Faculty Working Papers

THE SINGLE VS. SIMULTANEOUS EQUATION MODEL IN CAPITAL ASSET PRICING

Cheng F. Lee, Associate Professor of Finance
Joseph D. Vinso, Assistant Professor of Finance, University of Pennsylvania

#492

College of Commerce and Business Administration
University of Illinois at Urbana-Champaign

FACULTY WORKING PAPERS

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

THE SINGLE VS. SIMULTANEOUS EQUATION MODEL IN CAPITAL ASSET PRICING

Cheng F. Lee, Associate Professor of Finance
Joseph D. Vinso, Assistant Professor of Finance, University of Pennsylvania

#492

Summary:

In this paper, alternative capital asset pricing models (CAPM) are first reviewed and criticized. Then a new simultaneous equation CAPM is derived to take the essences of the existing capital asset pricing models into account. Some data are also used to show the usefulness of the new CAPM derived in this study.

The Single Vs. Simultaneous
Equation Model in Capital
Asset Pricing

I. INTRODUCTION

The security market model (SML) of Sharpe (1964), Lintner (1965) and Mossin (1966) has been the foundation for much of the research for the last decade. In that model, it is suggested that the return generating process of an asset is a function of the variations in a market index and is known as the capital asset pricing model (CAPM). Recently, it has been suggested that introducing additional factors into the single-index model could improve the power of the model. In particular, the introduction of firm-specific information may be important in the returns generating process. For example, King (1966) demonstrated the importance of industry factors in determining stock price behavior. Likewise, Cohen and Pogue (1967) have suggested that an industry factor (I_t) could be included in Sharpe's model to increase the explanatory power of that model. Beaver (1972) and Downes and Dyckman (1973) argue that certain types of accounting information are taken into account in security pricing and, thus, should be included in a model of capital asset pricing. Similarly, Rosenberg (1974) has shown that there exist some extra-market components in the covariance of the market model.

Several approaches have been suggested to provide such a multifactor model of the return generating process. Sharpe (1977) has given the SLM model a "Multi-Beta" interpretation. Similarly, Ross (1976,1977) uses a arbitrage approach in

the same spirit of Sharpe's multi-beta interpretation. Likewise, Rosenberg and McKibben (1973) and Rosenberg and Guy (1976) add additional explanatory variables to better explain factors affecting beta and residual variance.[1] All of these approaches employ a single-equation approach to the CAPM.

One problem with these single equation approaches is the inability to directly introduce firm specific variables. Using a different approach, Simkowitz and Logue (S-L) (1973) describe the return generation process as an interdependent structure of security returns which include industry data as well as certain accounting information. They then use a two-stage least squares (2SLS) estimation procedure to solve for the appropriate estimators. While this approach is an improvement over the single equation methods in that the firm-related variables are entered directly into the CAPM, it can be shown that the estimation procedure is inefficient due to multicollinearity and identification problems. As a result, the inherent attractiveness of a simultaneous equation system describing the return generating process has not been exploited. The purpose of this paper is to provide an alternative modelling system which more adequately describes the return generating process than the usual single equation CAPM by introducing firm-related variables but allows the exploitation of the simultaneous equation system of S-L without the estimation problems of S-L. While not empirically testing this system with the single-equation "multi-beta" models, the relationships between the alternative processes are explored.

Section II describes the model and demonstrates the relationship with the single equation models. The next section provides some empirical investigation into alternative estimation procedures. The role of firm-related variables is also included. A summary and conclusions follows.

II. MODEL SPECIFICATION

Following S-L, the returns generating process can be specified as a simultaneous equation system as follows:

$$\text{(i)} \quad R_{1t} = \alpha_1 + \gamma_{12} R_{2t} + \gamma_{13} R_{3t} + \ldots\ldots + \gamma_{1n} R_{nt} + b_{11} X_{11t} + b_{12} X_{12t} + b_{13} X_{13t} + \beta_{1m} R_{mt} + E_{1t}$$

$$\vdots \qquad \vdots \qquad (1)$$

$$\text{(vii)} \quad R_{nt} + \alpha_n + \gamma_{n1} R_{1t} + \gamma_{n2} R_{2t} + \ldots\ldots + \gamma_{nn-1} R_{n-1t} + b_{n1} X_{n1t} + b_{n2} X_{n2t} + b_{n3} X_{n3t} + \beta_{nm} R_{mt} + E_{nt}$$

Where R_{jt} = the return on the j^{th} security over time interval t, (j = 1, 2, ..., n)

R_{mt} = the return on a market index over time interval t,

X_{j1t} = the profitability index of j^{th} firm over time interval t, (j = 1, 2, ..., n)

X_{j2t} = the leverage index of j^{th} firm over time period t, (j = 1, 2, ..., n)

X_{j3t} = the dividend policy index of j^{th} firm over time period t, (j = 1, 2, ..., n)

b_{jk} = the coefficient of the k^{th} firm related variable in the j^{th} equation, (k = 1, 2, 3)

γ_{ji} = the coefficient of the i^{th} endogenous variable in the j^{th} equation, (i = 1, 2, ..., n; j = 1, 2, ..., n)

β_{mj} = the coefficient of market rate of return in the j^{th} equation,

E_{jt} = the disturbance term for the j^{th} equation

Equation (1) is a simultaneous equation system with n endogenous and 3n + 1 exogenous variables.[2] It can easily be shown that every equation in the system is over-identified.

Following S-L (1973, p. 260), Equation (1) can be rewritten in terms of matrix notation as:

$$\Gamma R_i' = B^*X^* + B_m R_m' + E \qquad (2)$$

Premultiplying both sides of equation (2) by Γ^{-1}, the reduced form of the structural equation obtains:

$$R_i' = \Gamma^{-1}B^*X^* + \Gamma^{-1}B_m R_m' + \Gamma^{-1}E \qquad (3)$$

In order to arrive at an appropriate model two restrictions are imposed on equation (3):

(i) $\Gamma^{-1}E$ is spherical normal

(ii) $\Gamma^{-1}B^*$ is block diagonal.[3]

The justification of imposing the second constraint is that the change of firm-related variables for other firms will not affect the rate of return for firm i. This model can be explicitly written as:

$$R_{jt} = \alpha_j' + \beta_j' R_{mt} + b_{j1}X_{j1t} + b_{j2}X_{j2t} + b_{j3}X_{j3t} + \varepsilon_{jt} \qquad (j = 1, 2, \ldots, n) \qquad (4)$$

where α'_j, β'_j, b_{j1}, b_{j2} and b_{j3} are regression parameters and ε_{jt} is disturbance term.

If the residual terms among different firms within a group are highly correlated, then the equations in the system only appear to be unrelated but are, in fact, related through the disturbances. When such a condition exists, a method developed

by Zellner (1962) and extended by Zellner and Huang (1962) and Zellner (1963) can be used to improve the efficiency of the estimators in equation (4). This technique is known as the method of seemingly-unrelated regression (SUR) and is used to pool the cross-section information with the time series information.[4] Since this estimation procedure will be used to solve the system of equations described by equation (4), this model will be referred to as the SUR market model.[5]

Before testing this definition of the return generating process, it would be appropriate to show the relationship between this model and the "multi-beta" interpretation of the SML model. In terms of the regression parameters of equation (1), the systematic risk of Sharpe's model of β_{jy} can be written as:

$$\beta_j = \gamma_{j1}D_1 + \gamma_{j2}D_2 + \dots \gamma_{jm}D_n + b_{j1}\,D_{n+1} + b_{j2}D_{n+2} + b_{j3}D_{n+3} + \beta_{jm} \qquad (5)$$

$$(j = 1, 2, \dots, n \text{ and } \gamma_{jj} = 0)$$

where the D_i represent auxiliary regression coefficients from regressing each explantory variable on R_m respectively. In addition, D_1 to D_n are the systematic risk obtained from Sharpe's model.

The relationship between the firm related variables and market rate of return is generally expected to be trivial; therefore, equation (5) can approximately be rewritten as

$$\beta_j \doteq \gamma_{j1}D_1 + \dots \gamma_{jn}D_n + \beta_{jm} \qquad (6)$$

This expression implies that the measure of systematic risk from SML is a weighted average of the coefficients of all endogenous variables and the coefficient of R_m in the simultaneous equation system.[6] Now it is shown that the relationships of equations (4), (5), and (6) can be related to the Sharpe (1977), Ross (1976, 1977) and Rosenberg and McKibben's (1973), multibeta theories.

Using the multi-beta interpretation, Sharpe (1977, 128-131) has shown that the systematic risk obtained from the SLM model can be defined as follows using Sharpe's notation:

$$\beta_{im} = \sum_{j=1}^{M} \frac{W_j \ \mathrm{Var}\ (\tilde{R}_j)}{\mathrm{Var}\ (\tilde{R}_m)} \beta_{ij} \tag{7}$$

where β_{im} = the beta value for security i relative to the market portfolio

β_{ij} = the beta value for security i relative to the j^{th} portfolio

$\mathrm{Var}(\tilde{R}_j)$ = the variance of the j^{th} portfolio rates of return

$\mathrm{Var}(\tilde{R}_m)$ = the variance of market rates of return

W_j = the contribution of $\tilde{R}_j$ to $\tilde{R}_m$.

Equation (7) can be rewritten as

$$\beta_{im} = \sum_{j=1}^{M} \left(\frac{W_j}{\beta_{mj}}\right) (\beta_{ij}\beta_{jm}) \tag{8}$$

where $\beta_{mj} = \mathrm{Cov}(\tilde{R}_m,\tilde{R}_j)/\mathrm{Var}(\tilde{R}_j)$

$\beta_{jm} = \mathrm{Cov}(\tilde{R}_m,\tilde{R}_j)/\mathrm{Var}(\tilde{R}_m)$

If we let $W_j/\beta_{mj} = 1$, then

$$\beta_{im} = \sum_{j=1}^{M} (\beta_{ij}\beta_{jm}) \tag{9}$$

It is clear that the relationship of (9) is similar to that of equation (6), therefore, the theories used by Sharpe (1977) can be extended to interpret the relationship as defined in equation (6).

Rosenberg and McKibben (1973) and Rosenberg and Guy (1976) have derived a model which the firm related variables are used as decriptors for both systematic and specific work. In other words, they argued that the multi-factor instead of the single factor rates of return generating process should be used for capital asset pricing. Essentially, they have added explanatory variables to better explain factors affecting beta and residual variance. The additional explanatory variables used by Rosenberg and McKibben are the cross products between market factor and the different firm related variables; the additional explanatory variables used in this study are the firm related variables themselves.

Ross (1976, 1977) has used the arbitrage argument to derive the multi-factor rates of return generating process. In his 1977 paper, Ross has explicitly derived the basic arbitrage condition is similar to that of Sharpe's multi-beta interpretaion of CAPM, the relationship between the model defined in equation (4) and the Ross multi-facotr model does not require further explanation.

III. EMPIRICAL RESULTS AND IMPLICATIONS

To investigate the returns generating process presented here, relative to the usual SLM model as well as the simultaneous equation system of S-L and to illustrate the problems with the S-L formulation, annual data of stock price and firm related variables from the period 1945-1973 for seven oil companies are used to calculate the related rates of return, the profitability index, the leverage index and the dividend policy index.[7] The appropriate rates of return for each company are adjusted for dividends and stock splits. The annual Standard and Poor index (S & P) with dividends is used to calculate the annual rate of return on the market.[8]

First, to investigate the difficulties with the S-L model, a traditional 2SLS method is used to estimate the simultaneous relationship of security returns for the seven oil companies. These results are listed in Table I. Then, the usual SLM model is used to calculate systematic risks for seven oil companies which are shown in Table II. From Tables I and II, it is found that the adjusted coefficients of determination ($\bar{R}^2$) of S-L model are consistently higher than those of Sharpe's model. This result implies that the explanatory power of S-L model is higher than that of Sharpe's model. However, it is found from the reduced form estimates that the market rate of return is always the most important exogenous variable in estimating the endogenous variables. In other words, the estimated endogenous variables used in the second stage are highly correlated with the market rate of return which appears in every second stage regression. These

TABLE I

2SLS Structure Estimates of Oil Industry; R_m Included
(t=values appear in parentheses beneath coefficient)

Dependent Variables (R_i)	R_1	R_2	R_3	R_4	R_5	R_6	R_7
Endogenous Variables							
R_1		4.3251	-.1172	.6199*	-.3461	-.0323	-.2089
	1.0	(1.3504)	(.4711)	(3.5029)	(.8766)	(.1506)	(1.0269)
R_2	.1034		-.1228	.1640	.7604*	-.0921	-.1893
	(.4875)	1.0	(.4594)	(.7610)	(2.9124)	(.5674)	(1.0468)
R_3	-.0898	.2105		-.0750	-.2341	.2934	.0404
	(.4203)	(.7578)	1.0	(.4101)	(.6692)	(1.6715)	(.2134)
R_4	1.0012*	-.0504	.1120		.2702	.3970	.6824*
	(3.4242)	(.1113)	(.3054)	1.0	(.4267)	(1.3637)	(2.5069)
R_5	.0529	.4704*	.2552	-.1723		.1805	-.0156
	(.2930)	(2.8126)	(.1531)	(1.0571)	1.0	(1.4404)	(.1105)
R_6	-.1792		.5125	.2818	1.0234*		.1824
	(.5281)	--	(1.6127)	(1.0447)	(1.8031)	1.0	(.6254)
R_7	-.1618	-.3657		.1519	.1945	-.1330	
	(.5419)	(1.0446)	--	(.6284)	(.3923)	(.4571)	1.0
Exogenous Variables†							
Constant	-.1168	-.2300	-.2781	.3241	.1825	-.0529	-.0025
X_{j1}	1.4595	3.6608	2.8658*	-5.4350*	-3.2714	-.1330	-.4648
	(.4939)	(1.0009)	(2.0256)	(1.8298)	(.9801)	(.0990)	(.2658)
X_{j2}	1.6809	-1.0986	2.3367*	.8283	2.4936*	.8941	1.1500*
	(1.2933)	(1.3711)	(2.6637)	(.8969)	(2.1135)	(1.2919)	(2.3707)
X_{j3}		-17.5875	-6.7611	30.5213*	1.4031	59.5394*	2.5121
	--	(1.0725)	(.7694)	(1.8208)	(1.0286)	(1.8322)	(.4229)
R_m	.2486		.6470*	-.2720	.2774	.1446	.6498*
	(.6626)	--	(2.2035)	(.7610)	(.4428)	(.4362)	(2.3336)
$\bar{R}^2$ ††	.4576	.3934	.5982	.6341	.5500	.559	.6206

†Subscript j, j = 1, ..., 7 denotes variables in second stage of relevant equation.
††Second stage estimates for the single equation.
*Denotes significant at .10 level of significance or better for two-tailed test.

results imply that indeed the methodology developed by S-L can be subject to the multi-collinearity associated with the 2SLS estimation method and suggests that the SUR method should be useful to increase estimation efficiency.[9]

A. Estimation Efficiency

To test the validity of the SUR market model specified in equation (4), first OLS is used to estimate the necessary parameters of seven oil companies. (See Table III). It can be seen that five of the seven $\bar{R}^2$ associated with OLS estimates of the SUR market model are higher than those of the Sharpe model shown in Table II. These results indicate that the firm-related variables increase the explanatory power of CAPM. However, since the motivation for the SUR approach is the multicollinearity problem of 2SLS the interrelationship among these firms must be investigated by examining the correlation of the residuals. The residual correlation coefficient matrix for these seven companies (shown in Table IV) indicates these firms are highly interrelated in that ten residual correlation coefficients involving all seven firms are significantly different from zero at the .05 level. This result implies that the SUR estimation method can improve the efficiency of some estimators.[10]

Parameter estimates utilizing the SUR method are also provided in Table III. It can be observed that when the SUR estimation method is applied to the market model, in most cases the efficiency of the estimators appears to be increased.[11] Thus, the SUR market model developed here can result in the efficient estimators while also increasing the explanatory

TABLE II

OLS Parameter Estimates of Oil Industry - SLM Model†

	α	β	$\bar{R}^2$
R_1 Imperial Oil	.04421	.7421 (2.6699)	.1851
R_2 Phillips Petroleum	-.0141	.6578 (1.6758)	.0626
R_3 Shell Oil	-.0043	1.2353 (4.2429)	.3864
R_4 S. O. of IN	.0366	.6869 (2.8619)	.2082
R_5 S. O. of OH	-.0161	.8720 (1.8160)	.0782
R_6 Sun Oil	.0049	.6240 (2.7130)	.1906
R_7 Union Oil of CA	.0058	1.0228 (4.6704)	.4353

† - t-values appear in parentheses beneath the corresponding coefficients

power of the CAPM.

It would now be useful to demonstrate that the statistical efficiency of the SUR market model is superior to the 2SLS method used in the S-L model. First, the reduced form of the S-L model associated with seven oil companies is estimated in accordance with the specification of equation (3). Then, the approximated standard errors of the reduced form coefficients are compared to the standard error of the corresponding variables in the SUR market model.[12] It is found that 21 out of 28 standard errors from the SUR market model are significantly smaller than those from the reduced form. Several reasons suggest themselves to explain why the statistical efficiency of the estimated coefficients associated with the SUR model is higher than that of the estimated coefficients associated with the reduced form of the S-L model. First, the degrees of freedom of the estimated residual variance associated with the SUR market model is larger than that of the reduced form of the S-L model. Secondly, the generalized least square method cannot be used to improve the efficiency of the estimated coefficients associated with the reduced form of the S-L model {see Theil (1971), p. 453}.[13]

B. Role of Firm-related Variables

Results of the SUR market model have a great deal of intuitive appeal. For example, Imperial Oil which is a Canadian firm shows the lowest correlation with other firms as might be expected. It is also found in Table III that the residuals of Phillips Petroleum are highly correlated with those of Standard Oil of Ohio and Sun Oil. However, the SUR estimation

TABLE XII

OLS and SUR Estimates of Oil Industry - Synthesis Model†

	α_j	β_j	b_{j1}	b_{j2}	b_{j3}	$\bar{R}^2$
OLS	-.4479	-.7602	6.043	.4582	-2.0840	.2202
	(1.6480)	(2.4770)**	(1.7270)	(.2932)	(-.1749)	
SUR	-.2361	.7663**	4.2020	.8655	-6.9720	
	(1.1390)	(2.5560)	(1.3600)	(.6268)	(-.6648)	
OLS	-.8203	.5413	1.0680	-1.2170	5.2960	.0248
	(-.2580)	(1.3140)	(.2359)	(-1.2180)	(.6786)	
SUR	-.1600	.5822	2.4550	-.8193	-8.509	
	(-.8001)	(1.4340)	(.8517)	(-1.2710)	(.6768)	
OLS	-.2251	1.1200	2.0550	2.5110	-.5165	.4540
	(-1.7230)	(3.9800)**	(1.4220)	(2.1510)**	(-.0677)	
SUR	-.3003	1.0880	2.9140	2.665	-2.532	
	(2.633)**	(3.8870)**	(2.393)**	(2.812)**	(-.4126)	
OLS	-.1550	.7283	2.1070	-.2395	25.2900	.3040
	(.8648)	(3.0540)**	(.7852)	(-.2707)	(1.8170)*	
SUR	-.0538	.6458	-.9832	.4262	26.3500	
	(-.3519)	(2.7530)**	(-.4340)	(.5884)	(2.337)**	
OLS	-.0029	1.1280	-.7520	1.8370	15.600	.0558
	(-.0111)	(2.0820)**	(-.2295)	(1.3410)	(.8942)	
SUR	-.0047	1.1640	-.8607	1.8860	.9103	
	(-.0271)	(2.3090)**	(-.4143)	(2.4970)**	(.9555)	
OLS	-.1472	.7098	.6888	.9155	62.2000	.2755
	(1.4350)	(3.0960)**	(.5501)	(1.3050)	(1.7140)	
SUR	-.1509	.7070	.8508	.9016	54.1100	
	(1.9960)*	(3.1751)**	(1.0101)	(2.0230)**	(2.4020)**	
OLS	-.1140	1.0141	1.347	.6427	4.5300	.4693
	(-.9173)	(4.7590)**	(.7899)	(1.2400)	(.7579)	
SUR	-.0661	1.0190	.6500	.9632	3.9660	
	(-.5823)	(4.6800)**	(.4206)	(2.0890)**	(.7431)	

†t-values appear in parentheses beneath the corresponding coefficients.
*denotes significant at .10 level of significance or better for two-tailed test.
**denotes significant at .05 level of significance or better for two-tailed test.

TABLE IV

Residual Correlation Coefficient Matrix

	R_1	R_2	R_3	R_4	R_5	R_6	R_7
R_1	1.0000	.1725	.1687	.4422*	.0571	.1129	.1450
R_2		1.0000	.2062	.2312	.7487	.4420*	-.0770
R_3			1.0000	.1634	.3542*	.5748*	.2183
R_4				1.0000	.1789	.3645*	.3329
R_5					1.0000	.6697*	.6234*
R_6						1.0000	.3154*
R_7							1.0000

*Denoted significant at .05 level of significance.

method does not improve the efficiency of estimators for Phillips Petroleum. One possible reason is that the financial management policies of this company may be highly correlated with those of other companies in the oil industry. As previously discussed, when the explanatory variables of a regression become more similar to those of other regressions in the same industry, the gain of SUR estimation method will be smaller.[14]

Now that the validity of the SUR market model has been shown, it would be of interest to investigate the importance of the three firm related variables used by Simkowitz and Logue (1973) in capital asset pricing. They have shown that the roles played by three firm related variables are to identify the simultaneous equation system of security market and to improve the explanatory power of the diagonal security market model. These same firm related variables also are explicitly included in the SUR market model indicated in equation (4). Using the SUR estimates of the market model, the importance of these firm related variables in the return generation process can be analyzed. The profitability index is significant in explaining the rates of return of Shell Oil; the dividend policy index is significant in explaining the rates of return of Standard Oil of Indiana, Standard Oil of Ohio and Sun Oil; and the leverage index is significant in explaining the rates of return of Shell Oil, Standard Oil of Ohio, Sun Oil and Union Oil of California. These results imply that both leverage and dividend policies can be additional factors important in capital asset pricing. From a financial management viewpoint both leverage and dividend

policies are unique factors of an industry so the market index itself can hardly be used to take care of the change of these two policies associated with a particular industry.

Thus the SUR market model is formulated by introducing such accounting information as indices of probability, leverage and dividend policy into Sharpe's model. It has explicitly taken into account the arguments of the possible impacts of accounting information on the bahavior of security price. This multi-index model differs with other multi-index models from several aspects. First, the additional indices employed in the SUR market model are the accounting information of an individual firm rather than general economic activity indicators. Secondly, the indices of accounting information are relatively orthogonal to the market rate of return and the multicollinearity problem is much less essential relative to that of other multi-index models. Finally, the SUR estimation method can be used to take care of the interdependent relationship among securities of a particular industry. As quarterly data instead of annual data is employed to estimate the synthesis model for a particular industry, then the gain associated with the SUR estimation method will become much more important.[15] It can be expected that since the SUR model has been shown to be consistent with the multibeta interpretations of Sharpe and others, the results obtained here should be consistent with empirical tests of those models.

IV. SUMMARY

This paper proposed a model of the returns generating process of capital asset pricing which introduces firm-related variables directly into the analysis using a simultaneous equation modelling so as to increase the power of the usual single equation capital asset pricing model. It is shown that the process described here does not suffer from the estimation problems encountered in previous attempts yet it is shown to be consistent with recent "multibeta" interpretations of the single equation methodology. Unlike these multibeta approaches, however, the process described here introduces firm specific variables directly which aids in the interpretation.

Empirical results of seven oil firms are used to demonstrate that some accounting information - leverage and dividend policy indices might be used to increase the explanatory power of the diagonal security market model in capital asset pricing. The methodology developed here will give additional information in order to expand the understanding of the rates of return in the capital market.

NOTES

* Associate Professor of Finance, University of Illinois and Assistant Professor of Finance, The Wharton School, University of Pennsylvania, respectively. We would like to thank Irwin Friend for his helpful comments on an earlier draft as well as the cogent comments of an anonymous referee.

1. For example Rosenberg and McKibben use the cross products between the market factor and the different firm related variables.

2. It should be noted that S-L chose indices of profitability, leverage, and dividend policy as firm specific accounting information to include in the model as it is assumed that these factors will have the greatest impact on determining security returns. While others could be added or substituted, these same variables will be used in the present study to maintain continuity with the S-L study.

3. When formulating their model, S-L restrict equation (3) to E being spherical normal and B* being block diagonal. These restrictions allowed use of the 2SLS estimation procedure. That procedure can be subject to the problems of multicollinearity and identification. Aber (1973), for example, has pointed out that multi-index models are generally complicated by the problem of multicollinearity. In using 2SLS, S-L regressed R_j's on 22 (or 21) exogenous variables to obtain the estimated R_j's. They then use the estimated R_j's as regressors in the second stage regression. Their results indicate that market rate of return, R_m, is the most important explanatory variable for each first stage regression. However, R_m also appears in each second stage regression within the system. Since the correlation between R_m and the estimated R_j is very strong, it is possible that the problem of multicollinearity faced by S-L is not negligible. This type of multicollinearity problem associated with the 2SLS was first pointed out by Klein and Nakamura (1962) and discussed in detail by Fox (1968). Recently, Lee (1976) truncated the market rates of return, R_m, in the first stage and found that the multicollinearity problem associated with 2SLS method in the S-L type model significantly affects the efficiency of the estimated parameter associated with R_m. Similarly Klein and Nakamura (1962) demonstrates that 2SLS can also suffer from an identification problem. As a result, while the model developed by S-L may theoretically introduce additional information to CAPM, failing to specify appropriate restrictions requires the use of an estimation procedure which suffers some severe problems.

4. Kamenta (1971) demonstrates why the SUR estimation method can be used to pool the time series information to the cross-section information.

5. It is easily shown that the SUR model is consistent with other models. If we impose two conditions on equation (3).

   (i) $\Gamma^{-1}B^*$ is a zero matrix

   (ii) the variance-covariance matrix of $\Gamma^{-1}E$ is a diagonal, then we obtain Sharpe's model as:

$$R_{jt} = \alpha''_j + \beta''_j R_{mt} + E'_{jt}$$

   where α''_j, β''_j are regression parameters and E'_{jt} is a disturbance term. As the Sharpe market model has identical regressors for every firm, the SUR technique cannot be used to improve the efficiency of related estimates.

   It can be shown that the SUR market model developed here is also consistent with the Rosenberg (1974) model. That model uses the extra-market components as the descriptors of the regression coefficient associated with the market model; i.e., it is assumed that the cross-product terms between market rates of return and firm related variables should be read as additional explanatory variables. The model described here has explicitly included some observable extra-market components, and the excluded extra-market components are implicitly taken care by the SUR estimation technique. Finally, the relationship with the S-L model must be explored. Although the SUR market model developed here is theoretically a specific case of the S-L model, the empirical problems associated with the S-L model as previously discussed do not exist with the SUR methodology. In sum, the SUR market model preserves most of the strengths of the S-L model and yet overcomes most of the empirical problems associated with the 2SLS estimation procedure.

6. Results in the S-L paper demonstrates that the relationship of (6) approximately holds true for the sample they consider. Since the multicollinearity problem would not effect this test, it can be assumed that these results also hold for the SUR model.

7. Following Simkowitz and Logue (1973), the profitability index is defined as annual retained profit (retained earning plus interest and preferred divident divided by total assets; the leverage index is defined as annual change of long term debt plus annual change of outstanding preferred stock divided by total assets; and the dividend policy index is defined as annual change of total dividends divided by the book value of equity. Since annual instead of quarterly data are used in this study, the annualizing

procedure used by S-L is not applicable here. It should be reemphasized that other firm-related variables can be added or substituted for those chosen by S-L. For example, recently Kusyner (1977) used the methodology developed here to investigate the impact of other firm-related variables, specifically level of oil reserves, refining capacity and production capacity on the returns for a sample of oil companies. Similarly, other firm-related variables can be used depending on the nature of the industry under investigation.

8. Using such a long time period raises the question of stationarity as parameter estimation assumes a stationary distribution. Tests using the Box Pierce Q-statistic, however, show that the hypothesis that the time series is white noise cannot be rejected. As no discussion of this test is better than a necessarily brief one, the reader is referred to the original work by Box and Jenkins (1970) for a complete discussion of the Q-statistic.

9. It should be noted that in Table I, using the usual 2SLS method, R_m is significant at the 10% level in only two regressions. However, if the modified 2SLS developed by Klein and Nakamura (1962) is used to estimate the simultaneous relationship, there exist five regression coefficients associated with R_m which are significant at the 10% level (See Lee (1976) for details of this modification as well as the empirical results). Thus, R_m does play an important role in the second stage regression of the S-L model.

10. Besides the problem with multicollinearity, it can also be shown that the parameter estimates associated with the 2SLS model are inefficient due to the identification problem previously discussed. Results available on request.

11. The gain associated with the SUR estimation method is measured using the t-statistic of the regression coefficient as the coefficient of determination for the SUR estimation method is not provided by the SUR computer program. It is obvious, however, that the efficiency of SUR if greater than with OLS.

12. The standard errors of the reduced form estimates are calculates from an approximate covariance matrix and have been adjusted for the degree of freedom.

13. Another advantage of the market model developed here concerns the problem of sample size. While the S-L model has explicitly specified the full structural simultaneous relationship of capital asset pricing for a particular industry, the multicollinearity and the identification problems explored here generally makes the statistical results of S-L model

become less meaningful. As the number of equations in the system becomes larger, the number of regressors in the first stage of 2SLS generally gets too large to be handled. Since the regressor of the model developed here is not affected by the number of equations, the problem of undersized sample does not exist in the SUR market model. For a complete discussion of this problem, see Brundy and Jorgenson (1971).

14. While these results are interesting, they can be viewed with confidence only if the assumptions of the regression model are fulfilled. Besides the stationarity assumption previously discussed, the homoscedasticity assumption of the regression residuals is an additional condition required to ascertain the stability of the estimated systematic risk as discussed by Blume (1971). It is even more important to investigate this assumption in light of the recent work by Rogalski and Vinso (1978) who found that the OLS estimation of the CAPM for over 15% of all securities show heteroscedasticity. To test for homoscedasticity of the regression residuals for each equation associated with the synthesis model, the Goldfield-Quandt (1965) test is used. To test whether the variance-covariance matrix obtained for the SUR equation system is stable over time, Anderson's (1958, Chapter 10) approximate χ^2 statistic is used. The results shows that the assumption of homoscedasticity cannot be rejected at the .05 level of significance for every firm except Imperial Oil. Likewise, the assumption of a constant covariance matrix cannot be rejected either at the .05 significance level.

15. In this circumstance, the sample size increases sharply and the gain associated with the SUR estimation method is substantiated. See Zellner (1962) for details.

REFERENCES

1. Aber, J.W. (1973), Beta Coefficients and Models of Security Return. Lexington: C.C. Heath and Company.

2. Anderson, T.W. (1958). An Introduction to Multivariate Statistical Analysis. New York: John Wiley and Sons, Inc.

3. Beaver, W.H. (1972), "The Behavior of Security Price and Its Implications for Accounting Research (Methods)," supplement to The Accounting Review, pp. 407-437.

4. Blume, M. (1971), "On the Assessment of Risk," Journal of Finance, 29, pp. 1-10.

5. Box, B.E.P., and Jenkins, G. (1970). Time Series Analysis, Forecasting and Control. San Francisco: Holden Day.

6. Brundy, J. and D. Jorgenson,(1971). "Efficient Estimation of Simultaneous Equations by Instrumental Variables". Review of Economic and Statistics, 53, pp. 207-224.

7. Cohen, K., and J. Pogue (1967), "An Empirical Evaluation of Alternative Portfolio Selection Models," Journal of Business, 40, pp. 166-193.

8. Dowens, D. and T. Dyckman (1973), "Efficient Market Research and Accounting Information," Accounting Review, XLVIII, p. 300-317.

9. Fox, K.A. (1968). Intermediate Economic Statistics. New York: John Wiley and Sons, Inc.

10. Goldfield, S.M. and R.E. Quandt (1965), "Some Tests for Homescedasticity," Journal of the American Statistical Association, 60, pp. 539-57.

11. Kamenta, J. (1971), Element of Econometric, New York: The MacMillan Company.

12. King, B.F. (1966), "Market and Industry Factors in Stock Price Bahavior," Journal of Business, 39, pp. 139-190.

13. Klein, L.R. and Mitsugu Nakamura (1962), "Singularity in the Equation Systems of Econometrics: Some Aspects of the Problem of Multicollinearity," International Economic Review, 3, pp. 274-299.

14. Lee, Cheng F. (1976), "A Note on the Interdependent Structure of Security Returns", Journal of Financial and Quantitative Analysis, 11, pp. 73-86.

15. Lintner, J. (1965), "Security Prices, Risk and Maximal Gains from Diversification", Journal of Finance, 20, pp. 587-616.

16. Mossin, J. (1966), "Equilibrium in a Capital Asset Market," Econometrica, 34, pp. 768-783.

17. Rogalski, Richard and Joseph D. Vinso, "Heteroscedasticity and the Market Model," Financial Review, forthcoming, 1978.

18. Rosenberg, B. (1974), "Extra-Market Components of Covariance in Security Returns," Journal of Financial and Quantitative Analysis, 9, pp. 263-274.

19. Rosenberg, B. and J. Guy (1976), "Beta and Investment Fundamentals," Financial Analysts Journal, May/June (pp. 60-72) and July/August (pp. 62-70).

20. Rosenberg, B. and W. McKibben (1973), "The Prediction of Systematic and Specific Risk in Common Stocks," Journal of Financial and Quatitative Analysis, 8, pp. 317-333.

21. Ross, S.A. (1976), "Arbitrage Theory of Capital Asset Pricing," Journal of Economic Theory, 8, pp.

22. Ross, S.A. (1977) "Return, Risk and Arbitrage," In I. Friend and J.L. Bicksler, Risk and Return in Finance, Vol. 1 (edited), Ballinger Publishing Company.

23. Sharpe, W.F. (1964), "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risks," Journal of Finance, 19, pp. 425-442.

24. Sharpe, W. (1977), "The Capital Asset Pricing Model: A Multi-Beta Interpretation," in Levy, H. and M. Sarnat's Readings book from Academic Press.

25. Simkowitz, M.A. and D.E. Logue (1973), "The Interdependent Structure of Security Return," Journal of Financial and Quantitative Analysis, 8, pp. 259-272.

26. Theil, H. (1971), Principles of Econometrics, New York: John Wiley and Sons, Inc.

27. Zellner, A. (1962), "An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias," Journal of the American Statistical Association, 57, pp. 348-368.

28. Zellner, A., (1973) "Estimators for Seemingly Unrelated Regression Equations: Some Exact Finite Sample Results," Journal of the American Statistical Association, 58, pp. 977-992.

29. Zellner, A. and D.S. Huang, "Further Properties of Efficient Estimators for Seemingly Unrelated Regression Equations", *International Economic Review*, 3, 1962, pp. 300-313.