 \cdot 5 \cdot 7}{4 \cdot 8 \cdot 12} + \frac{3 \cdot 5 \cdot 7 \cdot 9}{4 \cdot 8 \cdot 12 \cdot 16} + \ldots \textit{ ad. inf.} \text{ is } \sqrt{8}.$$

10. The base of the Napierian system of logarithms being defined as the limiting value of $\left(1 + \frac{1}{x}\right)^x$ when x is infinitely large, calculate its value to 3 decimal places.

11. Prove that $\log_b N = \frac{1}{\log_a b} \log_a N$.

Given $\log_{10} 2 = {\cdot}30103$, $\log_{10} 3 = {\cdot}47712$, find the values of

$$\log_{10} (5{\cdot}4)^{\frac{1}{7}} \text{ and } \log_5 (5{\cdot}4)^{\frac{1}{7}}.$$

12. The three conterminous edges of a rectangular block are $9\frac{3}{5}$, $13\frac{1}{5}$, and $14\frac{3}{10}$ inches; find the length of its diagonal.

13. If a cubic foot of cast iron weigh 450 lbs., what will be the weight of a cast iron spherical shell whose external diameter is 6 inches and thickness $\frac{1}{2}$ an inch ($\pi = \frac{22}{7}$)?

III.—EUCLID AND TRIGONOMETRY.

[*Ordinary abbreviations may be used in answers to the first six questions, but the method of proof must be geometrical. Great importance will be attached to accuracy.*]

1. If a straight line be divided into any two parts, prove that the squares on the whole line and on one of the parts are equal to twice the rectangle contained by the whole and that part, together with the square on the other part.

Prove that the rectangle contained by the diagonals of the squares on the whole line and on one of the parts is equal to twice the rectangle contained by the whole line and that part.

2. Enunciate and prove a proposition which gives the excess of the square on the longest side of an obtuse-angled triangle over the sum of the squares on the other sides.

3. Prove that the sum of either pair of opposite angles of a quadrilateral inscribed in a circle is equal to two right angles.

Two circles intersect in the points P and Q. A straight line MPN is drawn, terminated by the circles in M and N. Through M and N tangents are drawn to the circles intersecting in T. Prove M, N, Q, and T all lie on the circumference of a circle.

4. Describe an isosceles triangle having each of the angles at the base double of the third angle.

Describe an isosceles triangle having each of the angles at the base one-third part of the remaining angle.

5. If the vertical angle of a triangle be bisected by a straight line which likewise cuts the base, prove that the rectangle contained by the sides of the triangle is equal to the rectangle contained by the segments of the base, together with the square on the straight line which bisects the angle.

6. Draw a circle through two given points (1) to touch a given circle, (2) to touch a straight line given in position.

7. The tangent of an angle is 2·4. Find the cosecant of the angle, the cosecant of half the angle, and the cosecant of the supplement of double the angle.

8. Simplify the expression

$$2 \sec^2 A - \sec^4 A - 2 \operatorname{cosec}^2 A + \operatorname{cosec}^4 A,$$

giving the result in terms of $\tan A$.

9. Prove the following

(1) $\tan 2A = (\sec 2A + 1)\sqrt{\sec^2 A - 1}$.

(2) $\sec\left(\frac{\pi}{4} + \theta\right)\sec\left(\frac{\pi}{4} - \theta\right) = 2 \sec 2\theta$.

(3) $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \frac{\pi}{4} = \sin^{-1}\frac{1}{\sqrt{5}} + \cot^{-1} 3$.

10. Show that in any plane triangle

(1) $4 \times \text{area} = (b^2 + c^2 - a^2) \tan A$.

(2) $\frac{\sin 2A}{a^2(b^2 + c^2 - a^2)} = \frac{\sin 2B}{b^2(c^2 + a^2 - b^2)} = \frac{\sin 2C}{c^2(a^2 + b^2 - c^2)}$.

11. If A', B', C' are the points in which the sides BC, CA, AB respectively of the triangle ABC are touched by the three escribed circles, and Δ is the area of the triangle ABC, prove that the area of the triangle $A'B'C'$ is $2\Delta \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$.

12. Two sides of a triangle are 540 yards and 420 yards, and the included angle is 52° 6′. Find the remaining angles.

$$\log_{10} 2 = \cdot 3010300.$$
$$L \tan 26^\circ\ 3' = 9\cdot 6891430.$$
$$\left.\begin{array}{l} L \tan 14^\circ\ 20' = 9\cdot 4074189. \\ L \tan 14^\circ\ 21' = 9\cdot 4079453. \end{array}\right\}$$

13. The angle of elevation of the top of a tower rising out of a horizontal plane is $\cot^{-1}\frac{2}{5}$ at a point A in the plane.

AB (= 32 feet) is drawn in the plane at right angles to the line joining A to the base of the tower; and the angle of elevation of the top of the latter is observed at B to be $\cot^{-1}\frac{2}{3}$. Find the height of the tower.

HIGHER MATHEMATICS.

IV.—CONIC SECTIONS AND DIFFERENTIAL CALCULUS.

[*Full marks may be obtained for about* four-fifths *of this paper. Great importance will be attached to accuracy.*]

1. In the parabola the subnormal is of constant length.

The ordinate PN and normal PG are drawn at the point B of a parabola whose vertex is A and axis ANG. Prove that GP produced through P touches the parabola whose vertex is N and focus at A' such that A bisects NA'.

2. If CN and NP be the coordinates of a point of the hyperbola, with centre C, referred to the asymptotes as axes, prove that $CN \,.\, NP$ is constant.

If CY be the perpendicular from C upon the tangent at P, prove that $CP \,.\, CY$ is also constant when the hyperbola is rectangular.

3. Find the equation of a straight line (1) in rectangular coordinates, (2) in polar coordinates.

Taking the polar equations of two straight lines, find the condition among the constants that the lines may be at right angles to one another.

4. Through the point (3, 4) two straight lines are drawn each making the angle 45° with the line $x - y = 2$. Find their equations.

Find also the area of the triangle between the two lines and the given line.

5. Find the equation of the circle of radius (a) which cuts the axis of x in the points A and B whose abscissæ are $-b$ and $+b$.

6. Find the equations of the diameter through A, and the tangent at B, and prove that this diameter and tangent intersect on the curve

$$x^2 - y^2 = b^2.$$

7. Find the equation of the chord of contact of the two tangents to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

from the point h, k.

If h, k be situated on the parabola $y^2 = \frac{2b^2}{a}x$, prove that the chord of contact will touch the parabola $y^2 = -\frac{2b^2}{a}x$.

8. Obtain from first principles the different coefficients of e^x and $\sin x$.

9. Differentiate

$$\frac{\sqrt{1+x}}{1+\sqrt{x}} \text{ and } \left(\frac{1}{x}\right)^x.$$

10. Prove Maclaurin's theorem for all functions capable of expansion in positive integral powers of x.

Expand $\log(1 + 2x + 3x^2)$ as far as x^3.

11. Show how to find by differentiation the maxima and minima values of a given function of x.

Prove that the greatest value of $(3 - x)\left\{\sqrt{x^2+1} + x\right\}$ is 5.

12. If ϕ be the angle between the radius vector and tangent at the point (r, θ) in a polar curve, prove that

$$\cot\phi = -r\frac{d}{d\theta}\left(\frac{1}{r}\right).$$

13. Define the point of contrary flexure, and prove that in a polar curve $\frac{d\phi}{d\theta} = -1$ at such a point.

Find the points of contrary flexure in the curve

$$r = \frac{a}{\theta^2 - 2m\theta}.$$

14. Find the forms of the curve $ay^2 - bxy + x^3 = 0$ near the origin and at infinity respectively, and trace the curve.

V.—STATICS AND DYNAMICS.

[*Full marks may be obtained for about* four-fifths *of this paper. Great importance will be attached to accuracy.*]

1. Define the moment of a force about a point as to magnitude and sign.

If two forces X and Y act at the origin O in the rectangular axes Ox and Oy, find the algebraical sum of their moments about the point P, whose coordinates are x and y.

2. State the conditions of equilibrium when three forces, P, Q, and R, in one plane act at a point.

If in this case a triangle ABC be drawn with A, B, and C upon the lines of action of P, Q, and R respectively, and the force P be replaced by its equivalent forces in AB and AC, Q by its equivalent forces in BA and BC, and R by its equivalent forces in CA and CB; prove that the pair of forces so formed in each side of the triangle are separately in equilibrium.

3. What is the condition of equilibrium when a heavy body rests upon a horizontal plane?

The quadrilateral lamina $ABCD$ is formed of two uniform isosceles triangles ABC and ADC, whose vertices are B and D, on opposite sides of the common base AC, the angle ABC being a right angle. Prove that it will rest in a vertical plane with BC upon a horizontal plane provided the area of ADC be not greater than four times that of ABC.

4. Three weightless strings AC, BC, and AB are knotted together to form an isosceles triangle ABC with vertex C. If a weight W be suspended from C, and the whole be supported with AB horizontal by two forces bisecting the angles at A and B, find the tension in AB.

5. State the laws of statical friction.

Three equal circular disks A, B, and C are placed in contact with each other upon a smooth horizontal plane, B and C being also in contact with a rough vertical wall. If the coefficient of friction between the circumferences of the disks, and also that between the circumferences of the disks and the wall be equal to $2 - \sqrt{3}$ prove that no motion will ensue when A is pushed perpendicularly towards the wall with any force P.

6. How is velocity measured?

A man on board ship walks round a deck-cabin upon a square track 200 feet in perimeter, in 40 secs. The ship is moving $8\frac{2}{11}$ miles an hour. Supposing the man to start in the direction of motion of the ship, how far will he have travelled in absolute space in 50 minutes?

7. Prove that the times of descent down all chords of a circle passing through the highest point of the circle are equal to each other.

A number of smooth rods meet in a point A, and rings placed on them slide down the rods, starting simultaneously from A. Prove that after the time (t) the rings are all on the surface of a sphere with radius $\frac{gt^2}{4}$.

8. Find the horizontal range of a projectile when the magnitude and direction of the initial velocity are given.

A ball is projected upwards in a room, and after hitting the ceiling rebounds with perfect elasticity, prove that the distance from the point of projection (A) at which the ball again meets the horizontal plane through A is equal to twice the horizontal distance from A of the point of collision with the ceiling.

9. Find the velocities of two elastic spheres of equal radii but not of equal masses after direct impact.

10. A ball is dropped from the top of a tower of height h and at the same instant another ball of equal mass is projected vertically upwards from the base with the velocity due to falling down h.

If the balls collide directly, and the elasticity be perfect, prove that each will be in its original position after a time from starting equal to that of falling down h.

If the elasticity be e, prove that the falling body rebounds to a height short of the top of the tower by $\frac{h}{4}(1 - e^2)$.

11. Find the acceleration when two unequal weights are suspended by a cord over a smooth pulley.

A rope hangs over a smooth pulley, and a man of 12 stone lets himself down the portion of rope on one side of the pulley with an acceleration unity. Find with what uniform acceleration a man of $11\frac{1}{2}$ stone must pull himself up by the other portion of rope in order that the rope may remain at rest. ($g = 32$.)

12. Define the terms *Force*, *Energy*, *Power*, and state how the quantities so defined are measured.

A train weighing 50 tons is kept moving at the uniform rate of 30 miles an hour on the level, the resistance of air, friction, &c. being 40 lbs. per ton. Find the Horse-power of the Engine.

Express the *Energy* of the train in any units previously defined.

December 1888.

I.—ALGEBRA AND MENSURATION.

[*Great importance will be attached to accuracy.*]

1. Divide

$$(\sqrt{a} + \sqrt{b})^3 - (\sqrt{a} - \sqrt{b})^3 \text{ by } 3a + b;$$

and resolve into factors the expressions

$$x^4 + x + \frac{x^3 + 1}{x + 1}, \quad 4x^4 - 5x^2 + 1, \quad \text{and} \quad x^2 + 4xy + 4y^2 - 4z^2.$$

2. Find the value of

$$\sqrt{\frac{5}{x}} - \sqrt[3]{-x}, \text{ when } x = \cdot 008;$$

and simplify the expressions

(1) $\sqrt[12]{12} \times \sqrt[18]{18} \div \sqrt[36]{\frac{3}{16}}$.

(2) $a + b - \dfrac{1}{a + \frac{1}{b}} - \dfrac{1}{b + \frac{1}{a}}$.

(3) $\dfrac{x^3y - y^4}{xy^2 + x^2y} \div \left\{ \dfrac{x^4 + x^3y + x^2y^2}{(x^2 - y^2)^3} \times \left(1 + \dfrac{y}{x}\right)^2 \right\}$.

3. Find the highest common factor and the lowest common multiple of $6x^4 - 13x^3 + 6x^2$ and $8x^4 - 36x^3 + 54x^2 - 27x$; and express $\dfrac{x}{x^2 - 4} - \dfrac{1}{x - 2}$ as a single fraction, with $(2 - x)(2 + x)^2$ as denominator.

4. Solve the equations

(1) $1 - \dfrac{x^3 - 8\frac{1}{2}}{x} = 5\frac{1}{4} - x^2.$

(2) $2x - y = 2y - \dfrac{1}{z} = \dfrac{3}{z} - 7x = 1.$

5. *A* starts to walk from *P* to *Q*, a distance of one mile, while *B* starts simultaneously to run from *Q* to *P* and back. If *B*'s speed be to *A*'s as 9 is to 4, where will *A* be overtaken?

6. Find the square roots of

(1) $a^{-2} + 2a^{-1}(2 - b^{-2}) + b^{-4} + 4(1 - b^{-2}).$

(2) $44 - 16\sqrt{7}.$

7. Solve the equations

(1) $\dfrac{x}{x^2 - 1} + \dfrac{x^2 - 1}{x} = 2\frac{1}{6}.$

(2) $z^x = y^{2x}, \quad 2^z = 2 \times 4^x, \quad x + y + z = 16.$

8. The sum of the first 10 terms of an arithmetical series is to the sum of the first 5 terms as 13 is to 4. Find the ratio of the first term to the common difference.

9. Ten men are chosen in every possible way out of 16. In how many of the groups do two particular men both occur?

10. Prove the binomial theorem for negative integral indices.

Find the greatest term in the expansion of $(1 - x)^{-\frac{4}{3}}$ when $x = \frac{12}{13}$.

11. Prove that a^x may be expanded in the form of

$$1 + (\log_e a)x + (\log_e a)^2 \frac{x^2}{\underline{|2}} + (\log_e a)^3 \frac{x^3}{\underline{|3}} + \dots$$

12. Given

log 5·76 = ·7604226, log 2 = ·3010300, and log ·0105 = $\bar{2}$·0211893, find the logarithms of the digits above 2.

13. The area of an equilateral triangle is 17320·5 square feet. About each angular point, as centre, a circle is described with radius equal to half the length of a side of the triangle. Find the area of the space included between the three circles. ($\pi = 3·1416$, $\sqrt{3} = 1·73205$.)

14. A hollow cone, the length of whose slant side is twice the radius of its base, is held with its vertex vertically downwards and completely filled with water. A sphere of greater density than water is gradually immersed, and it is found that, when it rests upon the sides of the interior of the cone, it is just submerged. Find the amount of water displaced by the sphere, and also the amount contained between the sphere and the vertex of the cone. Consider radius of base of cone as 1·73205 inches.

II.—EUCLID AND TRIGONOMETRY.

[*Ordinary abbreviations may be used in answers to the first six questions, but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.*]

1. Divide a given straight line into two parts, so that the rectangle contained by the whole and one of the parts may be equal to the square on the other part.

Divide a given straight line into two parts, so that the rectangle contained by the whole and one part may be equal to the rectangle contained by the other part and another given straight line.

2. Draw a straight line from a given point, either without or in the circumference, which shall touch a given circle.

If AB, AC are the tangents at the points B, C of a circle, and if D is the middle point of the arc BC, prove that D is the centre of the circle inscribed in the triangle ABC.

3. Prove that the opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles.

If the exterior angles of any quadrilateral be bisected by four straight lines, prove that the quadrilateral formed by these lines can be inscribed in a circle.

4. A and B are fixed points, and AC, AD are fixed straight lines, such that BA bisects the angle CAD; if any circle passing through A and B cut off the chords AK and AL from AC and AD, prove that the sum of the lengths of AK and AL is always the same.

5. Prove that similar triangles are to one another in the duplicate ratio of their homologous sides.

6. Prove that the rectangle contained by the diagonals of a quadrilateral figure inscribed in a circle is equal to both the rectangles contained by its opposite sides.

What does this theorem become when the quadrilateral is a rectangular parallelogram?

7. Prove that $\tan(\pi + \theta) = \tan\theta$, and find a general expression for all the angles which have the same tangent as a given angle.

Solve the equation

$$\tan(\pi \sin\theta) = \tan(\pi \cos\theta).$$

8. Prove geometrically the formula

$$\sin(A + B) = \sin A \cos B + \cos A \sin B;$$

and, if $\tan x + \tan y + \tan z = 0$, prove that

$$\sin 2x \sin 2y \sin 2z + 8 \sin(y + z) \sin(z + x) \sin(x + y) = 0.$$

9. Establish the relations

$$(1) \quad \cot A - \cot 2A = \operatorname{cosec} 2A,$$

$$(2) \quad \frac{1 + \sin\theta - \cos\theta}{1 + \sin\theta + \cos\theta} = \tan\frac{\theta}{2};$$

and solve the equation

$$\sec^{-1}\frac{x}{a} - \sec^{-1}\frac{x}{b} = \sec^{-1} b - \sec^{-1} a.$$

10. Prove that the distances between the centre of the inscribed circle of the triangle ABC and the centres of the escribed circles are respectively

$$a \sec\frac{A}{2}, \quad b \sec\frac{B}{2}, \quad c \sec\frac{C}{2}.$$

Prove also that, if R be the radius of the circumscribing circle of the triangle ABC, the area of the triangle formed by the centres of the escribed circles is equal to

$$8R^2 \cos\frac{A}{2} \cos\frac{B}{2} \cos\frac{C}{2}.$$

11. If the two sides a, b and the angle A of the triangle ABC are given, find the third side; and state under what conditions there will be two solutions.

If $2b = 3a$, and $5 \tan^2 A = 3$, prove that there are two values of the third side, one of which is double of the other.

12. Prove that, for a triangle ABC,

$$(b + c) \tan \frac{1}{2} (B - C) = (b - c) \cot \frac{A}{2} \cdot$$

If $b = 14$, $c = 11$, $A = 60^\circ$, find B and C, having given

$\log 2 = \cdot 3010300$, $\quad L \tan 11^\circ 44' = 9 \cdot 3174299$.
$\log 3 = \cdot 4771213$, $\quad L \tan 11^\circ 45' = 9 \cdot 3180640$.

HIGHER MATHEMATICS.

III.—CONIC SECTIONS AND DIFFERENTIAL CALCULUS.

[*Full marks may be obtained for about* four-fifths *of this paper. Great importance will be attached to accuracy.*]

1. In the ellipse find the mean proportional between the two perpendiculars from the foci upon any tangent, and prove that

$$SY^2 : CB^2 :: SP : HP,$$

where H, S are the foci, and Y the foot of the perpendicular from S upon the tangent at P.

2. State, and prove, the geometrical property which enables you to trace the form of the hyperbola at considerable distances from the centre.

A small piece of a curve, which is a conic section, is drawn upon paper. Explain what measurements you would make to determine if it is a hyperbola, parabola, or ellipse.

3. If PQ be the tangent at the point P of an ellipse, whose focus is S, and semi-minor axis CB, and if the angle PSQ be equal to the angle BSC, prove that the locus of Q for all positions of P is a parabola having the same focus and directrix as the ellipse.

4. Find the equation to the straight line drawn through C, the middle point of the line AB, and perpendicular to that line, the coordinates of A and B being x', y' and x'', y'' respectively, and the axes rectangular.

If A and B be situated on the rectangular axes Ox and Oy respectively, and the perpendicular through C meet the line bisecting the angle xOy in P, prove that CP is equal to half AB.

5. If (x', y') (x'', y'') be the coordinates of the extremities of any diameter of a circle, prove that the equation of the circle may be written in the form

$$(x - x')(x - x'') + (y - y')(y - y'') = 0.$$

If ρ be the distance of the point x, y on the circumference from the extremity x', y' of the diameter, prove that

$$\rho^2 = (x - x')(x'' - x') + (y - y')(y'' - y'.)$$

6. If any normal to the parabola

$$y^2 = 4ax,$$

be inclined at the angle whose tangent is m to the axis, prove that its equation may be written in the form

$$y = mx - 2am - am^3.$$

If normals be drawn from h, k to the parabola, prove that they meet it in points situated on the curve

$$y^2(2a - h) + 4ax^2 = 2aky.$$

7. Prove that the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

and the hyperbola

$$\frac{x^2}{a^2 - \lambda} - \frac{y^2}{\lambda - b^2} = 1$$

have coincident foci, and that they intersect at right angles.

8. Find, from first principles, the differential coefficients of $\sin^{-1} x$ and $\log_a x$, and differentiate

(1) $\dfrac{1 - x}{\sqrt{1 + x^2}}$; (2) $x^a \log_a x$.

9. Find the limiting value of $(\cos x)^{\cot^2 x}$ when $x = 0$.

10. Show how the maxima and minima values of $\phi(x)$ may be ascertained by differentiation.

Apply to $3x^4 + 8x^3 - 24x^2 - 96x + 112$.

11. If ψ be the angle between the tangent and radius vector at any point of a polar curve, prove that

$$\cos\psi = \frac{dr}{ds}; \qquad \sin\psi = r\frac{d\theta}{ds}.$$

If p be the perpendicular from the pole upon the tangent at the point, prove that

$$\frac{d\psi}{d\theta} = -p\frac{d}{dr}\left(\frac{r}{p}\right).$$

12. Find an expression for the radius of curvature of a polar curve. Prove that the chord of curvature through the pole at any point of the curve

$$r = a\cos^2\frac{\theta}{2}$$

is equal to $\frac{4}{3}$rds of the radius vector at the point.

13. Find the asymptote of, and trace the curve

$$ax^2 - xy^2 - y^3 = 0.$$

IV.—STATICS AND DYNAMICS.

[*Full marks may be obtained for* four-fifths *of this paper. Great importance will be attached to accuracy.*]

1. Find the resultant of two parallel forces in the same direction, and prove that its moment about any point in their plane is equal to the algebraical sum of the moments of the forces about the same point.

2. State the conditions of equilibrium when any number of forces in one plane act on a rigid body.

Four forces acting in the sides of the quadrilateral $ABCD$ are in equilibrium. If the straight lines representing the forces in the sides AB, BC, CD, DA are to the lengths of the respective sides as $p : q : r : s$, prove, by considering the moments of the forces about the angular points, that $pr = qs$.

3. A parallelogram is formed of four strings knotted together at their extremities, and is kept at rest by four forces at the angles. Use the triangle of forces to determine each of these four forces in terms of the tensions of the strings; and, hence, show that, if a quadrilateral be drawn whose sides are parallel and proportional to the forces, the diagonals of this quadrilateral will be parallel to the sides of the parallelogram.

4. Find the relation of the power to the weight in the wedge.

A uniform rod, of weight W, is suspended horizontally from two nails in a wall, by two vertical strings, each of length (l), attached to its ends. A smooth weightless wedge, of vertical angle 30°, is pressed down with a vertical force $\frac{W}{2}$ between the wall and rod (so as not to touch the strings), its lower edge being kept horizontal, and one face kept touching the wall.

Find the distance through which the rod is thrust from the wall.

5. A circular disc BCD, of radius (a) and weight W, is supported by a smooth band of inappreciable weight and thickness, which surrounds the disc along the arc BCD, and is fastened at its extremities to the point A in a vertical wall, the portion AD touching the wall, and the plane of the disc being at right angles to the wall.

If the length of the band not in contact with the disc be $2b$, prove that the tension of the string is $\frac{W}{2}\frac{a^2+b^2}{b^2}$, and find the pressure at D.

6. Define the coefficient of friction, and limiting angle of resistance.

The two planes AB and AC are hinged together at A, and AC is horizontal, while AB slopes downwards at the angle (α) to the horizon. Two equal weights are connected by a string, and placed one on AB and the other on AC, the limiting angle of friction between each weight and plane being (ϵ) and less than (α). Find through what angle AC may be slowly tilted round the hinge, always sloping towards A, before motion ensues.

7. State how velocity is measured—(1) linear, (2) angular.

A sportsman covers a bird flying in a straight line at the rate of 6 miles an hour, moving his gun round his shoulder as a fixed point. If when the bird is nearest to him it be 20 yards distant, find the angular velocity of the gun at that instant.

8. A very small marble is thrown horizontally from a given point, so as to hit another very small marble hanging from a string in a given position. Find the magnitude of the velocity with which it is projected.

9. If two bodies impinge upon each other, prove that the motion of their centre of gravity is unaffected by the impact.

Is it possible for a rider to influence the leap of his horse by any action of his own after the horse has taken off? Give reasons for your answer.

10. Find the velocities of two perfectly elastic spheres after oblique impact.

If two billiard balls were equal to each other in all respects, and were, as well as the table, perfectly smooth, prove that the direction taken by the striker's ball after hitting the object ball would be the same for all velocities of the former, and would depend only upon the point at which the latter was hit.

11. Find the acceleration in the case of two unequal weights connected by a weightless string over a smooth pulley.

If a weight W be connected by a weightless string hanging over a smooth pulley with a scale pan containing two weights, each equal to W, lying one upon the other, find the pressures during free motion between these weights—the weight of the pan being neglected.

12. A man sculling does E foot-pounds of work, usefully applied, at each stroke. If the total resistance of the water when the boat is moving uniformly n miles an hour be R lbs., find the number of strokes he must take per minute to maintain the speed.

July 1889.

I.—ALGEBRA AND MENSURATION.

[*Great importance will be attached to accuracy.*]

1. Simplify the expressions

$$(\sqrt{a} + \sqrt{b})^3 - (\sqrt{a} - \sqrt{b})^3 - 2\sqrt{b}(a + b);$$

$$(5x + 4y)(5x - 4y) - 5(7x - 3y)(7x + 3y) + 25(3x - y)(3x + y).$$

2. Multiply

$$1 - \frac{3}{x} - \frac{5}{x^2} \text{ by } 1 + \frac{3}{x} - \frac{5}{x^2},$$

and divide

$$x^5 + 3x^4 - 5x^3 - 7x^2 + 12x - 4 \text{ by } x^2 + 3x - 2.$$

3. Resolve into factors the expressions

$$x^2 - 10x + 24, \qquad 27x^2 - 48x - 512,$$

$$(a + b - c - d)^2 - (a - b + c - d)^2, \text{ and } x^4 + x^2 + 1.$$

4. Prove that the expression $x^3(y - z) + y^3(z - x) + z^3(x - y)$ is divisible by $y - z$, by $z - x$, and by $x - y$; and find the other factor.

5. Express

$$\frac{4b - c}{bc} + \frac{2c - 3a}{ca} + \frac{a - 2b}{ab}$$

as a single fraction; and simplify the expression

$$\frac{x - 1}{x + 1}\left\{\frac{x - 1}{\sqrt{x} - 1} + \frac{1 - x}{x + \sqrt{x}}\right\}.$$

6. Find the highest common factor and the lowest common multiple of the expressions

$$6x^3 - 19x^2 - 9x + 36 \text{ and } 12x^3 - 35x^2 - 23x + 60.$$

Also find the highest common factor and the lowest common multiple of the expressions

$$18(x + y)^2(x^3 - y^3),\ 24(x - y)^2(x^3 + y^3), \text{ and } 36(x^2 - y^2)^2.$$

7. Solve the equations

(1) $\frac{1}{2}(x+5)+\frac{1}{3}(x-5)=\frac{1}{6}(x+7)+\frac{1}{7}(5x-4)$;

(2) $\left(\frac{2x-1}{x-2}\right)^3=\frac{8x-1}{x-8}$;

(3) $2^x=8^{y+1}$ and $9^y=3^{x-9}$.

8. A man leaves by his will the sum of £5040 to be divided between his four sons *A, B, C, D,* as follows:

C is to have £720, *B* as much as *C* and *D* together, and *A* £2000 less than twice as much as *B*. How was the money divided?

9. Find the sum of 14 terms of an arithmetical progression whose first term is 11, and common difference 9.

Find five numbers in geometrical progression such that their sum shall be 124, and that the quotient of the sum of the first and last by the middle term shall be $4\frac{1}{4}$.

10. If $(n)_r$ represent the number of combinations of n things taken r together, prove that

$$(n+1)_r=(n)_r+(n)_{r-1}.$$

Each of two bags contains twelve different coins; find how many different combinations of ten coins can be made by taking five out of each bag.

11. Write down the coefficients of x^r in the expansions of $(1+x)^n$, $(1-x)^{-3}$, and $(1-2x)^{-\frac{1}{2}}$; and find the sum of the coefficients in the expansion of $(1+x)^{10}$.

Also prove that the coefficient of x^r in the expansion, in powers of x, of the expression

$$\frac{1}{(1-x)(1-2x)} \text{ is } 2^{r+1}-1.$$

12. Having given log 2 = ·3010300, log 3 = ·4771213, and log 7 = ·8450980, find the logarithms of 28000, ·000036, and (by a series) 3·001.

13. Assuming that $\pi=3·1416$, find the perimeter and the radius of a circle the area of which is 5·309304 square feet.

14. A solid sphere fits closely into the inside of a closed cylindrical box, the height of which is equal to the diameter of the cylinder. Having given the radius of the sphere, write down the expressions for the volume of the sphere, the surface of the sphere, and the volume of the empty space between the sphere and the cylinder.

If the volume of this empty space is 134·0416 cubic inches, what is the radius of the sphere?

II.—EUCLID AND TRIGONOMETRY.

[*Ordinary abbreviations may be used in answers to the first six questions, but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.*]

1. Prove that, if a straight line be divided into any two parts, the squares on the whole line and on one of the parts are together equal to twice the rectangle contained by the whole line and that part together with the square on the other part.

2. Describe a square that shall be equal to a given rectilineal figure.

Describe an isosceles right-angled triangle that shall be equal to a given rectilineal figure.

3. Define similar segments of a circle. Prove that on the same side of the same chord there cannot be two segments similar and non-coincident.

4. Give the construction of the problem to describe about a given circle a triangle equiangular with a given triangle.

If the angles of a triangle circumscribing a circle are α, β, and γ, what are the angles of the triangle formed by joining the points of contact?

5. If a straight line be drawn parallel to one of the sides of a triangle, prove that it will cut the other sides, or the sides produced, proportionally.

Straight lines are drawn from two of the vertices of a triangle to the middle points of the opposite sides respectively. Prove that these two lines divide each other in the same ratio, and find that ratio.

6. If a straight line bisect the vertical angle of a triangle, or its exterior angle, and also cut the base, prove that the segments of the base have to one another the same ratio as the sides of the triangle have.

7. Find $\cos (A + B)$, where $A < 90$ and $> B$; and write down the expressions for $\sin 2A$, $\cos 2A$, $\tan 2A$ in terms of $\sin A$, $\cos A$, $\tan A$ respectively.

8. Prove that

(1) $\sec^2 A \operatorname{cosec}^2 A = \sec^2 A + \operatorname{cosec}^2 A$;

(2) $\operatorname{cosec} A (\sec A - 1) + \sin A = \cot A(1 - \cos A) + \tan A$;

(3) $\cos 3A = 4 \cos^3 A - 3 \cos A$.

9. Simplify

$$\frac{\sin 7A - \sin A}{\sin 8A - \sin 2A};$$

and show that $\tan^{-1} \frac{2}{3} = \frac{1}{2} \tan^{-1} \frac{12}{5}$.

10. Prove that in any triangle

$$\sin A = \frac{2}{bc} \sqrt{s(s-a)(s-b)(s-c)}.$$

Perform the reduction of the right-hand member which is possible when $a^2 = b^2 + c^2$.

11. The sides of a triangle are 9 and 3, and the difference of the angles opposite to them is 90°. Find, by using logarithms, the base, and all the angles.

$$\begin{aligned} \log 2 &= \cdot 3010300. \\ \log 3 &= \cdot 4771213. \\ \log 75894 &= 4\cdot 8802074. \\ \log 75895 &= 4\cdot 8802132. \\ L \tan 26^\circ\ 33' &= 9\cdot 6986847. \\ L \tan 26^\circ\ 34' &= 9\cdot 6990006. \end{aligned}$$

12. Find the area of the triangle formed by joining the centres of the three circles escribed to the sides of a given triangle.

HIGHER MATHEMATICS.

III.—CONIC SECTIONS AND DIFFERENTIAL CALCULUS.

[*Full marks may be obtained for about* four-fifths *of this paper. Great importance will be attached to accuracy.*]

1. Define a "parabola," and state how it can be obtained by the section of a right circular cone by a plane.

Prove that the length of the subnormal in a parabola is constant.

2. Prove that the sum of the focal distances of a point on an ellipse is constant. How may the ellipse be practically constructed?

3. Describe the methods of rectangular and polar coordinates, explaining how to pass from the former system to the latter. Find the equation of the straight line which passes through the point whose rectangular co-ordinates are (1, 2) and makes equal intercepts on the axes.

4. Find the polar equation of the circle.

A circle is described on the line joining the points whose polar coordinates are $(1, 60^\circ)$, $(2, 30^\circ)$, as diameter; show that its polar equation is

$$r^2 - r\{\cos(\theta - 60^\circ) + 2\cos(\theta - 30^\circ)\} + \sqrt{3} = 0.$$

5. Find the rectangular equation of the tangent to a parabola at a given point.

Prove that the two parabolas

$$y^2 = 8x, \qquad x^2 = 27y$$

cut one another at an angle

$$\tan^{-1}\tfrac{9}{13}.$$

6. Define the "eccentric angle" of a given point on an ellipse. The sum of the eccentric angles of two points on an ellipse is equal to 2β; show that the straight line joining them is parallel to the tangent at the point whose eccentric angle is β.

State the converse of this proposition.

7. Define a "rectangular hyperbola."

P, P' are the extremities of a fixed diameter of a circle, and Q, Q' are the extremities of any chord perpendicular to this diameter; find the locus of the intersection of the straight lines PQ and $P'Q'$.

8. Explain the fundamental principle of the Differential Calculus.

Differentiate from first principles

$$(1)\ \frac{mx+n}{px+q}, \qquad (2)\ \frac{1}{x^n}, \qquad (3)\ \cos\frac{x}{a}.$$

9. Enunciate Leibnitz' theorem concerning successive differentiation, and give a rigid inductive proof of it.

If $y = e^{x \cos a} \cos (x \sin a)$, show that

$$\frac{d^4y}{dx^4} = e^{x\cos a} \cos (x \sin a + 4a).$$

10. What different forms do indeterminate expressions assume?

Evaluate

$$(1)\ \frac{1 - \cos x}{x^2} \text{ when } x = 0.$$

$$(2)\ (\sin x)^{\tan x} \text{ when } x = \tfrac{1}{2}\pi.$$

11. Give the ordinary rules for the determination of the maximum and minimum values of a function of one variable, illustrating them as far as possible geometrically.

Determine the volume of the greatest cylinder which can be inscribed in a given sphere.

12. If the equation of a curve be $u = f(x, y) = 0$, prove that at the point (x, y) the equation of the tangent is

$$(X - x)\frac{du}{dx} + (Y - y)\frac{du}{dy} = 0.$$

Thence prove that the conics

$$\frac{x^2}{3} - y^2 = 1, \qquad \frac{x^2}{5} + y^2 = 1$$

intersect at right angles.

13. Find the equation of the asymptote of the Folium of Descartes, whose equation is

$$x^3 + y^3 - 9xy = 0.$$

Trace the curve, and state the significance of the circumstance that the equation is unaltered by the interchange of x and y.

IV.—STATICS AND DYNAMICS.

[*Full marks may be obtained for* four-fifths *of this paper. Great importance will be attached to accuracy.*]

1. Define the moment of a force—(1) about a point, (2) about a line.

The rod $AB\,(2a)$ is inclined (α) to the vertical, and to C, its middle point, another rod $CD\,(a)$ is fastened in the vertical plane through AB and at right angles to it. If a weight W be suspended from D, find its moment about the point A.

2. Enunciate the triangle of forces.

In the sides CB and CA of the triangle ABC, right-angled at C, whose lengths are 4 yards and 3 yards, points D and E are taken distant 4 feet and 1 yard respectively, from C. Prove that the resultant of the two forces represented by the lines AD and BE will be a force represented by a line of 10 feet and parallel to CO, where O bisects AB.

3. Find the C.G. of any number of weights rigidly connected.

Any closed polygon is drawn with $2n$ sides, and E, F, G, &c., are the points of bisection of n alternate sides, and e, f, g, &c., are the points of bisection of the remaining n sides. Prove that the C.G. of any n equal weights at E, F, G, &c., coincides with that of any n equal weights at e, f, g, &c.

4. The rectangle $ABCD$ has its sides AB and AD equal to 3 and 4 feet respectively, and stands with AB upon a rough horizontal plane. A string CPQ, fastened to the angle C, passes over a small smooth pulley at P, so that CP is perpendicular to AC and supports the weight Q.

If Q be gradually increased, find its value when the point B is just raised from the plane. Find also the least value of the coefficient of friction between the plane and rectangle that there may be no sliding.

5. The string $ABCD$, attached to two fixed points A and D, has two equal weights knotted to it at B and C, and rests with the portions AB and CD inclined 30° and 60° respectively to the vertical. Prove that the tension in the portion BC is equal to either weight, and that BC is inclined 60° to the vertical.

6. Find the relation of the power to the weight in the smooth screw.

If the axis of the screw be vertical and the distance between the threads 2 inches, and a door (weight 100 lbs.) be attached to the movable screw, as to a hinge, find the work done in turning the door through a right angle.

7. A wedge, whose weight may be neglected, stands upon a rough plane. Prove that, if the coefficient of friction between the base of the wedge and the plane exceed a certain quantity, no force, however great, acting perpendicular to the slant face of the wedge in a downward direction can cause the wedge to move.

8. Define angular velocity.

A circular disk of radius (a) lies on a smooth horizontal plane, and revolves round a vertical axis through its centre with angular velocity (ω).

If particles fly off simultaneously from different points of its edge, prove that, after any interval (t), they will all lie on a circle, and find the radius of this circle.

9. A number of particles slide down smooth chords of a sphere which meet in the highest point of the sphere. Prove that the times of descent down all the chords is the same, and that the velocities acquired in the descent are proportional to the lengths of the chords.

10. Find the horizontal range and time of flight of a projectile.

A particle projected with a velocity (v) at the angle 45° to the horizontal plane reaches the plane (a) feet due North of the point of projection; if, at the instant of projection, a horizontal blow were given to the particle, which would, if it acted alone, impart the velocity (v) due East, find where the particle would now meet the plane.

11. Two guns are pointed at each other, one upwards at the angle of elevation 30°, and the other downwards at the same angle of depression, the muzzles being 100 feet apart.

If the charges leave the muzzles with velocities 2200 and 1800 feet per second, find when and where they will meet.

12. Prove that kinetic energy is unaltered by the impact of perfectly elastic balls.

Two perfectly elastic balls equal in all respects are in contact, and are impinged upon simultaneously by a third ball, in all respects equal to each of the former, moving with velocity (u) perpendicular to the line of centres of the two former. Find the velocity of the balls after the impact.

13. In a railway train the resistance and friction of the rails is 1 lb. per ton. What is the horse-power of an engine which will maintain a speed on the level of 30 miles an hour in a train of 60 tons?

December 1889.

I.—ALGEBRA AND MENSURATION.

[*Great importance will be attached to accuracy.*]

1. Divide

$$a^3 + b^3 - 3ab(a + b) \text{ by } a + b;$$

and find the value of

$$(x - a)^3 + (x - b)^3 \text{ when } 2x = a + b.$$

2. Multiply

$$1 - x + x^2 - \frac{x^3}{1 + x} \text{ by } 1 + x;$$

and resolve

$$(a^2 - b^2 - c^2 + d^2)^2 - 4(ad - bc)^2$$

into four factors.

3. Find the G.C.M. and L.C.M. of

$$2x^4 + x^3 - 9x^2 + 8x - 2 \text{ and } 2x^4 - 7x^3 + 11x^2 - 8x + 2.$$

4. Simplify

(1) $$\left\{\frac{a+b}{2(a-b)}-\frac{a-b}{2(a+b)}+\frac{2b^2}{a^2-b^2}\right\}\cdot\frac{a-b}{2b}.$$

(2) $$\frac{a^2}{(a-b)(a-c)}+\frac{b^2}{(b-c)(b-a)}+\frac{c^2}{(c-a)(c-b)}$$

5. Solve the equations

(1) $a(x-a)^2+b(x-b)^2=ab(a+b)$.

(2) $\sqrt{3x-3}+\sqrt{5x-19}=\sqrt{3x+4}$.

6. A commercial undertaking costs £8000, which was to have been divided among the promoters; but four withdraw without paying, and the rest have each to contribute £100 more in consequence: how many promoters were there originally?

7. a, b, and c are in Arithmetical, and a, b, and $c+1$ are in Geometrical Progression. Prove that

$$a=(a-b)^2=(b-c)^2.$$

Sum

$$1-\frac{1}{2}+\frac{1}{2\cdot 3}-\frac{1}{2\cdot 3^2}+\ldots \text{ ad infin.}$$

8. Expand to 5 terms $(1+x)^{-\frac{1}{2}}$; and find the magnitude of the greatest term or terms of $(1-x)^{-6}$, when $x=\frac{1}{2}$.

9. In how many ways can a party of 6 be chosen from 10 persons; and in how many of these parties will a particular individual appear?

10. Show that the logarithms to base 10 of numbers consisting of the same significant figures in the same order differ only in the characteristic. If M is the modulus for converting Napierian to ordinary logarithms, prove that

$$2\log_{10}x-\log_{10}(x+1)-\log_{10}(x-1)$$
$$=2M\left\{\frac{1}{2x^2-1}+\frac{1}{3(2x^2-1)^3}+\ldots\right\}$$

11. Calculate by logarithms $\sqrt[3]{510}$, and $(3{\cdot}14159)^7$, being given

log 1·7 = ·2304489	log 3·14159 = ·4971495
log 3·0 = ·4771213	log 7·9895 = ·9025196
log 3·0202 = ·4800460	log 7·9896 = ·9025250
log 3·0203 = ·4800501	

12. A right prism on a triangular base—each of whose sides is 21 inches—is such that a sphere, described within it, touches its five faces; find the volume of the sphere, and of the space between it and the surface of the prism ($\pi = \frac{22}{7}$ and $\sqrt{3} = 1{\cdot}732$).

13. Find the expense of paving a circular court 80 feet in diameter, at 3*s.* 4*d.* per square foot, leaving in the centre a space for a fountain in the shape of a hexagon, each side of which is a yard.

II.—EUCLID AND TRIGONOMETRY.

[*Ordinary abbreviations may be used in answers to the first six questions, but the method of proof must be geometrical. Proofs other than Euclid's must not violate Euclid's sequence of propositions. Great importance will be attached to accuracy.*]

1. In every triangle, the square on the side subtending an acute angle is less than the squares on the sides containing that angle, by twice the rectangle contained by either of these sides, and the straight line intercepted between the perpendicular let fall from the opposite angle and the acute angle.

From two angular points of an acute-angled triangle *ABC* perpendiculars *AD*, *BE* are let fall upon the opposite sides. Show that the rectangle contained by *AC*, *CE* is equal to the rectangle contained by *BC*, *CD*.

2. If any point be taken on the circumference of a circle, of all the straight lines which can be drawn from it to the circumference, the greatest is the diameter through the point; and of any others, that which is nearer to the diameter is always greater than one more remote.

3. On a given straight line describe a segment of a circle containing an angle equal to a given rectilineal angle.

4. Inscribe a square in a given circle.

Show that the straight lines which touch the circle at the angular points of the square are parallel to the diagonals of the square.

5. The sides about the equal angles of equiangular triangles are proportionals.

The diagonal BD of a parallelogram $ABCD$ is divided in E so that BE is one-third of ED, and AE, DC produced meet in F. Show that FC is double of AB.

6. If the vertical angle of a triangle be bisected by a straight line which likewise cuts the base, the rectangle contained by the sides of the triangle is equal to the rectangle contained by the segments of the base, together with the square on the straight line which bisects the angle.

7. Explain how angles are measured (1) by degrees, minutes, and seconds, (2) by circular measure.

What is the circular measure of an angle containing a degrees, $6a$ minutes, and $9a$ seconds?

8. Write down the *sine*, *cosine*, and *tangent* of 60°, and deduce the values of cos 120°, sin 150°, tan 240°.

AB the diameter of a semicircle $APQB$ is 2 inches long, and AP, AQ make angles of 60° and 30° respectively with AB. Find the lengths of BQ, BP, PQ.

9. Express tan $2A$ and sin $2A$ in terms of tan A.

Find all the values of x which satisfy the equation

$$\cot x - \tan x = 2.$$

10. Show that in every plane triangle

(1) $c = a \cos B + b \cos A$.

(2) $\cos 2A + \cos 2B + 4 \cos C \sin A \sin B = 1 + \cos 2C$.

11. Express the sine of half an angle of a triangle in terms of the three sides.

Find the greatest angle of a triangle whose sides are 5, 8, 11 respectively, having given

$$\log 7 = \cdot 845098,$$
$$L \sin 56^\circ 47' = 9\cdot 9225205,$$
$$L \sin 56^\circ 48' = 9\cdot 9226032.$$

12. The vertical angle of an isosceles triangle is 120°, and its perpendicular altitude is an inch. Find (1) the area of the triangle, (2) the radius of the circumscribing circle, (3) the radius of an escribed circle touching the base produced.

HIGHER MATHEMATICS.

III.—CONIC SECTIONS AND DIFFERENTIAL CALCULUS.

[*Full marks may be obtained for about* two-thirds *of this paper. Great importance will be attached to accuracy.*]

1. In the parabola prove that $ST = SP$ where S is the focus and T the point in which the tangent at P meets the axis.

Prove that two confocal coaxial parabolas whose vertices are on opposite sides of the common focus intersect at right angles.

2. In the hyperbola prove that the difference of the focal distances of any point on the curve is constant.

A point B is taken without a circle whose centre is C, and straight lines are drawn bisecting at right angles the lines joining B with different points on the circle; prove that the locus of the intersection of the lines so drawn with the radii to these points, produced when necessary, is an hyperbola with foci at B and C.

3. Find the rectangular and polar equations of the line joining the points $(a, 2a)$ and $(3a, 4a)$.

4. Find the condition that the circles

$$(x-a)^2 + (y-b)^2 = c^2$$
$$(x-b)^2 + (y-a)^2 = c^2$$

should intersect, and prove that if θ be the angle between the two tangents to one of the circles at the points of intersection

$$\sin\frac{\theta}{2} = \frac{a-b}{c\sqrt{2}}.$$

5. Prove that the equation of the tangent at any point of the parabola $y^2 = 4a(x+a)$ may be expressed in the form

$$y = mx + a\left(m + \frac{1}{m}\right).$$

Prove that if two points be taken on the axis equidistant from the origin the difference of the squares of the perpendiculars from them upon any tangent is constant.

6. Find the equation of the ellipse referred to a vertex as origin and the major axis as the axis of x.

The extremities of the axes of an ellipse are at A and B, and a chord PQ parallel to AB cuts the axis major in O, prove that if M and N be the feet of the ordinates at P and Q,

$$2AM \cdot AN = AO^2.$$

7. Define, and find the equation of, the polar of the point h, k with reference to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Prove that the coordinates of the pole (T) of the line whose equation is

$$x \cos \theta + y \sin \theta = p,$$

with reference to the above ellipse, are $\frac{a^2}{p} \cos \theta$ and $\frac{b^2}{p} \sin \theta$.

8. If T' be the pole of the line in the last question with reference to the ellipse

$$\frac{x^2}{a^2 + \lambda^2} + \frac{y^2}{b^2 + \lambda^2} = 1,$$

prove that $TT' = \frac{\lambda^2}{p}$.

9. Find the differential coefficients of $\sin x$ and ϵ^x.

10. Differentiate

(1) $\sqrt{\frac{ax + b}{cx + d}}$; (2) $\tan^{-1} (m \tan x)$.

11. Find the limiting values of

(1) $\frac{\tan^2 mx - \tan^2 ma}{\sin^2 mx - \sin^2 ma}$ $(x = a)$,

(2) $(1 - x)^2 \sec \frac{\pi x}{2} \tan \frac{\pi x}{2}$ $(x = 1)$.

12. Find the conditions that $f(a)$ shall be a maximum or minimum value of $f(x)$.

Find the maximum and minimum value of the expression

$$2x^3 - 17x^2 + 44x - 30.$$

13. Prove the equation

$$p = x\frac{dy}{ds} - y\frac{dx}{ds} = r^2\frac{d\theta}{ds}$$

in plane curves, x, y being rectangular, r, θ polar coordinates, p the perpendicular from the origin on the tangent, and s the arc.

14. Find the linear oblique asymptotes to the curve

$$y^2 = \frac{x^3}{x - a}.$$

IV.—STATICS AND DYNAMICS.

[*Full marks may be obtained for about* two-thirds *of this paper. Great importance will be attached to accuracy.*]

1. Enunciate the triangle, and deduce therefrom the polygon, of forces.

If two forces in one plane be represented in magnitude, direction, and lines of action, by the two sides AB and DC of the quadrilateral $ABCD$, and E and F be the middle points of AD and BC, prove that the force necessary for equilibrium will be represented in magnitude and direction by $2FE$.

If AD and BC be parallel, prove that the line of action of the equilibrating force will coincide with FE.

2. Three uniform heavy rods AB, BC, and CA of lengths 5, 4, and 3 feet respectively are hinged together at their extremities to form a triangle. Prove that the whole will balance with AB horizontal about a fulcrum distance $1\frac{1}{5}$ of an inch from the middle point of AB towards A.

3. If W be the total weight of the three rods in the last question, prove that the vertical components of the actions at the hinges A and B, when the rod is balanced, will be

$$\frac{187}{600}W \quad \text{and} \quad \frac{163}{600}W$$

respectively.

4. Prove that the centre of gravity of a uniform triangular lamina coincides with that of three equal heavy particles at the angles.

If one angle, C, of such a lamina, ABC, be 120°, and the sides, CA and CB, be to one another as 4 to 1, find the weight which must be attached at B in order that the whole may rest with its plane vertical, and CB upon a horizontal plane.

5. State the principal laws of Statical friction.

A chest in the form of a rectangular parallelepiped whose weight without the lid is 200 lbs., and width from back to front 1 foot, has a lid weighing 50 lbs., and stands with its back 6 inches from a smooth wall and parallel to it.

If the lid be open and leans against the wall find the least coefficient of friction between the chest and ground that there may be no motion.

6. Show how to graduate the common steelyard.

What will be the effect upon the distance between successive graduations of changing the movable weight from W to W'?

7. A ladder of length l feet and weight W lbs., and uniform in every respect throughout, is raised by two men, A and B, from a horizontal to a vertical position. A stands at one end, and B getting underneath the ladder, walks from the other end towards A, holding successive points of the ladder vertically above his head, at the height of d feet from the ground, the force he exerts being vertical. Find the force exerted by B when thus supporting a point n feet from A, and prove that the work done by him in passing from the n^{th} to the $\overline{n-1}^{\text{th}}$ foot is

$$\frac{Wld}{2n(n-1)}.$$

When must A press his feet *downwards* against his end of the ladder?

8. How is the relative velocity of two bodies determined when their absolute velocities are known?

Two men, A and B, are walking in two roads which meet at right angles in C, A approaching and B receding from C; prove that if they are always at the same distance apart, *A*'s velocity must be to *B*'s velocity at each instant as *CB* is to *CA* at that instant.

9. What units are employed when the numerical value of the acceleration of gravity in this latitude is taken to be very nearly 32?

What would it become if the unit of space were one yard and the unit of time the time of falling from rest down a yard?

10. Find the acceleration down a smooth inclined plane.

A body starting from rest on such a plane describes 40 feet in the third second, find the inclination of the plane.

11. A body is projected from the point *A* on the deck of a steamer going 15 miles an hour due east, at an angle of elevation 45°, and reaches the deck again at *B*, 16 feet north-east from *A*.

Find (*a*) the velocity of projection relative to the steamer, (*b*) the time of flight, and (*c*) the length and direction of the range in fixed space.

12. Find the velocity of *G*, the C.G. of two spheres of masses, m and m', and moving with velocities u and u' in the same straight line, and find also the relative velocities of m and m' with respect to *G*.

Prove that after collision each of these relative velocities is reversed in direction and diminished in magnitude in the ratio of e to 1 where e is the coefficient of elasticity.

13. A man weighing 12 stone and a sack weighing 10 stone are suspended over a smooth pulley by a rope whose weight may be neglected; find their common acceleration.

If the man pulls himself up the rope so as to diminish his downward acceleration by one-half, find the upward acceleration of the sack in this case, and prove that the acceleration upwards of the man relative to the rope will be 3.2 (foot-seconds).

ANSWERS.

July 1881.

I.—ALG. MENS. 1. (a) 16. (b) $x^3 - 2x^2y + 2xy^2 - y^3$.

(c) $a + \frac{b}{2} + \frac{c}{3}$. 2. $a^3bc^{\frac{1}{8}}$; x. 3. $(2x + 1)(x^2 - 1)$.

4. $2a^2 - ab + 3b^2$.

5. Sides, 3, 5, 7 inches. Volumes, 27, 125, 343 cub. in.

7. (1) $\frac{ac}{b}$. (2) $x = -1, \frac{1}{2}, 2$; $y = 0, \frac{1}{8}, 1$.

(3) $x = 2, y = 0, z = -2$.

8. When $m = 4r - 1$, real and positive value; when $m = 4r + 1$, real and negative value; when $m = 2r$, no real value (r being any integer). 9. $d - (a - b)t$; $\frac{d}{a - b}$.

(1) when $a = b$, they are always d miles apart. (2) when $d = 0$ also, they are always together. 10. 7·870 ... sq. inches.

11. 1 : ·9226. 12. 31416 oz. 13. 1·6268.

14. Between 139 and 140 years. 15. 20; $n(n + 1) = \frac{2 \log p}{\log a}$.

II.—EUC. TRIG. 1. $(x + y)y = xy + y^2$. 13. $55° 46' 16''$.

December 1881.

I.—ALG. MENS. 1. (1) $\frac{z(x + y - z)}{x(y + z - x)}$. (2) 7·382 ... (3) $\frac{1}{b^3}$.

2. $x - ax^{\frac{2}{3}} - a^2x^{\frac{1}{3}} + a^3$; $x^2 - (\sqrt[3]{2} - 1)x + \sqrt[3]{4} + \sqrt[3]{2} + 1$.

3. $2(a^4 - 6a^2b^2 + b^4)$. 4. $(x - 3)(2x - 1)(3x - 2)$.

5. $a - \frac{3}{4}\sqrt{a} + 1$; $x^2 - 2x + 1$. 6. (1) $\pm 3, -2$.

(2) $x = 1, y = -1, z = 0$. (3) 3.

7. 6 hrs. $16\frac{4}{11}$ m., and 6 hrs. $49\frac{1}{11}$ m. 8. $a^2 - 4ab + 2b^2 = 0$.

9. A son's share : widow's : a brother's = 9700 : 3201 : 1100.

10. $x^2 - ax + \frac{1}{4}(1 + a + a^2)^2 = 0$. 11. 11·628 in.

12. 156025 : 11664 ; 4094·9. 14. 1·991226, $\bar{2}$·066766 ; − ·8899.

II.—Euc. Trig. 4. Straight line through points of intersection.

6. 3 : 4. 9. ·3927 of a mile. 14. 61° 17′ 22″.

15. $$\frac{2q}{\sqrt{\left\{\cot^2\frac{\beta}{2} - \cot^2\frac{a}{2}\right\}}},\quad \frac{q\cot\frac{a}{2}}{\sqrt{\left\{\cot^2\frac{\beta}{2} - \cot^2\frac{a}{2}\right\}}}.$$

July 1882.

I.—Alg. Mens. 1. $\frac{5}{24}$. 2. $-2\sqrt{-1}$; $a^2 + a + 1 = 0$.

3. 1 ; $\frac{1}{2}(2 + \sqrt{2} + \sqrt{6})$. 4. $(a - b)(x + a)$.

5. 1·732050, $a + \sqrt{8b^2 - 3a^2}$. 6. $\frac{1}{2}(a \pm b)$; 0. 7. (1) 8.

(2) 4. (3) 15, and $-\frac{3}{5}$. (4) $x = \pm 5, y = \pm 4$.

8. 12, $10\frac{10}{11}$, and $10\frac{10}{23}$ miles an hour. 9. ·5, and 1·292.

10. ·3979400, ·3521826, $\bar{1}$·3467875, ·9824394.

11. $128\frac{5}{8}$, and $2\frac{1}{24}$ cub. in. ; 166·67 and 10·703 sq. in.

12. $38\frac{1}{2}$, $9\frac{5}{8}$ sq. in. 13. 11000 oz.

II.—Euc. Trig. 1. The line must be bisected.

7. $\sin 495° = \frac{1}{\sqrt{2}}$, $\cos 495° = -\frac{1}{\sqrt{2}}$, $\tan 495° = -1$. The angles are 60°, 120°, 240°, 300°, 420°, 480°.

8. $\frac{n\pi}{5}, \frac{n\pi}{3}$; $\frac{1}{47}$. 10. B, 5° 55′ 10″·6 : C, 167° 27′ 25″·4.

December 1882.

I.—Alg. Mens. 1. $\dfrac{a^2+y^2}{a-y}$; 6·635...

2. $x^6+x^{-6}+3(x^2+x^{-2})$; $\dfrac{x^2-2x+1}{x^2-3x+1}$. 3. $1-\sqrt{1-x}$.

4. 502·953178...

5. $4\sqrt{-b^2}-3(a^2+4b^2)^{\frac{1}{3}}\left\{(a+2\sqrt{-b^2})^{\frac{1}{3}}+(a-2\sqrt{-b^2})^{\frac{1}{3}}\right\}$.

6. (1) $\dfrac{a}{3}$. (2) $x=1+\sqrt{-2},\ -3,\ 1,\ \frac{1}{2}(-3+\sqrt{17})$;

$y=1-\sqrt{-2},\ 1,\ 2,\ \frac{1}{2}(-3-\sqrt{17})$ and *vice versâ.*

(3) $x=1,\ y=4,\ z=27$. 7. £50.

9. $\dfrac{b^3-3abc}{a^3}$; $a^2x^2-a(b+\sqrt{b^2-4ac})x+b\sqrt{b^2-4ac}=0$.

10. $-2,\ 3$; 2·0782... 11. $\bar{3}$·6989700, ·2184267, ·544068.

12. 254 yds. 2 ft., 3458939 cub. yds. 7 ft. 13. 1·6205 in.

II.—Euc. Trig. 3. 1 : 2.

5. A circle on the line joining the points as diameter.

6. $-2,\ -\dfrac{2}{\sqrt{3}}$. 10. ·469296, 71° 44′ 29″, 48° 15′ 31″.

11. $l\sqrt{\dfrac{\sqrt{5}-1}{8}}$.

July 1883.

I.—Alg. Mens. 1. 1. 2. $2(a+b)x$; $x-3$.

3. $\sqrt{b(a-x)}-\sqrt{a(b-x)}$. 4. 3·43; $1-x-x^2-\dfrac{5}{3}x^3$.

5. (1) $a^2-2,\ a^3-3a,\ a^4-4a^2+2$;

(2) $4a^2-2,\ 8a^3-6a,\ 16a^4-16a^2+2$; $7\sqrt{3}$. 6. (1) -1.

(2) $x=3,\ -3+\sqrt{3}$; $y=2,\ -3-\sqrt{3}$; and *vice versâ.*

(3) $x = 3, y = 7, 5, z = 5, 7$; $x = 12, y = \frac{1}{2}(3 \pm \sqrt{409})$, $z = \frac{1}{2}(3 \mp \sqrt{409})$. 7. $4\frac{1}{2}$ and 4 miles an hour.

8. Gold 24 oz., price £4 10*s.* per oz.; silver 12 oz., price 5*s.* 3*d.* per oz.

10. (1) ·43152311. (2) $\bar{4}$·86028767. (3) $\bar{1}$·82124095.

11. $\frac{3}{2}$; 4; 1·062. 12. 15360 sq. ft.; $8\frac{7}{16}$ ft. 13. $\frac{138369}{600640}$ lbs.

II.—Euc. Trig. 8. 435·908 miles. 9. minus sign.

13. 64° 39′ 23″.

December 1883.

I.—Alg. Mens. 1. [Read in question $a = 6, b = 3$] 3.

2. (1) $\dfrac{2ab}{(a - x)(b - x)}$.

(2) $\dfrac{x^2 - y^2}{x^2 + y^2}$. 3. $x^2 + (2m - 3)x - 6m$; $(x^2 + a^2)(x + a)^2(x - a)$.

4. -4; $\frac{1}{3}$. 5. $\dfrac{2}{x + 1} - \dfrac{1}{x - 2}$; -3.

6. $-\dfrac{1 + \sqrt{3}}{\sqrt[3]{2}} = -2·168\ldots$; $x + \sqrt{8y^2 - 3x^2}$.

7. (1) $\frac{4}{3}$. (2) $x = a, y = 2a, z = 3a$.

(3) $x = \dfrac{1}{2}\sqrt{\dfrac{3b^2 - a^2}{2}} + \dfrac{1}{2}\sqrt{\dfrac{3a^2 - b^2}{2}}$,

$y = \dfrac{1}{2}\sqrt{\dfrac{3b^2 - a^2}{2}} - \dfrac{1}{2}\sqrt{\dfrac{3a^2 - b^2}{2}}$, and *vice versâ.*

8. 4 days. 9. $\dfrac{b^2 - 2ac}{a^2c^2}$; $\dfrac{1}{c}$.

10. $\sqrt{2(a + 1)}$; ·1556302, ·2100296, 1·1810658.

11. 1949·7 sq. in., 1949·7 cub. in. 12. 379·7 sq. yds. ($\pi = \frac{22}{7}$).

II.—Euc. Trig. 8. 60°, 36°, 84°; $66\frac{2}{3}^g$, 40^g, $93\frac{1}{3}^g$; $\dfrac{\pi}{3}, \dfrac{\pi}{5}, \dfrac{7\pi}{15}$.

10. (1) $n\pi, 2n\pi \pm \dfrac{\pi}{3}$. (2) $(2n + 1)\dfrac{\pi}{2}$.

(3) $n\pi \pm \dfrac{\pi}{10}, n\pi \pm \dfrac{3\pi}{10}$.

12. 79° 6′ 23″, 40° 53′ 37″. 14. 18° 26′ 5″·8.

July 1884.

I.—ALG. MENS. 1. (1) $5xy(x-y)(x^2-xy+y^2)$.
(2) $(a+b+c)(a^2+b^2+c^2-bc-ac-ab)$.
Condition is $(p+q)^3+(p+q)^2-a=0$. 2. $\frac{x^2+y^2}{x^2-y^2}$.

3. $2x+3(x^2+y^2)^{\frac{1}{3}}\{(x+\sqrt{-y^2})^{\frac{1}{3}}+(x-\sqrt{-y^2})^{\frac{1}{3}}\}$; $2a-\frac{3}{2}b+\frac{1}{3}c$.

4. $3x^2+4ax-a^2$. 5. $5\sqrt{a-b}-1$. 6. (1) $\pm\frac{\sqrt{5}}{2}$.

(2) $\pm\sqrt{\left\{a^2-\frac{(m^3-2a)^3}{27m^3}\right\}}$. (3) $x=1,-3$; $y=3,-5$, $z=3\frac{1}{2},-5\frac{1}{2}$.

7. $16p^4-16p^2q+2q^2$; $4x^2-16x+9=0$.

8. 20·728, 4·728 sq. yds. 9. $x^{-\frac{1}{2}}+y$; $\frac{1}{2}$; $\frac{1}{\sqrt{2}}\pm\sqrt{\frac{5}{2}}$.

10. ·23456. 11. $\bar{3}$·69897, ·218426; $x=2$, $y=3$.

12. $a^2\left(\pi-\frac{6\sqrt{3}}{4}\right)\left(1+\frac{3}{4}+\frac{9}{16}+\ldots\right)$. 13. (1) ·5198 in.

(2) ·8284 in. 14. 19 : 7 : 1.

II.—EUC. TRIG. 2. (1) 2, one internal one external.

(2) 2, both internal. 8. $\frac{54\pi}{121}, \frac{57\pi}{121}, \frac{67\pi}{121}, \frac{64\pi}{121}$. 9. $\theta=\frac{5}{\pi}\cdot\frac{\text{arc}}{\text{radius}}$.

11. (1) $\frac{(2n+1)\pi}{2}$. (2) $2n\pi$, $2n\pi+\frac{\pi}{2}$. (3) $\frac{n\pi}{12}$.

12. 0. 14. 106 ft. 15. AB, 1263·581; AC, 767·721.

December 1884.

I.—ALG. MENS. 1. $a^{\frac{3}{2}}+b+c^{\frac{3}{4}}-3a^{\frac{1}{2}}b^{\frac{1}{3}}c^{\frac{1}{4}}$; $\frac{x^2}{2}-\frac{3x}{4}+6$.

2. $\frac{x+3}{x^4-1}$. 3. x^6-1. 4. 1·732050; $x^2+xy+xz+yz$.

5. (1) $x=4$, $y=5$, $z=6$. (2) 1. (3) $x=\pm 1$, $y=\pm 27$.

6. $-2\sqrt{-1}$, $\sqrt{n}+\sqrt{-n}$. 7. 56.

8. $x^2 - 2(p^2 - 2q)x + p^2(p^2 - 4q) = 0$. 9. $\cdot 5875158$.

10. $\log_a \frac{1 + \sqrt{5}}{2}$; -1. 11. $\frac{\pi}{12} = \cdot 2619$ sq. ft.

12. $18\pi = 56\cdot 57$ sq. in. 13. $\cdot 3318$ cub. ft.

II.—Euc. Trig. 8. $31\cdot 831$. 9. $\pm \frac{\sqrt{3}}{2}$, and $\pm \frac{1}{\sqrt{2}}$.

10. $n\pi \pm \frac{\pi}{6}$. 13. $81° 45' 2''$ and $23° 2' 58''$.

July 1885.

I.—Alg. Mens. 1. $a^{\frac{2}{3}} + 2a^{\frac{1}{3}}b^{\frac{1}{3}} + 2a^{\frac{1}{3}}c^{\frac{1}{3}} + b^{\frac{2}{3}} + c^{\frac{2}{3}} + 2b^{\frac{1}{3}}c^{\frac{1}{3}}$.

2. $a^{x+2} - a^x b^2 + a^2 b^y - b^{y+2}$; $x^5 + x^4 - 5x^3 - 5x^2 + 4x + 4$.

3. $\frac{ab(a+b)}{ab+1}, \frac{2a}{\sqrt{x+a}}$.

4. $x^2 - 2x + 3$; $x^5 + 10x^3 + 40x + \frac{80}{x} + \frac{80}{x^3} + \frac{32}{x^5}$.

5. $\sqrt{8} + \sqrt{10}$; $2\sqrt[4]{5} = 2\cdot 99\ldots$

6. (1) 7. (2) 16, $\frac{5\sqrt[3]{5}}{4\sqrt[3]{4}}$. (3) $x = 3, 2$; $y = 2, 3$.

8. £40. 9. 0, $\cdot$3010300, $\cdot$4771213, $\cdot$6020600, $\cdot$6989700, $\cdot$7781513, $\cdot$8450980, $\cdot$9030900, $\cdot$9542426.

10. $\cdot$9895, log $\cdot$2000088 = $\bar{1}\cdot 3010491$

Diff. 217 $\begin{cases} 1\ldots 22, & 2\ldots 43, & 3\ldots 65, & 4\ldots 87, & 5\ldots 108 \\ 6\ldots 130, & 7\ldots 152, & 8\ldots 174, & 9\ldots 195 \end{cases}$

11. 18 ft. 12. $4\cdot 732$ sq. ft. 13. $\frac{\pi}{3} = 1\cdot 04719$ cub. ft.

II.—Euc. Trig. 9. $\frac{484}{945}$. 13. *B*, $51° 18' 21''$, *C*, $88° 41' 39''$; or *B*, $128° 41' 39''$, *C*, $11° 18' 21''$.

December 1885.

I. Alg. Mens. 1. $x^2 - 2x + 3$. 2. $6x + 3$.

3. (1) 0, 5, $2\frac{3}{5}$. (2) $-1\frac{2}{3}$, $-2\frac{1}{3}$.

(3) $x = 9, 4 + \sqrt{15}$; $y = 4, 4 - \sqrt{15}$, and *vice versa*.

4. $p^2 - 2q, 3pq - p^3$. 6. 27, 54, 81. 7. $\frac{2(2a+d)}{3d}$ and $\frac{2a+d}{3d}$.

8. $n = 3$. The condition that the coefficient of the r^{th} term = sum of $\overline{r-1}$ preceding coefficients is $n = r - 1$. 9. 2·69897, 4·90309.

11. 3·337728, 3·343262, mean 3·340595 acres. 12. 9·537 tons.

13. 2·535 in.

II.—Euc. Trig. 9. $\frac{7\pi}{30} = \cdot 733 \ldots$, $47^\circ\ 44'\ 47''\cdot 4$.

11. $n\pi \pm \frac{\pi}{4}$. 12. 100 ft.

III.—Conics, Diff. Cal. 4. $2ac$.

5. (1) $ay - bx = 0$. (2) $by - ax = \frac{b^2 - a^2}{2}$.

6. (1) $3a$; $a, 2a$. (2) $\frac{5}{2}a$; $\frac{3a}{2}$, $-2a$. Common chord, $x - 8y = 4a$.

7. $\frac{x}{a}\cos\frac{\alpha+\beta}{2} + \frac{y}{b}\sin\frac{\alpha+\beta}{2} = \cos\frac{\alpha-\beta}{2}$.

10. $\cos x$, $a^x \log_e a$, $-\frac{1}{x\sqrt{x^2-1}}$. 12. 1.

13. max. $-\frac{1}{2}a$; min. $\frac{1}{2}a$. 14. $x \pm a = 0$, $y + x = 0$.

IV.—Statics, Dynamics. 4. $\frac{W}{n+1}$, when n = number of strings attached to the lower block. 5. $\sqrt{3}$ feet.

7. $\frac{5}{7}$ mile an hour. 8. 900 feet.

12. Lengthen by $\frac{2}{17281}$ of its length.

13. $\sqrt{\frac{122}{13}} = 3''\cdot 063 \ldots$, $20\cdot 89 \ldots$ f. s.

July 1886.

I.—ALG. MENS. 1. $a^2(b-c)+b^2(c-a)+c^2(a-b)$.

2. $ax^3+b(x^2+x)+a$. 3. (1) 11, and $-8\frac{3}{13}$.

(2) $x=5,\ 2,\ \pm\sqrt{6}+1$; $y=-2,\ -5,\ \pm\sqrt{6}-1$.

5. £2000, 5%. 7. First term, 7, common ratio $\frac{1}{2}$.

8. $\dfrac{n(n-1)\ldots(n-r+3)}{\lfloor r}\{n^2-(4r-3)n+4r^2-8r+2\}$;

$1-x+\frac{1}{6}x^2+\frac{1}{54}x^3+\frac{1}{216}x^4$. 9. 1·69897, 2·544068, ·9116518.

11. 38·105, and 76·21 yds.; 30·403 yds. 12. 386425 lbs.

13. 6·48 m.

II.—EUC. TRIG.

8. $\tan A=\sqrt{\dfrac{1-\cos 2A}{1+\cos 2A}}$, $\sec A=\sqrt{\dfrac{2}{1+\cos 2A}}$,

$\operatorname{cosec} A=\sqrt{\dfrac{2}{1-\cos 2A}}$;

$\tan 22°30'=\sqrt{2}-1$, $\sec 22°30'=\sqrt{4-2\sqrt{2}}$, $\operatorname{cosec} 22°30'=\sqrt{4+2\sqrt{2}}$.

11. (1) $\frac{1}{2}ab\sin C$. (2) $\dfrac{c^2\sin A\sin B}{2\sin(A+B)}$.

12. 1·4649733, $\bar{1}$·3173938.

13. Base $4\sqrt{2}$, angles 90°, and 51° 3′ 27″·2.

III.—CONICS, DIFF. CAL. 1. A parabola. 5. 9·8.

6. 4·8. 10. $\frac{1}{2}\sec^3 ma \operatorname{cosec} ma$.

11. $\sin^{-1}\sqrt[3]{\dfrac{2y'}{l}}$, ($y'$ being the ordinate of the fixed point, and l the length of the rod).

IV.—STATICS, DYNAMICS.

4. $\dfrac{W\cos\theta}{2\cos(a-\theta)}$, $\dfrac{W\sqrt{\cos^2\theta+4\sin a\sin\theta\cos(a-\theta)}}{2\cos(a-\theta)}$.

5. $\tan\theta=\dfrac{1}{2\mu}(1-3\mu\cot a)$, $\dfrac{\tan a}{3+2\tan\theta\tan a}$. 6. $4W$.

7. $\dfrac{5\pi\sqrt{3}}{4}$ f.s. 11. $\frac{1}{2}t(t+1)\dfrac{g}{n}$.

December 1886.

I.—ALG. MENS. 1. $\frac{1}{a+c}$. 2. $x^2 - \frac{xy}{\sqrt{2}} + y^2$.

3. (1) $8, -2$. (2) $x = 5 \pm \sqrt{6}, y = 1 \pm \sqrt{\frac{2}{3}}$. 4. 1.

5. B overtakes A in 3 days, and 7 days later A overtakes B.

6. $8x = (yz)^{\frac{1}{2}} + 2(yz)^{\frac{1}{3}}$. 7. $\frac{|\underline{21}\ |\underline{5}\ |\underline{6}}{|\underline{17}\ |\underline{4}\ |\underline{3}\ |\underline{2}} = 43092000$.

8. (2) 5·0396842. (3) $\left(\frac{2}{3}\right)^{r+1} - \frac{1}{2^{r+1}}$.

9. 1·26765, 2·49555, 1·285... 11. 13403 sq. yds.

12. 784 sq. ft.; nearly 23 ft. 13. £5 10*s*.

II.—EUC. TRIG. 12. Base $2\sqrt{10}$, angles $90°$, and $54° 54' 11''$.

13. $\frac{1}{2}\sqrt{(\sqrt{5}+1)}$.

III.—CONICS, DIFF. CAL. 4. A rhombus, whose sides = 1.

9. e^x, $\sec x \tan x$. 10. (1) $-\frac{1+x}{(1+x^2)^{\frac{3}{2}}}$. (2) $x^{c-1} \log_a ex^c$.

11. 1. 13. Polar subtangent $= -a$. 14. $x + y = a$.

IV.—STATICS, DYNAMICS. 3. At the centre of the hexagon.

4. A distance equal to $1\frac{5}{6}$ of radius of a coin. 5. $\frac{\sqrt{3}}{2}W$.

6. $\frac{\sqrt{3}}{4}$. 8. $\frac{11\sqrt{257}}{28 \cdot 60^2} = \cdot 00174$ f.s. 9. $\frac{V}{g \sin a}$ secs.

10. $\frac{2V^2 \sin 2a}{dng}$ from prow. 11. $3\frac{3}{4}$ f.s. 12. 4 ft.; $3\frac{3}{4}$ secs.

July 1887.

I.—ALG. MENS. 1. (1) 2. (2) $ab - ac - bd + cd$.

2. $x^{\frac{3}{n}} - x^{-\frac{3}{n}}$, $x^6 + \frac{1}{x^6} + 3\left(x^2 + \frac{1}{x^2}\right)$. 3. $3; x - \frac{b}{2} + \frac{a}{3}$.

4. (1) $3, -3\frac{1}{2}, \frac{-3 \pm \sqrt{1357}}{12}$. (2) $x = \frac{3}{4}, 1, 0; y = \frac{3}{4}, 0, 1$.

6. $\frac{\lfloor 100}{\lfloor 10 \lfloor 90}, \frac{\lfloor 98}{\lfloor 8 \lfloor 90}$.

8. Yes. For the motion 300, against 260. Total 560.

9. $x + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \frac{x^4}{\lfloor 4} + \ldots$ 10. ·02086675. 11. 4·686 ft.

12. $59\frac{11}{21}$ cub. ft. 13. £466 7*s.* 1·14*d.*

II.—EUC. TRIG. 3. $\angle ABD = 2B$, $\angle BDE = A + B$, $\angle DEC = A + C$, and $\angle ECA = 2C$. 11. 108° 12′ 26″, and 49° 27′ 34″. $\log a = \cdot 00001$ or $a = 1$ very nearly.

III.—CONICS, DIFF. CAL. 3. A circle.

8. The given lines being $y = 0$ and $y = x \tan\theta$, the ellipse is $y^2(1 + \cos^2\theta) + x^2 \sin^2\theta - xy \sin 2\theta = c^2$.

9. (1) $\frac{a^2 - 2x^2}{\sqrt{a^2 - x^2}}$. (2) $\frac{1}{1 - x^2} + \frac{x \sin^{-1} x}{(1 - x^2)^{\frac{3}{2}}}$.

11. 2. 12. 16 min. 8 secs. 13. $2a \operatorname{cosec}\theta$.

IV.—STATICS, DYNAMICS. 5. $\frac{W}{2} \cot \frac{A}{2}$. 6. $9\frac{15}{17}$ lbs.

8. With velocity $\sqrt{2gl}$, at an angle of 45°. 9. $\frac{\sin a - \sin a'}{\cos a}$.

10. 31 lbs. 11. $\frac{8mm' \sin a}{m + m' \sin a}$; $\frac{8m \sin a}{m + m' \sin a}$.

12. $\frac{m - m' \cos a}{m + m'} \cdot g \sin a$. 13. $21W$, $15W$. A circle round A or B with initial velocity $3\sqrt{lg\sqrt{3}}$.

December 1887.

I.—Alg. Mens. 1. $\dfrac{a+b+c}{a-b-c}$; 1.

2. $2x^2 - 3x + 5$, $(x+2y)(x+7y)(x-3y)(x-5y)$.

3. $\dfrac{x}{2}$; $\dfrac{3}{2} + 2x - 7x^2$.

4. (1) 3. (2) $x = \dfrac{2(b-1)}{2ab-a-b}$; $y = \dfrac{2(a-1)}{2ab-a-b}$.

5. (1) 2, $3\frac{3}{4}$. (2) $x = \pm 3, \pm 2\sqrt{-3}, \pm 2\sqrt{3}, \pm 3\sqrt{-1}$,
$\pm\sqrt{\{\frac{3}{2}(\sqrt{-1} \pm \sqrt{47})\}}, \pm\sqrt{\{\frac{3}{2}(-\sqrt{-1} \pm \sqrt{47})\}}$; $y = \pm 1, \pm\frac{2}{3}\sqrt{-3}$,
$\mp\frac{2}{3}\sqrt{3}, \mp\sqrt{-1}, \pm\dfrac{1}{3\sqrt{-1}}\{\frac{3}{2}(\sqrt{-1} \pm \sqrt{47})\}, \mp\dfrac{1}{3\sqrt{-1}}\{\frac{3}{2}(-\sqrt{-1} \pm \sqrt{47})\}$.

6. 12*s*. 7. 27 and 48. 9. $\frac{704}{625} a^{\frac{21}{15}} x^{-\frac{4}{3}}$.

10. 1·537 . . . , $\frac{16}{3}$. 11. 1·027 . . . sq. in. 12. 648000.

13. Area of surface, 2130·7 sq. ft.; volume 12016·58 cub. ft.

II.—Euc. Trig. 10. 3·68 and 17·1. 12. $\dfrac{45\sqrt{3}}{2} = 38{\cdot}97 \ldots$ ft.

III.—Conics, Diff. Cal. 4. $3x - y = 7$ and $x + 3y = 19$.

6. (1) A straight line. (2) A circle, radius $\dfrac{a}{2}$. (3) A parabola, latus rectum $4a$. (4) A parabola, latus rectum $4a$.
(5) An ellipse, semi-axes a, b. 7. $3x^2 + 5y^2 = 32$.

9. (1) $\dfrac{2(1-x^2)}{(x^2-x+1)^2}$. (2) $2e^{\sin 2x} . \cos 2x$.

(3) $(\sin 2x)^{\sin} . \{\cos x \log \sin 2x + 2 \sin x \cot 2x\}$.

10. Maximum, none; minimum, 1. 12. 0, $-a$; and a, $\dfrac{5a}{9}$.

13. (1) $y - x = 0$. (2) $r \sin\left(\dfrac{\pi}{3} - \theta\right) - \dfrac{a\pi}{9} = 0$

IV.—STATICS, DYNAMICS.

5. Tension $2W$, stress at B or D, $\frac{W}{2}$; where W = weight of a rod.

6. $W(\sin a + \mu \cos a)$, and $W(\sin a - \mu \cos a)$. 8. 576.

9. Ratio of the masses $e : 1$; velocities e^2u, $(1 - e)e^2u$, where u is the original velocity.

12. Assuming the energy to vary as the number of lbs. of powder, the velocity would be doubled.

July 1888.

I.—ALG. MENS. 1. (1) $\frac{a^2 + b}{2(a + b)\sqrt{ab}}$. (2) $\frac{a^2 + b^2}{2(a + b)}$.

4. $2a + a^{\frac{1}{2}}b^{\frac{1}{2}} - 3b$, $\frac{5}{2}x + \frac{3}{2}y + z$.

5. (1) 2, $1\frac{1}{2}$. (2) $x = 0, 8$; $y = -5, 3$. 6. 469.

7. $2\frac{2}{5}$, 3, 4, 6. 8. 675. 9. $-\frac{5b^2}{128a^{\frac{3}{2}}}$.

11. $\bar{3}\cdot698970$, $\cdot799340$, $\bar{1}\cdot785248$; $x = \frac{22}{17}$.

12. (1) 7031250 tons. (2) 3750000 yds. 13. 804·78 cub. in.

II.—ALG. MENS. 1. (1) $3(x^2 + y^2 + z^2)$. (2) $\frac{13a + 5x}{5a + 13x}$.

2. $(a^2 + b^2)(a + b)(a - b)(2a - b)(a + 2b)$. 3. 5.

4. (1) $1 - \frac{3}{4}a^{\frac{1}{2}} + a$. (2) $\sqrt{2x + 1} + \sqrt{x - 4}$.

5. (1) 20. (2) $x, \pm 2$; $y, \pm 1$. 6. 18 gallons.

7. $\frac{a + b}{2}$. Arith. Series, 4th term $= -\frac{1}{2\sqrt{2} + 3}$.

9. $1 + 4x + 10x^2 + 20x^3 + 35x^4 + \ldots + \frac{1}{6}(r+6)(r+5)(r+4)x^{r+3} + \ldots$

10. 2·718. 11. ·10462, ·14967. 12. $21\frac{7}{10}$ in.

13. 12 lbs. $6\frac{11}{18}$ oz.

III.—EUC. TRIG. 7. $\frac{13}{12}$, $\frac{\sqrt{13}}{2}$, $\frac{169}{120}$. 8. $\frac{1 - \tan^8 A}{\tan^4 A}$.

12. 78° 17′ 39·6″, 49° 36′ 20·4″. 13. 60 feet.

IV. CONICS, DIFF. CAL. 4. $x - 3 = 0$, $y - 4 = 0$. Area $= \frac{9}{2}$.

5. $x^2 + y^2 - 2\sqrt{a^2 - b^2} \cdot y = b^2$ (or $x^2 + y^2 + 2\sqrt{a^2 - b^2} \cdot y = b^2$).

6. $\frac{x}{b} - \frac{y}{\sqrt{a^2 - b^2}} + 1 = 0$, $bx - \sqrt{a^2 - b^2} \cdot y = b^2$.

9. $\frac{\sqrt{x} - 1}{2\sqrt{x} \cdot \sqrt{1 + x}\,(1 + \sqrt{x})^2}$, $-\left(\frac{1}{x}\right)^x (1 + \log x)$.

10. $2x + x^2 - \frac{10}{3}x^3$. 13. $-\frac{a}{2}$, $m \pm \sqrt{m^2 - 2}$.

V.—STATICS, DYNAMICS. 4. $\frac{1}{2} W \sec \frac{C}{2}$. 6. 12500 yds.

11. $\frac{8}{23}$. 12. HP, 160; 3388000 ft.-lbs.

December 1888.

I.—ALG. MENS. 1. $2\sqrt{b}$; $(x^2 + x + 1)(x^2 - x + 1)$, $(2x + 1)(2x - 1)(x + 1)(x - 1)$, $(x + 2y + 2z)(x + 2y - 2z)$.

2. 25·2. (1) $\sqrt[6]{12}$. (2) $\frac{ab(a + b)}{ab + 1}$. (3) $\frac{(x - y)^4}{x}$.

3. H.C.F. $= 2x^2 - 3x$, L.C.M. $= x^2(2x - 3)(3x - 2)(4x^2 - 12x + 9)$; $\frac{2(2 + x)}{(2 - x)(2 + x)^2}$. 4. (1) 2. (2) $x = 2$, $y = 3$, $z = \frac{1}{5}$.

5. 352 yds. from *Q*. 6. (1) $a^{-1} - b^{-2} + 2$. (2) $4 - 2\sqrt{7}$.

7. (1) $2, -\frac{1}{2}, \frac{1 \pm \sqrt{10}}{3}$.

(2) $x = 4, 6\frac{2}{9}$; $y = 3, -3\frac{2}{3}$; $z = 9, 13\frac{4}{9}$. 8. 2 : 1.

9. $30c_3$. 10. 4th and 5th terms $= \frac{8960}{6591}$.

12. ·4771213, ·6020600, ·6989700, ·7781513, ·8450980, ·9030900, ·9542426. 13. 1612·5 sq. ft. 14. 4·1888, ·5236 cu. in.

II.—EUC. TRIG. 6. Sq. on diagonal = sum of sqq. on the two sides. 7. $\theta = \frac{n\pi}{2}$ and $n\pi + \frac{\pi}{4}$. 9. $\pm ab$.

11. $\frac{\sqrt{5}}{2\sqrt{2}}a$, and $\frac{\sqrt{5}}{\sqrt{2}}a$. 12. $B\ 71^\circ\ 44'\ 29\frac{1}{2}''$, $C\ 48^\circ\ 15'\ 30\frac{1}{2}''$.

III.—Conics, Diff. Cal.

4. $2(x'' - x')x + 2(y'' - y')y = x''^2 - x'^2 + y''^2 - y'^2$.

8. (1) $\dfrac{x^2 - 2x - 1}{(1 + x^2)^2}$. (2) $x^{a-1} \log_a ex^a$. 9. $e^{-\frac{1}{2}}$.

10. max. none; min. -64. 13. $y + x = a$.

IV.—Statics, Dynamics. 4. $\frac{1}{2}l$. 5. $\dfrac{a}{b} \cdot W$.

6. $2\epsilon - a$. 7. $\frac{11}{15}$. 8. $l\sqrt{\dfrac{g}{2h}}$, where h = depth of marble below, and l its distance from the 1st. 11. $\frac{4}{3}W$. 12. $\dfrac{88nR}{E}$.

July 1889.

I. Alg. Mens. 1. $4a\sqrt{b}$; $5x^2 + 4y^2$.

2. $1 - \dfrac{19}{x^2} + \dfrac{25}{x^4}$, $x^3 - 3x + 2$.

3. $(x - 4)(x - 6)$, $(9x + 32)(3x - 16)$, $4(a - d)(b - c)$, $(x^2 + x + 1)(x^2 - x + 1)$.

4. Expression $= -(x - y)(y - z)(z - x)(x + y + z)$.

5. $\dfrac{1}{c}$; $\dfrac{x - 1}{\sqrt{x}}$.

6. (1) H.C.F. $3x^2 - 5x - 12$; L.C.D. $24x^4 - 106x^3 + 59x^2 + 189x - 180$.

(2) H.C.F. $6(x^2 - y^2)$;
L.C.D. $72(x^2 - y^2)^2(x^2 + xy + y^2)(x^2 - xy + y^2) = 72(x^8 - x^6y^2 - x^2y^6 + y^8)$.

7. (1) 5. (2) 0, ± 1. (3) $x = 21$, $y = 6$.

8. A £1520, B £1760, C £720, D £1040.

9. 973; 4, 8, 16, 32, 64. 10. $\left(\dfrac{\lfloor 12}{\lfloor 5 \lfloor 7}\right)^2 = 627264$.

11. $\dfrac{n(n-1)(n-2)\dots(n-r+1)}{\lfloor r}$, $\dfrac{3.4.5\dots(r+2)}{\lfloor r}$, $\dfrac{1.3.5\dots(2r-1)}{\lfloor r}$; $2^{10} = 1024$.

12. 4·4471580; $\bar{5}$·5563026; ·4772661.

13. 8·16816, 1·3 ft. 14. $\frac{4}{3}\pi r^3$, $4\pi r^2$, $\frac{2}{3}\pi r^3$; 4 in.

II.—Euc. Trig. 4. $\dfrac{\pi-\alpha}{2}, \dfrac{\pi-\beta}{2}, \dfrac{\pi-\gamma}{2}$. 5. 1 : 2.

9. $\dfrac{\cos 4A}{\cos 5A}$. 10. $\sin A = 1$.

11. 7·589466; 18° 26′ 6″, 108° 26′ 6″, 53° 7′ 48″.

12. $\dfrac{(a+b+c)a}{2\sin A}$.

III. Conics, Diff. Cal. 3. $x + y = 3$.

7. $x^2 - y^2 = a^2$ (referred to centre of circle as origin).

8. (1) $\dfrac{mq - pn}{(px+q)^2}$. (2) $-\dfrac{n}{x^{n+1}}$. (3) $-\dfrac{1}{a}\sin\dfrac{x}{a}$.

10. (1) $\frac{1}{2}$. (2) 1. 11. $\dfrac{4\pi r^3}{3\sqrt{3}}$. 13. $x + y + 3 = 0$.

IV. Statics, Dynamics. 1. $Wa(\cos\alpha \pm \sin\alpha)$.

4. $\frac{3}{10}W$; $\frac{12}{41}$. 6. $4\frac{1}{6}$ ft.-lbs. 10. At distance $a\sqrt{3}$ feet along a line making an angle $\tan^{-1}\sqrt{2}$ with line due N.

11. Time $\frac{1}{40}$ sec., at distance 54·6 ft. from A.

12. $\dfrac{2u\sqrt{3}}{5}$, and $-\dfrac{u}{5}$. 13. $4\frac{4}{5}$.

December 1889.

I.—Alg. Mens. 1. $a^2 - 4ab + b^2$; 0.

2. 1; $(a+b+c+d)(a-b-c+d)(a+b-c-d)(a-b+c-d)$.

3. $2x^2-3x+1$; $(x^2+2x-2)(x^2-2x+2)(2x^2-3x+1) = 2x^6-3x^5-7x^4 + 28x^3 - 36x^2 + 20x - 4$.

4. (1) 1. (2) 1.

5. (1) $a+b$, and $\dfrac{(a-b)^2}{a+b}$. (2) 4. 6. 20. 7. $\frac{5}{8}$.

8. $1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \frac{35}{128}x^4 - \ldots$; 5th and 6th.

9. 210, 126. 11. 7·98957; 3020·212. 12. 933·548 cu. in.; 1381·702 cu. in. 13. £834 3*s.* 11½*d.* (to nearest farthing).

II.—Euc. Trig. 7. ·006125 $a\pi$.

8. $\cos 120° = -\frac{1}{2}$, $\sin 150° = \frac{1}{2}$, $\tan 240° = \sqrt{3}$; $BQ = 1$, $BP = \sqrt{3}$, $PQ = 1$ inch.

9. $\sin 2A = \frac{2\tan A}{1 + \tan^2 A}$; $x = \frac{n\pi}{2} + \frac{\pi}{8}$. 11. 113° 34′ 41″·3.

12. (1) $\sqrt{3}$ sq. in. (2) 2 in. (3) 1 in.

III.—Conics, Diff. Cal.

3. $y = x + a$, $r(\sin\theta - \cos\theta) = a$. 4. $a - b < c\sqrt{2}$.

10. (1) $\frac{ad - bc}{2(ax+b)^{\frac{1}{2}}(cx+a)^{\frac{3}{2}}}$. (2) $\frac{m(1 + \tan^2 x)}{1 + m^2\tan^2 x}$.

11. (1) $\sec^4 ma$. (2) $\frac{4}{\pi^2}$. 12. max. 6; min. $1\frac{19}{27}$.

14. $y = x + \frac{a}{2}$, $y + x + \frac{a}{2} = 0$.

IV.—Statics, Dynamics. 4. $\frac{1}{3}$ weight of triangle.

5. $\frac{1}{10\sqrt{3}}$. 7. $\frac{Wl}{2n}$; when B has reached a point $\frac{l}{2}$ feet from A.

9. 2. 10. 30°. 11. (*a*) $16\sqrt{2}$. (*b*) 1 sec.

(*c*) 35·18... ft., $\tan^{-1}\frac{4\sqrt{2}}{11 + 4\sqrt{2}}$ with direction of ship.

12. $\frac{mu + m'u'}{m + m'}$; $\frac{m'(u - u')}{m + m'}$, $\frac{m(u - u')}{m + m'}$. 13. $\frac{g}{11}$; $\frac{8g}{55}$.

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