International Library of Technology

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A SERIES OF TEXTBOOKS FOR PERSONS ENGAGED IN ENGINEERING PROFESSIONS, TRADES, AND VOCATIONAL OCCUPATIONS OR FOR THOSE WHO DESIRE INFORMATION CONCERNING THEM. FULLY ILLUSTRATED

SHOP CALCULATIONS
READING WORKING DRAWINGS
MEASURING INSTRUMENTS
PRECISION MEASURING INSTRUMENTS
GENERAL APPLIANCES AND PROCESSES

SCRANTON
INTERNATIONAL TEXTBOOK COMPANY
1920

PRESS OF
INTERNATIONAL TEXTBOOK COMPANY
SCRANTON, PA.

78289

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PREFACE

The volumes of the International Library of Technology are made up of Instruction Papers, or Sections, comprising the various courses of instruction for students of the International Correspondence Schools. The original manuscripts are prepared by persons thoroughly qualified both technically and by experience to write with authority, and in many cases they are regularly employed elsewhere in practical work as experts. The manuscripts are then carefully edited to make them suitable for correspondence instruction. The Instruction Papers are written clearly and in the simplest language possible, so as to make them readily understood by all students. Necessary technical expressions are clearly explained when introduced.

The great majority of our students wish to prepare themselves for advancement in their vocations or to qualify for more congenial occupations. Usually they are employed and able to devote only a few hours a day to study. Therefore every effort must be made to give them practical and accurate information in clear and concise form and to make this information include all of the essentials but none of the non-essentials. To make the text clear, illustrations are used freely. These illustrations are especially made by our own Illustrating Department in order to adapt them fully to the requirements of the text.

In the table of contents that immediately follows are given the titles of the Sections included in this volume, and under each title are listed the main topics discussed. At the end of the volume will be found a complete index, so that any subject treated can be quickly found.

INTERNATIONAL TEXTBOOK COMPANY

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SHOP CALCULATIONS

(PART 1)

INTRODUCTION

NOTATION AND NUMERATION

1. Necessity for Calculations.—The worker in the shop frequently meets with problems that require figuring. For example, the machinist may wish to know how far he must set the lathe centers out of line to turn a desired taper on a piece of work, or he may wish to know the sizes of the change gears required to cut a certain screw thread. The blacksmith may have to find how long a piece of straight bar must be cut off, so that, when it is bent, it will form a ring of a certain size. The patternmaker may wish to know how to set his dividers so that they will space off a certain number of equal divisions on a circle. The foundryman may need to know the amount of metal required to pour a casting whose dimensions are given on a drawing. In each of these cases it is necessary to make calculations in order to obtain the desired information. Sometimes the calculations are short and simple, and at other times they are long and difficult.

2. Use of Arithmetic.—Before it is possible to make calculations of any kind, it is necessary to know something about figures and numbers, because all calculations bring figures and numbers into use. The study of numbers, or the art of reckoning, is commonly called **arithmetic**. Thus, the various calculations that may be made in the shop really rest on an understanding of arithmetic. For this reason, it will be necessary to begin with a study of those arithmetical principles and processes that are to be used later in making shop calculations.

3. Numbers.—A **number** is simply one or more units or things. It answers the question "How many?" For example, let the question be "How many bolt holes are there in that cylinder head?" If the answer is "Eight holes," then *eight* is a number, because it tells how many. A **unit** is *one*, or a single thing, as *one* inch, *one* dozen. A number, however, may be either *one* or more than *one*, as *one* hour, *six* feet, *ten* dollars.

4. Numbers are written by using certain signs, or characters, called **figures.** The figures that are used in making arithmetical calculations are ten in number. These ten figures, with their names below them, are as follows:

Figures	0	1	2	3	4	5	6	7	8	9
Names	*Cipher*	*One*	*Two*	*Three*	*Four*	*Five*	*Six*	*Seven*	*Eight*	*Nine*

The first character, or figure, 0, has no value when it stands by itself; that is, it is nothing, or naught. Sometimes it is also called *naught*, or *zero*. Each of the other nine figures, which are called **digits,** has its own value, which is different from the values of the others.

5. The method of counting above ten and up to one thousand, and the names of the various numbers, are given in the following list:

11 *eleven*	21 *twenty-one*	31 *thirty-one* and so	100 *one hundred*
12 *twelve*	22 *twenty-two*	on up to	200 *two hundred*
13 *thirteen*	23 *twenty-three*	40 *forty;* then	300 *three hundred*
14 *fourteen*	24 *twenty-four*	41 *forty-one* and so	400 *four hundred*
15 *fifteen*	25 *twenty-five*	on up to	500 *five hundred*
16 *sixteen*	26 *twenty-six*	50 *fifty;* then to	600 *six hundred*
17 *seventeen*	27 *twenty-seven*	60 *sixty*	700 *seven hundred*
18 *eighteen*	28 *twenty-eight*	70 *seventy*	800 *eight hundred*
19 *nineteen*	29 *twenty-nine*	80 *eighty*	900 *nine hundred*
20 *twenty*	30 *thirty*	90 *ninety*	1,000 *one thousand*

6. The value represented by a figure depends on its position in relation to other figures; thus, figures may have *simple values* and *relative values*. The **simple value** of a figure is the value it has when it stands alone; for example, the figure 2 standing alone has a value that is one greater than 1 and one less than 3. But if a figure 1 is placed to the right of the 2, making 21, the first figure no longer has the value it had before. This new value that is given to it by placing another figure to the right of it is called its **relative value.** The difference between simple and relative values may be explained as follows:

If the figure 8 stands alone, thus.................... 8
it is simply *eight units*, or **eight.**

Place a 2 to the right of it; thus.................... 82
The 2 is now *two units*, but the 8 has moved one place to the left, so that it is no longer *eight units*. Instead, it is *eight* **tens,** or *ten times* 8.

Now place a 5 to the right; thus................... 825
The 8 is now moved another place to the left and its value is again increased ten times, or *ten times eight tens*, making 8 **hundreds.** At the same time the 2 is moved one place to the left and its value is increased to 2 **tens.**

Add a 6 to the right; thus......................... 8256
The 8 is now another place to the left and its value is increased ten times more, or to 8 **thousands.** The 2 is increased to 2 **hundreds,** and the 5 to 5 **tens.**

The last number of four figures is read *eight thousand two hundred fifty-six.*

7. The preceding illustration shows that by moving a figure one place to the left in a number its value is made ten times as great as before. The last position at the right in a number is called the *units* place. Take the number 417,385,926 as an example. The figure 6 is in the units place and is simply *six*. The next place, occupied by the 2, is the *tens of units* place, so that the 2 in this position has a value of 2 tens of units, or *twenty*. The 9 is in the *hundreds of units* place, and its value is 9 hundreds of units, or *nine hundred*. This right-hand group of three figures, therefore, has a value of nine hundreds, two tens, and six units, or *nine hundred twenty-six*, as it would be read.

The next group of three figures, or 385, is at the left of 926; therefore, the figure 5 is in the fourth place from the right end, and its value is ten times as great as it would be in the third position. The third place is the hundreds place, and as the fourth place is ten times as great, it must be the *thousands* place, so that the figure 5 in this position represents *five thousands*. The figure 8 is in the *tens of thousands* place, and its value is 8 tens of thousands, or *eighty thousand*. The figure 3 is in the *hundreds of thousands* place, and its value is 3 hundreds of thousands, or *three hundred thousand*. The middle group of three figures, therefore, has a value of three hundred thousand, eighty thousand, and five thousand, which would be read *three hundred eighty-five thousand*.

The group of figures at the left, or 417, refer to millions, a *million* being ten times as great as a hundred thousand. The 7 is in the *millions* place and has a value of *seven million*. The 1 is in the *tens of millions* place and has a value of *ten million*. The 4 is in the *hundreds of millions* place and has a value of *four hundred million*. This group of figures, therefore, has a value of four hundred million, one ten million, and seven million, or *four hundred seventeen million*, as it would be read.

The entire number 417,385,926, made up of the three groups of figures, would be read *four hundred seventeen million three hundred eighty-five thousand nine hundred twenty-six*. The following table shows the positions of the figures, the groups, and the name of each of the places, or positions:

Millions			*Thousands*			*Units*		
Hundreds of Millions	Tens of Millions	Millions	Hundreds of Thousands	Tens of Thousands	Thousands	Hundreds of Units	Tens of Units	Units
4	1	7,	3	8	5,	9	2	6

8. The cipher, 0, has no value in itself, because it represents nothing or zero; but it is useful in determining the position of other figures. Suppose, for example, that the number *two hundred five* is to be written. It would not be correct to write it 25, because that is *twenty-five*. The 2 must be in the hundreds place and the 5 in the units place, because *two hundred five* means *two hundreds* and *five units;* therefore, it is written by placing a cipher between the 2 and the 5, giving 205. The 2 is then in the hundreds place and the 5 in the units place, as required, and the cipher indicates that there are no *tens*. In the same way, *three thousand twenty-six* would be written 3026, and *six thousand four* would be written 6004. In the last case, two ciphers are needed, because the 6 must be in the thousands place, which is the fourth from the right. If the number to be written is *five thousand nine hundred eighty*, it could *not* be written 598 because that is only *five hundred ninety-eight*. The 5 must be in the fourth, or thousands place, the 9 in the hundreds place, and the 8 in the tens place; consequently, a cipher is added at the right, giving 5980, which is the correct way of writing *five thousand nine hundred eighty*.

9. In writing numbers that contain more than three figures, it is common to divide them into groups of three figures, counting from the *right*. This is called **pointing off,** because a comma (,) is used to point off, or mark, each group of three figures. The object of doing this is to enable the number to be read easily and accurately. The first group at the right is the units group, the next the thousands group, and the next the millions group. This method of pointing off and the naming of the groups is shown in the table at the end of Art. **7**.

10. The four fundamental processes, or operations, in arithmetic are *addition*, *subtraction*, *multiplication*, and *division*. Every calculation that is made must use one or more of these processes; for this reason, each is explained fully and examples are given to show how the operation is carried out.

EXAMPLES FOR PRACTICE

Point off and read the following numbers:

(1) 31072; (2) 317020; (3) 1007; (4) 6051; (5) 28970093.

Write the following numbers, using figures:

(6) Seven thousand seventeen; (7) One thousand nine hundred fourteen; (8) Ten million eighty two thousand thirty-six.

Ans. (1) 31,072 or thirty one thousand seventy-two; (2) 317,020 or three hundred seventeen thousand twenty; (3) 1,007 or one thousand seven; (4) 6,051 or six thousand fifty-one; (5) 28,970,093 or twenty-eight million nine hundred seventy thousand ninety-three; (6) 7,017; (7) 1,914; (8) 10,082,036.

FUNDAMENTAL PROCESSES

ADDITION

11. Addition is the *process of finding* the *sum of two or more numbers*. The sign of addition is +. It is read *plus*, and means *more*. Thus, 5+6 is read *5 plus 6*, and means that 5 and 6 are to be added.

12. The sign of equality is =. It is read *equals* or *is equal to*. Thus, 5+6=11 may be read *5 plus 6 equals* 11.

13. Numbers expressed in the same units can be added, but numbers expressed in different units cannot be added. Thus, 6 dollars can be added to 7 dollars and the sum will be 13 dollars; but 6 dollars cannot be added to 7 feet. Table I gives the sum of any two numbers from 1 to 12. This table should be carefully committed to memory. As 0 has no value, the sum of any number and 0 is the number itself; thus, 17 and 0 is 17, or 17+0=17.

14. For addition, place the numbers to be added directly under each other, taking care to place *units* under *units*, *tens* under *tens*, *hundreds* under *hundreds*, and so on. When the numbers are thus written, the *right-hand figure of one number is placed directly under the right-hand figure of the one above it*, thus bringing units under units, tens under tens, etc. Proceed as in the following example:

EXAMPLE.—What is the sum of 131, 222, 21, 2, and 413?

SOLUTION.—

```
       131
       222
        21
         2
       413
       ---
   sum 789  Ans.
```

TABLE I

ADDITION TABLE

1 and 1 is 2	2 and 1 is 3	3 and 1 is 4	4 and 1 is 5
1 and 2 is 3	2 and 2 is 4	3 and 2 is 5	4 and 2 is 6
1 and 3 is 4	2 and 3 is 5	3 and 3 is 6	4 and 3 is 7
1 and 4 is 5	2 and 4 is 6	3 and 4 is 7	4 and 4 is 8
1 and 5 is 6	2 and 5 is 7	3 and 5 is 8	4 and 5 is 9
1 and 6 is 7	2 and 6 is 8	3 and 6 is 9	4 and 6 is 10
1 and 7 is 8	2 and 7 is 9	3 and 7 is 10	4 and 7 is 11
1 and 8 is 9	2 and 8 is 10	3 and 8 is 11	4 and 8 is 12
1 and 9 is 10	2 and 9 is 11	3 and 9 is 12	4 and 9 is 13
1 and 10 is 11	2 and 10 is 12	3 and 10 is 13	4 and 10 is 14
1 and 11 is 12	2 and 11 is 13	3 and 11 is 14	4 and 11 is 15
1 and 12 is 13	2 and 12 is 14	3 and 12 is 15	4 and 12 is 16
5 and 1 is 6	6 and 1 is 7	7 and 1 is 8	8 and 1 is 9
5 and 2 is 7	6 and 2 is 8	7 and 2 is 9	8 and 2 is 10
5 and 3 is 8	6 and 3 is 9	7 and 3 is 10	8 and 3 is 11
5 and 4 is 9	6 and 4 is 10	7 and 4 is 11	8 and 4 is 12
5 and 5 is 10	6 and 5 is 11	7 and 5 is 12	8 and 5 is 13
5 and 6 is 11	6 and 6 is 12	7 and 6 is 13	8 and 6 is 14
5 and 7 is 12	6 and 7 is 13	7 and 7 is 14	8 and 7 is 15
5 and 8 is 13	6 and 8 is 14	7 and 8 is 15	8 and 8 is 16
5 and 9 is 14	6 and 9 is 15	7 and 9 is 16	8 and 9 is 17
5 and 10 is 15	6 and 10 is 16	7 and 10 is 17	8 and 10 is 18
5 and 11 is 16	6 and 11 is 17	7 and 11 is 18	8 and 11 is 19
5 and 12 is 17	6 and 12 is 18	7 and 12 is 19	8 and 12 is 20
9 and 1 is 10	10 and 1 is 11	11 and 1 is 12	12 and 1 is 13
9 and 2 is 11	10 and 2 is 12	11 and 2 is 13	12 and 2 is 14
9 and 3 is 12	10 and 3 is 13	11 and 3 is 14	12 and 3 is 15
9 and 4 is 13	10 and 4 is 14	11 and 4 is 15	12 and 4 is 16
9 and 5 is 14	10 and 5 is 15	11 and 5 is 16	12 and 5 is 17
9 and 6 is 15	10 and 6 is 16	11 and 6 is 17	12 and 6 is 18
9 and 7 is 16	10 and 7 is 17	11 and 7 is 18	12 and 7 is 19
9 and 8 is 17	10 and 8 is 18	11 and 8 is 19	12 and 8 is 20
9 and 9 is 18	10 and 9 is 19	11 and 9 is 20	12 and 9 is 21
9 and 10 is 19	10 and 10 is 20	11 and 10 is 21	12 and 10 is 22
9 and 11 is 20	10 and 11 is 21	11 and 11 is 22	12 and 11 is 23
9 and 12 is 21	10 and 12 is 22	11 and 12 is 23	12 and 12 is 24

EXPLANATION.—After placing the numbers in proper order, begin at the bottom of the right-hand, or units, column, and add, mentally repeating the different sums. Thus, three and two is five and one is six and two is eight and one is nine, the sum of the numbers in units column. Place the 9 directly beneath as the first, or units, figure in the sum.

The sum of the numbers in the next, or tens, column equals 8 tens, which is the second, or tens, figure in the sum.

The sum of the numbers in the next, or hundreds, column equals 7 hundreds, which is the third, or hundreds, figure in the sum.

The sum, or answer, is 789. The word *answer* is abbreviated to *Ans.*

15. Rule.—I. *Begin at the right, add each column separately, and write the sum, if it is only one figure, under the column added.*

II. *If the sum of any column consists of two or more figures, put the right-hand figure of the sum under that column, and add the remaining figure or figures to the next column.*

Proof.—*To prove addition, add each column from top to bottom. If the same result is obtained as by adding from bottom to top, the work is probably correct.*

EXAMPLE 1.—What is the sum of 425, 36, 9,215, 4, and 907?

SOLUTION.—

$$\begin{array}{r} 425 \\ 36 \\ 9215 \\ 4 \\ 907 \\ \hline 27 \\ 60 \\ 1500 \\ 9000 \\ \hline \textit{sum}\ 10587 \end{array} \text{ Ans.}$$

EXPLANATION.—The sum of the numbers in the first, or units, column is seven and four is eleven and five is sixteen and six is twenty-two and five is twenty-seven, or 27 units; that is, two tens and seven units. Write 27 as shown. The sum of the numbers in the second, or tens, column is six tens, or 60. Write 60 underneath 27, as shown. The sum of the numbers in the third, or hundreds, column is 15 hundreds, or 1,500. Write 1,500 under the two preceding results as shown. There is only one number in the fourth, or thousands, column, 9, which represents 9,000. Write 9,000 under the three preceding results. Adding these four results, the sum is 10,587, which is the sum of 425, 36, 9,215, 4, and 907.

NOTE.—It frequently happens, when adding a long column of figures, that the sum of two numbers, one of which does not occur in the addition table, is required. Thus, in the first column of example 1, the sum of 16 and 6 was required. We know from the table that $6+6=12$; hence, the first figure of the sum is 2. Now, the sum of any number less than 20 and of any number less than 10 must be less than 30, since $20+10=30$; therefore, the sum is 22. Consequently, in cases of this kind, add the first figure of the larger number to the smaller number, and, if the result is greater than 9, increase the second figure of the larger number by 1. Thus, $44+7=?$ $4+7=11$; hence, $44+7=51$.

The addition may also be performed as follows:

```
  425
   36
 9215
    4
  907
-----
10587 Ans.
```

EXPLANATION.—The sum of the numbers in the units column is 27 units, or 2 tens and 7 units. Write the 7 units as the first, or right-hand, figure in the sum. Carry the two tens and add them to the figures in the tens column. The sum of the figures in the tens column, plus the 2 tens carried from the units column, is 8, which is written down as the second figure in the sum. There is nothing to carry to the next column, because 8 is less than 10. The sum of the numbers in the next column is 15 hundreds, or 1 thousand and 5 hundreds. Write down the 5 as the third, or hundreds, figure in the sum and carry the 1 to the next column. $1+9=10$ which is written down at the left of the other figures.

The second method saves space and figures, but the first is to be preferred when adding a long column.

EXAMPLE 2.—The number of pieces of work turned out by thirteen workmen were, respectively, 890, 82, 90, 393, 281, 80, 770, 83, 492, 80, 383, 84, and 191. What was the total number of pieces turned out by all?

SOLUTION.—To find the total, we must add these thirteen numbers, as follows:

```
 890
  82
  90
 393
 281
  80
 770
  83
 492
  80
 383
  84
 191
----
3899 Ans.
```

EXPLANATION.—The sum of the figures in the first column is 19 units, or 1 ten and 9 units. Write down the 9 and carry 1 to the next column. The sum of the figures in the second column +1 is 109 tens, or 10 hundreds and 9 tens. Write down the 9 and carry the 10 to the next column. The sum of the figures in this column plus the 10 reserved is 38. The entire sum is 3,899, which is the total number of pieces. Ans.

EXAMPLES FOR PRACTICE

1. Find the sums of the following numbers:

(*a*) 104+203+613+214.	Ans.	(*a*) 1,134
(*b*) 1,875+3,143+5,826+10,832.		(*b*) 21,676
(*c*) 4,865+2,145+8,173+40,084.		(*c*) 55,267
(*d*) 14,204+8,173+1,065+10,042.		(*d*) 33,484

2. If four castings weigh 3,265, 1,092, 748, and 2,587 pounds, respectively, what is their combined weight? Ans. 7,692 pounds

3. The monthly output of a shop manufacturing a line of small tools was as follows: January, 8,502; February, 8,748; March, 9,215; April, 9,770; May, 10,269; June, 12,184. What was the total output in the six months? Ans. 58,688

SUBTRACTION

16. In arithmetic, **subtraction** is the process of finding how much greater one number is than another.

The greater of the two numbers is called the **minuend.**

The smaller of the two numbers is called the **subtrahend.**

The number left after subtracting the subtrahend from the minuend is called the **difference,** or **remainder.**

17. The sign of subtraction is −. It is read *minus*, and means *less*. Thus, 12−7 is read *12 minus 7*, and means that 7 is to be taken from 12.

EXAMPLE.—From 7,568 take 3,425.

SOLUTION.—The larger number is written above the smaller number, and a line is drawn below them. The remainder is placed below this line, thus:

minuend	7 5 6 8	
subtrahend	3 4 2 5	
remainder	4 1 4 3	Ans.

Explanation.—Begin at the right-hand, or units, column and subtract in succession each figure in the subtrahend from the one directly above it in the minuend, and write the remainders below the line. The result is the entire remainder.

18. When there are more figures in the minuend than in the subtrahend, and when some figures in the minuend are less than the figures directly under them in the subtrahend, proceed as in the following example:

Example 1.—From 8,453 take 844.

Solution.—

minuend	8 4 5 3
subtrahend	8 4 4
remainder	7 6 0 9 Ans.

Explanation.—Begin at the right-hand, or units, column to subtract. We cannot take 4 from 3, and must, therefore, take 1 from 5 in tens column and annex it to the 3 in units column. The 1 ten = 10 units, which added to the 3 in units column = 13 units. 4 from 13 = 9, the first, or units, figure in the remainder.

As we took 1 from the 5, only 4 remains; 4 from 4 = 0, the second, or tens, figure. We cannot take 8 from 4, and must, therefore, take 1 from 8 in thousands column. Since 1 thousand = 10 hundreds, 10 hundreds + 4 hundreds = 14 hundreds, and 8 from 14 = 6, the third, or hundreds, figure in the remainder.

As we took 1 from 8, only 7 remains, from which there is nothing to subtract; therefore, 7 is the next figure in the remainder, or answer.

The operation of taking 1 away is performed by mentally placing 1 before the figure following the one from which it is taken. In example 1, the 1 taken from 5 is placed before 3, making it 13, from which we subtract 4. The 1 taken from 8 is placed before 4, making 14, from which 8 is subtracted.

Example 2.—Out of 306 pieces of work inspected, 14 were rejected as being too small. How many pieces were passed by the inspector?

Solution.—The number of pieces passed by the inspector is the difference between 306 and 14. The subtraction is as follows:

```
 306
  14
 ---
 292  Ans.
```

Explanation.—The first step is to subtract 4 from 6, leaving 2 as a remainder, which is set down. Now, 1 cannot be taken from 0; so, 1 is taken from the 3 and brought over to add to the 0. But, the 1 taken from 3 has a value 10 times as great as it would have in the position of the zero, according to Arts. 6 and 7. Therefore, 10 is added to the zero, making 10, and the 3 becomes decreased to 2. The subtraction might then be written

$$\begin{array}{r} {\scriptstyle 10} \\ 206 \\ 14 \\ \hline 292 \end{array}$$

The remainder of the solution is easy; 1 from 10 leaves 9, and nothing from 2 leaves 2. Hence, 292 pieces were passed by the inspector. It is not customary to rewrite the subtraction as here shown; instead, the taking of 1 from the 3 is done mentally.

EXAMPLE 3.—From a stock of 20,000 small machine parts 8,763 were used. How many remained?

SOLUTION.—To find how many remained, subtract the number used from the total stock, thus:

$$\begin{array}{r} 20000 \\ 8763 \\ \hline 11237 \end{array} \text{ Ans.}$$

EXPLANATION.—We cannot take 3 from 0 in the units column, so we go to the tens column to get 1 ten, or 10 units. But there are no tens, so we go to the hundreds column to take 1 hundred, or 10 tens. Here again is a cipher, so we go to the thousands place. There are no thousands, so we go to the ten-thousands place, where we find 2. We take 1 ten-thousand, or 10 thousands from the 2 and add it to the 0 thousands, giving $10+0=10$ thousands; but, it was necessary to take 1 thousand, or 10 hundreds, away to add to the hundreds place; so there is left $10-1=9$ thousands. Adding 10 hundreds to 0 hundreds, the sum is 10 hundreds. But 1 hundred, or 10 tens, must be taken away to add to 0 tens, leaving $10-1=9$ hundreds. Adding 1 hundred, or 10 tens, to 0 tens, the sum is 10 tens. But 1 ten, or 10 units, must be taken away to be added to 0 units, so 9 tens remain. The 1 ten or 10 units added to the 0 units gives 10 units as the sum. The subtraction therefore might be written as follows:

$$\begin{array}{r} {\scriptstyle 10} \\ 19990 \\ 8763 \\ \hline 11237 \end{array}$$

Then, 3 from 10 leaves 7; 6 from 9 leaves 3; 7 from 9 leaves 2; 8 from 9 leaves 1; and nothing from 1 leaves 1. Usually the taking of 1 unit from the several places is done mentally, and not written down as shown.

19. Rule.—*Place the subtrahend, or smaller number, under the minuend, or larger number, in the same manner as for addition, and proceed as in Art.* **18.**

Proof.—*To prove an example in subraction, add the subtrahend and the remainder. The sum should equal the minuend. If it does not, a mistake has been made, and the work should be done over.*

Proof of example 3, Art. **18.**

$$\begin{array}{r} 8763 \\ \underline{11237} \\ 20000 \end{array}$$

20. General Remarks on Subtraction.—Subtraction is a much simpler and easier operation than addition; nevertheless, the student should study the subject thoroughly. Its very simplicity is deceiving, and it is probable that more mistakes are made in subtraction than in addition. The student should practice subtraction until he feels that he has thoroughly mastered the subject. He should not try to subtract rapidly at first, but endeavor to make as few mistakes as possible. When he has attained proficiency, a rapid calculator will subtract as fast as he can write the figures of the result.

It will be a great help to the student if he will reverse the addition table. At odd moments, when walking or working, let him say to himself, 6 from 11 is how much? 9 from 17 is what? etc., and in a short time the right answer will present itself without any mental effort whatever.

EXAMPLES FOR PRACTICE

1. From:

		Ans.
(*a*)	94,278 take 62,574.	(*a*) 31,704
(*b*)	53,714 take 25,824.	(*b*) 27,890
(*c*)	71,832 take 58,109.	(*c*) 13,723
(*d*)	20,804 take 10,408.	(*d*) 10,396

2. A shop employing 3,214 hands was forced to lay off 736 of them. What number was left? Ans. 2,478

3. A casting weighing 2,785 pounds was poured from a ladle containing 4,210 pounds of metal. How many pounds remained in the ladle to be used for other castings? Ans. 1,425

MULTIPLICATION

21. To **multiply** a number is to *add* it to itself a certain number of times.

22. Multiplication is the process of multiplying one number by another.

The number thus added to itself, or the number to be multiplied, is called the **multiplicand.**

The number that shows how many times the *multiplicand* is to be taken, or the number by which we *multiply*, is called the **multiplier.**

The result obtained by multiplying is called the **product.**

23. The sign of multiplication is ×. It is read *times* or *multiplied by*. Thus, 9×6 is read *9 times 6*, or *9 multiplied by 6*. It matters not in what order the numbers to be multiplied together are placed; thus, 6×9 is the same as 9×6.

24. In Table II, the product of any two numbers (neither of which exceeds 12) may be found. This table should be carefully committed to memory. Since 0 has no value, the product of 0 and any number is 0.

25. To multiply by a number of one figure only.

EXAMPLE.—Multiply 425 by 5.

SOLUTION.—

multiplicand	4 2 5	
multiplier	5	
product	2 1 2 5	Ans.

EXPLANATION.—For convenience, the multiplier is generally written under the right-hand figure of the multiplicand. On looking in the multiplication table, we see that 5×5=25. Multiplying the first figure at the right of the multiplicand, or 5, by the multiplier 5, it is seen that 5×5 units =25 units, or 2 tens and 5 units. Write the 5 units in units place in the product, and reserve the 2 tens to add to the product of tens. Looking in the multiplication table again, we see that 5×2=10. Multiplying the second figure of the multiplicand by the multiplier 5, we see that 5×2 tens =10 tens, and 10 tens plus the 2 tens reserved is 12 tens, or 1 hundred plus 2 tens. Write the 2 tens in tens place, and reserve the 100 to add to the product of hundreds. Again, we see by the multiplication table that 5×4=20. Multiplying the third, or last, figure of the multiplicand by the multiplier, 5, we see that 5×4 hundreds=20 hundreds, and 20 hundreds plus the 1 hundred reserved, is 21 hundreds, or 2 thousands and 1 hundred, which we write in thousands and hundreds places, respectively.

Hence, the product is 2,125.

This result is the same as adding 425 five times; thus,

```
       425
       425
       425
       425
       425
     -----
sum   2125  Ans.
```

26. To multiply by a number of two or more figures.

Example.—Multiply 475 by 234.

Solution.—

```
                  multiplicand      475
                  multiplier        234
                                 ------
                  (                1900
partial products  {               1425
                  (              950
                                 ------
                  product        111150  Ans.
```

Explanation.—For convenience, the multiplier is generally written under the multiplicand, placing units under units, tens under tens, etc.

We cannot multiply by 234 at one operation; we must, therefore, multiply by the *parts* and then *add* the partial products.

The parts by which we are to multiply are 4 units, 3 tens, and 2 hundreds. 4×475=1,900, the *first partial product;* 3 tens×475=1,425 tens, the *second partial product, the right-hand figure of which is written directly under the figure multiplied by*, or 3; 2 hundreds×475=950 hundreds, the *third partial product*, the right-hand figure of which is written directly under the figure multiplied by, or 2.

The sum of these three partial products is 111,150, which is the entire product.

27. Rule.—I. *Write the multiplier under the multiplicand, so that units are under units, tens under tens, etc.*

II. *Begin at the right and multiply each figure of the multiplicand by each successive figure of the multiplier, placing the right-hand figure of each partial product directly under the figure used as a multiplier.*

TABLE II

MULTIPLICATION TABLE

1 times 1 is 1	2 times 1 is 2	3 times 1 is 3	4 times 1 is 4
1 times 2 is 2	2 times 2 is 4	3 times 2 is 6	4 times 2 is 8
1 times 3 is 3	2 times 3 is 6	3 times 3 is 9	4 times 3 is 12
1 times 4 is 4	2 times 4 is 8	3 times 4 is 12	4 times 4 is 16
1 times 5 is 5	2 times 5 is 10	3 times 5 is 15	4 times 5 is 20
1 times 6 is 6	2 times 6 is 12	3 times 6 is 18	4 times 6 is 24
1 times 7 is 7	2 times 7 is 14	3 times 7 is 21	4 times 7 is 28
1 times 8 is 8	2 times 8 is 16	3 times 8 is 24	4 times 8 is 32
1 times 9 is 9	2 times 9 is 18	3 times 9 is 27	4 times 9 is 36
1 times 10 is 10	2 times 10 is 20	3 times 10 is 30	4 times 10 is 40
1 times 11 is 11	2 times 11 is 22	3 times 11 is 33	4 times 11 is 44
1 times 12 is 12	2 times 12 is 24	3 times 12 is 36	4 times 12 is 48
5 times 1 is 5	6 times 1 is 6	7 times 1 is 7	8 times 1 is 8
5 times 2 is 10	6 times 2 is 12	7 times 2 is 14	8 times 2 is 16
5 times 3 is 15	6 times 3 is 18	7 times 3 is 21	8 times 3 is 24
5 times 4 is 20	6 times 4 is 24	7 times 4 is 28	8 times 4 is 32
5 times 5 is 25	6 times 5 is 30	7 times 5 is 35	8 times 5 is 40
5 times 6 is 30	6 times 6 is 36	7 times 6 is 42	8 times 6 is 48
5 times 7 is 35	6 times 7 is 42	7 times 7 is 49	8 times 7 is 56
5 times 8 is 40	6 times 8 is 48	7 times 8 is 56	8 times 8 is 64
5 times 9 is 45	6 times 9 is 54	7 times 9 is 63	8 times 9 is 72
5 times 10 is 50	6 times 10 is 60	7 times 10 is 70	8 times 10 is 80
5 times 11 is 55	6 times 11 is 66	7 times 11 is 77	8 times 11 is 88
5 times 12 is 60	6 times 12 is 72	7 times 12 is 84	8 times 12 is 96
9 times 1 is 9	10 times 1 is 10	11 times 1 is 11	12 times 1 is 12
9 times 2 is 18	10 times 2 is 20	11 times 2 is 22	12 times 2 is 24
9 times 3 is 27	10 times 3 is 30	11 times 3 is 33	12 times 3 is 36
9 times 4 is 36	10 times 4 is 40	11 times 4 is 44	12 times 4 is 48
9 times 5 is 45	10 times 5 is 50	11 times 5 is 55	12 times 5 is 60
9 times 6 is 54	10 times 6 is 60	11 times 6 is 66	12 times 6 is 72
9 times 7 is 63	10 times 7 is 70	11 times 7 is 77	12 times 7 is 84
9 times 8 is 72	10 times 8 is 80	11 times 8 is 88	12 times 8 is 96
9 times 9 is 81	10 times 9 is 90	11 times 9 is 99	12 times 9 is 108
9 times 10 is 90	10 times 10 is 100	11 times 10 is 110	12 times 10 is 120
9 times 11 is 99	10 times 11 is 110	11 times 11 is 121	12 times 11 is 132
9 times 12 is 108	10 times 12 is 120	11 times 12 is 132	12 times 12 is 144

III. *The sum of the partial products will equal the required product.*

Proof.—*Review the work carefully, or multiply the multiplier by the multiplicand; if the results agree, the work is probably correct.*

28. When there is a cipher in the multiplier, multiply by it the same as with the other figures. Thus,

```
 (a)         (b)         (c)          (d)
  0           2          15           708
 ×0          ×0         × 0         ×   0
 ---         ---        ---         -----
  0 Ans.      0 Ans.     00 Ans.      000 Ans.

   (e)            (f)              (g)
   3114           4008             31264
    203            305              1002
   ----          -----             -----
   9342          20040             62528
  0000           0000             00000
 6228           12024            00000
 ------         -------         31264
 632142 Ans.    1222440 Ans.    --------
                                31326528 Ans.
```

29. When multiplying by a number containing one or more ciphers, the work may be shortened by simply writing one cipher in its proper place and the next partial product alongside it. For example, the solutions to (*e*) and (*g*) in the preceding article may be written as follows:

```
   3114              31264
    203               1002
   ----              -----
   9342              62528
 62280             3126400
 ------            --------
 632142 Ans.       31326528 Ans.
```

Each partial product is written in the same position as before, since the product of any number and 0 is 0.

30. If there are ciphers at the right-hand end of the multiplier, they need not be used in the multiplication, but may be carried down to the product.

EXAMPLE 1.—Multiply 2,675 by 3,900.

SOLUTION.—

```
   2675
     3900
 ---------
  24075
  8025
 ---------
 10432500  Ans.
```

EXPLANATION.—In a case like this, the multiplier is written so that the ciphers at its right extend to the right of the units figure in the multiplicand. These two ciphers are brought down vertically to the end of the product of 2675 and 39.

If there are ciphers at the end of the multiplicand, the procedure is similar in all respects; thus, to multiply 4,907,600 by 487 proceed as follows:

```
    4907600
        487
   ---------
    343532
   392608
  196304
  ----------
  2390001200  Ans.
```

If both multiplicand and multiplier end in ciphers, place the right-hand digits under each other, as above, and add to the product as many ciphers as are contained in both multiplicand and multiplier on the right of their right-hand digits.

EXAMPLE 2.—Multiply 590,000 by 420.

SOLUTION.—

```
   590000
   420
  ---------
  118
 236
 ---------
 247800000  Ans.
```

EXAMPLES FOR PRACTICE

1. Find the product of the following:

(*a*) 61,483×6.	Ans.	(*a*) 368,898
(*b*) 12,375×5.		(*b*) 61,875
(*c*) 4,836×47.		(*c*) 227,292
(*d*) 3,257×246.		(*d*) 801,222
(*e*) 2,875×302.		(*e*) 868,250
(*f*) 17,819×1,004.		(*f*) 17,890,276

2. A certain machine is capable of turning out 48 finished pieces of work in a day. At this rate, find the output of the machine in one year of 296 working days. Ans. 14,208

3. What is the total weight of cast iron in 649 pumps, if the amount of iron in each pump weighs 37 pounds? Ans. 24,013 pounds

---

DIVISION

31. Division is the process of finding how many times one number is contained in another of the same kind.

The number to be *divided* is called the **dividend.**

The number by which we *divide* is called the **divisor.**

The number that shows how many times the *divisor is contained in the dividend* is called the **quotient.**

32. The sign of division is ÷. It is read *divided by.* 54÷9 is read *54 divided by 9.* Another way to write 54 divided by 9 is $\frac{54}{9}$. Thus, 54÷9=6, or $\frac{54}{9}=6$.

In both of these cases, 54 is the *dividend,* and 9 is the *divisor.*

Division is the reverse of multiplication.

33. To divide when the divisor consists of but one figure, proceed as in the following example:

EXAMPLE.— What is the quotient of 875÷7?

```
                 divisor dividend quotient
SOLUTION.—          7 ) 8 7 5 ( 1 2 5   Ans.
                        7
                        ‾
                        1 7
                        1 4
                        ‾‾‾
                          3 5
                          3 5
                          ‾‾‾
            remainder       0
```

EXPLANATION.— 7 is contained in 8 hundreds, 1 hundred times. Place the one as the first, or left-hand, figure of the quotient. Multiply the divisor, 7, by the 1 hundred of the quotient, and place the product, 7 hundreds, under the 8 hundreds in the dividend, and subtract. Beside the remainder, 1, bring down the next, or tens, figure of the dividend, in this case 7, making 17 tens; 7 is contained in 17, 2 times. Write the 2 as the second figure of the quotient. Multiply the divisor, 7, by the 2 in the quotient, and subtract the product from 17. Beside the remainder, 3, bring down the units figure of the dividend, making 35 units. 7 is contained in 35, 5 times, which is placed in the quotient. Multiplying the divisor by the last figure of the quotient, $5 \times 7 = 35$, which subtracted from 35, under which it is placed, leaves 0. Therefore, the quotient is 125.

34. The method of solving the preceding example is called **long division.** In **short division,** only the divisor, dividend, and quotient are written, the operations being performed mentally.

```
              dividend
divisor  7 ) 8 ¹7 ³5
            ---------
quotient     1  2  5   Ans.
```

The mental operation is as follows: 7 is contained in 8, once and one remainder; imagine 1 to be placed before 7, making 17; 7 is contained in 17, 2 times and 3 over; imagine 3 to be placed before 5, making 35; 7 is contained in 35, 5 times. These partial quotients, placed in order as they are found, make the entire quotient 125.

35. Divisor Consists of Two or More Figures.—Proceed as in the following example:

EXAMPLE.—Divide 2,702,826 by 63.

```
            divisor  dividend    quotient
SOLUTION.—    63)2702826(42902   Ans.
                 252
                 ---
                  182
                  126
                  ---
                   568
                   567
                   ---
                     126
                     126
                     ---
```

EXPLANATION.—As 63 is not contained in the first two figures, 27, we must use the first three figures, 270. Now, by trial, we must find how many times 63 is contained in 270; 6 is contained in the first two figures of 270, 4 times. Place the 4 as the first, or left-hand, figure in the quotient. Multiply the divisor, 63, by 4, and subtract the product, 252, from 270. The remainder is 18, beside which we write the next figure of the dividend, 2, making 182. Now, 6 is contained in the first two figures of 182, 3 times, but on multiplying 63 by 3, we see that the product, 189, is too great, so we try 2, as the second figure of the quotient. Multiplying the divisor, 63, by 2, and subtracting the product, 126, from 182, the remainder is 56, beside which we bring down the next figure of the dividend, making 568; 6 is contained in 56 about 9 times. Multiply the divisor, 63, by 9 and subtract the product, 567, from 568. The remainder is 1, and bringing down the next figure of the dividend, 2, gives 12. As 12 is smaller than 63, we write 0 in the quotient and bring down the next figure, 6, making 126. 63 is contained in 126, 2 times, without a remainder. Therefore, 42,902 is the quotient.

36. Rule.—**I.** *Write the divisor at the left of the dividend with a line between them.*

II. *Find how many times the divisor is contained in the lowest number of the left-hand figures of the dividend that will contain it, and write the result at the right of the dividend, with a line between, for the first figure of the quotient.*

III. *Multiply the divisor by this quotient; write the product under the partial dividend used and subtract, annexing to the remainder the next figure of the dividend. Divide as before, and thus continue until all the figures of the dividend have been used.*

IV. *If any partial dividend will not contain the divisor, write a cipher in the quotient, annex the next figure of the dividend, and proceed as before.*

V. *If there is at last a remainder, write it after the quotient, with the divisor underneath.*

Proof.—*Multiply the quotient by the divisor, and add the remainder, if there is any, to the product. The result will be the dividend.* Thus,

divisor *dividend* *quotient*
63) 4235 ($67\frac{14}{63}$ Ans.
378
455
441
remainder 14

PROOF.— *quotient* 67
divisor 63
201
402
4221
remainder 14
dividend 4235

EXAMPLES FOR PRACTICE

1. Divide the following:

(*a*) 126,498 by 58.	Ans.	(*a*) 2,181
(*b*) 3,207,594 by 767.		(*b*) 4,182
(*c*) 11,408,202 by 234.		(*c*) 48,753
(*d*) 2,100,315 by 581.		(*d*) 3,615
(*e*) 969,936 by 4,008		(*e*) 242
(*f*) 7,481,888 by 1,021.		(*f*) 7,328

2. A lot of castings weigh 11,060 pounds. If they are alike, and one weighs 28 pounds, how many are there in the lot? Ans. 395

3. If the driving shaft of a machine makes 9,730 turns in 35 minutes, how often does it turn in 1 minute? Ans. 278 times

---

SYMBOLS OF AGGREGATION

37. The **vinculum** $\overline{\quad}$, **parenthesis** ( ), **brackets** [ ], and **brace** { } are called **symbols of aggregation**, and are used to include numbers that are to be considered together; thus, $13\times\overline{8-3}$, or $13\times(8-3)$, shows that 3 is to be taken from 8 before multiplying by 13.

$$13\times\overline{8-3}=13\times 5=65$$
$$13\times(8-3)=13\times 5=65$$

When the vinculum or parenthesis is not used, we have

$$13\times 8-3=104-3=101$$

38. In any series of numbers connected by the signs $+$, $-$, $\times$, and $\div$, the operations indicated by the signs must be performed in order from left to right, *except* that no addition or subtraction may be performed if a sign of multiplication or division *follows* the number on the *right* of a sign of addition or subtraction until the indicated multiplication or division has been performed.

EXAMPLE 1.—What is the value of $4\times 24-8+17$?

SOLUTION.—Performing the operations in order from left to right,

$$4\times 24=96;\ 96-8=88;\ 88+17=105.\ \text{Ans.}$$

EXAMPLE 2.—What is the value of the expression $1{,}296 \div 12 + 160 - 22 \times 3$?

SOLUTION.— $1{,}296 \div 12 = 108$; $108 + 160 = 268$; here we cannot subtract 22 from 268 because the sign of multiplication *follows* 22; hence, multiplying 22 by 3, we get 66, and $268 - 66 = 202$. Ans.

EXAMPLES FOR PRACTICE

Find the values of the following expressions:

(*a*) $(8+5-1) \div 4$.
(*b*) $5 \times 24 - 32$.
(*c*) $144 - 5 \times 24$.
(*d*) $2{,}080 + 120 - 80 \times 4 - 1{,}670$.

Ans. (*a*) 3; (*b*) 88; (*c*) 24; (*d*) 210

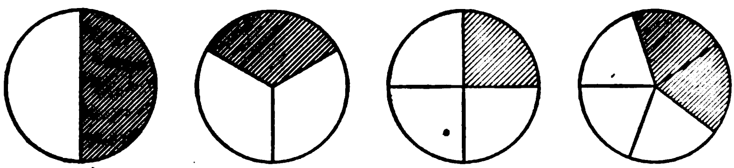
SHOP CALCULATIONS
(PART 2)

FRACTIONS

DEFINITIONS

1. A **fraction** is a part of something. In Fig. 1 (*a*) is shown a circle divided into two equal parts by a straight line, with one of the parts shaded. Each of these parts is a half of the whole circle; that is, each is a fraction of a circle. When the circle is divided into two equal parts, as shown, each part, or fraction, is *one-half*, which is written $\frac{1}{2}$.

Another circle is shown in (*b*), divided into three equal parts. Each of these equal parts is a third, or *one-third*, of the entire



(*a*) (*b*) (*c*) (*d*)

FIG. 1

circle. This fraction is written $\frac{1}{3}$. The circle in (*c*) is divided into four equal parts, each of which is a fourth, or *one-fourth*, of the whole circle. The fraction *one-fourth* is written $\frac{1}{4}$. The circle in (*d*) is divided into five equal parts, each of which is *one-fifth* of the whole circle. This fraction is written $\frac{1}{5}$. Two of the equal parts in (*d*) are shaded; consequently, the part of the whole circle that is shaded is *two-fifths*, which is written $\frac{2}{5}$.

Similarly, in (*b*), there are two parts not shaded, so that the part or fraction not shaded is *two-thirds*, written $\frac{2}{3}$. In (*c*), three of the equal parts are not shaded; consequently, the fraction of the circle that is not shaded is *three-fourths*, or $\frac{3}{4}$.

2. From what has just been stated it must be plain that two numbers are needed to write a fraction. The numbers are written one above the other, with a line between them, and each has its own name. The number above the line is called the **numerator** of the fraction and the number below the line is called the **denominator** of the fraction. Every fraction must have a numerator and a denominator. The denominator shows how many equal parts a thing is divided into, and the numerator shows how many of those equal parts are taken, or considered.

For example, in Fig. 1 (*a*) the shaded part of the circle is $\frac{1}{2}$ of the circle. In the fraction $\frac{1}{2}$, the numerator is 1 and the denominator is 2. The denominator 2 shows that the circle is divided into two equal parts, and the numerator 1 shows that one of those parts is considered. In (*d*), the fraction of the circle that is shaded is $\frac{2}{5}$. The denominator 5 of the fraction shows that the circle is divided into five equal parts, and the numerator 2 shows that the two parts that are shaded are considered. If the unshaded part had been considered, the numerator would have been 3, and the fraction would have been $\frac{3}{5}$, because three of the equal parts are not shaded.

The numerator and the denominator of a fraction are called the **terms** of a fraction.

3. The larger the denominator of a fraction, the smaller is the fraction, the numerator being the same. This may easily be shown by referring to Fig. 1 (*b*) and (*c*). The fraction represented by one of the equal parts in (*b*) is $\frac{1}{3}$ and that represented by one of the equal parts in (*c*) is $\frac{1}{4}$. The denominator 4 is greater than the denominator 3, but the fraction $\frac{1}{4}$ is smaller than the fraction $\frac{1}{3}$. This can be seen by comparing one of the equal parts in (*c*) with one in (*b*). One of the parts in (*c*), or $\frac{1}{4}$ of the circle, is much smaller than one of the parts in (*b*), or $\frac{1}{3}$ of the circle. Hence, if the numerators of two fractions are equal, the fraction with the smaller denominator is the greater. Thus, of the two fractions $\frac{3}{8}$ and $\frac{3}{4}$, the latter is the greater. But if the denominators are equal, the one with the larger numerator is the greater. Take $\frac{3}{8}$ and $\frac{4}{8}$, for example; in this case the denominators are equal, and $\frac{4}{8}$ is greater than $\frac{3}{8}$ because 4 is greater than 3. This is the same thing as saying that, in Fig. 1 (*d*), four of the equal parts, or $\frac{4}{8}$ of the circle, are greater than three parts, or $\frac{3}{8}$ of the circle. Similarly, $\frac{2}{3}$ is greater than $\frac{1}{3}$, as may be seen in (*b*), where the unshaded part is $\frac{2}{3}$ of the circle and the shaded part is $\frac{1}{3}$.

4. A fraction may also be used to express division; for example, $4 \div 5$ may be written $\frac{4}{5}$, which is a fraction; and similarly, $3 \div 16$ may be written $\frac{3}{16}$. The **value** of a fraction is the result obtained by dividing the numerator by the denominator.

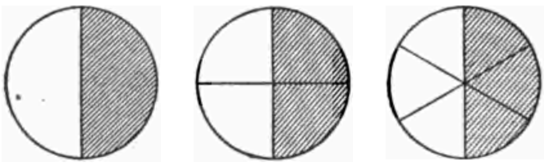
If the numerator of a fraction is less than the denominator, the fraction is called a **proper fraction**; thus, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{16}$, $\frac{3}{10}$ are proper fractions. If the numerator is equal to or greater than the denominator, the fraction is an **improper fraction**; thus, $\frac{3}{2}$, $\frac{5}{5}$, $\frac{16}{13}$, $\frac{12}{4}$, $\frac{28}{7}$ are improper fractions.

A fraction whose numerator and denominator are equal is equal to 1; thus, $\frac{4}{4}=1$, $\frac{2}{2}=1$, $\frac{8}{8}=1$. If the numerator is less than the denominator, the value of the fraction is less than 1; thus, the value of each of the fractions $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{16}$ is less than 1. If the numerator of the fraction is larger than the denominator, the value of the fraction is more than 1; thus, $\frac{12}{2}=6$, $\frac{10}{5}=2$, $\frac{3}{2}=1\frac{1}{2}$.

5. A **whole number** is a number that does not contain a fraction. For example, 2, 36, 185, 4,063 are whole numbers. A **mixed number** is a number composed of a whole number and a fraction united. For example, $3\frac{3}{8}$ is a mixed number, being composed of a whole number 3 and a fraction $\frac{3}{8}$. This number is read *three and three-eighths*. It is equal to $3+\frac{3}{8}$, but for convenience the plus sign is omitted in writing it and it appears simply as $3\frac{3}{8}$. The mixed number $10\frac{5}{16}$ is read *ten and five-sixteenths*. A whole number is very frequently called an **integer**. It is also occasionally referred to as an *integral number*.

REDUCTION OF FRACTIONS

6. The form of a fraction may be changed without changing its value; that is, a certain part of a thing may be expressed by more than one fraction. For example, $\frac{1}{2}$ has the same value as $\frac{2}{4}$, and $\frac{3}{6}$ has the same value as $\frac{1}{2}$ or $\frac{2}{4}$. This may easily be understood by referring to Fig. 2, in which are three circles of the same size. The circle in (*a*) is divided into two equal parts, one of which is shaded; that is, the part shaded is $\frac{1}{2}$ of the whole circle. The circle in (*b*) is divided into four equal parts, two of which are shaded; that is, the shaded portion is $\frac{2}{4}$ of the whole. The circle in (*c*) is divided into six equal parts, three of which are shaded; that is, the shaded portion is $\frac{3}{6}$ of the whole. But, the three circles are of the same size, and it may be seen by comparing them that the same amount is shaded in each;



(*a*) (*b*) (*c*)

FIG. 2

therefore, $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{3}{6}$ must be equal to one another, because each one represents the same amount, or half of the circle.

7. If both terms of a fraction, that is, both numerator and denominator, are multiplied or divided by the same number, the value of the fraction is not changed. For example, suppose that in the fraction $\frac{1}{2}$, both terms are multiplied by 2. Then, $\frac{1\times 2}{2\times 2}=\frac{2}{4}$. The fraction $\frac{2}{4}$ has the same value as $\frac{1}{2}$, as was shown in the preceding article. Again, take $\frac{1}{2}$ and multiply both terms by 3. Then, $\frac{1\times 3}{2\times 3}=\frac{3}{6}$, which has the same value as $\frac{1}{2}$, according to the preceding article. Now take the fraction $\frac{3}{6}$ and divide both terms by 3. Then, $\frac{3\div 3}{6\div 3}=\frac{1}{2}$. But it was shown that $\frac{1}{2}=\frac{3}{6}$, in the preceding article; therefore, by dividing both terms by the same number, the value of the fraction is not changed. This process of changing the form of a fraction without changing its value is called **reduction of fractions.**

8. *A fraction is reduced to higher terms by multiplying both terms of the fraction by the same number.* For example, $\frac{3}{4}$ is reduced to $\frac{6}{8}$ by multiplying both terms by 2; thus,

$$\frac{3\times2}{4\times2}=\frac{6}{8}$$

9. *A fraction is reduced to lower terms by dividing both terms by the same number.* For example, $\frac{8}{10}$ is reduced to $\frac{4}{5}$ by dividing both terms by 2; thus,

$$\frac{8\div2}{10\div2}=\frac{4}{5}$$

A fraction is reduced to its *lowest terms* when both its numerator and its denominator cannot be divided by the *same* number without a remainder; for example, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{11}{24}$, $\frac{8}{15}$ are fractions reduced to their lowest terms.

10. To reduce a fraction to an equal fraction having a given denominator.

Rule.—*Divide the given denominator by the denominator of the given fraction, and multiply both terms of the fraction by the result.*

EXAMPLE 1.—Reduce $\frac{7}{8}$ to an equal fraction having 96 for a denominator.

SOLUTION.—Both the numerator and the denominator must be multiplied by the same number in order not to change the value of the fraction. The denominator must be multiplied by some number which will, in this case, make the product 96; this number is evidently $96\div8=12$, because $8\times12=96$. Hence,

$$\frac{7\times12}{8\times12}=\frac{84}{96}. \text{ Ans.}$$

EXAMPLE 2.—Reduce $\frac{3}{4}$ to 100ths; that is, to a fraction having 100 for a denominator.

SOLUTION.— $100\div4=25$; hence,

$$\frac{3\times25}{4\times25}=\frac{75}{100}. \text{ Ans.}$$

EXAMPLES FOR PRACTICE

Reduce the following:

(*a*) $\frac{7}{16}$ to 128ths.
(*b*) $\frac{24}{132}$ to its lowest terms.
(*c*) $\frac{64}{1000}$ to its lowest terms.
(*d*) $\frac{5}{7}$ to 49ths.
(*e*) $\frac{13}{16}$ to 10,000ths.

Ans.
(*a*) $\frac{56}{128}$
(*b*) $\frac{2}{11}$
(*c*) $\frac{8}{125}$
(*d*) $\frac{35}{49}$
(*e*) $\frac{8125}{10000}$

11. To reduce a whole number or a mixed number to an improper fraction.

Rule.—*Multiply the whole number by the denominator of the fraction, add the numerator to the product, and place the denominator under the result. If it is desired to reduce a whole number to a fraction, multiply the whole number by the denominator of the given fraction, and write the result over the denominator.*

EXAMPLE 1.—How many fourths in 5?

SOLUTION.—There are 4 fourths in 1, because $\frac{4}{4}=1$, and in 5 there will be 5×4 fourths, or 20 fourths; that is, $5\times\frac{4}{4}=\frac{20}{4}$. Ans.

EXAMPLE 2.—Reduce $8\frac{3}{4}$ to an improper fraction.

SOLUTION.—According to the rule,

$$8\tfrac{3}{4}=\frac{8\times4+3}{4}=\frac{32+3}{4}=\tfrac{35}{4}.\quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

Reduce to improper fractions:

(*a*) $4\frac{1}{8}$.
(*b*) $5\frac{3}{16}$
(*c*) $10\frac{2}{10}$.
(*d*) $37\frac{3}{4}$.
(*e*) $50\frac{4}{5}$.
(*f*) Reduce 7 to a fraction whose denominator is 16.

Ans. (*a*) $\frac{33}{8}$; (*b*) $\frac{83}{16}$; (*c*) $\frac{102}{10}$; (*d*) $\frac{151}{4}$; (*e*) $\frac{254}{5}$; (*f*) $\frac{112}{16}$

---

12. To reduce an improper fraction to a whole or a mixed number.

Rule.—*Divide the numerator by the denominator and write the result as in ordinary division.*

EXAMPLE.—Reduce $\frac{21}{4}$ to a mixed number.

SOLUTION.— 4 is contained in 21, 5 times and 1 remaining, which is written over the 4, giving the fraction $\frac{1}{4}$, and added to the 5. Therefore, $5+\frac{1}{4}$, or $5\frac{1}{4}$, is the number. Ans.

EXAMPLES FOR PRACTICE

Reduce to whole or mixed numbers:

(*a*) $\frac{145}{6}$.	Ans.	(*a*) $24\frac{1}{6}$
(*b*) $\frac{185}{3}$.		(*b*) $61\frac{2}{3}$
(*c*) $\frac{701}{6}$.		(*c*) $116\frac{5}{6}$
(*d*) $\frac{149}{3}$.		(*d*) $49\frac{2}{3}$
(*e*) $\frac{76}{19}$.		(*e*) 4
(*f*) $\frac{125}{25}$.		(*f*) 5

COMMON DENOMINATOR OF FRACTIONS

13. A **common denominator** of two or more fractions is a number that may be divided by the denominator of each of the given fractions without a remainder. The **least common denominator** is the least number that may be divided by each denominator of the given fractions without a remainder. Suppose, for example, that the fractions $\frac{1}{4}$ and $\frac{3}{8}$ are to be changed to fractions having a common denominator. We know that $\frac{1}{4}=\frac{2}{8}$, because $\frac{1\times 2}{4\times 2}=\frac{2}{8}$. Then, instead of $\frac{1}{4}$ we can write $\frac{2}{8}$, and the two fractions $\frac{1}{4}$ and $\frac{3}{8}$ then become $\frac{2}{8}$ and $\frac{3}{8}$, which have the common denominator 8. Again, suppose that the fractions $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ are to be changed to forms that have the same denominator. We know that 2, 3, or 4 can be divided into 12 without a remainder, so we take 12 as the common denominator. Then, $\frac{1}{2}=\frac{6}{12}$, $\frac{2}{3}=\frac{8}{12}$, and $\frac{3}{4}=\frac{9}{12}$, and the fractions $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ therefore become $\frac{6}{12}$, $\frac{8}{12}$, and $\frac{9}{12}$. The number 24 could also be used as a common denominator, because it can be divided by 2, 3, or 4 without a remainder. The three fractions would then become $\frac{12}{24}$, $\frac{16}{24}$, and $\frac{18}{24}$, and these values would be the same as $\frac{6}{12}$, $\frac{8}{12}$, and $\frac{9}{12}$. But the point to be noticed is that although 12 and 24 can both be used as common denominators for these three fractions, 12 is the *least* common denominator, because it is the *smallest* number that can be divided by 2, 3, and 4 without leaving a remainder. A series of fractions may have a great many different common denominators, but they can have only *one* least common denominator. In adding or subtracting fractions, they are generally reduced to the *least* common denominators.

14. The rules used by mechanics in English-speaking countries to take measurements are divided into equal parts called *inches*, and each inch is further divided into equal parts called fractions of an inch. A part of a rule showing a common way of dividing the inch is illustrated in Fig. 3. The long line *a* at the middle of the inch divides it into halves. At the middle of the halves are shorter lines *b* and *c* that divide the half-inches into halves, making quarter inches, and the quarter inches are still further divided by the lines *d*, *e*, *f*, and *g* into eighths. Finally, the shortest lines divide the eighths, so that there are sixteen small divisions in 1 inch, and these are called sixteenths. Suppose that the rule is used to measure the length of a block *h*. The end of the rule is put in line with the end of the block, and the other end of the block comes just to the line *a*, which is the half-inch mark; therefore, the block is said to be $\frac{1}{2}$ inch long.

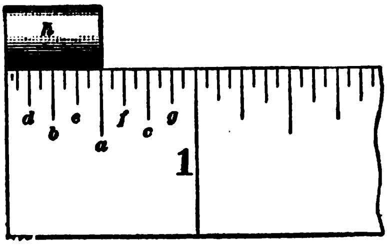


FIG. 3

15. Now, suppose that the length of the block *h*, Fig. 3, is to be found in quarters, eighths, or sixteenths of an inch. By looking at the rule, it is seen that there are just two quarter inches between the line *a* and the end of the rule, so the length of the block is two quarter inches or two-fourths of an inch, which is written $\frac{2}{4}$ inch. This is exactly equal to $\frac{1}{2}$ inch, because $\frac{1\times2}{2\times2}=\frac{2}{4}$. If the length of the block is expressed in eighths of an inch, it is $\frac{4}{8}$ inch long, because there are four eighth-inch divisions between the line *a* and the end of the rule. Also, the block is $\frac{8}{16}$ inch long, because there are eight sixteenth-inch divisions from the line *a* to the end of the rule. This simply shows that $\frac{1}{2}=\frac{2}{4}=\frac{4}{8}=\frac{8}{16}$. Sometimes rules are divided into thirty-seconds and sixty-fourths of an inch.

16. Because the rule he uses in measuring is divided as shown in Fig. 3, most of the fractions that a mechanic has to use are halves, fourths, eighths, sixteenths, thirty-seconds, and sixty-fourths. So, if he has to find the common denominator of several such fractions, he can simply use the greatest denominator in the series of fractions. For example, suppose that the fractions are $\frac{5}{16}$, $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{1}{2}$. The largest denominator is 16, which would be selected as the least common denominator. Then, $\frac{3}{4}=\frac{12}{16}$, $\frac{7}{8}=\frac{14}{16}$, and $\frac{1}{2}=\frac{8}{16}$. Again, suppose that the fractions are $\frac{27}{64}$, $\frac{3}{8}$, $\frac{7}{32}$, and $\frac{1}{4}$. The largest of these denominators is 64, which is taken as the least common denominator, and then $\frac{3}{8}=\frac{24}{64}$, $\frac{7}{32}=\frac{14}{64}$, and $\frac{1}{4}=\frac{16}{64}$. It is therefore a very simple matter to choose the least common denominator for fractions of an inch as found by a rule divided like that in Fig. 3. This method of finding the least common denominator by merely looking at the denominators of the given fractions is called finding the least common denominator *by inspection.*

17. It is not always so easy to find the least common denominator as in the preceding article. For this reason, the following method is given, showing how to find the least common denominator of *any* set of fractions:

EXAMPLE 1.—Find the least common denominator of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{9}$, and $\frac{1}{16}$.

SOLUTION.—We first place the denominators in a row, separated by commas.

$$\begin{array}{r|l} 2 & 4,\ 3,\ 9,\ 16 \\ \hline 2 & 2,\ 3,\ 9,\ 8 \\ \hline 3 & 1,\ 3,\ 9,\ 4 \\ \hline & 1,\ 1,\ 3,\ 4 \end{array}$$

$2\times2\times3\times3\times4=144$, the least common denominator. Ans.

EXPLANATION.—Divide by some prime* number other than 1 that will divide at least two of them without a remainder (if possible), bringing down those denominators to the row below which will not contain the divisor without a remainder. Dividing by 2, the second row becomes 2, 3, 9, 8, because 2 will not divide 3 and 9 without a remainder. Dividing again by 2, the result is 1, 3, 9, 4, Dividing the third row by 3, the result is 1, 1, 3, 4. There is no number except 1 that will divide both 3 and 4 without a remainder. The product of the divisors and the 3 and 4, or $2\times2\times3\times3\times4=144$, is the least common denominator.

*A *prime number* is one that cannot be divided by any other number except 1, without a remainder; thus, 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, etc. are prime numbers.

EXAMPLE 2.—Find the least common denominator of $\frac{4}{9}$, $\frac{5}{12}$, $\frac{7}{18}$.

SOLUTION.—

$$\begin{array}{r|rrr} 3 & 9, & 12, & 18 \\ \hline 3 & 3, & 4, & 6 \\ \hline 2 & 1, & 4, & 2 \\ \hline & 1, & 2, & 1 \end{array}$$

$3\times3\times2\times2=36$. Ans.

18. After the least common denominator has been found, the fractions may be reduced to forms having this least common denominator by using the following rule:

Rule.—*Divide the least common denominator by the denominator of the given fraction, and multiply both terms of the fraction by the quotient.*

EXAMPLE 1.—Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{2}$ to fractions having a least common denominator.

SOLUTION.—The least common denominator is first found as follows:

$$\begin{array}{r|rrr} 2 & 3, & 4, & 2 \\ \hline & 3, & 2, & 1 \end{array}$$

Then, $2\times3\times2=12$, the least common denominator

Now, each of the three fractions must be changed to a form in which it will have 12 for its denominator. First take the fraction $\frac{2}{3}$, whose denominator is 3. By the rule, the common denominator is to be divided by the denominator of the fraction. This gives $12\div3=4$. Both terms of the fraction are then to be multiplied by the result, or 4, that is, $\frac{2\times4}{3\times4}=\frac{8}{12}$. Take the next fraction $\frac{3}{4}$, whose denominator is 4. Following the same method, $12\div4=3$, and $\frac{3\times3}{4\times3}=\frac{9}{12}$. The third fraction $\frac{1}{2}$ has 2 for a denominator. Then, $12\div2=6$, and $\frac{1\times6}{2\times6}=\frac{6}{12}$. Hence, the fractions $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{2}$, when reduced to the least common denominator, become $\frac{8}{12}$, $\frac{9}{12}$, and $\frac{6}{12}$. Ans.

EXAMPLE 2.—Reduce $\frac{13}{16}$, $\frac{5}{7}$, and $\frac{11}{12}$ to the least common denominator.

SOLUTION.—First find the least common denominator.

$$\begin{array}{r|rrr} 2 & 16, & 7, & 12 \\ \hline 2 & 8, & 7, & 6 \\ \hline & 4, & 7, & 3 \end{array}$$

Least common denominator $=2\times2\times4\times7\times3=336$.

In the case of the fraction $\frac{13}{16}$, $336 \div 16 = 21$, and $\frac{13 \times 21}{16 \times 21} = \frac{273}{336}$. In the case of $\frac{5}{7}$, $336 \div 7 = 48$, and $\frac{5 \times 48}{7 \times 48} = \frac{240}{336}$. Finally, in the case of $\frac{11}{12}$, $336 \div 12 = 28$, and $\frac{11 \times 28}{12 \times 28} = \frac{308}{336}$. Therefore, the three fractions reduced to the least common denominator are $\frac{273}{336}$, $\frac{240}{336}$, and $\frac{308}{336}$. Ans.

EXAMPLES FOR PRACTICE

Reduce to fractions having the least common denominator:

		Ans.
(a)	$\frac{3}{4}, \frac{5}{8}, \frac{7}{8}$.	(a) $\frac{6}{8}, \frac{5}{8}, \frac{7}{8}$
(b)	$\frac{3}{16}, \frac{3}{4}, \frac{7}{32}$.	(b) $\frac{6}{32}, \frac{24}{32}, \frac{7}{32}$
(c)	$\frac{7}{8}, \frac{7}{88}, \frac{10}{11}$.	(c) $\frac{77}{88}, \frac{7}{88}, \frac{80}{88}$
(d)	$\frac{3}{5}, \frac{5}{8}, \frac{11}{40}$.	(d) $\frac{24}{40}, \frac{25}{40}, \frac{11}{40}$
(e)	$\frac{4}{10}, \frac{6}{40}, \frac{9}{20}$.	(e) $\frac{16}{40}, \frac{6}{40}, \frac{18}{40}$
(f)	$\frac{7}{15}, \frac{17}{30}, \frac{7}{10}$.	(f) $\frac{14}{30}, \frac{17}{30}, \frac{21}{30}$

ADDITION OF FRACTIONS

19. The reason for reducing fractions to a common denominator is to enable them to be added or subtracted. Fractions cannot be added unless they have the same denominator; hence, before adding several fractions, it is necessary to reduce them to a common denominator. If a machinist measures off three distances of $\frac{1}{16}$ inch, $\frac{5}{16}$ inch, and $\frac{7}{16}$ inch, he knows that the sum is $\frac{13}{16}$ inch. This is obtained by adding the numerators, thus, $1+5+7 = 13$, and placing that sum over the common denominator 16, giving $\frac{13}{16}$. But if he measures off three distances like $\frac{1}{4}$ inch, $\frac{3}{8}$ inch, and $\frac{7}{16}$ inch, he cannot correctly find the sum by adding the numerators, because the fractions have different denominators. He must first change all the fractions to forms in which they have the same denominator, that is, a common denominator. The rule to be used in the addition of fractions is as follows:

Rule.—*To add several fractions, first reduce them to a common denominator. Then add the numerators and write the sum over the common denominator. If there are whole numbers and mixed numbers, add the whole numbers and the fractions separately, and then add the results.*

EXAMPLE 1.—Find the sum of $\frac{1}{2}$ inch, $\frac{3}{4}$ inch, and $\frac{5}{8}$ inch.

SOLUTION.—The least common denominator of these fractions is 8. Then, $\frac{1}{2}=\frac{4}{8}$, $\frac{3}{4}=\frac{6}{8}$, and $\frac{5}{8}=\frac{5}{8}$. The numerators are now added, giving $4+6+5=15$, which is placed over the common denominator 8; thus, $\frac{15}{8}$. On reducing the improper fraction $\frac{15}{8}$ to a mixed number, it is found that $\frac{15}{8}=1\frac{7}{8}$. Therefore, $\frac{1}{2}$ inch $+\frac{3}{4}$ inch $+\frac{5}{8}$ inch $=1\frac{7}{8}$ inches. Ans.

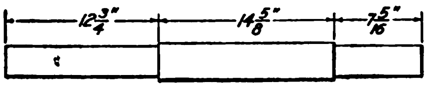


FIG. 4

EXAMPLE 2.—What is the total length, or overall length, of the piece of shafting shown dimensioned in Fig. 4?

NOTE—The marks " used on the drawing, mean *inches;* thus, $12\frac{3}{4}''$ means $12\frac{3}{4}$ inches.

SOLUTION.—The total length is the sum of the three parts, measuring $12\frac{3}{4}$ inches, $14\frac{5}{8}$ inches, and $7\frac{5}{16}$ inches. The least common denominator of the three fractions is 16. Then,

$$12\frac{3}{4}=12\frac{12}{16}$$
$$14\frac{5}{8}=14\frac{10}{16}$$
$$7\frac{5}{16}=\ \ 7\frac{5}{16}$$
$$\overline{\textit{sum}=33+\frac{27}{16}=33+1\frac{11}{16}=34\frac{11}{16}.}\quad \text{Ans.}$$

The sum of the fractions $=\frac{27}{16}$ or $1\frac{11}{16}$, which added to the sum of the whole numbers $=34\frac{11}{16}$; that is, the total, or overall, length of the shaft is $34\frac{11}{16}$ inches.

EXAMPLE 3.—A workman has five pieces of belt whose lengths are $9\frac{1}{2}$, $6\frac{1}{4}$, $3\frac{3}{4}$, $3\frac{1}{3}$, and $1\frac{5}{6}$ feet, respectively, and he needs a belt 24 feet long for some special work. Can he make up a belt of the required length by lacing the pieces together, end to end?

SOLUTION.—If the sum of the lengths of the five pieces is equal to or greater than 24 feet, the required belt can be made. The least common denominator in this case is 12. Now, $\frac{1}{2}=\frac{6}{12}$; $\frac{1}{4}=\frac{3}{12}$; $\frac{3}{4}=\frac{9}{12}$; $\frac{1}{3}=\frac{4}{12}$; and $\frac{5}{6}=\frac{10}{12}$. Therefore, $9\frac{1}{2}+6\frac{1}{4}+3\frac{3}{4}+3\frac{1}{3}+1\frac{5}{6}=9\frac{6}{12}+6\frac{3}{12}+3\frac{9}{12}+3\frac{4}{12}+1\frac{10}{12}$ $=22\frac{32}{12}=22+2\frac{8}{12}=24\frac{8}{12}=24\frac{2}{3}$. As the total length is $24\frac{2}{3}$ feet, and the length required is only 24 feet, the pieces can be joined to form the required belt. Ans.

20. The mechanic most frequently is required to add dimensions expressed in inches and fractions of an inch. This he can do by the methods just outlined for the addition of fractions; or, if he desires, he may perform the addition on his rule. To illustrate, suppose that the dimensions $1\frac{1}{2}$, $\frac{5}{8}$, $2\frac{3}{16}$, $\frac{3}{4}$, $\frac{15}{16}$, and $1\frac{3}{8}$ inches are to be added. The several fractions $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{16}$, $\frac{3}{4}$, $\frac{15}{16}$, and $\frac{3}{8}$ are first added, by using a rule graduated in sixteenths of an inch, as shown in Fig. 5. Beginning at the end *a*, count off $\frac{1}{2}$ inch, to the point *b*. From *b* count off five $\frac{1}{8}$-inch divisions, or $\frac{5}{8}$ inch, to *c*. Next count three $\frac{1}{16}$-inch divisions, or $\frac{3}{16}$ inch, to *d*. From *d* count off three $\frac{1}{4}$-inch spaces, or $\frac{3}{4}$ inch, which is the same as twelve $\frac{1}{16}$-inch divisions, because

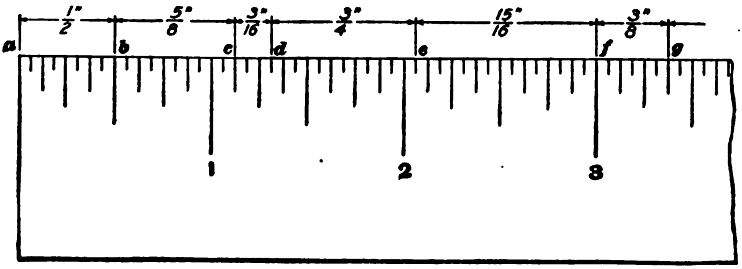


FIG. 5

$\frac{3}{4}=\frac{12}{16}$. This locates the mark *e*. From *e* count off $\frac{15}{16}$ inch, or fifteen $\frac{1}{16}$-inch spaces, to *f*. Finally, from *f* count off three $\frac{1}{8}$-inch divisions, to *g*. The point *g* thus found is at a distance of $3\frac{3}{8}$ inches from *a*; hence, $\frac{1}{2}+\frac{5}{8}+\frac{3}{16}+\frac{3}{4}+\frac{15}{16}+\frac{3}{8}=3\frac{3}{8}$ inches. To this add the whole numbers of inches, 1, 2, and 1, and the total is $3\frac{3}{8}+1+2+1=7\frac{3}{8}$ inches; that is, $1\frac{1}{2}+\frac{5}{8}+2\frac{3}{16}+\frac{3}{4}+1\frac{15}{16}+1\frac{3}{8}=7\frac{3}{8}$ inches.

EXAMPLES FOR PRACTICE

1. Find the sum of the following:

(*a*) $\frac{4}{6}$, $\frac{7}{24}$, $\frac{5}{8}$.
(*b*) $\frac{2}{3}$, $\frac{5}{15}$, $\frac{24}{45}$.
(*c*) $\frac{1}{2}$, $\frac{3}{8}$, $\frac{5}{16}$.
(*d*) $\frac{5}{6}$, $\frac{11}{72}$, $\frac{13}{18}$.

Ans. (*a*) $1\frac{7}{12}$; (*b*) $1\frac{8}{15}$; (*c*) $1\frac{3}{16}$; (*d*) $1\frac{17}{24}$

2. The bolt shown in Fig. 6 has the end threaded for a distance of $1\frac{15}{16}$ inches, is plain for $3\frac{3}{4}$ inches, has a collar $\frac{17}{32}$ inch long, and has a head $\frac{7}{8}$ inch thick. What is the overall length of the bolt? Ans. $7\frac{3}{32}$ inches

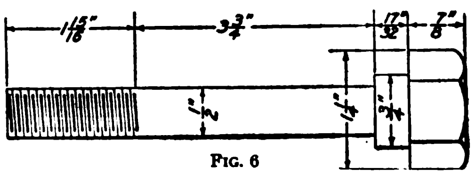


FIG. 6

3. A helper works $2\frac{3}{4}$ hours on one job, $1\frac{1}{3}$ hours on another, and $\frac{5}{12}$ hour on a fourth. What is the total time that he worked? Ans. $4\frac{1}{2}$ hours

SUBTRACTION OF FRACTIONS

21. Fractions cannot be subtracted without first reducing them to a common denominator. This can be shown in the same manner as in the case of addition of fractions. The rule to be used is as follows:

Rule.—I. *Reduce the fractions to fractions having a common denominator. Subtract one numerator from the other and place the remainder over the common denominator.*

II. *When there are mixed numbers, subtract the fractions and whole numbers separately, and place the remainders side by side.*

III. *When the fraction in the subtrahend is greater than the fraction in the minuend, take 1 from the whole number in the minuend and add it to the fraction in the minuend, from which subtract the fraction in the subtrahend.*

IV. *When the minuend is a whole number, take 1 from it, reduce the 1 to a fraction whose denominator is the same as the denominator of the fraction in the subtrahend, and place it over that fraction for subtraction.*

EXAMPLE 1.—Subtract $\frac{3}{8}$ from $\frac{13}{16}$.

SOLUTION.—The least common denominator is 16, and $\frac{3}{8}=\frac{6}{16}$. Then, according to the rule,

$$\frac{13}{16}-\frac{3}{8}=\frac{13}{16}-\frac{6}{16}=\frac{13-6}{16}=\frac{7}{16}. \text{ Ans.}$$

EXAMPLE 2.—From 7 take $\frac{5}{8}$.

SOLUTION.—There is no fraction in the minuend 7, so it is necessary to take 1 from 7 to form a fraction whose denominator is the same as that of the fraction to be subtracted, or 8. Now, $1=\frac{8}{8}$; so, instead of 7 we can write $6+1$, or $6+\frac{8}{8}$, or simply $6\frac{8}{8}$. Then, $6\frac{8}{8}-\frac{5}{8}=6\frac{3}{8}$; that is, $7-\frac{5}{8}=6\frac{3}{8}$. Ans.

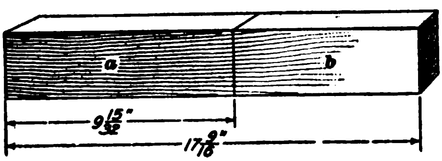


FIG. 7

EXAMPLE 3.—A block of wood $17\frac{9}{16}$ inches long, as shown in Fig. 7, has a piece *a* $9\frac{15}{32}$ inches long sawed off. What is the length of the remaining piece *b*, neglecting the thickness of the saw cut?

SOLUTION.—The common denominator of the fractions is 32. $17\frac{9}{16} = 17\frac{18}{32}$.

minuend	$17\frac{18}{32}$
subtrahend	$9\frac{15}{32}$
difference	$8\frac{3}{32}$ Ans.

That is, the remainder *b* is $8\frac{3}{32}$ inches long.

EXAMPLE 4.—A piece $4\frac{7}{16}$ inches long is cut from a bar $9\frac{1}{4}$ inches long. What is the length of the remaining piece?

SOLUTION.—The common denominator of the fractions is 16. $9\frac{1}{4} = 9\frac{4}{16}$.

minuend	$9\frac{4}{16}$	or $8\frac{20}{16}$	
subtrahend	$4\frac{7}{16}$	$4\frac{7}{16}$	
difference	$4\frac{13}{16}$	$4\frac{13}{16}$	Ans.

That is, the remainder is $4\frac{13}{16}$ inches long.

EXPLANATION.—As the fraction in the subtrahend is greater than the fraction in the minuend, it cannot be subtracted; therefore, take 1, or $\frac{16}{16}$, from the 9 in the minuend and add it to the $\frac{4}{16}$; $\frac{4}{16}+\frac{16}{16}=\frac{20}{16}$. $\frac{7}{16}$ from $\frac{20}{16}=\frac{13}{16}$. Since 1 was taken from 9, 8 remains; 4 from $8=4$; $4+\frac{13}{16}=4\frac{13}{16}$.

EXAMPLE 5.—From 9 take $8\frac{3}{16}$.

SOLUTION.—

minuend	9	or $8\frac{16}{16}$	
subtrahend	$8\frac{3}{16}$	$8\frac{3}{16}$	
difference	$\frac{13}{16}$	$\frac{13}{16}$	Ans.

EXPLANATION.—As there is no fraction in the minuend from which to take the fraction in the subtrahend, take 1, or $\frac{16}{16}$, from 9. $\frac{3}{16}$ from $\frac{16}{16}=\frac{13}{16}$. Since 1 was taken from 9, only 8 is left. 8 from $8=0$.

EXAMPLES FOR PRACTICE

1. In the following examples, subtract:

		Ans.
(*a*)	$\frac{4}{30}$ from $\frac{5}{10}$.	(*a*) $\frac{11}{30}$
(*b*)	$\frac{15}{35}$ from $\frac{45}{70}$.	(*b*) $\frac{3}{14}$
(*c*)	$\frac{15}{16}$ from $\frac{57}{48}$.	(*c*) $\frac{1}{4}$
(*d*)	$13\frac{1}{4}$ from $30\frac{1}{2}$.	(*d*) $17\frac{1}{4}$
(*e*)	$12\frac{1}{8}$ from 27.	(*e*) $14\frac{7}{8}$
(*f*)	$5\frac{1}{4}$ from 30.	(*f*) $24\frac{3}{4}$

2. A bar of iron $22\frac{1}{8}$ inches long, as shown in Fig. 8, has three pieces cut from it, measuring $6\frac{1}{2}$ inches, $4\frac{7}{8}$ inches, and $2\frac{5}{32}$ inches in length.

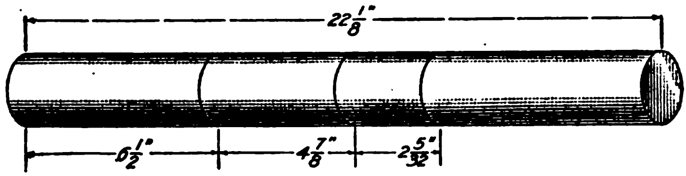


FIG. 8

What length of the original bar remains? Ans. $8\frac{19}{32}$ inches

SUGGESTION.—Add the lengths of the three pieces that are cut off and subtract their sum from the total length.

3. A piece of cast iron $4\frac{1}{8}$ inches thick was planed down to a thickness of $3\frac{31}{32}$ inches. What thickness of metal was removed? Ans. $\frac{5}{32}$ inch

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MULTIPLICATION OF FRACTIONS

22. In multiplication of fractions, it is not necessary to reduce the fractions to fractions having a common denominator. Multiplying the numerator or dividing the denominator multiplies the fraction. The following rule is used:

Rule.—I. *To multiply proper fractions, divide the product of the numerators by the product of the denominators.*

II. *To multiply one mixed number by another, reduce them both to improper fractions and then multiply them as in the case of proper fractions.*

III. *To multiply a mixed number by a whole number, first reduce the mixed number to an improper fraction; then multiply the numerator of this fraction by the whole number, and divide the product by the denominator. Or, multiply the whole number part of the mixed number by the whole number multiplier, and then the fractional part by the whole number multiplier, and add the products.*

EXAMPLE 1.—Multiply $\frac{3}{4}$ by 4.

SOLUTION.— $\frac{3}{4}\times4=\frac{3\times4}{4}=\frac{12}{4}=3$. Ans.

Or $\frac{3}{4}\times4=\frac{3}{4\div4}=\frac{3}{1}=3$. Ans.

The word "of" when placed between two fractions, or between a fraction and a whole number, means the same as ×, or *times;* thus,

$$\frac{3}{4} \text{ of } 4 = \frac{3}{4} \times 4 = 3$$

$$\frac{1}{8} \text{ of } \frac{5}{16} = \frac{1}{8} \times \frac{5}{16} = \frac{5}{128}$$

EXAMPLE 2.—Multiply $\frac{3}{8}$ by 2.

SOLUTION.— $\frac{3}{8} \times 2 = \frac{3 \times 2}{8} = \frac{6}{8} = \frac{3}{4}$. Ans.

Or $\frac{3}{8} \times 2 = \frac{3}{8 \div 2} = \frac{3}{4}$. Ans.

EXAMPLE 3.—Three cuts are taken on a piece 10 inches in diameter in a lathe. The first cut is $\frac{1}{8}$ inch deep, the second $\frac{1}{16}$ inch, and the third a finishing cut $\frac{1}{64}$ inch deep. What is the diameter of the finished piece?

SOLUTION.—A cut on the outside of a circular piece reduces the diameter by *twice* the depth of the cut. Therefore, the first cut reduces the diameter $2 \times \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$ inch; the second cut reduces it $2 \times \frac{1}{16} = \frac{2}{16} = \frac{1}{8}$ inch; and the last cut reduces it $2 \times \frac{1}{64} = \frac{2}{64} = \frac{1}{32}$ inch. The total amount by which the diameter is reduced is $\frac{1}{4} + \frac{1}{8} + \frac{1}{32}$. The common denominator is 32; $\frac{1}{4} = \frac{8}{32}$, $\frac{1}{8} = \frac{4}{32}$, and $\frac{1}{32} = \frac{1}{32}$. Then, $\frac{1}{4} + \frac{1}{8} + \frac{1}{32} = \frac{8}{32} + \frac{4}{32} + \frac{1}{32} = \frac{13}{32}$. Then, if the diameter was 10 inches and it was reduced $\frac{13}{32}$ inches, the diameter of the finished piece is $10 - \frac{13}{32} = 9\frac{19}{32}$ inches. Ans.

23. The operation of multiplying fractions or mixed numbers together may often be shortened. For example, suppose that the product of $\frac{3}{4} \times \frac{8}{9} \times \frac{10}{16}$ is required. According to the rule, this is equal to $\frac{3 \times 8 \times 10}{4 \times 9 \times 16}$, which is simply a fraction. Now, instead of multiplying the numerators together and then multiplying the denominators together, let us divide both numerator and denominator by the *same* number, wherever we can do so without a remainder, just as in reducing a fraction to its lowest terms. Doing this will change the form of the fraction, but not its value, as was proved in Art. **6.** There is a 3 in the numerator and a 9 in the denominator, so divide each by 3, draw lines through them, and write the quotient of each division near the dividend. This first step will then appear

$$\frac{\overset{1}{\cancel{3}} \times 8 \times 10}{4 \times \underset{3}{\cancel{9}} \times 16}$$

because 3 is contained 1 time in 3 and 3 times in 9. Again, 8 is above the line and 16 below it, and both of these can be divided by 8. The next step then is

$$\frac{\overset{1}{\cancel{3}}\times\overset{1}{\cancel{8}}\times 10}{4\times\underset{3}{\cancel{9}}\times\underset{2}{\cancel{16}}}$$

Now, there is a 10 above the line and a 4 below it, and both of these can be divided by 2 without any remainder. The next step then is

$$\frac{\overset{1}{\cancel{3}}\times\overset{1}{\cancel{8}}\times\overset{5}{\cancel{10}}}{\underset{2}{\cancel{4}}\times\underset{3}{\cancel{9}}\times\underset{2}{\cancel{16}}}$$

This process cannot be carried any farther, because there are no numbers above and below the line that can be divided by the same number without a remainder. Therefore, we multiply together all the uncrossed numbers in the numerator, or $1\times1\times5=5$; next we multiply together the uncrossed numbers in the denominator, or $2\times3\times2=12$; and then we write the first product over the second, giving $\frac{5}{12}$, which is the desired result. In other words, $\frac{3}{4}\times\frac{8}{9}\times\frac{10}{16}=\frac{5}{12}$. This can be proved very simply; for example, $\frac{3\times8\times10}{4\times9\times16}=\frac{240}{576}$, which can be reduced to lowest terms by dividing both terms by 48. $\frac{240\div48}{576\div48}=\frac{5}{12}$. This shows that the process of dividing numbers above and below the line by the *same* number will give the same result as taking the product of all the numerators and dividing it by the product of all the denominators.

The operation of dividing the numbers above and below the line by the same number is called **cancelation**, and when a line is drawn through one of the numbers, as shown, the number is said to be *canceled*. In cancelation, if the quotient is 1, it is not written down at all. The preceding example, therefore, would commonly be written

$$\frac{\cancel{3}\times\cancel{8}\times\overset{5}{\cancel{10}}}{\underset{2}{\cancel{4}}\times\underset{3}{\cancel{9}}\times\underset{2}{\cancel{16}}}=\frac{5}{12}$$

EXAMPLE 1.—What is the product of $\frac{4}{16}$ and $\frac{7}{8}$?

SOLUTION.— $\frac{4}{16}\times\frac{7}{8}=\frac{4\times 7}{16\times 8}=\frac{28}{128}=\frac{7}{32}$. Ans.

Or, by cancelation, $\frac{\not{4}\times 7}{\underset{4}{\not{16}}\times 8}=\frac{7}{4\times 8}=\frac{7}{32}$. Ans.

EXAMPLE 2.—What is $\frac{4}{8}$ of $\frac{3}{4}$ of $\frac{16}{32}$?

SOLUTION.— $\frac{\not{4}\times 3\times\not{16}}{8\times\not{4}\times\underset{2}{\not{32}}}=\frac{3}{8\times 2}=\frac{3}{16}$. Ans.

24. The multiplication of mixed numbers and of mixed numbers and whole numbers may be illustrated by the following examples:

EXAMPLE 1.—What is the product of $9\frac{3}{4}$ and $5\frac{5}{8}$?

SOLUTION.—The first step is to reduce the mixed numbers to improper fractions; thus, $9\frac{3}{4}=\frac{9\times 4+3}{4}=\frac{36+3}{4}=\frac{39}{4}$, and $5\frac{5}{8}=\frac{5\times 8+5}{8}=\frac{40+5}{8}=\frac{45}{8}$.

Then, $\frac{39}{4}\times\frac{45}{8}=\frac{39\times 45}{4\times 8}=\frac{1{,}755}{32}=54\frac{27}{32}$. Ans.

EXAMPLE 2.—What is the product of $32\frac{5}{8}$ and 24?

SOLUTION.—The mixed number $32\frac{5}{8}$ when reduced to an improper fraction becomes $\frac{32\times 8+5}{8}=\frac{261}{8}$. Then, by the first part of the third section of the rule of Art. 22,

$$\frac{261}{8}\times 24=\frac{261\times 24}{8}=\frac{261\times\overset{3}{\not{24}}}{\not{8}}=261\times 3=783.\quad\text{Ans.}$$

If the second part of the third section of the rule mentioned is used, the solution is as follows: The mixed number $32\frac{5}{8}$ is divided into the whole number part 32 and the fractional part $\frac{5}{8}$. Each of these is now multiplied by the whole number multiplier 24. Thus, $32\times 24=768$, and $\frac{5}{8}\times 24=\frac{5\times\overset{3}{\not{24}}}{\not{8}}=15$. Then, according to the rule, it is necessary to add the two products, giving $768+15=783$. That is,

$$32\tfrac{5}{8}\times 24=783.\quad\text{Ans.}$$

EXAMPLES FOR PRACTICE

1. Find the product of:

(*a*)	$7 \times \frac{3}{19}$.	Ans. (*a*)	$1\frac{2}{19}$
(*b*)	$14 \times \frac{5}{16}$.	(*b*)	$4\frac{3}{8}$
(*c*)	$\frac{21}{32} \times \frac{5}{14} \times \frac{8}{3}$.	(*c*)	$\frac{5}{8}$
(*d*)	$\frac{16}{27} \times 4$.	(*d*)	$2\frac{10}{27}$
(*e*)	$\frac{19}{18} \times 9$.	(*e*)	$9\frac{1}{2}$
(*f*)	$17\frac{18}{21} \times 7\frac{1}{2}$.	(*f*)	$133\frac{13}{14}$

2. How long must a piece of stock be to make 4 bolts if each bolt requires a piece of stock $3\frac{7}{8}$ inches long? Ans. $15\frac{1}{2}$ inches

3. Eleven holes are spaced equally in a straight line, as in Fig. 9.

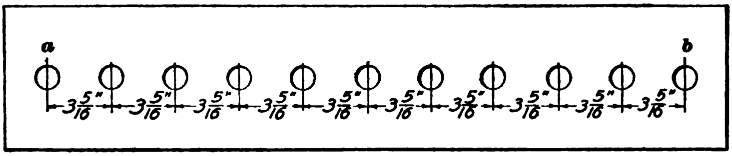


FIG. 9

What is the distance between the centers of the end holes *a* and *b*, if the spacing from center to center of two adjacent holes is $3\frac{5}{16}$ inches? Ans. $33\frac{1}{8}$ inches

NOTE.—The number of *spaces* is one less than the number of holes.

DIVISION OF FRACTIONS

25. In dividing one fraction by another it is not necessary to reduce them both to a common denominator. The rule to be used in dividing a fraction or a mixed number by another fraction or a mixed number is as follows:

Rule.—I. *To divide one fraction by another or by a whole number, take the reciprocal of the divisor and then proceed as in multiplication of fractions.*

II. *If mixed numbers are used, reduce them to improper fractions and then proceed as before.*

The **reciprocal** (pronounced *rec-sip′-ro-kal*) of a number is 1 divided by that number; thus, the reciprocal of 2 is $1 \div 2 = \frac{1}{2}$, and the reciprocal of 16 is $1 \div 16 = \frac{1}{16}$. The reciprocal of any whole number is always a proper fraction.

The reciprocal of a fraction is the fraction inverted. To *invert* a fraction, or to find its reciprocal, simply turn it upside down; that is, put the denominator *above* the line and the numerator *below*. For instance, the fraction $\frac{3}{4}$ when inverted becomes $\frac{4}{3}$, and $\frac{11}{16}$ inverted is $\frac{16}{11}$; this means that the reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$, and of $\frac{11}{16}$ is $\frac{16}{11}$.

EXAMPLE 1.—Divide $\frac{3}{4}$ by $\frac{5}{16}$.

SOLUTION.—In this case, the divisor is $\frac{5}{16}$, and its reciprocal is $\frac{16}{5}$. Then, according to the rule,

$$\frac{3}{4} \div \frac{5}{16} = \frac{3}{4} \times \frac{16}{5} = \frac{3 \times \overset{4}{\cancel{16}}}{\cancel{4} \times 5} = \frac{12}{5} = 2\tfrac{2}{5}. \quad \text{Ans.}$$

EXAMPLE 2.—Divide $\frac{3}{4}$ by 3.

SOLUTION.—The divisor is 3, and its reciprocal is $\frac{1}{3}$. According to the rule, the fraction is to be multiplied by this reciprocal; then,

$$\frac{3}{4} \div 3 = \frac{3}{4} \times \frac{1}{3} = \frac{\cancel{3} \times 1}{4 \times \cancel{3}} = \frac{1}{4}. \quad \text{Ans.}$$

EXAMPLE 3.—Divide 48 by $\frac{3}{16}$.

SOLUTION.—The divisor is $\frac{3}{16}$ and its reciprocal is $\frac{16}{3}$. Then,

$$48 \div \frac{3}{16} = 48 \times \frac{16}{3} = \frac{\overset{16}{\cancel{48}} \times 16}{\cancel{3}} = 256. \quad \text{Ans.}$$

EXAMPLE 4.—Divide $13\frac{3}{4}$ by $2\frac{1}{2}$.

SOLUTION.—First reduce both mixed numbers to improper fractions; thus, $13\frac{3}{4} = \frac{55}{4}$ and $2\frac{1}{2} = \frac{5}{2}$. Then

$$13\tfrac{3}{4} \div 2\tfrac{1}{2} = \frac{55}{4} \div \frac{5}{2} = \frac{55}{4} \times \frac{2}{5} = \frac{\overset{11}{\cancel{55}} \times \cancel{2}}{\underset{2}{\cancel{4}} \times \cancel{5}} = \frac{11}{2} = 5\tfrac{1}{2}. \quad \text{Ans.}$$

EXAMPLE 5.—A bar of steel $23\frac{5}{8}$ inches long is divided into 7 equal parts. What is the length of each part?

SOLUTION.—The length of each part must be $23\frac{5}{8} \div 7$. Now, $23\frac{5}{8} = \frac{189}{8}$. The reciprocal of 7 is $\frac{1}{7}$. Then, applying the rule,

$$\frac{189}{8} \div 7 = \frac{189}{8} \times \frac{1}{7} = \frac{\overset{27}{\cancel{189}} \times 1}{8 \times \cancel{7}} = \frac{27}{8} = 3\tfrac{3}{8}$$

Therefore, the length of each part is $3\frac{3}{8}$ inches. Ans.

EXAMPLES FOR PRACTICE

1. In the following examples, divide:

(*a*) 15 by $6\frac{3}{7}$.	Ans.	(*a*) $2\frac{1}{3}$
(*b*) 172 by $\frac{4}{5}$.		(*b*) 215
(*c*) $\frac{103}{6}$ by $14\frac{2}{3}$.		(*c*) $1\frac{15}{88}$
(*d*) $\frac{142}{27}$ by $17\frac{1}{9}$.		(*d*) $\frac{71}{231}$
(*e*) $15\frac{3}{8}$ by 6.		(*e*) $2\frac{9}{16}$

2. How many gears, each $1\frac{1}{8}$ inches thick, can be set side by side in a space 45 inches long? Ans. 40

3. A distance measuring $34\frac{7}{32}$ inches is divided into 15 equal parts. What is the length of each part? Ans. $2\frac{9}{32}$ inches

FACTORS AND AVERAGES

FACTORS

26. In some calculations it is necessary to *factor* a fraction or a whole number, that is, to divide the fraction or the number into its factors. The **factors** of a number are simply those numbers which, when multiplied together, will equal the number.

The factors of 6 are 2 and 3, because $2\times3=6$. The factors of 10 are 2 and 5, because $2\times5=10$. The factors of 12 are 3 and 4, because $3\times4=12$, but 6 and 2 are also factors, because $6\times2=12$; also, 3, 2, and 2 are factors, because $3\times2\times2=12$.

A fraction is factored by finding the factors of its numerator and denominator. Thus, the factors of $\frac{10}{16}$ are $\frac{5}{4}$ and $\frac{2}{4}$, because $\frac{5}{4}\times\frac{2}{4}=\frac{5\times2}{4\times4}=\frac{10}{16}$; also, $\frac{5}{2}$ and $\frac{2}{8}$ are factors of $\frac{10}{16}$, because $\frac{5}{2}\times\frac{2}{8}=\frac{10}{16}$; and $\frac{5}{8}$ and $\frac{2}{2}$ also are factors, as $\frac{5}{8}\times\frac{2}{2}=\frac{10}{16}$.

In other words, a number is factored by dividing it into two or more numbers whose product is equal to the original number. Some numbers have no factors except themselves and 1; thus, the factors of 5 are 1 and 5, because $1\times5=5$. The factors of 7 are 1 and 7; of 11, 1 and 11; of 13, 1 and 13; and so on.

Such numbers are called *prime* numbers (see Art. **17**). It is always understood that all prime numbers are integers, and the right-hand figure is 1, 3, 7, or 9, as, for example, 41, 53, 67, 79, etc.

The same number may be often factored in several different ways; thus, $24=6\times4$, or $3\times2\times4$, or 3×8, or $6\times2\times2$, or $3\times2\times2\times2$, because each of these products is equal to 24. Similarly, the fraction $\frac{24}{30}$ has the factors $\frac{4}{6}\times\frac{6}{5}$, $\frac{3}{10}\times\frac{8}{3}$, $\frac{2}{3}\times\frac{12}{10}$, and many others.

AVERAGES

27. It often becomes necessary to find the average of several numbers; for instance, a number of men will require different lengths of time in which to do a piece of work, and it may be required to find the average time.

The **average** of several terms is equal to their sum divided by the number of terms.

EXAMPLE 1.—Find the average of the numbers 6, 12, 8, 10, 11, and 7.

SOLUTION.—There are six numbers, and their sum is $6+12+8+10+11+7=54$. Then, the average is $54\div6=9$. Ans.

EXAMPLE 2.—If five gear-wheels weigh 28, 12, 36, 25, and 14 pounds, respectively, what is the average weight per wheel?

SOLUTION.—There are five gears, and the sum of their weights is $28+12+36+25+14=115$ pounds. The average is therefore $115\div5=23$ pounds. Ans.

It will be noted that if the average is multiplied by the number of things it will give the sum. Thus, in example 1 the average of six numbers is 9; hence, the sum of the six numbers is $9\times6=54$. In example 2, the average weight of the gears is 23 pounds; hence, the weight of the five gears is $23\times5=115$ pounds.

EXAMPLES FOR PRACTICE

1. A workman finishes 23 pieces of work on Monday, 26 on Tuesday, 19 on Wednesday, 27 on Thursday, and 20 on Friday. What is the average per day? Ans. 23

2. Six workmen engaged on the same class of work require 48, 42, 56, 50, 45, and 51 minutes, respectively. What is the average time for the work? Ans. $48\frac{2}{3}$ minutes

3. If the average weight of eight castings is $334\frac{1}{2}$ pounds each, what is the total weight of the castings? Ans. 2,676 pounds

4. Six rivets measure, respectively, $2\frac{1}{4}$, $2\frac{15}{16}$, $2\frac{7}{8}$, $2\frac{3}{8}$, $2\frac{7}{16}$, and $2\frac{1}{2}$ inches in length. What is the average length? Ans. $2\frac{9}{16}$ inches

5. What prime numbers are factors of: (*a*) 210? (*b*) 1,001?

Ans. (*a*) $2\times3\times5\times7$; (*b*) $7\times11\times13$

SHOP CALCULATIONS

(PART 3)

DECIMALS

WRITING AND READING DECIMALS

1. A **decimal,** sometimes called a *decimal fraction,* is a fraction whose denominator is 10, 100, 1,000, etc.; but a decimal differs from an ordinary fraction because its denominator is not written down. For example, .3 is a decimal, and it is read *three tenths.* Its value is the same as the fraction $\frac{3}{10}$; that is, $.3 = \frac{3}{10}$. The (.) placed in front of the figure 3 is called the **decimal point** and it is always the sign of a decimal. Every decimal consists of a decimal point followed by one or more figures.

2. The number that follows the decimal point is the ***numerator*** of the decimal fraction, and the ***denominator*** is the figure 1 followed by as many ciphers as there are figures following the decimal point; the value of the decimal is then equal to this numerator divided by this denominator. To illustrate, take the decimal .3. The number following the decimal point is 3, which is the numerator; and as there is only one figure after the decimal point, or one *decimal place,* as it is usually stated, the decimal represents tenths, or a denominator of 10. Then, the value of .3 is $\frac{3}{10}$, and it is read *three tenths.*

3. The reading of a decimal number depends on the number of decimal places in it, or the number of figures to the right of the decimal point.

One decimal place expresses *tenths*
Two decimal places express *hundredths*
Three decimal places express *thousandths*
Four decimal places express *ten-thousandths*
Five decimal places express *hundred-thousandths*
Six decimal places express *millionths*

Thus:

.3 = $\frac{3}{10}$ = 3 tenths
.03 = $\frac{3}{100}$ = 3 hundredths
.003 = $\frac{3}{1000}$ = 3 thousandths
.0003 = $\frac{3}{10000}$ = 3 ten-thousandths
.00003 = $\frac{3}{100000}$ = 3 hundred-thousandths
.000003 = $\frac{3}{1000000}$ = 3 millionths

4. From the examples given in the preceding article it may be seen that to change a decimal to an ordinary fraction, the number in the decimal is made the numerator, and the denominator is made up of a figure 1 followed by as many ciphers as there are decimal places in the decimal. These facts should be remembered, as they will make it easy to read decimals. For example .5625 is equal to $\frac{5625}{10000}$; there are four ciphers after the 1 in the denominator because there are four decimal places in the decimal. Similarly, the decimal .52 is equal to $\frac{52}{100}$, two ciphers being used after the 1 in the denominator because the decimal has two decimal places, or two figures after the decimal point.

5. Very often there are figures written to the left of the decimal point as well as to the right of it, as, for example, 12.5, 7.25, and 18.125. These are called *mixed numbers*, also, because each consists of a whole number and a fraction combined. The decimal point simply separates the whole number at its left from the decimal at its right. The number 12.5 is read *twelve and five tenths;* 7.25 is read *seven and twenty-five hundredths;* 18.125 is read *eighteen and one hundred twenty-five thousandths.* Sometimes decimals are written with a cipher to the left of the decimal point, as 0.6. The cipher in this case merely shows that there is no whole number. The relation of whole numbers and decimals in relation to the decimal point may be shown as follows:

hundreds of millions	tens of millions	millions	hundreds of thousands	tens of thousands	thousands	hundreds	tens	units	decimal point	tenths	hundredths	thousandths	ten-thousandths	hundred-thousandths	millionths	ten-millionths	hundred-millionths
9	8	7	6	5	4	3	2	1	.	2	3	4	5	6	7	8	9

6. Both whole numbers and decimals *decrease* on the scale of ten to the *right*, and both *increase* on the scale of ten to the *left*. The first figure to the left of units is *tens*, and the first figure to the right of units is *tenths*. The second figure to the left of units is *hundreds*, and the second figure to the right is *hundredths*. The third figure to the left is *thousands*, and the third to the right is *thousandths*, and so on. The figures equally distant from units place correspond in name, the decimals having the ending *ths*, to distinguish them from the whole numbers. The number in the preceding table is read *nine hundred eighty-seven million, six hundred fifty-four thousand, three hundred twenty-one and twenty-three million, four hundred fifty-six thousand, seven hundred eighty-nine hundred-millionths.*

The decimals increase to the *left*, on the scale of ten, the same as whole numbers; for, beginning at the 4 in thousandths place in the table, the next figure to the left is *hundredths*, which is ten times as great, and the next *tenths*, or ten times the *hundredths*, and so on through both decimals and whole numbers.

7. *Annexing or taking away a cipher at the right of a decimal does not affect its value.* For example, .5 is $\frac{5}{10}$ and .50 is $\frac{50}{100}$. But, $\frac{5}{10}=\frac{50}{100}$; therefore, $.5=.50$.

8. *Inserting a cipher between a decimal and the decimal point divides the decimal by 10.* For example, $.5=\frac{5}{10}$; but, $\frac{5}{10}\div 10=\frac{5}{100}=.05$. Therefore, $.05=.5\div 10$.

9. *Taking away a cipher from the left of a decimal multiplies the decimal by 10.* For example, $.05=\frac{5}{100}$; but, $\frac{5}{100}\times 10=\frac{5}{10}=.5$. Therefore, $.5=10\times .05$.

10. In some cases, a mixed number in the form of a decimal may be more conveniently expressed in the form of a common improper fraction. To do so, it is only necessary to write the entire number, omitting the decimal point, as the numerator of the fraction, and the denominator of the decimal part as the denominator of the fraction. Thus, $127.483 = \frac{127483}{1000}$; for,

$$127.483 = 127\tfrac{483}{1000} = \frac{127000+483}{1000} = \tfrac{127483}{1000}.$$

ADDITION OF DECIMALS

11. In the addition of decimals tenths are placed under tenths, hundredths under hundredths, etc.; this, of course, brings the decimal points in line, one directly under the other. Then addition is performed exactly as in the case of whole numbers. Hence, in placing the numbers to be added, it is only necessary to take care that the *decimal points are in line.* In adding whole numbers, the right-hand figures are always in line; but in adding decimals, the right-hand figures will not be in line unless each decimal contains the same number of figures.

whole numbers	*decimals*	*mixed numbers*
342	.342	342.032
4234	.4234	4234.5
26	.26	26.6782
3	.03	3.06
sum 4605 Ans.	*sum* 1.0554 Ans.	*sum* 4606.2702 Ans.

Rule.—*Place the numbers to be added so that all the decimal points will stand in the same column. Add as in whole numbers, and place the decimal point in the sum directly under the decimal points above.*

EXAMPLE 1.—What is the sum of 242, .36, 118.725, 1.005, 6, and 100.1?

SOLUTION.—

```
242.
   .36
118.725
  1.005
  6.
100.1
-------
468.190  Ans.
```

EXAMPLE 2.—A bar is marked into 8 parts measuring 1.25, 4.3125, 2.305, 7.6, 10.4375, 5.5625, .875, and 3.0625 inches in length. What is the total length of the bar?

SOLUTION.—The length of the bar is the sum of the lengths of the parts. Set down the numbers with the decimal points in a vertical line, and add, thus:

$$\begin{array}{r@{.}l} 1 & 25 \\ 4 & 3125 \\ 2 & 305 \\ 7 & 6 \\ 10 & 4375 \\ 5 & 5625 \\ & 875 \\ 3 & 0625 \\ \hline 35 & 40500 \end{array} \text{ Ans.}$$

The total length, therefore, is 35.405 inches.

EXAMPLES FOR PRACTICE

1. In the following examples, find the sum of:
   - (a) .2143, .105, 2.3042, and 1.1417.
   - (b) 783.5, 21.473, .2101, and .7816.
   - (c) 21.781, 138.72, 41.8738, .72, and 1.413.
   - (d) .3724, 104.15, 21.417, and 100.042.

   Ans. (a) 3.7652; (b) 805.9647; (c) 204.5078; (d) 225.9814

2. Four round pieces of bar steel, when measured accurately, are found to have lengths of 11.25, 7.625, 1.3125, and 5.4375 inches. What is their total length when placed end to end? Ans. 25.625 inches

SUBTRACTION OF DECIMALS

12. In the subtraction of decimals, tenths are placed under tenths, hundredths under hundredths, etc., bringing the decimal points under each other, as in addition of decimals.

Rule.—*Place the subtrahend under the minuend, so that the decimal point of the subtrahend will be directly under that of the minuend. Subtract as in whole numbers, and place the decimal point in the remainder, directly under the decimal points above.*

When the figures in the decimal part of the subtrahend extend beyond those in the minuend, place ciphers in the minuend above them, and subtract as before.

EXAMPLE 1.—Subtract .132 from .3063.

SOLUTION.—

minuend	.3 0 6 3
subtrahend	.1 3 2
difference	.1 7 4 3 Ans.

EXAMPLE 2.—A piece 7.895 inches long has .725 inch cut from one end. What length remains?

SOLUTION.—The length remaining must be the difference between 7.895 inches and .725 inch.

minuend	7.8 9 5
subtrahend	.7 2 5
difference	7.1 7 0 or 7.17 Ans.

That is, the length remaining is 7.17 inches.

EXAMPLE 3.—A block 11 inches thick has .625 inch planed from it. What is its thickness then?

SOLUTION.—The resulting thickness is the difference between 11 and .625 inches.

minuend	1 1.0 0 0
subtrahend	.6 2 5
difference	1 0.3 7 5 Ans.

That is, the thickness is then 10.375 inches.

EXAMPLES FOR PRACTICE

1. In the following examples, from:
   (*a*) 407.385 take 235.0004.
   (*b*) 22.718 take 1.7042.
   (*c*) 1,368.17 take 13.6817.
   (*d*) 70.00017 take 7.000017.

   Ans. (*a*) 172.3846; (*b*) 21.0138; (*c*) 1,354.4883; (*d*) 63.000153

2. A block of steel 1.0625 inches thick is cut down to a thickness of .9375 inch. What thickness of metal is removed? Ans. .125 inch

3. If a bar 3.25 inches long has a piece 1.625 inches long cut away, what is the length of the remainder? Ans. 1.625 inches

MULTIPLICATION OF DECIMALS

13. In multiplication of decimals, the decimal points are not placed directly under each other, as in addition and subtraction. No attention for the time being is paid to the decimal points. Place the multiplier under the multiplicand, so that the *right-hand* figure of the one is under the *right-hand* figure of the other, and proceed exactly as in multiplication of whole numbers.

Rule.—*Place the multiplier under the multiplicand, disregarding the position of the decimal points. Multiply as with whole numbers, and in the product point off as many decimal places as there are decimal places in both multiplier and multiplicand beginning at the right, and prefixing ciphers if necessary.*

EXAMPLE 1.—Multiply .825 by 13.

SOLUTION.—

multiplicand	.8 2 5
multiplier	1 3
	2 4 7 5
	8 2 5
product	1 0.7 2 5 Ans.

In this example, there are 3 decimal places in the multiplicand and none in the multiplier; therefore, 3 decimal places are pointed off in the product, beginning to count at the *right.*

EXAMPLE 2.—What is the product of 426 and the decimal .005?

SOLUTION.—

multiplicand	4 2 6
multiplier	.0 0 5
product	2.1 3 0 or 2.13 Ans.

In this example, there are 3 decimal places in the multiplier and none in the multiplicand; therefore, 3 decimal places are pointed off in the product, counting from the right.

14. It is not necessary to multiply by the ciphers on the *left* of a decimal; they merely determine the number of decimal places. Ciphers to the *right* of a decimal should be dropped, as they only make more figures to deal with, and do not change the value.

EXAMPLE 1.—Multiply 1.205 by 1.15.

SOLUTION.—

```
    1.2 0 5
      1.1 5
    -------
    6 0 2 5
  1 2 0 5
1 2 0 5
-----------
1.3 8 5 7 5  Ans.
```

In this example, there are 3 decimal places in the multiplicand, and 2 in the multiplier; therefore, 3+2=5 decimal places must be pointed off in the product.

EXAMPLE 2.—Multiply .232 by .001.

SOLUTION.—

```
   .232
   .001
.000232  Ans.
```

In this example, we multiply the multiplicand by the digit in the multiplier, which gives 232 for the product; but since there are 3 decimal places each in the multiplier and multiplicand, we must put three ciphers in front of the 232, to make 3+3=6 decimal places in the product.

EXAMPLES FOR PRACTICE

1. In the following examples, find the product of:

(*a*) .000492×4.1418.	Ans.	(*a*) .0020377656
(*b*) 4,003.2×1.2.		(*b*) 4,803.84
(*c*) 78.6531×1.03.		(*c*) 81.012693
(*d*) .3685×.042.		(*d*) .015477

2. Four equal distances of 2.375 inches are marked off, end to end, on a piece of work. What is the total distance marked off? Ans. 9.5 inches

3. If 1 cubic inch of cast brass weighs .295 pound, what is the weight of a brass casting containing 768 cubic inches? Ans. 226.56 pounds

---

DIVISION OF DECIMALS

15. In division of decimals, we pay no attention to the decimal point until after the division has been performed. If the *number of decimal places in the dividend is less than the number of decimal places in the divisor, annex ciphers to the dividend until it has the same number of decimal places as the divisor. Divide exactly as with whole numbers*. The rule to be used is as follows:

Rule.—I. *Place the divisor to the left of the dividend, and proceed as in division of whole numbers; in the quotient, point off as many decimal places as the number of decimal places in the dividend exceed those in the divisor, prefixing ciphers to the quotient, if necessary.*

II. *If in dividing one number by another there is a remainder, the remainder can be placed over the divisor, as a fractional part of the quotient, but it is generally better to annex ciphers to the remainder, and continue dividing until there are 3 or 4 decimal places in the quotient, and then if there still is a remainder, terminate the quotient by the plus sign (+), which shows that it can be carried farther.*

EXAMPLE 1.—Divide .625 by 25.

```
                  divisor dividend quotient
SOLUTION.—          25 ) .625 ( .025  Ans.
                          50
                          ---
                          125
                          125
                          ---
                remainder   0
```

In this example, there are no decimal places in the divisor, and 3 decimal places in the dividend; therefore, there are $3-0=3$ decimal places in the quotient. One cipher has to be prefixed to the 25, to make the 3 decimal places.

EXAMPLE 2.—Divide 6.035 by .05.

```
                  divisor dividend  quotient
SOLUTION.—          .05 ) 6.035 ( 120.7  Ans.
                          5
                          -
                          10
                          10
                          --
                            35
                            35
                            --
                 remainder   0
```

In this example, we divide by 5, as if the cipher were not before it. There is one more decimal place in the dividend than in the divisor; therefore, 1 decimal place is pointed off in the quotient. Short division should be used in this case.

EXAMPLE 3.—Divide .125 by .005

```
                 divisor  dividend  quotient
SOLUTION.—       .005).125(25  Ans.
                       10
                       --
                        25
                        25
                        --
               remainder 0
```

In this example, there are the same number of decimal places in the dividend as in the divisor; therefore, the quotient has no decimal places, and is a whole number. Short division should be used in this case.

EXAMPLE 4.—Divide 326 by .25.

```
SOLUTION.—       .25)326.00(1304  Ans.
                     25
                     --
                      76
                      75
                      --
                       100
                       100
                       ---
                         0
```

In this example, two ciphers were annexed to the dividend, to make the number of decimal places equal to the number in the divisor. The quotient is a whole number.

EXAMPLE 5.—Divide .0025 by 1.25.

```
SOLUTION.—       1.25).00250(.002  Ans.
                        250
                        ---
                          0
```

EXPLANATION.—In this example, we are to divide .0025 by 1.25. Consider the dividend as a whole number, or 25 (disregarding the two ciphers at its left, for the present); also, consider the divisor as a whole number, or 125. The dividend 25 will not contain the divisor 125; we must therefore annex one cipher to the 25, thus making the dividend 250. 125 is contained twice in 250, so we place the figure 2 in the quotient. In pointing off the decimal places in the quotient, it must be remembered that there were only 4 decimal places in the dividend; but one cipher was annexed, thereby making $4+1=5$ decimal places. Since there are 5 decimal places in the dividend and 2 decimal places in the divisor, we must point off $5-2=3$ decimal places in the quotient. In order to point off 3 decimal places, two ciphers must be prefixed to the figure 2, thereby making

.002 the quotient. It is not necessary to consider the ciphers at the left of a decimal when dividing, except when determining the position of the decimal point in the quotient.

16. In the foregoing examples, the divisor was contained in the dividend without any remainder. The following example shows a case in which there is a remainder:

EXAMPLE.—What is the quotient of 199 divided by 15?

SOLUTION.—

```
           15)199(13+4/15 Ans.
              15
               49
               45
remainder       4
```

Or,

```
           15)199.000(13.266+ Ans.
              15
               49
               45
                40
                30
                100
                 90
                 100
                  90
remainder         10
```

$13\frac{4}{15} = 13.266+$

$\frac{4}{15} = .266+$

17. It frequently happens, as in the preceding example, that the division will never end. In such cases, decide to how many decimal places the division is to be carried, and carry the work one place farther. If the last figure of the quotient thus obtained is 5 or a greater number, increase the preceding figure by 1, and write after it the minus sign (−), thus indicating that the quotient is not quite so large as indicated; if the figure thus obtained is less than 5, write the plus sign (+) after the quotient, thus indicating that the number is slightly greater than as indicated. In the last example, had it been desired to obtain the answer correct to 4 decimal places, the work would have been carried to 5 places, obtaining 13.26666, and the answer would have been given as 13.2667−.

This remark applies to any other calculation involving decimals, when it is desired to omit some of the figures in the decimal. Thus, if it is desired to retain 3 decimal places in the number .2471253, it would be expressed as .247+; if it was desired to retain 5 decimal places, it would be expressed as .24713−. Both the + and − signs are frequently omitted; they are seldom used outside of arithmetic, except in exact calculations, when it is desired to call particular attention to the fact that the result obtained is not *quite* exact.

EXAMPLES FOR PRACTICE

1. In the following examples, divide:

(*a*) 101.6688 by 2.36.
(*b*) 187.12264 by 123.107.
(*c*) .08 by .008.
(*d*) .0003 by 3.75.

Ans. (*a*) 43.08; (*b*) 1.52; (*c*) 10; (*d*) .00008

2. A bar 24.375 inches long is divided into equal parts measuring 1.625 inches in length. How many parts are there? Ans. 15

3. In the manufacture of a number of machines 612 pounds of bronze was required. If each machine used 12.75 pounds, how many machines were there? Ans. 48

REDUCING FRACTIONS AND DECIMALS

REDUCING FRACTIONS TO DECIMALS

18. The workman in the shop often finds it necessary to change common fractions to decimals, as, for example, changing fractions of an inch to equivalent decimals. The following rule is used:

Rule.—*Annex ciphers to the numerator, place a decimal point before the ciphers, and divide by the denominator. Point off as many decimal places in the quotient as there are ciphers annexed.*

Example 1.— $\frac{3}{4}$ equals what decimal?

Solution.—

$$\begin{array}{r} 4\,)\,\underline{3.0\,0} \\ .7\,5 \end{array}$$

or $\frac{3}{4} = .75$ Ans.

EXAMPLE 2.—What decimal is equivalent to $\frac{7}{8}$?

SOLUTION.—

```
8) 7.000(.875
   64
   --
    60
    56     or 7/8 = .875  Ans.
    --
     40
     40
     --
      0
```

EXAMPLES FOR PRACTICE

Reduce the following common fractions to decimals:

(a) $\frac{15}{32}$.		(a) .46875
(b) $\frac{3}{8}$.		(b) .375
(c) $\frac{21}{32}$.		(c) .65625
(d) $\frac{51}{64}$.	Ans.	(d) .796875
(e) $\frac{3}{16}$.		(e) .1875
(f) $\frac{5}{8}$.		(f) .625
(g) $\frac{10}{200}$.		(g) .05
(h) $\frac{4}{1000}$.		(h) .004

REDUCING DECIMALS TO FRACTIONS

19. In case it is desired to change a decimal into a common fraction of the same value, the following rule may be used:

Rule.—*Under the figures of the decimal, place 1 with as many ciphers at its right as there are decimal places in the decimal, and reduce the resulting fraction to its lowest terms by dividing both numerator and denominator by the same number.*

EXAMPLE 1.—Reduce .125 to a fraction.

SOLUTION.— $.125 = \frac{125}{1000} = \frac{25}{200} = \frac{5}{40} = \frac{1}{8}$. Ans.

EXAMPLE 2.—Reduce .875 to a fraction.

SOLUTION.— $.875 = \frac{875}{1000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$. Ans.

20. Sometimes, however, it is necessary to change a decimal to a fraction having a certain denominator. In such a case, the following rule is used:

Rule.—*Reduce 1 to a fraction having the given denominator. Multiply the given decimal by the fraction so obtained, and the result will be the fraction required.*

When 1 is reduced to a fraction having a given denominator, the numerator and denominator of that fraction must be equal; for example, if 1 is reduced to a fraction whose denominator is 16, the fraction is $\frac{16}{16}$, because $\frac{16}{16}=1$.

EXAMPLE 1.—A machinist finds that a certain piece is .5827 inch thick. How many 64ths is this equal to?

SOLUTION.—This means that the decimal .5827 is to be changed to a fraction whose denominator is 64. Applying the rule, $1=\frac{64}{64}$; then,

$$.5827\times\frac{64}{64}=\frac{.5827\times 64}{64}=\frac{37.2928}{64}$$

37.2928 is closer to 37 than to 38, so we call it 37, approximately, and drop the small decimal. Then we say that $.5827=\frac{37}{64}$, nearly. Ans.

EXAMPLE 2.—Change .3917 to 12ths.

SOLUTION.—This means that .3917 is to be changed to an equal fraction whose denominator is 12. Following the rule, $1=\frac{12}{12}$; then,

$$.3917\times\frac{12}{12}=\frac{.3917\times 12}{12}=\frac{4.7004}{12}$$

The number 4.7004 is nearer 5 than 4, so we call it 5, nearly. That is, $.3917=\frac{5}{12}$, nearly. Ans.

EXAMPLES FOR PRACTICE

1. Reduce the following to common fractions:

		Ans.
(*a*)	.25.	(*a*) $\frac{1}{4}$
(*b*)	.625.	(*b*) $\frac{5}{8}$
(*c*)	.3125.	(*c*) $\frac{5}{16}$
(*d*)	.04.	(*d*) $\frac{1}{25}$
(*e*)	.06.	(*e*) $\frac{3}{50}$

2. Express:

		Ans.
(*a*)	.625 in 8ths.	(*a*) $\frac{5}{8}$
(*b*)	.3125 in 16ths.	(*b*) $\frac{5}{16}$
(*c*)	.15625 in 32ds.	(*c*) $\frac{5}{32}$
(*d*)	.77 in 64ths.	(*d*) $\frac{49}{64}$
(*e*)	.81 in 48ths.	(*e*) $\frac{39}{48}$
(*f*)	.923 in 96ths.	(*f*) $\frac{89}{96}$

DECIMAL CURRENCY

21. In the United States, the **dollar** is the unit in which money values are expressed. The dollar is 100 **cents**; that is, 1 cent is one-hundredth of a dollar. The sign $ is the *dollar mark*, and is usually placed in front of a number representing dollars or cents. For example, $12 is read *twelve dollars*, and $.25 is read *twenty-five cents*. The expression $12.25 is read *twelve dollars and twenty-five cents*. The decimal point is used to separate the dollars from the cents. All to the left of the decimal point represents dollars, and the two figures to the right of the decimal point represent cents, the cents being hundredths of a dollar.

Calculations in which dollars and cents are used are made in the same way, and according to the same rules, as calculations using decimals, because an expression like $1.15 is a decimal, and represents $1\frac{15}{100}$ dollars. The following examples, with their solutions, will serve to illustrate the methods of using dollars and cents in calculations:

EXAMPLE 1.—A workman's wages are $2.85 a day. How much does he he earn in 26 days?

SOLUTION.—In 26 days he will earn 26 times as much as in 1 day, or 26×$2.85=$74.10. Ans.

EXAMPLE 2.—The total cost of making 95 pieces of work was $235.60. What was the cost per piece?

SOLUTION.—The cost per piece is equal to the total cost divided by the number of pieces, or $235.60÷95=$2.48. Ans.

EXAMPLE 3.—If a man receives $36.94 on payday and immediately pays bills amounting to $19.67, how much has he left?

SOLUTION.—He will have left the difference between what he received and what he paid out, or $36.94−$19.67=$17.27. Ans.

EXAMPLE 4.—A certain piece of work requires four men to complete it. If these men are paid $12.80, $21.16, $13.54, and $6.15, what is the cost of the labor?

SOLUTION.—The labor cost must be the sum of the four amounts, or $12.80+$21.16+$13.54+$6.15=$53.65. Ans.

EXAMPLES FOR PRACTICE

1. A workman is paid $27.60 for 6 days' work. How much does he earn per day? Ans. $4.60

2. If a certain tool costs $.45, what will 48 of these tools cost? Ans. $21.60

3. Three different grades of pig iron were purchased at $14.25, $19.65, and $17.40 a ton. What was the average cost per ton? Ans. $17.10

4. A machinist works 112 hours at $.46 an hour, and draws $28.75 on his pay. How much is still due him? Ans. $22.77

PERCENTAGE

22. The term **per cent.** is very frequently used in connection with calculations of various kinds, as, for example, in expressing the composition of certain brands of iron and steel. It is an abbreviation of the Latin words *per centum*, and it means *per hundred*. The **sign** of per cent. is %; thus, 2% means 2 per cent. and is read *two per cent.* The figure before the sign % indicates how many per cent. is meant, or the number of *hundredths*. Thus, 1% means 1 hundredth or $\frac{1}{100}$; 5% means $\frac{5}{100}$; 33% means $\frac{33}{100}$; and so on. The process of calculating by hundredths, or by per cent., is called **percentage.** One per cent. of a number means $\frac{1}{100}$ of that number; 5 per cent. of a number is $\frac{5}{100}$ of the number; and so on. Thus, 1% of 200 is 2, because $\frac{1}{100}$ of 200 is $\frac{1}{100}\times200=2$. If a man is receiving 60 cents an hour for his work and he gets an increase of 10% in his wages, his increase amounts to 10% of 60, or $\frac{10}{100}\times60=6$ cents; then, his new rate of pay is $60+6=66$ cents an hour.

23. If a number is followed by the sign %, the sign may be dropped and the per cent. expressed in a number of other ways. For example, 12% may be written 12 per cent., or $\frac{12}{100}$, or .12. When the per cent. is changed to a common fraction, the sign is simply dropped and the number in front of the sign is written over 100. Thus, $6\%=\frac{6}{100}$; $25\%=\frac{25}{100}$; $12\frac{1}{2}\%=\frac{12\frac{1}{2}}{100}=\frac{12.5}{100}=\frac{125}{1000}$; $51.6\%=\frac{51.6}{100}=\frac{516}{1000}$.

24. If the per cent. is to be changed to a decimal instead of to a common fraction, the sign % is dropped, and the decimal point is moved two places to the left in the number that represents the per cent.. For example, suppose that 25% is to be changed to a decimal having the same value. The sign % is dropped, and the decimal point is moved two places to the left in the number, giving .25; that is, 25% = .25. It is easy to prove that this method is correct. Take the case just given, of changing 25% to a decimal. According to the preceding article, $25\% = \frac{25}{100}$; but, $\frac{25}{100} = .25$. Therefore, $25\% = \frac{25}{100} = .25$.

Suppose that 2.5% is to be changed to a decimal. There is already one decimal place in the number, and the two more that must be added when the sign % is dropped make a total of three decimal places; then, 2.5% = .025, a cipher being used to give the necessary number of places. Again, .25% = .0025. The number of decimal places to be pointed off in this case is 2+2=4, and so two ciphers must be used.

25. When a per cent. is used in a calculation, it is always changed to a fraction or to a decimal, but more usually to a decimal; therefore, the student should thoroughly understand the rules illustrated in the preceding articles, explaining the method of changing a per cent. to a fraction and to a decimal. The following table also shows how a number of different expressions are changed:

Per Cent.	Decimal	Fraction	Per Cent.	Decimal	Fraction
1%	.01	$\frac{1}{100}$	150%	1.50	$\frac{150}{100}$ or $1\frac{1}{2}$
2%	.02	$\frac{2}{100}$ or $\frac{1}{50}$	500%	5.00	$\frac{500}{100}$ or 5
5%	.05	$\frac{5}{100}$ or $\frac{1}{20}$	$\frac{1}{4}$%	.0025	$\frac{\frac{1}{4}}{100}$ or $\frac{1}{400}$
10%	.10	$\frac{10}{100}$ or $\frac{1}{10}$	$\frac{1}{2}$%	.005	$\frac{\frac{1}{2}}{100}$ or $\frac{1}{200}$
25%	.25	$\frac{25}{100}$ or $\frac{1}{4}$	$1\frac{1}{2}$%	.015	$\frac{1\frac{1}{2}}{100}$ or $\frac{3}{200}$
50%	.50	$\frac{50}{100}$ or $\frac{1}{2}$	$8\frac{1}{3}$%	$.08\frac{1}{3}$	$\frac{8\frac{1}{3}}{100}$ or $\frac{1}{12}$
75%	.75	$\frac{75}{100}$ or $\frac{3}{4}$	$12\frac{1}{2}$%	.125	$\frac{12\frac{1}{2}}{100}$ or $\frac{1}{8}$
100%	1.00	$\frac{100}{100}$ or 1	$16\frac{2}{3}$%	$.16\frac{2}{3}$	$\frac{16\frac{2}{3}}{100}$ or $\frac{1}{6}$
125%	1.25	$\frac{125}{100}$ or $1\frac{1}{4}$	$62\frac{1}{2}$%	.625	$\frac{62\frac{1}{2}}{100}$ or $\frac{5}{8}$

26. One of the most usual calculations using per cent. is that of finding a certain per cent. of a given number. The rule to be used is as follows:

Rule.—*To find a certain per cent. of a number, multiply the number by the per cent. expressed as a decimal.*

EXAMPLE 1.—What is 36% of 125?

SOLUTION.—According to the rule, 36% is first changed to a decimal, thus becoming .36. Then, $125 \times .36 = 45$. Therefore, 36% of 125 is 45. Ans.

EXAMPLE 2.—From a stock of 300 machines, 76% were sold. How many were sold?

SOLUTION.—The number sold was 76% of 300. Expressed as a decimal, 76% = .76. Then, $300 \times .76 = 228$; that is, 228 machines were sold. Ans.

EXAMPLE 3.—A brass casting weighs 348 pounds. The brass is made up of 65% of copper and 35% of zinc. (*a*) What is the weight of copper in the casting? (*b*) What is the weight of zinc in the casting?

SOLUTION.—(*a*) The weight of copper is 65% of the whole, or 65% of 348. Expressed as a decimal, 65% = .65. The weight of copper is then,

$$348 \times .65 = 226.2 \text{ pounds. Ans.}$$

(*b*) The weight of zinc is

$$348 \times .35 = 121.8 \text{ pounds. Ans.}$$

Another way of finding the amount of zinc is to subtract the weight of copper from the weight of the casting, or $348 - 226.2 = 121.8$ pounds of zinc.

27. Another calculation that frequently must be made is to find what per cent. one number is of another. For example, if a certain weight of copper and a certain weight of tin are melted together to form bronze, it may be desired to know what per cent. is copper and what per cent. is tin. The rule to be used is as follows:

Rule.—*To find what per cent. of the whole is represented by a given part, divide the part by the whole, and multiply the result by 100.*

EXAMPLE 1.—What per cent. of 64 is 16?

SOLUTION.—In this example, 64 is the whole, and 16 is the part. Then, $16 \div 64 = .25$. Finally, $.25 \times 100 = 25\%$. Therefore, 16 is 25% of 64. Ans.

EXAMPLE 2.—A bronze casting weighing 98 pounds contains 83.3 pounds of copper. What per cent. of copper is there in the casting?

SOLUTION.—The whole weight is 98 pounds and the part that is copper is 83.3 pounds. Then, following the rule, $83.3 \div 98 = .85$, and $.85 \times 100 = 85\%$. Ans.

EXAMPLE 3.—From a stock of 300 machines 228 were sold. What per cent. was sold?

SOLUTION.—The whole is 300 and the part sold is 228. Then, $228 \div 300 = .76$, and $.76 \times 100 = 76\%$; that is, 76% was sold. Ans.

EXAMPLE 4.—A certain alloy contains 144 pounds of copper, 27 pounds of tin, and 9 pounds of zinc. Find (*a*) the per cent. of copper, (*b*) the per cent. of tin, and (*c*) the per cent. of zinc in the alloy.

SOLUTION.—The total weight of metal in the alloy is $144+27+9=180$ pounds.

(*a*) There are 144 pounds of copper in 180 pounds of the alloy. Therefore, by the rule, $144 \div 180 = .8$, and $.8 \times 100 = 80\%$; that is, there is 80% of copper in the alloy. Ans.

(*b*) There are 27 pounds of tin. Hence, by the rule, $27 \div 180 = .15$, and $.15 \times 100 = 15\%$; that is, there is 15% of tin in the alloy. Ans.

(*c*) There are 9 pounds of zinc. Hence, by the rule, $9 \div 180 = .05$, and $.05 \times 100 = 5\%$; that is, there is 5% of zinc in the alloy. Ans.

EXAMPLE 5.—The coke used in melting iron in a cupola contains 1% of sulphur. If $\frac{1}{2}$ of the sulphur in the coke goes into the iron during the melting, how many pounds of sulphur does the iron absorb from 100 pounds of coke?

SOLUTION.—The coke contains 1% of sulphur; or, $\frac{1}{100}$ of the coke is sulphur. Therefore, the amount of sulphur in 100 pounds of coke is $\frac{1}{100} \times 100 = 1$ pound. Now, $\frac{1}{2}$ of the sulphur enters the iron; hence, the amount of sulphur entering the iron from each 100 pounds of coke is $\frac{1}{2} \times 1 = \frac{1}{2}$ pound. Ans.

28. It should be noted that when the composition of a body is given in per cent. of its parts, the sum of the per cents. is equal to 100. For instance, take example 4 in the preceding article. There is 80% of copper, 15% of tin, and 5% of zinc, or a total of $80+15+5=100\%$. Also, if a certain per cent. of a thing is taken away, the remainder is the difference between 100 per cent. and the per cent. taken away. Thus, suppose that a water tank holding 8,000 gallons loses 12% by leakage. The amount remaining is then $100-12=88\%$. This can easily be proved. If 12% leaks away, the amount that leaks away is $8{,}000 \times .12 = 960$ gallons, and the amount left in the tank is

8,000 − 960 = 7,040 gallons. Now, 7,040 ÷ 8,000 = .88, or 88%. In other words, the water remaining is 88% of the original amount, which is exactly the same result as was obtained by subtracting 12% from 100%.

EXAMPLES FOR PRACTICE

1. A certain bronze casting weighing 150 pounds contains 78% of copper and 22% of tin. Find the weight (*a*) of copper and (*b*) of tin in the casting. Ans. { (*a*) 117 pounds; (*b*) 33 pounds }

2. Out of a lot of 240 castings, 5% were found to be defective and were scrapped. How many were scrapped? Ans. 12

3. A grade of pig iron contains 2.75% of silicon, .04% of sulphur, 1% of phosphorus, and 3.6% of carbon. The remaining per cent. is iron. What is the per cent. of iron? Ans. 92.61%

4. A ton of manganese steel contains 8 pounds of manganese. What is the per cent. of manganese in the steel? A ton is 2,000 pounds. Ans. .4%

POWERS OF NUMBERS

29. In certain shop calculations it is necessary to find the *power* of a number. The **power** of any number is the product obtained by multiplying that number by itself one or more times. In the calculations that follow, the only powers required are the second and third powers of numbers.

30. The *second power* of a number, usually called the **square** of the number, is the product obtained by using that number *twice* as a factor. For example, the square of 3 is $3 \times 3 = 9$, because the number 3 is used twice as a factor in the multiplication. The square of 5 is 25, because $5 \times 5 = 25$. The square of 2.4 is 5.76, because $2.4 \times 2.4 = 5.76$. The square of a number is usually indicated by a small figure 2 written to the right of the number and above it; thus, 4^2 means the second power of 4, or the square of 4 and is read *four square*. This little figure shows that 4 is to be used twice as a factor in finding the power; that is, $4^2 = 4 \times 4 = 16$. The reason for calling this product the square of the number may easily be understood. In connection with square measure it will be shown that the area of a square is equal to the product of its equal sides; that is, the area is the product of two equal numbers. Thus, the product of two equal numbers came to be known as the square of the number.

31. The *third power* of a number is found by using that number three times as a factor in multiplication; thus, the third power of 3 is $3 \times 3 \times 3 = 27$. The third power is usually called the **cube** of a number, because the volume of a cube is found by using the length of one edge three times as a factor. The cube of a number is usually indicated by writing a small figure 3 to the right of the number and above it; for example, 3^3 indicates the cube of 3, or $3 \times 3 \times 3 = 27$, and is read *three cube*.

32. The small figure above and to the right of the number is called an **exponent**; thus, in the expression 6^2 the exponent is 2, and in the expression 8^3 the exponent is 3. When a fraction is to be raised to a power, the exponent is written outside parentheses put around the fraction; for example, the square of $\frac{1}{2}$ is written $(\frac{1}{2})^2$, and the cube of $\frac{3}{4}$ is written $(\frac{3}{4})^3$. The reason for using parentheses is to separate the exponent from the numerator of the fraction. If the exponent were written beside the numerator, it might be mistaken for a part of the numerator: thus, $\frac{13^2}{32}$ might be mistaken for $\frac{132}{32}$, whereas it is intended to represent the square of $\frac{13}{32}$, or $(\frac{13}{32})^2$.

If the parenthesis is omitted, the exponent should be written after both numerator and denominator. Thus, $(\frac{13}{32})^2 = \frac{13^2}{32^2} = \frac{169}{1024} = .1650390625$; $(\frac{3}{8})^3 = \frac{3^3}{8^3} = \frac{27}{512} = .052734375$. That these results are correct is easily proved: $\frac{13}{32} = .40625$, and hence, $(\frac{13}{32})^2 = .40625^2 = .1650390625$; $\frac{3}{8} = .375$, and $(\frac{3}{8})^3 = .375^3 = .052734375$.

33. To find any power of a number, the following rule should be used:

Rule.—I. *To raise a whole number or a decimal to any power, use it as a factor as many times as is indicated by the exponent.*

II. *To raise a fraction to any power, raise both the numerator and the denominator to the power indicated by the exponent.*

EXAMPLE 1.—What is the square of 7?

SOLUTION.—The square of a number is found by using the number twice as a factor; therefore, the square of 7 is $7 \times 7 = 49$. Ans.

EXAMPLE 2.—What is the value of 13^2?

SOLUTION.—The exponent is 2, which shows that the number 13 is to be used twice as a factor; hence,

$$13^2 = 13 \times 13 = 169. \quad \text{Ans.}$$

EXAMPLE 3.—Find the value of 4.8^2.

SOLUTION.—The exponent is 2; therefore,

$$4.8^2 = 4.8 \times 4.8 = 23.04. \quad \text{Ans.}$$

EXAMPLE 4.—What is the value of $(\frac{5}{8})^2$?

SOLUTION.—The exponent is 2, so the numerator and the denominator must each be used twice as a factor. Then,

$$(\tfrac{5}{8})^2 = \frac{5 \times 5}{8 \times 8} = \tfrac{25}{64}. \quad \text{Ans.}$$

EXAMPLE 5.—Find the value of 35^3.

SOLUTION.—The exponent is 3; therefore,

$$35^3 = 35 \times 35 \times 35 = 42{,}875. \quad \text{Ans.}$$

EXAMPLE 6.—What is the value of $(\frac{7}{8})^3$?

SOLUTION.—The exponent is 3; therefore,

$$(\tfrac{7}{8})^3 = \frac{7 \times 7 \times 7}{8 \times 8 \times 8} = \tfrac{343}{512}. \quad \text{Ans.}$$

EXAMPLE 7.—What is the cube of .12?

SOLUTION.—The cube is found by using the number three times as a factor; therefore, the cube of .12 is

$$.12 \times .12 \times .12 = .001728. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

Raise the following to the powers indicated:

(a) 85^2.	Ans.	(a) 7,225
(b) $(\frac{12}{13})^2$.		(b) $\frac{144}{169}$
(c) 6.5^2.		(c) 42.25
(d) 14^3.		(d) 2,744
(e) $(\frac{3}{4})^3$.		(e) $\frac{27}{64}$
(f) $(\frac{5}{6})^3$.		(f) $\frac{125}{216}$

SHOP CALCULATIONS

(PART 4)

ENGLISH MEASURES

DEFINITIONS

1. A **measure** is a standard unit, established by law or by custom, by which the amount or quantity of a thing can be found. For example, the shop man gives the lengths of pieces of work in inches and feet; therefore, the inch and the foot are *measures of length*. In measuring the weights of things, he uses the ounce, the pound, and the ton; therefore, these are *measures of weight*. The pint, the quart, and the gallon are *measures of capacity*, used for liquids. The *measures of time* are the minute, the hour, the year, and so on.

2. In writing numbers expressing measures of various kinds, it is convenient to use *abbreviations* instead of writing out the name of the unit in full. For example, 5 *inches* may be abbreviated to 5 *in.*, the expression *in.* meaning *inch* or *inches*. Similarly, 8 *ft.* means 8 *feet*. The abbreviations commonly used for the various units are given in connection with the following tables showing the relation between measures of the same kinds.

There are two kinds of numbers used in connection with measures, namely, *simple numbers* and *compound numbers*. A **simple number** consists of units of one kind, or grade, as, for example, 5 inches. A **compound number** consists of units of two or more grades, as 2 feet 5 inches.

3. Lengths or distances are measured by the units of **linear measure.** The table that follows shows the values of the various units, and the right half gives the equivalent of each of the larger units in the smaller units.

In the shop, measurements of lengths and distances are made in feet and inches. The yard is used in measuring cloth, and the rod and the mile in measuring great distances.

LINEAR MEASURE

		Abbreviation	in.	ft.	yd.	rd.	mi.
12 inches (in.)	= 1 foot	ft.					
3 feet	= 1 yard	yd.	36 =	3 =	1		
5.5 yards	= 1 rod	rd.	198 =	16½ =	5.5 =	1	
320 rods	= 1 mile	mi.	63,360 =	5,280 =	1,760 =	320 =	1

EXAMPLE 1.—A piece of shafting is 7 feet long. What is its length in inches?

SOLUTION.—According to the table, 1 ft. = 12 in.; therefore, 7 ft. must be equal to

$$7 \times 12 = 84 \text{ in. Ans.}$$

EXAMPLE 2.—The distance between two machines is 48 inches. What is the distance in feet?

SOLUTION.—According to the table, 12 in. = 1 ft.; therefore, the number of feet in 48 in. is

$$48 \div 12 = 4 \text{ ft. Ans.}$$

4. Square measure is used to find areas, or amount of surface. For example, a flat sheet of brass represents a surface, and the amount or extent of this surface is the area. Small areas are measured in square inches or square feet, and larger areas in square yards, square rods, acres, and square miles.

A **square inch** is a square that is 1 inch long on each side. A **square foot** is a square that measures 1 foot on each side, as shown in Fig. 1. According to the table of linear measure, 1 foot is equal to 12 inches. Suppose, then, that each side of the square in Fig. 1 is divided into twelve equal parts, and that lines are drawn across the square from these points of division. The square will then appear as in Fig. 2, cut up into a number of small squares. As each side of the square was divided into twelve equal parts, each part is $\frac{1}{12}$ foot long, or $\frac{1}{12} \times 12 = 1$ inch long, and each little square therefore measures 1 inch on each side; that is, each little square is 1 square inch. Now, if the total number of little squares is counted, it will be found

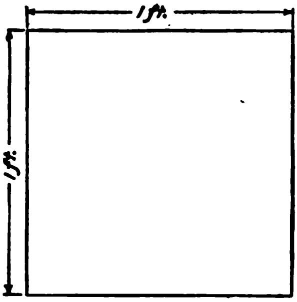


FIG. 1

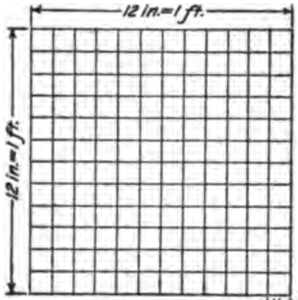


FIG. 2

to be 144. In other words, there are 144 square inches in 1 square foot.

SQUARE MEASURE

144	square inches (sq. in.)	= 1 square foot	sq. ft.
9	square feet	= 1 square yard	sq. yd.
30¼	square yards	= 1 square rod	sq. rd.
160	square rods	= 1 acre	A.
640	acres	= 1 square mile	sq. mi.

sq. mi.		A.		sq. rd.		sq. yd.		sq. ft.		sq. in.
1	=	640	=	102,400	=	3,097,600	=	27,878,400	=	4,014,489,600

5. The square shown in Fig. 2 is 12 inches long and 12 inches wide. If the length and the width are multiplied together, the product is 12×12=144, which is the same result as was found by dividing the square foot into square inches. This gives a new and a quicker way of finding the area of a square; that is, *multiply the length by the width.* But when this rule is used, the length and the width must be in the *same* units. In the case just mentioned, both the length and the width were in inches, and the area as a result is square inches. If the length and the width had been taken in feet, the area would have been 1 foot × 1 foot = 1 square foot.

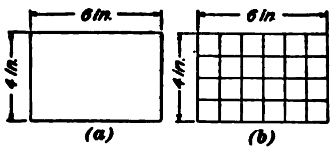


FIG. 3

Sometimes the area to be measured is a **rectangle**; that is, it is a figure like that shown in Fig. 3 (*a*), having square corners but longer than it is wide. In a case like this, the area may be found in the same way as that of a square; hence, the following rule:

Rule.—*To find the area of a square or a rectangle, multiply the length by the width, both being expressed in the same units.*

Thus, if the rectangle in Fig. 3 (*a*) is 6 inches long and 4 inches wide, its area is $6\times4=24$ square inches. This can be proved as shown in (*b*), by dividing each side into inches, drawing lines across the figure from the points of division, and counting the number of squares thus formed. Each of these squares is 1 inch on each side, or 1 square inch in area, and there are 24 in all, or 24 square inches.

EXAMPLE 1.—How many square inches are there in a rectangle 28 inches long and 13 inches wide?

SOLUTION.—According to the rule, the area is

$$28\times13=364 \text{ sq. in. Ans.}$$

EXAMPLE 2.—A sheet of brass is 4 feet long and 3 inches wide. What is the area of its surface?

SOLUTION.—Both dimensions must be expressed in the same units; therefore, change 4 ft. to inches, which gives $12\times4=48$ in. Then, applying the rule, the area is

$$48\times3=144 \text{ sq. in., or 1 sq. ft. Ans.}$$

EXAMPLE 3.—The base of a gear-cutting machine is $6\frac{1}{4}$ feet long and $2\frac{1}{2}$ feet wide. What area of floor space does it cover?

SOLUTION.—According to the rule, the area is

$$6\tfrac{1}{4}\times2\tfrac{1}{2}=\frac{25}{4}\times\frac{5}{2}=\frac{125}{8}=15\tfrac{5}{8} \text{ sq. ft. Ans.}$$

6. Cubic measure is used to measure the volumes of bodies or the amount of space occupied by them. The following table gives the units of cubic measure:

CUBIC MEASURE

1728 cubic inches (cu. in.)	= 1 cubic foot	cu. ft.
27 cubic feet	= 1 cubic yard	cu. yd.

cu. yd.		cu. ft.		cu. in.
1	=	27	=	46,656
		1	=	1,728

The cubic inch and the cubic foot are used to measure small volumes, and the cubic yard is used for large volumes.

A **cube** is a solid whose length, breadth, and thickness are equal. In Fig. 4 is shown a cube that measures 1 yard, or 3 feet, on each edge. The volume of this cube is therefore a cubic yard. Now, as 1 yard is equal to 3 feet, suppose that each edge is divided into three equal parts, and division lines are drawn across the cube, as shown. The large cube will then be cut up into a number of small cubes, each of which measures 1 foot on each edge; that is, each small cube is 1 cubic foot. There are three layers of these smaller cubes, and there are 9 cubes in each layer, making $3 \times 9 = 27$ in all. In 1 cubic yard, then, there are 27 cubic feet; or, 27 cubic feet make 1 cubic yard. In a similar way, it can be shown that a cubic foot, which is a cube measuring 12 inches on each edge, contains 1,728 cubic inches; that is, 1,728 cubic inches make 1 cubic foot.

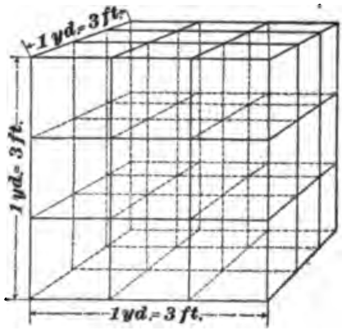


FIG. 4

7. If the length, breadth, and thickness, in feet, of the cube in Fig. 4 are multiplied together, the product is $3 \times 3 \times 3 = 27$, which is exactly equal to the number of small cubes, or cubic feet, in 1 cubic yard; therefore, the volume of a cube is found by multiplying the length, breadth, and thickness together. The same thing is true of a box-shaped figure like that shown in Fig. 5, called a **parallelopipedon;** hence the following rule:

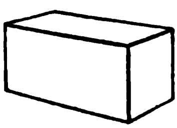


FIG. 5

Rule.—*To find the volume of a cube or a box, multiply together the length, breadth, and thickness, all expressed in the same units.*

If the dimensions are expressed in inches, the volume is given in cubic inches; if in feet, the volume is given in cubic feet; and so on.

EXAMPLE 1.—A sand bin is 14 feet long, 8 feet wide, and 6 feet high. How many cubic feet does it contain?

SOLUTION.—According to the rule, it contains

$$14 \times 8 \times 6 = 672 \text{ cu. ft.} \quad \text{Ans.}$$

EXAMPLE 2.—How many cubic inches of metal are there in a block $12\frac{1}{2}$ inches long, $8\frac{1}{2}$ inches wide, and 4 inches thick?

SOLUTION.—Applying the rule, the volume is

$$12\tfrac{1}{2} \times 8\tfrac{1}{2} \times 4 = \frac{25}{2} \times \frac{17}{2} \times \frac{4}{1} = 425 \text{ cu. in.} \quad \text{Ans.}$$

8. The **measures of weight** are given in the following table:

AVOIRDUPOIS WEIGHT

16 ounces (oz.)	=1 pound	lb.
100 pounds	=1 hundredweight	cwt.
20 cwt., or 2,000 lb.	=1 ton	T.

T.		cwt.		lb.		oz.
1	=	20	=	2,000	=	32,000

Small weights are usually expressed in pounds and ounces, and larger ones in hundredweights and tons. The ton of 2,000 pounds is called the *short ton*. There is another kind of ton, called the *long ton;* it contains 2,240 pounds, and is used in weighing coal at the mines, and other wholesale work. Unless the long ton is particularly stated, the short ton of 2,000 pounds will always be meant when the word *ton* is used in the following pages.

9. Liquid measure is used for measuring liquids. The standard gallon in the United States contains 231 cubic inches. The standard barrel, as used in measuring capacity, contains $31\frac{1}{2}$ gallons, but the barrels ordinarily used vary greatly in size, and their capacities or volumes must be found by measurement or by gauging.

LIQUID MEASURE

4 gills (gi.)	=1 pint	pt.
2 pints	=1 quart	qt.
4 quarts	=1 gallon	gal.
$31\frac{1}{2}$ gallons	=1 barrel	bbl.

bbl.		gal.		qt.		pt.		gi.
1	=	$31\frac{1}{2}$	=	126	=	252	=	1,008

10. Dry articles, such as fruit, grain, vegetables, etc. are measured by **dry measure.** The standard unit in the United States is the *Winchester bushel*, which contains 2,150.42 cubic inches. The quart in dry measure is not the same as the quart in liquid measure. The dry quart contains 67.2 cubic inches and the liquid quart only 57¾ cubic inches. In Canada the standard is the *imperial bushel*, which contains 2,219.7 cubic inches, and the same measure is used for liquids and solids.

DRY MEASURE

2 pints (pt.)	= 1 quart	qt.
8 quarts	= 1 peck	pk.
4 pecks	= 1 bushel	bu.

bu.		pk.		qt.		pt.
1	=	4	=	32	=	64

MEASURE OF TIME

60 seconds (sec.)	= 1 minute	min.
60 minutes	= 1 hour	hr.
24 hours	= 1 day	da.
7 days	= 1 week	wk.
365 days 12 months	= 1 common year	yr.
366 days	= 1 leap year.	
100 years	= 1 century.	

NOTE.—It is customary to consider 1 month as 30 days.

MISCELLANEOUS TABLE

12 things are 1 dozen.	1 hand is 4 inches.
12 dozen are 1 gross.	1 palm is 3 inches.
12 gross are 1 great gross.	1 span is 9 inches.
2 things are 1 pair.	24 sheets are 1 quire.
20 things are 1 score.	20 quires, or 480 sheets, are 1 ream.

1 U. S. standard gallon of water weighs 8.355 pounds, nearly.
1 cubic foot of water contains 7.481 U. S. standard gallons, nearly.
1 cubic foot of water weighs 62½ pounds, nearly.
1 imperial gallon (British) contains 277.463 cubic inches.
5 imperial gallons are very nearly equal to 6 U. S. standard gallons.
1 imperial gallon of distilled water at 62° F. weighs 10 pounds.

REDUCTION OF COMPOUND NUMBERS

11. The **reduction of compound numbers** consists in changing their class or denomination, without changing their value. For example, suppose that the compound number 1 foot 6 inches is to be changed to inches. As 1 foot is 12 inches, 1 foot 6 inches must be 12 inches+6 inches = 18 inches. Therefore, 1 foot 6 inches, when reduced to inches, is 18 inches. In this case, the *higher* denomination, or the foot, is changed to the *lower* denomination, or the inch.

12. In changing a number from *higher to lower denomination*, the following rule is used:

Rule.—*To change a compound number to a lower denomination, multiply the number representing the higher denomination by the number of units in the next lower denomination required to make one unit of the higher denomination, and to the product add the number of units of the lower denomination.*

EXAMPLE 1.—How many inches in 3 feet $6\frac{1}{2}$ inches?

SOLUTION.—The number 3 is the higher denomination. According to the rule, this must be multiplied by 12, because it takes 12 of the lower units, or inches, to make one unit of the higher denomination, or 1 foot. Multiplying, $3\times12=36$ in. To this product is now added the $6\frac{1}{2}$ in. in the given number, making a total of $36+6\frac{1}{2}=42\frac{1}{2}$ in. Therefore, 3 ft. $6\frac{1}{2}$ in. $=42\frac{1}{2}$ in. Ans.

EXAMPLE 2.—Reduce 6 tons 268 pounds to pounds.

SOLUTION.—One ton contains 2,000 lb. Then, by the rule, $6\times2{,}000=12{,}000$ lb., and $12{,}000+268=12{,}268$ lb. Ans.

EXAMPLE 3.—Change 5 hours 24 minutes to minutes.

SOLUTION.—There are 60 min. in 1 hr. Therefore, according to the rule, $5\times60=300$ min., and $300+24=324$ min. Ans.

EXAMPLE 4.—How many square inches in a surface whose area is 3 square feet 28 square inches?

SOLUTION.—In 1 sq. ft. there are 144 sq. in. Then, by the rule, $3\times144=432$ sq. in., and $432+28=460$ sq. in. Ans.

13. In changing a number from *lower to higher denomination*, the following rule is used:

Rule.—*To change a number to a higher denomination, divide the number representing the denomination given by the number of units of this denomination required to make one unit of the higher denomination. The remainder will be of the same denomination, but the quotient will be of the higher. The quotient and the remainder together are the required result.*

EXAMPLE 1.—How many weeks in 18 days?

SOLUTION.—It takes 7 da. to make 1 wk.; that is, 7 units of the lower denomination to make 1 unit of the higher. By the rule, 18 is to be divided by 7 to get the result. Now, 7 is contained in 18 twice, or 2 times, with a remainder of 4. The quotient 2 is of the higher denomination, and is therefore 2 wk. The remainder 4 is of the lower denomination or 4 da. The result is therefore 2 wk. 4 da.; that is, 18 da. is equal to 2 wk. 4 da. Ans.

EXAMPLE 2.—Reduce 14,728 pounds to tons.

SOLUTION.—One ton is equal to 2,000 lb. Then, according to the rule, 14,728 is to be divided by 2,000; thus,

$$\begin{array}{r} 2000)\,14728\,(7\text{ T.} \\ \underline{14000} \\ 728\text{ lb.} \end{array}$$

Therefore, 14,728 lb. = 7 T. 728 lb. Ans.

ADDITION OF COMPOUND NUMBERS

14. Compound numbers formed of like units may be added. The rule to be used is as follows:

Rule.—*Place the numbers so that like denominations are in the same column. Begin at the right-hand column, and add. Divide the sum by the number of units of this denomination required to make one unit of the higher denomination. Place the remainder under the column added, and carry the quotient to the next column.*

EXAMPLE 1.—If a bar is broken into four pieces measuring 1 foot $3\frac{1}{2}$ inches, 2 feet $7\frac{3}{4}$ inches, 1 foot $11\frac{1}{8}$ inches, and 3 feet $8\frac{7}{8}$ inches, what was its whole length?

SOLUTION.—

ft.	in.	
1	$3\frac{1}{2}$	
2	$7\frac{3}{4}$	
1	$11\frac{1}{8}$	
3	$8\frac{7}{8}$	
9	$7\frac{1}{4}$	Ans.

EXPLANATION.—The sum of the numbers in the right-hand column is 31½ in., or 2 ft. 7½ in. The 7½ in. is put down and the 2 ft. is added to the numbers in the other column, giving 9 ft. The whole length of the bar, therefore, was 9 ft. 7½ in.

EXAMPLE 2.—A workman spends 1 hour 40 minutes on one piece of work, 2 hours 35 minutes on another, and 3 hours 10 minutes on a third. What is his time on all three?

SOLUTION.—

hr.	min.	
1	40	
2	35	
3	10	
7	25	Ans.

EXPLANATION.—The sum of the numbers in the right-hand column is 85 min., which is equal to 1 hr. 25 min. The 25 min. is set down and the 1 hr. is carried over and added to the numbers in the other column, making 7 hr. Therefore, the total time is 7 hr. 25 min.

EXAMPLES FOR PRACTICE

1. Find the sum of 7 feet 6 inches, 3 feet 7½ inches, 1 foot 9¾ inches, and 2 feet 2 inches. Ans. 15 ft. 1¼ in.

2. Find the total weight of three lots of pig iron that weigh 2 tons 765 pounds, 4 tons 1,240 pounds, and 1 ton 800 pounds. Ans. 8 T. 805 lb.

3. A man works 2 hours 45 minutes on one job, 1 hour 5 minutes on another, and 3 hours 25 minutes on another. What is the total time worked? Ans. 7 hr. 15 min.

4. Along the end of a machine shop, as shown by the diagram, Fig. 6, four machines *a*, *b*, *c*, and *d* are set in line. The distances between the

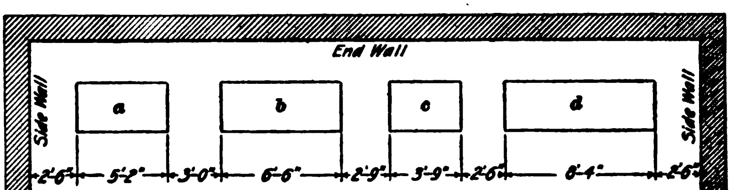


FIG. 6

machines and between the end machines and the wall, and the lengths of the machines are as marked. What is the distance from one side wall to the other? Ans. 37 ft.

SUBTRACTION OF COMPOUND NUMBERS

15. In the subtraction of compound numbers the following rule should be used:

Rule.—*Place the smaller quantity under the larger quantity, with like denominations in the same column. Beginning at the right, subtract the number in the subtrahend in each denomination from the one above, and place the differences underneath. If the number in the minuend of any denomination is less than the number under it in the subtrahend, one must be taken from the minuend of the next higher denomination, reduced, and added to it.*

EXAMPLE 1.—A pile of scrap iron containing 3 tons 728 pounds has 1 ton 566 pounds taken away. How much remains?

SOLUTION.—Setting down the numbers as told in the rule, and subtracting, the remainder is found; thus,

T.	lb.
3	728
1	566
2	162

That is, 2 T. 162 lb. remain. Ans.

EXAMPLE 2.—A bar 4 feet 6 inches long has a piece 1 foot 8 inches long broken off. What length remains?

SOLUTION.—The remainder is found thus:

ft.	in.	
4	6	
1	8	
2	10	Ans.

EXPLANATION.—It is impossible to take 8 from 6, because 8 is greater than 6. So we take 1 ft. from the 4 ft. and add it to the 6 in. Now, 1 ft. = 12 in., and so 6+12 = 18 in. The 1 ft. taken away from the 4 ft. leaves 3 ft. The problem can then be written as follows:

ft.	in.
3	18
1	8
2	10

In this form, 8 can be taken from 18, leaving 10, and 1 from 3 leaves 2. The remainder is therefore 2 ft. 10 in. The operation of taking 1 ft. away from the 4 ft., reducing it to inches, and adding it to the 6 in. would usually be done mentally, and would not be written down as shown.

EXAMPLES FOR PRACTICE

1. From a bar 10 feet 8 inches long two pieces are broken off, one measuring 2 feet 6 inches and the other 3 feet 8 inches. What length of the bar remains? Ans. 4 ft. 6 in.

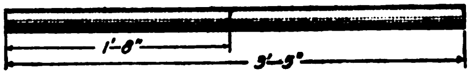


FIG. 7

2. A rod 3 feet 5 inches long, as shown in Fig. 7, is marked to be cut into two parts, the shorter of which is 1 foot 8 inches. What is the length of the longer part? Ans. 1 ft. 9 in.

MULTIPLICATION OF COMPOUND NUMBERS

16. Compound numbers may be multiplied by other numbers. The rule to be used is as follows:

Rule.—*Multiply the number representing the lower denomination by the given multiplier and reduce the product to the higher denomination. Write the remainder under the lower denomination, and add the quotient to the product obtained by multiplying the higher denomination by the multiplier.*

EXAMPLE.—What length of stock is needed to make 16 bolts, if each bolt requires a length of 1 foot $3\frac{1}{8}$ inches?

SOLUTION.—The stock required must be 16 times the length required for one bolt.

ft.	in.	
1	$3\frac{1}{8}$	
	16	
20	2	Ans.

EXPLANATION.—First, $3\frac{1}{8}$ in. is multiplied by 16; thus, $3\frac{1}{8}\times 16 = \frac{25}{8}\times 16 = \frac{400}{8} = 50$ in. But, 50 in. = 4 ft. 2 in. The remainder 2 in. is put down and the 4 ft. is carried over to add to the next product. Multiplying 1 ft. by 16, we get 16 ft., and adding the 4 ft. carried over, 16+4 = 20 ft. The length of stock must therefore be 20 ft. 2 in.

EXAMPLES FOR PRACTICE

1. Twelve pieces each 1 foot $3\frac{1}{2}$ inches long are sawed from a bar of iron. What is the total length sawed off, neglecting the width of the saw cut? Ans. 15 ft. 6 in.

2. If 18 boxes each 2 feet $4\frac{1}{2}$ inches wide are set side by side, what will be the total width? Ans. 42 ft. 9 in.

3. Five loads of pig iron, each consisting of 4 tons 650 pounds, are purchased. What is the total amount of iron purchased? Ans. 21 T. 1,250 lb.

DIVISION OF COMPOUND NUMBERS

17. The rule to be used in dividing a compound number by another number is as follows:

Rule.—*Find how many times the divisor is contained in the first or higher denomination of the dividend; reduce the remainder, if any, to the lower denomination, and add to it the number in the given dividend expressing that denomination; divide this new dividend by the divisor, and the quotient will be the next denomination in the quotient required. Or, reduce the dividend to units of the lower denomination and divide by the given divisor.*

EXAMPLE.—A piece of bar iron 20 feet 2 inches long is used in making 16 bolts, all of the same length. What is the length used for each bolt?

FIRST SOLUTION.—

```
      ft. in.
16 ) 20  2  ( 1 ft. 3 1/8 in.  Ans.
     16
     --
      4 ft. rem.
     12
     --
     48 in.
      2 in.
     --
16 ) 50 in. ( 3 1/8 in.
     48
     --
     2/16 = 1/8
```

EXPLANATION.—The total length of the bar, or 20 ft. 2 in., is divided by 16. First, 16 is contained once in 20 ft., with 4 ft. as a remainder. This 4 ft. is now reduced to inches by multiplying it by 12, which gives 48 in. Adding the 2 in. in the original number, we have 50 in. Then, $50 \div 16 = 3\frac{2}{16} = 3\frac{1}{8}$ in. Thus, the amount used for each bolt is 1 ft. $3\frac{1}{8}$ in.

SECOND SOLUTION.—Another way to solve this example is to reduce the compound number to its lowest units and then divide by the given divisor. For example, 20 ft. 2 in. is equal to $(12\times20)+2=240+2=242$ in. Then, $242\div16=15\frac{1}{8}$ in., or 1 ft. $3\frac{1}{8}$ in. Ans.

EXAMPLES FOR PRACTICE

1. A bar 5 feet 3 inches long is divided into four equal parts. What is the length of each? Ans. 1 ft. $3\frac{3}{4}$ in.

2. If it takes a workman 23 hours 50 minutes to make 11 pieces of work, how long will it take him to make one of these pieces? Ans. 2 hr. 10 min.

MEASUREMENT OF TEMPERATURE

18. A **thermometer** is an instrument used to measure temperature, or to tell how hot or how cold a thing is. It consists of a glass tube closed at the upper end and having at the lower end a bulb filled with mercury. When the thermometer is placed in contact with a hot body, the heat causes the mercury to expand, or take up more space, and so it rises in the hollow tube. If the body is cold, the mercury contracts, and occupies less space than before, and the column then descends or grows shorter. The greater the temperature, the higher the mercury rises in the tube; and the lower the temperature, the lower the mercury descends in the tube.

19. Thermometric Scales.—The degree of hotness or coldness of a body, that is, its *temperature*, is indicated by the height to which the mercury rises or falls on a scale to which the mercury tube is attached. A thermometer with two scales, one on each side of the tube, is shown in Fig. 8. The one on the left, with F at the top, is the **Fahrenheit scale**, and the one at the right, with C above it, is the **centigrade scale**. The Fahrenheit thermometer, named after its inventor, is the one most commonly used in the United States. The centigrade thermometer is used largely in scientific work. The difference between the two is that the boiling point of water is marked 212 on the Fahrenheit scale and 100 on the centigrade; and the freezing point is 32 on the Fahrenheit scale and 0 on the centigrade. Each small division on each scale is a *degree* of temperature. The abbreviations for Fahrenheit and centigrade are F. and C., respectively; thus, 180° F. means 180 degrees on the Fahrenheit scale, and 65° C. means 65 degrees on the centigrade scale.

20. In making his thermometer, Fahrenheit put the bulb in melting ice, and marked the height of the mercury as the freezing point; similarly, the boiling point was found by putting the bulb in boiling water. The distance between these two points he divided into 180 equal parts each called a degree. Then he laid off a number of these equal divisions below the freezing point. In his experiments, the lowest temperature he could get corresponded to the thirty-second division below the freezing point, or 32° below freezing; and as he thought this was the coldest that could be obtained, he marked it zero. That is why the freezing point is 32 and the boiling point 212 on the Fahrenheit thermometer. On many Fahrenheit thermometers, the space between the freezing point and the boiling point is divided into 90 equal parts, so that the division marks will not be so close together. In such a case, each division corresponds to 2 degrees.

FIG. 8

21. Conversion of Thermometric Readings.—It is oftentimes necessary to find the equivalent of Fahrenheit temperature in centigrade degrees, or vice versa. This may be done by the use of the following rules:

Rule I.—*To find the Fahrenheit temperature, multiply the centigrade temperature by $\frac{9}{5}$ and add 32 to the product.*

Rule II.—*To find the centigrade temperature, subtract 32 from the Fahrenheit temperature and multiply the remainder by $\frac{5}{9}$.*

EXAMPLE 1.—What is the equivalent of 85° C. on the Fahrenheit scale?

SOLUTION.—Applying rule I. the temperature is

$$(85\times\tfrac{9}{5})+32=185° \text{ F. Ans.}$$

Example 2.—If a liquid has a temperature of 167° F., what is its centigrade temperature?

Solution.—Applying rule II, the temperature is

$$(167-32)\times\frac{5}{9}=75° \text{ C. Ans.}$$

METRIC MEASURES

GENERAL REMARKS

22. The **metric system** of measures is in general use in Europe and is used to some extent in the United States. The unit of length is the *meter*, which has a length equal to 39.37 inches. The unit of capacity is the *liter*, pronounced *lee'ter*, and the unit of weight is the *gram*.

In the United States, the man employed in the shop is likely to meet with metric units in constructing machinery that has been designed abroad and is being built to order in this country. The units used on the drawings are usually the *centimeter* and the *millimeter*.

Although the metric system has units of length, area, volume, capacity, and weight, the complete tables will not be given. Instead, only those units that are likely to be used by the shop man will be explained.

23. The lengths of the **millimeter** and the **centimeter** can be seen in Fig. 9, which shows 10 centimeters, each divided

Fig. 9

into 10 equal parts called millimeters; in other words, 10 millimeters is equal to 1 centimeter. It requires 100 centimeters, or 10 times the length shown in the illustration, to make 1 meter. A millimeter is equal to .03937 inch and a centimeter is equal to .3937 inch; that is, 1 inch is equal to $\frac{1}{.03937}$ $=25.4$ millimeters, or 2.54 centimeters. The **kilometer** is

1,000 meters and is used in measuring great distances. It is equal to about $\frac{5}{8}$ mile.

10 millimeters (mm.)	= 1 centimeter	cm.
100 centimeters	= 1 meter	m.
1 millimeter	= .03937 inch	
1 inch	= 25.4 millimeters	

24. Volumes are measured in **cubic centimeters**, a cubic centimeter being the volume of a cube measuring 1 centimeter on each edge. A cubic centimeter is equal to .06102 cubic inch.

The **liter** is used in measuring capacity, either dry or liquid. It is the volume of 1,000 cubic centimeters, or the volume of a cube measuring 10 centimeters on each edge. It is very nearly equal to a United States liquid quart. One cubic inch is equal to 16.388 cubic centimeters.

1 cubic centimeter (cc.)	= .06102 cubic inch
1 cubic inch	= 16.388 cubic centimeters

25. The metric unit of weight is the **gram**, which is equal to the weight of 1 cubic centimeter of pure distilled water at a temperature of 39.1° F. The gram is equal to .03527 ounce, avoirdupois, and is used in weighing small weights. For greater weights the **kilogram** is used. This is 1,000 grams, and is equal to 2.2046 pounds avoirdupois. One pound is equal to .4536 kilogram.

1 gram (g.)	= .03527 ounce	
1,000 grams	= 1 kilogram	Kg.
1 kilogram	= 2.2046 pounds	
1 pound	= .4536 kilogram	

OPERATIONS INVOLVING METRIC UNITS

26. Numbers expressed in metric units may be added, subtracted, multiplied, and divided. The rules given in connection with these operations as applied to numbers expressed in English units of measurement can also be used with metric numbers. Only like units can be added or subtracted; that is, meters must be added to or subtracted from meters, millimeters must be added to or subtracted from millimeters, and so on.

27. Conversion of Metric Units to English Units. In changing metric units to English units, the general rule to follow is to multiply the number of metric units by the equivalent of that unit in the desired English units. The method can most easily be illustrated by examples, as follows:

EXAMPLE 1.—A machine part has a diameter of 115 millimeters. What is its diameter in inches?

SOLUTION.—The equivalent of 1 mm. is .03937 in. Therefore, the equivalent of 115 mm. is

$$115 \times .03937 = 4.52755 \text{ in., or } 4\tfrac{17}{32} \text{ in., nearly. Ans.}$$

EXAMPLE 2.—If a vessel has a volume of 248 cubic centimeters, what is its volume in cubic inches?

SOLUTION.—The equivalent of 1 cc. is .06102 cu. in. The volume of the vessel in cubic inches, therefore, is

$$248 \times .06102 = 15.13 \text{ cu. in. Ans.}$$

28. Conversion of English Units to Metric Units. In order to change English units to metric units, divide the number of English units by the equivalent of the desired metric unit in English units; or, as multiplication is simpler than division, multiply the number of English units by the number of metric units equivalent to the English unit.

EXAMPLE 1.—If a bolt is $6\frac{3}{8}$ inches long, what is its length in millimeters?

SOLUTION.—According to Art. **23**, 1 in. contains 25.4 mm. Therefore,

$$6\tfrac{3}{8} \times 25.4 = 162 \text{ mm., very nearly. Ans.}$$

EXAMPLE 2.—If a casting weighs 280 pounds, what is its weight in kilograms?

SOLUTION.—By Art. **25**, 1 lb. = .4536 Kg.; hence,

$$280 \times .4536 = 127 \text{ Kg. Ans.}$$

EXAMPLES FOR PRACTICE

1. A shaft has a diameter of $3\frac{1}{4}$ inches. What is its diameter in millimeters? Ans. 82.55 mm.

2. Find the length in inches of a piece that is 280 millimeters long. Ans. 11 in.

3. A box containing $12\frac{1}{2}$ cubic inches contains how many cubic centimeters? Ans. 205 cc., nearly

4. A tool weighs 12 kilograms. What is its weight in pounds? Ans. $26\frac{1}{2}$ lb., nearly

RATIO AND PROPORTION

RATIO

29. A **ratio** is simply a comparison of the values of two numbers. Suppose that the two numbers to be compared are 20 and 4. The value of 20 is 5 times the value of 4, because $20=5\times4$; that is, 20 is 5 times as large as 4. On the other hand, if 4 is compared with 20, it is said that 4 is $\frac{1}{5}$ of 20 because 4 is $\frac{1}{5}$ as large as 20. This comparing of one number with another is called *finding the ratio* of the numbers. The two numbers compared must always be of the same kind. For example, 20 inches could properly be compared with 4 inches, or 4 pounds with 20 pounds; but two unlike numbers, such as 20 inches and 4 pounds, could not be compared. To find a ratio, therefore, the numbers must be of the same kind.

30. A ratio may be written in two different ways, both of which are correct. Thus, the ratio of 20 to 4, or the value of 20 compared to the value of 4, may be written $20:4$ or $\frac{20}{4}$. Each of these expressions is read *the ratio of 20 to 4*. The ratio of 4 to 20 would be written either $4:20$ or $\frac{4}{20}$. The method most commonly used in writing ratios is the first one shown; that is, the two numbers are separated by a colon(:), which has the same meaning as $\div$, the sign of division. Hence, $20:4$ is equal to $20\div4=5$. In calculations, ratios are frequently written in the form of fractions; thus, the ratio of 20 to 4 may be written $\frac{20}{4}$.

31. The **value** of a ratio is the result obtained by dividing the first number of the ratio by the second; thus, the value of the ratio $20:4$ is 5, because $20\div4=5$.

If both numbers of a ratio, called the **terms** of the ratio, are multiplied or divided by the same number, the value of the ratio is not changed. For example, suppose that both terms of the ratio 20 : 4 are multiplied by 5. The ratio then becomes $20\times5 : 4\times5=100 : 20$. But this has the same value as the ratio 20 : 4, because $100\div20=5$, just as $20\div4=5$. Again, suppose that both terms of the ratio 20 : 4 are divided by 4. The ratio then becomes $\frac{20}{4} : \frac{4}{4}=5 : 1$. But the value of the ratio 5 : 1 is $5\div1=5$, which is the same as the value of the ratio 20 : 4.

A ratio is **reduced to its lowest terms** by dividing both terms by the same number and repeating the operation until no number except 1 can be found that will divide both terms without a remainder; thus, the ratio 84 : 48, when reduced to its lowest terms, becomes 7 : 4, which is obtained by dividing each number by 12. The operation may be written $\frac{84}{48}\div\frac{12}{12}=\frac{7}{4}$, which is the ratio 7 : 4.

32. A ratio may be **inverted** by simply changing the positions of its terms; thus, the ratio 20 : 4, when inverted, becomes 4 : 20. The ratio 4 : 20 is then called the **inverse ratio** of 20 : 4. The value of an inverse ratio is the reciprocal of the value of the original ratio, or **direct ratio,** as it is called. For example, the direct ratio 20 : 4 has a value of 5; but the inverse ratio 4 : 20 equals $4\div20=\frac{4}{20}=\frac{1}{5}$, which is the reciprocal of 5.

EXAMPLE 1.—A pair of gears contain 60 and 35 teeth, respectively. (*a*) What is the ratio of the number of teeth in the larger to the number of teeth in the smaller? (*b*) What is the ratio of the smaller number to the larger?

SOLUTION.—(*a*) The ratio of the larger number of teeth to the smaller is the ratio of 60 to 35, or 60 : 35, and reducing this to lowest terms by dividing both terms by 5, we get 12 : 7. Ans.

(*b*) The ratio of the smaller number to the larger is 35 : 60, which, reduced to its lowest terms, is 7 : 12. Ans.

EXAMPLE 2.—A lathe spindle makes 32 turns while the lead screw makes 24. What is the value of the ratio of the spindle speed to the lead-screw speed?

SOLUTION.—The ratio of the spindle speed to the screw speed is 32 : 24 which, expressed in the fractional form, is $\frac{32}{24}$. The value of the ratio is therefore $\frac{32}{24}=\frac{4}{3}=1\frac{1}{3}$. Ans.

EXAMPLE 3.—Two pulleys connected by a belt are 12 and 20 inches in diameter, respectively. (*a*) What is the ratio of the larger diameter to the smaller? (*b*) What is the ratio of the speed of the larger to that of the smaller, if it is the inverse ratio of the diameters?

SOLUTION.—(*a*) The ratio of the larger diameter to the smaller is 20 : 12, or 5 : 3. Ans.

(*b*) The speed ratio between the larger and the smaller is the inverse of the ratio 5 : 3, the ratio of the diameters; therefore, it must be 3 : 5. Ans.

EXAMPLES FOR PRACTICE

1. What is the ratio of 126 to 18, reduced to lowest terms? Ans. 7 : 1

2. Two gears have 39 and 54 teeth, respectively. What is the value of the ratio of the larger number to the smaller? Ans. $1\frac{5}{13}$

3. What is the value of the ratio of 6.25 to .75? Ans. $8\frac{1}{3}$

4. One pulley is 24 inches in diameter and another is 60 inches in diameter. What is the inverse ratio of the diameter of the smaller pulley to that of the larger? Ans. 5 : 2

PROPORTION

33. A **proportion** is simply an equality of two ratios. Two ratios that have the same value are required to form a proportion, and the proportion is written by placing an equality sign (=) or a double colon (: :) between the two ratios. For example, the ratios 8 : 6 and 12 : 9 are of the same value, and the proportion formed by them is 8 : 6 = 12 : 9, or 8 : 6 : : 12 : 9. The equality sign is used more frequently than the double colon, so the proportions in this and other Sections will be written with the equality sign. The proportion 8 : 6 = 12 : 9 is read *8 is to 6 as 12 is to 9*, or *the ratio of 8 to 6 is equal to the ratio of 12 to 9*. This same proportion can also be written $\frac{8}{6}=\frac{12}{9}$, each of the two ratios being given as a fraction.

34. Proportion is used in stating the relative sizes, weights, or dimensions of bodies. For example, if a piece of shafting 4 feet long weighs 60 pounds, another piece of the same diameter but 8 feet long will weigh 120 pounds, because, by doubling the length, the weight also is doubled. Expressed as a proportion this would be $4 : 8 = 60 : 120$. In the same way, if a tree 64 feet high throws a shadow 20 feet long, another tree only 32 feet high will throw a shadow 10 feet long at the same time; that is, the shadows bear the same ratio to each other that the heights of the trees bear to each other. Written as a proportion, it becomes $20 : 10 = 64 : 32$.

A proportion consists of four numbers, which are called the **terms** of the proportion. The first and last terms are called the **extremes,** and the second and third terms are called the **means.** Thus, in the proportion $25 : 10 = 40 : 16$, the extremes are 25 and 16 and the means are 10 and 40.

35. In any proportion, *the product of the extremes is equal to the product of the means.* For example, in the proportion $17 : 51 = 14 : 42$, the extremes are 17 and 42 and the means are 51 and 14; also, $17 \times 42 = 51 \times 14$, because $17 \times 42 = 714$ and $51 \times 14 = 714$. This important fact makes it possible to find one unknown term of a proportion when the three others are known.

36. If one mean and the two extremes of a proportion are known, the other mean may be found by the following rule:

Rule I.—*Either mean is equal to the product of the extremes divided by the other mean.*

If one extreme and both means are known, the other extreme may be found by the following rule:

Rule II.—*Either extreme is equal to the product of the means divided by the other extreme.*

When stating a proportion in which one of the terms is unknown, represent the missing term by a letter, as x. Thus, if a proportion is written $x : 51 = 14 : 42$, it is meant that the first term or extreme is not known, but that it bears the same ratio to 51 that 14 does to 42.

EXAMPLE 1.—The weights of two pine patterns are in the ratio of 3 to 4, and the iron casting made from the second weighs 36 pounds. What is the weight of an iron casting made from the first?

SOLUTION.—Call the weight of the casting made from the first pattern x, because it is not known. Then, x bears the same ratio to 36 that 3 bears to 4. Forming a proportion, $x : 36 = 3 : 4$. In this case, one of the extremes is not known. Hence, applying rule II,

$$x = \frac{36 \times 3}{4} = \frac{108}{4} = 27$$

That is, the casting made from the first pattern weighs 27 lb. Ans.

PROOF.—As x has been found to be 27 lb., the proportion can now be written $27 : 36 = 3 : 4$, which can easily be proven to be correct because the product of the extremes, or $27 \times 4 = 108$, is equal to the product of the means, or $36 \times 3 = 108$.

EXAMPLE 2.—The number of teeth in a gear is 36, which bears a ratio of 2 to 5 as compared with the number of teeth in a second gear. How many teeth are there in the second gear?

SOLUTION.—Call the unknown number of teeth x; then, $36 : x = 2 : 5$. Now, applying rule I,

$$x = \frac{36 \times 5}{2} = \frac{180}{2} = 90$$

That is, the second gear has 90 teeth. Ans.

EXAMPLE 3.—If a piece of shafting 18 feet long weighs 216 pounds, what will 7 feet of the shafting weigh?

SOLUTION.—Let x represent the unknown weight of the 7-ft. piece of shafting. Now, the unknown weight x bears the same ratio to the weight of the entire shaft as the length of the piece bears to the entire length of shafting; that is, $x : 216 = 7 : 18$. Then, applying rule II, the value of x is

$$x = \frac{216 \times 7}{18} = \frac{1{,}512}{18} = 84 \text{ lb. Ans.}$$

37. Relative Speeds of Pulleys and Gears.—If two pulleys are connected by a belt, or if two gears are in mesh, the one that gives motion is called the **driver** and the one that receives motion from it is called the **driven**, or the *follower*. The smaller of the two will always run at the greater speed. If the driver is larger in diameter than the driven, the driven pulley will run faster than the driver; but if the driver is the smaller of the two, it will run faster than the driven. The product of the diameter of the driver and its number of revolutions per minute is equal to the product of the diameter of the driven and its number of revolutions per minute, in the case of pulleys; and in the case of gears, the product of the number of teeth of the driver and its number of revolutions per minute is equal to the product of the number of teeth of the driven and its number of revolutions per minute. From this it is easy to find the size of one pulley when its speed and the speed and size of the other pulley are known, or to find the speed of one pulley when its size and the speed and size of the other pulley are known. The rule to be used is as follows:

Rule.—*To find the diameter* (*or number of revolutions per minute*) *of one pulley, multiply together the diameter and the number of revolutions per minute of the other pulley and divide the product by the number of revolutions per minute* (*or the diameter*) *of the first pulley.*

This rule may be used to find the number of teeth or the number of revolutions per minute of one of two gears, by using the number of teeth in each gear instead of the diameter of each pulley. The speeds of wheels, pulleys, and gears are usually expressed in *revolutions per minute*, which is abbreviated R. P. M.

EXAMPLE 1.—Two pulleys are 36 inches and 12 inches in diameter, respectively, and the larger makes 120 revolutions per minute. How many revolutions per minute does the smaller make?

SOLUTION.—The 36-in. pulley makes 120 R. P. M. Applying the rule, therefore, the speed of the smaller pulley is

$$\frac{36\times120}{12}=360 \text{ R. P. M. Ans.}$$

EXAMPLE 2.—Two gears have speeds of 96 and 288 revolutions per minute, respectively, and the smaller has 24 teeth. How many teeth has the larger?

SOLUTION.—By the statements just made, the smaller gear must run the faster; therefore, the 24-tooth gear makes 288 R. P. M. Applying the rule, but using the number of teeth instead of the diameter, the larger gear is found to have

$$\frac{24\times288}{96}=72 \text{ teeth. Ans.}$$

EXAMPLE 3.—A pulley 54 inches in diameter runs at a speed of 100 revolutions per minute and drives another pulley at a speed of 450 revolutions per minute. What is the diameter of the other pulley?

SOLUTION.—Applying the rule, its diameter is

$$\frac{54\times100}{450}=12 \text{ in.}\quad \text{Ans.}$$

EXAMPLE 4.—A gear with 35 teeth runs at a speed of 144 revolutions per minute and meshes with a gear having 56 teeth. What is the speed of the latter?

SOLUTION.—Applying the rule and using numbers of teeth instead of pulley diameters, the speed of the larger gear is

$$\frac{35\times144}{56}=90 \text{ R. P. M.}\quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. The numbers of teeth in two gears are in the ratio of 4 to 7 and the smaller gear has 28 teeth. Find the number of teeth in the larger.
Ans. 49

2. Two castings have weights in the ratio of 3 to 2 and the heavier one weighs 3,750 pounds. What is the weight of the lighter?
Ans. 2,500 lb.

3. The numbers of teeth in two gears are 15 and 36, respectively. If the large gear makes 180 revolutions, how many does the small gear make in the same time? Ans. 432

4. The diameters of two pulleys connected by a belt are, respectively, 12 and 45 inches, and the smaller makes 270 revolutions per minute. Find the number of revolutions per minute of the larger pulley.
Ans. 72 R. P. M.

5. A 9-inch pulley is driven at a speed of 312 revolutions per minute by a 24-inch pulley. What is the speed of the driver? Ans. 117 R. P. M.

6. A 28-tooth gear running at 144 revolutions per minute drives another at a speed of 96 revolutions per minute. How many teeth are there in the driven gear? Ans. 42

SHOP CALCULATIONS
(PART 5)

LINES AND ANGLES

LINES

1. In laying out his work, the mechanic must make frequent use of lines and angles. Methods of measuring lines and angles and some of the relations between them, with practical applications, will now be described.

FIG. 1

2. A **straight line,** Fig. 1, is one that does not change its direction throughout its whole length. A straight line is frequently called a *right line.*

FIG. 2

3. A **curved line,** Fig. 2, changes its direction at every point.

FIG. 3

4. **Parallel lines,** Fig. 3, are right lines which are equally distant from each other throughout their whole length. When every point of a line is the same distance from another line it is said to be *parallel to the line.*

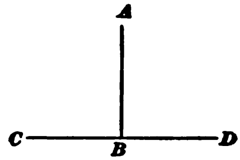


FIG. 4

5. A line is **perpendicular** to another when it meets that line so as not to incline toward it on either side. Thus, in Fig. 4, the line *A B* is perpendicular to the line *C D*.

6. A **horizontal line** is a line parallel to the horizon, or water level, Fig. 5.

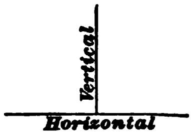


FIG. 5

7. A **vertical line**, Fig. 5, is a line perpendicular to a horizontal line; consequently, it has the direction of a plumb-line.

ANGLES

8. When two lines cross or cut each other, as in Fig. 6, they are said to **intersect**, and the point at which they intersect is called the *point of intersection*, as at *A*.

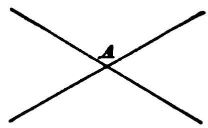


FIG. 6

9. An **angle**, Fig. 7, is the opening between two lines that intersect or meet; the point of meeting is called the **vertex** of the angle.

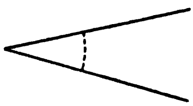


FIG. 7

10. In writing an angle, three letters are commonly used to denote it. The *middle* letter of the three must be the letter at the vertex of the angle. Thus, in Fig. 8, the angle formed by the lines *A B* and *C B* is written, the angle *A B C*. Here the middle letter *B* denotes the vertex of the angle meant, and the other letters are those at the ends of the lines that form the angle. This angle could also be stated as the angle *C B A*. The angle formed by the lines *A B* and *B D* is called the angle *A B D* or the angle *D B A*.

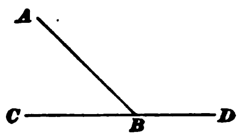


FIG. 8

When an angle stands alone so that it cannot be mistaken for any other angle, only the vertex letter need be given; thus, the angle *E*, Fig. 15, the angle *B*, Fig. 20, etc.

11. Two angles having the same vertex and a common side are called **adjacent angles**. Angles *A B C* and *A B D*, Fig. 8, are adjacent angles.

12. When one straight line meets another so that the adjacent angles formed are equal, as *A B C* and *A B D*, Fig. 4, the angles are called **right angles.** The line *A B* is then perpendicular to *C D*.

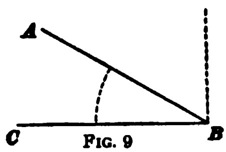


Fig. 9

13. An **acute angle** is less than a right angle. *A B C*, Fig. 9, is an acute angle.

14. An **obtuse angle** is greater than a right angle. *ABD*, Fig. 10, is an obtuse angle.

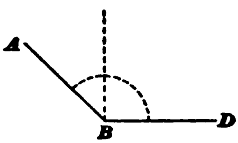


Fig. 10

15. There are many examples of angles to be seen in a shop. The edges of a square box or of a square iron block form angles at the corners where they meet. These angles are right angles. The two faces of a chisel that meet to form the cutting edge form an angle with each other. The sharper the cutting edge, the smaller is the angle of these faces.

16. Angles are measured in degrees and parts of a degree. A **degree** is $\frac{1}{90}$ of a right angle; that is, if a right angle were divided into 90 equal parts, each of these smaller parts, or angles, would be 1 degree. Another way of stating the same fact is to say that a degree is $\frac{1}{360}$ part of a circle; that is, if a circle were divided into 360 equal wedge-shaped pieces by drawing lines from the center outwards, each of the angles thus formed would be 1 degree. In Fig. 11 is shown a circle divided into 36 equal

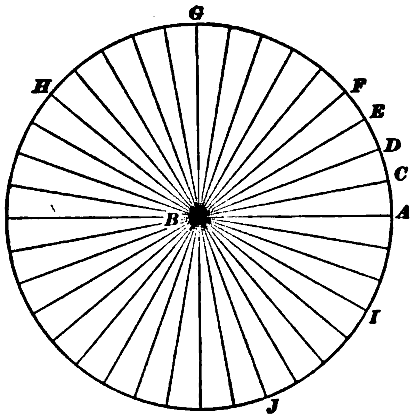


Fig. 11

parts; hence, each of the small angles formed, as *ABC*, *CBD*, *DBE*, etc., is $\frac{360}{36} = 10$ degrees. The angle *A B F* is therefore

4×10=40 degrees; the angle ABG is 9×10=90 degrees, or a right angle; and the angle ABH is 14×10=140 degrees.

17. Fractional parts of a degree are measured in minutes and seconds. A **minute** is $\frac{1}{60}$ of a degree, and a **second** is $\frac{1}{60}$ of a minute, or $\frac{1}{3600}$ of a degree. The table of angular measure may then be written as follows:

ANGULAR MEASURE

60 seconds (″)	=1 minute	′
60 minutes	=1 degree	°
360 degrees	=1 circle	cir.

The sign for the degree is °; thus, 36° is read *36 degrees.* The signs for minutes and seconds are ′ and ″, the same as for feet and inches; but as they usually occur in connection with the sign for degrees, there is little danger of their being misread for feet and inches. The expression 27° 13′ 45″ is read *27 degrees 13 minutes 45 seconds.*

18. In some calculations it may be necessary to reduce degrees and minutes to degrees and a fraction of a degree. To do this, divide the number of minutes by 60, and add the fraction or the decimal to the whole number of degrees. For example, 12° 45′ is equal to $12\frac{45}{60}$ degrees, or $12\frac{3}{4}$ degrees, which may be expressed decimally as 12.75°. If the expression contains both minutes and seconds, reduce these to seconds, divide the result by 3,600, and add the fraction to the whole number of degrees. For example, suppose that 39° 16′ 15″ is to be reduced to degrees. The 16′ 15″, reduced to seconds, becomes (16×60) +15=975 seconds. Dividing this product by 3,600, the fraction is $\frac{975}{3600}=\frac{13}{48}=.271$. Therefore, 39° 16′ 15″ may be written $39\frac{13}{48}$ degrees, or 39.271°.

19. Angles may be added and subtracted in the same way as other compound numbers. Suppose that the angles $A\ B\ I$ and $I\ B\ J$, Fig. 11, are to be added. The angle $A\ B\ I=30°$ and the angle $I\ B\ J=40°$; hence, their sum is 30°+40°=70°. Usually the mechanic will not have to deal with smaller divisions of a degree than minutes; that is, the values of the angles he may have to use in calculations will ordinarily be given in degrees and minutes, and not in degrees, minutes, and seconds.

EXAMPLE 1.—Find the sum of 12° 34′, 7° 48′, and 36° 11′.

SOLUTION.—

$$\begin{array}{rr} 12^\circ & 34' \\ 7 & 48 \\ 36 & 11 \\ \hline 56^\circ & 33' \end{array} \text{ Ans.}$$

EXPLANATION.—Place the compound numbers as shown, with like units in the same column. On adding, the sum of the right-hand column is 93′, which is equal to 1° 33′. Put down the 33′ and carry the 1° over, and add it to the numbers in the remaining column, giving a total of 56°. The sum is then 56° 33′.

EXAMPLE 2.—From 45° 40′ subtract 12° 25′.

SOLUTION.—Place the smaller value under the larger, with like units in the same column, and subtract in the usual way; thus:

$$\begin{array}{rr} 45^\circ & 40' \\ 12 & 25 \\ \hline 33^\circ & 15' \end{array} \text{ Ans.}$$

EXAMPLE 3.—Subtract 27° 56′ from 39° 4′.

SOLUTION.—Place the smaller value below the larger, and proceed as usual.

$$\begin{array}{rr} 39^\circ & 4' \\ 27 & 56 \\ \hline 11^\circ & 8' \end{array} \text{ Ans.}$$

EXPLANATION.—In the right-hand column, it is not possible to take 56′ from 4′, so 1°, or 60′ is taken from the 39° and added to the 4′. The upper number may then be written 38° 64′, and the subtraction becomes

$$\begin{array}{rr} 38^\circ & 64' \\ 27 & 56 \\ \hline 11^\circ & 8' \end{array}$$

The upper number is not usually rewritten as shown here, but the process of taking 1° from 39° and adding its equivalent, or 60′, to the minutes in the upper number is done mentally.

20. Angles are commonly measured by the use of a **protractor,** a form of which is shown in Fig. 12. It consists of a piece of celluloid or of metal of a semicircular shape and very thin. Along the curved edge are a number of divisions, the smallest representing $\frac{1}{2}°$, or 30′, the next larger representing 1°, and the larger ones representing 5° and 10°, respectively. To

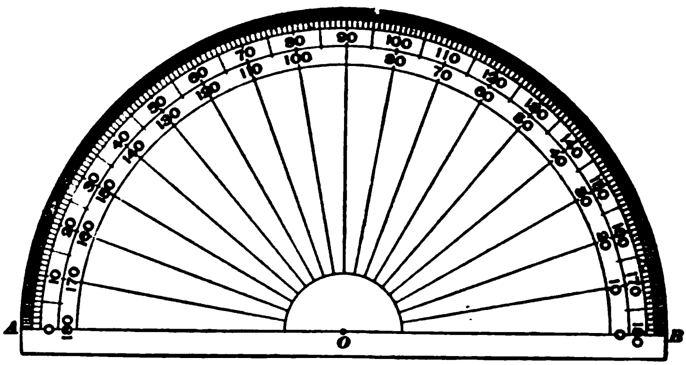


FIG. 12

use the protractor, it is laid flat on the angle to be measured, with the point *O* directly on the vertex of the angle and the line *A B* directly over one side of the angle. The point where the other side of the angle crosses the scale shows the angle. The process of reading the value of a given angle is just the opposite of laying off a given angle, which will now be described. Suppose that it is required to draw a line from the point *C*, Fig. 13, so as to make an angle of 54° with the line *E F*. The protractor

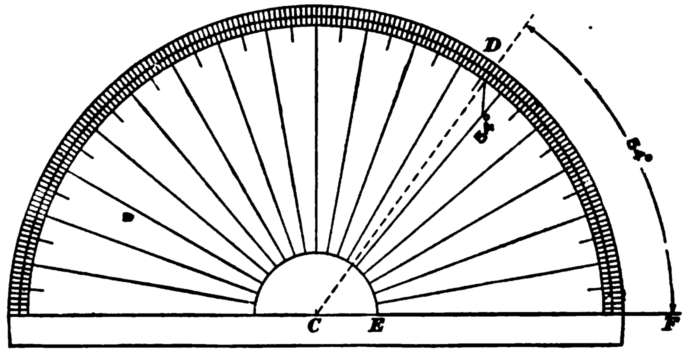


FIG. 13

is first laid on the line *E F* so that its center comes directly over the point *C* and so that its lower line, as *A B*, Fig. 12, lies directly on the line *E F*, Fig. 13. Then, beginning at the end of the scale that is against the line *E F*, 54° is counted off. This means 5 large divisions, each of which represents 10°, and 4 small divisions, each of which represents 1°. Opposite the end of the 54° mark the point *D* is located with a sharp pencil or a scriber. Then the protractor is removed and a straight line is drawn or scribed so as to pass through *C* and *D*. This line *C D* will make an angle of 54° with the line *E F*; that is, the angle *D C F* will be 54°.

PLANE FIGURES

POLYGONS

DEFINITIONS

21. A **surface** has only two dimensions: *length* and *breadth*. A **plane surface**, usually called a *plane*, is a flat surface, like a surface plate. If a straightedge is laid on a plane surface, every point along the edge of the straightedge will touch the surface, no matter in what direction it is laid. **Parallel surfaces** are surfaces that are the same distance apart at all points, like opposite sides of parallel bars.

22. A **plane figure** is any part of a plane surface bounded by straight or curved lines.

23. One of the simplest and most common of plane figures is the *square*, shown in Fig. 14. It is a figure that has four equal sides, *AB*, *BD*, *DC*, and *CA*, and four equal angles, *ABD*, *BDC*, *DCA*, and *CAB*.

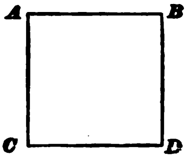


FIG. 14

24. When a plane figure is bounded by *straight* lines only, it is called a **polygon**. The square in Fig. 14 is therefore a polygon. The bounding lines are called the **sides**, and the line that bounds it (or the whole distance around it) is called the **perimeter** of the polygon.

The angles formed by the sides are called the angles of the polygon. Thus, *A B C D E*, Fig. 15, is a polygon. *A B*, *B C*, etc. are the sides; *E A B*, *A B C*, etc. are the angles; and the line *A B C D E A* is the perimeter.

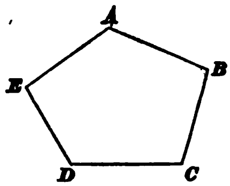


FIG. 15

25. Polygons are classified according to the number of their sides: One of three sides is called a **triangle**; one of four sides, a **quadrilateral**; one of five sides, a **pentagon**; one of six sides, a **hexagon**; one of seven sides, a **heptagon**; one of eight sides, an **octagon**; one of ten sides, a **decagon**; one of twelve sides, a **dodecagon**; etc.

26. A **regular polygon** is one in which all the sides and all the angles are equal. Thus, in Fig. 14, $AB = BD = DC$

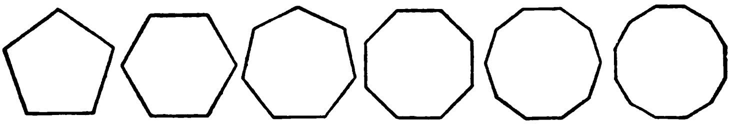


FIG. 16

$= CA$, and angle A = angle B = angle D = angle C; hence, the square *A B D C* is a regular polygon.

Some regular polygons are shown in Fig. 16.

TRIANGLES

27. Triangles are named according to their sides as *isosceles*, *equilateral*, and *scalene triangles*, and according to their angles as *right-angled* and *oblique triangles*.

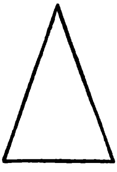


FIG. 17

28. An **isosceles triangle**, Fig. 17, is one having two of its sides equal.

29. When the three sides are equal, as in Fig. 18, it is called an **equilateral triangle**. An equilateral triangle is also isosceles.

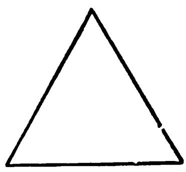


FIG. 18

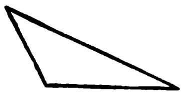


FIG. 19

30. A **scalene triangle**. Fig. 19, is one having no two of its sides equal.

31. A **right-angled triangle**, Fig. 20, is any triangle having one right angle. The side *A B* opposite the right angle *C* is called the **hypotenuse.** For brevity, a right-angled triangle is now termed a *right triangle.*

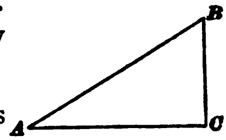


FIG. 20

32. An **oblique triangle**, Fig. 19, is one that has no right angle.

33. The **base** of any triangle is the side on which the triangle is supposed to stand; any side may be considered to be the base. In Figs. 21 and 22, *A C* is the base.

34. The **altitude** of any triangle is a line drawn from the vertex of the angle opposite the base perpendicular to the base, or to the base prolonged. Thus, in Figs. 21 and 22, *B D* is the altitude of the triangles *A B C*.

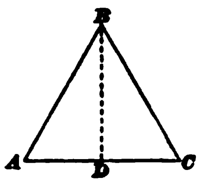


FIG. 21

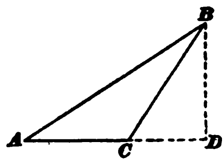


FIG. 22

35. Two triangles are *similar* when the angles of one are equal to the angles of the other.

QUADRILATERALS

36. A **parallelogram** is a quadrilateral whose opposite sides are parallel. There are four kinds of parallelograms: the *rectangle*, the *square*, the *rhomboid*, and the *rhombus*.

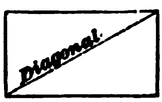


FIG. 23

37. A **rectangle**, Fig. 23, is a parallelogram whose angles are all right angles.

38. A **square**, Fig. 24, is a rectangle, all of whose sides are equal.

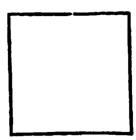


FIG. 24


FIG. 25

39. A rhomboid, Fig. 25, is a parallelogram whose opposite sides only are equal, and whose angles are not right angles.

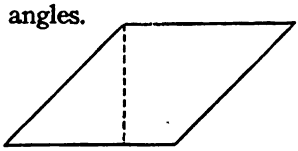


FIG. 26

40. A rhombus, Fig. 26, is a parallelogram having equal sides, but whose angles are not right angles.


FIG. 27

41. A trapezoid, Fig. 27, is a quadrilateral which has only two of its sides parallel.

42. A trapezium, Fig. 28, is a quadrilateral having no two sides parallel.

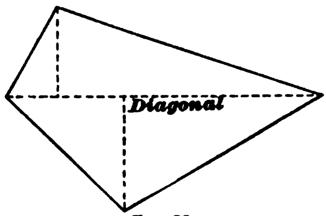


FIG. 28

43. The **altitude** of a parallelogram, or of a trapezoid, is the perpendicular distance between the parallel sides. It is indicated by a dotted line in Figs. 25, 26, and 27.

44. A **diagonal** is a straight line drawn from the vertex of any angle of a quadrilateral to the vertex of the angle opposite; a diagonal divides a quadrilateral into two triangles, as shown in Figs. 23 and 28. A diagonal divides a parallelogram into two *equal* and *similar* triangles.

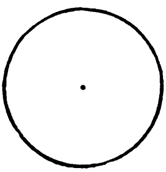


FIG. 29

CIRCLES

45. A circle, Fig. 29 is a plane figure bounded by a curved line, called the **circumference**, every point of which is equally distant from a point within, called the **center**.

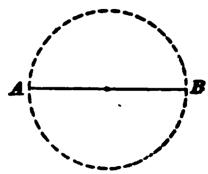


FIG. 30

46. The **diameter** of a circle, as *A B*, Fig. 30, is a straight line passing through the center and ending at each end in the circumference.

47. The **radius** of a circle, as $O\ A$, Fig. 31, is a straight line drawn from the center to the circumference. It is equal in length to one-half the diameter. The plural of radius is **radii**. All radii of any circle are equal in length.

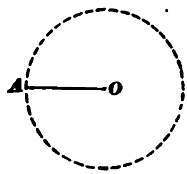


Fig. 31

48. An **arc** of a circle, as $a\ e\ b$, Fig. 32, is any part of its circumference.

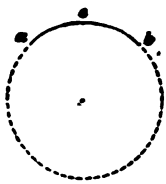


Fig. 32

49. A **chord** is a straight line joining any two points in a circumference; or, it is a straight line joining the extremities of an arc. Thus, in Fig. 33, $a\ b$ is the chord of the arc $a\ e\ b$.

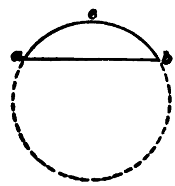


Fig. 33

50. A **segment** of a circle is the space included between an arc and its chord. Thus, in Fig. 33, the portion of the circle included between the chord $a\ b$ and arc $a\ e\ b$ is a segment.

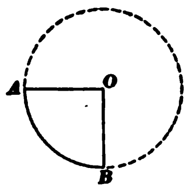


Fig. 34

51. A **sector** of a circle is the space included between an arc and two radii drawn to the extremities of the arc. Thus, in Fig. 34, the space included between the arc $A\ B$ and the radii $O\ A$ and $O\ B$ is a sector of the circle.

52. Two circles are equal when the radius or diameter of one equals the radius or diameter of the other.

Two arcs are equal when the radius and chord of one equal the radius and chord of the other.

53. If $A\ D\ B\ C$, Fig. 35, is a circle in which two diameters $A\ B$ and $C\ D$ are drawn at right angles to each other, then, $A\ O\ D$, $D\ O\ B$, $B\ O\ C$, and $C\ O\ A$ are right angles. The circumference is thus divided into four equal parts; each of these parts is called a **quadrant**.

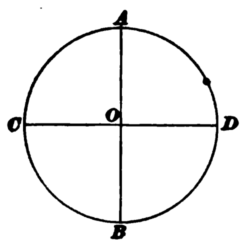


Fig. 35

54. If a circle is divided into halves, each half is called a **semicircle**, and each half circumference is called a **semicircumference.**

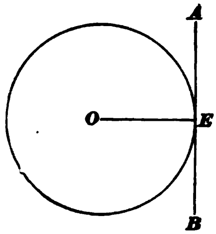


FIG. 36

55. A **tangent** to a circle is a straight line that touches the circle at one point only; it is always perpendicular to a radius drawn to that point. Thus, in Fig. 36, $A\ B$ drawn perpendicular to the radius $O\ E$ at its extremity E is a *tangent* to the circle.

If a straight line is perpendicular to a radius at its extremity, it is tangent to the circle. Thus, if $A\ B$ is perpendicular to the radius $O\ E$ at E, $A\ B$ is tangent to the circle.

56. One circle is said to be **tangent** to another circle when they touch each other at one point only, as in Fig. 37. This point is called the **point of tangency,** or the *point of contact.*

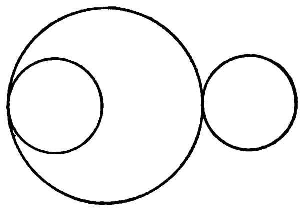


FIG. 37

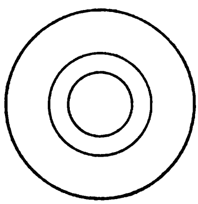


FIG. 38

57. When two or more circles are drawn from the same center, as in Fig. 38, they are called **concentric circles.**

58. Circumference, Diameter, and Radius of Circle.—To find the circumference, diameter, or radius of a circle, the following rules are used:

Rule I.—*The circumference of a circle equals the diameter multiplied by 3.1416.*

Rule II.—*The diameter of a circle equals the circumference divided by 3.1416.*

Rule III.—*The radius of a circle equals the circumference divided by 2×3.1416.*

EXAMPLE 1.—What is the circumference of a circle whose diameter is 15 inches?

SOLUTION.—Applying rule I, the circumference is $15 \times 3.1416 = 47.12$ in. Ans.

EXAMPLE 2.—(*a*) What is the diameter of a circle whose circumference is 65.9736 inches? (*b*) What is the radius?

SOLUTION.—(*a*) Applying rule II, the diameter is $\frac{65.9736}{3.1416} = 21$ in. Ans.

(*b*) Applying rule III, the radius is $\frac{65.9736}{2 \times 3.1416} = 10\frac{1}{2}$ in. Ans.

The number 3.1416 is the ratio of the circumference of a circle to its diameter; it is represented very frequently by the Greek letter π, pronounced "pi." Its value has been calculated to over 700 decimal places, but the value here given is the one most generally used, four decimal places being sufficient for all practical purposes. The values $\frac{1}{4}\pi$, or .7854, and $\frac{1}{6}\pi$, or .5236, are frequently used. In case the calculation does not need to be very accurate, the value of π may be taken as $3\frac{1}{7}$, because $3\frac{1}{7} = 3.1429$, which is fairly close to 3.1416.

59. Dividing Circle Into Equal Parts.—The usual way of dividing the circumference of a circle into a number of equal parts is to take a pair of dividers and step them around the circle, changing the amount of opening, until an adjustment is found that will go around exactly and give the desired number of parts. For example, suppose the circle in Fig. 39 is to be divided into five equal parts. The dividers are adjusted until they will step off five equal chords *AB*, *BC*, *CD*, *DE*, and *EA* in going around exactly once. The values given in Table I are the lengths of the chords, per inch of diameter, for all divisions from 3 to 100, inclusive. By the use of this table, a circle of any diameter can be divided into from 3 to 100 equal parts. The length of the chord, or the distance between the points of the dividers, is found by multiplying the diameter

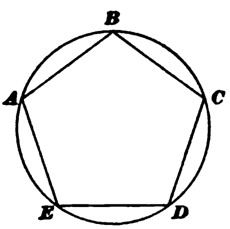


FIG. 39

of the circle by the multiplier corresponding to the number of divisions desired.

TABLE I
LENGTH OF CHORDS

Number of Divisions	Multiplier	Number of Divisions	Multiplier	Number of Divisions	Multiplier	Number of Divisions	Multiplier
3	.86603	28	.11197	53	.05924	77	.04079
4	.70711	29	.10812	54	.05815	78	.04027
5	.58779	30	.10453	55	.05709	79	.03976
6	.50000	31	.10117	56	.05607	80	.03926
7	.43388	32	.09802	57	.05509	81	.03878
8	.38268	33	.09506	58	.05414	82	.03830
9	.34202	34	.09227	59	.05322	83	.03784
10	.30902	35	.08964	60	.05234	84	.03739
11	.28173	36	.08716	61	.05148	85	.03695
12	.25882	37	.08480	62	.05065	86	.03652
13	.23932	38	.08258	63	.04985	87	.03610
14	.22252	39	.08047	64	.04907	88	.03569
15	.20791	40	.07846	65	.04831	89	.03529
16	.19509	41	.07655	66	.04758	90	.03490
17	.18375	42	.07473	67	.04687	91	.03452
18	.17365	43	.07300	68	.04618	92	.03414
19	.16460	44	.07134	69	.04552	93	.03377
20	.15643	45	.06976	70	.04487	94	.03342
21	.14904	46	.06824	71	.04423	95	.03306
22	.14232	47	.06679	72	.04362	96	.03272
23	.13617	48	.06540	73	.04302	97	.03238
24	.13053	49	.06407	74	.04244	98	.03205
25	.12533	50	.06279	75	.04188	99	.03173
26	.12054	51	.06156	76	.04133	100	.03141
27	.11609	52	.06038				

EXAMPLE.—A circle 14 inches in diameter is to have 24 holes spaced equally around it. What must be the distance between the points of the dividers?

SOLUTION.—According to Table I, the multiplier for 24 divisions is .13053. Therefore, the dividers must be set to a distance of

$$14 \times .13053 = 1.82742, \text{ say } 1.83 \text{ in. Ans.}$$

60. One of the most common problems in the division of a circle into equal parts is that of dividing the circumference into six equal parts. In this case the dividers are set so that the distance between their points is exactly equal to the radius of the circle to be divided. This distance stepped off around the circumference will divide it into exactly six equal parts.

GEOMETRICAL CONSTRUCTIONS

61. Drawing Perpendicular Lines.—In laying out some classes of work it becomes necessary to draw one line perpendicular to another; therefore, methods of doing this will be explained. Suppose, for example, that it is desired to draw a line perpendicular to the line *A B*, Fig. 40, at the point *P* near the middle. Take a pair of dividers and open them to any convenient distance.

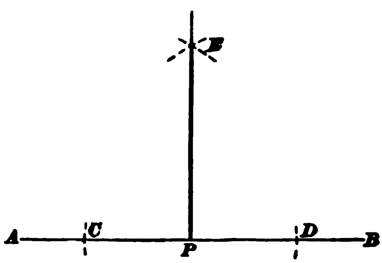


FIG. 40

Set one leg at *P* and with the other leg make marks *C* and *D* on the line *A B* at equal distances on opposite sides of *P*. Now open the dividers still farther so that they will span a distance greater than *P C*, set one leg of the dividers at *C* and strike a short arc above *P*, and then do the same with one leg at *D*, with the dividers opened the same distance in both cases. The two short arcs will cross at a point *E*. Draw a straight line from *E* to *P* and the line *E P* will be perpendicular to *A B*. The distance between the divider points is, of course, the radius of the arcs.

62. If the point at which the perpendicular is to be drawn lies near the end of the line, as at the edge of a piece of work, the foregoing method cannot be used; but the method shown in

Fig. 41 may be employed. Let $A B$ be the line and P the point where the perpendicular is to be drawn. Open the dividers to any convenient distance, and with one leg at O and the other on the point P, draw the greater part of a circle, as shown dotted. This will cross the line $A B$ at D and at P. Through D and O draw a straight line till it crosses the circle at C. Then, a straight line from C to P will be perpendicular to $A B$.

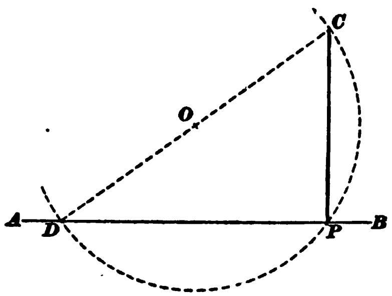


FIG. 41

63. If the perpendicular must be drawn to the line from a point not on the line, the method shown in Fig. 42 is used. Let $A B$ be the line to which a perpendicular must be drawn from the point P. Set one leg of the dividers at P and open them wide enough so that an arc of a circle can be drawn, as shown dotted, to cross the line $A B$ at two points C and D well apart. Now, with one leg at C, make a short arc below the line $A B$, and with the dividers open the same amount, set one leg at D and make another arc crossing the first one at E. Then join the points E and P, and the line $E P$ will be perpendicular to $A B$.

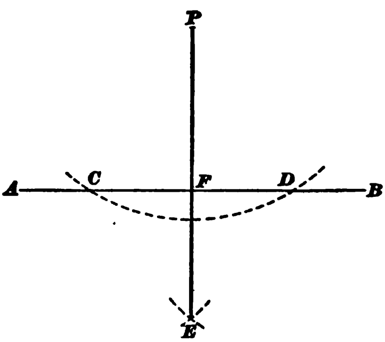
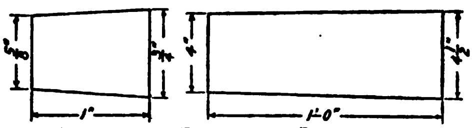


FIG. 42

CALCULATION OF TAPERS

64. Definition of Taper.—The term *taper* means a gradual and regular increase or decrease in the width or diameter of a piece from one end to the other. The taper is usually expressed by stating how much the width or diameter of the piece increases or decreases per inch or per foot of length.

Thus, if the piece shown in Fig. 43 (*a*) is 1 inch long, $\frac{3}{4}$ inch wide at one end, and $\frac{5}{8}$ inch wide at the other, the taper is $\frac{3}{4}-\frac{5}{8}=\frac{1}{8}$ inch in 1 inch, or $\frac{1}{8}$ inch per inch. If the piece shown in (*b*) is 1 foot long and its diameters at the ends are $4\frac{1}{2}$ inches and 4 inches, the taper is $4\frac{1}{2}-4=\frac{1}{2}$ inch per foot. The taper may be expressed in inches per inch or in inches per foot, as desired.



(*a*) FIG. 43 (*b*)

65. Finding the Taper.—If the length and the end dimensions of a piece are known, the taper may be found by the following rules:

Rule I.—*To find the taper in inches per foot, divide the difference of the end dimensions, in inches, by the length of the piece in feet.*

Rule II.—*To find the taper in inches per inch, divide the difference of the end dimensions, in inches, by the length of the piece in inches.*

EXAMPLE 1.—A tapered bar $2\frac{1}{2}$ feet long is $4\frac{5}{8}$ inches wide at one end and 4 inches wide at the other. What is the taper per foot?

SOLUTION.—Applying rule I, the taper is

$$(4\tfrac{5}{8}-4)\div 2\tfrac{1}{2}=\tfrac{5}{8}\times\tfrac{2}{5}=\tfrac{1}{4} \text{ in. per ft.} \quad \text{Ans.}$$

EXAMPLE 2.—A round tapered piece 8 inches long has diameters of $1\frac{5}{8}$ and $2\frac{1}{8}$ inches at the ends. What is the taper per inch?

SOLUTION.—Applying rule II, the taper is

$$(2\tfrac{1}{8}-1\tfrac{5}{8})\div 8=\tfrac{1}{2}\times\tfrac{1}{8}=\tfrac{1}{16} \text{ in. per in.} \quad \text{Ans.}$$

66. Finding Dimensions From Taper.—Sometimes the dimension at one end of the piece and the taper are given and the dimension at the other end is to be calculated. This may be done by the following rules:

Rule I.—*To find the dimension at the large end, multiply the length of the piece in feet (or in inches) by the taper per foot (or per inch) and to the product add the dimension at the small end.*

Rule II.—*To find the dimension at the small end, multiply the length of the piece in feet (or in inches) by the taper per foot (or per inch) and subtract the product from the dimension at the large end.*

EXAMPLE 1.—The piece shown in Fig. 44 has a taper of $\frac{1}{8}$ inch per foot, is $4\frac{3}{4}$ inches wide at the small end, and is 2 feet 3 inches long. What is the width at the large end?

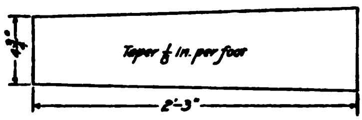


FIG. 44

SOLUTION.—Applying rule I, and remembering that 2 ft. 3 in. = $2\frac{1}{4}$ ft.,

$$2\frac{1}{4} \times \frac{1}{8} = \frac{9}{4} \times \frac{1}{8} = \frac{9}{32}$$

Then, the width at the large end is

$$\frac{9}{32} + 4\frac{3}{4} = 5\frac{1}{32} \text{ in. Ans.}$$

EXAMPLE 2.—A piece 16 inches long has a taper of $\frac{1}{4}$ inch per inch and is $7\frac{3}{4}$ inches in diameter at the large end. Find the diameter at the small end.

SOLUTION.—Applying rule II,

$$16 \times \frac{1}{4} = 4$$

Then, the diameter at the small end is

$$7\frac{3}{4} - 4 = 3\frac{3}{4} \text{ in. Ans.}$$

SHOP CALCULATIONS
(PART 6)

MENSURATION

MEASUREMENT OF LINES, AREAS, AND VOLUMES

LENGTHS OF LINES

1. In connection with his work the mechanic is often required to calculate the lengths of lines, the areas of surfaces, and the volumes of solids. Calculations of this nature belong to that branch of mathematics called **mensuration.**

2. Length of Arc of Circle.—The length of an arc of a circle may be found by the following rule:

Rule.—*The length of an arc of a circle equals the circumference of the circle of which the arc is a part, multiplied by the number of degrees in the arc, and the product divided by 360.*

EXAMPLE.—What is the length of an arc of 24°, the radius of the circle being 18 inches?

SOLUTION.— $18 \times 2 = 36$ in., the diameter of the circle, and the circumference is 3.1416×36. Then, applying the rule, the length of the arc is

$$\frac{3.1416 \times 36 \times 24}{360} = 7.54 \text{ in.} \quad \text{Ans.}$$

AREAS OF PLANE SURFACES

3. Area of Triangle.—The area of a triangle may be found by the following rule, if the length of the base and the altitude are known:

Rule.—*The area of any triangle equals one-half the product of the base and the altitude.*

If the triangle is a right triangle, one of the short sides may be taken as the base, and the other short side as the altitude; hence, *the area of a right triangle is equal to one-half the product of the two short sides.*

EXAMPLE.—What is the area of a triangle whose base is 18 inches and whose altitude is $7\frac{3}{4}$ inches?

SOLUTION.—Applying the rule, the area is

$$\tfrac{1}{2}\times(18\times7\tfrac{3}{4})=69\tfrac{3}{4} \text{ sq. in. Ans.}$$

4. Area of Quadrilateral.—To find the area of a parallelogram the following rule may be used:

Rule.—*The area of any parallelogram equals the product of the base and the altitude.*

EXAMPLE.—What is the area of a parallelogram whose base is 12 inches and whose altitude is $7\frac{1}{2}$ inches?

SOLUTION.—Applying the rule, the area is $12\times7\frac{1}{2}=90$ sq. in. Ans.

If the area and one dimension are given, the other dimension may be found by dividing the area by the known dimension. If the parallelogram is a square, and its area is given, the length of a side is found by extracting the square root of the area.

5. To find the area of a trapezoid the following rule may be used:

Rule.—*The area of a trapezoid equals one-half the sum of the parallel sides multiplied by the altitude.*

EXAMPLE.—What is the area of a trapezoid whose parallel sides are 9 inches and 15 inches, and whose altitude is 6 inches?

SOLUTION.—The sum of the parallel sides is $9+15=24$ in., and half of this is 12 in. Then,

$$12\times 6=72 \text{ sq. in. Ans.}$$

6. Area of Circle.—The area of a circle may be found by the following rule:

Rule.—*The area of a circle is equal to .7854 times the square of the diameter, or 3.1416 times the square of the radius.*

EXAMPLE.—What is the area of a circle whose diameter is 15 inches?

SOLUTION.—Applying the rule, the area is

$$.7854\times 15^2=.7854\times 225=176.72 \text{ sq. in. Ans.}$$

In practice it is found very convenient to use tables giving the diameters, circumferences, and areas of circles, as they make it unnecessary to perform the calculation just described, and thus save much time.

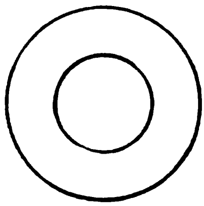


FIG. 1

7. Area of Flat Circular Ring.—The area of a flat circular ring like that in Fig. 1 may be found by the following rule:

Rule.—*The area of a flat circular ring is equal to the area of the outer circle minus the area of the inner circle.*

EXAMPLE.—The diameters of the outer and inner circles of a flat ring are $6\frac{1}{2}$ inches and 4 inches, respectively. What is the area of the ring?

SOLUTION.—The area of the outer circle is $.7854\times(6\frac{1}{2})^2=33.183$ sq. in. and of the inner circle is $.7854\times 4^2=12.566$ sq. in. Then, the area of the ring is $33.183-12.566=20.617$ sq. in. Ans.

8. Area of Sector.—The area of a sector may be found by the following rule:

Rule.—*Divide the number of degrees in the arc of the sector by 360, and multiply the result by the area of the circle of which the sector is a part. The product is the area of the sector.*

EXAMPLE.—The angle formed by drawing radii from the center of a circle to the extremities of the arc of the circle is 75° and the diameter of the circle is 12 inches. What is the area of the sector?

SOLUTION.—The area of a circle 12 in. in diameter is $.7854 \times 12^2 = 113.1$ sq. in., nearly. Applying the rule, the area of the sector is

$$\frac{75}{360} \times 113.1 = 23.56 \text{ sq. in. Ans.}$$

9. If the length of the arc and the radius of a sector are given, the following rule may be used to find the area:

Rule.—*The area of a sector is equal to one-half the product of the radius and the length of the arc.*

EXAMPLE.—If the radius of an arc is 5 inches, and the length of arc is 4 inches, what is the area of the sector?

SOLUTION.—Applying the rule, the area is

$$\frac{1}{2} \times (5 \times 4) = 10 \text{ sq. in. Ans.}$$

10. Area of Segment.—A segment of a circle is the part that is included between a chord and its arc, as the area $A\ B\ C$, Fig. 2. Now, suppose that lines are drawn from A and B to the center D of the circle. Then the area of the segment $A\ B\ C$ is equal to the area of the sector $A\ D\ B\ C$ minus the area of the triangle $A\ B\ D$. Hence, the following rule is used to find the area of a segment of a circle:

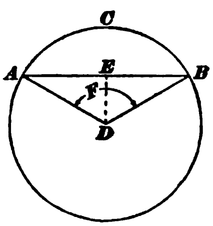


FIG. 2

Rule.—*Draw radii from the center of the circle to the ends of the arc of the segment; find the area of the sector thus formed, subtract from this the area of the triangle formed by the radii and the chord of the arc of the segment; the result is the area of the segment.*

The area of the sector is found by the rule of Art. **8**, the angle F being determined by making a full-sized drawing of the segment and measuring the angle with a protractor. The area of the triangle is equal to half of the product of the length $A\ B$ of the chord and the height $E\ D$. The distance $E\ D$ can be measured directly from the drawing.

EXAMPLE.—Find the area of the segment $A\ B\ C$, Fig. 2, if the chord $A\ B$ is 8.66 inches, $E\ D$ is $2\frac{1}{2}$ inches, the radius $A\ D$ is 5 inches, and the angle F is 120°.

SOLUTION.—Applying the rule of Art. 8, the area of the sector is

$$\frac{120}{360}\times(3.1416\times5^2)=\frac{1}{3}\times78.54=26.18 \text{ sq. in.}$$

The area of the triangle is $\frac{1}{2}\times(8.66\times2\frac{1}{2})=10.83$ sq. in. Therefore, by the foregoing rule, the area of the segment is

$$26.18-10.83=15.35 \text{ sq. in. Ans.}$$

11. Area of Regular Polygon.—The area of a regular polygon may be found by the following rule:

Rule.—*To find the area of a regular polygon, square the length of a side and multiply this by the proper multiplier taken from Table I.*

EXAMPLE.—A regular octagon has sides 8 inches long. What is the area?

SOLUTION.—The multiplier for an octagon, taken from Table I, is 4.8284. Applying the rule, the area is

$$8^2\times4.8284=64\times4.8284=309.02 \text{ sq. in. Ans.}$$

TABLE I

MULTIPLIERS FOR AREAS OF REGULAR POLYGONS

Name	Number of Sides	Multiplier	Name	Number of Sides	Multiplier
Equilateral triangle..	3	0.4330	Octagon...	8	4.8284
Square.....	4	1.0000	Nonagon..	9	6.1818
Pentagon..	5	1.7205	Decagon..	10	7.6942
Hexagon...	6	2.5981	Undecagon	11	9.3656
Heptagon..	7	3.6339	Dodecagon	12	11.1960

12. Area of Ellipse.—An **ellipse** is a figure having a curved outline, but longer than it is wide, as shown in Fig. 3 (*a*). The distance *A B* is called the *long diameter* of the ellipse and the distance *C D* the *short diameter;* also, *A B* is sometimes called the *major axis* and *C D* the *minor axis.* To draw an ellipse having a long and a short diameter of certain length, first draw the two diameters at right angles to each other, as shown by *a b* and *c d* in (*b*). These must cross each other at the middle point *e* of each diameter. Now, with *c* as a center and a radius equal to *a e*, or half of the long diameter, draw an arc of a circle, cutting the long diameter at *f* and *g*. Stick pins at these two

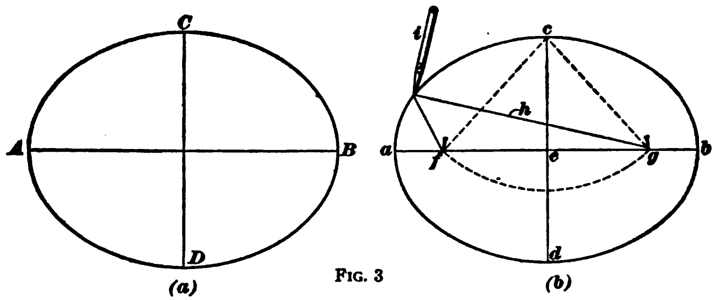


FIG. 3

points, as shown, and around them tie a loop of cord *h*, of such length that when a pencil *i* is placed in it and drawn taut, the point of the pencil will come just to the point *c*. Now pass the pencil around the pins, keeping the cord stretched taut, and it will trace an ellipse that will pass through the ends *a*, *b*, *c*, and *d* of the diameters. The area of an ellipse may be found by the following rule:

Rule.—*The area of an ellipse is equal to .7854 times the product of its long and short diameters.*

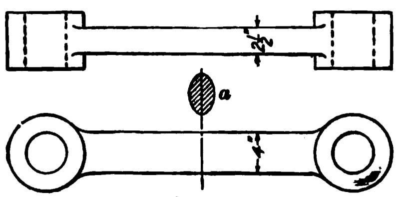


FIG. 4

EXAMPLE.—The link shown in Fig. 4 has its central part elliptical; that is, the cross-section, as shown at *a*, is an ellipse 4 inches long and $2\frac{1}{2}$ inches wide. What is the area of this cross-section?

SOLUTION.—The long diameter is 4 in. and the short diameter is $2\frac{1}{2}$ in. Applying the rule, the area is

$$.7854\times4\times2\tfrac{1}{2}=7.854 \text{ sq. in. Ans.}$$

13. Areas of Irregular Figures.—The foregoing rules for finding areas apply only to regular figures, such as squares, rectangles, circles, etc. If the figure or surface is of irregular shape, it can generally be divided into simple forms, such as squares, rectangles, triangles, circles, etc. The areas of these parts may then be calculated by the rules given, and the sum of the results will be the total area of the figure. However, if the figure is of very irregular shape, there is no general rule that will give the exact area. In such a case it is necessary to find the area by an approximate method. There are several approximate methods, all of which will give fairly accurate results. The following one will be found easy to apply:

Suppose that the area of an irregular figure like that shown by the curved line in Fig. 5 is to be found. The first thing to do is to draw a straight line *A B* below the figure. Then, from the ends of the figure, draw the lines *C A* and *D B* perpendicular to *A B*. The distance *A B* then is equal to the length of the figure. Now divide the distance *A B* into 20 equal parts. This can be done by using a pair of dividers, setting their points a certain distance apart, and stepping off from *A* to *B*. The

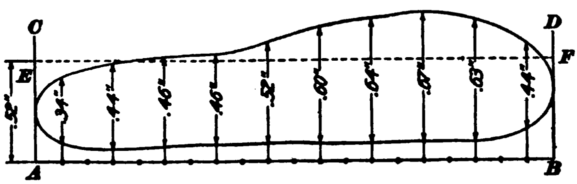


FIG. 5

distance between the points should be altered at each trial, until a setting is found that will give exactly 20 equal spaces on *A B*. Mark each division point with a dot, as shown. Now, beginning at the dot next to one end of the figure, draw a line upwards from it, parallel to the end lines *C A* and *D B*, so as to cross the figure. Do the same thing from each alternate point, skipping one point each time. There will then be 10 lines drawn across the figure from top to bottom, as shown. Now measure the length of each of these 10 lines between the top and the bottom of the figure, as indicated by the arrowheads. On the figure these lengths are marked in decimals of an inch. When all the lengths are measured, add them and divide the sum by 10, or the number of lines. The result is the *average* length of the lines, or the average height of the figure. In this case the sum of the lengths is $.34+.44+.46+.46+.52+.60+.64+.67+.63+.44=5.2$ inches, and $5.2 \div 10 = .52$ inch; that is, the average height of the figure is .52 inch. The length of the figure is the length *A B*, which measures $2\frac{1}{2}$ inches. The area is therefore equal to the length times the average height, or $2\frac{1}{2} \times .52 = 1.3$ square inches.

The area of the figure is equal to the area of a rectangle *A E F B* whose length *A B* is equal to the length of the figure and whose height *A E* is equal to the *average* height of the figure.

14. Although the method just described for finding areas is most commonly applied to irregular figures, it can also be used for finding the approximate area of any regular figure, in case the formula for finding such area cannot be remembered. Also, it is not absolutely necessary to divide the base line of the figure into 20 equal parts. Any number of equal parts may be used; but the greater the number, the greater will be the accuracy of the calculation. Not less than 20 divisions should be used, so that 10 lines of measurement will be obtained, if fairly accurate results are expected.

It is not necessary to measure each of the 10 lines separately, as was done in the preceding article. A narrow strip of paper with one straight edge may be laid across the figure and the lengths of the 10 lines may be marked off on it, one after the other, beginning at the end of the strip. The total distance from this end of the strip to the last mark made is then measured, and this is the sum of the 10 lengths.' The remainder of the calculation is then performed as already explained.

EXAMPLES FOR PRACTICE

1. What is the area in square feet of a parallelogram whose base is 84 inches, and whose altitude is 3 feet? Ans. 21 sq. ft.

2. One side of a shop is 16 feet long. If the floor space is 240 square feet, what is the length of the other side? Ans. 15 ft.

3. How many square feet in a board 12 feet long, 18 inches wide at one end and 12 inches wide at the other end? Ans. 15 sq. ft.

4. What is the area of a triangle whose base is 10 feet 6 inches long, and whose altitude is 18 feet? Ans. 94.5 sq. ft.

5. Find the area of a circular plate 2 feet 3 inches in diameter.
Ans. 3.976 sq. ft.

6. What is the area of an ellipse whose axes are 15 inches and 9 inches?
Ans. 106.03 sq. in.

VOLUMES OF SOLIDS

15. A **solid,** or body, has three dimensions: length, breadth, and thickness. The sides that enclose it are called the **faces,** and their intersections are called **edges.**

16. The **volume** of a solid is expressed by the number of times it will contain another volume, called the unit of volume. Instead of the word *volume*, the expression **cubical contents** is frequently used.

17. A **prism** is a solid whose ends are equal polygons and parallel to each other, and whose sides are parallelograms.

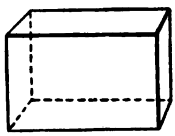


FIG. 6

18. A **rectangular prism,** Fig. 6, is a prism whose bases (ends) are rectangles.

19. A **cube,** Fig. 7, is a rectangular prism whose faces and ends are squares.

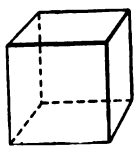


FIG. 7

20. Volume of Rectangular Prism.—The volume of a solid like that shown in Fig. 6 or in Fig. 7 may be found by the following rule:

Rule.—*The volume of a rectangular prism is equal to the product of the length, width, and thickness.*

The three dimensions must be in the same units—as inches or feet—before they are multiplied together.

EXAMPLE 1.—A rectangular block of cast iron is 20 inches long, 11 inches wide, and $6\frac{1}{2}$ inches thick. What is its volume?

SOLUTION.—Applying the rule, the volume is

$$20 \times 11 \times 6\frac{1}{2} = 1,430 \text{ cu. in. Ans.}$$

EXAMPLE 2.—A cube is 16 inches on each edge. What is its volume?

SOLUTION.—The three dimensions are the same. Therefore, by the rule, the volume is

$$16 \times 16 \times 16 = 4{,}096 \text{ cu. in. Ans.}$$

21. Prisms.—Prisms take their names from their bases. Thus, a *triangular prism* is one whose bases are triangles; a *pentagonal prism* is one whose bases are pentagons, etc.

22. A **cylinder,** Fig. 8, is a round body of uniform diameter with circles for its ends.

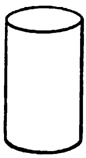


FIG. 8

23. A **right prism** or a **right cylinder** is one whose center line (axis) is perpendicular to its base. In this Section all of the solids will be considered as having their center lines perpendicular to their bases.

24. The **altitude** of a prism or a cylinder is the perpendicular distance between its two ends.

25. Volume of Right Prism or Cylinder.—The volume of a right prism or of a cylinder may be found by the following rule:

Rule.—*The volume of any right prism or cylinder equals the area of the base multiplied by the altitude.*

If the given prism is a cube, the three dimensions are all equal, and the volume equals the cube of one of the edges.

If the volume and the area are given, the altitude equals the volume divided by the area. If the cylinder or prism is hollow, the volume is equal to the area of the ring or base multiplied by the altitude.

EXAMPLE 1.—What is the volume of a rectangular prism whose base is 6 inches by 4 inches, and whose altitude is 12 inches?

SOLUTION.—The base of a rectangular prism is a rectangle. Hence, $6 \times 4 = 24$ sq. in., the area of the base. Applying the rule, the volume is $24 \times 12 = 288$ cu. in. Ans.

EXAMPLE 2.—What is the volume of a cube whose edge is 9 inches?

SOLUTION.—The volume is $9^3 = 9 \times 9 \times 9 = 729$ cu. in. Ans.

EXAMPLE 3.—What is the volume of a cylinder whose base is 7 inches in diameter, and whose altitude is 11 inches?

SOLUTION.— $.7854 \times 7^2 = 38.48$ sq. in., the area of the base. Applying the rule, the volume is $38.48 \times 11 = 423.28$ cu. in. Ans.

26. A problem met by the blacksmith is to find how much stock of a given size is required to make a longer piece of smaller size. For instance, a piece of square stock like that shown in

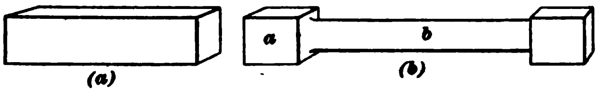


FIG. 9

Fig. 9 (*a*) may have to be made into the form shown in (*b*). The ends *a* of the finished piece are of the same size as the original square bar in (*a*), but the middle part *b* is round, instead of square, and is considerably longer and smaller than the middle part of the square stock. The problem then is to find how much square stock to allow, so that, when it is hammered out into the long, round part *b*, this part will be of the required length. The rule to be used is as follows:

Rule.—*Calculate the volume, in cubic inches, of the part that is to be made from the stock and divide it by the area of the end of the bar stock, in square inches. The quotient is the length of stock required.*

If the finished piece is to be shorter and thicker than the bar stock, the same rule can be used.

EXAMPLE.—A forging like that shown in Fig. 9 (*b*) is to be made from a piece of bar stock 3 inches square. If the dimensions of the forging are to be as shown in Fig. 10, what length of stock is required?

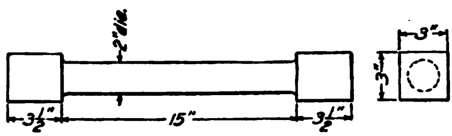


FIG. 10

SOLUTION.—The ends of the forging are of the same size as the bar stock, that is, 3 in. square, and each is 3½ in. long; therefore, it will take $2 \times 3\frac{1}{2}$ in. = 7 in. of stock for the ends. The middle part is a cylinder 2 in. in diameter and 15 in. long. The volume of this part, therefore, is, by the rule of Art. **25**,

$$.7854 \times 2^2 \times 15 = 47 \text{ cu. in.}$$

The area of the end of the bar stock, which is 3 in. square, is $3 \times 3 = 9$ sq. in. Then, applying the rule of this article, the length of stock required for the middle section of the forging is

$$47 \div 9 = 5.22 \text{ in., or } 5\tfrac{1}{4} \text{ in., very nearly}$$

That is, a piece of 3-in. square stock $5\frac{1}{4}$ in. long, when hammered down to a round piece 2 in. in diameter, will be 15 in. long. The total length of bar stock required for the forging, therefore, is the sum of that needed for the ends and that needed for the middle part, or

$$7 + 5\tfrac{1}{4} = 12\tfrac{1}{4} \text{ in. Ans.}$$

27. Board Measure.—Boards and squared timber are usually measured in board feet. One foot, board measure, or 1 *board foot*, is a piece of timber 1 inch thick and 1 square foot in area. The number of feet, board measure, may therefore be found by the following rule:

Rule.—*To find the number of feet, board measure, in a board or a piece of squared timber, multiply the length, in feet, by the breadth, in feet, and that product by the thickness, in inches; except that a board less than 1 inch thick is always taken as being 1 inch thick.*

The usual abbreviation for board measure is B. M.; thus, 150 ft. B. M. means 150 feet, board measure.

EXAMPLE.—Find the number of feet, board measure, in 268 boards, each 12 feet long, 8 inches wide, and $1\frac{1}{2}$ inches thick.

SOLUTION.—The length of each board is 12 ft.; the width is $\frac{8}{12}$ ft., or $\frac{2}{3}$ ft.; and the thickness is $1\frac{1}{2}$ in., or $\frac{3}{2}$ in. Applying the rule, one board contains

$$12 \times \tfrac{2}{3} \times \tfrac{3}{2} = 12 \text{ ft. B. M.}$$

As there are 268 boards, they contain

$$268 \times 12 = 3{,}216 \text{ ft. B. M. Ans.}$$

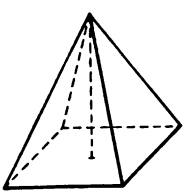


FIG. 11

28. Pyramid and Cone.—A **pyramid**, Fig. 11, is a solid whose base is a polygon, and whose sides are triangles uniting at a common point, called the **vertex**.

A **cone**, Fig. 12, is a solid whose base is a circle, and which tapers uniformly to a point called the **vertex**.

The **altitude** of a pyramid or a cone is the perpendicular distance from the vertex to the base, shown by a dotted line in

Figs. 11 and 12. If this line passes through the center of the base, the solid is called a right pyramid or a right cone.

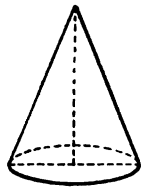


FIG. 12

29. Volume of Right Pyramid or Right Cone.—The volume of a right pyramid or of a right cone may be found by the following rule:

Rule.—*The volume of a right pyramid or a right cone equals the area of the base multiplied by one-third of the altitude.*

If the base of the pyramid is a regular polygon, its area may be found by the rule of Art. **11.**

EXAMPLE 1.—What is the volume of a triangular right pyramid, the edges of whose base each measure 6 inches, and whose altitude is 8 inches?

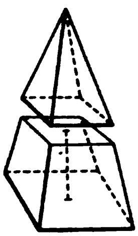


FIG. 13

SOLUTION.—The base is an equilateral triangle; hence, applying the rule of Art. **11**, its area is $6^2 \times .433 = 15.59$ sq. in. Applying the rule, the volume is

$$15.59 \times \frac{8}{3} = 41.57 \text{ cu. in. Ans.}$$

EXAMPLE 2.—What is the volume of a right cone whose altitude is 18 inches, and whose base is 14 inches in diameter?

SOLUTION.— $.7854 \times 14^2 = 153.94$ sq. in., the area of the base. Applying the rule, the volume is

$$153.94 \times \frac{18}{3} = 923.64 \text{ cu. in. Ans.}$$

30. Frustums.—If a pyramid is cut off parallel to the base, as in Fig. 13, so as to form two parts, the lower part is called the **frustum** of the pyramid. If a cone is cut in a similar manner, as in Fig. 14, the lower part is called the **frustum** of the cone. The upper end of the frustum of a pyramid or cone is called the **upper base**, and the lower end the **lower base.** The **altitude** of a frustum is the perpendicular distance between the bases, indicated by the dotted lines in Figs. 13 and 14.

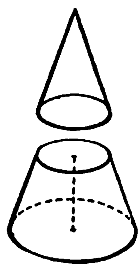


FIG. 14

31. Volume of Frustum of Cone.—The volume of the frustum of a cone may be found by using the following rule:

Rule.—*Multiply together the diameters of the two bases and .7854 and to the product add the sum of the areas of the upper and* *lower bases; then multiply the total sum by one-third the altitude of the frustum. The result will be the volume of the conical frustum.*

EXAMPLE.—What is the volume of a frustum of a cone whose upper base is 8 inches in diameter, whose lower base is 12 inches in diameter, and whose altitude is 15 inches?

SOLUTION.—The area of the upper base is $.7854 \times 8^2 = 50.27$ sq. in., and that of the lower base is $.7854 \times 12^2 = 113.1$ sq. in. Their sum is $50.27 + 113.1 = 163.37$ sq. in. Applying the rule, the volume of the frustum is

$$[(8 \times 12 \times .7854) + 163.37] \times \tfrac{15}{3} = (75.4 + 163.37) \times \tfrac{15}{3}$$
$$= 238.77 \times 5 = 1{,}193.85 \text{ cu. in. Ans.}$$

32. Volume of Frustum of Pyramid Having Regular Polygon for a Base.—The volume of a frustum of a pyramid may be found by the following rule:

Rule.—*Multiply together the lengths of one side of the upper and lower bases and the corresponding multiplier taken from Table I, and to the product add the sum of the areas of the upper and lower bases; then multiply the total sum by one-third the altitude of the frustum. The result will be the volume of the frustum.*

EXAMPLE.—In a frustum of a pyramid whose bases are regular hexagons, each edge of the upper base is 5 inches long and of the lower base is 8 inches long. If the altitude is 14 inches, what is the volume?

SOLUTION.—The area of the upper base, by the rule of Art. **11**, is $5^2 \times 2.5981 = 64.95$ sq. in. By the same rule, the area of the lower base is $8^2 \times 2.5981 = 166.28$ sq. in. The sum of these areas is $64.95 + 166.28 = 231.23$ sq. in. Then, applying the rule, the volume of the frustum is

$$[(5 \times 8 \times 2.5981) + 231.23] \times \tfrac{14}{3} = (103.92 + 231.23) \times \tfrac{14}{3}$$
$$= 335.15 \times \tfrac{14}{3} = 1{,}564 \text{ cu. in. Ans.}$$

FIG. 15

33. Volume of Sphere.—A **sphere**, Fig. 15, is a solid bounded by a uniformly curved surface, every point of which is equally distant from a point within, called the center. The word *ball* is commonly used instead of sphere. The volume of a sphere may be found as follows:

Rule.—*The volume of a sphere equals the cube of the diameter multiplied by .5236.*

EXAMPLE.—What is the volume of a cannon ball 12 inches in diameter?

SOLUTION.—Applying the rule, the volume is $12^3\times.5236=12\times12\times12\times.5236=904.78$ cu. in. Ans.

The volume of a spherical shell, or hollow sphere, is equal to the difference in volume between two spheres having the outer and inner diameters of the shell.

CALCULATION OF WEIGHTS

34. Method of Estimating Weight.—It often becomes necessary to calculate the weight of a body whose dimensions are known but which cannot conveniently be put on scales to be weighed. For instance, suppose that it is desired to estimate the weight of a casting that has not been made. From the drawing it is possible to obtain all the dimensions of the casting. Then, by the principles of mensuration given previously, the volume of the casting can be found. After the volume is known, it is multiplied by the weight of a unit volume of the material, and thus the weight is obtained. Sometimes, castings or forgings have intricate shapes; but usually it is possible to consider them as made up of a combination of cylinders, spheres, rectangular prisms, cones, etc. The volume of each of these is calculated and the total is the approximate volume of the piece.

35. Weights of Materials.—In finding the weights of machine parts, the volumes are usually calculated in cubic inches. The volume in cubic inches must then be multiplied by the weight of the material per cubic inch, to obtain the weight of the body. If the volumes are calculated in cubic feet, the weight per cubic foot is then used. To enable the weights of bodies to be calculated, the weights of a number of different materials are given in Table II, in pounds per cubic foot and per cubic inch. The method of using the table is shown in the following examples:

TABLE II

AVERAGE WEIGHTS OF MATERIALS

Material	Weight per Cubic Foot Pounds	Weight per Cubic Inch Pound	Material	Weight per Cubic Foot Pounds	Weight per Cubic Inch Pound
Dry Woods			*Metals—(Cont.)*		
Ash	45	.0260	Brass castings	510	.295
Beech	46	.0266	Brass, rolled	518	.300
Birch	41	.0237	Bronze	527	.305
Box	70	.0405	Copper	550	.318
Cedar	39	.0226	Iron, cast	450	.260
Cherry	41	.0237	Iron, wrought	480	.278
Chestnut	35	.0203	Lead	710	.411
Cork	14	.0081	Steel castings	480	.278
Elm	38	.0220	Steel, forged	490	.283
Fir	37	.0214	Tin	458	.265
Hemlock	24	.0139	Zinc castings	432	.250
Hickory	48	.0278	Zinc sheets	450	.260
Lignum vitæ	62	.0359	*Miscellaneous*		
Mahogany	51	.0287	Cement, loose	92	
Maple	42	.0243	Cement in barrel	115	
Oak, red	46	.0266			
Oak, white	48	.0278	Concrete	120 to 155	
Pine, white	28	.0162			
Pine, yellow	38	.0220	Earth, loose	72 to 80	
Poplar	30	.0174			
Spruce	28	.0162	Earth, rammed	90 to 110	
Walnut	36	.0208			
Willow	34	.0197	Sand	90 to 110	
Metals					
Aluminum	167	.096	Sand, wet	118 to 130	
Antimony	420	.243			
Babbitt metal	466	.270			

EXAMPLE 1.—A hollow box-shaped casting has outside dimensions of 18, 10, and 6 inches, and inside dimensions of 16, 8, and 4 inches. What is its weight, if it is a steel casting?

SOLUTION.—The first step is to calculate the volume of the casting. This is the difference between the volume represented by the outside and inside dimensions. The volume of a solid 18 in. by 10 in. by 6 in. is $18 \times 10 \times 6 = 1{,}080$ cu. in., and that of a solid 16 in. by 8 in. by 4 in. is $16 \times 8 \times 4 = 512$ cu. in. Then, the volume of the casting is $1{,}080 - 512 = 568$ cu. in. According to Table II, a cubic inch of steel casting weighs .278 lb. Therefore, the weight of the casting is

$$568 \times .278 = 158 \text{ lb., nearly. Ans.}$$

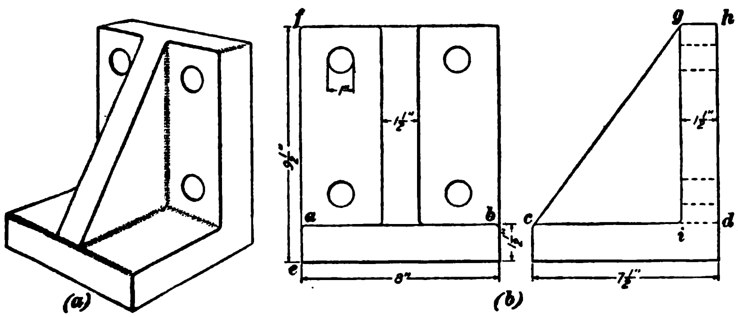


FIG. 16

EXAMPLE 2.—Find the weight of a cast-iron bracket like that in Fig. 16 (*a*), having the dimensions shown in (*b*).

SOLUTION.—The bracket may be considered as being made up of three parts, two of which are rectangular prisms, and the other of which is a triangular prism. One rectangular prism has a length *a b* of 8 in., a width *c d* of $7\frac{1}{2}$ in., and a thickness *a e* of $1\frac{1}{2}$ in. Its volume, therefore, is $8 \times 7\frac{1}{2} \times 1\frac{1}{2} = 90$ cu. in. The other rectangular prism has a width *a b* of 8 in., a height *a f* of $9\frac{1}{2} - 1\frac{1}{2} = 8$ in., and a thickness *g h* of $1\frac{1}{2}$ in., so its volume is $8 \times 8 \times 1\frac{1}{2} = 96$ cu. in. The bracing web has the shape of a right triangle whose base *c i* is $7\frac{1}{2} - 1\frac{1}{2} = 6$ in., and whose altitude *i g* is $9\frac{1}{2} - 1\frac{1}{2} = 8$ in. Its area is therefore $\frac{6 \times 8}{2} = 24$ sq. in. As the thickness is $1\frac{1}{2}$ in., the volume of the triangular prism, according to the rule of Art. 25, is $24 \times 1\frac{1}{2} = 36$ cu. in. There are four holes each 1 in. in diameter and $1\frac{1}{2}$ in. long; so the amount of metal cut out by each is the volume of a cylinder 1 in. in diameter and $1\frac{1}{2}$ in. long. The area of the base is the area of a 1-in. circle, or $.7854 \times 1^2 = .7854$ sq. in. Then, by the rule of Art. 25, the volume of one cylinder is $.7854 \times 1\frac{1}{2} = 1.1781$ cu. in. As there are four holes, their volume is $4 \times 1.1781 = 4.7$ cu. in., approximately. The actual volume of the bracket, therefore, is 90+96+36−4.7=217.3 cu. in. According to Table II, a cubic inch of cast iron weighs .260 lb. The weight of the bracket, therefore, is

$$217.3 \times .260 = 56\tfrac{1}{2} \text{ lb., very nearly. Ans.}$$

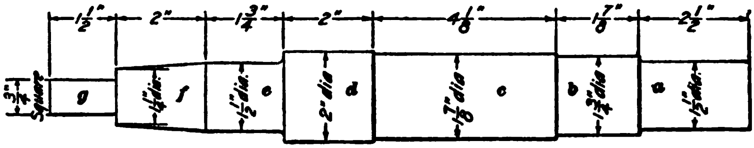


FIG. 17

EXAMPLE 3.—What is the weight of the forged steel shaft shown in Fig. 17?

SOLUTION.—The shaft may be considered as being made up of a cylindrical piece a that is $1\frac{1}{2}$ in. in diameter and $2\frac{1}{2}$ in. long, a cylinder b $1\frac{3}{4}$ in. in diameter and $1\frac{7}{8}$ in. long, a cylinder c $1\frac{7}{8}$ in. in diameter and $4\frac{1}{8}$ in. long, a cylinder d 2 in. in diameter and 2 in. long, a cylinder e $1\frac{1}{2}$ in. in diameter and $1\frac{3}{4}$ in. long, a frustum of a cone with bases $1\frac{1}{4}$ in. and $1\frac{1}{2}$ in. in diameter and 2 in. long, and a prism $\frac{3}{4}$ in. square and $1\frac{1}{2}$ in. long. Applying the rule of Art. **25**,

$$\begin{aligned}
\text{volume of part } a &= .7854 \times (1\tfrac{1}{2})^2 \times 2\tfrac{1}{2} = 4.4 \text{ cu. in.}\\
\text{volume of part } b &= .7854 \times (1\tfrac{3}{4})^2 \times 1\tfrac{7}{8} = 4.5 \text{ cu. in.}\\
\text{volume of part } c &= .7854 \times (1\tfrac{7}{8})^2 \times 4\tfrac{1}{8} = 11.4 \text{ cu. in.}\\
\text{volume of part } d &= .7854 \times 2^2 \times 2 = 6.3 \text{ cu. in.}\\
\text{volume of part } e &= .7854 \times (1\tfrac{1}{2})^2 \times 1\tfrac{3}{4} = 3.1 \text{ cu. in.}
\end{aligned}$$

The part f is a frustum of a cone. The area of the smaller base is $.7854 \times (1\frac{1}{4})^2 = 1.23$ sq. in. and that of the larger base is $.7854 \times (1\frac{1}{2})^2 = 1.77$ sq. in. Then, applying the rule of Art. **31**, the volume of the frustum is

$$[(1\tfrac{1}{4} \times 1\tfrac{1}{2} \times .7854) + 1.23 + 1.77] \times \tfrac{2}{3} = (1.47 + 1.23 + 1.77) \times \tfrac{2}{3}$$
$$= 2.98 \text{ cu. in., or approximately 3 cu. in.}$$

The volume of the rectangular solid end g, according to the rule of Art. **20**, is $\frac{3}{4} \times \frac{3}{4} \times 1\frac{1}{2} = .8$ cu. in.

The total volume of the shaft, therefore, is 4.4+4.5+11.4+6.3+3.1+3+.8=33.5 cu. in.

According to Table III, the weight of a cubic inch of forged steel is .283 lb. The weight of the shaft, then, is

$$33.5 \times .283 = 9\tfrac{1}{2} \text{ lb., nearly. Ans.}$$

36. To make allowance for the expansion of the mold, rapping of the pattern in removing it from the sand, and the rounding of corners by the use of fillets, it is customary to add a small percentage to the calculated weight of the casting as found by the method outlined in the previous article. The fillets, the expansion of the mold, and the rapping all tend to increase the weight of the casting; so the calculated weight is increased by from 2 to 5 per cent. of itself. In example 1 of the previous article, suppose that the allowance was 3 per cent. The amount to be added is then $158 \times .03 = 4.74$ pounds, or 5 pounds, nearly. The weight of the casting, therefore, would be $158 + 5 = 163$ pounds, including the allowance.

EXAMPLES FOR PRACTICE

1. A rectangular tank 12 feet long, 8 feet wide, and 10 feet deep, inside measurements, is half full of water. How many cubic feet of water does it contain? Ans. 480 cu. ft.

2. A box filled with sand has the dimensions 4 feet, 3 feet 6 inches, and 2 feet 6 inches on the inside. How many pounds of sand does it hold, if the sand weighs 96 pounds per cubic foot? Ans. 3,360 lb.

3. A forged steel shaft 9 inches in diameter and 10 feet long has a hole $4\frac{1}{2}$ inches in diameter through it from end to end. What is the weight of the shaft? Ans. 1,620 lb.

4. A cast-iron pulley has six straight elliptical arms, each 15 inches long. The ellipse has axes of $2\frac{1}{2}$ and $1\frac{1}{2}$ inches. What is the weight of metal in the arms? Ans. 57.4 lb.

SHOP CALCULATIONS

(PART 7)

ROOTS OF NUMBERS

METHOD OF FINDING SQUARE ROOT

1. The **root** of a number is one of the equal factors into which that number may be divided, and the method of finding the root is exactly the opposite of finding the power. It was shown that the square of 3 is $3\times3=9$. Now, suppose that the process is reversed, and the number 9 is split up into two equal factors. Then, $9=3\times3$; that is, 9 is split up into the two equal factors 3 and 3. The number 3 is therefore said to be the *square root* of 9.

The **square root** of any number is that number which, when multiplied by itself, will produce the first number. The square root of 25 is 5, because $5\times5=25$; the square root of 49 is 7, because $7\times7=49$; the square root of 1.21 is 1.1, because $1.1\times1.1=1.21$.

The **cube root** of a number is one of three equal factors of which the number is composed. The cube root of 27 is 3, because $27=3\times3\times3$; and the cube root of 64 is 4, because $64=4\times4\times4$.

2. The fact that the root of a number is to be found is usually indicated by writing the **radical sign** $\sqrt{}$ in front of it, with the vinculum $\overline{}$ over it; thus, $\sqrt{36}$.

The particular root to be found may be shown by the use of an **index**, which is a small figure placed *above* the radical sign; thus, $\sqrt[2]{100}$ indicates the square root of 100, and $\sqrt[3]{1728}$ indicates the cube root of 1,728. When the square root of a number is to be found, the index figure is usually omitted; thus, $\sqrt{81}$ means the square root of 81; $\sqrt{.25}$ means the square root of .25; $\sqrt{\frac{121}{144}}$ means the square root of $\frac{121}{144}$.

In shopwork, it is seldom necessary to find a higher root than the square root. The process of finding the cube root is long and difficult, and is not needed in any of the following calculations; therefore, it will not be described. The process of finding the square root of a number is one with which many students find trouble. This seems to be due to the fact that the process makes use of a large number of separate operations that must be followed in a regular order. The method can best be explained by giving several examples with full explanations of each step. In order to make the work clearer to the student and easier to follow, the figures in the root and the successive numbers resulting from their use are printed in Roman and Italic type alternately.

EXAMPLE 1.—Find the square root of 31,505,769.

SOLUTION.—

			root
			3 1'5 0'5 7'6 9 (5 *6* 1 *3* Ans.
(*a*)	5	(*b*)	2 5
	5	(*c*)	6 *5 0*
(*d*)	1 0 *0*		*6 3 6*
	6	(*e*)	*1 4* 5 7
	1 0 6		1 1 2 1
	6		3 3 6 *6 9*
	1 1 2 0		*3 3 6 6 9*
	1		
	1 1 2 1		
	1		
	1 1 2 2 *0*		
	3		
	1 1 2 2 3		

EXPLANATION.—First point off into periods of two figures each. Now, find the largest single number whose square is less than or equal to 31, the first period. This is evidently 5, since $6^2=36$, which is greater than 31. Write it to the right, as in long division, and also to the left, as shown at (*a*). This is the first figure of the root. Now, multiply the 5 at (*a*) by the 5 in the root, and write the result under the first period, as shown at (*b*). Subtract and obtain 6 as a remainder.

Add the root already found to the 5 at (*a*), getting 10, and annex a cipher to this 10, thus making it 100, as shown at (*d*), which call the **first trial divisor.** Bring down the next period, 50, and annex it to the remainder 6, as shown at (*c*), which call the **first dividend.** Divide the first dividend (*c*) by the first trial divisor (*d*) and obtain 6, which is *probably* the next figure of the root. Write 6 in the root, as shown, and also add it to 100, the trial divisor, making it 106. This is called the **first complete divisor.**

Multiply the first complete divisor, 106, by 6, the second figure in the root, and subtract the result from the first dividend (*c*); the remainder is 14. Add the second figure of the root to the complete divisor, 106, and annex a cipher, thus getting 1120, which call the **second trial divisor.** Annex the next period to the remainder in the second column, making it 1457, as shown at (*e*), which call the **second dividend.** Dividing 1457 by 1120, we get 1 as the next figure of the root. Adding this last figure of the root to 1120, the result is 1121, the **second complete divisor.** Multiplying the second complete divisor by the third figure of the root and subtracting from the second dividend, 1457, the remainder is 336.

Now, adding the last figure of the root to 1121 and annexing a cipher as before, the result is 11220, the **third trial divisor.** Annexing the next and the last period, 69, to the remainder in the second column the result is 33669, the **third dividend.** Dividing 33669 by 11220, the result is 3, the fourth figure of the root. Adding the fourth figure of the root to 11220, the result is 11223, the **third complete divisor.** Multiplying the third complete divisor by the fourth figure of the root, the result is 33669. Subtracting the product from the third dividend, there is no remainder; hence, $\sqrt{31{,}505{,}769} = 5{,}613$. When there is no remainder, as in this case, the number whose root is found is said to be a *perfect power.*

EXAMPLE 2.—What is the square root of .000576?

```
                              root
SOLUTION.—          .0 0'0 5'7 6 ( .0 2 4   Ans.
             2       4
             2       1 7 6
             4 0     1 7 6
               4
             4 4
```

EXPLANATION.—Beginning at the decimal point and pointing off the number into periods of two figures each, it is seen that the first period is composed of ciphers; hence, the first figure of the root must be a cipher. The remaining portion of the solution should be perfectly clear from what has preceded.

3. If the number is not a perfect power, the root will consist of an interminable number of decimal places. The result may be carried to any required number of decimal places by annexing periods of two ciphers each to the number.

EXAMPLE 1.—What is the square root of 3? Find the result to five decimal places.

```
                                                  root
SOLUTION.—               3.0 0'0 0'0 0'0 0'0 0 ( 1.7 3 2 0 5+  Ans.
           1             1
           1             2 0 0
           2 0           1 8 9
             7             1 1 0 0
           2 7             1 0 2 9
             7                 7 1 0 0
           3 4 0               6 9 2 4
               3                 1 7 6 0 0 0 0
           3 4 3                 1 7 3 2 0 2 5
               3                     2 7 9 7 5
           3 4 6 0
                 2
           3 4 6 2
                 2
           3 4 6 4 0 0
                     5
           3 4 6 4 0 5
```

EXPLANATION.—Annex five periods of two ciphers each to the right of the decimal point. The first figure of the root is found to be 1. To get the second figure, we find that, on dividing 200 by 20, it is 10. This is evidently too large.

Trying 9, we add 9 to 20 and multiply 29 by 9; the result is 261, a result which is considerably larger than 200; hence, 9 is too large. In the same way, it is found that 8 is also too large. Trying 7, 7 times 27 is 189, a result smaller than 200; therefore, 7 is the second figure of the root. The next two figures, 3 and 2, are easily found. The fifth figure in the root is a cipher, since the trial divisor, 34640, is greater than the new dividend, 17600. In a case of this kind, we annex another cipher to 34640, thereby making it 346400, and bring down the next period, making the 17600, 1760000. Dividing the dividend, 1760000, by the trial divisor, 346400, the result is 5. Hence, the next figure of the root is 5, and as we now have five decimal places, we stop.

The square root of 3 is then 1.73205+. In this case, the seventh figure of the root is 0, as may be proved by carrying the division one place farther. In case the seventh figure had been 5 or more than 5, the sixth figure of the root, or 5, would have been increased to 6.

EXAMPLE 2.—What is the square root of .3 to five decimal places?

SOLUTION.—

	.3 0'0 0'0 0'0 0'0 0'0 0 (.5 4 7 7 2+ Ans. (root)
5	25
5	500
100	416
4	8400
104	7609
4	79100
1080	76629
7	247100
1087	219084
7	28016
10940	
7	
10947	
7	
109540	
2	
109542	

EXPLANATION.—In the preceding example we annex a cipher to .3, making the first period .30, since every period of a decimal, as was mentioned before, must have two figures in it. The remainder of the work should be perfectly clear.

4. If it is required to find the square root of a mixed number, begin at the decimal point and point off the periods both to the right and to the left. The manner of finding the root will then be exactly the same as in the previous cases.

EXAMPLE.—What is the square root of 258.2449?

SOLUTION.—

```
                                     root
                2'5 8.2 4'4 9 ( 1 6.0 7   Ans.
   1            1
   1            -
   -            1 5 8
   2 0          1 5 6
     6          -----
   ---            2 2 4 4 9
   2 6            2 2 4 4 9
     6            ---------
   -
   3 2 0 0
         7
   -------
   3 2 0 7
```

EXPLANATION.—In the preceding example, since 320 is greater than 224, we place a cipher for the third figure of the root and annex a cipher to 320, making it 3200. Then, bringing down the next period, 49, 7 is found to be the fourth figure of the root. Since there is no remainder, the square root of 258.2449 is 16.07.

5. PROOF.—To prove square root, square the result obtained. If the number is an exact power, the square of the root will equal it; if it is not an exact power, the square of the root will very nearly equal it.

6. Rule.—I. *Begin at units place and separate the number into periods of two figures each, proceeding from left to right with the decimal part, if there are any.*

II. *Find the greatest number whose square is contained in the first, or left-hand, period. Write this number as the first figure in the root; also, write it at the left of the given number.*

Multiply this number at the left by the first figure of the root, and subtract the result from the first period.

III. *Add the first figure of the root to the number in the first column on the left and annex a cipher to the result; this is the first trial divisor. Annex the second period to the remainder in the second column; this is the first dividend. Divide the dividend by the trial divisor for the second figure in the root and add this figure to the trial divisor to form the complete divisor. Multiply the complete divisor by the second figure in the root and subtract this result from the dividend. (If this result is larger than the dividend, a smaller number must be tried for the second figure of the root.) Add the second figure of the root to the complete divisor. Annex a cipher for a new trial divisor, and bring down the third period and annex it to the last remainder for the second dividend.*

IV. *Continue in this manner to the last period, after which, if any additional places in the root are required, bring down cipher periods and continue the operation.*

V. *If at any time the trial divisor is larger than the dividend, place a cipher in the root, annex a cipher to the trial divisor, and bring down another period.*

VI. *If the root contains an interminable decimal and it is desired to terminate the operation at some point, say, the fourth decimal place, carry the operation one place farther, and if the fifth figure is 5 or greater, increase the fourth figure by 1 and omit the sign +.*

7. Square Roots of Fractions.—If the given number is in the form of a fraction, and it is required to find the square root of it, the simplest and most exact method is to reduce the fraction to a decimal and extract the square root of the decimal. If, however, the numerator and denominator of the fraction are perfect powers, extract the square root of each separately, and write the root of the numerator for a new numerator, and the root of the denominator for a new denominator.

Rule.—*Extract the square root of the numerator and denominator separately; or, reduce the fraction to a decimal, and extract the square root of the decimal.*

EXAMPLE 1.—What is the square root of $\frac{9}{64}$?

SOLUTION.—The square root of $\frac{9}{64}$ is $\sqrt{\frac{9}{64}}$; then,

$$\sqrt{\frac{9}{64}} = \frac{\sqrt{9}}{\sqrt{64}} = \frac{3}{8}. \quad \text{Ans.}$$

EXAMPLE 2.—What is the square root of $\frac{5}{8}$?

SOLUTION.—Reduce the fraction to a decimal; thus, $\frac{5}{8} = .625$. Then the square root of $\frac{5}{8}$ is the square root of .625.

$$\sqrt{.625} = .79057. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

Find the square root of:

		Ans.
(*a*)	186,624.	(*a*) 432
(*b*)	2,050,624.	(*b*) 1,432
(*c*)	29,855,296.	(*c*) 5,464
(*d*)	.0116964.	(*d*) .10815
(*e*)	198.1369.	(*e*) 14.0761
(*f*)	994,009.	(*f*) 997
(*g*)	2.375 to four decimal places.	(*g*) 1.5411
(*h*)	1.625 to three decimal places.	(*h*) 1.275
(*i*)	.3025.	(*i*) .55
(*j*)	.571428.	(*j*) .75593
(*k*)	.78125.	(*k*) .88388
(*l*)	$\frac{25}{256}$.	(*l*) $\frac{5}{16}$
(*m*)	$\frac{9}{16}$.	(*m*) $\frac{3}{4}$

APPLICATIONS OF SQUARE ROOT

8. Finding Diameter of Circle From Area.—The process of finding the square root of a number is very useful in making many calculations, as for example, in finding the diameter of a circle when the area is known, or in finding the length of one side of a right triangle when the lengths of the two remaining sides are known. To find the diameter of a circle, the following rule may be used:

Rule.—*Divide the area of the circle by .7854 and take the square root of the quotient. The result will be the diameter of the circle.*

EXAMPLE 1.—The area of a circle is 38.4846 square inches. What is its diameter?

SOLUTION.—According to the rule, the diameter is

$$\sqrt{\frac{38.4846}{.7854}} = \sqrt{49} = 7 \text{ in.} \quad \text{Ans.}$$

EXAMPLE 2.—What is the diameter of a circle whose area is 1.7671 square feet?

SOLUTION.—According to the rule, the diameter is

$$\sqrt{\frac{1.7671}{.7854}} = \sqrt{2.25} = 1.5 \text{ ft., or } 1\tfrac{1}{2} \text{ ft.} \quad \text{Ans.}$$

9. Principle of Right Triangle.—In any right triangle, the sum of the squares of the lengths of the two sides is equal to the square of the length of the hypotenuse. This may be shown very easily by means of Fig. 1. The triangle *ABC* is a right triangle. The side *AB* is 3 inches long, the side *BC* is 4 inches long, and the hypotenuse *AC* is 5 inches long. Now, suppose that the side *AB* is divided into parts 1 inch long, and then the square *ABIK* is drawn, with *AB* as one side. This square will contain 9 equal squares measuring 1 inch on each side, or 9 square inches.

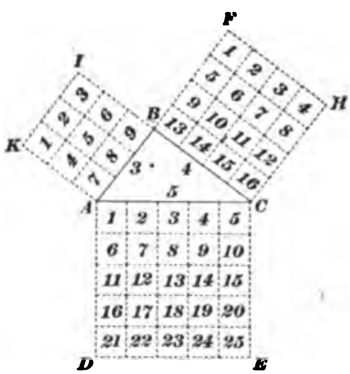


FIG. 1

The square *BCHF* on the side *BC* will contain 16 squares 1 inch on each side, or 16 square inches. The square *ACED* on the hypotenuse will contain 25 square inches, as shown. But, 9+16=25; that is, the sum of the squares of the lengths of the sides is equal to the square of the hypotenuse.

10. The principle stated in the preceding article is a very important one; for, if the lengths of any two lines, as *AB* and BC, Fig. 2, are known, and these lines are at right angles, the distance AC between their ends may be found. This distance is simply the hypotenuse of the right triangle ABC formed by the lines and may be calculated by the following rule:

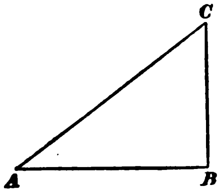


FIG. 2

Rule.—*To find the hypotenuse of a right triangle, square the length of each side, add these squares, and take the square root of the sum.*

EXAMPLE.—In Fig. 2, suppose that the side AB is 8 inches and the side BC is 6 inches long. What is the length of the hypotenuse AC?

SOLUTION.—The square of the length of AB is $8 \times 8 = 64$, and of BC is $6 \times 6 = 36$. Then, applying the rule, $64 + 36 = 100$, and $\sqrt{100} = 10$; that is, the length of AC is 10 in. Ans.

11. It is also possible to calculate the length of one side of a right triangle if the lengths of the other side and the hypotenuse are known. The following rule is used:

Rule.—*To find the length of either side of a right triangle, square the length of the hypotenuse, subtract from it the square of the length of the known side, and take the square root of the remainder.*

EXAMPLE 1.—In Fig. 2, suppose that the hypotenuse AC is 25 inches long, and that the side AB is 20 inches long. Find the length of the side BC.

SOLUTION.—The square of AC is $25^2 = 625$, and the square of AB is $20^2 = 400$. Then, $625 - 400 = 225$, and $\sqrt{225} = 15$; that is, BC is 15 in. long. Ans.

EXAMPLE 2.—In a right triangle, suppose that the hypotenuse is 12 inches long and that one side is 4 inches long. What is the length of the other side?

SOLUTION.—The square of the length of the hypotenuse is $12^2 = 144$, and the square of the length of the known side is $4^2 = 16$. Then, $144 - 16 = 128$, and $\sqrt{128} = 11.31$; hence, the other side is 11.31 in. long. Ans.

EXAMPLE 3.—The hypotenuse and one side of a right triangle measure 15 inches and 5 inches, respectively. Find the length of the other side.

SOLUTION.—Applying the rule, the length of the other side is $\sqrt{15^2 - 5^2} = \sqrt{225 - 25} = \sqrt{200} = 14.14$ in. Ans.

TRIGONOMETRIC CALCULATIONS

12. Tangent of Angle.—The protractor used for measuring and laying off angles usually is not graduated into parts smaller than half a degree; consequently, it is not possible with this instrument to measure or lay off accurately an angle having a fraction of a degree greater or less than half a degree. For example, if the angle to be laid off were 26° 34′, or $26\frac{17}{30}°$, a protractor graduated to half degrees could not be used to get the correct angle. In such a case, a different method would have to be used. One such method is to draw a straight line AB, Fig. 3 (*a*), to represent one side of the angle. This line can be of any length; but in this case it is made 2 inches

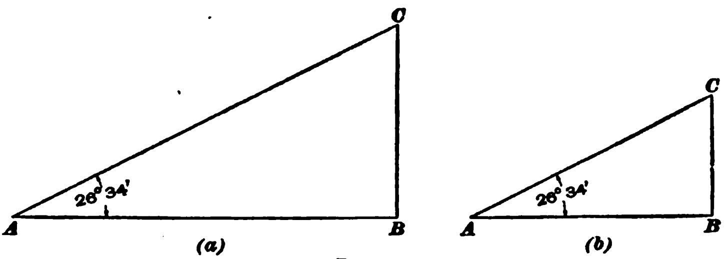


FIG. 3

long. At the end B a perpendicular is drawn, and the distance BC is made equal to .5000 times AB, or $.5000 \times 2 = 1$ inch, and the points A and C are joined by the straight line AC. Then the angle BAC is 26° 34′, as desired. If the line AB had been made $1\frac{1}{4}$ inches long, as shown in (*b*), and BC had been made $.5000 \times 1\frac{1}{4} = .625$ inch, the angle BAC would still have been 26° 34′. In other words, *every* right triangle that has one side .5000 times as long as the other has an acute angle of 26° 34′.

13. In each of the triangles shown in Fig. 3 (*a*) and (*b*), the quotient obtained by dividing the length of the side BC by the length of the side AB is .5000, because $1.0000 \div 2 = .5000$

and $.625 \div 1\frac{1}{4} = .5000$. The side BC is the *side opposite* the angle BAC and the side AB is the *side adjacent*, or next, to the angle BAC. The value of this quotient or ratio of the side opposite to the side adjacent to the angle is called the **tangent** of the angle. It has a certain value for every different angle. For an angle of 26° 34′ it is .5000, as just shown. For an angle of 72° 54′ it is 3.2506; that is, if one angle of a right triangle is 72° 54′, the side opposite the angle has a length equal to the length of the side adjacent multiplied by 3.2506. Thus, for every angle there is a definite value of the tangent, and this value is always the *same* for the *same angle*, no matter how large or how small the right triangle may be.

The values of the tangents of different angles have been found accurately to four decimal places and have been arranged in the form of a table, given at the end of this Section, so that, if the size of an angle is known, its tangent can quickly be found by referring to the table.

EXAMPLE.—In Fig. 3, if AB is 5 inches and BC is $1\frac{7}{8}$ inches, what is the tangent of the angle A?

SOLUTION.—The tangent is the ratio of the side opposite to the side adjacent, or the ratio of BC to AB; hence, it is $1\frac{7}{8} \div 5 = 1.875 \div 5 = .375$. Ans.

14. Cotangent of Angle.—A ratio can be formed between the length of the side adjacent to an angle and the length of the side opposite. Such a ratio is called the **cotangent** of the angle. In Fig. 3 (*a*), for example, the ratio of the side adjacent to the side opposite the angle BAC is the ratio of AB to BC, or 2 : 1, the value of which is $2 \div 1 = 2$; therefore, the cotangent of the angle BAC, or the cotangent of 26° 34′, is 2. But, the ratio of AB to BC, or the cotangent, is the reciprocal of the ratio of BC to AB, or the tangent; in other words, the cotangent is the reciprocal of the tangent, or 1 divided by the tangent. In the same way, the tangent is the reciprocal of the cotangent. For every angle there is a corresponding value of the cotangent, also, and the values of the cotangents have likewise been found and put in the form of a table.

EXAMPLE.— Find the cotangent of an angle if the side adjacent is 3.45 inches and the side opposite is 1.625 inches.

SOLUTION.—The cotangent is the ratio of the side adjacent to the side opposite, or the ratio of 3.45 to 1.625; hence, it is

$$3.45 \div 1.625 = 2.1231. \quad \text{Ans.}$$

15. Sine of Angle.—The two ratios known as the tangent and cotangent are not the only ones that can be used. The ratio of the side opposite the angle to the hypotenuse may also be used. This ratio is called the **sine** of the angle. In the right triangle ABC, Fig. 2, the hypotenuse is AC and the side opposite the angle A is BC. The ratio of the length of the side BC to the length of the hypotenuse AC, or $BC \div AC$, is the sine of the angle A. For every angle there is a corresponding value of the sine, and the values of the sines, also, have been calculated and arranged in the form of a table. For example, the sine of 21° 6′ is .3600. Therefore, if AC were made 5 inches long, and BC were made 1.8 inches long, the angle A would be 21° 6′. For, if $AC = 5$ inches and $BC = 1.8$ inches, then the ratio of BC to AC, or the sine of the angle A, is 1.8 : 5, the value of which is $1.8 \div 5 = .36$, which is the sine of 21° 6′. Therefore, the angle A would be 21° 6′.

EXAMPLE.—In a certain right triangle, the hypotenuse measures 3 inches and the side opposite the angle measures 1.875 inches. Find the sine of the angle.

SOLUTION.—The sine is equal to the ratio of the side opposite to the hypotenuse; hence, it is

$$1.875 \div 3 = .625. \quad \text{Ans.}$$

16. Cosine of Angle.—Still another ratio that is useful in measuring or laying off angles accurately is that called the **cosine,** which is the ratio of the side adjacent to the angle to the hypotenuse. In Fig. 2, for example, the cosine of the angle A is equal to the ratio of AB to AC, or the length of AB divided by the length of AC. For every angle there is a definite value of the cosine, so that, by arranging the values of the cosine in a table, the value for any angle can quickly be found; or, the size of the angle corresponding to a certain cosine can easily be determined from the table.

Example.—If the hypotenuse of a right triangle is 5 inches long and the side adjacent to the angle is 2.4375 inches long, find the cosine of the angle.

Solution.—The cosine is equal to the ratio of the side adjacent to the hypotenuse; hence, it is

$$2.4375 \div 5 = .4875. \quad \text{Ans.}$$

17. To have the various ratios in one group, where they may be found quickly and easily made use of, the following collection is given, the letters referring to the triangle in Fig. 2:

$$\sin A = \frac{BC}{AC} \qquad BC = AC \times \sin A$$

$$\cos A = \frac{AB}{AC} \qquad BC = AB \times \tan A$$

$$\tan A = \frac{BC}{AB} \qquad AB = AC \times \cos A$$

$$\cot A = \frac{AB}{BC} \qquad AB = BC \times \cot A$$

$$AC = \frac{BC}{\sin A}$$

$$AC = \frac{AB}{\cos A}$$

18. Trigonometric Table.—The four ratios of sine, cosine, tangent, and cotangent are the ones most commonly used in shop calculations, and at the end of this Section is given a table of natural sines, cosines, tangents, and cotangents. When writing the names of these various ratios, it is customary to use abbreviations, so as to save time and space; thus, the word *sine* is abbreviated **sin,** *cosine* is abbreviated **cos,** *tangent* is abbreviated **tan,** and *cotangent* is abbreviated **cot.** Then, sin 26° is read *sine of 26°*; cos 15° is read *cosine of 15°*; tan 3° 27′ is read *tangent of 3° 27′*. It should be observed that these abbreviations are *not* followed by periods.

The study of the relations of the sides and angles of triangles is called **trigonometry,** and hence the table giving the values of the sines, cosines, tangents, and cotangents of the various angles is called a *trigonometric table.* It is also termed a *table of trigonometric functions*, because the sine, cosine, tangent, and cotangent are functions of angles; that is, they are quantities whose values depend on the value of the angle.

19. The table at the end of this Section gives the values, to four decimal places, of the sines, cosines, tangents, and cotangents of all angles from 0° to 90°, inclusive. Angles less than 45° are given in the first column at the left-hand edge of the page and the names of the functions are given at the top of the page. Angles greater than 45° are given in a column near the right-hand edge of the page and the names of the functions for these angles are given at the bottom of the page. Thus, the second column on each page gives sines of angles less than 45° and cosines of angles greater than 45°. The fourth column gives tangents of angles less than 45° and cotangents of angles greater than 45°. The eighth column gives cosines of angles less than 45° and sines of angles greater than 45°. The successive angles, given in the first and tenth columns, differ by 10′. They increase downwards in the first column and upwards in the tenth column. Thus, for angles less than 45°, read downwards from the top of the page, and for angles greater than 45°, read upwards from the bottom of the page.

To find some function of an angle less than 45°, look in the first column for the angle and at the top of the page to find the name of the function; and to find some function of an angle greater than 45°, look in the tenth column for the angle and at the bottom of the page to find the name of the function.

20. The third, fifth, seventh, and ninth columns, headed d, contain the differences between the successive values of the sine, cosine, etc. For example, in the second column, the sine of 32° 10′ is .5324 and the sine of 32° 20′ is .5348; the difference is $.5348-.5324=.0024$, and the 24 is written in the third column, just opposite the space between .5324 and .5348. In the same manner, the fifth column gives the differences between successive values of the tangents; and the differences for the cotangents and cosines are given in the seventh and ninth columns. In each case, the difference corresponds to a difference of 10′ in the angle; thus, when the angle 32° 10′ is increased by 10′, that is, to 32° 20′, the increase in the value of the sine is .0024, or, as given in the table, 24. It should be observed that in the table no attention is paid to the decimal point, the difference being simply the number obtained by subtracting the smaller value from the larger, neglecting the ciphers and the decimal point that may precede it.

21. These differences are used to obtain the sines, cosines, etc. of angles not given in the table; the method employed may be illustrated by an example.

EXAMPLE.—Find the tangent of 27° 34′.

SOLUTION.—Looking in the table, we see that the tangent of 27° 30′ is .5206, and, in column 5, the difference for 10′ is 37. The difference for 1′ is $37 \div 10 = 3.7$, and the difference for 4′ is $3.7 \times 4 = 14.8$. Adding this difference to the value of the tan 27° 30′, we have

$$\begin{aligned} \tan 27° 30' &= .5206 \\ \text{difference for } 4' &= \underline{14.8} \\ \tan 27° 34' &= .52208 \text{ or } .5221, \text{ to four places. Ans.} \end{aligned}$$

Since only four decimal places are retained, the 8 in the fifth place is dropped and the figure in the fourth place is increased by 1, because 8 is greater than 5.

22. To avoid multiplication, the column of proportional parts, headed P. P., at the extreme right of the page, is used. At the head of each table in this column is the difference for 10′, and below are the differences for any intermediate number of minutes from 1′ to 9′. In the preceding example, the difference for 10′ was 37; looking in the table with 37 at the head, the difference opposite 4 is 14.8; that opposite 7 is 25.9; and so on. For want of space, the differences for the cotangents for angles less than 45°, or the tangents of angles greater than 45°, have been omitted from the tables of proportional parts. The use of these functions should be avoided, if possible, since the differences change very rapidly, and the computation is therefore likely to be inexact. The method to be employed when dealing with these functions may be shown by an example.

EXAMPLE.—Find the tangent of 76° 34′.

SOLUTION.—Since this angle is greater than 45°, we look for it in the column at the right, and read up; opposite 76° 30′, we find, in the sixth column, the number 4.1653, and corresponding to it in the seventh column is the difference 540. Since 540 is the difference for 10′, the difference for 4′ is $540\times\frac{4}{10}=216$. Adding this difference,

$$\begin{array}{rl} \tan 76°\ 30' = & 4.1653 \\ \text{difference for } 4' = & 216 \\ \hline \tan 76°\ 34' = & 4.1869 \quad \text{Ans.} \end{array}$$

The tangent of an angle is the same as the cotangent of 90° minus that angle; and vice versa. Hence, if the tangent of an angle greater than 45° is desired, there will be a smaller chance of error by finding the cotangent of 90° minus that angle; and if the cotangent of an angle less than 45° is desired, by finding the tangent of 90° minus that angle.

23. Angles greater than 45° will now be considered.

EXAMPLE 1.—Find the sine of 68° 47′.

SOLUTION.—In obtaining the *difference*, it must be remembered to choose the one between the sine of 68° 40′ and the next angle above it, namely, 68° 50′.

$$\begin{array}{rl} \sin 68°\ 40' = & .9315 \\ \text{difference for } 7' = & 7 \\ \hline \sin 68°\ 47' = & .9322 \quad \text{Ans.} \end{array}$$

Difference for 10′ = 10

The tangent is found in the same manner.

EXAMPLE 2.—Find cos 68° 47′.

SOLUTION.—The cosine decreases as the angle increases; therefore, we subtract the successive sine values corresponding to the increments in the angle.

$$\begin{array}{rl} \cos 68°\ 40' = & .3638 \\ \text{difference for } 7' = & 18.9 \\ \hline & .36191 \end{array}$$

Difference for 10′ = 27

Therefore, cos 68° 47′ = .3619, to four decimal places. Ans.

The cotangent is found in the same way.

24. In finding the functions of an angle, the only difficulty likely to be encountered is to determine whether the difference obtained from the table of proportional parts is to be added or subtracted. This can be told in every case by observing whether the function is increasing or decreasing as the angle increases. For example, take the angle 21°; its sine is .3584, and the following sines, reading downwards, are .3611, .3638, etc. It is plain, therefore, that the sine of say 21° 6′ is greater than that of 21°, and that the difference for 6′ must be added. On the other hand, the cosine of 21° is .9336, and the following cosines, reading downwards, are .9325, .9315, etc.; that is, as the angle grows larger the cosine decreases. The cosine of an angle between 21° and 21° 10′, say 21° 6′, must therefore lie between .9336 and .9325; that is, it must be smaller than .9336, which shows that in this case the difference for 6′ must be subtracted from the cosine of 21°.

25. We will now consider the case in which the function, that is, the sine, cosine, tangent, or cotangent, is given and the corresponding angle is to be found.

EXAMPLE 1.—Find the angle whose sine is .4943.

SOLUTION.—The operation is arranged as follows:

	.4943		Difference for 10′=26
	.4924	=sin 29° 30′	
remainder	19		
	18.2=difference for 7′		

29° 30′+7′=29° 37′; therefore, .4943=sin 29° 37′. Ans.

EXPLANATION.—Looking down the second column, we find the sine next *smaller* than .4943 to be .4924, and the difference for 10′ to be 26. The angle corresponding to .4924 is 29° 30′. Subtracting the .4924 from .4943, the remainder is 19; looking in the table of proportional parts, the part nearest this difference is 18.2, opposite which is 7′. Hence, the angle is 29° 30′ +7′=29° 37′.

EXAMPLE 2.—Find the angle whose tangent is .8824.

SOLUTION.—

	.8824		Difference for 10′=51
	.8796	=tan 41° 20′	
remainder	28		
	25.5=difference for 5′		

Therefore, .8824=tan 41° 25′. Ans.

In the two examples just given, the minutes corresponding to the remainders are added to the angle taken from the table. Thus, in the first example, an inspection of the table shows that the angle increases as the sine increases; hence, the angle whose sine is .4943 must be greater than 29° 30′, whose sine is .4924. For this reason the number of minutes corresponding to the difference between .4943 and .4924 must be *added* to 29° 30′. The same reasoning applies to the second example.

EXAMPLE 3.—Find the angle whose cosine is .7742.

SOLUTION.— .7742 Difference for 10′ = 18
.7735 = cos 39° 20′
remainder 7
7.2 = difference for 4′
39° 20′ − 4′ = 39° 16′, which is the angle whose cosine is .7742. Ans.

EXPLANATION.—Looking down the eighth column, headed Cos, the next smaller cosine is .7735, to which corresponds the angle 39° 20′. The difference for 10′ is 18. Subtracting, the remainder is 7, and the nearest number in the table of proportional parts is 7.2, which is the difference for 4′. Hence, the correction for the angle 39° 20′ is 4′. From the table, it appears that, as the cosine increases, the angle grows smaller; therefore, the angle whose cosine is .7742 must be smaller than the angle whose cosine is .7735, and the correction for the angle must be subtracted.

26. The application of trigonometry to the solution of practical problems is illustrated by the following example:

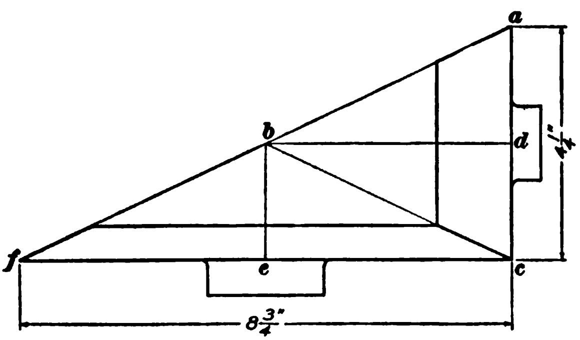


FIG. 4

EXAMPLE.—In Fig. 4 is shown a sketch of portions of two cones that roll together, with some of the leading dimensions. What is the size of the angle *a b c*?

SOLUTION.—The angle *a b c* is twice as large as the angle *a b d*, so we shall find the size of the angle *a b d*, and then multiply it by 2. The triangle *a b d* is a right triangle. The side *a d* opposite the desired angle *a b d* is half the length of *a c*, or half of $4\frac{1}{4}$ in., which is $2\frac{1}{8}$ in. The side *b d* adjacent to the angle is of the same length as *c e*, which is half of *c f*, or $\frac{1}{2}\times 8\frac{3}{4}=4\frac{3}{8}$ in. Now, the side opposite divided by the side adjacent is the tangent of the angle; therefore,

$$\tan a\,b\,d=\frac{a\,d}{b\,d}=\frac{2\frac{1}{8}}{4\frac{3}{8}}=.4857$$

In the table, in the column of tangents, the number .4857 does not appear but .4841 and .4877 are given, and .4857 lies between them. The difference between .4841 and .4857 is .0016 and between .4841 and .4877 is .0036. In the table of proportional parts headed 36, the number nearest 16 is 14.4, which corresponds to 4′. The angle increases as the tangent increases. Then, as .4857 is greater than .4841, the angle corresponding to .4841, or 25° 50′, must be increased by 4′, giving 25° 54′ as the angle *a b d*. The required angle *a b c* is twice the angle *a b d* and is therefore $2\times 25°\ 54'=51°\ 48'$. Ans.

°	′	Sin	d.	Tan	d.	Cot	d.	Cos	d.	′	°
0	0	0.0000		0.0000		infinit.		1.0000		0	90
	10	0.0029	29	0.0029	29	343.7737		1.0000	0	50	
	20	0.0058	29	0.0058	29	171.8854		1.0000	0	40	
	30	0.0087	29	0.0087	29	114.5887		1.0000	0	30	
	40	0.0116	29	0.0116	29	85.9398		0.9999	1	20	
	50	0.0145	29	0.0145	29	68.7501		0.9999	0	10	
1	0	0.0175	30	0.0175	30	57.2900		0.9998	1	0	89
	10	0.0204	29	0.0204	29	49.1039	81861	0.9998	0	50	
	20	0.0233	29	0.0233	29	42.9641	61398	0.9997	1	40	
	30	0.0262	29	0.0262	29	38.1885	47756	0.9997	0	30	
	40	0.0291	29	0.0291	29	34.3678	38207	0.9996	1	20	
	50	0.0320	29	0.0320	29	31.2416	31262	0.9995	1	10	
2	0	0.0349	29	0.0349	29	28.6363	26053	0.9994	1	0	88
	10	0.0378	29	0.0378	29	26.4316	22047	0.9993	1	50	
	20	0.0407	29	0.0407	29	24.5418	18898	0.9992	1	40	
	30	0.0436	29	0.0437	30	22.9038	16380	0.9990	2	30	
	40	0.0465	29	0.0466	29	21.4704	14334	0.9989	1	20	
	50	0.0494	29	0.0495	29	20.2056	12648	0.9988	1	10	
3	0	0.0523	29	0.0524	29	19.0811	11245	0.9986	2	0	87
	10	0.0552	29	0.0553	29	18.0750	10061	0.9985	1	50	
	20	0.0581	29	0.0582	29	17.1693	9057	0.9983	2	40	
	30	0.0610	29	0.0612	30	16.3499	8194	0.9981	2	30	
	40	0.0640	30	0.0641	29	15.6048	7451	0.9980	1	20	
	50	0.0669	29	0.0670	29	14.9244	6804	0.9978	2	10	
4	0	0.0698	29	0.0699	29	14.3007	6237	0.9976	2	0	86
	10	0.0727	29	0.0729	30	13.7267	5740	0.9974	2	50	
	20	0.0756	29	0.0758	29	13.1969	5298	0.9971	3	40	
	30	0.0785	29	0.0787	29	12.7062	4907	0.9969	2	30	
	40	0.0814	29	0.0816	29	12.2505	4557	0.9967	2	20	
	50	0.0843	29	0.0846	30	11.8262	4243	0.9964	3	10	
5	0	0.0872	29	0.0875	29	11.4301	3961	0.9962	2	0	85
	10	0.0901	29	0.0904	29	11.0594	3707	0.9959	3	50	
	20	0.0929	28	0.0934	30	10.7119	3475	0.9957	2	40	
	30	0.0958	29	0.0963	29	10.3854	3265	0.9954	3	30	
	40	0.0987	29	0.0992	29	10.0780	3074	0.9951	3	20	
	50	0.1016	29	0.1022	30	9.7882	2898	0.9948	3	10	
6	0	0.1045	29	0.1051	29	9.5144	2738	0.9945	3	0	84
	10	0.1074	29	0.1080	29	9.2553	2591	0.9942	3	50	
	20	0.1103	29	0.1110	30	9.0098	2455	0.9939	3	40	
	30	0.1132	29	0.1139	29	8.7769	2329	0.9936	3	30	
	40	0.1161	29	0.1169	30	8.5555	2214	0.9932	4	20	
	50	0.1190	29	0.1198	29	8.3450	2105	0.9929	3	10	
7	0	0.1219	29	0.1228	30	8.1443	2007	0.9925	4	0	83
	10	0.1248	29	0.1257	29	7.9530	1913	0.9922	3	50	
	20	0.1276	28	0.1287	30	7.7704	1826	0.9918	4	40	
	30	0.1305	29	0.1317	30	7.5958	1746	0.9914	4	30	
	40	0.1334	29	0.1346	29	7.4287	1671	0.9911	3	20	
	50	0.1363	29	0.1376	30	7.2687	1600	0.9907	4	10	
8	0	0.1392	29	0.1405	29	7.1154	1533	0.9903	4	0	82
	10	0.1421	29	0.1435	30	6.9682	1472	0.9899	4	50	
	20	0.1449	28	0.1465	30	6.8269	1413	0.9894	5	40	
	30	0.1478	29	0.1495	30	6.6912	1357	0.9890	4	30	
	40	0.1507	29	0.1524	29	6.5606	1306	0.9886	4	20	
	50	0.1536	29	0.1554	30	6.4348	1258	0.9881	5	10	
9	0	0.1564	28	0.1584	30	6.3138	1210	0.9877	4	0	81
		Cos	d.	Cot	d.	Tan	d.	Sin	d.	′	°

P. P.

	30	29	28	5	4
1	3.0	2.9	2.8	0.5	0.4
2	6.0	5.8	5.6	1.0	0.8
3	9.0	8.7	8.4	1.5	1.2
4	12.0	11.6	11.2	2.0	1.6
5	15.0	14.5	14.0	2.5	2.0
6	18.0	17.4	16.8	3.0	2.4
7	21.0	20.3	19.6	3.5	2.8
8	24.0	23.2	22.4	4.0	3.2
9	27.0	26.1	25.2	4.5	3.6

° ′	Sin	d.	Tan	d.	Cot	d.	Cos	d.	
9 0	0.1564		0.1584		6.3138		0.9877		0 **81**
		29		30		1168		5	
10	0.1593		0.1614		6.1970		0.9872		50
		29		30		1126		4	
20	0.1622		0.1644		6.0844		0.9868		40
		28		29		1086		5	
30	0.1650		0.1673		5.9758		0.9863		30
		29		30		1050		5	
40	0.1679		0.1703		5.8708		0.9858		20
		29		30		1014		5	
50	0.1708		0.1733		5.7694		0.9853		10
		28		30		981		5	
10 0	0.1736		0.1763		5.6713		0.9848		0 **80**
		29		30		949		5	
10	0.1765		0.1793		5.5764		0.9843		50
		29		30		919		5	
20	0.1794		0.1823		5.4845		0.9838		40
		28		30		890		5	
30	0.1822		0.1853		5.3955		0.9833		30
		29		30		862		6	
40	0.1851		0.1883		5.3093		0.9827		20
		29		31		836		5	
50	0.1880		0.1914		5.2257		0.9822		10
		28		30		811		6	
11 0	0.1908		0.1944		5.1446		0.9816		0 **79**
		29		30		788		5	
10	0.1937		0.1974		5.0658		0.9811		50
		28		30		764		6	
20	0.1965		0.2004		4.9894		0.9805		40
		29		31		742		6	
30	0.1994		0.2035		4.9152		0.9799		30
		28		30		722		6	
40	0.2022		0.2065		4.8430		0.9793		20
		29		30		701		6	
50	0.2051		0.2095		4.7729		0.9787		10
		28		31		683		6	
12 0	0.2079		0.2126		4.7046		0.9781		0 **78**
		29		30		664		6	
10	0.2108		0.2156		4.6382		0.9775		50
		28		30		646		6	
20	0.2136		0.2186		4.5736		0.9769		40
		28		31		629		6	
30	0.2164		0.2217		4.5107		0.9763		30
		29		30		613		6	
40	0.2193		0.2247		4.4494		0.9757		20
		28		31		597		7	
50	0.2221		0.2278		4.3897		0.9750		10
		29		31		582		6	
13 0	0.2250		0.2309		4.3315		0.9744		0 **77**
		28		30		568		7	
10	0.2278		0.2339		4.2747		0.9737		50
		28		31		554		7	
20	0.2306		0.2370		4.2193		0.9730		40
		28		31		540		6	
30	0.2334		0.2401		4.1653		0.9724		30
		29		31		527		7	
40	0.2363		0.2432		4.1126		0.9717		20
		28		30		515		7	
50	0.2391		0.2462		4.0611		0.9710		10
		28		31		503		7	
14 0	0.2419		0.2493		4.0108		0.9703		0 **76**
		28		31		491		7	
10	0.2447		0.2524		3.9617		0.9696		50
		29		31		481		7	
20	0.2476		0.2555		3.9136		0.9689		40
		28		31		469		8	
30	0.2504		0.2586		3.8667		0.9681		30
		28		31		459		7	
40	0.2532		0.2617		3.8208		0.9674		20
		28		31		448		7	
50	0.2560		0.2648		3.7760		0.9667		10
		28		31		439		8	
15 0	0.2588		0.2679		3.7321		0.9659		0 **75**
		28		32		430		7	
10	0.2616		0.2711		3.6891		0.9652		50
		28		31		421		8	
20	0.2644		0.2742		3.6470		0.9644		40
		28		31		411		8	
30	0.2672		0.2773		3.6059		0.9636		30
		28		32		403		8	
40	0.2700		0.2805		3.5656		0.9628		20
		28		31		395		7	
50	0.2728		0.2836		3.5261		0.9621		10
		28		31		387		8	
16 0	0.2756		0.2867		3.4874		0.9613		0 **74**
		28		32		379		8	
10	0.2784		0.2899		3.4495		0.9605		50
		28		32		371		9	
20	0.2812		0.2931		3.4124		0.9596		40
		28		31		365		8	
30	0.2840		0.2962		3.3759		0.9588		30
		28		32		357		8	
40	0.2868		0.2994		3.3402		0.9580		20
		28		32		350		8	
50	0.2896		0.3026		3.3052		0.9572		10
		28		31		343		9	
17 0	0.2924		0.3057		3.2709		0.9563		0 **73**
		28		32		338		8	
10	0.2952		0.3089		3.2371		0.9555		50
		27		32		330		9	
20	0.2979		0.3121		3.2041		0.9546		40
		28		32		325		9	
30	0.3007		0.3153		3.1716		0.9537		30
		28		32		319		9	
40	0.3035		0.3185		3.1397		0.9528		20
		27		32		313		8	
50	0.3062		0.3217		3.1084		0.9520		10
		28		32		307		9	
18 0	0.3090		0.3249		3.0777		0.9511		0 **72**
	Cos	d.	Cot	d.	Tan	d.	Sin	d.	′ °

P. P.

	32	31	30
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

	29	28	27
1	2.9	2.8	2.7
2	5.8	5.6	5.4
3	8.7	8.4	8.1
4	11.6	11.2	10.8
5	14.5	14.0	13.5
6	17.4	16.8	16.2
7	20.3	19.6	18.9
8	23.2	22.4	21.6
9	26.1	25.2	24.3

	9	8
1	0.9	0.8
2	1.8	1.6
3	2.7	2.4
4	3.6	3.2
5	4.5	4.0
6	5.4	4.8
7	6.3	5.6
8	7.2	6.4
9	8.1	7.2

	7	6
1	0.7	0.6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

	5	4
1	0.5	0.4
2	1.0	0.8
3	1.5	1.2
4	2.0	1.6
5	2.5	2.0
6	3.0	2.4
7	3.5	2.8
8	4.0	3.2
9	4.5	3.6

P. P.

°	′	Sin	d.	Tan	d.	Cot	d.	Cos.	d.		
18	0	0.3090	28	0.3249	32	3.0777	302	0.9511	9	0	**72**
	10	0.3118	27	0.3281	33	3.0475	297	0.9502	10	50	
	20	0.3145	28	0.3314	32	3.0178	291	0.9492	9	40	
	30	0.3173	28	0.3346	32	2.9887	287	0.9483	9	30	
	40	0.3201	27	0.3378	33	2.9600	281	0.9474	9	20	
	50	0.3228	28	0.3411	32	2.9319	277	0.9465	10	10	
19	0	0.3256	27	0.3443	33	2.9042	272	0.9455	9	0	**71**
	10	0.3283	28	0.3476	32	2.8770	268	0.9446	10	50	
	20	0.3311	27	0.3508	33	2.8502	263	0.9436	10	40	
	30	0.3338	27	0.3541	33	2.8239	259	0.9426	9	30	
	40	0.3365	28	0.3574	33	2.7980	255	0.9417	10	20	
	50	0.3393	27	0.3607	33	2.7725	250	0.9407	10	10	
20	0	0.3420	28	0.3640	33	2.7475	247	0.9397	10	0	**70**
	10	0.3448	27	0.3673	33	2.7228	243	0.9387	10	50	
	20	0.3475	27	0.3706	33	2.6985	239	0.9377	10	40	
	30	0.3502	27	0.3739	33	2.6746	235	0.9367	11	30	
	40	0.3529	28	0.3772	33	2.6511	232	0.9356	10	20	
	50	0.3557	27	0.3805	34	2.6279	228	0.9346	10	10	
21	0	0.3584	27	0.3839	33	2.6051	225	0.9336	11	0	**69**
	10	0.3611	27	0.3872	34	2.5826	221	0.9325	10	50	
	20	0.3638	27	0.3906	33	2.5605	219	0.9315	11	40	
	30	0.3665	27	0.3939	34	2.5386	214	0.9304	11	30	
	40	0.3692	27	0.3973	33	2.5172	212	0.9293	10	20	
	50	0.3719	27	0.4006	34	2.4960	209	0.9283	11	10	
22	0	0.3746	27	0.4040	34	2.4751	206	0.9272	11	0	**68**
	10	0.3773	27	0.4074	34	2.4545	203	0.9261	11	50	
	20	0.3800	27	0.4108	34	2.4342	200	0.9250	11	40	
	30	0.3827	27	0.4142	34	2.4142	197	0.9239	11	30	
	40	0.3854	27	0.4176	34	2.3945	195	0.9228	12	20	
	50	0.3881	26	0.4210	35	2.3750	191	0.9216	11	10	
23	0	0.3907	27	0.4245	34	2.3559	190	0.9205	11	0	**67**
	10	0.3934	27	0.4279	35	2.3369	186	0.9194	12	50	
	20	0.3961	26	0.4314	34	2.3183	185	0.9182	11	40	
	30	0.3987	27	0.4348	35	2.2998	181	0.9171	12	30	
	40	0.4014	27	0.4383	34	2.2817	180	0.9159	12	20	
	50	0.4041	26	0.4417	35	2.2637	177	0.9147	12	10	
24	0	0.4067	27	0.4452	35	2.2460	174	0.9135	11	0	**66**
	10	0.4094	26	0.4487	35	2.2286	173	0.9124	12	50	
	20	0.4120	27	0.4522	35	2.2113	170	0.9112	12	40	
	30	0.4147	26	0.4557	35	2.1943	168	0.9100	12	30	
	40	0.4173	27	0.4592	36	2.1775	166	0.9088	13	20	
	50	0.4200	26	0.4628	35	2.1609	164	0.9075	12	10	
25	0	0.4226	27	0.4663	36	2.1445	162	0.9063	12	0	**65**
	10	0.4253	26	0.4699	35	2.1283	160	0.9051	13	50	
	20	0.4279	26	0.4734	36	2.1123	158	0.9038	12	40	
	30	0.4305	26	0.4770	36	2.0965	156	0.9026	13	30	
	40	0.4331	27	0.4806	35	2.0809	154	0.9013	12	20	
	50	0.4358	26	0.4841	36	2.0655	152	0.9001	13	10	
26	0	0.4384	26	0.4877	36	2.0503	150	0.8988	13	0	**64**
	10	0.4410	26	0.4913	37	2.0353	149	0.8975	13	50	
	20	0.4436	26	0.4950	36	2.0204	147	0.8962	13	40	
	30	0.4462	26	0.4986	36	2.0057	145	0.8949	13	30	
	40	0.4488	26	0.5022	37	1.9912	144	0.8936	13	20	
	50	0.4514	26	0.5059	36	1.9768	142	0.8923	13	10	
27	0	0.4540		0.5095		1.9626		0.8910		0	**63**
		Cos	d.	Cot	d.	Tan	d.	Sin	d.	′	°

P. P.

	37	**36**	**35**
1	3.7	3.6	3.5
2	7.4	7.2	7.0
3	11.1	10.8	10.5
4	14.8	14.4	14.0
5	18.5	18.0	17.5
6	22.2	21.6	21.0
7	25.9	25.2	24.5
8	29.6	28.8	28.0
9	33.3	32.4	31.5

	34	**33**	**32**
1	3.4	3.3	3.2
2	6.8	6.6	6.4
3	10.2	9.9	9.6
4	13.6	13.2	12.8
5	17.0	16.5	16.0
6	20.4	19.8	19.2
7	23.8	23.1	22.4
8	27.2	26.4	25.6
9	30.6	29.7	28.8

	28	**27**	**26**
1	2.8	2.7	2.6
2	5.6	5.4	5.2
3	8.4	8.1	7.8
4	11.2	10.8	10.4
5	14.0	13.5	13.0
6	16.8	16.2	15.6
7	19.6	18.9	18.2
8	22.4	21.6	20.8
9	25.2	24.3	23.4

	13	**12**
1	1.3	1.2
2	2.6	2.4
3	3.9	3.6
4	5.2	4.8
5	6.5	6.0
6	7.8	7.2
7	9.1	8.4
8	10.4	9.6
9	11.7	10.8

	11	10	9
1	1.1	1.0	0.9
2	2.2	2.0	1.8
3	3.3	3.0	2.7
4	4.4	4.0	3.6
5	5.5	5.0	4.5
6	6.6	6.0	5.4
7	7.7	7.0	6.3
8	8.8	8.0	7.2
9	9.9	9.0	8.1

P. P.

° ′	Sin	d.	Tan	d.	Cot	d.	Cos	d.	
27 0	0.4540	26	0.5095	37	1.9626	140	0.8910	13	0 63
10	0.4566	26	0.5132	37	1.9486	139	0.8897	13	50
20	0.4592	25	0.5169	37	1.9347	137	0.8884	14	40
30	0.4617	26	0.5206	37	1.9210	136	0.8870	13	30
40	0.4643	26	0.5243	37	1.9074	134	0.8857	14	20
50	0.4669	26	0.5280	37	1.8940	133	0.8843	14	10
28 0	0.4695	25	0.5317	37	1.8807	131	0.8829	13	0 62
10	0.4720	26	0.5354	38	1.8676	130	0.8816	14	50
20	0.4746	26	0.5392	38	1.8546	128	0.8802	14	40
30	0.4772	25	0.5430	37	1.8418	127	0.8788	14	30
40	0.4797	26	0.5467	38	1.8291	126	0.8774	14	20
50	0.4823	25	0.5505	38	1.8165	125	0.8760	14	10
29 0	0.4848	26	0.5543	38	1.8040	123	0.8746	14	0 61
10	0.4874	25	0.5581	38	1.7917	121	0.8732	14	50
20	0.4899	25	0.5619	39	1.7796	121	0.8718	14	40
30	0.4924	26	0.5658	38	1.7675	119	0.8704	15	30
40	0.4950	25	0.5696	39	1.7556	119	0.8689	14	20
50	0.4975	25	0.5735	39	1.7437	116	0.8675	15	10
30 0	0.5000	25	0.5774	38	1.7321	116	0.8660	14	0 60
10	0.5025	25	0.5812	39	1.7205	115	0.8646	15	50
20	0.5050	25	0.5851	39	1.7090	113	0.8631	15	40
30	0.5075	25	0.5890	40	1.6977	113	0.8616	15	30
40	0.5100	25	0.5930	39	1.6864	111	0.8601	14	20
50	0.5125	25	0.5969	40	1.6753	110	0.8587	15	10
31 0	0.5150	25	0.6009	39	1.6643	109	0.8572	15	0 59
10	0.5175	25	0.6048	40	1.6534	108	0.8557	15	50
20	0.5200	25	0.6088	40	1.6426	107	0.8542	16	40
30	0.5225	25	0.6128	40	1.6319	107	0.8526	15	30
40	0.5250	25	0.6168	40	1.6212	105	0.8511	15	20
50	0.5275	24	0.6208	41	1.6107	104	0.8496	16	10
32 0	0.5299	25	0.6249	40	1.6003	103	0.8480	15	0 58
10	0.5324	24	0.6289	41	1.5900	102	0.8465	15	50
20	0.5348	25	0.6330	41	1.5798	101	0.8450	16	40
30	0.5373	25	0.6371	41	1.5697	100	0.8434	16	30
40	0.5398	24	0.6412	41	1.5597	100	0.8418	15	20
50	0.5422	24	0.6453	41	1.5497	98	0.8403	16	10
33 0	0.5446	25	0.6494	42	1.5399	98	0.8387	16	0 57
10	0.5471	24	0.6536	41	1.5301	97	0.8371	16	50
20	0.5495	24	0.6577	42	1.5204	96	0.8355	16	40
30	0.5519	25	0.6619	42	1.5108	95	0.8339	16	30
40	0.5544	24	0.6661	42	1.5013	94	0.8323	16	20
50	0.5568	24	0.6703	42	1.4919	93	0.8307	17	10
34 0	0.5592	24	0.6745	42	1.4826	93	0.8290	16	0 56
10	0.5616	24	0.6787	43	1.4733	92	0.8274	16	50
20	0.5640	24	0.6830	43	1.4641	91	0.8258	17	40
30	0.5664	24	0.6873	43	1.4550	90	0.8241	16	30
40	0.5688	24	0.6916	43	1.4460	90	0.8225	17	20
50	0.5712	24	0.6959	43	1.4370	89	0.8208	16	10
35 0	0.5736	24	0.7002	44	1.4281	88	0.8192	17	0 55
10	0.5760	23	0.7046	43	1.4193	87	0.8175	17	50
20	0.5783	24	0.7089	44	1.4106	87	0.8158	17	40
30	0.5807	24	0.7133	44	1.4019	85	0.8141	17	30
40	0.5831	23	0.7177	44	1.3934	86	0.8124	17	20
50	0.5854	24	0.7221	44	1.3848	84	0.8107	17	10
36 0	0.5878		0.7265		1.3764		0.8090		0 54

	Cos	d.	Cot	d.	Tan	d.	Sin	d.	′ °

P. P.

	44	43	42	41	40	39	38	37
1	4.4	4.3	4.2	4.1	4.0	3.9	3.8	3.7
2	8.8	8.6	8.4	8.2	8.0	7.8	7.6	7.4
3	13.2	12.9	12.6	12.3	12.0	11.7	11.4	11.1
4	17.6	17.2	16.8	16.4	16.0	15.6	15.2	14.8
5	22.0	21.5	21.0	20.5	20.0	19.5	19.0	18.5
6	26.4	25.8	25.2	24.6	24.0	23.4	22.8	22.2
7	30.8	30.1	29.4	28.7	28.0	27.3	26.6	25.9
8	35.2	34.4	33.6	32.8	32.0	31.2	30.4	29.6
9	39.6	38.7	37.8	36.9	36.0	35.1	34.2	33.3

	26	25	24	23	17	16	15	14	13
1	2.6	2.5	2.4	2.3	1.7	1.6	1.5	1.4	1.3
2	5.2	5.0	4.8	4.6	3.4	3.2	3.0	2.8	2.6
3	7.8	7.5	7.2	6.9	5.1	4.8	4.5	4.2	3.9
4	10.4	10.0	9.6	9.2	6.8	6.4	6.0	5.6	5.2
5	13.0	12.5	12.0	11.5	8.5	8.0	7.5	7.0	6.5
6	15.6	15.0	14.4	13.8	10.2	9.6	9.0	8.4	7.8
7	18.2	17.5	16.8	16.1	11.9	11.2	10.5	9.8	9.1
8	20.8	20.0	19.2	18.4	13.6	12.8	12.0	11.2	10.4
9	23.4	22.5	21.6	20.7	15.3	14.4	13.5	12.6	11.7

° ′	Sin	d.	Tan	d.	Cot	d.	Cos	d.	
36 0	0.5878		0.7265		1.3764		0.8090		0 54
10	0.5901	23	0.7310	45	1.3680	84	0.8073	17	50
20	0.5925	24	0.7355	45	1.3597	83	0.8056	17	40
30	0.5948	23	0.7400	45	1.3514	83	0.8039	17	30
40	0.5972	24	0.7445	45	1.3432	82	0.8021	18	20
50	0.5995	23	0.7490	45	1.3351	81	0.8004	17	10
37 0	0.6018	23	0.7536	46	1.3270	81	0.7986	18	0 53
10	0.6041	23	0.7581	45	1.3190	80	0.7969	17	50
20	0.6065	24	0.7627	46	1.3111	79	0.7951	18	40
30	0.6088	23	0.7673	46	1.3032	79	0.7934	17	30
40	0.6111	23	0.7720	47	1.2954	78	0.7916	18	20
50	0.6134	23	0.7766	46	1.2876	78	0.7898	18	10
38 0	0.6157	23	0.7813	47	1.2799	77	0.7880	18	0 52
10	0.6180	23	0.7860	47	1.2723	76	0.7862	18	50
20	0.6202	22	0.7907	47	1.2647	76	0.7844	18	40
30	0.6225	23	0.7954	47	1.2572	75	0.7826	18	30
40	0.6248	23	0.8002	48	1.2497	75	0.7808	18	20
50	0.6271	23	0.8050	48	1.2423	74	0.7790	18	10
39 0	0.6293	22	0.8098	48	1.2349	74	0.7771	19	0 51
10	0.6316	23	0.8146	48	1.2276	73	0.7753	18	50
20	0.6338	22	0.8195	49	1.2203	73	0.7735	18	40
30	0.6361	23	0.8243	48	1.2131	72	0.7716	19	30
40	0.6383	22	0.8292	49	1.2059	72	0.7698	18	20
50	0.6406	23	0.8342	50	1.1988	71	0.7679	19	10
40 0	0.6428	22	0.8391	49	1.1918	70	0.7660	19	0 50
10	0.6450	22	0.8441	50	1.1847	71	0.7642	18	50
20	0.6472	22	0.8491	50	1.1778	69	0.7623	19	40
30	0.6494	22	0.8541	50	1.1708	70	0.7604	19	30
40	0.6517	23	0.8591	50	1.1640	68	0.7585	19	20
50	0.6539	22	0.8642	51	1.1571	69	0.7566	19	10
41 0	0.6561	22	0.8693	51	1.1504	67	0.7547	19	0 49
10	0.6583	22	0.8744	51	1.1436	68	0.7528	19	50
20	0.6604	21	0.8796	52	1.1369	67	0.7509	19	40
30	0.6626	22	0.8847	51	1.1303	66	0.7490	19	30
40	0.6648	22	0.8899	52	1.1237	66	0.7470	20	20
50	0.6670	22	0.8952	53	1.1171	66	0.7451	19	10
42 0	0.6691	21	0.9004	52	1.1106	65	0.7431	20	0 48
10	0.6713	22	0.9057	53	1.1041	65	0.7412	19	50
20	0.6734	21	0.9110	53	1.0977	64	0.7392	20	40
30	0.6756	22	0.9163	53	1.0913	64	0.7373	19	30
40	0.6777	21	0.9217	54	1.0850	63	0.7353	20	20
50	0.6799	22	0.9271	54	1.0786	64	0.7333	20	10
43 0	0.6820	21	0.9325	54	1.0724	62	0.7314	19	0 47
10	0.6841	21	0.9380	55	1.0661	63	0.7294	20	50
20	0.6862	21	0.9435	55	1.0599	62	0.7274	20	40
30	0.6884	22	0.9490	55	1.0538	61	0.7254	20	30
40	0.6905	21	0.9545	55	1.0477	61	0.7234	20	20
50	0.6926	21	0.9601	56	1.0416	61	0.7214	20	10
44 0	0.6947	21	0.9657	56	1.0355	61	0.7193	21	0 46
10	0.6967	20	0.9713	56	1.0295	60	0.7173	20	50
20	0.6988	21	0.9770	57	1.0235	60	0.7153	20	40
30	0.7009	21	0.9827	57	1.0176	59	0.7133	20	30
40	0.7030	21	0.9884	57	1.0117	59	0.7112	21	20
50	0.7050	20	0.9942	58	1.0058	59	0.7092	20	10
45 0	0.7071	21	1.0000	58	1.0000	58	0.7071	21	0 45
	Cos	d.	Cot	d.	Tan	d.	Sin	d.	′ °

P. P.

	58	57	56	55
1	5.8	5.7	5.6	5.5
2	11.6	11.4	11.2	11.0
3	17.4	17.1	16.8	16.5
4	23.2	22.8	22.4	22.0
5	29.0	28.5	28.0	27.5
6	34.8	34.2	33.6	33.0
7	40.6	39.9	39.2	38.5
8	46.4	45.6	44.8	44.0
9	52.2	51.3	50.4	49.5

	54	53	52	51
1	5.4	5.3	5.2	5.1
2	10.8	10.6	10.4	10.2
3	16.2	15.9	15.6	15.3
4	21.6	21.2	20.8	20.4
5	27.0	26.5	26.0	25.5
6	32.4	31.8	31.2	30.6
7	37.8	37.1	36.4	35.7
8	43.2	42.4	41.6	40.8
9	48.6	47.7	46.8	45.9

	50	49	48
1	5.0	4.9	4.8
2	10.0	9.8	9.6
3	15.0	14.7	14.4
4	20.0	19.6	19.2
5	25.0	24.5	24.0
6	30.0	29.4	28.8
7	35.0	34.3	33.6
8	40.0	39.2	38.4
9	45.0	44.1	43.2

	47	46	45
1	4.7	4.6	4.5
2	9.4	9.2	9.0
3	14.1	13.8	13.5
4	18.8	18.4	18.0
5	23.5	23.0	22.5
6	28.2	27.6	27.0
7	32.9	32.2	31.5
8	37.6	36.8	36.0
9	42.3	41.4	40.5

	24	23	22	21
1	2.4	2.3	2.2	2.1
2	4.8	4.6	4.4	4.2
3	7.2	6.9	6.6	6.3
4	9.6	9.2	8.8	8.4
5	12.0	11.5	11.0	10.5
6	14.4	13.8	13.2	12.6
7	16.8	16.1	15.4	14.7
8	19.2	18.4	17.6	16.8
9	21.6	20.7	19.8	18.9

	20	19	18	17
1	2.0	1.9	1.8	1.7
2	4.0	3.8	3.6	3.4
3	6.0	5.7	5.4	5.1
4	8.0	7.6	7.2	6.8
5	10.0	9.5	9.0	8.5
6	12.0	11.4	10.8	10.2
7	14.0	13.3	12.6	11.9
8	16.0	15.2	14.4	13.6
9	18.0	17.1	16.2	15.3

P. P.

READING WORKING DRAWINGS

REPRESENTATION OF OBJECTS

WORKING DRAWINGS

GENERAL PRINCIPLES

1. A **drawing** is a representation of an object by means of lines. A **mechanical drawing** consists of one or more views of an object, so made and arranged as to show the shape of the object, its dimensions, the way in which its parts are put together, and the material or materials of which it is made. Mechanical drawings form a universal language, so far as engineers and mechanics are concerned. A mechanical drawing that is used as a guide in making the parts of a machine or in putting those parts together properly is called a **working drawing.**

2. Comparison of Perspective and Mechanical Drawings.—A drawing made to show the shape of an object by a single view is called a **perspective drawing**. It represents the object as it would appear to the eye, or as in a photograph; but though it shows clearly the general form of the object, it cannot be used successfully as a working drawing, because it does not show all the dimensions of the object in their true lengths. A mechanical drawing does not represent an object as it would appear to the eye, but it is the kind of drawing that is universally used as a working drawing. The reasons for its adoption are that it may be made more easily than a perspective drawing, it shows the correct shape of the object, and it gives the true dimensions of the various parts. Perspective drawings, however, show the form of an object so well that they are used considerably in this Section in connection with working drawings.

3. Reading of Drawings.—The reading of working drawings must not be confused with the art of making the drawings. In order to read a working drawing, that is, to understand clearly just what the drawing is intended to show, it is necessary to know what is meant by each different kind of line, by the arrangement of the different views, and by the lettering, abbreviations, signs, etc. used on the drawing. This is rendered easier if the reader is also familiar with mechanical terms and shop methods. In order to make a drawing, however, the draftsman must know, in addition to all the foregoing matters, how to use drawing instruments. The purpose of this Section is to explain how to read working drawings, and not how to make them; hence, it is not necessary to know anything about the use of drawing instruments.

BLUEPRINTS

4. Purpose of Blueprints.—The working drawing prepared by the draftsman is made on heavy paper, with a lead pencil. If this original drawing were sent to the shop to be used as a working drawing from which to construct the piece of work, it would soon become soiled or torn, and the pencil lines would be rubbed out to such an extent that it would be useless for the desired purpose. It would then be necessary to repeat the work of the draftsman and make a fresh drawing. To avoid this unnecessary labor and expense it is customary to use blueprints as the working drawings in the shop, and to preserve either the original drawing or a copy of it. The common form of blueprint is a sheet of paper one side of which is blue and on which side, in white lines, is an exact duplicate of the original working drawing. Blueprints of this kind have the advantage of being cheaply and quickly made; thus, if one becomes so soiled or torn that it is hard to read, it can easily be replaced by a new print.

5. Making a Blueprint.—The first step in making a blueprint of a drawing is to make a *tracing* of the drawing. The pencil drawing is covered with a sheet of transparent paper or cloth, known as *tracing paper* or *tracing cloth*, and on this transparent sheet a copy, or tracing, of the original drawing is made with black ink. This tracing is then an exact duplicate of the drawing, and contains all the views of the object, the notes, title, dimensions, scale, and any other necessary marks. The tracing is next laid flat on a sheet of blueprint paper, the face of which is coated with a chemical solution that, when dry, gives the paper a yellowish color. The chemical coating is very easily affected by light. Therefore, the blueprint sheet, with the tracing on it, is exposed for a short time to electric light or to the sun after which the tracing is removed and the blueprint paper is washed in water, and dried. Wherever a black line was shown on the tracing, a white line now appears on the blueprint, and the blank, or white, spaces as seen on the tracing, appear blue on the print. This print is in every detail an exact duplicate of the tracing. Any desired number of prints may be made from the tracing.

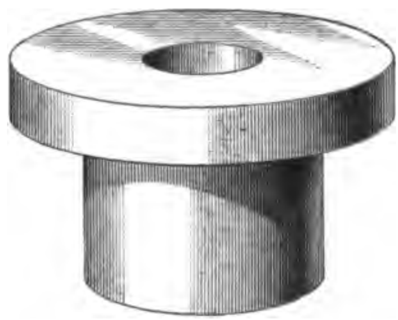


FIG. 1

VIEWS OF AN OBJECT

6. Elementary Working Drawing.—A perspective drawing of a plain bushing is shown in Fig. 1, and a working drawing of the same piece, in Fig. 2. The working drawing shows two views, that in (*a*) being a *side view* and that in (*b*) a *plan*, or *top view*. The side view, sometimes called the *side elevation*, shows the piece as it would appear when looked at squarely from one side; or, it shows the outlines of the piece as they would appear if the piece were stood upright against a sheet of paper held vertically and its form were traced against the sheet. The plan, or top view, shows the appearance of the piece when looked at from a point directly above.

7. The dimensions are marked on the working drawing, Fig. 2, so that the two views and the dimensions tell the exact shape of the piece and the size of each part of it. The sign " written above and to the right of each dimension means inches; thus, $1\frac{1}{2}''$ means $1\frac{1}{2}$ inches. The plan shows three circles, one of which is dotted. The circle *a* is the top of the hole that runs through the piece from top to bottom, and in the side view this hole is indicated by the two parallel dotted lines *b* and *c*, which run from top to bottom and are marked as being $\frac{5}{8}$ inch apart; therefore, it is understood that the hole is round and is $\frac{5}{8}$ inch in diameter. The circle *d* is the top view of the flange, or collar, *e* and shows that the flange is round; it is 2 inches in diameter.

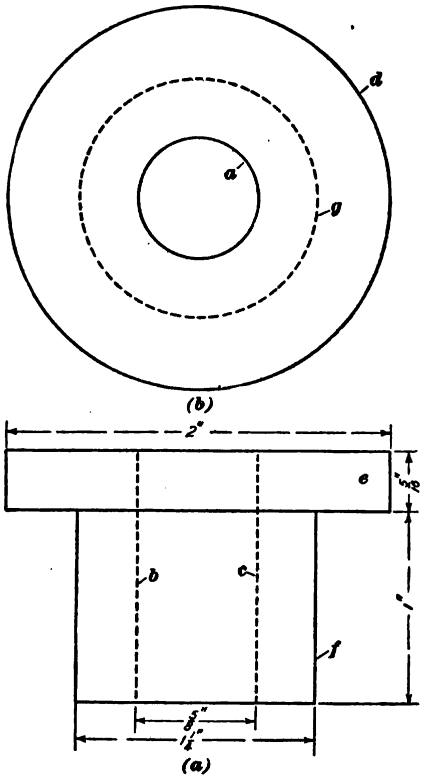


FIG. 2

The thickness of the flange is $\frac{5}{16}$ inch, marked in the side view. The part *f* that projects below the flange is 1 inch long, as marked in the side view, and is indicated in the plan by the dotted circle *g*. This circle indicates that the part *f* is round, also, and the side view gives $1\frac{1}{4}$ inches as the diameter of it.

Thus, this working drawing gives all the information needed to enable the workman to make the bushing.

8. Number of Views on a Working Drawing.—The purpose of the working drawing is to furnish enough information as to the shape and size of an object to enable the workman to make it correctly. The number of views to be given on the working drawing depends on the nature of the object. Usually two or three views are enough to show the form and all the necessary dimensions, and in some cases only one view is needed; on the other hand, if the object is intricate or of peculiar shape, four or more views may be required. In Fig. 2, for example, two views show all that is required. In Fig. 3, one view gives the necessary information for making a pin having the dimensions indicated. The use of the abbreviation *Dia.*, meaning diameter, shows that the head of the pin and the shank are round, and not square or of some other shape.

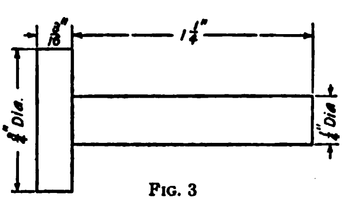


FIG. 3

9. Arrangement of Views.—The different views of the object are arranged on the working drawing according to either the American or the British system. In the American system, one of the views is taken as the principal view and the others are grouped around it, each being placed next to the side that it represents. This may be shown more clearly by an example. In Fig. 4 is illustrated a wedge-shaped block, and in Fig. 5, a mechanical drawing of the block showing five views arranged according to the American system. The front view (*a*) is taken as the central view, and the others are placed around it. Thus, the view (*b*) of the right-hand face is placed at the right, and the view (*c*) of the left-hand face at the left. The top view, or plan, (*d*) is

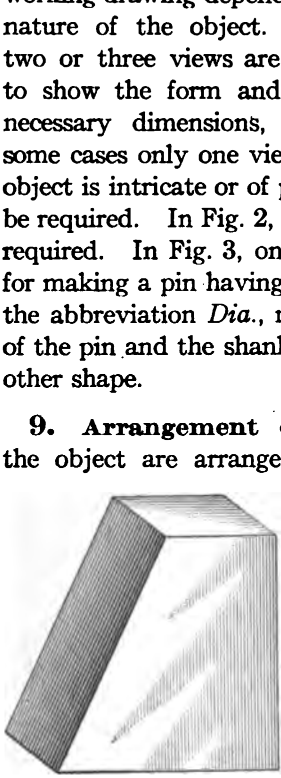


FIG. 4

put above the front view and the bottom view (*e*) is placed below. In this way, each view is at that side, or face, which it represents.

10. The American system of arranging the views on a mechanical drawing is often called the *third-angle method*, and the British system is called the *first-angle method*. The British method, which is used in Great Britain and on the European continent, is the opposite of the American practice. For instance, in the British system, the view (*b*) of the right-hand face of the block, Fig. 5, would be placed at the left of the front view (*a*), and the view (*c*) of the left-hand face would be placed at the right of view (*a*). Similarly, the top view would be placed below the front view and the bottom view would be placed above it. The five views shown in this drawing are not necessary, as the shape of the block could be fully indicated by two views, namely, the front view (*a*) and the top view (*d*). However, the illustration serves to show the positions in which the several views should be placed when more than one or two are necessary.

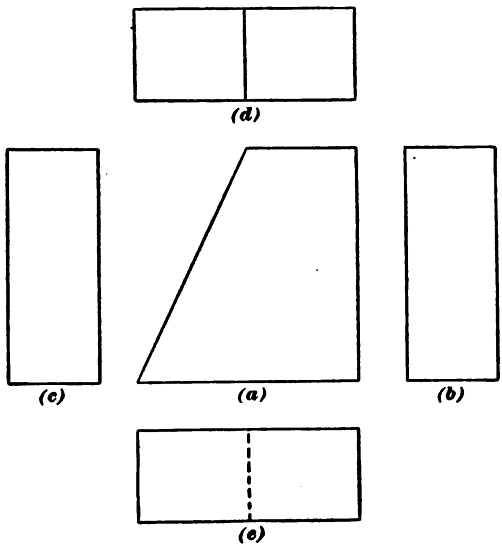


FIG. 5

11. Relation of Views.—The relation that the different views of an object bear to one another, when arranged according to the American system, may be made clearer by the method illustrated in Fig. 6. In (*a*) is shown a plate of glass *a* beveled at the edges and hinged to four similar glass plates *b*, *c*, *d*, and *e*.

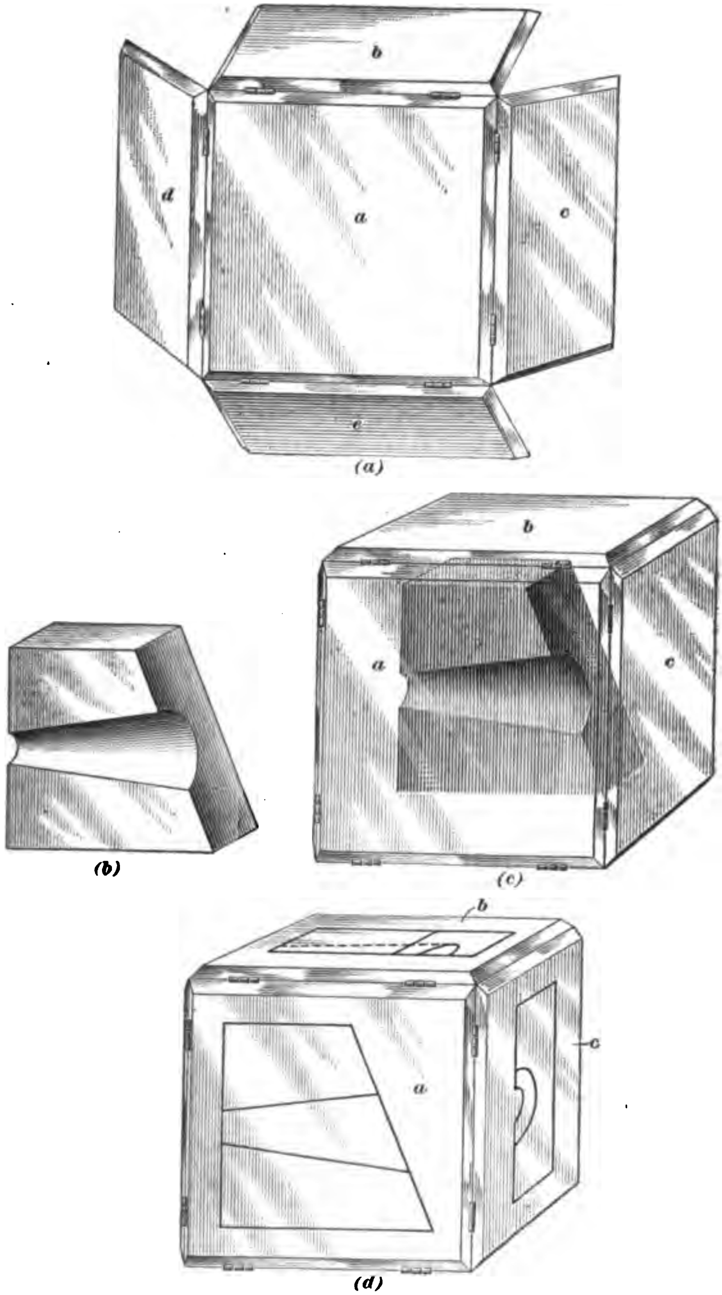
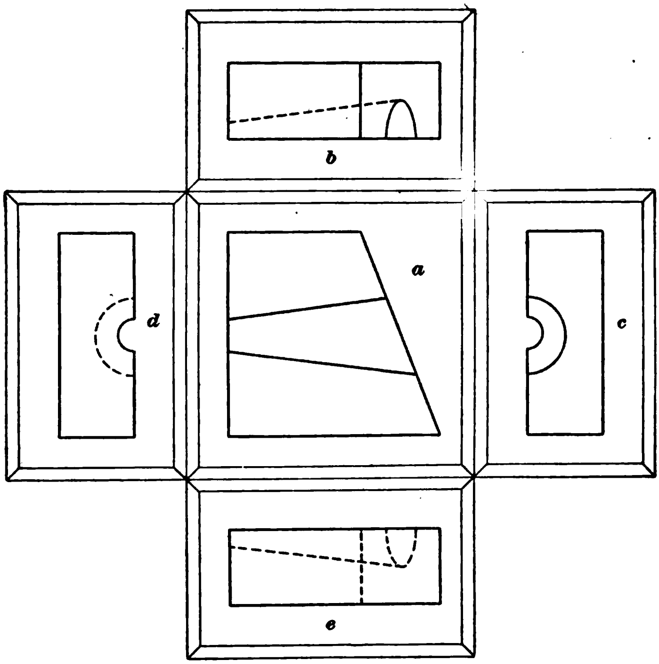


Fig. 6

These five plates may be folded so as to form a box, or they may all be opened out flat. In (*b*) is shown a wedge-shaped block with a tapered groove in one side. Suppose that this block is placed inside the box formed by the framed plates in (*a*), as shown in (*c*). Next, suppose that the wedge-shaped block is viewed through each of the plates *a*, *b*, *c*, etc., from the front, top, sides, and bottom, and that on each plate is drawn the



(*e*)

FIG. 7

outline of the block as it appears when viewed squarely through that plate. The views of the block as seen through the plates *a*, *b*, and *c* are shown in (*d*). Now, when all the views have been drawn, suppose that the four hinged plates *b*, *c*, *d*, and *e* are folded out until they come flush with the central plate *a*, as shown in Fig. 7. The five views of the object will then be in their correct relative positions.

12. Projectors.—The draftsman, of course, does not obtain the various views by the method shown in Figs. 6 and 7. Instead, he uses a series of horizontal and vertical lines, called *projectors*, to transfer points and lines from one view to another. This is more clearly illustrated in Fig. 8. Suppose that a drawing is to be made of the grooved wedge-shaped block shown in (*a*). The drawing may then be done as shown in the front view (*b*), the two end views (*c*) and (*d*), the top view (*e*), and the bottom view (*f*). The outline *a b c d*, view (*b*), is the front face *a b c d* of the block shown in (*a*), and the lines *e f* and *g h* are the straight edges of the groove. To construct the end view (*c*), the lines *a b* and *d c* are extended to the right, and between them is drawn the vertical line *b c*, which represents the edge *b c* of the original block. Then, along the top horizontal line a distance *b i* is laid off equal to the thickness of the block, and the vertical line *i j* is then drawn. From the points *e*, *f*, *g*, and *h*, in the view (*b*), horizontal projectors are drawn toward the right until they cut the vertical line *b c*, view (*c*), at *e*, *f*, *g*, and *h*. Then the half-circles *e g* and *f h* are drawn, completing the end view. The semicircles *e g* and *f h* are full lines because both of the curved edges at the ends of the groove can be seen when viewing the block from the right. The end view (*d*) is drawn in the same way as the view (*c*), but by extending projectors to the left. Also, in the view (*d*), the semicircle *f h* is dotted, because this curved edge is not seen when viewing the block from the left side.

13. To draw the view (*e*), Fig. 8, vertical projectors are extended upwards from the points *a*, *b*, *c*, *f*, and *h* in view (*b*), and across them the line *a c* is drawn, locating the points *a*, *b*, *c*, *f*, and *h*, as shown in view (*e*). The distance *a k* is then laid off equal to the thickness of the block, and the horizontal line *k j* is drawn, completing the outline *a k j c*. From the point *b*, the vertical line *b i* is drawn. Then the outlines *a k i b* and *b i j c* represent the faces *a k i b* and *b i j c* of the block, as shown in (*a*). The curved edge of the large end of the groove can be seen when the block is viewed from the top, and this curved line is shown by the full line *f l h* in the top view (*e*). The inner line of the groove is not visible, and it is therefore represented by the dotted line *l m*. The point *m* is located above *a* at a distance equal to the depth of the groove

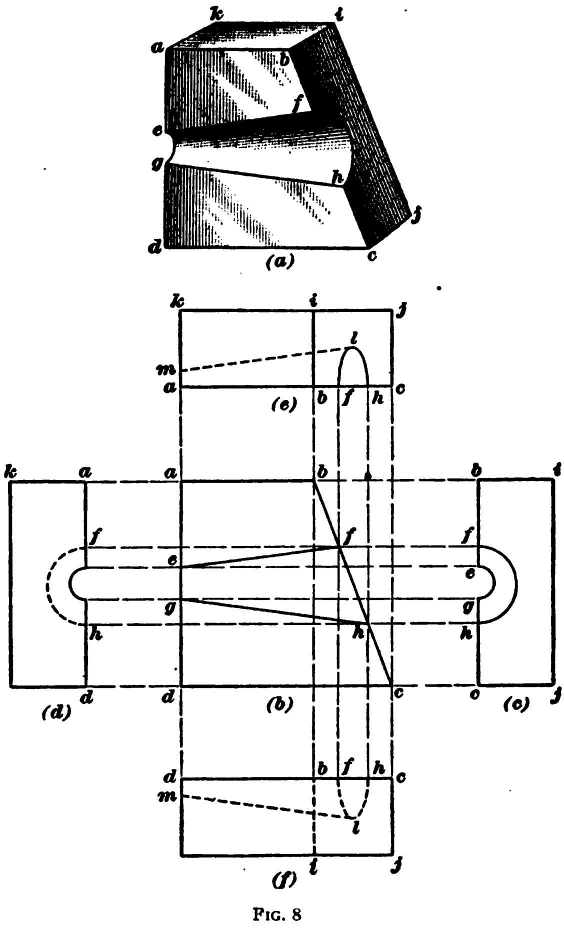


Fig. 8

at the small end. The point *l* is at a distance above the line *a c* equal to the depth of the groove at the large end; also, it is directly above a point midway between *f* and *h*. A smooth curve through the points *f*, *l*, and *h* gives the outline of the large end of the groove. The bottom view (*f*) is made in the same way as the top view, except that the projectors are drawn downwards; also, as the edge *b i* and the outline of the groove cannot be seen from the bottom, they are shown dotted in the bottom view.

14. From the foregoing example, it should be clear that the use of the projectors is simply to find the correct positions of points and lines in one view from the same points and lines in another view. After the different views are drawn, the projectors are of no value whatever; they are valuable only while the drawing is being made. Therefore, when the tracing of the drawing is being made, the projectors are omitted; consequently, they do not appear on the blueprint. It is an advantage to omit them from the blueprint, for their presence would only add lines that would confuse the drawing and make it harder to read; besides, the projectors are not needed in order to read a blueprint or a drawing properly.

LINES USED ON DRAWINGS

15. Markings on Drawings.—In addition to the several views that are needed to show the form of the object represented, the working drawing contains other lines, marks, figures, and signs, such as dimension lines and figures, lettering showing the names of parts or the title of the drawing, written or printed directions as to the materials to be used or the shop processes to be followed, abbreviations, and so on. The purpose and meaning of these various marks are dealt with more fully later.

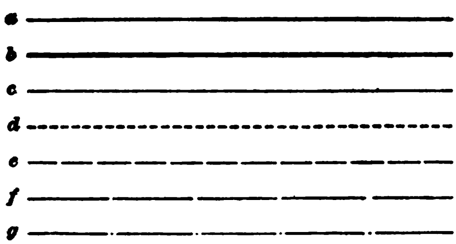


FIG. 9

16. Full Lines.—The several different kinds of lines used on working drawings are shown in Fig. 9, and in order to read working drawings easily and correctly it is necessary to know the use and meaning of each of these lines. The *medium full line a* is used more than any other kind of line and shows the outlines of the visible parts of an object. The *heavy full line b* is also used to mark the visible outline of an object, but it is employed only in the positions where shading is to be indicated, and for this reason it is frequently called a *shade line*. Shade lines are not used on all working drawings, and when they are not used, the outline is indicated wholly by medium full lines. The *light full line c* is used in cross-sectioning and in making *extension lines*, which are the light lines between which dimension lines are frequently drawn. It is also used on drawings that are to be shade-lined. The pencil lines made in the construction of a drawing are light full lines, also.

17. Dot and Dash Lines.—The *dot line d*, Fig. 9, sometimes called the *short-dash line*, is used to show the outlines and extent of the invisible parts of an object, as, for example, holes or recesses inside a piece of work. The dots are made of the same breadth, or weight, as the line *a*. Dotted lines on drawings that are shade-lined are made of the same weight as the line *c*. The line *e* is the *dash line*. It is used to indicate *projection lines*, or lines that join corresponding points in different views of an object. This line is made of the same weight as the line *c*. Although projection lines are used considerably by the draftsman, they are generally omitted from the finished drawing, so as to avoid confusion of lines. The *long-dash line f* is of the same weight as the line *c* and is used for the dimension lines of drawings. The *dot-and-dash line g* is formed by the regular repetition of a long dash and a dot, and is of the same weight as the line *c*. It is used to indicate center lines and the position of sections taken through an object.

18. Center Lines.—Center lines are reference lines drawn by the draftsman to aid him in making the drawing. They usually, though not always, denote the center of the object and the centers of its parts. The draftsman first draws such center lines as he requires and then constructs the various views by laying off the dimensions from these lines. Not all the center lines are allowed to remain on the completed drawing in every instance, because they are not necessary and would only make the drawing less clear and less readable. But such center lines as are needed and those which do not confuse the drawing are kept on the finished drawing. They are required particularly where details have been located, but not drawn in full, where measurements are to be laid off from them, and where the work to be built must be laid off from the drawing. In some kinds of patternmaking, for example, the working drawing must be copied directly on the wood or on sheet metal; consequently, the workman in laying out the work must consider the center lines first of all.

SHADING ON DRAWINGS

19. Shading of Holes.—On some mechanical drawings a number of the lines are made much heavier than the others. These heavy lines are *shade lines* and are used to indicate by their positions whether the outline is that of a hole in the work or of a raised part projecting above the surface. For example, Fig. 10 (*a*) shows a perspective view of a rectangular block in which are a round hole *a* and a long slot *b*. A mechan-

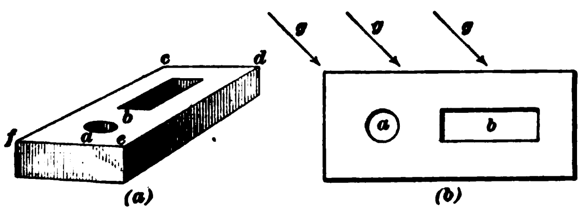
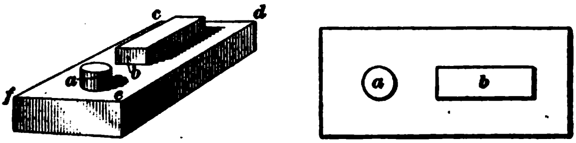


FIG. 10

ical drawing of the block, showing a top view, is given in (*b*), with the proper shade lines. In the case of the round hole, the upper left-hand part of the circle is made heavy; and in the case of the slot, the upper horizontal line and the left-hand vertical line are made heavy. This is the usual way of shading holes. Consequently, if the shade lines forming part of the outline of a piece are located at the upper and left-hand edges, they indicate that there is a hole or a depression below and to the right of them. The reason for drawing the shade lines in this way may be explained simply. The light by which the piece is seen is supposed to come from above and to the left, and to fall on the top face *c d e f* of the block in the direction of the arrows *g*. The result is that the upper and left-hand edges of the holes *a* and *b* cast shadows in the holes, just below those edges; hence, the shade lines are drawn at those points.

20. Shading of Raised Parts.—A block having a round boss *a* and a long rib *b* on its upper face is shown in perspective in Fig. 11 (*a*), and a plan of the block, properly shaded, is shown in (*b*). In this case, the parts *a* and *b* of the object project above the surface of the work, and the shade lines are at the right and the bottom of the outlines representing these parts in the plan (*b*). Consequently, if the shade lines



(*a*) (*b*)

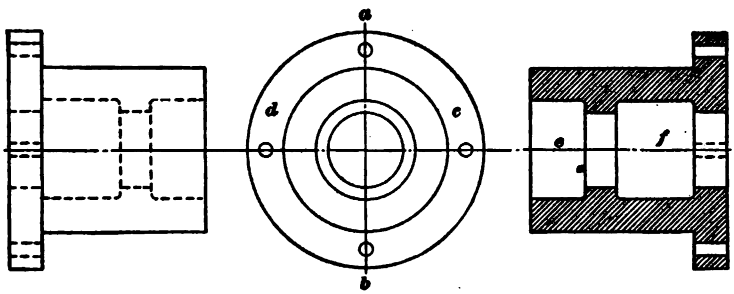
FIG. 11

forming part of the outline of a piece are at the bottom and right-hand edges, they indicate that the part bounded by them is raised above the surface. The right-hand and bottom lines of the drawing are also shade lines, because they represent the edges *c d* and *d e*, which are two of the top edges of the block. They must be shade lines because the surface *c d e f* is raised above the lower surface by an amount equal to the thickness of the block. These latter statements refer also to the block shown in Fig. 10 (*b*).

21. Differences in Use of Lines.—All manufacturing companies do not use the same forms of lines for representing various parts of an object in a drawing. For example, some companies use the medium full line for all the outlines of a part and do not use shade lines at all. Others use the heavy full line, with no shade lines. Again, it is the practice in some drafting rooms to use the light full line instead of the dot-and-dash line for center lines. Each shop has its own rules in these matters, and the workman should make it his business to become familiar with the usual practice of the company by which he is employed.

SECTIONS

22. Sectional Views.—The various kinds of views considered in preceding articles are external views and they show the appearance of the object as seen from different positions outside it. If the object is not complicated, outside views are sufficient; but if the inside is of peculiar form, or if one part fits into another, it is usually found that external views alone will not be enough. In such a case, one or more *sectional views*, or *sections*, of the object are given also. In making a sectional view, the object is assumed to be cut into



(a) (b) (c)

FIG. 12

two parts, the part nearer the observer is supposed to be removed, and the appearance of the remaining part is shown as a section. Thus, while it would be possible to determine from Fig. 12 (*a*) and (*b*) the shape of the object indicated, the sectional view (*c*) makes the drawing much clearer. The object is supposed to be cut in two along the line *a b*, and the right half *c* is assumed to be removed, so that the other half *d* may be viewed on the surface along which it is cut. This view of the half *d* is the section, or sectional view, shown in (*c*).

23. By means of the sectional view, Fig. 12 (*c*), the interior construction of the work is shown clearly. The thickness of the material at various points is plainly indicated, the positions of the recesses *e* and *f* are shown, and all this is done by full lines instead of by the somewhat confusing dotted lines shown in the side view (*a*). One of the most noticeable features of the section (*c*), however, is the series of evenly spaced slanting lines drawn on it. These lines are called *section lines*, or *cross-hatching*, and indicate the surface along which the object is supposed to be cut in making the section. Every sectional view of an object is indicated by cross-hatching or by some similar marking, and when a drawing shows views thus marked, the workman knows at once that such views are sections. Sections are usually taken along center lines, as in the example given, but a section may be taken in any desired direction through an object.

24. Forms of Cross-Hatching.—As many different materials are used in the construction of machinery, it has become the custom to indicate these differences on the drawings by using a particular form of cross-hatching to denote each material. The practice is not the same in all manu-

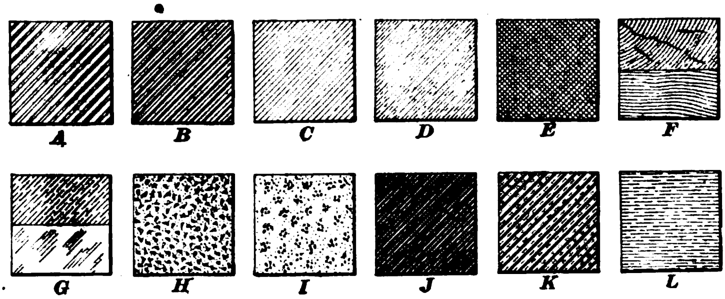


FIG. 13

facturing shops, as each adopts its own standards; in a general way, however, the forms of section lining shown in Fig. 13 represent usual practice. *A* indicates the sectioning used for all forms of steel; *B* shows that used for wrought iron; *C* is the form for cast iron; *D* indicates brass, bronze, and all other alloys containing copper; *E* is the form of section lining for lead, Babbitt, and soft metal; the upper half of the section *F* is for wood cut across the grain and the lower half for wood cut along the grain; *G* in the upper half, is the sectioning for glass and stone, but when not in section these materials are represented as in the lower half; *H* indicates concrete; *I*, leather; *J*, rubber or wood fiber; *K*, firebrick; and *L*, water. It is becoming the practice in many shops to use the cross-hatching shown in *C* for *all* materials, and to state the kind of material in a note below each piece. Sometimes the materials to be used are given in a ***material list***, which is in the form of a table placed either in one corner of the drawing sheet or on a separate sheet. These practices not only lessen the time required for making the drawing, but also diminish the chances of error in using wrong materials.

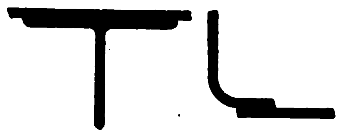


FIG. 14

25. In case the sectional view is very narrow, so that difficulty would be encountered in distinguishing the style of cross-hatching, the section is usually indicated by blackening it to form a solid black outline, as shown by the examples in Fig. 14. Blackening is done also if the section is to show prominently on the drawing. When two or more sections of this kind meet, a white line is left between them, to show that they are separate, and not in one piece. Solid black sections are most commonly used for sectional views of work composed of plates, angle iron, I beams, channels, and so on.

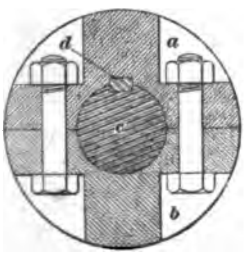


FIG. 15

26. In a sectional view of two or more pieces that are fitted together, the section lines of adjacent pieces are slanted in different directions, so that the difference between the parts may readily be seen. Thus, in Fig. 15 is shown a section of two cast-iron pieces *a* and *b* that are bolted together about a shaft *c* and held from turning by a key, shown in section at *d*.

The section lines of adjacent pieces should be slanted differently, no matter whether the pieces are of the same or of different materials. In Europe it is common practice to indicate sections by various colors, instead of by lines; but this practice is rarely followed in the United States.

27. Classes of Sections.—There are several kinds of sections, and these are named according to the manner or the direction in which the object is assumed to be cut in making the section. A section taken by cutting across a body in the direction of its smallest dimension is called a *cross-section*. If the section is taken in the direction of the greatest length

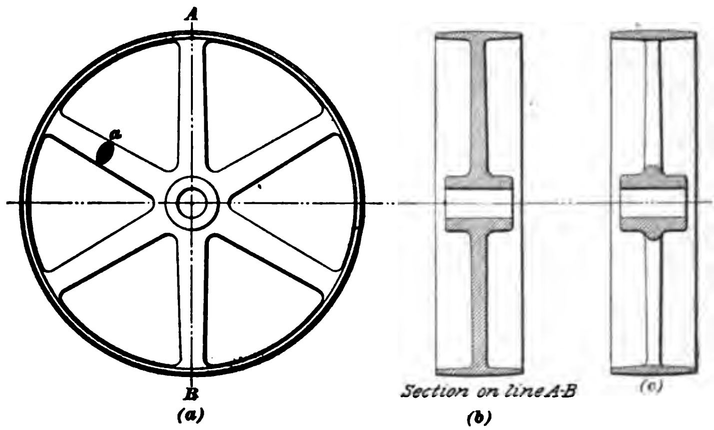


FIG. 16

of the body, it is called a *longitudinal section*. If a cross-section or a longitudinal section is made to show only part of the way through an object, it is then known as a *partial section* or a *half-section*. Sometimes two or more sections of the object are given to show its construction at different points; but if two or more partial sections are combined in one view, they form what is called a *conventional section* or a *conventionalism*.

28. The nature of a conventional section may be illustrated more clearly by reference to Fig. 16, which shows views of a six-armed pulley. The side view (*a*) indicates the center line *A B* along which the sectional view (*b*) is taken. In this sectional view the arms are shown in longitudinal section, and from a study of this view alone it would be impossible to tell whether the pulley had arms or whether it had a continuous solid web instead. The fact that arms are used, and not a web, may be indicated very clearly by the use of a conventional section (*c*). In this view the rim and the hub of the pulley are shown in section, but an external view of the arms is added. The result is not a true sectional view of the pulley, but a view that is clearer than a true section and is easier to make because it does not require so many short section lines. The cross-section of the arm, shown at *a* in the side view, is the customary way of indicating the shape of the section of the arm.

SCALES AND DIMENSIONS

29. Scales Used on Drawings.—It would be very difficult, and in some cases wholly impossible, to make full-size drawings of all machines or other work to be manufactured. Also, even if such drawings could be made, they would be of so many different sizes that it would be very inconvenient to file them away systematically. For these and other reasons, the usual practice is to make drawings to a reduced scale, that is, to make them some convenient fraction of the full size. Thus, if a piece of work is very large, a drawing of reasonable size may be obtained by making all the dimensions one-eighth of the actual full-size dimensions, or one-fourth, or one-half, as the case may be.

30. The scale to which a drawing is made is usually denoted by stating the number of inches of length on the drawing corresponding to 1 foot of length of the actual piece. For example, if a drawing is made to a scale of 3 inches to the foot, a part of the object that is 1 foot long will be represented on the drawing by a line 3 inches long; or a part 2 feet long will be denoted by a length of 6 inches on the drawing. In other words, as 3 inches is one-fourth of a foot, the drawing will show every part exactly one-fourth of its true size.

The scales to which drawings are made are numerous, the selection of a scale being dependent on such matters as the size of the drawing sheet used and the clearness and accuracy with which the various parts must be shown. The scale is invariably written or printed on the drawing, in the following style: *Scale 3″ = 1 ft.* The scales commonly used are 6″ = 1 ft., 3″ = 1 ft., $1\frac{1}{2}$″ = 1 ft., and $\frac{3}{4}$″ = 1 ft., but other scales may be employed when special conditions require.

31. Dimensions on Drawings.—The several views on a drawing will show the shape of the object, but in order to indicate its size, dimensions must be marked on the drawing. The dimension figures denote the lengths, diameters, etc. of the various parts in inches, in feet and inches, or in decimal or fractional parts of an inch, according to the size of the work and the accuracy with which it is to be constructed. As shown in Fig. 2, the dimension figures are written at or near the middle points of the dimension lines, which are drawn either across the views of the object or at the sides of the views. An arrowhead is placed at each end of the dimension line, to show just how far the dimension extends. When the part is very short, so that there is no room for a dimension line on the drawing, the dimension figures are written between two arrowheads that point toward each other, as in the case of the $\frac{3}{16}$-inch dimension shown in Fig. 3; and if there is so little space between the arrowheads that the dimension figures cannot be written in, they are placed at one side or the other, on a line with the arrowheads. When the dimension figures are placed on dimension lines at the side or the end of a view, extension lines are drawn outwards from the view and the dimension lines are drawn between the extension lines.

32. The dimensions marked on a working drawing always indicate the actual dimensions of the completed work, no matter what the scale of the drawing may be. Thus, two different draftsmen might make drawings of the same piece of work, one using a scale of 3″ = 1 ft. and the other a scale of $1\frac{1}{2}$″ = 1 ft., but the drawings of both would show exactly the same dimensions for the various parts. If the dimension is 2 feet or smaller, it is usually expressed wholly in inches, as, for example, $\frac{3}{16}$″, $5\frac{1}{8}$″, $11\frac{1}{4}$″, $23\frac{11}{16}$″, the sign ″ being used to indicate inches. If the dimension is greater than 2 feet, however, it is usually expressed in feet and inches, the sign ′ being used to denote feet; thus, a drawing might show such dimensions as 2′ 3½″, 4′ 6″, etc. Frequently the feet and inches are separated by a short dash, as 3′– 8¼,″ to avoid any chance of error. In some shops, all dimensions greater than 1 foot are expressed in feet and inches; whereas, on United States government drawings, dimensions less than 4 feet are expressed in inches. There is no fixed rule in this respect.

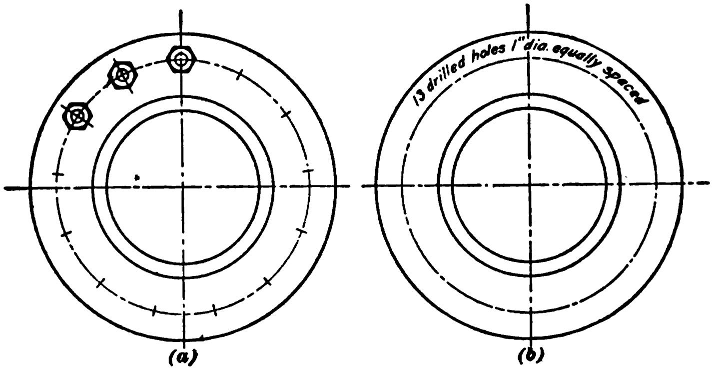
ABBREVIATIONS USED ON DRAWINGS

33. Written Abbreviations.—In addition to the dimensions, working drawings frequently contain written or printed abbreviations and directions, and the workman must be able to read these correctly in order to understand the construction and method of making the piece. If he does not understand the meaning of one or more of these written or printed marks, he should consult some one in authority, as the foreman or the superintendent. The most common written abbreviations are *D.*, *Dia.*, *Diam.*, *d.*, *dia.*, and *diam.*, each of which indicates the *diameter* of a part. The *radius* of a circular part, which is equal to half of the diameter, is abbreviated *R.*, *Rad.*, *r.*, or *rad.* Thus, if a part is marked *1½″ dia.*, it is 1½ inches in diameter; and if a curved outline is marked *3′–10″ R.*, it is understood to have a radius of 3 feet 10 inches. If a drawing is made according to the metric system, the dimensions are usually stated in *millimeters*, abbreviated *mm.*; thus, *110 mm.* as a dimension would indicate 110 millimeters. Similarly, the abbreviation *m.* is used for *meters*. Occasionally, the sign ℄ is employed as an abbreviation for *center line*. *Thds.* or *thds.*, preceded by a number, is used to show the number of *threads* per inch; thus, a screw marked *14 Thds.* is to have 14 threads per inch.

34. Various machine-shop operations are often indicated on the working drawing by means of abbreviations and notes. Thus, *f* or *fin.* denotes that the surface marked by that abbreviation is to be finished by machining, grinding, polishing, etc.

Drill indicates that a hole is to be made by drilling. *Bore* or *Bored* is the term used to show that a hole is finished by boring. *Cored* implies that a hole is to be left as it is cast in the foundry. *Black*, which is a term used in connection with forged work, means that the work is to be left as it comes from the smith, without finish. *Tap*, preceded by a number, indicates that the hole is to be tapped with a tool of the size indicated by the number, as, $\frac{5}{8}''$ *Tap*. Sometimes another number is added to signify the number of threads per inch on the tap, as, for example, $\frac{9}{16}''$*–12 Tap;* in this case the tap is understood to be $\frac{9}{16}$ inch in diameter and to have 12 threads per inch. *Planed* calls for finishing by planing. *Mill* for finishing by milling, and *Scraped* for finishing by scraping. A finish by a reamer is marked *Ream* or *Reamed*. *Tool finish* means that the surface, after machining, is not to be finished further. *Shrink Fit*, *Forced Fit*, or *Driving Fit* shows the workman that, in machining the part, he must make the proper allowance for the kind of fit named.

35. Repetition of Details.—When an object has several details, or parts, that are exactly alike, the draftsman saves time by showing only one or two on the drawing and then



(a) (b)

FIG. 17

indicating the positions of the others by marking their center lines; or, he places on the drawing a note stating that certain parts are to be duplicated. These points are clearly illustrated in Fig. 17, which shows two ways of indicating the positions of the bolt holes in a pipe flange. In view (*a*) are shown three bolts, and the positions of the other bolts are indicated by

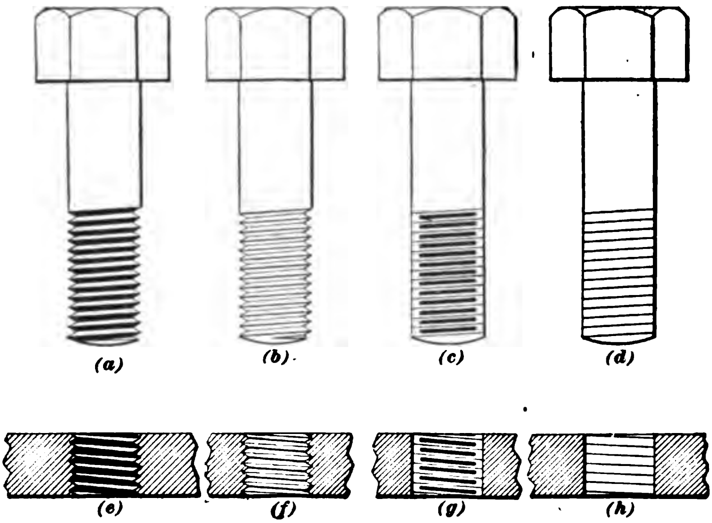


FIG. 18

short lines drawn across the bolt circle. In view (*b*) no bolts are drawn and no positions are indicated, but the note written along the bolt circle explains clearly that thirteen 1-inch holes

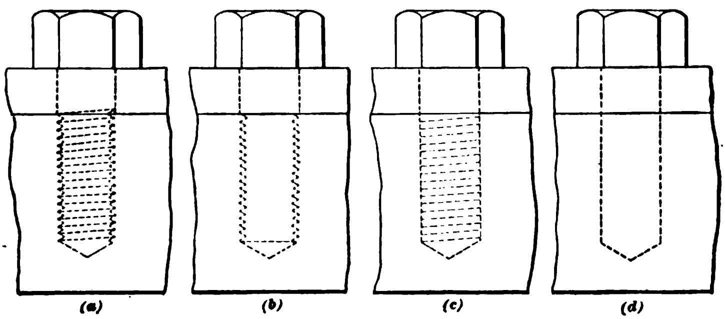


FIG. 19

are to be drilled in the flange, along the bolt circle and at equal distances from one another. These methods of abbreviation are used on drawings of gear-wheels, riveted joints, and other work having repeated details. In this way the time and expense incurred in making the drawing are lessened, yet the clearness is not sacrificed.

36. Abbreviation of Screw Threads.—Various methods of indicating screw threads are shown in Fig. 18. That in (*a*) shows the thread shaded, and that in (*b*) omits all shading. Because of the time required to draw all the threads in either case, the forms shown in (*a*) and (*b*) are not commonly used on drawings; instead, some conventional method of abbreviation, such as that shown in (*c*) or (*d*), is used. The latter forms are easily and quickly drawn and are quite clear. The views (*e*), (*f*), (*g*), and (*h*) show sections of holes tapped to receive the bolts shown in (*a*), (*b*), (*c*), and (*d*), respectively. The slant of the thread on the drawing of a tapped hole is opposite that on the bolt for that hole.

FIG. 20

When the threaded parts are hidden, so that they must be shown on the drawing by dotted lines, any one of the methods illustrated in Fig. 19 (*a*) to (*d*) may be used. The diameter of the screw should be written on the drawing, and the shape of the thread, the number of threads per inch, and whether the thread is right-hand or left-hand, may be given.

37. A right-hand screw, that is, a screw with a right-hand thread, will advance into a nut or other part when turned in

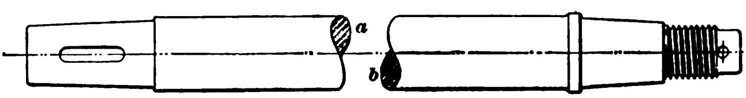


FIG. 21

the direction of the hands of a watch. A left-hand screw will advance when turned in the opposite direction. In Fig. 18 right-hand threads are shown. A view of a left-hand screw and a section of a hole tapped for this screw are shown in Fig. 20.

It may be noticed that the threads slant in the opposite direction from those of right-hand threads. It is not difficult to determine whether a drawing shows a right-hand or a left-hand thread. If it is a drawing of a screw, and the thread slants upwards to the right, the thread is right-hand; and if it slants upwards to the left, it is a left-hand thread. If the drawing is a section of a tapped hole, a right-hand thread is indicated by lines sloping upwards to the left, and a left-hand thread by lines sloping upwards to the right.

38. Breaks.—A break is used to reduce the space occupied by a drawing of a long, slender part, without making use of a small scale. An example of a break is shown in Fig. 21, which represents a cylindrical shaft. A part of the cylindrical portion is assumed to be cut out, and the ends are brought close to each other, the curved lines indicating the break. The surfaces at the break, as at *a* and *b*, are cross-hatched to indicate the material of which the piece is made. Always, when a break is thus used, it is understood that the part cut away is of the same shape and diameter as the parts adjacent to the break. Sometimes, one end of a piece is broken away, instead of the middle.

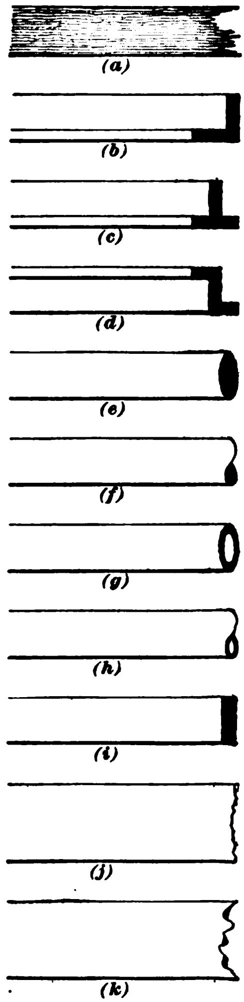


FIG. 22

39. Breaks are indicated in various ways, but usually they show the shape of the cross-section of the object. Some forms of breaks are illustrated in Fig. 22. A break in a wooden piece is shown in (*a*). A bar of angle iron is broken as in (*b*), a bar of **T** iron as in (*c*), and **Z** bars as in (*d*). Cylindrical objects are sometimes shown broken as in (*e*), but the usual method is that in (*f*). Hollow cylinders, such as pipes, are broken as in (*g*), in some cases, but the more common form is shown in (*h*). Square or rectangular parts have breaks indi-

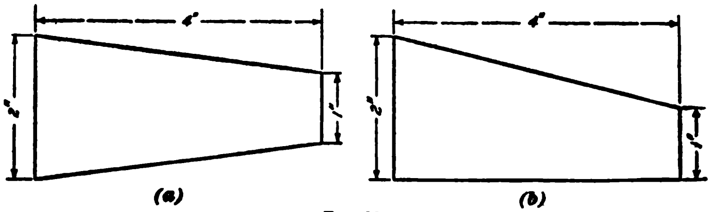


FIG. 23

cated as in (*i*), and plates are shown broken by a wavy line, as in (*j*) or (*k*).

40. Tapers.—The word *taper* means that the piece of work to which it applies grows smaller in cross-section from one point to another. The reduction in size may be made equal on both sides of the piece, as in Fig. 23 (*a*), or it may be made wholly on one side, as in (*b*); each shows a flat wedge.

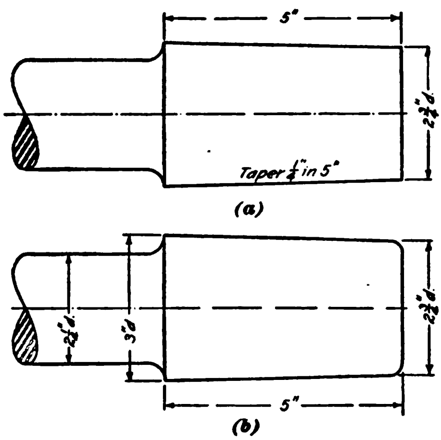


FIG. 24

If a cylindrical piece is turned down so as to diminish in size toward one end, as in Fig. 24, a turned taper is produced. The clearest way of expressing a taper is to mark on the drawing the length of the tapered part and the size at each end of this part, as is done in Figs. 23 and 24 (*b*). Sometimes however, the taper is indicated as in Fig. 24 (*a*); that is, the size at one end and the length of the tapered part are given, and a note indicating the taper is added on the drawing. In this particular case, the note indicates a taper of $\frac{1}{4}$ inch in 5 inches; that is, in 5 inches of length, the piece changes $\frac{1}{4}$ inch in diameter. As the diameter is $2\frac{3}{4}$ inches at the small end, and the tapered part is 5 inches long, the diameter at the large end is $2\frac{3}{4}+\frac{1}{4}=3$ inches. The taper is sometimes given in inches per foot, as, for example, 1 inch per foot, $\frac{3}{16}$ inch per foot, and so on.

INTERPRETATION OF DRAWINGS

POINTS TO BE OBSERVED

41. Order of Reading Drawings.—The first thing for a workman to do, on receiving a drawing, is to study the views in order to learn the shape of the piece as a whole, after which he should observe the details, or parts, to see their forms and the way in which they are put together. Next, he should read the notes, so as to determine the materials used, the number of pieces required to be made, the operations to be performed, and so on. He may then proceed to lay out the work. In doing this, he will discover whether any necessary dimensions have been omitted and whether any given dimensions are wrong. In either case he should report the matter to some one in authority, and the drawing office should then be notified. No dimensions should be added or changed without consulting the drawing office; also, no workman should begin operations on a piece of work until he fully understands what the drawing requires. Unless special orders are given to him, the workman should never scale a blueprint to obtain a dimension; that is, he should not measure the parts on the blueprint. The wetting and drying necessary in making a blueprint are apt to cause it to shrink, and any measurements taken from it would be incorrect.

42. Sizes Indicated by Dimensions.—In some shops, the dimensions given on the working drawing represent the size that the object is to be when finished; hence, the blacksmith or the patternmaker must make the necessary finishing allowances. In other shops, the finishing allowance is made by the draftsman and the dimensions given are, then, those of the pattern or forging. If the workman is in doubt about the practice followed, he should find out by inquiry what system is used in the particular shop in which he is employed. In the best modern practice, a note is put on the drawing, calling attention to the fact that the sizes given are those of the finished piece. For example, the note may be *All Finished Sizes*, or some phrase of similar meaning.

43. Accuracy of Dimensions.—The accuracy with which a workman is to work to a given dimension may be stated in a number of different ways. One of these is to indicate the limits of accuracy by smaller figures written above and below the dimension figures, and to the right. Thus, a dimension like $2.25^{+.001''}_{-.001''}$ means that the piece is to be $2\frac{1}{4}$ inches thick, but that it may be $\frac{1}{1000}$ inch large, as indicated by +.001″, or $\frac{1}{1000}$ inch small, as indicated by −.001″, without being rejected; that is, the thickness may be anything between 2.251 and 2.249 inches.

Sometimes only one limit is given. Thus, $2.25^{-.001''}$ means that the part may be as much as .001 inch under size, but not over 2.25 inches; and $2.25^{+.001''}$ means that the piece may be exact or .001 inch large, but not under size. Another method is to indicate the accuracy by the number of decimal places in the dimension. Thus, a dimension of 2.50″ indicates that the piece is to be accurate to a hundredth of an inch; 4.375″ shows that the piece is to be within a thousandth of an inch of the stated size; and 1.1875″ indicates accuracy to a ten-thousandth. There is no universal system adopted, and the methods given are meant only to illustrate the idea of working to limits.

EXAMPLES OF WORKING DRAWINGS

44. General Drawing.—A general drawing shows the workman the relation between the different parts of a machine and indicates the position occupied by these parts. It is used largely in assembling, or putting together in proper relation, the different parts that make up the completed machine. For this reason it is frequently called an *assembly drawing.* The general drawing usually has only a few of the principal dimensions on it, and in some cases no dimensions at all are shown. In Fig. 25, for example, is a general drawing of a connecting-rod for a steam engine. It will be seen that the only dimensions given are the diameters and lengths of the bearings and the distance between their centers. Sometimes, however, the general drawing shows also the leading dimensions of the different parts.

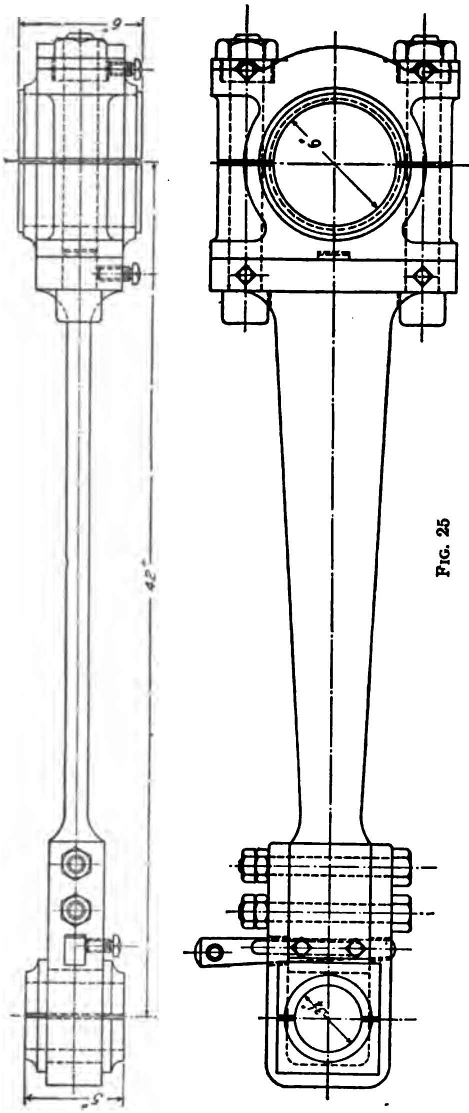
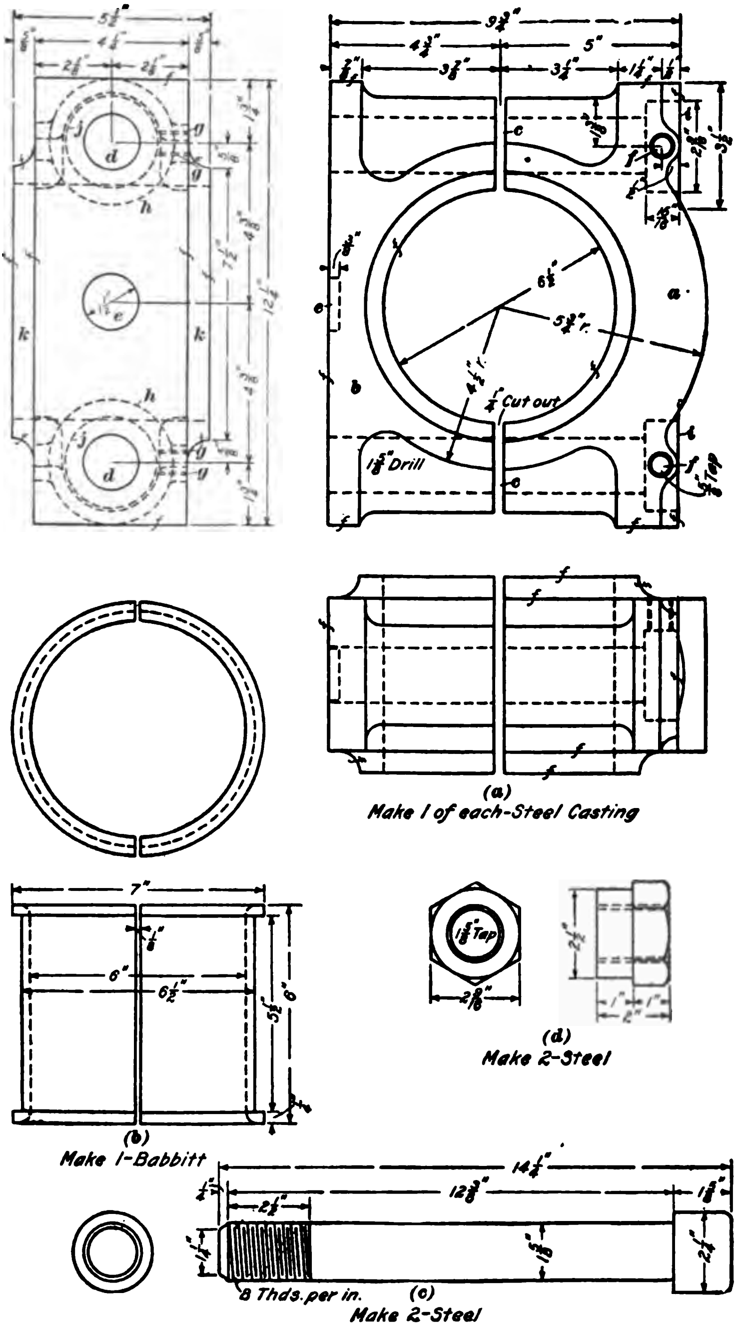


FIG. 25

45. Detail Drawing.—A detail drawing shows the necessary views of each separate part, or detail, of a machine, gives the dimensions, and contains the notes or directions relating to the actual construction of the parts. It may be made so that the same working drawing may be used by the pattern-maker, the blacksmith, and the machinist; or, if desired, separate working drawings may be made for each, in which case the dimensions, notes, etc. given on the drawing are only those required by the particular workman for whom it is intended. A detail drawing showing the details of the large end of the connecting-rod, Fig. 25, is illustrated in Fig. 26. Views of the box are shown in (*a*) and the Babbitt lining that fits in the box is shown in (*b*). The bolt and the nut shown in (*c*) and (*d*) are used to hold the parts of the box together and to fix them to the rod.

46. The steel casting forming the large end of the connecting-rod, shown in Fig. 26 (*a*), consists of two parts *a* and *b* that are first finished in one piece, and are then cut in two, as at *c*. The amount of metal cut away is $\frac{1}{4}$ inch thick. The center is bored out to a diameter of $6\frac{1}{2}$ inches, and two holes *d* are drilled through with a $1\frac{5}{8}$-inch drill. These holes receive the bolts shown in (*c*). As shown in (*a*), a recess *e*, $1\frac{1}{2}$ inches in diameter and $\frac{3}{8}$ inch deep, is cut in the face of the part *b* that fits against the body of the rod, and the outer ends of the holes *d* are countersunk to a depth of $\frac{15}{16}$ inch, with a diameter of $2\frac{9}{16}$ inches, to receive the projections on the nuts. The nuts are kept from turning by setscrews inserted at *f*, and the holes for the setscrews are tapped with a $\frac{5}{8}$-inch tap. The two small circles marking each of these holes represent the tops and bottoms of the threads. In the end view they are represented by the pairs of dotted lines *g*. The dotted circles *h* represent the finished surfaces *i*, and the dotted circles *i* denote the large countersunk ends of the holes *d*. A boss *k* $\frac{5}{8}$ inch thick and $7\frac{1}{2}$ inches in diameter is formed at each side and is joined to the main part by fillets of $\frac{5}{8}$-inch radius. The surfaces that are to be finished are marked with the symbol *f*.

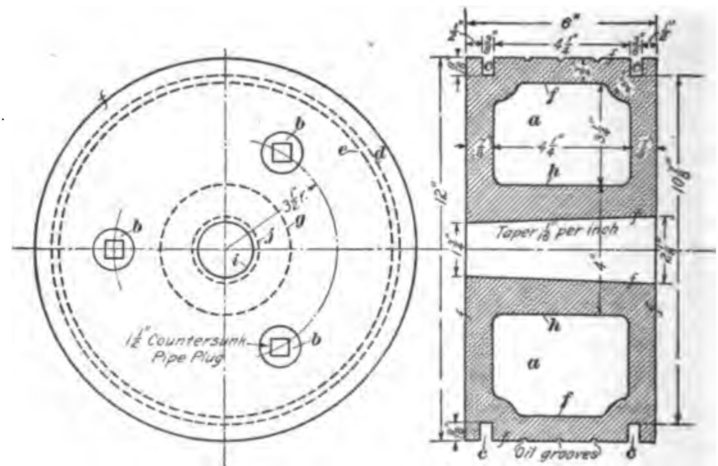
47. The Babbitt-metal lining for the bearing, shown in Fig. 26 (*b*), is first made in one piece having an overall length of 6 inches and an overall diameter of 7 inches. It is turned down to a diameter of $6\frac{1}{2}$ inches at the middle, thus forming a flange at each end, and the whole is bored out to a



(a)
Make 1 of each-Steel Casting
(b)
Make 1-Babbitt
(c)
Make 2-Steel
(d)
Make 2-Steel
1/4" Cut out
8 Thds. per in.

FIG. 26

diameter of 6 inches inside. A slot $\frac{1}{8}$ inch wide is then cut out, dividing the lining into two parts. This lining fits inside the $6\frac{1}{2}$-inch hole bored in the casting shown in (*a*). The bolt shown in (*c*) has an overall length of $14\frac{1}{4}$ inches, and is threaded at the end for a distance of $2\frac{1}{2}$ inches with 8 threads per inch. The bolt is $1\frac{5}{8}$ inches in diameter and has a round head $1\frac{5}{8}$ inches high and $2\frac{1}{4}$ inches in diameter. In the end view, the two inner circles represent the tops and bottoms of the threads, and the outer circle represents the outside diameter of the head. The nut for the bolt is shown in (*d*). It is a hexagonal



(a) (b)

Make 1-Cast Iron

FIG. 27

nut $2\frac{9}{16}$ inches wide across the flats and is tapped with a $1\frac{5}{8}$-inch tap to suit the threaded end of the bolt. The total thickness of the nut is 2 inches, 1 inch of which is formed and chamfered to the hexagonal shape of the nut. The remainder forms an extension 1 inch long and $2\frac{1}{2}$ inches in diameter that fits into the countersunk end of the hole *d* in view (*a*), and against which the end of the setscrew bears. Two of these bolts and nuts are required, and they are to be made of machinery steel.

48. Hollow Piston.—The piston shown in front elevation in Fig. 27 (*a*) and in section in (*b*) has an outside diameter of 12 inches when finished and is 6 inches thick from front to back. It is finished all over, as indicated by the finish marks *f*; consequently, the patternmaker, in making the pattern for this casting, must allow for the metal cut away in finishing and for the shrinkage of the casting while cooling. The inside of the piston is cored out, as at *a*, and a core must be made for this. It is held in place in the mold by three supports; thus, when the casting is finished, there are in it three holes through which the supports passed. The core is broken up and shaken out through these holes, and they are then tapped and plugged with pipe plugs *b*. The holes are $3\frac{1}{2}$ inches from the center and are equally spaced. The center of the piston is bored with a taper of $\frac{1}{16}$ inch per inch, being $2\frac{1}{8}$ inches in diameter at one end and $1\frac{3}{4}$ inches at the other. The metal is $\frac{7}{8}$ inch thick, and on the circumference are turned two grooves *c* $\frac{3}{8}$ inch wide and $\frac{9}{16}$ inch deep to receive the piston rings. Three oil grooves are also cut in this face. The grooves *c* are $\frac{1}{2}$ inch from the end faces of the piston, but the oil grooves are not definitely located, being cut at about equal distances between the grooves *c*. The dotted circle *d* represents the bottoms of the grooves *c*, and the circle *e* the face *f* of the cored hole. The circle *g* denotes the surface *h* of the cored part, and the circles *i* and *j* represent the ends of the tapered hole.

49. Belt Pulley.—A working drawing of a belt pulley is shown in Fig. 28. In this case, a single view, consisting of a section of the hub and the rim, is sufficient to show the construction, and only a little more than half of the pulley is shown. The face of the pulley is 18 inches wide and is *crowned*, or curved, so that the pulley is greater in diameter at the center of the rim than at the edges. The face is curved to a radius of 80 inches, as marked, and the pulley is 54 inches in diameter at the middle of the face. The dimension lines for these dimensions are not drawn in their full lengths, because the upper half of the pulley is not shown. The dimension line may be broken off and left unfinished, as in the case of the 80-inch radius; or, a double arrowhead may be used to indicate that the line should extend farther, as in the case of the 54-inch dimension.

50. The outer edges of the rim and the side surfaces of the hub of the pulley, Fig. 28, are faced, as is indicated by the notes marked thereon. The hub has a 5-inch hole bored through it to receive the shaft, and the interior of the hub is cored out to a diameter of 5½ inches and a width of 6 inches.

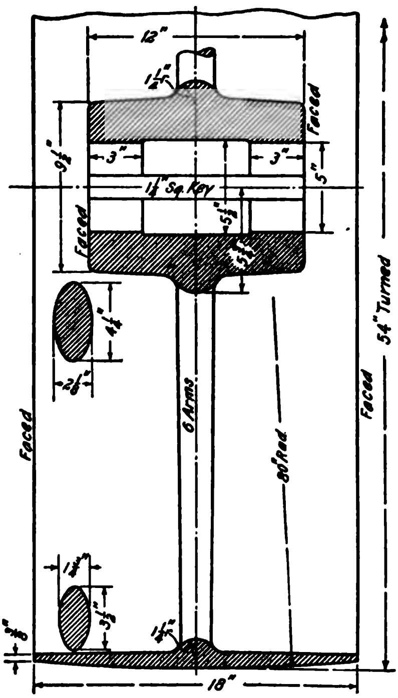


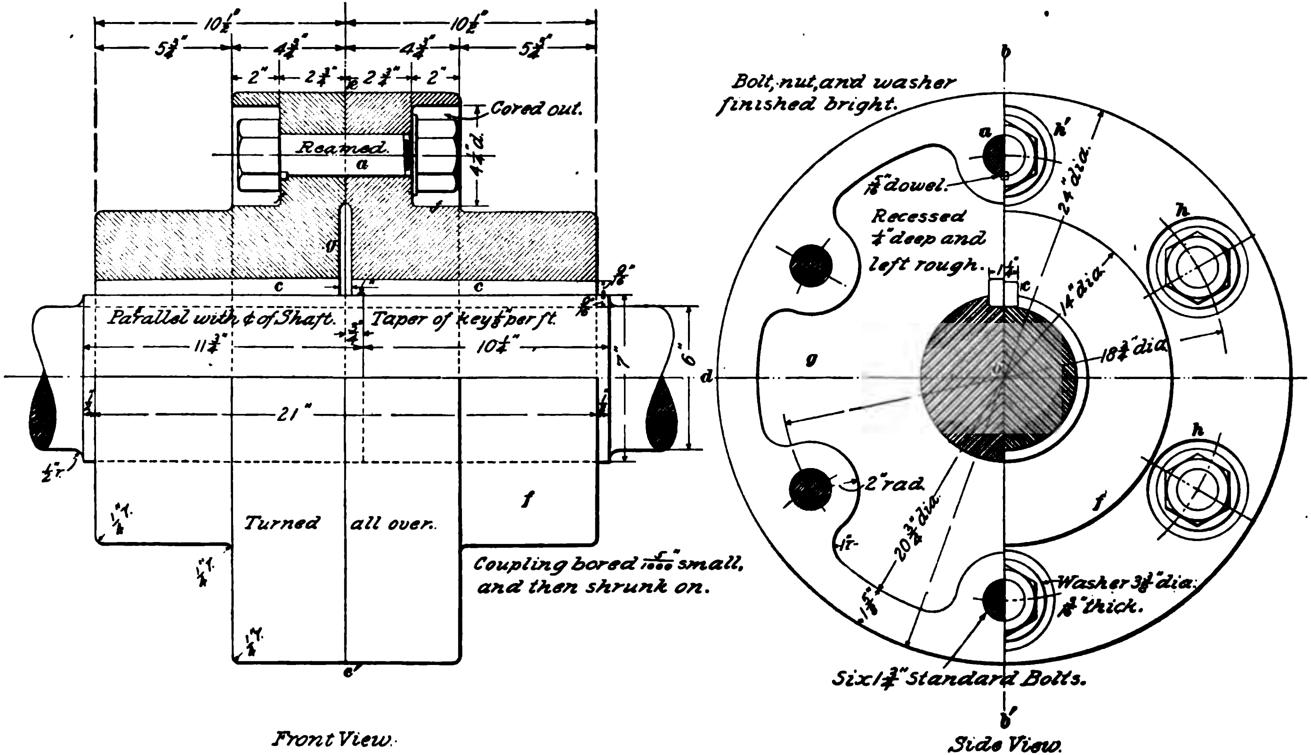
Fig. 28

The note *54" Turned* indicates that the pulley is to be 54 inches in diameter when finished by turning. As this is the finished size, the patternmaker understands at once that the pattern must be made larger by ⅛ inch or so, to allow for the metal removed when turning the face. The number of arms is clearly stated to be six, and it is understood that these are equally spaced. Sections of the arm at points near the hub and near the rim are given to show the shape and dimensions of the arm at these points. Such sections are often placed directly on the arm instead of at the side of it. It has been stated that dimensions of more than 24 inches are usually expressed in feet and inches; but in the case of round work the dimensions are often given wholly in inches, as has been done with the 80-inch and 54-inch dimensions on the drawing.

51. Flange Coupling.—A working drawing of a flange coupling is given in Fig. 29, and each of the two views shows a half-section, in order to indicate the construction as clearly as possible. The purpose of the coupling is to join two pieces of shafting that come together, end to end, so that they will turn as one piece. To do this, a circular disk is keyed to each piece of shafting and the disks are then bolted together firmly with six $1\frac{3}{4}$-inch bolts, as stated in the note. In the front view, the upper part is shown in section; as the bolt *a* and the keys *c* are shown, it is understood that the section has been taken along the center line *b o* of the side view, and that the part included between *d o* and *o b* is removed. As is customary, the shafts, the keys, and the bolt *a* are not shown in section. The cross-hatching on the coupling to the right and left of the line *e e'* runs in opposite directions, indicating that there are two parts to the coupling and that the line *e e'* shows the joint.

52. In the side view, Fig. 29, are two half views. In the one at the right of the center line *b b'*, the semicircle *f* is shaded so as to show a projection that, by referring to the front view, is seen to be the hub *f*. Furthermore, as the nut on the end of the bolt *a* is shown, it is to be understood that the right-hand half view is a side view drawn in full. In the left half view, nothing is shown in section except the bolts and the shaft. Consequently, the only place where this view could have been taken is along the line *e e'*; for, if the section had been taken at some point on either side of this line, part of the coupling itself would have been sectioned. The side view, by its outline, shows that the coupling is circular; this would also be inferred from the note *Turned all over* placed on the front view, as it is well understood by mechanics that only work of circular cross-section can be finished by turning.

53. The front view, Fig. 29, shows that there is a recess *g* in each half of the coupling, and the shape of the recess is given by the left-hand half of the side view. In the absence of any note or other indication to the contrary, it is to be assumed that the shape of the recess is the same in the other half view. The total depth of the recess is indicated in the front view, and the depth in each half of the coupling is given by the note



Cored out.
Reamed.
Parallel with ¢ of Shaft.
Taper of key per ft.
21"
Turned
all over.
Coupling bored small,
and then shrunk on.
Front View.
Bolt, nut, and washer
finished bright.
dowel.
Recessed
deep and
left rough.
2" rad.
Washer
thick.
Standard Bolts.
Side View.

Fig. 29

Recessed $\frac{1}{4}''$ *deep and left rough*, placed on the side view. The fact that there are six bolts is clearly specified by the note, which also gives the size and states that they are to be proportioned in accordance with the United States standard system of screw threads. The hole through which each bolt passes is to be reamed, as stated by the note on the upper half of the front view.

54. The circles *h*, Fig. 29, indicate that the nuts on the ends of the bolts are placed in recesses that have a circular cross-section; if these circles were not shade-lined, it would be necessary to look at the front view to see whether the circles indicate projections or recesses. Neither of the two circles *h* can be seen in the front view, but reference to the side view shows a semicircle *h'* occupying the same position with respect to the center, and, when projected over to the front view, it is seen to be the edge of a recess marked *Cored out*. It will be noticed that the finish mark *f* is placed on the surfaces of the cored recess for the bolt head and nut. This indicates to the machinist that the recess is to be finished and that the finishing should be done by counterboring, which is the most practical way of finishing the recess.

55. As far as the bolts in Fig. 29 are concerned, no bolt circle has been drawn to locate all of them definitely at the same distance from the center. A short arc has, however, been drawn through the center of each bolt; and as the dimension *18*$\frac{3}{4}''$ *dia.* is placed between two opposite arcs, the workman infers from this that all bolts are located on the same circle, there being no note or other indication to the contrary. The position of the key *c* is fixed by the center line *b b'*, which is seen to pass through two of the coupling bolts. On the front view appears a note *Parallel with* ℄ *of Shaft*. This note indicates that the keyway is to be cut parallel to the center line of the shaft.

56. Jig for Holding Round Work.—The three views shown in Fig. 30 (*a*) form a general drawing of a jig for holding round pieces. The jig consists of a fixed jaw *a* that is supported on four posts *b* screwed into the corners on the under

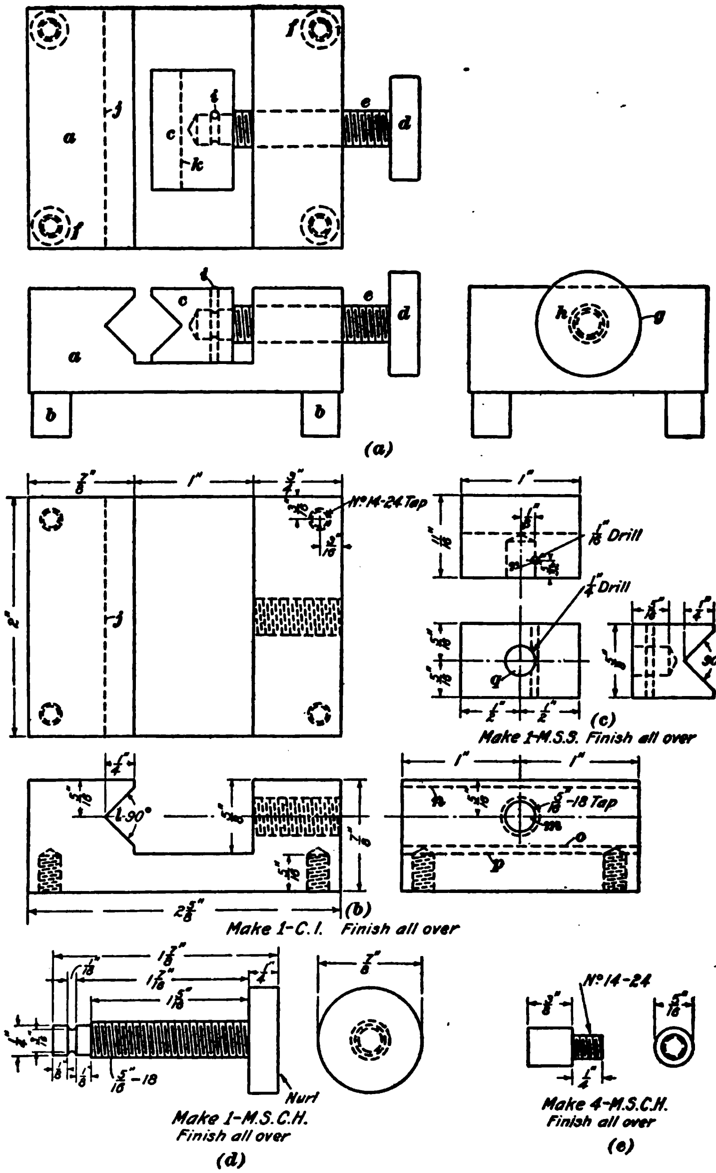

(a)
Nº 14-24 Tap
Drill
Drill
(c)
Make 1-M.S.S. Finish all over
(b)
Make 1-C.I. Finish all over
-18 Tap
Nurl
Make 1-M.S.C.H.
Finish all over
(d)
Nº 14-24
Make 4-M.S.C.H.
Finish all over
(e)

FIG. 30

side, and a jaw *c* that is moved by turning the nurled head *d* of the screw *e*. Each jaw has cut in it a 90° groove, in which the work is held. The dotted circles *f* in the corners of the plan view represent the posts *b*, each of which has three different diameters, namely, the diameter of the body, the diameter over the top of the thread, and the diameter at the bottom of the thread. Similarly, in the end view, the circle *g* represents the nurled head *d* and the dotted circles *h* represent the screw *e*. The end of the screw *e* fits in a drilled hole in the jaw *c* and has a groove cut in it. A pin *i* is driven down through a hole in the jaw and fits in the groove. In this way the screw *e* may turn freely in the jaw, but it cannot pull out. The dotted lines *j* and *k* represent the sharp bottoms of the grooves in the jaws.

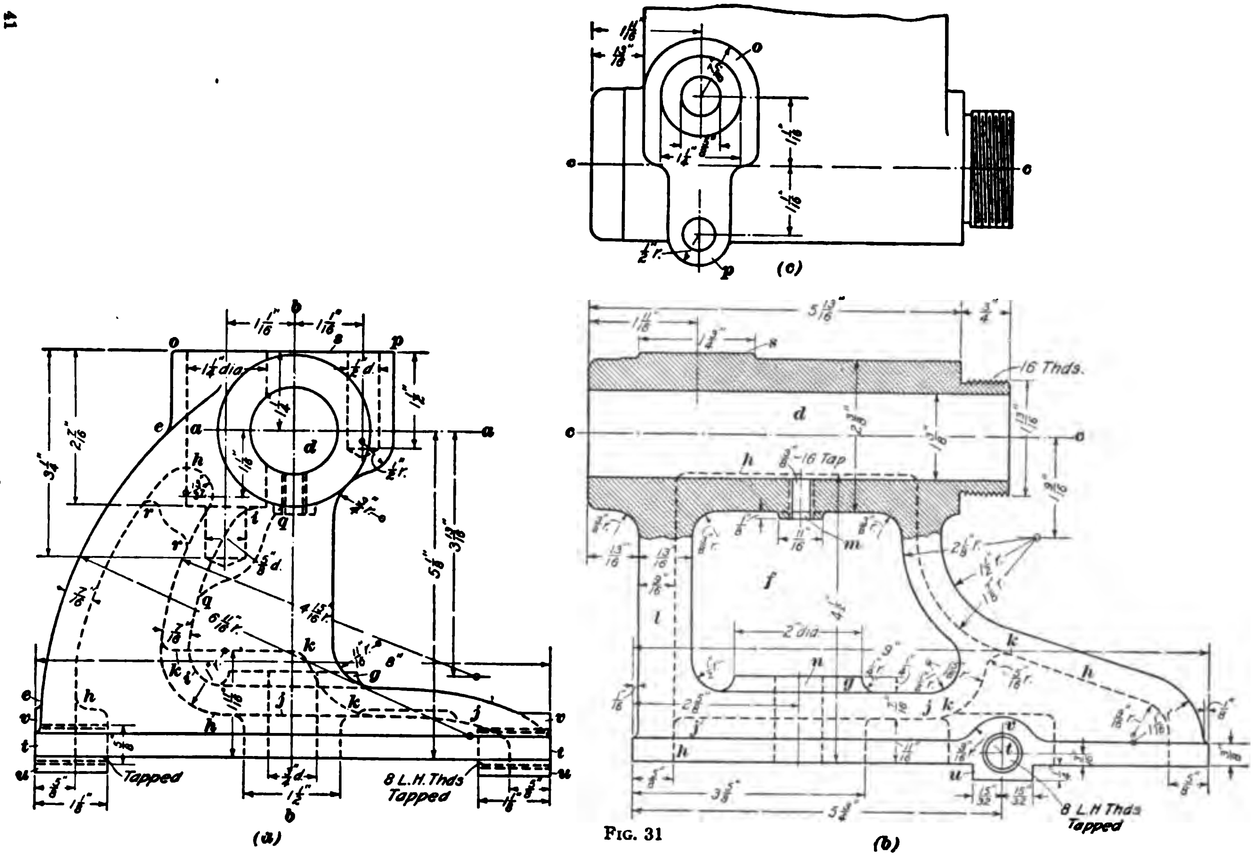
57. A detail drawing of the fixed jaw is shown in Fig. 30 (*b*). According to the note, the piece is to be of cast iron and finished all over. The block from which this jaw is formed is $2\frac{5}{8}$ inches long, 2 inches wide, and $\frac{7}{8}$ inch thick. At a distance of $\frac{3}{4}$ inch from one end, a slot 1 inch wide and $\frac{5}{8}$ inch deep is cut across the block, forming the recess in which the work is placed. A right-angled groove *l* is cut in the inner face of the large end of the piece at such a depth that the point of the groove is $\frac{5}{16}$ inch below the top surface. A hole *m* is tapped in the middle of the smaller end $\frac{5}{16}$ inch below the top surface, and a $\frac{5}{16}$-inch tap having 18 threads per inch is used. The four holes for the supporting posts are located so that the center of each is $\frac{3}{16}$ inch from each edge of the bottom of the block, and each hole is $\frac{5}{16}$ inch deep. On the side and end views, the threaded holes for the posts are shown by dotted lines, and the dotted line *j* represents the bottom of the groove. The lines *n* and *o* represent the top and bottom edges of the groove, and the line *p* represents the bottom surface on which the movable jaw slides.

58. A detail drawing of the movable jaw is shown in Fig. 30 (*c*). It is to be made of machinery steel, soft, and finished all over. A block 1 inch long, $\frac{11}{16}$ inch wide, and $\frac{5}{8}$ inch high is first made, and then along one side is cut a 90° groove

$\frac{1}{4}$ inch deep, so that the bottom of the groove will be $\frac{5}{16}$ inch from the top and bottom faces. In the center of the side opposite the groove is drilled a $\frac{1}{4}$-inch hole *q* $\frac{5}{16}$ inch deep. On the top face, at a distance of $\frac{1}{8}$ inch to the side of the center line of the hole *q* and $\frac{5}{32}$ inch from the front face, is located the center of the hole *r*. At this point, a $\frac{1}{16}$-inch hole is drilled squarely through the block, as shown by the vertical dotted lines. The horizontal dotted line is the bottom of the groove.

The screw, shown in view (*d*), is of machinery steel, case-hardened, and finished all over. The head is $\frac{7}{8}$ inch in diameter and $\frac{1}{4}$ inch thick, and is nurled. The threaded part is $\frac{5}{16}$ inch in diameter, $1\frac{5}{16}$ inches long, and it has 18 threads per inch. At the end, the screw is turned down to a diameter of $\frac{1}{4}$ inch for a length of $\frac{5}{16}$ inch, and a round groove $\frac{1}{32}$ inch deep is cut in the middle of this plain part. The post is shown in view (*e*). Four posts, each of machinery steel, case-hardened, are required. Each post has a body $\frac{3}{8}$ inch long and $\frac{5}{16}$ inch in diameter, with a shank $\frac{1}{4}$ inch long and threaded to fit the holes in the fixed jaw.

59. Tailstock Casting.—Views of the casting forming the body of the tailstock of a lathe are shown in Fig. 31 (*a*) and (*b*). The main center lines *a a*, *b b*, and *c c* pass through the center of the $1\frac{3}{8}$-inch hole *d* bored in the barrel to receive the spindle, and from these lines the principal dimensions are laid off. The body is of irregular shape, being curved at the back, as indicated by the line *e e*, and a recess *f* is formed beneath the middle of the barrel to receive the nut by which the tailstock is clamped to the bed of the lathe. This nut rests on the top of a boss *g* on the lower surface of the recess. Between the back *e e* and the recess *f* there is an opening cored out inside the casting, as is indicated by the dotted line *h h h*. The back surface of the recess *f* is denoted by the dotted line *i i*, and the thickness of metal between the recess and the cored opening is $\frac{7}{16}$ inch, as marked; also, the thickness of the metal at the back is $\frac{7}{16}$ inch. Beneath the boss *g* the metal is cored out as indicated by the line *j j*, and beyond the recess the cored part is carried higher, as shown by the dotted



Tapped
8 L.H. Thds
Tapped
(a)
(c)
16 Thds.
3/8"-16 Tap
2" dia.
8 L.H Thds
Tapped
(b)

FIG. 31

line *k k*. The barrel is thus supported by a number of metal webs, or walls, varying from $\frac{7}{16}$ to $\frac{5}{8}$ inch in thickness.

60. The barrel has an overall length of $6\frac{9}{16}$ inches, as shown in Fig. 31 (*b*), and at the end it is turned down for a distance of $\frac{3}{4}$ inch to a diameter of $1\frac{13}{16}$ inches. On this part is cut a screw thread with 16 threads per inch. The opposite end of the barrel overhangs the supporting web *l* by $\frac{13}{16}$ inch. A hole *m* for a setscrew is drilled in the bottom of the barrel, on a line with the center of the hole *n* for the anchor bolt. The hole *m* is threaded with a $\frac{3}{8}$-inch tap having 16 threads per inch, and the threads are indicated by the double lines at each side of the hole. The threads are understood to be United States standard threads, as no definite note is given to state their shape.

A top view, or plan, of the barrel, is shown in (*c*). Two bosses *o* and *p* are formed at opposite sides, on the same center line, at a distance of $1\frac{11}{16}$ inches from the end of the barrel, and the center of each is $1\frac{1}{16}$ inches from the center line *c c*. At the center of the boss *o*, a $\frac{5}{8}$-inch hole is first drilled to a depth of $3\frac{1}{4}$ inches, and this is then counterbored $1\frac{1}{4}$ inches in diameter to a depth of $2\frac{7}{16}$ inches. As these holes would extend through into the cored spaces, metal is added, as shown by the dotted lines *q q* and *r r*, view (*a*), beneath the holes. The boss *p*, view (*c*), has a $\frac{1}{2}$-inch hole drilled at its center to a depth of $1\frac{1}{2}$ inches. The surface *s* of these bosses is faced off so as to be $1\frac{1}{4}$ inches above the center line *c c*.

61. The base of the tailstock casting, Fig. 31, has a length of 9 inches and a width of 8 inches, and projects slightly beyond the unfinished casting all around. The vertical faces of this projecting part of the base are all finished. Two holes *t* are tapped through the base at the sides with 8 left-hand threads per inch. The center is located $5\frac{3}{4}$ inches from the heavy end of the casting, and $\frac{3}{16}$ inch above its lower surface. There is a lug *u* beneath each of the holes *t*, extending $\frac{1}{4}$ inch below the lower surface and $\frac{15}{32}$ inch on either side of the center line through the hole. Each lug is $1\frac{1}{8}$ inches long and is finished on the sides and the bottom. A raised part *v* having a

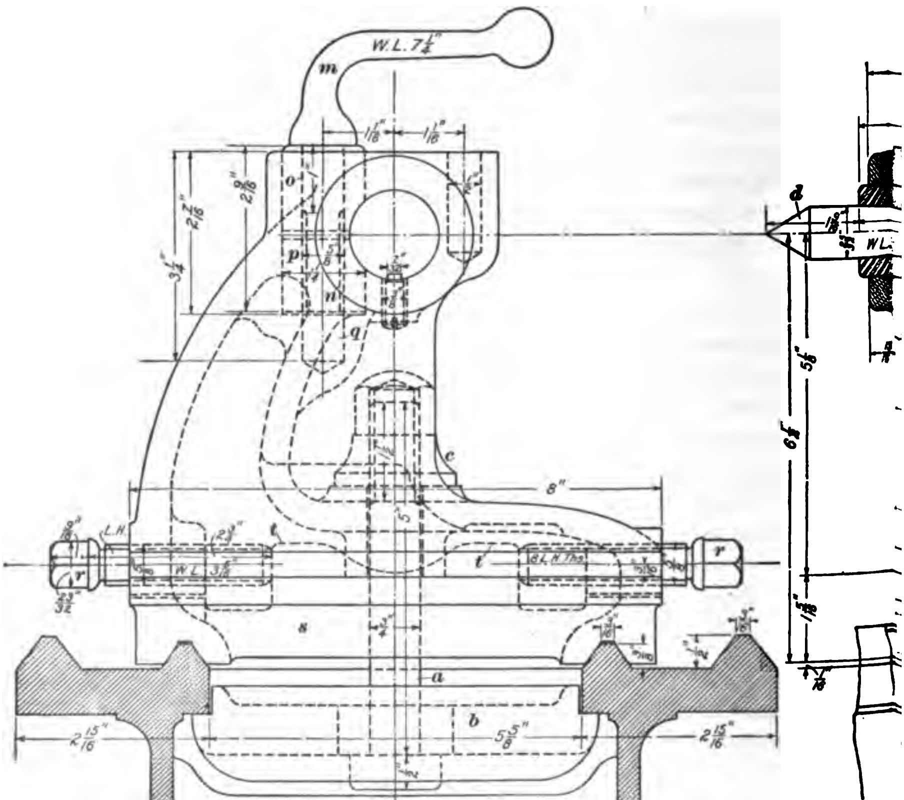
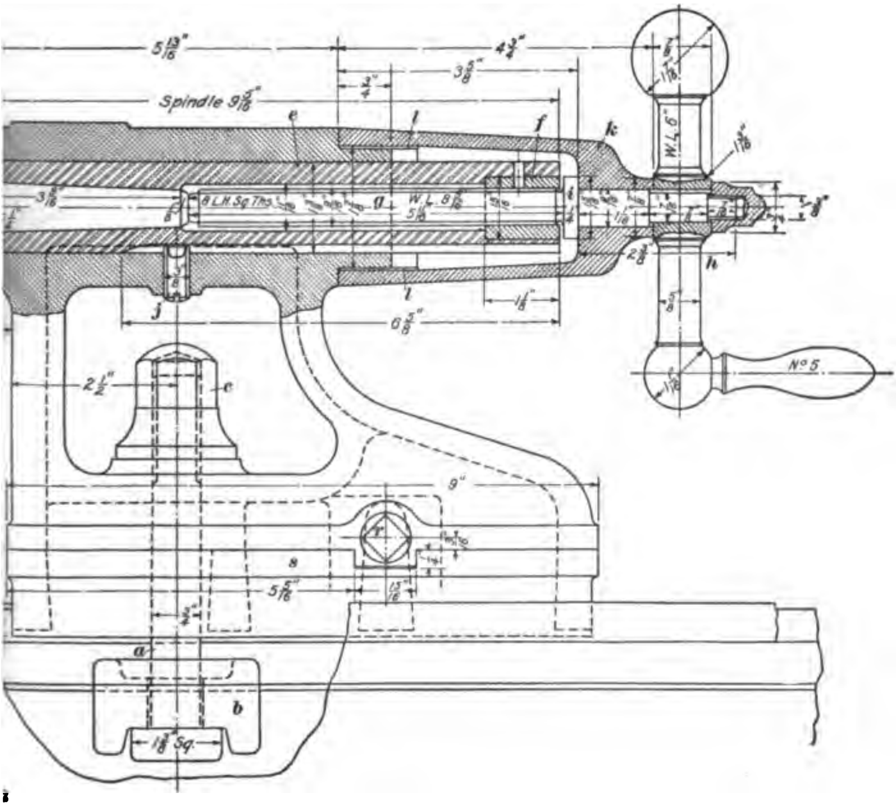


FIG. 8



Spindle
W.L. 6"
8 L.H. Sq. Ths.
9"
Nº 5
e
f
k
l
g
h
j
c
r
s
a
b
Sq.

radius of $\frac{9}{16}$ inch is formed above each hole *t* to give enough metal around the hole. The radii of the several curved surfaces of the casting, and the centers from which the curves are drawn, are clearly indicated.

62. General Drawing of Tailstock.—A general drawing of a tailstock is shown in Fig. 32. This is the same tailstock casting that is shown in Fig. 31, with the addition of all the parts that are attached to it and a part of the lathe bed on which it is clamped. The anchor bolt *a*, Fig. 32, passes up through a slot in the yoke *b* and is fitted with a capnut *c* that rests on a boss on the casting. When the nut *c* is screwed down, the yoke is drawn up against the bed and the tailstock is firmly held on the V's. The dead center *d* is held in a steel spindle *e* that fits inside the bored barrel of the casting. A nut *f* is pinned to the end of the spindle, and through it passes the screw *g*, to the outer end of which is fixed the handle *h*. The collar *i* and the hub of the handle *h* prevent the screw from moving endwise, and when it is turned by the handle the spindle *e* is moved out of or into the barrel, thus moving the dead center. The spindle *e* has a keyway along its under side, in which fits the flatted end of the setscrew *j*. The spindle is thus prevented from turning, but is free to be moved endwise. The part *k* that carries the screw *g* is threaded inside and is screwed tightly on the threaded end of the barrel, the threads being indicated by the pairs of parallel lines *l*. The thread on the screw *g* is left-hand, so that, when the handle is turned right-handed, the dead center will be moved outwards, away from the tailstock.

63. When the spindle *e*, Fig. 32, has been moved outwards as far as it should go, it may be locked in position by turning the handle *m*. This handle has a threaded stem *n* that passes through the collar *o* and the nut *p* and fits into the drilled hole *q*. Both the collar *o* and the nut *p* are in place when the barrel is bored, and they are cut away at one side to the curve of the spindle. When the handle *m* is turned in the proper direction, the nut *p* is drawn upwards and the collar *o* is forced downwards, and the spindle is clamped firmly between

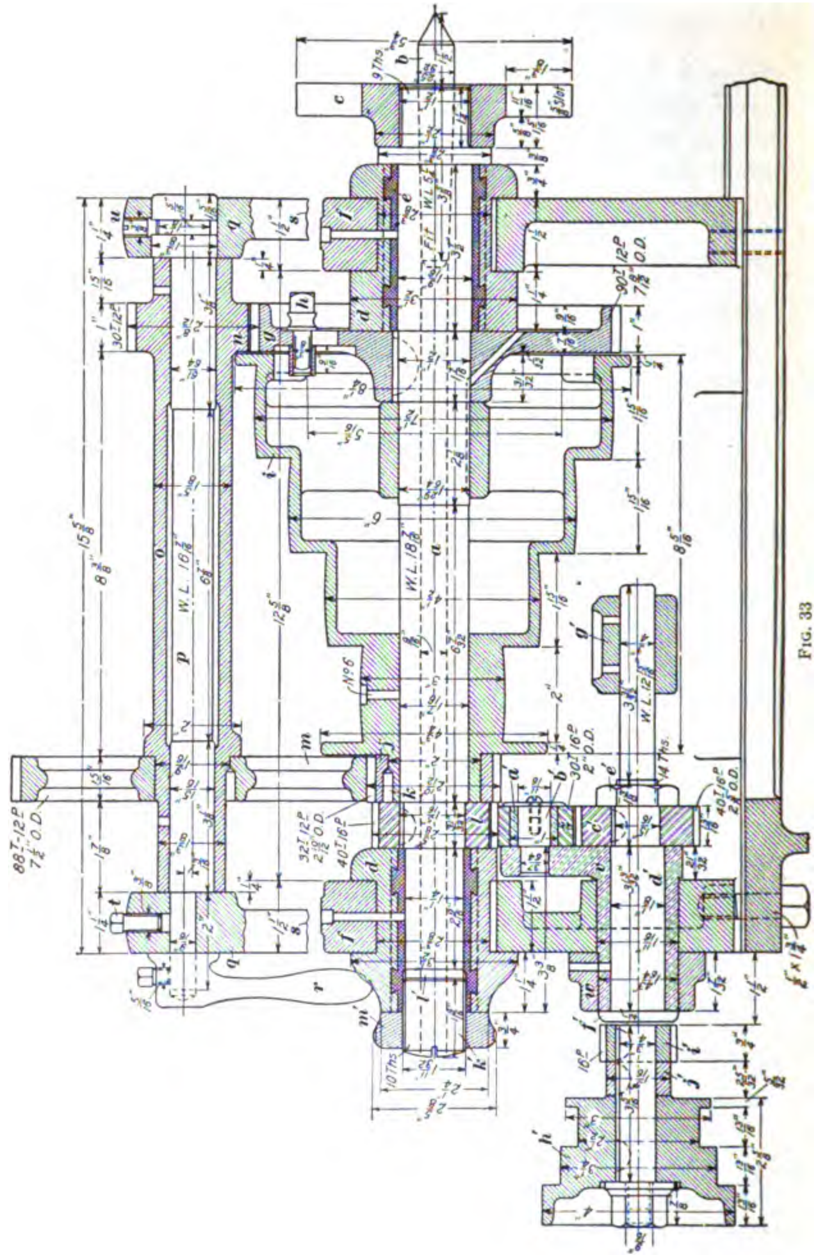


FIG. 33

the two. The adjusting screws *r* furnish a means of changing the position of the dead center sidewise on its base *s*. The base has grooves that fit the ways on the bed, and on its upper side it has two lugs *t* against which the screws *r* bear. The screws *r* are made with 8 left-hand threads per inch. The abbreviation *W. L.* is used frequently on the drawing to indicate the whole length, or overall length, of a part.

64. Lathe Headstock.—A longitudinal section of a headstock for a lathe is shown in Fig. 33, and in Fig. 34 is an end view showing the center lines of several parts, but omitting

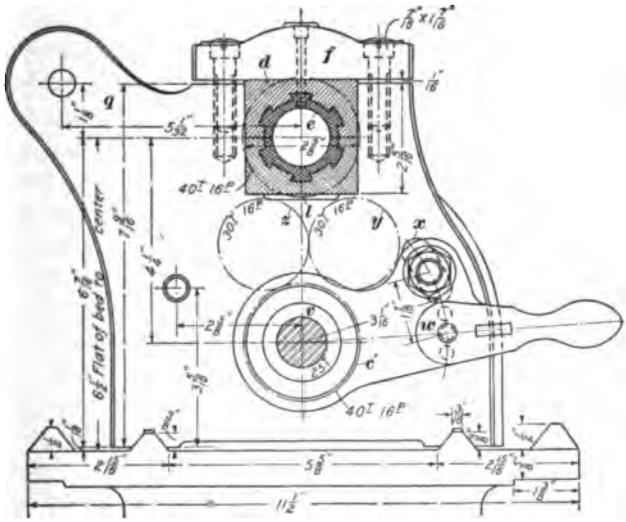


FIG. 34

details. The spindle *a* is of different outside diameters at various points and is drilled with a $\frac{9}{16}$-inch hole except at the inner end, where the hole is tapered to receive the live center *b*. The large end is made $1\frac{1}{2}$ inches in diameter and a screw of 9 threads per inch is cut on it, so that the face plate *c* may be screwed on. The face plate is screwed up until it comes firmly against the collar, which is $\frac{3}{8}$ inch wide. The threaded hole in the face plate is beveled at the ends, to allow the face plate to come solidly against the collar and to give a good start to the threading tool. Slots $\frac{5}{8}$ inch wide and $1\frac{3}{8}$ inches deep are cut on opposite sides of the center of the plate, to receive the tail of the dog that holds the work. The spindle turns in cast-iron bearing boxes *d* lined with Babbitt metal *e* that is held in by means of dovetailed slots. The boxes are split in halves, as is indicated by the fact that the section lining, or cross-hatching, runs in opposite directions in the upper and lower parts of the boxes. These boxes are fitted into square slots in the frame of the headstock and are held in place by the caps *f*. Oil holes are drilled through the centers of the caps and the upper half of each box to enable oil to be supplied to each bearing.

65. To the spindle *a*, Fig. 33, is keyed the gear *g*, which has 90 teeth of 12 pitch and an outside diameter of $7\frac{8}{12}$ inches, as indicated by the abbreviation *O. D.* This gear has a short slot near the outer rim, in which is a screw *h* that may be moved so as to lock the gear to the cone pulley *i*. The cone pulley is loose on the spindle, and at its smaller end there is fastened to it a small gear *j* by a pin key *k*. This small gear, or pinion, as it is called, has 32 teeth of 12 pitch, and an outside diameter of $2\frac{10}{12}$ inches. Beside it, also keyed to the spindle, is another pinion *l*, made of steel, as shown by the cross-hatching, and having 40 teeth of 16 pitch. When the work is to be rotated rapidly, the screw *h* is set so as to lock the cone pulley and the gear *g*, and the driving belt is put on one of the steps of the cone pulley. The spindle is thus caused to turn at the same rate as the cone pulley and the gear *g*, thus rotating the work by means of the face plate *c*.

66. When the work in the lathe is to be turned slowly, the back gearing is brought into action. It consists of a pair of gears *m* and *n*, Fig. 33, on a sleeve *o* that is loose on a shaft *p* held in brackets *q*. The gear *m* is held to the sleeve by a pin key, but the pinion *n* is in one piece with the sleeve. The shaft *p* is eccentric; that is, its center line does not pass through the centers of the holes in the brackets *q*. As a result, when the handle *r* fixed on the smaller end of the shaft *p* is pushed over away from the spindle, the gears *m* and *n* are swung backwards, and are drawn out of gear with the wheels *j* and *g*. When the handle is drawn forwards, the gears *m* and *n* are caused to engage with the gears *j* and *g*. When the back gears are thrown into action, the screw *h* is set so that the cone is free from the gear *g*. The cone then turns freely on the spindle, rotating the pinion *j*, which turns the gear *m* on the sleeve *o*. As the pinion *n* must turn with the sleeve *o*, motion is given by it to the gear *g*, which is fast to the spindle, and thus the spindle and the work are rotated; but the speed is much slower than when the cone pulley drives the gear *g* directly.

67. The back gearing in Fig. 33 is shown directly above the spindle *a*, but this is not its actual position. As may be seen in Fig. 34, the brackets *q* supporting the shaft *p* are behind the spindle instead of directly above it. In Fig. 33, therefore, the brackets *q* are not shown joined to the frame of the headstock, but are left with broken lines at *s*, indicating that a conventional section is shown, rather than a true section. The setscrew *t* is screwed down when the shaft *p* is to be held firmly, so as to keep the back gears in action or out of action. A disk of copper or brass between the setscrew and the shaft prevents the latter from being injured. The setscrew *u* is flattened, and its end fits a groove turned in the enlarged end of the shaft *p*. In this way the shaft cannot move endwise, but may turn in its supports *q*. Oil holes are drilled through the sleeve *o* to admit oil to those parts of the shaft *p* on which the sleeve turns.

68. Passing through a bored hole in the outer part of the headstock frame is a bronze sleeve *v*, Figs. 33 and 34, to which is pinned a lever *w* that may be used to turn the sleeve through a small angle, and that may be locked in position by tightening the nut of the clamp bolt in the slot *x*. At the inner end of the sleeve is formed an arm carrying two steel pinions *y* and *z* having 30 teeth each. These two pinions mesh with each other, and either one may be swung into mesh with the gear *l* on the spindle *a* by moving the handle *w*. Each of the pinions *y* and *z* turns on a bronze bushing *a′* fitted on a pin *b′* that is riveted to the arm of the sleeve *v*. A washer held in place by a screw prevents the pinion from running off. The pinion *z* gears with a small 40-tooth wheel *c'* that is keyed to the shaft *d'* and is held in place by the nut *e'*. This nut, in connection with the collar *f'*, keeps the shaft *d'* from moving endwise. The shaft rotates in the bronze sleeve *v* and the cast-iron bearing *g'* fixed to the headstock casting. At the outer end of the shaft are keyed the cone pulley *h'* and the pinion *i'*, separated by a short sleeve *j'* that is also keyed to the shaft.

69. A plug *k'*, Fig. 33, is threaded over its entire length and is screwed into the end of the box forming the outer bearing of the spindle *a*. A fiber washer or ring *l'* is placed between the plug and the spindle, and the plug is then screwed up against the washer. In this way the fiber washer and the plug take the end thrust of the spindle. The nut *m'* acts as a locknut to keep the plug from turning when it has been correctly set.

The section shown is a conventional one in several respects. The matter of showing the back gearing above the spindle has already been mentioned. The pinion *z* is shown between the pinions *l* and *c'* as if it were on a vertical line between their centers, whereas, Fig. 34 shows that the pinion *z* is not on the vertical center line. The keys shown in the sectional view, Fig. 33, are semicircular in shape and are known as Woodruff keys. In the end view, Fig. 34, none of the gear-wheels are shown in full, but they are indicated by circles on which the number of teeth and the pitch are written.

MEASURING INSTRUMENTS

MEASUREMENTS AND MEASURING DEVICES

METHODS OF MEASUREMENT

1. A **measuring instrument** may be defined as any tool, device, appliance, or instrument that can be used to measure or compare a linear, angular, superficial, or cubic dimension, a manifestation of force, physical property, or temperature, with some established unit of measurement. A **gauge**, in its broadest sense, is any kind of measuring instrument; however, among mechanics it is understood to be any reasonably unchangeable device or appliance that is used to establish or define some particular linear or angular dimension, but that cannot be used to measure, in principal or secondary units, any other dimension.

2. **Secondary units** are integral parts or multiples of the principal units. For example, the yard is a principal unit of linear measurement, and the foot and the inch are secondary units, because they are integral parts of the yard; also, the mile is a secondary unit, because it is a multiple of the yard. The minute and the second are secondary units of angular measurement, as they are integral parts of the degree, which is the principal unit. The difference between a measuring instrument and a gauge, then, is that the former measures a dimension by comparing it with a unit, whereas the latter defines a particular unit or a part of a unit.

3. Kinds of Measurements.—Measurements made by comparing the coincidence or agreement of lines with the aid of a microscope, a simple magnifying glass, or the naked eye, are called **line measurements.** An example of a line measurement is illustrated in Fig. 1, in which the calipers *a* are set to a distance of 2½ inches on the scale *b* by noting when the line *c* of the scale coincides with the inner edge of the leg *d* of the calipers. Measurements made by comparing lengths by means of instruments that are brought in contact with the opposite ends or faces of the pieces to be compared are called **end measurements.** A measurement of this kind is shown in Fig. 2, in which the diameter of the piece *a* is determined when the inner surfaces of the jaws of the gauge *b* just touch the piece *a*. Thus, a line measurement is made by eye, and an end measurement by touch.

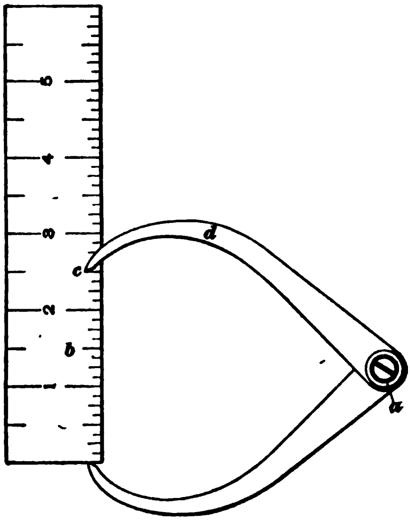


FIG. 1

4. When measurements are made to determine an angle, or the amount of opening between two lines that meet or cross, they are called **angular measurements.** Such measurements are made by comparison of the coincidence of lines, by end measurements, or by a combination of these two methods. In many cases the measurements made by a mechanic are such a combination. Thus, the setting of a pair of calipers to a given dimension by the use of a steel rule is a line measurement, whereas, the measuring of a piece of work by the

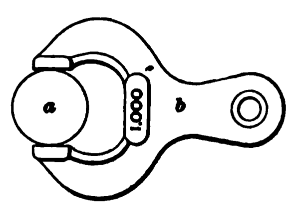


FIG. 2

calipers thus set is an example of end measurement. In using a bevel protractor, the mechanic sets it to the given angle by line measurement; but when he brings the protractor in contact with the work so as to compare the angle of the work, he makes an end measurement. The angular measurement thus becomes a combined line and end measurement.

STANDARDS OF MEASUREMENT

5. English Standard.—In the United States of America, in Great Britain, and in most of the British colonies, the principal unit of length is the **imperial yard,** which is the distance between two marks near the opposite ends of a bronze bar kept within the walls of the Houses of Parliament, London. This original bar is known as Bronze No. 1. Several copies of this bar were made at various times, and in 1856 two copies were delivered to the United States Government.

6. American Standard.—In 1866, Congress passed a law declaring a bar in the possession of the Government, defining a length of 1 meter, to be the legal standard. Thus, the only standard of length recognized by law in the United States is the meter, which is the unit of length in the French system of measurement, and which is equivalent to a distance of 39.37 inches. This metric bar is now the basic or ultimate standard of length in the United States. By the act of 1866, the legal yard was established as $\frac{3600}{3937}$ of the length of the meter marked on the metric bar. This yard corresponds to the yard denoted on the two bars presented to the United States by the British Government. Although these two bars were never declared the legal standard, they are used as standard bars whenever great precision of measurement is required.

7. Working copies of the standard bars in the possession of the United States Government have been constructed by private firms engaged in the manufacture of measuring instruments and gauges. These copies have been carefully compared with the standard bars, and form the working standards of reference of the manufacturers, but the standard bars in the possession of the government form the ultimate standard of reference.

8. **Angular Standard.**—So far as measurements of angular dimensions are concerned, no ultimate standard of reference is required, as the degree, which is the principal unit of angular measurement, can always be found by dividing a circle into 360 equal parts.

9. **Units of Measurement.**—For ordinary work, the English system of measures is in general use, but for scientific purposes the metric system is preferred. The latter system is also frequently employed in connection with various forms of engineering work, and for this reason the corresponding values of the units in the English and metric systems should be learned. The meter has a length of 39.37 inches; hence, the following *conversion factors* may be used to change the units of one system into the units of the other system:

METRIC TO ENGLISH		ENGLISH TO METRIC	
1 meter (m.)	=39.37 inches	1 foot	=.3048 meter
1 meter	=3.28 feet	1 inch	=.0254 meter
1 centimeter (cm.)	=.3937 inch	1 inch	=2.54 centimeters
1 millimeter (mm.)	=.03937 inch	1 inch	=25.4 millimeters

EXAMPLE 1.—Reduce 1.5 meters to inches.

SOLUTION.—As 1 m. = 39.37 in., a length of 1.5 m. is equal to $1.5 \times 39.37 = 59.055$ in. Ans.

EXAMPLE 2.—How many inches are there in a length of 20 centimeters?

SOLUTION.—As 1 cm. = .3937 in., a length of 20 cm. is equal to $20 \times .3937 = 7.874$ in. Ans.

EXAMPLE 3.—Convert $3\frac{1}{2}$ inches to centimeters.

SOLUTION.—As 1 in. = 2.54 cm., $3\frac{1}{2}$ in. is equal to $3\frac{1}{2} \times 2.54 = 8.89$ cm. Ans.

EXAMPLE 4.—How many millimeters are there in a length of $7\frac{1}{2}$ inches?

SOLUTION.—There are 25.4 mm. in 1 in.; hence, $7\frac{1}{2}$ in. is equal to $7\frac{1}{2} \times 25.4 = 190.5$ mm. Ans.

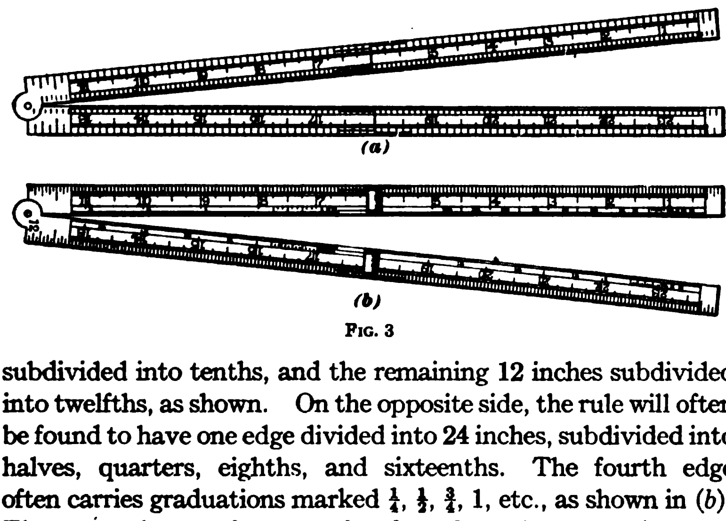
EXAMPLE 5.—A certain piece of work is 125 millimeters long. What is its length in inches?

SOLUTION.—As 1 mm. = .03937 in., a length of 125 mm. is equal to $125 \times .03937 = 4.92$ in. Ans.

FORMS OF MEASURING INSTRUMENTS

INSTRUMENTS FOR LINEAR MEASUREMENT

10. Two-Foot Rule.—The most familiar measuring instrument for the comparison of linear dimensions is the two-foot rule, shown in Fig. 3 (*a*), which is usually made up of four leaves hinged together to allow it to be folded so as to be carried easily. For the sake of convenience, the edges on both sides are usually graduated with different kinds of subdivisions; frequently, two or more kinds of subdivisions appear on the same edge. Thus, the rule being 24 inches long, one edge may be divided into 24 inches, and each inch subdivided into halves, quarters, and eighths. The opposite edge may have 12 inches



(*a*)

(*b*)

FIG. 3

subdivided into tenths, and the remaining 12 inches subdivided into twelfths, as shown. On the opposite side, the rule will often be found to have one edge divided into 24 inches, subdivided into halves, quarters, eighths, and sixteenths. The fourth edge often carries graduations marked $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1, etc., as shown in (*b*). These numbers refer to *reduced scales*; thus, on the part marked $\frac{1}{2}$, an actual length of $\frac{1}{2}$ inch represents a length of 1 foot; on the part marked $\frac{3}{4}$, an actual length of $\frac{3}{4}$ inch represents 1 foot; and so on.

11. Two-foot rules are usually made of boxwood; the most expensive ones are made of ivory. Divisions smaller than

$\frac{1}{16}$ inch are rarely marked on them, for they become so soiled in use that fine divisions could not be read. When divided into sixteenths, the smallest fraction of an inch that can be directly measured is $\frac{1}{16}$, but with a little practice the middle point between the sixteenth-inch marks can be located with a fair degree of accuracy, thus making it possible to measure distances as small as $\frac{1}{32}$ inch. Usually, each inch graduation is marked by a number that represents its distance in inches from the right-hand end; that is, the numbers increase from right to left. The numbers increase from right to left because the two-foot rule is mostly used in such a manner that its right-hand end forms a stop from which the required dimension is transferred by a lead-pencil mark or by scribing to the work. The two-foot rule is well adapted for rough work, where accuracy of measurement is not absolutely necessary.

12. **Engineer's Pocket Rule.**—Very frequently it is convenient to have a rule more than 2 feet long. Mea-

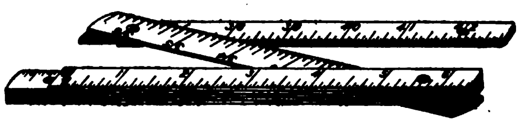


FIG. 4

surements of more than 2 feet can be made with tapes, but when a tape is used both ends of it must be held; whereas, on account of the greater stiffness of the rule, approximate measurements can be made when only one end is supported. On this account, the form of rule shown in Fig. 4 has been brought out. It consists of a number of short wooden pieces joined by pins and provided with special spring joints that keep the rule in a straight line when open. The ends are provided with brass tips. This rule is made in lengths of from 2 feet to 8 feet, and is divided into inches and sixteenths, or into millimeters, if in the metric system.

13. **Steel Tapes.**—In measuring distances greater than a few feet, steel tapes are very convenient. They are thin bands of steel made in lengths varying from 2 to 100 feet for shop use. Instead of being cut in as in other rules, the graduations on a

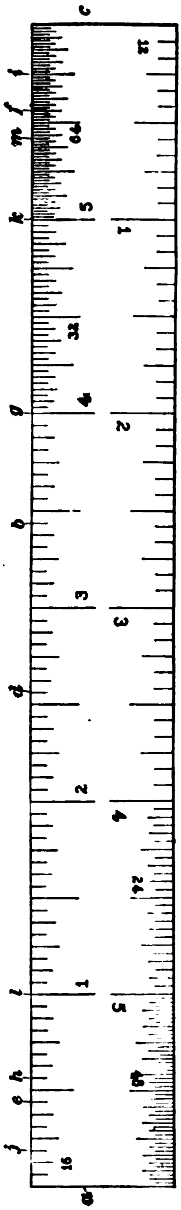


Fig. 5

steel tape are raised. The divisions are not very fine; hence, tapes are suitable only for approximate measurements.

14. Standard Steel Rule.—For the use of machinists and for more accurate measurements in general, so-called steel rules are used. In the better class of these instruments, all graduations are cut in a dividing machine. Standard steel rules bought in the open market are always graduated on both edges of each side, and a large choice of different kinds of graduation is offered by the makers. In Fig. 5 are shown the graduations of one side of a six-inch standard steel rule having several kinds of graduation along each edge. For convenience, and also for the purpose of an extensive range, the different kinds of graduation on each edge are usually made multiples of one another; thus, on one edge are given divisions of the inch into 12, 24, and 48 parts, and on the other edge into 16, 32, and 64 parts. The figures denoting inches almost invariably increase from left to right, or in a direction opposite to that in which they are given on a two-foot rule.

15. Setting Calipers on Steel Rule. In the use of the steel rule, several cases may arise, each of which requires a different method of procedure. Suppose, for example, that a pair of calipers is to be set to a measurement of $3\frac{7}{16}$ inches by the use of the rule shown in Fig. 5. One leg of the calipers is placed against the left end *a* of the rule, in line with the series of graduations marked by the number 16 in small figures. The calipers are then opened, by moving the other leg to the right along the rule, until this leg comes in line with the mark *b*, which is the seventh mark beyond the 3-inch mark, counting from the left end. The calipers are then set to a measurement of $3\frac{7}{16}$ inches. If it is found more convenient, the measurement may be made from the right end *c* of the rule. In this case, one leg of the calipers is placed against the right end of the rule, and is opened by drawing the other leg toward the left until the mark *d* is reached. This mark *d* is $3\frac{7}{16}$ inches from the right end, for, in opening the calipers to this point, the movable leg passes over 3 whole inch divisions, and 7 of the small divisions denoting $\frac{1}{16}$ inch each, or a total distance of $3\frac{7}{16}$ inches.

16. It will be observed that in the second case no attention was paid to the figures denoting the inch divisions. As these figures become smaller in value toward the left, the operation that was performed was distinctly a case of mechanical subtraction; that is, $3\frac{7}{16}$ inches was subtracted from the total length of the scale, which is 6 inches. Then, the line denoting a distance of $3\frac{7}{16}$ inches from the right end is $6-3\frac{7}{16}=2\frac{9}{16}$ inches from the left end. Thus, the subtraction may be performed arithmetically, in order to find the position of a line denoting a given distance from the right end, or it may be performed mechanically. In the latter case, the actual subtraction is saved; there is also less danger of making an error.

17. As a further illustration, suppose that the calipers are to be set to a measurement of $5\frac{9}{16}$ inches on the steel rule, Fig. 5. This may easily be done by working from the right end *c* and counting off to the left 5 whole inch divisions and 9 sixteenth divisions to the mark *e*. But if the left end of the rule is used, the movable leg of the calipers will finally be brought to some point between the line 5 and the end *c* of the rule. This last inch division at the right is subdivided into sixty-fourths of an inch, and $\frac{9}{16}$ inch $=\frac{36}{64}$ inch. Therefore, with one leg of the calipers placed against the end *a* of the rule, the other leg is moved to the right over 5 whole inch divisions, and then over 36 of the sixty-fourth divisions, to the mark *f*. The calipers are then properly set to $5\frac{9}{16}$ inches.

18. Setting Dividers on Steel Rule.—When setting a pair of dividers on a rule, the greatest accuracy can be obtained by placing them so that their points coincide with division lines that are the required distance apart. Thus, suppose that dividers are to be set to $3\frac{7}{16}$ inches on the rule shown in Fig. 5. Then, if one point is placed in the division mark *g* of the fourth inch, it is necessary to count to the left 3 whole inches, and 7 sixteenth divisions, to the mark *h*, and to adjust the other point until it just drops fairly into the mark *h*. Sometimes it is more convenient to work to both right and left of a whole inch mark. Thus, it is possible to start at the 1-inch mark, and count 7 sixteenth divisions to the left of it, to the mark *h*, and 3 whole inches to the right of it, to the mark *g*, to get the positions of the division marks denoting a distance of $3\frac{7}{16}$ inches.

19. If a pair of dividers is to be set to $5\frac{9}{16}$ inches on the rule, Fig. 5, it is not possible to put one point of the dividers in one of the whole inch marks, as the greatest measurement that can then be obtained is only 5 inches, or less than the required dimension. Neither is it possible to use the first half-inch mark, either on the right end or on the left end, as the largest measurement that may then be obtained is only $6-\frac{1}{2}=5\frac{1}{2}$ inches, or less than the required dimension. But it is possible to find the location of a graduation mark from which to start the measurement by choosing a graduation that is nearer the end of the rule than the difference between the length of the rule and the given dimension. The difference is $6-5\frac{9}{16}=\frac{7}{16}$ inch. Choosing the graduation mark *i* representing the first quarter-inch from the right end, one point of the dividers is placed in it. Now, considering this line as the starting point of the graduations, and disregarding the whole-inch marks entirely, 5 whole inches and then 9 of the sixteenth divisions are counted off, locating the division mark *j*. The other point of the dividers is then brought fairly into the division line *j*, which in this case is $6-(\frac{1}{4}+5\frac{9}{16})=\frac{3}{16}$ inch from the end of the rule, and the dividers are then correctly set to $5\frac{9}{16}$ inches.

20. When a measurement is to be made that has a fraction expressed in multiples of the smallest number of divisions given on the rule, it is usually most convenient to work to both right and left of the mark that defines the beginning of the graduations representing the denominator of the given fraction. For instance, referring again to Fig. 5, let it be required to find the division lines that define $4\frac{27}{64}$ inches. As the divisions representing sixty-fourths commence at the 5-inch mark *k*, it is necessary to count off 4 inches to the left of this mark, locating the mark *l*, and then to count off 27 sixty-fourth divisions to the right of it, to the mark *m*; the distance *lm* is then the required length, or $4\frac{27}{64}$ inches.

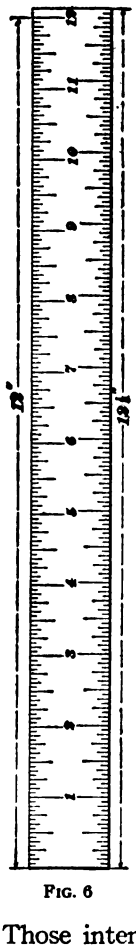


Fig. 6

21. Shrink Rule.—All metals shrink somewhat in cooling; allowance must therefore be made in forming the pattern or the mold, in order that the casting may be of the correct dimensions. The average amount of shrinkage is $\frac{1}{8}$ inch per foot for cast iron; that is, if the mold for a rectangular bar is $12\frac{1}{8}$ inches long, the casting will be about 12 inches long. To make a pattern with the aid of an ordinary rule would require a calculation for each separate dimension, in order to obtain the proper shrink allowance; hence, the patternmaker or the molder uses a special shrink rule, in which $12\frac{1}{8}$ inches is taken as 1 foot. This lengthened foot is subdivided into 12 equal parts, called inches, and each inch is subdivided into sixteenths. By the use of this rule the proper shrink allowance for an iron casting is then made.

22. Shrink rules are made of steel or of boxwood; they can be obtained for cast iron, brass, and other metals. In shrink rules intended for cast iron, $12\frac{1}{8}$ inches represents a length of 1 foot. Those intended for brass castings have a shrink allowance of $\frac{3}{16}$ inch per foot; that is, $12\frac{3}{16}$ inches represents 1 foot. Many shrink rules, for convenience, have a standard graduation on one edge and a shrink graduation on the other edge. Thus, the shrink rule illustrated in Fig. 6 has the standard graduation on one edge; the opposite edge has a length of 12⅛ inches, divided into 12 parts subdivided into 16 parts each.

23. Graduation of Shrink Rule.—Where patterns are made for castings poured from special alloys, it is often advantageous to have a shrink rule that exactly corresponds with the amount of shrink required for the alloy in use. Any such special rule may be made by the method illustrated in Fig. 7. A standard steel rule is laid down, as shown at *a b*. The amount of shrink per foot of length is laid off as shown by *b c*. Then, with *a* as a center, the arc *c d* is drawn with a pair of dividers. The point *d* at which the arc intersects the line *b d*, *b d* being square with *a b*, represents one end of the required shrink rule.

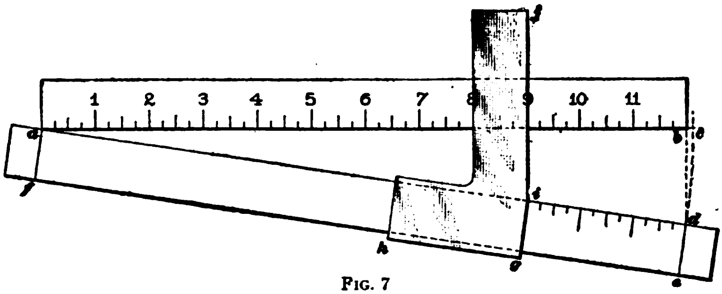


FIG. 7

The rule is usually made from a piece of hardwood. The stock is planed to the same thickness as the steel rule by which it is to be graduated.

24. The stock should be left somewhat longer than the required rule, and placed with one edge as shown at *a d*, Fig. 7. A special templet or square for graduating is necessary. This can be made of a piece of sheet brass or steel, or it may be made of wood. The general form is shown in the illustration. The edge *g h* is bent down so that it will be guided by the edge *e f* of the required rule. The edges *a d* and *e f* must be parallel. The side *i g* is perpendicular to *e f*, and *i j* is perpendicular to *a b*. By sliding the special graduating piece along the edge *e f*, the edge *i j* may be brought in line with any graduation on the standard rule, and corresponding lines may be drawn on the special rule along the edge *i g* by using a knife or any suitable scribing tool. Care must be taken to hold the scribing tool in the same position while drawing each line. Shrink rules made by this process are accurate enough for any ordinary shop work.

25. Key-Seat Rule.—A useful tool for key-seat work is the key-seat rule shown in Fig. 8. It consists of two straightedges *c* and *d* clamped or formed together so as to make a box square. In the form shown, a graduated scale *c* is clamped to

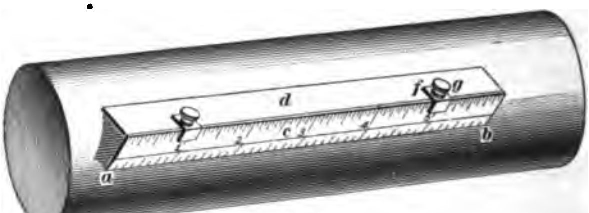


FIG. 8

the straightedge *d* with two hook clamps *f* that are tightened by means of eccentric pins controlled by the milled heads *g*. When the rule is placed in contact with a shaft, the edges, as *a b*, are always parallel to the center-line of the shaft; hence, in laying out a key seat, the side lines can be drawn by scribing along the edge *a b*, the width laid off with a pair of dividers, and the length laid off from the graduations along *a b*.

INSTRUMENTS FOR COMPARING AND TRANSFERRING MEASUREMENTS

26. Inside and Outside Calipers.—Calipers are of many shapes and sizes, and are perhaps more extensively used than any other tool in the machine shop. They are used to measure either the diameter or length of a piece of work, from a small fraction of an inch to several feet. The simplest forms of calipers are the **firm-joint calipers,** shown in Fig. 9. The one illustrated in (*a*) is an *outside caliper* used for taking outside measurements of shafts, wheels, and similar pieces, and that shown in (*b*) is its companion tool, the *inside caliper*, which, as its name indicates, is used to measure the diameters of holes or the distance between two objects.

Additional forms of outside and inside calipers, shown in Fig. 10 (*a*) and (*b*), are the **spring calipers**, so called because the pressure of the spring *a* always keeps the leg *b* snug against the nut *c*. The nut *c* may be either a solid or a spring nut. If it is a spring nut, and the pressure is relieved by pressing the legs together, the nut will slide freely on the screw. The thread of the spring nut will reengage with the screw on the slightest pressure.

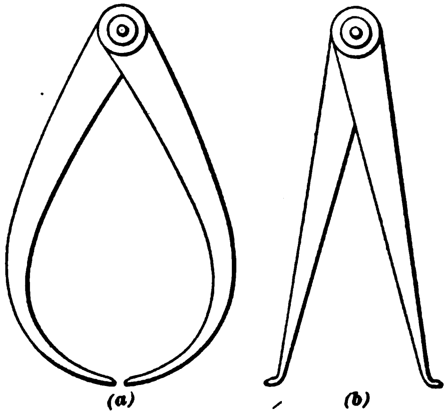


FIG. 9

The spring calipers shown in Fig. 11 are used for measuring the outside diameter of a threaded piece of work, such as a screw, and are called **thread calipers**. The ends *a* and *b* of the legs are made wide, so that they will extend across the tops of two or more threads.

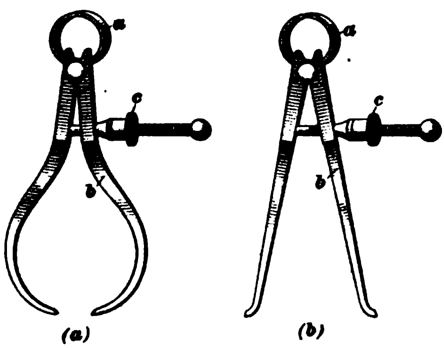


FIG. 10

27. The forms of calipers shown in Fig. 12 are called **transfer calipers**. They are used for making inside and outside measurements under such conditions that the calipers must be closed or opened in order to be removed from the place where the measurement is taken. For example, in calipering the dimension *a*, Fig. 13, the calipers must be opened to be removed from the work; and in calipering the dimension *b* they must be closed in order to be removed. Ordinary calipers cannot be used in such cases; but the dimensions may easily be measured by the use of transfer calipers. The legs *a* and *b*, Fig. 12 (*a*), of the calipers are first clamped together by pressing the slot *c* over the shank of the screw *d* and then turning the screw down tightly. The calipers are then applied to the piece of work and are set to the exact measurement, which may easily be done, as the leg *e* is free to move. After the calipers are set, the screw *d* is loosened, and the legs *a* and *e* are pressed apart so that the calipers may easily be removed. The legs *a* and *b* are then clamped together again by the screw *d*, as at first; when this has been done, the distance between the points of the legs *a* and *e* will correctly measure the dimension taken from the piece of work.

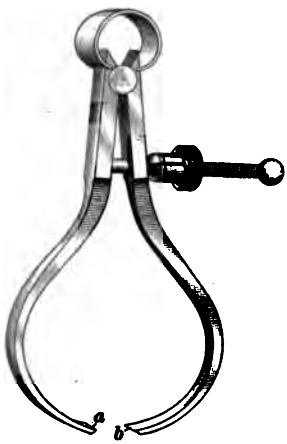


FIG. 11

28. The calipers shown in Fig. 12 (*a*) have screw adjustments in order to enable them to be set accurately to the desired dimension. The inside calipers illustrated in (*b*) are used in the same way as the outside calipers, and show this adjustment clearly. In making a measurement, the calipers are first set roughly to the required dimension, and then the screw *f* is turned until the exact measurement is obtained. Turning the screw *f* right-handed moves the legs *a* and *e* together against the action of the spring *g*, and turning it left-handed allows the spring to press the legs apart. Outside calipers may be set directly to a graduated rule, or they may be set to standard gauges or to pieces to be duplicated and then tried on the work. Inside calipers are

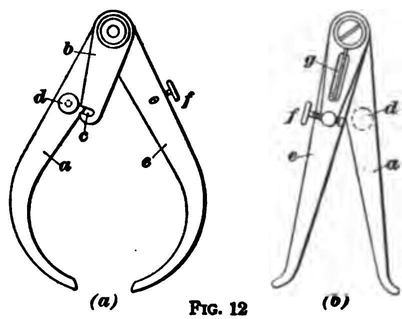


FIG. 12

also set directly to holes either in gauges or in the work, and outside calipers are in turn set to them and tried on the piece under operation. These calipers are used in connection with standard gauges or rules, or for the comparison of one size with another.

29. Care in Setting Large Calipers.—Unless proper care is taken in setting large calipers, an error may be made because of the flexibility of the legs. The amount of this error may be found by making a simple test. A pair of large calipers is opened as wide as possible and is laid flat on the bench, after which an iron bar is placed between the points, so that the calipers measure exactly the length of the bar. If the calipers are now picked up and held by the joint, with the legs hanging down, and are again applied to the bar, it will be found that the points will not go over the ends of the bar without springing the legs apart. The shortening of the distance between the points of the calipers is due to the flexibility of the legs and the change of position. It follows, therefore, that in setting large calipers, they should be held in the same position as that in which they are to be applied to the work.

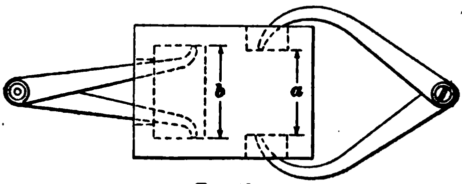


FIG. 13

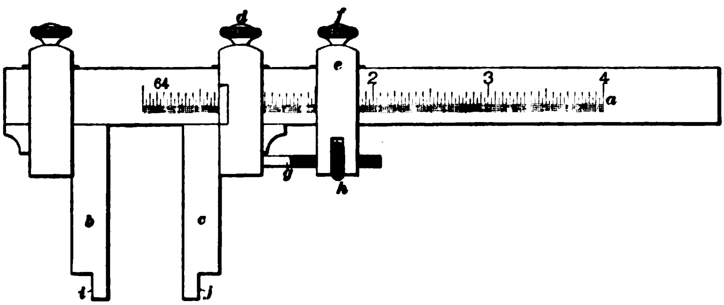


FIG. 14

30. Caliper Square.—The caliper square, shown in Fig. 14, is a measuring instrument with which line and end measurements are made at the same time, to obtain the size of work. It consists of a graduated beam *a* fitted with a head, or jaw, *b*, whose inner surface is straight and at right angles to the beam. A sliding head *c* is carefully fitted to the beam, to which it can be clamped by means of a small thumbscrew *d*. The sliding head carries a jaw whose inside surface is exactly parallel to that of the fixed jaw. On the better grade of instruments is fitted another sliding head *e*, which can be clamped to the beam by means of a screw *f*. The two sliding heads are connected by a fine-threaded screw *g*, which is fixed to the head *c* and passes through a clearance hole in the head *e*. A nurled nut *h* is set into a slot cut in the head *e*, and fits over the screw *g*. A very sensitive adjustment is thus made possible, as, by first clamping the head *e* and slightly turning the nut *h*, the sliding head *c* can be moved a very small amount with the greatest ease. The sliding head *c* must always be unclamped when it is to be moved.

31. The edge of the sliding head *c*, Fig. 14, forms the zero line of the instrument. To read the opening of the jaws, therefore, it is necessary to note the position of the edge of the sliding head with reference to the graduation on the beam. Most caliper squares are adapted for both inside and outside measurements; thus, the jaws *i* and *j* are used for inside measurements. The outside edges of these jaws are parallel to each other and to the inside edges of the jaws *b* and *c*. Evidently, the measurement desired would be greater than the reading obtained by an amount equal to the thickness of the jaws *i* and *j*. Hence, for inside measurements, it is necessary to add to the reading obtained an amount equal to the thickness of the jaws. In most cases this amount is $\frac{1}{4}$ inch, and denotes the smallest inside measurement possible with the caliper square.

INSTRUMENTS FOR LAYING OUT WORK

32. Dividers and Trammels.—One type of dividers is shown in Fig. 15, although there are many forms and sizes of this instrument. Dividers are used for drawing circles, spacing off distances, and subdividing circles or lines. The legs *a* and *b* are pointed at the ends and are held apart by the action of the spring *c*. The points are adjusted accurately by means of the nut *d*.

Trammels, shown in Fig. 16, are merely large dividers. They differ from ordinary dividers in that they consist of two movable heads *a* and *b* that are generally clamped to a bar of wood *c*. If the work is varied, several bars of different lengths should be provided. The tool illustrated is provided with inside and outside caliper points *d* and *e*, and long and short divider points *f* and *g*, the latter being shown in place in the movable heads. The ball points *h*, *i*, *j*, *k* shown in the center are for use in case circles must be drawn or spacing done from holes in partly finished work, the ball making it possible to locate one point in the center of the hole. The heads *a* and *b* are secured to the bar *c* by the clamp screws *l* and *m*.

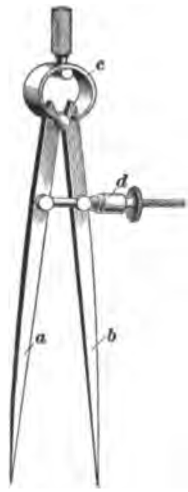


FIG. 15

The **universal dividers** shown in Fig. 17 consist of a short bar *a* carrying a scriber *b* whose holder *c* is bent. The point *d* is held in a holder *e* that is free to slide on the bar *a* when the adjusting nut *f* on the screw *g* is turned. The nut *f* is held in a slot in a clamp *h* that is fixed on the bar by a screw *i*. The nut *f* and the screw *g* are used when setting the dividers very accurately. The holder *c* may be reversed, thus allowing lines to be scribed close to a shoulder. The point *j* may be substituted for the point *d* when a circle is to be scribed around a drilled hole.

The **hermaphrodite calipers** are a combination of the outside calipers and the dividers, in that one leg is an outside-caliper leg and the other is a divider leg. This tool is used for laying out work on which lines must be drawn parallel to an edge of a piece.

33. Scratch Gauge.—The scratch gauge, a convenient form of which is shown in Fig. 18, is generally constructed from a round bar *a*, and has a pin through one end or a thin steel star *b* screwed to the end, as shown in the illustration. A movable guide *c*, hardened on its face and held by a thumbscrew, may be located in any position on the bar. With this tool a variety of operations may be performed in laying out work in which lines must be drawn parallel to an edge. The guide *c* is pressed against the edge of the piece, and the gauge is drawn along, when the pin or star *b* will scribe the desired line.

34. Center Square.—The center square, shown in Fig. 19 (*a*), derives its name from the fact that it can be used con-

Fig. 16

veniently for finding the centers on the ends of cylindrical work. The stock is so placed that its two legs *a* each make an angle of 45° with the blade *c*, and consequently make an angle of 90° with each other; the cross-bar *b* is placed at right angles to the blade. In using the center square it is placed so that both legs are in contact with the work. Then a line is drawn along the blade *c*, which, as it bisects the right angle formed by the square, will pass through the center of the circle. The center square is then placed in a different position, but still in contact with the work, and the process is repeated. The second line will also pass through the center of the circle; hence, the point where the two lines cross is the center of the circle.

35. Another useful form of center square is illustrated in Fig. 19 (*b*). The body is made of $\frac{1}{16}$-inch sheet steel. The two pins *d* are $\frac{3}{8}$ inch in diameter and $\frac{3}{8}$ inch long, and are so placed that a line *b b* drawn so as to touch them will be square with the face *a a* and with the inner sides *c* equidistant from *a a*; the face *e e* is made at an angle of 45° with the face *a a*. This tool is used in centering work for the lathe; it is easily applied to either the inside or the outside of rings, and is a good square for drawing lines perpendicular to any surface or edge.

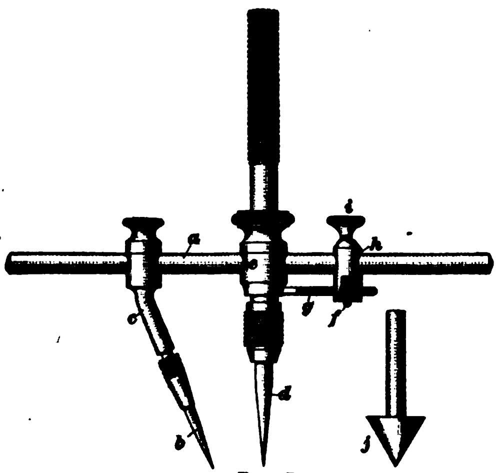


FIG. 17

36. Surface Gauges.—A surface gauge consists of a flat base to which is attached a spindle that carries an adjustable scriber. Fig. 20 shows a good form of surface gauge, which

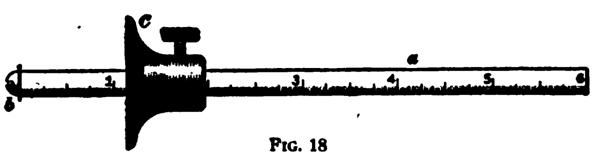


FIG. 18

consists of a rectangular base *b* carrying a spindle *a*, to which is attached the scriber *d*. The scriber can be set at any height or angle within the capacity of the instrument, and then clamped by means of the nut *e* and the clamp *c*. The spindle *a* can be set at any desired angle and clamped by means of the nut *f*. A fine adjustment for swinging the spindle is provided by means of the lever *g* and the thumbscrew *h*, the lever *g* being pivoted on the inner end of the screw *i*, so that it can be moved by the screw *h*.

The surface gauge may be used for drawing lines parallel to a given flat surface, or for setting a given line or surface parallel to another surface. In the former case, the base *b* is placed on the given surface, as a planer table, and the point of the scriber *d* is set to the required distance from the surface

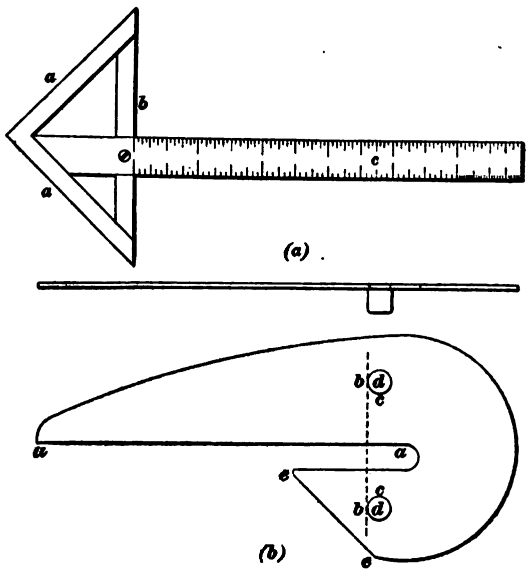


Fig. 19

and used to draw the required line. For setting a line parallel to a given surface, the piece on which the line is drawn is adjusted until the point of the scriber will follow the line as the surface gauge is moved along the guiding surface.

37. Plumb-Bobs.—The plumb-bob serves to determine a vertical straight line; it is made in a variety of forms, and has many uses in the machine shop. The plumb-bob consists of a weight hung from a string or a wire. For some purposes, a nut tied to the end of a string is used, but for most work a special form of weight is employed. A form that takes little space and has many advantages is shown in Fig. 21 (*a*). It is made by drilling out a steel rod *a* and filling the space *b* with mercury, which is held in place by the screw *c*. Through the cap *d* is a small hole, which is counterbored in the bottom to hold the knot in the line, which should be of the braided or woven variety. The small diameter of this weight makes it particularly useful when hung near a partition or an upright, or in windy places. The point is ground true.

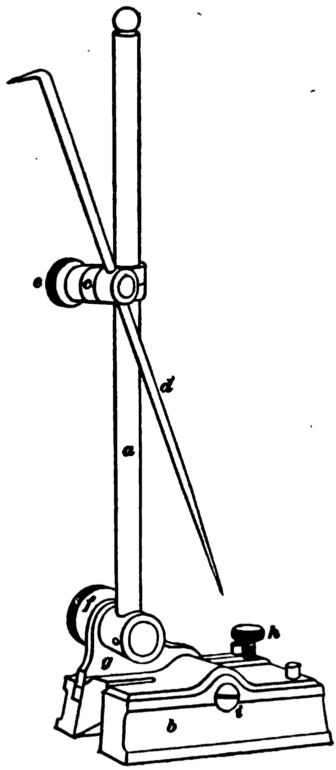


FIG. 20

38. A very common form of plumb-bob is that shown in Fig. 21 (*b*); it is often made with a brass body *e* having an inserted steel point *j* and the usual cap *f* for holding the line. This plumb-bob has most of its weight at the upper end, and therefore is somewhat unsteady. A better form for use where great accuracy is desired is that shown in (*c*). This plumb-bob is made with a brass body *g* having its greatest weight at the lower end to insure steadiness. The point *h* fits the tapered hole through the body *g* and is held in position by the cap *i*. The point is readily removed for repairs, and may be used without the body by simply screwing on the cap. The point may be ground by providing a sleeve about 1¼ inches long, reamed to the same taper as the body and turned parallel outside, so that it may be held in the chuck of a grinding machine. If the line is very long, so that the plumb-bob tends to swing, it may be allowed to hang in a pail of water or oil, in order to steady it.

39. Spirit Levels.—Spirit levels are used for testing horizontal and vertical lines and surfaces. The essential feature of a level is a glass tube or vial nearly filled with alcohol or ether and sealed at both ends. The air remaining in the tube forms a small bubble that is always found at the highest point of the glass tube. The tube is slightly bent, as shown in Fig. 22 (*a*),

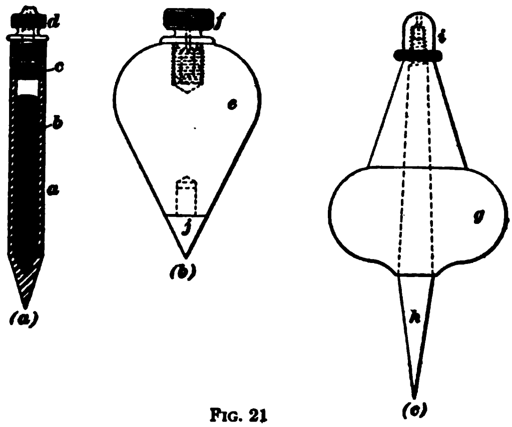


FIG. 21

and is mounted with its convex side up, so that the bubble will come to rest quickly. A line is scratched on the glass at *a*, coinciding with the center of the bubble when the level is truly horizontal. For work requiring the greatest accuracy, use is made of a ground-glass tube, that is, a tube ground on the inside to a barrel shape, as shown somewhat exaggerated in (*b*). The less curve there is in the glass, the more sensitive will be the

FIG. 22

level, and the longer will be the bubble. The bubble in the ground glass is usually much longer than that in the bent glass. The body, or frame, of the carpenter's level is generally made of hardwood, the best levels being built up of several well-seasoned strips glued together. Fig. 23 shows the ordinary form of iron level often used in the machine shop.

40. When great accuracy is required, the form of level shown in Fig. 24 may be used. The base *a* is from 12 to 18 inches long and from $1\frac{1}{4}$ to $1\frac{3}{4}$ inches wide, with a V groove $\frac{3}{4}$ inch wide extending its entire length, as shown at *b* in the end view. The base is scraped perfectly flat and parallel, and the V is carefully tested to insure that it is exactly parallel with the edges.

FIG. 23

The glass *c*, which is preferably ground and graduated, is held in a casing *d* that may be adjusted as to height by means of the nuts *e*. A second glass *f* is set at right angles to the glass *c*, to be used for testing vertical surfaces, the base in this case being laid against the vertical surface to

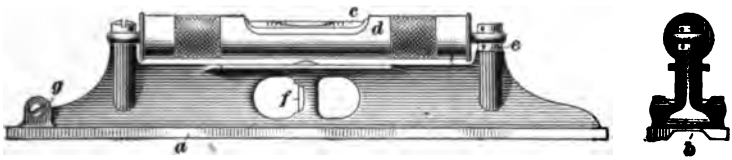


FIG. 24

be tested. The glass *g* is of service when the level is used for testing horizontal cylindrical work. The work fits into the groove in the base, as illustrated in Fig. 25, and the glass *g*, Fig. 24, serves to show when the level is held horizontal, as indicated by the full lines, Fig. 25. It should not be tilted to one side or the other as shown by the dotted lines, or it will not indicate a true level. In some makes, the crosswise glass is omitted.

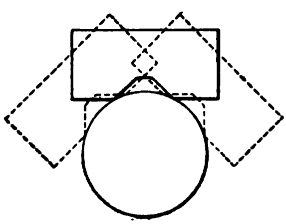


FIG. 25

41. Testing of Levels. Levels of the best makes are very sensitive to changes of position. If a good level is laid on a surface plate and a piece of tissue paper is slipped under one end, the bubble will move about $\frac{1}{32}$ inch. In order to test its accuracy, a level is placed on a surface plate that is known to be truly horizontal, and is then turned end for end. If the level is accurate, the bubble will not change its position, whatever that of the level. The level should next be tilted on one edge and then on the other; if the bubble remains in the center during these operations, the glass is correctly set. Two short rollers of exactly the same diameter should now be laid on the surface plate, and the level should be placed on them, with about ½ inch of the end of the V on each roller. The level should then be rocked from side to side, and if the bubble remains central the level is correct; if it does not remain central, the groove should be scraped until it does. If, in selecting or setting a glass, it is found to be perfect in the horizontal position, and when set up edgewise will always show high on one end and is the same when reversed, the fault is due to a slight defect in the glass, probably caused when the opening was sealed, one end having been made a little larger than the other. It can be noticed only when the level is rotated on the rollers or on a shaft, as shown in Fig. 25.

INSTRUMENTS FOR MEASURING DEPTH

42. Measuring Depth With Rule and Straightedge. The depth of a cavity in a piece of work may conveniently be measured by the use of a standard rule and a straightedge. The straightedge is laid across the cavity and the rule is then inserted in the cavity, and at right angles to the edge of the straightedge. The depth of the cavity, reckoning from the bottom of the straightedge, may then be read directly from the rule at a point even with the lower edge of the straightedge. If the bottom of the cavity is flat and level, the rule will rest evenly on it and stand perpendicular to the straightedge. If the bottom of the cavity is

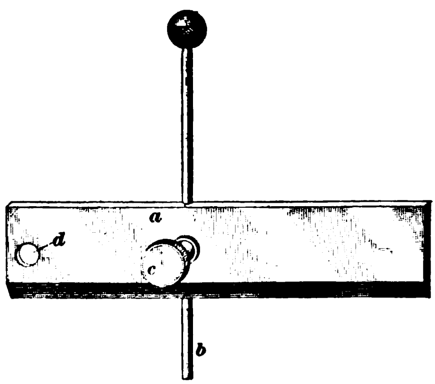


FIG. 26

curved, however, a small rod must be used instead of the rule. The rod is ground to a point and is held perpendicular to the straightedge with the pointed end resting on the bottom of the cavity. A line is then scratched on the rod, even with the lower edge of the straightedge, and the rod is removed. The distance from this line to the point is then the required depth.

43. Depth Gauge.—For testing the depth of holes or grooves, the depth gauge is frequently used. One form of this instrument is shown in Fig. 26. It consists of a straightedge *a*, to which is attached a wire *b*. The amount that the wire projects beyond the straightedge can be regulated, and the wire then may be clamped to the straightedge by means of a clamp controlled by the nut *c*. Sometimes the wire is replaced by a small graduated bar or scale, so that the depth can be read directly, without being compared with a rule. If the hole whose depth is to be measured lies close to a shoulder, it may be necessary to shift the clamp *c* to the hole *d* at the end of the straightedge and to attach the wire *b* to the clamp at that point.

INSTRUMENTS FOR ANGULAR MEASUREMENT

44. Combination Bevel.—The combination bevel, shown in Fig. 27, is an instrument that may be used for measuring angles. The stock *a* and the split blade *b* form the ordinary bevel, and the slotted blade *c* may be removed, when desired. The numbers 30°, 45°, and 60° represent the angles the respective edge make with the adjacent sides. These angles make the tool convenient for measuring in corners, and it may be used for measuring angles of 30°, 45° and 60° direct. The instrument is set either to an angle desired to be transferred, or by means of a protractor, and is a light and convenient tool to use.

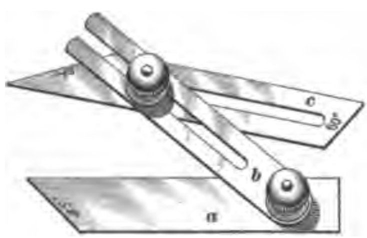


FIG. 27

45. Protractor.—For the measurement of angles, an instrument called a protractor is used. The unit of angular measurement is $\frac{1}{360}$ part of a circle, which is called a degree. The degree is subdivided into 60 parts, called minutes. A minute is subdivided into 60 parts, called seconds. Smaller divisions of the degree are expressed as decimal parts of the second.

46. Protractor With Guide Pin.—The protractor shown in Fig. 28 consists of a thin plate of steel or brass, semicircular in shape, with the center of the semicircle at *a*. The curved edge is divided into 180 equal parts, and each division thus represents 1°. For convenience the degrees are numbered from 0 to 90, from each end of the semicircle. A center guide pin *b* is so placed that, if a straightedge is set against the flat surface of the pin, the edge in contact with the pin will pass through the center of the protractor. This guide pin is a rivet with half of the head cut away, and is loose enough to conform readily to the edge of a straightedge. Should the angle to be measured be larger than 90°, the reading must be subtracted from 180°. Thus, the angle *c a d* is read off directly as 55°; whereas, the angle *c a e*, being greater than 90°, is equal to 180° − 55° = 125°. This form of protractor is very convenient for setting the combination bevel. The stock of the bevel is placed against the straight bottom edge of the protractor. The blade is adjusted so as to lie flat against the center guide pin and cross the graduated edge at the correct division.

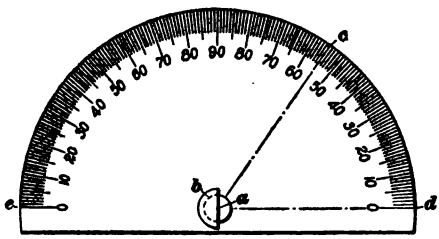


FIG. 28

47. Bevel Protractor.—The protractor as used by the mechanic makes measurements by a combination of line and end measurements. In this modified form, it is commonly known as a *bevel protractor*. To suit different purposes and individual preferences, the bevel protractor is made in a number of different forms by the various manufacturers. All these forms embody the same principle of construction, which is the combination of two straightedges with a graduated circle or part of a circle.

48. All the main features of the bevel protractor are well exhibited in Fig. 29. The arm *a* is attached to the graduated

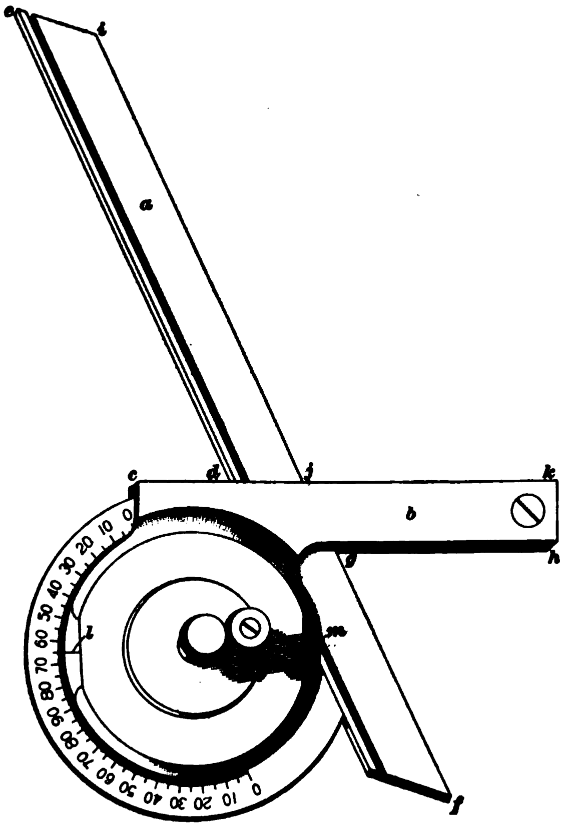


FIG. 29

semicircle, and the straightedge *b* forms one piece with the circular part on which the lines *l* and *m* are marked. A line passing through *l* and *m* passes also through the center of the protractor and is parallel to the edge *c k*. The arm and the straightedge are pivoted at the center of the graduated semicircle. The zero lines *l* and *m* are so located that they exactly coincide with the zeros of the graduated semicircle when the arm *a* and the straightedge *b* are in the same straight line. For convenience, the straightedge *b* is slotted, to allow the arm *a* to pass through it; in consequence of this, either the arm or the straightedge can be rotated through 360°. The arm *a* may be slipped back and forth its full length, and can be clamped at any point to the part on which the graduations are marked.

49. To make a measurement with the bevel protractor shown in Fig. 29, the arm *a* and the straightedge *b* are brought

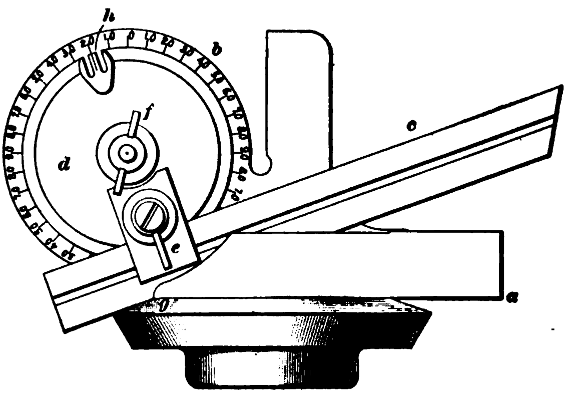


FIG. 30

in contact with the work; the angle is then read off by observing the position of the zero line *l* with regard to the graduations. Some judgment is required to read the angle correctly; that is, if the angle measured is smaller than 90°, it can be read off directly, but if it is larger than 90°, the reading must be subtracted from 180°. When the arm *a* stands in such a position that the angles *c d e* and *i j k* are equal, each of these angles is 90°, and the mark *l* stands at 90 on the scale. The angle *f g h*, also, is 90° when the mark *l* is at 90. But when the arm *a* is swung to the left, as shown, the angle *i j k* is made greater than 90° and the angles *c d e* and *f g h* are made less than 90°. The mark *l* then stands opposite 65 on the scale. Therefore, as the angle *i j k* is greater than 90°, its value must be found by subtracting the scale reading from 180°; that is, the angle *i j k*=180°−65°=115°. As the angles *c d e* and *f g h* are smaller than 90°, their values are read directly from the scale; that is, each is 65°.

50. Another form of bevel protractor is shown in Fig. 30. It has a stock *a* fixed to a graduated circle *b*. In the center of the graduated circle is a pin on which the disk *d* rotates. The arm *c* is fastened to the disk by the thumbnut and clamp *e*, and the disk can be held in any position by the thumbnut *f*. The protractor is shown measuring the angle of the face of a bevel-

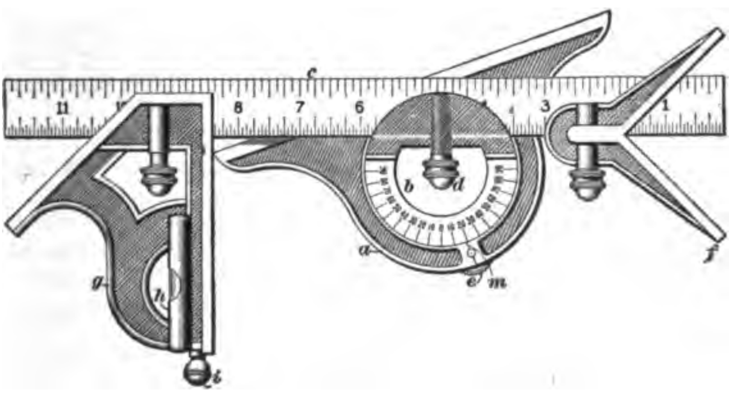


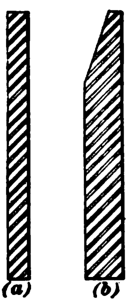
FIG. 31

gear blank. The edge of the stock *a* is placed across the work, and the arm *c* is brought in contact with the face *g* of the gear and locked by means of the nut *f*. The line *h* on the disk *d* coincides with the twentieth division on the graduated circle; hence, the angle of the face is 20°.

51. A simpler protractor is shown in Fig. 31. It consists of a stock *a* having a graduated rotating holder *b* for the blade *c*, which after setting may be clamped in any position by the nut *d*. The holder *b* is graduated in degrees through one-half of its circumference, or 180°, and is clamped in any desired position by the thumbscrew *e*. With this instrument the same care is to be taken that is necessary in reading the bevel protractor shown in Fig. 29. A center-square head *f*, Fig. 31, with which the protractor is supplied, furnishes an excellent means for centering lathe work. The head *g* makes an adjustable square and a 45° angle, and is provided with a level *h* and a scriber *i*. These various fittings are not all used at one time. Each is removable, and may be attached easily when required. The straight edge of the stock *a* is in line with the upper edge of the blade *c* when the mark *m* is opposite the zero of the scale. This scale is graduated from 0° to 90° in each direction; hence, the angle between the edge of the stock and the top edge of the blade is always 180° minus the reading on the scale. In the illustration the reading is about 19°, and consequently the angle at the upper edge of the blade is $180° - 19° = 161°$. The angle between the lower edge of the blade and the edge of the stock is equal to the reading on the scale, and is about 19° in this case.

FIXED MEASURING INSTRUMENTS

52. Straightedges.—The straightedge is a bar of metal having its edges true to a straight line. It is used for testing surfaces, edges, etc. Straightedges are made in several forms, to suit individual preferences. They either have a rectangular cross-section, as shown in Fig. 32 (*a*), or are beveled on one edge, as in (*b*). For very accurate work, straightedges are sometimes made as shown in Fig. 33. When thus made, they are extremely sensitive, owing to the fact that they touch the work along a line only, and a very small deviation will show daylight between the work and the straightedge in contact with it.


(*a*) (*b*)
FIG. 32

53. Try Square, or Steel Square.—A gauge made in either of the forms shown in Fig. 34, is called a try square, or a steel square. It is used for testing right angles, that is, angles of 90°. The hardened-steel squares made by several manufacturers are especially fitted for such work as tool making, jig making, etc. They are made in various sizes from 1½ inches to 24 inches or over, the size being measured on the inside of the blade. The smaller sizes are usually made as shown in (*a*), with a blade *a* fastened by soldering and rivets to the stock *b*. In the larger sizes, the stock and the blade are usually united by screws, as shown in (*b*); this allows them to be repaired more readily.

54. Graduated Square.—A form of square similar to the steel square described in the previous article, graduated on the back edge, but not hardened, is known as the graduated square. It is an excellent substitute for the hardened instrument, especially on the laying-out table. The well-known carpenter's square, which is of the graduated type, is used for a large variety of work in the machine shop. For many ordinary uses it answers the purpose perfectly, but it is not sufficiently accurate for the best grades of work.

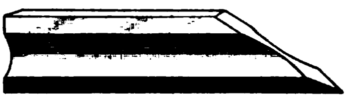


FIG. 33

55. Use of Squares.—Only the edges of the blade of a try square and not the corners or sides should be employed in testing work. In using the square, the beam or stock is placed at right angles to the surface to be tested, and the blade is brought

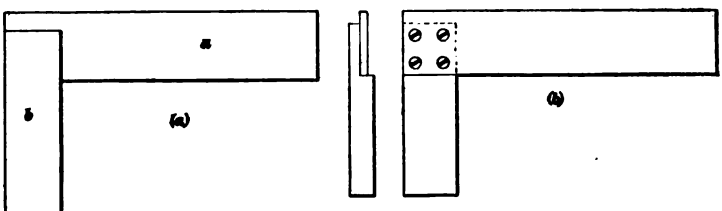


FIG. 34

against it. If the surface touches the edge of the blade along its whole length, the surface is at right angles to that on which the stock of the square rests. Sometimes the square must be used in such a position that it is impossible to see by the eye whether the work is true or not. In such a case, the sense of touch may be used. Two strips of paper of the same thickness are placed between the blade of the square and the surface being tested, one strip being put at each end of the blade. The strips are then pulled gently. If the same strength of pull is required to draw both strips out from behind the blade, the surface is true; but if one strip draws out more easily, the surface slants away from the blade under that strip.

56. Adjustable Square.—The adjustable square, illustrated in Fig. 35, forms a combination having many useful applications. The blade *a* is held in the stock *b* by a hook clamp that enters the groove *c* in the blade, and is tightened by means of the nut *d*. The groove *c* runs the full length of the blade, so that the stock may be set at any point throughout its length.

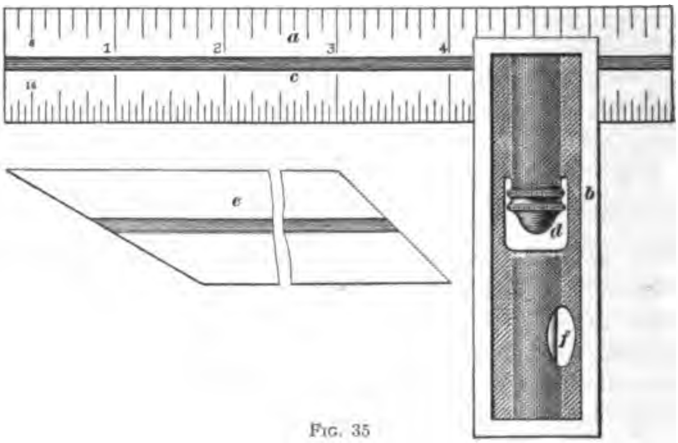
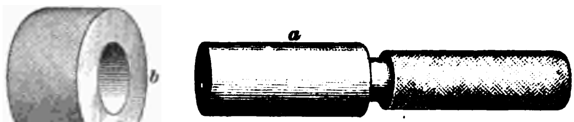


Fig. 35

The square may thus be adjusted to the work in hand, which could not be done with an ordinary try square having a fixed blade. A special bevel blade *e*, with a 45° angle at one end and a 30° angle at the other, may be substituted for the graduated blade whenever these angles are to be laid off or tested. This special blade provides the angles required in laying out either a hexagon or an octagon. The stock is also provided with a level at *f* that may be used in testing either a vertical or a horizontal surface. It should be borne in mind that a square is perfect only when the blade and the stock are exactly at right angles to each other, and that a fall or any careless use is liable to destroy its accuracy. The adjustable square is not so accurate as the steel square.

57. Classes of Gauges.—Gauges may be divided into two general classes, namely, *reference gauges* and *working gauges*. The first class includes all gauges that represent ultimate standards of reference, accurate subdivisions of these standards, or some arbitrary size or shape that must be preserved. The bars representing the standard yard, kept by the Government, are reference gauges. The second class includes all gauges that are used in the shop for making linear and other measurements. As reference gauges are used very rarely, and then not in the shop, this Section will deal only with working gauges.

58. Cylindrical Gauges.—For testing the inside and outside dimensions of cylindrical work of relatively small diameter, the cylindrical gauges shown in Fig. 36 have a wide range of application. These are also known as *plug and ring*

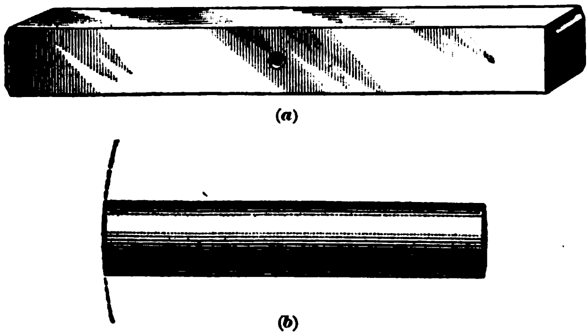


Internal

External Fig. 36

gauges. The plug gauge *a* is a hardened-steel cylinder that is ground and accurately lapped to size and provided with a suitable handle. The ring gauge *b* is a hardened-steel ring ground and lapped inside to fit the plug gauge accurately. The plug gauge is used for testing inside dimensions, and the ring gauge for outside dimensions.

59. End-Measuring Gauges.—A form of end-measuring rod is shown in Fig. 37 (*a*), consisting of a square hardened-steel bar whose ends are ground and lapped until they are plane surfaces parallel to each other. For testing the size of holes beyond the range to which a plug gauge is limited, because of its weight and cost, an end-measuring rod of the form shown in (*b*) may be used. The ends of the rod are made part of a sphere equal in diameter to the length of the rod; consequently, the rod can be used for any kind of internal measurement without danger of cramping. It is also useful for setting calipers, comparing other working gauges, measuring between parallel sur-



(a)

(b)

Fig. 37

faces, either plain or curved, and similar work. The form of rod shown in (a) cannot be used for internal measurements, as it would cramp, because its ends are not spherical. End-measuring gauges are also made in the form of circular disks, like that shown in Fig. 38. This gauge is made with a circular hole, for ease of manufacture, and also to allow it to be used with a handle inserted therein. The circular form makes the disk very convenient for setting calipers. When used for testing purposes, a very delicate contact can be obtained, since it is only in line contact with the measuring instrument.

60. Caliper Gauge.—The standard caliper gauge, two forms of which are shown in Fig. 39 (*a*) and (*b*), is preferable to any other style of gauge for many classes of work. In the smaller sizes it is usually made with one end for inside and the other for outside measurements, as shown in (*a*); in the larger sizes, in order to keep its weight within a reasonable limit, the gauge for internal measurement is made separate from that intended for outside measurement, as shown in (*b*). The gauge intended for outside measurement has its measuring surfaces ground and lapped to be plane surfaces parallel to each other. When the gauge

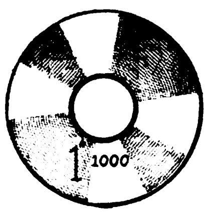


Fig. 38

intended for internal measurement is to be used for circular holes, its measuring surfaces are formed as parts of the surface

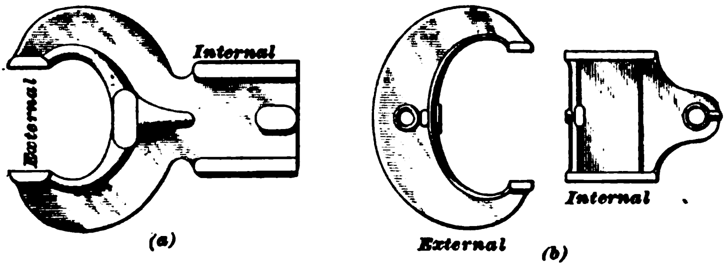


FIG. 39

of a cylinder. For measuring between parallel plane surfaces, its measuring surfaces should be planes parallel to each other.

61. Limit Gauges.—The working gauges thus far described simply determine whether the work is of the true size; they do not show the amount of variation from this size, nor do they tell whether the variation is sufficient to condemn the work. Suppose that two working gauges are employed, of which the one is made to the greatest allowable size the work can be, and the other one is made to correspond to the smallest

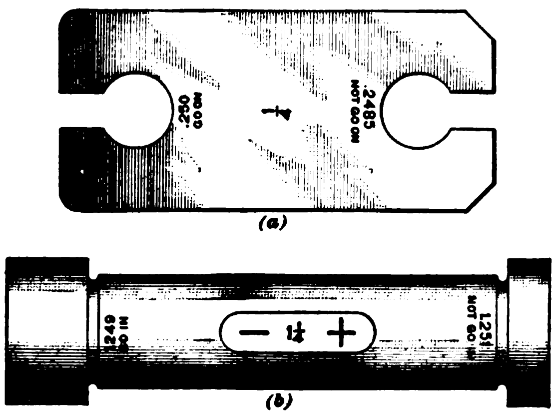


FIG. 40

allowable size. Then, if the larger gauge goes over the work and the smaller one does not, it follows that the amount the work varies from the true size must lie somewhere between the limit set by the two gauges; hence, the variation is not sufficient to condemn the work. A pair of such gauges forms a limit gauge. A limit gauge for external measurements is shown in Fig. 40 (*a*). If the end marked GO ON will fit over the work and the end marked NOT GO ON will not fit over the work, it follows that the piece of work has a size somewhere between .2485 inch and .250 inch. A gauge for internal measurements is shown in (*b*) and is applied in a similar manner to that shown in (*a*). It must not be inferred that the two forms shown are the only forms limit gauges can have; they may be made in many ways, to suit the character of the work for which they are to be used.

62. Screw-Thread Gauges.—Special gauges for measuring the sizes of screws and screw threads are shown in Fig. 41.

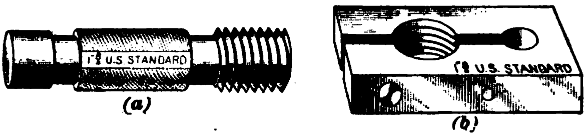


FIG. 41

The internal gauge shown in (*a*) consists of a threaded cylinder made to standard size at one end, and on the other end a true cylinder of a diameter equal to that over the bottom of the thread. The threaded end is used to test the thread on the inside of a hole, as in a nut, whereas the plain end forms a gauge for testing the size to which the hole is bored or drilled before the thread is cut in it. The external gauge shown in (*b*) is threaded to fit the internal gauge, and is adjustable for wear within reasonable limits; it is used for testing the size of screws, bolts, taps, etc. For shop use, these gauges are made of hardened steel; for reference purposes they are left soft.

63. Screw-Pitch Gauge.—The advance of a screw thread in one complete turn, measured in a direction parallel to the axis of the screw, is the pitch of the thread; it is equal to 1 divided by the number of threads per inch. The screw-pitch gauge is a gauge used to measure the number of threads per inch of a screw, and is made in various shapes and sizes to suit threads of different forms and pitches. A screw-pitch gauge intended for the measurement of 60° sharp V threads is shown in Fig. 42. It consists of a number of thin leaves pinned at each end of the handle *a* so that all may be folded inside the handle when not in use and any one leaf may be swung out when needed. This particular gauge may be used for screws having from 9 to 40 threads per inch, and is suitable for inside or outside threads. Each leaf is stamped with an integral number denoting the number of threads per inch for which it is intended; also, it bears a decimal number that denotes twice the depth of the thread. By subtracting this decimal from the outside diameter of the screw, the diameter at the bottom, or root, of the thread is obtained.

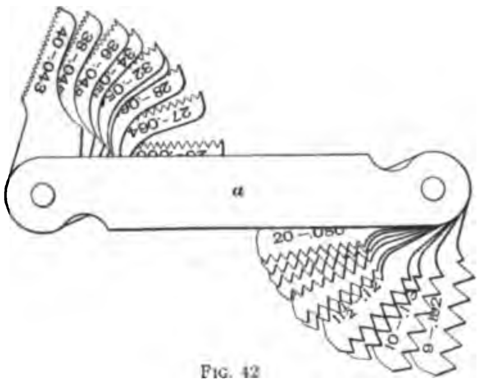


FIG. 42

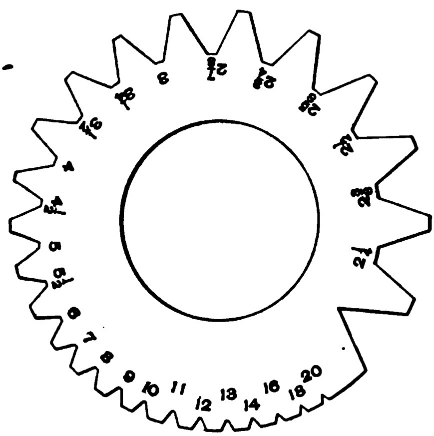


FIG. 43

64. Tool Gauges. A tool gauge is a gauge that is used to determine the proper size and shape to which a tool must be ground in order that it may cut a correct thread of the desired pitch. As threads are of different forms and pitches, tool gauges must differ in shape and size. A tool gauge to be used to determine the form of a tool intended for cutting United States standard threads is shown in Fig. 43. It is a ring of metal bearing a series of 60° notches, each of which is marked with a number. The depth and form of the notch determines the size and shape to which the point of the tool must be ground, and the number denotes the number of threads per inch of the screw to be cut by that tool. A worm-thread tool gauge is shown in Fig. 44. It is used to test the shape and size of a tool for cutting worm threads. The numbers denote the number of threads per inch.

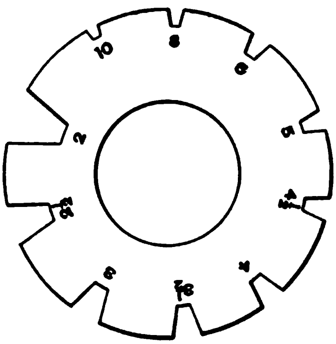


FIG. 44

65. Center Gauge.—The gauge shown in Fig. 45 is a center gauge, and is also known as a *thread gauge*. It defines the standard angle for lathe centers, 60°, which is almost universally used in America; this is also the standard angle for the sharp or V screw thread and the Sellers, or United States standard, thread. Notches *a* and *b*, having an angle of 60°, are convenient for grinding thread-cutting tools. These notches are cut so that their sides make the same angle with the edge in which they are cut; they can therefore be used for setting thread-cutting tools square with the work.

66. Nearly every center gauge has stamped on it a table, as shown in Fig. 45. Each of the whole numbers from 4 to 26 denotes the number of threads per inch, and the corresponding decimal denotes twice the depth of the sharp or V thread. To find the size of tap drill for a given number of threads per inch, the decimal corresponding to this number of threads per inch is subtracted from the outside diameter of the screw. Center gauges are usually graduated on both sides, along both edges, with divisions of the inch that

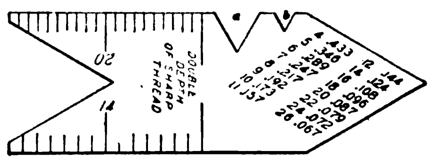


FIG. 45

are convenient for measuring the number of threads per inch on screws. Thus, the side shown in Fig. 45 has 20 divisions to the inch on one edge, and 14 divisions on the other edge. The different numbers of threads per inch for which each graduation is most suitable are those by which the graduation is divisible without a remainder. Thus, the twentieth graduation is suitable for measuring 1, 2, 4, 5, 10, and 20 threads per inch, and also for any multiple of 20, as 40, 60, 80, etc.

67. Decimal Gauge.—The decimal gauge, a form of which is illustrated in Fig. 46, is used for measuring the thickness of wire and sheet metal directly in decimal parts of an inch. It is a thin oval plate of metal having a series of slots along its outer edge. The distance between the parallel sides of each slot is indicated by the number stamped at the inner end of the slot. The thickness of wire or sheet metal is determined by finding the smallest slot that will fit over the wire or the sheet.

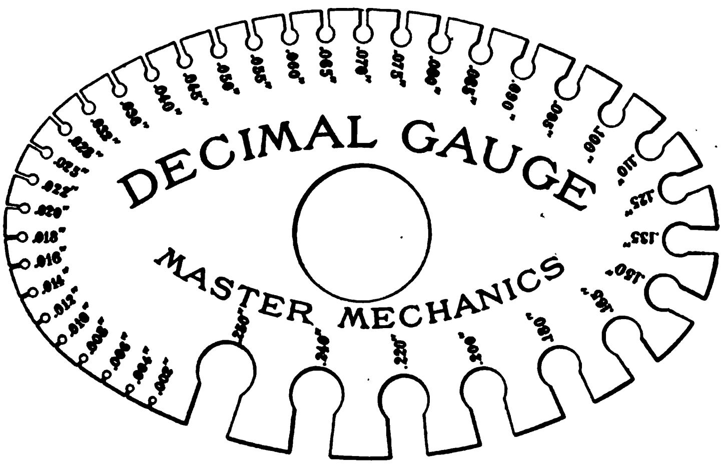


Fig. 46

68. Numbered Gauges.—Numbered gauges express sizes by arbitrarily selected numbers or letters, and are used for gauging metal plate, wire, and small sizes of twist drills. There are a large number of numbered gauges on the market, among which may be mentioned the United States, American or Brown & Sharpe, Birmingham or Stubs' iron, American Steel and Wire, Russia iron, steel music-wire, Stubs' steel-wire, twist-drill and steel-wire; and machine and wood screw standard. Owing to the confusion caused by the use of so many different gauges, it is best to state the thickness of sheet metal and the diameters of wire or drills by the decimal parts of an inch. Each gauge number or letter denotes some particular thickness, and Tables I to IV are given in order to show the numbers or letters used and the corresponding thicknesses in decimal parts of an inch.

69. Three common forms of numbered gauges for sheet metal and wire are shown in Fig. 47. The notched gauges are

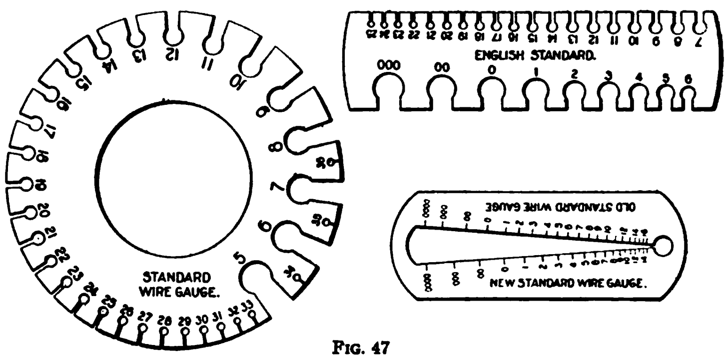


FIG. 47

made like the decimal gauge in Fig. 46, with the exception that they have the sizes of the slots marked by numbers instead of by decimals of an inch. They are used in the same manner as the decimal gauge. The gauge with the triangular opening, Fig. 47, is adapted only for stock of circular cross-section, such as wire and the shanks of screws. It is passed over the wire until it touches on both sides; the division at the point of contact indicates the gauge number of the wire.

70. United States Standard Gauge.—The United States standard gauge, given in Table I, is a gauge established by Act of Congress in 1893 for use in determining duties and taxes levied by the United States on sheet iron, plate iron, and steel. It is used to a limited extent by American rolling mills

TABLE I

SHEET-METAL AND WIRE GAUGES

Gauge Number	Dimension in Decimal Parts of an Inch				Gauge Number
	U. S. Standard	American or Brown & Sharpe	Birmingham or Stubs'	American Steel & Wire	
0000000	.500000000			.49000	0000000
000000	.468750000			.46150	000000
00000	.437500000			.43050	00000
0000	.406250000	.460000	.454	.39380	0000
000	.375000000	.409640	.425	.36250	000
00	.343750000	.364800	.380	.33100	00
0	.312500000	.324860	.340	.30650	0
1	.281250000	.289300	.300	.28300	1
2	.265625000	.257630	.284	.26250	2
3	.250000000	.229420	.259	.24370	3
4	.234375000	.204310	.238	.22530	4
5	.218750000	.181940	.220	.20700	5
6	.203125000	.162020	.203	.19200	6
7	.187500000	.144280	.180	.17700	7
8	.171875000	.128490	.165	.16200	8
9	.156250000	.114430	.148	.14830	9
10	.140625000	.101890	.134	.13500	10
11	.125000000	.090742	.120	.12050	11
12	.109375000	.080808	.109	.10550	12
13	.093750000	.071961	.095	.09150	13
14	.078125000	.064084	.083	.08000	14
15	.070312500	.057068	.072	.07200	15
16	.062500000	.050820	.065	.06250	16
17	.056250000	.045257	.058	.05400	17
18	.050000000	.040303	.049	.04750	18
19	.043750000	.035890	.042	.04100	19
20	.037500000	.031961	.035	.03480	20
21	.034375000	.028462	.032	.03175	21
22	.031250000	.025347	.028	.02860	22
23	.028125000	.022571	.025	.02580	23
24	.025000000	.020100	.022	.02300	24
25	.021875000	.017900	.020	.02040	25
26	.018750000	.015940	.018	.01810	26
27	.017187500	.014195	.016	.01730	27
28	.015625000	.012641	.014	.01620	28
29	.014062500	.011257	.013	.01500	29
30	.012500000	.010025	.012	.01400	30
31	.010937500	.008928	.010	.01320	31
32	.010156250	.007950	.009	.01280	32
33	.009375000	.007080	.008	.01180	33
34	.008593750	.006304	.007	.01040	34
35	.007812500	.005614	.005	.00950	35
36	.007031250	.005000	.004	.00900	36
37	.006640625	.004453		.00850	37
38	.006250000	.003965		.00800	38
39		.003531		.00750	39
40		.003144		.00700	40

for sheet iron, plate iron, and steel, and also for galvanized sheet iron and American planished iron. The gauge numbers are

TABLE II

STUBS' STEEL-WIRE GAUGE

Gauge Letter	Size in Decimals of an Inch	Gauge Number	Size in Decimals of an Inch	Gauge Number	Size in Decimals of an Inch	Gauge Number	Size in Decimals of an Inch
Z	.413	1	.227	28	.139	55	.050
Y	.404	2	.219	29	.134	56	.045
X	.397	3	.212	30	.127	57	.042
W	.386	4	.207	31	.120	58	.041
V	.377	5	.204	32	.115	59	.040
U	.368	6	.201	33	.112	60	.039
T	.358	7	.199	34	.110	61	.038
S	.348	8	.197	35	.108	62	.037
R	.339	9	.194	36	.106	63	.036
Q	.332	10	.191	37	.103	64	.035
P	.323	11	.188	38	.101	65	.033
O	.316	12	.185	39	.099	66	.032
N	.302	13	.182	40	.097	67	.031
M	.295	14	.180	41	.095	68	.030
L	.290	15	.178	42	.092	69	.029
K	.281	16	.175	43	.088	70	.027
J	.277	17	.172	44	.085	71	.026
I	.272	18	.168	45	.081	72	.024
H	.266	19	.164	46	.079	73	.023
G	.261	20	.161	47	.077	74	.022
F	.257	21	.157	48	.075	75	.020
E	.250	22	.155	49	.072	76	.018
D	.246	23	.153	50	.069	77	.016
C	.242	24	.151	51	.066	78	.015
B	.238	25	.148	52	.063	79	.014
A	.234	26	.146	53	.058	80	.013
		27	.143	54	.055		

given in the first column and the corresponding thicknesses, according to the United States standard, are given in the second column. For example, a wire having a gauge number of 18 according to the United States standard gauge, is .05 inch in diameter.

71. American, or Brown & Sharpe, Gauge.—The American, or Brown & Sharpe, standard gauge, given in the third column of Table I, is the gauge used almost exclusively for sheet brass, sheet aluminum, sheet German silver, brazed brass tubing, brass wire, copper wire, and German-silver wire of American manufacture.

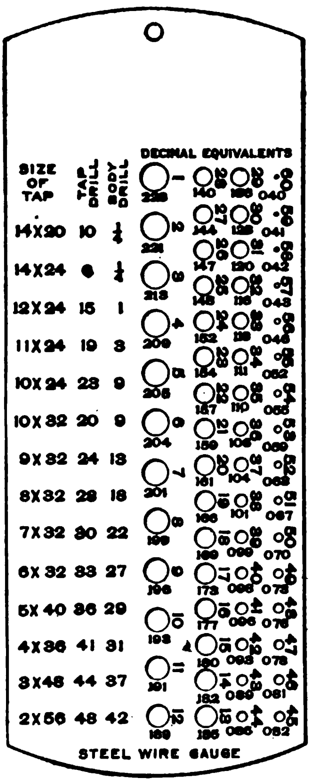


FIG. 48

72. Birmingham, or Stubs', Iron Gauge.—The Birmingham, or Stubs', iron gauge, given in the fourth column of Table I, is the gauge used by the trade for sheet iron, sheet steel, sheet copper, iron wire, Bessemer-steel wire, and seamless tubing of all kinds, practically to the exclusion of all other gauges.

73. American Steel and Wire Company Gauge.—The American Steel and Wire gauge, given in the fifth column of Table I, is a gauge used by the American Steel and Wire Company to gauge its products. This gauge was formerly known as the Washburn & Moen, and this name is still extensively used.

74. Stubs' Steel-Wire Gauge.—The Stubs steel-wire gauge, given in Table II, is the gauge used almost exclusively for tool-steel wire in the form of straight rods, commonly called drill rods. It is also used by some makers for tool-steel wire sold in coils and on spools. Drill rods and wire gauged by this gauge are usually sold annealed; hence, if such drill rods and wire are used for helical or other coiled springs, the springs must be hardened and tempered in order to retain their elasticity.

75. Twist-Drill Gauge.—The twist-drill gauge, given in Table III, is the gauge used entirely by American manufacturers

TABLE III

TWIST-DRILL AND STEEL-WIRE GAUGE

Gauge Number	Size in Decimals of an Inch	Gauge Number	Size in Decimals of an Inch	Gauge Number	Size in Decimals of an Inch	Gauge Number	Size in Decimals of an Inch
1	.2280	21	.1590	41	.0960	61	.0390
2	.2210	22	.1570	42	.0935	62	.0380
3	.2130	23	.1540	43	.0890	63	.0370
4	.2090	24	.1520	44	.0860	64	.0360
5	.2055	25	.1495	45	.0820	65	.0350
6	.2040	26	.1470	46	.0810	66	.0330
7	.2010	27	.1440	47	.0785	67	.0320
8	.1990	28	.1405	48	.0760	68	.0310
9	.1960	29	.1360	49	.0730	69	.0292
10	.1935	30	.1285	50	.0700	70	.0280
11	.1910	31	.1200	51	.0670	71	.0260
12	.1890	32	.1160	52	.0635	72	.0250
13	.1850	33	.1130	53	.0595	73	.0240
14	.1820	34	.1110	54	.0550	74	.0225
15	.1800	35	.1100	55	.0520	75	.0210
16	.1770	36	.1065	56	.0465	76	.0200
17	.1730	37	.1040	57	.0430	77	.0180
18	.1695	38	.1015	58	.0420	78	.0160
19	.1660	39	.0995	59	.0410	79	.0145
20	.1610	40	.0980	60	.0400	80	.0135

for designating the smaller sizes of twist drills, excepting twist drills known as jobbers' drills. Jobbers' drills are sold directly by their actual size. The twist-drill gauge is used to some extent for steel wire and also for drill rods. A drill and tap-drill gauge is illustrated in Fig. 48. The tap-drill feature of this gauge consists of the table stamped on the left of the gauge, giving the size of tap drill and body drill for the small sizes of taps in use. A 14×20 tap, for example, means a tap with an outside diameter equal to gauge number 14, Table III, and hav-

TABLE IV

SCREW GAUGE FOR MACHINE AND WOOD SCREWS

Gauge Number	Size in Decimals of an Inch	Gauge Number	Size in Decimals of an Inch	Gauge Number	Size in Decimals of an Inch
000	.03152	16	.26840	34	.50528
00	.04468	17	.28156	35	.51844
0	.05784	18	.29472	36	.53160
1	.07100	19	.30788	37	.54476
2	.08416	20	.32104	38	.55792
3	.09732	21	.33420	39	.57108
4	.11048	22	.34736	40	.58424
5	.12364	23	.36052	41	.59740
6	.13680	24	.37368	42	.61056
7	.14996	25	.38684	43	.62372
8	.16312	26	.40000	44	.63688
9	.17628	27	.41316	45	.65004
10	.18944	28	.42632	46	.66320
11	.20260	29	.43948	47	.67636
12	.21576	30	.45264	48	.68952
13	.22892	31	.46580	49	.70268
14	.24208	32	.47896	50	.71584
15	.25524	33	.49212		

ing 20 threads to the inch. It should be observed that the decimal equivalents stamped on the gauge are carried to only three decimal places, whereas Table III gives the equivalents to four places.

76. American Screw Gauge.—The American screw gauge, given in Table IV, is the gauge used exclusively for

American machine screws and wood screws made of iron, steel, or brass. The table gives the diameter of the unthreaded cylindrical part or shank of the screw, and is applicable to round-headed, flat-headed, and fillister-headed screws. The difference between consecutive sizes is .01316 inch.

77. Special Gauges.—The gauges hitherto described are those in general use in the shop; but there are a great many special gauges of different forms, made for duplicate work and to facilitate manufacturing.

INSTRUMENTS FOR MEASURING SPEED

78. Speed Indicator.—The speed indicator, shown in Fig. 49 (*a*), is an instrument used to determine either the number of revolutions made by a rotating body in a given time, or the surface speed of a body. It consists of a spindle *a* that is free to turn easily in the body *b* of the instrument, and that is geared to the graduated ring *c* in such a way that 100 turns of the spindle cause the ring to make exactly one complete revolution. The ring is subdivided into 100 equal parts, and is numbered both right-handed and left-handed, so that the indicator may be used on a body turning in either direction. Each division on the graduated ring thus represents one turn of the spindle. A disk *d* fits inside the ring and is marked with 50 notches evenly spaced. The finger *e* is a spring that is screwed fast to the body *b* and that prevents the disk *d* from turning. However, every time the ring *c* makes a complete turn, the pin *f* passes under the spring *e*, lifts it for an instant, and allows the disk to revolve freely with the ring. The disk is thus rotated through one division at each complete turn of the ring, and each division on the disk represents 100 revolutions of the spindle *a*.

79. To use the speed indicator, shown in Fig. 49 (*a*), one of the tips illustrated in (*b*) or (*c*) is slipped over the end of the spindle *a*. If the shaft whose speed is to be found has a conical hole in the end, the tip shown in (*b*) is used; if the shaft is pointed, the tip in (*c*) is employed, and the end of the shaft then fits in a hole in the end of the tip. The ends of these tips are made of rubber, so that there will be no slipping when they are pressed against the shaft. The ring *c* is next set to zero, which may be done by turning the spindle until the pin *f* comes against the spring *e*. The disk *d* may be set to zero by lifting the spring *e* and turning the disk until the pin *g* comes opposite the end of the spring. The indicator is held by the handle *h*, and a watch is taken in the other hand. At an observed time, the tip on the spindle *a* is pressed against the end of the rotating shaft and is held there for a minute, at the end of which time it is sharply withdrawn. The number of revolutions recorded on the ring, or on the disk and the ring, is the number of revolutions per minute of the shaft. If the indicator is held against the shaft only half a minute, the reading must be doubled in order to find the number of revolutions per minute.

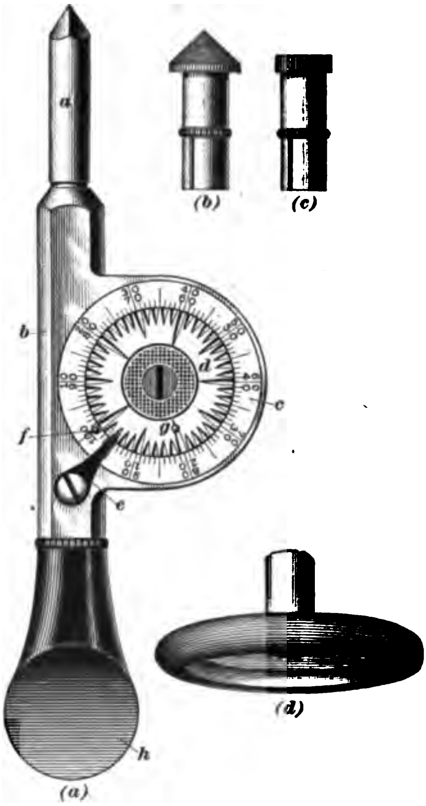


Fig. 49

80. If the spindle of the speed indicator, Fig. 49 (*a*), is fitted with the rubber-faced attachment shown in (*d*), it may then be used to find the speed, in feet per minute, at which the surface of a piece of work is moving. The rubber-faced wheel is slipped over the spindle in the same way as the tips shown in (*b*) and (*c*), and the indicator is set to zero, as already explained. The wheel is then pressed against the face of the work whose surface speed is to be found, and is held there for an observed time, as, for example, one minute. In doing this the spindle *a* must be held parallel to the face of the work, and the time must be carefully noted. The wheel is of such size that every revolution represents a movement of 6 inches, or ½ foot, of the surface of the work. Consequently, the total reading of the indicator, in revolutions per minute, is divided by 2, and the result is the speed of the surface of the work, in feet per minute.

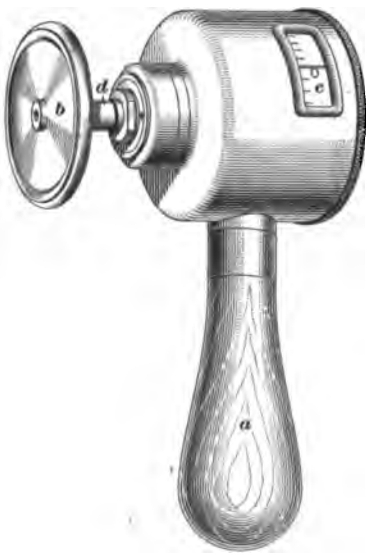


FIG. 50

81. Cut Meter.—The cut meter, illustrated in Fig. 50, is used to measure the surface speed of a body, in feet per minute, without either using a watch to time the operation or making calculations. The instrument is grasped by the handle *a* and the wheel *b* is held against the face of the work whose surface speed is to be found. The surface speed, in feet per minute, may then be read directly from the graduated scale *c*. It is necessary to hold the instrument in such a position that the shaft *d* is parallel to the face of the work.

INSTRUMENTS FOR MEASURING HARDNESS

82. Principle of Scleroscope.—The scleroscope, illustrated in Fig. 51 (*a*) and (*b*), is an instrument for measuring the hardness of materials, particularly of metals. The hardness is determined by dropping a small diamond-faced hammer from a fixed height on the piece to be tested, and observing the height to which the hammer rebounds. The harder the body tested, the higher will be the rebound of the hammer. The height of the rebound is noted on a vertical scale divided into 140 equal parts, called *degrees of hardness*, and the number of the division to which the hammer rises on the rebound denotes the degree of hardness. This scale is purely arbitrary, so that the

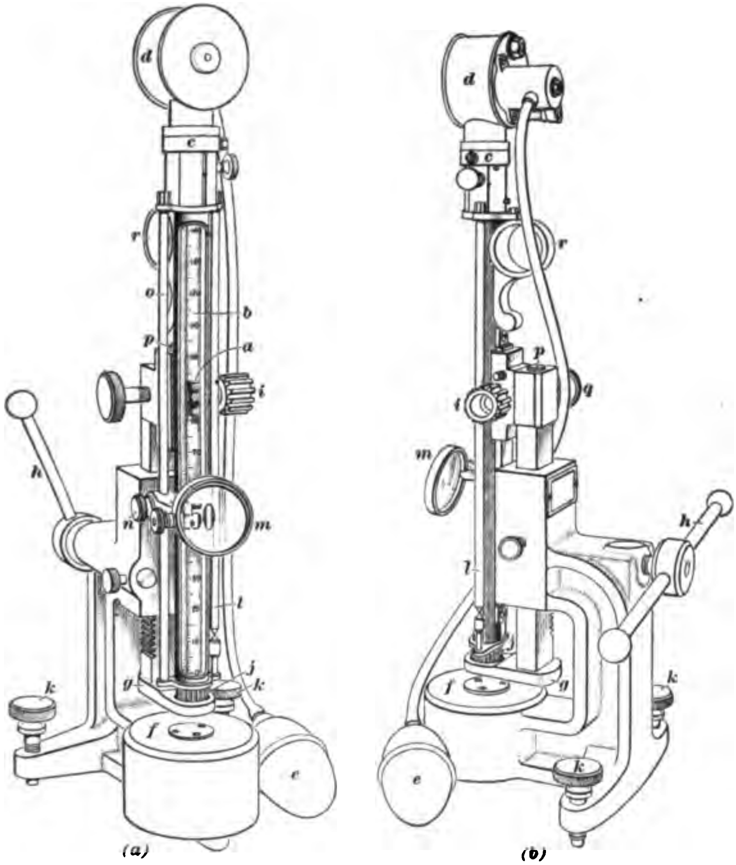


FIG. 51

various hardnesses determined by the instrument are only relative. Metals that can be machined have less than 50 degrees of hardness. The values for some of the common forms of metal are as follows: Gray cast iron, 25 to 50; annealed tool steel, 30 to 35; annealed high-speed steel, 35 to 40; chilled cast iron, 50 to 80; cutting tools, 75 to 110; brass castings, 7 to 25; soft brass, 9 to 10; and hard brass, 45.

83. Operation of Scleroscope.—Front and side views of the Shore scleroscope are shown in Fig. 51 (*a*) and (*b*). The diamond-faced hammer *a* is allowed to fall inside the vertical glass tube *b*, at the back of which is the graduated hardness scale. The hammer is held by a pair of hooks contained inside the casing *c*, and the hooks are connected with valves inside the casing *d*. When the valves are moved by air pressure, caused by pressing the rubber bulb *e*, the hooks are forced to release the hammer, and it falls. It may be returned to its original position by squeezing the bulb a second time. The piece of work to be tested is placed on the base *f* and is firmly held down by the clamp *g*, which may be raised or lowered by turning the arm *h*. Pressure should always be put on this arm when a test is being made, so that the work will be held firmly. The glass tube is held in a barrel that may be raised or lowered by turning the knob *i*. The lower end *j* of this barrel projects through a hole in the clamp *g* and touches the work when a test is being made. The glass tube must be set vertically, which may be done by adjusting the screws *k* in the base; the plumb-rod *l* will show when the instrument stands truly vertical.

84. The result of a scleroscope test is obtained by observing to what height the top of the hammer rebounds, as indicated on the scale; consequently, the top of the hammer should be watched closely. It may be necessary to try several times before an accurate reading can be obtained. In such cases, the first few tests will show the approximate height, and will indicate to the observer the point at which he should look. It is advisable to fix the eyes on a point a trifle lower than the height of the first rebound, as it is easier to note the next reading in this way than by looking at a higher point. Under no circumstances should the hammer be allowed to fall twice on the same spot, for readings thus obtained are not accurate. If exceedingly close readings are desired, the magnifying lens *m*, Fig. 51, should be used; it may be adjusted to the necessary point by loosening the screw *n* and moving the bracket along the rod *o*. The barrel of the scleroscope is fixed on the post *p* and is held by the screw *q*. When desired, the barrel may be removed and carried to the piece of material to be tested, being held in the hand by the grip ring *r*; or it may be fixed to a swinging arm, so as to be brought over the work. The latter arrangement is very convenient when a large number of similar pieces are to be tested.

INSTRUMENTS FOR MEASURING TEMPERATURE

85. Fahrenheit Thermometer.—The thermometer is an instrument that is used to measure the temperatures of various substances. A common form, known as a Fahrenheit thermometer, is illustrated in Fig. 52. It consists of a glass tube of small inside diameter having a bulb at its lower end and sealed at the top. The bulb is filled with mercury, or quick-silver, which possesses the property of expanding when heated. Thus, when the thermometer is placed near a heated body, or in contact with such a body, the mercury expands and rises inside the glass tube; when the body is cooled, the mercury contracts and moves downwards in the tube.

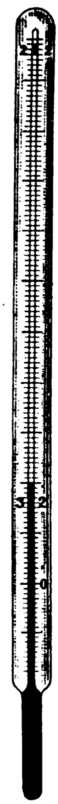


FIG. 52

86. A graduated scale is formed on the back of the glass tube. The height at which the mercury stands when the thermometer is placed in melting snow and ice is the *freezing point*, and is marked 32; the point to which the mercury rises when the instrument is placed in boiling water is the *boiling point*, and is marked 212. The space between these marks is divided into 180 equal parts, called degrees, and 32 of these divisions are laid off below the freezing point, thus locating the zero point. The height on the scale at which the mercury comes to rest when the thermometer is applied to a body, is the Fahrenheit temperature of the body, in degrees, and is usually abbreviated °F. Thus, a temperature of 62° F. means a temperature of 62 degrees indicated on the scale of a Fahrenheit thermometer. The graduations may be extended above the boiling point and below the zero as far as may be desired. In the illustration, only every other degree mark is shown; that is, each division represents 2° of temperature.

87. Centigrade Thermometer.—In scientific work the centigrade thermometer is used very largely. In principle and construction it is like the Fahrenheit thermometer; but its graduated scale is different. On the centigrade thermometer, the freezing point of water is marked 0 and the boiling point is marked 100, and the space between these two marks is divided into 100 equal parts, or degrees. Consequently, a degree on the centigrade scale is larger than a degree on the Fahrenheit scale. Temperatures on the centigrade scale are usually abbreviated °C.; thus, a temperature of 24° C. indicates 24 degrees on the centigrade thermometer.

88. Use of Pyrometer.—A thermometer using mercury cannot be used to measure very high temperatures, such as those of furnaces, molten metal, and so on, because the mercury would boil and become a gas, and the glass tube would melt. To measure high temperatures it is necessary to use a pyrometer, which is an instrument specially designed for high-temperature work. Pyrometers are of many kinds and act on different principles. It would be impossible to describe each of the various types; consequently, only a few forms will be illustrated here.

89. Le Chatelier Pyrometer.—An outside view and a sectional view of a Le Chatelier pyrometer are given in Fig. 53 (*a*) and (*b*). The instrument consists of a pipe *a*, at one end of which is a clamp *b* holding a hollow fire-clay cone *c* that is capable of resisting great heat. Inside the cone are two wires *d* and *e*, one of platinum and the other of an alloy of platinum and rhodium. These two wires are joined at *f*, in the tip of the cone, and their ends are carried back through the pipe *a* to a box *g* at the other end, where they are firmly fastened to the posts *h* and *i*. Two wooden handles *j* and *k* are provided, the first being fixed to the box *g* and the other being loose on the pipe, so that it can be moved nearer to or farther from the clamp *b*. A metal screen *l*, lined on the inside with asbestos, is attached to the handle *k* to protect the hand of the workman from the heat when the pyrometer is being used. From the posts *h* and *i*

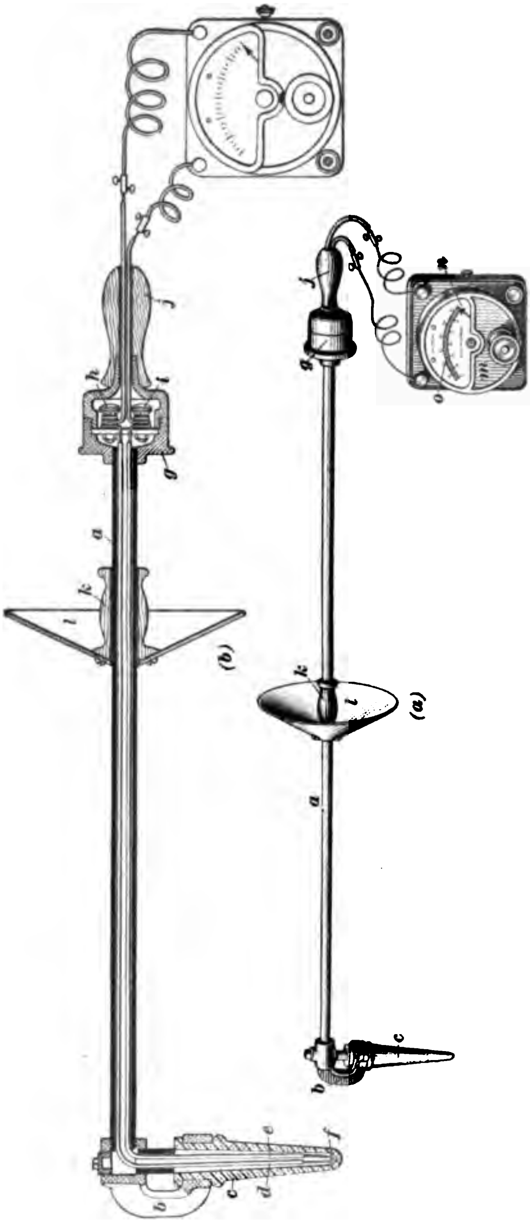


FIG. 53

separate wires are led out through the handle *j*, and are connected to an electrical instrument *m*, called a galvanometer, having a pointer *n* that moves over a graduated scale *o*.

90. The pyrometer shown in Fig. 53 acts on the principle that when the joined ends of the two wires *d* and *e* are heated, an electric current is set up in the wires, and the higher the temperature, the stronger is the current. Thus, if the pyrometer is taken by the handles and the fire-clay tip *c* is plunged into a ladle of molten metal whose temperature is to be found, the hot metal will heat the joint *f* and a current will be set up in the wires. This current will cause the pointer of the galvanometer to swing over the scale, its stopping point depending on the strength of the current. The scale is usually graduated to show the temperature, although it may also indicate the strength of the current. The reading at which the pointer comes to rest, therefore, indicates the temperature of the molten metal. The galvanometer may be located at any convenient point, either near or far away from the pyrometer. The connecting wires must, of course, be of suitable lengths.

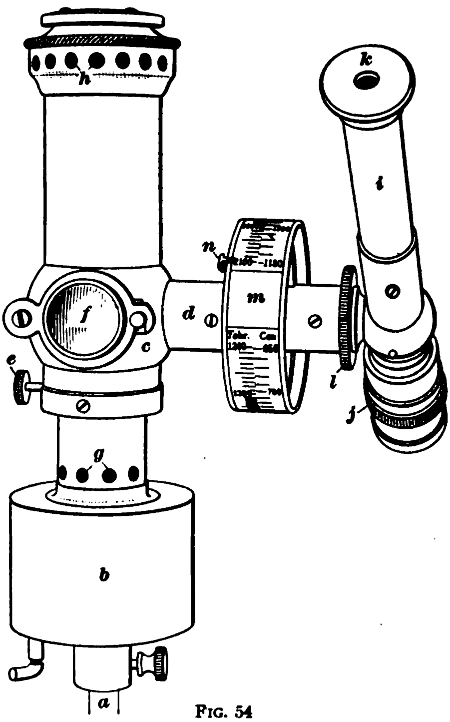


FIG. 54

91. Shore Pyroscope.—The Shore pyroscope, shown in Fig. 54, is used to determine the temperature of heated metal by comparing the color of the hot piece with the reflected light of a kerosene burner. The instrument is mounted on a rod *a* that is fixed in a three-legged base, not shown. The tank *b* contains kerosene, and inside the casing *c* is a burner, on a level with the tube *d*. The wick is adjusted by the button *e* and may be lighted by opening the glass door *f*. The air for the burner enters the holes *g*, and the gases escape through the holes *h* at the top of the chimney. When the burner shows a flame about $\frac{3}{4}$ inch high, the telescope *i* is swung so as to point directly at the piece of metal whose temperature is to be found, and the nurled ring *j* is turned until the focus is correct and the work is clearly visible through the eyepiece *k*. The work may have a color ranging from a dull red to a brilliant white, depending on the degree to which it is heated.

92. Inside the telescope, opposite the end of the tube *d*, Fig. 54, is a small round reflector on which the light of the kerosene burner falls and is reflected toward the eyepiece, so that the eye of the observer sees the light from the hot work and that from the reflector. The nurled ring *l* is now turned, which revolves a colored diaphragm inside the tube *d*, and thus alters the amount of light that falls on the small reflector from the oil burner. As the ring *l* is turned, the drum *m* is turned with it, and the graduations of the temperature scales marked on the drum pass under the end of the stationary pointer *n*. When a point is reached at which the color of the heated work and the color of the light reflected from the small reflector are the same, the reading on the graduated scale, opposite the pointer *n* is taken, this being the temperature of the work. There are two scales on the drum, one denoting the temperature in Fahrenheit degrees, and the other the temperature in centigrade degrees.

PRECISION MEASURING INSTRUMENTS

KINDS OF INSTRUMENTS

CLASSIFICATION

1. For the ordinary work of the machinist, patternmaker, foundryman, or blacksmith, the steel or wooden rules described in *Measuring Instruments* are accurate enough and measurements that cannot be made with the rules directly are transferred by the calipers and dividers there described. For the finer measurements necessary in machine-shop work and for toolmaking or gauge making, very much finer measurements must be made than those that are possible with the ordinary rule or calipers. Such measurements are called **precision measurements,** and the instruments used in making them are commonly called **precision measuring instruments.**

2. Precision measuring instruments are classified as follows: *Vernier instruments*, *micrometers*, *vernier micrometers*, *measuring machines*, *instruments for differential measurements*, *indicators*, *dial calipers*, and *thickness gauges*.

Precision measurements are generally read by shop men in thousands and fractions of thousandths of an inch. Thus, .0001 inch would be read $\frac{1}{10}$ of a thousandth of an inch; similarly, .00005 inch would be read $\frac{5}{100}$, or $\frac{1}{20}$, of a thousandth of an inch; .0783 inch would be read $78\frac{3}{10}$ thousandths of an inch; .5432 inch would be read $543\frac{2}{10}$ thousandths of an inch, etc.

VERNIER INSTRUMENTS

GENERAL INFORMATION

3. A vernier instrument is a precision measuring instrument in which measurements are made by means of the vernier scale. The **vernier, or vernier scale,** is a scale which, by means of its graduation, makes possible fine subdivisions of dimensions without placing the graduation lines so close together that they will prove confusing. The scale to be subdivided by means of the vernier scale is called the **true scale.** A vernier is used on a measuring instrument for measuring shorter lengths than those represented by the smallest divisions of the true scale. The vernier consists of a short scale arranged to slide along the true scale. By its application to shop measuring instruments it is possible to measure lengths as short as

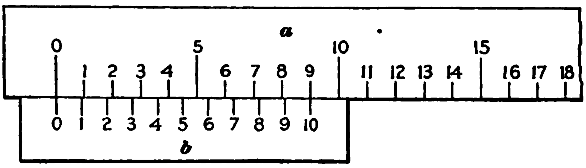


FIG. 1

$\frac{1}{10000}$ inch, the graduations on both the true and vernier scales being large enough to be easily read.

The lines of a vernier are numbered from the zero and are read from that point; thus, the line that has *0* written below it is called the *zero line*, the line next to it at the end of the first space is called the *first line*, the line at the end of the second space is called the *second line*, etc.

In the discussion that follows, unless otherwise stated, a division of the true scale, in order to simplify the text, will be spoken of as *a division*.

4. Principle of the Vernier.—In Fig. 1, let the vernier scale *b* have 10 divisions, the sum of which is equal in length to 9 divisions on the true scale *a*. One division of the vernier scale will then be equal to $\frac{9}{10}$ division of the true scale. If the zeros on the two scales coincide, the line numbered *1* on the vernier falls $\frac{1}{10}$ division of the true scale short of coinciding with the line numbered *1* on the true scale; line *2* of the vernier falls short $\frac{2}{10}$ division of the true scale of coinciding with line *2* of the true scale; line *3* falls $\frac{3}{10}$ division of the true scale short of coinciding with line *3* of the true scale; and so on. In like manner each successive line will fall short $\frac{1}{10}$ division more than the preceding line.

The number on the vernier scale indicates the number of tenths of a division of the true scale that that line falls short of coinciding with the line having the same number on the true scale.

5. Since, when the zeros coincide, the distance between the lines numbered *3* of each scale is $\frac{3}{10}$ division of the true scale,

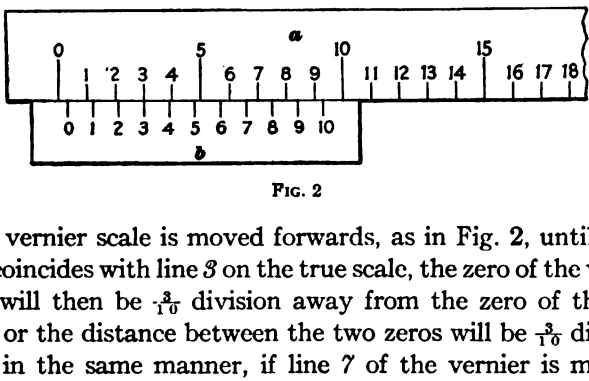


FIG. 2

if the vernier scale is moved forwards, as in Fig. 2, until line *3* on it coincides with line *3* on the true scale, the zero of the vernier scale will then be $\frac{3}{10}$ division away from the zero of the true scale, or the distance between the two zeros will be $\frac{3}{10}$ division. Also, in the same manner, if line *7* of the vernier is made to coincide with line *7* of the true scale, the distance between the two zeros will be $\frac{7}{10}$ division of the true scale; if the two *9*'s coincide, the distance between the two zeros will be $\frac{9}{10}$ division of the true scale; if the two *10*'s coincide, the zero of the vernier scale will also coincide with line *1* of the true scale. Clearly, then, when the zero of the vernier is between zero and line *1* of the true scale, the number of tenths of a division of the true scale between the two zeros can be told at once by finding the number of the line of the vernier scale that coincides with one of the lines of the true scale.

6. The vernier and true scales are often made, as illustrated in Fig. 3, so that 25 divisions of the vernier scale are equal to 24 divisions of the true scale. In this case 1 division of the vernier scale is equal to $\frac{24}{25}$ division of the true scale. Should the zero of the vernier scale coincide with any line of the true scale, line *1* on the vernier scale would fall short $\frac{1}{25}$ division of coinciding with the first line on the true scale to the right of the zero of the vernier scale. Similarly, line *24* on the vernier scale would fall short $\frac{24}{25}$ division of coinciding with the twenty-fourth line on the true scale to the right of the zero line of the vernier scale.

7. If the vernier scale is now moved forwards, as in Fig. 3, until the line numbered *10* on the vernier coincides with a line on the true scale, the zero of the vernier scale will then be

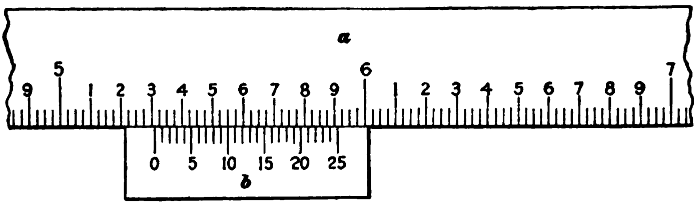


FIG. 3

$\frac{10}{25}$ division from the first line to the left of it on the true scale. When the zero of the vernier is between any two lines on the true scale, the number of twenty-fifths of a division of the true scale between the zero of the vernier scale and the line immediately to the left of it on the true scale, can be told at once by finding the number of the line of the vernier scale that coincides with one of the lines of the true scale.

8. The vernier scale on some measuring instruments is also made with 20 divisions and with 24 divisions. The vernier readings will then be in twentieths or twenty-fourths, respectively, of the divisions to which the true scale is graduated.

9. In the cases illustrated in Figs. 1 and 2, let each division of the true scale be $\frac{1}{10}$ inch. By means of the vernier scale, readings may then be made to $\frac{1}{10}$ division of the true scale or $\frac{1}{10}$ of $\frac{1}{10}$, or $\frac{1}{100}$ inch. Should each division of the true scale represent

$\frac{1}{1000}$ inch, which occurs in instruments to be described later, readings could be made to $\frac{1}{10}$ division of the true scale or $\frac{1}{10}$ of $\frac{1}{1000}$, or $\frac{1}{10000}$ inch.

If, in the case illustrated in Fig. 3, each division of the true scale is $\frac{1}{40}$ inch, which occurs in instruments to be described later, readings may be made to $\frac{1}{25}$ division of the true scale, or $\frac{1}{25}$ of $\frac{1}{40}$, or $\frac{1}{1000}$ inch.

If each division of the true scale represents $\frac{1}{50}$ inch and the vernier is divided into 20 divisions, readings may be made to $\frac{1}{20}$ division of the true scale, or $\frac{1}{20}$ of $\frac{1}{50}$, or $\frac{1}{1000}$ inch.

10. If each division of the true scale represents 1 degree, and the vernier is divided into 24 divisions, readings may be made to $\frac{1}{24}$ division of the true scale, or $\frac{1}{24}$ degree, or $\frac{1}{24}$ of 60 minutes, or $2\frac{1}{2}$ minutes.

The vernier scale need not be confined to reading hundredths, thousandths, or ten-thousandths, but may read to any subdivision required for the work.

11. It will be observed that in every case so far considered, the lengths of the divisions on the vernier scales are less than the lengths of the divisions on the true scale. Vernier instruments in which this rule prevails are known as **direct-vernier instruments.** All vernier instruments considered in this Section are direct.

The number of divisions on the true scale equal to the total length of the vernier divisions are usually equal to the number of divisions on the vernier scale less one. This is not always the case, as will be seen later, when studying the vernier protractor. It will, however, be understood to be the case in interpreting the rules of this Section.

12. Subdivisions Obtainable by a Vernier.—The number of subdivisions obtainable by a vernier may be found by the application of one of the following rules:

Rule I.—*To find the number of subdivisions of an inch which may be measured by the vernier instrument, multiply the number of divisions on 1 inch of the true scale by the number of divisions on the vernier scale.*

EXAMPLE 1.—In a vernier each inch of the true scale is divided into 16 parts. The vernier scale contains 8 divisions. What subdivision of the inch can be obtained?

SOLUTION.—Applying the rule just given, the number of parts into which the inch is divided is equal to $16 \times 8 = 128$. Ans.

EXAMPLE 2.—The unit of measurement on a true scale is not subdivided, while the vernier scale has 12 divisions or spaces. What subdivisions of the unit can be obtained?

SOLUTION.—Since the unit of measurement is not subdivided, the number of parts into which it may be conceived to be divided is 1. Then, by the rule just given, the number of subdivisions is $1 \times 12 = 12$. Ans.

Rule II.—*To find the smallest subdivision of a degree, expressed in minutes, which may be measured by a vernier, divide sixty by the number of subdivisions on the vernier.*

EXAMPLE 3.—The true scale of a protractor is divided into degrees, and the vernier scale is divided into 24 divisions, whose total length is equal to 23 degree divisions on the true scale. How close may readings be made with this instrument?

SOLUTION.—By the rule, the smallest subdivision of a degree, expressed in minutes, which may be measured by the vernier is $\frac{60}{24}$, or $2\frac{1}{2}$. Hence, the instrument will read as close as $2\frac{1}{2}$ minutes. Ans.

13. Reading the Vernier.—Vernier instruments may be read by applying the following rules:

Rule I.—*The vernier must always read from its zero point onwards in the same direction as the numbers of the true scale increase.*

The reading of the instrument will be the reading of the true scale, as indicated by the zero of the vernier scale, plus the reading of the vernier.

Rule II.—*To read a vernier instrument, proceed as follows: Observe the location of the zero point of the vernier scale, to obtain the number of whole divisions it is from the zero line of the true scale. From the zero point of the vernier scale, advance until a line is found on it that coincides with a line of the true scale. The number of spaces included between this line on the vernier scale and the zero point of the vernier scale is the number of the subdivisions obtained by the vernier that is to be added to the number of whole divisions previously found on the true scale.*

14. The following rule may be used for reading any vernier instrument designed for linear measurements, and measuring thousandths of an inch.

Rule.—*To read a vernier instrument designed for linear measurements in thousandths of an inch, count the number of whole inches between the zero of the true scale and the zero of the vernier scale. Count the number of tenths-of-an-inch spaces between the last whole-inch mark and the zero line of the vernier scale. This is the first decimal figure of the reading. Observe how many subdivisions of the tenth part of the inch are between the last tenth-of-an-inch mark on the true scale and the zero of the vernier scale. Express the value of these subdivisions in thousandths of an inch and add it to the value already found. Next count the number of spaces between the zero of the vernier scale and the line that coincides with one of the true scale; this number, if the instrument reads to thousandths, is the number of thousandths to be added to the true-scale reading for the complete reading.*

Thus, if the line at the end of the third space is the coinciding one, add 3 one-thousandths to the true-scale reading;

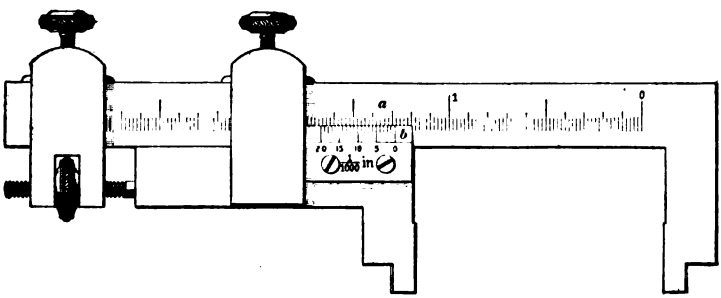


FIG. 4

if the line at the end of the ninth space coincides with one of the true scale, add 9 one-thousandths, etc. With a little practice the reading can be taken very rapidly by this rule, since the calculations are so simple that they can be performed mentally.

EXAMPLE.—What is the reading of the vernier caliper square shown in Fig. 4?

SOLUTION.—The number of whole inches is 1. Hence, write 1 as the integral part of the reading. There are two $\frac{1}{10}$-inch spaces between the graduation mark denoting 1 in. and the zero line of the vernier scale *b*. Hence, write 2 as the first decimal figure of the reading, giving 1.2 in. By inspection of the caliper square, it is seen that there are 4 subdivisions of the tenth part of an inch between the last $\frac{1}{10}$-in. mark and the zero line of the vernier scale. Now, in this case, each space represents $\frac{1}{50}$ in., or, expressed in thousandths, .020 in. Hence, 4 spaces represent $.02 \times 4 = .08$ in. Adding this to the previous reading, the reading of the true scale *a* is $1.2 + .08 = 1.28$ in. Inspection shows that the line at the end of the seventh space of the vernier scale coincides with a line on the true scale. Hence, as the instrument reads to thousandths, 7 one-thousandths should be added to the true-scale reading, giving $1.280 + .007$, or 1.287 in., as the complete reading. Ans.

Vernier readings may usually be made with the naked eye. However, should the graduations be close together, a magnifying glass is sometimes used.

APPLICATIONS OF THE VERNIER

15. Vernier Caliper Square.—The vernier is most frequently applied to caliper squares in order to obtain fine

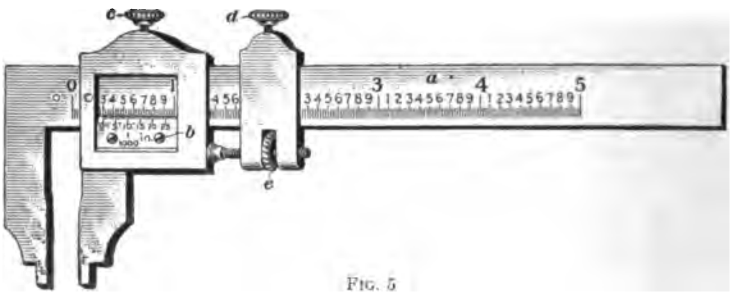


FIG. 5

measurements. The vernier caliper square, illustrated in Fig. 5, as used in the United States, is commonly made to read to thousandths of an inch. Fig. 6 shows the vernier scale and a portion of the instrument enlarged. The true scale *a*, which reads to fortieths of an inch, is engraved on the beam; while the vernier scale *b*, which has 25 divisions, is attached to the sliding head. For convenience in manufacturing, the vernier scale is in this case engraved on a separate plate, which is then fastened by small screws, as shown.

When adjusting the instrument, the screws *c* and *d*, Fig. 5, are loosened and the sliding head is moved approximately to the dimension to be measured. The screw *d* is then tightened,

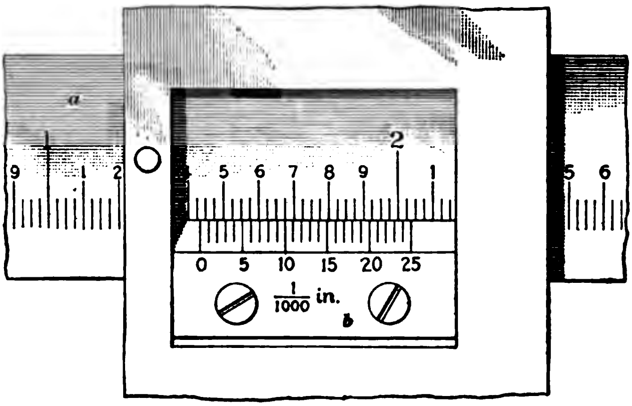


FIG. 6

and fine adjustment made by the nut *e*. The screw *c* is now tightened.

Applying the rule of Art. **14**, the reading of the instrument in the position shown in Fig. 6 is:

	INCHES
Whole number of inch......................	1.000
Additional tenths of inch...................	.400
Additional subdivision of tenths-of-an-inch space is 1, or $\frac{1}{4}$ of $\frac{1}{10}$, or $\frac{1}{40}$, or............	.025
Vernier reading..........................	.010
	1.435

16. It is to be observed that the reading obtained with this instrument is the measurement between the jaws; that is, an outside measurement. For inside measurements, add to the reading an amount equal to the total width of the jaws when closed.

17. The back side of the square, shown in Fig. 7, is graduated to read to sixty-fourths of an inch. By means of the lines marked *in* and *out* on the sliding head, the distance between them being equal to that across the points of the closed jaws, inside and outside measurements may be made directly.

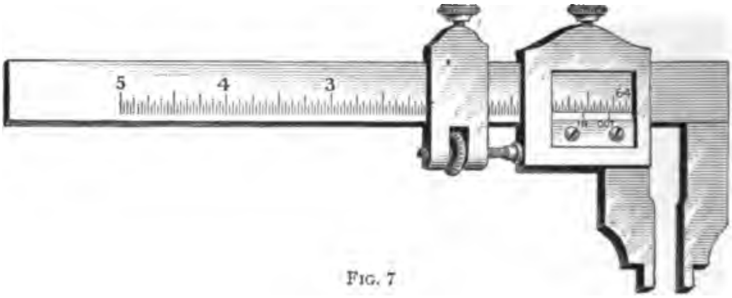


FIG. 7

18. Vernier caliper squares are usually made in 3-inch, 6-inch, 12-inch, and 24-inch sizes. A 3-inch size will make measurements from zero to 3 inches; a 6-inch size from zero to 5 inches, etc.

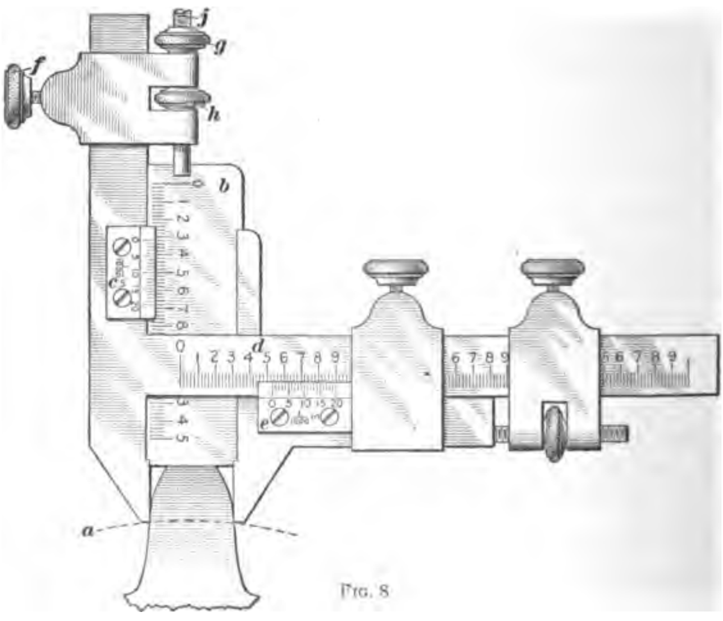


FIG. 8

19. Gear-Tooth Vernier Caliper.—The gear-tooth vernier caliper illustrated in Fig. 8 is designed to measure accurately the thickness of a gear-tooth on the pitch circle. The instrument is first set to the correct distance from the top of the tooth to the pitch circle, which is represented in the figure by the dotted line *a*, by means of the tongue *b* and the vernier *c*. The thickness of the tooth is then measured by means of the scale *d* and the vernier *e*. Both verniers read to thousandths of an inch.

The horizontal head of this instrument is adjusted precisely as the sliding head of the caliper square just described. When

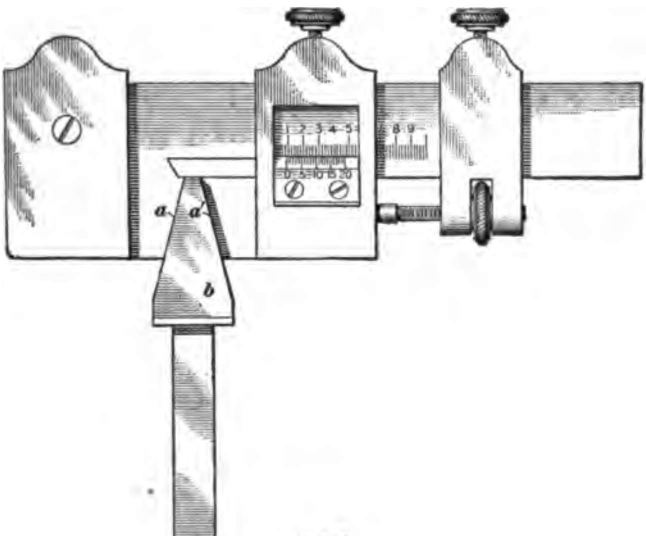


FIG. 9

adjusting the vertical head, the screw *f* and the nut *g* are loosened. The instrument is then set approximately to the desired dimension. Finer adjustment is made by means of the nut *h*, the setting being locked by the nut *g*. When using the instrument care should be taken that the recess in the tongue *b* bears firmly against the end of the screw *j*.

20. Vernier Thread-Tool Caliper.—The vernier thread-tool caliper shown in Fig. 9 is designed to gauge thread tools when grinding them. All widths of tool points up to about $\frac{6}{16}$ inch may be measured with this caliper, thus doing away with separate gauges for grinding tools having different widths of point. The angle between the hardened jaws *a* and *a'* is 60° for United States standard threads, and 29° for the 29° standard threads. In use, the tool *b* is held against the jaw *a* and the jaw *a'* is then brought against the tool and the measurement made as with the ordinary vernier caliper. The vernier reads to thousandths of an inch.

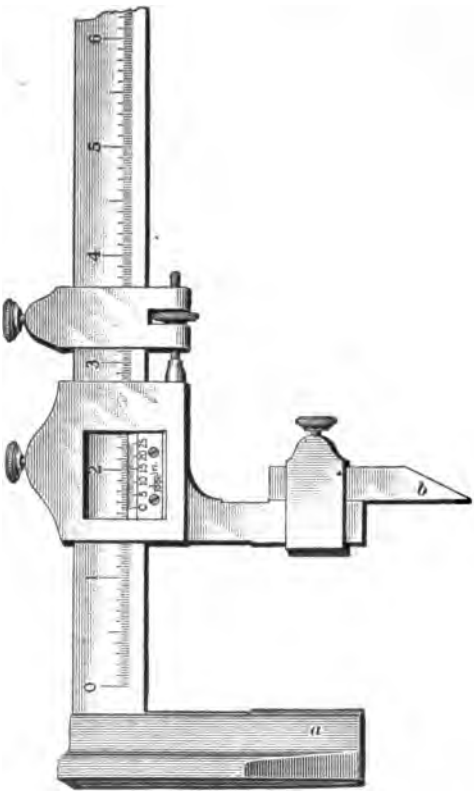


FIG. 10

21. Vernier Height Gauge. The vernier height gauge, illustrated in Fig. 10, is designed to measure heights accurately. It is used to obtain the heights of projections from a plane surface, to locate bushings in jigs, and to lay out work on vertical surfaces. The base *a* is made wide enough to allow the gauge to stand upright. The distance across the base and upper jaw when closed is 1 inch; hence, 1 inch must be added to the reading of the instrument to get the correct measurement. By the use of the extension piece *b*, it is possible to reach over projections in order to lay out or measure work as desired. The vernier reads to thousandths of an inch.

22. Vernier Depth Gauge.—The vernier depth gauge illustrated in Fig. 11 is a depth gauge with a vernier scale.

It reads to thousandths of an inch with the vernier scale, and to sixty-fourths of an inch with the scale on the opposite side.

23. Vernier Bevel Protractor.—Fig. 12 illustrates a bevel protractor with a vernier scale attached for the purpose of measuring subdivisions of the degree. Fig. 13 shows a portion of the disk, or true scale *a*, Fig. 12, and the vernier scale *b* enlarged. The vernier and true scales are designed so that 24 divisions of the vernier scale equal 23 degree divisions on the true scale. By rule II of Art. **12**, the smallest subdivision of the degree, expressed in minutes, obtainable is $\frac{60}{24}=2\frac{1}{2}$ minutes. In practice, however, it is not found desirable to work as close as $2\frac{1}{2}$ minutes, 5 minutes being sufficiently close. Every other division is therefore omitted on the vernier scale and the closest reading obtainable is 5 minutes.

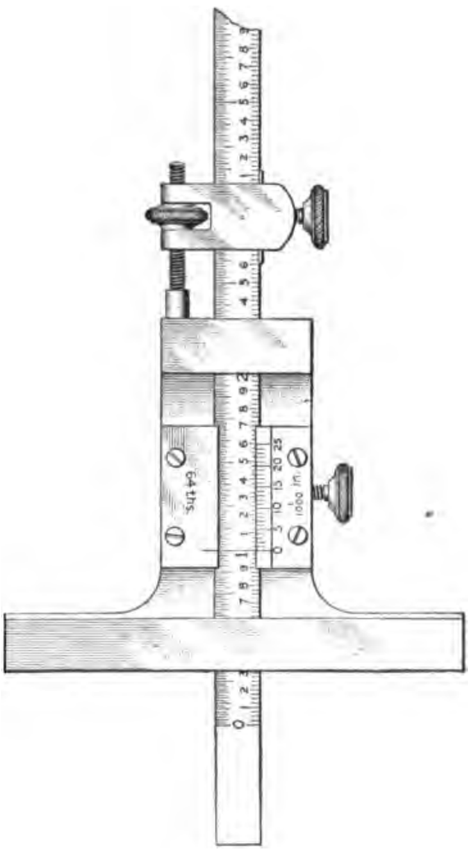


Fig. 11

24. The instrument could have been designed to read to 5 minutes by making 12 divisions on the vernier equal to 11 degree divisions on the true scale. The distance between the lines of the vernier would then be about half as large as that obtained by the design used.

By applying rule I of Art **13**, the numbers to the left of zero on the vernier scale *b*, Fig. 13, would be used to take the reading in the position shown and the numbers to the right of zero on

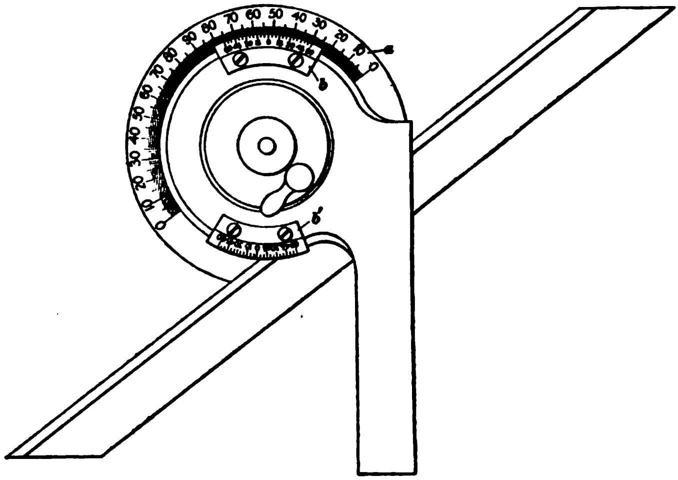


FIG. 12

the vernier scale would be used to take the reading when the vernier-scale zero corresponds with any position on the left half of the true scale *a*, Fig. 12. The vernier scale *b'* would be used

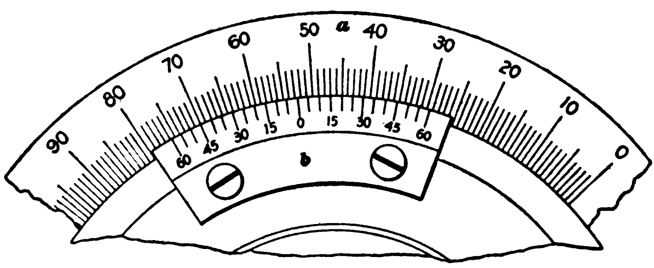


FIG. 13

similarly to *b* when its zero mark corresponds with some position of the true scale.

25. Reading Vernier Bevel Protractor.—It will be observed that in Fig. 13 the line at the end of the ninth space on the vernier is marked *45*, which represents the number of minutes to be added to the degree reading. Should the vernier not be marked in this manner, the number of spaces from the zero of the vernier scale to a line coinciding with a line on the true scale multiplied by 5 will be the number of minutes to be added to the degree reading of the protractor.

The following rule will be found useful for reading protractors of the type described:

Rule.—*To read any vernier protractor, read off directly the number of whole degrees between the zero of the true scale and the zero of the vernier scale. Add to this reading the number of minutes shown by the number under that line of the vernier scale which coincides with a line of the true scale.*

EXAMPLE.—Read the vernier protractor shown in Fig. 13 to degrees and minutes.

SOLUTION.—There are 52 whole degrees between the zeros of the true scale and the vernier scale. The line numbered 45 of the vernier scale coincides with a line of the true scale. Hence, the reading of the protractor is 52° 45′. Ans.

26. Setting Vernier Protractor.—The operation of setting the protractor is the converse of that of reading it. For example, if it were required to set the instrument to 52° 45′, the operation would be as follows: Move the true scale to the right until the 52° mark is opposite the zero of the vernier; then, looking at the vernier scale, move the true scale carefully to the right until the line numbered *45* on the vernier scale coincides with a line of the true scale.

MICROMETERS

PRINCIPLE OF THE MICROMETER

27. A **micrometer** is a precision measuring instrument in which an accurate screw is used for determining the measurements. Micrometers are usually constructed to read to thousandths of an inch for ordinary use and to $\frac{1}{100}$ millimeter for scientific measurements. The principle and method of using are identical in both systems.

28. Micrometer Screw.—The screws used on micrometers have right-handed threads; that is, the screw moves forward when turned to the right. In order to study the principle of the micrometer, the simple outside micrometer caliper illustrated in Fig. 14 may be considered. In order to determine easily through what part of a revolution the screw has been turned, the *micrometer screw a* is supplied with a *thimble b*, which is graduated on one edge. The cut shows the screw uncovered, for the purpose of more clearly illustrating the principle. Micrometers of recent manufacture have the screw covered, in order to protect it from wear. A line *c* engraved lengthwise on the *barrel d*, which is attached to the frame, serves as a zero line from which to read the part of a revolution through which the micrometer screw has been turned. In order to adapt the micrometer screw for end measurements, an *anvil e* is provided, the surface of which is a plane surface parallel to that on the end of the screw. The anvil is so adjusted by means of the screw *f* that when the end of the micrometer screw is in contact with it, the end of the thimble will coincide with the zero line on the barrel and the zero line on the thimble will coincide with the line *c* on the barrel.

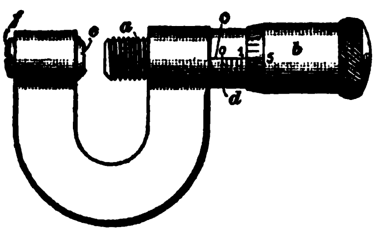


FIG. 14

29. The advance in one turn, which the thread makes, in a line parallel to the axis of the screw is called the **lead.** If a screw of known lead is fitted in a fixed nut so that the screw can be turned while the nut remains stationary, and the screw is turned one complete revolution, it will advance in the direction of its axis a distance exactly equal to the lead of the thread. But if it is turned exactly one-half of a complete revolution, it will advance a distance equal to one-half the lead of the thread. Following this line of reasoning, it is seen that the amount the screw advances in the direction of its axis in a fraction of one revolution is equal to the product of the lead and the fraction expressing the part of the revolution through which the screw has been turned.

30. Assume that the micrometer screw has 40 right-handed threads to the inch, and a lead of $\frac{1}{40}$ inch. Then, if the screw is turned $\frac{1}{25}$ of a complete revolution, the amount advanced will be $\frac{1}{40} \times \frac{1}{25} = \frac{1}{1000}$ inch. It is to be observed that the thimble *b* and the screw *a*, Fig. 14, are rigidly connected. Hence, if there are 25 equal graduations on the circumference of the thimble *b*, the screw *a* will advance lengthwise $\frac{1}{1000}$ inch when the thimble *b* is turned 1 division to the right. Every 5 divisions of the thimble are marked *0*, *5*, *10*, *15*, *20*, and are read thousandths. One inch of the lengthwise line *c* is divided into 40 equal divisions, every division being equal to $\frac{1}{40}$ or .025 inch, and every fourth division representing tenths of an inch. Every fourth division of the line *c* is marked as follows and read tenths: *0, 1, 2, 3, 4, 5, 6, 7, 8, 9.*

31. Reading the Micrometer.—Practically all micrometers for measuring thousandths of an inch use a screw having forty threads per inch, and have the thimble graduated so that each space is equal to one twenty-fifth of a revolution. Hence, they read directly to thousandths of an inch. Assuming that the micrometer screw and the anvil are in contact, let the micrometer screw be turned back until the graduations on the thimble show, by reference to the zero line on the barrel, that it has made seven twenty-fifths of a revolution. Then, the opening between the end of the micrometer screw and the anvil will, if the lead of the thread is $\frac{1}{40}$ inch, be $\frac{7}{1000}$ inch. Now, in order to measure sizes in excess of $\frac{1}{40}$ inch, or .025 inch, the screw must make more than one turn. When the micrometer screw is turned one complete revolution, the second graduation line on the barrel will be even with the end of the thimble, so that the thimble is 1 space from the beginning of the graduation on the barrel. If the micrometer screw is turned through another revolution, there will be 2 spaces visible; hence, the number of spaces shows how many complete turns the micrometer screw has made. As each space represents $\frac{1}{40}$ inch, or $\frac{25}{1000}$ inch, to find the distance represented by an unknown number of whole turns, count the number of spaces and multiply by .025. Then read off the fractional part of a turn and add it to the first value.

32. Hence, to read a micrometer measuring thousandths multiply the number of whole spaces visible on the barrel by .025. Add the number of spaces between the zero line of the thimble and the zero line on the barrel multiplied by .001.

As each graduation on the barrel equals .025 inch, four of these spaces must equal $4 \times .025 = .1$ inch. Therefore, every fourth line of the graduations stands for .1 inch, and the figures *0*, *1*, *2*, *3*, etc. are stamped on the barrel to indicate these tenth-inch graduations. They stand for .0, .1, .2, .3, etc. inch. As each space represents a distance of $\frac{25}{1000}$, or .025 inch, the line at the end of the first space denotes a .025-inch opening; the line at the end of the second space, .050 inch; the line at the end of the third space, .075 inch.

33. In order to save multiplication, the following rule may be used:

Rule.—*To read a micrometer measuring thousandths of an inch, look for the last figure that is exposed on the barrel. This is the first decimal figure of the dimension. Count the number of whole spaces beyond the last figure and between it and the end of the thimble. For one space, annex 25; for two spaces, annex 50; and for three spaces annex 75 to the first figure. Mentally add the number of spaces, expressed in thousandths of an inch, between the zero line of the thimble and the zero line on the barrel, counting from the zero line of the thimble forwards.*

EXAMPLE.—What is the reading of the micrometer shown in Fig. 14?

SOLUTION.—The last figure that is exposed is 1. There is one whole space between this figure and the end of the thimble, but the line representing it cannot be distinguished in the figure, as it is so near the end of the thimble. Since there is one whole space between this figure and the end of the thimble, annex 25, giving .125. The line at the end of the fifth space of the thimble coincides with the zero line on the barrel, hence add .005 inch, making the reading $.125 + .005 = .130$ inch. Ans.

34. Since the micrometer screw is made only long enough to give dimensions from 0 to 1,000 thousandths, any whole number of inches included in the measurement must be prefixed to the reading of the micrometer. Thus, in a 2-inch micrometer caliper, the micrometer screw will give readings only between 1 and 2 inches when the anvil and screw are arranged in such a manner that they are 1 inch apart when the micrometer is closed. In that case, 1 inch must always be prefixed to the reading. For instance, if the reading is .376 inch, the opening between screw and anvil is 1+.376=1.376 inches.

EXAMPLE.—What is the reading of the micrometer caliper shown in Fig. 15? In this instrument the whole number of inches is shown by the number opposite the line on the bar *a* with which the line *b* on the slide coincides.

SOLUTION.—Since the line *b* on the slide coincides with line 3 on the bar, the whole number of inches is 3. There are 5 tenths-of-an-inch spaces

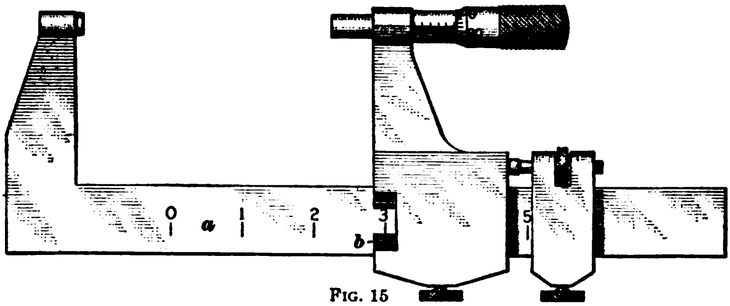


FIG. 15

exposed on the micrometer barrel, and hence 5 is the first decimal figure of the answer. There are no whole spaces between the last tenth-of-an-inch space and the end of the thimble. The line at the end of the twenty-first space of the thimble coincides with the zero line of the barrel; hence, add .021 in., making the reading 3+.5+.021=3.521 in. Ans.

35. Setting the Micrometer.—The process of setting the micrometer is as follows: If the given dimension is .1 inch or greater, open the thimble until the figure in the tenths place is visible on the barrel. Subtract this amount from the required dimension and divide the remainder by .025 to find how many more spaces on the barrel should be shown. Subtract the amount now shown upon the barrel from the required setting and obtain the thousandths remaining by opening the thimble still farther until the line showing the proper graduation on the thimble coincides with the zero line of the barrel.

In order to set the instrument to .462 inch, open the thimble until the figure *4* appears on the barrel and 2 whole spaces beyond. This gives .45. Subtracting .45 from .462 leaves .012, which is still to be found. Then continue turning the thimble until there are 12 spaces between the zero line on the thimble and the zero line on the barrel. The instrument is now set to .462 inch.

36. Should the given dimension be less than .1 inch, proceed as illustrated in the following example: Let it be required to set the micrometer to .093 inch. There is no figure in the tenths place, hence no figure should be exposed on the barrel. Dividing .093 by .025, 3 is obtained. Hence, with the micrometer set at zero, open up the thimble 3 whole spaces. As each space gives 25 one-thousandths, the setting will then be 75 one-thousandths. Subtracting 75 one-thousandths from 93 one-thousandths leaves 18 one-thousandths as a remainder. Next turn the thimble until there are 18 spaces between the zero line on the thimble and the zero line on the barrel. The instrument is now set to .093 inch.

APPLICATIONS OF THE MICROMETER

37. Outside Micrometer Calipers.—Micrometer calipers are made in a great variety of types and sizes. Fig. 16

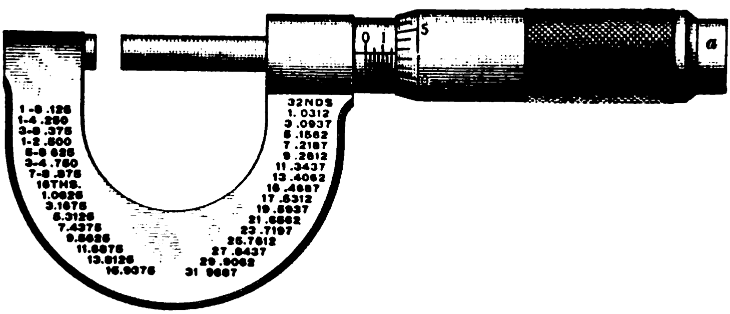


FIG. 16

illustrates a 1-inch micrometer of the *quick-adjusting* type. This instrument can be opened or closed instantly to any point within its capacity. Pressing the thumb against the plunger *a* disengages the nut from the screw, when the screw and thimble may be moved to any desired position. When the pressure is released, the nut will immediately engage the screw. Fine adjustments are then made.

When a measurement is to be made with a micrometer caliper, the operator places the work between its measuring points, that is, between the anvil and the end of the micrometer screw. The micrometer screw is then slowly revolved until the sense of touch tells the operator that the micrometer screw is in contact with the work. The reading of the micrometer caliper is then taken.

38. Before making measurements with a new or unfamiliar micrometer caliper, the anvil and micrometer screw should be

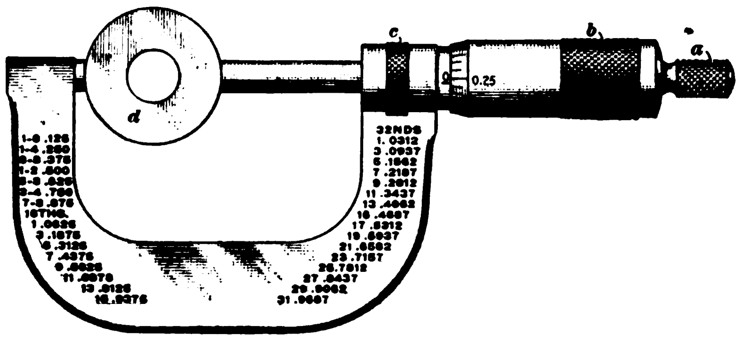


FIG. 17

brought together, and the amount of force required to bring the zero line of the thimble in line with the zero line of the barrel should be noted. The operator should then bring the micrometer screw against the work with the same force, in order to get a correct measurement.

While the micrometer caliper obtained from a reliable maker will indicate sizes correctly within an extremely small limit of variation, this fact does not imply that every one can measure within that limit. The accuracy with which a size can be measured with an accurate micrometer caliper depends almost entirely on the sense of touch of the operator and his amount of training.

39. Nearly all the modern micrometer calipers that measure in accordance with the English system of measurements are stamped on the frame with a table of the decimal equivalents of the divisions of the inch. Thus, in the micrometer calipers shown in Figs. 16, 17, and 18, on one side are stamped the decimal equivalents of eighths, sixteenths, and thirty-seconds; on the other side the decimal equivalents of sixty-fourths. This arrangement is very convenient, since, in many cases, the micrometer is used for measuring work to be finished to a

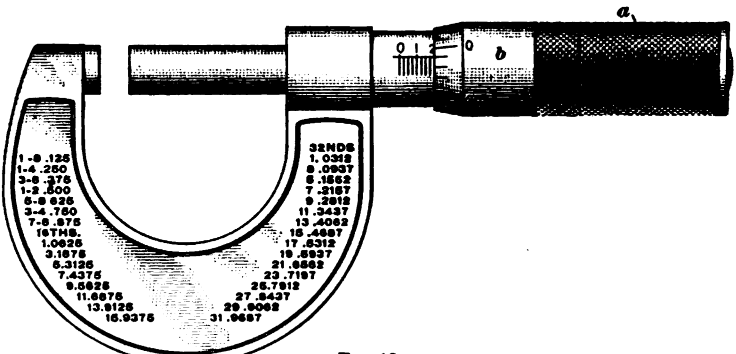


FIG. 18

dimension given in sixty-fourths. When micrometer calipers are made to measure in accordance with the metric system no decimal equivalents are needed.

For special purposes, other tables are occasionally stamped on the frame, and sometimes certain useful formulas. Thus, a special micrometer caliper intended for measuring the thickness of tubing has stamped on it the gauge numbers and the decimal equivalents of the gauge used by the tube makers.

40. A *locknut c*, Fig. 17, is frequently applied to micrometer calipers by means of which the micrometer screw may be prevented from rotating; the micrometer caliper is thus transformed into a fixed gauge adjustable for size. The circumference of the nurled extension *a*, being less than the circumference of the thimble *b*, it follows that the screw may be advanced lengthwise in less time by means of this nurled extension, which is known as a *speeder*.

41. Sometimes the speeder is attached to the thimble by a ratchet and spring in such a way that when more than a

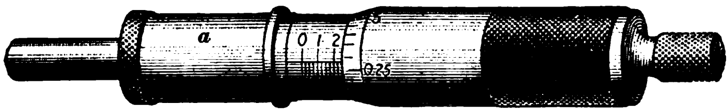
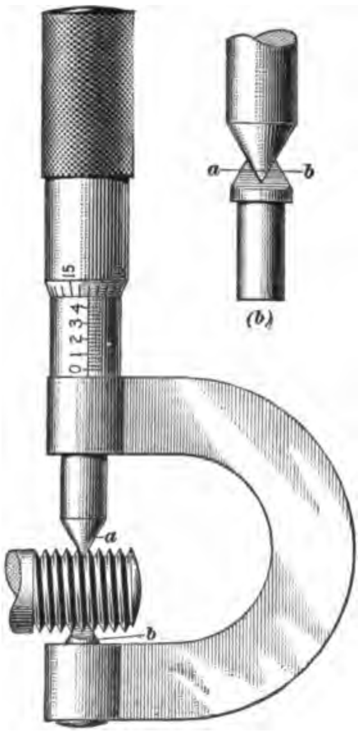


Fig. 19

certain amount of pressure is applied the ratchet will slip, thus preventing the measuring spindle from turning further. This attachment, called the ***ratchet stop***, is used for accurate measurements. Thus, inaccuracies due to the varying sense of touch of different persons are avoided.

42. An attachment known as the ***friction thimble***, illustrated in Fig. 18, is applied to some micrometers. When more than a certain pressure is exerted against the measuring point of the spindle, the sleeve *a* rotates about the thimble *b* and the spindle stops advancing. This attachment serves the same purpose as the ratchet stop; its advantage is that the hand that holds the tool can also work the friction thimble.



(a) Fig. 20

43. A method of testing the setting of a 2-inch caliper is illustrated in Fig. 17. The instrument is set to the outside diameter of a standard 1-inch disk *d*. Should the reading not be 1 inch, adjustment may be made by means of a nut under the cover of the thimble *b*, which can be turned by means of a spanner wrench.

44. Micrometer Heads.—A micrometer head, Fig. 19, consists of a micrometer screw, nut, spindle, thimble, barrel, and a shank *a*. The head is applied to tools or machines when fine measurements are required, the shank *a* being held by a part of the tool or machine. For very large work, say from 12 to 48 inches in diameter, which would not warrant the expense of large metal micrometers, rigidly constructed wood frames are sometimes arranged to hold a micrometer head. Very accurate work may be done with a properly constructed instrument of this kind.

45. Micrometer Thread Caliper.—The micrometer thread caliper illustrated in Fig. 20 (*a*) is an instrument for the

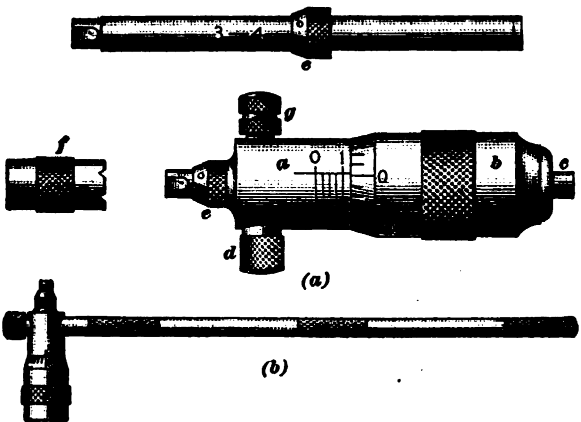


Fig. 21

accurate measurement of threads and screws. The instrument shown measures standard V threads, although this form of caliper is also made to measure United States standard and Whitworth standard threads. For threads not finer than 20 threads per inch, the same caliper may be used for both the V and United States standard threads. The cone-shaped end *a* of the screw is ground to the same angle as the angle of the thread to be measured, and the anvil *b* is V shaped. When the point and anvil are in contact, the zero represents a line *a b*, Fig. 20 (*b*). Hence, the measurement shown on the micrometer is the distance between the top of the thread on one side [see (*a*)], and the bottom of the thread on the other side; that is, the outside diameter of the screw less the depth of one thread. Consequently, the depth of one thread must be added to the reading to obtain the outside diameter of the screw measured. These instruments are necessarily limited in capacity and different anvils are required for different ranges of threads.

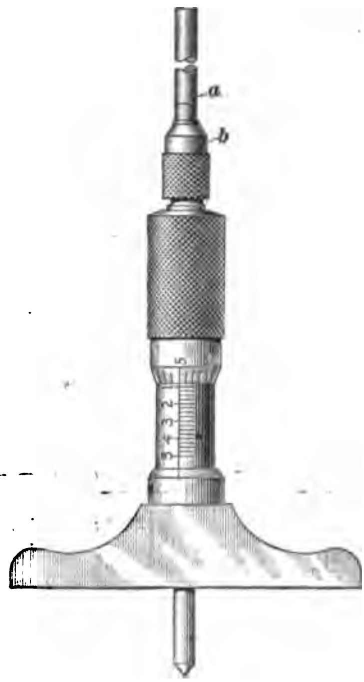


FIG. 22

46. Inside Micrometer Calipers.—An inside micrometer caliper, illustrated in Fig. 21 (*a*), is an instrument used for making close inside measurements by means of a micrometer screw. It is of considerable value in measuring for shrink or forced fits. It consists of a micrometer barrel *a* and thimble *b*, which are graduated and read just like the micrometers already described, and extension rods. The point *c* is permanently attached to the end of the thimble *b*, and the micrometer screw has a movement of ½ inch. The extension rods, which are held in position by the clamp *d*, are provided with collars *e*, which fit snugly against the barrel *a*. An adjustment of ½ inch is obtained by means of the standard ring *f*, which slips over the rods, thus extending the collars *e* ½ inch from the barrel. An auxiliary handle, shown in (*b*), screws into the side of the micrometer after the screw *g*, in (*a*), has been removed, thereby adapting the tool for use in places too small for the hand. The instrument when set as shown in (*a*) will measure from 2 to 2½ inches, in thousandths of an inch, and with the ring *f* in position from

2½ to 3 inches. Other measurements may be made by the use of other extension rods.

47. **Micrometer Depth Gauge.**—A micrometer depth gauge, Fig. 22, is an instrument used for making close measurements of depths by means of a micrometer screw. The screw in the gauge shown has a movement of ½ inch. The rod is graduated to ½ inches by grooves *a*. These grooves are of such depth that clamping fingers, under the cover *b*, spring into them, thus permitting ½-inch adjustments to be quickly made. This instrument will measure depths, from zero to 2½ inches, in thousandths of an inch.

VERNIER MICROMETERS

48. A **vernier micrometer caliper,** shown in Fig. 23, is a micrometer with a vernier scale attached for the purpose of subdividing the smallest measurement made by the micrometer itself. The true scale is represented by the graduations on the thimble, while the vernier scale is engraved on the barrel.

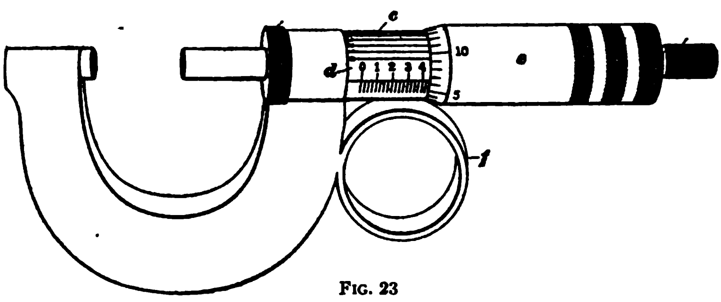


FIG. 23

Usually the micrometer reads to thousands and the vernier to ten-thousandths of an inch. Fig. 24 shows a development of the thimble and barrel; that is, the surfaces of the barrel and the thimble are shown in the illustration as if they were rolled out flat. The distance from zero to zero on the vernier scale, 10 divisions, is equal in length to 9 divisions on the true scale. The graduation lines of the vernier scale are parallel to the zero line of the barrel and extend its whole length; the zero line of the vernier scale may be a line separate from the zero line of the barrel, or the latter may at the same time form the zero line of the vernier scale. The reading of the vernier is the same in any case.

49. Reading Vernier Micrometer Calipers.—To read a vernier micrometer, read to three decimal places as directed by the rule of Art. **33.** To obtain the fourth decimal figure, find out which line of the vernier scale coincides with a line on the thimble. Count the number of spaces from the zero line of the vernier scale to this line, counting on the vernier scale. This number is the fourth decimal figure of the reading.

EXAMPLE.—Read the vernier micrometer shown in Fig. 24.

SOLUTION.—Applying the rule of Art. **33,** the micrometer reading to three decimal places is .4+.05+.006 = .456 in. The line at the end of the third space of the vernier scale coincides with a line of the thimble. Hence, the fourth decimal figure is 3, and the complete reading of the instrument is .4563 in. Ans.

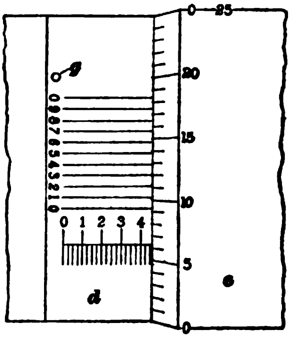


FIG. 24

50. Setting Vernier Micrometer Caliper.—In setting the vernier micrometer caliper, the thimble *e*, Fig. 23, is run back as in the regular micrometer, so as to expose the graduations on the barrel *d*, and the micrometer is set to thousandths in the ordinary manner. In order to obtain the additional ten-thousandths, or the fourth figure of the decimal, the thimble *e* is turned such a fraction of a thousandth of an inch that a line on the thimble comes opposite the desired line on the vernier scale. Thus, to set the instrument to an additional 3 ten-thousandths the line at the end of the third space on the vernier must be set to coincide with a line on the thimble.

51. Applications of the Vernier Micrometer.—The vernier scale may be applied to all the instruments described under the heading Micrometers. The instruments will then read to ten-thousandths of an inch instead of to thousandths. Fig. 23 illustrates a representative type of vernier micrometer caliper. This micrometer is adjusted for correct zero reading of the vernier by turning the barrel *d* by means of a spanner wrench that fits in the hole *g*, Fig. 24. In addition to the vernier scale, a new feature shown on this instrument, which may also be had on the other micrometers described, is the finger ring *f*, to hold the caliper while setting and measuring.

MISCELLANEOUS MEASURING DEVICES

MEASURING MACHINES

52. A measuring machine is used to measure sizes beyond the range of the portable micrometer caliper. In most cases it is arranged to measure with a degree of accuracy not obtainable with an ordinary micrometer caliper. There are a number of measuring machines in the market that differ only in the design of the details.

53. The measuring machine shown in Fig. 25 illustrates the type that is provided with a device for noting the degree of contact with which the work is held by the measuring points. This type of machine is used for making measurements within the closest possible limits of variation. By the use of the machine illustrated, measurements may be readily made within a limit of variation of .00001 inch. This type of machine does not depend on the sense of touch of the operator for the accuracy of the measurement. It may be considered as a very refined combination of a caliper square and a micrometer, to which a special device for noting the degree of contact of the measuring points with the work has been added.

54. The machine consists of a rigid bed *a*, Fig. 25, supported on three points to obviate flexure; a stationary head *b* that carries the stationary measuring point *b′*; a sliding head *c* that carries the movable measuring point *c′* and the micrometer mechanism for moving the point and reading its movement; and finally a test bar *d*, which is graduated usually to whole inches or to 25 millimeters. The sliding head *c* is set to coincide with the graduations on the test bar *d* by means of a fine-

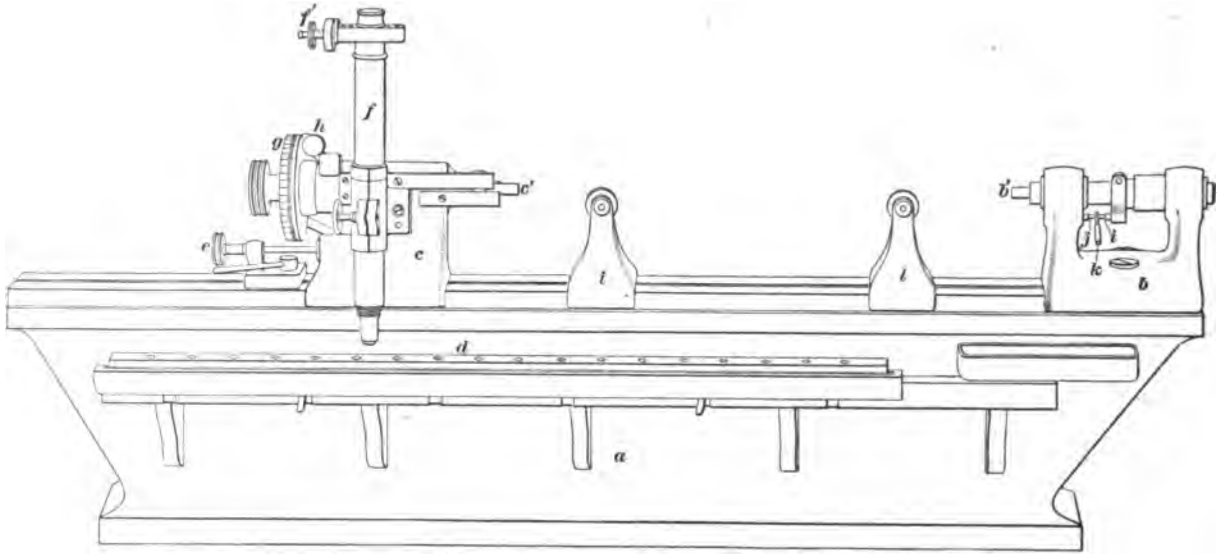


Fig. 25

threaded screw *e*; a powerful microscope *f* shows when the sliding head is set correctly in reference to the graduations on the bar. In the machine shown, the micrometer screw has fifty threads per inch, or a lead of $\frac{1}{50}$ inch, and it is advanced by turning the index wheel *g*. The index wheel *g* rotates, but cannot move forwards or backwards, the screw being prevented from rotating by a suitable device. As the index wheel does not move from the arm *h* when turned, but merely rotates, the circle carrying the graduations is always in contact with the zero on the arm *h*. The index wheel is divided into 400 spaces, so that each space represents a forward movement of the micrometer screw amounting to $\frac{1}{50}\times\frac{1}{400}=\frac{1}{20000}$ inch. The complete number of turns made by the index wheel is recorded on a scale, similar to that on the barrel of an ordinary micrometer, on the side of the instrument not shown in the illustration. The fractional part of a turn is shown by the graduation on the index wheel *g*, opposite the zero on the arm *h*. The fifth part of the space can readily be estimated by the eye, which would be measuring to $\frac{1}{5}\times\frac{1}{20000}=\frac{1}{100000}$ inch. The zero line from which the readings are obtained is engraved on an arm *h*, which can be slightly rotated about the axis of the micrometer screw by means of a suitable screw.

55. The measuring point *b'* of the stationary head is free to slide in its bearing; it is pushed outwards to the left by a light helical spring. An auxiliary measuring point *i* is rigidly fastened to the measuring point *b'* and moves with it. A second, but stationary, auxiliary measuring point *j* is fastened to the frame of the stationary head in line with *i*. A small, light feeling piece *k* is placed between the auxiliary measuring points, and determines by its behavior the degree of contact of the primary measuring points at zero and with the work. When no pressure is exerted against *b'*, tending to push it inwards to the right, the feeling piece is subjected to the whole pressure. The friction due to the pressure exerted by the helical spring will hold the feeling piece in its horizontal position. Now let a pressure that tends to push *b'* inwards be exerted against *b'*. This has the effect of lessening the pressure on the feeling piece until finally the friction has become small enough for the feeling piece to rotate under the influence of its own weight about the end by which it is held. Then, if the work is always measured just when the feeling piece rotates under its own weight, the degree of contact is the same, within extremely small variations, for each measurement made.

56. To adjust the machine for use, the micrometer screw in the sliding head is run all the way out and the zero line on its index wheel *g* placed opposite the zero line on *h*. The sliding head *c* is brought forwards and pushed against *b'* until the feeling piece just rotates. To get this adjustment roughly, the screw *e* may be used; but for final adjustment the index wheel is turned slightly. The arm *h* is rotated until its zero line coincides exactly with the zero of the index wheel. The microscope is next adjusted so that the line on the glass coincides with the zero line on the test bar *d*. This adjustment is made by moving the glass by means of the micrometer screw *f'*, which is supplied with a graduated index wheel. The adjustment is now complete, giving the zero for the entire capacity of the machine.

57. To make a measurement the sliding head is run back and adjusted by means of the screw *e* until the line on the glass of the microscope coincides with a graduation on the bar corresponding to the whole number of inches of the measurement to be made. The micrometer screw is now loosened sufficiently to admit the work between *c'* and *b'*; with the work resting on the supports *l*, the micrometer screw is then moved until the behavior of the feeling piece indicates the correct degree of contact; the reading of the micrometer is then taken and added to that given by the test bar.

DIFFERENTIAL RULE

58. The differential rule illustrated in Fig. 26 is a rule that, by its peculiar arrangement of lines and dots, is suitable for setting dividers to thousandths of an inch. One edge of the scale, the lower one in this instance, is divided into inches and hundredths of an inch. At the end of the hundredth graduations there are an additional nine lines, making 9 spaces *a*, the width of each space being $\frac{11}{1000}$ inch. Beyond the last line there is a row of eight dots, shown at *b*, so placed that the first dot is $\frac{12}{1000}$ inch to the right of the last of the lines *a* and each succeeding dot is $\frac{1}{1000}$ inch further to the right.

59. If the dimension is less than $\frac{1}{10}$ inch and a multiple of 11, use the lines *a*, Fig. 26, for setting the dividers. Dimensions coming under this head are easily recognized, as the first decimal digit is zero, and the second and third decimal digits are alike.

EXAMPLE.—Set the dividers to .066 inch.

SOLUTION.—Since the dimension is less than .1 in. and a multiple of 11, use the lines *a*. Put one leg of the dividers in the line at the left, and adjust the dividers until the other leg coincides with the line at the end of the sixth space. The dividers are now set to .066 in. Ans.

60. If the dimension is less than $\frac{1}{10}$ inch and the sum of multiples of 10 and 11, use the lines *a* and the hundredth spaces at the left of the lines *a* for setting the dividers, by dividing the dimension into two parts; one a multiple of .011, and the other a multiple of .010. The part of the dimension that is a multiple of .011 is laid off on the $\frac{11}{1000}$-inch spaces and the part that is a multiple of .010 is laid off on the hundredth spaces. Dimensions coming under this head may be recognized from the facts that the first decimal digit is zero, and the third decimal digit is less than the second.

61. The number of $\frac{11}{1000}$-inch spaces to be used is equal to the number in the third decimal place of the dimension to which the dividers are to be set. The part of the dimension to be laid off on the hundredth spaces is the difference between the original number and the part to be laid off on the $\frac{11}{1000}$-inch spaces.

EXAMPLE.—Set the dividers to .072 inch.

SOLUTION.—Since the first decimal digit is zero and the third decimal digit is less than the second, use the lines *a* and the hundredth spaces at the left of the lines *a*. Since the third decimal digit is 2, use two $\frac{11}{1000}$-in. spaces. The result obtained by subtracting the product of 2 and .011, or .022 in., from .072 in. is .050 in. The number of hundredth spaces to

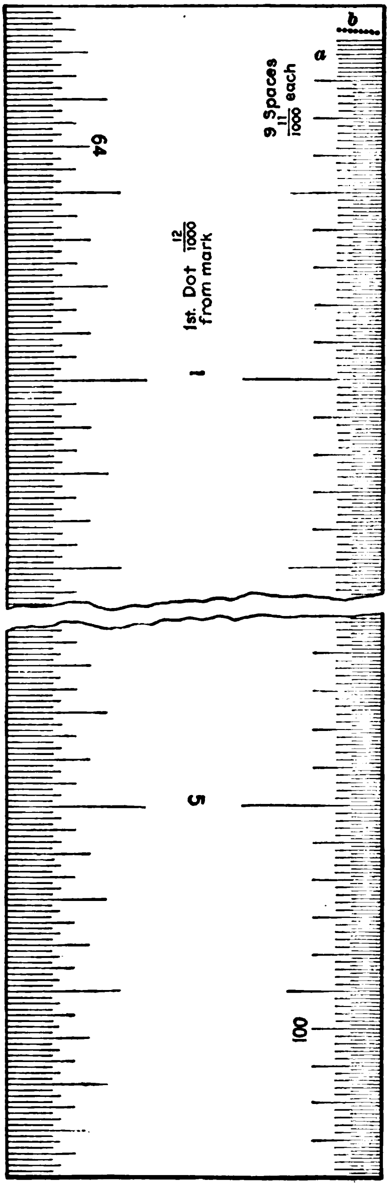


FIG. 26

be used is therefore five. One leg of the dividers is now set in the line at the end of the second $\frac{11}{1000}$-in. space, counting from the left, and the other leg adjusted to the line at the end of the fifth hundredth space, counting from the left of the lines *a*. The dividers are now set to .072 in. Ans.

62. If the dimension is less than $\frac{1}{10}$ inch and neither a multiple of 11 nor the sum of multiples of 10 and 11, use the lines *a* and the dots *b* to set the dividers. The first dot is $\frac{12}{1000}$ inch to the right of the lines *a* and should be considered as an additional $\frac{11}{1000}$-inch space plus $\frac{1}{1000}$ inch. Dimensions under this head may be recognized from the facts that the first decimal digit is zero and that the third decimal digit is greater than the second. When the given dimension is divided by .011 inch, the quotient will be the number of $\frac{11}{1000}$-inch spaces to be used and the remainder the number of dots to be used.

EXAMPLE.—Set the dividers to .079 inch.

SOLUTION.—Since the first decimal digit is zero and the third decimal digit is greater than the second, use the lines *a* and the dots *b*. Dividing .079 by .011 the quotient is found to be 7 and the remainder 2. Hence, use seven $\frac{11}{1000}$-in. spaces and go two dots beyond. One leg of the dividers is now set in the second dot, counting from the lines *a*, and the other leg is adjusted to coincide with the seventh of the lines *a*, counting from the right. It will be remembered that the first dot is $\frac{12}{1000}$ in., or one $\frac{11}{1000}$-inch space plus $\frac{1}{1000}$ in., from the nearest of the lines *a*. The dividers will then be set to .079 in. Ans.

63. If the dimension is greater than $\frac{1}{10}$ inch, the lines *a* and the hundredth spaces to the left of them are used. The number of $\frac{11}{1000}$-inch spaces to be used is equal to the number in the third decimal place of the given dimension. The number of hundredth spaces to be used is equal to 100 times the result obtained by subtracting the product of the number in the third decimal place and .011 inch from the dimension to which the dividers are to be set.

EXAMPLE.—Set the dividers to .547 inch.

SOLUTION.—Since the dimension is greater than $\frac{1}{10}$ in., use the lines *a* and the hundredth spaces to the left of them. The third decimal digit is 7; hence, seven of the $\frac{11}{1000}$-in. spaces are used. The number of hundredth spaces to be used equals

$$100\,[.547-(7\times.011)]=47 \text{ spaces}$$

One leg of the dividers is now placed in the line at the end of the seventh $\frac{11}{1000}$-in. space, counting from the left, and the other leg is adjusted to coincide with the line at the end of the forty-seventh hundredth space. The dividers will then be set to .547 in. Ans.

MACHINISTS' INDICATORS

64. A machinist's indicator is a precision measuring instrument used to indicate inaccuracies of alinement by means of a pointer or dial.

65. Center Tester.—In Fig. 27 is shown a center tester, that is, an indicator designed to magnify the amount any point, in a piece of work held in a lathe chuck or on a face plate, runs out of true. The shank *a* of the tool is secured in the tool post of the lathe; the rod *b* is held rigidly in the universal joint *c* by the nurled nut *d*; and the rod *e* is fastened to the rod *b* at *f*.

By means of the universal joint the point *g* is free to swing in any direction. In operation, the point *g*, which is ground to a 60° angle, is brought in contact with the work to be tested. If the work does not run true, a slight variation will be magnified

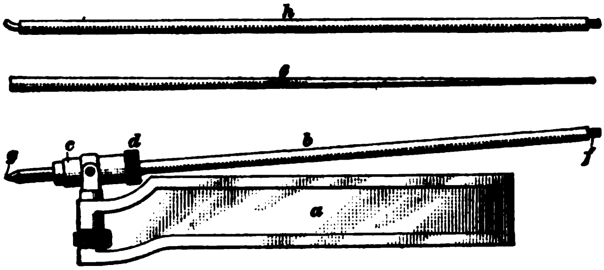


FIG. 27

many times at the end of the rod *e*. A rod *h* with a bent point may be used instead of *b*, thereby adapting the tool to inside and outside testing.

66. Lathe-Test Indicator.—In Fig. 28 is illustrated a lathe-test indicator that shows the amount in thousandths of an inch that the lathe center, or a piece of work held in a lathe chuck, on a lathe face plate, or between the lathe centers, runs

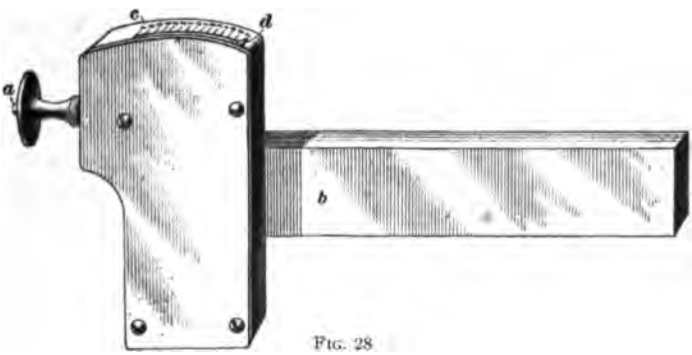


FIG. 28

out of true. In operation, the point *a* is made to bear against the part of the work whose truth is to be tested, the shank *b* being held in the tool post. The zero is in the center of the scale *c* and the graduations are in thousandths of an inch, reading to 7 in both directions. The amount that the work is out of true is shown by the pointer *d*.

Fig. 29 shows how this indicator may be applied to work between the centers of a lathe. The indicator is held in the

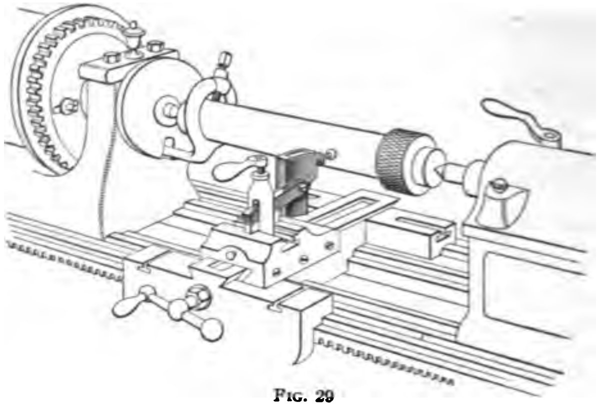


Fig. 29

tool post; the cross-slide is then fed forwards until the point touches the work. When the latter revolves, the indicating lever shows how much, if any, the work is out of true.

67. Dial-Test Indicator.—One form of indicator, known as the dial-test indicator and illustrated in Fig. 30, is used chiefly in erecting and inspecting machines. By its use the degree of inaccuracy of a plane surface on the top, bottom, or side of a piece of work may be readily determined, or the amount of end movement for example, of a spindle, or the extent to which a spindle may run out of true, easily ascertained.

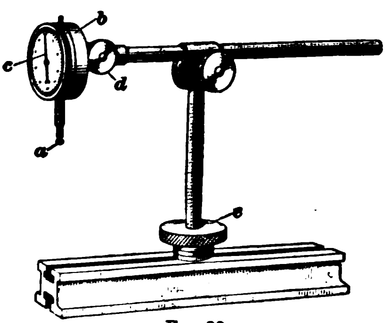


Fig. 30

The arm turns in the sleeve and may be set at any angle relative to the base, or it may be inverted so that the point *a*, which is brought in contact with the work, will point upwards. The movement of this point is magnified a number of times and is shown on the dial *b*, which is graduated to read to thousandths of an inch. The index finger *c* may be adjusted to the zero point by means of the joint *d*, at any position of the arm. The upright post, or stand, may be clamped at any point on the base by the nurled nut *e*. The sleeve that carries the arm may be fastened at any height on the post, or may be turned around the post to bring the arm on either side.

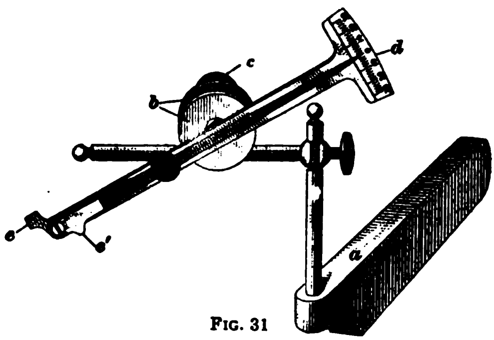


FIG. 31

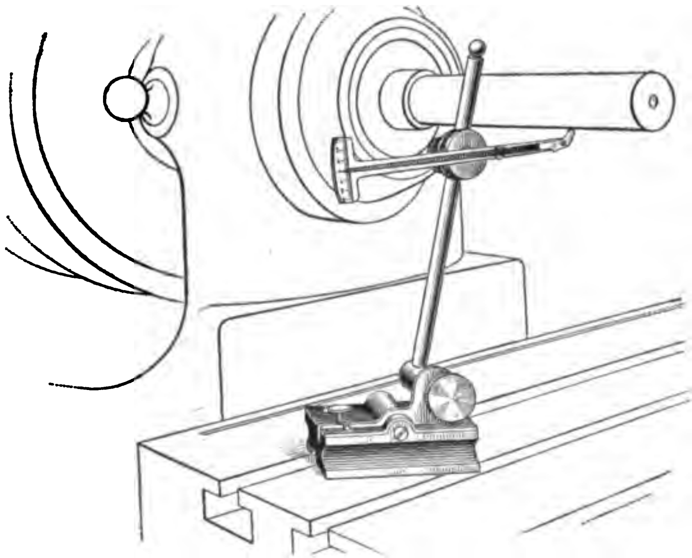


FIG. 32

68. Universal Test Indicator.—The universal test indicator, shown in Fig. 31, is designed to show imperfections of truth of inside, outside, or surface work. It may be used to indicate the accuracy with which lathe work is turned, chucked, or centered on the face plate. When used for this purpose, the indicator is clamped to the holder *a* by means of the disk *b* and the nurled nut *c* when used for lathe work, or to the spindle of the surface gauge when used for surface work. In operation, one of the working points *e*, or *e'*, both equally distant from its fulcrum, is brought in contact with the work, causing the needle to vibrate. The amount of variation is magnified by the lever and shown on the scale *d* in thousandths of an inch. This scale is composed of thirty $\frac{2}{1000}$-inch divisions, fifteen on each side of the zero, imperfections of truth up to .03 inch being shown by the indicator. Fig. 32 shows the indicator attached to a surface gauge and used to test the truth of the arbor of a milling machine. As the arbor revolves, any imperfection in truth will be shown on the scale *b*.

DIAL CALIPERS

69. A dial caliper is a precision measuring instrument in which a pinion and rack are used to determine the measure-

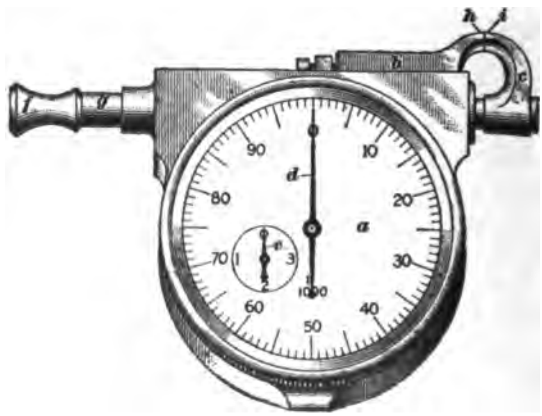


FIG. 33

ments, which are recorded on a dial, usually graduated to read to thousandths of an inch. One form of dial caliper, sometimes called a *jaw gauge* and shown in Fig. 33, is adapted for outside measurements in thousandths of an inch up to .4 inch. It consists of a dial *a*, a fixed jaw *b*, a movable jaw *c*, a pointer *d* indicating thousandths of an inch, a pointer *e* indicating tenths of an inch, a spindle *g* to which the movable jaw *c* is attached, a finger piece *f* by means of which the jaws *b* and *c* are opened, and a spring, not shown in the illustration, which maintains a uniform pressure on the work to be measured. This work is placed between the measuring points *h* and *i* of the jaws; the pointer *e* indicates the first decimal figure of the reading and the pointer *d* the second and third figures. This caliper is much used by inspectors for testing small work as it may be adjusted and read instantly.

THICKNESS GAUGE

70. A thickness gauge, or *feeler*, illustrated in Fig. 34, is a precision measuring instrument to measure small inside dimensions, as the width of narrow gaps, slots, or spaces between surfaces. It consists of a number of leaves accurately made of

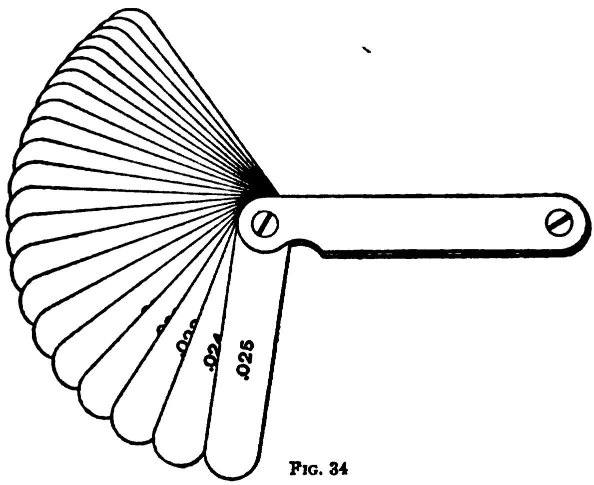


Fig. 34

hardened steel and varying in thickness from .002 to .025 inch. The thickness of each leaf is indicated by the number stamped on it. Each leaf is .001 inch thicker than the preceding one, and may be used either singly or in combination with others, thus securing a large range of measurements. For example, by combining the leaves marked .012, .011, .010, .009, and .008, a measurement of .050 inch may be made.

CARE OF MEASURING INSTRUMENTS

71. Great care should be taken in the use of tools and instruments lest they become injured and their accuracy impaired. Instruments designed for accurate measurements are necessarily delicate in construction and hence easily disarranged. A place should be provided for every tool, and the habit of putting each tool away in its proper place after being used should be acquired. Instruments of precision are frequently kept in plush-lined cases; when no case is provided they are sometimes wrapped in chamois skin when not in use. It is of the utmost importance that the tools should not be dropped, or permitted to remain where other tools or work may be carelessly knocked against them. While the effect of dropping an accurate try square, vernier instrument, or micrometer may not be noticeable at once, yet investigation will probably show that the accuracy of the instrument has been either impaired or destroyed.

Before tools are put away, they should be examined to make sure that they are not damp, as moisture will cause steel to rust. It is well to wipe them off occasionally with an oily cloth in order to lubricate the bearings and to prevent rusting. Except in the case of instruments especially designed to work at high temperatures, they should not be allowed to come in contact with anything that is made much above the normal temperature, as the heat would tend to distort them and make them inaccurate. The practice of stamping the name of the owner on tools is not good, as it tends to spring the tools and make them unreliable. Adjustable instruments should be frequently tested to be sure the zero readings are correct, and fixed measuring instruments should be tested at intervals to make sure that the wear on the parts has not been sufficient to render them not sufficiently accurate for the purposes for which they are used.

72. Cultivating Accuracy in Use of Calipers.—The use of calipers is something that needs much practice to insure good results. The workman must accustom himself to the use of these tools and cultivate a delicacy of touch in order to use them to the best advantage. Mechanics have been known to detect a variation of $\frac{1}{20000}$ inch in the size of two cylindrical pieces with a pair of ordinary spring calipers, while others could scarcely feel any difference between two pieces that varied as much as $\frac{1}{64}$ inch. If the workman will set his calipers so that he can just feel them pass over a cylindrical piece and will then hold a piece of thin tissue paper against the side of the piece and try the caliper over both, he will soon learn to note a variation of $\frac{1}{1000}$ inch in size. Thin tissue paper varies in thickness, but may ordinarily be called $\frac{1}{1000}$ inch thick, and common newspaper, $\frac{3}{1000}$ inch thick.

Experience in calipering may be attained by centering a piece of soft steel about 1 inch in diameter and 1 foot long, and turning and grinding 10 inches carefully so that one end will be $\frac{1}{100}$ inch larger than the other; then marking ten lines 1 inch apart on the piece with a sharp-pointed tool. The diameter of the piece will now vary $\frac{1}{1000}$ inch at each line, and by trying the calipers in the various divisions one may familiarize himself with any variation that may occur between $\frac{1}{1000}$ inch and $\frac{1}{100}$ inch.

73. Opening and Closing Micrometer Calipers.—The micrometer screw should always be revolved between the thumb and finger and not by holding the thimble firmly in the hand and swinging the frame. This latter method, while an easy one, causes the hole in the frame around the smooth portion of the screw to wear out of shape, making a loose fit for the screw that impairs the accuracy of the micrometer. Whenever a micrometer is used in a dusty place, care should be taken to wipe the dust from the smooth portion of the screw.

74. Lubricating Micrometer Calipers.—As the accuracy of a micrometer or any other measuring instrument using a screw depends on the accuracy of the screw, care should be taken to keep it in the best possible condition. If thin oil is used upon a micrometer screw, it is liable to work down upon the smooth portion of the screw where it will collect dust, and this dust will easily be carried back into the threads, thus causing undue wear, which will soon destroy the accuracy of the instrument. For lubricating the screws of measuring instruments a heavy, high-grade lubricating oil—or, better still, vaseline—should be employed. Care should be taken to use as little lubricant as will insure easy working of the parts. Any excess of the lubricant is sure to work out and collect dust.

75. Temperature Changes.—As is well known, all metals expand if their temperature is raised, hence it follows that a piece measured at a temperature above the normal will measure less when cooled. The average temperature at which measurements of length are usually supposed to be correct is about 62° F.; all gauge manufacturers adjust their gauges to be correct at that temperature. Although the temperature of the work may be normal while measuring, yet the heat from the hand will cause the instrument itself to expand. To lessen this expansion the frames of large micrometers are often wholly or partly covered with hard rubber or wood. Work should not be measured while it is warm from machining, as the contraction in cooling may be sufficient to cause a serious error.

GENERAL APPLIANCES AND PROCESSES

(PART 1)

GENERAL SHOP APPLIANCES

OILS AND LUBRICATION

KINDS AND USES OF LUBRICANTS

1. When two bodies move against each other, friction is produced; and when metal is removed by cutting, heat is generated. A **lubricant** is a substance, such as oil, grease, or graphite, used to diminish the friction between the working parts of machinery or to carry away heat. Oils are also used for cleaning purposes.

2. The lubricants used in shops are generally the *mineral* and *animal oils*. **Mineral oils** are obtained by refining crude petroleum, taken from the earth. In the refining process many different grades of oil are produced varying from the light, volatile gasoline to the heavy cylinder oil. Intermediate forms of mineral oils are kerosene, naphtha, benzine, and machine oil. **Animal oils** are obtained from animals. Lard and sperm oils are the animal oils used in shop practice. Turpentine and mineral substitutes for lard and sperm oil are also used for lubrication.

OILS FOR CLEANING

3. Benzine, naphtha, and **turpentine** are used considerably in shops for cleaning purposes; these oils evaporate very rapidly and form highly inflammable vapors. If these vapors are mixed with air in certain proportions, they form explosive mixtures that need but a spark to ignite them. For this reason, great care should be taken not to have a naked light or any fire close to any of these volatile oils.

4. Gasoline, kerosene, or **coal oil,** and **sperm** oil are among the most fluid commercial oils. They will flow into smaller spaces than heavier oils, but have the disadvantage that they lack body; that is, they evaporate quickly, and consequently are of little value as lubricants. They are of great value, however, when used to clean rubbing surfaces, as they will dissolve or thin down almost any heavier oil, and can be employed for cleaning bearings, etc., where it is suspected that the oil channels have become clogged by the gumming of the regular oil. A copious and constant supply of kerosene or mineral sperm oil may be applied to the bearing until the oil comes out clear; it must then be followed immediately by a plentiful application of the heavier oil generally used for lubrication, in order to prevent any cutting of the rubbing surfaces owing to their becoming dry through the rapid evaporation of the light oil. These light oils are also employed for lapping operations.

5. The lighter oils can often be used advantageously for thinning down the heavier oils in order to make a grade suitable for some special purpose. Most of the lighter oils are quite inflammable, and consequently due care must be taken to prevent their ignition.

REDUCING FRICTION BY LUBRICATION

6. Selection of Lubricant.—A lubricant reduces friction by interposing itself in the form of a thin film, which may be considered as being composed of a large number of minute globules, between the rubbing surfaces of the moving bodies.

These globules act as rollers or balls, and convert the sliding friction into a rolling friction to an extent depending on their deformation under the load they carry. The deformation of the globules of the lubricant depends on its consistency, and is greater for a thin lubricant than for a heavy thick one. A thick oil should therefore be selected for heavy pressures, while for light pressures a thin oil may be used. The contact between the rubbing surfaces must also be duly considered in connection with the selection of a lubricant, and one used that is fluid enough to flow in between the surfaces. Thus, in machine tools and fine machinery, the rubbing surfaces are usually fitted very closely to each other, and hence fluid mineral oil having sufficient body to last a reasonable length of time must generally be used.

7. Machine Oil.—Machine oil is a mineral oil having considerable body; it is well adapted to general shop use, and should be thin enough to run freely through the oil holes and oil channels of bearings. Animal oils are generally objectionable as machine oils, for decomposition by age is liable to develop fatty acids that attack most metals; they are also very liable to gum; that is, some of their constituent parts will collect into a sticky mass and close the oil channels of bearings. Machine oil, and all oil intended for lubrication, should be entirely free from grit. By examining a drop of the oil with a strong magnifying glass, the presence of grit may be readily discovered.

8. Cylinder Oil.—A special grade of heavy oil known to the trade as *cylinder oil*, is used for the lubrication of parts subjected to fairly high temperatures, as the valves and pistons of steam engines. It has the property of standing considerable heating without volatilizing or being decomposed. Owing to its heavy body, it is used sometimes for bearings subjected to heavy pressures.

9. Greases.—Oils in the solid state are known as *greases*. They may be either of the mineral or animal variety, although usually of the latter, and are soft and plastic. For very heavy work and relatively low rubbing speeds, one of the many forms of manufactured grease is frequently used. The bearings must then be fitted loosely enough to admit the grease. The great body, which is the characteristic feature of a grease, prevents its being crushed or squeezed out by the weight of the moving parts.

10. Grease of the best quality may be used to advantage in putting ball-bearing work together, when difficulty is experienced in keeping the balls in place while assembling the bearing. The ball races, or seats, are filled with the grease and the balls are then pressed into it. The grease will hold the balls in place while the parts are being put together, and will serve to lubricate them for a long time afterwards. This is a very convenient aid in assembling the ball bearings of the pneumatic drilling machines now so commonly used in large shops.

11. Bearings of shaftings and machines that are subject to great wear and are not always under the eye of the attendant, or easily within reach, and hence are liable to run dry with the ordinary methods of oiling, are thus provided for: Grease cups are screwed, or grease pockets are cast, on places where bearings are liable to heat; these are filled with a grease that will not melt until the bearing becomes warm, when it runs down through the oil holes to the surfaces needing lubrication.

12. Graphite.—*Graphite*, *black lead*, or *plumbago* is a solid mineral substance that forms an excellent lubricant. When ground fine it may be used either dry or mixed with some fluid lubricant or grease to a consistency considered suitable for the work. Graphite is one of the most refractory substances known, and is therefore an invaluable lubricant for bearings subjected to high temperatures. Its lubricating qualities at all temperatures are so good that it forms a very valuable addition to almost any oil.

13. Lubricating Hot Bearings.—A bearing will get hot by reason of friction due to an insufficient or interrupted supply of the lubricant, or because of the journal fitting so close that the lubricant cannot pass between the rubbing surfaces. When a bearing gets hot, it should at once be supplied with a liberal quantity of oil, and the application repeated frequently until the bearing commences to cool. If the bearing becomes so hot that it smokes before the hot bearing is discovered, and it is not advisable to stop the machine, water may be poured down the oil hole on the bearing, or a hose played on the bearing until it is cool. When a hot bearing is discovered, the cap may be slacked back somewhat so as to allow a free circulation of the lubricant. As soon as the bearing is cool, a copious and constant supply of oil, which may have some graphite mixed with it, should be provided and the results noted. If the bearing refuses to keep cool after this, the rubbing surfaces are probably in such a bad condition as to need refitting.

If the hot bearing is rigid—that is, not self-adjusting to the shaft—observe whether one end is hotter than the other; also test the shaft for alinement, as the heating of the bearing may not be caused by a defect in the hot box, but by the bearing next to it getting out of line, thus bringing all the load on one end of the bearing that is heating.

14. Oil Holes and Oil Channels.—Various means are provided to make sure that the lubricant reaches the place or surface it is intended to cover. In the first place, oil holes are drilled through the metal from the high side so that the oil will reach its proper place by gravity. The size of the oil holes should vary with the kind of lubricant that is to be used, drilling small holes in small work and for a fluid lubricant, and larger ones as the heaviness of the oil and the length of the hole increase. Bearings that are not easily reached must have tubing or pipe run to them as directly as possible; this oil piping should be supplied with fittings that allow it to be easily taken down and cleaned.

15. Oil channels should be cut so as to distribute the oil over the whole length of the bearing; also, such other channels should be provided as may be needed to insure an even distribution of the lubricant. To insure thorough lubrication, the oil channels must have a liberal width and must be so deep as not to become filled too rapidly with the impurities some lubricants contain. Furthermore, the direction in which the oil channels run from the point of supply (the bottom of the oil hole) should be the same as the direction of rotation of the journal, in order that the latter may tend to draw in the oil rather than to repel it. A lubricant will not flow up hill any more than any other liquid; hence, it should always be applied at the highest point permitted by circumstances.

LUBRICATION OF CUTTING TOOLS

16. The cutting speed of a tool may often be considerably increased by the application of some kind of lubricant, such as oil or water. When oil is used, it reduces the friction between the shaving and the face of the tool, and thus reduces the heating. If a sufficient quantity is used, it also carries off much of the heat generated by the cutting operation and keeps the tool from getting as hot as it otherwise would; consequently, the cutting speed may be increased without overheating the cutting edge.

Cast iron is usually worked dry. The dirt caused by mixing fine cast-iron turnings with oil or water on the machine more than overbalances the possible increase in cutting speed. Furthermore, it is difficult to take a light cut on cast iron when it is oily. The oil soaks into the surface of the iron for a short distance and seems to form a skin that is not easily broken. If the cut is deeper than a finishing cut, the oil on the surface will not impede the cutting. Brass, copper, and Babbitt metal are generally cut without a lubricant, although work composed of these metals is sometimes flooded with lard oil in automatic screw-machine work, thereby reducing friction and prolonging the life of the tools.

17. Lubricants Used in Cutting Steel and Wrought Iron.—The best lubricants to use when cutting steel or wrought iron are the best grades of lard oil and sperm oil. Lard oil is a much heavier oil than sperm oil and is generally used for this class of work. Sperm oil is employed chiefly for very light work. It is a good oil to mix with the abrasive when lapping. One of these oils should be used for all thread-cutting or reaming operations. For turning shafts, soda water is used, or in some cases a mixture of soft soap and water. Soda water is an excellent medium for absorbing heat; the soda also keeps the water from rusting the machines or the work. Soft soap dissolved in water is used in some shops instead of soda water and possesses some lubricating quality. When a finishing cut is taken on soft iron or steel with a keen tool, and a supply of water is kept on the tool, a very bright smooth surface is produced. Such a cut is called a *water cut;* some kinds of work are thus finished with sufficient smoothness to make polishing unnecessary.

18. Cheap Lubricant for Tools.—For some classes of work, a cheap and satisfactory lubricant may be made by combining oil with other ingredients. There are many such mixtures in use in which an oil is first thinned down by mixing it with a cheap soda water, and then adding some ingredient that will give body to the lubricant—that is, thicken it enough to make it somewhat adhesive. A good lubricant may be made by mixing together $\frac{1}{4}$ pound of sal soda, $\frac{1}{2}$ pint of lard oil, $\frac{1}{2}$ pint of soft soap, and enough water to make 10 quarts, boiling $\frac{1}{2}$ hour, and stirring well. When cool, it is ready for use. This mixture can easily be handled by a pump, and is quite satisfactory for general use.

19. Lubricants for Cutting Babbitt Metal.—In most cases, Babbitt metal may be worked dry; but when bushings of this material are being bored in the lathe or when boxes are machined in position, a lubricant is often found necessary. This is especially true of bushings that are being bored in the lathe, since the chip has a tendency to wind around the boring tool and form a compact ball. Boxes that have been bored and are to be reamed will sometimes be scored or roughened in the reaming if the work is done dry. Lard oil is sometimes used in working Babbitt metal, but a copious supply of kerosene oil will give far better results than any other lubricant.

20. Lubricants for Drilling Rawhide.—If a twist drill must be used to drill rawhide, drilling in general will be found a trying and tedious job, owing to a tendency to clog the flutes of the drill when the drilling is done dry. If a cake of ordinary laundry soap is held against the drill every little while, however, no trouble will be experienced. The drill should be run quite fast for drilling rawhide.

21. Lubricants for Fitting.—Fitters who are working on cast-iron work must sometimes use a lubricant, other than the marking material, when they are rubbing two parts together in order to obtain bearing marks. Oil will prevent the seizing and cutting of the surfaces; but it will leave no bearing marks, and, besides, it will interfere with the scraping. Turpentine may be used freely on such work, however, and will prove beneficial.

22. Pipe System of Lubrication.—When a large number of machine tools are used on work where constant lubrication is necessary, the tank containing the lubricant is often placed in some warm out-of-the-way locality, as in the boiler room. A system of piping having branch pipes leading to the different machines may then be laid through the shops. A force pump should have its suction pipe connected to the tank and its discharge pipe connected with the pipe system. All the drippings may be automatically returned to another tank near the first through a separate pipe system so arranged that they will flow back by gravity. By grouping together all the machines requiring lubrication, the piping system can be made relatively inexpensive. The placing of the tank in the boiler room is especially convenient when mixtures require boiling, since a steam coil can then be placed in the tank at small expense.

Sometimes, the supply tank is located at such a height that the lubricant will flow to the machines by gravity, and the pump is used to raise it from the lower to the upper tank. The oil should always pass through a strainer or a filter before being used again.

23. Oil Separator.—Shops in which a great deal of screw-machine work, milling, and tapping of wrought iron or steel is done, use correspondingly large quantities of oil to lubricate the cutting tools. This oil becomes mixed with the cuttings or chips from the work, and while most of the oil can be drained off, a large amount adheres firmly to the chips and is usually thrown away. Much of it may be saved by collecting the oily chips, and running them through a separator. This separator consists of a circular tank that is open at the top and is provided with a cock in the bottom, to allow the extracted oil to be drained off. A vertical spindle passing up through the center of the tank carries a strong conical steel pan provided with an equally strong cover that is held on, when in use, by a locknut. The edge of the pan has small openings for the escape of the oil. A pulley is provided on the lower end of the spindle to drive the extracting pan, and is belted to an overhead countershaft.

The pan is filled with the oily chips, the cover securely fastened, and the machine started slowly and allowed to come up to its full speed, which should be about 7,000 feet per minute at the periphery of the pan. The oil is thrown from the chips by the centrifugal force and finds its way out through the small openings in the top edge of the pan. As the oil flies from the pan it is caught by the wall of the tank and flows down to the oil well.

24. Oil Filter.—Oil that has been used a number of times is liable to be filled with very fine chips that separators will fail to remove. Such oil may be filtered through an oil filter; in its absence blotting paper will be a fair substitute. Some of the heavier particles of metal in the oil can be gotten rid of by letting the oil stand in a quiet place for some time, when the heavy foreign matter will settle to the bottom of the vessel. The clear oil may then be poured off into another vessel. This settling process will not clean the oil as effectually as an oil filter. Oil cleaned by the settling process should never be used for lubricating bearings, but only for the cutting tools. An oil filter or settling tank will not work well if kept in a cold place.

MISCELLANEOUS DEVICES

25. Boxes, Pans, and Trays.—All shops and manufacturing establishments doing small work have more or less trouble in moving small parts from place to place. This is generally done by using such boxes, kegs, and barrels as happen to be at hand. These soon become dirty or are broken, and must then be replaced.

An excellent substitute for these makeshift devices is found in the metallic articles illustrated in Fig. 1. The one shown in (*a*) is a steel box that can be used for many shop purposes. The pressed-steel pan illustrated in (*b*) may be employed instead of the box, and has the advantage that it will hold water or oil. These pans, when not in use, may be stacked up, so as to occupy very little space. They are commonly called **tote boxes.**

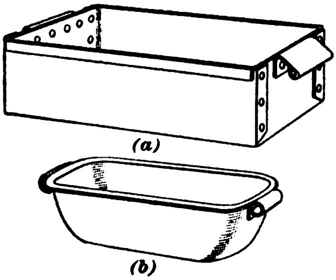


FIG. 1

26. Another useful and cleanly device is the **tray rack,** illustrated in Fig. 2. It consists of three iron trays, the upper one of which carries a drawer. It is especially useful where a number of operations have to be performed on pieces by different machines. The trays may be used by the machine-tool man to hold both the tools and work, while the drawer may contain his individual tools. Casters may be added to this rack, so that it may be moved from place to place with ease.

27. Waste Cans.—All waste or greasy material should be put in sheet-iron cans or barrels located at convenient points throughout the shop. These cans should be made of heavy galvanized iron or steel, and should have legs to keep their bottoms 2 or 3 inches above the floor. They should be riveted together, instead of being soldered, so that if the material in them should catch fire, they will not come apart and set fire to the building. A good tight-fitting cover should always be kept on the can, so that if fire does start in the waste it will be smothered before gaining much headway. These cans should be taken out and emptied at stated times.

In some shops the dirty waste is washed and used again. The cleaning is done by putting the waste into a tank of water with soda, cheap soap, or some washing compound, and boiling it for a few hours by the use of either live or exhaust steam that enters the tank through a suitably arranged pipe.

28. Compressed-Air Hose.—In shops having compressed-air service, a $\frac{1}{2}$-inch rubber hose with a $\frac{3}{8}$-inch nozzle attached to it forms a convenient means of cleaning many pieces of work that are so shaped that it is difficult to reach every part with the hand. The blast is simply turned on the piece to be cleaned, and most, if not all, of the loose dirt is blown off. An air hose will be found very useful in the tool room for cleaning the shelves, racks, and drawers, and may even be used advantageously for rapid cleaning of some tools. The disadvantage of the air blast lies in the fact that it scatters the dirt all over the vicinity.

FIG. 2

29. Storing Devices.—Various methods are followed in caring for shop tools. Sometimes the tools are left where used until they are wanted for another job, when they are hunted for until found, and are then cleaned and made ready for use. This method is probably the worst that could possibly be devised, and is a direct evidence of mismanagement. The modern and proper plan is to require all tools to be cleaned by the user and returned to such a place as may be designated for their storage and care.

30. Tool rooms are built in most shops for the storage and care of all the tools used in the place; or, if the shop is divided into departments, each department may have its own tool room and a man to care for it, who, in addition, does such other work as he may have time for. The tool room may be used only as a storeroom for tools, or it may be equipped with such a varied selection of machines that any tool or appliance needed on the work may be made there, and tools and light machinery may be repaired. Large shops usually have, in addition to the tool room, such storerooms or vaults as may be needed for the storage of any large and valuable jigs, tools, or fixtures that are seldom needed, but require protection from fire. Tools should be so kept that they may be taken out and returned in the least time, and should also, while in their places, be well protected from dust and rust.

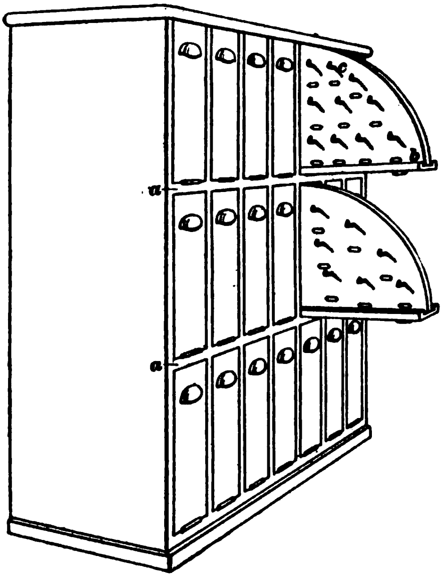


FIG. 3

31. Drawers are extensively used for holding tools, and for many purposes they answer admirably. They are, however, very liable to be overloaded, which treatment soon racks them to pieces. This may be avoided by making them extra heavy, or providing rollers for them to run on. They may also be easily handled if the sliding surfaces are of hardwood or are metal-faced, and the contact surfaces are greased occasionally with a good lubricating grease. Drawers are used to the best advantage for tools which are seldom needed but require protection from injury and dirt.

32. Shelves or pigeon-holes furnish the most ready means of keeping tools that are much used. These should be as shallow as possible in order that the tools may not be pushed in out of sight, and that they may be easily brushed out, or blown out if an air hose is used for cleaning. Cupboards containing numerous shelves are useful for special tools used less frequently than standard ones, since the cupboard doors protect them from dirt and the atmosphere.

33. The walls of tool storerooms are often covered with boards, which are painted and have hardwood pegs put into them on which to hang milling cutters and similar tools; in some cases, nails are used instead of the wooden pegs. It is better to keep the cutters in the form of cabinet shown in Fig. 3, which cabinet has a series of shelves *a* to which boards *b* are hinged. These boards are provided with hooks *c* on which to hang the cutters. This cabinet provides a clean, convenient, and space-economizing place for a large number of milling cutters and gear cutters.

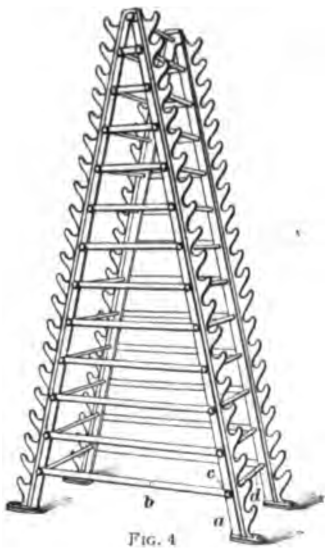


FIG. 4

34. Racks of various kinds furnish a convenient and clean place for keeping a large class of tools, such as pipe stocks, wrenches, long taps, reamers, drills, boring bars, cutter bars, sockets, and other similar long tools, in such a manner that they are easily put away or gotten out, and are kept clean when in their places. A rack of this description is shown in Fig. 4. It consists of four uprights *a* that are braced by wrought-iron tie-bars *b*. The tie-bars are fastened to the uprights by long bolts *c*, which pass through them at each end. The bolts are surrounded by distance pieces *d* made of iron pipe, which keep the uprights separated.

35. Racks that are constructed as shown in Fig. 4 may be made 7 feet high and 3½ feet wide at the base, with the uprights spaced at such distances as will accommodate the shortest tools that may be kept on them. Light and frequently used tools are piled on the arms, while less used and heavier tools are placed on the cross-pieces *b*. These racks may stand against the wall, but are preferably placed on the floor where they can be reached from all sides. Racks of special design are usually provided for such tools as ratchets, tapping attachments, air drills, and other portable drilling and grinding fixtures.

Boxes are sometimes used for storing tools, and when so used they should be plainly marked; a convenient record should also be kept of their exact contents.

36. Rams.—When taking old machinery apart—as, for instance, when trying to remove an old shaft from a wheel or crank—the heaviest blow possible must sometimes be struck. The heavy blow carries the object struck before it, while lighter blows will simply upset the end of the piece and thus rivet it into place. When heavy sledge hammers are used on light work, the surfaces hammered should be protected by a piece of Babbitt metal or copper held or laid on them.

Where heavier blows are required than can be struck with a sledge, a ram is used. This device is a long bar of iron suspended at its center of gravity, in order that it may hang in a horizontal position, and hung in front of the piece to be rammed. The rope suspending the ram is made fast to an overhead point, after which the operators draw the bar, or ram, backwards as far as possible, and then run with it toward the piece to be struck. The ram is often used when a hydraulic press is not available or suitable for the work. Care should be taken in using the ram not to upset the face of the part that is being rammed, as this will tighten the parts in their places. Several men are required to operate a heavy ram, which is expensive and should not be resorted to if a press can be used.

37. Soda Kettle.—All shops have more or less work that must be cleaned so as to be free from grease. This is often a troublesome task, involving the expenditure of time and energy. The greater the irregularity of the pieces, the more trouble it is to clean them.

A very convenient method of quickly and easily cleaning small parts of machines, tools, or machined parts is to wash them in hot soda water. The most convenient receptacle for this mixture is known in the shop as a soda kettle, which is often a shop-made affair, but embodies the main features of the kettle illustrated in Fig. 5. This soda kettle consists of a cast-iron kettle *a* containing a coil of steam pipe for heating the soda water. Live steam enters the coil when the globe valve *b* is opened, and the exhaust steam leaves through the pipe *c*, which is provided with a globe valve. A by-pass pipe *d* having a globe valve connects the exhaust pipe to the kettle; if the globe valve in the exhaust pipe is closed and the valve in the by-pass pipe is opened, the pressure of the live steam will force the water of condensation in the bottom of the coil into the kettle. By opening the drain cock *e* the kettle may be emptied.

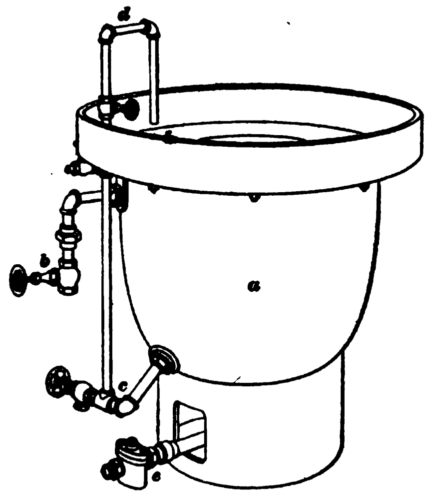


Fig. 5

38. In use, the kettle is filled about three-fourths full of clean water to which is added about $\frac{1}{20}$ its volume of sal soda; the mixture is then heated as hot as the steam will heat it. A wire basket, or an iron pail, or bucket, having the bottom punched full of holes, is provided for holding small pieces while dipping them into the soda mixture. Suitable hooks made of small iron rod may be used to dip single pieces into the kettle.

A pair of pick-up tongs and one or two hooks should be kept near the kettle, since pieces are sometimes dropped into it and must be fished out. Work covered with soft grease or oil and chips is cleaned by putting it into the basket, which is then dipped into the hot liquid. Work covered with oil that has dried on it often has to be soaked in the solution for some time, and a part of the dried oil then has to be scraped off; the work is now given a further soaking, which is generally sufficient to remove the rest of the dried oil. Work cleaned in hot soda water dries quickly and the soda water will not rust it.

GENERAL SHOP PROCESSES

BELTING AND SHAFTING

PULLEY SIDE OF BELT

39. Oak-tanned, fulled leather belting is the kind commonly used in shop practice. It is made of ox hide by a tanning process. The side of the hide that was next to the flesh of the animal is known as the *flesh side*, and the other side as the *hair*, or *grain*, *side*. The grain side of the belt is harder and weaker and will crack easier than the flesh side. It will, however, stand more crimping than the flesh side, and the grain side is run next to the pulleys. By running the grain side next to the pulleys the tendency is to cramp, or compress, it as it passes over the pulley, while if it ran on the outside, the tendency would be for it to stretch and crack. Moreover, as the flesh side is the stronger, the life of the belt will be longer if the wear comes on the weaker or grain side.

40. Belts may be *single*, *double*, *triple*, or *quadruple*. A single belt consists of a single ply of leather, a double belt of two plies, etc. The two plies of a double belt are placed with the flesh sides together, and consequently either side of a double belt may be run against the pulley. Similarly, the triple and quadruple belts are so arranged that the grain sides of the outer plies are exposed and they may be run with either side in contact with the pulleys. When crossed belts are used, care must be taken to put on the belt so that the grain side will be in contact with both pulleys, it being possible to put a crossed belt on in such a way that the grain side will be in contact with one pulley and the flesh side with the other.

FASTENING AND CARE OF BELTING

41. Cemented Belt Splices.—The cemented splice, Fig. 6 (*a*), is the most durable of all belt-fastening methods.

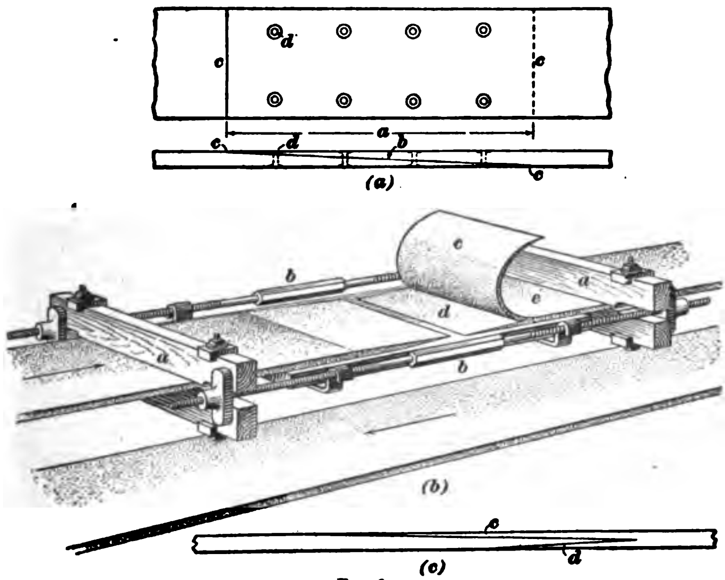


Fig. 6

As belts usually give out at the point where they are fastened together, it is thought by some shop men that all belts should be joined by the cemented splice. Unless, however, some provision is made for adjusting the center distance between pulleys as the belt stretches, at least one other form of splice must be used in the belt.

The length *a* of the cemented splice varies from about 9 to 18 inches. Belts 9 inches in width and narrower have splices 9 inches long, and belts greater than 9 inches in width have splices as long as the belt is wide. However, no splice is made longer than 18 inches.

The belt is first cut off the required length, making the proper allowance for the length of splice, after which one end of the belt is tapered, as shown at *b*, using a small block plane. The ends are tapered in the same direction as the corresponding parts of other splices, if there are any in the belt. If there are no other splices in the belt, the end is tapered in such a way that the feather edge *c* of the pulley side of belt will point away from the pulley it is approaching and the grain side of the belt will be in contact with the pulleys. The position of the lap is marked off on the other end of the belt and this end is cut similarly. Both ends are now scraped smooth with a piece of glass, called a *slicker*.

42. A thin coating of belt glue, known as *sizing*, is next applied to the surfaces to be cemented, after which the cement is put on rapidly, and the laps are immediately brought together and hammered all over with a shoemaker's hammer. Rubber-faced shoes—that is, flat boards faced with rubber—are now brought against the splices, hand clamps are applied to them, and the splice is squeezed hard between the shoes.

The splice is allowed to set for from 15 minutes to 1 hour, when the clamps are removed, and shoemaker's nails are driven through the splice about ½ inch apart. The cement used should be rather hot. It may be made by mixing Le Page's liquid fish glue and Russian liquid isinglass in the proportions of 2 to 1, by measure. If the cementing is done in a cold room, burning paper may be passed back and forth over the laps to warm them before applying the cement, which should be about as thick as ordinary glue. Instead of shoemaker's nails, iron or copper rivets *d*, preferably iron, are sometimes used to reinforce the splice.

43. Sometimes a belt must be cemented together in position on the pulleys. In that case a belt tightener, Fig. 6 (*b*), must be used to hold the belt in position while making the splice. The clamps *a* of the tightener are secured to the belt, which is tightened to the desired tension by means of the screws *b*. The belt is located in the center of the clamps, which are square with the belt. The belt is drawn a little tighter than a running tension. The joint is spliced and cemented in the usual way, or one end may be spliced V-shaped as shown at *c*, to receive the other end of the belt which is wedged into the V. This is an alternate method for making a cemented splice. A little longer time is required to complete the work; but it has the advantage over the preceding method in that when the feather edge *d* is pointing away from the pulley to which it is advancing, the feather edge *c* on the outside of the belt is not running against the air, which would tend to force the splice open. The finished splice is shown in (*c*).

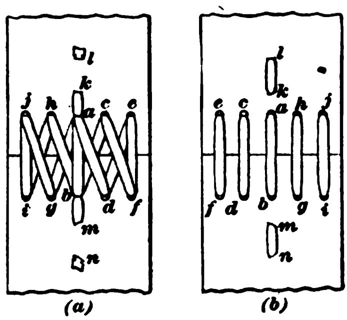
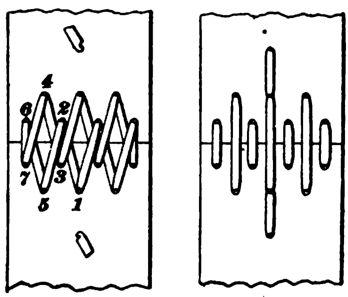


FIG. 7

44. Laced Joint.—There are many ways of making a laced joint. Whichever method is used, the usual practice is to place all holes at least from ½ to 1 inch from the edge and end of the belts, to space the holes uniformly, about 1 inch apart, and to lace the joint so that the pull on the lace will be in line with the center line of the belt. The lace is cut into strips of the necessary length and varying in width from ¼ inch up, depending on the size and the form of the joint. The holes are made in the belt with a lace punch, and the lace is pushed through the hole with an awl or pulled through with a pair of pincers. The smoother side of the joint is generally run next to the pulley and the stronger side on the outside, the greatest force acting on this part of the joint. In all cases it is essential that the ends of the belt be squared up with a try square before lacing.

45. The outside of a standard form of laced joint is shown in Fig. 7 (*a*), and the pulley side of the same joint in (*b*). An odd number of holes is required in each end of the belt for this joint, the total number depending on the width of the belt to be laced. The holes on each end must be exactly opposite the corresponding holes in the other end. To lace the joint, the lacing is put through the middle holes *a* and *b* from the pulley side of the belt and drawn taut against the belt in such a manner that the extending ends will be equal. The lower end of the lace in (*a*) is now passed through *c*, *d*, *e*, *f*, *e* again, *f*, and *c*. The other end of the lace is next passed through *g*, *h*, *i*, *j*, *i* again, *j*, *g*, *h*, *b*, and *a*. The first end of the lace is then passed through *d*, *a*, and *b*. The lacing is now adjusted until the ends of the belt butt tightly against each other and all the looseness in the lace is taken up, after which the lacing is passed through the holes *k*, *l*, *m*, and *n* as shown. The ends are cut off near the belt, and then cut slightly close to the belt, which makes a notch, so that the part beyond the belt will catch and be held from moving farther.



(*a*) (*b*)
FIG. 8

46. In Fig. 8 is illustrated a method of lacing, in which double rows of holes are used, (*a*) being the outside and (*b*) the pulley side of the belt. The lacing for the left side is begun at *1*, and continues through *2*, *3*, *4*, *5*, *6*, *7*, *4*, *5*, etc. A 6-inch belt should have seven holes, four in the row nearest the end; and a 10-inch belt should have nine holes, five in the row nearest the end.

47. Wire-Laced Joint.—Belts are frequently joined with wire lacing. A joint made in this way is shown in Fig. 9 (*a*). It consists of coiled wire *a* inserted in each end of the belt and a rawhide pin *b* that holds the wires together. The wire is helical, having a pitch of about $\frac{1}{6}$ inch, is flattened to a thickness not greater than the thickness of the belt, and has its ends bent back into the belt, so as not to catch and tear the hands in shifting. The lacing is wound into the belt by a special machine.

The wire-laced joint is strong and flexible, though not so strong as the cemented splice; also, it has the advantage that it may be quickly connected and disconnected, the insertion or removal of the rawhide pin being all that is required to make or break the joint.

New belts stretch considerably. For this reason, the belt may be cut shorter than the desired length by about 4 inches and a 4-inch filler *c* inserted in this space. When the belt stretches, a 2-inch filler is substituted and then a 1-inch filler, until the belt is stretched its nominal amount and the main ends come together. The fillers may be kept in the stock room and used repeatedly.

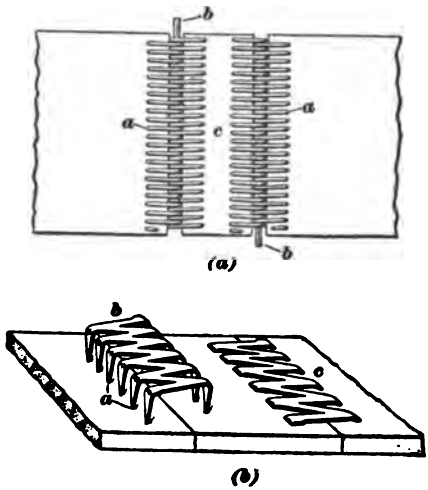


Fig. 9

48. Metal Belt Fasteners.—Many types of metal belt fasteners are in use. Their chief advantage lies in the fact that they can be quickly attached to a belt, as well as quickly removed from it. In using any type of metal fastener, the abutting ends of the belt should be carefully squared before the fasteners are applied. In Fig. 9 (*b*) is shown one form of metal belt fastener. When applying this fastener, no holes are needed in the belt, as the prongs, or points, *a* are so shaped that they will cut their way through the belt material. The fastener is shown ready to apply at *b*, and the finished joint is shown at *c*. After driving the fasteners through the belt, the prongs are clinched on the opposite, or pulley, side of the belt.

49. Tension of Belting.—Good oak-tanned belting about $\frac{3}{16}$ inch thick should be tightened to a tension of about 40 to 50 pounds per inch of width. The tension to which belts of different thicknesses should be tightened may be found by dividing the thickness of the belt by $\frac{3}{16}$ and multiplying the result by 40 or 50, as desired. If belts $\frac{3}{16}$ inch thick are tightened to a tension of 50 pounds, a greater load may be carried than could be done if tightened to a tension of 40 pounds; but the wear and tear on the belt will also be greater and the belt will not last so long.

Tension scales by the use of which belts can be set to the desired tension are to be had. Belts are most commonly fastened without the use of tension scales. In this case, care should be taken not to have the belt too tight, as serious wear on the pulley bearings and injury to the belt might ensue. On the other hand, the belt should be tight enough to carry its normal load. A good rule to follow is to tighten the belt so that it will give slightly under pressure from the hand.

50. Length and Retightening of Belts.—When putting on new belting, the length to which the belt is to be cut may be found by measuring the distance around the pulleys with a tape and subtracting from this result $\frac{1}{8}$ inch for every foot of length. New belts must usually be tightened every day for a time, after which the intervals may be gradually increased to 2 days a week, and finally to from 3 to 6 months, depending on the severity of the service. The time between retightening may be greatly lengthened by throwing the belt off its pulleys when not in use.

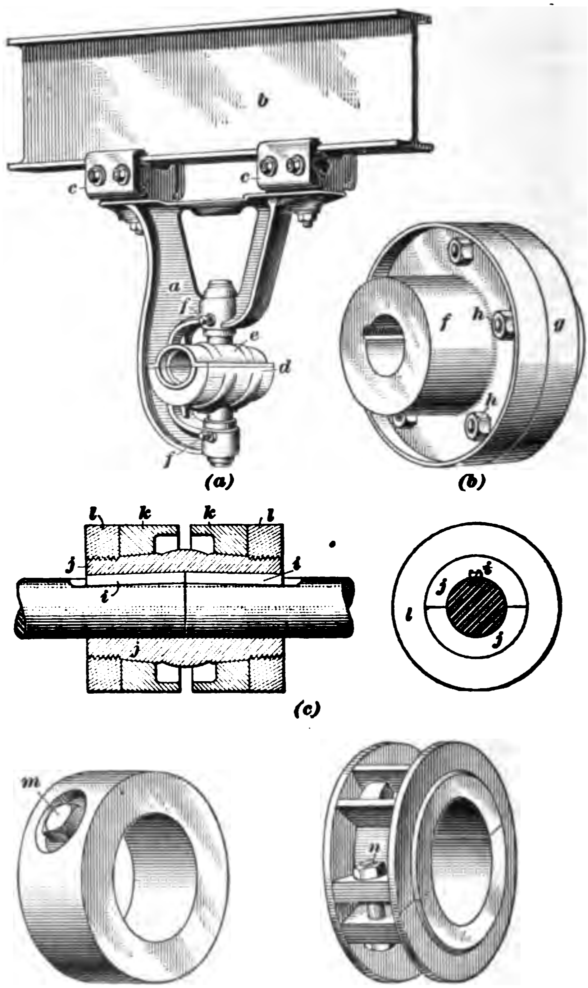
51. Greasing of Belts.—A new belt contains a certain amount of oil that makes it pliable and gives it a clean, pulling surface. After the belt is in use for some time, this oil becomes spent and to keep the belt in good working condition some form of oil or grease must be supplied. A very serviceable grease is a mixture of 2 parts, by weight, of pure beef tallow and 1 pint of cod-liver oil. In preparing the grease, the tallow is melted and allowed to cool until one can put his finger in it without being burned; then the cod-liver oil is added and the mixture stirred until cold. A number of good belt greases under various names are on the market. Belts should be scraped clean and greased about every 6 months.

52. Dressing and Cleaning of Belts.—From various causes, belts often slip when in use. In these cases, resin is sometimes applied to the surface of the belt; but it should never be used, as it ruins the belt, which in a short time will slip worse than before. Belt dressings should be applied sparingly, too much being often worse than none at all. They are not preservatives, and should be used only in emergencies.

53. Extreme care should be taken to prevent belting from becoming saturated with oil, which is liable to work its way to the belting from the shafting and bearings. When belts become oil-soaked, they should be cleaned by wiping with waste and scraping off the oil, after which they may be run through a pair of rollers under pressure. The rollers will squeeze out the oil, which should be absorbed by powdered chalk. If no rollers are available, the belt may be packed in sawdust, for from 2 to 7 days, after which it may be cleaned and used.

LINING OF SHAFTING

54. Line shafting may be either suspended from roofs, ceilings, walls, or columns, or supported from floors; by far the larger portion is suspended from roofs and ceilings. The methods of erecting are, in general, the same for all; but the details usually vary more or less with each installation and the care in carrying out these details determines the subsequent smooth running of the shafting. Aside from the structural work, the various parts entering into a section of shafting are *hangers*, *bearings* and *boxes*, *shafting*, *couplings*, *collars*, *girder clamps*, *pulleys*, and *belt-shifting devices*. Some of these parts are shown in Fig. 10. In (*a*) is shown a hanger *a* attached to the I beam *b* by means of the girder clamps *c*. The bearing box *d* and cover *e* are shown in position. To adjust the height of the bearing, the screws *f* are loosened; the box and cover are then raised or lowered as desired and secured in position by tightening the screws *f*. Hangers are, as a rule, attached to I beams only when used to support main shafts, owing to the rather long distance between them. When employed to support countershafts, secondary beams are generally connected to adjoining I beams and the hangers are bolted to these in any convenient manner.



(d) FIG. 10 (e)

55. Couplings are used to join sections of shafting. The *flanged coupling* is shown in Fig. 10 (*b*). The part *f* of this coupling is keyed to the end of one shaft, the part *g* to the other, and both parts are clamped together in position by means of the bolts and nuts *h*. The *compression coupling* is shown in (*c*). It consists of two keys *i*, two half clamps *j*, two tapered rings *k*, and two threaded rings *l*. When setting this coupling in position, the keys are first fitted to the shafting, the half clamps are then placed over them, and the taper rings are slid over the clamps. The threaded rings are next screwed on the threaded ends of the half clamps, forcing the tapered rings home and completing the connection.

56. Collars are put on the shafting to prevent it from working endwise in the bearings. A *solid collar* is shown at Fig. 10 (*d*). It is secured to the shaft by tightening the set-screw *m*. A *split collar* is illustrated at (*e*). It is made in two pieces and can be put on any part of the shaft without slipping it over the end. It is secured in position by means of the bolts and nuts *n*. The collars may be separated from the bearing boxes by a fiber washer.

57. Erection of Main Shafts.—A method of supporting shafting from wooden girders is shown in Fig. 11. This is the method that is used to support shafting in the older types of buildings and in new buildings where a low first cost is of prime importance. In modern structures, however, the hangers are supported by metal girders.

58. A straight line *a b*, Fig. 11, is first made along the ceiling of the building or on the bottom of the horizontal girders *c* by stretching a chalked line tightly and then snapping it on the ceiling or girders. This line is stretched parallel to the nearest wall or to the nearest row of vertical girders and as nearly as possible at the distance from the wall the shaft is to be located. The position of the line may be located readily by measuring off at right angles to the wall or girders an amount at the beginning and end of the section the shaft is to occupy equal to the distance the shaft is to be from the wall or girders. The stringers *d* are now bolted to the horizontal girders *c*. These stringers are located on opposite sides of, and parallel to, the line *a b*, and are spaced a distance apart sufficient to accommodate the hangers.

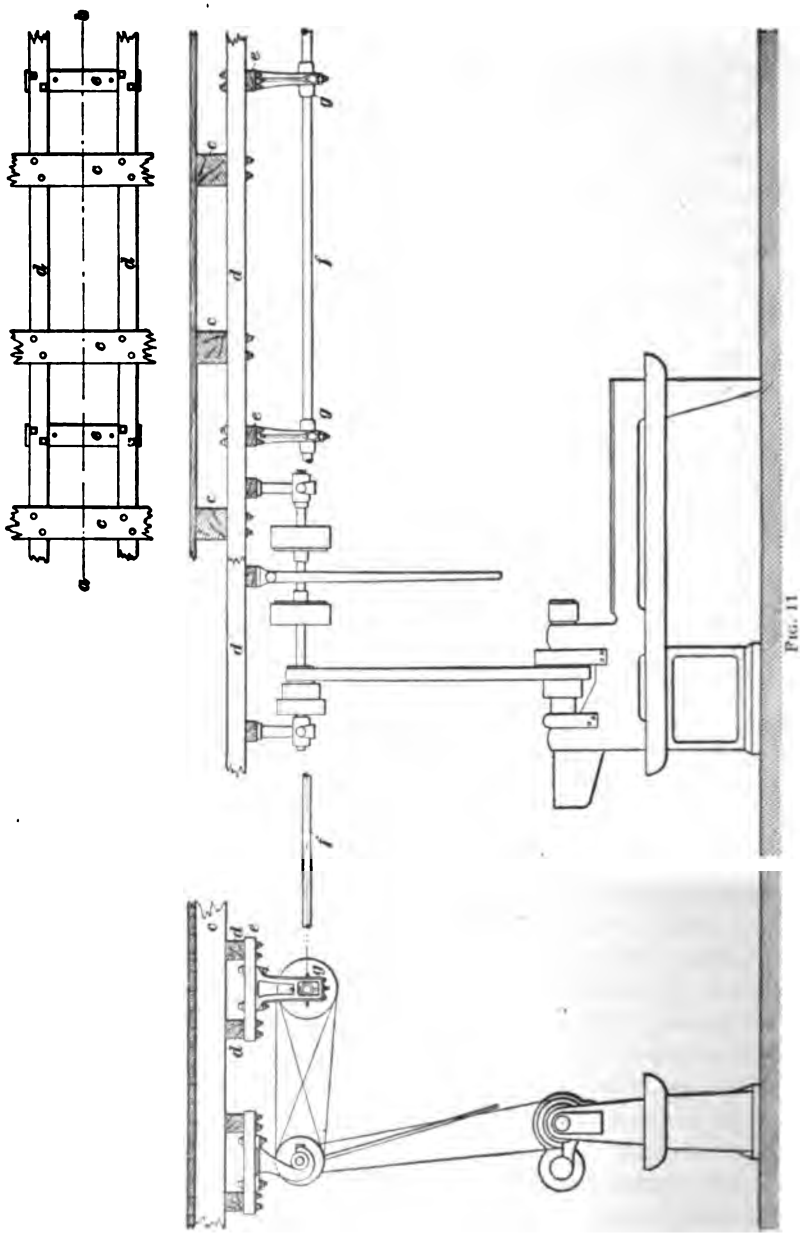


FIG. 11

The footing pieces *e* for the support of the hangers are next bolted to the stringers, the distance between them for ordinary work being from 8 to 10 feet. The hangers are bolted quite loosely to the footing pieces. The shafting *f*, which is usually in 24-foot lengths, is then lifted into the hangers. The bearing boxes *g* are now adjusted and, if necessary, the hangers are moved horizontally either way from their original position to accommodate the shafting properly, after which the couplings on adjoining lengths of shafting are connected up, thus forming a continuous piece. A line is now stretched parallel to two lengths of the shafting, in a horizontal plane with it, and about 12 inches from it. This line is secured to pins or nails set near the ends of short sticks, which are nailed to the ends of the footing pieces. The line is so set that it is the same distance from the extreme ends of two adjoining pieces of shafting. The intermediate portion of these parts of shafting is then made parallel to this line by adjustment of the intermediate hanger. This operation is repeated for every length of shafting, each hanger being bolted tightly in position after adjustment.

When all the shafting is in place, the various lengths are leveled by placing a spirit level along the top of each length and moving the adjusting screws on the bearing boxes up or down, as may be necessary, locking the screws into position when finally adjusted. Precaution must be taken to place the required collars, if the solid or flanged type of coupling is used, or the rings, if the compression type of coupling is employed, on the shafting before erecting.

59. Erection of Countershafts.—The countershaft is erected in practically the same way as the main-line shafting. The machine is, however, first set in its proper position on the floor and the countershaft is placed temporarily on the floor directly under the position it is to occupy overhead. The machine and countershaft are now shifted until a position is found for the countershaft in which there will be no interference overhead by other shafting. A plumb-line is then dropped from the roof or ceiling to the machine and the point on the ceiling from which it is dropped is marked. From this point the erection of the countershaft is then proceeded with. The countershaft is erected parallel to the main-line shaft in much the same manner as the latter was originally made parallel with the wall of the building; and the machine is plumbed from the main-line shaft, or it is alined by sighting along some suitable objects such as a line of columns, other machines in the vicinity, or in various other ways.

Whether the belting connecting the machine with the countershaft is exactly vertical, at an angle, or with a twist, depends almost entirely on the construction of the machine tool, it being largely a matter of the belting clearing all parts of the machine.

BABBITTING

PRINCIPLES, PROCESSES, AND DEVICES

60. Babbitting, as is generally understood, is the process of lining a wearing surface or journal, for the purpose of reducing friction, with **Babbitt,** which is a soft, metal alloy containing varying quantities of lead, copper, tin, zinc, and antimony. Babbitts, or Babbitt metals, are divided into two classes, for heavy and light service, respectively, tin predominating in the former and lead in the latter.

The babbitting process is also used in the keying, or holding, of one piece of apparatus to another, the balancing of rotating parts, the plugging of holes, etc. The lining of journal bearings, however, is the most common application of the process of babbitting and, while exceedingly simple in itself, certain steps must be carefully followed to insure success.

61. Composition of Babbitt Metal.—There are many combinations of Babbitt, each having some particular feature to recommend it. The prices of Babbitts vary greatly, depending on its percentages of tin and copper, these two metals being the highest in value of its constituents. It is therefore well to base the selection of a Babbitt on a knowledge of the purpose for which it is intended, and it is usually the part of wisdom to purchase the Babbitt already made instead of attempting to make this metal, since there are many points in its manufacture requiring the knowledge of experts.

62. In the best grades of Babbitt, tin is the principal constituent, antimony is next in importance, and copper follows. Antimony gives hardness to the Babbitt, while tin gives it its antifriction qualities. In the cheaper grades of Babbitt, lead is used. Lead is unsurpassed for antifriction qualities, but is too soft to be used alone; for light, fast-running journals it may be employed when alloyed with antimony or antimony and tin.

For hard service, the following composition may be used: Tin 90 per cent., copper 2 per cent., and antimony 8 per cent., by weight.

For light service, the following composition may be used: Lead $83\frac{1}{3}$ per cent., tin $8\frac{1}{3}$ per cent., and antimony $8\frac{1}{3}$ per cent., by weight. The first alloy should be poured at about 800° F. and the second at 900° F., but where both are in constant use they are poured at about 860° F. The melted lead base Babbitt is sluggish, sets quickly, and must be poured at a high temperature.

63. Melting Babbitt Metal.—Care must be taken in melting Babbitt to heat it slowly. The surface of the melting metal is covered with powdered charcoal, to prevent oxidation of the tin and antimony, and the Babbitt is stirred with a dry pine stick, which is a guide to the temperature of the Babbitt, since the metal must not become so hot as to char the pine. An occasional job can be babbitted by metal melted in an iron ladle over a blacksmith's fire, but in larger establishments the steady employment of several men with special fires and appliances is often required. Babbitt metal should never be melted over a blacksmith's fire that is to be used for welding, for if a little of it gets into the fire, it is likely to spoil the welds.

64. A fire should be set apart for melting Babbitt. A portable forge is especially useful for this work, as it can be moved to the place where the work is to be done. Coke is preferable to coal for melting Babbitt, as it makes less smoke. Gas is also an excellent fuel for this purpose. A form of Babbitt pot heated by gas is shown in Fig. 12. The gas passes through the pipe *a* to a burner, the crucible *b* receives the Babbitt to be melted, and the gas is lighted through the door *c*.

65. The ladles may be either of wrought iron, steel, or cast iron, and may be of either the plain or the self-skimming type. They should be discarded when worn thin at the bottom, as metal may be lost by an unnoticed leak. Two forms of self-skimming ladles are shown in Fig. 13. The one shown at (*a*) has the spout located at the bottom of the ladle, while that shown

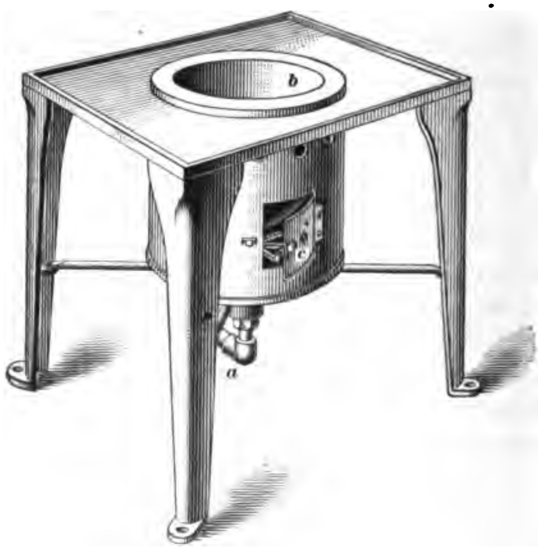
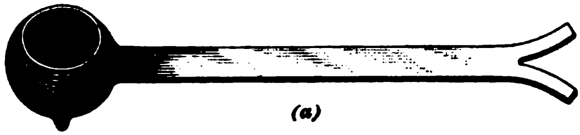


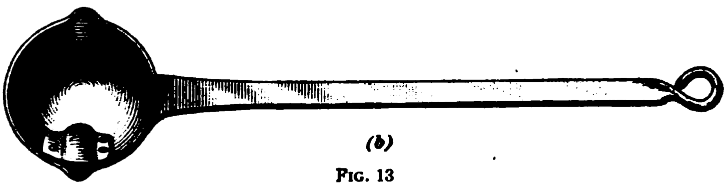
FIG. 12

in (*b*) is provided with a covered spout. These spout constructions tend not only to keep out the dross when the ladles are being filled by immersion in the Babbitt pot, but also to hold back within themselves any dross that may have been collected in the filling and to prevent it from entering the casting during the pouring. By *dross* is meant the oxides that accumulate on the surface of the molten metal, owing to its contact with the air.

66. For large work, the melting is usually done in either a cast-iron kettle or a boiler-iron ladle set in a brick furnace. The surface of the metal when melting should be covered with powdered charcoal to exclude the air in order to prevent excessive oxidation. The melted metal is dipped out of the melting pot in hand ladles, from which it is poured into the boxes. When dipping the ladle into the pot, it should be done slowly in order to bring the ladle gradually to the temperature of the Babbitt, thus precluding chilling of the Babbitt if the pot be a small one, as well as avoiding any risk of spattering the Babbitt if there should be any moisture or grease on the ladle. A little powdered resin should be scattered on the surface of the



(a)



(b)

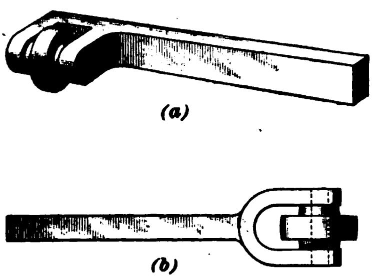
FIG. 13

metal and the metal stirred with a stick just before pouring. The rosin acts as a flux and leaves the metal cleaner and more fluid.

67. Form of Box for Babbitting.—The Babbitt metal in boxes or bearings is generally held in place by raised strips or projections cast in the box, which enclose it on all sides, for the purpose of restraining the tendency of the metal to stretch or flow under the pressure or pounding of the shaft. The strips should be cut below the surface of the bearing, to avoid contact with the journal. In the case of large boxes, dovetail grooves are sometimes cast in the surface of the casting to aid the strips in holding the Babbitt; in some case the strips are omitted, and the dovetail grooves only are relied on to hold the metal.

68. Compressing Babbitt Metal Into Place.—All metals expand when heated and contract when cooled; but Babbitt metal expands and contracts a greater amount than the boxes in which it is usually cast. Heating the box before pouring the Babbitt in it will reduce the difference between the amounts the Babbitt and box will contract after cooling. For this reason, when a good bearing is desired, it is babbitted smaller than the finished size, and the Babbitt is compressed into its place in the box. Important journal-boxes like those in the pillow-blocks of an engine are generally babbitted from $\frac{1}{8}$ to $\frac{1}{4}$ inch smaller than the finished bore. The metal is then hammered into the box by using the round peen of a hammer to either compress or expand it firmly into place, after which the bearing is bored to the required diameter.

69. Roller tools, illustrated in Fig. 14, may be used to advantage for compressing Babbitt into place. If the Babbitt bearing is chucked in a lathe, the tool (*a*) is used in the tool post and fed through the work after the manner of a boring tool; or, if a boring bar is used, the tool (*b*) may be inserted in the bar or cutter head and takes the place of a boring tool. The feed and speed of the roll may be considerably faster than in the case of a boring tool. The surface to be rolled may be lubricated with soda water, soap water, or oil. In the case of small bearings, the metal is compressed by driving or forcing one or more polished steel drift plugs through the bearing.



(*a*) (*b*)

FIG. 14

70. Mandrels for Babbitting.—A mandrel is placed in the box when the Babbitt is poured. If the Babbitt is to be compressed and bored, this mandrel is made smaller in diameter than the shaft for which the box is constructed; otherwise, it is made of the same diameter as the shaft. The mandrels are often made hollow, as shown in Fig. 15, and consist of a cylindrical portion *a* with a bar *b* across each end. The bars *b* carry the centers *c* for turning the mandrel in a lathe. The hollow mandrel is not only cheaper than a solid one, but is also lighter to handle and quicker to heat in case it must be warmed before pouring the Babbitt. Mandrels are also made of wood. Iron mandrels should always be warmed before the metal is poured into the box.

The mandrels are often made a little larger than the journal that is to be used in the box, both to allow for the shrinkage of the box and to insure that the bearing will not bind the shaft sidewise; a box that bears in the bottom is less likely to heat than one that pinches the shaft sidewise. Sometimes the journal is used instead of a mandrel, paper being wrapped around it; this is not advisable except in repair work which must be done quickly.

71. Babbitting Process.—A proper thickness of Babbitt may be obtained by supporting the mandrel on pieces of pasteboard or wood, which are placed on the strips that retain the Babbitt in the box. To prevent the Babbitt from running out at the ends and joints of the box, the openings should be closed with clay or putty; care should be taken that they are not too wet, as water in the mold is likely to form steam and blow out the metal. A pouring basin leading to the box may also be made of clay or putty; large boxes are sometimes poured from several ladles simultaneously. In all cases, ample vents should be left for the air to escape from the box, and the metal should be poured at a low heat and as rapidly as possible. The surface of the mandrel should be slightly oiled.

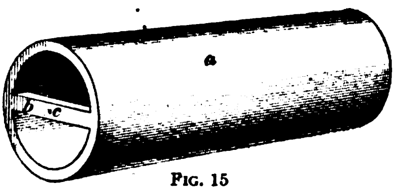


FIG. 15

72. When babbitting cast- or malleable-iron bearings, the rough casting must first be thoroughly cleaned and the anchor holes well freed from foundry sand, which may be accomplished either by *brushing*, *sand blasting*, or *pickling*. When sand blasting, a jet of sand is directed on the work with great force; and when pickling, the work is washed in a dilute acid, as for example, 1 part of sulphuric acid to 4 or 5 parts of water, and the castings are permitted to soak until all the scale or sand has become loosened and has fallen off. The casting must next be washed in water, preferably hot, and no trace of acid allowed to remain; otherwise, it will continue to eat its way into the casting and eventually loosen the Babbitt in the finished bearing. Another point to be carefully observed is to dry the casting thoroughly, especially in the anchor holes. Even the smallest amount of moisture is liable to cause an explosion, scattering the molten metal and perhaps burning the workmen, when, in pouring, the hot Babbitt comes in contact with the wet casting.

73. Whenever possible, both the casting and the mandrel should be heated to approximately the temperature of the Babbitt before pouring. The casting may either be placed in an oven especially provided for this purpose, or it may be set on the rim of the Babbitt pot until well heated, then immersed in the pot for a few minutes, after which it is withdrawn and made ready for babbitting. The mandrel may be treated in like manner, though after once heating, the contact with the molten metal, as each bearing is poured, will keep it sufficiently hot. This preliminary heating tends to reduce shrinkage of the Babbitt when it comes in contact with the cooler walls of the casting and the mandrel. Any cored holes in the casting should be fitted before pouring, with metal or wooden plugs and the crevices closed by a mud of fireclay, so as to retain the Babbitt during the pouring. After babbitting, the plugs and the mandrel are removed, and all fins or surplus Babbitt trimmed away with a hot soldering iron. The bearing is then further machined, being sometimes turned or broached on the inside, though machining is not always necessary. It is, however, usually placed on a mandrel or arbor and turned on the outside as well as faced at both ends. The oil grooves, if they have not been cast in the Babbitt at the time of pouring, are next cut and the dowel-pin holes drilled or the keyway slotted.

EXAMPLES OF BABBITTING

74. Babbitting a Plain Box.—Suppose a plain box is to be babbitted. A mandrel of the form shown in Fig. 16 may then be used when pouring its upper and lower parts. This mandrel consists of a cylinder *a* of the required diameter and length, with a disk *b* at each end to fit against the machined ends of the box. One disk is held in place by a cap screw *c* and is removable. The mandrel is first put in position in the bottom of the box and Babbitt is poured in the lower half of it. The top of the lower half is then finished, and the work is set up again, liners of pasteboard or sheet iron being placed between the halves of the box to prevent the Babbitt of the upper half from running into that of the lower half when poured. The Babbitt is poured through the oil holes or slot in the cap. Should blowholes be found in the Babbitt lining, they may be filled with the aid of a hot iron and a stick of Babbitt of the same composition as that used in the bearing.

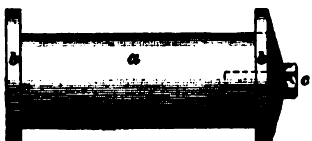
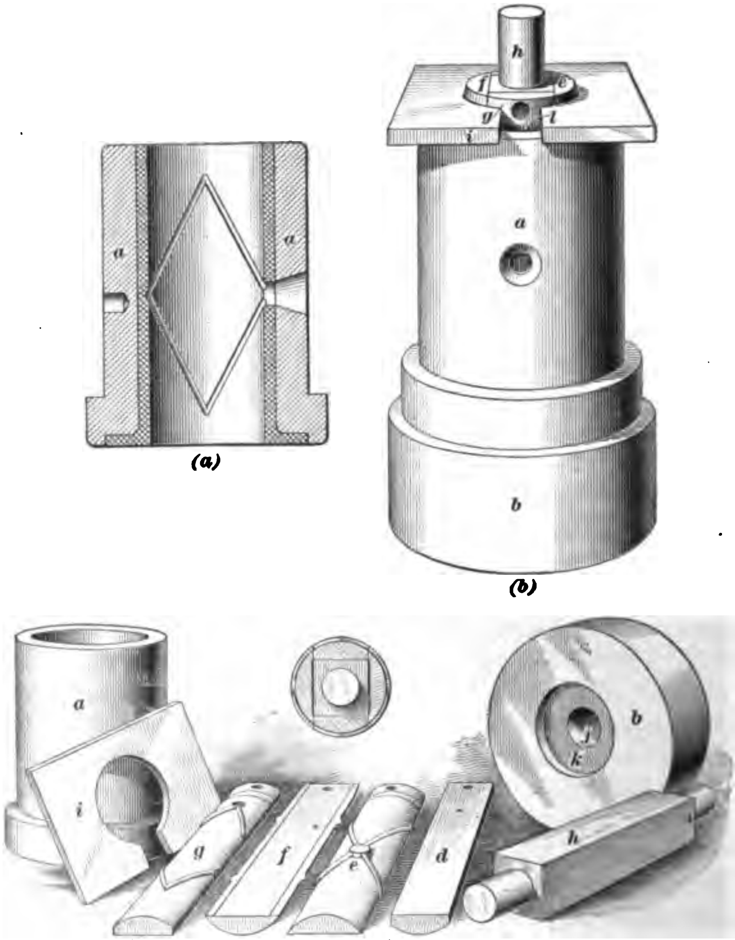


FIG. 16

75. Jig Method of Babbitting a Box.—The bearing box *a*, Fig. 17 (*a*), may be babbitted by the use of a babbitting jig (*b*) and (*c*). The jig consists of a base *b*, four side pieces *d*, *e*, *f*, and *g*, which fit on the sides of a central core *h*, and a collar *i*. The core and the four side pieces are tapered on the abutting sides, so that after pouring, the core may be withdrawn, and the side pieces allowed to collapse and be also withdrawn. The purpose of the collar is simply to hold the side pieces to the core before pouring. When setting up the jig, one end of the core *h* is placed in the bore *j* of the base *b*, the side pieces *d*, *e*, *f*, and *g* are placed in the bore *k*, the bearing *a* is put over the core and side pieces, and the collar *i*, by which the upper ends of the side pieces are held against the core, is placed in position as shown in (*b*). The Babbitt is poured around the core through the opening *l*.

76. Babbitting Pillow-Blocks of Engine.—In Fig. 18 is shown a rig for babbitting the pillow-blocks of a center-crank engine, (*a*) being a side view and (*b*) an end view. The engine frame *a* rests on the parallels *b*, which are supported by a cast-iron floor plate *c*. The parallel *b* under the pillow-block *d* is



(*a*) (*b*) (*c*)

Fig. 17

set square across the plate *c* and bolted to it, and the engine frame *a* is set to a center line on the plate *c*. Standards *e* rest on the parallel *b* under each end of the engine shaft or mandrel *f*. The bottoms of the standards *e* have projections *g* that fit in the grooves in the parallel *b*, and brackets *h* with vertical adjustment are bolted to the top of the standards. These brackets have V-shaped tops that support the shaft of

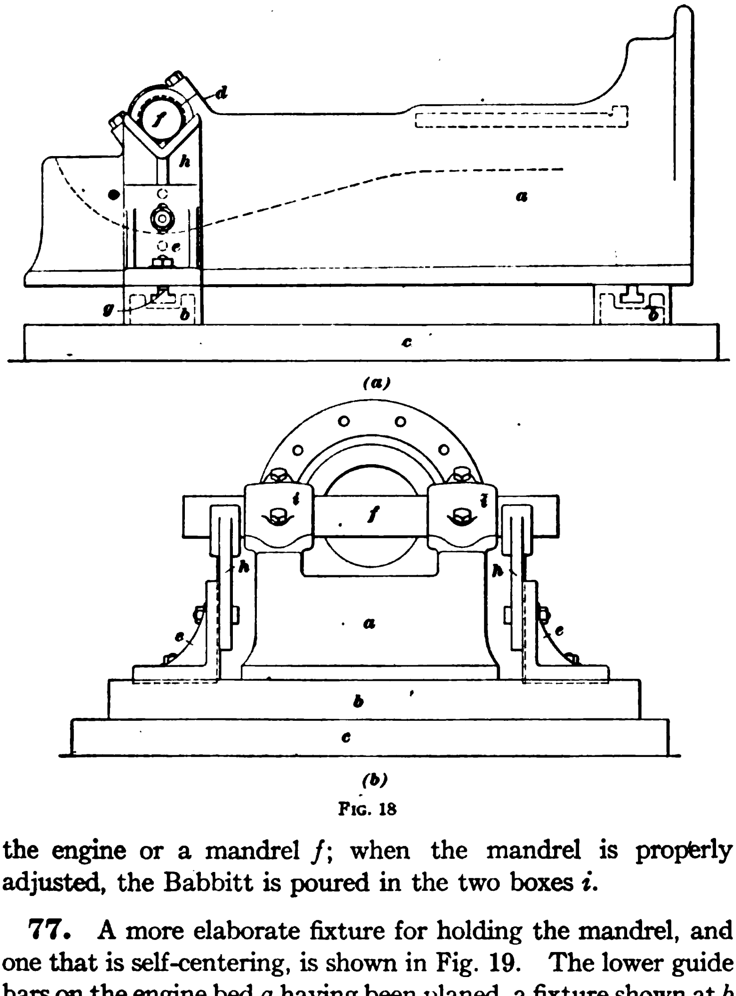


FIG. 18

the engine or a mandrel *f*; when the mandrel is properly adjusted, the Babbitt is poured in the two boxes *i*.

77. A more elaborate fixture for holding the mandrel, and one that is self-centering, is shown in Fig. 19. The lower guide bars on the engine bed *a* having been planed, a fixture shown at *b* is placed at each end of the engine guides and these support a bar *c* with a casting *d* on the end to hold the babbitting mandrel *e*.

Boxes of different sizes may be babbitted by having several mandrels all the same size in the center *d*, but of different diameter in the journals *f*. The center of the hole in *d* for holding the mandrel *e* can be bored, if desired, ⅛ inch or so higher than the center of the hole for the bar *c*. This will allow for the spring of the bar *c* and also bring the center of the journals a little above the center line of the engine, so that the wear will be down toward the center line and not away from it. Keys may be fitted in the bar *c* and the castings *b* and *d*

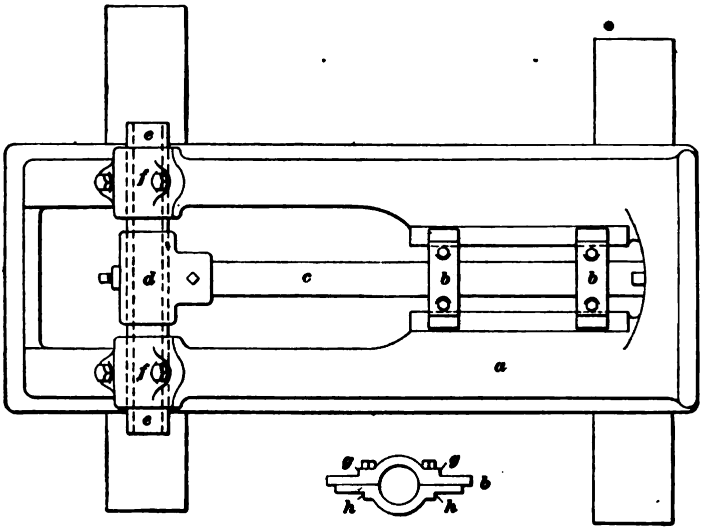


FIG. 19

to level the mandrel *e*, or the keys may be omitted and the mandrel *e* leveled by a surface gauge, or a spirit level may be used on the mandrel *e*. The rig shown in Fig. 19 can be made to serve for babbitting two sizes of engine by making the casting *b* with two sets of shoulders *g* and *h* to fit the guides of two sizes of engine.

78. Jig Method of Babbitting Pillow-Blocks of Engine.—When a large number of engines are to be built, a babbitting jig may be made for each size, as shown in Fig. 20, which is a single casting *a* with a rib *b* to stiffen it. The casting is planed under the lugs *c* to fit the engine guides and bored at the end *d* so as to receive and support the babbitting mandrel. Many other forms of mandrels may be designed to meet the requirements of the case in hand.

79. Rebabbitting of Box.—In case a babbitted box is so worn down as to require renewal, first chip out the old Babbitt metal and then proceed to rebabbitt as nearly in the way described for a new box as the appliances at hand will permit.

80. Babbitting Journal Brasses.—Journal-box brasses are sometimes lined with Babbitt metal. It is necessary to *tin*—that is, wipe with Babbitt—the surfaces of the brass so that the Babbitt will adhere to it. In tinning a bearing, it is first

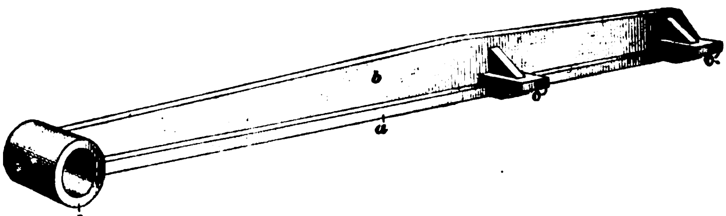


Fig. 20

heated as a precaution to drive off any moisture, after which every part not to be tinned is coated with a mixture of graphite and water of sufficient body to leave a coat of graphite upon drying. In case graphite is not available, a mud of fireclay and water may be similarly used with equally good results. When dried, the part of the casting to be tinned is cleaned of the graphite and well rubbed over with a swab dipped in a tinning solution. It is then immersed in the *tin pot*, containing a melted mixture, usually composed of approximately 42 parts tin and 58 parts lead, by weight, though these proportions are sometimes slightly varied. After the casting has reached a temperature sufficiently high to cause the metal to run readily over its surface, it is removed and immediately gone over again with the swab to insure that all parts are properly tinned. It is finally rubbed with a bunch of clean waste to smooth the surface, and the graphite is also brushed off. The tinning solution is made by dissolving zinc in muriatic acid; sometimes sal ammoniac is added. Tinning salts are also on the market. After the surface is thoroughly tinned, the Babbitt is poured in the usual way; but in this case unites with the tinning and is held firmly to the brass.

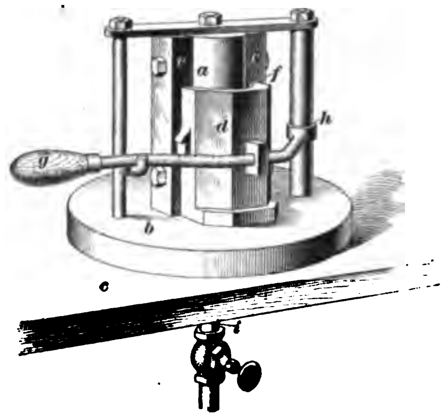


Fig. 21

81. A special mandrel for babbitting brasses is shown in Fig. 21. The mandrel consists of a hollow cast-iron cylinder *a* resting on a base *b* bolted to a table *c*. The cylinder *a* is turned to the proper diameter and the edges of the brasses *d* rest against two lugs *e* of the cylinder, leaving a space *f* between the mandrel and the surface of the brass that is to be filled with Babbitt. The brass *d* stands on the base *b* and is held against the lugs *e* of the mandrel *a* by means of a curved lever *g* hinged to the frame at *h*. The Babbitt is poured from a dipper into the space *f*, and the brass is removed as soon as the metal sets. The cast-iron mandrel *a* is cooled by means of a circulation of water that enters through a pipe *i* attached to the center at the bottom.

GENERAL APPLIANCES AND PROCESSES

(PART 2)

GENERAL SHOP PROCESSES—(Continued)

SOLDER AND SOLDERING

1. Soldering is the process of uniting metals by means of solder. **Solder** is an alloy of lead and tin or other metals, that melts at a comparatively low temperature. The solder is applied to the work in the molten state. It then adheres to the surfaces to be joined, provided they are properly prepared, and unites solidly with them while cooling. To assist the flow of the solder and to unite it with the metal, the surfaces to be joined must be covered with a *flux*, by which is meant a substance that promotes the fusing of metals.

Solder is usually composed of tin and lead, or tin, lead, and bismuth. It is cast in short blocks, or sticks that can be easily handled, and is sometimes made in the form of wire. The proportions of the metals used in making a solder will affect its hardness; the greater the proportion of lead employed, in general, the softer the solder will be.

2. Solders are sometimes classified as soft and hard solders, the term *soft solder* being applied to solders used when soldering, and *hard solder* to the spelter employed when brazing. In this Section soft solders are called *solders* and hard solders, *spelter*.

Table I gives the proportions of the constituents of solder and the fluxes commonly used for various kinds of work. In interpreting the table, either tin 50 per cent. lead 50 per cent., or tin 64 per cent. and lead 36 per cent. may be used for copper, brass, or general work; and either chloride of zinc, resin, alcohol, or sal ammoniac may be employed as a flux, no matter which of the metals named are to be soldered or which percentage of lead and tin is used.

3. Preparing Solder.—Solder is usually bought ready for use, but occasionally it may be found necessary to make it.

TABLE I

PROPORTIONS OF CONSTITUENTS OF SOLDERS AND THEIR CORRESPONDING FLUXES

Metals to Be Soldered	Solders			Fluxes
	Tin Per Cent.	Lead Per Cent.	Bismuth Per Cent.	
Lead-wiped joints	50	50		Tallow
Galvanized iron or steel	50	50		Chloride of zinc or sal ammoniac
Tin plate or tinned iron	50 64	50 36		Chloride of zinc, or resin, or alcohol
Copper, brass, or general work	50 64	50 36		Chloride of zinc, resin, alcohol, or sal ammoniac
Iron or steel	50	50		Chloride of zinc
Pewter	20	40	40	Resin or olive oil

When preparing solder, the tin is first melted in a crucible, preferably of clay instead of iron, in order that no trace of the iron may enter, as its presence tends to harden the solder. The lead is then added in small quantities, the mixture being constantly stirred with a wooden stick until all the lead has been put in and the solder has become thoroughly mixed. It is then poured into molds of any desired shape, usually of iron. Although lead has a higher melting point than tin, it oxidizes more readily when melted alone and for this reason the tin is

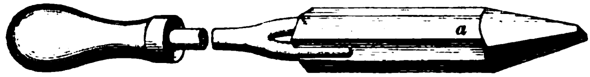


FIG. 1

melted first. If bismuth is also used, as is sometimes done to lower the melting point of a solder, it should be added last and in powdered form; and the solder should first be removed from the fire; otherwise, the bismuth will evaporate.

4. Equipment for Soldering.—A very simple equipment is required for ordinary soldering; it consists of a *copper bit* and a *fire-pot* in which to heat it. The copper bit, or *soldering iron*, Fig. 1, is a piece of copper *a* drawn to a point or edge and fastened to an iron rod having a wooden handle. The bits used for soldering must be of sufficient weight to hold the heat necessary to heat the metal and fuse the solder during a reasonable length of time. If they are too light, the soldering is apt to be very uneven in quality, and the bits will require such frequent reheating that they will be troublesome. If they are

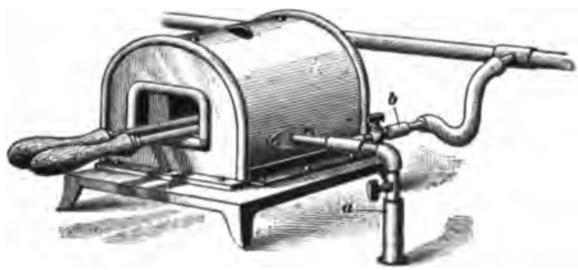


FIG. 2

too heavy, the work of handling them will be laborious, and the time required to reheat them will be excessive. An average weight of bit is 6 or 7 pounds.

5. The soldering iron may be heated in any kind of fire. In Fig. 2 is shown one type of gas furnace for this purpose.

The gas and air, which enter through the pipes *a* and *b*, must be so proportioned as to give a blue flame. A lack of air will cause the flame to burn yellow and soot the iron, while too much air will tend to reduce the flame and perhaps extinguish it entirely. Portable gasoline-fired torches and charcoal furnaces are also used for soldering, the latter more particularly by plumbers. When a soft-coal fire is used to heat the iron, the soldering iron should be protected from direct contact with the coal, by means of a thin sheet of iron laid between it and the coal. The iron need only be heated to such a temperature as will melt the solder.

6. Soldering Flux.—The least film of either oxide, grease, or dirt on the surface of the metal will usually prevent the adhesion of the solder; therefore, the surfaces to be joined must be thoroughly cleaned, or coated with some substance that will reduce the oxides to the metallic state or that will destroy the grease and deposit a thin film of tin on the surface to be soldered. For this purpose, a soldering fluid or some other kind of flux is used.

Of all the fluxes used, the *chloride-of-zinc soldering fluid* possesses the greatest range of usefulness. It is made by placing small clippings of zinc in hydrochloric, commonly known as muriatic, acid that has been diluted with an equal quantity of water. The acid vigorously attacks the zinc, causing bubbles of gas to rise and forming chloride of zinc. Zinc should be added until the bubbles cease to rise while a small amount of the undissolved metal remains in the liquid. When the acid has dissolved all the zinc with which it will unite, the liquid is strained and thinned by adding an equal quantity of water. A few small pieces of zinc are then placed in the liquid to neutralize any free acid that may remain.

7. The same flux cannot be used on all metals. Sal ammoniac is commonly used on copper or brass, chloride of zinc on iron, resin on tinned iron, and resin or tallow on lead. The soldering fluid, however, is the best all-around flux. By adding ½ ounce of sal ammoniac to 4 ounces of the liquid, it can be used in soldering iron or steel without first having to tin the surfaces to be joined.

Copper, brass, or iron not galvanized may be prepared to receive solder by cleaning the surfaces and applying the chloride-of-zinc soldering fluid. A stronger joint is assured by tinning the metal before soldering, in which case resin is the proper flux.

8. Tinning.—In copper-bit work, and also in blowpipe work, there is a preliminary operation known as *tinning*, in which the metals to be united are properly prepared for soldering. This operation consists in spreading a thin layer of solder on the surfaces of the metals and causing it to adhere and make a firm metallic union therewith. Its object is so to prepare the surfaces of the metals that they will readily unite with the melted solder that is applied to them in the process of soldering. All the common metals become tarnished when exposed to the air and this tarnished surface must be removed and the bare, clean, metal exposed to the influence of the solder; otherwise, the solder will not adhere to the metal. Great care should be taken to give the tinning a uniform thickness and have it free from imperfections. Small lumps or ridges of solder in the tinning coat must be carefully taken out, as they will interfere with the proper closing of joints and seams. Any superfluous solder can be shaken off or wiped off with clean waste or cloth.

9. Copper bits must be tinned before they can be used for soldering purposes. One method of doing this is to heat the bit until it melts solder, but not until it is red hot, then lay it on a brick or other suitable material, and file the flat sides at the point to a distance of about 1 inch, or as far back as it may be desirable to tin the bit. When thoroughly clean, rub the filed surfaces on a piece of solder over which some pulverized resin has been sprinkled. The hot copper will quickly melt the resin, which prevents the copper from tarnishing before the solder melts. The resin also facilitates the adhesion of the solder to the cleaned copper. If the bit is red hot, it will oxidize the instant that the file leaves it, and tinning cannot be done with resin as a flux. Another way of tinning a bit that is to be used for soldering is to rub it, while hot, on a block of sal ammoniac having a few drops of solder spattered over its surface. The sal ammoniac reduces any oxide that may be present on the bit, and the solder adheres to the clean copper instantly on coming in contact with it.

Another quick way is to dip the point of the bit, while hot, in a saturated solution of sal ammoniac and water for an instant before rubbing it on the solder. But all four sides are tinned by this method and this is not always desirable.

When a bit is overheated, the coating of solder, or the tinning, as it is called, is reduced to a yellow powder and is destroyed. The bit must be tinned before it can again be used.

10. Making Soldered Joint.—In soldering, the parts to be joined are heated by a copper bit, by a blowpipe flame, or by some equivalent means, to the fusing point of the solder.

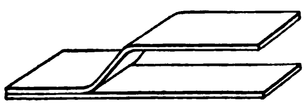


FIG. 3

Solder flows best at high temperatures, provided the temperature is not so high as to oxidize it, and will flow into a joint until it is chilled; therefore, it will flow farthest when it possesses a large excess of heat above that which is necessary to maintain it in the fluid condition. In soldering, bits that are barely hot enough to melt the solder should not be used, because the solder will unite only at the edges of the metal and will not flow into the joint properly.

The metal to be soldered may be heated by contact with the hot bit, and by moving the bit just fast enough to cause a little melted solder to follow the point. This body of solder increases the area of contact and conducts heat from the bit to the metal with great rapidity. In working with the blowpipe, the necessary heat is applied directly to the metal by the flame which must be so handled as to avoid overheating or oxidizing either the metal or the solder.

11. Examples of Soldering.—If two pieces of sheet brass, like those shown in Fig. 3, are to be soldered together, the surfaces to be soldered are first rubbed clean and a little of the soldering fluid is applied with a small brush or a feather. The bit is then heated to the proper temperature, which can be determined by striking it a quick glancing blow with the hand. Any dirt is thus removed and a clean, tinned surface exposed. When hot enough, the molten tin on the bit has a streaked appearance when the hand is brushed over it. With a little experience, the proper temperature can be easily told by this method.

When the bit is properly heated, a drop of solder is melted from the stick with the point of the bit, allowed to fall on one of the fluxed surfaces, and spread over it with the hot bit, care being taken to get the surface well tinned. The other piece is then prepared in the same way, and both pieces are placed in position and pressed together with the hot bit. The heat from the bit heats the pieces of brass and melts the solder on the surfaces; the bit is then removed and the pieces are held together until the solder has become hard. The soldering fluid that still remains on the pieces must be washed off, so as not to corrode the brass.

The ability to make neat and fast joints is a matter of practice. The method of soldering is varied slightly to suit conditions, as, for instance, when two pieces are laid edge to edge, and the molten solder is drawn along by means of the hot copper bit.

Iron articles may be tinned by thoroughly cleaning the surfaces and treating them with chloride of zinc or sal ammoniac before the solder is applied.

12. Sometimes, as in seam soldering, the soldering iron is rubbed back and forth along the work, slowly advancing, the stick of solder being applied to the point of the iron from time to time. The heat from the iron melts the solder at once and it runs into the seam.

Where the soldering iron cannot be used, the work may be heated by means of a blow torch as shown in Fig. 4, in which *a* is the work, *b* the torch, and *c* the blast, or flame, from the torch. Small chips of solder are dropped on the pieces to be soldered before the blow torch is applied. When the solder has melted, it should be spread around by means of a small copper or iron wire, the two pieces to be soldered being, of course, suitably held together during the soldering.

13. Soldering may also be accomplished with two ladles, one being filled with solder and held over the pieces to be soldered, the other being empty and held beneath. The upper ladle is then tilted and the solder caused to spill over the pieces into the lower ladle. The soldering of cables for electrical work thus is very effectively accomplished. The pouring of the solder back and forth through the strands of the cables removes all dirt most thoroughly, and should be continued until the solder has by cooling, become sufficiently thick to allow the joint to

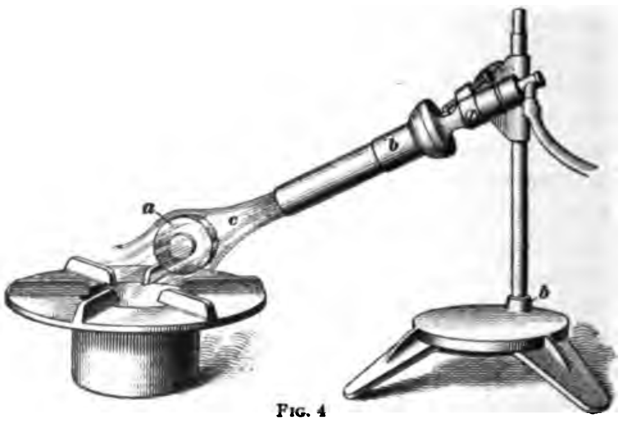


FIG. 4

be wiped. A pad for wiping may be made from a number of layers of heavy duck well soaked in tallow.

14. Another method of soldering applied largely to electric terminals and similar work consists in setting the terminals, which are of brass or copper, in line and directing a torch on several of them at once. When sufficiently heated, stick solder of small diameter is pressed into them and at once melts, filling the hole in each. The terminals, which may then be considered as small cups containing molten solder, are gripped in a pair of tongs, one at a time, and the end of the cable or wire to which each belongs is forced into the cup and thus held until the solder has cooled enough to solidify and hold the terminal and the cable or wire together. The solder that is displaced when the cable or wire is pressed into position of course falls to the floor.

15. Soldering Aluminum.—All copper and tin alloys, and, in fact, most metals, have oxides that can readily be dissolved in some flux, thus leaving the surface of the metal clean, so that the solder may adhere to it. The oxide of aluminum, however, is not soluble in any known flux; hence, the surface must be covered with the melted solder and the oxide rubbed off with the point of the copper bit. In this case, the bit need not be tinned, but should be rather heavy, so that it will hold a large amount of heat. A good solder for aluminum has the following composition:

	Parts
Aluminum	1
Phosphor tin	1
Zinc	11
Tin	29

16. In making the solder, the aluminum should be melted first; the zinc should then be added in small pieces, taking care not to solidify the melted aluminum; the tin should then be added in the same way; and last of all, the phosphor tin, and the mixture thoroughly stirred with a brass rod. This solder should be made in a graphite crucible. The reason for melting the metals in the order given is that, if the metals with the lower melting point were heated to the melting point of aluminum, they would be partly vaporized, thus destroying the proper proportion of the alloy. To solder aluminum, the bit is heated to a red heat, some solder is placed on one of the surfaces to be united, melted with the copper bit, and then the aluminum oxide rubbed from the surface beneath the molten solder until the solder adheres evenly to the entire surface. The surface of the other piece is then treated in the same way, the two pieces placed together, and heated with the copper bit or with a torch until they unite.

17. Sweating.—Sweating is a process of soldering metals without using a copper bit, the surfaces to be joined being cleaned and tinned with solder and then brought together and heated until the solder flows and unites the pieces, which may be pressed together while cooling. The process of *sweating on*, as it is usually called, is frequently adopted for temporarily holding in place pieces of work to be turned or otherwise finished to shape, after which the parts may easily be separated by heating and melting the solder that holds them together.

18. In boring out boxes for bearings, the pieces are sometimes sweated together and then bored and finished. After this they are again heated, in order to melt the solder, and the pieces taken apart. When brass boxes *b*, Fig. 5, are sweated together, liners *a* are sometimes placed between them to allow for wear when in service. The faces of the brasses and the liners are planed smooth and rubbed bright. They are then heated in the forge, and, when hot, the brasses are fluxed with sal ammoniac or cleaned with acid, and tinned by the method employed in tinning the copper bit. The liners, if of iron, are fluxed with borax and tinned. The pieces are then put together and heated so as to melt the solder. If too light to make a tight joint, they are weighted down until cold. After being bored out and finished in the machine shop, they are melted apart and the liners taken out.

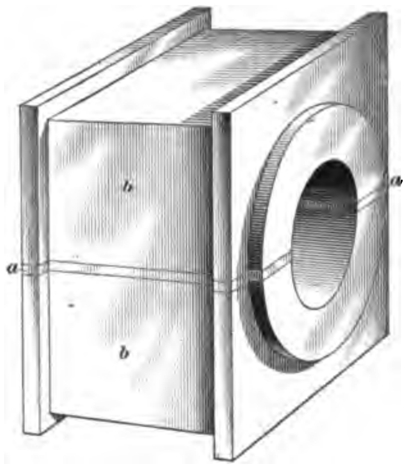


FIG. 5

SPELTER AND BRAZING

19. Brazing is a process for joining two or more metals by means of **spelter,** whose temperature of fusion is much higher than that of solders, and whose strength is greater. Spelters are composed of copper, zinc, tin, silver, etc. in proportions that are varied to suit the requirements. A good soft spelter is made with 1 part of copper and 1 part of zinc; 65 parts of copper and 35 parts of zinc make a good spelter for general work; 13 parts of copper, 5 parts of zinc, and 82 parts of silver make a good spelter for soldering band saws. Coin silver is sometimes used as a brazing solder.

20. Only such metals as iron, copper, and brass, whose temperature of fusion exceeds that of spelters, can be brazed. In brazing, the temperature required to fuse the spelter is so high that soldering bits cannot be used.

In preparing the spelter, the metals used are fused together, then filed to a powder, and sometimes made into a paste by the addition of calcined borax and water.

The proportions of copper and zinc in the alloy vary greatly, depending on whether a hard or a soft joint is desired. If a very hard and tough joint is wanted, the proportions may be as much as 3 parts copper to 1 part zinc; while if a soft joint is wanted, the zinc may exceed the copper. Occasionally the brazing material may consist simply of copper filings or copper ribbon; again, other metals, especially silver, may enter into the brazing alloy with copper and zinc.

21. Preparing Spelter.—When used in granular form, spelters may be readily made simply by melting the several constituent parts together, then filing into fairly coarse grains and mixing with the flux. Sometimes, just before using, the mixture is made into a paste by the addition of water.

Brass wire or rod may be used in place of the granular form of spelter. The wire or rod should be dipped into boracic acid or covered with a paste of borax and water, then applied to the joint and held there until it commences to melt, when it should be moved slowly along the joint.

22. Table II gives the proportions of the constituents of spelters and their corresponding fluxes, as commonly used for various kinds of work.

23. Brazing Equipment.—The heat is usually applied to the parts to be brazed, by means of an intensely hot blowpipe flame. Large work, having a considerable weight of metal, may be heated in a forge fire. For brazing collars, etc. on 2- or 3-inch tubing, the fire is arched over with coke, thus making a hot chamber in which the work may be heated uniformly. A gasoline torch may be used on small work to good advantage.

TABLE II

PROPORTIONS OF CONSTITUENTS OF SPELTERS AND THEIR CORRESPONDING FLUXES

Metals to Be Brazed	Spelters				Fluxes
	Copper Per Cent.	Zinc Per Cent.	Silver Per Cent.	Tin Per Cent.	
Wrought iron, steel, and general work	64	36			Borax, sal ammoniac, or mixture of borax and sal ammoniac
Cast iron	55	45			Cuprous oxide
Brass tubing	17	33	50		Borax
Steel band saws	13	5	82		Borax
Copper	50	50			Borax
	55	40		5	
Brass, soft	22	78			Borax
Brass, hard	45	55			Borax

In Fig. 6 is shown a very convenient form of *blowpipe* to be used in connection with a bellows. The blowpipe consists of a gas pipe *a* having a controlling cock attached; an air or blast pipe *b*, also having a controlling cock attached; and an iron pipe nozzle *c* joined by a special casting to the pipes *a* and *b*.

24. Fig. 7 shows a form of *brazing table*, or forge, operated by gas and air. It is provided with two blowpipes *a*, which are

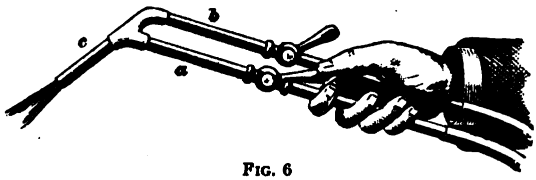


FIG. 6

adjustable in any direction. The top *b* of the table, or forge, consists of a fireclay slab on which are placed any suitable number of bricks *c* so that the work may be properly supported

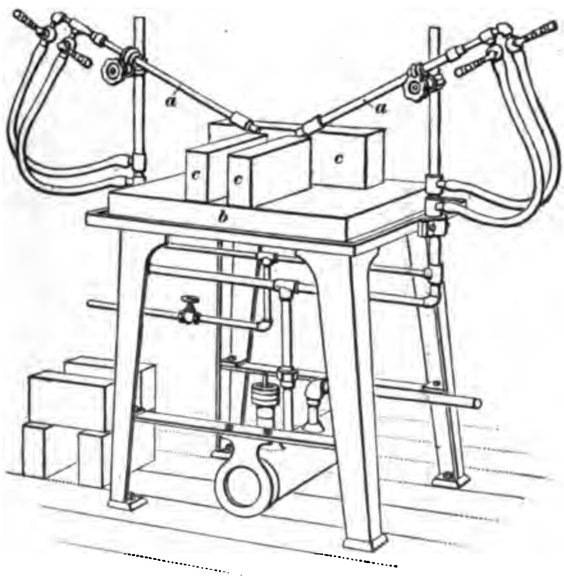


FIG. 7

or enclosed. The flame when properly adjusted should be of a blue color; if there is too little air, it will be yellow, while too much air will be indicated by a short flame accompanied by noise and a tendency to be blown out.

25. In Fig. 8 is shown a form of *blower* suitable for supplying air to the blowpipe. It consists of a single-acting bellows having an air inlet check-valve on the inside of the bottom board *a*, and another on the upper side of the pressure board *b* and within the rubber storage bag *c*, which is enclosed by a cord network to prevent it from bursting.

The bellows is operated as follows: The top board, which is hinged at the lower end and supported by a spring within the bellows, on being pushed down with the foot compresses the air within the bellows and forces a portion of it through the upper check-valve into the rubber bag. When the pressure of the foot is removed from the pressure board, the bellows will again be filled with air by the spring, which raises the pressure board. This operation is continued, thus filling the rubber bag with compressed air, which flows to the blowpipe through the rubber tube *d* when the air cock of the blowpipe is open. The elasticity of the rubber bag serves to equalize the pressure of the blast. This form of blower is capable of furnishing a strong and nearly continuous blast through a jet ⅛ inch in diameter.

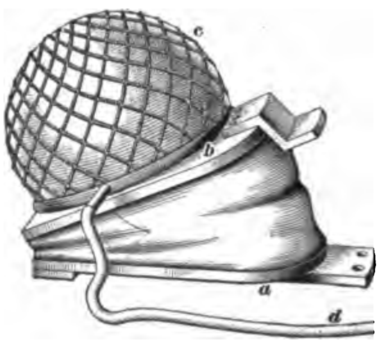


FIG. 8

26. The blowpipe should be connected by rubber tubing to a gas burner or other supply and to the blower. The bore of the tubing should be large enough to avoid excessive friction.

Air is mixed with the gas before it is consumed, as otherwise the flame is low in temperature and gives off products of combustion that not only tarnish the metal, but also cover it with a coating that keeps the flame from coming in contact with it. The gas should be turned on first and should be lighted at the jet; air is then admitted gradually until the flame is brought to the proper size and color. If too much gas is admitted, the flame will be yellow and will blacken the work by depositing a film of carbon on it. If too much air is admitted, the flame will be short, ragged, and noisy, and the temperature will be too low to heat the metal properly. The flame is hottest and at its best condition when it burns with a pale-blue or bluish-green color, with no white or yellow parts.

27. Brazing Flux.—The flux for brazing is almost always borax, though a mixture of 10 parts by volume of borax and 1 part of sal ammoniac is sometimes employed when brazing wrought iron or steel. In using borax, it should be fused to a

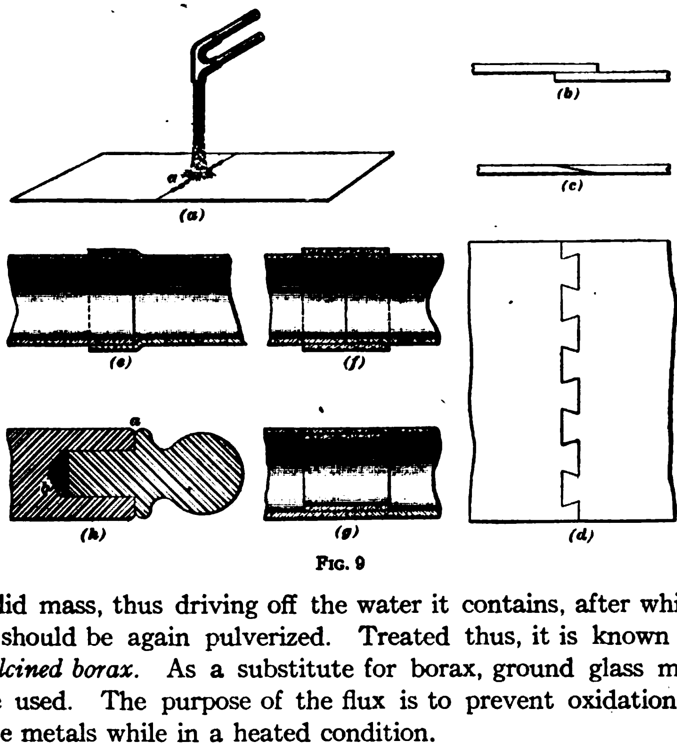


FIG. 9

solid mass, thus driving off the water it contains, after which it should be again pulverized. Treated thus, it is known as *calcined borax*. As a substitute for borax, ground glass may be used. The purpose of the flux is to prevent oxidation of the metals while in a heated condition.

28. Types of Brazed Joints.—In Fig. 9 are shown a number of joints, suitable for different classes of work. The joint shown in (*a*) is called a **butt joint**; the lumps of spelter at *a* are placed in position ready for fusion. The strength of this joint is slight, depending on the area of the surfaces that are united by the spelter. The strength is greatly increased by lapping the plates, as in (*b*). An equal amount of strength may be secured and the appearance greatly improved by beveling, or splaying, the edges, as in (*c*), provided that the plates are thick enough to permit the beveling to be extended to a sufficient width. The strongest joint for sheet metals is made by dovetailing the edges together before brazing, as in (*d*).

Thin tubing may be joined by a **slip joint,** as shown in Fig. 9 (*e*), by first annealing one of the ends and forming it into a socket. The end is flared out by means of a drift pin or plug, care being taken not to split the pipe, after which the metal is expanded by hammering until the other end will enter properly.

Circular butt joints may be strengthened by means of a band put on externally, as in Fig. 9 (*f*); or by an internal ferrule, as in (*g*).

29. Making a Brazed Joint.—The surfaces of the metals to be united are first thoroughly cleaned, after which the spelter, generally in granular form, is mixed with a flux and placed on the metals to be brazed and which have been previously brought in contact. Heat is then applied, causing the spelter to melt and flow between the abutting surfaces. Upon cooling, a solid mass is obtained.

The metals to be brazed should, as already stated, be thoroughly cleaned and brightened either by filing, grinding, scraping, or rubbing with emery paper. They should next be nicely fitted, scarfed if necessary, clamped, riveted, tied with iron wire, or forced together, and then placed in position and covered with the spelter that has been mixed with the flux, usually wet with water.

30. The metals, after being placed in position upon the brazing table or forge, should be brought slowly to such a temperature as will melt the spelter, and kept at this temperature until the spelter has penetrated the entire space between the surfaces. The joined metals should then be allowed to cool slowly to avoid warping or possible cracks. As little space as possible should be left between the surfaces of the uniting metals, for the less alloy permitted, the stronger will be the union. Equally important is slow and uniform heating; if the heating be too fast or too high, burning, as well as unequal heating, is liable to occur, resulting in an imperfect union. An iron wire or rod of small diameter and suitable length will be found most convenient to assist the alloy in finding its way into the space between the metals. A long-handled iron spoon should likewise be available to add more spelter or flux if more of either is needed.

If the pieces are small and easily handled they may be manipulated during the brazing process by means of a pair of tongs to assist the spelter, after it has melted, to flow in the desired direction.

31. Sometimes the spelter is used as a very thin ribbon, which is placed between the surfaces to be brazed. These surfaces are then tied, riveted, or otherwise held together and heat is applied. In brazing brass tubing, for example, the abutting ends are tapered and reamed respectively, so as to telescope into each other, being separated by a thin ribbon of silver alloy. This process has been found to give excellent results. Certain kinds of brass tubing have a tendency to open at the seam when being brazed, but this danger is practically eliminated by the use of silver alloy.

32. Brazing a Knob to a Rod.—A knob brazed to the end of a rod is shown in Fig. 9 (*h*). To do this job properly, the spelter must flow into the socket and secure the shank of the knob. A good joint cannot be made by merely securing the edges at *a*. The rod should be held vertically in a suitable fire or flame until both the socket and the knob are well heated. Borax and spelter are then placed in the socket, and as soon as the spelter is melted, the shank of the knob should be inserted and pressed into place. The spelter will flow outwards as it is displaced by the shank, filling the entire joint; or the space *b* at the end of the shank may be filled with spelter, as shown, and the knob inserted. If the knob and socket are then heated in an inverted position, the spelter in *b* will flow around the shank and sweat down to the rim *a*.

33. Brazing the Joint of a Pair of Tweezers.—The brazing of a pair of tweezers, shown in Fig. 10, is a good example of flat brazing. The surfaces to be brazed are cleaned; then some of the spelter is applied to each surface, and the pieces are tied together with a fine iron wire and heated sufficiently to melt the spelter. The heat may be applied with a blowpipe or by holding the pieces in a pair of hot tongs. When the spelter is melted, the piece is cooled and the iron wire is removed. The wire may be omitted when hot tongs are used, the pieces being placed in their proper position and held there by the tongs.

34. Brazing Tempered-Steel Articles.—A tempered-steel article should be heated carefully in brazing, so as to draw the temper as little as possible. The selection of the proper

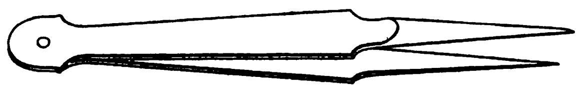


FIG. 10

spelter is important. If an article tempered to a dark-blue color is to be brazed without spoiling the temper, a spelter that will melt below 600° F. must be used. As this spelter is weaker than the harder kinds, the brazed surfaces must be larger, so as to make the joint equally strong.

When brazing steel articles that are to be tempered, the pieces are sometimes held together by snapping a small metal clip over the joint. This clip is left on after the brazing is completed and while the piece is being tempered, provided the tempering is done after the brazing. By this means, the pieces may be brazed with spelter or silver, and subsequently tempered, the clip or clamp being removed after the work is finished.

35. Butt Brazing.—If two thin pieces are to be butt-brazed—that is, brazed end to end, as in making a butt weld—the pieces must be held in position in a bench vise, hand vise, or clamp, and the heat applied with a pair of tongs or with a blowpipe. The surfaces to be brazed are fluxed with borax and then clamped in position, and a little spelter is sprinkled on the side over the joint. The heat is then applied by means of a blowpipe, a Bunsen burner, or a hot iron, until the pieces are hot enough to melt the spelter, which will then flow into the joint. By giving one of the pieces a slight tap on the end, the pieces are brought tightly together. They are then allowed to cool and the remaining spelter is scraped off.

36. Lap Brazing.—Band saws are always lap-brazed, the two ends being filed to make an accurate joint. Silver solder is generally used, being applied between the two surfaces; or the surfaces may be coated with borax and the solder allowed to flow into the joint. Fig. 11 shows the two ends of a band saw filed for brazing. The pieces are clamped together

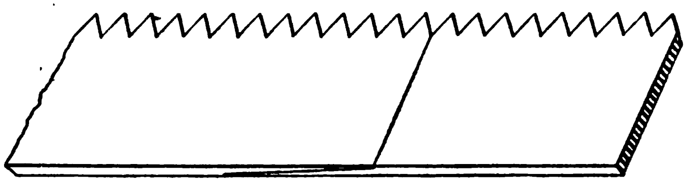


FIG. 11

or tied with a wire after having been fluxed. The spelter is either laid over the joint, or put between the pieces. When heat is applied, the spelter melts, and the pieces must be squeezed tightly together. An alloy containing 90 per cent. of silver and 10 per cent. of copper makes a good spelter.

37. Brazing Cast Iron.—The carbon contained in cast iron prevents most metals from adhering to it. The surface to be brazed should therefore be first coated with a metallic oxide—usually oxide of copper—made into the consistency of varnish and applied with a brush. The metallic oxide when heated acts as a reducer of the carbon on the surface of the cast iron to be brazed and really decarbonizes the surface of the metal for a short distance below it. The removal of the carbon leaves the surface of the metal with an open structure, since the spaces that formerly contained carbon are left empty.

The surface of the metal to be brazed is next brought to a red heat by means of gas torches or blast lamps. The oxide of copper is reduced to metallic copper and unites with the metal at the surface of the break. After the metal is brought to the required temperature, about 1,800° F., ordinary brass filings are put into the fracture and melted, as in the ordinary brazing process. The fact that the carbon is extracted from the iron for a short distance below the surface allows the brazing material to secure a firm grip on the surface of the metal, and results in very strong brazed joints. A brazing spelter as strong as cast iron or stronger than it is usually employed; hence, the joint is as strong as the original casting. The only difficulty with this brazing process is that the heating of the iron expands it permanently; hence, in some cases, finished machine parts so united must afterwards be brought to the proper size.

38. Brazing Broken Castings.—Commonly, a broken iron casting is consigned to the scrap heap; but this loss in many cases may be prevented by brazing the broken parts together. A coating of copper oxide is spread evenly over the fractured surfaces. The surfaces are then placed together properly, and brought to a cherry-red heat, which melts the flux. Spelter or brazing material is added, after which the piece is allowed to cool in the air. This method is used in cases where the castings are not subjected to extraordinary strains and are not excessively large. Castings should, however, be neither repaired in this way nor in any other when the repairing will cost more than would a new casting.

GAS WELDING

39. Oxygen-acetylene welding, commonly called *oxyacetylene welding*, is a process of uniting two metals by means of an oxyacetylene flame; that is, a flame produced by the burning of a mixture of oxygen and acetylene gases. When united by this process, metals are fused together without pressure, being allowed to flow together when in the molten state. A *welding stick* of the same material as the metals being welded is used to supply the necessary metal to flow into the open spaces, the torch being directed on the end of this stick as well as on the parts to be joined. Both the oxygen and acetylene gases may be bought compressed in steel bottles or cylinders, ready for use, or they may be generated by the user. Because of the difficulty of manufacturing oxygen and the ease of preparing acetylene, oxygen is usually bought compressed to about 1,800 pounds per square inch, and acetylene is generated in the home plant. When the welding cannot all be done in one central place, the compressed form of acetylene, though more expensive, is much to be preferred. Portable outfits that generate the acetylene are also used.

40. Oxyacetylene Welding Apparatus.—The apparatus for oxyacetylene welding consists of a blowpipe or a blow torch and generators for producing the two gases, acetylene and oxygen, though it is the more common practice to use compressed oxygen instead. In Fig. 12 (*a*) is shown the apparatus for generating the acetylene gas, which is carried to the place where it is used by the pipe *a*. This apparatus is usually kept in a separate room. In (*b*) is illustrated a welding table *b*, the torch *c*, a hose *d* connecting the torch to the oxygen bottle *e*, a hose *f* connecting the torch to the back-pressure valve *g*, which is connected to the pipe *a* leading from the acetylene generator.

In Fig. 12 (*c*) is shown a portable oxyacetylene welding outfit. In this case, both the oxygen and the acetylene are compressed in steel bottles, the oxygen being stored in *h* and the acetylene in *i*. As the apparatus is mounted on a truck, it may be readily moved from one part of the shop to another.

41. In operating a welding set, the pressure of the oxygen is varied from 9 to 30 pounds, the acetylene from a few ounces to several pounds, depending on the class of work being done. An exterior view (*a*) and an interior view (*b*) of one style of torch are shown in Fig. 13. The capacity of a torch is measured by the size of orifice in the nozzle. Each torch is equipped with a set of tips *j*, Fig. 12 (*c*), which may be so fitted to the nozzle that the same torch is made available both for large and for

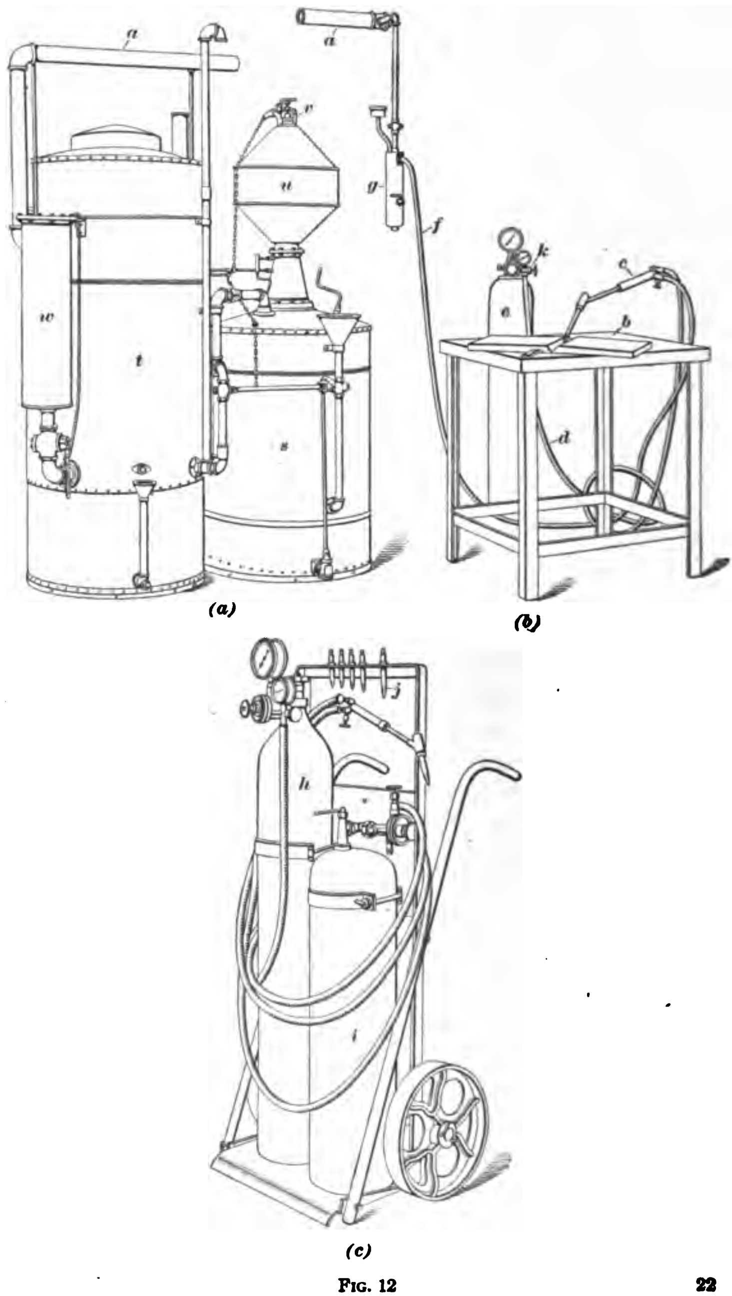
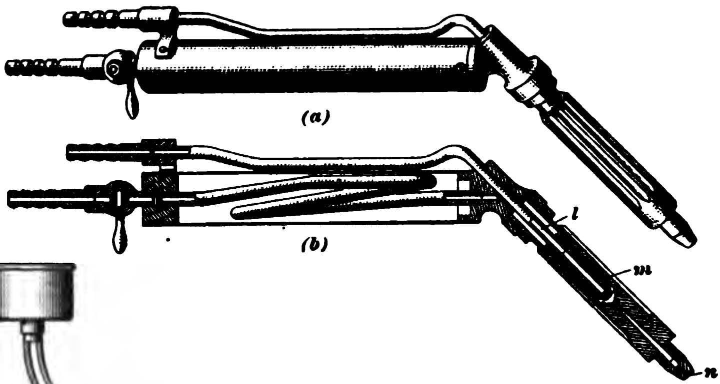


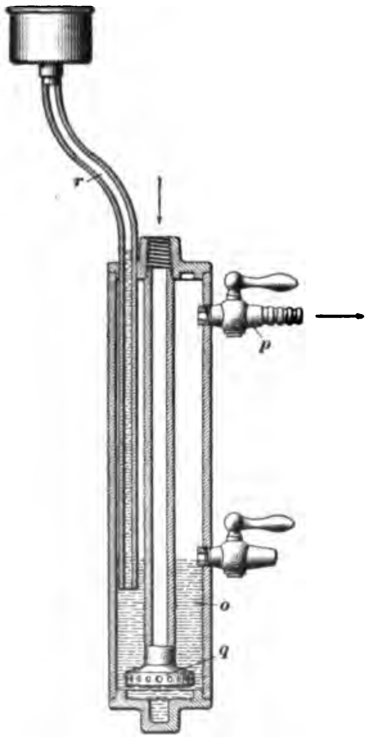
Fig. 12

small work. Referring to Fig. 12 (*b*), the oxygen flows from its bottle *e* through a regulating valve *k*, where it is reduced to the proper pressure, and from there to the mixing chamber *l*, Fig. 13 (*b*), in the head of the torch. The acetylene similarly flows from its source through a back-pressure valve *g*, Fig. 12 (*b*), and into the mixing chamber *l*, Fig. 13 (*b*), of the torch, where it thoroughly mixes with the oxygen, after which the mixed gases flow through the nozzle *m* and are ignited at the tip *n*. While oxygen is a non-combustible gas, though a supporter of combustion, acetylene is combustible, and there is a considerable tendency for the flame to travel back through the hose to the source of supply. However, the acetylene pipe in the torch itself is made as small and as long as practicable, thus tending to smother any flare back through the pipe. The velocity of the oxygen gas is also made very high, approximating 500 feet per second, and as the velocity of the explosion wave of acetylene is but 300 feet per second, the tendency for the flame to travel back is effectually guarded against. Additional precautions must still be taken to prevent air and oxygen from working back into the acetylene supply when the apparatus is not in use or if the nozzle of the torch should become clogged by a particle of molten metal, as sometimes happens.

42. The back-pressure valve *g*, Fig. 12 (*b*), is used to prevent air and oxygen from working back into the acetylene. A sectional view of this valve is shown in Fig. 13 (*c*). The flow of the acetylene through this valve is, under normal conditions, as indicated by the arrows; but if either of the two abnormal conditions just mentioned occurs at any time, either air or oxygen might flow back and produce an explosive mixture that would require but a spark to ignite it. However, the water *o* with which the valve is filled as shown, prevents such an outcome. Any back pressure forcing its way through the cock *p* would increase the pressure in the valve, thus driving some water up the central tube *q* and maintain a seal in the acetylene supply pipe, while a considerable increase in back pressure would increase this central column of water, and drive it entirely out of the pipe *r*. The oxygen would then blow off into the air.

43. Preparation of Acetylene.—As stated, the prevailing custom is to make the acetylene gas as needed, instead of





(c)

FIG. 13

using the compressed form. In order to explain clearly the principles involved in oxy-acetylene welding, a brief statement will be given of the theory of the formation and subsequent combustion of acetylene gas. Lime and coke, when intimately mixed and fused in the electric furnace, produce calcium carbide, a dark blue-gray slag, or stone, that is easily broken into lumps of various sizes as desired. When kept dry and away from air, it remains inactive; but when exposed to moisture it gradually slakes, giving off acetylene, a gas with a peculiar, onion-like odor. While the gas is not poisonous when inhaled in small quantities, it is exceedingly dangerous if mixed with air in a confined space, as any flame brought near it is likely to produce a heavy explosion. To produce acetylene gas from calcium carbide, therefore, water should be brought into contact with it. When this is done, the resulting by-product is slaked lime. About 4½ cubic feet of acetylene gas is obtained from 1 pound of calcium carbide.

44. Generators for the manufacture of acetylene gas are of two types, built respectively on the carbide-to-water or water-to-carbide principle. In the former, or carbide-feed type, the carbide is dropped into the water; in the latter, or water-feed type, the water is dropped on the carbide. The carbide-feed type is the more widely used, as it has closer pressure regulation. The generator shown in Fig. 12 (*a*) will be seen to consist of two main parts, the tank *s* being the generator, and the tank *t* the reservoir, or gas holder. On top of the generator tank is located a hopper *u*, which is filled with carbide from time to time and which is fitted with a controlling mechanism *v* operated by the reservoir.

This mechanism permits the carbide, a few lumps at a time, to feed down into the water in the tank, thus resulting in the formation of acetylene gas. The reservoir is of the ordinary gas-storage type, consisting of a hollow cylinder open at the top that holds the water, and a hollow cylinder open at the bottom in which the gas gathers. The gas passes over into the water and up into the inner cylinder, causing it to rise gradually as the pressure increases. The rise in pressure in turn shuts off the feeding of the carbide until such time as enough gas has been withdrawn from the reservoir to reduce the pressure, thereby causing the inner cylinder to sink gradually and to commence again the feeding of the carbide. The acetylene gas as it leaves the reservoir passes through a small tank *w* filled with felt, which serves as a filter.

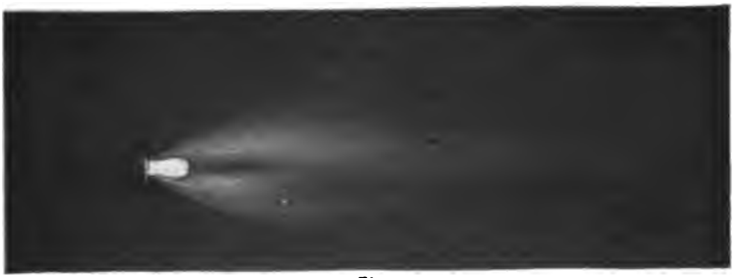
45. In generators of the water-feed type, the carbide is placed in trays arranged in tiers or rows and the water is caused to flood them gradually as required.

With the formation of acetylene gas, heat is given off. If the rise in temperature is excessive, there is not only a decrease in the quantity of acetylene gas produced, but there are also formed certain tar products, which tend to clog the apparatus and otherwise interfere with its proper working.

46. Dissolved Acetylene.—When acetylene in its compressed form is to be used, certain precautions must be taken, because when compressed to a pressure of more than 30 pounds, acetylene gas is liable to explode simply by concussion. In order, therefore, to make it commercially serviceable, it is dissolved in acetone, which is a constituent of wood alcohol and which has the remarkable property of absorbing at ordinary temperatures, about twenty-five times its own volume for every increase in pressure of 15 pounds. The volume of the acetone at the same time increases with the absorption of acetylene, being approximately one and one-half times greater at 180 pounds than at 15 pounds or atmospheric pressure. Accordingly, the acetylene is compressed into the acetone at a pressure of 150 to 180 pounds, and is then known as *dissolved acetylene*. It is, however, not yet ready to be used. If a steel bottle were to be filled with this liquid at the pressure indicated and placed in service, the pressure would, of course, gradually decrease as the gas was used and the volume of the liquid would likewise be reduced, although no liquid had actually been taken from the bottle. An ever-increasing space would thus be left in the bottle into which the gas contained in the liquid would expand, thereby forming a pocket of gas liable to explode by concussion. The acetylene bottle must therefore be filled with a porous cement made of charcoal or asbestos. This cement must be so arranged in the bottle that no shrinkage of this packing can occur. While acetylene gas made under such conditions costs more than when produced by and used directly from a generator, it has a compensating feature, extremely valuable at times, in that it has a freezing point of −49° F. It can therefore be used where the generator cannot; in fact, all generators must be kept at temperatures above 32° F., to prevent the water from freezing.

47. Preparation of Oxygen.—Owing to the great difficulty in the manufacture of oxygen, it is customary to purchase the compressed form. However, the place where it is to be used may be so remote from its place of production, and

(a)



(b)

(c)

27

(d)

Fig. 14

the cost of transportation so high, as to compel its manufacture by the consumer. There are several ways of making oxygen by chemical processes, though but one will be considered here; namely, a so-called *dry process*, in which chlorate of potash is mixed with manganese dioxide and burned in a furnace. The manganese dioxide apparently undergoes no change in this process, but its presence is necessary.

48. The apparatus for making oxygen by this process consists of a tank or furnace in which the oxygen is generated, a scrubber through which it is passed to cleanse it of impurities, a storage reservoir, and a compressor. A mixture of crystallized chlorate of potash and manganese dioxide in the proportion of about eight to one, is placed in the furnace and heat applied, usually from the outside. The generation of the oxygen commences at once, and the oxygen immediately passes into the scrubber, which is a steel tank containing pebbles or pumice stone and which has been further filled with a solution of caustic soda. The oxygen gas bubbling through the scrubber is freed of its impurities and then passes into the storage tank. From here it is drawn by the compressor and pumped into suitable steel bottles at any desired pressure.

As oil forms an explosive compound when brought in contact with oxygen, it must not be employed as a lubricant on the compressor or gauges, castile-soap water being used instead.

49. Flame of Torch.—When the torch is to be operated, the acetylene is first turned on and ignited, then the oxygen, both gases being regulated as may prove necessary. Acetylene burned alone in air gives a flaring brilliant white light, as shown in Fig. 14 (*a*), accompanied by heavy soot. With the two gases properly proportioned a small, very intense blue cone of flame with a temperature approximating 6,000° F. is produced at the tip of the nozzle, as shown in (*b*). This flame is surrounded by a semiluminous pink flame. An excess of acetylene results in a double cone, the larger being the less distinct of the two as shown in (*c*). With oxygen in excess the cone is very short, as illustrated by (*d*). Too much acetylene causes the metal about the weld to carbonize, while too much oxygen results in oxidizing the metal, giving it also a rather spongy appearance. As a rule, therefore, the flame should be a neutral one, leaning, if at all, to an excess of acetylene.

50. Making Welded Joint.—In oxyacetylene welding, the necessity for absolute cleanliness cannot be too strongly emphasized, not only with respect to the surfaces of the metals to be welded and to the torch, but also with regard to the gas-generating apparatus and the chemicals used. Failure to secure good welds may sometimes be due to lack of appreciation of this fact.

In Fig. 15, a variety of ways of making joints are shown. The parts are placed in position and welded, the joints being

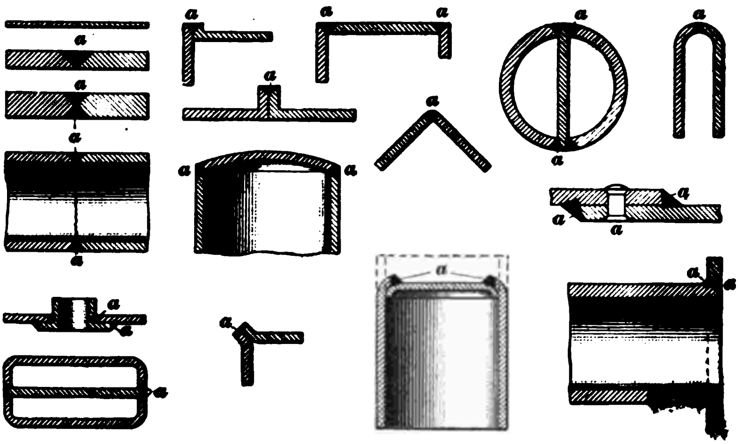


FIG. 15

made at the points *a*. The making of an oxyacetylene-welded joint is illustrated in Fig. 16. No protection is needed for the operator other than a pair of heavily smoked glasses. The torch *a* is held in one hand and the welding stick *b* in the other. The two almost touch within about $\frac{1}{32}$ to $\frac{1}{4}$ inch of the metal being welded. As the metal melts, both the torch and the welding stick are, in the case of seam welding, moved along the seam, always retaining their relative positions. Added strength is sometimes secured by piling up the metal at the weld. A closer grain and toughness may be imparted by hammering the weld while it is still red hot. Annealing is also beneficial, though not at all necessary for the usual run of work.

When repairing cracks, it is very essential to chip, machine, or melt away the metal around the edges before commencing the weld. When welding hollow work, it should be packed with dry molding sand to hold the metal in its softened condition, as well as to decrease any tendency toward warping.

51. Both sheet iron and steel are readily welded. All sheet-metal work is butt-welded, and when the thickness of the

FIG. 16

sheet exceeds $\frac{3}{16}$ inch the edges should be beveled on one or both sides, as shown in Fig. 17 (*a*) and (*b*), either by machine or by means of the torch. If this is not done, an imperfect weld will result, being more or less superficial and not extending the full depth of the metal. When the sheets are of appreciable thickness, they must be spread apart at one end, as seen in (*c*), and held thus rather loosely. If not spread apart, the plates will overlap as the welding progresses.

Wrought iron and steel present no difficulty in welding unless it be owing to some special shape or section.

Cast and malleable iron are rather difficult to weld. When welding these materials, the metal is frequently preheated and while in this condition the weld is made. Subsequent cooling should be done slowly, so that shrinkage strains and cracks may be avoided.

Brass is exceedingly difficult to weld because of its zinc component, which vaporizes at a low temperature. The metal should be carefully supported so as to retain its shape and the flame should be applied intermittently.

52. Strength of Joints Welded by Oxyacetylene. Good welding should result in the metal at the joint being about 80 per cent. as strong as the original stock. The gas-welding process is best adapted to light work or work requiring a fine

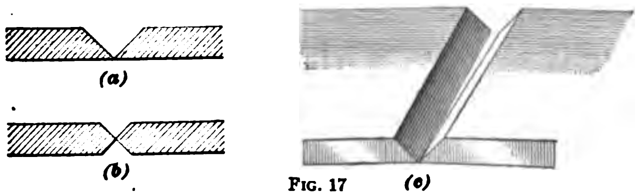


FIG. 17

appearance or great strength, in all of which features it excels the electric-welding process, to be described later.

53. Cutting Metals With Oxyacetylene Apparatus. The oxyacetylene torch is employed very successfully in the cutting of steel or wrought iron. The part of the metal surface to be cut is first heated either by an oxyacetylene or oxygas flame until a state of incandescence is reached, when a fine, powerful flame of oxygen alone is directed on the spot. The cutting torch resembles the welding torch in appearance, but has an additional pipe either inside of it or on the outside for the passage of the oxygen for the cutting jet. Immediate combustion commences with the formation of iron oxide, which is blown away by the pressure of the gas, leaving a narrow slot with clean-cut edges. The cut may be given any shape desired, curved or straight.

54. Blau Gas-Welding Processes.—Various other gases are used in connection with oxygen when welding, one of these being *Blau gas*, named after its inventor. **Blau gas** is a compressed, liquefied, distillation gas produced from mineral oils. In the Blau gas-welding system, both the oxygen and the Blau gas are bought compressed in steel bottles, which, together with a mixing tank, torches, and their appurtenances, comprise the system. The oxy-Blau gas system is used for the same purposes as oxyacetylene and has certain advantages. Blau gas is non-poisonous, has a small explosive range, and seldom flashes back. The oxy-Blau gas apparatus is compact and easily transported.

ELECTRIC WELDING

PRINCIPLES

55. Electrical Terms.—The flow of an electric current in a wire is, in many ways, like the flow of water through a pipe. If a pipe connects two vessels, one of which is placed much higher than the other, the water will flow from the higher to the lower, and will exert considerable pressure at the lower end of the pipe. It is this pressure, which is measured in pounds per square inch, that causes the water to flow. In the case of an electric current pressure is necessary to cause the flow; but this pressure is measured in **volts** and not in pounds.

The amount of water that flows through a pipe is usually measured in gallons, and the rate at which the water flows is measured by the number of gallons per minute. In the case of the electric current, the rate of the flow is measured in **amperes**.

56. When water flows through a pipe, the bends, valves, and rough sides of the pipe tend to prevent the flow. If a fine wire netting is placed over the end of the pipe, the flow is much reduced because the netting hinders the passage of the water. Likewise, the flow of electric current is hindered by the resistance of the wire. The smaller the wire, the greater its resistance, just as a decrease in the size of a pipe will increase the resistance to the flow of water. The current will heat the wire, and the larger the number of amperes, the greater is the heating effect. By increasing the flow of current, the wire may be heated to its melting point.

Electric power is measured in **watts**, and is equal to the product of the rate of flow of current in amperes by the pressure of the current in volts; that is, watts = amperes × volts. One thousand watts equal 1 **kilowatt.**

57. There are two kinds of electric current in general use, *alternating* and *direct*. An **alternating current** is one in which the flow of the current changes from one direction to the opposite and back rapidly. A **direct current** is one that always flows in the same direction. The free end of a direct-current conductor from which, when the circuit is closed, the current flows is called the *positive terminal* and the end to which the current flows is called the *negative terminal*. An **electrode** is the end of a conductor from which the current flows to another electrode through a liquid or the air. A **switch** is a device for opening and closing the circuit.

58. Principle of Electric Welding.—The fact that the temperature of a body is raised by passing a current of electricity through it is the principle on which electric welding depends. The heating is greatest at the point where the resistance is greatest, which is at the contact surface between the ends of the pieces to be welded.

BERNARDOS ELECTRIC-WELDING PROCESS

59. One of the most common of the many different processes by which electric welding is done is the Bernardos process, in which the welding is accomplished by fusion. By means of the electric current, the parts to be welded are heated until they melt, when they flow together, no pressure being required.

The Bernardos process is one form of *electric-arc welding*. The classes of work handled by these processes are usually quite different from those to which the oxyacetylene process is adapted; in fact, the processes may be said not to infringe on one another in any way, the electric processes being used for rough work, the oxyacetylene process for fine work.

60. Apparatus.—In the Bernardos process, the metal *a*, Fig. 18, that is to be welded constitutes the positive terminal of a direct-current electrical circuit and a carbon electrode *b* the negative terminal. In operating, the carbon is first brought in contact with the metal and then separated, thus striking the arc, which is then played upon the metal electrode. The temperature of the flame is between 6,500° and 7,000° F., and all metals can be melted in it.

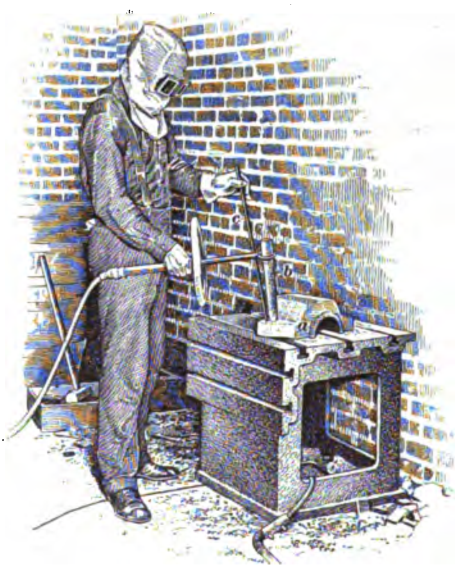


FIG. 18

The apparatus required consists of a source of direct current of the proper voltage, switches, resistances, carbon electrodes of various diameters, flux, material for filling cavities, covering for the head and hands of the operator, and an enclosure for the work.

61. Current.—The current for the Bernardos process must always be direct, not alternating, and while usually obtained from special dynamos, it may also be taken from public-supply mains. While the capacity of the dynamos may be for small work—as low as 15 kilowatts—250 amperes at 60 volts, it is better to be liberal in this respect; therefore, not less than 30 kilowatts is recommended. The pressure may vary from 60 to 100 volts; but in the latter case it must be reduced somewhat when welding by inserting resistance in the circuit, the higher voltage only being necessary for cutting.

62. Controlling Apparatus.—One of the most common working arrangements of the circuit is shown in Fig. 19. The current from the lead *a*—that is, the wire through which the current flows—passes through the steel terminal plates *b* immersed in barrels of water *c*, through the water, and thence through the wire *d* to the metal table *e* and casting *f* to be welded. The other lead *g* connects directly with the carbon electrode *h*. A *voltmeter*, an instrument that measures voltage, and an *ammeter*, an instrument that measures amperes, are generally placed in the circuit to indicate the voltage and the amperes of the current that are being consumed. A circuit-breaker is usually placed in the circuit to break the circuit

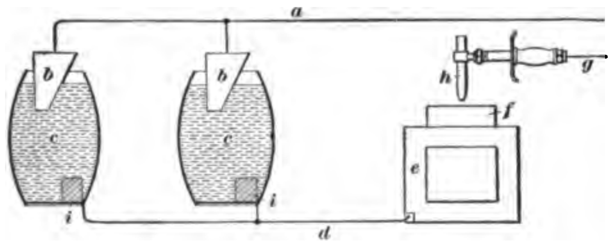


FIG. 19

if the current should become greater than desired. The steel castings *i* serve as weights to hold steel plates down in the barrel. The leads are connected to the steel plates. The controlling apparatus consists of the terminal plates *b* and the water barrels *c*. These terminal plates may be raised or lowered at will, raising them increasing the resistance and decreasing the voltage at the electrodes, and lowering them decreasing the resistance and increasing the voltage at the electrodes.

The water in the barrels will boil over occasionally, necessitating a temporary stop in the work to allow the water to cool or be renewed. For this reason, instead of controlling the apparatus with the resistance offered by the water, cast-iron resistances are often employed, the principle being the same.

63. Carbon-Electrode Holder.—Fig. 20 shows a common type of holder for the carbon electrode used in the Bernardos process. It consists of a piece of pipe *a* threaded and split at one end *b* to receive the eyepiece which holds the electrode and over the other end of which is fitted a wooden handle *c* with a protecting shield *d* of fiber or asbestos. The electrode *e* is driven lightly into a special metal eyepiece *f*, which, in turn, fits into the threaded pipe, being firmly held by a nut *g*. The sizes of the carbon electrodes will vary from ¼ inch to 1½ inches in diameter; they are about 6 inches in length and should be made solid, and preferably of graphite

64. Covering for Operator.—Owing to the intense brightness of the arc in the Bernardos electric-welding process, the operator must be well covered, not only about the head, but

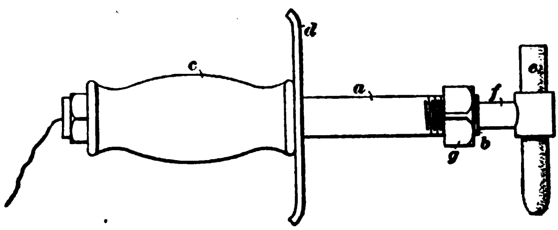


FIG. 20

over the hands and the body as well. Exposure to the rays for even a few moments will result in sunburning the exposed surfaces, with the subsequent irritation and peeling of the skin, though fortunately with no other serious consequences. The clothing will be sufficient covering for the body; the hands may be protected by gauntleted gloves, and a hood of canvas, wood, or sheet iron, fitted with a small projecting window of colored glass, should be worn to protect the eyes. The glass should be of several thicknesses, blue and red forming the best combination.

65. Flux and Filling Material.—Electric welding may largely be done without any flux whatever. Borax is, however, very good, especially if the water of crystallization has been removed. Removal of the water may be readily accomplished by heating the borax in a crucible of graphite until bubbling has ceased, then allowing it to cool and crushing it. The purpose of the borax is simply to cover the melted surface of the metal with a thin film, thus keeping the air from it, and preventing oxidation. An excellent flux for the general run of work in cast, wrought or malleable iron and steel consists of a mixture of 75 per cent. of borax and 25 per cent. of red oxide of iron. Any carbon that has entered the weld from the electrode unites with the oxygen in the flux and is burned, leaving pure iron in its place. Should the gas not be entirely dissipated, the metal will show a fine, spongy appearance, which, however, can be largely removed by hammering the metal while still white hot.

The material used to fill the openings in the work to be welded is known as *filling material*. When welding steel, wrought, cast or malleable iron, the filling material may be soft Norway or Swedish iron rod, boiler-plate scrap, or bits of steel castings, though for cast and malleable iron, a special cast iron with a high percentage of silicon is recommended.

66. Making a Weld.—In making a weld by the Bernardos process, the piece to be welded may be connected either directly to the terminal, or laid on the metal table shown in Figs. 18 and 19, if one is used. The resistance in the circuit should next be adjusted by raising or lowering the terminal plates if water barrels, Fig. 19, are used, or by closing the proper switch if grids are employed. The circuit-breaker should next be closed and finally the single-pole switch. With the flux and the filling material close by, the operator now places himself in the proper position, taking the carbon electrode in one hand and pulling the head-gear over his head with the other. The carbon electrode is next touched to the metal and immediately pulled away again, thus closing the circuit and striking the arc. A rotary motion is then given to the arc so that the metal in the vicinity of the weld will also be heated, thus facilitating the work. The arc should be fairly long, from 2 to 4 inches, as in this case particles of carbon are less likely to find their way into the molten metal. If the arc proves troublesome, either by going out or by being too violent, the resistance in the circuit should be decreased in the first instance or increased in the second. After the metal commences to boil, small quantities of flux and filling material should be added from time to time, the arc being stopped only long enough to allow them to be added. When soft-iron rods are used for filling material, they are sometimes first dipped in a paste made of the flux mixed with water which is allowed to dry on the rods. The operator then feeds the rod *c*, Fig. 18, directly into the weld, the arc being played on the end and so causing it to melt and at once become a part of the boiling mass. The instant welding ceases, the weld should be hammered to give the metal a closer texture or grain, as well as to eliminate sponginess.

67. Not only must precautions be taken to start with clean cavity and clean materials, but throughout the operation all dirt must be kept away. Before commencing to weld, therefore, the cavity should be chipped clean. This cleaning may be accomplished with the aid of the arc by placing the piece in such a position that when the arc is played upon it, gravity will cause the molten slag to drop free. As oxide of iron or scale readily forms on exposure of the heated metal to the air, the weld should be made, whenever possible, by a single application of the arc. Preliminary heating of the metal is not essential with wrought iron or steel, but is almost always advisable with cast or malleable iron. Annealing after welding is strongly recommended in all cases, if soft welds are to be obtained. One of the greatest criticisms of the Bernardos process of arc welding is that the welds are sometimes so hard as to make machining impossible. When this proves to be the case, the only solution is to remelt and remake the weld.

68. When building a projection or lug on a casting it is usually necessary to make a retaining wall of fireclay to hold the molten mass. When welding brass, the piece should be carefully supported and the arc applied but intermittently, otherwise warping will take place. Welds made by the Bernardos process may be taken as averaging 70 per cent. of the strength of the original material. As in oxyacetylene welding, the section of the metal at the weld may sometimes be increased over that of the other portions, thus compensating for its lesser strength.

69. Cutting of Metals.—The Bernardos process may be used for cutting work of the rougher kinds, as in cutting away surplus metal, such as the sink heads on steel castings, for opening the tap holes in iron cupolas, etc. About the same methods are employed as in welding, except that the current and voltage are usually higher, sometimes 800 to 1,200 amperes at 100 or more volts.

THOMSON ELECTRIC-WELDING PROCESS

70. Another method of electric welding is by the *incandescent process*, in which the ends of the pieces to be welded are brought to a welding heat by passing a large current through them, and when they are heated pressing them together firmly, thus forming the weld. The heating power of a current depends on the rate of flow, or the number of amperes. In electric welding, the pressure may be only a few volts or even a fraction of a volt, while the current is large; that is, a large number of amperes is required to obtain the necessary heating effect. The **Thomson electric-welding process** is one form of incandescent electric welding.

71. Apparatus.—An electric-welding machine is used for welding by the Thomson process. This machine holds the pieces to be welded, provides means for heating them by the electric current and for forcing them together to form the weld. A welding machine used in welding flat iron hoops, shown in Fig. 21, consists of a heavy cast-iron base *a* that supports two bronze clamps *b*, *c*, which are operated by the handles *d* and *e*. The clamp *b* is fixed, but the clamp *c* can be moved toward the clamp *b* by means of the lever *f*. Water flows through the pipes *h* into the clamps, which are hollow, and prevents them from becoming overheated. The rubber tube *h'* allows water to circulate from one clamp to the other, and permits the clamp *c* to be moved, which could not be done if *h'* were a solid pipe.

Since water and iron are both conductors of electricity, a small part of the current will flow from one clamp to the other through the water in the tube h' and through the lower solid part of the hoop. But since the resistance along each of these paths is much greater than that directly between the clamps, the small amount of current thus lost does not affect the action of the machine. By changing the form of the clamps, this machine can be used for other classes of work.

72. Making a Joint.—To form a weld by the Thomson process, the ends of the hoop are fixed firmly in the clamps,

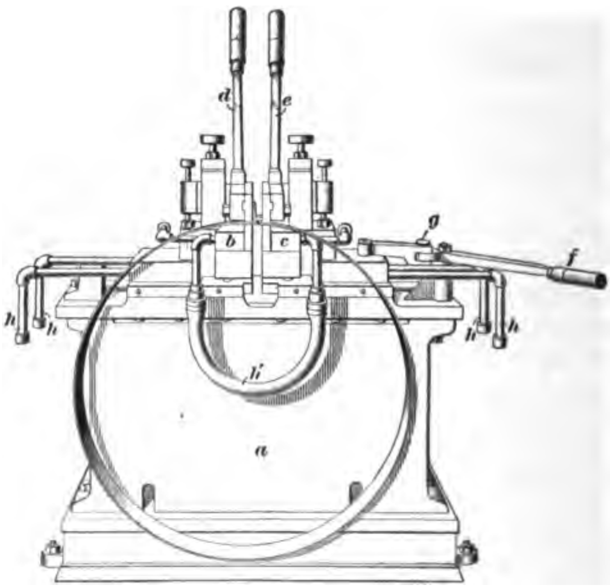


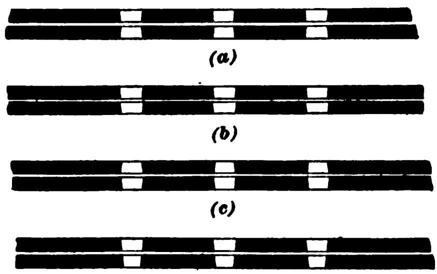
FIG. 21

as shown, the ends are then brought together, and the current is turned on. The current passes into one end of the hoop from the clamp *b*, Fig. 21, across the minute air gap between the ends of the hoop, and then back through the clamp *c*. But the resistance of this small air space, or gap, between the ends is great enough to bring the metal ends to a welding temperature. As soon as the welding temperature is reached, the lever *f* is pulled toward the left, when the joint at *g* straightens and moves the clamp *c* to the left. The two ends are thus forced together and the weld is completed.

RIVETING

73. Hand Riveting.—It is often desired to fasten together two pieces of iron or steel by riveting, as, for example, in joining the plates that form a steam boiler. Holes are punched or drilled through both pieces, and a *rivet*, which is a pin having a head at one end, is put through both holes. The small end of the rivet is then upset so as to form a second head. Riveting may be done cold, but if a tight joint is required the rivet is heated and headed while red hot; in cooling, it will contract and draw the heads together, thus making the joint tighter.

74. The rivet holes in plates are generally punched; this makes them slightly tapered, and if two punched plates are joined the holes may come together in any one of three ways, as shown in Fig. 22 (*a*), (*b*), and (*c*). If the rivets are hot enough, they will readily upset so as to fill the holes in any case; but passing them through the cold plates frequently chills them so that they will not upset properly. For this reason, when the holes are not in line, they should be smoothed with a *drift pin*, or, better still, reamed out after the plates have been clamped in position, as shown in (*d*). The reaming also removes some of the injured metal from the sides of the holes. The *drift pin* is a smooth, slightly tapered pin that is driven into the holes to expand them in places where they are too small, thus giving them an even taper. The reamer gives the same result by cutting away the rough edges of the hole. Drifting strains or distorts the metal, and should never be done on any riveted joint that must bear a heavy load. The hot rivet is put in the hole, and a heavy piece of iron or a hammer is held against the head while the other head is being formed. The body of



(a) (b) (c) (d)

FIG. 22

the rivet should completely fill the holes in the plates after the rivet is headed.

75. Machine Riveting.—Where a large number of rivets are to be headed, the work is done by machine, which does it much faster and better than it can be done by hand, because the rivet is headed before it has had time to cool. The hammer of the riveting machine is sometimes driven by compressed air at the rate of several hundred blows per minute, the rapidity of the blows being more necessary than their force. In forming the head, quick blows with a light hammer upset and spread the iron, while heavy blows tend to bend it. Another form of riveting machine presses the end of the rivet into a head by forcing a forming die against it. This machine is operated by compressed air or by water pressure, and gives satisfactory results, as the flow of the metal under the heavy pressure completely fills the rivet holes. In taking riveted plates apart, it is better to drill the rivets out than to drive them through, since it is less liable to injure the plates.

BENDING BRASS AND COPPER PIPE

76. Annealing Brass and Copper Pipe.—Whenever a piece of brass or copper pipe or tubing is to be bent or shaped, it must be first annealed, which should make it so soft that the smaller sizes can be bent by hand. This *annealing*, or softening, is done by heating the metal evenly to a dull-red heat and then plunging it in cold water.

77. Bending Small Tubing.—The simplest way to make a bend in a small tube is to turn a block of hardwood to the radius of the desired curve and then bend the pipe about the block. When the radius is small, this may be done as shown in Fig. 23, *a* being the block about which the pipe is to be bent and *d* a square block of the same thickness, clamped in a vise *b*, so as to hold the end of the pipe during the bending. After the two blocks *a* and *d* are so placed that the pipe can just be slipped between them, the end of the pipe *c* is slipped through to the point where it is desired to form the bend, and the other end is carried around, as indicated by the dotted lines, to the desired angle. If a greater bend than 180° is made, it is sometimes difficult to remove the wooden block from the tubing.

In some cases, a groove is turned about the block *a*, the radius of the groove being equal to the radius of the outside of the pipe, so that the pipe will bed istelf in the groove while being bent. This process aids in keeping the pipe from flattening. This simple device will serve to bend pipe up to ¾ inch in diameter, and is sometimes used for larger sizes.

78. Support of Tubing During Bending Process.—To prevent the tubing from kinking or flattening while being bent, the inside must be filled with some substance. Sometimes, when there is a thread on each end of the tube, it is filled with sand and a cap screwed on each end; or the tube may be filled with water and the ends capped. When water is used as a filling material, care must be taken to fill the pipe completely, for if it contains any air the latter may be compressed and allow the pipe to flatten at some point. The more common practice is to fill the tube with melted resin and allow this to harden. During bending, the resin will be pulverized, but it will prevent the tube from flattening. After the bend is made, the resin can easily be melted and run out. Pipes less than ⅝ inch in diameter are bent without filling. Occasionally, if the metal is thick, larger pipes may be bent in this manner, but no risk should be taken if a good bend is required.

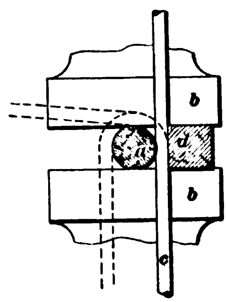


FIG. 23

79. Bending Large Tubing.—To bend large tubes, a special device like the one shown in Fig. 24 is used. It consists of two wheels *a* and *b* arranged as shown. The wheel *a* is clamped in a vise or by means of a special clamp. If it is required to bend greater angles than 90°, the vise or clamp must be so located that the lever *c* can make the desired portion of a revolution. The lever *c*, which is forked at the end and carries the wheel *b*, is pivoted to the pin *g* passing through the center of the wheel *a*. Attached to the wheel *a* is a clamp *d* that holds the tube in its proper position. The radius of the wheel *a* must be equal to that of the desired curve, and the outside of each wheel is turned to such a form that when the wheels are in position they barely allow the tube to pass between them, which prevents any tendency of the metal to flatten or buckle. The clamp *e* is placed on the tube, so that it will locate the point at which the bend is to begin. After the tube is in place,

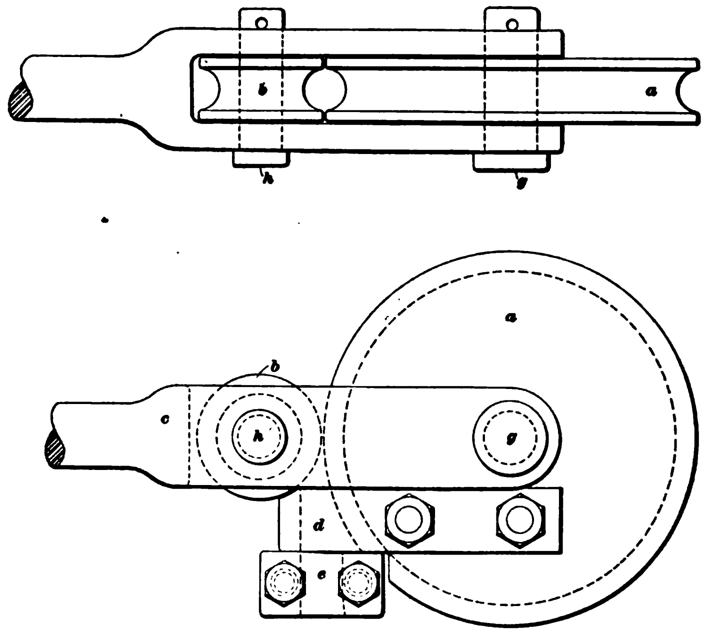


FIG. 24

the lever *c* is carried around the wheel *a* and the bend is formed. To remove the bent tube, the pin *h* may be taken out to allow the removal of the wheel *b*; or the pin *g* may be removed, thus allowing the entire lever *c* to be taken away from the wheel *a*. The radius of the larger wheel *a* is made from $\frac{1}{32}$ to $\frac{1}{16}$ inch less than that of the corresponding radius of the pipe, to allow for the spring when the pipe is released. The wheel *b* is made as small as the stresses on it will permit.

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NOTE.—In this volume, each Section is complete in itself and has a number. This number is printed at the top of every page of the Section in the headline opposite the page number, and to distinguish the Section number from the page number, the Section number is preceded by a section mark (§). In order to find a reference, glance along the inside edges of the headlines until the desired Section number is found, then along the page numbers of that Section until the desired page is found. Thus, to find the reference "Acute angle, §5, p3," turn to the Section marked §5, then to page 3 of that Section.