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ABSTRACT

Key Words: Skewness, Sampling Risk, Diversification

Recent papers have extended portfolio theory to include skewness along with mean return and variance to explain security preferences. Because the positive skewness which characterizes many assets is rapidly reduced through diversification, several authors have suggested that a preference for positive skewness can lead to antidiversification as investors attempt to capture the greatest positive skew. However, these analyses ignore the sampling risk present when selecting assets from skewed distributions. Because the mean of a positively skewed distribution is biased upward, an investor who ignores sampling risk may hold a smaller portfolio than required to achieve a desired level of expected utility.

The purpose of this paper is to further examine the question of diversification for security populations whose returns are characterized by positive or negative skewness. Our empirical results indicate that even though diversification reduces positive expected skewness, the sampling risk for small portfolios may be so large as to motivate investors to further diversify. Only with diversification can the investor achieve a level of confidence that actual skewness will be within prescribed limits of its expected value. While some degree of antidiversification may be warranted, the number of securities producing optimal diversification may be substantially greater than indicated in prior studies.

Skewness, Sampling Risk and
the Importance of Diversification

Traditional portfolio theory and the principle of diversification have been developed within the mean-variance Capital Asset Pricing Model (CAPM). In the context of the CAPM under perfect market assumptions, investors should hold in their portfolios all risky securities available in the market.

There has been recent interest in extending portfolio theory to include the third moment, skewness.[1] This interest is motivated, in part, by the inadequacy of the CAPM in explaining security returns (Friend and Westerfield (1980), Kraus and Litzenberger (1976)), as well as developments in the options and futures market which enable investors to create portfolio returns which are distinctively skewed (Merton, et. at. (1978), Sears and Trennepohl (1983)). Of particular interest are those studies (Beedles (1979), Conine and Tamarkin (1981), Kane (1982) and Simkowitz and Beedles (1978)) which argue that a preference for positive skewness may lead to antidiversification. Simkowitz and Beedles (1978) present empirical evidence that the mean level of positive skewness in common stock portfolios is quickly eliminated as portfolio size increases. Conine and Tamarkin (1981) demonstrate theoretically that the consideration of skewness may cause investors to antidiversify and hold as few as two securities.

While Scott and Horvath (1980) have shown that a preference for positive skewness is consistent behavior for rational investors, the antidiversification implications of Conine and Tamarkin, and Simkowitz and Beedles apply only if investors ignore the sampling risk that exists for portfolio skewness, where sampling risk refers to the likelihood that the skewness of a particular portfolio chosen of size n will be near its expected value.[2] Because the skewness of a particular portfolio chosen by the investor may differ significantly from its expected (mean) value, it seems inconsistent to assert that an investor seeks positive skewness, yet ignores the risk associated with obtaining its value.

The purpose of this paper is to further examine the question of the appropriate level of diversification for security populations whose returns are characterized by positive or negative skewness. Our empirical results indicate that even though diversification reduces positive expected skewness, the sampling risk for small portfolios may be so large as to motivate investors to further diversify. Only with diversification can the investor achieve a level of confidence that actual skewness will be within prescribed limits of its expected value. Thus, while some degree of antidiversification may be warranted, the number of securities producing "optimal" diversification may be greater (perhaps substantially) than indicated in prior studies. To be useful for investor decision-making, analyses of diversification and skewness should examine not only the expected value of skewness, but also the level of confidence regarding its estimate.

In Part II, the literature dealing with utility, skewness and sampling risk is briefly reviewed. Part III illustrates the behavior of portfolio skew and sampling risk for three diverse security populations, while Part IV contains conclusions and implications of our analysis.

II. Utility, Skewness and Sampling Risk

The recognition of sampling risk has its origin in the widely quoted study of Evans and Archer (1968), who constructed an upper confidence limit around a regression equation relating expected portfolio standard deviation to portfolio size. Elton and Gruber (1977) extended the work of Evans and Archer by developing analytical measures for the sampling risks of variance (the variance in variance) and mean return (average return variance)(see Elton and Gruber (1977) equations (B17) and (B18)). Sears and Trennepohl (1982) illustrate the importance of sampling risk in measuring the return and variance of option portfolios.[3]

In a mean-variance setting under perfect capital markets with normally distributed asset returns, the implication of sampling risk for diversification is straight-forward. Since expected return is constant for all portfolio sizes, complete diversification is optimal because an investor who holds the market minimizes expected variance while eliminating the sampling risks from small portfolios associated with mean return and variance.

Sampling risk assumes importance when distribution moments beyond mean and variance are incorporated into the investor's utility function.[4] If asset return distributions are non-normal and investor utility is other than quadratic, skewness is considered by assuming that investors possess either (1) a cubic utility function or (2) a utility function of log, power, exponential or other non-polynomial form (see Kallberg and Ziemba (1983)) which can be approximated by the first three terms of a Taylor series expansion.[5]

Letting $\bar{R}$ denote the mean of the random variable for return (R), expected utility of end of period wealth (W) can be expressed using a Taylor series as $E[U(W)] = \sum_{n=0}^{\infty}\{U^n(W\bar{R})\ E[WR - W\bar{R}]^n\}/n!$. For widely used utility functions such as $U(R)=\ln(R)$ and $U(R)=WR^p$, $0<p<1$, wealth can be separated out and returns analyzed as $E[U(R)] = \sum_{n=0}^{\infty}\{U^n(\bar{R})/\ E(R - \bar{R})^n/n!\}$, where $U^{(n)}(\bar{R})$ is the nth derivative of the utility function at point R. Expected utility is expressed as a function composed of constants, a_n, determined from the selected utility function where $a_n = U^{(n)}(\bar{R})/n!$ and the distribution moments, $E(R - \bar{R})^n$. Equation (1) describes an investor whose utility is described by mean return, variance (σ^2) and skewness (M^3), where the constants $a_0 = U(\bar{R})$, $a_2 = U''(\bar{R})/2!$, and $a_3 = U'''(\bar{R})/3!$, reflect the relative importance of each moment.

$$E[U(R)] = a_0 + a_2\,\sigma^2 + a_3M^3 \qquad (1)$$

Given risk aversion, $a_2<0$, and positive skewness preference, $a_3>0$, the ratio $a_3/(-a_2)$ indicates the investors skewness-risk trade-off. Increasing values of $a_3/(-a_2)$ describe investors willing to accept greater return dispersion in exchange for positive skewness.

Previous studies have evaluated the impact of diversification on expected utility by substituting population (average) values of R, σ^2 and M^3 at selected portfolio sizes into equation (1). Because the mean values of σ^2 and M^3 decline with diversification for many security populations, it has been argued that a skewness preference will cause investors to antidiversify. However, these studies have not considered the uncertainty about distribution moments for portfolios smaller than the market.[6] Inparticular, the more uncertainty concerning the mean level of skew, the greater is the motivation to diversify and increase the confidence about the actual level of skewness which will be obtained.

Under a naive or random investment policy of equal investment in each security, the mean level of skewness for a portfolio of n securities (see Conine and Tamarkin, (1981)[7,8] can be determined by equation (2).

$$E(M_n^3) = (\frac{1}{n})\,\bar{M}^3 + [\frac{3(n-1)}{n^2}]\bar{M}_{ij} + [\frac{(n-1)(n-2)}{n^2}]\bar{M}_{ijk} \tag{2}$$

where: $E(M_n^3)$ = mean skewness on a portfolio of n securities

n = number of securities in the portfolio

$\bar{M}^3$ = average skewness for a one security portfolio

$\bar{M}_{iij}$ = average curvilinear relationship for the population

$\bar{M}_{ijk}$ = average triplicate product for the population also equal to the systematic skewness of an equally weighted market portfolio, M

It is important to realize that (2) is a measure of only the mean level of portfolio skewness at portfolio size n. For example, in a population containing 100 securities, 4950 (i.e., 100 x 99/2) unique two security portfolios can be formed; equation (1) is the cross-sectional arithmetic average of the skewness found in these portfolios.

Whereas mean-variance studies have employed the variance (e.g., variance in variance) as a measure of sampling risk, the comparable measure for skewness, the variance in skewness, is insufficient to evaluate the sampling risk associated with skewness because the distribution of portfolio skews at any given size is itself markedly skewed.[9] For securities such as stocks and long options, the distribution of skews is positively skewed; for portfolios of covered calls, the distribution of skews is negatively skewed. Thus, positive (negative) outliers in individual security return distributions produce positive (negative) outliers in the structure of portfolio skews. These cases are displayed graphically in Figures 1 and 2.

INSERT FIGURES 1 AND 2

The asymmetry of the skewness distribution has interesting implications about the conclusions presented in prior studies concerning diversification and skewness. Focusing on the mean as a "typical value" in a skewed distribution is misleading (see Winkler and Hayes (1975)),because the mean is "pulled" in the direction of the tail. For positively skewed distributions the mean will overstate the typical value (in a probability sense) and mislead the investor to hold a smaller portfolio than would be selected if the investor was aware of the sampling risk present. For a negatively skewed distribution there is motivation for complete diversification since mean skewness becomes less negative as portfolio size increases.

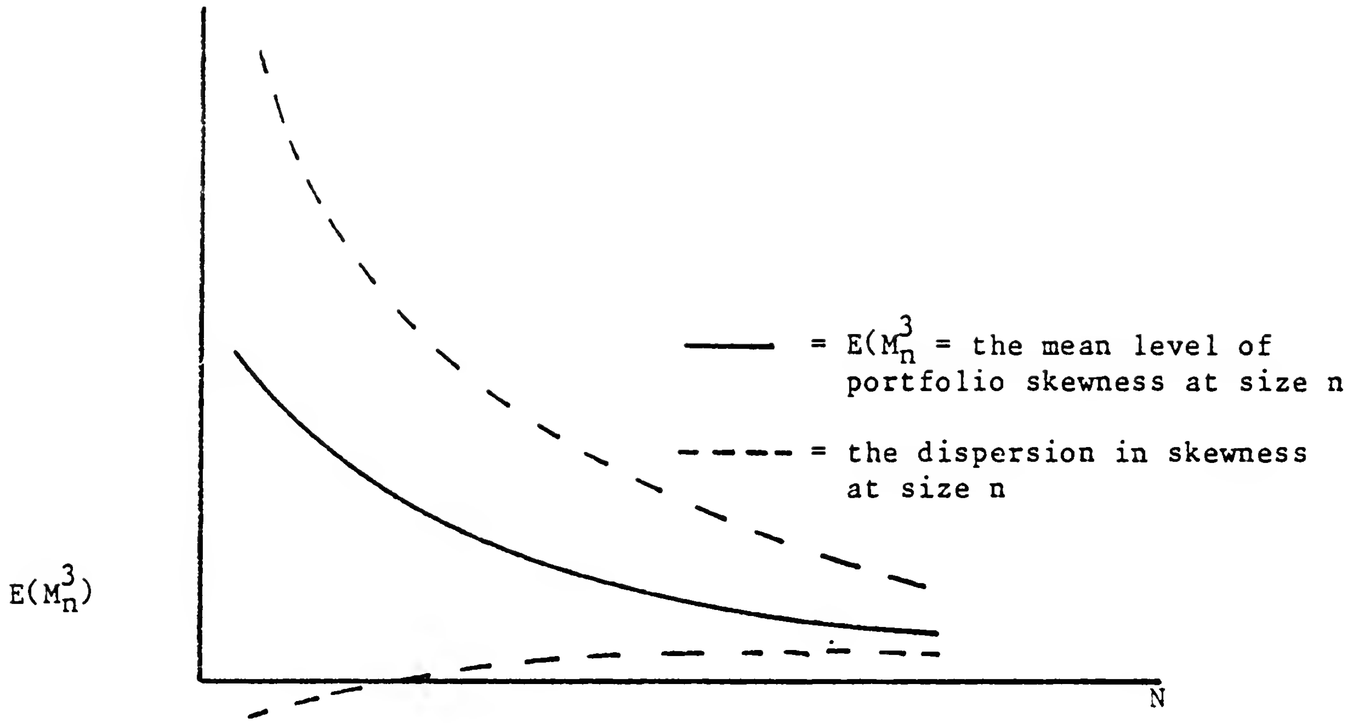

Figure 1: Diversification and its effects upon the dispersion dispersion about a declining mean level of portfolio skewness

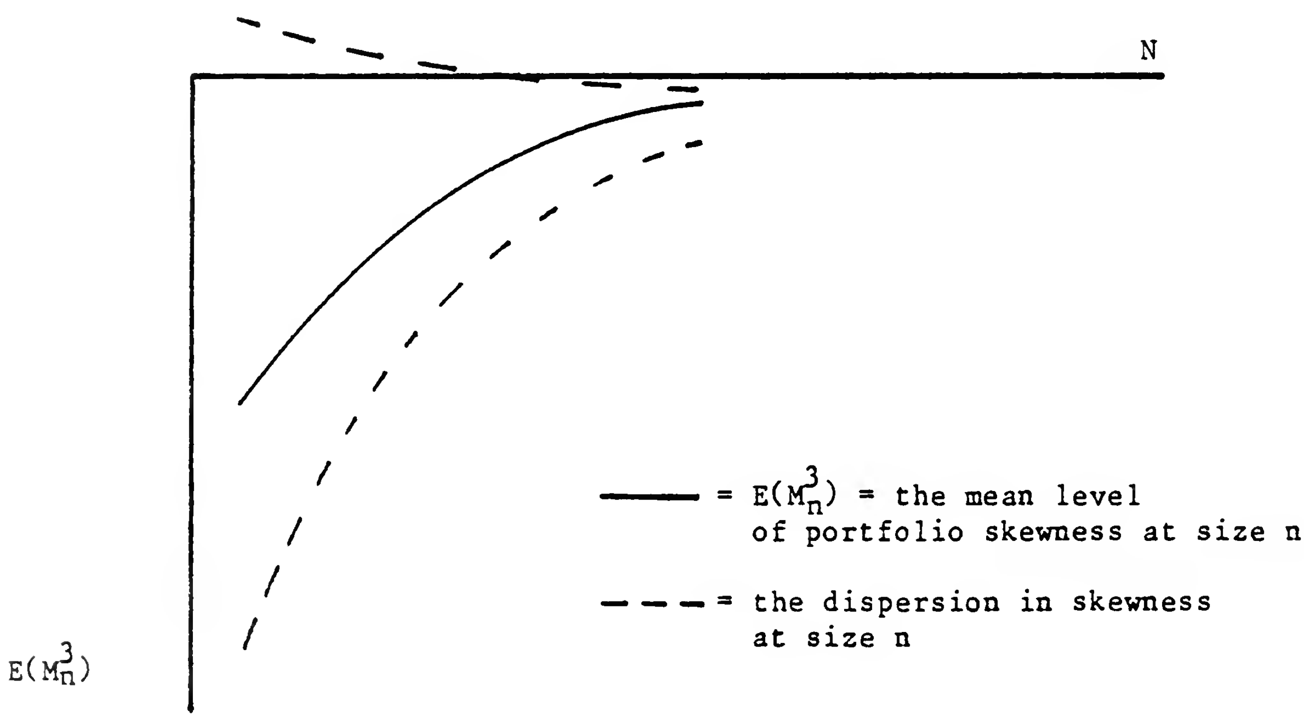

Figure 2: Diversification and its effects upon the dispersion about an increasing mean level of portfolio skewness

III. Portfolio Skewness and Sampling Risk

In this section we illustrate the differences in magnitude and behavior of portfolio skewness and sampling risk for different security populations. Three types of assets are chosen as samples, primarily because of the diverse nature of their return distributions; these include common stocks, at-the-money covered option writing and at-the-money long option positions.

A. The Data and Methodology

The sample chosen includes the 136 stocks having listed options available on December 31, 1975. Securities not having complete price data on the Compustat tapes over the period July 1, 1963 to December 31, 1978, were eliminated, resulting in 102 sample securities for analysis. Although the choice of this particular group introduces a selection bias in the study, these securities represent almost one-half of the population of listed option securities; thus, these results may be inferred to the current universe of optionable stocks.

Since listed options were not available until 1973, six month at-the-money premiums for the 102 stock sample were generated for the 15 1/2 year sample period using the Black and Scholes option pricing model adjusted for dividends.[10] Use of Black-Scholes beginning-of-period option premiums is believed necessary to generate a sample period of sufficient length and to standardize stock price/exercise price ratios.[11] The similarity between Black-Scholes model prices and actual premiums has been demonstrated in Bhattachrya (1980) and Merton, et. al. (1978).

Semi-annual returns (gross of commissions) on each long option position for the thirty-one six month holding periods were calculated by dividing the beginning of period call value as determined by the Black-Scholes option pricing model into the intrinsic value of the option at maturity. Intrinsic value is the maximum of zero or the difference between stock price and striking price at option maturity.

Semi-annual returns on each covered writing position were calculated by dividing the beginning stock price less the option premium received into the sum of stock price at the end of the period plus dividends, less the option's intrinsic value at maturity. Semiannual holding period returns for stocks include price appreciation plus dividends. Commissions are ignored in all transactions.

B. Return Distribution Statistics for Alternative Portfolios

Table I presents return distribution statistics for the three security groups examined. Line 1 reveals that average returns increase (5.01% to 17.60%) as one goes from option writing strategies, to stocks, to long call options while total risk as measured by the average security variance, $\bar{\sigma}^2$, increases from 130.51 to 31,869.90. The large amounts of systematic risk in long option positions compared to stocks and covered writing portfolios is shown by the market variance, σ_N^2 (line 3).

Average security skewness ($\bar{M}^3$) and systematic (market portfolio) skewness, ($\bar{M}_N^3$), data presented in lines 4 and 6 exhibit a wide range of values and behavior. Average one-security skewness for covered call options is negative while one-security common stock skewness is positive; one-security long call positions exhibit extreme positive skewness. The average curvilinear product, $\bar{M}_{ijk}$, and market portfolio skewness values, $\bar{M}_N^3$, reveal that for stocks and long option positions, diversification will reduce the positive skewness ($M_N^3 < \bar{M}_{iij} < \bar{M}^3$ and M_N^3, $\bar{M}_{iij}$ and $\bar{M}^3 > 0$) whereas for option writing strategies, increasing portfolio size will lower (a benefit) the negative skewness ($M_N^3 > \bar{M}_{iij} > \bar{M}^3$ and M_N^3, $\bar{M}_{iij}$ and $\bar{M}^3 < 0$). Since investors can diversify their holdings, it is instructive to examine the behavior of the portfolio skewness and the sampling risk for these security populations in response to changes in portfolio size.

INSERT TABLE 1

C. Diversification and Changes in Portfolio Skewness

Using the summary skewness and coskewness data from Table 1, equation (2) allows the traditional time series average skewness measures for any portfolio size to be analytically determined. Sampling procedures such as those used by Evans and Archer (1968) and Simkowitz and Beedles (1978) are unnecessary and not as precise. Table 2 presents relationships between portfolio size and mean skewness for the three security populations examined.

INSERT TABLE 2

The results reveal a wide spectrum of portfolio size-skewness relationships. First, for the option writing strategy, increasing portfolio size is beneficial because it eliminates much of the negative skewness present in these security positions, with over 90% being potentially diversifiable (1-[-156.96/-2423.13] = 93.52%). On the other hand, for stocks and long call options, the diversification process reduces the desired positive skewness. This is particularly evident for the long call portfolio, where over 98% of the skewness is unsystematic and thus can be destroyed with diversification. Slight increases in portfolio size of long call options or stock portfolios rapidly reduces the mean level of positive skewness.

D. Diversification and the Uncertainty about Skewness

Previous studies have focused on data similar to that presented in Table 2 and have concluded that investors in stocks or long options could be expected to hold small portfolios in an effort to capture the greatest positive skew. However, this approach ignores the sampling risk in these portfolios and the upward bias in the expected level of skewness caused by the positively skewed distributions. While a t-statistic has been developed for certain skewed distributions (see Johnson (1978)) we believe sampling risk best can be illustrated by examining the sampling distributions of portfolio skews at various portfolio sizes.

One thousand portfolios were randomly selected with security replacement for n = 3, 5, 10 20 and 40 and time series skewness values calculated for each portfolio. The size 1 values were computed directly from the data. The portfolios at each n were ranked by skewness; the deciles and extreme values of skewness in these distributions are presented in Table 3. In Table 4 the relationship of each skewness decile is expressed as a fraction of the mean (i.e., $M^3/E(M^3)$.

INSERT TABLES 3 AND 4

The upward bias in the "typical" skewness as measured by the mean level of skewness is illustrated by noting that for the stock sample (panel A) at portfolio size 1, approximately 75% of the distribution of portfolio skews lies below the mean value. As diversification proceeds the distribution of skews becomes more normal and the probability that an investor will draw a portfolio of stocks whose positive skewness is below the expected value falls to 51% at portfolio size 40. Because the right tail of the distribution collapses with diversification, the mean ($E(M^3)$) changes more dramatically than the median (50%) value of skew. For example, for stocks, $E(M^3)$ goes from 29,001 to 5,518 as n moves from 1 to 40, but the median (50%) value only falls from 6,681 to 5,537. In fact, there is very little change in the median until n equals 10. These results imply that previous studies of stocks and skewness which used $E(M^3)$ have overstated the change in a "typical" portfolios skew with increased diversification and thus understated the "optimal" portfolio size. Similar behavior is evidenced by the option portfolios, which, because of their greater skewness, exhibit a greater proportion of portfolio skews below the mean value. For example, at a portfolio size of 1 (see panel B), over 80% of the skewness distribution lies below the expected value. For portfolio size 40, this number is reduced to 55%. The covered option portfolio skews shown in panel C of Table 3 reveal that the probability of holding a portfolio which is less negatively skewed than the mean is about 65% for one-security portfolios and 52% for 40-security portfolios.

The importance of sampling risk to the diversification decision can be illustrated further by examining the lowest decile and lowest extreme value of skewness at each portfolio size as shown in Table 4. For stocks, the minimum decile of skewness increases with diversification and represents a value 62% as large as the mean for a 40 stock portfolio. However, it is possible to select a stock portfolio with negatively skewed returns as shown by the lowest observed values, even when holding five securities. Because the downside risk of holding a portfolio with a skewness below the expected level is significantly reduced with diversification, even investors having a preference for positive skewness may choose to diversify. Investors in covered call portfolios also benefit from diversification as the minimum decile level of negative skewness approaches zero as portfolio size increases.

For the option buying strategy the diversification implications are less clear. These assets contain such extreme levels of positive skewness that even the lowest decile at portfolio size one is almost three times as large as the lowest decile of all other portfolio sizes. It should be noted, however, that the smallest observed skewness, 542,100 is less than 3% as large as the mean value. Furthermore, Tables 3 and 4 illustrate the wide range of skewness values possible for small option portfolios and the data suggest that investors may choose to engage in some diversification to increase the certainty of the skewness estimate.

IV. Conclusions and Implications

Recent papers have extended portfolio theory to include skewness along with mean return and variance to explain security preferences. Because the positive skewness present in many assets is rapidly reduced through diversification, several authors have suggested that a preference for positive skewness can lead to antidiversification as investors attempt to capture the greatest amount of positive skew. However, these analyses ignore the sampling risk present when selecting assets from skewed distributions. Because the mean value of a positively skewed distribution is biased upward, an investor who ignores sampling risk may hold a smaller portfolio than required to achieve a desired level of expected utility.

Our data illustrates the extreme differences in distributions of portfolio skews and sampling risk for stocks, long calls and covered option writing portfolios. Without having knowledge of an individual's preference function, it is impossible to specify which types of securities and what portfolio sizes can be expected to maximize investor utility. However, some general observations can be made based on the data. First, investors who hold covered call positions should follow a policy of complete diversification because larger portfolios possess lower variance, less negative skewness and less sampling risk. While as strong a statement cannot be made about common stock portfolios, diversification will dramatically increase the probability of achieving a portfolio skewness value at least as great as the expected value for any portfolio size. Because of sampling risk, it appears that diversification beyond the levels suggested in previous skewness literature may be appropriate. Finally, the extreme skewness uncertainty present in call option portfolios provides motivation for diversification, thereby improving the investor's chances of holding a portfolio which will obtain a level of skewness near its expected value.

FOOTNOTES

[1]In this paper the term "skewness" will refer to a distribution's third moment. Many authors use the term "skewness" to denote the third moment divided by the cube of the standard deviation.

[2]Failure to consider sampling risk in the diversification process implies that all portfolios of a given size have the same distribution (e.g., the same mean, variance, skewness. . . .) Because portfolio distributions do differ, the investor is faced with sampling risk -- the probability that a particular portfolio will have return characteristics different from the averages.

[3]"Sampling risk" should not be confused with "estimation risk", a phrase popularized in work of Bawa, et. al. (1979) and others. The "estimation risk" literature deals with the uncertainty of measuring individual security returns and the resultant implications for optimal decision-making. Sampling risk, on the other hand, measures the uncertainty that distribution moments for a particular portfolio of a given size n will differ from the average values of all portfolios of size n. Even if estimation risk is assumed to be 0, sampling risk still is present because the moments of portfolios of a given size will differ from the expected values.

[4]Because sampling risk is a function of the cross-sectional dispersion among individual asset returns (see Elton and Gruber (1977), its magnitude becomes increasing larger with higher distribution moments (e.g., skewness). that is, the dispersion of portfolio variances is greater than the variance in portfolio mean returns, the dispersion among portfolio skews is greater than the variance dispersion and so on. For any distribution moment, the sampling risk is greatest when only one security is held and is eliminated when the investor is fully diversified, since full diversification results in only one possible portfolio -- the market portfolio.

[5]Care must be exercised when using a truncated Taylor series to represent expected utility. The truncation after three moments transforms the original function into a cubic expression whose values may diverge significantly from the original utility function (see Hasset, et. al. (1984), Levy (1969).

[6]Because portfolios of a given size possess different levels of R, σ^2 and M^3, they will also have different expected utilities. In particular, because the third moment will have the greatest amount of sampling risk, differences in portfolio expected utilities will be especially sensitive to differences in M^3.

[7]The assumption of an equal weighting scheme is consistent with the diversification literature (for example, see Beedles (1979), Conine and Tamarkin (1981), Elton and Gruber (1977), Evans and Archer (1968), Sears and Trennepohl (1982, 1983) and Simkowitz and Beedles (1978).

[8]Equation (1) is developed as follows. The skewness of any equally-weighted portfolio containing n securities is:

$$M^3 = \sum_{i=1}^{N} \left(\frac{1}{N}\right) M^3 + \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} \left(\frac{1}{N}\right)^3 \bar{M}_{iij} + \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} \left(\frac{1}{N}\right)^3 \bar{M}_{ijk}$$

where $M^3 = E(r_i - \bar{r}_i)^3$

$M_{iij} = E[(r_i - \bar{r}_i)^2(r_i - \bar{r}_j)]$ and

$M_{ijk} = E[(r_i - \bar{r}_i)(r_j - \bar{r}_j)(r_k - \bar{r}_k)]$.

There are n terms like M^3, 3n(n-1) terms like M_{iij} and n(n-1)(n-2) terms like M_{ijk} for a total of N^3 term. Taking expected values:

$$E(M_n^3) = (\frac{1}{n})\bar{M}^3 + [\frac{3(n-1)}{n^2})]\bar{M}_{iij} + [\frac{(n-1)(n-2)}{n^2}]\bar{M}_{ijk} \quad (1)$$

[9] Evans and Archer (1968) found the distribution of risks (standard deviation) to be approximately normal, thus justifying their F test analysis. The authors have derived the analytical expression for the variance in skewness. While it is a reasonable approximation of sampling risk at large portfolio sizes, it is inadequate at small n, because of the asymmetry in the distribution.

[10] Sterk (1983) has compared the Black and Roll adjustment procedures for dividends and found that the Roll Technique produces slightly better results. However, we do not believe that the Roll method would produce any significant differences in our data due to the extreme skewness in option portfolios. Furthermore, the Roll technique is only applicable in the strictest sense for short-lived options having only one dividend payment.

[11] While it would be informative to use actual premiums, we believe that deficiencies in the historical data base could provide misleading results. These data problems include:

a. A short time period for analysis. The CBOE began trading listed options in 1973 on only sixteen securities.

b. Nonavailability of listed contracts for desired stock price/exercise price ratios. It has not been until the last few years that sufficient varieties of stock price/exercise price ratios have been available on most securities.

Further research can incorporate actual premiums once the listed option market becomes more complete and the historical data base has been generated. The objective of our analysis was to select a sample of reasonable size and sufficient duration so as to provide meaningful measures of portfolio skewness.

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TABLE 1

Return Distribution Statistics for Alternative Samples*

	Symbol	Writing P/K=1.0	Stocks	Calls P/K=1.0
1. Sample mean return	r_N	5.01%	6.61%	17.60%
2. Average security variance	$\bar{\sigma}^2$	130.51	696.90	31,869.90
3. Market variance	σ_N^2	42.51	277.90	8,735.90
4. Average security skewness	$\bar{M}^3$	-2423.13	29,001.52	18,311,019.90
5. Average curvilinear relationship	$\bar{M}_{iij}$	- 600.52	8,696.80	2,606,978.33
6. Market skewness	$\bar{M}_{ijk}$ or (M_N^3)	- 156.96	5,354.31	875,729.92

*Each sample consists of 102 securities and all statistics relate to six month differencing intervals over the period examined.

TABLE 2

The Effects of Portfolio Size Upon Mean Skewness for Alternative Samples

Portfolio Size	Writing P/K=1.0	% Change in $M^3-M^3_N$ **	Stocks*	% Change in $M^3-M^3_N$ **	Calls, P/K=1.0*	% Change in $M^3-M^3_N$ **
1 (M^3_1)	-2423.13	--	29,001.52	--	18,311,019.90	--
2	-1056.17	60.32	13,772.98	64.40	6,532,988.72	67.55
3	- 701.45	75.97	10,187.30	79.56	3,955,224.70	82.34
4	- 543.02	82.96	8,673.93	85.96	2,919,138.88	88.28
5	- 454.02	86.89	7,855.34	89.42	2,378,382.17	91.38
10	- 289.64	94.15	6,419.37	95.50	1,478,882.01	96.54
20	- 214.26	97.47	5,801.99	98.11	1,120,138.56	98.60
40	- 178.28	99.06	5,518.46	99.31	963,518.49	99.50
102	- 156.96	100.00	5,354.31	100.00	875,729.92	100.00
$1-(M^3_1/M^3_N)$	93.52%		81.54%		95.22%	

*Calculated using equation (1) and lines 46 of Table I.

** $= \frac{\bar{M}^3_1 - E(M^3_n)}{\bar{M}^3_1 - M^3_N}$, any sample's % may differ slightly due to rounding.

TABLE 3

Skewness at Various Portfolio Sizes
By Deciles

A. Stocks

Portfolio Size =	1	3	5	10	20	40
Lowest Value	-3,500	- 500	- 100	800	1,500	2,200
10%	478	1,058	1,476	2,180	2,740	3,435
20%	1,693	2,006	2,425	2,904	3,506	4,013
30%	2,693	3,083	3,479	3,737	4,202	4,558
40%	4,087	4,318	4,600	4,571	4,812	5,090
50%	6,681	6,024	6,042	5,602	5,350	5,537
60%	10,156	8,743	7,587	6,499	6,041	6,025
70%	19,772	11,986	9,595	7,867	6,808	6,569
80%	38,976	16,901	12,405	9,595	8,016	7,276
90%	121,991	25,272	16,976	12,076	10,015	8,330
100%	289,400	68,800	40,100	23,501	16,780	11,650
mean skewness, $E(M^3)$	29,001	10,187	7,855	6,419	5,810	5,518

Table 3 (Continued)

B. Options

Portfolio Size =	1	3	5	10	20	40
Lowest Value	542,100	131,800	117,700	156,700	259,800	376,000
10%	1,409,852	653,915	515,759	512,759	524,383	583,680
20%	2,194,081	953,267	758,343	643,117	651,117	694,297
30%	2,732,349	1,231,072	939,377	807,879	778,229	779,011
40%	3,313,758	1,517,406	1,137,860	957,607	887,332	870,266
50%	4,775,718	1,860,970	1,390,659	1,128,896	1,011,086	956,032
60%	6,957,450	2,445,777	1,729,505	1,327,474	1,149,467	1,048,345
70%	10,669,606	3,124,141	2,279,615	1,662,350	1,318,871	1,161,596
80%	18,180,405	4,050,750	3,021,235	2,120,337	1,562,357	1,341,368
90%	30,153,076	6,582,599	4,951,161	3,072,596	1,994,303	1,660,283
100%	701,365,479	207,808,212	33,002,579	24,240,601	11,287,101	3,962,992
mean skewness, $E(M^3)$	18,311,091	3,955,225	2,378,382	1,478,882	1,120,138	963,518

Table 3 (Continued)

C. Covered Option Writing

Portfolio Size =	1	3	5	10	20	40
Lowest Value	-16,400	-3,700	-2,600	-1,300	-800	-600
10%	-6,243	-1,620	-1,098	-694	-466	-379
20%	-4,338	-1,153	- 748	-480	-363	-301
30%	-3,121	- 811	- 571	-371	-294	-251
40%	-2,094	- 597	- 423	-292	-245	-215
50%	-1,264	- 439	- 311	-231	-204	-187
60%	-1,017	- 327	- 221	-176	-167	-155
70%	- 746	- 226	- 155	-123	-130	-129
80%	- 499	- 141	- 89	- 74	- 88	- 96
90%	- 336	- 56	- 26	- 14	- 31	- 58
100%	- 164	- 37	- 24	- 13	- 8	- 7
mean skewness, $E(M^3)$	-2,423	- 701	- 459	-289	-214	-178

TABLE 4

Skewness Deciles as a Fraction of the Mean

A. Stocks

Portfolio Size =	1	3	5	10	20	40
Lowest Value	-.1207	-.0491	-.0127	.1246	.2583	.3987
10%	.0165	.1039	.1879	.3396	.4716	.6225
20%	.0584	.1969	.3087	.4524	.6034	.7273
30%	.0929	.3026	.4429	.5822	.7232	.8260
40%	.1409	.4239	.5856	.7121	.8282	.9224
50%	.2366	.5913	.7692	.8727	.9209	1.0034
60%	.3502	.8582	.9659	1.0125	1.0398	1.1905
70%	.6818	1.1766	1.2215	1.2256	1.1717	1.1905
80%	1.3440	1.6099	1.5792	1.4948	1.3797	1.3186
90%	4.2064	2.4808	2.1611	1.8813	1.7238	1.5096
100%	9.9789	6.7537	5.1050	3.6612	2.8881	2.1113

Table 4 (Continued)

B. Options

Portfolio Size =	1	3	5	10	20	40
Lowest Value	.0296	.0333	.0495	.1060	.2319	.3902
10%	.0770	.1653	.2169	.3467	.4679	.6058
20%	.1198	.2410	.3188	.4348	.5813	.7206
30%	.1492	.3113	.3950	.5463	.6946	.8085
40%	.1810	.3836	.4784	.6477	.7920	.9032
50%	.2608	.4705	.5847	.7627	.9027	.9922
60%	.3800	.6184	.7271	.8979	1.0259	1.0880
70%	.5827	.7899	.9585	1.127	1.1778	1.2056
80%	.9929	1.0242	1.2703	1.4334	1.3946	1.3922
90%	1.6467	1.6643	2.0817	2.0770	1.7804	1.7231
100%	38.3028	5.2540	13.8761	16.3895	10.0777	4.1130

Table 4 (Continued)

C. Covered Option Writing

Portfolio Size =	1	3	5	10	20	40
Lowest Value	6.7681	5.2781	5.6645	4.4983	3.7383	3.3708
10%	2.5766	2.3110	2.4185	2.4014	2.1775	2.1292
20%	1.7903	1.6448	1.6475	1.6609	1.6963	1.6910
30%	1.2881	1..1569	1.2577	1.2838	1.3738	1.4101
40%	.8642	.8516	.9317	1.0104	1.1449	1.2079
50%	.5217	.6262	.6850	.7993	.9533	1.0506
60%	.4197	.4665	.4868	.6090	.7804	.8708
70%	.3079	.3224	.3414	.4256	.6075	.7247
80%	.2059	.2011	.1960	.2561	.4112	.5393
90%	.1387	.0799	.0572	.0484	.1448	.3258
100%	.0677	.0529	.0529	.0450	.0374	.0373