SELF-TAUGHT

MECHANICAL DRAWING

AND ELEMENTARY

MACHINE DESIGN

A Treatise
Comprising the First Principles of Geometric and Mechanical Drawing, Workshop Mathematics, Mechanics, Strength of Materials, and the Design of Machine Details, including Cams, Sprockets, Gearing, Shafts, Pulleys, Belting, Couplings, Screws and Bolts, Clutches, Flywheels, etc. Prepared for the Use of Practical Mechanics and Young Draftsmen.

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With Additions

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PREFACE

THE demand for an elementary treatise on mechanical drawing, including the first principles of machine design, and presented in such a way as to meet, in particular, the needs of the student whose previous theoretical knowledge is limited, has caused the author to prepare the present volume. It has been the author's aim to adapt this treatise to the requirements of the practical mechanic and young draftsman, and to present the matter in as clear and concise a manner as possible, so as to make "self-study" easy. In order to meet the demands of this class of students, practically all the important elements of machine design have been dealt with, and, besides, algebraic formulas have been explained and the elements of trigonometry have been treated in a manner suited to the needs of the practical man.

In arranging the material, the author has first devoted himself to mechanical drawing, pure and simple, because a thorough understanding of the principles of representing objects greatly facilitates further study of mechanical subjects; then, attention has been given to the mathematics necessary for the solution of the problems in machine design presented later, and to a practical introduction to theoretical mechanics and strength of materials; and, finally, the various elements entering in machine design, such as cams, gears, sprocket wheels, cone pulleys, bolts, screws, couplings, clutches, shafting, fly-wheels, etc., have been treated. This arrangement makes it possible to present a continuous course of study which is easily comprehended and assimilated even by students of limited previous training.

Portions of the section on mechanical drawing was published by the author in *The Patternmaker* several years ago. These articles have, however, been carefully revised to harmonize with the present treatise, and in some sections amplified. In the preparation of the material, the author has also consulted the works of various authors on machine design, and credit has been given in the text wherever use has been made of material from such sources.

Several important additions have been made by Mr. Erik Oberg, Associate Editor of *Machinery*. In the preparation of these additions, use has partly been made of material published from time to time in *Machinery*.

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APRIL, 1910.

CONTENTS

CHAPTER IV

PROJECTION

CHAPTER V

WORKING DRAWINGS

CHAPTER VI

ALGEBRAIC FORMULAS

CHAPTER VII

ELEMENTS OF TRIGONOMETRY

CHAPTER VIII

ELEMENTS OF MECHANICS

CHAPTER IX

FIRST PRINCIPLES OF STRENGTH OF MATERIALS

CHAPTER X

CAMS

CHAPTER XI

SPROCKET WHEELS

CHAPTER XII

GENERAL PRINCIPLES OF GEARING

CHAPTER XIII

CALCULATING THE DIMENSIONS OF GEARS

CHAPTER XIV

CONE PULLEYS

CHAPTER XV

BOLTS, STUDS AND SCREWS

CHAPTER XVI

COUPLINGS AND CLUTCHES

CHAPTER XVII

SHAFTS, BELTS AND PULLEYS

CHAPTER XVIII

FLY-WHEELS FOR PRESSES, PUNCHES, ETC.

CHAPTER XIX

TRAINS OF MECHANISM

CHAPTER XX

QUICK RETURN MOTIONS

SELF-TAUGHT MECHANICAL DRAWING

CHAPTER I

INSTRUMENTS AND MATERIALS

ONE who is to study the subject of drawing should not merely read a book on the subject, but should prepare sheets of exercises. This will fix the principles which he learns in his mind in a way as reading alone will not do, and will give him practical experience in the use of the tools. The geometrical problems given in this book make perhaps the best of subjects for a beginning, as their proper execution will require careful work. Later, the student may make dimensioned free-hand sketches of some machine with which he is familiar, and from these sketches he may make up a set of finished working drawings. In all of this work, care should be taken to have it so laid out, with proper margins and spaces between different parts, that the drawing when finished shall present an appearance of neatness and methodical arrangement.

For the purposes of the student, a drawing board about 15 by 18 inches will be large enough. With this should be an 18-inch T-square, a pair of 6-inch triangles, and a set of three or four irregular curves.

For drawing full-size work, a good flat beveled-edge rule will answer ordinary requirements, but for making half- or quarter-size drawings some kind of a "scale" will be found desirable. The triangular scale shown in Fig. 1 is perhaps the one mostly used, and it has the advantage of possess-

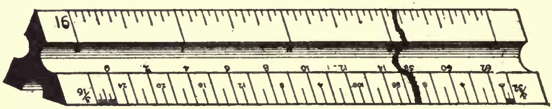


FIG. 1.—The Triangular Scale.

ing six surfaces for graduations, giving variety enough for all sorts of conditions, but it has the disadvantage of persistently presenting the wrong edge, and putting one to the trouble of turning it over and over to get the desired edge. This trouble may, of course, be overcome by using a scale guard such as is shown in Fig. 2, but the guard is itself often in the way. As but two or three different scales, aside from full size, will be likely to be required, it will be found much more convenient to have a separate flat scale for each graduation. Such scales may be purchased, or, if one is satisfied with the open graduation system shown in Fig. 3, he may make them without much trouble himself. In this system, only one inch is divided, this inch being numbered 0; and measurements which include a

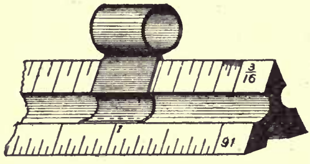


FIG. 2.—Scale Guard or Holder used on Triangular Scale.

fractional part of an inch are reckoned from the required whole number to the proper place on the divided inch.

The drawing instruments themselves, while not necessarily of the highest price, should be of a good serviceable quality of German silver. The cheap brass or nickel plated school sets should not be considered, as they will prove unsatisfactory for regular work. It is not necessary to have a large number of instruments. A very good set, sufficient for all ordinary requirements, might be as follows: First a pair of about 4½- or 5-inch com-

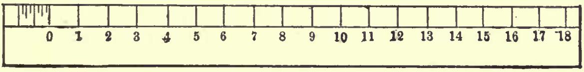


FIG. 3.—Inexpensive Type of Scale.

passes with fixed needle points (bayonet points are useless) and interchangeable pin and pencil points, with lengthening bar. Then, a pair of hair-spring spacers of about the same size. These resemble ordinary plain compasses, but the steel end of one leg is made adjustable by means of a thumb screw. Next, a pair of ruling pens, one large and one small, and, lastly, a set of three spring instruments, pen, pencil and spacers, for small work. Rather than to get cheap instruments, it would be advisable to obtain a set gradually by getting the large instruments and one pen first, and adding the second pen and the spring instruments later. The large compasses can, if necessary, be used to make circles of from about ¼ inch to about 18 or 20 inches in diameter, so that they will do very well for a beginning. For making larger circles, beam compasses, in which separate heads for the needle point and for the pen or pencil point are attached to a wooden bar, after the manner of workmen's trammels, are used.

A convenient case for the instruments, when they are bought separately, is shown in Fig. 4, and is made as follows: Take two pieces of chamois skin or thin broadcloth, one of them about one-half longer than the longest instrument, and somewhat wider than all of them when they are

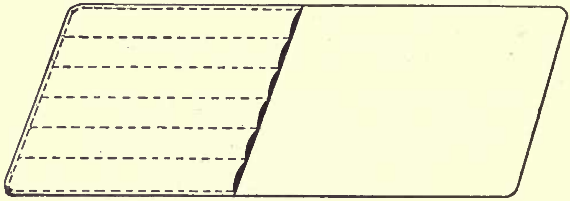


FIG. 4.—Home-made Instrument Case.

laid out side by side, and the second one of the same width as the first, but somewhat shorter than the longest instrument. This second piece is sewed onto the large piece at one end by the outer edges. Pockets for the reception of the instruments are then made as shown, and when the free end of the large piece is folded over, the instruments are rolled up together.

The pencils, which to avoid scratching particles, should be of best quality, should not be sharpened to a round point, but to a flat oval point, as such a shape will wear longer than a round point; the leads used in the compasses, however, should be only slightly flattened. It will be found desirable to have two grades of pencils, one quite hard, about "4H," to be used for laying out work, and a softer one, about "2H," to be used for going over the lines of work which is not to be inked in. In laying-out work where the hard pencil is used, only a moderate pressure should be applied, so as to permit of erasures at any time, whether for the purpose of making alterations, or to free the drawing of pencil marks after inking.

The drawing pens should be kept sharp, though not so sharp as to cut the paper, and their ends should present a neat oval shape. The needle points of the compasses should also be kept sharp to avoid the tendency to slip when doing work where it is undesirable to prick through the paper. A small Arkansas stone will be found useful for this purpose. Where much use is made of a given center, it may be desirable to employ a horn or metal center, such as are kept in stock by dealers in artists' supplies, to avoid the troublesome enlargement of the center in the paper which the points of the compasses would otherwise make.

In making a drawing, care should be taken to have the preliminary pencil work done correctly. It is a mistake which beginners are likely to make, to think that errors in the pencil work may be readily corrected in the inking. This, however, is usually another case where "haste makes waste." It is much better to spend a little extra time on the pencil work, than to have to throw away a nearly finished ink drawing and do the work all over again. In locating the various views of a drawing upon the paper, it will frequently be found to be well to make rough sketches of it on scrap paper. These sketches can then be moved around on the drawing paper until the best arrangement is secured.

In making a drawing, it will be found most convenient, ordinarily, to limit the use of the T-square to horizontal lines, the head of the square being kept pressed firmly against the left-hand end of the drawing board. Vertical lines are then made

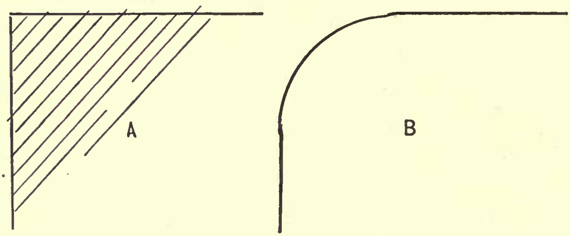


FIG. 5.—Appearance of Carelessly made Drawing.

with the aid of the triangles resting against the blade of the T-square. Vertical lines which are too long to be made in this way, are, of course, made with the T-square itself. In inking in a drawing, it is best to draw all curved or circular lines first, as it is easier to join straight lines onto curved lines than to join curved lines onto straight lines. Care should also be taken to have meeting lines just meet, whether they meet end to end or at an angle. Carelessness in this respect gives a drawing a very bad appearance, as shown by Fig. 5, *A* and *B*.

In using the pens, whether the ruling or the compass pens, care should be taken to see that both nibs rest upon the paper, otherwise lines such as shown in Fig. 6 may result. If the pen does rest squarely upon the paper, and such lines continue to appear, it is fair to infer that the paper has become somewhat greasy, perhaps from too much handling. This trouble may be avoided, and the work kept cleaner, by having a piece of thin paper interposed between the hands and the drawing paper.

The cross hatching work, such as is shown at *A* in Fig. 5, is frequently done by simply using one of the triangles resting against the blade of the

FIG. 6.—Line Resulting from not Having both Pen Points or Nibs Resting on the Paper when Inking.

T-square, the same as is done for vertical lines, the spacing being done entirely by the eye; but unless one is doing a good deal of this work, so as to keep in practice, he will find it very difficult to make the spacing regular. There are various section-lining devices on the market for doing this work, some of them quite expensive. Fig. 7 shows a simple device for cross-sectioning, which serves the purpose as well as any of the more elaborate ones, and possesses the additional advantage that anyone may readily make it for himself. This instrument was shown by Mr. E. W. Beardsley in *Machinery*, September, 1905. An old instrument screw, *B*, is screwed into a slightly smaller hole in a piece of wood, *A*, shaped as shown, and of a thickness a little in excess of the diameter of the screw-head. This combination is then used in the central hole in a triangle, as shown. Then, with one finger on the triangle itself, and with another one on *A*, the two may be moved along, first one and then the other, for section lining, the desired width of space being secured by the adjustment given to *B*.

For making erasures of ink lines on paper, a steel scraping eraser or a sharp knife blade is usually the best, the roughened surface being afterwards rubbed down smooth with some hard substance. When making erasures of either pencil or ink with a rubber eraser, an erasing shield, such as is shown in Fig. 8, is useful for preventing rubbing out more than is intended. These shields are made both of thin sheet metal and of celluloid; the metal ones, being the thinner, are the more convenient to use.

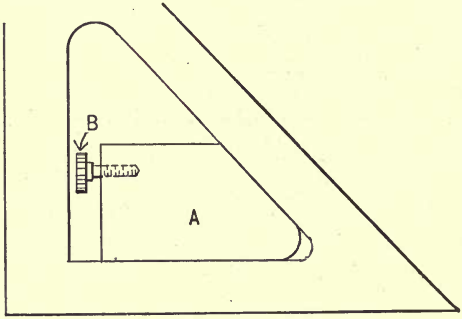


FIG. 7.—Simple Cross-section Liner.

The paper used, if good work is desired, should be regular drawing paper, whether it be white or brown. This has an unglazed surface, and will be found much more satisfactory in every way than common paper. The glazed surface of the cheaper paper does not take pencil marks well, and is torn up badly in making erasures. Such paper, if used at all, should be used only on the most temporary

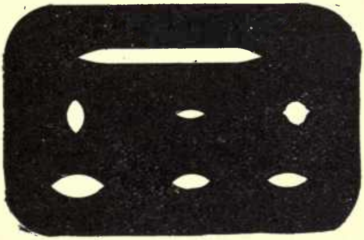


FIG. 8.—Erasing Shield made from Sheet Metal or Celluloid.

work. Of white drawing papers, the smooth surfaced kinds should be selected. For making ink drawings, it will be found most satisfactory to use the prepared drawing inks, rather than to go to the trouble of preparing it oneself from the stick India ink.

For fastening the paper on to the board, common one-half-ounce copper tacks are as good, if not preferable, to other fastening means.

CHAPTER II

DEFINITIONS OF TERMS USED IN GEOMETRICAL AND MECHANICAL DRAWING

1. A *Point* has position, but not magnitude.

2. A *Line* has length, but neither breadth nor thickness.

3. A *Surface* has length and breadth, but not thickness.

4. A *Solid* has length, breadth and thickness.

5. A *Plane* is a surface which is straight in every direction; that is, one which is perfectly flat.

6. *Parallel* lines are such as are everywhere equally distant from each other. Circular lines which answer to this condition are also said to be *concentric.*

7. An *Angle* is the difference in the direction of two lines. If the lines meet, the point of meeting is called the *vertex* of the angle, and the lines *ab* and *ac*, Fig. 9, are its *sides.*

8. If a straight line meets another so that the adjacent angles are equal, each of these angles is a *right angle,* and the two lines are *perpendicular* to each other. Thus the angles *acd* and *dcb,* Fig. 10, are right angles, and the lines *ab* and *dc* are perpendicular to each other. A distinction is to be made here between the words *perpendicular*

and *vertical.* A vertical line is one which is perpendicular to the plane of the earth's horizon; that is, to the surface of still water.

9. An *Obtuse Angle* is one which is greater than a right angle, as *ace*, Fig. 10.

10. An *Acute Angle* is one which is less than a right angle, as *ecb*, Fig. 10.

11. It is obvious that the sum of all the angles which may be formed about the point *c*, Fig. 10, above the line *ab* will be equal to the two right angles *acd* and *dcb*.

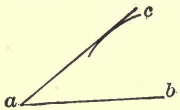


FIG. 9.—Angle.

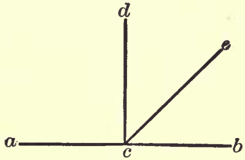


FIG. 10.—Illustration for Making Clear the Terms Right, Acute and Obtuse Angles.

12. The *Complement* of an angle is a right angle, less the given angle. Thus *bce*, Fig. 10, is the complement of *dce*.

13. The *Supplement* of an angle is two right angles less the given angle. Thus *bce*, Fig. 10, is the supplement of *ace*.

14. A *Circle* is a continuous curved line, Fig. 11, or the space enclosed by such line, every point of which is equally distant from a point within called the *center*.

15. The distance across a circle, measured through the center, is the *diameter*. The distance around the circle is the *circumference*. The dis-

tance from the center to the circumference is the *radius*.

16. The ratio between the circumference and the diameter, that is, the circumference divided by the diameter, is 3.1416. While this is not exact (Bradbury's *Geometry* states that it has been carried out to two hundred and fifty places of decimals), it is near enough for practical purposes. This ratio is frequently represented by the Greek letter π (pi).

17. A circle is considered as being equally divided

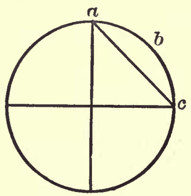


FIG. 11.—Illustration for Making Clear the Terms Relating to the Circle.

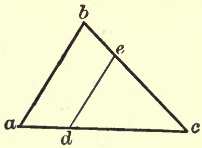


FIG. 12.—Similar Triangles.

into three hundred and sixty *degrees* (360°), each degree into sixty *minutes* (60′), and each minute into sixty *seconds* (60″).

18. If two diameters cross each other at right angles, the circle is divided into four equal parts; hence a right angle contains ninety degrees.

19. An *Arc* of a circle is any part of its circumference, as *abc*, Fig. 11.

20. A *Chord* is a straight line joining the ends of an arc, as *ac*, Fig. 11.

21. Two triangles, as *abc* and *dec*, Fig. 12, having like angles are *similar* triangles. The corre-

sponding sides of similar triangles have the same ratio. Thus if *ac* were twice as long as *dc*, *ab* would be twice as long as *de*, and *bc* would be twice as long as *ec*.

22. The sum of the angles of a triangle is equal to two right angles. Let *abc*, Fig. 13, represent any triangle. Extend one side, *ac*, as shown, and make *cd* parallel with *ab*. Then the angle *dce* is equal to the angle *bac*, for their sides have the same direction, and the angle *bcd* is equal to the

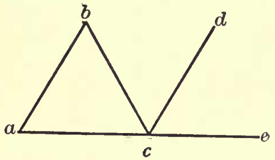


FIG. 13.—Illustration for Showing that the Sum of the Angles in a Triangle equals Two Right Angles.

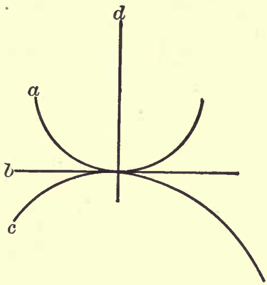


FIG. 14.—Tangent and Normal to a Curve.

angle *abc*, for their sides have opposite directions; hence the sum of the three angles formed about the point *c* is equal to the sum of the three angles of the triangle *abc*, and these are equal to two right angles (11).

23. A *Tangent* is a line which touches another, but does not, though extended, cross it. Thus, *a*, *b* and *c*, Fig. 14, are tangent lines. A line, *d*, perpendicular to the straight line *b*, at the point of tangency, is called a *normal*. If one of the

lines, as *a*, is circular, the normal will pass through its center.

24. A *Parallelogram* is a figure whose opposite sides are parallel, as *ab* and *cd*, or *eb* and *fd* in Fig. 15. The sides may all be of equal length,

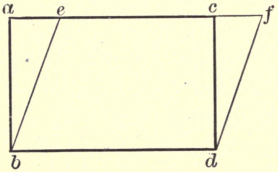


FIG. 15.—Parallelograms.

FIG. 16.—Square.

when the parallelogram is called a square. (See Fig. 16.)

25. Figures having five, six or eight sides are called respectively *Pentagon, Hexagon* and *Octagon.* These, and all figures having more than four sides, are called *Polygons.* If the sides in a polygon are

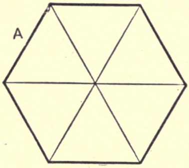


FIG. 17.—Regular Polygon.

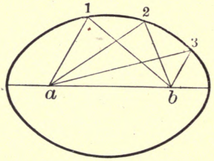


FIG. 18.—Ellipse.

all of equal length, and all the angles equal, the polygon is called a *regular polygon.* (See Fig. 17.)

26. An *Ellipse*, Fig. 18, is a continuous curved line, or the space enclosed by such line, of such shape that the sum of the distances from two

points within, as *a* and *b*, called the *foci* (singular: *focus*), to any point upon its circumference is constant. Thus *a1* plus *b1* equals *a2* plus *b2* or *a3* plus *b3*.

27. An *Involute* is a line of such shape (as *a* in Fig. 19) as might be made by a pencil at the end of a string which is unwound from a circle.

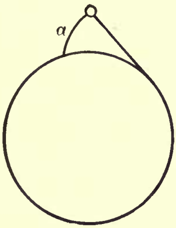


FIG. 19.—Involute.

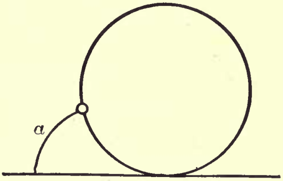


FIG. 20.—Cycloid.

28. A *Cycloid* is a line of such shape (as *a* in Fig. 20) as might be made by a pencil fastened to the circumference of a circle which is being rolled upon a straight line. If the circle was being rolled upon the convex side of a circular line the line traced by the pencil would be an *epicycloid*. If it was being rolled upon the concave side of a circular line, the line traced by the pencil would be a *hypocycloid*. The involute and cycloidal curves are used in gear outlines.

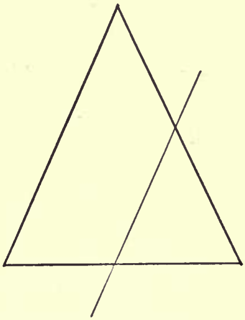


FIG. 21. Method of Sectioning a Cone to Obtain a Parabola.

29. A *Parabola* is a curve which may be ob-

tained by cutting a cone so that the exposed sectional surface will be parallel with one of the sides of the cone, as shown in Fig. 21. This curve, as shown in Fig. 22, is of such shape that lines drawn to it from a certain point within, called the *focus*, shown at *f* in the illustration, make the same angle with it as lines drawn from

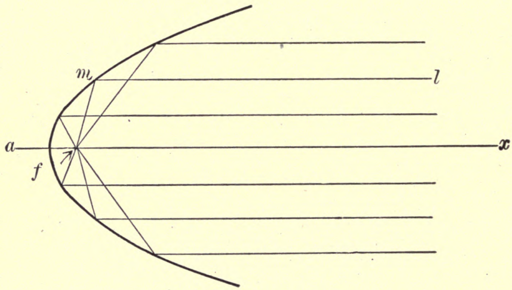


FIG. 22. Parabola.

the intersection points parallel with the axis *ax*. Thus the line *fm* makes the same angle with the parabola, at the point of intersection, as the line *ml*. Because of this property of the parabola, mirrors of this shape are used in headlights of locomotives, in search lights, and in many lighthouses; because, if a light be placed at the focus, its rays, when reflected from the mirror, will be thrown out in parallel lines.

CHAPTER III

GEOMETRICAL PROBLEMS

Prob. 1, Fig. 23. To bisect a line, either curved as abc, or straight as ac.—With centers at *a* and *c* and with a radius somewhat greater than half the length of the line, describe the arcs *d* and *e*. A line passing through the intersections of these arcs bisects either line. It will also pass through the center of the circle of which the arc *abc* is a part.

Prob. 2, Fig. 24. To bisect an angle.—With center at *a*, and with any convenient radius, describe the arc *bc*. With centers at *b* and *c*, and with a radius greater than half the arc, describe the arcs *d* and *e*. A line from *a* through the intersection of these arcs bisects the angle.

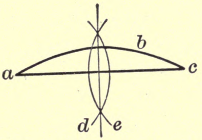


FIG. 23.—Bisecting a Line.

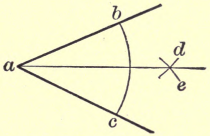


FIG. 24.—Bisecting an Angle.

Prob. 3, Fig. 25. To make an angle equal to a given angle.—Let *a* be the given angle, and let it be desired to make an angle equal to it on the line *dg*. With center at *a* make the arc *bc*, and then with center at *d* make the arc *eh* with the same radius. Then with a radius equal to *bc*, and with center at *h*, make the arc *f*. A line from *d* through the intersection of the arcs gives the required angle.

Prob. 4, Fig. 26. To erect a perpendicular at the end of a line, ab.—With any convenient center, *c*,

FIG. 25.—Making an Angle Equal to a Given Angle.

and with radius *cb*, draw a semicircle intersecting *ab* at *d*. Draw a line from *d* through *c* intersecting the semicircle at *e*. A line from *b* passing through *e* is the required perpendicular.

Prob. 5, Fig. 27. To drop a perpendicular from a point a, to a given line bc.—With *a* as a center,

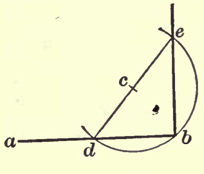


FIG. 26.—Erecting a Perpendicular Line.

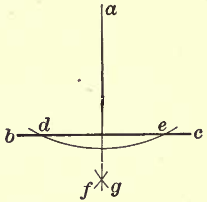


FIG. 27.—Drawing a Perpendicular Line.

draw an arc intersecting *bc* at *d* and *e*. With *d* and *e* as centers draw the intersecting arcs *f* and *g*. A line from *a* through the intersection of these arcs is the required perpendicular. If *a* were over one end of the line *bc* the process shown in the preceding problem might be reversed by drawing a line from *a* corresponding to *de*, Fig. 26, and upon this line drawing a semicircle, when its intersection with the base line would give the point to which the perpendicular from *a* should be drawn.

Prob. 6, Fig. 28. To draw a tangent to a circle at a given point.—Draw a radius of the circle to the required point, and erect a perpendicular to it, which will be the required tangent. To find the point of tangency of a line to a circle, drop a perpendicular to the tangent from the center of the circle.

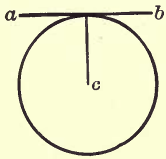


FIG. 28.—Drawing a Tangent to a Circle.

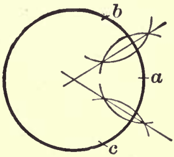


FIG. 29.—Finding the Center of a Circle.

Prob. 7, Fig. 29. To find the center of a circle.—Mark off two arcs as *ab* and *ac* upon the circumference, and bisect these arcs as in Prob. 1. Where these bisecting lines cross each other will be the required center.

Prob. 8, Fig. 30. To draw a regular hexagon upon a given base, ab.—With a radius equal to the length of *ab* draw the arcs *c* and *d*. The intersection of these arcs will be the center of a circumscribing circle upon which the other sides may be marked off.

Prob. 9, Fig. 31. To draw a regular octagon in a square.—Draw the diagonals of the square, *ad* and *bc*, and with a radius equal to half of a diagonal, and with centers at *a*, *b*, *c* and *d*, draw the arcs *e*, *f*, *g* and *h*. The intersections of these arcs

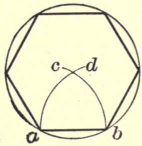


FIG. 30.—Drawing a Regular Hexagon.

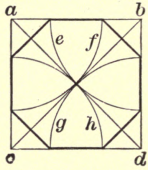


FIG. 31.—Drawing a Regular Octagon.

with the sides of the square give the corners of the required octagon.

Prob. 10, Fig. 32. To draw a circle about a triangle, as abc.—Bisect any two of the sides as in Prob. 1. Where the bisecting lines cross each

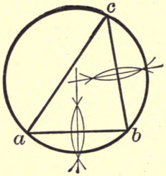


FIG. 32.—Drawing a Circle about a Triangle.

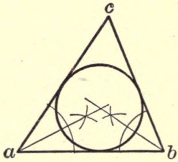


FIG. 33.—Inscribing a Circle in a Triangle.

other will be the center of the required circle. In a similar manner a center may be found from which to draw a circle through any three given points, the given points in this case being the corners of the triangle.

Prob. 11, Fig. 33. To draw a circle within a given triangle, as abc.—Bisect any two of the angles as in Prob. 2. Where the bisecting lines cross, will be the center of the required circle. In a similar manner a center may be found from which to draw a circle tangent to any three given straight lines.

Prob. 12, Fig. 34. To find the foci of an ellipse.—Draw the long and the short diameters of the ellipse, *ab* and *cd*, and with a radius equal to half of the long diameter, and with a center at *c* or *d*

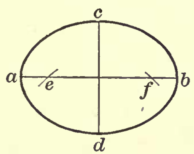


FIG. 34.—Finding the Foci of an Ellipse.

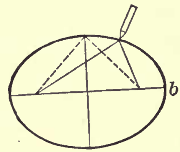


FIG. 35.—Simplified Method of Drawing an Ellipse.

draw the arcs *e* and *f*. Where these arcs intersect the long diameter will be the required foci.

Prob. 13, Fig. 35. To draw an ellipse with a pencil and thread.—Having found the foci of the ellipse, stick a pin firmly into each focus, and looping a thread around them, allow it to be slack enough so that the pencil will draw it out to the end of the short diameter. The thread will then guide the pencil so that it will draw an ellipse. A groove should be cut around the pencil lead to prevent the thread from slipping off.

Prob. 14, Fig. 36. To draw an ellipse with a trammel.—Lay out the long and the short diameters of the ellipse, *ab* and *cd*, and on a strip of paper, *A*, mark off *1-3* equal to half of the long diameter, and *2-3* equal to half of the short diameter. Then, keeping point *1* on the short diameter, and point *2* on the long diameter, mark off any desired number of points at *3*. A curved line passing through these points will be the required ellipse. The ellipsograph, an instrument for drawing ellipses, is made on this principle, points at *1* and *2* traveling in grooves which coincide with *ab* and *cd*.

Prob. 15, Fig. 37. To draw an ellipse by tangent lines.—Make *ab* equal to one-half of the long di-

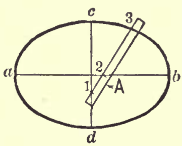


FIG. 36.—Another Method of Drawing an Ellipse.

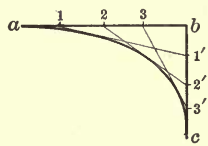


FIG. 37.—Drawing an Ellipse by Tangents.

ameter of the required ellipse, and *bc* equal to one-half its short diameter. Divide *ab* and *bc* into the same number of equal parts, and, numbering them as indicated, connect *1* and *1′*, *2* and *2′* and so forth. A curved line starting at *a*, tangent to these lines, and ending at *c*, is one-quarter of the required ellipse.

Prob. 16, Fig. 38. To draw an approximate ellipse with compasses, using four centers.—Lay out the long diameter *ab*, and the short diameter *cd*, crossing each other centrally at *o*. From *b* measure off *be* equal to *co*, one-half of the short diameter. The length *ae* will then be the radius *gh* for forming the part *hk* of the ellipse. From *e* mark off the point *f*, making *ef* equal to one half of *oe*. The point *f* will be the center, and *fb* the radius for forming the end of the ellipse. Lines drawn from the centers *g* through the points *f* determine the points at which the different curves meet. This method is not considered applicable when the short diameter is less than two-thirds of the long diameter.

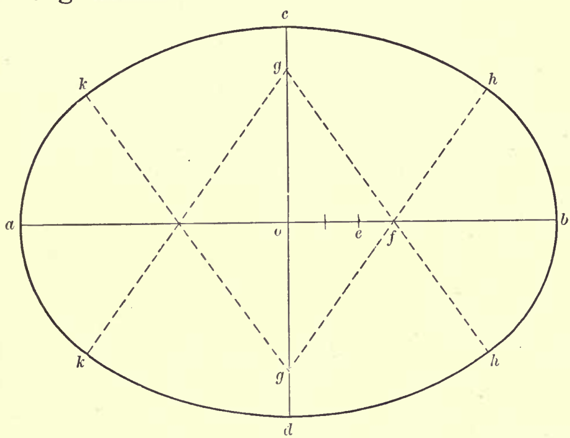


FIG. 38.—Drawing an Approximate Ellipse by Four Circular Arcs.

Prob. 17, Figs. 39 and 39a. To draw an approximate ellipse with compasses, using eight centers.—Lay out the long diameter *ab*, and the short diameter *cd* crossing each other centrally at *f*. Construct the parallelogram *aecf*, and draw the diagonal *ac*. From *e* draw a line at right angles to *ac*, crossing the long diameter at *h*, and meeting the short diameter, extended, at *g*. Point *g* is the center from which to strike the sides of the ellipse, and *h* will be the center, subject to certain modifications for narrow ellipses, from which to strike the ends of the ellipse. To get the radius of the third curve for connecting the side and end curves, lay off a base line *ab*, Fig. 39A, of any convenient length, and divide it into five equal parts by the points *1*, *2*, *3* and *4*. At one end of the line erect the perpendicular *ac*, equal to the end radius *ah*, and at the other end erect the perpendicular *bd* equal to the side radius *cg*. Connect the ends of these perpendiculars by the line *cd*, and at point *2* erect a perpendicular, meeting *cd* at *e*. The length *e2* will be the desired third radius. With the compasses set to this radius, find a center *i* from which a curve can be struck which will be just tangent to the side and end curves. From other centers similarly located the remainder of the ellipse is drawn. Lines drawn from *i* through *h*, and from *g* through *i* determine the meeting points of the different curves.

For narrow ellipses the length of the end radius, *ah*, should be increased as follows: For an ellipse having its breadth equal to one-half of its length, make *ah* one-eighth longer. For an ellipse having its breadth one-third of its length, make *ah* one-fourth longer. For an ellipse having its breadth equal one-quarter of its length, make *ah* one-half longer. For intermediate breadths lengthen *ah* proportionately. With this modification of the length of the end radius, this method gives curves which blend well together so as to satisfy the eye, and gives a figure which conforms quite closely to the actual outlines of an ellipse.

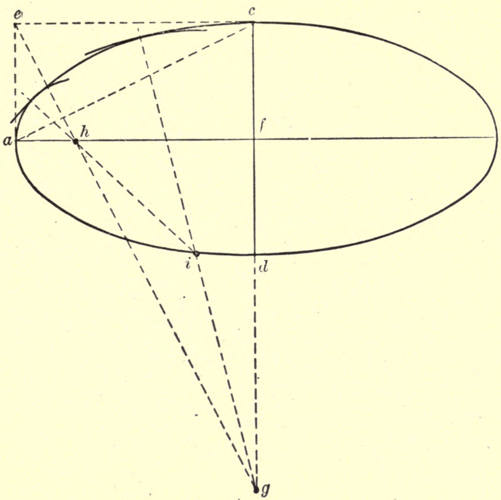


FIG. 39.

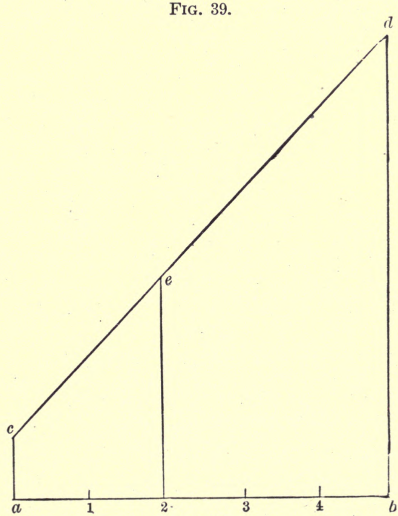


FIG. 39a.

FIGS. 39 and 39a.—Drawing an Approximate Ellipse by Eight Circular Arcs.

Prob. 18, Fig. 40. To draw a regular polygon of any number of sides on a given base, ab.—Extend *ab* as shown, and on it with one end as a center and a radius equal to the length of the given side, draw a semicircle. Divide this semicircle into as many equal spaces as there are to be sides to the polygon. A line from *b* to the *second* space, reckoning from where the semicircle meets the extension of *ab*, will be a second side of the required polygon. Lines are then drawn from *b* through the remaining divisions of the semicircle, and the remaining

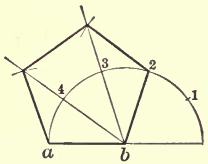


FIG. 40.—Drawing a Regular Pentagon.

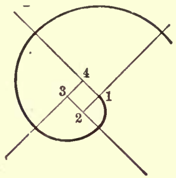


FIG. 41.—Drawing a Spiral about a Square.

sides of the polygon are marked off upon them as indicated. If the polygon is to have many sides, as an additional precaution against error, bisect *ab* and *b2*, thus getting the center of a circumscribing circle upon which the remaining sides may be marked off.

Prob. 19, Fig. 41. To draw a spiral about a square.—Lay out a square, *1-2-3-4*, having the length of each side equal to one-quarter of the desired distance between the successive convolutions of the spiral, and extend each side in one direction as shown. With a center at *2*, and with a radius *1-2* draw a quarter of a circle. With a center at *3* draw another quarter of a circle, continuing the first one, and so continue with successive corners of the square for centers.

Fig. 42 shows how, by similarly extending one end of each side, a spiral may be drawn about a regular polygon of any number of sides. A curve so formed determines the shape of the teeth of sprocket wheels.

Prob. 20, Fig. 43. To draw an involute.—Upon the circumference of the given circle mark off any

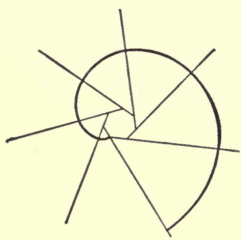


FIG. 42.—Drawing a Spiral about a Regular Polygon.

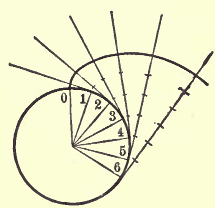


FIG. 43.—Drawing an Involute.

number of equally distant points, as *0-1-2-3*, etc., and draw lines tangent to the circle at these points, beginning at point *1*. Then with the compasses set the same as for marking off the spaces on the circle, mark off one space on line *1*, two spaces on line *2*, three spaces on line *3*, and so forth. A curved line starting at *0* and passing through these points will be the required involute. This curve is used for the shape of the teeth of involute gears.

Prob. 21, Fig. 44. To draw a cycloid.—Upon the base line *ab* mark off any number of equally distant points, as *0-1-2-3*, etc., the distance between them being made, for convenience sake, about one-sixth of half the circumference of the generating circle. Beginning at *1* erect perpendiculars from these points, and with centers on these lines draw arcs of circles tangent to the base line to represent

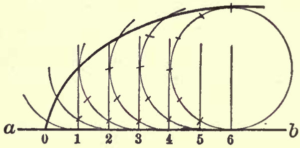


FIG. 44.—Drawing a Cycloid.

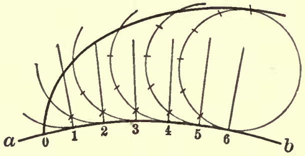


FIG. 45.—Drawing an Epicycloid.

successive positions of the generating circle as it is rolled along. With the compasses set as for spacing off the base line, mark off one space on the arc which starts from point *1*, two spaces on arc *2*, three spaces on arc *3*, and so forth. A curved line starting at *0* and passing through the points thus obtained will be the required cycloid.

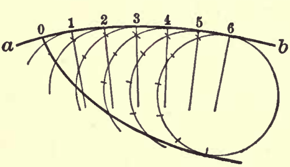


FIG. 46.—Drawing a Hypocycloid.

An epicycloid, Fig. 45, or a hypocycloid, Fig. 46, is formed in precisely the same way, excepting that as the base line, *ab*, is an arc of a circle, the center lines from points *1-2-3*, etc., are made radial.

These three cycloidal curves are used for the shape of the teeth of epicycloidal gears, sometimes called simply cycloidal gears.

Prob. 22, Fig. 47. To draw a parabola by means of intersecting lines.—Draw the axis *ax*, and on it mark the focus *f* and the vertex *v*, and at right angles to it draw the line *bc* at a distance from *v* equal to the distance of *v* from *f*. Across the axis, and at right angles to it, draw a number of lines, *1, 2, 3, 4, 5, 6*. Then with radius *a1*, and with center at the focus *f*, draw arcs intersecting line *1;* with radius *a2*, and with center again on *f* draw arcs intersecting line *2*, and so on. A curved line

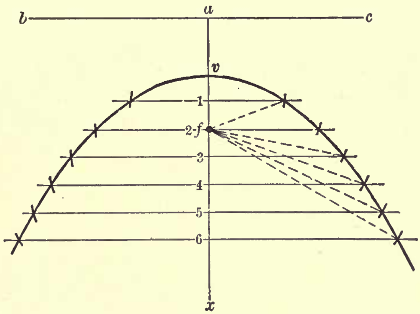


FIG. 47.—Drawing a Parabola.

passing through these intersections will be a parabola. It will be seen from this method of drawing a parabola that any point on it is equally distant from the focus, and from the line *bc*, called the *directrix*.

Prob. 23, Fig. 48. To draw a parabola with a pencil and string.—Lay out the axis, the focus, the vertex and the directrix as before. Attach one end of a thread to the focus, *f*, by means of a pin, and attach the other end of the thread to the square shown at *d*, having the thread of such length that when the inner edge of the square is on the axis, *ax*, the thread if drawn down with a pencil will just reach to the vertex, *v*. Now slide the square along *bc* in the direction of the

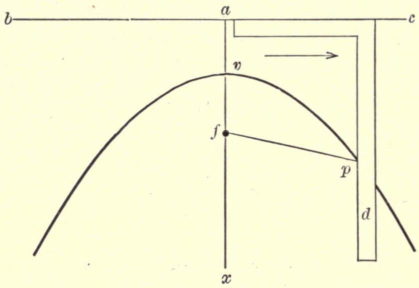


FIG. 48.—Simplified Method of Drawing a Parabola.

arrow, keeping the pencil against the square; the thread will cause the pencil to move along so as to describe a parabola as shown.

Prob. 24, Fig. 49. To draw a parabola of a given

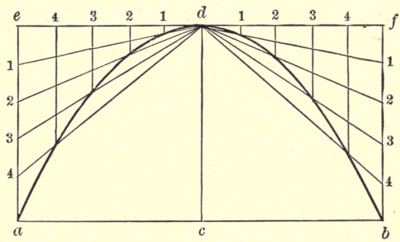


FIG. 49.—Another Method of Drawing a Parabola.

breadth of opening, ab, and of a given depth, cd.—Draw *ef* parallel with *ab*, and draw *ae* and *bf* parallel with *cd*, having *ac* and *bc* equal. Space off *dc* and *df* into any number of equal parts, and also space off *ea* and *fb* into the same number of equal parts, as shown. From *d* draw lines to the divisions on *ea* and *fb*, and from *1, 2, 3* and *4* on *de* and *df* draw perpendicular lines to intersect the lines drawn from *d* to *1, 2, 3* and *4* on lines *ca* and *fb*. A curved line passing through these intersections will be the required parabola.

Prob. 25, Fig. 50. To find the focus of a parabola.—Let *abcd* be the given parabola, *ef*, being its

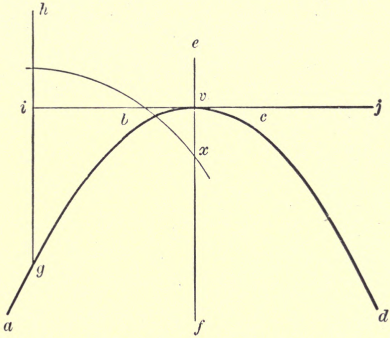


FIG. 50.—Finding the Focus of a Parabola.

axis. Across the parabola at its vertex, *v*, draw the line *ij* at right angles to the axis. From any point, *g*, on the parabola, draw the line *gh* parallel to the axis. With center at *g* find a radius, by trial, which will cut the axis as much inside the vertex, *v*, as it cuts the line *gh* beyond the line *ij*. The intersection at *x* will be the required focus.

CHAPTER IV

PROJECTION

Mode of Representing Objects.—In mechanical drawing, machines, or parts of machines, are represented by views, generally three, in which perspective is ignored, and which show the object in different positions at right angles to each other. The mode of representing these views, and their positions with regard to one another, which experience has shown to be most convenient is perhaps best shown by means of the familiar cardboard illustration. Let *abcdefgh*, Fig. 51, represent a piece of cardboard, which we will suppose to be transparent, creased on the dotted lines to permit of the outer portions being turned back. Let us now suppose that we have a prism shaped as shown at *C*, and of the length shown at *A*. If the prism is stood upright with its broad side facing the observer, and the cardboard, being blank, is held up in front of it, the prism will appear, if all its lines are brought perpendicularly forward to the cardboard, as it is shown at *A*, lines on the prism which would be hidden by its body, as the further corner, being dotted. If section *C* of the cardboard is now turned backward through an angle of 90 degrees over the top of the prism we would get the view shown in that part, all lines being brought perpendicularly forward from the prism to the cardboard as before. Likewise if part D of the cardboard were turned backward through an angle of 90 degrees, and the lines of the prism were brought perpendicularly forward onto it, we would get the view shown in that part. The view shown at A is called the *elevation*, that shown at C is called the *plan*, and that shown at D is called the *side view*. Occasionally a piece is so shaped, or

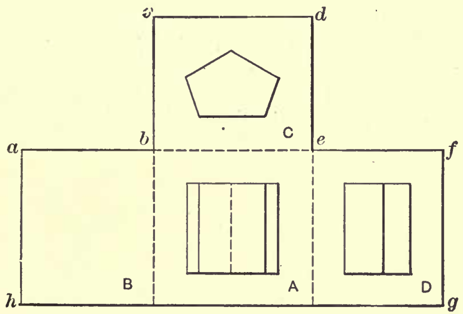


FIG. 51.—Principle of Projection.

has so much of detail to it as to make another side view desirable; such a view would be placed at B. In many other cases, as in the case of the prism here shown, the plan and elevation views alone will fully show the object.

The production of these views from one another is called *projection;* and by the use of connecting lines, and also at times of temporary construction views, objects may be shown at any desired angle, irregular or curved lines may be traced, and surfaces may be developed.

An Upright Prism.—Fig. 52 shows a prism in its simplest position. A moment's examination will show that the elevation cannot be drawn directly, as the distance apart of the vertical lines which represent the corners of the prism, cannot be determined without other aid; hence it is necessary to draw the plan view first. Horizontal lines having been made to give the height of the prism in the elevation, the vertical lines may then be drawn in

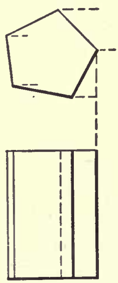


FIG. 52.—Projections of Prism.

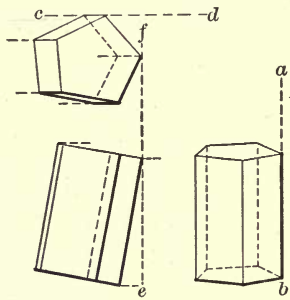


FIG. 53.—Projections of Tilted Prism.

from the plan, as indicated by the vertical dotted line.

The Prism Inclined at One Angle.—Fig. 53 shows the prism inclined to the right. A brief examination of these views will show that none of them can be drawn directly, as the distance apart of the vertical lines in the elevation and side views is not known, and the lines of the plan view are foreshortened; but the views can be developed from Fig. 52. It is evident that as the prism is tipped, the elevation view will remain unchanged, hence the first step will be to reproduce that view inclined at the desired angle. As the prism is tipped it is also evident that all points in the plan view of Fig. 52 will move in horizontal lines to the right, hence horizontal lines are drawn from these points through the position which the plan will occupy in Fig. 53. The intersection of these lines with vertical lines from the corresponding points in the elevation will determine the position of each point in the plan. The points so determined *one by one* being then connected by straight lines, gives the plan view as shown. To make the side view, horizontal lines are first drawn from the various points of the prism as seen in the elevation through the position which the side view will occupy. Then, bearing in mind that each point of the prism in the side view will be as much to the left of the vertical line *ab* as the same point in the plan is below the line *cd*, the position of each point on the horizontal lines is marked off from *ab*.

The Prism Inclined at Two Angles.—Fig. 54 shows the prism tipped forward after having been tipped to the right as shown in Fig. 53. An examination of these views will show that not only can they not be drawn directly, but they cannot be developed from Fig. 52. They may, however, be developed from Fig. 53. It is evident that as the prism is tipped forward, the side view of Fig. 53 will remain unchanged; hence the first step will be to reproduce that view inclined at the desired angle. Next, horizontal lines are drawn from the corners of the prism as seen in this view through the place which the elevation is to occupy, and the perpendicular line *gh* is drawn. It is evident that as the prism is tipped forward, the different points of it as seen in the elevation of Fig. 53 do not move any to the right or left, but forward only. Hence, the distance of the corners of the prism from the line *ef* may be taken by the compasses and marked off from the line *gh* upon the proper horizontal line. The new position of all of the corners having thus been determined, the connecting straight lines are drawn, giving the elevation as shown in Fig. 54. Vertical lines are then drawn from the different points of the prism, as seen in this view, through the position which the plan is to occupy, and the exact position of each point upon these lines is marked off from *mn* at the same distance which it is from the line *jk* in the side view.

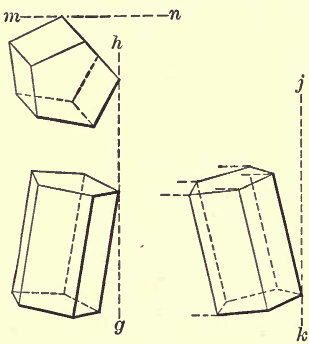


FIG. 54.—Projections of Prism Tilted in Two Directions.

An Upright Rectangular Prism.—The upright rectangular prism shown in Fig. 55 is, of course, drawn in the same way as was the prism shown in Fig. 52.

The Prism of Fig. 55 Tipped Forward on One Edge.—It is evident that if the prism were to be tipped on its edge in the direction of the arrow No. *1*, the result would be the same as though it had been tipped first to the right, and then directly forward, as was done to produce Fig. 54; but as those angles are not given, the method employed in that case is not readily available.

Fig. 56 shows the prism tipped to its new position, and shows, also, the method employed to produce the views. Draw the line *cd* at the same

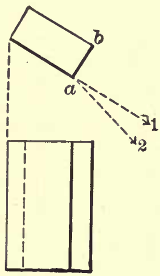


FIG. 55.—Upright Rectangular Prism.

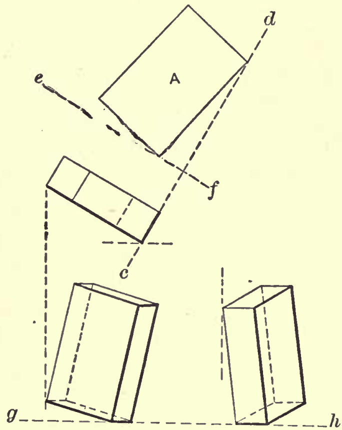


FIG. 56.—Rectangular Prism Tipped Forward.

angle to the horizontal as the edge *ab* of the prism in Fig. 55, and make *ef* at right angles to it. Upon these lines draw the temporary side view of the prism, *A*, tipped at the desired angle. With the aid of this view the plan view is readily drawn. Vertical lines are then drawn from the various points of the plan view through the place which the elevation is to occupy, and the exact location of each point is marked off on these lines at the same height above the base line *gh* that it is above the line *ef* in the temporary side view, *A*. The permanent side view is then developed from the plan and elevation in the same way as was the side view of Fig. 53.

Let it now be required to tip the prism of Fig. 55 forward on one corner in the direction of arrow No. 2.

It will be seen that tipping it in this direction

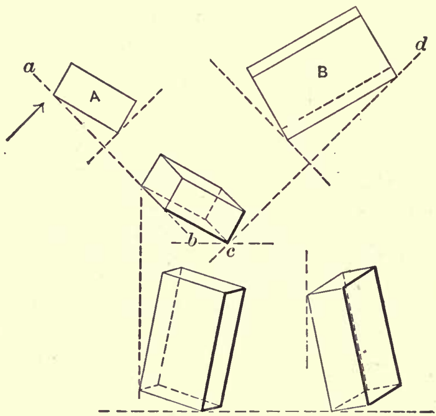


FIG. 57.—Rectangular Prism Tipped in Two Directions.

will cause a foreshortening of all of the lines in the plan, hence the use of a single temporary view such as was used in Fig. 56 will not solve the problem; but it may be solved by the use of two temporary views as shown in Fig. 57. Draw the line *ab* in the direction in which the prism is to be tipped, and the line *cd* at right angles to it. At *A* reproduce the plan view of Fig. 55, and at *B* draw a side view of the prism as it would appear if *A* were viewed in the direction of the arrow, but inclined to *cd* at the required angle. The intersection of lines drawn from the corners of *A*, parallel with *ab*, with lines drawn from the same corners of *B*, parallel with *cd*, will give their location

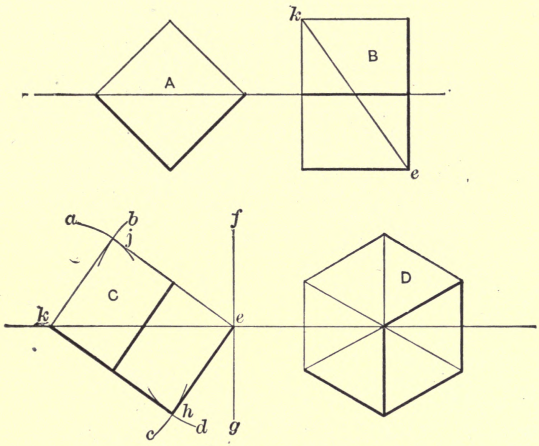


FIG. 58.—Projections of a Cube.

in the permanent plan view. This view being finished, the elevation and the permanent side views are drawn in the same way as were those of Fig. 56.

Let a cube be set on one corner so that a diagonal of it shall be horizontal; required to show the angle which the edges that meet at that forward corner make with a plane perpendicular to the diagonal, the angle which the sides that have corners coming together at the same point make with the plane, and *also the amount of foreshortening of the lines which will be caused.*

In Fig. 58, *A* shows a face view of the cube set on edge, *B* shows a side view of the same, and *C* shows *B* inclined until the diagonal *ke* becomes horizontal. The length of *ke* being laid out on the center line, the position of the other corners is obtained as indicated by the arcs *a*, *b*, *c* and *d*. The angle *geh* is the required angle which the edges which meet at *e* make with a plane perpendicular to *ek*, of which *fg* is an edge view; the angle *fej* is the angle which the sides having corners meeting at *e* make with the plane. *D* is a face view of *C*, and any of its lines, when compared with any of the lines of *A*, will show the foreshortening caused by the cube being put into this position.

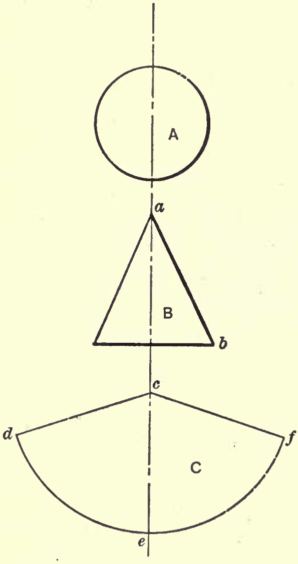


FIG. 59.—Development of a Cone.

The Surface Development of a Cone.—Let *A* and *B*, Fig. 59, be the plan and elevation views of a cone. With a radius equal to *ab*, and with a center at *c*, draw the arc *def*, making it equal in length to the circumference of the base of the cone, as shown at *A*. This may be most conveniently done by spacing it off. Draw the lines *cd* and *cf*, and the figure *C* thus formed will be the required surface development.

The Surface Development of a Pyramid Having Its Top Cut Off Obliquely.—In Fig. 60, *A*, *B* and *C* show, respectively, the plan, elevation, and side

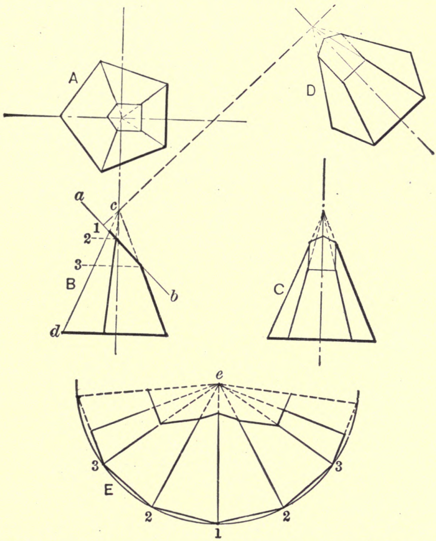


FIG. 60.—Development of a Frustum of a Pyramid.

views of the pyramid, the top of which is cut off by the plane *ab*. These views may be made by the principles already explained, as may also the view at *D*, which shows the pyramid as though *B* were viewed in the direction of the connecting dotted line, which is at right angles to *ab*, thus showing the shape of the section exposed by cutting off the top.

To get the surface development, take a radius equal to the length of one edge of the pyramid as shown at *cd* in the elevation, this being the only one which shows at full length, the others being more or less foreshortened, and with a center at *e* in view *E*, draw an arc of a circle upon which the sides of the base are to be marked off. These points are connected with one another and with *e;* this gives the shape of the surface of the whole pyramid. Upon the lines connecting the points with *e*, as *e1*, *e2* and *e3*, the lengths of the different edges of the cut off pyramid are marked off. As the edge which is seen at the left in the elevation shows full length, its length, *d1*, may be taken directly and marked off on the line *e1*. As the other edges are seen foreshortened, their lengths cannot be taken directly, but by horizontally transferring the upper end of each edge to the line *cd*, their actual lengths *d2* and *d3* may be obtained and then marked off on the lines *e2* and *e3*. The points so obtained being connected, and the outer half sections being finished, gives the required surface development.

If the cone shown in Fig. 59 were to have its top cut off obliquely, the views of it corresponding to *A*, *B*, *C* and *D*, Fig. 60, and its surface development, would be obtained by dividing off its base, as seen in the plan, into any number of sides, and then proceeding as though it were a pyramid of that number of sides, until the points corresponding to those of Fig. 60 had been located, but then connecting them with curved lines instead of straight lines.

Intersecting Cylinders, Fig. 61.—*Required the line of the intersection, the surface development of the branch, and the shape which the end of the branch would appear to have as seen in the view at the right.*

First draw the elevation, *A*, in outline, and as much of the end view, *B*, as can be directly drawn.

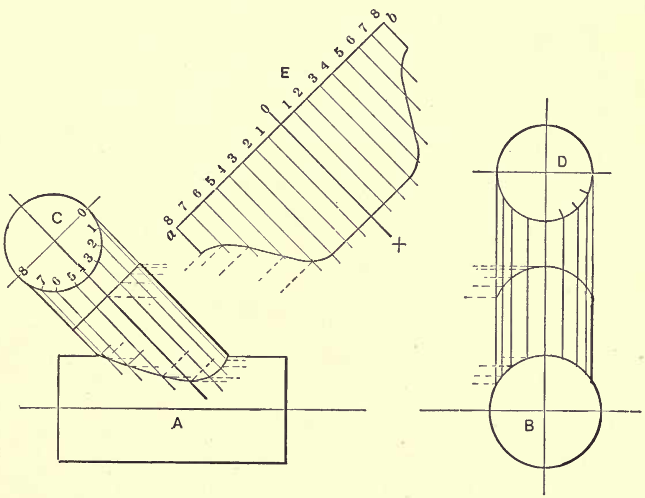


FIG. 61.—Intersecting Cylinders.

Opposite the end of the branch in each of these views, and in line with it, draw circles of the same diameter as the branch, and space off the semi-circumference nearest to it into a number of equal parts, the same number in both cases. From the points so obtained draw lines parallel with the center line of the branch, as shown. From the points where these lines in the view *B* meet the circle representing the end of the large cylinder, draw horizontal lines intersecting the lines drawn from *C*. These intersections will be points through which the line of the intersection of the cylinders is to be drawn. From the points where the lines drawn from *C* cross the end of the branch, draw horizontal lines intersecting those drawn from *D*. These intersections will be points through which the line representing the end of the branch is to be drawn.

To get the surface development of the branch, first draw the line *ab*, in *E*, having it in line with the end of the branch. Make this line equal in length to the circumference of the branch, spacing it off equally each way from the center line *0X* into the same number of spaces as the semi-circumference of *C* was divided into. From these points draw lines parallel with *0X*, and from the points in the intersection of the two cylinders, previously obtained, draw lines parallel with *ab*, intersecting these lines. These intersections will be points through which a curved line is to be drawn, thus giving the completed surface development of the branch.

In drawing these curved lines through the points of intersection, the irregular curves mentioned in the early part of the chapter on instruments and materials are used.

Intersecting Cylinder and Frustum of Cone, Fig. 62.—*Required line of intersection and surface development of branch, as before.*

Draw the elevation, *A*, in outline, continuing the sides of the conical branch either way until they meet at their vertex, *a*, on the one hand, and to any convenient points, *c* and *d*, on the other. In a similar manner draw as much of the end view, *B*,

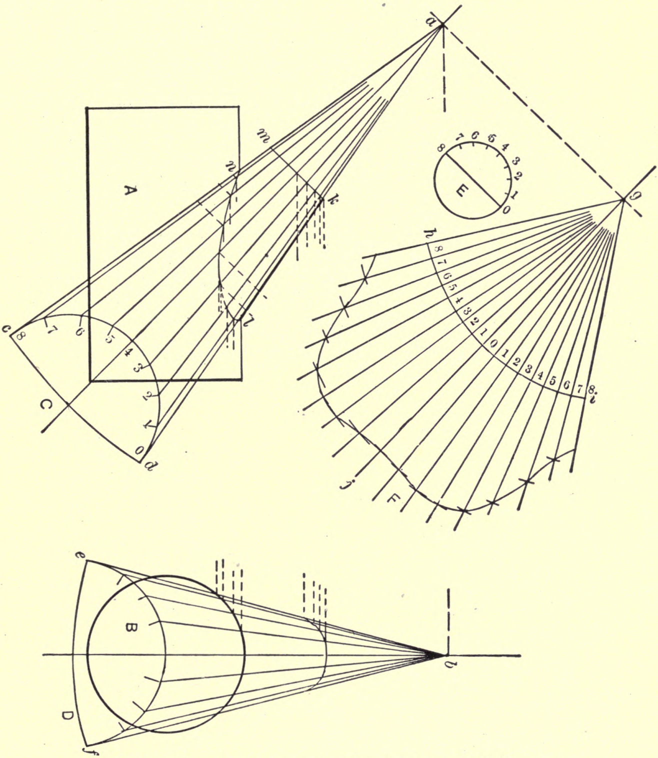


FIG. 62.—Intersecting Cylinder and Cone.

as can be made directly. With centers at *a* and at *b*, and with any convenient radius, draw the arcs *cd* and *ef*, intersecting the extended sides of the conical branch. Then, with centers at the intersection of these arcs with the center line of the branch, draw the half cicrles shown, tangent to the extended sides of the branch, and space them off into a number of equal parts, the same number in each case. From these points draw lines to the vertices *a* and *b*. From the points where these lines in the end view, *B*, intersect the circle representing the end of cylinder, draw horizontal lines to the elevation, *A*, intersecting the lines drawn from the vertex *a* to the half-circle *cd*. The intersections will be points through which the line representing the intersection of the cylinder and its conical branch is to be drawn. The shape of the end of the branch as seen in the end view, *B*, is now obtained in the same manner as in the case of the intersecting cylinders. From the points where the lines drawn from the vertex, *a*, of the side elevation *A*, to the half-circle at *cd*, cross the end of the branch, draw horizontal lines intersecting the lines drawn from the vertex *b*. These intersections will give points through which the line representing the end of the branch in view *B* is to be drawn.

To get the development of the branch as shown at *F* take a radius equal to the distance from the apex *a* to the end of the branch as seen in the side elevation, *A*, and with a center at *g* draw an arc *hi*, making the length of the arc equal to the circumference of the end of the branch as shown at *E*, spacing equally each way from the center line *gj*, the length and number of the spaces each way being the same as those obtained in spacing off the semicircle at *E*. Through these points draw lines radiating from g, as shown. On these lines distances are marked off from the arc hi through which the irregular curved line is drawn which gives the development of the branch. The lengths at the middle and at the extremities may, of course, be taken directly from the elevation A, the length kl being the length on the center line, and the length mn being the length at the extremities. The other lengths, being foreshortened, as seen in the elevation A, cannot be taken directly, but are obtained by transferring the points to either kl or mn as shown by the dotted lines, as was done in the case of the pyramid, Fig. 60.

To Draw a Helix.—A helix is a line of such shape as would be made by winding a thread around a

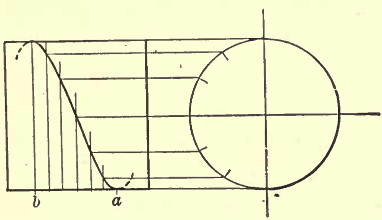


FIG. 63.—Drawing a Helix.

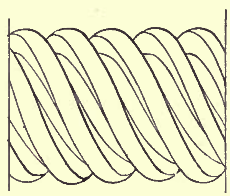


FIG. 64.—The Helix as it Appears in a Screw Thread.

cylinder, and having it advance lengthwise on the cylinder at a uniform rate as it is wound around it. In Fig. 63 we have the side and end views of a cylinder upon which it is desired to draw a helix, which shall advance from a to b in making a half turn around it. Divide the space from a to b into any number of equal parts, and at the points so obtained erect perpendicular lines. Divide the semi-circumference of the end view of the cylinder, toward the side view, into the same number of equal parts, and from these points draw horizontal lines to meet the perpendiculars previously erected. Where these lines meet will be points through which the helix is to be drawn.

The outlines of a screw thread are helices. Fig. 64 shows a double threaded Acme standard, or 29 degree threaded screw, the outline of which, on its outside diameter, is the helix of Fig. 63.

Isometric Projection.—If a cube is tipped over on one corner, so that the diagonal of it is horizontal as shown at *D*, Fig. 58, and also in Fig. 65, the

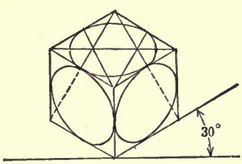


FIG. 65.—Principle of Isometric Projection.

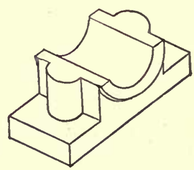
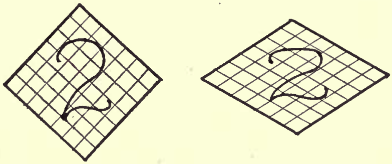


FIG. 66.—An Example of Isometric Projection.

lines of it will all appear of equal length. Drawings made on this principle, as Fig. 66, are called *isometric* drawings. Vertical lines remain vertical. Horizontal lines become inclined to the horizontal of the paper at an angle of 30 degrees. Circles appear as ellipses, which may be drawn as shown in the upper square of Fig. 65. From the ends of the "short" diagonals, lines are drawn to the middle of the opposite sides. Where these lines cross the "long" diagonals are located the centers from which the ends of the ellipse may be drawn. The ends of the short diagonals will be centers from which to draw the sides of the ellipse.

Irregular curves may be drawn as indicated in Figs. 67 and 68. The figure 2 there shown is first drawn in the desired position in a naturally shaped square, which is then divided off by equally spaced lines into smaller squares. The isometric square is then similarly divided off, and the figure is



FIGS. 67 and 68.—Method of Transferring Irregular Lines in Isometric Projection.

made to pass through the corresponding intersections.

Isometric drawings differ from perspective drawings in that receding lines remain parallel, instead of converging to a vanishing point. They may be measured the same as ordinary drawings in any one of the three directions indicated by the lines of the cube. The foreshortening of the lines caused by tipping the cube into this position is generally ignored. If an isometric drawing is to be shown in connection with ordinary views, however, it should be made on a scale of about 8-10 of an inch to the inch, otherwise it would appear too large.

CHAPTER V

WORKING DRAWINGS

As the object of working drawings is to convey to the workman a clear idea of the appearance and construction of the piece to be made, and as the whole "science" of mechanical drawing has been developed primarily for the purpose of conveying the ideas and thoughts of the designer and draftsman to the men who carry out these ideas in wood and metal, the subject of working drawings is of supreme importance to all mechanics. A working drawing should be as complete as possible, so complete, in fact, that when it has once passed out of the draftsman's hand into the shop, no further questions will be necessary. In order to accomplish this, all necessary information, of whatever kind, should be included, and, if required, short notes and directions may be written on the drawing to prevent eventual misunderstandings.

The number of views necessary to properly represent an object must be left for the draftsman's judgment to determine. Usually two views are sufficient, when the object is simple, but when at all complicated, three or more views will be found necessary. Cylindrical pieces can often be adequately represented by a single view, on which the various diametral and length dimensions are given.

While it is customary to put the plan view of an object above the elevation, it frequently becomes necessary, in order to present the objects shown in as clear a manner as possible, to deviate from this rule. A case of this kind is shown in Fig. 69, where the shaft hanger illustrated has been selected as an example of the methods employed in working drawings.

An examination of the hanger will show that if the plan were placed above the elevation, and if it were represented according to the methods already explained, the box and the yoke with its adjusting screws and check-nuts would have to be shown mostly by dotted lines. Such a multiplicity of dotted lines would tend to confusion; hence the object in view, that of presenting the hanger in as clear a manner as possible, is best accomplished in a case like this by having the plan underneath the elevation, and letting it be a bottom view instead of a top view.

In designing a machine detail of this kind, the starting point would of necessity be the shaft itself, and the first step would be to design the box; next would come the yoke, and lastly, the frame. Much of the preliminary work may frequently be done on scrap paper; having determined the size and proper proportions of the various parts, the position which the different views will occupy in the finished drawing is easily ascertained. The center lines are then laid out as shown, and the drawing built up about these lines as a base.

When a drawing is for temporary use only, and the mechanism represented on it of a simple nature, the *assembly* drawing, corresponding to the three views in Fig. 69, will answer all purposes, the dimensions being given directly on this drawing. In

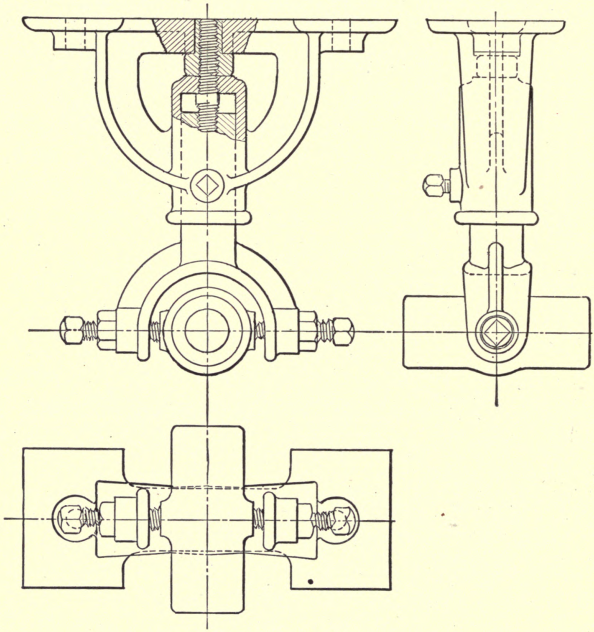


FIG. 69.—Shaft Hanger.

some cases only the most important dimensions would be given, those of secondary consequence being left for the workman to be obtained by "scaling" the drawing. This procedure, however, is possible only when the drawing is made carefully to scale, and is not one that should be encouraged. In general, a drawing should be so dimensioned that it can be worked to without the workman obtaining any measurements by "scaling" the drawing.

In most cases it is not possible to show the details of a mechanism clearly enough in an assembly drawing; for if the device shown is more or less complicated, a hopeless confusion results from the attempt to put in all the lines necessary to fully show all the details; neither would it be possible, for the same reason, to give more than the principal dimensions.

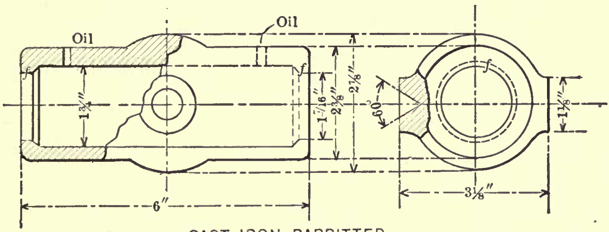


FIG. 70.—Example of Working Drawing.

pal dimensions. In such cases it is, therefore, customary, after the assembly drawing has been completed, and the proper sizes and proportions of the various parts of the mechanism thus ascertained, to make a separate drawing of each detail, either on the same sheet of paper, or on separate sheets. This permits the parts of the mechanism to be clearly and completely shown and fully dimensioned. Figs. 70 and 71 show two pieces of the hanger in Fig. 69 detailed in this manner. These detail drawings give all the required information for the making of the pieces, and the assembly drawing merely shows, in a general way, how the parts are to be assembled when completed.

In the case of jig and fixture drawings, it is the practice in a great many large drafting-rooms to show assembled views only, and to put all dimen-

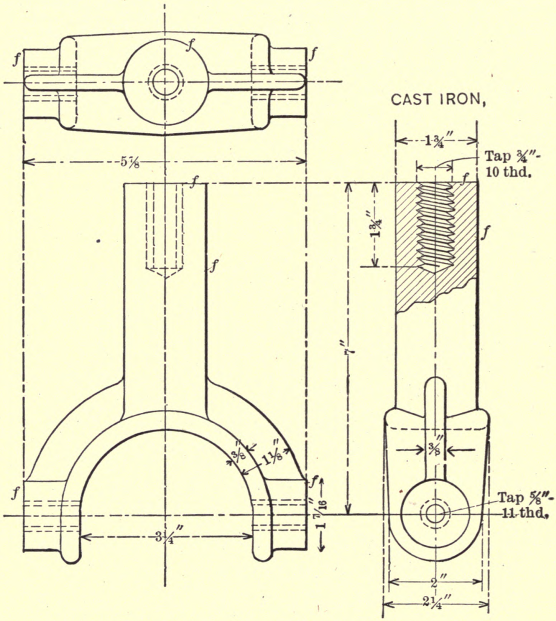


FIG. 71.—Example of Working Drawing.

sions directly on the assembly drawing; the argument advanced in favor of this practice is that experienced pattern-makers and tool-makers, who are, as a rule, the only mechanics who will work on the making of these tools, will find no difficulty in reading the assembly drawing; besides, it is said, as a drawing of this kind is, in most cases, used but once, it would be waste of time to have the draftsman detail the different parts of the tool.

While these arguments are undoubtedly true in the case of very simple jigs and fixtures, there can be little doubt that in the case of more complicated ones, the comparatively short time required by the draftsman to make detail drawings will be saved many times over in the shop; for the pattern-maker and tool-maker will not have to spend, in the total, a number of hours puzzling over the drawing, and even then being liable to make a mistake.

In making drawings, it is always a rule to work from the center lines, when the outline of the piece is such that it has a definite center line. Dimensions in either direction from the center line can be best marked off with the compasses. This insures a symmetrical appearance to the finished drawing, such as might not be secured if the dimensions are set off on either side of the center line from the rule, it always being easy to then introduce small errors which show plainly in the finished work. If the piece is of such shape as to have no center line, some one principal line may be selected, one in each direction in each view, and the remaining points and lines may be located from these lines.

The various styles of lines ordinarily used in working drawings are shown in Fig. 72. The regular "full" line *AA* is used for the outlines of objects, and when drawn rather "fine," for cross-hatching or cross-sectioning. The heavy shade line *BB* is used to represent lines assumed to separate the light surfaces of an object from the dark, as will be explained in the following. The dotted line *CC*, as has already been explained in the previous chapter, is used to represent lines obscured or hidden from view. The line *DD*, called a "dash" line, is used by a great many draftsmen for dimension lines. Finally, the line *EE*, the "dash and dot," or, simply, the "dash-dotted"

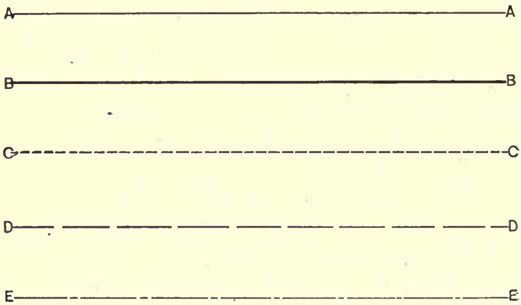


FIG. 72.—Styles of Lines Used on Working Drawings.

line, is used in common practice for center lines, to indicate sections, etc. This line is also commonly used for construction lines, in laying out mechanical movements.

The dimension lines may be made either fine full lines or "dash" lines, the dashes being about $\frac{3}{8}$ inch long. A space is left open for the figures giving the dimension. The witness points or arrow heads, showing the termination of the dimension, are made free hand. Many draftsmen draw the extension and dimension lines in red ink, the arrow heads, however, still being made black. It is well to avoid, as far as possible, having the dimension lines cross each other, as such crossing tends to confusion; the difficulty can usually be avoided by having at least one set of dimensions placed outside or between the views, the larger dimensions being placed farther from the outline of the object than the shorter ones, to avoid having the extension lines of the latter cross the dimension lines of the former. Dimensions under 24 inches are most conveniently given in inches; larger dimensions are given in feet and inches. The usual practice is to indicate feet and inches on drawings by short marks, "prime" marks (′), placed at the right, and a little above the figure, one mark (′) indicating feet, and two marks, "double prime" marks (″), indicating inches, so that 5′ 7″ would read 5 *feet* 7 *inches.* Some draftsmen do not consider this method of marking safe enough to eliminate mistakes, and prefer to write dimensions of this kind in the form 5 ft. 7″. A method equally satisfactory in preventing possible mistakes is to place a short dash between the figure giving the number of feet and that giving the number of inches, at the same time retaining the "prime" marks; thus, 5′—7″. When feet only are given, it is well, for the sake of uniformity and to prevent any misunderstanding, to give the dimension in the form 5′—0″.

A few examples showing the principles of the usual methods of dimensioning drawings may be of value. In Fig. 73 is shown a simple bushing. The diameter of the hole or bore is given as 2 inches by a dimension line passing through the center of the circles in the end view. It is confusing, however, to have more than one dimension line passing through the same center, and, therefore, the outside diameters of the bushing have been given on the side view. The lengths of the various steps or shoulders of the bushing are given below the side view, as is also the total length. It will be noticed that the dimensions of the three steps are slightly offset—that is, the dimension

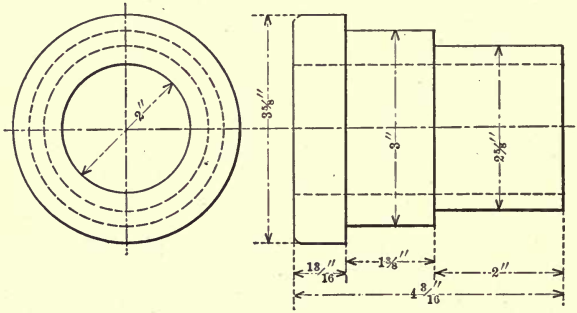


FIG. 73.—Simple Example of Dimensioning a Drawing.

lines do not extend in one straight line; this makes a very clear arrangement.

The method of dimensioning holes drilled in a circle is shown in Fig. 74. Outside of the dimension for the holes themselves only the diameter of the circle passing through the centers of the holes is given, together with the number of holes. As the holes, of course, are to be equally spaced, that is all that is required. When a great many bolt holes or bolts occur around a flange, it is not necessary to draw them all in on the working drawing; a common method is to show a few, and to draw the circle passing through their centers, the *pitch* circle. The total number of bolts around the flange is, of course, also given. A case of this kind is illustrated in Fig. 75. When a great many holes are drilled in a row, a similar expedient may

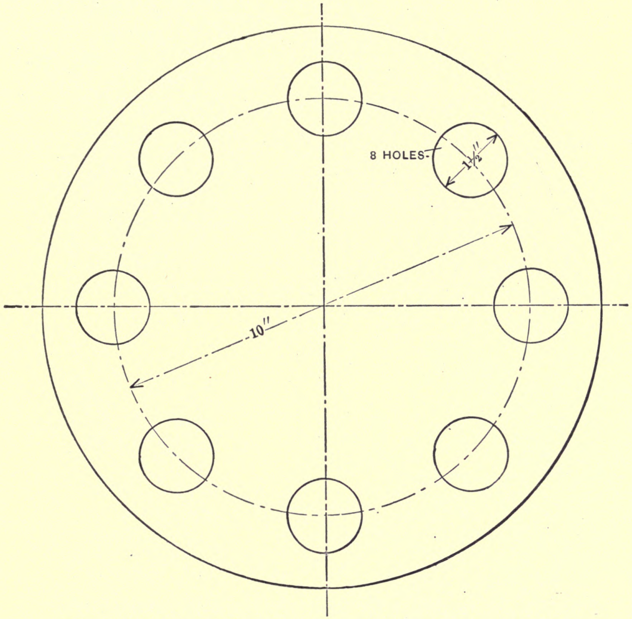


FIG. 74.—Dimensioning Holes Drilled in a Circle.

be adopted to avoid showing and dimensioning all the holes; an illustration of this is shown in Fig. 76.

In Fig. 77 are shown the common methods of dimensioning screws and bolts. At *A* is shown a hexagon head bolt, so drawn that three sides of the head are visible. Hexagon bolt-heads are usually drawn in this manner in all views, irrespective of the fact that the rules of projection would call for only two sides to be visible in one view. The reason for this is partly that the bolt-

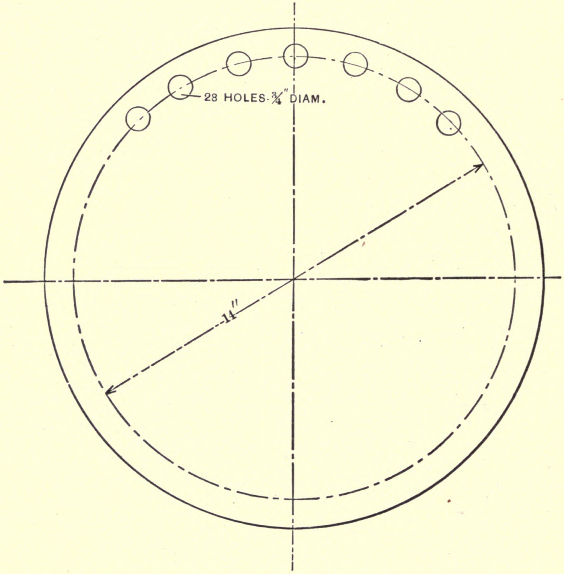


FIG. 75.—Simplified Method of Dimensioning Holes Drilled in a Circle.

head looks better when three sides are visible, and partly that when so drawn there can be no confusion whether a hexagon or a square head is meant. If only two sides were shown, as at *B*, the head, especially if carelessly drawn, might be mistaken for a square bolt-head. As a rule, the dimensions of bolt-heads are standard for given diameters of bolts, and no dimensions are required for the head. In some cases, however, the head may be required to fit a given size of wrench, or for some other reason be required to be made different from the standard size; in such cases dimensions may be given as shown at *C*, Fig. 77, the dimension "1″ hex." indicating that the head is one inch

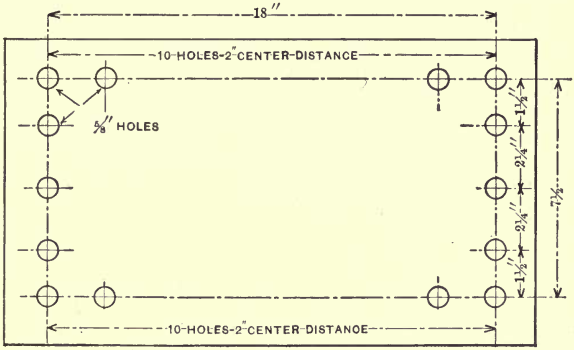


FIG. 76.—Dimensioning Holes Drilled in a Row.

"across flats." In the same way, "¾″ sq." would indicate that the head should be square, and three-quarters inch "across flats."

The length of the bolt should be given as shown in the lower view in Fig. 77. The dimensions should be given "under the head," both the total dimension, and the distance to the beginning of the thread.

In general, full circles should be dimensioned by their diameters; an arc of a circle, again, should be dimensioned by its radius. The center from which the arc is struck should preferably be indicated by a small circle drawn around it. In small dimensions, the arrow points are frequently placed outside of the lines between which the dimension is given, as shown in Fig. 71 in dimensioning the narrow ribs; sometimes, the figures giving the

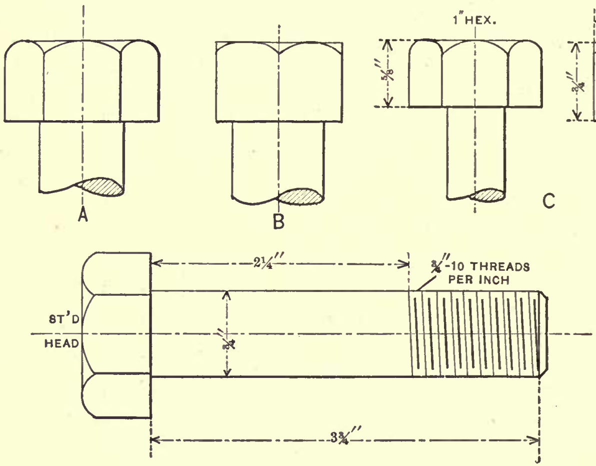


FIG. 77.—Dimensioning Screws and Bolts.

dimension are themselves placed outside of the space between the arrow heads, because the space is too small to permit the dimension to be clearly written within it.

The principal dimensions should be so given that the workman will not have to add a number of other dimensions to get them. When the dimensioning of a piece naturally divides itself into several measurements, an over-all dimension should always be given for verification. If, however, the piece terminates with a round end, as the yoke in Fig. 71, the over-all dimension may properly terminate at the center of curvature of the end, the distance beyond being of entirely secondary importance, and being taken care of by its radius. If a dimension has been given in one view, there is usually no reason for repeating it in the other views; sometimes such repetitions would cause too many dimensions to be given in each view, so that confusion would arise, and instead of making the drawing plainer, the repetition of dimensions might cause mistakes which otherwise would have been avoided.

Drawings should always be dimensioned the *full* size of the *finished* article, regardless of the scale to which the drawing is made. If a drawing is made to any other scale than full size, it is customary to state on the drawing the scale to which it is made, as "Scale, $\frac{1}{4}$ inch=1 ft."

A drawing should be so marked as to tell the workman what surfaces are to be finished; a finished surface is usually indicated by the letter "f" placed either upon the line representing the surface, or in close proximity to it. While the amount and kind of finish is usually left to the workman to determine, the best modern methods require that the draftsman should indicate on the drawing how closely the various parts are to be machined. A very commendable method is to give dimensions in thousandths of an inch, where accuracy is required, and in common fractions in cases where there is no need of working to thousandths. In very highly systematized establishments, the limits of variation between which any measurement is allowed to vary, are given with each dimension, or, at least, with dimensions for diameters which are to fit the holes or bores of other pieces. The determination of the limits of accuracy required calls for good judgment on the part of the draftsman. Limits may be expressed in two ways. For instance, a running fit on a shaft to go into a $1\frac{1}{2}$ inch standard size hole may be marked

$$1\tfrac{1}{2}\begin{matrix}-0.0005 \text{ max.}\\-0.0015 \text{ min.}\end{matrix}$$

or it may be expressed

1.4995 max.
1.4985 min.

which means that the shaft must not be larger than 1.4995 inch, and not smaller than 1.4985 inch.

On drawings, the tap drill size and the depth of tapped holes should always be shown. Surfaces to be ground to size should be marked "grind." If the surface is to be filed, the words "file finish" are substituted for the letter "f." Finishing marks, as a rule, are used on castings and forgings only. On work made from bar stock, every surface is nearly always finished, so that here the finishing marks are omitted. When a casting or forging is finished on every surface, it is not necessary to show finish marks, but the words "finish all over" may be written in a conspicuous place, so as to readily catch the eye of the workman. If, on work made from bar stock, it is desired that the piece be left rough at any point, the words "stock size" may be applied to the figures giving that particular dimension. For instance, on a 1½-inch cold rolled shaft, turned for journals for a short distance at each end, the central part would be dimensioned "1½-inch stock size."

While the practice of indicating finished surfaces by the letter "f" is by far the most frequently met with, it is by no means universal. In some shops the words "polish," "ream," "finish," etc., are written near the lines representing the surfaces to be thus treated. Still another method much in use is to draw a red line outside of the line representing each surface to be finished. If a blue-print is made from a tracing thus prepared, the red lines will print fainter than the black ones, and the finish lines on the blue-prints are traced over with a red pencil or red ink before being sent out in the shop. This method, however, is more expensive than that of indicating the finished surfaces by the letter "f," and on complicated drawings, the many additional red lines tend to cause confusion. By whatever method the finish is indicated, the finishing marks should always be shown fully in every view of the object.

It frequently happens that the representation of an object is made clearer by the use of sectional views, representing the object as having been cut in two, either wholly or in part. Examples of this are shown in Figs. 69, 70 and 71. From these illustrations it is apparent that the construction of the various pieces is much more clearly exhibited when a section is shown. The surface "cut" or shown in section is cross-hatched or cross-sectioned with fine lines at a distance apart varying from a thirty-second to an eighth of an inch, according to the size of the drawing and the piece. The cross-sectioning brings the parts in section into bold contrast with the remainder of the drawing, and prevent all confusion as to what parts are in section and what parts shown in full. All lines beyond the sectional surface which are exposed to view, should be shown in the drawing as usual. Should it be deemed necessary, which it seldom is, to show any parts that have been cut away for the purpose of showing a section, such parts may be drawn in by dash-dotted lines, this indicating that the parts thus shown are in front of the section and actually cut away.

When a mechanism is shown in section, the different parts of the same pieces should always be cross-sectioned by lines inclined in the same direction, while separate pieces adjoining each other should always, when possible, be cross-sectioned by lines running in different directions. When a solid round piece is exposed to view by a section, it is customary to show it solid, and not to section it; the screw stud in Fig. 69 is an example of this practice.

Sectional views may also be used for many purposes where a slight deviation from the theory of projection will tend to simplify the representation of certain machine details. The shape of the arm of a pulley or gear, or of any other part of a casting, may be conveniently represented in this way. The cutting plane may be assumed to lie at any angle necessary to bring out the details most clearly. A sectional view, for instance, may represent a casting as though it were cut through partly on one plane and partly on another. In all such cases, however, it should be indicated in another view of the object just where the sectional views are sup-

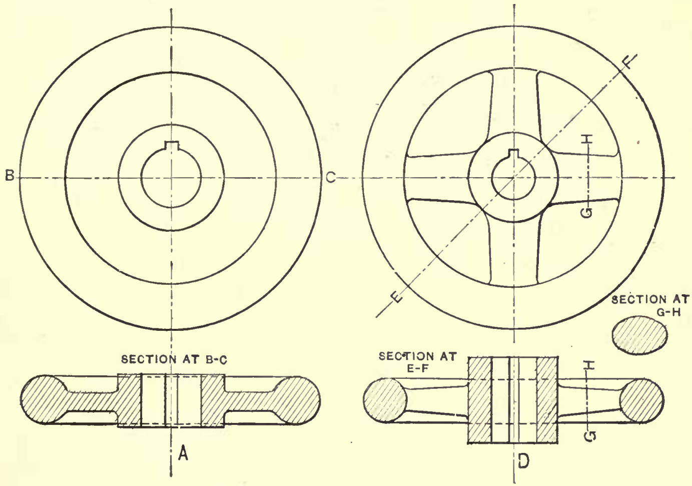


FIG. 78.—Methods of Showing Sections.

posed to be taken, so that no confusion may arise on this account. The examples in the following will serve to make clear the principles laid down.

In Fig. 78 are shown sections of two hand-wheels. When an object is symmetrical it is unnecessary to show more than one half in section, although it is quite common to section gears, pulleys, etc., completely on working drawings. The hand-wheel at *A* in Fig. 78 is represented as though cut in two along its diameter *BC*. When the section is taken along the center line, it is not absolutely necessary to explain where the section is taken; but it can do no harm to make a practice of in all cases to state where the section is made, except when perfectly obvious. In this case it would be clear that the section is taken through

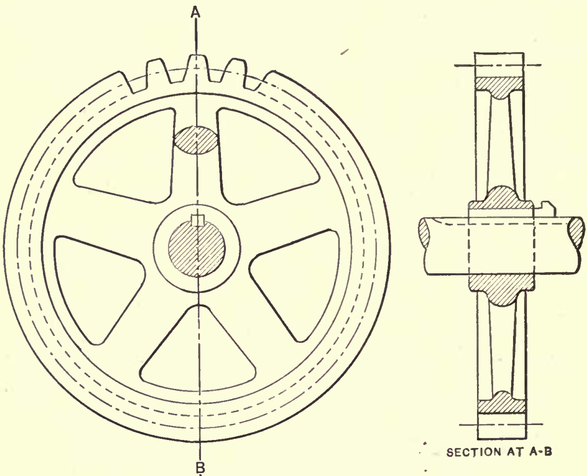


FIG. 79.—A Gear-wheel in Section.

the center, and the legend "Section at *BC*" is given only to show the principle. The hand-wheel at *D* is provided with four arms, and the method of representing the shape of the arms, hub and rim are clearly indicated.

In Fig. 79 are shown two views of a gear-wheel, indicating the conventional method of representing gears on drawings. The view on the left side is the side view, and as all the teeth are, of course, alike, it is unnecessary to draw more than a few of them. The pitch line of the teeth is represented by a dash-dotted line. In the part of the gear-wheel rim where the teeth are not shown, the face of the gear is indicated by a solid line, and the bottom of the teeth by a dotted line. In the case of machine-cut gearing, where the teeth are cut by standard formed cutters, it is unnecessary to show any teeth at all on the rim of the gear, it being sufficient to state the pitch and the number of teeth, as will be more fully explained later in the chapter on gearing. To show the shape to which the arms are formed, a sectional view of one of the arms is drawn in the side view; the ends of the shaft are supposed to be broken off, and are, therefore, sectioned as shown. The right-hand view of the gear is a section taken along the line *AB*. It will be noted that the shaft and key are not sectioned, usual practice being followed in this respect. The gear shown has five arms, and the line *AB* cuts through one of them only. This arm, however, is not sectioned in the right-hand view, and two opposite arms are drawn as though both of them lay in the plane of the paper. While this is not theoretically correct, it is the method usually followed because of simplicity in drawing and clearness of representation. The method of representing the gear teeth in the sectional view is the one commonly employed.

Sectional and top views of a cylinder end with flange and cover are shown in Fig. 80. This cylinder cover has only five bolts, and the plane through which the section is taken cuts through only one of the bolts. It is common practice, however, to draw the section as shown at the left. The bolts are shown as if two of them were in the plane of the section. The bolts are not sectioned,

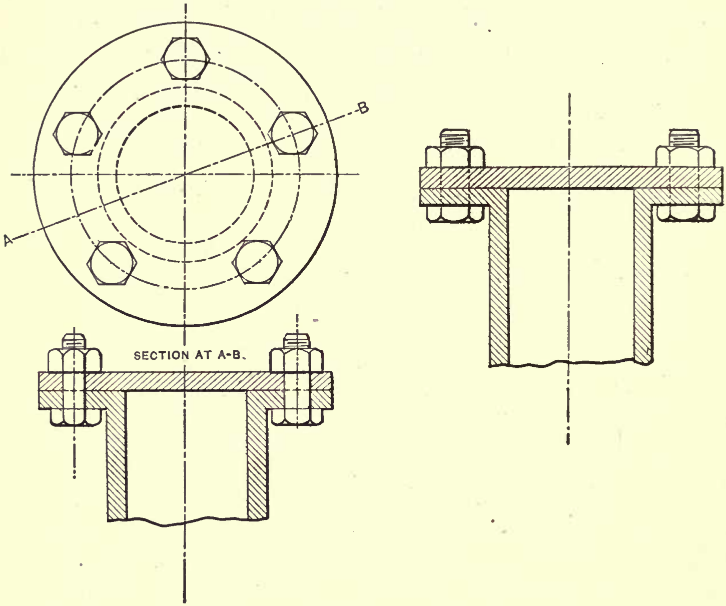


FIG. 80.—Section of Cylinder End with Flange and Cover.

but are drawn in full, as explained previously. Dotted lines of the remaining bolts, or full lines of their nuts, should not be shown, because this detracts from the clearness of the drawing; the top view shows clearly the number of the bolts and their arrangement, and that is all that is necessary. Some draftsmen prefer to draw sections of this kind as indicated at the right in Fig. 80. This method, however, is not as commonly used.

In a case where the object is rather unsymmetrical, as, for instance, in Fig. 81, the draftsman's judgment must often be relied upon to decide how

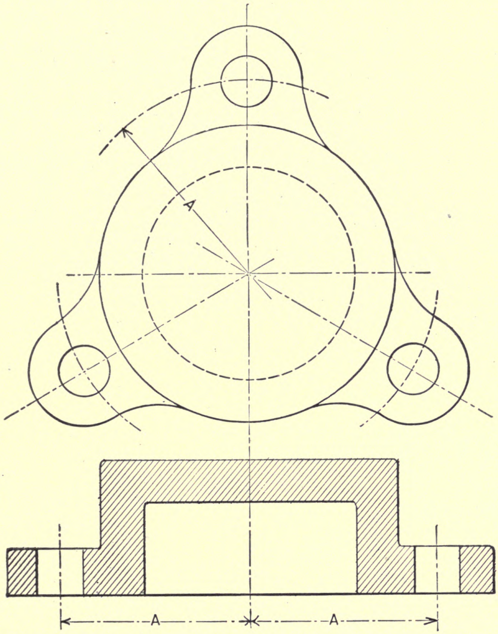


FIG. 81.—Another Method of Showing Sections.

it shall best be shown in section. Usually the sectional view is made symmetrical as shown, the distances *A* in the lower view being made equal to the radius *A* in the top view.

The materials for the various details making up a complete mechanism are usually cross-sectioned in such a way as to indicate the material from which each piece is made. There is, however, no universally adopted or recognized standard for cross-sectioning for the purpose of indicating different materials. In Fig. 82 is shown a chart,

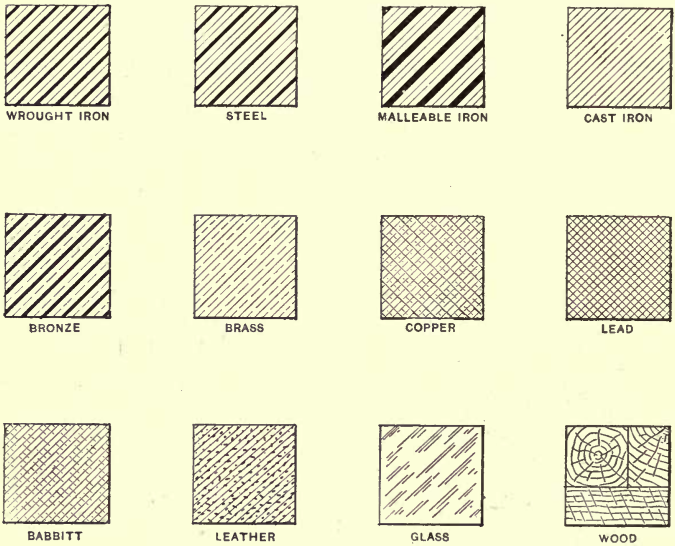


Fig. 82.—Cross-sectioning used for Indicating Different Materials.

published by Mr. I. G. Bayley in *Machinery*, October, 1906, which represents average practice, although it must be distinctly understood that there is no agreement in all respects between the numerable charts in use in various drafting-rooms. For this reason, cross-sectioning alone should never be depended upon for indicating to the workman the kind of material to be used. Written directions should also be given, the kind of material for each part being plainly marked. Tool steel may be abbreviated "T. S.", machine steel, "M.

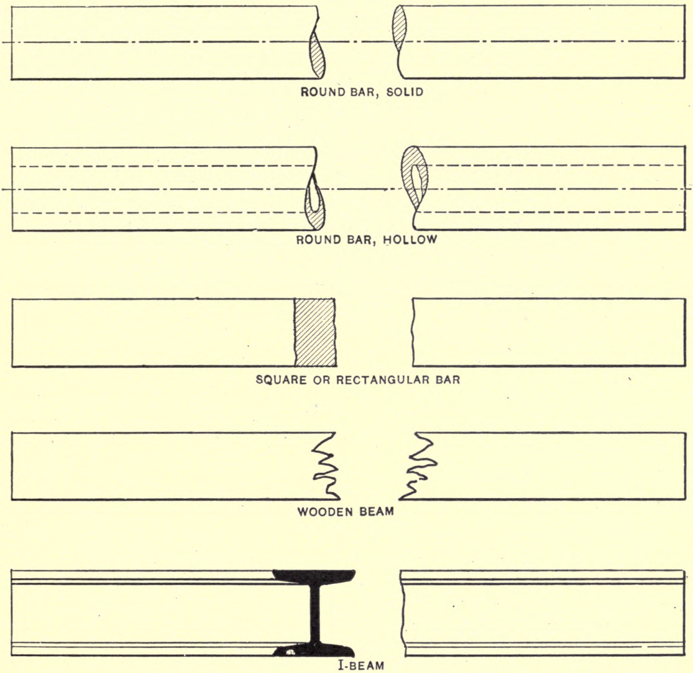


FIG. 83.—"Broken" Drawings of Long Objects.

S."; wrought iron, "W. I."; cast iron, "C. I.", etc. The less common materials in machine construction, such as bronze, brass, copper, etc., should preferably be written out in full, in order to avoid any chances for confusion. It is better to be too explicit as regards the information on the drawing, than to risk misunderstandings and consequent errors.

Long bars, shafting, structural beams, etc., cannot conveniently be shown for their full length on the drawing. In such cases the pieces are drawn as long as the drawing and the adopted scale permit, and are broken as shown in Fig. 83, a part between the two end portions shown being imagined as broken out. The dimensions, of course, are given for the full length of the piece, as if not broken.

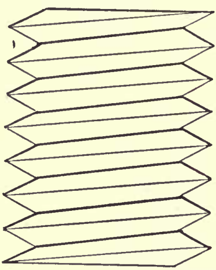


FIG. 84.—Method of Drawing a Screw, Giving Correct Helix Effect.

There are several conventional methods for showing screw threads; these methods are adopted largely for saving of time, as it would be out of the question to spend the time required for drawing a true helical screw thread on a working drawing. A method for very nearly approximating the appearance of a theoretically correct screw drawing is shown in Fig. 84, where the projection of the screw helix is drawn by straight lines. The V-shaped outline is first laid out, and the connecting lines are then drawn. It will be noticed that the lines representing the roots of the threads are not parallel with those representing the tops or points. This aids in making the drawing resemble that of a true helix.

Usually, however, much simpler methods are employed for indicating screw threads. In Fig. 85, *A*, *B* and *C*, some of these methods are shown. When a long piece is threaded the entire length, this fact can be indicated as at *D*, which saves drawing the conventional thread for the full length of the piece. The lines indicating the thread are

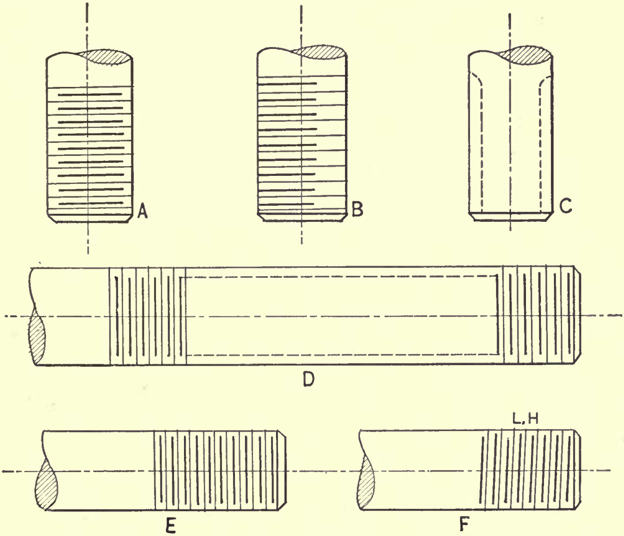


FIG. 85.—Simplified Methods for Showing Screw Threads.

inclined, the same as would be the lines representing the true helix. At *E* in Fig. 85 is shown a right-hand thread and at *F* a left-hand thread, the different direction of inclination of the thread indicating this fact. However, if a thread is to be left-hand, it should always be so marked on the drawing. It is usual to abbreviate left-hand, writing "L. H."

Three methods of indicating tapped holes are shown in Fig. 86, these being used when the holes are obscured from view, and shown by dotted lines. When a tapped hole is shown in section, and looked upon from the top, it is shown as indicated at *D*, while if seen from the side, in section, it is repre-

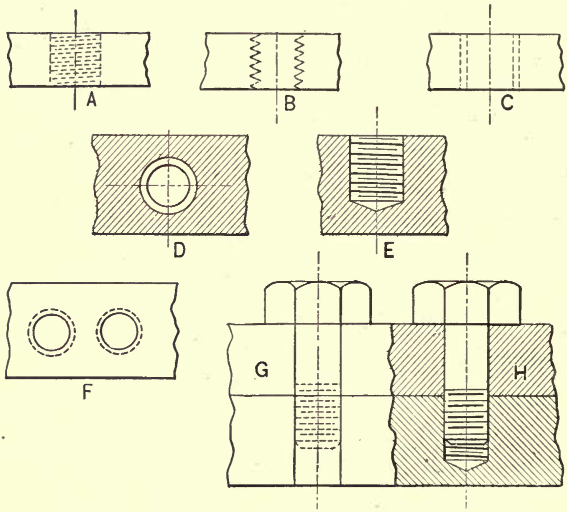


FIG. 86.—Simplified Methods for Indicating Tapped Holes.

sented as at *E*. A surface having tapped holes in it, seen from above, is shown at *F*. At *G* and *H* are shown the methods of representing bolts or screws inserted in place in tapped holes. It will be noted that when the threads of a tapped hole are exposed to view by section, the lines representing the screw helix will be seen to slope in the opposite direction to those of the screw, it being the back side that is exposed to view. An example of this is shown in Fig. 71 as well as in Fig. 86.

In drawings made for use in the shop it is customary to make the lines of uniform thickness. For shop use such drawings are as good as any. When, however, the purpose of a drawing is chiefly to show up the object which it represents, its effectiveness may be considerably enhanced by the use of *shade lines* as shown in Fig. 87. In shade line work, the light is usually assumed to come from the upper left hand corner, and to shine diagonally across the paper at an angle of forty-five degrees. Lines on the side of the object away from the light, or lines separating light from dark surfaces, are made extra heavy. This gives to the drawing a suggestion of relief. An examination of the lines of Fig. 87 taken in connection with the direction from which the light is supposed to come will show, without the aid of any other view, that the hexagonal part is raised above the surface of the square, and that the circle in the center represents a depression.

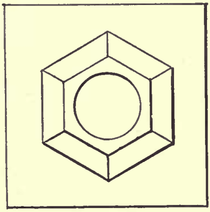


FIG. 87.—Use of Shade Lines.

When a drawing is intended for permanent use it is customary to make only a pencil layout on paper, usually on brown paper, and from this to make a tracing from which any number of blue print copies may be made. The tracing is usually made on the regular tracing cloth. This has one glazed and one unglazed surface. Either surface may be used. The tracing cloth is drawn tightly over the pencil drawing, and its surface is cleaned of any greasiness with dry powdered chalk. This insures a good flow to the ink. In doing the ink work curved lines should be made first, straight lines afterwards, as mentioned in Chapter I.

The blue prints are made in the same manner as photographs are printed, the tracing taking the place of the photographic negative. An exposure of from three to ten minutes may be required, depending on the freshness of the blue print paper and the brightness of the sun. After the proper exposure has been given, which may require some experimenting at first, until one gets accustomed to the change in the paper which the light makes, the print is thoroughly rinsed out in clear water and dried, by being hung up by one edge.

White writing may be made on a blue print with saleratus water, the water being given all the saleratus it will dissolve.

CHAPTER VI

ALGEBRAIC FORMULAS

In order to be able to carry out the calculations required in simple machine design, it is necessary that a general understanding of the use of formulas, such as are used in mechanical hand-books and in articles in the technical press, is acquired. Knowledge of algebra or so-called "higher mathematics" is by no means necessary, although, of course, such knowledge is very valuable; but simple formulas can be used, and the results of scientific results employed in practical work to a very great extent, by any man who understands how to use the formulas given by the various autnorities; and the knowledge required for an intelligent use of algebraic formulas can be very easily acquired. All the mathematical knowledge necessary as a foundation is a clear understanding of the fundamental rules and processes of arithmetic.

A formula is simply a rule expressed in the simplest and most compact manner possible. By using letters and signs in the formula instead of the words in the rule, it is possible to condense, in a very small space, the essentials of long and cumbersome rules. The letters used in formulas simply stand in place of the figures which would be used for solving any specific problem; the signs used are the ordinary arithmetical signs used in all kinds of calculations. As each letter stands for a certain number or quantity, whenever a specific problem is solved the figures for that case are put into the formula in place of the letters, and the calculation is carried out as in ordinary arithmetic. This may, perhaps, be made clearer by means of a few examples.

The circumference of a circle equals the diameter times 3.1416. This rule may be written as a formula as follows:

$$C = D \times 3.1416.$$

In this formula C = circumference, and D = diameter. No matter what the diameter is, this formula says, the circumference is always equal to the diameter (D) times 3.1416. Assume that the diameter is 5 inches. Then, to find the circumference, place 5 in the formula in place of D.

$$C = 5 \times 3.1416 = 15.708 \text{ inches.}$$

If the diameter of a circle is 12 feet, then

$$C = 12 \times 3.1416 = 37.6992 \text{ feet.}$$

This, of course, is the very simplest kind of a formula, but it illustrates the principle involved, and indicates how easily formulas may be employed.

One of the most well-known formulas in steam engineering is that giving the horse-power of an engine, when the average or mean effective pressure of the steam on the piston, the length of the stroke of the piston in feet, the area of the piston in square inches, and the number of strokes per minute, are known. Let

$H.P.$ = horse-power,
P = mean effective pressure in pounds per square inch,
L = length of stroke in feet,
A = area of piston in square inches, and
N = number of strokes per minute.

Then

$$H.P. = \frac{P \times L \times A \times N}{33,000}.$$

The rule conveying this information expressed in words would require considerable space, and be difficult to grasp immediately; but the meaning of the formula is quickly understood. If the pressure (P) equals 75 pounds, the stroke (L) 2 feet, the area of the piston (A) 125 square inches, and the number of strokes per minute (N) 60, then

$$H.P. = \frac{75 \times 2 \times 125 \times 60}{33,000} = 34.1.$$

It will be seen that the values for the different quantities are merely inserted in the formula in place of the corresponding letters, and then the calculation is carried out as usual. It will be remembered that the line between numerator and denominator in a fraction also means a division; that is

$$\frac{1,125,000}{33,000} = 1,125,000 \div 33,000 = 34.1.$$

It is very common in formulas to leave out, entirely, the sign of multiplication (×) between the letters expressing the values of the various quantities that are to be multiplied. Thus, for example,

PL means simply $P \times L$, and if $P = 21$ and $L = 3$, then $PL = P \times L = 21 \times 3 = 63$. If the multiplication signs are left out in the formula for the horse-power of engines just referred to, the formula

$$\frac{P \times L \times A \times N}{33{,}000} \text{ could be written } \frac{PLAN}{33{,}000}.$$

As a further example of the leaving out of the multiplication sign in a formula, assume that $D = 12$, $R = 3$, and $r = 2$, then

$$\frac{DRr}{9} = \frac{D \times R \times r}{9} = \frac{12 \times 3 \times 2}{9} = \frac{72}{9} = 8.$$

It must be remembered that no other signs, except the multiplication sign, may thus be left out between the letters in a formula.

From the examples given, the use of simple formulas is clear; each letter stands for a certain number or quantity which must be known in order to solve the problem; when the formula is used for the solution of a problem, the letters are simply replaced by the corresponding number, and the result is found by regular arithmetical operations.

The expressions "square" and "square root" and "cube" and "cube root" are frequently used in engineering hand-books and technical journals. It would seem, to one unfamiliar with these names and their mathematical meaning, as well as the signs by which they are indicated, that difficult mathematical operations are involved; but this is not necessarily always the case. The square of a number is simply the product of that number multiplied by itself. Thus the square of 3 is $3 \times 3 = 9$, and the square of 5 is $5 \times 5 = 25$. In the same way, the square of 81 is $81 \times 81 = 6561$. Instead of writing 81×81, it is common practice in mathematics to write 81^2, which is read "81 square," and indicates that 81 is to be multiplied by itself. Similarly, we may write $7^2 = 7 \times 7 = 49$, and $12^2 = 12 \times 12 = 144$. The little "2" in the upper right-hand corner of these expressions is called "exponent." Nearly all mechanical and engineering hand-books are provided with tables which give the squares (and also the square root, cube and cube root) of all numbers up to 1000, so that it is usually unnecessary to calculate these values by actual multiplication.

As the squares of numbers are frequently used in formulas for solving problems occurring in machine design and machine-shop calculations, a few examples will be given below of formulas containing squares.

The area of a circle equals the square of the radius multiplied by 3.1416. Expressed as a formula, if A = area of circle, R = radius, and the Greek letter π (Pi) = 3.1416, we have:

$$A = R^2 \pi.$$

If we want to know the area of a circle having a 5-foot radius, we have:

$$A = 5^2 \pi = 5 \times 5 \times 3.1416 = 78.54 \text{ square feet.}$$

As a further example, assume a formula to be given as follows:

$$A = \frac{D^2N + R^2\pi}{DR}$$

Assume that D = 3, $N = 5$, $R = 4$, and π (as usual) = 3.1416. What is the value of A? Inserting the values of the various letters in the formula, we have:

$$A = \frac{3^2 \times 5 + 4^2 \times \pi}{3 \times 4} = \frac{3 \times 3 \times 5 + 4 \times 4 \times \pi}{3 \times 4} =$$

$$\frac{9 \times 5 + 16 \times \pi}{12} = \frac{45 + 50.2656}{12} = \frac{95.2656}{12} = 7.9388.$$

It will be seen in the example above that all the multiplications are carried out before any addition is made. This is in accordance with the rules of mathematics. When several numbers or expressions are connected with signs indicating that additions, subtractions, multiplications or divisions are to be made, the multiplications should be carried out before any of the other operations, because the numbers that are connected by the multiplication sign are actually only factors of the product thus indicated, and consequently this product must be considered as one number by itself. The other operations are carried out in the order written, except that divisions when written in line with additions and subtractions, precede these operations. A number of examples of these rules are given below:

$$12 \times 3 + 7 \times 2\tfrac{1}{2} - 1\tfrac{1}{2} = 36 + 17\tfrac{1}{2} - 1\tfrac{1}{2} = 52.$$
$$5 + 13 \times 7 - 2 = 5 + 91 - 2 = 94.$$
$$9 \div 3 + 9 \times 3 = 3 + 27 = 30.$$
$$9 + 9 \div 3 - 2 = 9 + 3 - 2 = 10.$$

Sometimes, however, in formulas, it is desired that certain operations in addition and subtraction precede the multiplications. In such cases use are made of the parenthesis ( ) and bracket [ ]. These mathematical auxiliaries indicate that the expression inside of the parenthesis or bracket should be considered as one single expression or value, and that, therefore, the calculation inside the parenthesis or bracket should be carried out by itself complete before the remaining calculations are commenced. If one bracket is placed inside of another, the one inside is first calculated, and when completed the other one is carried out. Some examples will illustrate these rules and principles:

$(6 - 2) \times 3 + 4 = 4 \times 3 + 4 = 12 + 4 = 16.$

$3 \times (12 + 7) \div 28\frac{1}{2} = 3 \times 19 \div 28\frac{1}{2} = 57 \div 28\frac{1}{2} = 2.$

$3 + [5 \times 3\,(5 + 2) - 3] \times 6 = 3 + [5 \times 3 \times 7 - 3] \times 6 = 3 + [105 - 3] \times 6 = 3 + 102 \times 6 = 3 + 612 = 615.$

Without the parentheses and brackets, the calculations above would have been as follows:

$6 - 2 \times 3 + 4 = 6 - 6 + 4 = 4.$

$3 \times 12 + 7 \div 28\frac{1}{2} = 36 + 0.2456 = 36.2456.$

$3 + 5 \times 3 \times 5 + 2 - 3 \times 6 = 3 + 75 + 2 - 18 = 62.$

These examples should be carefully studied until thoroughly understood.

We are now ready to return to the question of square roots. The square root of a number is that number which, if multipled by itself, would give the given number. Thus, the square root of 9 is 3, because 3 multiplied by itself equals 9. The square root of 16 equals 4, of 36 equals 6, and so forth. It will be seen at once that the square root may be considered, or, rather, actually is the reverse of the square, so that if the square of 20 is 400, then the square root of 400 is 20. In the same way, as the square of 100 is 10,000, so the square root of 10,000 is 100. The sign used in mathematical formulas for the square root is $\sqrt{\ }$. Thus $\sqrt{9} = 3$, $\sqrt{49} = 7$, and so forth. The process of actually calculating the square root is rather cumbersome, and it is very seldom required, because, as already mentioned, the engineering hand-books usually give tables of square roots for all numbers up to 1000, and for larger numbers the tables can also be used for obtaining the square root approximately correct, or at least near enough so for almost all practical calculations.

The cube of a number is the product resulting from repeating the given number as a factor three times. Thus, the cube of 3 is $3 \times 3 \times 3 = 27$, and the cube of 17 is $17 \times 17 \times 17 = 4913$. In the same way as we write $2^2 = 2 \times 2 = 4$, for the square of 2, so we can write $2^3 = 2 \times 2 \times 2 = 8$, for the cube of 2. The exponent (^{3}) indicates how many times the given number is to be repeated as a factor. The cube of 4, for example, may be written $4^3 = 4 \times 4 \times 4 = 64$. Similarly $17^3 = 4913$. The expression 17^3 may be read "the cube of 17," "17 cube," or "the third power of 17." In the same way as the square root means the reverse of square, so the cube root (or "third root") means the reverse of cube or "third power"; that is, the cube root of a number is the number which, if repeated as factor three times, would give the given number. For example, the cube root of 64 is 4, because $4 \times 4 \times$

$4 = 64$. It is evident that if the cube of a number, say 6, is 216 ($6 \times 6 \times 6 = 216$), then the cube root of 216 is 6. The sign used in formulas for the cube root is $\sqrt[3]{\ }$. For example, $\sqrt[3]{8} = 2$ (because $2 \times 2 \times 2 = 8$), and $\sqrt[3]{125} = 5$ (because $5 \times 5 \times 5 = 125$). Similarly, $\sqrt[3]{3,723,875} = 155$.

The use of the square and square root, and cube and cube root in formulas may be shown by a few examples:

$$A = \frac{\sqrt[3]{B} \times \sqrt{C}}{C^2 + D^2}.$$

Assume that $B = 27$, $C = 25$, and $D = 2$. Insert these values in the formula. Then

$$A = \frac{\sqrt[3]{27} \times \sqrt{25}}{25^2 + 2^2} = \frac{3 \times 5}{125 + 4} = \frac{15}{129} = 0.116.$$

As another example:

$$A = \frac{B^2 + C^2 + D^2}{B^3 \times \sqrt{C}}.$$

Assume $B = 2$, $C = 9$, and $D = 4$. Then

$$A = \frac{2^2 + 9^2 + 4^2}{2^3 \times \sqrt{9}} = \frac{4 + 81 + 16}{8 \times 3} = \frac{101}{24} = 4.208.$$

In the same way as $2^2 = 2 \times 2 = 4$, so $2^4 = 2 \times 2 \times 2 \times 2 = 16$, and $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$.

The expression 2^4 is read the "fourth power of 2," and 2^5 the "fifth power of 2." The exponents (^{4}) and (^{5}) indicate how many times the given number is to be repeated as factor.

If, again, it is required to find the number which, if repeated as factor four times, gives the given number, we must obtain the "fourth root" or $\sqrt{\ }$.

Thus, $\sqrt{16} = 2$, because $2 \times 2 \times 2 \times 2 = 16$. In the same way $\sqrt{256} = 4$. The fifth root is written $\sqrt{\quad}$; and $\sqrt{243} = 3$, because $3 \times 3 \times 3 \times 3 \times 3 = 243$.

These explanations, when fully understood, will eliminate all difficulties with formulas of a simple nature, and with such expressions as cube root, exponents, etc.

An important method facilitating the use of formulas, is commonly known as the transposition of formulas. A formula for finding the horse-power which can safely be transmitted by a gear of a given size, running at a given speed, is:

$$H.P. = \frac{D \times N \times P \times F \times 200}{126{,}050}.$$

In this formula $H.P.$ = horse-power,
D = pitch diameter,
N = revolutions per minute,
P = circular pitch of gear,
F = width of face of gear.

Assume, for example, that the pitch diameter of a gear is 31.5 inches, the number of revolutions per minute 200, the circular pitch $1\frac{1}{2}$ inch, and the width of the face 3 inches. Then, if these values are inserted in the formula, we have:

$$H.P. = \frac{31.5 \times 200 \times 1\frac{1}{2} \times 3 \times 200}{126{,}050} = 45 \text{ horse-}$$
power, very nearly.

Assume, however, that the horse-power required to be transmitted is known, and that the pitch of the gear is required to be found. Assume that

$H.P = 30$; $D = 31.5$; $N = 200$; $F = 3$; and that P is the unknown quantity; then, inserting the known values in the formula, gives us:

$$30 = \frac{31.5 \times 200 \times P \times 3 \times 200}{126,050}.$$

In order to be able to find P, we want it given on one side of the equals sign, with all the known quantities on the other side. If we multiply the expressions on both sides of the equals sign by the same number we do not change the conditions; thus

$$30 \times 126,050 = \frac{31.5 \times 200 \times P \times 3 \times 200 \times 126,050}{126,050}.$$

By canceling the number 126,050 on the right-hand side we have:

$$30 \times 126,050 = 31.5 \times 200 \times P \times 3 \times 200.$$

If we now divide on both sides of the equals sign with $31.5 \times 200 \times 3 \times 200$, we have:

$$\frac{30 \times 126,050}{31.5 \times 200 \times 3 \times 200} = \frac{31.5 \times 200 \times P \times 3 \times 200}{31.5 \times 200 \times 3 \times 200}.$$

We can now cancel all numerical values in the fraction on the right-hand side; then:

$$\frac{30 \times 126,050}{31.5 \times 200 \times 3 \times 200} = P.$$

This is then the transposed formula giving P, and from this we find that $P = 1$ inch.

In general, any formula of the form

$$A = \frac{B}{C}.$$

can be transposed as below:

$$A \times C = B; \; C = \frac{B}{A}.$$

It will be seen that the quantities which are in the denominator on one side of the equals sign, are transposed into the numerator on the other side, and *vice versa*.

Examples:

$$A = \frac{B \times C}{D}.$$

Then:

$$D = \frac{B \times C}{A}; \; B = \frac{A \times D}{C}; \; C = \frac{A \times D}{B}.$$

$$A = \frac{E \times F \times G}{K \times L}.$$

Then:

$$E = \frac{A \times K \times L}{F \times G}; \; F = \frac{A \times K \times L}{E \times G}; \; G = \frac{A \times K \times L}{E \times F};$$

$$K = \frac{E \times F \times G}{A \times L}; \; L = \frac{E \times F \times G}{A \times K}.$$

The principles of transposition of formulas can best be grasped by a careful study of the examples given. Note that the method is only directly applicable when all the quantities in the numerator and denominator are *factors of a product*. If connected by + or − signs, the transposition cannot be made by the simple methods shown unless the *whole sum* or *difference* is transposed. Example:

$$A = \frac{B + C}{D}; \text{ then } D = \frac{B + C}{A} \text{ and } B + C = A \times D.$$

The most usual caclulations, perhaps, in some classes of machine design, are those involving the finding of the strength of certain machine members; and, in order to find the strength of these members, it is necessary to first find the cross-sectional area of the part subjected to stress. For this reason, the remainder of this chapter will be largely taken up with rules and formulas for finding the areas and other properties of various geometrical figures. Rules and formulas for volumes of solids will also be given. Examples have been given in some cases merely to show the applications of the formulas.

The area of a triangle equals one-half the product of its base and its altitude. The base may be any side of the triangle, and the altitude is the length of the line drawn from the angle opposite the base, perpendicular to it.

Assume that A = area of triangle,
B = base,
H = altitude.

Then the rule above may be expressed as a formula as follows:

$$A = \frac{B \times H}{2}.$$

Let the base (B) of a triangle be 5 feet, and the altitude (H) 8 feet. Then the area

$$A = \frac{5 \times 8}{2} = \frac{40}{2} = 20 \text{ square feet.}$$

The area of a square equals the square of its side. If A = the area, and S the side of the square, then

$$A = S^2.$$

If the side is 9.7 inches long, then

$A = 9.7^2 = 9.7 \times 9.7 = 94.09$ square inches.

The area of a rectangle equals the product of its long and short sides. If A = area, L = length of the longer side, and H = length of the shorter side, then

$$A = L \times H.$$

The area of a parallelogram equals the product of the base and the altitude.

The area of a trapezoid equals one-half the sum of the parallel sides multiplied by the altitude. If A = area, B = length of one of the parallel sides, C = length of the other parallel side, and H = altitude, then

$$A = \frac{B + C}{2} \times H.$$

Assume that the lengths of the two parallel sides are 12 and 9 feet, respectively, and that the altitude is 16 feet. Then

$$A = \frac{12 + 9}{2} \times 16 = 10.5 \times 16 = 168 \text{ square feet.}$$

To find the area of an irregular figure bounded by straight lines, divide the figure into triangles, and find the area of each triangle separately. The sum of the areas of all the triangles equals the area of the figure.

The circumference of a circle equals its diameter multiplied by 3.1416.

The diameter of a circle equals the circumference divided by 3.1416.

The area of a circle equals the square of the diameter multiplied by 0.7854.

The diameter of a circle equals the area divided by 0.7854, and the square root extracted of the quotient.

If D = diameter, C = circumference, and A = area, these last rules may be expressed in formulas as follows:

$$C = D \times 3.1416. \qquad D = \frac{C}{3.1416}.$$

$$A = D^2 \times 0.7854. \qquad D = \sqrt{\frac{A}{0.7854}}.$$

The length of a circular arc equals the circumference of the circle, multiplied by the number of degrees in the arc, divided by 360. If L = length of arc, C = circumference of circle, and N = number of degrees in the arc, then

$$L = \frac{C \times N}{360}.$$

The area of a circular sector equals the area of the whole circle multiplied by the quotient of the number of degrees in the arc of the sector divided by 360. If a = area of sector, A = area of circle, and N = number of degrees in sector, then

$$a = A \times \frac{N}{360}.$$

The area of a circular segment equals the area of the circular sector formed by drawing radii from the center of the circle to the extremities of the arc of the segment, minus the area of the triangle formed by these radii and the chord of the arc of the segment.

The area of a pentagon (regular polygon having five sides) equals the square of the side times 1.720.

The area of a hexagon (regular polygon having six sides) equals the square of the side times 2.598.

The area of a heptagon (regular polygon having seven sides) equals the square of the side times 3.634.

The area of an octagon (regular polygon having eight sides) equals the square of the side times 4.828.

The volume of a cube equals the cube of the length of its side.

The volume of a prism equals the area of the base multiplied by the altitude.

The volume of a cylinder equals the area of its base circle multiplied by the altitude.

The volume of a pyramid or cone equals the area of the base times one-third the altitude.

The area of the surface of a sphere equals the square of the diameter multiplied by 3.1416.

The volume of a sphere equals the cube of the diameter times 0.5236.

The volume of a spherical sector equals two-thirds of the square of the radius of the sphere multiplied by the height of the contained spherical segment, multiplied by 3.1416. If V = volume of sector, R=radius of sphere, and H=height of the contained spherical segment, then

$$V = \frac{2}{3} R^2 \times H \times 3.1416.$$

Assume that the length of the radius of a spherical sector is 6 inches, and the height of the contained segment 2 inches. Then

$$V = \frac{2}{3} \times 6^2 \times 2 \times 3.1416 = 150.7968 \text{ cubic inches.}$$

The volume of a spherical segment equals the radius of the sphere less one-third the height of the segment, multiplied by the square of the height of the segment, multiplied by 3.1416. If R = radius, H = height, and V = volume of segment, then

$$V = \left(R - \frac{H}{3}\right) \times H^2 \times 3.1416.$$

Assume that the length of the radius is 4 inches, and the height of the segment 3 inches. Then

$$V = \left(4 - \frac{3}{3}\right) \times 3^2 \times 3.1416 = 84.8232 \text{ cubic inches.}$$

The area of an ellipse equals the long axis multiplied by the short axis, multiplied by 0.7854. If the area $= A$, the long axis $= B$, and the short axis $= C$, then

$$A = B \times C \times 0.7854.$$

If the long axis is 12 inches and the short axis $8\frac{1}{3}$ inches, then

$$A = 12 \times 8\tfrac{1}{3} \times 0.7854 = 78.54.$$

Formulas and application of formulas have not been given for such rules which are so simple and easy to understand that the reader without difficulty can formulate his own formula.

CHAPTER VII

ELEMENTS OF TRIGONOMETRY

TRIGONOMETRY is a very important part of the science of mathematics, and deals with the determination of angles and the solution of triangles. In order to fully understand the subjects treated of in the following, it is necessary that the reader is fully familiar with the usual methods of designating the measurements or sizes of angles. While mathematicians employ also another method, in mechanics angles are measured in degrees and subdivisions of a degree, called minutes. The minute is again subdivided into seconds, but these latter subdivisions are so small as to permit of being disregarded in general practical machine design.

A degree is 1-360 part of a circle, or, in other words, if the circumference of a circle is divided into 360 parts, then each part is called one degree.

If two lines are drawn from the center of the circle to the ends of the small circular arc which is 1-360 part of the circumference, then the angle between these two lines is a 1-degree angle. A quarter of a circle or a 90-degree angle is called a right angle. The meaning of obtuse and acute angles has already been explained in Chapter II. Any angle which is not a right angle is called an oblique angle.

A minute is 1-60 part of a degree, and a second 1-60 part of a minute. In other words, one circle = 360 degrees, one degree = 60 minutes, and one minute = 60 seconds. The sign (°) is used for indicating degrees; the sign (′) indicates minutes, and the sign (″) seconds. A common abbreviation for degree is "deg."; for minute, "min."; and for second, "sec."

Two angles are equal when the number of degrees they contain is the same. If two angles are both 30 degrees, they are equal, no matter how long the sides of the one may be in relation to the other.

Of all triangles, the right-angled triangle occurs most frequently in machine design. A right-angled triangle is one having the angle between two sides a right angle; the angles between the other sides may be of any size. In the calculations involved in solving right-angled triangles, a useful application of the squares and square roots of numbers is also presented. Assume that the lengths of the sides of a right-angled triangle, as shown in Fig. 88, are 5 inches, 4 inches, and 3 inches, respectively. Then

$$5^2 = 4^2 + 3^2, \text{ or } 25 = 16 + 9.$$

This relationship between the three sides in a right-angled triangle holds good for all right-angled triangles. The square of the side opposite the right angle equals the sum of the squares of the sides including the right angle. Assume, for example, that the lengths of the two sides including the right angle in a right-angled triangle are 12 and 9 inches long, respectively, as shown in Fig. 89, and that the side opposite the right angle, the *hypotenuse,* is to be found. We then first square the two given sides, and from our rule, just given, we have that the sum of the squares equals the square of the side to be found. The square root

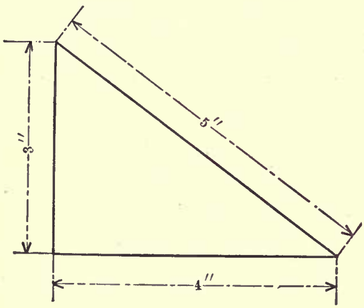


FIG. 88.

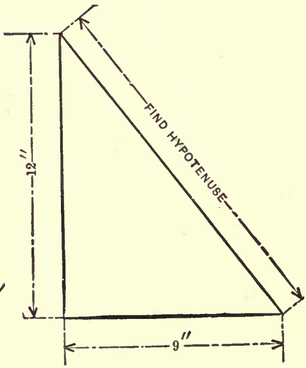


FIG. 89.

of the sum must then equal the side itself. Carrying out this calculation we have:

$$12^2 + 9^2 = 144 + 81 = 225$$

$$\sqrt{225} = 15 \text{ inches} = \text{length of hypotenuse.}$$

Similar methods may be employed for finding any of the sides in a right-angled triangle if two sides are given. If the hypotenuse were known to be 15 inches, and one of the sides including the right angle 9 inches, as shown at *D* in Fig. 90, then the other side including the right angle can be found. In this case, however, we must subtract the square of the known side including the right angle from the square of the hypotenuse to obtain the square of the remaining including side. We, therefore, have:

$$15^2 - 9^2 = 225 - 81 = 144$$
$$\sqrt{144} = 12 \text{ inches} = \text{length of unknown side.}$$

In the same way, if the lengths 15 and 12 were

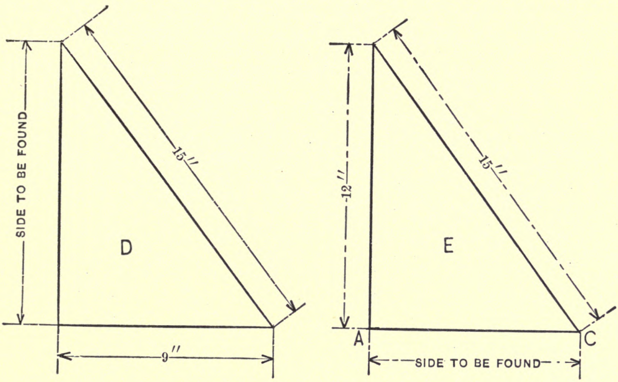


FIG. 90.

known, we could find the side *AC*, as shown at *E*, Fig. 90:

$$15^2 - 12^2 = 225 - 144 = 81$$
$$\sqrt{81} = 9 \text{ inches} = \text{length of } AC.$$

From these examples we may formulate rules and general formulas for the solution of right-angled triangles when two sides are known. In Fig. 91, at *F*, the square of *AB* plus the square of *AC* equals the square of *BC;* the square of *BC* minus the square of *AC* equals the square of *AB;* and the square of *BC* minus the square of *AB* equals the square of AC. These rules written as general formulas would take the form:

$$AB^2 + AC^2 = BC^2$$
$$BC^2 - AC^2 = AB^2$$
$$BC^2 - AB^2 = AC^2$$

From these formulas we have, by extracting the square root on each side of the equal sign:

$$BC = \sqrt{AB^2 + AC^2}$$
$$AB = \sqrt{BC^2 - AC^2}$$
$$AC = \sqrt{BC^2 - AB^2}$$

These formulas make it possible to find the third side when two sides are given, no matter what the

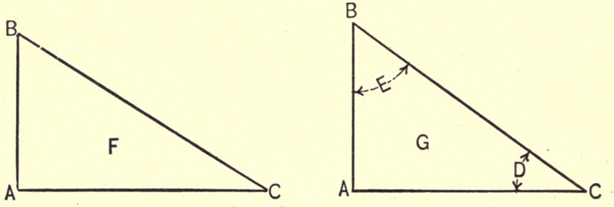


FIG. 91.

numerical values of the length of the sides may be. Assume $AB = 12$, and $BC = 20$; find AC. According to the formula:

$$AC = \sqrt{20^2 - 12^2} = \sqrt{400 - 144} = \sqrt{256} = 16.$$

Assume that $AB = 15$ and $AC = 20$. Find BC.

$$BC = \sqrt{15^2 + 20^2} = \sqrt{225 + 400} = \sqrt{625} = 25.$$

The rules and formulas given make it possible to find the length of the sides in a right-angled triangle. To find the angles, however, use must be made of the *trigonometric functions*, the meanings of which will be presently explained. The trigonometric functions are the *sine*, *cosine*, *tangent*, *cotangent*, *secant* and *cosecant* of angles. While these functions are used in the solution of all kinds of triangles, they refer directly to right-angled triangles, and the meaning or value of each function can be explained by reference to a right-angled triangle as shown in Fig. 91, at G, where the side BC is the hypotenuse, AC the side adjacent to angle D, and AB the side opposite angle D. Of course, if reference is made to angle E, then AB is the side adjacent and AC the side opposite.

The sine of an angle is the length of the opposite side, if the hypotenuse is assumed to equal 1. The sine of angle D, then, is the length of AB if BC equals 1. To find the sine of D when BC is any other length, divide AB by the length of BC. To find the sine of D, if BC equals 5, for example, it is necessary to divide the length of AB by 5.

Find the sine of D, when $AB = 15$ and $BC = 20$. The sine of $D = 15 \div 20 = 0.75$.

The cosine of an angle is the length of the adjacent side, if the hypotenuse is assumed to equal 1. The cosine of angle D, then, is the length of AC if BC equals 1. To find the cosine of D when BC is any other length, divide AC by the length of BC. To find the cosine of D, if BC equals 8, for example, it is necessary to divide the length of AC by 8.

Find the cosine of D, when $AC = 12$ and $BC = 30$. The cosine of $D = 12 \div 30 = 0.4$.

The tangent of an angle is the length of the opposite side, if the adjacent side is assumed to equal 1. The tangent of angle D is the length of AB if AC equals 1. To find the tangent of D when AC equals any other length, divide AB by the length of AC. To find the tangent of D when AC equals 3, for example, it is necessary to divide the length of AB by 3.

Find the tangent of D, when $AB = 16$ and $AC = 12$. The tangent of $D = 16 \div 12 = 1.333$.

The cotangent of an angle is the length of the adjacent side, if the opposite side is assumed to equal 1. The cotangent of angle D is the length of AC if AB equals 1. To find the cotangent of D when AB equals any other length, divide AC by the length of AB. To find the cotangent of D when AB equals 12, for example, divide AC by 12.

Find the cotangent of D when $AB = 3$ and $AC = 36$. The cotangent of $D = 36 \div 3 = 12$.

The secant of an angle is the length of the hypotenuse, if the adjacent side is assumed to equal 1. The secant of angle D is the length of BC when AC equals 1. To find the secant of D when AC is any other length, divide BC by the length of AC.

Find the secant of D when $BC = 24$ and $AC = 9$. The secant of $D = 24 \div 9 = 2.666\ldots$

The cosecant of an angle is the length of the hypotenuse if the opposite side is assumed to equal 1. The cosecant of angle D is the length of BC when AB equals 1. To find the cosecant of D when AB is any other length, divide BC by the length of AB.

Find the cosecant of D when $BC = 30$ and $AB = 3.75$. The cosecant of $D = 30 \div 3.75 = 8$.

The expressions sine, cosine, tangent, cotangent, secant and cosecant are abbreviated as follows: sin, cos, tan, cot, sec, and cosec. Instead of writing tangent of *D*, for example, it is usual to write tan *D*. By means of these functions, tables of which are given in the following, the values of angles can be introduced in the calculations of triangles. The tables here used give the values of the functions of angles for every degree and for every ten minutes. Only three decimal places are given, as that is enough for the great majority of shop calculations. When very accurate calculations are required, tables can be procured giving the functions for every minute, and with five decimal places. From the tables given, when the angle is known, the corresponding angular function can be found, and when the function is known, the corresponding angle can be determined by merely reading off the values in the table. The tables include sines, cosines, tangents and cotangents only, as these are most commonly used, and all problems can be solved by the use of them. When the secant is required, it can be found by dividing 1 by the cosine. The cosecant is found by dividing 1 by the sine.

The tables of sines, cosines, etc., are read the same as any other table. It will be seen that the four tables given are headed Sines, Cosines, Tangents, and Cotangents, respectively. At the bottom of the table headed "Sines" is read the word "Cosines," and at the bottom of the table headed "Cosines" is read the word "Sines." In the same way, at the bottom of the table headed "Tan-

Deg.	Minutes. 0′	10′	20′	30′	40′	50′	60′	Deg.
0	0.000	0.003	0.006	0.009	0.012	0.015	0.017	89
1	0.017	0.020	0.023	0.026	0.029	0.032	0.035	88
2	0.035	0.038	0.041	0.044	0.047	0.049	0.052	87
3	0.052	0.055	0.058	0.061	0.064	0.067	0.070	86
4	0.070	0.073	0.076	0.078	0.081	0.084	0.087	85
5	0.087	0.090	0.093	0.096	0.099	0.102	0.105	84
6	0.105	0.107	0.110	0.113	0.116	0.119	0.122	83
7	0.122	0.125	0.128	0.131	0.133	0.136	0.139	82
8	0.139	0.142	0.145	0.148	0.151	0.154	0.156	81
9	0.156	0.159	0.162	0.165	0.168	0.171	0.174	80
10	0.174	0.177	0.179	0.182	0.185	0.188	0.191	79
11	0.191	0.194	0.197	0.199	0.202	0.205	0.208	78
12	0.208	0.211	0.214	0.216	0.219	0.222	0.225	77
13	0.225	0.228	0.231	0.233	0.236	0.239	0.242	76
14	0.242	0.245	0.248	0.250	0.253	0.256	0.259	75
15	0.259	0.262	0.264	0.267	0.270	0.273	0.276	74
16	0.276	0.278	0.281	0.284	0.287	0.290	0.292	73
17	0.292	0.295	0.298	0.301	0.303	0.306	0.309	72
18	0.309	0.312	0.315	0.317	0.320	0.323	0.326	71
19	0.326	0.328	0.331	0.334	0.337	0.339	0.342	70
20	0.342	0.345	0.347	0.350	0.353	0.356	0.358	69
21	0.358	0.361	0.364	0.367	0.369	0.372	0.375	68
22	0.375	0.377	0.380	0.383	0.385	0.388	0.391	67
23	0.391	0.393	0.396	0.399	0.401	0.404	0.407	66
24	0.407	0.409	0.412	0.415	0.417	0.420	0.423	65
25	0.423	0.425	0.428	0.431	0.433	0.436	0.438	64
26	0.438	0.441	0.444	0.446	0.449	0.451	0.454	63
27	0.454	0.457	0.459	0.462	0.464	0.467	0.469	62
28	0.469	0.472	0.475	0.477	0.480	0.482	0.485	61
29	0.485	0.487	0.490	0.492	0.495	0.497	0.500	60
30	0.500	0.503	0.505	0.508	0.510	0.513	0.515	59
31	0.515	0.518	0.520	0.522	0.525	0.527	0.530	58
32	0.530	0.532	0.535	0.537	0.540	0.542	0.545	57
33	0.545	0.547	0.550	0.552	0.554	0.557	0.559	56
34	0.559	0.562	0.564	0.566	0.569	0.571	0.574	55
35	0.574	0.576	0.578	0.581	0.583	0.585	0.588	54
36	0.588	0.590	0.592	0.595	0.597	0.599	0.602	53
37	0.602	0.604	0.606	0.609	0.611	0.613	0.616	52
38	0.616	0.618	0.620	0.623	0.625	0.627	0.629	51
39	0.629	0.632	0.634	0.636	0.638	0.641	0.643	50
40	0.643	0.645	0.647	0.649	0.652	0.654	0.656	49
41	0.656	0.658	0.660	0.663	0.665	0.667	0.669	48
42	0.669	0.671	0.673	0.676	0.678	0.680	0.682	47
43	0.682	0.684	0.686	0.688	0.690	0.693	0.695	46
44	0.695	0.697	0.699	0.701	0.703	0.705	0.707	45
Deg.	60′	50′	40′	30′	20′	10′	0′ Minutes.	Deg.

COSINES

Deg.	Minutes.							Deg.
	0′	10′	20′	30′	40′	50′	60′	
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	89
1	1.000	1.000	1.000	1.000	1.000	0.999	0.999	88
2	0.999	0.999	0.999	0.999	0.999	0.999	0.999	87
3	0.999	0.998	0.998	0.998	0.998	0.998	0.998	86
4	0.998	0.997	0.997	0.997	0.997	0.996	0.996	85
5	0.996	0.996	0.995	0.995	0.995	0.995	0.995	84
6	0.995	0.994	0.994	0.994	0.993	0.993	0.993	83
7	0.993	0.992	0.992	0.991	0.991	0.991	0.990	82
8	0.990	0.990	0.989	0.989	0.989	0.988	0.988	81
9	0.988	0.987	0.987	0.986	0.986	0.985	0.985	80
10	0.985	0.984	0.984	0.983	0.983	0.982	0.982	79
11	0.982	0.981	0.981	0.980	0.979	0.979	0.978	78
12	0.978	0.978	0.977	0.976	0.976	0.975	0.974	77
13	0.974	0.974	0.973	0.972	0.972	0.971	0.970	76
14	0.970	0.970	0.969	0.968	0.967	0.967	0.966	75
15	0.966	0.965	0.964	0.964	0.963	0.962	0.961	74
16	0.961	0.960	0.960	0.959	0.958	0.957	0.956	73
17	0.956	0.955	0.955	0.954	0.953	0.952	0.951	72
18	0.951	0.950	0.949	0.948	0.947	0.946	0.946	71
19	0.946	0.945	0.944	0.943	0.942	0.941	0.940	70
20	0.940	0.939	0.938	0.937	0.936	0.935	0.934	69
21	0.934	0.933	0.931	0.930	0.929	0.928	0.927	68
22	0.927	0.926	0.925	0.924	0.923	0.922	0.921	67
23	0.921	0.919	0.918	0.917	0.916	0.915	0.914	66
24	0.914	0.912	0.911	0.910	0.909	0.908	0.906	65
25	0.906	0.905	0.904	0.903	0.901	0.900	0.899	64
26	0.899	0.898	0.896	0.895	0.894	0.892	0.891	63
27	0.891	0.890	0.888	0.887	0.886	0.884	0.883	62
28	0.883	0.882	0.880	0.879	0.877	0.876	0.875	61
29	0.875	0.873	0.872	0.870	0.869	0.867	0.866	60
30	0.866	0.865	0.863	0.862	0.860	0.859	0.857	59
31	0.857	0.856	0.854	0.853	0.851	0.850	0.848	58
32	0.848	0.847	0.845	0.843	0.842	0.840	0.839	57
33	0.839	0.837	0.835	0.834	0.832	0.831	0.829	56
34	0.829	0.827	0.826	0.824	0.822	0.821	0.819	55
35	0.819	0.817	0.816	0.814	0.812	0.811	0.809	54
36	0.809	0.807	0.806	0.804	0.802	0.800	0.799	53
37	0.799	0.797	0.795	0.793	0.792	0.790	0.788	52
38	0.788	0.786	0.784	0.783	0.781	0.779	0.777	51
39	0.777	0.775	0.773	0.772	0.770	0.768	0.766	50
40	0.766	0.764	0.762	0.760	0.759	0.757	0.755	49
41	0.755	0.753	0.751	0.749	0.747	0.745	0.743	48
42	0.743	0.741	0.739	0.737	0.735	0.733	0.731	47
43	0.731	0.729	0.727	0.725	0.723	0.721	0.719	46
44	0.719	0.717	0.715	0.713	0.711	0.709	0.707	45
	60′	50′	40′	30′	20′	10′	0′	
Deg.	Minutes.							Deg.

SINES

Deg.	Minutes.							Deg.
	0′	10′	20′	30′	40′	50′	60′	
0	0.000	0.003	0.006	0.009	0.012	0.015	0.017	89
1	0.017	0.020	0.023	0.026	0.029	0.032	0.035	88
2	0.035	0.038	0.041	0.044	0.047	0.049	0.052	87
3	0.052	0.055	0.058	0.061	0.064	0.067	0.070	86
4	0.070	0.073	0.076	0.079	0.082	0.085	0.087	85
5	0.087	0.090	0.093	0.096	0.099	0.102	0.105	84
6	0.105	0.108	0.111	0.114	0.117	0.120	0.123	83
7	0.123	0.126	0.129	0.132	0.135	0.138	0.141	82
8	0.141	0.144	0.146	0.149	0.152	0.155	0.158	81
9	0.158	0.161	0.164	0.167	0.170	0.173	0.176	80
10	0.176	0.179	0.182	0.185	0.188	0.191	0.194	79
11	0.194	0.197	0.200	0.203	0.206	0.210	0.213	78
12	0.213	0.216	0.219	0.222	0.225	0.228	0.231	77
13	0.231	0.234	0.237	0.240	0.243	0.246	0.249	76
14	0.249	0.252	0.256	0.259	0.262	0.265	0.268	75
15	0.268	0.271	0.274	0.277	0.280	0.284	0.287	74
16	0.287	0.290	0.293	0.296	0.299	0.303	0.306	73
17	0.306	0.309	0.312	0.315	0.318	0.322	0.325	72
18	0.325	0.328	0.331	0.335	0.338	0.341	0.344	71
19	0.344	0.348	0.351	0.354	0.357	0.361	0.364	70
20	0.364	0.367	0.371	0.374	0.377	0.381	0.384	69
21	0.384	0.387	0.391	0.394	0.397	0.401	0.404	68
22	0.404	0.407	0.411	0.414	0.418	0.421	0.424	67
23	0.424	0.428	0.431	0.435	0.438	0.442	0.445	66
24	0.445	0.449	0.452	0.456	0.459	0.463	0.466	65
25	0.466	0.470	0.473	0.477	0.481	0.484	0.488	64
26	0.488	0.491	0.495	0.499	0.502	0.506	0.510	63
27	0.510	0.513	0.517	0.521	0.524	0.528	0.532	62
28	0.532	0.535	0.539	0.543	0.547	0.551	0.554	61
29	0.554	0.558	0.562	0.566	0.570	0.573	0.577	60
30	0.577	0.581	0.585	0.589	0.593	0.597	0.601	59
31	0.601	0.605	0.609	0.613	0.617	0.621	0.625	58
32	0.625	0.629	0.633	0.637	0.641	0.645	0.649	57
33	0.649	0.654	0.658	0.662	0.666	0.670	0.675	56
34	0.675	0.679	0.683	0.687	0.692	0.696	0.700	55
35	0.700	0.705	0.709	0.713	0.718	0.722	0.727	54
36	0.727	0.731	0.735	0.740	0.744	0.749	0.754	53
37	0.754	0.758	0.763	0.767	0.772	0.777	0.781	52
38	0.781	0.786	0.791	0.795	0.800	0.805	0.810	51
39	0.810	0.815	0.819	0.824	0.829	0.834	0.839	50
40	0.839	0.844	0.849	0.854	0.859	0.864	0.869	49
41	0.869	0.874	0.880	0.885	0.890	0.895	0.900	48
42	0.900	0.906	0.911	0.916	0.922	0.927	0.933	47
43	0.933	0.938	0.943	0.949	0.955	0.960	0.966	46
44	0.966	0.971	0.977	0.983	0.988	0.994	1.000	45
	60′	50′	40′	30′	20′	10′	0′	
Deg.	Minutes.							Deg.

COTANGENTS

Deg.	Minutes. 0′	10′	20′	30′	40′	50′	60′	Deg.
0	∞	343.8	171.9	114.6	85.94	68.75	57.29	89
1	57.29	49.10	42.96	38.19	34.37	31.24	28.64	88
2	28.64	26.43	24.54	22.90	21.47	20.21	19.08	87
3	19.08	18.07	17.17	16.35	15.60	14.92	14.30	86
4	14.30	13.73	13.20	12.71	12.25	11.83	11.43	85
5	11.43	11.06	10.71	10.39	10.08	9.788	9.514	84
6	9.514	9.225	9.010	8.777	8.556	8.345	8.144	83
7	8.144	7.953	7.770	7.596	7.429	7.269	7.115	82
8	7.115	6.968	6.827	6.691	6.561	6.435	6.314	81
9	6.314	6.197	6.084	5.976	5.871	5.769	5.671	80
10	5.671	5.576	5.485	5.396	5.309	5.226	5.145	79
11	5.145	5.066	4.989	4.915	4.843	4.773	4.705	78
12	4.705	4.638	4.574	4.511	4.449	4.390	4.331	77
13	4.331	4.275	4.219	4.165	4.113	4.061	4.011	76
14	4.011	3.962	3.914	3.867	3.821	3.776	3.732	75
15	3.732	3.689	3.647	3.606	3.566	3.526	3.487	74
16	3.487	3.450	3.412	3.376	3.340	3.305	3.271	73
17	3.271	3.237	3.204	3.172	3.140	3.108	3.078	72
18	3.078	3.047	3.018	2.989	2.960	2.932	2.904	71
19	2.904	2.877	2.850	2.824	2.798	2.773	2.747	70
20	2.747	2.723	2.699	2.675	2.651	2.628	2.605	69
21	2.605	2.583	2.560	2.539	2.517	2.496	2.475	68
22	2.475	2.455	2.434	2.414	2.394	2.375	2.356	67
23	2.356	2.337	2.318	2.300	2.282	2.264	2.246	66
24	2.246	2.229	2.211	2.194	2.177	2.161	2.145	65
25	2.145	2.128	2.112	2.097	2.081	2.066	2.050	64
26	2.050	2.035	2.020	2.006	1.991	1.977	1.963	63
27	1.963	1.949	1.935	1.921	1.907	1.894	1.881	62
28	1.881	1.868	1.855	1.842	1.829	1.816	1.804	61
29	1.804	1.792	1.780	1.767	1.756	1.744	1.732	60
30	1.732	1.720	1.709	1.698	1.686	1.675	1.664	59
31	1.664	1.653	1.643	1.632	1.621	1.611	1.600	58
32	1.600	1.590	1.580	1.570	1.560	1.550	1.540	57
33	1.540	1.530	1.520	1.511	1.501	1.492	1.483	56
34	1.483	1.473	1.464	1.455	1.446	1.437	1.428	55
35	1.428	1.419	1.411	1.402	1.393	1.385	1.376	54
36	1.376	1.368	1.360	1.351	1.343	1.335	1.327	53
37	1.327	1.319	1.311	1.303	1.295	1.288	1.280	52
38	1.280	1.272	1.265	1.257	1.250	1.242	1.235	51
39	1.235	1.228	1.220	1.213	1.206	1.199	1.192	50
40	1.192	1.185	1.178	1.171	1.164	1.157	1.150	49
41	1.150	1.144	1.137	1.130	1.124	1.117	1.111	48
42	1.111	1.104	1.098	1.091	1.085	1.079	1.072	47
43	1.072	1.066	1.060	1.054	1.048	1.042	1.036	46
44	1.036	1.030	1.024	1.018	1.012	1.006	1.000	45
Deg.	60′	50′	40′	30′	20′	10′	0′ Minutes.	Deg.

TANGENTS

gents," we read "Cotangents," and at the bottom of the table headed "Cotangents," we read "Tangents." The object of this will be presently explained. The extreme left-hand column, we find, is headed "Deg.," and the following seven columns are headed 0′, 10′, 20′, 30′, 40′, 50′ and 60′, respectively, these columns indicating the minutes. At the bottom of the pages the same numbers are found but reading from the right to the left. The values of the functions marked at the top are read in the table opposite the degrees in the left-hand column and under the minutes at top. The values of the functions marked at the bottom are read opposite the degrees in the right-hand column and over the minutes at the bottom. For example, the sine of 39° 40′ or sin 39° 40′, as it is written in formulas, is thus found to be 0.638, and the sine of 64° 10′ is 0.900, this latter value being read off in the second table, reading it from the bottom up, and locating the number of degrees in the right-hand column.

As further examples, we find

$$\tan 37^\circ\ 40' = 0.772$$
$$\cot 37^\circ\ 40' = 1.295$$
$$\tan 80^\circ\ \ 0' = 5.671$$
$$\cos 75^\circ\ 30' = 0.250$$

We are now ready to proceed to solve right-angled triangles with regard both to the sides and the angles. In any right-angled triangle, if either two sides, or one side and one of the acute angles are known, the remaining quantities can be found. As a general rule, in *any* triangle, all the quantities can be found when three quantities, at least one of which is a side, are given. In a right-angled triangle the right angle is always known, of course, so that here, therefore, only two additional quantities are necessary. If all the three angles are known, the length of the sides cannot be determined; *one* side, at least, must also always be known in order to make possible the solution of the triangle.

The following rules should be used for solving right-angled triangles.

Case 1. Two sides known.—Use the rules already given in this chapter for finding the third

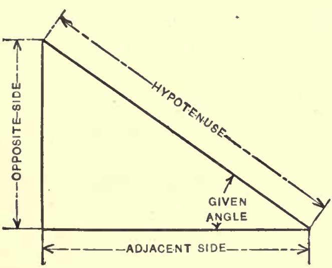


FIG. 92.

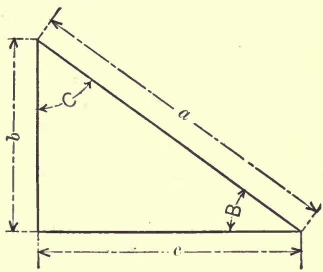


FIG. 93.

side when two sides in a right-angled triangle are given. To find the angles use the rules already given for finding sines, cosines, etc., and the tables.

Case 2. Hypotenuse and one angle given.—Call the side adjacent to the given angle the adjacent side, and the side opposite the given angle the opposite side (see Fig. 92.) Then the adjacent side equals the hypotenuse multiplied by the cosine of the given angle; the opposite side equals the hypotenuse multiplied by the sine of the given angle; and the unknown angle equals 90 degrees minus the given angle.

Case 3. One angle and its adjacent side given. —The hypotenuse equals the adjacent side divided by the cosine of the given angle; the opposite side equals the adjacent side multiplied by the tangent of the given angle; and the unknown angle is found as in Case 2.

Case 4. One angle and its opposite side known. —The hypotenuse equals the opposite side divided by the sine of the given angle; the adjacent side equals the opposite side multiplied by the cotangent of the given angle; and the unknown angle is found as in Case 2.

These rules may be written as formulas as follows (see Fig. 93):

Case 1. For formulas for the sides see the first part of this Chapter. For the angles we have:

$$\sin B = \frac{b}{a} \qquad \sin C = \frac{c}{a}.$$

Case 2. Here, when a and B are given, we have:

$$c = a \cos B;\ b = a \sin B;\ C = 90° - B.$$

When a and C are given, we have:

$$b = a \cos C;\ c = a \sin C;\ B = 90° - C.$$

Case 3. Here, when B and c are given, we have:

$$a = \frac{c}{\cos B};\ b = c \tan B;\ C = 90° - B.$$

When C and b are given, we have:

$$a = \frac{b}{\cos C};\ c = b \tan C;\ B = 90° - C.$$

Case 4. Here, when B and b are known, we have:

$$a = \frac{b}{\sin B}; \; c = b \cot B; \; C = 90° - B.$$

When C and c are known, we have:

$$a = \frac{c}{\sin C}; \; b = c \cot C; \; B = 90° - C.$$

These rules and formulas, while not including all possible combinations for the solution of right-angled triangles, give all the information necessary for the solution of any kind of a right-angled triangle. A few examples of the use of these rules and formulas will now be given, so as to clearly indicate the mode of procedure in practical work.

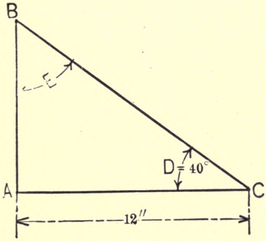


FIG. 94.

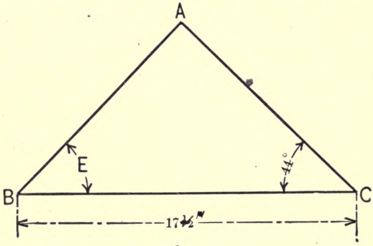


FIG. 95.

Example 1.—In the triangle in Fig. 94, side AC is 12 inches long and angle D is 40 degrees. Find angle E and the two unknown sides.

This is an example of Case 3, one angle and its adjacent side being given. Angle E equals 90 degrees minus the given angle, or

$$E = 90° - 40° = 50°$$

The hypotenuse BC equals the adjacent side divided by the cosine of D, or

$$BC = \frac{12}{\cos 40°} = \frac{12}{0.766} = 15.666 \text{ inches.}$$

Side AB equals the adjacent side multiplied by the tangent of D, or

$$AB = 12 \times \tan 40° = 12 \times 0.839 = 10.068 \text{ inches.}$$

The cosine and tangent of 40 degrees are found in the tables of trigonometric functions as already explained.

Example 2.—In the triangle in Fig. 95, the hypotenuse $BC = 17\frac{1}{2}$ inches. One angle is 44 degrees. Find angle E and the sides AB and AC.

This is an example of Case 2, the hypotenuse and one angle being given. Using the rules or formulas given for Case 2, we have:

$$AC = 17\tfrac{1}{2} \times \cos 44° = 17.5 \times 0.719 = 12.5825 \text{ inches.}$$

$$AB = 17\tfrac{1}{2} \times \sin 44° = 17.5 \times 0.695 = 12.1625 \text{ inches.}$$

$$E = 90° - 44° = 46°.$$

Example 3.—In the triangle in Fig. 96, side $AC = 208$ feet, and the angle opposite this side $= 38$ degrees. Find angle E, and the two remaining sides.

This is an example of Case 4, one side and the angle opposite it being known. From the rules or formulas given for Case 4, we have:

$$BC = 208 \div \sin 38° = 208 \div 0.616 = 337.66 \text{ feet.}$$

$$AB = 208 \times \cot 38° = 208 \times 1.280 = 266.24 \text{ feet.}$$

$$E = 90° - 38° = 52°.$$

Example 4.—In the triangle in Fig. 97, side $AC = 3$ inches, and the hypotenuse $BC = 5$ inches. Find side AB and angles D and E.

This is an example of Case 1. According to a formula previously given in this chapter

$AB = \sqrt{BC^2 - AC^2} = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4.$

$$\sin E = \frac{AB}{BC} = \frac{4}{5} = 0.800.$$

From the tables we find that the angle corre-

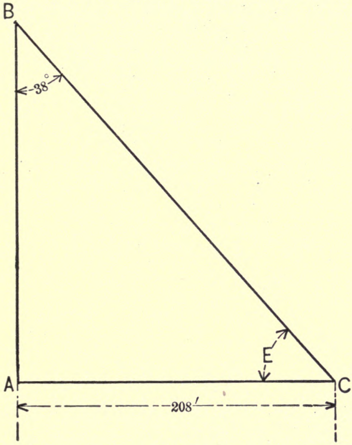


FIG. 96.

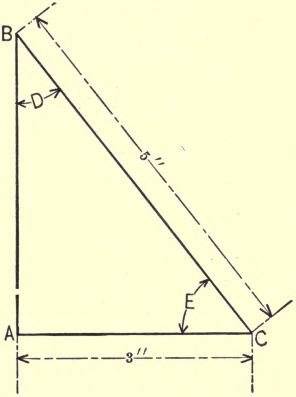


FIG. 97.

sponding to a sine which equals 0.800 is $53° 10'$. Consequently:

$E = 53° 10'$, and $D = 90° - 53° 10' = 36° 50'$.

Example 5.—In the triangle in Fig. 98, side BC, the hypotenuse, is $1\frac{3}{8}$ inch long. One angle is 65 degrees. Find angle E and the remaining sides.

This is an example of Case 2. We have:

$E = 90° - 65° = 25°.$

$AB = 1\frac{3}{8} \times \cos 65° = 1.375 \times 0.423 = 0.5816$ inch.

$AC = 1\frac{3}{8} \times \sin 65° = 1.375 \times 0.906 = 1.2457$ inch.

Example 6.—In the triangle in Fig. 99, side $AB = 0.706$ inch, and the angle adjacent to this side is 60 degrees. Find angle E and the sides AC and BC.

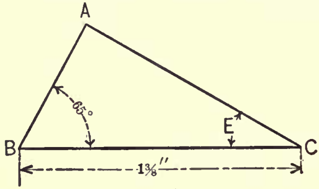


FIG. 98.

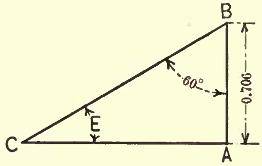


FIG. 99.

This is an example of Case 3. We have:

$E = 90° - 60° = 30°.$

$BC = 0.706 \div \cos 60° = 0.706 \div 0.500 = 1.412$ inch.

$AC = 0.706 \times \tan 60° = 0.706 \times 1.732 = 1.2228$ inch.

The previous examples, carefully studied, will give a comprehensive idea of the methods used for solving right-angled triangles, no matter which parts are given or unknown.

A triangle which does not contain a right angle is called an oblique triangle. Any such triangle can be solved by the aid of the formulas given for the right triangle, by dividing it into two right-angled triangles by means of a line drawn from the vertex of one angle perpendicular towards the opposite side. Formulas can be deduced which do not require that the triangle be so divided, but for elementary purposes, the method indicated is the most easily understood.

In Fig. 100, for example, a triangle is given as shown. One angle is 50 degrees, and the sides including this angle are 4 and 5 inches long, respectively. Draw a line from A perpendicular to the

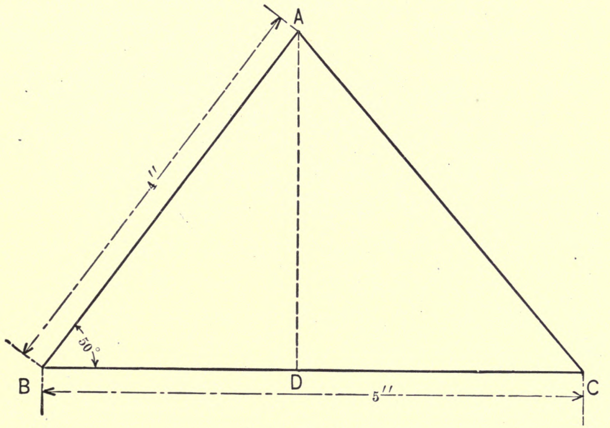


FIG. 100.

side BC. We have here two right-angled triangles, and can now proceed by using the formulas previously given. In triangle ADB, the hypotenuse AB and one angle are given. We then find side AD by means of the formulas for Case 2, and also angle BAD and side BD. Next we find $CD = 5 - BD$. We then, in the triangle ACD know two sides AD and CD, and can thus find side AC as in Case 1, as well as angles ACD and CAD. The angle BAC finally is found by adding angles

BAD and *CAD* and, then, all the angles and sides in the triangle are found.

The successive calculations would be carried out as follows:

$AD = 4 \times \sin 50° = 4 \times 0.766 = 3.064.$
$BD = 4 \times \cos 50° = 4 \times 0.643 = 2.572.$
Angle $BAD = 90° - 50° = 40°.$
$DC = 5 - BD = 5 - 2.572 = 2.428.$
$AC = \sqrt{AD^2 + DC^2} = \sqrt{3.064^2 + 2.428^2} = 3.91.$

$$\text{Sine of angle } ACD = \frac{AD}{AC} = \frac{3.064}{3.91} = 0.784.$$

Angle $ACD = 51° 40'.$
Angle $CAD = 90° - 51° 40' = 38° 20'.$
Angle $BAC = 40° + 38° 20' = 78° 20'.$

In order to check the results obtained, add angles *ABC*, *BAC* and *ACD*. The sum of these angles must equal 180 degrees if the results are correct:

$50° + 78° 20' + 51° 40' = 180°.$

This method, with such modifications as are necessary to meet the different requirements in each problem, may be used for solving all oblique-angled triangles, except in the case where no angle is known, but only the lengths of all the three sides. In this case the use of a direct formula will prove the best and most convenient. Let the three known sides be a, b and c, and the angles opposite each of them A, B and C, respectively, as in Fig. 101; then we have:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}; \quad \sin B = \frac{b \sin A}{a};$$
$$C = 180° - (A + B).$$

As an example, assume that the three sides in a triangle are $a = 4$, $b = 5$, and $c = 6$ inches long. Find the angles.

$$\operatorname{Cos} A = \frac{5^2 + 6^2 - 4^2}{2 \times 5 \times 6} = \frac{45}{60} = 0.750.$$

$$A = 41^\circ 25'.$$

$$\operatorname{Sin} B = \frac{5 \times \sin 41^\circ 25'}{4} = \frac{5 \times 0.662}{4} = 0.827.$$

$$B = 55^\circ 50'.$$

$$C = 180^\circ - (41^\circ 25' + 55^\circ 50') = 82^\circ 45'.$$

As only the first principles of trigonometry have here been treated, some of the more advanced

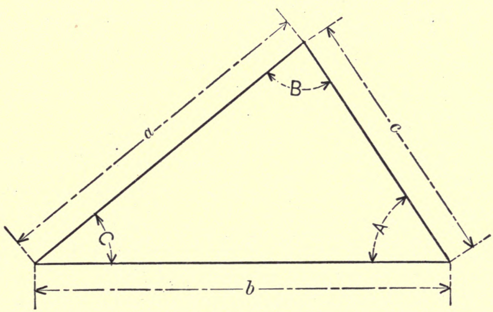


FIG. 101.

problems have, by necessity, been omitted. For ordinary shop calculations the present treatment will, however, be found more satisfactory, as some of the matter which would unnecessarily burden the mind has been left out. If the student only first acquires a thorough understanding of the first principles of mathematics and their application to machine design, it is comparatively easy to broaden the field of one's knowledge; it is, therefore, of extreme importance that these first principles be thoroughly understood and digested. The application will then be found comparatively easy.

The trigonometric functions afford a convenient means for laying out angles; and when the sides

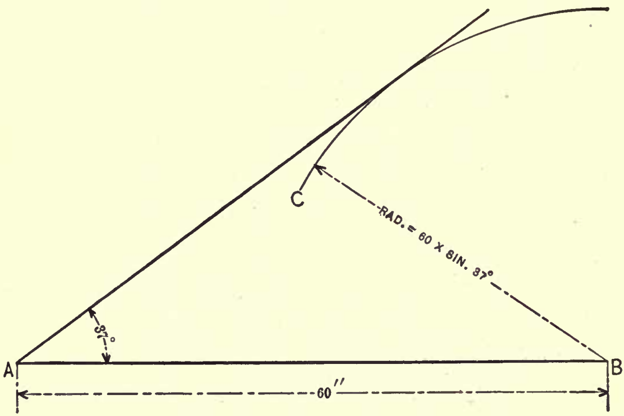


FIG. 102.—Method of Laying Out Angles by Means of Natural Functions.

of the angle laid out are much extended, it can be laid out more accurately in this manner than by the use of an ordinary protractor. Let it be required, for instance, to lay out an angle of 37 degrees, one side of the angle being 60 inches long. Lay out the side *AB*, Fig. 102, 60 inches long. Then with a radius equal to the sine of 37 degrees multiplied by 60, and with a center at *B*, draw an arc *C*. Then draw a line from *A*, tangent to arc *C*. This line forms an angle of 37 degrees with line *AB*. If the required angle is over 45 degrees, then it is preferable to lay out the complement angle from a line perpendicular to the original

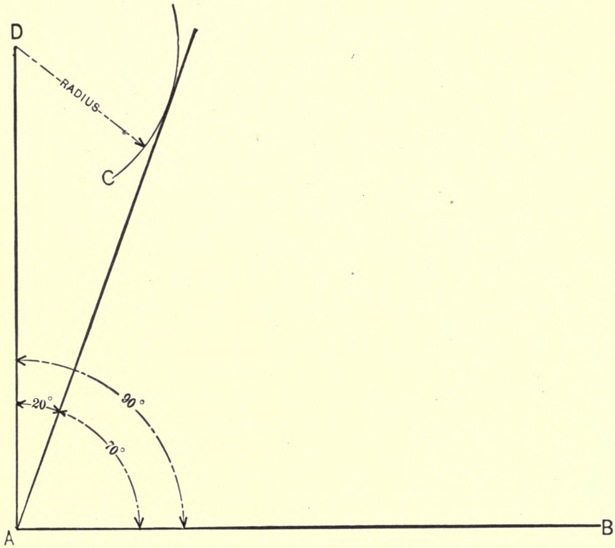


FIG. 103.—Laying Out an Angle Greater than 45 Degrees.

line, as shown in Fig. 103, where an angle of 70 degrees is to be laid out, but the 20-degree complement angle is actually constructed. Many other methods for use in laying out angles, arcs, etc., will readily suggest themselves to the student who thoroughly understands the relation of the trigonometric functions in a right-angled triangle.

CHAPTER VIII

ELEMENTS OF MECHANICS

MECHANICS is defined as that science, or branch of applied mathematics, which treats of the action of forces on bodies. That part of mechanics which considers the action of forces in producing rest or equilibrium is called *statics;* that which relates to such action in producing motion is called *dynamics;* the term *mechanics* includes the action of forces on all bodies whether solid, liquid or gaseous. It is sometimes, however, and formerly was often, used distinctively of *solid* bodies only. The mechanics of *liquid* bodies is called also *hydrostatics* or *hydrodynamics*, according as the laws of rest or motion are considered. The mechanics of *gaseous bodies* is called also *pneumatics*. The mechanics of fluids in motion, with special reference to the methods of obtaining from them useful results, constitutes *hydraulics*.

The Resultant of Two or More Forces.—When a body is acted upon by several forces of different magnitudes in different directions, a single force may be found, which in direction and magnitude will be a resultant of the action of the several forces. The magnitude and direction of this single force may be obtained by what is known as the *parallelogram of forces*. Let *A* and *B*, Fig. 104, represent the direction of two forces acting simultaneously upon *P*, and let their lengths represent the relative magnitude of the forces; then, to find a force which in direction and magnitude shall be a resultant of these two forces, draw the line *C* parallel with *B*, and draw the line *D* parallel with *A*. A diagonal of the parallelogram thus formed, drawn from *P* to *E*, will give the direction, and its

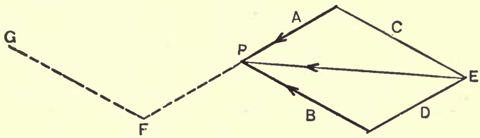


FIG. 104.—Parallelogram of Forces.

length as compared with *A* and *B*, the relative magnitude, of the required force.

That this is so may be seen by considering the two forces as acting separately upon *P*. Let *A* be considered as acting upon *P* to move it through a distance equal to its length. Then *P* would be moved to *F*. If the force *B* is now caused to act upon *P* to move it through a distance equal to its length, *P* will arrive at *G*. As *FP* has the same length and direction as *A*, and as *GF* has the same length and direction as *B*, the distance from *G* to *P* would be the same as the distance from *P* to *E*; therefore, *PE*, the diagonal of the parallelogram formed by the lines *A*, *B*, *C*, and *D*, represents the required new force or resultant.

If there are more than two forces acting upon the point *P*, first find a resultant of any two of the forces; then consider this resultant as replacing the first two, and find the resultant of it and another of the original forces; continue this process until a force is obtained which will be the resultant of all of the original forces. Thus, in Fig. 105, if *A*, *B* and *C* be considered as representing in direction and magnitude three forces which are acting simultaneously upon *P;* then, if we draw a parallelogram upon *A* and *B*, we have its diagonal *PD* as the resultant of *A* and *B*. A parallelogram is now drawn upon *PD* and *C*, giving *PE*, its diagonal, as the resultant of these two, and, consequently, of the three original forces.

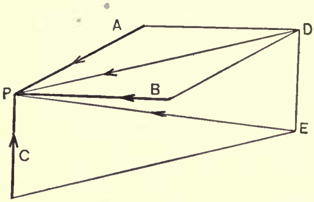


FIG. 105.—Resultant of Three Forces.

This principle holds true whether the original forces are acting in the same plane or not. Thus, in Fig. 106, let *A*, *B* and *C* be three forces acting simultaneously upon *P*. Then the resultant of *A* and *B* would be the diagonal *PD*. Considering this as replacing *A* and *B*, a resultant of it and *C* would be a diagonal drawn from *P* to the further corner *E; PE* would then be the resultant of *A*, *B* and *C*.

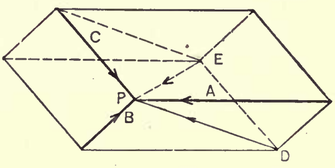


FIG. 106.—Resultant of Three Forces in Different Planes.

This operation may, of course, be reversed to allow of finding two or more forces in different directions which in magnitude shall be equivalent to a single known force. Thus in Fig. 107, if *PA* represents the direction and magnitude of a given force which it is desired to replace by two others acting in the direction of *PB* and *PC*, respectively, then draw a line from *A* to *PB* parallel with *PC*, and draw another from *A* to *PC* parallel with *PB*. The lengths *Pa* and *Pb* thus determined will represent the relative magnitudes, as compared with *PA*, of the required new forces.

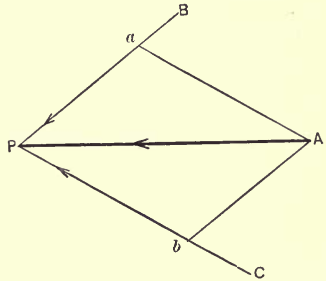


FIG. 107.—Resolution of Forces.

Parallel Forces.—Let *A* and *B*, Fig. 108, represent the direction and magnitude of two parallel forces acting together upon the bar *DE*. These two forces may be replaced or counterbalanced by a single force, equal in magnitude to *A* and *B* combined. To determine the point of application of this new force produce *A* to *a*, making *Da* equal in length to *B*. Also make *bE* equal in length to *A*. The intersection of the line connecting *a* and *b* with *DE*, at *F*, will be the required point of application. The lengths *DF* and *FE* will be inversely proportional to the forces *A* and *B*. That is, the length *FE* will be to the force *A* as the length *DF* is to the force *B*. The product of *DF* multiplied by *A* will be equal to the product of *FE* multiplied by *B*.

Fig. 109 shows how several parallel forces, acting in the same direction, may be replaced or counterbalanced by a single force. Let *A*, *B* and *C* represent the relative magnitudes of the forces. A resultant of *B* and *C* would be *D*, equal in

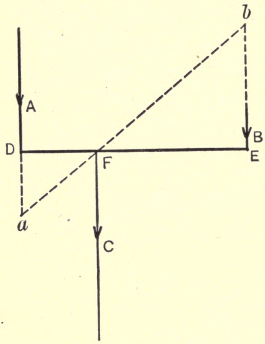


FIG. 108.—Parallel Forces.

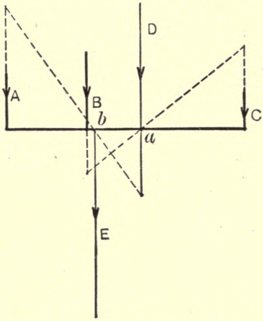


FIG. 109.—Resultant of Several Parallel Forces.

magnitude to *B* and *C* combined, and its point of application, determined in the manner previously described, would be at *a*. Regarding *D* as a single force replacing *B* and *C*, would give *E*, equal in magnitude to *A* and *D* combined, as the resultant of these two, and its point of application, determined as before, would be at *b*.

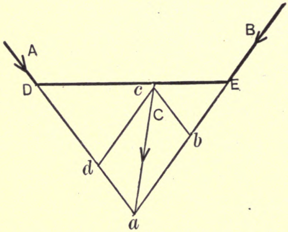


FIG. 110.—Oblique Forces Acting at Different Points on a Bar.

Oblique Forces.—Let *A* and *B*, Fig. 110, represent the directions and relative magnitudes of two forces acting simultaneously upon the bar *DE*. These two forces may be either replaced or counterbalanced by a single force, which in direction and magnitude shall be a resultant of them. Produce *A* and *B* until they meet at *a*. Draw the parallelogram *abcd*, making *da* equal to *A*, and *ba* equal to *B*. The diagonal of this parallelogram will give the direction and relative magnitude of the new force, and if extended its intersection with *DE* will give the point of application.

Opposing Forces.—Let *A* and *B*, Fig. 111, represent the directions and relative magnitudes of two forces acting upon opposite sides of the bar *DE*. These two forces may be replaced by a single force, which in direction and magnitude will be a resultant of them. Produce *A* and *B* until they meet at *a*. Lay off *ac* equal to the length of *B*, and make *bc* equal to and parallel with *A*. A line drawn from *a* to *b* will give the direction of the new force, and the length of *ab*, as compared with *A* and *B* will give its relative magnitude. Its application on bar *DE* may be determined by extending *ab* until it intersects *DE*.

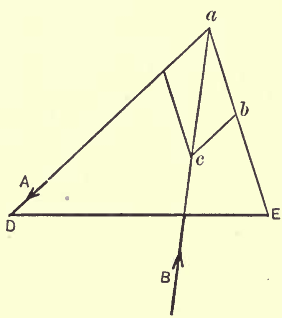


FIG. 111.—Opposing Oblique Forces.

Levers.—When a workman wishes to raise a heavy object, he may insert one end of a bar under it, and lift on the other end; or, pushing a block of wood or iron in under the bar as close to the object to be raised as he can, he presses down upon the free end of the bar. A bar so used constitutes a *lever*, and the point where the bar rests when the lever is doing its work, the end of the bar in under the heavy object in the first case, or the block on which the bar rests in the second case, is the *fulcrum* of the lever.

Levers are of three kinds, as shown in Fig. 112: First, where the fulcrum is between the power

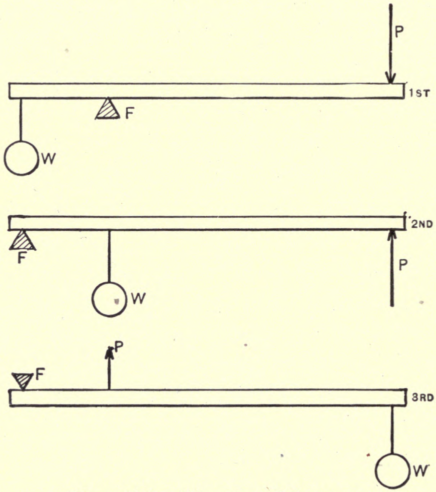


FIG. 112.—Classes of Levers.

and the weight; second, where the weight is between the fulcrum and the power; and, third, where the power is between the fulcrum and the weight. A man's forearm furnishes a good illustration of a lever of the third class, the fulcrum being at the elbow, the weight at the hand, and the muscle, being attached to the bone of the arm, at a short distance from the elbow, furnishing the power.

In all of these cases the gain in power is exactly proportional to the loss in speed, or the gain in speed is exactly proportional to the loss in power. Also, in every case the product of the weight multiplied by its distance from the fulcrum, will equal the product of the power multiplied by its distance from the fulcrum, or, the weight and power will balance each other when the weight multiplied by the distance through which it moves, equals the power multiplied by the distance through which it moves.

If in Fig. 108 the bar *DE* is a lever, the fulcrum will be at *F*, and the methods used in that figure and in Figs. 109, 110 and 111 give solutions of different lever problems.

The length of the lever arm is independent of the form of the lever. In Fig. 113 is shown a lever

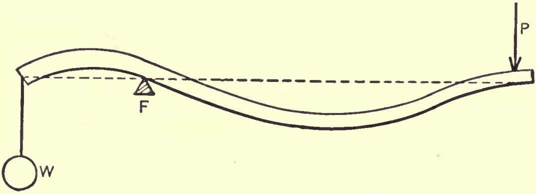


FIG. 113.—Lever of Curved Shape.

of curved shape; but the lever arms on which the calculation as to the work that the lever is doing, will be based, will be straight lines connecting the point where the power is applied, or the point which supports the weight, with the fulcrum.

The length of the lever arm is always at right angles to the direction in which the power is being applied, or to the direction of the resistance of the weight or load.

In Fig. 114 two cases are shown where the power is applied obliquely on the lever; but the lever arm on which the calculation is based will be the dis-

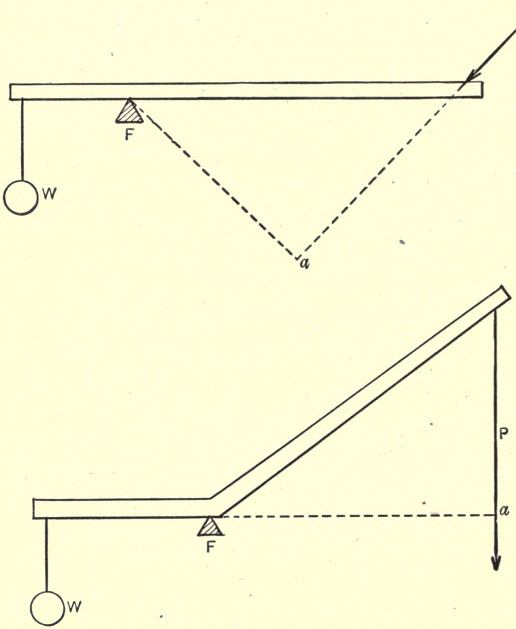


FIG. 114.—Power Applied Obliquely on Lever.

tance *Fa* measured from the fulcrum, at right angles to the direction of the power.

Compound Levers.—In Fig. 115 is shown a case where the power gained with one lever is further increased by the use of a second lever, acting on the first one. The weight and power will balance each other when the product of the weight and the lever arms *ab* and *ef*, multiplied together, equals the product of the power and the lever arms *gf* and *bc* multiplied together. Thus, to find the weight

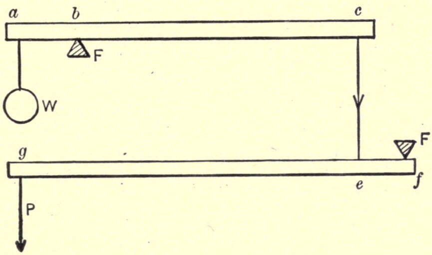


FIG. 115.—Compound Levers.

which a given power will lift, divide the product of the power and its lever arms *gf* and *bc*, multiplied together, by the product of the lever arms of the weight, *ab* and *ef*, multiplied together. To find

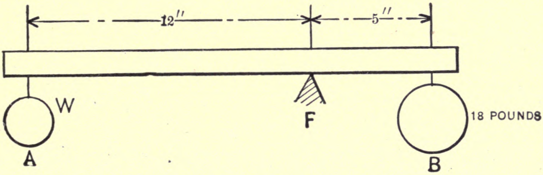


FIG. 116.—Diagram for Lever Problem.

the power necessary to lift a given weight, divide the product of the weight and its lever arms, *ab* and *ef*, multiplied together, by the product of the lever arms of the power, *gf* and *bc*, multiplied together.

A few examples will illustrate these principles. Assume that in Fig. 116 a weight at *A* must balance the 18-pound weight at *B*. The lever arms are given as 12 and 5 inches, respectively. How much must the weight *W* be, in order to balance the weight at *B*?

The weight at *B* (18 pounds) times its lever arm (5 inches) must equal the weight *W* times its lever arm (12 inches). In other words:

$$18 \times 5 = W \times 12.$$
$$90 = 12\ W.$$
$$W = \frac{90}{12} = 7\tfrac{1}{2} \text{ pounds.}$$

In Fig. 117, two weights, 4 and 2 pounds, respectively, are balanced by a weight *W*. Find what

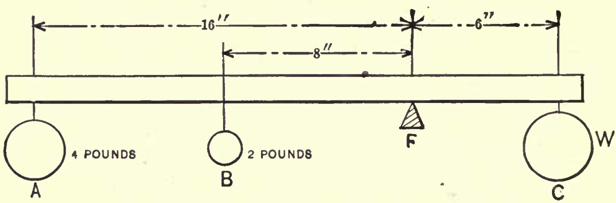


FIG. 117.—Diagram for Lever Problem.

the weight of *W* must be with the lever arms given in the engraving.

In this case the weight at *A* times its lever arm *plus* the weight at *B* times its lever arm, will equal weight *W* times its lever arm. The sum of the products of the weights and leverages of the weight at *A* and *B* is taken, because both these weights are on the same side of the fulcrum *F*.

Carrying out the calculation outlined above, we have:

$$4 \times 16 + 2 \times 8 = 6\ W.$$
$$64 + 16 = 80 = 6\ W.$$
$$W = \frac{80}{6} = 13\tfrac{1}{3} \text{ pounds.}$$

The product of a weight or force and its lever arm is commonly called the *moment* of the force. The moment of the force at A, for example, is 4 pounds × 16 inches = 64 *inch-pounds.* If the lever arm were 16 feet instead of 16 inches, the result would be 64 *foot-pounds.*

An interesting application of the lever, and the moments of forces, is presented in calculations of

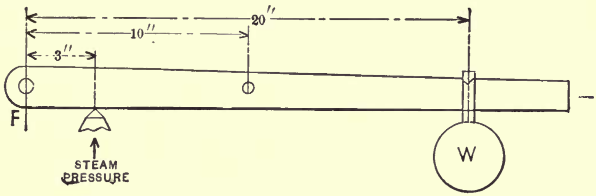


FIG. 118.—Diagram for Lever Problem.

weights for safety valves. A diagrammatical sketch of a safety valve lever is shown in Fig. 118. Assume that the total steam pressure, acting on the *whole* area of the safety valve, is 300 pounds when it is required that the steam should "blow off." Find the weight W required near the end of the lever to keep the valve down until the total pressure is 300 pounds on the valve. Assume the weight of the lever itself to be 6 pounds, concentrated at its center of gravity, 10 inches from the fulcrum F.

In this case we have that the moment of the steam pressure, which acts upward, should equal the sum of the moments of the weight of the lever and the weight W. Therefore:

$$300 \times 3 = 6 \times 10 + 20\ W.$$
$$900 = 60 + 20\ W.$$
$$900 - 60 = 20\ W.$$
$$840 = 20\ W.$$
$$W = \frac{840}{20} = 42 \text{ pounds.}$$

The calculation above has been carried out step by step, so that students unfamiliar with the algebraic solution of equations may be able to understand the principles involved in simple examples of this kind. In the following, the calculations have been carried out more directly, but the student should use the "step by step" method until thoroughly familiar with the subject.

Fixed and Movable Pulleys.—A fixed pulley is frequently used to change the direction of the power, as shown in Fig. 119, but there is no gain in power with such a pulley, as there is no compensating loss of speed; the weight will move upward at the same rate of speed as the power moves downward.

If now a movable pulley be used in connection with the fixed pulley as shown in Fig. 120, then as the end of the rope to which the power is applied is drawn downward, each of the two strands of rope between the pulleys will take half of the stress of the suspended weight, and the weight will be raised only one-half the distance that the power descends. The power will therefore need to be only one-half of the weight. In Fig. 121, there are three strands of rope between the pulleys, each of which will be equally shortened when the free end of the rope is pulled; the power, therefore, is only one-third of the weight. In Fig. 122, with

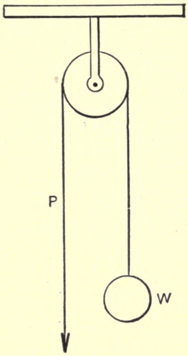


FIG. 119.—Fixed Pulley.

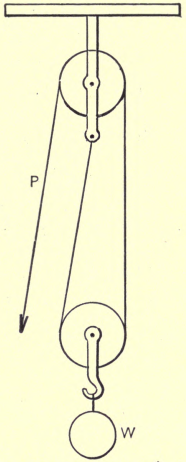


FIG. 120.—Fixed and Movable Pulleys.

four strands of rope between the pulleys, each furnishing an equal amount to the free end as it is drawn out, the power need be only one-fourth of the weight.

The law of the pulley, then, where a single rope is employed, is that the power will be increased as many times as there are lines of rope between the pulleys to participate in the shortening. In a system using more than one rope, as shown in Fig.

123, each additional movable pulley doubles the power, as it will move at only half the rate of the preceding pulley.

Differential Pulleys.—Another form of pulley, known as the differential pulley, much used in machine shops, is shown in Fig. 124. In this form of

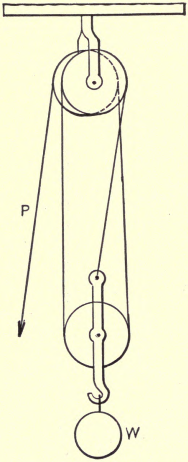


FIG. 121.—Tackle where Load is Taken on Three Strands of Rope.

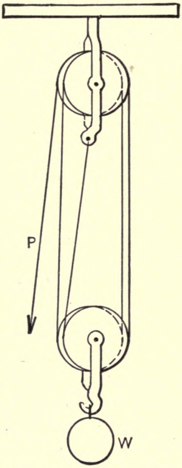


FIG. 122.—Tackle where Load is Taken on Four Strands of Rope.

pulley an endless chain replaces the rope, the pulleys themselves being grooved and toothed like sprocket wheels. The two pulleys at the top are of slightly different diameters, but rotate together as one piece. In operation, as the chain is drawn up by the large wheel it passes around in a loop to the small wheel from which it is unwound, causing the loop in which the movable pulley rests to be shortened by an amount equal to the difference in the pitch circumferences of the two upper wheels, when they have made one revolution. This would cause the weight to be raised one-half of that amount. If in a given case the two upper pulleys had respectively 20 and 19 teeth, then as the ap-

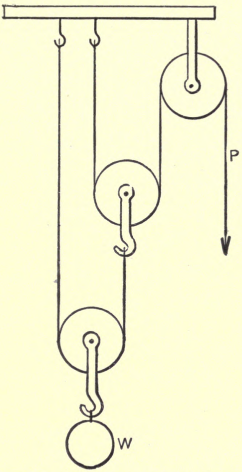


FIG. 123.—A Special Arrangement of Movable Pulleys.

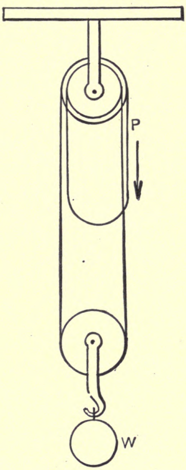


FIG. 124.—Differential Pulley.

plied power was being moved through a distance of 20 inches the small pulley would unwind 19 inches of the chain, causing a shortening of the loop in which the movable pulley rests of one inch, which would raise the weight one-half of an inch, giving a ratio of load to power of 40 to 1.

In all of these cases the results actually attained in practice will be somewhat modified from the theoretical results given by calculations, by the losses occasioned by friction.

Inclined Planes.—In raising heavy weights through short distances, as for instance in loading barrels onto wagons, a plank may be used to facilitate the work by placing one end of it on the ground and the other end on the wagon, and rolling the barrel up the plank onto the wagon. Such an arrangement is called an *inclined plane.* When the force which is being applied to the rolling

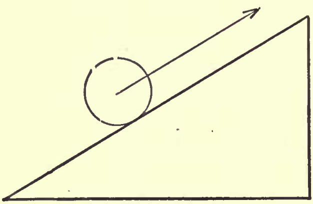


FIG. 125.—Inclined Plane.

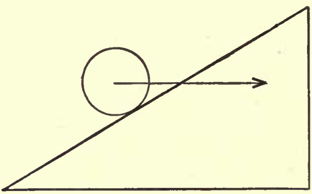


FIG. 126.—Power Applied Parallel to Base.

object is exerted in a direction parallel to the inclined surface, as in Fig. 125, it is evident that the power must move through a distance equal to the length of the incline in order to raise the weight the desired height. The gain in power will then be equal to the length of the incline divided by the height.

If the power is applied in a direction parallel with the base, as in Fig. 126, the power will have to advance through a distance equal to the length of the base to raise the object the desired height. The gain in power will then be equal to the base divided by the height. By considering Fig. 126

further, it will be seen that in rolling the object up the incline the power will have to advance from the beginning of the incline to a point from which a line may be drawn perpendicular to its direction to the top of the incline. In any case where the power is applied in any direction other than parallel with the incline, in rolling the object to the top, the power will have to advance to a point from which a line may be drawn perpendicularly to its direction to the top of the

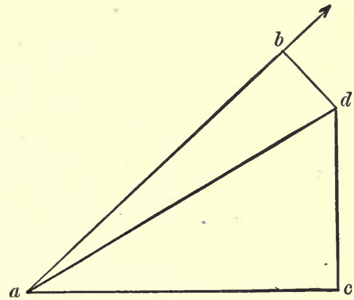


FIG. 127.—Power Applied Obliquely to Surface of Incline.

incline. In Figs. 127 and 128 are shown two other cases where the power is applied in a direction obliquely to the surface of the incline. In either of these cases, as in the other two cases, the gain in power will be found by dividing the distance through which the force moves, *ab*, by the distance through which the object is raised, *cd*.

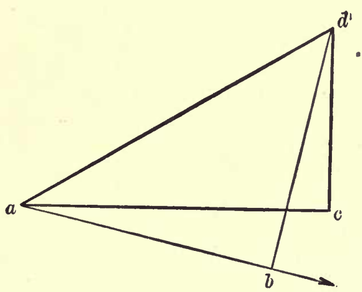


FIG. 128.—Another Case where Power is Applied Obliquely to Surface of Incline.

It will be further seen that the gain in power is greatest when the direction in which the force is being applied is parallel with the incline. When the direction of the force is upward from the incline, as in Fig. 127, part of the force is expended in lifting the weight off from the incline, until, when its direction is made vertical, it is all expended in this way. When the direction of the force is downward from the incline, as in Figs. 126 and 128, part of it is lost in pressing the object against the incline.

The Screw.—The screw is a modified form of inclined plane, the lead of the screw, the distance

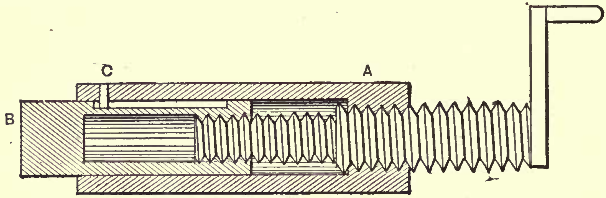


FIG. 129.—Differential Screw.

that the thread advances in going around the screw once, being the height of the incline, and the distance around the screw, measured on the thread, being the length of the incline.

The Differential Screw.—The differential screw is a compound screw having a coarse thread part of its length, and a somewhat finer thread the rest of its length, the object being to get a slow motion combined with the strength of a coarse thread. Fig. 129 shows such a screw. The piece *A* is a fixed part of some machine. The piston *B* slides within *A*, being prevented from turning by the pin *C* which enters a groove in *B*. If that part of the screw which engages in *A* has eight threads to the inch, and that part of it which engages in *B* has ten threads, then when the screw makes one revolution, it will advance into *A* one-eighth of an inch, and into *B* one-tenth of an inch; the piston *B* will therefore advance through a distance equal to the difference between one-eighth of an inch and one-tenth of an inch, or twenty-five one-thousandths of an inch, requiring forty turns of the screw to make the piston advance one inch.

Newton's Laws of Motion.—The relation which exists between force and motion is stated by the three fundamental laws of motion formulated by Newton.

Newton's first law says that if a body is at rest it will remain at rest, or, if it is in motion, it will continue to move at a uniform velocity in a straight line, until acted upon by some force and compelled to change its state of rest or of straight-line uniform motion. In a general way, this law is self-evident, and based on daily experience. However, the part of the law stating that a body in motion will continue indefinitely to move if not acted upon by resisting forces, may not be so self-evident; yet whenever a body is brought to a standstill after it has been in motion, such forces as frictional resistance, gravity, etc., always have in some way influenced the motion of the body.

Newton's second law of motion says that a change in the motion of a body is proportional to the force causing the change, and takes place in the direction in which the force acts. If several forces act on a body, the change is proportional to the *resultant* of the several forces, and takes place in the direction of the resultant. This has been clearly explained in the previous pages, in connection with the resolution and composition of forces. The most important point to note in regard to the second law of motion is that when two or more forces act on a body at the same time, each causes a motion exactly the same as if it acted alone; each force produces its effect independently, but the total effect on the motion of the body, of course, is a combination of all these independent motions.

Newton's third law says that for every action there is an equal reaction. This means that if a force or weight presses downward on a support with a certain pressure, the reaction, or resistance in the support, must equal the same pressure. If a bullet is shot from a rifle with a certain force, there is a reaction, or "recoil," in the rifle, equal to the force required to give the velocity to the bullet. This law is very important, and many failures in machine design have been due to ignorance of the real meaning of the law of action and reaction.

Newton's third law may be illustrated by a locomotive drawing a train of cars. The driving wheels give as much of a backward push on the rails as there is of forward pull exerted on the train; and it is only because the rails are held in place by their fastenings, and by the weight resting on them, that the locomotive is able to pull the train forward. This principle of action and reaction being equal and opposite is also an effectual bar to any perpetual-motion machine, as such a machine in order to work would have to produce a greater action in one direction than the reaction in the other direction.

The Pendulum.—A body or weight suspended from a fixed point by a string or rod, and free to oscillate back and forth is called a *pendulum. The center of oscillation* is the point which, if all of the material composing the pendulum, including the sustaining string or rod, were concentrated at it (the material so concentrated being considered as being suspended by a line of no weight) would vibrate in the same time as the actual pendulum. The length of the pendulum is the length from the point of suspension to the center of oscillation.

When the length of the pendulum is unchanged, its time of vibration will be the same, if its angle of vibration does not exceed three or four degrees, and its time of vibration will be but slightly increased for larger angles.

The time of vibration of a pendulum is not affected by the material of which it is made, whether light or heavy, except as the light material will offer greater resistance to the air, by presenting a greater surface in proportion to its weight, than a heavy material.

The time of vibration of a pendulum of a given length is inversely as the square root of the intensity of gravity. As the intensity of gravity decreases with the distance from the center of the earth it follows that a pendulum will vibrate faster at the poles or at sea level than it will at the equator or at an elevation.

The time of vibration of a pendulum varies directly as the square root of its length. That is, a pendulum to vibrate in one-half or one-third the time of a given pendulum will need to be only one-quarter or one-ninth of its length.

Example 1.—A pendulum in the latitude of New York will require to be 39.1017 inches long to beat seconds. Required the length of a pendulum to make 100 beats per minute.

A pendulum to make 100 beats per minute will have to make its vibrations in 60-100 of the time of one which is making 60 beats per minute, and its length will be equal to the length of one which beats seconds, multiplied by the square of 60-100, or:

$$\frac{39.1017 \times 60^2}{100^2} = \frac{39.1017 \times 3600}{10{,}000} = 14.076 \text{ inches.}$$

Example 2.—Required the time of vibration of a pendulum 120 inches long. Letting x represent the required time, we have the proportion $\sqrt{120} : \sqrt{39.1017} = x : 1$, or $10.954 : 6.253 = x : 1$.

$$x = \frac{10.954}{6.253} = 1.75 \text{ second.}$$

A short pendulum may be made to vibrate as slowly as desired by having a second "bob" placed above the point of suspension, which will partially counteract the weight of the lower bob.

Falling Bodies.—A falling body will have acquired a velocity at the end of the first second of 32.16 feet per second, under ordinary conditions. If the body is of such shape or material as to present a large surface to the air in proportion to its weight, its velocity will, of course, be lessened, and as its velocity depends upon the force of gravity, its velocity will be affected somewhat by the latitude of the place, and its distance above sea level. During the next second it will acquire 32.16 feet additional velocity, giving it a velocity of 64.32 feet at the end of the second second. Each succeeding second will add 32.16 feet to the velocity the body had at the end of the preceding second.

To find the velocity of a falling body at the end of any number of seconds, therefore, multiply the number of seconds during which the body has fallen by 32.16. This rule, expressed as a formula, would be:

$$v = 32.16 \times t$$

in which v = velocity in feet per second, t = time in seconds.

The acceleration due to gravity, 32.16 feet, is often, in formulas, designated by the letter g. As an example, find the velocity of a falling body at the end of the twelfth second:

$$v = 32.16 \times 12 = 385.92 \text{ feet.}$$

As the body falling starts from a state of rest, its average velocity will be one-half of its final velocity; the distance through which it falls equals the average velocity multiplied by the number of seconds during which it has been falling. This rule, expressed as a formula, is:

$$h = \frac{v}{2} \times t$$

in which h = distance or height through which body falls, and v and t have the significance given above. But $v = 36.16 \times t$; if this value of v is inserted in the formula just given, we have:

$$h = \frac{32.16 \times t \times t}{2} = 16.08\, t^2.$$

This last formula, expressed in words, gives us the rule that the distance through which a body falls in a given time equals the square of the number of seconds during which the body has fallen, multiplied by 16.08.

How long a distance will a body fall in 10 seconds? Inserting $t = 10$ in the formula, we have:

$$h = 16.08\, t^2 = 16.08 \times 10^2 = 16.08 \times 100 = 1608 \text{ feet.}$$

The time, in seconds, required for a body to fall a given distance equals the square root of the distance, expressed in feet, divided by 4.01. Expressed as a formula, this rule would be:

$$t = \frac{\sqrt{h}}{4.01}.$$

As an example, assume that a stone falls through a distance of 3600 feet. How long time is required for this?

Inserting $h = 3600$ in the formula, we have:

$$t = \frac{\sqrt{3600}}{4.01} = \frac{60}{4.01} = 15 \text{ seconds, very nearly.}$$

The velocity of a falling body after it has fallen through a given distance equals the square root of the distance through which it has fallen multiplied by 8.02.

This rule, expressed as a formula, is:

$$v = 8.02\sqrt{h}.$$

What is the velocity of a falling body after it has fallen through a distance of 3600 feet?

Inserting $h = 3600$ in the formula, we have:

$$v = 8.02 \times \sqrt{3600} = 8.02 \times 60 = 481.2 \text{ feet.}$$

The height from which a body must fall to acquire a given velocity equals the square of the velocity divided by 64.32. As a formula, this rule is:

$$h = \frac{v^2}{64.32}$$

From what height must a body fall to acquire a velocity of 500 feet per second? Inserting $v = 500$ in the formula given, we have:

$$h = \frac{500^2}{64.32} = \frac{500 \times 500}{64.32} = 3887 \text{ feet.}$$

If a body is thrown upward with a given velocity, its velocity will diminish during each second at the same rate as it increases when the body falls. A body thrown up into the air in a vertical direction will return to the ground with exactly the same velocity as that with which it was thrown into the air. At any point, the velocity on the upward journey will be equal to the velocity on the downward journey, except that the direction is reversed.

The acceleration of a falling body, 32.16 feet per second, is the value at the latitude of New York, at sea level.

The force required to give to a falling body its acceleration of 32.16 feet per second is the weight of the body itself. The force required to give any acceleration to a body, then, is to the weight of the body as that acceleration is to the acceleration produced by gravity. Therefore, to find the force required to produce a given rate of acceleration to a body, divide the weight of the body by 32.16, and multiply the quotient by the required rate of acceleration.

Example.—A body weighing 125 pounds is to be lifted with an acceleration of 10 feet per second. Required the strain on the sustaining rope.

$\frac{125}{32.16} \times 10 = 38.8$, the tension necessary to produce the acceleration.

To this must be added the pull necessary to lift the weight without acceleration, or the weight of the body itself. Thus $38.8 + 125 = 163.8$ is the required tension on the rope.

The rate of acceleration which a continuously acting force will produce is equal to the force divided by the weight of the body, multiplied by 32.16.

Energy and Work.—The unit of work, the standard by which work is measured, is the *foot-pound*, or the amount of work done in lifting a weight or overcoming a resistance of one pound through one foot of space.

"Energy is the product of a force factor and a space factor. Energy per unit of time, or *rate of doing work*, is the product of a force factor and a velocity factor, since velocity is space per unit of time. Either factor may be changed at the expense of the other; *i.e.*, velocity may be changed, if accompanied by such a change of force that the energy per unit of time remains constant. Correspondingly force may be changed at the expense of velocity, energy per unit of time being constant. *Example.*—A belt transmits 6000 foot-pounds per minute to a machine. The belt velocity is 120 feet per minute, and the force exerted is 50 pounds. Frictional resistance is neglected. A cutting tool in the machine does useful work; its velocity is 20 feet per minute, and the resistance to cutting is 300 pounds. Then the energy received per minute $= 120 \times 50 = 6000$ foot-pounds; and energy delivered per minute $= 20 \times 300 = 6000$ foot-pounds. The energy received therefore equals the energy delivered. But the velocity and force factors are quite different in the two cases." (Prof. A. W. Smith.)

Force of the Blow of a Steam Hammer or Other Falling Weight.—The question, "With what force does a falling hammer strike?" is often asked. This question can, however, not be answered directly. The energy of a falling body cannot be expressed in pounds, simply, but must be expressed in foot-pounds. The energy equals the weight of the falling body multiplied by the distance through which it falls, or, expressed as a formula:

$$E = W \times h,$$

in which E = energy in foot-pounds,
W = weight of falling body in pounds,
h = height from which body falls in feet.

The energy can also be found by dividing the weight of the falling body by 64.32 and then multiplying the quotient by the square of the velocity at the end of the distance through which it falls. This rule, expressed as a formula, is:

$$E = \frac{W}{64.32} \times v^2$$

in which E and W denote the same quantities as before, and v = the velocity of the body at the end of its fall.

Both of these formulas give, of course, the same results. That the second method gives the same result as multiplying the weight by the height through which it falls, is evident from the fact, stated under the head of "Falling Bodies," that the square of the velocity of a falling body, divided by 64.32, gives the height through which it has fallen.

This second method allows of determining the energy of any weight or force moving at a given velocity, whether its velocity has been acquired by falling, or is due to other causes.

Now assume that we wish to find the force of the blow of a 300-pound drop hammer, falling 2 feet before striking the forging, and compressing it 2 inches.

The energy of the falling hammer when reaching the forging is:

$$E = W \times h = 300 \times 2 = 600 \text{ foot-pounds.}$$

This energy is used during the act of compressing the forging 2 inches or 0.166 of a foot. Consequently, the average force of the hammer with which it compresses the forging is 600 ÷ 0.166 + the weight of the hammer, or

$$\text{Average force of blow} = \frac{600}{0.166} + 300 =$$
$$3600 + 300 = 3900 \text{ pounds.}$$

The general formula for the force of a blow is:

$$F = \frac{W \times h}{d} + W$$

in which F = average force of blow in pounds,
W = weight of hammer in pounds,
h = height of drop of hammer in feet,
d = penetration of blow in feet.

A *horse-power*, in mechanics, is the power exerted, or work done, in lifting a weight of 33,000 pounds one foot per minute, or 550 pounds one foot per second. The power exerted by a piston driven by steam or other medium during one stroke, in foot-pounds, is equal to the area of the piston, multiplied by the pressure per square inch, multiplied by the stroke in feet, the product of the area by the pressure giving the force, and the stroke giving the distance through which the force is exerted. In the case of steam engines, where the steam is cut off at one-quarter, one-third or one-half of the stroke, the piston being driven the rest of the way by the expansion of the steam, the average pressure for the entire stroke, the "mean effective pressure" (M.E.P.), as it is called, is the basis of calculations. As each revolution of the engine equals two strokes of the piston, the number of foot-pounds per minute an engine is developing will be the product of the area of the piston in square inches, multiplied by the mean effective pressure, multiplied by the stroke in feet, multiplied by the number of revolutions per minute times 2. This product, divided by 33,000, gives the *indicated* horse-power (I.H.P.) of the engine; this name being derived from the fact that the mean effective pressure is determined by the use of the steam engine indicator. Therefore:

$$I.H.P. = \frac{\text{Area} \times M.E.P. \times \text{stroke} \times \text{rev. per min.} \times 2}{33,000}.$$

This formula may be transposed in various ways to give other information. For instance, if the piston area for a given horse-power is desired, then

$$\text{Area} = \frac{I.H.P. \times 33,000}{M.E.P. \times \text{stroke} \times \text{rev. per min.} \times 2.}$$

If the volume of the cylinder is desired, then

$$\text{Area} \times \text{stroke} = \frac{I.H.P. \times 33000}{M.E.P. \times \text{rev. per min.} \times 2.}$$

If the pressure to produce a given horse-power is desired, then

$$M.E.P. = \frac{I.H.P. \times 33000}{\text{Area} \times \text{stroke} \times \text{rev. per min.} \times 2.}$$

The mean effective pressure in the cylinder of the engine is, of course, considerably less than the boiler pressure as shown by the steam gauge. The indicated horse-power of an engine does not take into account the losses caused by the friction of the working parts. The power which the engine actually delivers as shown by a brake dynamometer or other contrivance at the flywheel is called the *brake horse-power*.

CHAPTER IX

FIRST PRINCIPLES OF STRENGTH OF MATERIALS

Factor of Safety.—It is obvious that it would be unsafe in designing a piece of construction work to allow a strain of anywhere near the breaking limit of the material it is to be made from. It is, therefore, customary in making any calculations for the size of the parts to use what is called a *factor of safety*, by making the part from three or four to ten or even more times the strength necessary to just resist breaking with a steady load. The factor of safety used will depend upon several considerations. It will depend, first, upon the nature of the material used. A wrought or drawn metal, for instance, will be likely to be more uniform in its nature than a cast metal which may contain air holes, or which may be more or less spongy, or which may be under unequal strains in cooling. The matter of strains in a casting due to unequal cooling is to a considerable extent a matter of proper or improper design; still it is not possible to entirely avoid them.

Again the factor of safety to be used will depend upon the nature of the work which will be required of the part. If the part has to simply sustain a steady load it will not need to be as strong as though the load was applied and reversed, or even as strong as though the load was applied and released. To illustrate, it is a familiar fact that a piece of wire which may be bent a given amount without apparent injury, may be broken by repeatedly bending it back and forth the same amount at one point. And, similarly, in machine parts, rupture may be caused not only by a steady load which exceeds the carrying strength, but by repeated applications of stresses none of which are equal to the carrying strength. Rupture may also be caused by a succession of shocks or impacts, none of which alone would be sufficient to cause it. Iron axles, the piston rods of steam hammers and other pieces of metal subjected to repeated shocks, invariably break after a certain length of service.

The factor of safety used will therefore vary widely with the nature of the work required of the part. For a steady or "dead" load, Prof. A. W. Smith says: "In exceptional cases where the stresses permit of accurate calculation, and the material is of proven high grade and positively known strength, the factor of safety has been given as low a value as $1\frac{1}{2}$; but values of 2 and 3 are ordinarily used for iron or steel free from welds; while 4 to 5 are as small as should be used for cast iron on account of the uncertainty of its composition, the danger of sponginess of structure, and indeterminate shrinkage stresses." Others would make 3 the lowest factor of safety that should be used for wrought iron and steel.

Where the load is variable, but well within the elastic limit of the material, that is where the load is not so great but so that the part will immediately resume its original shape when the load is removed, a factor of safety of 5 or 6 might be used. The part will need to be made stronger if the load or force acts first in one direction and then in the opposite direction, that is, if it acts back and forth, than it will need to be if the same force is simply applied and then released. Where the part is subjected to shock, the factor of safety is generally made not less than 10. A factor of safety as high as 40 has been used for shafts in mill-work which transmit very variable powers.

In cases where the forces are of such a nature that they cannot be determined, then Prof. Smith says: "Appeal must be made to the precedent of successful practice, or to the judgment of some experienced man until one's own judgment becomes trustworthy by experience. * * * In proportioning machine parts, the designer must always be sure that the stress which is the basis of calculation or the estimate, is the maximum possible stress; otherwise the part will be incorrectly proportioned." And he cites the case of a pulley where if the arms were to be designed only to resist the belt tension they would be absurdly small, because the stresses resulting from the shrinkage of the casting in cooling are often far greater than those due to the belt pull.

In many cases the practical question of feasibility of casting will determine the thickness of parts, independent of the question of strength. For instance, on small brass work, such as plumbers' supply, and small valve work, a thickness of about 3-32 of an inch is as little as can be relied on to make a good casting on cored out work; or in the case of partitions in such work where the metal has to flow in between cores, a thickness of about $\frac{1}{8}$ of an inch is as small as should be used; yet such thicknesses may be much greater than are required to give the necessary strength. On larger cast iron work, the thickness to be allowed to insure a good casting will, of course, depend upon the size of the piece. The judgment of the pattern-maker or foundry-man will naturally determine the thickness in such cases.

Shape of Machine Parts.—While the size of machine parts will vary greatly with the nature of the work required of them, their shape will depend very much on the manner or direction in which the load or strain is brought to bear upon them. If the part is subjected to simple tension, that is, merely resists a force tending to pull it apart, then the shape of the member which serves this purpose is not very material, though a round rod, being most compact and cheapest, is best. Almost any shape will answer, however, though it is well to avoid using thin and broad parts, as a strain, though not greater than that which the part as a whole might bear safely, might be brought upon one edge, producing a tearing effect beyond the safe limit. For resisting simple tension the part should be made of uniform size its entire length, of a size to be determined by the tensile strength of the material and the factor of safety used.

If the part is to resist *compression*, then when the proportion of its length to its diameter or thickness is such that it will "buckle" or bend, instead of crushing, that is when its length exceeds five or six times its diameter, it becomes desirable to use a hollow or cross-ribbed form of construction, so as to get the metal as far from the axis of the piece as possible. The hollow cylindrical form, by getting all of the metal equally distant from the axis is, of course, most effective, but considerations of appearance may make a hollow square form more desirable, while considerations of cost may make a cross-ribbed form to be preferred, as such a form can be cast without the use of cores. In cases where a wrought metal must be used a solid form is often the only practicable one. When it becomes important to keep the weight down to the lowest point, it is common to have the piece slightly enlarged in the middle of its length, as in the case of connecting rods of steam engines. In the case of steam engine connecting-rods, the tendency to buckle is least sideways, as the cross-head and crank-pins tend to hold it in line this way, while the rotary motion of the crank-pin tends to produce buckling the other way. Connecting rods are therefore frequently made somewhat flat, of a breadth about twice their thickness.

When a piece is designed to resist *bending*, it becomes desirable to get a good depth of material in the direction in which the force is applied, as the capacity of a piece to resist bending increases as the square of its thickness or depth in the direction of the force, but only directly as its breadth or width, so that to increase the thickness of a piece two or three times in the direction of the force would increase its capacity to resist bending four or nine times; while to increase its breadth two or three times would only increase its strength two or three times. The proportion of depth to breadth which can be used will, of course, depend upon the length of the piece, as if the piece is long and its depth is made large in proportion to its thickness the tendency will be for the piece to buckle, or yield sideways. To resist this tendency it is customary to put ribs on the edges of such a

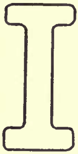


FIG. 130.

FIG. 131.

FIGS. 130 and 131.—Beam Cross-sections of Different Types.

piece, giving it the form shown in Fig. 130. The hollow box-form shown in Fig. 131 is, of course, equally effective to resist combined bending and buckling stresses, and in some cases may be preferable as a matter of appearance on account of the impression of solidity which it gives.

A projecting beam, like that shown in Fig. 132, designed to resist a force or sustain a load at its end, would need to have its lower edge made of the form of a parabola, if made of uniform thickness. If the edges were ribbed to prevent buckling, then material might be taken out of the middle portion, as shown in Fig. 133, without weakening it.

Strength of Materials as Given by Kirkaldy's Tests.—A very large number of tests of cast iron made by Kirkaldy gave results as follows: Tensile strength per square inch, necessary to just tear asunder, from about 10,000 or 12,000 pounds to about 28,000 or 32,000 pounds, or an average strength of about 20,000 pounds. Tests on the ability of cast iron to resist crushing gave results varying from about 50,000 to about 150,000 pounds,

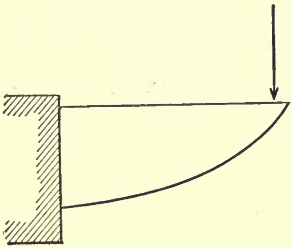


FIG. 132.—Cantilever of Uniform Strength, when Loaded at End.

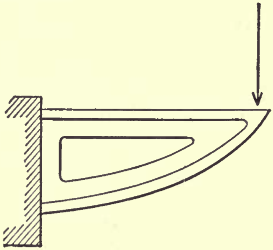


FIG. 133.—Common Design of Cantilever of Uniform Strength.

or an average strength of about 100,000 pounds per square inch. These tests indicate that cast iron has about five times the capacity to resist crushing that it has to resist tension. They also indicate that cast iron is a somewhat uncertain material.

Tests of wrought iron indicated a tensile strength of between 40,000 and 50,000 pounds per square inch, the elastic limit being reached at about one-half the tensile strength. Tests on steel castings gave results for tensile strength ranging from 55,000 to about 64,000 pounds per square inch, the elastic limit being reached at about 30,000 pounds.

Tests of wire gave results as follows: Brass, from 81,000 to 98,000 pounds per square inch of area. Iron, from 59,000 to 97,000 pounds. Steel, from 103,000 to 318,000 pounds.

The tensile strength of regular machine steel (low carbon steel) is generally given at about 60,000 pounds per square inch.

Size of Parts to Resist Stresses.—To resist tension it is, of course, only necessary to have the piece of such a size that each square inch shall not have a stress greater than the average strength of the material (as 20,000 pounds for cast iron) divided by whatever factor of safety may be selected.

To Resist Crushing.—Prof. Hodgkinson's rule for the strength of hollow cast iron pillars is as follows: To ascertain the crushing weight in *tons* multiply the outside diameter by 3.55; from this subtract the product of the inside diameter multiplied by 3.55, and divide by the length multiplied by 1.7. Multiply this quotient by 46.65. Expressed as a formula this rule would be:

$$S_c = 46.65 \times \frac{(D \times 3.55) - (d \times 3.55)}{L \times 1.7}$$

in which

S_c = ultimate compressive (crushing) strength of hollow column, in tons,

D = outside diameter in inches,

d = inside diameter in inches,

L = length of column in feet.

Any desired factor of safety may be introduced in the above formula by dividing the factor 46.65 by the factor of safety. In this case the formula would be:

$$S_s = \frac{46.65 \times [(D \times 3.55) - (d \times 3.55)]}{F \times L \times 1.7}$$

in which

S_s = safe compressive strength in tons,
F = factor of safety, and
D, d and L have the same meaning as above.

This rule and formula assumes that the ends of the column are perfectly flat and square, and that the load bears evenly on the whole surface.

If the ends are rounded, the column yields at about one-half the stress of one with fixed square ends.

To Resist Bending.—In the following commonly given rules for the strength of beams or bars to

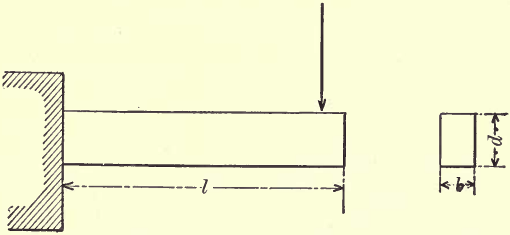


FIG. 134.—Rectangular Cantilever.

resist breaking by transverse stresses, the tensile strength of cast iron is assumed at 20,000 pounds per square inch. Divide 20,000 in the formulas by the desired factor of safety. The breadth and depth of rectangular bars, the diameter, if the bar is round, and the length, are all in inches.

For rectangular bars fixed at one end with the force applied at the other, Fig. 134, the breaking load equals

$$\frac{1}{6} \times \frac{b \times d^2 \times 20{,}000}{l}.$$

For round bars under the same conditions, Fig. 135, the breaking load equals

$$\frac{1}{6} \times \frac{0.59 \times d^3 \times 20{,}000}{l}.$$

If the rectangular bar is hollow, as shown in

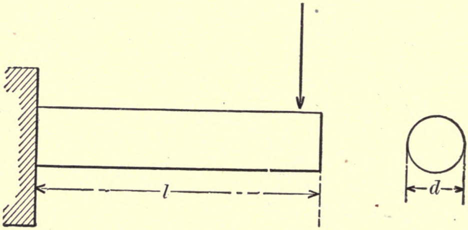


FIG. 135.—Circular Section Cantilever.

Fig. 131, subtract the internal $b \times d^2$ from the external $b \times d^2$.

If the round bar is hollow subtract the internal d^3 from the external d^3.

The case of a bar of the I-section shown in Fig. 130 is similar to that of the hollow rectangular bar of Fig. 131, the depressions in its sides corresponding to the hollow part of Fig. 131, the sum of their depths corresponding with the internal width *b* of the hollow rectangular bar.

If a beam is fixed at one end and the load is evenly distributed throughout its entire length, it will bear double the weight it will if the load is supported at the outer end.

If the beam is supported at the ends and loaded in the middle it will bear four times the weight of the beam of Fig. 134, or, if the load is evenly distributed throughout the length of the beam, eight times.

If the beam, instead of being simply supported at the ends, has the ends fixed and is loaded at the center, its ability to resist breaking will be doubled as compared with that when loaded at the center and with the ends only supported.

Regarding the safe load that beams or bars of different material may bear Griffin says that "with but a general knowledge of the elastic limit, ordinary steel is good for from between 12,000 to 15,000 pounds per square inch non-reversing stress, and from 8000 to 10,000 pounds reversing stress. Cast iron is such an uncertain metal, on account of its variable structure, that stresses are always kept low, say from 3000 to 4000 for non-reversing stress, and 1500 to 2500 for reversing stress."

Again, though the tests of wrought iron show it to have a much higher tensile strength than cast iron, Nyström, in formulas for lateral strength, gives wrought iron but little more than three-quarters the value of cast iron, probably because it bends so readily.

A table is appended giving the average breaking strength, in pounds per square inch, of some commonly used materials in engineering practice.

	Tension.	Compression.
Aluminum	15,000	12,000
Brass, cast	24,000	30,000
Copper, cast	24,000	40,000
Iron, cast	15,000	80,000
Iron, wrought	48,000	46,000
Steel castings	70,000	70,000
Structural steel	60,000	60,000

Stresses in Castings.—Reference has been previously made to stresses in castings, due to shrinkage in cooling. If all parts of a casting could be made to cool equally fast there would not be much trouble in this respect, but as different parts of a casting vary in thickness, the time they require to cool will vary, and the thick parts remaining fluid the longest, will, on cooling, cause a strain on the already cool thin parts. In the case of a pulley, where the rim and arms are much lighter than the hub, the hub on cooling will tend to draw the arms to itself and away from the rim, and if the difference in thickness is great, they may be even found to be pulled away so as to show a crack where they join the rim. The remedy in such a case would, of course, be first, to take out as much of the metal from the center of the hub as possible by means of a core, and second, to keep the outside of the hub as small as would be consistent with strength, getting necessary thickness for set screws by having a raised place or boss at that point.

As these strains are primarily due to unequal cooling, it is evident that in order to reduce them to the lowest point the first thing to do is to make the different parts of the casting of as nearly uniform thickness as possible. Where different parts of the casting vary in thickness, the change from one thickness to the other should be made as gradual as possible. Sharp internal corners should also be avoided, as such places are very liable to be spongy; the sand from the sharp corner in the mould is also very liable to wash away when the metal is poured in, and lodge in some other place, causing a defective casting. A good "fillet," as an internal round corner is called, which the pattern-maker may put into the pattern with wax, putty or leather, will not be very expensive, and will save much trouble in the casting.

Besides possessing a knowledge of factors of safety, proportioning parts to resist various stresses and the like, a general knowledge of the principles of foundry and machine shop practice is essential to properly design machine work. If one does not understand foundry work, he will be constantly designing castings which it will be impracticable to mould; if not actually impossible of moulding, they will be needlessly expensive. And in like manner, unless he understands the general principles of machine shop practice, his work will be giving trouble at that end of the line.

CHAPTER X

CAMS

General Principles.—In designing machinery it is frequently desirable to give to some part of the mechanism an irregular motion. This is often done by the use of *cams*, which are made of such form that when they receive motion, either rotary or reciprocating, they impart to a follower the desired irregular motion.

The follower is sometimes flat, and sometimes round. When the follower is round it is usually made in the form of a wheel or roller, so as to lessen the wear and the friction. The follower may work upon the edge of the cam, or if round, it may work in a groove formed either on the face or on the side of the cam.

The working surfaces of cams with round followers are laid out from a *pitch line*, so called, which passes through the center of the follower. The shape of this pitch line determines the work which the cam will do. The working surface of the cam is at a distance from the follower equal to one-half the diameter of the follower. This principle of a pitch line holds good whether the cam works only upon its edge like the one shown in Fig. 139, or whether it has an outer portion to insure the positive return of the follower. This outer portion is frequently made in the form of a rim of uniform thickness around the groove.

Design a Cam Having a Straight Follower Which Moves Toward or From the Axis of the Cam, as Shown in Fig. 136.—Let it be required that the follower shall advance at a uniform rate from *a* to

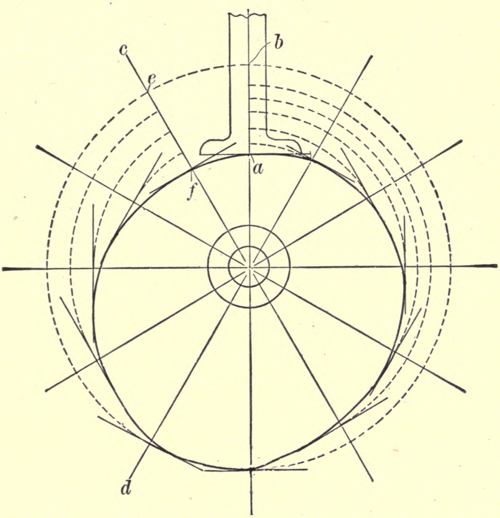


FIG. 136.—Cam with Straight Follower having Uniform Motion.

b as the cam makes a half revolution, this advance being preceded and followed by a period of rest of a twelfth of a revolution of the cam.

Divide that half of the cam during the revolution of which the follower is to be raised from *a* to *b*, in this case the half at the right of the vertical center line, into a number of equal angles, and divide the distance from *a* to *b* into the same number of equal spaces. Mark off the points so obtained onto the successive radial lines as indicated by the dotted lines, and at the points where these dotted lines intersect the radial lines draw lines at right angles to the radial lines to represent the position of the follower when these radial lines become vertical as the cam revolves.

A period of rest in a cam is represented by a circular portion, having the axis of the cam as its center. In order, therefore, to obtain the required periods of rest, the distances of *a* and *b* from the center are marked off upon the radial lines *c* and *d*, these lines being made a twelfth of a revolution from the vertical center line, and lines representing the follower are drawn at these points as before. To get the return of the follower the space from *c* to *d* is divided into a number of equal angles, and the distance from *e* to *f* is divided off to represent the desired rate of return of the follower. In this case the rate of return is made uniform, so the distance *ef* is spaced off equally. The distance of these points from the axis is marked off upon the radial lines between *c* and *d*, and lines representing the follower are drawn.

A curved line, which may be made with the aid of the irregular curves, which is tangent to all of the lines representing the follower, gives the shape of the cam.

Fig. 137 shows a cam having the conditions as to the rise, rest and return of the follower the same as the one shown in Fig. 136, the follower, however, being pivoted at one end.

Draw the arc *ab* representing the path of a point in the follower at the vertical center line, and divide that part of the arc through which the follower rises into the same number of equal spaces as the half circle at the right of the vertical center line is divided into angles. Through these

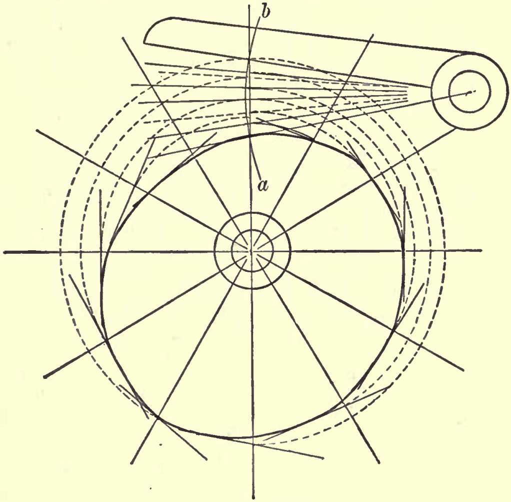


FIG. 137.—Cam with Pivoted Follower.

points draw lines, as shown, representing consecutive positions of the working face of the follower. The various distances of the follower from the axis of the cam are now marked off upon the corresponding radial lines as before. Lines to represent the follower are now drawn across each of these radial lines, at the same angle to them that the follower makes with the vertical center line when at that part of its stroke corresponding to the particular radial line across which the line representing the follower is being drawn. A curved line passing along tangent to all of these lines gives the shape of the cam as before.

Design a Cam with a Round Follower Rising Vertically.—In Fig. 138 the follower has the same uniform rise, and the same periods of rest as before.

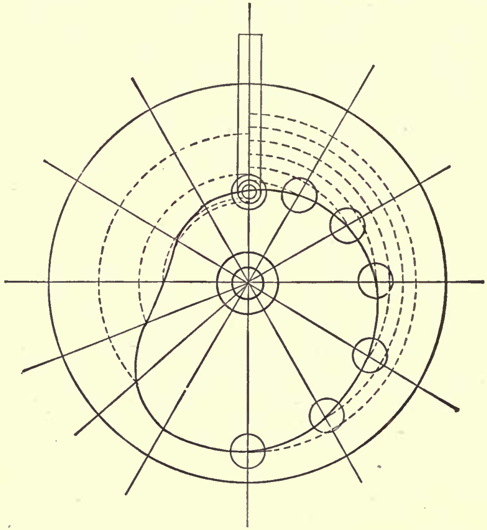


FIG. 138.—Cam with Roller Follower.

A cam with a round follower is less limited in its capabilities than one with a straight follower; in the one here shown the follower on its return drops below the position in which it is shown. That part of the cam during which the conditions are the same as in the others is divided off and the position of the *center* of the follower upon the radial lines is obtained in the same manner as before. That part of the cam representing the return of the follower is divided into such angles as desired, and the distance through which the follower is to drop as the cam revolves through each of these angles is marked off upon the proper radial line. A curved line which is now made to pass through all of the points so obtained gives the pitch line of the cam.

In drawing such a cam it is not always necessary to fully draw the working faces. The pitch line and the method of obtaining it being shown, a number of circles representing consecutive positions of the follower may be drawn. This will usually be sufficient. The side view of the cam, which in a case like this would naturally be made in section, will give opportunity to show any further detail that may be desired.

Design a Cam with a Round Follower Mounted on a Swinging Arm.—Fig. 139 shows such a cam, all of the conditions as to rise, rest and return of the follower being the same as in the cam shown in Fig. 138. The cam is divided into the same angles as before, and the position of the follower is laid out on these radial lines as though it moved vertically. These positions are then modified in the following manner: Draw the arc *ab* representing the path of the center of the follower as it rises, and extend the dotted circular lines, which represent successive heights of the follower, from the vertical center line to this arc. The distance of each of the intersections of the dotted circular lines with the arc *ab*, from the vertical center line is then taken with the compasses and is marked off upon the same dotted line from the radial line at which it terminates, or, where the follower has a period of rest, from both of the radial lines

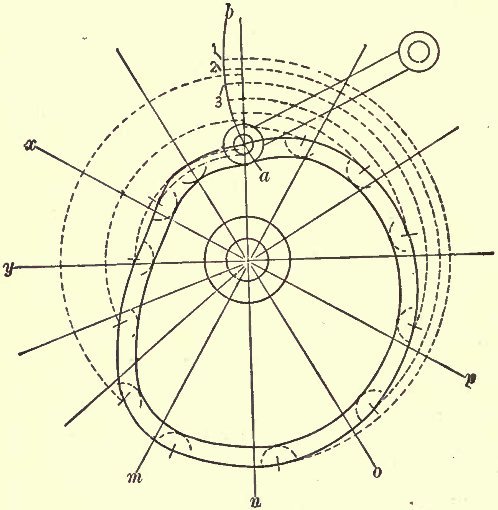


FIG. 139.—Cam with Roller Follower Mounted on Swinging Arm.

where the period of rest takes place. Thus the distance of the point *1* from the vertical center line is marked back upon the dotted circular line from the radial lines *m* and *n*. Point *2* is marked back from the radial line *o*. Point *3* is marked back from the line *p*. By this means the position which the follower will occupy, when each of the radial lines has become vertical, as the cam revolves, is determined. A curved line which is made to pass through all of these points will be the required pitch line of the cam. The method of getting the working face of the cam is indicated by the small dotted circular arcs, which are drawn with a radius equal to that of the follower. It will be noticed that, as the follower, on its return, drops below the position in which it is shown, it passes to the other side of the vertical center line, so that in marking off its position from the radial lines x and y this must be borne in mind. The question as to

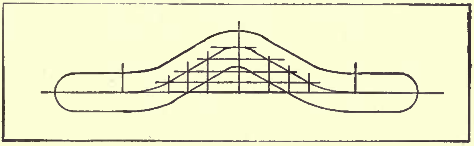


FIG. 140.—Reciprocating Motion Cam.

on which side of a radial line the new position of the follower will be, may be readily determined by imagining the cam to revolve so as to bring that particular line vertical.

Reciprocating Cams.—Fig. 140 shows a straight cam, which by a reciprocating motion imparts a sideways motion to its follower. The pitch line of such a cam may be determined by intersecting lines at right angles to each other. As here shown the distance through which the follower is to be raised is divided into a number of equal spaces by horizontal lines, and the distance through which it is desired to have the cam move in order to raise the follower from one horizontal line to the next one is indicated by vertical lines. A curved line which is made to pass through the intersections of these lines will be the required pitch line of the cam.

If the follower, instead of rising vertically, rose at an angle, or if it were mounted on a swinging arm, the pitch line would be modified in the same manner as that of the cam shown in Fig. 139.

Cams With a Grooved Edge.—It is sometimes desired to have a revolving cam impart a sideways

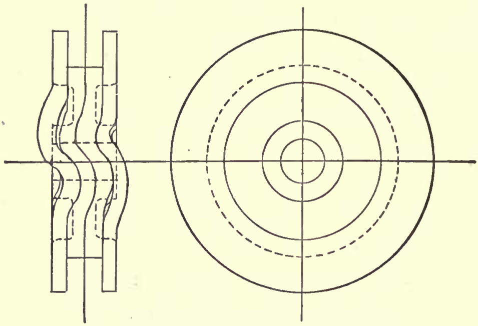


FIG. 141.—Cam with Grooved Edge.

motion to a follower. This is done by having a groove in the edge of the cam, as shown in Fig. 141. Such a cam may be considered as a modified form of a reciprocating cam, and its pitch line may be determined in the same way.

By laying out a development of the pitch line, or of that part of it which is to operate the follower, as shown in Fig. 142, horizontal lines, that is, lines parallel with the pitch line, may be drawn to indicate successive stages in the movement of the follower, and lines at right angles to these to indicate the desired movement of the cam. The pitch line is then drawn through the intersections of these lines as before.

A Double Cam Providing Positive Return.—In a cam like that shown in Fig. 138, where the return

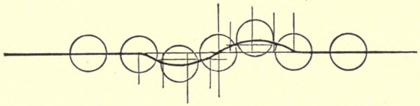


FIG. 142.—Development of Cam Action of Grooved-Edge Cam in Fig. 141.

of the follower is insured by a groove in the face of the cam, the groove must be slightly broader than the diameter of the cam roller to insure freedom of action, as, when the cam is forcing the rol-

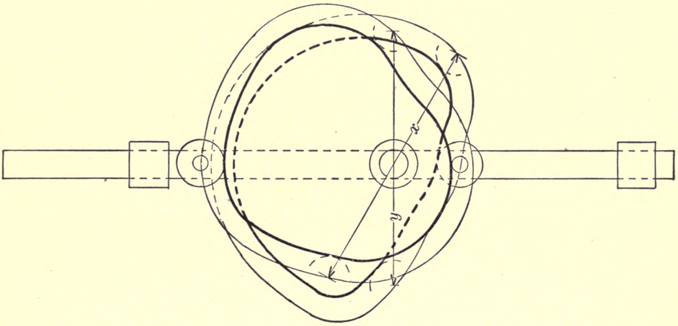


FIG. 143.—Double Cam Providing Positive Return.

ler away from the center, the roller will revolve in the opposite direction to that in which it revolves when the other face of the cam groove acts on it to draw it toward the center, so that unless clearance is provided, there will be a grinding action between the roller and the faces of the cam groove. This clearance, however, causes the cam to give a knock or blow on the roller each time its action is reversed, and the reversal of the direction of the revolution of the roller itself causes a temporary grinding action. These actions may become ob-

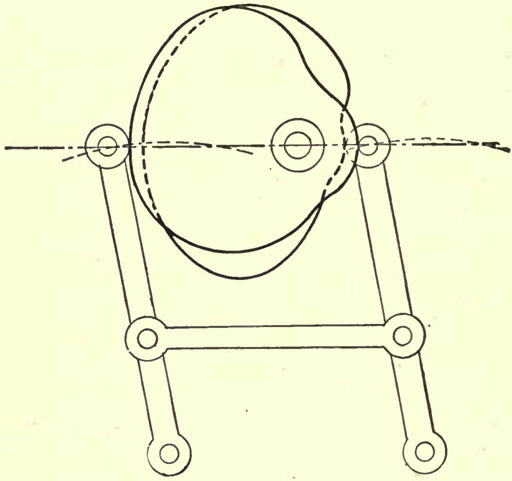


FIG. 144.—Positive Return Cam with Rollers Mounted on Swinging Arms.

jectionable, especially at high speeds. A method which overcomes these objections, and which is preferred by some for such work, is shown in Fig. 143, where the return is secured by a secondary cam mounted on the same shaft as the primary cam, but acting on a roller of its own. In this case there is no reversal of the direction of the revolution of the rollers, so that the necessity of providing clearance does not exist. Where the forward and backward motion of the rollers is in a straight line passing through the center of the cam shaft, as in this case, it is only necessary in designing the secondary cam to preserve the distance between its pitch line and the pitch line of the primary cam constant, measuring through the center of the cam shaft, as shown at x and y.

If, however, the rollers are mounted on swinging arms, as shown in Fig. 144, so that their forward and backward motion is not in such a straight line, then the shape of the secondary cam will be subject to modification on principles previously explained. It is obviously necessary where this method of operation is used, that provision be made to absolutely prevent any change in the relative position of the two cams, as by bolting them together, or, better still, by having them cast together in one piece.

Cams for High Velocities.—In machinery working at a high rate of speed, it becomes very important that cams are so constructed that sudden shocks are avoided when the direction of motion of the follower is reversed. While at first thought it would seem as if the uniform motion cam would be the one best suited to conditions of this kind, a little consideration will show that a cam best suited for high speeds is one where the speed at first is slow, then accelerated at a uniform rate until the maximum speed is reached, and then again uniformly retarded until the rate of motion of the follower is zero or nearly zero, when the reversal takes place. A cam constructed along these lines

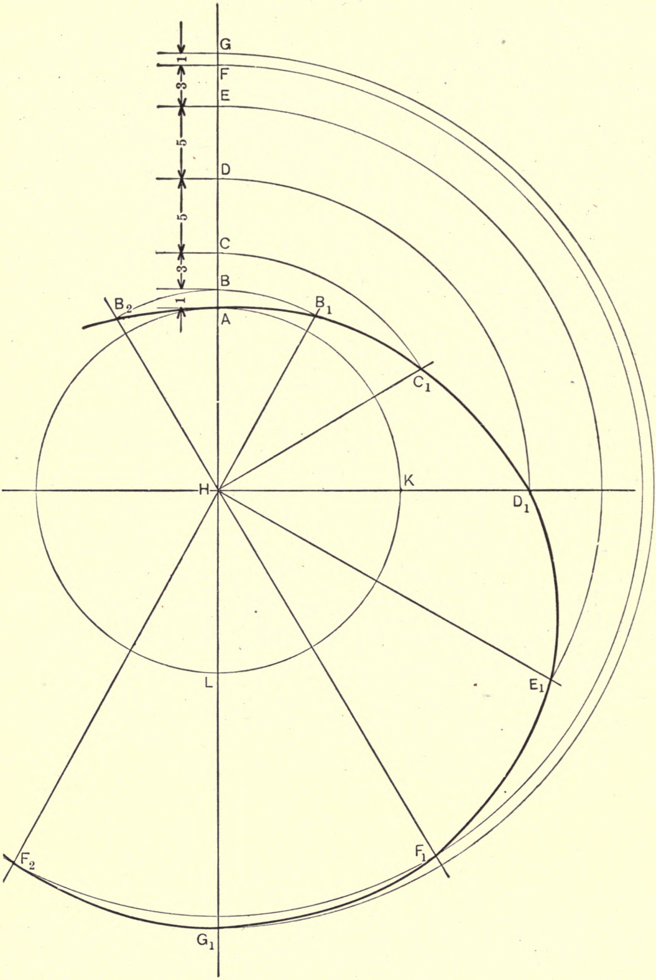


FIG. 145.—Uniformly Accelerated Motion Cam.

is called a uniformly accelerated motion cam. The distances which the follower passes through during equal periods of time increase uniformly, so that, if, for instance, the follower moves a distance equal to 1 length unit during the first second, and 3 during the second, it will move 5 length units during the third second, 7 during the fourth, and so forth. When the motion is retarded, it will move 7, 5, 3 and 1 length units during successive seconds, until its motion becomes zero at the reversal of the direction of motion of the follower.

In Fig. 145 is shown a uniformly accelerated motion plate cam. Only one-half of the cam has been shown complete, the other half being an exact duplicate of the half shown, and constructed in the same manner. The motion of the follower is back and forth from *A* to *G*, the rise of the cam being 180 degrees, or one-half of a complete revolution. To construct this cam, divide the half-circle, *AKL*, in six equal angles, and draw radii HB_1, HC_1, etc. Then divide *AG* first in two equal parts *AD* and *DG*, and then each of these parts in three divisions, the length of which are to each other as 1:3:5, as shown. Then with *H* as a center draw circular arcs from *B*, *C*, *D*, etc., to B_1, C_1, D_1, etc. The points of intersection between the circles and the radii are points on the cam surface.

If the half-circle *AKL* had been divided into 8 equal parts, instead of 6, then the line *AG* would have been divided into 8 parts, in the proportions 1:3:5:7:7:5:3:1, each division being the same amount in excess of the previous division while the motion is accelerated, and the same amount less than the previous division while the motion is being retarded. With a cam constructed on this principle the follower starts at *A* from a velocity of zero; it reaches its maximum velocity at *D*; and at *G* the velocity is again zero, just at the moment when the motion is reversed.

A graphical illustration of the shape of the uniformly accelerated motion curve is given in Fig.

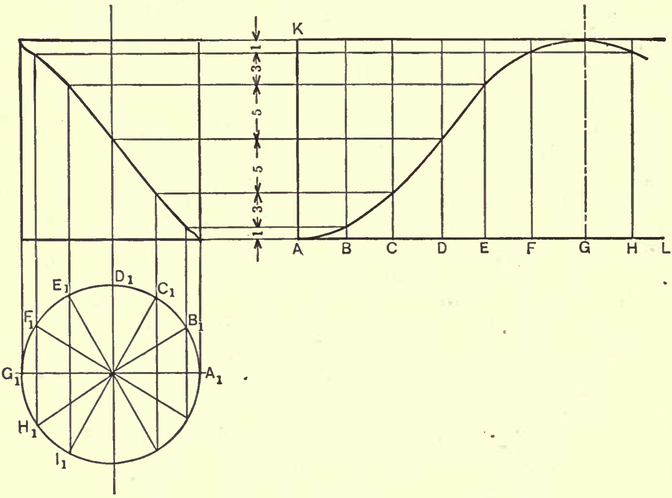


FIG. 146.—Development and Projection of Uniformly Accelerated Motion Cam Curve.

146. To the right is shown the development of the curve as scribed on the surface of a cylindrical cam. This development is necessary for finding the projection on the cylindrical surface, as shown at the left. To construct the curve, divide first the base circle of the cylinder in a number of equal

parts, say 12; set off these parts along line *AL*, as shown; only one division more than one-half of the development has been shown, as the other half is the same as the first half, except that the curve to be constructed here is falling instead of rising. Now divide line *AK* in the same number of divisions as the half-circle, the divisions being in the proportion 1:3:5:5:3:1. Draw horizontal lines from the divisions on *AK* and vertical lines from *B*, *C*, *D*, etc. The intersections between the two sets of lines are points on the developed cam curve. These points are transferred to the cylindrical surface at the left simply by being projected in the usual manner.

In order to show the difference between the uniform motion cam curve, and that illustrating the uniformly accelerated motion, a uniform motion cylinder cam has been laid out in Fig. 147. The base circle is here divided in the same number of equal parts as the base circle in Fig. 146. The divisions are set off on line *AL* in the same way. The line *AK*, however, is divided into a number of *equal* parts, the number of its divisions being the same as the number of divisions in the half-circle. By drawing horizontal lines through the division points on *AK*, and vertical lines through points *B*, *C*, *D*, etc., points on the uniform motion cam curve are found. It will be seen that this curve is merely a straight line *AM*. The curve is transferred to its projection on the cylinder surface at the left, as shown.

It is evident from the developments of the two curves in Figs. 146 and 147, that the uniform motion curve, Fig. 147, causes the follower to start very abruptly, and to reverse from full speed in one direction to full speed in the opposite direction. The uniformly accelerated motion curve, Fig. 146, permits the follower to start and reverse very smoothly, as is clearly shown by the graphical

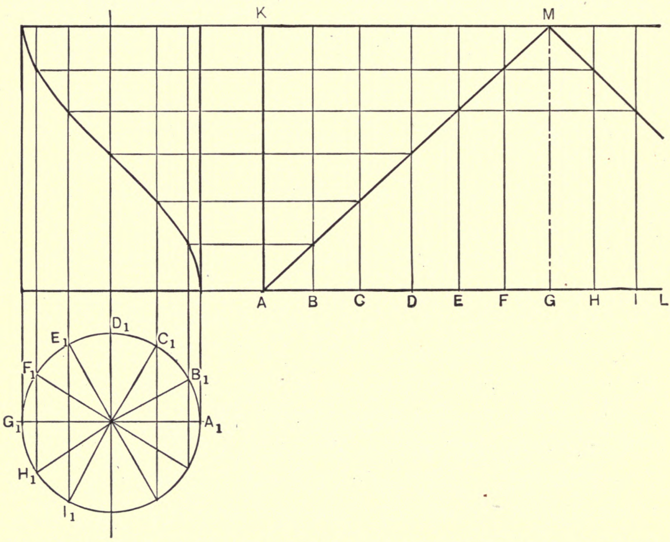


FIG. 147.—Development and Projection of Uniform Motion Cam Curve.

illustration of the curve. The abrupt starting and reversal of the follower in the uniform motion curve is the cause why this form of cam, while the simplest of all cams to lay out and cut, cannot be used where the speed is considerable, without a perceptible shock at both the beginning and the end of the stroke.

Besides the uniformly accelerated motion cam curve, quite commonly called the gravity curve, on account of it being based on the same law of acceleration as that due to gravity, there is another curve, the harmonic or crank curve, which is quite often used in cam construction. The harmonic motion curve provides for a gradual increase of speed at the beginning, and decrease of speed at the end, of the stroke, and in this respect resembles

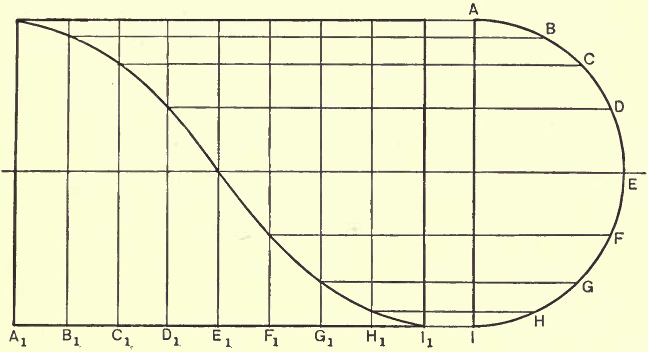


FIG. 148.—Lay-out of Harmonic Motion Cam Curve.

the uniformly accelerated motion curve; but the acceleration, not being uniform, does not produce so easy working a cam as the gravity curve provides for. The harmonic motion curve is, however, very simple to lay out, and for ordinary purposes, where excessively high speeds are not required of the mechanism, cams laid out according to this curve are very satisfactory.

The harmonic curve is laid out as shown in Fig. 148. Draw first a half-circle *AEI*. Divide the circle in a certain number of equal parts. Draw a line $A_1 I_1$, and divide this line in a number of equal parts, the number of divisions of $A_1 I_1$ being the same as that of the half-circle. Now draw horizontal lines from the divisions *A*, *B*, *C*, etc., on the half-circle, and vertical lines from the divisions on line $A_1 I_1$. The points where the lines from corresponding division points intersect, are points on the required harmonic cam curve.

An approximation of the uniformly accelerated motion or gravity curve can be drawn as shown in

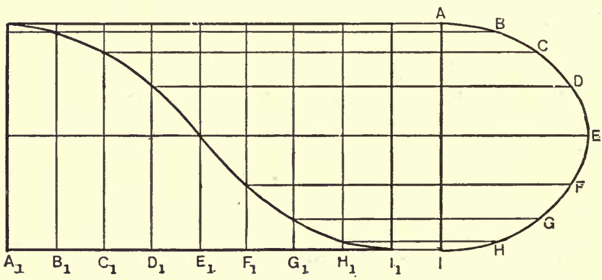


FIG. 149.—Approximation of Uniformly Accelerated Motion Curve.

Fig. 149. By using this approximate method, any degree of accuracy can be attained without the necessity of dividing the vertical line *AK*, Fig. 146, in an excessively great number of parts. The approximate curve in Fig. 149 is constructed as follows: Draw a half-ellipse *AEI*, in which the minor axis is to the major axis as 8 to 11. Divide this half-ellipse in any number of equal parts, and divide the line $A_1 I_1$ in the same number of equal parts. Now draw horizontal lines from the division

points on the ellipse, and vertical lines from A_1, B_1, C_1, etc. The points of intersection between corresponding horizontal and vertical lines, are points on the cam curve. This cam curve, as well as the one in Fig. 148, can be transferred to the cylindrical surface of a cylinder cam by ordinary projection methods, as shown in Figs. 146 and 147.

In Figs. 150 and 151 are shown two plate cams for comparison. The one in Fig. 150 is a uniform

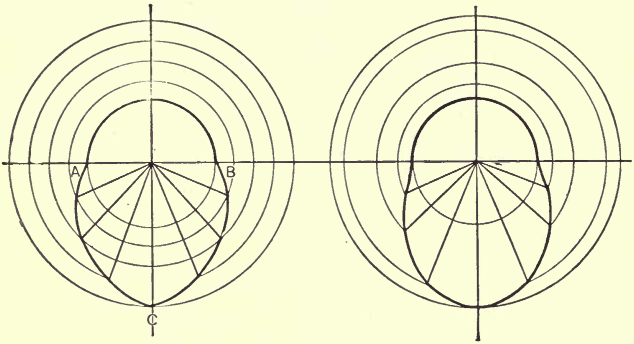


FIG. 150.—Plate Cam Laid out for Uniform Motion.

FIG. 151.—Plate Cam Laid out for Uniformly Accelerated Motion.

motion cam. The dwell is 180 degrees, the rise, 90 degrees, and the fall, 90 degrees. As shown by the sudden change of direction of the cam curve at A and B, there is considerable shock when the follower passes from its "dwell" to the "rise," as well as at the end of the "fall." A sudden reversal takes place at C, which also causes a shock in the mechanism connected with the follower. In the uniformly accelerated motion cam, Fig. 151, the passing from "dwell" to "rise," the reversal of the direction of motion, and the return to the "dwell" position, is accomplished by means of smoothly acting curves, and, even at high speeds, no perceptible shock will be noticed.

The examples given will show the necessity of careful analysis of conditions, before a certain type of cam curve is selected. In machinery which works at a low rate of speed, it is not important whether the follower moves with a uniform, harmonic, or uniformly accelerated motion; but when the cam has a high rotative speed, and the follower a reciprocating motion, it often becomes practically impossible to make use of the uniform motion curve in the cam. In such cases, as already mentioned, the harmonic, or, preferably, the uniformly accelerated motion curve should be used in laying out the cam.

CHAPTER XI

SPROCKET WHEELS

When it is desired to transmit power from one shaft to another one quite near to it, especially if the power to be transmitted is considerable, so as to preclude the use of belting, sprocket wheels with chain are frequently used, if the speed is not high. Bicycles afford a familiar illustration of this sort of power transmission.

Fig. 152 shows a sprocket wheel of a type similar to those used on bicycles and shows the method of getting the shape of the teeth. The chain is shown with the links (on the side toward the observer) removed so as to allow of showing the teeth without dotted lines. The size of a sprocket wheel to fit a given chain may be determined graphically as follows: A circle, not shown in the illustration, is first drawn of a diameter about equal to that of the desired wheel, and this circle is spaced off into as many divisions as the wheel is to have teeth. Lines corresponding to the dotted radial lines in the upper half of the wheel shown, are drawn from these division points to the center of the circle. A templet, similar in shape to that shown in Fig. 154, is next cut out of paper, the lines *ab* and *cd* being at right angles to each other, and the length of a link of the chain, measured from center to center of the pins as shown at *a*, Fig. 152, is marked off upon the line *ab*, measuring equally each way from the center line *cd*. In getting the length of the link in the chain it will be best, for the sake of accuracy, to measure off the length of a considerable portion of the chain, and with the spacing compasses divide this length into twice as many spaces as there are links in the measured portion of the

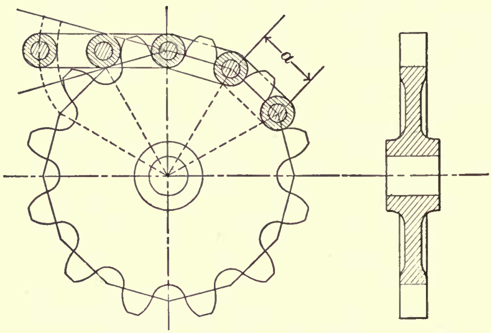


FIG. 152.—Sprocket Wheel and Chain.

chain. The compasses, being then set to exactly half the length of a link, may be used to mark off the length of the link, *1—2*, upon the templet. Now letting the angle *abc*, Fig. 155, represent one of the angles into which the circle has been divided, bisect it to get a center line *bd*, and placing the templet so that its line *cd* shall coincide with this center line move it along until the points *1—2* shall coincide with the lines *ab* and *cb* of the angle. These points being now marked off upon the lines, give the location of the centers of the pins in the chain, and a line connecting them will be one side of the polygon which forms the pitch line of the wheel. A spiral may now be formed upon this polygon (see geometrical problem 19, Figs. 41 and 42), and will give the path of the pin as the chain

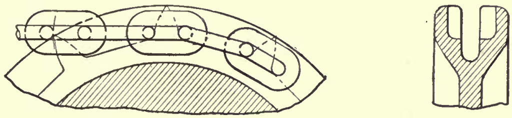


FIG. 153.—Sprocket Wheel Designed for Common Link Chain.

unwinds from the wheel when the latter revolves, as shown in Fig. 152. The working face of that part of the tooth in the wheel lying outside of the pitch polygon is now struck from such a center as will cause it to fall slightly within the path of the chain, as just obtained, so that the link may fall

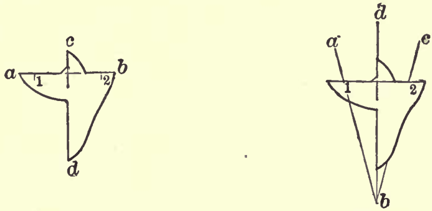


FIG. 154. FIG. 155.

FIGS. 154 and 155.—Graphical Method of Laying Out Sprocket Wheel.

freely into place as it enters upon the tooth. Of course allowance must be made all around for the natural roughness of the casting if the wheel is to be left unfinished. The length of the tooth is usually made about equal to the width of the chain.

If a wheel is to have many teeth, it will generally be accurate enough to consider the pitch line as a circle of a circumference equal to the number of the teeth multiplied by the length of the link. Its diameter will then, of course, be found by dividing the circumference by 3.1416.

In the case of the wheel shown in Fig. 152, should the pitch line be regarded as such a circle it would have a diameter a little over a thirty-second of an inch too small, if the length of the link is taken at three-quarters of an inch. If the wheel were to be made twice as large, the error would be a little *less* than a sixty-fourth of an inch, as it would decrease at a slightly faster rate than that at which the number of the teeth increased. An error of a sixty-fourth of an inch in the diameter of such a sprocket would be of but very little moment. Where a sprocket has but few teeth, however, it will be on the side of safety to always give to the pitch line its true polygonal form, and the only way by which its diameter could be ascertained with any greater accuracy than by the method here given would be to calculate it, as may be done by trigonometry. When the pitch line of a sprocket is regarded as a circle, the path of the chain as it unwinds will be regarded as an involute (see geometrical problem 20).

The shape of the rim of a sprocket wheel will be governed by the style of the chain for which it is designed. Fig. 153 shows a portion of the rim of a wheel which is designed for a common link chain; but whatever the general shape of the rim may be, the working faces of the teeth, or of the projections which correspond to teeth, will always be made on the principles here explained.

The speed ratio of the two wheels of a pair of sprockets will be inversely as the number of teeth in each. For instance, if the large and the small wheels have respectively 13 and 7 teeth, then the speed of the large wheel will be to the speed of the small wheel as 7 to 13.

CHAPTER XII

GENERAL PRINCIPLES OF GEARING

Friction and Knuckle Gearing.—In machinery it is frequently necessary to transmit power from one shaft to another near to it. For this purpose gears are generally employed. Let *a* and *b*, Fig. 156, be two such shafts. If now disks *c* and *d* are mounted upon these shafts, of such diameters as

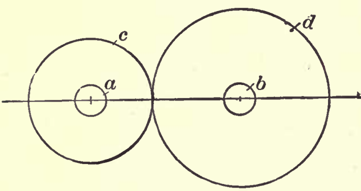


FIG. 156.—Friction Wheels.

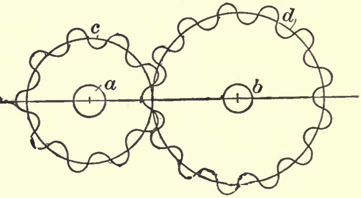


FIG. 157.—Knuckle Gears.

to give the required speed ratio, we will have gearing in its simplest form. Such disks, having their edges covered with leather or other equivalent material, are called *friction* gears and are sometimes employed on light work. At best, however, they will transmit but little power.

If now we make semi-circular projections at equal distances apart upon the outside of the circles *c* and *d*, and cut out corresponding depressions inside of the circles, as shown in Fig. 157, we will have a simple form of toothed gearing and the circles *c* and *d* will be the *pitch* circles. Such gears, called *knuckle* gears, are sometimes employed on slow-moving work where no special accuracy is required. They will not transmit speed uniformly. If the driver of such a pair of gears rotated at a uniform rate, the driven gear would have a more or less jerky movement as the successive teeth came into contact, and if run at high speed they would be noisy. Various curves may be employed to give to gear teeth such an outline that the driver of a pair of gears will impart a uniform speed to the driven one, but in common practice only two kinds are used, the cycloidal, or, as it is sometimes called, epicycloidal, and the involute.

Epicycloidal Gearing.—Let the circles *a*, *b* and *c*, Fig. 158, having their centers on the same straight line, be made to rotate so that their circumferences roll upon each other without slipping. If the circle *c* has tracing points *1*, *2*, *3* upon its circumference, and when we start to rotate the circles point *1* is half way around from the position in which it is shown, then in rotating the circles sufficiently to bring the tracing points to the position in which they are shown, point *1* will trace the line *1′* inwardly from the circle *a*, and the line *1″* outwardly from the circle *b*. Point *2* will trace the two lines which are shown meeting at that point, one inwardly from the circle *a*, and one outwardly from the circle *b*. Point *3* will similarly trace the two lines which met at that point. Inasmuch as these lines were traced simultaneously by points at a fixed distance apart, it is evident that if the circle *c* were to be removed, and the circles *a* and *b* were rolled back upon each other, these lines would work smoothly together, being in contact and tangent to each other at all times upon the line of the circle *c*. If the circle *c* is now placed beneath the circle *b* in the position shown, and the three circles are rolled together as before, the tracing points would trace lines inwardly from *b*, and

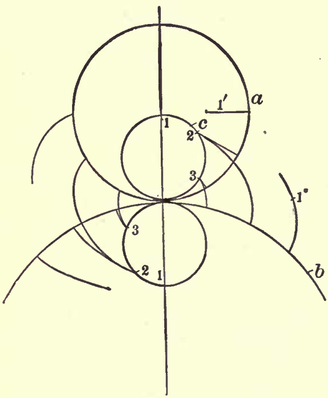


FIG. 158.—Principle of Epicycloidal Gearing.

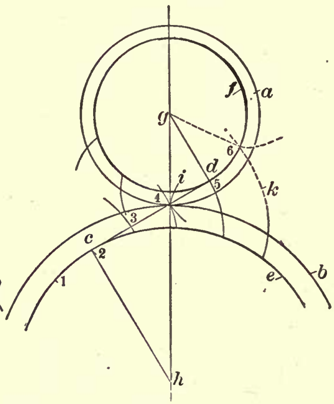


FIG. 159.—Principle of Involute Gearing.

outwardly from *a*, which would also work together smoothly if the circle *c* were removed and the circles *a* and *b* were rolled back upon each other. It is evident that as the three circles are rolled together the lines formed by the tracing points are the same as though either *a* or *b* were taken by itself, and the circle *c* were rolled either within or upon it, hence the lines formed by the tracing points are either epicycloids or hypocycloids as the case may be, and so could be formed by the plotting method described in the geometrical problems.

If these two sets of lines are now joined together so that the lines which extend inwardly from a or b form a continuation of those which extend outwardly and reverse curves are made at a distance from the first set equal to the thickness of a gear tooth, and they are then cut off at such a distance both outside and inside of the circles a and b as to give to the teeth the proper length, it is evident that we will have a pair of perfectly working gears. The circles a and b would roll upon each other without slipping and hence would be true pitch circles. The teeth would work smoothly together in constant contact, the point of contact being always on the line of the generating circle.

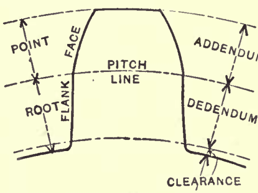


FIG. 160.—Definitions of Gear Tooth Terms.

The length of the point of the gear tooth, that is the portion lying outside of the pitch line, is usually made one-third of the circular pitch—the latter being the distance between the teeth measured from center to center on the pitch line. The distance below the pitch line is made somewhat greater for the sake of clearance. For the names of the various parts of a gear tooth see Fig. 160. Cast gears have some *backlash* between the teeth to allow for the roughness of the castings, as shown in Figs. 161 and 163.

It is evident that if another circle, either larger or smaller, were substituted for b in Fig. 158, the lines formed by the generating circle *c* either within or upon the circle *a* would remain unchanged. Or if a different circle were substituted for *a*, the curves formed within or upon *b* would remain unchanged. Hence it follows that all gears in the epicycloidal system, having their teeth formed by the same generating circle and made of the same

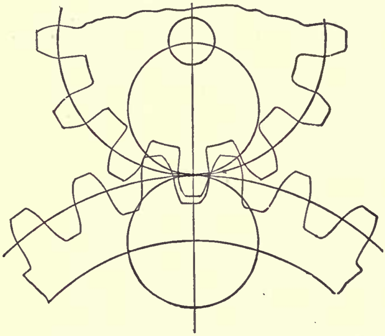


FIG. 161.—Gears with Epicycloidal Teeth.

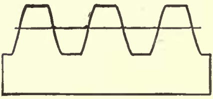


FIG. 162.—Rack with Epicycloidal Teeth.

size, will work together correctly, or, as it is commonly expressed, are interchangeable.

In standard interchangeable gears the generating circle is made one-half the diameter of the smallest gear of the set, which has twelve teeth. This smallest gear will have radial flanks, as that part of the working surface lying within the pitch line is called, because the hypocycloid of a circle formed by a generating circle of half its size will be a straight line passing through its center.

Fig. 161 shows a portion of a pair of such gears, Fig. 162 showing the rack.

Gears with Strengthened Flanks.—A further examination of Fig. 158 will show that the curves formed by the generating circle when it is in the upper of the two positions in which it appears, work together by themselves, and those formed when it is in the lower position work similarly, so that it is not necessary that the same sized gener-

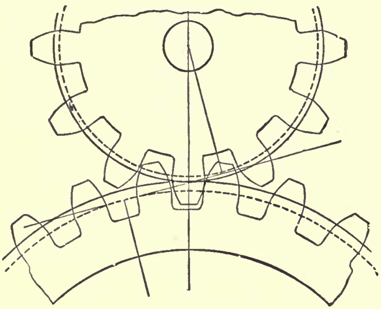


FIG. 163.—Gears with Involute Teeth.

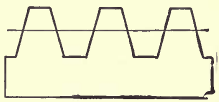


FIG. 164.—Rack with Involute Teeth.

ating circle should be used in both positions, unless the gears are to be members of an interchangeable set of gears. Advantage may be taken of this fact to strengthen the roots of the teeth in a pinion.

If, for instance, in Fig. 161, a smaller generating circle were used in the upper position, the effect would be to broaden out the roots of the teeth in the pinion, and to correspondingly round off the points of the teeth of the other gear.

Gears with Radial Flanks.—Another modification which may be made is to have the teeth of both gears with radial flanks. If, for instance, in Fig. 161 a generating circle were to be used in the lower portion, of half the pitch diameter of the large gear, the effect would be to give to that gear radial flanks, and to make the points of the teeth of the small gear broader in order to work properly with them. Then both gears would have radial flanks. Such gears have been considerably used. They are not as strong as gears of the standard shape, and the only advantage is that it is easier to make the pattern, the teeth being all worked out with a flat-faced plane; but as the teeth of involute gears, described in the next section, can be worked out in the same way, and as such gears are interchangeable, the advantage is obviously in favor of the involute system for such work.

Involute Gears.—In involute gears the working surfaces of the teeth are involutes, formed not upon the pitch circles, but upon base circles lying within the pitch circles and tangent to a line, called the line of action, which passes obliquely through the point where the pitch circles cross the line connecting their centers. Let *a* and *b*, Fig. 159, be pitch circles, and let the line *cd* be the line of action. Then *e* and *f*, being made tangent to the line *cd*, will be the base circles upon which the involutes are to be formed. If now this line of action be considered as part of a thread which unwinds from one base circle and winds up on the other, as the pitch circles are revolved back and forth upon each other, then if tracing points were attached to the thread at points *1*, *2*, *3*, *4*, *5* and *6*, these points would describe involutes outwardly from the base circles, which, being formed simultaneously in pairs and each pair being formed by a common point, would work together smoothly like those formed by the generating circles of the epicycloidal system. That the base circles are of such size as to just pass the thread as the pitch circles roll upon each other is proven by the fact that their radii, *gd* and *gi*, and *hc* and *hi*, the radii *gd* and *hc* being made at right angles to the line of action, are corresponding sides of similar triangles, the segments into which the line of action is divided by the line of centers being the other sides, and hence have the same ratio. It would only then

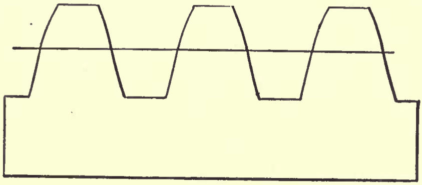


FIG. 165.—Modified Form of Involute Rack Teeth.

be necessary to reverse the direction of the thread to get curves for the other side of the teeth, and to give to the teeth their proper length inside and outside of the pitch line to obtain a pair of correctly working involute gears. That part of the tooth of an involute gear which may lie within the base line is made radial.

In the standard interchangeable involute gears the line of action is given an obliquity of 15 degrees (cut gears, 14½ degrees). This angle may be readily obtained by the combination of the triangles resting against the blade of the T-square shown in Fig. 166. The point of contact of the teeth is always upon the line of action and the push of one tooth against another is in its direction, hence its name.

The teeth of the 15-degree involute rack have straight sides, inclined to the pitch line at an angle of 75 degrees as shown in Fig. 164. This shape, however, is subject to a slight modification to avoid interference of the points of the teeth with the radial flanks of small gears.

Interference in Involute Gears.—The points *c* and *d*, Fig. 159, where the line of action is tangent to the base circles, are called the limiting points. If the involutes which spring from either base circle are so long as to reach beyond these limits on the other base circle, they will interfere with the radial flanks of the mating teeth. At *k* is shown an elongated involute interfering with the radial flank of the mating tooth. This is, of course, a highly exaggerated case. The interference will occur sooner as the line of action is made to cross the line of centers at a less oblique angle, as in standard gears, and still earlier as the pitch circle *b* is made larger. In gearing of standard proportions, a gear of 30 teeth is the smallest that will work correctly with a straight toothed rack. In the gears shown in Fig. 163, the teeth of the large gear pass beyond the limiting point of the small gear, and hence, if made of true involute shape,

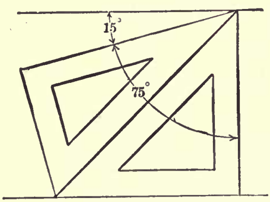


FIG. 166.—Obtaining a 15- or 75-degree Angle by 30- and 45-degree Triangles.

their extremities will not work properly with the flanks of the small gear.

There are three methods available to overcome this interference. First, to hollow out the flanks of the teeth of the small gear. Second, to round off the points of the teeth of the large gear. This is the method usually adopted, in interchangeable gears, the point being rounded off enough to clear the flanks of the smallest gear of the set. Fig. 165 shows the teeth of the rack so corrected in larger scale. Third, to cut off that part of the tooth in the large gear which extends beyond the limiting point of the small gear. This is done in special cases.

The Two Systems Compared.—The great point in favor of epicycloidal gearing would appear to be in its freedom from interference. It is necessary, however, in order to have epicycloidal gears run well, to have the pitch circles of the two gears of a pair just coincide, as shown in Fig. 161; but with involute gears the distance between centers may be varied somewhat without affecting their smoothness of operation, though where the points of the teeth are rounded off to avoid interference, as previously explained, the amount of variation which can be allowed is not great. As no value has been given to the angle at which the line of action crosses the line of centers in Fig. 159, it is evident that whether the base circles are brought nearer together or are carried further apart, circles which might then be drawn through the point where the line of action crosses the line of centers, would roll upon each other while the base circles passed the thread as before, and hence would be true pitch circles for the time being. The amount of backlash, that is, the space between the faces of the teeth, would vary, but the smoothness of operation would not be affected. This property of involute gears is very valuable in cases where the distance between centers is variable, as in rolling mill gearing. In such cases, however, interference must be avoided by the first of the three methods explained, that of hollowing out the flanks of the teeth of the mating gear.

The epicycloidal system is the older of the two, and cast gears are still quite largely made to this system, there being so many patterns of that system on hand. But though the epicycloidal system once had the field to itself, the fact that the involute system has so largely replaced it, having almost wholly superseded it for cut gearing, shows the trend of modern practice. It is sometimes urged against the involute system that the thrust on the shaft bearings is greater than with the epicycloidal system, on account of the obliquity of its line of action. But though the line of action is at an angle to the direction of the motion of the teeth when they are on the line connecting their centers, it is a constant angle; while it is never less, it is never more. With the epicycloidal system, on the other hand, though the teeth of the driver give a square push to the teeth of the driven gear when they are in contact on the line of centers, yet the direction of this pushing action being on the line of the generating circle, is variable, so that when the teeth are first coming into contact with one another they have an obliquity of action fully as great, if not greater, than standard involute gears. For this reason such authorities as the Brown & Sharp Co., Grant and Unwin, do not consider this objection as being of great weight.

Twenty-Degree Involute Gears.—It has been already shown how the teeth of epicycloidal gears may be considerably strengthened where it is not necessary to have them interchangeable. In involute gearing, when a stronger gear is desired than the standard 15-degree tooth provides for, recourse may be had to increasing the obliquity of the line of action. This makes the tooth considerably broader at the base, and correspondingly narrower at the point. The angle usually adopted in such cases is 20 degrees, and some makers report an increasing demand for such gears.

Shrouded Gears.—When it is desired to strengthen the teeth of cast gears without increasing their size, or without using any other than a standard shape or tooth, the practice of shrouding them is sometimes resorted to. This consists in casting a flange on one or both sides of the gear. Full shrouding consists in having the flanges extend to the points of the teeth as shown in Fig. 167; half shrouding is where the flanges extend only to the pitch line as shown in Fig. 168. When the two gears of a pair are of nearly equal size so that their teeth would be of about the same strength it would be natural to use half shrouding on both gears as shown.

When, however, there is much difference in the size of the gears, as shown in Fig. 167, it would be natural to use full shrouding on the small gear, as otherwise its teeth would be weaker than those of the large gear. Shrouding is estimated to strengthen the teeth from 25 to 50 per cent.

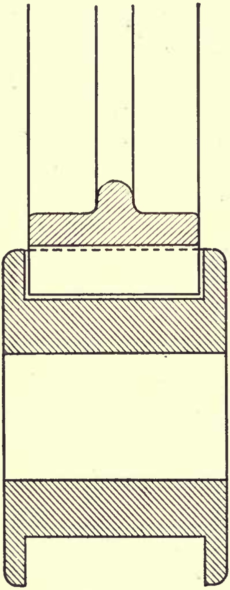


FIG. 167.

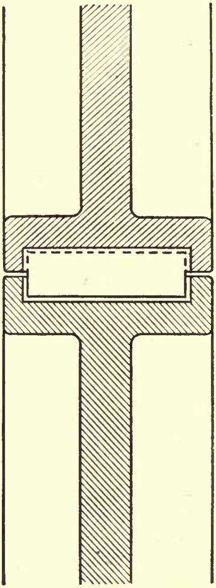


FIG. 168.

FIGS. 167 and 168.—Shrouded Gears.

Bevel Gears.—In cylindrical or spur gears the pitch surfaces are cylinders of a diameter equal to the pitch circle; in bevel gears the pitch surfaces are cones, having their apices coinciding.

In designing a pair of bevel gears as shown in Fig. 169, the center lines *ab* and *cd* are first drawn, and the pitch diameters then laid out from these lines as indicated. From the point where the lines of the pitch diameters meet at *e*, a line is drawn to the point where the center lines intersect at *h*. This gives one side of the pitch cone of each gear and from this the other sides of the cones are

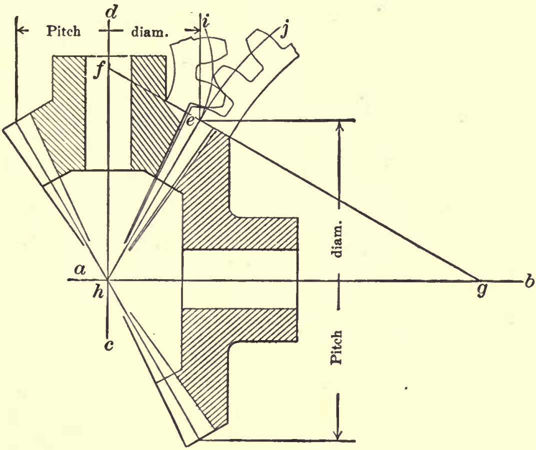


FIG. 169.—Bevel Gears.

readily drawn. All lines of the working surfaces of the gears meet at the point *h*.

To lay out the teeth, the line *fg* is first drawn through the point *e* and at right angles to *eh*. This gives the outside face of the teeth, and the points *f* and *g* become the apices of cones upon the development of which the teeth are laid out. With centers at *f* and *g* the pitch line developments *ei* and *ej* are drawn, and upon these lines the teeth are laid out the same as for ordinary gears. When the two gears of a pair are of the same size they are called *miter* gears.

Worm Gearing.—In worm gearing, as shown in Fig. 170, a screw having its threads shaped like the teeth of a rack engages with the teeth of a gear having a concave face and teeth of such shape as to fit the threads of the screw. If the screw is single threaded, one rotation of it will cause the gear to revolve the distance of one tooth; if double threaded, the gear will turn two teeth, and so on.

In worm gearing, the worm wears much faster than the gear; it is, therefore, frequently made of

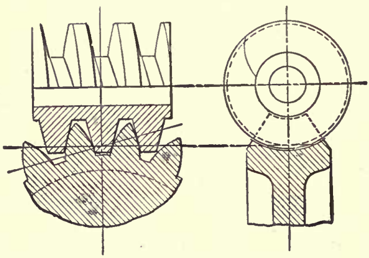


FIG. 170.—Worm and Worm-Gear.

steel while the worm-wheel is made of bronze, to give the combination increased durability.

In involute worm gearing interference is commonly avoided by the last of the three methods already mentioned. The points of the thread of the screw in Fig. 170 project but little beyond the pitch line, the root spaces of the gear being made correspondingly shallow. At the same time, the points of the teeth in the gear are made long enough to preserve their total length the same as usual, and the depth of the screw thread inside the pitch line is made sufficient for clearance. But unless the worm-gear has less than 30 teeth, the standard shape of tooth will be satisfactory.

Circular Pitch.—In designing gearing, the old method (the one which is given in the older treatises on the subject) is to use the circular pitch; that is, the distance between the teeth, measured from center to center on the pitch circle. This method has many disadvantages. For instance, if it is required to make a pattern of a gear to mesh with one already on hand, the natural thing to do in measuring up the old gear is to first guess at where the pitch line is, and then measure straight across from one tooth to the next. This leads to two errors in the result; first, the probably incorrect location of the pitch line, and, second, the distance measured is the chordal pitch instead of the circular pitch. A noisy pair of gears would quite likely be the result.

Again, as the ratio between the circumference and the diameter of a circle is not an even number, but a troublesome fraction, the use of the circular pitch method will give the pitch diameter of the gear in inconvenient fractions of an inch, unless an equally inconvenient circular pitch is used. This method has so many disadvantages that it has been largely replaced by the more convenient "diametral pitch" method. For cut gears the diametral pitch method is used almost exclusively; but for cast gears there are so many patterns on hand, made by the circular pitch method, that that method is still used considerably on such work, especially on the larger sizes of gears.

Where one is designing new work, however, where no old gear patterns made by the circular pitch method are used, the diametral pitch method will be by far the most convenient to use, whichever style of tooth, whether involute or epicycloidal, may be adopted.

PITCH DIAMETERS OF GEARS FROM 10 TO 100 TEETH, OF 1-INCH CIRCULAR PITCH.

No. of Teeth	Diam. in Inches	No. of Teeth	Diam. in Inches	No. of Teeth	Diam. in Inches	No. of Teeth	Diam. in Inches
10	3.183	33	10.504	56	17.825	79	25.146
11	3.501	34	10.823	57	18.144	80	25.465
12	3.820	35	11.141	58	18.462	81	25.783
13	4.138	36	11.459	59	18.781	82	26.101
14	4.456	37	11.777	60	19.099	83	26.419
15	4.775	38	12.096	61	19.417	84	26.738
16	5.093	39	12.414	62	19.735	85	27.056
17	5.411	40	12.732	63	20.054	86	27.375
18	5.730	41	13.051	64	20.372	87	27.693
19	6.048	42	13.369	65	20.690	88	28.011
20	6.366	43	13.687	66	21.008	89	28.329
21	6.685	44	14.006	67	21.327	90	28.648
22	7.003	45	14.324	68	21.645	91	28.966
23	7.321	46	14.642	69	21.963	92	29.285
24	7.639	47	14.961	70	22.282	93	29.603
25	7.958	48	15.279	71	22.600	94	29.921
26	8.276	49	15.597	72	22.918	95	30.239
27	8.594	50	15.915	73	23.236	96	30.558
28	8.913	51	16.234	74	23.555	97	30.876
29	9.231	52	16.552	75	23.873	98	31.194
30	9.549	53	16.870	76	24.192	99	31.512
31	9.868	54	17.189	77	24.510	100	31.831
32	10.186	55	17.507	78	24.828		

When the pitch of a gear is given in inches or fractions of an inch, the circular pitch is always meant; as, for instance, where a gear is said to be of 1-inch pitch, or 1¼-inch pitch. To get the pitch diameter in such a case, it is necessary to multiply this pitch by the number of teeth in the gear, and then divide this product by 3.1416, the ratio between the circumference and the diameter. For ascertaining the pitch diameter of gears when using the circular pitch, the accompanying table will save much time. If the gear is of any other than 1-inch circular pitch, multiply the diameter here given for the required number of teeth, by the circular pitch to be used.

Proportions of Teeth.—The proportions of the teeth of gears where the circular pitch method is used, are given slightly different by various writers. The length of the teeth is entirely arbitrary and therefore this discrepancy is quite natural. It is also unimportant, excepting as uniformity is desirable. The proportions as given by Grant are as follows: The addendum and dedendum are each made one-third of the circular pitch; the clearance, the distance of the root line below the dedendum line, is made one-eighth of the addendum; the backlash, the space which is allowed between the sides of the teeth in cast gears, is made about the same as the clearance. This presents the proportions in fractions which are convenient to use, and at the same time makes the proportions practically the same as those of the diametral pitch method. Cut gears are made without backlash.

Diametral Pitch.—In the diametral pitch method the gear is considered as having a given number of teeth for each inch of pitch diameter. Gears having three, four, or five teeth to each inch of their pitch diameters are said to be of three, four, or five pitch. With this method the addendum (the distance which the teeth project beyond the pitch line) is made equal to one divided by the pitch, so that the addendum on gears of three, four or five pitch would be, respectively, one-third, one-fourth or one-fifth of an inch. The advantages of this method are numerous.

To get the diametral pitch of a gear it is only necessary to divide the number of teeth by the pitch diameter, or to divide the number of teeth plus two, by the outside diameter. A complete set of rules, as well as formulas and examples for calculating spur gear dimensions, will be given in the next chapter.

It is quite a common practice in figuring gears made by diametral pitch to give only the pitch and the number of teeth, as 4 pitch, 18 teeth, or 4 D. P., 18 T. The letters D. P. stand for diametral pitch, the letters P. D. standing for pitch diameter. The pitch diameter is then found by dividing the number of teeth by the diametral pitch. When this method is used, the circular pitch becomes of secondary importance, but may be found by dividing 3.1416 by the diametral pitch. When the circular pitch is given and the diametral pitch is desired, divide 3.1416 by the circular pitch. The diameter of a gear, unless otherwise specified, is always understood to be the pitch diameter. With the diametral pitch method, the pitch diameter, unless in even inches, will be in fractions of an inch corresponding to the pitch, so that the fractional parts of the diameter of gears of three, four or five pitch, for instance, would be thirds, fourths or fifths of an inch.

The Hunting Tooth.—It is a common practice in making gear patterns to have the teeth of the two gears of a pair of such numbers that they do not have a common divisor. For instance, instead of having 25 and 35 teeth in the gears of a pair, one may give to one of them one more or one less tooth, so as to insure all of the teeth of one gear coming into contact with all of the teeth of the other as they run together.

This practice is condemned by some, however, on the ground that if any of the teeth are of bad shape it would be better to confine their injurious action within as narrow limits as possible, rather than to have them ruin all of the teeth of the other gear; but the shape of badly formed teeth should be corrected as soon as the error is discovered.

Approximate Shapes for Cycloidal Gear Teeth.—That part of the cycloidal curve which is used in the formation of gear tooth outlines is so short that it may be replaced with a circular arc which will very closely approximate it, and such arcs are generally used in the practical construction of gear patterns. In the following is given a table of such arcs with the location of the centers from which they are struck. The center from which that part of the tooth lying outside of the pitch line is drawn, the *face* of the tooth, will be inside of the pitch line, while the center from which that part of the tooth lying inside of the pitch line is drawn, the *flank* of the tooth, will be outside of the pitch line. These radii and center locations were obtained directly from a set of tooth outlines of 3-inch circular pitch, formed by rolling a generating circle, drawn upon tracing paper, upon a set of pitch circles, correct rotation being assured by the use of needle points pricked through the generating circle into the pitch circle, the needle points serving as pivots upon which the generating circle was swung through short successive stages, the forward movements of the tracing point in forming the cycloidal curves being also pricked through. Needle points were also used in the instruments which were used for tracing this curve when the radius and center location were determined.

CYCLOIDAL TOOTH OUTLINES

Radii and center locations for one-inch circular pitch. For any other pitch multiply the given figure by the required pitch.

Number of Teeth.	Face Radius.	Inside of Pitch Line.	Flank Radius.	Outside of Pitch Line.
12	0.625 ins.	0.016 ins.	Radial	
14	0.666 "	0.021 "	4.00 ins.	2.35 ins.
16	0.697 "	0.026 "	2.80 "	1.33 "
18	0.724 "	0.031 "	2.37 "	0.96 "
20	0.750 "	0.036 "	2.14 "	0.73 "
25	0.802 "	0.042 "	1.91 "	0.58 "
30	0.844 "	0.052 "	1.79 "	0.48 "
40	0.906 "	0.062 "	1.64 "	0.375 "
60	0.958 "	0.083 "	1.50 "	0.29 "
100	1.010 "	0.095 "	1.33 "	0.21 "
200	1.040 "	0.120 "	1.23 "	0.177 "
Rack	1.080 "	0.127 "	1.08 "	0.127 "

If the diametral pitch method is being used, the corresponding circular pitch may be found by dividing 3.1416 by the diametral pitch, as already mentioned.

Involute Teeth.—The construction of a correct involute tooth outline is so simple a matter as to make the use of tables of approximate circular arcs unnecessary. An involute may be formed by the plotting method given in the geometrical problems, but in most cases it may be more readily formed by the use of a sharply pointed pencil guided by a strong thread as shown in Fig. 171, where *ab* represents the pitch line of a gear, and *cd* represents the base circle, having a number of pins stuck into it at short distances apart. The thread being doubled, forms a loop to hold the pencil point. The thread being drawn tightly around the pins, the pencil is swung outward from the

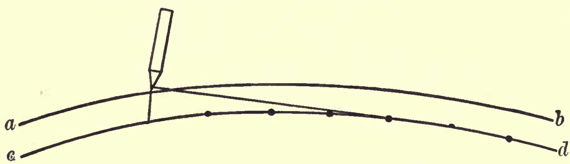


FIG. 171.—Laying out an Involute Gear Tooth.

base circle, forming the required involute. When gears of over thirty teeth are to mesh into others of less than that number, it will be necessary to slightly round over the points of the teeth to avoid interference with the radial flanks of the mating gear. For this purpose use a radius of 2.10 inches divided by the diametral pitch, with a center on the pitch line as shown in Fig. 172. This radius, 2.10 inches divided by the diametral pitch, is the same as that given by Grant for rounding off the points of the teeth of racks; but actual trial on teeth of large size shows it to be correct for gear wheels also, giving a curve which coincides very closely with the epicycloidal shape which the point should have to work correctly with the radial flank of the mating gear.

That part of an involute tooth lying within the base circle is made radial, as previously stated, and a good fillet should be drawn in at the root. For this purpose use a radius of one-twelfth of the circular pitch. A templet which is fitted to this

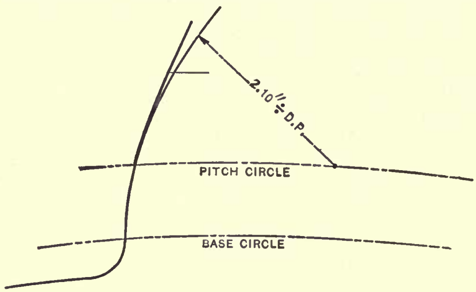


FIG. 172.—Modified Tooth Form to Avoid Interference.

outline is used to finish the drawing, and to mark out the teeth on the pattern.

On large work the size of the base circle may be obtained by calculation more readily than by the use of the triangle, as shown in Fig. 166. When the line of action has an obliquity of 15 degrees, the diameter of the base circle will be equal to 0.966 of the pitch diameter. For 20-degree involute gears the diameter of the base circle will be 0.94 of the pitch diameter.

With the 20-degree involute system the teeth of the rack have an inclination of 70 degrees to the pitch line. With this system there will be no necessity for rounding off the points of the teeth of the rack or of a large gear unless it meshes with a gear of less than 18 teeth. When, to avoid interference, it does become necessary to round off the points of the teeth of the rack or of large gears, the same radius, 2.10 inches divided by the diametral pitch, is to be used, as in the 15-degree system, the center being on the pitch line as before.

Proportions of Gears.—A somewhat common rule is to make the rim and the arms of about the same thickness as the teeth at the root, though some make the thickness of the rim equal to the height of the tooth; and to make the diameter and length of the hub about equal to about twice the diameter of the shaft. On spoked gears, the rim is also stiffened by ribbing it between the arms. On a light gear mounted on a relatively large shaft it would be natural to lighten the hub somewhat. The width of the face of cast gears is usually made from two to three times the circular pitch. The face of bevel gears should not exceed one-fifth of the diameter of the large gear, and the face of worm gears should not exceed one-half of the diameter of the worm.

Strength of Gear Teeth.—When a gear is to be designed for a given work, the first question is how large to make the teeth to give the required strength. On their size will also depend the general proportions of the gear.

It is comparatively easy to determine the work which the teeth are doing, that is, the strain or load which they are bearing, when the power which the gear transmits is known. A horse-power being the power required to lift 33,000 pounds one foot in one minute, the load on the teeth will be 33,000 multiplied by the horse-power which is being transmitted, and divided by the velocity of the pitch line of the gear in feet per minute; or, what is the same thing, 126,050 multiplied by the horse-power, and divided by the product of the pitch diameter in inches multiplied by the number of revolutions per minute. This latter figure, 126,050, takes into account the fact that in the first case the velocity is expressed in feet, while in this case the diameter is in inches, and also the fact that the velocity is a factor of the circumference instead of the diameter.

While the load on the teeth may be readily determined, the question of how large they should be made to bear it is one where authorities have differed very much on account of the number of factors involved. First of all is the question of the material, usually cast iron, which is a variable quantity, both on account of the nature of the material itself, different grades varying greatly as to strength, and the liability of defects in the casting. Then there is the question of whether the load should be considered as divided between two or more teeth or carried by one tooth, or the corner of a tooth.

Then there is the nature of the work: whether the load will be uniform or whether the teeth will be subject to severe strain or shock. There are questions of the shape of the tooth, and the velocity at which the gear is running, the teeth having greater strength at slow speeds than at high speeds due to the shocks accompanying high velocities.

To show the different results given by different writers we may take the case of a gear 24 inches diameter, 2 inches circular pitch, 4 inches face, running at 100 revolutions per minute. A rule given by Box in his treatise on mill gearing, and quoted by Grant and Kent, would make the gear safe for 9.4 horse-power. The rule in Nyström's *Mechanics* gives 12.2 horse-power. Rules by other writers, quoted by Kent, give results as follows: Halsey, 22.6; Jones & Laughlin, 35; Harkness, 38; Lewis, 65.2. The rule by Prof. Harkness is the result of investigations conducted by him in 1886. He examined a great many rules, largely, however, for common cast gears. Mr. Lewis's method, the result of his investigations of modern machine molded and cut gears, though giving much higher results than the others, is said to have proved satisfactory in an extensive practice, and so may be considered reliable for gears which are so well made that the pressure bears along the face of the teeth instead of upon the corners.

It is customary in calculating gears to proceed on the assumption that the load is borne by one tooth, and in ordinary work, the size of the tooth may be determined by the load it may safely bear per inch of face and per inch of circular pitch.

In 1879, J. H. Cooper selected an old English rule giving the breaking load of the tooth as $2000 \times$ pitch $\times$ face, which, allowing a factor of safety of 10, would give us a safe load of $200 \times$ pitch $\times$ face. Kent says of this rule that for rough ordinary work it "is probably as good as any, except that the figure 200 may be too high for weak forms of tooth, and for high speeds." Lewis also considers this rule as a passably correct expression of good general averages.

The value given by Nyström and those given by Box for teeth of small pitch, are so much smaller than those of other authorities that Kent says they may be rejected as giving unnecessary strength. Accepting the factor 200 as a good average would leave one room for the exercise of individual judgment for the particular case in hand. If the speed were slow and the teeth were of strong shape, as where both the gears of a pair, or all of the gears of a train, have a reasonably large number of teeth, a higher figure, perhaps 225 or more, might be taken; while if the speed were higher and one of the gears had but few teeth, giving them a weak form, or if they were to be subject to much vibration or shock, a lower figure, perhaps as low as 125, might be taken.

To ascertain the horse-power safely transmitted by an existing gear, we would then multiply together its diameter, pitch (circular) and face, taken in inches, and the number of revolutions per minute, and multiply their product by 200, or whatever figure is selected, and divide the total product by 126,050. This may, perhaps, be expressed clearer, as follows:

$$\text{Horse-power} = \frac{\text{diam.} \times \text{rev.} \times \text{circ. pitch} \times \text{face} \times 200}{126{,}050.}$$

The figure 200 would give to the 24-inch gear previously considered 30.5 horse-power. The figure 125 would give 19.0 horse-power.

To ascertain the size of the teeth to transmit a given horse-power we may transpose the above rule and say that the product of the pitch multiplied by the face would be equal to 126,050 multiplied by the horse-power, and divided by the product of the diameter in inches, the number of revolutions per minute, and 200, or the figure selected; that is:

$$\text{Circ. pitch} \times \text{face} = \frac{126,050 \times \text{horse-power}}{\text{diam.} \times \text{rev.} \times 200.}$$

Assuming some pitch and dividing this result by it would give the breadth of face. A few trials will give the desired ratio between pitch and breadth of face. If one has a table of square roots at hand, the work may be simplified by assuming some desired ratio, when the pitch will be the square root of the quotient of this figure, pitch multiplied by the face, divided by the ratio. If, for instance, the pitch multiplied by the face were found to be 12, and we desired them to be in the ratio of $2\frac{1}{2}$ to 1, the pitch would be equal to the square root of the quotient of 12 divided by $2\frac{1}{2}$, or 2.191, which would be about the same as $1\frac{1}{2}$ diametral pitch.

Example.—Required the size of the teeth of a gear 18 inches in diameter, to run 120 revolutions per minute, which shall transmit five horse-power, allowing 200 pounds load per inch of face, and inch of pitch. Then:

$$\text{Pitch} \times \text{face} = \frac{126,050 \times 5}{18 \times 120 \times 200} = \frac{630,250}{432,000} = 1.46$$

nearly. A circular pitch of 0.785 inch, corresponding to 4 diametral pitch, would give a breadth of face of about $1\frac{7}{8}$ inches. For bevel gears take the diameter and pitch at the middle of the face.

Mr. Lewis's method differs from the preceding in that instead of using a single constant, as 200 pounds per inch of pitch and inch of face, two constants are used, one, *Y*, a factor of strength depending on the number of teeth in the gear, and another, *S*, a safe working stress for different speeds of the pitch line, in feet per minute. The values of these constants are given in the accompanying tables.

The rule to get the horse-power of a given gear is:

$$\text{H.P.} = \frac{\text{circ. pitch} \times \text{face} \times \text{velocity} \times S \times Y}{33{,}000}$$

the velocity being that at the pitch line in feet per minute, and the values of *S* and *Y* being taken from the tables. The velocity is, of course, the diameter in feet × 3.1416 × number of revolutions. If the diameter were taken in inches then the total product would be divided by 12. The product of the pitch multiplied by the face, to determine the size of teeth to transmit a given power, would then be

$$\text{Circ. pitch} \times \text{face} = \frac{33{,}000 \times \text{H. P.}}{\text{velocity} \times S \times Y.}$$

The calculation should be made for the gear of the pair or train having the fewest teeth, as it would be the weakest, unless it were made of some stronger material as steel, or unless it were

WORKING STRESS, S, FOR DIFFERENT SPEEDS AT PITCH LINE IN FEET PER MINUTE, FOR CAST IRON.

Speed.	S.	Speed.	S.
100 or less	8000	900	3000
200	6000	1200	2400
300	4800	1800	2000
600	4000	2400	1700

shrouded. If made of steel S might be taken $2\frac{1}{2}$ times the tabulated values.

As a gear with cut teeth has from two to three times the strength of one with cast teeth, because of the more perfect contact, Mr. Lewis's method might be adapted to common cast gears by taking the value of S at from one-half to one-third of the tabulated value. By so doing one could bring into the calculation the question of shape of teeth and

FACTOR FOR STRENGTH, Y, TO BE USED IN LEWIS'S FORMULAS.

No. of teeth.	20 degree involute	15 degree involute and cycloidal.	No. of teeth.	20 degree involute	15 degree involute and cycloidal.	No. of teeth	20 degree involute.	15 degree involute and cycloidal
12	0.078	0.067	20	0.102	0.090	43	0.126	0.110
13	0.083	0.070	21	0.104	0.092	50	0·130	0.112
14	0.088	0.072	23	0.106	0.094	60	0.134	0.114
15	0.092	0.075	25	0.108	0.097	75	0.138	0.116
16	0.094	0.077	27	0.111	0.100	100	0.142	0.118
17	0.096	0.080	30	0.114	0.102	150	0.146	0.120
18	0.098	0.083	34	0.118	0.104	300	0.150	0.122
19	0.100	0.087	38	0.122	0.107	Rack	0.154	0.124

speed, which would be especially desirable if the speed were high or the teeth of weak form. Taking S at one-half the tabulated value would give to the 24-inch gear previously considered about the same power as allowing 200 pounds per inch of pitch and face, which Mr. Lewis considers a fair value. With cast gears where interchangeability is not a necessary feature, the teeth of a small gear could of course be considerably strengthened in the manner previously indicated for epicycloidal gears; or the 20-degree system might be used if the teeth have the involute form.

Thurston's Rule for Shafts.—The size of shaft which the gear will require may be found by the rule given by Thurston. Multiply the horse-power to be transmitted by 125 for iron, or by 75 for cold rolled iron, and divide the product by the number of revolutions per minute. The cube root of the quotient will be the size of the shaft.

The size of gear to give a required speed may be readily determined from the fact that the product of the speed of the driving shaft multiplied by the size of the driving gear or gears, should be equal to the product of the speed of the driven shaft, multiplied by the size of the driven gear or gears. This, perhaps, may be made clearer by placing the driving members on one side of a line, and the driven members on the other side, as in the following example.

A shaft making 75 turns per minute has on it a gear of 200 teeth. Required the size of gear to mesh with it which shall drive its shaft 120 revolutions per minute. Letting x represent the size of the required gear we have

Rev. driving shaft	= 75	x	= size driven gear.
Size driving gear	= 200	120	= rev. driven shaft.

Then as the product of the numbers on one side of the line equals the product of those on the other side, $75 \times 200 \div 120$ will give the value of x, the number of teeth in the driven gear. This method applies to a train of gears as well as a pair.

CHAPTER XIII

CALCULATING THE DIMENSIONS OF GEARS

In the previous chapter, the general principles of gearing have been explained. The three kinds of gearing most commonly in use, spur gearing, bevel gearing and worm gearing, have been touched upon, and the fundamental rules for the dimensions of gear teeth have been given. In this chapter it is proposed to give in detail the rules and formulas for these three classes of gears, so as to enable the student to calculate for himself any general problem in gearing with which he may meet.

Spur Gearing.—In the following, machine cut gearing is, in particular, referred to; but the general formulas are, of course, of equal value for use when calculating cast gears. The expressions pitch diameter, diametral pitch and circular pitch have already been explained, and rules have been given for transferring circular pitch into diametral pitch, and *vice versa*. These rules, expressed as formulas, would be:

$$P = \frac{3.1416}{P'}, \text{ and } P' = \frac{3.1416}{P}$$

in which P = diametral pitch, and
P' = circular pitch.

Assume as an example that the diametral pitch of a gear is 4. What would be the circular pitch of this gear?

Using the formula given, we have:

$$P' = \frac{3.1416}{4} = 0.7854 \text{ inch.}$$

When the diametral pitch and the pitch diameter are known, the number of teeth may be found by multiplying the pitch diameter by the diametral pitch, as already mentioned in the previous chapter. This rule, expressed as a formula, would be:

$$N = P \times D$$

in which N = number of teeth,
D = pitch diameter, and
P = diametral pitch.

Assume that the diametral pitch of a gear is 4 and the pitch diameter $6\frac{1}{4}$ inches. What would be the number of teeth in this gear?

By inserting the given values in the formula above, we would have:

$$N = 4 \times 6\tfrac{1}{4} = 25 \text{ teeth.}$$

If the number of teeth and pitch diameter of the gear are known, and the diametral pitch is to be found, a rule and formula for this may be arrived at by merely transposing the rule and formula just given. The diametral pitch equals the number of teeth divided by the pitch diameter, or, expressed as a formula:

$$P = \frac{N}{D}$$

in which P, N and D signify the same quantities as in the previous formula.

Assume, for an example, that the number of teeth in a gear equals 35 and that the pitch diameter is $3\frac{1}{2}$ inches. What is the diametral pitch?

If we insert the known values in the given formula, we have:

$$P = \frac{35}{3\frac{1}{2}} = 10 \text{ diametral pitch.}$$

Finally, if the diametral pitch and the number of teeth are known, the pitch diameter is found by dividing the number of teeth by the diametral pitch, which rule expressed as a formula, would be:

$$D = \frac{N}{P}.$$

As an example, assume that the number of teeth in a gear is 58 and the diametral pitch 6. What is the pitch diameter of this gear?

By inserting the known values in the formula, we find:

$$D = \frac{58}{6} = 9.667 \text{ inches.}$$

If it now be required to find the outside diameter of the gear, that is, the diameter of the gear blank, we make use of the following rule: The outside diameter equals the number of teeth plus 2, divided by the diametral pitch. Expressed as a formula, this rule is:

$$D' = \frac{N + 2}{P}$$

in which D' = outside diameter of gear, and N and P have the same significance as before.

As an example, assume that the number of teeth is 58 and the diametral pitch 6. By inserting these values in the formula, we find the outside diameter:

$$D' = \frac{58 + 2}{6} = \frac{60}{6} = 10 \text{ inches.}$$

When the pitch diameter and the diametral pitch are known, the outside diameter is found as follows: Add the quotient of 2 divided by the diametral pitch to the pitch diameter; the sum is the outside diameter. This rule, expressed as a formula, is:

$$D' = D + \frac{2}{P}$$

in which the letters have the same significance as before.

Assume that the pitch diameter of a gear is 9.667 inches, and the diametral pitch 6. Find the outside diameter.

By inserting the given values in the formula, we have:

$$D' = 9.667 + \frac{2}{6} = 9.667 + 0.333 = 10 \text{ inches.}$$

By a transposition of the rule and formula just given, we find that the pitch diameter equals the outside diameter minus the quotient of 2 divided by the diametral pitch. This rule, written as a formula, is:

$$D = D' - \frac{2}{P}$$

Assume that the diametral pitch of a gear is 8, and the outside diameter 12 inches. What is the pitch diameter?

$$D = 12 - \frac{2}{8} = 12 - \tfrac{1}{4} = 11\tfrac{3}{4} \text{ inches.}$$

When the number of teeth and outside diameter are known, the diametral pitch may be found by adding 2 to the number of teeth and dividing the sum by the outside diameter; or, expressed as a formula:

$$P = \frac{N + 2}{D'}.$$

If the number of teeth in a gear is 96 and the outside diameter is 14 inches, what is the diametral pitch?

If the known values are inserted in the given formula, we have:

$$P = \frac{96 + 2}{14} = \frac{98}{14} = 7 \text{ diametral pitch.}$$

When the outside diameter and the number of teeth are known, the pitch diameter may be found by multiplying the outside diameter by the number of teeth, and dividing the product by the sum of 2 added to the number of teeth; or, as a formula:

$$D = \frac{D' \times N}{N + 2}.$$

Find the pitch diameter for the gear having 96 teeth and an outside diameter of 14 inches.

$$D = \frac{14 \times 96}{96 + 2} = \frac{1344}{98} = 13.714 \text{ inches.}$$

When it is required to find the center distance C between two gears in mesh with each other, we must first know the pitch diameters of, or the number of teeth in, the two gears. The center distance equals one-half of the sum of the pitch diameters of the two gears:

$$C = \frac{D + d}{2}$$

in which D and d denote the pitch diameters in the large and small meshing gears, respectively.

The pitch diameters of two gears equal 9.5 and 7 inches, respectively. Find the center distance between them when in mesh.

$$C = \frac{9.5 + 7}{2} = \frac{16.5}{2} = 8.25 \text{ inches.}$$

The center distance is also equal to the sum of the numbers of teeth in the two gears divided by two times the diametral pitch; or, as a formula:

$$C = \frac{N + n}{2P}$$

in which N and n denote the numbers of teeth in the meshing gears.

As an example, assume that the number of teeth in each two gears equals 95 and 75. The diametral pitch is 10. What is the center distance?

$$C = \frac{95 + 75}{2 \times 10} = \frac{170}{20} = 8.5 \text{ inches.}$$

We will now find the dimensions of the tooth parts. The addendum (see Fig. 160) equals 1 divided by the diametral pitch. Expressed as a formula:

$$A = \frac{1}{P}$$

in which A = addendum.

What is the addendum or height above the pitch line of a 5 diametral pitch gear tooth?

$$A = \frac{1}{5} = 0.2 \text{ inch.}$$

The dedendum (see Fig. 160) equals the addendum.

The clearance, c, equals 0.157 divided by the diametral pitch, or:

$$c = \frac{0.157}{P.}$$

What is the clearance at the bottom of the gear tooth (see Fig. 160) of a 4 diametral pitch gear?

$$c = \frac{0.157}{4} = 0.039 \text{ inch.}$$

The full depth of the tooth equals the sum of the addendum, dedendum, and clearance, or

$$d' = \frac{1}{P} + \frac{1}{P} + \frac{0.157}{P} = \frac{2.157}{P}$$

in which d' = full depth of gear tooth.

What is the full depth of a 4 diametral pitch tooth?

$$d' = \frac{2.157}{4} = 0.539 \text{ inch.}$$

The thickness of a cut gear tooth at the pitch line equals 1.5708 divided by the diametral pitch; or, as a formula:

$$T = \frac{1.5708}{P}$$

in which T = thickness of tooth at pitch line.

What is the thickness at the pitch line of a 4 diametral pitch gear tooth?

$$T = \frac{1.5708}{4} = 0.3927 \text{ inch.}$$

As a general example, let it be required to determine the various dimensions for a pair of gears, the one having 36 and the other 27 teeth. The gears are of 8 diametral pitch.

By using the formulas given, we have:

For the larger gear:

$$\text{Pitch diameter} = \frac{N}{P} = \frac{36}{8} = 4.5 \text{ inches.}$$

$$\text{Outside diameter} = \frac{N+2}{P} = \frac{36+2}{8} = 4.75 \text{ inches.}$$

For the smaller gear:

$$\text{Pitch diameter} = \frac{n}{P} = \frac{27}{8} = 3.375 \text{ inches.}$$

$$\text{Outside diameter} = \frac{n+2}{P} = \frac{27+2}{8} = 3.625 \text{ inches.}$$

For both gears:

$$\text{Addendum} = \frac{1}{P} = \frac{1}{8} = 0.125 \text{ inch.}$$

$$\text{Dedendum} = \frac{1}{P} = \frac{1}{8} = 0.125 \text{ inch.}$$

$$\text{Clearance} = \frac{0.157}{P} = \frac{0.157}{8} = 0.0196 \text{ inch.}$$

$$\text{Full depth of tooth} = \frac{2.157}{P} = \frac{2.157}{8} = 0.2696 \text{ inch.}$$

$$\text{Center distance} = \frac{N+n}{2P} = \frac{36+27}{2 \times 8} = \frac{63}{16} = 3\tfrac{15}{16} \text{ inch.}$$

This concludes the required calculations necessary for a pair of spur gears.

Bevel Gears.—Bevel gears are used for transmitting motion between shafts whose shafts are not parallel, but whose center lines form an angle with each other. In most cases this angle is a

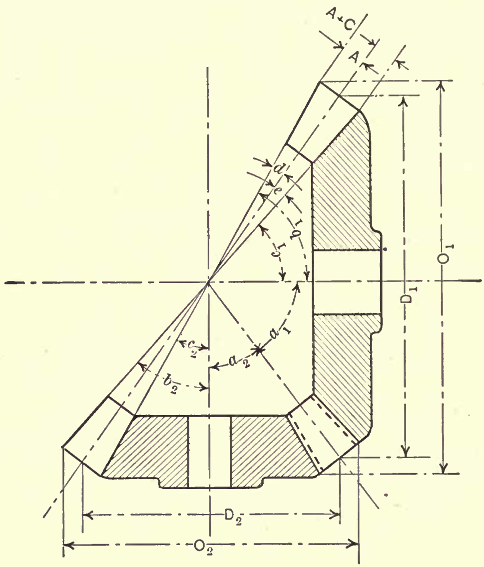


FIG. 173.—Diagram for Calculation of Bevel Gearing.

right, or 90-degree, angle. The formulas for the dimensions of bevel gears are not as simple as those for spur gears, and an understanding of the trigonometrical functions, explained in Chapter VII, is necessary, as well as the use of trigonometrical tables. As bevel gears with a 90-degree angle between their center lines are the most common, formulas will be given for this case only, in the following.

In Fig. 173 a pair of bevel gears are shown, the dimensions of which are to be determined. The letters in the formulas below denote the following quantities:

P = diametral pitch,
D_1 = pitch diameter of large gear,
D_2 = pitch diameter of small gear,
O_1 = outside diameter of large gear,
O_2 = outside diameter of small gear,
N_1 = number of teeth in large gear,
N_2 = number of teeth in small gear,
N_1' = number of teeth for which to select cutter for large gear,
N_2' = number of teeth for which to select cutter for small gear,
a_1, b_1, c_1, a_2, b_2, c_2, d and e = angles as shown in Fig. 173.
A = addendum,
$A + C$ = dedendum = addendum plus clearance.

If the pitch diameter and diametral pitch are known, the number of teeth equals the pitch diameter multiplied by the diametral pitch, or:

$$N_1 = D_1 \times P$$
$$N_2 = D_2 \times P$$

If the number of teeth and the diametral pitch are known, the pitch diameter equals the number of teeth divided by the diametral pitch, or:

$$D_1 = \frac{N_1}{P}$$

$$D_2 = \frac{N_2}{P}$$

Angles a_1 and a_2 can be determined if either the numbers of teeth or the pitch diameters of both gears are known. The tangent for these angles, the pitch cone angles, equals the number of teeth in one gear divided by the number of teeth in the other, or the pitch diameter in one gear divided by the pitch diameter in the other, according to the following formulas:

$$\tan a_1 = \frac{N_1}{N_2} = \frac{D_1}{D_2}$$

$$\tan a_2 = \frac{N_2}{N_1} = \frac{D_2}{D_1}$$

Angle a_2 also equals $90^\circ - a_1$.

The outside diameter equals the pitch diameter plus the quotient of 2 times the cosine of a_1 or a_2, respectively, divided by the diametral pitch, or:

$$O_1 = D_1 + \frac{2 \cos a_1}{P}$$

$$O_2 = D_2 + \frac{2 \cos a_2}{P}$$

Angles d and e are determined by the formulas:

$$\tan d = \frac{2 \sin a_1}{N_1} = \frac{2 \sin a_2}{N_2}$$

$$\tan e = \frac{2.314 \sin a_1}{N_1} = \frac{2.314 \sin a_2}{N_2}$$

Angles b_1, c_1, b_2 and c_2 are determined by the formulas:

$$b_1 = a_1 + d$$

$$c_1 = a_1 - e$$

$$b_2 = a_2 + d$$

$$c_2 = a_2 - e$$

The number of teeth for which the cutter for cutting the teeth should be selected is found as follows:

$$N_1' = \frac{N_1}{\cos a_1}$$

$$N_2' = \frac{N_2}{\cos a_2}$$

Finally the addendum, dedendum and clearance are found as in spur gears.

As a practical example, assume now that two bevel gears are required, 8 diametral pitch, with 24 and 36 teeth, respectively. Find the various dimensions.

$$D_1 = \frac{N_1}{P} = \frac{36}{8} = 4.5 \text{ inches.}$$

$$D_2 = \frac{N_2}{P} = \frac{24}{8} = 3 \text{ inches.}$$

$$\tan a_1 = \frac{N_1}{N_2} = \frac{36}{24} = 1.5; \; a_1 = 56° \; 20'.$$

$$\tan a_2 = \frac{N_2}{N_1} = \frac{24}{36} = 0.667; \; a_2 = 33° \; 40'.$$

$$O_1 = D_1 + \frac{2 \cos a_1}{P} = 4.5 + \frac{2 \times 0.554}{8} = 4.638 \text{ inches.}$$

$$O_2 = D_2 + \frac{2 \cos a_2}{P} = 3 + \frac{2 \times 0.832}{8} = 3.208 \text{ inches.}$$

$$\tan d = \frac{2 \sin a_1}{N_1} = \frac{2 \times 0.832}{36} = 0.046; \; d = 2° \; 40'.$$

$$\tan e = \frac{2.314 \sin a_1}{N_1} = \frac{2.314 \times 0.832}{36} = 0.053; \; e = 3° \; 0'.$$

$$b_1 = a_1 + d = 56° \, 20' + 2° \, 40' = 59° \, 0'.$$
$$c_1 = a_1 - e = 56° \, 20' - 3° \, 0' = 53° \, 20'.$$
$$b_2 = a_2 + d = 33° \, 40' + 2° \, 40' = 36° \, 20'.$$
$$c_2 = a_2 - e = 33° \, 40' - 3° \, 0' = 30° \, 40'.$$

$$N_1' = \frac{N_1}{\cos \ a_1} = \frac{36}{0.554} = 65 \text{ approximately.}$$

$$N_2' = \frac{N_2}{\cos \ a_2} = \frac{24}{0.832} = 29 \text{ approximately.}$$

$$A = \frac{1}{P} = \frac{1}{8} = 0.125 \text{ inch.}$$

$$C = \frac{0.157}{8} = 0.0196 \text{ inch.}$$

$$\text{Whole depth of tooth} = \frac{1}{P} + \frac{1}{P} + \frac{0.157}{P} = 0.2696 \text{ inch.}$$

Worm Gearing.—Worms and worm gears are used for transmitting power in cases where great

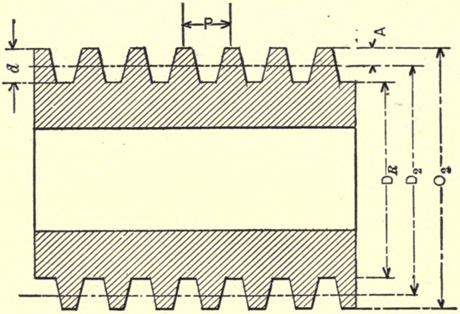


FIG. 174.—Worm.

reduction in velocity and smoothness of action are desired. They are also used when a self-locking power transmission is desirable, that is, when it is required that the mechanism itself, due to the friction between the worm and worm-wheel, should support the load without slipping if the driving power be rendered inoperative.

In Figs. 174 is shown a worm and in Fig. 175 a worm-wheel; the dimensions to be found are, in most cases, given in these illustrations. The following notation has been used in the formulas given below for worm and worm-wheels:

P = circular pitch of worm-wheel = pitch of the worm thread,
N = number of teeth in worm-wheel,
D_1 = pitch diameter of worm-wheel,
D_T = throat diameter of worm-wheel,
O_1 = outside diameter of worm-wheel (to sharp corners),
R = radius of worm-wheel throat,
C = center distance between worm and worm-wheel axes,
D_2 = pitch diameter of worm,
O_2 = outside diameter of worm,
D_R = root diameter of worm,
A = addendum, or height of worm tooth above pitch line,
d = depth of worm tooth,
a = face angle of worm-wheel.

If the pitch of the worm and the number of teeth in the worm-wheel are known, the pitch diameter of the worm-wheel may be found by multiplying the pitch of the worm by the number of teeth, and dividing the result by 3.1416, or, as a formula:

$$D_1 = \frac{P \times N}{3.1416}$$

The outside diameter of the worm, O_2, is usually assumed. To find the pitch diameter of the worm, the addendum must first be found. The addendum equals the pitch of the worm thread multiplied by 0.3183, or:

$$A = P \times 0.3183.$$

Now the pitch diameter of the worm equals the outside diameter minus 2 times the addendum, or:

$$D_2 = O_2 - 2A.$$

The root diameter of the worm can be found first after the full depth of the worm-wheel thread has been found. The full depth of the worm-wheel thread equals the pitch multiplied by 0.6866, or:

$$d = P \times 0.6866.$$

Now the root diameter of the worm thread equals the outside diameter of the worm minus 2 times the depth of the thread, or:

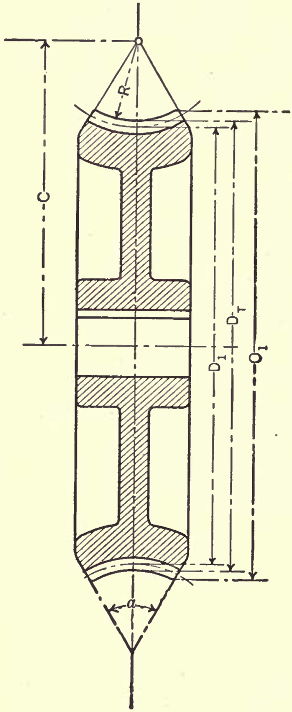


FIG. 175.—Worm-wheel.

$$D_R = O_2 - 2d.$$

The throat diameter of the worm-wheel is found by adding 2 times the addendum of the worm thread to the pitch diameter of the worm-wheel, or:

$$D_T = D_1 + 2A.$$

The radius of the worm-wheel throat is found by subtracting 2 times the addendum from the outside diameter of the worm divided by 2, or:

$$R = \frac{O_2}{2} - 2A.$$

The outside diameter of the worm-wheel (to sharp corners) is found by the formula below:

$$O_1 = D_T + 2\left(R - R \cos \frac{a}{2}\right)$$

The angle a is usually 75 degrees.

Finally, the center distance between the center of the worm and the center of the worm-wheel equals the sum of the pitch diameter of the worm plus the pitch diameter of the worm gear, and this sum divided by 2, or:

$$C = \frac{D_1 + D_2}{2}$$

Find, for an example, the required dimensions for a worm and worm-wheel, in which the worm-wheel has 36 teeth, the pitch of the worm thread is ½ inch, and the outside diameter of the worm is 3 inches. We have given $P = \frac{1}{2}$; $N = 36$; $O_2 = 3$.

$$D_1 = \frac{P \times N}{3.1416} = \frac{\frac{1}{2} \times 36}{3.1416} = 5.730 \text{ inches.}$$

$$A = P \times 0.3183 = \tfrac{1}{2} \times 0.3183 = 0.15915 \text{ inch.}$$

$$D_2 = O_2 - 2A = 3 - 0.3183 = 2.6817 \text{ inches.}$$

$d = P \times 0.6866 = \frac{1}{2} \times 0.6866 = 0.3433$ inch.

$D_R = O_2 - 2d = 3 - 0.6866 = 2.3134$ inches.

$D_T = D_1 + 2A = 5.730 + 0.3183 = 6.0483$ inches.

$$R = \frac{O_2}{2} - 2A = \frac{3}{2} - 0.3183 = 1.1817 \text{ inch.}$$

$$O_1 = D_T + 2\left(R - R \cos \frac{a}{2}\right) = 6.0483 + 2 \times$$

$(1.1817 - 1.1817 \times \cos 37° \, 30') = 6.5375$ inches.

$$C = \frac{D_1 + D_2}{2} = \frac{5.730 + 2.6817}{2} = 4.2058 \text{ inches.}$$

CHAPTER XIV

CONE PULLEYS

WHEN it is desired to have a variable speed ratio between two shafts which are belted together, the method of having reversed conical cylinders or drums mounted on the shafts, as shown in Fig. 176 and 177, is sometimes used. These permit any

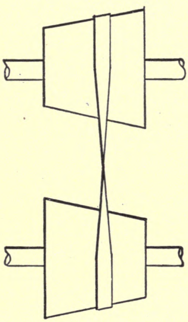


FIG. 176.—Simplest Form of "Cone-Pulley."

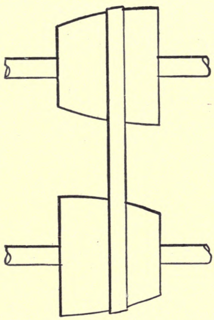


FIG. 177.—An Improved Form of "Cone-Pulley."

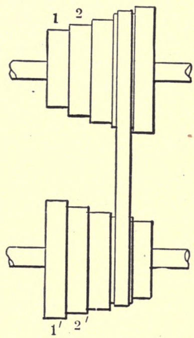


FIG. 178.—The Modern Type of Stepped Cone Pulley.

desired change of speed, but they have disadvantages which on most work offset this advantage. It would be necessary, in the first place, to use a narrow belt to avoid undue stretching at the edges. Then, as the tendency of a belt is to mount to the largest part of a pulley, this tendency, acting in the same way on the cones, would produce undue tension on the belt. If a crossed belt is used on such cones their faces would be made straight, as the belt would be equally tight in any position. This may be seen by an inspection of Fig. 179, where circles *A* and *B* represent sections of such cones on one line, and circles *C* and *D* represent sections on another line. If the cones have the

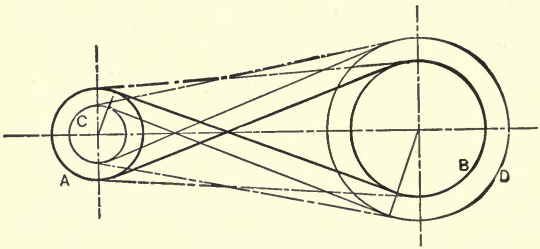


FIG. 179.—Diagram Showing relative Influence of Open and Crossed Belt on Pulley Sizes.

same taper it is evident that the circle *D* will be as much larger than *B* as *C* is smaller than *A*, the gain in one diameter being offset by the loss in the other. Then, as the circumferences of circles vary directly as their diameters (the circumference of a circle having twice the diameter of another, for instance, will be twice as long as the circumference of the other), whatever is gained on one circumference will be lost on the other. For a crossed belt then, it is only necessary that the cones have the same taper.

When, however, an open belt is used, it becomes necessary to have the cones slightly bulging in the middle as shown in Fig. 177. By again inspecting Fig. 179 it will be seen that it is only when the belt is crossed that one cone gains as fast in size as the other loses, because it is only when the belt is crossed that the arc of contact of the belt on the pulleys is the same on all steps of the cone.

In practice these cones are usually replaced by stepped or cone pulleys as shown in Fig. 178, so as to avoid the troubles with the belt previously mentioned.

Applying the principles mentioned to cone pulleys, we see that when a crossed belt is used, all that is necessary is that the sum of the diameters of any pair of steps shall be equal to the sum of the diameters of any other pair of steps. For instance, the sum of the diameters of steps 1 and 1′ must be equal to the sum of the diameters of steps 2 and 2′. When, however, an open belt is used, as is usually the case, the sum of the diameters of the steps at or near the middle of the cone will have to be somewhat greater than the sum of the diameters of those at or near the ends.

What is generally considered to be the best method of determining the size of the various steps of cone pulleys is that given by Mr. C. A. Smith in the "Transactions of the American Society of Mechanical Engineers," Vol. X, page 269. Make the distance *C*, Fig. 180, equal to the distance between the centers of the shafts, and draw the circles *A* and *B* equal to the diameters of a known pair of steps on the cones. At a point midway between the shaft centers erect the perpendicular *ab*. Then, with a center on *ab* at a distance from *a* equal to the length of *C* multiplied by 0.314, draw the arc *c* tangent to the belt line of the given pair of steps. The belt line of any other pair of steps will then be tangent to this arc.

If the angle which the belt makes with the line of centers, *de*, exceeds 18 degrees, however, a slight modification of the above is made as follows: Draw a line tangent to the arc at *c* at an angle of 18 degrees with *de*; and with a center on *ab*, at

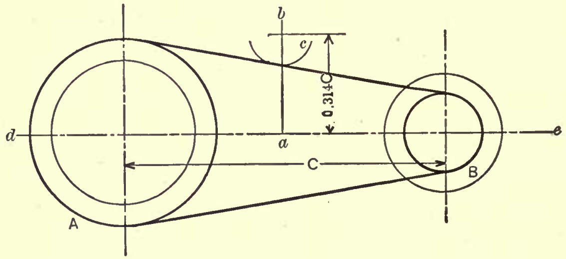


FIG. 180.—Method of Laying out Cone Pulleys.

a distance from *a* equal to the length *C* multiplied by 0.298 draw an arc tangent to this 18-degree line.

All belt lines which make an angle with *de* greater than 18 degrees are made tangent to this new arc.

The sizes of the steps so obtained may be verified by measuring the belt lengths of each pair. For this purpose a fine wire may be used, the wire being held in place by pins placed at close intervals on the outer half circumference of each pulley of the pair.

CHAPTER XV

BOLTS, STUDS AND SCREWS

SCREWS for clamping work together are of three classes: through bolts, Fig. 181; studs, Fig. 182; cap screws, Fig. 183. In Fig. 181 the bolt is put entirely through both of the two pieces to be

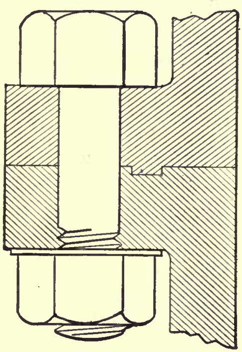


FIG. 181.—Through Bolt for Holding two Pieces together.

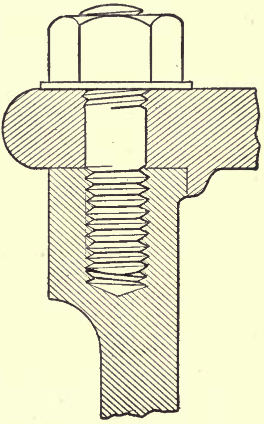


FIG. 182.—Stud used for Clamping one Piece to another.

clamped together, and a nut is put onto the threaded end. This is considered to be the best method on cast iron work, both as regards efficiency and cheapness, as there is no tapping of any holes in the cast iron. A tapped hole in cast iron is to be avoided, if possible, as, on account of the brittle nature of the material, the threads are liable to crumble or wear away easily.

In many cases, however, it is not practicable to avoid tapping holes in cast iron, or questions of appearance may make the broad flange which is necessary when through bolts are used, undesirable. In such cases studs should be used. A stud consists of a piece of round stock threaded on both ends, and having a plain portion in the middle. The studs are screwed firmly into the tapped holes, which should be deep enough to prevent the studs from bottoming in them, the studs instead binding or coming to a bearing at the end of the threaded portion. The loose piece is then put on over the studs, and is held in place by the nuts. By using studs, any further wear of the tapped hole is avoided, as, when removing the loose part, the nuts only are taken off, the studs being left in the body piece.

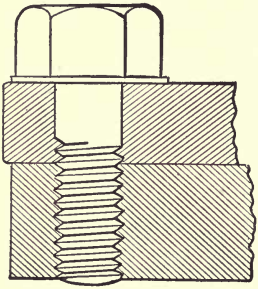


FIG. 183.—Cap Screw used for Clamping Purposes.

When the material of the parts which are being clamped together is of such a nature that threads formed in it are not liable to crumble or to rapid wear, then cap screws, Fig. 183, may be used to advantage. They give a neat appearance to a piece of work, and the nut is entirely eliminated.

United States Standard Screw Thread.—The most commonly used of all screw threads is the United States standard thread. A section, indicating the form of this thread, is shown in Fig. 184. The thread is not sharp neither at the top nor at the bottom, but is provided with a flat at both of these points, the width of the flat being one-eighth of the pitch of the thread. The sides of the thread

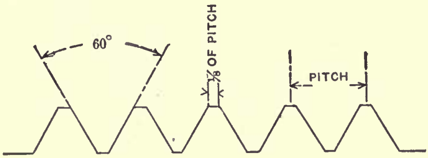


FIG. 184.—Form of the United States Standard Thread.

form an angle of 60 degrees with each other. The "pitch" and the "number of threads per inch" should not be confused. The pitch is the distance from the top of one thread to the top of the next. If the number of threads is 8 per inch, then the pitch would be $\frac{1}{8}$ inch; and the flat on the top of a United States standard thread, which, as mentioned, is one-eighth of the pitch, would be 1-64 inch. If the number of threads per inch is known, the pitch may be found by dividing 1 by the number of threads per inch, or

$$\text{Pitch} = \frac{1}{\text{No. of threads per inch.}}$$

If, again, the pitch is known and the number of threads per inch required, then

$$\text{No. of threads per inch} = \frac{1}{\text{Pitch.}}$$

U. S. STANDARD SCREW THREADS.

Bolts and Threads						Hex. Nuts and Heads.					Square Nut and Head.
Diam. of Bolt.	Threads per Inch.	Diam. at Root of Thread.	Width of Flat.	Area of Bolt Body.	Area at Root of Thread.	Width Across Flats, Rough.	Width Across Flats, Finished	Across Corners, Rough.	Thickness, Rough.	Thickness, Finished.	Across Corners, Rough
1/4	20	0.185	0.0062	0.049	0.027	1/2	7/16	37/64	1/4	3/16	45/64
5/16	18	0.240	0.0074	0.077	0.045	19/32	17/32	11/16	5/16	1/4	53/64
3/8	16	0.294	0.0078	0.110	0.068	11/16	5/8	51/64	3/8	5/16	63/64
7/16	14	0.344	0.0089	0.150	0.093	25/32	23/32	29/32	7/16	3/8	1 7/64
1/2	13	0.400	0.0096	0.196	0.126	7/8	13/16	1	1/2	7/16	1 15/64
9/16	12	0.454	0.0104	0.249	0.162	31/32	29/32	1 1/8	9/16	1/2	1 23/64
5/8	11	0.507	0.0113	0.307	0.202	1 1/16	1	1 7/32	5/8	9/16	1 1/2
3/4	10	0.620	0.0125	0.442	0.302	1 1/4	1 3/16	1 7/16	3/4	11/16	1 49/64
7/8	9	0.731	0.0138	0.601	0.420	1 7/16	1 3/8	1 21/32	7/8	13/16	2 1/32
1	8	0.837	0.0156	0.785	0.550	1 5/8	1 9/16	1 7/8	1	15/16	2 19/64
1 1/8	7	0.940	0.0178	0.994	0.694	1 13/16	1 3/4	2 3/32	1 1/8	1 1/16	2 9/16
1 1/4	7	1.065	0.0178	1.227	0.893	2	1 15/16	2 5/16	1 1/4	1 3/16	2 53/64
1 3/8	6	1.160	0.0208	1.485	1.057	2 3/16	2 1/8	2 17/32	1 3/8	1 5/16	3 3/32
1 1/2	6	1.284	0.0208	1.767	1.295	2 3/8	2 5/16	2 3/4	1 1/2	1 7/16	3 23/64
1 5/8	5 1/2	1.389	0.0227	2.074	1.515	2 9/16	2 1/2	2 21/32	1 5/8	1 9/16	3 5/8
1 3/4	5	1.491	0.0250	2.405	1.746	2 3/4	2 11/16	3 3/16	1 3/4	1 11/16	3 57/64
1 7/8	5	1.616	0.0250	2.761	2.051	2 15/16	2 7/8	3 13/32	1 7/8	1 13/13	4 5/32
2	4 1/2	1.712	0.0277	3.142	2.302	3 1/8	3 1/16	3 5/8	2	1 15/16	4 27/64
2 1/4	4 1/2	1.962	0.0277	3.976	3.023	3 1/2	3 7/16	4 1/16	2 1/4	2 3/16	4 61/64
2 1/2	4	2.176	0.0312	4.909	3.719	3 7/8	3 13/16	4 1/2	2 1/2	2 7/16	5 31/64
2 3/4	4	2.426	0.0312	5.940	4.620	4 1/4	4 3/16	4 29/32	2 3/4	2 11/16	6
3	3 1/2	2.629	0.0357	7.069	5.428	4 5/8	4 9/16	5 3/8	3	2 15/16	6 17/32
3 1/4	3 1/2	2.879	0.0357	8.296	6.510	5	4 15/16	5 13/16	3 1/4	3 3/16	7 1/16
3 1/2	3 1/4	3.100	0.0384	9.621	7.548	5 3/8	5 5/16	6 7/64	3 1/2	3 7/16	7 39/64
3 3/4	3	3.317	0.0413	11.045	8.641	5 3/4	5 11/16	6 21/32	3 3/4	3 11/16	8 1/8
4	3	3.567	0.0413	12.566	9.963	6 1/8	6 1/16	7 3/32	4	3 15/16	8 41/64
4 1/4	2 7/8	3.798	0.0435	14.186	11.32[illegible]	6 1/2	6 7/16	7 9/16	4 1/4	4 3/16	9 3/16
4 1/2	2 3/4	4.028	0.0454	15.904	12.753	6 7/8	6 13/16	7 31/32	4 1/2	4 7/16	9 3/4
4 3/4	2 5/8	4.256	0.0476	17.721	14.226	7 1/4	7 3/16	8 13/32	4 3/4	4 11/16	10 1/4
5	2 1/2	4.480	0.0500	19.635	15.763	7 5/8	7 9/16	8 27/32	5	4 15/16	10 49/64
5 1/4	2 1/2	4.730	0.0500	21.648	17.572	8	7 15/16	9 9/32	5 1/4	5 3/16	11 23/64
5 1/2	2 3/8	4.953	0.0526	23.758	19.267	8 3/8	8 5/16	9 23/32	5 1/2	5 7/16	11 7/8
5 3/4	2 3/8	5.203	0.0526	25.967	21.262	8 3/4	8 11/16	10 5/32	5 3/4	5 11/16	12 3/8
6	2 1/4	5.423	0.0555	28.274	23.098	9 1/8	9 1/16	10 19/32	6	5 15/16	12 15/16

For example, assume that the pitch is 0.0625 inch. Then

$$\text{No. of threads per inch} = \frac{1}{0.0625} = 16.$$

The accompanying table of United States standard screw threads gives the standard number of threads per inch, corresponding to given diameters, the diameter at the root of the thread, the width of the flat at the top and bottom of the thread, the area of the full bolt body, and the area at the bottom of the thread. These dimensions are, of course, always the same with all manufacturers. As regards the sizes for hexagon nuts and heads, and square nuts and heads also given in the table, it may be said that all makers do not conform strictly to the sizes as given. The catalog of one large bolt manufacturing concern, which is at hand, gives the width across flats of finished bolt heads and nuts the same as the rough sizes given in the table, which, it will be seen, are founded on the rule that the width across the flats of the heads and nuts should equal one and one-half times the diameter of the body of the bolt, plus one-eighth of an inch. It will also be noticed that the thickness of the head or nut is the same as the diameter of the body of the bolt.

With cap screws, although the length of the head is made the same as for bolts, or equal to the diameter of the bolt body, the diameter of the head, and the distance across flats, is made different as shown in table on the following page:

CAP SCREW SIZES.

(*From catalog of Boston Bolt Co.*)

Size of Screw	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1
Width Across Flats Hex. Head	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$
Width Across Flats Square Head	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{7}{8}$	$1\frac{1}{8}$	$1\frac{1}{4}$

Check or Lock Nuts.—When a bolt is subjected to constant vibrations there is a tendency for the nut to work loose. To overcome this tendency it is customary to employ a second nut, called a check or lock nut, which is screwed down upon the first one as shown in Fig. 185. When the first nut is screwed down to a bearing, the upper surfaces of its thread are in contact with the under surfaces of the bolt thread. When the check nut is screwed down, however, it forces the first nut down so that the under surfaces of its thread come into contact with the upper thread surfaces of the bolt. This means that the check nut has to bear the entire load. When, therefore, the two nuts are of unequal thickness, as is frequently the case, the thick nut should be on the outside.

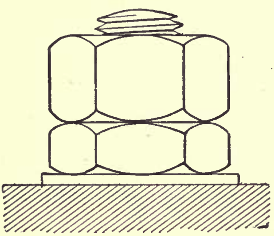


Fig. 185.—Correct Arrangement when Using Check or Lock Nut.

Bolts to Withstand Shock.—When a bolt which is subjected to shocks fails, it breaks, of course, at the part having the least cross sectional area, that is, at the bottom of the thread. If now the body of the bolt be reduced so that its cross section is of the same area as the area at the bottom of the thread, a slight element of elasticity is introduced, and the bolt is likely to yield somewhat instead of breaking. This is considered very important in some classes of work. The reduction of area may be accomplished by turning down the body of the bolt, or, according to some authorities, the same object is attained by removing stock from the inside by drilling into the bolt from the head end.

Either method, it is stated, gives the same degree of elasticity to the bolt, but as the drilling method takes the stock from the center, the bolt is left stiffer to resist bending or twisting than when the stock is taken off the outside by turning.

Wrench Action.—When bolts or any form of screws are used to hold machine parts together, they must be strong enough not only to withstand the strain which is put upon them by the operation of the machine, but also to withstand the strain which is put upon them by the wrench in setting or screwing them up. In the case of a cylinder head, for instance, the strain upon the bolts due to the working of the engine will be the exposed area of the head, multiplied by the pressure per square inch. This divided by the number of bolts used will give the proportional part of this strain which each bolt must sustain. But in order to insure a tight joint, it is necessary that the bolts be not merely brought up to a bearing, but that they be set up hard enough so as to press the cylinder and cylinder head surfaces firmly together. The force which the wrench exerts in doing this work will be equal to the circumference of the circle through which the hand moves in turning the wrench through one revolution, multiplied by the force in pounds exerted at the handle, and this product divided by the distance through which the nut advances in one revolution, that is, by the lead of the screw. This theoretical result is, of course, modified by the friction between the nut and the bolt, and between the nut and washer. In addition to this direct strain, there is also a twisting strain in the bolt, caused by the friction between the bolt and nut.

To insure the bolts being sufficiently strong to resist these various forces, it is customary to make them somewhat more than double the strength that would be necessary to enable them to safely resist the pressure of the steam or other fluid in the cylinder; that is, they are made about double strength to enable them to resist the direct strain of the wrench action, and then this amount is increased about 15 or 20 per cent. to allow for the twisting action of the wrench. Allowing that a factor of safety of 4 would be sufficient to allow for the steam pressure only, a factor of safety of not less than about 9 or 10 would therefore be used to provide for the added strain on the bolt due to the wrench action. In the case of small bolts, where the workman might set them up much harder than is really necessary, a factor of safety of about 15 may be used.

The distance apart which bolts can be spaced without danger of leakage is given by Prof. A. W. Smith as between 4 or 5 times the thickness of the cylinder flange for pressures between 100 and 150 pounds per square inch.

In the case of bolts which are not under strain as a result of the wrench action, as in the case of

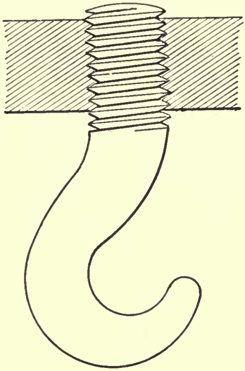


FIG. 186.—Example of Thread not under Stress due to Wrench Action.

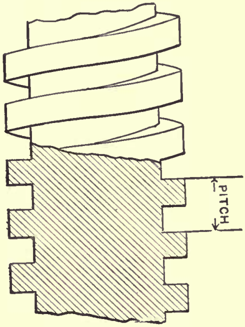


FIG. 187.—Square Threaded Screw, such as is Generally used for Power Transmission.

the hook bolt shown in Fig. 186, a factor of safety as low as 4 might be properly used, if the load is steady.

Assuming that the material of which the bolts are made has an ultimate strength of 40,000 to 60,000 pounds per square inch, the factors of safety previously indicated would give allowable working stresses of from 4000 to 15,000 pounds per square inch.

Screws for Power Transmission.—In Fig. 187 is shown a square threaded screw such as is generally used for power transmission. In such a screw the depth of the thread is made one-half of the pitch. The size of the body of the screw, assuming that the work which the screw is doing brings a tensional stress on the screw, will be determined by the tensile strength of the material of which it is made and the factor of safety which is used. As a screw which is used for power transmission is subjected to constant wear when in use, the question of the proper amount of bearing surface in the threads of the nut is of first importance, in order that it may not wear out too rapidly. The area of the thread surface in the nut on which the pressure bears will be equal to the difference in area of a circle of a diameter equal to the outside diameter of the screw, and one of a diameter equal to the diameter at the root of the thread of the screw, multiplied by the number of threads; or, letting D represent the outside diameter of the screw, and d represent the diameter of the body, the area will be:

$$(D^2 - d^2) \times 0.7854 \times \text{No. of threads in the nut.}$$

The allowable pressure per square inch of working surface will vary with the nature of the service required, whether fast or slow, and also with the lubrication, and with the material used. Where the speed is slow, say not over 50 feet per minute, and the service is infrequent, as in lifting screws, a pressure of 2500 pounds for iron or 3000 pounds for steel is allowable, while for more constant service some authorities limit the pressure to about 1000 pounds per square inch even when the lubrication is good. For high speeds a pressure of about 200 or 250 pounds is considered to be as much as should be allowed.

For a screw which, fitting loosely in a well lubricated nut, is to sustain a load without danger of running down of itself, the pitch of the screw should not, according to Professor Smith, be greater than about one-tenth of its circumference.

Efficiency of Screws.—A square-threaded screw has a greater efficiency than a V-threaded one, as the sloping sides of the V-thread cause an increase of friction. Square threads are therefore preferable for power transmission. Experiments show that in the case of bolts used for fastenings, the friction of the nut on the bolt and washer may absorb 90 per cent. of the power applied to the wrench, leaving only 10 per cent. for producing direct compression. For square-threaded screws an efficiency of about 50 per cent. is considered fair, if the screws are well lubricated.

Acme Standard Thread.—While the square thread gives the greatest efficiency in a screw it is not as strong as one having sloping sides. Fig. 188 shows a section of a screw thread called the Acme or 29-degree thread, which is often used for replacing the square thread for many purposes, such as in screws for screw presses, valve stems, and the like. The use of such a screw permits the employment of a split nut, when such construction is desirable, which would not be practicable with a perfectly square thread, and for this reason, as well as for the reason that it can be cut with greater ease than the square thread, it has of late become widely used. In the Acme standard thread system the threads on the screw and in the nut are not exactly alike. A clearance of 0.010 inch is provided at the top and at the bottom of the thread, so that if the screw is 1 inch in diameter, for example, then the largest diameter of the thread in the nut would be 1.020 inch. If the root diameter of the same screw were 0.900 inch, then the smallest diameter of the thread in the nut would be 0.920 inch. The sides of the threads, however, fit perfectly.

The depth of an Acme thread equals one-half the pitch of the thread plus 0.010 inch. The width

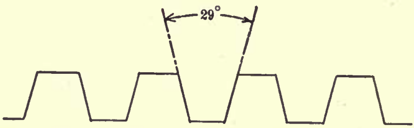


FIG. 188.—Shape of Acme Screw Thread.

of the flat at the top of the screw thread equals 0.3707 times the pitch; and the width of the flat at the bottom of the thread equals 0.3707 times the pitch minus 0.0052 inch.

Miscellaneous Screw Thread Systems.—Besides the screw thread systems already mentioned, a great many other systems are in more or less common use. Leading among these is the sharp V-thread, which, previous to the introduction of the United States standard thread, was the most commonly used thread in this country. This thread is, theoretically at least, sharp at both the top and the bottom of the thread, the angle between the sides of the thread being the same as in the United States standard system, or 60 degrees. In ordinary practice, however, a small flat is provided on the top of the thread, because it would be almost impossible to commercially produce the thread otherwise; and even if the thread could be produced, the sharp edge at the top would rapidly wear away. The sharp V-thread is being more and more forced out of use by the United States standard thread, although it must be admitted that it will probably long hold its own in steam fitting work, because of being especially adapted for making steam-tight joints. It answers this purpose probably better than any of the other common forms of threads.

The Whitworth standard thread is not used to a very great extent in the United States, but it is the recognized standard thread in Great Britain. In this form of thread the sides of the thread form an angle of 55 degrees with each other, and the tops and bottoms of the threads are rounded to a radius equal to 0.137 times the pitch. This rounding of the thread at the top provides for a thread which does not wear rapidly, and screws and nuts made according to this thread system will work well together in continuous heavy service for a longer period than would screws and nuts with any of the other standard thread forms. The fact that the threads are rounded in the bottom is advantageous on account of the elimination of sharp corners from which fractures may start. The main disadvantage of the thread, and the reason why the United States standard thread was adopted in this country in preference to the Whitworth standard, which is the older of the two, is to be found in the fact that it is more difficult to produce than a 60-degree thread with flat top and bottom. The Whitworth form of thread is used in this country mostly on special work and on stay-bolts for locomotive boilers.

A thread perhaps more commonly used than any of the others, with the exception of the United States standard thread, is the Briggs standard pipe thread, which is used, as the name indicates, for pipe fittings. This thread is similar to the sharp V-thread, having an angle of 60 degrees between the sides, and nearly sharp top and bottom; instead of being exactly sharp at the top and bottom, however, it is slightly rounded off at these points. The difficulty of producing these slightly rounded surfaces has brought about a modification, at least in the United States, so that a small flat is made at the top, and the thread made to a sharp point at the bottom. It appears that a thread cut with these modifications serves its purpose equally as well as a thread cut according to the original thread form.

Besides these systems, there are the metric screw thread systems. These use the same form of thread as the United States standard system, but the thread diameters and the corresponding pitches are, of course, made according to the metric system of measurement.

Other Commercial Forms of Screws.—Set-screws, shown in Fig. 189, are usually made with square heads, and have either round or cup-shaped points, and are generally case hardened. They are used for such work as fastening pulleys onto shafts, etc. Some set-screws are made headless, and are slotted for use with a screw-driver in places where it is undesirable that the screw projects beyond the work.

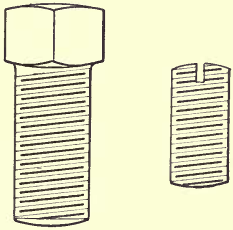


FIG. 189.—Forms of Set-screws.

The term machine screws covers a number of styles of small screws made for use with a screw-driver. Fig. 190 shows the principal styles. Machine screw sizes are usually designated by numbers, the size and the number of threads per inch being usually given together, with a "dash" between; thus a 10—24 screw would be a number 10 screw with 24 threads per inch. There are two standard systems for machine screw

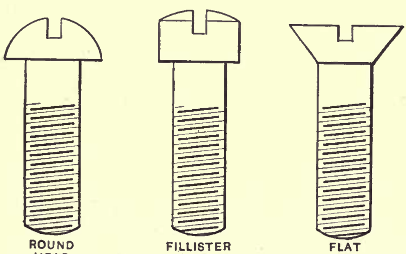


FIG. 190.—Forms of Machine Screws.

threads, the old, which until recently was the only system, and the new, which was approved in 1908 by the American Society of Mechanical Engineers. The standard thread form of the old system was the sharp V-thread, with a liberal but arbitrarily selected flat on the top. The basic thread form of the new system is that of the United States standard thread.

The accompanying tables give the numbers and corresponding diameters and number of threads per inch of the old as well as the new system for machine screw threads.

MACHINE SCREW THREADS, OLD SYSTEM.

Number.	Diameter.	Threads per inch.	Number.	Diameter.	Threads per inch.
1	0.071	64	12	0.221	24
1½	0.081	56	13	0.234	22
2	0.089	56	14	0.246	20
3	0.101	48	15	0.261	20
4	0.113	36	16	0.272	18
5	0.125	36	18	0.298	18
6	0.141	32	20	0.325	16
7	0.154	32	22	0.350	16
8	0.166	32	24	0.378	16
9	0.180	30	26	0.404	16
10	0.194	24	28	0.430	14
11	0.206	24	30	0.456	14

MACHINE SCREW THREADS, NEW SYSTEM.

Number.	Diameter.	Threads per inch.	Number.	Diameter.	Threads per inch.
0	0.060	80	12	0.216	28
1	0.073	72	14	0.242	24
2	0.086	64	16	0.268	22
3	0.099	56	18	0.294	20
4	0.112	48	20	0.320	20
5	0.125	44	22	0.346	18
6	0.138	40	24	0.372	16
7	0.151	36	26	0.398	16
8	0.164	36	28	0.424	14
9	0.177	32	30	0.450	14
10	0.190	30			

CHAPTER XVI

COUPLINGS AND CLUTCHES

A COUPLING is a device for connecting together the ends of two shafts or axles for the purpose of making a longer shaft, the term being usually limited to those devices which are intended for permanent fastening. The term clutch is used to designate a disengaging coupling.

The simplest form of coupling consists simply of a sleeve or muff, made of a length about three times the diameter of the shaft, bored out to fit the shaft, and provided with a keyway its entire length, made to receive a tapering key. The ends of the shafting are, of course, also provided with keyways, and are inserted into the sleeve; then the key is driven in. In some couplings the sleeve is made tapering on the outside at both ends, and, being split, is clamped upon the shafts by means of rings or hollow conical sleeves which are driven onto the tapered ends, or drawn together by means of bolts.

One of the most common forms of coupling is the flange coupling shown in Fig. 191. In this case a flanged hub is keyed to each of the shaft ends, and the flanges are then held together and prevented from turning relative to each other by bolts, as shown. In some cases the bolt heads and nuts are provided with a guard by having the rim on the outer edge of the flange made deep as shown by the dotted lines on one side. This construction also allows the coupling to be used as a pulley, if necessary. In a coupling of this kind, the chief problem is to get the bolts of such size that their combined strength to resist the shearing action to which they are subjected equals the twisting

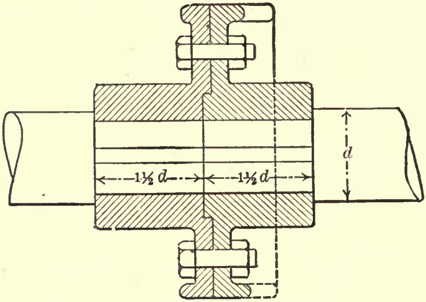


FIG. 191.—Flange Coupling.

strength of the shaft. Letting d represent the diameter of the shaft in inches, its internal resistance to twisting is given by the formula

$$T = \frac{d^3 S}{5.1}$$

in which T equals the internal resistance to twisting, or the twisting moment, and S the shearing strength per square inch of area in pounds.

Regarding the shearing strength of materials Kent says: "The ultimate torsional shearing resistance is about the same as the direct shearing resistance, and may be taken at 20,000 to 25,000 pounds per square inch for cast iron, 45,000 pounds for wrought iron, and 50,000 to 150,000 pounds for steel according to its carbon and temper."

The torsional and direct shearing resistance being the same, this quantity may be neglected if the shaft and coupling bolts are of the same material, and

$$\frac{d^3}{5.1}$$

the internal resistance factor or torsion modulus of the shaft, should be equal to the product of the radius of the bolt circle of the coupling, the number of bolts used, and the area of each bolt. Or, letting a represent the area of each bolt, R the radius of the bolt circle of the coupling, and n the number of bolts used, we would have:

$$a = \frac{d^3}{5.1} \div (R \times n).$$

Example.—Required the size of the bolts for a flange coupling for a 2-inch shaft. The radius of the bolt circle is 3 inches, four bolts being used.

Using the notation in the formula given, our known values are:

$$d = 2 \text{ inches},$$
$$R = 3 \text{ inches},$$
$$n = 4 \text{ bolts}.$$

If we insert these values in the formula we have:

$$a = \frac{2^3}{5.1} \div (3 \times 4) = \frac{8}{5.1} \div 12 = 0.13 \text{ square inches.}$$

This area corresponds to a diameter of about $\frac{7}{16}$ of an inch. To allow for the strain on the bolt caused by the action of the wrench, the next size larger bolt, at least, or a ½ inch bolt, will be selected. The capacity of the bolt to resist shearing will be considerably increased by having the corners of the holes at those faces of the flanges which come together, somewhat rounded. If this is not done, the action of the flanges on the bolts will be like that of a pair of sharp shears. Experiments have shown that with the corners rounded, the capacity of the bolt to resist shearing may be increased 12 per cent.

If the shaft and bolts are of different materials then the modulus

$$\frac{d^3}{5.1}$$

should be multiplied by the shearing strength of the shaft in pounds per square inch and the product

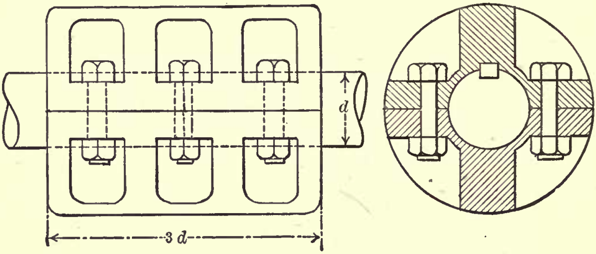


FIG. 192.—Clamp Coupling.

$R \times n$ should be multiplied by the shearing strength of the bolts per square inch, before dividing in the formula to get the bolt area.

In Fig. 192 is shown another form of coupling much used. It consists of two parts bolted together over the joint in the shafting, a key and keyway being provided to prevent the slipping of the shafts.

By having a thickness of heavy paper interposed between the two parts of the coupling when it is bored out, it may be made to clamp very tightly onto the shafts.

With either form of coupling, the length is made such that each shaft end is held by the coupling by a length of about one and one-half times its diameter, as indicated in the engravings.

Oldham's Coupling.—Fig. 193 shows a form of coupling which may be used for shafts which are

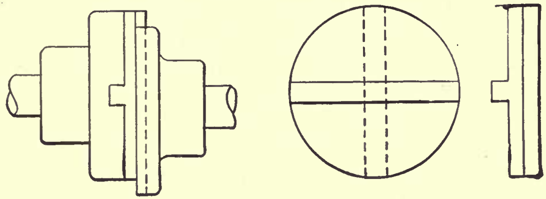


FIG. 193.—Oldham's Coupling.

parallel, but slightly out of line. In this coupling each shaft end has a flanged hub attached to it.

Across the face of each flange is planed a single groove passing through its center. Interposed between the two flanges is a disk, shown at the right, having tongues on both faces at right angles to each other, to engage in the grooves in the flanges.

Hooke's Coupling or Universal Joint is used for connecting two shafts whose axes are not in line with each other, but merely intersect. The shafts *A* and *B*, and *B* and *C*, in Fig. 194, are thus connected by universal joints. If the shaft *B* is made telescoping, as is very often the case, a solid part entering into and being keyed in a sleeve so as to prevent independent rotation, but yet permit a sliding action, then the two shafts *A* and *C* may move independently of each other within certain limits, the distance between their ends being capable of variation. The arrangement shown in Fig. 194 is used on various machine tools, notably on milling machines, flange drilling machines, etc. Many designs of flexible shafts are really only a combination of a great number of universal joints.

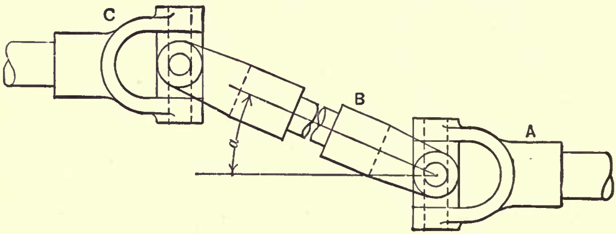


FIG. 194.—Application of Universal Joints and Telescoping Shaft.

When this coupling is employed for driving only one shaft at an angle with another, as if shaft *A* simply drove shaft *B* which, of course, is the fundamental type of universal coupling, then, if the driving shaft has a uniform motion, the driven shaft will have a variable motion, and so cannot be used in such cases where uniformity of motion of the driven shaft is necessary; but where there are three shafts, as shown in the illustration, *A* will impart a uniform motion to *C* provided the axes of *A* and *C* are parallel with each other, as shown; for if *A*, having a regular motion, imparts an irregular motion to *B*, then if *B*, with its irregular motion, is made the driver, it will impart a regular motion to *A*, and as *C* is parallel with *A* it will also impart a regular motion to *C*.

This form of coupling does not work very well if the angle *a* is more than 45 degrees.

Clutches are of two general classes, toothed clutches and friction clutches. An example of a toothed clutch is shown in Fig. 195. In this clutch the part at the left is fastened to its shaft; the part at the right is free to slide back and forth upon

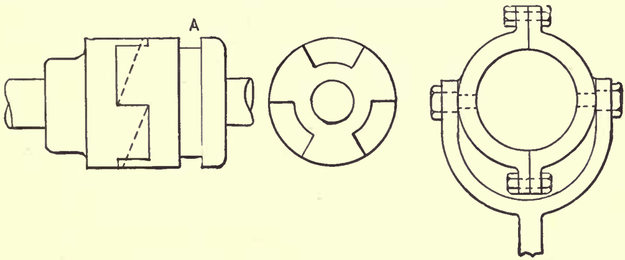


FIG. 195.—A simple Form of Toothed Clutch.

its shaft, but is prevented from turning on the shaft by a key. The sliding motion for engaging or disengaging this part of the clutch is accomplished by means of the forked lever and jointed ring, shown at the right, which latter engages in the groove *A*. Such a clutch, while giving a positive drive, cannot, of course, be thrown in or out while the driving shaft is running at a high rate of speed. By having the back faces of the teeth beveled off as shown by the dotted lines, this difficulty is partly overcome, although the shock caused by the sudden engaging of the teeth still renders the clutch unsuitable for operating at very high speed. To facilitate uncoupling, the driving faces may also be given an angle of about 10 or 12 degrees.

Friction Clutches are generally made in one of the two styles shown in Figs. 196 and 197. The power which a clutch of the type shown in Fig. 196 will transmit, depends upon the power which is applied to force the sliding part against the fixed part,

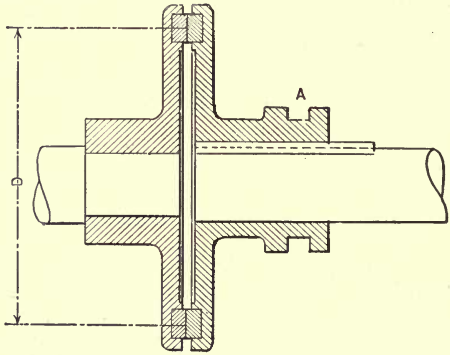


FIG. 196.—Friction-Disk Clutch.

and the efficiency of the frictional force between the rubbing surfaces. As to the efficiency of the clutch, therefore, much depends upon the nature of the engaging surfaces, whether metal comes in contact with metal, or whether one of the surfaces has a facing of leather or wood. The efficiency is, of course, much increased by either a leather or wood facing. Professor Smith gives the efficiency of these different surfaces as follows: Cast iron on cast iron, 10 to 15 per cent.; cast iron on leather, 20 to 30 per cent.; cast iron on wood, 20 to 50 per cent.

The horse-power which such a clutch will transmit will be found by multiplying the velocity of the parts in contact, in feet per minute, taken at their mean diameter as indicated at *D*, by the force which is being applied at this diameter in the direction of revolution, and dividing this product by 33,000. The force which is acting at the diameter *D* to produce revolving motion is equal to the pressure which is being applied to force the two parts of the clutch together, multiplied by the coefficient of friction (as the frictional efficiency between the surfaces in contact, as given above, is called) of the materials which form the driving surfaces.

Example.—What power will a clutch of the type shown in Fig. 196 transmit if running at a speed of 250 revolutions per minute? The diameter *D* is 18 inches, and a pressure of 50 pounds is exerted to force the two clutch faces together. One of the clutch parts has a leather facing, and the coefficient of friction is 0.25.

The general formula for finding the horse-power of a clutch of this type is:

$$H.P. = \frac{D \times 3.1416 \times n \times P \times f}{33,000}$$

in which $H.P.$ = horse-power transmitted,
D = mean diameter of friction surfaces *in feet*,
n = revolutions per minute,
P = pressure between clutch surfaces in pounds,
f = coefficient of friction.

The values to be inserted in the formula, which are given in this problem, are as follows:

$$D = \frac{18}{12} = 1.5 \text{ foot},$$
$$n = 250 \text{ revolutions},$$
$$P = 50 \text{ pounds},$$
$$f = 0.25.$$

Inserting these values in the formula we have:

$$H.P. = \frac{1.5 \times 3.1416 \times 250 \times 50 \times 0.25}{33,000} = 0.45.$$

The formula given may be transposed in various ways according to the requirements of the problem; if, for instance, it is desired to know what pressure must be applied to transmit a given horse-power, then:

$$P = \frac{H.P. \times 33,000}{D \text{ (in feet)} \times 3.1416 \times n \times f.}$$

If the pressure is known, and it is required to find what diameter the clutch must be made to transmit a given power, then:

$$D \text{ (in feet)} = \frac{H.P. \times 33,000}{3.1416 \times n \times P \times f.}$$

If the pressure and diameter are both known, then the number of revolutions which the clutch must make per minute to transmit a given horse-power will be:

$$n = \frac{H.P. \times 33,000}{D \text{ (in feet)} \times 3.1416 \times P \times f.}$$

It may be said that the capacity of the clutch to transmit power is independent of the area of the friction surfaces; for, if the friction surface is increased the pressure which is applied to force the two parts of the clutch together is simply distributed over a much greater area, giving a smaller pressure per square inch. The durability would be increased, but the horse-power capacity would remain unchanged.

The conical clutch shown in Fig. 197 may be made to run metal to metal, or the hollow part may

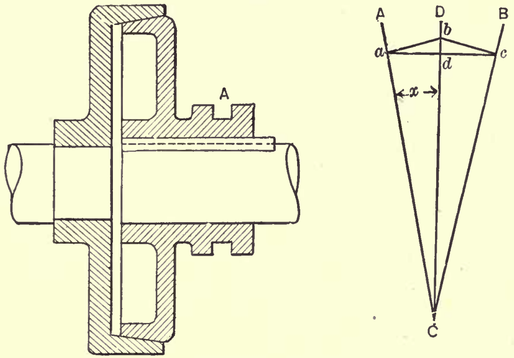


FIG. 197.—Friction Cone Clutch.

be made larger to allow of the insertion of wooden blocks. This would increase the efficiency, but at the expense of the durability. The principle of this form of clutch may be explained by referring to the diagrammatical sketch at the right of Fig. 197, where the angle ACB represents the angle which the opposite sides of the clutch make with each other, the line DC representing the axis of the shaft. If now line bd of the small triangle abd be considered as representing the magnitude of the force acting in the direction of the axis of the shaft to force the two parts of the clutch together, then if ab is at right angles to AC, ab will represent the resultant magnitude of the force acting on the face of the clutch at right angles to its surface, according to the principles explained in the chapter on the elements of mechanics. The efficiency of the clutch will therefore be as much greater than that of a flat-faced clutch as ab is greater than bd. The horse-power of such a clutch, using the same notation as before, would, therefore, be:

$$H.P. = \frac{D \text{ (in feet)} \times 3.1416 \times n \times P \times f}{33{,}000} \times \frac{ab}{bd}.$$

But from the chapter on the solution of triangles we know that

$$\frac{bd}{ab} = \text{sine of angle } bad.$$

Hence

$$\frac{ab}{bd} = \frac{1}{\sin bad.}$$

But angle bad equals angle x, the angle which the conical surface of the clutch makes with the axis of the shaft.

Therefore

$$\frac{ab}{bd} = \frac{1}{\sin x}$$

and our original formula takes the form:

$$H.P. = \frac{D \text{ (in feet)} \times 3.1416 \times n \times P \times f}{33{,}000 \times \sin x.}$$

Transposing this formula as before for the flat-faced clutch, gives us:

$$P = \frac{H.P. \times 33{,}000 \times \sin x}{D \text{ (in feet)} \times 3.1416 \times n \times f.}$$

$$D \text{ (in feet)} = \frac{H.P. \times 33{,}000 \times \sin x}{3.1416 \times n \times P \times f.}$$

$$n = \frac{H.P. \times 33{,}000 \times \sin x}{D \text{ (in feet)} \times 3.1416 \times P \times f.}$$

The sine of x may be taken from the tables of trigonometric functions previously given in the chapter on the solution of triangles, or it may be found by dividing the length *bd* (Fig. 197) by the length *ab*.

The power necessary to force the two parts of the clutch together may be neglected, as the slipping which occurs as they are engaging allows them to come together with but little pressure beyond what is required for power transmission purposes. The angle which the face of the clutch makes with the shaft (the angle x in the diagram at the right in Fig. 197) should be such that the clutch does not grip too quickly when thrown into gear, nor require too much pull to release. Making this angle between 7 and 12 degrees conforms to the average given by different authorities.

CHAPTER XVII

SHAFTS, BELTS AND PULLEYS

Shafts.—The twisting strength of a shaft, as stated in the preceding chapter, is given by the formula

$$T = \frac{d^3 \times S}{5.1}$$

in which T = twisting moment, or force which acting at a distance of one inch from the center of the shaft would produce in it a torsional shearing stress of S pounds per square inch,

d = diameter of shaft in inches,

S = torsional shearing stress in pounds per square inch.

Expressing this formula in words we may say that the cube of the diameter in inches multiplied by the torsional shearing stress, and this product divided by 5.1, gives the force which acting at a distance of one inch from the center of the shaft would produce in it the given torsional shearing stress.

The twisting moment T equals, therefore, the force F_1, acting at a distance of one inch from the center of the shaft, times 1; it also equals any other force F exerting a twisting action on the shaft multiplied by its distance from the center of the shaft. The formula given can hence be written

$$T = F \times r = \frac{d^3 \times S}{5.1}$$

in which F = any force acting at a distance r from the center of the shaft.

Transposing this formula to obtain the distance from the center (r) at which a given force would have to act to set up a torsional shearing stress S in the shaft, we would have:

$$r = \frac{d^3 \times S}{5.1 \times F.}$$

The force which would be necessary to set up a stress S in the shaft when acting at a given distance would be:

$$F = \frac{d^3 \times S}{5.1 \times r.}$$

The diameter of shaft to resist a given force acting at a given distance would be:

$$d = \sqrt{\frac{F \times r \times 5.1}{S}}$$

The torsional shearing strength of ordinary shafting is about 45,000 pounds to the square inch, and of steel shafting from about 50,000 to 150,000 pounds, according to its quality; these figures should be divided by five or six to give a safe working stress.

The above formulas, however, are based on the assumption that the force acting is of a purely twisting nature, as if a hand-wheel were put onto the end of the shaft, and the tendency to bend the shaft, caused by the pull of one hand, were counteracted by the push of the other hand. In the case of a shaft actuated by a rocker arm, as sometimes occurs in machines, the tendency to bend the shaft caused by the push on the arm could be provided for by using a somewhat higher factor of safety. If the arm were placed at some distance from the bearing, however, the tendency to bend the shaft might be greater than the twisting effect.

The methods of calculating the size of shafts for transmitting a given power, so as to take into account both the twisting and bending effects produced by the pull of the belt are quite complicated, and the beginner will ordinarily find it best to use some of the empirical formulas for that purpose which are intended to take into account both of these effects.

The following rules by Thurston are considered to afford ample margin for strength for shafts which are well supported against springing:

To find the diameter of a cold rolled iron shaft to transmit a given horse-power, multiply the horse-power to be transmitted by 75, and divide the product by the number of revolutions per minute that the shaft is to make. The cube root of this quotient will be the diameter of the shaft.

If the shaft is to be of turned iron, proceed as above, except that the horse-power to be transmitted is to be multiplied by 125 instead of 75.

This rule is "for head shafts, supported by bearings close to each side of the main pulley or gear, so as to wholly guard against transverse strain." If the main pulley is at a distance from the bearing, the size of the shaft will need to be increased, while for ordinary line shafting, with hangers 8 feet apart, the size may be reduced, figures of 90 for turned iron, and 55 for cold rolled iron shafting being substituted for those given in the rule; or, in the case of shafting for transmission only, without pulleys, figures of 62.5 for turned iron, and 35 for cold rolled iron are substituted.

To find the horse-power which a given shaft will transmit, multiply the cube of its diameter by the number of revolutions per minute, and divide the product by 125 for turned iron, or by 75 for cold rolled iron.

For line shafting substitute the figures given by 90 and 55, respectively.

The horse-power which is being transmitted is determined by multiplying the pull in pounds which the belt exerts (or the push which the teeth of the driving gear exert, if gears are used) by the diameter of the pulley in inches (or the pitch diameter of the gear in inches) and multiplying this product, again, by the number of revolutions per minute of the shaft; then divide this product by 126,050, and the quotient gives the horse-power transmitted.

Expressed as a formula this rule would be:

$$H.P. = \frac{P \times D \times N}{126{,}050}$$

in which P = pull on belt or push on gear teeth in pounds,

D = diameter of pulley or pitch diameter of gear in inches,

N = number of revolutions per minute of pulley or gear.

Belts.—The theoretical horse-power which a belt will transmit is equal to the pull which the belt exerts in pounds, multiplied by its velocity in feet per minute, and this product divided by 33,000. The question then arises as to what is the allowable stress to be put upon a belt.

A common rule of practice for ordinary belting is that for single thickness belts the horse-power transmitted equals the breadth of the belt in inches, multiplied by its velocity in feet per minute, this product being divided by 1,000. This rule assumes a belt pull of 33 pounds per inch of width. Many authorities, however, would allow a much higher tension. The higher the tension, however, the narrower the belt for a given horse-power, and the greater the stretch, the more frequent the necessity for relacing, and the shorter the life of the belt.

Allowing 33 pounds tension per inch in width for the thinnest commercial single belt, and allowing the tensions for increased thicknesses given by a large belt manufacturing concern, would give the following formulas for the transmission capacities of given belts:

Single belt, $\frac{3}{16}$ inch thick, $H.P. = \frac{\text{Breadth} \times \text{velocity.}}{1000}$

Single belt, $\frac{1}{4}$ inch thick, $H.P. = \frac{\text{Breadth} \times \text{velocity.}}{800}$

Light double, $\frac{17}{64}$ inch thick, $H.P. = \frac{\text{Breadth} \times \text{velocity.}}{733}$

Heavy double, $\frac{9}{32}$ inch thick, $H.P. = \frac{\text{Breadth} \times \text{velocity.}}{687}$

Heavy double, $\frac{5}{16}$ inch thick, $H.P. = \frac{\text{Breadth} \times \text{velocity.}}{660}$

Heavy double, $\frac{3}{8}$ inch thick, $H.P. = \frac{\text{Breadth} \times \text{velocity.}}{550}$

Heavy double, $\frac{13}{32}$ inch thick, $H.P. = \frac{\text{Breadth} \times \text{velocity.}}{500}$

In these formulas the breadth of the belt is understood to be in inches, and its velocity in feet per minute, the letters *H.P.* meaning horse-power. Transposing the above formulas to ascertain the breadth of belt required to transmit a given power, we would have:

Single belt, $\frac{3}{16}$ inch thick, Breadth $= \frac{H.P. \times 1000}{\text{Velocity}}$

Single belt, $\frac{1}{4}$ inch thick, Breadth $= \frac{H.P. \times 800}{\text{Velocity}}$

Light double, $\frac{17}{64}$ inch thick, Breadth $= \frac{H.P. \times 733}{\text{Velocity}}$

Heavy double, $\frac{9}{32}$ inch thick, Breadth $= \frac{H.P. \times 687}{\text{Velocity}}$

Heavy double, $\frac{5}{16}$ inch thick, Breadth $= \frac{H.P. \times 660}{\text{Velocity}}$

Heavy double, $\frac{3}{8}$ inch thick, Breadth $= \frac{H.P. \times 550}{\text{Velocity}}$

Heavy double, $\frac{13}{32}$ inch thick, Breadth $= \frac{H.P. \times 500}{\text{Velocity}}$

These formulas are all for laced belts. A belt made endless by being lapped and cemented or riveted is considered to be nearly 50 per cent. stronger than a laced belt, and is thus capable of transmitting nearly 50 per cent. more power; or the breadth of an endless belt to transmit a given power would not need to be more than between two-thirds to three-quarters of the breadth of a laced belt. Metal fastenings are not considered to make as strong a belt as lacings.

If the foregoing formulas had been made on the basis of an allowable stress of 45 pounds for each inch in width of a single belt, a figure which many consider perfectly safe for a belt in good condition, they would have shown the belts as being capable of transmitting one-third more power than at 33 pounds stress per inch; to transmit a given power a belt would then need to be not more than three-quarters of the width.

It will be seen from these formulas that the power transmitting capacity of a belt depends upon its breadth (a wide belt allowing an increased tension) or on its velocity. Increasing the width of the belt without increasing the tension to correspond would not give any increase of power transmitting capacity, as the given tension would simply be distributed over so much more pulley surface; but a tight belt means more side strain on shaft and journal. Therefore, according to Griffin, from the standpoint of efficiency, *use a narrow belt under low tension at as high a speed as possible.* The desired high speed is, of course, secured by simply putting on large pulleys.

Speed of Belting.—The most economical speed is somewhere between 4000 and 5000 feet per minute. Above these values the life of the belt is shortened; "flapping," "chasing," and centrifugal force also cause considerable loss of power at higher speeds. The limit of speed with cast iron pulleys is fixed at the safe limit for the bursting of the rim, which may be taken at one mile surface speed per minute.

The formulas given for the horse-power transmitted assume that the belt is in contact with just one-half of the pulley; or, in other words, that the arc of contact is 180 degrees. If the arc of contact is increased, as it might be in the case of a crossed belt, until it becomes 240 degrees, or two-thirds of the circumference of the pulley, it is stated that the adhesion of the belt to the pulley, and consequently the efficiency of the belt, will be increased 50 per cent. If, on the other hand, the arc of contact should be reduced to 120 degrees, or one-third of the circumference of the pulley, as might be the case with open belts where the shafts were near together, and the pulleys were very unequal in size, the efficiency is stated to be only 60 per cent. of what the formulas would show; if the arc of contact should be reduced to 90 degrees, the efficiency is stated to be only 30 per cent.

From these percentages one can form a fairly good idea of what percentage to allow for varying arcs of contact. In most cases, however, it will probably be correct enough to assume the arc of contact to be 180 degrees.

In all cases of open horizontal belting the lower run of the belt should be made the working part, so that the sag of the upper run will increase the arc of contact.

In the location of shafts that are to be connected with each other by belts, care should be taken to secure them at a proper distance from one another. It is not easy to give a definite rule what this distance should be. Some authorities give this rule: Let the distance between the shafts be ten times the diameter of the smaller pulley; but while this is correct for some cases, there are many other cases in which it is not correct. Circumstances generally have much to do with the arrangement; and the engineer or machinist must use his judgment, making all things conform, as far as may be, to general principles. The distance should be such as to allow a gentle sag to the belt when in motion. The Page Belting Co. states that if too great a distance is attempted, the weight of the belt will produce a very heavy sag, drawing so hard upon the shafts as to produce considerable friction in the bearings, while at the same time the belt will have an unsteady, flapping motion, which will destroy both the belt and the machinery.

As belts increase in width they should be made thicker. It is advisable to use double belts on pulleys 12 inches in diameter and larger. If thin belts are used at very high speed, or if wide belts are thin, they almost invariably run in waves on the slack side, or "travel" from side to side of the pulleys, especially if the load changes suddenly. This waving and snapping that occurs as the belts straighten out, wears the belts very fast, and frequently causes the splices to part in a very short time, all of which is avoided by the employment of suitable thickness in the belts. The Page Belting Co. states that driving pulleys on which are to be run shifting belts should have a perfectly flat surface. All other pulleys should have a convexity in the proportion of about $\frac{3}{16}$ of an inch to one foot in width. The pulleys should be a little wider than the belt required for the work.

Pulley Sizes.—The sizes of pulleys to give a required speed, or the speed which will be obtained with given pulleys may be readily found from the fact that the product of the speed of the driving shaft, in revolutions per minute, and the diameters of all driving pulleys, on the main and on countershafts, multiplied together, will be equal to the product of the diameters of all driven pulleys and the speed of the last driven shaft, in revolutions per minute, multiplied together; so that if the size of one driven pulley, for instance, is required, its size may be found by dividing the product of the speed of the driving shaft and all driving pulleys multiplied together, by the product of speed of the final driven shaft and the diameters of such driven pulleys as are given, multiplied together. The result will be the required pulley size.

Example.—A shaft making 200 revolutions per minute has mounted on it a pulley 18 inches in diameter which belts onto a 6-inch pulley on a countershaft. The countershaft has mounted on it a 20-inch pulley which belts to a pulley on the spindle of a machine which is to make 3000 revolutions per minute. What size pulley will be required on the spindle.

Placing the speed of the driving shaft, and the sizes of all driving pulleys on one side of a vertical line, for convenience sake, and the sizes of all driven pulleys and the speed of the last driven shaft (or spindle) on the other side, and letting x represent the required size we would have:

Speed of shaft	= 200	6	= Driven pulley on countershaft.
Pulley on shaft	= 18	x	= Required size of pulley on spindle.
Driving pulley on countershaft	= 20	3000	= Speed of spindle.

Then $200 \times 18 \times 20 = 6 \times x \times 3000$

$$x = \frac{200 \times 18 \times 20}{6 \times 3000} = \frac{72{,}000}{18{,}000} = 4.$$

The diameter of the pulley on the spindle would therefore have to be 4 inches. If this size had been given, and the speed of the spindle had been required, x might have been taken to represent the required speed, when the same process would have given the desired information.

Twisted and Unusual Cases of Belting. — It frequently happens that, in transmitting power, conditions present themselves in which ordinary straight belting, either open or crossed, will not serve the purpose, and recourse must be had to some form of twisted belting, either quarter turn belting or belting guided by idler pulleys. In the following are given some of the principal conditions.

Fig. 198 shows a quarter turn belt, by which power can be transmitted from one shaft to another at right angles to it. The condition necessary for the successful working of this arrangement is that the middle of the face of the pulley toward which

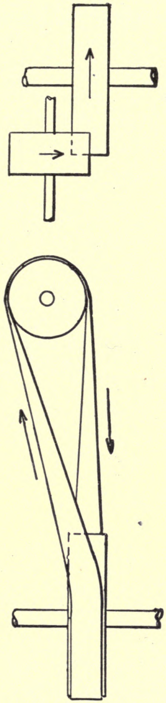


FIG. 198.—Arrangement of Pulleys for Quarter-Turn Belt.

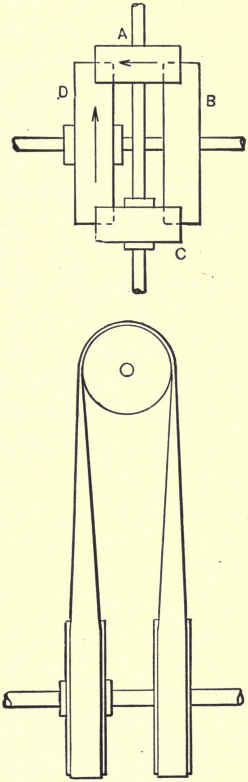


FIG. 199.—Another Arrangement for Transmitting Power between Shafts at Right Angles.

the belt is advancing shall be in line with the edge of the pulley that the belt is leaving. An examination of both the plan and elevation views will make this clear.

While this is the simplest arrangement for this purpose, it has several drawbacks. The edgewise stress on the belt as it is leaving either pulley is very severe on the belt. It also causes a considerable loss of contact with the pulley face, with corresponding loss of power transmission capacity. The edgewise stress also makes it necessary, if durability is to be considered, to have the belt relatively narrow. Incidentally, also, any reversal of the motion will cause the belt to immediately run off the pulleys.

Fig. 199 shows another arrangement for transmitting power from one shaft to another at right angles to it, which overcomes all of the objections mentioned to the arrangement shown in Fig. 198, but at the expense of a double length belt and an extra pair of pulleys.

As shown in the illustration, *A* and *B* are tight pulleys, and *C* and *D* are loose pulleys. The belt, as it leaves the tight pulley *A*, passes down under the loose pulley *D*, up over the loose pulley *C*, down under the tight pulley *B*, and then up over the tight pulley *A*, making a complete circuit. The loose pulleys, it will be seen, revolve in an opposite direction to the shafts on which they are mounted.

Fig. 200 shows an arrangement by which, by employing loose guide pulleys, power may be transmitted from one shaft to another so close to it as to prohibit direct belting. If the main pulleys are of the same size, and their shafts are in the same plane, the guide pulleys may be mounted on a single straight shaft at right angles to a plane passing through the axes of the shafts on which the main pulleys are mounted. If, however, the main pulleys are of unequal size, as shown in the illustration, the guide pulleys will have to be inclined to such an angle that the center of the face

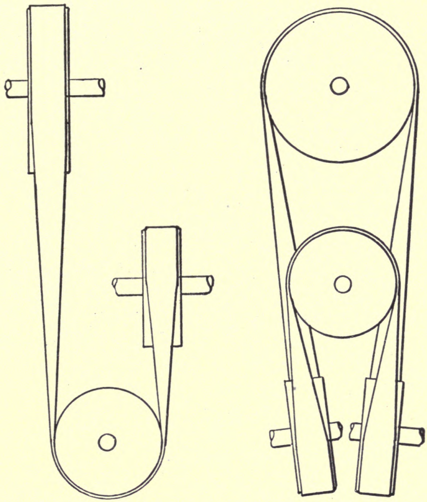


FIG. 200.—Arrangement of Belt Transmission Using Loose Guide Pulleys.

of the pulley toward which the belt is advancing shall be in line with the edge of the pulley that the belt is leaving, the same as in the case of the quarter turn belt shown in Fig. 198.

It is not necessary that the shafts on which the main pulleys are mounted be in the same plane; their direction may be such that their relation to each other is similar to that of those shown in Fig. 198, or at any intermediate angle.

Again, if they are in the same plane, it is not

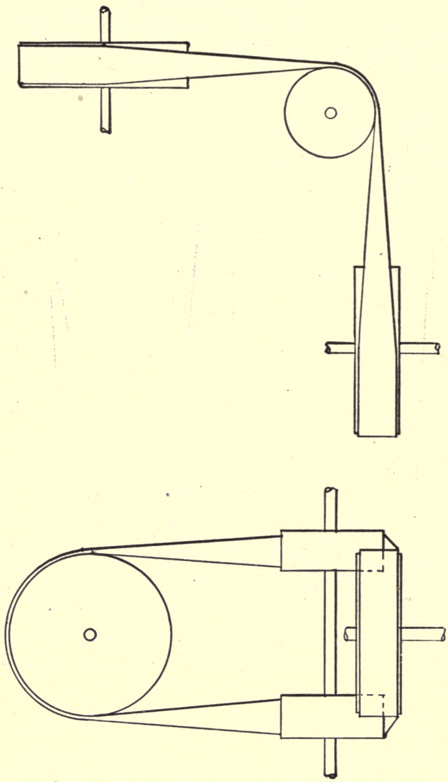


FIG. 201.—A Case Permitting the Guide Pulleys to be Mounted on the Same Shaft.

necessary that they should be parallel with each other; they may be at any angle with each other.

Fig. 201 shows a case which is a modification of Fig. 200. The main shafts are at right angles to each other. The main pulleys, being of the same size, permit the guide pulleys to be mounted on a single shaft. This arrangement is a common method of transmitting power around a corner.

Fig. 202 shows a case where the direction of the shafts with regard to each other is the same as in

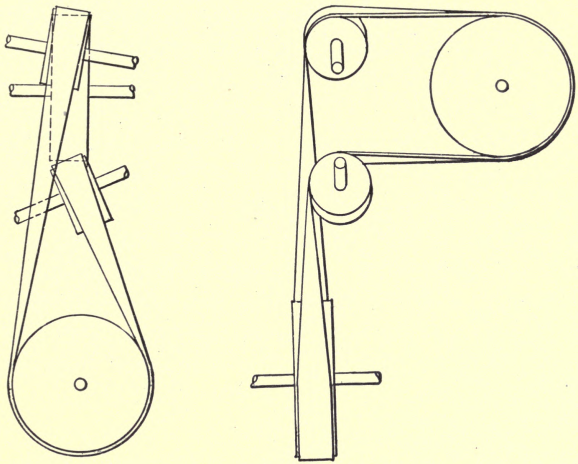


FIG. 202.—A Case where the Guide Pulleys would be Mounted in an Adjustable Frame.

Fig. 198, but where shop conditions are such that it is not practicable to bring the lower shaft under the upper one to permit of belting by either of the methods shown in Figs. 198 or 199. The guide pulleys are, therefore, mounted on a frame which can be raised or lowered in guides by means of an adjusting screw, permitting of an easy adjustment of the belt tension.

Fig. 203 shows a case which is similar to Fig. 200 in that it permits the belting together of shafts which are at angle to each other, but accomplishes this result by the use of only one guide pulley. The shafts, though at an angle to each other, are in the same plane. This, however, is not necessarily so. The shafts may be twisted around until they are at right angles to each other, as in Fig. 198. As shown in Fig. 200, the belt may be run in either direction as long as the shafts are in the same plane; but as shown in Fig. 203, it is necessary that the belt should be run in the direction indicated by the arrows.

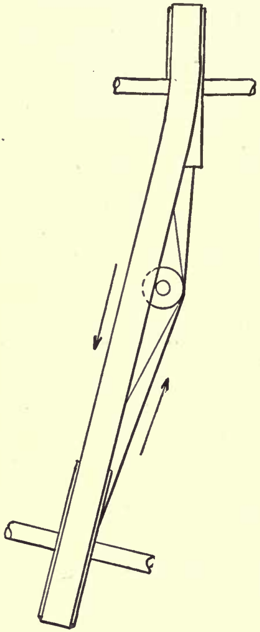


FIG. 203.—An Arrangement in Which but One Guide Pulley is Used.

An examination of the engravings will show that the condition necessary for the proper working of guide pulleys is that the shaft on which the guide pulley is mounted shall be at right angles to a line drawn from the edge of the pulley that the belt is leaving in its advance toward the guide pulley, to the middle of the guide pulley face.

CHAPTER XVIII

FLY-WHEELS FOR PRESSES, PUNCHES, ETC.

In a great many different classes of machinery, the work that the machine performs is of a variable or intermittent nature, being done, in the case, for example, of punches and presses, during a small part of the time required for the driving shaft or spindle of the machine to make a complete revolution. If this work could be distributed over the entire period of the revolution, a comparatively narrow belt would be sufficient to drive the machine; but a very broad and heavy belt would otherwise be necessary to overcome the resistance, if the belt only be depended on to do the work. It is, of course, in a sense, impossible to distribute the work of the machine over the entire period of revolution of the driving shaft of the machine, but by placing a large, heavy-rimmed wheel, a *fly-wheel*, on the shaft, the belt is given an opportunity to perform an almost uniform amount of work during the whole revolution. During the greater part of the revolution of the driving shaft the power of the belt is devoted to accelerating the speed of the fly-wheel. During that brief period of the revolution of the shaft when the work of the machine is being done, the energy thus stored up in the fly-wheel is given out at the expense of its velocity. The energy a fly-wheel would give out if brought to a standstill would be (neglecting the weight of the arms and hub, as the efficiency of the wheel depends chiefly on the weight of the rim), expressed in foot-pounds, equal to the weight of the rim in pounds multiplied by the square of its velocity at its mean diameter in feet per second, and this product divided by 64.32, the same as in the case of a falling body moving at the same velocity, as explained in the section on mechanics.

Expressed as a formula this rule is:

$$E = \frac{W v^2}{2g} = \frac{W v^2}{64.32}$$

in which E = total energy of fly-wheel,
W = weight of fly-wheel rim in pounds,
v = velocity at mean radius of fly-wheel in feet per second,
g = acceleration due to gravity = 32.16.

If the speed of the fly-wheel is only reduced, the energy which it would give out would be equal to the difference between the energy which it would give out if brought to a full stop, and that which it would still have stored up in it at its reduced velocity. Therefore, *to find the energy in foot-pounds which a fly-wheel will give out with an allowable loss of speed*, subtract the square of the velocity of the rim in feet per second at its reduced speed from the square of its velocity in feet per second at full speed, multiply this difference by the weight in pounds, and divide the product by 64.32. The result will give the loss of energy in foot-pounds.

This long and cumbersome rule is expressed much more simply by the formula:

$$E_1 = \frac{(v_1^2 - v_2^2)\ W}{64.32}$$

in which E_1 = energy, in foot-pounds, fly-wheel gives out while speed is reduced from v_1 to v_2,

v_1 = speed before any energy has been given out, in feet per second,

v_2 = speed at end of period during which energy has been given out, in feet per second,

W = weight of fly-wheel rim in pounds.

This rule and formula may be transposed as follows: *To find the weight of a fly-wheel to give out a required amount of energy with an allowable loss of speed,* multiply the required amount of energy in foot-pounds by 64.32, and divide the product by the difference between the square of the velocity of the rim, at its mean diameter, in feet per second at full speed, and the square of its velocity in feet per second at its reduced speed; or, expressed as a formula, using the same notation as above:

$$W = \frac{E_1 \times 64.32}{v_1^2 - v_2^2}$$

When the mean diameter of the fly-wheel is known, the velocity of the rim at its mean diameter in feet per second will be

$$\frac{\text{Diameter in feet} \times 3.1416 \times \text{rev. per minute}}{60}$$

It is evident that in designing a fly-wheel for a machine, there is an opportunity for a wide range in the weight, from a wheel heavy enough, when once it has been brought to its full speed, to do, by means of the energy stored in it, the work without assistance from the belt, the belt being only just wide enough to restore the speed of the wheel in time for the next operation, to a wheel where the belt is wide enough to do the most of the work directly, the stored energy in the fly-wheel merely assisting it somewhat. Perhaps the best way would be to have the wheel heavy enough so that its stored energy could do the bulk of the work, the belt assisting it, and at the same time have the latter wide enough to quickly restore the speed of the wheel, so that, in case its velocity should be reduced beyond that calculated, there would be a margin of available power in the belt.

Example.—Let it be required to design a fly-wheel for a press to cut off one-inch round bar steel, the press making 30 strokes per minute.

Soft steel having a shearing resistance of about 50,000 pounds per square inch, and a one-inch bar having an area of cross-section of 0.7854 square inch, the shearing resistance of the bar will be $50{,}000 \times 0.7854 = 39{,}270$ pounds, or practically 40,000 pounds. This resistance varies, however, during the process of shearing, being greatest near the beginning of the cut, and decreasing as the cutting progresses. In the case of a round bar it could not decrease uniformly, because of the shape of the cross-section. For the sake of getting the decrease in resistance as nearly uniform as possible, we will assume that the work of cutting off a one-inch round bar is the same as the work of cutting off a square bar of the same area; though this may not be quite exact, it would probably not be far out of the way. The length of the sides of a square of the same area as a given circle, is equal to the diameter of the circle multiplied by 0.886. Therefore, our equivalent square bar will be 0.886 of an inch square. The mean resistance to cutting, assuming that the resistance decreases uniformly as the cutting progresses, would be $40,000 \div 2 = 20,000$ pounds. As the cutting operation continues through a space of 0.886 of an inch, the power required would be $20,000 \times 0.886 = 17,720$ inch-pounds, or 1476.6 foot-pounds. Let us plan to have the belt do one-fifth of the work of cutting directly, leaving four-fifths to be done by the stored up energy of the fly-wheel. One-fifth of 1476.6 equals 295.3. Subtracting this from 1476.6 leaves 1181.3 foot-pounds to be supplied by the energy of the fly-wheel. As a preliminary calculation let us find what would have to be the weight of the wheel if it were to be placed upon the crank-shaft, the shaft which operates the plunger of the press. Assuming the mean diameter of the fly-wheel rim to be 4 feet, the circumference would be $4 \times 3.14 = 12.56$ feet, and, as the shaft makes 30 revolutions per minute, the velocity of the rim in feet *per second* would be:

$$\frac{12.56 \times 30}{60} = 6.28 \text{ feet.}$$

If we expect the fly-wheel to suffer a loss of, say, 10 per cent. while doing its work, then its velocity at its reduced speed will be $6.28 - 0.628 =$

5.65 feet. The weight of the fly-wheel to give out 1181.3 foot-pounds under these conditions will then be, according to the rule and formula already given:

$$\frac{1181.3 \times 64.32}{6.28^2 - 5.65^2} = \frac{75,981.2}{39.44 - 31.92} = \frac{75,981.2}{7.52} = 10,100$$

nearly.

A wheel weighing 10,100 pounds would, of course, be out of the question; but as the energy increases as the square of the velocity, the weight may be very rapidly reduced by mounting the wheel upon a higher-speeded secondary shaft, connected with the crank-shaft by reducing gears. If the speed of the secondary shaft is to the speed of the crank-shaft as 6 to 1, the weight of the wheel, if the mean diameter be kept the same, will need to be only about one thirty-sixth of what it would need to be if mounted on the crank-shaft. At this higher speed, however, it might be desirable to somewhat reduce the diameter of the wheel. Let us assume that the mean diameter be made 3 feet. If the ratio of speeds is 6 to 1, the wheel will make 180 revolutions per minute, and the velocity of the rim in feet per second will be:

$$\frac{3 \times 3.14 \times 180}{60} = 28.3 \text{ feet.}$$

If the wheel suffers a loss of 10 per cent., its velocity at its reduced speed will be 28.3 − 2.83 = 25.5 nearly. The weight of the wheel will then be:

$$\frac{1181.3 \times 64.32}{28.3^2 - 25.5^2} = \frac{75,981.2}{150.64} = 504 \text{ pounds.}$$

As a cubic inch of cast iron weighs 0.26 pound, the wheel will contain $504 \div 0.26 = 1938$ cubic inches. The mean circumference of the rim in inches will be $3 \times 12 \times 3.14 = 113$ inches. The cross-section of the rim will then be:

$$1938 \div 113 = 17.1 \text{ square inches.}$$

This would mean a rim about 4 by 4¼ inches. The outside diameter of the wheel would then be 40 inches.

We planned to have the belt do one-fifth of the work, and this we found to be 295.3 foot-pounds. If the crank has a radius of 1¼ inch, the cutter will have a stroke of 2½ inches, and if the cutters overlap each other one-quarter of an inch at the end of the stroke, the crank will have to swing through an angle of about 54 degrees in order to make the cutters advance the one inch necessary to cut off the one-inch bar, as a simple lay-out will show. The belt must then develop 295.3 foot-pounds while the crank swings through 54 degrees. It will then develop $295.3 \div 54 = 5.5$ foot-pounds, nearly, in one degree, and in a complete revolution it will develop $5.5 \times 360 = 1980$ foot-pounds. As the press makes 30 strokes per minute, the belt will develop $30 \times 1980 = 59{,}400$ foot-pounds per minute. If a driving pulley 18 inches in diameter is used, the belt speed in feet per minute will be:

$$\frac{18 \times 3.14 \times 180}{12} = 848 \text{ feet.}$$

If a single thickness belt, one-inch wide, at 1000 feet per minute, transmits 33,000 foot-pounds per minute, the same belt at 848 feet per minute will transmit $\frac{848}{1000}$ as much, or $33,000 \times 0.848 =$ 27,984 foot-pounds. The width of belt necessary to transmit 59,400 foot-pounds per minute at this speed will then be $59,400 \div 27,984 = 2.1$ inches. No account has so far been taken of the power necessary to drive the machine itself. To allow for this the belt should evidently be not less than $2\frac{1}{2}$ inches wide. A 3-inch belt would allow considerable of a margin of safety, and further calculation will show that such a belt would develop, during about one-third of a revolution of the crank, the amount of energy which the fly-wheel had lost, so that, as the cutting operation takes about one-sixth of a revolution, the fly-wheel would be running at full speed for about one-half of a revolution of the crank, previous to the beginning of the cut, provided that it had not suffered any greater reduction of velocity than the 10 per cent. planned for.

If the press was employed doing punching the same method of procedure would be employed in the calculations, the area in shear in such a case being equal to the circumference of the hole multiplied by the thickness of the plate. The end of a punch is usually made slightly conical or slightly beveling, the effect in either case being to increase the shearing action, and make the work of punching easier.

CHAPTER XIX

TRAINS OF MECHANISM

FOR obtaining high speeds without the use of unduly large driving pulleys or gears, for securing gain in power by sacrificing speed, for securing reversal of direction, or for obtaining some particular velocity ratio between the driver and some part of the mechanism, pulleys, gears, worm-gears, or the like, may be substituted for direct acting driving-mechanisms.

To Secure Increase of Speed.—Let a shaft making 100 revolutions per minute be required to drive the spindle of a machine at 2000 revolutions per minute, the pulley on the spindle being 3 inches in diameter. If a direct drive were to be used, the pulley on the shaft would have to be as many times greater than the pulley on the spindle as 2000 is greater than 100, or 20 times.

This would mean a pulley on the shaft 60 inches in diameter. Practical considerations, such as the weight of the pulley, size of hangers and the like, would make such a pulley out of the question.

By interposing an intermediate countershaft between the first shaft and the spindle of the machine, however, having pulleys of such size that the product of the ratio of the pulley on the first shaft and the one to which it is belted on the countershaft, multiplied by the ratio of the second pulley on the countershaft and the pulley on the spindle to which it is belted is equal to the ratio which it is desired to have between the first shaft and the spindle, the same speed may be secured by the use of pulleys of convenient size. Thus, if the ratio between the pulley on the first shaft and the one on the countershaft is as 1 to 4, and the ratio between the driving pulley on the countershaft and the one on the spindle of the machine is as 1 to 5, the product of these two ratios, 1 to 4 and 1 to 5, is 1 to 20, and the arrangement will give the

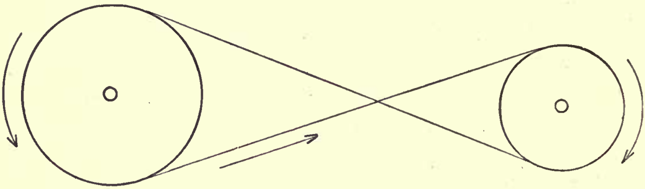


FIG. 204.—Reversal of Direction Obtained by Crossed Belt.

required speed. The pulley on the spindle being 3 inches in diameter, the driving pulley on the countershaft will be 15 inches in diameter, and if the driven pulley on the countershaft is 4 inches in diameter the pulley on the first shaft to which it is belted will be 16 inches in diameter, instead of 60 inches, as would be required with direct belting.

If the spindle of the machine, instead of being driven were made the driver, as it would be if it were the armature shaft of a motor, then this arrangement would give gain in power with consequent loss of speed.

To Secure Reversal of Direction.—In cases where shafts are belted together, reversal of direction of rotation is secured by simply using a crossed belt instead of an open one, as shown in Fig. 204. When gears are used, reversal of direction of rotation follows as a natural condition of their meshing together, as shown in Fig. 205. In order that the two gears *A* and *B* shall rotate in the same direction, it is necessary to separate them slightly, and interpose an intermediate gear, or *idler*, between

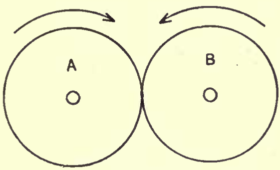


FIG. 205.—Relative Direction of Rotation in a Pair of Gears.

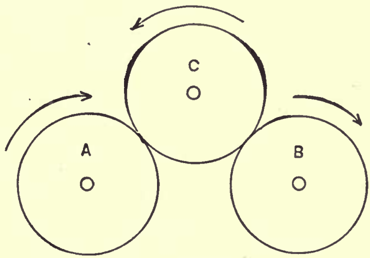


FIG. 206.—Influence of Idler on Direction of Rotation.

them as shown in Fig. 206. The rates of rotation of *A* and *B* with regard to each other is not affected by the idler gear, whether the idler be large or small. That this is so may be seen by direct examination. If *A* is the driver, its circumference will impart to the circumference of *C* its own rate of motion, and *C* will in turn impart to *B* the same rate of motion, which is the same as it would have if in direct connection with *A*.

If, now, another idler be interposed between *A* and *B*, making four gears in the train, *A* and *B* will again rotate in opposite directions. From this it will be seen that when a train is composed of an *even* number of gears, the first and last members rotate in opposite directions; but when the train is composed of an *odd* number of gears, the first and last members rotate in the same direction.

In Fig. 207 is shown the mechanism used in engine lathes to secure either direct or reversed motion, by having the working train consist of either an even or an odd number of gears. In this

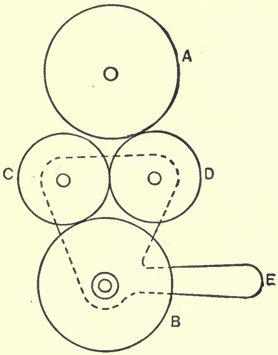


FIG. 207.—Principle of Tumbler Gear.

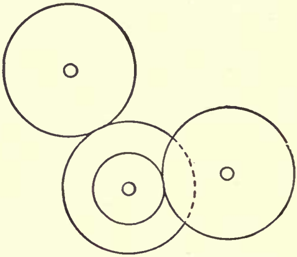


FIG. 208.—Principle of Compound Idler.

arrangement *A* is a gear on the head-stock spindle, and *B* is a gear on a stud below. Pivoted on the axis of *B* is a triangular piece of metal, or bracket, shown in dotted lines, which can be swung back and forth by the handle *E*. Mounted on this bracket are the idler gears *C* and *D*, *C* being constantly in mesh with *B*, and *D* being in mesh with *C*. When it is required that *B* shall rotate in the same direction as *A*, the handle *E* is lowered until *C* meshes with *A*. The working train then consists of three gears, *A*, *C* and *B*, *D* being out of mesh with *A*, revolving by itself, but not forming a part of the working train. When it is desired that *B* shall rotate in the opposite direction to *A*, the handle *E* is raised until *D* meshes with *A*, *C* being thrown out of mesh with it. The working train then consists of four gears, *A*, *D*, *C* and *B*, and the desired reversal is secured.

The Compound Idler.—It has been shown that when a train consists of simple gears the relative rates of rotation of the first and last members remain unchanged, regardless of the number or size of the idlers that may be interposed. When it is desired to secure a different rate of rotation between two members of a train than that which they would have if meshing directly together, a compound idler is used, as shown in Fig. 208. Such a gear is used on many screw cutting lathes. For cutting threads up to a certain number per inch the screw cutting train consists of simple gears.

A compound idler may then be introduced into the train, when without other change additional threads may be cut. If with screw cutting trains of simple gears a lathe will cut all whole numbers of threads up to 13 threads per inch, then, by adding a compound idler to the train, having its two steps in the ratio of 2 to 1, threads from 14 to 26 per inch (except odd numbers) may be cut with the same gears as previously used for cutting up to 13 threads per inch. If the compound idler forms an additional member of the train, the reversal of direction of rotation which would take place in the motion of the lead-screw of the lathe may be taken care of by the reversing gears between the spindle of the head-stock and the stud, previously described, and shown in Fig. 207.

The Screw Cutting Train.—In Fig. 209 is shown the screw cutting mechanism found on engine lathes. The reversing mechanism shown in Fig.

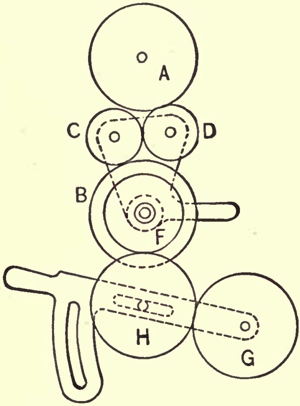


Fig. 209.

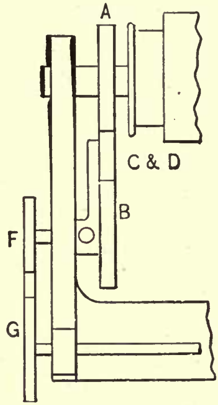


Fig. 210.

Figs. 209 and 210.—Arrangement of Lathe Change Gearing.

207 is reproduced entire, and these gears—the gear *A* on the lathe spindle, the gear *B* on the stud, which is connected with *A* by the idlers *C* and *D*—are all permanent gears. These gears are usually on the inside of the head-stock as shown in Fig. 210. The stud reaches through the head-stock, and on its outer end is the change gear *F*, connecting with the change gear *G* on the lead-screw of the lathe by means of the intermediate idler *H*. The idler *H* is mounted on a slotted swinging arm as shown, so as to allow of gears *F* and *G* being

replaced by others of such size as may be required to cut the particular screw desired. The carriage of the lathe, carrying the screw cutting tool, is driven directly by the lead-screw. On large lathes this screw is quite coarse, four threads per inch being common, while on smaller lathes a finer thread is used. The gear *A* on the spindle and the fixed gear *B* on the stud are sometimes of the same size, and sometimes of different sizes.

The problem met with in screw cutting is to find what sizes change gears, *F* and *G*, must be used so that the lead-screw shall drive the carriage along one inch while the spindle of the lathe is making a number of revolutions equal to the number of threads to be cut per inch. Let us take as an example the assumed case of a lathe in which the lead-screw has 9 threads per inch, and in which the number of teeth in the gear on the spindle is to the number of teeth in the fixed gear on the stud as 3 to 4; required the size of change gears to cut 23 threads per inch. Then, as the lead-screw has 9 threads per inch, the spindle of the lathe must make 23 revolutions while the lead-screw is making 9 revolutions. The method used in a previous chapter for obtaining the size of pulleys to give required speeds will give us the solution of this problem; if the speed of the first driving member of the train, together with the number of teeth or relative sizes of all other driving members be placed on one side of a vertical line, and the speed of the last driven member, together with the number of teeth or relative sizes of all other driven members be placed on the other side of the line, the product of the numbers on one side of the line multiplied together will equal the product of the numbers on the other side of the line multiplied together. The spindle of the lathe is, of course, the first driving member of the train, and the lead-screw is the last driven member. As the spindle is to make 23 revolutions while the lead-screw makes 9 revolutions, 23 will be the first number on the side of the line on which the driving members are placed, and 9 will be the last number on the side of the line on which the driven members are placed. Next, as the ratio between the sizes of the driving gear on the lathe spindle and the fixed gear on the stud below which it drives is as 3 to 4, these numbers will be placed against each other on opposite sides of the line.

The ratio between the numbers of teeth or sizes of the two change gears, *F* and *G*, whose sizes it is required to find, being unknown, may be said to be as 1 to the unknown number x. These numbers, 1 and x, are now placed on their proper sides of the line, and the problem appears as shown below. The size of the idler gear *H* does not enter into the question, because, as has been previously shown, a simple idler gear does not affect the relative rates of rotation of the gears between which it transmits motion.

Speed of spindle	23		
Ratio of size of spindle gear	3	4	to size of fixed stud gear.
Ratio of number of teeth in change gear *F*	1	x	to number of teeth in change gear *G*
		9	speed of lead-screw.

$$69 = 36x$$

Multiplying together the numbers on both sides of the line gives the equation $69 = 36x$. It is evident that if 69 equals $36x$, x must be equal to 69 divided by 36, or $\frac{69}{36}$. The ratio between sizes of the gear F and the gear G is then as 1 to $\frac{69}{36}$. Eliminating the fraction by multiplying both terms of the ratio by 36 gives the ratio as 36 to 69. If, then, F has 36 teeth, and G has 69 teeth, the lathe will cut the required number of 23 threads per inch.

In Fig. 211 is shown how a compound idler gear is sometimes used in a screw cutting train. The

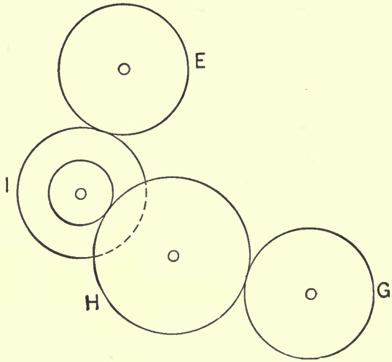


Fig. 211.—Compound Gearing.

change gear G and the idler H have long hubs on one side. When it is desired to cut finer threads than what the gears E and G with the idler H will give, H and G, are put on with the long hubs toward the lathe, throwing them out of line with E. The gear E then meshes into the large step of I, the small step of I meshes into H, and H meshes into *G*. The ratio between the large and the small steps of *I* must then be taken into account in the calculation. For cutting the coarser threads *H* and *G* are put on with the short hubs toward the lathe, bringing them into line with *E*. The idler *I* is also turned over, so that its large step is on the outside and out of line with *E* and *H*. It is then swung back out of the way.

When the gearing is fully compounded the two gears at *I* are separate from each other but keyed together on the same stud and mounted in the same manner as shown in Fig. 211. By varying the sizes of these gears, almost any screw thread may be cut within reasonable limits. In this case, of course, there are four gears to be determined in our calculations. Simplified rules are given in the following for this case, as well as for the regular simple trains.

Large lathes are provided with change gears for cutting threads from about 2 to about 20 threads per inch, smaller lathes being provided with gears for cutting from about 3 or 4 to 40 or 50 threads per inch, in either case including a pair of gears for cutting $11\frac{1}{2}$ threads per inch, this being the standard thread for iron pipes from one to two-inch sizes inclusive. The smaller lathes would also naturally be provided with gears for cutting 27 threads per inch, this being the number of threads on $\frac{1}{8}$-inch iron pipes.

Simplified Rules for Calculating Lathe Change Gears.—The following rules for calculating change gears for the lathe have been published by *Machinery* (Reference Series Book No. 35, *Tables* *and Formulas for Shop and Draftingroom*), and are here given because of their concise form and simplicity.

Rule 1.—To find the "screw-cutting constant" of a lathe, place equal gears on spindle stud and lead-screw; then cut a thread on a piece of work in the lathe. The number of threads cut with equal gears is called the "screw-cutting constant" of *that* particular lathe.

Rule 2.—To find the change gears used in simple gearing, when the screw-cutting constant as found by Rule 1, and the number of threads per inch to be cut are given, place the screw-cutting constant of the lathe as numerator and the number of threads per inch to be cut as denominator in a fraction, and multiply numerator and denominator by the same number until a new fraction is obtained representing suitable numbers of teeth for the change gears. In the new fraction, the numerator represents the number of teeth in the gear on the spindle stud, and the denominator, the number of teeth in the gear on the lead-screw.

Rule 3.—To find the change gears used in compound gearing, place the screw-cutting constant as found from Rule 1 as numerator, and the number of threads per inch to be cut as denominator in a fraction; divide up both numerator and denominator in two factors each, and multiply each pair of factors (one factor in the numerator and one in the denominator making a pair) by the same number, until new fractions are obtained, representing suitable numbers of teeth for the change gears. The gears represented by the numbers in the new numerators are driving gears, and those in the denominators driven gears.

Two examples, showing the application of these rules, will be given in the following.

Example 1.—Assume that 20 threads per inch are to be cut in a lathe having a "screw-cutting constant," as found by the method explained in Rule 1, equal to 8. The numbers of teeth in the available change gears for this lathe are 28, 32, 36, 40, 44, etc., increasing by 4 up to 96.

By applying Rule 2, we have then:

$$\frac{8}{20} = \frac{8 \times 4}{20 \times 4} = \frac{32}{80}$$

By multiplying both numerator and denominator by 4 we obtain two available gears having 32 and 80 teeth. The 32-tooth gear goes on the spindle stud and the 80-tooth gear on the lead-screw. It will be seen that if we had multiplied by 3 or by 5 instead of by 4, we would not have obtained available gears in both numerator and denominator, as 8×3 would have given 24 and 20×5 would have given 100, both of which gears are not in our given set of gears. The proper number by which to multiply can be found by trial only.

Example 2.—Assume that 27 threads per inch are to be cut on the same lathe as assumed in Example 1.

In this case the calculation must be made for compound gearing, as so fine a pitch could not be cut by simple gearing in this lathe. By applying Rule 3 we have:

$$\frac{8}{27} = \frac{2 \times 4}{3 \times 9} = \frac{(2 \times 20) \times (4 \times 8)}{(3 \times 20) \times (9 \times 8)} = \frac{40 \times 32}{60 \times 72}$$

The four numbers in the last fraction give the numbers of teeth in the required gears. The gears in the numerator (40 and 32) are the driving gears, and those in the denominator (60 and 72) are the driven gears.

It makes no difference which one of the driving gears is placed on the spindle stud or which one of the driven gears is placed on the lead-screw.

Back-Gears.—Nearly all engine lathes and many other machine tools are provided with a set of re-

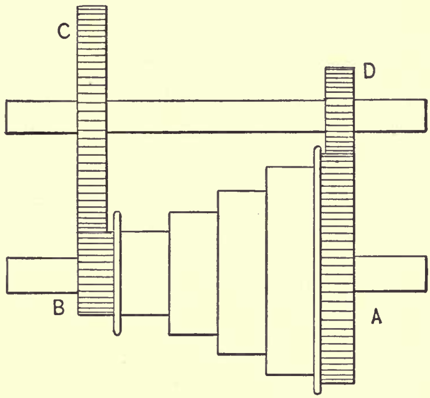


FIG. 212.—Principle of Back-Gearing.

ducing gears, called *back-gears*, by means of which double the range of speeds that can be obtained by direct driving may be given to the spindle of the machine. Fig. 212 illustrates such a set of gears, and the method of applying them to the machine. The large gear *A* is fastened to the spindle of the machine, but the cone pulley, with the gear *B* attached to it, is loose on the spindle. The back-gear shaft with gears *C* and *D* is mounted in brackets on the back side of the head-stock, and is provided with eccentric bearings, by means of which the gears on it can be thrown into or out of mesh with the gears on the head-stock spindle. When direct driving is desired, the back-gears are thrown back, out of the way, and the cone pulley and the large gear are clamped together by means of a screw pin or stud passing through the gear into the cone. They then revolve together as one piece.

Let us assume the case of a lathe having a cone with four steps, the largest step being 6 inches in diameter, and the smallest 4 inches in diameter, with the intermediate steps in proper proportion. If the cone pulley on the countershaft is of the same size as the one on the spindle, then, if the countershaft runs 300 revolutions per minute, direct driving will give about the following speeds to the spindle: 450, 345, 260 and 200. Let it now be required to find the sizes of gears to be used so that with the back-gear driving, a proportionately slower rate of speeds may be obtained. We may solve the problem by giving to the gears some arbitrary sizes, and finding what speeds such sizes will give, and then modify these sizes until the required speeds are obtained. For trial purposes let us make the pitch diameter of the gear *A* the same as the diameter of the large step of the cone pulley, or 6 inches, and the pitch diameter of the gear *B* the same as the diameter of the small step of the cone pulley, or 4 inches. Arranging driving and driven members on opposite sides of a vertical line, the speed of the first driving member of the train, the countershaft, being 300, the required speed of the last member, the lathe spindle, being represented by x, and having the belt on the largest step of the countershaft cone so as to obtain the highest speed with back-gears, gives an arrangement of the case as below. The sizes of the back-gears are the same as those on the lathe spindle, the gear C being 6 inches in pitch diameter, and the gear D 4 inches in pitch diameter.

Speed of countershaft	300		
Pulley on countershaft	6	4	Pulley on lathe
Gear B on lathe	4	6	Back-gear C
Back-gear D	4	6	Gear A on lathe
		x	Speed of spindle

$$28{,}800 = 144x$$

$$x = \frac{28{,}800}{144} = 200.$$

From this it is seen that with the sizes of the gears as above, the highest speed with back-gears would be the same as the lowest speed without the back-gears. This, of course, would be useless duplication of speeds.

For another trial we will make the sizes of the gears B and D each $3\frac{1}{2}$ inches in pitch diameter. The calculation then becomes:

Speed of countershaft	300		
Pulley on countershaft	6	4	Pulley on lathe
Gear B on lathe	3.5	6	Back-gear C
Back-gear D	3.5	6	Gear A on lathe
		x	Speed of spindle

$$22{,}050 = 144x$$

$$x = \frac{22{,}050}{144} = 153, \text{ nearly.}$$

A speed of 153 revolutions per minute for the fastest back-gear speed follows quite regularly the series of speeds which the direct drive gives.

Instead of using the pitch diameters of the gears in making the calculations the number of teeth which the gears would have, the pitch being first decided on, might be used. In this manner it is possible to make slight changes in the diameters of the gears without bringing troublesome fractions into the calculations.

Many lathes and other machine tools have trains of mechanism much more complicated than any here shown, but the method of procedure here outlined can be applied to all of them.

CHAPTER XX

QUICK RETURN MOTIONS

In a large class of machinery the work is done during the forward motion of a reciprocating part; the return of the part to its starting point is then a question of time. The quicker the part can be returned to its starting point, the more efficient becomes the machine. When the stroke is long, as in the case of the bed of an iron planer for large work, this rapid return motion is usually obtained by means of shifting the driving belt onto a return pulley so arranged that a higher ratio of speed is procured; but in other cases, where the reciprocating motion is shorter, and the stroke is actuated by means of a crank, the actuating mechanism is made such that the crank gives a slow forward and a quick return motion to the reciprocating part. Iron planers for small work, shapers, and the like, and some classes of engines and pumps, use such quick return motions. Below are described the principal devices used for such purposes.

Fig. 213 shows a method of securing a quick return by having the axis of the crank outside of the path of the reciprocating end of the connecting-rod. Let A be a crank, the crank-pin of which, a, acting upon the connecting-rod B represented by the heavy line, causes the block b to move back and forth in the path *CD*. When the crank is in the position shown the block is at the extreme left of its stroke, the connecting-rod and crank being in the same straight line, the center line of the connecting-rod coinciding with the axis of the crank. As the crank swings downward, the block *b* is driven to the right; but an examination of the illustration will show that the crank must make

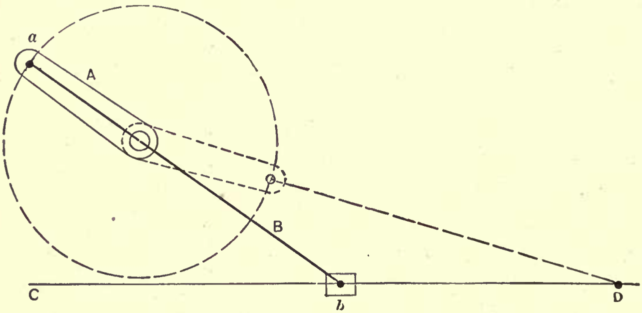


FIG. 213.—Simple Quick Return Motion.

more than a half revolution before it again forms a straight line with the connecting-rod, which it will do when the block has reached its extreme position to the right. As, therefore, the block makes its movement to the right while the crank is swinging through the lower angle included between these two positions, and as it makes its return stroke while the crank is swinging through the upper angle included between these same two positions, the time of the forward stroke of the block will be to the time of its return stroke as the lower angle is to the upper angle.

The upper angle being the smaller of the two, the block has a quick return motion. To secure ease of motion to the block as it starts on its stroke to the right, the angle *abC*, the angle which the connecting-rod makes with the path of the block, should not be more than about 45 degrees.

To design a quick return motion of this type, lay out a horizontal line *ab*, Fig. 214, and on it mark off *cb* equal to the required length of stroke. From *c* draw the line *cd* of indefinite length at such an

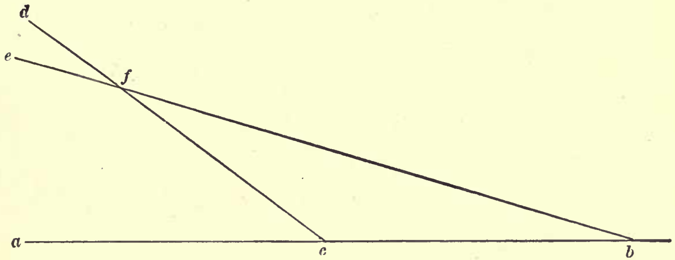


FIG. 214.—Lay-out of Quick Return Motion in Fig. 213.

obliquity that the angle *acd* shall not be more than 45 degrees. From *b* draw the line *be* at the angle required to give the desired quick return. The intersection of these two lines at *f* will be the axis of the crank. The length *bf* will be seen by referring back to Fig. 213 to be equal to the length of the crank plus the length of the connecting-rod. The length of *cf* will be seen to be equal to the length of the connecting-rod minus the length of the crank. If in a given case the length *cb* is made 12 inches, and *cf* is found to be 10 and *bf* 21 inches, which they would be if the angles were as shown in Fig. 214, then, letting x represent the length of the connecting-rod and y the length of the crank, we would have $x + y = 21$ inches, and $x - y = 10$ inches. Adding the left-hand and the right-hand members, respectively, of these two equations, we would have $x + y + x - y = 21 + 10 = 31$ inches. As $+ y - y = 0$ we may eliminate these expressions, and the equation will read $2x = 31$ inches, and x, the length of the connecting-rod, will thus be $15\frac{1}{2}$ inches. The length of the crank will then be 21 inches (the length of bf) minus $15\frac{1}{2}$ inches, or $5\frac{1}{2}$ inches.

It will be seen that if the length of the stroke is made variable by having the crank-pin, a, adjustable to different positions on the crank A, Fig. 213, the difference between the time of the forward and of the return stroke of the sliding block b will be lessened, because the two positions which it will occupy at the extremes of its stroke will be nearer together, and the lower and upper angles which the crank passes through in giving to the block its forward and return movements will be more nearly equal.

Fig. 215 shows a quick return motion device especially adapted to cases where the horizontal space is limited, and which is much used on shapers. The illustration shows a shaper in outline. The ram of the shaper is given its forward and return motion by means of the rocking arm A, which swings on a fulcrum at B. The rocking arm is given its motion by means of a crank-pin on the disk C, the pin engaging in a sliding block which travels in a slot in the arm A.

Let *BC* and *BD*, Fig. 216, represent the extreme positions of the rocker arm *A*. Draw the lines *OF* and *OG* from the center of the crank disk at *O* at right angles to *BC* and *BD*. It is evident that in order that the crank, on its upper sweep, shall

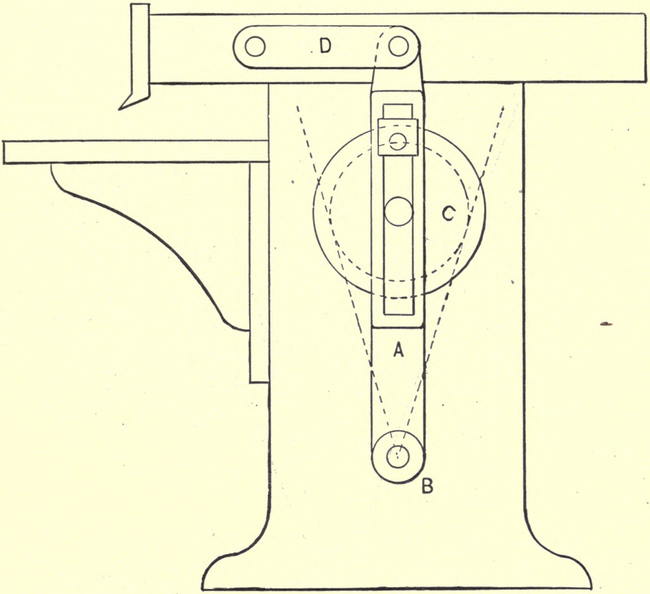


FIG. 215.—Diagram of Quick Return Arrangement in a Shaper.

move the rocker arm from *C* to *D*, it must move through the arc *FAG*, while to return the arm from *D* to *C*, on its lower sweep, it must move only through the lower arc *FG*. The time of the return motion will therefore be to the time of the forward motion as the lower arc or angle *FG* is to the arc or angle *FAG*. If the crank is shortened so as to give a shorter stroke to the ram of the shaper, then the rocker arm will swing through a smaller angle, as from *H* to *I*, and lines drawn from *O* at

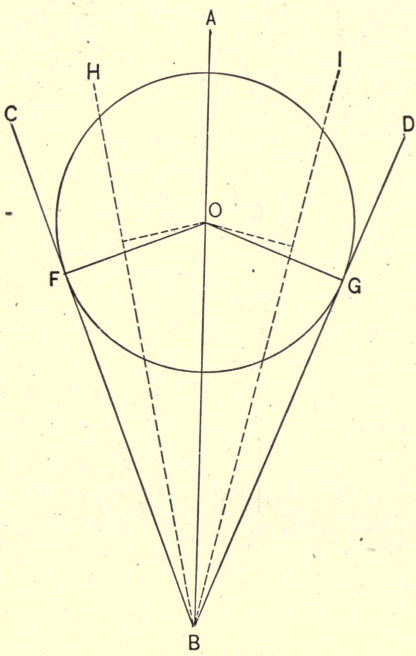


FIG. 216.—Diagram of Speed Ratios in Shaper Motion.

right angles to *HB* and *IB* will be more nearly in a straight line than *OF* and *OG*. There will, therefore, be less difference between the time of forward and return motions on short strokes than on long ones.

The Whitworth Quick Return Device.—Let *A*, Fig. 217, be a slotted arm revolving on its axis at *B*. Above *A* is the driving crank *C*, having a pin engaging in the slot at the left in the arm *A*. The slot at the right in the arm *A* is provided for an adjustable stud which drives the reciprocating parts, through the medium of the connecting-rod

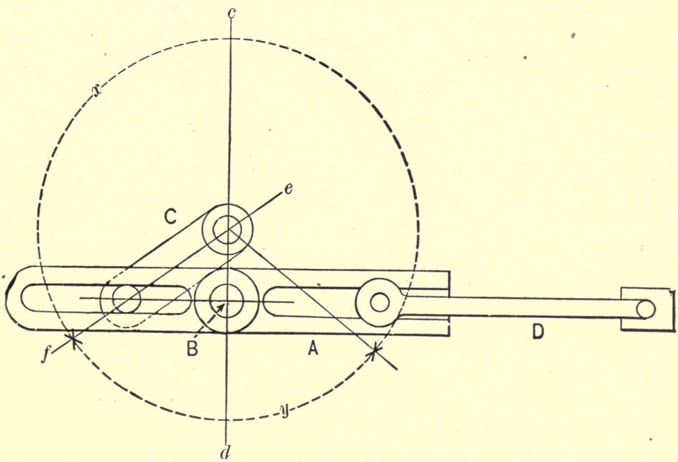


FIG. 217.—Whitworth Quick Return Motion.

D. It will be seen that, as shown, the connecting-rod is at the extreme right of its motion, forming as it does a straight line with the revolving arm *A*, which latter is at the same time at right angles with the center line *cd*. It will be seen that in order that the arm *A* may move through half a revolution so as to bring the connecting-rod to the extreme left of its motion, it will be necessary for the actuating crank *C* to revolve either through the upper angle x or through the lower angle y, so as to form again the same angle with the center line cd, but at the right of it, as it is now shown forming with it at the left. The forward and return motions will, therefore, be to each other as the angle x is to the angle y. To design a quick return motion of this type it is, therefore, necessary to first lay out the angles x and y of such relative sizes that x is as many times greater than y as the time of the forward motion is to be greater than the time of the return motion, having them, of course, central on the line cd. The distance apart of the fulcrums of the crank C and of the revolving arm A will be partly determined by the sizes of their shafts. The location of the crank-pin, determining the length of the crank, will then be at the intersection of the horizontal center line of the revolving arm A with the dividing line ef between the angles x and y. The length of the crank must, of course, be sufficient so that the crank pin will swing under the hub of the arm A, and the length of the crank-pin slot in A must be sufficient for the motion of the pin relative to the arm.

It will be noticed that, unlike the two preceding quick return devices, varying the stroke of the reciprocating parts does not alter the relative time of the forward and return motions; for such change does not affect the angles x and y upon which the time of the forward and return motions depends. If, however, the length of the crank C is varied, then the angles x and y are altered, and the time of the forward and return motions will be affected.

It will be seen upon examination that with the construction shown the revolving arm *A* must be made in two parts, one at each end of its shaft, in order to avoid interference of the parts of the mechanism with one another as they revolve. This trouble is overcome by replacing the crank *C* with a crank disk which fits over and revolves upon a fixed stud or hub large enough to receive the stud at *B* upon which the arm *A* revolves.

The Elliptic Gear Quick Return.—If two ellipses of equal size, Fig. 218, having foci at *w* and *x* and

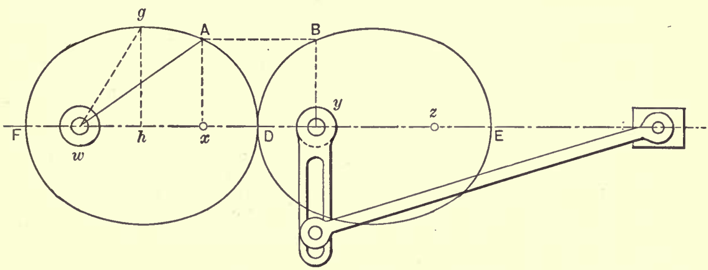


Fig. 218.—Quick Return Motion by Means of Elliptic Gears.

at *y* and *z*, be placed in contact with each other with their long diameters forming a continuous straight line as shown; then if the ellipses are caused to revolve freely upon their corresponding foci, *w* and *y*, they will roll upon each other perfectly, without slipping. From the nature of an ellipse as shown by its construction with a thread and pencil (see Chapter III, Problem 13) it will be seen that if the ellipse at the left were being formed in this manner and the pencil were at *D*, the intersection of the circumference of the ellipse with the long diameter, the length of the thread would be equal to the sum of the distances *wD* and *Dx*. But the distance *Dx* is the same as the distance *Dy*; therefore, the length of the thread would be equal to the distance *wy*, the distance between the foci upon which the ellipses are revolving. If, now, the ellipses are revolved until the points *A* and *B*, vertically over the foci *x* and *y*, are in contact with each other, the sum of the distances *wA* and *By* will be equal to the distance between the foci *w* and *y*, for their sum is equal to the length of the thread, and the length of the thread is equal to *wA* plus *Ax*, and *Ax* is equal to *By*, as points *A* and *B* are both vertically over the foci of the ellipses. In a similar manner any pair of points may be selected on the two ellipses equally distant from the point *D*. The distance from the point on the ellipse at the left, to the focus *w*, will be equal to the length of the thread at the left of the pencil, and the distance from the point on the ellipse at the right, to the focus *y*, will be equal to the length of the thread at the right of the pencil, and their sum will be equal to the distance between the foci *w* and *y*. This distance between the foci *w* and *y* will be seen on further examination to be equal to the long axis of the ellipse. This property of the ellipse has been taken advantage of to secure a quick return motion to a reciprocating part of a machine. If in Fig. 218 the two ellipses represent the pitch lines of elliptic gears; with the gear at the left as the driver with a uniform motion, the one at the right will have an ununiform motion. If, now, a crank is mounted on the same shaft as the driven elliptic gear, the crank having its center line at right angles to the long axis of the ellipse, and this crank actuates a sliding block back and forth in the direction of the center line of the two gears, then this block will have a slow motion in one direction, and a quick motion in the other direction. If, now, the gears are revolved from the position in which they are shown until *A* and *B* are in contact, the gear at the right will have made a quarter of a revolution and the sliding block will be at the extreme right of its stroke; but while this gear has made a quarter of a revolution, the driving gear has revolved through the angle *AwD* only. If, now, the gear at the right is revolved another quarter of a turn, the points *E* and *F* will be in contact, and the crank will be directed vertically upward. The driving gear will, however, have revolved through the angle *AwF*. The forward and return motions of the sliding block will, therefore, be to each other as the angle *AwF* is to the angle *AwD*. In designing a pair of elliptic gears, therefore, the first thing to do is to determine the size of the angle *Awx*. To find the distance between the foci *w* and *x* first lay out on a large scale a triangle similar to the triangle *Awx*. Then the sum of its hypothenuse and the perpendicular will be to the length of its base as the sum of *wA* and *Ax* (the long axis of the ellipse) is to *wx*, the distance between the foci of the ellipse. The length of the short axis may then be found by reversing Problem 13, Chapter III. The problem may be solved even more accurately by the rules given for the solution of right-angled triangles. The length of wA will be to Ax as 1 is to the sine of the angle Awx. Dividing the long axis of the ellipse into two parts in this proportion gives the length of wA and Ax. The length of wx will then be equal to the length of Aw multiplied by the cosine of the angle Awx. Then to find the short axis of the ellipse, divide the distance wx into two equal parts and construct the triangle wgh. The length wh will be half of the distance between the foci, and the length of wg will be half of the long axis. The length gh, half of the short axis, may then be found.

Calculations made in this manner give the following proportions to ellipses for quick return ratios as indicated in the first column:

Ratio of Forward to Return Motion.	Long Axis.	Short Axis.	Distance Between Foci.
2 to 1	1.000	0.963	0.268
2½ to 1	1.000	0.936	0.351
3 to 1	1.000	0.910	0.414
4 to 1	1.000	0.860	0.509
5 to 1	1.000	0.817	0.577

There appear to be two difficulties with elliptic gearing. The first is that if a high quick return ratio is attempted, so as to make considerable difference between the long and the short axes, the obliquity of the action of the teeth upon each other, and the consequent great amount of friction between the teeth as they come together, becomes so great as to be troublesome. This may, to a considerable extent at least, be overcome by using a train of gears, each gear but slightly elliptic, in place of one pair of decidedly elliptic form. Thus a train of three gears having their long and short axes in the proportion required to give a quick return of 3 to 1, with one pair of gears, will give a quick return of 9 to 1. If three gears of the 4 to 1 proportion are used, a quick return of 16 to 1 will result.

The second difficulty is that of correctly cutting the teeth. To work properly, the teeth should be cut on a machine having a special elliptic gear cutting attachment, otherwise the gears are likely to be expensive and unsatisfactory. Such an elliptical gear cutting arrangement is described, and the subject of elliptic gearing is quite fully discussed, in Grant's treatise on gearing. Not being within the territory of this elementary treatise on machine design, the subject cannot here be dealt with in detail.

INDEX

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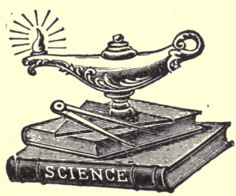

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