SELECTION OF FRESHENING DATES THAT WILL MAXIMIZE DAIRY PROFITS

An Application of Linear Programming Using DHIA Data From Northern Illinois

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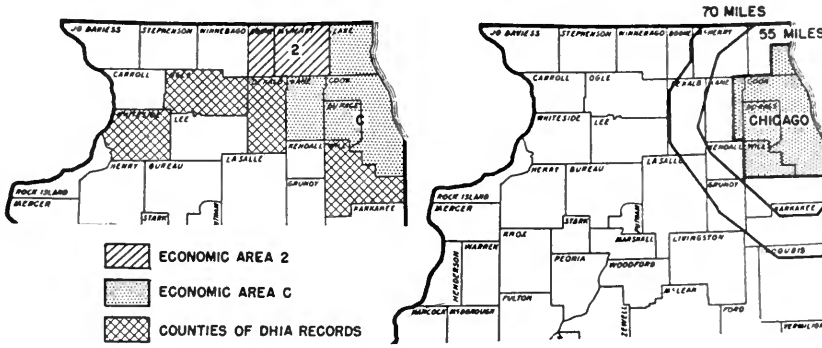

(Left) location of Illinois economic areas 2 and C and of DHIA records from which lactation curves were derived; *(right)* Chicago milk market. (Fig. 1)

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DAIRY HERD MANAGEMENT is affected by numerous seasonal variations — quantity and quality of milk per cow, quantity and quality of feed supplied by the farm, labor available for the farm as a whole and for the dairy enterprise, and prices for milk and milk products, as well as resources bought for the dairy enterprise.

The results of a study to select freshening dates that maximize annual income from a dairy herd are reported here.[1] The results apply most specifically to a herd in the Chicago milkshed (Fig. 1). They would differ little elsewhere, however, in the presence of similar seasonal price movements, provided that production conditions were not too different from those in the Chicago milkshed. These conditions, with details of method and findings, are described later.

REVIEW OF LITERATURE

In an early study of 10,870 cows, over the period 1910-1920, McPowell (7)[2] found that cows freshened in fall months produced the highest average annual milk output. Lowest production came from cows freshened in spring months. Neither breed nor age was significantly related to the influence of freshening dates on milk production.

Similar results were found by Morrow, Keener, and Hall (8) in a study of dairy herd management in New Hampshire, and by Dow, Johnson, and Parsons (2), a regional group studying dairying in a 12-state area in the northeast, including New York. Production variation by season of calving was ascribed generally by these investigators to climate, feeding rates, management practices, and other related factors. This regional group found that older cows showed a greater output response to month of calving than did younger cows.

From 15,442 records of cows in Dairy Herd Improvement Associations, T. E. Woodward (9) found November fresheners to yield the highest total production; July fresheners, the lowest. Highest peaks of production were exhibited by April fresheners. High points on their lactation curves coincided with the lush pasture season of May and June. Cows calving in August produced the lowest peaks of production.

These seasonal variations accord well with the findings of Baum, Mey, and Shaw (1) who fitted monthly lactation curves to milk-production data from 244 individual yearly records of cows in western

[1] Selection was made with use of linear programming, a mathematical technique that assures a maximum net income, given stated conditions. (For further discussion, see Appendix B, p. 30).

[2] Numbers in parentheses refer to Literature Cited.

Washington. The Washington investigators, however, made no attempt to relate total volume of milk to freshening dates.

Fat content of milk is affected by temperature and stage of lactation. Woodward (9) found fat content to be lowest in the third month, and to increase thereafter with a more rapid rise toward the end of the lactation period. He concluded that fat content is related inversely to milk quantity except after peak production, when both decline for a short period. Fat content tends to be lowest during the hot months of summer and highest in the cool months of fall and winter.

Baum, Mey, and Shaw (1) found that in western Washington cows freshened in August and through September were cheapest to feed. Cows freshened in January and through March tended to be the most expensive. These results originate from seasonal variation in feed prices and in farm supplies of feed, especially pasture. Cows in lush pasture months (freshened the preceding July and August) convert pasture feed into body maintenance and weight gain. On the other hand, pasture feed is converted into peak period milk production for cows freshened in late winter. However, grain prices are lowest in early fall months, benefiting the fall-freshened cows, then in peak production. Cows freshened in March and April, on the other hand, required the most grain when grain prices were highest. Similar results are given in studies by McPowell (7), Dow, Johnson, and Parsons (2), and by Morrow, Keener, and Hall (8).

PRODUCTION CONDITIONS

This study is based on 42,747 monthly records of cows in all-Holstein herds in Ogle, Whiteside, Will, and DeKalb counties, in northern Illinois. The records were taken in DHIA (Dairy Herd Improvement Associations) between January, 1955, and August, 1957. The lactation period is assumed to continue for 305 days from the middle of the month shown as "month of freshening" (Table 1).

The records accord with the findings of others in indicating that the length of lactation period is largely independent of month of calving. The records indicate that December fresheners produced the highest annual output of milk, an average of 11,905 pounds per cow. August fresheners produced the lowest annual output, 10,450 pounds. When monthly production is expressed as a percent of annual production, the highest peak, 15 percent, is yielded by May fresheners in June; the lowest peak, 12 percent, by January fresheners in April and May.

The fact that the cows were in DHIA creates selectivity in the observations with respect to numerous management factors. The annual production reported in Table 1 far exceeds the average for the state as

Table 1. — Pounds Produced per Month and Total per Year, by Month of Freshening: Cows From All-Holstein Herds in the Dairy Herd Improvement Associations, Northern Illinois[a]

Month of production	Month of freshening											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Jan.	741	0	323	738	842	927	976	1,028	1,147	1,311	1,464	1,561
Feb.	1,565	764	0	343	772	857	900	942	1,052	1,223	1,324	1,504
Mar.	1,498	1,531	782	0	348	741	824	888	982	1,147	1,235	1,394
Apr.	1,366	1,452	1,562	761	0	345	741	820	927	1,052	1,128	1,257
May	1,357	1,461	1,626	1,693	819	0	368	814	927	1,049	1,131	1,272
June	1,232	1,324	1,470	1,613	1,668	817	0	349	769	939	1,052	1,171
July	1,095	1,159	1,290	1,424	1,549	1,623	753	0	326	720	875	1,034
Aug.	976	1,049	1,168	1,284	1,366	1,504	1,473	692	0	323	711	894
Sept.	857	939	1,061	1,211	1,269	1,366	1,397	1,415	714	0	326	769
Oct.	695	772	891	1,043	1,095	1,193	1,223	1,287	1,446	720	0	323
Nov.	297	634	741	891	961	1,037	1,080	1,135	1,293	1,467	738	0
Dec.	0	294	695	817	906	982	1,022	1,080	1,223	1,424	1,513	726
Annual total	11,679	11,379	11,609	11,818	11,595	11,392	10,757	10,450	10,806	11,375	11,497	11,905

[a] Derived from 42,747 records of cows from Holstein herds in Dairy Herd Improvement Associations in Ogle, Whiteside, Will, and DeKalb counties; 305-day lactation periods started from middle of month of freshening.

a whole and for that of the area included in the study. Yet much evidence suggests that the response to most management factors, such as improved feeding, is more in the level of annual production than in difference in seasonal distribution of milk production. However, selectivity, originating in the source of the data, must be kept in mind in considering the results of this study.

Records in this study indicate variations in fat content similar to Woodward's findings (Table 2). They indicate a tendency toward a seasonal decline in fat content associated with the high temperatures of summer months. Indeed, this effect seems strong enough partly to offset the influence of stage of lacation on fat content.

The study relates to an owner-operated farm within a 55- to 70-mile radius of the Chicago metropolitan milk market, in Illinois Economic Areas 2 and C, the latter being the Chicago metropolitan area (Fig. 1). Dairy farms are the most frequent type of commercial farms in the areas, most farmers selling Grade A milk to the Chicago market.

To represent a dairy farm typical of the area, 168 acres of farmland[1] were assumed used to produce 40 acres of corn for grain, 10 acres of corn for silage, 31 acres of oats, 32 acres of alfalfa hay, and 39 acres of pasture. Crop yields are based on averages for dairy farms in the areas. Since dairying is the only livestock enterprise, the entire crop output is available to support it.

Based on Census estimates for 1954,[2] the farm family was assumed to supply a total of 350 hours of man-equivalent labor per month from October through May; 450 hours from June through August; and 400 in September.[3] Deducting the labor required for nondairy enterprises left for dairying the labor supplies reported in Table 3 (p. 8).

Total labor required per cow was fixed at about 110 hours per year, distributed differently over the year, according to month of freshening. The largest use of labor occurs in the month of freshening and during the winter months. Thus, cows freshened from November through March require the largest amounts of labor in the month of freshening. It seems likely that the total, as well as distribution, of the annual labor required would vary by month of freshening, but data available did not provide a basis for making any distinctions. (For monthly labor required by month of freshening, see Table 4, p. 9.)

[1] Arithmetic mean for farms in Illinois State Economic Areas 2 and C, as reported by the U. S. Census of Agriculture, 1954.

[2] Source: U. S. Census of Agriculture, 1954.

[3] A survey made in 1958 in essentially the same area revealed that when hired labor is added to family labor, the total available on similar farms ranges from 365 hours per month in November and January through March, and from between 453 to 456 from June through August. For further information, see M. R. Langham and C. B. Baker, *Optimum Plans for Farms in Northeastern Illinois;* Ill. Agr. Exp. Sta. AERR-40.

Table 2. — Percent of Fat Content of Milk, by Month of Freshening: Cows From All-Holstein Herds in Dairy Herd Improvement Associations, Northern Illinois[a]

Month of production	Month of freshening											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Jan.	4.29	0	3.97	3.89	3.81	3.74	3.69	3.65	3.63	3.59	3.64	4.00
Feb.	3.84	4.14	0	3.93	3.91	3.77	3.74	3.68	3.62	3.55	3.55	3.65
Mar.	3.57	3.86	4.17	0	3.92	3.86	3.81	3.74	3.64	3.56	3.51	3.56
Apr.	3.52	3.62	3.90	4.12	0	3.87	3.90	3.79	3.70	3.63	3.56	3.56
May	3.45	3.50	3.55	3.69	3.81	0	3.84	3.73	3.68	3.59	3.51	3.51
June	3.34	3.38	3.35	3.49	3.61	3.81	0	3.75	3.70	3.52	3.40	3.38
July	3.43	3.43	3.37	3.27	3.40	3.59	3.66	0	3.78	3.69	3.51	3.44
Aug.	3.48	3.42	3.41	3.37	3.37	3.38	3.51	3.82	0	3.70	3.63	3.53
Sept.	3.58	3.53	3.48	3.42	3.43	3.34	3.39	3.54	3.85	0	3.77	3.68
Oct.	3.81	3.75	3.68	3.60	3.59	3.51	3.46	3.50	3.69	4.05	0	3.82
Nov.	3.94	3.96	3.87	3.79	3.74	3.68	3.63	3.60	3.64	3.85	4.30	0
Dec.	0	4.02	3.94	3.88	3.78	3.70	3.67	3.63	3.62	3.66	3.87	4.35
Annual average	3.62	3.64	3.64	3.61	3.61	3.61	3.62	3.65	3.67	3.66	3.64	3.66

[a] Derived from 42,747 records of cows from Holstein herds in Dairy Herd Improvement Associations in Ogle, Whiteside, Will, and DeKalb counties; 305-day lactation periods started from middle of month of freshening.

Table 3. — Feed and Family Labor Available to the Dairy Enterprise: Typical Family-Operated Dairy Farm, Chicago Milkshed[a]

	Quantity available		Quantity available
Farm-produced feed supply	*lb.*	Family labor supply	*hr.*
Grain, corn equivalent	161,000	May	232
Alfalfa hay	160,000	June	307
Corn silage, green weight	336,000	July	214
May and June pasture, TDN	24,500	Aug.	407
July–September pasture, TDN	21,000	Sept.	204
		Oct.	242

[a] Sources: U. S. census of agriculture. Vol. 1. Counties and state economic areas. Part 5. 1954. And *Illinois Farm and Home Development Reference Book;* Ext. Serv. in Agr. and Home Econ., Univ. of Ill. 1955.

The farm-produced feed supply was yielded by the cropping system already described and is given in Table 3. Protein supplement was assumed to be purchased without physical limit. Pasture, silage, and hay were allowed to substitute in the forage portion of the dairy ration. Details of the substitution will be developed later.[1]

The records furnishing information concerning production also provided estimates of grain consumption per cow. Total requirements in pounds of total digestible nutrients were established by reference to standards of the National Research Council (5), taking into account body weight maintenance, milk production, and reproduction requirements. It was assumed that requirements not met by grain would be furnished from pasture, hay, or silage. Pasture was assumed to be used to the extent seasonally available. Consequently, it was assumed that no hay or silage was fed during the months of May through August.

Grain consumption per cow was highest for cows freshened between October and January and lowest for those freshened from February to May (Table 5). Grain consumption during the seasonally expensive months of August and September was especially high for cows freshened in June and July. No hay or silage was fed during May through August. A sample of hay and silage requirements is given for each month, by month of freshening (Table 6). These figures are best regarded as "samples" because the method used for estimating optimum freshening dates allows the roughage to be supplied from any suitable source — hay, silage, or pasture. An example of computations used to obtain roughage requirements is given in Appendix A.

[1] Limiting the supply of grain in this way might be considered too restrictive. Yet grain did not prove to be a limiting factor in any of the solutions described later.

Table 4. — Hours of Monthly Labor Required per Cow for Cows Freshening in Each Calendar Month

Calendar month	Month of freshening											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Jan.	13.9	7.4	9.0	10.6	10.6	10.6	10.6	10.6	10.6	11.1	10.6	10.6
Feb.	10.6	13.9	7.4	9.0	10.6	10.6	10.6	10.6	10.6	10.6	11.1	10.6
Mar.	10.6	10.6	13.9	7.4	9.0	10.6	10.6	10.6	10.6	10.6	10.6	11.1
Apr.	10.7	10.2	10.2	13.5	7.0	8.6	10.2	10.2	10.2	10.2	10.2	10.2
May	7.7	8.2	7.7	7.7	11.0	4.5	6.1	7.7	7.7	7.7	7.7	7.7
June	7.7	7.7	8.2	7.7	7.7	11.0	4.5	6.1	7.7	7.7	7.7	7.7
July	7.7	7.7	7.7	8.2	7.7	7.7	11.0	4.5	6.1	7.7	7.7	7.7
Aug.	7.7	7.7	7.7	7.7	8.2	7.7	7.7	11.0	4.5	6.1	7.7	7.7
Sept.	8.3	8.3	8.3	8.3	8.3	8.8	8.3	8.3	11.6	5.1	6.7	8.3
Oct.	8.5	8.5	8.5	8.5	8.5	8.5	9.0	8.5	8.5	11.8	5.3	6.9
Nov.	9.0	10.6	10.6	10.6	10.6	10.6	10.6	11.1	10.6	10.6	13.9	7.4
Dec.	7.4	9.0	10.6	10.6	10.6	10.6	10.6	10.6	11.1	10.6	10.6	13.9
Annual total	109.8	109.8	109.8	109.8	109.8	109.8	109.8	109.8	109.8	109.8	109.8	109.8

Table 5. — Pounds of Grain Consumed per Cow, by Month of Freshening: Dairy Herd Improvement Associations Records of Cows From All-Holstein Herds, Northern Illinois[a]

Month of consumption	Month of freshening											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Jan.	**141**	**149**	**183**	**237**	**259**	**286**	**295**	**312**	**326**	**376**	**385**	**378**
Feb.	351	**153**	**150**	**181**	**201**	**239**	**239**	**265**	**266**	**306**	**318**	**328**
Mar.	347	324	**158**	**176**	**192**	**243**	**264**	**282**	**295**	**354**	**360**	**391**
Apr.	360	356	335	**160**	**164**	**193**	**224**	**262**	**270**	**322**	**328**	**353**
May	279	282	283	288	**140**	**148**	**178**	**217**	**239**	**286**	**303**	**303**
June	269	275	261	289	259	**123**	**126**	**170**	**189**	**250**	**261**	**266**
July	260	256	270	297	287	293	**119**	**142**	**156**	**207**	**241**	**265**
Aug.	264	262	276	293	289	300	300	**132**	**128**	**172**	**203**	**247**
Sept.	238	240	249	263	266	283	283	280	**110**	**137**	**155**	**212**
Oct.	201	219	249	278	274	300	297	305	318	**139**	**123**	**175**
Nov.	171	199	226	273	283	295	305	312	334	363	**120**	**137**
Dec.	143	162	212	249	273	291	294	310	326	384	371	**138**
Annual consumption	3,024	2,877	2,852	2,984	2,887	2,994	2,924	2,989	2,957	3,296	3,168	3,193

[a] Ogle, Whiteside, Will, and DeKalb Counties, January, 1955, through August, 1957. Average freshening dates were mid-month; grain fed is for period to middle of each succeeding month. Boldfaced numbers indicate quantities for months in the calendar year following year of freshening date.

Table 6. — Pounds of Hay and Silage Required per Cow, by Month of Freshening: Dairy Herd Improvement Associations Records of Cows From All-Holstein Herds, Northern Illinois[a]

Month of freshening	Month of feeding													
	January		February		March		April		October		November		December	
	Hay	Silage	Hay	Silage	Hay	Silage	Hay	Silage	Hay	Silage	Hay	Silage	Hay	Silage
Jan.	440	1,321	496	1,488	434	1,302	435	1,240	353	1,060	360	1,079	322	1,116
Feb.	336	1,008	414	1,243	431	1,294	403	1,210	325	1,000	314	1,000	325	1,000
Mar.	357	1,070	363	1,088	456	1,367	500	1,525	375	1,225	347	1,042	360	1,079
Apr.	336	1,008	339	1,117	342	1,026	508	1,332	364	1,116	339	1,017	357	1,071
May	157	1,060	372	1,116	357	1,070	444	1,088	400	1,200	357	1,079	366	1,097
June	345	1,035	348	1,022	333	9,990	363	1,008	393	1,179	372	1,080	354	1,062
July	363	1,088	366	1,097	350	1,151	336	1,079	415	1,246	372	1,116	375	1,125
Aug.	364	1,097	353	1,060	347	1,042	344	1,032	431	1,293	372	1,116	378	1,135
Sept.	375	1,125	363	1,089	354	1,062	360	1,044	465	1,395	402	1,206	384	1,152
Oct.	391	1,201	388	1,163	363	1,088	344	1,079	443	1,330	446	1,339	425	1,274
Nov.	437	1,269	399	1,197	366	1,098	354	1,062	378	1,124	438	1,314	450	1,350
Dec.	490	1,469	465	1,395	400	1,100	378	1,135	360	1,079	375	1,125	440	1,321

[a] Ogle, Whiteside, Will, and DeKalb Counties, January, 1955, through August, 1957. Average freshening dates were mid-month; hay and silage required is for period to middle of each succeeding month.

MARKET CONDITIONS

To encourage milk production seasonally consistent with demand, a "base-excess" plan was introduced for milk in the Chicago market in 1954.[1] The months of September through November are termed "base periods"; March through June "excess periods." During the excess period, each producer receives each month two prices for his milk. He receives a "base price" for the quantity that does not exceed his average monthly production in the preceding base period. He receives a "surplus price" for the quantity in excess of this base quantity. Historically, the surplus price has been less than the base price by 40 cents per 100 pounds.[2]

Since monthly production is known for cows freshening in each calendar month (Table 1), quantities of milk to be sold at the "base" and "surplus" prices can easily be computed for any combination of freshening dates. For example, a cow freshening in May establishes a fall base equal to 9.56 percent of her annual production and 3 percent of the annual production was produced in March, none in April, 7.06 in May, and 14.39 in June. Hence, if all the herd had been freshened in May, 4.83 percent of the annual production would be sold at "surplus price" during June as the "fall base" is less than the monthly production in June. But the entire amount of milk produced in March, April, and May would be sold at the "base price" as the "fall base" is greater than monthly production during these months.

The price paid for milk is modified further by three additional factors.[3] The first is a price premium negotiated for Class I milk sold each calendar month. The premium applies only to nonexcess milk. Hence, the premium tends to accentuate the difference between base and surplus prices. The second factor is locational. Two cents per 100 pounds are deducted from the uniform price of milk for each 15-mile-wide zone outside a 55-mile zone. Within the 55-mile zone, producers receive a small premium above the uniform price. The third factor is a butterfat differential. Between 1955 and 1958, the butterfat differential varied from 6.9 and 7.2 cents for each tenth of a pound above or below 3.5 pounds per 100 pounds of milk.

The "base price," adjusted for butterfat test, is given for a cow freshened for each month (Table 7). The quantity-weighted mean

[1] Federal Milk Order No. 41 *Reporter,* "Grade A uniform price for base milk," January, 1959.

[2] Had the negotiated price premium been taken into account, the difference between "base" and "excess" price would have been 65 cents per 100 pounds. When this differential was used in solving for optimal freshening dates, no difference of substance was found in the freshening pattern.

[3] A fourth factor is a premium paid for bulk-handled milk. However, this factor in no way influences seasonal variation of price.

Table 7. — "Base Price" Received for Milk From Holstein Cows, Adjusted for Differential Due to Butterfat Content of Milk: 55- to 70-mile Zone of Chicago Milk Market, 1955 through 1958[a]

Month of sale	Month of freshening											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
	dollars per 100 lb.											
Jan.	4.10	0	3.88	3.82	3.77	3.72	3.68	3.66	3.64	3.61	3.65	3.90
Feb.	3.76	3.97	0	3.82	3.81	3.71	3.69	3.65	3.60	3.56	3.56	3.62
Mar.	3.54	3.74	3.96	0	3.78	3.74	3.71	3.66	3.59	3.53	3.50	3.53
Apr.	3.51	3.58	3.78	3.93	0	3.76	3.78	3.70	3.64	3.59	3.54	3.54
May	3.44	3.48	3.52	3.61	3.70	0	3.72	3.64	3.61	3.54	3.49	3.49
June	3.37	3.40	3.38	3.47	3.56	3.70	0	3.66	3.62	3.49	3.41	3.40
July	3.43	3.43	3.39	3.39	3.41	3.54	3.59	0	3.68	3.61	3.49	3.44
Aug.	3.69	3.64	3.64	3.61	3.61	3.62	3.71	3.93	0	3.84	3.79	3.72
Sept.	3.80	3.76	3.73	3.68	3.69	3.62	3.66	3.77	3.99	0	3.39	3.87
Oct.	3.96	3.92	3.87	3.81	3.80	3.75	3.71	3.74	3.88	4.13	0	3.97
Nov.	4.04	4.06	3.99	3.94	3.90	3.86	3.82	3.80	3.83	3.98	4.30	0
Dec.	0	3.94	3.88	3.84	3.77	3.71	3.69	3.66	3.66	3.68	3.83	4.18
Average	3.63	3.66	3.67	3.67	3.68	3.69	3.71	3.72	3.70	3.67	3.65	3.65

[a] Four-year weighted average base prices for the Chicago market; Federal Milk Order No. 41 *Reporter*, "Grade A Uniform Price for Base Milk"; January, 1955, through December, 1958. The butterfat content and the distribution of monthly milk production were derived from 42,747 monthly records of cows from all-Holstein herds in Ogle, Whiteside, Will, and DeKalb Counties, January, 1955, through August, 1957.

base price for the lactation output of the cow is shown across the bottom of the table. The price so reported for March through June would apply only to the quantity of milk, each month, equal to or less than average production in the previous months of September through November. Quantities above these figures are actually priced at 40 cents per 100 pounds below the figures shown in the table.[1]

Farmers actually receive for milk a "blend" price, a price averaged for all classes of milk, for the eight months excluding March through June. The prices reported in Table 7 are comprised only of those elements of price that vary by month of sale. Thus the income levels resulting from problem solutions reported below are different from actual experience. However, the seasonal distribution of freshening dates are in no way affected by raising or lowering milk prices. Hence, these "partial" prices were used for computational simplicity.

Besides milk, the dairy cow produces meat in the form of veal calf and cull-cow beef on herd replacement. Compared with the value of milk, these by-products add little to the net value of output. Annual production per cow is taken as 342 pounds,[2] 60 pounds,[3] 120 pounds[4] per cow, respectively, for cull-cow beef, veal beef, and replacement heifer, regardless of month of freshening. There appears to be little reason for assuming that the price varies with respect to month of freshening for herd replacement or for veal-calf beef. Utility grade heifers averaged $13 per hundred pounds live weight (4) and veal calves, $13.50.[5] Cull-cow beef is valued at prices two months prior to freshening month, averaged over the period, 1953-1957 (4).

A summary of prices used for products of dairy cows is given in Table 8. For each product, the price is a quantity-weighted estimate for cows freshened in the month indicated. In the last row, the value of total cow production per 100 pounds of milk is shown. In the actual computation, however, these prices varied according to the amount of milk sold at excess prices. This quantity is influenced by the freshening pattern of the entire herd.

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[1] Federal Milk Order No. 41 *Reporter,* "Grade A Uniform Price for Base Milk"; January, 1959.

[2] Average weight of a Holstein cow at replacement age was 1,489 pounds (6). Replacement was estimated at 23 percent per year (based on unpublished data of R. W. Touchberry, Department of Dairy Science).

[3] Bull calves are ordinarily sold for veal from dairy farms in northern Illinois (based on observations from Farm Bureau Farm Management Service Records, 1957-1958).

[4] About half the replacement heifers are sold as culls before first freshening. The estimate includes total pounds of cull heifer sold from birth to freshening date.

[5] Farm Bureau Farm Management Service Records, 1957-58.

Table 8. — Prices of Products From Dairy Cows, by Month of Freshening: Northern Illinois and the Chicago Milk Market; Averages, 1953-1958

Item	Month of freshening											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Milk[a]	$ 3.63	$ 3.66	$ 3.67	$ 3.67	$ 3.68	$ 3.69	$ 3.71	$ 3.72	$ 3.70	$ 3.67	$ 3.65	$ 3.65
Cull cow beef[b]	10.45	10.92	11.43	11.34	11.44	11.07	10.51	9.96	9.66	9.67	9.24	9.91
Cull cow replacement[c]	13.00	13.00	13.00	13.00	13.00	13.00	13.00	13.00	13.00	13.00	13.00	13.00
Veal calf[d]	13.30	13.30	13.30	13.30	13.30	13.30	13.30	13.30	13.30	13.30	13.30	13.30
Total value per 100 lb. of milk[e]	3.66	3.71	3.73	3.73	3.74	3.72	3.73	3.72	3.70	3.66	3.63	3.65

[a] Four-year weighted average base prices for the Chicago market; Federal Milk Order No. 41 *Reporter*, "Grade A Uniform Price for Base Milk"; January, 1955, through December, 1958. The butterfat content and the distribution of monthly milk production were derived from 42,747 monthly records of cows from all-Holstein herds in Ogle, Whiteside, Will, and DeKalb Counties, January, 1955, through August, 1957.

[b] Five-year average prices, two months preceding month of freshening, 1953-1957; U. S. Dept. Agr., Agr. Mktg. Serv., Livestock and Meat Statistics, Stat. Bul. 230, p. 227. 1958.

[c] Estimated average price for utility grade heifers, 1956-1957. U. S. Dept. Agr., Agr. Mktg. Serv., Livestock and Meat Statistics, Stat. Bul. 230, p. 227. 1958.

[d] Unpublished data relating to 120-pound calves sold from northern Illinois dairy farms in Farm Bureau Farm Management Service, 1957-1958.

[e] Total value excludes the following components of milk price; negotiated price premiums, locational price adjustments, deductions from production in excess of fall base, and any premiums resulting from bulk-handled milk.

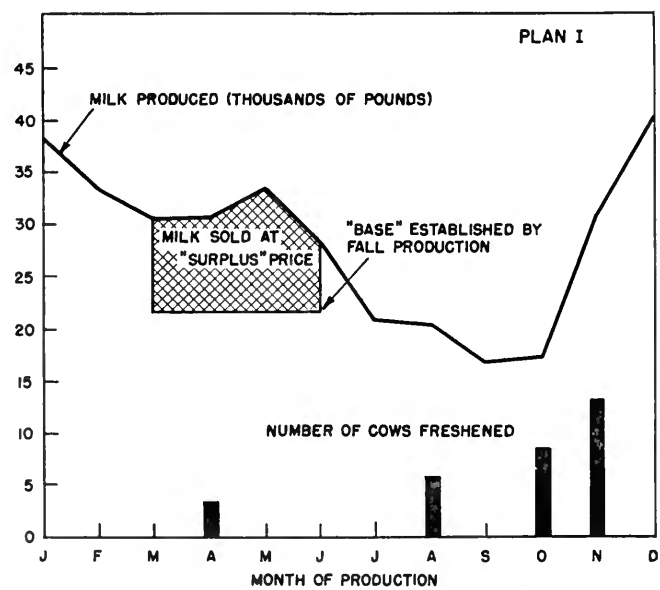
PLANS FOR MAXIMUM PROFITS

All or any part of the herd can be freshened in any month. The number of cows milked and the distribution of freshening dates are assumed limited only by the feed and labor supply as already described. Any hay beyond requirements of the dairy herd is assumed to be sold at $21.20 per ton; any grain at $2.35 per 100 pounds of corn equivalent. The method used to select a profit-maximizing plan yields estimates on value of adding a unit of any resource that limits the number of cows to be freshened. The usefulness of this added information will be demonstrated later.

Varied Labor Supply

Freshening dates that maximize returns were selected under two conditions: (1) that the operator hires no labor to supplement the supply furnished by the farm family; and (2) that labor is hired to the extent found profitable. Labor was priced alternately at two wage rates, 80 cents and $1.70 per hour.[1] Results of the three sets of conditions are given in Tables 9 and 10 under headings of Plan I (no labor hired), Plan II (labor hired at 80 cents an hour), and Plan III (labor hired at $1.70 an hour).

[1] Other plans were estimated for higher wage rates. Until wages reached $3.60 an hour, however, no substantial change was observed in freshening patterns that maximize returns.



Optimum distribution of milk production; no hired labor. (Fig. 2)

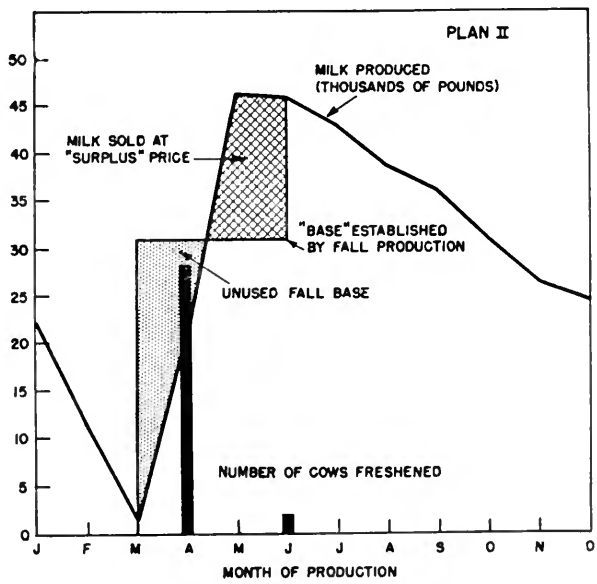
Plan I

With no supplemental labor, the maximum-profit plan is dominantly one of fall freshening, 27 out of 30 cows (Fig. 2). A substantial volume of annual milk production, over 10 percent, is sold at surplus price. Hay produced on the farm is practically used up in this plan, though some pasture is in excess during June and July (enough perhaps for three more cows in these months).

An appreciable amount of grain, about 1,000 bushels of corn or corn equivalent, is available for sale. Slightly less than 20 percent of family labor available for dairying is unused for dairying in June; slightly more than 40 percent in August. The plans differ little in income generated from the dairy enterprise. Income is based on milk prices that exclude all but seasonally variable components.

Plan II

When labor is hired at 80 cents an hour, the freshening pattern (Fig. 3) shifts from a dominantly fall freshening program to one of spring freshening. The consequence is to reduce milk sold at surplus to about 8½ percent of the annual total. Little change occurs in the use or nonuse of family labor. The equivalent of about 3 man days are hired in July and about 4 in September and October. As additional labor becomes available during the summer months, the optimal-freshening program shifts to spring fresheners that use pasture as a



Optimum distribution of milk production; labor hired at 80 cents per hour. (Fig. 3)

Table 9. — Profit-Maximizing Plans for Freshening Cows in All-Holstein Herds: 55- to 70-Mile Zone From Chicago Milk Market, at Selected Wage Rates for Hired Labor

Item	Plan I, family labor only	Plan II, labor hired, 80 cents per hour[a]	Plan III, labor hired, $1.70 per hour[a]
Number of cows freshened in[b]:			
Apr.	3	28	26
June		2	1
Aug.	6		
Oct.	8		
Nov.	13		
Dec.			3
Products sold			
Milk at base price, pounds	304,763	318,774	316,141
Milk at surplus price, pounds	35,179	29,711	33,816
Grain, 100 lb. of corn equivalent	638	694	685
Hay, ton	2.5		
Unused resources			
Pasture, June and July, TDN pounds	3,836		
Family labor, man hours			
May		11	8
June	85	72	74
Aug.	168	179	179
Labor hired, man hours			
July		27	27
Sept.		42	43
Oct.		37	
Income, dollars[c]	13,849	14,445	14,379

[a] To limit found profitable.
[b] Rounded to nearest unit. The solutions can and do indicate fractions of cows. For this reason the following milk production is not exactly the same as would be estimated by summing products of cow numbers and the annual production indicated in Table 1.
[c] Total annual income from the dairy enterprise with no deduction for the cost of resources with quantities fixed as shown in Tables 3 and 4.

Table 10. — Values of Added Units of Resources Found to Limit the Number of Cows Freshened in All-Holstein Herds: 55- to 70-Mile Zone From Chicago Milk Market, at Selected Wage Rates for Hired Labor

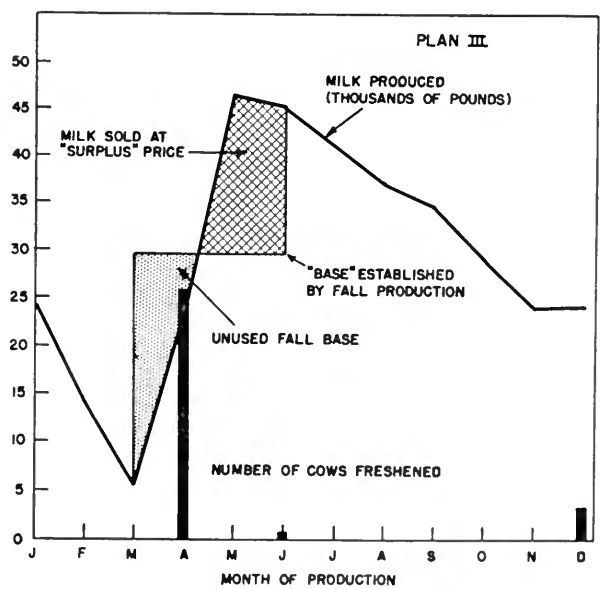
Limiting resource	Value of added unit under—		
	Plan I[a]	Plan II[b]	Plan III[b]
Silage, dollars per ton	.60	11.60	10.00
Hay, dollars per ton	0	34.00	32.00
Pasture, cents per pound, TDN			
May–June	0	.047	.057
July–Sept.	.04	.065	.045
Family labor, dollars per man hour			
May	1.56	0	0
July	11.21	.80	1.70
Sept.	12.33	.80	1.70
Oct.	4.17	.80	1.59

[a] For description of plans, see Table 9.
[b] To limit found profitable.

substantial part of their feed requirements during months of high milk production. Pasture unused in Plan I was used by the spring fresheners in Plan II. The value of increased corn sales in Plan II was greater than the loss in income from the decrease in hay sales. In summary, additional labor in certain summer months allows spring fresheners to make up the optimal-freshening program and these fresheners use cheaper feeds than did those in Plan I.

Plan III

With a wage rate of $1.70 an hour, the freshening pattern becomes slightly less extreme than that in Plan II, though it is still dominantly a spring-freshening program. There is also virtually no change in total milk production (Fig. 4), though a slight increase did occur in the percentage sold at surplus. As between Plans II and III, the shift is primarily from fewer April fresheners to more December fresheners. Cows freshening in December produced the largest amounts of milk annually. December fresheners have a "second freshening" from lush spring pastures. These pastures are a cheap source of feed during months of relatively high milk production. When the price of labor was increased to $1.70 an hour, it became more profitable to shift to the high-producing December fresheners even though a larger quantity of milk had to be sold at surplus price. No labor is hired in October. The amount of labor hired in July and September remains at about 3 and 4 man days, respectively.



Optimum distribution of milk production; labor hired at $1.70 per hour. (Fig. 4)

Effect of adding one unit of resource

For each plan, Table 10 shows: (1) resources that limit further expansion of the dairy enterprise, and (2) the value, in terms of income, of adding one more unit of the resource that thus limits the plan.

In Plan I, although silage was limiting, the value of an added pound (60 cents per ton) was very little. But, an added pound of total digestible nutrients in the form of pasture during July through September would have added 4 cents to income. An hour of labor added by the family would have added $1.56 to income. Values were still higher for July, $11.21; September, $12.33; and October, $4.17. The size of these values indicates that labor was the most restrictive resource in Plan I. In reality, farmers add to their labor supply in seasons of peak requirements by working longer hours and by exchanging work with neighbors. The value of such measures is clearly indicated by these estimates of increases created in income.

These figures must be used with considerable caution. The method of computation provides no basis for estimating the quantity of the resources for which these estimates would be valid. For labor, we know from the results of hiring it in Plans II and III that the quantity that would produce such high returns is quite small.

In Plans II and III, hiring labor removes labor as a limiting resource and adds as limiting resources hay and pasture in May and June. In Plan II, an added ton of silage would be worth $34.00 and hay $11.60; an added pound of total digestible nutrient would be worth 4.7 cents in May and June and 6.5 cents in July and September. The value of an added man hour of labor from the family during July, September, and October is, of course, equal to the rate for hired labor. Since surplus family labor is found in May and June, a man hour of labor added in these months would have no value.

In Plan III, the various types of roughage are again the only restrictive resources, as labor may be hired for $1.70 per hour. As in Plan II, surplus family labor is found in May and June. But labor was hired only in July and September and the value of an added man hour of labor was equal to the rate for hired labor ($1.70). The value of an added man hour of labor in October was $1.59. Since the cost of an added man hour of labor was $1.70, no labor was hired in October even though there was a labor shortage.

Varied Seasonal Price Movement

The seasonal price pattern for dairy-cow products can change for many reasons. The consequence of such a change was examined by arbitrarily assuming that cull-cow beef is priced at a constant over the

year at the average of prices given in Table 8, $10.47 per 100 pounds. The effect on total value of production per 100 pounds of milk is shown in Table 11. In general, prices are reduced for cows freshened in late winter, spring, and early summer and are increased for fall fresheners.

Table 11. — Value of Dairy Products per 100 Pounds of Milk Produced; by Month of Freshening, Under Alternate Assumptions on Price of Cull-Cow Beef

Month of freshening	Price of cull cow beef		Change
	Varied	Constant	
Jan.	$3.66	$3.66	$ .00
Feb.	3.71	3.70	−.01
Mar.	3.73	3.70	−.03
Apr.	3.73	3.70	−.03
May	3.74	3.71	−.03
June	3.72	3.70	−.02
July	3.73	3.73	−.00
Aug.	3.72	3.74	.02
Sept.	3.71	3.74	.03
Oct.	3.66	3.68	.02
Nov.	3.63	3.67	.04
Dec.	3.65	3.67	.02

With the changed pattern of monthly prices, new solutions were found under conditions of: (1) no labor added to family-supplied labor; and (2) labor hired as long as profitable at 80 cents an hour. Under the first condition, the solution differed in no significant way from Plan I (Table 9). Operator's returns were increased slightly, but the freshening dates were unchanged as were the milk quantities

Table 12. — Profit-Maximizing Freshening Dates for Cows in All-Holstein Herds: 55- to 70-Mile Zone From Chicago Milk Market; Labor Hired at 80 Cents per Hour and Cull-Cow Beef Constant Over the Year

Item	Amount	Item	Amount
Number of cows freshened in[a]		Unused family labor, man hours	
Apr.	23	May	3
Dec.	7	June	78
Products sold		Aug.	178
Milk at base price, pounds	312,459	Oct.	5
Milk at surplus price, pounds	39,577	Labor hired, man hours	
Grain, per 100 pounds of corn equivalent	674	July	26
		Sept.	43

[a] See footnote b in Table 9.

sold at base and surplus prices. Therefore the results are not reported in detail.

Under condition 2 when labor is hired, however, the results (Table 12) are comparable with Plan II (Table 9). Five fewer cows are freshened in April. Instead of two cows freshened in June, seven are freshened in December. Milk sold at surplus price increased by about one-third. The grain surplus is about the same, as is the surplus of family labor. No hired labor is required in October and operator's returns are reduced only slightly.

Optimum and Actual Freshening Dates

The computed optima for freshening dates rest on a specifically defined production situation, as previously described. The sensitivity of the results to changes in labor supply suggests that the dates would differ substantially for differences in nondairy enterprises. For these and other reasons, it would be surprising to find much accord among dairy farmers generally in the area and especially over larger or different areas. This would be true even though all dairy farmers so bred their cows as to attain profit-maximizing dates in their own specific production situations.

Nevertheless it is of interest to compare results of this study with average freshening dates in somewhat similar herds in the area. Such comparison is possible by reference to the first five columns of Table 13. In the first four columns data are summarized from solutions previously reported. In column five the percentages of cows freshened each month are given for DHIA herds of 30 cows or more in northern Illinois. Plan I, using nonsupplemented family labor, most closely approximates the actual situation. Any single plan could be expected to be far more extreme than an aggregate of plans for many herds. In Plan I, no cows are freshened in months reported low in DHIA herds, either all or only those of 30 cows or more. The fall freshenings of Plan I coincide roughly with the months of heavy freshening in all DHIA herds in Illinois. The same relation seems generally to hold for herds reported for Wisconsin, Michigan, and New York. The plans using hired labor, however, seem distinctly at odds with the freshening pattern in either northern Illinois or in the three-state area.

In recent years, the seasonal variation in the Chicago area seems to have been reduced materially. Such an observation is consistent with findings in this study. Little response was found to the small change introduced in the seasonal price pattern for output of the dairy enterprise. However, from varied labor conditions, wide differences were noted in the profit-maximizing freshening pattern, though with relatively little effect on income generated by the dairy enterprise. Thus

Table 13. — Optimum Freshening Dates Compared With Dates Reported for Dairy Herds in Northern Illinois, Wisconsin, Michigan, and New York

Month	Percent of cows freshened						
	Optimum plans						
	Plan I, family labor only	Plan II, labor hired, 80 cents per hour; price of cull-cow beef constant	Plan II A, labor hired, 80 cents per hour; price of cull-cow beef varied monthly	Plan III, labor hired, $1.70 per hour; price of cull-cow beef varied monthly	DHIA herds, northern Illinois[a]		Wisconsin, Michigan, and New York[b]
					30 cows or more	All herds	
Jan.....					9.6	9.3	9.0
Feb.....					8.8	7.6	7.9
Mar.....					7.4	8.7	7.9
Apr.....	10.0	93.1	76.7	86.2	7.0	7.9	8.0
May.....					6.2	6.8	6.2
June.....		6.9		3.5	6.7	6.0	4.5
July.....					6.8	5.0	4.0
Aug.....	20.0				8.4	7.5	6.1
Sept.....					10.4	11.0	9.1
Oct.....	26.7				10.4	10.3	10.6
Nov.....	43.3				9.6	11.0	10.9
Dec.....			23.3	10.3	8.7	8.9	9.7

[a] Unpublished data from DHIA records for 12-month period ended June, 1959.
[b] U. S. Dept. of Agr., *Milk Production*, March, 1960, for 1959; DHIA herds in Wisconsin, crop reporter data in Michigan, and a combination of DHIA and crop reporter data in New York. The authors are indebted for this reference to J. Russell Ives, Director, Department of Marketing, American Meat Institute, Chicago, Illinois.

over a range of farms, the study results would lead one to expect to find widely varied freshening patterns. On each farm, moreover, the total supply of labor and the nondairy demands for its use appear to be important determinants of the freshening pattern.

CONCLUSIONS

When labor for dairying is limited from May to October, a fall-freshening program seems most profitable. Such a limitation might be found on farms with large labor demands from crop enterprises. Yet with sufficient labor or with labor hired up to $1.70 an hour, income can be increased by changing to a spring-freshening program. The relatively small change in income suggests that the freshening program might be better adjusted to the seasonal variation in labor supply rather than the reverse on farms where dairying is not the sole enterprise.

These conclusions depend heavily on the production situation assumed for the dairy herd. For example, fall fresheners require most

feed during months of low feed prices. Hence, if all feed is purchased, fall fresheners would be favored relative to nonfall fresheners. In the study situation, however, all but protein supplement is supplied from the farm. Thus, no seasonal pricing was used for feeds and the feed supplies are expressed, except for pasture, as annual supplies. These conclusions are especially dependent on the amount of pasture available, as no cost was assigned to pasture. Hence, a farmer with a large supply of pasture might differ from a farmer with a rather limited supply of pasture in a profit-maximizing seasonal freshening pattern.

Milk prices so vary seasonally as to favor fall fresheners, producing a relatively large proportion of annual milk output in the high-price fall and winter months. Yet after taking into account the seasonal variation in price of cull cows, inversely related to milk price, and adjustments for butterfat content, the actual seasonal variation in price of dairy-cow products was substantially reduced. In other production situations, especially in different culling programs, the results might have been different.

The results seem to be fairly stable in the presence of changes, at least small changes, in the seasonal pattern of prices for products of dairy cows unless the labor supply also is changed. In this event, the profit-maximizing freshening dates can change markedly.

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APPENDIX A: ROUGHAGE REQUIREMENTS OF DAIRY COWS

To estimate roughage required monthly for cows freshened in each month, the following data were available: (1) milk produced per cow during each of the first 10 months after freshening; (2) average body weight of cows; and (3) grain consumed during each of the first 12 months after freshening. From (1) and (2) total-digestible-nutrient requirements were estimated. An example of this computation is given in Table 14 for a cow freshened on February 15.

The DHIA records furnished evidence on grain consumed per cow per day. Mean estimates and standard deviations about means are given in Table 15. From these data, it is a simple extension to compute the monthly grain consumed per cow.

The amount of feed required from nongrain sources is the difference between total feed required and the amount supplied by grain. As an example of this computation, the roughage required for cows freshened on February 15 is shown in Table 16.

The nongrain feed requirement is supplied by roughage — pasture, hay, silage, or hay and silage. Pasture was assumed to be used in season to the limit supplied by the farm or required by the herd, whichever limit is reached first. The total digestible nutrients furnished daily per cow from pasture are given in Table 17.

The remaining roughage requirements must come from silage or hay, or both. Estimates of hay and silage required to balance the ration of a cow freshened on February 15 are given in Table 18. Requirements of hay and silage for cows freshened in other months were estimated similarly and are reported in Tables 19 and 20, respectively.

To accord more closely with farm-feeding practice, both hay and silage were allowed to replace pasture as needed, if the value of either exceeded in feeding its value if sold ($21.20 per ton for hay; $8.50 per ton for silage). Unused pasture was assumed to have a value of zero. In the programming model, the system of transfers necessary to effect the required substitutions was as follows:

Resource	*Initial supply, pounds*	*Value if sold, cents per pound*	*Substitute for pasture, pounds*	
			Hay	*Hay and silage*
Hay	160,000	.0106	1	1
Silage	336,000	.0043		3
Pasture				
May–June	24,500	0	−.25	−.5
July–September	21,000	0	−.25	−.5

When one pound of hay was substituted for pasture, the supply of

Table 14. — Pounds of Total Digestible Nutrients Required per Cow per Day Not Furnished by Concentrates for Cows Freshening on February 15; 305-Day Lactation Period

Month of feeding	Average body weight[a]	Average daily milk production per cow[b]	Average butterfat test[b]	Daily TDN required for[c]			Total TDN required
				Milk	Body maintenance	Reproduction	
	lb.	*lb.*	*perct.*	*lb.*	*lb.*	*lb.*	*lb.*
Feb. 1–15	1434	0	0	0	10.1	6.0	20.8
Feb. 16–28		50.1	4.14	16.5	9.0	0	
Mar.	1334	50.2	3.86	15.6	9.0	0	24.6
Apr.	1334	47.6	3.62	14.5	9.0	0	23.5
May	1344	47.9	3.50	14.4	9.1	0	23.5
June	1356	43.4	3.38	12.7	9.1	0	21.8
July	1365	38.0	3.43	13.3	9.2	0	20.5
Aug.	1376	34.4	3.42	10.2	9.2	0	19.4
Sept.	1407	30.8	3.53	9.3	9.4	0	18.7
Oct.	1435	25.3	3.75	7.8	9.6	0	17.4
Nov.	1465	20.8	3.96	6.6	9.8	0	16.4
Dec. 1–15	1499	19.3	4.02	6.2	9.9	0	....
Dec. 16–31		0	0	0	9.9	6.0	16.0
Jan.	1534	0	0	0	10.1	6.0	16.1

[a] Derived from Matthews and Fohrman.
[b] Derived from DHIA records of Holstein herds in Whiteside, Will, Ogle, and DeKalb counties; January, 1955, through August, 1957.
[c] Adopted from National Research Council, *Nutrient requirements of domestic animals*: No. III, Nutrient requirements of dairy cattle. Pub. 464 (rev.) 1956.

hay was decreased one pound, but the substitution added to the supply of pasture the equivalent of a half-pound of pasture feed, one-fourth pound in May and June and one-fourth in July through September. The substitution of hay and silage can be interpreted similarly.

Table 15. — Pounds of Grain Consumed per Day per Cow During Each of the First 12 Months After Freshening; by Cows Freshening in Each Calendar Month*

Month of freshening	Cow records in the first month	Calendar month after freshening											
		1	2	3	4	5	6	7	8	9	10	11	12
		Open figures are means; figures in parentheses are standard deviations											
Jan.	323	11.32	12.38	11.60	9.29	8.67	8.66	8.50	7.67	6.71	5.52	4.77	4.55
		(3.8)	(4.5)	(3.1)	(7.1)	(3.6)	(3.4)	(2.6)	(2.9)	(3.1)	(3.4)	(3.2)	(3.4)
Feb.	343	11.57	11.47	9.40	8.88	8.52	8.46	7.75	7.31	6.42	5.40	4.81	4.93
		(3.9)	(3.3)	(4.0)	(3.4)	(3.3)	(2.8)	(2.9)	(3.2)	(3.4)	(3.5)	(3.6)	(3.4)
Mar.	306	11.17	9.14	8.69	8.72	8.91	8.31	8.03	7.53	6.83	5.89	5.37	5.08
		(3.4)	(4.4)	(3.5)	(3.5)	(3.0)	(3.0)	(3.1)	(3.1)	(3.6)	(3.8)	(3.4)	(3.1)
Apr.	263	9.30	9.63	9.57	9.44	8.78	8.95	9.09	8.03	7.65	6.47	5.67	5.32
		(4.1)	(3.5)	(3.4)	(2.9)	(3.2)	(2.8)	(2.9)	(2.9)	(3.1)	(3.3)	(3.1)	(3.3)
May	309	8.64	9.25	9.33	8.87	8.85	9.42	8.80	8.36	7.16	6.18	5.48	4.52
		(4.1)	(3.9)	(3.6)	(3.4)	(3.2)	(3.1)	(3.4)	(3.0)	(3.0)	(3.2)	(3.5)	(3.4)
June	382	9.46	9.68	9.43	9.67	9.82	9.39	9.22	8.54	7.84	6.43	4.78	4.09
		(3.8)	(3.4)	(3.4)	(2.9)	(2.8)	(3.0)	(2.8)	(3.0)	(3.1)	(3.5)	(3.6)	(3.9)
July	442	9.66	9.44	9.57	10.17	9.49	9.52	8.52	8.51	7.46	5.73	4.20	3.84
		(3.1)	(3.4)	(3.3)	(2.9)	(3.0)	(2.9)	(3.1)	(3.0)	(3.1)	(3.5)	(3.6)	(3.7)
Aug.	586	9.34	9.83	10.39	10.00	10.06	9.46	9.10	8.72	7.00	5.67	4.57	4.25
		(3.1)	(3.2)	(2.9)	(2.8)	(2.5)	(2.6)	(2.6)	(2.9)	(3.2)	(3.4)	(3.6)	(3.6)
Sept.	444	10.25	11.12	10.50	10.51	9.50	9.50	9.01	7.72	6.30	5.02	4.13	3.67
		(3.2)	(3.1)	(3.0)	(2.9)	(3.1)	(3.1)	(3.1)	(3.1)	(3.3)	(3.5)	(3.3)	(3.1)
Oct.	259	12.10	12.37	12.14	10.91	11.42	10.73	9.23	8.32	6.69	5.56	4.56	4.49
		(2.9)	(3.0)	(2.7)	(3.7)	(2.9)	(2.9)	(3.4)	(3.1)	(3.3)	(3.3)	(3.4)	(3.3)
Nov.	200	11.98	12.41	11.36	11.60	10.93	9.76	8.69	7.76	6.55	5.16	3.95	4.00
		(3.5)	(3.1)	(3.8)	(3.0)	(3.2)	(3.4)	(2.8)	(2.7)	(3.1)	(3.4)	(3.3)	(2.8)
Dec.	268	12.20	11.72	12.60	11.76	9.77	8.85	8.55	7.95	7.08	5.64	4.55	4.45
		(3.0)	(3.1)	(4.1)	(4.0)	(3.1)	(4.0)	(3.4)	(3.0)	(2.9)	(2.8)	(3.2)	(3.3)

* Derived from 42,747 DHIA monthly records of Holstein herds in four Illinois counties — Ogle, Whiteside, Will, and DeKalb — January 1955 through August 1957. The average freshening dates were in the middle of each calendar month of freshening.

Table 16. — Pounds of Roughage Required per Cow per Day in Each Calendar Month for Cows Freshening on February 15; 305-Day Lactation Period

Month of feeding	TDN consumed daily from concentrates[a]				Total TDN required from roughage	Total TDN required[b]
	Corn	Oats	Protein supplement	Total		
Feb.	2.9	1.5	.9	5.3	15.5	20.8
Mar.	4.7	2.5	1.3	8.5	16.1	24.6
Apr.	4.6	2.5	1.3	8.4	15.1	23.5
May	4.1	2.7	0	6.8	16.7	23.5
June	3.9	2.5	0	6.4	15.4	21.8
July	3.7	2.1	.3	6.1	14.4	20.5
Aug.	3.7	1.8	.6	6.1	13.4	19.4
Sept.	3.1	1.6	.9	5.6	13.1	18.7
Oct.	2.9	1.5	.9	5.3	12.1	17.4
Nov.	2.6	1.3	.8	4.7	11.7	16.4
Dec.	2.2	1.1	.6	3.9	12.1	16.0
Jan.	1.9	1.0	.6	3.5	12.6	16.1

[a] Corn was 73.2 percent TDN; oats, 70.1 percent; protein supplement, 78.6 percent.
[b] Daily TDN required per cow were taken from Table 14.

Table 17. — Total Digestible Nutrients Required From Pasture per Cow per Day in Certain Months for Cows Freshening in Each Calendar Month[a]

Month of freshening	Months of pasture[b]					Total TDN May–June[c]	Total TDN July–Sept.[c]
	May	June	July	Aug.	Sept.		
	Daily TDN requirement, pounds					*lb.*	*lb.*
Jan.	15.8	14.6	13.8	12.9	6.4	1,160	1,089
Feb.	16.7	15.4	14.4	13.4	6.8	1,212	1,134
Mar.	18.4	16.9	15.1	14.2	7.0	1,310	1,186
Apr.	19.5	18.0	15.7	14.4	7.7	1,377	1,232
May	16.4	19.1	17.5	15.3	7.9	1,316	1,320
June	12.7	16.5	18.1	16.9	7.9	1,124	1,387
July	12.6	13.1	15.8	16.5	8.2	1,017	1,314
Aug.	13.1	12.4	12.8	15.0	8.7	1,011	1,192
Sept.	13.4	13.0	12.5	13.2	8.0	1,038	1,107
Oct.	13.2	12.8	12.4	12.1	6.6	1,026	1,027
Nov.	13.3	13.5	12.6	12.1	6.6	1,050	1,034
Dec.	14.7	13.3	13.5	12.6	6.4	1,087	1,070

[a] Total digestible nutrients required from pasture daily for each calendar month of freshening were computed by the method shown in Table 16.
[b] Pasture was utilized only from May 1 through September 15.
[c] Total amount of TDN required in these months is the daily TDN requirement in each month multiplied by 30.5 days plus 233 and 79 pounds of TDN required for replacements during May-June and July-September, respectively.

Table 18. — Pounds of Hay and Silage Required per Cow per Day to Balance the Ration of a Cow Freshened February 15; 305-Day Lactation Period

Month of feeding	Pasture, pounds of TDN		Hay[a]	Silage[a]
	May–June	July–Sept.		
Jan.	0	0	14.8	44.4
Feb.	0	0	15.4	46.2
Mar.	0	0	14.4	43.2
Apr.	16.7	0	0	0
May	15.4	0	0	0
June	0	14.4	0	0
July	0	13.4	0	0
Aug.	0	6.8	0	34.7
Sept.	0	0	11.6	34.8
Oct.	0	0	11.2	33.6
Nov.	0	0	11.6	34.8
Dec.	0	0	12.0	36.0

[a] Alfalfa hay is estimated at 50.3 percent TDN; corn silage at 18.1. See F. B. Morrison, *Feeds and Feeding,* 22nd ed., pp. 1000-1030; The Morrison Pub. Co.; Ithaca, N. Y.; 1956.

Table 19. — Pounds of Hay Required per Cow per Day in Certain Months for Cows Freshening in Each Calendar Month[a]

Month of freshening	Month of feeding[b]							Annual total[c]
	Jan.	Feb.	Mar.	Apr.	Oct.	Nov.	Dec.	
Jan.	14.2	16.0	14.0	13.4	11.4	11.6	12.0	3,824
Feb.	12.0	14.8	15.4	14.4	11.6	11.2	11.6	3,776
Mar.	11.5	11.7	14.7	16.4	12.1	11.2	11.6	3,721
Apr.	11.2	11.3	11.4	14.8	12.4	11.3	11.9	3,571
May	11.5	12.0	11.5	11.7	12.9	11.6	11.8	3,532
June	11.5	11.6	11.1	11.2	13.1	12.0	11.8	3,510
July	11.7	11.8	11.3	11.6	13.4	12.0	12.1	3,559
Aug.	11.8	11.4	11.2	11.1	13.9	12.0	12.2	3,550
Sept.	12.5	12.1	11.8	11.6	15.5	13.4	12.8	3,736
Oct.	12.6	12.5	11.7	11.6	14.3	14.4	13.7	3,769
Nov.	14.1	13.3	12.2	11.8	12.6	14.6	15.0	3,855
Dec.	15.8	15.0	12.9	12.2	11.6	12.1	14.2	3,861

[a] Daily hay requirements for each calendar month of freshening were computed by the method shown in Tables 16, 17, and 18.
[b] Hay was not fed from May 1 through September 30.
[c] The total hay required annually is the sum of the daily hay requirements multiplied by 30.5 days plus the pounds of hay required for replacement.

Table 20. — Pounds of Silage Required per Cow per Day in Certain Months for Cows Freshening in Each Calendar Month[a]

Month of freshening	Month of feeding[b]								Annual total[c]
	Jan.	Feb.	Mar.	Apr.	Sept.	Oct.	Nov.	Dec.	
Jan.	42.6	48.0	42.0	40.2	35.7	34.2	34.8	36.0	11,129
Feb.	36.0	44.4	46.2	43.2	34.7	34.8	33.6	34.8	11,035
Mar.	34.5	35.1	44.1	49.2	38.7	36.3	33.6	34.8	10,902
Apr.	33.6	33.9	34.2	44.4	42.3	37.2	33.9	35.7	10,555
May	34.2	36.0	34.5	35.1	43.2	38.7	34.8	35.4	10,461
June	34.5	34.8	33.3	33.6	43.2	39.3	36.0	35.4	10,397
July	35.1	35.4	33.9	34.8	45.3	40.2	36.0	36.3	10,603
Aug.	35.4	34.2	33.6	33.3	46.2	41.7	36.0	36.6	10,601
Sept.	37.5	36.3	35.4	34.8	44.1	46.5	40.2	38.4	11,101
Oct.	37.8	37.5	35.1	34.8	36.3	42.9	43.2	41.1	10,981
Nov.	42.3	39.9	36.6	35.4	36.0	37.8	43.8	45.0	11,228
Dec.	47.4	45.0	38.7	36.6	35.4	34.8	36.3	42.6	11,230

[a] Daily silage requirements for each calendar month of freshening were computed by the method shown in Table 16.
[b] Silage was not fed from May 1 through August.
[c] Total silage required annually per cow is the sum of the daily silage requirements multiplied by 30.5 days plus the pounds of silage required for replacements.

APPENDIX B. A LINEAR PROGRAMMING MODEL FOR A TWO-PRICE MILK MARKET

The monthly price received for milk produced by a cow cannot be entirely determined by the month she freshens. The price received for one cow's milk production is partially influenced by the dairy-herd freshening program. Regardless of the monthly distribution of milk production of one cow, the total quantity of milk sold at the "base price" during the months of surplus[1] cannot exceed the average monthly sales from the herd during the "base period."[2]

To express this pricing system in the linear-programming model requires several selling activities in addition to one producing activity for each month of freshening. A milk-selling activity at "base price" was constructed for each of the surplus months. The C_j values for these activities were equal to zero. A positive value assigned to these activities would create double accounting as each of the 12-producing activities was assigned C_j values that represented the "base prices."

A milk-selling activity at "surplus price" was constructed for each of the surplus months. The C_j values assigned to these activities were equal to minus 40 cents. The activities thus express the penalty assessed against excess milk sales in March, April, May, and June.

[1] These months are March, April, May, and June.
[2] This period is from September 1 to November 30.

Four additional activities were necessary to account for the unused "fall base" established during the previous "base period." Activities for the unused fall base were constructed to account for the additional amount of milk that could have been sold at the "base price" during March, April, May, and June. C_j values assigned to each of these activities were zero. A hundred pounds of fall base has a positive value only when used.

By adding these 12 activities to the 12 milking-producing activities linear programming can be used to select the most profitable freshening program without violating the conditions of the pricing system for milk in the Chicago market. In the programming model, the relevant restrictions, activities, and coefficients are as follows:

						Milk sales in surplus period								*Dairy sales with base price milk*
		Unused base				*At "surplus" price*				*At "base" price*				
Resource	*Initial supply*	*M*	*A*	*M*	*J*	*M*	*A*	*M*	*J*	*M*	*A*	*M*	*J*	*Jan....Dec.*
Base established by fall output:														
March.....	0	1								1				−.053
April.....	0		1								1			−.053
May.....	0			1								1		−.053
June.....	0				1								1	−.053
Milk to sell in "surplus" period:														
March.....	0					1				1				−.128
April.....	0						1				1			−.117
May.....	0							1				1		−.116
June.....	0								1				1	−.106

The resource supplies for milk are furnished by each of the 12 (monthly) producing activities. They contribute to income as they are sold. The sales are distributed among (1) dairy output sold in each of the 12 months, with milk priced at "base" value, as shown at the right of the above table, (2) milk sales at base price (zero, to avoid double counting) in the surplus period, and (3) milk sales at "surplus" price in the surplus period. The latter "price" is −.40 per hundred pounds to reflect the deduction imposed on the price used in (1) for milk in excess of the base established by fall output.

Production coefficients are given for a January freshener to illustrate how the base is established by fall output and how milk is supplied at "surplus" price in the surplus period. A cow freshening in January establishes a "fall base" equal to 5.27 percent of her annual production

for each of the surplus months. Milk to sell at "surplus" price in the surplus months is supplied at rates that are also percents of annual production, but the size of the coefficients decreases from March to June. A January freshener produces 12.83 percent of her annual production in March, 11.70 in April, 11.62 in May, and 10.55 in June. The fall base is less than monthly production in each of the surplus months. If all the herd had been freshened in January, 7.56 percent (12.83 minus 5.27) of the annual production would have been sold at "surplus" price in March, 6.43 in April, 6.35 in May, and 5.28 in June. All of the "fall base" would have been used and milk sales at the "base" price in each surplus month would have been equal to 5.27 percent of the annual production.

No coefficients are given for the February-December activities listed at the right of the tabular material given on page 31. The coefficients related to resources are not included in this abbreviated table.

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