SHOP MATHEMATICS

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SHOP MATHEMATICS

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PREFACE

This book is the result of twelve years' experience as draftsman and shop foreman combined with an equal length of service as instructor in day and evening classes in technical and trades schools.

The main feature of the book is the collection of practical shop problems, the most of them being either actual problems which have arisen in the author's experience or those suggested by that experience, and were first collected for use in his own classes.

The formulas given are those usually found in mechanics' hand books, and the author acknowledges his indebtedness to Wm. Kent and P. Lobbin for permission to use their formulas; also to Brown & Sharpe Mfg. Co. for courtesies extended, and especially to Mr. C. S. Bragdon of the Technical High School, Springfield, Massachusetts, for assistance in collecting and arranging material.

EDWARD E. HOLTON.

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INTRODUCTION

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The loudest note sounded in all recent educational discussions is the call for more practical methods in presenting the fundamental subjects of study. It is claimed that language, science, and mathematics have been taught with too little reference to their utility in the vocations and in the ordinary affairs of life. The methods commonly employed in presenting mathematical subjects have been especially open to criticism. This is true even in some schools designed to give vocational training. Mathematical books of the kind traditional in the older schools have been continued in use in the newer and more practical schools. These books were written almost entirely from the point of view of the teacher of pure mathematics with little reference to concrete problems of life and having no reference whatever to the actual problems of the drafting-room and the shop. As a natural consequence the class room work in mathematics, in many of our most practical schools, has failed to utilize the material afforded by the shops and science laboratories to fix the knowledge of mathematical principles by concrete illustration and by practice.

There is much truth in the criticism. But what are we going to do about it? It will not do to cast aside the old-time algebras and geometries unless something really better can be found to take their place. The effort to make the applications of mathematics more easily understood might lead to the substitution of a practical course in which the mathematical element is too much diluted. This would be folly. The real object should be to strengthen,—not to weaken—the teaching of mathematics in practical schools. What is needed is to purge the old books of useless material and put in place of it practical mathematical work distinctly planned to make up for the short-comings of the old methods when measured by the practical demands of modern times.

The author of "Shop Mathematics" has had many years' experience in designing and making machine tools and in a wide range of practical shop work. In addition to this he has had a long experience as a teacher of drawing and of machine shop practice and tool-making in schools for boys and for adults. This has given him an unusual opportunity not only to find out what is needed, but to discover the facilities for supplying that need. The book contains a selection of problems that actually arise in shop practice. This is what is needed by the young mathematical student and for two reasons,—first, that he may know what the shop problems are, and, second, that he may learn how to apply mathematical principles, rules, and formulas in the solution of such problems. No attempt is made in this book to teach mathematical theory or principles. That would be a needless repetition of countless books already in existence. "Shop Mathematics" may be used to supplement the course in elementary algebra, geometry, and trigonometry, and if used in this way it will be found of great value in technical schools. But abundant rules and formulas are given under each subject so that the book will also find a place in brief practical courses which do not admit of the use of the ordinary mathematical text-book.

SIGNS, SYMBOLS, AND ABBREVIATIONS USED IN THIS BOOK

$+$, *plus*, is the sign of addition.

$-$, *minus*, is the sign of subtraction.

Plus and minus are also used to indicate positive and negative quantities.

$\times$, *times*, or *multiplied by*, is the sign of multiplication. A dot . is sometimes used for $\times$ where the quantities are expressed by letters, but is usually omitted in algebraic formulas; thus $a \times b$, or $a \cdot b$ is ordinarily written ab.

$\div$ *divided by*, is the sign of division.

: without the dash between also indicates division, being used as the ratio sign; as, $a : b$, means the ratio of a to b, or a divided by b.

$=$ is the sign of equality and indicates that the two quantities between which it is placed are of equal value.

$\therefore$, *therefore*.

$>$, *greater than*; as, $6 > 4$, read 6 is greater than 4.

$<$, *less than*; as, $4 < 6$, read 4 is less than 6.

$\angle$, *angle*.

$\perp$, *perpendicular to;* as $A \perp B$, read A is perpendicular to B.

$\|$, *parallel to*; as, $C \| D$, read C is parallel to D.

., *the decimal point*; as, *0.2*, read two tenths; or *0.004*, four thousandths.

$^\circ$, the symbol for the *degree* in the measurement of angles.

$'$ and $''$, the accents, denote *minutes* and *seconds* in the measurement of angles; as, $5^\circ\ 10'\ 15''$, read *5* degrees *10* minutes *15* seconds.

The above symbols, $'\ ''$, are also used for feet and inches in indicating dimensions; as, $3'\ 6''$, read *3* feet *6* inches.

The subscript is a small figure written at the lower right of a letter; as, A_1, A_2, A_3, read *A* sub one, *A* sub two, *A* sub three. It is used to denote corresponding parts of related objects, or sometimes to avoid using too many different letters.

$\sqrt{\ }$, the *radical sign*, denotes the square root.

Other roots are indicated by a small figure, called the index, written at the upper left of the radical sign; as, $\sqrt[3]{\ }$, $\sqrt[5]{\ }$, $\sqrt[n]{\ }$, read *cube root, fifth root, nth root.*

The exponent is a small figure written at the upper right of a quantity to indicate a power, or the number of times the quantity is used as a factor; as, 5^2 means 5×5, $3^4 = 3 \times 3 \times 3 \times 3$.

( ), *parentheses,* { } *braces,* [ ] *brackets,* —— *vinculum,* signify that the inclosed quantities are to be considered as one quantity.

sin, sine.

cos, cosine.

tan, tangent.

cot, cotangent.

sec, secant.

cosec, or *csc.*, cosecant.

π, *Pi*, the ratio of the circumference of a circle to its diameter, $= 3.1416$.

D, or *dia.*, diameter of a circle.

R, or r, radius of a circle.
P or F, power or force.
W, weight or resistance.
H. P., horse power.
R. P. M., revolutions per minute.
F. P. M., feet per minute.
f, coefficient of friction.
ft.-lb., foot pound.
ft., foot or feet.
lb., pound.
in., inch or inches.
pi., pitch.

MECHANICS

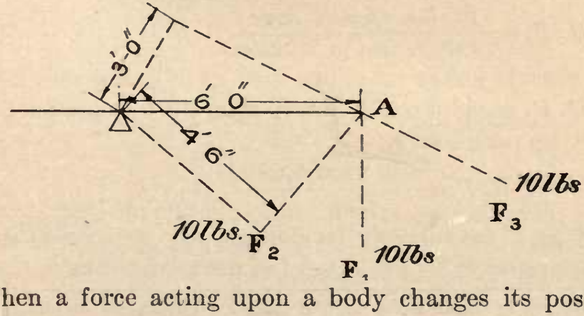
Mechanics treats of forces and of the effects of forces. **Force** is the action between two bodies tending to produce a change of position or shape; as when a horse pulls a load, a motor drives an electric car, elasticity causes the action of a steel spring, etc.

The moment of a force. If a bar of uniform size is pivoted at its center and a weight placed on one end, the bar will rotate about the pivot. The numerical value of the importance of the force in producing motion about a pivot is called its *moment* and is equal to the product of the force by the distance from its line of action to the pivot. By reference to the accompanying sketch, the moment of the force F_1, applied at point A, is seen to be $6 \times 10 = 60$ *ft.-lbs.*

of F_2, $4\frac{1}{2} \times 10 = 45$ *ft.-lbs.*

of F_3, $3 \times 10 = 30$ *ft.-lbs.*

Diagram for Moments of Force



When a force acting upon a body changes its position, *work* is done upon the body. The amount of work done depends upon the force applied, also upon the distance through which it acts; that is, *work is measured* by the resistance overcome, multiplied by the distance through which it is overcome; as a 10 pound weight that is lifted to a height of 5 feet requires an amount of work equal to the product of *10 times 5 = 50 ft.-lbs.*

The unit of measure in the above example is expressed by the term *foot-pound*, that is, a force of one pound acting through a distance of one foot, or its equivalent; as 4 pounds acting through a distance of $\frac{1}{4}$ foot, or $\frac{1}{10}$ pound acting through 10 feet, etc.

The unit can be expressed not only in ft-lbs. but in inch-pounds, foot-tons, centimeter-grams,* etc.

The amount of work done in lifting a body a given distance is the same whether done in 5 seconds or 5 minutes, but it is often necessary also to denote the *rate* of doing work.

This is expressed in terms of ***horse power***. An engine of one horse power (*1 H. P.*) means an engine capable of doing *33,000 ft.-lbs. of work in one minute.*

Therefore to find the horse power of any machine the formula used is,

$$H.P. = \frac{\text{Ft.-lbs. of work done}}{33000 \times \text{time in minutes}}$$

In electric power machines such as dynamos and motors one *H. P.* is equal to *746 watts*; then the formula for power of electric machines is:

$$H.P. = \frac{\text{volts} \times \text{amperes}}{746}$$

Energy is the capacity for doing work; as a coiled spring has the capacity to set in motion the mechanism of a clock

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*Note. For dimensions in the metric system see tables, pages 173 to 176.

or watch. The coiled spring of a watch possesses energy because at some previous time work has been performed upon it in the winding.

The amount of energy in a body is measured in *ft.-lbs.*, the unit used for measuring work. *Mechanical* energy is of two kinds, *potential, and kinetic. Potential* energy is due to the *position* of a body; as, for example, if a pile driver head weighing 50 pounds is suspended 20 feet above the ground, it has a potential energy of $20 \times 50 = 1000$ *ft.-lbs.* If now the weight is released and falls, the energy is of a different kind because of the *motion* of the falling weight. This is called *kinetic* energy. In either case the *weight times the height* equals the measure of the energy of the body or $K = Wh$.

When the *velocity* of the falling body is given instead of the height from which it falls,

Then by the laws of falling bodies

$$h = \frac{v^2}{2g}$$

Where $v =$ velocity in feet per second

$g = 32.16$ feet

Then $$K = \frac{Wv^2}{2g}$$

$K =$ the kinetic energy

$W =$ weight of the body in pounds.

PROBLEMS

1. A drop hammer weighing 400 lbs. falls from a height of 36 in. What kind and how much energy will be exerted?

2. A weight of 500 lbs. is used for breaking up old car wheels and is suspended 15 ft. above the anvil block. Calculate the kind and amount of energy.

3. An elevator car weighing 2 tons requires how much energy to lift it a distance of 20 feet?

4. A tank contains 5,000 gals. of water, 50 ft. being the average height above the ground. What is the energy of the water at the ground?

5. A pile driver head weighing 175 lbs. falls from a height of 18 ft. What is the energy at the end of the fall?

6. A man weighing 180 lbs. jumps up to a platform 42 inches above the ground. How much energy was expended?

7. If the man in problem 6 should climb a distance of 100 ft. above a certain point, how much energy would be exerted?

8. If a hammer that weighs 1 lb. has a velocity of 22 ft. per sec., what is its energy?

9. If an iron ball weighs 100 lbs., what is its energy when suspended at a height of 28 ft. from the ground?

10. A rock drill is operated by a 10 lb. sledge hammer with a striking velocity of 30 ft. per sec. Find the energy used in the drilling.

11. What is the *H. P.* of a pump that raises 100,000 gals. of water per hour to a reservoir 25 ft. above the level of the lake from which it is pumped?

Note. One cu. ft. of water weighs approximately $62\frac{1}{2}$ lbs. and contains about $7\frac{1}{2}$ gals.

12. A motor is able to lift an elevator, which with load weighs 5 tons, to the top of a tower 500 ft. high in two minutes. What *H. P.* is required to do the work?

13. What must be the *H. P.* of the engine required to raise a block of granite weighing 10 tons to the top of a wall 35 ft. from starting point when it takes 20 minutes to do the work?

14. If it takes $1\frac{1}{2}$ hours to raise a weight of 20 tons 100 ft., what *H. P.* engine will be required?

15. If a workman carries 5 tons of pig iron up a flight of steps 14 ft. high in 10 hours, how much work does he accomplish expressed in ft.-lbs.?

16. A rope turns a pulley 48 in. dia. with a pull of 10 lbs. at the rim at the rate of 2500 ft. per minute. How many ft.-lbs. of work are done in 5 hours consecutive movement?

17. A 12 lb. ball falls 2500 ft. What is its velocity when it strikes and what is its kinetic energy?

18. A 500 volt electric motor runs 50 machines using 22 amperes of current. What is its *H. P.*?

19. What is the *H. P.* of a dynamo which will run 300 110-volt incandescent lamps if each lamp consumes $\frac{1}{2}$ ampere of current?

20. An electric motor has a voltage of 250 and supplies 30 amperes to run a certain set of machines. Find the *H. P.* of motor.

21. What *H. P.* will be required to run a 250 lamp circuit, the lamps being the same as in problem 19?

22. A direct connected dynamo delivers power at 110 volts to a 1400 electric light circuit, also to 9 motors with normal amperage as follows: 1 at 144, 1 at 110, 1 at 80, 3 at 76, 1 at 57 and 2 at 20 amperes. What *H. P.* will be required to run the above equipment?

MECHANICAL POWERS

An appliance by which force can be used to do useful work is called a *machine.*

The mechanical powers or elements of machines are six in number, as follows:

1. Lever.
2. Wheel and Axle.
3. Pulley.
4. Inclined plane.
5. Screw.
6. Wedge.

The mechanical advantage of all kinds of machines, whether simple or compound, may be computed in accordance with the following

GENERAL LAW

The force multiplied by the distance through which it moves is equal to the resistance or weight multiplied by the distance through which it moves, or $P \times dP = W \times dW$.

Each class of machines permits of a special statement of this law by substituting for the general terms the special terms used with that class of machines.

1. THE LEVER

The *lever* is an inflexible bar or rod supported at some point, the bar being free to move about that point as a pivot.

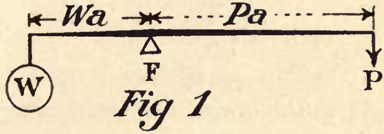
This pivotal point is the *fulcrum*, usually represented by F.

The force applied to the lever is represented by P.

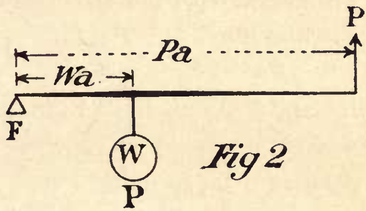
The weight lifted, or the resistance to the force is represented by W.

The lever is classed according to the position of the three points P, F, W.

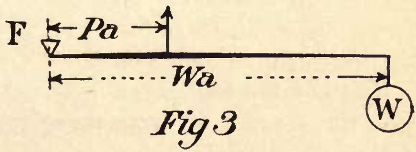
A lever of the *first class*, Fig. 1, has F between P and W.



A lever of the *second class*, Fig. 2, has W between F and P.



A lever of the *third class*, Fig. 3, has P between F and W.



The machinists' and the tinsmiths' pliers are examples of levers of the first class.

The nut cracker and lemon squeezer, are levers of the second class.

The sheep shears and firm joint calipers are good examples of levers of the third class.

The three classes of levers are operated and controlled by the following:

LAW FOR LEVERS. The force multiplied by its distance from the fulcrum is equal to the weight multiplied by its distance from the fulcrum.

Let $P =$ force

$W =$ weight or resistance.

$Pa =$ distance from fulcrum to point where force is applied.

$Wa =$ distance from fulcrum to point where weight is applied.

Then the law of Levers becomes $P \times Pa = W \times Wa$.

From this equation the following are readily derived.

$$P = \frac{W \times Wa}{Pa}; \quad W = \frac{P \times Pa}{Wa}; \quad Pa = \frac{W \times Wa}{P}; \quad Wa = \frac{P \times Pa}{W}$$

Example. What force 18 in. from fulcrum will balance a weight of 870 lbs. 3 in. from the fulcrum?

Solution. By formula:

$$P = \frac{W \times Wa}{Pa}, \; P = \frac{870 \times \cancel{3}}{\underset{6}{\cancel{18}}} = \frac{870}{6} = 145 \text{ lbs., Ans.}$$

The law for bent levers is the same as for straight levers but the lengths of arms are computed on lines from the fulcrum at right angles to the directions in which P and W act.

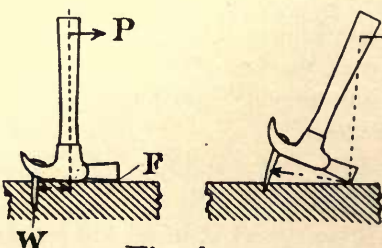


Fig 4

The lengths of the arms of a bent lever are continually changing as the force and weight move into new positions. The case of pulling out a nail with the common claw hammer is a good illustration of the bent lever, Fig. 4.

The *moving strut* in Fig. 5 is a bent lever with one arm lacking. The force is applied at the same end at which the resistance is to be overcome. The resistance in this case is not the weight, W, but its resistance to being moved. The ratio between force and resistance changes as the angle A changes.

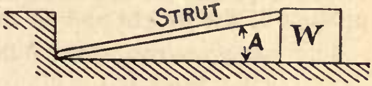


Fig 5

Then

$$force : resistance = \sin A : \cos A$$

or

$$P \times \cos A = W \times \sin A$$

and $$P = \frac{W \times \sin A}{\cos A}; \quad W = \frac{P \times \cos A}{\sin A}$$

The *toggle joint* is a double strut, Figs. 6 and 7.

The statement is

$$P : W = 2 \sin A : \cos A$$

$$\therefore P = \frac{W \times 2 \sin A}{\cos A}$$

$$W = \frac{P \times \cos A}{2 \sin A}$$

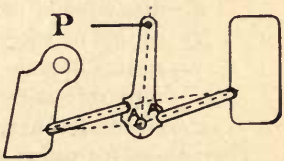


Fig 6

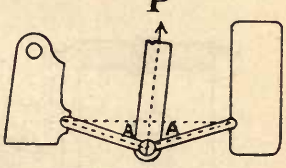


Fig 7

A *compound lever* is a combination of two or more levers which may be of the first, second or third classes. The calculations for compound levers can be made by taking each lever separately by the formula for single levers. These separate operations, however, are usually condensed into one, in accordance with the following:

LAW OF COMPOUND LEVERS. The continued product of the force and all the force arms is equal to the continued product of the weight and all the weight arms.

This law gives rise to the following formulas:

$$P = \frac{W \times Wa \times Wa_1 \times Wa_2 \ldots}{Pa \times Pa_1 \times Pa_2 \ldots}$$

$$W = \frac{P \times Pa \times Pa_1 \times Pa_2 \ldots}{Wa \times Wa_1 \times Wa_2 \ldots}$$

II. WHEEL AND AXLE

The *wheel and axle* may be considered a continuous lever. By its use a continuous motion is obtained for raising a weight. The two arms of the lever are the diameters of the wheel and axle, or the radii of wheel and axle.

If D is diameter of wheel:

d = diameter of axle
R = radius of wheel
r = radius of axle
C = circumference of wheel
c = circumference of axle

Then these formulas apply: $P : W = d : D$
$P : W = r : R$
$P : W = c : C$

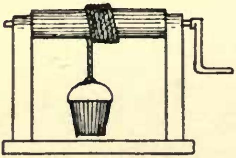


Fig. 8.

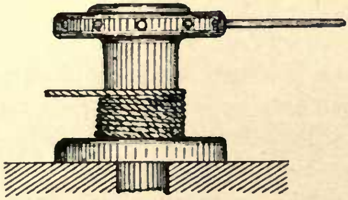


Fig. 9.

Fig. 8 shows sketch of simple *windlass*.

Fig. 9 shows sketch of *capstan*, which is a wheel and axle where the axle has a vertical position. It is used on ships for raising the anchor.

The mechanical advantage of the wheel and axle may be increased by making the diameter of wheel larger or the diameter of axle smaller.

The windlass is a modification of the wheel and axle where a crank is substituted for the wheel. It is used especially where the power is applied by hand.

The *differential windlass*, Fig. 10, is a device for increasing the mechanical advantage of the axle by unwinding rope from a small axle and winding it on to an axle of larger diameter; for with one turn of crank C, the section of rope supporting the weight will be shortened a distance equal to circumference of the large axle minus the circumference of the small axle and W will be raised half this distance.

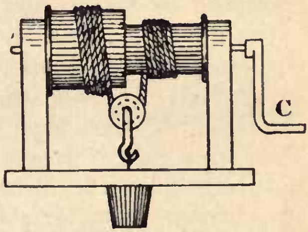


Fig. 10.

By the law of the wheel and axle,

$$P : W = r : R$$

then in the differential windlass

$$P \times R = W \times \frac{(r_1 - r)}{2} \quad \text{or} \quad P \times 2R = W \times (r_1 - r)$$

r_1 = radius of large axle

r = radius of small axle

When motion is transmitted from one body to another by direct or by indirect contact the body that produces the motion is called the *driver*, the body that receives the motion is called the *follower*.

In a combination of wheels and axles each pair may be determined separately, but the shorter way is to use a formula similar to that used for compound levers, as follows:

$$P=\frac{W\times r\times r_1\times r_2 \ldots .}{R\times R_1\times R_2 \ldots ..}$$

Where R, R_1, R_2, are the radii of the wheels and r, r_1, r_2, are the radii of the axles.

These problems may also be calculated by the following:

RULE FOR DRIVERS AND FOLLOWERS. The speed of the first driver multiplied by the product of the sizes of all the drivers is equal to the speed of the last follower multiplied by the product of the sizes of all the followers.

For the sizes of followers and drivers may be taken the circumferences, diameters, radii or number of teeth (if a gear wheel); only whatever dimension is taken for any driver the corresponding dimension must be taken for its follower. Thus for a train of wheels having four drivers and followers, the diameters can be taken for one pair, the radii for another, the circumferences for a third and number of teeth for the fourth, to solve the problem.

Let N = the $R. P .M.$ of first driver
n = the $R. P. M.$ of last follower
D, D_1, D_2 = size of drivers
F, F_1, F_2 = size of followers

Then by formula,

$$n=\frac{N \times D \times D_1 \times D_2}{F \times F_1 \times F_2}$$

The size of any driver or follower can be found by the formula:

$$F=\frac{D \times N}{n}$$

$$D=\frac{F \times n,}{N}$$

where, $N = R. P. M.$ of the driver
$n = R. P. M.$ of the follower

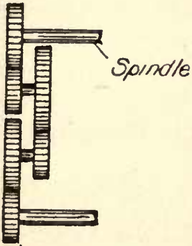


Fig. 12.

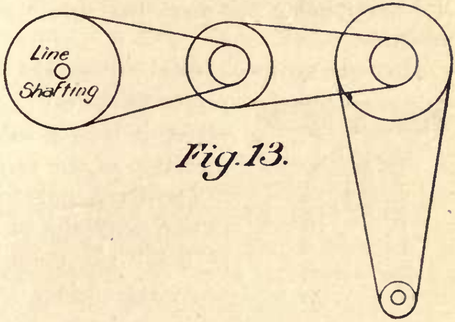


Fig. 13.

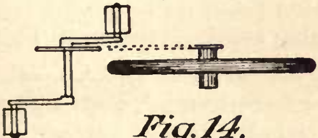


Fig. 14.

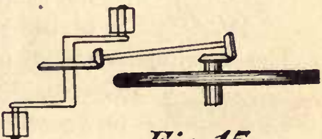


Fig. 15.

Figures 12, 13, 14 and 15 are good illustrations of the above rule in shop practice. Fig. 12, shows a combination of the stud and change gears used on the engine lathe for thread cutting, and on the milling machine for cutting spiral flutes in cutting tools. Fig. 13, shows the belting from main line drive pulley through two countershafts to spindle pulley of machine. Fig. 14 shows sketch of chain and sprocket drive for bicycle or automobile, which form of drive is also being used extensively for positive drives in some kinds of machine construction. Fig. 15 is the bevel gear drive.

III. THE PULLEY

The *pulley* is a wheel over which a cord, band or chain is passed to transmit the force applied to the cord in another direction. The wheel is introduced to diminish the friction, the band being the part that gives practical effect to the machine.

There is no mechanical advantage gained with the *fixed* pulley as shown in Fig. 16, but as stated above it is of great use in changing the direction of the force.

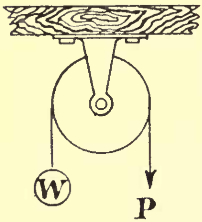


Fig. 16.

The fixed pulley when used in combination with the *movable pulley* as shown at B in Figs. 17 and 18 has mechanical advantage, since the weight is carried by strands of the cord on either side of the movable pulley.

The usual arrangement of pulley blocks is shown in Figs. 17 and 18 where one or more wheels or sheaves (grooved pulleys) are placed in suitable pivots and bearings, and cords are passed over the pulleys connecting the force and weight.

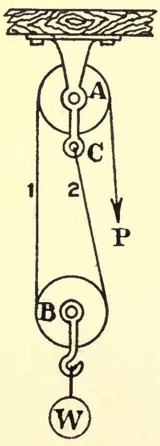


Fig. 17.

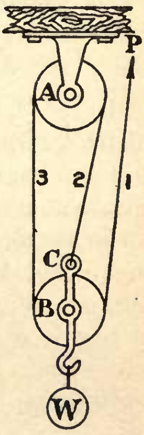


Fig. 18.

In Fig. 17, if there is a single pulley at A and B and a rope is passed over A and around B and fastened at C, then a pull of 1 pound at P will transmit a pull of 1 pound to the rope on the other side of pulley A; this will transmit the same amount of force at B, which in turn will transmit the same force to the other side of pulley B. Thus 1 pound pull at P

will balance 1 pound at *1* and *2* or the combined pull of *1* and *2* at W, so that 1 pound at P will support 2 pounds at W. Again, if rope at P moves down 1 foot the rope must move up 1 foot on the other side of A: but the end of rope is fixed at C, so that when rope moves up at *1* and remains stationary at *2*, one-half the motion will be given to the movable pulley between *1* and *2*, which is in accordance with the general law of machines $\therefore W = 2\ P$.

If there are two sheaves on each block A and B, each turning independently of the other in the bearings, the pull of 1 pound at P will be transmitted as in the first case and will also transmit the pull to the third and fourth strands, so that a pull of 1 pound at P will balance a weight of 4 pounds at W. Similarly it can be shown that 1 foot downward travel at P will give an upward travel of $\frac{1}{4}$ foot at W. From these principles is obtained the following:

RULE FOR PULLEYS. The force multiplied by the number of strands from the movable pulley will equal the weight that can be raised,

or $P \times N = W$,

where N is the number of strands from the movable pulley.

Whenever possible the pulleys should be arranged so as to pull in the direction in which the weight is to be moved, as shown in Fig. 18, for whereas in Fig. 17, $N = 2$, in Fig. 18, $N = 3$.

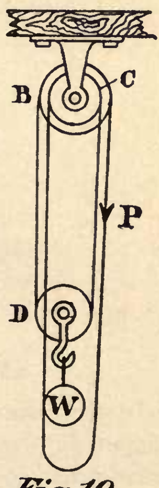


Fig. 19.

The *differential pulley*, Fig. 19, is quite generally used in shop practice, the band for transmitting the force being an endless chain of iron links. The principle of the differential pulley is very similar to that of the differential windlass. The radius of the movable pulley F must be an exact mean between the radii of B and C, in order to have a uniform velocity ratio, so that chain links may fit the pockets in sprocket wheels.

Let R = radius of pulley B,

r = radius of pulley C.

While the pull at P moves the chain between B and D up a distance R, the chain at C will move down a distance r, the loop around the pulley D will be shortened the distance R-r and W will be raised one-half this amount. From this is obtained the statement as follows:

$$P \times R = W \times \frac{R-r}{2}$$

$$P = \frac{W \times \frac{R-r}{2}}{R}$$

$$W = \frac{P \times R}{\frac{R-r}{2}}$$

MISCELLANEOUS PROBLEMS

In the following problems the weight of the lever and friction of the bearings will not be considered in the calculations.

1. What force 36 in. from the fulcrum will balance a weight of 500 lbs., 9 in. from the fulcrum in a lever of the first class?

2. If a pull of 75 lbs. is made on the end of a bar 6 ft. long with the fulcrum 6 in. from the other end, what weight in a lever of the first class will just balance the pull?

3. If the lever of problem 2 had been of the second class and the weight 6 in. from the end, what weight would have been required?

4. When a weight of 324 lbs. is balanced on the end of a lever of first class by a force of 62 lbs. 27 in. from fulcrum, what distance is the weight from fulcrum?

5. When a weight of 685 lbs. $2\frac{3}{4}$ in. from the fulcrum is balanced on a bar by a pull of 75 lbs., what distance is the force from the fulcrum?

6. What weight $4\frac{1}{2}$ in. from the fulcrum will be balanced on a bar by a force of 96 lbs. 48 in. from the fulcrum?

7. What force 10 ft. from the fulcrum will just raise a weight of 2465 lbs. 8 in. from the fulcrum with a lever of the second class?

8. A bar of iron was placed with one end against the wall of a shop, the other end being placed against the leg of a machine that weighed 1500 lbs. If the bar lay at an angle of 10° to the floor, what weight would be required on the end next to the machine to move the machine along the floor assuming that the resistance is equal to the weight of the machine?

9. A quartz crusher jaw requires a force of 10,000 lbs. to crack a certain stone. If the angle through which toggle arms move is $5\frac{1}{2}$°, find the force that must be put on the toggle arms to break the stone.

10. The power on the end of a strut is 124 lbs., the angle is 5°. What resistance could it overcome?

11. A baling press requires a pressure of 6000 lbs. to bale a bundle of steel chips. The platen is operated by a toggle joint with arms at an angle of 10°. What force will be required to operate the press?

12. The knife of a paper shear is operated by a toggle joint, the arms beginning the cut at an angle of $4\frac{1}{2}$°. If a pull of 650 lbs. is required to start the arms to obtain a cut, what is the resistance on the knife blade?

Fig. 20 shows a practical example of a lever of the first class. The long end of the pointer at *C* multiplies the error in the position of point *A* with respect to the axis of the lathe spindle, in proportion to the length of the arms *A B* and *B C*. If the distance from *A* to *B* is $\frac{1}{2}$ inch and from *B* to *C* is 8 inches the amount of motion of *C* will be in the ratio of 8 to $\frac{1}{2}$ or 16 times as much as *A*.

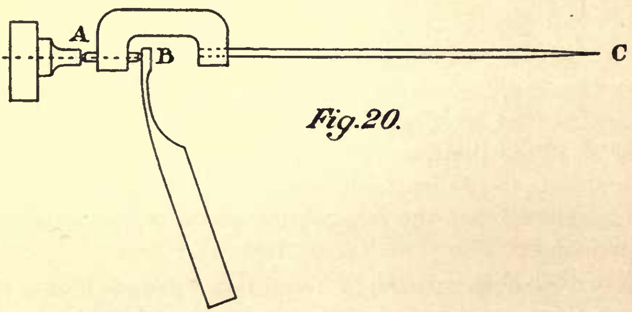


Fig. 20.

13. The pointer of a lathe test indicator is 15 in. long over all; if the short arm of lever is .375 in. and point *c* moves in a path $\frac{1}{32}$ in. across, what amount of error in position of the point *A* is indicated?

14. The bar of a lathe test indicator is 13 in. from the end to a point in the work to be tested and the short arm is $\frac{9}{16}$ in. How large a circle will be described at long end when short end is $\frac{1}{1000}$ in. out of line with the axis of lathe spindle?

15. The axis of a windlass is 6 in. dia. and the crank is 20 in. long. If a pull of 56 lbs. is given at the end of the crank what weight will be raised?

16. Six men each exert a force of 125 lbs. on the ends of 3 ft. capstan bars, the barrel of capstan is 16 in. and the bars enter 6 in. into the cap of capstan, which is 24 in. dia. What weight will be lifted?

17. The steering wheel of a ship is 6 ft. dia. and the drum is 12 in. dia. What resistance can be overcome on ropes from drum by a force of 200 lbs. at the rim of the steering wheel?

18. The crank of a differential windlass is 30 in. long and the force applied at the end of the crank is 75 lbs. When the diameters of the axles are 12 in. and 14 in., what weight will be raised?

19. A weight of 10,000 lbs. is to be raised by a differential windlass with a force of 100 lbs. What length of crank will be required with axles 12 in. and 13 in. diameters?

20. What is the force required on a crank 18 in. long on a differential windlass, with axles 18 in. and 20 in. diameters, to raise a weight of 8,000 lbs?

21. A compound lever of first, second and third classes has levers each 4 ft. long, the short arm in each lever being 12 in. long. What weight can be lifted with a force of 10 lbs.?

22. If the short arm of a lathe test indicator is 1 in. long, and the length of the pointer over all is 14 in., what is the amount of motion at the end when indicated point is $\frac{1}{1000}$ in. from axis of the lathe spindle?

23. A windlass is to raise a weight of 1000 lbs. with axle 3 in. dia. What length of crank is required when a force of 120 lbs. is to be used on end?

24. A windlass is to raise 1200 lbs. with a 24 in. crank. If a force of 120 lbs. is applied at the end of crank, what is the dia. of the axle required?

25. If a wheel and axle, 24 in. dia. of wheel and 3 in. dia. of axle, required 25 lbs. force to raise a weight of 200 lbs., what size wheel will be required to raise 500 lbs. with an axle 2 in. dia. and a force of 36 lbs.?

26. A windlass with an 18 in. crank and 3 in. axle will require how much force to raise a weight of 1500 lbs.?

27. What dia. of axle will be required for a windlass with an 18 in. crank to raise a weight of 1000 lbs. with a force of 35 lbs.?

28. What dia. of axle will be required for a windlass to raise 1600 lbs. with a 20 in. crank and a force of 45 lbs.?

29. With a wheel 30 in. dia. and axle 5 in. dia., what force will be required to raise a weight of 500 lbs.?

30. In a combination of three wheels and axles, when R is 10 in. for each wheel and r is 4 in. for each axle, what weight at W can be supported by a pull at P of 25 lbs.?

31. If P moves a distance of 5 ft. in problem 30, how far will W move in the same time?

32. Find the ratio of speeds between P and W in problem 31.

33. How many *R. P. M.* will a grinder spindle make that is driven through two counter shafts, Fig. 13, main line pulley

30 in. dia. and 150 *R. P. M.* with drivers on counters each 24 in. dia., followers 6 in. dia. and spindle pulley 3 in. dia.?

34. If it is desired to drive the spindle of problem 33 20,000 *R. P. M.*, what size drive pulley will be required on main line if all the other pulleys remain the same?

35. A spindle is driven by a train of gears, as in Fig. 12; the first driver on shaft has 50 teeth, the two stud driving gears each 50 teeth, and the followers each 125 teeth with 150 teeth in gear on spindle. What is the number of revolutions of spindle with one turn of the shaft?

36. If one set of stud gears were thrown out of mesh in the train of problem 35 and the remaining gears moved into mesh, what is the number of revolutions of spindle for one turn of the shaft?

37. If 60 teeth gears were substituted in the place of the 50 teeth in problem 35, what would be the number of revolutions of the spindle for one turn of the shaft?

38. Find the "*gear*" of a bicycle, Fig. 14, having a rear wheel 28 in. dia. with 18 teeth in front and 7 teeth in rear sprocket.

Note. The term *gear* as used in problem 38 means that the forward motion due to one revolution of the crank is the same as would be produced by one revolution of a wheel whose diameter in inches is equal to the "*gear*."

39. What is the "*gear*" with 17 teeth in front, 8 teeth in rear sprocket, with 30 in. dia. of wheel?

40. What is the "*gear*" with 30 teeth front, 10 teeth rear and 28 in. dia. of wheel?

41. What is the "*gear*" with 16 teeth front, 7 teeth rear with 30 in. dia. of wheel?

42. Find the "*gear*" with 50 teeth front, 15 teeth rear and 28 in. wheel.

43. Find the "*gear*" with 22 teeth front, 7 teeth rear and 26 in. wheel.

44. What is the "*gear*" with 32 teeth front, 12 teeth rear and 26 in. wheel?

45. What is the "*gear*" with 24 teeth front and 7 teeth rear sprocket and 24 in. dia. of wheel?

46. A man weighing 150 lbs. has to raise a weight of 1,200 lbs. How many sheaves must be placed in each pulley block as arranged in Fig. 17 to raise the weight?

47. What force would be required in problem 46 if a pulley is used as in Fig. 19, when sheave B is 8 in. dia. and C is 7 in. diameter?

48. With the pulley blocks as in Fig. 17, with 4 sheaves in both A and B, what weight can be raised with a 15 lb. pull at P?

49. What weight can be lifted with a pair of 6 sheave pulley blocks, as in Fig. 17, with an 85 lb. pull at P?

50. When the pull of 85 lbs. is in the direction shown in Fig. 18, what weight can be raised with 6 sheave pulley blocks?

51. What is the ratio of the efficiency of the pulley blocks of problems 49 and 50?

52. What force will be required to lift a 1,200 lbs. machine with the pulley blocks of problem 48?

53. How many men weighing 150 lbs. each can raise, by their own weight, a block of stone that weighs 10,000 lbs. with a pulley block as in Fig. 19, when sheaves B and C are 10 in. and 9 in. dia. respectively?

54. Find the size of a differential pulley, to raise 6 tons with a force of 150 lbs. when the difference of the diameters is $\frac{1}{4}$ inch.

IV. INCLINED PLANE

The *inclined plane* is a flat surface sloping or inclined to the horizontal. With the inclined plane a weight can be raised by a force of less magnitude as the following illustration will show.

Illustration. Suppose the weight W is to be raised from a horizontal AC to a point D, Fig. 21. If the weight is raised in a vertical line, as CD, then P and W act through the same distance $\therefore$ by the general law of machines $P = W$. But if the weight is pushed up the incline AD, then the force acts through the distance AD while the weight is lifted only the distance CD; and the statement becomes

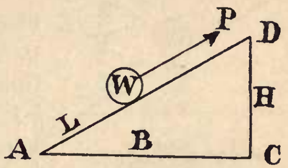


Fig. 21.

$$P : W = CD : AD$$

or $$P : W = \textit{height of plane} : \textit{length of plane},$$

from which are obtained the formulas

$$P = \frac{W \times H}{L}$$

and $$W = \frac{P \times L}{H}$$

If the force acts along a line parallel to the base, AC then

$$P : W = \textit{height} : \textit{base}$$

from which is obtained

$$P = \frac{W \times H}{B}$$

and $$W = \frac{P \times B}{H}$$

If the force acts at any angle to the plane as Y Fig. 21A then $P : W = sin\ x : cos\ Y$

and $$P = \frac{W \times sin\ x}{cos\ Y} \qquad W = \frac{P \times cos\ Y}{sin\ x}$$

Example. A horse pulling a load on the level has only the friction to overcome, but the moment that it starts up an incline it has a part of the weight of the load added to the force required to overcome friction. If the pull was 500 lbs. on the level and the load weighs 1,200 lbs., what extra force is required on an incline of 1 foot in 20 feet?

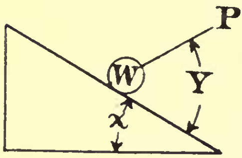


Fig. 21A.

Solution. By formula:

$$P = \frac{W \times H}{L} = \frac{1200 \times 1}{20} = 60. \quad \text{Then } 60 + 500 = 560 \text{ lbs.}$$

V. THE SCREW

The *screw* is an inclined plane wrapped or wound around a cylinder. If the incline is long in comparison with the diameter of the cylinder, it may extend more than once around the cylinder forming the *threads* of the screw; the height of the incline in going once around is called the *lead* of the *screw*. The term *pitch* is used to designate either the *number of threads per inch* or the *distance from the top of one thread to the top of the next;* hence in a single thread screw the lead is the same as the pitch, but in a double or triple thread screw the lead is two or three times the pitch.

When the screw is turned on its axis through one revolution, the nut being stationary, the screw is raised or lowered a distance equal to the lead of the thread; the force moves in the same time a distance equal to the circumference of twice the length of the lever or bar that is used to turn the screw on its axis. From this is obtained the following:

RULE. The force multiplied by the circumference of the circle through which the force arm moves equals the weight multiplied by the lead of the screw.

From this rule is obtained the statement:

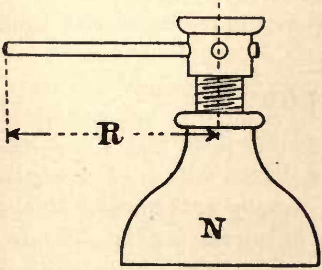


Fig. 22.

$$P : W = L : 2\pi R$$

where L = lead of screw

R = length of bar or wrench used to operate the screw

then,
$$P = \frac{W \times L}{2\pi R}$$

$$W = \frac{P \times 2\pi R}{L}$$

The screw can be compounded like the other elements of machines, as shown in Fig. 23, the differential screw. This is made with two screws of different pitches, or leads of threads, either both right or both left hand threads.

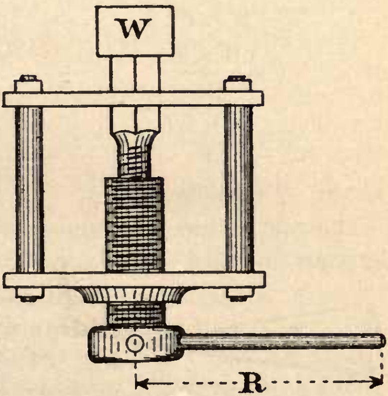


Fig 23.

It will be seen from Fig. 23 that one turn of the large screw lifts the weight only the difference between the leads of the large and small screws; then by the *General Law*,

Note. The nut N, *Fig. 22*, is the part that must be used with a screw to make it effective.

$$P : W = L - l : 2\pi R$$

and
$$P = \frac{W \times (L - l)}{2\pi R}$$

$$W = \frac{P \times 2\pi R}{L - l}$$

Note. Owing to the amount of friction in the differential screw its practical use is limited.

VI. WEDGE

The *wedge* is a pair of inclined planes placed back to back. It is used in two ways; by being driven with a blow of the hammer, and by pressure which usually acts parallel to the base of the planes. The difficulty in calculating the effectiveness of the first kind is to determine the force of the hammer blow, otherwise, the statement and rule is the same for either kind.

$$\therefore P : W = T : L$$

or
$$P = \frac{W \times T}{L}$$

$$W = \frac{P \times L}{T}$$

where T = thickness.

The rule is thus the same as for the inclined plane where force acts parallel to the base of incline.

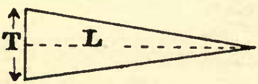


Fig. 25.

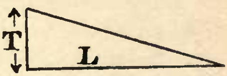


Fig 24

Example. A sliding wedge, Fig. 24, has a pressure of 100 lbs., the width of back end is 4 in., the length is 20 in. What weight can be raised by its use?

Solution. By formula:

$$W = \frac{P \times L}{T} = \frac{100 \times 20}{4} = 500 \text{ lbs.}$$

MISCELLANEOUS PROBLEMS

Note. In the following problems friction of moving parts will not be considered.

1. An iron ball weighing 398 lbs. rests on a surface which is inclined 16° 45′ to the horizontal. What force, acting at an angle of 14° 30′ to the incline, is required to hold the ball in position?

2. A weight of 3,500 lbs. is to be drawn up an incline 640 ft. long, 85 ft. above the horizontal. What force acting parallel to incline will be required to keep the load on incline?

3. A cylinder of cast iron 24 in. dia., 30 in. long is to be rolled up an incline of 18° 15′. What force, acting at 8° 15′ to the incline, will be required to hold the cylinder from rolling down?

4. What weight will be raised with a screw of $\frac{1}{4}$ in. lead when 100 lbs. of force is applied at the end of a lever 18 in. from the center of screw?

5. When the lead of a screw is $\frac{3}{4}$ in., R is 20 in. and a weight of 12,000 lbs. is to be raised, what is the force required?

6. In Fig. 22, a screw of $\frac{1}{16}$ in. lead is turned with a bar $1\frac{1}{2}$ in. long, with 1 lb. of force on end of R; what weight can be raised?

7. A sliding wedge, Fig. 24, is used to raise the knife on a shearing press that weighs 100 lbs., the wedge is to move 18 in. and is 3 in. thick at back end. What force will be required?

8. A truck loaded with an engine weighing 6 tons is to be drawn up an incline 12 ft. long and 5 ft. above the horizontal. What force will be required when the pull is parallel to the incline?

9. What weight can be drawn up an incline 10 ft. long and 4 ft. high with a pull of 300 lbs.?

10. Two men each pulling 125 lbs. can pull what weight up an incline 8 ft. long and 6 ft. high?

11. What force will be required on a single thread screw having 3 threads per inch, with a bar 18 in. long from center of screw to raise a block of granite that weighs 5 tons?

12. Two jack screws are to be used to raise a block 10 ft. long, weighing 10,000 lbs. One is a third more powerful than the other. Make sketch showing the position of screws to give the proportionate load on each.

13. How many jack screws with $\frac{1}{4}$ in. lead and having R 16 in. long, will be required to raise a building weighing 50 tons, if the pull on each lever is 50 lbs.?

14. How many jacks will be required with screws of $\frac{1}{6}$ in. lead, and 12 in. levers, with 25 lbs. pull on each lever to raise the building of problem 13?

15. A wedge 8 in. long, $1\frac{3}{4}$ in. thick at end will require how many lbs. of a hammer blow to drive it into a log that has a resistance of 2,000 lbs. against splitting?

16. When the screws of a differential are 8 and 12 pitch single threads respectively, with a pull of 5 lbs., what length of lever will be required to raise a weight of 5,000 lbs.?

17. What length of bar will be required to raise a building of 100 tons weight with 10 lbs. pull each on 100, $\frac{1}{4}$ in. lead jack screws?

18. A cylinder 25 in. dia. weighs 5,000 lbs., Fig. 26. What force at P will hold the cylinder on incline?

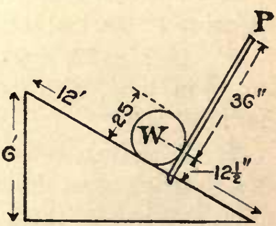


Fig. 26.

19. If the cylinder of problem 18 weighs 6,575 lbs., the lever is $36\frac{1}{2}$ in. from P to fulcrum, $12\frac{1}{2}$ in. from weight to incline, and incline is 10 ft. long, 4 ft. high, what force on the end of lever P will prevent cylinder from rolling back?

SCREW THREADS

The formula for finding tap drill sizes is based on the depth from point to bottom of thread, as follows: Fig. 27 shows outline of the sharp, or V thread. The depth A of a thread of 1 inch pitch is found by trigonometry and is equal to .8660 inch. If then the thread is taken on both sides of a cylinder the double depth for 1 inch pitch $= .8660 \times 2 = 1.732$.

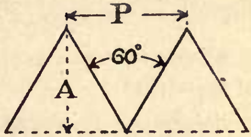


Fig. 27.

For any other pitch of thread the double depth $= \dfrac{1.732}{N}$ where N is the number of the threads per inch of the required screw.

Then the formula for tap drill size is

$$S = T - \frac{1.732}{N}$$

S = diameter of drill.
T = outside diameter of bolt.

Fig. 28 shows the outline of the United States Standard (U. S. S.) thread. Here one-eighth of the total depth of the sharp V is flattened on the points, and the same amount filled in at the bottom of the V, thus making the flattened parts of the U. S. S. thread each equal to one-eigth of the pitch so that the double depth for a

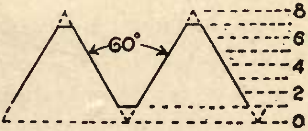


Fig. 28.

U. S. S. thread of 1 inch lead will be

$$1.732-2\times\tfrac{1}{8} \text{ of } 1.732$$

or

$$1.732-.432=1.3$$

For any other pitch the double depth will be $\frac{1.3}{N}$
where T = outside diameter of bolt.
Then S is the size of tap drill to be used before tapping threads in the nut. The formula is $S=T-\frac{1.3}{N}$.

Example. For a $\frac{1}{2}\times 13$ pi. U. S. S. bolt, what is the size of tap drill?

Solution. By formula:

$$S=T-\frac{1.3}{N}=\frac{1}{2}-\frac{1.3}{13}=\frac{1}{2}-\frac{1}{10}=.400 \textit{ inch} \text{ tap drill.}$$

It is sometimes necessary to use the *British Standard thread*, or *Whitworth Standard*, as it is usually called, after the name of the man who established it.

In this thread the top is rounded over one-sixth of the depth and the bottom is filled in the same amount, so that one-third of the depth is subtracted for actual depth of thread, Fig. 29.

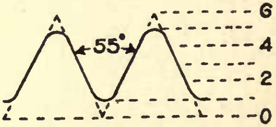


Fig. 29.

The formula for tap drill is, $S=T-\frac{1.28}{N}$.

The included angle of the thread is 55° instead of 60° as in the V and U. S. S. threads.

PROBLEMS

1. Find the depth of a V thread of 12 pitch.
2. Find the depth of an 18 pi. V thread.
3. What is the depth of a U. S. S. thread of 12 pitch?

4. What is the double depth for a Whitworth thread of 11 pitch?

5. What is the tap drill size for a 1 in. by 8 pi. U. S. S. tap?

6. What is the tap drill size for $\frac{1}{4}$ in. by 20 pi. V thread nut?

7. What size drill will be needed to allow a full thread on a $\frac{1}{2}$ in. by 12 pi. Whitworth tap?

8. Give the width of the flat on the top of a $\frac{7}{8}$ in. by 9 pi. U. S. S. thread.

9. Find tap drill size for 18 pi. double V thread nut $\frac{11}{16}$ in. diameter.

10. What is the bottom dia. of a $\frac{5}{8}$ in. by 11 pi. U. S. S. thread?

11. Find bottom dia. of a $\frac{3}{4}$ in. by 10 pi. Whitworth thread.

12. Find bottom dia. of a $\frac{1}{2}$ in. by 13 pi. V thread.

13. What is the double depth for a $\frac{3}{8}$ in. by 16 pi. U. S. S. bolt?

14. What is the double depth for a $\frac{1}{4}$ in. by 20 pi. V thread?

15. If there are 16 threads per inch on a 1 in. dia. bolt and the nut advances $\frac{1}{8}$ in. for each turn of the bolt, what is the pitch of the thread?

16. One turn of the feed screw on a milling machine moves the table $\frac{1}{4}$ in., but the threads measure 8 per in. How should the threads be designated?

Answer. $\frac{1}{4}$ *in. lead or 8 pitch double thread.*

17. If there were 12 threads per in., and one turn of screw moves nut $\frac{1}{4}$ in., how would the thread be designated?

TOOTHED GEAR WHEELS

A *toothed gear wheel* is one with projections on its periphery; these projections, or teeth, engage with the teeth of a similar wheel and the engagement or meshing of these teeth imparts a *positive* rotary motion from driver to follower.

The ratio of the speeds of two gears that run together is called their *velocity ratio*, and is in inverse proportion to their size.

Of two gears, if one revolves once while the other revolves twice, their velocity ratio is as 1 to $2 = \frac{1}{2}$, which indicates that the first gear is twice the size of the other.

The *pitch circle* of a gear is the circle near the center between top and bottom on the face of the teeth, such that if the teeth were to be made infinitely small the gear wheel would become a cylinder, Fig. 31.

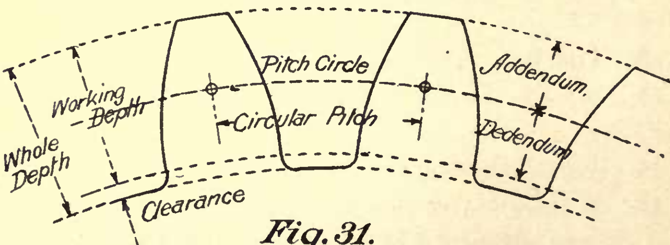


Fig. 31.

The *diameter of a gear wheel* is always the diameter of the pitch circle, unless otherwise stated.

The *outside diameter* is the diameter of the blank in which the teeth of a gear are cut.

The pitch of the teeth of a gear, usually called the diametral pitch, is the number of teeth for each inch of the diameter of the gear, unless the circular pitch is stated, which is the distance *from center of one tooth to center of the next tooth measured on the pitch circle.*

When a pair of gears are running in mesh, the smaller of the pair is called the *pinion.*

Let D = diameter of pitch circle of gear, and OD = diameter of blank.

P = diametral pitch.

N = number of teeth in large gear and n = number of teeth in pinion.

C = circumference of pitch circle.

CP = circular pitch.

T = thickness of tooth on pitch circle.

a = addendum.

x = distance between centers of two gears in mesh with each other.

Then $P = \frac{N}{D} = \frac{N+2}{OD} = \frac{\pi}{CP}$

$$D = \frac{N}{P} = OD - \frac{2}{P}$$

$$OD = \frac{N+2}{P} = D + \frac{2}{P}$$

$$N = D \times P = (OD \times P) - 2$$

$$C = D \times \pi$$

$$CP = \frac{C}{N} = \frac{\pi}{P} = 2T$$

$$T = \frac{CP}{2} = \frac{1.57}{P}$$

$$a = \frac{1}{P}$$

$$\textit{Clearance} = \frac{T}{10} = \frac{.157}{P}$$

$$\textit{Working depth} = \frac{2}{P}$$

$$\textit{Whole depth} = \frac{2.157}{P}$$

$$x = \frac{(N+n)}{2P}$$

The thickness of cast gears with cut teeth $= \dfrac{8}{P}$

To find the diameters of two gears in mesh with each other when x and velocity ratio of the two gears are given:

RULE. Divide the distance between the centers by the sum of the terms of the ratio; the pitch diameters will be twice the quotient multiplied by each term of the ratio.

Example. Find the diameter of two gears with centers 10 in. apart and a velocity ratio of 2 to 5.

Solution. By rule:

$2+5=7$

$10 \div 7 = 1.42857$

$1.42857 \times 2 \times 2 = 5.72$

and $1.42857 \times 2 \times 5 = 14.28$

5.72 in. and *14.28 in.* are the pitch diameters of gears.

PROBLEMS

1. What is the circular pitch of an 8 pi. gear of 40 teeth?

2. What is the circular pitch of a 6 pi. gear of 108 teeth?

3. What is the pitch of a 75 tooth gear when the circular pitch is .7854?

4. A gear of 45 teeth has a circular pitch of .3142 in. Find the diametral pitch of gear.

5. What is the dia. of a gear blank that has 6 pi. teeth and 18 in. diameter?

6. A gear blank is $3\frac{3}{4}$ in. dia. and is to have 13 teeth. What is the pitch?

7. What is the number of teeth in a 6 pi. gear 18 in. diameter?

8. What is the number of teeth in a 4 pi. gear $19\frac{1}{4}$ in. blank diameter?

9. What is the thickness of tooth for a 75 tooth gear, .7854 in. circular pitch?

10. What is the thickness of tooth for a 4 pi. gear of 40 teeth?

11. Find the whole depth of tooth for a 4 pi. gear of 40 teeth.

Note. Unless otherwise stated the depth of tooth means the whole depth.

12. What is the depth of tooth for a 6 pi. gear with 30 teeth?

13. Find dia. of blank for a 16 pi. gear with 32 teeth.

14. What is the dia. of blank of a 10 pi. gear with 28 teeth?

15. What is the pitch of a 75 tooth gear $19\frac{1}{4}$ in. outside diameter?

16. Find the number of teeth in a 16 pi. gear 8 in. diameter.

17. Find the number of teeth in a 10 pi. gear 64 in. diameter.

18. Find the number of teeth in a 20 pi. gear 10.1 in. outside diameter.

19. Find the sizes of two gears when the velocity ratio is 1 to 2 and the distance between centers is 12 inches.

20. Find the diameters of two gears 12 in. between centers, with velocity ratio 2 to 4.

21. Find the diameters of two gears 28 in. between centers, when the velocity ratio is 3 to 5.

22. When the distance between centers of two gears is 27 in. and the velocity ratio is 4 to 5, find the diameters of the gears.

23. When velocity ratio of two gears is 4 to 5, and the distance between the centers is 36 in., find the diameters of the gears.

24. The velocity ratio of two gears is 2 to 3 and distance between the centers 30 in. Find the diameters of gears.

25. What is the clearance of a 4 pi. gear of 75 teeth?

26. Find the clearance of teeth for a 2 in. dia. gear with 32 teeth.

27. Find the dia. of a 6 pi. gear with 108 teeth.

28. What is the circumference of a gear blank with 32 teeth 16 pitch?

29. Two gears in mesh have 30 and 24 teeth respectively and are 6 pitch. What is the distance between centers?

30. Two gears running together have 120 and 80 teeth respectively and are 10 pitch. Find the distance between the centers.

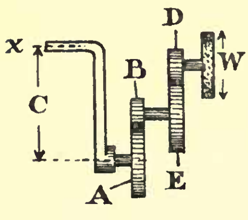


Fig. 32.

Emery wheels are designed to run at 5,500 *F. P. M.* (*surface feet per minute*), but the speeds of wheels are usually given in *R. P. M.*;

$$F.\,P.\,M. = R.\,P.\,M. \times \pi D,$$

D = diameter of wheel in feet.

$$R.\,P.\,M. = \frac{F.\,P.\,M.}{\pi D}$$

31. A hand power grinding wheel, *W*, Fig. 32, is 6 in. dia. and is to run 5,500 *F. P. M.* What is the *R. P. M.* of crank *C* when *A* has 200 teeth, *B* 20 teeth, *D* 15 teeth, and *E* 250 teeth?

32. How many *R. P. M.* of *W.* in problem 31, when crank makes 25 *R. P. M.*?

33. How many *F. P. M.* for x in problems 31 and 32 when C is 12 in. long?

34. If there is a resistance of 3 lbs. on wheel W, problem 31, what force will be required at x to turn wheel 5,000 *F. P. M.*?

35. What is the velocity ratio between the *R. P. M.* of W and x in problem 31?

Polishing wheels, with wood center and leather covered rim, are allowed to run 7,000 *F. P. M.*

Cloth buffing wheels also run 7,000 *F. P. M.*

Hair-brush cleaning wheels run 12,000 *F. P. M.*

Ordinary grindstones (Ohio) should *not* run over 2,500 *F. P. M.* and Huron stones 3,500 *F. P. M.*

36. If a Huron stone of same size as in problem 31 was substituted for the emery wheel, what *R. P. M.* of C will give 3,500 *F. P. M.* of W?

37. If a brush wheel $4\frac{1}{2}$ in. dia. was substituted for wheel of problem 31 to run 10,000 *F. P. M.*, find *R. P. M.* of C.

38. What force will be required at x if there is 1 lb. resistance at surface of W in problem 37?

39. What is the *R. P. M.* of C in problem 31 when A, B, D, and E are changed to friction wheels 9 in., $2\frac{1}{2}$ in., 2 in., and 11 in. diameters respectively?

BEVEL GEARS

Positive rotary motion can be transmitted from driver to follower, when the shafts stand at an angle to each other, and in the same plane, by using a pair of bevel gears.

When the angle of the shafts is 90° and the velocity ratio is 1 to 1, both gears are of the same size and are called *miter gears*. When the pair run at a velocity ratio other than 1 to 1, the smaller gear is called the *pinion*, the same as in *spur* gears.

The pitch diameter of a bevel gear is the diameter of the base of the pitch cone, AB. All calculations for sizes, as in spur gears, are made on this base circle. The teeth of bevel gears vanish or become infinitely small at the apex of the pitch cone. O_1AM is the normal cone, that is, one whose convex surface stands at right angles to the pitch cone OAM. The shape and size of teeth are found on the normal cone. The addendum $AS=\frac{1}{P}$ and P, D, N, C, CP, and T also clearance, whole and working depths of teeth are all found by the formulas on page 33 for spur gears.

The calculations for sizes and shapes of blanks are found by the following formulas:

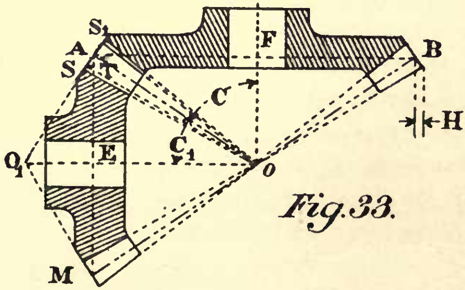


Fig. 33.

D = diameter of gear.
D_1 = diameter of pinion.
C = center angle of gear.
C_1 = center angle of pinion,

then, $\quad tan\ C = \frac{AF}{OF}$ *by (5) page 179*

and $\quad C_1 = 90\text{-}C$ or $tan\ C_1 = \frac{AE}{OE}$

$AS = a = \frac{1}{P}$

and $\quad AS_1 = a + clearance.$

$Angle\ FOS = \angle C + \angle AOS$

and $\quad tan\ AOS = \frac{AS}{AO}$

Angle AOS is called the angle of increment, angle AOS_1 is the angle of decrement for the gear.

$tan\ AOS_1 = \frac{AS_1}{AO}$. This angle is used in the setting of the blank at the proper angle for the depth of tooth.

The OD for the bevel gear $= D + 2H$

and $\quad H = cos\ C \times AS$ *(by 4) page 179.*

Example. Two shafts at an angle of 90° to each other are connected by a pair of 16 pitch bevel gears with a velocity ratio of 3 to 4. If the pinion has 48 teeth find all the other dimensions of the gears.

Solution. If the pinion has 48 teeth and the velocity ratio is 3 to 4 then $3 : 4 = 48 : X$ which makes $X = 64$, the number of teeth in the gear.

$$D = \frac{64}{16} = 4 \text{ inches.} \quad D_1 = \frac{48}{16} = 3 \text{ inches.}$$

$$tan\ of\ \angle C = \frac{AF}{OF} = \frac{2}{1.5} = 1.3333 \therefore \angle C = 53^\circ\ 8'.$$

$$\angle C_1 = 90 - \angle C = 36^\circ\ 52'.$$

$$AS = \frac{1}{p} = \frac{1}{16} = .0625;\ AS_1 = \frac{1}{p} + \frac{.157}{p} = .0723.$$

$$AO = \sqrt{2^2 + (1.5)^2} = 2.5$$

$$\tan \text{ of } \angle AOS = \frac{AS}{AO} = \frac{.0625}{2.5} = .025 \therefore \angle AOS = 1° \; 25'.$$

$$\text{Face angle of gear} = \angle FOS = \angle C + \angle AOS = 54° \; 34'.$$

$$\tan \text{ of } \angle AOS_1 = \frac{AS_1}{AO} = \frac{.0723}{2.5} = .0289 \therefore \angle AOS_1 = 1° \; 39'.$$

$$\angle FOS_1 = \angle C - \angle AOS_1 = 51° \; 29'.$$

This is the angle of working depth, and is the angle at which to set the spiral head for cutting the teeth of the gear.

OD of gear blank $= D + 2H$.

$$H = ST; \cos \angle AST = \frac{ST}{AS} = \cos \angle C \therefore H = AS \times \cos \angle C$$

then $OD = 4 + 2 \times .0375 = 4.075$ *inches.*

The dimensions of the pinion can be found in a similar manner.

PROBLEMS

1. Two bevel gears, shafts at 90°, have a velocity ratio of 2 to 3 with 40 6 pi. teeth in the pinion. What are the diameters and angles of blanks?

2. In a pair of bevel gears with shafts at 90°, having a velocity ratio 4 to 5, the gear is 15 in. dia. and has 8 pi. teeth. Find diameters and angles to cut teeth of gear and pinion.

3. The circular pitch of a miter gear is .31416 in. and it has 40 teeth. Find sizes and angles required to machine the gear.

4. The thickness of the tooth of a bevel gear pinion of 32 teeth is .19635 inches. What are the finished sizes and angles of a pair of gears with velocity ratio of 3 to 5, when shafts stand at 90° to each other?

5. The center lines of a pair of bevel gears of 6 pitch stand at 90° to each other and the velocity ratio is 3 to 4. Find finished sizes and angles of gears when pinion has 30 teeth.

6. Two bevel gears with shafts at right angles to each other have a velocity ratio of 4 to 5; the pinion is 15 in. dia. with 8 pitch teeth. What are the diameters and angles of the pair?

WORM GEARING

When two shafts stand at 90° to each other but not in the same plane, rotary motion can be transmitted between the shafts by the use of worm gearing. This is generally used where a great difference is required in the velocity ratio, combined with positive action between the teeth.

There is one great objection to worm gearing on account of the friction due to the sliding action between the teeth. The worm is a screw with threads shaped on the plan of the U. S. S. except that the flats at the top are much wider and the included angle of the sides is 29°. The thread of a worm can be spoken of as having a lead and it can be of single, double, triple, quadruple, etc., thread as in screw threads.

The tooth of the worm wheel, which meshes with the thread of the worm, advances the distance of one tooth at each revolution of a single thread worm; therefore if a worm wheel has 30 teeth, the worm will have to revolve thirty times for one complete revolution of the worm wheel.

The velocity ratio in the case of double, triple, etc., thread worms will then be the number of teeth in the worm wheel divided by 1, 2, 3, etc., according as it is a single, double, triple, etc., thread worm.

PROBLEMS

1. A, Fig. 34, is a double thread worm and B a worm wheel with 80 teeth, $12\frac{3}{4}$ in. dia. fast to an axle, a, 4 in. dia. The crank C is 12 in. long. What weight can be balanced at W when 75 lbs. of force is applied at x?

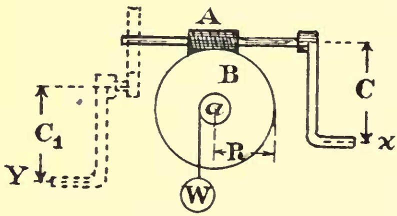


Fig. 34.

2. If A, Fig. 34, had been a triple thread worm and B had 45 teeth and 18 in. dia. with 5 in. axle, a, what length of crank C will be required to lift 4,000 lbs. with a force of 60 lbs. at x?

3. If a, Fig. 34, is a differential axle with 5 in. and 7 in. diameters, what length of crank C will be required to lift 24,000 lbs. at W, with 60 lbs. at x, when B is 16 in. dia. with 45 teeth and A is a single thread worm?

4. If on left end of A, Fig. 34, a gear of 60 teeth is in mesh with a gear of 24 teeth which is turned by an 18 in. crank C_1, when A is single thread worm, B 15 in. dia. with 42 teeth, a 4 in. dia., what weight can be raised with a 50 lb. pull at Y?

5. With mechanism the same as in problem 4 except a differential axle at a, having $4\frac{1}{2}$ in. and $6\frac{1}{2}$ in. diameters, what weight can be raised?

SPIRAL GEARING

Worm gearing is the limiting case of what is called spiral or helical gearing with the velocity ratio expressed, in a single thread worm, as $\dfrac{1}{\textit{No. of teeth in wheel.}}$ The other extreme is where the velocity ratio is 1 to 1, with the shafts in different planes and at any angle to each other from 90° to 0°. When the shafts make an angle of 90° with each other, the angle of spiral for each wheel will be 45°, the spiral being either right or left hand. If, however, the shafts are parallel, the teeth of one gear will be right hand spiral and the other left hand spiral and the angle of spiral needs to be only large enough to have each tooth begin its contact with the tooth of the mating gear before the next tooth ends its contact with the next tooth of the mating gear.

The size of the blank is found in a different manner from that of the spur wheel formulas (although cutters of the same pitch may be used to cut the teeth), and must be calculated as follows:

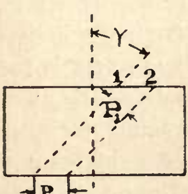


Fig. 35.

Let Fig. 35 represent the top view of a blank for a helical gear with lines *1* and *2* drawn through the centers of two adjacent teeth at the angle which the teeth make with the axis of the gear. The distance P_1 measured at right angles to the lines *1* and *2* is called the normal circular pitch and is calculated as for ordinary spur wheels according to the pitch of cutters used to form the teeth. The distance P on the face of the gear, called the circular pitch determines the size of the gear, after the number of teeth is given, then by trigonometry $P = \frac{P_1}{\cos Y}$

Example. Two helical gears, with velocity ratio 1 to 1 shafts at 90° are to be cut with a Brown & Sharpe 16 pitch cutter. Find the size of blanks required for 20 teeth.

Solution. As angle of shafts is 90° and gears are the same size, angle $Y = 45°$, and may be either right or left hand. The normal circular pitch P_1 for 16 pitch cutter $= .19636$, then,

by formula

$$P = \frac{.19636}{.70711} = .277.$$

Now as the gears are to have 20 teeth,

$.277 \times 20 = 5.554$ *inches*, which is the circumference of the pitch circle

and $5.554 \div \pi = 1.768$, the diameter of pitch circle.

Two addendums $= \frac{1}{8}$ *inch*, which added to D makes

$$OD = 1.768 + .125 = 1.893 \text{ inches.}$$

When the velocity ratio of two spiral gears is other than 1 to 1, it cannot be determined from the pitch diameters, as in spur gearing, but must be calculated from the helical angle of the teeth of each gear. If the helical angle is the same in each the velocity ratio will be inversely as the pitch diameters, but if the helical angles are not the same, the number of teeth per inch will vary. In the case of a single pitch worm and wheel, the worm is a spiral gear of one tooth, the velocity ratio being 1 to the number of teeth in the wheel; increasing or decreasing the pitch diameter of the worm will change the helical angle of the teeth in both worm and wheel, but will not affect the velocity ratio so long as the number of teeth remains the same in the wheel.

Note. The term helix is used to denote the path of a point moving parallel to and at equal distances from the axis of a cylinder, while the spiral is a point moving in a path that gradually increases its distance from the axis. As the helix is a special case of the spiral it is customary to class helical work as *spirals*. The angle Y which the spiral makes with the axis of the gear, is called the spiral angle.

The velocity ratio in all cases depends upon the number of teeth and their helical angle, from which is derived the formula:

$$v : V = D \times \cos A : d \times \cos a$$

V = velocity of large gear.

v = velocity of small gear.

D = pitch diameter of large gear.

d = pitch diameter of small gear.

A and a = the angles that the teeth make with the axes of the respective gears.

From the above formula the velocity ratio of two spiral gears is proportional to the pitch diameters only when the spiral angles of the gears are the same, or 45° when axes are at 90° to each other. The sum of the spiral angles of the two gears must always equal the angle between the shafts, and if the end thrusts on the bearings of the gears are to be the same the angle of the teeth must be the same in both. When the angle of the spiral to the axis of one gear is greater than the other, that gear should be the driver.

Example. Two spiral gears with axes at 90°, velocity ratio 2 to 3, are to be cut with a 16 pitch cutter. The pitch dia. of the driving gear is $1\frac{1}{2}$ in., and the teeth are at an angle of 60° with the axis. Calculate the other dimensions of the gears.

Solution. The normal circular pitch of driver

$$P_1 = \frac{3.1416}{16} = .19635$$

and $$P = \frac{.19635}{\cos 60°} = \frac{.19635}{.5} = .3927.$$

Then $$No.\ of\ teeth = \frac{1\frac{1}{2} \times \pi}{.3927} = \frac{4.7124}{.3927} = 12.$$

The velocity ratio is *2 to 3*,

then $2 : 3 = 12 : X \therefore X = 18$, the number of teeth in follower.

To find the normal circular pitch of follower $P_1 = .19635$,

then $$P = \frac{.19635}{\cos 30°} = \frac{.19635}{.86603} = .2267$$

and $$D = \frac{.2267 \times 18}{\pi} = \frac{4.0806}{\pi} = 1.298.$$

The outside diameters of the gears are found by adding $\frac{2}{P}$ to each D, the resulting blank diameters being

$$1.298 + \tfrac{1}{8} \text{ inch} = 1.423$$

and $$1.5 + \tfrac{1}{8} = 1.625.$$

The formula for finding the number of the cutter to use for the teeth of spiral gears is,

$$\frac{\textit{No. of teeth in gear}}{\textit{cube of the cosine of the spiral angle}}$$

In the above example it is

$$\frac{12}{\cos^3 60°} = \frac{12}{.125} = 96 \text{ tooth cutter}$$

and $$\frac{18}{\cos^3 30°} = \frac{18}{.65} = 28 \text{ tooth cutter.}$$

PROBLEMS

1. Two helical gears are required with a ratio of 1 to 1, the shafts to be at 90° to each other, a 5 pi. cutter to be used and the *OD* to be as near 4 in. as can be obtained. Find their diameters.

2. Two gears with shafts parallel to each other are to be cut with a 10 pi. cutter and to have 10 teeth. What are the sizes for the helical gear with a velocity ratio of 1 to 1?

3. Find the blank diameter of two helical gears to have 12 teeth when the velocity ratio is 1 to 1, teeth to be cut with an 8 pi. cutter and shafts at 90°.

4. What will be the distance between the centers of two helical gears with velocity ratio of 1 to 1, each having 20 teeth cut with a 12 pi. cutter?

5. Find centers of two helical gears with 20 teeth, and 20 pi. cutter, with a velocity ratio of 1 to 1.

6. Two spiral gears with axes at 90° to each other and velocity ratio 1 to 2 are to be cut with a 14 pi. cutter; the outside diameter of driving gear is 1.9 in. and angle of teeth is 63° 26′ to axis of gear. Find the other dimensions for the gears.

PULLEYS

Cast iron pulleys should not run much over a mile per minute at the surface of rim, or $\frac{5280}{60}=88$ *feet per second.*

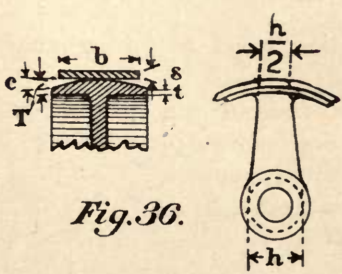


Fig. 36.

The following dimensions may be used for cast iron pulleys.

When b, Fig. 36 = width of belt for required power,

B = width of rim = $\frac{9}{8}(b+.4)$.

c = crown of rim, with radius varying from three to five times the width of rim.

Note. When the crown of pulley is tapered each way from center of rim instead of a curved surface, the taper for 6 in. wide and under is $\frac{3}{4}$ in. per ft. 6 in. to 12 in. wide is $\frac{1}{2}$ in. per ft. 12 in. to 18 in. wide is $\frac{3}{8}$ in. per ft.

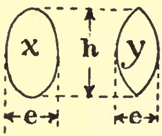


Fig. 37.

D = diameter of pulley.

S = thickness of belt, calculated from the *H. P.* required, usually $\frac{3}{16}$ inch. for single and $\frac{3}{8}$ inch for double belts.

t = thickness of rim at edge $= .7S + .005D$.

T = thickness of rim at center $= 2t + C$.

L = length of hub $= \frac{2B}{3}$.

D_1 = diameter of hub $= 2 \times$ *diameter of shaft.*

N = number of arms = 4 to 6, up to 60 inches D.

h = width of arm at center of pulley and approximately $= \frac{D-12}{16} + \frac{1}{4}$ *inch.*

$h = .633 \sqrt[3]{\frac{BD}{N}}$ for single and

$h = .798 \sqrt[3]{\frac{BD}{N}}$ for double belts.

$\frac{h}{2}$ = width of arm at rim of pulley.

$e = .4h$ for elliptic section x and $.5h$ for segmental section Y, Fig. 37.

The *H. P.* that a pulley will transmit can be calculated from the same formula as the one for belts provided that it has been designed with the right proportions.

The following formula is the one generally used:

$$H.\ P. = \frac{C \times R.\ P.\ M. \times b}{600}$$

where b = width of belt in inches.

C = circumference of pulley in feet.

Note. Diameters of pulleys are designed to be not less than 18 times the thickness of the belt that is to run over them.

PROBLEMS

1. A main line shafting runs 170 *R. P. M.;* a driver 36 in. dia. is belted to 12 in. pulley on counter; a 16 in. driver on same counter is belted to 4 in. pulley on grinder. What is the *R. P. M.* of grinder spindle?

2. A main line running 150 *R. P. M.*, has a 42 in. driver belted to an 8 in. follower on first counter; an 18 in. driver on first counter to a 6 in. follower on second counter; a 16 in. driver on second counter to a 3 in. pulley on machine spindle. Find the *R. P. M.* of spindle.

3. Compute the maximum thickness of belt that should be used on the machine in problem 2.

4. A driving pulley 30 in. dia. makes 150 *R. P. M.* What is the size of the follower making 175 *R. P. M.?*

5. If a follower is 6 in. dia. and makes 1,000 *R. P. M.* what is the size of driver making 150 *R. P. M.?*

6. Compute the width of pulley face for a 3 in. belt.

7. What is the width of pulley for a 7 in. belt?

8. What is the width of belt for a pulley 13 in. wide?

9. Find c, t, and T when $S = \frac{1}{4}$ in., $D = 6$ in. and $B = 4$ in.

10. Find h for 60 in. D and 14 in. B for single belt where $N = 6$.

11. A pulley is 60 in. dia. by 16 in. face, for a double belt, 4 in. dia. shaft and elliptic cross section for the arms. What are the dimensions for b, c, S, N, h, e, t, T, D_1 and L?

MISCELLANEOUS PROBLEMS

1. A helical gear with 50 teeth is to be cut with a 20 pi. Brown and Sharpe cutter. What are the blank and pitch diameters and lead of helix, when helical angle is 45°?

2. The angle of a helical gear is 45° and is to have 30 teeth cut with B. & S. 10 pi. cutter. What is the D, OD and lead of helix calculated on pi. circle?

3. A helical gear is to have 25 teeth, and 45° angle, and to be cut with a B. & S. 12 pi cutter. Find D, OD and lead of helix at bottom of teeth.

Note. For best results in cutting these gears it has been found that the lead of helix should be calculated at the bottom of tooth, and not on the pitch circle.

4. A double thread worm, Fig. 34, with 16 in. crank, meshes with a 40 tooth wheel 16 in. dia. Fast to worm wheel is a 3 in. dia. axle. When a force of 75 lbs. is applied at x, what weight can be raised?

5. The same as problem 4 except a triple thread worm in mesh with 75 tooth worm wheel.

6. The same as problem 4 except a single thread worm in mesh with 80 tooth worm wheel.

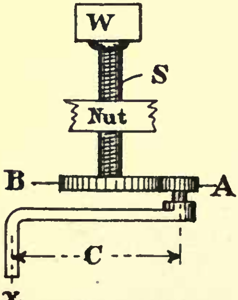


Fig. 38.

7. In Fig. 38, S is a screw of $\frac{3}{8}$ in. lead, B a gear with 75 teeth fast to S, A, a gear of 15 teeth fast to a crank C 12 in. long. When a pull of 75 lbs. is applied at x, what weight can be raised at W?

8. With a single pitch screw, Fig. 38, having $\frac{1}{4}$ in. lead fast to gear B with 80 teeth, and driven by gear A with 20 teeth that is fast to a crank C 15 in. long, what weight can be raised at W with a pull at x of 100 lbs.?

9. The same as problem 8, except with a double pitch screw and 50 lbs. pull at x.

10. The same as problem 8, except follower gear has 100 teeth and driver 10 teeth and crank 25 in. long.

11. The same as problem 8, except W is 9,975 lbs. to find P.

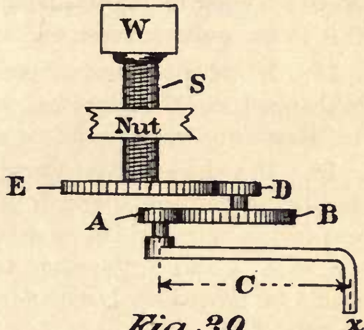


Fig. 39.

12. In Fig. 39, S is a screw of $\frac{1}{2}$ in. lead, gear E, fast to S, has 100 teeth, D has 40 teeth, gears B and D fast to same stud, and B has 120 teeth, A has 30 teeth fast to crank C, 27 in. long. Find W when the force applied at x is 60 lbs.

13. The same as problem 12, except gears D and B have 30 and 150 teeth respectively.

14. What power applied at x, problem 12, will raise a weight of 250,000 lbs.?

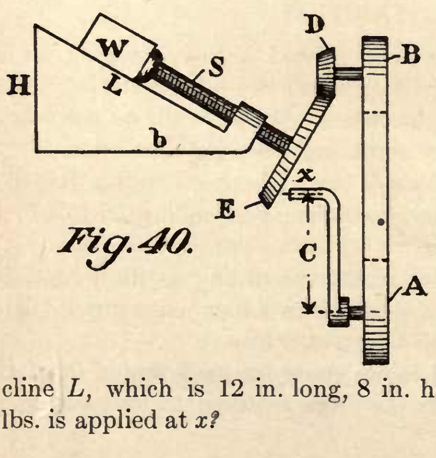


Fig. 40.

15. In Fig. 40, C is a crank 12 in. long fast to pulley A 8 in. dia. A is belted to B which is 6 in. dia. D is a gear of 10 teeth fast to B, and D is in mesh with gear E having 50 teeth; screw S is $\frac{1}{2}$ in. lead, fast to gear E. What weight can be moved up incline L, which is 12 in. long, 8 in. high, when a force of 75 lbs. is applied at x?

16. Find the force that will be required at x, Fig. 40, to move a weight of 75,000 lbs. up L, when S is $\frac{3}{8}$ in. lead, and C is 16 in., pulleys, gears and incline same as in problem 15.

17. If the positions of pulleys A and B, Fig. 40, were exchanged and C 24 in. long, otherwise same as in problem 16, what force will be required at x?

18. If a single thread worm was used in place of gear D and belted pulleys A and B of Fig. 40, and a 12 in. crank was fast to worm which meshes with a 50 tooth wheel 16 in. dia. at E, L and S, the same as in problem 15, what weight would be moved up L with force of 75 lbs. at end of crank on worm?

19. If in Fig. 40, L is 8 ft. and H is 3 ft. and S is $\frac{1}{4}$ in. lead double thread, gears E and D 15 in. and 6 in. diameters respectively, and A and B 15 in. and 12 in. diameters respectively, what force applied at x of a 12 in. crank will move 60,000 lbs. up the incline at W?

FRICTION

Friction is the resistance offered a body moving on a surface. It is *sliding* friction when one surface slides on the other, and *rolling* friction when one body rolls on the other so that new surfaces are continually coming into contact.

A wagon moving along a road illustrates rolling friction between the wheels and roadway, and sliding friction between wheels and axles.

Sliding friction varies greatly according to the materials used. For example, a sleigh drawn over bare ground has more friction than when drawn over ice.

f is the symbol used to designate the *coefficient* of friction. It is the ratio between the force required to overcome the resistance due to friction, and the weight or pressure of the moving body on a horizontal plane. From this is obtained

$f = \frac{P}{W}$, and $P = f \times W$; $W = \frac{P}{f}$.

Any pressure at right angles to the line of a moving body may be considered as part of its weight.

Example. The slide valve of a steam chest may weigh only 15 lbs., but a steam pressure of 100 lbs. per sq. in. on the valve may bring a weight of 5,000 lbs. on the valve seat to resist the sliding of the valve, so that pressure and weight may thus be equivalent terms. If, in this example, $f = .10$, then $f \times 5{,}000 = .10 \times 5{,}000 = 500$ *lbs.*, which will be the force required to slide the valve on its seat.

Example. If a block of granite weighs 6,000 lbs. and $f = .1666$, what force will be required to slide the block along a platform?

Solution. $6{,}000 \times .166 = 999$ lbs.

Example. If the weight of a block is 300 lbs. and the force necessary to slide it along is 50 lbs., find f.

Solution. $f = 50 \div 300 = .166$.

Axle friction is sliding friction, but the bearings are made very smooth and f is much smaller than for ordinary sliding friction.

The value for f on well lubricated bearings is from *.01* to *.05*, when not well lubricated f is usually taken at *.075*.

The values for f on well lubricated flat slides have been established by experiment as *.08* to *.10*, when not so well lubricated f is given as *.16* to *.20*.

The friction between a shaft bearing and its journal is axle friction.

Example. If a turning lathe with driving spindle 3 in. dia. making 50 *R. P. M.* has a pressure on bearings of 400 lbs., what power will be required to run the lathe with a 24 in. drive pulley when $f = .08$?

Solution. The resistance due to friction is

$400 \times .08 = 32$ *lbs.*, and the force necessary to overcome friction $= \dfrac{32 \times 1\frac{1}{2}}{12} = 4$ *lbs.*

The travel of the force at the rim of a 24 in. dia. pulley is

$$\frac{50 \times 2\pi r}{12} = 50 \times 6.2832 = 314.16 \text{ ft. per min.}$$

and the power required is

$$\frac{314.16 \times 4}{33{,}000} = \frac{1{,}256.64}{33{,}000} \text{ ft.-lbs.} = .038 \text{ H. P.}$$

Then the power necessary to overcome friction in fly wheels or other heavy axle bearings is found by taking the product of the frictional resistance and the distance of travel of this force. Or $H.\ P. = \dfrac{Force \times travel}{33{,}000}$. The following formula can be taken as a close approximation to find the *H. P.* absorbed by friction in heavy shaft bearings:

$$H.\ P. = \frac{W \times f \times N \times \pi \times d}{33{,}000 \times 12}.$$

The above formula becomes

$$H.\ P. = W \times f \times N \times d \times .000008.$$

W = load on bearing in pounds.

d = dia. of shaft in inches.

$N = R.\ P.\ M.$

PROBLEMS

1. A fly wheel for a steam engine weighing 6,000 lbs. makes 74 *R. P. M.* Find power absorbed by friction if the fly wheel shaft is 10 in. dia. and $f = .08$.

2. A fly wheel weighs 12 tons, the shaft at bearing is 12 in. dia., makes 72 *R. P. M.* and $f = .08$. Find power absorbed by friction.

LAWS OF FRICTION

1. Friction is in direct proportion to the pressure with which bodies bear against each other.

2. Friction depends upon the quality of the surfaces in contact.

3. Velocity within ordinary limits has no influence on the value of f.

4. The area of surfaces of contact does not affect the value of f within ordinary limits, but if the surfaces are unproportionately large or small the friction will be increased. f for rolling friction such as that between the car wheels and rails, on railways, is .002, for iron tired wheels on hard roads .02, on soft roads, .06.

ANGLE OF FRICTION OR REPOSE

The angle which a plane makes with the horizontal when a body just begins to slide on that plane is called the angle of friction. It can be demonstrated that f is equal to the tangent of the angle of friction.

FRICTION IN PULLEY BLOCKS

Motion between two bodies in contact always produces friction, and in determining the efficiency of any appliance or machine the friction must be taken into account. In a simple bar or lever there will be very little force lost in friction, but in the pulley there is the friction of the rope bending over the sheaves as well as the friction of the axles in the blocks.

The amount of force required to overcome friction can be readily found by experiment. The following formula has been found to be right for the effective pull obtained by use of pulley blocks. When W is the weight that can be raised by blocks as arranged in Fig. 18,

$$W = F(1 + .9 + .9^2 + .9^3 + .9^4 + etc.),$$

according to the number of sheaves in both blocks; and for pulleys as arranged in Fig. 17,

$$W = F(.9 + .9^2 + .9^3 + .9^4 + etc.).$$

The force required to overcome the friction of a body moving up an inclined plane and to balance the weight, by formula, page 23, will be

$$F = \frac{W \times H}{L} + friction.$$

By trigonometry this becomes

$$F = W \times sin\ A + friction;$$

but the friction is equal to the perpendicular pressure of the body on the inclined plane multiplied by the coefficient of friction.

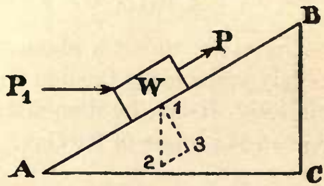


Fig. 41.

Let the pressure of a body on an inclined plane be represented by line *1—3*, Fig. 41, perpendicular to AB, while the whole weight of the body will be represented by the line *1—2* perpendicular to the base AC of the plane.

Draw *2—3* parallel to AB and perpendicular to *1—3*, to complete the triangle. Then *2—3* represents the force required to balance W on the plane.

$$2—3 = W \times \sin A$$
$$1—3 = W \times \cos A$$

since triangle *123* is similar to triangle ABC ∴ *Friction* = $W \times \cos A \times f$, and $F = W \times \cos A \times f + W \times \sin A$.

Example. A weight of 300 lbs. rests on a plane inclined at 30° to the horizontal; what force will be required to balance the weight on plane?

Solution. Friction not considered,

$$F = W \times \sin A = 300 \times .5 = 150 \text{ lbs.}$$

Now the perpendicular pressure on AB, Fig. 41, is

$$300 \times (\cos A).86603 = 259.8 \text{ lbs.},$$

and the total force required to pull W up the incline if $f = .15$, is $259.8 \times .15 + 150 = 188.97$ *lbs.*

If W is moving down the incline then the force due to $W \times \sin A$ will help to move it. If $W \times \sin A$ is more than $W \times \cos A \times f$, the body will slide of itself. Thus in the above example it will require *150 lbs.—38.97 lbs.* = *111.03 lbs.* to keep the weight from sliding.

When the force acts parallel to the base of the incline, as P_1, Fig. 41, as it does in screw threads and helical cams, $F = W \times \tan A$ and when both weight and friction are considered the formula is:

$$F = W \times \frac{\sin A + (f \times \cos A)}{\cos A - (f \times \sin A)}.$$

Example. When $W = 300$ *lbs.*, $A = 30°$, $f = .15$, find F.

$$F = 300 \times \frac{.5 + .15 \times .86606}{.86606 - (.15 \times .5)} = 300 \times \frac{.6299}{.791} = 300 \times .795 =$$
238.5 lbs.

The mechanical advantage for the screw has been given, as $P : W = Lead : 2\pi R$, but the screw is an inclined plane of which the hypotenuse is the middle circumference of the thread, and the lead is the altitude of the triangle, with the force acting parallel to the base; hence when friction is considered the formula is:

$$F = W \times \frac{Lead + (f \times 2\pi r)}{2\pi r - (f \times Lead)} \times \frac{r}{R},$$

where F = total force.

W = weight.

R = radius through which power acts.

r = middle radius of screw thread.

$2\pi r$ = middle circumference of screw thread.

For V thread screw, the frictional resistance will be increased as $\frac{1}{\cos A}$ = the secant of half the angle of thread, which for U. S. S. thread = $\sec$ *of* $30° = 1.15$. The formula for V threads is,

$$F = W \times \frac{L + (2\pi r \times f \times 1.15)}{2\pi r - (L \times f \times 1.15)} \times \frac{r}{R},$$

where the letters have the same meaning as in formula above for square threads.

MISCELLANEOUS PROBLEMS

1. What force will be required to raise a weight of 1 ton with a double sheave pulley as in Fig. 18?

2. What force will be required to raise 1,675 lbs. with a double sheave pulley as arranged in Fig. 17?

3. How much extra weight would a 180 lbs. man require to raise 2,000 lbs. with a 3 sheave pulley as arranged in Fig. 17?

4. What is the value of f when the pull is 75 lbs. and the weight is 600 lbs.?

5. What is the value of f when the pull is 80 lbs. and the weight is 1,000 lbs.?

6. What is the pull when f is .15 and the weight is 800 lbs.?

7. What force will be required to haul a machine weighing 1,500 lbs. up an incline with the horizontal length 12 ft. and height 5 ft. when f is .2?

8. What force will be required to roll a cast iron cylinder weighing 1,150 lbs. up an incline 12 ft. long and 6 ft. high when f is .01?

9. An engine fly wheel and shaft 10 in. dia. weighs 7 tons. What is the power required to move wheel when f is .08 and shaft makes 62 *R. P. M.*?

10. A truck loaded with a box of castings weighs 875 lbs. If the two bearings of truck wheels $1\frac{1}{4}$ in. dia. have $f = .15$, and rims of wheels have $f = .02$ on floor of shop, what pull will be required to move truck?

11. A 5 in. pi. dia. worm and a 2 pi. worm wheel of 36 teeth are used to lift an elevator weighing, with the maximum load, 2 tons; the drum, carrying the load is 30 in. dia. and is fastened to worm wheel shaft which makes 10 *R. P. M.* The shaft bearings are 3 in. dia. with $f = .05$; the f for sliding of teeth of worm and wheel is .15 and for step bearing 3 in. dia. $f = .10$. What force will be required at the rim of an 18 in. dia. pulley on worm shaft to start the elevator?

12. If an elevator weighing 475 lbs. is to be raised a distance of 9 ft. with 4 sheave pulley blocks, what pull will be required, and what is the movement of the rope, with pulley arranged as in Fig. 17?

13. How much force will be required to roll a barrel of sugar weighing 300 lbs. up a plank 8 ft. long inclined at 15° when $f = .005$?

14. What force will be required at the end of a 26 in. lever to raise 20,000 lbs. with a square thread screw 3 in. dia. and $\frac{3}{8}$ in. lead when $f = .15$?

15. What weight can be lifted with a square thread screw of $\frac{1}{4}$ in. lead $2\frac{1}{2}$ in. dia. with $f = .15$, when a force of 25 lbs. is put on end of a 20 in. lever?

16. 25 tons weight is to be lifted with 50 jack screws of $\frac{3}{8}$ in. lead, and 3 in. dia. of screw, with $f = .10$. What force will be required, distributed evenly at ends of 12 in. levers?

17. A hand screw press is used to punch holes in a sheet of iron, the pressure required being 1,250 lbs. What force will be required at the end of a 16 in. lever on a triple square thread screw $2\frac{1}{2}$ in. dia. $\frac{3}{4}$ in. lead and $f = .25$, to punch the holes?

18. A $\frac{1}{4}$ in. lead square thread screw 2 in. dia. with $f = .10$ is used to pull a broach through the $\frac{1}{2}$ in. square hole of an end wrench. A 12 in. pulley with 200 lbs. pull at the rim was required to revolve the screw. What was the resistance of the pull?

19. A hand press with screw $2\frac{1}{2}$ in. dia. and $\frac{1}{4}$ in. lead V thread is used to force small pins into small discs. A pull of 40 lbs. is used on the rim of a 16 in. hand wheel. If $f = .15$, what is the resistance against the pull?

20. If 100 ft. of shafting 2 in. dia. equipped with the usual number of drive pulleys and belting, at 100 *R. P. M.* requires 1 *H. P.* to keep it moving, what *H. P.* will be required to move the same shaft at 170 *R. P. M.*, when the power required increases directly as the speed?

21. If the power required to revolve a line of shafting increases as the cube of its dia., what power will be required to turn a line of shafting 3 in. dia., 150 ft. long when other requirements are the same as in problem 20?

22. What power is absorbed in friction in a lathe spindle $2\frac{1}{2}$ in. dia. with 830 lbs. weight on bearings due to belt pressure, etc., on a 10 in. dia. pulley at 100 *R. P. M.*, when f is taken at .01?

23. What is the power absorbed in running a large lathe with a 10 in. dia. pulley making 20 *F. P. M.*, if the weight on bearings is equal to 3,600 lbs., the dia. of spindle is 5 in. and f equals .01?

BELTING

A common method for the transmission of rotary motion is with leather belting over pulleys. The adhesion of a leather belt to the surface of a pulley rim is given in terms of the amount of weight it is capable of lifting; the speed in feet per minute times the weight lifted equals the foot pounds of work it is capable of doing.

Cooper, in his treatise on belting, considers a safe velocity for leather belts to be fifty square feet per minute; for a belt one inch wide this would be equivalent to a linear velocity of 600 *F. P. M.*

The breaking strain for leather belting is given as 3,200 pounds per square inch of cross section; using a factor of safety of 11, $\frac{3,200}{11} = 290$ *lbs.* = *the safe strain.* Now for belting $\frac{3}{16}$ inch thick and 1 inch wide, $290 \times \frac{3}{16} = 55$ *lbs. strain per inch.* Double belts, that is, two single belts cemented together may have about one-half more added for the allowable pull on the belt, or $82\frac{1}{2}$ pounds per inch of width.

Modern shop practice allows less strain on the belt than that given above, as low as 30 pounds being given by some belt makers and users. This would correspond to a linear velocity of over *1,100 F. P. M.* per *H. P.* for each inch of width, but about *800 F. P. M.* per *H. P.* is a fair average for single belts, and *550 F. P. M.* per *H. P.* for double belts.

In selecting the size and speed for belts general shop conditions must be taken into account.

Belts should not run faster than 6,000 *F. P. M.* on account of reducing the tension of the belt on the pulley by centrifugal force; between 3,000 and 4,000 *F. P. M.* is considered the better practice.

The speed of a belt is found by multiplying the *R. P. M.* of driver by πD or by formula: $F.\ P.\ M. = R.\ P.\ M. \times \pi D$. D is diameter of driver in feet.

In the calculations for speeds of pulleys for belt transmission, 2% may be allowed for slip, or creep, of belt on the pulleys.

The following rule is approximately correct for lengths of belts over driver and follower pulleys.

RULE 1. Add twice the distance between centers of shafts to half the sum of the diameters of the two pulleys multiplied by π,

or by formula:

$$l = 2L + \frac{(D+d)\pi}{2}.$$

l = whole length of belt.

L = distance between centers of pulleys.

D = diameter of larger pulley.

d = diameter of smaller pulley.

The *exact lengths* for belts for two mating pulleys are found as follows:

RULE 2. Find arc of contact of belt on driver and follower and to their sum add twice the distance between points of contact on driver and follower.

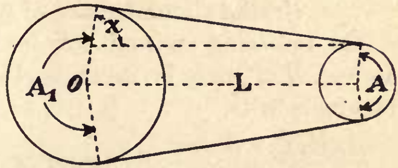


Fig. 42.

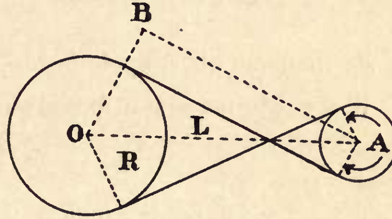


Fig. 43.

This distance may be found by the graphical method, or as follows:

$\frac{\cos A}{2}=\frac{R-r}{L}$, Fig. 42, for arc of contact on small pulley.

L = distance between centers of shafts in inches.

R = radius of large pulley.

r = radius of small pulley.

A = arc of contact on small pulley.

A_1 = arc of contact on large pulley $=360°-A$.

The arc of contact on cross belts, Fig. 43, is the same for both pulleys and formula is:

$$\cos\left(180-\frac{A}{2}\right)=\frac{R+r}{L}.$$

The length of arc of contact of belt on pulley is found by the formula

$$L=\frac{A°C}{360},$$

where L = length of arc of contact.

A = angle of arc.

C = the circumference of pulley.

The following approximate formula for width of belt for a given $H. P.$ may be taken as close enough for practical use for single belts.

$$b = \frac{H. P. \times 50{,}000}{d \times W \times N}, \text{ by transposing } d = \frac{H. P. \times 50{,}000}{b \times W \times N}$$

b = width of belt in inches, $\quad W = \dfrac{H. P. \times 50{,}000}{d \times b \times N}$

d = diameter of pulley in inches, $\quad N = \dfrac{H. P. \times 50{,}000}{d \times b \times W}$

W = weight per square foot in ounces,

$$H. P. = \frac{W \times b \times d \times N}{50{,}000}$$

$N = R. P. M.$

1 square foot of leather belt $\frac{3}{16}$ inch thick, weighs 16 ounces.

For double belts, $b = \dfrac{H. P. \times 67{,}500.}{d \times W \times N}$

PRESSURE ON BEARINGS

Let P = pressure on bearings from pull of belt.

V = travel of belt in $F. P. M.$

Then approximately $P = \dfrac{3 \times 33{,}000 \times H. P.}{V}$

PROBLEMS

1. A single belt $\frac{3}{16}$ in. thick, 12 in. wide runs 5,000 $F. P. M.$ If there is a strain of 55 lbs. per in. in width, what $H. P.$ can be transmitted by the belt?

2. A double belt 20 in. wide runs over a 4 ft. dia. pulley at 180 $R. P. M.$ Find $H. P.$ that the belt will transmit.

3. A single belt 24 in. wide runs on 14 ft. dia. pulley at 75 *R. P. M.* Find the *H. P.* it is capable of transmitting.

4. Two pulleys 60 in. and 24 in. diameters respectively are on separate shafts 12 ft. between centers. What is the approximate length of belt required?

5. What is the exact length of a cross belt running over two pulleys 60 in. and 24 in. diameters, when the shafts are 16 ft. between centers?

6. How many *H. P.* can be transmitted by a 3 in. belt at 55 lbs. strain per in. of width at 3,500 *F. P. M.?*

7. How many *H. P.* can be transmitted by a 4 in. belt, 35 lbs. strain, at 4,800 *F. P. M.?*

8. How many *H. P.* can be transmitted by a 24 in. double belt, over an 8 ft. driver at 115 *R. P. M.?*

9. How many *H. P.* can be transmitted by a 26 in. single belt at 33 lbs. strain on a 14 ft. driver, at 80 *R. P. M.?*

10. If the belt in problem 9 is running over a 6 ft. follower pulley, 16 ft. between centers, what is the weight for a belt $\frac{3}{16}$ in. thick?

11. Find the length of a cross belt 16 ft. between the centers of pulleys that are 14 ft. and 6 ft. diameters respectively?

12. Find the exact length for an open belt 12 ft. between centers of pulleys that are respectively 84 in. and 48 in. diameters.

13. What is the length of a cross belt 16 ft. between centers of pulleys that are respectively 7 ft. and 3 ft. diameters?

14. Find exact length for an open belt 18 ft. between the centers of pulleys that are respectively 30 in. and 70 in. diameters.

15. What is the width of a double belt to transmit 112 *H. P.* on a 4 ft. diameter pulley at 180 *R. P. M.?*

16. What *H. P.* will a double belt 30 in. wide weighing 36 oz. per sq. ft., transmit on a 60 in. dia. pulley at 200 *R. P. M.?*

17. A 16 ft. dia. pulley runs at 75 *R. P. M.;* over this is running a 24 in. double belt that weighs 32 oz. per sq. ft. What *H. P.* will the belt transmit?

18. What is the pressure on bearings of a belt that is running 900 *F. P. M.* and transmits 60 *H. P.?*

19. What will be the pressure on the caps of lathe spindle bearings, when a 4 in. belt on 10 in. dia. pulley making 100 *R. P. M.* does 2 *H. P.* of useful work?

20. What is the pressure on the bearings for a 4 in. grinder belt at 3,000 *F. P. M.* transmitting $\frac{1}{4}$ *H. P.?*

21. What is the pressure on the bearings of a drop hammer pulley with 5 in. belt running at a velocity of 2,000 *F. P. M.* when 5 *H. P.* is to be transmitted?

22. What dia. of pulley would be required to increase the velocity of a belt from 900 *F. P. M.* over a pulley 20 in. dia. to 3,000 *F. P. M.* with the same *R. P. M.?*

23. Find the *H. P.* of a 24 in. double belt, 36 oz. weight per sq. ft. with 36 in. dia. drive pulley making 155 *R. P. M.*

24. Find *H. P.* of 40 in. double belt with 4 reinforcing strips each 4 in. wide. The strips weigh 16 oz. per sq. ft., the belt 32 oz. per sq. ft. and runs on a 20 ft. fly wheel pulley at 65 *R. P. M.*

25. What is the *H. P.* of a 16 in. single belt over an 8 ft. drive pulley at 110 *R. P. M.?*

26. What is the *H. P.* of a 24 in. double belt on 48 in. driver at 275 *R. P. M.* when the weight is 24 oz. per sq. ft.?

27. What width of double belt $\frac{3}{8}$ in. thick will be required for a 28 *H. P.* drive at 3,500 *F. P. M.*?

28. What width of double belt at 27 oz. per sq. ft. on a 24 in. dia. driver at 150 *R. P. M.* will be required for transmitting $62\frac{1}{2}$ *H. P.*?

29. Find b when $w=16$, $N=125$, $d=20$ in., $H.\ P.=50$.

30. Find w when $b=20$ in., $d=36$ in., $N=260$, $H.\ P.=75$.

31. What weight of belt per sq. ft. will be required to transmit 250 *H. P.* over a 72 in. dia. pulley at 250 *R. P. M.*, when belt is 36 in. wide?

32. What *R. P. M.* will a 34 in. dia. drive pulley make to transmit 100 *H. P.* with a 24 in. belt weighing 32 oz. per sq. ft.?

33. What *R. P. M.* will a 96 in. dia. drive pulley be required to make in order to transmit 175 *H. P.* with a belt 36 in. wide and weighing 36 oz. per sq. ft.?

34. Find the dia. of a drive pulley making 125 *R. P. M.* for a 28 in. belt weighing 32 oz. per sq. ft. to transmit 185 *H. P.*

35. What is the dia. of pulley required at 250 *R. P. M.* for a belt 30 in. wide weighing 32 oz. per sq. ft. to transmit 185 *H. P.*?

36. What weight per sq. ft. will be required for a belt 12 in. wide running 3,600 *F. P. M.* on a 24 in. dia. pulley, to transmit 75 *H. P.*?

37. What weight per sq. ft. will be required for a 24 in. belt on a 46 in. dia. pulley making 225 *R. P. M.* to transmit 125 *H. P.*?

38. Find w when $R.\ P.\ M.=750$, $d=15$ in., $b=7$ in., $H.\ P.=25$.

ROPES

Rope drives are used for long distance transmissions and for drives leading off to several shafts at different angles to each other and not parallel to the driving shaft. The effectiveness of a rope drive depends upon the friction in the grooves of the pulleys.

The velocity of rope transmission is from 1,500 to 5,000 *F. P. M.* and should not exceed 5,000 *F. P. M.* for then the loss due to centrifugal force will offset the gain in speed. The diameter of sheave should not be less than 40 times the diameter of rope in inches.

The breaking strain for hemp rope is 6,000 pounds per square inch of cross section area; for manila rope 3,000 pounds per square inch.

To find the breaking strength, multiply the cross section area of the rope by 6,000 for hemp and by 3,000 for manila rope.

The horse power which a rope is capable of transmitting may be found by the formula:

$$H.\ P. = \frac{d^2 \times V \times 12\frac{1}{2}}{5000} \text{ or } H.\ P. = d^2 \times V \times .0025.$$

d = diameter of rope in inches.

$V = F.\ P.\ M.$

The weight of 1 inch diameter rope is $\frac{3}{10}$ of a pound per foot in length, the weights of other sizes can be found by formula:

$$W = d^2 \times .3.$$

PROBLEMS

1. Find the weight of 400 ft. of $1\frac{1}{4}$ in. dia. rope.

2. Find the weight per ft. of $1\frac{1}{2}$ in. dia. rope.

3. What weight will be required to break a $1\frac{1}{2}$ in. dia. manila rope?

4. What weight would be likely to break a $\frac{3}{4}$ in. dia. hemp rope?

5. What weight would be apt to break a $1\frac{3}{4}$ in. dia. manila rope?

6. What *H. P.* will a $1\frac{1}{4}$ in. dia. manila rope transmit, running over two 8 ft. sheaves at 150 *R. P. M.?*

7. If the pulleys in problem 6 are 100 ft. apart on centers, what is the weight of the rope?

8. What strain will be put on a $1\frac{3}{8}$ in. manila rope running at 4,500 *F. P. M.*, to transmit 150 *H. P.?*

9. What *H. P.* will be transmitted with a $1\frac{3}{4}$ in. manila rope at 5,500 *F. P. M.?*

10. What is the minimum size of sheaves and *R. P. M.* of pulley for a $1\frac{5}{8}$ in. dia. manila rope at 3,500 *F. P. M.?* Also, what *H. P.* can be transmitted by this rope?

11. What weight would be required to break a $\frac{7}{8}$ in. dia. manila rope?

WIRE CABLE TRANSMISSION

With a *wire cable drive* there should be no binding or wedging of the cable in the groove of the sheave. The cable should run on wood or leather packing in the bottom of the groove, and the sheave for iron wire should not be less than 150 times the diameter of the cable. The cable should not run over 6,000 *F. P. M.* on account of centrifugal force, and 3,000 to 5,000 *F. P. M.* is preferable.

The distance between centers of pulleys should not be less than 60 feet nor over 400 feet. Carrying pulleys are used when the spans are over 400 feet.

The *H. P.* transmitted by an iron wire cable is found by the formula:

$$H.\,P. = \frac{d^2 \times V \times 275}{5{,}000}$$

or $H.\,P. = d^2 \times V \times .055,$

where the letters have the same significance as in the formulas for rope drives. A steel cable will transmit double the above amount.

The breaking strain for iron cables is 40,000 pounds per square inch of cross section area, and for steel cables 80,000 pounds.

A *factor of safety of 10* is generally allowed for rope and cable transmission.

A *factor of safety* is the number which expresses the ratio of the ultimate strength of a body to the working load.

The weights of wire cables per foot in length are as follows:

$\frac{3}{8}$ in. dia. = .21 lbs.	$\frac{5}{8}$ in. dia. = .57 lbs.
$\frac{7}{16}$ in. dia. = .23 lbs.	$\frac{3}{4}$ in. dia. = .92 lbs.
$\frac{1}{2}$ in. dia. = .31 lbs.	$\frac{7}{8}$ in. dia. = 1.20 lbs.

1 in. dia. = 1.50 lbs.

Cable Transmission Problems

1. What is the weight of a $\frac{7}{8}$ in. dia. cable at 1.2 lbs. per ft. $837\frac{1}{2}$ ft. long, and what *H. P.* can be transmitted at 100 *R. P. M.* over 12 ft. dia. sheaves?

2. What is the difference in *H. P.* of problem 1 when figured by the formula and by ft.-lbs. of work?

3. What is the weight of a $\frac{5}{8}$ in. iron cable 700 ft. long, and what *H. P.* can be transmitted over 10 ft. dia. sheaves at 80 *R. P. M.?*

4. What $H. P.$ can be transmitted by a 1 in. cable at 95 $R. P. M.$ over a $12\frac{1}{2}$ ft. dia. sheave?

5. The length of a $1\frac{1}{8}$ in. iron cable is 800 ft. and it weighs 2 lbs. per ft. Find the $F. P. M.$ required to transmit 200 $H. P.$

6. Find the dia. of sheave at 125 $R. P. M.$ and length of a $\frac{3}{4}$ in. steel cable to transmit 90 $H. P.$ and be inside the safe limit.

CHAIN TRANSMISSION

One form of chain used for hoisting machinery is made of round wire links as short as possible for strength. Let *d = diameter of wire,* then the *length of link inside = 2.5 d; width of link inside = 1.5 d.*

The following table gives sizes of wire for hoists:

Capacity in tons,	$\frac{1}{8}$	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	3	4	5	6	8	10
Diam. of wire in inches,	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{9}{32}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{13}{16}$

Another form of chain used for giving a positive motion between two shafts by means of sprocket wheels, is the flat link and block chain; here the strain on the chain is limited by the shearing stress on the pivots.

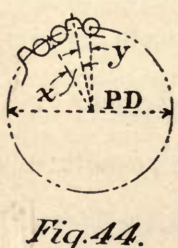


Fig. 44.

The sprockets should not have less than 7 teeth; the larger the sprocket the less will be the strain on chain and consequently less wear on the pivots of the rivets.

The formula for sizes of sprockets is as follows:

$$x = \frac{180^\circ}{N},$$

$$tan\ \ Y = \frac{sin\ x}{\frac{B}{A} + cos\ x}$$

$$PD = \frac{A}{sin\ Y}$$

N = number of teeth in sprocket,

A = distance between centers of rivets in link,

B = distance between centers of rivets in block,

b = diameter of end of block,

for 1 inch pitch of chain b is usually .325 inch when A = .6 inch and B = .4 inch.

The bottom diameter of sprocket wheel is the important dimension, therefore size b must be taken accurately.

Then $OD = PD + b$

and *Bottom diameter* $= PD - b$.

PROBLEMS

1. What is the dia. of an 8 tooth sprocket wheel with a 1 in. pi. chain?

2. What is the *OD* and bottom dia. of a 20 tooth sprocket with 1 in. pi. chain?

3. What are the distances A, B, and *dia. of* b, for a $1\frac{1}{2}$ in. pi. chain, when proportionally the same as for 1 in. pitch?

4. What are the *OD* and *PD* for a 20 tooth sprocket, with $1\frac{1}{2}$ in. pi. chain?

5. Find the diameters for a 24 tooth sprocket for a $1\frac{1}{2}$ in. pi. chain.

6. Find A, B, and b, for $1\frac{3}{4}$ in. pi. when proportionally the same as for a 1 in. pi. chain.

7. Find the diameters for a 28 tooth sprocket with a $1\frac{3}{4}$ in. pi. chain.

8. What are the *PD* and *OD* for a sprocket of 28 teeth and 1 in. pi. chain?

9. Find the bottom dia. for a sprocket of 30 teeth and 1 in. pi. chain.

SHAFTING

The *H. P.* which a line shaft can impart to connected machinery is limited by the strength of the material of which it is made. The principal strain is in the twisting of the round bar when the pulleys are made to revolve carrying driving belts to the various machines.

The twisting of the shaft is called its *torsional strain*, and the formula which determines the amount of torsion which a shaft will safely stand is

$$d=\sqrt[3]{\frac{rw}{C\times\frac{1}{10}}},$$

where $d=$ the diameter of shaft in inches,

$w=$ the pull of the belt in pounds,

$r=$ the radius of the pulley in feet,

$C=$ the constant for breaking moment which is found by experiment for cold rolled steel to be 660 pounds.

10 is the factor of safety.

Then the above formula may be written

$$d=\sqrt[3]{\frac{H\times 80}{N}}$$

where $H=H.\ P.$

$N=R.\ P.\ M.$

PROBLEMS

1. Find values for H and N by transposing the formula for d, given above.

2. A 4 in. dia. shaft runs 150 *R. P. M.* What is the safe load in *H. P.* to put on the shaft?

3. Find the *H. P.* that can be transmitted by a shaft $2\frac{1}{2}$ in. dia. at 200 *R. P. M.*

4. What *H. P.* can be transmitted by a 5 in. dia. shaft running at 275 *R. P. M.?*

5. What dia. shaft will be required to transmit 500 *H. P.* running at 150 *R. P. M.?*

6. What dia. of shaft will be required to transmit 250 *H. P.* running at 74 *R. P. M.?*

7. How fast should a shaft revolve that is to transmit 1,000 *H. P.* and is 12 in. diameter?

8. What is the dia. of a shaft that is to transmit 150 *H. P.* at 150 *R. P. M.?*

9. Find the *R. P. M.* of a 5 in. shaft to transmit 300 *H. P.*

10. Find the dia. of a shaft to transmit $12\frac{1}{2}$ *H. P.* at 500 *R. P. M.*

11. Find the dia. of a shaft to transmit 400 *H. P.* at 200 *R. P. M.*

12. What dia. of shaft will be required to transmit 1,000 *H. P.* at 36 *R. P. M.?*

13. What dia. of shaft will be required to transmit 200 *H. P.* at 180 *R. P. M.?*

14. Find the *H. P.* transmitted by an 8 in. dia. shaft at 115 *R. P. M.*

15. Find the *H. P.* transmitted by a 6 in. dia. shaft at 150 *R. P. M.*

16. What *H. P.* can be transmitted by an 18 in. dia. shaft at 50 *R. P. M.?*

17. How many *R. P. M.* should an 8 in. dia. shaft make to transmit 200 *H. P.?*

18. How many *R. P. M.* should a 3 in. dia. shaft make to transmit 200 *H. P.?*

19. How many *R. P. M.* should a $1\frac{15}{16}$ in. dia. shaft make to transmit 28 *H. P.?*

A *jack shaft* is a short shaft between the engine and main line of shafting, and is used to regulate the speed of the main line, and prevent the heavy strains on the main line shafting due to the engine drive belt. The bearings must be as near the pulleys as possible.

The strain on shaft being greater, the constant is increased from 80 to 120. The formula is $d=\sqrt[3]{\frac{H\times 120}{N}}$, the same letters having the same meaning as in formula for shafting given on page 73.

Idler shafts are used to change the direction in which the belts run and have only a bending or shearing strain.

The distance between the centers of bearings for line shafting varies in proportion to the size of shaft.

Dia. of shaft	$1\frac{1}{2}$ to $1\frac{3}{4}$ in.	2 to $2\frac{1}{2}$ in.	$2\frac{1}{2}$ to 4 in.
Center of boxes	7 ft.	8 ft.	10 ft.

PROBLEMS

1. What dia. of jack shaft will be required at 225 *R. P. M.* to transmit 225 *H. P.?*

2. What dia. of jack shaft will be required at 500 *R. P. M.* to transmit 250 *H. P.?*

3. How many *R. P. M.* should a 6 in. dia. jack shaft make to transmit 500 *H. P.?*

4. How many *R. P. M.* should a 2 in. dia. jack shaft make to transmit $87\frac{1}{2}$ *H. P.?*

5. What *H. P.* will a 6 in. jack shaft transmit at 250 *R. P. M.?*

6. What *H. P.* will a 2 in. dia. jack shaft transmit at 250 *R. P. M.?*

7. Find *H. P.* transmitted by a $2\frac{1}{2}$ in. dia. jack shaft at 170 *R. P. M.*

8. Find dia. of jack shaft designed for a 250 *H. P.* engine, when the shaft is to make 84 *R. P. M.*

9. What should be the distance between the centers of hangers for a line shaft that will transmit 100 *H. P.* at 160 *R. P. M.?*

10. Find the distance between the centers of hangers on a line shaft to transmit 50 *H. P.* at 150 *R. P. M.*

JOURNAL BEARINGS

The lengths of journal bearings vary with the work to be done, lubrication of bearing, kind of materials used for the boxes, and diameter of journal.

Line shaft bearings are designed from the formula:

$$L = 6\sqrt{d},$$

where L = length of bearing,

d = dia. of shaft in inches.

The projected area of the journal is the size used in the calculations for the bearing surface; thus the area of bearing surface for a 4 inch diameter by 12 inch journal is $4 \times 12 = 48$ square inches.

The length of bearings for machine spindles vary according to the service required, from $1\frac{1}{2}$ to 6 times the diameter of journal. For journals under 2 inches diameter having 100 to 1,000 *R. P. M.* use the formula:

$$L = d\left(1 + \frac{R.\ P.\ M.}{200}\right).$$

The allowable pressure on the ordinary shaft bearings is 40 pounds per square inch of projected area.

BALL BEARINGS

Ball bearings are used especially on high speed and light running machinery. The diameter of the ball race may be calculated from the size and number of balls as follows:

In Fig. 45 by trigonometry

$$AO = \frac{AD}{\sin AOD}$$

let $2 \times AO = D$

$$\angle AOD = \frac{360^\circ}{2n} = \frac{180^\circ}{n}$$

where n = number of balls in race
D = dia. of circle at center of balls
d = dia. of ball

then $AD = \frac{d}{2}$

and $D + d$ = dia. of race ring

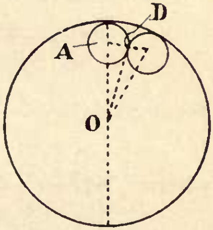


Fig. 45.

Example. Find the diameter of ball race for 15, $\frac{1}{2}$ inch dia. balls.

Solution. $AD = \frac{d}{2} = .250$ *inches.*

$$\frac{180^\circ}{n} = 12^\circ$$

$$\sin 12^\circ = .20791$$

then $$AO = \frac{.25}{.20791} = 1.20244$$

$$D = 1.20244 \times 2 = 2.405 \text{ inches.}$$

$$D + d = 2.405 + .5 = 2.905$$

A clearance of about $\frac{1}{100}$ inch is usually added to the diameter of the ball race to allow for slight oversize in the diameter of the balls.

A modification of the ball bearing shown in Fig. 45 is sometimes made in order to reduce the number of balls used in the race ring.

This is shown in Fig. 45A where fewer balls are held from contact with each other by a separator ring or cage.

Then the formula above

$$AO = \frac{AD}{\sin AOD}$$

becomes $$AO = \frac{r + \frac{s}{2}}{\sin AOD}$$

where $$AD = r + \frac{s}{2}$$

$$\angle AOD = \frac{360^\circ}{2n} = \frac{180^\circ}{n}$$

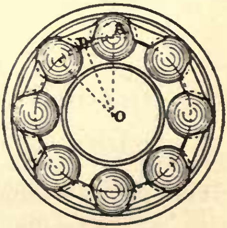


Fig. 45 A.

then r = radius of balls and $d = 2r$

n = number of balls

s = clearance between two balls

$D = 2 \times AO$ = diameter of circle at center of ball

By transposition four statements are obtained as follows:

1. Given the diameter and number of balls and the clearance, to find D.

$$\frac{D}{2}=\frac{r+\frac{s}{2}}{\sin\frac{180°}{n}} \quad or \quad D=\frac{d+s}{\sin\frac{180°}{n}}$$

2. Given the number of balls, the clearance and D, to find r.

$$r=\frac{D}{2}\left(\sin\frac{180°}{n}\right)-\frac{s}{2} \text{ or } d=D\left(\sin\frac{180°}{n}\right)-s$$

3. Given the diameter and number of balls and D, to find s.

$$s=D\left(\sin\frac{180°}{n}\right)-d$$

4. Given D, the diameter of balls and clearance, to find n.

Find $\sin\frac{180°}{n}$ from $\frac{2r+s}{D}$

but $\frac{180°}{n}=\angle AOD$

then $n=\frac{180°}{\angle AOD}$

PROBLEMS

1. Find the dia. of ball race for 12, $\frac{1}{4}$ in. dia. balls.
2. Find the dia. of ball race for 15, $\frac{1}{4}$ in. dia. balls.
3. What is the dia. of ball race for 3, $\frac{1}{2}$ in. dia. balls?
4. What is the dia. of roll race for 12, $1\frac{1}{2}$ in. dia. rolls?
5. Find dia. of ball race for 22, $\frac{3}{16}$ in. dia. balls.
6. What is the dia. of ball race for 26, $\frac{5}{16}$ in. dia. balls?
7. What is the length of box for $2\frac{7}{16}$ in. dia. shafting?

8. What is the length of box for $1\frac{15}{16}$ in. dia. shaft?

9. Find length of box for $2\frac{15}{16}$ in. dia. shafting.

10. Find length of hanger bearing for $3\frac{7}{16}$ in. dia. shaft.

11. What is the dia. of shafting, when bearing for boxes is 12 in. long?

12. What length of shaft bearing will be required for $5\frac{7}{16}$ in. dia. shaft?

13. Find length of box for $6\frac{7}{16}$ in. dia. shafting.

14. Find dia. of shaft for hanger box 16 in. long.

15. What dia. of shaft will be required for box $9\frac{1}{2}$ in. long?

16. When 40 lbs. is allowed per sq. in. on bearings, what is the length of box for $4\frac{3}{16}$ in. dia. shaft, that carries 9 pulleys, with $3\frac{1}{2}$ in. belts at 55 lbs. per in. strain, each pulley weighing 35 lbs.?

Note. Use approximate formula in finding weight of shaft. Page 172.

17. What length of bearing will be required for a $3\frac{7}{16}$ in. dia. shaft, carrying 6 pulleys for $3\frac{1}{2}$ in. belts at 55 lbs. per in. strain and weighing 35 lbs. each, allowable pressure 40 lbs. per sq. in.?

18. How many 12 in. dia. pulleys weighing 40 lbs. each for 4 in. belts at 55 lbs. per in. strain, can be put on 3 in. shaft with allowable pressure 40 lbs. per sq. in.?

19. A jack shaft has 3 pulleys each weighing 1,500 lbs. The pulleys carry 26 in. double belts at $82\frac{1}{2}$ lbs. strain. Find length of 3 boxes used, at 8 ft. on centers, when shaft is 5 in. dia. and projects 6 in. beyond centers of end boxes, with 75 lbs. per sq. in. pressure on bearing.

20. What is the length of crank shaft bearing for $7\frac{1}{2}$ in. dia. of journal when shaft is 8 ft. long, fly wheel weighs 8 tons, belt 26 in. wide with $82\frac{1}{2}$ lbs. per in. strain and 80 lbs. is allowable pressure per sq. in. on bearing?

21. Find number of 3 in. belt pulleys 55 lbs. strain, each pulley weighing 100 lbs., that can be placed on a 4 in. dia. line shaft, 8 ft. between the centers of hangers.

22. What lengths of boxes will be required for $4\frac{1}{2}$ in. dia. jack shaft, 8 ft. long, to carry 2 drive pulleys, weighing 1,000 lbs. each with 24 in. single belt at 55 lbs. strain per in., allowable pressure being 65 lbs.?

23. A jack shaft with 5 in. journals, with 75 lbs. pressure per sq. in. will require 2 boxes of what length, when the total weight and pull of belt is 15,000 lbs.?

24. What is the bearing surface required for a $3\frac{1}{2}$ in. dia. line shaft with 40 lbs. allowable pressure, when there is a weight including shaft of 2,000 lbs. on each bearing?

25. Find the dia. of race ring for 10 $\frac{5}{16}$ in. dia. balls when the clearance between the balls is $\frac{1}{4}$ inch.

26. What is the clearance between 8 $\frac{1}{2}$ in. dia. balls when the race ring is $2\frac{1}{4}$ in. diameter?

27. Find the dia. of the ball to be used in a 15 ball race ring when the clearance is .114 in. and the circle at the center of the balls is $1\frac{3}{4}$ in. diameter.

28. How many $\frac{1}{4}$ in. dia. balls will be required for a race ring $1\frac{1}{4}$ in. dia. when the clearance between the balls is .1365 inch?

29. Find D when $n=8$, $d=\frac{3}{8}$ in. and $s=.250$ in.

30. Find n when $D=1\frac{1}{2}$ in., $d=\frac{3}{8}$ in. and $s=.199$ in.

31. Find s when $d=\frac{1}{2}$ in., $n=20$ and $D=4$ in.

32. Find d when $D=1.82$ in., $n=6$ and $s=.41$ in.

33. Find s when $n=9$, $d=\frac{1}{4}$ in. and $D=1\frac{3}{8}$ in.

34. Find s when $D=1\frac{1}{2}$ in., $d=\frac{5}{16}$ in. and $n=11$.

MACHINE KEYS

Machine keys are short bars of metal, usually either square or rectangular in cross section, used to prevent wheels, pulleys, cranks, or other pieces from rotating on the shaft.

Proportion of Machine Keys

Let d = diameter of shaft
b = breadth of key
t = thickness of key, then $b=\frac{d}{4}$ and $t=\frac{2b}{3}$

Feather keys are used so that a piece can slide along a shaft and yet have a positive rotation with the shaft. The key is usually fastened in the sliding piece, and is square in cross section outline

then $$t=b=\frac{d}{4}.$$

PROBLEMS

1. A 6 in. dia. shaft is to be coupled to a 5 in. shaft with flange couplings. What are the sizes of keys required for each flange?

2. What is the thickness of key required for fastening pulley to a 2 in. dia. shaft?

3. What size of key will be required to fasten a 24 in. crank to an 8 in. dia. shaft?

4. What size key should be used on feed worm of an engine lathe, when feed rod is 1 in. diameter?

LINEAR MEASURING INSTRUMENTS

The *meter* is the only unit for linear measure legalized by the United States government, but the bronze bar No. 11, given to the United States by the British government, has been generally accepted as the standard unit for length of the yard at a temperature of 61.79° F. From this bar the makers of measuring instruments have compared their standards, and have fixed lengths of bars, from which their instruments are made with as fine a degree of accuracy as it is possible for a trained and skilled mechanic to attain.

When it is remembered that the bronze No. 11 changes in length over $\frac{3}{100000}$ inch for each degree Fahrenheit, it will be seen that extreme accuracy and close measurements depend not only upon the accuracy with which the standard bar is made, but also upon the temperature of the bar at the time that measurements are taken or comparisons made. Usually each shop has some standard bar, which is referred to as a standard gauge, with which all other measuring instruments are compared.

The *micrometer* is the measuring instrument that is now generally used to record dimensions of any body in thousandths of an inch. The principle of construction and operation of the micrometer is comparatively simple. It consists of a screw C, Fig. 46, having forty threads per inch, that turns through a stationary nut B, which is part of a curved bar to which is attached a round pivot E called the anvil located in line with the screw C. F is a lock nut which sets the screw C firmly in any desired position for taking dimensions. The barrel or thimble A is a part of, or fast to the screw C, and is divided into twenty-five divisions on the beveled end at a, each five divisions being marked with figures *0—5—10—15—20*. A cylindrical part of the nut B has a line parallel to the axis of the longitudinal motion of the screw, and at right angles to this line are short lines which represent the position of the end of thimble A for each revolution of the screw C. The first one marked O shows the position of screw when it just touches the end of anvil E, and at this position the O line on thimble A, should coincide with this O point at its intersection with line parallel to axis of screw C. As one turn of a forty pitch screw is equal to $\frac{1}{40}$ of 1 inch, or $\frac{25}{1000}$ inch, the motion of thimble A through the space of one of the twenty-five divisions on end of thimble will give a longitudinal movement of the screw C of one-thousandth inch ($\frac{1}{1000}$ inch). Each four turns of the screw C will be $\frac{25}{1000} \times 4$, or $\frac{100}{1000}$ *inch*, these points being marked *1*, *2*, *3*, *4*, etc., up to *9*, indicating *100*, *200*, *300*, etc., thousandths of an inch between end of screw C and anvil E, according to the position of the beveled end of thimble. Suppose a dimension of *.5625 inch* is to be taken, the screw C is opened by revolving A until *0* on thimble corresponds with the axial line at *500* thousandths. As each turn of C is equal to *25* thousandths two turns are made with A which gives *550* thousandths. Now turning A through *12* spaces on thimble will give *562* thousandths, and *5* tenths or the half thousandth will be obtained when the axial line is half way between the twelfth and thirteenth division lines on the thimble. In this way *.5625 inch* can be obtained. Other dimensions are obtained in like manner, getting the number of thousandths required between the hundredths by adding

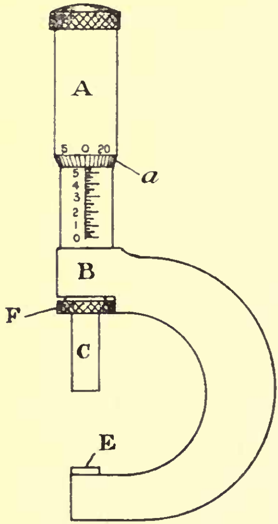


Fig. 46.

25, *50*, or *75* for one, two, or three complete turns of the screw, and then turning the thimble any number of spaces required between the complete turns to make up the required number of thousandths for getting the dimension wanted.

When a vernier scale is made a part of the graduations of a micrometer caliper, it is unnecessary to get the fraction of a thousandth of an inch by the judgment of setting the line of axis between two spacing lines on thimble, as the setting can be found directly by the vernier reading.

THE VERNIER

The *vernier* principle is used on the slide caliper for setting the jaws of the caliper to $\frac{1}{1000}$ *inch* and is as follows: the bar *A*, Fig. 47, which has solid jaw *B* at one end graduated the whole length into fortieths of an inch; on the sliding jaw *C* is a scale *D*, which has twenty-five divisions on the beveled edge equal in length to twenty-four of the $\frac{1}{40}$ *inch* spaces on the bar. It is evident that the spaces on the scale *D* will be smaller than the spaces on the bar by $\frac{1}{25}$ of a space on the bar, as *24* of them are equal to *25* spaces on the scale. Now $\frac{1}{25}$ of $\frac{1}{40}$ *inch* $=\frac{1}{1000}$ *inch.*

The great difficulty in setting the vernier slide caliper is to determine which lines of the scale and bar coincide as, where in this case the variations in width of spaces are actually $\frac{1}{1000}$ *inch,* in the micrometer the space that represents $\frac{1}{1000}$ *inch* is usually $\frac{1}{16}$ *inch* or more in width on the thimble of the screw.

The vernier calipers are set to any dimensions as follows: if a dimension of *1.128 in.* is to be taken the *O* mark on scale *D* is moved until it coincides with the *1 inch* mark of the bar

A. As each division on the bar *A* is $\frac{1}{40}$ *inch* or $\frac{25}{1000}$ *inch,* the *O* line on the scale is moved until it coincides with the *fifth* line from the *1* inch line on bar *A*, the set screw *H* is then tightened, fastening slide *F* securely to bar *A*, *E* being left loose to allow sliding jaw *C* to move freely along bar *A* by the turning of thumb nut *G*, which is revolved until the third line from *O* on the scale is exactly in line with the eighth line from the *1* inch line on bar *A*. This gives the required dimension of *1.128 inch.* Other dimensions are taken in the same way.

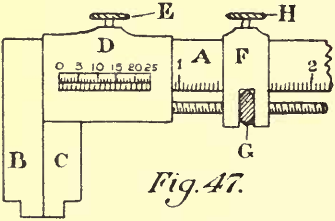


Fig. 47.

The vernier scale on the micrometer used to get the tenths of thousandths of an inch is made as follows: Fig. 48 shows the developed projection of the barrel of nut *B* of Fig. 46; to the left of the axial line are eleven lines, also drawn parallel to the axis line of the screw, forming ten spaces equal to nine of the twenty-five divisions on the thimble of screw *C*, each of these spaces being smaller by one-tenth than one division on the thimble. These lines are so placed on the barrel of the nut that the first one coincides with a line on the thimble when any line of the thimble is in line with the first axial line *b*, Fig. 48. Now if a dimension of one-tenth of some even one-thousandth inch is required the second parallel line on the barrel will be brought into line with the next division line on thimble, if two-tenths over are required, the thimble will be turned until the third line coincides

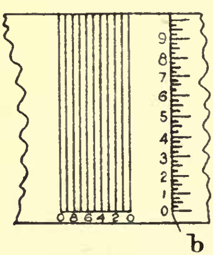


Fig. 48.

with the third parallel line on the barrel, and so on to obtain any number of tenths over the required dimension wanted.

Suppose the reading *.7658* is to be found on the micrometer that has a vernier attachment. The micrometer is set first to the dimension *.765* as given above. The reading for *.7658* will be obtained when the line *8* on the vernier scale exactly coincides with a line on the thimble.

The principle of the micrometer screw thread is used on different machines to gauge the depth of a cut to thousandths of an inch. The method of measuring the movement of the screw is the same as for the micrometer and the formula is as follows:

Let x = endwise movement of the screw or nut for the rotation through one division on the disc or thimble.

N = number of divisions on the disc or thimble that is fastened on screw.

L = lead of screw thread.

Then $x = L \times \frac{1}{N}$;

thus in the micrometer,

$$x = \tfrac{1}{40} \times \tfrac{1}{25} = \tfrac{1}{1000} \textit{ inch}.$$

Example. If the feed screw of a universal grinder bed is $\frac{1}{8}$ *inch* lead and the disc fast to the screw has *125* divisions, how far will movement of disc through one division move bed of grinder up to the wheel?

Solution. By formula:

$$x = L \times \frac{1}{N} \therefore \tfrac{1}{8} \times \tfrac{1}{125} = \tfrac{1}{1000}.$$

PROBLEMS

1. What is the lead required on the screw of grinder bed to move bed $\frac{1}{1000}$ in., when the disc on screw is turned through an arc of 3 degrees?

2. How far will a screw of $\frac{1}{16}$ in. lead move a nut when turned through $\frac{1}{100}$ of a turn?

3. When the cross slide on a lathe is operated with a screw of $\frac{1}{6}$ in. lead, what is the movement of slide when a disc of 100 divisions on the screw is moved through an arc of one division?

4. Find the lead of feed screw on a slide that will move the slide $\frac{1}{500}$ in. when a disc with 100 divisions is turned through an arc of one division.

5. If the cross slide feed screw of a certain machine is $\frac{1}{4}$ in. lead, what ratio of gears on an auxiliary crank rod geared to screw will be required on rod and screw to move the slide $\frac{1}{1000}$ in., when the graduated disc having 100 divisions on the crank moves through an arc of one division?

6. If the adjustable knee of a shaper is to be raised $\frac{1}{1000}$ in. by turning crank through arc of one space of a disc fast to crank rod and having 100 divisions, and the screw that stands at right angles to crank rod has $\frac{1}{12}$ in. lead, what will be the ratio between the number of teeth in the beveled gears running in mesh on crank rod and screw?

7. If the screw that moves the bed of a milling machine is $\frac{1}{4}$ in. lead, what number of spaces should be made on a disc to move the bed $\frac{1}{1000}$ in. when the screw is moved through the arc of one space on the disc that is fast to screw?

8. If the cross feed screw of a lathe is $\frac{1}{4}$ in. lead, how far will the movement of one space on a disc of 25 divisions that is fast to screw move the cross slide?

9. The screw that raises the knee on a milling machine is $\frac{3}{16}$ in. lead, and is turned by a crank rod with a pair of bevel gears on rod and screw. On crank rod is a disc having 100 divisions. What ratio of gears will be needed to raise the knee $\frac{1}{1000}$ in. when the disc fast to the crank is turned through an arc of one division?

10. The cross feed screw for a milling machine bed is 20 pitch. Into how many divisions should the disc that is fast to the screw be divided to move the bed $\frac{1}{1000}$ in. when the disc is moved through the arc of one division?

11. If the table of a horizontal boring mill is moved with an 8 pitch single thread screw, into how many divisions should a disc fast to screw be divided to give $\frac{1}{600}$ in. movement to the table when the screw is moved through the arc of one division on the disc?

MACHINES

The various machines used for manufacturing purposes are generally classified as follows:

Lathes; as, engine, turning, speed, grinding, cutting off, axle, wheel, screw machine, etc.

Planers; as, shapers, slotters, surface grinders, etc.

Milling machines; as, power, hand, duplex, universal, vertical, profiles, etc.

Drills; as, vertical or horizontal spindles, chucking and boring machines, etc.

Punches; as, presses, shears, etc.

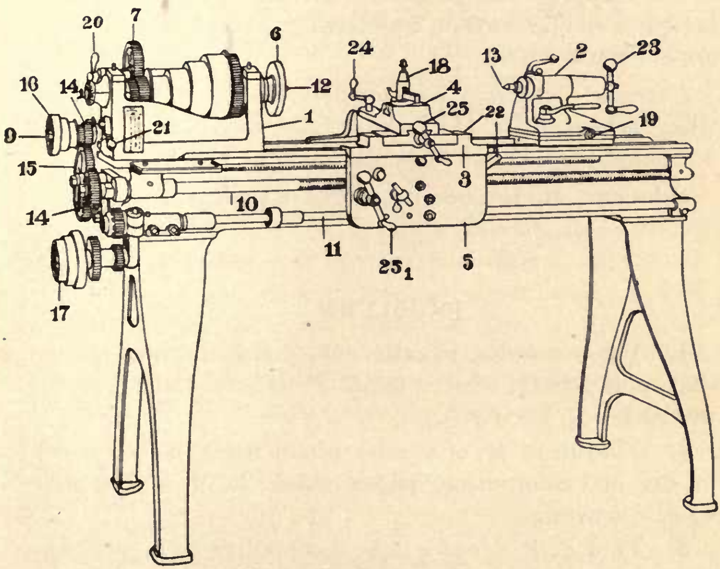
Power hammers; as, steam, helve, trip, drop, etc.

There are numerous *special machines* which can be classed as modifications of some one of the above classes.

LATHE

By far the most important and the one which embodies the general principles of the other classes is the *lathe*. The figure on page 91 shows outline of a 12 inch by 5 foot engine lathe.

The size of a lathe is stated by the largest diameter it will swing on centers and the length of the bed. The carriage is moved along the length of the Vs of the lathe bed parallel to the axis of the driving spindle; the motion is imparted to carriage from the driving spindle either by the feed rod *11*, through a belt transmission that is used when doing plain turning, or by the lead screw *10*, and a system of change gears which is used when cutting screw threads.



12″ x 3′0″ Engine Lathe

1 Head stock
2 Foot or tail stock
3 Carriage
4 Elevating tool rest
5 Apron
6 Face plate
7 Back gears
9 Stud of feed spindle
10 Lead screw
11 Feed rod
12 Live center
13 Dead center
14 and 14_1 Change gears
15 Intermediate gear
16 Upper belt feed cone
17 Lower belt feed cone
18 Tool post
19 Set over screw for tail stock
20 Back gear lever
21 Reverse feed lever for carriage
22 Screw feed nut lever
23 Ball crank handle for tail spindles
24 Ball crank handle for elevating rest
25 Ball crank handle for cross feed
25_1 Ball crank handle for carriage feed

The stepped cone pulley on driving spindle of lathe is belted to similar cone on countershaft except that the steps are in reverse order.

Example. When a countershaft makes 160 *R. P. M.*, what is the *R. P. M.* of a lathe spindle, with pulley $7\frac{5}{8}$ in. dia. on counter, belted to a $4\frac{5}{8}$ in. dia. on lathe spindle?

Solution. By formula for speeds of pulleys,

$$n=\frac{160\times 7\frac{5}{8}}{4\frac{5}{8}}=263+$$

PROBLEMS

1. When the dia. of lathe pulley is $7\frac{5}{8}$ in. and countershaft pulley is $4\frac{5}{8}$, what is the *R. P. M.* of spindle if counter makes 160 *R. P. M.?*

2. Find *R. P. M.* of a lathe spindle when the pulley is $6\frac{1}{8}$ in. dia. and countershaft makes 155 *R. P. M.* with a pulley $6\frac{7}{8}$ in. diameter.

3. Find *R. P. M.* of a lathe cone pulley $8\frac{1}{4}$ in. dia. when the countershaft makes 170 *R. P. M.* and the pulley on countershaft is 5 in. diameter.

4. When a lathe cone pulley is $3\frac{1}{4}$ in. dia. and countershaft pulley is $11\frac{1}{4}$ in. dia., find *R. P. M.* of spindle when countershaft makes 165 *R. P. M.*

5. When a lathe spindle pulley is $8\frac{1}{2}$ in. dia. and countershaft runs 170 *R. P. M.* with a pulley $4\frac{1}{8}$ in. dia., what is the *R. P. M.* of the lathe spindle?

6. When a cone pulley is $3\frac{1}{2}$ in. dia. and countershaft runs 195 *R. P. M.* with pulley $9\frac{1}{2}$ in. dia. on countershaft, what is the *R. P. M.* of the lathe spindle?

7. The cone pulley of a certain lathe has steps $7\frac{5}{8}$ in., $6\frac{1}{8}$ in., $4\frac{5}{8}$ in., and $3\frac{1}{8}$ in. diameters respectively, belted to the steps of a cone on countershaft, the diameters of which are as follows: $4\frac{5}{8}$ in., $6\frac{1}{8}$ in., $7\frac{5}{8}$ in., and $9\frac{1}{8}$ in. When the countershaft runs 170 *R. P. M.* find the different speeds at which the work on the spindle may be driven.

8. A lathe has on its driving spindle a four step cone pulley with diameters as follows: $8\frac{1}{4}$ in., $6\frac{3}{4}$ in., 5 in., and $3\frac{1}{4}$ in. Belted to this cone is the cone on countershaft with steps $5\frac{1}{4}$ in., $6\frac{3}{4}$ in., $8\frac{1}{2}$ in., and $10\frac{1}{4}$ in. diameters respectively. Find the different *R. P. M.* at which work can be turned in the lathe, when the countershaft runs 180 *R. P. M.*

9. A lathe has a cone on the head spindle with steps $8\frac{1}{2}$ in., $6\frac{3}{4}$ in., $5\frac{1}{8}$ in., and $3\frac{1}{2}$ in. diameters respectively. If the countershaft runs 175 *R. P. M.* with a cone having steps $4\frac{1}{2}$ in., $6\frac{1}{4}$ in., $7\frac{7}{8}$ in., and $9\frac{1}{2}$ in. diameters respectively, what different *R. P. M.* may be given the work on the lathe centers?

BACK GEARS

The *back gears* are used to reduce the speed of lathe spindle, thus allowing a heavier cut to be taken at reduced speed.

On the back gear spindle are two gears fast to the same sleeve; on the lathe cone pulley is fastened a small gear which may be thrown into mesh with large gear on back gear sleeve; on the other end of the cone pulley fastened securely to lathe spindle is a large gear which is thrown into mesh with the smaller gear of back gears. When the back gears are not in mesh, a clamp nut fastens spindle gear securely to cone pulley.

Example. If a countershaft running 160 *R. P. M.* has a 5 in. dia. pulley belted to 8 in. pulley on lathe spindle, with gear fast to cone having 32 teeth and gear fast to spindle having 90 teeth, these being in mesh with back gears of 88 and 30 teeth respectively, what is the *R. P. M.* of lathe driving spindle?

Solution. By formula for driver and follower,

$$n = \frac{\overset{20}{\cancel{160}} \times 5 \times \overset{4}{\cancel{32}} \times \cancel{30}}{\cancel{8} \times \underset{11}{\cancel{88}} \times \underset{3}{\cancel{90}}} = \frac{400}{33} = 12 + R.\ P.\ M.$$

PROBLEMS

1. When a countershaft runs 165 *R. P. M.* and pulley on countershaft is $4\frac{5}{8}$ in. dia. belted to a $7\frac{5}{8}$ in. dia. pulley on lathe spindle; and when the gears are the same as in above example, what is the *R. P. M.* of lathe spindle?

2. When the pulley on a countershaft is $7\frac{5}{8}$ in. dia. and the cone pulley is $4\frac{5}{8}$ in. dia. other sizes same as in problem 1 find *R. P. M.* of spindle.

3. When a countershaft makes 155 *R. P. M.* and the pulley is $8\frac{1}{2}$ in. dia. belted to lathe cone pulley $6\frac{3}{4}$ in. dia. with back gears 85 and 32 teeth, in mesh with 31 on cone and 84 on spindle, what is the number of *R. P. M.* of lathe spindle?

4. When a countershaft runs 185 *R. P. M.* and the pulley on countershaft is 5 in. dia. belted to an $8\frac{1}{4}$ in. dia. pulley on spindle with gears same as in problem 3, what is the number of *R. P. M.* of lathe spindle?

5. When a countershaft pulley 6 in. dia. running 160 *R. P. M.* is belted to a cone pulley $6\frac{3}{4}$ in. dia. and back gears with 75 and 20 teeth in mesh with 25 tooth gear on cone and 69 tooth gear on spindle, what is the number of *R. P. M.* of spindle?

6. When a countershaft pulley $5\frac{1}{8}$ in. dia. and running 225 *R. P. M.* is belted to a cone pulley on the lathe spindle $7\frac{3}{4}$ in. dia. with gears on back gear quill of 75 and 25 teeth in mesh with 25 teeth on cone pulley and 65 teeth on spindle, what is the *R. P. M.* that a piece will make on lathe centers?

SCREW THREAD CUTTING

One of the most important operations on the lathe is the *cutting of screw threads* with a tool formed to make the exact shape of thread. In order to cut the required number of threads per inch, the work must revolve an *exact number* of times, equal to the number of threads to be cut, while the carriage which holds the thread cutting tool firmly during the cut is moved along the bed of lathe *exactly one inch.* Now as the carriage feed is liable to variable motion with the belt feed owing to slip of the belt a *positive uniform* motion is given to the cutting tool by a system of *change* gears and *lead screw* arranged to give any desired feed of carriage for the required number of revolutions of lathe spindle, or work to be cut.

The illustration on page 91 shows details of lathe: *10* is the lead screw and *14* is the gear that imparts a positive motion to *10* by means of a feather key. This gear is held in place with a nut on the end of lead screw; *9* is the change gear spindle on which any gear may be fastened with a feather key and nut. Inside of the head casting *1* is a gear in mesh with the gear on the driving spindle of the lathe. If these two gears are in the ratio of *1* to *1*, then when the work makes one revolution *9* will also make one revolution imparting motion to lead screw *10* with the velocity ratio equal to the ratio of the number of teeth in change gears *14* and 14_1. For example, suppose lead screw *10* has *5* threads per inch, or $\frac{1}{5}$ *inch* lead, and a screw of the same number of threads is to be cut on the lathe centers, what gears will be required on stud *9* and end of lead screw *10?*

Solution. Drive spindle must revolve five times while lathe carriage moves along the bed *1* inch. As lead screw *10*, which gives it a positive motion is *5* pitch single thread, it must revolve five times to give *1* inch motion to carriage; therefore *9* and *10* must each make five turns in the same time and gears *14* and 14_1 will be the same size, or will have a velocity ratio of *1* to *1*. Therefore any two gears having the same number of teeth as *25* and *25* may be used.

A general formula for change gears is as follows:

$$\frac{\textit{Pitch of lead screw}}{\textit{Pitch of screw to be cut}} = \frac{\textit{No of teeth of stud gear}}{\textit{No. of teeth of lead screw gear}}$$

Example. If the pitch of the lead screw is *5* and it is required to cut a *10* pitch screw.

Then $\frac{5}{10} = \frac{1}{2}$ or gear on lead screw must have double the number of teeth of the gear on the stud. Then any two gears which have this ratio may be used.

The above formula may also be stated in the form of a rule as follows:

RULE. The product of number of threads to be cut, by the number of teeth in spindle (or stud) gear, is equal to the product of the number of teeth in the gear on lead screw by the number of threads in the lead screw.

If the gears on stud *9* and driving spindle of the lathe are not in the ratio of *1* to *1* the ratio can be found by the following method: putting two gears of the same size on *9* and *10*, cut a trial piece and count the threads per inch. If on trial it is found that the number of threads cut per inch is twice the number per inch of the lead screw then the velocity ratio of the spindle and stud is *2* to *1*. Then in calculating the ratio of the change gears required, the above formula could be changed to read as follows:

$$\frac{\textit{Pitch of lead screw} \times 2}{\textit{Pitch of screw to be cut}} = \frac{\textit{No. of teeth in stud gear}}{\textit{No. of teeth in lead screw gear}}$$

If the velocity ratio is not given, *1* to *1* will be understood in the problems for screw cutting on the lathe.

PROBLEMS

1. What change gears can be used to cut an $11\frac{1}{2}$ pi. thread, when lead screw is 5 pitch?

2. What change gears can be used to cut a 12 pi. thread, when lead screw is 5 pi. and stud gear has 25 teeth?

3. Find the stud gear to use to cut 16 threads per in. when lead screw is 5 pi. and screw gear has 80 teeth.

4. Find the screw gear to cut 18 pi. screw thread when lead screw is 6 pi. and stud gear has 24 teeth.

5. When the velocity ratio of driving spindle and stud is 2 to 1 and lead screw is 8 pi., what is the number of teeth in screw gear to cut 11 threads per inch when stud gear has 96 teeth?

Note. The gears on inside end of spindle are arranged on a pivoted lever so that a train of three gears shall be in mesh, or by throwing over lever 21 a train of four gears can be put into mesh, thus giving a feed motion of carriage in either direction according as lever is thrown up or down; this mechanism is called the *reversing feed gears.*

6. When the reversing feed gears are in the ratio of 2 to 1 with driving spindle, what stud gear will be required to cut 16 threads per inch, if lead screw is 8 pi. and screw gear has 48 teeth?

7. What stud gear will be used to cut a 9 pi. thread, when lead screw is 6 pi. and the screw gear has 36 teeth?

8. What screw gear will be used to cut a 16 pi. thread, when lead screw is 6 pi. and stud gear has 24 teeth?

9. What change gears can be used to cut a 5 pi. thread when lead screw is 6 pitch?

10. What change gears can be used to cut 36 pi. threads, when lead screw is 6 pi. and the smallest gear of the set has 24 teeth?

COMPOUND GEARING

To save making so many change gears in a set for thread cutting and especially to avoid using gears with a large number of teeth, a pivot is placed between the stud and intermediate gear *15*, on which revolves two gears fast together called a compound gear. These gears are usually in the ratio of *2* to *1* but the principle of operation would be the same if gears in any other ratio were used. Thus the principle of compound gearing comes under the rule for drivers and followers, page 12, where another driver and follower has been brought into the calculations.

When the velocity ratio is *2* to *1* the number of teeth in followers can be multiplied by *2* and the drivers by *1* rather than to count up the number of teeth in the compound gears.

Example. With a *2* to *1* compound gear having *24* teeth and lead screw *6* pi., what gear will be required on lead screw to cut *36* threads per inch?

Solution. With simple change gears the proportion will be $\frac{6}{36}=\frac{24}{x} \therefore x=144$, which makes a gear of a large number of teeth.

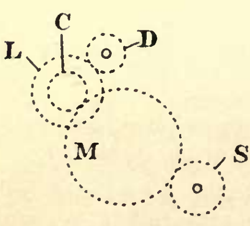


Fig. 49.

With a compound gear arranged as in Fig. 49, the proportion will be as follows: $\frac{6\times 2}{36}=\frac{24}{x} \therefore x = 72$ teeth in lead screw gear. By throwing stud gear out of mesh an *18* pi. thread can be cut. The *intermediate* gear *M* serves the purpose of a *belt* except that it imparts a positive motion, therefore it is not brought into the calculations.

D = the gear on stud that meshes with lathe spindle through the reversing gear train.

L = the large gear of the compound.

C = the small gear of the compound.

S = gear on end of lead screw.

D and C are the drivers.

L and S are the followers.

It frequently happens that the velocity ratio between the lead screw and the head spindle is such that no two gears of the set furnished with the lathe have the required ratio. In such cases, two sets of gears may be used, the product of whose ratios gives the ratio required.

Example. Find the gearing required to cut a *30* pitch thread on a lathe having a *5* pitch lead screw.

Solution. The ratio $=\frac{5}{30}=\frac{1}{6}=\frac{1}{3}\times\frac{1}{2}$. Therefore one set of gears in the ratio of *1* to *3* and a second set in the ratio of *1* to *2* would be needed; and the gears *24*, *48*, *30* and *90* would be correct, *24* and *30* being the drivers and *48* and *90* the followers.

Example. Find the compound gearing required to cut a *35* pitch thread on a lathe whose lead screw is *6* pitch.

$$\frac{6}{35}=\frac{3\times2}{7\times5}=\frac{3\times10}{7\times10}\times\frac{2\times20}{5\times20}=\frac{30}{70}\times\frac{40}{100}$$

Therefore gears *30* and *40* will be used as drivers and *70* and *100* as followers.

These principles are summarized in the following:

RULE FOR COMPOUND GEARING.

1. Find the ratio between the pitch of the lead screw and the pitch of the screw to be cut.

2. Separate each term of this ratio into two factors and express the result as the product of two fractions.

3. Multiply both numerator and denominator of each fraction by such a number as will make each product represent the number of teeth found in one of the gears furnished with the lathe. The numerators of these two fractions will denote the driving gears, and the denominators the follower gears.

PROBLEMS IN COMPOUND GEARING

1. The gears furnished with a 14 in. Reed lathe have 24, 32, 40, 44, 48, 52, 56, 60, 64, 72, and 110 teeth, and the lead screw is 6 pitch. Find the compound gears required for cutting a $3\frac{1}{2}$ pitch thread.

2. With the same set of gears and same pitch lead screw as given in problem 1, find compound gearing required for cutting a 25 pitch screw. A 28 pitch screw. A $7\frac{1}{2}$ pitch screw. A screw of $\frac{2}{9}$ in. lead.

3. The change gears furnished with a 13 in. Reed lathe have 25, 30, 35, 40, 45, 50, 55, 60, 65, 69, 70, 80, 90, 100, 110 and 120 teeth, and lead screw has 5 threads to the inch. Find the compound gearing required for cutting an $11\frac{1}{2}$ pitch pipe thread.

4. With the same lathe and same set of gears, find the compound gearing required for cutting a 40 pitch screw. A 23 pitch screw. A 27 pitch screw. A screw of $\frac{3}{11}$ inch lead. A screw of $\frac{3}{16}$ inch lead.

TURNING TAPERS

Lathes are also used to *turn tapers* on round stock, and although there are several attachments in use for taper turning, a common method is that of *offsetting* the tail or foot stock center so that it is out of line with driving center the necessary amount to produce the required taper.

When the taper is to be turned the whole length of the bar, the dead center is offset one-half the difference between the diameters at each end of piece. The formula is,

$$x=\frac{D-d}{2}$$

$x=$ offset of center.
$D=$ diameter at large end of taper.
$d=$ diameter at small end of taper.

When the taper runs only a part of the length of piece, the amount of offset is determined by the length of piece between the centers, and the diameters at each end of the tapered part. The formula becomes

$$x=\frac{L\left(\frac{D-d}{2}\right)}{l}$$

where $x=$ offset of center.
$D=$ diameter at large end of taper.
$d=$ diameter at small end of taper.
$L=$ whole length of piece.
$l=$ length of the part tapered.

Example. What is the offset of dead center, when a bar *36* inches long is to be turned *9* inches of its length, tapered from *2* inch dia. at end to *4* inch diameter?

Solution. By formula

$$x=\frac{L\left(\frac{D-d}{2}\right)}{l}=\frac{36\left(\frac{4-2}{2}\right)}{9}=\frac{36}{9}=4 \text{ in.}$$

The taper per inch in a turned piece is found by subtracting the small diameter from the large and dividing the remainder by the length; as a piece *8* inches long with diameters at ends *1* and $1\frac{1}{8}$ inches is tapered $\frac{1}{64}$ inch per inch in length or $Y=\frac{D-d}{l}=\frac{\frac{1}{8}}{8}=\frac{1}{8}\times\frac{1}{8}=\frac{1}{64}$.

When it is desired to find the taper per foot, the formula is

$$y=\frac{12(D-d)}{l},$$

where $y=$ the taper per foot and other values same as given above.

Example. What is the taper per ft. in the example given above?

Solution. By formula:

$$y=\frac{12(D-d)}{l}=\frac{12(4-2)}{9}=\frac{24}{9}=2\tfrac{2}{3} \textit{ inches}$$ which is the taper per foot.

PROBLEMS

1. A bar of steel is tapered $1\frac{1}{4}$ in. per ft. and diameters at end of taper are 1 in. and $1\frac{3}{4}$ in. Find length of taper.

2. A bar is tapered 16 in. of its length and diameters at ends of taper are 1 in. and $2\frac{1}{2}$ in. Find taper per foot.

3. A bar of steel is tapered 7 in. of its length, the diameters at ends of taper are $2\frac{1}{4}$ in. and $3\frac{5}{8}$ in. What is the taper per foot?

4. A bar is turned on a taper 24 in. of its length. The diameters at each end of taper are $\frac{7}{8}$ in. and $\frac{7}{16}$ in. What is the taper per foot?

5. A shank mill has a tapered end $5\frac{1}{2}$ in. long. The taper is $\frac{9}{16}$ in. and $\frac{15}{16}$ in. diameters at the ends. What is the taper per foot?

6. A small cutter has shank tapered 3 in. long, the dia. at small end is .365 in., at large end the taper is .525 in. dia. What is the taper per foot?

7. The taper shank on a cutter is $3\frac{1}{2}$ in. long, the small dia. is .573 in., and the large dia. is .749 in. What is the taper per foot?

8. The taper on a reamer is $8\frac{1}{2}$ in. long, the small dia. is $2\frac{1}{16}$ in., the large dia., is $2\frac{9}{16}$ in. What is the taper per foot?

9. Find the taper per ft. of a pin $2\frac{1}{2}$ in. long, and $\frac{1}{4}$ in. and $\frac{3}{8}$ in. diameters respectively at ends.

10. What is the offset of dead center for turning a taper 16 in. long on the end of bar 40 in. long, when the diameters at ends of the taper are $1\frac{1}{2}$ in. and 4 inches?

11. What is the offset of center for turning a taper 19 in. long on a bar 25 in. long, if the diameters at ends of taper are 1 in. and $1\frac{5}{8}$ inches?

12. A bar is 39 in. long; a taper 11 in. long turned on one end is $2\frac{3}{8}$ in. dia. at small end and $3\frac{3}{16}$ in. dia. at large end. What is the offset of the center for turning the taper?

13. Find the offset of the center for turning a taper 30 in. long on a bar 34 in. long, when the diameters of taper are $1\frac{1}{4}$ in. and $1\frac{3}{4}$ inches.

14. Find the offset of the center for turning a taper 33 in. long on a bar 44 in. long, with the diameters $3\frac{1}{4}$ in. and $4\frac{1}{2}$ in. on ends of taper.

15. What is the offset for a lathe center to turn a taper 17 in. on a bar 19 in. long, if the diameters of taper are $1\frac{1}{8}$ in. and $1\frac{7}{8}$ in. on ends?

16. How much should the dead center be moved over to turn a taper bar 4 in. long, when one end of piece is .779 in. dia. and the other end is .982 in. diameter?

17. What is the taper per ft. in problem 16?

Note. The formula for offset of lathe center for turning tapers, is very helpful. It is, however, only a close approximation as the calculation is only exact between the ends of centers, whereas, the stock to be turned is supported a little way up the countersunk part on each end.

CUTTING SPEEDS

The speed at which any piece of work should revolve in taking a cut on the lathe depends on the following conditions:

(*1*) *The kind and condition of the material of which the work is made.*

(*2*) *The kind and condition of the cutting tool.*

(*3*) *The lubrication of the tool while making the cut.*

(*4*) *The size of the chip.*

Therefore any rule for speeds will be only approximate, but from experience with the carbon steels which are used most in the toolmaking department for the finishing cuts, the following speeds have been found practical as a basis for trial:

Soft brass	*80 F. P. M.*
Gray iron castings	*40 F. P. M.*
Machinery steel	*30 F. P. M.*
Annealed tool steel	*20 F. P. M.*

Note. The high speed steel cutting tools can be used at about double the above speed.

The *F. P. M.* can be found for lathe turning from the formula:

$$C=\frac{\pi R d}{12}$$

C = cutting speed in *F. P. M.*

R = *R. P. M.* of the work.

d = diameter of work in inches.

The approximate rule for finding the *F. P. M.* is to *count* the revolutions for a *quarter of a minute* and multiply this number by the diameter of the work in inches. This rule is based on the fact that if in formula just given $d=1$ inch,

then $$\frac{\pi d}{12}=\frac{1}{4} \text{ foot approximately.}$$

Then approximately,

$$C=\frac{Rd}{4}$$

and $$R=\frac{4C}{d}$$

PROBLEMS

In the following problems the approximate formula for R will be used unless otherwise stated.

1. A bar of 4 in. dia. cold rolled machinery steel is to be turned in lathe. What approximate *R. P. M.* should it revolve to turn a chip at 30 *F. P. M.?*

2. What is the approximate *R. P. M.* for turning a chip on a tool steel reamer 1 in. diameter?

3. A cast iron pulley 10 in. dia. is to have the rim face turned in the lathe. How many *R. P. M.* should it make?

4. How many *R. P. M.* should a brass rod $\frac{3}{4}$ in. dia. make in taking a cut in the lathe?

5. The cast iron cylinder of a gasolene motor 5 in. dia. is to be bored in lathe. How many *R. P. M.* should a boring bar make to do the work?

6. If a ring gauge of tool steel is to be bored in a chuck to 2 in. dia., what is the approximate *R. P. M.* that it should make?

7. The hub of a cast iron pulley is to be bored for finishing with a $3\frac{7}{16}$ in. dia. reamer. How many *R. P. M.* should it make approximately in the chuck?

8. A brass nut is to be threaded in the lathe for a $1\frac{1}{4}$ in. bolt. What would be the maximum speed in *R. P. M.?*

9. A machinery steel collar is to be shrunk onto a $1\frac{1}{2}$ in. arbor. What speed should it revolve when boring to size?

10. A crank pin $4\frac{1}{8}$ in. dia. is to be shrunk into place. What approximate *R. P. M.* should it make in turning to size, if made of annealed tool steel?

11. A cast iron pulley 16 in. dia. should turn approximately at what *R. P. M.* for turning the rim?

12. A $1\frac{15}{16}$ in. cold rolled machinery steel shaft is to be turned one chip in depth near the end to fit a flange coupling. How many *R. P. M.* should it make?

13. A crank shaft made of cold rolled machinery steel is to have the bearings finished $7\frac{1}{2}$ in. dia. What speed in *R. P. M.* should it have?

MISCELLANEOUS PROBLEMS

14. When a main line of shafting runs at 200 *R. P. M.* with a 10 in. dia. driver belted with a 14 in. pulley on countershaft, what *R. P. M.* will a lathe spindle make that has a 9 in. dia. cone pulley belted to a 6 in. pulley on counter?

15. When main line shafting runs 150 *R. P. M.* with a 16 in. driver belted to a 12 in. follower on countershaft, how many *R. P. M.* will the spindle make with a 3 in. dia. pulley belted to a 7 in. pulley on counter?

16. When a main line runs 150 *R. P. M.* from a 14 in. driver belted to 10 in. dia. pulley on counter, cone pulley on counter $8\frac{1}{2}$ in. belted to $6\frac{1}{4}$ in. dia. on lathe spindle, find *R. P. M.* of spindle when gear fast to cone has 28 teeth in mesh with 88 teeth on back gear and 36 teeth on back gear in mesh with 96 teeth gear fast to spindle?

17. If 2% is allowed for slip on each belt in problem 16, what *R. P. M.* will spindle make?

18. What dia. of pulley should be put on a countershaft to make 175 *R. P. M.* when the pulley is to be belted to a 15 in. dia. on main line making 160 *R. P. M.?*

19. A lathe has cone pulley 9 in. dia. belted to a countershaft cone 12 in. dia. with back gears 39 and 88 teeth in mesh with 65 teeth on spindle and 27 teeth on cone; the lathe is to bore a $4\frac{1}{2}$ in. dia. cast iron disc at 40 *F. P. M.* When the dia. of drive pulley on main line is 15 inches, what dia. of follower pulley on the countershaft will be required if main line runs 150 *R. P. M.?*

20. An axle lathe is to turn $4\frac{1}{8}$ in. dia. crucible steel axles at 9 *F. P. M.*; the back gears have 66 and 22 teeth in mesh with 26 and 58 teeth on cone and spindle respectively. When main line runs 160 *R. P. M.* and dia. of driver on main line is 10 in., what dia. of follower on counter will be required, if a 12 in. dia. pulley on lathe spindle is belted to 11 in. dia. pulley on countershaft?

21. If 2% were to be allowed for slip in each of the two belts of problem 20, how much larger dia. of drive pulley will be required on main line?

22. A crank pin is to have a taper seat in crank $4\frac{1}{2}$ in. long. The diameters at each end of taper are $4\frac{1}{8}$ in. and $4\frac{9}{16}$ in. What offset will be required for dead center to turn the taper when whole length of pin is $10\frac{9}{16}$ inches?

23. The reamer to finish seat for the pin in crank of problem 22, was 8 in. long. What was the offset of the dead center to turn the taper on a reamer when the small dia. of taper was $4\frac{3}{32}$ in. dia. and taper was $6\frac{1}{4}$ in. long?

24. An axle turning lathe was equipped with high speed tools for turning at 100 *F. P. M.* What *R. P. M.* should the spindle revolve for turning an axle $3\frac{1}{2}$ in. diameter?

25. A lathe is fitted to turn heavy cast iron pulleys at 60 *F. P. M.* What approximate *R. P. M.* will spindle make for a 30 in. dia. pulley?

METAL PLANER

A sketch of the driving mechanism for a Powell planer is shown in Fig. 50. Two belts connect the countershaft with

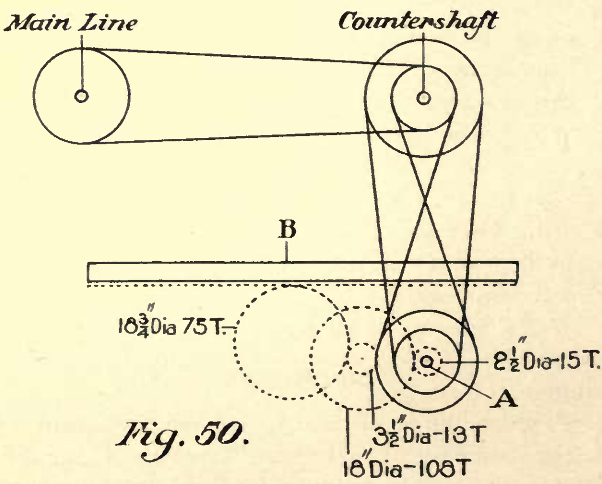


Fig. 50.

the driving shaft, the cross belt driving the platen forward for the cut and the straight belt causing the return. The motion is transmitted from the driving shaft A to the platen by means of a system of gearing as shown in the cut, the last gear meshing with a rack on the under side of the platen.

The cutting speed for planer tools is about the same as for lathe tools but heavier cuts can be taken on account of the rigid support of work on the platen. One great disadvantage with doing work on a planer is that the return stroke is time lost for the tool in the cut, although belted to return about three times as fast as the cutting stroke. The calculations for the planer are to determine sizes of pulleys to give required travel of work in *F. P. M.* The formula is essentially the one for drivers and followers.

Example. What is the travel of platen *B*, Fig. 50, for one revolution of shaft *A?*

Solution. By formula:

$$n = \frac{Drivers \times R \text{ of 1st driver}}{Followers},$$

$n =$ *revolution of last follower,*

then $$n = \frac{1 \times 15 \times 13}{108 \times 75} = \frac{13}{540} = .024 \text{ R. P. M.}$$

The travel $= R.\ P.\ M. \times \pi \times \text{diameter of last follower}$

or, $.024 \times 3.1416 \times 18.75 = 1.4137$ *inches.*

PROBLEMS

1. Find the *F. P. M.* for the cut on a planer when the 16 in. dia. pulley on main line runs 170 *R. P. M.* and is belted to a 12 in. follower on countershaft on which is another 12 in. dia. pulley belted to an 18 in. dia. pulley on shaft *A*, the gearing from *A* to platen being the same as given in example illustrated above.

2. What is the return stroke in *F. P. M.* when the pulley on countershaft is changed from 12 in. to 20 in. dia. belted to an $11\frac{1}{2}$ in. pulley on shaft A in place of the 18 in. dia. pulley of problem 1, with the gearing from A to platen remaining the same.

3. When the drive pulley on main line is changed from 16 in. dia. to 20 in. dia., otherwise the same as problem 1, what is the *F. P. M.* of the cut?

4. If the belt from driver on main line to countershaft has a slip of 2% and the one from countershaft to shaft A has a creep of 4%, what is the speed of problem 1 in *F. P. M.?*

5. What is the time required to take 150 strokes on a plate 24 in. long when the travel in *F. P. M.* is the same as in problems 1 and 2, and 3 in. travel is allowed at the beginning and end of stroke for shipping the belt?

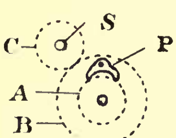


Fig. 51.

Figure 51 shows an outline of the gearing for feeding the tool carrier across the plane of the work. The tool carrier is moved by a $\frac{1}{4}$ inch lead square thread screw S, to which is fastened a *19* tooth gear C in mesh with a gear B, having *78* teeth. The ratchet wheel A with *80* teeth is connected by a rod to the adjustable cam that imparts a rotary motion to it at each stroke of the planer; fast to gear B is the ratchet P which holds the gear B from moving back with the rod when it returns back at end of stroke for the next movement for the feed. The feed motion is calculated by the formula used for gear trains, page 12. For one tooth movement of ratchet wheel A the feed of carrier $= \frac{78 \times .250 \text{ in.} \times 1}{19 \times 80} =$ *.01283 inch.*

For two teeth movement, the feed $= \frac{78 \times .250 \times 2}{19 \times 80} =$ *.02566 inch.*

6. When a planer mechanism is arranged as in problems 1 and 2 and the feed is for a two teeth movement of ratchet as shown in Fig. 51, what is the time required to plane a plate 12 in. square when the time allowance for shipping is the same as in problem 5?

7. What is the time required to finish a surface 24 in. long by 18 in. wide when the gearing is the same as in problems 1 and 2 with 8 tooth movement of *A?*

8. What gears should be used at *B* and *C* to give a .01 in. feed to the tool for 1 tooth movement of *A?*

9. What changes will be required in the sizes of two driving pulleys on main line and countershaft if pulleys are in the same ratio as in problem 1, to give 40 *F. P. M.* for planing cast iron, when the other mechanism remains the same?

UNIVERSAL MILLING MACHINE

The *universal milling* machine is used with various attachments for taking special cuts on work, and more especially for cutting the teeth of mills, cutters, gears, reamers, drills, etc.

The most important attachment is the *index* or *spiral head* which is used with a dead center in tail stock, both fastened to a bed or platen that can be moved back or forward in a horizontal plane under a revolving cutter carried by the main driving spindle of the machine. The feeding mechanism for the platen is shown in Fig. 52. A feed belt connects the main driving spindle with an intermediate shaft on which is a stepped cone belted to a second cone pulley in reverse order of steps allowing for changes in speeds of the feed gearing; from the second cone, the feed is connected

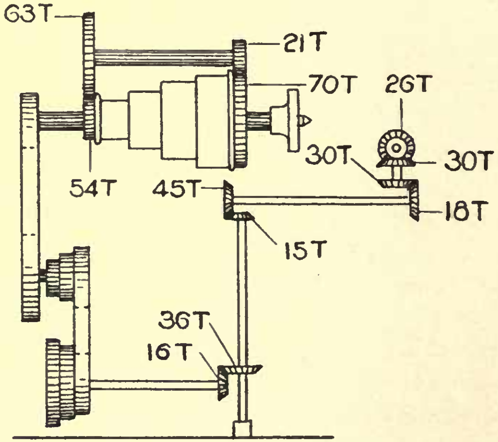


Fig. 52.

to the bed by a system of shafts and bevel gears. This shows a simple system of connecting the feeding of the bed from the driving spindle but it is done in a variety of ways. The calculations for the speeds and feeds of the milling machine are almost identical with those for the lathe, therefore the mathematical work of the miller will be confined to the index head attachment, in connection with such parts of the feeding mechanism as are related to it.

THE INDEX HEAD

The *index head* is used to divide the periphery of a piece of work into any number of equal parts and to hold the work at these positions to allow cuts to be taken at equal intervals, as in gear cutting, etc. The mechanism for indexing is shown in Fig. 53, with the body and supporting parts removed.

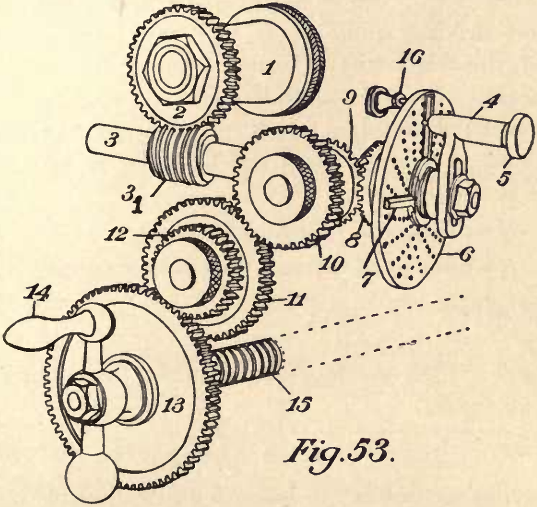


Fig.53.

The work which is to be divided into equal divisions is supported and revolved on centers in much the same manner as work is turned in a lathe.

A *latch pin 5* is held by the tension of a spring in one of the holes of the *index plate 6;* when the pin is pulled out of the plate, *crank 4,* can be turned, transmitting motion through *worm shaft 3*, and its *worm,* 3_1 to *worm wheel 2* fast to *spindle 1,* in which is the *live center* that supports the work to be cut. The *worm wheel 2* has *40* teeth driven by a *single thread worm* 3_1, so that one revolution of *worm* 3_1, which is also the same as one revolution of *index crank 4,* will move the *spindle 1* carrying the work through $\frac{1}{40}$ of a revolution; then forty revolutions of *crank 4* will give the work *one* *complete revolution.* If then *40* teeth, or divisions, are required on the work, one revolution of the crank will be made between each cut as the work is moved under the cutter on the main driving spindle. If *20* teeth, or divisions, are required, the crank will be turned twice around for each cut since *20* is one-half of *40, 10* divisions will require four turns of the crank; *8* divisions five turns; *5* divisions eight turns, etc. From this is obtained the formula for any required number of divisions.

Let N = number of divisions required.

R = number of turns of the crank for each cut.

Then $R=\frac{40}{N}$.

Example. Find the indexing required for cutting a gear having 60 teeth.

Solution. By formula: $R=\frac{40}{N}=\frac{40}{60}=\frac{2}{3}$; therefore any plate having the number of holes in a row divisible by *3* may be used. In this case take the *39 hole index* and for each one of the *60* teeth cut move the *index pin 5* around *26* holes, since $26=\frac{2}{3}$ *of 39.*

The length of the crank *4* is adjustable to any size of the plate *6*, but to avoid using a plate of a large diameter, three detachable plates are supplied with each machine so that they can be changed easily to allow any simple indexing to be made. The following list gives the usual number of circles of holes on the three plates.

Plate Number of holes in rows:

1 = 15—16—17—18—19—20

2 = 21—23—27—29—31—33

3 = 37—39—41—43—47—49

A *sector* 7 is adjustable so that its two arms can be moved to take the space between any two holes of a series, thus avoiding the necessity of counting the number of holes for each division as it is spaced on the work.

PROBLEMS

1. Find the indexing for 3 divisions.

2. If a mill is to have 32 teeth, what is the indexing required?

3. If a tap is to have 4 flutes cut in the universal miller, what indexing can be used?

4. A gear is to have 24 teeth. What is the required indexing?

5. A mill is to have 35 teeth cut on miller. What is the indexing required?

6. Find the indexing required for 45 divisions.

7. Find the indexing required for 72 divisions.

8. Find the indexing required for 68 divisions.

9. If 82 teeth were required to be cut on a gear in the universal miller, what indexing would be made?

10. A gear is to have 108 teeth. Find the indexing required.

11. An index plate with 116 holes is to be drilled on the milling machine with horizontal drilling attachment. What is the indexing required?

12. A ratchet wheel is to have 148 teeth to be cut on its periphery. Find the required indexing.

13. A circular plate is to be marked on the circumference with 164 divisions. What indexing will allow the work to be done on the universal miller?

14. A knurl is to have teeth $\frac{1}{16}$ in. circular pitch at bottom of cut on the circumference; if the blank is 3.3706 in.

dia. and the teeth are $\frac{3}{32}$ in. deep, what indexing will be necessary for cutting the teeth in the universal miller?

15. A thin disc is to have 165 notches cut on its edge at equal intervals around its circumference. If the work is done in universal miller, what indexing will be necessary?

16. A worm wheel blank 3.6330 in. dia. is to mesh with a $\frac{1}{18}$ in. lead V thread screw. What indexing will allow the wheel to be cut on miller?

17. An indexing disc blank 4.1253 in. dia. for the planer centers has V cuts in the circumference .06 in. wide from point to point. Find the indexing to cut the teeth in a universal miller.

18. A disc 31.2 in. in periphery has lines $\frac{1}{10}$ in. apart on the circumference. What indexing will allow the marking to be done in miller?

19. A disc 1 in. in periphery has lines at $\frac{1}{69}$ in. intervals around the circumference. Find the indexing necessary to do the work in the universal milling machine.

COMPOUND INDEXING

When simple indexing will not give the required spacing *compound* indexing may be used as follows:

Example. Find the indexing required to cut *69* teeth in a gear.

Solution.
(1) $69=23\times3$
(2) $33-23=10=2\times5$
(3) ―――――――
(4) $40=2\times2\times2\times5$
(5) $33=11\times3$
(6) $23=23\times1$

(1) Set down the number of divisions required and resolve into factors.

(2) Choose an index plate as *33* and *23* holes and set their difference under the required number and factor as in (1).

(3) Draw a line under the two sets.

(4) Set down the number representing teeth in worm wheel and factor as in (1).

(5) Set down the number of the larger index and factor as in (1).

(6) Set down the number of the smaller index and factor as in (1).

(7) Cancel factors above the line with the factors below and *if all the factors above the line cancel*, then the right index plate has been chosen. If the factors above the line do not all cancel, other numbers representing sets of holes in the index plates must be chosen until a number is found that will permit cancellation of all factors above the line. The product of the factors below the line remaining uncancelled, $2\times2\times11=44$, will be the number required to give the right indexing fraction; that is, turning the *33* hole index *44* holes in one direction and the *23* hole index *44* holes in the other direction will make the movement of the index

$$\frac{44}{23}-\frac{44}{33}=\frac{40}{69} \text{ of a turn.}$$

If any whole number is subtracted from both of these fractions the resulting fraction is unchanged. In this case subtracting one turn from each fraction the indexing will be $\frac{21}{23}$ of a turn in one direction and $\frac{11}{33}$ of a turn in the opposite direction. It does not matter in which direction the indexing is done, as it will simply turn the work in either direction throughout the cut.

It will be seen that *pin 16* is not adjustable and is in line only with the outside row of holes in index plates, therefore the outside row of holes of one of the three plates must be chosen for the larger number for the compounding.

PROBLEMS

20. Find compound indexing required for 77 divisions.

21. What indexing will be required to cut a gear of 91 teeth in the universal milling machine?

22. A ratchet wheel is required to have 96 teeth spaced at equal intervals on its circumference. What indexing will be required to do the work in the universal milling machine?

Note. In problem 22 it is possible to use simple indexing for 48 divisions and cut around once, then turn the work one-half the distance between two cuts and cut around again.

23. A gear with 99 teeth is to be cut in milling machine. Find the required indexing.

24. What compound indexing can be made for 147 divisions?

25. A gear is to be cut in the milling machine to have 154 teeth. What indexing will be required to do the work?

26. What indexing in the milling machine will be required to do the cutting of 174 equal divisions?

27. Find the indexing required for 182 equal spaces on the circumference of a disc.

28. What indexing will cut 186 teeth in a gear?

DIFFERENTIAL INDEXING

The method of compound indexing has been to a large extent displaced by the *differential system of indexing* that is now a special feature of all the Brown and Sharpe Manufacturing Company's universal milling machines. With the differential indexing any number of divisions from *1* to *382* may be obtained in the same manner as for plain indexing except that properly selected change gears are used.

In plain indexing the index plate is held firmly in position by means of pin *16*, Fig. 53, so that $\frac{1}{3}$ of a turn of the index crank moves the work on centers $\frac{1}{3}$ *of* $\frac{1}{40} = \frac{1}{120}$ of a turn. In differential indexing the index plate is connected with the work spindle through a train of gears.

Every movement of the index crank causes a movement of the index plate, and this motion may be either forward or backward as desired.

The effects of these motions may be seen from the following illustrations: Fig. A represents the method of plain indexing, with index plate fixed.

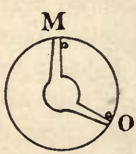


Fig. A.

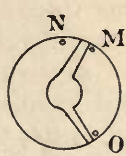


Fig. B.

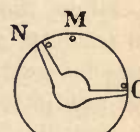


Fig. C

$\frac{1}{3}$ of a turn of the index crank from *M* to *O* between the arms of the sector produces $\frac{1}{3}$ of $\frac{1}{40} = \frac{1}{120}$ of a turn of the work on *work spindle 1,* Fig. 53. Fig. B represents the result when index plate is geared to move forward. While the index crank is being moved through $\frac{1}{3}$ of a turn, from

N to O of sector, the index plate is moving forward the distance $N\ M$ so that the work is actually turned through more than $\frac{1}{3}$ of $\frac{1}{40}$ of a turn.

Fig. C represents the result when the index plate is geared to move backward. While the index crank is being moved through $\frac{1}{3}$ of a turn as indicated by the distance $N\ O$ between the arms of the sector, the index plate is being moved in the opposite direction, so that the movement of the work on the work spindle centers is thus less than $\frac{1}{120}$ of a turn.

The amount of rotation of the index plate may be regulated by the difference in the velocity ratios of the change gears.

The gears furnished with the Brown and Sharpe Mfg. Co. milling machine for differential indexing and spiral milling cuts are as follows: 3 gears of 24 teeth and one each with 28, 32, 40, 44, 48, 56, 64, 72, 86, and 100 teeth respectively.

Example. Find the indexing required for 53 divisions.

Solution. By simple indexing the index crank would be revolved through $\frac{40}{53}$ of a turn for each division, but as there is no index plate having *53* holes this spacing is impossible, therefore another fraction is selected whose value is near $\frac{40}{53}$, say $\frac{40}{56}=\frac{5}{7}$, then the *21* or *49* hole index plate can be used, as *15* holes in the *21* row or *35* holes in the *49* row gives the required fraction. Indexing in this way for *53* divisions gives $53\times\frac{5}{7}=\frac{265}{7}=37\frac{6}{7}$ complete turns of the index crank or $2\frac{1}{7}$ turns less than the *40* required for one complete turn of the work. By using gears in the ratio of $2\frac{1}{7}$ to *1*, however, the index plate will make $2\frac{1}{7}$ revolutions which, with the $37\frac{6}{7}$ turns of the crank will make the *40* turns required. Hence the gears will be in the ratio of

$$\frac{15}{7}=\frac{5\times 3}{7\times 1}\ \text{or}\ \frac{40\times 72}{56\times 24}.$$

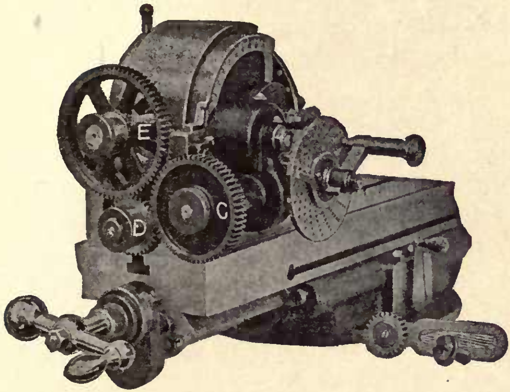


Fig. 54

Then place gear of *56* teeth *on worm* as *C*, Fig. 54
gear of *40* teeth *first on stud*
gear of *24* teeth *second on stud*
gear of *72* teeth *on spindle* as *E*, Fig. 54

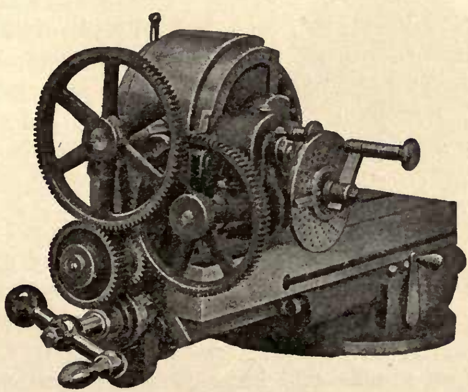


Fig. 55

As the motion of the index plate is to be in the same direction as the motion of the crank, no idler gear will be required, and the setting to use the compound stud gears is shown in Fig. 55.

Example. Find the indexing for 352 divisions.

Solution. By simple indexing the spacing number $=\frac{40}{352}$. As this fraction cannot be obtained directly from the index plate, select $\frac{40}{360}=\frac{1}{9}$. This may be found by using *3* holes in the *27* hole row or *2* holes in the *18* hole row; then $\frac{1}{9}\times 352=39\frac{1}{9}$, which is the number of turns of the index crank, or $\frac{8}{9}$ less than the required *40* for one complete turn of the work. The ratio of gears used will therefore be $\frac{8}{9}=\frac{8\times 1}{9\times 1}=\frac{64\times 24}{72\times 24}$.

Example. Find the indexing required for 51 divisions.

Solution. The spacing number $=\frac{40}{51}$. As this fraction cannot be taken directly from the index plate, select $\frac{42}{51}=\frac{14}{17}$; then the *17* hole index may be used by taking *14* holes; $\frac{14}{17}\times 51=42$, which is the number of turns of the index crank or *2* more than the required *40* for one turn of the work. The ratio of gears must then be as 2 to 1, $\frac{2}{1}=\frac{48}{24}$.

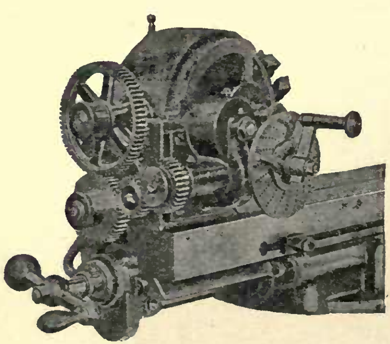


Fig. 56

Then gear on spindle will have *48* teeth, gear on worm will have *24* teeth.

As the motion of the index plate must be in the direction opposite to the movement of the index crank, two idler gears must be used, as shown in Fig. 56.

PROBLEMS

29. What is the indexing required for 227 divisions?

30. What is the indexing required for 57 divisions?

31. Find the indexing by the differential method for 149 divisions.

32. Find the indexing required for 159 divisions.

33. What is the differential indexing for 161 divisions?

34. What is the differential indexing for 162 spaces?

35. When the circumference of a disc is to be marked into 359 equal spaces, what is the indexing required on the index head of milling machine?

THE SPIRAL HEAD

The index head is used in connection with the feeding of the platen carrying the work, when cutting spiral or helical flutes in reamers and drills or the teeth of spiral mills, gears, etc., as well as in indexing for the equal spaces. The object is to impart a rotary motion to the work as it is fed along under the cutter, which is done by the action of a train of gears in mesh with the *feed screw 15*, and the *worm* 3_1, Fig. 53. This action is very similar to that of the gearing used in thread cutting in the lathe; in fact, the flute of a drill is a thread of quite long lead, and is spoken of as of so many inches to one turn, instead of so many threads per inch as in the case of a screw thread; thus a drill is said to have a flute *one turn in six inches*, or it might be said to be a flute of *six inch lead*.

It is usually the custom to call the cutting of the helical flutes of straight reamers and drills and teeth in mills, gears, etc., spiral cuts, whereas a spiral is the winding cut on the cone, but the action of feeding the work is the same in the case of a straight or taper reamer, hence the custom. The principle is as follows: Feed screw *15*, is a $\frac{1}{4}$ *inch* lead double square thread screw. Removing pin *16* from plate *6* allows it to revolve through the action of the gears *10* and *13*, *8*, *9*, etc. Now, if gears *10* and *13* are the same size and gears *11* and *12* are the same size, or are replaced by a single idler gear, then forty turns of *15* will give forty turns to shaft *3*, which gives the work carried by *1* one complete turn, since gears *8* and *9* are miter gears. Now forty turns of *15* will feed the platen carrying the work a distance of *40* multiplied by the lead of screw *15*, or $40 \times \frac{1}{4} = 10$ *inches*. This motion of platen when the ratio of gears is *1* to *1* is called the lead of the machine and is the factor used to get the ratio of gears for any other lead required.

Let $R =$ the lead to be cut.

$F =$ product of follower gears of the train.

$D =$ product of driver gears of the train.

Then $R = \dfrac{10 \times F}{D}$

or $R : 10 = F : D$

and $\dfrac{R}{10} = \dfrac{F}{D}$.

In the arrangement of gearing shown at Fig. 53, gear *13* is the first driver, motion coming from the feed screw *15*, the follower in mesh with it is the second gear that is put on the compound stud; the second driver is the first gear on the compound stud and the follower in mesh with it, gear *10*, the gear that drives the worm 3_1 through the miter gears *8* and *9*.

Example. Find the gearing required to cut a spiral flute with a lead of 12 inches.

Solution. The gears *10, 11, 12,* and *13* must be, by formula above, in the ratio of *12* to *10*, then $\frac{12}{10}=\frac{F}{D}$.

To use the compound stud as shown, $\frac{12}{10}=\frac{4\times 3}{5\times 2}$ and any gears in these ratios can be used, $\frac{4}{5}\times\frac{8}{8}=\frac{32}{40}$ and $\frac{3}{2}\times\frac{24}{24}=\frac{72}{48}$ then $\frac{72\times 32}{48\times 40}=\frac{F}{D}$. It makes no difference whether *48* or *40* is first driver, as long as they are made the drivers for 3×4 is the same as 4×3; also the same principle applies to the followers, for 2×5 is the same as 5×2.

PROBLEMS

1. Find the gearing required for the spiral head to cut a flute of 1 turn in 13⅓ inches.

2. What gears will be required to cut the teeth in a spiral mill when the teeth have a lead of 28 inches?

3. What gearing will be required to cut the teeth in a mill when the spiral has 1 turn in 22½ inches?

4. Find the gears required to cut 26¼ in. lead.

5. What gears are necessary to cut a 48 in. lead?

6. What is the lead of spiral that will be cut with gears 64 on worm, 48 first on stud, 40 second on stud, and 56 on screw?

7. Find the lead when gears 72 and 64 are followers and 32 and 40 are drivers.

8. What is the lead of the spiral in inches for 1 turn that will be cut with a 24 tooth gear on worm, 64 first on stud, 48 second on stud, 72 on screw?

Note. The platen must be set at the right angle for the cut in the work otherwise the mill will cut on the back of the center of revolution. The angle at which the platen is turned is determined by the diameter of the work to be cut and the lead of spiral, and can be found graphically or can be calculated by trigonometry, as the circumference of the work represents the side opposite the angle and the feed of bed is the side adjacent to angle.

Example. Find the angle through which to swing the platen of milling machine to cut the flute in a $1\frac{1}{2}$ in. dia. drill with a lead of *12* inches.

Solution. $Tan\ A = \frac{a}{b} = \frac{4.7124}{12} = .3927$ and this is found in table, page 183, under tangents as $21° \ 30'$ for the nearest angle.

PROBLEMS

9. At what angle should the platen of a universal miller be set to cut $\frac{1}{2}$ in dia. and 4 in. lead?

10. Find the angle at which to set the platen of miller for $1\frac{1}{2}$ in. dia. and 9 in. lead.

11. At what angle should the platen of miller be set to cut $1\frac{3}{4}$ in. dia. and 8 in. lead?

12. What is the angle at which to set the miller platen for a 3 in. dia. mill to cut a spiral tooth 28 in. lead?

13. Find the angle for 3 in. mill, one turn in 40 inches.

14. If the spiral teeth in a 3 in. dia. roll are at an angle of 30°, what would be the length of the roll for one complete tooth?

15. A spiral gear 10 in. dia. has teeth at 45° angle. What is length for feed of platen in miller for one complete turn of gear on centers of spiral head?

16. A milling machine platen has a feed of 2 in. per minute and is cutting a spiral flute reamer $\frac{7}{8}$ in. dia. with 8 in. lead. Find angle at which to set the platen and the time required to cut a flute of $1\frac{1}{2}$ turns.

SPEEDS AND FEEDS

The amount of feed per revolution of a milling cutter depends somewhat on the number of teeth in the cutter. For backed off mills the following table is considered good practice:

Brass, 120 *F. P. M.*, feed $4\frac{2}{3}$ inches per minute, $\frac{1}{4}$ inch deep.

Cast Iron, 60 *F. P. M.*, feed $4\frac{2}{3}$ inches per minute, $\frac{1}{4}$ inch deep.

Wrought Iron, 48 *F. P. M.*, feed 2 inches per minute, $\frac{1}{4}$ inch deep.

Steel, 36 *F. P. M.*, feed $1\frac{1}{2}$ inches per minute, $\frac{1}{4}$ inch deep.

In the following problems use the approximate formula $R=\frac{4C}{d}$ for *R. P. M.* unless otherwise stated.

PROBLEMS

17. What speed is required in *R. P. M.* for a $3\frac{1}{2}$ in. dia. milling cutter to be used in cutting a steel forging?

18. Find the *R. P. M.* of a mill $4\frac{1}{2}$ in. dia. for milling the teeth of steel sprocket wheel forgings, when cut is $\frac{3}{8}$ in. deep, and *F. P. M.* is in inverse proportion to depth of cut as allowed in table of feeds given above.

19. What *R. P. M.* will a mill 6 in. dia. make at 60 *F. P. M.* when cutting cast iron journal boxes?

20. Find the *R. P. M.* of a 3 in. dia. cutter for cast iron surface plates.

21. What *R. P. M.* should a $1\frac{1}{4}$ in. T slot cutter revolve for cutting cast iron planer beds?

22. What speed in *R. P. M.* will a $\frac{3}{4}$ in. end mill make while slotting cast iron planer beds with $\frac{5}{8}$ in. depth of cut if the *F. P. M.* is in inverse proportion to depth of cut?

23. What speed in *R. P. M.* will a 5 in. dia. mill make on composition step bearings at 100 *F. P. M.?*

24. If babbitt metal bars can be machined at 130 *F. P. M.* find the *R. P. M.* for a 6 in. dia. mill for cutting these bars.

25. An Ingersoll milling cutter 5 in. dia. is finishing milling machine beds at 1 in. feed per minute and $\frac{1}{2}$ in. depth of cut. If the mill makes 60 *R. P. M.*, exactly how much faster in *F. P. M.* is it cutting than is ordinarily allowed for cast iron?

26. What *F. P. M.* and *R. P. M.* should a $5\frac{1}{2}$ in. dia. milling cutter feed when taking a $\frac{5}{8}$ in. depth of cut in soft brass, and the *F. P. M.* is in inverse proportion to depth of cut?

27. How deep a cut can a 4 in. dia. mill ordinarily take for steel when running 25 *F. P. M.* if the feed is $\frac{7}{8}$ in. per minute, and *F. P. M.* is in inverse proportion to depth of cut?

28. What depth of cut can be made with a 5 in. mill at 48 *R. P. M.* in cast iron feeding 4 in. per minute when feed and depth of cut are in inverse proportion to the *F. P. M.?*

29. What is the feed per minute for a 6 in. dia. mill, $\frac{5}{8}$ in. depth of cut, for milling cast iron lathe cross slides at 60 *F. P. M.* if feed is in inverse proportion to the depth of cut?

DRILL PRESS

The feeding mechanism of the drill press is very similar to that of the lathe and milling machine, and the cutting feeds and speeds for different materials are about the same as for milling cutters, the speeds of drills being in proportion to their diameters. The *R. P. M.* is found by dividing the constant for each material by the diameter of drill.

Then $$R.\,P.\,M. = \frac{x}{D},$$

from which the formulas are as follows:

for steel $$R.\,P.\,M. = \frac{80}{D},$$

for cast iron $$R.\,P.\,M. = \frac{110}{D},$$

for brass $$R.\,P.\,M. = \frac{180}{D},$$

D = diameter of drill in inches.

Tables for speeds of drills have been made, but, as in all cutting operations, the kind and condition of material and tools, and the depth of cut must always be taken into account. This can only be determined by experience and trial. The depth of cut for small drills is usually *.002* to *.005 inch* per revolution, on large drills, from *.005* to *.015 inch* per revolution, *.010 inch* being a fair average for machinery steel.

PROBLEMS

1. Find the speed and time required to drill 1 in. cast iron at .005 in. feed per revolution with drill, $\frac{3}{8}$ in. diameter.

2. Find the depth of cut for 1 in. drill in a steel plate for 1 minute at .008 in. feed per revolution.

3. What is the time required to drill 1 in. deep in cast iron with $1\frac{1}{4}$ in. dia. drill at .012 in. feed per revolution?

4. What is the time required to drill $1\frac{1}{8}$ in. deep in brass bearings with $1\frac{1}{8}$ in. dia. drill at .009 in. feed per revolution?

5. Find the time and speed of a $\frac{7}{8}$ in. drill to cut a hole $\frac{7}{8}$ in. deep in cast iron at .008 in. feed per revolution.

6. Find the time and speed of $\frac{1}{2}$ in. drill to cut $1\frac{1}{2}$ in. deep in cast iron at .005 in. feed per revolution.

7. What is the time required for a $1\frac{1}{2}$ in. dia. drill if point is $\frac{3}{16}$ in. longer than body size for cutting steel with .012 in. feed per revolution when the work is $2\frac{1}{2}$ in. thick?

8. What is the time required to drill a steel plate $1\frac{3}{8}$ in. thick with a 2 in. dia. drill when the point is $\frac{1}{4}$ in. longer than the body size with a feed of .01 in. per revolution?

9. Find the time required to drill a cast iron bed plate 4 in. thick with a 2 in. dia. drill if point of drill is $\frac{4}{10}$ in. longer than body size and .02 in. feed per revolution.

HAMMER BLOW

The force of a hammer blow is calculated from the amount of work done. In the case of a drop hammer, the work is equal to the product of the weight of the hammer head and the distance it falls. Neglecting friction losses, the force of the blow may be found by the following formula:

Let F = the striking force in pounds.

W = the weight of the hammer.

H = the distance through which hammer moves in feet.

D = the amount of compression of the material or penetration expressed in feet.

Then $F = \frac{WH}{D}$.

Example. A 400 lb. drop hammer falling *4 ft.* compresses a bar of steel $\frac{1}{32}$ in. at each blow. What is the force of the blow?

Solution. By formula:

$$F = \frac{WH}{D} = \frac{400 \times 4}{\frac{.03125}{12}} = 614{,}400 \text{ pounds.}$$

PROBLEMS

1. A drop hammer weighing 1,000 pounds falls 3 feet. If the stock is reduced in thickness $\frac{1}{8}$ inch at the first blow, what is the force of the blow?

2. If the compression of the material in problem 1 is lessened $\frac{1}{64}$ inch at each succeeding blow, what is the force of the second, third and fourth blows?

3. A drop hammer weighing 575 pounds falls from a height of 30 inches. If the forging is compressed $\frac{1}{16}$ inch, what is the force of the blow?

4. A forging is made $\frac{1}{32}$ inch thinner at one blow of a 425 pound drop hammer falling a distance of 24 inches. What is the force of the blow?

5. A 10 pound hammer falls a distance of 20 inches and compresses the material $\frac{1}{100}$ inch. What is the force of the blow?

6. Find the force of the blow from a weight of 400 pounds falling a distance of 15 feet that bends a bar of steel $\frac{1}{4}$ inch.

7. A pile driver has a weight of 500 pounds which falls 20 feet and drives a pile 12 inches into the ground. Find the force of the blow.

8. What is the force of the blow in problem 7 when the pile is driven 6 inches?

9. When $W = 1{,}250$ pounds, $H = 3\frac{1}{2}$ feet and $D = 4$ inches, what is the striking force in pounds?

HORSE POWER OF MACHINES

The horse power required to run the different machines while taking the various cuts is calculated in several ways. One way is by finding the speed and pull of belt, thus getting directly the *ft.-lbs.* of work done in a specified time. This calculation assumes that the cut taken is just enough to use all the force of the belt and not have the pulley slip. The formula will be:

$$H.\ P. = \frac{W \times P \times R.\ P.\ M. \times \pi d}{33{,}000 \times 12},$$

when W = width of belt in inches,
d = diameter of pulley in inches,
P = pull of belt in pounds per inch wide, which is generally taken at about *33* pounds for single belts and *50* pounds for double belts.

Example. A 3 in. double belt runs 50 *R. P. M.* on a 10 in. dia. pulley of lathe and the strain of belt is 60 lbs. per in. in width. What is the *H. P.* of lathe?

Solution. By formula:

$$H.\ P. = \frac{\cancel{3} \times \cancel{60} \times \cancel{50} \times 10 \times 3.1416}{\underset{11}{\cancel{33{,}000}} \times \underset{4}{\cancel{12}}} = \frac{31.416}{44} = .714\ H.\ P.$$

Another, and perhaps a surer way, is based on the actual *amount of metal* removed from the work in chips. Exhaustive experiments have been made with the *carbon* steel cutting tools under average conditions, by which a constant has been determined for the various materials.

Let W = weight of the chips in pounds removed per hour.
C = constant which for cast iron = *.03*.
For wrought iron and soft steel $C = .032$.
For crucible and tool steel $C = .047$.
Then formula is $H.\ P. = C \times W$.

Example. What is the *H. P.* of a milling machine that cuts 50 lbs. of chips per hour from a lot of castings?

Solution. By formula:

$$H.\ P. = C \times W = 50 \times .03 = 1.5\ H.P.$$

PROBLEMS

1. A 3 in. single belt with an effective pull of 55 lbs. per inch on 9 in. step pulley of a milling machine runs 50 *R. P. M.* What is the *H. P.* of the machine?

2. A $1\frac{1}{2}$ in. single belt with an effective pull of 55 lbs. per inch runs 142 *R. P. M.* on a 10 in. dia. pulley of a planer. Find the *H. P.* of machine if $\frac{1}{4}$ the time is on the return stroke.

3. A 2 in. single belt on a 10 in. dia. pulley of a drill press runs 60 *R. P. M.* Find the *H. P.* required to run the press, if the effective pull of the belt is taken at 55 lbs. per inch.

4. The average *F. P. M.* for cutting work in a planer is 10 ft. If a cut $\frac{1}{8}$ in. deep, $\frac{1}{32}$ in. feed, is made on castings 12 in. long, what is the *H. P.* of the machine?

5. A lathe is turning a 2 in. tool steel bar from 2 in. to $1\frac{3}{4}$ in. dia. in one cut at 25 *R. P. M.* with a $\frac{1}{64}$ in. feed per revolution. Find the *H. P.* of the machine.

6. A milling cutter runs 55 *R. P. M.* with .02 in. feed per revolution on cast iron plates; the cut is 1 in. deep and $3\frac{1}{2}$ in. wide. Find the *H. P.* required to operate the machine.

7. A drill press with a 2 in. dia. drill runs 55 *R. P. M.* with feed .012 in. per revolution in a wrought iron draw bar plate. What *H. P.* will be required to run the press?

8. What *H. P.* will be required to run a lathe at 30 *R. P. M.* turning a shaft to $1\frac{15}{16}$ in. dia. with a $\frac{1}{32}$ in. feed and $\frac{1}{8}$ in. depth of cut?

9. What *H. P.* will be required to turn soft steel shafting to 6 in. dia. with cut $\frac{3}{16}$ in. deep and $\frac{1}{32}$ in. feed at 12 *R. P. M.?*

10. A drill press cutting soft steel runs 85 *R. P. M.* with a drill $1\frac{1}{8}$ in. dia. and the feed .012 in. per revolution. What is the *H. P.* required to run the drill press?

11. If the cutting speed of a planer is 24 *F. P. M.* what *H. P.* will be required to run the planer on cast iron with a feed .03 in. per stroke, the length of cut being 36 in. and the depth of cut $\frac{1}{8}$ inch?

Note. In calculating the *H. P.* for a planer the weight of chips is taken as though the cut was continuous.

12. Find *H. P.* to run a milling machine in cast iron 1 in. deep, 1 in. wide and 4 in. feed per minute.

13. A milling machine takes a cut 6 in. wide, $\frac{1}{4}$ in. deep and $2\frac{5}{8}$ in. feed per minute. If the material is soft steel, what *H. P.* will be required to operate the machine?

14. A drill press makes 36 *R. P. M.* in a drop forging with a drill 3 in. dia. feeding .015 in. per revolution. What *H. P.* will be required to run the drill press?

15. Find the *H. P.* of a drill press running 100 *R. P. M.* with a $2\frac{1}{2}$ in. single belt on a 12 in. dia. pulley.

16. Find the *H. P.* of a lathe at 70 *R. P. M.* with a 4 in. single belt running on a 12 in. dia. pulley.

17. Find the *H. P.* of a milling machine at 90 *R. P. M.* with a 12 in. dia. pulley having a 3 in. double belt.

18. What is the *H. P.* for operating a punch press with a 4 in. double belt on a 24 in. dia. pulley running 75 *R. P. M.?*

DYNAMOMETER

Another method for finding the power required to operate machinery is by using a dynamometer, which is a power measuring instrument. The accuracy in determining the power is in proportion to the kind and condition of the measuring instrument.

The *Prony Brake* is one of the most simple and familiar examples of the dynamometer. Let *A*, Fig. 57, represent a pulley *126.04 inches* diameter that makes *200 R. P. M.*, then $\frac{\pi \times 126.04 \times 200}{12} = 6600$ *F. P. M.* If a pull of *5* pounds is made at *P* the work is $6600 \times 5 = 33{,}000$ *ft.-lbs. per minute* or *1 H. P.* To record the amount of pull at *P* is the office of the dynamometer, as the diameter and *R. P. M.* of pulley can be readily obtained.

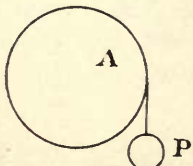


Fig. 57.

Fig. 58 shows the general principle of the prony brake and the way the apparatus is commonly constructed. Shoes *a* and *b* can be clamped to pulley with bolts *c, c;* when the pulley is revolved in the direction of the arrow, the tendency is for the

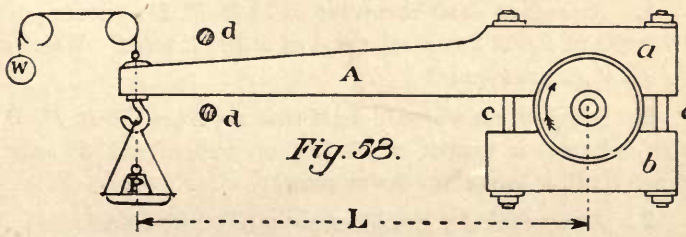


Fig. 58.

entire brake and lever to rotate in the same direction, which is prevented by weights *P* in the scale pan (the weight W is to counterbalance the weight of lever arm A when pulley is at rest). When the pulley revolves at its normal *R. P. M.*, sufficient weight P is put in the pan to balance the lever between pins d, d, which are placed to prevent lever from revolving. The power absorbed by the brake shoes a, b, is equal to the amount of work which is accomplished in *ft.-lbs.* per minute by the revolving shaft. This work in $ft.\text{-}lbs. = P \times N \times 2\pi \times L$

then $H.\ P. = \dfrac{2\pi LPN}{33{,}000}$.

The brake must be well lubricated to prevent seizing.

Example. An engine shaft makes 150 *R. P. M.* What is the *H. P.* developed when a weight of $10\frac{1}{2}$ lbs. is just balanced at the end of an 8 ft. lever attached to a pair of brake shoes as in Fig. 58?

Solution:

$$H.\ P. = \frac{2\pi LPN}{33{,}000} = \frac{6.2832 \times 8 \times 10\frac{1}{2} \times 150}{33{,}000} = 2.4\ H.\ P.$$

PROBLEMS

1. An engine shaft revolving at 74 *R. P. M.* will support a weight of 2,000 lbs. at the end of a 10 ft. lever. What is the *H. P.* of the engine?

2. A pulley on a motor shaft that revolves 750 *R. P. M.* just balances a weight of 25 lbs. on end of a 5′ 3″ lever. What *H. P.* is the motor developing?

3. A two cylinder gasolene motor has a fly wheel making 1,000 *R. P. M.* When a $5\frac{1}{4}$ ft. lever arm balances 25 lbs., what is the *H. P.* of the motor?

4. What length of arm will be required to balance a 25 lb. weight on a shaft making 150 *R. P. M.* doing 2 *H. P.* of work, when the brake shoes are clamped to the fly wheel pulley?

5. Find the weight required to balance the brake shoe lever 5′ 3″ long on a pulley at 200 *R. P. M.* and transmitting 10 *H. P.*

ENGINE FLY WHEELS

Fly wheels are used to regulate the motion in machinery by storing energy during increasing velocity and giving it out during decreasing velocity. There is no power gained in the use of a fly wheel, in fact, power is used in overcoming friction in bearings when the shaft is loaded with the extra weight of the wheel. Fly wheels are usually made of cast iron and the safe velocity is one mile per minute

or $\frac{5280}{60} = 88$ *feet per second.*

The formula for centrifugal force in fly wheels is

$$F = \frac{WV^2}{gR} = \frac{4W\pi^2 RN^2}{3{,}600g} = W \times R \times N^2 \times .000341.$$

W = weight of rim in pounds.

R = mean rad. of rim in feet.

$N = R.\ P.\ M.$ of wheel.

$g = 32.16.$

V = velocity in feet per second $= 2\pi RN \div 60.$

The rotating parts of machinery must be well balanced on their axes on account of the centrifugal force which would cause wear on the bearing and vibration to the machinery.

Example. What strain will be put on the bearing of a fly wheel shaft with an unbalanced weight of 5 lbs. on the rim of fly wheel which is 10 ft. dia. making 100 *R. P. M.?*

Solution:

$$\frac{10 \times 3.1416 \times 100}{60} = 52.36 \text{ feet per second.}$$

$$\text{Then } c.\,f. = \frac{5 \times 52.36 \times 52.36}{32.16 \times 5} = \frac{2741.5696}{32.16} = 85\tfrac{1}{4} \text{ lbs.}$$

The tensile strain (S) on the cross section of rim is found by dividing the force by 2π, then $S = W \times R \times N^2 \times .00005427$.

The maximum tension per square inch for cast iron, allowing factor of safety of *10*, is 1,000 pounds, corresponding to a velocity of *6,085 F. P. M.* so that one mile per minute is within the safe limit. The diameter of a fly wheel is found from formula

$$D = \frac{5280}{\pi N}$$

$N = R.\ P.\ M.$ of the wheel.

$D =$ diameter of the wheel in feet.

PROBLEMS

1. When $R = 7\frac{1}{2}$ ft., find the strain in a 15 ft. cast-iron fly wheel rim that is $1\frac{1}{4}$ in. thick and 16 in. face, running 6,000 *F. P. M.*

2. What extra strain will be put on the rim in problem 1 if a weight of 10 lbs. is placed at some point on the rim?

3. What strain will be found in a cast-iron fly wheel 4 ft. dia. when the cross section outline of rim is 4 in. square running at 6,000 *F. P. M.?*

4. What is the strain at *a* speed of 5,000 *F. P. M.* of a 4 ft. dia. cast-iron fly wheel with a cross section outline 4 in. dia.?

5. If $R = 8$ ft., what is the tensile strain in a cast-iron fly wheel 16 ft. dia. at 74 *R. P. M.* with a rim which is $1\frac{1}{2}$ in. thick and 26 in. wide?

6. What is the tensile strain in a cast-iron fly wheel, 4 ft. dia. at 400 *R. P. M.* with a cross section outline of rim 4 in. square?

7. If $R = 15$ ft., what is the strain of a cast-iron fly wheel 30 ft. dia. with bolted rim, 36 in. width of face and $2\frac{1}{2}$ in. thick, running 60 *R. P. M.?*

8. What is the strain on the rim of an 8 ft. fly wheel at 115 *R. P. M.* with rim of cast iron 16 in. wide and 1 in. thick?

9. If 6,000 *F. P. M.* is allowable for the wheel of problem 7 what is the difference between the actual and the allowable strain in the rim?

10. Find the *R. P. M.* of a cast-iron fly wheel 8 ft. dia. at the safe velocity.

11. Find the *R. P. M.* of a $3\frac{1}{2}$ ft. dia. fly wheel at one mile per minute of rim speed.

12. What is the *R. P. M.* of a 14 ft. fly wheel running 6,000 ft. per minute?

13. What is the *F. P. M.* in an 18 ft. fly wheel rim running at 100 *R. P. M.?*

14. How much over the mile limit is a 22 ft. fly wheel rim moving, when making 90 *R. P. M.?*

POWER OF THE STEAM ENGINE

There are *three* kinds of *horse power* spoken of by the engineer.

Nominal horse power is a term used by makers of engines as a convenient way of stating the dimensions of an engine. It really means nothing, is misleading, and should not be used.

Indicated horse power is the actual measure of work done by the steam in the cylinder; this is not based on any assumption, but is actually calculated from the steam pressure and the travel of the moving parts.

Effective horse power is the amount of work which an engine is capable of transmitting. It is the difference between the indicated *H. P.* and the amount of power required to run the engine without any load.

The indicated *H. P.* is found by calculation and is the one generally meant when the term *H. P.* of the engine is used.

The *H. P.* of any mechanism depends on the *ft.-lbs.* of work done in a specified time, therefore the power of the steam engine can be calculated in the same manner, and the usual formula taken is

$$H.\ P. = \frac{PLAN}{33{,}000},$$

where P = the steam pressure on the piston called the *mean effective pressure*, or *M. E. P.*

L = twice the length of the piston stroke and is the travel of the piston in *feet* for the motion out and back.

A = the area of the piston in square inches.

N = the *R. P. M.* of the crank.

The *M. E. P.* can be found only by using an indicator, but where it is not possible to use an indicator the *M. E. P.* is sometimes taken at one-half the gauge pressure on the boiler.

Since the *area of the piston* equals the *square of the diameter* $\times .7854$, then by *dividing .7854 by 33,000*, $H.\ P. = D^2 \times P \times T \times .0000238$. There are many formulas given for finding the *H. P.* of the steam engine, but all are based on the one first given. A short formula is sometimes used as follows:

$$H.\ P. = \frac{D^2 \times T \times M.\ E.\ P.}{42{,}000}$$

D = diameter of piston in inches.

$T = L \times N$ of first formula given, which is the distance in feet of piston travel per minute.

In low pressure engines, that is, condensing engines, it is usually the custom to add *10* pounds to P for the increased efficiency of the vacuum.

Example. Find the horse power of a steam engine when the piston is 18 in. dia., the crank is 15 in. long and makes 115 $R.\ P.\ M.$ and the $M.\ E.\ P.$ is 40 lbs.

Solution. By formula:

$$H.\ P. = \frac{PLAN}{33{,}000}, \text{ or } H.\ P. = \frac{40 \times 5 \times 254.47 \times 115}{33{,}000} = 177.35.$$

The length of crank is the distance from the center of crank shaft to the center of pin. In a complete revolution, the stroke being twice the length of the crank, the piston travel is forward and back for each stroke or *5* feet per revolution for a 15 inch crank.

PROBLEMS

1. The dia. of a cylinder is 16 in. and the stroke is 24 in.; when the crank makes 120 $R.\ P.\ M.$ and the $M.\ E.\ P.$ is 45 lbs. what is the $H.\ P.$ that will be developed by a 2-cylinder engine?

2. Find the $H.\ P.$ of an engine when the cylinder is 13 in. dia., 8 in. crank, 300 $R.\ P.\ M.$ and $M.\ E.\ P.$ 67 lbs.

3. Use the formula $D^2 \times P \times T \times .0000238$ to find the $H.\ P.$ of an engine, with 18 in. dia. of cylinder, 30 in. stroke, 115 $R.\ P.\ M.$ and 40 lbs. $M.\ E.\ P.$

4. What is the $H.\ P.$ developed by an engine with 24 in. dia. cylinder, 60 in. stroke, 75 $R.\ P.\ M.$ of crank and 70 lbs. $M.\ E.\ P.$?

5. Find the $H.\ P.$ developed in an engine with a 6 in. crank, 5 in. dia. cylinder and 400 $R.\ P.\ M.$

Note. When the steam pressure is not known, the $H.\ P.$ will be given per lb. of $M.\ E.\ P.$

6. Use short formula for the following: Find the *H. P.* of a pair of 24 in. dia., 48 in. stroke, horizontal high pressure cylinders at 120 *R. P. M.* of crank.

7. Find the *H. P.* of two 12 in. dia. cylinders, 20 in. stroke and 60 *R. P. M.* of crank, when the *M. E. P.* is to be found with an indicator.

8. Find the *H. P.* of an engine with 9 in. dia. cylinder, 30 in. stroke, 90 *R. P. M.* of crank, when *M. E. P.* is to be found by an indicator.

9. Find the *H. P.* developed by a compound engine when the high pressure cylinder is 16 in. dia., *M. E. P.* 70 lbs., and low pressure is 27 in. dia. with *M. E. P.* 12 lbs., stroke is 16 in. and 250 *R. P. M.* for each cylinder.

Note. The *H. P.* of a compound engine is found as though each cylinder was for a separate engine, by taking the sum of the *H. P.* of both cylinders. By the term low pressure is meant that the steam is exhausted from the first cylinder to the second at a lower pressure.

10. Find the *H. P.* of a compound engine when the high pressure cylinder is $27\frac{1}{2}$ in. dia., has *M. E. P.* of 36.75 lbs., low pressure cylinder 48 in. dia., *M. E. P.* of 7.25 lbs., length of crank 25 in. and 75 *R. P. M.*

POWER OF THE LOCOMOTIVE

The term horse power is seldom used in speaking of the work of the locomotive as the service is entirely different from the work done by the stationary engine.

The *power of the locomotive* is measured at the point where the wheel touches the rail and is the weight on the drivers and their adhesive power on the rails, in other words it is its *tractive* force, or capacity to pull a load by steam pressure through mechanism similar to the stationary engine. Thus a great deal of the efficiency of the locomotive depends on the speeds, grades, curves, weights on drivers, etc.

It has been proved by actual test that only 43% of the effective power at the cylinders is available at the draw bar of a locomotive.

Example. A locomotive with 14 in. dia. and 24 in. length of cylinders and 80 lbs. *M. E. P.* with 6 ft. dia. of drive wheels running at one mile per minute has what effective power at the draw bar?

Solution. The *H. P.* at the piston by formula:

$$H.\ P. = \frac{PLAN}{33{,}000} = \frac{80 \times 4 \times 153.94 \times 280 \times 2}{33{,}000} = 836\ H.\ P.$$

$$43\%\ of\ 836\ H.\ P. = 359.5\ H.\ P.$$

The usual formula for the tractive power of the locomotive is as follows:

Let $T =$ the tractive power of the locomotive.

$W =$ the diameter of the wheel in feet.

$S =$ the length of stroke in feet.

$D =$ diameter of piston in inches.

$M.\ E.\ P. =$ the pressure found with an indicator which may be taken as found by experiment, to be about 40% of the boiler gauge pressure,

then
$$T = \frac{D^2 \times M.\ E.\ P. \times S}{W}.$$

The tractive power is found to be equal to the load that the locomotive can lift from a pit by means of a rope over a pulley from the circumference of the drive wheel.

PROBLEMS

11. What is the tractive power of a locomotive having 19 in. dia. cylinders and 12 in. cranks with 68 in. drive wheels and gauge pressure 160 lbs.?

12. What is the tractive power of a locomotive having 16 in. dia. cylinders with 12 in. cranks, 72 in. drive wheels at 180 lbs. gauge pressure of which 50% is *M. E. P.?* Also what *H. P.* can be applied at the draw bar if only 45% of the indicated *H. P.* of the piston is effective?

GASOLENE ENGINES

The difficulty of computing the horse power of the gasolene engine with the formula $H.\,P. = \dfrac{PLAN}{33{,}000}$ is to establish the *M. E. P.* and to keep the piston speed at a uniform rate.

The A. L. A. M. (Association of Licensed Automobile Manufacturers) have adopted the following standard formula:

Let D = diameter of cylinder in inches.
N = number of cylinders.

Then $H.\,P. = \dfrac{D^2N}{2.5}$.

Example. Find the *H. P.* of a 6 cylinder gasolene engine when the cylinders are 4 in. diameter.

Solution. By formula:

$$H.\,P. = \frac{16 \times 6}{2.5} = \frac{96}{2.5} = 38.4\ H.\,P.$$

The following formulas are used by the mechanical engineers in some of the leading automobile factories as giving close results under certain conditions.

$$H.\ P. = \frac{S\ C\ A\ N}{12{,}000}.$$

S = stroke in inches.
C = number of cylinders.
A = area of piston.
N = number of $R.\ P.\ M.$

This formula is based on *20* cubic inches piston displacement and 600 $R.\ P.\ M. = 1\ H.\ P.$

$$H.\ P. = \frac{P\ L\ D^2\ N}{1{,}000}.$$

$P = M.\ E.\ P.$ taken as 87 pounds per square inch.
L = length of stroke in inches.
D = diameter of cylinder in inches.
N = number of cylinders.

All of the formulas given are based on the effective work that a certain size of engine should accomplish and have been found correct in actual practice.

PROBLEMS

13. Find the $M.\ E.\ P.$ of a gas engine from formula $H.\ P. = \frac{P\ L\ A\ N}{33{,}000}$, when the dia. of cylinder is $7\frac{7}{8}$ in., length of stroke $15\frac{3}{4}$ in., making 200 $R.\ P.\ M.$, when a brake test of the engine shows $8\frac{1}{2}$ H. P.

14. A gas engine does 25 horse power in a test with a 6 in. dia. cylinder, 12 in. stroke, at 600 $R.\ P.\ M.$ What is the $M.\ E.\ P.$?

15. A single cylinder motorcycle $2\frac{3}{4}$ in. dia. and 3 in. length of stroke makes 2,500 $R.\ P.\ M.$ What is the $H.\ P.$ when $M.\ E.\ P.$ is 10 lbs.?

16. If the engine in problem 15 has two cylinders, and dimensions the same, what is the *H. P.?*

17. Find the *H. P.* by formula, $H.\ P. = \dfrac{S\ C\ A\ N}{12{,}000}$, of a two cylinder gasolene engine, 5 in. stroke, 4 in. dia., 1,000 *R. P. M.*

18. Find the *H. P.* by formula, $H.\ P. = \dfrac{P\ L\ D^2\ N}{1{,}000}$, of a four cylinder automobile engine making 750 *R. P. M.*, $4\frac{1}{2}$ in. dia. and stroke $4\frac{3}{4}$ in. long with *M. E. P.* 87 lbs.

19. What is the *H. P.* of a six cylinder automobile gasolene motor if $D = 4\frac{3}{4}$ in. and $L = 4\frac{1}{4}$ inches.

20. Find the *H. P.* of a four cylinder gasolene engine $3\frac{3}{4}$ in. diameter.

21. What is the *H. P.* of a six cylinder automobile engine with cylinder $4\frac{3}{8}$ in. dia. and length of stroke $4\frac{3}{4}$ inches?

22. What is the *H. P.* of a single cylinder gasolene motor $3\frac{7}{8}$ in. dia., $5\frac{3}{4}$ in. length of stroke?

ENGINE CYLINDERS

The size of the cylinder can be found by transposing the formula for $H.\ P. = \dfrac{P\,L\,A\,N}{33{,}000}$ and getting the *diameter* from the *area*.

PROBLEMS

23. Find the *H. P.* of an engine when the cylinder is 18 in. dia., 26 in. stroke, *R. P. M.* 200, and *M. E. P.* of 48 lbs.

24. What is the *H. P.* of an engine per lb. *M. E. P.* when piston is 20 in. dia., 36 in. stroke, 175 *R. P. M.?*

25. A locomotive's cylinders are 16 in. dia., 26 in. stroke, drivers 6 ft. dia. What is the tractive power when boiler pressure is 200 lbs., *M. E. P.* taken at 50% of boiler pressure, as shown by the steam gauge?

26. The same as problem 25 except boiler pressure is 175 lbs. and *M. E. P.* taken at 60%.

27. What size cylinder will be required for an engine to develop 250 *H. P.* when the boiler pressure is 100 lbs., and *M. E. P.* taken at 50%, 24 in. crank, at 74 *R. P. M.?*

28. What is the *H. P.* of an engine per lb. *M. E. P.* when the piston is 18 in. dia., 30 in. stroke, crank making 112 *R. P. M.?*

29. Find the size of a cylinder to develop 75 *H. P.* with 18 in. crank at 115 *R. P. M.*, boiler pressure of 85 lbs. and *M. E. P.* taken at 45% of the boiler pressure.

30. What dia. of cylinder will be required for a 150 *H. P.* engine with 24 in. crank, at 85 *R. P. M.*, 80 lbs. boiler pressure if *M. E. P.* is taken at 40%?

31. Find the size of a cylinder required for a 75 *H. P.* engine with 16 in. crank at 150 *R. P. M.* and *M. E. P.* $37\frac{1}{2}$ lbs.

32. Find the length of an engine crank when *R. P. M.* is 85, dia. of cylinder 16 in., *M. E. P.* $37\frac{1}{2}$ lbs. when the engine is to develop 62 *H. P.*

33. What is the length and *R. P. M.* of a crank for a 50 *H. P.* engine, when the piston is 12 in. dia., and the length of crank is $\frac{3}{4}$ of the dia. of piston and the *M. E. P.* is 30 lbs.?

34. Find the *H. P.* of an engine with a 20 in. crank at 140 *R. P. M.* with cylinder 20 in. dia. and *M. E. P.* 49 lbs.

35. Find the *H. P.* per lb. *M. E. P.* with an 18 in. crank at 130 *R. P. M.* and dia. of cylinder 16 inches.

INDICATOR DIAGRAM

The *M. E. P.* on the piston of an engine can be found accurately with an indicator. By the mechanism of the instrument the exact pressure on the piston during the entire length of the stroke is obtained in diagram on a card.

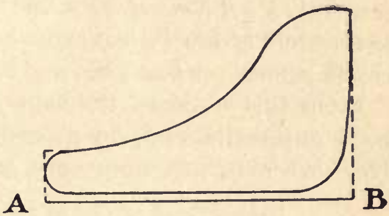


Fig 59

A sketch of an indicator card diagram from one end of cylinder is shown at Fig. 59.

To find the *M. E. P.* from the diagram, the line *AB*, which is the atmospheric line, is drawn by the pencil of the indicator when the connections with the engine are closed and each side of the piston is open to the atmosphere. Divide the length of a base line as *AB* into *any even number* of equal parts, say *10*, setting off half a part from ends *A* and *B* with *9 parts* between and from these points of division erect perpendiculars to the base line *AB* crossing the diagram at top and bottom; add together the lengths of these lines cut off by the diagram and divide by the number of lines. This will give the mean height, which multiplied by the scale of the spring used to get the diagram will give the *M. E. P.*

The area can also be found by Simpsons' rule, or more accurately by using a planimeter.

STEAM BOILER

The unit of *H. P.* for the steam boiler is derived from the number of pounds of water evaporated or converted into steam in one hour. The American Society of Mechanical

Engineers gives the following rule which has been adopted as the standard unit for *H. P.* of boilers. *1 H. P. = The evaporation per hour of 30 pounds of water from a feed water temperature of 100°F. into steam at 70 pounds gauge pressure, or 34½ pounds per hour from and at 212°.*

From this standard the capacity of a boiler, fired with good anthracite coal, to give the above evaporation per *H. P.* with maximum economy, is given by Kent as follows:

Proportions of Grate and Heating Surface

1. The *heating surface* per *H. P. = 11½ square feet.*

2. The *grate surface* per *H. P. = ⅓ square foot.*

3. The *ratio of heating to grate surface = 34½ to 1.*

4. The *water evaporated* per square foot of *heating surface* from *212°F. = 3 pounds per hour.*

5. The *combustible burned* per *H. P.* per hour = *3 pounds.*

6. The *combustible burned* per square foot of *grate surface* per hour = *9 pounds.*

7. The water evaporated at a temperature of 212°F. per pound of *combustible = 11½ pounds.*

Example. What heating and grate surface will be required for a 200 *H. P.* boiler?

Solution. The heating surface by (1) is *11½* sq. ft. per *H. P.*, then for *200 H. P.* the heating surface will be *11½ × 200 = 2,300 sq. ft.* If ⅓ *sq. ft.* of grate surface is required per *H. P.* then for *200 H. P.* it will require *200 × ⅓ sq. ft = 66⅔ sq. ft.* of grate surface.

Example. How many lbs. of good coal will be required to run a 200 *H. P.* boiler for 10 hours?

Solution. If *3 lbs.* of coal are required per *H. P.* for *1 hr.*, for *200 H. P.* it will require *200 × 3 = 600 lbs. per hr.;* and if *600 lbs.* of coal are required for *1 hr.*, for *10 hrs.* it will require *600 × 10 = 6,000 lbs.*

TO FIND HEATING SURFACE FOR VERTICAL TUBULAR BOILER

RULE. Multiply the circumference of fire box in inches by the height above the grate; multiply the combined circumference of all the tubes by the length of one tube, in inches, and to the sum of these two products add the area of the lower tube sheet, first subtracting the area of all the tubes in inches; divide the sum of these products by 144 to find the square feet of heating surface.

Example. What is the heating surface of a vertical tubular boiler having twenty-four $3\frac{1}{2}$ in. dia. and 8 ft. length of tubes, when the fire box is 16 in. high and boiler 30 in. diameter?

Solution:

By rule $c \times h = 30 \times \pi \times 16 = 1508$ *sq. in.*

and $3\frac{1}{2} \times \pi \times 24 \times 96 = 25{,}334$ *sq. in.*

Area $= 30 \times 30 \times .7854 = 707$ *sq. in.*

then $1508 + 25{,}334 + 707 = 27{,}549$ *sq. in.*

and $27{,}549 \div 144 = 191.3$ *sq. ft.*

TO FIND HEATING SURFACE FOR HORIZONTAL TUBULAR BOILER

RULE. Take all dimensions in inches. Multiply two-thirds of the circumference of the shell by its length; multiply the combined circumference of all the tubes by the lengths of one tube, to the sum of these two products add two-thirds of the area of both tube sheets and subtract twice the combined area of all the tubes; divide this remainder by 144 to find the square feet of heating surface.

Example. What is the heating surface of a horizontal tubular boiler 5 ft. dia. and 16 ft. long with fifty $2\frac{1}{2}$ in. dia. tubes?

Solution:

By rule $\dfrac{2\times C\times l}{3}=\dfrac{2\times\pi\times 60\times 192}{3}=24127.5$ *sq. in.*

then $2\frac{1}{2}\times 50\times\pi\times 192=75398.4$ *sq. in.*

$\frac{2}{3}$ area of heads minus twice the area of all the tubes

or $\dfrac{2\times 2\times 60^2\times .7854}{3}-2\times 50\times(2\frac{1}{2})^2\times .7854=$

$3769.92-490.875=3279$ *sq. in.*

Then $\dfrac{24127.5+75{,}398.4+3279}{144}=713.9$ *square feet.*

TO FIND THE HEATING SURFACE FOR ANY NUMBER OF TUBES

RULE. Multiply the number of tubes by the diameter of one tube in inches, this product by its length in feet, and this product by .2618. The final product will give the square feet of heating surface.

Example. Find the heating surface of fifty-six 3 in. dia. tubes, 18 ft. long.

Solution. By the rule, $56\times 3\times 18\times .2618=791.68$ *square feet.*

TO FIND THE WATER CAPACITY OF A BOILER

RULE. Multiply two-thirds of the area of the head in square inches, by the length of the boiler in inches, and subtract the volume of the tubes in cubic inches; divide the remainder by 231, to find the capacity in gallons.

Example. What is the water capacity of a horizontal tubular boiler 72 in. dia. and 18 ft. long with 66 tubes $3\frac{3}{4}$ in. outside diameter?

Solution:

By rule $V = \frac{2}{3}\ A \times L$—*volume of tubes.*

Then $$\frac{2 \times 4{,}071.5 \times 216}{3} - 11.045 \times 216 \times 66 =$$

$$586{,}297 - 157{,}457 = 428{,}840 \text{ cubic inches.}$$

then $$\frac{428{,}840}{231} = 1856.4 \text{ gallons.}$$

TO FIND THE STEAM CAPACITY OF A BOILER

RULE. Multiply one-third area of head in square inches by the length of the boiler in inches, and divide this product by 1728 to find capacity in cubic feet.

Example. What is the steam capacity of a horizontal tubular boiler 6 ft. dia., 18 ft. long with sixty-six $3\frac{1}{2}$ in. dia. tubes?

Solution:

By rule $$\frac{A \times l}{3} \div 1{,}728,$$

then $$\frac{4{,}071.5 \times 216}{3} \div 1{,}728 = \frac{293{,}148}{1{,}728} = 169.6 \text{ cu. ft.}$$

TO FIND THE PRESSURE CARRIED BY STAY BOLTS

RULE. Find the area of the space between any set of adjacent bolts as A, B, C, D, Fig. 60, and multiply this area in square inches by the pressure of steam gauge. This gives the pressure carried by one bolt; to find the pressure for any number of bolts, multiply by that number.

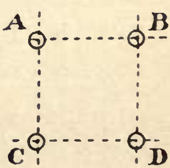


Fig. 60.

The area of bolt must be subtracted for accurate results.

TO FIND THE BURSTING STRENGTH OF A BOILER

$$B=\frac{2t\,T\,c}{d}.$$

$B=$ bursting strength of boiler.

$d=$ diameter of boiler in inches.

$t=$ thickness of plate.

$T=$ tensile strength.

$c=$ coefficient of strength of the riveted joint and is the ratio of the strength of the joint to the solid plate.

For double riveted joints $c=.7$.

For single riveted joints $c=.5$.

The safe working pressure for a boiler may be taken as follows:

$$P=\frac{2t\,T\,c}{fd}$$

$P=$ safe working pressure.

$f=$ factor of safety usually taken as 6.

The *horizontal* seams in a boiler are double riveted unless otherwise stated.

Example. Find the bursting strength and safe working pressure of a 78 in. dia. boiler 18 ft. long with $\frac{5}{8}$ in. thickness of plates tested at 62,000 lbs.

Solution. By formula:

$$B=\frac{2t\,T\,c}{d}=\frac{2\times\frac{5}{8}\times 62{,}000\times .7}{78}=\frac{54{,}250}{78}=695.5\ lbs.$$

and dividing by *6* for the safe working pressure

$$\frac{695.5}{6}=115.9\ lbs.$$

The length of a boiler is approximately *3½ times its diameter.*

PROBLEMS

1. Find the cost for coal at $3.50 per ton to run a 200 *H. P.* boiler for 20 days of 10 hrs. at the average coal supply for a well equipped boiler.

2. What is the cost of water and coal for running an economically equipped power plant of 150 *H. P.* for a year of 300 days of 10 hrs. each, when coal is $3.50 per ton and water is 8c per 100 gallons?

3. Find *H. P.* of a vertical tubular boiler 48 in. dia. having thirty $2\frac{1}{2}$ in. dia. tubes, fire box 20 in. high and tubes 8 ft. long.

4. What is the *H. P.* of a horizontal tubular boiler 72 in. dia., 18 ft. long with one hundred $3\frac{1}{2}$ in. dia. tubes?

5. What is the heating surface of a horizontal tubular boiler 20 ft. long, 5 ft. 6 in dia., with ninety $3\frac{1}{2}$ in. dia. tubes?

6. Find the heating surface of a horizontal tubular boiler, $18\frac{1}{2}$ ft. long, 5 ft. 3 in. dia., having eighty 3 in. tubes.

7. What will be the heating surface of a horizontal tubular boiler, $18\frac{1}{2}$ ft. long, 5 ft. 2 in. dia., with seventy-two 3 in. tubes?

8. Find the steam capacity for the boiler in problem 4.

9. Find the heating surface of a horizontal tubular boiler 18 ft. long, 5 ft. dia., with sixty-six 3 in. tubes.

10. What is the grate surface in sq. ft. of problem 6?

11. Find the sq. ft. of grate surface of problem 7.

12. How much more was the cost of coal per week of 56 hrs. at 4 dollars per ton of 2,000 lbs. for running a 100 *H. P.* boiler with 4,800 lbs. of coal per day of 10 hrs. than if the boiler had been operated on the economical basis?

13. If the grate area had been 24 sq. ft. in problem 12, how much should it be increased to bring it up to the economical measurement?

14. What is the water capacity for a boiler 18 ft. long, 72 in. dia., with ninety $3\frac{1}{2}$ in. tubes?

15. What is the water capacity for a boiler $21\frac{1}{2}$ ft. long, 6 ft. dia., with ninety-six $3\frac{1}{2}$ in. tubes?

16. Find the steam capacity for a boiler $16\frac{1}{2}$ ft. long with one hundred 3 in. tubes when boiler is 6 ft. diameter.

17. Find the steam space for a boiler 21 ft. long by 78 in. dia. with one hundred $3\frac{1}{2}$ in. tubes.

18. What is the bursting strain for a boiler 48 in. dia., single riveted, with a $\frac{1}{4}$ in. thickness of plate tested at 60,000 lbs. per square inches?

19. Find the bursting pressure of a 72 in. dia. boiler for $\frac{9}{16}$ in. boiler plate tested at 55,000 lbs.

20. What should be the thickness of plates for a 66 in. dia. boiler to carry 125 lbs. steam pressure with 6 for factor of safety and plates to be tested at 60,000 lbs.?

21. What is the heating surface and bursting strength of a horizontal tubular boiler $18\frac{1}{2}$ ft. long, 66 in. dia. with seventy-two 3 in. tubes, when made of $\frac{5}{8}$ in. plates tested at 65,000 lbs.?

22. Find the bursting strength of a boiler 66 in. dia. with $\frac{1}{2}$ in. plates tested at 65,000 lbs.

23. What is the bursting strength of a boiler 10 ft. dia. with $1\frac{1}{4}$ in. thickness of steel plates at 80,000 lbs. tensile strength?

24. What is the allowable pressure on a boiler $7\frac{1}{2}$ ft. dia. made of 1 in. steel plates tested at 65,000 lbs. tensile strength?

25. What thickness of plate with a test of 65,000 lbs. per sq. in. should be used for a 6 ft. dia. boiler, if 6 is the factor of safety and 100 lbs. per in. boiler pressure is to be used?

26. Find the thickness of the plate to use for a boiler 66 in. dia. to carry 80 lbs. gauge pressure when plates are tested at 65,000 lbs.

Note. In all calculations for strength of boilers it is assumed that the riveting is of right proportions and good workmanship throughout. A boiler plate is supposed to be tested at not over one-third the breaking strain of the plate. The tensile strength of a stay bolt is not over 6,000 lbs. per square in. and area is taken at the bottom dia. of thread.

Example. What pressure is carried on forty $\frac{7}{8}$ in. stay bolts that are spaced 4 in. on centers, when steam gauge shows 75 lbs.?

Solution:

Area of pressure $=4 \times 4=16$ *sq. in.*

Pressure on one bolt $=16 \times 75=1{,}200$ *lbs.*

Pressure on 40 bolts $=1{,}200 \times 40=48{,}000$ *lbs.*

Area of bolt $=.6875 \times .6875 \times .7854=.371$ *sq. in.*

Area of 40 bolts $=.371 \times 40=14.84$ *sq. in.*

$14.84 \times 75=1{,}113$ *lbs.*

Actual total pressure on 40 bolts $=48{,}000-1{,}113=46{,}887$ *lbs.*

PROBLEMS

27. What is the total pressure carried on thirty-six $\frac{7}{8}$ in. stay bolts spaced $4\frac{1}{2}$ in. between centers, (see Fig. 60,) when the steam gauge shows 87 lbs. pressure?

28. Find the pressure on forty-eight $\frac{3}{4}$ in. $\times$ 10 pi. stay bolts with U. S. S. thread when boiler pressure is 95 lbs. and area between a set of bolts is 12 square inches.

SAFETY VALVE

The lever safety valve, Fig. 61, is a lever of the third class, and calculations for lengths of arms and weights required for given boiler pressure are made from the formulas of the lever except that weights of lever and valve must be taken into account.

When the lever and valve connected to it will just balance over a knife edge, this point is called the center of gravity of the lever; the fulcrum is at the center of pivot on which the lever swings.

Then let g = the distance between center of gravity and fulcrum.

w = weight of ball in pounds.

VL = weight of valve and lever in pounds.

A = area of safety valve in square inches.

a = distance between ball and fulcrum in inches.

b = distance between center of valve and fulcrum in inches.

P = pressure per square inch on steam gauge.

The formula for weight to balance the pressure is

$$W=\frac{A\times P\times b-(VL\times g)}{a}$$

and g, A, P, a, etc., can be found by transposing terms and solving by algebra; the above formula can be written

$$W=\frac{APb-VLg}{a}$$

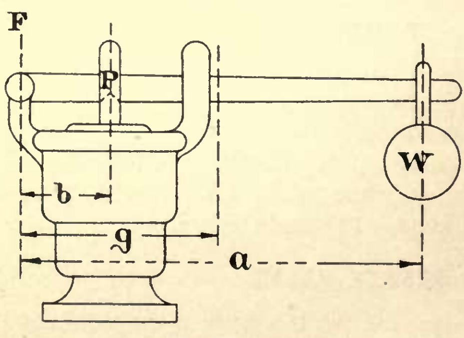


Fig 61

Example. What distance from the center of fulcrum will a weight be placed, if the steam gauge shows 95 lbs., weight is 15 lbs., area of valve 3 square in., and valve and lever weigh 18 lbs., center of valve $2\frac{1}{2}$ in. from fulcrum, $g=12$ inches?

Solution. Substituting the values given in the formula

$$15 = \frac{3 \times 95 \times 2\frac{1}{2} - 18 \times 12}{a},$$

$$\text{then } a = \frac{3 \times 95 \times 2\frac{1}{2} - 18 \times 12}{15} = 33.1 \text{ in.}$$

PROBLEMS

1. From the law of levers show how the formula for W is derived.

2. What is the weight for a ball on a lever safety valve 5 in. dia., if the ball is placed 30 in. from center of fulcrum, the center of gravity is 12 in. from fulcrum, valve and lever weigh 20 lbs., steam gauge registers 80 lbs., and centre of valve is 3 in. from fulcrum?

3. What is the dia. of a cast iron ball for safety valve, when the center of gravity is 12 in. from fulcrum, lever and valve weigh 14 lbs., distance from center of ball to fulcrum is 27 in., center of $2\frac{1}{2}$ in. dia. valve is 3 in. from fulcrum, and the steam gauge shows 85 lbs. pressure?

4. When the center of a safety valve 3.75 in. dia. is $2\frac{1}{2}$ in. from fulcrum, what distance from the fulcrum must a $6\frac{1}{2}$ in. iron ball be placed, if the valve and lever weigh 16 lbs., boiler pressure is 70 lbs., and center of gravity is 16 in. from fulcrum?

5. What weight placed 30 in. from the fulcrum of a safety valve $4\frac{1}{2}$ in. dia. will just balance 80 lbs. boiler pressure, when the valve and lever weigh 12 lbs., the center of gravity is 13 in. from fulcrum, and the center of valve is $4\frac{1}{4}$ in. from fulcrum?

HYDRAULICS

Hydraulics treats of liquids in motion, especially of the action of water in canals, pipes, machinery for raising water and the use of water as a source of power.

Pressure varies directly as the depth from the free surface. This depth from the free surface is called the *head*. If the weight of a cubic foot of water is taken as $62\frac{1}{2}$ pounds, then the weight of a column of water 1 foot high and 1 square inch in cross section $= 62.5 \div 144 = .434$ *pound*.

Therefore the pressure per square inch at any point in a body of water = the depth below the surface, or the head, $\times .434$,
then let

P = pressure per square inch.
H = head.

Then $$P = H \times .434$$

and $$H = \frac{P}{.434}.$$

To find the head when pressure is given

RULE. Divide the pressure by .434 or multiply the pressure by 2.302.

The following laws apply to liquids:

The pressure does not depend upon the size or shape of the vessel.

The pressure increases with the depth below the free surface.

At any point in a liquid the upward, downward and lateral pressures are equal.

The pressure which a body of liquid exerts on the containing vessel such as the walls of a tank, is subject to the following:

RULE. Pressure is equal to the product of the head, the area of the surface on which the liquid presses and the weight of a cubic foot of the contained liquid.

From this rule is obtained the formula:

$$P = H\ A\ W$$

Let $W = 62.5$ pounds = weight of a cubic foot of water.

A = area in square feet of surface in contact with liquid.

H = the head, which is the distance from the free surface of the liquid to the center of the surface in contact with the liquid.

Archimedes' discovery, that a solid body immersed in a liquid displaces the same volume as itself, furnishes an excellent method of finding the volume of any irregular shaped body, by immersing it in water and measuring the volume of the water displaced. The weight of the body can then be found by multiplying the volume of water displaced by the weight of a cubic inch of the substance of which the body is composed.

Example. An iron casting when immersed displaces half a gallon of water. Find the weight of casting.

Solution. A cu. in. of cast iron weighs *.2604* lb. and there are *115.5* cu. in. in a half gal., therefore

$$115.5 \times .2604 = 30.076 \text{ lbs.}$$

The *specific gravity* of a substance is its weight as compared with the weight of an equal volume of water.

PROBLEMS

1. Find the specific gravity of lead when a cu. in. weighs .4106 lb.

2. Find the specific gravity of coal when a cu. ft. weighs 57 lbs.

3. Find the specific gravity of southern pine timber when a cu. ft. weighs 60 lbs.

4. A cu. in. of a certain composition metal weighs .358 lb. What is its specific gravity?

5. What pressure will be shown on the water gauge of a boiler if the supply tank has a head of 200 feet?

6. What head will show 120 lbs. pressure on the water gauge of a receiving tank?

7. A tank that is 3 ft. square is filled to a depth of 3 ft. with water. What is the pressure of the water on the bottom of the tank?

8. What is the pressure of water on one side of tank of problem 7?

9. A tank is 12 ft. square on its base and 5 ft. high. What is the pressure of the water on one side, if the tank is filled to a depth of 5 feet?

10. When the tank of problem 9 is a closed tank and a pipe 5 in. dia. runs 5 ft. above top of tank, what is the pressure on top surface of tank when the water is filled to the top of pipe?

11. A 5 in. dia. pipe closed on lower end is sunk in a vertical position in a tank of water to a depth of 30 in. What is the upward pressure on the closed end of the pipe?

12. A sluice gate is 6 ft. high and 4 ft. wide. When closed the water is up to the top on one side and 2 ft. high on the other. What is the pressure on the gate?

13. When a cast iron ball displaces $1\frac{1}{2}$ gal. of water, what is the weight of the ball?

14. If an iron casting displaces 3 qts. of water, how much does it weigh?

15. A steel bar was sunk in a tank full of water and it was found that 15 qts. of water ran out to allow space for the bar. What was the weight of the bar?

16. An irregular shaped plate of steel was immersed in a tank of water and found to displace 6 gal. What was the weight of the plate?

17. A steel forging was found to displace 6 qts. of water. How much did it weigh?

18. How many feet of lead pipe weighing 10 lbs. per ft. in length, will displace 10 gal. of water?

HYDRAULIC MACHINES

Elevators and pumps are the most common examples of hydraulic machines used in shop practice.

The hydraulic elevator is based on the principle known as *Pascal's law* to the effect that pressure exerted upon a liquid in a containing vessel is transmitted equally and undiminished throughout the liquid.

If pipe *a*, Fig. 62, has an area of *1 sq. in.* and the pipe *b* has an area of *100 sq. in.* and water is pumped into *b* through pipe *a* at a pressure of *200 lbs. per sq. in.*, then for each *sq. in.* of surface on piston *B* there will be a pressure of *200 lbs.*, and on the whole surface of *B* there will be a pressure of *100×200 lbs.* or *20,000 lbs.*

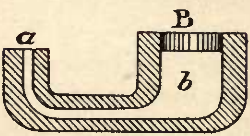


Fig. 62.

Example. A supply pipe for a 12-in. plunger hydraulic elevator piston is 1 in. area, and the pressure into supply pipe is pumped up to 150 lbs. per sq. in. What is the total pressure on bottom of piston?

Solution. Since pressure on a liquid is transmitted undiminished in all directions, a pressure of n lbs. per sq. in. on small pipe will produce the same pressure per sq. in. on large pipe at the piston, therefore

$$150 : x = 1 \text{ in.} : 113.10 \text{ in.} \qquad x = 16{,}965 \text{ lbs.}$$

Pumps are of several kinds and are operated by hand or power. The formula for lifting or forcing water either under pressure or head is as follows:

$$P = H\ A\ W,$$

where H is the distance from the level of the source of supply to the point of discharge.

Example. What is the pull on a pump rod, when dia. of bucket is 5 in. and water is raised 24 feet?

Solution. By formula:

$$P = H\ A\ W = 24 \times \frac{5^2 \times .7854}{144} \times 62.5 = 204.45 \text{ lbs.}$$

which is the pull on rod necessary to operate the pump; to this must be added the amount of power required to overcome friction in the moving parts.

The steam pump is often used to supply the feed water to the boiler. The pump in such cases is usually made with a steam piston at one end of a connecting rod and the water piston at the other end. In this case the steam piston must be enough larger in diameter than the water piston to overcome the friction of the mechanism and leakage in the valves, besides the steam pressure in boiler, against which the pump is working. From 5% to 40% is allowed according to the condition of the mechanism of pump.

TO FIND THE CAPACITY OF A PUMP PER HOUR

RULE. Find the cubical contents of the water cylinder per stroke in cubic inches, multiply by number of strokes per hour and divide the product by 231 to find the number of gallons or by 1,728 to find the capacity in cubic feet.

TO FIND THE H. P. REQUIRED TO PUMP WATER TO A GIVEN HEIGHT

RULE. Multiply the weight in pounds of water to be raised per minute by the height in feet and divide by 33,000; the quotient will be the H. P. required.

The formula is $H.\ P. = \dfrac{W \times H}{33,000}$.

Example. What is the capacity per hour of a pump with water piston 6 in. dia. and 8 in. stroke, when the piston makes 75 strokes per minute?

Solution. The contents of water cylinder, if cylinder is filled at each stroke is $A \times L = 28.274 \times 8 = 226.2$ *cu. in.*

At 75 strokes per minute there will be $75 \times 60 = 4,500$ strokes per hour.

If the piston pumps *226.2 cu. in.* per stroke then for one hour it will pump

$$226.2 \times 4,500 = 1,017,900 \text{ cu. in.},$$

and $$1,017,900 \div 231 = 4,406.4 \text{ gal. per hr.}$$

Example. Find the *H. P.* required to pump the water of above example to a height of 50 ft. above source of supply.

Solution. If a pump will raise *4,406.33 gal. of water per hour*, it will raise $4,406.33 \div 60 = 73.438$ *gal. per minute* and *73.438 gal. weighs* $73.438 \times 8\frac{1}{3} = 611.983$ *lbs.* This weight of water is to be pumped *50 ft.* high per minute; then by formula

$$H.\,P.=\frac{W\times H}{33{,}000}=\frac{611.983\times 50}{33{,}000}=\frac{611.983}{660}=.927\ H.\,P.$$

PROBLEMS

1. Find the number of gal. of water delivered per hour by a single action pump 6 in. dia. with a 12 in. stroke of the water piston at 100 strokes per minute.

2. What *H. P.* would be required to operate the pump of problem 1 if the discharge is 80 ft. above the source of supply?

3. How many gals. per hour will be delivered by a 6 in. dia. water piston with 10 in. length of stroke and making one stroke per second, if the cylinder only fills three-fourths full at each stroke?

4. If the plunger of a hydraulic elevator is 16 in. dia. and the pressure of supply pipe is 120 lbs. per sq. in., what weight can be lifted if car and plunger weigh 400 lbs. and the friction of moving parts takes 5% of the power?

5. How many cu. ft. of water per hour will a single action pump 12 in. dia., 15 in. stroke deliver at 50 double strokes per minute, when water cylinder is three-fourths full per stroke?

6. What *H. P.* will be required to raise the water of problem 5, 40 ft. above source to a delivery tank?

7. Find the amount of water in gals. that will be pumped with a 4 in. dia. single action pump with water piston moving at 100 *F. P. M.* and cylinder drawing only seven-eighths full per stroke.

8. What power will be required to pump water 200 ft. above source with a 12 in. dia. double action water cylinder moving 100 *F. P. M.?*

9. Find the power required to pump from a river to a tank at the top of a factory building 67 ft. from river to top of tank with an 8 in. dia. single action water piston moving 100 *F. P. M.?*

10. What steam pressure will be required on the steam piston of a direct acting pump to deliver water 70 ft. above source, when the steam and water pistons are both 8 in. dia. if 25% is allowed for leakage and friction in the mechanism?

11. A tank 10 ft. dia. and 10 ft. high is set on the top of a building 70 ft. above the level of a water supply from which an 8 in. dia. single action piston pump moving 100 ft. per minute is delivering to tank at 200 *F. P. M.* How long will it take to fill the tank if 30% is allowed for loss by leakage, etc.?

Note. The velocity of water through a pipe at *200 F. P. M.* requires a pipe to be of *diameter* $.35\sqrt{\textit{gal. per minute}}$. Between *100 and 200 F. P. M.*, the pipe should be of a *diameter* =

$$4.95\sqrt{\frac{\textit{gal. per minute}}{\textit{velocity in F. P. M.}}}$$

12. What size pipe will be required for delivery pipe of problem 11?

13. When water is pumped through a delivery pipe at 150 *F. P. M.*, what dia. of pipe will be required from a 12 in. dia. by 18 in. stroke piston of a single action pump making 60 strokes per minute?

14. When the piston travel is 125 *F. P. M.* what is the number of cu. ft. of water pumped per minute by a double action pump with 10 in. dia. of piston?

15. What is the *H. P.* required to run the pump of problem 14, when it delivers to a tank 72 ft. above the source?

16. Find the size of a delivery pipe required for problem 15 if it were to discharge into the tank at 150 *F. P. M.*

17. When the bucket of a hand suction pump is 3 in. dia. and the supply is drawn from a depth of 30 ft., what pressure is required on the handle 26 in. from the fulcrum when the center of bucket rod is 5 in. from the fulcrum?

18. When the piston travel of a double action steam pump is 150 *F. P. M.* and 15 in. dia., find the *H. P.* required to fill a supply tank $62\frac{1}{2}$ ft. above the source.

19. What dia. of pipe will be required to deliver the water of problem 18 at a speed of 200 *F. P. M.?*

STANDARD UNITS OF WEIGHTS AND MEASURES

Measure is that by which extension, capacity, force, duration, or value is estimated or determined.

The **weight** of a body is the measure of the force of the earth's attraction for that body, commonly called the force of gravity.

Linear or Long measure is used in measuring distances.

Table of Linear Measure

12	inches	= 1 foot (ft.).
3	feet	= 1 yard (yd.).
$5\frac{1}{2}$	yds. or $16\frac{1}{2}$ ft.	= 1 rod (rd.).
320	rods	= 1 mile.
1	mile	= 320 rds. = 1,760 yds. = 5,280 ft. = 63,360 in.

Surface measure is used in measuring in two dimensions, *length and breadth.*

Table of Square Measure

144	square inches	= 1 square foot.
9	square feet	= 1 square yard.
$30\frac{1}{4}$	sq. yds. or $272\frac{1}{4}$ sq. ft.	= 1 square rod.
160	square rods	= 1 acre.
640	acres	= 1 square mile.
1	sq. rd.	= $30\frac{1}{4}$ sq. yds. = $272\frac{1}{4}$ sq. ft. = 39,204 sq. in.

Pressures of liquids and gases are usually given in pounds per square inch or square foot.

Cubic measure is used in measuring the *volume of solids.*

Table of Solid or Cubic Measure

1,728 cubic inches	= 1 cubic foot.
27 cubic feet	= 1 cubic yard.
1 cu. yd.	= 27 cu. ft. = 46,656 cu. in.

The measure of *volume for liquids.*

Table of Liquid Measure

4 gills	= 1 pint.
2 pints	= 1 quart.
4 quarts	= 1 gallon.
$31\frac{1}{2}$ gallons	= 1 barrel.

Liquids are sometimes measured with cubic measure, the U. S. gal. contains 231 cu. in. so that a cu. ft. of water contains approximately $7\frac{1}{2}$ gallons.

One gallon of water taken at the temperature of maximum density, 39.1° F., weighs 8.3389 lbs., avoirdupois, which is approximately $8\frac{1}{3}$ lbs.

A cubic foot of water weighs 62.355 lbs., approximately $62\frac{1}{2}$ lbs. Water freezes at 32° F. or 0° C., and boils at 212° F (Fahrenheit) or 100° C. (Centigrade).

The standard unit for weight in shop practice is the *avoirdupois pound.*

Table of Avoirdupois Weight

16 ounces	= 1 pound.
100 pounds	= 1 hundred weight (cwt.).
20 cwt.	= 1 ton.
1 ton	= 20 cwt. = 2,000 lbs. = 32,000 oz.

Time is the measure of duration.

Table of Measure for Time

60 seconds = 1 minute.
60 minutes = 1 hour.
24 hours = 1 day.
7 days = 1 week.
365 days = 1 year.
366 days on leap years.

Angular measure is used in measuring angles.

Table of Angular Measure

60 seconds (60″) = 1 minute (1′).
60 minutes (60′) = 1 degree (1°).
360 degrees (360°) = 1 circumference.

Money is the measure for values. It is used as a medium of exchange.

Table of U. S. Money

10 mills = 1 cent.
10 cents = 1 dime.
10 dimes = 1 dollar.
10 dollars = 1 eagle.
1 eagle = $10 = 100 dimes = 1,000 cents = 10,000 mills.

MISCELLANEOUS MEASURES

Table

12 units = 1 dozen.
12 dozen = 1 gross.
12 gross = 1 great gross.
1 gross = 12 doz. = 144 units.

Weights of Materials

1 cu. in. of cast iron	weighs .2604 lb.	melts at 2,500° F.
1 cu. in. of wr't. iron	weighs .2779 lb.	melts at 3,100° F.
1 cu. in. of steel	weighs .2833 lb.	melts at 3,000° F.
1 cu. in. of copper	weighs .3195 lb.	melts at 1,930° F.
1 cu. in. of brass { cop. 65 / zinc 35 }	weighs .3029 lb.	melts at
1 cu. in. of lead	weighs .4106 lb.	melts at 625° F.
1 cu. in. of aluminum	weighs .0963 lb.	melts at 1,160° F.

The *approximate* weight of round bar iron or steel can be found by the formula

$$L=\frac{(d\times 4)^2}{6},$$

where L = weight of one foot in length of the bar
and d = diameter of the bar in inches.

Horse Power

The *horse power* is an expression for foot-pounds of useful work accomplished in a specified time. *1 H. P.=33,000 pounds* raised *1 foot* in one minute, or *1 pound* raised *33,000 feet* in one minute.

Electric Units

The *volt* is the unit of electrical pressure.

The *ampere* is the unit of current strength or rate of flow.

The *ohm* is the unit of resistance.

The *watt* is the unit of power.

The *ohm, ampere* and *volt* are defined in terms of one another as follows:

Ohm, the resistance of a conductor through which a current of *one ampere* will pass when the electro-motive force is *one volt.*

Ampere, the quantity of current which will flow through a resistance of *one ohm* when the electro-motive force is *one volt.*

Volt, the electro-motive force required to cause a current of *one ampere* to flow through a resistance of *one ohm.*

The relation which these quantities bear to one another is expressed by *Ohm's Law.*

$$Current\ in\ amperes = \frac{E\ M\ F\ in\ volts}{Resistance\ in\ ohms}.$$

Then $\quad C = \dfrac{E}{R}.$

where $\quad E =$ the electro-motive force in volts,
$R =$ resistance in ohms,
$C =$ current in amperes.

The electric $H.\ P. = 746$ *watts.*

METRIC SYSTEM OF MEASURES

The *metric* system is used in almost all the European countries and is authorized by law in the United States and is in general use for all scientific treatises.

Metric Linear Measure

10 millimeters (mm.)	= 1 centimeter (cm.).
10 centimeters	= 1 decimeter (dm.).
10 decimeters	= 1 meter (m.).
10 meters	= 1 dekameter (Dm.).
10 dekameters	= 1 hektometer (Hm.).
10 hektometers	= 1 kilometer (Km.).
10 kilometers	= 1 myriameter (Mm.).

1 Mm. = 10 Km. = 100 Hm. = 1,000 Dm. = 10,000 meters.
1 m. = 10 dm. = 100 cm. = 1,000 mm. = 39.37 inches.

The *unit* for metric linear measure is the *meter.*

Metric Square Measure

100 sq. millimeters	= 1 sq. centimeter (sq. cm.).
100 sq. centimeters	= 1 sq. decimeter (sq. dm.)
100 sq. decimeters	= 1 sq. meter (sq. m.).
100 sq. meters	= 1 sq. dekameter (sq. Dm.).
100 sq. dekameters	= 1 sq. hektometer (sq. Hm.).
100 sq. hektometers	= 1 sq. kilometer (sq. Km.).

The *unit* for metric square measure is the *square meter* for small surfaces, and the *are* for land measurements.

100 centares = 1 are   1 sq. meter = 1.196 sq. yard
100 ares = 1 hectare

Metric Cubic Measure

1,000 cu. millimeters	= 1 cu. centimeter (cu. cm.).
1,000 cu. centimeters	= 1 cu. decimeter (cu. dm.).
1,000 cu. decimeters	= 1 cu. meter (cu. M.).

The *unit* for cubic measure is the *cubic meter* and *1 cu. m. = 1.308 cu. yard.*

Metric Measure of Capacity

10 milliliters	= 1 centiliter (cl.).
10 centiliters	= 1 deciliter (dl.).
10 deciliters	= 1 liter (l.).
10 liters	= 1 dekaliter (Dl.).
10 dekaliters	= 1 hektoliter (Hl.).
10 hektoliters	= 1 kiloliter (Kl.).

The *unit* for capacity for both liquid and dry measure is the *liter*.

1 liter = .908 of a dry quart. 1 liter = 1.0567 of a liquid quart.

Metric Measure of Weight

10 milligrams	= 1 centigram (cg.).
10 centigrams	= 1 decigram (dg.).
10 decigrams	= 1 gram (g.).
10 grams	= 1 dekagram (Dg.).
10 dekagrams	= 1 hektogram (Hg.).
10 hektograms	= 1 kilogram (Kg.).
10 kilograms	= 1 myriagram (Mg.).
10 myriagrams	= 1 quintal (Q.).
10 quintals	= 1 tonneau (T.).

The *unit* of weight is the *gram.* 1 gram = .03527 oz. Avoirdupois.

EQUIVALENTS OF COMMON AND METRIC SYSTEMS

Linear Measure

1 inch	= 2.54 cm.	1 cm.	= 0.3937 in.
1 foot	= 0.3048 m.	1 dm.	= 0.328 ft.
1 yard	= 0.9144 m.	1 m.	= 1.0936 yds.
1 rod	= 5.029 m.	1 Dm.	= 1.9884 rds.
1 mile	= 1.6093 Km.	1 Km.	= 0.62137 mile

Square Measure

1 sq. in.	= 6.452 sq. cm.	1 sq. cm.	= 0.155 sq. in.
1 sq. ft.	= 0.0929 sq. m.	1 sq. dm.	= 0.1076 sq. ft.
1 sq. yd.	= 0.8361 sq. m.	1 sq. m.	= 1.196 sq. yds.
1 sq. rd.	= 25.293 sq. m.	1 are	= 3.954 sq. rds.
1 acre	= 40.47 ares	1 hectare	= 2.471 acres
1 sq. mile	= 259 hectares	1 sq. Km.	= 0.3861 sq. mile

Cubic Measure

1 cu. in. = 16.387 cu. cm.	1 cu. cm. = 0.061 cu. in.
1 cu. ft. = 28.317 cu. dm.	1 cu. dm. = 0.0353 cu. ft.
1 cu. yd. = 0.7645 cu. m.	1 cu. m. = 1.308 cu. yds.
1 cord = 3.624 steres	1 stere = 0.2759 cord

Measures of Capacity

1 liquid qt. = 0.9463 liters	1 liter = 1.0567 liquid qts.
1 dry qt. = 1.101 liters	1 liter = 0.908 dry qts.
1 liquid gal. = 0.3785 Dl.	1 Dl. = 2.6417 liquid gals.
1 peck = 0.881 Dl.	1 Dl. = 1.135 pecks
1 bushel = 0.3524 Hl.	1 Hl. = 2.8375 bushels

Measures of Weight

1 grain Troy = 0.0648 gram.	1 g. = 0.03527 oz. Avoir.
1 oz. Avoir. = 28.35 g.	1 g. = 0.03215 oz. Troy
1 oz. Troy = 31.104 g.	1 g. = 15.432 grains Troy
1 lb. Avoir. = 0.4536 Kg.	1 Kg. = 2.2046 lbs. Avoir.
1 lb. Troy = 0.3732 Kg.	1 Kg. = 2.679 lbs. Troy

1 ton (2000 lbs.) = 0.9072 tonneau,

1 tonneau = 1.1023 tons (2000 lbs.)

DECIMAL EQUIVALENTS OF PARTS OF AN INCH

1/64	.01563	33/64	.51563
1/32	.03125	17/32	.53125
3/64	.04688	35/64	.54688
1/16	.0625	9/16	.5625
5/64	.07813	37/64	.57813
3/32	.09375	19/32	.59375
7/64	.10938	39/64	.60938
1/8	.125	5/8	.625
9/64	.14063	41/64	.64063
5/32	.15625	21/32	.65625
11/64	.17188	43/64	.67188
3/16	.1875	11/16	.6875
13/64	.20313	45/64	.70313
7/32	.21875	23/32	.71875
15/64	.23438	47/64	.73438
1/4	.25	3/4	.75
17/64	.26563	49/64	.76563
9/32	.28125	25/32	.78125
19/64	.29688	51/64	.79688
5/16	.3125	13/16	.8125
21/64	.32813	53/64	.82813
11/32	.34375	27/32	.84375
23/64	.35938	55/64	.85938
3/8	.375	7/8	.875
25/64	.39063	57/64	.89063
13/32	.40625	29/32	.90625
27/64	.42188	59/64	.92188
7/16	.4375	15/16	.9375
29/64	.45313	61/64	.95313
15/32	.46875	31/32	.96875
31/64	.48438	63/64	.98438
1/2	.5		
		1	1.00000

USE OF FORMULAS

A *formula*, as used in technical books and papers, may be defined as a rule in which letters and symbols are used in place of numbers or words. A formula, then, is a short way of expressing a rule, and is more convenient, as it shows at a glance the operations to be performed.

For example, the *volume* of a rectangular solid is equal to the product of its three dimensions, *length*, *width*, *and height.*

The same statement would be expressed by the formula, $V = LWH$.

The signs of addition, subtraction, multiplication and division are used in formulas in the same way as in arithmetic except that the sign of multiplication between two letters is usually omitted; as $V = LWH$ means $V = L \times W \times H$.

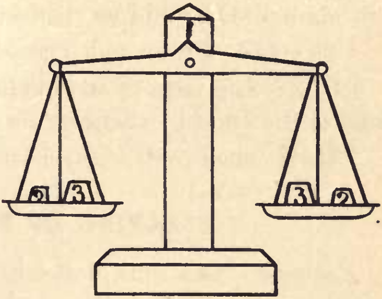
In working out or solving a formula, the numerical values of all but one of the letters must usually be known. These values are substituted for their respective letters and the value of the remaining letter may then be easily found.

Example. In the formula $S = \frac{1}{2}\ gt^2$. Let $g = 32$, $t = 5$. Find S.

Solution. By formula, $S = \frac{1}{2}$ of $32 \times 5^2 = 16 \times 25 = 400$.

Every formula is in reality an *algebraic equation*, and a knowledge of some of the simpler principles of equations will be of great assistance in working out formulas.

The equation may be illustrated by reference to a scale pan with equal arms. If equal weights, say five pounds, are put in each pan, they will just balance. If two pounds is



added to one pan, two pounds must be added to the other pan, and if three pounds is taken from one pan, three pounds must also be taken from the other or the pans will not balance.

The same is true of the equation; whatever operation is performed on one side, the same operation must be performed on the other side.

The following operations may be performed on an equation without changing its value.

I *The same number may be added to each side.*
II *The same number may be subtracted from each side.*
III *Each side may be multiplied by the same number.*
IV *Each side may be divided by the same number.*
V *The same root may be taken of each side.*
VI *Each side may be raised to the same power.*

TRANSPOSING TERMS

Example. Five added to five times a certain number is equal to 15. What is the number?

Solution. Let x = the unknown number. Then the statement of the equation will be $5x+5=15$.

But since by (II) 5 may be taken from each side, the statement then becomes, $5x=10$.

By (IV) $x=2$.

In the above example the statement:

$5x+5=15$ could be changed to read as follows:

$5x=15-5$ from which could be obtained the following.

RULE: Any term of an equation can be moved from one side to the other by changing its sign.

This is called *transposing* a term.

CLEARING OF FRACTIONS

Example. The sum of the half and the fifth parts of a number is 7. What is the number?

Solution. Let x = the unknown number.

Then the statement will be as follows:

$$\frac{x}{2}+\frac{x}{5}=7.$$

By (III) it is possible to multiply both sides of the equation by the same number, in this case 10, which is the *L. C. M.* of 2 and 5.

Then the equation becomes:

$$5x+2x=70$$
$$7x=70$$

By (IV) $\quad x=10$

To prove that the result is correct, the value found for x when substituted in the original equation should make both sides identical.

Then $\frac{10}{2}+\frac{10}{5}=7$, or $7=7$.

This is called *checking the problem.*

A *coefficient* is any factor of the rest of the term; as, in ab, a is the coefficient of b and b is the coefficient of a, but in an algebraic formula the coefficient is usually understood to mean the *numerical* coefficient of the given term; as, *5* is the coefficient of x in the expression $5x$.

Terms that differ only in their numerical coefficients are called *similar* and may be added or subtracted by finding the sum or difference of their coefficients and affixing to the result the common letters.

Examples. $7a+3a+6a=16a.$

$2xm+12xm-8xm=6xm.$

$8abc-3abc=5abc.$

The law for precedence of signs. When the signs $+$, $-$, $\times$, and $\div$ occur in the same formula, the operations of multiplication and division must be performed before those of addition and subtraction; as,

$$9\times3-6\div2=27-3=24.$$

The law of signs for addition. In adding similar terms having $+$ and $-$ signs, subtract the less number from the greater (arithmetically) and prefix the sign of the greater number; as, $5x+(-8x)=-3x$.

The law of signs for subtraction. Change the sign of every term in the subtrahend and then proceed as in addition; as, $5x-(-8x)=5x+(+8x)=13x$.

The law of signs for multiplication. The product of two terms of like signs has the plus sign; the product of two terms of unlike signs has the minus sign; as,

$$(+a)\times(+b)=ab.$$
$$(+a)\times(-b)=-ab.$$
$$(-a)\times(-b)=ab.$$
$$(-a)\times(+b)=-ab.$$

An exponent denotes the number of times a quantity is used as a factor; as, $a\times a\times a=a^3$.

Then multiplying a^2 by a^3 means, $(a\times a)\times(a\times a\times a)$ or a used as a factor five times. The result is written a^5.

From the above follows:

The law for exponents in multiplication. The exponent of a letter in the product is equal to the sum of its exponents in the factors.

The law of signs for division. If the numbers to be divided have the same sign, the sign of the quotient will be plus; if the numbers to be divided have unlike signs the sign of the quotient will be minus; as,

$$6ab \div 2ab = 3$$
$$8x \div (-2) = -4x$$
$$-12b \div (-4b) = 3.$$

The law for exponents in division. The exponent of a letter in the quotient is equal to its exponent in the dividend minus its exponent in the divisor.

Example. Divide a^4 by a^2.

Solution. $\frac{a^4}{a^2} = \frac{a\times a\times a\times a}{a\times a}$. Two factors of the dividend will be canceled by the two factors of the divisor and the quotient $= a^2$, or $a^4 \div a^2 = a^{4-2} = a^2$.

PROBLEMS INVOLVING USE OF FORMULAS

Example. Find the value of V in the formula $V=L\,W\,H$. When $L=20$, $W=15$ and $H=5$.

Solution. Substituting the known values for the letters L, W, and H in the formula:

Then $V=20\times15\times5=1500$.

Example. What is the value for H in the formula $V=L\,W\,H$, when $V=3000$, $L=45$ and $W=20$?

Solution. Substituting the known values for the letters V, L, and W in the formula:

Then $3000=45\times20\times H$.

The quantities L, W and H are factors of the quantity V. Therefore the values of one of the factors of V must be the quotient obtained by dividing V by the product of the other factors;

Then by transposing, $H=\dfrac{V}{L\times W}=\dfrac{3000}{45\times20}=3\frac{1}{3}$.

In like manner any other term can be found, as,

$$L=\frac{V}{WH} \text{ and } W=\frac{V}{L\,H}$$

1. $C=\dfrac{E}{R}$. Find E when $C=10$ and $R=5$.

2. $\dfrac{F}{D}=\dfrac{L}{10}$. Find F when $D=10$ and $L=20$.

3. $V=\dfrac{H}{3}\times B$. Find B when $V=216$ and $H=6$.

4. $\dfrac{P}{W}=\dfrac{L}{C}$. Find C when $P=6$, $W=96$ and $L=7$.

5. $P \times a = W \times b$. Find the value of each letter in terms of the other letters.

6. $Sin\ A = \frac{a}{c}$. Find a in terms of c and $sin\ A$. Also find c in terms of a and $sin\ A$.

7. $Tan\ A = \frac{a}{b}$. Find tan A when $a = 3$ and $b = 6$.

8. $C = \frac{5}{9}(F^\circ - 32^\circ)$. Find F° in terms of C°.

Note. In the solution of this problem the result could be obtained by first taking $\frac{5}{9}$ of F° and from this product subtracting $\frac{5}{9}$ of 32°; but the parenthesis indicates that the enclosed terms are to be treated as one; 32° should first be subtracted from F° and then $\frac{5}{9}$ of the difference should be found.

From the formula $C^\circ = \frac{5\ (F^\circ - 32^\circ)}{9}$ a formula for F° may be derived as follows:

Clearing of fractions,	$9C^\circ = 5\ (F^\circ - 32^\circ)$
Removing parenthesis,	$9C^\circ = 5F^\circ - 160^\circ$
Transposing,	$5F^\circ = 9C^\circ + 160^\circ$
Dividing by the coefficient of the unknown term,	$F^\circ = \frac{9C^\circ}{5} + 32^\circ$

The operation of removing the parenthesis from a formula should be performed first and the resulting number treated according to the signs of operation that affect the whole parenthesis; thus, $\frac{2}{3}\ (3+5)^2 = \frac{2}{3}\ (8)^2 = \frac{2}{3}\ (64) = 42\frac{2}{3}$.

Again $6a - (3b - 2c + 1) = 6a - 3b + 2c - 1$.

Also, $6a + (3b - 2c + 1) = 6a + 3b - 2c + 1$.

From the above it follows: that

To remove a parenthesis. When the plus sign precedes it, the signs of all the terms within the parenthesis remain unchanged. But if the minus sign precedes, the sign of every term within the parenthesis must be changed; this is but an illustration of the law of signs in subtraction where the signs of all the terms in the subtrahend are changed before adding.

When a parenthesis or other sign of union occurs within a parenthesis, *the inner ones are usually removed first;* as,

$$5a-[-(3a+4a)-(11a+9a)]=5a-[-3a-4a-11a-9a]=$$
$$5a+3a+4a+11a+9a=32a.$$

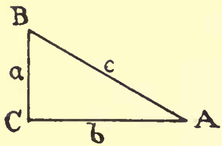
9. Change 77° F to the equivalent Centigrade reading.

10. Change 18° F to the equivalent Centigrade reading.

11. Change 10° C to the equivalent Fahrenheit reading.

12. Change $-10°$ C to the equivalent Fahrenheit reading.

SIMPLE TRIGONOMETRIC FUNCTIONS



The right triangle ABC has six parts or elements; three sides a, b, c, and three angles A, B, and C. When three of these elements are given one of the known parts being a side the three unknown parts can be determined by computation.

The process of finding the unknown parts is called *solving* the triangle.

The *ratios* of the sides of the right triangle are called *trigonometric functions* and are named as follows:

The *sine* of an angle is the ratio of the side opposite the angle to the hypotenuse, as $sin\ A = \frac{a}{c}$, $sin\ B = \frac{b}{c}$.

The *cosine* of an angle is the ratio of the adjacent side to the hypotenuse, as $cos\ A = \frac{b}{c}$, $cos\ B = \frac{a}{c}$.

The *tangent* of an angle is the ratio of the side opposite to the side adjacent, as $tan\ A = \frac{a}{b}$, $tan\ B = \frac{b}{a}$.

The *cotangent* of an angle is the ratio of the adjacent side to the opposite side, as $cot\ A = \frac{b}{a}$, $cot\ B = \frac{a}{b}$.

The *secant* of an angle is the ratio of the hypotenuse to the adjacent side, as $sec\ A = \frac{c}{b}$, $sec\ B = \frac{c}{a}$.

The *cosecant* of an angle is the ratio of the hypotenuse to the opposite side, as $csc\ A = \frac{c}{a}$, $csc\ B = \frac{c}{b}$.

The following arrangement of the functions of an angle will be found convenient, and with the tables of natural functions given on pages 189 to 193 furnish all the data necessary for the solution of right triangles.

(1). Sine $= \dfrac{\text{side opposite}}{\text{hypotenuse}}$

(2). Side opposite $= \text{hypotenuse} \times \text{sine}$

(3). Cosine $= \dfrac{\text{side adjacent}}{\text{hypotenuse}}$

(4). Side adjacent $= \text{hypotenuse} \times \text{cosine}$

(5). Tangent $= \dfrac{\text{side opposite}}{\text{side adjacent}}$

(6). Side opposite $= \text{side adjacent} \times \text{tangent}$

(7). Cotangent $= \dfrac{\text{side adjacent}}{\text{side opposite}}$

(8). Side adjacent $= \text{cotangent} \times \text{side opposite}$

(9). Hypotenuse $= \dfrac{\text{side opposite}}{\text{sine}}$

(10). Hypotenuse $= \dfrac{\text{side adjacent}}{\text{cosine}}$

How to Use the Tables

Example. If $a=47$ ft., and $c=63$ ft., find angle A.

Solution. By proposition (1) $\sin A = \dfrac{a}{c} = \dfrac{47}{63} = .74603$. Refer to page 193 in column marked sines at bottom on right, find .74606 just above the figures 48 in the column marked D for degrees and on the same line with figures 15 in column marked M for minutes. It is evident that the angle A is very close to $48° \; 15'$, as the next smaller angle given in the table is $48° \; 0'$ for which the *sine* is *.74314* and as the machinists' protractor has *quarter degrees* for the smallest graduations the nearest approximation is taken to the size of the angle as found in the table, that is *48° 15′*.

Example. Find *cosine* of angle A when $A = 35° 30'$.

Solution. On page 192 in column headed D on left side, follow down the column until the figure *35* is reached, in the next column headed M on the same line is *o*, under this is *15* and under *15* is *30* which is the angle required, that is, *35° 30′*. On this line at the right under column headed *cosines* (at top) will be found the decimal *.81412* which is the *cosine of angle 35° 30′*.

Note. For angles between 0° and 45° the names of the functions are found at the *top* of the page and the angles at the *left;* for all angles between 45° and 90° the names of the functions are found at the *bottom* of the page and the angles at the *right.*

NATURAL TRIGONOMETRICAL FUNCTIONS

D	M	Sines	Cosines	Tangents	Cotangents	Secants	Cosecants		
0	0	.00000	1.0000	.00000	Infinite	1.0000	Infinite	90	0
	15	.00436	.99999	.00436	229.182	1.0000	229.18		45
	30	.00873	.99996	.00873	114.589	1.0000	114.59		30
	45	.01309	.99991	.01309	76.3900	1.0001	76.397		15
1	0	.01745	.99985	.01746	57.2900	1.0001	57.299	89	0
	15	.02181	.99976	.02182	45.8294	1.0002	45.840		45
	30	.02618	.99966	.02619	38.1885	1.0003	38.202		30
	45	.03054	.99953	.03055	32.7303	1.0005	32.746		15
2	0	.03490	.99939	.03492	28.6363	1.0006	28.654	88	0
	15	.03926	.99923	.03929	25.4517	1.0008	25.471		45
	30	.04362	.99905	.04366	22.9038	1.0009	22.926		30
	45	.04798	.99885	.04803	20.8188	1.0011	20.843		15
3	0	.05234	.99863	.05241	19.0811	1.0014	19.107	87	0
	15	.05669	.99839	.05678	17.6106	1.0016	17.639		45
	30	.06105	.99813	.06116	16.3499	1.0019	16.380		30
	45	.06540	.99786	.06554	15.2571	1.0021	15.290		15
4	0	.06976	.99756	.06993	14.3007	1.0024	14.336	86	0
	15	.07411	.99725	.07431	13.4566	1.0028	13.494		45
	30	.07846	.99692	.07870	12.7062	1.0031	12.745		30
	45	.08281	.99657	.08309	12.0346	1.0034	12.076		15
5	0	.08716	.99619	.08749	11.4301	1.0038	11.474	85	0
	15	.09150	.99580	.09189	10.8829	1.0042	10.929		45
	30	.09585	.99540	.09629	10.3854	1.0046	10.433		30
	45	.10019	.99497	.10069	9.93101	1.0051	9.9812		15
6	0	.10453	.99452	.10510	9.51436	1.0055	9.5668	84	0
	15	.10887	.99406	.10952	9.13093	1.0060	9.1855		45
	30	.11320	.99357	.11394	8.77689	1.0065	8.8337		30
	45	.11754	.99307	.11836	8.44896	1.0070	8.5079		15
7	0	.12187	.99255	.12278	8.14435	1.0075	8.2055	83	0
	15	.12620	.99200	.12722	7.86064	1.0081	7.9240		45
	30	.13053	.99144	.13165	7.59575	1.0086	7.6613		30
	45	.13485	.99087	.13609	7.34786	1.0092	7.4156		15
8	0	.13917	.99027	.14054	7.11537	1.0098	7.1853	82	0
	15	.14349	.98965	.14499	6.89688	1.0105	6.9690		45
	30	.14781	.98902	.14945	6.69116	1.0111	6.7655		30
	45	.15212	.98836	.15391	6.49710	1.0118	6.5736		15
9	0	.15643	.98769	.15838	6.31375	1.0125	6.3924	81	0
	15	.16074	.98700	.16286	6.14023	1.0132	6.2211		45
	30	.16505	.98629	.16734	5.97576	1.0139	6.0589		30
	45	.16935	.98556	.17183	5.81966	1.0147	5.9049		15
10	0	.17365	.98481	.17633	5.67128	1.0154	5.7588	80	0
		Cosines	Sines	Cotangents	Tangents	Cosecants	Secants	D	M

From 80° to 90° read from bottom of table upwards.

NATURAL TRIGONOMETRICAL FUNCTIONS

D	M	Sines	Cosines	Tangents	Cotangents	Secants	Cosecants		
10	0	.17365	.98481	.17633	5.67128	1.0154	5.7588	80	0
	15	.17794	.98404	.18083	5.53007	1.0162	5.6198		45
	30	.18224	.98325	.18534	5.39552	1.0170	5.4874		30
	45	.18652	.98245	.18986	5.26715	1.0179	5.3612		15
11	0	.19081	.98163	.19438	5.14455	1.0187	5.2408	79	0
	15	.19509	.98079	.19891	5.02734	1.0196	5.1258		45
	30	.19937	.97992	.20345	4.91516	1.0205	5.0158		30
	45	.20364	.97905	.20800	4.80769	1.0214	4.9106		15
12	0	.20791	.97815	.21256	4.70463	1.0223	4.8097	78	0
	15	.21218	.97723	.21712	4.60572	1.0233	4.7130		45
	30	.21644	.97630	.22169	4.51071	1.0243	4.6202		30
	45	.22070	.97534	.22628	4.41936	1.0253	4.5311		15
13	0	.22495	.97437	.23087	4.33148	1.0263	4.4454	77	0
	15	.22920	.97338	.23547	4.24685	1.0273	4.3630		45
	30	.23345	.97237	.24008	4.16530	1.0284	4.2837		30
	45	.23769	.97134	.24470	4.08666	1.0295	4.2072		15
14	0	.24192	.97030	.24933	4.01078	1.0306	4.1336	76	0
	15	.24615	.96923	.25397	3.93751	1.0317	4.0625		45
	30	.25038	.96815	.25862	3.86671	1.0329	3.9939		30
	45	.25460	.96705	.26328	3.79827	1.0341	3.9277		15
15	0	.25882	.96593	.26795	3.73205	1.0353	3.8637	75	0
	15	.26303	.96479	.27263	3.66796	1.0365	3.8018		45
	30	.26724	.96363	.27732	3.60588	1.0377	3.7420		30
	45	.27144	.96246	.28203	3.54573	1.0390	3.6840		15
16	0	.27564	.96126	.28675	3.48741	1.0403	3.6280	74	0
	15	.27983	.96005	.29147	3.43084	1.0416	3.5736		45
	30	.28402	.95882	.29621	3.37594	1.0429	3.5209		30
	45	.28820	.95757	.30097	3.32264	1.0443	3.4699		15
17	0	.29237	.95630	.30573	3.27085	1.0457	3.4203	73	0
	15	.29654	.95502	.31051	3.22053	1.0471	3.3722		45
	30	.30071	.95372	.31530	3.17159	1.0485	3.3255		30
	45	.30486	.95240	.32010	3.12400	1.0500	3.2801		15
18	0	.30902	.95106	.32492	3.07768	1.0515	3.2361	72	0
	15	.31316	.94970	.32975	3.03260	1.0530	3.1932		45
	30	.31730	.94832	.33460	2.99868	1.0545	3.1515		30
	45	.32144	.94693	.33945	2.94591	1.0560	3.1110		15
19	0	.32557	.94552	.34433	2.90421	1.0576	3.0715	71	0
	15	.32969	.94409	.34922	2.86356	1.0592	3.0331		45
	30	.33381	.94264	.35412	2.82391	1.0608	2.9957		30
	45	.33792	.94118	.35904	2.78523	1.0625	2.9593		15
20	0	.34202	.93969	.36397	2.74748	1.0642	2.9238	70	0
		Cosines	Sines	Cotangents	Tangents	Cosecants	Secants	D	M

From 70° to 80° read from bottom of table upwards.

NATURAL TRIGONOMETRICAL FUNCTIONS

D	M	Sines	Cosines	Tangents	Cotangents	Secants	Cosecants		
20	0	.34202	.93969	.36397	2.74748	1.0642	2.9238	70	0
	15	.34612	.93819	.36892	2.71062	1.0659	2.8892		45
	30	.35021	.93667	.37388	2.67462	1.0676	2.8554		30
	45	.35429	.93514	.37887	2.63945	1.0694	2.8225		15
21	0	.35837	.93358	.38386	2.60509	1.0711	2.7904	69	0
	15	.36244	.93201	.38888	2.57150	1.0729	2.7591		45
	30	.36650	.93042	.39391	2.53865	1.0748	2.7285		30
	45	.37056	.92881	.39896	2.50652	1.0766	2.6986		15
22	0	.37461	.92718	.40403	2.47509	1.0785	2.6695	68	0
	15	.37865	.92554	.40911	2.44433	1.0804	2.6410		45
	30	.38268	.92388	.41421	2.41421	1.0824	2.6131		30
	45	.38671	.92220	.41933	2.38473	1.0844	2.5859		15
23	0	.39073	.92050	.42447	2.35585	1.0864	2.5593	67	0
	15	.39474	.91879	.42963	2.32756	1.0884	2.5333		45
	30	.39875	.91706	.43481	2.29984	1.0904	2.5078		30
	45	.40275	.91531	.44001	2.27267	1.0925	2.4829		15
24	0	.40674	.91355	.44523	2.24604	1.0946	2.4586	66	0
	15	.41072	.91176	.45047	2.21992	1.0968	2.4348		45
	30	.41469	.90996	.45573	2.19430	1.0989	2.4114		30
	45	.41866	.90814	.46101	2.16917	1.1011	2.3886		15
25	0	.42262	.90631	.46631	2.14451	1.1034	2.3662	65	0
	15	.42657	.90446	.47163	2.12030	1.1056	2.3443		45
	30	.43051	.90259	.47698	2.09654	1.1079	2.3228		30
	45	.43445	.90070	.48234	2.07321	1.1102	2.3018		15
26	0	.43837	.89879	.48773	2.05030	1.1126	2.2812	64	0
	15	.44229	.89687	.49315	2.02780	1.1150	2.2610		45
	30	.44620	.89493	.49858	2.00569	1.1174	2.2412		30
	45	.45010	.89298	.50404	1.98396	1.1198	2.2217		15
27	0	.45399	.89101	.50953	1.96261	1.1223	2.2027	63	0
	15	.45787	.88902	.51503	1.94162	1.1248	2.1840		45
	30	.46175	.88701	.52057	1.92098	1.1274	2.1657		30
	45	.46561	.88499	.52613	1.90069	1.1300	2.1477		15
28	0	.46947	.88295	.53171	1.88073	1.1326	2.1300	62	0
	15	.47332	.88089	.53732	1.86109	1.1352	2.1127		45
	30	.47716	.87882	.54296	1.84177	1.1379	2.0957		30
	45	.48099	.87673	.54862	1.82276	1.1406	2.0790		15
29	0	.48481	.87462	.55431	1.80405	1.1433	2.0627	61	0
	15	.48862	.87250	.56003	1.78563	1.1461	2.0466		45
	30	.49242	.87036	.56577	1.76749	1.1490	2.0308		30
	45	.49622	.86820	.57155	1.74964	1.1518	2.0152		15
30	0	.50000	.86603	.57735	1.73205	1.1547	2.0000	60	0
		Cosines	Sines	Cotangents	Tangents	Cosecants	Secants	D	M

From 60° to 70° read from bottom of table upwards.

NATURAL TRIGONOMETRICAL FUNCTIONS

D	M	Sines	Cosines	Tangents	Cotangents	Secants	Cosecants		
30	0	.50000	.86603	.57735	1.73205	1.1547	2.0000	60	0
	15	.50377	.86384	.58318	1.71473	1.1576	1.9850		45
	30	.50754	.86163	.58904	1.69766	1.1606	1.9703		30
	45	.51129	.85941	.59494	1.68085	1.1636	1.9558		15
31	0	.51504	.85717	.60086	1.66428	1.1666	1.9416	59	0
	15	.51877	.85491	.60681	1.64795	1.1697	1.9276		45
	30	.52250	.85264	.61280	1.63185	1.1728	1.9139		30
	45	.52621	.85035	.61882	1.61598	1.1760	1.9004		15
32	0	.52992	.84805	.62487	1.60033	1.1792	1.8871	58	0
	15	.53361	.84573	.63095	1.58490	1.1824	1.8740		45
	30	.53730	.84339	.63707	1.56969	1.1857	1.8612		30
	45	.54097	.84104	.64322	1.55467	1.1890	1.8485		15
33	0	.54464	.83867	.64941	1.53986	1.1924	1.8361	57	0
	15	.54829	.83629	.65563	1.52525	1.1958	1.8238		45
	30	.55194	.83389	.66188	1.51084	1.1992	1.8118		30
	45	.55557	.83147	.66818	1.49661	1.2027	1.7999		15
34	0	.55919	.82904	.67451	1.48256	1.2062	1.7883	56	0
	15	.56280	.82659	.68087	1.46870	1.2098	1.7768		45
	30	.56641	.82413	.68728	1.45501	1.2134	1.7655		30
	45	.57000	.82165	.69372	1.44149	1.2171	1.7544		15
35	0	.57358	.81915	.70021	1.42815	1.2208	1.7434	55	0
	15	.57715	.81664	.70673	1.41497	1.2245	1.7327		45
	30	.58070	.81412	.71329	1.40195	1.2283	1.7220		30
	45	.58425	.81157	.71990	1.38909	1.2322	1.7116		15
36	0	.58779	.80902	.72654	1.37638	1.2361	1.7013	54	0
	15	.59131	.80644	.73323	1.36383	1.2400	1.6912		45
	30	.59482	.80386	.73996	1.35142	1.2440	1.6812		30
	45	.59832	.80125	.74673	1.33916	1.2480	1.6713		15
37	0	.60181	.79864	.75355	1.32704	1.2521	1.6616	53	0
	15	.60529	.79600	.76042	1.31507	1.2563	1.6521		45
	30	.60876	.79335	.76733	1.30323	1.2605	1.6427		30
	45	.61222	.79069	.77428	1.29152	1.2647	1.6334		15
38	0	.61566	.78801	.78129	1.27994	1.2690	1.6243	52	0
	15	.61909	.78532	.78834	1.26849	1.2734	1.6153		45
	30	.62251	.78261	.79543	1.25717	1.2778	1.6064		30
	45	.62592	.77988	.80258	1.24597	1.2822	1.5976		15
39	0	.62932	.77715	.80978	1.23490	1.2868	1.5890	51	0
	15	.63271	.77439	.81703	1.22394	1.2913	1.5805		45
	30	.63608	.77162	.82434	1.21310	1.2960	1.5721		30
	45	.63944	.76884	.83169	1.20237	1.3007	1.5639		15
40	0	.64279	.76604	.83910	1.19175	1.3054	1.5557	50	0
		Cosines	Sines	Cotangents	Tangents	Cosecants	Secants	D	M

From 50° to 60° read from bottom of table upwards.

NATURAL TRIGONOMETRICAL FUNCTIONS

	M	Sines	Cosines	Tangents	Cotangents	Secants	Cosecants		
40	0	.64279	.76604	.83910	1.19175	1.3054	1.5557	50	0
	15	.64612	.76323	.84656	1.18125	1.3102	1.5477		45
	30	.64945	.76041	.85408	1.17085	1.3151	1.5398		30
	45	.65276	.75756	.86165	1.16056	1.3200	1.5320		15
41	0	.65606	.75471	.86929	1.15037	1.3250	1.5242	49	0
	15	.65935	.75184	.87698	1.14028	1.3301	1.5166		45
	30	.66262	.74896	.88472	1.13029	1.3352	1.5092		30
	45	.66588	.74606	.89253	1.12041	1.3404	1.5018		15
42	0	.66913	.74314	.90040	1.11061	1.3456	1.4945	48	0
	15	.67237	.74022	.90834	1.10091	1.3509	1.4873		45
	30	.67559	.73728	.91633	1.09131	1.3563	1.4802		30
	45	.67880	.73432	.92439	1.08179	1.3618	1.4732		15
43	0	.68200	.73135	.93251	1.07237	1.3673	1.4663	47	0
	15	.68518	.72837	.94071	1.06303	1.3729	1.4595		45
	30	.68835	.72537	.94896	1.05378	1.3786	1.4527		30
	45	.69151	.72236	.95729	1.04461	1.3843	1.4461		15
44	0	.69466	.71934	.96569	1.03553	1.3902	1.4396	46	0
	15	.69779	.71630	.97416	1.02653	1.3961	1.4331		45
	30	.70091	.71325	.98270	1.01761	1.4020	1.4267		30
	45	.70401	.71019	.99131	1.00876	1.4081	1.4204		15
45	0	.70711	.70711	1.00000	1.00000	1.4142	1.4142	45	0
		Cosines	Sines	Cotangents	Tangents	Cosecants	Secants	D	M

From 45° to 50° read from bottom of table upwards.

FORMULAS

ANSWERS

Mechanics, Page 3

1. 1,200 ft.-lbs.
2. 7,500 ft.-lbs.
3. 80,000 ft.-lbs.
4. 2,083,350 ft.-lbs.
5. 3,150 ft.-lbs.
6. 630 ft.-lbs.
7. 18,000 ft.-lbs.
8. $7\frac{1}{2}$ ft.-lbs.
9. 2,800 ft.-lbs.
10. 139.9 ft.-lbs.
11. 10.52 H. P.
12. 75.76 H. P.
13. 1.06 H. P.
14. 1.35 H. P.
15. 140,000 ft.-lbs.
16. 7,500,000 ft.-lbs.
17. $\begin{cases} V = 400 \text{ ft. per sec.} \\ K = 30{,}000 \text{ ft.-lbs.} \end{cases}$
18. 14.75 H. P.
19. 22.12 H. P.
20. 10.05 H. P.
21. 18.43 H. P.
22. 200.38 H. P.

Miscellaneous Problems, Page 16

1. 125 lbs.
2. 825 lbs.
3. 900 lbs.
4. $5\frac{1}{6}$ in.
5. 25.117 in.
6. 1,024 lbs.
7. $164\frac{1}{3}$ lbs.
8. 264.49 lbs.
9. 1,925.86 lbs.
10. 1,417.25 lbs.
11. 2,115.94 lbs.
12. 8,258.96 lbs.
13. 0.0004 in.
14. 0.0442 in.
15. $373\frac{1}{3}$ lbs.
16. $3{,}937\frac{1}{2}$ lbs.
17. 1,200 lbs.
18. 4,500 lbs.
19. 25 in.
20. $222\frac{2}{9}$ lbs.
21. 30 lbs.
22. 0.026 dia. circle
23. $12\frac{1}{2}$ in.
24. 4.8 in.
25. $27\frac{7}{9}$ in.
26. 125 lbs.

27. 1.26 in.
28. $1\frac{1}{8}$ in.
29. $83\frac{1}{3}$ lbs.
30. $390\frac{5}{8}$ lbs.
31. $\frac{8}{25}$ ft.
32. $15\frac{5}{8}$
33. 24,000 R. P. M.
34. 25 in. dia.
35. $\frac{4}{75}$ rev.
36. $\frac{2}{15}$ rev.
37. $\frac{288}{3125}$ rev.
38. 72
39. $63\frac{3}{4}$
40. 84
41. $68\frac{4}{7}$
42. $93\frac{1}{3}$
43. $81\frac{5}{7}$
44. $69\frac{1}{3}$
45. $82\frac{2}{7}$
46. 4 sheaves
47. 75 lbs.
48. 120 lbs.
49. 1,020 lbs.
50. 1,105 lbs.
51. 92.2%
52. 150 lbs.
53. 4 men
54. 10 in. and $9\frac{3}{4}$ in.

Problems, Page 27

1. 118.477 lbs.
2. 464.84 lbs.
3. 1,118.3 lbs.
4. 45,239.04 lbs.
5. 71.62 lbs.
6. 150.8 lbs.
7. $16\frac{2}{3}$ lbs.
8. 5,000 lbs.
9. 750 lbs.
10. $333\frac{1}{3}$ lbs.
11. 29.47 lbs.
12. $\left.\begin{cases} 2\frac{1}{7} \text{ ft.} \\ 2\frac{6}{7} \text{ ft.} \end{cases}\right\}$ 3 to 4
13. 5
14. 9
15. $437\frac{1}{2}$ lbs.
16. 6.631 in.
17. 7.957 in.
18. 644.33 lbs.
19. 900.68 lbs.

Screw Threads, Page 30

1. 0.0722 in.
2. 0.0481 in.
3. 0.0542 in.
4. 0.1164 in.
5. 0.8375 in.
6. 0.1634 in.
7. 0.3933 in.
8. 0.0139 in.

9. 0.5913 in.
10. 0.5068 in.
11. 0.622 in.
12. 0.3668 in.
13. 0.0813 in.
14. 0.0866 in.
15. 16 pitch double
17. 12 pitch triple

Gears, Page 34

1. 0.3927 in.
2. 0.5236 in.
3. 4 pitch
4. 10 pitch
5. $18\frac{1}{3}$ in.
6. 4 pitch
7. 108 teeth
8. 75 teeth
9. 0.3927 in.
10. 0.3925 in.
11. 0.5392 in.
12. 0.3595 in.
13. $2\frac{1}{8}$ in.
14. 3 in.
15. 4 pitch
16. 128 teeth
17. 640 teeth
18. 200 teeth
19. $\begin{cases}\text{8 in.}\\ \text{16 in.}\end{cases}$
20. $\begin{cases}\text{8 in.}\\ \text{16 in.}\end{cases}$
21. $\begin{cases}\text{21 in.}\\ \text{35 in.}\end{cases}$
22. $\begin{cases}\text{24 in.}\\ \text{30 in.}\end{cases}$
23. $\begin{cases}\text{32 in.}\\ \text{40 in.}\end{cases}$
24. $\begin{cases}\text{24 in.}\\ \text{36 in.}\end{cases}$
25. 0.0393 in.
26. 0.0098 in.
27. 18 in.
28. 6.6759 in.
29. $4\frac{1}{2}$ in.
30. 10 in.
31. 21.008 R. P. M.
32. $4166\frac{2}{3}$ R. P. M.
33. $\begin{cases}\text{132 F. P. M.}\\ \text{157.08 F. P. M.}\end{cases}$
34. 125 lbs.
35. $166\frac{2}{3}$
36. 13.368 R. P. M.
37. 50.93
38. $31\frac{1}{4}$ lbs.
39. 176.84 R. P. M.

Bevel Gears, Page 40

1.
- N $\begin{cases} 40 \\ 60 \end{cases}$
- D $\begin{cases} 6\frac{2}{3} \text{ in.} \\ 10 \text{ in.} \end{cases}$
- O D $\begin{cases} 6.944 \text{ in.} \\ 10.185 \text{ in.} \end{cases}$
- Cutting Angles Index head $\begin{cases} 31° \ 51' \\ 54° \ 29' \end{cases}$
- Angles C_1 and C $\begin{cases} 33° \ 41' \\ 56° \ 19' \end{cases}$
- Increment 1° 35′
- Decrement 1° 50′
- Center face angles of blank $\begin{cases} 70° \ 32' \\ 115° \ 48' \end{cases}$

2.
- N $\begin{cases} 96 \\ 120 \end{cases}$
- D $\begin{cases} 12 \\ 15 \end{cases}$
- Cutting Angles to set index head $\begin{cases} 37° \ 49' \\ 50° \ 29' \end{cases}$
- Angles C_1 and C $\begin{cases} 38° \ 40' \\ 51° \ 20' \end{cases}$
- Increment 0° 45′
- Decrement 0° 51′
- Center face angles of blank $\begin{cases} 77° \ 4' \\ 104° \ 30' \end{cases}$

3.
- N 40
- D 4 in.
- O D 4.1414 in.
- Cutting angle to set head 42° 45′
- Angle C 45°
- Center face angle of blank 94° 2′
- Increment 2° 1′
- Decrement 2° 15′

4.
- N $\begin{cases} 32 \\ 54 \end{cases}$
- D $\begin{cases} 5 \text{ in.} \\ 6\frac{2}{3} \text{ in.} \end{cases}$
- O D $\begin{cases} 4.234 \text{ in.} \\ 6.795 \text{ in.} \end{cases}$
- Cutting angles $\begin{cases} 28° \ 43' \\ 56° \ 47' \end{cases}$
- Angle C_1 and C $\begin{cases} 30° \ 58' \\ 59° \ 2' \end{cases}$
- Center face angles blank $\begin{cases} 65° \ 37' \\ 121° \ 45' \end{cases}$
- Decrement 2° 15′
- Increment 1° $50\frac{1}{2}'$

5.
- N: 30 / 40
- D: 5 in. / $6\frac{2}{3}$ in.
- O D: 5.267 in. / 6.867 in.
- Cutting angles: 34° 7′ / 50° 23′
- Angles C_1 and C: 36° 52′ / 53° 8′
- Center face angles of blank: 78° 20′ / 110° 52′
- Increment 2° 18′
- Decrement 2° 45′

6.
- N: 120 / 150
- D: 15 in. / $18\frac{3}{4}$ in.
- O D: 15.195 in. / 18.906 in.
- Cutting angles: 38° 0′ / 50° 40′
- Angles C_1 and C: 38° 40′ / 51° 20′
- Center face angles of blank: 78° 30′ / 103° 50′
- Increment 0° 35′
- Decrement 0° 40′

Worm Wheels, Page 42

1. 18,000 lbs.
2. $11\frac{1}{9}$ in.
3. $4\frac{4}{9}$ in.
4. 47,250 lbs.
5. 189,000 lbs.

Helical Gears, Page 46

1. D 3.677 in. / O D 4.077 in.
2. D 1.414 in. / O D 1.614 in.
3. D 2.121 in. / O D 2.371 in.
4. Centers 2.357 in. / O D 2.524 in.
5. 1.414 in.
6. No teeth 11 and 22 / D for both gears 1.754

Pulleys, Page 49

1. 2,040 R. P. M.
2. 12,600 R. P. M.
3. 0.167 in.
4. $25\frac{5}{7}$ in.
5. 40 in.
6. 3.825 in.
7. 8.325 in.
8. 11.156 in.

9. t 0.205 in.; c 0.125 in.; T 0.535 in.
10. 3.287 in.
11. b 13.82 in.; c 0.250 in.; s 0.375 in.
11. N 6; h 4.332 in.; e 1.733 in.; t 0.5625 in.; T 1.375 in.; D_1 8 in.; L 10.667

Miscellaneous Problems, Page 49

1. lead 11.107 in.; D 3.535 in.; O D 3.635 in.
2. lead 13.329 in.; D 4.243 in.; O D 4.443 in.
3. lead 8.650 in.; D 2.946 in.; O D 3.113 in.
4. 16,000 lbs.
5. 20,000 lbs.
6. 64,000 lbs.
7. 75,398.4 lbs.
8. 150,796.8 lbs.
9. 75,398.4 lbs.
10. 628,320 lbs.
11. 6.615 lbs.
12. 203,575.68 lbs.
13. 339,292.8 lbs.
14. 73.68 lbs.
15. 63,617.4 lbs.
16. 49.7 lbs.
17. 18.65 lbs.
18. 848,232 lbs.
19. 37.30 lbs.

Friction, Page 55

1. 2.842 H. P.
2. 13.271 H. P.

Miscellaneous Problems, Page 58

1. 488.39 lbs.
2. 541.18 lbs.
3. 474.27 lbs.
4. 0.125
5. 0.08
6. 120 lbs.
7. 853.85 lbs.
8. 584.96 lbs.

9. 5.6 H. P.
10. $148\frac{3}{4}$ lbs.
11. 542.87 lbs.
12. 92.67 lbs.
13. 79.09 lbs.
14. 209.5 lbs.
15. 2,282.95 lbs.
16. 16.76 lbs.
17. 33.36 lbs.
18. 8,948.02 lbs.
19. 1,300.52 lbs.
20. 1.7 H. P.
21. 8.606 H. P.
22. 543.235 ft.-lbs. per min.
23. 360 ft.-lbs. per min.

Belting, Page 64

1. 100 H. P.
2. 81.92 H. P.
3. 96.77 H. P.
4. 34.996 ft.
5. 43.764 ft.
6. $17\frac{1}{2}$ H. P.
7. 20.36 H. P.
8. 125.61 H. P.
9. 91.48 H. P.
10. 139.58 lbs.
11. 69.9 ft.
12. 41.5 ft.
13. 49.3 ft.
14. 49.25
15. 24.3 in.
16. 192 H. P.
17. 163.84 H. P.
18. 6,600 lbs.
19. 756.3 lbs.
20. 8.25 lbs.
21. 247.5 lbs.
22. $66\frac{2}{3}$ in.
23. 71.42 H. P.
24. 408.23 H. P.
25. 54.1 H. P.
26. 112.64 H. P.
27. 4.4 in.
28. 43.4 in.
29. $62\frac{1}{2}$ in.
30. 20 + oz.
31. 26 + oz.
32. 258 R. P. M.
33. 95 R. P. M.
34. 111.5 in. dia.
35. 52 in.
36. 30 oz.
37. 34 oz.
38. 15.8 oz.

Rope Drives, Page 68

1. $187\frac{1}{2}$ lbs.
2. 0.675 lbs.
3. 5,301.45 lbs.
4. 2,650.725 lbs.
5. 7,215.86 lbs.
6. 14.73 H. P.
7. 105.53 lbs.
8. 1,100 lbs.
9. 42.11 H. P.
10. 23.11 H. P.; 205+ R. P. M.; 65 in.
11. 1,804 lbs.

Wire Cables, Page 70

1. 1,005 lbs.; 158.34 H. P.
2. 43.53 H. P.
3. 399 lbs.; 54 H. P.
4. 205.19 H. P.
5. 2,873.15 F. P. M.
6. 7.41 ft. dia.; 1,110 ft.

Chains, Page 72

1. P D 2.565 in.; O D 2.890 in.
2. O D 6.699 in.; B D 6.049 in.
3. A 0.9 in.; B 0.6 in.; b 0.4875 in.
4. P D 9.561 in.; O D 10.049 in.
5. B D 10.974 in.; P D 11.461 in.; O D 11.949 in.
6. A 1.05 in.; B 0.7 in.; b 0.569 in.
7. B D 15.038 in.; P D 15.607 in.; O D 16.176 in.
8. P D 8.917 in.; O D 9.242 in.
9. 9.229 in.

Shafting, Page 74

1. $H=\frac{d^3N}{80}$, $N=\frac{H\times 80}{d^3}$
2. 120 H. P.
3. 39 H. P.
4. 429.7 H. P.
5. 6.4 in.
6. $6\frac{1}{2}$ in.
7. 46 + R. P. M.
8. 4.3 in.
9. 192 + R. P. M.
10. $1\frac{1}{4}$ in.
11. 5.4 in.
12. 13.05 in.
13. $4\frac{1}{4}$ in.
14. 736 H. P.
15. 405 H. P.
16. 3,645 H. P.
17. 31 + R. P. M.
18. 592 + R. P. M.
19. 308 + R. P. M.

Jack Shaft, Page 76

1. 4.93 in.
2. 3.92 in.
3. 277.8 R. P. M.
4. 1,312.5 R. P. M.
5. 450 H. P.
6. $16\frac{2}{3}$ H. P.
7. 22.14 H. P.
8. 7.1 in.
9. 10 ft.
10. 10 ft.

Journal Bearings, Page 79

1. 1.226 in.
2. 1.462 in.
3. 1.087 in.
4. 7.306 in.
5. 1.515 in.
6. 2.915 in.
7. 9.367 in.
8. 8.352 in.
9. 10.283 in.
10. 11.124 in.
11. 4 in.
12. 13.991 in.
13. 15.223 in.
14. 7.111 in.
15. 2.507 in.
16. 11.567 in.
17. 12.219 in.
18. 3 pulleys
19. 10.727 in.
20. 16.121 in.
21. 5 pulleys
22. 8.67 in.

23. 20 in.
24. 14.29 in.
25. 1.82 in.
26. 0.17 in.
27. $\frac{1}{4}$ in.
28. 10 balls
29. 1.633 in.
30. 8 balls
31. .126 in.
32. $\frac{1}{2}$ in.
33. .343 in.
34. .109 in.

Machine Keys, Page 82

1. $\begin{cases} \text{b } 1\frac{1}{2} \text{ \& } 1\frac{1}{4} \text{ in.} \\ \text{t } 1 \text{ \& .833 in.} \end{cases}$
2. $\begin{cases} \text{b } \frac{1}{2} \text{ in.} \\ \text{t .333 in.} \end{cases}$
3. $\begin{cases} \text{b 2 in.} \\ \text{t } 1\frac{1}{3} \text{ in.} \end{cases}$
4. b = t = $\frac{1}{4}$ in.

Micrometer, Page 88

1. $\frac{3}{25}$ in.
2. $\frac{1}{1600}$ in.
3. $\frac{1}{600}$ in.
4. $\frac{1}{5}$ in.
5. $\frac{2}{5}$
6. $\frac{5}{6}$
7. 250 divisions
8. $\frac{1}{100}$ in.
9. $\frac{8}{15}$
10. 50 divisions
11. 75 divisions

Lathe Work, Page 92

1. 97 + R. P. M.
2. 173 + R. P. M.
3. 103 + R. P. M.
4. 571 + R. P. M.
5. 82 + R. P. M.
6. 529 + R. P. M.

Back Gears, Page 94

1. 12 + R. P. M.
2. 32 + R. P. M.
3. 27 + R. P. M.
4. 15 + R. P. M.
5. 13 + R. P. M.
6. 19 + R. P. M.

Screw Cutting, Page 97

1. Ratio of $\frac{5}{11\frac{1}{2}}$
2. 60 teeth
3. 25 teeth
4. 72 teeth
5. 66 teeth
6. 48 teeth
7. 24 teeth
8. 64 teeth
9. Ratio of $\frac{6}{5}$
10. Ratio of $\frac{1}{6}$

Taper Turning, Page 102

1. $7\frac{1}{5}$ in.
2. $1\frac{1}{8}$ in. per ft.
3. $2\frac{5}{14}$ in. per ft.
4. $\frac{7}{32}$ in. per ft.
5. $\frac{9}{11}$ in. per ft.
6. 0.64 in. per ft.
7. 0.603 in. per ft.
8. 0.706 in. per ft.
9. 0.6 in. per ft.
10. $3\frac{1}{8}$ in.
11. 0.411 in.
12. 1.44 in.
13. 0.283 in.
14. 0.833 in.
15. 0.419 in.
16. 0.102 in.
17. 0.609 in. per ft.

Miscellaneous Lathe Problems, Page 105

1. 30 R. P. M.
2. 80 R. P. M.
3. 16 R. P. M.
4. 426 + R. P. M.
5. 32 R. P. M.
6. 40 R. P. M.
7. 46 + R. P. M.
8. 256 R. P. M.
9. 80 R. P. M.
10. 19 + R. P. M.
11. 10 R. P. M.
12. 61 + R. P. M.
13. 16 R. P. M.
14. 95 + R. P. M.
15. 466 + R. P. M.
16. 34 R. P. M.
17. 32 R. P. M.
18. 13.716 + in.

19. $\begin{cases} 16.266 \text{ in.} \\ 15.533 \text{ in. approx.} \end{cases}$

20. $\begin{cases} 26.297 \text{ in.} \\ 25.112 \text{ in. approx.} \end{cases}$

21. 10.412 in.

22. 0.513 in.

23. 0.389 in.

24. 114 + R. P. M.

25. 8 R. P. M.

Planer, Page 109

1. 17.864 F. P. M.
2. 46.584 F. P. M.
3. 22.317 F. P. M.
4. 16.788 F. P. M.
5. 29 + min.
6. 54 + min.
7. 34 + min.
8. $\frac{16}{5}$
9. $\begin{cases} 24 \text{ in. dia.} \\ 18 \text{ in. dia.} \end{cases}$

Milling Machine, Page 115

Simple Indexing

1. $13\frac{1}{3}$ turns
2. $1\frac{1}{4}$
3. 10
4. $1\frac{2}{3}$
5. $1\frac{1}{7}$
6. $\frac{8}{9}$
7. $\frac{5}{9}$
8. $\frac{10}{17}$
9. $\frac{20}{41}$
10. $\frac{10}{27}$ turn
11. $\frac{10}{29}$
12. $\frac{10}{37}$
13. $\frac{10}{41}$
14. $\frac{1}{4}$
15. $\frac{8}{33}$
16. $\frac{1}{5}$
17. $\frac{5}{27}$
18. $\frac{5}{39}$

Compound Indexing, Page 118

20. $\frac{30}{21} - \frac{30}{33}$ or
    $\frac{9}{21} + \frac{3}{33}$
21. $\frac{6}{39} + \frac{14}{49}$
22. $\frac{3}{18} + \frac{5}{20}$ or
    $\frac{10}{15} - \frac{5}{20}$
23. $\frac{6}{27} + \frac{6}{33}$ or
    $\frac{15}{27} - \frac{5}{33}$
24. $\frac{13}{39} - \frac{3}{49}$ or
    $\frac{15}{45} - \frac{3}{49}$
25. $\frac{8}{21} - \frac{4}{33}$ or
    $\frac{15}{21} - \frac{15}{33}$
26. $\frac{11}{33} - \frac{3}{29}$
27. $\frac{3}{39} + \frac{7}{49}$
28. $\frac{17}{31} - \frac{11}{33}$

Differential Indexing

So many combinations possible that answers are omitted.

The Spiral Head, Page 125

1. Ratio of $\frac{4}{3}$
2. Ratio of $\frac{14}{5}$
3. Ratio of $\frac{9}{4}$
4. Ratio of $\frac{21}{8}$
5. Ratio of $\frac{24}{5}$
6. 9.524 in.
7. 36 in.
8. $2\frac{1}{2}$ in.
9. 21° 26′
10. 27° 38′
11. 34° 30′
12. 18° 36′
13. 13° 15′
14. 16.327 in.
15. 31.416 in.
16. $\begin{cases} 18° \ 58' \\ 6 \text{ min.} \end{cases}$
17. 41 + R. P. M.
18. 21 + R. P. M.
19. 40 R. P. M.
20. 80 R. P. M.
21. 192 R. P. M.
22. 128 R. P. M.
23. 80 R. P. M.
24. 86 + R. P. M.
25. 18.54 F. P. M.
26. $\begin{cases} 50.4 \text{ F. P. M.} \\ 35+ \text{ R. P. M.} \end{cases}$
27. 0.617 in.
28. 0.278 in.
29. 1.867 in.

Drill Press, Page 129

1. $\begin{cases} 293 \text{ R. P. M.} \\ 41 \text{ sec.} \end{cases}$
2. 0.64 in.
3. 57 sec.
4. 47 sec.
5. $\begin{cases} 52+ \text{ sec.} \\ 125+ \text{ R. P. M.} \end{cases}$
6. $\begin{cases} 1 \text{ min. } 22 \text{ sec.} \\ 220 \text{ R. P. M.} \end{cases}$
7. 4 min. 12 sec.
8. 4 min. 4 sec.
9. 4 min.

Hammer Blow, Page 131

1. 288,000 lbs.
2. 329,143 lbs.; 384,000 lbs.; 460,800 lbs.
3. 276,000 lbs.
4. 326,400 lbs.
5. 20,000 lbs.
6. 288,000 lbs.
7. 10,000 lbs.
8. 20,000 lbs.
9. 13,125 lbs.

Horse Power of Machines, Page 133

1. 0.589 H. P.
2. 0.929 H. P.
3. 0.524 H. P.
4. 0.220 H. P.
5. 0.23 H. P.
6. 1.805 H. P.
7. 1.106 H. P.
8. 0.413 H. P.
9. 0.743 H. P.
10. 0.551 H. P.
11. 0.506 H. P.
12. 1.875 H. P.
13. 2.142 H. P.
14. 2.076 H. P.
15. .785 H. P.
16. .880 H. P.
17. 1.276 H. P.
18. 2.856 H. P.

Dynamometers, Page 136

1. 281 + R. P. M.
2. 18.75 H. P.
3. 25 H. P.
4. 2.801 ft.
5. 50 lbs.

Fly Wheels, Page 139

1. 19,432.7 lbs.
2. 414.6 lbs.
3. 13,063 + lbs.
4. 7,124 + lbs.
5. 14,563 + lbs.
6. 9,167 + lbs.
7. 77,676 + lbs.
8. 3,607 + lbs.
9. 9,770 + lbs.
10. 210 + R. P. M.
11. 480 + R. P. M.
12. 136 + R. P. M.
13. 5,654.88 F. P. M.
14. 940.368 F. P. M.

Horse Power of Engines, Page 142

1. 263.2 H. P.
2. 215.6 H. P.
3. 177.4 H. P.
4. 719.7 H. P.
5. 0.476 H. P. per lb. M. E. P.
6. 26.33 H. P. per lb. M. E. P.
7. 1.37 H. P. per lb. M. E. P.
8. 0.8675 H. P. per lb. M. E. P.
9. 423.1 H. P.
10. 415.9 H. P.
11. 15,402 lbs.
12. 15,360 lb. 552.7 H. P. at 1 mile per min.
13. 10.95 lbs. M. E. P.
14. 24.3 lbs. M. E. P.
15. 2.67 + H. P.
16. $5\frac{1}{3}$ H. P.
17. $11\frac{1}{2}$ H. P.
18. 33.47 H. P.
19. 52 + H. P.
20. $22\frac{1}{2}$ H. P.
21. 25.9 H. P.
22. 6 + H. P.
23. 320.8 H. P.
24. 9.996 H. P. per lb. M. E. P.
25. 18,488.89 lbs.
26. 19,413.33 lbs.
27. 18.838 in.
28. 4.318 H. P. per lb. M. E. P.
29. 10.927 in.
30. 17.019 in.
31. 10.249 in.
32. 9.577 in.
33. 8 in. crank 182 + R. P. M.
34. 435.4 H. P.
35. 4.752 H. P. per lb. M. E. P.

Steam Boiler, Page 155

1. $210
2. $3,852.90
3. 16.48 H. P.
4. 165.2 H. P.
5. 1899.4 sq. ft.
6. 1386.8 sq. ft.
7. 1267.2 sq. ft.
8. 169.65 cu. ft.
9. 1141.2 sq. ft.
10. 40.2 H. P.

11. 36.7 H. P.
12. $20.16
13. $9\frac{1}{3}$ sq. ft.
14. 1,728 gals.
15. 2,000 gals.
16. 174.36 cu. ft.
17. 232.3
18. $312\frac{1}{2}$ lbs.
19. 601.5 lbs.
20. .589 in.
21. $\begin{cases} 861 + \text{lbs.} \\ 1283 + \text{sq. ft.} \end{cases}$
22. 689 lbs.
23. 1,167 lbs.
24. 168 lbs.
25. .474 in.
26. .348 in.
27. 62,261 lbs.
28. 53,342 lbs.

Safety Valve, Page 159

2. 149.08 lbs.
3. 6.652 + in.
4. 45 in.
5. $175\frac{1}{2}$ lbs.

Hydraulic Machines, Page 161

1. 11.38
2. .914
3. .962
4. 9.92
5. 86.8 lbs.
6. 276.5 ft.
7. 1,687.5 lbs.
8. $843\frac{3}{4}$ lbs.
9. 9,375 lbs.
10. 44,957.4 lbs.
11. 21.3 lbs.
12. 4,000 lbs.
13. 90.228 lbs.
14. 14.114 lbs.
15. 245.41 lbs.
16. 392.65 lbs.
17. 98.16 lbs.
18. 94.85 ft.

Steam Pumps, Page 166

1. 8,812.68 gals.
2. 1.48 H. P.
3. 3,304.8 gals.
4. 22,521 lbs.
5. 2,208.9 cu. ft.
6. 2.79 H. P.

7. 3,427.2
8. 29.75 H. P.
9. 2.22 H. P.
10. 37.975 lbs. per sq. in.
11. 64 min. 37 sec.
12. 3.346 in.
13. 6.632 in.
14. 68.177 cu. ft.
15. 9.29 H. P.
16. 10 in. nearly
17. 17.7 lbs.
18. 21.8 H. P.
19. 12.988 in.

Formulas, Page 183

1. 50
2. 20
3. 108
4. 112
5. $\begin{cases} a=\dfrac{W\times b}{P};\ P=\dfrac{W\times b}{a} \\ b=\dfrac{P\times a}{W};\ W=\dfrac{P\times a}{b} \end{cases}$
6. $c=\dfrac{a}{\sin A},\ a=c\times\sin A$
7. $\frac{1}{2}$ or .5
9. 25° C
10. $-7\frac{7}{9}$° C
11. 50° F
12. 14° F

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