SECTOR AND SPATIAL ANALYSES OF THE UNITED STATES FEED ECONOMY

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Definitions

Feed concentrates, as used in this study, include feed grains, wheat fed, rye fed, and byproduct feeds whether fed as such or used in mixed or formula feeds.

Feed grains consist of corn, oats, barley, sorghum grain, wheat fed and rye fed; corn silage, considered a roughage, is not included.

Byproduct feeds is a term generally applied to byproducts of other industries, such byproducts being used for livestock feed; byproduct feeds are important as a source of protein for livestock rations and consist mainly of three major groups — high-protein feeds, medium-protein feeds, and other relatively low-protein feeds.

High-protein feeds consist of: (1) oilseed meals and cakes, such as meals and cakes of soybean, cottonseed, linseed, peanut, and copra; (2) animal and marine proteins, such as meat scraps, tankage, fish byproducts, and commercial and noncommercial milk byproducts; and (3) grain byproducts, such as gluten feed and meal, brewers' dried grains, distillers' dried grains, and dried solubles. Fish byproducts include meal and scrap, condensed fish solubles (on solid basis), and homogenized condensed fish (on solid basis). Commercial milk byproducts include dried and concentrated skim milk and buttermilk, dried whey, and concentrated and condensed whey. Noncommercial milk byproducts cover the dry-weight equivalent of skim milk, buttermilk, whole milk, and whey fed on farms where produced. Byproduct feeds with a protein content of 20 percent or more are generally classified as high-protein feeds.

Medium-protein feeds and low-protein feeds — byproduct feeds with a protein content of less than 20 percent but more than 15 percent are regarded as medium-protein feeds; they include wheat millfeeds, alfalfa meal, and oatmeal. Byproduct feeds with a protein content of less than 15 percent are classified as low-protein feeds and include dried and molasses beet pulp, rice millfeeds, oat millfeeds, hominy feed, screenings, and molasses.

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Y. H. Chuang and G. G. Judge[1]

THIS STUDY is concerned with a quantitative analysis of the feed grains and high-protein feed sectors of the economy. As such it will: (1) develop a sector model of the feed economy and estimate the parameters of the demand relations for feed grains and high-protein feeds; and (2) develop and analyze alternative spatial price equilibrium models to determine the regional consumption and geographical flows of feed grains and high-protein feeds under several conditions and time periods.

The study is presented in four parts. In Part I, as a base for the analyses to follow, the time-spatial characteristics of the feed-livestock economy is reviewed. In Part II factors that appear to condition fluctuations in the prices and quantities of feed grains and high-protein feeds produced and sold over time are enumerated, a feed sector model is constructed, and the parameters of the demand relations are estimated. Part III presents the regional data underlying the spatial equilibrium models for feed grains and high-protein feeds for the feeding years 1947-48, 1954-55, and 1957-58. It presents the results of a 37 state or regional model that: (1) ascertains a set of regional equilibrium prices and the regional quantities of feed grains and high-protein feeds consumed under a joint market equilibrium situation; (2) determines quantities of feed grains and high-protein feeds exported and imported from each state or region; (3) estimates the aggregate volume of trade and the corresponding total transport cost for each commodity; (4) ascertains the direction and volume of trade between each possible pair of states or regions that will maximize net returns to all sources and permit geographical distribution of both feed grains and high-protein feeds at a minimum transport cost; and (5) evaluates the effects on the results of changes in the basic data, such as equal regional consumption on a grain-consuming animal unit basis, or total available supply exceeding total demand. Part IV gives the implications of the results and discusses possibilities for further research.

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Part I. — Review of Time-Spatial Characteristics of the Livestock-Feed Economy

Production and Availability

Feed grains

The principal feed grains are corn, oats, barley, and sorghum grain. Corn ranks first in volume of production, in livestock feed, and in farm value. It is followed by oats, barley, and sorghum grain. These four major feed grains plus wheat fed and rye fed comprise more than 80 percent of annual total production and available supply of feed concentrates. Corn alone accounts for more than one-half the annual production and available supply of feed concentrates (Tables 1 and 2).

Table 1. — Total Available Supply of Feed Grains and Byproduct Feeds; Selected Years, 1926-1958; Year Beginning October 1

Kinds of feed	1926	1936	1946	1956	1957	1958
			1,000 tons			
Feed grains, total	109,273	75,744	137,591	175,021	192,546	216,681
Corn (excluding silage)	79,262	49,987	94,905	129,408	135,629	147,602
Oats	22,596	17,175	28,304	34,431	25,056	28,008
Barley	4,249	5,783	7,870	12,492	14,136	15,674
Sorghum grain	1,770	757	2,651	7,190	16,095	23,105
Wheat fed and rye fed	1,396	2,042	3,861	1,500	1,630	2,292
Byproduct feeds	14,836	14,801	20,177	26,069	26,689	28,118
High-protein feeds, total	7,427	7,346	10,455	15,622	15,760	17,215
Oilseed meals and cakes	3,939	3,654	6,278	10,941	11,287	12,620
Soybean meal and cake	32	531	4,086	7,621	8,340	9,556
Cottonseed meal	3,050	2,298	1,499	2,485	2,204	2,355
Linseed meal and cake	750	602	374	590	507	484
Copra meal	87	137	195	181	188	148
Peanut meal	10	66	124	64	48	77
Animal protein feeds, total	2,747	2,796	2,538	3,138	2,932	2,983
Meat scrap and cake	640	634	741	1,485	1,388	1,415
Fish products	72	277	202	403	409	498
Milk products	2,035	1,885	1,595	1,250	1,135	1,070
Grain product feeds, total	751	896	1,639	1,543	1,541	1,612
Gluten feed and meal, and corn oil meal	657	547	1,097	1,011	1,030	1,025
Brewers' dried grains	8	117	232	240	229	236
Distillers' dried grains, and dried solubles	86	232	310	292	282	351
Medium-protein feeds, total	5,133	5,057	7,271	6,240	6,432	6,507
Low-protein feeds	2,276	2,398	2,451	4,207	4,497	4,396
Total feed concentrates	124,109	90,545	157,768	201,090	219,235	244,799

Sources: For feed grains, data obtained in 1,000 bushels from *Agricultural Statistics*, U. S. Dept. Agr. for the respective years and then converted into tons. Data also taken from *Grain and Feed Statistics*, U. S. Dept. Agr. Stat. Bul. 159, May, 1957, and its *Supplement*, March, 1960.

For byproduct feeds, data for 1926-1956 obtained from G. A. King, *The Demand and Price Structure for Byproduct Feeds*, U. S. Dept. Agr. Tech. Bul. 1183, August, 1958, and for 1957 and 1958 from *The Feed Situation*, U. S. Dept. Agr., November, 1959, pp. 21, 45, and February, 1960, p. 36.

Table 2. — Percentages That Various Feeds Were of Total Available Supply and of Quantity Fed to Livestock; Selected Years, 1926-1958[a]; Year Beginning October 1

	1926	1936	1946	1956	1957	1958
Available supply						
Corn to feed grains	72.54	65.99	68.90	73.94	70.44	68.12
Corn to feed concentrates	63.86	55.23	60.15	64.35	61.86	60.30
Feed grains to concentrates	88.05	83.65	87.21	87.04	87.83	88.51
High-protein feeds to byproduct feeds	50.06	49.63	51.82	51.82	59.05	61.22
High-protein feeds to concentrates	5.98	8.11	6.63	7.77	7.19	7.03
Byproduct feeds to concentrates	11.95	16.35	12.79	12.96	12.17	11.49
Livestock feed						
Corn to feed grains	73.66	67.44	71.16	71.16	69.51	69.52
Corn to concentrates	63.58	54.02	59.23	57.11	55.82	56.53
Feed grains to concentrates	86.31	80.11	83.24	80.25	80.20	81.31
High-protein feeds to byproduct feeds	45.69	47.70	51.32	59.62	54.88	57.15
High-protein feeds to concentrates	6.26	9.49	8.60	11.78	10.81	10.68
Byproduct feeds to concentrates	13.69	19.89	16.76	19.75	19.70	18.69

[a] Computed.

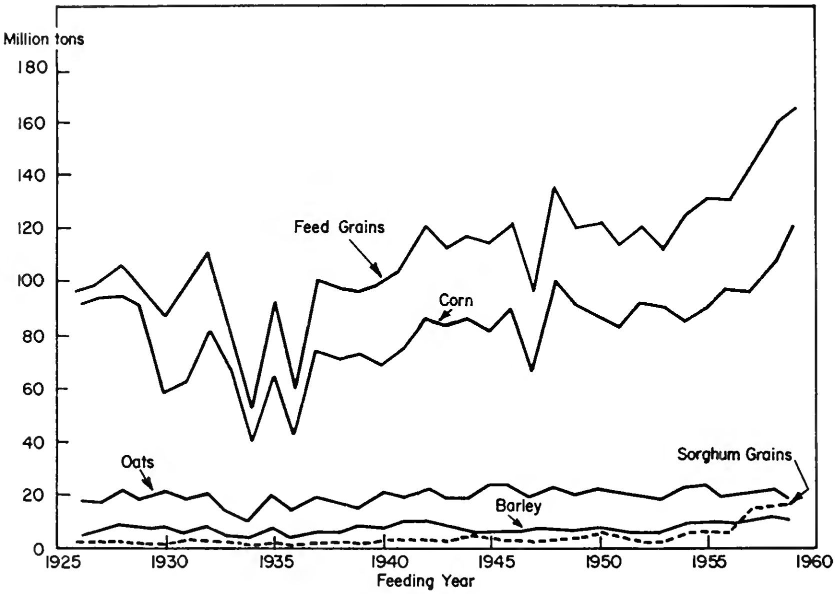
Total production and consumption of feed grains since 1925 are given in Figs. 1 and 2. Consumption of feed grains per animal unit is given in Fig. 3.

Every state raises at least one kind of feed grain. The soil and climate requirements, however, limit production of each feed grain in large commercial quantities to a relatively few states. For corn, oats, and barley, the North Central Region is the primary commercial producing area. For grain sorghum, the most important producing states are Texas, Kansas, Oklahoma, and Arkansas.

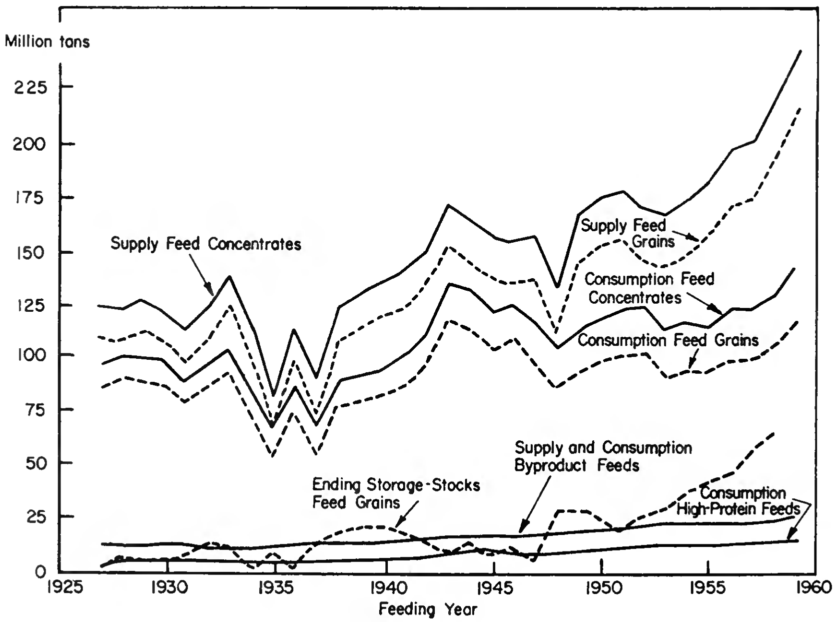
Byproduct feeds

Practically all domestic production of byproduct feeds goes into livestock feeding each year. In magnitude of annual production and as a source of protein for livestock rations, high-protein feeds rank first among byproduct feeds. High-protein feeds average about 10 percent of the feed concentrates fed annually and 50 percent of the byproduct feeds fed annually (Table 1). High-protein feeds provide roughly 35 percent of the total digestible protein fed to livestock each year. Other byproduct feeds supply about 8 percent, and feed grains the remaining 57 percent.[1]

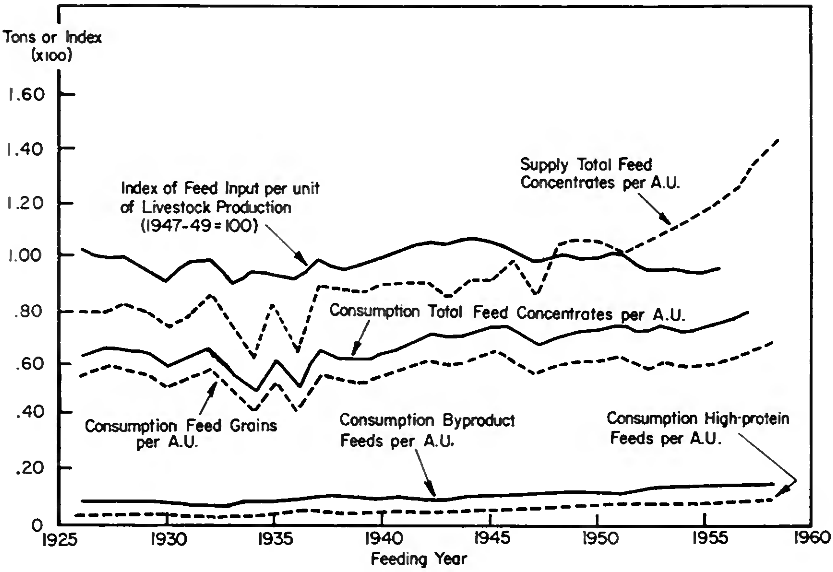
[1] G. A. King, *The Demand and Price Structure for Byproduct Feeds.* U.S. Dept. Agr. Tech. Bul. 1183, August, 1958, p. 2.



Annual production of four major feed grains — corn, oats, barley, and sorghum grain, 1926-1958. (Fig. 1)



Total available supply, consumption, and stocks of total feed concentrates, feed grains, and byproduct feeds, 1926-1959. (Fig. 2)



Per animal unit available supply, consumption of total feed concentrates, feed grains, and high-protein feeds, and feed inputs per unit of livestock production, 1926-1958. (Fig. 3)

Sources: Supply of total feed concentrates per animal unit, *Grain and Feed Statistics*, U. S. Dept. Agr. Stat. Bul. 159, May, 1957, Table 1, p. 2, and its *Supplement for 1959*, March, 1960, Table 1, p. 2. Index of feed input per unit of livestock production, Ralph Jennings, *Consumption of Feed by Livestock, 1909-1956*, U. S. Dept. Agr. Prod. Res. Rprt 21, November, 1958, Table 77, p. 128. All other data computed.

The production and consumption of high-protein feeds and byproduct feeds are shown in Fig. 2. Increases in production of oilseed cake and meal and animal and marine byproducts since 1926 (Table 1) have been primarily responsible for increases in production of high-protein feeds. The two most important components of high-protein feeds are soybean meal and cake and cottonseed meal and cake. Together they comprise more than 90 percent of the total annual production of oilseed cake and meal.

For a given feeding year, production of each high-protein feed is somewhat restricted geographically. In general, production is limited to a few states where oil seeds are produced and where oil mills and processing plants are located. Since World War II, nearly 90 percent of the total United States production of soybean meal and cake has been in the North Central Region. The Cotton Belt provides almost all the total United States production of cottonseed cake and meal. (For a discussion of major producing and processing areas for high-protein feeds, see U. S. Dept. Agr. Tech. Bul. 1183, pp. 40-60, cited on page 5.)

Livestock Feed

Supplies of feed grains and byproduct feeds available at the start of a given feeding year are largely fed to domestic red-meat livestock and poultry. About 64 percent of the annual supply of available feed grains and about 95 percent of the annual production of byproduct feeds are fed to livestock. Feed grains provide more than 80 percent of the total feed concentrates fed each season, while byproduct feeds provide the remainder. Corn alone supplies nearly 60 percent of the total concentrates fed (Tables 2, 3, and 4).

Table 3. — Quantities of Feed Fed to Livestock; Selected Years, 1926-1958

Type of feed	1926	1936	1946	1956	1957	1958
			1,000 tons			
Feed grains, total	85,915	57,354	97,825	100,155	105,736	117,987
Corn	63,289	38,678	69,610	71,273	73,493	82,030
Oats	16,249	13,310	19,270	17,336	17,554	19,628
Barley	2,978	2,528	2,973	4,652	5,629	5,680
Sorghum grain	2,003	796	2,110	4,870	7,430	8,357
Wheat fed and rye fed	1,396	2,042	3,862	2,024	1,630	2,292
High-protein feeds, total	6,229	6,793	10,110	14,697	14,234	15,497
Byproduct feeds, total	13,632	14,241	19,699	24,653	25,936	27,114
Total feed concentrates	99,547	71,595	117,524	124,808	121,672	145,101

Sources: For feed grains, data for 1926-1956 obtained from Ralph Jennings, *Consumption of Feed by Livestock, 1909-1956,* U. S. Dept. Agr. Prod. and Res. Rprt 21, Table 1, pp. 5-6; data for 1957 and 1958 obtained directly from U. S. Dept. Agr.

For byproduct feeds, data for 1926-1956 obtained from U. S. Dept. Agr. Tech. Bul. 1183 (*op. cit.* in Table 1) Table 74, pp. 142-147; data for 1957 and 1958 obtained directly from Earl F. Hodges, Agricultural Economist, U. S. Dept. Agr.

Table 4. — Percentages That the Total of Various Feeds Fed to Livestock Were of Their Respective Total Available Supplies; Selected Years, 1926-1958; Year Beginning October 1

Type of feed	1926	1936	1946	1956	1957	1958
Corn	79.85	77.38	73.35	55.08	54.19	55.58
Feed grains	78.62	75.72	71.10	57.22	54.91	54.45
High-protein feeds	83.87	92.47	96.70	94.08	90.32	90.02
Byproduct feeds	91.88	96.22	97.63	94.57	97.18	96.43
Total feed concentrates	80.21	79.07	74.49	62.07	60.06	59.27

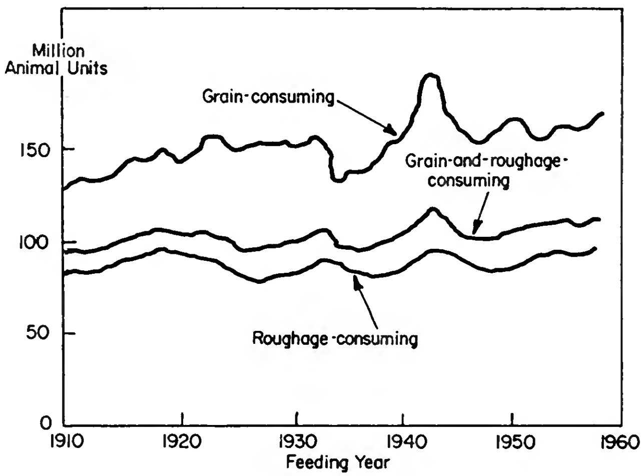
Source: Tables 1 and 4.

The percentage of specified feeds consumed by livestock varies from one class of livestock to another. For example, hogs and poultry together consume more than 60 percent of the feed grains fed each year. Since 1940, poultry have consumed on the average about 37 percent of the commercial byproduct feeds fed annually, while dairy cattle have consumed 29 percent, and hogs 15 percent.

Number of Grain-Consuming Animal Units Fed Annually

During the last half century, the total number of grain-consuming livestock has increased. This increase has ranged from 127 million units in 1909 to 172 million units in 1958. Roughage-consuming animal units increased somewhat less (Fig. 4). The year-to-year changes in total number of grain-consuming units mainly reflect variations in units of hogs, beef cattle, and poultry. In the last ten years, animal units of beef cattle have increased 51 percent, hogs 13 percent, and poultry 6 percent. Animal units of sheep increased by roughly 16 percent. Animal units of horses and mules, and dairy cattle, on the other hand, have decreased steadily.

One or more types of livestock are raised in every state. Although beef cattle are raised throughout the United States, the heaviest production is concentrated in the western part of the Corn Belt, the West,



Number of animal units of livestock and poultry fed annually, 1910-1958. (Fig. 4)

Source: *Animal Units of Livestock Fed Annually, 1909-1958,* U. S. Dept. Agr. Stat. Bul. 255, October, 1959, p. 6.

and Southwest. Dairy cattle are widely dispersed over the country with areas of concentration in the Northeast and in the northwestern part of the Corn Belt. The North Central Region is the primary hog producing region. Sheep production is primarily concentrated in the eleven western states. The leading areas for broiler production are the South Atlantic, Eastern, Southern, Central, and North Atlantic Regions. The North Central and the Middle Atlantic Regions are the leading areas in egg production. Primary areas of turkey production are concentrated in the west North Central Region and the Northwest.

Surplus or Deficit Position of Feed Grains and High-Protein Feeds

Before proceeding to the sector and spatial analyses, the surplus and deficit positions of specified regions in terms of feed grains and high-protein feeds will be examined under two restrictive assumptions for the feeding year 1957-1958. These assumptions are: (1) absence of interregional trade; and (2) equal regional per animal-unit consumption of feed grains and high-protein feeds. The terms "surplus" and "deficit" refer to the difference between the regional total supply available for current livestock feeding and total regional consumption. When the total regional supply available for current feeding is greater than total regional consumption, supplies are in surplus; when total regional supply is less than total regional consumption, supplies are in deficit.

For each of 37 regions, estimates were made of: (1) quantities of feed grains and high-protein feeds produced in and available for current livestock feeding and numbers of animal units fed; (2) total consumption of feed grains and high-protein feeds; and, (3) the United States average consumption of feed grains and high-protein feeds per animal unit. These estimates were then compared to the available supplies in each region.

The comparison showed both feed grains and high-protein feeds were surplus in 13 and deficit in 24 regions (Table 5). Feed grains and high-protein feeds, however, were not surplus and deficit in the same regions. Regions in which feed grains were surplus were concentrated mainly in the Corn Belt. Large surpluses of high-protein feeds were available for interregional movement in the eastern part of the North Central Region and Texas. Although this analysis of regional demand and supply is very rough, it does point up the dimensions of the distributional problem.

Table 5. — Surplus or Deficit Supplies of Feed Grains for Livestock Feeding by States or Regions, 1957-1958

State or region	Surplus or deficit; feed grains[a]	Surplus or deficit; high-protein feeds[a]	State or region	Surplus or deficit; feed grains[a]	Surplus or deficit; high-protein feeds[a]
	1,000 tons	*1,000 tons*		*1,000 tons*	*1,000 tons*
New England....	−2,814[b]	−212[b]	West Virginia....	− 417[b]	− 81[b]
New York.......	−1,876	− 49	Virginia.........	− 773	− 75
Pennsylvania....	−1,234	−292	Tennessee.......	− 784	− 10
Maryland, New Jersey, Delaware	−3,287	− 91	North Carolina...	− 969	−293
Ohio............	−1,255	375	South Carolina...	− 78	− 70
Indiana.........	562	188	Georgia.........	−1,774	−356
Illinois..........	8,135	2,319	Alabama........	− 795	−172
Michigan.......	1,229	−178	Florida..........	− 758	−120
Wisconsin.......	− 924	−504	Arkansas........	−1,542	127
Minnesota......	3,540	160	Louisana and Mississippi....	−3,022	14
Iowa...........	1,382	−350	Montana and Idaho.........	811	− 98
Missouri........	−1,333	−216	Wyoming........	− 31	− 26
Kentucky.......	− 528	255	Colorado........	303	− 81
North Dakota...	2,920	−141	Nevada and Utah	− 269	− 57
South Dakota...	1,274	−337	New Mexico and Arizona.......	163	131
Nebraska.......	3,736	−493	Washington......	− 538	− 74
Kansas.........	2,265	36	Oregon..........	− 204	− 82
Oklahoma.......	− 18	−118	California.......	−2,865	301
Texas..........	− 742	550	Total surplus or deficit....	27,575	4,466

[a] Average consumption of feed grains per grain-consuming animal unit for 37 states or regions in 1957-1958 was 0.8016 tons. Average per capita consumption of high-protein feeds for all states or regions in the same year was 0.0943 tons.

[b] A state or region with a negative number is classified as deficit, while one with a positive number is classified as surplus.

Part II. — Sector Analysis for Feed Grains and High-Protein Feeds

Estimates of the parameters of demand for feed grains and high-protein feeds are a prerequisite to the spatial equilibrium analysis to follow. The following discussion will therefore be devoted to the specification of a model for the feed sector of the economy, the estimation of the attendant parameters of the postulated relationships, and the statistical and economic significance of the results.

The Model

The model postulated represents only the set of equations and variables considered most significant for the livestock-feed economy. The equations specified in the model are: (1) demand for feed grains; (2) supply of feed grains; (3) demand for high-protein feeds; and (4) supply of high-protein feeds.

Demand for feed grains

The total availability of feed grains for a feeding year is channeled into four major demands: (1) livestock feeding; (2) exports; (3) human food, seed, and industrial purposes; and (4) storage stocks. In this study attention is directed to the demand for feed grains for livestock feeding purposes. The total availability of feed grains for a given feeding year is considered predetermined. Since feed grains are one of the major inputs in livestock production, the demand for feed grains is a derived demand that depends on the demand for livestock and livestock products.

Following traditional firm-and-factor-market-equilibrium theory, the demand for a factor is postulated as being a function of its price, the prices of all other competing and complementary factors of production, and the price of the finished product. In this conceptual framework, high-protein feeds are considered the most important substitutes for feed grains. Livestock and livestock products are thought of as the finished forms of feed inputs fed. Given the number of animal units of livestock and poultry fed, consumption of feed grains per grain-consuming animal unit of livestock and poultry is specified as being a function of the price of feed grains, the price of high-protein feeds, and the price of livestock and livestock products. In equation form this may be expressed as:

$$Y_{1t} = \alpha_{12} Y_{2t} + \alpha_{13} Y_{3t} + \beta_{11} Z_{1t} + \alpha_{10} + U_{1t} \tag{3.1}$$

where Y_{1t} = quantity of feed grains consumed in livestock feeding per grain-consuming animal unit in a feeding year, t; Y_{2t} = the nondeflated average price of feed grains received by farmers in a year, t; Y_{3t} = the nondeflated average price of high-protein feeds at wholesale level in a year, t; Z_{1t} = the nondeflated average price index of livestock and livestock products received by farmers in a year, t; and U_{1t} = the disturbance in the first equation.

Supply of feed grains

Farmers' decisions to sell or use feed grains for feeding purposes were postulated to depend on the price of feed grains, the support price of feed grains, and the total availability of feed grains. A trend variable was introduced to reflect possible changes in technologies over time and to adjust for any other economic factors that change at a constant rate per unit of time. The average price of feed grains set by the Commodity Credit Corporation is introduced as an institutional variable in this study. All feed grains supplied for feeding purposes in a feeding year are assumed to be entirely consumed by livestock and poultry within the same year.

This specification may be expressed in equation form as:

$$Y_{1t} = \alpha_{22} Y_{2t} + \beta_{23} Z_{3t} + \beta_{24} Z_{4t} + \beta_{26} Z_{6t} + \alpha_{20} + U_{2t} \qquad (3.2)$$

where Z_{3t} = the nondeflated national average support price of corn for feeding year, t, as announced by the Commodity Credit Corporation; Z_{4t} = total availability of feed grains at the beginning of a feeding year, t; Z_{6t} = a time trend variable; and U_{2t} is the disturbance in the second equation. The variables Y_{1t} and Y_{2t} have been defined previously.

Demand for high-protein feeds

The same logic used in specifying the demand for feed grains is used in the construction of this relation. Symbolically, the demand for high-protein feeds is expressed as follows:

$$Y_{4t} = \alpha_{32} Y_{2t} + \alpha_{33} Y_{3t} + \beta_{31} Z_{1t} + \alpha_{30} + U_{3t} \qquad (3.3)$$

where Y_{4t} = quantity of high-protein feeds consumed in livestock feeding on a per grain-consuming animal unit basis in feeding year, t, and U_{3t} = the disturbance in the third equation. The other variables appearing in the relation have been previously defined.

Supply of high-protein feeds

The per animal unit supply of high-protein feeds available for current feeding is postulated as being functionally associated with the price of high-protein feeds, the total availability of all byproduct feeds, and a time trend variable. This may be expressed in equation form as:

$$Y_{4t} = \alpha_{43} Y_{3t} + \beta_{45} Z_{5t} + \beta_{46} Z_{6t} + \alpha_{40} + U_{4t} \qquad (3.4)$$

where Z_{5t} = the total availability of byproduct feeds for a feeding year, t, and U_{4t} = the disturbance in the fourth equation. All other variables have been previously defined.

Discussion of the model

The model outlined in the foregoing section comprises a complete system of equations. It involves 4 equations, 4 random disturbances denoted by U_{it}, 4 endogenous variables denoted by Y_{it}, and 5 exogenous variables denoted by Z_{it}. The α_{ij}'s and the β_{ij}'s are the parameters attached to the endogenous and the exogenous variables of *i*th equation, respectively.

Following the enumeration of the structural relations and the classification of variables specified, and by use of the order condition, all four equations are overidentified.

Algebraic Form and Data

A functional form linear in the logarithms of the observed variables was used except for the variable Z_{6t} (time). The time variable was specified as linear in natural units. In the empirical estimates to follow, capital letters will be used to denote logarithms of the observed variables; i.e., $Y_{it} = \log y_{it}$, $Z_{it} = \log z_{it}$.

The sample period included the years 1927 through 1959 with the war years 1942-1946 omitted because the prevalence of rationing and because the price-setting policies in effect for livestock products affected the normal market mechanism of the feed sector.

The unobserved random disturbances U_{it} are assumed to have a multivariate normal distribution with zero means and a finite variance and covariance matrix. They are assumed to be independent over time and their distribution is assumed to remain constant over the observation period. The Z_{it} are also assumed to be fixed and independent of the U_{it}. The joint distribution of the endogenous variables Y_{it} for any given time period t is conditioned by the joint probability distribution of the disturbances U_{it}. All these specifications form the basis for the estimation of the unknown parameters, the α_{ij} and the β_{ij}.

Secondary data from various publications of the federal government were used for the variables specified in the model. The appropriate data, however, were not available for some variables. Ideally, for demand and supply functions at each market level, the appropriate price variables defined at each corresponding level should be used. The demand and the supply relations of feed grains and high-protein feeds for current feeding as specified in the model, for example, should be based on the market prices of feed grains and high-protein feeds paid by farmers. Since such prices were not available throughout the sample period, the season average prices received by farmers for corn and the weighted season average prices of eleven principal high-protein feeds at wholesale level were used to reflect the desired prices. Estimates of the parameters so obtained, however, require careful interpretation.[1] The basic unit used for measuring the quantity variables

[1] For a discussion of this problem see R. J. Foote, *Analytical Tools for Studying Demand and Price Structure,* U. S. Dept. Agr. Handbook 146, August, 1958, pp. 23-24.

relating to feed is 1,000 tons. All price variables specified in the model are nondeflated. The number of animal units of livestock and poultry fed annually is measured in thousand units.

Empirical Results

Out of the set of equations specified, only the parameters of the demand equations (3.1) and (3.3) were estimated. Since both of the demand equations to be estimated were overidentified, the limited information and Theil-Basmann estimation methods were employed.[1] The classical least squares method was employed for comparative purposes.

In the following discussion the resulting estimates are examined to determine their agreement or disagreement with theory in regard to sign and an economic interpretation of the estimates is given. Results of the statistical tests are presented with standard errors of estimates directly below the estimates and the other tests below the equation to which they apply. The results of applying various estimation methods to particular relationships are also compared.

Demand for feed grains as livestock feed

Limited information estimates. The parameters associated with this relationship are estimated by the limited information method as:

$$Y_{1t} = \underset{(.1381)}{-.6725\,Y_{2t}} + \underset{(.2988)}{.4528\,Y_{3t}} + \underset{(.1842)}{.3712Z_{1t}} - 1.3916 + U_{1t} \quad (3.5)$$

$$\hat{\sigma}^2 = .000656 \qquad d' = 1.85478, \qquad 4 - d' = 2.14522 \quad (3.6)$$

$$T \log_e (1 + v) = .3668 \text{ where } T = 28,\ v = .01317 \quad (3.7)$$

Estimates of the parameters α and β do not conflict with theoretical preconception in regard to sign and are quite consistent with those given in other studies.[2] The estimate of the parameter associated with Y_{2t} is statistically significant at the 5-percent probability level. The parameter estimates associated with Y_{3t} *and* Z_{1t} are found to be nonsignificant. Since the computed Durbin-Watson statistics d' and $4 - d'$ exceed the upper bound of the critical values, the residuals of this

[1] For a discussion of these methods see T. D. Wallace and G. G. Judge, *Discussion of the Theil-Basmann Method for Estimating Equations in a Simultaneous System,* Okla. Agr. Exp. Sta. Pro. Ser. P-301, August, 1959, pp. 2-7.

[2] R. J. Foote, *Statistical Analyses Relating to the Feed-Livestock Economy,* U. S. Dept. Agr. Tech. Bul. 1070, June, 1953.

R. J. Foote, *The Demand and Price Structure for Corn and Total Feed Concentrates,* U. S. Dept. Agr. Tech. Bul. 1061, October, 1952.

C. Hildreth, and F. G. Jarrett, *A Statistical Study of Livestock Production and Marketing,* Cowles Commission for Research in Economics, Monograph 15, John Wiley and Sons, New York, 1955, p. 72.

W. A. Cromarty, "An Econometric Model for U. S. Agriculture," *J. Amer. Stat. Assoc.* 54:563, September, 1959.

relationship are concluded to be serially uncorrelated.[1] Given a 5-percent significance level and 2 degrees of freedom, the computed statistic, $T \log_e (1 + v)$ is found to be less than the corresponding tabled chi-square value. Thus, the hypothesis that the overidentifying restrictions were valid can not be rejected.

Since the postulated form of this relationship is linear in logarithms, the coefficients may be interpreted directly as elasticities. Other things being equal, the estimates may be interpreted in the form of the following economic statements.

1. A 1-percent decrease in the price of corn received by farmers would result in approximately a 0.6725-percent increase in the per animal unit consumption of feed grains.
2. A 1-percent increase in the price of high-protein feeds at wholesale level would result in a 0.4528-percent increase in the per animal unit consumption of feed grains.
3. A 1-percent increase in the price of livestock and livestock products received by farmers would result in 0.3712-percent increase in the per animal unit consumption of feed grains.

Theil-Basmann estimates. Application of the Theil-Basmann method to the demand equation for feed grains resulted in the following parameter estimates:

$$Y_{1t} = \underset{(.1316)}{-.6459\, Y_{2t}} + \underset{(.2881)}{.4166\, Y_{3t}} + \underset{(.1782)}{.3796 Z_{1t}} - 1.3362 + U_{1t} \quad (3.8)$$

$$\hat{\sigma}^2 = .00062 \qquad d' = 1.88643 \qquad 4 - d' = 2.11357 \quad (3.9)$$

Again, estimates obtained using this method do not conflict with *a priori* reasoning in regard to sign. The parameter estimates associated with Y_2 and Z_1 are found to be significantly different from zero at the 5-percent probability level. The parameter estimate attached to Y_3, however, is insignificant. The hypothesis that the residuals of the equation are not serially correlated can not be rejected at the 5-percent significance level.

The estimates may be interpreted in a manner similar to that for relation (3.5).

Least squares estimates. Results of the application of the classical least squares method to the demand for feed grains are as follows:

$$Y_{1t} = \underset{(.082)}{-.420\, Y_{2t}} + \underset{(.170)}{.090\, Y_{3t}} + \underset{(.118)}{.4672 Z_{1t}} - .8451 + U_{1t} \quad (3.10)$$

$$R^2 = .897 \qquad d' = 2.02197 \qquad 4 - d' = 1.97803 \quad (3.11)$$

[1] J. Durbin and G. S. Watson, "Testing for Serial Correlation in Least Squares Regression," *Biometrika* 37:409-428 (1950) and 38:159-177 (1951).

Estimates of the parameters, again, do not conflict with *a priori* reasoning regarding sign. As in the second analysis, the coefficients of Y_2 and Z_1 are found to be significantly different from zero at a 5-percent probability level. The coefficient associated with Y_3 is not significant. The coefficient of multiple determination, denoted by R^2, indicates a rather high degree of explanation. Application of the Durbin-Watson test indicates that the hypothesis that the residuals of the equation are independent over time can not be rejected.

Except for the numerical values, the resulting estimates of the parameters may be interpreted as in the preceding analyses.

Summary of demand for feed grains as high-protein feed

The parameter estimates for the demand equation for feed grains obtained by the limited information, the Theil-Basmann, and the least squares methods agree as to sign. The Theil-Basmann method, in magnitude, yields estimates that fall between the least squares and the limited information estimates.

Demand for high-protein feeds

Limited information estimates. Applying the limited information technique to the demand equation for high-protein feeds resulted in the following estimates:

$$Y_{4t} = \underset{(8.4622)}{6.9603\,Y_{2t}} - \underset{(19.3499)}{16.4528\,Y_{3t}} + \underset{(10.4118)}{9.5551 Z_{1t}} - 4.11345 + U_{3t} \quad (3.12)$$

$$\hat{\sigma}^2 = .188484 \qquad d' = .01588 \qquad 4 - d' = 3.98412 \quad (3.13)$$

$$T \log_e (1 + v) = 13.3252 \text{ where } T = 28 \text{ and } v = .60964 \quad (3.14)$$

No divergence in sign is discerned between the empirical estimates and theory. All parameter estimates are found to be not significantly different from zero at the 5-percent significance level. In magnitude, the estimates are rather consistent with those obtained by Gordon A. King.[1] The Durbin-Watson statistic, d', concerning the independence over time of the residuals, was estimated at 0.01588. Since d' is less than the lower bound of the critical values, the nonautocorrelation hypothesis about the residuals of the demand equation for high protein-feeds is rejected. There are 2 degrees of freedom associated with the overidentifying restrictions test statistic and, when compared to the appropriate Chi square value, the hypothesis that the overidentifying restrictions are valid is rejected at a 5-percent probability level.

[1] G. A. King, *The Demand and Price Structure for Byproduct Feeds,* U. S. Dept. Agr. Tech. Bul. 1183, August, 1958, p. 87.

The resulting estimates may be interpreted as in the preceding analyses.

Theil-Basmann estimates. Results of applying the Theil-Basmann method to equation (3.3) of the model are as follows:

$$Y_{4t} = \underset{(.4075)}{.3216\,Y_{2t}} - \underset{(.8924)}{1.2842\,Y_{3t}} + \underset{(.5519)}{1.5263 Z_{1t}} - 2.87849 + U_{3t} \qquad (3.15)$$

$$\hat{\sigma}^2 = .00558 \qquad d' = .62119 \qquad 4 - d' = 3.37881 \qquad (3.16)$$

The signs of all coefficients are again compatible with *a priori* reasoning. At a 5-percent probability level, a positive serial correlation of the residuals over time is concluded. Of all resulting estimates only the one associated with Z_1, the price of livestock and livestock products received by farmers, is concluded to be significantly different from zero at the 5-percent probability level. The estimates may be interpreted as before.

Least squares estimates. The parameters associated with the demand equation for high-protein feeds were estimated by the method of the classical least squares as follows:

$$Y_{4t} = \underset{(.276)}{.056\,Y_{2t}} - \underset{(.576)}{.627\,Y_{3t}} + \underset{(.399)}{1.162 Z_{1t}} - 2.8213 + U_{3t} \qquad (3.17)$$

$$R^2 = .88 \qquad d' = .38693 \qquad 4 - d' = 3.61307 \qquad (3.18)$$

Again estimates of the parameters do not conflict with theoretical preconception in regard to sign. The Durbin-Watson statistic, d', was estimated at 0.38693. It is smaller than the lower bound of the critical values. The residuals of the equation are thus concluded to be positively correlated over time. The coefficient of multiple determination is approximately the same size as the one obtained for the estimated demand equation for feed grains. Only the coefficient attached to Z_1 is found to be statistically significant at a 5-percent probability level.

Summary of demand for high-protein feeds

Generally, the parameter estimates for the demand equation for high-protein feeds obtained by the three methods agree as to sign. Again the Theil-Basmann method, in magnitude, yields estimates that tend to fall between the classical least squares and the limited information estimates.[1]

[1] The limited information estimates presented in this study are rather suspect since, after these results were computed, the Limited Information Estimation Program for the Illiac Computer has been found to give incorrect answers for some problems. It has not been possible to determine to what extent the LIE estimates presented were affected by the coding error.

Part III. — Spatial Analyses of Feed Grains and High-Protein Feeds

Although livestock and feed are produced in virtually every state, the geographical intensities of annual production in each differ, so that interregional shipments of feed become necessary. In addition, the regional densities of feed production and livestock numbers vary over time. These considerations, along with other factors such as transportation charges, condition the structure of regional feed prices and the geographical flows of feed.

Spatial Equilibrium Model

To date, most of the empirical studies concerning spatial price equilibrium analyses have dealt with a single commodity.[1] The purpose of this part of the study, however, is to develop and analyze a spatial model that will permit: (1) simultaneous quantitative assessment of the equilibrium geographical prices and flows of both feed grains and high-protein feeds under a particular set of conditions; and (2) will then permit ascertainment of the economic consequences of specified disturbances in the initial set of data. In the model to be presented, feed grains and high-protein feeds are considered as two competitive and substitutable input factors in livestock feeding. Because of this consideration, the spatial analyses of both commodities will be carried out jointly.

Assumptions required

In all analyses to follow, conditions of feed grains and high-protein feed supplies and conditions of the livestock sector will be taken as

[1] G. G. Judge, *A Spatial Equilibrium Model for Eggs,* Conn. Agr. Exp. Sta. Bul. 318, January, 1956.

G. G. Judge and T. D. Wallace, *Spatial Price Equilibrium Analyses of the Livestock Economy,* Okla. Agr. Exp. Sta. Tech. Bul. TB-78, June, 1959.

K. A. Fox, "A Spatial Equilibrium Model of the Livestock-Feed Economy," *Econometrica* 21:40-48, October, 1953.

K. A. Fox and R. C. Taeuber, "Spatial Equilibrium: Livestock-Feed Economy," *Amer. Econ. Rev.* 45:584-608, September, 1955.

W. R. Henry and C. E. Bishop, "North Carolina Broilers in Inter-regional Competition," Agr. Econ. Inf. Ser. No. 56, North Carolina State College, February, 1957.

M. M. Snodgrass and C. E. French, *Linear Programming Approach to Inter-regional Competition in the Dairy Industry,* Ind. Agr. Exp. Sta. Bul. S. B. 637, May, 1958.

C. C. Dennis, *Inter-regional Competition in the Frozen Strawberry Industry,* Unpub. Ph.D. thesis, Univ. Calif., Berkeley, May, 1959.

predetermined. Prices of livestock and livestock products received by farmers and the number of grain-consuming animal units fed annually will then be used to reflect conditions in the livestock sector of the economy.

Perfect competition is assumed to dictate the requirements for the regional pattern of prices and flows of the commodities in question. Individual producing or selling units have no influence on market prices and accept them as data. Firms are assumed to possess perfect knowledge of technologies and market conditions and to have complete freedom of entry and exit. Given a consistent transport cost structure, they may ship their excess supplies of specified commodities in any direction as long as delivery costs permit. No governmental or other institutional barriers are placed on the direction and the volume of trade within or between regions. Consumers of specified commodities are assumed to be indifferent as to supply sources and the products are regarded as being homogeneous in all regions.

For purpose of analysis, the supply source and market for a region is represented by a selected basing point. Regional market demand relationships are assumed to be some known linear functions and regional supplies are taken as predetermined. Transport rates are assumed to be independent of the direction and volume of trade. Shipment of feed grains and high-protein feeds within a region is not considered. Deficit regions can not export and surplus regions can ship only to all possible deficit regions.

Definitions

The following terms appear frequently in spatial analyses.

A basing point is a geographical point chosen to represent the location of supply and consumption in a region.

A base region is a region from which price differentials of other regions are measured. The state of Illinois is regarded as the base region in the analyses.

The base region equilibrium prices of feed grains and high-protein feeds are a set of prices that satisfy simultaneously the market equilibrium conditions of both feed grains and high-protein feeds in the base region. These prices are simultaneously determined from a system of two regional market demand equations of feed grains and high-protein feeds.

Estimated price differentials are a set of numbers simultaneously derived from a given optimum shipment program and represent the differences between the prices in other regions and that in the base region under equilibrium conditions.

Regional supplies are the quantities of feed available in a region for current feeding of livestock in a feeding year before interregional shipments take place.

Regional market demand refers to the equilibrium quantity of feed required by a region for current livestock feeding in a feeding year. It is either assumed known or estimated from a regional linear demand function.

Surplus and deficit: surplus is a positive difference between a regional supply and an estimated regional demand, whereas deficit is a negative difference between a regional supply and an estimated demand.

Optimum basic feasible solution in a transportation program is a basic feasible solution that minimizes the total transport cost involved in the shipments of feed.

The formal model

Spatial price equilibrium analyses in this study may be divided for simplicity into the following three steps: (1) general, simultaneous solution for determining the prices and consumption of feed grains and high-protein feeds under going market equilibrium conditions, and the resulting regional surpluses and deficits; (2) determination of the minimum-transport cost flows of feed grains and high-protein feeds; and (3) estimation of regional price differentials for feed grains and high-protein feeds and determination of the final spatial price equilibrium solution.

Determination of regional prices, consumption, and surpluses and deficits. The first task is to determine simultaneously the prices of feed grains and high-protein feeds in the base and all other regions under joint market equilibrium conditions from observations relating to the number of grain-consuming animal units, prices of livestock products, quantities of feed grains and high-protein feeds available for current livestock feeding, and market demand relationships of feed grains and high-protein feeds by regions for a feeding year.

Assume that regional market demand relationships for feed grains and high-protein feeds are some exact linear functions and can be represented as:

$$y_{1i} = A_{12i}(y_{20} + d_{0i}) + A_{13i}(y_{30} + c_{0i}) + B_{11i}z_{1i} + A_{10i} \qquad (4.1)$$

and

$$y_{4i} = A_{22i}(y_{20} + d_{0i}) + A_{23i}(y_{30} + c_{0i}) + B_{21i}z_{1i} + A_{20i} \qquad (4.2)$$

where y_{1i} and y_{4i} = the quantity of feed grains and high-protein feeds consumed per grain-consuming animal unit in *i*th region in a given feeding year; y_{20} = the price of feed grains received by farmers at the base region in a given year; y_{30} = the price of high-protein feeds at the base region in a given year; z_{1i} = the price of livestock and livestock products received by farmers in *i*th region *in a given year;* d_{0i} = the price differential of feed grains between *i*th region and the base region in a given year. In addition, A_{10i} = the intercept of the *i*th regional market demand equation for feed grains in a given feeding year; A_{20i} = the intercept of the *i*th regional market demand equation for high-protein feeds in a given year. A_{12i} and A_{22i} are the coefficients associated with y_{2i} in equations (4.1) and (4.2), respectively. A_{13i} and A_{23i} are the coefficients associated with y_{3i} in equations (4.1) and (4.2). Likewise, B_{11i} and B_{21i} are the parameter estimates attached to z_{1i} in equations (4.1) and (4.2), respectively. All variables in these two regional market demand relationships are expressed in natural units.

Because of the perfectly competitive nature of the markets, it is further assumed that

$$y_{2i} = y_{20} + d_{0i} \qquad (4.3)$$

and

$$y_{3i} = y_{30} + c_{0i} \qquad (4.4)$$

where y_{2i} = the price of feed grains received by farmers at *i*th region in a given year and y_{3i} = the wholesale price of high-protein feeds in *i*th region in a given year.

Since there are only two equations, (4.1) and (4.2), in six unknowns, y_{1i}, y_{4i}, y_{20}, y_{30}, d_{0i}, and c_{0i}, the system is indeterminate. Provided a set of initially assumed regional price differentials, d_{0i} and c_{0i}, and the restriction that the total supply of each product equal total demand, the base region prices of feed grains and high-protein feeds, however, can be obtained by solving the following two equations in two unknowns simultaneously:

$$A_{12i}\left(\sum_i p_i\right) y_{20} + A_{13i}\left(\sum_i p_i\right) y_{30} = \sum_i \left(p_i \, y_{1i}\right) - \left(\sum_i p_i\right) A_{10i}$$
$$- A_{12i}\left(\sum_i p_i \, d_{0i}\right) - A_{13i}\left(\sum_i p_i \, c_{0i}\right) - B_{11i}\left(\sum_i p_i \, z_{1i}\right) \qquad (4.5)$$

$$A_{22i}\left(\sum_i p_i\right) y_{20} + A_{23i}\left(\sum_i p_i\right) y_{30} = \sum_i\left(p_i\, y_{4i}\right) - \left(\sum_i p_i\right) A_{20i}$$
$$- A_{22i}\left(\sum_i p_i\, d_{0i}\right) - A_{23i}\left(\sum_i p_i\, c_{0i}\right) - B_{21i}\left(\sum_i p_i\, z_{1i}\right) \tag{4.6}$$

where p_i = the number of animal units fed annually in the ith region in a given year; $\sum_i (p_i\, y_{1i})$ = the sum of all regional total consumptions of feed grains and $\sum_i (p_i\, y_{4i})$ = the sum of all regional total consumptions of high-protein feeds in a given year. Since the sum of the consumption of each product in each region is equal to the national total available supply for current livestock feeding in a given year, $\sum_i (p_i\, y_{1i})$ and $\sum_i (p_i\, y_{4i})$ are known.

After having solved for y_{20} and y_{30}, prices of feed grains and high-protein feeds in other regions, y_{2i} and y_{3i}, can be obtained by substituting values of Y_{20} and Y_{30} into equations (4.3) and (4.4), respectively.

In the initial stage, values of d_{0i} and c_{0i} may be assigned arbitrarily. At every successive stage, however, they will be derived from the linear programming transportation solution.

Given the equilibrium prices of feed grains and high-protein feeds in all regions, the per animal unit consumption of feed grains and high-protein feeds for each region is estimated from equations (4.1) and (4.2). Multiplication of the per animal unit consumption by the number of animal units in a region will result in the total consumption of each product in a given region. The total consumption so obtained for a product in each region is then compared to the predetermined available supply of the region. This comparison will result in the breakdown of all regions into two groups, surplus and deficit, and consequently the amount of surplus or deficit in terms of each of the commodities for each region can be calculated.

Determination of minimum transport cost flows. Given the breakdown and the quantities of regional surpluses and deficits for feed grains and high-protein feeds, the problem then is to determine two separate optimum shipment programs, one for feed grains and the other for high-protein feeds. For this determination, linear programming transportation models of Hitchcock-Koopmans type will be employed. Algebraically, a transportation model of the above type for a given commodity can be formulated as follows:

$$\sum_{j=1}^{m}\sum_{i=1}^{n} X_{ij}\, C_{ij} = \text{minimum, subject to} \tag{4.7}$$

$$\sum_{j=1}^{m} X_{ij} = a_i;\ i = 1, 2, 3, \ldots, n \tag{4.8}$$

$$\sum_{i=1}^{n} X_{ij} = b_j;\ j = 1, 2, 3, \ldots, m \tag{4.9}$$

$$\sum_{i=1}^{n} a_i = \sum_{j=1}^{m} b_j, \text{ and } X_{ij} \geq 0 \text{ for all } i \text{ and } j \tag{4.10}$$

where X_{ij} = the quantity of a commodity to be shipped from the ith surplus region to the jth deficit region; a_i = the total excess supply of the commodity at the ith surplus region; b_j = the total excess demand for the commodity at the jth deficit region; C_{ij} = the unit transportation cost from the ith surplus region to the jth deficit region for the commodity; n = the number of surplus regions, while m = the number of deficit regions. In order to be economically meaningful, all X_{ij}'s must take non-negative values.[1]

Generally, an optimum shipping pattern resulting from the above transportation model will make use of at most $(m + n - 1)$ of the (mn) possible routes. In this transportation model, total outshipments of a given commodity from all surplus regions must equal the total receipts of the commodity at all deficit regions. By introducing dummy variables, the same formulation may also apply where the total outshipments are not equal to the total receipts.[2]

Determination of regional price differentials. Since it is not definitely known which surplus states or regions ship to each deficit state or region at the initial stage of a spatial analysis, a set of price differentials for both feed grains and high-protein feeds is arbitrarily assumed. For example, all regional price differentials, d_{0i} and c_{0i}, may be initially assigned zero value. Given the initially assumed set of regional price differentials and the attendant optimum geographical flows of feed grains and high-protein feeds, an objective, unique set of regional price differentials, d_{0i} and c_{0i}, may then be estimated. This set is used to check the approximate differentials of the initial formulation and also determine the final spatial price equilibrium solution.

Algebraically, the dual problem of an original cost-minimization

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[1] S. I. Gass, *Linear Programming, Methods and Applications,* McGraw-Hill Book Co., New York, 1958, pp. 137-156.

R. Dorfman, P. A. Samuelson, and R. M. Solow, *Linear Programming and Economic Analysis,* McGraw-Hill Book Co., New York, 1958, pp. 106-129.

E. O. Heady and W. Chandler, *Linear Programming Methods,* Iowa State College Press, Ames, Iowa, reprinted 1960, pp. 332-376.

G. G. Judge and T. D. Wallace, Okla. Agr. Exp. Sta. Tech. Bul. T-78, *op. cit.,* p. 13.

[2] G. G. Judge and T. D. Wallace, *Spatial Price Equilibrium Analyses of the Livestock Economy, Application to Quarterly Models,* Okla. Agr. Exp. Sta. Tech. Bul. T-79, December, 1959, pp. 20-26.

problem may be written as follows: Let V_j be the variables associated with deficit regions and U_i be the variables associated with surplus regions. The problem then becomes:

$$f = \sum_j b_j V_j - \sum_i a_i U_i = \text{maximum, subject to} \quad (4.11)$$

$$V_j \leq U_i + C_{ij} \text{ or alternatively } V_j - U_i \leq C_{ij} \quad (4.12)$$

$$U_i \geq 0, \text{ and } V_j \geq 0 \quad (4.13)$$

Since $f =$ total transport cost calculated in the original minimization problem, the maximization problem just described may be thought of as finding the values of the U_i's and the V_j's that will maximize the total gain in the value of the quantities shipped subject to nonpositive profits on each shipment. Following this concept, it is then possible to interpret the U_i as the value of a product at the ith surplus region and the V_j as the value of the same product at the jth deficit region. In order to be economically meaningful, all U_i's and V_j's take a non-negative value. In the relationship (4.12) for any origin-destination pair, the value of the product at the deficit region must be no greater than the value at the surplus region plus the cost necessary for transportation. For those origin-destination pairs that are in a basic solution, the relation $V_j = U_i + C_{ij}$ holds. This means that whenever a shipping route is chosen, selling units or producers in the surplus region deliver a certain amount of their excess supplies to the deficit region neither at a positive profit nor at a loss. For those origin-destination pairs that are not in the basic solution, the relation (4.12) holds. Therefore, these relationships are consistent with the assumption of perfectly competitive markets.

Once the shipping routes are chosen, all the U_i's and the V_j's can be found from the relation, $V_j - U_i = C_{ij}$. This relation is a set of $(m + n - 1)$ linear equations in $(m + n)$ unknowns. In order to have a unique solution for every U_i and V_j, it is necessary to assign an arbitrary value to either a U_i or a V_j. For example, in the analysis to follow, a zero value is assigned for the U_i value in Illinois, the base region. Given $U_7 = 0$, all other U_i's and V_j's may then be interpreted as the price differentials between other regions and Illinois.[1]

The process outlined previously for determining regional equilibrium prices, consumption, and regional surpluses and deficits of feed

[1] For more detailed discussion see G. G. Judge and T. D. Wallace, Tech. Bul. T-78, *op. cit.*, pp. 26-31; and R. Dorfman, P. A. Samuelson, R. M. Solow, *op. cit.*, pp. 124-126.

grains and high-protein feeds is repeated, using regional price differentials provided by the U_i and V_j until the regional price differentials estimated agree with those at the immediately preceding solution.

The Data Basic for Spatial Analyses

In order to convert the foregoing framework into an operational spatial model, the area under study must first be demarcated into a number of meaningful geographical units or regions and the data basic to spatial analyses must then be specified. The basic data are: (1) regional market demand relationships of feed grains and high-protein feeds; (2) observed regional values of the chosen predetermined variables; and (3) a structure of transportation costs between all possible pairs of regions for both commodities in question.

Regional demarcation

The partitioning of a geographical area into a number of meaningful regions is a somewhat subjective endeavor. The objectives of a study, availability of data, and the computational facilities available for an analysis are some of the factors that condition the final choice.

Given these considerations, the continental United States was partitioned into 37 states or regions (Figs. 5a and b). A region is composed of two or more states. Each state or regional market or source of supply is represented by a point that is identified with a certain city near the geographical center of each area. When the regional basing points were selected, it was assumed that cities used as distribution centers for other commodities are also distribution centers for feed grains and high-protein feeds. In this connection, a map, "Trading Areas of the United States," and tables of 65 major trading areas and basic trading centers, published by Rand McNally and Company, were used as the main references.[1] All basing points chosen are cities or towns that characterize an area and have access to transportation facilities.

Regional market demand relationships

Since the types of feed and livestock produced and thus consumption behavior with respect to feed vary from region to region, it is perhaps more realistic to construct a sector model for each region and derive the market demand relationships for feed from these data. This is not possible, however, since the data on feed consumption by the regions specified have not been adequately recorded. Alternatively,

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[1] Rand McNally and Company, *Rand McNally Commercial Atlas and Marketing Guide,* Chicago, 1947, 1955, and 1959.

the regional market demand relationships are derived from the U.S. demand functions for which data on feed consumption are available.

In order to derive the regional market demand relationships, the two U.S. aggregated market demand relations of feed grains and high-protein feeds, logarithmically estimated by the Theil-Basmann method and discussed in Part II, are first converted to a functional form linear and exact in natural units. The converted functions differ from one time period to another, since observed values of the variables per animal unit of consumption of feed grains and high-protein feeds, prices of feed grains, high-protein feeds, and livestock and livestock products vary from year to year. The following are the converted functions for three different feeding years:

1957-58 feeding year

$$y_1 = .681149 - .014212y_2 + .004253y_3 + .001121z_1 \quad (4.15)$$
(feed grains)

$$y_4 = .041187 + .000832y_2 - .001542y_3 + .000530z_1 \quad (4.16)$$
(high-protein feeds)

1954-55 feeding year

$$y_1 = .55221 - .008277y_2 + .003193y_3 + .001031z_1 \quad (4.17)$$
(feed grains)

$$y_4 = .03545 + .000515y_2 - .001230y_3 + .000518z_1 \quad (4.18)$$
(high-protein feeds)

1947-48 feeding year

$$y_1 = .72288 - .007123y_2 + .003348y_3 + .001024z_1 \quad (4.19)$$
(feed grains)

$$y_4 = .02918 + .000279y_2 - .000812y_3 + .000324z_1 \quad (4.20)$$
(High-protein feeds)

For a designated feeding year, an appropriate set of the foregoing converted functions is then assumed for all regions in question. The adjustment of the constant terms of these functions for each region is ignored since the geographical variations in feeding rates for various classes of livestock and poultry have already been reflected in the number of animal units calculated.

Regional values of the predetermined variables

For a given feeding year *t*, regional data relating to: (1) the number of grain-consuming animal units of livestock and poultry fed; (2) the prices of livestock and livestock products received by farmers;

State or regional demarcation and selected basing points for the Western United States; 14 of 37 states or regions. (Fig. 5a)

The number of the state or region located in the western part of the United States is shown on the above map. The only regions in this portion of the country are: 30, Montana and Idaho; 33, Utah and Nevada; and 34, New Mexico and Arizona. The remainder are single states. The cities shown are the basing points for each state or region.

and (3) feeds available for current feeding are specified as predetermined. The regional values for these three predetermined variables in 1947-48, 1954-55 and 1957-58 were either obtained from records published by the U.S. Department of Agriculture or computed by authors. These observed values are assumed to be free of errors of measurement

State or regional demarcation and selected basing points for the Eastern United States; 23 of 37 states or regions. (Fig. 5b)

The number of the state or region located in the eastern part of the United States is shown on the above map. The only regions in this portion of the country are: 1, New England; 4, New Jersey, Delaware, Maryland, and District of Columbia; and 29, Mississippi, and Louisiana. The remainder are single states. The cities shown are the basing points for each state or region.

and are accepted as accurate portrayals of the variables they are supposed to reflect. The regional values of the number of grain-consuming animal units fed for the years analyzed are given in Table 6.

Table 6. — Number of Grain-Consuming Animal Units Fed Annually, by States or Regions; 1947-48, 1954-55, and 1957-58

State or region	1947-48	1954-55	1957-58
		1,000 units	
1	3,125	3,750	3,660
2	3,939	3,728	3,390
3	4,998	5,030	4,829
4	3,368	3,888	4,101
5	7,475	7,130	6,759
6	8,120	8,924	8,747
7	12,082	13,402	13,778
8	3,436	3,464	3,146
9	6,817	7,482	7,332
10	8,681	9,945	9,547
11	19,458	23,784	22,844
12	8,229	8,066	8,390
13	3,447	2,913	3,043
14	1,738	1,662	1,627
15	3,580	4,028	3,896
16	6,671	7,792	6,795
17	4,090	3,461	3,179
18	2,657	1,890	1,649
19	6,383	5,172	5,135
20	1,016	926	856
21	2,562	2,551	2,580
22	3,280	2,919	3,062
23	3,190	3,675	4,484
24	1,605	1,348	1,401
25	3,107	4,215	5,308
26	2,526	2,766	3,287
27	1,142	1,157	1,277
28	2,204	2,032	2,449
29	4,014	3,472	3,769
30	1,408	1,327	1,293
31	361	281	278
32	1,248	1,104	1,045
33	675	636	683
34	507	500	507
35	1,123	1,101	1,126
36	1,001	1,015	1,037
37	3,805	4,921	5,210
Total	153,098	161,458	161,499

Sources: *Animal Units of Livestock Fed Annually,* U. S. Dept. Agr. Stat. Bul. 215, July, 1957, and 255, October, 1959.

Prices of livestock and livestock products show considerable geographical differences. Accordingly, a weighted average price index was computed for each of the regions for each feeding year in question. The regional quantities of feed grains and high-protein feeds available for feeding for the years analyzed are given in Table 7. The regional data on export, import, and nonfeed uses of feed grains and high-protein feeds are not available for the years in question. As a result,

Table 7. — Feed Grains, High-Protein Feeds, and Total Feed Concentrates Available for Current Livestock Feeding; by States or Regions; Feeding Years, 1947-48, 1954-55, and 1957-58

State or region	1947-48		1954-55		1957-58	
	Feed grains	High-protein feeds	Feed grains	High-protein feeds	Feed grains	High-protein feeds
			1,000 tons			
1	207	88	184	119	120	133
2	1,196	298	1,059	222	841	271
3	2,211	125	1,842	158	2,637	163
4	.....	195	.....	177	.....	296
5	7,363	716	7,066	781	6,673	1,012
6	8,843	547	8,150	896	7,574	1,012
7	16,666	2,341	13,265	3,265	19,179	3,618
8	3,026	67	3,272	112	3,751	119
9	4,245	140	4,879	171	4,953	187
10	10,991	613	10,086	898	11,193	10,060
11	23,879	1,180	14,706	1,480	19,694	1,803
12	6,167	490	4,887	464	5,392	575
13	2,595	279	2,050	312	1,912	542
14	3,047	13	3,518	13	4,224	12
15	6,435	11	4,085	18	4,397	30
16	8,979	57	3,062	77	9,183	148
17	3,607	126	2,020	165	4,813	336
18	1,417	97	731	79	1,304	137
19	2,037	369	2,681	843	3,374	1,034
20	403	...	216	...	269	...
21	1,526	84	1,021	101	1,295	168
22	2,128	218	1,695	287	1,670	299
23	2,056	162	2,226	156	2,265	130
24	921	103	1,023	107	1,045	62
25	1,463	485	1,889	245	2,481	144
26	1,677	168	1,801	173	1,840	138
27	130	...	228	42	266	...
28	1,077	269	795	343	421	358
29	846	383	311	476	.....	368
30	1,326	8	1,774	7	1,847	24
31	213	...	172	...	192	...
32	897	10	823	15	1,141	18
33	266	6	275	6	278	7
34	430	56	617	179	569	179
35	188	11	101	79	365	32
36	494	10	424	13	627	16
37	1,289	466	1,894	631	1,311	792
Total	130,241	10,239	104,912	13,108	129,456	15,223

Sources: Production of feed grains obtained in 1,000 bushels from *Agricultural Statistics*, U. S. Dept. Agr. and then converted into tons. Stocks of feed grains obtained from *Stocks of Grains, Oilseeds, and Hay, Farm and Off-Farm Stocks by States, 1949-1955, Revised Estimates*, U. S. Dept. Agr. Stat. Bul. 203, January, 1957; *Crops and Markets*, U. S. Dept. Agr.; *Stocks of Grains in All Positions*, U. S. Dept. Agr. Misc. Rprts GrLg 11-1 and related issues. Production and stocks of soybean meal and cake, cottonseed meal and cake, linseed meal and cake obtained from *Facts for Industry*, Fats and Oils Issues, Bur. Census. Foreign trade and nonfeed uses of feed grains obtained by allocating U. S. data to each individual state according to the proportions set up by Jennings in *Feed Consumed by Livestock*, U. S. Dept. Agr. Stat. Bul. 145. All U. S. related data obtained from Stat. Bul. 159; Tech. Bul. 1183; Res. Rprt 21, all U. S. Dept. Agr. Some data obtained directly from U. S. Dept. Agr.

the U.S. data on each item for each of the years specified was allocated to each individual state according to the proportion of a state total to the U.S. total for the item estimated by Jennings for the 1949-50 feeding year.[1]

Structure of transportation costs

Given data relating to the regional demands for feed grains and high-protein feeds and the corresponding regional supply availabilities, the estimated equilibrium regional prices and flows are tied together by a specific structure of transportation rates. Therefore, for purposes of analysis it is necessary to obtain estimates of the transport costs between market and supply points that represent each pair of regions. To obtain these, the following procedure was used. Since feed grains and high-protein feeds are two collective terms and there are no specific rates constructed for them, observations on the commodity rail freight rates for corn, between selected origins and destinations, were obtained from the Transportation and Storage Service Division of the Commodity Stabilization Service of the U.S. Department of Agriculture. Mileage estimates between the selected origins and destinations were obtained and this information was combined with the commodity rate quotations and the following rail transport cost function was estimated:

$$T_{ij} = .142316 + \underset{(.0000561168)}{.0009356527 M_{ij}} - \underset{(.00000002391406)}{.00000016550142 M_{ij}^2}$$

$$r = .9206$$

where T_{ij} = the transport cost by rail for corn from the ith region to the jth region in dollars per 100 pounds; M_{ij} = the distance in miles between the ith region and the jth region; and r = the index of multiple correlation. The numbers in parentheses are the standard errors of their respective parameter estimates of the function. A total of 273 observations was used in estimating the equation (4.21). Since this function reached a maximum at approximately 2,825 miles, adjustment was made for distances exceeding this mileage.

[1] Ralph Jennings, *Feed Consumed by Livestock, Supply and Disposition of Feeds, 1949-50, by States,* U. S. Dept. Agr. Stat. Bul. 145, June, 1954.

Empirical Results: Regional Equilibrium Prices, Consumption, Surpluses and Deficits, and Final Optimum Flows of Feed Grains and High-Protein Feeds

Analyses were carried out for various periods of time and for alternative formulations by following the process outlined for determining regional equilibrium prices, consumption, regional surpluses and deficits, and the resulting geographical flows of feed grains and high-protein feeds.

Equal per animal unit consumption assumed for all regions, 1957-58

In this particular analysis, no regional market demand relationships (in functional form) were specified to estimate regional consumptions of feed grains and high-protein feeds. Rather, quantities of feed grains and high-protein feeds consumed per grain-consuming animal unit of livestock were assumed to be the same for each region and equal to the U.S. average consumption for each commodity. The main objective of this initial analysis was to provide some preliminary observations on the structure of price differentials, consumption patterns, surplus and deficit positions of regions, and the resulting optimum flow patterns for feed grains and high-protein feeds, and to provide an alternative way of generating regional consumption estimates.

Because of restrictions on the availability of data at the time the study was started, the 1957-58 feeding year is the most recent one analyzed. For this year, the quantities of feed grains and byproduct feeds available for livestock feeding and their actual consumption, both total and per animal unit, were largest among the years in the postwar period. The number of grain-consuming animal units of livestock and poultry fed was 161,499,000 (Table 6). During the postwar period, the number of grain-consuming animal units of livestock and poultry fed annually varied from a low of 153,098,000 units in 1947-48 to a high of 168,095,000 units in 1950-51. The yearly average price of livestock and livestock products received by farmers in the United States for 1957-58 was slightly below the average of the twelve postwar years considered. For the United States as a whole, the actual average price of corn received by farmers for 1957-58 was $36.40 per ton. This price was the lowest of the postwar years. The total quantity of feed concentrates, including feed grains and byproduct feeds, available for current livestock feeding in all thirty-seven states or regions for 1957-58 was 155,084,000 tons. Average per animal unit available supply of total feed concentrates was approximately 0.96 of a ton for the year.

The quantity of feed grains available for current livestock feeding in the 1957-58 season totaled 129,456,000 tons and the quantity of high-protein feeds available for feeding, 15,223,000 tons. For the 1957-58 feeding season, the average consumption of feed grains per animal unit assumed for each of the specified regions was 0.8016 ton, whereas the average per animal unit consumption of high-protein feeds was 0.0943 ton.

Since the per animal unit consumption of feed grains and high-protein feeds was assumed to be independent of geographical prices, regional total consumption depended entirely upon the regional numbers of grain-consuming animal units fed.

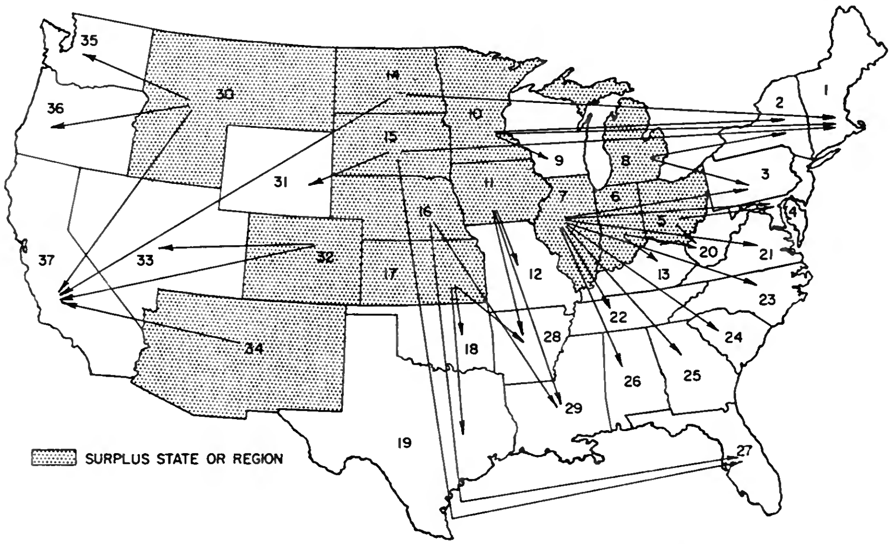
From predetermined availabilities of feed grains and high-protein feeds for feeding by regions and estimates of the regional total consumptions of both commodities, surplus and deficit regions were then derived (Table 5). The classification resulted in 13 surplus and 24 deficit regions for feed grains and 12 surplus and 25 deficit regions for high-protein feeds. The major surplus area for feed grains was the North Central Region and Kentucky which includes 13 states. No such concentrated surplus area for high-protein feeds was evident.

Given the estimates of the regional surpluses and deficits and a specific set of the rail transport costs, the problem is one of determining the spatial flow pattern for each commodity that would have minimized the total cost of transportation.

The final optimum shipment programs for feed grains and high-protein feeds for 1957-58 are presented in Figs. 6 and 7. The specific quantity of each type of feed that could have been feasibly shipped from every surplus area to each possible deficit area is given in tabular form under the maps in Figs. 6 and 7. Regional price differentials are given in the last column and in the bottom row.

As shown in Fig. 6, most of the surplus areas in the North Central Region except North Dakota and South Dakota shipped feed grains to the eastern and southeastern parts of the country. In the Dakotas, the eastward and westward shipments of feed grains were divided. Surplus areas in the Mountain Region shipped feed grains westward. Since the surplus areas of high-protein feeds in the 1957-58 feeding season were not as heavily concentrated as those of feed grains, excess quantities available at each surplus area were largely absorbed by the neighboring deficit regions (Fig. 7). The total volume of net shipments for feed grains and high-protein feeds was 27,575,000 and 4,466,000 tons, respectively. The transport costs that correspond to the total net shipments of these two commodities totaled $444,630,843 and $52,457,002.

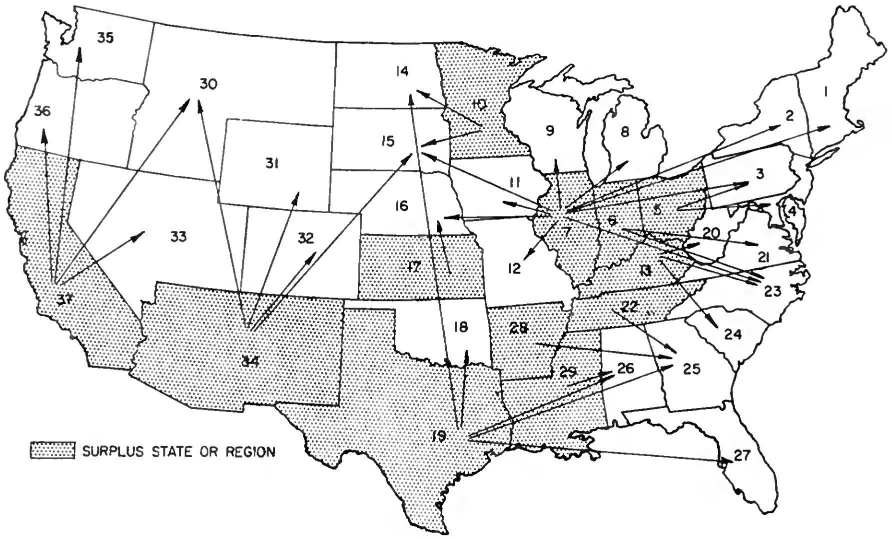
Final Optimum Geographical Flows of Feed Grains; Same per Animal Unit Consumption Assumed for all 37 States or Regions; 1957-58. (Fig. 6)



On the map, numbers indicate an area; arrows indicate an economically feasible route for shipment. Total transport cost would be $444,630,843. Data for surpluses, deficits, and differentials are given below.

Deficit state or region	Quantities that could be shipped from surplus to deficit states or regions (1,000 tons): 5	6	7	8	10	11	14	15	16	17	30	32	34	State or regional deficit, 1,000 tons	Differential per ton between deficit area and Illinois
1					1,282		321	1,211						2,814	$22.0
2				542	1,334									1,876	20.3
3			547	687										1,234	14.8
4	872		2,415											3,287	15.3
9					924									924	7.1
12						1,333								1,333	8.2
13		528												528	9.1
18										18				18	8.2
19										742				742	12.9
20	383	34												417	11.5
21			773											773	16.1
22			784											784	10.1
23			969											969	16.3
24			78											78	16.1
25			1,774											1,774	13.9
26			795											795	13.4
27								32	726					758	22.7
28						37				1,505				1,542	13.0
29						12			3,010					3,022	16.1
31								31						31	7.0
33												269		269	14.5
35											538			538	14.4
36											204			204	17.3
37							2,599				69	34	163	2,865	21.1
Area surplus, 1,000 tons	1,255	582	8,135	1,229	3,540	1,382	2,920	1,274	3,736	2,265	811	303	163	.....	....
Total shipment and total deficit, 1,000 tons (shipment = surplus)	....	...	.....	.....	.....	.....	.....	.....	.....	.....	...	...	...	27,575	....
Price differential per ton between surplus area and Illinois (dollars)	5.5	3.1	0	3.0	−.60	.45	4.1	−3.5	−1.4	1.2	1.3	−.40	5.4	....	....

Final Optimum Geographical Flows of High-Protein Feeds; Same per Animal Unit Consumption Assumed for all 37 States or Regions; 1957-58. (Fig. 7)



On the map, numbers indicate an area; arrows indicate an economically feasible route for shipment. Total transport cost would be $52,457,002. Data for surpluses, deficits, and differentials are given below.

Deficit state or region	Quantities that could be shipped from surplus to deficit states or regions: 5	6	7	10	13	17	19	22	28	29	34	37	State or regional deficit, 1,000 tons	Differential per ton between deficit area and Illinois
							1,000 tons							
1			212										212	$19.4
2			49										49	17.3
3	284		8										292	14.8
4	91												91	15.1
8			178										178	8.8
9			504										504	6.4
11			350										350	7.4
12			216										216	7.2
14				106			35						141	15.7
15			269	54							14		337	15.1
16			457			36							493	11.6
18							18						18	4.8
20					81								81	11.1
21		75											75	16.1
23		113	76		104								293	16.3
24					70								70	15.3
25							219	10	127				356	13.3
26							158			14			172	11.1
27							120						120	16.3
30											10	88	98	17.7
31											26		26	15.2
32											81		81	11.9
33												57	57	10.8
35												74	74	15.5
36												82	82	11.4
Area surplus, 1,000 tons	375	188	2,319	160	255	36	550	10	127	14	131	301	.....	....
Total shipment and total deficit, 1,000 tons (shipment = surplus)	...	...	.....	...	...	...	...	...	...	...	...	...	4,466	....
Price differential per ton between surplus area and Illinois (dollars)	5.3	2.9	0	5.2	4.7	4.3	−3.4	5.9	1.6	3.0	.43	−2.2	....	....

From the optimum shipment program for each commodity, a set of regional price differentials of feed grains and high-protein feeds was derived (Figs. 6 and 7). The resulting price differentials for feed grains varied from a low of −$4.10 per ton in North Dakota to a high of +$22.70 per ton in Florida, and for high-protein feeds varied from a low of −$3.40 per ton in Texas to a high of +$19.40 per ton in New England. Negative price differentials (relative to Illinois) of feed grains resulted in heavily surplus areas of the North Central Region, while negative price differentials of high-protein feeds resulted only in Texas and California.

Joint spatial model, 1957-58

In the previous analysis, per animal unit consumption of feed grains and high-protein feeds was assumed to be the same for all regions and no functional relation was used to estimate the regional demands. In this analysis, all regional prices and consumptions of feed grains and high-protein feeds are postulated as being simultaneously determined. They will be derived from the simultaneous system of the regional market demand relationships for feed grains and high-protein feeds specified in Part II of the study. The regional market demand relationships employed in this analysis were relations (4.15) and (4.16) of this section. The setting for this problem is the same as in the previous analysis.

Under this simultaneous framework, the resulting equilibrium prices of corn and high-protein feeds for the base region, Illinois, were $30.10 per ton and $70.70 per ton, respectively (Table 8). Regional equilibrium prices of feed grains varied from a low of $22.90 per ton in North Dakota to a high of $49.90 per ton in Florida. High-protein feed prices derived from the joint solution ranged from a low of $67.90 per ton in Texas to a high of $90.10 per ton in New England. As compared to Illinois, the base region, feed grain prices again were lower at the heavily surplus areas of the North Central Region and high-protein feed prices again were lower in Texas and California.

With the estimated regional equilibrium prices of corn and high-protein feeds and regional livestock prices, the regional consumption of feed grains and high-protein feeds per animal unit was estimated. In general, relatively more feed grains and high-protein feeds were fed to each livestock unit during the 1957-58 feeding season in the heavily surplus areas. The estimated per animal unit consumption of feed grains and high-protein feeds at the base region was 0.88 ton and 0.11 ton, respectively.

Table 8. — Regional Equilibrium Prices, Consumption, and Surpluses and Deficits of Feed Grains and High-Protein Feeds; Joint Spatial Model, 1957-58

State or region	Equilibrium prices		Equilibrium consumption		Surpluses and deficits	
	Feed grains	High-protein feeds	Feed grains	High-protein feeds	Feed grains	High-protein feeds
	dollars per ton		*1,000 tons*		*1,000 tons*	
1	49.0	90.1	2,408	293	− 2,288	− 160
2	47.3	84.1	1,798	99	− 957	172
3	44.8	84.8	3,154	315	− 517	− 152
4	45.0	85.1	2,911	386	− 2,911	− 90
5	35.5	75.3	5,180	557	1,403	455
6	33.7	73.2	3,240	911	334	101
7	30.1	70.7	12,177	1,545	7,002	2,073
8	33.0	79.5	2,402	151	1,349	− 32
9	34.1	79.7[a]	5,107	187	− 154	...
10	26.4	75.9	8,607	723	2,586	337
11	33.8	78.1	20,273	2,654	− 579	− 851
12	37.2	77.9	6,696	841	− 1,304	− 266
13	39.6	75.1	2,195	281	− 283	261
14	22.9	86.4	1,747	152	2,477	− 140
15	24.0	85.8	4,155	391	242	− 361
16	26.5	82.3	7,048	798	2,135	− 650
17	29.0	75.0	2,912	332	1,901	4
18	36.0	76.1	1,386	193	− 82	− 56
19	40.7	67.9	3,729	652	− 355	382
20	41.5	81.3	643	84	− 374	− 84
21	46.1	86.1	1,758	216	− 463	− 48
22	40.1	77.2	2,175	255	− 505	44
23	46.4	86.7	3,128	406	− 503	− 276
24	42.3[a]	85.6	1,045	118	.....	− 56
25	43.9	84.6	3,924	530	1,405	− 386
26	43.4	82.4	2,388	253	− 548	− 115
27	49.9	87.6	779	93	− 513	− 93
28	40.8	72.9	1,742	254	− 1,321	104
29	44.0	77.2[a]	2,535	368	− 2,535	...
30	30.0	90.0	1,252	108	595	− 84
31	34.5	87.6	272	36	− 80	− 36
32	28.5	84.3	1,071	125	70	− 107
33	43.4	83.1	510	71	− 232	− 64
34	32.4	72.8	456	65	113	114
35	43.1	87.9	797	77	− 432	− 45
36	46.0	83.8	720	100	− 93	− 84
37	48.1	70.1	3,129	603	− 1,818	189
Total			129,456	15,223	±20,297	±4,236

[a] These numbers were determined independently.

From the regional numbers of grain-consuming animal units and per animal unit consumption estimates, total regional consumption estimates of feed grains and high-protein feeds were derived. Iowa required the largest quantities of both feed grains and high-protein feeds for livestock feeding, 20,273,000 tons of feed grains and 2,654,000 tons of high-protein feeds. It was followed by Illinois which, for current feeding purposes, demanded 12,177,000 tons of feed grains and 1,545,000 tons of high-protein feeds.

Given the regional availabilities and demands for feed grains and high-protein feeds, the equilibrium solution for feed grains resulted in 12 surplus, and 24 deficit states or regions, and 1 self-sufficient state, and 12 surplus, 23 deficit, and 2 self-sufficient states or regions for high-protein feeds. South Carolina was self-sufficient for feed grains and Wisconsin and Louisiana and Mississippi were self-sufficient for high-protein feeds. As expected, the surplus areas of feed grains were mostly in the North Central Region. However, the surplus areas for high-protein feeds were more or less scattered over every section of the country.

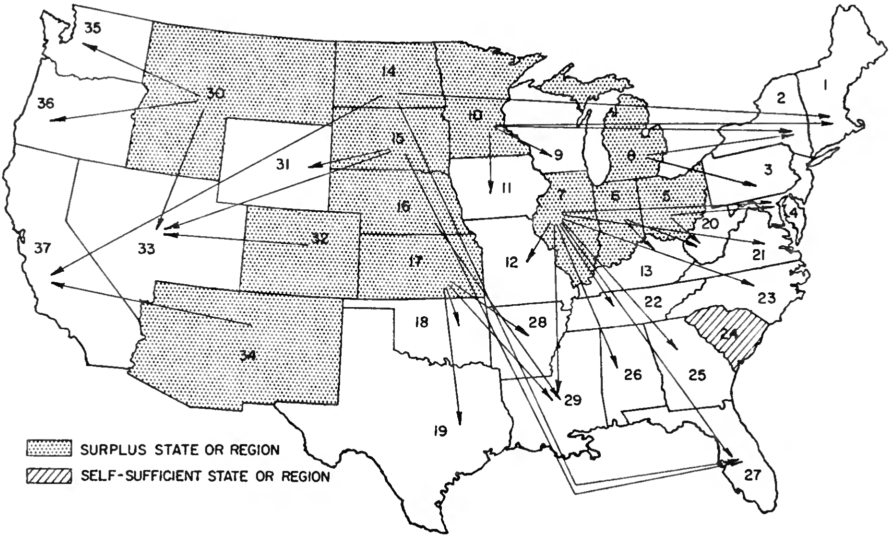
Given estimates of the regional surpluses and deficits and a specific set of rail transport costs, an optimum shipment program was derived for each of the specified commodities (Figs. 8 and 9). As in the spatial flow pattern in the previous analysis, all surplus areas except for North Dakota and South Dakota in the North Central Region shipped their feed grains eastward and southeastward. In the Dakotas, the eastward and westward shipments of feed grains were again divided. The surplus areas in the Mountain Region shipped feed grains westward. For high-protein feeds, no clear-cut shipping pattern was observable. The net trade involved totaled 20,297,000 tons of feed grains and 4,236,000 tons of high-protein feeds. The corresponding total transport costs required were $311,624,271 and $44,914,844, respectively. When these results were compared with those given in the previous analysis, it was noted that the net trade in feed grains and high-protein feeds and their attendant transport costs were much smaller under a simultaneous framework than under fixed regional demands.

The simple correlation coefficient between the actual regional prices of corn and those derived from the model was approximately 0.73.

Feed grains for livestock feed, for export, and for storage under predetermined regional prices of high-protein feeds, 1957-58

In the preceding analyses of this section, only feed grains and high-protein feeds for current livestock feeding were considered. For this analysis, the regional export and storage demands of feed grains are also taken into consideration. From the regional equilibrium prices of high-protein feeds, livestock prices, and regional export demands of feed grains, the purpose was to obtain simultaneously an optimum spatial solution for feed grains used both for livestock feed and for

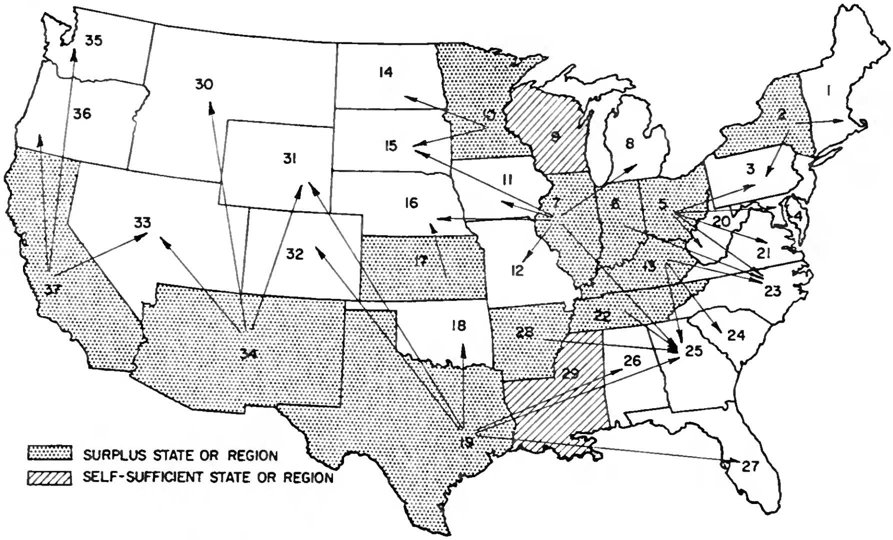
Final Optimum Geographical Flows of Feed Grains; Joint Spatial Model, 1957-58. (Fig. 8)



On the map, numbers indicate an area; arrows indicate an economically feasible route for shipment. Total transport cost would be $311,624,271. Data for surpluses, deficits, and differentials are given below.

Deficit state or region	Quantities that could be shipped from surplus to deficit states or regions (1,000 tons) 5	6	7	8	10	14	15	16	17	30	32	34	State or regional deficit, 1,000 tons	Differential per ton between deficit area and Illinois
1					1,728	560							2,288	$19.0
2				832	125								957	17.2
3				517									517	14.8
4	1,170		1,741										2,911	15.2
9			154										154	4.1
11			579										579	3.8
12			1,304										1,304	7.2
13		283											283	9.6
18									82				82	6.0
19									355				355	10.7
20	323	51											374	11.5
21			463										463	16.1
22			505										505	10.1
23			503										503	16.3
25			1,450										1,450	13.9
26			548										548	13.4
27			231			212	70						513	20.0
28									1,321				1,321	10.7
29			257					2,135	143				2,535	14.0
31							80						80	4.5
33							92			70	70		232	13.4
35										432			432	13.1
36										93			93	16.0
37						1,705						113	1,818	18.1
Area surplus, 1,000 tons	1,493	334	7,002	1,349	2,586	2,477	242	2,135	1,901	595	70	113	.....	....
Total shipment and total deficit, 1,000 tons (shipment = surplus)	.....	...	.....	.....	.....	.....	.....	.....	.....	...	...	...	20,297	....
Price differential per ton between surplus area and Illinois (dollars)	5.5	3.6	0	3.0	−3.6	−7.1	−6.0	−3.5	−1.0	0	−5	2.3	.....	....

Final Optimum Geographical Flows of High-Protein Feeds; Joint Spatial Model, 1957-58. (Fig. 9)



On the map, numbers indicate an area; arrows indicate an economically feasible route for shipment. Total transport cost would be $44,914,844. Data for surpluses, deficits, and differentials are given below.

Deficit state or region	Quantities that could be shipped from surplus to deficit states or regions (1,000 tons) 2	5	6	7	10	13	17	19	22	28	34	37	State or regional deficit, 1,000 tons	Differential per ton between deficit area and Illinois
1.	160												160	$19.4
3.	12	140											152	14.1
4.		90											90	14.4
8.				32									32	8.8
11.				851									851	7.4
12.				266									266	7.2
14.					140								140	15.7
15.				164	197								361	15.1
16.				646			4						650	11.6
18.								56					56	5.4
20.		84											84	10.6
21.		48											48	15.4
23.		93	101			82							276	16.0
24.						56							56	15.0
25.				114		123		1	44	104			386	14.0
26.								115					115	11.7
27.								93					93	17.0
30.											84		84	19.3
31.								10			26		36	17.0
32.								107					107	13.6
33.											4	60	64	12.4
35.												45	45	17.1
36.												84	84	13.1
Area surplus, 1,000 tons	172	455	101	2,073	337	261	4	382	44	104	114	189	.....	....
Total shipment and total deficit, 1,000 tons (shipment = surplus)	...	...	...	....	...	...	...	...	...	...	...	...	4,236	....
Price differential per ton between surplus area and Illinois (dollars)	13.4	4.6	2.5	0	5.2	4.4	4.3	−2.8	6.5	2.2	2.1	−.55	....	....

export and optimum allocation of feed grain stocks among all surplus regions that would minimize the total transport costs for shipments of feed grain surpluses among regions.

Since the setting in this analysis is somewhat different from those employed in other analyses, certain terms related to the analysis have to be redefined. *The available supply* in this analysis refers to the sum of the storage-stocks on October 1 of year t and the production for year t. This term, accordingly, does not account for the net changes in the yearly stocks. The available supply so defined may be channeled into exports, and livestock feed, or remain unused. Nonfeed uses of feed grains were not considered in this analysis. The term *demand* used here is defined as composed of the quantity consumed by domestic livestock and exports. Definitions of these two terms apply to both the national and regional levels.

The total available supply of feed grains in all 37 states or regions for 1957-58 was estimated to be 189,477,000 tons while the total demand was 140,746,000 tons. The total export demand of feed grains for all the regions in the year amounted to roughly 9,851,000 tons. The difference between the total available supply and the total demand of feed grains then gave the total amount of feed grains required for storage at the close of the 1957-58 feeding season. The ending stocks of feed grains totaled 48,731,000 tons for 1957-58.

Stocks of feed grains at the close of a feeding year in the regions were assumed to have some price-depressing effects for the next feeding year, but not to influence the regional prices of feed grains of the current year. Therefore, feed grains used for current purposes and exports only were assumed to affect the current regional prices. Stocks of feed grains under the market equilibrium were assumed to be held in surplus regions only. Regional equilibrium prices of high-protein feeds resulting from the previous simultaneous model were taken as given in this analysis.

Composed of the regional livestock feed demand and export demand of feed grains, the regional market demand relationships specified in this analysis may be expressed functionally as follows:

$$(y_{1i} + e_i) = (0.681149 + e_i) - 0.014212y_{2i} + 0.004253y_{3i} + 0.001121z_{1i} \qquad (4.22)$$

where e_i = the quantity of feed grains exported per grain-consuming animal unit in the ith region for 1957-58; y_{1i} = the per animal unit consumption of feed grains by domestic livestock in the ith region for 1957-58; and all other variables are as previously defined.

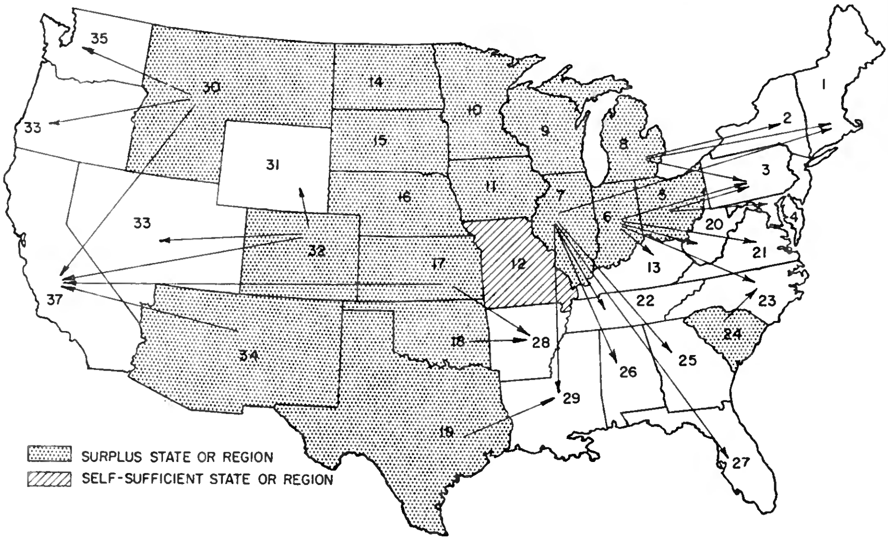
Since the regional exports of feed grains for 1957-58 were not known, they were estimated in the following manner: from the U.S. Department of Agriculture, *Grain and Feed Statistics, Supplement* for 1959, issued March 1960, the total exports of the four major feed grains for 1957-58 were known to be approximately 9,851,000 tons. The total quantity of inspection for export of feed grains by all ports for the year was derived from the monthly data in several issues of U.S. Department of Agriculture, *Grain Market News,* and estimated at 8,823,000 tons. The difference between the total exports by all means and the total quantity of inspection by ports, 1,028,000 tons, was then assumed to be the quantities exported by truck and rail to Canada and Mexico and some unreported inspections by ports.

Corn was assumed to be the main feed grain shipped to Canada and Mexico by rail and truck. The total quantities of corn exported to Mexico and Canada were 1,126,000 tons and 321,000 tons, respectively. The total quantities of corn inspected for export by ports to Mexico and Canada in 1957-58 were 427,000 tons and 271,000 tons, respectively (see U.S. Department of Agriculture, *Feed Situation,* November, 1960, p. 36). Given these quantities, the total exports of corn to Mexico and Canada by rail and truck were estimated at 699,000 tons and 50,000 tons, respectively.

In 1957-58, no shipments of oats, barley, and sorghum grain were made to Canada and approximately 32,000 tons of oats, barley, and sorghum grain were exported to Mexico via Texas by truck and rail. Unreported inspections for export of corn by ports in the United States for 1957-58 were estimated at 247,000 tons. These magnitudes were then allocated to each surplus area in the Southern Plain Region, the Lake Region, and the North Central Region, according to the relative size of inspections for export by ports in the area. Given the quantities inspected for export by ports by regions and the quantities estimated for the shipments by rail and truck to Mexico and Canada, the regional export demands for feed grains were then derived.

Given the purpose of the analysis, along with the assumptions and definitions discussed, a spatial analysis was carried out. The resulting regional equilibrium prices of feed grains varied from a low of $30.30 per ton for the heavily surplus areas of the North Central Region to a high of $54.20 per ton in California. No price differential between any of the heavily surplus areas in the North Central Region and the base region (Illinois) resulted, since feed grain surpluses in heavily surplus areas other than Illinois were not shipped and went to fulfill regional storage demands (Fig. 10).

Final Optimum Geographical Flows and Allocation of Feed Grains for Livestock Feed, for Export, and for Storage all Considered as the Regional Prices of High-Protein Feeds, 1957-58. (Fig. 10)



On the map, numbers indicate an area; arrows indicate an economically feasible route for shipment. Total transport cost would be $249,851,885. Data for surpluses, deficits, and differentials are given below.

Deficit state or region	5	6	7	8	9	10	11	14	15	16	17	18	19	24	30	32	34	State or regional deficit, 1,000 tons	Differential per ton between deficit area and Illinois
	Quantities that could be shipped from surplus to deficit states or regions (1,000 tons)																		
1			691	1,794														2,485	$19.4
2				260														260	17.1
3	122	180		256														558	14.6
4	2,730																	2,730	14.8
13		152																152	8.6
20		367																367	11.0
21		2,137																2,137	15.8
22			421															421	10.0
23		304												40				344	16.1
25			1,197															1,197	13.9
26			1,343															1,343	13.4
27			298															298	19.9
28											984	129						1,113	11.8
29			1,555										903					2,458	14.0
31																11		11	10.5
33																129		129	17.3
35															1,102			1,102	17.2
36															284			284	20.0
37											385				104	361	291	1,141	23.9
Amount stored, 1,000 tons			5,560		1,239	10,170	12,593	4,028	4,142	8,350	2,649								
Area surplus, 1,000	2,852	3,176	11,065	2,310	1,239	10,170	12,593	4,028	4,142	8,350	4,018	129	903	40	1,490	501	291	.....	....
Total shipment and total deficit, 1,000 tons (shipment = surplus)	....	....	......	....	.....	......	......	.....	.....	.....	.....	...	...	..	.....	...	...	18,566	....
Price differential per ton between surplus area and Illinois (dollars)	5.0	2.6	0	2.8	0	0	0	0	0	0	0	2.8	3.9	9.4	4.0	2.4	8.1	.....	....

The equilibrium solution for the 37 states or regions resulted in 17 being surplus, 19 deficit, and 1 self-sufficient. Missouri was a self-sufficient region in the analysis. Major surplus areas of feed grains were in the North Central Region.

Given the estimates of regional feed grain requirements for livestock feeding, export, and regional available supplies, a final optimum flow program for feed grains as livestock feed and a final optimum allocation program for feed grain stocks between surplus regions that would minimize total transport costs of shipments of feed grain surpluses were simultaneously derived (Fig. 10).

Surplus areas in the eastern half of the North Central Region and the Southern Plain Region provided almost all of the feed grains required by the deficit areas in the eastern and southeastern portions of the nation. All the feed grain surpluses of the surplus areas in the western half of the North Central Region were held in their own regions as unutilized stocks, since all deficit regions in the country could satisfy their excess demands from their neighboring or closer surplus areas at lower costs. Just as in the other analyses, the surplus areas in the Mountain Region shipped their feed grain surpluses westward. Kansas was the point connecting the eastward and westward shipments of feed grains.

In this analysis, the total net trade for feed grains as livestock feed was 18,566,000 tons and the attendant transport costs required for the shipments totaled $249,851,885. As compared to other analyses of this chapter, it was noted that the total net trade and the total transport costs required for the shipments were much less in this analysis than in the simultaneous framework analyses and in the fixed regional demand analysis.

In determining an optimum flow program for feed grains where total available supply exceeds total demand, feed grain stocks were treated as a dummy destination and transport cost between any surplus origin and this dummy destination was assumed to be zero. As previously assumed, the feed grain stocks would be held in surplus regions only.

With the introduction of a dummy destination in the flow program, the optimum allocation of feed grain stocks among the surplus regions resulted. From the solution, the feed grain stocks were held in Illinois, Wisconsin, Minnesota, Iowa, North Dakota, South Dakota, Nebraska, and Kansas. Minnesota and Iowa were estimated to be the leading holders of feed grain stocks in 1957-58.

The simple correlation coefficient between the actual regional price of corn and those given in this analysis was 0.78. The degree of this conformity is higher than those in other analyses.

Joint spatial model, 1954-55

So far, spatial analyses have been carried out for the 1957-58 feeding year under various formulations. The purpose of the following analyses was to extend the investigation to years reflecting a different set of characteristics. Therefore the objective of this analysis was to obtain simultaneously regional equilibrium prices, consumption, regional surpluses and deficits, and the resulting optimum spatial flows of feed grains and high-protein feeds from a system of regional market demand relationships for 1954-55.

The 1954-55 feeding year was regarded in the study as an average year for the postwar period in terms of availabilities and consumption of feed grains and high-protein feeds. For this year, total quantities of feed grains and high-protein feeds available for current livestock feeding in all 37 states or regions were 104,912,000 tons and 13,108,000 tons, respectively. The number of grain-consuming animal units of livestock and poultry fed was 161,458,000. The annual average price index of livestock and livestock products received by farmers in the United States was 239.2 (1910-1914 = 100). For the United States as a whole, the actual average price of corn received by farmers for 1954-55 was $50.70 per ton, while the average price of high-protein feeds at wholesale level was $84.80 per ton.

The basic data for the spatial analysis are changed, because characteristics of availabilities, consumption patterns, and prices of feed, and livestock numbers vary from year to year. While regional demarcation and rail transport costs remained the same as those for 1957-58, new regional market demand relationships for feed grains and high-protein feeds and new regional values of the predetermined variables were employed for the 1954-55 analysis.

Under a simultaneous framework, the resulting joint equilibrium prices of feed grains and high-protein feeds for the base region, Illinois, in 1954-55 were $44.40 and $78.30 per ton. Regional equilibrium prices for feed grains varied from a low of $39.80 per ton in North Dakota to a high of $65.10 per ton in California, and for high-protein feeds from a low of $76.90 per ton in Texas to a high of $104.80 per ton in Washington. As in 1957-58, the equilibrium prices of feed grains and high-protein feeds in their respective heavily surplus areas were low relative to the base region prices.

The quantities of feed grains and high-protein feeds consumed per grain-consuming animal unit were estimated from the regional equilibrium prices of feed grains and high-protein feeds and livestock prices. In the base region, the resulting per animal unit of consumption of feed grains was 0.683 ton and of high-protein feeds was 0.088 ton. As in 1957-58, Illinois and Iowa were estimated to be the two largest areas of total consumption of feed grains and high-protein feeds for the 1954-55 feeding season.

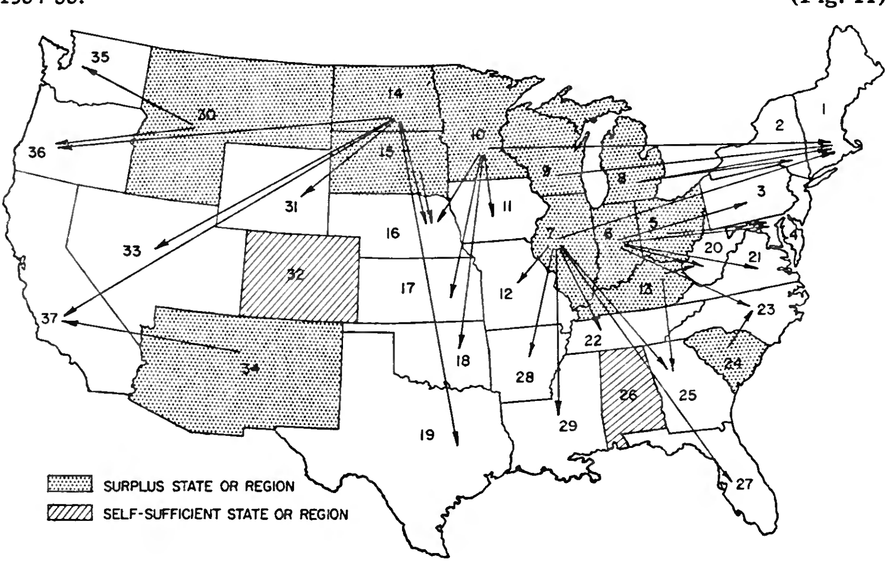
Comparisons between the regional availabilities for livestock feeding and estimated regional equilibrium consumptions led to the classification of regions into 12 surplus, 23 deficit, and 2 self-sufficient regions for feed grains and 12 surplus, 24 deficit regions, and 1 self-sufficient region for high-protein feeds. Self-sufficient areas were Alabama and Colorado for feed grains, and Wisconsin for high-protein feeds. The breakdown of regional surpluses and deficits for 1954-55 was slightly different from that for 1957-58.

From the regional surpluses and deficits for both feed grains and high-protein feeds and a specific set of the rail transport costs, a final optimum flow program for each commodity was obtained, Figs. 11 and 12.

Surplus areas in the Mountain Region moved their excess surpluses of feed grains westward, whereas most surplus areas in the North Central Region shipped their excess supplies east and southeast. Nebraska and Kansas were deficit areas in feed grains for the 1954-55 feeding season, while they were surplus in 1957-58. This change in the classification of regions from the 1957-58 season made Minnesota the connecting link between the eastward and westward shipments of feed grains. Analogous to the 1957-58 situation, no clear-cut picture for high-protein feeds in direction of shipments was observed. Texas, however, still played an important role in connecting the eastward and westward shipments of high-protein feeds. In 1954-55, Lousiana and Mississippi became a surplus area for high-protein feeds, while it was classified as a self-sufficient region in 1957-58. Consequently, Louisiana and Mississippi shipped to most of regions in the east previously supplied by Texas. Illinois shipped its surpluses of high-protein feeds in almost all directions. The shipping pattern of high-protein feeds for the western half of the country for 1954-55 was similar to that for 1957-58.

Net trade among regions totaled approximately 18,282,000 tons for feed grains, and 3,930,000 tons for high-protein feeds in the 1954-55 season. Total transport costs that correspond to these net trades were $241,405,415 for feed grains and $43,357,391 for high-protein feeds. As compared to the 1957-58 feeding season, total net trades and their attendant transport costs were much less in 1954-55.

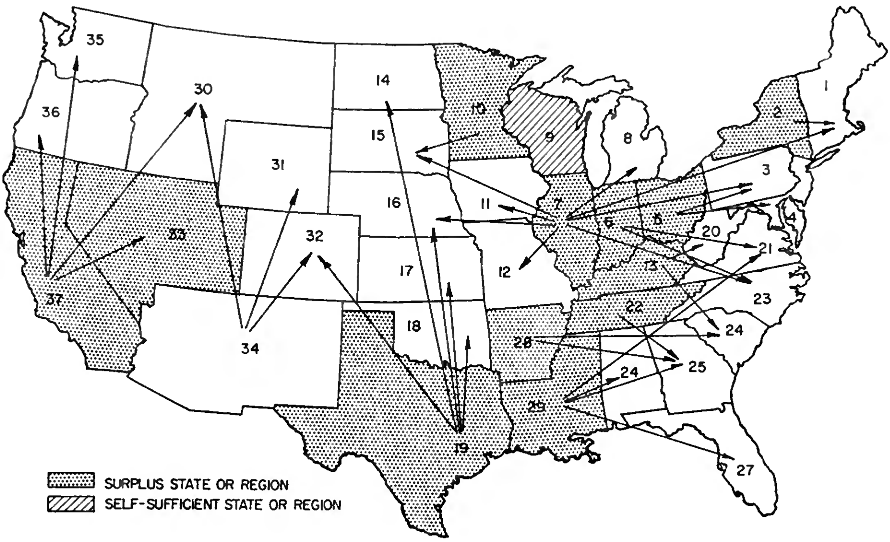
Final Optimum Geographical Flows of Feed Grains; Joint Spatial Model, 1954-55. (Fig. 11)



On the map, numbers indicate an area; arrows indicate an economically feasible route for shipment. Total transport cost would be $241,405,415. Data for surpluses, deficits, and differentials are given below.

Deficit state or region	Quantities that could be shipped from surplus to deficit states or regions												State or regional deficit, 1,000 tons	Differential per ton between deficit area and Illinois
	5	6	7	8	9	10	13	14	15	24	30	34		
							1,000 tons							
1..........			540	359	438	771							2,108	$19.4
2..........				767									767	17.1
3..........		1,137											1,137	14.4
4..........	2,558	29											2,587	14.9
11..........						1,764							1,764	4.2
12..........			190										190	7.2
16..........						120		1,206	1,016				2,343	7.0
17..........						113							113	10.5
18..........						419							419	12.7
19..........								261					261	14.5
20..........		293					111						404	10.8
21..........		584											584	15.6
22..........			26										26	10.1
23..........		117								86			203	15.8
25..........			939				80						1,019	13.9
27..........			406										406	19.9
28..........			473										473	11.9
29..........			1,624										1,624	14.0
31..........								32					32	7.4
33..........								131					131	15.9
35..........											641		641	15.6
36..........								47			152		199	18.5
37..........								574				278	852	20.6
Area surplus, 1,000 tons...	2,558	2,160	4,198	1,126	438	3,187	191	2,251	1,016	86	793	278	.....	....
Total shipment and total deficit, 1,000 tons (shipment = surplus)....	.....	.....	.....	.....	.....	.....	.....	.....	.....	...	...	...	18,282	....
Price differential per ton between surplus area and Illinois (dollars)....	5.1	2.4	0	2.8	−.37	−3.2	4.4	−4.6	−1.4	49.2	2.5	4.9	....	....

Final Optimum Geographical Flows of High-Protein Feeds; Joint Spatial Model, 1954-55. (Fig. 12)



On the map, numbers indicate an area; arrows indicate an economically feasible route for shipment. Total transport cost would be $43,357,391. Data for surpluses, deficits, and differentials are given below.

Deficit state or region	Quantities that could be shipped from surplus to deficit states or regions												State or regional deficit, 1,000 tons	Differential per ton between deficit area and Illinois
	2	5	6	7	10	13	19	22	28	29	34	37		
						1,000 tons								
1	101			111									212	$19.4
3		29		156									185	14.8
4		239											239	15.0
8				42									42	8.8
11				715									715	7.4
12				140									140	7.2
14							91						91	15.7
15				40	260								300	15.0
16				647			9						656	11.6
17							94						94	10.3
18							94						94	6.8
20						93							93	11.2
21			123							3			126	16.1
23			6	227									233	16.3
24						7			13	61			20	15.4
25								82	109	98			252	13.9
26										31			98	11.2
27													31	17.8
30											46	30	76	23.3
31											26		26	18.3
32							22				62		84	14.9
33												47	47	20.1
35												11	11	26.5
36												65	65	22.4
Area surplus, 1,000 tons	101	268	129	2,078	260	100	310	82	122	193	134	153	.....	....
Total shipment and total deficit, 1,000 tons (shipment = surplus)	....	...	....	....	....	....	....	....	....	...	...	...	3,930	....
Price differential per ton between surplus area and Illinois (dollars)	13.4	5 3	2.9	0	5.2	4.8	−1.4	6.5	2.2	3.2	6.1	8.8	....	....

The simple correlation coefficient between the actual regional prices of corn and those given by the model for the 1954-55 feeding season was 0.64. Accordingly, this suggests that merely 41 percent of the total variation in the actual regional prices of corn received by farmers for 1954-55 was explained by the model employed. The degree of conformity was therefore not as high as that for 1957-58.

Joint spatial model, 1947-48

The purpose of this analysis was to obtain simultaneously the regional equilibrium prices, consumption, the regional surpluses and deficits, and the resulting optimum spatial flows of feed grains and high-protein feeds from a system of the regional market demand relationships for 1947-48.

The 1947-48 feeding year was characterized by a severe drought. For this year, the total available supply and consumption of feed concentrates in the United States were the lowest among the postwar years. Compared to other postwar years, average prices of corn, of high-protein feeds, and of livestock and livestock products were relatively high and the number of grain-consuming livestock units fed was relatively low. In 1947-48 for current feeding of domestic livestock total available quantities of feed grains were about 130,241,000 tons and of high-protein feeds, 10,239,000 tons. The number of grain-consuming animal units fed annually in the 37 states or regions was 153,098,000. National average prices of corn received by farmers were $77.10 per ton and of high-protein feeds at wholesale level, $105.90 per ton. The national average price index (1910-14 = 100) of livestock and livestock products received by farmers was approximately 315.3.

As previously stated, characteristics of availabilities, consumption patterns, prices of feed, and numbers of livestock differ from one year to another. A new set of basic data, therefore, was employed for the 1947-48 analysis. More specifically, new regional market demand relationships for feed grains and high-protein feeds and new regional values of predetermined variables were specified for the analysis. The regional demarcation and rail transport costs used for the 1947-48 analysis, however, were the same as those for the 1954-55 analysis.

Given the simultaneous framework, the resulting equilibrium prices for feed grains varied from a low of $67.00 per ton in North Dakota to a high of $92.70 per ton in Florida and for high-protein feeds from a low of $100.60 per ton in Illinois to a high of $124.10 per ton in Montana and Idaho. In Illinois, the base region, equilibrium prices of feed grains were $72.90 per ton and high-protein feeds, $100.60 per ton.

From estimates of the regional equilibrium prices for feed grains and high-protein feeds, the regional consumption of feed grains and high-protein feeds per animal unit was derived from the specified market demand relationships. For the base region, per animal unit consumption of feed grains was 0.88 ton and of high-protein feeds, 0.075 ton.

The joint equilibrium solution resulted in 10 surplus, 25 deficit, and 2 self-sufficient states or regions for feed grains and 12 surplus, and 24 deficit states or regions, and 1 self-sufficient state for high-protein feeds. Self-sufficient areas so determined were Kansas and New Mexico and Arizona for feed grains and Alabama for high-protein feeds.

From estimates of regional surpluses and deficits and transport costs, the optimum geographical flows of feed grains and high-protein feeds were determined (Figs. 13 and 14).

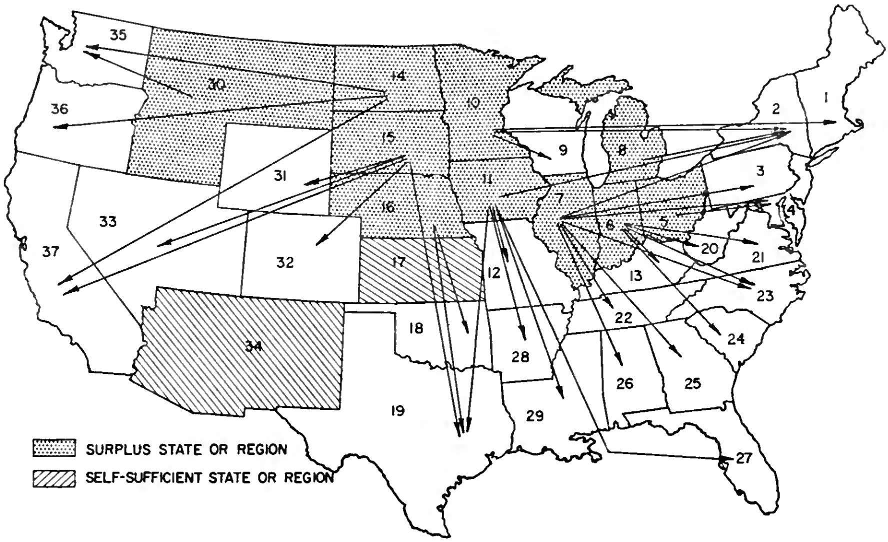
The final optimum shipment programs for feed grains and high-protein feeds in 1947-48 differ considerably from those for 1954-55 and 1957-58. For the 1947-48 feeding year, North Dakota shipped its feed grain surpluses westward only. In this analysis, unlike the analyses for the other years, Iowa became a surplus and Colorado became a deficit state for feed grains. Moreover, New Mexico and Arizona changed from a surplus to a self-sufficient area. South Dakota was the point where movements of feed grains to the eastern and western half of the country were connected. For high-protein feeds, Texas became a deficit state. This change made Illinois the point where the eastward and westward shipments of high-protein feeds met.

In 1947-48, total net trade of feed grains was estimated at 25,354,000 tons and high-protein feeds at 2,589,000 tons. The corresponding transport costs were about $404,597,433 and $29,296,995. Total volume of net shipments and attendant costs for feed grains were large relative to the other years analyzed. Net trade and attendant transport costs for high-protein feeds, however, were smaller.

The simple correlation coefficient between the actual regional prices and those generated by the model was unusually low. It was computed to be 0.41. As would be expected, the severe drought year, 1947-48, showed an abnormal price pattern of corn which was slightly related to that given by the model. A previous study for 1947-48 by Karl A. Fox also resulted in a very low coefficient of determination of the actual regional prices of corn.[1]

[1] Karl A. Fox, *Econometric Analysis for Public Policy* (1958), Ch. 9, p. 188.

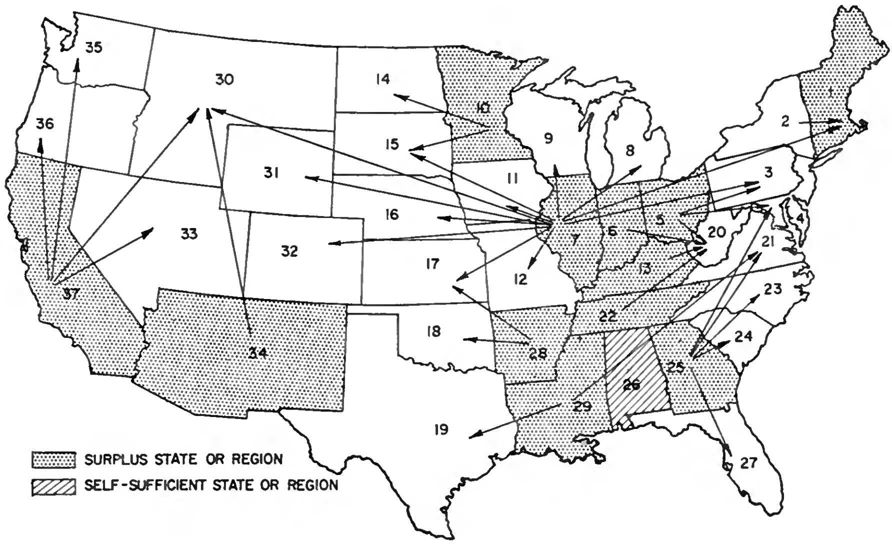
Final Optimum Geographical Flows of Feed Grains; Joint Spatial Model, 1947-48. (Fig. 13)



On the map, numbers indicate an area; arrows indicate an economically feasible route for shipment. Total transport cost would be $404,597,433. Data for surpluses, deficits, and differentials are given below.

Deficit state or region	Quantities that could be shipped from surplus to deficit states or regions										State or regional deficit, 1,000 tons	Differential per ton between deficit area and Illinois
	5	6	7	8	10	11	14	15	16	30		
	1,000 tons											
1...........					2,394						2,394	$19.1
2...........			96	222	153	1,138					1,609	17.3
3...........			1,868								1,868	14.8
4...........	1,105		1,874								2,979	15.3
9...........					755						755	4.2
12...........						917					917	5.1
13...........		264									264	8.8
18...........									789		789	8.7
19...........						269		1,069	1,845		3,183	13.1
20...........		496									496	11.2
21...........		645									645	16.1
22...........			365								365	10.1
23...........		34	560								594	16.3
24...........		408									408	15.9
25...........			1,057								1,057	13.9
26...........			268								268	13.4
27...........						653					653	19.8
28...........						728					728	9.9
29...........						2,155					2,155	13.0
31...........								121			121	4.9
32...........								239			239	6.1
33...........								327			327	13.9
35...........							725		7		732	15.3
36...........							314				314	17.3
37...........							325	1,169			1,494	19.4
Area surplus, 1,000 tons...	1,105	1,847	6,088	222	3,302	5,860	1,364	2,925	2,634	7	.....	....
Total shipment and total deficit, 1,000 tons (shipment = surplus)....	.....	.....	.....	.....	.....	.....	.....	.....	.....	..	25,354	....
Price differential per ton between surplus area and Illinois (dollars)....	5.5	2.9	0	3.1	−3.5	−2.6	−5.9	−5.6	−1.9	2.2	.....	....

Final Optimum Geographical Flows of High-Protein Feeds; Joint Spatial Model, 1947-48. (Fig. 14)



On the map, numbers indicate an area; arrows indicate an economically feasible route for shipment. Total transport cost would be $29,296,995. Data for surpluses, deficits, and differentials are given below.

Deficit state or region	2	5	6	7	10	13	22	25	28	29	34	37	State or regional deficit, 1,000 tons	Differential per ton between deficit area and Illinois
	Quantities that could be shipped from surplus to deficit states or regions (1,000 tons)													
1.	134			7									141	$19.4
3.		89		116									205	14.8
4.		97						7					104	15.0
8.				95									95	8.8
9.				51									51	6.3
11.				192									192	7.4
12.				84									84	7.2
14.					91								91	15.7
15.				218	6								224	15.1
16.				426									426	11.6
17.				129					19				148	12.6
18.									73				73	9.8
19.										74			74	11.0
20.		31	8			25	23						87	11.2
21.								69		46			115	15.8
23.								82					82	13.6
24.								27					27	10.7
27.								43					43	14.6
30.				15							21	35	71	23.4
31.				24									24	18.2
32.				77									77	17.1
33.												48	48	16.6
35.												52	52	21.3
36.												55	55	17.2
Area surplus, 1,000 tons. . .	134	217	8	1,434	97	25	23	228	92	120	21	190	. . . . .	. . . .
Total shipment and total deficit, 1,000 tons (shipment = surplus). . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . .	. . .	. . .	2,589	. . . .
Price differential per ton between surplus area and Illinois (dollars). . . .	13.4	5.3	2.9	0	5.2	4.9	2.3	3.7	.88	.97	6.2	3.6	. . . .	. . . .

Part IV. — Implications of the Results and Possibilities for Further Research

Knowledge of the structural relationships of a particular segment of the general economy is not only a prerequisite for spatial analyses, but also useful to decision makers at both government and firm levels. The construction of structural relationships and the attendant connecting parameter estimates, such as those in this study, provide a basis for: (1) describing the connection between certain relevant variables that existed in the past; and (2) assessing systematically the potential probable quantitative impact of various economic policy actions (changes in the levels of certain variables that are under the policy maker's control) on the future time path of certain economic variables in a particular sector of the economy. With knowledge of the estimates of future prices and demands, a firm could adjust production plans or resource allocation to more nearly meet profit and efficiency objectives.

The specification and solution of spatial models such as those employed in this study yield implications for many areas of research involving economic choice. By employing this type of model, the efficiency and competitive structure of individual sectors may be investigated and knowledge may be obtained relative to problems of structure and comparative statics when the consequences of changes or actions are desired.

Methodologically, this type of spatial model treats the space factor explicitly and offers an efficient approach to the determination of regional prices, consumption patterns, and the resulting geographical flows of commodities. From the point of view of economic policy, the specified spatial model offers an operational tool to the policy maker for answering questions of a comparative static nature, and thus indicates the consequences and repercussions of certain policy actions or changes in the data on the optimum values of geographical prices, differentials, and flows of commodities. As a basis for policy action, the perfectly competitive market concept used in formulating the spatial price equilibrium model provides a criterion of comparison whereby the pricing and distribution of a commodity can be judged as efficient or inefficient relative to this base.

From the standpoint of producing firms, spatial price equilibrium analysis of this type provides information about how changes in transportation, and geographical distribution, and level of livestock numbers, and feed supply might alter prices and flows of the commodities in question. These expected prices could then be used as a basis for resource adjustments. By introducing the time dimension, insights into the changing character of the industry in question may be discerned and the long-term competitive position of one region relative to another can be analyzed.

From the standpoint of the distributing firm, the analysis specified provides a rationale for the choice among alternative geographical destinations of product shipments. Corresponding to a given set of product flows, a unique set of regional price differentials can be derived with the aid of the duality theorem of linear programming. The result of the spatial price equilibrium analysis may then be utilized by the distributing firm in ascertaining the comparative or location advantage of one region relative to others.

Currently, alternative programs for feed grains are being considered by the Secretary of Agriculture. In evaluating alternative programs, the sector and spatial models, such as those that have been employed in this study, could be used as a basis for assessing the total and regional consequences of such actions. For example, what would be the impact on the consumption of feed grains of a 10-percent increase in their price? How would this change in price affect regional prices, consumption, and flows? In addition, since weather is an important factor in determining the total and regional output of feed grains, analyses of this type give some indications where stocks should be accumulated in good years so as to optimally satisfy the demands in drought or other years when output is relatively low. The impact of the changing structure of cattle feeding on the regional prices and flows of feed grains is another area in which analyses of this type may be employed. Similar analyses also apply to the high-protein feed sector.

For the sector and spatial equilibrium models presented in this study, certain restrictive assumptions which condition the interpretation of the results were made. In order to make the study more realistic, the relaxation of certain assumptions and the possibilities of extending the present models may now be considered.

Since feed is the key input factor for producing livestock, a more realistic sector model for the feed economy should probably include the structural relationships of the livestock sector. Moreover, inclusion of the export and storage demand relationships for various major feeds in the models is desirable.

For spatial equilibrium models, the predetermined nature of the regional supplies of feeds may be relaxed and the construction of a separate demand function for each major feed or a more disaggregated group of feeds in each region may be considered. The differences in the transport costs for each major feed and for each freight territory should be recognized. Allowance for accumulation and depletion of inventories and the fact that all demands are not entirely met from current production suggest that the dynamic aspects of the spatial equilibrium models for feed need to be investigated. As a first step toward the dynamic approach, the quarterly or semi-annual models should be constructed.

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