THE SCIENCE OF ILLUMINATION

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AN OUTLINE OF THE PRINCIPLES OF ARTIFICIAL LIGHTING

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AUTHOR'S PREFACE

In treatises dealing with the science of illumination, very complete descriptions of the various sources of artificial light are to be found, together with full particulars of the measuring instruments and methods necessary for exact research. Technical questions in the true sense, involving the estimation, measurement, and calculation of illumination, are, however, almost always very briefly treated.

It often happens that the realization of the desired result on the completion of an installation is a matter of chance, chiefly owing to the lack of data and the want of a simple method of calculation.

Hitherto, also, artificial lighting has not been of the great importance which it is to-day, when, with the introduction of newer and more economical sources of light, and with the competition between different lighting systems, illumination requirements are increasing to an astonishing extent.

In view of the large sums now spent on artificial lighting, very careful examination of any projected scheme becomes advisable, in order that the desired illumination may be attained without unnecessary expense.

During the last few years the author has devoted himself to the task of making actual measurements of illumination, and to the question of its predetermination. The results are presented in the principal chapters of this book. These deal with the estimation, calculation, and measurement of illumination, branches of the subject which have hitherto received the least attention. They are the embodiment of articles published by the author from time to time in the *Journal für Gasbeleuchtung* and the *Elektrotechnische Zeitschrift*, special attention having been paid in the compilation to practical requirements.

The first part of the book is devoted to an explanation of fundamental principles, as a clear understanding of these is necessary before any extended investigation becomes possible. Vagueness in fundamentals is very prevalent in this connection, and an effort has been made to facilitate comprehension of these by the use of a simple analogy.

Methods for determining mean spherical and mean hemispherical candle-power are then considered in some detail, as much time is often wasted over long and laborious constructions for their determination.

There is a final chapter on indirect lighting, in which the methods of predetermination already given for direct lighting are shown to be applicable, provided that the necessary experimental data are available. Indirect lighting has recently attracted much favourable notice, although for a long time it was regarded as only practicable with arc lamps, and was even then considered as a costly luxury.

The greater part of the experimental and practical data incorporated in the book have been obtained in the experimental lighting-station of the Berliner Elektrizitäts-Werke. I am under obligation to the Directors of that establishment, and desire to express to them my best thanks for the impetus they have given to this branch of the work, and for their permission to publish the results. In this connection I must not omit the names of Dr. Rathenau of the engineering staff, and Dr. Passavant, a Director of the Company.

L. BLOCH.

BERLIN,
May, 1907.

TRANSLATOR'S PREFACE

WITH the consent of the author, I have made certain alterations and additions to his original work, rendered necessary by the difference between the English and German units and standards, and by the lapse of time. These are indicated in the text by enclosure in brackets. Otherwise, I have endeavoured to follow the author's admirable exposition as closely as possible. My thanks are due to Mr. G. B. Dyke, B.Sc., and to Mr. A. P. Thurston, B.Sc., for their careful revision of the proofs.

W. C. CLINTON.

UNIVERSITY COLLEGE,
July, 1912.

CONTENTS

CHAPTER I

FUNDAMENTAL UNITS IN ILLUMINATION

CHAPTER II

MEASUREMENT AND CALCULATION OF LUMINOUS INTENSITY, OR CANDLE-POWER

CHAPTER III

THE ESTIMATION OF ILLUMINATION

CHAPTER IV

THE CALCULATION OF ILLUMINATION

CHAPTER V

THE MEASUREMENT OF ILLUMINATION

CHAPTER VI

INDIRECT LIGHTING

APPENDIX

LIST OF ILLUSTRATIONS

ILLUMINATION

CHAPTER I

FUNDAMENTAL UNITS IN ILLUMINATION

1. Introduction.

The most important fundamental magnitudes in the science of illumination are luminous flux, illumination, and luminous intensity, or candle-power. The meaning of each one of these terms should be thoroughly familiar to all who are called upon to deal with the subject, whereas they are often confounded with one another, or else wrong relations are established among them. In order to avoid these errors, fundamental ideas and definitions will be first set forth as simply as possible, and comprehension of them assisted by the use of an analogy.

For the present purpose, the propagation of light-rays may be realized by likening a luminous source, radiating in all directions with different intensities, to a sand-blast, sending sand in all directions in varying quantity. Let such a sand-blast be placed at the centre of a large sphere, the internal surface of which is covered with gum, so that the sand impinging on it in a given time at a given place accumulates there. Neglecting its weight, the sand issuing from the centre follows radial paths, and strikes the internal surface of the sphere normally.

If the intensity of the sand-blast varies in different directions, the thickness of the layer of sand formed on the internal surface of the sphere will vary in the same way in these directions. Thus the total quantity of sand transferred in a given time corresponds to the total quantity of light emitted from a source during this time.

The conception of the total quantity of light is rarely used, as the radiation from a luminous source is almost always independent of time, or depends upon it only to a very slight extent.

The emission of light per unit time, known as the *luminous flux*, requires much more consideration.

2. Flux of Light.

The total luminous flux emitted by a source, corresponds, on this analogy, to the total quantity of sand from the sand-blast received over the whole surface of the sphere during

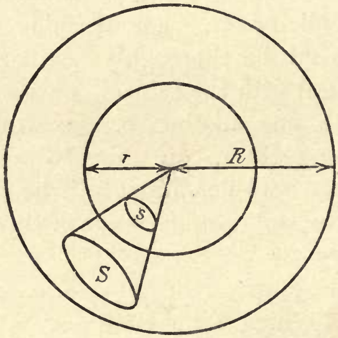


Fig. 1.

unit time. Luminous flux in general will be denoted by the symbol Φ, and the total flux emitted from a source by Φ_0 (spherical Φ).*

Considering for example, only a portion *S* of the internal surface of a sphere of radius *R* (Fig. 1), the quantity of sand which it receives in unit time corresponds to the luminous flux Φ within the solid angle defined by the area *S* and the

* [The symbols used for intensity, flux of light, and illumination, are those given in a "Report on Photometric Units," by A. Blondel, and adopted by the International Electrical Congress at Geneva in 1896.]

radius R of the sphere. Upon a concentric sphere of smaller radius r, this solid angle defines a smaller area s, the ratio of the two areas being the ratio of the squares of their radii.

$$\frac{s}{S} = \frac{r^2}{R^2}.$$

On account of its radial direction of emission, the quantity of sand ejected within a given solid angle always remains the same, just as does the luminous flux from a source of light. The small area s therefore receives the same quantity of sand as the larger one S. The thickness of the layer which fixes itself on s will then be greater, and exactly in the inverse ratio of the areas or the squares of the radii R and r.

3. Illumination.

The thickness of the layer of sand deposited per unit of time on each part of the spherical surface corresponds with the illumination (E) due to a luminous source at the centre of the sphere. Just as the thickness of this layer is inversely proportional to the square of the distance from the centre, so the illumination will vary according to the same law of distance from the source.

Again, if the luminous radiation varies in different directions over the surface of a given sphere, there will be different illuminations in those directions corresponding to the different thicknesses of sand. The layer of sand of variable thickness over a given area S can be replaced by a layer of uniform thickness over the same area, under the condition that the total quantity of sand remains the same. This mean thickness represents the mean illumination (E_m) of the area S, and its value could be obtained by dividing the total quantity of sand by the numerical value of the area. This total quantity corresponds to the luminous flux Φ and the mean thickness of the layer to the mean illumination (E_m); hence this last quantity is obtained by dividing the total flux Φ by the area of the surface S on which this flux is incident.

$$E_m = \frac{\Phi}{S}.$$

This relation is quite general, and applies to any surface whatever, whether part of a sphere concentric with the source or not.

In the case just considered, the luminous rays are incident perpendicularly to the surface of the sphere, producing what is known as *normal illumination.* If, however, the direction of the light is not perpendicular to the illuminated surface, but makes an angle α with the normal to the surface, the same solid angle defines an area in a plane perpendicular to the direction of the luminous rays, and at the same distance from the source, such that the ratio of the areas of the two surfaces is, as—

$$1 : \cos \alpha.$$

Both the thickness of the layer of sand received and the illumination are inversely proportional to the area of surface, so that the illumination is $\cos \alpha$ times the normal illumination at the same distance, and therefore diminishes as α increases.

4. Luminous Intensity, or Candle-Power.

The meaning of luminous flux and of illumination having been made clear, the conception of most frequent use—that of luminous intensity, or candle-power (*C.P.*)—can be simply dealt with. Taking a sphere surrounding the sand-blast, concentric with it, and of unit radius—1 metre for example—the thickness of the layer at each point of the surface of the sphere corresponds to the normal illumination at unit distance from the luminous source, and this illumination is to be taken as measuring the luminous intensity (I) of the lamp in the direction considered. To each direction in space there corresponds in the same fashion a luminous intensity determined by the normal illumination due to this source at unit distance.

According to the definition given on p. 18, the normal illumination (E_n) at any distance r, due to a luminous source of intensity I in that direction, will be given by the relation—

$$E_n = I\frac{1}{r^2}.$$

If radii are drawn from a point source of light in every direction, each having a length proportional to the corresponding intensities in those directions, a surface is defined by the ends of these radii known as the *photometric figure* of the source. This surface gives an image of the distribution of light in all directions, yet neither its volume nor its area can be turned to account. If the figure is cut by a plane passing through the vertical axis of the source, the section produced is known as the *polar curve of distribution of light.* This curve shows, by means of a system of plane polar co-ordinates, the variation of luminous intensity for all directions in the plane.

If, for a given source, all such polar curves are alike, then the photometric figure is a solid of revolution, and the source is said to be symmetric. If, however, the polar curves are different for different sections through the vertical axis, the source is said to be unsymmetric.

For a symmetric source the luminous intensity is the same in all directions making the same angle with the vertical axis, a condition not fulfilled by unsymmetric sources.

Instead of beginning with a simple explanation of the meaning of fundamental conceptions and of their relations with one another, as set forth above, the tendency in practice is to consider the intensity of a source as of the first importance, as it is this property which makes the most direct appeal to the senses. The eye judges the effect of a source, not by its total luminous flux or by the illumination resulting, but rather according to the intensity in the direction in which the source is seen. In general, the comparison of sources on the basis of their luminous intensity should be made in those directions in which the intensity developed is a maximum.

Before the advent of the electric arc, this direction was in almost all cases in the neighbourhood of the horizontal, and as the measurement of intensity in this direction was the most convenient one to make, such sources were usually compared and judged by their horizontal intensity.

The attention paid to more accurate measurement has shown that differences exist in the horizontal intensity of most sources, depending on the direction in which the test is made. The mean value of the horizontal intensity in different directions should therefore be taken. It is known as the *mean horizontal intensity.*

The electric arc cannot be judged by this criterion, as its horizontal intensity is relatively small; the conclusion is therefore unavoidable—that measurements of intensity taken in a single direction fail as a basis for the comparison of sources of different type. A quantity is required having the same meaning and dimensions as luminous intensity, but taking account of the total luminous flux emitted by the source. The conception involved is that of the *mean spherical luminous intensity*, or *mean spherical candle-power.*

5. Mean Spherical Candle-Power.

It has already been pointed out in the analogy with the sand-jet, that luminous intensity in a given direction corresponds to the thickness of the layer of sand deposited in this direction per second on a sphere of unit radius. If the whole of the sand received upon the surface of the sphere were redistributed so that the thickness of the layer became uniform, this mean thickness would then correspond to the mean illumination of the spherical surface of unit radius. This quantity is the mean spherical candle-power (I_0).

As the area of the surface of a sphere of unit radius is 4π, and as the whole of the sand on the surface corresponds to a total luminous flux (Φ_0), it follows at once that the relation between luminous flux and mean spherical candle-power is—

$$\Phi_0 = 4\pi I_0 \quad . \quad . \quad . \quad . \quad (1).$$

The mean spherical candle-power is a measurable magnitude of the first importance in the theoretical comparison of different luminous sources, as it gives a measure of the sum total of energy emanating from a source as light.

6. Mean Hemispherical Candle-Power.

For practical purposes, it is very often not a question of the total quantity of light emitted by a source, but only of that part of it really available for lighting. In streets, for instance, it is only the light emitted in the lower hemisphere of which the source is the centre that is used. For the comparison of sources from this point of view, the quantity known as the *mean hemispherical candle-power* has been introduced.

Placing the source at the centre of a sphere of unit radius, the mean hemispherical candle-power corresponds to the mean illumination of the lower hemisphere, which is, in terms of the sand analogy, the mean thickness of the layer on the lower half of the sphere. If the luminous flux emitted in this lower hemisphere is denoted by $\Phi_{\triangledown}$, then, as the area of the surface of the hemisphere is 2π,

$$\Phi_{\triangledown} = 2\pi I_{\triangledown} \quad . \quad . \quad . \quad . \quad (2).$$

The highest possible value of the ratio of the mean hemispherical intensity to the mean spherical intensity is 2, and occurs when the luminous flux is entirely in the lower hemishere. $\Phi_{\triangledown}$ is then equal to Φ_0. The lower limit of this ratio is zero in the case when no light reaches the lower hemisphere.

Usually, by the mean hemispherical intensity is understood the mean *lower* hemispherical intensity. There are, however, cases—as, for instance, with indirect lighting—where the luminous intensity in the upper hemisphere ($I_{\vartriangle}$) is of importance. This quantity is related to the corresponding flux by the equation—

$$\Phi_{\triangle} = 2\pi I_{\triangle} . \quad . \quad . \quad . \quad (3).$$

The total luminous flux over the whole sphere is then—

$$\Phi_{o} = \Phi_{\triangledown} + \Phi_{\triangle}$$

and it follows that—

$$4\pi I_{o} = 2\pi I_{\triangledown} + 2\pi I_{\triangle}$$

$$\text{or } 2I_{o} = I_{\triangledown} + I_{\triangle} \quad . \quad . \quad . \quad . \quad (4).$$

7. Surface Brightness.

There is still one more fundamental quantity to consider, and that is *surface brightness*, sometimes known as *intrinsic brightness*, or *intrinsic brilliance*.

By this is understood the ratio of the luminous intensity of the source to the area of the surface from which light is radiated. The greater the surface brightness of a source, the greater is the dazzling effect produced on the eye. In order to avoid such disturbance, very bright sources must be enclosed in diffusing envelopes, which diminish the brightness by increasing the surface from which light is emitted.

[This dazzling effect, or *glare* as it is sometimes called, has been the subject of much discussion, and various means of estimating it have been proposed. Professor L. Weber * of Kiel defines a system of illumination as "glaring" when it exceeds any of the following limits :

(*a*) If the ratio of the intrinsic brilliance or surface brightness of the source of light to that of the illuminated surroundings exceeds a certain limit. This ratio should not exceed a value of about 100.

(*b*) If the absolute intrinsic brilliance of the source exceeds a certain value. The brilliancy of the open candle-flame (about 2·5 candles per square inch) might be taken as a safe limit.

(*c*) If the angle between the direction of vision of the eye, when applied to the work it is called upon to do, and the line

* *The Illuminating Engineer,* 1910, vol. iii., p. 116.

from the eye to the source of light, is too small. This minimum angle may be provisionally assumed to be 30°.

(*d*) When the extent (apparent area) of the illuminating body is too large. The source should not subtend an angle of more than 5° at the eye.

Values of the intrinsic brilliance of various sources are given in Table I. below.*]

TABLE I.

SURFACE BRIGHTNESS OF DIFFERENT ILLUMINANTS.

Source.	Candle-Power per Square Centimetre.	Candle-Power per Square Inch.
Moore tube	0·04–0·25	0·2–1·5
Gas arc lamp, with alabaster globe	0·15–0·3	1–2
Candle	0·3 –0·6	2–4
Cooper-Hewitt lamp (approximate)	0·45–0·9	3–6
Incandescent electric lamp, with bulbs frosted to different degrees	0·3 –1·25	2–8
Kerosene oil lamp	0·45–1·25	3–8
Mantle gas-burner unshaded ...	3–4	20–25
Acetylene flame	12–18	75–120
Enclosed arc lamp, depending on globe used	15–30	100–200
Incandescent electric lamp bare ...	15–45	100–300
Nernst filament bare	125–150	800–1,000

8. Units and Standards.

The system of units on which illumination measurements are based is not absolute, but arbitrary, as no relation has been established between the magnitudes of the absolute system of units and those that are here dealt with. A beginning is made by selecting an arbitrary unit of one of the photometric quantities, the one chosen being a unit of luminous intensity, or candle-power.

[The distinction between a unit and a standard may perhaps

* *Zeitschrift für Beleuchtungswesen,* 1903, p. 245; [J. E. Woodwell, *The Western Electrician,* 1906, vol. xxxix., p. 309].

be emphasized here. The actual material standard may or may not be of the same size as the abstract unit.

Thus the International Candle, as the common unit of intensity in England, France, and America is called, is a *unit* and not a *standard*. It is maintained by the continued interchange of standards, reckoned as containing so many units, between the National Laboratories of the countries concerned.

It is essential for any standard that it should be reproducible by following a strict specification at any time and place, and this is the reason why at present all standards of luminous intensity are flame standards.

In England, France, and America, the Harcourt Pentane standard* is in use, and is rated as of 10 candle-power when burning at normal barometric pressure (76 centimetres or 29·92 inches) in an atmosphere containing 8 litres of water-vapour per cubic metre or 13·8 cubic inches to the cubic foot.

In Germany the "Hefner Kerz" (*H.K.*) is the standard, and its value is exactly 1 Hefner unit when burning at normal barometric pressure (76 centimetres) in an atmosphere containing 8·8 litres of water-vapour per cubic metre, or 15·2 cubic inches per cubic foot. The liquid used in the lamp is amyl-acetate, burnt through a cotton wick of circular section. The ratio of the Hefner unit to the international candle has been fixed as 0·9.†

1 Hefner unit = 0·9 international candle.]

All the other units are derived from the unit of luminous intensity (*I*).

The unit of illumination is the normal illumination on a

* A. C. V. Harcourt, British Association Reports, 1877, 1883, 1885, and 1898; a complete account of all the flame standards is given by J. A. Fleming, *Journ. Inst. Elec. Eng.*, 1903, vol. xxxii., p. 119.

† C. C. Paterson, "The Proposed International Unit of Candle-Power," *Journal of Physical Society of London*, vol. xxi., p. 867; E. Liebenthal, "Ueber die Abhängigkeit der Hefner lampe und der Pentanlamp von der Beschaffenheit der umgebunden luft," *Zeitschrift für Instrumentenkunde Jahrgang* xvi., 1895, p. 157.

surface at a distance of 1 metre from a point source of unit intensity. It is called 1 *lux* (E).

[Sometimes the distance is taken as 1 foot instead of 1 metre. The unit is then known as 1-foot candle (*f.c.*), and the relation between the two units is—

$$\text{1-foot candle} = 10{\cdot}76 \text{ lux.}]$$

The unit of light flux is the flux emitted per unit solid angle from a source of uniform unit intensity. It is the light coming through every square metre of a sphere of 1 metre radius concentric with the source. This unit is called 1 *Lumen* (Φ).

The unit of quantity of light is the quantity of light emitted per second or per hour when the flux is 1 Lumen. The unit may be either the *Lumen second* or the *Lumen hour*.

The maintenance of a given illumination during a given time has led to the use of the unit, the *Lux second*.

The unit of surface brightness or intrinsic brightness is an intensity of 1 candle per square centimetre or per square inch, and has received no special name.

[The latest tabulation of the photometric quantities and units is due to E. B. Rosa * and is given on p. 26.

In this tabulation the symbol F is used instead of Φ for illumination, and quantity of light Q is taken as the surface integral of b, instead of the time integral of F or Φ.

The definitions and symbols of these quantities, as given above, will, however, be retained throughout the rest of the book.]

* E. B. Rosa, "Photometric Units and Nomenclature," *Bulletin of the Bureau of Standards,* No. 4, vol. vi.; Washington, 1911.

TABLE II.

Photometric Magnitude.	Symbol.	Unit.	Equation of Definition.
1. Intensity of light	I	Candle	$I = \frac{F}{\omega}$
2. Luminous flux	F	Lumen	$F = I\omega = \frac{IS}{r^2} = ES = \pi Q$
3. Illumination	E	$\frac{\text{Lumens}}{\text{cm}^2}$ or $\frac{\text{milli lumens}}{\text{cm}^2}$	$E = \frac{F_e}{S} = \frac{I}{r^2}$
4. Radiation ...	E'	Lux = meter-candle	$E' = \frac{F_i}{S} = \pi b = mE$
5. Brightness	b	$\frac{\text{Candles}}{\text{cm}^2}$	$b = \frac{I}{S \cos e}$
6. Quantity ...	Q	Candles	$Q = bS$
7. Lighting ...	L	Lumen hours	$L = FT$

$\left.\begin{array}{l} i = \text{angle of incidence of light} \\ e = \text{angle of emergence of light} \end{array}\right\}$ measured from normal. F_i = incident flux. F_e = emergent flux.
m = coefficient of diffuse reflection or transmission.
$(1 - m)$ = coefficient of absorption.

CHAPTER II

MEASUREMENT AND CALCULATION OF LUMINOUS INTENSITY, OR CANDLE-POWER

THIS subject is usually treated in great detail. It will, however, only be dealt with here in so far as the explanation of the necessary principles in the measurement and calculation of illumination require it.

9. THE DISTRIBUTION OF LIGHT.

The distribution of light from a luminous source is usually sufficiently well known if measurements are made in a plane passing through the vertical axis of the source at intervals of 10°. Observations at closer intervals are only useful in precise researches or for a very irregular distribution. Measured values are plotted as polar co-ordinates from the luminous source as centre, and the line joining these points gives its curve of light distribution. In this way, for instance, the curve shown in Fig. 33, p. 128, has been drawn for an ordinary continuous current arc lamp.

There are two distribution curves in the same vertical plane, one on each side of the centre, but for quite symmetrical sources only one of these need be drawn (p. 19). With the moderate asymmetry usually met with, both sets of measurements should be taken, and the mean of the two observations made under the same angle on either side of the source should be used to plot an average curve. Lamp-holders have been constructed which allow of the experimental determination of this mean value in one observation. For very unsymmetric sources, measurements taken in one vertical plane only are not sufficient, but should be supplemented by another series in a plane perpendicular to the first, also containing the vertical axis of the lamp. The intensity at a given angle from the vertical is then given as the mean of four readings, and the mean distribution curve is plotted from these values.

In practical calculations, only the mean distribution curve of such unsymmetric sources will be taken into account, as a more detailed examination of such sources demands excessive and unnecessary labour.

10. Mean Spherical and Mean Hemispherical Candle-Power.

In the analogy used above, the mean spherical intensity or mean spherical candle-power corresponds to the mean thickness of the layer that would be formed by the whole of the sand received per second upon a sphere of unit radius, the actual thickness of the layer existing at any given place, corresponding to the luminous intensity I in that particular direction. The total quantity of sand on the surface of the sphere can be considered as the integral or sum of all the separate quantities, IdS, accumulated on the infinitely small elements of area, dS. The mean spherical intensity will then be—

$$I_0 = \frac{1}{4\pi}\int IdS \quad . \quad . \quad . \quad . \quad (5).$$

Let O be the source at the centre of a sphere of radius r (Fig. 2), and having a vertical axis, GOK. Let α be the angle between this axis and a line joining the element R with O, and β the angle between the vertical plane containing R and GOK, and the plane of the paper ; then—

$$dS = rd\alpha \cdot r \sin \alpha \cdot d\beta = r^2 \sin \alpha \, d\alpha d\beta.$$

Making the radius of the sphere unity, and substituting for dS in the expression for I_0 given above, this becomes—

$$I_0 = \frac{1}{4\pi}\int I \sin \alpha \, d\alpha d\beta.$$

As symmetric sources only need be considered, luminous intensity will vary only with the angle α, and will be independent of β. Hence the zone of the sphere containing the point R, and having a width $rd\alpha$, can be taken as the element of surface dS, giving at once—

$$dS = rd\alpha\ r \sin\alpha\ 2\pi = 2\pi r^2 \sin\alpha\ d\alpha.$$

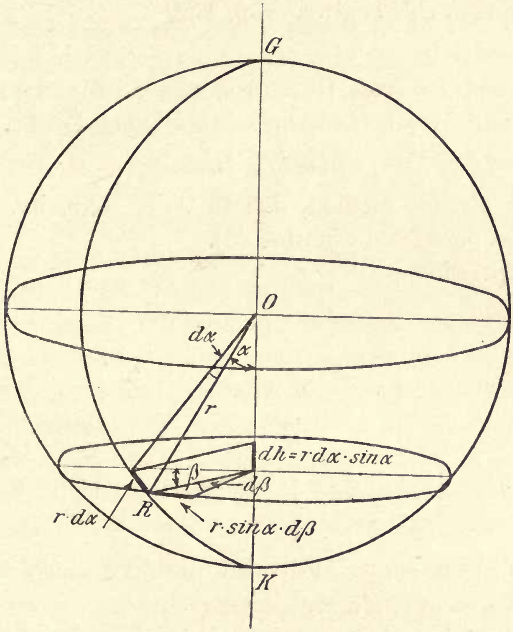


FIG. 2.

The mean spherical intensity is then given by—

$$I_\circ = \frac{1}{4\pi}\int_{\alpha=0}^{\alpha=\pi} I\ 2\pi \sin\alpha\, d\alpha = \frac{1}{2}\int_0^\pi I \sin\alpha\ d\alpha \quad . \qquad . \quad (6).$$

To obtain the corresponding expression for the mean hemispherical luminous intensity, the integration need only be extended over the lower hemisphere—that is, from O to $\frac{\pi}{2}$, the area of the hemisphere of unit radius being used to determine the mean intensity instead of that of the whole sphere. Thus—

$$I_{\smile} = \frac{1}{2\pi}\int_{\alpha=0}^{\alpha=\frac{\pi}{2}} I\, 2\pi \sin\alpha\, d\alpha = \int_{0}^{\frac{\pi}{2}} I \sin\alpha\, d\alpha \quad . \quad . \quad (7).$$

These formulæ are chiefly useful when the relation between luminous intensity I and the direction of radiation is known. An example of this is given on p. 132.

11. Methods for the Determination of Mean Spherical and Mean Hemispherical Candle-Power.

[*Kennelly's Method.*

A very elegant method, due to A. E. Kennelly,* can be deduced at once from equation (6).

The expression—

$$I_{o} = \frac{1}{2}\int_{0}^{\pi} I \sin\alpha\, d\alpha.$$

can be written as a series of terms—

$$2I_{o} = \int_{0}^{\frac{\pi}{n}} I \sin\alpha\, d\alpha + \int_{\frac{\pi}{n}}^{\frac{2\pi}{n}} I \sin\alpha\, d\alpha \;.\;.\;.\; \int_{\frac{(n-1)\pi}{n}}^{\pi} I \sin\alpha\, d\alpha.$$

if the sphere be supposed divided up into n zones, each subtending the same angle at the centre.

In each of the above n terms, I can differ but little from its value at the mid-zone. Hence the summation can be written—

$$2I_{o} = I_{\frac{\pi}{2n}}\int_{0}^{\frac{\pi}{n}} \sin\alpha\, d\alpha + I_{\frac{3\pi}{2n}}\int_{\frac{\pi}{n}}^{\frac{2\pi}{n}} \sin\alpha\, d\alpha \;.\;.\;.\; + I_{\frac{\pi}{2}-\frac{\pi}{2n}}\int_{\frac{\pi}{2}-\frac{\pi}{n}}^{\frac{\pi}{2}} \sin\alpha\, d\alpha$$

$$+ I_{\frac{\pi}{2}+\frac{\pi}{2n}}\int_{\frac{\pi}{2}}^{\frac{\pi}{2}+\frac{\pi}{n}} \sin\alpha\, d\alpha \;.\;.\;.\; + I_{\pi-\frac{\pi}{2n}}\int_{\frac{(n-1)\pi}{n}}^{\pi} \sin\alpha\, d\alpha,$$

* American Illuminating Engineering Society, February 18, 1908.

Performing the integration—

$$I_{\triangledown}=I_{\frac{\pi}{2n}}\left(1-\cos\frac{\pi}{n}\right)+I_{\frac{3\pi}{2n}}\left(\cos\frac{\pi}{n}-\cos\frac{2\pi}{n}\right)\ldots+I_{\frac{\pi}{2}-\frac{\pi}{2n}}\cos\left(\frac{\pi}{2}-\frac{\pi}{n}\right)$$

and—

$$I_{\triangle}=I_{\frac{\pi}{2}+\frac{\pi}{2n}}\sin\frac{\pi}{n}+I_{\frac{\pi}{2}+\frac{3\pi}{2n}}\left(\sin\frac{2\pi}{n}-\sin\frac{\pi}{n}\right)\ldots+I_{\pi-\frac{\pi}{2n}}\left(1-\cos\frac{\pi}{n}\right).$$

Each term in these two expressions is the difference between the vertical projections of the mean value of I for the sector at the boundaries of the sector.

The resulting graphical construction is given in Fig. 3.

Let $OU'R'P'C$ be any polar curve of distribution. Draw radii at equal angular intervals of 15° from 0° to 180°. This gives a value of 12 to n in the above equations, and is a sufficiently close division for most work. Describe the arc AP'' with O as centre and radius OP'' equal to the mid-zone value of the intensity for the first 15°. The projection AB of AP'' on the vertical through O is the first term in the equation for $I_{\triangledown}$. Describe the arc $P''Q''$ with P as centre and a radius PP'' equal to the mid-zone value of the intensity for the second interval of 15°. The projection BC of this arc on the vertical is the second term. Repeating this construction in the manner shown, AG is arrived at as the value of $I_{\triangledown}$ and GL as the value of $I_{\triangle}$.

From the relation $I_{\triangledown}+I_{\triangle}=2I_{\circ}$, it follows that $\frac{AL}{2}=I_{\circ}$.]

*Rousseau's Method.**

This method is much more widely known than the preceding, but, as it involves the estimation of an area, it is somewhat longer.

From Fig. 2 the width of the zone of the sphere containing R, and having an area dS, is $rd\alpha$, and the height of this zone is—

$$dh=rd\alpha\sin\alpha.$$

* Rousseau, "Lumière Électrique, vol. xxvi., p. 60.

If this value is substituted in the expression for dS (p. 29), that relation is transformed to—

$$dS = 2\pi r^2 \sin \alpha \, d\alpha = 2\pi r dh.$$

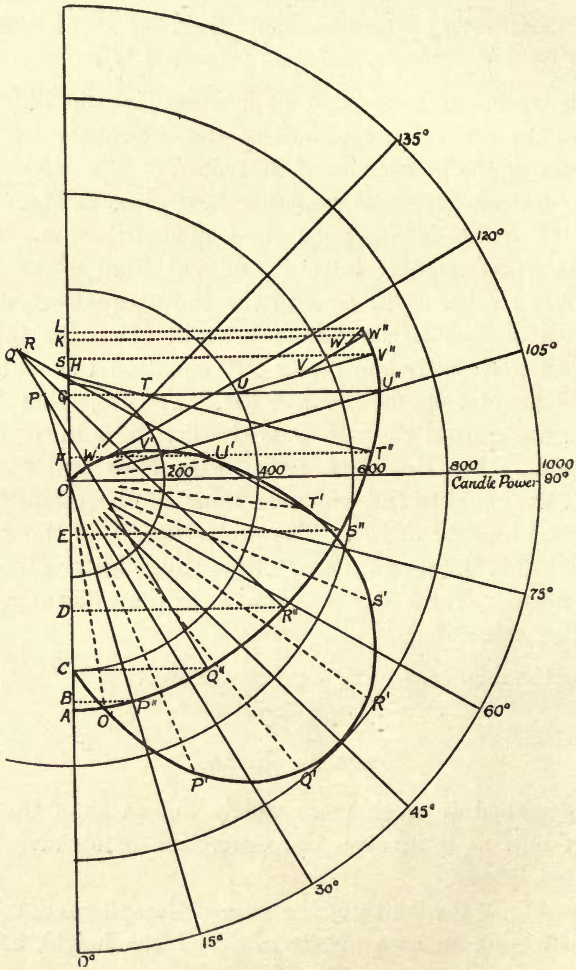


FIG. 3.—KENNELLY'S CONSTRUCTION FOR MEAN SPHERICAL AND MEAN HEMISPHERICAL INTENSITY.

Taking a sphere of radius r, and replacing 4π and 2π by $4\pi r^2$ and $2\pi r^2$ in equations (6) and (7),

$$I_0 = \frac{1}{4\pi r^2}\int I 2\pi r dh = \frac{1}{2r}\int_{h=+r}^{h=-r} I dh ;$$

or

$$\frac{1}{2}\int_{h=+1}^{h=-1} I dh \text{ if } r = \text{unity} ;$$

and

$$I_{\triangledown} = \frac{1}{2\pi r^2}\int I 2\pi r dh = \frac{1}{r}\int_{h=+r}^{h=o} I dh ;$$

or

$$\int_{h=+1}^{h=o} I dh \text{ if } r = \text{unity}.$$

Rousseau's method follows very simply from the formula obtained in this shape. Let the mean spherical intensity be required from the curve of distribution given in the right-hand half of Fig. 4, p. 34. The curve is the same as that used in demonstrating Kennelly's method. A circle is described about O as centre, with a radius of length corresponding to some convenient value of candle-power, say 1,000. Every section SS_1 of the vertical diameter KG through the source corresponds to the height dh of a spherical zone. The appropriate luminous intensity is set up as an ordinate from S_1, so that KG being the axis of abscissæ,

$$I = OT = SU.$$

The line joining the extremities of all such ordinates is the Rousseau curve, and defines an area which represents $\int I dh$ in the expressson for I_0.

The value of I corresponding to the point S is determined

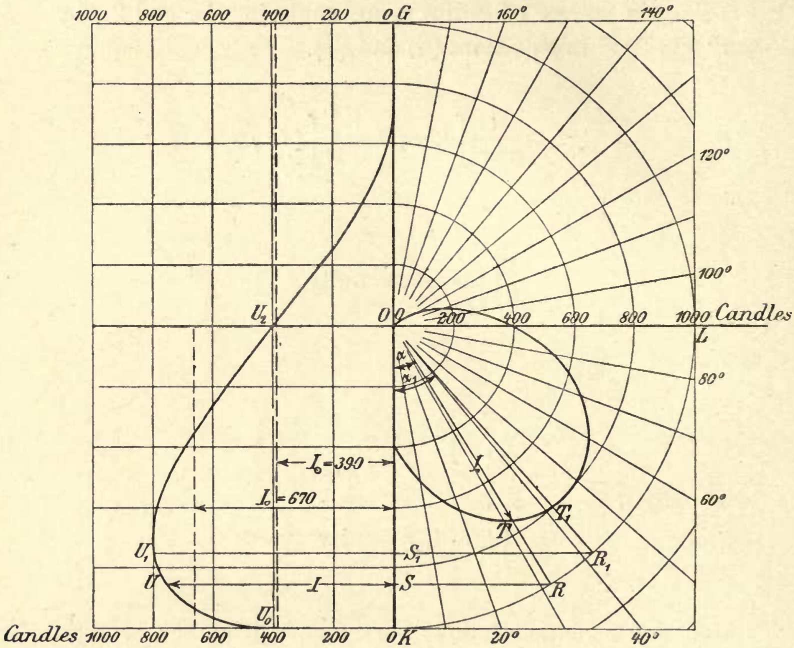


FIG. 4.—ROUSSEAU'S METHOD FOR THE DETERMINATION OF MEAN SPHERICAL AND MEAN HEMISPHERICAL INTENSITY.

by projecting S back to R on the semicircle KLG, and taking the intercept on OR made by the distribution curve.

The whole area KU_0U_2G is then divided by $2r$, and the result is the mean spherical intensity. Thus,

$$I_{\circ} = \frac{1}{2r}\int_{h=+r}^{h=-r} I dh = \frac{\text{Area } KU_0U_2G}{2r}.$$

Also, the mean hemispherical intensity is given by dividing the area KU_0U_2O by $r = OK$. Thus,

$$I_{\triangledown} = \frac{1}{r}\int_{h=r}^{h=0} I dh = \frac{KU_0U_2O}{r}.$$

The number of points taken between K and G may be the same as the number of observations made to determine the distribution curve (about 20 for I_0, and 10 for $I_\smile$), the area of the curve being then taken with a planimeter.

12. Approximate Methods for Determining Mean Spherical and Mean Hemispherical Candle-Power.

I.

The construction and planimetry of the Rousseau curve can be avoided in the following way: Divide the diameter KG (Fig. 5) into twenty equal parts. Determine the appropriate luminous intensity at the mid-point of each part. The twenty values so obtained are given to the right of Fig. 5, p. 36; add these up, divide by twenty, and the result is the mean ordinate of the Rousseau curve, and the value of the mean spherical intensity.

Take only the sum of the ten values of the luminous intensity in the lower hemisphere and divide by ten, and the result is the mean lower hemispherical intensity. The same procedure in the upper hemisphere gives the mean upper hemispherical intensity. These values have been worked out in Fig. 5 for the distribution curve given there, and the agreement between these and the values obtained by the two graphical methods will be noted.

The accuracy of the values arrived at in this way is of the same order as that which is possible with the photometric measurements. When the curve of distribution is fairly regular, as in incandescent lamps, ten parts only need be taken instead of twenty.

II.

An estimate of the mean spherical or hemispherical intensity is often required without tracing the curve of distribution in detail. Such a measurement is best carried out with an Ulbricht globe photometer; but, failing this, L. W. Wild has given a method which allows of the calculation of mean spherical or hemispherical intensity from measurements made at intervals of 30° without drawing the distribution curve.* The method gives results which are

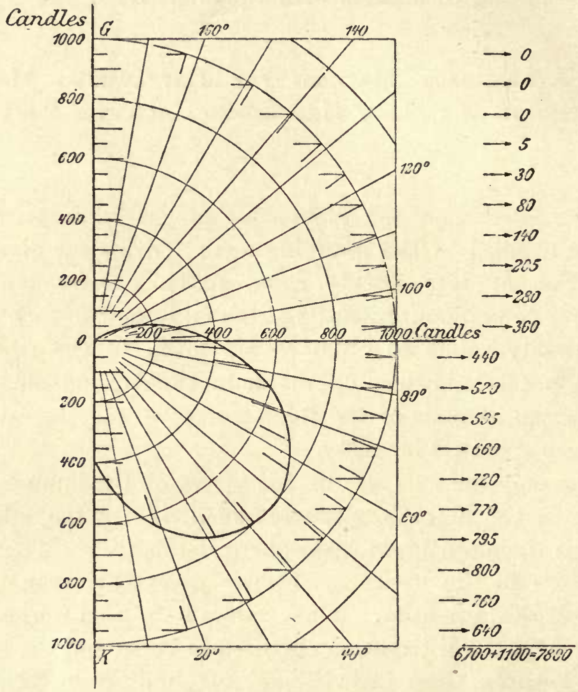


FIG. 5.—APPROXIMATE METHOD FOR THE DETERMINATION OF MEAN SPHERICAL AND MEAN HEMISPHERICAL INTENSITY.

$$I_{\cup} = \frac{6700}{10} = 670 \ C.P.$$

$$I_{\cap} = \frac{1100}{10} = 110 \ C.P.$$

$$I_{\circ} = \frac{7800}{2} = 390 \ C.P.$$

accurate to about ½ per cent. for fairly uniform sources, such as incandescent lamps. The accuracy is smaller with such unsymmetric sources as arc lamps, but the error seldom exceeds 3 per cent.

* *Electrician,* 1905, p. 396; *Elektrotechnische Zeitschrift,* 1906, p. 122.

The method may be explained by reference to Fig. 4, p. 34. The area of the slice SUU_1S_1 of the Rousseau curve KU_0U_2G is given by—

$$Idh = I(SS_1) = Ir(\cos\alpha - \cos\alpha_1),$$

where I is the mean of the intensities between the angles α and α_1, and practically agrees with the luminous intensity for the angle $\frac{\alpha+\alpha_1}{2}$. The mean spherical intensity is then given by the expression—

$$I_o = \frac{1}{2r}\int_{h=+r}^{=-r} Idh = \frac{\Sigma Ir(\cos\alpha - \cos\alpha_1)}{2r} = \frac{1}{2}\Sigma I(\cos\alpha - \cos\alpha_1).$$

The angle $(\alpha - \alpha_1)$ may have any value, provided that the number of sections be great enough. When the interval is 30° the equation becomes—

$$\begin{aligned} I_o = \tfrac{1}{2}\{ & I_{0^\circ}(\cos 0^\circ - \cos 15^\circ) + I_{30^\circ}(\cos 15^\circ - \cos 45^\circ) \\ & + I_{60^\circ}(\cos 45^\circ - \cos 75^\circ) + I_{90^\circ}(\cos 75^\circ - \cos 105^\circ) \\ & + I_{120^\circ}(\cos 105^\circ - \cos 135^\circ) + I_{150^\circ}(\cos 135^\circ - \cos 165^\circ) \\ & + I_{180^\circ}(\cos 165^\circ - \cos 180^\circ)\}. \end{aligned}$$

It would be more exact to take the values of luminous intensity for 7·5° and 172·5° in place of those for 0° and 180°, but these terms contribute so little to the sum that this irregularity is quite permissible. Substituting the numerical values of cosines in the above from trigonometric tables,

$$\begin{aligned} I_o = {} & 0{\cdot}017 I_{0^\circ} + 0{\cdot}1295 I_{30^\circ} + 0{\cdot}224 I_{60^\circ} + 0{\cdot}259 I_{90^\circ} \\ & + 0{\cdot}224 I_{120^\circ} + 0{\cdot}1295 I_{150^\circ} + 0{\cdot}017 I_{180^\circ}; \end{aligned}$$

or

$$\begin{aligned} I_o = {} & 0{\cdot}017(I_{0^\circ} + I_{180^\circ}) + 0{\cdot}1295(I_{30^\circ} + I_{150^\circ}) \\ & + 0{\cdot}224(I_{60^\circ} + I_{120^\circ}) + 0{\cdot}259 I_{90^\circ} \quad . \quad . \quad (8). \end{aligned}$$

The mean hemispherical intensity can be calculated in the same way, given the values of I at 0°, 30°, 60°, and 82·5°.

The equation is—

$$\begin{aligned} I_{\triangledown} = {} & I_{0^\circ}(\cos 0^\circ - \cos 15^\circ) + I_{30^\circ}(\cos 15^\circ - \cos 45^\circ) \\ & + I_{60^\circ}(\cos 45^\circ - \cos 75^\circ) + I_{82\cdot5^\circ}(\cos 75^\circ - \cos 90^\circ); \end{aligned}$$

or

$$I_{\overline{\smile}} = 2\{0{\cdot}017 I_{0^\circ} + 0{\cdot}1295(I_{30^\circ} + I_{82\cdot5^\circ}) + 0{\cdot}224 I_{60^\circ}\} \quad . \quad (9).$$

As an example of the use of this approximate method, the calculation of the mean spherical and hemispherical intensity from the distribution curve of Figs. 4 and 5 is as follows :

I_{0°	$= 400$ *C.P.*	I_{90°	$= 400$ *C.P.*
I_{30°	$= 745$ „	I_{120°	$= 55$ „
I_{60°	$= 745$ „	I_{150°	$= 0$ „
$I_{82\cdot5^\circ}$	$= 505$ „	I_{180°	$= 0$ „

From these figures,

$$I_{\circ} = 0{\cdot}017(400+0) + 0{\cdot}1295(745+0) + 0{\cdot}224(745+55) + 0\;259 \times 400 = \underline{386\ C.P.}$$

and

$$I_{\overline{\smile}} = 2\{0{\cdot}017 \times 400 + 0{\cdot}1295(745+505) + 0{\cdot}259 \times 400\} = \underline{672\ C.P.}$$

A comparison of these results with those obtained by the more exact method, viz.—

$$\underline{I_{\circ} = 390\ C.P.}, \text{ and } \underline{I_{\overline{\smile}} = 670\ C.P.},$$

shows that there is an error of only 1 per cent. in I° and of 0·3 per cent. in $I_{\overline{\smile}}$.

[This method is the numerical expression of Kennelly's graphic construction.]

III.

With the object of lessening the experimental work and rendering the calculation simpler still, the author has deduced two formulæ from the approximate methods already described, which admit of the calculation of mean spherical intensity as the mean of six measurements, and the mean hemispherical intensity as the mean of three measurements only. They are easy formulæ to remember, and give values exact to 5 per cent. for quite irregular sources.

Luminous intensities are measured at angles of 30°, 60°, and 80° from the vertical axis for determinations of mean hemispherical intensity, and, in addition, at 100°, 120°, and 150°, if the mean spherical intensity is required.

The following formulæ are then employed:

$$I_{\circ} = \tfrac{1}{8}(I_{30^\circ} + 2I_{60^\circ} + I_{80^\circ} + I_{100^\circ} + 2I_{120^\circ} + I_{150^\circ}) \quad . \quad (10).$$
$$I_{\triangledown} = \tfrac{1}{4}(I_{30^\circ} + 2I_{60^\circ} + I_{80^\circ}) \quad . \quad . \quad (11).$$

Applying this method to the same distribution curve as before, the values of I are—

$I_{30^\circ} = 745$ *C.P.* $\qquad I_{100^\circ} = 265$ *C.P.*
$I_{60^\circ} = 745$ „ $\qquad I_{120^\circ} = 55$ „
$I_{80^\circ} = 535$ „ $\qquad I_{150^\circ} = 0$ „

and the equation is—

$$I_{\circ} = \tfrac{1}{8}(745 + 2 \cdot 745 + 535 + 265 + 2 \cdot 55 + 0) = \underline{393 \; C.P.}$$
$$I_{\triangledown} = \tfrac{1}{4}(745 + 2 \cdot 745 + 535 = \underline{692 \; C.P.}$$

The errors are $+0 \cdot 8$ per cent. and $+3 \cdot 3$ per cent. respectively from the exact values of mean spherical intensity and mean hemispherical intensity ($I_{\circ} = 390$, $I_{\triangledown} = 670$).

13. Comparison of Intensities of Commercial Sources of Light.

The special characteristics of light sources in common use are not dealt with here, but as luminous intensity and energy consumption form the basis of all lighting calculations, it appeared advisable to include data of this kind relative to commercial sources. These are collected together in Tables XIV., XV., and XVI., pp. 147, 148, and 149.

First there is given the range of the normal consumption generally met with in a given type of lamp; then, limiting values for normal efficiency—that is, the consumption per candle. The luminous intensities of different sources are not always capable of direct comparison, because the type of source usually settles whether the maximum, horizontal, mean horizontal, or mean hemispherical intensity is given. The next three columns of the table, however, contain specific consumptions for all sources in relation to mean horizontal, spherical, and hemispherical intensity. The last three columns contain reciprocal values of these quantities, the numbers giving the return in candle-power for a consumption of 1,000 watts, 1,000 litres of gas per hour, or 1 litre of fluid combustible per hour.

Comparisons of the cost of illumination from different sources are very often set forth in tables of this kind, but such comparisons can only give conclusive information when made between intensities and distributions not differing very widely from one another. For example, exact comparison can be made between carbon filament lamps and metal filament lamps, or between incandescent mantles supplied with gas or petrol vapour, either in regard to economy or production of light; and in either case the comparison could be based indifferently on the mean horizontal, spherical, or hemispherical intensity. But if it is desired to compare together entirely different light sources, such as incandescent electric lamps with arc lamps, or incandescent gas lamps with Nernst lamps, the conclusions arrived at are never free from errors, and are often entirely untrustworthy, no matter what kind of luminous intensity the comparison may be based upon.

It is evident that horizontal intensity is not adapted to estimates of this kind, neither can reliable figures of cost be extracted from a comparison of mean spherical intensities. It is true that the total fluxes of light emitted are contrasted by this means, but this comparison has already been shown (p. 21) to have a value more theoretical than practical.

The best figure to use in making these comparisons practically is the mean hemispherical intensity. It is then necessary to take into account the effect of globes or reflectors, as these accessories are usually present, and can materially modify its value. For this reason the values of luminous intensity, specific consumption, and luminous efficiency, in Tables XIV., XV., and XVI., are all given for uncovered sources without globe or reflector, the effect of these being so various that no sufficiently general mean value can be quoted for covered sources. The influence of globes and reflectors is, however, given separate treatment in the next section, because of its great importance in determining illumination values.

14. Influence of Globes and Reflectors.

(1) *Globes.*

When light sources are to be simply protected from moisture, dust, currents of air, etc., it is sufficient to surround them with globes of transparent glass. But if, in addition, it is desired to lower the surface brightness by radiating from a larger surface, then various forms of opal, opaline, alabaster, ground or frosted glass globes are used.*

The effect of such globes in increasing the uniformity of distribution is great or small, depending upon whether the initial distribution is irregular or regular. The holophane globes, due to Mr. A. P. Trotter,† and developed by Professor A. Blondel,‡ are exceptional, as by means of prismatic ridges arranged in correct positions on the surface of the globe almost any desired distribution of light can be obtained.

The use of a globe always diminishes the available luminous intensity of a source, as a part of the light is absorbed in its passage through the glass. The diminution is known as the loss due to absorption, or simply the absorption, and can be expressed as so much per cent., either of the total luminous flux, or of the mean spherical intensity without globe. The value of this absorption varies very much with the shape of

* Monasch, *Elektrische Beleuchtung,* p. 166.

† *Proc. Inst. Civ. Eng.,* 1884, vol. lxxviii., p. 346.

‡ *Bulletin de la Soc. Française de Physique,* March 17, 1893, p. 84; *L'Éclairage Électrique,* October 27, 1894, p. 308.

globe and the kind of glass. The following are limiting values of this quantity expressed as a percentage:

Clear glass globes . . .	3–10	per cent.	loss.
Holophane globes . . .	5–15	,,	,,
Opal and opaline globes . .	10–20	,,	,,
Ground or frosted glass globes .	15–30	,,	,,
Alabaster globes . . .	20–40	,,	,,
Milk-glass globes . . .	30–50	,,	,,

The absorption loss is often deduced from the ratio of the mean hemispherical intensities with and without globes, particularly with arc lamps. This, however, does not give the true absorption loss in any case of irregular initial distribution, because of the rearrangement of light emission in the two hemispheres. Due to the use of a dispersive globe, the distribution is rendered more regular, and in the case of an arc lamp, a relatively larger part of the light is sent into the upper hemisphere. For that reason, absorption loss reckoned on this basis always appears greater than it really is when referred to mean spherical intensity, the apparent increase certainly amounting in general to as much as 50 per cent. The figures for absorption losses given above should therefore be multiplied by 1·5 in order to obtain approximately correct values of the diminution of mean hemispherical intensity of arc lamps produced by enclosure in dispersive globes.

(2) *Reflectors.*

As distinct from ordinary globes, the chief use of reflectors is to send the light from the source into those directions where it will be most useful, usually in the lower hemisphere. On the one hand, therefore, reflectors increase the mean hemispherical intensity by reflection, while, on the other, just as with globes, they diminish the mean spherical intensity by absorption of part of the incident light.

The action of a reflector depends greatly on its design, as this determines the quantity of light which it can direct from the upper into the lower hemisphere. Reflectors are not a great advantage to sources which already send the greater part of their light to the ground—as, for instance, in the case of continuous current arc lamps—but the use of suitable reflectors can largely increase the mean lower hemispherical intensity of such sources as electric incandescent lamps or upright gas-mantles, where, without a reflector, a large part of the light emitted is sent into the upper hemisphere. The percentage of total luminous flux sent into the lower hemisphere may be increased in such cases from about 40 or 55 per cent. without a reflector to 60 or 80 per cent. with one; in fact, a suitably chosen reflector, properly fixed, and allowing no light whatever to escape vertically, can make the whole of the reflected light available in the lower hemisphere.

The loss by absorption in reflectors varies very much with the construction and material forming the reflecting surface. The reduction effected in the mean spherical intensity due to this source of loss generally varies from 5 to 25 per cent. of its value without a reflector, although sometimes, due to bad reflecting surfaces, this loss may amount to as much as 40 per cent.

If a given source sends p per cent. of the total luminous flux into the lower hemisphere, then, in accordance with the principles given on p. 22, the value of the mean hemispherical intensity ($I'_{\triangledown}$) in terms of the mean spherical intensity with reflector ($I'_{\circ}$) is given by—

$$I'_{\triangledown} = \frac{2p}{100} I'_{\circ}.$$

If this is effected by a reflector which produces a diminution of a per cent. in the mean spherical intensity without a reflector ($I_{\circ}$), then the mean hemispherical intensity with reflector ($I'_{\triangledown}$) is given by—

$$I'_{\triangledown} = \frac{2p}{100} \cdot \frac{100 - a}{100} I_{\circ}.$$

An example of the use of this formula is given on p. 74. Another example on p. 83 brings into prominence the great influence which the choice of an efficient reflector can have on the desired illumination.

Reflectors are very often found in use that fulfil their functions very imperfectly. As an instance, the efficiency of the sheet-metal reflectors often fitted in street-lanterns above upright gas-mantles, is very small.* They increase the mean hemispherical intensity by 6·5 per cent. only, although, without the reflector, about 55 per cent. of the total luminous flux is emitted in the upper hemisphere.

* Wedding, *Journal für Gasbeleuchtung*, 1904, p. 563 ; Drehschmidt, *ibid.*, 1906, p. 767.

CHAPTER III

THE ESTIMATION OF ILLUMINATION

The comparison of different methods of lighting on the basis of luminous intensity is imperfect, because the resultant illumination depends not only on this intensity, but also, and in great measure, on the distribution of the light and the arrangement of the sources.

Conclusive comparisons can only be drawn, having regard to the actual illumination produced. To make matters simple, and to avoid assumptions, it is of prime importance to check predetermined values by actual measurement.

The first part of this chapter is devoted to a discussion of the general principles on which such comparisons can be made.

15. Street Lighting.

The question of the correct method of dealing with street lighting has been agitated for years in the literature of the subject, without any definite conclusion being reached. In a memoir entitled "Public Lighting by Arc Lamps," Professor Blondel* has put forward, as a result of extended experiments, no less than five criteria as governing the design of street lighting, and since then many others have been proposed. In order to arrive at estimates as free as possible from objection, it is essential to keep down the number of constants used, and to choose these so that they can be easily predetermined, and afterwards verified by experiment.

In street lighting the quantities to be considered are the desired vertical and horizontal illuminations. These are the

* *Le Génie Civil,* 1894-95, vol. xxvi., p. 279.

illuminations upon a vertical and upon a horizontal plane respectively. The horizontal illumination of a street is deemed sufficient if it is possible to distinguish with ease objects lying on the ground, or to read a letter or fairly large print anywhere without difficulty. The recognition of passers-by, and the legibility of street notices and directions, is, on the other hand, determined by the vertical illumination. The two points of view are of equal importance. The question then arises as to whether there are other grounds for choosing one of these as a standard in preference to the other, or whether one alone can be considered as sufficient, as it would be laborious and tedious if both values were always to be measured and calculated.

At any point of the street surface the horizontal illumination has a unique value. On the other hand, the vertical illumination at any place depends on the orientation of the particular vertical plane chosen, and on whether the side of it is considered facing the nearest lantern, or the opposite side. These two values are, in general, very different. Thus, a person's face might be well lit when looking in the direction of the nearest lantern; but on his turning round with his back to this source, and with the other lamps far away, his face might not be lit at all. Vertical illumination can then present at one and the same place in a street much greater variations than occur with the horizontal illumination anywhere over the whole surface of the street, without the particular scheme being considered as useless or even defective.

A correct estimate of the vertical illumination at a given point means, therefore, the evaluation of this quantity in a number of different vertical planes through the point—for example, at every 90°—thus giving four observations for each point, four times as many as are necessary for an estimate of the horizontal illumination. Naturally, this multiplication of observations would be tedious, and measurements a hindrance instead of a help to correct judgment.

So far, then, the evidence is decidedly in favour of the horizontal illumination as being the only quantity which is perfectly definite at every point of the street surface, and preference is therefore given to this, on the understanding that a sufficient horizontal illumination also provides for the claims of vertical illumination. This is usually the case. In those instances where there are vertical planes not receiving light from the source—as, for example, in the central lighting of open spaces, where the vertical planes passing through the source are not illuminated—the specification for a minimum vertical illumination naturally would not apply, especially as in such cases the horizontal illumination may have been made as great again as usual.

Agreement on this question has not hitherto been arrived at. In particular, gas engineers still regard the criterion of vertical illumination as the basis of their lighting schemes.*

For the reasons just given, the horizontal illumination will be treated as of first importance in the discussion following, reserving the consideration of vertical illumination for those cases where it is obviously necessary to do so.

The question also arises as to whether horizontal illumination should be measured and calculated at the level of the ground itself, or at a given height above it. For easy recognition of objects lying on the ground, the first should be taken; for the ability to read easily, the reference is best made to a plane at a height of 1·5 metres (4 feet 11 inches) above the ground. It is also much easier to make measurements at this height than directly on the ground.

It may, therefore, be stated that street lighting is best estimated on the basis of the horizontal illumination at a height of 1·5 metres, or, in round numbers, 5 feet above the ground.

16. Interior Lighting.

In this connection, the lighting of tables and work-benches is the chief thing to be considered. Horizontal surfaces are almost exclusively dealt with, so that for interiors the

* Bloch, Drehschmidt, Krüss, and Uppenborn, *Journal für Gasbeleuchtung,* 1906.

horizontal illumination will be the standard of reference. In exceptional cases the vertical illumination becomes the important quantity—as in art galleries, for the illumination of the walls, and in studios with nearly vertical easels.

As then, for interiors, the horizontal illumination of tables has usually to be considered, the height of the horizontal plane of reference for measurement and calculation is chosen at 0·8 to 1·0 metre (2 feet 7½ inches to 3 feet 3½ inches) above the floor.

17. Mean Horizontal Illumination.

The distribution of light over an illuminated surface is, in general, irregular, the horizontal illumination varying at different places between a minimum and a maximum. The knowledge of these two quantities does not, however, give a correct idea of the value of the illumination. It is easy to attain to quite large maxima by suspending the lamps at very small distances above the tables, but then the general illumination of the room may be wholly insufficient.

On the other hand, the minimum illumination may be excessively reduced owing to chance shadows of trees or the like, although the lighting as a whole may be quite good. Also, in street lighting, the minimum values of horizontal illumination may be so small that they are not easy to measure with the available photometric apparatus.

The mean horizontal illumination (E_m) is, however, a really practical quantity by which to judge a lighting scheme. The exact expression for it is given by—

$$E_m = \frac{1}{S}\int E dS = \frac{\Phi}{S} \quad . \quad . \quad . \quad (12).$$

in which E is the horizontal illumination at a given point of the surface illuminated, and S is the area of this surface. The integral is equal to the total luminous flux received, divided by the surface S. This compares with the derivation of the conception of illumination from the flux of light given on p. 18.

The mean horizontal illumination can therefore be written approximately as—

$$E_m = \frac{\Sigma(Es)}{\Sigma s},$$

s being the area of an element of the surface, and E the horizontal illumination at the centre of it. Let each of these elements be of unit area, and let z be the number of units in the surface s, then:

$$E_m = \frac{\Sigma(Es)}{zs} = \frac{\Sigma(E)}{z}.$$

An approximate integration of this kind permits of the determination of mean horizontal illumination from a finite number of measured or calculated values of horizontal illumination, it being impossible to measure the mean value directly. Later on approximate methods will be given for its determination, applicable to both calculated and measured figures, which will greatly simplify the task of arriving at the most suitable values in every case.

18. Uniformity of Lighting.

A criterion of the uniformity of lighting is the ratio of the maximum value to the minimum value of the horizontal illumination. The greater this ratio, the more irregular the lighting. A still better estimate of the uniformity of illumination can be made from the ratios of the maximum and minimum values to the mean value of the horizontal illumination. The more uniform the lighting, the nearer are these two ratios to unity. Particular care should be taken that the ratio of minimum to mean horizontal illumination is not allowed to become too small, as otherwise dark areas are certain to occur. On the other hand, the ratio of maximum to mean horizontal illumination need not be brought so nearly to unity, as in many cases specially bright areas within the limits of the general illumination are required to read certain type or to see certain objects with great distinctness.

19. Constancy of Light Sources.

While the limits of local variation of illumination are fixed by the demand for a certain amount of uniformity, the allowable variations with time will depend upon requirements as to constancy. These naturally vary very much, according to the special purpose of the lighting. In general, constancy of illumination is not so necessary in the street as in interiors, less visual disturbance resulting from time variation in the first case than in the second.

It is almost always sufficient to rely simply on the eye in estimating the amount of this variation, and it is seldom that the discrepancies in a series of readings due to this cause are great enough to warrant the taking of time curves of illumination.

20. The Colour of Light.

Colour plays an important part in judging of the value of an illumination. The best colour is evidently that which approaches most nearly to daylight, no definite tint being predominant. The light of an ordinary arc lamp very nearly satisfies this condition, while most other artificial sources have some characteristic colour. Thus for a long time the greenish tint of the mantle gas-burner was regarded as objectionable, although users gradually became quite accustomed to it, and to-day great dislike is often taken to the yellowish or reddish tint of certain flame arc lamps. Special carbons are now, however, available for these lamps, from which a nearly white light is obtained, thus getting over the difficulty. There is no doubt that the strangeness of the colour of these sources is chiefly responsible for the disagreeable sensations provoked.

This applies with particular force in the case of mercury-vapour lamps, and has greatly hindered their adoption. There are hardly any red rays in the light from these lamps, the result being that objects of a reddish colour appear altogether different; in particular, the colour of the human skin is very disagreeable by this light. Such a light is thus quite unsuitable where æsthetic considerations are paramount. There are, however, many cases where these lamps might well be applied, in which the unusual colour of the light is of no consequence.

It is often preferable to use a coloured light. For instance, in restaurant and shop lighting, the warm, slightly reddish colour of incandescent electric lamps is particularly suitable. The red colour obtainable with flame arc lamps using impregnated carbons is of use sometimes in enhancing the appearance of perishable articles of a reddish tinge when exposed for sale.

21. Efficiency of Illumination.

What is usually known as the *efficiency* of an illumination is important in making comparisons between different methods of lighting. It is the ratio between the energy used, or its cost, to the illumination produced by it, and really measures the *inefficiency* of the lighting, although the opposite usage is sanctioned by custom. Using the definition of mean horizontal illumination given on p. 48, the efficiency can be expressed as the consumption in watts or in cubic metres or cubic feet of gas per hour, or in litres or gallons of combustible liquid per hour, for each square metre or square foot of surface maintained at 1 lux or 1 foot-candle of mean horizontal illumination. As soon as these "efficiencies" or *economic coefficients* are known in a sufficient number of cases, they afford a simple means of performing approximate calculations of illuminating values, as will be explained in more detail on p. 95.

CHAPTER IV

THE CALCULATION OF ILLUMINATION

22. Normal, Horizontal, and Vertical Illumination.

Consider a lamp of which the distribution of light in the lower hemisphere is given by Fig. 6.

The lamp is suspended at the point O at a height h above the horizontal plane; the vertical axis of the lamp cuts the horizontal plane in the point H, often called the *foot* of the lamp. The problem is to calculate the normal, horizontal,

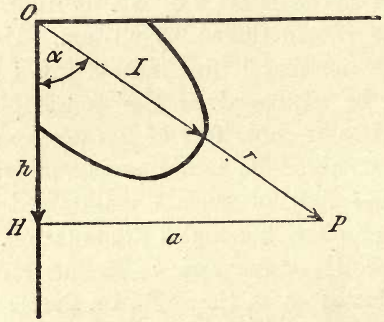


Fig. 6.—Normal and Horizontal Illumination.

and vertical illumination at the point P distant a from the foot of the lamp, and r from the lamp itself.

The intensity in the direction OP, making an angle α with the vertical axis, is given by I on the distribution curve of Fig. 6., above. The normal illumination (E_N) at P—that is, the illumination on a plane passing through P and perpendicular to the direction of the ray OP—is given by (p. 19)—

$$E_N = \frac{I}{r^2} = \frac{I}{a^2 + h^2} \quad . \quad . \quad . \quad (13).$$

The horizontal illumination (E_H) at P is then obtained by multiplying the normal illumination just arrived at by the cosine of the angle which the horizontal plane makes with the normal plane. This angle is α, the angle between the normals to the two planes.

The relation is—

$$E_H = E_N \cos\alpha = \frac{I\cos\alpha}{r^2} = \frac{Ih}{r^3} = \frac{Ih}{\sqrt{(a^2+h^2)^3}} = \frac{I\cos^3\alpha}{h^2} \qquad (14).$$

The vertical illumination at the point P has different values according to the angle which the vertical plane makes with the normal plane at that point. To calculate the vertical illumination (E_V) at P for the plane f (Fig. 7), the

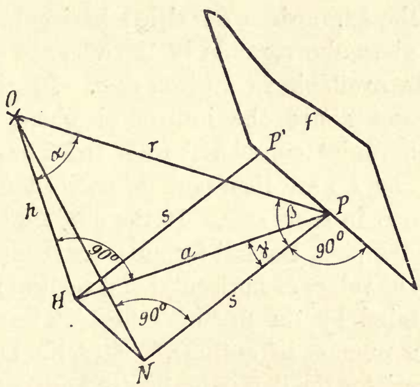


Fig. 7.—Diagram illustrating the Calculation of Vertical Illumination.

normal to which, PN, makes an angle β with the direction of the luminous ray OP and an angle γ with the direction HP, the relation is (p. 18)—

$$E_V = E_N \cos\beta = \frac{I}{r^2}\cos\beta.$$

The perpendicular distance from the foot of lamp H to the vertical plane f is—

$$s = r\cos\beta = r\sin\alpha\cos\gamma = a\cos\gamma;$$

and on substitution—

$$E_V = \frac{I}{r^2} \sin \alpha \cos \gamma = \frac{Is}{r^3} = \frac{Ia \cos \gamma}{\sqrt{(a^2+h^2)^3}}.$$

The vertical plane perpendicular to the direction PH is the one that is nearly always considered, for which—

$$\gamma = 0, \cos \gamma = 1, s = a, \text{ and } \beta = (90° - \alpha),$$

and the vertical illumination is a maximum, having the value—

$$E_V = E_N \cos (90° - \alpha) = E_N \sin \alpha = \frac{I \sin \alpha}{r^2} = \frac{Ia}{r^3}$$

$$= \frac{Ia}{\sqrt{(a^2+h^2)^3}} = \frac{I \sin \alpha \cos^2\alpha}{h^2} \quad . \quad . \quad (15).$$

Which of the expressions for either horizontal or vertical illumination, given above, would be the easier to use, depends upon the data available in a given case. In general, h the height of lamp above the horizontal plane, and a the distance of the point considered from the foot of lamp, are known. E_H and E_V are then best determined from the last formula but one in each case. If the angle α is given, and the height of the post, the last formulæ are the most suitable.

Calculation of values of horizontal and vertical illumination can be facilitated by the use of tables. These have been compiled by a number of authors.* Graphic methods have also been given† for the determination of horizontal illumination, but the use of a slide rule is generally easier and quicker.

A purely numerical method will be described later (p. 70), which allows of the determination of illumination values without the aid of a distribution curve.

The three different kinds of illumination having been

* A. P. Trotter, "Illumination: its Distribution and Measurement," pp. 275-278; Uppenborn, *Deutscher Kalender für Elektrotechniker*, 1907, part i., p. 303; Monasch, "Elektrische Beleuchtung," Table I; Hogner, "Lichtstrahlung und Beleuchtung" (eighth vol. of "Elektrotechnik in Einzeldarstellungen," edited by G. Benischke Braunschweig, 1906, Tables IV.-VII., p. 28.

† Uppenborn, *Kalender für Elektrotechniker*, 1907, vol. i., p. 304.

determined, curves may be drawn showing their relation to either the angle above the horizontal, or the distance from the foot of lamp, as in Fig. 8. These curves are derived from the curve of distribution of light of Fig. 43 (p. 168), and for a height of lamp of 8·5 metres (27 feet 10·5 inches) above the horizontal plane. They give the illumination as a function of the distance from the foot of lamp, and show clearly that

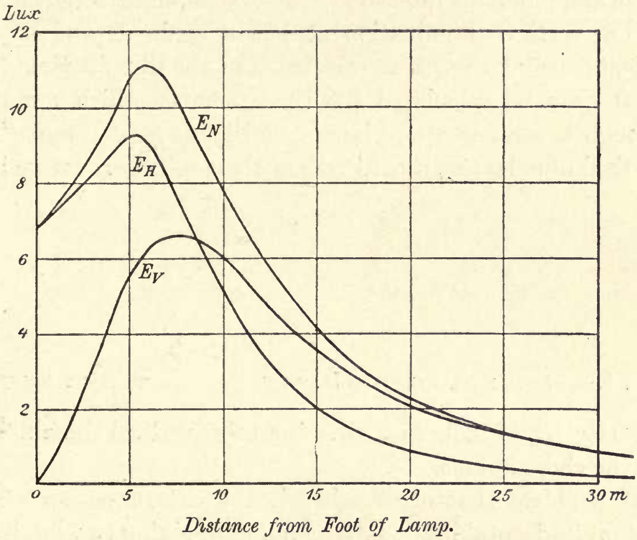


FIG. 8.—E_N=NORMAL ILLUMINATION, E_H=HORIZONTAL ILLUMINATION, E_V=VERTICAL ILLUMINATION.

horizontal illumination approximates to normal illumination at short distances, while the vertical illumination does so at long distances.

23. ILLUMINATION DUE TO DISTRIBUTED SOURCES.

If a plane is lit by a number of separate sources, L_I, L_{II}, L_{III} (Fig. 9), at a height h above it, the normal illumination at any point P in the plane has no meaning, as there is no plane passing through P which can be so placed as to be illuminated normally at one and the same time by all the sources. There remains, however, a definite horizontal illumination, no matter how many sources there may be. The horizontal illumination at P is evidently equal to the sum of the horizontal illuminations due to each source separately, as these quantities are all related to the same plane. The separate horizontal illuminations due to each source can then be calculated, as explained above (p. 53), from the luminous intensity of the source and its distance.

The vertical illumination at P will again depend on the orientation of the plane selected, and the illumination on it need only be calculated for those sources which are near enough to send an appreciable quantity of light there. The vertical illumination due to each of these sources is determined

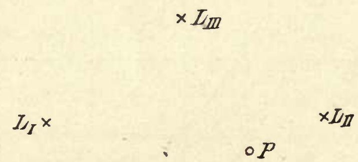


FIG. 9.—DIAGRAM ILLUSTRATING ILLUMINATION DUE TO MANY SOURCES.

separately, and their sum gives the total vertical illumination on the chosen plane.

A problem that often arises is the determination of resultant horizontal illumination on a line joining two luminous sources. This is arrived at by superimposing the curves of illumination proceeding from each source separately. Suppose, for example, the curve of horizontal illumination furnished by a source is that given in Fig. 8, and that two such sources, L_I and L_{II}, are placed at a distance of 30 metres (98 feet 5 inches). The curves for each are superimposed as is done in Fig. 10, and the sum of the ordinates E_{HI} and E_{HII} for a given point gives the ordinate of the resultant curve at that point. This curve has been drawn in on the figure.

[The variation of the horizontal illumination over a horizontal instead of a vertical plane can be shown by drawing lines on the plane, along which the horizontal illumination is constant. Such lines are known as iso-lux lines, and simple methods of drawing these, with examples, have been given by Trotter.*]

24. Mean Linear Illumination.

The mean ordinate of the curve of horizontal illumination in the line joining two lamps is often taken as the mean horizontal illumination. This is true only for the line between the two lamps, and not for any other portion of the plane. Hogner † has introduced the expression "mean linear

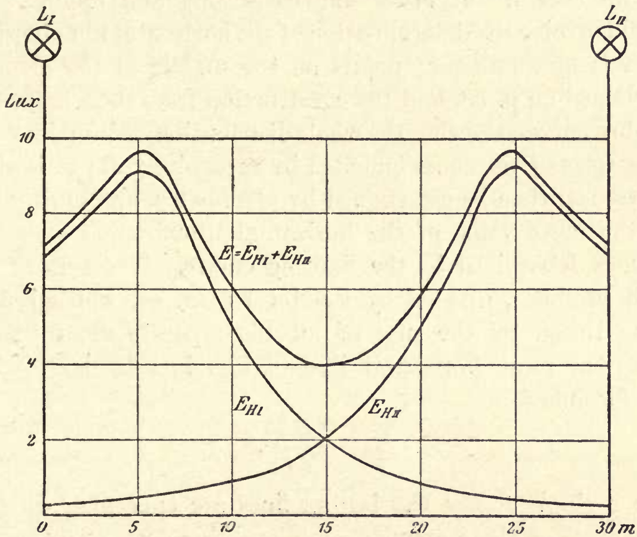


Fig. 10.—Variation of the Horizontal Illumination between Two Light Sources.

illumination" for this quantity, and recommends its use for narrow streets and roadways.

This mean linear illumination is, however, misleading, and unsuitable for forming a correct judgment of an illumination. The ground immediately under the lamps is taken into account in a much greater proportion than its area warrants. If the

* A. P. Trotter, "Illumination : its Distribution and Measurement," pp. 45-51.

† Hogner, "Lichtstrahlung und Beleuchtung," p. 59.

illumination is increased by lowering the lamps, the rise in the value of the mean linear illumination is much greater than the rise in the actual mean horizontal illumination. It is better, therefore, to avoid the use of such inconclusive quantities, and deal only with the mean horizontal illumination as defined on p. 48.

25. Calculation of Mean Horizontal Illumination.

The exact calculation of mean horizontal illumination on an area lit by many sources is long and minute. It would involve the determination of the horizontal illumination at a large number of points on the surface in the manner explained on p. 53, and the construction from these figures of iso-lux curves, showing the whole distribution of illumination. The areas of the zones bounded by successive pairs of iso-lux curves can then be ascertained by planimetry, and multiplied by the mean value of the horizontal illumination over the zone as determined by the limiting curves. The sum of all such products, divided by the total area, or, which is the same thing, by the sum of all the separate areas taken, gives the mean horizontal illumination in accordance with the formula —

$$E_m = \frac{\Sigma(Es)}{\Sigma s}.*$$

For a single source the iso-lux lines are concentric circles. The areas of these annular zones are, therefore, easily calculated, and the determination of the mean horizontal intensity is made correspondingly simple.

For many sources, the mean horizontal illumination due to each source separately over the whole area can be found in this manner, and the total arrived at by summing these values. The method given by Zeidler † is based on this principle.

* For completely worked examples of this nature, see Maréchal, "L'Éclairage à Paris," Baudry et Cie.

Lux, "Die öffentliche Beleuchtung von Berlin," 1896, pp. 439-457.

† "Die Elektrischen Bogenlampen" (part six of "Elektrotechnik in Einzeldarstellung"), edited by Dr. Benischke, Braunschweig, 1905, p. 100.

Blondel* gives some examples of the calculation of mean horizontal illumination for areas of any shape. He determines the luminous flux received by each zone of the surface instead of its mean horizontal illumination, and thus arrives at the whole flux over the area considered. He then divides this by the total area, and obtains the mean horizontal illumination.

The construction and planimetry of iso-lux curves, and the calculations involved, are always very tedious and troublesome. Results are more quickly obtained, and a clear idea of the luminous distribution formed, by dividing the illuminated surface into a number of equal squares or rectangles, and calculating the horizontal illumination for the centre of each square. This quantity is most easily got from a curve of horizontal illumination for a single lamp, such as that given in Fig. 8 (p. 55). The distance from the centre of each square to the foot of every lamp that contributes sensibly to the lighting at the point considered is measured from the plan. The value of the horizontal illumination for each of these distances is determined from the curve of horizontal illumination, and the sum of these values gives the desired illumination at the centre of each square. The mean of all these will give the mean horizontal illumination. An example is worked out in this way on p. 109.

This somewhat lengthy procedure can usually be shortened, as in most cases the symmetry of the distribution renders it necessary only to investigate a small portion of the illuminated area. In Fig. 11, for example, where a customary arrangement of lamps is shown, it is only necessary to divide a portion of the area into squares, in order to be able to make a complete calculation for the whole, the lighting being arranged symmetrically.

With from ten to twenty squares or rectangles, sufficient accuracy is attained without rendering the work too laborious.

For the practical use of specialists having little time at their disposal, even the above described method is rather

* Blondel, "L'Éclairage par les Lampes à Arc," *Le Génie Civil*, 1894-95, vol. xxvi., p. 230.

long. A result is required without much preliminary drawing or calculation, and a suitable method to this end is given in the next section.*

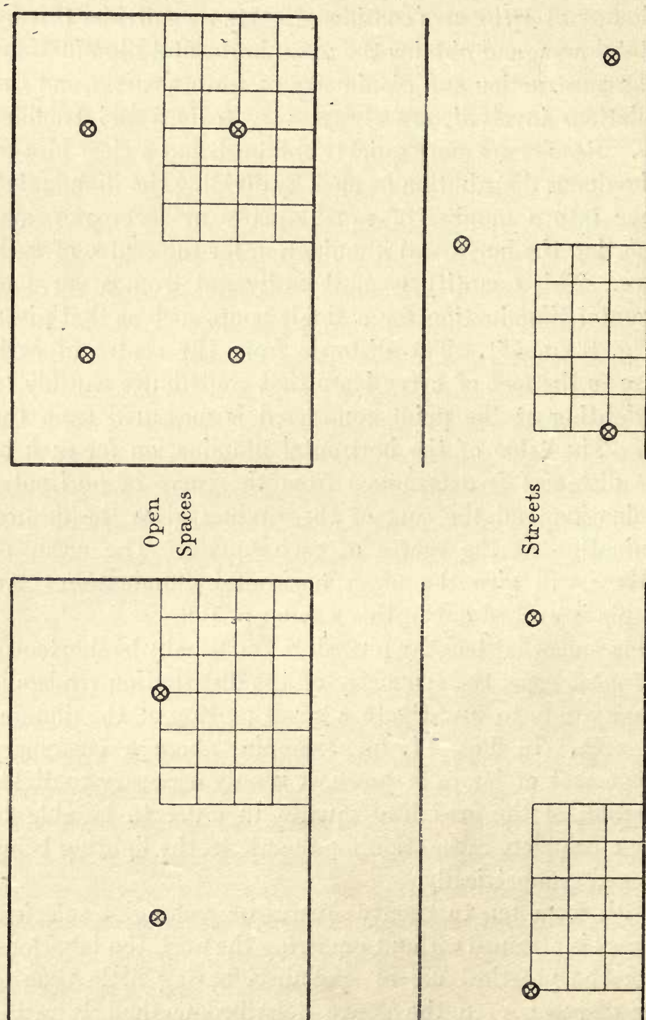


Fig. 11.—Division of Streets and Open Spaces into Rectangles

* The Author, *Elektrotechnische Zeitschrift*, 1906, pp. 493, 1129.

26. Simplified Method for the Calculation of Mean Horizontal Illumination.

The principle of the method will be demonstrated, first in a special case that can be very easily dealt with, and then in its general application to the problems arising in practice.

Let a circular area S of radius $HP=a$ be lighted from a luminous source O placed at a height h above the area, so that HP subtends an angle $HOP=\alpha$ from O (Fig. 12). The source having the luminous distribution shown, the mean horizontal illumination over the area S is required.

This would be given by dividing the total luminous flux Φ that falls on S by its area (p. 18), as expressed by the relation—

$$E_m = \frac{\Phi}{S}.$$

Hence Φ must be determined, and this is done as follows:

The relation between mean lower hemispherical intensity $I_{\triangledown}$ and the corresponding luminous flux Φ is given on p. 21 as—

$$\Phi_{\triangledown} = 2\pi I_{\triangledown}.$$

The mean hemispherical intensity is given as the area of the Rousseau curve between the limits specified on p. 29. To determine the area under this curve for any given solid angle α between 0° and 90°, a number of ordinates are drawn, dividing the area under the curve into parallel strips, and the light flux corresponding to each strip is calculated. Starting from zero at the point K, the integral curve of the Rousseau curve can then be drawn by plotting the sums of the light flux up to each point along the corresponding abscissæ. The ordinate of the integral curve for $\alpha=90°$ or cos $\alpha=0°$ corresponds to the mean hemispherical intensity. If this is multiplied by 2π, the total luminous flux is obtained for the solid angle given by $\alpha=90°$. In the same way for any other angle α, the total luminous flux up to that point is 2π times the corresponding ordinate Ψ.

This integral curve of the Rousseau curve is therefore

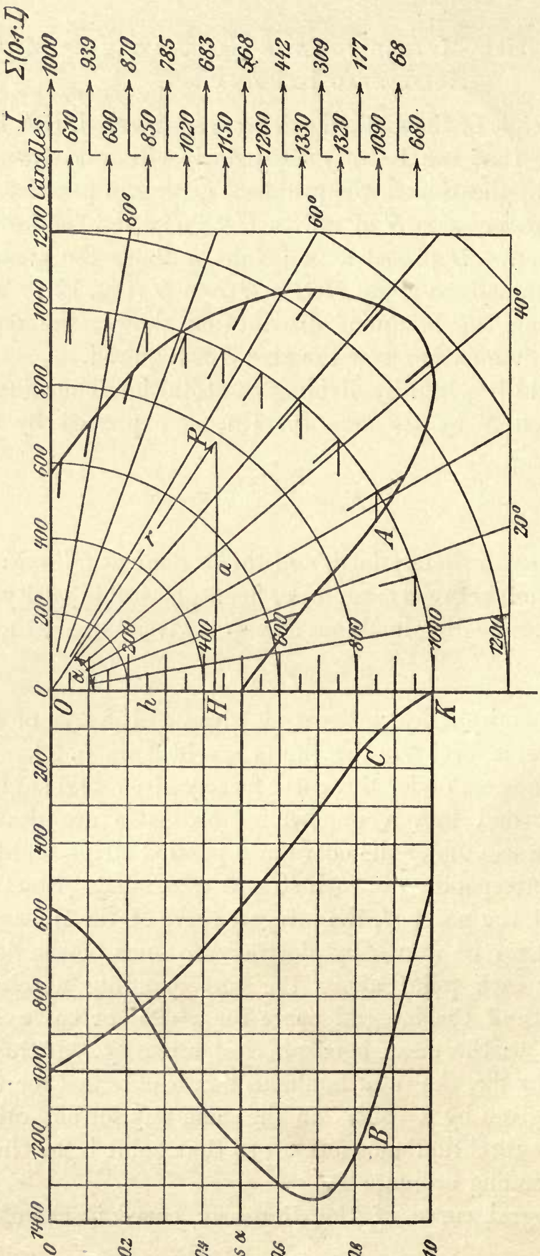


FIG. 12.—METHOD FOR THE DETERMINATION OF THE LIGHT-FLUX CURVE.

A, Curve of Distribution of intensity; *B*, Rousseau curve; *C*, Light-flux curve.

referred to in the following as the curve of luminous flux.

If the distance OK is made equal to unity, the abscissæ of the curve of luminous flux have the values $\cos\alpha$ or $(1-\cos\alpha)$, depending on whether O or K is taken as origin. As in general the angle α is not given, but the surface illuminated, $S=\pi a^2$, and the height of suspension h, of the source, it is convenient to express $\cos\alpha$ in terms of these quantities.

From the diagram (Fig. 12, p. 62),

$$\cos\alpha=\frac{h}{\sqrt{h^2+a^2}}=\frac{1}{\sqrt{1+\frac{a^2}{h^2}}}=\frac{1}{\sqrt{1+\frac{S}{\pi h^2}}}.$$

The quantity $1-\cos\alpha$, a knowledge of which is necessary in every case for determining the mean horizontal illumination, is thus easily calculated from this formula, S and h being given. The numerical work may be lightened by using the curve connecting $(1-\cos\alpha)$ and $\frac{S}{h^2}$ drawn in Fig. 35 (p. 154), for values of $\frac{S}{h^2}$ between 0 and 100, only one setting of a slide rule being necessary to determine $1-\cos\alpha$. The first part of this curve has been drawn to a larger scale in the next diagram (Fig. 36, p. 154), and the same range of values is also given in the table on p. 155, the use of this being often more expeditious than the curve.

27. Curves and Tables of Luminous Flux.

In order to use the simplified method of estimating mean horizontal illumination just given, a knowledge of the curve of luminous flux of the source used is necessary. The determination of such a curve in every case would be very laborious, but practical use of the above-described method has shown that it is convenient, and quite sufficient to draw average flux curves for each type to a fixed scale, say for a mean hemispherical intensity of 1,000 *C.P.* The curves for the same type of lamp would be similar, and available for the calculation of illumination values, whatever the value of the mean hemispherical intensity of the particular lamp used might be. A series of these curves is given (pp. 156-176) for sources usually met with.

The method used in constructing them is the first approximate one given on p. 35 for the calculation of mean spherical and mean hemispherical intensity, followed by the construction of Fig. 12, p. 62. In this figure the polar curve of intensity *A* is on the right, and is drawn for a mean hemispherical intensity of 1,000 *C.P.* The numerical values shown to the right are, as in Fig 5, p. 36, the ordinates of the Rousseau curve *B* at the midpoints of ten equal divisions of the line *OK*. They are, then, the mean heights of ten rectangles, into which the area bounded by the axis of abscissæ and the Rousseau curve is divided. If *OK* is unity, the bases of these rectangles are each equal to 0·1. On multiplying the numerical values of each ordinate by 0·1, the area of the corresponding rectangle is arrived at, and the sum of these products from top to bottom gives the second series of numbers in Fig. 12. These are the ordinates Ψ of the sum curve of the Rousseau curve—that is, the ordinates of the light-flux curve—for values of $1 - \cos \alpha$ equal to 0·1, 0·2, 0·3, up to 1·0. The curve of total flux *C* is drawn to the left of Fig. 12 from these values. The last ordinate of this curve at *O* should have the value 1,000 *C.P.*, since the distribution curve *A* was constructed for $I_{\triangledown} = 1{,}000$ *C.P.*

The equivalent numerical values of light flux are given (Tables XXII. to XXXII., pp. 157-177), with the curves, as being sometimes more useful. They can be used as ordinary trigonometric tables, the two lateral columns giving values of $1 - \cos \alpha$, and of $\cos \alpha$, Ψ being indicated for every hundredth part of these. Intermediate values can be obtained by interpolation.

These curves and tables are given for the following types:

1. *A Small Straight Luminous Rod.*—This curve is often

useful in theoretical calculation, and is derived from the equation—

$$I = I_{90^\circ} \sin \alpha$$

(Fig. 37, Table XXII., pp. 156, 157).

2. *An Element of Surface Luminous on One Side.*—This is again of theoretical interest, the curve being obtained from the equation—

$$I = I_{0^\circ} \cos \alpha$$

(Fig. 38, Table XXIII., pp. 158, 159).

3. *Naked Carbon or Metal Filament Glow Lamps.*—The distribution here is only slightly different from Case 1 (Fig. 39, Table XXIV., pp. 160, 161).

4. *Carbon or Metal Filament Glow Lamps, with Reflectors or Holophane Globes*, sending more light below the horizontal than in Case 3 (Fig. 40, Table XXV., pp. 162, 163).

5. *Carbon or Metal Filament Glow Lamps, with Reflectors producing Strong Downward Concentration of light*, there being practically no lateral or upward illumination (Fig. 41, Table XXVI., pp. 164, 165).

6. *Ordinary Continuous Current Arc Lamps without Globes*, or with new clear glass globes (Fig. 42, Table XXVII., pp. 166, 167).

7. *Ordinary Continuous Current Arc Lamps with Opal or Alabaster Globes.*—The curve and table is also available for flame arc lamps with vertical carbons in opal or alabaster globes, and for lamps working on alternating current when fitted with a reflector (Fig. 43, Table XXVIII., pp. 168, 169).

8. *Continuous Current and Alternating Current Long-burning Enclosed Arcs in Opal or Alabaster Globes* (Fig. 44, Table XXIX, pp. 170, 171).

9. *Flame Arc Lamps with Inclined Carbons and Horizontal Arc in Clear, Opal, or Alabaster Globes* (Fig. 45, Table XXX., pp. 172, 173).

10. *Ordinary Upright Gas Mantles with Reflectors*, as usually arranged in street lanterns (Fig. 46, Table XXXI., pp. 174, 175).

11. *Inverted Gas Mantles* (Fig. 47, Table XXXII, pp. 176, 177).

Each of these curves is the mean of from six to twelve separate curves taken from the author's practice or from the literature of the subject.

No more data than those given were available in reference to incandescent gas lamps, the figures quoted being taken from a memoir by Professor Drehschmidt.*

One or other of these curves will be found to approximate sufficiently closely to the distribution curve of any light source not specifically illustrated in this collection.

An inspection of these distribution curves shows that they vary very greatly among themselves. These variations are, however, much less marked for curves of luminous flux. For rough calculations, therefore, the use of particular curves of luminous flux can be avoided by treating them all as straight lines. These lines would pass through zero, each being the integral of a straight Rousseau curve parallel to the axis of abscissæ. For a mean hemispherical intensity of 1,000 *C.P.*, Ψ then becomes the abscissa $1-\cos\alpha$ multiplied by 1,000.

Curves have not been given for Nernst lamps, as these are now seldom installed.

28. Application of Simplified Method to the Calculation of the Mean Horizontal Illumination of Streets and Open Spaces.

The case of a single lamp placed in the middle of a circular area, already dealt with by this method, is very rare in practice. In general, it is a question of calculating the illumination of streets and open spaces of any form whatever, lit by a large number of lamps. One way of dealing with this problem would be to consider each lamp, or group of lamps, as at the centre of a circular area, and then to determine the mean horizontal illumination on that area by

* Drehschmidt, "Uber Hängendes Gasglühlicht," *Journal für Gasbeleuchtung,* 1905, p. 816.

the above-mentioned method. Two sources of error would, however, be introduced by this procedure.

The portion of street or area lit by one lamp has nearly always a rectangular form. In replacing such a rectangle by an equal circular area, there will be portions of the rectangle outside the circle, compensated for by those places where the opposite is the case—that is, there will be some places on the periphery of the rectangle where the illumination will be weaker, and others where it will be stronger, than that on the periphery of the corresponding circle. In the result, the mean horizontal illumination, as deduced from the circular area, will be too high. On the other hand, illumination coming from neighbouring lamps is not considered, an error which makes the estimated illumination smaller than the actual. The two errors tend to compensate one another, and frequently the final error is very small.

In street-lighting this resultant error depends principally on the ratio between the distance of the lamps apart and the width of the street. It can be made not to exceed about 5 per cent. by the application of a correction factor k. The value of k has been deduced from a large number of instances covering very different types and arrangements of lamps.

If λ is the ratio of distance of lamps apart to width of street, then the correction factor is given by—

$$k = 1{\cdot}2 - 0{\cdot}1\,\lambda.$$

The distance between lamps in a straight street should be measured along the street, even when the lamps are arranged as in the top right-hand diagram of Fig. 11 (p. 60).

In the determination of mean horizontal illumination in open spaces, there is equally room for the use of the correction factor k. λ is then the ratio of the mean distance of neighbouring lamps to the greatest dimension of the illuminated area. Errors arising from the use of this approximation may amount to 5 per cent., and only exceptionally to 10 per cent., as has been shown in the greater number of cases examined.

The determination of the mean horizontal illumination E_m can then be undertaken as follows:

Let there be a group of z lamps on each standard lighting an area S of a street or place, let each have a mean hemispherical intensity of $I_{\bigtriangledown}$, and let their height above the horizontal plane considered be h. The ratio $\frac{S}{h^2}$ is first determined, and the corresponding value of $1 - \cos\alpha$ is found either from curves (Figs. 35 and 36) or from Table XXI (p. 155). A reference to the curves or tables of luminous flux for the type of lamp used will give, for the particular value of $1 - \cos\alpha$, the value of Ψ corresponding to a mean hemispherical intensity of 1,000 *C.P.* The luminous flux received by the surface S is then—

$$\Phi = 2\pi\Psi\frac{I_{\bigtriangledown}z}{1{,}000},$$

and the mean horizontal illumination will be, taking account of the correction factor—

$$E_m = \frac{2\pi\Psi}{S} \cdot \frac{I_{\bigtriangledown}z}{1{,}000}k \quad . \quad . \quad . \quad (16).$$

29. Application to Interiors.

With interiors, every lamp must be considered as affecting the illumination of the whole room, as long as the area does not exceed about 1,000 square metres or 11,000 square feet. The condition, of course, can only apply when the shape of the room and the disposition of the lamps is such as to allow of any lamp lighting any part. On this assumption, the surface S can be taken as the total surface of the room, both in the calculation of $\frac{S}{h^2}$ and in Equation 16 above, for the mean horizontal illumination, where z is the total number of lamps alight in the room, each having a mean hemispherical intensity of $I_{\bigtriangledown}$.

It is only in the case of very large interiors that it becomes necessary to divide up the surface into separate areas, according to its shape, and the distribution of the lamps, taking for z the number of lamps in each area. This treatment must be adopted when the conditions approximate to those obtaining in streets or open spaces, such as occur in railway-stations and markets.

The factor k in Equation 16 has a different meaning for interiors from that which it has for streets and open spaces. In the lighting of rooms the increase of the mean illumination due to reflection from the ceiling and walls must be taken into account. This increase is very often over-estimated, because the floor and tables in a room are almost always illuminated at the expense of the walls and ceiling, either by the use of a source which in itself does this, as an inverted gas-mantle, or by a combination of globes and reflectors, as is common with upright gas-mantles or electric incandescent lamps. The effect of walls and ceiling, painted or papered in dark colours, can, under these circumstances, be neglected, and k can be taken as unity. With moderately light walls and ceiling, k varies from 1·1 to 1·2, while if colours approaching white are used, and if a relatively large amount of light reaches the walls and ceiling, k can vary from 1·2 to 1·5.

The special question of indirect lighting, in which all the illumination is derived from light reflected from the ceiling, is dealt with in the last chapter (p. 127).

30. Application to the Determination of Necessary Illumination, or to the Number of Lamps.

Hitherto the number of lamps and the luminous intensity of each have been assumed as known, and the mean horizontal illumination calculated from these data. Very often, however, the problem presents itself in this way:—Given fixed positions for the sources of light, to determine the luminous intensity ($I_{\triangledown}$) of the lamps to use, or the number (z) of these lamps in each group, in order that a given mean horizontal illumination may be produced on a given area. This problem may be immediately solved by the method already given. Equation 16 gives—

$$I_{\triangledown} = \frac{\dfrac{E_m S}{\Psi}}{2\pi \dfrac{\Psi}{1{,}000} zk}, \text{ or } z = \frac{\dfrac{E_m S}{\Psi}}{2\pi I_{\triangledown} \dfrac{\Psi}{1{,}000} k} \quad . \quad (17).$$

The quantities in these equations may refer either to the lighting of streets or of interiors.

Lastly, there is the problem that arises in street-lighting in which the mean horizontal illumination and the luminous intensity of the lamps to be used are given, and it is required to calculate the permissible distance between two consecutive lamps, and consequently the area S that can be dealt with. In this case it is convenient to proceed in an indirect manner by determining the mean horizontal illumination for an assumed distance between the lamps. It is then easy to modify the distance in the desired direction, and arrive rapidly at the result.

It must not be forgotten that in most cases this method does not give a nearer approximation than 5 per cent. Reasonably wide variations must, however, be allowed for in nearly all lamps, and this will produce corresponding variations in the illumination, so that the application of such an approximate method as that described is fully justified.

The application of this simplified method to the lighting of streets, open spaces, and interiors is demonstrated below in a number of examples.

31. Use of Flux Tables for the Determination of a Given Illumination.

The tables at the end of the book are not only useful in the calculation of mean horizontal illumination as hitherto described, but they also afford a convenient means of calculating any desired illumination values with sufficient exactitude for practical purposes without reference to distribution curves.

As the curve of luminous flux is the integral of the Rousseau curve (p. 61), the value $\frac{d\Psi}{d \cos \alpha}$ is equal to the ordinate of the latter, and is also the luminous intensity I for the corresponding angle α.

That is—

$$\frac{d\Psi}{d\cos\alpha} = I.$$

The values of the flux of light are given in the tables for every hundredth part of $\cos\alpha$. If $d\cos\alpha$ is taken as 0·01, then $d\Psi$ or Δ becomes the difference between two successive values of Ψ, and may be written as—

$$\Delta = Id\cos\alpha = 0{\cdot}01\ I, \text{ or } I = 100\ \Delta.$$

Further, as the tables are all drawn up for a mean hemispherical intensity of 1,000 *C.P.*, the intensity I under an angle α of any lamp having a mean hemispherical intensity of $I_\triangledown$ is given by—

$$\frac{I}{I_\triangledown} = \frac{100\ \Delta}{1{,}000}, \text{ or } I = I_\triangledown \frac{\Delta}{10},$$

Δ being selected from the appropriate table and for the given angle α. Where Δ is different on either side of the value of $\cos\alpha$, the mean of the two values should be used.

As an example, let it be required to find by calculation the candle-power of a carbon filament glow lamp at an angle of 45° from the vertical, the horizontal candle-power being 32. From Fig. 39 (p. 160), the mean hemispherical candle-power is approximately—

$$0{\cdot}8 \times 32 = 25{\cdot}6$$
$$\cos\alpha = 0{\cdot}707,$$

and from Table XXIV. (p. 161)—

$$\Delta = 221 - 212 = 9,$$

so that the required candle-power is—

$$I_{45^\circ} = 25{\cdot}6 \times \tfrac{9}{10} = 23.$$

The intensity having been found, the horizontal illumination (E_H) at any point can be determined. Take a point P at a distance α from the foot of a lamp whose height is h. Then—

$$r = \sqrt{a^2 + h^2} \text{ and } \cos\alpha = \frac{h}{r}.$$

Determine the value of Δ from the appropriate flux table, and (p. 53)—

$$E_H = \frac{I}{r^2}\cos\alpha = I_{\triangledown}\frac{\Delta}{10}\frac{\cos\alpha}{r^2}.$$

The maximum value of the vertical illumination at P also follows as—

$$E_V = \frac{I}{r^2}\sin\alpha = E_H\frac{a}{h},$$

and the normal illumination is—

$$E_N = \frac{I}{r^2} = \frac{I_{\triangledown}}{r^2}\frac{\Delta}{10}.$$

32. Group Lighting.

A number of lamps grouped together can, as a rule, be treated as a single unit having the same kind of distribution curve as the single lamp, in spite of small differences due to their disposition. The flux curves and tables will then apply to the group as to the single lamp.

This is not the case for the total luminous intensity of the group, which is not equal to the sum of the separate intensities, but is less than this, as the lamps shade one another, and thus reduce the total effective luminous intensity. This can be allowed for by multiplying the sum of the separate intensities by a factor varying from 0·9 to 0·8, according to the number and disposition of the lamps.

33. Uniformity of Illumination.

If a definite amount of uniformity is desired in a lighting scheme, the maximum and minimum values of the horizontal illumination, as well as the mean, must be determined. The maximum value can be taken from a curve of horizontal illumination, as described on p. 55, the effect of other lamps in the neighbourhood being imperceptible unless they are very close indeed.

It is generally unnecessary to draw the complete distribution curve, as a few trial calculations will give the desired result sufficiently well.

The point of minimum horizontal illumination must obviously be the farthest from any of the lamps, and is easy to find; its value is at once calculated, knowing the distances of the nearest lamps. If the given conditions of uniformity of illumination are not fulfilled, either the height of the lamps must be increased, or, if this gives an insufficient mean horizontal illumination, they must be placed closer together.

34. Examples on the Use of the Simplified Method in Calculations of Illumination.

Example 1.—*Street Lighting.*

A street, 16 metres (52 feet 6 inches) in width, is to be lit by groups of metal filament lamps placed along the kerb on each side of the street, and 30 metres (98 feet 5 inches) apart

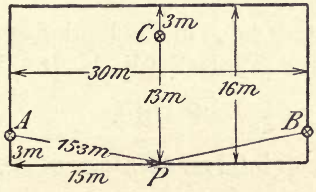


Fig. 13.

along either kerb (Fig. 13). Each group, consisting of two lamps, is enclosed in a transparent glass globe, and is fitted with a reflector. The height above the ground is 4 metres (13 feet 1 inch), and each lamp has a horizontal luminous intensity of 100 *C.P.*, and takes 1·1 watt per candle.

From Fig. 39, p. 160, the factor connecting I_{hor} and $I_{\triangledown}$ for these lamps is 0·8. Due to the mutual shielding action of the two lamps, about 10 per cent. of the light is lost, giving a second reduction factor of 0·9. Absorption due to globe and reflector amounts to about 15 per cent., and gives a third factor of 0·85. Due to the reflector, 70 per cent. of the total light is sent into the lower hemisphere, so that the mean hemispherical intensity is 1·4 times as great as it would be without a reflector (p. 43).

The resultant intensity is—

$$I_{\triangledown} = 2 \times 100 \times 0{\cdot}8 \times 0{\cdot}9 \times 0{\cdot}85 \times 1{\cdot}4 = 172 \; C.P.$$

The area of street lit per group is—

$$S = \frac{30 \times 16}{2} = 240 \text{ square metres,}$$

or—

2,582 square feet.

The plane on which the mean horizontal illumination is to be determined is taken at a height of 1·5 metres (4 feet 11 inches) above the ground, giving—

$$h = 4 - 1{\cdot}5 = 2{\cdot}5 \text{ and } \frac{S}{h^2} = 38{\cdot}4.$$

From Table XXI., p. 155, $(1 - \cos \alpha) = 0{\cdot}725$.

The corresponding value of the light flux for incandescent lamps with reflector (Table XXV., p. 163) is—

$$\Psi = 810.$$

The ratio of lamp interval to width of street is—

$$\lambda = \frac{30}{16} = 1{\cdot}87,$$

and the correction factor for street lighting is (p. 67)—

$$k = 1{\cdot}2 - 0{\cdot}1 \times 1{\cdot}87 = 1{\cdot}01.$$

Substituting these values, the mean horizontal illumination is—

$$E_m = \frac{2\pi \cdot 810}{240} \cdot \frac{172}{1{,}000} \cdot 1{\cdot}01 = 3{\cdot}7 \text{ lux};$$

or—

$$\frac{3{\cdot}7}{10{\cdot}76} = 0{\cdot}35 \text{ f.c. (p. 25).}$$

The economic coefficient σ (p. 51), which expresses the consumption of energy in watts per lux and square metre, or in watts per foot candle and square foot, will be—

$$\frac{2\times 1{\cdot}1\times 100}{3{\cdot}7\times 240}=0{\cdot}247.$$

In addition, the maximum and minimum values of the horizontal illumination can be determined.

The maximum value for a single lamp (p. 72) is—

$$E_H=I_{\triangledown}\frac{\Delta}{10}\,\frac{\cos\alpha}{r^2},$$

$\dfrac{\Delta\cos\alpha}{r^2}$ being given its maximum value. This can easily be determined by trial from the tables of luminous flux. In the case considered, the maximum value occurs when $\cos\alpha=1$; that is, the point is immediately under the lamp. The influence of neighbouring lamps on the magnitude and position of the resultant maximum illumination is here quite negligible.

When— $\cos\alpha=1,\ \Delta=13,$

and—

$$E_{max}=172\cdot\frac{13}{10}\cdot\frac{1}{(2{\cdot}5)^2}=36\text{ lux},$$

or—

$$3{\cdot}35\text{ f.c.}$$

The minimum value of the horizontal illumination occurs at the point P in Fig. 13—that is, at 15·3 metres (50 feet 2 inches) from the foot of lamps A and B. Hence—

$$r_{PA}=\sqrt{15{\cdot}3^2+2{\cdot}5^2}=15{\cdot}5\text{ metres (50 feet 11 inches)},$$

and— $$\cos\alpha_{PA}=\frac{2{\cdot}5}{15{\cdot}5}=0{\cdot}161.$$

For this value, flux Table XXV. gives $\Delta=7$; and the horizontal illumination at P due to lamp A is—

$$E_{PA}=172\times\frac{7}{10}\times\frac{0{\cdot}161}{15{\cdot}5^2}=0{\cdot}08\text{ lux},$$

or— $$0{\cdot}0075\text{ c.f.}$$

The distance from foot of lamp C to P is 13 metres (42 feet 8 inches). Hence—

$$r_{PC} = \sqrt{13^2 + 2{\cdot}5^2} = 13{\cdot}2 \text{ metres (43 feet 4 inches)},$$

and—

$$\cos \alpha_{PA} = \frac{2{\cdot}5}{13{\cdot}2} = 0{\cdot}19,$$

for which $\Delta = 8$, and the horizontal illumination at P due to C is—

$$E_{PC} = 172 \times \frac{8}{10} \times \frac{0{\cdot}19}{13{\cdot}2^2} = 0{\cdot}15 \text{ lux},$$

or—

$$0{\cdot}014 \text{ f.c.}$$

The minimum value of the horizontal illumination is then—

$$E_{min} = E_{PA} + E_{PB} + E_{PC} = 2 \times 0{\cdot}08 + 0{\cdot}15 = 0{\cdot}31 \text{ lux},$$

or—

$$0{\cdot}029 \text{ f.c.}$$

The coefficients of uniformity will be—

$$\frac{E_{max}}{E_{min}} = \frac{36}{0{\cdot}31} = 121, \frac{E_{max}}{E_{mean}} = \frac{36}{3{\cdot}7} = 9{\cdot}7, \text{ and } \frac{E_{min}}{E_{mean}} = \frac{0{\cdot}31}{3{\cdot}7} = 0{\cdot}08.$$

The two last ratios are very different from unity, so that this illumination would be considered as rather irregular. This is to be expected from the relatively short distance of the lamps above the plane of reference.

Example 2.—Street Lighting.

A street, 20 metres (65 feet 7 inches) wide, is to be lit by alternate current flame arc lamps with vertical carbons. The lamps are run three in series on 120 volts, and have globes of opal glass. They are hung over the middle of the street. It is desired to have a mean horizontal illumination of 3 lux (0·28 foot candles) at 1·5 metres (4 feet 11 inches), but no coefficient of uniformity is specified. The problem is to determine the correct distance apart of the lamps.

As no data for the luminous intensity of these lamps has been tabulated, an approximate estimate will be made from the values given in Table XVI., p. 149. It is there stated that continuous current flame arcs with vertical carbons have a mean hemispherical intensity of 2,250 *C.P.* for a consumption of 1,000 watts, and that these values should be diminished by about 30 per cent. in order to apply to alternate current arc lamps of the same type. This is the same thing as multiplying the figures in the table by 0·7. Opal glass globes diminish the mean spherical intensity by absorption to the extent of from 10 to 20 per cent. (p. 42), and diminish the mean hemispherical intensity by about half as much again. The globe may then be taken to absorb 25 per cent. of the light, giving an absorption factor of 0·75. Each lamp takes 12 amperes at 40 volts, with a power factor of about 0·95. The consumption of each lamp will be—

$$W = 12 \times 40 \times 0{\cdot}95 = 455 \text{ watts},$$

and the mean hemispherical intensity per lamp is—

$$I_{\triangledown} = \frac{2{,}250}{1{,}000} \times 0{\cdot}7 \times 0{\cdot}75 \times 455 = 540 \ C.P.$$

As no coefficients of uniformity are specified, any reasonable value of the height of lamp above the ground may be taken. If the very usual value of 8 metres (26 feet 3 inches) is chosen, then—

$$h = 8 - 1{\cdot}5 = 6{\cdot}5 \text{ metres (21 feet 4 inches)},$$

as the vertical distance from lamps to the plane on which the mean horizontal illumination is to be 3 Lux. As indicated above (p. 70), the procedure is indirect, and consists in calculating the mean horizontal illumination for different distances between the lamps. The best distance to use is then sufficiently nearly determined by an inspection of the results. These calculations have been made for lamp intervals of 30, 40, and 50 metres, and the results given in Table III. The area S is that served by one lamp, and the height h being fixed at 6·5 metres, the next column is immediately deducible. The corresponding values of $1 - \cos\alpha$ are then found in Table XXI. (p. 155). The figures for Ψ are taken from flux Table XXVIII. (p. 169), and the mean horizontal illumination E_m follows from Equation 16 given on p. 68. As the ratio λ of lamp interval to width of street varies with α, the factor k is different in each case.

TABLE III.

a. Distances.			S.		$\frac{S}{h^2}$	$(1-\cos\alpha)$	Ψ.	λ.	k.	E_m.	
Metres.	Ft.	In.	Square Metres.	Square Feet.						Lux.	Foot Candles.
30	98	5	600	6,460	14·2	0·574	654	1·5	1·05	3·87	0·36
40	131	2	800	8,610	18·9	0·623	708	2·0	1·00	3·02	0·28
50	164	0	1,000	10,750	23·7	0·658	744	2·5	0·95	2·39	0·22

The correct distance apart is evidently 40 metres, the data given not being exact enough to warrant a closer calculation.

Example 3.—Lighting of an Open Space.

The Potsdamer Platz in Berlin has an area of about 7,300 square metres (78,500 square feet), and is lit by continuous current flame arc lamps arranged in two groups of four lamps each, the distance between the groups being about 45 metres (147 feet 7 inches), and their height 18 metres (59 feet). Each lamp takes 20 amperes at 55 volts, and is fitted with a transparent glass globe, the combination having a mean hemispherical intensity of 3,600 *C.P.* When grouped together, their mutual shadowing action reduces this intensity by 15 per cent., or a reduction factor of 0·85. The area lit per group is—

$$S = \frac{7{,}300}{2} = 3{,}650 \text{ square metres (39,250 square feet).}$$

Height of lamps above reference plane is—

$$h = 18 - 1{\cdot}5 = 16{\cdot}5 \text{ metres (54 feet 1 inch),}$$

and—

$$\frac{S}{h^2} = 13{\cdot}4,$$

whence from Table XXI. (p. 155)—

$$1 - \cos\alpha = 0{\cdot}564,$$

and from Table XXX. (p. 173) for intensive flame arc lamps $\Psi = 680$. The greatest dimension of the area is 160 metres (524 feet 10 inches), and with the lamp interval of 45 metres (147 feet 7 inches)—

$$\lambda = \tfrac{45}{160} = 0{\cdot}28, \text{ and } k = 1{\cdot}2 - 0{\cdot}1 \times 0{\cdot}28 = 1{\cdot}17.$$

Hence, the mean horizontal illumination—

$$E_m = \frac{2\pi \cdot 680}{3{,}650} \cdot \frac{3{,}600 \times 0{\cdot}85 \times 4}{1{,}000} \times 1{\cdot}17 = 16{\cdot}7 \text{ lux.}$$

This calculated result has been confirmed by direct measurement on the Platz itself.

The efficiency of the illumination is—

$$\sigma = \frac{20 \times 55 \times 8}{7{,}300 \times 16{\cdot}7} = {\cdot}072 \text{ watt}$$

per lux and square metre, or per foot candle and square foot.

Example 4.—Lighting of an Open Space.

A square, with sides of 40 metres (131 feet 2 inches) and an area of 1,600 square metres (17,200 square feet), is to be

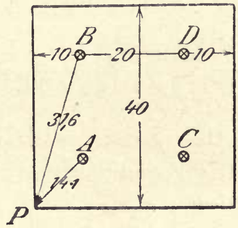


FIG. 14.

lit by inverted gas-mantles. A mean horizontal illumination of 10 lux (0·93 f.c.) is required at a height of 1·5 metres (4 feet 11 inches) above the ground. The minimum horizontal illumination is not to be below 1 lux (0 093 f.c.) at the same height.

The shape of the area suggests the arrangement of the lamps in four groups, as indicated in Fig. 14. The height of lamps above ground is to be 5 metres (16 feet 5 inches), or 3·5 metres (11 feet 6 inches) above the reference plane. The correct luminous intensity of the lamps to be used is required, and the efficiency of the lighting.

The area corresponding to a group of lamps is—

$$S = \frac{1{,}600}{4} = 400 \text{ square metres (4,300 square feet),}$$

and for $h = 3{\cdot}5$ metres—

$$\frac{S}{h^2} = 32{\cdot}5.$$

Hence, $1 - \cos\alpha = 0{\cdot}703$ (Table XXI., p. 155), and $\Psi = 717$ (Table XXXII., p. 177).

The lamp interval being 20 metres, and the greatest dimension of the area being $40 \times \sqrt{2} = 57$ metres,

$$\lambda = \frac{20}{57} = 0{\cdot}35, \text{ and } k = 1{\cdot}2 - 0{\cdot}1 \times 0{\cdot}35 = 1{\cdot}165.$$

As more than one mantle will be required in each lantern, the factor due to mutual shadowing can be taken as 0·8 (Equation 17, p. 69), and the luminous intensity of the group is—

$$I_{\triangledown}z = \frac{10 \times 400}{2\pi \times 0{\cdot}717 \times 1{\cdot}165 \times 0{\cdot}8} = 950 \; C.P.$$

Two lanterns to the group, with six mantles in each lantern, each of 80 *C.P.*, would give the desired result with sufficient approximation.

From Table XV., p. 148, it will be found that a luminous intensity of 690 *C.P.* can be obtained by the consumption of 1,000 litres of gas per hour (35·3 cubic feet per hour); 950 *C.P.* will therefore require a consumption of—

$$\frac{95}{69} \times 1{,}000 = 1{,}380 \text{ litres per hour (48·7 cubic feet per hour).}$$

The consumption per burner is $\frac{1{,}380}{12} = 115$, and comes within the limits given in Table XV., p. 148.

The economic coefficient of the lighting is then—

$$\frac{1{,}380}{10\times 400}=0{\cdot}345 \text{ litre } (0{\cdot}012 \text{ cubic foot})$$

of gas per hour per lux and square metre, or per foot candle and square foot.

The minimum horizontal illumination occurs in the corner of the area at the point P (Fig. 14), and is found by adding together the horizontal illumination produced there by each lamp separately.

The distance PA is—

$$PA=10\sqrt{2}=14{\cdot}1 \text{ metres,}$$

and with—

$$h=3{\cdot}5 \text{ metres,}$$

$$r_{PA}=\sqrt{14{\cdot}1^2+3{\cdot}5^2}=14{\cdot}6, \cos\alpha=\frac{3{\cdot}5}{14{\cdot}6}=0{\cdot}24.$$

From the appropriate flux Table XXXII. (p. 177), $\Delta=10$ for this value of cos α. Allowing for the mutual shadowing effect of the lamps, the effective mean hemispherical intensity is—

$$I_{\triangledown}=950\times 0{\cdot}8=760 \; C.P.$$

The horizontal illumination at P coming from A is therefore—

$$E_{PA}=760\times\frac{10}{10}\times\frac{0{\cdot}24}{14{\cdot}6^2}=0{\cdot}86.$$

The distance PB is—

$$PB=\sqrt{10^2+30^2}=31{\cdot}6 \text{ metres.}$$

Hence—

$$r_{PA}=\sqrt{31{\cdot}6^2+3{\cdot}5^2}=31{\cdot}8, \cos\alpha=\frac{3{\cdot}5}{31{\cdot}8}=0{\cdot}11,$$

and $\Delta=9$ (Table XXXII., p. 177), giving—

$$E_{PB}=760\frac{9}{10}\cdot\frac{0{\cdot}11}{31{\cdot}8^2}=0{\cdot}075.$$

For the lamp C, $E_{PC}=E_{PB}$, and the lamp D is so far away that E_{PD} is negligible.

Finally, $E_{min}=E_{PA}+2E_{PB}=0{\cdot}86+2\times 0{\cdot}075=1{\cdot}01$ lux (0·093 f.c.), and the condition demanded is just realized.

Example 5.—Interior Lighting.

A room with a floor area of 25 square metres (269 square feet), with moderately light walls and ceiling, is lit by a ring of electric incandescent lamps, each of 32 *C.P.* The lamps are suspended at a height of 2·5 metres (8 feet 2·5 inches) above the floor, and are enclosed in globes which absorb 10 per cent. of the light, without sensibly modifying its distribution. The mean hemispherical intensity of a single lamp with globe is then (Fig. 39, p. 160)—

$$I_{\triangledown} = 32 \times 0{\cdot}8 \times 0{\cdot}9 = 23 \; C.P.$$

The mean horizontal illumination at a table height of 0·8 metre (2 feet 7·5 inches) is to be 30 lux (2·8 f.c.). The number of lamps in the ring to produce this result is required.

$$S = 25 \text{ square metres, and } h = 2{\cdot}5 - 0{\cdot}8 = 1{\cdot}7 \text{ metres};$$

hence—

$$\frac{S}{h^2} = 8{\cdot}65, \; (1 - \cos\alpha) = 0{\cdot}484 \text{ (Table XXI., p. 155)},$$

and $\Psi = 405$ (Table XXIV., p. 161).

With moderately light walls and ceiling, the coefficient of reflection can be taken as $k = 1{\cdot}1$, and using Equation 17 (p. 69)—

$$z = \frac{30 \times 25}{2\pi \times 23 \times 0{\cdot}405 \times 1{\cdot}1} = 11{\cdot}7.$$

Twelve lamps are therefore necessary.

Allowing 110 watts per 32 *C.P.* carbon filament lamp, the efficiency is—

$$\sigma = \frac{12 \times 110}{30 \times 25} = 1{\cdot}8 \text{ watts}$$

per lux and square metre (foot candle and square foot).

This unfavourable value of the economic coefficient is partly due to the large amount of energy used by carbon filaments, and partly to the bad use made of the light, as lamps without reflectors send a large part of their light towards the walls and ceiling. If their illumination is not a consideration, the mean hemispherical intensity of these lamps can be increased by about 40 per cent. by the use of proper reflectors. Taking the same value of $1 - \cos\alpha$—viz., 0·484, but referring for the value of the flux to Table XXV., p. 163, $\Psi = 582$, a value about 40 per cent. greater than that found for lamps without reflector. The mean illumination at the height of the tables is now almost doubled, and the economic coefficient reduced to about half its former value. If, in addition, metallic filament lamps taking 40 watts for 32 *C.P.* be substituted for carbon filament lamps taking 110 watts for 32 *C.P.*, the economic coefficient can be finally brought down to—

$$\sigma = 1{\cdot}8 \times \frac{405}{582} \times \frac{40}{110} = 0{\cdot}46 \text{ watt}$$

per lux and square metre (foot candle and square foot).

Example 6.—Interior Lighting.

The room considered is one designed for exhibition purposes, with light-coloured walls, and with a floor area of 150 square metres (1,615 square feet). An abundant illumination is provided by means of Nernst lamps, Type B,* distributed uniformly over the whole ceiling at the rate of one lamp per square metre (per 1·2 square yards). The lamps are provided with opal glass globes without reflector, and are fixed immediately under the ceiling, 4 metres (13 feet 1 inch) above the ground. Each lamp with its globe has a mean hemispherical intensity of 21·5 *C.P.*, and takes 55 watts. It is required to calculate the mean horizontal illumination on the tables (0·8 metre or 2 feet 7·5 inches high).

$S = 150$ square metres, and $h = 4 - 0{\cdot}8 = 3{\cdot}2$ metres; hence—

$$\frac{S}{h^2} = 14{\cdot}6, \text{ and } (1 - \cos\alpha) = 0{\cdot}579 \text{ (Table XXI., p. 155).}$$

* The B type of Nernst lamp has a flat coil-heater, with the straight luminous rod mounted horizontally underneath it.]

The curve of luminous flux for these lamps can be considered as a straight line, so that—

$$\Psi = 579.$$

As the room is painted in light colours, the coefficient of reflection from the walls can be taken as 1·2. The mean horizontal illumination is then—

$$E_m = \frac{2\pi \times 579}{150} \times \frac{21{\cdot}5 \times 150}{1{,}000} \times 1{\cdot}2 = 94{\cdot}5 \text{ lux};$$

and the economic coefficient of the lighting is—

$$\sigma = \frac{150 \times 55}{94{\cdot}5 \times 150} = 0{\cdot}58 \text{ watt}$$

per lux and square metre (per foot candle and square foot).

No further calculations need be made in connection with this kind of distributed lighting, as it is found that the results obtained by means of this simple and rapid method are in very good agreement with actual measurements made in rooms lit in this manner.

Example 7.—Lighting of a Large Interior.

The booking-hall of a railway-station is rectangular, with sides of 80 metres (262 feet 5 inches) and 20 metres (65 feet 7 inches), giving 1,600 square metres (17,215 square feet) of floor area. It is to be lit by eight ordinary continuous current arc lamps in opal glass globes. The lamps are arranged in two parallel rows of four lamps each, with 10 metres (32 feet 9·5 inches) between two successive lamps in a row, and are suspended 6 metres (19 feet 8 inches) above the floor. It is required to determine the necessary luminous intensity per lamp, in order that the mean horizontal illumination at 1·5 metres (4 feet 11 inches) above the floor may reach 25 lux, and to find the economic coefficient of the lighting.

As the area is much greater than 1,000 square metres (11,000 square feet), this problem should be treated as one in street lighting. The effect of reflection from the walls is quite negligible. The lamps can be taken in pairs along the length of the hall, the area per pair being—

$$S = \frac{1{,}600}{4} = 400 \text{ square metres};$$

$h = 6 - 1{\cdot}5 = 4{\cdot}5$ metres, $\frac{S}{h^2} = 19{\cdot}7$, $(1 - \cos\alpha) = 0{\cdot}63$ (Table XXI., p. 155); and $\Psi = 716$ (Table XXVIII., p. 169).

The lamp interval along the hall being 20 metres, and the width of the hall being also 20 metres—

$$\lambda = \frac{20}{20} = 1, \text{ and } k = 1{\cdot}2 - 0{\cdot}1 \times 1 = 1{\cdot}1.$$

Applying Equation 17, p. 69—

$$I_{\triangledown} = \frac{25 \times 400}{2\pi \times 0{\cdot}716 \times 2 \times 1{\cdot}1} = 1{,}010 \ C.P.$$

Ordinary continuous current arc lamps taking more than 8 amperes give a mean hemispherical intensity of 1,500 *C.P.* per 1,000 watts (Table XVI., p. 149). Allowing a loss of 20 per cent. in the globe, and a further loss of 10 per cent. on the mean hemispherical intensity owing to redistribution of the light (p. 42), the effective mean hemispherical intensity per 1,000 watts is $1{,}500 \times 0{\cdot}7 = 1{,}050$ *C.P.* This is sufficiently near to the value found as to be taken the same, and with a supply voltage of 110 and two lamps in series, the current is $\frac{1{,}000}{55} = 18$ amperes. The economic coefficient of the lighting is—

$$\sigma = \frac{1{,}000 \times 2}{25 \times 400} = 0{\cdot}2 \text{ watt}$$

per lux and square metre (per foot candle and square foot).

35. Influence of Height and Spacing of Lamps.

The effect of the height of suspension of lamps on the vertical illumination E_V is evident from Equation 15, p. 54, which is—

$$E_V = \frac{Ia}{\sqrt{(a^2+h^2)^3}}.$$

As the height of suspension of lamp is increased, the vertical illumination diminishes at every point whose distance a from the foot of lamp is fixed. The vertical illumination just above a given horizontal plane is therefore improved by bringing the lamps as near to that plane as possible, provided that their distribution is approximately uniform.

The effect on the horizontal illumination is, however, quite different. The relation in Equation 14 (p. 53), giving this quantity in its simplest form, is—

$$E_H = \frac{Ih}{\sqrt{(a^2+h^2)^3}}.$$

The horizontal illumination at a point whose distance a from foot of lamp is fixed increases to a maximum as h is increased, and then diminishes. The maximum value is reached when—

$$h = \frac{a}{\sqrt{2}} = 0{\cdot}707a,$$

as the result of differentiating the expression for E_H and equating to zero.* This relation is, however, only true if the luminous intensity is constant—that is, if there is a uniform distribution. The variable distribution curves that are common in practice give the maximum horizontal illumination for other values of the ratio $\frac{h}{a}$. These ratios have been the subject of research by M. Blondel, and he gives the following values for continuous current arc lamps:†

* Uppenborn, *Centralblatt für Elektrotechnik*, vol. ii., p. 383.

† Uppenborn, *Deutscher Kalender für Elektrotechniker*, 1907, vol. i., p. 305.

Arc lamps without globe . . . $\frac{h}{a}=0{\cdot}95$

Arc lamps with fairly clear glass globe $\frac{h}{a}=0{\cdot}85$

Arc lamps with opaline globe . . $\frac{h}{a}=0{\cdot}85$

Arc lamps with opal globe . . . $\frac{h}{a}=0{\cdot}5\text{-}0\ 8$

Values for gas lamps have also been given.*

The height of suspension may often be settled by the consideration that at the point of minimum illumination, that minimum should be as large as possible. The calculation is then reduced to the determination of this minimum value, and in many cases, as with open spaces, this is sufficient. The results will, however, always be incomplete unless the mean and maximum values are given.

It is much more to the point to examine the effect of the height of suspension on the uniformity of the illumination and upon the value of the mean horizontal illumination, and then to fix the height with reference to these.

Uniformity of illumination is particularly affected by the height of the lamps. The higher the lamps are raised above the ground or reference plane, the more uniform will be the illumination as the ratio of maximum to minimum value of the horizontal illumination becomes smaller and smaller. The height of the lamps is then particularly dependent on the specified conditions of uniformity of illumination.

The luminous intensity is also important, as the greater it is, the greater should be the height of the lamps in order that people may not be compelled to use their eyes at very short distances from intense sources.

The value of the mean horizontal illumination is also dependent on the height of suspension. This quantity, contrary to the minimum horizontal illumination, diminishes as the height of suspension increases, since the higher the

* Schaars, "Kalender für das Gas und Wasserfach," hsgb. von Dr. E. Schilling, München, 1907, vol. i., p. 156.

lamps, the smaller will be the total flux sent on to a given horizontal area.

An example will give a clear idea of the variation of these important quantities with the height of the lamps.

A street, 20 metres (65 feet 7 inches) wide, is lit by lamps having a mean spherical intensity of 1,000 *C.P.* The lamps are hung in the middle of the street, 40 metres (131 feet 2 inches) apart. Light is radiated uniformly from each lamp, giving a circular arc for the distribution curve, and a straight line for the curve of luminous flux.

The values of E_{mean}, E_{max}, and E_{min} have been calculated by the simplified method already given, for six different heights of suspension, varying from 4 metres (13 feet 1·5 inches) to 15 metres (49 feet 2·5 inches), and the results are given in Table IV. (p. 89).

The maximum horizontal illumination with lamps having uniform distribution occurs immediately under the lamp, and diminishes almost inversely as the square of the height of suspension.

The minimum horizontal illumination on the other hand, increases as the height of suspension is increased, and has not yet reached its maximum at 15 metres (49 feet 2·5 inches), a height beyond which it is not customary to go in practice.

The mean horizontal illumination diminishes with the height of suspension, but only inversely proportionally to the square root of the height.

The last column of Table IV., which contains the ratio of maximum to minimum values, characterises the uniformity of the illumination, and it is evident that uniformity depends a great deal on the height chosen. In the example taken, heights up to 6 metres (19 feet 8 inches) give too small a coefficient of uniformity to be recommended in practice, while for heights above 10 metres (32 feet 9·5 inches) the mean horizontal illumination suffers. The right height to use lies between these limits.

In this way trial calculations enable a decision to be made as to the height that will give the best results, as much from the point of view of uniformity as of mean horizontal illumination. Suitable values for the height of suspension can usually be deduced from the more extended considerations given on pp. 90, 91.

TABLE IV.

VARIATION OF ILLUMINATION VALUES WITH HEIGHT OF SUSPENSION.

Height of Suspension.		Horizontal Illumination in Lux.			Want of Uniformity.
		E_{mean}.	E_{max}.	E_{min}.	$\frac{E_{max}}{E_{min}}$.
Metres.	Ft. In.				
4	13 1·5	5·95	62·5	0·7	89
6	19 8	5·1	28	1·0	28
8	26 3	4·35	15·8	1·2	13·2
10	32 9·5	3 7	10·2	1·35	7·6
12	39 4	3·15	7·1	1·45	4·9
15	49 2·5	2·45	4·65	1·55	3

In contrast with the effects produced by variation of the height of lamp, variation of their distance apart produces a greater alteration in the values of mean horizontal illumination, and only to a minor degree affects the uniformity.

Again, taking the above example, Table V. has been calculated for distances varying from 20 metres (65 feet 7 inches) to 80 metres (262 feet 5 inches), the height of suspension being constant and equal to 8 metres (26 feet 3 inches). The figures show clearly the effect of alteration of distance between lamps on each of the magnitudes considered.

The mean horizontal illumination is almost inversely proportional to the distance between the lamps. The maximum horizontal illumination varies very little as the spacing is increased, while the minimum decreases very rapidly, and, as a consequence, the uniformity of the lighting becomes less and less as the ratio between maximum and minimum values increases.

TABLE V.

VARIATION OF ILLUMINATION VALUES WITH LAMP INTERVAL.

Lamp Interval a.		Horizontal Illumination in Lux.			Want of Uniformity.
		E_{mean}.	E_{max}.	E_{min}.	$\frac{E_{max}}{E_{min}}$.
Metres.	Ft. In.				
20	65 7	7·95	17·2	4·2	4·1
30	98 5	6·0	16·1	2·3	7
40	131 2·5	4·35	15·8	1·2	13·2
50	164 0	3·55	15·7	0·7	22·5
60	196 9·5	2·95	15·7	0·45	35
70	229 7	2·45	15·7	0·3	52·5
80	262 5	2·1	15·6	0·2	78

36. Working Values for Height of Suspension and Spacing of Lamps.

In current practice for street lighting with upright gas-mantles or electric incandescent lamps, the posts are from 3 metres (9 feet 10 inches) to 4 metres (13 feet 1·5 inches) high, and are placed from 25 metres (82 feet) to 50 metres (164 feet) apart, measured along the street.

With inverted gas-mantles better uniformity is obtained by making the height between 4 metres (13 feet 1·5 inches) and 6 metres (19 feet 8 inches), the spacing being rather less than with upright mantles. A height of from 5 metres (16 feet 5 inches) to 6 metres (19 feet 8 inches) above the ground is used in most installations of high pressure gas. The lamp interval should then be 50 metres (164 feet) to 60 metres (196 feet 9·5 inches).

Greater regularity of illumination can be obtained by using arc lamps than by using gas in most cases, on account of the greater permissible height of suspension, which may be from 6 metres (19 feet 8 inches) to 12 metres (39 feet 4 inches) above the ground, with a spacing of from 30 metres (98 feet 5 inches) to 80 metres (262 feet 5 inches) along the street. Flame arc lamps send a great quantity of light immediately under the lamp, so that they can with advantage be used at a greater height than ordinary arcs. A suitable range is from 8 metres (26 feet 3 inches) to 14 metres (45 feet 11 inches).

[Prismatic reflectors immediately under the horizontal arc of a flame arc lamp, with inclined carbons, have recently been introduced with very good results. A greater uniformity of illumination can be secured in this way without using excessively high posts.*]

In lighting very large open spaces a height of from 10 metres (32 feet 10 inches) to 15 metres (46 feet 2 inches) with ordinary arcs, and from 12 metres (39 feet 4 inches) to 18 metres (59 feet) with flame arcs, is used in order to get a sufficient minimum on the borders of the area.

For interior illumination no rules can be given either for height or spacing. The shape and size of the room, and the use to which it is put, determine the lighting. It is usual for the height of suspension to be about two-thirds of the total height of the room.

The coefficients of uniformity vary in practice within very wide limits, as may be seen by reference to Tables VIII. and IX. (pp. 123-126).

37. Working Values for Mean Horizontal Illumination.

In illumination calculations the value of the necessary horizontal illumination is usually fixed in advance. It then becomes a question of arrangement; or with a given arrangement, of determining whether the desired mean horizontal illumination can be attained. In any case the customary practical values are required as a starting-point, and these will now be considered.

For street lighting, the mean horizontal illumination is regulated by the type of street and the prevailing traffic. In

[* *Electrician,* London, vol. lxvi., p. 365.]

side streets with little traffic, a mean horizontal illumination of 0·5 lux (0·05 f.c.) to 1·0 lux (0·1 f.c.) at 1·5 metres (4 feet 11 inches) above the ground is sufficient. This, however, would not be enough to see clearly by except close to the lamps. These figures must be trebled if the traffic is at all considerable, and on main roads 3 lux (0·28 f.c.) to 6 lux (0·56 f.c.) must be given. [The dense, fast, motor traffic in the central districts of large towns calls for values from three to four times greater even than these.] Thus, the Königstrasse in Berlin is lit by pressure-gas lamps ("Millennium Light") that give a mean horizontal illumination of 12 lux (1·1 f.c.), while in the Potsdamerplatz a figure of 19 lux (1·75 f.c.) is attained by means of flame arc lamps.*

The value of the mean horizontal illumination for interiors depends on the use to which they are put. Detailed data for different kinds of interiors are given in Tables XVIII. and XIX., pp. 151, 152. It might appear that the values given in Table XVIII. for living-rooms are low, but it must be remembered that these refer to the *mean* horizontal illumination of the whole area. The illumination on a table underneath a source will naturally be much greater.

38. Working Values for the Economic Coefficient of Commercial Sources of Light.

As already defined on p. 51, the economic coefficient σ is taken to be the power or its equivalent used to maintain a mean horizontal illumination of 1 lux over every square metre, the figure being the same for 1 f.c. over every square foot. Its value is easily arrived at as follows:

The mean horizontal illumination is—

$$E_m = \frac{2\pi\Psi}{S} \cdot \frac{I_{\triangledown} z}{1{,}000} k,$$

this being Equation 16 on p. 68, where the definitions of the quantities involved are also given.

[* For more recent data on street lighting in Berlin, see *Journal für Gasbeleuchtung,* 1909, p. 385; *The Gas World,* 1909, p. 419; and *The Electrician,* 1909, vol. lxii., pp. 460, 506.]

If A is the equivalent power consumption per lamp used either singly or as one of a number forming a group, the specific consumption per lamp per unit of mean hemispherical intensity may be written as—

$$\eta = \frac{A}{I_{\triangledown}},$$

and the economic coefficient includes this factor as follows:

$$\sigma = \frac{zA}{E_m S} = \frac{zAS\,1{,}000}{2\pi\Psi I_{\triangledown} zkS} = \frac{\eta\,1{,}000}{2\pi\Psi k} \quad . \quad . \quad (18).$$

From this it is evident that the economic coefficient of lighting depends not only on the specific consumption of the lamps used, but also on their disposition, and on the kind of surface that is illuminated, as is shown by the presence in the preceding expression of the terms Ψ and k. If the values of η, Ψ, and k, are known for any given lighting scheme, then the economic coefficient can be determined. Its value, however, for a given method of lighting is confined within fixed limits. Table XX., p. 153, contains limiting values of the economic coefficient for different types of lighting, deduced from direct measurements or arrived at by calculation.

39. Comparison of Cost of Different Types of Illumination.

The economic coefficient is of service in the first place, in comparing together the cost of different methods of lighting, with reference, however, only to the cost of the energy consumed (electric energy, gas, or combustible liquid), and leaving out of account accessory expenses (renewal of electric glow lamps and mantles of incandescent gas-burners, carbons of arc lamps, cleaning, repairs, etc.).

A comparison can be made, for example, between electric lighting with a consumption of 0·20 watt per lux and square metre (per foot candle and square foot), and gas lighting, with an hourly consumption of 0·45 litre (0·016 cubic foot) per lux and square metre, the price of a kilowatt hour, or Board of Trade Unit (B.T.U.), being 5d., and that of a cubic metre of gas $1\frac{1}{4}$d. (2s. 11d. per 1,000 cubic feet).

The ratio of costs of energy in the two cases is given by—

$$\frac{\text{Electric lighting}}{\text{Gas lighting}} = \frac{0{\cdot}2 \times 0{\cdot}005}{0{\cdot}016 \times 0{\cdot}035} = \frac{10}{5{\cdot}6} = 1{\cdot}8.$$

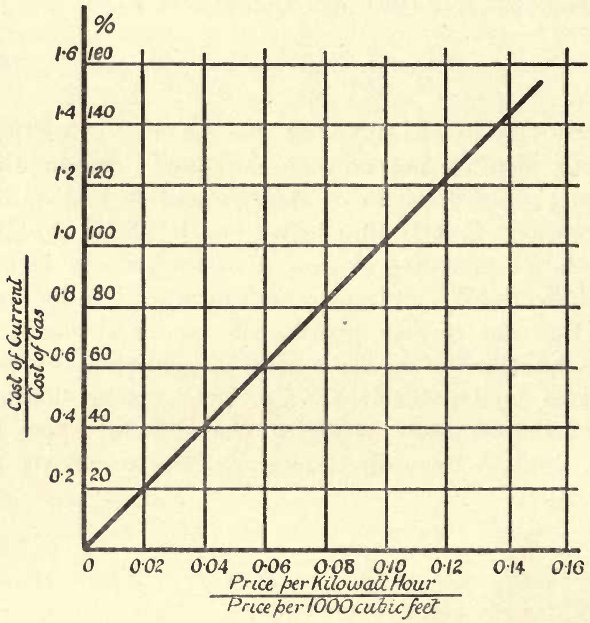


FIG. 15.—RATIO OF COST OF CURRENT TO COST OF GAS FOR A GIVEN ILLUMINATION OVER A GIVEN AREA.

Electric lighting by ordinary arc lamps at a height of 10 metres (32·8 feet).
Gas lighting by "Millennium Light" (high-pressure gas) at a height of 5·7 metres (18·7 feet).

The cost of energy is, in this case, nearly double with electricity than with gas.

A comparison between the economic coefficients for two different types of lighting can conveniently be made graphically, when any ratio of prices per unit is taken instead of fixed values, as the variation of the ratio of cost of energy to ratio of price per unit is given by a straight line. A case dealt with by the author can serve as an example, in which an illumination by means of arc lamps is compared with the same illumination by means of gas under pressure.*

The lighting by arc lamps involved a consumption of 0·18 watt per lux and square metre, and gas under pressure an hourly consumption of 0·5 litre per lux and square metre. These data allow of the construction of Fig. 15, which gives the ratio of cost of current to cost of gas as a function of the ratio of price of a kilowatt hour to the price of gas per 1,000 cubic feet. For example, when the price of a kilowatt hour is about one-eighth of the price of gas per 1,000 cubic feet, the cost of electric lighting is 20 per cent. more than the cost of gas lighting; or, if the costs are to be equal, the price of 1 B.T.U. must be one-tenth of the price of 1,000 cubic feet of gas.

40. Rough Calculation of Lighting Costs.

The values given in Table XX., p. 153, for the economic coefficients of the chief systems of illumination in use, can be applied in the preparation of rough estimates of illumination when there is neither time nor opportunity to go through the more exact precedure already indicated.

If the mean horizontal illumination necessary is E_m, and the total area to be lit is S, then the economic coefficient is—

$$\sigma = \frac{A}{E_m S},$$

and the consumption per unit of area is—

$$\frac{A}{S} = \sigma E_m,$$

or the total consumption for the area S is—

$$A = \sigma E_m S \quad . \quad . \quad . \quad . \quad (19).$$

* L. Bloch, "Vergleichende Verteilung moderner Strassen beleuch tungen," *Journal für Gasbeleuchtung,* 1906, p. 93, and *Elektrotechnische Zeitschrift,* 1906, p. 843.

Inversely, if the total consumption A, the total area S, and the economic coefficient σ, are known, E_m, the mean horizontal illumination, follows, as—

$$E_m = \frac{A}{\sigma S} \quad . \quad . \quad . \quad . \quad (20).$$

Let it be required, for example, to light an office of 50 square metres (538 square feet) area by means of metallic filament lamps taking 30 watts each. To begin with (Table XVIII., p. 151), the mean horizontal illumination necessary can be taken as 40 lux (3·7 f.c.), and the economic coefficient for such lamps (Table XX., p. 153) used in interior lighting is 0·30 watt per lux and square metre. The total consumption is then—

$$A = 0{\cdot}3 \times 40 \times 50 = 600 \text{ watts};$$

that is, it would be necessary to install twenty incandescent lamps of 30 watts each.

Here is another example dealing with street lighting. Along a roadway, 18 metres (59 feet) wide, lanterns are placed on the edge of the pavement, on both sides of the way, 30 metres (98 feet 5 inches) apart. Each lantern is fitted with two upright gas-mantles, and each mantle consumes 125 litres (4·4 cubic feet) per hour. A rough estimate of the mean horizontal illumination is required.

The mean value for the economic coefficient of incandescent gas lamps (Table XX., p. 153) is a consumption of 0·5 litre (0·017 cubic foot) of gas per hour per lux and square metre. The street area corresponding to each lantern is—

$$S = 9 \times 30 = 270 \text{ square metres (2,905 square feet)},$$

and the hourly consumption is 250 litres (8·8 cubic feet).

Hence the mean horizontal illumination of the street will be—

$$E_m = \frac{250}{0{\cdot}5 \times 270} = 1{\cdot}85 \text{ lux};$$

or making the calculation in British units—

$$E_m = \frac{8{\cdot}8}{0{\cdot}017 \times 2{,}905} = 0{\cdot}178 \text{ f.c.}$$

The selection of a value of the economic coefficient for different types of lighting between the limits given in Table XX., p. 153, is determined in the first place by the specific consumption of the lamps used, and then by the probable height of suspension. The highest values would be used for small and dark interiors, and the lowest for large light rooms.

If there is no determinative condition given, the mean is taken.

For modes of lighting not included in Table XX., an approximate economic coefficient can be arrived at by comparison of specific consumptions of the respective sources with those of known sources having luminous intensity of the same order, and a similar distribution curve. Tables XIV., XV., and XVI., pp. 147-149, permit of these comparisons being made.

CHAPTER V

THE MEASUREMENT OF ILLUMINATION

41. Illumination Photometers.

The measurement of candle-power by the ordinary photometer is accomplished by comparing the unknown luminous intensity of a source with the known luminous intensity of another source. Actually, the two sources themselves are not compared, but the illuminations which they produce when they are at known distances from the photometer screen, the value of the unknown intensity being calculated from these data. Any photometer based on this principle can be used for the measurement of illumination as long as the surface in it that is usually lit by the unknown source can be brought to the place where a measurement of illumination is desired. The usual Bunsen, or Lummer-Brodhun photometer head that is moved along a bench, is not available for this service, as in general the bench is not portable, and cannot be put in any desired position.

To meet practical requirements, a large number of portable instruments have been developed, a detailed account of which would be outside the scope of this work.* It will be sufficient here to describe the arrangement used in the streets of Berlin by the author, together with a few more recent photometers.

Fig. 16, opposite, shows the apparatus as used by the testing staff of the Berliner Elektrizitäts Werke, mounted on a small cart for quick and easy transport from one measuring-point to another. The photometer itself is a Brodhun street photo-

[* Illumination photometers are thoroughly dealt with by A. P. Trotter, "Illumination: its Distribution and Measurement," pp. 199-249.]

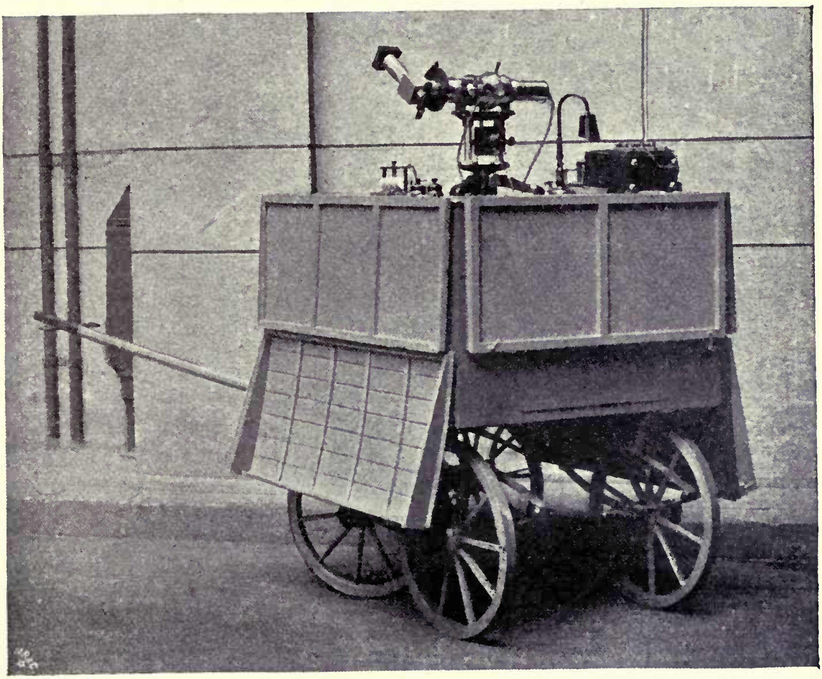


FIG. 16.—TESTING-WAGGON FOR STREET PHOTOMETRY.

To face p. 98.

meter.* To the right of it is an ammeter, behind which is a small rheostat, the combination being used to keep the current constant through the standard lamp. This is a small metallic filament electric lamp, which is found to be better as a standard than a benzine lamp, as this last is quite unsatisfactory unless fixed and sheltered from draughts. The low specific consumption of the metal filament lamp makes it easy for small accumulators to keep the voltage on the lamp terminals constant for a long time at a stretch, the regulation, however, being better effected by current observation, as in this way error due to bad contact is eliminated.

To the left of the photometer is the switch and rheostat for the motor driving the revolving prism, as well as a switch controlling a small metal filament lamp used to illuminate the ammeter and photometer scale. Two 6-volt accumulators are placed in the body of the truck, one to run the motor and the other to supply the scale lamp and standard. A cover is fitted to protect the photometer and all its accessories in case of rain, and during transport.

[Fig. 17 gives a diagram, and Fig. 18 an external view of the latest pattern of Trotter illumination photometer. The illumination to be measured is received on a white screen at the top of the photometer, and is looked at from above by the observer. In the middle of the screen is an elongated hole, through which can be seen another white screen illuminated by the standard lamp, and capable of being tilted in order to get photometric balance. As a tilting screen gives a crowded scale for low illuminations, when the light from the standard is nearly at grazing incidence, a special snail-shaped cam is used to open out the scale for low values. This renders the instrument very suitable for measuring the low illumination values frequently met with in street lighting.

A 4-volt standard lamp is used, fed from a battery in a separate case, the lamp being connected by one switch directly to the battery terminals, or by another through a resistance

[* L. Bloch, *Elektrotechnische Zeitschrift,* 1905, p. 1051 ; E. Liebenthal, "Praktische Photometrie," p. 224.]

which is adjusted so that when it is in circuit the scale readings must be divided by ten to get the actual illumination measured. It is found unnecessary to provide any electrical measuring instrument continuously in circuit, as the voltage

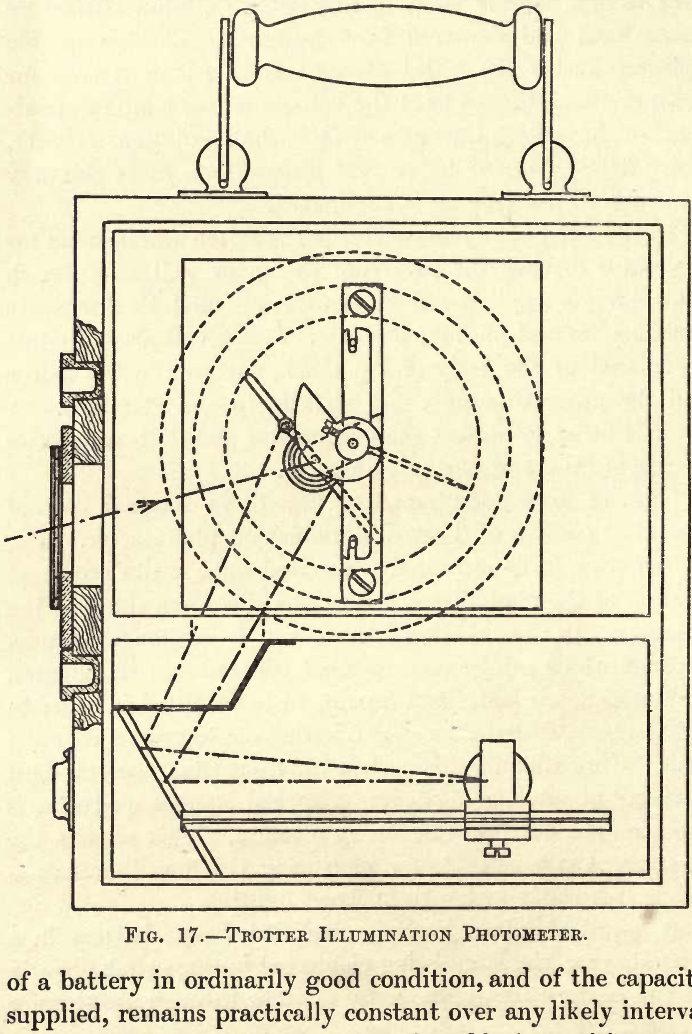


FIG. 17.—TROTTER ILLUMINATION PHOTOMETER.

of a battery in ordinarily good condition, and of the capacity supplied, remains practically constant over any likely interval of use. The standard lamp is adjustable in position on a scale to allow for any permanent difference in voltage.

Attachments are provided for the measurement of daylight illumination and surface brightness.*

The whole instrument can be mounted on a light tripod, as shown in Fig. 18, and can be set in any desired direction from the vertical. The weight with battery, but without the daylight attachment, is about 7 kilogrammes (15·2 pounds).

A further development of this instrument in the direction of greater portability and lightness has been made in the

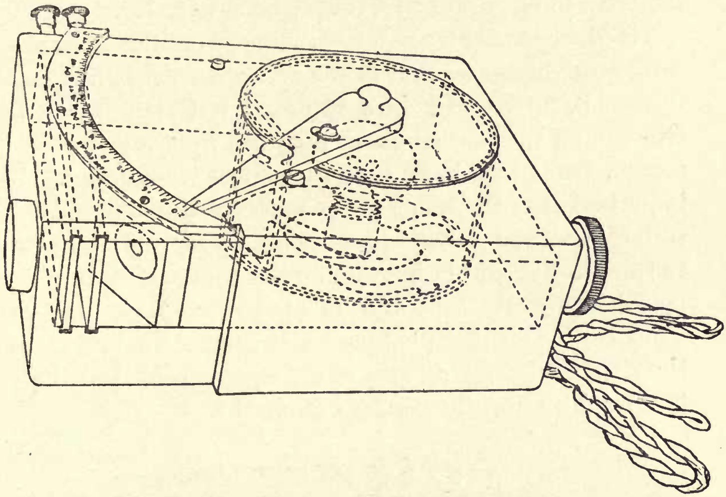


FIG. 20.—THE DOW AND McKINNEY LUMETER.

luxometer,† which is, as shown in Fig. 19, little more than pocket size.

In the lumeter of Messrs. Dow and McKinney the illumination from the standard lamp on the test surface in the photometer is varied by sliding a slotted plate over a window of translucent glass in the chamber containing the standard lamp. This window suffers a sudden reduction of area at one point, thus opening out the scale for the lower readings. The

[* A complete description is given by A. P. Trotter, "Illumination: its Distribution and Measurement," p. 211.

† The *Illuminating Engineer*, London, vol. iv., p. 656, and the *Electrician*, Industrial Supplement to vol. lxvii., p. 174.]

arrangement is shown diagrammatically in Fig. 20, p. 101,* and an external view is given in Fig. 21, opposite.

The modified Weber photometer due to Messrs. C. H. Sharp and P. S. Millar has been redesigned by them with a view to securing greater portability. They have thought it necessary to keep the standard lamp-current constant by means of an instrument always in circuit, and, in order to save weight, an ingenious Wheatstone bridge arrangement is used with the lamp in one arm and a telephone as detector.†

The difference between a measurement of illumination and surface brightness is, that in the first, the light falling on the surface is intercepted on a standard test-plate which may either be on the photometer or detached from it; while in the second, the light coming from the surface is examined. It is important that the whole of the aperture through which the surface is viewed should appear filled by it, and the area of the aperture should be known in order that surface brightness may be correctly expressed in candle-power per unit area. Thus, supposing that photometric balance is obtained at 4 f.c., the aperture having an area of 0·5 square inch, then the surface brightness of the surface examined is—

$$\frac{4}{0{\cdot}5} = 8 \ \mathit{C.P.} \text{ per square inch.}]$$

Portable photometers should have their scales frequently tested by measuring a known illumination, and then adjusted if necessary, especially if often used in the open.

42. On the Making of Illumination Measurements, and their Application.

Experimental determinations of illumination on completed installations are desirable, mainly for two reasons: In the first place, an objective estimate of the amount of illumination

[* The *Illuminating Engineer*, London, vol. iii., p. 581; the *Electrician*, vol. lxviii., p. 686.

† A complete description is given in the *Illuminating Engineer*, 1912, vol. v., p. 7.]

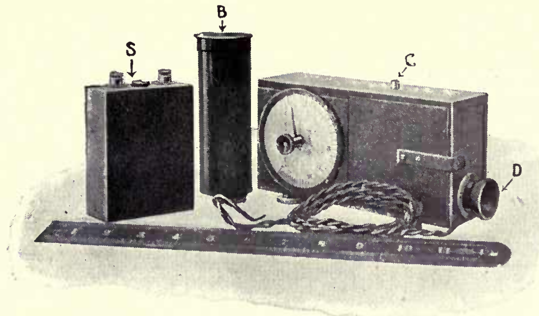


FIG. 19.—THE LUXOMETER.

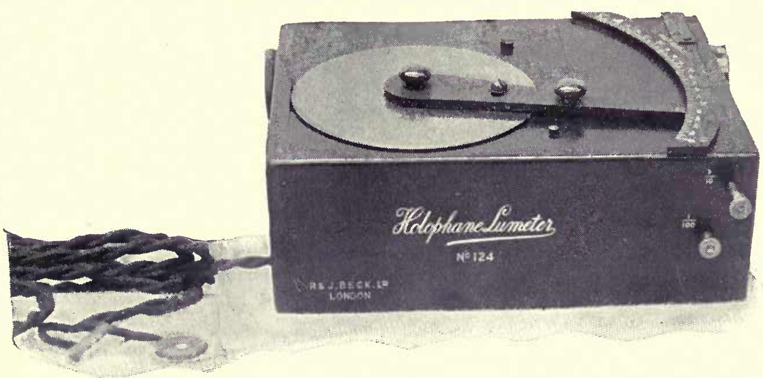


FIG. 21.--THE DOW AND McKINNEY LUMETER.

FIG. 18.

provided is arrived at, as appearances only are apt to be very deceptive, and as values have already been assigned to the illumination necessary for most purposes (Tables XVII., XVIII., and XIX., pp. 150-152), actual measurements decide whether the illumination obtained is reasonably sufficient for the given conditions; secondly, valuable material can be collected in this way, available for the design and execution of new installations of a similar character.

It is to be borne in mind that measurements should be so made as to give the result really required. It is not sufficient to measure the illumination at points on a vertical or horizontal plane chosen at random, but the measuring stations should be so selected, and the measurements themselves must be so conducted, as to render a true estimate of the illumination possible. It has been shown (pp. 45-49) that the best basis on which to found a judgment of most kinds of lighting is the mean horizontal illumination. As this cannot be obtained by a direct measurement, it must be deduced from those that can be made. The maximum and minimum illumination should also be determined in a complete test, in order that the uniformity of the lighting may be estimated.

If a knowledge of the vertical illumination is also desired, it can be conveniently measured at the same points as the horizontal illumination, the number of vertical planes considered being settled beforehand.

It will now be shown how to undertake a series of measurements from which the mean horizontal illumination can be deduced. An accurate method will first be discussed in connection with some examples of street and interior lighting, and afterwards an approximate method will be given, developed on an experimental basis.

43. The Measurement of Street Illumination.

The least complex method of mapping the distribution of illumination over the whole area of a street or place, and so deducing the mean horizontal illumination, is to divide the surface into squares or rectangles, and measure the horizontal illumination in the middle of each, the mean horizontal illumination being arrived at in the manner described on p. 58. To get results independent of local peculiarities in the street, measurements should be extended over two or three lamp intervals. The method then becomes somewhat tedious, and in a large number of cases it will not be very easy to extend the measurements to all the assigned positions.

The work can be carried out more simply by restricting the measurements to certain lines, and not extending them to all parts of the area. From such data the mean horizontal illumination can still be calculated, as will be explained. The best line to select is the one joining two lamps along the street. In addition, a second line can be taken parallel to the first, and on the kerb if the lamps are suspended over the middle of the road, or in the middle of the road if the lamps are on the kerb. If measurements are made at from 8 to 12 points on a line between two lamps, a sufficiently exact curve can be drawn without extending the experimental work too much. It is always better to repeat the tests over two or three lamp intervals, in order to minimise errors due to abnormal positions of lamps, troublesome shadows, or the like. If the lamp intervals are fairly equal, the mean values for points similarly situated in the intervals can be taken; but if the intervals are very unequal, then each must be treated separately.

44. Worked Example on the Measurement of Street Illumination.

As an example of the use that can be made of illumination measurements, those carried out by the author in the Friedrichstrasse in Berlin will be discussed.*

The street was then lit by ordinary continuous current arc lamps, each taking 15 amperes, fitted with globes of opal

* L. Bloch, "Vergleichende Beurteilung moderner Strassenbeleuchtungen," *Journal für Gasbeleuchtung*, 1906, p. 90.

glass, and erected at a height of about 10 metres (32 feet 10 inches) over the middle of the street. The lamp interval was about 30 metres (98 feet 5 inches), and the width of the street 22 metres (72 feet 2 inches). The horizontal illumination was measured at 1·5 metres (4 feet 11 inches) above the ground, along a line joining the lamps in the middle of the street, and also along a line on the kerb parallel to this. The measurements were extended over two intervals of 24 metres (78 feet 9 inches) and 34 metres (111 feet 6 inches).

Two curves of horizontal illumination are thus obtained, as given in Fig. 22. From these curves it is possible to deter-

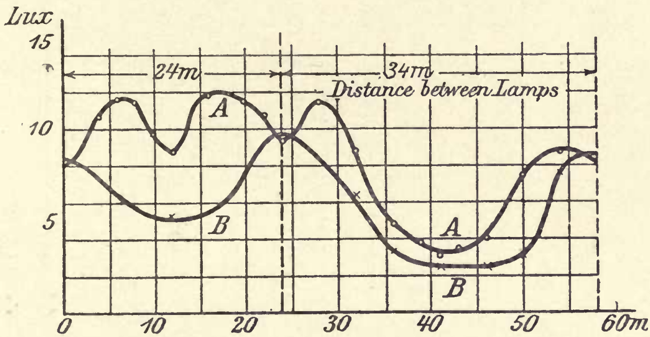


FIG. 22.—LIGHTING OF THE FRIEDRICHSTRASSE IN BERLIN WITH ELECTRIC ARC LAMPS.

Measured values of the horizontal illumination :
Curve *A*, Along the middle of the street.
Curve *B*, Along the edge of the pavement.

mine the distribution of light over the whole of the street, and, as a consequence, the mean horizontal illumination. The lamp interval for a part of curve *A* is 24 metres (78 feet 9 inches). This portion of the curve is redrawn in Fig. 23, the three lamps concerned in the calculation being shown, with the normal distance of 30 metres (98 feet 5 inches) between lamps *I.* and *III.*

The first thing to do is to determine *the curve of horizontal illumination due to one lamp.* An auxiliary curve is used for this purpose, derived from the distribution curve of a similar type of lamp, and, as long as this curve is of the same character as that given by the source in question, the scale to which it is drawn may have any convenient value.

In this case a suitable auxiliary curve is the normal curve of distribution for the type of lamp indicated, drawn for a mean hemispherical intensity of 1,000 *C.P.* in Fig. 43, p. 168. Taking a height of 8·5 metres (27 feet 10 inches) above the plane of reference, luminous intensities for different distances a (0–40 metres) from the foot of lamp are measured off

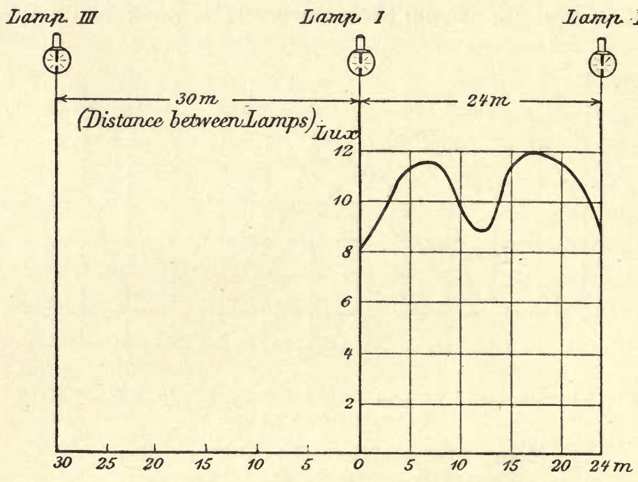


FIG. 23.—LIGHTING OF THE FRIEDRICHSTRASSE.

Measured values of the horizontal illumination along the line joining two lamps.

from the curve. From these values and the height h of the suspension, the horizontal illumination can be calculated by Equation 14 (p. 53)—

$$E_H = \frac{Ih}{\sqrt{(a^2+h^2)^3}},$$

and the auxiliary curve A (Fig. 24) obtained by plotting E_H against a.

Table VI. is then made up as follows: The three first columns give for 7 points on the median line their distances a_I, a_{II}, a_{III}, from the foot of lamps *I*, *II*, and *III*, in metres and feet. Column 4 gives the values of horizontal illumination E_l at these points taken from the left half of the curve in Fig. 23, while column 5 gives values E_r taken from points symmetrically placed on the right half of the curve. The mean values E of these two quantities are given in column 6. This figure E is the resultant horizontal illumination due to

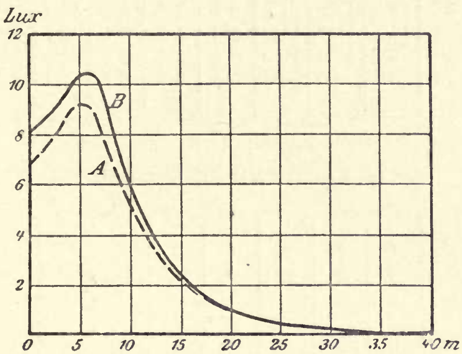


FIG. 24.—LIGHTING OF THE FRIEDRICHSTRASSE.

Horizontal illumination due to a single lamp:
A, Auxiliary curve.
B, Actual curve.

lamps *I*, *II*, and *III*, all other lamps further away having a negligible effect.

Values of the horizontal illumination due to each lamp separately, and corresponding to a_I, a_{II}, and a_{III}, are taken from the auxiliary curve *A*. These are denoted by E'_I, E'_{II}, E'_{III}, in columns 7, 8, and 9, their sum being—

$$E' = E'_I + E'_{II} + E'_{III}$$

in column 10.

The curve of horizontal illumination due to a single lamp must be similar in every respect to the auxiliary curve *A* (Fig. 24).

TABLE VI.

IN METRES AND LUX (METRE CANDLES).

1	2	3	4	5	6	7	8	9	10	11	12	13
a_I	a_{II}	a_{III}	E_l	E_r	E	E'_I	E'_{II}	E'_{III}	E'	E_I	E_{II}	E_{III}
0	24	30	8·1	9·3	8·7	6·9	0·5	0·2	7·6	7·9	0·6	0·2
2	22	32	9·0	9·8	9·4	7·9	0·7	0·2	8·8	8·4	0·8	0·2
4	20	34	10·8	11·4	11·1	8·9	0·9	0·2	10·0	9·9	1·0	0·2
6	18	36	11·6	11·8	11·7	9·1	1·2	0·1	10·4	10·2	1·4	0·1
8	16	38	11·4	11·8	11·6	7·0	1·6	0·1	8·7	9·4	2·1	0·1
10	14	40	9·8	10·6	10·2	5·1	2·4	0·1	7·6	6·9	3·2	0·1
12	12	42	8·8	8·8	8·8	3·5	3·5	0·1	7·1	4·35	4·35	0·1

IN FEET AND FOOT CANDLES

1	2	3	4	5	6	7	8	9	10	11	12	13
a_I	a_{II}	a_{III}	E	E_r	E	E'_I	E'_{II}	E'_{III}	E'	E_I	E_{II}	E_{III}
0	78·7	98·4	0·75	0·86	0·81	0·64	0·046	0·019	0·71	0·73	0·56	0·019
6·6	72·2	105·0	0·84	0·91	0·87	0·73	0·065	0·019	0·82	0·78	0·74	0·019
13·1	65·6	111·5	0·10	0·106	0·103	0·83	0·084	0·019	0·93	0·92	0·093	0·019
19·7	59·0	118	0·108	0·11	0·109	0·84	0·111	0·009	0·97	0·95	0·13	0·009
26·2	52·5	125	0·106	0·11	0·108	0·65	0·149	0·009	0·81	0·87	0·195	0·009
32·8	45·9	131	0·91	0·98	0·95	0·47	0·22	0·009	0·71	0·64	0·30	0·009
39·4	39·4	138	0·82	0·82	0·82	0·325	0·325	0·009	0·66	0·40	0·40	0·009

Hence, its unknown ordinates E_I, E_{II}, E_{III}, must each bear to the ordinates E'_I, E'_{II}, E'_{III}, of auxiliary curve, the ratio of the actual illumination—

$$E = E_I + E_{II} + E_{III} \text{ to } E' = E'_I + E'_{II} + E_{III}.$$

Hence—

$$E_I = E'_I \frac{E}{E'},\quad E_{II} = E'_{II} \frac{E}{E'},\ \text{and}\ E_{III} = E'_{III} \frac{E}{E'}.$$

E_I, E_{II}, and E_{III}, are tabulated in columns 11, 12, and 13 of Table VI., and give the horizontal illumination due to a single lamp for distances up to 42 metres from the foot of lamp. The curve *B* of Fig. 24 can then be constructed from these figures. Other values derived by this method from measurements made on the lateral line in the 24-metre interval have been used in drawing curve *B*, besides those given in Table VI., with the result that curve *B* is a little lower than the numbers in the table would give by themselves. Curve *B* differs very little from curve *A*, showing that the lamps used in the Friedrichstrasse happen to have a mean hemispherical intensity of about 1,000 *C.P.*

The accuracy of the result could be increased by repeating the calculation with curve *B* as auxiliary instead of *A*, but this refinement is usually unnecessary.

The horizontal illumination coming from a single lamp having been determined from values measured in the above described manner, the distribution of illumination over the whole street can be arrived at. For this purpose a plan of the portion of street dealt with is divided up into a number of squares or rectangles of equal area, as described on p. 59. In this case it will be sufficient to consider one-fourth of the area comprised between two lamps, as in the other three-fourths the distribution of light is repeated symmetrically. The lamp interval is assigned the mean value of 30 metres (98 feet 5 inches), and the number of rectangles, usually from 10 to 20, is here fixed at 12. Their positions with regard to the lamps is that shown in the bottom right-hand diagram of Fig. 11 (p. 60). The distances from the middle points of these rectangles to the foot of each of the lamps *I*, *II*, and *III*, are measured off from Fig. 25, and set out in Table VII., along with the corresponding values of horizontal illumination derived from curve *B*, Fig. 24. The sum *E* of the three individual values, E_I, E_{II}, and E_{III}, obtained for each point gives the resultant horizontal illumination at this point of the street. Finally, the mean horizontal illumination of the whole street is given as the mean of the twelve values of *E*, thus arriving at the most important factor in the estimation of a system of lighting.

TABLE VII.

IN METRES AND LUX (METRE CANDLES).

Rectangle No.	a_I	a_{II}	a_{III}	E_I	E_{II}	E_{III}	E
1	9·3	29·6	33	6·8	0·2	0·1	7·1
2	10·6	26	37	5·3	0·4	0·1	5·8
3	13·0	22·5	40	3·0	0·7	0·1	3·8
4	16·0	19·2	44	1·8	1·1	—	2·9
5	14·2	17·7	44	2·4	1·3	—	3·7
6	10·8	21·3	40	5·0	0·8	0·1	5·9
7	7·8	25	36	8·8	0·4	0·1	9·3
8	5·8	28·6	32	10·5	0·2	0·2	10·9
9	2·6	28·2	32	9·2	0·2	0·2	9·6
10	5·9	24·4	36	10·5	0·5	0·1	11·1
11	9·6	20·6	39	6·4	0·9	0·1	7·4
12	13·2	17·0	43	3·0	1·6	—	4·6

$$\Sigma = 82·1$$

$$E_{mean} = \frac{82·1}{12} = 6·8$$

IN FEET AND FOOT CANDLES.

Rectangle No.	a_I	a_{II}	a_{III}	E_I	E_{II}	E_{III}	E
1	30·5	97·1	108	0·63	0·019	0·009	0·658
2	34·8	85·3	121	0·49	0·037	0·009	0·536
3	42 6	73·8	131	0·28	0·065	0·009	0·354
4	52 5	63·0	144	0·17	0·102	—	0·272
5	46·6	58·0	144	0·22	0·128	—	0·348
6	35·4	69·9	131	0·46	0·074	0·009	0·543
7	25·6	82·0	118	0·82	0·037	0·009	0·866
8	19·0	93·8	105	0·98	0 019	0·019	1·018
9	8·5	92·5	105	0·85	0·019	0·019	0·888
10	19·35	80·0	118	0·98	0·046	0·009	1·035
11	31·5	67·5	128	0·60	0·084	0·009	0·693
12	43·3	55·7	141	0·28	0·15	—	0·43

$$\Sigma = 7·64$$

$$E_{mean} = \frac{7·64}{12} = 0·637 \text{ f.c.}$$

Curve B also enables the maximum and minimum values of the horizontal illumination to be determined. The maximum occurs at the point M (Fig. 25), 5 metres (16 feet 5 inches) from I, 25 metres (82 feet) from II, and 35 metres (114 feet 10 inches) from III. Its value is—

$$E_{max} = 10{\cdot}5 + 0{\cdot}5 + 0{\cdot}1 = 11{\cdot}1 \text{ lux } (1{\cdot}03 \text{ f.c.}).$$

The minimum value occurs at m, 18·6 metres (61 feet) from I and II, and its value is—

$$E_{min} = 1{\cdot}3 + 1{\cdot}3 = 2{\cdot}6 \text{ lux } (0{\cdot}24 \text{ f.c.}).$$

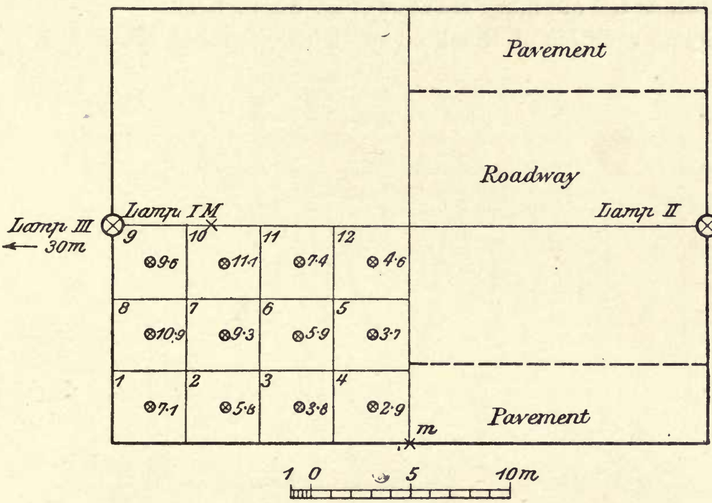


FIG. 25.—LIGHTING OF THE FRIEDRICHSTRASSE IN BERLIN WITH ELECTRIC ARC LAMPS.

Distribution of illumination over the surface of the street :

Maximum	horizontal	illumination,	11·1 lux	(1·03 f.c.).
Minimum	,,	,,	2·6 ,,	(0·24 ,,).
Mean	,,	,,	6·8 ,,	(0·64 ,,).

Hence the coefficient of uniformity is—

$$\frac{E_{max}}{E_{min}} = \frac{11{\cdot}1}{2{\cdot}6} = 4{\cdot}3.$$

The illumination is sensibly more uniform than is usually found in street lighting.

Complete data for the formation of a judgment on the lighting scheme in question have thus been obtained.

45. Worked Example on the Measurement of Illumination of an Open Space.

When many lamps are on one post, or grouped together in a single lantern, they can be considered as a single source, and the curve of horizontal illumination due to a lantern or group, drawn in the manner described on p. 71, a distribution curve appropriate to a single lamp of the type used being chosen as the auxiliary curve A (Fig. 24, p. 107).

As an example of this kind of lighting, and of the way in

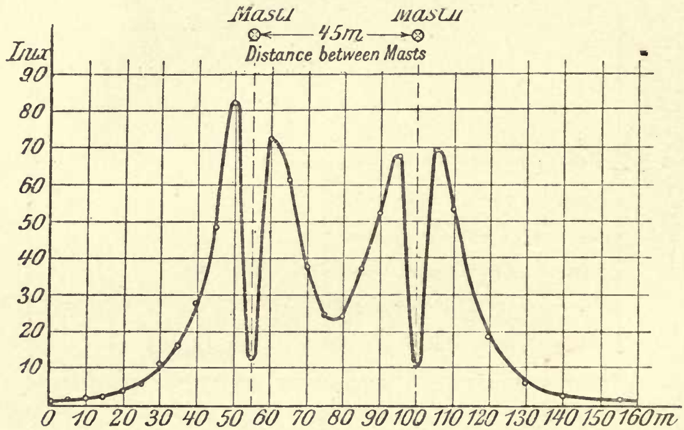


Fig. 26.—Lighting of the Potsdamer Platz in Berlin with Intensive Flame Arc Lamps.

Measured values of the horizontal illumination.

which measurements of illumination in an open space can be treated, the lighting of the Potsdamer Platz in Berlin will be considered. (See also third example of Calculation of Illumination, p. 78).

Two groups, each consisting of four intensive flame arc lamps, are installed, 45 metres (148 feet) apart. Each lamp takes 20 amperes, is fitted with a transparent glass globe, and the group is hung at a height of 18 metres (59 feet) above the ground. Measurements of horizontal illumination were made every 5 metres (16 feet 5 inches) along a line joining the lamp standards, at a height of 1·5 metres (4 feet 11 inches). The results obtained are given as a curve in Fig. 26. The auxiliary curve for one group has been drawn in Fig. 27 from the distribution curve of that kind of lamp (Fig. 45, p. 172) in the manner already described on pp. 104-109. From this curve the distribution of illumination over the whole space due to one source can be determined, the area being divided up into squares, as shown in Fig. 28. The value of the horizontal illumination at the middle point of each square is written within it, but it was only necessary to

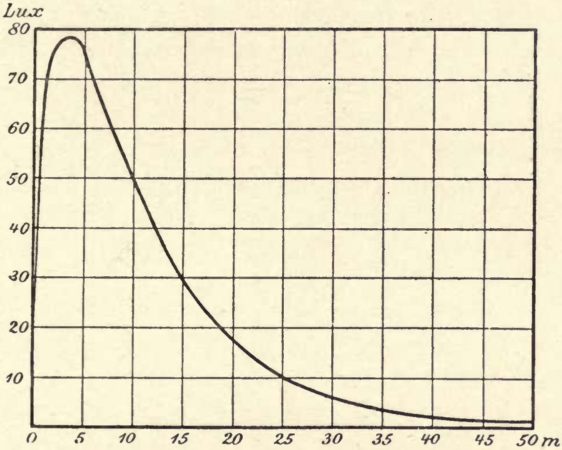


FIG. 27.—LIGHTING OF THE POTSDAMER PLATZ.

Horizontal illumination due to the lamps on a single mast.

actually make the calculation for one-fourth of the number. The mean value of all these gives the mean horizontal illumination as—

$$E_{mean} = 18{\cdot}8 \text{ lux } (1{\cdot}75 \text{ f.c.}).$$

In this example the maximum value of the horizontal illumination reaches 89 lux (7·6 f.c.), the minimum value at the boundary of the area 1·3 lux (0·12 f.c.), and the ratio—

$$\frac{E_{max}}{E_{min}} = \frac{82}{1{\cdot}3} = 63.$$

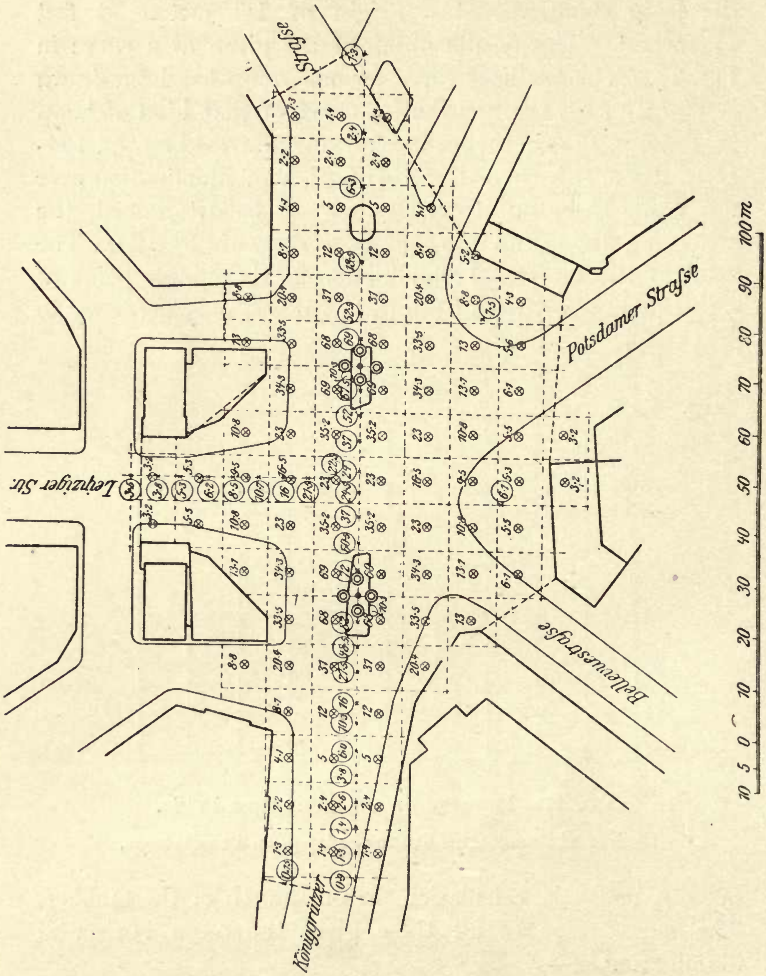


FIG. 28.—LIGHTING OF THE POTSDAMER PLATZ IN BERLIN WITH INTENSIVE FLAME ARC LAMPS.

Distribution of illumination over the whole area.

(5.7) Measured values of the horizontal illumination.

⊗5.1 Predetermined values of the horizontal illumination.

◎ Position of arc lamps.

Maximum horizontal illumination,		82	lux	(7·6 f.c.).
Minimum	,,	,, 1·3	,,	(0·12 ,,).
Mean	,,	,, 18 8	,,	(1·75 ,,).

This figure shows that the lighting is rather irregular. There is not, however, a demand for so great a uniformity in this kind of lighting as in that of streets, and it is justifiable to allow a much smaller value round the boundary of the area than in the middle. It is sufficient that the minimum illumination of the place should be greater than the mean illumination of the streets running into it.

46. Measurement of Interior Illumination.

In dealing with interiors, there are cases in which it is possible to divide the illuminated surface into a suitable number of rectangles, and at once measure the horizontal illumination at the centre of each of these rectangles. The mean of all these measurements then gives the mean horizontal illumination. The disposition of furniture and other causes sometimes prevents this method from being used, and recourse must be had to a somewhat different procedure than that adopted in street lighting. Then, again, interiors are often lit from a large number of sources rather close together, while reflection from the walls and ceiling also becomes important. From these considerations it will be seen that it is no longer possible to deduce the distribution curve of a single lamp from the auxiliary curve and a few observations taken between two lamps, as in street lighting.

It follows that the measurements to be made and their application will depend upon the nature of the room and the arrangement of lamps.

When the room is approximately square or circular, with the lamps disposed symmetrically, either in a group in the centre or distributed regularly over the whole room, the mean horizontal illumination can be deduced from measurements made along two or three diagonals or diameters. The height of the reference plane is usually taken at from 0·8 to 1·0 metre (2 feet 7·5 inches to 3 feet 3·4 inches), the height of an ordinary table, but sometimes 1·5 metres (4 feet 11 inches) is used, as in street lighting.

The points at which measurements are made are marked on a plan of the room with the observed values of the horizontal illumination, and the distance of each point from the centre of the room is determined. A curve can then be deduced, showing the variation of horizontal illumination along the selected lines through the centre. This is always possible for square or round rooms symmetrically illuminated. One-fourth of the total floor area is then divided into ten or twenty squares or rectangles, and the distances from the centre of the room to the centre of each of these areas is measured off from the plan. The values of the horizontal illumination corresponding to these distances are then taken from the illumination curve, and the mean of these gives the mean horizontal illumination.

If the room is a long rectangle, measurements should be made along two or three lines parallel to the direction of the greatest length. Curves of horizontal illumination can be constructed along these lines, from which values can be read off for the centres of rectangles dividing up the whole area, or a corresponding portion of it if the lighting is symmetrical. When the centre of a given rectangle is not actually on one of the reference lines, the value of the horizontal illumination can be found by interpolation as before, the mean of all these giving the mean horizontal illumination.

47. Worked Examples on the Measurement of Interior Illumination.

The method just described was applied to measurements of illumination taken over the floor of a large restaurant. The floor area was 490 square metres (5,270 square feet), and the lighting was carried out by Nernst lamps, type A,* fixed in the ceiling at a height of 7 metres (23 feet) above the floor and distributed, as shown in Fig. 29. The horizontal illumination was measured at 1·5 metres (4 feet 11 inches) above

* The A type of Nernst lamp has a loop filament projecting vertically downward with the heater inside it.

the ground, along two straight lines parallel to the long side of the room. The results are indicated at the corresponding

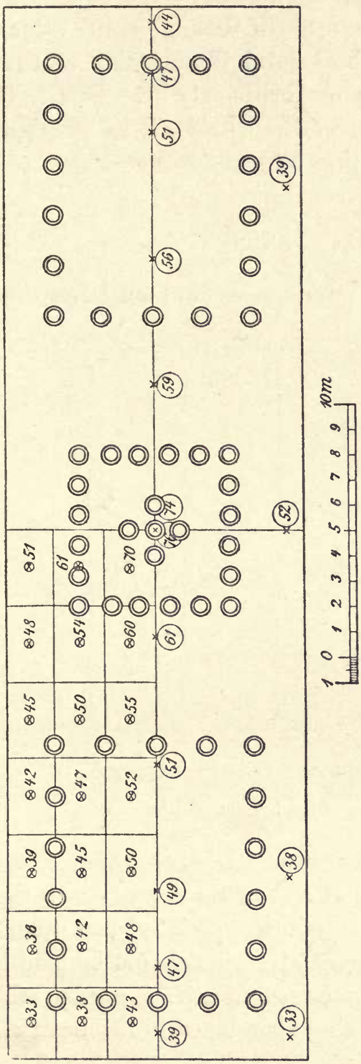


Fig. 29.—Lighting of a Restaurant with Nernst Lamps.

×(33) Measured values of the horizontal illumination.

⊗43 Predetermined values of the horizontal illumination.

◎ Positions of the Nernst lamps.

Maximum horizontal illumination, 76 lux (7·05 f.c.).

Minimum ,, ,, 33 ,, (3·1 ,,).

Mean ,, ,, 56 ,, (5·2 ,,).

points of Fig. 29, and from them the curves of distribution of horizontal illumination along the reference lines have been drawn in Fig. 30. These curves permit of the determination of the horizontal illumination at the centre of each of twenty-one rectangles, into which one-fourth of the floor of the hall has been divided, and the mean of these gives the value 56 lux (5·2 f.c.) as the mean horizontal illumination of the whole area. This hall is therefore brilliantly lit. Due to the distribution of the lamps over the whole ceiling the lighting is also very uniform, as is shown by the ratio—

$$\frac{E_{max}}{E_{min}} = \frac{76}{33} = 2 \cdot 3.$$

In the illumination of interiors, account must often be taken,

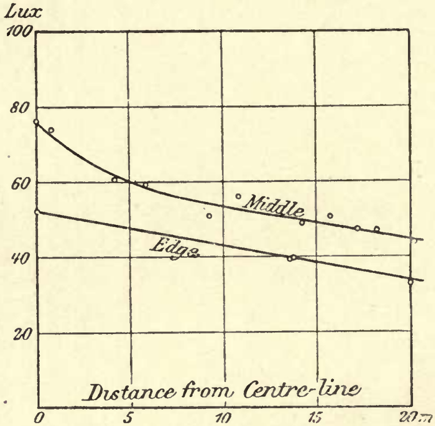


Fig. 30.—Lighting of a Restaurant.

Measured values of the horizontal illumination.

not only of the arrangement of lamps, the kind of lighting, and the shape of room, but also of the use to which the room is to be put, when this necessitates specially good illumination at particular points. Great attention should be paid to such conditions when making measurements, a purely schematic series of measurements being abandoned in favour of observations to suit each case.

An example in which the measurements and their application were made to suit the particular circumstances of use is that of the lighting of an art exhibition. The hall is nearly square, with an area of 330 square metres (3,550 square feet), and is lit by four continuous current arc lamps, each

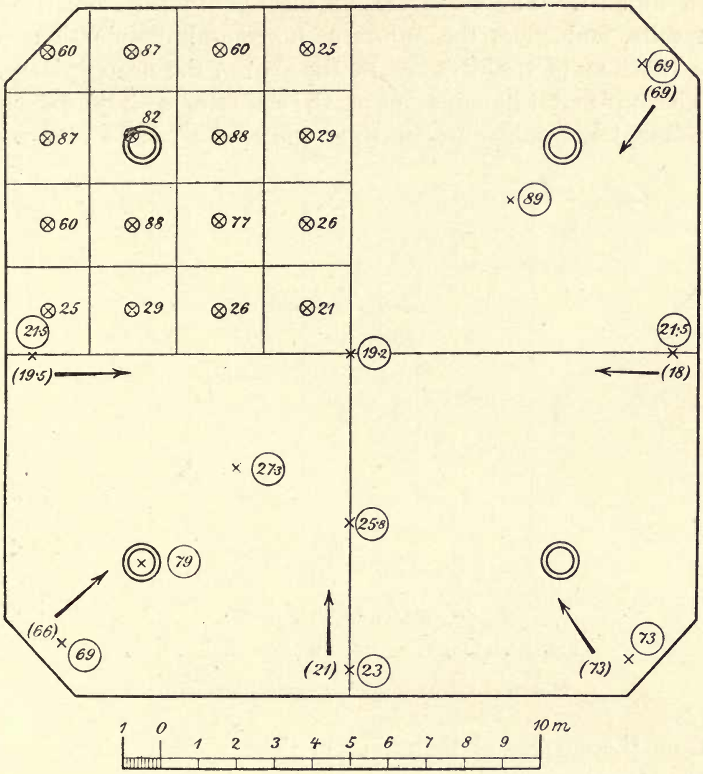


FIG. 31.—LIGHTING OF AN ART EXHIBITION WITH ELECTRIC ARC LAMPS.

x (51)→ Measured values of the vertical illumination.
x(57) Measured ,, ,, horizontal ,,
⊗51 Pre-determined values of the horizontal illumination.
Maximum horizontal illumination, 89 lux (8·3 f c.).
Minimum ,, ,, 19 ,, (1·75 ,,).
Mean ,, ,, 54 ,, (5·0 ,,).

taking 12 amperes at 55 volts, and each fitted with a diffusing globe. The lamps are 5 metres (16 feet 5 inches) above the ground, and measurements of illumination were made at a height of 1·5 metres (4 feet 11 inches). The positions of the lamps, the several points at which measurements were made and the values found, are shown on the ground-plan in Fig. 31. The curve of Fig. 32 is obtained from these results, and gives the values of horizontal illumination as a function of the distance to the foot of the nearest lamp. The horizontal illumination at the centre of each of sixteen rectangles, forming one-fourth of the square, can be read off

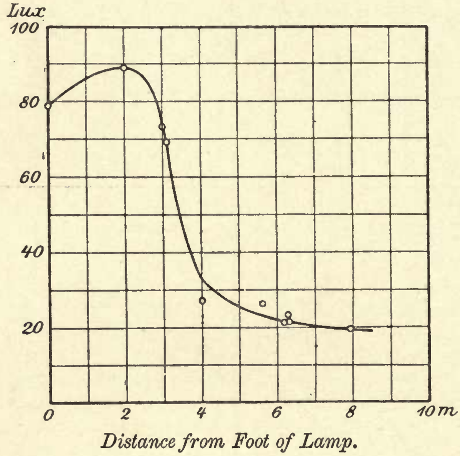


FIG. 32.—LIGHTING OF AN ART EXHIBITION.

Measured values of the horizontal illumination.

from the curve, and the means of these sixteen values gives the mean horizontal illumination at 54 lux (5·0 f.c.).

The coefficient of uniformity is given by the ratio—

$$\frac{E_{max}}{E_{min}} = \frac{89}{19} = 4 \cdot 7,$$

and is not particularly good for an interior. In this case, however, the vertical illumination of the walls is of more importance, as a good illumination is required on the pictures placed there. Measurements of vertical illumination were made close to the walls, the points chosen and the values obtained being indicated on Fig. 31 by the numbers in brackets. The arrows perpendicular to the walls indicate the direction in which measurements were made. A comparison of values at the points considered show that the differences between vertical and horizontal illumination are very small.

48. Simple Methods of Making and Applying Illumination Measurements.

The method just described and illustrated by examples gives exact results, and can be used without any particular preliminary knowledge. It often happens in practice that there is not much time for these measurements, however desirable it might be to make them, and in those cases a simplified method can be used involving the least possible amount of measurement and calculation, yet giving results which can be relied upon in judging of the illumination.*

The examination of a very large number of measurements of illumination has shown that the value of the mean horizontal illumination can be calculated approximately if the maximum and minimum horizontal illumination have been correctly determined. The two values should be measured in as many places as possible, say from four to six, and the mean taken in order to get a result independent of accidental circumstances. These points can be fixed by eye, or by a few trial readings, positions of special illumination due to shadows, for instance, being carefully avoided.

The calculation of the mean horizontal illumination by this short method depends on the uniformity of the lighting as defined by the ratio of the maximum value (E_{max}) to the minimum value (E_{min}). For interiors this ratio is nearly always less than three, and in such cases the value of E_{mean} is given at once by taking the arithmetic mean of the maximum and minimum.

* L. Bloch, "Die Verwertung von Beleuchtungsmessungen" (*Journal für Gasbeleuchtung,* 1907, p. 152).

For—

$$\frac{E_{max}}{E_{min}} < 3, \; E_{mean} = \frac{E_{max} + E_{min}}{2} \quad . \quad . \quad (21).$$

For street lighting the ratio $\frac{E_{max}}{E_{min}}$ is generally much greater. The conditions and formulæ are, then,

For—

$$\frac{E_{max}}{E_{min}} \begin{cases} > 3 \\ < 70 \end{cases} \; E_{mean} = 0{\cdot}18(E_{max} + 10E_{min}) \quad . \quad (22).$$

And for—

$$\frac{E_{max}}{E_{min}} > 70 \; E_{mean} = 0{\cdot}10(E_{max} + 10E_{min}) \quad . \quad (23).$$

Approximate figures can thus be obtained very simply and rapidly.

The experimental data leading to these empirical formulæ are collected together in Tables VIII. and IX., pp. 123-126.

They contain actual measurements of illumination of streets, squares, and interiors, varying according to the type and arrangement of the lamps, the current taken, or the gas burnt. Some data have also been given on the economic coefficient, showing the consumption in watts or litres per hour to get a mean horizontal illumination of 1 lux and and square metre (1 f.c. and square foot).

Column 14 of the tables contains the values of mean horizontal illumination as the result of exact calculation, and column 15 gives the approximate values, calculated from equations 21, 22, and 23, the errors tabulated in the last column being seldom over 10 per cent., and in no case above 16 per cent. The coefficient of uniformity is given in column 13.

This simplified method is of especial value wherever great exactitude is not required, and is available in most practical cases.

TABLE VIII.

EXPERIMENTAL RESULTS IN STREET LIGHTING: METRIC UNITS.

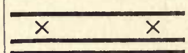
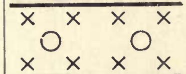
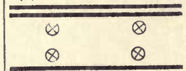
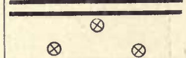
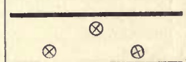
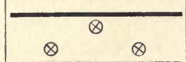
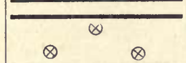
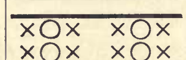
Kind of Lamp.	Locality. B = Berlin, C = Charlottenburg.	Arrangement of Lamp × and Pole ○.	Consumption per Lamp.	Height of Suspension (Metres).	Lamp Interval (Metres).	Width of Street (Metres).	Area of Street per Pole (Square Metre).	Consumption per Square Metre.	Consumption per Lux and Square Metre.	Horizontal Illumination in Lux.					Error per Cent.
										E_{max}	$E_{min.}$	$\frac{E_{max.}}{E_{min.}}$	E_{mean} (exact).	E_{mean} (approx)	
Ordinary continuous current arc lamps	Friedrichstrasse (B)		Watts. 825	10	30	22	660	Watts. 1·25	Watts. 0·205	10·0	2·3	4·3	6·1	6·0	−1·5
	Unter den Linden (B)		935	7·2–8·0	{39, 70}	61	930	1·00	0·19	15·3	1·0	15·3	5·3	4·5	−15
Continuous current flame arc lamps	Potsdamer Platz (B)		1,100	18	45	—	3,640	1·20	0·072	73	1·2	6·1	16·9	15·4	−9
	Alsenbrücke (B)		440	8	22	17	187	2·35	0·140	30·5	6·6	4·6	16·7	17·4	+4
Alternate current flame arc lamps	Hardenbergstrasse (C)		455	9	40	38·5	770	0·59	0·17	10·1	1·0	10·1	3·5	3·6	+2·5
Upright gas-mantles	Schumannstrasse (B)		Litres per hr. about 125	3·5	50	18·5	462	Litres per hr. 0·27	Litres per hr. 0·46	5·2	0·07	74	0·585	0·605	+1·5
High-pressure gas-mantles (millennium light)	Alexanderstrasse (B)		about 2 × 600	5·7	42	21	441	2·72	0·58	17·0	0·7	24	4·7	4·35	−7
	Königstrasse (B)		about 2 × 1,200	5·7	42	20·5	431	5·57	0·53	36·9	1·8	20·5	10·5	9·9	−6
Inverted gas-mantles	Invalidenstrasse (B)		about 2 × 114	4·5	28	17	238	1·92	0·39	52·2	0·54	97	4·95	5·75	+16

TABLE VIII.—*Continued.*

EXPERIMENTAL RESULTS IN STREET LIGHTING: ENGLISH UNITS.

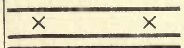
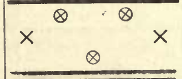
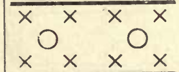
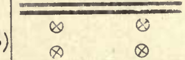
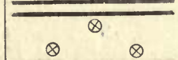
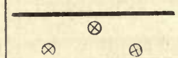
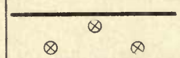
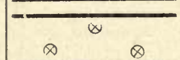
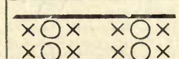
Kind of Lamp.	Locality. B=Berlin, C=Charlottenburg.	Arrangement of Lamp × and Pole ○.	Consumption per Lamp.	Height of Suspension.	Lamp Interval in Feet.	Width of Street in Feet.	Area of Street per Pole (Square Foot).	Consumption per Square Foot.	Consumption per f.c. and Square Foot.	Horizontal Illumination in Foot Candles.					Error per Cent.
										$E_{max.}$	$E_{min.}$	$\frac{E_{max.}}{E_{min.}}$	E_{mean} (exact).	E_{mean} (approx.).	
			Watts.	Ft. In.				Watts.	Watts.						
Ordinary continuous current arc lamps	Friedrich-strasse (B)		825	32 9·5	98	72	7,100	0·116	0·205	0·93	0·21	4·3	0·57	0·56	– 1·5
	Unter den Linden (B)		935	{23 7 / 26 3}	128–230	200	10,000	0·092	0·19	1·42	0·09	15·3	0·49	0·42	– 15
Continuous current flame arc lamps	Potsdamer Platz (B)		1,100	59 0	147	—	39,150	0·111	0·072	6·79	0·11	6·1	1·57	1·43	– 9
	Alsen-brücke (B)		440	26 3	72	56	2,010	0·218	0·14	2·83	0·61	4·6	1·55	1·62	+ 4
Alternate current flame arc lamps	Hardenberg-strasse (C)		455	20 6	131	126	8,280	0·055	0·17	0·94	0·09	10·1	0·32	0·33	+ 2·5
			Cubic ft. per hr.					Cubic ft. per hr.	Cubic ft. per hr.						
Upright gas-mantles	Schumann-strasse (B)		about 4·5	11 6	164	61	4,970	0·0009	0·017	0·483	0·006	74	0·054	0·056	+ 1·5
High - pressure gas-mantles (millennium light)	Alexander-strasse (B)		about 2 × 22	18 8·5	138	69	4,745	0·009	0·020	1·58	0·065	24	0·44	0·405	– 7
	König-strasse (B)		about 2 × 44	18 8·5	138	67	4,640	0·018	0·0185	3·43	0·17	20·5	0·97	0·92	– 6
Inverted gas-mantles	Invaliden-strasse (B)		about 2 × 4	14 9	92	61	2,560	0·0063	0·0137	4·85	0·05	97	0·46	0·535	+ 16

TABLE IX.

EXPERIMENTAL RESULTS IN INTERIOR LIGHTING: METRIC UNITS

Kind of Lamp.	Kind of Room.	Arrangement of Lamps.	Number of Lamps.	Consumption per Lamp in Watts.	Height of Lamps in Metres.	Floor Area of Room in Square Metres.	Floor Area per Lamp in Square Metres.	Consumption per Square Metre in Watts.	Consumption per Lux and Square Metre in Watts	Horizontal Illumination in Lux.					Error per Cent.
										$E_{max.}$	$E_{min.}$	$\frac{E_{max.}}{E_{min.}}$	E_{mean} (exact).	E_{mean} (approx.).	
Nernst lamps Type B (p. 83)	Show-room	Distributed over the whole ceiling	136	55	4	145	1·07	51·5	0·64	88	73	1·21	80	80·5	+0·6
Nernst lamps (Type A (p. 116)	Restaurant	Distributed over the whole ceiling	61	200	7	490	8·0	25·0	0·49	68	30	2·3	50	49	−3
Continuous current arc lamps	Art gallery	Semi-indirect lighting	4	660	5	330	82·5	8·0	0·164	80	17	4·7	49	49	0
Continuous current arc lamps with normal arrangement of carbons	Drawing office (coloured yellowish white	Indirect lighting	2	550	3	72	36	15·3	0·336	63	26·5	2·4	45·5	45	−1
Continuous current arc lamps with inverted carbons	Drawing office (coloured yellowish white)	Indirect lighting	2	650	2·5	72	36	18·0	0·24	119	39	3·0	74	79	+6·5

TABLE IX.—*Continued.*

EXPERIMENTAL RESULTS IN INTERIOR ILLUMINATION: ENGLISH UNITS.

Kind of Lamp.	Kind of Room.	Arrangement of Lamps.	Number of Lamps.	Consumption per Lamp in Watts.	Height of Lamps in Feet.	Floor Area of Room in Square Feet.	Floor Area per Lamp in Square Feet.	Consumption per Square Foot in Watts.	Consumption per Foot Candle and Square Foot in Watts.	Horizontal Illumination in Foot Candles.					Error per Cent.
										$E_{max.}$	$E_{min.}$	$\frac{E_{max.}}{E_{min.}}$	E_{mean} (exact).	E_{mean} (approx.).	
Nernst lamps Type B (p. 83)	Show-room	Distributed over the whole ceiling	136	55	13·1	1,560	11·5	4·8	0·64	8·2	6·8	1·21	7·45	7·5	+0·4
Nernst lamps Type A (p. 116)	Restaurant	Distributed over the whole ceiling	61	200	23	5,270	86·5	2·3	0·49	6·3	2·8	2·3	4·65	4·55	−2
Continuous current arc lamps	Art gallery	Semi-indirect Lighting	4	660	16·4	3,550	890	0·75	0·164	7·45	1·6	4·7	4·55	4·55	0
Continuous current arc lamps with normal arrangement of carbons	Drawing office (coloured yellowish white)	Indirect lighting	2	550	9·8	775	387	1·4	0·336	5·9	2·5	2·4	4·2	4·15	−1
Continuous current arc lamps with inverted carbons	Drawing office (coloured yellowish white)	Indirect lighting	2	650	8·2	775	387	1·7	0·24	11·1	3·6	3·0	6·85	7·35	+6·5

CHAPTER VI

INDIRECT LIGHTING

49. Light Sources for Indirect Illumination.

Indirect lighting is a special branch of the science of illumination. Its essential feature is that all, or nearly all, of the light emitted by the sources is sent on to the ceiling or walls of the room, and illumination is effected by the light reflected from these surfaces. A very uniform illumination is thus secured, with an almost complete suppression of troublesome shadows, while the invisibility of the actual sources is in itself an aid to clear vision.

Electric arc lamps, incandescent gas mantles, and in some cases metal filament lamps provide the best sources for indirect lighting. Arc lamps should be worked with ordinary, unimpregnated carbons, and they can be arranged as for direct illumination—that is, with the highly luminous positive carbon at the top, and the less luminous negative carbon underneath. The naked lamp under these conditions sends nearly all its light into the lower hemisphere, as shown in curve A, Fig. 33 (p. 128), where it will be seen that the mean hemispherical intensity ($I_{\triangledown}$) is almost double of I_0. For the purpose of indirect lighting, this light must be sent upwards, and this can be done by fixing a reflector coated with white enamel underneath the lamp. A reflector of this kind absorbs about 40 per cent. of the total light, the resulting distribution being given by curve B, Fig. 33. An examination of these curves, and a comparison of the new values of the mean spherical intensity with the mean upper hemispherical intensity, shows that practically all the light is sent into the upper hemisphere, and used to light the ceiling.

Curve *B* is almost circular, and is very much like the curve given for an element of luminous area radiating from one face (Fig. 38, p. 158).

If the enamelled iron reflector is replaced by one of opal or

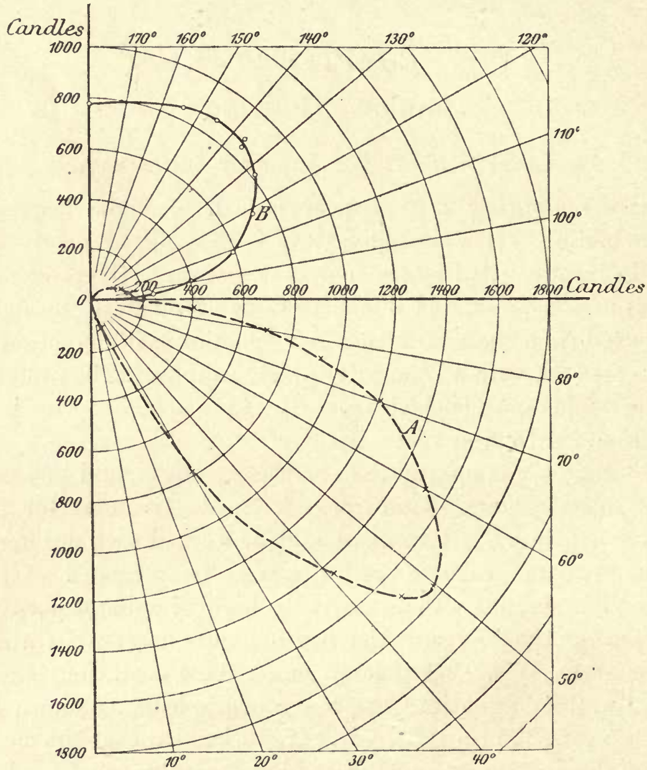


FIG. 33.—LIGHT DISTRIBUTION CURVES FROM AN ORDINARY CONTINUOUS CURRENT ARC LAMP, TAKING 12 AMPERES AT 42 VOLTS.

Curve *A*, Without reflector—

$I_0 = 560$ *C.P.* $I_{\triangledown} = 1{,}035$ *C.P.*

Curve *B*, With enamelled sheet-metal reflector for indirect lighting—

$I_0 = 340$ *C.P.* $I_{\triangle} = 660$ *C.P.*

milk glass, allowing some light to traverse it, the so-called semi-indirect lighting is obtained (p. 146), in which some of the light passes directly below.

Arc lamps can also be used in which the relative positions of the carbons are reversed, the positive carbon being underneath, and the negative carbon above. The distribution in the upper hemisphere is then almost exactly the same as that given in the lower hemisphere with the usual arrangement of carbons; the light of the arc goes directly to the ceiling, and loss by absorption in the reflector is avoided. A small reflector must, however, be placed under the arc to prevent a direct view of it from below. The output of light under these conditions is almost the same as for an ordinary arc, but the working of the lamp is less regular, and the shadows cast on the ceiling by accessories become more marked. The inverse arrangement of carbons is to be preferred where the highest possible efficiency is aimed at, but the normal arrangement is better if it is desired to create a good general impression of the lighting, and to have the lamps working in the best way.*

Incandescent gas lamps for this purpose should also be fitted with large enamelled reflectors in order to divert the downwardly-directed radiation upwards and on to the ceiling.

The type of gas lamp with which the best results have been achieved is that in which the gas is burnt under pressure in large units grouped together. The advantage of uniform illumination can be secured in this way with a small number of lamps, the same degree of illumination by indirect means from ordinary burners only being possible with a much larger number.

Metallic filament lamps are finding increasing use for indirect and semi-indirect lighting. The cost is greater than with arc lamps, but not so much greater as to be more than made up for by the greater convenience. The lamps can be placed, for example, in a channel along the wall, and concealed from view by a cornice, from whence they send their light on to the walls above them and on to the ceiling. A very fine luminous effect can be obtained in this way, particularly

* Zeidler, "Die Elektrischen Bogenlampen," pp. 54 and 57; Monasch, *Elektrische Beleuchtung,* pp. 87 and 89.

if no source is directly visible from the room below. There is, however, a great loss of light, rendering the method an uneconomical one, and only to be regarded as a luxury.

50. Effect of the Colour of Walls and Ceiling.

With indirect lighting, the original light is only of use after reflection from the ceiling and walls, and it is therefore evident that the colour of these surfaces should be such that loss of light by absorption is kept as small as possible.

The following figures have been given by Sumpner* for the reflecting power of different surfaces :

TABLE X.

Kind of Surface.	Reflecting Power. Per Cent.
White blotting-paper	82
White cartridge-paper	80
Tracing-cloth	35
Tracing-paper	22
Ordinary foolscap	70
Newspapers	50–70
Tissue-paper (one thickness)	40
Tissue-paper (two thicknesses)	55
Yellow wall-paper	40
Blue paper	25
Dark brown paper	13
Deep chocolate paper	4
Plane deal (clean)	40–50
Plane deal (dirty)	20
Yellow cardboard	30
Parchment (one thickness)	22
Parchment (two thicknesses)	35
Yellow painted wall (dirty)	20
Yellow painted wall (clean)	40
Black cloth	1·2
Black velvet	0·4

[In the majority of cases the above numbers are approximate only. The first four surfaces referred to were, however, carefully tested, and the numbers obtained represent the mean of many observations. The source of light was a carbon filament glow lamp.]

[* W. E. Sumpner, *Philosophical Magazine*, Series 5, vol. xxxv., February, 1893.]

The surfaces tested in these experiments not being of the kind commonly used in indirect lighting, further tests were carried out in the Laboratory of the Berliner Elektrizitäts-Werke.

Seventeen different kinds of ceiling surface and three different kinds of paper were prepared on sheets of cardboard 20×25 centimetres ($7{\cdot}9 \times 9{\cdot}8$ inches), and their reflecting power investigated. The way in which this was done is shown in Fig. 34.

The test-sheets were illuminated by an ordinary arc lamp without a globe. The luminous intensity on the sheet was that given by the arc lamp in the direction making an angle

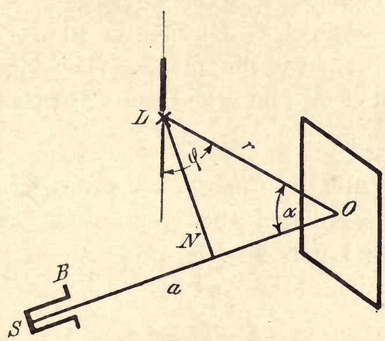


Fig. 34.—Diagram of Arrangements for the Measurement of Reflecting Power.

ϕ with the vertical, and was determined by direct measurement. The distance of the source L from the centre O of the test-sheet being r, and the direction of the luminous ray arriving at O making an angle α with the normal ON to the sheet, the illumination at the centre O is given by—

$$E_0 = \frac{I \cos \alpha}{r^2}.$$

As LN is perpendicular to NO, measurement of the distances LO and NO gives—

$$\cos \alpha = \frac{NO}{LO}.$$

The light reflected from the test-surface was measured by a photometer. The photometer screen was placed parallel to the test surface at a distance a along the normal, as shown in Fig. 34, from which it is also evident that the window B defines a circular area on the test-sheet from which alone luminous flux can reach the photometer screen. The value s of this area can be obtained by direct measurement of the diameter of the circle seen when the eye is placed at S. The surface s receives from the arc lamp a mean illumination E_0, and therefore a luminous flux—

$$\Phi = E_0 s \ (cf.\ \text{p. } 17).$$

To get the luminous flux coming back from the surface s, it must be considered as a source of which the maximum intensity is I_0 normal to the surface. This luminous intensity is proportional to E_0 and s, and can be written—

$$I_0 = \chi E_0 s.$$

The luminous intensity from such a surface element, assuming that the light is radiated wholly on one side of the test-sheet is, according to Lambert's law, proportional to cos α. The law of distribution of reflected light will then be—

$$I_r = I_0 \cos \alpha,$$

and the corresponding distribution curve is a circle of diameter I_0 (p. 65, and Fig. 38, p. 158). The mean hemispherical intensity is for this curve of distribution (Eq. 7, p. 30)—

$$I_{\triangledown} = \int_0^{\frac{\pi}{2}} I_r \sin \alpha \, d\alpha = \int_0^{\frac{\pi}{2}} I_0 \cos \alpha \sin \alpha \, d\alpha = \frac{I_0}{2}.$$

This result can also be seen at once from the normal distribution curve (Fig. 38, p. 158), where—

$$I_0 = 2{,}000, \text{ and } I_{\triangledown} = 1{,}000 \ C.P.$$

The mean spherical intensity is, then, half of the mean hemispherical intensity, as the light is distributed all on one side.

Hence— $$I_\circ = \frac{I_0}{4},$$

and the total luminous flux is—

$$\Phi_r = 4\pi I_\circ = \pi I_0 = \chi\pi E_0 s = \chi\pi\Phi$$

The reflecting power ρ of the surface is expressed as the ratio of luminous flux reflected to luminous flux received, thus—

$$\rho = \frac{\Phi_r}{\Phi} = \chi\pi.$$

I_0 is measured by the photometer, and if p is the photometer reading, and c the photometer constant—

$$I_0 = ca^2 p,$$

and the reflecting power is given by—

$$\rho = \frac{\Phi_r}{\Phi} = \frac{\pi I_0}{E_0 s} = \frac{\pi c a^2 p}{\frac{I \cos \alpha}{r^2} s} = \frac{\pi c a^2 r^2}{I s \cos \alpha} \times p.$$

Multiplying the values of ρ by 100, we get the figures in per cent. The values of the reflecting power of the various surfaces tried, given in the table below, have been arrived at in this way.

TABLE XI.

Kind of Surface.	Reflecting Power. Per Cent.
Coated Surfaces:	
Pure white stone	75·0
Zinc white (pure)	76·0
Limewash	66·5
,, with chrome-yellow (light)	66·5
,, ,, chrome-yellow (dark)	64·5
,, ,, ochre (light)	66·5
,, ,, ochre (dark)	52·5
,, ,, green (light)	66·5
,, ,, green (dark)	57·0
,, ,, ochre and green (light)	55·5
,, ,, ochre and green (dark)	51·5
,, ,, red (light)	63·5
,, ,, red (dark)	50·5
,, ,, blue (light)	60·0
,, ,, blue (dark)	53·0
,, ,, umber (light)	56·0
,, ,, umber (dark)	40·5
Papers:	
White writing-paper	68·0
Yellowish paper	67·0
Yellow paper	60·0

An inspection of the figures brings out the fact that pure white stone or zinc white is a better reflector than white paper. The limewash commonly used on ceilings has, on the contrary, a reflecting power 10 per cent. less than the two first. Usually, the whiteness of limewash is modified to a yellowish, greenish, or bluish tint, as a dead white appears cold and unpleasant. Experiments were therefore made on two samples of each colour, one slightly and the other heavily tinted. The lightly-tinted test-sheets matched the colourings usually met with very well. The experiments with the darker colours were useful, as these are sometimes used. The results show that the reflecting powers of the dark colours are decidedly smaller than those of the corresponding light colours, with the single exception of chrome-yellow. The smallest addition of colour produces some diminution in the reflecting power, except with chrome-yellow, ochre and green, which give the same figure as pure limewash. The ceilings of rooms in which it is intended to use indirect lighting should, therefore, either be white or, at the most, tinted a light yellow or green.

Experiments were also made to find out whether the colour of incident light exercised any sensible effect on reflecting power. For this purpose a mercury vapour lamp in a quartz tube was used instead of the arc lamp, thus replacing a nearly pure white light by one giving chiefly green and yellow rays with only a slight admixture of red. The differences found in the reflecting power were, at the most, 5 per cent. The chrome-yellow surfaces, both dark and light, gave a reflecting power 5 per cent. higher with this light, while the red and blue surfaces, on the other hand, gave values 5 per cent. lower. The differences being so small over so great a range of colour, it may be taken that, with usual sources, the reflecting power is independent of the colour of the light.

From these and other results, the reflection factor ρ can be taken to have the values—

Pure white ceilings (new) . . . $\rho = 0{\cdot}65$
Lightly-tinted ceilings (new) . . $\rho = 0{\cdot}6$
Lightly-tinted ceilings (after long use) $\rho = 0{\cdot}5\text{–}0{\cdot}4$

51. Calculation of Indirect Illumination.

In the nature of things, it would be much more difficult and more complicated to calculate an indirect than a direct illumination—in fact, accurate predetermination has to be abandoned in favour of approximations based on the results of experience. Suitable experimental values for such approximate calculations will be given later, the method followed being similar to that set forth on p. 61, for the calculation of direct illumination.

In the first place, the mean horizontal illumination of the ceiling must be determined, this being the first surface lit by the lamps, and this calculation can be made in a manner that has already been completely dealt with. The total ceiling surface being S, and the distance from sources to ceiling h', $\frac{S}{h'^2}$ can be determined. Table XXI. (p. 155), gives $1-\cos\alpha'$, and Ψ' can then be obtained from the table of flux for the source used—viz., Table XXVII. (p. 167) for arc lamps with the normal carbon arrangement, and Table XXXI. (p. 175), for upright gas-mantles with reflector. The mean upper hemispherical intensity $I_{\triangle}$ must be taken instead of $I_{\triangledown}$. The factor k due to reflection from the walls can be neglected to begin with. The mean horizontal illumination of the ceiling is (Eq. 16, p. 68)—

$$E'_m = \frac{2\pi\Psi'}{S} \cdot \frac{I_{\triangle} z}{1{,}000}.$$

The procedure is then similar to that given on p. 132 in connection with the measurement of reflecting power. Each element of ceiling area dS itself becomes a luminous source, and if the illumination at dS is E', then the maximum luminous intensity of such a source is—

$$dI_0 = XE'dS.$$

The distribution of light is like that given in Fig. 88 (p. 158), and the mean hemispherical intensity of an element of area will be (p. 132)—

$$dI_{\triangle} = \frac{dI_0}{2}.$$

The whole ceiling behaves then as a source of which the mean hemispherical intensity is—

$$I_{\triangledown} = \int \frac{dI_0}{2} = \tfrac{1}{2}\int \chi E' dS = \tfrac{1}{2}\chi E'_m S,$$

the mean horizontal illumination E'_m of the ceiling being defined by the relation—

$$E'_m = \frac{1}{S}\int E' dS \text{ (Eq. 12, p. 48).}$$

$(1 - \cos \alpha)$ can be determined from the surface S and the height h of the ceiling above the plane for which the useful mean horizontal illumination E_m is to be calculated, and the corresponding value of Ψ taken from Table XXIII. (p. 159). The mean horizontal illumination then follows in the same way as above, viz.—

$$E_m = \frac{2\pi\Psi}{S} \frac{I_{\triangledown}}{1{,}000} k = \frac{2\pi\Psi}{S} \cdot \frac{\tfrac{1}{2}\chi E'_m S}{1{,}000} k$$

$$= \chi\pi \frac{\Psi}{1{,}000} E'_m k = \rho \frac{\Psi}{1{,}000} E'_m k$$

as on p. 133 it has been shown that—

$$\chi\pi = \rho,$$

the reflection factor for the ceiling surface.

The factor Ψ depends very greatly on the height of the room, and the factor k on the reflection from the walls, as well as on the increase of illumination due to multiple reflection.

The procedure in these calculations will be made plain by the investigation of two examples, in the course of which it will become evident that it is possible to make approximate calculations of indirect illumination, although these may be more complicated than with direct. More factors are required in order to arrive at correct estimates, such as ρ, the reflection factor for the ceiling, and k, the reflection factor for the walls; further, the mean upper hemispherical intensity of the combination of lamp and reflector used is not often well known, and has to be estimated. These preliminary calculations are useful in cases where there are no practical data to go upon, as some indication is then afforded of the results to be expected before actually testing the installation.

52. Examples on the Calculation of Indirect Illumination.

(1) *Electric Lighting.*

The Commission on Indirect Lighting of Schoolrooms and Drawing Offices, appointed by the German Union of Water and Gas Engineers, carried out some experiments at Munich on a drawing office 156 square metres (1,680 square feet) in area and 4·8 metres (15 feet 9 inches) high, indirectly lit by three continuous current arc lamps with inverted carbons.* The total power taken by the lamps was 2,137 watts. From Table XVI. (p. 149), ordinary arc lamps taking more than 8 amperes give a mean hemispherical intensity of 1,500 *C.P.* per kilowatt, which could be considered in this case with carbons inverted, as mean upper hemispherical intensity. Part of the light sent into the lower hemisphere is returned by a reflector; this light may be taken as an approximate compensation for the somewhat lower efficiency of the lamp with inverted carbons, giving as the mean effective upper hemispherical intensity—

$$I_{\triangle}z = \frac{1{,}500}{1{,}000} \times 2{,}137 = 3{,}205 \ C.P.$$

The lamps were placed 0·75 metre (2 feet 6 inches) below the ceiling, and the illumination was measured at 1 metre (3 feet 3 inches) above the ground. The calculated value at this height is arrived at as follows—

From $S = 156$ square metres and $h' = 0{\cdot}75$ metre,

$$\frac{S}{h'^2} = 278,$$

and—

$$1 - \cos \alpha' = 1 - \frac{1}{\sqrt{1 + \frac{S}{\pi h'^2}}} = 0{\cdot}895.$$

* *Journal für Gasbeleuchtung,* 1904, p. 709, Experiment 11.

From Table XXVII. (p. 167) $\Psi = 958$. The mean horizontal illumination on the ceiling will then be—

$$E'_m = \frac{2\pi \times 958}{156} \times \frac{3{,}205}{1{,}000} = 124 \text{ lux } (11\cdot5 \text{ f.c.}).$$

The height of the ceiling above reference plane is—

$$h = 4\cdot8 - 1 = 3\cdot8 \text{ metres},$$

and—

$$\frac{S}{h^2} = 10\cdot8;$$

whence—

$$1 - \cos\alpha = 0\cdot526 \text{ (Table XXI., p. 155)},$$

and—

$$\Psi = 775 \text{ (Table XXIII., p. 159)}.$$

As the ceiling had been given a wash of pure white just before the experiments, the reflection factor from the ceiling can be taken as $\rho = 0\cdot65$.

The mean horizontal illumination at 1 metre (3 feet 3 inches) above the ground, neglecting the reflection factor for the walls, is—

$$E_m = 0\cdot65 \times 0\cdot775 \times 124 = 62\cdot5 \text{ lux } (5\cdot8 \text{ f.c.}).$$

Direct measurement made by the Munich Commission gave a mean horizontal illumination of 84·4 lux (7·8 f.c.). Hence the reflection factor for the walls, which has hitherto been neglected, is—

$$k = \frac{84\cdot4}{62\cdot5} = 1\cdot35.$$

The values given for this factor on p. 69 for walls of a light tint, with direct lighting, lie between 1·2 and 1·5. The above calculation confirms values of the reflection factor of this order as available also for indirect lighting.

(2) *Lighting by Gas.*

Experiments with indirect lighting by gas under pressure have been described by Schumann,* from whom the following example is taken:

* *Journal für Gasbeleuchtung*, 1907, p. 112.

A classroom in a secondary school in Munich, 77 square metres (828 square feet) in area, and 3·85 metres (12 feet 7·5 inches) high, was lit indirectly by six pressure gas lamps with mantle-burners. The light radiated below was sent to the ceiling by reflectors painted white. The total consumption for the six lamps was 1,470 litres (52 cubic feet) of gas per hour. From Table XV. (p. 148) "Millennium" lamps give a mean spherical intensity of 640 *C.P.*, and a mean lower hemispherical intensity of 570 *C.P.* for a consumption of 1,000 litres (35·3 cubic feet) per hour. The mean upper hemispherical intensity without reflector for this hourly consumption will then be (Eq. 4, p. 22)—

$$I'_{\bigtriangleup} = 2I_0 - I_{\bigtriangledown} = 1{,}280 - 570 = 710 \ C.P.$$

About 40 per cent. of the light sent below is absorbed by the reflector, and 60 per cent. returned to the upper hemisphere. Under these conditions the mean upper hemispherical intensity with reflector becomes—

$$I''_{\bigtriangleup} = 710 + 0{\cdot}6 \times 570 = 1{,}050 \ C.P.$$

The hourly consumption being 1,470 litres, the mean upper hemispherical intensity of the six lamps is—

$$I_{\bigtriangleup}z = \frac{1{,}050}{1{,}000} \times 1{,}470 = 1{,}550 \ C.P.$$

The distribution of light in the upper hemisphere for these lamps fitted with reflectors is almost the same as that of the normal lamp in the lower hemisphere. The normal curve of distribution given for this type can be used to complete the calculation.

The lamps are hung at a height of 2·7 metres above the ground, hence—

$$h' = 3{\cdot}85 - 2{\cdot}7 = 1{\cdot}15 \text{ metres};$$

and it follows that—

$$\frac{S}{h'^2} = \frac{77}{1{\cdot}32} = 58,$$

$$1 - \cos \alpha' = 0{\cdot}774 \text{ (Table XXI., p. 155)},$$

and—

$$\Psi' = 788 \text{ (Table XXXII., p. 177)}.$$

The mean horizontal illumination of the ceiling is then—

$$E'_m = \frac{2\pi \times 788}{77} \times \frac{1,550}{1,000} = 99{\cdot}5 \text{ lux } (9{\cdot}25 \text{ f.c.}).$$

The plane of reference being 0·9 metres above the ground—

$$h = 3{\cdot}85 - 0{\cdot}9 = 2{\cdot}95 \text{ metres},$$

$$\frac{S}{h^2} = \frac{77}{8{\cdot}7} = 8{\cdot}8,$$

$$1 - \cos\alpha = 0{\cdot}488 \text{ (Table XXI., p. 155)},$$

and—

$$\Psi = 738 \text{ (Table XXIII., p. 159)}.$$

This ceiling also had been newly whitewashed immediately before the experiments, so that the factor ρ for the ceiling reflection can be taken as 0·65. Leaving the factor for reflection from the walls out of account, the mean horizontal illumination is—

$$E_m = 0{\cdot}65 \times 0{\cdot}738 \times 99{\cdot}5 = 48 \text{ lux } (4{\cdot}5 \text{ f.c.}).$$

Direct measurement in the room gave 65 lux (6·0 f.c.).

The reflection factor for the walls is then—

$$k = \frac{65}{48} = 1{\cdot}36,$$

a value very nearly the same as before.

53. Measurements and Data for the Approximate Calculation of Indirect Lighting.

Practical data are obviously best derived from measurements made on existing installations. A summary of measurements of indirect illumination made and published by different experimenters, which may be regarded as giving standard figures for this kind of lighting, are therefore collected together in Table XII. (pp. 142, 143). Investigation shows that there is sufficient agreement among these figures to warrant their use in the preparation of new schemes, the most important values being those of the economic coefficient (consumption per lux and square metre) and the ground area per lamp. The measurements in this table coming from the Berliner Elektrizitäts-Werke give more unfavourable values for the consumption per lux and square metre than the others, because the colouring of the walls in the rooms that were tested was unsuitable for indirect lighting.

Limiting values of the economic coefficient for different methods of indirect lighting are given in Table XIII. (p. 144), deduced from the figures already quoted, and from a large number of other experimental results. By means of these limiting values and Tables XVII., XVIII., and XIX. (pp. 150-152), which give figures for the illumination necessary in the lighting of interiors according to the use to be made of them, simple and rapid approximate calculations of indirect illumination can be made in the same way as has been indicated (p. 95) for direct lighting.

The smaller values of the economic coefficient quoted in Table XIII. apply to freshly-coated ceilings. As the colouring gradually becomes darker with time, an allowance must be made for this in the preparation of lighting estimates, with the object of securing a sufficient illumination after lapse of time has diminished the reflecting power of the ceiling. The lower values given in the Table should not therefore be used, but numbers approximating to the mean. In a well-kept installation the ceilings should be white-washed at least every three years, if electric light is used, and every two years if gas light is used.

When the total consumption for the lighting of the room has been arrived at by means of the economic coefficient, the approximate number of lamps necessary can be determined from the values given in Table XIII. for the normal consumption per lamp. The greater the number of lamps, the more uniform will be the lighting. The numbers given in Table XII., referring to the area per lamp, could also serve as starting-points in estimating.

Complete tables relating to the number and intensity of

TABLE XII.

EXPERIMENTAL RESULTS ON THE INDIRECT LIGHTING OF SCHOOLROOMS AND DRAWING OFFICES (METRIC UNITS).

Kind of Lamp Used.	Measurements made by—	No. of Lamps.	Consumption per Lamp.	Height of Suspension (Metres).	Height of Reference Plane (Metres).	Height of Room (Metres).	Floor Area of Room (Sq. M.).	Floor Area per Lamp (Sq. M.).	Consumption per Sq. M.	Consumption per Lux and Sq. M.	Horizontal Illumination in Lux. E_{mean}	E_{max}	E_{min}	$\frac{E_{max}}{E_{min}}$
			Watts.						Watts.	Watts.				
Continuous current arc lamps with normal arrangement of carbons	Munich Commission *	3	770	4	1	4·8	156	52	14·8	0·24	61	74	49	1·5
	Uppenborn, Munich †	2	550	3	—	4	72	36	15·3	0·27	56	75	38·5	1·95
	Lehmann-Richter, Nürnberg ‡	2	605	—	—	—	95	47·5	12·8	0·207	61	—	—	—
	B.E.W. Testing Station	4	540	3	1·2	3·6	152	38	14·2	0·325 ‖	43	58·5	26·5	2·2
	B.E.W. Testing Station	2	550	2·9	1·2	3·5	72	36	15·3	0·333 ‖	45·5	63	26·5	2·4
Continuous current arc lamps with inverted carbons	Munich Commission *	3	735	4	1	4·8	156	52	14·1	0·150	93·5	120	72	1·67
	Munich Commission *	3	715	4	1	4·8	156	52	13·7	0·161	84·5	104	68	1·54
	B.E.W. Testing Station	2	650	2·5	1·2	3·5	72	36	18·0	0·242 ‖	74	119	39	3·0
			Litres per hr.						Litres per hr.	Litres per hr.				
Mantle gas-burners at ordinary pressure	Munich Commission *	52	110	4	1	4·8	156	3	36·8	0·455	79·5	93	63·5	1·45
	Lehmann-Richter, Nürnberg ‡	14	119	—	—	—	95	6·8	17·6	0·395	44	—	—	—
Mantle gas-burners at higher pressure ("Selas Light")	Munich Commission *	10	375	4	1	4·8	156	15·6	24·0	0·314 ‖	75·5	85·5	66	1·30
Mantle gas-burners at higher pressure ("Millennium Light")	Munich Commission *	8	480	4	1	4·8	156	19·5	24·6	0·33	74	91	62·5	1·45
	Schumann, Munich §	6	245	2·7	0·9	3·85	77	12·8	19·1	0·289	65	72	57	1·27

* *Journal für Gasbeleuchtung*, 1904, pp. 713-715.
† *Elektrotechnische Zeitschrift*, 1906, p. 360.
‡ *Journal für Gasbeleuchtung*, 1904, p. 349.
§ *Ibid.*, 1907, p. 113.
‖ Higher values than normal, as the ceiling colouring was unsuitable for indirect lighting.

TABLE XII.—*Continued.*

EXPERIMENTAL RESULTS ON THE INDIRECT LIGHTING OF SCHOOLROOMS AND DRAWING OFFICES (ENGLISH UNITS).

Kind of Lamp Used.	Measurements made by—	No. of Lamps.	Consumption per Lamp.	Height of Suspension (Feet).	Height of Reference Plane (Feet).	Height of Room (Feet).	Floor Area of Room (Sq. Ft.).	Floor Area per Lamp (Sq. Feet.).	Consumption per Sq. Ft.	Consumption per F.C. and Sq. Ft.	Horizontal Illumination in Foot Candles. E_{mean}.	E_{max}.	E_{min}.	$\frac{E_{max}}{E_{min}}$.
			Watts.						Watts.	Watts.				
Continuous current arc lamps with normal arrangement of carbons	Munich Commission*	3	770	13·1	3·28	15·75	1,678	559	1·375	0·24	5·7	6·9	4·55	1·5
	Uppenborn, Munich†	2	550	9·8	—	13·1	775	387	1·42	0·27	5·25	6·95	3·6	1·95
	Lehmann-Richter, Nürnberg‡	2	605	—	—	—	1,022	511	1·19	0·207	5·7	—	—	—
	B.E.W. Testing Station	4	540	9·8	3·93	11·8	1,635	409	1·32	0·325‖	4·0	5·45	2·45	2·2
	B.E.W. Testing Station	2	550	9·5	3·93	11·48	775	387	1·42	0·333‖	4·25	5·85	2·45	2·4
Continuous current arc lamps with inverted carbons	Munich Commission*	3	735	13·1	3·28	15·75	1,678	559	1·31	0·150	8·7	11·15	6·7	1·67
	Munich Commission*	3	715	13·1	3·28	15·75	1,678	559	1·275	0·161	7·85	9·65	6·3	1·54
	B.E.W. Testing Station	2	650	8·2	3·93	11·48	775	387	1·675	0·242‖	6·9	11·05	3·6	3·0
			Cubic ft. per hr.						Cubic ft. per hr.	Cubic ft. per hr.				
Mantle gas-burners at ordinary pressure	Munich Commission*	52	3·88	13·1	3·28	15·75	1,678	32·3	0·121	0·016	7·4	8·65	5·9	1·45
	Lehmann-Richter, Nürnberg‡	14	4·20	—	—	—	1,022	73	0·058	0·014	4·1	—	—	—
Mantle gas-burners at higher pressure ("Selas Light")	Munich Commission*	10	13·2	13·1	3·28	15·75	1,678	168	0·079	0·011	7·0	7·95	6·1	1·30
Mantle gas-burners at higher pressure ("Millennium Light")	Munich Commission*	8	16·9	13·1	3·28	15·75	1,678	210	0·081	0·0115	6·9	8·45	5·8	1·45
	Schumann, Munich§	6	8·65	8·85	2·95	12·6	828	138	0·063	0·010	6·05	6·7	5·3	1·27

* *Journal für Gasbeleuchtung*, 1904, pp. 713-715.
† *Elektrotechnische Zeitschrift*, 1906, p. 360.
‡ *Journal für Gasbeleuchtung*, 1904, p. 349.
§ *Ibid.*, 1907, p. 113.
‖ Higher values than normal, as the ceiling colouring was unsuitable for indirect lighting.

TABLE XIII.

PRACTICAL VALUES FOR THE ECONOMIC COEFFICIENT IN INDIRECT LIGHTING.

Lighting by Means of—	Normal Consumption per Lamp.	Consumption per Lux and Square Metre (per f.c. and sq. ft).
Continuous current arc lamp with normal arrangement of carbons	400–900 watts	0·22–0·39 watt
Continuous current arc lamp with inverted carbons	400–900 watts	0·16–0·28 watt
Ordinary gas-mantles	100–180 litres per hour 3·5–6·5 cubic feet per hour	0·39–0·56 litre per hour 0·014 – 0·02 cubic foot per hour
Mantles using high-pressure gas "Selas" and "Millennium" light	200–500 litres per hour 7–17 cubic feet per hour	0·28–0·44 litre per hour 0·1–0·15 cubic foot per hour

lamps used in indirect lighting by electricity and gas have been published.*

An example of the rough estimation of illumination by indirect lighting is as follows:

A drawing office of 100 square metres (1,075 square feet) area is to be lit by ordinary arc lamps, and is to have a mean horizontal illumination of 70 lux (6·5 f.c.). The total power and the number of lamps is to be determined.

In Table XIII. the consumption per lux and square metre for the type of lamp to be used is given as a mean value of 0·31 watt. The total power taken to light the room will then be—

$$W = \sigma E_m S = 0{\cdot}31 \times 70 \times 100 = 2{,}170 \text{ watts.}$$

* Uppenborn, *Deutscher Kalender für Elektrotechniker*, 1907, pp. 306, 308; Hogner, "Lichstrahlung und Beleuchtung," Tables XXII. and XXIII., p. 50; Schaars, *Kalender für das Gas und Wasserfach*, 1907, p. 161.

Four lamps might be used, each lamp taking 550 watts, or 10 amperes at 55 volts, thus giving two in series on a 110 volt circuit.

54. Comparison of Different Kinds of Lighting.

A comparison of different methods of indirect lighting can also be made on the basis of the economic coefficient and the price per unit of the power supply. The procedure is the same as with direct lighting (p. 39). In so far as establishment charges cannot be assumed as approximately the same, they must be taken into account in making comparisons, their effect on the result often being very great. The Munich Commission of the German Union of Gas and Water Engineers (p. 142), in an addendum to their report on indirect lighting, has made comparisons of cost between different systems.* The numerical results obtained are naturally very variable, as the prices of units of gas and electricity are very different, as are also the conditions of use.

If the values of the economic coefficient for indirect lighting (Table XIII., p. 144) are compared with those of direct lighting for sources of the same type (Table XX., p. 153), it will be found that they are not very different. It would appear, at first sight, an extraordinary claim to make that indirect could be as economical as direct lighting, because of the obvious losses by absorption in reflectors and ceilings. There are, however, absorption losses in direct lighting due to globes and reflectors, although on a somewhat smaller scale. Further, the total light in direct schemes is not made use of in the same proportion as in indirect lighting, the reflecting properties of walls and ceiling not being usually so good, thereby resulting in the loss of a considerable part of the light.

The result is that for interiors the energy consumption is not essentially different for the two systems using lamps of the same type. Evidently, in particular cases, as, for instance,

* *Journal für Gasbeleuchtung,* 1904, pp. 720-722.

10

when a concentrated light is desired on a table, and the illumination of the rest of the room is not important, direct lighting is much more economical than any indirect system could be.

55. Semi-Indirect Lighting.

It often happens that it is neither necessary nor agreeable to light a room without producing any shadows. The visibility of the source may also not be of great importance, although an illumination as uniform as possible, without too heavy shadows, is desired; resort can then be had to semi-indirect lighting. The opaque reflectors of the indirect method are replaced by others made of opal or frosted glass. Some portion of the light then passes directly below, while a great part is sent to the ceiling and distributed as before. In general, a more favourable performance can be obtained in this way with arc lamps having the normal arrangement of carbons, the consumption per lux and square metre being about 15 to 25 per cent. less than with indirect lighting. Taking account of this difference, numbers in Table XIII. could still serve as a basis for the calculation of such an illumination.

APPENDIX

ENERGY CONSUMPTION AND YIELD OF LIGHT FOR COMMERCIAL SOURCES.

TABLE XIV.

LAMPS USING LIQUID COMBUSTIBLE.

METRIC UNITS.

Kind of Lamp.	Normal Consumption per Lamp in Cubic Centimetres per Hour.	Normal Economic Coefficient in Cubic Centimetres per Candle Hour of Horizontal Intensity.	Mean Values.					
			Cubic Centimetres per Candle Hour.			Intensity per Litre per Hour.		
			Horizontal.	Spherical.	Hemispherical.	$I_{hor.}$	I_0.	$I_{\triangledown}$.
Petroleum lamps	50–150	5·0–3·3	3·9	4·4	6·1	260	225	160
Petroleum mantle lamps	50–100	1·7–1·1	1·3	1·8	2·3	750	560	430
Alcohol mantle lamps	50–150	2·8–1·7	2·1	3·1	4·1	480	320	240

ENGLISH UNITS.

Kind of Lamp.	Normal Consumption per Lamp in Gallons per Hour.	Normal Economic Coefficient in Cubic Inches per Candle Hour of Horizontal Intensity.	Mean Values.					
			Cubic Inches per Candle Hour.			Intensity per Gallon per Hour.		
			Horizontal.	Spherical.	Hemispherical.	$I_{hor.}$	I_0	$I_{\triangledown}$.
Petroleum lamps	0·01–0·03	0·30–0·20	0·24	0·27	0·37	1,180	1,020	730
Petroleum mantle lamps	0·01–0·02	0·10–0·07	0·08	0·11	0·14	3,400	2,550	1,950
Alcohol mantle lamps	0·01–0·03	0·17–0·10	0·13	0·19	0·25	2,180	1,450	1,090

REMARKS ON TABLES XIV., XV., XVI.

1. All the figures given for intensity refer to lamps without globe or reflector. For the effect of these on the intensity, see p. 42.

2. The data on the consumption of electric arc lamps and vapour lamps are given for operation at normal voltage, and include losses in control resistances.

3. The yield of light from alternating current arc lamps is from 25–50 per cent. less on the mean hemispherical intensity, and from 20–40 per cent. less on the mean spherical intensity than with continuous current. Inversely, the economic coefficient is greater with alternating current arc lamps than with the corresponding continuous current arc lamps.

TABLE XV.*

LAMPS USING GASEOUS COMBUSTIBLE.

Metric Units.

Kind of Lamp.	Normal Consumption per Lamp in Litres per Hour.	Normal Economic Coefficient in Litres per Candle of Horizontal Intensity.	Mean Values. Litres per Candle Hour.			Mean Values. Intensity per 1,000 Litres per Hour.		
			Horizontal.	Spherical.	Hemispherical.	$I_{hor.}$	I_o.	$I_{\bigtriangledown}$.
Two-hole acetylene burner	20–60	1·0–0·7	0·8	1·2	1·3	1,290	820	750
Lighting Gas at Normal Pressure.								
Slotted burner (Sugg)	140–500	13–9	11	18	19	90	57	53
Argand burner	100–500	11–8	9	12	14	112	81	72
Upright gas-mantle	80–180	1·8–1·3	1·5	2·1	2·4	640	480	410
Inverted gas-mantle	60–120	2·0–0 9	1·4	1 7	1·4	690	600	690
Lucas Light ...	250–650	1·3–1·1	1·2	1·7	1·9	820	600	530
Pressure Gas.								
Millennium Light and Pharos Light	200–1,200	1·3–0 9	1·1	1·5	1·8	900	640	570
Selas Light ...	200–1,500	1·1–0·9	1·0	1·3	1·5	1,000	750	640

English Units.

Kind of Lamp.	Normal Consumption per Lamp in Cubic Feet per Hour.	Normal Economic Coefficient in Cubic Feet per Candle of Horizontal Intensity.	Mean Values. Cubic Feet per Candle Hour.			Mean Values. Intensity per 10 Cubic Feet per Hr.		
			Horizontal.	Spherical.	Hemispherical.	I hor.	I_o.	$I_{\bigtriangledown}$.
Two-hole acetylene burner	0·7–2·1	0·035–0·025	0·028	0·042	0·045	365	230	210
Lighting Gas at Normal Pressure.								
Slotted burner (Sugg)	5·0–17·5	0·45–0 3	0·4	0·65	0·67	25	16	15
Argand burner ...	3·5–17·5	0·4–0 28	0·3	0·4	0·5	32	23	21
Upright gas-mantle	2·8–6·4	0·065-0·045	0·05	0·075	0·085	180	135	115
Inverted gas-mantle	2·1–4·2	0·07–0·03	0·05	0·06	0·05	195	170	195
Lucas Light ...	8·8-23	0·045–0·04	0·04	0·06	0·07	230	170	150
Pressure Gas.								
Millennium Light and Pharos Light	7–42	0·045–0·03	0·04	0·05	0·065	255	180	160
Selas Light ...	7–52	0·04–0·03	0·035	0·045	0·05	285	215	180

* See Remarks on p. 147.

TABLE XVI.*

ELECTRIC LAMPS.

Kind of Lamp.	Normal Consumption per Lamp in Watts.	Normal Economic Coefficient in Watts per Candle of Mean Horizontal Intensity.	Mean Values.					
			Watts per Candle.			Intensity in Candles per Kilowatt.		
			Horizontal.	Spherical.	Hemispherical.	I_{hor}.	$I_{\circ}$.	$I_{\cup}$.
Glow Lamps:								
Carbon filament	20–350	4·5–3·5	3·7	4·7	4·4	270	220	225
Metallized carbon filament ...	10–100	2·8–2·2	2·4	3·1	3·0	405	325	335
Nernst lamp	50–200	2·0–1·7	2·0	2·7	2·2	530	380	450
Tantalum lamp	20–80	2·0–1·8	2·0	2·4	2·3	530	405	430
Osmium lamp	25–50	1·8–1·7	1·7	2·1	2·0	605	470	495
Tungsten and Osram lamp ...	20–400	1·4–1·1	1·2	1·6	1·4	820	640	695
Vapour Lamps:		Watts per Candle of Maximum Intensity.						
Mercury in ordinary glass ...	200–500	1·0–0·8	0·9	1·1	1·0	1,125	900	1,000
Mercury in quartz glass ...	300–900	0·45–0·28	0·4	0·7	0·45	2,565	1,485	2,250
Continuous Current Arc Lamps:		Watts per Candle of Hemispherical Intensity.						
Ordinary arc lamps under 8 amperes	200–450	1·1–0·8	—	1·7	0·95	—	605	1,060
Ordinary arc lamps over 8 amperes	450–1,500	0·8–0·55	—	1·2	0·7	—	820	1,500
Miniature arc lamps	150–350	1·6–1·1	—	2·0	1·2	—	530	820
Enclosed arc lamps	400–1,000	1·3–1·0	—	1·8	1·1	—	560	900
Flame arc lamps with vertical impregnated carbons ...	300–1,000	0·5–0·3	—	0·85	0·45	—	1,195	2,250
Flame arc lamps with inclined impregnated carbons ...	300–1,100	0·3–0·2	—	0·45	0·22	—	2,300	4,500
Flame arc lamps with inclined ordinary carbons	600–1,200	1·2–0·9	—	1·8	1·0	—	560	990

* See Remarks on p. 147.

TABLE XVII.

WORKING VALUES FOR MEAN HORIZONTAL ILLUMINATION IN STREET LIGHTING.

Type of Street.	Mean Horizontal Illumination 1·5 Metre (4 Feet 11 Inches) above the Ground.	
	Lux.	Foot Candles.
Side streets with little traffic ...	0·5–1·0	0·05–0·1
Side streets with heavier traffic	1·5–3·0	0·15–0·3
Main roads	3·0–6·0	0·3 –0·6
Centres of heavy traffic ...	10·0–30·0	1·0 –3·0

TABLE XVIII.

WORKING VALUES FOR MEAN HORIZONTAL ILLUMINATION IN INTERIORS.

Type of Interior.	Mean Horizontal Illumination 1·0 Metre (3 Feet 3·5 Inches) above Floor Level.	
	Lux.	Foot Candles.
Bedrooms and corridors ...	5–10	0·5–1·0
Ordinary living rooms and hotels	10–20	1·0–2·0
Hotel lighting in special cases	20–30	2·0–3·0
Offices, sale-rooms, school and lecture rooms, restaurants ...	25–50	2·5–5·0

TABLE XIX.

[WORKING VALUES FOR HORIZONTAL ILLUMINATION IN FACTORIES AND WORKSHOPS.

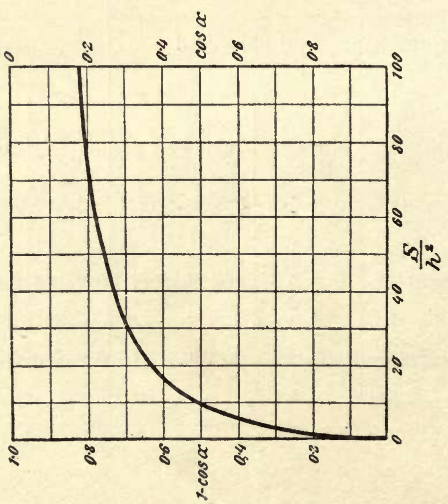
Nature of Work.	Kind of Lighting.	How Modified.	Number of Observations.	Horizontal Illumination. Lux. Min.	Lux. Max.	Lux. Mean.	Foot Candles. Min.	Foot Candles. Max.	Foot Candles. Mean.
Cotton-weaving	Carbon electric glow lamps	—	86	14·3	21·3	17·7	1·33	1·98	1·65
	Ordinary gas-jets	—	4	5·2	6·45	5·7	0·48	0·60	0·53
	Upright gas-mantles	—	54	6·8	17·2	11·6	0·63	1·60	1·08
	Inverted gas-mantles	—	42	18·2	36·6	26·9	1·69	3·4	2·5
	High-pressure gas-mantles (Selas Light)	—	12	19·9	61·8	39·3	1·85	5·75	3·65
Linen-weaving	Metallic electric glow lamps	—	39	22·5	35·0	27·9	2·09	3·25	2·59
	High-pressure gas-mantles (Selas Light)	—	18	67·8	269	141	6·3	25·0	13·1
Cotton-spinning, all operations	Carbon electric glow lamps	—	130	1·65	8·93	4·4	0·153	0·83	0·41
	Metallic electric glow lamps	—	3	7·75	21·5	14	0·72	2·0	1·3
	Ordinary gas-jets	—	8	0·64	6·7	3·0	0·059	0·62	0·28
	Inverted gas-mantles	—	27	6·7	47·3	23·7	0·62	4·4	2·2
Flax spinning and preparing	Carbon electric glow lamps	—	31	2·26	6·7	4·3	0·21	0·62	0·40
	Metallic electric glow lamps	—	29	19·7	30·7	23·8	1·83	2·85	2·21
Composing-rooms in letterpress printing-works	Carbon electric glow lamps	With reflectors	6	40·9	107·5	62·4	3·8	10·0	5·8
	Carbon electric glow lamps	Partially shaded	40	19·4	47·3	34·4	1·8	4·4	3·2
	Metallic electric glow lamps	Opaque shades	26	51·1	115	78·5	4·75	10·7	7·3
	Inverted electric arc lamps	Opaque shades	6	52·7	172	100	4·9	16·0	9·3
	Mercury vapour lamps	—	2	—	177	—	—	16·5	—
	Upright gas-mantles	Partially shaded	44	34·4	62·4	44·1	3·2	5·8	4·1
	Inverted gas-mantles	Opaque shades	10	59	215	120·5	5·4	20·0	11·2
	High-pressure gas-mantles (Keith Light)	Partially shaded	22	51·6	94·7	65·6	4·8	8·8	6·1
	High-pressure gas-mantles (Keith Light)	Opaque shades	18	63·5	239	134	5·9	22·2	12·5

The figures given in this table are taken from the Annual Report of the Chief Inspector of Factories for Great Britain and Ireland for the year 1911. All measurements were taken on a horizontal plane at a height of 3 to 3·5 feet from the floor, with the exception of those in composing-rooms, where the measurements were made on the lower edge of the lower case.]

TABLE XX.

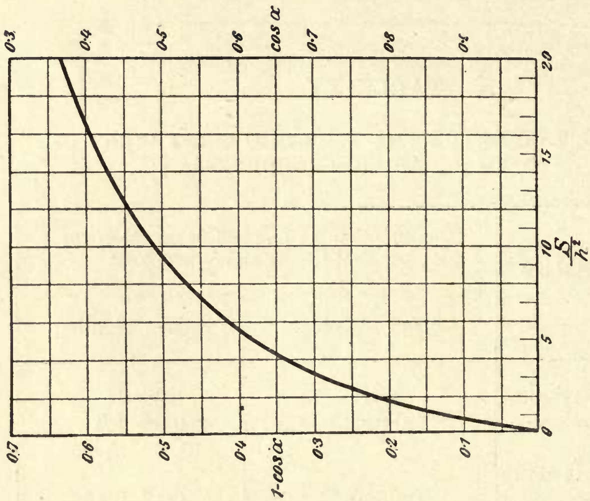
WORKING VALUES FOR THE ECONOMIC COEFFICIENT OF COMMERCIAL LIGHT SOURCES.

Lighting carried out with—	Consumption per Lux and Square Metre or per Foot Candle and Square Foot.			
	Street Lighting.		Interior Lighting.	
	Watts.		Watts.	
Carbon electric glow lamps	0·9–1·3		0·55–1·3	
Metallic electric glow lamps ...	0·28–0·45		0·17–0·45	
Ordinary continuous current arc lamps	0·17–0 28		0·17–0·33	
Continuous current flame arc lamps ...	0·055–0·13		—	
	Litres per Hour.	Cubic Feet per Hour.	Litres per Hour.	Cubic Feet per Hour.
Upright gas-mantles	0·4–0·6	0·014–0·021	0·3–0·6	0·0105–0·021
High-pressure upright gas-mantles	0·3–0·6	0·0105–0·021	0·2–0·4	0·007–0·014
Inverted gas-mantles	0·25–0·45	0·009–0·016	0·2–0·4	0·007–0·014



Curve for $1 - \cos\alpha = 1 - \dfrac{1}{\sqrt{1 + \dfrac{S}{\pi h^2}}}$

FIG. 35.



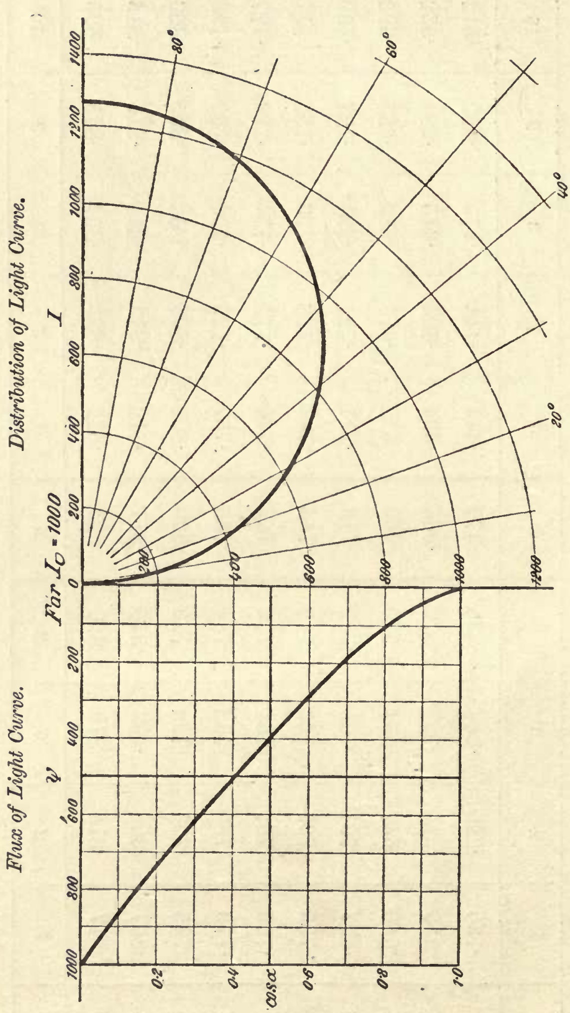
Curve for $1 - \cos\alpha = 1 - \dfrac{1}{\sqrt{1 + \dfrac{S}{\pi h^2}}}$.

FIG. 36.

TABLE XXI.

VALUES OF $1 - \cos \alpha = 1 - \frac{1}{\sqrt{1 + \frac{S}{\pi h^2}}}$.

$\frac{S}{h^2}$	0	1	2	3		5	6	7	8	9	10	$\frac{S}{h^2}$
0	0	·130	·220	·284	·336	·379	·414	·444	·470	·492	·512	0
10	·512	·530	·545	·559	·572	·583	·594	·605	·615	·624	·632	10
20	·632	·640	·647	·654	·660	·666	·672	·677	·682	·687	·692	20
30	·692	·697	·701	·705	·709	·713	·717	·721	·724	·727	·730	30
40	·730	·733	·736	·739	·742	·744	·747	·750	·753	·755	·757	40
50	·757	·760	·762	·764	·766	·768	·770	·772	·774	·776	·777	50
60	·777	·779	·780	·782	·784	·785	·787	·789	·790	·792	·793	60
70	·793	·795	·796	·798	·799	·800	·802	·803	·804	·805	·806	70
80	·806	·807	·808	·809	·810	·811	·812	·813	·814	·815	·816	80
90	·816	·817	·818	·819	·820	·821	·822	·823	·824	·825	·825	90
	0	1	2	3	4	5	6	7	8	9	10	



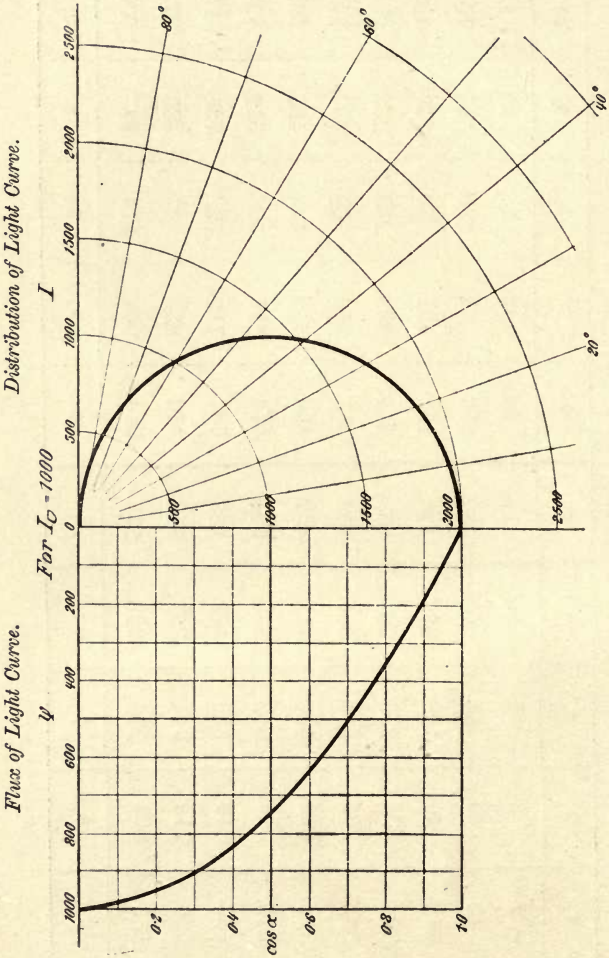
$I = I_{90} \sin \alpha.$

FIG. 37.

TABLE XXII.

LIGHT FLUX VALUES FOR $I = I_{90} \sin \alpha$.

$1 - \cos \alpha$	0	1	2	3	4	5	6	7	8	9	10	
0	0	1	3	6	10	14	18	22	27	32	37	0·9
0·1	37	43	49	55	62	69	76	83	90	97	104	0·8
0·2	104	112	120	128	136	144	152	161	170	179	188	0·7
0·3	188	197	206	215	224	233	243	253	263	273	283	0·6
0·4	283	293	303	314	325	336	347	358	369	380	391	0·5
0·5	391	402	413	424	435	447	458	469	481	493	505	0·4
0·6	505	516	528	540	552	564	576	588	600	612	624	0·3
0·7	624	636	648	660	672	685	697	710	722	734	747	0·2
0·8	747	759	772	784	797	810	822	835	847	860	873	0·1
0·9	873	885	898	910	923	936	948	961	974	987	1,000	0
	10	9	8	7	6	5	4	3	2	1	0	$\cos \alpha$



$I = I_0 \cos \alpha.$

Fig. 38.

TABLE XXIII.

LIGHT FLUX VALUES FOR $I = I_0 \cos \alpha$.

$1 - \cos \alpha$	0	1	2	3	4	5	6	7	8	9	10	
0	0	20	40	59	78	97	116	135	154	172	190	0·9
0 1	190	208	226	243	260	277	294	311	328	344	360	0·8
0·2	360	376	392	407	422	437	452	467	482	496	510	0·7
0·3	510	524	538	551	564	577	590	603	616	628	640	0·6
0·4	640	652	664	675	686	697	708	719	730	740	750	0·5
0·5	750	760	770	779	788	797	806	815	824	832	840	0·4
0·6	840	848	856	863	870	877	884	891	898	904	910	0·3
0·7	910	916	922	927	932	937	942	947	952	956	960	0·2
0·8	960	964	968	971	974	977	980	983	986	988	990	0·1
0·9	990	992	994	995	996	997	998	999	1,000	1,000	1,000	0
	10	9	8	7	6	5	4	3	2	1	0	$\cos \alpha$

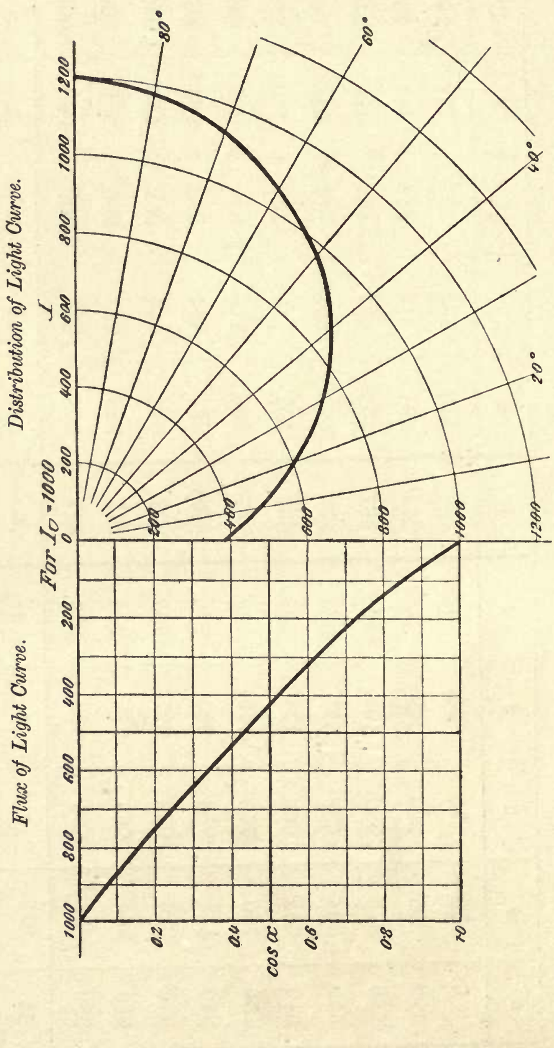


Fig. 39.—Carbon Filament and Metal Filament Glow Lamps.

TABLE XXIV.

LIGHT FLUX VALUES FOR CARBON FILAMENT AND METAL FILAMENT GLOW LAMPS.

1—cos α	0	1	2	3	4	5	6	7		9	10	
0	0	4	8	13	18	24	30	36	42	49	56	0·9
0·1	56	63	70	77	85	93	101	109	117	125	133	0·8
0·2	133	141	149	158	167	176	185	194	203	212	221	0·7
0·3	221	230	239	249	259	269	279	289	299	309	319	0·6
0·4	319	329	339	349	359	370	380	390	400	411	422	0·5
0·5	422	432	442	453	464	475	486	497	508	519	530	0·4
0·6	530	541	552	563	574	586	597	608	619	631	643	0·3
0·7	643	654	665	677	689	701	712	724	736	748	760	0·2
0·8	760	771	783	795	807	819	831	843	855	867	879	0·1
0·9	879	891	903	915	927	939	951	963	975	987	1,000	0
	10	9	8	7	6	5	4	3	2	1	0	cos α

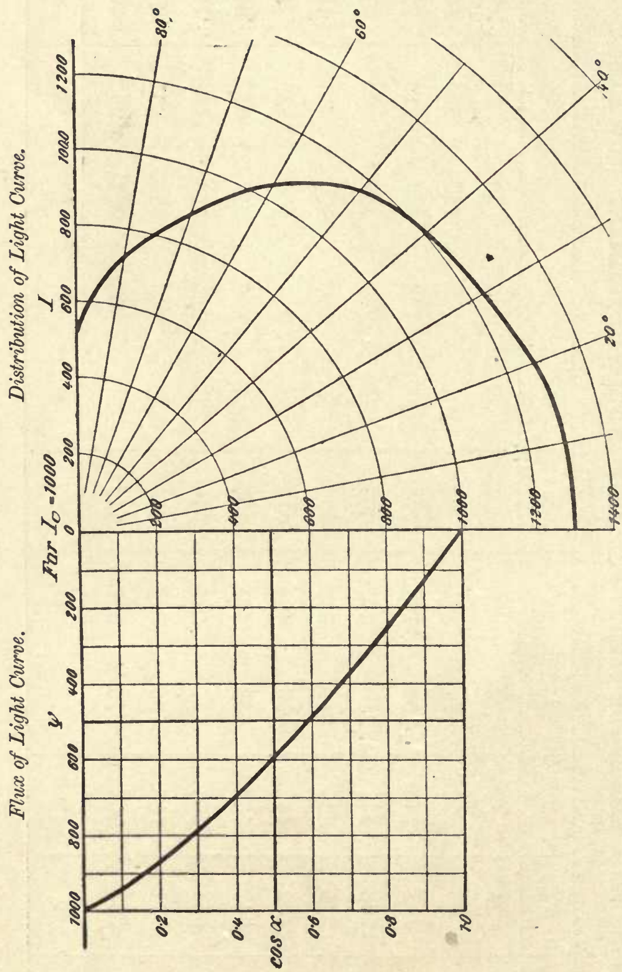


FIG. 40.—CARBON FILAMENT AND METAL FILAMENT GLOW LAMPS, WITH REFLECTORS AND HOLOPHANE GLOBES.

TABLE XXV.

LIGHT FLUX VALUES FOR CARBON FILAMENT AND METAL FILAMENT GLOW LAMPS, WITH REFLECTORS AND HOLOPHANE GLOBES.

– cos	0	1	2	3	4	5	6	7	8	9	10	
0	0	13	26	39	52	65	78	91	104	116	128	0·9
0·1	128	141	154	166	178	190	203	215	227	239	251	0·8
0·2	251	264	276	288	300	312	324	336	348	360	372	0·7
0·3	372	384	396	408	420	431	443	455	466	477	488	0·6
0·4	488	500	511	522	533	544	555	566	577	588	598	0·5
0·5	598	609	619	629	639	649	659	669	679	689	698	0·4
0·6	698	708	718	727	736	745	754	763	772	781	789	0·3
0·7	789	798	806	814	822	830	838	846	854	862	869	0·2
0·8	869	877	885	892	899	906	913	920	927	934	940	0·1
0·9	940	947	954	960	966	972	978	984	990	995	1,000	0
	10	9	8	7	6	5	4	3	2	1	0	cos α

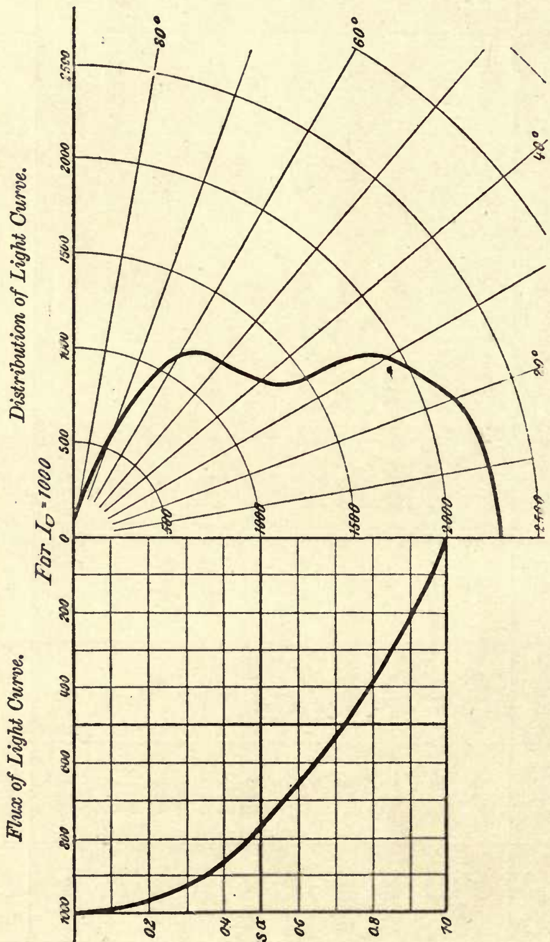


FIG. 41.—CARBON FILAMENT AND METAL FILAMENT GLOW LAMPS, WITH REFLECTORS PRODUCING GREAT CONCENTRATION OF LIGHT DOWNWARDS.

TABLE XXVI.

LIGHT FLUX VALUES FOR CARBON FILAMENT AND METAL FILAMENT GLOW LAMPS, WITH REFLECTORS PRODUCING GREAT CONCENTRATION OF LIGHT DOWNWARDS.

$1-\cos\alpha$	0	1	2	3	4	5	6	7	8	9	10	
0	0	23	45	67	89	111	133	155	176	197	217	0·9
0·1	217	237	257	277	296	315	334	352	369	384	398	0·8
·2	398	411	424	437	450	463	476	489	502	515	527	0·7
0·3	527	540	553	565	577	589	602	614	626	638	650	0·6
0·4	650	662	674	686	698	710	722	734	745	756	767	0·5
0·5	767	778	789	800	810	820	830	840	849	858	866	0·4
0·6	866	874	882	889	896	902	908	914	919	924	929	0·3
0·7	929	934	939	943	947	951	955	958	961	964	967	0·2
0·8	967	970	972	974	976	978	980	982	984	985	986	0·1
0·	986	988	990	992	993	994	996	997	998	999	1,000	0
	10	9	8	7	6	5	4	3	2	1	0	$\cos\alpha$

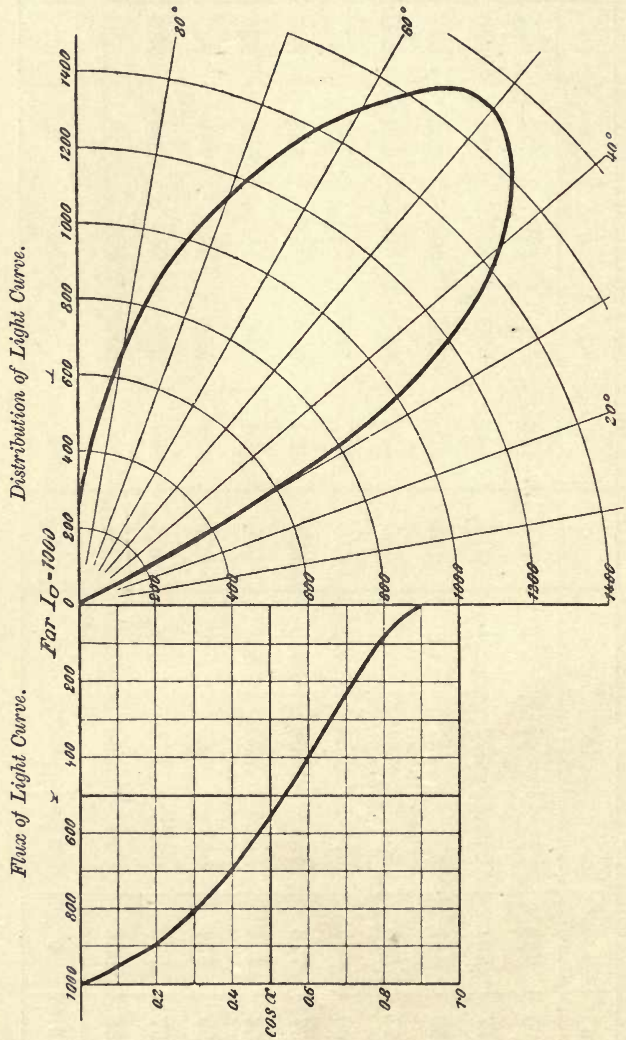


Fig. 42.—Ordinary Continuous Current Arc Lamps without Globes, or with New Clear Glass Globes.

TABLE XXVII.

LIGHT FLUX VALUES FOR ORDINARY CONTINUOUS CURRENT ARC LAMPS WITHOUT GLOBES, OR WITH NEW CLEAR GLASS GLOBES.

1 − cos α	0	1	2	3	4	5	6	7	8	9	10	
0	0	0	0	1	1	1	2	2	3	3	4	0·9
0·1	4	9	15	22	29	37	46	56	67	79	92	0·8
0·2	92	105	119	133	147	162	177	193	209	225	241	0·7
0·3	241	257	273	289	306	323	340	357	374	391	408	0·6
0·4	408	425	441	457	473	489	505	520	535	550	565	0·5
0·5	565	580	594	608	622	636	650	663	676	689	702	0·4
0·6	702	715	727	739	751	762	773	784	795	805	815	0·3
0·7	815	825	834	843	852	861	870	878	886	894	901	0·2
0·8	901	908	915	922	928	934	940	946	951	956	961	0·1
0·9	961	966	971	975	979	983	987	991	994	997	1,000	0
	10	9	8	7	6	5	4	3	2	1	0	cos α

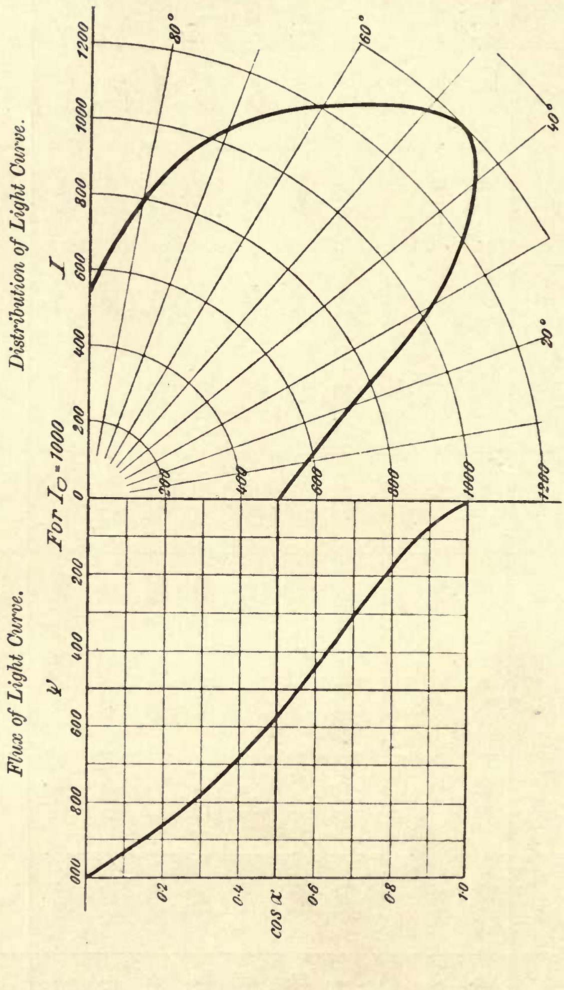


FIG. 43.—ORDINARY AND FLAME ARC LAMPS, WITH OPAL GLASS OR ALABASTER GLOBES FOR CONTINUOUS CURRENT, AND WITH REFLECTORS FOR ALTERNATING CURRENT.

TABLE XXVIII.

LIGHT FLUX VALUES FOR ORDINARY AND FLAME ARC LAMPS, WITH OPAL GLASS OR ALABASTER GLOBES FOR CONTINUOUS CURRENT, AND WITH REFLECTORS FOR ALTERNATING CURRENT.

1—cos α	0	1	2	3	4	5	6	7	8	9	10	
0	0	5	10	15	21	28	35	43	51	59	68	0·9
0·1	68	77	87	97	107	118	129	140	152	164	177	0·8
0·2	177	190	203	216	229	242	255	268	281	295	309	0·7
0·3	309	323	337	350	363	376	390	403	416	429	442	0·6
0·4	442	455	468	481	494	506	519	532	544	556	568	0·5
0·5	568	580	592	604	616	627	639	650	661	672	683	0·4
0·6	683	694	705	716	726	736	746	756	766	776	785	0·3
0·7	785	795	804	813	822	830	838	846	854	862	870	0·2
0·8	870	878	885	892	899	906	913	920	926	933	939	0·1
0·9	939	946	952	958	964	970	976	982	988	994	1,000	0
	10	9	8	7	6	5	4	3	2	1	0	cos α

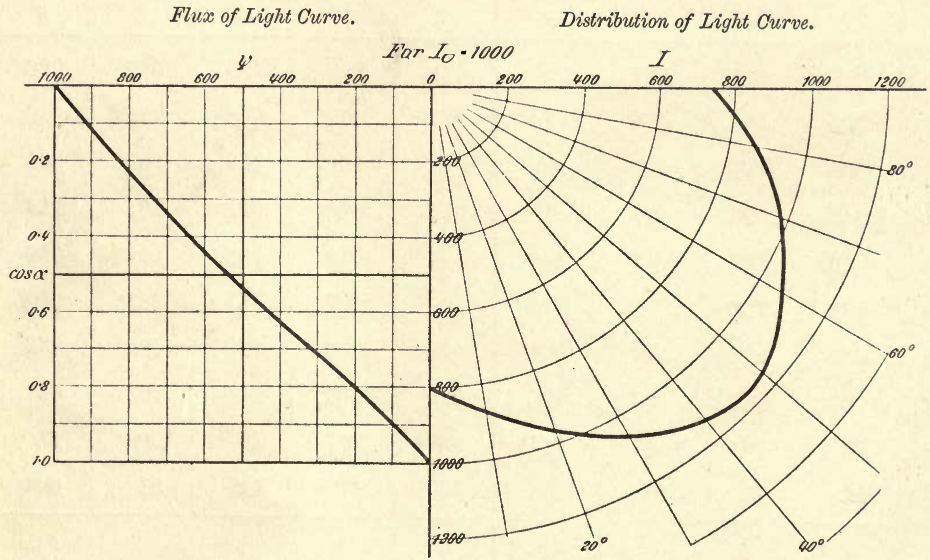


Fig. 44.—Enclosed Arc Lamps for Continuous and Alternating Current, with Opal Glass and Alabaster Globes.

TABLE XXIX.

LIGHT FLUX VALUES FOR ENCLOSED ARC LAMPS USING CONTINUOUS AND ALTERNATING CURRENT, WITH OPAL GLASS AND ALABASTER GLOBES.

1 – cos α	0	1	2	3	4	5	6	7	8	9	10	
0	0	8	17	26	35	45	55	65	75	85	95	0·9
0·1	95	105	115	125	135	146	157	168	179	190	201	0·8
0·2	201	212	223	234	245	256	267	278	289	301	313	0·7
0·3	313	325	337	349	361	373	384	395	406	417	428	0·6
0·4	428	439	450	461	472	483	494	505	516	527	537	0·5
0·5	537	548	559	570	580	590	601	612	622	632	642	0·4
0·6	642	652	662	672	682	692	702	712	722	732	741	0·3
0·7	741	751	761	771	780	789	799	808	817	826	835	0·2
0·8	835	844	853	862	871	879	888	896	904	912	920	0·1
0·9	920	928	936	944	952	960	968	976	984	992	1,000	0
	10	9	8	7	6	5	4	3	2	1	0	cos α

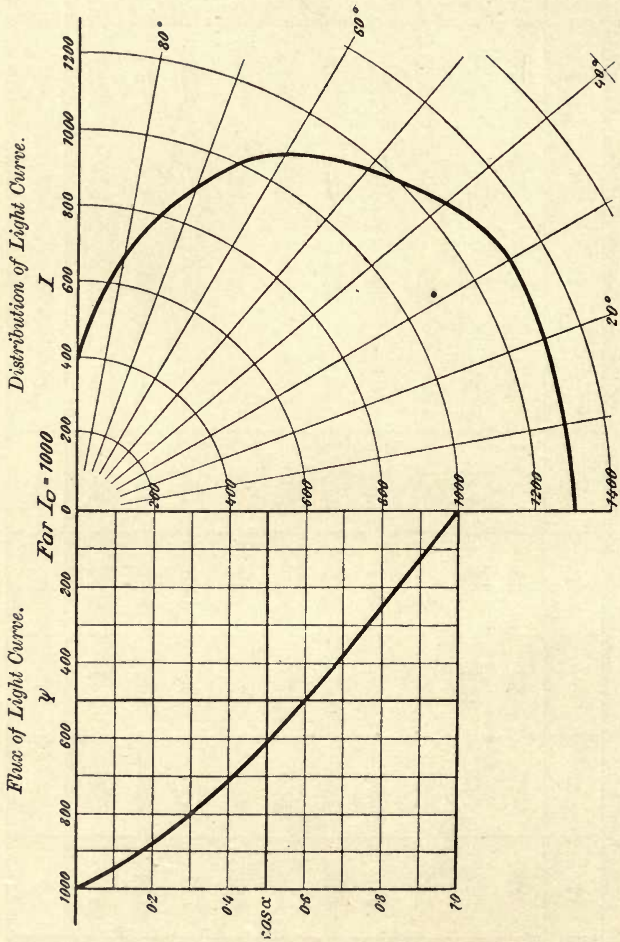


FIG. 45.—FLAME ARC LAMPS WITH INCLINED CARBONS AND ECONOMIZERS FOR CONTINUOUS CURRENT AND ALTERNATING CURRENT, WITH CLEAR GLASS AND OPAL GLASS GLOBES.

TABLE XXX.

LIGHT FLUX VALUES FOR FLAME ARC LAMPS, WITH INCLINED CARBONS AND ECONOMIZERS FOR CONTINUOUS CURRENT AND ALTERNATING CURRENT, WITH CLEAR GLASS AND OPAL GLASS GLOBES.

$1-\cos\alpha$	0	1	2	3	4	5	6	7	8	9	10	
0	0	12	25	38	51	64	77	90	103	116	129	0·9
0·1	129	142	156	169	182	195	208	221	234	247	260	0·8
0·2	260	273	286	299	312	324	337	350	362	374	386	0·7
0·3	386	398	410	422	434	446	458	470	482	493	504	0·6
0·4	504	516	527	538	549	560	571	582	593	604	614	0·5
0·5	614	625	636	646	656	666	676	686	696	706	716	0·4
0·6	716	726	736	745	754	763	772	781	790	799	808	0·3
0·7	808	816	824	832	840	848	856	864	871	878	885	0·2
0·8	885	892	899	906	912	918	924	930	936	942	947	0·1
0·9	947	953	959	964	969	975	980	985	990	995	1,000	0
	10	9	8	7	6	5	4	3	2	1	0	$\cos\alpha$

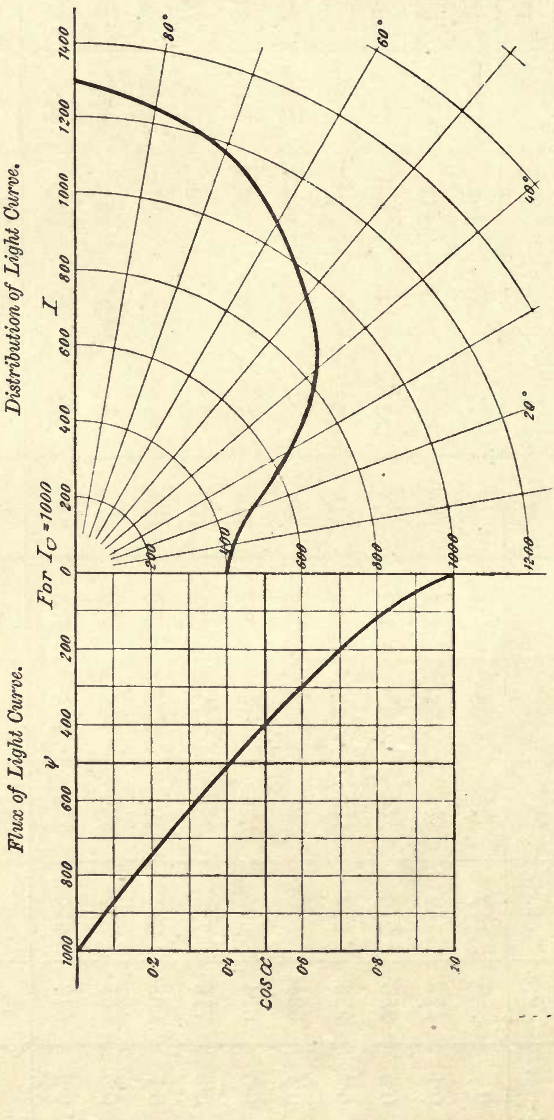


FIG. 46.—UPRIGHT GAS-MANTLE BURNERS, WITH REFLECTORS, FOR STREET LANTERNS.

TABLE XXXI.

LIGHT FLUX VALUES FOR UPRIGHT GAS-MANTLE BURNERS, WITH REFLECTORS, FOR STREET LANTERNS.

1 − cos α	0	1	2	3	4	5	6	7	8	9	10	
0	0	4	8	12	16	21	26	31	36	42	48	0·9
0·1	48	54	60	66	73	80	87	94	101	108	116	0·8
0·2	116	124	132	140	148	156	165	174	183	192	201	0·7
0·3	201	210	220	229	239	249	258	268	278	288	298	0·6
0·4	298	308	318	328	338	348	358	368	378	389	400	0·5
0·5	400	410	421	432	443	454	465	476	487	498	510	0·4
0·6	510	521	533	544	556	568	579	591	603	615	627	0·3
0·7	627	639	651	663	675	687	699	711	723	735	748	0·2
0·8	748	760	773	785	797	810	822	835	847	860	873	0·1
0·9	873	885	898	910	923	936	948	961	974	987	1,000	0
	10	9	8	7	6	5	4	3	2	1	0	cos α

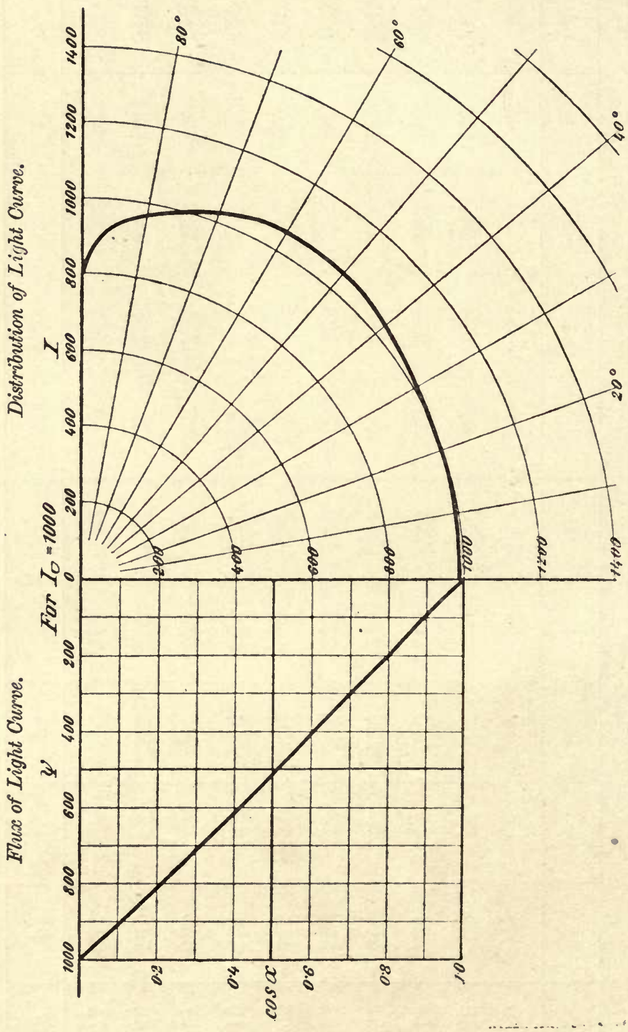


FIG. 47.—INVERTED GAS-MANTLE BURNERS.

TABLE XXXII.

LIGHT FLUX VALUES FOR INVERTED GAS-MANTLE BURNERS.

$1 - \cos\alpha$	0	1	2	3	4	5	6	7	8	9	10	
0	0	9	19	29	39	49	59	69	79	89	99	0·9
0·1	99	109	119	129	139	149	159	169	179	189	200	0·8
0·2	200	210	220	230	240	251	261	271	281	291	302	0·7
0·3	302	312	322	332	342	353	363	374	384	394	405	0·6
0·4	405	415	426	436	446	457	467	478	488	498	509	0·5
0·5	509	519	530	540	550	561	571	582	592	602	612	0·4
0·6	612	623	633	643	653	663	674	684	694	704	714	0·3
0·7	714	724	734	744	754	764	774	784	794	804	814	0·2
0·8	814	824	834	844	854	864	874	884	894	903	912	0·1
0·9	912	922	931	940	949	958	967	976	985	993	1,000	0
	10	9	8	7	6	5	4	3	2	1	0	$\cos\alpha$

INDEX

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