INDUSTRIAL SERIES

SHOP MATHEMATICS

PART I

SHOP ARITHMETIC

PREPARED IN THE
EXTENSION DIVISION OF
THE UNIVERSITY OF WISCONSIN

BY

EARLE B. NORRIS, M. E.
ASSOCIATE PROFESSOR OF MECHANICAL ENGINEERING, IN CHARGE OF MECHANICAL ENGINEERING COURSES IN THE UNIVERSITY EXTENSION DIVISION

AND

KENNETH G. SMITH, A. B., B. S.
ASSOCIATE PROFESSOR OF MECHANICAL ENGINEERING, DISTRICT REPRESENTATIVE IN CHARGE OF FIRST DISTRICT, THE UNIVERSITY EXTENSION DIVISION, MILWAUKEE

FIRST EDITION
SECOND IMPRESSION

McGRAW-HILL BOOK COMPANY
239 WEST 39TH STREET, NEW YORK
6 BOUVERIE STREET, LONDON, E. C.
1912

THE · MAPLE · PRESS · YORK · PA

PREFACE

The aim of this book is to teach the fundamental principles of mathematics to shop men, using familiar terms and processes, and giving such applications to shop problems as will maintain the interest of the student and develop in him an ability to apply the mathematical and scientific principles to his every day problems of the shop. The problems and applications relate largely to the metal working trades. It has, however, been the aim in preparing this volume not to apply the work to these particular trades so closely but that it shall be of interest and value to men in other lines of industry.

This volume presents the first half of the instruction papers in Shop Mathematics as developed and used by the Extension Division of the University of Wisconsin. As here offered, it embodies the point of view obtained through apprenticeship and shop experience as well as the experience gained through its use during the past four years as a text for both correspondence and class room instruction. It is believed that the book will be found suitable for home study and for use as a text in trade, industrial, and continuation schools.

The instruction in arithmetic ends with Chapter XII. The remaining chapters are introduced to give further practice in calculation and to develop an ability to handle simple formulas, as well as to impart a knowledge of the principles of machines. The second volume will take up more fully the use of formulas and will teach the principles of geometry and trigonometry as applied to shop work.

The authors are indebted to Mr. F. D. Crawshaw, Professor of Manual Arts in The University of Wisconsin, for a careful reading of the proof and for valuable criticisms and suggestions.

E. B. N.

THE UNIVERSITY OF WISCONSIN,
MADISON, WIS.
June 1, 1912.

CONTENTS

CHAPTER I

Common Fractions

CHAPTER II

Addition and Subtraction of Fractions

CHAPTER III

Multiplication and Division of Fractions

CHAPTER IV

Money and Wages

CHAPTER IX

Pulley and Gear Trains; Change Gears

CHAPTER X

Areas and Volumes of Simple Figures

CHAPTER XI

Square Root

CHAPTER XII

Mathematical Tables (Circles, Powers, and Roots)

CHAPTER XIII

Levers

CHAPTER XIV

Tackle Blocks

CHAPTER XV

The Inclined Plane and Screw

CHAPTER XVI

Work Power and Energy; Horse-power of Belting

CHAPTER XVII

Horse-power of Engines

CHAPTER XVIII

Mechanics of Fluids

CHAPTER XIX

Heat

HAPTER XX

Strength of Materials

SHOP ARITHMETIC

CHAPTER I

COMMON FRACTIONS

1. Why We Use Fractions.—When we find it necessary to deal with things that are less than one unit, we must use fractions. A machinist cannot do all his work in full inches because it is generally impossible to have all measurements in exact inches. Consequently, for measurements less than 1 in., he uses fractions of an inch; he also makes use of fractions for measurements between one whole number of inches and the next whole number. If a bolt is wanted longer than 4 in. but shorter than 5 in., it would be 4 in. and a fraction of an inch. This fraction of an inch might be nearly a whole inch or it might be a very small part of an inch. The system used to designate parts of a unit is

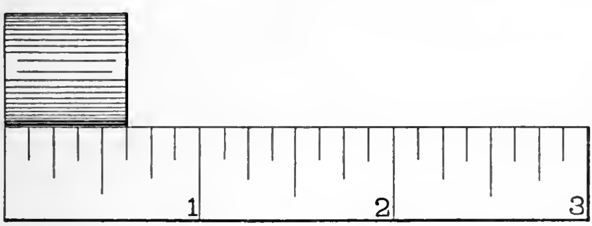


Fig. 1.

easily seen by looking at a machinist's scale or at a foot-rule of any sort. Each inch on the scale is divided into a number of equal parts. A wooden foot-rule usually has eight or sixteen parts to each inch, while a machinist's steel scale has much finer divisions. Now, if we want to measure a piece of steel which is not an inch long, we hold a scale against it, as in Fig. 1, and find out how many of these divisions of an inch it takes to equal the length of the piece. The scale in Fig. 1 is 3 in. long and each inch is divided into eight parts. We see that this piece is as long as *five* of these *eight* parts of an inch, or we say that it is "*five-eighths*" of an inch long.

2. Definition of a Fraction.—A Fraction is one or more of the equal parts into which anything may be divided. Every fraction must contain two numbers, a numerator and a denominator. These are called the *terms* of a fraction.

3. The Denominator.—The Denominator tells into how many equal parts the unit is divided. In the case shown in Fig. 1, 1 in. was the unit and it was divided into eight equal parts. The denominator in this case was eight.

4. The Numerator.—The Numerator shows how many of these parts are taken. In giving the length of the piece of steel in Fig. 1, we divided the inch into eight parts and took five of them for the length. Five is the numerator and eight is the denominator.

5. Writing and Reading Fractions.—In writing fractions, the numerator is placed over the denominator and either a slanting line, as in 5/8, or a horizontal line, as in $\frac{5}{8}$, drawn between them. The horizontal line is the better form to use, as mistakes are easily made when a whole number and a fraction with a slanting line are written close together.

$\frac{1}{4}$ is read *one-fourth* or *one-quarter.*

$\frac{1}{2}$ is read *one-half.*

$\frac{3}{4}$ is read *three-fourths* or *three-quarters.*

$\frac{5}{8}$ is read *five-eighths.*

$\frac{3}{7}$ is read *three-sevenths.*

We can have fractions of all sorts of things besides inches. An *hour* of time is divided into sixty equal parts called *minutes*. A minute is merely $\frac{1}{60}$ of an hour. Likewise, 20 minutes is $\frac{20}{60}$ of an hour. In the same way, 1 second is $\frac{1}{60}$ of a minute.

In the early days, before we had the unit called the inch, the foot was the common unit for measuring lengths. When it was necessary to measure lengths less than 1 ft., fractions of a foot were used. This got to be too troublesome, so one-twelfth of a foot was given the name of *inch* to avoid using so many fractions. For instance, where formerly one said $\frac{5}{12}$ of a foot, we can now say 5 in. This shows how the use of a smaller unit reduces the use of fractions. In Europe, a unit called the *millimeter* is used in nearly all shop work. This is so small, being only about $\frac{4}{100}$ of an inch, that it is seldom necessary in shop work to use fractions of a millimeter.

6. Proper Fractions.—If the numerator and denominator of a fraction are equal, the value of the fraction is 1, because there are just as many parts taken as there are parts in one unit.

$$\frac{4}{4}=1 \quad \frac{8}{8}=1 \quad \frac{10}{10}=1$$

In each of these cases, the numerator shows that we have taken the full number of parts into which the unit has been divided. Consequently, each of the fractions equals a full unit, or 1.

A Proper Fraction is one whose numerator is less than the denominator. The value of a proper fraction, therefore, is always less than 1.

$$\frac{3}{4}, \frac{5}{16}, \frac{7}{8}, \frac{27}{32} \text{ are all proper fractions.}$$

7. Improper Fractions.—An Improper Fraction is one whose numerator is equal to or larger than the denominator. Therefore, an improper fraction is equal to, or more than 1.

$$\frac{24}{12}, \frac{14}{8}, \frac{17}{16}, \frac{64}{64} \text{ are all improper fractions.}$$

8. Mixed Numbers.—A Mixed Number is a whole number and a fraction written together: for example, $4\frac{1}{2}$ is a mixed number. $4\frac{1}{2}$ is read *four and one-half* and means four whole units and one-half a unit more.

9. Reduction of Fractions.—Quite often we find it desirable to change the form of a fraction in order to make certain calculations; but, of course, the real value of the fraction must not be changed. The operation of changing a fraction from one form to another without changing its value is called *Reduction*.

By referring to the scale in Fig. 2 it will be seen that, if we take the first inch and divide it into 8 parts, each $\frac{1}{8}$ in. will contain 4 of these parts. Hence, $\frac{1}{2}$ in. $=\frac{4}{8}$ in. In this case, we make the denominator of the fraction 4 times as large, by making 4 times as many parts in the whole. It then takes a numerator 4 times as large to represent the same fractional part of an inch. This relation holds whether we are dealing with inches or with any other thing as a unit.

Fig. 2.

10. Reduction to Higher Terms.—When we raise a fraction to higher terms, we increase the number of parts in the whole, as just shown, and this likewise increases the number of parts taken. Therefore, the numerator and denominator both become larger numbers.

$$\frac{1}{2}\text{ in.} = \frac{4}{8}\text{ in.} \qquad \frac{5}{16}\text{ in.} = \frac{10}{32}\text{ in.}$$

A fraction is raised to higher terms by multiplying both numerator and denominator by the same number.

Examples:

$$\frac{1}{2}=\frac{1\times 4}{2\times 4}=\frac{4}{8}$$

Similarly,

$$\frac{5}{16}=\frac{5\times 2}{16\times 2}=\frac{10}{32}$$

Suppose we want to change $\frac{3}{16}$ of an inch to 64ths. To get 64 for the denominator, we must multiply 16 by 4 and, therefore, must multiply 3 by the same number.

$$\frac{3}{16}=\frac{3\times 4}{16\times 4}=\frac{12}{64}$$

11. Reduction to Lower Terms.—When we reduce a fraction to lower terms, we reduce the number of parts into which the whole unit is divided. This likewise reduces the number of parts which are taken.

$$\frac{4}{8}\text{ in.}=\frac{1}{2}\text{ in.} \qquad \frac{2}{16}\text{ in.}=\frac{1}{8}\text{ in.}$$

A fraction is reduced to lower terms by dividing both numerator and denominator by the same number. When there is no number which will exactly divide both numerator and denominator, the fraction is already in its *lowest* terms.

Example:

Reduce $\frac{36}{128}$ to its lowest terms.

$$\frac{36}{128}=\frac{36 \div 2}{128 \div 2}=\frac{18 \div 2}{64 \div 2}=\frac{9}{32}$$

There is no number that will exactly divide both 9 and 32 and, therefore, the fraction is reduced to its lowest terms.

12. Reduction of Improper Fractions.—When the numerator of a fraction is just equal to the denominator, we know that the value of the fraction is 1 (see Art. 6):

$$\frac{8}{8}=1 \qquad \frac{64}{64}=1 \qquad \frac{10}{10}=1$$

In each of these cases we have taken the full number of parts into which we have divided the unit. Consequently, each of these fractions is one whole unit, or 1.

When the numerator is greater than the denominator, the value of the fraction is one or more units, plus a proper fraction, or a whole plus some part of a whole.

Examples:

$$\frac{12}{8}=\frac{8}{8}+\frac{4}{8}=1\frac{4}{8} \text{ or } 1\frac{1}{2}$$

$$\frac{47}{12}=\frac{36}{12}+\frac{11}{12}=3\frac{11}{12}$$

From these examples we may see that to reduce an improper fraction to a whole or mixed number the simplest way is as follows:

Divide the numerator by the denominator. The quotient will be the number of whole units. If there is anything left over, or a remainder, write this remainder over the denominator since it represents the number of parts left in addition to the whole units. We now have a mixed number, or an exact whole number, in place of the improper fraction.

Examples:

$$\frac{27}{7}=27 \div 7=3\frac{6}{7}$$

```
7)27(3
  21
  --
   6
```

$$\frac{45}{6}=45 \div 6=7\frac{3}{6}=7\frac{1}{2}$$

```
6)45(7
  42
  --
   3
```

These show that a fraction represents unperformed division. In fact, division is often indicated in the form of a fraction. The numerator is the dividend and the denominator is the divisor.

$$24 \div 3 \text{ can be written } \frac{24}{3}$$

$$2 \div 8 \text{ can be written } \frac{2}{8}$$

13. Reduction of Mixed Numbers.—It is often necessary or desirable to change mixed numbers to improper fractions. The method of doing this may be seen from the following examples.

Examples:

Reduce $5\frac{1}{2}$ to an improper fraction.

$$5\frac{1}{2}=5+\frac{1}{2}$$

In one unit there are two halves. Therefore,

$$5=\frac{5\times 2}{2}=\frac{10}{2}$$

$$5\frac{1}{2}=\frac{10}{2}+\frac{1}{2}=\frac{11}{2}$$

If $7\frac{1}{4}$ were to be reduced to an improper fraction we would say: "Since there are 4 fourths in 1, in 7 there are 4×7, or 28 fourths. 28 fourths plus 1 fourth equals 29 fourths."

$$7\frac{1}{4}=\frac{28}{4}+\frac{1}{4}=\frac{29}{4}$$

The rule which this gives us is very simple: Multiply the whole number by the denominator of the fraction and write the product over the denominator. This reduces the whole number to a fraction. Add to this the fractional part of the mixed number. The sum is the desired improper fraction.

In working problems like the above, the work should be arranged as in the following example.

Example:

Reduce $5\frac{1}{8}$ in. to eighths of an inch.

$$5\frac{1}{8}=\frac{40}{8}+\frac{1}{8}=\frac{41}{8}, \textit{ Answer.}$$

USEFUL TABLES

Measures of Length

12	inches (in.)	=1 foot (ft.)
3	ft. or 36 in.	=1 yard (yd.)
$5\frac{1}{2}$	yd. or $16\frac{1}{2}$ ft.	=1 rod (rd.)
320	rd. or 5280 ft.	=1 mile (mi.)

Measures of Time

60	seconds (sec.)	=1 minute (min.)
60	minutes	=1 hour (hr.)
24	hours	=1 day (da.)
7	days	=1 week (wk.)
$365\frac{1}{4}$	days	=1 average year (yr.)
100	years	=1 century

Note.—Thirty days are generally considered as one month, though the number of days differs for different months.

Miscellaneous Units

12 things	=1 dozen (doz.)
12 dozen or 144 things	=1 gross (gr.)
12 gross	=1 great gross
20 things	=1 score

QUESTIONS AND PROBLEMS

1. What is a fraction?

2. Name some fractions of an inch commonly used.

3. Write the following as fractions or mixed numbers.

Five-sixteenths
Nine thirty-seconds
Twenty and one-eighth
Twenty-one eighths
Three and three-fourths

4. Write out in words the following:

$$3\frac{1}{6},\ \frac{5}{8},\ \frac{7}{16},\ \frac{8}{8},\ \frac{5}{24},\ 20\frac{1}{4},\ \frac{21}{4}$$

5. Indicate the proper fractions, the improper fractions, and the mixed numbers among the following:

$$\frac{3}{4},\ 3\frac{1}{8},\ \frac{16}{16},\ \frac{21}{16},\ 1\frac{5}{16},\ \frac{7}{8},\ \frac{9}{16}$$

6. Change $\frac{10}{16}$ of an inch to eighths of an inch.

Change $\frac{4}{16}$ of an inch to fourths of an inch.

7. How many sixteenths of an inch in $\frac{3}{4}$ in.?

How many thirty-seconds of an inch in $\frac{3}{4}$ in.?

8. Which is greater, $\frac{13}{16}$ in. or $\frac{7}{8}$ in.? $\frac{3}{10}$ or $\frac{3}{12}$?

9. Reduce the following mixed numbers to improper fractions:

$$4\frac{1}{8},\ 7\frac{1}{2},\ 3\frac{3}{4},\ 2\frac{1}{6},\ 1\frac{1}{16}$$

10. Reduce the following improper fractions to whole or mixed numbers:

$$\frac{21}{16},\ \frac{8}{8},\ \frac{24}{3},\ \frac{7}{2},\ \frac{121}{12}$$

11. I want to mix up a pound of solder to be made of 5 parts zinc, 2 parts tin, and 1 part lead. What fraction of a pound of each metal—zinc, tin and lead—must I have?

12. If a train is running at the rate of a mile a minute, how many feet does it go in 1 second?

13. An apprentice who is drilling and tapping a cylinder for $\frac{7}{8}$ in. studs, tries a $\frac{3}{4}$ in. drill but the tap binds, so he decides to use a drill $\frac{1}{64}$ in. larger. What size drill does he ask for?

14. The tubes in a certain boiler are 15 ft. 11 in. long. How many inches long are they?

15. How many seconds in an hour? 40 seconds is what fraction of an hour?

16. An 8-ft. bar of steel is cut up into 16 in. lengths. What fraction of the whole bar is one of the pieces?

17. When a man runs 100 yd. in 10 seconds, how many feet does he go in 1 second?

18. Wood screws are generally put up in boxes containing one gross. If 36 screws are taken from a full box for use on a certain job, what fraction of the gross is used on this job and what fraction is left in the box? Reduce both fractions to their lowest terms.

19. A steel plate 2 ft. 6 in. wide is to be sheared into four strips of equal width. How wide will each strip be in inches?

20. In one plant all drawings are dimensioned in inches, while in another all dimensions above 2 ft. are given in feet and inches. If a dimension is given as 89 in. in the first plant, how would the same dimension be stated in the other plant?

CHAPTER II

ADDITION AND SUBTRACTION OF FRACTIONS

14. Common Denominators.—Fractions cannot be added unless they contain the same kind of parts, or, in other words, have the same denominator. When fractions having different denominators are to be added, they must first be reduced to fractions having a common denominator. A number of fractions are said to have a *common* denominator when they all have the same number for their denominators. $\frac{3}{4}$ and $\frac{5}{8}$ cannot be added as they stand, any more than can 3 bolts and 5 washers. Both the fractions must be of the same kind, that is, must have the same denominator. $\frac{3}{4}$ may be changed to $\frac{6}{8}$. By making this change, the fractions are given a common denominator and can now be added. 6 eighths plus 5 eighths equals 11 eighths, in just the same manner as 6 inches plus 5 inches equals 11 inches. The work of this example would be written as follows:

$$\frac{3}{4}+\frac{5}{8}=?$$

$$\frac{3\times 2}{4\times 2}=\frac{6}{8}$$

$$\frac{6}{8}+\frac{5}{8}=\frac{11}{8}=1\frac{3}{8},\ \textit{Answer.}$$

8 is called the *Least* Common Denominator (L. C. D.) of $\frac{3}{4}$ and $\frac{5}{8}$, because it is the smallest number that can be used as a common denominator for these two fractions. In this case, the least common denominator is apparent at a glance; in many other cases it is more difficult to find, especially if there are several fractions to be added. In the case just given, the denominator of one fraction can be used for the common denominator. When we have two denominators like 5 and 8, neither of them is an exact multiple of the other number, and so neither can be the common denominator. In such a case, the product of the two numbers can be used as a common denominator.

Example:

$$\frac{3}{5}+\frac{5}{8}=?$$

$5\times8=40$, the L. C. D.

$$\frac{3}{5}=\frac{3\times8}{5\times8}=\frac{24}{40}$$

$$\frac{5}{8}=\frac{5\times5}{8\times5}=\frac{25}{40}$$

$$\frac{24}{40}+\frac{25}{40}=\frac{49}{40}=1\frac{9}{40}, \text{ Answer.}$$

We can always be sure that the product of the denominators will be a common denominator, to which all the fractions can be reduced, but it will not always be the *least* common denominator. For example, if we wish to add $\frac{5}{12}$ and $\frac{7}{16}$, we can use $12\times16=192$ for the common denominator, but we readily see that 48 will serve just as well and not make the fractions so cumbersome. In this case 48 is the least common denominator.

15. To Find the L. C. D.—If the L. C. D. cannot be easily seen by examining the denominators, it may be found as follows: Suppose we are to find the L. C. D. of $\frac{1}{4}$, $\frac{2}{3}$, $\frac{5}{9}$, and $\frac{3}{16}$. First place the denominators in a row, separating them by commas.

```
2)4, 3, 9, 16
2)2, 3, 9,  8
3)1, 3, 9,  4
  1, 1, 3,  4
```

L. C. D. $=2\times2\times3\times3\times4=144$

Select the smallest number (other than 1) that will exactly divide two or more of the denominators. In this case, 2 will exactly divide 4 and 16. Divide it into all the numbers that are exactly divisible by it, that is, may be divided by it without leaving a remainder. When writing the quotients below, also bring down any numbers which are not divisible by the divisor and write them with the quotients. Now proceed as before, again using the smallest number that will divide two or more of the numbers just obtained. Continue this process until no number (except 1) will exactly divide more than one of the remaining numbers. The product of all the divisors and all the numbers (except 1's) left in the last line of quotients is the Least Common Denominator.

16. To Reduce to the L. C. D.—Having found the least common denominator of two or more fractions, the next step is to reduce the given fractions to fractions having this least common denominator. Let us take $\frac{1}{3}$, $\frac{2}{5}$, $\frac{1}{4}$, and $\frac{3}{8}$. We first find the L. C. D., which turns out to be 120. We next proceed to reduce the fractions to fractions having the L. C. D. Divide the common denominator by the denominator of the first fraction. Multiply both numerator and denominator of the fraction by the quotient thus obtained. Do this for each fraction, as illustrated here.

$$\begin{array}{l} 2\underline{)3,\ 5,\ 4,\ 8} \\ 2\underline{)3,\ 5,\ 2,\ 4} \\ \quad 3,\ 5,\ 1,\ 2 \end{array}$$

$$\text{L. C. D.} = 2 \times 2 \times 3 \times 5 \times 2 = 120$$

$$120 \div 3 = 40 \quad \frac{1}{3} = \frac{1 \times 40}{3 \times 40} = \frac{40}{120}$$

$$120 \div 5 = 24 \quad \frac{1}{5} = \frac{1 \times 24}{5 \times 24} = \frac{24}{120}$$

$$120 \div 4 = 30 \quad \frac{1}{4} = \frac{1 \times 30}{4 \times 30} = \frac{30}{120}$$

$$120 \div 8 = 15 \quad \frac{3}{8} = \frac{3 \times 15}{8 \times 15} = \frac{45}{120}$$

17. Addition of Fractions.—Addition of fractions is very simple after the fractions have been reduced to fractions with a common denominator. Having done this it is only necessary to add the numerators and place this sum over the common denominator. The sum should always be reduced to lowest terms and if it turns out to be an improper fraction it should be reduced to a mixed number.

Example:

Find the sum of $\frac{5}{16} + \frac{3}{4} + \frac{9}{32} + \frac{7}{32}$

Common denominator $= 32$

$$\frac{5}{16} = \frac{5 \times 2}{16 \times 2} = \frac{10}{32}$$

$$\frac{3}{4} = \frac{3 \times 8}{4 \times 8} = \frac{24}{32}$$

$$\frac{10}{32} + \frac{24}{32} + \frac{9}{32} + \frac{7}{32} = \frac{50}{32}$$

$$\frac{50}{32} = \frac{25}{16} = 1\frac{9}{16}, \textit{ Answer.}$$

If there are mixed numbers and whole numbers, add the whole numbers and fractions separately. If the sum of the fractions is an improper fraction, reduce it to a mixed number and add this to the sum of the whole numbers.

Example:

How long a steel bar is needed from which to shear one piece each of the following lengths:

$7\frac{1}{2}$ in., $5\frac{7}{8}$ in., $4\frac{3}{4}$ in., $6\frac{1}{8}$ in. ?

$$\begin{array}{ll} 7\frac{1}{2} & \frac{4}{8} \\ 5\frac{7}{8} & \frac{7}{8} \\ 4\frac{3}{4} & \frac{6}{8} \\ 6\frac{1}{8} & \frac{1}{8} \\ \hline 22 & \frac{18}{8}=2\frac{1}{4} \\ 2\frac{1}{4} & \\ \hline 24\frac{1}{4}, & \textit{Answer.} \end{array}$$

Explanation: The sum of the whole numbers is 22. The sum of the fractions is $\frac{18}{8}$, which reduces to $2\frac{1}{4}$. Adding this to the sum of the whole numbers (22), gives $24\frac{1}{4}$ as the sum of the mixed numbers. Hence we must have a bar $24\frac{1}{4}$ inches long.

18. Subtraction of Fractions.—Just as in addition, the fractions must first be reduced to a common denominator. Then we can subtract the numerators and write the result over the common denominator.

Example:

Subtract $\frac{5}{8}$ from $\frac{15}{16}$.

Common demoninator = 16

$$\frac{5}{8}=\frac{10}{16}$$

$$\frac{15}{16}-\frac{10}{16}=\frac{5}{16}, \textit{ Answer.}$$

In subtracting mixed numbers, subtract the fractions first and then the whole numbers.

Example:

How much must be cut from a $15\frac{1}{2}$ in. bolt to make it $12\frac{3}{16}$ in. long?

L. C. D. = 16

$$15\frac{1}{2}=15\frac{8}{16}$$

$$\begin{array}{r} 12\frac{3}{16} \\ \hline 3\frac{5}{16}, \end{array} \textit{ Answer.}$$

Sometimes, in subtracting mixed numbers, we find that the fraction in the subtrahend (the number to be taken away) is larger than the fraction in the minuend (the number from which the subtrahend is to be taken). In this case, we borrow 1 from the whole number of the minuend and add it to the fraction of the minuend. This makes an improper fraction of the fraction in the minuend and we can now subtract the other fraction from it.

Example:

Take $9\frac{3}{4}$ from $12\frac{1}{8}$

$$12\frac{1}{8} = 11\frac{9}{8}$$
$$9\frac{3}{4} = 9\frac{6}{8}$$
$$2\frac{3}{8}, \text{ Answer.}$$

Explanation: $\frac{3}{4}$ cannot be substracted from $\frac{1}{8}$, so we borrow 1 $\left(\text{or } \frac{8}{8}\right)$ from 12 and write the minuend $11\frac{9}{8}$.

If the minuend happens to be a whole number, borrow 1 from it and write it as a fractional part of the minuend. Then subtract as before.

Example:

$$10-8\frac{5}{16}=?$$

$$10=9\frac{16}{16}$$
$$8\frac{5}{16}$$
$$1\frac{11}{16}, \text{ Answer.}$$

PROBLEMS

21. Reduce to the L. C. D. $\frac{5}{16}$, $\frac{3}{4}$, and $\frac{9}{32}$. *Answer,* $\frac{10}{32}$, $\frac{24}{32}$, $\frac{9}{32}$.

22. Reduce to the L. C. D. $\frac{3}{4}$, $\frac{3}{8}$, $\frac{7}{10}$, and $\frac{5}{16}$.

23. Add $\frac{10}{24}$, $\frac{11}{12}$, and $\frac{15}{16}$. *Answer,* $\frac{109}{48}=2\frac{13}{48}$.

24. $\frac{5}{6}+\frac{1}{8}+\frac{2}{3}+\frac{7}{16}=?$

25. Add $2\frac{3}{8}$, $5\frac{1}{4}$, and $7\frac{1}{16}$.

26. Find the sum of 8, $3\frac{1}{2}$, $4\frac{5}{8}$, and $\frac{3}{16}$.

27. Subtract $\frac{4}{30}$ from $\frac{7}{10}$.

28. $\frac{1}{16}-\frac{1}{24}=?$

29. Find the difference between $13\frac{3}{4}$ and $20\frac{1}{2}$.

30. $15-11\frac{7}{8}=?$

31. The weights of a number of castings are: $412\frac{1}{2}$ lb., $270\frac{1}{2}$ lb., 1020 lb., $75\frac{1}{2}$ lb., $68\frac{1}{2}$ lb. What is their total weight?

32. Four studs are required: $2\frac{3}{4}$ in., $1\frac{7}{8}$ in., $2\frac{5}{16}$ in., and $1\frac{13}{32}$ in. long; how long a piece of steel will be required from which to cut them allowing $\frac{3}{4}$ in. altogether for cutting off and finishing their ends?

33. Monday morning an engineer bought $48\frac{1}{2}$ gallons of cylinder oil; on Monday, Tuesday, and Wednesday he used $\frac{3}{4}$ gallon per day; on Thursday he used $\frac{7}{8}$ gallon; and on Friday $\frac{1}{2}$ gallon. How much oil had he left on Saturday?

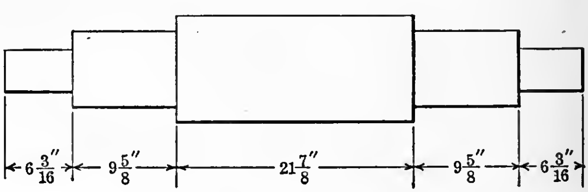


FIG. 3.

34. Find the total length of the roll shown in the sketch in Fig. 3.

35. A piece of work on a lathe is 1 ft. in diameter; it is turned down in five cuts; in the first step the tool takes off $\frac{3}{32}$ in. from the diameter; then

$\frac{1}{16}$ in.; then $\frac{1}{32}$ in.; then $\frac{1}{32}$ in., and the fifth time $\frac{1}{64}$ in. What is the diameter of the finished piece?

36. How long must a machine shop be to accommodate the following machines installed in a single line: lathe, $8\frac{1}{2}$ ft. long; planer, $14\frac{1}{3}$ ft. long; milling machine, $4\frac{1}{8}$ ft. long; engine, $7\frac{7}{8}$ ft. long; tool room, $12\frac{1}{6}$ ft. long? Allow $3\frac{1}{4}$ ft. between a wall and a machine, and $3\frac{1}{2}$ ft. between two machines. The tool room is to be placed at the end of the shop.

37. In doing a certain piece of work one man puts in $1\frac{1}{3}$ hours, a second man $\frac{1}{2}$ hour, a third works $2\frac{1}{6}$ hours, and a fourth man works $1\frac{1}{4}$ hours. How long would it take one man to do the work?

38. By mistake, the draftsman omitted the thickness of the flange on the drawing of a gas engine cylinder in Fig. 4. From the other dimensions given, calculate the thickness of the flange.

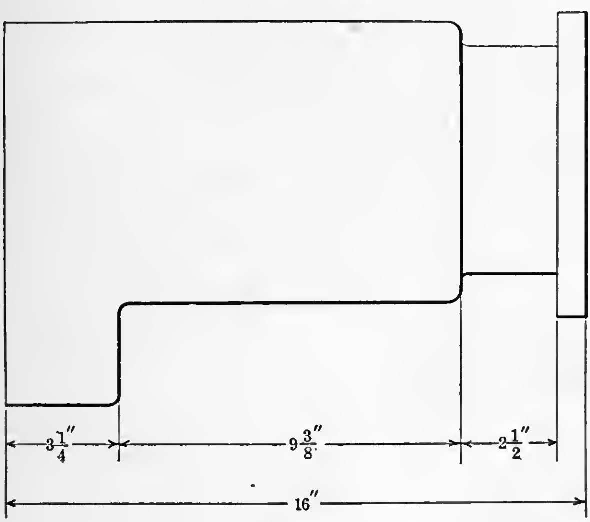


FIG. 4.

39. A millwright has to rig up temporarily a 6 in. belt to be $367\frac{5}{8}$ in. long. In looking over the stock of old belting he finds the following pieces of the right width; one piece $126\frac{1}{2}$ in. long, one $142\frac{7}{8}$ in. long, and one $133\frac{3}{8}$ in. long.

How many inches must be cut from one of the pieces so that these pieces can be laced together to give the right length?

40. The time cards for a certain piece of work show 2 hours and 15 minutes lathe work, 3 hours and 10 minutes milling, 1 hour and 10 minutes planing, and 1 hour and 15 minutes bench work; what is the total number of hours to be charged to the job?

CHAPTER III

MULTIPLICATION AND DIVISION OF FRACTIONS

19. A Whole Number Times a Fraction.—In the study of multiplication, we learn that multiplying is only a short way of adding. 4×7 is the same as four 7's added together. Either 4×7, or $7+7+7+7$ will give 28. If we apply this same principle to the multiplying of fractions, we see that $4 \times \frac{7}{8}$ is the same as four of these fractions added together.

$$4 \times \frac{7}{8} = \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} = \frac{28}{8}$$

This shows that multiplying a fraction by a whole number is performed by multiplying the numerator by the whole number and placing the product over the denominator of the fraction.

In other words, the *size* of the parts is not changed, but the *number* of parts is increased by the multiplication. After multiplying, the product should be reduced to lowest terms and, if an improper fraction, should be reduced to a whole or mixed number.

Example:

What would be the total weight of 12 brass castings each weighing $\frac{3}{4}$ of a pound?

$$12 \times \frac{3}{4} = \frac{36}{4} = 9 \text{ lb., } \textit{Answer.}$$

20. "Of" Means "Times."—The word "of" is often seen in problems in fractions, as for instance, "What is $\frac{3}{4}$ of 5 in.?" In such a case, we work the problem by multiplying, so we say that "of" means "times." You can see that this is so by taking a piece of wood 5 in. long and cutting it into four equal parts and then taking three *of* these parts. These three parts will be $\frac{3}{4}$ of 5 in., and by actual measurement will be $3\frac{3}{4}$ in long, so we know that $\frac{3}{4}$ *of* $5 = 3\frac{3}{4}$. Now see what $\frac{3}{4}$ *times* 5 is·

$$\frac{3}{4} \times 5 = \frac{15}{4} = 3\frac{3}{4}$$

which is the same value. Therefore, we see that the word "of" in such a case signifies multiplication.

21. A Fraction Times a Fraction.—To multiply two or more fractions together, multiply the numerators together for the numerator of the product and multiply the denominators together for the denominator of the product.

Example:

Multiply $\frac{7}{8}\times\frac{2}{3}$.

$$\frac{7}{8}\times\frac{2}{3}=\frac{14}{24}=\frac{7}{12}$$

Explanation: The numerator of the product is obtained from multiplying the numerators together: $7\times2=14$. The denominator of the product, in the same manner, is $8\times3=24$. This gives the product $\frac{14}{24}$, which can be reduced to $\frac{7}{12}$.

Let us see what multiplication of fractions really means, and why the work is done as just shown. Suppose we are to find $\frac{3}{4}$ of $\frac{7}{8}$ in. This means that $\frac{7}{8}$ of an inch is to be divided into 4 equal parts and 3 of these parts are wanted. If we divide $\frac{7}{8}$ in. into 4 equal parts, each part will be one-fourth as large as $\frac{7}{8}$ in. and, therefore, can be considered as being made up of 7 parts, each one-fourth as large as $\frac{1}{8}$ in. Then $\frac{1}{4}$ of $\frac{7}{8}=\frac{7}{32}$. Three of these parts will naturally contain three times as many thirty-seconds, or $\frac{3\times7}{32}=\frac{21}{32}$. Therefore:

$$\frac{3}{4} \text{ of } \frac{7}{8}=\frac{3}{4}\times\frac{7}{8}=\frac{3\times7}{4\times8}=\frac{21}{32}.$$

22. Multiplying Mixed Numbers.—This is one of the most difficult operations in the study of fractions, unless one adopts a fixed rule and follows it in all cases. The student will have no trouble if he will first reduce the mixed numbers to improper fractions, and then multiply these like any other fractions.

Example:

Find the product of $3\frac{1}{4}$ and $2\frac{1}{2}$.

$$3\frac{1}{4}=\frac{13}{4}; \text{ and } 2\frac{1}{2}=\frac{5}{2}$$

$$3\frac{1}{4}\times2\frac{1}{2}=\frac{13}{4}\times\frac{5}{2}=\frac{13\times5}{4\times2}=\frac{65}{8}=8\frac{1}{8}, \textit{ Answer.}$$

To multiply a mixed number by a whole number, we can reduce the mixed number to an improper fraction and then multiply it; or we can multiply the fractional part and the whole number part separately by the number and then add the products.

Example:

What would be the cost of ten $\frac{1}{2}$ in. by 6 in. machine bolts at $1\frac{3}{8}$ cents a piece?

First method:

$$10\times 1\frac{3}{8}=10\times\frac{11}{8}=\frac{110}{8}=13\frac{6}{8}=13\frac{3}{4}\text{ cents.}$$

Second Method:

$$\begin{array}{r} 10 \\ 1\frac{3}{8} \\ \hline 3\frac{3}{4} \\ 10 \\ \hline 13\frac{3}{4} \end{array}\text{ cents, } \textit{Answer.}$$

Explanation: First multiply 10 by $\frac{3}{8}$. This gives $\frac{30}{8}$, or $3\frac{6}{8}$, or $3\frac{3}{4}$. Set this down. Then multiply 10 by 1. This gives 10, and we add this to the $3\frac{3}{4}$, giving a total of $13\frac{3}{4}$.

23. Cancellation.—Very often the work of multiplying fractions may be lessened by cancellation, as it avoids the necessity of reducing the product to lowest terms. To get an idea of cancellation we must first understand what a "factor" is. A Factor of a number is a number which will exactly divide it. Thus, 2 is a factor of 8, 3 is a factor of 27, 5 is a factor of 35, etc. When the same number will exactly divide two or more numbers it is called a *common factor* of those numbers. Thus, 2 is a common factor of 8 and 12, because it will divide both 8 and 12 without leaving a remainder. 4 is also a common factor of 8 and 12. Similarly, 7 is a common factor of 14 and 21.

This idea of common factors we have already used in reducing fractions to lowest terms. Thus, when we have $\frac{8}{12}$ we divide both 8 and 12 by 4 and get

$$\frac{8}{12}=\frac{8\div 4}{12\div 4}=\frac{2}{3}$$

Cancellation is a process of shortening the work of reduction by removing or cancelling the equal factors from the numerator and denominator.

Example:

Suppose we have several fractions to multiply together, as $\frac{3}{4}\times\frac{2}{3}\times\frac{3}{14}\times\frac{21}{32}$.

$$\text{Their product is }\frac{3\times 2\times 3\times 21}{4\times 3\times 14\times 32}=\frac{378}{5376}.$$

This is not in its lowest terms so we divide both numerator and denominator by 2, 3, and 7, and get

$$\frac{378 \div 2}{5376 \div 2} = \frac{189 \div 3}{2688 \div 3} = \frac{63 \div 7}{896 \div 7} = \frac{9}{128}. \quad \textit{Answer.}$$

Now, if we had struck out the common factors from the numerator and denominator before multiplying the fractions, we would have shortened the work and our answer would have been in its lowest terms without reducing. Thus:

$$\frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{2}} \times 3 \times \overset{3}{\cancel{21}}}{\underset{2}{\cancel{4}} \times \underset{1}{\cancel{3}} \times \underset{2}{\cancel{14}} \times 32} = \frac{1 \times 1 \times 3 \times 3}{2 \times 1 \times 2 \times 32} = \frac{9}{128}$$

Explanation: First the 3 in the numerator is cancelled with the 3 in the denominator. This merely divides the numerator and denominator by 3 at the outset instead of waiting until the terms are all multiplied together; and, as $3 \div 3 = 1$, we cancel a 3 from both numerator and denominator and place 1's in their stead. Next we divide both terms by 2. The gives 1 in place of the 2 in the numerator and 2 in place of the 4 in this denominator. Next we see that 7 is a common factor of the numerator and denominator, so we divide the 21 and 14 each by 7 and place 3 and 2 in their places. There are no more common factors; so we multiply together the numbers we now have and get $\frac{9}{128}$.

Another Example:

$$\frac{\overset{\overset{1}{\cancel{2}}}{\cancel{500}} \times \overset{4}{\cancel{36}} \times \overset{6}{\cancel{42}} \times \overset{5}{\cancel{50}}}{\underset{1}{\cancel{250}} \times \underset{\underset{1}{\cancel{7}}}{\cancel{63}} \times \underset{\underset{1}{\cancel{2}}}{\cancel{20}}} = \frac{120}{1} = 120$$

Explanation: First we cancel 250 out of 500 and 250; and then 9 out of 36 and 63; then 7 out of 42 and 7; then 10 out of 50 and 20; and finally 2 out of 2 and 2. This removes all the common factors and we get 120 for the answer.

PROBLEMS IN MULTIPLICATION

41. $7 \times \frac{3}{16} = ?$

42. $8 \times \frac{17}{32} = ?$

43. Multiply $\frac{23}{32}$ by $\frac{5}{16}$.

44. Find the product of $\frac{3}{4}$ and $\frac{5}{8}$.

45. What is $\frac{3}{8}$ of $2\frac{2}{3}$?

46. What is $\frac{2}{3}$ of $\frac{3}{8}$ of $\frac{5}{16}$ of $\frac{4}{5}$?

24. Division—The Reverse of Multiplication.—Division is just the opposite of multiplication and this fact gives us the cue to a very simple method of dividing fractions.

To divide one fraction by another, invert the divisor and then multiply. To invert means to turn upside down. Invert $\frac{3}{4}$ and we get $\frac{4}{3}$; invert $\frac{1}{2}$ and we get .

Example:

Divide $\frac{27}{32}$ by $\frac{3}{4}$.

$$\frac{27}{32} \div \frac{3}{4} = \frac{\overset{9}{\cancel{27}}}{\underset{8}{\cancel{32}}} \times \frac{\cancel{4}}{\cancel{3}} = \frac{9}{8} = 1\frac{1}{8}, \text{ *Answer.*}$$

Explanation: The divisor is $\frac{3}{4}$. Inverting this gives $\frac{4}{3}$. In multiplying, we make use of cancellation to simplify the work, and we get $\frac{9}{8}$ or $1\frac{1}{8}$ for the result.

Suppose we have a fraction to divide by a whole number; as $\frac{14}{16} \div 2 = ?$ 2 is the same as $\frac{2}{1}$. If we invert this we get $\frac{1}{2}$. Therefore,

$$\frac{14}{16} \div \frac{2}{1} = \frac{\overset{7}{\cancel{14}}}{16} \times \frac{1}{\cancel{2}} = \frac{7}{16}, \text{ *Answer.*}$$

25. Compound Fractions.—Sometimes we see a fraction which has a fraction for the numerator and another fraction for the denominator. This is called a Compound Fraction. If we remember that a fraction indicates the division of the numerator by the denominator, we will see that a compound fraction can be simplified by performing this division.

Example:

What is $\dfrac{\frac{27}{32}}{\frac{3}{4}}$?

This means the same as $\frac{27}{32} \div \frac{3}{4}$ and, therefore, would be solved as follows:

$$\dfrac{\frac{27}{32}}{\frac{3}{4}} = \frac{27}{32} \div \frac{3}{4} = \frac{\overset{9}{\cancel{27}}}{\underset{8}{\cancel{32}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{3}}} = \frac{9}{8} = 1\frac{1}{8}, \text{ *Answer.*}$$

PROBLEMS IN DIVISION

47. Divide 175 by $\frac{3}{4}$.

48. Divide $\frac{15}{16}$ by $1\frac{7}{8}$.

49. Divide $3\frac{3}{4}$ by $5\frac{4}{5}$.

50. $\frac{27}{32} \div \frac{9}{10} = ?$

51. Find the quotient of $21 \div \frac{7}{8}$.

52. $\dfrac{\frac{7}{8}}{\frac{3}{4}} = ?$

26. How to Analyze Practical Problems.—The chief trouble that students have in working practical problems is in analyzing the problems to find out just what operations they should use to work them. Problems in multiplication or division of fractions will fall in one of the three following cases:

1. Given a whole; to find a part (multiply).
2. Given a part; to find the whole (divide).
3. To find what part one number is of another (divide).

Example of Case 1:

The total weight of a shipment of steel bars is 3425 lb. $\frac{7}{25}$ of this consists of $\frac{3}{4}$ in. round bars and the balance is $\frac{1}{2}$ in. round. What weight is there of each size?

$\frac{7}{\cancel{25}} \times \frac{\overset{137}{\cancel{3425}}}{1} = 959$ lb. of $\frac{3}{4}$ in. bars.

$3425 - 959 = 2466$ lb. of $\frac{1}{2}$ in. bars.

or

$\frac{18}{\cancel{25}} \times \frac{\overset{137}{\cancel{3425}}}{1} = 2466$ lb. of $\frac{1}{2}$ in. bars.

Explanation: In this example we have the whole (3425 lb.); to find a part $\left(\frac{7}{25}\right)$. If the whole is 3425, then $\frac{7}{25}$ of $3425 = 959$ lb. The balance, which consists of $\frac{1}{2}$ in. bars, will be $3425 - 959$, or it will be $1 - \frac{7}{25} = \frac{25}{25} - \frac{7}{25} = \frac{18}{25}$ of the whole. $\frac{18}{25}$ of $3425 = 2466$ lb.

Example of Case 2:

The base of a dynamo weighs 270 lb.; the base is $\frac{3}{11}$ of the total weight; find the total weight.

$\frac{3}{11}$ of the whole $= 270$ lb.

whole $= 270 \div \frac{3}{11}$

$$270 \div \frac{3}{11} = \frac{\overset{90}{\cancel{270}}}{1} \times \frac{11}{\cancel{3}} = 990 \text{ lb., } \textit{Answer.}$$

Explanation: Here we have a part given to find the whole. $\frac{3}{11}$ of the whole is 270 lb. This means that if the whole machine were divided into eleven equal parts, three of these parts together would weigh 270 lb. Then one part would weigh $270 \div 3 = 90$ lb. Since there are 11 of these equal parts, the whole machine weighs

$$11 \times 90 = 990 \text{ lb.}$$

This is the same as dividing 270 by the fraction $\frac{3}{11}$.

Example of Case 3:

A molder who is on piece work sets up 91 flasks, but the castings from 7 of them are defective. What fractional part of his work does he get paid for?

$91 - 7 = 84$ sound castings.

$\frac{84}{91} = \frac{84 \div 7}{91 \div 7} = \frac{12}{13}$, *Answer.*

Explanation: The problem is: What part of 91 is 84? There are 91 parts in his whole work and he gets paid for 84 parts, or $\frac{84}{91}$ of the whole. This can be reduced to $\frac{12}{13}$, which is the answer.

PROBLEMS

53. A gallon is about $\frac{2}{15}$ of a cubic foot. If a cubic foot of water weighs $62\frac{1}{2}$ lb., how much does a gallon of water weigh?

54. Aluminum is $2\frac{2}{3}$ times as heavy as water; and copper is $8\frac{4}{5}$ times as heavy as water. Copper is how many times as heavy as aluminum?

55. If a certain sized steel bar weighs $2\frac{1}{5}$ lb. to the foot, how long must a piece be to weigh $8\frac{1}{4}$ lb.?

56. What is the cost of a casting weighing $387\frac{1}{2}$ lb. at $4\frac{1}{4}$ cents a pound?

57. How many steel pins to finish $1\frac{1}{8}$ in. long can be cut from an 8 ft. rod if we allow $\frac{3}{16}$ in. to each pin for cutting off and finishing?

58. A certain piece for a machine can be made of steel or of cast iron. If drop forged from steel it would weigh $7\frac{3}{4}$ lb. and would cost $6\frac{1}{8}$ cents per pound. If made of cast iron, it would have to be made much heavier and would weigh 14 lb. and cost $2\frac{5}{8}$ cents per pound. Which would be the cheaper and how much?

59. I want to measure out $2\frac{1}{4}$ gallons of water, but I have no measure at hand. However, there are some scales handy and I proceed to weigh out the proper amount in a pail that weighs $1\frac{7}{8}$ lb. What should be the total weight of the pail and the water, if one gallon of water weighs $8\frac{1}{3}$ lb.?

60. I want to cut 300 pieces of steel, each 112 in. long for wagon tires. I have in stock a sufficient number of bars of the same size, but they are 120 in. long; and I also have a sufficient number 235 in. long. Which length should I use in order to waste the least material? Calculate the total number of inches of stock that would be wasted in each case.

CHAPTER IV

MONEY AND WAGES

27. U. S. Money.—Nearly every country has a money system of its own. The unit of money in the United States is the *dollar.* To represent parts of a dollar, we use the *cent,* which is $\frac{1}{100}$ of a dollar. Fifty cents is $\frac{50}{100}$ of a dollar; it is also one-half dollar ($\frac{50}{100}=\frac{1}{2}$). Likewise, twenty-five cents is $\frac{25}{100}$ dollar, or one-quarter dollar.

In writing United States money, the dollar sign ($) is written before the number; a period called the *decimal point,* is placed after the number of dollars; following this decimal point is placed the number of cents.

Two dollars and seventy cents is written	$ 2.70
Fifteen dollars and seven cents is written	$ 15.07
One Hundred twenty-five dollars is written	$125.00
One dollar and twenty-five cents is written	$ 1.25
Thirty-five cents is written	$ .35
Eight cents is written	$ .08

Since one cent is $\frac{1}{100}$ dollar, it follows that the figures to the right of the decimal point represent a fraction of a dollar. These figures are the numerator, and the denominator is 100.

$ 2.70 is the same as $ $2\frac{70}{100}$
$15.07 is the same as $$15\frac{7}{100}$
$ .08 is the same as $ $\frac{8}{100}$

The first figure following the decimal point can be said to indicate the number of dimes, because 1 dime = 10 cents, and this figure indicates the number of tens of cents. Also this number represents tenths of a dollar, because 1 dime = 10 cents $= \frac{10}{100}$ dollar $= \frac{1}{10}$ dollar. The second figure after the decimal point indicates cents, or hundredths of a dollar.

This decimal system of writing amounts of money has great advantages in performing the operations of addition, subtraction, multiplication, and division, because we can perform these operations just as if we were dealing with whole numbers, which makes the work much simpler than if we had fractions to deal with.

28. Addition.—We can add numbers made up of dollars and cents and carry forward just as in simple addition. The number of *tens* of cents will represent dimes (10 cents = 1 dime; 30 cents = 3 dimes, etc.) and thus can be carried forward and added into the dime column. Likewise, the number of *tens* of dimes will represent dollars (10 dimes = 1 dollar) and, therefore, this number can be added into the dollar column.

Example:

Add \$5.20, \$2.65, \$3.25, and \$.35.

```
 $5.20
  2.65
  3.25
   .35
------
$11.45, Answer.
```

Explanation: Adding the cents column, we get 5 + 5 + 5 + 0 = 15 cents. Put down the 5 and crary the 1 into the next column (since 15 cents = 1 dime and 5 cents.) Adding the dimes, we get 1 + 3 +. 2 + 6 + 2 = 14 dimes. Put down the 4 and carry the 1 into the dollar column (since 14 dimes = \$1.4). 1 + 3 + 2 + 5 = 11 dollars, which we put down complete. The decimal point we now place in the sum exactly as it was in the numbers added, so that it properly separates dollars from cents.

The only precaution to be observed is to see that the dollars, dimes, and cents are properly lined up vertically before adding. To do this it is only necessary to see that the decimal points are kept in a straight vertical line.

Example:

What is the total cost of three articles priced as follows: \$2.25, \$1, and \$1.75?

```
$2.25
 1
 1.75
-----
$5.00, Answer.
```

Explanation: Here the \$1 does not have any decimal point or cents after it, and care should be taken to see that it is put down in the dollar column and not in the cents column. \$1 can be written \$1.00, if desired, to avoid any danger of a mistake.

29. Subtraction.—The same rules should be followed in subtracting. If any figure in the subtrahend is larger than the corresponding figure in the minuend, we can borrow 1 from the figure next to the left, just as in ordinary subtraction.

Example:

A man draws \$24.75 on pay day and immediately pays bills amounting to \$8.86. How much does he have left to put in the bank?

```
 $24.75
   8.86
 $15.89, Answer.
```

Explanation: Having set down the numbers properly, we subtract just as if we were subtracting whole numbers. The only difference is that, after subtracting, we put the decimal point in the remainder directly below the other decimal points, to separate the dollars and cents in the remainder.

30. Multiplication.—In multiplying an amount of money by any number, the process is the same as in simple multiplication, remembering, however, to keep the decimal point to separate the dollars from the cents. The reasoning for this is just the same as in addition. Multiplying cents, gives cents; multiplying dimes gives dimes, and multiplying dollars gives dollars. The figures left over are carried forward just as in plain multiplication, because 10 cents = 1 dime, and 10 dimes = 1 dollar.

Examples:

1. What should a machinist receive for finishing 48 gas engines pistons at 25 cents each?

```
  $.25
    48
   200
   100
$12.00, Answer.
```

Explanation: The 25 cents is put down as \$.25. The multiplication of 25 by 48 is performed as usual. Then the decimal point is placed in the product to separate the dollars and cents by leaving the two places, counting from the right, to represent cents.

2. If you owe 2½ weeks board at \$3.25 a week, how much money will it take to settle the bill?

```
 $3.25
    2½
  162½
  650
$8.12½ Answer.
```

Explanation: First, we put down the numbers and then multiply as if we were simply multiplying 325×2½. The product is 812½, but since we are dealing with dollars and cents, we must put the decimal point in the product to show what part of it represents dollars and what part cents.

31. Division.—In dividing an amount of money by any number, the division is carried out as in ordinary division. The decimal point is then placed in the quotient in the same position (from the right) that it had in the dividend.

Example:

The weekly pay roll of a company employing 405 men is $4880.25. What is the average amount paid to each man?

```
405)$4880.25($12.05, Answer.
     405
      830
      810
       2025
       2025
```

Explanation: The division is carried out just as if we were dividing 488025 by 405. The quotient obtained is 1205. Then, since two of the figures in the dividend were cents, we place the decimal point in the quotient so as to point off the cents, making the quotient read $12.05.

Another way of locating the decimal point is to place it in the quotient as soon as the number of dollars in the dividend has been divided. Taking the same example; we first divide 405 into 488 and get 1, with a remainder of 83. Annexing the next figure 0 and dividing again, we get 2 for the quotient. We have now divided the number of whole dollars (4880) and have $12 for the quotient, with a remainder of 20. The 12 is, therefore, the number of whole dollars in the quotient. We now bring down the next figure (2) from the dividend and find that 405 will not go into 202, so we have 0 dimes. Then, bringing down the five cents, we get 2025 cents, which, divided by 405, gives just 5 cents. The men, therefore, get an average of $12.05 each, per week.

32. Reducing Dollars to Cents.—Sometimes we find it desirable to change a number of dollars and cents all into cents. To do this, merely remove the decimal point from between the dollars and cents and you will have the number of cents. Every one knows that:

$1.00 is 100 cents
$1.25 is 125 cents
$ .25 is 25 cents

Likewise:

$ 12.75 is 1275 cents
$ 247.86 is 24786 cents
$1000.00 is 100000 cents

What we have really done in making these changes is to multiply the dollars by 100 to get the equivalent cents. We have taken a mixed number and multiplied it by 100 because there are 100 cents in a dollar. This operation is performed by moving the decimal point two figures to the right, or placing it after the cents, where it is, of course, useless and is seldom written.

In many problems it is quite desirable to change the dollars to cents and carry the work through as cents. The following example shows clearly such a case.

Example:

During one month a foundry turned out 312,000 lb. of iron castings. The total cost of the iron used, including the cost of melting and pouring was \$3900. What was the cost, in cents, of 1 lb. of iron, melted and poured?

$\$3900 = \$3900.00 = 390000$ cents.

$390000 \div 312000 = 1\frac{78}{312} = 1\frac{1}{4}$ cents, *Answer.*

Explanation: Since the cost of iron, melted and poured, is but 1 or 2 cents, we might as well change the total cost to cents before we divide by the number of pounds. Then we will get the cost directly in cents per pound, as we want it.

33. Reducing Cents to Dollars.—The reduction of cents to dollars is really performed by dividing the number of cents by 100, since there are 100 cents in 1 dollar.

$$217 \text{ cents} = \frac{217}{100}\text{dollars} = \$2\frac{17}{100} = \$2.17$$

Hence, 217 cents = \$2.17

This shows us that the following simple rule can be adopted for this reduction:

To reduce cents to dollars, place a decimal point in the number so as to have two figures to the right of the decimal point.

34. The Mill.—There is another division of U. S. money called the *mill.* A Mill is one-tenth of a cent or one one-thousandth of a dollar.

10 mills = 1 cent
100 mills = 10 cents = 1 dime
1000 mills = 100 cents = 10 dimes = 1 dollar

There is no coin smaller than the cent and, therefore, the mill is merely a name applied in calculations where it is desirable to have some unit smaller than the cent. For example, tax rates are usually given in mills per dollar. A tax rate of 15 mills on the dollar would mean that a person would have to pay 15 mills (or $1\frac{1}{2}$ cents) on each dollar of assessed valuation. Cost accountants generally figure costs down as fine as mills and even, in some cases, to tenths of mills or finer.

In sums of money containing mills, there are three figures following the decimal point. The first and second figures after the decimal point indicate dimes and cents, as before. The third figure indicates mills.

$.014 is 1 cent and 4 mills
It is also 14 mills (since 1 cent = 10 mills).

In multiplying or dividing numbers containing mills, we must place the decimal point in the answer in the same position, that is, three places from the right of the number.

Example:

If your house and lot were assessed at $2000, and the tax rate was 15 mills on the dollar, what would be the amount of your taxes?
15 mills = $.015.

```
$.015
  2000
$ 30.000, Answer.
```

Explanation: 15 mills is 1 cent and 5 mills, or $.015. This is the amount you must pay on each dollar of assessed value. For an assessment of $2000, you would pay 2000 times $.015, which is $30.

35. Wage Calculations.—The chief use that shop men have, within the shop, for calculations concerning money is in connection with their time and wages. With some wage systems the calculations of one's earnings is comparatively simple; with others it seems rather complicated until the systems are thoroughly understood. The simplest systems are, of course, the well-known day-rate and piece-rate systems. By the old day-rate system, the men are paid according to the time put in, without any reference to the work accomplished. The rate may be so much per hour, per day, or per week, but the method of calculating is the same, and the time keeper will use exactly the same process in each case. The pay roll calculations consist merely in multiplying the number of units of time which each man has to his credit by his rate per unit of time; hours by rate per hour, or days by rate per day.

Examples:

1. A machinist puts in 106 hours at 32½ cents per hour. How much money is due him from the company?

```
   106
   .32½
    53
   212
  318
$34.45, Answer.
```

Explanation: The amount due is the product of 106×32½. Since the amount will run into dollars, we write the 32½ cents as dollars, $.32½. The product is $34.45, which is the amount due.

2. The tool-room foreman gets $6.00 a day and has worked 12 days. What is the amount due him?

12
6.00
$72.00, *Answer*.

Explanation: The same process is carried out here except that we have the rate per day times the number of days.

The piece-work system, by setting prices for certain pieces of work and paying according to the work done, rewards the man in exact proportion to the work that he does. In this system, an account is kept of the amount of a man's work and his pay is calculated by multiplying the numbers of pieces by the piece-rates.

Example:

The price for assembling a certain sized commutator is 55 cents each. If a man assembles an order of 14 of them, how much does he receive? What will he average per day if he does the job in three days?

55¢ = $ .55
$ .55 × 14 = $7.70 Amount he receives for the job.
$7.70 ÷ 3 = 2.56$\frac{2}{3}$ Amount he averages per day.

There are many other wage systems, too numerous to be all explained here. They all are planned to take into account both the amount of work a man does and the time that he puts in to do it. One of these systems, the Premium System, is so well known and so successful as to warrant brief mention here. In its most common form this system is as follows:

The men are all placed on a time rate, usually a rate per hour.

A record is kept of every man's time and also of the work done by him.

Every job has a standard time (sometimes called "the limit") allowed for its completion.

On each job, the man's actual time is recorded and also the limit for the job.

The man is paid straight wages for the time put in on the job and, in addition, is paid a premium of usually one-half of the time that he saves below the limit.

If it takes a man a longer time than the limit, he is paid full wages for the time he puts in.

This system is planned to satisfy both parties by giving the efficient man an increased earning, and by giving the firm a share of the time saved, thus giving them a reduced cost whenever they pay higher wages.

Let us take an example to see how one's pay would be figured on this plan.

Example:

In one day, a man whose rate is $27\frac{1}{2}$ cents an hour, does the following premium jobs: the first has a limit of 8 hours and is done in 5 hours; the second has a limit of 5 hours and is done in 3 hours; the third has a limit of 3 hours and is finished in 2 hours. What is his pay for the day?

$5+3+2=10$ hours, actually put in.
$8-5=$ 3 hours saved on the first job.
$5-3=$ 2 hours saved on the second job.
$3-2=$ 1 hour saved on the third job.
$3+2+1=$ 6 hours saved on the days work.

He will get paid for the 10 hours and, in addition, for half of the 6 hours that he saved. Altogether he will be paid for $10+\frac{1}{2}$ of $6=13$ hours.

$$13\times \$.27\frac{1}{2}=\$3.57\frac{1}{2},\ \textit{Answer.}$$

He gets $3.58 for his 10 hours work and, therefore, makes a premium of 83 cents. Meanwhile, the company gets the work done for $3.58, instead of paying $4.40, which it would have cost if the workman had taken the full limit for the work.

PROBLEMS

61. Write in figures the following sums of money:
One dollar and twelve cents
Two dollars and twenty-five cents
Eight cents
Fifteen dollars and thirty-seven and one-half cents
Twenty-five mills

62. Read the following and write them out in words:

$ 2.75	$ .008
$ .03½	$1.08
$16.25	

63. A young man makes the following purchases: Suit of clothes $25, shoes $3.75, hat $2.25, necktie 50 cents. What is the total cost of his purchases?

64. A certain job calls for four $\frac{3}{4}$ in. by 3 in. machine bolts, two $\frac{5}{8}$ in. by $1\frac{3}{4}$ in. set screws, and two $\frac{1}{2}$ in. by 2 in. cap screws. What would be the total cost of the bolts and screws, if the machine bolts are worth $2\frac{1}{4}$ cents each, the set screws $1\frac{1}{4}$ cents each, and the cap screws $1\frac{3}{4}$ cents each?

65. If you bought a house for $3000 and it was assessed at two-thirds of what it cost you, what taxes would you have to pay if the tax rate was 14 mills on the dollar?

66. How much would a man who is paid $4.25 a day earn in a month of 26 working days?

67. If the piece price for a certain job is 4 cents, how many pieces must a man do in one day to make $5.00?

68. An apprentice and a machinist are working together on a piece-work job and they earn $30. They are prorated on the job, which means that the money is divided according to their day-work rates. If the apprentice's rate is 10 cents an hour and the machinist's is 30 cents, what fraction of the money does each get, and how much money would each get?

69. A man rated at 25 cents an hour is working under the premium system. In one day of $9\frac{1}{2}$ hours he performs 3 operations the limits of which are $4\frac{1}{2}$ hours each. If he gets paid a premium of half the time saved, how much will he make for the day?

70. The following figure represents a page from a time book of a shop working entirely on day work. Calculate the total time of each man, the amount of money due him, and the total pay roll for the shop.

Drop Forge · SHOP – TWO WEEKS ENDING Jan. 26th. 1911

NAME	SUN.	MON.	TUES.	WED.	THUR.	FRI.	SAT.	SUN.	MON.	TUES.	WED.	THUR.	FRI.	SAT.	TOTAL HOURS	RATE PER HR.	AMOUNT FOR THE 2 WEEKS
Randall, J. F.		$9\frac{1}{2}$	10	10	10	10	8		10	10	10	10	10	9		.60	
Nelson, O. C.		10	10	10	10	10	9		5	—	10	10	10	9		.45	
Baker, J. C.		8	10	10	10	10	9		10	10	10	10	10	9		.40	
Carter, L. B.		10	10	10	10	10	5		10	10	10	10	10	9		.40	
Mullen T. F		10	10	10	10	10	9		10	10	10	10	10	9		.40	
Holmes, Wm		10	10	10	10	10	9		10	10	10	10	10	9		.40	
Murphy, P. J.		10	10	10	10	10	9		10	10	10	10	10	9		.35	
Kelly, T. P.		10	10	10	10	10	9		10	10	10	10	10	5		.30	
Bodenstab, J.		10	10	10	5	10	9		8	10	10	10	10	9		.25	
Schaub, F. T.	6	10	10	10	10	10	10	5	10	10	10	10	10	9		.20	
Strozinski, P.		10	10	10	10	10	9		10	10	10	10	10	9		.15	
Rynkiewicz J. Z.		10	10	10	10	10	9		10	10	10	5	10	9		.15	
Skawski G. J.		10	10	10	10	10	5		10	10	10	10	10	9		.15	
																TOTAL	

Fig. 5.

CHAPTER V

DECIMAL FRACTIONS

36. What are Decimals?—In the old days, when no machinist pretended to work much closer than $\frac{1}{64}$ in. and the micrometer was unknown, the mechanic had little use for decimals except in figuring his pay. Now, however, we find that micrometer measurements are used so generally that a knowledge of decimal fractions is essential.

A Decimal Fraction is merely a fraction having a denominator of 10, 100, 1000, or some similar multiple of 10. The denominator is never written, however, but a system similar to that used in writing U. S. money is used. A decimal fraction is written by first putting down a period or "decimal point" and then writing the numerator of the fraction after the decimal point in such a manner that the denominator can be understood. Everything that comes after the decimal point (to the right of it) is a fraction, or part of a unit.

In writing sums of money, the first figure after the decimal point indicates dimes or tenths of a dollar; the second figure indicates cents, or hundredths of a dollar; the third figure, if any, indicates mills or thousandths of a dollar. This system has proved so handy that it has been extended to representing fractions of any sort of a unit (not necessarily dollars).

$$.08 \text{ in. means } \frac{8}{100} \text{ in.}$$

$$.25 \text{ in. means } \frac{25}{100} \text{ in.}$$

$$1.256 \text{ in. means } 1\frac{256}{1000} \text{ in.}$$

Let us take a decimal, say .253, and find out its meaning. We said that the first figure was tenths; the second, hundredths; the third, thousandths, and so on. Then .253 would be $\frac{2}{10} + \frac{5}{100} + \frac{3}{1000}$. This is not a very handy system unless there is some easier way to read it. If we reduce these to a common denominator and add them, we get:

$$.253 = \frac{2}{10} + \frac{5}{100} + \frac{3}{1000} = \frac{200}{1000} + \frac{50}{1000} + \frac{3}{1000} = \frac{253}{1000}$$

$$\text{Then } .253 \text{ is } \frac{253}{1000}$$

This shows that one place to the right of the decimal point indicates a denominator of 10, two places a denominator of 100, three places a denominator of 1000, and so on. The number at the right of the decimal point can, therefore, be taken as the numerator, and the denominator obtained as follows: Put down a 1 below the decimal point and a cipher (0) after it for each figure in the numerator. This will give the denominator. In the case just given, we would have $\frac{;253}{1000}$ showing that the denominator is 1000.

In the same manner:

$.2$ is $\frac{2}{10}$, or *two-tenths.*

$.37$ is $\frac{37}{100}$, or *thirty-seven one-hundredths.*

$.526$ is $\frac{526}{1000}$, or *five hundred twenty-six one-thousandths.*

$.2749$ is $\frac{2749}{10000}$, or *two thousand seven hundred forty-nine ten-thousandths.*

$.042$ is $\frac{42}{1000}$, or *forty-two one-thousandths.*

The last case (.042) presents an interesting problem. Here we have a numerator so small in respect to the denominator that it is necessary to have a cipher, or zero (0) between it and the decimal point, in order that the denominator can be indicated correctly. Let us see how we would go about writing such a common fraction into a decimal. Take $\frac{5}{1000}$. If we merely wrote .5 that would be $\frac{5}{10}$ and would, therefore, not be right. From the rule for finding denominators of decimals we see that there must be as many figures after the decimal point as there are ciphers in the denominator. In this case the denominator (1000) has 3 ciphers, so we must have three figures in our decimal. We, therefore, put two ciphers to the left of the 5 and then put down the decimal point. We now have .005, which can be easily seen to be $\frac{5}{1000}$.

One thing that must be carefully borne in mind is that adding ciphers *after* a decimal does not change the value of the fraction. .5 is the same in value as .50 or .500 because $\frac{5}{10}$ is the same in value as $\frac{50}{100}$ or $\frac{500}{1000}$. On the other hand, ciphers immediately following the decimal point do affect the value of the fraction, as has just been shown.

Mixed numbers are especially easy to handle by decimals, because the whole number and the fraction can be written out in a horizontal line with the decimal point between them. We read mixed decimals just as we would any mixed number—first the whole number, then the numerator, and lastly, the denominator.

Example:

42137.24697

In this example, 42137 is the whole number, and .24697 is the fraction. The number reads "forty-two thousand one hundred thirty-seven *and* twenty-four thousand six hundred ninety-seven *hundred-thousandths.*

The names and places to the right and left of the decimal point are as follows:

Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units	Decimal Point	Ten*ths*	Hundred*ths*	Thousand*ths*	Ten-thousand*ths*	Hundred-thousand*ths*	Million*ths*
7	**6**	**5**	**4**	**3**	**2**	**1**	**.**	**1**	**2**	**3**	**4**	**5**	**6**

37. Addition and Subtraction.—Knowing that all figures to the right of the decimal point are decimal parts of 1 thing and that all figures to the left are whole numbers and represent whole things, it will be seen readily that in addition and subtraction the figures must be so placed that the decimal points come under each other. As was shown under U. S. Money, the operations can then be carried out just as if we were dealing with whole numbers.

Examples:

Addition	Subtraction
783.5	22.7180
21.473	1.7042
804.973	21.0138

Pay no attention to the number of figures in the decimal. Place the decimal points in line vertically. You can, if you desire, add ciphers to make the number of decimal places equal in the two numbers. Remember, however, that the ciphers must be added to the *right* of the figures in the decimal. Proceed as in ordinary addition and subtraction, carrying the tens forward in addition and borrowing, where necessary, in subtraction just as with whole numbers.

38. Multiplication.—In multiplication forget all about the decimal point until the work is finished; multiply as usual with whole numbers. Then point off in the product as many decimal places, counting from the right, as there are decimal places in the multiplier and multiplicand together.

Example:

6.685	Multiplicand	(3 places)
5.2	Multiplier	(1 place)
13370		
33425		
34.7620	Product	(4 places)

Since there are three decimal places in one number, and one in the other, we count off in the product four (3+1) places from the right and place the point between the 7 and the 4. The last 0 can be dropped after pointing off the product, giving the result 34.762 (or $34\frac{762}{1000}$). The reason for this can be seen from the following: The whole numbers are 6 and 5. The result must be a little more than $6\times5=30$, and less than $7\times6=42$, since the numbers are more than 6 and 5, and less than 7 and 6. The actual result is 34.762.

The position of the decimal point can be reasoned out in this way for any example, but the quickest way is to point off from the right a number of decimal places equal to the sum of the numbers of decimal places in the multiplier and multiplicand.

Examples:

(a) $.0045 \times 2.7$

```
  .0045 (4 places)
    2.7 (1 place)
  -----
    315
    90
 ------
 .01215 (5 places)
```

(b) $.000402 \times 4.26$

```
  .000402 (6 places)
     4.26 (2 places)
  -------
     2412
     804
    1608
---------
.00171252 (8 places)
```

In the above examples it was necessary to put ciphers before the product in order to get the required number of decimal places. To see the reason for this take a simple example such as $.2 \times .3$ The product is .06 or $\frac{6}{100}$, as can be readily seen if they are multiplied as common fractions $(\frac{2}{10} \times \frac{3}{10} = \frac{6}{100})$. This checks with the rule of adding the number of decimal places in the two numbers to get the number in their product. The product of two proper fractions is always less than either of the fractions, because it is part of a part.

39. Short Cuts.—If we want to multiply or divide a decimal by 10, 100, 1000, or any similar number, the process is very simple. Suppose we had a decimal .145 and then moved the decimal point one place to the right and made it 1.45. The number would then be $1\frac{45}{100}$ or $\frac{145}{100}$ instead of $\frac{145}{1000}$; so we see that moving the decimal point one place to the right has multiplied the original number by 10. Therefore, we see that:

To multiply by 10 move the decimal point *one* place to the *right.*

To multiply by 100 move the decimal point *two* places to the *right.*

For other similar multipliers move the decimal point one place to the right for each cipher in the multiplier. This process is reversed in division, the rules being:

To divide by 10 move the decimal point *one* place to the *left.*

To divide by 100 move the decimal point *two* places to the *left,* etc.

Example:

Reduce 10275 cents to dollars.

$10275 \div 100 = \$102.75$ (Decimal point moved two places to left).

40. Division.—The division of decimals is just as easy as the multiplication of them after one learns to forget the decimal point entirely until the operation of dividing is finished. Divide as in simple numbers. Then point off from the right as many decimal places in the quotient as the number of decimal places in the dividend *exceeds* that in the divisor. In other words, we subtract the number of decimal places in the divisor from the number in the dividend and point this number off from the right in the quotient.

Example:

Divide 105.587 by .93

```
.93)105.587(113.5+, Answer.
     93
     125
      93    Dividend, 3 places
     328     Divisor,  2 places
     279    Quotient, 1 place
      497
      465
       32    Remainder.
```

Explanation: The number of places in the dividend is 3, and in the divisor 2. Hence, when the division is completed, we point off $3-2=1$ place in the quotient. The + sign after the quotient means that the division did not come out even, but that there was a remainder and that the quotient given is not complete. If desired, ciphers could be placed after the dividend and the division carried farther, giving more decimal places in the quotient.

It makes no difference if the divisor is larger than the dividend, as in the following example. In such a case the quotient will be entirely a decimal.

Example:

$22.762 \div 84.25 = ?$

```
84.25)22.762000(.2701+ or .2702, Answer.
      16 850
       5 9120
       5 8975
         14500
          8425
          6075
```

Explanation: The divisor being larger than the dividend, the quotient turns out to be an entire decimal. In this case we will presume that we wanted the answer to four decimal places. We have, therefore, added ciphers to the dividend until we have six decimal places. When these have all been used in the division, we have $6-2=4$ places in the quotient. The remainder is more than half of the divisor, showing that if we had carried the division to another place, the next figure would have been more than 5. We, therefore, raise the last figure (1) of the quotient to 2, because this is nearer the exact quantity.

In stopping any division this way, if the next figure of the quotient would be less than 5, let the quotient stand as it is, but, if the next figure would be 5 or more, as in the example just worked, raise the last figure of the quotient to the next higher figure.

Sometimes the decimal places are equal in dividend and divisor, as for instance, if we divide .28 by .07.

```
.07).28
      4
```

As the numbers of decimal places in the dividend and divisor are the same, the difference between them is zero, and there are no decimal places in the quotient. The answer is simply 4. The decimal point would come after the 4 where it would, of course, be useless.

If there are more decimal places in the divisor than in the dividend, add ciphers at the right of the decimal part of the dividend as far as necessary. In counting the decimal places, be sure to count *only the ciphers actually used.*

Examples:

```
      1. ÷ .025 = ?
.025)1.000(40, Answer.
     1 00
        0
        0
```

```
        4.2 ÷ 38.25 = ?
38.25)4.200000(.1098+, Answer.
      3 825
        37500
        34425
         30750
         30600
           150
```

41. Reducing Common Fractions to Decimals.—Common fractions are easily reduced to decimals by dividing the numerator by the denominator. In the case of $\frac{1}{2}$, we divide 1.0 by 2 and get .5 All that is necessary is to take the numerator and place a decimal point after it, adding as many ciphers to the right of the decimal point as are likely to be needed, four being a common number to add, as four decimal places (ten thousandths) are accurate enough for almost any calculations.

If $\frac{1}{32}$ is to be reduced to a decimal, the work is simply an example in long division, the placing of the point being the main thing to consider. Simply divide 1.00000 by 32. This gives .03125 or 3125 one hundred-thousandths.

```
32)1.00000(.03125
     96
      40
      32
       80
       64
       160
       160
```

42. Complex Decimals.—A complex decimal is a decimal with a common fraction after it, such as .12½, .0312½, etc. The fraction is not counted in determining the number of places in the decimal. .12½ is read "twelve and one-half hundredths." .0312½ is read "three hundred twelve and one-half ten-thousandths." To change a complex decimal to a straight decimal, reduce the common fraction to a decimal and write it directly after the other decimal, leaving out any decimal point between them.

Examples:

$$.06\frac{1}{2} = .065 \quad .8\frac{3}{4} = .875 \quad .03\frac{1}{8} = .03125$$

43. The Micrometer.—The micrometer is a device to measure to the thousandth of an inch and is best known to shop men in the form of the micrometer caliper shown in Fig. 6. The whole principle of the micrometer, as generally made, can be said to depend on the fact that $\frac{1}{25}$ of $\frac{1}{40} = \frac{1}{1000}$. The micrometer, as shown in Fig. 6, is made up of the frame or yoke *b*, the anvil *c*, the screw or spindle *a*, the barrel *d*, and the thimble *e*. The

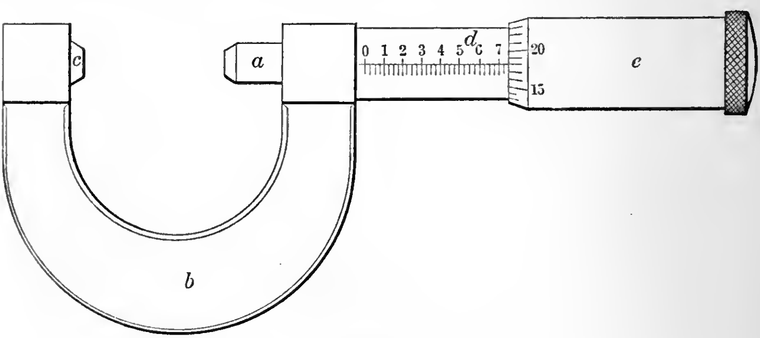


Fig. 6.

spindle *a* is threaded inside of *d*. The thimble *e* is attached to the end of the spindle *a*. The piece to be measured is inserted between *c* and *a*, and the caliper closed on it by screwing *a* against it. The screw on *a* has 40 threads to the inch, so if it is open one turn, it is open $\frac{1}{40}$ in., or $\frac{25}{1000}$, or .025. Along the barrel *d* are marks to indicate the number of turns or the number of fortieths inch that the caliper is open. Four of these divisions ($\frac{4}{40}$) will represent one-tenth of an inch, so the tenths of an inch are marked by marking every fourth division on the barrel.

Around the thimble *e* are 25 equal divisions to indicate parts of a turn. One of these divisions on *e* will, therefore, indicate $\frac{1}{25}$ of a turn, and the distance represented will be $\frac{1}{25}$ of $\frac{1}{40}=\frac{1}{1000}$ in.

To read a micrometer, first set down the number of tenths inch as shown by the last number exposed on the barrel. Count the number of small divisions on the barrel which are exposed between this point and the edge of the barrel. Multiply this number by .025 and add to the number of tenths. Then observe how far the thimble has been turned from the zero point on its edge. Write this number as thousandths of an inch and add to the reading already obtained. The result is the reading in thousandths of an inch.

Example:

Let us read the micrometer shown in Fig. 6.

$$\begin{array}{r} .7 \\ .025 \\ .018 \\ \hline .743 \end{array}$$ in., *Answer.*

Explanation: First we find the figure 7 exposed on the barrel, indicating that we have over $\frac{7}{10}$ in. This we put down as a decimal. In addition, there is one of the smaller divisions uncovered. This is .025 in more. And on the thimble, we find that it is 3 divisions beyond the 15 mark toward the 20 mark. This would be 18, and indicates .018 in. more. Adding the three, $.7+.025+.018=.743$ in., *Answer.* This can perhaps be better understood as being 7 thousandths less than $\frac{3}{4}$ in. Lots of men locate a decimal in their minds by its being just so far from some common fraction.

Most micrometers have stamped in the frame the decimal equivalents of the common fractions of an inch by sixty-fourths from $\frac{1}{64}$ in. to 1 in. A table of these decimal equivalents is given in this chapter, and will be found very useful. Everyone should know by heart the decimal equivalents of the eighths, quarters, and one-half, or, at least, that one-eighth is .125. Then $\frac{5}{8}=5\times.125=.625$; and $\frac{7}{8}=7\times.125=.875$, etc. Also, if possible, learn that $\frac{1}{16}=.062\frac{1}{2}$, or .0625. To get the decimal equivalent of a number of sixteenths, add $.062\frac{1}{2}$ to the decimal equivalent of the eighths next below the desired sixteenths.

Example:

What is the decimal equivalent of $\frac{13}{16}$ in.?

$$\frac{13}{16}=\frac{3}{4}+\frac{1}{16}=.750+.062\frac{1}{2}=.812\frac{1}{2}\text{ in., or }.8125\text{ in.}$$

To set a micrometer to a certain decimal, first unscrew the thimble until the number is uncovered on the barrel corresponding to the number of tenths in the decimal. Divide the remainder by .025. The quotient will be the additional number of the divisions to be uncovered on the barrel and the remainder will give the number of divisions that the thimble should be turned from zero.

Example:

Calculate the setting for $\frac{7}{16}$ in. (= .4375 or .437½).

First unscrew the micrometer until the 4 is uncovered on the barrel. Then divide the remainder .0375 by .025. This gives 1 and leaves a remainder of .0125. The thimble should, therefore, be unscrewed one full turn or 1 division beyond 4 on the barrel, plus 12.5 divisions on the thimble.

TABLE OF DECIMAL EQUIVALENTS FROM $\frac{1}{64}$ TO 1.

Fraction	Decimal Equivalent	Fraction	Decimal Equivalent
$\frac{1}{64}$	.015625	$\frac{33}{64}$	.515625
$\frac{1}{32}$	.03125	$\frac{17}{32}$	.53125
$\frac{3}{64}$	.046875	$\frac{35}{64}$	.546875
$\frac{1}{16}$	.0625	$\frac{9}{16}$	.5625
$\frac{5}{64}$	.078125	$\frac{37}{64}$	.578125
$\frac{3}{32}$	.09375	$\frac{19}{32}$	.59375
$\frac{7}{64}$	.109375	$\frac{39}{64}$	.609375
$\frac{1}{8}$	.125	$\frac{5}{8}$	.625
$\frac{9}{64}$	.140625	$\frac{41}{64}$	.640625
$\frac{5}{32}$	.15625	$\frac{21}{32}$	.65625
$\frac{11}{64}$	.171875	$\frac{43}{64}$	.671875
$\frac{3}{16}$	.1875	$\frac{11}{16}$	.6875
$\frac{13}{64}$	.203125	$\frac{45}{64}$	.703125
$\frac{7}{32}$	.21875	$\frac{23}{32}$	.71875
$\frac{15}{64}$	.234375	$\frac{47}{64}$	.734375
$\frac{1}{4}$	.25	$\frac{3}{4}$	.75
$\frac{17}{64}$	.265625	$\frac{49}{64}$	.765625
$\frac{9}{32}$	.28125	$\frac{25}{32}$	.78125
$\frac{19}{64}$	.296875	$\frac{51}{64}$	.796875
$\frac{5}{16}$	.3125	$\frac{13}{16}$	.8125
$\frac{21}{64}$	.328125	$\frac{53}{64}$	.828125
$\frac{11}{32}$	.34375	$\frac{27}{32}$	.84375
$\frac{23}{64}$	.359375	$\frac{55}{64}$	.859375
$\frac{3}{8}$	.375	$\frac{7}{8}$	.875
$\frac{25}{64}$	.390625	$\frac{57}{64}$	.890625
$\frac{13}{32}$	.40625	$\frac{29}{32}$	.90625
$\frac{27}{64}$	.421875	$\frac{59}{64}$	.921875
$\frac{7}{16}$	.4375	$\frac{15}{16}$	.9375
$\frac{29}{64}$	.453125	$\frac{61}{64}$	.953125
$\frac{15}{32}$	.46875	$\frac{31}{32}$	.96875
$\frac{31}{64}$	.484375	$\frac{63}{64}$	.984375
$\frac{1}{2}$	.5	1	1.00000

PROBLEMS

71. Write the following as decimals:

One and twenty-five one-hundredths.
Three hundred seventy-five one-thousandths.
Three hundred *and* seventy-five one-thousandths.
Sixty-two and one-half one-thousandths.
Seven hundred sixty-five and five one-thousandths.

72. Read the following decimals and write them out in words:

.075
.137
100.037
$.12\frac{1}{2}$
1.09375

73. Find the sum of .2143, 783.5, 138.72, and 10.0041.

74. From 241.70 take 215.875.

75. a. Find the product of 78.8763 $\times$.462.
b. Multiply 21.3 by .071.

76. a. Divide 187.2421 by 123.42.
b. Divide 25 by .0025.

77. Reduce $\frac{1}{16}$ in. to a decimal and compare with the table.

78. Reduce $\frac{17}{32}$ in. to a decimal and compare with the table.

79. Calculate the decimal equivalent of $\frac{1}{24}$ in.

80. Write .8125 as a common fraction and reduce it to the lowest terms.

81. If an alloy is .67 copper and .33 zinc, how many pounds of each metal would there be in a casting weighing 75 lb.?

82. A steam pump delivers 2.35 gallons of water per stroke and runs 48 strokes per minute; how many gallons will it deliver in one hour?

83. The diameter of No. 8 B. W. G. wire is .165 in. and of No. 12 wire is .109 in. What is the difference in diameter of the two wires? What do the letters B. W. G. stand for?

84. A machinist whose rate is 27.5 cents per hour puts in a full day of 10 hours and also 3 hours overtime. If he is paid "time and a half" for overtime, how much should he be paid altogether?

85. The depth of a thread on a $\frac{3}{4}$ in. bolt with U. S. Standard threads is .065 in. What is the diameter at the bottom of the threads?

86. I want 5000 ft. of $\frac{3}{4}$ in. □ (square) steel bars. I find from a table that this size weighs 1.914 lb. per foot of length. How many pounds must I order and what will it cost at $1.85 per 100 lb.?

87. Explain how you would set a micrometer for $\frac{2}{1000}$ in. over $\frac{7}{8}$ in.

88. A 28-tooth 7-pitch gear has an outside diameter of 4.286 in. The diameter at the bottom of the teeth is 3.67 in. How deep are the teeth cut?

89. A 2 in. pipe has an actual inside diameter of 2.067 in. The metal of the pipe is .154 in. thick. What is the outside diameter of the pipe?

90. Read the micrometer shown below in Fig. 7.

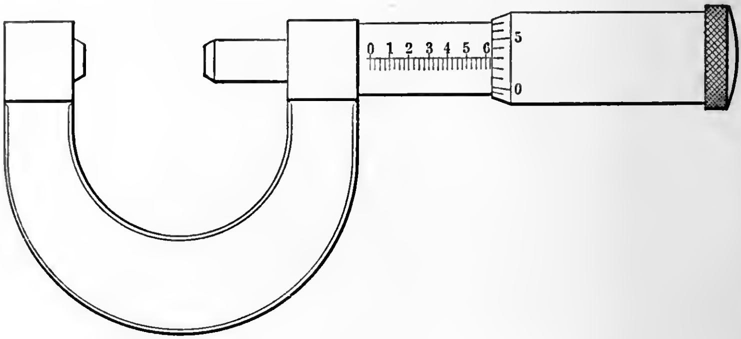


FIG. 7.

CHAPTER VI

PERCENTAGE

44. Explanation.—Percentage is merely another kind of fractions or, rather, a particular kind of decimal fractions, of which the denominator is always 100. Instead of writing the denominator, we use the term "per cent" to indicate that the denominator is 100. When we speak of "6 per cent" we mean $\frac{6}{100}$ or .06. These all mean the same thing; namely, six parts out of one hundred. Instead of writing out the words "per cent" we more often use the sign % after the number, as, for instance, 6%, which means "6 per cent." Since per cent means hundredths of a thing, then the whole of anything is 100% of itself, meaning $\frac{100}{100}$, or the whole. If a man is getting 40 cents an hour and gets an increase of 10%, this increase will be 10% (or $\frac{10}{100}$ or .10) of 40 cents and this is easily seen to be 4 cents, so his new rate is 44 cents. Another way of working this would be to say that his old rate is 100% of itself and his increase is 10% of the old rate, so that altogether he is to get 110% of the old rate. Now 110% is the same as 1.10 and $1.10 \times 40 = 44$ cents, the new rate.

Any decimal fraction may be easily changed to per cent.

$$.875=\frac{87.5}{100}=87.5\%.$$

Here we first change the decimal to a common fraction having 100 for a denominator. Then we drop this denominator and use, instead, the per cent sign (%) written after the numerator. This sign indicates, in this case, 87.5 parts out of 100, or

$$\frac{87.5}{100}$$

The change from a decimal to percentage can be made without changing to a common fraction as was just done. Having a decimal, move the decimal point *two* places to the right and write per cent after the new number.

$$.625=62.5\% \qquad .06=6\% \qquad 1.10=110\%$$

If it is desired to use a certain number of per cent in calculations, it is usually expressed as a decimal first and then the calculations are made. For example, when figuring the interest on $1250 at the rate of 6%, we would first change 6% to .06 and multiply $1250 by .06 which gives $75.00.

$$\begin{array}{r} \$1250 \\ .06 \\ \hline \$75.00 \end{array}$$

A common fraction is reduced to per cent by first reducing it to a decimal and then changing the decimal to per cent.

Example:

The force in a shop is cut down from 85 men to 62. What per cent of the original number of men are retained?

$$62 \text{ is } \frac{62}{85} \text{ of } 85.$$

$$\frac{62}{85}=.729=72.9\%$$

$$62 \text{ is } 72.9\% \text{ of } 85.$$

Therefore, the number of men retained is 72.9% or nearly 73% of the original number of men.

If we want to reduce the fraction $\frac{1}{8}$ to per cent, we first get $\frac{1}{8}=.125$ and then, changing this decimal to per cent, we have $.125=12.5\%$. Then $\frac{1}{8}$ of anything is the same as $12\frac{1}{2}\%$ of it, because $12\frac{1}{2}\%=\frac{12.5}{100}=\frac{1}{8}$.

The following table gives a number of different per cents with the corresponding decimals and common fractions:

Per cent	Decimal	Fraction	Per cent	Decimal	Fraction
1%	.01	$\frac{1}{100}$	25%	.25	$\frac{25}{100}=\frac{1}{4}$
2%	.02	$\frac{2}{100}=\frac{1}{50}$	$33\frac{1}{3}$%	$.33\frac{1}{3}$	$\frac{33\frac{1}{3}}{100}=\frac{1}{3}$
$2\frac{1}{2}$%	.025	$\frac{2\frac{1}{2}}{100}=\frac{1}{40}$	$37\frac{1}{2}$%	.375	$\frac{37\frac{1}{2}}{100}=\frac{3}{8}$
5%	.05	$\frac{5}{100}=\frac{1}{20}$	50%	.50	$\frac{50}{100}=\frac{1}{2}$
$6\frac{1}{4}$%	.0625	$\frac{6\frac{1}{4}}{100}=\frac{1}{16}$	75%	.75	$\frac{75}{100}=\frac{3}{4}$
10%	.10	$\frac{10}{100}=\frac{1}{10}$	90%	.90	$\frac{90}{100}=\frac{9}{10}$
$12\frac{1}{2}$%	.125	$\frac{12\frac{1}{2}}{100}=\frac{1}{8}$	100%	1.00	$\frac{100}{100}=1$
$16\frac{2}{3}$%	$.16\frac{2}{3}$	$\frac{16\frac{2}{3}}{100}=\frac{1}{6}$	200%	2.00	$\frac{200}{100}=2$

45. The Uses of Percentage.—In shop work, the chief use of percentage is to express loss or gain in certain quantities or to state portions or quantities that are used or unused, good or bad, finished or unfinished, etc. Very often we hear expressions like: "two out of five of those castings are bad;" or "nine out of ten of those cutters should be replaced." If, in the first illustration, we wanted to talk on the basis of a hundred castings instead of five, we would say "40 per cent of those castings are bad," because "two out of five" is the same as $\frac{2}{5}, =\frac{40}{100}, =40\%$. And in the second case: "90 per cent of those cutters should be replaced." Here, "nine out of ten" $=\frac{9}{10}, =\frac{90}{100}, =90\%$. If a piece of work is said to be 60% completed, it means that, if we divide the whole work on the job into 100 equal parts, we have already done 60 of these parts or $\frac{60}{100}$ of the whole.

If a shop is running with 50% of its full force, it means that $\frac{50}{100}$ or $\frac{1}{2}$ of the full force is working. If the full force of men is 1300, then the present force is 50% of $1300=.50\times1300=650$. If the full force were 700 men, then the 50% would be 350.

Another very common use of percentage is in stating the portions or quantities of the ingredients going to make up a whole. We often see formulas for brasses, bronzes, and other alloys in which the proportions of the different metals used are indicated by per cents. For example, brass usually contains about 65% copper and 35% zinc. Then, in 100 lb. of brass, there would be 65 lb. of copper and 35 lb. of zinc. Suppose, however, that instead of 100 lb. we wanted to mix a smaller amount, say 8 lb. The amount of copper needed would be 65% or .65 of 8 lb.

$.65\times8=5.20$ lb., or $5\frac{2}{10}$ lb., the copper needed.
$.35\times8=2.80$ lb., or $2\frac{8}{10}$ lb., the zinc needed.

Sometimes, in dealing with very small per cents, we see a decimal per cent such as found in the specifications for boiler steel, where it is stated that the sulphur in the steel shall not exceed .04%. Now this is not 4%; neither is it .04; but it is .04%, meaning four one-hundredths per cent, or *four one-hundredths of one one-hundredth.* This is $\frac{4}{100}$ of $\frac{1}{100}=\frac{4}{10000}$, so if we write this .04% as a decimal, it will be .0004. It is a very common mistake to misunderstand these decimal per cents, and the student should be very careful in reading them. Likewise, be careful in changing a decimal into per cent that the decimal point is shifted *two* places to the right.

46. Efficiencies.—Another common use of percentage is in stating the efficiencies of engines or machinery. The efficiency of a machine is that part of the power supplied to it, that the machine delivers up. This is generally stated in per cent, meaning so many out of each hundred units. If it requires 100 horse-power to drive a dynamo and the dynamo only generates 92 horse-power of electricity, then the efficiency of the dynamo is $\frac{92}{100}$ or 92%. If the engine driving a machine shop delivers 250 horse-power to the lineshaft, but the lineshaft only delivers 200 horse-power to the machines, then the efficiency of the lineshaft is $\frac{200}{250}=.80$ $=80\%$. The other 50 horse-power, or 20%, is lost in the friction of the shaft in its bearings and in the slipping of the belts. The efficiencies of all machinery should be kept as high as possible because the difference between 100% and the efficiency means money lost. The large amount of power that is often lost in line shafting can be readily appreciated when we try to turn a shaft by hand and try to imagine the power that would be required to turn it two or three hundred times a minute.

47. Discount.—In selling bolts, screws, rivets, and a great many other similar articles, the manufacturers have a standard list of prices for the different sizes and lengths and they give their customers discounts from these list prices. These discounts or reductions in price are always given in per cent. Sometimes they are very complicated, containing several per cents to be deducted one after another. Each discount, in such a case, is figured on the basis of what is left after the preceding per cents have been deducted.

Example:

The list price of $\frac{1}{4}$ in. by $1\frac{1}{4}$ in. stove bolts is $1 per hundred. If a firm gets a quotation of 75, 10 and 10% discount from list price, what would they pay for the bolts per hundred?

$100 \times .75 = 75$ cents
$100 - 75 = 25$ cents
$25 \times .10 = 2\frac{1}{2}$ cents
$25 - 2\frac{1}{2} = 22\frac{1}{2}$ cents
$22\frac{1}{2} \times .10 = 2\frac{1}{4}$ cents
$22\frac{1}{2} - 2\frac{1}{4} = 20\frac{1}{4}$ cents.

Explanation: 75, 10 and 10% discount means 75% deducted from the list price, then 10% deducted from that remainder, then 10% taken from the second remainder.

Starting with 100 cents, the list price, we deduct the first discount of 75%. This leaves 25 cents. The next discount of 10% means 10% off from this balance. Deducting this leaves $22\frac{1}{2}$ cents. Next, we take 10% from this, leaving $20\frac{1}{4}$ cents per hundred as the actual cost of these stove bolts.

48. Classes of Problems.—Nearly all problems in percentage can be divided into three classes on the same basis as explained in Article 26. There are three items in almost any percentage problem: namely, the *whole*, the *part*, and the *per cent*. For example, suppose we have a question like this: "If 35% of the belts in a shop are worn out and need replacing, and there are 220 belts altogether, how many belts are worn out?" In this case, the *whole* is the number of belts in the shop, 220. The *part* is the number of belts to be replaced, which is the number to be calculated. The *per cent* is given as 35%.

Any two of these items may be given and we can calculate the missing one. We thus have the three cases:

1. Given the whole and the per cent, to find the part.
2. Given the part and the per cent that it is of the whole, to find the whole.
3. Given the whole and the part, to find what per cent the part is of the whole.

The principles taught under common fractions will apply equally well in working problems under these cases, the only difference being that here a per cent is used instead of a common fraction. In working problems, the per cent should always be changed to a decimal.

One difficulty in working percentage problems is in deciding just what number is the whole (or the base, as it is often called). The following illustration shows the importance of this.

If I offer a man $2000 for his house, but he holds out for $3000, then his price is 50% greater than my offer, while my offer is 33⅓% less than his price. The difference is $1000 either way but, if we take my offer as the base, it would be necessary for me to raise it ½, or 50%, to meet his price. On the other hand, for him to meet my bid, he would only have to cut his price ⅓, or 33⅓%.

Examples of the three types of problems, before mentioned, may help somewhat in getting an understanding of the processes to be used.

Example of Case 1:

How many pounds of nickel are there in 1 ton of nickel-steel containing 2.85% nickel?

1 ton = 2000 lb.
2.85% = .0285
.0285 × 2000 = 57 lb., *Answer.*

Explanation: Here we have the whole (2000 lb.) and the per cent (2.85%) to find the part. After changing the 2.85% to a decimal fraction, the problem becomes a simple problem in multiplication of decimals.

Example of Case 2:

The machines in a small pattern shop require altogether 12 horse-power and are to be driven from a lineshaft by a single electric motor. If we assume that 20% of the power of the motor will be lost in the line shaft and belting, what size motor must we install?

100% − 20% = 80%
80% = .80, or .8
12 ÷ .8 = 15, *Answer.*

Explanation: Here the per cent given (20%) is based on the horse-power of the motor, which is, as yet, unknown. The horse-power of the motor is 100% of itself and, if 20% is lost, then the machines will receive 80%, or .8 of the power of the motor. This is 12 horse-power. The whole will be 12 ÷ .8 = 15 horse-power. This is the size of motor to install.

Example of Case 3:

If the force in a shop is increased from 160 to 200 men, what per cent is the capacity of the shop increased?

200 ÷ 160 = 1.25
1.25 = 125%
125% − 100% = 25%, *Answer.*

Explanation: The present force is 1¼ or 1.25 of the old force, because 200 is $\frac{200}{160}$ of 160, and $\frac{200}{160}$ = 1¼. This is the same as 125%. The increase is, therefore, 25% of the former force.

Note.—To reduce the present force back to the old number would require a reduction of only 20%, because now the base is different on which to figure the per cent. 40 ÷ 200 = .20, or 20%.

PROBLEMS

91. Write 4% as a decimal and as a common fraction.

92. Write 25% as a common fraction and reduce the fraction to its lowest terms.

4

93. What per cent of an inch is $\frac{17}{32}$ in.?

94. If there are 240 men working in a shop and 30% of them are laid off, how many men will be laid off and how many will remain at work?

95. Out of one lot of 342 brass castings, 21 were spoiled and out of another lot of 547, 32 were spoiled. Which lot had the larger per cent of spoiled castings?

96. 500 lb. of bronze bearings are to be made; the mixture is 77% copper, 8% tin, and 15% lead. How many pounds of copper, tin, and lead are required?

Note.—This is a standard bearing mixture used by the Pa. R. R. and by some steam turbine manufacturers.

97. The boss pattern maker is given a raise of 25% on Christmas, after which he finds that he is receiving $130 a month. How much did he get per month before Christmas?

98. In testing a shop drive it was found that the machines driven by one motor required horse-power as follows:

60 in. mill	3.31 horse-power
20 in. lathe	.75 horse-power
48 in. lathe	2.42 horse-power
42 in. by 42 in. by 12 ft. planer	4.82 horse-power
16 in. shaper	.33 horse-power

The total power delivered by the motor was 13.65 horse-power. What per cent of the total power was used in belting and lineshaft? What per cent by the machines?

99. A man who has been drawing $2.50 a day gets his pay cut 10% on May 1, and the following September he is given an increase of 10% of his rate at this time. How much will he get per day after September?

100. The following weights of metals are melted to make up a solder: 18 lb. of tin, 75 lb. of bismuth, 37.5 lb. of lead, and 19.5 lb. of cadmium. What per cent of the total weight is there of each metal?

CHAPTER VII

CIRCUMFERENCES OF CIRCLES; CUTTING AND GRINDING SPEEDS

49. Shop Uses.—In the running of almost any machine, judgment must be used in order to determine the speed which will give the best results. Lathes, milling machines, boring mills, etc., are provided with means for changing the speed, according to the judgment of the operator. Emery wheels and grindstones, however, are often set up and run at any speed which the pulleys happen to give, regardless of the diameter.

If an emery wheel of large size is put on a spindle that has been belted to drive a smaller wheel, the speed may be too great for the larger wheel and, if the difference is considerable, the large wheel may fly to pieces. Every mechanic should know how to calculate the proper sizes of pulleys to use for emery wheels or grindstones, the correct speed at which to run the work in his lathe, or the most economical speeds to use for belts and pulleys. A little data on this subject may be useful and will afford applications for arithmetical principles.

50. Circles.—To understand what has just been mentioned, it is necessary to get a knowledge of circles and their properties. The distance across a circle, measured straight through the center, is called the *Diameter*. Circles are generally designated by their diameters. Thus a 6 in. circle means a circle 6 in. in diameter. Sometimes the *radius* is used. The Radius is the distance from the center to the edge or circumference and is, therefore, just half the diameter. If a circle is designated by the radius, we should be careful to say so. Thus, there would be no misunderstanding if we said "a circle of 5-in. radius"; but unless the word radius is used, we always understand that the measurement given is the diameter. The *Circumference* is the name given to the *distance around* the circle, as indicated in Fig. 8. The circumference of any circle is always 3.1416 times the diameter. In other words, if we measure the diameter with a string and lay this off around the circle, it will go a little over three times.

This number 3.1416 is, without doubt, the most used in practical work of any figure in mathematics. In writing formulas, it is quite common to represent this decimal by the Greek letter π (pronounced "pi"), instead of writing out the whole number. For this reason, the number 3.1416 is given the name "pi."

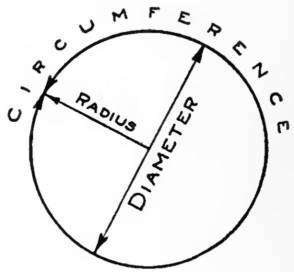


FIG. 8.

Where it is more convenient and extreme accuracy is not required, the fraction $\frac{22}{7}$ may be used for π instead of the more exact value 3.1416.

$$\frac{22}{7}=3\frac{1}{7}=3.1429$$

It, therefore, gives values of the circumference slightly too large, but in many cases it is sufficiently accurate and saves time.

Examples:

1. What length of steel sheet would be needed to roll into a drum 32 in. in diameter?

When rolled up, the length of the sheet will become the circumference of a 32-in. circle. The circumference must be π times 32.

$$3.1416\times 32=100.5+\text{ in.}$$

The length of the sheet must, therefore, be 100½ in. and, if it is to be lapped and riveted, we would have to add a suitable allowance of 1 in. or so for making the joint.

2. A circular steel tank measures 37 ft. 8½ in. in circumference. What is its diameter?

If the circumference of a circle is 3.1416 times the diameter, then the diameter can be obtained by dividing the circumference by 3.1416.

$$37\text{ ft. }8\frac{1}{2}\text{ in.}=37\frac{8.5}{1}\text{ ft.}=37.7+\text{ ft.}$$

$$37.7\div 3.1416=12\text{ ft., } \textit{Answer.}$$

51. Formulas.—A formula, in mathematics, is a rule in which mathematical signs and letters have been used to take the place of words. We say that "the circumference of a circle equals

3.1416 times the diameter." This is a rule. But suppose we merely write

$$C = \pi \times D$$

This is the same rule expressed as a *formula*. We have used C instead of the words "the circumference of a circle;" the sign = replaces the word "equals;" the symbol π is used instead of the number 3.1416; $\times$ stands for "times;" and D stands for "the diameter."

We found in the second example under Article 50 that, when the circumference is given, we can obtain the diameter by dividing the circumference by π. As a formula this would be written

$$D = \frac{C}{\pi} \quad \text{or} \quad D = C \div \pi$$

This arrangement is useful when we want to get the diameters of trees, chimneys, tanks, and other large objects. We can easily measure their circumferences and, by dividing by 3.1416, we get the diameters.

Formulas do not save much, if any space, because it is necessary usually to explain what the letters stand for. They have, however, the great advantage that intricate mathematical operations can be shown much more clearly than if they were written out in a long sentence or statement. One can usually see in one glance at a formula just what is to be done, with the numbers that are given in the problem, to find the quantity that is unknown.

Example:

What is the circumference in feet of a 16-in. emery wheel?

$C = \pi \times D$
$C = 3.1416 \times 16 = 50.2656$ in.
50.2656 in. ÷ 12 = 4.1888 ft., *Answer.*

Explanation: We have the diameter given and want to get the circumference. We, therefore, use the formula which says that $C = \pi \times D$. π is always 3.1416 and D in this case is 16 in. Then C comes out 50.2656 in. But the problem calls for the circumference in feet. This is $\frac{1}{12}$ of the number of inches, or it is the number of inches divided by 12.

In the work of this chapter, the circumferences of circles are always used in feet, and, consequently, should always be calculated in feet. If we use D in feet, we will get C in feet, while, if D is in inches, C will come out in inches. If the diameter can be reduced to exact feet, it is easiest to use the diameter in feet when multiplying by π, rather than to reduce to feet after multiplying.

Example:

What is the circumference of a 48-in. fly wheel?

48 in. ÷ 12 = 4 ft., the diameter.

$C = \pi \times D$

$C = 3.1416 \times 4 = 12.5664$ ft., *Answer.*

This is much shorter than it would be to multiply 3.1416 by 48 and then divide the product by 12.

52. Circumferential Speeds.—When a fly wheel or emery wheel or any circular object makes one complete revolution, each point on the circumference travels once around the circumference and returns to its starting-point. When the wheel turns ten times, the point will have travelled a distance of ten times the circumference. In one minute, it will travel a distance equal to the product of the circumference times the number of revolutions per minute. The distance, in feet per minute, travelled by a point on the circumference of a wheel is called its *Circumferential Speed*, *Rim Speed*, or *Surface Speed*. It is also sometimes called *Peripheral Speed*, because the circumference is sometimes given the name of *periphery*. It is the surface speed by which we determine how to run our fly wheels, belts, emery wheels, and grindstones, and what speeds to use in cutting materials in a machine.

Written as a formula:

$$S = C \times N$$

where:

S is the surface speed

C is the circumference

N is the number of revolutions per minute (R. P. M.).

Expressed in words this formula states that the surface speed of any wheel is equal to the circumference of the wheel multiplied by the number of revolutions per minute.

Example:

What would be the rim speed of a 7 ft. fly wheel when running at 210 revolutions per minute?

$C = \pi \times D$

$C = \frac{22}{7} \times 7 = 22$ ft.

$S = C \times N$

$S = 22 \times 210 = 4620$

4620 ft. per min., *Answer.*

Explanation: First we find the circumference of the wheel, by multiplying the diameter by π. Here is a case where it is much easier to use $\frac{22}{7}$ for π than to use the decimal 3.1416, and the result is sufficiently accurate for our purposes. We get 22 ft. for the circumference. We can now get the rim speed, which is equal to the product of the circumference times the number of revolutions per minute; or $S = C \times N$. C being 22 ft. and N being 210 revolutions per minute, we find that S is 4620 ft. per min. Hence, the rim of this fly wheel travels at a speed of 4620 ft. per minute.

If we have given a certain speed which is wanted and have the circumference of the wheel, then the R. P. M. (revolutions per minute) will be obtained by dividing the desired speed by the circumference. In the example just worked, if we want to give the fly wheel a rim speed of 5280 ft. per minute, it requires no argument to show that the wheel will have to run at $5280 \div 22 = 240$ revolutions per minute. In such a case, we would use our formula in the form

$$N = \frac{S}{C}$$

This formula expresses the same relation as $S = N \times C$, but now it is rearranged to enable us to find the R. P. M. when the rim speed and the circumference are given.

Sometimes, especially with emery wheels, we know the proper surface speed and we have an arbor belted to run a certain number of R. P. M. The problem then is to find the proper size of stone to order.

The desired speed divided by the number of R. P. M. will give the circumference, and from this we can figure the diameter of the stone.

$$C = \frac{S}{N}$$

Here again we have merely rearranged the formula $S = C \times N$ so as to be in more suitable form for finding the circumference when the surface speed and the R. P. M. are given.

53. Grindstones and Emery Wheels.—Makers of emery wheels and grindstones usually give the proper speed for the stones in feet per minute. This refers to the distance that a point on the circumference of the stone should travel in 1 minute and is called the "surface speed" or the "grinding speed."

The proper speed at which to run grindstones depends on the kind of grinding to be done and the strength of the stones. For heavy grinding they can be run quite fast. For grinding edge tools they must be run much slower to get smooth surfaces and to prevent heating the fine edges of the tools. The following surface speeds may be taken as representing good practice:

Grindstones:

For machinists' tools, 800 to 1000 ft. per minute.

For carpenters' tools, 550 to 600 ft. per minute.

Grindstones for very rapid grinding:

Coarse Ohio stones, 2500 ft. per minute.
Fine Huron stones, 3000 to 3400 ft. per minute.

Sometimes the rule is given for grindstones as follows: "Run at such a speed that the water just begins to fly." This is a speed of about 800 ft. per minute and would be a good average speed for sharpening all kinds of tools.

Examples:

1. A 36-in. grindstone, used for sharpening carpenters' and pattern-makers' tools, is run at 60 R. P. M. Is this speed correct?

We must first find the circumference and then the surface speed to see if it falls between the allowable limits.

36 in. $\div 12 = 3$ ft., the diameter
$C = \pi \times D$
$C = 3.1416 \times 3 = 9.4248$ ft.
$S = C \times N$
$S = 9.4248 \times 60 = 565.488$
$S = 565.488$ ft. per minute.

Explanation: First we find the circumference, which comes out 9.4248 ft. Using this and the R. P. M., we find S to be 565 F. P. M. (feet per minute). As this lies between the allowed limits (550 to 600 F. P. M.) the speed of the stone is correct.

2. At what R. P. M. should a 50-in. Huron stone be run if it is to be used for rough grinding?

$C = \pi \times D$
$C = 3.1416 \times 50 = 157.08$ in.
$C = \frac{157.08}{12}$ in. $= 13.09$ ft.
$N = \frac{S}{C}$
Take $S = 3200$ F. P. M.
$N = \frac{3200}{13} = 246. +$ R. P. M.

Explanation: First we find the circumference of the stone in feet, which turns out to be a little over 13 ft. The proper speed is given as 3000 to 3400 F. P. M. Trying 3200 we find that N comes out 246 R. P. M. The stone should, therefore, be belted to run about 240 or 250 R. P. M.

Emery wheels are usually run at a speed of about 5500 ft. per minute. A good, ready rule, easy to remember, is a speed of *a mile a minute.* Most emery wheel arbors are fitted with two pulleys of different diameters. When the wheel is new, the larger pulley on the arbor should be used and, when the wheel becomes worn down sufficiently, the belt should be shifted to the smaller pulley. Never shift the belt on an emery wheel, however, without first calculating the effect on the surface speed of the wheel. Many serious accidents have been caused by emery wheels bursting as a result of being driven at too great a speed. Before cutting a new wheel on an arbor the resultant surface speed should be calculated, to see if the R. P. M. is suitable for the size of the wheel.

Example:

What size wheel should be ordered to go on a spindle running 1700 R. P. M.?

$$C=\frac{S}{N}$$

$$C=\frac{5280}{1700}=3.106 \text{ ft.}$$

$$D=\frac{C}{\pi}$$

$$D=\frac{3.106}{3.1416}=1 \text{ ft., nearly.}$$

$D=$ 12 in. wheel, *Answer.*

Note.—A wheel of exactly 12 in. diameter would, at 1700 R. P. M., have a surface speed of 5340 F. P. M. ($1700\times\pi=5340$).

54. Cutting Speeds.—Cutting speeds on lathe and boring mill work may be calculated in the same way that grinding speeds are calculated. The life of a lathe tool depends on the rate at which it cuts the metal. This cutting speed is the speed with which the work revolves past the tool and is, therefore, obtained by multiplying the circumference of the work by the revolutions per minute. The same formulas are used as in the calculations for emery wheels and grindstones but, of course, the allowable speeds are much different. Tables of proper cutting speeds are given in many handbooks in feet per minute. To find the necessary R. P. M., divide the cutting speed by the circumference of the work.

The cutting speeds used in shops have increased considerably with the advent of the high speed steels. No exact figures can be given for the best speeds at which to cut different metals. The proper speed depends on the nature of the cut, whether finishing or roughing, on the size of the work and its ability to stand heavy cuts, the rigidity and power of the lathe, the nature of the metal being cut, and the kind of tool used. If the work is not very rigid it is, of course, best to take a light cut and run at rather high speed. On the other hand, it is generally agreed that more metal can be removed in the same time if a moderate speed is used and a heavy cut taken.

As nearly as any general rules can be given, the following table gives about the average cutting speeds.

CUTTING SPEEDS IN FEET PER MINUTE

Material	Kind of tool	
	Carbon steel	High speed steel
Cast iron	30 to 40	60 to 80
Steel or wrought iron	25 to 30	50 to 60
Tool steel	20 to 25	40 to 50
Brass	80 to 100	160 to 200

$$\frac{\text{Cutting speed per minute (in feet)}}{\text{Circumference of work (in feet)}} = \text{revolutions per minute.}$$

Example:

A casting is 30 in. in diameter. Find the number of R. P. M. necessary for a cutting speed of 40 ft. per minute.

$$C = \pi \times D = 3.1416 \times 30 = 94.248 \text{ in.}$$

$$\frac{94.248}{12} = 7.854 \text{ ft., circumference.}$$

$$N = \frac{S}{C} = \frac{40}{7.854} = 5.09 \text{ R. P. M., } \textit{Answer.}$$

The same principles apply to milling and drilling, except that in these cases the tool is turning instead of the work. Consequently, the cutting speeds are obtained from the product of the circumference of the tool times its R. P. M.

In calculating the cutting speed of a drill, take the speed of the outer end of the lip or, in other words, the speed of the drill circumference.

Example:

A ½-in. drill is making 300 revolutions per minute; what is the cutting speed?

$$3.1416 \times \frac{1}{2} = 1.5708, \text{ circumference in inches}$$

$$\frac{1.5708}{12} = .131 \text{ ft. (nearly)}$$

$$.131 \times 300 = 39.3 \text{ ft. per minute, cutting speed.}$$

55. Pulleys and Belts.—If the rim of a pulley is run at too great a speed, the pulley may burst. The rim speeds of pulleys are calculated in the same manner as are grinding and cutting speeds. A general rule for cast iron pulleys is that they should not have a rim speed of over a mile a minute (5280 ft. per minute).

This speed may be exceeded somewhat if care is taken that the pulley is well balanced and is sound and of good design.

The proper speeds for belts is taken up fully in a later chapter under the general subject of belting. It is well, however, to point out now that the speed at which any belt is travelling through the air is practically the same as that of the rim of either of the pulleys over which the belt runs; and, if we neglect the small amount of slipping which usually occurs between a belt and its pulleys, we can say that the speed of a belt is the same as the rim speed of the pulleys. It will be seen from this that if two pulleys are connected by a belt, their rim speeds are practically the same.

PROBLEMS

101. A stack is measured with a tape line and its circumference found to be 88 in. What is the diameter of the stack?

102. An emery wheel 16 in. in diameter runs 1300 R. P. M. Find the surface speed.

103. The Bridgeport Safety Emery Wheel Co., Bridgeport, Conn., build an emery wheel 36 in. in diameter and recommend a speed of 425–450 revolutions. Calculate the surface speeds at 425 and at 450 revolutions.

104. An emery wheel runs 1000 R. P. M. What should be its diameter to give a surface speed of 5500 ft.?

105. A grindstone $3\frac{1}{2}$ ft. in diameter is to be used for grinding carpenters' tools; how many R. P. M. should it run?

106. Calculate the belt speed on a high-speed automatic engine carrying a 48 in. pulley and running at 250 R. P. M.

107. How many revolutions will a locomotive driving wheel, 72 in. in diameter, make in going 1 mile?

108. What would be the rim speed in feet per minute of a fly wheel 14 ft. in diameter running 80 R. P. M.?

109. At how many R. P. M. should an 8 in. shaft be driven in a lathe to give a cutting speed of 60 ft. per minute?

110. At what R. P. M. should a $1\frac{1}{4}$ in. high speed drill be run to give a cutting speed of 80 ft. per minute? If the drill is fed .01 in. per revolution, how long will it take to drill through 2 in. of metal?

CHAPTER VIII

RATIO AND PROPORTION

56. Ratios.—In comparing the relative sizes of two quantities, we refer to one as being a multiple or a fraction of the other. If one casting weighs 600 lb., and another weighs 200 lb., we say that the first one is three times as heavy as the second, or that the second is one-third as heavy as the first. This *relation* between two quantities of the same kind is called a *Ratio.*

In comparing the speeds of two pulleys, one of which runs 40 revolutions per minute and the other one 160 revolutions per minute, we say that their speeds are "as 40 is to 160," or "as 1 is to 4." In this sentence, "40 is to 160" is a ratio, and so also is "1 is to 4" a ratio.

Ratios may be written in three ways. For example, the ratio of (or relation between) the diameters of two pulleys which are 12 in. and 16 in. in diameter can be written as a fraction, $\frac{12}{16}$; or, since a fraction means division, it can be written $12 \div 16$; or, again, the line in the division sign is sometimes left out and it becomes 12:16. The last method, 12:16, is the one most used and will be followed here. It is read "twelve *is to* sixteen."

A ratio may be reduced to lower terms the same as a fraction, without changing the value of the ratio. If one bin in the stock room contains 1000 washers, while another bin contains 3000, then the ratio of the contents of the first bin to the contents of the second is "as 1000 is to 3000." The relation of 1000 to 3000 can be reduced by dividing both by 1000. This leaves the ratio 1 to 3.

$$\frac{1000 \div 1000}{3000 \div 1000} = \frac{1}{3}$$

Hence, the ratio between the contents of the bins is also as 1 is to 3.

Likewise, the ratio 24:60 can be reduced to 2:5 by dividing both terms by 12. If we write it as a fraction we can easily see that

$$\frac{24}{60} = \frac{24 \div 12}{60 \div 12} = \frac{2}{5}$$

Therefore, $24:60 = 2:5$.

The ratio of the 1000 washers to the 3000 washers is 1000:3000 or 1:3.

The ratio of 8 in. to 12 in. is 8:12 or 2:3.

The ratio of \$1 to \$1.50 is $1:1\frac{1}{2}$ or 2:3.

The ratio of 30 castings to 24 castings is 30:24 or 5:4.

57. Proportion.—When two ratios are equal, the four terms are said to be *in proportion.* The two ratios 2:4 and 8:16 are clearly equal, because we can reduce 8:16 to 2:4 and we can therefore write 2:4 = 8:16. When written thus, these four numbers form a *Proportion.*

Likewise, we can say that the numbers 6, 8, 15, and 20 form a proportion because the ratio 6:8 is equal to the ratio of 15:20.

$$6:8 = 15:20$$

Now, it will be noticed that, if the first and fourth terms of this proportion be multiplied together, their product will be equal to the product of the second and third terms:

(First term)		(Second term)		(Third term)		(Fourth term)
6	:	8	=	15	:	20

$$6 \times 20 = 120$$
$$8 \times 15 = 120$$
$$\text{Therefore, } 6 \times 20 = 8 \times 15$$

This is true of any proportion and forms the basis for an easy way of working practical examples, where we do not know one term of the proportion, but know the other three. The first and fourth terms are called the *Extremes*, and the second and third are called the *Means*. Then we have the rule: "*The product of the means is equal to the product of the extremes.*"

This relation can be very nicely and simply expressed as a formula.

Let a, b, c, and d represent the four terms of any proportion so that

$$a : b = c : d$$

Then, according to our rule, we have

$$a \times d = b \times c$$

Let us now see of what practical use this is. We will take this example:

If it requires 137 lb. of metal to make 19 castings, how many pounds will it take to make 13 castings from the same pattern?

Now very clearly the ratio between the number of castings 19:13 is the same as the ratio of the weights, but one of the weights we do not know. Writing the proportion out and putting the word "answer" for the number which we are to find, we have

$$19:13 = 137:\text{Answer}$$

From our rule which says the product of the means equals the product of the extremes:

$13 \times 137 = 1781$, product of "means."

This must equal the product of the extremes which would be

$19 \times$ Answer.

Then: $19 \times \text{Answer} = 1781$

$\text{Answer} = 1781 \div 19$

$\text{Answer} = 93.7+$ lb.

In using proportion keep the following things in mind:

(1) Make the number which is the same kind of thing as the required answer the third term. Make the answer the fourth term.

(2) See whether the answer will be greater or less than the third term; if *less*, place the *less* of the other two numbers for the *second* term; if *greater*, place the *greater* of the other two numbers for the *second* term.

(3) Solve by knowing that the product of the means equals the product of the extremes, or by this rule: Multiply the means together and divide by the given extreme; the result will be the other extreme or answer.

Let us see how these rules would be applied to a practical example.

Example:

A countershaft for a grinder is to be driven at 450 R. P. M. by a lineshaft that runs 200 R. P. M. If the pulley on the countershaft is 8 in. in diameter, what size pulley should be put on the lineshaft? A proportion can be formed of the pulley diameters and their revolutions per minute. Applying the rules of proportion, we get the following analysis and solution to the problem.

(1) The diameter of the lineshaft pulley is the unknown answer. The other number of the same kind is the diameter of the countershaft pulley (8 in.). So we have the ratio.

8:Answer

(2) If the countershaft pulley is to run faster, its diameter must be smaller than the other one. Therefore, the answer is greater than 8. Hence, the greater revolutions (450) will be placed as the second term and the other R. P. M. (200) will be the first term. Therefore, we have the completed proportion:

200:450 = 8:Answer

(3) Solving this we get:

$450 \times 8 = 3600$, product of means.

$3600 \div 200 = 18$, *Answer.*

Hence, an 18-in. pulley should be put on the lineshaft to give the desired speed to the countershaft.

Sometimes the letter X is used to represent the unknown number whose value is sought. The following is an example of such a case.

$6:40=5:X$. Find what number X stands for.

$40\times 5=200$, product of the means.

Hence, $6\times X=200$

$$X=\frac{200}{6}=33.3+, \text{ Answer.}$$

58. Speeds and Diameters of Pulleys.—As shown in an example previously worked, if two pulleys are belted together, their diameters and revolutions per minute can be written in a proportion having diameters in one ratio and R. P. M. in the other ratio of the proportion. It will be noticed from the example which was worked, that the numbers which form the means apply to *the same pulley*, while the extremes both refer to the other pulley. Then, since the product of the means equals the product of the extremes, we obtain the following simple relation for pulleys belted together: The product of the diameter and revolutions of one pulley equals the product of the diameter and revolutions of the other. This gives us the following simple rule for working pulley problems.

Rule for Finding the Speeds or Diameters of Pulleys.—Take the pulley of which we know both the diameter and the R. P. M., and multiply these two numbers together. Then divide this product by the number that is known of the other pulley. The result is the desired number.

Examples:

1. A 36-in. pulley running 240 R. P. M. is belted to a 15-in. pulley. Find the R. P. M. of the 15-in. pulley.

$36\times 240=8640$, the product of the known diameter and revolutions.

$8640\div 15=576$, the R. P. M. of the 15-in. pulley, *Answer.*

2. A 36-in. grindstone is to be driven at a speed of 800 R. P. M. from a 6-in. pulley on the lineshaft which is running 225 R. P. M. What size pulley must be put on the grindstone arbor?

$$N=\frac{S}{C}$$

$$=\frac{800}{3.1416\times 3}=85 \text{ R. P. M., nearly.}$$

$6\times 225=1350$

$1350\div 85=16$ in., nearly.

Use a 16-in. pulley on the arbor.

Explanation: First we must find the R. P. M. for the grindstone as explained in Chapter VII. To get the required surface speed we find 85 R. P. M. necessary.

Now we have the R. P. M. and the size of the lineshaft pulley. The product of these two numbers is 1350. Dividing this by the R. P. M. of the grindstone arbor gives 16 in. as the nearest even size of pulley, so we will use that size.

59. Gear Ratios.—The same principles as are applied to pulleys can be applied to gears. If we have two gears running together as shown in Fig. 9, the product of the diameter and R. P. M. of one gear will be equal to the product of the diameter and R. P. M. of the other. In studying gearing, we do not deal with the diameters so much as we do with the numbers of teeth. We find that gears are generally designated by the numbers of teeth. For example, we talk of 16 tooth gears and 24 tooth gears, etc., but we seldom talk about gears of certain diameters.

In making these calculations for gears, we can use the numbers of teeth instead of the diameters. When a gear is revolving, the number of teeth that pass a certain point in one minute will be the product of the number of teeth times the R. P. M. of the gear.

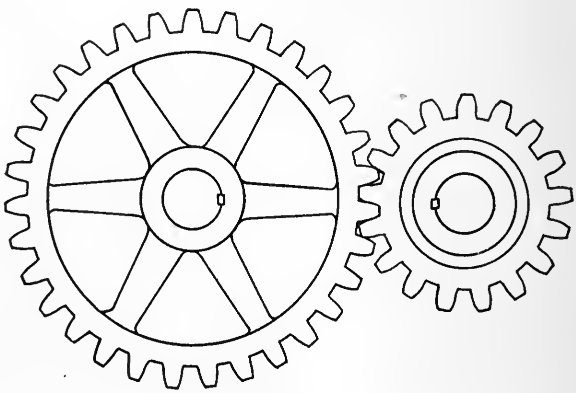


Fig. 9.

If this gear is driving another one, as in Fig. 9, each tooth on the one gear will shove along one tooth on the other one. Consequently, the product of the number of teeth times R. P. M. of the second gear will be the same as for the first gear. This gives us our rule for the relation of the speeds and numbers of teeth of gears.

Rule for Finding the Speeds or Numbers of Teeth of Gears.—Take the gear of which we know both the R. P. M. and the number of teeth and multiply these two numbers together. Divide their product by the number that is known about the other gear. The quotient will be the unknown number.

Example:

A 38 tooth gear running 360 R. P. M. is to drive another gear at 190 R. P. M. What must be the number of teeth on the other gear?

$38 \times 360 = 13{,}680$, the product of the number of teeth and revolutions of one gear.

$190 \times \text{Answer} = 13{,}680$

$$\text{Answer} = \frac{13{,}680}{190}$$

Answer = 72 teeth.

PROBLEMS

111. (a) If you draw $33.00 on pay day and another man draws $22.00, what is the ratio of *your* pay to *his*?

(b) What is the ratio of *his* pay to *yours*?

112. The speeds of two pulleys are in the ratio of 1:4. If the faster one goes 260 R. P. M., how fast does the slower one go?

113. Two castings are weighed and the ratio of their weights is 5:2. If the lighter one weighs 80 lb., what does the heavier one weigh?

114. Find the unknown number in each of the following proportions:

(a) 2:10 = 5:Answer
(b) 6:42 = 5:Answer
(c) 7:35 = 10:X
(d) 6:72 = 8:X

115. If it takes 72 lb. of metal to make 14 castings, how many pounds are required to make 9 castings?

116. A 14 tooth gear is driving a 26 tooth gear. If the 14 tooth gear runs 225 revolutions per minute, what is the speed of the 26 tooth gear?

117. A 12 in. lineshaft pulley runs 280 revolutions and is belted to a machine running 70 revolutions. What must be the size of the pulley on the machine?

118. A lineshaft runs 250 R. P. M. A grinder with a 6 in. pulley is to run 1550 R. P. M. Determine size of pulley to put on the lineshaft to run the grinder at the desired speed.

119. An apprentice was given 100 bolts to thread. He completed three-fifths of this number in 45 minutes and then the order was increased so that it took him 2 hours for the entire lot. How many bolts did he thread?

120. A 42 in. planer has a cutting speed of 30 ft. per minute and the ratio of cutting speed to return speed of the table is 1:2.8. What is the return speed in feet per minute?

CHAPTER IX

PULLEY AND GEAR TRAINS—CHANGE GEARS

60. Direct and Inverse Proportions.—A proportion formed of numbers of castings and the weights of metal required to make them is a *direct proportion*, because the amount of metal required increases directly as the number of castings increases.

When two pulleys (or gears) are running together, one driving the other, the larger of the two is the one that runs the slower. The proportion formed from their diameters and revolutions is, therefore, called an *Inverse Proportion,* because the larger pulley runs at the slower speed. The number of revolutions of one pulley is said to vary *inversely* as its diameter, since the greater the diameter, the less the number of revolutions it will make.

In every pair of gears one of them is driving the other, so the one can be called the *driving gear,* or the *driver,* and the other the *driven gear,* or the *follower.* These names are in quite general use to designate the gears and to assist in keeping the proportions in the right order. Accordingly, we have the proportion:

$$\begin{pmatrix}\text{R. P. M. of}\\ \text{driven}\end{pmatrix} : \begin{pmatrix}\text{R. P. M. of}\\ \text{driver}\end{pmatrix} = \begin{pmatrix}\text{No. of teeth}\\ \text{on driver}\end{pmatrix} : \begin{pmatrix}\text{No. of teeth}\\ \text{on driven}\end{pmatrix}$$

This is an inverse proportion because the driver and the driven are in the reverse order in the second ratio from what they are in the first ratio. Perhaps this can be seen better if the ratios are written as fractions.

$$\frac{\text{R. P. M. of driven}}{\text{R. P. M. of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$$

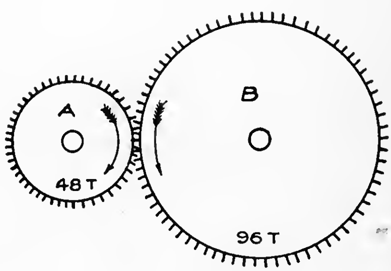


FIG. 10.

Here the reason for the name "*inverse* proportion" is easily seen. The second fraction has the driver and the driven inverted from what they are in the first fraction. This method of writing proportions as fractions is much used in solving problems in gears or pulleys.

61. Gear Trains.—A gear train consists of any number of gears used to transmit motion from one point to another. Fig.

10 shows the simplest form of gear train, having but two gears. Fig. 11 shows the same gears *A* and *B*, as in Fig. 10, but with a third gear, usually called an intermediate gear, between them. The intermediate gear *C* can be used for either of two reasons:

1. To connect *A* and *B* and thus permit of a greater distance between the centers of *A* and *B* without increasing the size of the gears; or

2. To reverse the direction of rotation of either *A* or *B*. If *A* turns in a clockwise direction, as shown in both Figs. 10 and 11, *B* in Fig. 10 will turn in the opposite, or counter-clockwise direction, but in Fig. 11, *B* will turn in the same direction as *A*.

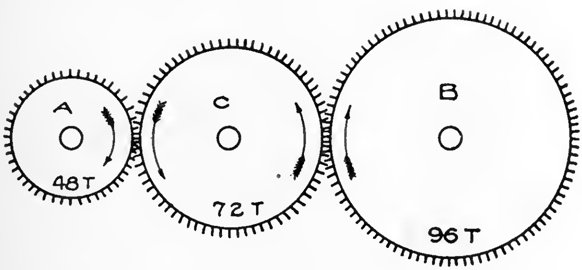


FIG. 11.

The introduction of the intermediate gear *C* has no effect on the speed ratio of *A* to *B*. If *A* has 48 teeth and *B* 96 teeth, the speed ratio of *A* to *B* will be 2 to 1 in either Fig. 10 or Fig. 11.

In Fig. 10 suppose *A* to be the driver.

$$\frac{\text{R. P. M. of driver}}{\text{R. P. M. of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$$

$$\frac{\text{R. P. M. of driver}}{\text{R. P. M. of driven}} = \frac{96}{48} = \frac{2}{1}$$

Hence, the speed ratio of *A* to *B* is 2 to 1.

In the case shown in Fig. 11 when *A* moves a distance of one tooth, the same amount of motion will be given to *C*, and *C* must at the same time move *B* one tooth. To move *B* 96 teeth, or one revolution, will require a motion of 96 teeth on *A*, or two revolutions of *A*. Hence, *A* will turn twice to each one turn of

B, or the speed ratio of *A* to *B* is 2 to 1, just as in the case of Fig. 10.

62. Compound Gear and Pulley Trains.—Quite often it is desired to make such a great change in speed that it is practically necessary to use two or more pairs of gears or pulleys to accomplish it. If a great increase or reduction of speed is made by a single pair of gears or pulleys, it means that the difference in the diameters will have to be very great. The belt drive of a lathe is an example of a compound train of pulleys, though here the train is used chiefly for other reasons. In the first step, the pulley on the lineshaft drives a pulley on the countershaft; then another pulley on the countershaft drives the lathe. The back gearing on a lathe is an example of compound gearing, two pairs of gears being used to make the speed reduction from the cone pulley to the spindle and face plate.

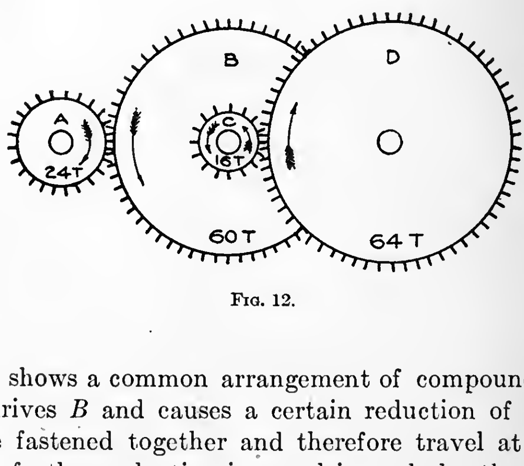


Fig. 12.

Fig. 12 shows a common arrangement of compound gearing. Here *A* drives *B* and causes a certain reduction of speed. *B* and *C* are fastened together and therefore travel at the same speed. A further reduction in speed is made by the two gears *C* and *D*. *A* and *C* are the driving gears of the two pairs and *B* and *D* are the driven gears.

In making calculations dealing with compound gear or pulley trains, we might make the calculations for each pair as explained in Chapter VIII and then proceed to the next pair, etc., but this can be shortened to form a much simpler process.

The speed ratio for a pulley or gear train is equal to the product of the ratios of all the separate pairs of pulleys or gears making up the train.

In using this principle for calculations, the ratios are written as fractions and we have the following formula:

$$\frac{\text{R. P. M. of last driven gear}}{\text{R. P. M. of first driver}} = \frac{\text{Product of Nos. of teeth of all drivers}}{\text{Product of Nos. of teeth of all driven gears}}$$

Or, if we want the ratio stated the other way around—

$$\frac{\text{R. P. M. of first driver}}{\text{R. P. M. of last driven gear}} = \frac{\text{Product of Nos. of teeth of all driven gears}}{\text{Product of Nos. of teeth of all drivers}}$$

Example:

Let us calculate the speed ratio for the train of gears in Fig. 12. This would be the ratio of the speed of A to the speed of D.

A is the first driver and D the last driven gear, and the ratio of their speeds is the ratio for the whole train.

$$\frac{\text{Speed of } A}{\text{Speed of } D} = \frac{\text{Teeth on } B \times \text{Teeth on } D}{\text{Teeth on } A \times \text{Teeth on } C}$$

$$= \frac{\overset{5}{\cancel{60}} \times \overset{\overset{2}{\cancel{4}}}{\cancel{64}}}{\underset{\underset{1}{\cancel{2}}}{\cancel{24}} \times \cancel{16}} = \frac{10}{1}$$

Hence,

Speed of A : Speed of $D = 10:1$

In other words, A revolves 10 times as fast as D.

Problems in getting the speed ratios of pulley trains are solved in the same way except that diameters are used instead of numbers of teeth.

$$\frac{\text{Speed of last driven pulley}}{\text{Speed of first driving pulley}} = \frac{\text{Product of diameters of all driving pulleys}}{\text{Product of diameters of all driven pulleys}}$$

Example:

Let us take the pulley train of Fig. 13 and calculate the ratio of the speeds of pulleys A and D. A and C are the drivers and B and D are the driven pulleys.

$$\frac{\text{Speed of } A}{\text{Speed of } D} = \frac{\text{Diameter of } B \times \text{Diameter of } D}{\text{Diameter of } A \times \text{Diameter of } C}$$

$$= \frac{\overset{5}{\cancel{10}} \times \cancel{12}}{\underset{3}{\cancel{30}} \times \underset{12}{\cancel{24}}} = \frac{5}{36} = \frac{1}{7.2}$$

Hence,

Speed of A : Speed of $D = 1 : 7.2$

Trains are frequently used having combinations of pulleys and gears. In nearly all machine tools, we will find both pulleys and gears between the lineshaft and the work. In wood-working machinery, on the other hand, we usually find only pulleys and belts, on account of the high speeds at which the machines are run. In calculating the speed ratios of these combined trains, we can use the diameters of the pulleys and the numbers of teeth of the gears in the same formula.

$$\frac{\left(\begin{array}{c}\text{R. P. M. of}\\ \text{first driver}\end{array}\right)}{\left(\begin{array}{c}\text{R. P. M. of}\\ \text{last driven}\end{array}\right)}=\frac{\left(\begin{array}{c}\text{Product of diameters}\\ \text{of all driven pulleys}\end{array}\right)\times\left(\begin{array}{c}\text{Product of teeth of}\\ \text{all driven gears}\end{array}\right)}{\left(\begin{array}{c}\text{Product of diameters}\\ \text{of all driving pulleys}\end{array}\right)\times\left(\begin{array}{c}\text{Product of teeth of}\\ \text{all driving gears}\end{array}\right)}$$

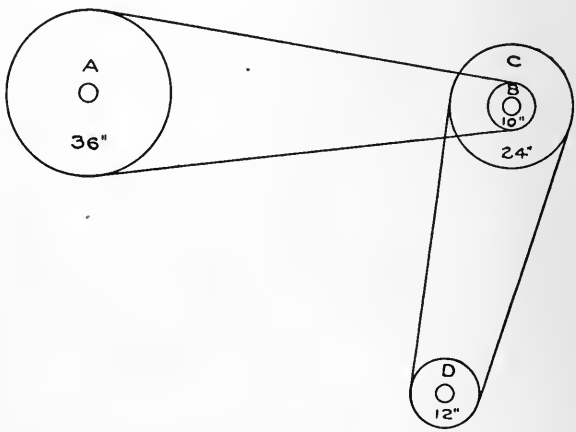


Fig. 13.

If a problem calls for the calculation of the size of one pulley or gear in a train, all the others and the speed ratio being known, start at one or both ends of the train and work toward the gear or pulley in question until you get a proportion which will give the desired quantity.

Example:

The punch shown in Fig. 14 is to be set up so that it will make 20 strokes per minute. (The 80 tooth gear must, therefore, run 20 R. P. M.) The punch is to be driven from a countershaft and we want to calculate the size of the pulley to put on the countershaft, to drive the punch at the desired speed. We find that the main lineshaft runs 240 R. P. M. and carries a 16-in. pulley which drives a 24-in. pulley on the countershaft.

Working from the lineshaft:

$$24 \text{ in.} \times \text{R. P. M. of countershaft} = 16 \times 240.$$

$$\text{R. P. M. of countershaft} = \frac{3820}{24} = 160 \text{ R. P. M.}$$

Working from the punch:

$20 \times$ R. P. M. of 20 T gear $= 80 \times 20$.

R. P. M. of 20 T gear $= \frac{1600}{20} = 80$

This is also the R. P. M. of the 24-in. pulley and this pulley is driven by the unknown pulley on the countershaft, which we have found runs 160 R. P. M.

$160 \times$ Diam. of pulley $= 80 \times 24$

Diam. of pulley $= \frac{1920}{160} = 12$ in., *Answer.*

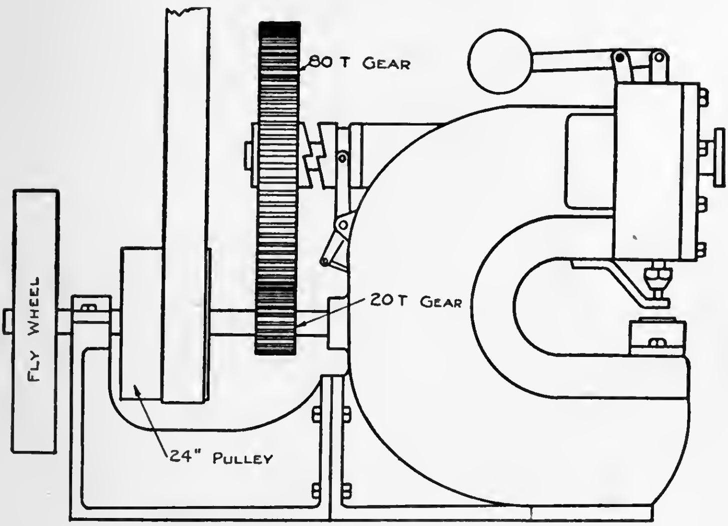


Fig. 11.

Another way to solve this would be to write out an equation for the entire train. using X to represent the pulley whose size we want to find.

$$\frac{20}{240}=\frac{16\times X\times 20}{24\times 24\times 80}$$

$$\frac{1}{12}=\frac{\overset{2}{\cancel{16}}\times X\times \overset{1}{\cancel{20}}}{\underset{3}{\cancel{24}}\times 24\times \underset{\underset{2}{\cancel{4}}}{\cancel{80}}}$$

$$\frac{1}{12}=\frac{1\times X}{6\times 24}=\frac{1}{6}\times\frac{X}{24}$$

Now, if $\frac{X}{24}\times\frac{1}{6}$ is to equal $\frac{1}{12}$, then $\frac{X}{24}$ must be $\frac{1}{2}$, which would be when X is 12.

Hence, $X = 12$ in., *Answer.*

63. Screw Cutting.—Most lathes are equipped with a small plate giving the necessary gears to use for cutting different threads, but every good machinist should know how to calculate the proper gear setting for such work. This is a simple problem in gear trains and should cause no difficulty for the man who understands the principles of gear trains.

The lathe carriage and tool are moved by a "lead screw" having usually 2, 4, 6, or 8 threads per inch. If a lathe has a lead screw having 6 threads per inch, each revolution of the lead screw will move the carriage $\frac{1}{6}$ in.; a 4 pitch screw would move the carriage $\frac{1}{4}$ in. for each revolution of the screw. Then, if the spindle of the lathe and the lead screw turn at the same speed, the lathe will cut a thread of the same pitch as that on the lead screw. If a finer thread is wanted than that on the lead screw, the spindle should make more turns than does the lead screw. Suppose we want to cut 24 threads per inch and have a 6 thread per inch lead screw. It will require 6 turns of the lead screw to move the carriage 1 inch. Meanwhile, the work should revolve 24 times. Then the ratio of spindle speed to lead screw speed should be 4:1

$$\frac{\text{Speed of spindle}}{\text{Speed of lead screw}} = \frac{\text{Threads per inch to be cut}}{\text{Threads per inch on lead screw}}$$

The first driving gear is that on the spindle, while the last driven gear is that on the end of the lead screw. Hence,

$$\frac{\text{Threads per inch to be cut}}{\text{Threads per inch on lead screw}} = \frac{\text{Product of Nos. of teeth on driven gears}}{\text{Product of Nos. of teeth on driving gears}}$$

PROBLEMS

121. In Fig. 10, if we removed the 48 tooth gear and put a 64 tooth gear in its place, what would be the speed ratio of *A* to *B*?

122. In Fig. 11, if *B* makes 6 revolutions, how many turns will *C* make and how many will *A* make?

123. What would be the speed ratio of the train of Fig. 12 if we put a 32T gear on at *C* and a 48T gear at *D*?

124. The lineshaft in Fig. 15 runs 250 R. P. M. Determine the size of lineshaft pulley to run the grinder at 1550 R. P. M. using the countershaft as shown in the figure.

125. A machinist wishes to thread a pipe on a lathe having 2 threads per inch on the lead screw. There are to be $11\frac{1}{2}$ threads per inch on the pipe. What is the ratio of the speeds of the spindle and the lead screw?

126. Two gears are to have a speed ratio of 4.6 to 1. If the smaller gear has 15 teeth, what must be the number of teeth on the larger gear?

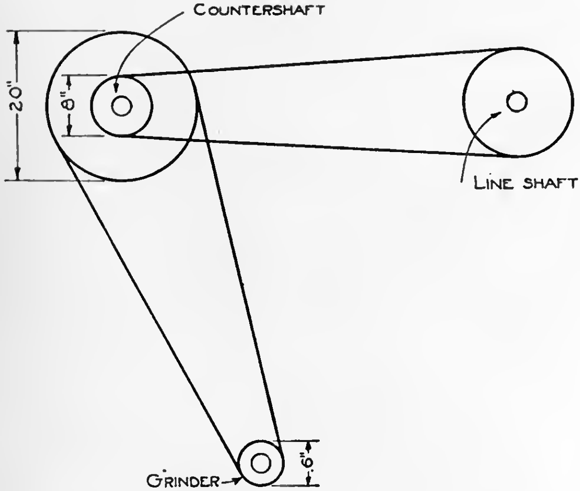


FIG. 15.

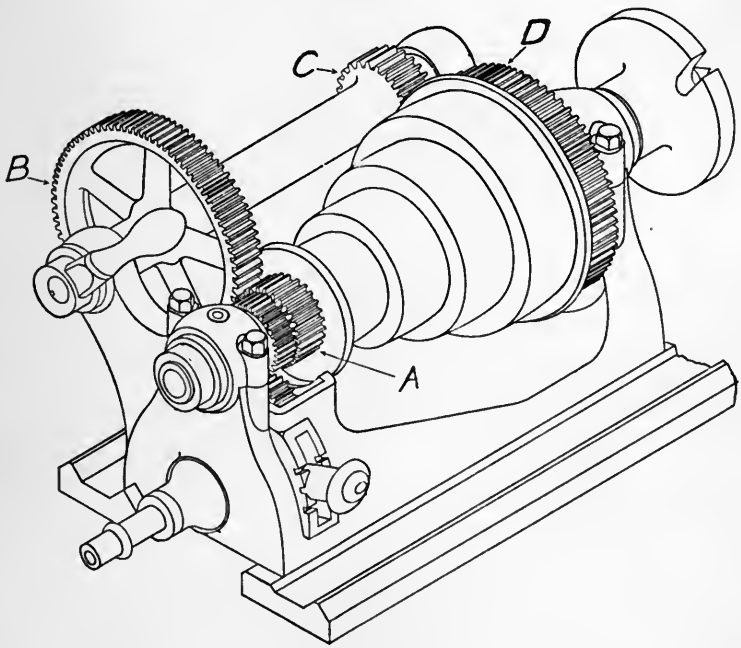


FIG. 16.

127. It has been decided to equip the punch in Fig. 14 with a motor drive by replacing the fly wheel with a large gear to be driven by a small pinion on the motor. If the motor runs 800 R. P. M., and has a 16 tooth pinion, what must be the number of teeth on the other gear? Speed of the punch to be 20 strokes per minute.

128. A street car is driven through a single pair of gears; a large gear on the axle being driven by a smaller one on the motor shaft. If a car has 33-in. wheels and a gear ratio of 1:4, how fast would the car go when the motor is running 1200 R. P. M.?

129. Fig. 16 shows the head stock for a lathe. The cone pulley carries with it the cone pinion *A*, which drives the back gear *B*. *B* is connected solidly with the back pinion *C* which drives the face gear *D*. If the gears have the following numbers of teeth, determine the back gear ratio (speed of *A*: speed of *D*):

Teeth on cone pinion,	*A*	28
Teeth on back gear,	*B*	82
Teeth on back pinion,	*C*	25
Teeth on face gear,	*D*	74

130. If you were to cut a 20 pitch thread on a lathe having a 4 pitch lead screw, what would be the ratio of the speeds of the spindle and the lead screw?

CHAPTER X

AREAS AND VOLUMES OF SIMPLE FIGURES

64. Squares.—In taking up the calculation of areas of surfaces and the volumes and weights of objects, the expressions "square" and "square root" will be met and must be understood. To one unfamiliar with these names and the corresponding operations the signs and operations themselves seem difficult. They are in reality very simple. The square of a number is simply the product of the number multiplied by itself; the square of 2 is $2\times2=4$; the square of 5 is $5\times5=25$; the square of 12.5 is $12.5\times12.5=156.25$. Instead of writing 2×2 or 5×5, it is customary to write 2^2 and 5^2. These are read "2 squared" and "5 squared." $12.5^2=12.5$ squared, and so on. The little 2 at the upper right hand corner is called the *Exponent*.

65. Square Root.—The square root of a given number is simply another number which, when multiplied by itself (or squared), produces the given number. Thus, the square root of 4 is 2, since 2 multiplied by itself (2×2) gives 4. The square root of 9 is 3, since $3\times3=3^2=9$. Square root is the reverse of square, so if the square of 5 is 25 the square root of 25 is 5. The mathematical sign of square root, called the radical sign, is $\sqrt{\ }$. Then $\sqrt{9}=3$; $\sqrt{25}=5$. These expressions are read "the square root of 9=3"; "the square root of 25=5". Square roots of larger numbers can usually be found in handbooks and the actual process of calculating them, which is somewhat complicated, will be taken up later on.

66. Cubes and Higher Powers.—In the same way that 2^2 (2 squared) $=2\times2=4$, 2^3 (2 cubed) $=2\times2\times2=8$. The exponent simply indicates how many times the number is used as a factor, or how many times it is multiplied together. $4^3=4\times4\times4=64$. $3^3=3\times3\times3=27$.

Just as square root is the reverse of square, so cube root is the reverse of cube. The sign for cube root is $\sqrt[3]{\ }$. So if $3^3=3\times3\times3=27$, then $\sqrt[3]{27}=3$. Sometimes a factor is repeated more than 3 times, in which case, the exponent indicates the number of times. 2^4 means $2\times2\times2\times2$ and is read "2 to *the fourth power*." $2^5=2\times2\times2\times2\times2$ and is read "2 to the fifth power," and so on. The roots are indicated in the same way. $\sqrt[4]{16}$ = fourth root of $16=2$; $5^4=5\times5\times5\times5=625$, etc.

67. Square Measure.—Before going further, it will be well to get clearly in mind just what the term "Square" means in terms of the things we see. Areas of figures are measured in terms of the "square" unit. For instance, if the dimensions of the base of a milling machine are 3 ft. by 5 ft., the floor space covered by this base is 15 *square feet*. In this case the area is

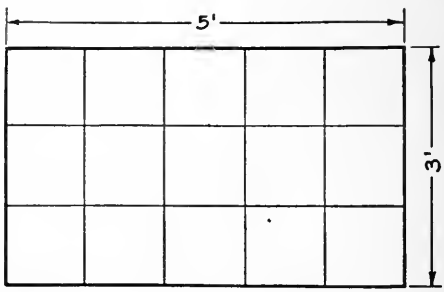


Fig. 17.

measured by the unit known as the square foot. A Square Foot is a surface bounded by a square having each side 1 ft. in length. In case of the milling machine base represented in Fig. 17, there are by actual count 15 sq. ft. in this surface and this is readily seen to be the product of the length and the breadth of the base, since $3\times5=15$.

The Square Inch is another common unit of area. This is much smaller than the square foot, being only one-twelfth as great each way. If a square foot is divided into square inches it will be seen to contain 12×12 or 144 sq. in. (see Fig. 18). It will be readily seen that the area of any square is equal to the product of the side of the square by itself. In other words, the area of a square equals the side "squared" (referring to the process explained in Article 64). Looking at it the other way around, the square of any number can be represented by the area of a square figure, one side of which represents the number itself. The actual things which the number represents makes no difference whatever. If the side of a square is 5 in., the area is

25 sq. in.; if the side is 5 ft., the area is 25 sq. ft. If we simply have the number 5, its square is 25, no matter what kind of things the 5 may refer to.

As mentioned before, 1 sq. ft. is the area of a square 1 ft. on each side and, if divided into square inches, will be found to contain 12^2 or 144 sq. in. Likewise, a square yard is 3 ft. on each side and, therefore, contains $3^2=9$ sq. ft. The following table gives the relation between the units ordinarily used in measuring areas:

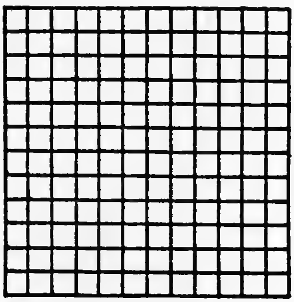


FIG. 18.

MEASURES OF AREA (SQUARE MEASURE)

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A)
640 acres = 1 square mile (sq. mi.)

68. Area of a Circle.—If a circle is drawn in a square as shown in Fig. 19, it is easily seen that it has a smaller area than the square because the corners are cut off. The area of the circle is always a definite part of the area of the square drawn on its diameter, the area of the circle being always .7854 times the area of the square. This number .7854 happens to be just one-fourth of the number 3.1416 given in Chapter VII. Just why this is so will be shown later on. If the diameter of the circle = 10 in., as in Fig. 19, the area of the square is 100 sq. in. and the area of the circle is $.7854\times100=78.54$ sq. in. You can prove this to your own satisfaction in the following manner. Cut a square of cardboard of any size, and from the center describe a circle as shown just touching on all four sides. Weigh the square, and then cut out the circle and weigh it. The circle will weigh .7854×weight of the square. A pair of balances such as are found in a drug store are the best for this experiment.

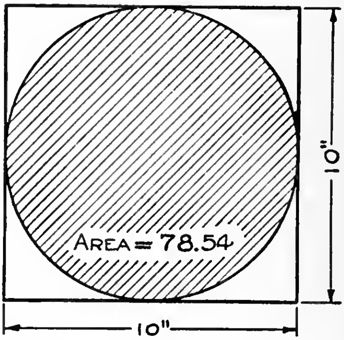


FIG. 19.

Rule for Area of Circle.—The area of any circle is obtained by squaring the diameter and then multiplying this result by .7854. If written as a formula this rule would read

$$A = .7854 \times D^2$$

where A = area of a circle of which D = the diameter.

Example:

Find the area of a circle 3 in. in diameter.

$$A = .7854 \times D^2$$
$$A = .7854 \times 3^2$$
$$= .7854 \times 9$$
$$= 7.0686 \text{ sq. in., } \textit{Answer.}$$

If you think a little you will see that, if the diameter is doubled, the area is increased four times. This can also be seen from Fig. 20. The diameter of the large circle is twice that of one of the small circles, but its area is four times that of one of the small circles. This is a very important and useful law and may be stated as follows: "The areas of similar figures are to each other as the squares of their like dimensions." A 2 in. circle contains $2^2 \times .7854 = 3.1416$ sq. in., while a 6 in. circle contains $6^2 \times .7854 = 28.2744$ sq. in., or nine times as much. This we can find by saying the 6 in. circle has three times the diameter of the 2 in. circle and, therefore, the area is 3^2, or nine times as great. A piece of steel plate 6 in. in diameter weighs nine times as much as a piece 2 in. in diameter of the same thickness. Likewise a 10 in. square has four times the area of a 5 in. square. If we

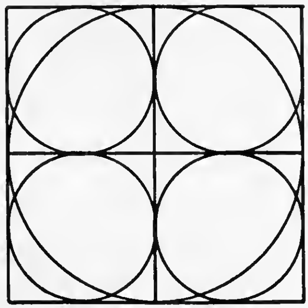


Fig. 20.

let A represent the area of the larger circle, a the area of the smaller circle, D the diameter of the larger circle, and d the diameter of the smaller circle, then we have the direct proportion:

$$A:a=D^2:d^2$$

69. The Rectangle.—When a four-sided figure has square corners it is a Rectangle. Each side of a brick is a rectangle.

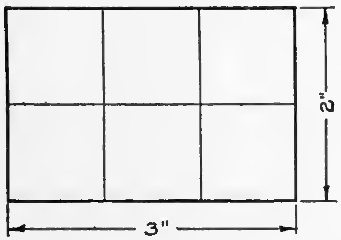


Fig. 21.

A Square is a special kind of rectangle having all the sides equal. The area of a rectangle is obtained by multiplying the length by the breadth. In Fig. 21 the area is $2 \times 3 = 6$ sq. in., as can be seen by counting the 1-in. squares, which each contain 1 sq. in.

70. The Cube.—Just as the square of a number is represented by the area of a square, one side of which represents the number, so the cube of a number is represented by the volume of a cubical block, each edge of which represents the number. The volume of a cube which is 10 in. on each edge is $10 \times 10 \times 10 = 1000$ *cubic inches*, and since this is obtained by "cubing" 10 ($10^3 = 10 \times 10 \times 10 = 1000$), we can see that the cube of a number can be represented by the volume of a cube, the edge of which represents the number. If the edge of the cube is one-half as long, that is 5 in., the volume is $5 \times 5 \times 5 = 125$ cubic inches, or only $\frac{1}{8}$ the volume of the 10 in. cube.

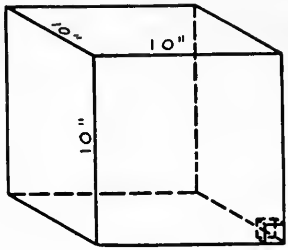


FIG. 22.

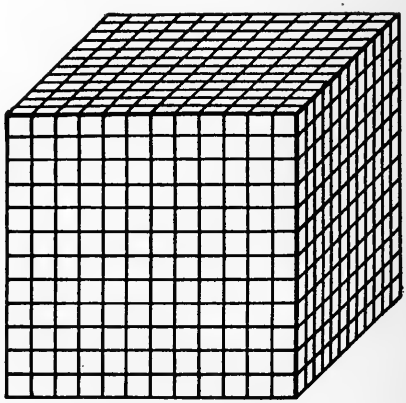


FIG. 23.

MEASURES OF VOLUME (CUBICAL MEASURE)

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
(Larger units than cubic yards are seldom, if ever, used.)

71. Volumes of Straight Bars.—A piece 1 in. long cut from a bar will naturally contain just as many cu. in. as there are sq. in. on the end of the bar. In the billet shown in Fig. 24, there are $3 \times 4 = 12$ sq. in. on the end of the bar, and a piece 1 in. long contains 12 cu. in. The entire billet contains 10 slices just like this one, so there are $12 \times 10 = 120$ cu. in. in the entire billet.

Therefore, we see that to find the number of cu. in. in any straight bar we proceed as follows:

Calculate the area of one end of the bar in square inches; then multiply this result by the length of the bar in inches; the result will be the number of cubic inches in the bar. For bars of square or rectangular section, the volume is the product of the three dimensions, length, breadth, and thickness. If L, B, and T represent the length, breadth, and thickness, and V stands for the volume, then

$$V = L \times B \times T$$

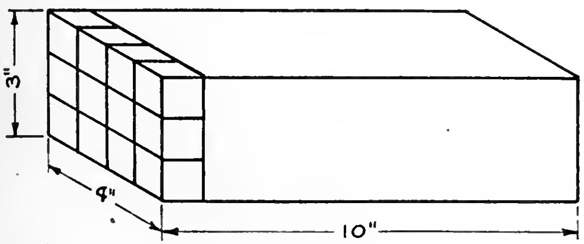


FIG. 24.

Example:

How many cubic inches of steel in a bar 2 in. square and 4 ft. long?

4 ft. = 48 in.

$V = L \times B \times T$

$= 48 \times 2 \times 2 = 192$ cu. in., *Answer.*

For round bars, the area on the end is .7854 times the square of the diameter, and this, multiplied by the length, gives the volume. Then, if D represents the diameter of the bar and L its length, the volume V will be

$$V = .7854 \times D^2 \times L$$

This will apply equally well to thin circular plates or to long bars or shafting. With thin plates, we would naturally speak of thickness (T) instead of length (L). Fig. 25 shows that the two objects have the same shape except that their proportions are different.

Examples:

1. How many cubic inches of steel in a shaft 2 in. in diameter and 12 ft. long?

12 ft. $= 12 \times 12 = 144$ in., length of bar.

$V = .7854 \times D^2 \times L$

$V = .7854 \times 2^2 \times 144$

$V = .7854 \times 4 \times 144 = 452.39$ cu. in., *Answer.*

2. How many cubic inches in a blank for a boiler head 60 in. in diameter and $\frac{7}{16}$ in. thick?

$$V = .7854 \times D^2 \times T$$

$$V = .7854 \times 60^2 \times \frac{7}{16}$$

$$V = .7854 \times 3600 \times \frac{7}{16} = 1237 \text{ cu. in., } \textit{Answer.}$$

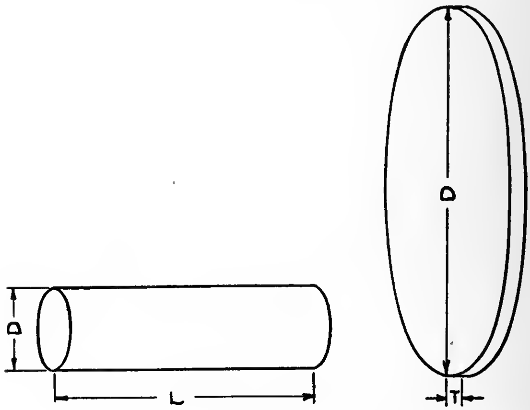


FIG. 25.

72. Weights of Metals.—The chief uses in the shop for calculations of volume are in finding the amount of material needed to make some object; in finding the weight of some object that cannot be conveniently weighed; or in finding the capacity of some bin or other receptacle. Having obtained the volume of an object, it is only necessary to multiply the volume by the known weight of a unit volume of the material to get the weight of the object. In the case of the shaft of which we just got the volume, 1 cu. in. will weigh about .283 lb., so the total weight of the shaft will be

$$452.39 \times .283 = 128.0 + \text{ pounds.}$$

The weight of the boiler head will be

$$1237 \times .283 = 350 + \text{ pounds.}$$

The following table gives the weights per cubic inch and per cubic foot for the most common metals and also for water:

WEIGHTS OF MATERIALS (Pounds)

Material	1 cu. in.	1 cu. ft.
Cast iron	.260 lb.	450 lb.
Wrought iron	.278 lb.	480 lb.
Steel	.283 lb.	489 lb.
Brass	.301 lb.	520 lb.
Copper	.318 lb.	550 lb.
Lead	.411 lb.	711 lb.
Aluminum	.094 lb.	162 lb.
Water	.036 lb.	62.4 lb.

73. Short Rule for Plates.—A flat wrought iron plate $\frac{1}{8}$ in. thick and 1 ft. square will weigh 5 lb., since $12\times12\times\frac{1}{8}=18$ cu. in., and $18\times.278=5$ lb. The rule obtained from this is very easy to remember and is very useful for plates that have their dimensions in exact feet.

Rule.—Weight of flat iron plates = area in square feet × number of eighths of an inch in thickness × 5. This rule can also be used for steel plates by adding 2 per cent. to the result calculated from the above rule.

Example:

Find the weight of a steel plate 30 in. × 96 in. × $\frac{3}{8}$ in.

30 in. $=2\frac{1}{2}$ ft., 96 in. = 8 ft., $\frac{3}{8}$ in. = 3 eighths.

$2\frac{1}{2}\times8\times3\times5=300$ (weight if it were of wrought iron).

2% of 300 = 6 lb.
300 + 6 = 306 lb.; weight of steel plate, *Answer.*

If this weight is calculated by first getting the cubic inches of steel, we get:

$30\times96\times\frac{3}{8}=1080$ cu. in.

$1080\times.283=305.64$ lb. weight of steel plate, *Answer.*

We see that the results check as closely as could be expected and, in fact, different plates of supposedly the same size would differ as much as this because of differences in rolling.

74. Weight of Casting from Pattern.—In foundry work, it is often desired to get the approximate weight of a casting in order to calculate the amount of metal needed to make it. The probable weight of the casting can be obtained closely enough by weighing the pattern and multiplying this weight by the proper number from the following table. In case the pattern contains core prints, the weight of these prints should be calculated and subtracted from the pattern weight before multiplying; or else the total pattern weight can be multiplied first and then the weight of metal which would occupy the same volume as the core print be subtracted from it.

PROPORTIONATE WEIGHT OF CASTINGS TO WEIGHT OF WOOD PATTERNS

For each 1 lb. weight of pattern when made of (less weight of core prints)	Casting will weigh if made of			
	Cast iron	Copper or bronze	Brass	Aluminum
White pine	16.	19.6	18.5	5.9
Mahogany	12.	14.7	14.	4.5
Pear wood	10.2	12.5	11.7	3.8
Birch	10.6	13.	12.3	3.9
Brass	0.84	1.	0.95	0.32
Aluminum	2.6	3.2	3.1	0.95
Cast iron	0.95	1.3	1.2	0.38

PROBLEMS

131. Find the weight of a piece of steel shafting 2 in. in diameter and 20 ft. long.

132. What is the weight of a billet of wrought iron 4 in. square and 2 ft. 8 in. long?

133. What would a steel boiler plate 36 in. by 108 in. by $\frac{1}{2}$ in. weigh?

134. A cast steel cylinder is 42 in. inside diameter, 4 ft. 6 in. long and $1\frac{1}{4}$ in. thick. Find its weight.

135. A steam engine cylinder $4\frac{5}{8}$ in. inside diameter has the cylinder head held on by four studs. When the pressure in the cylinder is 125 lb. per square inch, what is the total pressure on the cylinder head and what is the pull in each stud?

136. 50 studs $2\frac{1}{4}$ in. long and 1 in. in diameter are to be cut from cold rolled steel. Find the length and weight of bar necessary, allowing $\frac{1}{8}$ in. per stud for cutting off.

137. What would be the weight of a $\frac{1}{2}$ in. by 3 in. wagon tire for a 40 in. wheel? (Length of stock = circumference of a $39\frac{1}{2}$ in. circle.)

138. A copper billet 2 in. by 8 in. by 24 in. is rolled out into a plate of No. 10 B. & S. gage. The thickness of this gage is .1019 in. What would be the probable area of this plate in square feet?

139. The steel link shown in Fig. 26 is made of $\frac{3}{4}$ in. round steel (round steel $\frac{3}{4}$ in. in diameter). Find the length of bar necessary to make it and then find the weight of the link.

140. A steel piece is to be finished as shown in the sketch below (Fig. 27). The only stock available from which to make it is 4 in. in diameter. Compute the length of the 4 in. stock which must be upset to make the piece and have $\frac{1}{16}$ extra stock all over for finishing.

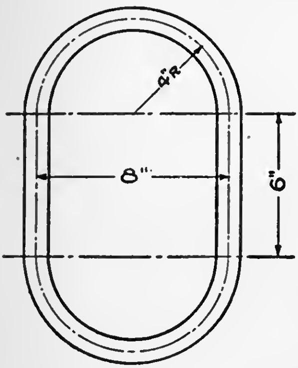


Fig. 26.

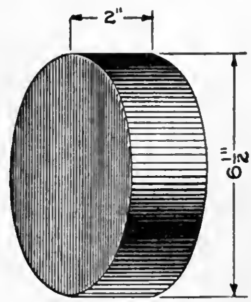


Fig. 27.

CHAPTER XI

SQUARE ROOT

75. The Meaning of Square Root.—The previous chapter showed the usefulness of squares in finding areas and of cubes in finding volumes. Problems often arise in which it is necessary to find one edge of a square or cube of which only the area or volume is given. For instance, what must be the side of a square so that its area will be 9 sq. in.? The length of the side must be such that when multiplied by itself it will give 9 sq. in. A moment's thought shows that $3\times3=9$, or $3^2=9$. Therefore, 3 is the necessary side of the square. Finding such a value is called *Extracting the Square Root,* and is represented by the sign $\sqrt{\ }$ called the square root sign or radical sign. Thus $\sqrt{9}=3$; $\sqrt{16}=4$. To make clear the idea of extracting square roots, the student should consider it as the reverse or "the undoing" of squaring, just as division is the reverse of multiplication or as subtraction is the reverse of addition.

$$5^2=25, \text{ and its reverse is: } \sqrt{25}=5.$$

The square roots of some numbers, like 4, 9, 16, 25, 36, 49, 64, 81, etc., are easily seen, but we must have some method that will apply to any number. There are several methods of finding square root, of which two are open to the student of shop arithmetic: (1) by actual calculation; (2) by the use of a table of squares or square roots. A third method which uses logarithms will be explained in the chapters on logarithms. In many handbooks will be found tables giving the square roots of numbers, but we must learn some method that can be used when a table is not available and the method that will now be explained should be used throughout the work in this chapter.

76. Extracting the Square Root.—The first step in finding the square root of any number is to find how many figures there are in the root. This is done by pointing off the number into periods or groups of two figures each, beginning at the decimal point and working each way.

$1^2=1$ $\qquad$ $10^2=1'00$ $\qquad$ $100^2=1'00'00$

From these it is evident that the number of periods indicates the number of figures in the root. Thus the square root of 103684 contains 3 figures because this number (10'36'84) contains three periods. Also the square root of 6'50'25 contains three figures since there are three periods. (The extreme left hand period may have 1 or 2 figures in it.) We must not forget that, for any number not containing a decimal, a decimal point may be placed at the extreme right of the number. Thus the decimal point for 62025 would be placed at the right of the number (as 62025.)

The method of finding the square root of a number can best be explained by working some examples and explaining the work as we go along. The student should take a pencil and a piece of paper and go through the work, one step at a time, as he reads the explanation.

Example:

Find the square root of 186624.

Point off into periods of two figures each (18'66'24) and it will be seen that there are 3 figures in the root. The work is arranged very similarly to division.

	18'66'24(432
	16
2×40=80	2 66
3	
83	2 49
2×430=860	17 24
2	
862	17 24

Explanation: First find the largest number whose square is equal to or less than 18, the first period. This is 4, since 5^2 is more than 18. Write the 4 to the right for the first figure of the root just as the quotient is put down in long division. The first figure of the root is 4. Square the 4 and write its square (16) under the first period (18) and subtract, leaving 2.

Bring down the next period (66) and annex it to the remainder, giving 266 for what is called the *dividend*. Annex a cipher to the part of the root already found (4) giving 40; then multiply this by 2, making 80, which is called the *trial divisor*. Set this off to the left. Divide the dividend (266) by the trial divisor (80). We obtain 3, which is probably the next figure of the root. Write this 3 in the root as the second figure and also add it to the trial divisor, giving 83, which is the *final divisor*. Multiply this by the figure of the root just found (3) giving 249. Subtract this from the dividend (266) leaving 17.

Bring down the next period (24) and annex to the 17, giving a new dividend 1724. Repeat the preceding process as follows: Annex a cipher to the part of the root already found (43), giving 430; and multiply by 2, giving 860, the trial divisor. Divide the dividend by this divisor and obtain 2 as the next figure of the root. Put this down as the third figure of the root and also add it to the trial divisor, giving 862 as the final divisor. Multiply this by the 2 and obtain 1724, which leaves no remainder when subtracted from the dividend. As there are no more periods in the original number, the root is complete.

77. Square Roots of Mixed Numbers.—If it is required to find the square root of a number composed of a whole number and a decimal, begin at the decimal point and point off periods to right and left. Then find the root as before.

Example:

Find the square root of 257.8623

2'57.86'23'00(16.058+, *Answer.*

	1
20	1 57
6	
26	1 56
3200	1 86 23 (a)
5	
3205	1 60 25
32100	25 98 00
8	
32108	25 68 64
	29 36

Explanation: 1 is the largest number whose square is equal to or less than 2, the first period. Proceeding as before, we get 6 for the second figure. After subtracting the second time (at a) we find that the trial divisor 320 is larger than the dividend 186. In this case, we place a cipher in the root, annex another cipher to 320 making 3200, annex the next period, 23, to the dividend and then proceed as before. If the root proves, as in this case, to be an interminable decimal (one that does not end) continue for two or three decimal places and put a + sign after the root as in division. In this example the decimal point comes after 16, because there must be two *figures* in the whole number part of the root since there are two *periods* in the whole number part of our original number.

78. Square Roots of Decimals.—Sometimes, in the case of a decimal, one or more periods are composed entirely of ciphers. The root will then contain one cipher following the decimal point for each full period of ciphers in the number.

Example:

Take .0007856 as an example.

Beginning at the decimal point and pointing off into periods of two figures each, we have .00'07'85'60. Hence, the first figure of the root must be a cipher. To obtain the rest of the root we proceed as before.

```
.00'07'85'60(.0280+, Answer.
      4
---------------
40  |  3 85
 8  |
--  |
48  |  3 84
---------------
560 |    1 60
```

It will be noticed that the square root of a decimal will always be a decimal. If we square a fraction, we will get a smaller fraction for its square, $(\frac{1}{4})^2 = \frac{1}{16}$; or as a decimal, $.25^2 = .0625$. Therefore, the opposite is true; that, if we take the square root of a number entirely a decimal, will get a decimal, but it will be larger than the one of which it is the square root. Notice the example just given: .0007856 is less than its square root .028.

79. Rules for Square Root.—From the preceding examples the following rules may be deduced:

1. Beginning at the decimal point separate the number into periods of two figures each. If there is no decimal point begin with the figure farthest to the right.

2. Find the greatest whole number whose square is contained in the first or left-hand period. Write this number as the first figure in the root; subtract the square of this number from the first period, and annex the second period to the remainder.

3. Annex a cipher to the part of the root already found and multiply by 2; this gives the trial divisor. Divide the dividend by the trial divisor for the second figure of the root and add this figure to the trial divisor for the complete divisor. Multiply the complete divisor by the second figure in the root and subtract this result from the dividend. (If this result is larger than the dividend, a smaller number must be tried for the second figure of the root.)

Bring down the third period and annex it to the last remainder for the new dividend.

4. Repeat rule 3 until the last period is used, after which, if any additional decimal places are required, annex cipher periods and continue as before. If the last period in the decimal should contain but one figure, annex a cipher to make a full period.

5. If at any time the trial divisor is not contained in the dividend, place a cipher in the root, annex a cipher to the trial divisor and bring down another period.

6. To locate the decimal point, remember that there will be as many *figures* in the root to the left of the decimal point as there were *periods* to the left of the decimal point in our original number.

80. The Law of Right Triangles.—One of the most useful laws of geometry is that relating to the sides of a right angled triangle. Fig. 28 shows a right angled triangle, or "right triangle," so

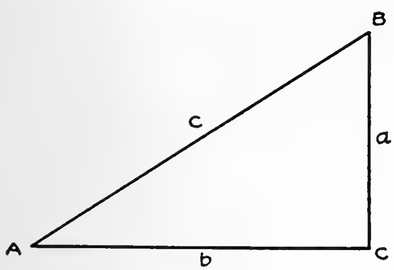


FIG. 28.

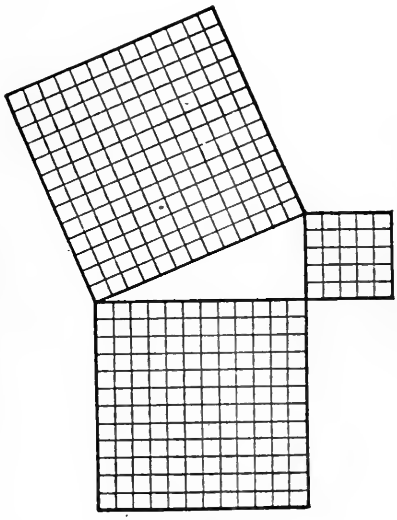


FIG. 29.

called because one of its angles (the one at C) is a right angle, or 90°. The longest side (c) is called the hypotenuse. "In any right triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides." Written as a formula this would read

$$c^2 = b^2 + a^2.$$

This can be illustrated by drawing squares on each side, as in Fig. 29, and noting that the area of the square on the hypotenuse is equal to the sum of the areas of the other two.

In using this rule, however, we do not care anything about these areas and seldom think of them except as being the squares of numbers. It is used to find one side of such a triangle when the other two are known.

Examples:

1. If the trolley pole in Fig. 30 is 24 ft. high, and the guy wire is anchored 7 ft. from the base of the pole, what is the length of the guy wire?

The guy wire is the hypotenuse of a right triangle whose sides are 24 ft. and 7 ft.

$$c^2 = b^2 + a^2$$
$$c^2 = 24^2 + 7^2$$
$$= 576 + 49 = 625$$
$$c = \sqrt{625} = 25 \text{ ft., } \textit{Answer.}$$

2. If the triangle of Fig. 31 is a right triangle having the hypotenuse $c = 13$ in. and the side $a = 5$ in., what is the length of the side b?

$$c^2 = b^2 + a^2$$
Hence, $$b^2 = c^2 - a^2$$
$$b^2 = 13^2 - 5^2$$
$$= 169 - 25 = 144$$
$$b = \sqrt{144} = 12 \text{ in., } \textit{Answer.}$$

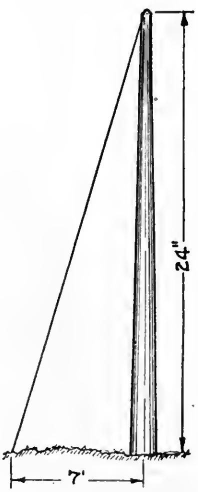


Fig. 30.

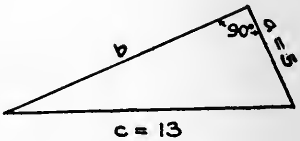


Fig. 31

This property of right triangles is also useful in laying out right angles on a large scale more accurately than it can be done with a square. This is done by using three strings, wires, or chains of such lengths that when stretched they form a right triangle. A useful set of numbers that will give this are 3, 4, and 5, since $3^2 + 4^2 = 5^2$ $(9 + 16 = 25)$.

Any three other numbers having the same ratios as 3, 4, and 5 can be used if desired. 6, 8, and 10; 9, 12, and 15; 12, 16, and 20; 15, 20, and 25; any of these sets of numbers can be used.

A surveyor will often use lengths of 15 ft., 20 ft., and 25 ft. on his chain to lay out a square corner; this method can also be used in aligning engines, shafting, etc.

81. Dimensions of Squares and Circles.—Square Root must be used in getting the dimensions of a square or a circle to have a given area. If the area of a square is given, the length of one side can be obtained by extracting the square root of the area. If we wish to know the diameter of a circle which shall have a certain area, we can find it by the following process:

The area is .7854 × the square of the diameter or, briefly,

$$A = .7854 \times D^2$$

If we divide the given area by .7854, we will get the area of the square constructed around the circle (see Fig. 19).

One side of this square is the same as the diameter of the circle and is equal to the square root of the area of the square.

Then, to find the diameter of a circle to have a given area: Divide the given area by .7854 and extract the square root of the quotient.

$$D = \sqrt{\frac{A}{.7854}}$$

82. Dimensions of Rectangles.—Occasionally one encounters a problem in which he wants a rectangle of a certain area and knows only that the two dimensions must be in some ratio. It may be that a factory building is to cover, say 40,000 sq. ft. of ground and is to be four times as long as it is wide, or some problem of a similar nature. Suppose we take the case of this factory and see how we would proceed to find the dimensions of the building.

Example:

Wanted a factory building to cover 40,000 sq. ft. of ground. Ratio of length to breadth, 4:1. Find the dimensions.

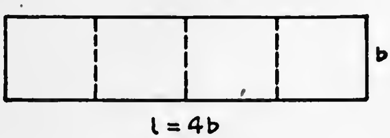


Fig. 32.

$$40000 \div 4 = 10000$$
$$b^2 = 10000$$
$$b = 100$$
$$l = 4 \times 100 = 400.$$

Explanation: If we divide the total area by 4, we get 10,000 as the area of a square having the breadth b on each side. From this we find the breadth or width b to be the square root of 10,000 or 100 ft. If the length is four times as great it will be 400 ft. and the dimensions of the building will be 400 by 100.

83. Cube Root.—The Cube Root of a given number is another number which, when cubed, produces the given number. In other words, the cube root is one of the *three* equal factors of a number. The cube root of 8 is 2, because $2^3=2\times2\times2=8$; also the cube root of 27 is 3 (since $3^3=27$) and the cube root of 64 is 4 (since $4^3=64$).

The sign of cube root is $\sqrt[3]{}$ placed over the number of which we want the root. Thus we would write

$$\sqrt[3]{8}=2 \qquad \sqrt[3]{64}=4 \qquad \sqrt[3]{1000}=10$$

If we consider the number of which we want the cube root as representing the volume of a cubical block, then the cube root of the number will represent the length of one edge of the cube. The cube root of 1728 is 12 and a cube containing 1728 cu. in. will measure 12 in. on each edge.

There are four ways of getting cube roots: (1) by actual calculation, (2) by reference to a table of cubes or cube roots, (3) by the use of logarithms, and (4) by the use of some calculating device like the slide rule.

The use of a table is the simplest way of finding cube roots, but its value and accuracy is limited by the size of the table. Tables of cubes or cube roots are to be found in many handbooks and catalogues and should be used whenever they give the desired root with sufficient accuracy.

Logarithms give us an easy way of getting cube roots, but here also a table is necessary and the accuracy is limited by the size of the table of logarithms. The use of logarithms will be explained in a later chapter. The ordinary pocket slide rule will give the first three figures of a cube root and for many calculations this is sufficiently accurate. The method of actually calculating cube roots is very complicated and is used so seldom that one can never remember it when he needs it. Consequently, if it is necessary to hunt up a book to find how to extract the cube root, one might just as well look up a table of cube roots or a logarithm table, either of which will give the root much quicker. The next chapter contains tables of cube roots and a chapter further on explains the use of logarithms.

PROBLEMS

141. Extract the square roots of

(*a*) 64516 *Answer* 254
(*b*) 198.1369 *Answer* 14.076+
(*c*) .571428 *Answer* .7559+

Note.—These answers are given so that the student can see if he understands the operations of square root before proceeding further.

142. The two sides of a right triangle are 36 and 48 ft.; what is the length of the hypotenuse?

143. A square nut for a 2 in. bolt is 3⅛ in. on each side. What is the length of the diagonal, or distance across the corners?

144. A steel stack 75 ft. high is to be supported by 4 guy wires fastened to a ring two-thirds of the way up the stack and having the other ends anchored at a distance of 50 ft. from the base and on a level with the base. How many feet of wire are necessary, allowing 20 ft. extra for fastening the ends?

145. What would be the diameter of a circular brass plate having an area of 100 sq. in.?

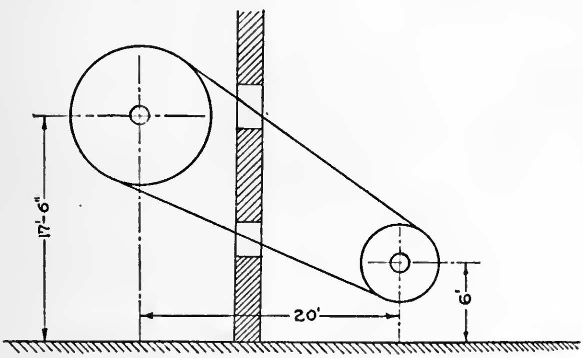


Fig. 33.

146. A lineshaft and the motor which drives it are located in separate rooms as shown in Fig. 33. Calculate the exact distance between the centers of the two shafts.

147. I want to cut a rectangular sheet of drawing paper to have an area of 235 sq. in. and to be one and one-half times as long as it is wide. What would be the dimensions of the sheet?

148. A 6 in. pipe and an 8 in. pipe both discharge into a single header. Find the diameter of the header so that it will have an area equal to that of both the pipes.

149. What would be the diameter of a 1 lb. circular cast iron weight ½ in. thick?

150. How long must be the boom in Fig. 34 to land the load on the 12 ft. pedestal, allowing 4 ft. clearance at the end for ropes, pulleys, etc.?

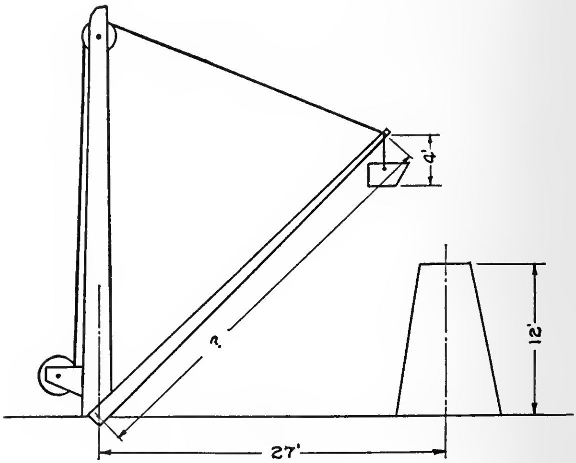


Fig. 34.

CHAPTER XII

MATHEMATICAL TABLES (CIRCLES, POWERS, AND ROOTS)

84. The Value of Tables.—There are certain calculations that are made thousands of times a day by different people in different parts of the world. For example, the circumferences of circles of different diameters are being calculated every day by hundreds and thousands of men. To save much of the time that is thus wasted in useless repetition, many of the common operations and their results have been "tabulated," that is, arranged in tables in the same way as are our multiplication tables in arithmetics. These tables are not learned, however, as were the multiplication tables, but are consulted each time that we have need for their assistance.

Just what tables one needs most, depends on his occupation. The machinist has use for tables of the decimal equivalents of common fractions, tables of cutting speeds, tables of change gears to use for screw cutting, etc. The draftsman would use tables of strengths and weights of different materials, safe loads for bolts, beams, etc., tables of proportions of standard machine parts of different sizes, etc. The engineer uses tables of the properties of steam, and of the horse-power of engines, boilers, etc.

There are certain mathematical tables that are of value to nearly everyone. Among these are the tables given in this chapter: Tables of Circumferences and Areas of Circles; Tables of Squares, Cubes, Square Roots, and Cube Roots of Numbers.

85. Explanation of the Tables.—The first table is to save the necessity of always multiplying the diameter by 3.1416 when we want the circumference of a circle, or of squaring the diameter and multiplying by .7854 when the area of a circle is wanted.

To find the circumference of a circle: Find, in the diameter column, the number which is the given diameter; directly across, in the next column to the right, will be found the corresponding circumference.

Examples:

Diameter $1\frac{1}{4}$	Circumference 3.9270
Diameter 27,	Circumference 84.823
Diameter $90\frac{1}{2}$,	Circumference 284.314

To find the area of a circle: Find, in the diameter column, the number which is the given diameter; directly across, in the *second* column to the right (the column headed "Area") will be found the area.

Examples:

Diameter 66,	Area 3421.2
Diameter 17,	Area 226.98
Diameter $\frac{5}{8}$,	Area 0.3067

If the area or circumference is known and we want to get the diameter, we find the given number in the area or circumference column and read the diameter in the corresponding diameter column to the left.

Examples:

Area	78.54,	Diameter 10
Area	706.86,	Diameter 30
Circumference	281.	Diameter $89\frac{1}{2}$

The second table, that of squares, cubes, square roots, and cube roots, is especially valuable in avoiding the tedious process of extracting square or cube roots. The table is read the same as the other one. Find the given number in the first column; on a level with it, in the other columns, will be found the corresponding powers and roots, as indicated in the headings at the tops of the columns.

Examples:

$$\sqrt[3]{25} = 2.924$$
$$\sqrt{260} = 16.1245$$
$$295^3 = 25,672,375$$
$$865^2 = 748,225$$

86. Interpolation.—This is a name given to the process of finding values between those given in the tables. For example, suppose we want the circumference of a $30\frac{1}{4}$ in. circle. The table gives 30 and $30\frac{1}{2}$ and, since $30\frac{1}{4}$ is half way between these, its circumference will be half way between that of a 30 in. and a $30\frac{1}{2}$ in. circle.

Circumference of $30\frac{1}{2}$ in. circle	$=95.819$
Circumference of 30 in. circle	$=94.248$
The Difference	$= 1.571$

Then the circumference of the $30\frac{1}{4}$ in. circle is just half this difference more than that of the 30-in. circle,

$$94.248+\frac{1}{2} \text{ of } 1.571=95.034$$

This method enables us to increase greatly the value of tables. For most purposes the interpolation can be done quickly, and while it requires some calculating, is much shorter than the complete calculation would be. This is especially true in finding square or cube roots.

Example:

Find from the table the cube root of 736.4

$$\sqrt[3]{737}=9.0328$$
$$\sqrt[3]{736}=9.0287$$
$$\text{Difference}=41$$
$$.4\times\text{the difference}=.4\times 41=16.4$$
$$9.0287$$
$$16$$
$$\sqrt[3]{736.4}=9.0303, \textit{ Answer.}$$

Explanation: The root of 736.4 will be between that of 736 and that of 737, and will be .4 of the difference greater than that of 736. In making this correction for the .4, we forget, for the minute, that the difference is a decimal and write it as 41 merely to save time. We then multiply it by .4, and, dropping the decimal part, add the 16 to the 90287. This gives 9.0303 as the cube root of 736.4

Hence, $\sqrt[3]{736.4}=9.0303$

87. Roots of Numbers Greater than 1000.—For getting the cube roots of numbers greater than 1000, the easiest and most accurate way is to look in the third column headed "cubes" for a number as near as possible to our given number. Now, we know that the numbers in the first column are the cube roots of these numbers in the third column. If we can find our number in the third column, there is nothing further to do because its cube root will be directly opposite it in the first column.

Examples:

$$\sqrt[3]{1728} = 12$$
$$\sqrt[3]{6967871} = 191$$
$$\sqrt[3]{166375} = 55$$

Likewise, the numbers in the first column are the *square* roots of the numbers in the *second* column. But suppose we want the cube root of a number which is not found in the third column, but lies somewhere between two consecutive numbers in that column. In this case we pursue the method shown in the following example:

Example:

Find $\sqrt[3]{621723}$

In the column headed "Cube" find two consecutive numbers, one larger and one smaller than 621723. These numbers are

636056 whose cube root is 86
and 614125 whose cube root is 85

Hence, the cube root of 621723 is more than 85 and less than 86; that is, it is 85 and a decimal, or 85+.

The decimal part is found as follows: Subtract the lesser of the two numbers found in the table from the greater and call the result the *First Difference.*

$$636056 - 614125 = 21931, \text{ First difference.}$$

Then subtract the smaller of the two numbers in the table from the given number and call the result the *Second Difference.*

$$621723 - 614125 = 7598, \text{ Second Difference}$$

Now the first difference, 21931, is the amount that the number increases when its cube root changes from 85 to 86. Our given number is only 7598 more than the cube of 85, so its cube root will be approximately $85\frac{7598}{21931}$. We do not want a fraction like this, so we reduce it to a decimal as follows:

Divide the second difference by the first difference and annex the quotient to 85. This will give us the cube root of our number, approximately.

$$\frac{\text{Second Difference}}{\text{First Difference}} = \frac{7598}{21931} = .346+.$$

This is the decimal part of the root sought and the whole root is 85.346+. Hence $\sqrt[3]{621723} = 85.346+$.

This method is not exact and the third decimal place will usually be slightly off, so it is best to drop the third decimal if less than 5, or raise it to 10, if more than 5. In this case we will call the root 85.35.

88. Cube Roots of Decimals.—In getting the cube root of either a number entirely decimal, or a mixed decimal number, it is best to move the decimal point a number of periods, that is, 3, 6, 9, or 12 decimal places, sufficient to make a whole number out of the decimal. After finding the cube root, shift the decimal point in the root back to the left as many *places* as the number of *periods* that we moved the decimal point in our original number. For example, suppose that we had .621723 of which to find the cube root. Moving the decimal point two periods (of three places each) to the right gives us 621723, of which we just found the cube root to be 85.35. We moved the decimal point of our original number two *periods* to the right, so we must move the decimal point back two *places* to the left in the root; we then have $\sqrt[3]{.621723} = .8535$. The following illustrations will show the principle:

$$\sqrt[3]{621723} = 85.35$$
$$\sqrt[3]{621.723} = 8.535$$
$$\sqrt[3]{.621723} = .8535$$
$$\sqrt[3]{.000621723} = .08535$$

Notice that there is no such similarity between the cube roots of numbers if we move the decimal point any other number of places than a multiple of *three*.

$$\sqrt[3]{6} = 1.817+ \qquad \sqrt[3]{60} = 3.915 \qquad \sqrt[3]{600} = 8.434$$
$$\text{But } \sqrt[3]{6000} = 18.17 \qquad \sqrt[3]{60,000} = 39.15 \qquad \sqrt[3]{600,000} = 84.34$$

Care should be taken, therefore, that, if necessary to move the decimal point in finding a cube root, it should be moved an exact multiple of 3 places. If we have a decimal such as .07462, it is necessary to attach a cipher at the right, making the decimal .074620, so we can shift the decimal point 2 periods or six places. We can now find $\sqrt[3]{74620}$ as follows:

$$43^3 = 79507 \qquad\qquad 74620$$
$$42^3 = 74088 \qquad\qquad 74088$$
$$5419 \text{ First Difference.} \qquad 532 \text{ Second Difference.}$$

$$\frac{532}{5419} = .01 \text{ nearly}$$

$$\text{Hence } \sqrt[3]{74620} = 42.01$$
$$\text{and } \sqrt[3]{.074620} = .4201$$

89. Square Root by the Table.—The same methods as have been explained for cube root can be applied to finding square roots by the use of the table. The only difference is in the case of decimals, in which case the decimal point is shifted by multiples of *two* places in the number; then, after we have the root, we shift the decimal point back one place for each period of two *places* that we moved the decimal point in our original number.

PROBLEMS

The tables are to be used wherever possible in working these problems.

151. What would be the length of a steel sheet from which to make a 28 in. circular drum, allowing $1\frac{1}{4}$ in. extra for lapping and riveting the ends? The circumference of the drum is measured in the direction of the length of the sheet.

152. The small sprocket of a bicycle contains 8 teeth, the large sprocket 24 teeth. The rear wheel is 30 in. in diameter. Find the distance travelled over by the bicycle for one revolution of the pedals.

153. The diameter of a $1\frac{3}{8}$ in. bolt at the bottom of the threads is 1.16 in. What is the sectional area at the bottom of the threads?

154. A No. 000 copper trolley wire has a diameter of 0.425 in. What would be the cost of 1 mile of the wire at 33 cents a pound?

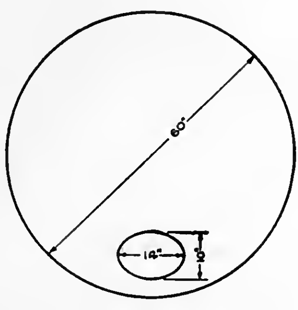


Fig. 35.

155. A circular piece of boiler plate (Fig. 35) $\frac{3}{8}$ in. thick and 60 in. diameter has an elliptical man hole in it 14 in. by 10 in. Find weight of plate.

Note.—Area of an ellipse $=.7854 \times a \times b$, where a and b are the long and short diameters of the ellipse.

156. The paint shop wants a cubical dip tank built to hold, when full, 400 gallons of varnish. What will be the dimensions of the tank? (There are 231 cu. in. in a gallon.)

157. How many square feet of galvanized iron will be needed to line the tank of problem 156 on four sides and the bottom, allowing 10% extra for the joints?

158. A high carbon steel contains the following items in the percentages given: Carbon, .60%; silicon, .10%; manganese, .40%; phosphorus, .035%; sulphur, .025%. The rest is pure iron. Calculate the weights of carbon, silicon, manganese, phosphorus, sulphur, and iron in one ton (2000 lb.) of the steel.

159. I want to get a cast iron block 18 in. long and of square cross-section so that it will weigh 200 lb. How many cubic inches of metal must be in the block and what will be its dimensions?

160. Fig. 36 shows a steam hammer having an 8000 lb. ram. If we assume that this ram is a rectangular block of steel, four times as high and twice as wide as it is thick, what will be the dimensions of the ram?

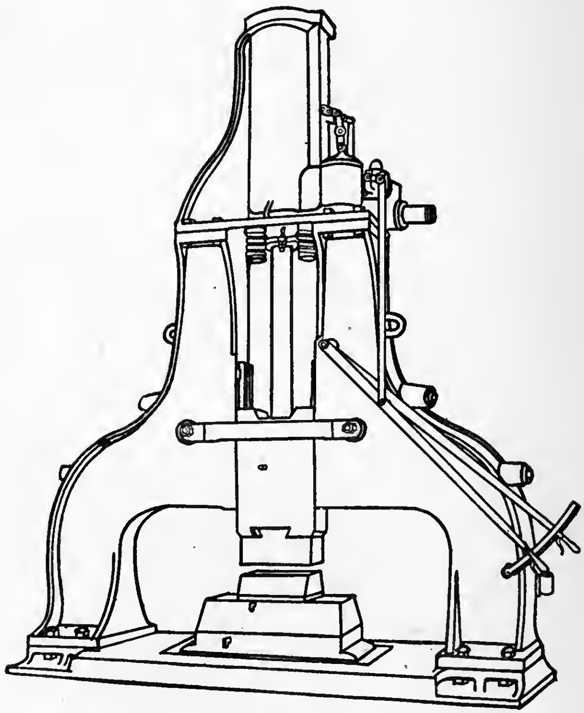


FIG. 36.

CIRCUMFERENCES AND AREAS OF CIRCLES

Diameter	Circumference	Area	Diameter	Circumference	Area
⅛	.3927	0.0123	19½	61.261	298.65
¼	.7854	0.0491	20	62.832	314.16
⅜	1.1781	0.1104	20½	64.403	330.06
½	1.5708	0.1963	21	65.973	346.36
⅝	1.9635	0.3067	21½	67.544	363.05
¾	2.3562	0.4417	22	69.115	380.13
⅞	2.7489	0.6013	22½	70.686	397.61
1	3.1416	0.7854	23	72.257	415.48
1⅛	3.5343	0.9940	23½	73.827	433.74
1¼	3.9270	1.227	24	75.398	452.39
1⅜	4.3197	1.484	24½	76.969	471.44
1½	4.7124	1.767	25	78.540	490.87
1⅝	5.1051	2.073	25½	80.111	510.71
1¾	5.4978	2.405	26	81.681	530.93
1⅞	5.8905	2.761	26½	83.252	551.55
2	6.2832	3.141	27	84.823	572.56
2¼	7.0686	3.976	27½	86.394	593.96
2½	7.8540	4.908	28	87.965	615.75
2¾	8.6394	5.939	28½	89.535	637.94
3	9.4248	7.068	29	91.106	660.52
3¼	10.210	8.295	29½	92.677	683.49
3½	10.996	9.621	30	94.248	706.86
3¾	11.781	11.044	30½	95.819	730.62
4	12.566	12.566	31	97.389	754.77
4½	14.137	15.904	31½	98.960	779.31
5	15.708	19.635	32	100.531	804.25
5½	17.279	23.758	32½	102.102	829.58
6	18.850	28.274	33	103.673	855.30
6½	20.420	33.183	33½	105.243	881.41
7	21.991	38.485	34	106.814	907.92
7½	23.562	44.179	34½	108.385	934.82
8	25.133	50.265	35	109.956	962.11
8½	26.704	56.745	35½	111.527	989.80
9	28.274	63.617	36	113.097	1017.9
9½	29.845	70.882	36½	114.668	1046.3
10	31.416	78.540	37	116.239	1075.2
10½	32.987	86.590	37½	117.810	1104.5
11	34.558	95.033	38	119.381	1134.1
11½	36.128	103.87	38½	120.951	1164.2
12	37.699	113.10	39	122.522	1194.6
12½	39.270	122.72	39½	124.093	1225.4
13	40.841	132.73	40	125.664	1256.6
13½	42.414	143.14	40½	127.235	1288.2
14	43.982	153.94	41	128.805	1320.3
14½	45.553	165.13	41½	130.376	1352.7
15	47.124	176.71	42	131.947	1385.4
15½	48.695	188.69	42½	133.518	1418.6
16	50.265	201.06	43	135.088	1452.2
16½	51.836	213.82	43½	136.659	1486.2
17	53.407	226.98	44	138.230	1520.5
17½	54.978	240.53	44½	139.801	1555.3
18	56.549	254.47	45	141.372	1590.4
18½	58.119	268.80	45½	142.942	1626.0
19	59.690	283.53	46	144.513	1661.9

CIRCUMFERENCES AND AREAS OF CIRCLES.—*Continued*

Diameter	Circumference	Area	Diameter	Circumference	Area
46½	146.084	1698.2	73½	230.907	4242.9
47	147.655	1734.9	74	232.478	4300.8
47½	149.226	1772.1	74½	234.049	4359.2
48	150.796	1809.6	75	235.619	4417.9
48½	152.367	1847.5	75½	237.190	4477.0
49	153.938	1885.7	76	238.761	4536.5
49½	155.509	1924.4	76½	240.332	4596.3
50	157.080	1963.5	77	241.903	4656.6
50½	158.650	2003.0	77½	243.473	4717.3
51	160.221	2042.8	78	245.044	4778.4
51½	161.792	2083.1	78½	246.615	4839.8
52	163.363	2123.7	79	248.186	4901.7
52½	164.934	2164.8	79½	249.757	4963.9
53	166.504	2206.2	80	251.327	5026.5
53½	168.075	2248.0	80½	252.898	5089.6
54	169.646	2290.2	81	254.469	5153.0
54½	171.217	2332.8	81½	256.040	5216.8
55	172.788	2375.8	82	257.611	5281.0
55½	174.358	2419.2	82½	259.181	5345.6
56	175.929	2463.0	83	260.752	5410.6
56½	177.500	2507.2	83½	262.323	5476.0
57	179.071	2551.8	84	263.894	5541.8
57½	180.642	2596.7	84½	265.465	5607.9
58	182.212	2642.1	85	267.035	5674.5
58½	183.783	2687.8	85½	268.606	5741.5
59	185.354	2734.0	86	270.177	5808.8
59½	186.925	2780.5	86½	271.748	5876.5
60	188.496	2827.4	87	273.319	5944.7
60½	190.066	2874.8	87½	274.889	6013.2
61	191.637	2922.5	88	276.460	6082.1
61½	193.208	2970.6	88½	278.031	6151.4
62	194.779	3019.1	89	279.602	6221.1
62½	196.350	3068.0	89½	281.173	6291.2
63	197.920	3117.2	90	282.743	6361.7
63½	199.491	3166.9	90½	284.314	6432.6
64	201.062	3217.0	91	285.885	6503.9
64½	202.633	3267.5	91½	287.456	6575.5
65	204.204	3318.3	92	289.027	6647.6
65½	205.774	3369.6	92½	290.597	6720.1
66	207.345	3421.2	93	292.168	6792.9
66½	208.916	3473.2	93½	293.739	6866.1
67	210.487	3525.7	94	295.310	6939.8
67½	212.058	3578.5	94½	296.881	7013.8
68	213.628	3631.7	95	298.451	7088.2
68½	215.199	3685.3	95½	300.022	7163.0
69	216.770	3739.3	96	301.593	7238.2
69½	218.341	3793.7	96½	303.164	7313.8
70	219.911	3848.5	97	304.734	7389.8
70½	221.482	3903.6	97½	306.305	7466.2
71	223.053	3959.2	98	307.876	7543.0
71½	224.624	4015.2	98½	309.447	7620.1
72	226.195	4071.5	99	311.018	7697.7
72½	227.765	4128.2	99½	312.588	7775.6
73	229.336	4185.4	100	314.159	7854.0

SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS OF NUMBERS

No.	Square	Cube	Square root	Cube root	No.	Square	Cube	Square root	Cube root
1	1	1	1.	1.	46	2116	97336	6.7823	3.5830
2	4	8	1.4142	1.2599	47	2209	103823	6.8557	3.6088
3	9	27	1.7321	1.4422	48	2304	110592	6.9282	3.6342
4	16	64	2.	1.5874	49	2401	117649	7.	3.6593
5	25	125	2.2361	1.7100	50	2500	125000	7.0711	3.6840
6	36	216	2.4495	1.8171	51	2601	132651	7.1414	3.7084
7	49	343	2.6458	1.9129	52	2704	140608	7.2111	3.7325
8	64	512	2.8284	2.	53	2809	148877	7.2801	3.7563
9	81	729	3.	2.0801	54	2916	157464	7.3485	3.7798
10	100	1000	3.1623	2.1544	55	3025	166375	7.4162	3.8030
11	121	1331	3.3166	2.2240	56	3136	175616	7.4833	3.8259
12	144	1728	3.4641	2.2894	57	3249	185193	7.5498	3.8485
13	169	2197	3.6056	2.3513	58	3364	195112	7.6158	3.8709
14	196	2744	3.7417	2.4101	59	3481	205379	7.6811	3.8930
15	225	3375	3.8730	2.4662	60	3600	216000	7.7460	3.9149
16	256	4096	4.	2.5198	61	3721	226981	7.8102	3.9365
17	289	4913	4.1231	2.5713	62	3844	238328	7.8740	3.9579
18	324	5832	4.2426	2.6207	63	3969	250047	7.9373	3.9791
19	361	6859	4.3589	2.6684	64	4096	262144	8.	4.
20	400	8000	4.4721	2.7144	65	4225	274625	8.0623	4.0207
21	441	9261	4.5826	2.7589	66	4356	287496	8.1240	4.0412
22	484	10648	4.6904	2.8020	67	4489	300763	8.1854	4.0615
23	529	12167	4.7958	2.8439	68	4624	314432	8.2462	4.0817
24	576	13824	4.8990	2.8845	69	4761	328509	8.3066	4.1016
25	625	15625	5.	2.9240	70	4900	343000	8.3666	4.1213
26	676	17576	5.0990	2.9625	71	5041	357911	8.4261	4.1408
27	729	19683	5.1962	3.	72	5184	373248	8.4853	4.1602
28	784	21952	5.2915	3.0366	73	5329	389017	8.5440	4.1793
29	841	24389	5.3852	3.0723	74	5476	405224	8.6023	4.1983
30	900	27000	5.4772	3.1072	75	5625	421875	8.6603	4.2172
31	961	29791	5.5678	3.1414	76	5776	438976	8.7178	4.2358
32	1024	32768	5.6569	3.1748	77	5929	456533	8.7750	4.2543
33	1089	35937	5.7446	3.2075	78	6084	474552	8.8318	4.2727
34	1156	39304	5.8310	3.2396	79	6241	493039	8.8882	4.2908
35	1225	42875	5.9161	3.2711	80	6400	512000	8.9443	4.3089
36	1296	46656	6.	3.3019	81	6561	531441	9.	4.3267
37	1369	50653	6.0828	3.3322	82	6724	551368	9.0554	4.3445
38	1444	54872	6.1644	3.3620	83	6889	571787	9.1104	4.3621
39	1521	59319	6.2450	3.3912	84	7056	592704	9.1652	4.3795
40	1600	64000	6.3246	3.4200	85	7225	614125	9.2195	4.3968
41	1681	68921	6.4031	3.4482	86	7396	636056	9.2736	4.4140
42	1764	74088	6.4807	3.4760	87	7569	658503	9.3274	4.4310
43	1849	79507	6.5574	3.5034	88	7744	681472	9.3808	4.4480
44	1936	85184	6.6332	3.5303	89	7921	704969	9.4340	4.4647
45	2025	91125	6.7082	3.5569	90	8100	729000	9.4868	4.4814

SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS OF NUMBERS.—*Continued*

No.	Square	Cube	Square root	Cube root	No.	Square	Cube	Square root	Cube root
91	8281	753571	9.5394	4.4979	136	18496	2515456	11.6619	5.1426
92	8464	778688	9.5917	4.5144	137	18769	2571353	11.7047	5.1551
93	8649	804357	9.6437	4.5307	138	19044	2628072	11.7473	5.1676
94	8836	830584	9.6954	4.5468	139	19321	2685619	11.7898	5.1801
95	9025	857375	9.7468	4.5629	140	19600	2744000	11.8322	5.1925
96	9216	884736	9.7980	4.5789	141	19881	2803221	11.8743	5.2048
97	9409	912673	9.8489	4.5947	142	20164	2863288	11.9164	5.2171
98	9604	941192	9.8995	4.6104	143	20449	2924207	11.9583	5.2293
99	9801	970299	9.9499	4.6261	144	20736	2985984	12.	5.2415
100	10000	1000000	10.	4.6416	145	21025	3048625	12.0416	5.2536
101	10201	1030301	10.0499	4.6570	146	21316	3112136	12.0830	5.2656
102	10404	1061208	10.0995	4.6723	147	21609	3176523	12.1244	5.2776
103	10609	1092727	10.1489	4.6875	148	21904	3241792	12.1655	5.2896
104	10816	1124864	10.1980	4.7027	149	22201	3307949	12.2066	5.3015
105	11025	1157625	10.2470	4.7177	150	22500	3375000	12.2474	5.3133
106	11236	1191016	10.2956	4.7326	151	22801	3442951	12.2882	5.3251
107	11449	1225043	10.3441	4.7475	152	23104	3511808	12.3288	5.3368
108	11664	1259712	10.3923	4.7622	153	23409	3581577	12.3693	5.3485
109	11881	1295029	10.4403	4.7769	154	23716	3652264	12.4097	5.3601
110	12100	1331000	10.4881	4.7914	155	24025	3723875	12.4499	5.3717
111	12321	1367631	10.5357	4.8059	156	24336	3796416	12.4900	5.3832
112	12544	1404928	10.5830	4.8203	157	24649	3869893	12.5300	5.3947
113	12769	1442897	10.6301	4.8346	158	24964	3944312	12.5698	5.4061
114	12996	1481544	10.6771	4.8488	159	25281	4019679	12.6095	5.4175
115	13225	1520875	10.7238	4.8629	160	25600	4096000	12.6491	5.4288
116	13456	1560896	10.7703	4.8770	161	25921	4173281	12.6886	5.4401
117	13689	1601613	10.8167	4.8910	162	26244	4251528	12.7279	5.4514
118	13924	1643032	10.8628	4.9049	163	26569	4330747	12.7671	5.4626
119	14161	1685159	10.9087	4.9187	164	26896	4410944	12.8062	5.4737
120	14400	1728000	10.9545	4.9324	165	27225	4492125	12.8452	5.4848
121	14641	1771561	11.	4.9461	166	27556	4574296	12.8841	5.4959
122	14884	1815848	11.0454	4.9597	167	27889	4657463	12.9228	5.5069
123	15129	1860867	11.0905	4.9732	168	28224	4741632	12.9615	5.5178
124	15376	1906624	11.1355	4.9866	169	28561	4826809	13.	5.5288
125	15625	1953125	11.1803	5.	170	28900	4913000	13.0384	5.5397
126	15876	2000376	11.2250	5.0133	171	29241	5000211	13.0767	5.5505
127	16129	2048383	11.2694	5.0265	172	29584	5088448	13.1149	5.5613
128	16384	2097152	11.3137	5.0397	173	29929	5177717	13.1529	5.5721
129	16641	2146689	11.3578	5.0528	174	30276	5268024	13.1909	5.5828
130	16900	2197000	11.4018	5.0658	175	30625	5359375	13.2288	5.5934
131	17161	2248091	11.4455	5.0788	176	30976	5451776	13.2665	5.6041
132	17424	2299968	11.4891	5.0916	177	31329	5545233	13.3041	5.6147
133	17689	2352637	11.5326	5.1045	178	31684	5639752	13.3417	5.6252
134	17956	2406104	11.5758	5.1172	179	32041	5735339	13.3791	5.6357
135	18225	2460375	11.6190	5.1299	180	32400	5832000	13.4164	5.6462

SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS OF NUMBERS.—*Continued*

No.	Square	Cube	Square root	Cube root	No.	Square	Cube	Square root	Cube root
181	32761	5929741	13.4536	5.6567	226	51076	11543176	15.0333	6.0912
182	33124	6028568	13.4907	5.6671	227	51529	11697083	15.0665	6.1002
183	33489	6128487	13.5277	5.6774	228	51984	11852352	15.0997	6.1091
184	33856	6229504	13.5647	5.6877	229	52441	12008989	15.1327	6.1180
185	34225	6331625	13.6015	5.6980	230	52900	12167000	15.1658	6.1269
186	34596	6434856	13.6382	5.7083	231	53361	12326391	15.1987	6.1358
187	34969	6539203	13.6748	5.7185	232	53824	12487168	15.2315	6.1446
188	35344	6644672	13.7113	5.7287	233	54289	12649337	15.2643	6.1534
189	35721	6751269	13.7477	5.7388	234	54756	12812904	15.2971	6.1622
190	36100	6859000	13.7840	5.7489	235	55225	12977875	15.3297	6.1710
191	36481	6967871	13.8203	5.7590	236	55696	13144256	15.3623	6.1797
192	36864	7077888	13.8564	5.7690	237	56169	13312053	15.3948	6.1885
193	37249	7189057	13.8924	5.7790	238	56644	13481272	15.4272	6.1972
194	37636	7301384	13.8284	5.7890	239	57121	13651919	15.4596	6.2058
195	38025	7414875	13.9642	5.7989	240	57600	13824000	15.4919	6.2145
196	38416	7529536	14.	5.8088	241	58081	13997521	15.5242	6.2231
197	38809	7645373	14.0357	5.8186	242	58564	14172488	15.5563	6.2317
198	39204	7762392	14.0712	5.8285	243	59049	14348907	15.5885	6.2403
199	39601	7880599	14.1067	5.8383	244	59536	14526784	15.6205	6.2488
200	40000	8000000	14.1421	5.8480	245	60025	14706125	15.6525	6.2573
201	40401	8120601	14.1774	5.8578	246	60516	14886936	15.6844	6.2658
202	40804	8242408	14.2127	5.8675	247	61009	15069223	15.7162	6.2743
203	41209	8365427	14.2478	5.8771	248	61504	15252992	15.7480	6.2828
204	41616	8489664	14.2829	5.8868	249	62001	15438249	15.7797	6.2912
205	42025	8615125	14.3178	5.8964	250	62500	15625000	15.8114	6.2996
206	42436	8741816	14.3527	5.9059	251	63001	15813251	15.8430	6.3080
207	42849	8869743	14.3875	5.9155	252	63504	16003008	15.8745	6.3164
208	43264	8998912	14.4222	5.9250	253	64009	16194277	15.9060	6.3247
209	43681	9129329	14.4568	5.9345	254	64516	16387064	15.9374	6.3330
210	44100	9261000	14.4914	5.9439	255	65025	16581375	15.9687	6.3413
211	44521	9393931	14.5258	5.9533	256	65536	16777216	16.	6.3496
212	44944	9528128	14.5602	5.9627	257	66049	16974593	16.0312	6.3579
213	45369	9663597	14.5945	5.9721	258	66564	17173512	16.0624	6.3661
214	45796	9800344	14.6287	5.9814	259	67081	17373979	16.0935	6.3743
215	46225	9938375	14.6629	5.9907	260	67600	17576000	16.1245	6.3825
216	46656	10077696	14.6969	6.	261	68121	17779581	16.1555	6.3907
217	47089	10218313	14.7309	6.0092	262	68644	17984728	16.1864	6.3988
218	47524	10360232	14.7648	6.0185	263	69169	18191447	16.2173	6.4070
219	47961	10503459	14.7986	6.0277	264	69696	18399744	16.2481	8.4151
220	48400	10648000	14.8324	6.0368	265	70225	18609625	16.2788	6.4232
221	48841	10793861	14.8661	6.0459	266	70756	18821096	16.3095	6.4312
222	49284	10941048	14.8997	6.0550	267	71289	19034163	16.3401	6.4393
223	49729	11089567	14.9332	6.0641	268	71824	19248832	16.3707	6.4473
224	50176	11239424	14.9666	6.0732	269	72361	19465109	16.4012	6.4553
225	50625	11390625	15.	6.0822	270	72900	19683000	16.4317	6.4633

SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS OF NUMBERS.—*Continued*

No.	Square	Cube	Square root	Cube root	No.	Square	Cube	Square root	Cube root
271	73441	19902511	16.4621	6.4713	316	99856	31554496	17.7764	6.8113
272	73984	20123648	16.4924	6.4792	317	100489	31855013	17.8045	6.8185
273	74529	20346417	16.5227	6.4872	318	101124	32157432	17.8326	6.8256
274	75076	20570824	16.5529	6.4951	319	101761	32461759	17.8606	6.8328
275	75625	20796875	16.5831	6.5030	320	102400	32768000	17.8885	6.8399
276	76176	21024576	16.6132	6.5108	321	103041	33076161	17.9165	6.8470
277	76729	21253933	16.6433	6.5187	322	103681	33386248	17.9444	6.8541
278	77284	21484952	16.6733	6.5265	323	104329	33698267	17.9722	6.8612
279	77841	21717639	16.7033	6.5343	324	104976	34012224	18.	6.8683
280	78400	21952000	16.7332	6.5421	325	105625	34328125	18.0278	6.8753
281	78961	22188041	16.7631	6.5499	326	106276	34645976	18.0555	6.8824
282	79524	22425768	16.7929	6.5577	327	106929	34965783	18.0831	6.8894
283	80089	22665187	16.8226	6.5654	328	107584	35287552	18.1108	6.8964
284	80656	22906304	16.8523	6.5731	329	108241	35611289	18.1384	6.9034
285	81225	23149125	16.8819	6.5808	330	108900	35937000	18.1659	6.9104
286	81796	23393656	16.9115	6.5885	331	109561	36264691	18.1934	6.9174
287	82369	23639903	16.9411	6.5962	332	110224	36594368	18.2209	6.9244
288	82944	23887872	16.9706	6.6039	333	110889	36926037	18.2483	6.9313
289	83521	24137569	17.	6.6115	334	111556	37259704	18.2757	6.9382
290	84100	24389000	17.0294	6.6191	335	112225	37595375	18.3030	6.9451
291	84681	24642171	17.0587	6.6267	336	112896	37933056	18.3303	6.9521
292	85264	24897088	17.0880	6.6343	337	113569	38272753	18.3576	6.9589
293	85849	25153757	17.1172	6.6419	338	114244	38614472	18.3848	6.9658
294	86436	25412184	17.1464	6.6494	339	114921	38958219	18.4120	6.9727
295	87025	25672375	17.1756	6.6569	340	115600	39304000	18.4391	6.9795
296	87616	25934336	17.2047	6.6644	341	116281	39651821	18.4662	6.9864
297	88209	26198073	17.2337	6.6719	342	116964	40001688	18.4932	6.9932
298	88804	26463592	17.2627	6.6794	343	117649	40353607	18.5203	7.
299	89401	26730899	17.2916	6.6869	344	118336	40707584	18.5472	7.0068
300	90000	27000000	17.3205	6.6943	345	119025	41063625	18.5742	7.0136
301	90601	27270901	17.3494	6.7018	346	119716	41421736	18.6011	7.0203
302	91204	27543608	17.3781	6.7092	347	120409	41781923	18.6279	7.0271
303	91809	27818127	17.4069	6.7166	348	121104	42144192	18.6548	7.0338
304	92416	28094464	17.4356	6.7240	349	121801	42508549	18.6815	7.0406
305	93025	28372625	17.4642	6.7313	350	122500	42875000	18.7083	7.0473
306	93636	28652616	17.4929	6.7387	351	123201	43243551	18.7350	7.0540
307	94249	28934443	17.5214	6.7460	352	123904	43614208	18.7617	7.0607
308	94864	29218112	17.5499	6.7533	353	124609	43986977	18.7883	7.0674
309	95481	29503629	17.5784	6.7606	354	125316	44361864	18.8149	7.0740
310	96100	29791000	17.6068	6.7679	355	126025	44738875	18.8414	7.0807
311	96721	30080231	17.6352	6.7752	356	126736	45118016	18.8680	7.0873
312	97344	30371328	17.6635	6.7824	357	127449	45499293	18.8944	7.0940
313	97969	30664297	17.6918	6.7897	358	128164	45882712	18.9209	7.1006
314	98596	30959144	17.7200	6.7969	359	128881	46268279	18.9473	7.1072
315	99225	31255875	17.7482	6.8041	360	129600	46656000	18.9737	7.1138

SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS OF NUMBERS.—*Continued*

No.	Square	Cube	Square root	Cube root	No.	Square	Cube	Square root	Cube root
361	130321	47045881	19.	7.1204	406	164836	66923416	20.1494	7.4047
362	131044	47437928	19.0263	7.1269	407	165649	67419143	20.1742	7.4108
363	131769	47832147	19.0526	7.1335	408	166464	67917312	20.1990	7.4169
364	132496	48228544	19.0788	7.1400	409	167281	68417929	20.2237	7.4229
365	133225	48627125	19.1050	7.1466	410	168100	68921000	20.2485	7.4290
366	133956	49027896	19.1311	7.1531	411	168921	69426531	20.2731	7.4350
367	134689	49430863	19.1572	7.1596	412	169744	69934528	20.2978	7.4410
368	135424	49836032	19.1833	7.1661	413	170569	70444997	20.3224	7.4470
369	136161	50243409	19.2094	7.1726	414	171396	70957944	20.3470	7.4530
370	136900	50653000	19.2354	7.1791	415	172225	71473375	20.3715	7.4590
371	137641	51064811	19.2614	7.1855	416	173056	71991296	20.3961	7.4650
372	138384	51478848	19.2873	7.1920	417	173889	72511713	20.4206	7.4710
373	139129	51895117	19.3132	7.1984	418	174724	73034632	20.4450	7.4770
374	139876	52313624	19.3391	7.2048	419	175561	73560059	20.4695	7.4829
375	140625	52734375	19.3649	7.2112	420	176400	74088000	20.4939	7.4889
376	141376	53157376	19.3907	7.2177	421	177241	74618461	20.5183	7.4948
377	142129	53582633	19.4165	7.2240	422	178084	75151448	20.5426	7.5007
378	142884	54010152	19.4422	7.2304	423	178929	75686967	20.5670	7.5067
379	143641	54439939	19.4679	7.2368	424	179776	76225024	20.5913	7.5126
380	144400	54872000	19.4936	7.2432	425	180625	76765625	20.6155	7.5185
381	145161	55306341	19.5192	7.2495	426	181476	77308776	20.6398	7.5244
382	145924	55742968	19.5448	7.2558	427	182329	77854483	20.6640	7.5302
383	146689	56181887	19.5704	7.2622	428	183184	78402752	20.6882	7.5361
384	147456	56623104	19.5959	7.2685	429	184041	78953589	20.7123	7.5420
385	148225	57066625	19.6214	7.2748	430	184900	79507000	20.7364	7.5478
386	148996	57512456	19.6469	7.2811	431	185761	80062991	20.7605	7.5537
387	149769	57960603	19.6723	7.2874	432	186624	80621568	20.7846	7.5595
388	150544	58411072	19.6977	7.2936	433	187489	81182737	20.8087	7.5654
389	151321	58863869	19.7231	7.2999	434	188356	81746504	20.8327	7.5712
390	152100	59319000	19.7484	7.3061	435	189225	82312875	20.8567	7.5770
391	152881	59776471	19.7737	7.3124	436	190096	82881856	20.8806	7.5828
392	153664	60236288	19.7990	7.3186	437	190969	83453453	20.9045	7.5886
393	154449	60698457	19.8242	7.3248	438	191884	84027672	20.9284	7.5944
394	155236	61162984	19.8494	7.3310	439	192721	84604519	20.9523	7.6001
395	156025	61629875	19.8746	7.3372	440	193600	85184000	20.9762	7.6059
396	156816	62099136	19.8997	7.3434	441	194481	85766121	21.	7.6117
397	157609	62570773	19.9249	7.3496	442	195364	86350888	21.0238	7.6174
398	158404	63044792	19.9499	7.3558	443	196249	86938307	21.0476	7.6232
399	159201	63521199	19.9750	7.3619	444	197136	87528384	21.0713	7.6289
400	160000	64000000	20.	7.3681	445	198025	88121125	21.0950	7.6346
401	160801	64481201	20.0250	7.3742	446	198916	88716536	21.1187	7.6403
402	161604	64964808	20.0499	7.3803	447	199809	89314623	21.1424	7.6460
403	162409	65450827	20.0749	7.3864	448	200704	89915392	21.1660	7.6517
404	163216	65939264	20.0998	7.3925	449	201601	90518849	21.1896	7.6574
405	164025	66430125	20.1246	7.3986	450	202500	91125000	21.2132	7.6631

SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS OF NUMBERS.—*Continued*

No.	Square	Cube	Square root	Cube root	No.	Square	Cube	Square root	Cube root
451	203401	91733851	21.2368	7.6688	496	246016	122023936	22.2711	7.9158
452	204304	92345408	21.2603	7.6744	497	247009	122763473	22.2935	7.9211
453	205209	92959677	21.2838	7.6801	498	248004	123505992	22.3159	7.9264
454	206116	93576664	21.3073	7.6857	499	249001	124251499	22.3383	7.9317
455	207025	94196375	21.3307	7.6914	500	250000	125000000	22:3607	7.9370
456	207936	94818816	21.3542	7.6970	501	251001	125751501	22.3830	7.9423
457	208849	95443993	21.3776	7.7026	502	252004	126506008	22.4054	7.9476
458	209764	96071912	21.4009	7.7082	503	253009	127263527	22.4277	7.9528
459	210681	96702579	21.4243	7.7138	504	254016	128024064	22.4499	7.9581
460	211600	97336000	21.4476	7.7194	505	255025	128787625	22.4722	7.9634
461	212521	97972181	21.4709	7.7250	506	256036	129554216	22.4944	7.9686
462	213444	98611128	21.4942	7.7306	507	257049	130323843	22.5167	7.9739
463	214369	99252847	21.5174	7.7362	508	258064	131096512	22.5389	7.9791
464	215296	99897344	21.5407	7.7418	509	259081	131872229	22.5610	7.9843
465	216225	100544625	21.5639	7.7473	510	260100	132651000	22.5832	7.9896
466	217156	101194696	21.5870	7.7529	511	261121	133432831	22.6053	7.9948
467	218089	101847563	21.6102	7.7584	512	262144	134217728	22.6274	8.
468	219024	102503232	21.6333	7.7639	513	263169	135005697	22.6495	8.0052
469	219961	103161709	21.6564	7.7695	514	264196	135796744	22.6716	8.0104
470	220900	103823000	21.6795	7.7750	515	265225	136590875	22.6936	8.0156
471	221841	104487111	21.7025	7.7805	516	266256	137388096	22.7156	8.0208
472	222784	105154048	21.7256	7.7860	517	267289	138188413	22.7376	8.0260
473	223729	105823817	21.7486	7.7915	518	268324	138991832	22.7596	8.0311
474	224676	106496424	21.7715	7.7970	519	269361	139798359	22.7816	8.0363
475	225625	107171875	21.7945	7.8025	520	270400	140608000	22.8035	8.0415
476	226576	107850176	21.8174	7.8079	521	271441	141420761	22.8254	8.0466
477	227529	108531333	21:8403	7.8134	522	272484	142236648	22.8473	8.0517
478	228484	109215352	21.8632	7.8188	523	273529	143055667	22.8692	8.0569
479	229441	109902239	21.8861	7.8243	524	274576	143877824	22.8910	8.0620
480	230400	110592000	21.9089	7.8297	525	275625	144703125	22.9129	8.0671
481	231361	111284641	21.9317	7.8352	526	276676	145531576	22.9347	8.0723
482	232324	111980168	21.9545	7.8406	527	277729	146363183	22.9565	8.0774
483	233289	112678587	21.9773	7.8460	528	278784	147197952	22.9783	8.0825
484	234256	113379904	22.	7.8514	529	279841	148035889	23.	8.0876
485	235225	114084125	22.0227	7.8568	530	280900	148877000	23.0217	8.0927
486	236196	114791256	22.0454	7.8622	531	281961	149721291	23.0434	8.0978
487	237169	115501303	22.0681	7.8676	532	283024	150568768	23.0651	8.1028
488	238144	116214272	22.0970	7.8730	533	284089	151419437	23.0868	8.1079
489	239121	116930169	22.1133	7.8784	534	285156	152273304	23.1084	8.1130
490	240100	117649000	22.1359	7.8837	535	286225	153130375	23.1301	8.1180
491	241081	118370771	22.1585	7.8891	536	287296	153990656	23.1517	8.1231
492	242064	119095488	22.1811	7.8944	537	288369	154854153	23.1733	8.1281
493	243049	119823157	22.2036	7.8998	538	289444	155720872	23.1948	8.1332
494	244036	120553784	22.2261	7.9051	539	290521	156590819	23.2164	8.1382
495	245025	121287375	22.2486	7.9105	540	291600	157464000	23.2379	8.1433

SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS OF NUMBERS.—*Continued*

No.	Square	Cube	Square root	Cube root	No.	Square	Cube	Square root	Cube root
541	292681	158340421	23.2594	8.1483	586	343396	201230056	24.2074	8.3682
542	293764	159220088	23.2809	8.1533	587	344569	202262003	24.2281	8.3730
543	294849	160103007	23.3024	8.1583	588	345744	203297472	24.2487	8.3777
544	295936	160989184	23.3238	8.1633	589	346921	204336469	24.2693	8.3825
545	297025	161878625	23.3452	8.1683	590	348100	205379000	24.2899	8.3872
546	298116	162771336	23.3666	8.1733	591	349281	206425071	24.3105	8.3919
547	299209	163667323	23.3880	8.1783	592	350464	207474688	24.3311	8.3967
548	300304	164566592	23.4094	8.1833	593	351649	208527857	24.3516	8.4014
549	301401	165469149	23.4307	8.1882	594	352836	209584584	24.3721	8.4061
550	302500	166375000	23.4521	8.1932	595	354025	210644875	24.3926	8.4108
551	303601	167284151	23.4734	8.1982	596	355216	211708736	24.4131	8.4155
552	304704	168196608	23.4947	8.2031	597	356409	212776173	24.4336	8.4202
553	305809	169112377	23.5160	8.2081	598	357604	213847192	24.4540	8.4249
554	306916	170031464	23.5372	8.2130	599	358801	214921799	24.4745	8.4296
555	308025	170953875	23.5584	8.2180	600	360000	216000000	24.4949	8.4343
556	309136	171879616	23.5797	8.2229	601	361201	217081801	24.5153	8.4390
557	310249	172808693	23.6008	8.2278	602	362404	218167208	24.5357	8.4437
558	311364	173741112	23.6220	8.2327	603	363609	219256227	24.5561	8.4484
559	312481	174676879	23.6432	8.2377	604	364816	220348864	24.5764	8.4530
560	313600	175616000	23.6643	8.2426	605	366025	221445125	24.5967	8.4577
561	314721	176558481	23.6854	8.2475	606	367236	222545016	24.6171	8.4623
562	315844	177504328	23.7065	8.2524	607	368449	223648543	24.6374	8.4670
563	316969	178453547	23.7276	8.2573	608	369664	224755712	24.6577	8.4716
564	318096	179406144	23.7487	8.2621	609	370881	225866529	24.6779	8.4763
565	319225	180362125	23.7697	8.2670	610	372100	226981000	24.6982	8.4809
566	320356	181321496	23.7908	8.2719	611	373321	228099131	24.7184	8.4856
567	321489	182284263	23.8118	8.2768	612	374544	229220928	24.7386	8.4902
568	322624	183250432	23.8328	8.2816	613	375769	230346397	24.7588	8.4948
569	323761	184220009	23.8537	8.2865	614	376996	231475544	24.7790	8.4994
570	324900	185193000	23.8747	8.2913	615	378225	232608375	24.7992	8.5040
571	326041	186169411	23.8956	8.2962	616	379456	233744896	24.8193	8.5086
572	327184	187149248	23.9165	8.3010	617	380689	234885113	24.8395	8.5132
573	328329	188132517	23.9374	8.3059	618	381924	236029032	24.8596	8.5178
574	329476	189119224	23.9583	8.3107	619	383161	237176659	24.8797	8.5224
575	330625	190109375	23.9792	8.3155	620	384400	238328000	24.8998	8.5270
576	331776	191102976	24.	8.3203	621	385641	239483061	24.9199	8.5316
577	332929	192100033	24.0208	8.3251	622	386884	240641848	24.9399	8.5362
578	334084	193100552	24.0416	8.3300	623	388129	241804367	24.9600	8.5408
579	335241	194104539	24.0624	8.3348	624	389376	242970624	24.9800	8.5453
580	336400	195112000	24.0832	8.3396	625	390625	244140625	25.	8.5499
581	337561	196122941	24.1039	8.3443	626	391876	245314376	25.0200	8.5544
582	338724	197137368	24.1247	8.3491	627	393129	246491883	25.0400	8.5590
583	339889	198155287	24.1454	8.3539	628	394384	247673152	25.0599	8.5635
584	341056	199176704	24.1661	8.3587	629	395641	248858189	25.0799	8.5681
585	342225	200201625	24.1868	8.3634	630	396900	250047000	25.0998	8.5726

SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS OF NUMBERS.—*Continued*

No.	Square	Cube	Square root	Cube root	No.	Square	Cube	Square root	Cube root
631	398161	251239591	25.1197	8.5772	676	456976	308915776	26.	8.7764
632	399424	252435968	25.1396	8.5817	677	458329	310288733	26.0192	8.7807
633	400689	253636137	25.1595	8.5862	678	459684	311665752	26.0384	8.7850
634	401956	254840104	25.1794	8.5907	679	461041	313046839	26.0576	8.7893
635	403225	256047875	25.1992	8.5952	680	462400	314432000	26.0768	8.7937
636	404496	257259456	25.2190	8.5997	681	463761	315821241	26.0960	8.7980
637	405769	258474853	25.2389	8.6043	682	465124	317214568	26.1151	8.8023
638	407044	259694072	25.2587	8.6088	683	466489	318611987	26.1343	8.8066
639	408321	260917119	25.2784	8.6132	684	467856	320013504	26.1534	8.8109
640	409600	262144000	25.2982	8.6177	685	469225	321419125	26.1725	8.8152
641	410881	263374721	25.3180	8.6222	686	470596	322828856	26.1916	8.8194
642	412164	264609288	25.3377	8.6267	687	471969	324242703	26.2107	8.8237
643	413449	265847707	25.3574	8.6312	688	473344	325660672	26.2298	8.8280
644	414736	267089984	25.3772	8.6357	689	474721	327082769	26.2488	8.8323
645	416025	268336125	25.3969	8.6401	690	476100	328509000	26.2679	8.8366
646	417316	269586136	25.4165	8.6446	691	477481	329929371	26.2869	8.8408
647	418609	270840023	25.4362	8.6490	692	478864	331373888	26.3059	8.8451
648	419904	272097792	25.4558	8.6535	693	480249	332812557	26.3249	8.8493
649	421201	273359449	25.4755	8.6579	694	481636	334255384	26.3439	8.8536
650	422500	274625000	25.4951	8.6624	695	483025	335702375	26.3629	8.8578
651	423801	275894451	25.5147	8.6668	696	484416	337153536	26.3818	8.8621
652	425104	277167808	25.5343	8.6713	697	485809	338608873	26.4008	8.8663
653	426409	278445077	25.5539	8.6757	698	487204	340068392	26.4197	8.8706
654	427716	279726264	25.5734	8.6801	699	488601	341532099	26.4386	8.8748
655	429025	281011375	25.5930	8.6845	700	490000	343000000	26.4575	8.8790
656	430336	282300416	25.6125	8.6890	701	491401	344472101	26.4764	8.8833
657	431649	283593393	25.6320	8.6934	702	492804	345948408	26.4953	8.8875
658	432964	284890312	25.6515	8.6978	703	494209	347428927	26.5141	8.8917
659	434281	286191179	25.6710	8.7022	704	495616	348913664	26.5330	8.8959
660	435600	287496000	25.6905	8.7066	705	497025	350402625	26.5518	8.9001
661	436921	288804781	25.7099	8.7110	706	498436	351895816	26.5707	8.9043
662	438244	290117528	25.7294	8.7154	707	499849	353393243	26.5895	8.9085
663	439569	291434247	25.7488	8.7198	708	501264	354894912	26.6083	8.9127
664	440896	292754944	25.7682	8.7241	709	502681	356400829	26.6271	8.9169
665	442225	294079625	25.7876	8.7285	710	504100	357911000	26.6458	8.9211
666	443556	295408296	25.8070	8.7329	711	505521	359425431	26.6646	8.9253
667	444889	296740963	25.8263	8.7373	712	506944	360944128	26.6833	8.9295
668	446224	298077632	25.8457	8.7416	713	508369	362467097	26.7021	8.9337
669	447561	299418309	25.8650	8.7460	714	509796	363994344	26.7208	8.9378
670	448900	300763000	25.8844	8.7503	715	511225	365525875	26.7395	8.9420
671	450241	302111711	25.9037	8.7547	716	512656	367061696	26.7582	8.9462
672	451584	303464448	25.9230	8.7590	717	514089	368601813	26.7769	8.9503
673	452929	304821217	25.9422	8.7634	718	515524	370146232	26.7955	8.9545
674	454276	306182024	25.9615	8.7677	719	516961	371694959	26.8142	8.9587
675	455625	307546875	25.9808	8.7721	720	518400	373248000	26.8328	8.9628

SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS OF NUMBERS.—*Continued*

No.	Square	Cube	Square root	Cube root	No.	Square	Cube	Square root	Cube root
721	519841	374805361	26.8514	8.9670	766	586756	449455096	27.6767	9.1498
722	521284	376367048	26.8701	8.9711	767	588289	451217663	27.6948	9.1537
723	522729	377933067	26.8887	8.9752	768	589824	452984832	27.7128	9.1577
724	524176	379503424	26.9072	8.9794	769	591361	454756609	27.7308	9.1617
725	525625	381078125	26.9258	8.9835	770	592900	456533000	27.7489	9.1657
726	527076	382657176	26.9444	8.9876	771	594441	458314011	27.7669	9.1696
727	528529	384240583	26.9629	8.9918	772	595984	460099648	27.7849	9.1736
728	529984	385828352	26.9815	8.9959	773	597529	461889917	27.8029	9.1775
729	531441	387420489	27.	9.	774	599076	463684824	27.8209	9.1815
730	532900	389017000	27.0185	9.0041	775	600625	465484375	27.8388	9.1855
731	534361	390617891	27.0370	9.0082	776	602176	467288576	27.8568	9.1894
732	535824	392223168	27.0555	9.0123	777	603729	469097433	27.8747	9.1933
733	537289	393832837	27.0740	9.0164	778	605284	470910952	27.8927	9.1973
734	538756	395446904	27.0924	9.0205	779	606841	472729139	27.9106	9.2012
735	540225	397065375	27.1109	9.0246	780	608400	474552000	27.9285	9.2052
736	541696	398688256	27.1293	9.0287	781	609961	476379541	27.9464	9.2091
737	543169	400315553	27.1477	9.0328	782	611524	478211768	27.9643	9.2130
738	544644	401947272	27.1662	9.0369	783	613089	480048687	27.9821	9.2170
739	546121	403583419	27.1846	9.0410	784	614656	481890304	28.	9.2209
740	547600	405224000	27.2029	9.0450	785	616225	483736625	28.0179	9.2248
741	549081	406869021	27.2213	9.0491	786	617796	485587656	28.0357	9.2287
742	550564	408518488	27.2397	9.0532	787	619369	487443403	28.0535	9.2326
743	552049	410172407	27.2580	9.0572	788	620944	489303872	28.0713	9.2365
744	553536	411830784	27.2764	9.0613	789	622521	491169069	28.0891	9.2404
745	555025	413493625	27.2947	9.0654	790	624100	493039000	28.1069	9.2443
746	556516	415160936	27.3130	9.0694	791	625681	494913671	28.1247	9.2482
747	558009	416832723	27.3313	9.0735	792	627264	496793088	28.1425	9.2521
748	559504	418508992	27.3496	9.0775	793	628849	498677257	28.1603	9.2560
749	561001	420189749	27.3679	9.0816	794	630436	500566184	28.1780	9.2599
750	562500	421875000	27.3861	9.0856	795	632025	502459875	28.1957	9.2638
751	564001	423564751	27.4044	9.0896	796	633616	504358336	28.2135	9.2677
752	565504	425259008	27.4226	9.0937	797	635209	506261573	28.2312	9.2716
753	567009	426957777	27.4408	9.0977	798	636804	508169592	28.2489	9.2754
754	568516	428661064	27.4591	9.1017	799	638401	510082399	28.2666	9.2793
755	570025	430368875	27.4773	9.1057	800	640000	512000000	28.2843	9.2832
756	571536	432081216	27.4955	9.1098	801	641601	513922401	28.3019	9.2870
757	573049	433798093	27.5136	9.1138	802	643204	515849608	28.3196	9.2909
758	574564	435519512	27.5318	9.1178	803	644809	517781627	28.3373	9.2948
759	576081	437245479	27.5500	9.1218	804	646416	519718464	28.3549	9.2986
760	577600	438976000	27.5681	9.1258	805	648025	521660125	28.3725	9.3025
761	579121	440711081	27.5862	9.1298	806	649636	523606616	28.3901	9.3063
762	580644	442450728	27.6043	9.1338	807	651249	525557943	28.4077	9.3102
763	582169	444194947	27.6225	9.1378	808	652864	527514112	28.4253	9.3140
764	583696	445943744	27.6405	9.1418	809	654481	529475129	28.4429	9.3179
765	585225	447697125	27.6586	9.1458	810	656100	531441000	28.4605	9.3217

SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS OF NUMBERS.—*Continued*

No.	Square	Cube	Square root	Cube root	No.	Square	Cube	Square root	Cube root
811	657721	533411731	28.4781	9.3255	856	732736	627222016	29.2575	9.4949
812	659344	535387328	28.4956	9.3294	857	734449	629422793	29.2746	9.4986
813	660969	537367797	28.5132	9.3332	858	736164	631628712	29.2916	9.5023
814	662596	539353144	28.5307	9.3370	859	737881	633839779	29.3087	9.5060
815	664225	541343375	28.5482	9.3408	860	739600	636056000	29.3258	9.5097
816	665856	543338496	28.5657	9.3447	861	741321	638277381	29.3428	9.5134
817	667489	545338513	28.5832	9.3485	862	743044	640503928	29.3598	9.5171
818	669124	547343432	28.6007	9.3523	863	744769	642735647	29.3769	9.5207
819	670761	549353259	28.6182	9.3561	864	746496	644972544	29.3939	9.5244
820	672400	551368000	28.6356	9.3599	865	748225	647214625	29.4109	9.5281
821	674041	553387661	28.6531	9.3637	866	749956	649461896	29.4279	9.5317
822	675684	555412248	28.6705	9.3675	867	751689	651714363	29.4449	9.5354
823	677329	557441767	28.6880	9.3713	868	753424	653972032	29.4618	9.5391
824	678976	559476224	28.7054	9.3751	869	755161	656234909	29.4788	9.5427
825	680625	561515625	28.7228	9.3789	870	756900	658503000	29.4958	9.5464
826	682276	563559976	28.7402	9.3827	871	758641	660776311	29.5127	9.5501
827	683929	565609283	28.7576	9.3865	872	760384	663054848	29.5296	9.5537
828	685584	567663552	28.7750	9.3902	873	762129	665338617	29.5466	9.5574
829	687241	569722789	28.7924	9.3940	874	763876	667627624	29.5635	9.5610
830	688900	571787000	28.8097	9.3978	875	765625	669921875	29.5804	9.5647
831	690561	573856191	28.8271	9.4016	876	767376	672221376	29.5973	9.5683
832	692224	575930368	28.8444	9.4053	877	769129	674526133	29.6142	9.5719
833	693889	578009537	28.8617	9.4091	878	770884	676836152	29.6311	9.5756
834	695556	580093704	28.8791	9.4129	879	772641	679151439	29.6479	9.5792
835	697225	582182875	28.8964	9.4166	880	774400	681472000	29.6648	9.5828
836	698896	584277056	28.9137	9.4204	881	776161	683797841	29.6816	9.5865
837	700569	586376253	28.9310	9.4241	882	777924	686128968	29.6985	9.5901
838	702244	588480472	28.9482	9.4279	883	779689	688465387	29.7153	9.5937
839	703921	590589719	28.9655	9.4316	884	781456	690807104	29.7321	9.5973
840	705600	592704000	28.9828	9.4354	885	783225	693154125	29.7489	9.6010
841	707281	594823321	29.	9.4391	886	784996	695506456	29.7658	9.6046
842	708964	596947688	29.0172	9.4429	887	786769	697864103	29.7825	9.6082
843	710649	599077107	29.0345	9.4466	888	788544	700227072	29.7993	9.6118
844	712336	601211584	29.0517	9.4503	889	790321	702595369	29.8161	9.6154
845	714025	603351125	29.0689	9.4541	890	792100	704969000	29.8329	9.6190
846	715716	605495736	29.0861	9.4578	891	793881	707347971	29.8496	9.6226
847	717409	607645423	29.1033	9.4615	892	795664	709732288	29.8664	9.6262
848	719104	609800192	29.1204	9.4652	893	797449	712121957	29.8831	9.6298
849	720801	611960049	29.1376	9.4690	894	799236	714516984	29.8998	9.6334
850	722500	614125000	29.1548	9.4727	895	801025	716917375	29.9166	9.6370
851	724201	616295051	29.1719	9.4764	896	802816	719323136	29.9333	9.6406
852	725904	618470208	29.1890	9.4801	897	804609	721734273	29.9500	9.6442
853	727609	620650477	29.2062	9.4838	898	806404	724150792	29.9666	9.6477
854	729316	622835864	29.2233	9.4875	899	808201	726572699	29.9833	9.6513
855	731025	625026375	29.2404	9.4912	900	810000	729000000	30.	9.6549

SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS OF NUMBERS.—*Continued*

No.	Square	Cube	Square root	Cube root	No.	Square	Cube	Square root	Cube root
901	811801	731432701	30.0167	9.6585	946	894916	846590536	30.7571	9.8167
902	813604	733870808	30.0333	9.6620	947	896809	849278123	30.7734	9.8201
903	815409	736314327	30.0500	9.6656	948	898704	851971392	30.7896	9.8236
904	817216	738763264	30.0666	9.6692	949	900601	854670349	30.8058	9.8270
905	819025	741217625	30.0832	9.6727	950	902500	857375000	30.8221	9.8305
906	820836	743677416	30.0998	9.6763	951	904401	860085351	30.8383	9.8339
907	822649	746142643	30.1164	9.6799	952	906304	862801408	30.8545	9.8374
908	824464	748613312	30.1330	9.6834	953	908209	865523177	30.8707	9.8408
909	826281	751089429	30.1496	9.6870	954	910116	868250664	30.8869	9.8443
910	828100	753571000	30.1662	9.6905	955	912025	870983875	30.9031	9.8477
911	829921	756058031	30.1828	9.6941	956	913936	873722816	30.9192	9.8511
912	831744	758550528	30.1993	9.6976	957	915849	876467493	30.9354	9.8546
913	833569	761048497	30.2159	9.7012	958	917764	879217912	30.9516	9.8580
914	835396	763551944	30.2324	9.7047	959	919681	881974079	30.9677	9.8614
915	837225	766060875	30.2490	9.7082	960	921600	884736000	30.9839	9.8648
916	839056	768575296	30.2655	9.7118	961	923521	887503681	31.	9.8683
917	840889	771095213	30.2820	9.7153	962	925444	890277128	31.0161	9.8717
918	842724	773620632	30.2985	9.7188	963	927369	893056347	31.0322	9.8751
919	844561	776151559	30.3150	9.7224	964	929296	895841344	31.0483	9.8785
920	846400	778688000	30.3315	9.7259	965	931225	898632125	31.0644	9.8819
921	848241	781229961	30.3480	9.7294	966	933156	901428696	31.0805	9.8854
922	850084	783777448	30.3645	9.7329	967	935089	904231063	31.0966	9.8888
923	851929	786330467	30.3809	9.7364	968	937024	907039232	31.1127	9.8922
924	853776	788889024	30.3974	9.7400	969	938961	909853209	31.1288	9.8956
925	855625	791453125	30.4138	9.7435	970	940900	912673000	31.1448	9.8990
926	857476	794022776	30.4302	9.7470	971	942841	915498611	31.1609	9.9024
927	859329	796597983	30.4467	9.7505	972	944784	918330048	31.1769	9.9058
928	861184	799178752	30.4631	9.7540	973	946729	921167317	31.1929	9.9092
929	863041	801765089	30.4795	9.7575	974	948676	924010424	31.2090	9.9126
930	864900	804357000	30.4959	9.7610	975	950625	926859375	31.2250	9.9160
931	866761	806954491	30.5123	9.7645	976	952576	929714176	31.2410	9.9194
932	868624	809557568	30.5287	9.7680	977	954529	932574833	31.2570	9.9227
933	870489	812166237	30.5450	9.7715	978	956484	935441352	31.2730	9.9261
934	872356	814780504	30.5614	9.7750	979	958441	938313739	31.2890	9.9295
935	874225	817400375	30.5778	9.7785	980	960400	941192000	31.3050	9.9329
936	876096	820025856	30.5941	9.7819	981	962361	944076141	31.3209	9.9363
937	877969	822656953	30.6105	9.7854	982	964324	946966168	31.3369	9.9396
938	879844	825293672	30.6268	9.7889	983	966289	949862087	31.3528	9.9430
939	881721	827936019	30.6431	9.7924	984	968256	952763904	31.3688	9.9464
940	883600	830584000	30.6594	9.7959	985	970225	955671625	31.3847	9.9497
941	885481	833237621	30.6757	9.7993	986	972196	958585256	31.4006	9.9531
942	887364	835896888	30.6920	9.8028	987	974169	961504803	31.4166	9.9565
943	889249	838561807	30.7083	9.8063	988	976144	964430272	31.4325	9.9598
944	891136	841232384	30.7246	9.8097	989	978121	967361669	31.4484	9.9632
945	893025	843908625	30.7409	9.8132	990	980100	970299000	31.4643	9.9666

SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS OF NUMBERS.—*Continued*

No.	Square	Cube	Square root	Cube root	No.	Square	Cube	Square root	Cube root
991	982081	973242271	31.4802	9.9699	996	992016	988047936	31.5595	9.9866
992	984064	976191488	31.4960	9.9733	997	994009	991026973	31.5753	9.9900
993	986049	979146657	31.5119	9.9766	998	996004	994011992	31.5911	9.9933
994	988036	982107784	31.5278	9.9800	999	998001	997002999	31.6070	9.9967
995	990025	985074875	31.5436	9.9833	1000	1000000	1000000000	31.6228	10.

CHAPTER XIII

LEVERS

90. Types of Machines.—All machines consist of one or more of the three fundamental types of machines—the Lever, the Cord, and the Inclined Plane or Wedge. Any piece of mechanism can be proved to be of one or more of these types. Pulleys, gears, and cranks will be shown to be forms of Levers; belts and chains come under the type called the Cord; while screws, worms, and cams are forms of Inclined Planes. They are all used to transmit power from one place to another and to modify it, as desired.

91. The Lever.—The lever is probably the most used and the simplest type of machine. We are all familiar with it in its simplest forms, such as crow bars, shears, pliers, tongs, and the numerous simple levers found on machine tools.

A lever is a rigid rod or bar so arranged as to be capable of turning about a fixed point. This fixed point about which the lever turns is called the *Fulcrum*. In Fig. 37 the fulcrum is

Fig. 37.

represented by the small triangular block *F*. The position of this fulcrum determines the effect which the force *P* applied at one end has toward lifting the weight *W* at the other end. If *F* is close to *W*, a comparatively small force *P* may be able to raise the weight *W*, but if *F* is moved away from *W* and placed close to *P*, then a greater force will be required at *P*. If *F* is in the middle, *P* and *W* will be just equal.

In every lever there are two opposing tendencies: first, that of the load or weight *W* tending to descend; and second, that of the force *P* tending to raise *W*. The ability of *W* to descend or to resist being lifted depends on two things—its weight and its distance from the fulcrum *F*. The product of these two is the measure of the tendency of *W* to descend. This product is called, in books on mechanics, *the Moment*. Likewise, the force P has a moment, which is the product of the force P and the distance from P to the fulcrum F. If the force and the weight just balance each other, their moments are equal.

The length from P to F is called the *force arm* and the length from W to F, the *weight arm*. Then, for balance, we have the equation:

Force × force arm = Weight × weight arm

If we let P stand for the force
a stand for the force arm
W stand for the weight
and b stand for the weight arm

as shown in Figs. 39, 40, and 41, we will have the formula

$$P \times a = W \times b$$

Although the force and weight are really *balanced* when this formula is fulfilled, still we use the formula for calculating the forces necessary to *lift* weights. The very slightest increase in the force above that necessary for balance will cause W to rise and, therefore, we can say practically that P will lift W if

$$P = \frac{W \times b}{a}$$

If it is the length a that is wanted, we can see that P would lift W when

$$a = \frac{W \times b}{P}$$

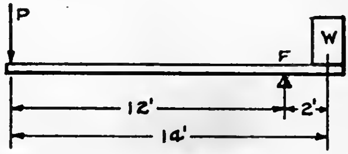


FIG. 38.

Example:

We have a lever 14 ft. long, with the fulcrum placed 2 ft. from the end, as shown in Fig. 38; how much force must we exert to lift 1800 lb.? In this problem a is 12 ft., b is 2 ft., and W is 1800 lb.

$P \times a = W \times b$
$P \times 12 = 1800 \times 2 = 3600$
Then, if $P \times 12$ (or $12 \times P$) is 3600
P will be $3600 \div 12$
$P = 3600 \div 12 = 300$ lb., *Answer.*

It will be seen that the relation between force, weight, force arm, and weight arm, can be written as an inverse proportion.

$$\text{Force} : \text{weight} = \text{weight arm} : \text{force arm}$$
or
$$P : W = b : a$$

This form of expressing the relation is not generally as useful as the other form, $P \times a = W \times b$. It is very useful, however, in cases where neither the force arm nor weight arm are known.

Example:

If a man wanted to lift a 750 lb. weight by means of a 12 ft. timber used as a lever, where would he place the fulcrum so that his whole weight of 150 lb. would just raise it?

$$P : W = b : a$$
$$150 : 750 = b : a$$
$$150 : 750 = 1 : 5$$
$$b : a = 1 : 5$$
$$a = \frac{5}{6} \text{ of } 12 = 10 \text{ ft.}$$
$$b = \frac{1}{6} \text{ of } 12 = 2 \text{ ft.}$$

Explanation: The total length of the timber (12 ft.) is the sum of a and b (see Fig. 39). We can find the ratio of b to a which is the same as P:W and reduces to 1:5. If the ratio is 1:5, then the whole length is 6 parts of which a is 5 parts and b, 1 part. Hence $a = 10$ ft. and $b = 2$ ft., and the fulcrum must be placed 2 ft. from the weight.

92. Three Classes of Levers.—Levers are divided into there kinds or classes according to the relative positions of the force, fulcrum, and weight.

Those shown so far are of the first class, Fig. 39, in which the fulcrum is between the force and the weight. The weight is lifted by pushing down at P.

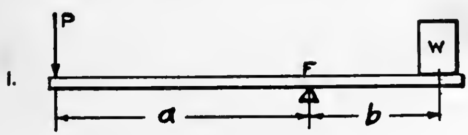


FIG. 39.

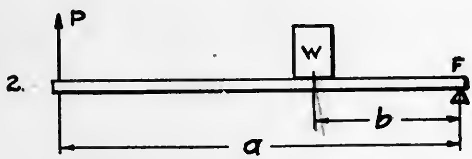


FIG. 40.

In the second class, Fig. 40, the weight is between the fulcrum and the force, and the weight is lifted by pulling up at P.

In the third class, Fig. 41, the force P is between the weight and the fulcrum and, therefore, P must be greater than the load that it lifts. The weight is lifted by an upward force at P.

In all these types the same rule holds that:

$$\text{Force} \times \text{force arm} = \text{weight} \times \text{weight arm}$$

or

$$P \times a = W \times b$$

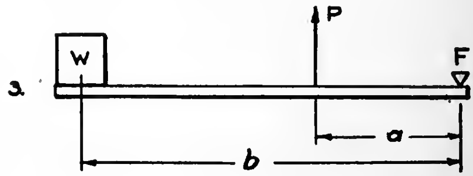


Fig. 41.

Particular attention should be given to the fact that the force arm and weight arm are always measured from the fulcrum. In levers of class 2, the force arm is the entire length of the lever. In class 3, the force arm is shorter than the weight arm. This type may be seen on the safety valves of many boilers and is used so that a small weight can balance a considerable pressure at P.

Quite often there appear to be two weights, or two forces, on a lever, and it is difficult to decide which to designate as the force and which as the weight. It really makes no difference which we call the force and which the weight; the relations between them would be the same in any case.

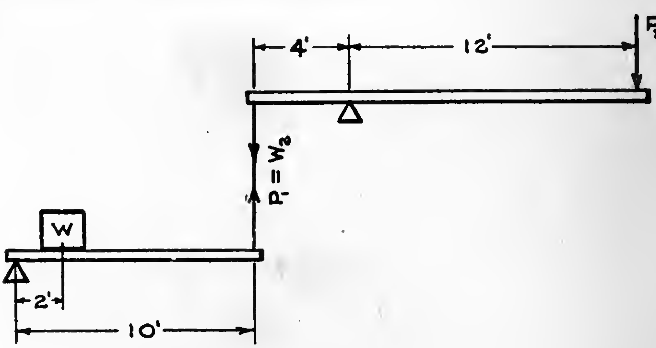


Fig. 42.

93. Compound Levers.—We frequently meet with compound levers; but problems concerning them are easily reduced to repeated cases of single levers, the force of one lever corresponding to the weight of the next, etc. To illustrate this we will solve the following example:

Example:

We wish to lift 8000 lb. with a compound lever as shown in Fig. 42, the first one being 10 ft. long with the weight 2 ft. from the end; the second 16 ft. long with the fulcrum 4 ft. from the end; what will be the necessary force, P_2?

$P \times a = W \times b$

$P_1 \times 10 = 8000 \times 2$

$P_1 = \frac{16000}{10} = 1600$ lb.

$W_2 = P_1 = 1600$ lb.

$P_2 \times a = W_2 \times b$

$P_2 \times 12 = 1600 \times 4$

$P_2 = \frac{6400}{12} = 533\frac{1}{3}$ lb., *Answer.*

Explanation: Taking the first, or lower lever, we find it to be an example of the second class. W has a weight arm b of 2 ft. The force has an arm equal to the whole length of the lever, or 10 ft. The necessary force on the end of this lever we find to be 1600 lb.

The second lever must pull upward through the connection with a force of 1600 lb. In other words, the 8000 lb. weight on the first lever is equivalent to a 1600 lb. weight on the short end of the second lever. The first lever pulls downward the same amount that the second pulls up, or $P_1 = W_2$. Having this 1600 as the weight, we find that a force of $533\frac{1}{3}$ lb. is needed on the end of the second lever.

94. Mechanical Advantage.—The ratio of the weight to the force is often called the Mechanical Advantage of the lever; this ratio is equal to the force arm ÷ the weight arm. In the compound lever of Fig. 42 the M. A. (mechanical advantage) of the first lever equals $10 \div 2$ or 5; of the second, $12 \div 4$ or 3; the M. A. of a compound lever is equal to the product of the M. A. of the separate single levers; hence, of the given compound lever the M. A. is 5×3 or 15. This means that a 1 lb. force will lift 15 lb.; 10 lb. will lift 150; or 100 lb. will lift 1500. The force multiplied by the M. A. gives the weight that can be lifted, or the weight divided by the M. A. gives the necessary force. In the case shown in Fig. 42, the mechanical advantage is 15 and consequently the necessary force is $8000 \div 15 = 533\frac{1}{3}$ lb.

If the mechanical advantage of a lever is 10, then 1 lb. will lift 10 lb., or 800 lb. will lift 8000 lb., etc.; but it must be remembered that the 1 lb. or the 800 lb. must travel 10 times as far as the 10 lb. or the 8000 lb.

If a lever has a mechanical advantage of 10, the force must travel 10 times as far as it lifts the weight, and consequently a lever effects no saving in *work*. Work is the product of force, or weight, times the distance moved, and is the same for either end of the lever. It is similar to carrying a lot of castings to the top floor of a building. If I carry half of them at a time, I must make two trips; if I carry one-fourth of them at a time, I must make four trips. The lighter the load, the more trips I must make. The work done is the same whatever way I carry them and is equal to the product of the total weight times the height to which the load must be carried.

95. The Wheel and Axle.—This is a name given in mechanics to the modification of levers that enables them to be rotated continuously. Fig. 43 shows the principle of this: By wrapping a belt or rope around each of the two circular bodies, we find that the pulls in the cords are in inverse proportion to the radii of the circles. A little consideration shows that the wheel and axle may easily be studied as a force, P, with lever arm R, equal to the radius of the wheel; and a weight, W, with a lever arm r, equal to the radius of the axle. Two pulleys on a countershaft

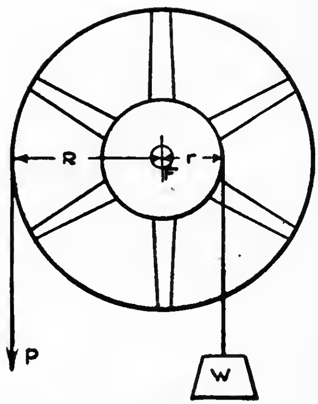


Fig. 43.

might be likened to a lever in the same way. The belt which drives the countershaft furnishes the force P. The radius of this pulley is the force arm. The radius of the other countershaft pulley, which transmits the power to the machine, is the weight arm and the pull in this belt is the weight. Gears also are levers that can be rotated continuously. The simplest example of the use of the axle is probably the windlass, which we see used for hoisting, house-moving, etc. See Figs. 48 and 50.

A geared windlass, such as shown in Fig. 50, is a case of compound levers. The crank and pinion form the first lever and the load on the gear teeth is transmitted to the teeth of the larger gear and becomes the force of the other lever, which consists of the large gear and the drum.

Example:

A geared windlass, such as shown in Fig. 50, has a crank 20 in. long; the small gear is 6 in. in diameter, the large gear is 30 in. in diameter, and the diameter of the drum is 6 in.

What load could be raised by a man exerting a force of 25 lb. on the crank?

$6 \div 2 = 3$ in., radius of pinion
$20 \div 3 = 6\frac{2}{3}$, M. A. of crank and pinion
$30 \div 2 = 15$ in., radius of gear
$6 \div 2 = 3$ in., radius of drum
$15 \div 3 = 5$, M. A. of gear and drum
$6\frac{2}{3} \times 5 = 33\frac{1}{3}$ in., total mechanical advantage
$25 \times 33\frac{1}{3} = 833+$ lb., *Answer.*

The solution of this problem might be shortened by writing all the work in a single equation:

$$\frac{25 \times 20 \times \frac{30}{2}}{\frac{6}{2} \times \frac{6}{2}} = \frac{25 \times 20 \times 15}{3 \times 3} = 833+ \text{ lb., } \textit{Answer.}$$

If we do not know the sizes of the gears in inches, but know the numbers of teeth, we can figure that the mechanical advantage of the pair of gears is the ratio of the numbers of teeth. In such a case with a hoist as shown in Fig. 50, we would first find the mechanical advantage of a simple windlass with the crank attached directly to the drum; then find the M. A. of the pair of gears, and by multiplying these two quantities together we would get the mechanical advantage of the entire hoist.

PROBLEMS

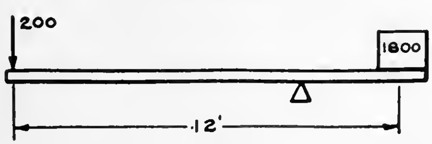


FIG. 44.

161. The lever shown in Fig. 44 is 12 ft. long. Where should the fulcrum be placed so that a weight of 200 lb. will lift a weight of 1800 lb.?

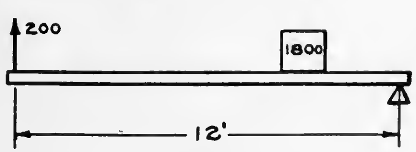


FIG. 45.

162. In Fig. 45 where should the weight of 1800 lb. be placed so that it can be lifted by a force of 200 lb.?

163. Fig. 46 shows a safety valve V loaded with a 50 lb. weight at W. Find the total steam pressure on the bottom of V necessary to lift the valve.

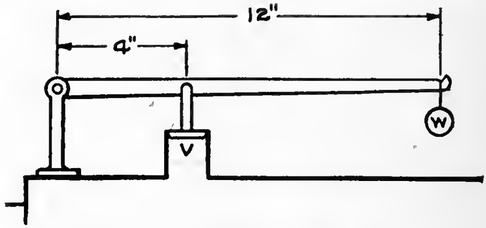


Fig. 46.

164. From the result of problem 163, find the steam pressure per square inch if V is $1\frac{1}{2}$ in. in diameter on the bottom where exposed to the steam.

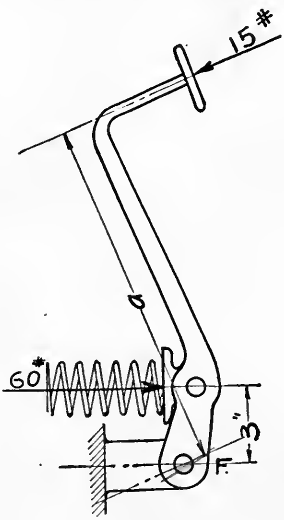


Fig. 47.

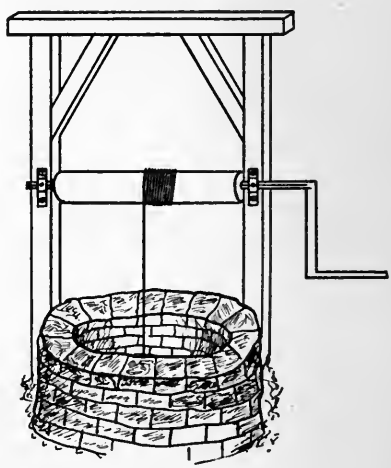


Fig. 48.

165. Fig. 47 shows the clutch pedal for an automobile. What must be the length of the power arm a in order that a foot pressure of 15 lb. can open the clutch against a spring pressure of 60 lb. having an arm of 3 in.?

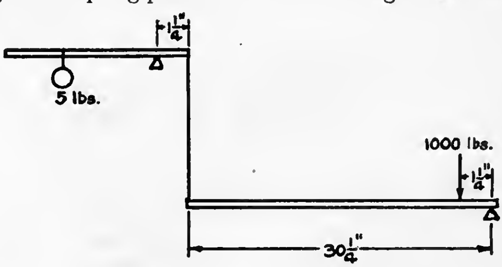


Fig. 49.

166. Fig. 48 shows an old fashioned windlass for raising water. If the crank is 15 in. long, and the drum is 5 in. in diameter, what pressure would be needed on the crank to raise a pail of water weighing 30 lb.?

167. Fig. 49 represents in an elementary way the levers of a pair of platform scales. How far from the fulcrum must the 5 lb. weight be placed to balance the 1000 lb. weight located as shown?

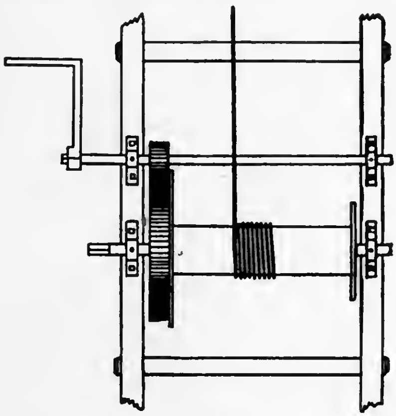


Fig. 50.

168. The hoist of Fig. 50 has an 18 in. crank; the drum is 10 in. in diameter; the diameter of the large gear is 30 in., and of the small gear 6 in. What weight can be raised by a force of 25 lb. on the crank?

CHAPTER XIV

TACKLE BLOCKS

96. Types of Blocks.—When a heavy weight is to be raised or moved through any considerable distance, either a windlass, such as described in Chapter XIII, or tackle blocks can be used. Referring to the figures in this chapter, the revolving part is called the Pulley or Sheave; the framework surrounding the pulleys is called the Block and, as generally used, includes both the frame and the sheaves contained in it.

In Fig. 51 we have a single pulley which serves merely to give a change of direction. There is no mechanical advantage in a single fixed pulley such as this. The pull on the rope at P is transmitted around the pulley and supports W on the other side. We can look at the pulley in this case as a lever with equal arms. P on one end must equal W on the other end. Such a block would be used solely for the convenience it affords, since it is usually easier to pull down than up.

In Fig. 52 the pull P is in the same direction that W is to be moved and is only one-half of W. As explained in Art. 94, the mechanical advantage of a machine can be obtained by comparing the distances moved by the force and the weight. If a force must move five times as far as it lifts the weight, then the mechanical advantage is 5, and the force is one-fifth of the weight. In Fig. 52 the mechanical advantage is 2. This can be seen by raising W a certain distance. The rope on each side will be slacked this same distances and, therefore, P must be drawn up twice this distance in order to remove the slack. Since P moves twice as far as W, the force P will be one-half of W, and the mechanical advantage will be 2.

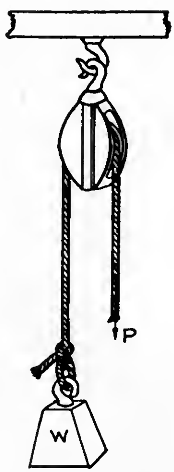


FIG. 51.

FIG. 52.

In Fig. 53 we have merely added, to the device of Fig. 52, a fixed block above to change the direction of P. It makes no change in the relation of P and W, except as to direction.

In Fig. 54 we have two pulleys in the fixed block and two in the movable block. Other cases might have even more pulleys, but the principle is the same, and a general rule for calculating their mechanical advantages will be worked out for all cases. Proceeding as before, let us imagine that W and the movable block of Fig. 54 are lifted 1 ft. The four ropes supporting W will each be slacked 1 ft., and it will be necessary to move P 4 ft. to remove this slack. Hence, the mechanical advantage of this system is 4, and P is $\frac{1}{4}$ of W, or W is 4 times P.

In general, we can say that the mechanical advantage is equal to the number of ropes supporting the movable block and the load. The best way to find the mechanical advantage is to draw a sketch of the blocks and to count the number of ropes that are pulling on the movable block. This number represents the mechanical advantage.

Whenever convenient, it is best to use as the movable block the one from which the free end of the rope runs. This means that *P* will pull in the same direction that *W* is to be moved. The

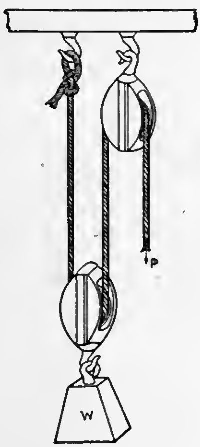


Fig. 53.

Fig. 54.

mechanical advantage is greater by 1 if *P* is pulling in the direction of motion. Notice in Fig. 54 that, if we turned these blocks around and pulled the other way, fastening *W* to what is in the figure the fixed block, the mechanical advantage would be 5 instead of 4.

In all problems where there is any doubt, draw a rough sketch and count the number of ropes pulling on the movable block (see Fig. 55).

Example:

How great a weight can be lifted by a pull of 150 lb. with a pair of pulley blocks, one being a three sheave and the other a two sheave block? Calculate, first, using the three sheave as the movable block and, second, using the two sheave block as the movable one.

Explanation: To avoid confusion, the sheaves are drawn one above the other, instead of parallel. The free end of the rope must run from the three sheave block. Starting from *P*, we wind the rope in and find that the inner end must be fastened to the two sheave block. We count the ropes pulling on each block and find that, with the three sheave block as the movable one, the mechanical advantage is 6, and the weight lifted would be

$150 \times 6 = 900$ lb., *First Answer.*

With the two sheave block as the movable one, the mechanical advantage is 5 and the weight would be

$150 \times 5 = 750$ lb., *Second Answer.*

In practice, about 60% of these theoretical weights would be raised, the rest being lost in overcoming friction. Likewise, to lift a certain load, the actual pull required will be about $\frac{100}{60}$ or $1\frac{2}{3}$ of the theoretical pull.

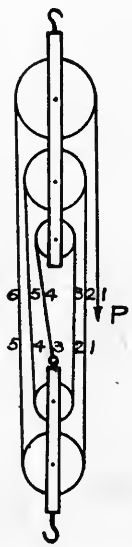


Fig. 55.

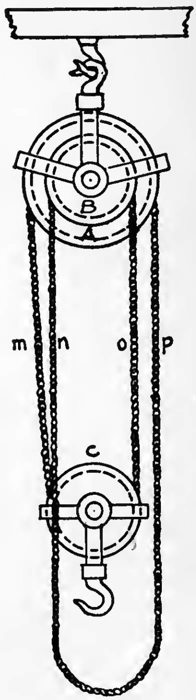


Fig. 56.

97. Differential Pulleys.—In lifting heavy weights by hand, a very satisfactory apparatus to use is a Differential Hoist. This is a very simple and cheap apparatus but it is not very efficient and is, therefore, not to be recommended for continuous use.

The pulleys are arranged as shown in Fig. 56. In the fixed block are two pulleys, *A* and *B*, *A* being somewhat larger than *B*.

These pulleys, A and B, are fastened solidly together and rotate as one about a fixed axis; the pulley C is in the movable block. An endless chain passes over the pulleys as shown, the rims of the pulleys being grooved and fitted with lugs to prevent the chain from slipping. The loop np hangs free and is the pulling loop.

From the figure it is easily seen that, if we pull down on p until pulley A is turned once around, the branch m will be *shortened* a length equal to the circumference of A. Since B is attached to A, it also will turn once around and the branch o will be *lengthened* a distance equal to the circumference of B.

Hence, the loop mo will be shortened by an amount equal to the difference of the circumferences of A and B; and the pulley C will rise one-half this amount.

We can express the difference in the distances moved by m and o as

$$\pi \times D - \pi \times d \text{ or } \pi \times (D-d)$$

where D and d represent the diameters of large and small pulleys, A and B. Hence C will move up one-half of this or $\frac{1}{2}$ of $\pi \times (D-d)$. To cause this motion of C upward, the chain p was moved a distance of $\pi \times D$.

The mechanical advantage of the hoist is obtained by dividing the motion of P by the motion of W.

$$\text{Mech. Adv.} = \frac{\pi \times D}{\frac{1}{2}\pi \times (D-d)}$$

This can be simplified by cancelling π out of both numerator and denominator of the fraction, leaving

$$\text{Mech. Adv.} = \frac{D}{\frac{1}{2} \text{ of } (D-d)}$$

This formula might be written as a rule in the following words: "The mechanical advantage of a differential hoist is obtained by dividing the diameter of the larger pulley in the upper block by half the difference between the diameters of the larger and smaller pulleys."

A differential hoist can actually lift about 30% of the theoretical load with a given pull; that is, the *efficiency* is about 30%. Likewise, to lift a given weight will require about $\frac{100}{30}$ or $3\frac{1}{3}$

times the theoretical force. In other words, the actual force must be such that the 30% that is really effective will equal the theoretical force.

Example:

Calculate the actual pull required to lift 600 lb. with a differential hoist having 10 and 8 in. pulleys and an efficiency of 30%.

$D=10$ in.
$d=8$ in.
$D-d=2$ in.
$\frac{1}{2}$ of $(D-d)=1$ in.

$$\text{Mech. Adv.}=\frac{D}{\frac{1}{2}\text{ of }(D-d)}=\frac{10}{1}=10$$

$600 \div 10 = 60$ lb. theoretical pull
$60 \div .30 = 60 \times \frac{100}{30} = 200$ lb. actual pull.

Explanation: In this case the letters D and d of the formula are 10 in. and 8 in., and we find the M. A. to be 10. It should, therefore, only require a force of 60 lb. to raise the 600 lb. weight. But we find that this type of hoist has only an efficiency of 30%, that is, it only does 30% of what we might expect it to do from our theories. Then to lift 600 lb. will require a force such that 30% of it will be 60 lb. This necessary force is 200 lb.

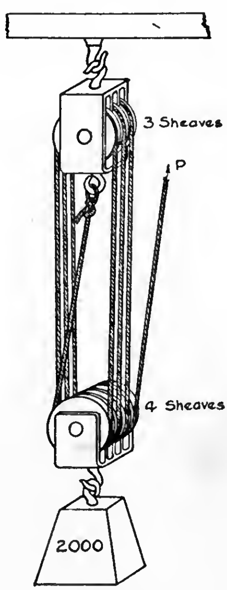


Fig. 57

PROBLEMS

169. A weight of 2000 lb. is to be lifted with a four sheave and three sheave pair of blocks, as shown in Fig. 57. The four sheave is used as the movable block. Neglecting friction and assuming each man to be capable of pulling 125 lb., how many men are necessary?

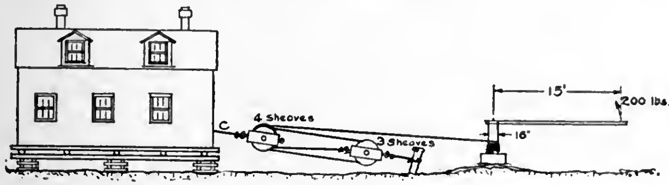


Fig. 58.

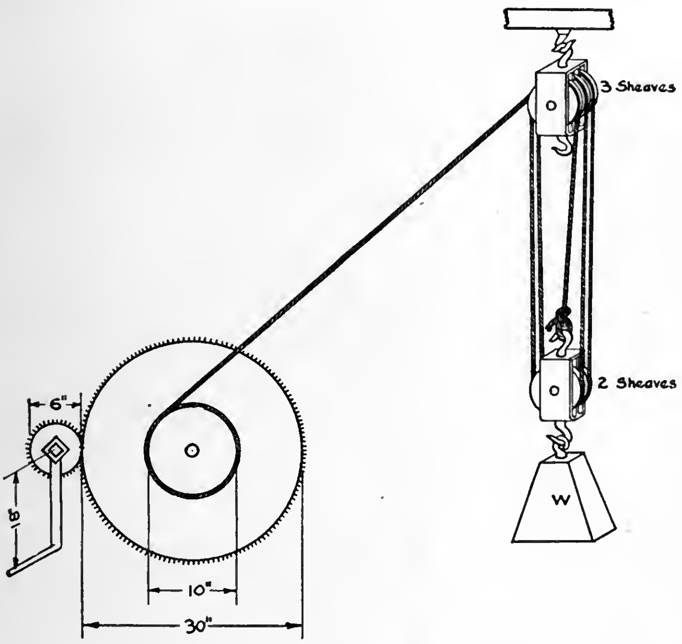


Fig. 59.

170. A windlass and tackle blocks, as shown in Fig. 58, are used for moving a house. If the team can exert a steady pull of 200 lb. at the end of the sweep, find the theoretical pull on the house. Also find the actual pull on the house if the efficiency of the whole mechanism is 65%.

171. Draw a sketch of a pair of blocks, each having 3 pulleys, and indicate which should be the movable block in order to secure the greatest mechanical advantage. What would the mechanical advantage be?

172. When the geared windlass of the dimensions shown in Fig. 59 is used with the pair of pulley blocks, find the weight at W that can be lifted by a force of 25 lb. on the crank.

173. A differential hoist has pulleys $7\frac{1}{2}$ in. and 6 in. in diameter. We attach a weight of 200 lb. to the hoist and find that a pull of 58 lb. is required to raise the weight.

(a) Find the theoretical force required to raise 200 lb. with this hoist.

(b) From this and the force actually required, calculate the efficiency of the hoist.

174. Three men pull 70 lb. apiece on a pair of pulley blocks, two sheaves above and one below. The single block is movable. Find the weight that can be lifted:

(a) Neglecting friction;

(b) Assuming that 40% of the work is lost in friction.

175. A load of 2 tons is to be lifted with a differential hoist. The pulleys are 12 in. and $10\frac{1}{2}$ in. in diameter.

(a) What is the theoretical pull required to lift the load?

(b) What is the actual pull required, if the efficiency of the hoist is 30%?

CHAPTER XV

THE INCLINED PLANE AND SCREW

98. The Use of Inclined Planes.—An Inclined Plane is a surface which slopes or is inclined from the horizontal. Any one who has had experience in raising heavy bodies from one level to another knows that inclined planes are very useful for such work. The Wedge is a form of an inclined plane, the powerful effect of which in splitting wood, quarrying stone, aligning machinery, and performing many other heavy duties is well known. The inclined plane, like the lever and the tackle block, enables us to lift a heavy weight with a smaller force.

99. Theory of the Inclined Plane.—The work done in moving a body up an inclined plane is merely the work of raising the body vertically. If we skid an engine base from the shop floor onto a flat car, the work accomplished is the raising of the base from the floor level to the car level, and is the same as if it was raised straight up by a crane, or by tackle blocks. The effect of the long incline is similar to that of a long force arm on a lever. It enables the force doing the work to use a greater distance, and hence the force will be smaller than the weight raised.

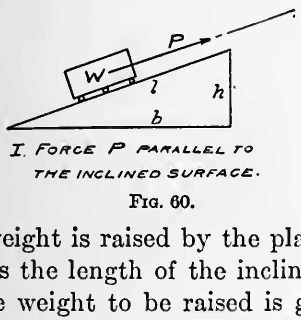
Neglecting friction or, in other words, supposing bodies to be perfectly smooth and hard, no work is done in moving the bodies in a horizontal direction; hence the work done upon a body when it is moved equals the weight of the body times the vertical height to which it is raised. In studying the theory of the inclined plane, we find that the force generally acts in one of two directions in raising the body: either parallel to the incline or parallel to the horizontal base.

In Case I (Fig. 60) the force P is exerted along the incline, and, in raising the weight to the top, will act through a distance l. Meanwhile, it will raise W a distance h. Consequently, the mechanical advantage will be $\frac{l}{h}$.

If we remember that the work put in equals the work got out of a machine (neglecting what is lost in friction) we see that we have the formula

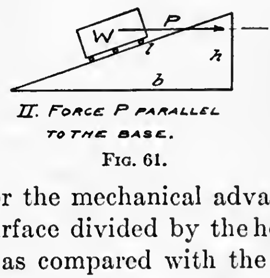
$$P \times l = W \times h$$

To sum up, when the force is exerted parallel to the surface of the inclined plane, the force times the length of the inclined plane equals the weight times the vertical height through which



I. FORCE P PARALLEL TO THE INCLINED SURFACE.

FIG. 60.



II. FORCE P PARALLEL TO THE BASE.

FIG. 61.

the weight is raised by the plane; or the mechanical advantage equals the length of the inclined surface divided by the height. If the weight to be raised is great as compared with the force available, a comparatively long incline must be used to give the necessary mechanical advantage.

In Case II (Fig. 61) the force acts parallel to the base of the inclined plane; that is, along the horizontal. This case is not often found in this elementary form, but is seen in jack screws, in worm gearing, in wedges, and in cams, all of which are modifications of inclined planes. When the force acts parallel to the base, the work expended by it is the product of the force times the length of the base; the work accomplished is, as before, the product of the weight times the height.

$$P \times b = W \times h$$

$$\text{Mechanical advantage} = \frac{b}{h}$$

A comparison of these formulas with those for Case I shows that the mechanical advantage is greatest where the force P is exerted along the incline, as in Case I, because l, the length of the incline or the hypotenuse of a right triangle, is greater than b, the length of the base.

There may be other cases where the force acts in some other direction, but they are seldom seen in practice.

100. The Wedge.—The Wedge consists of two inclined planes placed base to base, the force acting parallel to the base, as shown in Fig. 62, where the horizontal center line of the wedge is

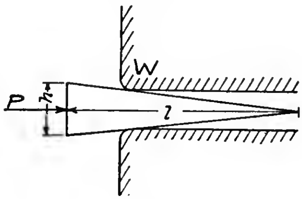


FIG. 62.

the common base of the two inclined planes. Usually the wedge is moved instead of the object to be raised, but the effect is the same and the force relations are the same as if the object itself were being moved up a stationary incline. In Fig. 62 it will be seen that the weight W will be raised a distance h when the wedge is driven a distance l. The work expended in driving the wedge is $P \times l$; the work accomplished in raising the weight is $W \times h$; and, neglecting friction, these are equal, or

$$P \times l = W \times h$$

From this we see that the mechanical advantage of the wedge is:

$$\text{Mechanical advantage} = \frac{l}{h}$$

The relation of P and W might, if desired, be written as a proportion, as follows:

$$P : W = h : l$$

Example:

Fig. 63 shows an adjustable pillow block for a Corliss engine, the bearing being raised or lowered by means of the wedge underneath. If the weight of the shaft and the fly-wheel upon this bearing is 6000 lb., and the wedge has a taper of 1 in. per foot of length, what pressure must be exerted on the wedge by the screw S in raising the shaft?

Mech. Adv. = 12
6000 ÷ 12 = 500 lb. *Answer.*

Explanation: If the taper of the wedge is 1 in. in 12 in., then a motion of 1 ft. would raise the bearing 1 in.; or a motion of 1 in. in the wedge would raise the bearing $\frac{1}{12}$ in. Hence, the Mech. Adv. of the wedge is 12 and $P = \frac{W}{12} = 500$ lb.

Note.—There would also be required, in addition to this 500 lb., a force sufficient to overcome the friction on the top and bottom of the wedge, which is neglected in this solution.

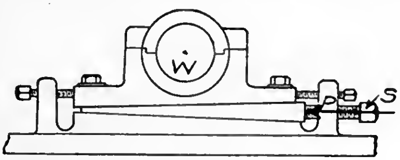


FIG. 63.

101. The Jack Screw.—A screw is nothing but an inclined plane which, instead of being straight, is wrapped around or cut into a round rod or bar. Turning the screw gives the same effect as giving a straight push on an inclined plane or wedge.

When we raise an object with a jack screw, such as shown in Fig. 64, the weight presses down on the screw and, consequently,

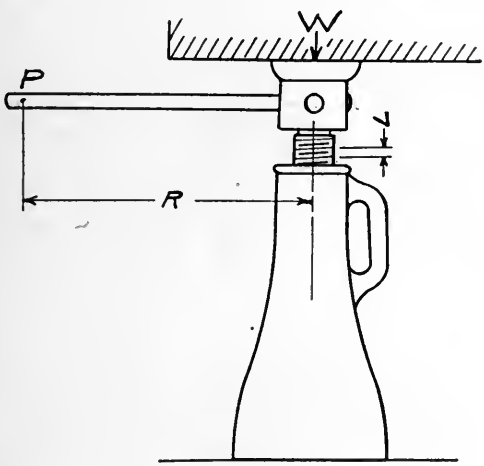


FIG. 64.

is borne by the threads (which are the inclined planes). The threads are advanced and the weight is raised by a pull (which we will call P) on the end of the rod whose length is marked R. The distance the weight W is moved for one revolution of the screw equals the lead of the screw expressed as a fraction of an inch. The lead of a screw is the distance it advances lengthwise in one turn or revolution. The force P moves through a distance equal to the circumference of a circle whose radius is the length of the handle; if we represent the length of the handle or lever by R, then the distance traversed by P in one revolution is $\pi \times 2 \times R$. If we let the letter L represent the lead of the screw then we will have the work accomplished in one revolution of the screw $= W \times L$. Meanwhile, the work expended in doing it

$$= P \times \pi \times 2 \times R.$$

Assuming that there is no friction in the screw, we have

$$P \times \pi \times 2 \times R = W \times L$$

$$\text{or}\quad \text{Mech. Adv.} = \frac{\text{Distance } P \text{ moves}}{\text{Distance } W \text{ is raised}} = \frac{\pi \times 2 \times R}{L}$$

If stated in words, these formulas would read: "The force multiplied by the circumference of the circle through which it moves equals the weight multiplied by the lead of the screw."

"The mechanical advantage of a jack screw equals the circumference of the circle through which the force moves divided by the lead of the screw" (the amount the screw advances in one turn).

Example:

With a $1\frac{1}{2}$ in. jack screw having 3 threads per inch and a pull of 50 lb. at a radius of 18 in., calculate:

(*a*) The theoretical load that can be lifted by the screw;
(*b*) The actual load if the efficiency of the screw is 18%.

$$\text{(a)}\quad \text{Mech. Adv.} = \frac{\pi \times 2 \times R}{L}$$

$$R = 18 \text{ in. and } L = \frac{1}{3} \text{ in.}$$

Hence,

$$\text{Mech. Adv.} = \frac{3.1416 \times 2 \times 18}{\frac{1}{3}}$$

$$= \frac{113.1}{\frac{1}{3}} = 339$$

$W = 339 \times 50 = 16050$ lb., *Answer.*

(*b*) 18% of 16950 = 3051 lb., *Answer.*

Explanation: If there are 3 threads per inch, the lead is $\frac{1}{3}$ in., and if the radius is 18 in., we have the Mech. Adv. = 339.3. In theory then we should be able to lift 50 lb. × 339 = 16950 lb. with this screw. But a screw has considerable friction and for this reason only 18% of the energy expended in this case is effective, the remaining 82% being all lost in friction. The actual weight lifted is, therefore, only 18% of 16950 lb. or 3051 lb.

102. Efficiencies.—In explaining the machines of this chapter and of Chapters XIII and XIV, it was assumed that no work is lost in friction within the machines. In a properly mounted lever there is little energy lost. In a tackle block the loss depends on the size of the pulleys as compared with the size of the rope and on the nature of the pulley bearings. The efficiency may vary from 60 to 95%. The more pulleys there are, the lower will be the efficiency, because each bend in the rope and each pulley means a loss in friction.

With inclined planes, the efficiency may vary all the way from 0 to nearly 100%. It will be lowest if the weight is merely slid on the plane and will be much higher if wheels or rollers are used.

In any machine, if the weight will start back of its own accord when the force is removed, the friction is less than 50% and the efficiency is greater than 50%. If the weight will not start back, the efficiency is less than 50%. This can be shown as follows: Of the force applied to a machine, part of it is absorbed in overcoming the friction within the machine. The balance goes through the machine and is effective in accomplishing the work to be done. Of the whole force applied, the per cent which this effective force represents is called the *Efficiency*. If the efficiency is less than 50%, it shows that the friction absorbs more than half of the total force and, therefore, that the friction is greater than the effective force. Now, suppose we had a simple machine such as a jack-screw, being used to raise a weight. If the applied force is removed, the friction will remain the same, but will now act to hold the weight from going back. If the friction is sufficient to hold the weight, it must at least equal the effective or theoretical force required to raise the weight. Therefore, if a machine does not run backward when the force is removed, the friction must be *more* than one-half of the total force required to raise the weight, and the effective force must be *less* than one-half of this total force. Hence, the efficiency in such a case is less than 50%. A jack-screw will not go down of its own accord when the force is removed and therefore its efficiency is less than 50%. In reality, for the usual dimensions of screws, it has been found to be only from 15 to 20%. Mr. Wilfred Lewis has derived, from experiment, a simple formula which gives the average efficiency for a jack-screw under ordinary conditions.

$$E = \frac{L}{L + D}$$

in which E is the efficiency, as a decimal,
where L is the lead of the screw,
and D is the diameter of the screw.

Example:

Find the probable efficiency of the screw given in the example under Article 101.

$$L=\frac{1}{3}\text{ in., and }D=1\frac{1}{2}\text{ in.}$$

$$E=\frac{.33}{.33+1.5}=.18+$$

$E=18\%$, *Answer.*

One can get an approximate idea of the efficiency of any machine by observing, as before explained, whether or not it will run backward of its own accord when the force is removed. This will tell whether the efficiency is above or below 50%. If it is above 50% and a considerable force is required to keep the weight from going back, then the efficiency is high. If, however, a very slight pull will hold it from going back, then the efficiency is not very much above 50%. If we find the efficiency to be under 50% but find that only a very small pull will start the weight down, then the efficiency is not far under 50%. On the other hand, if it seems as if almost as great a force is required to lower the weight as to raise it, this signifies that the efficiency of the machine is extremely low.

PROBLEMS

176. An engine weighing 5 tons is to be loaded onto a car, the floor of which is 6 ft. from the ground. If 16 ft. timbers are used for the runway, find the pull necessary to draw the engine up the slope, neglecting friction.

177. How many pounds must a locomotive exert to pull a train of 50 cars, each weighing 50 tons, up a grade of 3 in. in 100 ft.?

178. A building is to be raised by means of 4 jack-screws; the screws are 2 in. in diameter, with 4 threads to the inch. The lever is 20 in. long and a 30 lb. force is required on each handle. Calculate the theoretical weight which the four screws should lift under these conditions.

179. Calculate the probable efficiency of these jack-screws from Lewis' formula and estimate the probable weight of the building.

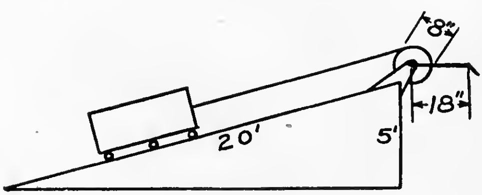


Fig. 65.

180. A windlass with an axle 8 in. in diameter and crank 18 in. long is used in connection with an inclined plane 20 ft. long and 5 ft. high, as shown in Fig. 65. Neglecting friction, what weight can be pulled up the slope with a force of 150 lb. on the crank?

CHAPTER XVI

WORK, POWER, AND ENERGY; HORSE-POWER OF BELTING

103. Work.—Whenever a force causes a body to move, work is done. Unless the body is moved, no work is accomplished. A man may push against a heavy casting for hours and, unless he moves it, he does no work, no matter how tired he may feel at the end of the time. It is evident that there are two factors to be considered in measuring work—*force* and *distance*. In the study of levers, tackle blocks, and inclined planes we dealt with the problem of work. In any of these machines the work accomplished in lifting a weight is measured by the product of the weight and the distance it is moved. The work expended or put into the machine to accomplish this is the product of the force exerted times the distance through which this force must act. We found that, if we neglect the work lost in friction, the work put into a machine is equal to the work accomplished by it. The actual difference between the work put in and the work accomplished is the amount that is lost in friction. The following expressions may make these relations clearer:

$$\text{Work lost in Friction} = \text{Work put in} - \text{Work got out.}$$

$$\text{Efficiency} = \frac{\text{Work got out}}{\text{Work put in}}$$

104. Unit of Work.—The unit by which work is measured is called the *Foot-pound*. This is the work done in overcoming a resistance of one pound through a distance of 1 ft.; that is, if a weight of 1 lb. is lifted 1 ft., the work done is equal to 1 foot-pound. All work is measured by this standard. The work in foot-pounds is the product of the force in pounds and the distance in feet through which it acts. In lifting a weight vertically, the resistance, and hence, the force that must be exerted, is equal to the weight itself in pounds. The work done is the product of the weight times the vertical distance that it is raised. If a weight of 80 lb. is lifted a distance of 4 ft., the work done is 80×4 or 320 foot-pounds. It would require this same amount of work to lift 40 lb. 8 ft., or to lift 20 lb. 16 ft.

When a body s moved horizontally, the only resistance to be overcome is the friction. When a team of horses pulls a loaded wagon, the only resistances which it must overcome are the friction between the wheels and the axles, and the resistance on the tires caused by the unevenness of the road.

The work necessary to pump a certain amount of water is the weight of the water times the height through which it is lifted or pumped (plus, of course, the work lost in friction in the pipes). The work necessary to hoist a casting is the weight of the casting times the height to which it is lifted. The work done by a belt is the effective pull of the belt times the distance in feet which the belt travels. The work done in hoisting an elevator is the weight of the cage and of the load it carries times the height of the lift. Numerous other illustrations of work will suggest themselves to the student.

105. Power.—Power is the *rate* of doing work; that is, in calculating power the time required to do a certain number of foot-pounds of work is considered. If 10,000 lb. are lifted 7 ft. the *work* done is 70,000 foot-pounds, regardless of how long it takes. But, if one of two machines can do this in one-half the time that the other machine requires, then the first machine has twice the *power* of the second.

The engineer's standard of power is the *Horse-power*, which may be defined as the ability to do 33,000 *foot-pounds of work per minute.* The horse-power required to perform a certain amount of work is found by dividing the foot-pounds done *per minute* by 33,000. If an engine can do 1,980,000 foot-pounds in a minute, its horse-power would be $1,980,000 \div 33,000 = 60$. An engine that can raise 66,000 lb. to a height of 10 ft. in 1 minute will do 66,000 lb. $\times$10 ft. $= 660,000$ foot-pounds per minute, and this will equal $\frac{660000}{33000} = 20$ horse-power. If another engine takes 4 minutes to do this same amount of work, it is only one-fourth as powerful; the work done per minute will be $\frac{660000}{4} = 165,000$ foot-pounds per minute; and its horse-power is $\frac{165000}{33000} = 5$ horse-power.

Example:

An electric crane lifts a casting weighing 3 tons to a height of 20 ft. from the floor in 30 seconds; what is the horse-power used?

3 tons = 6000 lbs.

6000 lb. $\times$20 ft. = 120,000 foot-pounds done.

120,000 foot-pounds done in 30 seconds = 240,000 foot-pounds per 1 minute.

$$\frac{240\not{0}\not{0}\not{0}}{33\not{0}\not{0}\not{0}} = 7.27 \text{ horse-power used.}$$

106. Horse-power of Belting.—A belt is an apparatus for the transmission of power from one shaft to another. The driving pulley exerts a certain pull in the belt and this pull is transmitted by the belt and exerted on the rim of the driven pulley.

The power transmitted by any belt depends on two things—the effective pull of the belt tending to turn the wheel, and the speed with which the belt travels. From the preceding pages, it is easily seen that these include the three items necessary to measure *power.* The pull of the belt is the *force.* The speed, given in feet per minute, includes both *distance* and *time.* Force, distance and time are the three items necessary for the measurement of power.

The total pull that a belt will stand depends on its width and thickness. It should be wide enough and heavy enough to stand for a reasonable time the greatest tension put upon it. This is, of course, the tension on the driving side. This tension, however, does not represent the force tending to turn the pulley. The force tending to turn the pulley (or the Effective Pull, as it is called) is the difference in tension between the tight and the slack sides of the belt.

The effective pull that can be allowed in a belt depends primarily on the width, thickness, and strength of the leather, or whatever material the belt is made of. Besides, we must consider that every time a belt causes trouble from breaking or becoming loose, it means a considerable loss in time of the machine, of the men who are using it, and of the men required to make the repairs and, therefore, it should not be loaded as heavily as might otherwise be allowed. Leather belts are called "single," "double," "triple," or "quadruple," according to whether they are made of one, two, three, or four thicknesses of leather. Good practice allows an effective pull of 35 lb. in a single leather belt per inch of width. In a double belt a pull of 70 lb. per inch of width may be allowed. The pull times the width gives the total effective pull or the *force* transmitted by the belt.

The force times the velocity, or speed, of the belt in feet per minute will give the foot-pounds transmitted by it in 1 minute. One horse-power is a rate of 33,000 foot-pounds per minute; hence, the horse-power of a belt is obtained by dividing the foot-pounds transmitted by it per minute by 33,000. The velocity of the belt is calculated from the diameter and revolutions per minute of either one of the pulleys over which the belt travels, as explained in Chapter VII. From these considerations, the formula for the horse-power that a belt will transmit may be written

$$H = \frac{P \times W \times V}{33000}$$

where H = horse-power
P = effective pull allowed per inch of width
W = width in inches
V = velocity in feet per minute

Stated in words, this formula would read as follows: "The horse-power that may be transmitted by a belt is found by multiplying together the allowable pull per inch of width of the belt, the width of the belt in inches, and the velocity of the belt in feet per minute and then dividing this product by 33,000.

Example:

Find the horse-power that should be carried by a 12-in. double leather belt, if one of the pulleys is 14 in. in diameter and runs 1100 R. P. M.

$P = 70$ lb.
$W = 12$ in.
$V = \pi \times \frac{14}{12} \times 1100$
$= 4032$ ft. per min.
$H = \frac{P \times W \times V}{33000}$
$= \frac{70 \times 12 \times 4032}{33000}$
$= 102+$ horse-power, *Answer.*

Explanation: To get the horse-power, we must first find the values of P, W, and V. We will take P as 70 lb. since this is a double belt. W is given, 12 in. V, the velocity, is obtained by multiplying the circumference of the pulley by the R. P. M., which gives us 4032. Multiplying these three together gives 3,386,880 foot-pounds per minute, and dividing by 33,000 we have 102+ as the horse-power that this belt might be required to carry.

107. Widths of Belts.—It is possible, also, to develop a formula with which to calculate the width of belt required to transmit a certain horse-power at a given velocity.

One horse-power is 33,000 foot-pounds per minute. Then the given number of horse-power multiplied by 33,000 gives the number of foot-pounds to be transmitted per minute.

$$\text{Foot-pounds per minute} = 33000 \times H$$

If we know the velocity in feet per minute, we can divide the foot-pounds per minute by the velocity; the quotient will be the force or the effective pull in the belt.

$$\text{Force} = \frac{33000 \times H}{V}$$

Now the force can be divided by the allowable pull per inch of width of belt. The result will be the necessary width.

$$W=\frac{33000\times H}{P\times V}$$

Stated in words, this formula would read: "To obtain the width of belt necessary for a certain horse-power; multiply the horse-power by 33,000 and divide by the product of the allowable pull per inch of width of belt times the velocity of the belt in feet per minute."

Example:

Find the width of a single belt to transmit 10 horse-power at a speed of 2000 ft. per minute.

$H=10$
$V=2000$
$P=35$

$$W=\frac{33000\times H}{P\times V}$$

$$=\frac{33000\times 10}{35\times 2000}=\frac{33}{7}=4\tfrac{5}{7}\text{ in.}$$

Use 5 in. belt, *Answer.*

Explanation: We have given the horse-power and the velocity, and we know that for a single belt a pull of 35 lb. per inch is allowable. This data is all that is needed to calculate the width, which comes out $4\frac{5}{7}$ in. The next larger standard width is 5 in., so that is the size that would be used.

108. Rules for Belting. 1. *Belt Thickness.*—It is generally advisable to use single belting in all cases where one or both pulleys are under 12 in. in diameter, and double belting on pulleys 12 in. or larger. Triple and quadruple belts are used only for main drives where considerable power is to be transmitted and where a single or double belt would have an excessive width. A triple belt should not be run on a pulley less than 20 in. in diameter, nor a quadruple belt on a pulley less than 30 in. in diameter.

2. *Tension per Inch of Width.*—An effective pull of 35 lb. per inch of width of belt is allowable for single belts. For double belts an effective pull of 70 lb. per inch is allowable unless the belt is used over a pulley less than 12 in. in diameter, in which case only 50 lb. per inch should be allowed. A prominent manufacturer of rubber belting recommends 33 lb. per inch of width of belt for 4-ply belts and 43 lb. for 6-ply rubber belts.

3. *Belt Speeds.*—The most efficient speed for belts to run is from 4000 to 4500 ft. per minute. Belts will not hug the pulley and therefore will slip badly if run at a speed of over one mile per minute. These figures are seldom reached in machine shops.

Belts for machine tool drives run from 1000 to 2000 ft. per minute, while main driving belts for line shafts are more often run about 3000 ft. per minute. On wood-working tools we find higher speeds, usually 4000 ft. per minute or over.

4. *Distance between Centers.*—The best distance to have between the centers of shafts to be connected by belting is 20 to 25 ft. For narrow belts and small pulleys this distance should be reduced.

5. *Arrangement of Pulleys.*—It is desirable that the angle of the belt with the floor should not exceed 45 degrees; that is, the belt should be nearer horizontal than vertical. Fig. 66 shows

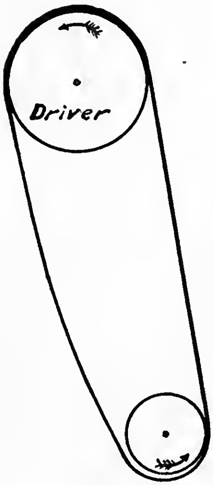


Fig. 66.

Fig. 67.

the effect of having a belt nearly vertical. Any sag in the belt causes it to drop away from the lower pulley and lose its grip on it. Fig. 67 shows the best arrangement. Have the belt somewhere near horizontal and have the tight side of the belt underneath, if possible. This will increase the wrap of the belt around the pulleys. If the lower side is the loose side, the wrap will be decreased by the sag.

It is also desirable, whenever possible, to arrange the shafting and machinery so that the belts will run in opposite directions from the shaft, as shown in Fig. 68. This arrangement balances somewhat the belt pulls, and reduces the friction and wear in the bearings.

For belts which are to be shifted, the pulley faces should be flat; all other pulleys should have the faces crowned (high in the center) about $\frac{3}{16}$ in. per foot of width.

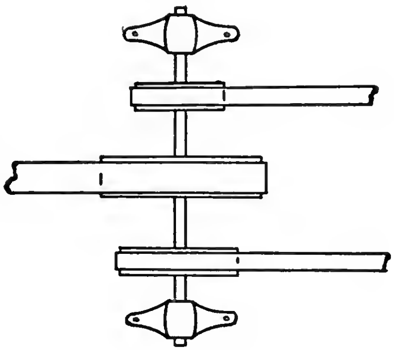


Fig. 68.

6. *Grain and Flesh Sides.*—The grain side of the leather is the side from which the hair is removed. It is the smoothest but weakest side of the leather, and should run next to the pulley surface. It will wrap closer to the pulley surface and thus get a better grip on the pulley. Furthermore, the flesh side, being stronger, is better able to stand the stretching which must occur in the outside of the belt in bending around a pulley.

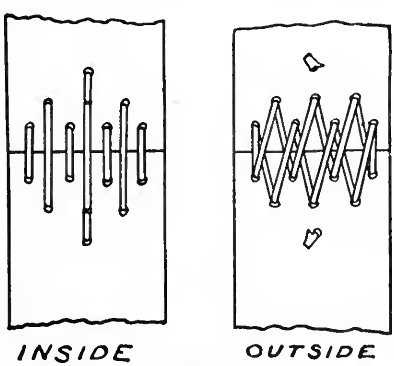


Fig. 69.

7. *Belt Joints.*—Whenever possible, the ends of belts should be fastened together by splicing and cementing. Never run a wide cemented belt onto the pulleys as one side is liable to be stretched out of true. Rather lift one shaft out of the bearings,

place the belt on the pulleys, and force the shaft back into place. Of other methods of fastening belts, the leather lacing is undoubtedly the best when properly done. In lacing a belt, begin at the center and lace both ways with equal tension. Fig. 69 shows an excellent method of lacing belts. The lacing should be crossed on the *outside* of the belt. On the inside, the lacing should lie in line with the belt. Holes should be about $1\frac{1}{2}$ in. apart and their edges should be at least $\frac{7}{8}$ in. from the ends of the belt. The holes should be punched, preferably with an oval punch, the long dimension of the oval running lengthwise of the belt so as not to weaken the belt too much.

PROBLEMS

181. A casting weighs 300 lb. How much work is required to place it on a planer bed 3 ft. 5 in. above the floor?

182. How much work is required to pump 5000 gallons of water into a tank 150 ft. above the pump?

183. Find the horse-power that may be transmitted per inch of width by a single belt running at 2500 ft. per minute. How does this compare with a double belt running at the same speed?

184. A 6 in. double belt is carried by a 48 in. pulley running 250 R. P. M. Find the horse-power that may be transmitted.

185. A shop requires 50 horse-power to run it. The main shaft runs 250 R. P. M. Select a main driving pulley and determine width of double belt to run the shop.

186. A foundry fan runs 3145 R. P. M., and requires 24 horse-power to run it. There are two single belts on the blower running over pulleys 7 in. in diameter. Determine the necessary width of belt.

Note.—(Each belt should be wide enough to drive the fan so that in case one breaks, the other will carry the load.)

187. A belt is carried by a 36 in. pulley running at 150 R. P. M. The effective pull in the belt is 240 lb. Find the horse-power.

188. A pumping engine lifts 92,500 gallons of water every hour to a height of 150 ft. What is the horse-power of the engine?

189. If a freight elevator and its load weigh 5000 lb., what horse-power must be exerted to raise the elevator at a rate of 2 ft. per second?

190. A touring car is travelling on a level road at a rate of 45 miles an hour. If it is shown by actual test that a force of 200 lb. is required to maintain this rate of speed, what horse-power must the engine deliver at the wheels?

CHAPTER XVII

HORSE-POWER OF ENGINES

109. Steam Engines.—In the last chapter, the meaning of the term horse-power was explained and its application to belting was discussed. We will now take up the calculations of the horse-powers of steam and gas engines.

One horse-power was given as the ability to do 33,000 foot-pounds of work in 1 minute. From this we see that the best way to get the horse-power of any engine is to find out how many foot-pounds of work it does in 1 minute and then to divide the number of foot-pounds delivered in a minute by 33,000.

Let us study the action of the steam in the cylinder of the ordinary double-acting steam engine. In Fig. 70 is shown a

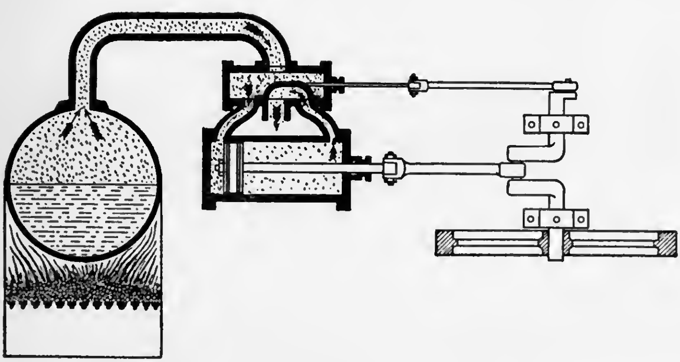


Fig. 70.

section of a very simple boiler and engine. We find that steam enters one end of the cylinder behind the piston and pushes the piston toward the other end of the cylinder. Meanwhile, the valve is moved to the other end of the valve chest. The operation is then reversed and the piston is pushed back to the starting-point. It has thus made two strokes, or one revolution. The steam pressure on the piston is not the same at all points in the stroke, but varies according to the action of the valve in cutting off the admission of steam into the cylinder. However, it is possible to obtain the average or "mean effective pressure" per square inch during a stroke, and, if we multiply this by the piston area in square inches, we will have the average total pressure or force exerted during one stroke. Now reduce the length of the stroke to feet and multiply this by the total pressure just found, and we have the number of foot-pounds of work done during one stroke. This result, when multiplied by the number of working strokes per minute, gives the foot-pounds per minute and this divided by 33,000 gives the horse-power.

The following are the symbols generally used:

$H.\ P.$ = Horse-power.
P = Mean pressure in pounds per square inch.
A = Area of piston in *square inches.*
L = Length of stroke in *feet.*
N = Number of working *strokes* per minute.
$P \times A$ = Total pressure on piston.
$P \times A \times L$ = ft. lb. of work done per stroke.
$P \times A \times L \times N$ = ft. lb. of work done per minute, and hence

$$H.\ P. = \frac{P \times A \times L \times N}{33000}$$ or, as usually written,

$$\frac{P \times L \times A \times N}{33000}.$$ In the latter form, the letters in the numerator spell the word *Plan* and the formula is thus easily remembered.

In the common steam engine, there are two *working* strokes for every revolution of the engine, that is, the engine is what is called *double acting,* and N is twice the revolutions per minute. A few steam engines, like the vertical Westinghouse engine, are single acting and, hence, have only one working stroke of each piston per revolution. Unless otherwise stated, it will be assumed in working problems that a steam engine is double acting.

Example:

Find the horse-power of a 32 in. by 54-in. steam engine running at 94 R. P. M. with an M. E. P. (Mean Effective Pressure) of 60 lb.

Note.—In giving the dimensions of an engine cylinder, the first number represents the diameter and the second number the stroke.

$P = 60$ lb.

$L = 54$ in. $= 4\frac{1}{2}$ ft.

A = Area of 32 in. piston = 804.25 sq. in.
N = Number of strokes per minute $= 94 \times 2 = 188$

$$H.\ P. = \frac{P \times L \times A \times N}{33000}$$

$$= \frac{60 \times 4.5 \times 804.25 \times 188}{33000} = 1237+$$ horse-power, *Answer.*

Notice particularly that the area of the piston is expressed in square inches, because the pressure is given in pounds per square inch; but that the stroke is reduced to feet because we measure work in foot-pounds and, consequently, must express in feet the distance which the piston moves.

If an engine has more than one cylinder, the horse-power of each can be calculated and the results added; or, if the cylinders are arranged to do equal amounts of work, we can find the horse-power of one cylinder and multiply this by the number of cylinders.

The mean effective pressure can be obtained for any engine by the use of a device called an "indicator," which draws a diagram showing just what the pressure is in the cylinder at each point in the stroke. From this diagram, we can calculate the average or mean effective pressure for the stroke. This pressure must not be confused with the boiler pressure or the pressure in the steam pipe. For instance, when the steam comes from the boiler to the engine at 100 lb. pressure, the mean pressure in the cylinder will not be 100 lb., as it would be very wasteful to use steam from the boiler for the full stroke. Instead, the M. E. P. (Mean Effective Pressure) will be from 20% to 85% of the boiler pressure depending on the type of the engine and the load it is carrying. Horse-power calculated as explained here is called Indicated Horse-power because an indicator is used to determine it. The indicated horse-power represents the power delivered to the piston by the steam.

110. Gas Engines.—The most common type of gas or gasoline engine works on what is called the *four stroke cycle.* Such an engine is called a *four-cycle* engine. Fig. 71 shows in four views the operation of such an engine. Four strokes, or two revolutions, are required for each explosion that occurs in the cylinder. Consequently, in calculating the horse-power of a single cylinder gas engine, the number of working strokes (or N in the horse-power formula) is one-half of the R. P. M. There is another type of gasoline engine called the *two-cycle* engine. A single cylinder two-cycle engine has one working stroke for each revolution of the crank shaft and N is therefore the same as the number of R. P. M.

The mean effective pressure of a gas engine is from 40 to 100 lb. per square inch, depending chiefly on the fuel used. For gasoline or natural gas or illuminating gas it is usually between 80 and 90 lb. per square inch.

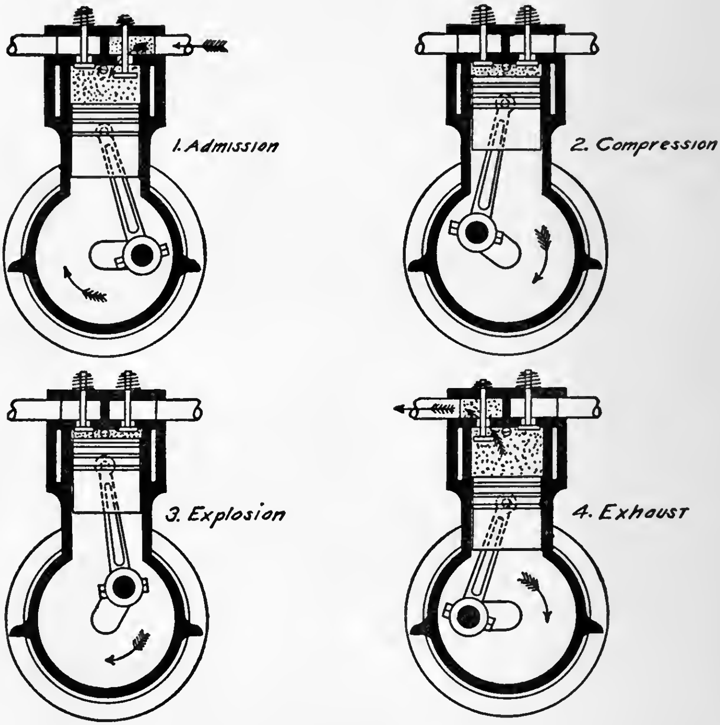


FIG. 71

Example:

What horse-power could be delivered by a single cylinder 5 in. by 8 in. four-cycle gasoline engine running 450 R. P. M.?

Note.—Use a value of $P=80$ lb. per square inch.

$$P=80$$

$$L=\frac{8}{12}=\frac{2}{3}\text{ ft.}$$

$$A=.7854\times 5^2=19.6\text{ sq. in.}$$

$$N=\frac{450}{2}=225$$

$$\text{Then, } H.\,P.=\frac{P\times L\times A\times N}{33000}$$

$$=\frac{80\times\frac{2}{3}\times 19.6\times 225}{33000}=7.13\text{ horse-power, } \textit{Answer.}$$

111. Air Compressors.—An air compressor is like a double acting steam engine in appearance; but, instead of delivering up power, it requires power from some other source to run it. This power is stored in the air and later is recovered when the air is used. An air compressor takes air into the cylinder, raises its pressure by compressing it, and then forces it into the air line or the storage tank. In calculating the horse-power of a compressor, the same formula can be used as for a steam engine. The value of P to use is not the pressure to which the air is raised, but is the average or mean pressure during the stroke. It is usually somewhat less than half the final air pressure; for example, when an air compressor is delivering air at 80 lb. pressure, the mean pressure on the piston is about 33 lb.

Most air compressors are double acting, though there are many small single acting ones.

Example:

A double acting 12 in. by 14 in. air compressor is running 150 R. P. M. It is supplying air at 100 lb. and the mean pressure in the cylinder is 37 lb. per square inch. Calculate the horse-power necessary to run it.

$$P = 37 \text{ lb.}$$

$$L = 14 \text{ in.} = \frac{14}{12} \text{ ft.}$$

$$A = \text{Area of 12 in. piston} = 113.1 \text{ sq. in.}$$

$$N = \text{Strokes} = 150 \times 2 = 300 \text{ per minute.}$$

$$\text{Then, } H.\,P. = \frac{P \times L \times A \times N}{33000}$$

$$= \frac{37 \times \overset{7}{\cancel{14}} \times 113.1 \times \cancel{300}}{\underset{6}{\cancel{12}} \times \underset{110}{\cancel{33000}}} = 44.4, \textit{ Answer.}$$

In this case 12 appears in the denominator in order to reduce the 14 *inches* to *feet.*

112. Brake Horse-power.—The Brake Horse-power of an engine is the power actually available for outside use. It, therefore, is equal to the indicated horse-power minus the power lost in friction in the engine. Brake horse-power can be readily determined by putting a brake on the rim of the fly-wheel and thus absorbing and measuring the power actually delivered. Fig. 72 shows such a brake arranged for use. This form is known as the "Prony Brake." It consists of a steel or leather band carrying a number of wooden blocks. By tightening the bolt at A, the friction between the blocks and the rim of the wheel can be varied at will. The corresponding pull which this friction gives at a distance R ft. from the shaft is weighed by a platform scale or spring balance. From the scale reading must be deducted the weight due to the unbalanced weight of the brake arms, which can be determined by reading the scales when the brake is loose and the engine is not running. If an engine is capable of maintaining a certain net pressure W on the scale, and meanwhile maintains a speed of N revolutions per minute, we can readily see that this is equivalent to an effective belt pull of W pounds on a pulley of radius R running at N revolutions; or it can be considered as being equivalent to raising a weight

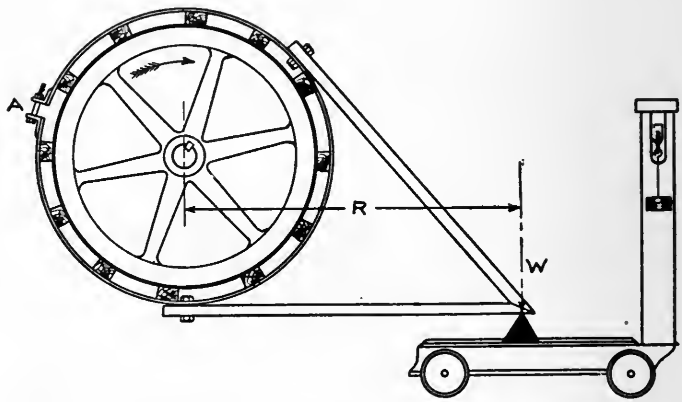


FIG. 72.

equal to W by means of a rope wound around a pulley of radius R turning at N revolutions per minute. This weight would be lifted at the rate of

$$3.1416 \times 2 \times R \times N \text{ ft. per minute}$$

and the brake horse-power will be

$$\text{B. H. P.} = \frac{3.1416 \times 2 \times R \times N \times W}{33,000}$$

The brake and wheel rim will naturally get hot during a test, as all of the work done by the engine is transformed back into heat at the rubbing surfaces of the pulley rim and the brake. It is necessary to keep a stream of water playing on the rim to remove this heat and it is best to have special brake wheels for testing. These have thin rims and inwardly extending flanges on the rims so that a film of water can be maintained on the inner surface of the rim.

Example:

Suppose that, at the time of testing the 5×8 gas engine in article 110, we also determined the brake horse-power by means of a Prony brake having a radius of 3 ft. and that a net pressure of 22 lb. was exerted on the scales (the speed of the engine was 450 R. P. M.). Let us calculate the brake horse-power.

3.1416×2×3×450 = 8482
22×8482 = 186,604 ft. lb.
186,604 ÷ 33,000 = 5.65, *Answer.*

Explanation: Our data is equivalent to that of hoisting a weight of 22 lb. by a rope winding upon a pulley of 3 ft. radius turning at 450 R. P. M. The 22 lb. weight would rise 8482 ft. per minute and the work done per minute would be 22 lb.×8482 ft. = 186604 foot-pounds per minute. Hence, the brake-horse power of the engine is 5.65.

113. Frictional Horse-power.—If this engine gave 7.13 indicated H. P. (I. H. P.), but the power available at the fly-wheel was only 5.65, it stands to reason that the difference, or 1.48 H. P., was lost between the cylinder and fly-wheel. The explanation is that this power is expended in simply overcoming the friction of the engine; and this horse-power is, therefore, called the Frictional Horse-power. At zero brake horse-power, the entire I. H. P. is used in overcoming friction.

114. Mechanical Efficiency.—The ratio of the Brake Horse-power to the Indicated Horse-power gives the mechanical efficiency, meaning the efficiency of the mechanism in transmitting the power through it from piston to fly-wheel. This is usually expressed in per cent. In the case of the engine of which we figured the I. H. P. and B. H. P., the mechanical efficiency was

$$E = \frac{5.65}{7.13} = .792 = 79.2\%$$

The mechanical efficiency of a gas engine is lower than that of a steam engine on account of the idle strokes which use up work in friction while no power is being generated, but at full load a well built gas engine should show over 80 per cent. mechanical efficiency. The mechanical efficiency of a steam engine should be above 90% at full load.

PROBLEMS

191. The cage in a mine weighs 2200 lb. and the load hoisted is 3 tons, The hoisting speed is 20 ft. per *second.* Calculate horse-power necessary. allowing 25% additional for friction and rope losses.

192. A 10 in. by 12 in. air compressor runs 150 R. P. M. The M. E. P. is 30 lb. Calculate the horse-power required to run it.

193. A pump lifts 2000 gallons of water per minute into a tank 150 ft. above it. Find the horse-power of the pump.

194. Find the horse-power of a 10 in. by 12 in. steam engine running 250 R. P. M. with a M. E. P. of 60 lb.

195. What will be the horse-power of a single cylinder, four cycle gas engine with the following data:

Size of cylinder, 12 in. by 16 in.
Rev. per minute, 225
Mean effective pressure, 78 lb. per square inch?
Number of working strokes $= \frac{1}{2}$ of the number of revolutions.

196. A body can do as much work in descending as is required to raise it. Knowing this fact, calculate the horse-power that could be developed by a water-power which discharges 800 cu. ft. of water per second from a height of 13.6 ft., assuming that 25% of the theoretical power is lost in the wheel and in friction.

197. What would be the brake horse-power of a steam engine which exerted a net pressure of 100 lb. on the scales, at a radius of 4 ft., when running at 250 R. P. M.?

198. How many foot-pounds of work per hour would be obtained from a 60 H. P. engine?

199. A centrifugal pump is designed to pump 3000 gallons of water per minute to a height of 70 ft. If the efficiency of the pump is 60%, what horse-power will be required to drive it?

200. The pump of problem 199 is to run 1500 R. P. M. and is to be belt-driven from a 48 in. pulley on a high speed automatic engine, running 275 R. P. M. What should be the diameter and width of face of the pulley on the pump, if the pulley is to be 1 in. wider than the belt?

CHAPTER XVIII

MECHANICS OF FLUIDS

115. Fluids.—Nearly every shop of any size contains some devices which are operated by water or air pressure, so every up-to-date mechanic should have a knowledge of how these machines work.

A Fluid is any substance which has no particular form, but always shapes itself to the vessel which contains it. Water, oil, air, steam, gas—all are fluids. In some ways water, oil, and similar substances are different from the lighter substances—air, steam, etc. To separate these, we give the name of *Liquids* to such substances as water and oil; while air, steam, etc., are given the general name of *Gases*. In some respects liquids and gases are alike and in others they are different. The chief difference is that liquids have definite volumes; they cannot be compressed or expanded any visible amount, while gases can be readily compressed or expanded to almost any extent. For all practical purposes we can say that liquids cannot be compressed. The third form of matter—*Solids*—needs no explanation. The differences in these three forms can be stated as follows:

A Solid has a definite shape and volume.

A Liquid has no definite shape, but has a definite volume.

A Gas has neither a definite shape nor volume.

There are some substances that exist in states in between the solid and the liquid form. Among these are tar, glue, putty, gelatine, etc.

116. Specific Gravity.—By Specific Gravity of a substance we mean its *relative* weight as compared with the same volume of water. Thus we say that the specific gravity of cast iron is 7.21, meaning that cast iron is 7.21 times as heavy as water. A cubic foot of water weighs 62.4 lb. and a cubic foot of cast iron weighs about 450 lb. The quotient $\frac{450}{62.4}=7.21$ is the specific gravity of the iron.

In many hand books we find tables of specific gravities and, when we wish to get the actual weight per cubic inch or per cubic foot of some substance, we must multiply the weight of the unit of water by the specific gravity of the substance.

Example:

The specific gravity of alcohol is .8. How many pounds would there be to a gallon of alcohol?

One gallon of water $= 8\frac{1}{3}$ lb.

One gallon of alcohol $= .8 \times 8\frac{1}{3} = 6\frac{2}{3}$ lb., *Answer.*

If a substance has a specific gravity less than 1, it will float in water, because it is lighter than the same volume of water. If the specific gravity is greater than 1, the substance is heavier than water and will sink. A substance that will float in water may sink in some other liquid if it has a greater specific gravity than the liquid in question. For example, a piece of apple-wood will float in water but will sink when placed in gasoline, the specific gravity of the wood being about .76 and that of gasoline about .71. A piece of iron will sink in water but will float when placed in mercury (quick silver), the specific gravity of mercury being 13.6 and that of iron about 7.21.

117. Transmission of Pressure Through Fluids.—One of the most useful properties of all fluids is the ability to transmit

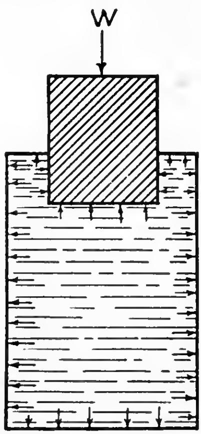


FIG. 73.

pressure in all directions. If we have a vessel filled with water, as shown in Fig. 73, and apply a pressure to the water by means of a piston, as shown, this pressure will be transmitted through the water in all directions. If the sides of the vessel are flat, they will bulge out, showing that there is a pressure on the sides; and if the piston is loose, the water will escape upward around it, showing that there is a pressure in this direction also.

If the total force on the piston is W lb. and the area on the bottom of the piston is A sq. in., then there will be a pressure of $\frac{W}{A}$ lb. exerted on each square inch of the water beneath the piston. This pressure will be transmitted equally in all directions and the pressure on each square inch of the top, sides, and bottom of the vessel will be $\frac{W}{A}$ lb.

Example:

If the piston of Fig. 73 is 6 in. in diameter, and has a total weight of 1000 lb., what would be the water pressure per square inch?

$$p=\frac{W}{A}=\frac{1000}{.7854\times 6^2}$$
$$=\frac{1000}{28.27}=35.4 \text{ lb. per sq. in., } \textit{Answer.}$$

Explanation: As the piston rests on the water, the pressure of the water on the bottom of the piston must be sufficient to support the weight. The area of the bottom is 28.27 sq. in., and the pressure on each sq. in. will be $\frac{1000}{28.27}$ or 35.4 lb. persquare inch. This pressure is transmitted throughout the water and is exerted by it with equal force in all directions.

118. The Hydraulic Jack.—This property of water of transmitting pressure in any direction is made use of in many ways. The same property is, of course, common to other fluids such as

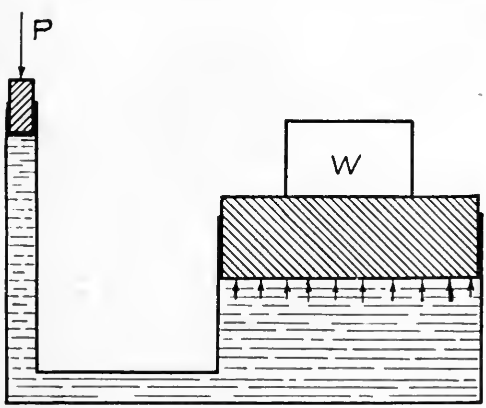


Fig. 74.

oil, air, etc. Wherever we find a powerful, slow-moving force required in a shop, we usually find some hydraulic machine. (The word "hydraulic" refers to the use of water but it is often applied to machines using any liquid—water, oil, or alcohol.)

Fig. 74 shows the principle of all these hydraulic machines.

A small force P is exerted on a small piston and this produces a certain pressure in the water. This pressure is transmitted to the larger cylinder where the same pressure per square inch is exerted on the under side of the large piston. If the large piston has 100 times the area of the small piston, the weight supported (W) will be 100 times P. If the water pressure produced by P is 100 lb. per square inch and the large piston has an area of 100 sq. in., then the weight W that can be raised will be 10,000 lb.

Like the lever, the jackscrew, and the pulley, this increase in force is obtained only by a decrease in the distance the weight is moved. The *work* done on the small piston is theoretically the same as the work obtained from the large piston. For example, suppose that the large piston has 100 times the area of the small one and we shove the small piston down 1 in.; the water that is thus pushed out of the small cylinder will have to spread out over the entire area of the large piston; the large piston will, therefore, be raised only one one-hundredth of the distance that the small piston was moved. Thus, we have, here also, an application of the law that the work put into a machine is equal, neglecting friction, to the work done by it.

$$\text{Force} \times \text{distance moved} = \text{weight} \times \text{distance raised.}$$

The Mechanical Advantage of such a machine will be seen to be the ratio of the areas of the pistons. In the case just mentioned, the ratio of the areas of the pistons was 100:1; hence, the mechanical advantage was 100.

In Fig. 74, the motion that can be given to W is very limited, but by using a pump with valves, instead of the simple plunger P, we can continue to force water into the large cylinder and thus secure a considerable motion to W.

Fig. 75 shows a common form of hydraulic jack which operates on this principle. The top part contains a reservoir for the liquid, and also has a small pump operated by a hand lever on the outside of the jack. By working the lever, the liquid is pumped into the lower part of the jack between the plunger and the casing, thus raising the load. The load may be lowered by slacking the lowering screw Y. This opens a passage to the reservoir, and the load on the jack forces the liquid to flow back through this passage to the reservoir.

In calculating the mechanical advantage of a hydraulic jack, we must consider the mechanical advantage of the lever which operates the pump as well as the advantage due to the relative sizes of the pump and the ram.

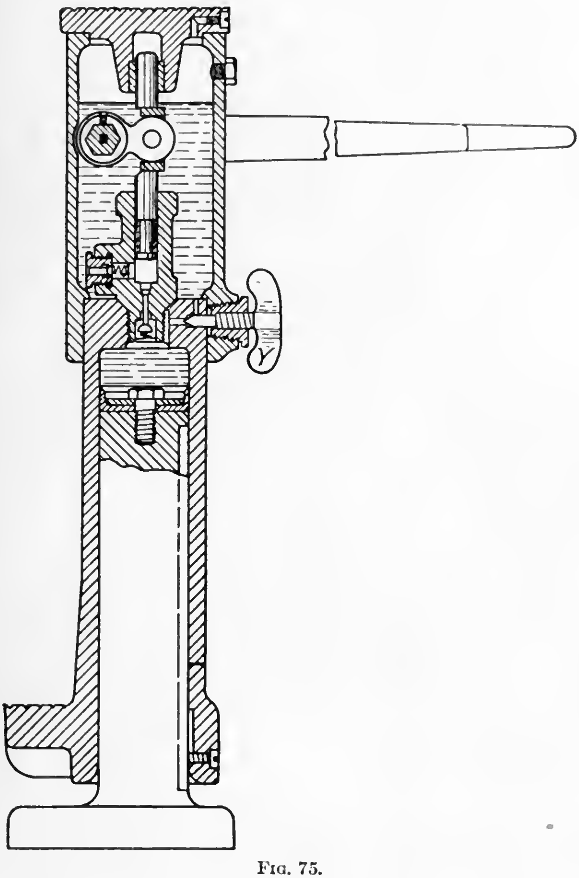


FIG. 75.

Example:

If the ram of Fig. 75 is 3 in. in diameter and the pump is 1 in. in diameter, while the lever is 15 in. long and is connected to the pump at a distance of 1½ in. from the fulcrum, what is the mechanical advantage of the entire jack?

$$\frac{.7854 \times 3^2}{.7854 \times 1^2} = \frac{9}{1} = 9$$

$$\frac{15}{1.5} = 10$$

$9 \times 10 = 90$, *Answer.*

Explanation: The areas of the ram and pumps are as 9:1, hence their mechanical advantage is 9. The lever has a mechanical advantage of 10. Hence, that of the whole jack is $9 \times 10 = 90$, and a force applied at the end of the lever would be multiplied 90 times. This force would, however, move through a distance 90 times as great as the distance the load would be raised.

The hydraulic jack has usually an efficiency of over 70% and is, therefore, a much more efficient lifting device than the jack screw. A mixture containing one-third alcohol and two-thirds water should be used in jacks. The alcohol is added to prevent freezing.

119. Hydraulic Machinery.—In the shop, we often find water pressure used to operate presses, punches, shears, riveters, hoists, and sometimes elevators. These machines are seldom operated by hand power but have water supplied under pressure from a central pumping plant. The admission of the water and the consequent motion of the machine is controlled by hand operated valves. Most of these hydraulic machines are used where tremendous forces are required. Therefore, very high water pressures are used, occasionally as high as 3000 lb. per square inch. 1500 lb. per square inch is a common working pressure for hydraulic machines.

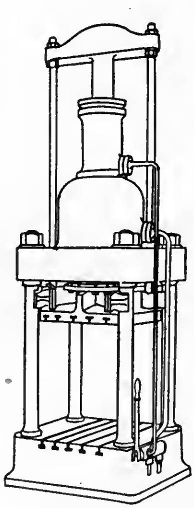


Fig. 76.

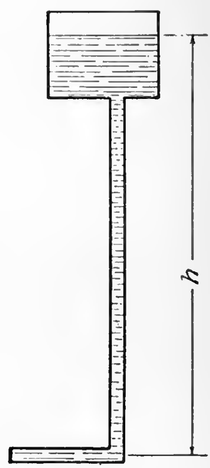


Fig. 77.

Fig. 76 shows a press operated by hydraulic pressure. It will be noticed that the movable head is connected to two pistons—a large one for doing the work on the down stroke, and a smaller one above, used only for the idle or return stroke of the press.

120. Hydraulic Heads.—Quite often we use a high tower or tank to secure a water pressure, or we make use of some natural source of water which is at some elevation. This is most often seen in the water supplies for towns and cities. Water tanks are put upon the roofs of some factories for the same purpose. The higher the tank, the greater will be the pressure which it will maintain in the system. Let Fig. 77 represent such a system. The water at the bottom has the weight of a column of water h ft. high to support and, consequently, will be under a pressure equal to the weight of this column of water. A volume of water 1 in. square and 1 ft. high weighs .434 lb., so the pressure per square inch at the base of the column in Fig. 77 will be $.434 \times h$. Notice particularly that the shape and size of the tank has no influence on the pressure, it being used merely for storage and to keep the pressure from falling too fast if the water is drawn off. The water in the tank on either side of the outlet is supported by the bottom of the tank and has no effect on the pressure in the pipe. The pressure at the bottom of the pipe would be the same if the pipe alone extended up to the height h without the tank. Also the size of the pipe has no effect on the pressure *per square inch.* The water in a large pipe will weigh more than in a small pipe, but the pressure will be spread over a larger area and if, the heights are the same, the pressure per square inch will be the same.

In pumping water to an elevated tank or reservoir, the pressure required per square inch is also determined in the same manner and is .434 times the height to which the water is raised, plus an allowance for friction in the pipes. Thus, to pump water to a height of 100 ft. requires a pressure somewhat greater than $.434 \times 100 = 43.4$ lb. per square inch.

121. Steam and Air.—Steam and air are likewise used to produce pressures in shop machinery. Being more elastic than water, they are preferred where the machines are to be operated quickly. Devices using air are called "pneumatic appliances," among the most common of which are pneumatic drills, hammers, and hoists. The air for operating these is supplied by air compressors located in the power house. These take the air from out of doors and compress it into a smaller volume. The resistance of the air to this compression causes it, in its effort to escape, to exert a pressure on the walls of the tank or pipe containing it. The more the air is compressed, the greater is the pressure exerted by it. The air pressure used in shop work is usually about

80 lb. per square inch. The air is conducted through pipes and hose to the point where it is to be used and there allowed to exert its pressure on the piston of the appliance which is to be driven.

PROBLEMS

201. The specific gravity of Lignum Vitæ (a hard wood) is 1.328. Will this wood float or will it sink in water?

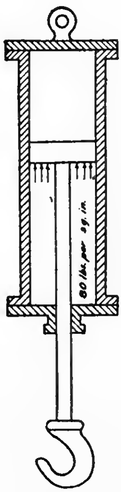


Fig. 78.

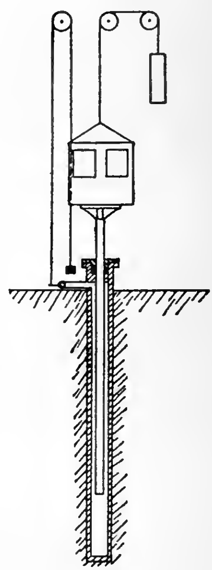


Fig. 79.

202. What weight on the small piston of Fig. 74 would support a weight of 30,000 lb. on the large piston if the small piston is 1 in. in diameter and the large one 12 in. in diameter?

203. If a hydraulic press works with a water pressure of 1500 lb. per square inch, what must be the diameter of the ram if a total pressure of 75,000 lb. is to be produced?

204. If the air hoist of Fig. 78 has a cylinder 10 in. in diameter inside, and the piston rod is $1\frac{1}{4}$ in. in diameter, and an air pressure of 80 lb. per square inch is exerted on the bottom of the piston, what weight can be lifted by the hoist?

205. If a city wishes to maintain a water pressure of 80 lb. per square inch at their hydrants, how high above the streets must be the water level in the stand pipe?

206. Fig. 79 shows the principle of one form of hydraulic elevator, the car being fastened directly to a long ram which is raised by water pressure. The weight of the ram and car are partially balanced by a counterweight. If this elevator is operated with water from the city mains at 80 lb. pressure per square inch and the ram has a diameter of 10 in., what load can be lifted allowing 30% for losses in friction, etc.?

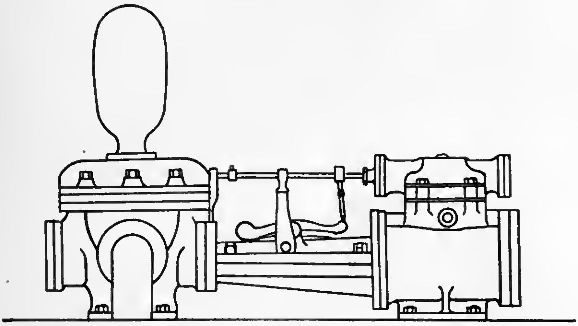


FIG. 80.

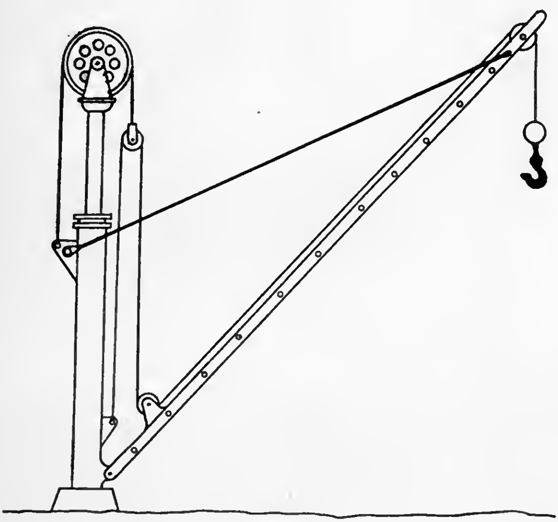


FIG. 81.

207. If a steam pump, such as shown in Fig. 80, has a 12 in. steam piston and an 8 in. water piston, what water pressure can be produced with a steam pressure of 90 lb. per square inch? How many gallons would be pumped per minute when the pump is running at 100 strokes per minute, the stroke of the pump being 12 in.? (1 gallon = 231 cu. in.)

208. What water pressure must a pump be capable of producing in order to force the water to a reservoir at an elevation of 300 ft. above the pump?

209. A gravity oiling system has an oil tank placed 10 ft. above the bearings to be lubricated. The tank is connected by small tubes to the various bearings. If the specific gravity of the oil is .88, what pressure will the oil have at the bearings?

210. In Fig. 81 we have a hoist operated by a hydraulic ram in the top of the crane post. The motion of the ram is multiplied by the system of pulleys shown in the figure. What size must the ram be that a load of 10,000 lb. can be lifted with a water pressure of 72 lb. per square inch, the efficiency of the whole apparatus being 70%?

CHAPTER XIX

HEAT

122. Nature of Heat.—Some of the effects of heat are very useful in shop work and every mechanic should know something of the nature of heat and of the laws which govern its applications to shop work.

Heat is a form of energy; that is, it is capable of doing work. This we see amply illustrated in the steam engine and the gas engine, where heat is used in producing work. The steam engine uses heat which has been imparted to the steam in the boiler. Part of the heat of the steam is changed to work in the engine and the rest is rejected in the exhaust. Heat is not a substance as was formerly supposed—it cannot be weighed and cannot exist by itself.

It is always found *in* some substances. We generally get heat by burning some fuel such as coal, wood, gas, or oil. In burning, the fuel unites with oxygen, one of the constituents of air, and this process, called combustion, generates the heat. We cannot get heat by this process, therefore, without air. No fuel will burn without a supply of air, and as soon as we shut off the air from a fire, combustion stops and no more heat is generated. A fire may continue to give off heat for some time after the air is cut off, but this heat comes from the cooling of the hot fuel in the fire. Of the heat generated during combustion, some of it goes through the furnace walls to the surrounding air; some goes to heat up the bed of coals and any object that may be placed in the fire to be heated; but the greater part of the heat goes off in the gases that are formed by the union of the fuel with the air. It is to save this heat that we sometimes see steam boilers set up in connection with the furnaces of large forge shops.

There are other ways of generating heat besides that of combustion. One method, that is coming into considerable use and which is especially interesting to shop men because of the ease with which it can be controlled, is by the use of electricity. We now have electric annealing and hardening furnaces for use in tool rooms, where a close regulation of the heat is very desirable. Then there are the electric furnaces by which aluminum and carborundum are produced. We also have electric welding as an example of the production of heat from electricity.

Another method of heat generation that is frequently encountered in shops, often where it is not desired, is the production of heat from work. We have seen how heat is turned into work, but here we have work returned into heat. One common case of this is in bearings, where heat is produced from the work that is spent in overcoming the friction. Another example is seen in the heating of a lathe tool when it is taking a heavy cut, or in the heating of the tool when it is being ground. In either event, the work spent in removing the metal goes into heat.

123. Temperatures.—Temperature is the indication of the height or intensity of the heat in a body. Lowering the temperature means a removal of heat from a body, and raising the temperature means the addition of more heat. The common method of measuring temperature is by means of an instrument known as a thermometer, which usually consists of a glass tube which is partly filled with mercury and which has the air exhausted from the other part of it. The mercury expands and contracts as the temperature rises or falls and, therefore, the height of the column of mercury is a measure of the temperature. Alcohol is often used instead of mercury for outdoor thermometers where the mercury might freeze.

There are two kinds of thermometer scales in common use—the Centigrade (abbreviated C.) and the Fahrenheit (abbreviated Fahr. or F.). On the Centigrade thermometer the space between the freezing-point of water and the boiling-point at atmospheric pressure is divided into 100 equal parts called Degrees (represented by °) the freezing-point being marked zero (0°) and the boiling-point 100°. The balance of the scale is then divided into spaces of equal length below zero and above 100° in order that temperatures higher than 100 and lower than zero may be read.

On the Fahrenheit scale the freezing-point of water is marked 32° and the boiling-point 212°, so the space between is divided into 180° ($212° - 32° = 180°$). This scale is also marked with divisions below 32° and above 212° in order to make the thermometer read through a wider range. The Fahrenheit thermometer is used more commonly in the United States than the Centigrade, which is used extensively in Europe. The Centigrade scale is, however, used in this country for most scientific work and is becoming so common that it is desirable to understand the relations between the two scales. Fig. 82 shows the relation of the two scales up to 212° F. or 100° C.

Since the same interval of temperature is divided into 100 parts in the Centigrade scale, and 180 parts in the Fahrenheit scale, each Centigrade degree is $\frac{180}{100}$, or $\frac{9}{5}$ Fahrenheit degrees. Similarly, one Fahrenheit degree is $\frac{5}{9}$ of a Centigrade degree. A change of 30° in temperature on the Centigrade scale would equal $\frac{9}{5}$ of 30, or 54° change on a Fahrenheit thermometer. Likewise, when the mercury moves 27° on a Fahrenheit thermometer, it would move only $\frac{5}{9}$ of $27 = 15°$ on a Centigrade scale. In changing a reading on one thermometer scale to the corresponding reading on the other, it is necessary to remember that the zero points are not the same. The Centigrade zero is at 32° F. In other words, the two zeros are 32 Fahrenheit degrees apart.

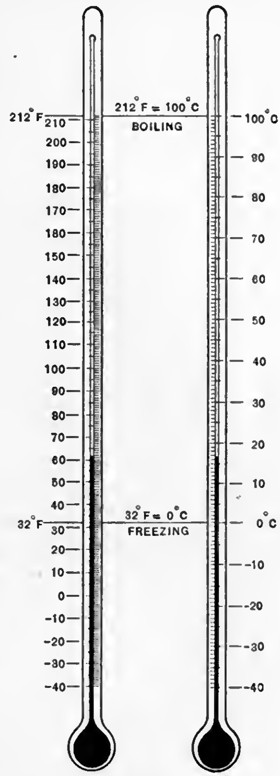


Fig. 82.

To change a reading on the Centigrade scale to the corresponding Fahrenheit reading: First multiply the degrees C. by $\frac{9}{5}$. This gives an equivalent number of degrees on the F. scale. To this add 32, in order to have the reading from the F. zero.

To change a reading on the Fahrenheit scale to the corresponding Centigrade reading: First subtract 32. This gives the number of F. degrees above freezing (which is the C. zero). Multiply the result by $\frac{5}{9}$, thus obtaining the desired C. reading.

Examples:

1. Change 30° C. to the corresponding Fahrenheit reading.

$$30 \times \frac{9}{5} = 54, \text{ the equivalent number of F. degrees.}$$

$$54 + 32 = 86° \text{ F., the reading on a F. thermometer.}$$

2. What would a Centigrade thermometer read when a Fahrenheit thermometer stood at 72°?

$72-32=40$, the number of F. degrees above freezing.

$40\times\frac{5}{9}=22\frac{2^\circ}{9}$ C., *Answer.*

These rules or relations are often expressed by the following formulas, in which C stands for a reading on the Centigrade thermometer and F for a reading on the Fahrenheit thermometer.

$$C=\frac{5}{9}\times(C-32)$$

$$F=\frac{9}{5}\times C+32$$

The parenthesis ( ) when used as above means that the work indicated inside of it is to be done *first* and then the result multiplied by $\frac{5}{9}$. In the second formula, the C is first to be multiplied by $\frac{9}{5}$ and then the 32 is added to the product. It is always to be understood that multiplications and divisions are to be performed before additions and subtractions unless the reverse is indicated, as was done in the first of these formulas, by the use of the parenthesis ( ).

When it comes to measuring the temperatures in furnaces, as is often desirable in fine tool work, a thermometer is clearly out of the question. As the mercury thermometer is ordinarily made, it should not be used for temperatures above 500°, but, by filling the glass tube above the mercury with nitrogen gas under pressure, a thermometer can be made that may be read up to 900°. For higher temperatures, devices called Pyrometers are used. There are numerous kinds of pyrometers, but the one most used in the shops for furnace temperatures is what is called the Le Chatelier pyrometer. In this pyrometer, one end of a porcelain tube about $\frac{5}{8}$ in. in diameter and from 12 in. to 40 in. long is thrust into the furnace and held there or, if frequent readings are to be taken, it may be placed there permanently. Inside this tube are some wires of special composition that generate an electric current when they get hot. From the other end of the tube a couple of wires run to a small box containing a "galvanometer," that is, a device for indicating the strength of the electric current generated. This has a needle swinging over a dial and the dial is usually laid off in degrees so the temperature is read direct. Most of these pyrometers have centigrade graduations, but one should be sure which scale a pyrometer has before he uses it.

For example, suppose we wanted to get 1000° F. and by mistake had 1000° C. instead.

$$\frac{9}{5}\times 1000+32=1832^\circ \text{ F.}$$
$$1000^\circ \text{ C.} \quad =1832^\circ \text{ F.}$$

This shows that it would be pretty serious to use the wrong scale.

It has for years been the practice of the older shop men to tell the temperature of steel or iron by its color. This method has its disadvantages, however, as so much depends on the sensitiveness of the man's eye and on whether the work is being done in bright sunlight or in a dark corner of the shop. A bar will show red in the dark when it would still be black in the sunlight.

For the lower range of temperatures (those used in tempering tools) we can judge the temperature by the color which will appear on a polished steel surface when heated in the air. These tempering colors and their uses for carbon tool steels are about as follows:

430° F.	Very pale yellow	Scrapers Hammer faces Lathe, shaper, and planer tools
460°	Straw yellow	Milling cutters Taps and dies
480°	Dark straw color	Punches and dies Knives Reamers
500°	Brownish-yellow	Stone cutting tools Twist drills
520°	Yellow tinged with purple	Drift pins
530°	Light purple	Augers Cold chisels for steel
550°	Dark purple	Hatchets Cold chisels for iron Screw drivers Springs
570°	Dark blue	Saws for wood
610°	Pale blue	
630°	Blue tinged with green	

More uniform results can be obtained if the steel is heated in a bath of sand or of oil, the bath being maintained at the desired temperature and a pyrometer being used to observe the temperature. For higher temperatures, molten lead or mineral salts such as common salt, barium chloride, potassium chloride, and potassium cyanide are used.

When steel and iron are heated to higher temperatures, they successively become red, orange, and white. These colors and the corresponding temperatures are about as follows:

957° F.	First signs of red
1290°	Dull red
1470°	Dark cherry
1655°	Cherry red
1830°	Bright cherry
2010°	Dull orange
2190°	Bright orange
2370°	White heat
2550°	Bright white—welding heat
2730° to 2910°	Dazzling white.

124. Expansion and Contraction.—Nearly all substances expand when heat is applied to them and contract when heat is removed. This phenomenon is greatest in gases and least in solids, but even in solids it is of enough moment to be extremely useful at times or to cause considerable trouble when allowance is not made for it.

There are a few metal alloys which, within certain limits, do not change their volumes with changes of temperature, and there are also some which between certain temperatures will even expand when cooled and contract when heated. A nickel steel containing 36% nickel has practically no expansion or contraction with changes in temperature and is, therefore, used in some cases for accurate measurements where expansion of the measuring instruments would introduce serious errors.

When a solid body is heated it expands in all directions, if free to do so, but as a rule we are concerned only with the change of one dimension and not with the change in volume. Thus, in the case of a steam pipe we do not care about the change in thickness or in diameter, but we are concerned with the change in length. On the other hand, when a bearing gets hot and seizes, it is the change in diameter that causes the trouble. There are few machinists who have not had the experience, in boring a sleeve to fit a certain shaft, of having a free fit when tested just after taking a cut through the sleeve, and then later of finding that the sleeve fitted so tightly that it had to be driven off the shaft. Of course, the explanation is that the sleeve becomes warm when being bored in the lathe, while the shaft is much cooler. When the sleeve cools to the temperature of the shaft, it contracts and seizes or "freezes" to the shaft. In accurate tool work the effect of differences in temperature between the measuring instruments and the work may become serious. For this reason many gages are provided with rubber or wooden handles which do not conduct heat readily. They thus prevent the heat of the hand from getting into the gages and expanding them.

But this is enough to give some idea of the troubles caused by this property of materials; let us now see of what benefit it is. We have already seen the use that is made of the expansion of mercury in thermometers. There are numerous heat regulating devices (called *thermostats*) which depend on the expansion or contraction of a bar to perform the desired operations. We find these used for regulating house heating boilers and furnaces, incubators, and other devices where uniform temperatures are required. Probably the greatest shop use of expansion and contraction is in making shrink fits. When we want to fasten securely and permanently one piece of metal around another, we generally shrink the first onto the second. This process is used for attaching all sorts of bands and collars to shafts, cylinders, and the like, for putting tires on locomotive wheels, and for similar work. The erecting engineer uses it to put in the links in a sectional fly-wheel rim or to draw up bolts in the hub or in any other place where he wants to make a rigid permanent joint.

The amount of linear expansion which a body undergoes depends upon the kind of material of which the body is made, upon the amount of the temperature change and, of course, upon the original length.

The coefficient of linear expansion of a substance is that part of its original length which a body will expand for each degree change in temperature. Coefficients for different metals have been determined for our use by careful experiments, and can be found in hand books or tables under the head of "Coefficients of Expansion." The values given in different books do not always agree. In fact, the exact compositions of the metals used in the tests were undoubtedly different for the different tests that are on record. Hence, different tables give slightly different rates of expansion. The following values are taken from the most reliable authorities and are sufficiently accurate for most purposes.

COEFFICIENTS OF EXPANSION

Metal	Coefficient
Aluminum	.00001234
Brass	.00001
Cast iron	.0000055 to .000006
Wrought iron and machine steel	.0000065
36% nickel steel	.0000003

The above values are based on a temperature rise of 1° F. For one Centigrade degree change in temperature the coefficients would be $\frac{9}{5}$ of those just given. The student is not expected to memorize these values. Remember that if the length is given in feet the expansion calculated will be in feet, and if the length is in inches the expansion calculated will be in inches. To get the actual expansion per degree for any certain length, multiply the coefficient of expansion by the length. If the temperature change is 100°, the expansion will be 100 times that for 1°.

Example:

The head of a gas engine piston in operation has a temperature of about 400° higher than the cylinder in which it is running. What allowance must be made for this expansion in a 12 in. piston? (The piston is made of cast iron.)

$.000006 \times 400 = .0024$ in. expansion per inch
$.0024 \times 12 = .0288$ in. expansion in 12 in., *Answer.*

The head of the piston must, therefore, be turned at least .0288 in. small to allow for the expansion to take place without the piston seizing in the cylinder.

The law of expansion and contraction may be expressed by a formula as follows:

$$E = T \times C \times L$$

where

E is the *change* in length
T is the *change* in temperature
C is the coefficient of linear expansion
L is the original length of the body.

Example:

What will be the expansion in a steam pipe 200 ft. long when subjected to a temperature of 300°, if erected when the temperature was 60°?

$$T = 300 - 60 = 240;\ C = .0000065;\ L = 200$$
$$E = T \times C \times L$$
$$= 240 \times .0000065 \times 200 = .312 \text{ ft., } \textit{Answer.}$$

Notice particularly that here we use L in feet and, consequently, the expansion E comes out in feet. This can be reduced to inches if desired, giving $12 \times .312 = 3.744$ in. or $3\frac{3}{4}$ in. nearly.

125. Allowances for Shrink Fits.—In making a shrink fit, the collar or band, or whatever is to be shrunk on, is bored slightly smaller than the outside diameter of the part on which it is to be shrunk. It is then heated and thus expanded until it can be slipped into place. When it cools, it cannot return to its original size but is in a stretched condition. It, therefore, exerts a powerful grip on the article over which is has been shrunk.

Practice differs considerably in the allowances that are made for shrink fits. A rule which has been widely and successfully used is to allow $\frac{1}{1000}$ in. for each inch of diameter. According to this rule, if we were shrinking a crank on a 6 in. shaft, the crank should be bored .006 in. small or else the shaft turned .006 in. oversize and the crank bored exactly 6 in. For a 10-in. shaft we would allow .010 in, and so on for other sizes.

This could be expressed by the following formulas:

$A = .001 \times D$, or, since $.001 = \frac{1}{1000}$, this could be written

$$A = \frac{D}{1000}$$

where A stands for "allowance"
and D for the diameter

Assuming that an allowance of $.001 \times D$ is made, let us see what temperature is necessary in order to give the necessary expansion so that a steel tire can be put over a locomotive driving wheel.

For each degree that the tire is heated, it will expand .0000065 in. per inch of diameter. We must have an expansion of at least .001 in. The number of degrees necessary to get this will be

$$\frac{.001}{.0000065} = 154$$

It would look as if a difference of 154° would be sufficient. However, a greater difference is necessary in practice. There must be sufficient clearance so that the tire can be slipped quickly into place before it has time to cool off or to warm the wheel. Once in place, the tire will grip the wheel when a temperature difference of 154° exists.

PROBLEMS

211. In testing direct current generators, it is customary to specify that under full load the temperature of the armature shall not rise more than 40° Centigrade above a room temperature of 25° C.; that is, the temperature of the armature under these conditions should not exceed 65° C.

In making a test a Fahrenheit thermometer was used. The room was at a temperature of 77° F. and the temperature of the armature at the end of the run was 180° F. Did the generator meet the specifications? What was the temperature change, Centigrade?

212. In erecting a long steam line that will have a variation in temperature of 320°, how far apart should the expansion joints be placed if each joint can take care of a motion of 3 inches?

213. If a brass bushing measures 2 in. just after boring, when its temperature is 95° F., what will it caliper when it has cooled to 65° F.?

214. A steel link 2 ft. long is made $\frac{1}{16}$ in. too short for the slot in the fly-wheel rim into which it is to be shrunk. How hot must the link be before it will go in?

215. If a hub bolt is heated until it just begins to show red and is immediately screwed up snug and allowed to cool, what shrinkage allowance per inch of length would we be allowing by such a plan?

216. If we wished to maintain a tempering bath at a temperature of 500° F., what should be the reading on a Centigrade pyrometer?

217. If the brass bearings for a 2 in. steel crank shaft are given a running clearance of .002 in. at a temperature of 60° F., what would be the clearance when running at a temperature of 100° F.?

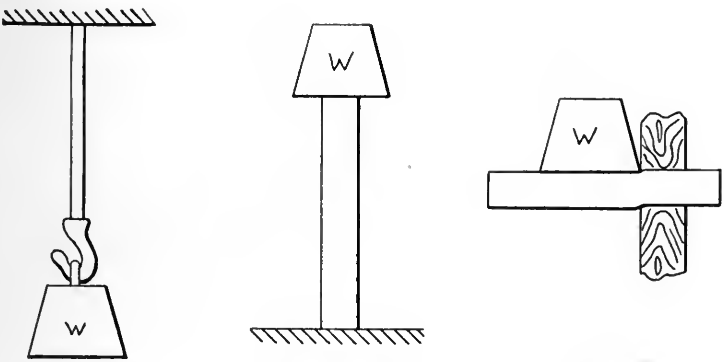
218. A horizontal steam turbine and dynamo are to be direct-connected, their shaft centers being $3\frac{1}{2}$ ft. above the bed plate. If the bearings are lined up at a temperature of 70°, how much will they be out of alignment under running conditions when the temperature of the dynamo frame is 80° F. and that of the turbine is 215° F., both frames being of cast iron?

CHAPTER XX

STRENGTH OF MATERIALS

126. Stresses.—When a load is put upon any piece of material, it tends to change the shape of the piece. The material naturally resists this and, therefore, exerts a force opposite to the load. If the load is not too heavy, the material may be able to exert a sufficient force to hold it, but often the strength of the material is exceeded and the piece breaks.

The resistance which is set up when a piece of material is loaded is called the *Stress*. For instance, if a casting weighing 3 tons or 6000 lb. is suspended by a single rope, the stress in the rope will be 6000 lb.



tension compression shear

Fig. 83.

There are three different kinds of stresses that can be produced, depending on the way the load is applied

(1) Tensile stress (pulling stress).

(2) Compressive stress (crushing or pushing stress).

(3) Shearing stress (cutting stress).

Fig. 83 shows how these different stresses are produced.

We sometimes recognize two other kinds of stresses, but these are really special cases of the three just given. These two others are:

(a) Bending Stress (really a combination of tension on one side and compression on the other).

(b) Torsional or Twisting Stress (a form of shearing stress).

127. Ultimate Strengths.—By taking specimens of the different materials and loading them until they break, it has been possible to find out just what each kind of material will stand. The load to which each square inch of cross-section must be subjected in order to break it, is called the *Ultimate* strength of the material. The strength of most materials differs for the different methods of loading shown in Fig. 83.

The Tensile Strength of a material is the resistance offered by its fibers to being pulled apart.

The Compressive Strength of a material is the resistance offered by its fibers to being crushed.

The Shearing Strength of a material is the resistance offered by its fibers to being cut off.

The following table gives the average values for the most used materials.

ULTIMATE STRENGTHS—POUNDS PER SQUARE INCH

Material	Tension	Compression	Shear
Timber..........	10000	8000	3000 (across grain)
Cast iron.........	20000	90000	20000
Wrought iron.....	50000	50000	40000
Machine steel.....	65000	65000	50000

128. Safe Working Stresses.—Having found how great a stress is required to break one square inch of material, we naturally would not allow anywhere near this stress to come on a piece of material in actual service. The Ultimate Strength is usually divided by some number, known as the *Factor of Safety*, and the quotient is used as the *Safe Working Stress*.

For example, if 60,000 lb. per square inch will break a piece of soft steel and we use a factor of safety of 5, this would give:

$$\text{Safe working stress} = \frac{60000}{5} = 12000 \text{ lb. per square inch.}$$

The following table gives the Safe Working Stresses of the most used materials.

SAFE WORKING STRESSES—POUNDS PER SQUARE INCH

Material	S_t Tension	S_c Compression	S_s Shear
Timber	700	700	500
Cast iron	3– 4000	15–18000	3– 4000
Wrought iron....	8–10000	8–10000	7– 9000
Machine steel....	10–16000	10–16000	8--12000

Instead of writing "safe loads in pounds per square inch" for tension, compression, or shear, the symbols S_t, S_c, and S_s are used. So if A = area in square inches, then the load W which can be carried safely = Area × safe load per square inch or

$$A \times S_t = W \text{ (tension)}$$
$$A \times S_c = W \text{ (compression)}$$
$$A \times S_s = W \text{ (shear)}$$

Or, in general, for all stresses

$$A \times S = W$$

Perhaps more often we would want to find the area necessary in order to support a certain weight or load. In this case, we would want a formula which would give A.

If we divide the total load by the safe stress, we will get the necessary area; or

$$A = \frac{W}{S}$$

This simply says, area of metal necessary = total weight to be carried divided by safe load in pounds per square inch. From the area of a bolt or rod, its diameter can be easily found.

129. Strengths of Bolts.—There is a well-known saying that "a chain is only as strong as its weakest link." This means, in general, that any mechanism must be so designed that its weakest part will be strong enough to stand the greatest load that may come on it. In figuring the size of a bolt to hold a certain load, we would not calculate the full diameter of the bolt and make the area there just sufficient, but we must see to it that the bolt has a cross-sectional area *at the root of the threads* large enough to support the load. Then the body of the bolt will have a surplus of strength.

Example:

'What size of steel eyebolt will support a weight of 5000 lb.? Take 12,000 lb. as the safe load in tension.

$$\text{Then, } A=\frac{W}{S}=\frac{5000}{12000}$$

$$A=\frac{5}{12}\text{ sq. in.}=.416\text{ sq. in.}$$

.416 sq. in. is then the necessary area to support the weight. Of course, the example could be completed by saying .7854 $D^2=.416$ sq. in., where D = diameter at the root of the thread. By then solving for D we would get the diameter at the root of the threads. But the Bolt Tables afford an easier method than this. In the following table, .4193 is given as the area of a $\frac{7}{8}$ in. bolt at the root of the thread. Therefore, a $\frac{7}{8}$ in. eyebolt would probably be used.

In figuring the allowable loads for steel bolts, it is best not to allow over 12,000 lb. stress per square inch and 10,000 lb. is perhaps even more usual on account of the sharp root of the threads, which makes a bolt liable to develop cracks at this point.

BOLT TABLE.—U. S. S. THREADS

Diam.	Threads to inch	Diam. at bottom of thread	Area of bolt	Area at bottom of thread
$\frac{1}{4}$ in.	20	.1850	.0491	.0269
$\frac{5}{16}$ in.	18	.2403	.0767	.0454
$\frac{3}{8}$ in.	16	.2938	.1104	.0678
$\frac{7}{16}$ in.	14	.3447	.1503	.0933
$\frac{1}{2}$ in.	13	.4001	.1963	.1257
$\frac{9}{16}$ in.	12	.4542	.2485	.1621
$\frac{5}{8}$ in.	11	.5069	.3068	.2018
$\frac{3}{4}$ in.	10	.6201	.4418	.3020
$\frac{7}{8}$ in.	9	.7307	.6013	.4193
1 in.	8	.8376	.7854	.5510
$1\frac{1}{8}$ in.	7	.9394	.9940	.6931
$1\frac{1}{4}$ in.	7	1.0644	1.2272	.8899
$1\frac{3}{8}$ in.	6	1.1585	1.4849	1.0541
$1\frac{1}{2}$ in.	6	1.2835	1.7671	1.2938
$1\frac{5}{8}$ in.	$5\frac{1}{2}$	1.3888	2.0739	1,5149
$1\frac{3}{4}$ in.	5	1.4902	2.4053	1.7441
2 in.	$4\frac{1}{2}$	1.7113	3.1416	2.3001
$2\frac{1}{4}$ in.	$4\frac{1}{2}$	1.9613	3.9761	3.0213
$2\frac{1}{2}$ in.	4	2.1752	4.9087	3.7163
$2\frac{3}{4}$ in.	4	2.4252	5.9396	4.6196
3 in.	$3\frac{1}{2}$	2.6288	7.0686	5.4277

130. Strength of Hemp Ropes.—It is quite common in calculating the strength of ropes and cables to assume that the section of the rope is a solid circle. Of course, the strands of the rope do not completely fill the circle but, if we find by test the allowable safe strength *per square inch* on this basis, it will be perfectly safe to make calculations for other sizes of ropes on the same basis. The safe working stress based on the full area of the circle is 1420 lb. per square inch. The Nominal Area (as the area of the full circle by which the rope is designated is called) is $A = .7854 \times D^2$. The safe stress is 1420 lb. per square inch and, consequently, the weight that can be supported by a rope of diameter D is

$$W = S \times A$$
$$= 1420 \times .7854 \times D^2$$

Here we have two constant numbers (1420 and .7854) that would be used every time we were to calculate the safe strength of a rope. If this were to be done often we would not want to multiply these together every time, so we can combine them now, once and for all.

$$1420 \times .7854 = 1120, \text{ approximately}$$

Hence

$$W = 1120 \times D^2$$

Example:

Find the safe load on a hemp rope of ½ in. diameter.

$$D = \frac{1}{2} \text{ and } D^2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$
$$W = 1120 \times D^2$$
$$= 1120 \times \frac{1}{4} = 280 \text{ lb., } \textit{Answer.}$$

131. Wire Ropes and Cables.—For wire ropes made of crucible steel, a safe working load of 15,000 lb. per square inch of nominal area is allowable. For cables of Swedish iron but half this value should be used.

132. Strength of Chains.—It has been demonstrated by repeated tests that a welded joint cannot be safely loaded as heavily as a solid piece of material. Of course, there are often welds that are practically as strong as the stock, but it is not safe to depend on them. For this reason, the safe working load per square inch for chain links is often given as 9000 lb., which is just $\frac{3}{4}$ of 12,000 lb.

If D = the diameter of the rod of which the links are made

$A = 2 \times .7854 \times D^2$

$W = S_t \times A$

$W = 9000 \times 2 \times .7854 \times D^2$

Combining the constant numbers, this can be simplified into

$W = 14{,}000 \times D^2$

This is used in the same way as the formula for a rope.

133. Columns.—The previous examples were cases of tension. The size of a rod or timber subjected to compression is computed in the same way unless it is long in comparison with its thickness. When a bar under compression has a length greater than ten times its least thickness, it is called a *Column* and must be considered by the use of complicated formulas which take account of its length. It can be seen by taking a yardstick, or similar piece, that it is much easier to break than a piece of shorter length but otherwise of the same dimensions. A long piece, when compressed, will buckle in the center and break under a light thrust or compression. An example of this can be found in the piston rod on a steam engine, where, on account of the length of the rod, it is necessary to use much lower stresses than those given in the tables. The compressive stress allowed in piston rods varies with the judgment of different designers but is generally about 5000 lb. per square inch, using a pressure on the piston of 125 lb. per square inch.

Example:

Find the size of rod for a 30 in. by 52 in. Corliss engine with 125 lb. steam pressure.

30 in. is the diameter of the cylinder and 52 in. is the stroke, which is not considered in the problem except in that it has reduced the allowable stress in the rod.

$.7854 \times 30^2 = 706.86$ sq. in., area of piston.

$706.86 \times 125 = 88357.5$ lb., total pressure on piston.

Using 5000 lb. per square inch, allowable stress in the rod.

$88358 \div 5000 = 17.67$ sq. in. sectional area of rod,

From the table of areas of circles, it is seen that this is the area of a circle nearly $4\frac{3}{4}$ in. in diameter, so we would use a $4\frac{3}{4}$ in. rod.

PROBLEMS

Note.—In all examples involving screw threads, to get areas at root of thread, use the table given in this chapter. Give sizes of bolts always as diameters.

219. If the generator frame shown in Fig. 84 weighs 3000 lb., what size steel eyebolt should be used for lifting it, allowing a stress of 10,000 lb. at the root of the thread?

220. What would be the safe load for a $\frac{3}{4}$ in. chain?

221. What size hemp rope would be necessary to lift a load of 4000 lb.?

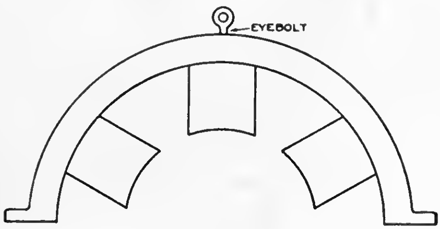


Fig. 84.

222. What force would be necessary to shear off a bar of machinery steel 2 in. in diameter?

223. A certain manufacturer of jack screws states that a $2\frac{1}{2}$ in. screw is capable of raising 28 tons. If the diameter of the screw at the base of the threads is 1.82 in., what is the stress per square inch at the bottom of the threads when carrying 28 tons?

224. A soft steel test bar having a diameter of .8 in. is pulled in two by a load of 31,500 lb. What was the breaking tensile stress per square inch?

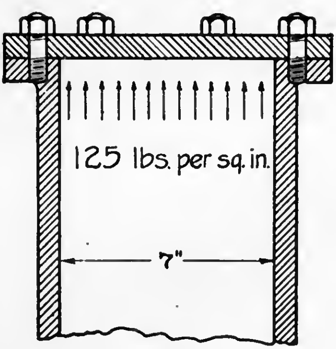


Fig. 85.

225. The cylinder head of a small steam engine (Fig. 85) having a cylinder diameter of 7 in. is held on by 6 studs of $\frac{3}{4}$ in. diameter. When there is a steam pressure of 125 lb. per square inch in the cylinder, what will be the pull on each stud? And what will be the stress per square inch in each stud, due to the steam pressure?

226. With a cylinder diameter of 10 in. and an air pressure of 100 lb. per square inch, find the greatest weight that can be lifted by the air hoist, shown in Fig. 86. Also find the size of piston rod necessary, assuming that it is screwed into the piston. Notice that this rod is subject only to tension and, therefore, a greater stress is allowable than in steam engine piston rods.

227. Work out a formula for the strength of crucible steel cables on the same plan as that given for hemp rope.

228. What is the greatest load that should be lifted with a pair of tackle blocks having 3 pulleys in the movable block and 2 in the fixed block, and having a $\frac{3}{4}$ in. rope.

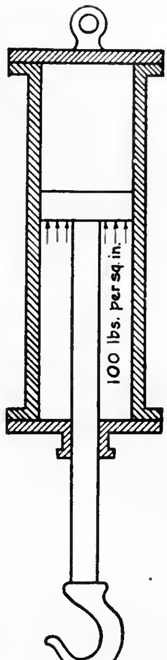


Fig. 86.

INDEX

2883