THE

SLIDE RULE:

A PRACTICAL MANUAL.

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EIGHTH EDITION.

MANCHESTER:
EMMOTT AND CO. LIMITED, NEW BRIDGE STREET; AND
C. N. PICKWORTH, FALLOWFIELD.

LONDON: EMMOTT AND CO. LIMITED, 118, CHANCERY LANE; AND
WHITTAKER AND CO., 2, WHITE HART STREET, E.C.

NEW YORK: D. VAN NOSTRAND CO., 23, MURRAY STREET.

[*Two Shillings.*]

PREFACE TO THE FIRST EDITION.

OF the several works on the Slide Rule which have appeared in this country, none, so far as the author is aware, relates to rules of the Gravêt or Mannheim type. This, the most modern design of instrument, undoubtedly possesses many advantages over the older forms, and its use is rapidly extending in consequence. A small manual of instruction exemplifying the utility of this form of slide rule has long been in request, not only by those who experience some difficulty in mastering the principle of slide-rule computation, but also by users of the instrument desirous of taking the fullest advantage of its numerous applications in technical calculations of all kinds. Hence the appearance of this little work.

The examples selected will, the author trusts, suffice to exhibit a few of the many and diverse uses of the instrument in the every-day work of the engineer—mechanical, electrical and civil,—as well as to indicate its more general application in a variety of other directions.

It cannot be denied that a useful knowledge of the slide rule may be obtained by rote. But it is nevertheless true that an acquaintance with the rudiments of logarithmic computation is necessary, to enable the operator to take that intelligent interest in the manipulation of the rule which is indispensable if confidence in its results and ability to extend its application are to be acquired. For this reason, the author has endeavoured to explain, in as simple a manner as possible, the general theory of the instrument, before proceeding to consider the various rules to be observed in its operation.

C. N. P.

MANCHESTER, *October, 1894.*

PREFACE TO THE EIGHTH EDITION.

CONCLUSIVE evidence of the success of this, the first English work on the modern Slide Rule, is afforded by the call for seven large editions of the book in but little more than as many years. In preparing a new and still larger edition for the press, the author has taken the opportunity of adding considerably to the contents, the new matter referring more particularly to such recent developments of slide rule methods as are rendered possible by the use of the additional log.-log. scales. New cursors and the solution of algebraic equations are also among the subjects now introduced, while the original matter has been revised wherever necessary.

The degree of accuracy which the slide rule offers is probably sufficient for nine-tenths of the ordinary calculations of the engineer, corresponding roughly to the precision obtainable with a three-figure logarithm table. When a closer result is necessary, recourse must be had either to the long-scale slide rules described in the later pages of this work, or to four, five, or seven-figure logarithm tables, according to the requirements of the calculation. With the ordinary slide rule, the accuracy obtainable will largely depend upon the precision of the scale spacings, the length of the rule, the speed of working, and the aptitude of the operator. With the lower scales it is generally assumed that the readings are accurate to within 0·5 per cent.; but with a smooth-working slide the practised user can work to within 0·25 per cent. With the upper scales the precision will be less—probably one-third of 1 per cent. would represent the proportional error. But much depends upon the operator. If he has good eyesight, and the ability to correctly estimate the value of proportional parts of the scale spacings, he will be able to improve upon the figures just given, even in fairly rapid working.

It may be remarked that with the ordinary logarithmic scales the percentage error due to a slight displacement of the slide is constant throughout the length of the rule. This fact may be roughly but convincingly shown by setting 1 on C to, say, 1·01 on D, giving an error of 1 per cent. Then the corresponding reading at 2, 6, 8, etc., will be 2·02, 6·06, 8·08, etc., each reading showing a uniform error of 1 per cent.

The author cannot conclude without tendering his thanks to the many who have evinced their appreciation of his efforts to popularise the subject; also for the many kind hints and suggestions which he has received from time to time, and with a continuance of which he trusts to be favoured in the future.

C. N. P.

FALLOWFIELD, MANCHESTER,
July, 1903.

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THE SLIDE RULE.

INTRODUCTORY.

THE slide rule may be defined as an instrument for mechanically effecting calculations by logarithmic computation. By its aid various arithmetical, algebraical and trigonometrical processes may be performed with ease and rapidity, the results obtained being sufficiently accurate for almost all practical requirements. The precision attained in slide-rule calculations depends chiefly upon the length of the instrument and the minuteness of the divisions, presuming that the several scales are accurately graduated—an assumption, unfortunately, not always warranted by facts. Rules of about 10in. in length are most generally employed, although for the pocket 5in., and for office use 20in. rules are sometimes adopted. The principle is, however, the same in all cases.

Although the slide rule has long been in general use on the Continent—notably in Germany and France,—it is not until within comparatively recent years that due attention has been given to this time and labour-saving device in this country. However, several excellent rules have been introduced within the last few years, principally of the Gravêt or Mannheim type, and this, together with a growing recognition of the extensive scope and varied capabilities of the instrument, have led to its greatly extended use for all kinds of calculations, but more particularly for those of a technical character. There can be no doubt that in the slide rule we have an exceedingly valuable instrument, of which much greater use will be made in the near future A grasp of the simple fundamental principles which underlie its operation, together with a little patient practice, are all that are necessary to acquire facility in the use of the instrument, and few who become proficient in this method of calculation will willingly revert to the laborious system of arithmetical computation.

THE MECHANICAL PRINCIPLE.

The mechanical principle involved in the slide rule is of the simplest possible character. In Fig. 1, A and B represent two uniformly graduated rules, divided into 10 equal parts, which are numbered consecutively as shown. If the lower rule B, while in edge contact with the upper one A, is moved to the right until 0 on B falls exactly opposite to 3 on A, it will be observed that any number on A is equal

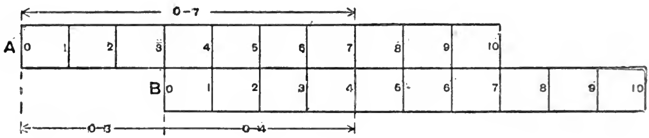

Fig. 1.

to the coinciding number on B, *plus* 3. Thus, opposite 4 on B is found 7 on A. The reason is obvious. By moving B to the right in the manner described, we add to a length 0-3, another length 0-4, the result read off on A being 7. Evidently, precisely the same result would have been obtained if a length 0-4 had been added, by means of a pair of dividers, to the length 0-3 on the scale A. By means of the slide B, however, the addition is more readily effected, and, what is of much greater importance, the result of adding 3 to *any one of the numbers* within range, on the lower scale, is *immediately* evident by reading the contiguous number on A.

It is scarcely necessary to observe that subtraction can be quite as readily performed by this means. Thus, if it is desired to subtract 4 from 7, we require to deduct from 0-7 on the A scale, a length 0-4 on B. This is accomplished by placing 4 on B under 7 on A, when over the zero of the B scale we find the result 3, on A. It is here evident that the *difference* of any pair of coinciding numbers on the scales is constantly equal to 3.

An important modification is effected in these results if the slide-scale B is inverted as indicated in Fig. 2. In this case, to find the sum of 4 and 3 we require to place the 4 of the A scale in juxtaposition with the 3 on the B scale, and the result is read off on A over the 0 of the B scale. Here it will be noted that the *sum* of any pair of coinciding numbers on the scales is constant and equal to 7. The method of operation in this case will be seen, therefore, to resemble that of the immediately preceding one, except in that the *sum*, instead of the *difference*, of any pair of coinciding numbers is constant.

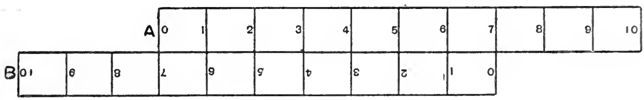

FIG. 2.

To find the difference of two factors, the converse operation is necessary. Thus, to ascertain the result of subtracting 4 from 7, the zero of B is placed opposite 7 on A, and over 4 on B is found 3 on A.

From these examples it will be seen that with the slide inverted the methods of operation are the reverse of those employed when the slide is in its normal position.

It will be understood that although in the foregoing only primary divisions of the scales have been considered, the remarks would equally apply to any subdivisions into which the primary spaces of the scales might be divided. Further, it is desirable to note that the length of scale taken to represent a unit is purely an arbitrary one.

THE MATHEMATICAL PRINCIPLE.

We have seen how a number of spaces on one of two contiguous scales may be added to or subtracted from a number of spaces on the other scale, and also that when one of the scales is inverted the methods of performing these operations are reversed. This is the mechanical portion of the theory of the slide rule; the mathematical portion, which is next to be considered, will be found to be almost equally simple.

By the aid of logarithms, an immense economy is effected in the labours of the mathematician, and as equal advantage is taken of this method of computation in the use of the slide rule, it becomes necessary to examine this process in some little detail.

Logarithms may be defined as a series of numbers in *arithmetical* progression, as 0, 1, 2, 3, 4, etc., which bear a definite relationship to another series of numbers in *geometrical* progression, as 1, 2, 4, 8, 16, etc. A more precise definition is:—The logarithm of a number to any base, is the *index of the power* to which the base must be raised in order to equal the given number. In the logarithms in general use, known as *common logarithms*, and with which we are alone concerned, 10 is the base selected. The general definition may therefore be presented in the following modified form:—The common logarithm of a number is the index of the power to which 10 must be raised in order to equal the given number. Applying this rule to a simple case, as $100=10^2$, we see that the base 10 must be squared (*i.e.*, raised to the 2nd power) in order to be equal to 100, the number selected for consideration. Therefore, as 2 is the index of the power to which 10 must be raised to equal 100, it follows from our definition that 2 is the common logarithm of 100. Similarly the common logarithm of 1000 is seen to be 3, while proceeding in the opposite direction the common log. of 10 must equal 1. Tabulating these results and extending the method, we have:—

Numbers	1	10	100	1000	10,000
Logarithms	0	1	2	3	4

It will now be evident that for numbers

between	1 and	10	the logs.	will be between	0	and	1
,,	10 ,,	100	,,	,,	1	,,	2
,,	100 ,,	1000	,,	,,	2	,,	3
,,	1000 ,,	10,000	,,	,,	3	,,	4

In other words, the logarithms of numbers between 1 and 10 will be wholly fractional (*i.e.*, decimal); the logs. of numbers between 10 and 100 will be 1 *plus a decimal quantity;* the logs. of numbers between 100 and 1000 will be 2 plus a decimal quantity, and so on. These decimal quantities for numbers from 1 to 10 (and which are the logarithms of this particular series) are as follows :—

Numbers...	1	2	3	4	5	6	7	8	9	10
Logarithms	0	0·301	0·477	0·602	0·699	0·778	0·845	0·903	0·954	1·000

Only the decimal part of the logarithm of a number (and which is called the *mantissa* of the logarithm) is given in the usual tables. The integral part (*characteristic*) of the logarithm will always be equal to the number of digits in the number *minus* 1, as a reference to the preceding tables will show. When the number is wholly decimal, the characteristic is equal to the number of cyphers following the decimal point *plus* 1, but in this latter case the characteristic is *negative*, and is to have the minus sign written over it. Thus, log. $0·003 = \bar{3}·477$.

An examination of the two rows of figures last given will reveal some striking peculiarities, and at the same time serve to illustrate the fundamental principle of logarithmic computation. In the first place, it will be noticed that the addition of any two of the logarithms gives a sum which is the logarithm of the product of these two numbers. Thus, the addition of log. 2 and log. $4 = 0·301 + 0·602 = 0·903$, and this is immediately seen to be the logarithm of 8, that is, of 2×4. Of necessity, the converse holds good equally, the difference of the logarithms of two numbers giving the logarithm of the quotient resulting from the division of these two numbers. Thus, log. 8 − log. $2 = 0·903 - 0·301 = 0·602$, and this is seen to be identical with the log. of 4, or $8 \div 2$.

One other important feature is to be noted. If the logarithm of any number is *multiplied* by 2, 3, or any other quantity, whole or fractional, the result is the logarithm of the original number, raised to the 2nd, 3rd, or other power respectively. Thus, multiplying the log. of 3 by 2, we obtain $0·477 \times 2 = 0·954$, and this is seen to be the log. of 9, that is, of 3 raised to the 2nd power, or 3 *squared.* Again, log. 2 multiplied by $3 = 0·903$—that is, the log. of 8, or of 2 raised to the 3rd power, or 2 *cubed.* Conversely, dividing the logarithm of any original number by any number *n*, we obtain the logarithm of the *n*th root of the original number. Thus, log. 8 ÷ 3 = 0·903 ÷ 3 = 0·301, and is therefore equal to log. 2 or to the log. of the *cube root* of 8.

In this brief explanation is included all that need now be said with regard to the properties of logarithms. The main facts to be borne clearly in mind are:—(1.) That to find the product of two numbers, the logarithms of the numbers are to be added together, the result being the logarithm of the product required, the value of which can then be precisely determined. (2.) That in finding the quotient resulting from the division of one number by another, the difference of the logarithms of the numbers gives the logarithm of the quotient, from which the value of the latter can be ascertained. (3.) That to find the result of raising a number to the *n*th power, we multiply the logarithm of the number by *n*, thus obtaining the logarithm, and hence the value, of the desired result. And (4.) That to find the *n*th root of a number, we divide the logarithm of the number by *n*, this giving the logarithm of the result, from which its value may be readily determined.

The obvious disadvantage of logarithmic computation lies in the fact that a book of logarithms is required. Another practical objection, perhaps not quite so evident, is that considerable time is occupied in looking out the logarithms, adding or otherwise dealing with them, and afterwards finding the number corresponding to the resulting logarithm. In the slide rule we have, so to speak, a small book of logarithms, which are, moreover, more conveniently arranged than the tables for the ordinary requirements of practice. In point of fact, all that the slide rule can be said to comprise, is a set of logarithms, together with a simple means of mechanically employing them to effect the various operations above referred to.

In order to illustrate these remarks, attention is directed to Fig. 3, which represents a scale divided into 10 equal parts, each of which is again assumed to be subdivided into 100 equal parts, thus forming a complete scale of 1000 equal parts. If this scale is regarded as a unit scale of logarithms, we may figure the 301st division, 2′, since 0·301 is the log. of 2. In the same way, the 477th division may be distinguished by 3′, the 602nd division by 4′, and so on, the final division, 1000, being figured 10′. If, now, the upper set of figures M be regarded as a series of whole numbers, it is clear that the logarithms of any of them can be readily ascertained by reading off the corresponding value on the scale of equal parts N. It will further be understood that although only the primary divisions of the upper scale M are shown, the intermediate figures of smaller subdivisions can also be readily inserted in the scale, in a precisely similar manner. We are thus furnished with a table of logarithms of numbers of from 1 to 10, and intermediate values, the possible number introduced of the latter depending upon the length of the whole scale, and on the extent to which it is physically possible to subdivide the primary spaces.

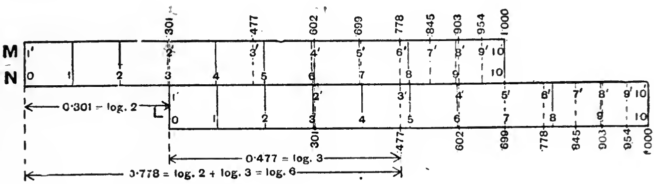

FIG. 3.

If we now take another and exactly similar scale L, and apply it to the edge of the first, we obtain all the essentials of the slide rule. To find the product of 2 and 3 by logarithms, we require to add 0·301, the log. of 2, to 0·477, the log. of 3, obtaining 0·778, or log. 6, as a result. To perform the operation with the slide rule, we place the end of the lower scale in agreement with the 301st division of the upper scale. Then opposite the 477th division of the lower or sliding scale, we read 778 on the upper scale, which, by reference to the upper set of figures, is seen to be the logarithm of 6. This method is precisely identical with that used in the simple case shown in Fig. 1, the only addition being the scale of numbers M, corresponding to their logarithms in N.

A brief consideration will show that no useful purpose would be served by retaining the scale of equal parts by which the *logarithms* are represented. It is with *numbers* that we have to deal, and in which we require the result to be expressed. Therefore, since each division of the number scale represents by its distance from 1, the log. of the number by which it is distinguished, it follows that the scale M can be directly employed in the various operations.

THE GRAVÊT OR MANNHEIM SLIDE RULE.

The modern form of slide rule, variously styled the Gravêt, the Tavernier-Gravêt, and the Mannheim rule, is frequently made of boxwood, but most of the leading instrument makers now supply rules made of mahogany or boxwood and faced with celluloid, the white surface of which brings out the graduations much more distinctly than

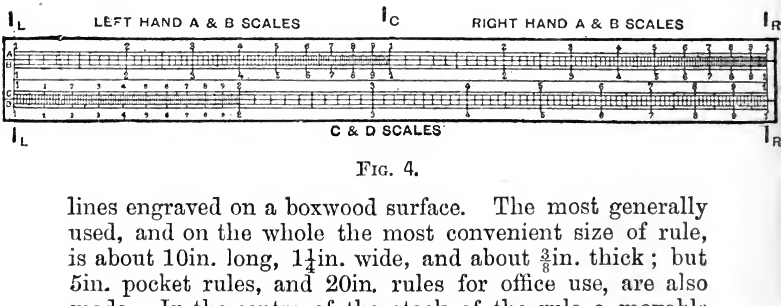

FIG. 4.

lines engraved on a boxwood surface. The most generally used, and on the whole the most convenient size of rule, is about 10in. long, $1\frac{1}{4}$in. wide, and about $\frac{3}{8}$in. thick; but 5in. pocket rules, and 20in. rules for office use, are also made. In the centre of the stock of the rule a movable slip is fitted, which constitutes the slide, and corresponds to the lower of the two rules used in the rudimentary examples previously given.

From Fig. 4, which is a representation of the face of a Gravêt or Mannheim slide rule, it will be seen that four series of logarithmic graduations or scale-lines are employed, the upper and lower being engraved on the stock or body of the rule, while the other two are engraved upon the slide. It will be noted that the two upper sets of graduations are exactly similar in every particular, and also that the lower sets are precisely alike. It is usual to designate the two upper scale-lines by the letters A and B, and the two lower by the letters C and D, as indicated in the figure at the left-hand extremities of the scales.

Referring in the first place to the scales C and D, these will each be seen to be a development of the elementary scale M, Fig. 3, being primarily divided into nine unequal parts in precisely the same manner; in this case, however, each principal space is subdivided, more or less minutely, into a number of smaller parts. As these graduations are arranged in logarithmic sequence, it follows, from what has been said in the preceding sections, that by moving the slide (carrying scale C), and so bringing the two scales into the required relative positions, multiplication and division can be mechanically performed in the manner already described.

The upper scale-line A will be seen to consist of two exactly similar scales, placed consecutively, the first lying between I_L and I_C, and the second between I_C and I_R. The first of these scales will be designated the *left-hand A scale*, and the second the *right-hand A scale*. Similarly the coinciding scales on the slide are the *left-hand B scale* and the *right-hand B scale*. It will be understood that each of these scales is divided (as far as physically possible) in precisely the same manner as are the C and D scales, the only difference being that they are exactly one half the length.

The two extreme divisions of the C and D scales, each of which is figured 1, are known as the left and right-hand indices of these scales. Similarly I_L and I_R are the left and right-hand indices of the A and B lines, while I_C is the centre index of these scales. The only other division lines usually found on the face of the rule are one on the left-hand A and B scales, indicating precisely the ratio of the circumference of a circle to its diameter:—$\pi = 3{\cdot}1416$; and a similar special line on the right-hand B scale marking the exact location of $\frac{\pi}{4} = 0{\cdot}7854$, and used in calculating the areas of circles. Reference will hereafter be made to the scales on the under-side of the slide, and we need now only add that one of the edges of the rule, usually bevelled, is generally graduated in millimetres, while the other edge has engraved on it a scale of inches divided into eighths or tenths. Inside the groove of the rule either one or the other of these scales is continued in such a manner that by drawing the slide out to the right and using the scale inside the rule, in conjunction with the corresponding scale on the edge, it is possible to measure 20 inches in the one case, or nearly 500 millimetres in the other.

THE NOTATION OF THE SLIDE RULE.

Hitherto our attention has been confined to a consideration of the primary divisions of the scales. The same principle of graduation is, however, used throughout; and after what has been said, this part of the subject need not be further enlarged upon. Some explanation of the method of reading the scales is, however, necessary, as facility in using the instrument depends in a very great measure upon the dexterity of the operator in assigning the correct value to each division on the rule. By reference to Fig. 4, it will be seen that each of the primary spacings in the several scales is invariably divided into ten subdivisions; but since the lengths of the successive primary divisions rapidly diminish, it is obviously impossible to subdivide each main space into the same number of parts that the space 1-2 can be subdivided. It must be admitted that some of the confusion which exists in the mind of the student is usually due to this variable spacing of the main intervals of the scales, but with a little practice this difficulty may be readily surmounted.

Turning attention first to the C or D scale, it will be noticed that the length of the interval 1-2 is here sufficient to allow each of the 10 subdivisions to be again divided into 10 parts, so that the whole interval 1-2 is thus divided into 100. The shorter main space 2-3, and the still shorter one 3-4, only allow of the 10 subdivisions of each being divided into five parts. Each of these main spaces is therefore divided into 50 parts. For the remainder of the scale each of the 10 subdivisions of each main space is divided into two parts only; so that from the main division 4 to the end of the scale the primary spaces are divided into 20 parts only.*

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* In the illustration some of the smaller division lines have been omitted, as they would have been indistinct.

Referring next to the upper scales A or B, it will be found that—as the space 1-2 is of only half the length of the corresponding space on C or D—the 10 subdivisions of this interval are again divided into five parts only. Similarly each of the 10 subdivisions of the intervals 2-3, 3-4, and 4-5 are further divided into two parts only, while for the remainder of the scale only the 10 subdivisions are rendered possible owing to the rapidly diminishing lengths of the primary spacings.

In assigning the correct value to each division of the several scales of the rule, it is important to observe that these values depend directly upon that given to the left index figure (1) of the C or D scale, and that this latter may be taken to represent any value that is a multiple or submultiple of 10. Thus I_L on the D scale may be regarded as 1, 10, 100, 1000, etc., or as 0·1, 0·01, 0·001, 0·0001, etc.; but once the initial value is assigned to the index, the ratio of value must be maintained throughout the whole scale. For example, if 1 on C is taken to represent 10, the main divisions 2, 3, 4, etc., will be read as 20, 30, 40, etc. On the other hand, if the fourth main division is read as 0·004, then the left index figure of the scale will be read as 0·001. The figured subdivisions of the main space 1-2 are to be read as 11, 12, 13, 14, 15, 16, 17, 18, and 19—if the index represents 10,—and as corresponding multiples for any other value of the index. Independently considered, these remarks apply equally to the A or B scale, but in this case the notation is continued through the second half of the scale, the figures of which are to be read as tenfold values of the corresponding figures in the first half of the scale. Thus, if the left index of A is read as 10, the scale will run 10, 20, 30, 40, 50, 60, 70, 80, 90, 100 (centre index), 200, 300, 400, etc., to 1000 at the right-hand index. The reading of the intermediate divisions will, of course, be determined by the values assigned to the primary divisions. Thus, if I_L on D is read as 1, then each of the smallest subdivisions of the space 1-2 will be read as 0·01, since they represent hundredths of the main interval. Similarly each of the smallest subdivisions of the spaces 2-3 or 3-4 on this scale are read as 0·02, while for the remainder of the scale the smallest subdivisions are read as

0·05, these being twentieths of the main intervals. In the A or B scale the subdivisions of the space 1-2 of the first half of the scale are (if $I_L=1$) read as 0·02, 0·04, etc., while for the divisions 2-3, 3-4, and 4-5, the smallest intervals are read as 0·05 of the primary spaces; while from 5 to the centre index of the scale the divisions represent 0·1 of each main interval. Passing the centre index, which is now read as 10, the smallest subdivisions immediately following are read 10·2, 10·4, etc., until 20·0—the next figured division line—is reached. After this the succeeding smallest intervals are read as 20·5, 21·0, 21·5, 22·0, etc., until the figured main division 5 is reached. The remainder of the scale is read 51, 52, 53, etc., up to 100, the right-hand index.

Further subdivision of any of the spaces of the rule can be effected by the eye, and after a little practice the operator will become quite expert in reading off any intermediate value by estimation.

The description of the method of reading the A and B scales, just given, applies only when these scales are regarded as altogether independent of the lower pair of scales C and D. As a matter of fact some operators prefer to use the A and B scales, and some the C and D scales, for the ordinary operations of proportion, multiplication, and division. Each method has its advantages, as will be presently shown. But in the more complex calculations, as involution and evolution, etc., the relation of the upper to the lower scales of the rule becomes a very important factor.

It will be remembered that among the properties of logarithms alluded to in a previous section, mention was made of the fact that if the logarithm of a number was multiplied by 2, the product would be the logarithm of the *square* of number, and that, *vice versâ*, if the logarithm of a number be divided by 2 we obtain the logarithm of the *square root* of the number. It is upon these properties of logarithms that the relation of the upper and lower scales of the slide rule directly depends.

It has been pointed out that each of the two similar scales contained in the A scale-line is only one-half the length of the D scale. Therefore, starting at the left index, the actual length representing on the D scale the log. of 2 would be twice that representing the log. of 2 taken on the A scale, and so on throughout the whole length of the scale In other words, the logs. of the numbers on the A scale are, considered comparatively, equal to those on the D scale multiplied by 2, and therefore equal to the logs. of the *squares* of the numbers on the latter scale. Since the numbers, and not the logarithms, are engraved on the rule, it consequently results that opposite any number on D we find its *square* on A, and conversely that under any number on A is its *square root* on D, this relationship holding for every division on the rule.

THE CURSOR OR RUNNER.

In order to accurately refer the divisions of the upper scales to those of the lower, use is made of the metal cursor or runner with which the rule is now invariably fitted. This attachment should be capable of being easily and smoothly traversed along the whole length of the rule; if it moves irregularly, the small grooves in which it slides should be carefully examined and any obstruction removed. Occasionally a little clean tallow may be applied to the retaining edges of the cursor, and also to the centre grooves in which the slide works. The cursor furnished with some instruments takes the form of an I-shaped piece of metal, from the body of which two small prongs project over the two pairs of scales. Fine reference lines on these prongs enable the divisions of the upper and lower scales to be compared; they are also of service in the setting of the graduations of one scale to those of another, more especially when the position of either or both of the values is dependent upon eye-estimation.

In a similar type of cursor sometimes employed two sets of projecting prongs, one on each side of the body, are provided. This, however, is not a good arrangement, as it tends to confuse the operator. The more general form of cursor consists of a light square frame of metal. In this is fixed a piece of glass or mica, about 1in. square, across the centre of which is a fine black line accurately placed at right angles to the scale-lines, and which reference line enables the readings on the two sets of scales to be readily compared. The cursor, as will appear later, greatly facilitates operations, especially when they are of a complex character.

PROPORTION.

Proportion is the simplest of all the many operations that can be so expeditiously performed by the aid of the slide rule. We therefore commence with this form of calculation, the C and D scales being used for this, as well as for multiplication and division. With the left-hand indices of C and D placed in coincidence, the ratio between all corresponding divisions of these scales is unity (1). If, however, the slide be moved to the right so that the index of C falls opposite 2 on D, it will be seen that the numbers on D will bear to the coinciding numbers on C, a ratio of 2 to 1. This of course follows from the fact that the logarithm of 2 has virtually been added to all the numbers on the slide, and the result is expressed by the coinciding numbers on the scale of the rule. The same relationship will obviously exist between the numbers on these two scales, no matter in what position the slide may be placed. Thus a most important feature becomes apparent. The rule for proportion, which is apparent from the foregoing, may be expressed as follows :—

Rule.—*Set the first term of a proportion on the C scale to the second term on the D scale, and opposite the third term on the C scale read the fourth term on the D scale.*

Ex.—Find the 4th term in the proportion of 20 : 27 : : 70 : x.

Place the slide so that 20 on C falls opposite 27 on D, and opposite 70 on C read 94·5 on D.

It will be evident that this method may be readily modified so as to give from *any three* known terms of a proportion, the remaining term of the expression. Thus, given the 2nd, 3rd, and 4th terms of the above example, the method of procedure would be :—Set the 3rd term (70) on C to the 4th term (94·5) on D, and opposite the 2nd term

(27) on D read the required 1st term (20) on C. All it is necessary to observe is that the 1st and 3rd terms are always found on one scale and the 2nd and 4th on the other.

It is useful to note that as a consequence of the constant proportion existing between the numbers on the two scales, the fraction $\frac{\text{1st term}}{\text{2nd term}}$ may be expressed as $\frac{1}{x}$, or as its reciprocal $\frac{x}{1}$, the value of x in the first case being read on the D scale, under the index of the slide, and in the second case on the C scale over the index of the D scale. This fact may be turned to advantage in reducing vulgar fractions to decimals. Thus, the decimal equivalent of $\frac{3}{16}$ is determined by placing 3 on C to 16 on D, when over the index or 1 of D we read 0·1875 on C. In this case the terms are $3 : 16 : : x : 1$. For the inverse operation—to find a vulgar fraction equivalent to a given decimal—the given decimal fraction on C is set to the index of D, and then opposite any numerator on C is the corresponding denominator of the fraction on D.

If the index of C be placed coincidentally with 3·1416 on D, it will be clear from what has been said that this ratio exists throughout between the numbers of the two scales. Therefore, against any *diameter* of a circle on C will be found the corresponding *circumference* on D. As the graduations of the rule, however, do not admit of a very exact setting being made in this way, it has become customary under this and similar circumstances to set the slide by some more convenient, but sufficiently exact, equivalent ratio. Thus, instead of the value of π just given, the ratio $\frac{22}{7}$ is frequently adopted. A more accurate value is $\frac{355}{113}$; but this is open to the objection that the setting cannot be exactly effected, since there is no graduation representing 355. But by multiplying both numerator and denominator by 2 we obtain the ratio $\frac{710}{226}$, a setting which is much more convenient than any other, since it is quite readily and exactly effected by bringing the graduation representing 226 on C to coincide with that representing 710 on D.

By the aid of the table of conversion factors given on the following pages, this exceedingly valuable property of the slide rule may be turned to very great advantage. As tabulated, scales C and D are intended to be used; but it will be evident that the A and B scales may be employed if desired, although the equivalent values cannot then be read off with the same exactitude. After what has been said in the preceding, the manner of using these factors requires little explanation. It will be understood, for example, that to convert feet into metres, or the converse, the graduation representing 292 on the C scale is to be placed opposite that representing 89 on the D scale (this latter graduation may be first located by setting the cursor to it as previously explained), and then against any number of feet on the C scale will be found the equivalent number of metres on the D scale, or *vice versâ*. All the equivalent ratios given, if used with the C and D scales, may be set line to line.

In the last column is given the ratio $\frac{\text{scale D}}{\text{scale C}}$ used in determining the corresponding conversion factors. This will enable those operators who so prefer, to work with the usual equivalent value on D to unit value on C.

If the A and B scales are used, a complete set of conversions will be at once obtained. In this case, however, the factors should be selected on the left-hand A and B scales, and any values read off on the right-hand A or B scales should be considered as being of tenfold value. With the C and D scales a portion of the one scale will project beyond the other. To read this portion of the scale, the cursor or runner is brought to whichever index (1) of the C scale falls within the rule. The slide is then carefully moved (without disturbing the cursor) until the other index of the C scale coincides with the cursor; the remainder of the equivalent values can then be read off. It must be remembered that if the slide is moved in the direction of notation (to the right), the values read thereon have a tenfold greater value; if the slide is moved to the left, the readings thereon are decreased in a tenfold degree.

TABLE OF CONVERSION FACTORS.

I.—Geometrical Equivalents.

Scale C.	Scale D.	Scale C.	Scale D.	If C = 1, D =
Diameter of circle ...	Circumference of circle..	226	710	3·1416
" " ...	Side of inscribed square..	99	70	0·707
" " ...	" equal square ...	79	70	0·886
" " ...	" " equilateral triangle.........	72	97	1·346
Circum. of circle ...	" inscribed square.	40	9	0·282
" "	" equal square ...	39	11	0·225
Side of square	Diagonal of square	70	99	1·414
Square inch	Circular inch	11	14	1·273
Area of circle.........	Area of inscribed square.	300	191	0·636

II.—Measures of Length.

Inches..................	Millimetres	5	127	25·40
"	Centimetres	50	127	2·54
8ths of an inch ...	Millimetres	40	127	3·175
16ths " " ...	"	80	127	1·587
32nds " " ...	"	160	127	0·794
64ths " " ...	"	320	127	0·397
Feet	Metres	292	89	0·3048
Yards	"	35	32	0·9144
Chains................ .	"	175	3520	20·116
Miles	Kilometres	87	140	1·609

III.—Measures of Area.

Square inches...	Square centimetres	31	200	6·45
Circular "	" "	434	2200	5·067
Square feet............	" metres	140	13	0·0929
" yards	" "	61	51	0·836
" miles	" kilometres	112	290	2·59
" "	Hectares	112	29,000	259·00
Acres	"	42	17	0·4046

IV.—Measures of Capacity.

Cubic inches	Cubic centimetres.........	36	590	16·38
" "	Imperial gallons	6100	22	0·00360
" "	U.S. gallons	6700	29	0·00432
" "	Litres........................	3600	59	0·01638
Cubic feet	Cubic metres.............	106	3	0·0283

TABLE OF CONVERSION FACTORS—(*Continued*).

IV.—Measures of Capacity—(*Continued*).

Scale C.	Scale D.	Scale C.	Scale D.	If C = 1, D =
Cubic feet	Imperial gallons	17	106	6·23
,, ,,	U.S. gallons	234	1750	7·48
,, ,,	Litres	3	85	28·37
,, yards	Cubic metres	51	39	0·764
Imperial gallons	Litres	262	1190	4·54
,, ,,	U.S. gallons	5	6	1·200
Bushels	Cubic metres	110	4	0·0363

V.—Measures of Weight.

Grains	Grammes	5000	324	0·0648
Ounces (Troy)	,,	9	280	31·103
,, (Avoird.)	,,	67	1900	28·35
,, ,,	Kilogrammes	670	19	0·02835
Pounds (Troy)	,,	75	28	0·3732
,, (Avoird.)	,,	280	127	0·4536
Hundredweights	,,	5	254	50·802
Tons	,,	62	63,000	1016·04
,,	Metric tonnes	62	63	1·016

VI.—Compound Equivalents.

(*a*) *Velocities.*

Feet per second	Metres per second	292	89	0·3048
,, ,,	,, minute	7	128	18·288
,, ,,	Miles per hour	22	15	0·682
,, minute	Metres per second	25,000	127	0·00508
,, ,,	,, minute	292	89	0·3048
,, ,,	Miles per hour	264	3	0·01136
Yards per ,,	,, ,,	88	3	0·0341
Miles per hour	Metres per minute	12	322	26·82
Knots	,, ,,	11	340	30·88
,,	Miles per hour	33	38	1·151

(*b*) *Pressures.*

Pounds per sq. inch	Grammes per sq. mm.	64	45	0·7031
,, ,,	Kilos. per sq. centimetre	128	9	0·0703
,, ,,	Atmospheres	485	33	0·068
,, ,,	Head of water in inches	57	1580	27·71

TABLE OF CONVERSION FACTORS—(*Continued*).

(*b*) *Pressures*—(*Continued*).

Scale C.	Scale D.	Scale C.	Scale D.	If C = 1, D =
Pounds per sq. inch..	Head of water in feet ...	13	30	2·309
,, ,,	,, ,, metres	33	250	7·573
,, ,,	Inches of mercury	25	51	2·04
,, ,,	Tons per square yard ...	270	156	0·578
Inches of water	Pounds per square inch .	360	13	0·0361
,, ,, ...	Inches of mercury	14	1	0·0714
,, ,, ...	Pounds per square foot..	5	26	5·20
Inches of mercury...	Atmospheres	30	1	0·0333
Atmospheres	Metres of water	29	300	10·34
,, ...	Kilos. per sq. cm..........	89	92	1·033
Feet of water.........	Pounds per square foot..	77	4800	62·35
,, ,, ...	Atmospheres...	1290	38	0·0294
,, ,, ...	Inches of mercury	106	91	0·883
Pounds per sq. foot..	,, ,, ...	1200	17	0·01417
,, ,,	Kilos. per square metre..	87	425	4·883
,, ,,	Atmospheres..............	91,000	43	0·000472
Pounds per sq. yard.	Kilos. per square metre..	94	51	0·5425
Tons per sq. inch ...	,, square mm. ...	40	63	1·575
,, sq. foot ...	Tonnes per square metre	80	875	10·936
,, sq. yard ...	,, ,, ,,	37	45	1·215

(*c*) *Weights, Capacities, etc.*

Pounds per lineal ft.	Kilos. per lineal metre...	41	61	1·488
,, lineal yd.	,, ,, ,, ...	500	248	0·496
,, lineal mile	Kilos. per kilometre......	71	20	0·2818
Tons per lineal mile.	Tonnes ,, ,, ...	450	284	0·6313
Feet ,, ,,	Metres ,, ,, ...	132	250	1·894
Pounds per cubic ft.	Kilos. per cubic metre...	39	625	16·02
,, cubic yd.	,, ,, ,, ...	700	415	0·593
Tons per cubic yard.	Tonnes ,, ,, ...	376	500	1·329
Cubic yds. per pound	Cubic metres per kilo....	210	354	1·685
,, ,, ton ...	,, ,, tonne..	206	155	0·7525
Cubic feet of water..	Weight in pounds	17	1060	62·35
,, ,,	,, kilos.	14	396	28·28
,, ,,	Imperial gallons	17	106	6·235
,, ,,	U.S. gallons	50	374	7·48
Gallons of water ...	Weight in kilos.	108	490	4·54
Grains per gallon ...	Grammes per litre	1500	214	0·01426
Pounds per gallon...	Kilos. per litre	240	23	0·0998
,, U.S. gal.	,, ,, ...	200	23	0·115
Pounds of fresh water	Pounds of sea water ...	38	39	1·026

TABLE OF CONVERSION FACTORS—(*Continued*).

(*d*) *Power Units, etc.*

SCALE C.	SCALE D.	SCALE C.	SCALE D.	If C = 1, D =
British heat units ...	Calories (Fr. heat units)	250	63	0·252
Heat units per sq. ft.	„ per square metre	129	350	2·713
„ „ pound	„ per kilogramme.	9	5	0·555
Foot-pounds	Kilogrammetres	340	47	0·1382
Horse-power	Force de cheval (Fr. H.P.)	72	73	1·0139
Pounds per H.P. ...	Kilos. per cheval	300	134	0·447
Square feet per H.P.	Square metres per cheval	2500	49	0·0196
Cubic „ „	Cubic „ „	1900	53	0·0279
Horse-power per hour	Kilowatts (B.T. electrical units)	134	100	0·746
Joules	Kilogrammetres	105	107	0·1019
Watts	Horse-power	5	0·0067	0·00134
„	Force de cheval (Fr. H.P.)	6	0·00815	0·001358

PROPORTION WITH THE SLIDE INVERTED.

Inverse Proportion.—If "more" requires "less," or "less" requires "more," the case is one of *inverse* proportion, and although from what has been said it will be seen that this form of proportion is quite readily dealt with by the preceding method, it may be observed that the working is simplified to some extent by inverting the slide so that the C scale is adjacent to the A scale. By the aid of the cursor, the values on the inverted C (or C′) scale, and on the D scale, can be then read off. These will now constitute a series of inverse ratios. Thus, in the proportion 4 : 3 : : 8 : 1·5, the 4 on the C′ scale is brought opposite 3 on D, when under 8 on C′ is found 1·5 on D.

MULTIPLICATION.

Multiplication and division may be justly regarded as modified forms of proportion, and are therefore the operations which come next in order. In the preliminary notes, it was shown that by mechanically adding two logarithmic space-lengths, the numbers represented thereby were multiplied together, while by subtracting one space-length from another the numbers represented by such space-lengths were divided the one by the other. Hence the rule for multiplication is :—

RULE.—*Set the index of the C scale to one of the factors on D, and under the other factor on C, find the product on D.*

Thus, to find the product of 2×4, the slide is moved to the right until the left index of C is brought over 2 on D, when under the other factor (4) on C, is found the required product (8) on D. A brief consideration will show that when in such a simple case as that selected, the product is less than 10, the result can be at once read off on the D scale. But if the product is more than 10, and less than 10 times the first factor, the slide must be moved to the left until the right-hand index of C is over the first factor. Thus, to find the product 2×7, the slide is moved to the left until the *right* index of C is brought over the first factor 2, when under 7 on C is found 14 on D. Since the product is greater than 10 and less than $2 \times 10 = 20$, it is evident that it is to be read as 14. This brings us to the rule for the number of figures in the product. It is obvious from the foregoing examples that when the product is read off with the slide projecting to the *left*, that portion of the scale D upon which the product is found is of tenfold value as compared with the same scale when the slide projects to the right. From this the rule for the position of the decimal point is deduced, as follows :—

RULE FOR NUMBER OF DIGITS IN A PRODUCT.—*The number of digits in the product is equal to the* SUM OF THE DIGITS IN THE TWO FACTORS *if the answer is obtained* WITH THE SLIDE PROJECTING TO THE LEFT ; *if, however, the slide projects to the* RIGHT, *the number of digits in the product is* EQUAL TO THE SUM OF THE DIGITS IN THE TWO FACTORS, LESS 1.

A few examples will best exemplify the application of these rules :—

Ex.—$36 \times 25 = 900$.

In this case the product is found with the slide projecting to the *right ;* the number of digits in the product therefore equals $2 + 2 - 1 = 3$.

Ex.—$25 \times 70 = 1750$.

Here the product is found with the slide projecting to the *left*, so that the number of digits in the product $= 2 + 2 = 4$.

Ex.—$3{\cdot}6 \times 25 = 90$.

The slide projects to the *right*, and the number of digits in the product is therefore $1 + 2 - 1 = 2$.

Ex.—$0{\cdot}025 \times 0{\cdot}7 = 0{\cdot}0175$.

The product is obtained with the slide projecting to the *left*, and the number of digits in the product is therefore $-1 + 0 = -1$.

Ex.—$0{\cdot}000184 \times 0{\cdot}005 = 0{\cdot}00000092$.

The sum of the number of digits in the two factors $= -3 + (-2) = -5$, but as the slide projects to the *right*, the number of digits in the product will be $-5 - 1 = -6$.

From the latter of these examples it will be seen that when the first significant figure of a decimal factor does not immediately follow the decimal point, the minus sign is to be prefixed to the number of digits, counting as many digits *minus* as there are 0's following the decimal point. Thus, 0·03 has -1 digit, 0·0035 has -2 digits, and so on. Some little care is necessary to ensure these minus values being correctly taken into account in determining the number of digits in the answer. For this reason many prefer to treat decimal factors as whole numbers, and to locate the decimal point according to the usual rules for the multiplication of decimals. Thus, in the last example we take $184 \times 5 = 920$, but as by the usual rule the product must contain $6 + 3 = 9$ decimal places, we prefix six cyphers, obtaining 0·00000092. When both factors consist of integers as well as decimals, the number of digits in the product, and therefore the position of the decimal point, can be determined by the usual rule for whole numbers.

Another method of determining the number of digits in a product deserves mention, which, not being dependent upon the position of the slide, is generally applicable to all calculating instruments.

General Rule for Number of Digits in a Product.—*When the first significant figure in the product is smaller than in* EITHER *of the factors, the number of digits in the product is equal to the* SUM *of the digits in the two factors.*

When the contrary is the case, the number of digits is 1 LESS *than the sum of the digits in the two factors. When the first figures are the same, those following must be compared.*

It frequently happens that the number of digits in the answer is self-evident from the character of the problem; but in any case, after a very little practice no difficulty will be experienced by the student in correctly estimating the position of the decimal point or the total number of figures in a product. With the ordinary 10in. Gravêt rule it will be found in general that the extent to which the C and D scales are subdivided is such as to enable not more than three figures in either factor being dealt with. For the same reason it is impossible to directly read more than the first three figures of any product, although it is often possible —by mentally dividing the smallest space involved in the reading—to correctly determine the fourth figure of a product. Necessarily this method is only reliable when used in the earlier parts of the C and D scales. However, the last numeral of a three-figure, and in some cases the last of a four-figure, product can be readily ascertained by an inspection of the factors.

Thus, $19 \times 27 = 513$. Placing the L.H. index of C to 19 on D, we find opposite 27 on C, the product, which lies between 510 and 515. A glance at the factors is sufficient to decide, however, that the third figure must be 3, since the product of 9 and 7 is 63, and the last figure of this product must be the last figure in the required answer.

Ex.—$79 \times 91 = 7189$.

In this case the division line 91 on C indicates on D that the answer lies between 7180 and 7190. As the last figure must be 9, it is at once inferred that the last two figures are 89.

When there are more than three figures in either or both of the factors, the fourth and following figures to the right must be neglected. It is well to note, however, that if the first neglected figure is 5, or greater than 5, it will generally be advisable to increase by 1 the third figure of the factor employed. Generally it will suffice to make this increase in only one of the two factors, but it is obvious that in some cases greater accuracy will be obtained by increasing both factors in this way.

Continued Multiplication.—By the aid of the metal cursor or runner, the continued multiplication of a number of factors is conveniently effected, this being one of the marked advantages, over the older forms, of the type of rule now under consideration. With the former it is necessary to read off the result of each separate multiplication, the product so found being used as one of the factors in the next operation, this process being repeated until all the factors have been taken into account. With the cursor, however, the operation is continuous, while not only is time saved, but much greater accuracy secured, since the errors incidental to the several separate settings are altogether avoided. From the rule already given (p. 27) for the number of digits in a product, it will be seen that in continued multiplication it becomes necessary to note the number of times the multiplication is effected with the slide projecting to the right. This number deducted from the sum of the digits of the several factors will then give the number of digits in the product. Ingenious devices have been adopted with the object of recording the number of times the slide projects to the right, but some of these are very inconvenient. The author's method is to record each time the slide projects to the right by a minus mark, thus –. These can be noted down in any convenient manner, and the sum of the marks so obtained deducted from the sum of the digits in the several factors, gives the number of digits in the product as before explained.

Ex.—$42 \times 71 \times 1{\cdot}5 \times 0{\cdot}32 \times 121 = 173{,}200$.

The product given, which is that read on the rule, is obtained as follows:—Set R.H. index of C to 42 on D, and bring the cursor to 71 on C. Next bring the L.H. index of C to the cursor, and the latter to 1·5 on C. This multiplication is effected with the slide to the right, and a memorandum of this fact is kept by making a mark –. The R.H. index of C is now brought to the cursor and the latter moved to 0·32 on C. Then the L.H. index of C is set to the cursor and the result, 1732, read off on D under 121 on C, while as the slide again projects to the right a second – memo-mark is recorded. There are $2+2+1+0+3=8$ digits in the factors, and as there were 2 – marks recorded during the operation, there will be $8-2=6$ digits in the product, which will therefore read 173,200. The true product is 173,194·56, showing that the result as obtained by the rule is in close agreement with the actual product.

DIVISION.

The somewhat detailed description given of the method of performing multiplication renders unnecessary any very full discussion of the inverse process of division.

RULE FOR DIVISION.—*Place the divisor on C, opposite the dividend on D, and read the quotient on D under the index of C.*

Ex.—225 ÷ 18 = 12·5.

Bringing 18 on C to 225 on D, we find 12·5 under the L.H. index of C.

As in multiplication, the factors are treated as whole numbers, and the position of the decimal point afterwards decided according to the following rule, which, as will be seen, is the reverse of that for multiplication :—

RULE FOR NUMBER OF DIGITS IN A QUOTIENT.—*When the quotient is obtained with the slide projecting to the* LEFT, *the number of digits in the result is found by subtracting the digits in the divisor from those in the dividend; but if the quotient is read with the slide projecting to the* RIGHT, 1 IS TO BE ADDED *to the difference of the number of digits found as above.*

In the above example the quotient is read off with the slide to the right, so that the number of digits in the answer = 3 − 2 + 1 = 2.

Ex.—0·000221 ÷ 0·017 = 0·013.

Here the number of digits in the dividend is − 3, and the number in the divisor − 1. The difference is therefore − 2 ; but as the result is obtained with the slide projecting to the right, this result must be increased by 1, so that the number of digits in the quotient is − 2 + 1 = − 1, giving the answer as 0·013.

This result can also be obtained in the manner referred to when considering the multiplication of decimals. Thus, treating the above as whole numbers, we find that the result of dividing 221 by 17 = 13, since the difference in the number of digits in the factors, which is 1, is, owing to the position of the slide, increased by 1, giving 2 as the number of digits in the answer. Then by the ordinary rules for the division of decimals we know that the number of decimal places in the quotient is equal to $6-3=3$, and showing that a cypher is to be prefixed to the result read on the rule.

As in multiplication, so in division, we have a

General Rule for Number of Digits in a Quotient.—*When the first significant figure in the* divisor *is greater than that in the* dividend, *the number of digits in the quotient is found by subtracting the digits in the divisor from those in the dividend. When the contrary is the case,* 1 is to be added *to this difference. When the first figures are the same, those following must be compared.*

The author's method of denoting the number of times division is performed with the slide to the right is by vertical memorandum marks, thus I. The full significance of these memo-marks will appear in the following section.

COMBINED MULTIPLICATION AND DIVISION.

Combinations of the rules for multiplication and division enable lengthy calculations of the form $\frac{a}{b}\times\frac{c}{d}\times\frac{e}{f}\times\frac{g}{h}=x$ to be very readily resolved. Many cases of this kind occur in engineering calculations, more especially in determining the ultimate velocity ratio in belt, rope, or wheel gearing. Such a simple case as $\frac{a\times b}{c}=x$ is readily seen to be an example of proportion, but it may be equally regarded as a combination of multiplication and division.

Ex.—$\frac{14{\cdot}45\times 60}{8{\cdot}5}=102.$

In this case we place 8·5 on C to 14·45 on D, and opposite 60 on C read the answer, 102 on D. The slide projects to the left, therefore the number of digits in the product is $2-1+2=3$. Practical applications of this form of calculation are found in questions relating to levers, bending moments, bursting pressures and strength of tubes and boilers, number of teeth or pitch of gear wheels, velocity ratios in gearing, etc.

Number of Digits in Result in Combined Multiplication and Division.—The author's method of determining the position of the decimal point in combined multiplication and division, will be found to reduce the operation to an exceedingly simple process. As before, each time *multiplication* is performed with the slide projecting to the *right*, a – mark is made; each time *division* is effected with the slide to the right, a | mark is made; *but the | marks are allowed to cancel the – marks as far as they will.* The sum of the digits in the denominator is subtracted from the sum of the digits in the numerator, and, if there are no uncancelled memo-marks remaining, this difference will represent the number of digits in the result. If, however, there remain any uncancelled memo-marks, they are to be *added* to the above difference if of | character, or *subtracted* if of – character.

An example will show the application of this rule to be far simpler than its enunciation.

$$\text{Ex.}—\frac{43{\cdot}5 \times 29{\cdot}4 \times 51 \times 32}{27 \times 3{\cdot}83 \times 10{\cdot}5 \times 1{\cdot}31} = 1468.$$

Set 27 on C to 43·5 on D, and as with this *division* the slide is to the right, make the first | mark. Bring cursor to 29·4 on C, and as in this *multiplication* the slide is to the right, make the first – mark, cancelling as shown. Setting 3·83 on C to the cursor, requires the second | mark, which, however, is cancelled in turn by the multiplication by 51. The division by 10·5 requires the third | mark, and after multiplying by 32 (requiring no mark) the final division by 1·31 requires the fourth | mark. Then, as there are 8 numerator digits, 6 denominator, and 2 uncancelled memo-marks (which, being |, are additive), we have

+
+
|
|

Number of digits in result $=8-6+2=4$.

Had the uncancelled marks been – in character, the number of digits would have been

$$8-6-2=0.$$

It will be seen that with this method all such expedients as ruled columns headed × and ÷, etc., are dispensed with, also that it is quite immaterial how the marks are noted down, since it is obvious that the preceding record may be equally well written + + | |

Those who are in the habit of using tables of logarithms will readily note that the rules for the position of the decimal point are based upon those used for determining the resultant characteristic in a calculation worked by logarithms. The usual rules for determining the characteristic are given on page 11, but many prefer the following method :—Fix upon the units place as the starting point (0), and for every space moved through to reach the first *significant* figure, add 1, calling the result *plus* for movements to the *left* and *minus* for movements to the *right*. Thus, for 5278·6, starting at the units place figure (8), we require to move 3 places to the *left* to reach 5, the first significant figure; the characteristic is therefore + 3. For 0·00026, we require to move from the units place, 4 places to the *right;* hence the characteristic is − 4. It will be seen that in the first case we express 5278·6 as $5{\cdot}2786 \times 10^3$; while in the second case, 0·00026 $=2{\cdot}6 \times 10^{-4}$, the indices of the multiplier 10 being the characteristics required. To convert a number so expressed to its original form, we move the decimal point through the number of places indicated by the characteristic, moving to the *right* for *positive*, and to the *left* for *negative* characteristics. The result is of course to be corrected by the addition or subtraction of the uncancelled memo-marks as before explained.

It will be evident that in many instances the number of digits in the answer can be obtained by inspection, or by making a rough calculation.

MULTIPLICATION AND DIVISION WITH THE SLIDE INVERTED.

If the slide be inverted in the rule so that the C scale lies adjacent to the A scale, and the right and left indices of the slide and rule are placed in coincidence, we find the product of any number on D by the coincident number on C′ (readily referred to each other by the cursor) is constantly 10. Hence by dividing the numbers on the C′ scale by 10 (*i.e.*, reading them as decimals) we obtain the *reciprocals* of those numbers coinciding with them on D, and *vice versâ*. Thus 2 on D is found opposite 0·5 on C′; 3 on D opposite to 0·333; while opposite 8 on C′ is 0·125 on D, etc. The reason of this is that the sum of the lengths of the slide and rule corresponding to the factors, is always equal to the length corresponding to the product—in this case, 10. Upon reflection it will be seen that if we attempt to apply the ordinary rule for multiplication, with the slide inverted, we shall actually be multiplying the one factor taken on D by the *reciprocal* of the other taken on C′. But multiplying by the *reciprocal of a number* is equivalent to *dividing* by that number. Further, *dividing* a factor by the *reciprocal* of a number is equivalent to *multiplying* by that number. It follows, therefore, that with the slide inverted the operations of multiplication and division are reversed, as are also the rules for the number of digits in the product and the position of the decimal point. In multiplying, therefore, with the slide inverted, we place (by the aid of the cursor) one factor on C′ opposite the other factor on D, and read the result on D under either index of C′. It follows that with the slide thus set, any pair of coinciding factors on C′ and D will give the same constant product found on D under the index of C′. One useful application of this fact is found in selecting the scantlings of rectangular sections of given areas or in deciding upon the dimensions of rectangular sheets, plates, cisterns, etc. Thus by placing the index of C′ to 72 on D, it is readily seen that a plate having an area of 72 sq. ft. may have sides 8 by 9ft., 6 by 12, 5 by 14·4, 4 by 18, 3 by 24, 2 by 36, with innumerable intermediate values. Many other useful applications of a similar character will doubtless suggest themselves.

THE USE OF THE UPPER SCALES FOR PROPORTION, MULTIPLICATION, AND DIVISION.

Many prefer to use the upper scales A and B, in preference to C and D, for the various operations discussed in the foregoing sections. The disadvantage is that as the scales are only one-half the length of C or D, the graduations do not permit of the same degree of accuracy being obtained as when working with the lower scales. On the other hand, a great advantage is obtained, inasmuch as the result, either in proportion, multiplication, or division, can always be read directly from the rule without—as is frequently necessary in calculations with the lower scales—having to change the position of the slide after it has been initially set. Hence, also, it obviates the uncertainty as to the direction in which the slide is to be moved in making a setting.

When the A and B scales are employed, it is understood that the left-hand pair of scales are to be used in the same manner as C and D, and so far the rules relating to the latter are entirely applicable. It is necessary to note, however, that in this case the slide is always moved to the right, and that in multiplication the product is in consequence found either upon the left A scale or on the right A scale. If it is found on the left A scale, the rule for the number of digits in the product is found as for the C and D scales, and is equal to the *sum of the digits in the two factors, minus 1.* But if the product is found on the right-hand A scale, the number of digits in the product is equal to the sum of the digits in the two factors. This, of course, follows from the fact that reading the result on the right scale A in this case corresponds to the slide being moved to the *left* when the C and D scales are used.

In division, similar modifications are necessary. If when moving the slide to the right the division can be completely effected by using the L.H. scale of A, the quotient (read on A above the L.H. index of B) has a number of digits equal to the number in the dividend, less the number in the divisor, plus 1. If, however, the division necessitates the use of both the A scales, the number of digits in the quotient equals the number in the dividend, less the number in the divisor. The rules for decimals are similar.

INVOLUTION AND EVOLUTION.

It has already been explained that the relation between the two scales of A and the lower scale D is such, that opposite any number on D is its *square* on A, and conversely that under any number on A is found its *square root* on D.

It is in the processes of involution and evolution that the upper scales, used in conjunction with the lower set, find their most important application.

To Find the Square of a Number.—It will be evident, from what has been said, that to square a number it is only necessary to refer the number on D, by means of the cursor, to the coinciding number on A, which latter will be the required square. The number of figures in the square of the number may be determined by a modification of the rule for multiplication. Thus, if the square is found on the *left* scale of A, the number of digits is equal to twice the number of digits in the number, *less 1*; but if the result is read on the *right* scale of A, the number of digits is equal to twice the number of digits in the original number.

Ex.—Find the square of 114.

Placing the cursor to 114 on D, it is seen that the coinciding number on A is 13. As the result is read off on the *left* scale of A, the number of digits will be $(3 \times 2) - 1 = 5$, and the answer will therefore be read 13,000. The true result is 12,996.

Ex.—Find the square of 0·0093.

The cursor being placed to 93 on D, the coinciding number on A is found to be 865. As the result is read on the *right* scale of A, the number of digits $= -2 \times 2 = -4$. The answer is therefore read as 0·0000865 [0·00008649].

Square Root.—The foregoing rules suggest the method of procedure in the inverse operation of extracting the square root of a given number, which will be found on the D scale opposite the number on the A scale. It is necessary to observe, however, that if the number consists of an *odd* number of digits, it is to be taken on the *left-hand* portion of the A scale, and the number of digits in the root is $= \frac{N+1}{2}$, N being the number of digits in the original number. When, however, there is an even number of digits in the given number, it is to be taken on the *right-hand* portion of the A scale, and the root contains *one-half* the number of digits in the original number.

Ex.—Find the square root of 36,500.

As there is an *odd* number of digits, the left scale of A is to be used; therefore, placing the cursor to 365 on A, we find the coinciding number on D to be 191. By the rule given there is $\frac{N+1}{2}=\frac{5+1}{2}=3$ digits in the required root, which is therefore read as 191 [191·05].

Ex.—Find $\sqrt{0·0098}$.

Placing the cursor to 98 on the right-hand scale of A (since − 2 is an even number of digits), it is seen that the coinciding number on D is 99. As the number of digits in the number is −2, the number of digits in the root will be $\frac{-2}{2}=-1$. It will therefore be read as 0·099 [0·09899 +].

Ex.—Find $\sqrt{0·098}$.

The number of digits is −1, so on the left scale of A, opposite 98, is found 313 on D. By the rule the number in the root will be $\frac{-1+1}{2}=0$. The root is therefore read as 0·313 [0·313049 +].

Ex.—Find $\sqrt{0·149}$.

As the number of digits (0) is *even*, the cursor is set to 149 on the right-hand scale of A, giving 386 on D. By the rule, the number of digits in the root will be $\frac{0}{2}=0$, and the root will be read as 0·386 [0·38605 +].

Another method of extracting the square root, and one by which more accurate readings may generally be obtained, is by using the C and D scales only with the slide inverted. If there is an *odd* number of digits in the number, the *right* index, or if an *even* number of digits the *left* index, of the inverted scale C′ is placed so as to coincide with the number on D of which the root is sought. Then by the aid of the cursor the number is found on D which coincides with the same number on C′, which number is the root sought.

Ex.—Find $\sqrt{22·2}$.

By the rule the left index of C′ is to be placed to 222 on D, when the two equal and coinciding numbers on C′ and D are found to be 4·71. The rule for the number of digits in the answer will be evident from those just given.

Many prefer to find the square of a number by the ordinary rules for multiplication, and this method is often very convenient. The determination of the square root by an inversion of this method is, however, not so readily effected. Taking the last example, the cursor would require to be set to 22·2 on D, and the slide moved until under the (L.H. in this case) index of C is found the same number on D as is found on C under the cursor, and which is seen in this case to be 4·71. It will be evident that this method of extracting the square root has nothing whatever to recommend its adoption.

To Find the Cube of a Number.—In raising a number to the third power, a combination of the preceding method and ordinary multiplication is employed.

RULE.—*Set the left or right-hand index of C to the given number on D, and opposite the given number* ON THE LEFT-HAND *scale of B read the required cube on the* L.H. *or* R.H. *scale of A.*

It will be noticed that by this rule four scales are brought into requisition. Of these, the D scale and the L.H. B scale are *always* employed, and are to be read as of equal denomination. The values assigned to the L.H. and R.H. scales of A will be apparent from the following considerations.

Commencing with the indices of C and D coinciding, and moving the slide to the right, it will be seen that, working in accordance with the above rule, the cubes of numbers from 1 to 2·154 ($=\sqrt[3]{10}$) will be found on the first or L.H. scale of A. Moving the slide still farther to the right, we obtain *on the* R.H. *A scale* cubes of numbers from 2·154 to 4·641 (or $\sqrt[3]{10}$ to $\sqrt[3]{100}$). If a *third* repetition of the L.H. A scale was available, the L.H. index of C could be still further traversed to the right, and the cubes of numbers from 4·641 to 10 read off on this prolongation of A. The same end can, however, be attained by making use of the R.H. index of C, when, traversing the slide to the right as before, the cubes of numbers from 4·641 to 10 on D can be read off *on the* L.H. *A scale* over the corresponding numbers on the L.H. B scale. From this it will be seen that using the L.H. index of C the readings on the L.H. A scale may be regarded comparatively as units, those on the R.H. A scale as tens; while for the hundreds we again make use of the L.H. A scale in conjunction with the *right-hand* index of C.

By keeping these points in view, the number of digits in the cube (N) of a given number (n) are readily deduced. Thus, if the unit scale is used, $N=3n-2$; if the ten scale, $N=3n-1$; while if the hundreds scale be used, $N=3n$. Placed in the form of rules :—

$N=3n-2$ when the product is read on the L.H. scale of A with the slide to the *right* (units scale).

$N=3n-1$ when the product is read on the R.H. scale of A; slide to the *right* (tens scale).

$N=3n$ when the product is read on the L.H. scale of A with the slide to the *right* (hundreds scale).

With decimals the same rule applies, but, as before, the number of digits must be read as -1, -2, etc., when one, two, etc., cyphers follow immediately after the decimal point.

Ex.—Find the value of $1{\cdot}4^3$.

Placing the L.H. index of C to 1·4 on D, the reading on A opposite 1·4 on the L.H. scale of B is found to be about 2·745 [2·744].

Ex.—Find the value of $26{\cdot}4^3$.

Placing the L.H. index of C to 26·4 on D, the reading on A opposite 26·4 on the L.H. scale of B is found to be about 18,400 [18,399·744].

Ex.—Find the value of $7{\cdot}3^3$.

In this case it becomes necessary to use the R.H. index of C, which is set to 7·3 on D, when opposite 7·3 on the L.H. scale of B is read 389 [389·017] on A.

Ex.—Find the value of $0{\cdot}073^3$.

From the setting as before it is seen that the number of digits in the number must be multiplied by 3. Hence, as there is -1 digit in 0·073, there will be -3 in the cube, which is therefore read 0·000389.

Cube Root (Direct Method).—The method of extracting the cube root of a number will be evident from the foregoing explanation. Employing the same scales, *the slide is* *moved either to the right or left until under the given number on A is found a number on the* L.H. *B scale, identical with the number simultaneously found on D under the right or left index of C.* This number is the required cube root. From what has already been said regarding the combined use of these scales in cubing, it will be evident that in extracting the cube root of a number, it is necessary, in order to decide which scales are to be used, to know the number of figures to be dealt with. It is therefore necessary (as in the arithmetical method of extraction) to point off the given number into sections of three figures each, commencing at the decimal point, and proceeding to the left for numbers greater than unity, and to the right for numbers less than unity. Then if the first section of figures reading from the left consists of—

1 figure, the number will evidently require to be taken on what we have called the "units" scale—*i.e.*, on the L.H. scale of A, using the L.H. index of C.

If of 2 figures, the number will be taken on the "tens" scale—*i.e.*, on the R.H. scale of A, using the L.H. index of C.

If of 3 figures, the number will be taken on the "hundreds" scale—*i.e.*, on the L.H. scale of A, using the R.H. index of C.

To determine the number of digits in cube roots it is only necessary to note that when the number is pointed off into sections as previously directed, there will be one figure in the root for every section into which the number is so divided, whether the *first* section consists of 1, 2, or 3 digits.

Of numbers wholly decimal, the cube roots will be decimal, and for every group of *three* 0's immediately following the decimal point, *one* 0 will follow the decimal point in the root. If necessary, 0's must be added so as to make up even multiples of 3 figures before proceeding to extract the root. Thus 0·8 is to be regarded as 0·800, and 0·00008 as 0·000080 before extracting the roots.

Ex.—Find $\sqrt[3]{14{,}000}$.

Pointing the number off in the manner described, it is seen that there are *two* figures in the first section—viz., 14. Setting the cursor to 14 on the R.H. scale of A, the slide is moved to the right until it is seen that 241 on the L.H. scale of B falls under the cursor, when 241 on D is under the L.H. index of C. Pointing 14,000 off into sections of three figures as directed, we have 14,000—that is, *two* sections. Therefore, there are two digits in the root, which in consequence will be read 24·1 [24·1014 +].

Ex.—Find $\sqrt[3]{0{\cdot}162}$.

As the divisional section consists of *three* figures, we use the "hundreds" scale. Setting the cursor to 0·162 on the L.H. A scale, and using the R.H. index of C, we move the slide to the left until under the cursor 0·545 is found on the L.H. B scale, while the R.H. index of C points to 0·545 on D, which is therefore the required cube root of 0·162.

Ex.—Find $\sqrt[3]{0{\cdot}0002}$.

To make even multiples of 3 figures requires the addition of 00; we have then 200, the cube root of which is found to be about 5·85. Then, since the first divisional group consists of 0's, one 0 will follow the decimal point, giving $\sqrt[3]{0{\cdot}0002} = 0{\cdot}0585$ [0·05848].

Cube Root with Inverted Slide.—When facility in reading the scales has been acquired, the most expeditious and convenient method of extracting the cube root is by inverting the slide. Several methods are used, but the following is preferred by the author. *Set the* L.H. *or* R.H. *index of the slide to the number on A, and the number on B′ (i.e., B inverted), which coincides with the same number on D, is the required root.*

Setting the slide as directed, and using first the L.H. index of the slide and then the R.H. index, it is always possible to find *three* pairs of coincident values. To determine which of the three is the required result is a very simple matter, as an example will show.

Ex.—Find $\sqrt[3]{5}$, $\sqrt[3]{50}$, and $\sqrt[3]{500}$.

Setting the R.H. index of the slide to 5 on A, it is seen that 1·71 on D coincides with 1·71 on B′. Then setting the L.H. index to 5 on A, further coincidences are found at 3·68 and at 7·93, the three values thus found being the required roots. It will be noted that the first root was found on that portion of the D scale lying under 1 to 5 on A; the second root on that portion lying under 5 to 50 on A; and the third root on that portion of D lying under 50 to 100 on A. In this connection, therefore, scale A may always be considered to be divided into three sections—viz., 1 to n, n to 10 n, and 10 n to 100. For all numbers consisting of 1, 1 + 3, 1 + 6, 1 + 9—*i.e.*, of 1, 4, 7, 10, or − 2, − 5, etc., figures—the coincidence under the first section is the one required. If the number has 2, 5, 8, or − 1, − 4, − 7, etc., figures, the coincidence under the second section is correct, while if the number has 3, 6, 9, or 0, − 3, etc., figures, the coincidence under the last section is that required. A little practice will soon reveal the simplicity and convenience of this method of working. The number of digits in the root is determined by marking off the number into sections, as already explained.

Fourth Powers and Roots.—By the use of the A and C scales in combination with the ordinary processes of multiplication or division, the fourth power or root of any number may be readily determined. To raise the number to the fourth power we have the following

RULE.—*Set the* R.H. *or* L.H. *index of C to the given number on D, and opposite the number on C read the required answer on A.*

Ex.—Find the value of 17^4.

Placing the L.H. index of C to 17 on D, the result, which is read on A opposite 17 on C, is found to be 83,500 [83,521].

The number of figures in the product may be determined by the rules for squaring a number.

The extraction of the fourth root may be effected by the reverse of the above process, but it will be found preferable to extract the square root of the square root of the given number, paying due regard to the number of figures in the intermediate result.

Involution and Evolution with any Power or Root.—When the power or root is other than those already dealt with, it will be necessary to adopt the usual logarithmic method. Thus to find $a^n = x$, we multiply the logarithm of a by n, and find the number x corresponding to the logarithm so obtained. Similarly, to find $\sqrt[n]{a} = x$ we divide the logarithm of a by n, and find the number x corresponding to the logarithm so obtained.

Upon the back of the slide of the Gravêt and similar slide rules there will be found three scales. One of these —usually the centre one of the three—is divided equally throughout its entire length, and figured from right to left. The whole scale is divided primarily into ten equal parts, and each of these is subdivided into 50 equal parts. In the newer forms of rules an aperture is formed in the right-hand end of the rule, and on the bevelled edge of this aperture is a fixed index or reference mark, to which any of the divisions of the evenly-divided scale can be set.

As this equally and decimally-divided scale is precisely equal in total length to the logarithmic scale D, and is figured in the reverse direction, it will be seen that when the slide is drawn to the right so that the L.H. index of C coincides with any number on D, the reading on the equally-divided scale will give the decimal of the logarithm of the number selected on D. Thus if the L.H. index of C is placed to agree with 2 on D, the reading of the equally-divided scale, taken at the reference mark, will be found to be 0·301, the logarithm of 2. It must be distinctly borne in mind that the number so obtained is the *decimal part* or *mantissa* of the logarithm of the number, and that to this the characteristic must be prefixed in accordance with the usual rule—viz., *The integral part, index, or characteristic of a logarithm is equal to the number of digits in the number, minus 1. If the number is wholly decimal, the index is equal to the number of cyphers following the decimal point, plus 1.* In the latter case the index is negative, and is so indicated by having the minus sign written over it.

In obtaining any given power or root of a number, the operation is as follows:—Set the L.H. index of C to the given number on D, and turning the rule over, read opposite the mark in the aperture* at the right-hand end of the rule, the decimal part of the logarithm of the number. Add the index according to the above rule, and multiply by the exponent of the given power, or divide by the exponent of the given root. Place the *decimal part* of the resultant, taken in the scale of equal parts, opposite the mark in the aperture of the rule, and read off the answer on D under

* Or opposite the extreme right-hand end of the rule when no aperture is provided.

the L.H. index of C, pointing off the number of digits in the answer in accordance with the number of the characteristic of the resultant.

Ex.—Find 70^5.

Placing the L.H. index of C to 70 on D we find (on the underside of the slide) the decimal part of the logarithm to be 0·845. As there are *two* digits in the number, the index or characteristic to be prefixed will be $2-1=1$. Placing, therefore, the index of C to 1·845 on D, we find the product of $1{\cdot}845 \times 5 = 9{\cdot}225$, which is therefore the logarithm of the answer. Bringing the *decimal* part or 225 on the scale of equal parts to the mark in the aperture, we find 168 on D under the L.H. index of C. The multiplied characteristic being 9, it follows that there must be $9+1$ or 10 digits in the answer, which latter is therefore read 1,680,000,000 [1,680,700,000]. This example will be sufficient to exemplify the method of procedure in determining the *n*th power or root of a number, it being, of course, understood that *n* may be either whole or fractional.

COMBINED OPERATIONS.

Thus far the various operations have been separately considered, and it now becomes necessary to investigate the methods of working to most simply effect the solution of the various formulæ met with in technical calculations. It is proposed to explain the methods of dealing with a few of the more generally used expressions, and this, it is thought, will suffice to suggest the mode of procedure in dealing with other and more intricate problems. In solving the following problems, both the upper and lower scales are brought into requisition, and it is therefore important that the relative value of the several scales be carefully kept in view. Thus, in solving such an expression as $\sqrt{\frac{745}{15{\cdot}8}}=6{\cdot}86$, the division is first effected by setting 15·8 on B to 745 on A. From what has been said regarding the relat on of the two parts of the upper scales, it will be clear that such values as 7·45, 745, etc., will be taken on the *left-hand* A and B scales, while values as 15·8, 1580, etc., will be taken on the *right-hand* A and B scales. In the example, therefore, 15·8 on the R.H. B scale is set to 745 on the L.H. A scale, and the result read on D under the index of C. Had both values been taken on the L.H. A and B scales, or both on the R.H. A and B scales, it will be seen that the results would have corresponded to $x=\sqrt{\frac{7\cdot45}{1\cdot58}}=2\cdot17$, or to $x=\sqrt{\frac{74\cdot5}{15\cdot8}}=2\cdot17$.

To solve $a \times b^2 = x$.

Place the index of C to b on D, and over a on B read x on A.

To solve $\frac{a^2}{b}=x$.

Set b on B to a on D by the cursor, and over index of B read x on A.

To solve $\frac{b}{a^2}=x$.

Set a on C to b on A, and over 1 on B read x on A.

To solve $\frac{a \times b^2}{c} = x$.

Set c on B to b on D, and over a on B read x on A.

To solve $(a \times b)^2 = x$.

Set 1 on C to a on D, and over b on C read x on A.

To solve $\left(\frac{a}{b}\right)^2 = x$.

Set b on C to a on D, and over 1 on C read x on A.

To solve $\sqrt{a \times b} = x$.

Set 1 on B to a on A, and under b on B read x on D.

To solve $\sqrt{\frac{a}{b}} = x$.

Set b on B to a on A, and under 1 on C read x on D.

To solve $c\sqrt{\frac{a}{b}} = x$.

Set b on B to a on A, and under c on C read x on D.

To solve $\frac{\sqrt{a}}{b} = x$.

Set b on C to a on A, and under 1 on C read x on D.

To solve $\frac{a}{\sqrt{b}}=x$.

Set b on B to a on D, and under 1 on C read x on D.

To solve $b\sqrt{a}=x$.

Set 1 on C to b on D, and under a on B read x on D.

To solve $\frac{a^3}{b}=x$.

Set b on B to a on A, and over a on C read x on A.

To solve $\sqrt{a^3}=x$.

Set 1 on C to a on D, and under a on B read x on D.

To solve $\frac{\sqrt{a^3}}{b}=x$.

Set b on C to a on D, and under a on B read x on D.

To solve $\sqrt{\frac{a^3}{b}}=x$.

Set b on B to a on A, and under a on C read x on D.

To solve $\sqrt{\frac{(a\times b)^2}{c}}=x$.

Set c on B to b on D, and under a on C read x on D.

To solve $\sqrt{\frac{a^2\times b}{c}}=x$.

Set c on B to a on D, and under b on B read x on D.

To solve $\frac{a^2\times b^2}{c}=x$.

Set c on B to a on D, and over b on C read x on A.

The following formulæ are more conveniently solved with the slide inverted :—

To solve $\frac{a\sqrt{b}}{c}=x$.

Set a on C′ to b on A, and under c on C′ read x on D.

To solve $\frac{\sqrt{a\times b}}{c}=x$.

Set a on B′ to b on A, and under c on C′ read x on D.

To solve $\sqrt{\frac{a^2 \times b^2}{c}} = x$.

Set b on C′ to a on D, and under c on B′ read x on D.

To solve $\left(\frac{a \times \sqrt{b}}{c}\right)^2 = x$.

Set a on C′ to b on A, and over c on C′ read x on A.

All of the foregoing solutions require one setting of the rule only. More complicated workings will be exemplified in the following section, in which the methods of dealing with various technical calculations are indicated.

EXAMPLES IN TECHNICAL COMPUTATION.

In order to illustrate the practical value of the slide rule, we now give a few examples which will doubtless be sufficient to suggest the methods of working other formulæ. A few of the rules give results which are approximate only, but in all cases the degree of accuracy obtained is well within the possible reading of the scales. In many cases the rules given may be modified, as may be desired, by varying the constants. In most of the examples the particular formula employed will be evident from the solution, but in a few of the more complicated cases a separate statement has been given.

I. Mensuration, Etc.

Given the chord c of a circular arc, and the vertical height h, to find the diameter d of the circle.

Set the height h on B to half the chord on D, and over 1 on B read x on A. Then $x + h = d$.

Ex.—$c = 6$; $h = 2$; find d.

Set 2 on B to 3 on D, and over 1 on B read 4·5 on A. Then $4{\cdot}5 + 2 = 6{\cdot}5 = d$.

Given the radius of a circle r, and the number of degrees n in an arc, to find the length l of the arc.

Set r on C to 57·3 on D, and over any number of degrees n on D read the (approximate) length of the arc on C.

Ex.—r = 24 ; n = 30 ; find l.
Set 24 on C to 57·3 on D, and over 30 on D read 12·56 = l on C.

Given the diameter d of a circle in *inches*, to find the circumference c in *feet*.
Set 191 on C to 50 on D, and under any diameter in inches on C read circumference c, in feet on D.

Ex.—Find the circumference in feet of a pulley 17in. in diameter.
Set 191 on C to 50 on D, and under 17 on C read 4·45ft. on D.

Given the diameter of a circle, to find its area.
Set 164 on C to 185 on D, and over any diameter on D read area on B.

When the rule has a special graduation line = 0·7854, on the right-hand scale of B, set this line to the R.H. index of A and read off as above.

On the C scale of Faber's calculating rule, a gauge point marked c will be found indicating $\sqrt{\frac{4}{\pi}} = 1{\cdot}128$. In this case, therefore, set gauge point c to the diameter on D, and over index of B read area on A. In one form of glass cursor supplied with Dennert and Pape's calculating rule, *two* lines are ruled on the glass, the interval between them being equal to $\frac{4}{\pi} = 1{\cdot}273$ on the A scale. In this case, if the right hand of the two cursor lines be set to the diameter on D, the *area* will be read on A under the *left*-hand cursor line. For diameters less than 1·11 it is necessary to set the middle index of B to the L.H. index of A, reading the areas on the L.H. B scale. The confusion which in general work is sometimes caused by the use of two cursor lines might be obviated by making the left-hand line in two short lengths, each only just covering the scales.

Given diameter of circle d in *inches*, to find area a in square *feet*.
Set 6 on B to 11 on A, and over diameter in inches on D read area in square feet on B.

To find the surface in square feet of boiler flues, condenser tubes, heating pipes, etc., having given the diameter in inches and length in feet.

Find the circumference in feet as above and multiply by the length in feet.

Ex.—Find the heating surface afforded by 160 locomotive boiler tubes $1\frac{3}{4}$in. in diameter and 12ft. long.

Set 191 on C to 50 on D; bring cursor to 1·75 on C, L.H. index of C to cursor; cursor to 12 on C; 1 on C to cursor; and under 160 on C read 880 sq. ft. of heating surface on D.

To find the side s of a square, equal in area to a given rectangle of length l and breadth b.

Set R.H. or L.H. index of B to l on A, and under b on B read s on D.

Ex.—Find the side of a square equal in area to a rectangle in which l = 31ft. and b = 5ft.

Set the (R.H.) index of B to 31 on A, and under 5 on B read 12·45ft. on D.

To find various lengths l and breadths b of a rectangle, to give a constant area a.

Invert the slide and set the index of C′ to the given area on D. Then opposite any length l on C′ find the corresponding breadth b on D.

Ex.—Find the corresponding breadths of rectangular sheets 16, 18, 24, 36, and 60ft. long, to give a constant area of 72 sq. ft.

Set the R.H. index of C′ to 72 on D, and opposite 16, 18, 24, 36, and 60 on C′ read 4·5, 4, 3, 2, and 1·2ft., the corresponding breadths on D.

To find the contents in cubic feet of a cylinder of diameter d in inches and length l in feet.

Find area in feet as before, and multiply by the length.

To find the area of an ellipse.

Set 205 on C to 161 on D; bring cursor to length of major axis on C, 1 on C to cursor, and under length of minor axis on C read area on D.

Ex.—Find the area of an ellipse the major and minor axes of which are 16in. and 12in. in length respectively.

Set 205 on C to 161 on D; bring cursor to 16 on C, 1 on C to cursor, and under 12 on C read 150·8in. on D.

To find the surface of spheres.

Set 3·1416 on B to R.H. or L.H. index of A, and over diameter on D read by the aid of the cursor, the convex surface on B.

To find the cubic contents of spheres.

Set 1·91 on B to diameter on A, and over diameter on C read cubic contents on A.

II. Weights of Metals.

To find the weight in lb. per lineal foot of square bars of metal.

Set index of B to weight of 12 cubic inches of the metal (*i.e.*, one lineal foot, 1 square inch in section) on A, and over the side of the square in inches on C read weight in lb. on A.

Ex.—Find the weight per foot length of 4½in. square wrought-iron bars.

Set middle index of B to 3·33 on A, and over 4½ on C read 67·5lb. on A.

(N.B.—For other metals use the corresponding constant in column (2), page 52.)

To find the weight in lb. per lineal foot of round bars.

Set R.H. or L.H. index of B to weight of 12 cylindrical inches of the metal on A (column (4), page 52), and opposite the diameter of the bar in inches on C, read weight in lb. per lineal foot on A.

Ex.—Find the weight of 1 lineal foot of 2in. round cast steel.

Set L.H. index of B to 2·68 on A, and over 2 on C read 10·7lb on A.

To find the weight of flat bars in lb. per lineal foot.

Set the breadth in inches on C to $\frac{1}{\text{weight of 12 cub. in.}}$ of the metal (column (3), page 52) on D, and above the thickness on D read weight in lb. per lineal foot on C.

Ex.—Find the weight per lineal foot of bar steel, 4½in. wide and ⅝in. thick.

Set 4·5 on C to 0·294 on D, and over 0·625 on D read 9·56lb. per lineal foot on C.

To find the weight per square foot of sheet metal, set the weight per cubic foot of the metal (col. 1) on C to 12 on D,

Metals.	(1) Weight in lb. per cubic ft.	(2) Weight of 12 cubic in.	(3) $\frac{1}{\text{Wt. of 12 cub. in.}}$	(4) Weight of 12 cylindrical in.
Wrought iron...	480	3·33	0·300	2·62
Cast iron	450	3·125	0·320	2·45
Cast steel	490	3·40	0·294	2·68
Copper	550	3·82	0·262	3·00
Aluminium ...	168	1·166	0·085	0·915
Brass	520	3·61	0·277	2·83
Lead	710	4·93	0·203	3·87
Tin	462	3·21	0·312	2·52
Zinc (cast)	430	2·98	0·335	2·34
„ (sheet) ...	450	3·125	0·320	2·45

and above the thickness of the plate in inches on D read weight in lb. per square foot on C.

Ex.—Find the weight in lb. per square foot of aluminium sheet $\frac{3}{8}$in. thick.

Set 168 on C to 12 on D, and over 0·375 on D read 5·25lb. on C.

To find the weight of pipes in lb. per lineal foot.

Set mean diameter of the pipe in inches (*i.e.*, internal diameter *plus* the thickness, or external diameter *minus* the thickness) on C to the constant given below on D, and over the thickness on D read weight in lb. per lineal foot on C.

Metals.	Constant for Pipes.	Constant for Spheres.
Wrought iron..............	0·0955	6·87
Cast iron....................	0·1020	7·35
Steel	0·0936	6·73
Brass	0·0882	6·35
Copper	0·0834	6·00
Lead	0·0646	4·65

Ex.—Find the weight per foot of cast-iron piping 4in. internal diameter and $\frac{1}{2}$in. thick.

Set 4·5 on C to 0·102 on D, and over 0·5 on D read 22·1lb. on C, the required weight.

To find the weight in lb. of spheres or balls of given diameter.

Set the constant for spheres (given on page 52) on B to diameter on A, and over diameter on C read weight in lb. on A.

Ex.—Find the weight of a cast-iron ball 7½in. in diameter.
Set 7·35 on B to 7·5 on A, and over 7·5 on C read 57·7lb. on A.

To find diameter in inches of a sphere of given weight.

Set the cursor to the given weight in lb. on A, and move the slide until the same number is found on C under the cursor that is simultaneously found on A over the constant for the sphere on B.

Ex.—Find diameter in inches of a sphere of cast iron to weigh 7½lb.
Set the cursor to 7·5 on A, and moving the slide, it is found that when 3·8 on C falls under the cursor, 3·8 on A is simultaneously found over 7·35 on B. The required diameter is therefore 3·8in.

The rules for cubes and cube roots (page 39) should be kept in view in solving the last two examples.

III. Falling Bodies.

To find velocity in feet per second of a falling body, given the time of fall in seconds.

Set index on C to time of fall on D, and under 32·2 on C read velocity in feet per second on D.

To find velocity in feet per second, given space fallen through in feet.

Set 1 on C to space fallen through on A, and under 64·4 on B read velocity in feet per second on D.

Ex.—Find velocity acquired by falling through 14ft.
Set (R.H.) index of C to 14 on A, and under 64·4 on B read 30ft. per second on D.

To find space fallen through in feet in a given time.

Set index of C to time in seconds on D, and over 16·1 on B read space fallen through in feet on A.

IV. Centrifugal Force.

To find the centrifugal force of a revolving mass in lb.

Set 2940 on B to revolutions per minute on D; bring cursor to weight in lb. on B; index of B to cursor, and over radius in feet on B read centrifugal force in lb. on A.

To find the centrifugal stress in lb. per square inch, in rims of revolving wheels of cast iron.

Set 61·3 on C to the mean diameter of the wheel in feet on D, and over revolutions per minute on C read stress per square inch on A.

Ex.—Find the stress per square inch in a cast-iron fly-wheel rim 8ft. in diameter and running at 120 revolutions per minute.

Set 61·3 on C to 8 on D, and over 120 on C read 245lb. per square inch on A.

V. The Steam Engine.

Given the stroke and number of revolutions per minute, to find the piston speed.

Set stroke in inches on C to 6 on D, and over number of revolutions on D read piston speed in feet per minute on C.

To find cubic feet of steam in a cylinder at cut-off, given diameter of cylinder and period of admission in inches.

Set 2200 on B to cylinder diameter on D, and over period of admission on B read cubic feet of steam on A.

Ex.—Cylinder diameter 26in., stroke 40in., cut-off at $\frac{5}{8}$ of stroke. Find cubic feet of steam (theoretically) used per stroke.

Set 2200 on B to 26 on D, and over 40 $\times$ $\frac{5}{8}$ or 25in. on B, read 7·68 cub. ft. on A, as the number of cubic feet of steam used per stroke.

Given the diameter of a cylinder in inches, and the pressure in lb. per square inch, to find the load on the piston in tons.

Set pressure in lb. per square inch on B to 2852 on A, and over cylinder diameter in inches on D read load on piston in tons on B.

Ex.—Steam pressure 180lb. per square inch; cylinder diameter, 42in. Find load in tons on piston.

Set 180 on B to 2852 on A, and over 42 on D read 111 tons, the gross load, on B.

Given admission period and absolute initial pressure of steam in a cylinder, to find the pressure at various points in the expansion period (isothermal expansion).

Invert the slide and set the admission period, in inches, on C′ to the initial pressure on D; then under any point in the expansion stroke on C′ find the corresponding pressure on D.

Ex.—Admission period 12in., stroke 42in., initial pressure 80lb. per square inch. Find pressure at successive fifths of the expansion period.

Set 12 on C′ to 80 on D, and opposite 18, 24, 30, 36, and 42in. of the whole stroke on C′ find the corresponding pressures on D:—53·3, 40, 32, 26·6, and 22·8lb. per square inch.

To find the mean pressure constant for isothermally expanding steam, given the cut-off as a fraction of the stroke.

Find the logarithm of the ratio of the expansion r, by the method previously explained (page 44). Add the characteristic, and to the number thus obtained on D, set 1 on C. Then under 2·302 on C read x on D. To $x + 1$ on D set r on C, and under index on C read mean pressure constant on D. The latter, multiplied by the initial pressure, gives the mean forward pressure throughout the stroke.

Ex.—Find the mean pressure constant for a cut-off of $\frac{1}{4}$th, or a ratio of expansion of 4.

Set (L.H.) index of C to 4 on D, and on the reverse side of the slide read 0·602 on the logarithmic scale. To 0·602 on D set (R.H.) index of C, and under 2·302 on C read 1·384 on D. Add 1, and to 2·384 thus obtained on D set r (= 4) on C, and under 1 on C read 0·596, the mean pressure constant required.

Mean pressure constants for the most usual degrees of cut-off are given below:—

Cut-off in fractions of stroke...	$\frac{3}{4}$	$\frac{7}{10}$	$\frac{2}{3}$	$\frac{5}{8}$	$\frac{3}{5}$	$\frac{1}{2}$
Mean pressure constant.........	0·968	0·952	0·934	0·919	0·913	0·846
Cut-off in fractions of stroke...	$\frac{2}{5}$	$\frac{3}{8}$	$\frac{1}{3}$	$\frac{3}{10}$	$\frac{1}{4}$	$\frac{1}{5}$
Mean pressure constant.........	0·766	0·750	0·699	0·664	0·596	0·522
Cut-off in fractions of stroke...	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{11}$
Mean pressure constant.........	0·465	0·421	0·385	0·355	0·330	0·309
Cut-off in fractions of stroke...	$\frac{1}{12}$	$\frac{1}{13}$	$\frac{1}{14}$	$\frac{1}{15}$	$\frac{1}{16}$	
Mean pressure constant.........	0·290	0·274	0·260	0·247	0·236	

To find mean pressure:—Set 1 on C to constant on D, and under initial pressure on C read mean pressure on D.

Given the absolute initial pressure, length of stroke, and admission period, to find the absolute pressure at any point in the expansion period, it being assumed that the steam expands adiabatically. ($P_2 = \frac{P_1}{R^{\frac{10}{9}}}$, in which P_1 = initial pressure and P_2 the pressure corresponding to a ratio of expansion R.)

Set L.H. index of C to ratio of expansion on D, and read on the back of the slide the decimal of the logarithm. Add the characteristic, and to the number thus obtained on D set 9 on C, and read off the value found on D under the index of C. Set this number on the logarithmic scale to the index mark, in the opening on the back of the rule, and under L.H. index of C read the value of $R^{\frac{10}{9}}$ on D. The initial pressure divided by this value gives the corresponding pressure due to the expansion.

Ex.—Absolute initial pressure 120lb. per square inch; stroke, 4ft.; cut-off $\frac{1}{4}$. Find the respective pressures when $\frac{1}{2}$ and $\frac{3}{4}$ths of the stroke have been completed.

In the first case $R=2$. Therefore, setting the L.H. index of C to 2 on D, we find the decimal of the logarithm on the back of the slide to be 0·301. The characteristic is 0, so placing 9 on C to 0·301 on D, we read 0·334 as the value under the R.H. index of C. (N.B.—In locating the decimal point it is to be observed that the log. of R has been multiplied by 10, in accordance with the terms of the above expression.) Setting this number on the logarithmic scale to the back index, the value of $R^{\frac{10}{9}}$ is found on D, under the L.H. index of C, to be 2·16. Setting 120 on C to this value, it is found that the pressure at $\frac{1}{2}$ stroke, read on C over the R.H. index of D, is 55·5lb. per square inch. In a similar manner, the pressure when $\frac{3}{4}$ths of the stroke is completed is found to be 35·4lb. per square inch.

For other conditions of expanding steam, or for gas or air, the method of procedure is similar to the above.

To find the horse-power of an engine, having given the mean *effective* pressure, the cylinder diameter, stroke, and number of revolutions per minute.

To cylinder diameter on D set 145 on C; bring cursor to stroke in feet on B, 1 on B to cursor, cursor to number of revolutions on B, 1 on B to cursor, and over mean effective pressure on B find horse-power on A.

(N.B.—If stroke is in inches, use 502 in place of 145 given above.)

Ex.—Find the indicated horse-power, given cylinder diameter 27in., mean effective pressure 38lb. per square inch, stroke 32in., revolutions 57 per minute.

Set 502 on C to 27 on D, bring cursor to 32 on B, 1 on B to cursor, cursor to 57 on B, 1 on B to cursor, and over 38 on B read 200 I.H.P. on A.

In determining the horse-power of compound, triple, or quadruple-expansion engines, invert the slide and set the diameter of the *high*-pressure cylinder on C′ to the cut-off in that cylinder on A. Use the number then found on A over the diameter of the *low*-pressure cylinder on C′ as the cut-off in that cylinder, working with the same pressure and piston speed.

To find the cylinder ratio in compound engines, invert the slide and set index of C′ to diameter of the low-pressure cylinder on D. Then over the diameter of the high-pressure cylinder on C′, read cylinder ratio on A.

Ex.—Diameter of high-pressure cylinder 7¾in., low-pressure 15in. Find cylinder ratio.

Set index on C′ to 15 on D, and over 7·75 on C′ read 3·75, the required ratio, on A.

The cylinder ratios of triple or quadruple-expansion engines may be similarly determined.

Ex.—In a quadruple-expansion engine, the cylinders are 18, 26, 37, and 54 inches in diameter. Find the respective ratios of the high, first intermediate, and second intermediate cylinders to the low-pressure.

Set (R.H.) index of C′ to 54 on D, and over 18, 26, and 37 on C′, read 9, 4·31, and 2·13, the required ratios, on A.

Given the mean effective pressures in lb. per square inch in each of the three cylinders of a triple-expansion engine, the I.H.P. to be developed in each cylinder, and the piston speed, to find the respective cylinder diameters.

Set 42,000 on B to piston speed on A; bring cursor to mean effective pressure in low-pressure cylinder on B, index of B to cursor, and under I.H.P. on A read low-pressure cylinder diameter on C. To find the diameters of the high-pressure and intermediate-pressure cylinders, invert the slide and place the mean pressure in the low-pressure cylinder on B′ to the diameter of that cylinder on D. Then under the respective mean pressures on B′ read corresponding cylinder diameters on D.

Ex.—The mean effective pressures in the cylinders of a triple-expansion engine are:—L.P., 10·32; I.M.P., 27·5; and L.P., 77·5lb. per square inch. The piston speed is 650ft. per minute, and the I.H.P. developed in each cylinder, 750. Find the cylinder diameters.

Set 42,000 on B to 650 on A, and bring cursor to 10·32 on B. Bring index of B to cursor, and under 750 on A read 68·5in. on C, the L.P. cylinder diameter. Invert the slide, and placing 10·32 on B′ to 68·5 on D, read, under 27·5 on B′, the I.M.P. cylinder diameter = 42in. on D; also under 77·5 on B′ read the H.P. cylinder diameter = 25in. on D.

To compute brake or dynamometrical horse-power.

Set 5252 on C to the total weight in lb. acting at the end of the lever (or pull of spring balance in lb.) on D; set cursor to length of lever in feet on C, bring 1 on C to cursor, and under number of revolutions per minute on C find brake horse-power on D.

Given cylinder diameter and piston speed in feet per minute, to find diameter of steam pipe, assuming the maximum velocity of the steam to be 6000ft. per minute.

Set 6000 on B to cylinder diameter on D, and under piston speed on B read steam pipe diameter on D.

Given the number of revolutions per minute of a Watt governor, to find the vertical height in inches, from the plane of revolution of the balls to the point of suspension.

Set revolutions per minute on C to 35,200 on A, and over index of B read height on A.

Given the weight in lb. of the rim of a cast-iron fly-wheel, to find the sectional area of the rim in square inches.

Set the diameter of the wheel in feet on C to 1280 on D, and under weight of rim on C find area on D.

Given the consumption of coal in tons per week of 56 hours, and the I.H.P., to find the coal consumed per I.H.P. per hour.

Set I.H.P. on C to 40 on D, and under weekly consumption on C read lb. of coal per I.H.P. per hour on D.

Ex.—Find coal used per I.H.P. per hour, when 24 tons is the weekly consumption for 300 I.H.P.

Set 300 on C to 40 on D, and under 24 on C read 3·2lb. per I.H.P. per hour on D.

(N.B.—For any other number of working hours per week divide 2240 by the number of working hours, and use the quotient in place of 40 as above.)

To find the tractive force of a locomotive.

Set diameter of driving wheel in inches on B to diameter of cylinder in inches on D, and over the stroke in inches on B read on A, tractive force in lb. for each lb. of effective pressure on the piston.

VI. Steam Boilers.

To find the bursting pressure of a cylindrical boiler shell, having given the diameter of shell and the thickness and strength of the material.

Set the diameter of the shell in inches on C to twice the thickness of the plate on D, and under strength of material per square inch on C read bursting pressure in lb. per square inch on D.

Ex.—Find the bursting pressure of a cylindrical boiler shell 7ft. 6in. in diameter, with plates $\frac{1}{2}$in. thick, and assuming an ultimate strength of 50,000lb. per square inch.

Set 90 on C to 1·0 on D, and under 50,000 on C find 555lb. on D.

To find working pressure for Fox's corrugated furnaces by Board of Trade rule.

Set the least outside diameter in inches on C to 14,000 on D, and under thickness in inches on C read working pressure on D in lb. per square inch.

To find diameter d in inches, of round steel for safety valve springs by Board of Trade rule.

Set 8000 on C to load on spring in lb. on D, and under the mean diameter of the spring in inches on C read d^3 on D. Then extract cube root as per rule.

VII. Speed Ratios of Pulleys, Etc.

Given the diameter of a pulley and number of revolutions per minute, to find the circumferential velocity of the pulley or speed of ropes, belts, etc., driven thereby.

Set diameter of pulley in inches on C to 3·82 on D, and over revolutions per minute on D read speed in feet per minute on C.

Ex.—Find the speed of a belt driven by a pulley 53in. in diameter and running at 180 revolutions per minute.

Set 53 on C to 3·82 on D, and over 180 on D read 2500ft. per minute on C.

Ex.—Find the speed of pitch line of a spur wheel 3ft. 6in. in diameter running at 60 revolutions per minute.

Set 42in. on C to 3·82 on D, and over 60 on D read 660ft. per minute on C.

Given diameter and number of revolutions per minute of driving pulley, and the diameter of driven pulley, to find number of revolutions of the latter.

Invert the slide and set diameter of driving pulley on C′ to given number of revolutions on D; then opposite diameter of driven pulley on C′ read number of revolutions of driven pulley on D.

Ex.—Diameter of driving pulley 10ft.; revolutions per minute 55; diameter of driven pulley 2ft. 9in. Find number of revolutions per minute of latter.

Set 10 on C′ to 55 on D, and opposite 2·75 on C′ read 200 revolutions on D.

VIII. Belts and Ropes.

To find the ratio of tensions in the two sides of a belt, given the coefficient of friction between belt and pulley μ, and the number of degrees θ in the arc of contact $\left(\log. R = \frac{\mu\,\theta}{132}\right)$.

Set 132 on C to the coefficient of friction on D, and read off the value found on D under the number of degrees in the arc of contact on C. Place this value on the scale of equal parts on the back of the slide, to the index mark in the aperture, and read the required ratio on D under the L.H. index of C.

Ex.—Find the tension ratio in a belt, assuming a coefficient of friction of 0·3 and an arc of contact of 120 degrees.

Set 132 on C to 0·3 on D, and under 120 on C read 0·273. Place this on the scale to the index on the back of the rule, and under the L.H. index C read 1·875 on D, the required ratio.

Given belt velocity and horse-power to be transmitted, to find the requisite width of belt, taking the effective tension at 50lb. per inch of width.

Set 660 on C to velocity in feet per minute on D, and opposite horse-power on D find width of belt in inches on C.

Given velocity and width of belt, to find horse-power transmitted.

Set 660 on C to velocity on D, and under width on C find horse-power transmitted on D.

(N.B.—For any other effective tension, instead of 660 use as a gauge point:—33,000 ÷ tension.)

Given speed and diameter of a cotton driving rope, to find power transmitted, disregarding centrifugal action, and assuming an effective working tension of 200lb. per square inch of rope.

Set 210 on B to 1·75 on D, and over speed in feet per minute on B read horse-power on A.

Ex.—Find the power transmitted by a $1\frac{3}{4}$in. rope running at 4000ft. per minute.

Set 210 on B to 1·75 on D, and over 4000 on B read 58·3 horse-power on A.

Find the "centrifugal tension" in the previous example, taking the weight per foot of the rope as $= 0{\cdot}33\ d^2$.

Set 350,000 on B to the diameter, 1·75in., on D, and over the speed, 4000ft. on C, read centrifugal tension = 140lb. on A.

IX. Spur Wheels.

Given diameter and pitch of a spur wheel, to find number of teeth.

Set pitch on C to π (3·1416) on D, and under any diameter on C read number of teeth on D.

Given diameter and number of teeth in a spur wheel, to find the pitch.

Set diameter on C to number of teeth on D, and read pitch on C opposite 3·1416 on D.

Given the distance between the centres of a pair of spur wheels and the number of revolutions of each, to determine their diameters.

To twice the distance between the centres on D, set the sum of the number of revolutions on C, and under the revolutions of each wheel on C find the respective wheel diameters on D.

Ex.—The distance between the centres of two spur wheels is 37·5in., and they are required to make 21 and 24 revolutions in the same time. Find their respective diameters.

Set $21+24=45$ on C to 75 (or $37{\cdot}5 \times 2$) on D, and under 21 and 24 on C find 35 and 40in. on D as the respective diameters.

To find the power transmitted by toothed wheels, given the pitch diameter d in inches, the number of revolutions per minute n, and the pitch p in inches, by the rule, H.P. $=\frac{n\,d\,p^2}{400}$.

Set 400 on B to pitch in inches on D ; set cursor to d on B, 1 on B to cursor, and over any number of revolutions n on B read power transmitted on A.

Ex.—Find the horse-power capable of being transmitted by a spur wheel 7ft. in diameter, 3in. pitch, and running at 90 revolutions per minute.

Set 400 on B to 3 on D ; bring cursor to 84in. on B, 1 of B to cursor, and over 90 revolutions on B read 170, the horse-power transmitted, on A.

X. Screw Cutting.

Given the number of threads per inch in the guide screw, to find the wheels to cut a screw of given pitch.

Set threads per inch in guide screw on C, to the number of threads per inch to be cut on D. Then opposite any number of teeth in the wheel on the mandrel on C, is the number of teeth in the wheel to be placed on the guide screw on D.

XI. Strength of Shafting.

Given the diameter d of a steel shaft, and the number of revolutions per minute n, to find the horse-power from :— $\text{H.P.} = d^3 \times n \times 0{\cdot}02$.

Set 1 on C to d on D, and bring cursor to d on B. Bring 50 on B to cursor, and over number of revolutions on B read H.P. on A.

Ex.—Find horse-power transmitted by a 3in. steel shaft at 110 revolutions per minute.

Set 1 on C to 3 on D, and bring cursor to 3 on B. Bring 50 on B to cursor, and over 110 on B read 59·4 horse-power on A.

Given the horse-power to be transmitted and the number of revolutions of a steel shaft, to find the diameter.

Set revolutions on B to horse-power on A, and bring cursor to 50 on B. Then move the slide until the same number is found on B under the cursor that is simultaneously found on D under the index of C. This number is the diameter required.

To find the deflection k in inches, of a round steel shaft of diameter d, under a uniformly distributed load in lb. w, and supported by bearings, the centres of which are l feet apart $\left(k \doteq \frac{w\,l^3}{78{,}800\,d^4}\right)$.

Modifying the form of this expression slightly, we proceed as follows:—Set *d* on C to *l* on D, and bring the cursor to the same number on B that is found on D under the index of C. Bring D on B to cursor, cursor to *w* on B, 78,800 on B to cursor, and read deflection on A over index of B.

Ex.—Find the deflection in inches of a round steel shaft 3½in. diameter, carrying a uniformly distributed load of 3200lb., the distance apart of the centres of support being 9ft.

Set 3·5 on C to 9 on D, and read off 2·57 on D, under the L.H. index of C. Set cursor to 2·57 on B, and bring 3·5 on B to cursor, cursor to 3200 on B, 78,800 on B to cursor, and over L.H. index of B read 0·197in., the required deflection on A.

To find the diameter of a shaft subject to twisting only, given the twisting moment in inch-lb. and the allowable stress in lb. per square inch.

Set the stress in lb. per square inch on B to the twisting moment in inch-lb. on A, and bring cursor to 5·1 on B. Then move the slide until the same number is found on B under the cursor that is simultaneously found on D under the index of C.

Ex.—A steel shaft is subjected to a twisting moment of 2,700,000 inch-lb. Determine the diameter if the allowable stress is taken at 9000lb. per square inch.

Set 9000 on B to 2,700,000 on A, and bring the cursor to 5·1 on B. Moving the slide to the left, it is found that when 11·51 on the R.H. scale of B is under the cursor, the L.H. index of C is opposite 11·51 on D. This, then, is the required diameter of the shaft.

(N.B.—The rules for the scales to be used in finding the cube root (page 41) must be carefully observed in working these examples.)

XII. Moments of Inertia.

To find the moment of inertia of a square section about an axis formed by one of its diagonals $\left(I=\frac{s^4}{12}\right)$.

Set index of C to the length of the side of square *s* on D; bring cursor to *s* on C, 12 on B to cursor, and over index of B read moment of inertia on A.

To find the moment of inertia of a rectangular section about an axis parallel to one side and perpendicular to the plane of bending.

Set index of C to the height or depth h of the section, and bring cursor to h on B. Set 12 on B to cursor, and over breadth b of the section on B read moment of inertia on A.

Ex.—Find the moment of inertia of a rectangular section of which $h=14$in. and $b=7$in.

Set index of C to 14 on D, and cursor to 14 on B. Bring 12 on B to cursor, and over 7 on B read 1600 on A.

XIII. Discharge from Pumps, Pipes, Etc.

To find the theoretical delivery of pumps, in gallons per stroke.

Set 29·4 on B to the diameter of the plunger in inches on D, and over length of stroke in feet on B read theoretical delivery in gallons per stroke on A.

(N.B.—A deduction of from 20 to 40 per cent. should be made to allow for slip.)

To find loss of head of water in feet due to friction in pipes (Prony's rule).

Set diameter of pipe in feet on B to velocity of water in feet per second on D and bring cursor to 2·25 on B; bring 1 on B to cursor, and over length of pipe in miles on B, read loss of head of water in feet, on A.

To find velocity in feet per second, of water in pipes (Blackwell's rule).

Set 2·3 on B to diameter of pipe in feet on A, and under inclination of pipe in feet per mile on B read velocity in feet per second on D.

To find the discharge over weirs in cubic feet per minute and per foot of width. (Discharge $=214\sqrt{h^3}$).

Set 0·00467 on C to the head in feet h on D, and under h on B read discharge on D.

To find the theoretical velocity of water flowing under a given head in feet.

Set index of B to head in feet on A, and under 64·4 on B read theoretical velocity in feet per second on D.

XIV. Horse-power of Water Wheels.

To find the effective horse-power of a Poncelet water wheel.

Set 880 on C to cubic feet of flow of water per minute on D, and under height of fall in feet on C, read effective horse-power on D.

For breast water wheels use 960, and for overshot wheels 775, in place of 880 as above.

XV. Electrical Engineering.

To find the resistance per mile, in ohms, of copper wire of high conductivity, at 60° F., the diameter being given in mils. (1 mil. = 0·001in.).

Set diameter of wire in mils. on C to 54,900 on A, and over R.H. or L.H. index of B read resistance in ohms on A.

Ex.—Find the resistance per mile of a copper wire 64mils. in diameter.
Set 64 on C to 54,900 on A, and over R.H. index of B read 13·4 ohms on A.

To find the weight of copper wire in lb. per mile.

Set 7·91 on C to diameter of wire in mils. on D, and over index on B read weight per mile on A.

Given electromotive force and current, to find electrical horse-power.

Set 746 on C to electromotive force in volts on D, and under current in ampères on C read electrical horse-power on D.

Given the resistance of a circuit in ohms and current in ampères, to find the energy absorbed in horse-power.

Set 746 on B to current on D, and over resistance on B read energy absorbed in H.P. on A.

Ex.—Find the H.P. expended in sending a current of 15 ampères through a circuit of 220 ohms resistance.
Set 746 on B to 15 on D. and over 220 on B read 66·3 H.P. on A.

XVI. Commercial.

To find selling price of goods when a given percentage on the *cost* price is added.

Set 100 on C to 100 + given percentage of profit on D, and opposite cost price on C read selling price on D.

To find selling price of goods when a given percentage of profit on the *selling* price is added.

Set 100 *minus* given percentage of profit on C to 100 on D, and under cost price on C read selling price on D.

To calculate simple interest.

Set 1 on C to rate per cent. on D; bring cursor to period on C and 1 on C to cursor. Then opposite any sum on C find simple interest on D.

For interest per annum.

Set R.H. index on C to rate on D, and opposite principal on C read interest on D.

Ex.—Find the amount with simple interest of £250 at 8 per cent., and for a period of 1 year and 9 months.

Set 1 on C to 8 on D; bring cursor to 1·75 on C, and 1 on C to cursor; then opposite 250 on C read £35, the interest, on D. Then 250 + 35 = £285 = the amount.

To calculate compound interest.

Set the L.H. index of C to the amount of £1 at the given rate of interest on D, and find the logarithm of this by reading on the reverse side of the rule, as explained on page 44. Multiply the logarithm, so found, by the period, and set the result, on the scale of equal parts, to the index on the under-side of the rule; then opposite any sum on C read the amount (including compound interest) on D.

Ex.—Find the amount of £500 at 5 per cent. for 6 years, with compound interest.

Set L.H. index of C to £1·05 on D, and read at the index on the scale of equal parts on the under-side of rule, 0·0212. Multiply by 6, obtaining 0·1272, which place on the scale of equal parts. Then opposite 500 on C read £670 on D, the amount required, including compound interest.

XVII. Miscellaneous Calculations.

To calculate percentages of compositions.

Set weight (or volume) of sample on C, to weight (or volume) of substance considered, on D; then under index of C read required percentage on D.

Ex.—A sample of coal weighing 1·25grms. contains 0·04425grm. of ash. Find the percentage of ash.

Set 1·25 on C to 0·04425 on D, and under index on C read 3·54, the required percentage of ash on D.

Given the steam pressure P and the diameter d in millimetres, of the throat of an injector, to find the weight W, of water delivered in lb. per hour from $W = \frac{d^2 \sqrt{P}}{0{\cdot}505}$.

Set 0·505 on C to P on A; bring cursor to d on C and index of C to cursor. Then under d on C read delivery of water on D.

To find the pressure of wind per square foot, due to a given velocity in miles per hour.

Set 2 on B to 9 on D, and over the velocity in miles per hour on D read pressure in lb. per square foot on B.

To find the horse-power of a windmill from H.P. = $\frac{aV^3}{1,080,000}$, in which a = total sail area in square feet, and V = velocity of wind in feet per second.

Set index of C to V on D, and bring cursor to V on B. Bring 1,080,000 on B to cursor, and over a on B read H.P. on A.

To find the kinetic energy of a moving body.

Set 64·4 on B to velocity in feet per second on D, and over weight of body in lb. on B read kinetic energy or accumulated work in foot-lb. on A.

To find in cotton spinning the draft of the rollers to spin any given counts of yarn, having given the hank roving, the delivery of the rollers, and the length of stretch in inches.

Set the number of the hank roving on C to the counts to be spun on D, and bring the cursor to the delivery of the rollers on C. Bring length of stretch in inches on C to cursor, and under 1 on C read draft required on D.

To find the number of turns per inch in the yarn, the speed of the spindles and the diameter and number of revolutions of the front rollers being given.

Set revolutions of the front roller on C to revolutions of spindles on D; bring cursor to 0·3183 on C, and diameter of front roller to cursor. Then under index on C read number of turns per inch on D.

TRIGONOMETRICAL APPLICATIONS.

Scales.—Not the least important feature of the modern slide rule is the provision of the special scales on the under-side of the slide, and by the use of which, in conjunction with the ordinary scales on the rule, a large variety of trigonometrical computations may be readily performed.

Three scales will be found on the reverse or under-side of the slide of the ordinary Gravêt or Mannheim rule.

One of these is the evenly-divided scale or scale of equal parts referred to in previous sections, and by which, as explained, the decimal parts or mantissæ of logarithms of numbers may be obtained. Usually this scale is the centre one of the three, but in some rules it will be found occupying the lowest position, in which case some little modification of the following instructions will be necessary. The requisite transpositions will, however, be self-evident when the objects of the scales are understood. The upper of the three scales, usually distinguished by the letter S, is a scale giving the logarithms of the sines of angles, and is used to determine the natural sines of angles of from 35 minutes to 90 degrees. The notation of this scale will be evident on inspection. The main divisions 1, 2, 3, etc., represent the degrees of angles; but the values of the subdivisions differ according to their position on the scale. Thus, if any primary space is subdivided into 12 parts, each of the latter will be read as 5 minutes (5′), since 1°=60′.

Sines of Angles.—To find the sine of an angle the slide is placed in the groove, with the under-side uppermost, and the end division lines or indices on the slide, coinciding with the right and left indices of the A scale. Then over the given angle on S is read the value of the sine of the angle on A. If the result is found on the left scale of A (1 to 10), the logarithmic characteristic is −2; if it is found on the right-hand side (10 to 100), it is −1. In other words, results on the right scale are prefixed by the decimal point only, while those on the left-hand scale are to be preceded by a cypher also. Thus:—

Sine 2° 40′ = 0·0465; sine 15° 40′ = 0·270.

Multiplication and division of the sines of angles are performed in the same manner as ordinary calculations, excepting that the slide has its under-face placed uppermost, as just explained. Thus to multiply sine 15° 40′ by 15, the R.H. index of S is brought to 15 on A, and opposite 15° 40′ on S is found 4·05 on A. Again, to divide 142 by sine 16° 30′, we place 16° 30′ on S to 142 on A, and over R.H. index of S read 500 on A.

The rules for the number of integers in the results are thus determined. Let N be the number of integers in the multiplier M or in the dividend D. Then the number of integers P, in the product or Q, in the quotient are as follows :—

When the result is found to the right of M or D, and in the same scale	P=N−2	Q=N
When the result is found to the right of M or D, and in the other scale	P=N−1	Q=N+1
When the result is found to the left of M or D, and in the other scale	P=N−1	Q=N+1
When the result is found to the left of M or D, and in the same scale	P=N	Q=N+2

If the division is of the form $\frac{20^\circ\ 30'}{50}$, the result cannot be read off directly on the face of the rule. Thus, if in the above example 20° 30′ on S, is placed coincidentally with 50 on the right scale of A, the result found on S under the R.H. index of A is 44° 30′. The required numerical value can then be found : (1) By placing the slide with all indices coincident when opposite 44° 30′ on S will be found 0·007 on A ; or (2) In the ordinary form of rule, by reading off on the scale B opposite the index mark in the opening on the under-side of the rule. The above rules for the number of integers in the quotient do not apply in this case.

If it is required to find the sine of an angle simply, this may be done with the slide in its ordinary position, with scale B under A. The given angle on scale S is then set to the index on the under-side of the rule, and the value of the sine is read off on B under the right index of A.

Owing to the rapidly diminishing differences of the values of the sines as the upper end of the scale is approached, the sines of angles between 60° and 90° cannot be accurately determined in the foregoing manner. It is therefore advisable to calculate the value of the sine by means of the formula :

$$\text{Sine }\theta = 1 - 2\sin^2\frac{90-\theta}{2}.$$

The manner of determining the value of $\sin^2\frac{90-\theta}{2}$ is as follows :—With the slide in the normal position, set the value of $\frac{90-\theta}{2}$ on S to the index on the under-side of the rule, and read off the value x on B under the R.H. index of A. Without moving the slide find x on A, and read under it on B the value required.

Ex.—Find value of sine 79° 40′.

Sine 79° 40′ $=1-2\sin^2$ 5° 10′.

But sine 5° 10′ $=0{\cdot}0900$, and under this value on A is 0·0081 on B. Therefore sine 79° 40′ $=1-0{\cdot}0162=0{\cdot}9838$.

The sines of very small angles, being very nearly proportional to the angles themselves, are found by direct reading. To facilitate this, some rules are provided with two marks, one of which, consisting of a single accent (′), corresponds to the logarithm of $\frac{1}{\text{sine } 1'}$ and is found at the number 3438. The other mark—a double accent (″)—corresponds to the logarithm of $\frac{1}{\text{sine } 1''}$ and is found at the number 206,265. In some rules these marks are either on the A or the B scales, sometimes on both, while in others they are found occupying the equivalent positions on scale S. In either case the angle on the one scale is placed so as to coincide with the significant mark on the other, and the result read off on the first-named scale opposite the index of the second.

In this connection it is to be observed that in sines of angles under 3″, the number of integers in the result is -5; while it is -4 for angles from 3″ to 21″; -3 from 21″ to 3′ 27″; and -2 from 3′ 27″ to 34′ 23″.

Ex.—Find sine 6′.

Placing the significant mark for minutes to coincide with 6, the value opposite the index is found to be 175, and by the rule above this is to be read 0·00175. For angles in seconds the other significant mark is used; while angles expressed in minutes and seconds are to be first reduced to seconds. Thus, 3′ 10″ = 190″.

Tangents of Angles.—There remains to be considered the third scale found on the back of the slide, and usually distinguished from the others by being lettered T. In most of the more recent forms of rule this scale is placed near the lower edge of the slide, but in some arrangements it is found to be the centre scale of the three. Again, in some rules this scale is figured in the same direction as the scale of sines—viz., from left to right,—while in others the T scale is reversed. In both cases there is now usually an aperture formed in the back of the left extremity of the rule, with an index mark similar to that already referred to in connection with the scale of sines. Considering what has been referred to as the more general arrangement, the method of determining the tangents of angles may be thus explained :—

The tangent scale will be found to commence, in some rules, at about 34′, or precisely at the angle whose tangent is 0·01. More usually, however, the scale will be found to commence at about 5° 43′, or at the angle whose tangent is 0·1. The other extremity of the scale corresponds in all cases to 45°, or the angle whose tangent is 1. This explanation will suggest the method of using the scale, however it may be arranged. If the graduations commence with 34′, the T scale is to be used in conjunction with the right and left scales of A ; while if they commence with 5° 43′ it is to be used in conjunction with the D scale.

In the former case the slide is to be placed in the rule so that the T scale is adjacent to the A scales, and, with the right and left indices coinciding, when opposite any angle on T will be found its tangent on A. It is to be noted, however, that from what has been said above, it follows that the tangents read on the L.H. scale of A have values extending from 0·01 to 0·1 ; while those read on the R.H. scale of A have values from 0·1 to 1·0. Otherwise expressed, to the values of any tangent read on the L.H. scale of A a cypher is to be prefixed ; while if found on the R.H. scale, it is read directly as a decimal.

Ex.—Find tan. 3° 50′.

Placing the slide as directed, the reading on A opposite 3° 50′ on T is found to be 67. As this is found on the L.H. scale of A, it is to be read as 0·067.

Ex.—Find tan. 17° 45′.

Here the reading on A opposite 17° 45′ on T is 32, and as it is found on the R.H. scale of A it is read as 0·32.

As in the case of the scale of sines, the tangents may be found without reversing the slide, when a fixed index is provided in the back of the rule for the T scale.

We revert now to a consideration of those rules in which a single tangent scale is provided. It will be understood that in this case the slide is placed so that the scale T is adjacent to the D scale, and that when the indices of both are placed coincidentally, the value of the tangent of any angle on T (from 5° 43′ to 45°) may be at once read off on D, the result so found being read as wholly decimal.

Thus tan. 13° 20′ is read 0·237.

If an index is provided, the slide is used in its normal position, when, setting the angle on the tangent scale to this index, the result can be read off on C over the L.H. index of D.

The tangents of angles above 45° are obtained by means of the formula: Tan. $\theta = \frac{1}{\tan.(90-\theta)}$. This operation is performed in a very simple manner for all angles from 45° to (90° − 5° 43′), as follows:—Place (90 − θ) on T to the R.H. index of D, and read tan. θ on D under the L.H. index of T. The first figure in the value thus obtained is to be read as an integer.

Thus, to find tan. 71° 20′ we place 90° − 71° 20′ = 18° 40′ on T to the R.H. index of D, and under the L.H. index of T read 2·96, the required tangent.

The tangents of angles less than 40′ are sensibly proportional to the angles themselves, and as they may therefore be considered as sines, their value is determined by the aid of the single and double accent marks on the S scale, as previously explained. The rules for the number of integers are the same as for the sines.

Multiplication and division of tangents may be quite readily effected.

Ex.—Tan. 21° 50′ × 15 = 6.

Set L.H. index of T to 15 on D, and under 21° 50′ on T read 6 on D.

Ex.—Tan. 72° 40′ × 117 = 375.

Set (90° − 72° 40′) = 17° 20′ on T to 117 on D, and under R.H. index of T read 375 on D.

Cosines of Angles.—The cosines of angles may be readily determined by placing the scale S, with its indices coinciding with those of A, and when opposite (90 − θ) on S is read cos. θ on A. If the result is read on the L.H. scale of A, a cypher is to be prefixed to the value read; while if it is read on the R.H. scale of A, the value is read directly as a decimal. Thus, to determine cos. 86° 30′ we find opposite (90° − 86° 30′) = 3° 30′ on S, 61° on A, and as this is on the L.H. scale the result is read 0·061. Again, to find cos. 59° 20′ we read opposite (90° − 59° 20′) or 30° 40′ on S, 51 on A, and as this is found on the R.H. scale of A, it is read 0·51.

In finding the cosines of small angles it will be seen that direct reading on the rule becomes impossible for angles of less than 20°. It is advisable in such cases to adopt the method described for determining the *sines* of the *large* angles of which the complements are sought.

Cotangents of Angles.—From the methods of finding the tangents of angles previously described, it will be apparent that the cotangents of angles may also be obtained with equal facility. For angles between 5° 45′ and 45°, the method of procedure is identical with that used for finding the tangents greater than 45°. Thus, the angle on scale T is brought to the R.H. index of D, and the cotangent read off on D under the L.H. index of T. The first figure of the result so found is to be read as an integer.

If the angle (θ) lies between 45° and 84° 15′, the slide is placed so that the indices of T coincide with those of D, and the result is then read off on D opposite (90 − θ) on T. In this case the value is wholly decimal.

Secants of Angles.—The secants of angles are readily found by bringing (90 − θ) on S to the R.H. index of A and reading the result on A over the L.H. index of S. If the value is found on the L.H. scale of A, the first figure is to be read as an integer; while if the result is read on the R.H. scale of A, the first *two* figures are to be regarded as integers.

Cosecants of Angles.—The cosecants of angles are found by placing the angle on S to the R.H. index of A, and reading the value found on A over the L.H. index of S. If the result is read on the L.H. scale of A, the first figure is to be read as an integer; while if the result is found on the R.H. scale of A, the first *two* figures are to be read as integers.

It will be noted that some of the rules here given for determining the several trigonometrical functions of angles apply only to those forms of rules in which a single scale of tangents T is used, reading from left to right. For the other arrangements of the scale, previously referred to, some slight modification of the method of procedure in finding the tangents and cotangents of angles will be necessary; but as in each case the nature and extent of this modification is self-evident, no further directions are required.

THE SOLUTION OF RIGHT-ANGLED TRIANGLES.

From the foregoing explanation of the manner of determining the trigonometrical functions of angles, the methods of solving right-angled triangles will be readily perceived, and only a few examples need therefore be given.

Let a and b represent the sides and c the hypothenuse of a right-angled triangle, and $a°$ and $b°$ the angles opposite to the sides. Then of the possible cases we will take

(1.) Given c and $a°$, to find a, b, and $b°$.

The angle $b° = 90 - a°$, while $a = c \sin a°$ and $b = c \sin b°$. To find a, therefore, the index of S is set to c on A, and the value of a read off on A opposite $a°$ on S. In the same manner the value of b is obtained.

Ex.—Given in a right-angled triangle $c = 9$ft. and $a° = 30°$. Find a, b, and $b°$.

The angle $b° = 90 - 30 = 60°$. To find a, set R.H. index of S to 9 on A, and over 30° on S read $a = 4{\cdot}5$ft. on A. Also, with the slide in the same position, read $b = 7{\cdot}8$ft. [7·794] on A over 60° on S.

(2.) Given a and c, to determine $a°$, $b°$, and b.

In this case advantage is taken of the fact that in every triangle the sides are proportional to the sines of the opposite angles. Therefore, as in this case the hypothenuse c subtends a right angle, of which the sine $= 1$, the R.H. index (or 90°) on S is set to the length of c on A, when under a on A is found $a°$ on S. Hence $b°$ and b may be determined.

(3.) Given a and $a°$, to find b, c, and $b°$.

Here $b° = (90 - a°)$, and the solution is similar to the foregoing.

(4.) Given a and b, to find a°, b°, and c.

To find a°, we have tan. $a^\circ = \frac{a}{b}$, which in the above example will be $\frac{4 \cdot 5}{7 \cdot 8} = 0 \cdot 577$. Therefore, placing the slide so that the indices of T coincide with those of D, we read opposite 0·577 on D the value of $a^\circ = 30^\circ$. The hypothenuse c is readily obtained from $c = \frac{a}{\sin a^\circ}$.

THE SOLUTION OF OBLIQUE-ANGLED TRIANGLES.

Using the same letters as before to designate the three sides and the subtending angles of oblique-angled triangles, we have the following cases :—

(1.) Given one side and two angles, as a, a°, and b°, to find b, c, and c°.

In the first place, $c^\circ = 180 - (a^\circ + b^\circ)$; also we note that, as the sides are proportional to the sines of the opposite angles, $b = \frac{a \text{ sine } b^\circ}{\text{sine } a^\circ}$ and $c = \frac{a \text{ sine } c^\circ}{\text{sine } a^\circ}$.

Taking as an example, $a = 45$, $a^\circ = 57^\circ$, and $b^\circ = 63^\circ$, we have $c^\circ = 180 - (57 + 63) = 60^\circ$. To find b and c, set a° on S to a on A, and read off on A above 63° and 60° the values of b (=47·8) and c (=46·4) respectively.

(2.) Given a, b, and a°, to find b°, c°, and c.

In this case the angle a° on S is placed under the length of side a on A, and under b on A is found the angle b° on S. The angle $c^\circ = 180^\circ - (a^\circ + b^\circ)$, whence the length c can be read off on A over c° on S.

(3.) Given the sides and the included angle, to find the other side and the remaining angles.

If, for example, there are given $a = 65$, $b = 42$, and the included angle $c^\circ = 55^\circ$, we have $(a + b) : (a - b) = \tan. \frac{a^\circ + b^\circ}{2}$: $\tan. \frac{a^\circ - b^\circ}{2}$. Then, since $a^\circ + b^\circ = 180^\circ - 55^\circ = 125^\circ$, it follows that $\frac{a^\circ + b^\circ}{2} = \frac{125}{2} = 62^\circ\ 30'$.

By the rule for tangents of angles greater than 45°, we find tan. 62° 30′ = 1·92. Inserting in the above proportion the values thus found, we have $107 : 23 = 1{\cdot}92 : \tan. \frac{a^\circ - b^\circ}{2}$. From this it is found that the value of the tangent is 0·412, and placing the slide with all indices coinciding, it is seen that this value on D corresponds to an angle of 22° 25′. Therefore, since $\frac{a^\circ + b^\circ}{2} = 62^\circ\ 30'$, and $\frac{a^\circ - b^\circ}{2} = 22^\circ\ 25'$, it follows that $a^\circ = 84^\circ\ 55'$, and $b^\circ = 40^\circ\ 5'$. Finally, to determine the side c, we have $c = \frac{a \sin c^\circ}{\sin a^\circ}$ as before.

PRACTICAL TRIGONOMETRICAL APPLICATIONS.

A few examples illustrative of the application of the methods of determining the functions of angles, etc., described in the preceding section, will now be given.

To find the chord of an arc, having given the included angle and the radius.

With the slide placed in the rule with the C and D scales outward, bring one-half of the given angle on S to the index mark in the back of the rule, and read the chord on B under twice the radius on A.

Ex.—Required the chord of an arc of 15°, the radius being 23in.

Set 7° 30′ on S to the index mark in the back of the rule, and under 46 on A read 6in., the required length of chord on B.

To find the area of a triangle, given two sides and the included angle.

Set the angle on S to the index mark on the back of the rule, and bring cursor to 2 on B. Then bring the length of one side on B to cursor, cursor to 1 on B, the length of the other side on B to cursor, and read area on B under index of A.

Ex.—The sides of a triangle are 5 and 6ft. in length respectively, and they include an angle of 20°. Find the area.

Set 20 on S to index mark, bring cursor to 2 on B, 5 on B to cursor, cursor to 1 on B, 6 on B to cursor, and under 1 on A read the area = 5·13 sq. ft. on B.

To find the number of degrees in a gradient, given the rise per cent.

Place the slide with the indices of T coincident with those of D, and over the rate per cent. on D read number of degrees in the slope on T.

As the arrangement of rule we have chiefly considered has only a single T scale, it will be seen that only solutions of the above problem involving slopes between 10 and 100 per cent. can be directly read off. For smaller angles, one of the formulæ for the determination of the tangents of sub-multiple angles must be used.

In rules having a double T scale (which is used with the A scale) the value in degrees of any slope from 1 to 100 per cent. can be directly read off on A.

To find the number of degrees, when the gradient is expressed as 1 in *x*.

Place the index of T to *x* on D, and over index of D read the required angle in degrees on T.

Ex.—Find the number of degrees in a gradient of 1 in 3·8.

Set 1 on T to 3·8 on D, and over R.H. index of D read 14° 45′ on T.

Given the lap, the lead and the travel of an engine slide valve, to find the angle of advance.

Set (lap + lead) on B to half the travel of the valve on A, and read the angle of advance on S at the index mark on the back of the rule.

Ex.—Valve travel $4\frac{1}{2}$in., lap 1in., lead $\frac{5}{16}$in. Find angle of advance.

Set $1\frac{5}{16}$ = 1·312 on B to 2·25 on A, and read 35° 40′ on S opposite the index on the back of the rule.

Given the angular advance θ, the lap and the travel of a slide valve, to find the cut-off in percentage of the stroke.

Place the lap on B to half the travel of valve on A, and read on S the angle (the supplement of the *angle of the eccentric*) found opposite the index in the back of the rule. To this angle, add the angle of advance and deduct the sum from 180°, thus obtaining the *angle of the crank* at the point of cut-off. To the cosine of the supplement of this angle, add 1 and multiply the result by 50, obtaining the percentage of stroke completed when cut-off occurs.

Ex.—Given the angular advance = 35° 40′, the valve travel = $4\frac{1}{2}$in., and the lap = 1in., find the angle of the crank at cut-off and the admission period expressed as a percentage of the stroke.

Set 1 on B to 2·25 on A, and read off on S opposite the index, the supplement of the angle of the eccentric = 26° 20′. Then 180 − (35° 40′ + 26° 20′) = 118° = the crank angle at the point of cut-off. Further, cos. 118° = cos. 62° = sin (90° − 62°) = sin 28°, and placing 28° on S to the back index, the cosine, read on B under R.H. index of A, is found to be 0·469. Adding 1 and placing the L.H. index of C to the result, 1·469, on D, we read off under 50 on C, the required period of admission = 73·4 per cent. on D.

SLIDE RULES WITH LOG.-LOG. SCALES.

For occasional requirements, the method described on page 44 of determining powers and roots other than the square and cube, is quite satisfactory. When, however, a number of such calculations are to be made, the process may be simplified considerably by the use of what are known as *log.-log.*, *logo.-log.*, or *logometric* scales, in conjunction with the ordinary scales of the rule. The principle involved will be understood from a consideration of those rules for logarithmic computation (page 12) which refer to powers and roots. From these it is seen that while for the multiplication and division of numbers we *add* their logarithms, for involution and evolution we require to *multiply* or *divide* the logarithms of the numbers by the exponents of the power or root as the case may be. Thus to find the 5th power of 3, we have (log. 3) $\times$ 5 = log. x, and by the ordinary method described on page 44 we should determine log. 3 by the aid of the scale L on the back of the slide, multiply this by 5 by using the C and D scales in the usual manner, transfer the result to scale L, and read the value of x on D under 1 on C. By the simpler method, first proposed by Dr. P. M. Roget,* the multiplication of log. 3 by 5 is effected in precisely the same manner as with any two ordinary factors—*i.e.*, by adding their logarithms and finding the number corresponding to the resulting logarithm. In this case we have log. (log. 3) + log. 5 = log. (log. x). The first of the three terms is obviously the *logarithm of the logarithm* of 3, the second is the simple logarithm of 5, and the third the *logarithm of the logarithm* of the answer. Hence it will be seen that if we have a scale so graduated that the distances from the point of origin represent the logarithms of the logarithms (the log.-logs.) of the numbers engraved upon it, then by using this in conjunction with the ordinary scale of logarithms, we can effect the required multiplication in a manner which is both expeditious and convenient. Slightly varying arrangements of the log.-log. scale, sometimes referred to as the "P line," have been introduced

* Philosophical Transactions of the Royal Society, 1815.

from time to time, but all follow more or less closely the form used by Dr. Roget, the principal differences being found in the limits to which the graduations have been extended. Latterly the increasing use which is being made of exponential formulæ in thermodynamic, electrical, and physical calculations has led to a revival of interest in Dr. Roget's invention, and various arrangements of rules with log.-log. scales are now available.

In the rule as arranged by Messrs. Dunlop and Jackson and introduced by Messrs. John Davis and Son Limited, Derby, the log.-log. scales are placed upon a separate slide—a plan which has the advantage of leaving the rule intact for all ordinary purposes, while providing a length of 40 inches for the log.-log. scales.

In the 10in. Davis rule this space is utilised in the following manner:—On one face of the slide, marked E, are two log.-log. scales for numbers greater than unity, the lower extending from 1·07 to 2, and the upper continuing the graduations from 2 to 1000. On the reverse face of the slide, marked − E, are two log.-log. scales for numbers less than unity, the upper extending from 0·001 to 0·5, and the lower continuing the graduations from 0·5 to 0·933. Both sets of scales are used in conjunction *with the lower or D scale of the rule*, which is to be primarily regarded as running from 1 to 10, and constitutes a scale of exponents. In the 20in. rule the log.-log. scales are more extensive, and are used in conjunction with the upper or A scale of the rule (1 to 100); in what follows, however, the 10in. rule is more particularly referred to.

It has been explained that on the log.-log. scale the distance of any graduation from the point of origin represents the log.-log. of the number by which that graduation is distinguished. The point of origin will obviously be that graduation whose log.-log. = 0. This is seen to be 10, since log. (log. 10) = log. 1 = 0. Hence, confining attention to the E scale, to locate the graduation 20, we have log. (log. 20) = log. 1·301 = 0·11897, so that if the scale D is 25cm. long, the distance between 10 and 20 on the corresponding log.-log. scale would be 113·97 ÷ 4 = 28·49mm. For numbers less than

10 the resulting log.-logs. will be negative, and the distances will be spaced off from the point of origin in a negative direction—*i.e.*, from right to left. Thus to locate the graduation 5, we have

$$\log.(\log. 5) = \log. 0{\cdot}699 = \bar{1}{\cdot}844; \textit{i.e.}, -1 + 0{\cdot}844 \text{ or } -0{\cdot}156;$$

so that the graduation marked 5 would be placed $156 \div 4 =$ 39mm. distant from 10 in a *negative* direction. Proceeding in a similar manner, the scale may be extended in either direction, until each of the sections which comprise it are about equal in length to the D scale of the rule. It may be noted that the position of the point of origin (10) is quite a matter of arbitrary selection; but as the spacings in the higher part of the scale rapidly increase in value, the accuracy obtainable is correspondingly lessened, and hence little is gained by extending the scale beyond 1000.

Turning to the – E scale, it will be seen that in this case the notation runs in the reverse direction to that of the E scale, but in all other respects it is arranged in a precisely analogous manner, the distance from the point of origin (0·1 in this case) to any graduation x representing log. [– log. x]. Thus to locate 0·2 we have

$$\log. [-\log. 0{\cdot}2] = \log. [-(\bar{1}{\cdot}301)] = \log. [-(-0{\cdot}699)]$$
$$= \log. 0{\cdot}699 = \bar{1}{\cdot}844 = -0{\cdot}156;$$

so that for a 25cm. rule, 0·2 would be $156 \div 4 = 39$mm. distant from 0·1 in a *negative* direction. This interval is seen to be precisely equal to that between 10 and 5 on the E scale, a fact which brings us to a simple method of expressing the relation between the two scales—viz., that of the similarly situated graduations on the two scales, those on the – E scale are the *reciprocals* of those on the E scale. This statement may be readily verified by setting, say, 10 on E to (R.H.) 1 on D, when turning to the back of the rule we find 0·1 on – E agreeing with the index mark in the aperture at the right-hand extremity of the rule. Similarly setting 1·25 on E to (L.H.) 1 on D we find 0·8 on – E at the left-hand aperture of the rule.

In using the log.-log. scales it is important to observe (1) that the values engraved on the scale are definite and unalterable (*e.g.*, 1·2 can only be read as 1·2 and not as 120, 0·0012, etc., as with the ordinary scales); (2) that the upper portion of each scale should be regarded as forming a prolongation to the right of the lower portion; and (3) that immediately above any value on the lower portion of the scale is found the 10th power of that value on the upper portion of the scale. Keeping these points in view, if we set 1·1 on E to 1 on D we find over 2 on D the value of $1{\cdot}1^2 = 1{\cdot}21$ on E. Similarly, over 3 we find $1{\cdot}1^3 = 1{\cdot}331$; over 5 we find $1{\cdot}1^5 = 1{\cdot}61$, and so on. Hence the rule:—*To find the value of x^n, set x on E to 1 on D, and over n on D read x^n on E.*

Before moving the slide, we may verify the statement (3) by noting that any graduation on the lower line is virtually separated from the graduation immediately above it, by the length of the D scale of exponents. As the length of the latter represents 10, it is seen that by reading across the slide we obtain the 10th power of the lower reading. Thus over 2 is found $1{\cdot}1^2$ or 1·21 on the lower line and $1{\cdot}21^{10} = 6{\cdot}73 = 1{\cdot}1^{2 \times 10} = 1{\cdot}1^{20}$ on the upper line.

With the slide set as above, the 8th, 9th, etc., powers of 1·1 cannot be read off; but it is seen that, according to (2) in the foregoing, the missing portion of the E scale is that part of the upper scale (2 to about 2·6) which is outside the rule to the left. Hence placing 1·1 to 10 on D, the 8th, 9th, etc., powers of 1·1 will be read off *on the upper part* of the E scale. As for the readings now found on the *lower* line, it is clear, from the relative value of the two scale lines, that over 8, 9, etc., on D we find $1{\cdot}1^{\frac{8}{10}}$, $1{\cdot}1^{\frac{9}{10}}$, etc.; in other words, the powers are $\frac{1}{10}$th those read on the upper scale line. In general, then,

If x on the *lower* line is set to 1 on D, then x^n is read directly on that line and x^{10n} on the upper line.

If x on the *upper* line is set to 1 on D, then x^n is read directly on that line and $x^{\frac{n}{10}}$ on the lower line.

If x on the *lower* line is set to 10 on D, then $x^{\frac{n}{10}}$ is read directly on that line and x^n on the upper line.

If x on the *upper* line is set to 10 on D, then $x^{\frac{n}{10}}$ is read directly on that line and $x^{\frac{n}{100}}$ on the lower line.

These rules are conveniently exhibited in the accompanying diagram (Fig. 5). They are equally applicable to both the E and — E scales of the 10in. rule, and include practically all the instruction required for determining the nth

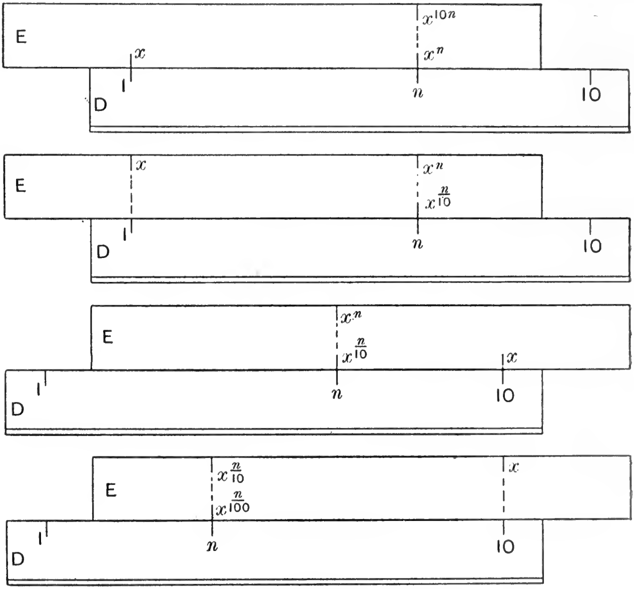

Fig. 5.

power or the nth root of a number. They do not apply directly to the 20in. rule, however, for here the relation of the lower and upper scales will be x^n and x^{100n}.

Ex.—Find $1{\cdot}167^{2{\cdot}56}$.

Set 1·167 on E to 1 on D, and over 2·56 on D read 1·485 on E.

Ex.—Find $4{\cdot}6^{1{\cdot}61}$.

Set 4·6 on upper E scale to 1 on D, and over 1·61 on D read 11·7 (11·67) on E.

Ex.—Find $1{\cdot}4^{0{\cdot}27}$ and $1{\cdot}4^{2{\cdot}7}$.

Set 1·4 on E to 10 on D, and over 2·7 on D read $1{\cdot}095 = 1{\cdot}4^{0{\cdot}27}$ on lower E scale and $2{\cdot}48 = 1{\cdot}4^{2{\cdot}7}$ on upper E scale.

Ex.—Find $46^{0{\cdot}0184}$ and $46^{0{\cdot}184}$.

Set 46 on upper E scale to 10 on D, and over 1·84 on D read 1·073 on lower E scale and 2·023 (2·0223) on upper E scale.

Ex.—Find $0{\cdot}074^{1{\cdot}15}$.

Using the $-$ E scale, set 0·074 to 1 on D, and over 1·15 on D read 0·05 on $-$ E.

The method of determining the root of a number will be obvious from the preceding examples.

Ex.—Find $\sqrt[1{\cdot}4]{17}$ and $\sqrt[14]{17}$.

Set 17 on E to 1·4 on D, and over 1 on D read 7·56 on upper E scale and 1·224 on lower E scale.

Ex.—Find $\sqrt[0{\cdot}031]{0{\cdot}914}$.

Set 0·914 on $-$ E to 3·1 on D, and over 10 on D read 0·055 on upper $-$ E scale.

When the exponent n is fractional it may be converted into the equivalent decimal and used as in the foregoing examples. In the majority of cases, however, it will be possible to directly obtain the result with one setting of the slide. Thus to determine $1{\cdot}135^{\frac{17}{16}}$ by the first method, we find $\frac{17}{16} = 1{\cdot}0625$, and placing 1·135 on E to 1 on D, read 1·144 on E over 1·0625 on D. By the direct method we place 1·135 on the E scale to 1·6 on D, and over 1·7 on D read 1·144 on E. It will be seen that since the scale D is assumed to run from 1 to 10 we are unable to read 16 and 17 on this scale; but it is obvious that the *ratios* $\frac{1{\cdot}7}{1{\cdot}6}$ and $\frac{17}{16}$ are identical, and it is with the ratio only that we are in effect concerned. Hence, if in a fractional index the ratio is seen to lie between 1 and 10, the second method is directly available, while obvious extensions in either direction are obtainable from the relative values of the E scale readings already explained (Fig. 5). These must be carefully attended to in extending this method of working; but if in doubt, the student will, as in all slide rule difficulties, reassure himself by working a simple example of which the result is self-evident. He should, however, assiduously cultivate the habit of roughly estimating the value he might expect to obtain from the conditions of the problem under consideration.

Since an expression of the form $x^{-n} = \dfrac{1}{x^n}$ or $\left(\dfrac{1}{x}\right)^n$, the required value may be obtained by first determining the reciprocal of x and proceeding as before. By using both the direct and reciprocal log.-log. scales (E and $-$ E) in conjunction, however, the required value could be read directly from the rule, and the preliminary calculation entirely avoided. In the Davis form of rule some slight constructional modification is necessary to enable this method to be employed, but with the rule thus modified the result could be read on the − E scale when used in conjunction with the D scale of the rule, x on E being set to the index mark in the aperture in the back of the rule.

Ex.—Find the value of $1·195^{-1·65}$.

Set 1·195 on E to the index in the left aperture in the back of the rule, and over 1·65 on D read 0·745 on the − E scale.

It may be noted in passing that the log.-log. scale affords a simple means for determining the logarithm or antilogarithm of a number to any base. For this purpose it is necessary to set the base of the given system on E to 1 on D, when *under* any number on E will be found its logarithm on D. Thus for common logs. we set the base 10 on E to 1 on D, and under 100 we find 2, the required log. Similarly we read log. 20 = 1·301; log. 55 = 1·74; log. 550 = 2·74, etc. Reading reversely, over 1·38 on D we find its antilog. 24 on E; also antilog. 1·58 = 38; antilog. 1·19 = 15·5, etc.

For logs. of numbers under 10 it is obviously necessary to set the base 10 to 10 on D; hence the readings on D will be read as one-tenth their apparent value. Thus log. 3 = 0·477; log. 5·25 = 0·72; antilog. 0·415 = 2·6; antilog. 0·525 = 3·35, etc.

The logs. of the numbers found on the lower half of the E scale will also be found on the D scale; but a consideration of Fig. 5 will show that this will be read as *one-tenth* its face value if the base is set to 1 on D, and as *one-hundredth* if the base is set to 10.

For natural, hyperbolic, or Napierian logarithms, the base is 2·718. A special line marked ϵ or e serves to locate the exact position of this value on the E scale, and placing this to 1 on D we read $\log._e$ 4·35 = 1·47; $\log._e$ 7·4 = 2·0; antilog._e 2·89 = 18, etc. The manner of reading the other parts of the scale is precisely similar to that already described for common logs. Calculations involving powers of e are frequently met with in electrical engineering, and these are facilitated by using the special graduation lines referred to, as will be readily understood.

If it is required to determine the power or root of a number which does not appear on either of the log.-log. scales, we may break up the number into factors. Usually it is convenient to make one of the factors a power of 10.

Ex.—$3950^{1 \cdot 97} = 3 \cdot 95^{1 \cdot 97} \times 10^{3 \times 1 \cdot 97} = 3 \cdot 95^{1 \cdot 97} \times 10^{5 \cdot 91}$.

Then $3 \cdot 95^{1 \cdot 97} = 15$, and $10^{5 \cdot 91}$ or antilog. 5·91 = 812,000. Hence 15 × 812,000 = 12,180,000 is the result sought.

Numbers which are to be found in the higher part of the log.-log. scale may often be factorised in this way, and greater accuracy obtained than is possible by direct reading.

The form of log.-log. rule which has been mainly dealt with in the foregoing gives a scale of comparatively long range, and the only objection to the arrangement adopted is the use of a separate slide.

In another form, arranged by Professor Perry, the log.-log. scale is substituted for the D scale of the ordinary rule, being used in conjunction with the C scale on the slide. The directions given will apply, with obvious modification, equally to this arrangement. The objection to this design is that it reduces by one-half the effective length of the rule for ordinary purposes, since only the shorter A and B scales are available for multiplication and division. In this instrument the graduations of the log.-log. scale extend from 2 to 1000. A point to be noted in this form of rule is that the T scale on the back of the slide is inverted, and is twice the usual range, being used in conjunction with the double A scale, since the D scale is no longer available.

The arrangement adopted by Mr. A. W. Faber comprises a double log.-log. scale, extending from 1·1 to 10^5, which is used in conjunction with the D scale as in the Davis form of rule, and substantially in the manner already described. This arrangement has the advantage of leaving the standard form of rule quite intact, while avoiding the use of a separate slide. On the other hand there is no — E scale for numbers less than unity, although the principle could be quite readily extended to include this feature.

With rules unprovided with a — E scale the value of x^{-n} is obtained indirectly by the usual rule for reciprocals. Thus, since $x^{-n} = \frac{1}{x^n} = \left(\frac{1}{x}\right)^n$, we may either first determine x^n and find its reciprocal, or alternatively, we may first determine the reciprocal of x and raise this to the power n. The first method should be followed when the number x is found on the E scale.

Ex.—$3{\cdot}45^{-1{\cdot}82} = 0{\cdot}105$.

Set 1 on C to 3·45 on E, and under 1·82 on C read 9·51 on C. Then set 1 on B to 9·5 on A, and under index of A read 0·105 on B.

When x is less than unity the second method is more suitable.

Ex.—$0{\cdot}23^{-1{\cdot}77} = \left(\frac{1}{0{\cdot}23}\right)^{1{\cdot}77} = 4{\cdot}35^{1{\cdot}77} = 13{\cdot}5$.

Set 1 on B to 0·23 on A, and under index of A read $\frac{1}{0{\cdot}23} = 4{\cdot}35$ on B. Set 1 on C to 4·35 on E, and under 1·77 on C read 13·5 on E.

As with the Davis rule, the exponent scale C will be read as $\frac{1}{10}$ its face value if its R.H. index (10) is used in place of 1.

OTHER METHODS OF OBTAINING POWERS AND ROOTS—THE RADIAL CURSOR.

A simple method of obtaining powers and roots, which may serve on occasion, is by scaling off proportional lengths on the D scale (or the A scale) of the ordinary rule. Thus, to determine the value of $1{\cdot}25^{1{\cdot}67}$ we take the actual length 1–1·25 on the D scale, and increase it by any convenient means in the proportion of 1 : 1·67. Then with a pair of dividers we set off this new length from 1, and obtain 1·44 as the result. A simple geometrical construction will serve to increase the scale-lengths in the desired ratio, but the author has found a pair of proportional compasses convenient when the numbers are small. Thus to obtain $1{\cdot}52^{\frac{17}{16}}$, the compasses would be set in the ratio of 16 to 17, and the smaller end opened out to include 1–1·52 on the D scale; the opening in the large end of the compasses will then be such that setting it off from 1 we obtain 1·56 on D as the result sought.

The converse procedure for obtaining the nth root of a number N will obviously resolve itself into obtaining $\frac{1}{n}$th of the scale length 1–N, and need not be further considered.

An ingenious device by which a scale length may be mechanically multiplied or divided by n has been introduced by Mr. F. W. Lanchester. This takes the form of a Radial Cursor, which can be used in conjunction with the A scale of an ordinary rule. As will be seen from Fig. 6, the body of the cursor P carries a graduated bar S which can be moved in a direction transverse to the rule, and adjusted to any desired position. Pivoted to the lower end of S is a radial arm R of transparent celluloid on which a centre line is engraved.

A reference to the illustration will show that the principle involved is that of similar triangles, the width of the slide being used as one of the elements. Thus, to take a simple case, if 2 on S is set to the index on P, and 1 on B is brought to N on A, then by swinging the radial arm until its centre line agrees with 1 on C, we can read N^2 on A. Evidently, since in the two similar triangles A O N^2

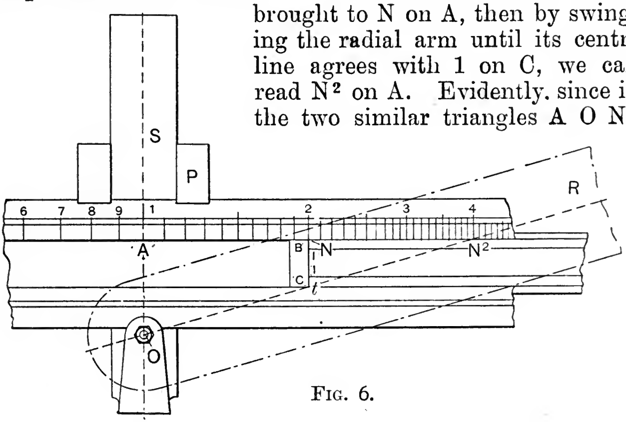

FIG. 6.

and N t N^2 the length of A O is double that of N t, it results that A $N^2 = 2$ A N. In general, then, to find the nth power of a number, we set the cursor to 1 or 10 on A, bring n on the cross bar S to the index on the cursor, and 1 on B to N on A. Then to 1 on C we set the line on the radial arm, and under the latter read N^n on A. The inverse proceeding for finding the nth root will be obvious.

An advantage offered by this and analogous methods of obtaining powers and roots is that the result is obtained on the ordinary scale of the rule, and hence it can be taken directly in any further calculation which may be necessary.

The importance of this feature is seen in various calculations pertaining to the expansion of steam, air compressors, etc., for which purposes the Lanchester attachment is more particularly suitable than for the higher powers and roots, which are best dealt with by the longer range log.-log. scales.

THE GOULDING AND OTHER SPECIAL CURSORS.

It has been pointed out that in order to obtain the third or fourth figure of a reading on the 10in. slide rule, it is frequently necessary to depend upon the operator's ability

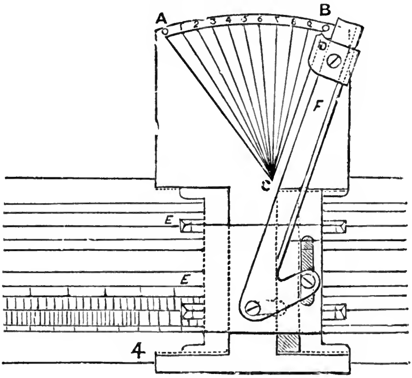

FIG. 7.

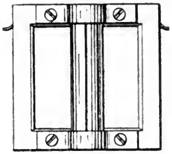

FIG. 8.

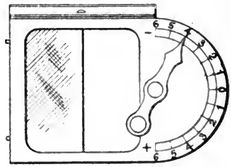

FIG. 9.

to mentally subdivide the space within which the reading falls. This subdivision can be mechanically effected by the aid of the Goulding Cursor (Fig. 7), which consists of a frame fitting into the usual grooves in the rule, and carrying a metal plate faced with celluloid, upon which is engraved a triangular scale A B C. The portion carrying the chisel edges E is not fixed to the cursor proper, but slides on the latter, so that the index marks on the projecting prongs can be moved slightly along the scales of the rule, this movement being effected by the short end of the bent lever F working in the slot as shown. D is a pointer which can be moved along F under spring control. As illustrating the method of use, we will assume the product of 155 × 28 to be required. This is seen to lie between 4150 and 4200, so setting the pointer D to the line B C—always the first operation,—we move the whole along the rule until the index line on the lower prong agrees with 4200. We then move F across the scale until the index line agrees with 4100, set the pointer D to the line A C, and move the lever back until the index line agrees with 28 on the slide. It will then be found that the pointer D gives 85 on A B as the value of the supplementary figures, and hence the complete reading is 4185.

Another device which is designed to assist the reading of the scales is the Magnifying Cursor (Fig. 8). In this, the cursor line is set in a strip of plano-convex glass, the magnifying effect of which is certainly helpful in a good and direct light. Both the Goulding and Magnifying Cursors are supplied by Messrs. J. Davis and Son Limited, Derby.

The Digit-registering Cursor, supplied by Mr. A. W. Faber, London, and shown in Fig. 9, is provided with a semi-circular scale running from 0 at the centre upward to -6 and downward to $+6$. A small finger enables the operator to register the number of digits to be added or subtracted at the end of a lengthy operation, in the manner explained at page 33.

THE SOLUTION OF ALGEBRAIC EQUATIONS.

The slide rule finds an interesting application in the solution of equations of the second and third degree; and although the process is essentially one of trial and error, it may often serve, as Messrs. Beghin, Dunlop, Jackson, and others have shown, as an efficient substitute for the more laborious algebraic methods, particularly when the conditions of the problem or the operator's knowledge of the theory of equations enables some idea to be obtained as to the character of the result sought. The principle may be thus briefly explained :—If 1 on C is set to x on D (Fig. 10), we find $x\,(x) = x^2$ on D under x on C. If, however, with the slide set as before, instead of reading under x, we read under $x + m$ on C, the result on D will now be $x\,(x + m) = x^2 + m\,x = q$. Hence to solve the equation

$x^2 + mx - q = 0$, we reverse the above process, and setting the cursor to q on D, we move the slide until the number on C under the cursor, and that on D under 1 on C, *differ by* m. It is obvious from the setting that the *product* of these numbers $= q$, and as their difference $= m$, they are seen to be the roots of the equation as required. For the equation $x^2 - mx + q = 0$, we require m to equal the

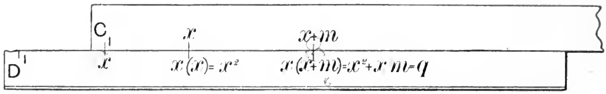

FIG. 10.

sum of the roots. Hence, setting the cursor as before to q on D, we move the slide until the number on C under the cursor, and that on D under 1 on C, are *together equal to* m, these numbers being the roots sought. The alternative equations $x^2 - mx - q = 0$, and $x^2 + mx + q = 0$ are deducible from the others by changing the signs of the roots, and need not be further considered.

Ex.—Find the roots of $x^2 - 8x + 9 = 0$.

Set the cursor to 9 on D, and move the slide to the right until when 6·64 is found under the cursor, 1·355 on D is under 1 on C. These numbers are the roots required.

The upper scales can of course be used; indeed, in general they are to be preferred.

Ex.—Find the roots of $x^2 + 12·8x + 39·4 = 0$.

Set the cursor to 39·4 on A, and move the slide to the right until we read 7·65 on B under the cursor, and 5·15 on A over 1 on B. The roots are therefore − 7·65 and − 5·15.

With a little consideration of the relative value of the upper and lower scales, the student interested will readily perceive how equations of the third degree may be similarly resolved. The subject is not of sufficient general importance to warrant a detailed examination being made of the several expressions which can be dealt with in the manner suggested; but the author gives the following example as affording some indication of the adaptability of the method to practical calculations.

Ex.—A hollow copper ball, 7·5in. in diameter and 2lb. in weight, floats in water. To what depth will it sink?

The water displaced $= 27{\cdot}7 \times 2 = 55{\cdot}4$ cub. in. The cubic contents of the immersed segment will be $\frac{\pi}{3}(3rx^2 - x^3)$, r being the radius and x the depth of immersion. Hence $\frac{\pi}{3}(3rx^2 - x^3) = 55{\cdot}4$, and

$$11{\cdot}25\,x^2 - x^3 = 52{\cdot}9.$$

To solve this equation we place the cursor to 52·9 on A, and move the slide until the reading on D under 1 and that on B under the cursor together amount to 11·25. In this way we find 2·45 on D under 1, with 8·8 on B under the cursor *c*, *c*, as a pair of values of which the sum is 11·25. Hence we conclude that $x = 2{\cdot}45$in. is the result sought.

With the rule thus set (Fig. 11) the student will note that the slide is displaced to the right by an amount which represents x on D, and therefore x^2 on A; while the length

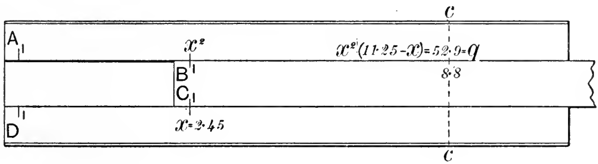

FIG. 11.

on B from 1 to the cursor line represents $11{\cdot}25 - x$. Hence the upper scale setting gives $x^2(11{\cdot}25 - x) = 11{\cdot}25\,x^2 - x^3 = 52{\cdot}9$ as required.

When in doubt as to the method to be pursued in any given case, the student should work synthetically, building up a simple example of an analogous character to that under consideration, and so deducing the plan to be followed in the reverse process.

In many cases the solution of quadratic and cubic equations can be more expeditiously, but less conveniently, effected by inverting the slide; but the student will be well advised to follow the direct method until he has gained confidence in such work. In any case, the proper relative value of the scales must be carefully observed throughout the operation, inconsistency in this respect leading to most of the beginner's difficulties.

CALCULATORS OF THE BOUCHER TYPE.

While as a calculating instrument of general utility the 10 or 20in. Gravêt or Mannheim slide rule is undoubtedly superior to any other arrangement, there have been introduced from time to time various modifications of this simple form of logarithmic calculator which require consideration. Of these, the Boucher circular calculator, which is sufficiently small to be carried in the pocket, somewhat resembles a stem-winding watch, being about 2in. in diameter and $\frac{9}{16}$in. in thickness. The instrument is furnished with two dials, the back one being fixed, while the front one, Fig. 12 (which shows the form

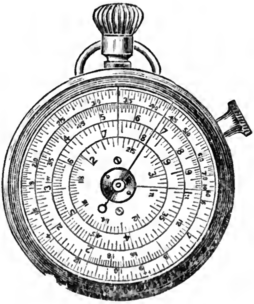

FIG. 12.

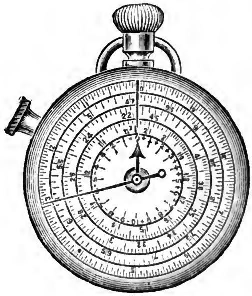

FIG. 13.

made by Mr. W. F. Stanley, London), turns upon the large centre arbor shown. This movement is effected by turning the milled head of the stem-winder. The small centre axis, which is turned by rotating the milled head at the side of the case, carries two fine needle pointers, one moving over each dial. These two pointers are so fixed on the axis that their positions on each dial are always coincident; in other words, one pointer always lies evenly over the other. A fine index or pointer fixed to the case in line with the axis of the winding stem, extends over the four scales of the movable dial as shown. Of these scales, the second from the outer is the ordinary logarithmic scale, which in this instrument corresponds to a straight scale of about $4\frac{3}{4}$in. in length. The two inner circles are those giving square roots of the numbers on the primary logarithmic scale, the smaller circle containing the square roots of values between 1 and 3·162 (the square root of 10), while the other section corresponds to values between 3·162 and 10. The outer circle is a scale of logarithms of sines of angles, the corresponding sines of which can be read off on the ordinary scale.

On the fixed or back dial there are also four scales, these being arranged as in Fig. 13. The outer of these is a scale of equal parts, while the three inner scales are separate sections of a scale giving the cube roots of the numbers taken on the ordinary logarithmic scale and referred thereto by means of the pointers. In dividing this cube-root scale into sections, the same method is adopted as in the case of the square-root scale. Thus, the smallest circle contains the cube roots of numbers between 1 and 10, and is therefore graduated from 1 to 2·154; the second circle contains the cube roots of numbers between 10 and 100, being graduated from 2·154 to 4·657; while the third section, in which are found the cube roots of numbers between 100 and 1000, carries the graduations from 4·657 to 10.

What has been said in an earlier section regarding the notation of the slide rule may in general be taken to apply to the scales of the Boucher calculator. The manner of using the instrument is, however, not quite so evident as is the case with the ordinary form of slide rule, although from what follows it will be seen that the operative principle—that of variously combining lengths of a logarithmic scale—is essentially similar. In this case, however, it is seen that in place of the straight scale-lengths shown in Fig. 3, we require to add or subtract arc-lengths of the circular scales, while, further, it is evident that in the absence of a fixed scale (corresponding to the stock of the slide rule) these operations cannot be directly performed as in the ordinary form of instrument. However, by the aid of the fixed index and the movable pointer, the desired combination of the scale-lengths may be readily effected in the following manner. Assuming it is desired to multiply 2 by 3, the dial is turned in a backward direction until the first factor 2 on the ordinary scale lies under the fixed index, after which the movable pointer is set to 1 on the scale. With this setting it is clear that the arc-length $1-2$ is spaced-off between the fixed index and the movable pointer, and it now only remains to add to this definite arc-length a further length of $1-3$. This is readily effected by a further backward turning of the dial until the arc $1-3$ has passed under the movable pointer, when the result, 6, is read under the fixed index. A little consideration will show that any other scale-length may be added to that included between the fixed and movable pointers, or, in other words, any number on the scale may be multiplied by 2 by bringing the number to the movable pointer and reading the result under the fixed index. The rule for multiplication is now evident.

Rule for Multiplication.—*Set one factor to the fixed index and bring the pointer to 1 on the scale; set the other factor to the pointer and read the result under the fixed index.*

The explanation just given renders a detailed description of the manner of performing the inverse operation of division unnecessary. It is clear that to divide 6 by 3, an arc-length $1-3$ is to be taken from a length $1-6$. To this end we set 6 to the index (corresponding in effect to passing a length $1-6$ to the left of that reference point) and set the pointer to the divisor 3. As now set, the arc $1-6$ is included between 1 on the scale and the index, while the arc $1-3$ is included between 1 on the scale and the pointer. It is clear that if the dial is now turned forward until 1 on the scale agrees with the pointer, an arc $1-3$ will have been deducted from the larger arc $1-6$, and the remainder, representing the result of this operation, will be read under the index as 2.

Rule for Division.—*Set the dividend to the fixed index, and the pointer to the divisor; turn the dial until 1 on the scale agrees with the pointer, and read the result under the fixed index.*

The foregoing method being an inversion of the rule for multiplication, is easily remembered and is generally advised. Another plan is, however, preferable when a series of divisions are to be effected with a constant divisor —*i.e.*, when b in $\frac{a}{b}=x$ is constant. In this case 1 on the scale is set to the index and the pointer set to b; then if any value of a is brought to the pointer, the quotient x will be found under the index.

Combined Multiplication and Division, as $\frac{a \times b \times c}{m \times n} = x$, can be readily performed, while cases of continued multiplication evidently come under the same category, since $a \times b \times c = \frac{a \times b \times c}{1 \times 1} = x$. Such cases as $\frac{a}{m \times n \times r} = x$ are expressed for the purpose of calculation, as $\frac{a \times 1 \times 1 \times 1}{m \times n \times r} = x$; while $\frac{a \times b \times c}{m} = x$ is similarly modified, taking the form $\frac{a \times b \times c}{m \times 1} = x$. In all cases the expression must be arranged so that there is *one more factor in the numerator* than *in the denominator, the factor 1 being introduced as often as required.* The simple operations of multiplication and division involve a similar disposition of factors, since from the rules given it is evident that $m \times n$ is actually regarded as $\frac{m \times n}{1}$, while $\frac{m}{n}$ becomes in effect $\frac{m \times 1}{n}$. It is important to note the general applicability of this arrangement-rule, as it will be found of great assistance in solving the more complicated expressions.

As with the ordinary form of slide rule, the factors in such an expression as $\frac{a \times b \times c}{m \times n} = x$ are taken in the order:— 1st factor of numerator; 1st factor of denominator; 2nd factor of numerator; 2nd factor of denominator, and so on; the 1st factor as a being set to the index, and the result x being finally read at the same point of reference.

Ex.—$\frac{39 \times 14{\cdot}2 \times 6{\cdot}3}{1{\cdot}37 \times 19} = 134.$

Commence by setting 39 to the index, and the pointer to 1·37; bring 14·2 to the pointer; pointer to 19; 6·3 to the pointer, and read the result 134 at the index.

It should be noted that after the first factor is set to the fixed index, the *pointer* is set to each of the *dividing* factors as they enter into the calculation, while the *dial* is moved for each of the *multiplying* factors. Thus the dial is first moved (setting the first factor to the index), then the pointer, then the dial, and so on.

NUMBER OF DIGITS IN RESULT.—For simple cases of multiplication and division, use the General Rules given on pages 28 and 32. For combined multiplication and division, modify the expression, if necessary, by adding 1's, as already explained, and subtract the sum of the denominator digits from the sum of numerator digits. Then proceed by the author's rule, as follows:—

Always turn dial to the LEFT; *i.e., against the hands of a watch.*

Note dial movements only; ignore those of the pointer.

Each time 1 *on dial agrees with or passes fixed index,* ADD 1 *to the above difference of digits.*

Each time 1 *on dial agrees with or passes pointer,* DEDUCT 1 *from the above difference of digits.*

Treat continued multiplication in the same way, counting the 1's used as denominator digits as one less than the number of multiplied factors.

Ex.—$\frac{8{\cdot}6 \times 0{\cdot}73 \times 1{\cdot}02}{3{\cdot}5 \times 0{\cdot}23} = 7{\cdot}95$ [7·95473 +].

Set 8·6 to index and pointer to 3·5. Bring 0·73 to pointer (noting that 1 on the scale passes the index) and set pointer to 0·23. Set 1·02 to pointer (noting that 1 on the scale passes the pointer) and read under index 7·95. There are $1+0+1=2$ numerator digits and $1+0=1$ denominator digits; while 1 is to be added and 1 deducted as per rule. But as the latter cancel, the digits in the result will be $2-1=1$.

When moving the dial to the left will cause 1 on the dial to pass *both* index and pointer (thus cancelling), the dial may be turned back to make the setting.

It will be understood that when 1 is the *first* numerator, and 1 on the dial is therefore set to the index, no digit addition will be made for this, as the actual operation of calculating has not been commenced.

In the Stanley-Boucher calculator (Fig. 13) a small centre scale is added, on which a finger indicates automatically the number of digits to be added or deducted; the method of calculating, however, differs from the foregoing. To avoid turning back to 0 at the commencement of each calculation, a circle on the glass face is ground, so that a pencil mark can be made thereon to show the position of the finger when commencing a calculation.

To Find the Square of a Number.—Set the number, on the square-root scale, to the index (using the inner circle for odd numbers, or the second circle for even numbers), and read the required square on the ordinary scale.

To Find the Square Root of a Number.—Set the number to the index, and if the number is *odd*, read the root on the inner circle; if even, on the second circle.

To Find the Cube of a Number.—Set 1 on the ordinary scale to the index, and the pointer (on the back dial) to the number on one of the three cube-root scales. Then under the pointer read the cube on the ordinary scale.

To Find the Cube Root of a Number.—Set 1 to index, and pointer to number. Then read the cube root under the pointer on one of the three inner circles on the back dial. If the number has

1, 4, 7, 10 or −2, −5, etc., digits, use the inner circle.
2, 5, 8, 11 or −1, −4, etc., ,, ,, second circle.
3, 6, 9, 12 or −0, −3, etc., ,, ,, third circle.

For Powers or Roots of Higher Denomination.—Set 1 to index, the pointer to the number on the ordinary scale, and read on the outer circle on the back dial the mantissa of the logarithm. Add the characteristic (see page 44), multiply by the power or divide by the root, and set the pointer to the mantissa of the result on this outer circle. Under the pointer on the ordinary scale read the number, obtaining the number of figures from the characteristic.

To Find the Sines of Angles.—Set 1 to index, pointer to the angle on the outer circle, and read under the pointer the *natural sine* on the ordinary scale; also under the pointer on the outer circle of the back dial read the *logarithmic sine.*

A simpler and less-expensive form of Boucher calculator, lately introduced by the Scientific Publishing Company, is shown in Fig. 14. As before, the dial is turned by the milled head A, and the pointer C by the milled head B.

D is a fixed index extending over all the scales. There is no back dial in this instrument.

The outer circle is an evenly-divided scale, on which the logarithms of the numbers on the second scale can be read off by means of the pointer. The second circle is a scale of ordinary numbers, and the third and fourth a scale giving square roots, arranged as in the previously-described instrument. The inner circle is a scale of sines of angles, the values being found by setting the pointer to the angle and reading the value of the sine under the pointer on the

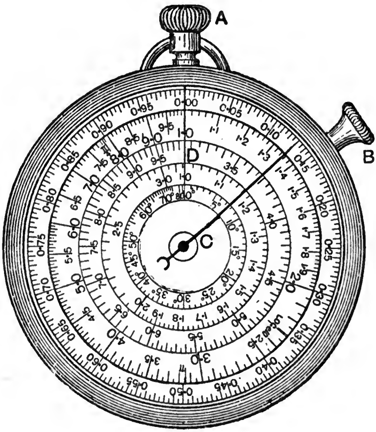

FIG. 14.

outer circle. Cubes may be obtained by multiplying the square of the number by the number, but for cube roots the method previously described for finding the higher powers and roots must be used. In other respects the general rules and instructions previously given apply equally to this instrument.

LONG-SCALE SLIDE RULES.

It has been shown in the earlier pages of this work that the degree of accuracy attainable in slide-rule calculations depends directly upon the length of the logarithmic scale employed. Considerations of general convenience, however, render simple straight-scale rules of more than 20in. in length inadmissible, so that inventors of long-scale slide rules, in order to obtain a high degree of precision, combined with convenience in manipulation, have been compelled to modify the arrangement of scales usually employed. The principal methods adopted may be classed under three varieties: (1) The use of a long scale in sectional lengths, as in Hannyngton's Extended Slide Rule and Thatcher's Calculating Instrument; (2) the employment of a long scale laid in spiral form upon a disc, as in Fearnley's Universal Calculator and Schuerman's Calculating Instrument; and (3) the adoption of a long scale wound helically upon a cylinder, of which Fuller's and the "R. H. S." calculating rules are examples.

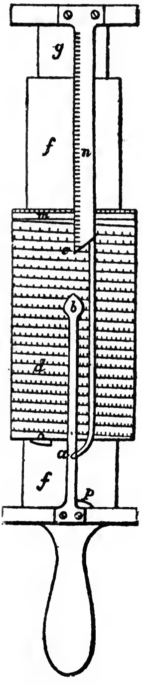

FIG. 15.

Fuller's Calculating Rule.—This instrument, which is shown in Fig. 15, consists of a cylinder *d* capable of being moved up and down and around the cylindrical stock *f*, which is held by the handle. The logarithmic scale-line is arranged in the form of a helix upon the surface of the cylinder *d*, and as it is equivalent to a straight scale of 500 inches, or 41ft. 8in., it is possible to obtain four, and frequently five, figures in a result.

Upon reference to the figure it will be seen that three indices are employed. Of these, that lettered *b* is fixed to the handle; while two others, *c* and *a* (whose distance apart is equal to the axial length of the complete helix), are fixed to the innermost cylinder *g*. This latter cylinder slides telescopically in the stock *f*, enabling the indices to be placed in any required position relatively to *d*. Two other scales are provided, one (*m*) at the upper end of the cylinder *d*, and the other (*n*) on the movable index.

In using the instrument a given number on *d* is set to the fixed index *b*, and either *a* or *c* is brought to another number on the scale. This establishes a ratio; and if the cylinder is now moved so as to bring any number to *b*, the fourth term of the proportion will be found under *a* or *c*. Of course, in multiplication, one factor is brought to *b*, and *a* or *c* brought to 100. The other factor is then brought to *a* or *c*, and the result read off under *b*. Problems involving continuous multiplication, or combined multiplication and division, are very readily dealt with; while by means of the scales *m* and *n*, logarithms of numbers, and hence powers and roots of any magnitude, may be obtained. The instrument illustrated is supplied by Mr. W. F. Stanley.

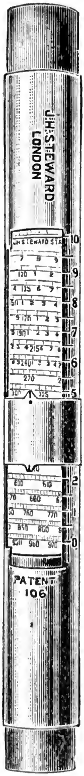

Fig. 16.

The "R. H. S." Calculator.—In this calculator, designed by Prof. R. H. Smith, the scale-line, which is 50in. long, is also arranged in a spiral form (Fig. 16), but in this case it is wrapped around the central portion of a tube which is about $\frac{3}{4}$in. in diameter and $9\frac{1}{2}$in. long. A slotted holder, capable of sliding upon the plain portions of this tube, is provided with four horns, these being formed at the ends of the two wide openings through which the scale is read. An outer ring carrying two horns completes the arrangement.

One of the horns of the holder being placed in agreement with the first factor, and one of the horns of the ring with the second factor, the holder is moved until the third factor falls under the same horn of the ring, when the resulting fourth term will be found under the same (right or left) horn of the holder, at either end of the slot. In multiplication, 100 or 1000 is taken for the second factor in the above proportion, as already explained in connection with Fuller's rule; indeed, generally, the method of using is essentially similar with both instruments.

The scale shown on one edge of the opening in the holder, together with the circular scale at the top of the spiral, enables the mantissæ of logarithms of numbers to be obtained, and thus problems involving powers and roots may be dealt with quite readily. This instrument is supplied by Mr. J. H. Steward, Strand, London.

Sectional Length or Gridiron Slide Rules.—The idea of breaking up a long scale into sectional lengths appears to have been due to Dr. J. D. Everett, who described such a gridiron type of slide rule in a paper read before the British Association in 1866. In Hannyngton's Extended Slide Rule the same principle is carried out in a more substantial manner. In both instruments, however, the lower scale is duplicated. Henry Cherry (1880) appears to have been the first to show that such duplication could be avoided by providing two fixed index points in addition to the natural indices of the scale. These additional indices are shown at 10′ and 100′ in Fig. 17, which represents the lower sheet of Cherry's Calculator on a reduced scale. The upper member of the calculator consists of a

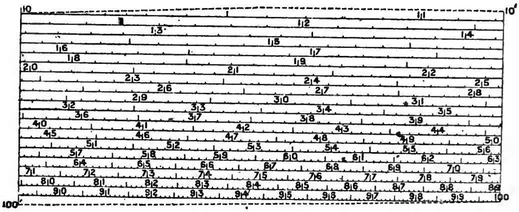

FIG. 17.

transparent sheet ruled with parallel lines, which coincide with the lines of the lower scale when the indices of both are placed in agreement. To multiply two numbers together, one of the indices on the upper sheet is placed to one of the factors, and the position of whichever index falls under the transparent sheet is noted on the latter. Bringing the latter point to the other factor, the result is found under whichever index lies on the card. In other arrangements the inventor used transparent scales, the graduations running in a reverse direction to those of the lower scale. In this case, a factor on the upper scale is set to the other factor on the lower, and the result read off at whichever index is available. A similar arrangement has recently been introduced in Germany bearing the title of Proell's Calculating Scale.

SLIDE RULES FOR SPECIFIC CALCULATIONS.

Of slide rules for special calculations there are already numerous examples, and as their advantages are now being more generally recognised, there is every reason to anticipate that a widely-extended use will be made of these very convenient calculators in the near future. Although based upon precisely the same principle as the ordinary form of slide rule, these special instruments possess advantages over the former, inasmuch as it becomes possible to arrange matters so that each division-line of the several scales represents a definite quantity, and thus all difficulty regarding the position of the decimal point is removed. Further,

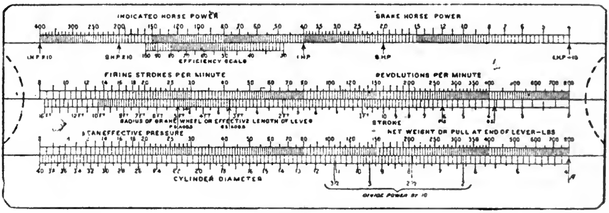

FIG. 18.

the graduations are distinguished by "plain figures" representing absolute factors in the formula dealt with, and the result is read off as an absolute quantity without mental effort or hesitation. Another advantage is that the method of using such instruments is usually so apparent that no specific instructions need be committed to memory, since it is generally only necessary to set the successive pairs of factors in coincidence, when the result can be at once read off. Moreover, the effect upon the result of varying in any way one or more of the factors, can be immediately seen.

Engine Power Computer.—An example of this type of special slide rule is shown in Fig. 18, which represents, on a scale of about half full size, the author's Power Computer for Steam, Gas, and Oil Engines. This, as will be seen, consists of a stock, on the lower portion of which is a scale of cylinder diameters, while the upper portion carries a scale of horse-powers. In the groove between these scales are two slides, also carrying scales, and capable of sliding in edge contact with the stock and with each other.

To find the *Brake Horse-power* of an engine by this instrument, the net weight in pounds acting on the brake strap (or the net weight at the end of the brake lever) is set to the arrow "A." Then setting the revolutions per minute to the effective radius of the brake wheel (or length of lever), the brake horse-power is read on the upper scale over the arrow "B.H.P."

In determining the *Indicated Horse-power* of gas or oil engines, the mean effective pressure, in pounds per square inch, is set to coincide with the cylinder diameter on the lower scale, after which, the number of firing strokes per minute being set to the length of stroke, the indicated horse-power is read over arrow "I.H.P." In the case of steam engines, the number of firing strokes is, of course, the number of working strokes. If in the latter calculation, the piston speed is given in place of stroke length and number of strokes, the given piston speed is set to 1ft. on the "stroke" scale and the result read off as before. Other calculations may also be very readily effected. Thus the dimensions of an engine to develop a given power may be found by setting the "I.H.P." arrow to the required indicated horse-power, and the length of stroke to the number of strokes per minute, when under the mean effective pressure is found the required cylinder diameter. To determine the mechanical efficiency of an engine, 100 on the efficiency scale is set to the indicated horse-power on the upper scale, when under the brake horse-power on the same scale, the percentage efficiency is read off. The calculation of piston speed, velocity ratios of pulleys and gear wheels, the circumferential speed of pulleys, and the velocity of belts and ropes driven thereby, are among the other principal purposes for which the computer may be advantageously employed.

RECENT IMPROVEMENTS IN SLIDE RULES.

Constructional Improvements.—The attention of instrument makers is now being given to the devising of means for ensuring the smooth and even working of the slide in the stock of the rule. In some cases very good results are obtained by slitting the back of the stock to give more elasticity. Mr. A. W. Faber provides a side spring which presses against one edge of the slide, and ensures smooth working throughout the whole length of the rule.

In a simple method of attaining the same end, recently adopted by Messrs. Dennert and Pape, the stock of the rule is formed of two strips of boxwood secured to a strip of sheet celluloid, the latter being slightly curved in cross section so as to cause the sides of the stock to close on the slide with an elastic grip. In the rule made by the Keuffel and Esser Company, of New York, one strip is made

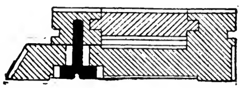

Fig. 19.

adjustable, allowing the fit of the slide to be regulated as desired (Fig. 19). Several makers now further secure the celluloid facings by screwing them to the stock; the latter being lengthened somewhat for this purpose, also provides a more effective support for the cursor when used near the ends of the rule.

Hall's Nautical Slide Rule consists of two slides fitting in grooves in the stock, and provided with eight scales, two on each slide, and one on each edge of each groove. While fulfilling the purposes of an ordinary slide rule, it is of especial service to the practical navigator in connection with such problems as the "reduction of an ex-meridian sight" and the "correction of chronometer sights for error in latitude." The rule, which has many other applications of a similar character, is made by Mr. J. H. Steward, Strand, London.

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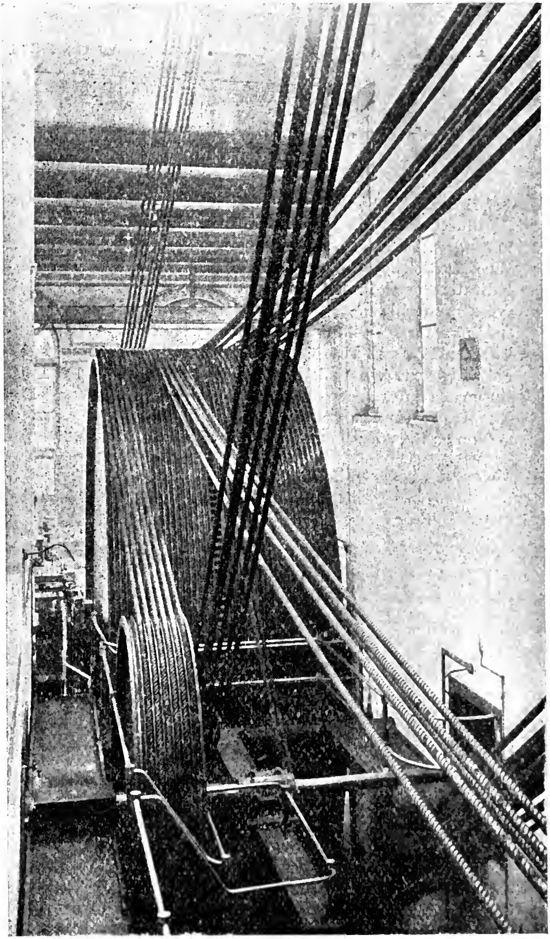

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