SMITHSONIAN

CONTRIBUTIONS TO KNOWLEDGE.

VOL. IX.

EVERY MAN IS A VALUABLE MEMBER OF SOCIETY, WHO, BY HIS OBSERVATIONS, RESEARCHES, AND EXPERIMENTS, PROCURES KNOWLEDGE FOR MEN.—SMITHSON.

CITY OF WASHINGTON:
PUBLISHED BY THE SMITHSONIAN INSTITUTION.
MDCCCLVII.

ADVERTISEMENT.

This volume forms the ninth of a series, composed of original memoirs on different branches of knowledge, published at the expense, and under the direction, of the Smithsonian Institution. The publication of this series forms part of a general plan adopted for carrying into effect the benevolent intentions of James Smithson, Esq., of England. This gentleman left his property in trust to the United States of America, to found, at Washington, an institution which should bear his own name, and have for its objects the "*increase* and *diffusion* of knowledge among men." This trust was accepted by the Government of the United States, and an Act of Congress was passed August 10, 1846, constituting the President and the other principal executive officers of the general government, the Chief Justice of the Supreme Court, the Mayor of Washington, and such other persons as they might elect honorary members, an establishment under the name of the "Smithsonian Institution for the increase and diffusion of knowledge among men." The members and honorary members of this establishment are to hold stated and special meetings for the supervision of the affairs of the Institution, and for the advice and instruction of a Board of Regents, to whom the financial and other affairs are entrusted.

The Board of Regents consists of three members *ex officio* of the establishment, namely, the Vice-President of the United States, the Chief Justice of the Supreme Court, and the Mayor of Washington, together with twelve other members, three of whom are appointed by the Senate from its own body, three by the House of Representatives from its members, and six persons appointed by a joint resolution of both houses. To this Board is given the power of electing a Secretary and other officers, for conducting the active operations of the Institution.

To carry into effect the purposes of the testator, the plan of organization should evidently embrace two objects: one, the increase of knowledge by the addition of new truths to the existing stock; the other, the diffusion of knowledge, thus increased, among men. No restriction is made in favor of any kind of knowledge; and, hence, each branch is entitled to, and should receive, a share of attention.

The Act of Congress, establishing the Institution, directs, as a part of the plan of organization, the formation of a Library, a Museum, and a Gallery of Art, together with provisions for physical research and popular lectures, while it leaves to the Regents the power of adopting such other parts of an organization as they may deem best suited to promote the objects of the bequest.

After much deliberation, the Regents resolved to divide the annual income into two equal parts—one part to be devoted to the increase and diffusion of knowledge by means of original research and publications—the other half of the income to be applied in accordance with the requirements of the Act of Congress, to the gradual formation of a Library, a Museum, and a Gallery of Art.

The following are the details of the parts of the general plan of organization provisionally adopted at the meeting of the Regents, Dec. 8, 1847.

DETAILS OF THE FIRST PART OF THE PLAN.

I. To increase Knowledge.—*It is proposed to stimulate research, by offering rewards for original memoirs on all subjects of investigation.*

1. The memoirs thus obtained, to be published in a series of volumes, in a quarto form, and entitled "Smithsonian Contributions to Knowledge."

2. No memoir, on subjects of physical science, to be accepted for publication, which does not furnish a positive addition to human knowledge, resting on original research; and all unverified speculations to be rejected.

3. Each memoir presented to the Institution, to be submitted for examination to a commission of persons of reputation for learning in the branch to which the memoir pertains; and to be accepted for publication only in case the report of this commission is favorable.

4. The commission to be chosen by the officers of the Institution, and the name of the author, as far as practicable, concealed, unless a favorable decision be made.

5. The volumes of the memoirs to be exchanged for the Transactions of literary and scientific societies, and copies to be given to all the colleges, and principal libraries, in this country. One part of the remaining copies may be offered for sale; and the other carefully preserved, to form complete sets of the work, to supply the demand from new institutions.

6. An abstract, or popular account, of the contents of these memoirs to be given to the public, through the annual report of the Regents to Congress.

II. To increase Knowledge.—*It is also proposed to appropriate a portion of the income, annually, to special objects of research, under the direction of suitable persons.*

1. The objects, and the amount appropriated, to be recommended by counsellors of the Institution.

2. Appropriations in different years to different objects; so that, in course of time, each branch of knowledge may receive a share.

3. The results obtained from these appropriations to be published, with the memoirs before mentioned, in the volumes of the Smithsonian Contributions to Knowledge.

4. Examples of objects for which appropriations may be made:—

(1.) System of extended meteorological observations for solving the problem of American storms.

(2.) Explorations in descriptive natural history, and geological, mathematical, and topographical surveys, to collect materials for the formation of a Physical Atlas of the United States.

(3.) Solution of experimental problems, such as a new determination of the weight of the earth, of the velocity of electricity, and of light; chemical analyses of soils and plants; collection and publication of articles of science, accumulated in the offices of Government.

(4.) Institution of statistical inquiries with reference to physical, moral, and political subjects.

(5.) Historical researches, and accurate surveys of places celebrated in American history.

(6.) Ethnological researches, particularly with reference to the different races of men in North America; also explorations, and accurate surveys, of the mounds and other remains of the ancient people of our country.

I. To diffuse Knowledge.—*It is proposed to publish a series of reports, giving an account of the new discoveries in science, and of the changes made from year to year in all branches of knowledge not strictly professional.*

1. Some of these reports may be published annually, others at longer intervals, as the income of the Institution or the changes in the branches of knowledge may indicate.

2. The reports are to be prepared by collaborators, eminent in the different branches of knowledge.

3. Each collaborator to be furnished with the journals and publications, domestic and foreign, necessary to the compilation of his report; to be paid a certain sum for his labors, and to be named on the title-page of the report.

4. The reports to be published in separate parts, so that persons interested in a particular branch, can procure the parts relating to it, without purchasing the whole.

5. These reports may be presented to Congress, for partial distribution, the remaining copies to be given to literary and scientific institutions, and sold to individuals for a moderate price.

The following are some of the subjects which may be embraced in the reports:—

I. PHYSICAL CLASS.

1. Physics, including astronomy, natural philosophy, chemistry, and meteorology.
2. Natural history, including botany, zoology, geology, &c.
3. Agriculture.
4. Application of science to arts.

II. MORAL AND POLITICAL CLASS.

5. Ethnology, including particular history, comparative philology, antiquities, &c.
6. Statistics and political economy.
7. Mental and moral philosophy.
8. A survey of the political events of the world; penal reform, &c.

III. LITERATURE AND THE FINE ARTS.

9. Modern literature.
10. The fine arts, and their application to the useful arts.
11. Bibliography.
12. Obituary notices of distinguished individuals.

II. To DIFFUSE KNOWLEDGE.—*It is proposed to publish occasionally separate treatises on subjects of general interest.*

1. These treatises may occasionally consist of valuable memoirs translated from foreign languages, or of articles prepared under the direction of the Institution, or procured by offering premiums for the best exposition of a given subject.

2. The treatises to be submitted to a commission of competent judges, previous to their publication.

DETAILS OF THE SECOND PART OF THE PLAN OF ORGANIZATION.

This part contemplates the formation of a Library, a Museum, and a Gallery of Art.

1. To carry out the plan before described, a library will be required, consisting, 1st, of a complete collection of the transactions and proceedings of all the learned societies in the world; 2d, of the more important current periodical publications, and other works necessary in preparing the periodical reports.

2. The Institution should make special collections, particularly of objects to verify its own publications. Also a collection of instruments of research in all branches of experimental science.

3. With reference to the collection of books, other than those mentioned above, catalogues of all the different libraries in the United States should be procured, in order that the valuable books first purchased may be such as are not to be found elsewhere in the United States.

4. Also catalogues of memoirs, and of books in foreign libraries, and other materials, should be collected, for rendering the Institution a centre of bibliographical knowledge, whence the student may be directed to any work which he may require.

5. It is believed that the collections in natural history will increase by donation, as rapidly as the income of the Institution can make provision for their reception; and, therefore, it will seldom be necessary to purchase any article of this kind.

6. Attempts should be made to procure for the gallery of art, casts of the most celebrated articles of ancient and modern sculpture.

7. The arts may be encouraged by providing a room, free of expense, for the exhibition of the objects of the Art-Union, and other similar societies.

8. A small appropriation should annually be made for models of antiquity, such as those of the remains of ancient temples, &c.

9. The Secretary and his assistants, during the session of Congress, will be required to illustrate new discoveries in science, and to exhibit new objects of art; distinguished individuals should also be invited to give lectures on subjects of general interest.

In accordance with the rules adopted in the programme of organization, each memoir in this volume has been favorably reported on by a Commission appointed for its examination. It is however impossible, in most cases, to verify the statements of an author; and, therefore, neither the Commission nor the Institution can be responsible for more than the general character of a memoir.

The following rules have been adopted for the distribution of the quarto volumes of the Smithsonian Contributions:—

1. They are to be presented to all learned societies which publish Transactions, and give copies of these, in exchange, to the Institution.

2. Also, to all foreign libraries of the first class, provided they give in exchange their catalogues or other publications, or an equivalent from their duplicate volumes.

3. To all the colleges in actual operation in this country, provided they furnish, in return, meteorological observations, catalogues of their libraries and of their students, and all other publications issued by them relative to their organization and history.

4. To all States and Territories, provided there be given, in return, copies of all documents published under their authority.

5. To all incorporated public libraries in this country, not included in any of the foregoing classes, now containing more than 7000 volumes; and to smaller libraries, where a whole State or large district would be otherwise unsupplied.

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SMITHSONIAN CONTRIBUTIONS TO KNOWLEDGE.

ON THE RELATIVE INTENSITY

OF THE

HEAT AND LIGHT OF THE SUN

UPON DIFFERENT

LATITUDES OF THE EARTH.

BY

L. W. MEECH, A. M.

[ACCEPTED FOR PUBLICATION, SEPTEMBER, 1855.]

COMMISSION

TO WHICH THIS PAPER HAS BEEN REFERRED.

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INTRODUCTION.

The regular and almost uniform variations which meteorological tables exhibit, indicate a periodical cause of change, which evidently resides in the sun. The inquiry then arises, may not these variations be determined by theory from the apparent course of the sun? The first part of the present investigation thus suggested by inspection of monthly temperatures, was published in *Silliman's Journal of Science* for 1850. Since then, considerable extensions have been made, including expressions for annual values; a view of the whole of which is given in the following pages. At some future time, the researches may be resumed in another series.

The object of the investigation here presented, is to resolve the problem of solar heat and light, to the extent of the principle, that the intensity of the sun's rays, like gravitation, varies inversely as the square of the distance, without resorting to any other hypothesis. The principle is but a geometrical consequence of the divergence of the rays. This elementary view thus presents the sun shining upon a distant planet, and indicates the sum of the intensities received at the planet's surface in all its various phases of position and inclination.

In relation to the earth especially, the sum of the intensities must be referred to the exterior limit of the atmosphere which surrounds the globe. This condition, which is perhaps necessary in the present state of science, has the advantage of rendering the formulas as rigorously accurate as are the propositions of geometry and the conic sections.

Poisson, in 1835, observed that, "for the completion of the theory of heat, it is necessary that it should comprise the determination of the movements produced in aeriform fluids, in liquids and even in solid bodies; but geometers have not yet resolved this order of questions, of great difficulty, with which are connected the phenomena of the trade-winds, of certain currents observed in the sea, and the diurnal variations of the barometer." The subject is believed to be now included among the prize questions of the French Academy, and in the increasing number of researches, it is hoped that its difficulties may at length be effectively obviated.

The laws of Solar Intensity here derived *à priori*, have a general accordance with physical phenomena, and will furnish instructive comparisons with analogous values obtained by meteorological observations. The changes of the sun's intensity upon the inaccessible regions of the Pole will be included, to which the late

Arctic explorations have given unusual interest. And, among other advantages, light will be thrown upon geological researches relating to changes of the heat of the globe at very remote epochs.

It will be proper also to observe, that the method of summation, of which examples are given in the fifth and ninth sections, is more simple and direct than the process of discontinuous functions. The general reader, however, passing over the algebraic analysis, which is but a means for carrying out the leading conception already stated, will find the conclusions which flow from it plainly discussed in the remaining paragraphs of the several sections, and illustrated by tables and the accompanying curves.

At the close, the course of investigation has led to the development of a peculiar inequality in the annual duration of sunlight. The like series of values for the duration of twilight is also new, and will not be devoid of interest. But the main design has been—distinguishing between the sun's intensity and terrestrial temperatures—to carry out one comprehensive principle, by which the laws of the sun's intensity of heat and light are obtained to some degree of completeness, as a system, embracing the following topics in order:—

Section I. *Irradiated Surface upon the Planets.*—Zone of Differential Radiation; its Breadth and Area; its Extension by Refraction; its Changes of Position.

Section II. *The Sun's Intensity upon the Planets, in relation to their Orbits.*—Intensity proportional to the true longitude described. Table of Relative Intensity in equal times and in entire revolutions. Resemblance of the Earth to the planet Mars. Equality of Intensities during the four Seasons.

Section III. *Law of the Sun's Intensity at any Instant during the day.*—It is proportional to the length of a perpendicular line from the Sun's Centre to the Horizon. The Atmosphere. Causes of Climate.

Section IV. *The Sun's Diurnal Intensity.*—It depends on the Latitude, the Sun's Declination, Hour-angle, and Distance. Intensity upon the North Pole, during Summer, greater than upon the Equator. Graphical comparison of Intensities with Temperatures. Average Rate of Solar Intensity per hour. Retardation of the effects of the Sun's Intensity. Indication of Equatorial, Tropical, and Polar Calms.

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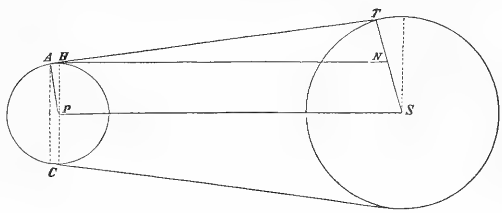
ON THE

RELATIVE INTENSITY OF THE HEAT AND LIGHT OF THE SUN.

SECTION I.

ON THE PROPORTION OF A PLANET'S SURFACE WHICH IS IRRADIATED BY THE SUN AT ANY GIVEN TIME.

It is evident that the extreme rays proceeding from the sun to the planet are *tangent* to the two spheres, as shown in the annexed diagram; where S denotes the centre of the sun, and P that of the planet.



Let $PS = \rho$, the radius-vector, or distance of the planet's centre from that of the sun.

Let $ST = R$, the radius of the sun, and $PA = r$, the radius of the planet, regarded as a sphere.

Through P, let a plane be drawn perpendicular to PS, and dividing the planet's surface into two equal hemispheres. The sun, being the greater body, illuminates not only the adjacent hemisphere of the planet, but also the zone or belt, AC, lying beyond; which may be called *the Zone of differential radiation.*

Let the angular breadth of this zone $APH = z$, and, drawing AN or ρ parallel to PS, the angle TAN is obviously equal to APH or z, since the including sides of the one angle are respectively perpendicular to those of the other, and, therefore, have the same relative inclination. Then, in the triangle ATN, which is right angled at T, by the condition of tangency,

$$\sin TAN = \frac{NT}{AN}, \quad \text{or } \sin z = \frac{R - r}{\rho}. \qquad (1.)$$

That is, *the sine of the angular breadth of the zone of differential radiation is equal to the difference of the radii of the sun and planet divided by the radius-vector of the planet's orbit.*

To express this value in another form, let A denote the semi-transverse axis of the planet's orbit, or its mean distance from the sun; let e denote the ratio of eccentricity, and θ the true anomaly estimated from the perihelion; then, by Analytical Geometry, $\rho = \frac{A\,(1 - e^2)}{1 + e\ cos\ \theta}$; and hence,

$$sin\ z = \frac{(R - r)\,(1 + e\ cos\ \theta)}{A\,(1 - e^2)}. \qquad (2.)$$

Here for special values, making $cos\ \theta$ successively equal to $-e$, $+1$, -1, and cancelling factors, we obtain for the values of $sin\ z$ in order:—

$$\left.\begin{array}{ll} \text{Average,} & sin\ z = \frac{R - r}{A}. \\ \text{Maximum,} & sin\ z = \frac{R - r}{A\,(1 - e)}. \\ \text{Minimum,} & sin\ z = \frac{R - r}{A\,(1 + e)}. \end{array}\right\} \qquad (3.)$$

Again, taking the length of arc z in the circle whose radius is 1, the breadth of the differential zone upon the planet will be rz; but since, for all the planets, z is less than 1°, its *sine* may be substituted from either of the former equations, and the same value essentially is represented by

$$\text{Linear breadth of zone} = r\ sin\ z. \qquad (4.)$$

It is also proved in Geometry that the surface of a sphere whose radius is r, is equal to $4 r^2 \pi$; π denoting 3.141592; and that the surface of a spherical zone is equal to its altitude multiplied by $2 r \pi$. Now, the altitude of the zone of differential radiation is, in ratio to that of the whole planet, as $sin\ z$ to 2, or $\frac{1}{2}\ sin\ z$ to 1. Hence, representing the whole area of the planet by 1.

$$\text{The } \textit{proportion} \text{ of irradiated surface} = \tfrac{1}{2} + \tfrac{1}{2}\ sin\ z. \qquad (5.)$$

$$\text{Whole surface irradiated} = (\tfrac{1}{2} + \tfrac{1}{2}\ sin\ z)\ 4 r^2 \pi. \qquad (6.)$$

$$\text{Surface of the zone} = 2 r^2 \pi\ sin\ z. \qquad (7.)$$

If r be taken in miles, the area will be given in square miles.

The following table exhibits some of the primary phases of solar intensity upon the planets; and was obtained by substituting the proper astronomic elements in formulæ (3), (4), and (5).

PLANET.	Average breadth of zone.	Greatest breadth of zone.	Least breadth of zone.	Proportion of surface irradiated.
	Miles.	Miles.	Miles.	
Mercury	17.89	22.32	14.96	.505991
Venus	61.12	61.54	60.70	.503190
Earth	18.29	18.60	17.98	.500231
Mars	6.42	7.07	5.87	.500152
Vesta	.26	.28	.24	.500980
Jupiter	34.87	36.62	33.28	.500404
Saturn	18.17	19.25	17.21	.500222
Uranus	4.01	4.20	3.83	.500117
Neptune	6.14	6.19	6.08	.500087

In obtaining these tabular results, the earth's mean distance from the sun was taken at 95,273,870 miles, and its radius at 3,962 miles.

It will be perceived that the vast *magnitude* of the sun brings advantages of temperature and sunlight similar to those which the preponderance of its *mass* gives to the steadiness and uniformity of the planetary revolutions. Were the same amount of heat and light, radiated from a smaller body like the Moon, the effects would be restricted to a smaller portion of the Earth's surface; and the zone of differential radiation would be reversed to one of cold and darkness. But in the present beneficent arrangement, light and heat preponderate, counteracting extremes of heat and cold with a warmer temperature. And this effect is further prolonged by atmospheric refraction and reflection of the rays, which, rendering the transitions more mild and gradual, lessens the reign of night.

To estimate this effect of the *Refraction of Light*, we have only to find two points on the spherical surface of the earth, at such distance that the inclination of the two tangent rays from the Sun falling on them, shall be just equal to the horizontal refraction. The terrestrial radii drawn to these points will evidently be inclined at the same angle as their tangents, which is 34′ nearly, or 40 English miles. Thus it appears that the effect of refraction in widening the irradiated zone of the earth is more than twice as great as that arising from the apparent semi-diameter, or the mere size of the sun. Uniting the two effects, the sun is found to illuminate more than half the Earth's surface by a belt or zone that is 58 miles in width, encircling the seas and continents of the globe.

The advantage of the vast size of the sun is most conspicuous upon the planet Venus, our evening and morning star, where the belt of illumination is sixty-one miles in width, as shown in the preceding table. The next in rank is Jupiter, whose belt of greater illumination is thirty-five miles wide; while those of Mercury, the Earth, and Saturn, are nearly eighteen miles in breadth. In the last column of the table, it will be observed that the asteroid Vesta, though situated beyond Mars, yet has, in consequence of its smaller size, a greater proportion of illuminated surface than the Earth.

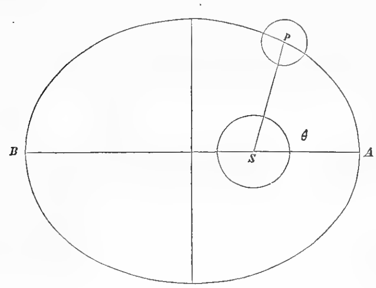
From formula (7), it is found that the zone of differential illumination upon the Earth extends over 455,400 square miles; or, including the additional area due to 34′ horizontal refraction, it comprehends an aggregate of 1,430,800 square miles of surface. The position of this great zone is continually changing, and in turn it overspreads every island, sea, and continent. At the vernal equinox, when the Sun is vertical to the Equator, it will readily be perceived that the larger base of this zone is a great circle passing through the Poles and having the Earth's axis for its diameter. From this position it gradually diverges, till at the summer solstice, one extremity of its diameter will be in the Arctic, and the other in the Antarctic circle. Thence it gradually returns to its former position at the Poles at the autumnal equinox, all the while revolving like a fringed circle around the globe, and accompanied with the lustrous tints and shadows which variegate the dawn and close of day.

SECTION II.

LAW OF THE SUN'S INTENSITY UPON THE PLANETS IN RELATION TO THEIR ORBITS.

The preceding Section represents the Sun's action upon a distant planet at a given distance, or at rest. It is here proposed to examine the effect when the distance is variable; that is, supposing the planet to commence its motion from a state of rest, in an elliptical orbit, to determine the intensity received during its passage through any part, or the whole of its orbit.

In the annexed figure, let S denote the Sun situated in one focus; P the Planet's position at a given time; A, the perihelion or point in the orbit nearest the sun, and B, the aphelion or point farthest from the sun.



Let SP or ρ denote the radius-vector; ASP or θ, the true anomaly; e, the ratio of eccentricity; and $a+nt$, the mean anomaly; n, being the mean motion in the unit of time.

If A denote the semi-transverse axis, it is well known that $A^2\pi\sqrt{1-e^2}$ will express the whole area of the ellipse, and $\int\frac{1}{2}\rho^2\,d\theta$, the area of the elliptic sector corresponding to θ, where π denotes 3.14 1592, or a semi-circumference. Hence by Kepler's law, that equal areas are described by the radius-vector in equal times,

$$A^2\pi\sqrt{1-e^2} : \int\tfrac{1}{2}\rho^2\,d\theta :: 2\pi : a+nt.$$

Reducing to an equation and differentiating,

$$\frac{1}{\rho^2}=\frac{d\theta}{A^2\,ndt\sqrt{1-e^2}}. \qquad (8.)$$

Since heat and light vary inversely as the square of the distance ρ, the second member evidently measures their intensity at any instant. Then, as pointed out in the Calculus, we may regard the second member as the ordinate, and the time t as the abscissa of a curve. Multiplying the equation by dt, therefore, and integrating between the limits of any two anomalies, θ and θ', we obtain for the sum of the intensities,

$$\int\frac{1}{\rho^2}\,dt=\frac{\theta-\theta'}{A^2 n\sqrt{1-e^2}}. \qquad (9.)$$

In interpreting this result, we know that the orbitual motion of a planet is not uniform, being accelerated in perihelion and retarded in aphelion. Hence, in the annual variations of radius-vector, the Earth does not receive equal increments of heat and light in equal times; but *the amount received in any given interval, is exactly proportional to the true anomaly or true Longitude described in that interval.*

This important law appears to have been first published in the Pyrometry of Lambert.

This point being established, let us, in the next place, compare the intensities received by the Planets during entire revolutions in their orbits. In the preceding formula, making $\theta - \theta'$ equal to an entire circumference, the sum of the intensities during a complete revolution, is found to be $u = \frac{2\pi}{A^2 n \sqrt{1-e^2}}$. Let this refer to the earth, and accenting the values for any other planet, $u' = \frac{2\pi}{A'^2 n' \sqrt{1-e'^2}}$. Now n, n', being inversely proportional to the planets' periodic times, we have by the third law of Kepler, $n^2 : n'^2 :: A'^3 : A^3$, or $n A^{\frac{3}{2}} = n' A'^{\frac{3}{2}}$. Whence by substitution and division, we obtain for the *relative intensity* upon any planet *in an entire revolution*,

$$\frac{u'}{u} = \frac{\sqrt{A(1-e^2)}}{\sqrt{A'(1-e'^2)}}. \qquad (10).$$

In like manner, the ratio of intensity *for equal times*, depending simply on the inverse square of the distances, will be represented by

$$\frac{u'}{u} = \frac{A^2}{A'^2}. \qquad (11).$$

With these last two formulas, the following table has been prepared from the usual astronomic elements:—

The Sun's Relative Intensity upon the Principal Planets.

Planet.	In a whole revolution.	In equal times.		
		Mean Distance.	Perihelion.	Aphelion.
Mercury	1.643	6.677	10.573	4.592
Venus	1.176	1.911	1.937	1.885
Earth	1.000	1.000	1.034	0.967
Mars	.813	.431	0.524	0.360
Jupiter	.439	.037	.041	.034
Saturn	.324	.011	.012	.010
Uranus	.228	.003	.003	.003
Neptune	.182	.001	.001	.001

It should be observed that the foregoing table does not take account of the different dimensions of the planets, but refers to a unit of plane surface upon their disks, which is exposed perpendicularly to the rays of the perpetual sun. Upon the disk of Mercury, the solar radiation appears to be nearly seven times greater than on the Earth; while upon Neptune, it is only as the one-thousandth part, in equal times. In entire revolutions, however, the intensities received will be seen to approach more nearly to equality.

The intensities are thus unequal; and, by a calculation founded on the apparent brightness of the planets as estimated by the eye, Prof. Gibbes has shown, in the Proceedings of the American Association for the Advancement of Science for 1850, that the reflective powers are also greater, according as the several planets are more distant from the Sun.

Another feature worthy of mention, is the resemblance of the earth to the planet Mars; upon which Sir W. Herschel has remarked: "The analogy between Mars and the Earth is, perhaps, by far the greatest in the whole solar system. The diurnal motion is nearly the same, the obliquity of their respective ecliptics not very different; of all the superior planets, the distance of Mars from the Sun is by far the nearest alike to that of the Earth; nor will the length of the Martial year appear very different from what we enjoy, when compared to the surprising duration of the years of Jupiter, Saturn, and Uranus. If we then find that the globe we inhabit has its polar region frozen and covered with mountains of ice and snow, that only partly melt when alternately exposed to the sun, I may well be permitted to surmise that the same causes may have the same effect on the globe of Mars; that the bright polar spots are owing to the vivid reflection of light from frozen regions; and that the reduction of those spots is to be ascribed to their being exposed to the sun."

Recurring now to equation (9) and the proposition following, it will readily be inferred that during each of the four astronomic seasons of Spring, Summer, Autumn, and Winter, the intensities received from the sun are precisely equal. For in each season, the earth passes over three signs of the zodiac, or a quadrant of longitude. The equality of intensities, however, applies to the entire globe regarded as one aggregate, and is consistent with local alternations, by which it is summer in the northern hemisphere when it is winter in the southern. Deferring the consideration of these local inequalities, however, we may here illustrate the connection of the seasons with the elliptic motion from an ephemeris. In the year 1855, for example, spring in the northern hemisphere, commencing at the vernal equinox March 20th, lasts eighty-nine days; summer, beginning at the summer solstice June 21, continues ninety-three days; autumn, commencing at the equinox, September 23, continues ninety-three days; and winter, beginning at the winter solstice, December 22, lasts ninety days; yet, notwithstanding their unequal lengths, the amounts of heat and light which the whole earth receives are equal in the several periods.[1]

At the present time the earth is in perihelion, or nearest the sun about the 1st of January, and farthest from the sun on the 4th day of July. A special cause must, therefore, be assigned for the striking fact which Professor Dove has shown by comparison of temperatures observed in opposite regions of the globe, namely: that the mean temperature of the habitable earth's surface in June considerably exceeds the temperature in December, although the earth in the latter month is nearer to the sun. This result is attributed by that meteorologist to the greater quantity of land in the northern hemisphere exposed to the rays of the sun at the summer solstice in June; while the ocean area has less power for this object, as it absorbs a large portion of the heat into its depths. Had land and water been equally distributed; in other words, were the earth a homogeneous sphere, the alleged inequality of temperature, it is obvious, would never have existed.

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[1] Since the earth is not strictly a sphere, but an oblate spheroid, it evidently presents its least section perpendicular to the rays of the sun at the equinoxes. As the sun's declination increases, the section also increases and attains its limit at the solstice. The variation, however, appears to be not material, and compensates itself in each season.

SECTION III.

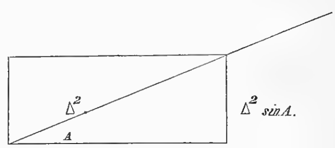
LAW OF THE SUN'S INTENSITY AT ANY INSTANT DURING THE DAY.

The rays which emanate from the Sun's disk into space proceed in diverging lines in the same manner as if they issued directly from the centre. And, on arriving at the Earth, their intensity as before stated will be inversely proportional to the square of the distance.

But the more obvious phenomena of solar heat and light are manifested to us under a secondary law. The Sun's intensity first becomes sensible in the eastern rays of morning; it gradually increases to a maximum during the day; it declines on the approach of the shades of evening, and becomes discontinuous during the night. On the morning following the same course is renewed, and continued successively through the year. Ordinary sensation and experience lead us to associate the degree of solar heat at any part of the day, with the apparent height which the sun has then attained above the horizon. Indeed, theory determines that at four in the afternoon, or any other instant during the day, *the Sun's intensity is proportional to the length of a perpendicular line dropped from the Sun to the plane of the apparent horizon, or varies as the size of the sun's altitude.*

The reason of this secondary law will be understood by regarding the beam of solar rays which traverses in a line from the sun to the observer, to be resolved, according to the parallelogram of forces, into a horizontal and a vertical component. The horizontal component running parallel to the earth's surface is regarded as inoperative, while the vertical component measures the direct heating effect.

This relation is more fully shown in the annexed figure, where A denotes the sun's apparent altitude above the horizon. The sun's intensity or impulse in an oblique direction will be measured by the inverse square of the distance, or the direct square of the sun's apparent semi-diameter Δ. If, therefore, Δ^2 denotes the intensity of the rays in a straight line from the sun, $\Delta^2 \sin A$, *will be the vertical component or heating force of the rays.* And these terms being in ratio as 1 to $\sin A$, the latter component will be represented by a perpendicular line from the sun's centre to the horizon.



Instead of thus decomposing the intensity after the manner of a force in Mechanics, as first proposed by Halley, in 1693, the same law may be obtained in an entirely different way from the principle of the inverse square of the distance. The latter mode appears to present it in a more evident light, and was suggested in the original beginnings of the present investigation, which were published in Silliman's *Journal of Science* for the year 1850.

It proceeds as follows:—

Let L = the 'apparent' Latitude of the place,
D = the sun's meridian Declination,
Δ = the sun's apparent semi-diameter,
A = the sun's Altitude, and
H = the Hour-angle from noon.

Also in reference to future applications, let
T = the sun's true Longitude, and
ω = the obliquity of the Ecliptic.

The horizontal section of a cylindrical beam of rays from the Sun's disk upon a plain on the Earth's surface, is well known to be an ellipse; and if 1 denote the sun's radius, 1 will likewise denote the semi-conjugate axis of this projected ellipse; while the horizontal projection, $\frac{1}{\sin A}$, will be the semi-transverse axis. The area of the elliptic projection is, therefore, $1 \times \frac{1}{\sin A} \times \pi$. But the intensity of the same quantity of heat being inversely as the space over which it is diffused, the reciprocal of this area, or $\sin A$, on rejecting the constant π, will express the sun's heating effect, supposing the distance to be constant for the same day. But, on comparing one day with another, the intensity further varies inversely as the square of the distance, that is, directly as the square of the apparent diameter or semi-diameter of the disk. Hence, generally, $\Delta^2 \sin A$, expresses the sun's intensity at any given instant during the day.

To determine the value of $\sin A$, by spherical trigonometry, the sun's angular distance from the pole, or co-declination, the arc from the pole to the zenith, or co-latitude, and the included hour-angle from noon are given to find the third side or co-altitude. Writing, therefore, sines instead of the cosines of their complements,

$$\begin{aligned} \sin A &= \sin L \sin D + \cos L \cos D \cos H. \\ \Delta^2 \sin A &= \Delta^2 \sin L \sin D + \Delta^2 \cos L \cos D \cos H. \end{aligned} \qquad (12.)$$

At the time of the equinoxes, D becomes 0, and the expression of the sun's intensity reduces to $\Delta^2 \cos L \cos H$. That is, the degree of intensity then decreases from the equator to each pole, and is *proportional to the cosine of the latitude.* At other times of the year, however, a different law of distribution prevails, as indicated by the formula.

The intensity at a fixed distance being as the sine of the altitude, it follows that the sun shining for sixteen hours from an altitude of 30°, would exert the same heating effect upon a plain, as when it shines during eight hours from the zenith; since $\sin 30°$ is 0.5, and $\sin 90°$ is 1. At least, such were the result independently of radiation.

By some writers, the measure of vertical intensity, as the sine of the sun's altitude, has been stated without limitation. Approximately it may apply at the habitable surface of the earth, when the influence of the atmosphere is neglected; yet it is strictly true only at the exterior of the atmospheric envelope which encompasses the globe, or at the outer limit where matter exerts its initial change upon the incident rays.

The distinction here explained has not only engaged the attention of the most eminent meteorologists of modern times, but was equally adopted in ancient philosophy, as appears in the following passage from Plato's Phædon, LVIII: "For around the earth are low shores, and diversified landscapes and mountains, to which are attracted water, the cloud, and air. But the earth, outwardly pure, floats in the pure heaven like the stars, in the medium which those who are accustomed to discourse on such things call ether. Of this ether, the things around are the sediment which always settles and collects upon the low places of the earth. We, therefore, who live in these terraqueous abodes, are concealed, as it were, and yet think we dwell above upon the earth. As one residing at the bottom of the sea might think he lived upon the surface, and, beholding the sun and stars through the water, might suppose the sea to be heaven. The case is similar, that through imperfection we cannot ascend to the highest part of the atmosphere, since, if one were to arrive upon its upper surface, or becoming winged, could reach there, he would on emerging look abroad, and, if nature enabled him to endure the sight, he would then perceive the true heaven and the true light."

In modern times, the researches of Poisson led him to the philosophic conclusion now generally received, that the highest strata of the air are deprived of elasticity by the intense cold; the density of the frozen air being extremely small, *Théorie de la Chaleur*, p. 460. An atmospheric column resting upon the sea may thus be regarded as an elastic fluid terminated by two liquids, one having an ordinary density and temperature, and the other a temperature and density excessively diminished.

Although the sun's intensity, which is here the subject of investigation, is the principal source of heat, yet its effects are modified by proximate causes of climate; of which, the following nine are enumerated by Malte Brun:—

1st.—Action of the sun upon the atmosphere.

2d.—The interior temperature of the globe.

3d.—The elevation above the level of the ocean.

4th.—The general inclination of the surface and its local exposure.

5th.—The position of mountains relative to the cardinal points of the compass.

6th.—The neighborhood of great seas and their relative situation.

7th.—The geological nature of the soil.

8th.—The degree of cultivation and of population to which a country has arrived.

9th.—The prevalent winds.

The same author observes, in relation to the fourth enumerated cause, that northeast situations are coldest; and southwest, warmest. For the rays of the morning which directly strike the hills exposed to the east, have to counteract the cold accumulated there during the night. The heat augments till three in the afternoon, when the rays fall direct upon southwest exposures, and no obstacle now prevents their utmost action.

SECTION IV.

DETERMINATION OF THE SUN'S HOURLY AND DIURNAL INTENSITY.

In the last Section, the sun's vertical intensity upon a given point of the earth's surface at any instant during the day, was proved to be measured by a perpendicular drawn from the centre of the Sun to the plane of the horizon. If perpendiculars be thus let fall at every instant during an hour, the sum of the perpendiculars will evidently represent the sum of the vertical intensities received during the hour, which sum may be termed the Hourly Intensity.

The Integral Calculus furnishes a ready means of obtaining this sum. For during any one day, the sun's distance or apparent semi-diameter, and the meridian Declination, may be regarded as constant, while H alone varies, and the deviations from the implied time of the sun's rising and setting will compensate each other. Therefore, multiplying the equation of instantaneous intensity (12) by $d\,H$, since astronomy shows that H varies uniformly with the time, and integrating between the limits of any two hour angles, H', H'', we obtain an expression for the hourly intensity.

In like manner let H denote the semi-diurnal arc, and integrating between the limits 0 and H, we obtain the intensity for a half day, which, on cancelling the constant multiplier 2, may be taken for the whole day, or Diurnal Intensity, as follows:—

$$\int \Delta^2 \sin A\ d\,H = \Delta^2 H \sin L\ \sin D + \Delta^2 \cos L \cos D \sin H. \qquad (13.)$$

The diurnal intensity is, therefore, proportional to the product of the square of the sun's semi-diameter into the semi-diurnal arc, multiplied by the sine of the latitude into the sine of the sun's declination, *plus* the like product of the square of the sun's semi-diameter into the sine of the semi-diurnal arc multiplied by the cosine of the latitude into the cosine of the declination. This aggregate obviously changes from day to day, according to the sun's distance and declination.

Introducing the astronomic equation, $\cos H = -\tan L \tan D$, or in another form, $\cos L \cos D = -\frac{\sin L\ \sin D}{\cos H}$; the expression reduces to the following:

$$\int \Delta^2 \sin A\, d\,H = \Delta^2 \sin L\ \sin D\,(H - \tan H).$$

It only remains to adopt a unit of intensity, the choice of which is entirely arbitrary. For the present, and in reference to Brewster's formula hereafter noticed, we will assume the intensity of a day on the equator at the time of the vernal equinox to be 81.5 units. For this case, where D and L are each 0, formula (13) reduces to Δ^2, which is $(965'')^2$; hence $81.5 \div (965'')^2$, or k, will be the multiplier for reducing all other values to the same scale; where the common logarithm of k is $\bar{5}.94210$. Denoting the annual intensity by u, and taking Δ in seconds of arc, we have in units of intensity,

$$u = k\,\Delta^2 \sin L\ \sin D\,(H - \tan H). \qquad (14.)$$

The following cases under the general formula may here be specified:—

First, at the time of the *Equinoxes*, D is 0, and consequently H is 6^h; substituting these values in (13) and converting into units,

$$u = k\,\Delta^2 \cos L. \qquad (15.)$$

Hence the sun's daily intensity for all places on the earth is then *proportional to the cosine of the latitude.* As the equinoxes in March and September lie intermedi ate between the extremes or maxima of heat and light in summer, and their minima in winter, the presumption naturally arises that the same expression will approximate to the mean annual intensity. The coincidence is accordingly worthy of note, that the best empirical expression now known for the annual temperature in degrees Fahrenheit, given by Sir David Brewster, in the *Edinburgh Philosophical Transactions*, Vol. IX, is $81.°5 \cos L$, being also proportional to the cosine of the latitude. It is remarkable that Fahrenheit, in 1720, should have adjusted his scale of temperature to such value, that this formula applies, without the addition of a constant term.

Secondly, for all places on the *Equator*, the latitude L is 0; and H is 6^h, or the sun rises and sets at six, the year round, exclusive of refraction. Consequently the Sun's diurnal intensity varies slowly from one day to another, being *proportional to the cosine of the meridian Declination*, or,

$$u' = k\,\Delta^2 \cos D. \qquad (16.)$$

Thirdly, at the *South or the North Pole*, the latitude L is 90°; and since $\tan 90°$ is infinite, the astronomic relation $\cos H = -\tan L \tan D$ is illusory, except when D is 0. The physical interpretation of this feature is, that at the North Pole, the sun rises only at the vernal equinox in March, and continues wholly above the horizon, till it sets at the autumnal equinox. Thus to either Pole, the sun rises but once, and sets but once in the whole year, giving nearly six months day, and six months night. Now suppose the six months day to be divided into equal portions of twenty-four hours each; then, in reference to formula (13), H is 12^h, and *the intensity during twenty-four hours of polar day is proportional to the sine of the Declination at the middle of the day;* or,

$$u'' = k\,\Delta^2 \pi \sin D.$$

This term varies much faster than the cotemporary value on the equator. And comparing the two expressions, it appears that during the summer season, in each twenty-four hours, the Sun's intensity upon the Equator is to that upon the Pole, in the following proportion:—

$$u' : u'' :: 1 : \pi \tan D. \qquad (17.)$$

Fourthly, at the summer solstice, when the intensity on the Pole is a maximum, D is 23° 28′, and the preceding ratio becomes as 1 to 1.25; or the Polar intensity is one-fourth part greater than on the Equator (Plate IV). The difference evidently arises from the fact that daylight in the one place lasts but twelve hours out of twenty-four, while at the Pole the sun shines on through the whole twenty-four hours.

It were interesting to find when this Polar excess begins and ends, which may be ascertained by equating the last two terms of (17). The condition $\pi \tan D = 1$, thus gives D equal to 17° 40′, which is the sun's Declination on May 10th, and again on August 3d. Therefore, *during this long interval of eighty-five days, comprehending nearly the whole season of summer, the Sun's vertical intensity over the North Pole is greater than upon the Equator.* To this subject we shall again recur in a subsequent Section.

Fifthly, having glanced at these particular cases of the formula, let a more complete survey be made for the northern hemisphere. And the same will equally apply to the southern hemisphere, allowing for the reversal of the seasons and change of the Sun's distance. In equation (14), when H exceeds 6^h, and when the declination D is south, a change of sign would be introduced; but the proper trigonometric signs will be observed simply by using the upper sign in summer, or when the declination is north, and the lower sign during the rest of the year, in the annexed formula of daily Intensity:—

$$u = [\bar{5}.94210]\, \Delta^2 \sin L \sin D\, (\tan H \pm H). \qquad (18.)$$

Here brackets include the logarithm of the co-efficient k; is to be taken in seconds of arc; H is the actual length of the semi-diurnal arc to radius 1, and $\tan H$ is the natural tangent. The subjoined table has been computed in this manner, for intervals of fifteen days, and expresses the results in *units of intensity*. In the last three columns for the Frigid Zone the braces include values for the days when the sun shines through the whole twenty-four hours; the blank spaces indicate periods of constant night.

The Sun's Diurnal Intensity at every Ten Degrees of Latitude in the Northern Hemisphere. (Plate I.)

A. D. 1853.	Lat. 0°.	Lat. 10°.	Lat. 20°.	Lat. 30°.	Lat. 40°.	Lat. 50°.	Lat. 60°.	Lat. 70°.	Lat. 80°.	Lat. 90°.
Jan. 1	77.1	67.2	55.8	42.8	30.1	16.5	5.1	. . .	. . .	. . .
" 16	78.1	68.9	58.2	45.8	32.7	19.3	7.2	. . .	. . .	. . .
" 31	79.6	71.7	61.9	49.7	38.6	25.0	11.9	1.4	. . .	. . .
Feb. 15	81.0	74.7	66.6	55.6	45.1	31.9	19.0	6.4	. . .	. . .
Mar. 2	81.6	78.0	71.3	62.9	52.7	41.1	27.9	14.5	2.1	. . .
" 17	82.0	80.2	76.0	69.6	61.1	50.2	37.1	25.5	11.6	. . .
April 1	80.8	81.4	79.5	75.3	68.9	60.2	49.9	38.0	25.6	20.5
" 16	79.0	81.7	82.0	79.5	75.1	68.6	61.1	51.4	44.0	44.6
May 1	76.9	81.5	83.7	83.6	80.8	77.1	70.9	64.6	64.3	65.3
" 16	74.7	80.8	84.7	86.7	85.7	83.3	79.7	76.8	80.3	81.5
" 31	73.0	80.1	85.1	87.8	88.9	87.8	85.7	86.8	91.0	92.4
June 15	72.0	79.6	85.2	88.4	90.1	89.9	88.8	91.7	96.1	97.6
July 1	72.0	79.5	85.0	88.5	90.4	89.5	88.4	90.8	95.1	96.6
" 16	73.0	79.8	84.7	87.5	87.6	86.5	84.1	84.3	88.3	89.7
" 31	74.7	80.4	83.9	85.1	84.5	81.6	77.3	73.4	76.2	77.4
Aug. 15	76.7	80.8	82.7	82.4	79.8	74.7	68.2	60.9	59.2	60.1
" 30	78.5	80.7	80.6	77.7	72.1	65.5	57.3	47.7	38.8	38.9
Sept. 14	79.8	79.8	77.5	72.6	65.6	58.8	46.9	34.5	21.9	14.7
" 29	80.5	78.4	73.8	67.0	57.8	47.0	36.2	22.5	9.0	. . .
Oct. 14	80.7	76.4	69.7	61.0	50.2	38.2	25.7	12.6	1.0	. . .
" 29	79.9	73.5	65.0	54.6	42.5	30.1	17.5	5.2	. . .	. . .
Nov. 13	78.8	70.7	60.8	49.8	37.1	23.8	11.0	0.9	. . .	. . .
" 28	77.5	68.3	57.3	45.3	31.8	18.9	6.8	. . .	. . .	. . .
Dec. 13	76.9	66.9	55.4	43.0	30.3	16.3	4.9	. . .	. . .	. . .

To indicate the law of the Sun's Diurnal Intensity to the eye also, I have taken the relative units in the table as ordinates, and their times for abscissas, and traced curves through the series of points thus determined, as shown in the accompanying diagram (Plate I).

The Equatorial curve will be observed to have two maxima at the Equinoxes in March and September, and two minima at the Solstices in June and December. Since the earth is nearer the sun in March than in September, the curve shows a greater intensity in the former month, other things being equal.

In the latitude of 10°, the Sun will not be vertical at the summer solstice, but only when the Declination is 10° N., which happens twice in the year. The curve corresponds in every particular with the known course of the sun. Above the latitude of 23° 28′, the tropical flexure entirely disappears; and there is only a single maximum at midsummer.

For comparison with the curves of *Intensity*, I have also traced curves of *Temperature* observed at Calcutta, in lat. 22° 33′ N.; at New Orleans, in lat. 29° 57′; at Philadelphia, in lat. 39° 57′; at London, in lat. 51° 31′; and at Stockholm, in lat. 59° 20′. The values for Stockholm represent the averages for every five days during fifty years, as given in the *Encyclopædia Metropolitana*, article Meteorology. The curve for Philadelphia is adjusted from the daily observations made at the Girard College Observatory from 1840 to 1845, under the direction of Prof. Bache. The rest are interpolated graphically from the mean monthly temperatures.

Retardation of the Effect.—In the Temperate Zone the Temperatures will be seen to attain their maximum about one month later than the sun's intensity would indicate. At Stockholm it is somewhat more than a month; and, during this interval the earth must receive, during the day, more heat than it loses at night; and, conversely, after the winter solstice, it loses more heat during the night than it receives by day. In illustration of this point, and to approximately verify the formula, I here insert a former computation of the sun's Intensity for the 15th day of each month, on the latitude of Mendon, Mass., and the results are found to agree very nearly with those observed at that place about *one month later*, as follows: (The observed values are taken from the *American Almanac* for 1849, and are derived from fifteen years' observations.)

Computed values.			Observed values.		Difference.
Jan. 15	5040	23°.3	24°.3	Feb. 15	+1°.0
Feb. "	7142	33°.1	33°.5	Mar. "	+ .4
Mar. "	9764	45°.2	45°.8	April "	+ .6
April "	12574	58°.3	55°.0	May "	—3°.3
May "	14482	67°.1	64°.5	June "	—2°.6
June "	15346	71°.1	71°.8	July "	+ .7
July "	15085	69°.9	68°.9	Aug. "	—1°.0
Aug. "	13437	62°.3	61°.0	Sept. "	—1°.3
Sept. "	10860	50°.3	48°.5	Oct. "	—1°.8
Oct. "	8080	37°.5	38°.9	Nov. "	+1°.4
Nov. "	5638	26°.1	27°.7	Dec. "	+1°.6
Dec. "	4510	20°.9	26°.0	Jan. "	+5°.1

It may be proper to observe that the formula was divided by *sin L*, a constant factor; and the numbers in the second column were then successively computed: their sum, divided by twelve, gave 10163 as the mean, to be compared with 47°.1, the observed mean at Mendon. Then as 10163 : 47°.1 : : 5040 : 20°.3, Jan. 15, etc.

Let it also be observed, that the Mendon values are the monthly means, which do not always fall on the 15th day, but nearly at that time.

Rate per Hour of the Sun's Intensity.—To glance at the subject from another point of view, let us consider the *Rate*, or the relative number of heating rays per hour. For any day, if we divide the computed Intensity by the length of the day, the quotient will express the average Hourly Intensity, denoted by R; thus,

$$R = \frac{u}{2H} = \tfrac{1}{2}\,[\bar{5}.94210]\,\Delta^2 \sin L \sin D \left(\frac{\tan H}{H} \pm 1\right). \qquad (19.)$$

In the accompanying table, the values of the rate R are exhibited at intervals of fifteen days, and for every ten degrees of latitude. From this, Plate II is constructed; and for comparison with the Daily Rate of Intensity, the Daily Range of the Thermometer is also delineated for Trinconomalee, on the coast of Ceylon (lat. 9° N.)—taking 5°.72 Fahr. plus $\frac{1}{9}$th of the mean daily ranges, as ordinates; also for Philadelphia (lat. 39° 57′), taking here 7° plus $\frac{1}{3}$rd of the daily ranges; for Göttingen (lat. 51° 32′), taking $\frac{1}{3}$rd of the daily ranges; and for Boothia Felix (lat. 70° N.), taking $\frac{5}{18}$ths of the daily ranges in degrees Fahrenheit as ordinates. These changes are arbitrary, but are analogous to the conversion of thermometric scales, and still preserve the original law of the curves. The peculiar inflexion at the vertex of the curve of Hourly Intensity for latitude 70°, evidently arises from the change to constant day. And apparently the hourly rates of Plate II, coincide more nearly with the temperatures of Plate I, than do the Diurnal Intensities, or absolute amounts.

Average Rate of the Sun's Hourly Intensity, or Relative Number of Vertical Rays per Hour.
(Plate II.)

A. D. 1853.	Lat. 0°.	Lat. 10°.	Lat. 20°.	Lat. 30°.	Lat. 40°.	Lat. 50°.	Lat. 60°.	Lat. 70°.	Lat. 80°.	Lat. 90°.
Jan. 1	6.43	5.89	5.16	4.24	3.26	2.08	0.88	. . .	. . .	. . .
" 16	6.51	5.99	5.32	4.44	3.44	2.32	1.12	. . .	. . .	. . .
" 31	6.63	6.20	5.56	4.66	3.86	2.75	1.56	0.34	. . .	. . .
Feb. 15	6.75	6.38	5.85	5.05	4.27	3.22	2.11	0.92	. . .	. . .
Mar. 2	6.80	6.59	6.11	5.50	4.71	3.78	2.70	1.56	0.35	. . .
" 17	6.83	6.70	6.38	5.85	5.15	4.25	3.17	2.21	1.03	. . .
April 1	6.73	6.71	6.50	6.09	5.51	4.73	3.82	2.76	1.64	0.86
" 16	6.58	6.67	6.56	6.21	5.70	5.02	4.24	3.22	1.83	1.86
May 1	6.40	6.59	6.57	6.33	5.86	5.32	4.50	3.52	2.68	2.72
" 16	6.23	6.48	6.53	6.40	6.01	5.46	4.71	3.55	3.35	3.40
" 31	6.08	6.39	6.49	6.36	6.07	5.54	4.78	3.62	3.79	3.85
June 15	6.00	6.33	6.45	6.34	6.07	5.57	4.81	3.82	4.00	4.07
July 1	6.01	6.32	6.44	6.36	6.12	5.58	4.83	3.78	3.96	4.03
" 16	6.08	6.38	6.46	6.37	6.01	5.51	4.75	3.51	3.68	3.74
" 31	6.22	6.46	6.50	6.32	5.98	5.42	4.65	3.55	3.18	3.22
Aug. 15	6.39	6.56	6.50	6.30	5.87	5.23	4.38	3.43	2.47	2.50
" 30	6.54	6.60	6.48	6.12	5.53	4.87	4.05	3.10	1.90	1.62
Sept. 14	6.64	6.60	6.37	5.92	5.45	4.70	3.68	2.61	1.50	0.61
" 29	6.70	6.57	6.21	5.68	4.93	4.05	3.16	2.04	0.89	. . .
Oct. 14	6.73	6.47	6.01	5.36	4.53	3.58	2.56	1.42	0.22	. . .
" 29	6.66	6.29	5.74	5.00	4.08	3.09	2.00	0.80	. . .	. . .
Nov. 13	6.56	6.12	5.48	4.72	3.75	2.66	1.48	0.25	. . .	. . .
" 28	6.46	5.95	5.26	4.42	3.36	2.28	1.08	. . .	. . .	. . .
Dec. 13	6.40	5.86	5.13	4.25	3.28	2.06	0.87	. . .	. . .	. . .

A close agreement, however, could not reasonably be expected; for the Intensities represent the sun's effect at the summit of the atmosphere, but the Temperatures, at its base. Indeed, the sun's intensity upon the exterior of the earth's atmosphere, like the fall of rain or snow, is a primary and distinct phenomenon. While passing through the atmosphere to the earth, the solar rays are subject to refraction, absorption, polarization and radiation; also to the effects of evaporation, of winds, clouds, and storms. Thus the heat which finally elevates the mercurial column of the Thermometer, is the resultant of a variety of causes, a single thread in the network of solar and terrestrial phenomena.

There is still a general agreement of the delineated curves of intensity with actual phenomena. Should the inquiry be made, in what part of the earth the sun's intensity continues most uniform for the longest period, an inspection of the flexures of the curves (Plate I), at once indicates the region intermediate between the Equator and the Tropic of Cancer, on the one side, and of Capricorn on the other.[1] Thus the curve for latitude 10° shows the solar intensity to be nearly stationary during half the year, from March to September. During October and November, it falls rapidly, and after remaining nearly unchanged for a few days in December, it again rises rapidly in January and February. As the sun's heat is the prime cause of winds, we might infer that this region would be comparatively calm during the half year mentioned, and that in the remaining months there would be greater atmospheric fluctuations.

Such were the general indications of Plate I, representing the *amounts;* and, on recurring to Plate II, representing the *rates* of diurnal intensity, the status is precisely similar, except that the region of summer calm is removed further from the equator, and nearer to the tropic. On referring to a recent work on the Physical Geography of the Sea, with respect to this circumstance, I find that "the variables," or calms of Cancer and of Capricorn, occur in the very latitudes thus indicated by the compound effect of the amount and rate of solar intensity. And further, the annual range of solar intensity, which is least upon the equator, has its counterpart in the belt of equatorial calms, or "doldrums." The same effect extends also to the ocean itself, and appears in the tranquillity of the Sargosso Sea. While the curves of intensity for the higher latitudes are significant hieroglyphs of the serenity of summer, and the more violent winds and storms of March and September. The entire deprivation of the sun's intensity during a part of the year, within the Arctic and Antarctic circles, may also produce a Polar calm, at least during the depth of winter. But the existence of such calm, though probable, can neither be disproved nor verified, as the pole appears not to have been approached nearer than within about five hundred miles. Parry and Barrow believed that a perfect calm exists at the Pole.

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[1] The connection of the curves of the Sun's Intensity with the lines of Equatorial and Tropical calms, was suggested by Prof. Henry.

SECTION V.

FORMULA AND TABLE OF THE SUN'S ANNUAL INTENSITY UPON ANY LATITUDE OF THE EARTH.

By the method explained in the last Section, the diurnal intensity, in a vertical direction, might be computed for each and every day in the year, and the sum total would evidently represent the Annual Intensity.

The sum of the daily intensities for a month, or monthly intensities, might be found in the same manner. But, instead of this slow process, we shall first find an analytic expression for the aggregate intensity during any assigned portion of the year, and then for the whole year. The summation is effected by an admirable theorem, first given by Euler; a new investigation of which, with full examples by the writer, may be found in the *Astronomical Journal* (Cambridge, Mass.), Vol. II, p. 121. Thus, let u denote the xth term of a series, where u is a function of x. Attributing to x the successive values 1, 2, 3, 4, ... x, and denoting the sum of the results by $\Sigma\, u$, it is shown that,

$$\Sigma u = \int u dx + \tfrac{1}{2}\, u + \tfrac{1}{12}\frac{du}{dx} - \tfrac{1}{720}\frac{d^3u}{dx^3} + \tfrac{1}{30240}\frac{d^5u}{dx} - \dots + C. \qquad (20.)$$

Since this important formula has not yet been introduced into any American treatise on the Calculus, I here insert one of the two demonstrations from the Journal referred to, which indeed was suggested by the present research:—

Imagine the several terms of the original series to be ordinates of a curve, and erected at a unit's distance from each other, along an "axis of X;" then, by the well-known formula of the Calculus, $\int u\, d\, x$ will represent the area of this curve.

Again, connecting the upper adjacent extremities of the ordinates by straight lines, there will be represented an inscribed semi-polygon made up of parallel trapezoids whose bases are each equal to unity, and their areas equal to $\frac{1}{2}\,(0 + F_{(1)}) + \frac{1}{2}\,(F_{(1)} + F_{(2)}) + \dots \frac{1}{2}\,(F_{(x-1)} + F_{(x)})$; adding the contiguous half terms, it becomes $\Sigma\, F_{(x)} - \frac{1}{2}\, F_{(x)}$, or $\Sigma\, u - \frac{1}{2}\, u$.

Between each trapezoid and the curved line above it, is a small segment; and if $f(x)$ or u' denote the area of the last or xth segment, then $\Sigma\, f\,(x)$ or $\Sigma\, u'$ will denote their collective area. The whole curve being made up of the inscribed semi-polygon and these segments, we have

$$\int u\, d\, x = \Sigma\, u - \tfrac{1}{2}\, u + \Sigma\, u',$$

or

$$\Sigma\, u = \int u\, d\, x + \tfrac{1}{2}\, u - \Sigma\, u'.$$

With respect to the last term, suppose u' to be referred to a new curve, as has already been done for u, and so on; then,

$$\Sigma\, u' = \int u'\, d\, x + \tfrac{1}{2}\, u' - \Sigma\, u'',$$

$$\Sigma\, u'' = \int u''\, d\, x + \tfrac{1}{2}\, u'' - \Sigma\, u''', \dots$$

Subtracting the last of these three equations from the preceding, and that result from the first, and cancelling,

$$\left.\begin{aligned}\Sigma u = \int u\, d\, x + \tfrac{1}{2} u - \int u'\, d\, x - \tfrac{1}{2} u' \\ + \int u''\, d\, x + \tfrac{1}{2} u'' \\ - \ldots\ldots - \ldots\end{aligned}\right\}$$

It is now necessary to determine u' in terms of u, or of x. Recurring to the last segment of the curve above referred to, it is evident that its area above the trapezoid, and denoted by u', is equal to

$$u' = \int F_{(x)}\, d\, x - \int F_{(x-1)}\, d\, x - \tfrac{1}{2}\left(F_{(x)} + F_{(x-1)}\right).$$

Developing by Taylor's theorem; since $u = F_{(x)}$,

$$F_{(x-1)} = F_{(x)} - \frac{d\, u}{d\, x} + \frac{d^2 u}{1.2\, d\, x^2} - \frac{d^3 u}{1.2.3\, d\, x^3} + \ldots$$

$$\int F_{(x-1)}\, d\, x = \int F_{(x)}\, d\, x - u + \frac{d\, u}{1.2\, d\, x} - \frac{d^2 u}{1.2.3\, d\, x^2} + \ldots$$

Substituting the two right-hand values in the former equation, the first terms will cancel each other, leaving

$$u' = -\tfrac{1}{12}\frac{d^2 u}{d\, x^2} + \tfrac{1}{24}\frac{d^3 u}{d\, x^3} - \tfrac{1}{80}\frac{d^4 u}{d\, x^4} + \ldots$$

That is, each derived function is equal to $-\frac{1}{12}$th of the second differential coefficient of the preceding, $+\frac{1}{24}$th of the third, &c.

$$u'' = \tfrac{1}{144}\frac{d^4 u}{d\, x^4} - \ldots$$

$$-\tfrac{1}{2} u' + \tfrac{1}{2} u'' - \ldots = \tfrac{1}{24}\frac{d^2 u}{d\, x^2} - \tfrac{1}{48}\frac{d^3 u}{d\, x^3} + \tfrac{7}{720}\frac{d^4 u}{d\, x^4} - \ldots$$

$$-\int u'\, d\, x + \int u''\, d\, x - \ldots = \tfrac{1}{12}\frac{d\, u}{d\, x} - \tfrac{1}{24}\frac{d^2 u}{d\, x^2} + \tfrac{7}{360}\frac{d^3 u}{d\, x^3} - \ldots$$

Substituting these last two values in the equation above,

$$\Sigma u = \int u\, d\, x + \tfrac{1}{2} u + \tfrac{1}{12}\frac{d\, u}{d\, x} - \tfrac{1}{720}\frac{d^3 u}{d\, x^3} \ldots + C,$$

as was to be demonstrated. Let it now be applied to different examples of series, whose xth term is a function of x.

I. To find the sum of the *arithmetical progression*,

$$d + 2\, d + 3\, d + \ldots + x\, d = \Sigma u.$$

Here $u = x\, d$; $\int u\, d\, x = \frac{1}{2} x^2 d$; $\frac{d\, u}{d\, x} = d$.

Whence $\Sigma u = \frac{1}{2} x^2 d + \frac{1}{2} x\, d + \frac{1}{12} d + C.$

If $x = 1, d = \frac{1}{2} d + \frac{1}{2} d + \frac{1}{12} d + C.$

Subtracting, $\Sigma u = \frac{1}{2} x\,(x\, d + d)$; which result coincides with the common arithmetical rule.

II. To find the sum of the *geometrical progression*,

$$a\, r + a\, r^2 + a r^3 + \ldots + a\, r^x.$$

$$\text{Here} \quad u = a\,r^x; \int u\,d\,x = \frac{a\,r^x}{\log r}.$$

$$\frac{d\,u}{d\,x} = a\,r^x \log r; \frac{d^3 u}{d\,x^3} = a\,r^x \log^3 r; \text{ \&c.}$$

The sum of the coefficients of $a\,r^x$ being constant, let it be denoted by B; then will

$$\Sigma\,u = B\,a\,r^x + C.$$

$$\text{If } x = 0, \quad a\,r = B\,a\,r + C.$$

$$\text{If } x = 1, \quad 0 = B\,a + C.$$

Whence $\Sigma\,u = \dfrac{a\,r^{x+1} - ar}{r - 1}$; which also agrees with the well known rule.

III. To find the sum of the *trigonometric series*,

$$\sin a + \sin 2\,a + \sin 3\,a + \ldots\ldots + \sin x\,a.$$

$$\text{Here} \quad u = \sin x\,a; \int u\,d\,x = -\frac{1}{a}\cos x\,a.$$

$$\frac{d\,u}{d\,x} = a \cos x\,a; \frac{d^3 u}{d\,x^3} = -a^3 \cos x\,a;$$

proceeding, therefore, as in II., we have

$$\Sigma\,u = \tfrac{1}{2} \sin x\,a + B \cos x\,a + C.$$

$$\text{If } x = 0, \quad 0 = B + C.$$

$$\Sigma\,u = \tfrac{1}{2} \sin x\,a + B(\cos x\,a - 1).$$

$$\text{If } x = 1, \sin a = \tfrac{1}{2} \sin a + B(\cos a - 1),$$

$$\text{And} \qquad B = \tfrac{1}{2}\,\frac{\sin a}{\cos a - 1} = -\tfrac{1}{2}\,\frac{\cos \frac{1}{2} a}{\sin \frac{1}{2} a}.$$

$$\Sigma\,u = \tfrac{1}{2} \sin x\,a - \frac{\cos \frac{1}{2} a (\cos x\,a - 1)}{2 \sin \frac{1}{2} a}.$$

Reducing to a common denominator, we have by Trigonometry,

$$\Sigma\,u = \frac{\cos \frac{1}{2} a - \cos (x + \frac{1}{2})\,a}{2 \sin \frac{1}{2} a} = \frac{\sin (x+1) \frac{1}{2}\,a \sin \frac{1}{2}\,x\,a}{\sin \frac{1}{2} a}.$$

The formula of summation has its failing cases; but these may be pointed out as plainly as those of Taylor's Theorem. Without entering here into a full discussion, it must apply in all cases where the summation is in its nature possible, and the differential co-efficients do not become infinite. It applies rigorously where the terms are all positive, and the differential co-efficient becomes zero, as in Example I.; also where the collective co-efficient can be represented by *a second constant*, denoted by B, and so can be eliminated, as in Example II. and III. Had not advantage been taken of this feature in the last Example, the sum were represented by the following series, which still converges rapidly when a does not much exceed unity:

$$\Sigma\,u, \text{ or } \Sigma \sin x\,a = -\frac{1}{a} \cos x\,a + \tfrac{1}{2} \sin x\,a + \tfrac{1}{12}\,a \cos x\,a - \tfrac{1}{720}\,a^3 \cos x\,a + \ldots\ldots + C.$$

Having now demonstrated the formula of summation, let it be applied to (13) where the diurnal intensity is measured by

$$u = \Delta^2 (\sin L \sin D.H + \cos L \cos D \sin H).$$

It may be remarked that the arc H can be developed in terms of its cosine; Δ^2 may be expressed in powers of $\cos\theta$; and thus u may be represented entirely in terms of the true longitude T; and ultimately in terms of the mean longitude or anomaly; as, $u = A + B \sin(b + a x) + C \sin(c + 2 a x) + D \sin(d + 3 a x) + \ldots$; where a or n denotes the Sun's daily motion in longitude, or arc 59′8″; which is .0172. This arc being so much less than unity, shows that the regular process of summation without a second constant, will converge with extreme rapidity, stopping at the first differential co-efficient, and leaves us at liberty to determine the sum $\int u\, d x + \frac{1}{2} u + \frac{1}{12} \frac{d u}{d x}$ in such manner as may be most convenient.

Therefore, let x or t denote the number of days elapsed after the beginning of the year or epoch; n being the mean daily motion in longitude; $a' + n t$ or $a' + n x$, the mean anomaly; T, the true longitude, and P the longitude of the perihelion, so that the true anomaly $\theta = T - P$, and $d\theta = d T$. Also, if ω denote the obliquity of the ecliptic, then by Astronomy, $\sin D = \sin\omega \sin T$.

Since $\cos H = -\tan L \tan D$, we have $\sin^2 H = 1 - \tan^2 L \tan^2 D$, or again $\cos^2 L \cos^2 D \sin^2 H = \cos^2 L \cos^2 D - \sin^2 L \sin^2 D$. Substituting in the last member, $1 - \sin^2 D$ for $\cos^2 D$, also 1 for $\cos^2 L + \sin^2 L$; then dividing by $\cos^2 L$, and taking the square root,

$$\cos D \sin H = \sqrt{1 - \frac{\sin^2 D}{\cos^2 L}} = \sqrt{1 - \left(\frac{\sin\omega}{\cos L}\right)^2 \sin^2 T}. \qquad (21.)$$

With respect to Δ^2, let us here write its values from equation (8), and another value given by the ordinary polar equation of the ellipse; assuming A to be 1; and c, a new constant such that, since $d\theta$ is equal to $d T$,

$$\Delta^2 = \frac{c}{\rho^2} = \frac{c\, d T}{n\, d x \sqrt{1 - e^2}} = \frac{c\,(1 + e \cos\theta)^2}{(1 - e^2)^2}. \qquad (22.)$$

Substituting now the third members of the last two equations in place of the first members which occur in the preceding expression for u, and multiplying by dx,

$$u\, d x = \frac{c\, d T}{n \sqrt{1 - e^2}} \left\{ \sin L \sin\omega \sin T . H + \cos L \sqrt{1 - \left(\frac{\sin\omega}{\cos L}\right)^2 \sin^2 T} \right\}. \quad (23.)$$

The next step is to integrate this equation, where in the first term, $\sin\omega \sin T$ has been substituted for its equal, $\sin D$. The integral of the last term is readily identified as the arc of an ellipse whose eccentricity is $\frac{\sin\omega}{\cos L}$; therefore let

$\int d T \sqrt{1 - \left(\frac{\sin\omega}{\cos L}\right)^2 \sin^2 T} = E$, *an elliptic function of the second species.*

Again, integrating the variable factors of the first term by parts, $\int \sin T . H\, d T = - H \cos T + \int \cos T\, d H$. To obtain $d H$ in a function of T, let us differentiate $\sin D = \sin\omega \sin T$, and $\cos H = - \tan L \tan D$, giving $\cos D\, d D = \sin\omega \cos T\, d T$, and $\sin H\, d H = \frac{\tan L\, d D}{\cos^2 D}$. Whence $d H = \frac{\tan L \sin\omega \cos T\, d T}{\sin H \cos D . \cos^2 D}$; or substituting for $\sin H \cos D$ its equal from (21), and for $\cos^2 D$ its equal $1 - \sin^2 \omega \sin^2 T$; then multiplying by $\cos T$, we have,

$$\int \cos T\, d\, H = \int \frac{\tan L \sin \omega \cos^2 T\, d\, T}{(1 - \sin^2 \omega \sin^2 T) \sqrt{1 - \left(\frac{\sin \omega}{\cos L}\right)^2 \sin^2 T}}.$$

Here in a changed form, $\sin \omega \cos^2 T = \sin \omega - \sin \omega \sin^2 T$, which is equal to $\frac{1}{\sin \omega}[(1 - \sin^2 \omega \sin^2 T) + \sin^2 \omega - 1]$; therefore writing $-\cos^2 \omega$ in place of $\sin^2 \omega - 1$, and then separating the expression into two parts, we obtain after cancelling the common factor,

$$\int \cos T d H = \int \frac{\tan L}{\sin \omega} \left\{ \frac{d\, T}{\sqrt{1 - \left(\frac{\sin \omega}{\cos L}\right)^2 \sin^2 T}} - \frac{\cos^2 \omega\, d\, T}{(1 - \sin^2 \omega \sin^2 T) \sqrt{1 - \left(\frac{\sin \omega}{\cos L}\right)^2 \sin^2 T}} \right\}. \quad (24.)$$

Now let $\int \frac{d\, T}{\sqrt{1 - \left(\frac{\sin \omega}{\cos L}\right)^2 \cdot \sin^2 T}} = F$, *an elliptic function of the first species;* and

$\int \frac{d\, T}{(1 - \sin^2 \omega \sin^2 T) \sqrt{1 - \left(\frac{\sin \omega}{\cos L}\right)^2 \sin^2 T}} = \Pi$, *an elliptic function of the third species,* according to Legendre and other geometers.

Adopting these designations, we have now defined the terms of $\int u\, d\, x$. Passing over $\frac{1}{2} u$, as already known, the next term of the general formula of summation (20), is $\frac{1}{12} \frac{d\, u}{d\, x}$; which is determined as follows: Taking the logarithmic differential of (13) in its simplified form,

$$u = \Delta^2 \sin L \sin \omega \sin T\, (H - \tan H).$$

$$\frac{d\, u}{u} = \frac{2 d\, \Delta}{\Delta} + \cot T\, d\, T + \left(1 - \frac{1}{\cos^2 H}\right) \frac{d\, H}{H - \tan H}.$$

Again, taking the logarithmic differential of the first and last members of (22), recollecting that $d\, \theta = d\, T$, we find $\frac{2 d\, \Delta}{\Delta} = \frac{-2\, e \sin \theta\, d\, T}{1 + e \cos \theta}$. Also equating the first and third members of (22), $\frac{1}{d\, x} = \frac{\Delta^2\, n \sqrt{1 - e^2}}{c\, d\, T}$. And the value of $d\, H$ has already been found; whence by making the indicated substitutions and changes,

$$\frac{1}{12} \frac{d\, u}{d\, x} = \frac{1}{12} \frac{u\, \Delta^2\, n \sqrt{1 - e^2}}{c} \left\{ \frac{-2\, e \sin \theta}{1 + e \cos \theta} + \cot T - \frac{\tan^2 H \tan L \sin \omega \cos T}{(H - \tan H) \cos^3 D \sin H} \right\}.$$

The last term may be further simplified by multiplying and dividing by $\sin T$, then substituting $\sin D$ for $\sin \omega \sin T$, and $-\cos H$ for $\tan L \tan D$, and so cancelling $\tan H$, as shown in the result which follows. Referring to (20), and collecting the terms of summation represented by $\Sigma\, u$, we obtain the annexed general expression of the Sun's intensity for any assigned part of the year; thus,

$$\left.\begin{array}{l}\int u\,dx = \dfrac{c.\cos L}{n\sqrt{1-e^2}}\left\{E - \tan L \sin\omega \cos T.H + \tan^2 L.F - \tan^2 L \cos^2\omega.\Pi\right\} \\ +\tfrac{1}{2}u = \tfrac{1}{2}\Delta^2(\sin L \sin D.H + \cos L \cos D \sin H) \\ +\tfrac{1}{12}\dfrac{du}{dx} = \tfrac{1}{12}.\dfrac{u\,\Delta^2 n\sqrt{1-e^2}}{c}\left(\dfrac{-2e\sin\theta}{1+e\cos\theta} + \cot T + \dfrac{\tan H \cot T}{(H-\tan H)\cos^2 D}\right) \\ \ldots + C = \ldots + C. \end{array}\right\} \quad (25.)$$

Having thus obtained Σu, we may regard it as an implicit function, varying continuously with the longitude T, which returns to the same value at the end of a tropical year. Taking, then, the sum of the above terms as an integral between the limits, $T = 360°$, and $T = 0$; the purely trigonometric terms and constant having the same values at the beginning and end of the year, will vanish, leaving only the three elliptic functions, multiplied as follows:—

$$\Sigma u' = \frac{c\cos L}{n\sqrt{1-e^2}}\left\{E'' + \tan^2 L.F'' - \tan^2 L \cos^2\omega.\Pi''\right\}. \quad (26.)$$

Here the eccentricity or common modulus is $\dfrac{\sin\omega}{\cos L}$.

The Sun's Annual Intensity upon any latitude of the Earth is thus proportional to the sum of two Elliptic circumferences of the first and the second order, diminished by an Elliptic circumference of the third order.

On the Equator, L and $\tan L$ are 0, $\cos L$ is 1, and the expression reduces to $\Sigma u' = \dfrac{c\,E''}{n\sqrt{1-e^2}}$.... (27.) This proves that *the Sun's annual Intensity on the Equator is represented by the circumference of an ellipse, whose ratio of eccentricity is equal to the sine of the obliquity of the ecliptic.*

In the Frigid Zones, where the regular interchange of day and night in every twenty-four hours, is interrupted, the formula will require modification, though the general enunciation of the elliptic functions remains the same. The year in the Polar regions is naturally divided into four intervals, the first of which is the duration of constant night at mid-winter. The second interval at mid-summer is constant day; the third and fourth are intermediate spring and autumnal intervals, when the sun rises and sets in every twenty-four hours. For a criterion of the beginning and end of the winter interval, we evidently have $H = 0$; and for the limits of the summer interval $H = 12^h$.

During the winter interval, there is of course no solar intensity. The intensity of the spring and autumn intervals will be found by integrating (25) between the including limits, which results, added to that of the summer interval, give the annual intensity. First, then, to examine the summer interval; H is 12 hours or π, $\sin H$ is 0, and consequently by (23),

$\int u\,dx = \int \dfrac{c\,d\,T}{n\sqrt{1-e^2}} \sin L \sin\omega \sin T.\pi = -\dfrac{c\sin L \sin\omega \cos T.\pi}{n\sqrt{1-e^2}}$, which is precisely equal to the second term of (25), at the end of the spring, and at the beginning of the autumnal interval; so that on integrating between these limits, it will entirely disappear; and the same will apply to $\tfrac{1}{2}u + \tfrac{1}{12}\dfrac{du}{dx}$. For at the beginning of the spring, and end of the autumn interval, when H is 0, $\frac{1}{2}u$ becomes 0; and u being a zero factor, $\frac{1}{12}\frac{du}{dx}$ in (25) reduces to 0. Then exclusive of the three elliptic functions, the intensity of the spring interval will be $\frac{1}{2}u + \frac{1}{12}\frac{du}{dx} - 0$; that of the autumnal interval, $0 - \frac{1}{2}u' - \frac{1}{12}\frac{du'}{dx}$; and for the summer interval, $\frac{1}{2}u' + \frac{1}{12}\frac{du'}{dx} - \frac{1}{2}u - \frac{1}{12}\frac{du}{dx}$; the sum of which is evidently 0.

The expression of annual intensity thus reduces to the three elliptic functions in (26) integrated between the limits of the spring and autumnal intervals. Their collective differential in (23) and the analysis subjoined to it, will give, by making $sin\, Z = \frac{sin\, \omega\, sin\, T}{cos\, L}$,

$$u\, d\, x = \frac{c\, d\, T}{n \sqrt{1 - e^2}} \left\{ cos\, L\, cos\, Z + \frac{sin\, L\, tan\, L}{cos\, Z} - \frac{sin\, L\, tan\, L\, cos^2\, \omega}{(1 - cos^2\, L\, sin^2\, Z)\, cos\, Z} \right\}.$$

Here $cos\, L\, cos\, Z + \frac{sin\, L\, tan\, L}{cos\, Z}$ may take the form $\frac{cos^2\, L\, cos^2\, Z + sin^2\, L}{cos\, L\, cos\, Z}$, or, $\frac{1 - cos^2\, L\, sin^2\, Z}{cos\, L\, cos\, Z}$, or $\frac{sin^2\, \omega}{cos\, L\, cos\, Z}\left[\frac{1}{sin^2\, \omega} - 1 + \left(1 - \frac{cos^2\, L}{sin^2\, \omega}\, sin^2\, Z\right)\right]$.

Again, differentiating the above value of $sin\, Z$,

$$d\, T = \frac{cos\, L}{sin\, \omega} \cdot \frac{cos\, Z}{cos\, T} \cdot d\, Z = \frac{cos\, L}{sin\, \omega} \cdot \frac{cos\, Z\, d\, Z}{\sqrt{1 - \left(\frac{cos\, L}{sin\, \omega}\right)^2 sin^2\, Z}};\ \text{whence,}$$

$$u\, d\, x = \frac{c\, d\, Z}{n\, sin\, \omega \sqrt{-e^2}} \left\{ \frac{cos^2\, \omega}{\sqrt{1 - \left(\frac{cos\, L}{sin\, \omega}\right)^2 sin^2\, Z}} + sin^2\, \omega \sqrt{1 - \left(\frac{cos\, L}{sin\, \omega}\right)^2 sin^2\, Z} - \frac{sin^2\, L\, cos^2\, \omega}{(1 - cos^2\, L\, sin^2\, Z)\sqrt{1 - \left(\frac{cos\, L}{sin\, \omega}\right)^2 sin^2\, Z}} \right\};\ \text{or,}$$

$$\int u\, d\, x = \frac{c\, .\, cos^2\, \omega}{n\, sin\, \omega \sqrt{1 - e^2}} \left\{ F + tan^2\, \omega\, .\, E - sin^2\, L\, .\, \Pi \right\}.$$

As before remarked, these three integrals are to be taken between the limits of the spring and autumnal intervals. At the beginning of the former and end of the latter, H is 0; whence $cos\, H$ or $1 = -\, tan\, L\, tan\, D$; and $D = 90° - L$ taken with an opposite sign for south Declination. In this case,

$$sin\, Z = \frac{sin\, \omega\, sin\, T}{cos\, L} = -\frac{sin\, D}{cos\, L} = -1,\ \text{or}\ Z = 270°.$$

At the end of the spring, and at the beginning of the autumnal interval H is 12^h; $cos\, H$ or $-1 = -\, tan\, L\, tan\, D$; whence D is $90° - L$, $sin\, Z = 1$, or $Z = 90°$. Now the elliptic functions integrated between the limits, $Z = 270°$, $Z = 90°$ give semi-circumferences for the spring interval, and the same for the autumnal interval, the sum of which will be entire circumferences. We have, therefore, for the Annual Intensity in the Frigid Zones,

$$\Sigma\, u' = \frac{4\, c\, cos^2\, \omega}{n\, sin\, \omega \sqrt{1 - e^2}} \left\{ F' + tan^2 \omega \,.\, E' - sin^2\, L \,.\, \Pi' \right\}. \qquad (28.)$$

Here the eccentricity or common modulus of the three elliptic integrals is $\frac{cos\, L}{sin\, \omega}$, being the reciprocal of the modulus in (26); but the intensity is still denoted by three entire elliptic circumferences.

At the Poles, where L is 90°, and $cos\, L$ is 0, the expression of annual intensity reduces to $\frac{2\, c\, \pi\, sin\, \omega}{n \sqrt{1 - e^2}}$. (29.)

The three species of elliptic functions are known to represent four equal and similar quadrants, as in the ellipse. Extensive tables have been published by Legendre, of the numeric values of E and F; and in his *Traité des Fonctions Elliptiques*, Vol. I. p. 141, the value of the quadrant Π' is given in terms of E and F. Thus, if x denote an axillary arc, such that $c^2\, sin^2\, x = n$, a negative quantity, less than 1; then,

$$\text{“}\Pi = \int \frac{d\, \phi}{(1 + n\, sin^2 \phi) \sqrt{1 - c^2\, sin^2\, \phi}}\, ;\ \Pi' = F' + \frac{tan\, x}{\sqrt{1 - c^2\, sin^2\, x}} (F'\, E_{(x)} - E'\, F_{(x)}).\text{”}$$

Comparing with (24), $sin^2\, x = cos^2\, L$; $\Pi' = F' + \frac{cot\, L}{cos\, \omega} (F'\, E_{(x)} - E'\, F_{(x)})$; substituting this value into (26), we find for the *Annual Intensity in the Torrid and Temperate Zones*,

$$\Sigma\, u' = \frac{4\, c}{n \sqrt{1 - e^2}} \left. \begin{array}{l} [E' (sin\, L\, cos\, \omega \,.\, F_{(90^\circ - L)} + cos\, L) + \\ F'.\, sin\, L\, (sin^2 \omega\, tan\, L - cos\, \omega \,.\, E_{(90^\circ - L)})] \end{array} \right\}. \qquad (30.)$$

We have heretofore denoted whole circumferences by the double accent, thus $E'' = 4\, E'$. In (30) E', and F' denote quadrants; $F_{(90^\circ - L)}$ and $E_{(90^\circ - L)}$ elliptic functions whose amplitude in Legendre's system is $90^\circ - L$; $\frac{sin\, \omega}{cos\, L}$ is the common modulus; L denoting the latitude of the place, and ω the obliquity of the ecliptic. The interpolation of Legendre's tables for second differences, is described in Vol. II. p. 202 of the *Fonctions Elliptiques.* From the Polar Circle to the Pole, $\frac{cos\, L}{sin\, \omega}$ will denote the common modulus, x becomes ω, and $\sqrt{1 - c^2\, sin^2\, x} = sin\, L$; hence by (28), *the Annual Intensity in the Frigid Zone is,*

$$\Sigma\, u' = \frac{4\, c}{n \sqrt{1 - e^2}} \left. \begin{array}{l} [E' (sin\, L\, cos\, \omega \,.\, F_{(\omega)} + sin\, \omega) + \\ F'.\, cos\, \omega\, cos\, L\, (cot\, \omega\, cos\, L - tan\, L \,.\, E_{(\omega)})] \end{array} \right\}. \qquad (31.)$$

With respect to the unit of measure for annual intensity, the mean tropical year contains 365.24 days; let this represent the annual number of vertical rays impinging on the equator; that is, *let the sun's intensity during a mean Equatorial day be taken as the thermal Unit*, and let the values for all the latitudes be converted in that proportion. Also denoting the annual intensity on the equator by 12, the mean equatorial *Month* may be used as another thermal unit. And taking the annual intensity on the equator as 81.5 *Units*, with reference to Brewster's formula, the intensity on other latitudes may be expressed in that proportion.

With the aid of Legendre's elliptical tables, and formulas (27), (30), (31), (29), the computation of annual intensities is entirely practicable. The results converted into units, with differences for every five degrees of latitude, have been carefully verified and tabulated as follows:—

The Sun's Annual Intensity.

Latitude.	Thermal units.	Thermal months.	Thermal days.	Diff. days.	Latitude.	Thermal units.	Thermal months.	Thermal days.	Diff. days.
0°	81.50	12.00	365.24	1.27	50°	55.73	8.21	249.74	20.92
5	81.22	11.96	363.97	3.78	55	51.06	7.52	228.82	21.06
10	80.38	11.83	360.19	6.28	60	46.36	6.83	207.76	19.91
15	78.97	11.63	353.91	8.70	65	41.92	6.17	187.85	14.81
20	77.03	11.34	345.21	11.01	70	38.61	5.69	173.04	9.82
25	74.57	10.98	334.20	13.20	75	36.42	5.36	163.22	6.59
30	71.63	10.55	321.00	15.30	80	34.95	5.15	156.63	3.80
35	68.21	10.04	305.70	17.15	85	34.10	5.02	152.83	1.24
40	64.39	9.48	288.55	18.76	90	33.83	4.98	151.59	0.00
45	60.20	8.86	269.79	20.05					

From this table it will be seen that, at the Tropic of Capricorn, or of Cancer, the Sun's annual Intensity is but eleven thermal months, being twelve on the Equator. In the latitude of New Orleans, the annual intensity in a vertical direction is ten and a half thermal months, and in the latitude of Philadelphia, nine and a half. At London the annual intensity is reduced to eight thermal months; and at the Polar Circle, to six months, being just one-half the value on the Equator. Thus the intensity irregularly decreases, till it terminates at the South or North Pole, where the annual intensity is but five thermal months.

Again, it will be interesting to note the analogy which the differences for every five degrees of latitude, in the last column of the table, bear to the corresponding differences of *height in the atmosphere which limit the region of perpetual snow.* It has been observed that the different heights of perpetual frost "decrease very slowly as we recede from the equator, until we reach the limits of the torrid zone, when they decrease much more rapidly. The average difference for every five degrees of latitude in the temperate zone is 1,318 feet, while from the equator to 30°, the average is only 664 feet, and from 60° to 80°, it is only 891 feet—important meteorological phenomena depend on this fact." (*Olmsted's Natural Philosophy.*) The differences of computed annual intensity in the table vary in a manner precisely similar. While in the Temperate Zone, the decrease for every five degrees of latitude is from 13 to 21 thermal days, yet it averages only about 6 thermal days within the Tropics and beyond the Polar circles. The line of congelation evidently rises in summer, and falls in winter, between certain limits.

With reference to the connection between these annual Intensities and the observed annual Temperatures, the analogy of the Centigrade scale shows that units of intensity may be converted into degrees Fahrenheit, by a multiplier and constants; thus, $d = (u - i)\, y + x$. Since the values of the multiplier y, and constants i, x are not precisely known, a graphical construction will be employed; and it is plain that if computed intensities and observed temperatures both follow the same law of change, their delineated curves will be symmetrical.

Therefore, taking the latitudes for ordinates, and the Annual Intensities in the table for abscissas, we obtain the curve of Annual Intensity (Plate III.); and, in the same manner, the curve of Annual Temperature. It will be seen, no doubt with interest, that the curve of annual intensity is almost symmetrical with that of European temperatures, observed mostly on the western side of that continent. But the curve of American temperature based on the U. S. Army Observations for places on the eastern portion of the continent, diverges from the curve of intensity, and indicates a special cause depressing these temperatures below the normal standard due to their latitudes.

At Key West, on the southern border of Florida, the divergence commences, and on proceeding northwardly, continually increases in magnitude; that is, so far as reliable observations have been made along the expanding breadth of the North American continent.

It were natural to suppose that the annual temperature would be defined by the annual number of heating rays from the sun. Indeed on and near the tropical regions, the curves of annual temperature and solar intensity are symmetrical. But in the polar regions, the irregularity of the intervals of day and night, and of the seasons, and various proximate causes, introduce a discrepancy, which the principle of annual average does not obviate. The laws of solar intensity, however, have been determined; the laws of terrestrial temperature will require a special and apparently more difficult analysis.

It has been inferred that there are two poles of maximum cold about the latitude of 80° north, and in longitudes 95° E. and 100° W. The fewness of the observations, however, in that remote Hyperborean region, leave this question still open to investigation. The more recent "isothermal lines of mean annual temperature" published by Prof. Dove of Berlin, in 1852, indicate but one pole of cold, and that is very near the geographical Pole.

SECTION VI.

AVERAGE ANNUAL INTENSITY OF THE SUN UPON A PART OR THE WHOLE OF THE EARTH'S SURFACE.

HAVING determined the value of Σu representing the Sun's vertical intensity upon a single unit or point of the Earth's surface, let us next ascertain the average annual intensity upon a larger area, a zone, or the entire surface of the globe. After which, we shall glance at some of the climatic alternations which are most clearly made known and interpreted by the mechanism of the heavens.

Regarding the earth as a sphere whose radius is unity, $\cos L$ will be the radius, and $2\pi \cos L$ the circumference of the parallel of latitude L. It is evident that the intensity upon a single point multiplied by the circumference $2\,\pi\,cos\,L$, will express the sum of the intensities received upon the whole parallel of latitude; consequently $\Sigma\,u\,.\,2\,\pi\,cos\,L\,.\,d\,L$ integrated between the limits of L and L' will denote the sum of the intensities upon the zone or surface between the latitudes L and L'. By Geometry, the surface of this zone is proved to be equal to $(sin\,L -$ $sin\,L')\,2\,\pi$. Therefore the sum of the annual intensities divided by the surface, will evidently give u, the *average annual intensity* of the Sun upon a unit of surface in that zone, as follows:—

$$u = \frac{\int_{L'}^{L} \Sigma\,u\,.\,cos\,L\,d\,L}{sin\,L - sin\,L'}. \qquad (32.)$$

To find the value of this integral, $\Sigma\,u$ must first be developed in terms of $cos\,L$. It is shown in (23), and in the analysis following that equation, that the annual intensity, exclusive of terms cancelled by the integration, is

$$\Sigma\,u = \frac{4\,c}{n\sqrt{1-e^2}}\left\{cos\,L\,.\,E' + \frac{sin^2 L\,sin^2\omega}{cos\,L}\int_0^{\frac{\pi}{2}} \frac{cos^2 T\,d\,T}{(1-sin^2\omega\,sin^2 T)\sqrt{1-\frac{sin^2\omega}{cos^2 L}\,sin^2 T}}\right\}. \,(33.)$$

From the well known formula for the rectification of the ellipse, we have in the first place,

$$cos\,L\,.\,E' = \frac{\pi}{2}\left(cos\,L - \tfrac{1}{4}\frac{sin^2\omega}{cos\,L} - \tfrac{3}{64}\frac{sin^4\omega}{cos^3 L} - \tfrac{5}{256}\frac{sin^6\omega}{cos^5 L} - \tfrac{175}{(128)^2}\frac{sin^8\omega}{cos^7 L} - \ldots\right). \quad (34.)$$

Next, to find the value of the last integral, let the radical of the denominator be first developed, and its terms multiplied into the other factors separately; then, preparatory to integration, let each numerator be divided by its denominator, as follows:—

$$\frac{cos^2 T}{(1-sin^2\omega\,sin^2 T)\sqrt{1-\frac{sin^2\omega}{cos^2 L}\,sin^2 T}} = \frac{1-sin^2 T}{1-sin^2\omega sin^2 T}\left(1+\tfrac{1}{2}\frac{sin^2\omega}{cos^2 L}sin^2 T + \tfrac{3}{8}\frac{sin^4\omega}{cos^4 L}sin^4 T + \ldots\right).$$

(a) (b) (c) ....

$$(a) = \frac{1 - sin^2 T}{1 - sin^2\omega\,sin^2 T} = \frac{1}{sin^2\omega} + \frac{1 - \frac{1}{sin^2\omega}}{1 - sin^2\omega\,sin^2 T}$$

$$(b) = \frac{1}{2\,cos^2 L} \times \frac{sin^2\omega\,sin^2 T\,cos^2 T}{1 - sin^2\omega\,sin^2 T} = \frac{1}{2\,cos^2 L}(-\,cos^2 T + (a)).$$

$$(c) = \frac{3}{8\,cos^4 L} \times \frac{sin^4\omega\,sin^4 T\,cos^2 T}{1 - sin^2\omega\,sin^2 T} = \frac{3}{8\,cos^4 L}(-\,sin^2\omega\,sin^2 T\,cos^2 T - cos^2 T + (a)).$$

$$(d) = \frac{5}{16\,cos^6 L}(-\,sin^4\omega\,sin^4 T\,cos^2 T - sin^2\omega\,sin^2 T\,cos^2 T - cos^2 T + (a)).$$

$$(e) = \frac{35}{128\,cos^8 L}(-\,sin^6\omega\,sin^6 T\,cos^2 T - sin^4\omega\,sin^4 T\,cos^2 T - sin^2\omega\,sin^2 T\,cos^2 T - cos^2 T + (a)).$$

Multiplying now each term by $d\,T$, and integrating between the limits of $T = \frac{\pi}{2}$, and $T = 0$, we obtain the following results:—

$\int (a)\, d\,T = \frac{\pi}{2\, sin^2 \omega} - \frac{1}{2} cot^2 \omega \int_0^{\frac{\pi}{2}} \frac{d\,(2\,T)}{1 - sin^2 \omega\, sin^2 T}$. Here substituting $\frac{1}{2} - \frac{1}{2} cos^2 T$ for its equal $sin^2 T$, the last term will take the known form of $\int \frac{d\,\theta}{p + q\, cos\,\theta}$, where θ represents $2\,T$; and by the Calculus its value between the proper limits, reduces to $\frac{\pi}{\sqrt{p^2 - q^2}}$ or $\frac{\pi}{cos\,\omega}$. Hence $\int (a)\, d\,T = \frac{\pi}{2} \left(\frac{1 - cos\,\omega}{sin^2 \omega} \right)$.

Since $\int_0^{\frac{\pi}{2}} cos^2 T\, d\,T = \frac{\pi}{4}$, we have $\int (b)\, d\,T = \frac{\pi}{2} \left(-\frac{1}{2} + \frac{1 - cos\,\omega}{sin^2 \omega} \right) \frac{1}{2\, cos^2 L}$.

Since $sin^2 T\, cos^2 T$ may take the form $cos^2 T - cos^4 T$; the formula of the Integral Calculus readily gives $\int (c)\, d\,T = \frac{\pi}{2} \left(\frac{-sin^2 \omega}{8} - \frac{1}{2} + \frac{1 - cos\,\omega}{sin^2 \omega} \right) \frac{3}{8\, cos^4 L}$.

In like manner $\int (d)\, d\,T = \frac{\pi}{2} \left(-\frac{1}{16} sin^4 \omega - \frac{1}{8} sin^2 \omega - \frac{1}{2} + \frac{1 - cos\,\omega}{sin^2 \omega} \right) \frac{5}{16\, cos^6 L}$.

And $\int (e)\, d\,T = \frac{\pi}{2} \left(-\frac{5}{128} sin^6 \omega - \frac{1}{16} sin^4 \omega - \frac{1}{8} sin^2 \omega - \frac{1}{2} + \frac{1 - cos\,\omega}{sin^2 \omega} \right) \frac{35}{128\, cos^8 L}$.

The general formula (33) may be written—

$$\Sigma u = \frac{4\,c}{n \sqrt{1 - e^2}} \left\{ cos\,L \,.\, E' + \frac{sin^2 L\, sin^2 \omega}{cos\,L} \left(\int (a)\, d\,T + \int (b)\, d\,T + \ldots \right) \right\}.$$

Here $\frac{sin^2 L}{cos\,L}$ can take the form of $-cos\,L + \frac{1}{cos\,L}$; substituting this value, and multiplying it into the series of terms denoted by $\int (a)\, d\,T + \int (b)\, d\,T + \ldots$, and adding the products to the series (34) for $cos\,L \,.\, E'$, we at length obtain,

$$\Sigma u = \frac{2\,c\,\pi}{n \sqrt{1 - e^2}} \left\{ cos\,\omega\, cos\,L + \frac{1 - cos\,\omega}{2\, cos\,L} + \frac{1 - cos\,\omega - \frac{1}{2} sin^2 \omega}{8\, cos^3 L} + \frac{1 - cos\,\omega - \frac{1}{2} sin^2 \omega - \frac{1}{8} sin^4 \omega}{16\, cos^5 L} \right.$$
$$\left. + \frac{5\,(1 - cos\,\omega - \frac{1}{2} sin^2 \omega - \frac{1}{8} sin^4 \omega - \frac{1}{16} sin^6 \omega)}{128\, cos^7 L} + \frac{7\,(N - \frac{5}{128} sin^8 \omega)}{256\, cos^9 L} + \ldots \right\}. \quad (35.)$$

In the last term, N denotes the series within the parenthesis of the preceding numerator. Now, taking the obliquity of the ecliptic ω at 23° 28′, and representing the particular value of $\Sigma\, u'$ on the equator by 365.24 *thermal days* as in the last Section, the *multiplier* for converting other values into thermal days, will evidently be $\frac{365.24}{\Sigma\, u'}$, or $365.24 \div \frac{2\,c\,\pi}{n \sqrt{1 - e^2}}$ (.9590919); the latter being the value of $\Sigma\, u'$ when L is 0, and $cos\,L$ is 1. In this manner, and denoting the logarithms of the co-efficients by brackets, we find for the present century,

$$\left. \begin{array}{l} \Sigma\, u = [2.543225]\, cos\,L + [1.197235]\, sec\,L + [\bar{1}.211695]\, sec^3 L + \\ \qquad [\bar{3}.819015]\, sec^5 L + [\bar{4}.616548]\, sec^7 L + [\bar{5}.509114]\, sec^9 L + \ldots \end{array} \right\}. \quad (36.)$$

Or in numbers,

$$\Sigma\, u = 349.322\, cos\,L + \frac{15.748}{cos\,L} + \frac{0.1628}{cos^3 L} + \frac{0.00659}{cos^5 L} + \frac{0.000414}{cos^7 L} + \ldots \quad (37.)$$

These formulas of Annual Intensity are applicable to the Torrid and Temperate Zones, and would have given those portions of the table in the last section with nearly the same facility as elliptic functions, but for the slow convergence of the series in the higher latitudes; the elliptic expressions are also preferred for the future case of secular values.

Denoting the co-efficients of (36) by $a, b, c, \ldots.$ and with reference to formula (32), multiplying by $\cos L\, d\, L$, and integrating,

$$\Sigma\, u \,.\, \cos L\, d\, L = a \cos^2 L \,.\, d\, L + b \,.\, d\, L + \frac{c \,.\, d\, L}{\cos^2 L} + \frac{d \,.\, d\, L}{\cos^4 L} + \frac{e \,.\, d\, L}{\cos^6 L} + \ldots.$$

$$\left.\begin{array}{l} \int \Sigma\, u \,.\, \cos L\, d\, L = a\left(\tfrac{1}{2} L + \tfrac{1}{2} \sin L \cos L\right) + b\, L + c \tan L + d\,\dfrac{\tan L}{3}\left(\dfrac{1}{\cos^2 L} + 2\right) \\ \qquad + e\left(\dfrac{\sin L}{5 \cos^5 L} + \dfrac{4}{5}\displaystyle\int \dfrac{d\, L}{\cos^4 L}\right) + \displaystyle\int\left(\dfrac{\sin L}{7 \cos^7 L} + \dfrac{6}{7}\displaystyle\int \dfrac{d\, L}{\cos^6 L}\right) + \ldots. C \end{array}\right\} . \quad (38.)$$

The last two integrals are given in the respective preceding terms. To determine the correction C, make L equal to 0; in this case, the surface being 0, the left hand member and all the other terms vanish, except C, which is, consequently, 0.

The next process is to find a similar formula for the Frigid Zone. Accordingly from (28), and the analysis preceding that equation, we have,

$$\Sigma\, u = \frac{4\, c}{n \sqrt{1 - e^2}} \left\{ \sin \omega . E' + \int_0^{\frac{\pi}{2}} \frac{\dfrac{\cos^2 \omega}{\sin \omega} . \cos^2 L \cos^2 Z\, d\, Z}{\left(1 - \cos^2 L \sin^2 Z\right) \sqrt{1 - \dfrac{\cos^2 L}{\sin^2 \omega} \sin^2 Z}} \right\} . \quad (39.)$$

This equation has precisely the form of (33); but ω there, corresponds to $90° - L$ here; and L there, corresponds to $90° - \omega$ here; T there, corresponds to Z here, and has the same limits of integration. Hence, by making the proper substitutions in (35) we may pass at once to the series for the annual intensity in the Frigid Zone, as here subjoined.

$$\begin{array}{l} \Sigma\, u = \dfrac{2\, c\, \pi}{n \sqrt{1 - e^2}} \left\{ \sin L \sin \omega + \dfrac{1 - \sin L}{2 \sin \omega} + \dfrac{1 - \sin L - \frac{1}{2} \cos^2 L}{8 \sin^3 \omega} + \dfrac{1 - \sin L - \frac{1}{2} \cos^2 L - \frac{1}{8} \cos^4 L}{16 \sin^5 \omega} \right. \\ \qquad \left. + \dfrac{5\left(1 - \sin L - \frac{1}{2} \cos^2 L - \frac{1}{8} \cos^4 L - \frac{1}{16} \cos^6 L\right)}{128 \sin^7 \omega} + \ldots. \right\} . \end{array} \quad (40.)$$

Multiplying this equation by $\cos L\, d\, L$, or $D \sin L$, and integrating,

$$\left.\begin{array}{l} \int \Sigma\, u \,.\, \cos L\, d\, L = \dfrac{2\, c\, \pi}{n \sqrt{1 - e^2}} \left\{ \dfrac{\sin \omega \sin^2 L}{2} + \dfrac{\sin L - \frac{1}{2} \sin^2 L}{2 \sin \omega} \right. \\ + \dfrac{\sin L - \frac{1}{2} \sin^2 L - \frac{1}{8}\left(\frac{\sin 3L}{3} + 3 \sin L\right)}{8 \sin^3 \omega} + \dfrac{N - \frac{1}{28}\left(\frac{\sin 5L}{5} + \frac{5}{3} \sin 3L + 10 \sin L\right)}{16 \sin^5 \omega} + \ldots C \end{array}\right\} . \quad (41.)$$

Here N denotes the numerator of the preceding fraction. Now, integrating between the limits $L = 90°$, and $L = 90° - \omega = 66°\ 32'$, also introducing the constant multiplier described after (35), we shall find for the Frigid Zone,

$$\int \Sigma\, u \,.\, \cos L\, d\, L = [2.580718] \left\{ \frac{\sin^3 \omega}{2} + \frac{\frac{1}{2} - \cos \omega + \frac{1}{2} \cos^2 \omega}{2 \sin \omega} + \ldots. \right\},$$ which is equal to 13.733.

Again, by formula (38), taking L between the limits 0 and 23° 28′, we find the like sum between the equator and tropic, for the Torrid Zone, to be 141.86.

And taking L between the limits 23° 28′ and 66° 32′, the like sum for the Temperate Zone is 143.46

Substituting these values in equation (32), dividing by the denominator, and then converting into the same thermal measures, which were employed in the last Section, we obtain these final results:—

The Sun's Average Annual Intensity.

	Thermal days.	Thermal months.	Thermal units.
Upon the Polar Zones	166.04	5.45	37.05
" " Temperate Zones	276.38	9.08	61.67
" " Torrid Zone	356.24	11.70	79.49
" " whole Earth	299.05	9.83	66.73

Thus it appears that the Sun's annual intensity upon the whole earth's surface from pole to pole, averages 299 thermal days, being about five-sixths of the value on the equator.

Though the figures in the last column are strictly units of *intensity*, yet as shown by the curves, they also approximately represent annual *temperatures*, except near the Poles. Following these indications, the mean annual temperature of the whole earth's surface must be somewhat below 66° Fahrenheit. In comparison with this result, the mean annual temperature found by Prof. Dove, from a vast number of observations, may be introduced, which is approximately 58°.1 Fahrenheit. The like value found from the formula of Brewster, is $\int_0^{\frac{\pi}{2}} 81^\circ.5 \cos^2 L \, d \, L$, which is 64°. 0 Fahrenheit.

SECTION VII.

ON SECULAR CHANGES OF THE SUN'S INTENSITY.

In relation to secular variations of intensity, we shall adopt the hypothesis that the physical constitution of the sun has remained constant. The secular changes here considered, therefore, are those which depend solely on position and inclination, according to the laws of physical astronomy.

The recurrence of Spots on the Sun's disc, has lately been discovered to observe a regular periodicity. But their influence upon temperature appears to be insufficient for taking account of them.[1] A writer in the *Encyclopædia Britannica*, article Astronomy, states that "in 1823 the summer was cold and wet, the thermometer at Paris rose only to 23°.7 of Reaumur, and the sun exhibited no spots; whereas, in

[1] M. R. Wolf, in the *Comptes Rendus*, XXXV, p. 704, communicates his discovery that the minima of solar spots occur in regular periods of 11.111 years, or nine cycles in a century—and that the years in which the spots are most numerous are generally drier and more productive than the others—the latter being more humid and showery. Counsellor Schwabe, after twenty-six years of observation, does not think that the spots exert any influence on the annual temperature.

the summer of 1807, the heat was excessive, and the spots of vast magnitude. Warm summers, and winters of excessive rigor have happened in the presence or absence of the spots."[1]

Proceeding now to investigation, our first inquiry will relate to *changes of the sun's annual intensity upon the earth's surface regarded as one aggregate.* In Section II, formula (10), let the accented letters refer to the earth at an antecedent or future epoch; then, since Astronomy proves that the semi-transverse axis A is invariable, we have for the proportion of intensity at the secular epoch $\sqrt{\frac{1-e^2}{1-e'^2}}$. (42.)

In the *Connaissance des Tems*, for 1843, Leverrier has exhibited the secular values of most of the elements of the planetary orbits during 100,000 years before and after Jan. 1, 1800. The eccentricity of the earth's orbit at the present time being .0168, the value 100,000 years ago, and the greatest in that interval was .0473. Substituting these in the preceding expression, we find that the sun's annual intensity at the former epoch was greater than at present by one-thousandth part. Now this fraction of 365.24 days, counting the days at twelve hours each in respect to solar illumination, amounts to *between four and five hours of sunshine in a year;* and by so small a quantity only has the sun's annual intensity, during 100,000 years past, ever exceeded the yearly value at the present time. Nor can it depart from its present annual value by more than the equivalent of five hours of average sunshine in a year, for 100,000 years to come.

The superior and *ultimate limit* given by Leverrier, to which the eccentricity of the earth's orbit may have approached at some very remote but unknown period or periods, is .0777. At such epoch, the annual intensity is computed, as before, to have exceeded the intensity of the present by *thirteen hours* of sunshine in a year. On the other hand, the inferior limit of eccentricity being near to zero, indicates only *four minutes* of average sunshine in a year, less than the present annual amount. Between these two extreme limits, all annual variations of the solar intensity, whether past or future, must be included, even from the primitive antediluvian era, when the sun was placed in his present relation to the earth. By the third law of Kepler, on which equation (10) is based, these results are rigorous for siderial years; and by reason of the slight but nearly constant excess, the same may be concluded of tropical or civil years. For the annual variation of the tropical year is only — 0d.000 000 066 86.

The preceding conclusions, it is proper again to observe, refer to the whole earth's surface collectively. Let us, in the next place, inquire concerning *changes of annual intensity upon the different Latitudes* of the earth. According to formulas (30) and (31), this variation will be a function of the eccentricity e, and the obliquity ω. For the present, let it be proposed to compute the annual intensity for an epoch 10,000

[1] Professor Henry was the first to show, by projecting on a screen in a dark room the image of the sun from a telescope with the eye glass drawn out, that the temperature of the spots was slightly less than that of the other parts of the solar disc. The temperature was indicated by a delicate thermo-electrical apparatus. Professor Sechi, of Italy, afterwards obtained the same result.—See *Silliman's Journal*, Vol. XLIX, p. 405.

years prior to A. D. 1800. The eccentricity of the orbit, e', was then .0187, according to Leverrier; and for the obliquity of the ecliptic, the most correct formula is probably that of Struve and Peters, quoted in the *American Nautical Almanac.* It is true, their formula may not strictly apply for so distant a period; but, since the value 24° 43′ falls within the maximum assigned by Laplace, it must be a compatible value, though its epoch may be somewhat nearer or more remote than 10,000 years. Therefore, substituting this value of ω, 24° 43′ in equations (30), (31) and multiplying by $\sqrt{\frac{1-e^2}{1-e'^2}}$, in order to substitute the proper eccentricity, and comparing the computed results with the table for 1850, given in Section V, as a standard, we find the annual intensity on the equator, at the former period, to have been 1.65 thermal days less than in 1850; the differences for every ten degrees of latitude are as follows:—

Change of the Sun's Annual Intensity 8,200 *Years B. C., from its Value in A.D.* 1850, *taken as the Standard.* (Plate III.)

Latitude	Difference in thermal days.	Latitude.	Difference in thermal days.	Latitude.	Difference in thermal days.
0°	—1.65	30°	—.96	60°	+2.11
10°	—1.58	40°	—.22	70°	+5.52
20°	—1.32	50°	+.68	80°	+7.18
				90°	+7.64

These results are exhibited graphically also on Plate III; from which it appears that the annual intensity within the Torrid Zone ten thousand years ago, averaged one thermal day and a half less than now; while from 35° of latitude to 50°, comprehending the whole area of the United States, it was virtually the same as at the present day. But above 50° of latitude, the annual intensity was then greater in an increasing rate towards the Pole, at which point it was between seven and eight thermal days greater than at the present time; in other words, the Poles both North and South, 10,000 years ago received twenty rays of solar heat in a year, where they now receive but nineteen. Owing to change in the obliquity of the ecliptic, the Sun may be compared to a swinging lamp; at the former period, it apparently moved farther to the north and to the south, passing more rapidly over the intermediate space.

The maximum variation of the obliquity of the ecliptic according to Laplace, without assigning its epoch, is 1° 22′ 34″, above or below the obliquity 23° 28′ in the year 1801.[1] Now the difference recognized in our calculation almost reaches this limit, being 1° 15′. As the secular perturbations are now understood, therefore, it follows that, since the Earth and Sun were placed in their present relation to each other, the annual intensity upon the Temperate zones has never varied (Plate III); between the Tropics, it has never departed from its present annual amount by more than about $\frac{1}{240}$th part, and is now very slightly increasing. The most perceptible

[1] Mécanique Céleste, Vol. II, p. 856, note, Bowditch's translation.

difference is in the Polar regions, where the secular change of annual intensity is more than four times greater than on the Equator; in its annual amount, the Polar cold is now very slowly increasing from century to century, which effect must continue so long as the obliquity of the ecliptic is diminishing. And thus, so far as relates to a decreased annual intensity, the celebrated "North-west passage" through the Arctic sea will be even more difficult in years to come than in the present age.

Having now considered the secular changes of annual intensity upon the earth and its different latitudes, let us next examine the *secular changes of intensity in relation to the Northern and Southern hemispheres.* The earth is now nearest the sun in winter of the northern hemisphere on January 1st, and farthest from the sun in summer, on July 4th. This collocation of times and distances has the advantage of rendering the extreme of summer cooler, and of winter, north of the equator, warmer than it would be at a mean distance from the sun. But south of the equator, on the contrary, it exaggerates the extremes by rendering the summer hotter and the winter colder. Before estimating this difference, we may observe that the perigee advances in longitude 11″.8 annually; by which the instant when the earth is nearest the sun, will date about five minutes in time later every year. The time of perihelion which now falls in January, will at length occur in February, and ultimately return to the southern hemisphere the advantage which we now possess. Indeed, it is remarkable that the perigee must have coincided with the autumnal equinox about 4,000 B. C., which is near the time that chronology assigns for the first residence of man upon the earth.

For ascertaining the difference of intensity, we know that the sun's declination goes through a nearly regular cycle of values in a year. The formula $\cos H = -\tan L \tan D$ then shows that the length of the day in the southern hemisphere is the same as in the northern hemisphere about six months earlier. Recurring to formula (18), it appears that the difference of intensities will then depend chiefly on the values of Δ^2. Now, for the northern winter on January 1st, Δ^2 is proportional to $\frac{1}{(1-e)^2}$; for winter in the southern hemisphere, July 4th, it is as $\frac{1}{(1+e)^2}$. The ratio of daily intensity of the northern, is to the southern then as one to $\left(\frac{1-e}{1+e}\right)^2$; or as 1 to $1-4e$ nearly; that is, 1 to $1-\frac{1}{15}$. And the like ratio for the summer intensities is as 1 to $1+\frac{1}{15}$. But $\frac{1}{15}$ is the extreme deviation for a few days only; the mean between this and 0, or $\frac{1}{30}$, would seem more correctly to apply to the whole seasons of summer and winter. Taking then $\frac{1}{30}$th of the greatest and least values of daily intensity, Section IV, for the temperate zone, it appears that winter in the southern hemisphere is now about 1° colder, and summer 3° hotter than in the northern hemisphere. The intensities during spring and autumn may be regarded as equal in both hemispheres. And the summer season of the south temperate zone being hotter, is also shorter by about eight days, owing to the rapid motion of the earth about the perihelion.

In confirmation of these last deductions, the younger Herschel refers to the glow and ardor of the sun's rays under a perfectly clear sky at noon, and observes, "one-fifteenth is too considerable a fraction of the whole intensity of sunshine, not to aggravate, in a serious degree, the sufferings of those who are exposed to it without shelter. The accounts of these sufferings in the interior of Australia, would seem far to exceed what have ever been experienced by travellers in the northern deserts of Africa. The author has observed the temperature of the surface soil in South Africa, as high as 159° Fahrenheit. The ground in Australia, according to Capt. Sturt, was almost a molten surface, and if a match accidentally fell upon it, it immediately ignited." (*Herschel's Astronomy.*)

The phenomenon is of sufficient interest to warrant a glance at the secular values. The eccentricity, 100,000 years ago, has already been stated at .0473; and the formula of the proportional general difference of the winter intensities, in the northern and southern hemispheres $1 - 2e$, becomes $1 - .0946$; and the maximum difference $1 - 4e$ becomes $1 - .1892$. Thus the difference of winter intensities between the northern and southern hemispheres, and likewise of summer intensities, was then about three times greater than at the present time. But this wide fluctuation of summer and winter intensities, in relation to the two hemispheres, scarcely affected the aggregate *annual* intensities, as before shown.

From occasional *Historic notices of climate*, it has been assumed that the winter season in Europe was formerly colder than at the present time. The rivers Rhine and Rhone were frozen so deep as to sustain loaded wagons; the Tiber was frozen over, and snow at one time lay forty days in the city of Rome; but the history of the weather presents winters of equal severity in modern times.[1] In the United States, likewise, since the period of our colonial history, the indications of an amelioration of climate are not conclusive. The great snow of February, 1717, rose above the lower doors of dwellings, and in the winters which closed the years 1641, 1697, 1740, and 1779, the rivers were frozen, and Boston and Chesapeake bays were at times covered with ice as far as the eye could reach; but the like occurs at similar intervals in our day. Mild winters, too, have intervened, and the other seasons are also very variable. The general indications, however, give rise to the question, whether there is a cause of change of climate in the course of the sun?

About two thousand years ago, in the time of Hipparchus, 128 B. C., the obliquity of the ecliptic, or the sun's greatest declination, was 23° 43′. It has now decreased to 23° 27½′; therefore, at the former epoch, the sun came farther north and rose to a higher altitude in summer; and went farther south and rose only to a lower altitude in midwinter. There is then an astronomic cause of change, of which we propose to determine more precisely the effect. For this purpose, the formula of daily intensity (18) may be written,

$$u = [1.90746] \left(\frac{1 + e \cos (T - P)}{1 - e^2}\right)^2 \sin L \sin D \, (\tan H \pm H).$$

[1] Thus, in the famous winter of 1709, thousands of families perished in their houses; the Arabic Sea was frozen over, and even the Mediterranean. The winter of 1740 was scarcely inferior, and snow lay ten feet deep in Spain and Portugal. In 1776 the Danube bore ice five feet deep below Vienna.

Here, for Δ, there is substituted its equal $\frac{1 + e \cos (T - P)}{1 - e^2} 960''.9$; also generally $\sin D = \sin \omega \sin T$, and $\cos H = -\tan L \tan D$. For secular values, if t denote the number of years after, and $-t$ before, the year 1800,

$$e = 0.0167836 - .0000004163\, t;\ P = 279°31'10'' + 1'.0315\, t;$$

$$\text{Mean obliquity}\ \omega = 23° 27' 54'' - 0''.4645\, t - 0''.0000014\, t^2.$$

At the solstices of summer and winter T is 90° or 270°, and D is ω; also let the latitude L be 40°, which is nearly the latitude of Philadelphia, also of southern Italy and Greece. Computing now for B. C. 128, and for A. D. 1850, the daily intensities at the summer solstice are 90.45 and 90.05 thermal units, and at the winter solstice 28.67 and 29.04 respectively. The differences .40 and .37 must correspond almost precisely to degrees of the thermometer; and halving them for the whole seasons as before described, we are conducted to the following conclusion. In the time of Hipparchus, or about a century before Julius Cesar, Virgil, Horace and Ovid flourished, *under the latitude of Italy and Greece the summer was two-tenths of a degree Fahrenheit hotter, and the winter as much colder, than at the present day.* The similar changes of solar intensity upon the United States in two hundred years, can only be made known by theory, and are evidently very slight. There has been, therefore, no sensible amelioration of climate in Europe or America from astronomical causes. The effect, however, of cutting down dense forests, of the drainage and cultivation of open grounds and woodlands admit of conflicting interpretation, and appear but secondary to the atmospheric fluctuations which are governed by the changes in the relative position of the earth and sun.

Before leaving the subject, the inquiry may arise respecting *Geological changes*, whether the secular inequalities have ever been of such value under the present order, as to admit of tropical plants growing in the temperate or frigid zones. In reply, as the annual intensity could never have varied in any considerable degree, the change must consist entirely in tempering the extremes of summer and winter to a perpetual spring. And this could not happen on both sides of the equator at once; for the same arrangement which made the daily intensities in the northern hemisphere equable, would subject those of the southern to violent alternations; and the wide breadth of the torrid zone would prevent the effects being conducted from one hemisphere to the other.

Let us then look back to that primeval epoch when the earth was in aphelion at midsummer, and the eccentricity at its maximum value—assigned by Leverrier near to .0777. Without entering into elaborate computation, it is easy to see that the extreme values of diurnal intensity, in Section IV, would be altered as by the multiplier $\left(\frac{1 \pm e}{1 \pm e'}\right)^2$, that is $1 - 0.11$ in summer, and $1 + 0.11$ in winter. This would diminish the midsummer intensity by about 9°, and increase the midwinter intensity by 3° or 4°; the temperature of spring and autumn being nearly unchanged. But this does not appear to be of itself adequate to the geological effects in question.

It is not our purpose, here, to enter into the inquiry, whether the atmosphere was once more dense than now, whether the earth's axis had once a different inclination to the orbit, or the sun a greater emissive power of heat and light. Neither shall we attempt to speculate upon the primitive heat of the earth nor of planetary space, nor of the supposed connection of terrestrial heat and magnetism; nor inquire how far the existence of coal fields in this latitude, of fossils, and other geological remains have depended upon existing causes. The preceding discussion seems to prove simply that, under the present system of physical astronomy, the sun's intensity could never have been materially different from what is manifested upon the earth at the present day. *The causes of notable geological changes must be other than the relative position of the sun and earth, under their present laws of motion.*

If we extend our view, however, to the general movement of the Sun and Planets in space we find here a *possible* cause for the remarkable changes of temperature traced in the geological periods. For as Poisson conjectured, *Théorie de la Chaleur*, p. 438, the phenomena may depend upon an inequality of temperature in the regions of space, through which the earth has passed. According to a calculation quoted by Prof. Nichol, the velocity of this great movement is six times greater than that of the earth in its orbit, or about 400,000 miles per hour.

In this motion, continued for countless ages, the earth may have traversed the vicinity of some one of the fixed stars, which are suns, whose radiance would tend to efface the vicissitudes of summer and winter, if not of day and night, with a more warm and equable climate. This may have produced those luxuriant forests, of which the present coal fields are the remains; and thus the existence of coal mines in Disco, and other Arctic islands, may be accounted for. If no similar traces exist in the Antarctic zone, the presumption will be strengthened, that the North Pole was presented more directly to the rays of such illuminating sun or star. Indeed, by this position, all possibility of conflict with Neptune, and the other planets which lie nearly in the plane of the ecliptic, was avoided.

The description of such period, with strange constellations and another sun gleaming in the firmament, their mysterious effects upon the growth of animals and vegetation, their untold vicissitudes of light, shadow and eclipse, belong to the romance of astronomy and geology. As in the ancient tradition described by Virgil in the sixth Eclogue:—

> Jamque novum terræ stupeant lucescere solem:
> Altiùs atque cadant submotis nubibus imbres:
> Incipiant silvæ quam primùm surgere, quumque
> Rara per ignotos errent animalia montes.

It is evident that, in receding from the sphere of intensity of such star, as a comet from the sun, the earth's annual temperature would very slowly decrease in process of time, according to the temperature of the space traversed. And, at a remote distance from the stars, the temperature of space ought to remain stationary; as the mean annual temperature of the earth has remained for at least two thousand years past, and without doubt will so continue for ages to come.

SECTION VIII.

ON LOCAL AND CLIMATIC CHANGES OF THE SUN'S INTENSITY.

As the principal topics under this head have been anticipated in the former portions of the work, they need not here be repeated. The inequality of winter, and especially of summer intensities in the northern and southern hemispheres, has already been discussed in the last Section, and ascribed to the changing position of the sun's perigee.

Let us now pass to another local inequality, which consists in the difference of daily intensities at two places situated on the same parallel of latitude, but separated by a considerable interval of longitude. This difference arises solely from hourly change of the Sun's Declination, while moving from the meridian of one place westward to the meridian of the other; the Sun in the interval attaining a higher or lower meridian altitude.

For example, the latitude of Greenwich, near London, is 51° 28′ 39″. Following this parallel west to a point directly north of San Francisco, in California, the difference of longitude is 122° 28′ 2″. At the time of the autumnal equinox, the daily change of the sun's declination is 23′ 23″. Consequently, in passing from the meridian of Greenwich to that of San Francisco, the declination is diminished by $\frac{122° \, 28' \, 2''}{360°} \times 23' \, 23''$, or by 7′ 57″.3.

When the Sun's Declination is 0, at apparent noon at Greenwich, on Sept. 21st, it will be 7′ 57″.3 S. at noon in the longitude of San Francisco on the same day; the semi-diameter being 15′ 59″ or 959″ for Greenwich, and 959″.1 for San Francisco. With these elements, let the sun's daily intensity be computed for both places by formulas (13), (18). The result is 50.13 thermal units for Greenwich, and 49.91 for the place north of San Francisco, on the same latitude. The difference is .22 corresponding to nearly + ¼° Fahrenheit; and by so much the intensity upon the zenith of Greenwich is greater, on the same day.

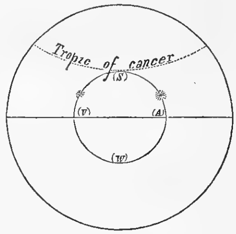
At the vernal equinox, March 20, the sun's daily change of declination would be in the opposite direction, and the difference would become — ¼° F. The inequality of this species thus compensates itself in theory, leaving the *yearly* intensity the same for all places having the same latitude.

For further reference on this point, the daily changes of declination, near the first of each month, are subjoined as follows:—

January,	5′	May,	18′	September,	22′
February,	18′	June,	8′	October,	23′
March,	23′	July,	5′	November,	18′
April,	23′	August,	17′	December,	9′

In this connection, it may be observed that Nervander, Buys Ballot, and Dove have developed a slight inequality of temperature dependent upon the Sun's rotation around his axis, and having the same period of about 27 days; but this result is not confirmed by Lamont, *Poggendorff's Annalen* for 1852.

With respect to maxima and minima, Plate I exhibits a resemblance to two summers and to two winters on the Equator—the sun being vertical at the two equinoxes. On receding from the equator, but still in the torrid zone, the sun will be vertical at equal intervals, before and after the summer solstice, which intervals diminish as the sun approaches the Tropic; the sun being vertical to each locality, when his declination is equal to the latitude of the place; as indicated in the annexed diagram.



On arriving at the Tropic in the yearly motion, the sun can be vertical but once in the year, namely at the summer solstice. At all places more distant from the equator the sun can never be vertical, but will approach nearest this position at the solstice in summer (*s*), and be farthest from it at the solstice of winter (*w*). Thus in the torrid zone, the sun's daily intensity has two maxima and two minima annually; in the temperate zones, one maximum and one minimum; and in the frigid zones, one maximum.

Owing to change of the sun's distance, the intensity is not precisely the same at the autumnal equinox as at the vernal; the difference, however, being small, may here be neglected. And for more full illustration, we exhibit a different projection of the Table in Section IV, showing (Plate IV) the Sun's Diurnal Intensity along the meridian at intervals of thirty days, from June to December, and approximately for the other months. The alternate curves will of course show the sun's changes of intensity in intervals of sixty days. It will be seen that the sun's least yearly range of intensity is not on the Equator, but about 3° of latitude from it north and south. Here the daily heat is most constant, and perpetual summer reigns through the year.

In like manner, the diverging curves show an increasing yearly range, which is greatest in the Polar regions. Also the changes from one day to another are most rapid in spring and autumn. The greatest intensity occurs at the summer solstice, June 21, and the least, at the winter solstice, December 21; so that the yearly range from minimum to maximum is a little wider than the drawn curves indicate. Near the Polar Circle, a singular inflection commences in summer, and the temperature rises rapidly to the Pole.

These laws of Intensity are subject to the retardation in time, mentioned in Section IV, when applied to temperatures; and thus will correspond, generally, with observations. For example, the thermometric column will, during the month of May, rise faster at Quebec than in Florida, and still more rapidly at the Arctic Circle.

It was proved, in Section IV, that the Sun's intensity upon the Pole during eighty-five days in summer, is greater than upon the Equator. Indeed, at the summer solstice it rises to 98.6 thermal units, corresponding nearly to 98° Fahrenheit, which singularly coincides with the temperature of the human body, or blood heat. Though this circumstance may invest the Hyperborean region with new interest, still we cannot assume a brief tropical summer with teeming forms of vegetable and animal life in the centre of the frozen zone. For the measured intensity refers to the outer limit of the atmosphere, upon which the sun shines continually, but from a low altitude which cannot exceed 23° 28′. Much of the heat must, therefore, be absorbed by the air, as happens near the hours of sunrise and sunset in our climate. Also "the vast beds of snow and fields of ice, which cover the land, and the sea in those dreary regions, absorb in the act of thawing or passing to the liquid form, all the surplus heat collected during the continuance of a nightless summer. But the rigor of winter, when darkness resumes her tedious reign, is likewise mitigated by the warmth evolved as congelation spreads over the watery surface." (*Encyc. Brit.*, article Climate.)

The sun's intensity may yet have a somewhat greater effect upon the pole, where it pierces a thinner stratum of the atmosphere than over another portion of the earth's surface. For, in consequence of the centrifugal force of the earth's diurnal motion, the particles of air in all other parts of the earth, being thrown outwards, tend to an increased thickness, in spheroidal strata. We might thence infer that a less proportion of the sun's rays would be absorbed, and a greater portion transmitted through the atmosphere, to the surface of the earth. However this may be in the immediate vicinity of the Pole, yet in the high latitudes hitherto visited by navigators, and which are not nearer than about five or six hundred miles from the North Pole, according to Dr. Kane and others,[1] a dense and lasting fog prevails after the middle of June, through the rest of the summer season, and effectually prevents the rise of temperature which the sun's intensity would otherwise produce.

The question of an *open, unfrozen sea* in the vicinity of the North Pole, has not yet been definitely settled. In this connection we shall only glance at some of the evidences on both sides, without discussing further a subject still unreclaimed from the domain of uncertainty.

"Of this I conceive we may be assured," says Scoresby, Vol. I, p. 46, "that the opinion of an open sea around the Pole is altogether chimerical. We must allow, indeed, that when the atmosphere is free from clouds, the influence of the sun, notwithstanding its obliquity, is, on the surface of the earth or sea, about the time of the summer solstice, greater at the Pole by nearly one-fourth part, than at the equator.[2] Hence it is urged that this extraordinary power of the sun destroys all the ice generated in the winter season, and renders the temperature of the Pole warmer and more congenial to the feelings than it is in some places lying near the equator. Now, it must be allowed, from the same principle, that the influence in the parallel of 78°, where it is computed in the same way to be only about one forty-fifth part

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[1] "The general obscurity of the atmosphere arising from clouds or fogs is such, that the sun is frequently invisible during several successive days. At such times, when the sun is near the northern tropic, there is scarcely any sensible quantity of light from noon to midnight." (*Scoresby's Arctic Regions*, Vol. I, p. 378.)

"The hoar-frost settles profusely in fantastic clusters on every prominence. The whole surface of the sea steams like a lime-kiln, an appearance called the *frost smoke*, caused, as in other instances of the production of vapors, by the waters being still relatively warmer than the incumbent air. At length the dispersion of the mist, and the consequent clearness of the atmosphere announce that the upper stratum of the sea itself has become cooled to the same standard; a sheet of ice quickly spreads, and often gains the thickness of an inch in a single night."

[2] See Section IV (17). The value was first determined by Halley, *Phil. Trans.*, 1693.

less than what it is at the Pole, must also be considerably greater than at the equator. But, from twelve years' observations on the temperature of the icy regions, I have determined the mean annual temperature in latitude 78° to be 16° or 17° F. [that is, about fifteen degrees below freezing point]; how then can the temperature of the Pole be expected to be so very different?"

After some further argument, the author remarks in a note: "Should there be land near the Pole, portions of open water, or perhaps even considerable seas might be produced by the action of the current sweeping away the ice from one side almost as fast as it could be formed. But the existence of land only, I imagine, can encourage an expectation of any of the sea northward of Spitzbergen being annually free from ice."

On the other hand, the following indications in favor of an open sea, are derived from a recent article upon Arctic Researches, announcing that "the existence of the long suspected unfrozen Polar Sea has been all-but proved."

First, it was found that the average annual temperature about the 80th parallel, was higher by several degrees, than that recorded farther south. At the island of Spitzbergen, for example, under the 80th parallel, the deer propagate, and on the northern coast the sea is quite open for a considerable time every year. But at Nova Zembla, five degrees further south, the sea is locked in perpetual ice, and the deer are rarely, if ever seen on its coast. This has led physical geographers to suppose that the milder temperature of Spitzbergen must be attributable to the well-known influence of proximity to a large body of water; while the contiguity of Nova Zembla to the continent was thought to account for the severity of its climate.

Secondly, Captain Parry reached Spitzbergen in May, 1827; from thence he went northward two hundred and ninety-two miles in thirty-five days, during which it rained almost all the time. The ice being much broken, and the current setting toward the south, he could not make way against it, and was compelled to return, which the current greatly facilitated. Besides the current here noticed by Parry, others had been determined before, and more have been ascertained since; so that powerful currents of the Arctic Ocean southward, may be considered as established.

Thirdly, in 1852, Captain Inglefield, while making his summer search for Sir John Franklin, in the northeast of Baffin's Bay, beheld with surprise "two wide openings to the eastward into *a clear and unencumbered sea*, with a distinct and unbroken horizon, which, beautifully defined by the rays of the sun, showed no signs of land, save one island." Further on he remarks, "the changed appearance of the land to the northward of Cape Alexander was very remarkable. South of this cape, nothing but snow-capped hills and cliffs met the eye; but to the northward an agreeable change seemed to have been worked by an invisible agency—here the rocks were of their natural black or reddish-brown color; and the snow which had clad with heavy flakes the more southern shore had only partially dappled them in this higher region, while the western shore was gilt with a belt of ice twelve miles broad, and clad with perpetual snows."

To these may be added the discovery of the southern boundary of an open Polar sea, in the expedition from which Dr. Kane has just returned, October, 1855. "There are facts," observes this distinguished explorer, "to show the necessity and certainty of a vast inland sea at the North. There must be some vast receptacle for the drainage of the Polar regions and the great Siberian Rivers. To prove that water must actually exist, we have only to observe the icebergs. These floating masses cannot be formed without *terra firma*, and it is a remarkable fact that, out of 360°, in only 30° are icebergs to be found, showing that land cannot exist in any considerable portion of the country. Again, Baffin's Bay was long thought to be a close bay, but it is now known to be connected with the Arctic Sea. Within the bay, and covering an area of ninety thousand square miles, there is an open sea from June to October. We find here a vacant space with water at 40° temperature—eight degrees higher than freezing point."[1]

SECTION IX.

ON THE DIURNAL AND ANNUAL DURATION OF SUNLIGHT AND TWILIGHT.

Having thus far considered the intensity of solar radiation upon any part of the earth, we shall lastly pass to examine its duration.

In several publications it has been stated that "the sun is, in the course of the year, the same length of time above the horizon at all places." On applying an accurate analysis, however, it appears, as will presently be shown, that the annual duration of sunlight is subject to a very considerable inequality. This annual inequality increases with the distance from the equator, and is proportional to the sine of the longitude of the sun's perigee.

The longitude of the perigee on Jan. 1, 1850, was 280° 21′ 25″, and increasing at the rate of 61″.47 annually; the sine of the longitude of the perigee is therefore decreasing in value every year, and with it, the inequality of sunlight. At the present time it amounts, in the latitude of 60°, to 36 hours—being additive in the northern, and subtractive in the southern hemisphere. That is, in the latitude of 60° north, the total duration of sunlight in a year is 36 hours more, and in the latitude of 60° south, 36 hours less than on the equator. At either Pole the inequality amounts to 92 hours, or more than seven and a half average days of twelve hours each.

The epoch when the inequality was at its last maximum, is found by dividing the present excess of the longitude of the perigee above three right angles, by the yearly change. The excess, in 1850, was 10° 21′ 25″, which divided by 61″.47 gives a quotient of 606.5 years; which refers back to the period of the middle ages, A. D. 1243.

At a still earlier epoch, this inequality must have entirely vanished. At that

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[1] A reference to Plate IV will confirm what was before known from observations that the extremes of summer and winter temperature range through wider and wider limits from the equator towards each Pole. The application of this general law favors an open Polar sea in summer, as actually seen by explorers, and more recently by Dr. Kane's party in the month of August. But it equally indicates that the sea is frozen over in winter, when there appears no assignable cause, but a calm atmosphere, to mitigate the most intense cold.

epoch, the line of the apsides evidently coincided with the line of the equinoxes, which is computed to have been about 4,000 years before the birth of Christ, at which time chronologists have fixed the first residence of man upon the earth. The luminous year was then of the same length, at all latitudes, from pole to pole.

Though the annual Duration of sunlight thus varies from age to age, and in the northern hemisphere differs from the southern; yet such is the law of the planet's elliptic motion, that the sun's annual Intensity at any latitude north, is precisely the same as at an equal latitude south of the equator. This immediately follows from formula (33), where the annual Intensity is developed in a series of powers of $\cos L$, which is always positive, whether the latitude L be south or north.

Proceeding now to direct investigation, the half day with its augments, may be represented under the general form,

$$H + \text{increase by Refraction} + \text{Twilight.}$$

The first term H is found from the astronomic equation,

$\cos H = -\tan L \tan D = -\dfrac{\tan L \sin \omega \sin T}{\sqrt{1 - \sin^2 \omega \sin^2 T}}$; and this may also take the form[1] of

$$H = \frac{\pi}{2} + \sin^{-1}\left(\frac{\tan L \sin \omega \sin T}{\sqrt{1 - \sin^2 \omega \sin^2 T}}\right). \qquad (43.)$$

Let $u = 2H$, or twice the semi-diurnal arc; then the sum of all the daily values of u through the year, may be found by the method of summation described in Section V. By (22) we have $dx = \dfrac{dT(1-e^2)^{\frac{3}{2}}}{n(1 + e\cos(T-P))^2}$; whence,

$$\int u\,dx = \pi x + \frac{2(1-e^2)^{\frac{3}{2}}}{n}\int_0^{2\pi} \frac{\sin^{-1}(\tan L \tan D)\,dT}{(1 + e\cos(T-P))^2}. \qquad (44.)$$

The general formula of summation, Section V, has the terms $\frac{1}{2}u + \frac{1}{12}\frac{du}{dx} + \ldots$ which in the present case vanish between the limits $T = 0$, and $T = 2\pi$; as will appear from developing u or $2H$ by (43), in terms of $\sin T$. For the annual value therefore, $\Sigma u = \int u\,dx$. Developing the denominator of (44), and substituting for D in the numerator,

$$\Sigma u = \pi x + \frac{2(1-e^2)^{\frac{3}{2}}}{n}\int_0^{2\pi} \sin^{-1}\left(\frac{\tan L \sin\omega \sin T}{\sqrt{1-\sin^2\omega \sin^2 T}}\right)dT\Big\{1 - 2e\cos(T-P)$$
$$+ 3e^2\cos^2(T-P) - \ldots\Big\}. \qquad (45.)$$

It is evident that $\sin^{-1}$ here would develope in *odd* powers of $\sin T$, which multiplied by dT, and integrated between the limits of 0 and 2π, will vanish, as appears from the formulas of the Integral Calculus; when multiplied by $dT.\cos T$, or $d\sin T$, and integrated between the same limits, they also will vanish, being powers of $\sin T$. Also developing $\cos(T-P)$ into $\cos T\cos P + \sin T\sin P$, and neglecting terms, which would so vanish by integration,

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[1] On this and the following pages, $\sin^{-1}\chi$ denotes the arc whose *sine* is χ; where χ represents any given quantity.

$$\Sigma u = \pi x + \frac{2(1-e^2)^{\frac{3}{2}}}{n} \int_0^{2\pi} \left(H - \frac{\pi}{2}\right) d\,T \left\{ -2\,e\,sin\,P\,sin\,T - 4\,e^3\,(sin^3\,P\,sin^3\,T \right.$$
$$\left. + 3\,sin\,P\,cos^2\,P\,sin\,T\,cos^2\,T) - \ldots \right\}. \qquad (46.)$$

Here $H - \frac{\pi}{2}$ has been substituted for its equal from (43); multiplying the $-\frac{\pi}{2}$ into the following term, and integrating between the limits 0 and 2π, the result vanishes. Also the terms multiplied by $4\,e^3$ being small; it will be sufficient for them to develope from (43), to the first power, $H - \frac{\pi}{2} = tan\,L\,sin\,\omega\,sin\,T - \ldots$ by which their integral is immediately found to be $-4\,e^3\,tan\,L\,sin\,\omega \times (sin^3\,P\,.\,\frac{3}{4}\,\pi + 3\,sin\,P\,cos^2\,P\,.\,\frac{\pi}{4})$, or $-3\,e^3\,\pi\,sin\,\omega\,sin\,P\,tan\,L$. Besides this, it only remains to integrate the first term depending on $H\,sin\,T\,d\,T$; but this corresponds to the first term of the formula of annual intensity (23); and if S denote thermal days in the Table of Section V, then $\int_0^{2\pi} H\,sin\,T\,d\,T = ([\bar{3}.61540]\,S\,sec\,L - E')\,4\,cot\,L\,cosec\,\omega$; whence finally, converting into hours,

$$\Sigma u = x\,12^h - \frac{16\,e\,(1-e^2)^{\frac{3}{2}}\,sin\,P}{.2618\,n\,sin\,\omega} \left\{ ([\bar{3}.61540]\,S\,sec\,L - E')\,cot\,L \right.$$
$$\left. + \tfrac{3}{8}\,e^2\,\pi\,sin^2\,\omega\,tan\,L + \ldots \right\}. \qquad (47.)$$

On the equator, L is 0, and the last part vanishes, leaving for the annual duration of sunlight, $x\,12^h$, where x denotes the number of days 365.24.

Therefore $x\,12^h$ represents *the mean value* of the annual duration of sunlight, and the following terms express *the Annual Inequality.* When the latitude is south, both *cot L* and *tan L* change sign; so that the inequality then becomes negative.

For A. D. 1850, *sin P* is negative; substituting the value of this and the other elements for that epoch,

$$\Sigma u = x\,12^h + [2.16700] \times \{[\bar{3}.61540]\,S\,sec\,L - E'\}\,cot\,L + [\bar{3}.88700]\,tan\,L + \ldots \quad (48.)$$

Here brackets include the logarithms of the co-efficients. By this formula the inequality may be readily computed for any latitude between the Equator and the Polar Circle.

In the frigid zone, the summer period of constant day will make another formula necessary. As explained in Section V, the year in that zone may be divided into four periods or intervals. At the beginning of the spring interval, H is 0, and $D = -(90° - L)$; at the end of the spring and beginning of the summer interval, H is 12^h, and $D = 90° - L$; at the end of the summer and beginning of the autumn interval, also, $D = 90° - L$; and at the end of the autumn interval $D = -(90° - L)$. With these data, the equation $sin\,D = sin\,\omega\,sin\,T$, or $T = sin^{-1}\left(\frac{sin\,D}{sin\,\omega}\right)$ enables us to define the lengths of the intervals. Thus the summer interval is measured by the sun's longitude passed over from $T = sin^{-1}\left(\frac{cos\,L}{sin\,\omega}\right)$, to $T = \pi - sin^{-1}\left(\frac{cos\,L}{sin\,\omega}\right)$, or the length of the arc is $\pi - 2\,sin^{-1}\left(\frac{cos\,L}{sin\,\omega}\right)$. This divided by n, the mean daily motion in longitude, and multiplied by 24, will give the number of hours of sunshine during the summer period, which is $\frac{24}{n}\left(\pi - 2\,sin^{-1}\left(\frac{cos\,L}{sin\,\omega}\right)\right)$.

For the spring and autumn intervals, when day and night alternate, the values must be found by the general formula of summation. Here $u = 2\,H = \pi + 2\,sin^{-1}(tan\,L\,tan\,D)$. (49.)

$$d\,u = \frac{2\,tan\,L\,sin\,\omega\,cos\,T\,d\,T}{(1 - sin^2\omega\,sin^2 T)\sqrt{1 - \frac{sin^2\omega}{cos^2 L}\,sin^2 T}}. \quad (50)$$

$$\frac{1}{d\,x} = \frac{n\,(1 + e\,cos\,P\,cos\,T + e\,sin\,P\,sin\,T)^2}{d\,T(1 - e^2)^{\frac{3}{2}}}.$$

Whence it will be seen that all the terms of $\frac{1}{2}\,u + \frac{1}{12}\frac{d\,u}{d\,x}$ vanish between the limits of $T = sin^{-1}\left(\frac{cos\,L}{sin\,\omega}\right), sin^{-1}\left(\frac{-\,cos\,L}{sin\,\omega}\right)$; and $T = sin^{-1}\left(\frac{-\,cos\,L}{sin\,\omega}\right), sin^{-1}\left(\frac{cos\,L}{sin\,\omega}\right)$; except the following, $\frac{4\,e\,n\,sin\,P\,sin\,\omega\,tan\,L\,cos\,T\,sin\,T}{3\,(1 - sin^2\omega\,sin^2 T)\sqrt{1 - \left(\frac{sin^2\omega}{cos^2 L}\right)sin^2 T}}$. Here make $\frac{sin\,\omega}{cos\,L}\,sin\,T = sin\,Z$, and the expression becomes $\frac{4\,e\,n\,sin\,P\,sin\,L\sqrt{1 - \frac{cos^2 L}{sin^2\omega}\,sin^2 Z}\,tan\,Z}{3\,(1 - cos^2 L\,sin^2 Z)}$. And this when taken between the proper limits of $Z = +\,90°, -\,90°$, evidently vanishes. It only remains to find $\int u\,d\,x$ between the same limits for the spring and the autumn interval. These limits show that the sun's longitude passed over in the two intervals is $4\,sin^{-1}\left(\frac{cos\,L}{sin\,\omega}\right)$ which divided by n gives the number of days. This multiplied by the first term of u which is π or 12^h, and added to the like result for the summer period, gives $12 \times \frac{2\,\pi}{n}$ or $x\,12^h$. And if $sin\,T = \frac{cos\,L}{sin\,\omega}\,sin\,Z$, or $sin\,D = cos\,L\,sin\,Z$; then for the whole year, in hours,

$$\Sigma\,u = x\,12^h + \frac{2\,(1 - e^2)^{\frac{3}{2}}}{.2618\,n}\int_0^{2\pi} sin^{-1}\left(\frac{sin\,L\,sin\,Z}{\sqrt{1 - cos^2 L\,sin^2 Z}}\right) \times \frac{d\,T}{(1 + e\,cos\,(T - P))^2}. \quad (51.)$$

It is here assumed that the integral will be taken successively between the limits of $Z = \frac{\pi}{2}, -\frac{\pi}{2}; -\frac{\pi}{2}, +\frac{\pi}{2}$; that is through a whole circumference.

But $d\,T = \frac{cos\,L\,cos\,Z}{sin\,\omega\,cos\,T}\,d\,Z = \frac{cos\,L}{sin\,\omega}.\frac{cos\,Z\,d\,Z}{\sqrt{1 - \left(\frac{cos\,L}{sin\,\omega}\right)^2 sin^2 Z}}$. As the whole function of Z by which $d\,Z$ is multiplied, would evidently develop in *odd* powers of $sin\,Z$, it follows as in the former operation, that terms which would vanish by integration may be neglected in advance, leaving for the last factor, precisely as in (46), $- 2\, e\, sin\, P\, sin\, T - 4\, e^3\, (sin^3\, P\, sin^3\, T + 3\, sin\, P\, cos^2\, P\, sin\, T\, cos^2\, T) - \ldots$

Substituting here for $sin\, T$ its equal $\frac{cos\, L}{sin\, \omega}\, sin\, Z$; the first term of the product is

$$- 2\, e\, sin\, P\, .\, sin^{-1} \left(\frac{sin\, L\, sin\, Z}{\sqrt{1 - cos^2 L\, sin^2 Z}}\right) . \frac{cos^2\, L}{sin^2\, \omega}\, \frac{sin\, Z\, cos\, Z\, d\, Z}{\sqrt{1 - \frac{cos^2\, L}{sin^2\, \omega}\, sin^2\, Z}}.$$

Integrating this by parts, we have

$$2\, e\, sin\, P .\, sin^{-1}\left(\frac{sin\, L\, sin\, Z}{\sqrt{1 - cos^2 L\, sin^2 Z}}\right)\sqrt{1 - \frac{cos^2 L}{sin^2\, \omega}\, sin^2 Z}$$

$$- 2\, e\, sin\, P \int \frac{sin L \sqrt{1 - \frac{cos^2 L}{sin^2 \omega}\, sin^2 Z} . d\, Z.}{1 - cos^2\, L\, sin^2\, Z}$$

Multiplying both numerator and denominator of the last term by the radical, it takes the form of

$$\int \frac{\frac{(sin\, L - sin\, L\, cos^2\, L\, sin^2\, Z)}{sin^2\, \omega}}{(1 - cos^2 L\, sin^2 Z)\sqrt{1 - \frac{cos^2\, L}{sin^2\, \omega}\, sin^2 Z}}, \text{ or } \int \frac{sin\, L\, d\, Z}{sin^2\, \omega\, \sqrt{1 - \frac{cos^2\, L}{sin^2\, \omega}\, sin^2\, Z}}$$

$+ \left(sin\, L - \frac{sin\, L}{sin^2\, \omega}\right) \Pi$; that is, $\frac{sin\, L}{sin^2\, \omega} . F - sin\, L\, cot^2\, \omega . \Pi$; where the letters F and Π designate elliptic functions. When integrated between the above named limits, or an entire circumference, the former term of the integral vanishes, leaving only $- \frac{8\, e\, sin\, P\, sin\, L}{sin^2\, \omega}\, (F' - cos^2\, \omega . \Pi')$.

Developing only to the first power, the next term to be integrated is $- \frac{4\, e^3\, sin^3\, P\, cos^4\, L}{sin^4\, \omega}\, sin\, L\, sin^4\, Z\, d\, Z$, which between 0 and $2\, \pi$, gives $- 3\, e^3\, \pi\, sin^3\, P\, sin\, L . \frac{cos^4\, L}{sin^4\, \omega}$. As the remaining terms are still smaller they may be omitted; whence

$$\Sigma\, u = x\, 12^h - \frac{16\, e\, (1 - e^2)^{\frac{3}{2}}\, sin\, P}{.2618\, n\, sin\, \omega} \left\{ \frac{sin\, L}{sin\, \omega}\, F' - \frac{sin\, L\, cos^2\, \omega}{sin\, \omega}\, \Pi' + \tfrac{3}{8}\, \frac{e^2\, \pi\, sin^2\, P\, sin\, L\, cos^4\, L}{sin^3\, \omega} - \ldots \right\} \qquad (52.)$$

The multiplier for converting $\Sigma\, u$ in Section V into thermal days S, is $\frac{n\, \sqrt{1 - e^2}}{4\, c} \times \frac{365.24}{1.5065}$; whence by (28), $- \frac{sin^2\, L\, cos^2\, \omega\, \Pi'}{sin\, \omega} = \frac{1.5065\, S}{365.24} - \frac{cos^2 \omega}{sin\, \omega}\, F' - sin\, \omega . E'$; and substituting this, we have for the year 1850,

$$\Sigma u = x\, 12^h + [2.16700]\, \frac{sin\, \omega}{sin\, L} \left\{ [\bar{3}.61540]\, \frac{S}{sin\, \omega} - E' + \left(1 - \frac{cos^2\, L}{sin^2\, \omega}\right) F' \right\} + [\bar{1}.87213]\, sin\, L\, cos^4\, L + \ldots \qquad (53.)$$

Here the modulus or eccentricity of the elliptic quadrant is $\frac{cos\, L}{sin\, \omega}$; and the brackets denote logarithms of the co-efficients. Such is the formula for the Frigid Zone.

By means of equations (48), (53), I have computed the annual duration of sunlight $\Sigma\,(2\,H)$, according to the rising and setting of the sun's centre, without regard to refraction. It is the half of 365.24 days, or 182.62 days increased by the quantities in the following table, for the northern hemisphere, and diminished by the same for the southern hemisphere:—

Annual Inequality of Sunlight.

Latitude.	Inequality.	Latitude.	Inequality.
0°	0 h. 00 m.	50°	24 h. 08 m.
10	3 25	60	36 51
20	7 07	70	66 52
30	11 23	80	86 02
40	16 40	90	92 01

Having thus discussed the duration of Sunlight, let us next consider its increase by Refraction, and by Twilight. The mean horizontal refraction, according to Mr. Lubbock's result, is 2075″, or 34′35″; the barometer standing at 30 inches, and the thermometer at 50° F. But as this is somewhat greater than what has been usually employed, we shall adopt 34′ as the mean value for determining the increase of daylight by direct refraction.

With respect to the duration of Twilight, A. Bravais, who has made extensive observations upon the phenomenon, observes, in the *Annuaire Météorologique de la France* for 1850, p. 34: "The length of twilight is an element useful to be known: by prolonging the day, it permits the continuance of labor. Unfortunately, philosophers are not agreed upon its duration. It depends on the angular quantity by which the sun is depressed below the horizon; but it is also modified by several other circumstances, of which the principal is the degree of serenity of the air. Immediately after the setting of the sun, the curve which forms the separation between the atmospheric zone directly illuminated by the sun, and that which is only illuminated secondarily, or by reflection, receives the name of the *crepuscular curve*, or *Twilight Bow*.[1] Some time after sunset, this bow, in traversing the heavens from east to west, passes the zenith; this epoch forms the end of *Civil Twilight*, and is the moment when planets and stars of the first magnitude begin to be visible. The eastern half of the heavens being then removed beyond solar illumination, night commences to all persons in apartments whose windows open to the east. Still later the Twilight bow itself disappears in the western horizon; it is then the end of *Astronomic Twilight*; it is closed night. We may estimate that civil twilight ends, when the sun has declined 6° below the horizon; and that a decline of 16° is necessary to terminate the astronomic twilight.

"I depart here from the general opinion, which fixes at 18° the solar depression at the end of twilight, and at 9° that which characterizes the end of civil twilight.

[1] The phenomenon is equally conspicuous in the west, before the rising of the sun, and in certain states of the atmosphere is scarcely less beautiful than the rainbow, for the symmetry and vivid tinting of its colors.

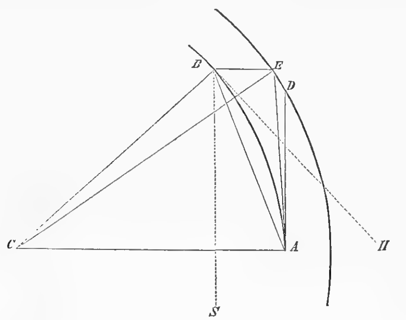
The numbers which I have adopted are derived from numerous observations." "The shortest civil twilight takes place on the 29th of September, and on the 15th of March; the longest on the 21st of June. The shortest astronomic twilight occurs on the 7th of October, and on the 6th of March; the longest on the 21st of June, in this latitude. Above the 50th degree of latitude, twilight lasts through the whole night at the summer solstice."

The analytic solution of the problem to find the time of *shortest twilight* was first given by John Bernoulli; the result, found in various works, is expressed by the two equations,

$$sin\ \frac{t}{2} = \frac{sin\ \frac{1}{2}\ m}{cos\ lat};\ sin\ Dec. = -\ tan\ \tfrac{1}{2}\ m\ sin\ lat.$$

Here t denotes the duration of shortest twilight, and m is the sun's depression 16° or 18° below the horizon.

To pursue the discussion of the physical details of twilight, would occupy too much space, and we shall here only glance at the method of Lambert for determining the height of the atmosphere from twilight. The demonstration is based upon the examples solved in *Gehler's Physikalisches Wörterbuch.*



Lambert found that when the true depression of the sun below the horizon was 8° 3′, (b), the height of the twilight arch was 8° 30, (a); and when the true depression of the sun was 10° 42′, the altitude of the bow was 6° 20′.

In the figure let C denote the centre and $A\,B$ the surface of the earth; $D\,E$, the outer limit of the atmosphere; B, the place of the observer; and E, the position of the bow.

Let r denote the refraction due to the altitude (a), then the angle $E\,B\,H = a - r = a'$. When the sun's centre is apparently in the horizon of A, it is really about 34′ below it. Denoting this horizontal refraction by r', and deducting it from b, leaves the angle $H\,B\,S = b - r' = b'$; where $B\,S$ is parallel to the tangent $A\,D$. But the ray of light after passing A is further refracted by r' or 34′ to E.

The angle $H\,B\,S$ or b', according to a proposition of geometry, is measured by the arc $A\,B$; whence the angle $A\,C\,B = b'$; then drawing the chord $A\,B$, and denoting the earth's radius by R, the isosceles triangle $A\,C\,B$ gives $A\,B = 2\,R\,sin\,\frac{1}{2}\,b'$.

In triangle $A\,B\,E$, the angle A is evidently $\frac{1}{2}\,b - r'$; and deducting the other three angles of the quadrilateral from four right angles, leaves the angle $E = 180° - a' - b' + r'$; then,

$$sin\ E \text{ or } sin\,(a' + b' - r') : sin\,(\tfrac{1}{2}\,b' - r') :: A\,B : B\,E = \frac{2\,R\,sin\,\frac{1}{2}\,b' \,.\, sin\,(\frac{1}{2}\,b' - r')}{sin\,(a' + b' - r')}. \quad (54.)$$

In the triangle $C\,B\,E$, two sides and the included angle $90° + a'$ are now given to find the third side $C\,E$; from which deducting R, leaves the required height of the atmosphere.

With this mode of calculation, the first observations of Lambert, before stated, determine the height to be 17 miles; and the second observations, 25 miles. And a still later observation would have given a still greater height, owing, perhaps, to the mingling of direct and reflected rays. The subject awaits further improvement; though some extensions have been made by M. Bravais, in the *Annuaire Météorologique de la France* for 1850.

If we regard only the appearance of the Twilight bow, the limits of the sun's depression assigned by M. Bravais are doubtless nearly correct, namely 16° for astronomical, and 6° for civil twilight. But, regarding only the actual intensity of light falling upon the eye, it appears that the effects of the bow are further increased by indefinite reflection among the particles of air, and this may increase the average limits to 9° for civil, and 18° for astronomical twilight. Without determining which view ought to be adopted, a mean has here been taken, and *the following tables have been calculated on the assumption that the sun is 7½° below the horizon at the end of civil twilight; and* 17°, *at the end of astronomic twilight.*

The increase of the day by Refraction and by the twilights, may all be comprehended in one general formula. Let m denote the sun's depression below the horizon at the end of either period; then the distance from the Pole to the zenith, $90° - L$, the distance from the Pole to the sun, $90° - D$, the distance from the zenith to the sun $90° + m$, or three sides of a spherical triangle are given to find the hour angle $H + T$, as in the following equation:—

$$\cos(H + \tau) = \frac{-\sin L \sin D - \sin m}{\cos L \cos D} = +\cos H - \frac{\sin m}{\cos L \cos D}. \qquad (55.)$$

Here τ denotes the increase by refraction or by Twilight, according as m is taken at 34′, at 7½°, or 17°.

When twilight lasts through the whole night, it is evident that at the commencement and at the end of such period, $\tau = 12^h - H$. Substituting this value in (55),

$$-1 = \frac{-\sin L \sin D - \sin m,}{\cos L \cos D}, \text{ or } \cos(L + D) = \sin m; \text{ that is, } D = 90° - L - m. \qquad (56.)$$

The corresponding yearly limit for constant sunlight has already been found to be indicated by $D = 90° - L$. The lowest latitude where this is possible is evidently $L = 90° - \omega$, or at the Polar Circle. In like manner, the lowest latitude where twilight through the whole night occurs, is $L = 90° - \omega - m = 49° 32'$ north or south of the equator.

During the long night in the Polar regions, twilight will be, for a time, impossible; that is, so long as the sun continues more than 17° below the horizon. The limits of this period will be defined by making $H + \tau$ equal to 0, in (55); whence $L - D = 90° + m$, or $D = -90° - m + L$. (57.)

The corresponding yearly limit of sunlight is indicated by $D = -90° + L$. But the application of these limits is reserved till after an expression for the annual duration of twilight has been found by the method of summation described in Section V. For this purpose, equation (55) may be put under the form of

$$\tau = -H + \cos^{-1}\left(\cos H - \frac{\sin m}{\cos L \cos D}\right). \qquad (58.)$$

Developing in powers of *sin m* by Maclaurin's Theorem,

$$\left.\begin{aligned}\tau = \frac{\sin m}{\sqrt{\cos^2 L - \sin^2 D}} - \frac{\frac{1}{2}\sin^2 m \sin L \sin D}{(\cos^2 L - \sin^2 D)^{\frac{3}{2}}} + \tfrac{1}{6}\sin^3 m \frac{(\cos^2 L + (3\sin^2 L - 1)\sin^2 D)}{(\cos^2 L - \sin^2 D)^{\frac{5}{2}}} \\ - \tfrac{1}{8}\sin^4 m\left(\frac{3\sin L \sin D}{(\cos^2 L - \sin^2 D)^{\frac{5}{2}}} + \frac{5\sin^3 L \sin^3 D}{\cos^2 L - \sin^2 D)^{\frac{7}{2}}}\right) + \ldots\end{aligned}\right\} . \quad (59.)$$

With respect to the yearly limits already assigned, (55), (56), (57), we know that in the lower latitudes, twilight recurs regularly, while the sun's longitude T varies from zero to an entire circumference; but in the Polar zone, this continuity is interrupted. Still, in integrating for the yearly duration of twilight between the proper limits, $\frac{1}{2}u + \frac{1}{12}\frac{du}{dx}$ being expressed in terms of $\sin D$ or $\sin T$ will vanish, even in the Polar zone, leaving only $\int u\,dx$. And with respect to dx, since $\cos T\,dT$ is $d \sin T$, which multiplied into the development of τ, would integrate in powers of $\sin T$ which vanish, we may reject all such factors in advance, leaving,

$$dx = \frac{(1-e^2)^{\frac{3}{2}}}{n} dT[1 - 2e\sin P \sin T + 3e^2(\cos^2 P - \cos 2P \sin^2 T) + \ldots]. \quad (60.)$$

Were this multiplied into (59), making $u = 2\tau$, and substituting for $\sin D$ its equal $\sin \omega \sin T$, then integrating between the proper limits, and dividing by $\frac{\pi}{12}$ in order to convert arc into hours of time, we should obtain the annual duration of twilight expressed in elliptic functions. It will be more convenient, however, to resort to circular functions.

To obtain the duration of Twilight in another form, let N denote the interval of *Night*, from the end of the evening twilight to midnight, or from midnight to the morning twilight, computed by the sun's midnight declination. The duration of N will correspond to any assumed depression or elevation of the crepusculum circle, or to any compatible value of m. Then $N = 12^h - (H + \tau)$;

$$\cos N = -\cos(H + \tau) = \frac{\sin L \sin D + \sin m}{\cos L \cos D}.$$

$$\frac{dN}{d \sin D} = \frac{-\sin L - \sin m \sin D}{\cos L \cos^3 D \sin N} = \frac{-\sin L - \sin m \sin D}{\cos^2 D \sqrt{\cos^2 L - \sin^2 m - \sin^2 D - 2\sin L \sin m \sin D}}.$$

Developing $\cos^2 D$ into the numerator under the form of $(1 - \sin^2 D)^{-1}$; also resolving the radical into two factors, one of which is $\sqrt{\cos^2 L - \sin^2 m}$, and developing the other into the numerator to the fifth power of $\sin D$; then multiplying the factors, and employing Maclaurin's Theorem, or integrating; also making $\cos^2 L - \sin^2 m = s$;

$$N = \cos^{-1}\left(\frac{\sin m}{\cos L}\right) - \frac{\sin L \sin D}{\sqrt{s}} - \frac{\frac{1}{2}\sin m \cos^2 m \sin^2 D}{s^{\frac{3}{2}}} - \frac{\sin L}{6 s^{\frac{3}{2}}}\left(\cos 2L + 2 + \right.$$
$$\left.\frac{3\sin^2 L \sin^2 m}{s}\right)\sin^3 D - \frac{\sin m}{8 s^{\frac{3}{2}}}\left(\cos 2m + 2 + \frac{3\sin^2 L(1 + \sin^2 m)}{s} + \frac{5\sin^4 L \sin^2 m}{s^2}\right)\sin^4 D$$
$$- \frac{\sin L}{10 s^{\frac{3}{2}}}\left\{\cos 2L + 2 + \frac{3\sin^2 m(1 + \sin^2 L) + \frac{3}{4}}{s} + \frac{5\sin^2 L \sin^2 m(\sin^2 m + \frac{3}{2})}{s^2} + \right.$$
$$\left.\frac{\frac{35}{4}\sin^4 L \sin^4 m}{s^3}\right\}\sin^5 D - \frac{\sin m}{12 s}\left\{\cos 2m + 2 + \frac{3\sin^2 L(1 + \sin^2 m) + \frac{3}{4}}{s} + \right.$$

$$\frac{5\ sin^2 L\,(sin^2 m\,(sin^2 L + \frac{3}{2}) + \frac{3}{4})}{s^2} + \frac{35\ sin^4 L\ sin^2 m\,(\frac{1}{2} + \frac{1}{4}\ sin^2 m)}{s^3} + \frac{315\ sin^6 L\ sin^4 m}{20\ s^4}\Big\}\ sin^6 D - \ldots \qquad (62.)$$

Here let us put $cos\ N' = \frac{sin\ m}{cos\ L}$; and denoting the co-efficients of $sin\ D, sin^2 D\ldots$ by $-N_1, -N_2, \ldots$ we have,

$$N = N' - N_1\, sin\ D - N_2\, sin^2 D - N_3\, sin^3 D - N_4\, sin^4 D - \ldots$$

Multiplying this by the former series for $d\,x$, and integrating the products, after substituting $sin\ \omega\ sin\ T$ for $sin\ D$, and dividing by $\frac{\pi}{12}$,

$$\begin{aligned}\Sigma\, 2\,N = \frac{24^{h}(1-e^2)^{\frac{3}{2}}}{n\,\pi}\Big\{ & N'[T(1 + 3e^2 cos^2 P) + 2\,e\,sin\,P\,cos\,T - 3\,e^2 cos\,2\,P\textstyle\int_2 + \ldots] \\ & + sin\,\omega\,N_1[(1 + 3\,e^2 cos^2 P)\,cos\,T + 2\,e\,sin\,P\textstyle\int_2 + 3\,e^2 cos\,2\,P\int_3 + \ldots] \\ & - \frac{sin^2\omega}{1.2} N_2\,[(1 + 3\,e^2 cos^2 P)\textstyle\int_2 - 2\,e\,sin\,P\int_3 - 3\,e^2 cos\,2\,P\int_4 - \ldots] \\ & - \frac{sin^3\omega\,N_3}{1.2.3}[(1 + 3\,e^2 cos^2 P)\textstyle\int_3 - 2\,e\,sin\,P\int_4 - 3\,e^2 cos\,2\,P\int_5 - \ldots] \qquad (63.) \\ & - \frac{sin^4\omega\,N_4}{1.2.3.4}[(1 + 3\,e^2 cos^2 P)\textstyle\int_4 - 2\,e\,sin\,P\int_5 - 3\,e^2 cos\,2\,P\int_6 - \ldots] \\ & - \frac{sin^5\omega\,N_5}{1.2.3.4.5}[(1+3\,e^2 cos^2 P)\textstyle\int_5 - 2\,e\,sin\,P\int_6 - 3\,e^2 cos\,2\,P\int_7 - \ldots] \\ & - \frac{sin^6\omega\,N_6}{1.2.3.4.5.6}[(1+3\,e^2 cos^2 P)\textstyle\int_6 - 2\,e\,sin\,P\int_7 - 3\,e^2 cos\,2\,P\int_8 - \ldots] \ldots + C\Big\}.\end{aligned}$$

The integral signs here designate the following quantities:—

$$\begin{aligned}\textstyle\int_2 &= \textstyle\int sin^2 T\,d\,T = -\frac{sin\,2\,T}{4} + \frac{T}{2}. \\ \textstyle\int_3 &= \textstyle\int sin^3 T\,d\,T = \frac{cos\,3\,T}{12} - \frac{3\,cos\,T}{4}. \\ \textstyle\int_4 &= \textstyle\int sin^4 T\,d\,T = \frac{sin\,4\,T}{32} - \frac{sin\,2\,T}{4} + \frac{3\,T}{8}. \qquad (64.) \\ \textstyle\int_5 &= \textstyle\int sin^5 T\,d\,T = -\frac{cos\,5\,T}{80} + \frac{5\,cos\,3\,T}{48} - \frac{5\,cos\,T}{8}. \\ \textstyle\int_6 &= -\frac{sin\,6\,T}{192} + \frac{3\,sin\,4\,T}{64} - \frac{15\,sin\,2\,T}{64} + \frac{5\,T}{16}. \ldots\end{aligned}$$

In the year 1850, $\omega = 23° 27\frac{1}{2}'$; $P = 280° 22\frac{1}{2}'$; $e = .01676$; $n = \frac{2\,\pi}{365.24}$; $1 + 3\,e^2 cos^2 P$ 1.000027; $2\,e\,sin\,P = -.032972$; $3\,e^2 cos\,2\,P = -.000788$; $\frac{24^h(1-e^2)^{\frac{3}{2}}}{n.\pi} = 443^h.89$.

For the lower and middle latitudes, where $2\,N$ and $2\,(H + \tau)$ alternate in every twenty-four hours through the year, we may integrate through an entire circumference. In this case, equation (63) is materially simplified; and denoting by brackets the common logarithms of the co-efficients,

$$\left.\begin{aligned}\Sigma\,2\,N = [3.44564]\,N' - [1.26253]\,N_1 - [2.04360]\,N_2 - [\bar{1}.55940]\,N_3 \\ - [0.03944]\,N_4 - [\bar{3}.37930]\,N_5 - [\bar{3}.68312]\,N_6 - \ldots\end{aligned}\right\}. \qquad (65.)$$

At the Pole, the duration of twilight is easily found by noting in the ephemeris the time at which the sun's declination south, is equal to the depression of the crepusculum circle below the horizon; this instant and the equinox being its limits of duration. As before indicated, the limit of refractional light is when the sun is 34′ below the horizon, or $m = 34'$; civil twilight, when $m = 7\frac{1}{2}°$; and common or astronomical twilight when $m = 17°$. Thus we shall find,

Annual Duration.

	Sunlight. $\Sigma\,(2\,H)$.	Refractional Light.	Civil Twilight.	Astronomic Twilight.	Darkness. $\Sigma\,(2\,N)$.
North Pole.	186d. 11h.	2d. 22h.	38d. 15h.	94d. 16h.	84d. 3h.
Lat. 40°.	183d. 8h.	1d. 14h.	21d. 6h.	49d. 2h.	132d. 20h.
Equator.	182d. 15h.	1d. 5h.	15d. 21h.	36d. 1h.	146d. 14h.

From this table, it appears that the annual length of darkness diminishes from the equator to the pole; while the duration of twilight increases from about one month on the equator to three months at the Pole. In this latitude, about thirty-eight hours of daylight, at the sun's rising and setting, are annually due to atmospheric refraction. The second, fifth, and sixth columns correspond to the formula $\Sigma\,(2\,H) + \Sigma\,(2\,\tau) + \Sigma\,(2\,N) = 365^{d}\,6^{h}$.

In further illustration of this subject, the duration from noon to midnight, or from midnight to noon, of Sunlight, Astronomic Twilight, and Darkness are exhibited to the eye in the accompanying Plate V, for every day in the year, on different latitudes. On the equator, it will be seen that Twilight has its least value, and is almost uniform through the year. In the latitude of 40°, the limiting curves of twilight bend upward in an arch-like form. The upper curve, at the same time recedes from the lower, and encroaches upon the duration of darkness, till, as shown for latitude 60°, twilight lasts through the whole night in summer. If the first and last extremities of the curves at January and December be united to complete the circuit of a year, darkness there, will be represented by an elliptic segment; the longest nights and shortest days being at mid-winter. In approaching the highest latitudes, the lines which form the limits continually change their inclination, till at the Pole, they become perpendicular to their position at the Equator.

The present Section contains formulæ and tables for determining both the diurnal and the yearly limits of twilight, with tabular examples for A. D. 1853, computed for 34′, 7° 30′, and 17°, depressions of the crepusculum circle below the horizon; the reasons for which have before been stated. Although these phenomena are varied by mists and clouds, and by the atmospheric temperature and density, still the assumption of mean depressions, has been necessary in order to obtain a general view of their laws of continuance. The duration of moonlight which is unattended by sensible heat, has not been discussed. From this source, the reign of night is still further diminished, till in this latitude, the remaining duration of total darkness after twilight and moonlight, can scarcely exceed three months in the year. The interval towards the close of astronomic or common twilight, corresponds to what is commonly termed, in the country, "early candle-light," when the glimmering landscape fades on the sight, and the stars begin to be visible. The end of civil twilight marks the time at which some city corporations in Europe are said to have made regulations for lighting the street lamps.

In conclusion, without entering into further details, the connection of solar heat and light has enabled us to exhibit, by the same formulæ and curves, the intensities of both in common. Indeed so close is the analogy that even the monthly height of the mercurial column, which shows the temperature, indicates generally the average intensity of sunlight in that locality.

Half Days, or Semi-Diurnal Arcs, in the Northern Hemisphere.

1853.	Lat. 0°.	Lat. 10°.	Lat. 20°.	Lat. 30°.	Lat. 40°.	Lat. 50°.	Lat. 60°.	Lat. 70°.	Lat. 80°.	Lat. 90°.
	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.
Jan. 1	6 00	5 43	5 24	5 03	4 37	3 58	2 51	0 00	0 00	0 00
" 16	6 00	5 44	5 28	5 09	4 45	4 11	3 13	0 00	0 00	0 00
" 31	6 00	5 48	5 35	5 18	5 00	4 33	3 49	2 04	0 00	0 00
Feb. 15	6 00	5 51	5 41	5 30	5 17	4 58	4 29	3 29	0 00	0 00
Mar. 2	6 00	5 55	5 50	5 44	5 36	5 26	5 10	4 40	3 00	0 00
" 17	6 00	5 59	5 58	5 57	5 56	5 54	5 51	5 46	5 32	0 00
April 1	6 00	6 04	6 07	6 11	6 16	6 22	6 32	6 51	7 49	12 00
" 16	6 00	6 07	6 15	6 24	6 35	6 50	7 12	7 59	12 00	12 00
May 1	6 00	6 11	6 22	6 36	6 53	7 15	7 52	9 12	12 00	12 00
" 16	6 00	6 14	6 29	6 46	7 08	7 38	8 28	10 50	12 00	12 00
" 31	6 00	6 16	6 34	6 54	7 19	7 55	8 58	12 00	12 00	12 00
June 15	6 00	6 18	6 36	6 58	7 25	8 04	9 14	12 00	12 00	12 00
July 1	6 00	6 17	6 36	6 57	7 23	8 02	9 09	12 00	12 00	12 00
" 16	6 00	6 16	6 33	6 52	7 17	7 51	8 51	12 00	12 00	12 00
" 31	6 00	6 13	6 28	6 44	7 04	7 32	8 19	10 20	12 00	12 00
Aug. 15	6 00	6 10	6 21	6 33	6 48	7 09	7 42	8 53	12 00	12 00
" 30	6 00	6 06	6 13	6 21	6 31	6 44	7 04	7 43	10 14	12 00
Sept. 14	6 00	6 02	6 05	6 08	6 11	6 16	6 23	6 37	7 18	12 00
" 29	6 00	5 59	5 57	5 54	5 52	5 48	5 43	5 33	5 03	0 00
Oct. 14	6 00	5 54	5 48	5 41	5 32	5 20	5 02	4 27	2 20	0 00
" 29	6 00	5 51	5 40	5 28	5 13	4 53	4 22	3 14	0 00	0 00
Nov. 13	6 00	5 47	5 33	5 17	4 57	4 29	3 43	1 46	0 00	0 00
" 28	6 00	5 44	5 27	5 08	4 43	4 09	3 09	0 00	0 00	0 00
Dec. 13	6 00	5 43	5 24	5 03	4 37	3 57	2 49	0 00	0 00	0 00

Increase of the Half Day at Sunrise, or Sunset, by Refraction.

1853.	Lat. 0°.	Lat. 10°.	Lat. 20°.	Lat. 30°.	Lat. 40°.	Lat. 50°.	Lat. 60°.	Lat. 70°.	Lat. 80°.	Lat. 90°.
	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.
January	2.5	2.5	2.6	2.8	3.3	4.4	6.7	0.00	0.00	0.00
February	2.4	2.4	2.5	2.7	3.1	3.8	5.1	9.0	0.00	0.00
March	2.3	2.3	2.5	2.7	3.0	3.8	4.6	7.0	14.0	0.00
April	2.3	2.4	2.5	2.8	3.2	3.8	5.0	8.0	0.00	0.00
May	2.4	2.5	2.6	3.2	3.5	4.5	6.1	22.0	0.00	0.00
June	2.5	2.6	2.8	3.1	3.7	4.9	7.6	0.00	0.00	0.00
July	2.5	2.5	2.7	3.0	3.5	4.7	6.7	0.00	0.00	0.00
August	2.4	2.5	2.5	2.8	3.2	4.0	5.2	9.7	0.00	0.00
September	2.3	2.4	2.5	2.7	3.1	3.7	4.6	7.0	14.7	0.00
October	2.3	2.4	2.5	2.7	3.1	3.7	4.9	7.5	24.3	0.00
November	2.4	2.5	2.6	2.8	3.2	3.9	5.9	16.3	0.00	0.00
December	2.5	2.5	2.7	2.9	3.5	4.6	7.5	0.00	0.00	0.00

Duration of Civil Twilight, Morning or Evening.

1853.	Lat. 0°.	Lat. 10°.	Lat. 20°.	Lat. 30°.	Lat. 40°.	Lat. 50°.	Lat. 60°.	Lat. 70°.	Lat. 80°.	Lat. 90°.
	m.	m.	m.	m.	m.	m.	h. m.	h. m.	h. m.	h. m.
January	32	33	34	37	43	57	1 16	3 21[2]	0 00	0 00
February	31	31	32	35	40	49	1 15	1 40	4 01[2]	0 00
March	30	30	32	35	39	50	1 03	1 29	3 04	12 00[2]
April	30	31	33	36	41	50	1 08	2 09	0 00	0 00
May	32	33	34	42	45	58	1 37	1 10[1]	0 00	0 00
June	33	34	36	40	48	64	2 46[1]	0 00	0 00	0 00
July	32	33	35	39	46	61	2 03	0 00	0 00	0 00
August	31	32	33	36	42	52	1 15	3 07[1]	0 00	0 00
September	30	31	32	35	40	47	1 02	1 35	4 42[1]	0 00
October	30	31	32	35	40	47	1 01	1 31	3 26	12 00[2]
November	31	32	34	37	42	51	1 10	2 16	0 00	0 00
December	33	33	35	38	44	60	1 22	2 42[2]	0 00	0 00

Duration of Astronomical Twilight, Morning or Evening.

1853.	Lat. 0°.	Lat. 10°.	Lat. 20°.	Lat. 30°.	Lat. 40°.	Lat. 50°.	Lat. 60°.	Lat. 70°.	Lat. 80°.	Lat. 90°.
	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.
January	1 13	1 13	1 17	1 24	1 39	1 56	2 38	5 29[2]	4 35[2]	0 00
February	1 10	1 10	1 14	1 20	1 30	1 43	2 20	3 32	7 49[2]	12 00[2]
March	1 08	1 09	1 12	1 19	1 30	1 48	2 21	3 44	6 29[1]	12 00[2]
April	1 09	1 11	1 15	1 24	1 36	2 01	3 06	4 01[2]	0 00	0 00
May	1 12	1 14	1 19	1 29	1 48	2 37	3 33[1]	1 10[2]	0 00	0 00
June	1 14	1 17	1 23	1 35	1 59	3 56[1]	2 46[1]	0 00	0 00	0 00
July	1 13	1 16	1 21	1 32	1 54	2 59	3 09[1]	0 00	0 00	0 00
August	1 10	1 12	1 16	1 25	1 40	2 11	4 18[1]	3 07[1]	0 00	0 00
September	1 08	1 09	1 13	1 18	1 31	1 51	2 30	5 23[1]	4 42[1]	0 00
October	1 09	1 10	1 13	1 19	1 29	1 47	2 18	3 25	7 48	12 00[2]
November	1 12	1 12	1 15	1 22	1 33	1 52	2 29	4 14	5 43[2]	0 00
December	1 14	1 15	1 18	1 25	1 37	2 00	2 47	5 03[2]	3 33[2]	0 00

[1] Twilight through the whole night. [2] Twilight without day.

NOTE.—Astronomical Twilight includes the duration of Civil Twilight.

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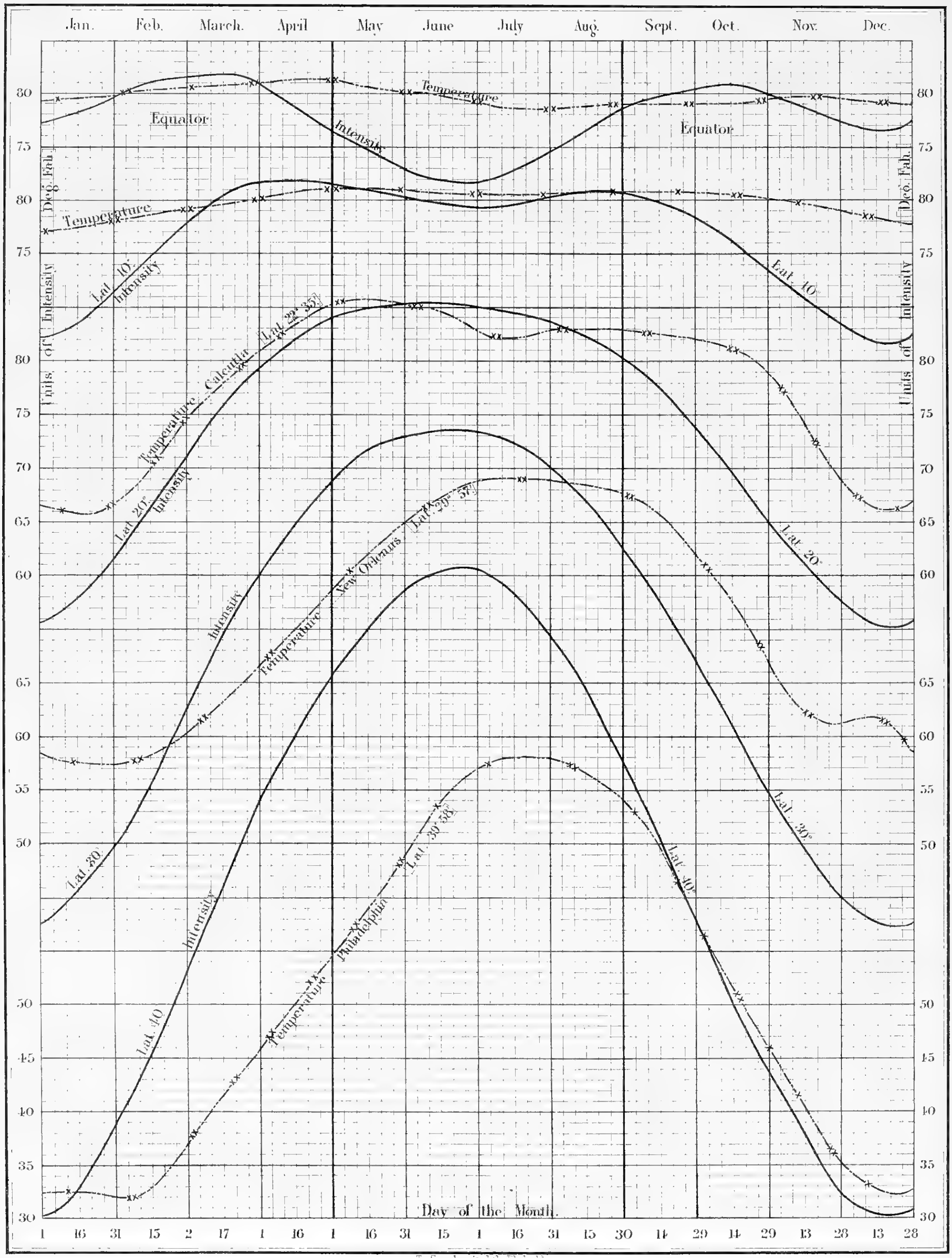
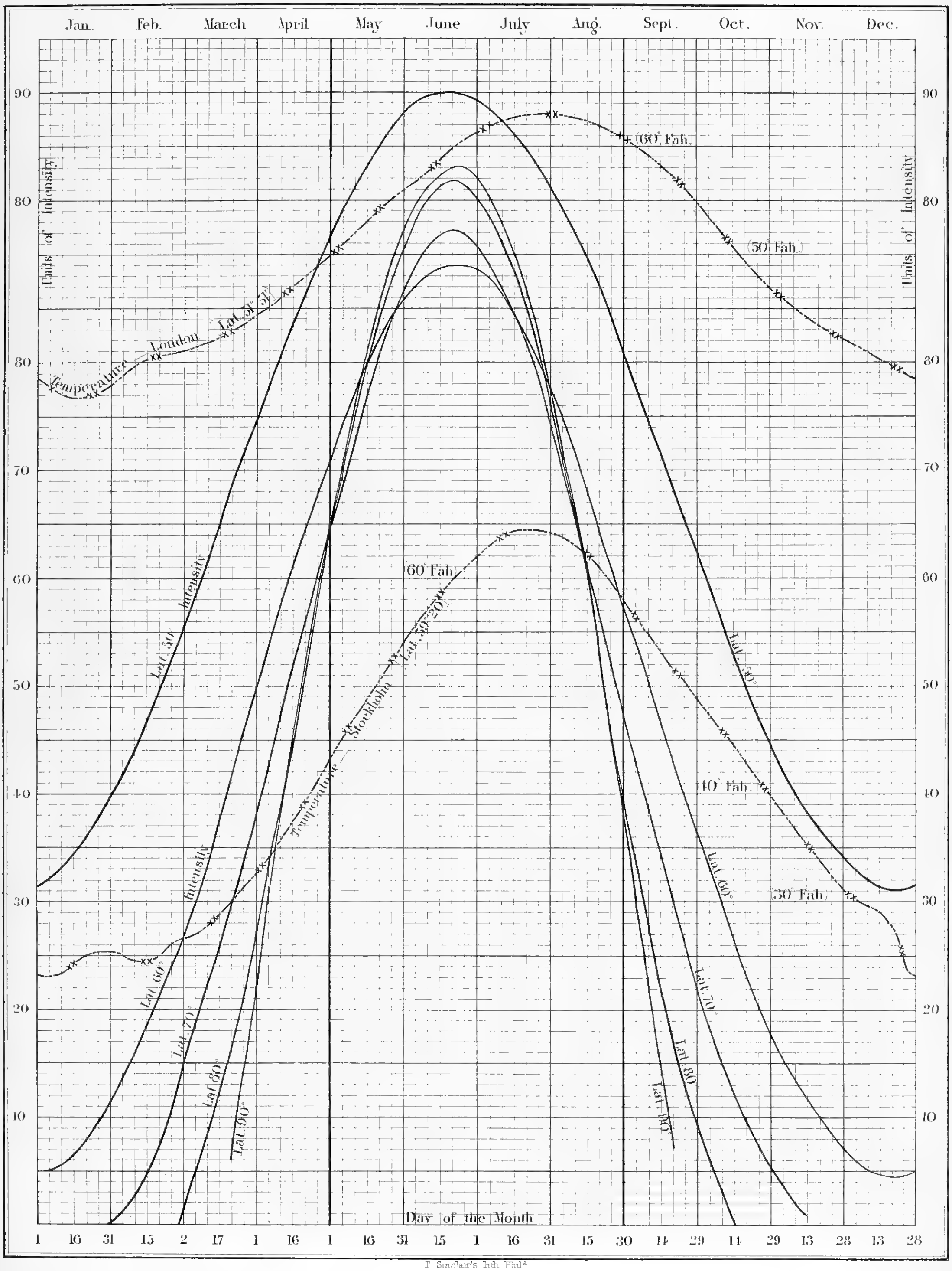
I.

THE SUN'S DIURNAL INTENSITY.



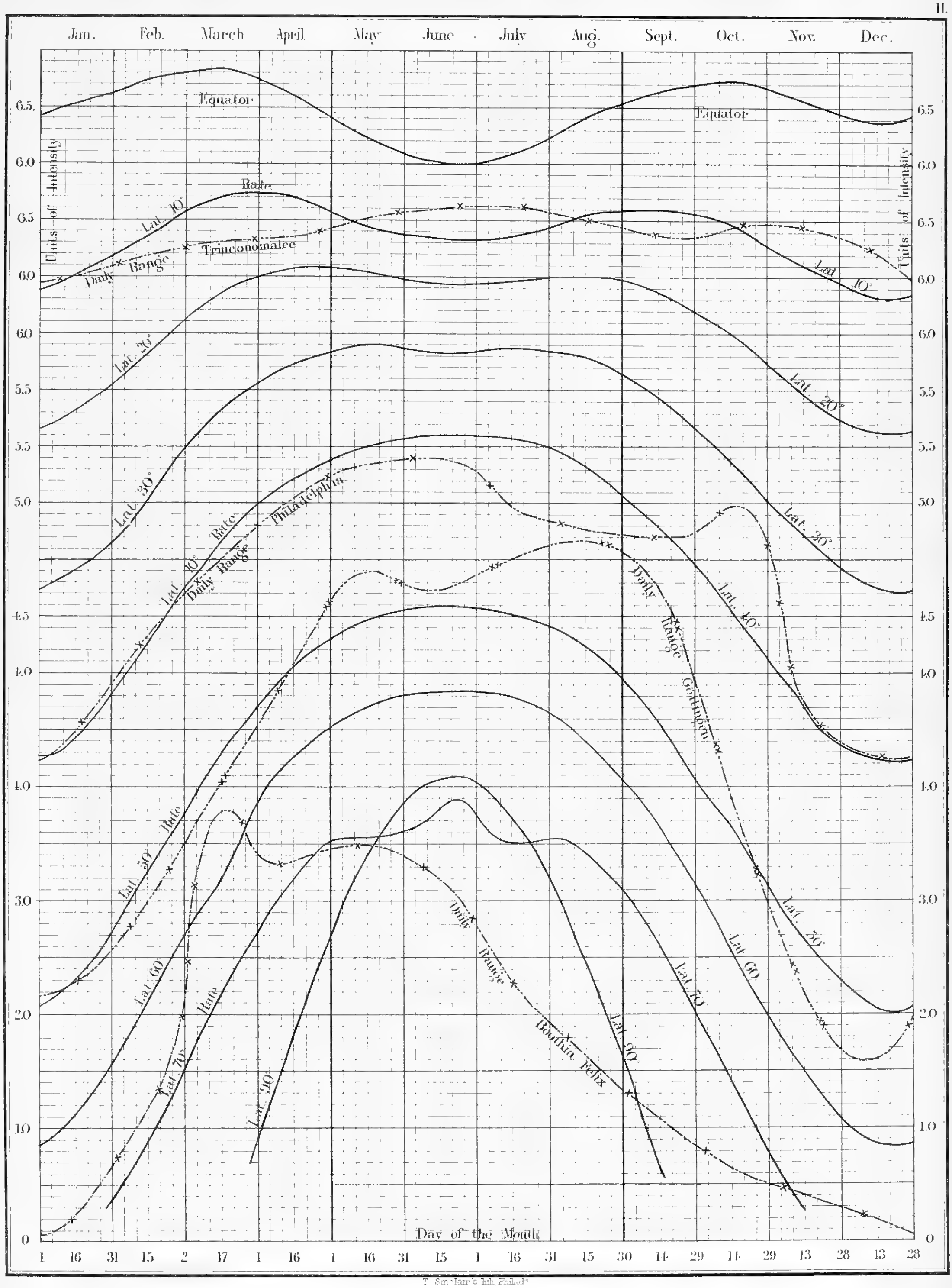
T. Sinclair's lith. Philad.

THE SUN'S DIURNAL INTENSITY.



T. Sinclair's lith. Phil.a

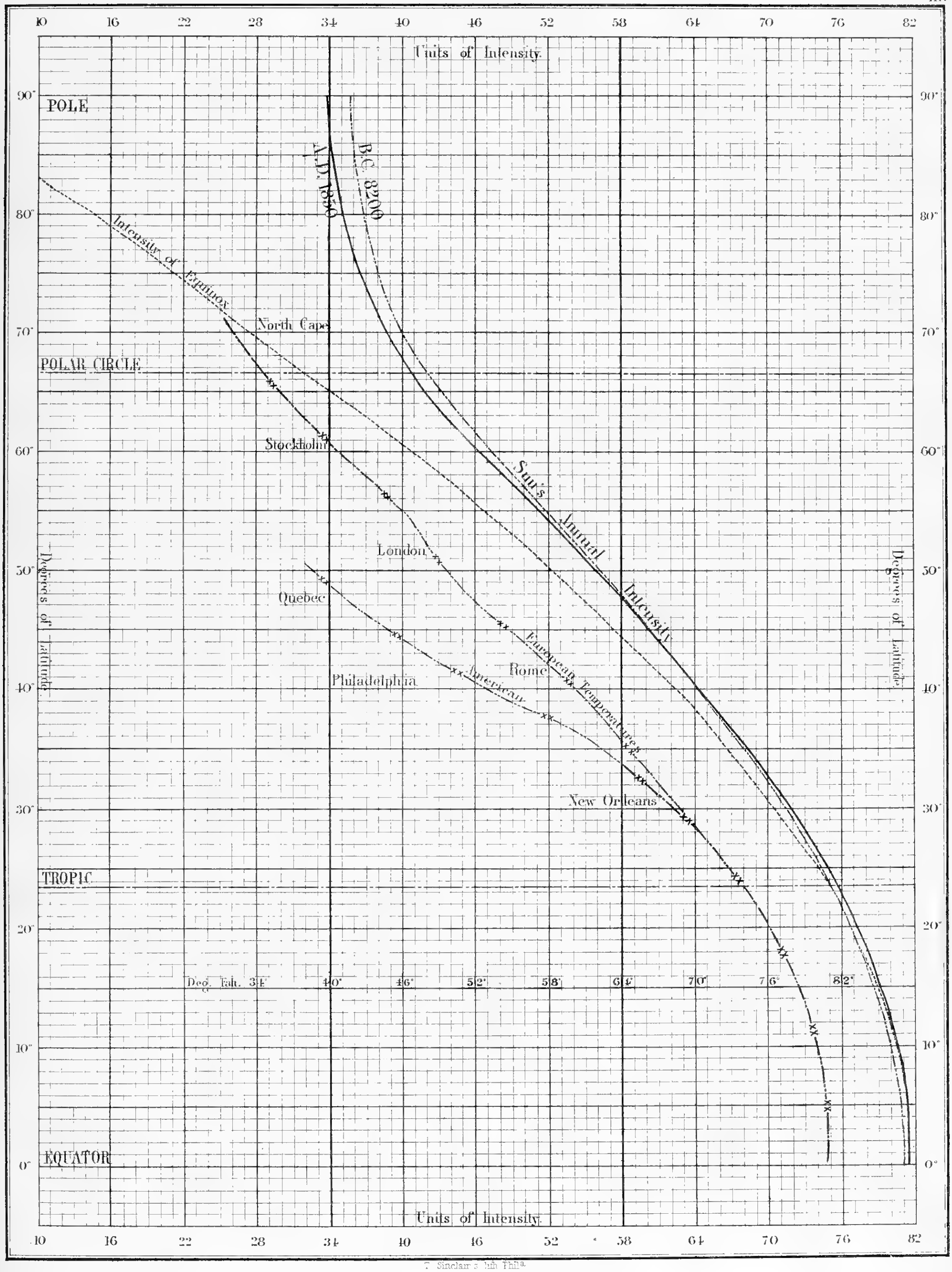
DAILY AVERAGE RATE PER HOUR OF THE SUN'S INTENSITY.



T. Sinclair's lith Philad.

THE SUN'S ANNUAL INTENSITY.

III.

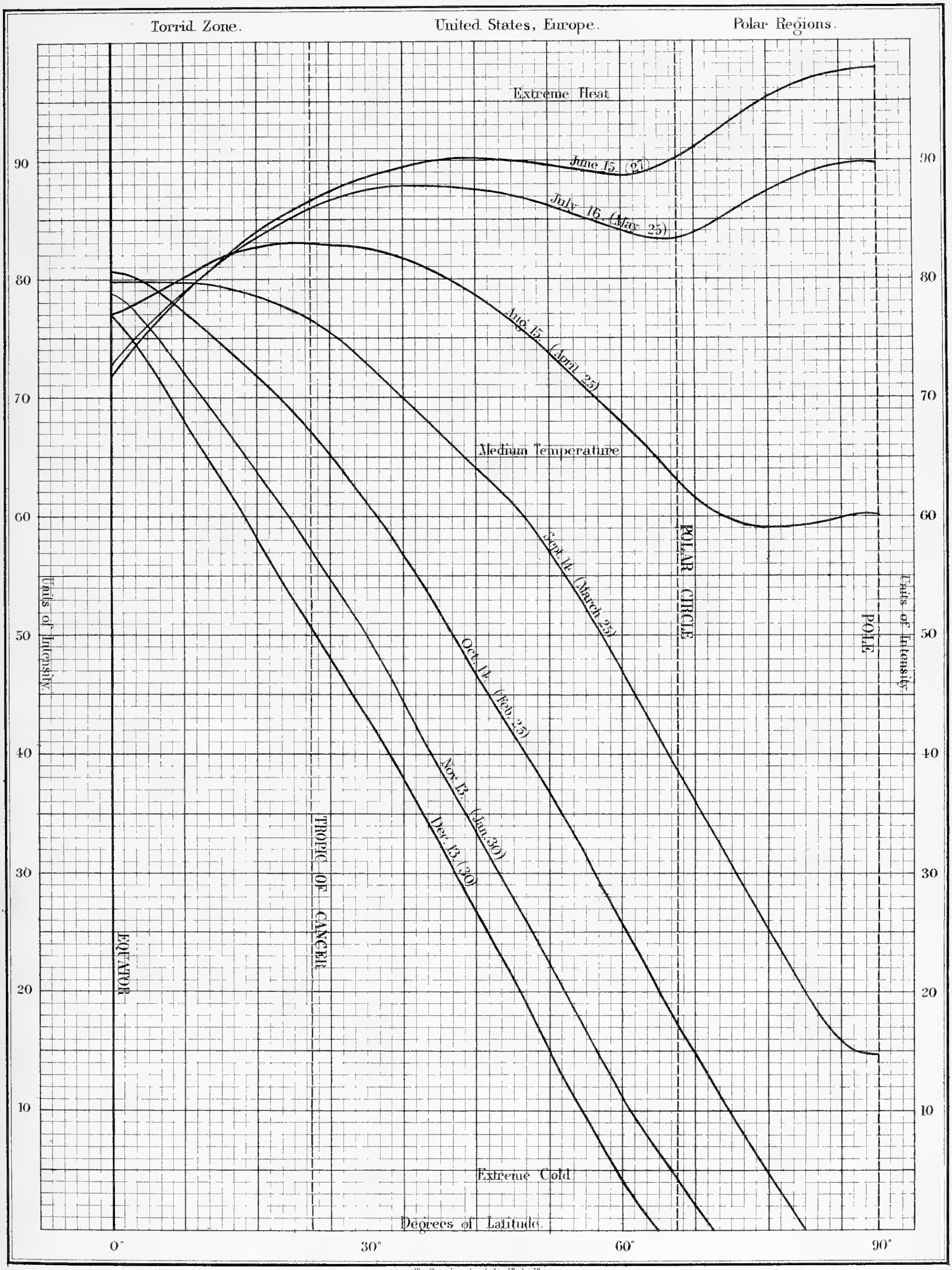


T. Sinclair's lith. Phil^a.

THE SUN'S DIURNAL INTENSITY ALONG THE MERIDIAN,

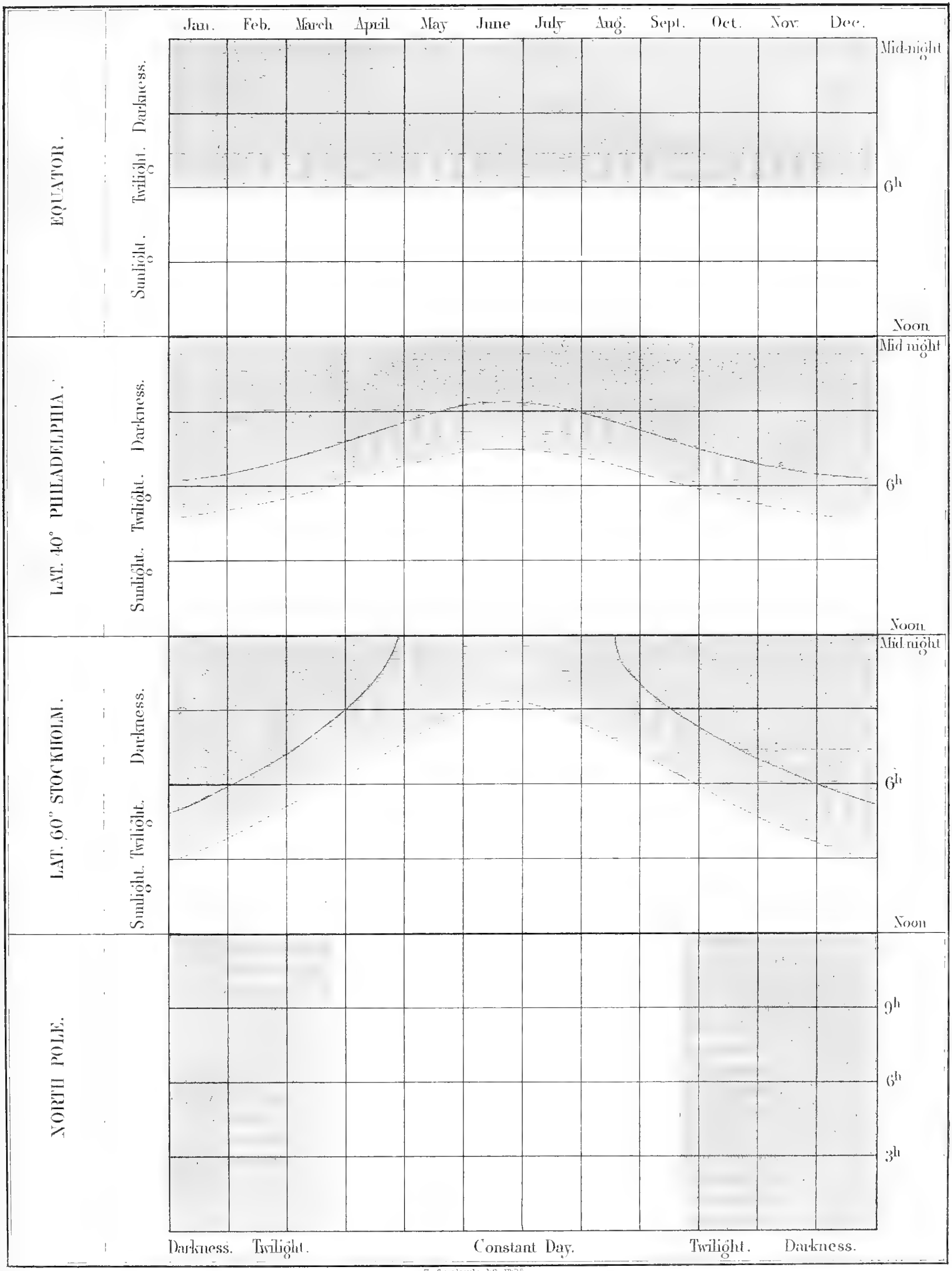
AT INTERVALS OF THIRTY DAYS.

IV.



T. Sinclair's lith. Philada.

DURATION OF SUNLIGHT, TWILIGHT AND DARKNESS.



T. Sinclair's lith Phila

ERRATUM.

Page 13, line 15, *for* size, *read* sine.

SMITHSONIAN CONTRIBUTIONS TO KNOWLEDGE.

ILLUSTRATIONS

OF

SURFACE GEOLOGY.

BY

EDWARD HITCHCOCK, LL. D.,

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[ACCEPTED FOR PUBLICATION, JANUARY, 1856.]

THIS paper has been submitted to a competent commission for critical examination, and has been recommended for publication.

JOSEPH HENRY,
Secretary S. I.

T. K. AND P. G. COLLINS, PRINTERS,
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PART I.

ON SURFACE GEOLOGY,

ESPECIALLY THAT OF THE

CONNECTICUT VALLEY IN NEW ENGLAND.

CORRECTIONS.

A change in the arrangement of the Plates, after a part of this Memoir was printed, makes a few corrections of reference necessary.

Wherever on pages 6, 7, and 8 Plate XI is referred to, it should read Plate XII.

Page 7, line 19 from bottom, *for* Fig. 3, *read* Fig. 2.

Page 7, line 16 from bottom, *for* Plate IX, Fig. 2, *read* Plate X, Fig. 1.

ON SURFACE GEOLOGY,

ESPECIALLY THAT OF THE

CONNECTICUT VALLEY IN NEW ENGLAND.

Introductory Remarks.

It has not unfrequently happened that those geological phenomena which lie nearest and most open to observation, have been the last to engage attention. The crystalline rocks were much earlier studied than the fossiliferous; and of the latter, the older and most deeply seated were well understood before Cuvier and Brogniart turned the attention of geologists to the tertiary deposits. It was not till a much later date, that the drift deposit, although so widely spread over the surface in northern regions, received any careful examination. And the subject of terraces and ancient beaches, is only at this late period beginning to call forth careful and thorough investigations; although these forms of gravel, sand, and loam, present themselves along nearly all our rivers, around our lakes, and towards the shores of the ocean.

I do not mean that these terraces, &c., have been entirely unnoticed by geological writers of the last quarter of a century. In the writings of Dr. Macculloch, more than thirty years ago, may be found some most beautiful delineations of these phenomena, and accurate descriptions of the very remarkable and peculiar terraces, called the Parallel Roads of Glen Roy, or Lochaber, which have engaged the attention of more subsequent writers than almost all other forms of the terrace. In the year 1833, the writer of this paper, in his Report to the Government of Massachusetts on its Geology, devoted some pages to a description of the river terraces; and gave a theory of their formation, different from that usually received. But no accurate details of facts accompanied these views.

Some elementary treatises on geology have, within a few years past, presented the subject of terraces and ancient beaches. This is especially the case in the writings of Sir Charles Lyell. That gentleman, also, has given to the public, through learned societies and journals, several detailed descriptions of these phenomena in particular localities.

The work, however, which seems to me to mark an era in this department of science, both in its presentation of facts and ability in reasoning, is Charles Darwin's *Geological Observations on South America.* It must have required extraordinary industry to collect the facts, and great familiarity with geological dynamics to arrive at the conclusions. This work was published in 1846, and directs geologists to the only true method of arriving at the truth on this subject, viz: by a careful investigation of the facts.

The work, however, which first awakened a more especial interest in my mind, probably because it came under my notice earlier than that of Mr. Darwin, was Robert Chambers' *Ancient Sea Margins,* published in 1847. Though dissenting from some of Mr. Chambers' theoretical views, I saw at once that he had given us an example of the true mode of getting at the truth on this subject. The numerous cases of the elevation of terraces and beaches in Scotland above the ocean, which this work contains, showed us that the same facts were needed in other countries. I felt desirous of throwing in my mite towards the work, so far as the valley of Connecticut River is concerned, though a bad state of health was a still stronger motive for engaging in it. But so many new views did my labors open upon me, that I have been stimulated to devote not a little time and labor to the subject of surface geology during the last seven or eight years. And I have been led to extend my observations beyond my expectations, not only in this country, but in Europe. I find the field to be a very large one; and that I have only begun to explore it. I have seen enough, however, greatly to modify, and as it seems to me to clarify, my views of the superficial deposits of the globe; and I venture to state my facts and conclusions before the scientific public.

I use the term *Surface Geology,* to embrace the results of all those geological agencies that have been in operation on the earth's surface since the tertiary period. All the changes that have taken place since that time, I regard as belonging to a single and uninterrupted formation, viz: the alluvial. The forces which were acting at its commencement are still in operation: but they have varied greatly in intensity at different times. Hence they have left various and peculiar products, of which the following are most worthy of note.

Drift unmodified.

Drift modified, which exhibits itself in the following forms:—

- Beaches, ancient and modern.
- Submarine Ridges.
- Sea Bottoms.
- Osars.
- Dunes.
- Terraces.
- Deltas.
- Moraines.

To which should be added the Erosions of the surface, from which the materials have been derived.

If we were to attempt to arrange these products in a chronological order, we might designate four periods, beginning with the oldest.

The Drift Period.

The period of Beaches, Osars, and Submarine Ridges.

The Terrace Period.

The Historic Period.

All the agencies, however, that have produced the above phenomena, are still in operation in some part of the globe; therefore, the above periods are intended to designate only the times when the different agencies were most intense, and produced their maximum effect. In a strict sense, they are contemporaneous. The Historic Period, however, merely designates the time since man and contemporary races have been upon the globe; and though it marks out an important zoological epoch, science has not yet been able to discover any correspondent geological change; though the presumption is, that one must have occurred, either local or general.

It is my purpose to go into a detailed description, in this paper, of only a part of the phenomena of surface geology, as enumerated above. I started with the intention of studying only the terraces and ancient sea-beaches in the vicinity of Connecticut River. I found these subjects, however, so closely related to other points, that to investigate a part would cast light upon the whole. The subject of erosions has specially attracted my attention, and, since these are not confined to the alluvial period, I shall treat of them in a separate paper. Unexpectedly, also, the marks of what I suppose to have been ancient glaciers, descending from the Hoosac and Green Mountains, fell under my notice; and I have devoted another short paper to an elucidation of the facts. In the present paper, I shall confine myself chiefly to beaches and terraces, with their associated phenomena, submarine ridges and old sea-bottoms. The subject of drift must, of course, receive some attention; since the other forms of detritus are mainly modified drift. But I assume that the general facts as to the phenomena of drift are understood by the reader.

At the first, I did not expect to extend my observations beyond the valley of Connecticut River. But, during the six years that have elapsed, I have travelled extensively, both in this country and in Europe, with an eye always open to surface geology, and usually with some kinds of instruments for measuring heights. The facts thus obtained, sometimes indeed but few and unimportant, I shall embrace in this paper.

It is well known that, usually, geological maps exhibit but little of surface geology; save where the drift or alluvium is so thick that the subjacent rocks cannot be ascertained. Were the surface geology well exhibited in such a region as New England, these subjacent rocks would occupy but a small space. I have appended to this paper, a few imperfect maps of this character. One represents, as far as I have been able to trace it out, the surface geology of the Connecticut valley; and others, certain spots, chiefly in that valley, much more limited. It has been an object of strong desire with me, to construct a similar map of the whole of Massachusetts; and the Legislature of the State have given me assistance to collect the facts. If life and ability to labor be continued to me long enough, I shall hope to accomplish this object. The present paper is a preliminary to such a work.

Several terms, mostly new, and necessary to a right understanding of surface geology, will need definition.

Drift is a mixture of abraded materials—such as boulders, gravel, sand, and mud—mixed confusedly together for the most part, but sometimes laminated, and occupying the lowest part of the unconsolidated strata, and lying immediately upon tertiary deposits, where they are present, or upon older rocks, where they are not.

Modified Drift.—When drift has been acted upon by waves, or currents of water, the boulders are reduced in size, they are smoothed and rounded, their striæ are generally obliterated, and all the materials are redeposited in regular layers, being sorted into finer and coarser deposits, according to the velocity of the currents. These I call modified drift, which constitutes nearly the whole of what usually goes by the name of alluvium, and assumes various forms, according to circumstances.

In this paper, the term alluvium includes not only modified but unmodified drift, for reasons which will appear in the sequel.

Sea-Bottoms.—The bottom of the ocean, along the coast, is in many places covered by deposits of sand and gravel, left there seemingly by tidal action, and presenting often numerous ridges and depressions. Often, too, bars are formed across the mouths of harbors, producing lagoons. Hooks, also, are produced, where the currents sweep around headlands. While these deposits are beneath the waters, they go by the name of shoals. If these shoals, bays, and harbors be raised out of the ocean, although they will be exposed to the modifying influence of rivers and rains, their essential characteristics will be long preserved; and my impression is, that these old sea-bottoms may still be traced in many parts of our country, to the height of 1,000 to 2,000 feet above the present ocean.

Submarine Ridges.—By this term, I intend to designate certain ridges of sand and fine gravel that must have been formed beneath the waters, and yet are different from those ridges called shoals, and, perhaps, from any other submarine deposit described by Lieutenant C. H. Davis, in his admirable paper, in the *Memoirs of the Academy of Arts and Sciences*, "On the Geological Action of the Tidal and other Currents of the Ocean." The great peculiarity of these submarine ridges is, that they slope in two directions—towards the lake or the ocean, on whose borders they lie, and towards the country; a fact which indicates subaqueous formation. The natural ridges around Lakes Ontario and Erie, are a fine example of the phenomenon I am describing. (See Charles Whittlesey's excellent paper, Am. Journ. Sci., N. S., X, 31.) Perhaps, also, I may be able to point out one or two examples on the sea-coast.

Osars.—These are similar ridges, formed beneath the waters, by currents piling up materials behind some obstruction. Their form is very much like that of a canoe turned over. I have not been able certainly to identify any ridges of sand or boulders in our country with the osars which I saw in Europe. But M. Desor, whose opportunities for observation upon this phenomenon have been very extensive, speaks of osars as occurring along the shores of Lake Superior. I have marked four on Map No. 1, (Plate III,) in N. H., viz: in Union, at the White Mountain Notch (at Fabyans), and a little south of Conway; but they are of doubtful character.

I use the terms dune and delta in their common acceptation. The same is true of moraine, excepting that I think I have found some ancient moraines that have been subsequently modified by the action of water, whereby the coarser detritus has been more or less covered by water-worn and sorted materials.

Terraces and beaches form, perhaps, the most important feature of surface geology; and, as I have directed my attention chiefly to these, I shall go into more details as to their nature and characteristics.

It is hardly necessary to say that, though the term terrace applies to any level-topped surface, with a steep escarpment, whether it be solid rock or loose materials, it is only the latter kind which are treated of in this paper; for I shall describe only those terraces which have been formed since the drift period—not even those which may be unconsolidated in the tertiary strata.

Terraces are of three kinds:—

1. *River Terraces.*

These are the most perfect of all, and are found along the shores of almost all rivers; but especially those passing through hilly countries, and forming narrow basins with a succession of gorges.

River terraces may be subdivided into four varieties, differing in position, and probably, also, in their mode of formation.

1. *The Lateral Terrace.*—This is the ordinary terrace, which we meet along the banks of a river, often many miles in length, and sometimes even miles in width.

2. *The Delta Terrace.*—This occurs at the mouths of tributary streams, and was most obviously a delta of the tributary; but, as the waters sunk, the delta was left dry, and the tributary cut a passage through it, so as to form a terrace of equal height on opposite banks.

3. *The Gorge Terrace.*—This occurs either above or below the gorges of a stream, and is intermediate between the lateral and delta terraces, graduating into both.

4. *The Glacis Terrace.*—This is not level topped, but slopes gradually both ways from its axis—on the side next the stream much more rapidly than on the other. Outwardly it resembles the *glacis* of a fortification, and hence the name. It is usually found in alluvial meadows, and might, perhaps, be regarded as merely the uneven surface of a lateral terrace, as it is seldom more than a few feet high. But in some of the high valleys of the Alps, I found broad terraces sloping very rapidly towards the stream to its very brink, as well as in the direction of the currents, and Mr. Darwin describes the same kind of terrace in the high valleys of the Andes. Such terraces, then, I should regard as the true type of the glacis terrace, rather than those undulations of surface which we see in alluvial meadows.

2. *Lake Terraces.*

These scarcely differ from the lateral terraces of rivers. Indeed, many small lakes, and even some of the larger ones, appear to have been merely expansions of rivers, such as are now seen in great numbers in the basin of the Upper Mississippi, west and southwest of Lake Superior. (See *Nicollet's Map.*) These were formerly retained by barriers at a higher level when the terraces were formed, and, as those barriers have been worn away, the terraces have been left on their borders.

3. *Maritime Terraces.*

Perhaps I ought not to speak of terraces as existing on the margin of the sea, but to regard all accumulations of sand and gravel there as beaches. Some of these accumulations, however, are so nearly level-topped as not to differ from genuine terraces, and this is the main distinction which I would make between terraces and beaches. It is not, however, a distinction of much practical importance. At the mouths of rivers, the two varieties are often seen running into each other.

Moraine Terrace.—I apply this term to a peculiar form, not unfrequently assumed by the more elevated terraces, exhibiting great irregularity of surface; elevations of gravel and sand, with correspondent depressions of most singular and scarcely describable forms. I prefix the name *moraine* terrace to such accumulations, under the impression that stranded ice, as well as water, was concerned in their production.

Sea Beaches.

The most perfect of these are seen along the sea-coast in the course of formation. They consist of sand and gravel, which are acted upon, rounded, and comminuted by the waves, and thrown up into the form of low ridges, with more or less appearance of stratification or lamination. As we rise above the terraces along our rivers, and often on the sides of our mountains, we find accumulations of a similar kind, evidently once deposited by water, and having the form of modern beaches, except that they have been often much mutilated, by the action of water and atmospheric agencies, since their deposition. These have hitherto been confounded with drift, but they nearly always lie above it, and show more evidently the effects of some comminuting, rounding, and sorting agency—of water, indeed, since this is the only agent that could produce such effects. They evidently belong to a period subsequent to the drift, and I cannot doubt that they once constituted the beaches of a retiring ocean. The proof of this will be given further on.

I have spoken of these beaches as lying above the terraces. I mean that they are at a higher level often, but geologically they are lower. When terraces occur as well as beaches, the latter always are seen at a higher level than the former; usually forming fringes along the sides of mountains. Yet in other places rivers may exist at a much higher level, which have terraces also; and usually above them we find beaches, still retaining the same relative position to the terraces.

General Lithological Character of the Terraces and Beaches.

As a general fact, I give the following description, applicable to the terraces and ancient beaches:—

1. The most perfect terrace is an alluvial meadow, annually more or less overflowed, and increased by a deposit of mud or sand. Rarely are the materials as coarse as pebbles, except on a small scale. Yet usually they are sorted, laminated, and stratified. (See A on Fig. 1, Plate XI, which is an ideal section across a valley.)

2. Ascending to a second terrace, we almost invariably find it composed of coarser materials; or, perhaps more frequently, of sand at the top and clay at the bottom; though sometimes the sand is all removed. (See B on Fig. 1, Plate XI.)

3. Rising to a third terrace, we usually find a mixture of sand and gravel; the latter not very coarse, the whole imperfectly stratified, and also sorted; that is, the fragments in each layer have nearly the same size; as if the waters that removed and deposited the materials, had a different transporting power for each stratum. (See C, Fig. 1, Plate X.)

4. A fourth terrace is sometimes found still higher, differing from the last only in being of coarser, but still of decidedly water-worn materials. (D, Fig. 1, Plate XI.) There is another important distinction. Hitherto the tops of the terraces have been for the most part level, unless worn away by agents subsequent to their formation. But now we find their surface not unfrequently piled up into rounded or curved masses with corresponding depressions, resembling what is called a *chopped sea*, or the eminences and anfractuosities on the surface of the human brain. The depressions are not valleys, which might have been made by currents of water, but irregular cavities, often a hundred feet deep, or more, usually not more than twenty or thirty, and perhaps more frequently not over ten or fifteen. Yet the materials forming the boundaries of these depressions are always water-worn and sorted, either sand or gravel. These irregular cavities and elevations do not always appear in connection with the fourth terrace, but sometimes with the fifth and sixth. Yet I believe there is never a level-topped terrace above them (that is, older) in the same series; and they are always below the beaches. They are a singular feature in the terrace landscape, and are among the most difficult of all the phenomena of these formations to account for satisfactorily. I shall of course recur to them again in a subsequent part of this paper. (See D, Fig. 1, Plate XI.) Plate IX, Fig. 3, is a sketch taken in the west part of Pelham, in which we see the more perfect lower terraces, succeeded by others having the peculiarity of outline above described. Such sketches, however, give but a faint idea of these moraine terraces, as I now call them. They are shown also imperfectly on Plate IX, Fig. 2, taken in Russell, on Westfield river, with the Pentagraph Delineator, by Mr. Chapin, its inventor.

5. Above the irregular terrace just described, we find other accumulations of decidedly water-worn materials, generally coarser, the fragments of rolled and smoothed rock being sometimes a foot or two in diameter; yet still more or less sorted, so as to bring together those of a determinate size, or rather those not exceeding a certain size. Coarse sand, however, constitutes the greater part of the deposit, and sometimes the whole of it. Its outline is rounded, rarely with a level top for any considerable distance. Yet in its longest direction it maintains essentially the same level, and often may be seen for many miles at the same height, and more or less worn away, as a fringe along the sides of the hills that bound a valley; appearing, in fact, as if these deposits once formed the beaches of estuaries that occupied those valleys; and such I suppose they were. (See Fig. 1, E, Plate XI.)

As we rise above the most recent ancient beach, we find others at different levels, of materials less water-worn, more irregular in their form, and less continuous in the direction of the valley. They seem to have constituted shores when the waters were higher, when less land was above the surface, and consequently the waves had less power to wear away and comminute the rocks.

6. Passing beyond and above the terraces and beaches, thus lying at the bottom, and along the sides of the valleys, we reach the genuine drift deposit (F, Fig. 1, Plate XI,) consisting of materials that are coarser, more angular, and less arranged in strata and laminæ. These are strewn promiscuously over the hills, except those quite steep and high. They are also seen occasionally in the valleys, wherever the terraces and beaches have been worn away or never existed. Yet it must be confessed that it is often not possible to draw a distinction between the oldest beaches and the drift. They pass insensibly the one into the other. The large blocks of the drift are indeed frequently angular, but they are mixed with finer materials that have been ground down and rounded, either by aqueous or glacial agency; and the oldest beaches seem to be of essentially the same materials, somewhat more modified.

It is important, also, to mention that what appears to be genuine drift, is sometimes found mixed with, and sometimes superimposed upon, the beach and terrace materials. This is especially true of large erratic blocks. And it shows us that the drift agency, whatever it was, occurred in some places, after the modifying agency that formed the older beaches and terraces had been for a time in operation. Or, more probably, it was the same agency in modified forms that produced all the phenomena. Below the drift we find the consolidated strata. (G, Fig. 1, Plate XI.)

The views that have now been presented I have attempted to exhibit to the eye on Fig. 1, which is an ideal section across a valley, showing the manner in which the terraces, beaches, and drift are usually found; the newer deposits being chiefly formed by the denudation and modification of the drift which lies beneath the others. But as to the number of terraces, their relative height, &c., we find in nature a great variety, and this section is intended only to give the general impression that has been made on my mind by all the cases which I have examined.

Origin of the Materials.

1. I have already said that the beaches and terraces appear to be mainly modified drift. The agency by which the former have been produced, commenced the process of separation and comminution, carrying it at first only far enough to form the higher and coarser beaches. The work still went on with another portion, till it was reduced into finer materials for the higher terraces—and still finer for the lower terraces, until, when it came to the lowest of all—our present alluvial meadows—the fragments had been brought into almost impalpable powder, so as to form fine loam.

2. Such a work could not go forward with fragments already detached from the ledges, as was drift, without subjecting the solid rocks to erosion, wherever exposed. Accordingly a part of the materials of the terraces and beaches must have been derived from this source. How deep in any place these erosions have

been made, may be learned by ascertaining how near the bed of the stream we find drift striæ and furrows. From some facts of this sort, I am satisfied that though fluviatile erosion has been considerable in some places, even as much as 200 or 300 feet, in general no great amount of the detritus of terraces has been thus produced, except in loose materials.

Arrangement of the Materials.

1. *Stratification and Lamination.*—All these deposits are more or less stratified, and most of the finer varieties are also laminated. The lamination is not unfrequently oblique to the stratification. The former is frequently inclined some 20° to the horizon, the latter usually quite horizontal, though the strata or laminæ of clay are sometimes plicated.

The *Loess* of the Germans, or *Limon* of the French, along the valley of the Rhine, is usually represented as neither stratified nor laminated. That it is a fresh-water deposit, all admit; and that the terraces along the Rhine are mainly composed of it, I was assured by Professor Noggerath, of Bonn, as I ascended that river in his company, in 1850. That it is also more or less stratified, I cannot doubt. Indeed, so it is represented by Sir Charles Lyell. But from its composition (fine calcareous clay), we might presume that lamination would be mostly absent.

The other deposit, apparently without stratification or lamination, is what in Scotland is called *bowlder clay;* that is, clay containing pebbles and frequently quite large bowlders. Some which goes by this name in Scotland may be unmodified drift: but where it was pointed out to me, by Dr. Fleming, in Edinburgh, it appeared to be drift modified by aqueous action and deposited in the turbulent waters of the ocean. In this country the clay sometimes so much predominates that it is used for making bricks. I cannot doubt that imperfect stratification may be found in it.

And here I ought to remark, that when a deposit has been exposed to the weather, even for a short time, all traces of stratification and lamination disappear: but when fresh excavations are made in it, both these structures are distinct. By examining many such cuts, made by canals and railroads, I have frequently found the structure beautifully developed where no trace of its parallel arrangement could be seen at the surface. Even beds of pebbles, apparently thrown promiscuously together, are often found to be arranged in a stratiform manner.

2. *Sorting.*—Wherever a section is made into a terrace, composed of clay, sand, and pebbles, we see that these varieties of material are usually arranged in distinct layers, the coarser together and the finer together. The impression is irresistible on the mind, that the water, which made the deposit at one time, had only velocity sufficient to move the finest sediment: at another, sand, finer or coarser; at another, small pebbles; at another, large pebbles; and sometimes to urge along masses of considerable size. In such cases the stream chose out and carried forward the largest pebbles or blocks, which its particular velocity would raise, leav-

ing other fragments for a time when its power should be increased. In this way have the materials been sorted out more nicely than any mechanical skill could do.

Details of the Facts.

I now proceed to give an account of the facts which I have collected respecting terraces and beaches, within the last six or seven years. I began their examination in 1849, and have since pursued it as diligently as my time and means would allow. And having, during that period, traversed several of the countries of Europe, I improved the opportunity to notice these phenomena, though it was out of my power to make very numerous measurements. I have also travelled somewhat extensively in our own country, to complete the comparisons. But it is along Connecticut river and its tributaries that I have made the most careful and consecutive observation. After reading Mr. Chambers' *Ancient Sea Margins*, I felt desirous of determining the true heights of the terraces in this valley, by mensuration. For a time I used the common levelling instruments, and thus obtained numerous sections. This method I found, however, to be so laborious, in a country like ours, where so few heights away from our railroads and canals have been ascertained, that some other method would be important, where the beach or terrace to be measured was distant from any such ascertained heights. I obtained an Aneroid Barometer; but my early trials with it were so unsatisfactory that I gave it up in despair. But when I reached Liverpool, and was desirous of visiting the mountains of Wales, I purchased another, and found the results so satisfactory in the measurement of Snowdon and Cader Idris, that I carried it with me in all my wanderings. In going to Ireland, however, the hair-spring that regulates the index, was broken by the rough usage of my luggage. It was mended in Edinburgh, but broken again before I reached Frankfort on the Main. Again I had it mended, and made use of it in Switzerland and Savoy. On my return to this country, I wished to ascertain whether the accidents to which it had been subject had affected its range. I soon discovered that they had. But instead of attempting to use the adjusting screws, I obtained from the Smithsonian Institution the loan of one of Green's Syphon Barometers, and commenced a series of observations in connection with the Aneroid. Those were at length reduced, and thus making the Syphon Barometer the standard, I ascertained the error of the Aneroid, and found that for every tenth of an inch it gave only 78.47 feet of altitude. Thus was I able to correct all my observations made in Europe, after the injury of the instrument, and the results I shall give below.

Having used the Aneroid Barometer so extensively, it might be desirable that I should go into details respecting the results, as compared with other measurements, in order to decide how much dependence can be placed upon the instrument. But these details would occupy too much space. If in my power, I hope to present them in some other form: for my own conviction is, that though this instrument cannot be depended on for nice observations, such as the mathematician needs, yet it is a most valuable help to the geologist. I think it can be depended on almost as confidently as the Syphon Barometer, except perhaps for very great altitudes.

In nearly forty observations, upon heights varying from 260 to 5000 feet, the difference between the two instruments rarely exceeded twenty feet, and in only one or two cases of great altitude, approached 100. Such an approximation to the truth, surely the geologist must regard as of great value, especially as the observations can be made with so little inconvenience and delay.

One of the most serious drawbacks upon this instrument, as appears to me, is the difficulty of adjusting it, or of ascertaining its range. In either case several observations must be made upon heights of several hundred feet. This is great labor for every turn of a screw. My experience leads me to conclude that to resort to the air-pump in such cases is not reliable.

Former Basins in the Connecticut Valley.

Originally, when the river stood at a higher level, this valley consisted of a succession of basins, or expansions in the stream, separated, or perhaps connected by ridges, through which gorges were cut, and deepened by the river alone, or with the aid of the ocean. At present, so deeply has the bed been worn down, that these narrow lakes or ponds have disappeared. But they have left evidence of their former existence by the terraces on their borders. The following ancient basins are well marked.

1. From the mouth of the river to Middletown, a distance of twenty-five miles, it is bounded by steep and rocky hills, with a narrow meadow occasionally. Where the river enters this mountainous region, just below Middletown, the gorge is the narrowest: but in its whole extent it has every appearance of having been formed by the joint action of the river and the ocean.

At Middletown the first, the longest, and the widest of these basins commences, and extends to Mount Holyoke, in Hadley, a distance of fifty-three miles. On the west side, however, the high land opens to the southwest of Hartford, so that on the line of the Hartford and New Haven Railroad, the summit is only a few feet above Connecticut river. It is certain, therefore, that when the river chose its present bed through the rocky region below Middletown, that bed must have been excavated nearly to its present depth; otherwise the water would have chosen the valley of the railroad in its way to the ocean. The passage through the mountains must have been lower than through Meriden, &c., to New Haven.

At Enfield, in this basin, the river has cut through a sandstone range of considerable height. The highest terraces, however, rise above the rocks in most places; yet, during the deposition of the lower terraces, the long basin above described must here have been divided into two of nearly equal size.

2. The second basin extends from Holyoke to Mettawampe (Toby,) in Sunderland, and Sugar-loaf, in Deerfield. From Holyoke, this basin must have extended southerly along the west side of Mt. Tom, and the other almost continuous trap ranges that extend to New Haven. Through this valley runs the canal railroad from New Haven to Northampton. But nowhere is this valley more than one hundred and thirty-four feet above the Connecticut at Northampton, and this is not so high as some of the terraces.

The second basin, also, extends northerly from Sunderland, on the west side of

Sugar-loaf and Deerfield Mountain, through Deerfield, Greenfield, and Bernardston. Here it joins the fourth basin of the Connecticut in the west part of Northfield; so that the second and fourth were one basin when the higher terraces were deposited. On its west side, this second basin must have been not less than one hundred and ten miles long.

3. The third basin extends from Mettawampe to the mouth of Miller's river, in the northeast part of Northfield. It is narrow, and not more than eight or ten miles long.

4. The next basin reaches from the mouth of Miller's river to Brattleborough. Some of its higher terraces extend across the barrier into No. 3, and also, as already stated, into No. 2, in Bernardston. Though seventeen miles long, it is narrow.

5. From Brattleborough to Westminster, seventeen miles, the bed of the river may be considered as a deep gorge through the mountains, similar to that south of Middletown. Through Westminster to Bellows Falls, embracing Walpole also, is a short, but very distinct basin, five miles long, with numerous terraces. Terraces also exist in most parts of the gorge, but they are narrow.

6. The next basin extends from Bellows Falls to North Charlestown, fourteen miles, where the mountains close in upon the river, as at Bellows Falls. Yet some of the highest terraces, at both extremities, pass over into the adjoining basins.

7. I regard the next basin as extending from Charlestown to Ascutney Mountain in the south part of Windsor, ten miles, although some of the terraces extend northerly into the next basin. Yet I cannot doubt, but that this mountain once formed a gorge.

8. The basins become less distinct as we ascend the valley, and I have not studied them as carefully in its upper part. I should say that we might regard an eighth basin as reaching as far as Fairlee, although the hills several times crowd closely upon the river south of that place.

9. From Fairlee, through Haverhill to Bath, the valley is wider, and the terraces numerous and distinct. This basin may extend beyond Bath, which is the northern limit of my examinations. This spot is two hundred and ten miles from the ocean in a direct line along the rivers.

On the map of the valley (Plate III,) accompanying this paper, I have marked the above basins more distinctly probably than facts will justify. But in the absence of all accurate delineations of our topography on the published maps of New England, I thought it would not be improper to represent elevations that do actually exist, in order to make myself better understood, even though they be more *prominent* than in nature.

I would here take occasion to remark, that the most serious obstacle to my progress in these investigations has been the want of accurate maps of the districts explored. Frequently have I spent the day (and the same experience is fresh in my mind as to older rocks) and have got a clear conception of the terraces, beaches, and hills in a considerable district. But on opening my map to delineate the same, I have often found, to my discomfiture, that no such region exists on the map as existed in my mind, and which I found in nature; and hence the greatest inaccuracy must be the result, and often total discouragement. For I should thus be charged with errors of observation by future geologists, when the fault lay solely in the maps. Massachusetts is the only State in New England that has constructed an accurate map of its surface. And in that State the topography was omitted till near the close of the survey, and then hastily observed; so that it only presents us with insulated hills and ranges, as if they rose out of a level surface; whereas, no idea is presented of the longer and broader features of the country; the comparatively low region, for instance, of twenty miles from the coast; the valley of Worcester, of the Merrimack, of the Connecticut, and the deep valleys of Berkshire. Imperfect maps are one of the great disadvantages under which American geologists labor, of which the European geologist knows but little. And it must be a long time before the matter is much mended.

Basins on the Tributaries.

The tributaries of the Connecticut exhibit successive basins of the same general character as those above described. But there are two of unusual importance, which I have examined. One is on the Agawam river, in Westfield, and the other on Deerfield river, in Deerfield. In the latter basin especially, we have an epitome of most of the facts concerning river terraces and changes in the beds of rivers. That spot I have, therefore, studied with care, and shall present a separate map of its features, and also of the Westfield basin.

Of some other peculiarly interesting places in respect to their terraces, I shall, also, present maps, on a larger scale than the general one. One will be given of the terraces at Bellows Falls, another of those in Brattleborough, and a third of those on Fort river, in Amherst and Pelham.

Mode of Representing the Terraces and Beaches.

On the general map of Connecticut river, from its mouth to Wells river in Vermont, a distance of two hundred and ten miles, I have attempted to exhibit the principal terraces by colors. There are many smaller ones, however, omitted; nor have I attempted to give the true width of the terraces with any degree of accuracy. Only where the basins are the widest, there I have represented a greater breadth of terraces. To give the terraces with entire accuracy, over so wide a region, would require a great amount of labor in observation, and then it would all be useless, because of the great imperfection of our present maps. All I have attempted, therefore, is an approximation to the truth. In the vicinity of my residence (Amherst) I have delineated the terraces with more accuracy, I hope. But in some parts of the river, especially its southern and northern limits, I have not been able to examine with the care which would have been desirable. I trust, however, that my maps will answer for all the purposes I have in view. This I believe is a first attempt of this kind, and I have been led to feel how desirable a work it would be to present a map, on a similar plan, of all the terraces, beaches, drift and other forms of surface geology in the northern parts of our country. In the vicinity of Amherst I have attempted to show what I conceive would be a desirable map for the whole country. But the work would be Herculean, even for New England. Yet if I were a younger man, I should have the ambition to attempt it.

The colors on all the maps are the same for the same terraces, reckoning upward from the river. The lowest meadow I call the first terrace, and then count them upward; thence it follows, that the same color does not always represent terraces of the same height, since they vary in this respect on different streams; and, in general, the size and height of the terrace correspond to the size and height of the river.

As to the beaches, I represent them all by one color, as I have not explored them with sufficient accuracy to enable me to make any correct distinction between the higher and the lower, nor do I know of any important object to be accomplished by such a distinction.

1. Sections of Terraces and Beaches.

The larger part of the terraces which I have measured, I have also shown by sections across them, down to the level of the rivers on which they are situated. This will give a clearer idea of their relative size than description can do.

Tables of their Heights.

To save prolix details, I have thrown together into a table at the end of this paper the heights of all the terraces and beaches which I have measured; their heights above the river on which they are situated; and usually, also, above the ocean. The manner in which the heights were obtained is also indicated. When measured by levelling, no mark is attached; when by the Aneroid Barometer, the letters A B; and when by the Syphon Barometer, the letters S B are added. The number of heights given is 219.

Details of Sections.

By means of maps, and sections, and tabulated heights, I hope to make facts on this subject understood without much detail. Yet the sections will require some explanations. I shall describe them by reference to the basins in which they occur.

1. *In Basin No.* 1, *from Middletown to Holyoke, commencing at the North End.*

1. In South Hadley. (See Section No. 1, Plate I.) The section commences at Mt. Tom, in Northampton, and runs east across Connecticut river. On the east side it strikes a high gorge terrace, which has been partially worn away by the river. The line of the section is only a few rods south of the gorge between Holyoke and Tom. East of the high terrace is a small stream, that seems to have been instrumental in forming the lower terrace, which runs along the south side of the Holyoke range to Belchertown, sloping towards the Connecticut. This section might have been more instructive if extended to that place; but I have not obtained the requisite data, and those which I have used are merely barometrical.

2. No. 2 extends from Connecticut river at Willimansett, in the north part of

Chicopee, at the foot of South Hadley Falls, to the high, sandy plain which extends easterly and southerly through South Hadley, Granby, Springfield, &c. This plain is a little short of two hundred feet above the river, and two hundred and seventy-four above the ocean. It is essentially composed of sand, and I think that it sinks as we go south. East of this plain we strike beds of gravel, with irregular elevations and depressions. Above these are accumulations of coarse materials, once beaches probably, but I have not measured their height. I am sure they may be found at different altitudes, even to the top of the hills lying east of this part of the Connecticut Valley and the ocean, as high as one thousand feet.

3. In Springfield, a little north of the centre of the city, and running from the river southeasterly, so as to cross the principal terraces in that place. The third terrace is the isolated remnant of one, probably of the same height as the first one we meet in ascending from the main street eastward, on which so many delightful residences have been chosen by the citizens. The intervening space, as shown on the section, was probably worn out by Connecticut river, which might formerly have run there, when at a higher level, or at least, a part of it. The terrace marked as one hundred and thirty-six feet above the river, is that on which the United States Armory is situated. I did not actually level to the top of these two right hand terraces; but have no doubt that their height is nearly as given in the section.

4. In the extreme northern part of Long Meadow, on the road to Springfield, commencing at the river, and running southeasterly to the level of the plain on which most of Long Meadow, and the higher part of Springfield, are situated. This upper terrace extends, with some irregularity of surface, eastward about nine miles to the railroad station in Wilbraham on the Western Railroad. Northward it reaches the foot of Holyoke in South Hadley, though broken by several streams. To the south, it reaches a ridge of sandstone, commencing at Enfield Falls in Connecticut, and extending easterly to the hypogene rocks of Monson and Stafford; though there may be places where the terraces overlie the sandstone, so as to connect with the upper terrace south of Enfield, that extends as far as Glastenbury. (See Terrace No. 2, Plate III.)

5. In East Windsor, commencing at the Connecticut river, and extending easterly to the broad plain, on which stands the Theological Seminary, past which the section runs.

6. In East Hartford, from Connecticut river, at the south part of the village, to the sandy plain a little eastward. This plain I have supposed to be the same as the upper terrace of all the previous sections. If so, it slopes southerly as follows, in a distance of forty or fifty miles, viz: at South Hadley (Mt. Holyoke) it is 292 feet above the ocean; at Willimansett, 268 feet; at Springfield and Longmeadow, 200 feet; at East Windsor, 96 feet; and at East Hartford, 61 feet. But this point demands more careful examination than I have given it.

7. In Glastenbury, south part of the town. Then the valley becomes narrower, and, indeed, Rocky Hill, a trap bluff, appears on the west side of the river; and we may regard these as gorge terraces, such as form on the up-stream side of a barrier. Hence, as I find is usual, they are higher than those in the central parts

of a basin. Yet the upper terrace of this section extends almost uninterruptedly to Middle Haddam, or Chatham. It is composed of sand, with coarse gravel, or even bowlders a foot or two in diameter. It is more irregular at the top than the lower terraces, and is, in fact, a moraine terrace.

8. In Wethersfield, a little north of the village, from Connecticut river westward, the highest terrace is probably the same as Main Street, in that village. It is sandy; the lower one loam.

9. This section begins near the mouth of Farmington river, on the bank of the Connecticut, and runs southwesterly to the level of the village, which stands on the highest terrace observable in that vicinity. This is sandy, the lower one loam.

2. *Sections in the Basin from Mount Holyoke to Mettawampe.* (*Toby.*)

To the surface geology in this basin I have devoted more time and attention than in any other, because I reside in it, and have lived in it most of my days.

10. The valley of the Connecticut, in the region of Northampton and Amherst, is not less than fifteen miles wide, from the old beaches on one side to those on the other. From the north part of Northampton, through Hatfield, Hadley, and Amherst, to the middle of Pelham, I have carried a level more than eleven miles, and the section, No. 10, presents the results. It shows, first, terraces on several existing small streams, besides the Connecticut; secondly, terraces and beaches on what I regard as two ancient beds of the Connecticut, one along the west side of Amherst, and the other along its eastern side. The ridge between is mainly composed of rearranged and water-worn materials; but the surface is too irregular for terraces, and I fancy that they might have formed beaches, though terraces occur on their sides; thirdly, as we approach Pelham, we come upon the upper part of a small stream, called Fort river, which descends from the hills of Pelham, almost in the direction of the section. On both sides of this stream I found numerous terraces, some of them delta terraces, and others, lateral terraces; although not all of them are very perfect, yet lying at a convenient distance from my residence, I have given them a good deal of attention, and regard them as very instructive. I have thought that they deserved a separate map, which I have given (Plate VI, Fig. 2,) as they could not be represented on the general map. The general section I have carried along the south side of the stream, as high as the terraces exist, and then it is continued across the south branch of the stream (a mere brook), so as to cross what I regard as three beaches; one of them more than 1,000 feet above Connecticut river. The highest of the terraces, No. 9, which is 383 feet above Connecticut river, occupies a gorge having Mount Hygeia[1] on the north, and a corresponding elevation, less bold, on the south. Above this spot is a depression, or basin, above which, on the north side of the stream, occur several distinct terraces, lying against Mount Hygeia; while at a still higher level, on the north, are large banks

[1] I apply this name to a bluff 706 feet above Connecticut river, rising directly above a fine mineral spring, of the chalybeate character, in a most romantic dell.

of coarse sand, which I regard as an ancient beach, and have so marked it on the map (Plate VI, Fig. 2).

11. On the north side of the stream (Fort river) the terraces are more numerous than on the south side, and in general they do not correspond in height. I have, therefore, been obliged to give another section (No. 11), extending from Fort river, in Amherst East Village, to the sandy sea-beach above described, 546 feet above Connecticut river. The course of this section, and also that on the south side of the stream, are indicated by the succession of figures on the map. On the north side the terraces rise highest at the southeast point of Mount Hygeia, evidently because there was once a barrier at this spot, at least a partial one, which would cause the materials drifted by the current to accumulate. The depression shown by the section, still further east, was doubtless made by the action of the small stream, as it wore away the barrier. In other words, it was a pond which was gradually drained, and so the terraces were formed.

It is not easy to say whether No. 17 be a terrace or a beach. It is coarse sand and gravel, and is somewhat level-topped: yet it passes into a decided beach further south, and I have marked it as such. When the ocean stood as high as 546 feet above Connecticut river at this spot, it must have produced a small bay opening to the north; Mount Hygeia forming the right hand side and Pelham Hill the left. Nearly 400 feet higher, we find another beach, which, on the general map, I have represented as extending through Shutesbury, several miles to the north. It can be traced a great distance, and probably might be found extending into New Hampshire. In Shutesbury it is very distinct, and more sandy than in Pelham, where, at its highest line, the rolled fragments are sometimes a foot in diameter. By carrying a level from Packard's Hill, in New Salem, the height of which has been accurately determined in the Trigonometrical Survey of Massachusetts, I found the most distinct beach in Shutesbury to be 1082 feet above Connecticut river. This corresponds nearly to a third beach on the east side of Pelham Hill, half a mile south of the Congregational Meeting-house, on the road to Enfield, which is 1049 feet above Connecticut river. Between these two highest beaches in Pelham, most of the surface is covered by ordinary drift, with rocks in places (gneiss) occasionally shooting through. Drift, also, appears between the lowest and the second beaches.

This section across the Connecticut valley I am convinced gives us a good idea of the character of a large part of the valleys of New England and New York, and perhaps of the whole country, with the exception of drift. Wherever I have travelled, since my attention was turned to the subject, I find terraces in the lower part of the valleys, and similar though usually coarser materials arranged beachwise, on the flanks of the mountains and hills, especially where spurs of the ridges form spots that might once have been bays, in which sand and gravel would naturally be accumulated on the shores of a lake, or the ocean, by winds and waves. There are scarcely any mountains of New England so high that this work has not reached their summits. But further on I shall have occasion to point out other particular examples.

The section of terraces on the north side of Fort river, passing most of the way through thick woods, I used barometers for getting their heights, except Nos. 11, 12, 13, and 14, which were obtained by levellings. Along this route the rock often projects through the terraces, and shows decided evidence of powerful erosion by aqueous agency, some hundreds of feet above the present stream.

12. This section is in Whately, on the west bank of Connecticut river, and extends only to the third terrace above the river. Had I followed up the side of the mountain in the west part of the town, no doubt I should have found beaches, and most likely one or two other terraces above No. 3. Indeed I know of one terrace, say 100 feet higher than No. 3, about two miles south of the line followed by No. 12, and I shall in the sequel point out a very high beach in the north part of Whately. The principal uses of this section, thus imperfect, are to show that the lowest terrace along the Connecticut is sometimes quite high (32 feet here), and that the height of the broadest terrace in the Connecticut valley, which is No. 3, is less than it is nearer to the gorges; a fact which shows the influence of those gorges in the accumulation of the materials of the terraces.

As already stated, there are two branches to this second basin, one extending north through Deerfield and Greenfield, and the other south through East Hampton, Westfield, Southwick, &c., nearly if not quite to Long Island Sound. These branches are separated from Connecticut river by an almost continuous ridge of trap and sandstone, as may be seen on the large accompanying map of the surface geology of the Connecticut valley. This ridge is breached in Deerfield by Deerfield river, in Westfield by Agawam river, and in Simsbury by Farmington river. On the two first of these rivers are two remarkable sub-basins, sunk some 80 or 100 feet below the general level of the valley, and exhibiting on their margins fine examples of terraces. As these cannot be well shown upon Plate III, I have devoted separate ones, but on a larger scale, to their exhibition. (See Plates IV and VII.) They both extend a considerable distance along the rivers, and show the surface geology, especially the terraces and old river beds.

The Deerfield Basin.

13. Where Deerfield river emerges from its long *Ghor*, between Shelburne and Conway, into the Connecticut valley, it has formed several terraces; a section of which No. 13 exhibits; though on the south side of the river I have failed to measure two small terraces. But on the north side of the stream a tongue of four or five terraces has been thrown forward, perhaps a mile long, forming a ridge a little over a hundred feet high, with regular terraces on its south side. The stream here descends rapidly, and so do the terraces slope in the same direction, although I did not measure the rate of descent. It is so obvious to the eye that I thought a measurement hardly necessary, especially as I find the same fact almost everywhere upon lateral terraces. They always have as great a slope as the stream on which they occur, and sometimes greater.

Until I discovered the tongue of terraces above described, I was of opinion that the basin of Deerfield was once occupied by terrace materials to the height of No. 3 (yellow) on Map No. 1, Plate III, which is the usual level of the Connecticut valley in that region, and is upon an average 173 feet above Deerfield river. This amount of sand and gravel (as I estimate it, 135,000 cubic yards) I supposed had been cut away by Deerfield river, and sent forward into the Connecticut. But I can hardly see why this ridge of terraces should in that case have been left. Yet some other facts seem to indicate strongly that most of the whole basin has been thus excavated; and upon the whole, I think this tongue of terraces has been formed by the river after it had excavated the basin, and sent its contents down Connecticut river.

The tongue of terraces above described was undoubtedly at first a delta terrace, though formed by the rapid stream as it issued from the mountains into the estuary, which is now the Connecticut valley. At present, the ice-floods in that stream, and at this very spot, exert an amazing power of erosion. In early times, such floods must have crowded along great masses of crushed and rounded materials, and piled them up along the margin just as lateral moraines are produced by glaciers. As the bed of the stream sunk, and also the waters of the estuary, successive terraces would be formed, looking like so many moraines, although of finer materials than the moraines of glaciers, and sorted too.

14. This section extends across the Deerfield basin, though not exactly on a right line. The eastern part starts at Deerfield river, just south of the village, and the western part from the meadows, a little north of the village. Yet there is no error in representing them as connected, since at their starting points they are nearly on the same level, differing in height only as the banks of the river differ. The terraces are very distinct till we reach the third, over which the railroad passes, on the east side of the valley. Above the third, the top of the deposits is only imperfectly level, and they may be regarded perhaps as beaches; for I am confident that such beaches may be traced all along the flanks of the Connecticut valley, at about the same height. But I have not measured them, save in a few places, as they did not attract my attention when I measured the terraces. The three lowest terraces on both sides of Deerfield river, were measured by levelling; the two highest, by the syphon and aneroid barometers. Yet the latter, on the west side of the river, have not been measured at all. As I saw them from the east side, they appeared to be at about the same height as those on the east side; still I know well how difficult it is to judge accurately in such cases by the eye alone, and actual measurement might show a considerable discrepancy in the heights. Hence, I have added an interrogation point to the heights on the west side of the river.

15. This section, of no great importance, shows the terraces at the north end of Deerfield meadows, to the top of Pettee's Plain, which lies southwest of the village of Greenfield, and corresponds to the general level of the Connecticut valley. The meadows, or lowest terrace, are here worn away, and the lowest terrace remaining is mostly clay; the upper one sand. The river would encroach still further upon this hill, had it not struck a ledge of red sandstone, which will at least retard its lateral erosion.

16. Pine Hill is an insulated eminence, apparently composed of two terraces, in the northern part of Deerfield meadows. These terraces do not correspond in height, as far as I can see, with any on the margin of the basin; yet they must have been once continuous, as I know of no instance where terraces have been formed

so perfect upon a small hill. This fact goes strongly to show that at least a large part of the Deerfield basin was once filled with terrace materials, which the river has subsequently worn away, and the reason why those on Pine Hill remain, I find to be that they rest on a protuberant mass of red sandstone. On the west side of the hill, as shown in the section, is an ancient bed of Deerfield river (crossed twice by the section), which was prevented from making any further lateral encroachments by the underlying rock. I shall have more to say hereafter concerning the ancient beds of Deerfield river, shown in such numbers upon Map No. 2 (Plate IV).

A few other terraces on Deerfield river, out of the Connecticut valley, will be noticed further on.

The Westfield Basin.

17. The major axis of the Deerfield basin lies north and south; that of the Westfield basin nearly east and west. The present section starts from Agawam river, near the east end of the basin, on the north side, and runs northerly. The height of the four lower terraces was obtained by levelling; that of the highest by estimation. All of them, except the lowest, which is loam, are sandy. The most elevated brings us to the general level of the Connecticut valley, though it is for the most part lower towards the east side, and not a little irregular on its top.

18. This section was but imperfectly measured, and only with the arenoid barometer; which, although very valuable where an error of twenty or thirty feet is not of much consequence, does not answer well for such small elevations as our river terraces. By looking at Map No. 6, it will be seen that between Westfield river and Little river, a tongue of terraces extends easterly from Middle Tekoa Mountain, almost to the village of Westfield. In one place on the north side of this tongue, perhaps a mile west of the village, I noticed five terraces, reckoning that on which the village stands as the lowest, although generally the highest terrace around Westfield is reached by three steps from the river. Commencing on the high sandy plain north of Westfield basin, I have carried this section southwesterly across these five terraces and over Little river to the plain of nearly equal height on its south bank; in other words, across the entire basin. I think the barometer has made the central terraces considerably too high. But the section will give an idea of this interesting valley. The materials of which all these terraces are formed are clay, sand, and gravel, though the red sandstone shows itself occasionally near the river.

19. On this section I have attempted to give an idea of what I suppose to be the remnants of gorge terraces, where Westfield river issues through the deep gorge between Tekoa and Middle Tekoa. The height (measured by the Aneroid), is very great for a stream of no larger size. Near the river on the same section are shown two other narrow terraces, produced at a vastly later period. On both sides of the river the mica slate ledges show themselves frequently as we ascend the mountains.

20. This section commences on the east side of Westfield river, opposite the station house of the Western Railroad, in Russell, and crosses the river, passing westerly through the flourishing village which has lately sprung up there. Its

western extremity is very near the place where an old river bed, about a mile long, unites with the present bed. I do not feel much confidence in the accuracy of the heights, since they were taken by the aneroid barometer. For the view of the terraces on which this village stands, accompanying this paper (Plate IX, Fig. 2), I am indebted to Mr. Franklin P. Chapin, who took it with his pentagraphic delineator from the east side of the river.

21. This section extends from the present bed of Westfield river over the hill on its west bank, and across the old river bed referred to in the last paragraph. The heights were obtained by the aneroid barometer; and, therefore, are liable to some uncertainty.

Many other terraces are shown along Westfield river on Plate VII, with three old river beds to be described in my paper on erosions. The heights of the terraces I have not measured, and therefore do not give sections of them.

3. *In the Basin extending from Mettawampe to the Mouth of Miller's River.*

22. This basin, though small, has many terraces, but none of them seem to me of special interest. I have measured only one section in it, and not the highest terrace upon that; as it lies at a distance from those which were measured. I commenced on the narrow alluvial plat just above Turner's Falls, on the Montague shore, and ascended the sandy hill that lies southeasterly. This was reached as the third terrace; and, except along its eastern margin, it constitutes the general surface of the basin. At its southern part, in the south part of Montague, I judge the surface to be higher than on the section, as is usually the case near gorges.

4. *Sections in the Basin extending from the Mouth of Miller's River to Brattleborough.*

I ought to repeat here, and make more general, a remark elsewhere made, that the upper terraces usually extend more or less from one basin into another; that is, these higher terraces were formed when the waters extended from one basin into another, and what now seem to have been barriers, were then only narrower places in the estuary. On the east side of the river, in this case, terrace No. 4, and perhaps No. 3, on Map No. 1, Plate III, were continued into Northfield from the basin next south.

23. This is in Northfield, two miles south of the village, running eastward from Connecticut river. The fourth terrace, or beach more properly, is irregular on its top, and was not measured.

24. This runs from the same river eastward in the north part of Northfield, only a short distance south of the State line.

25. At the mouth of Ashuelot river, in Hinsdale, the terraces are numerous and instructive. This river is a small but rapid stream, and where it debouches from the hills into the Connecticut valley, it has brought forward a large mass of terrace materials, mainly of gravel, which originally constituted a delta terrace; that is, the stream threw forward these materials into the lake, or estuary, and formed a bank along its mouth. But as the waters drained off, so as to bring this

bank above them, the Ashuelot cut through them, and formed lateral terraces along its margin. On the northern side of the stream, at its mouth, a rocky hill extends nearly or quite to the Connecticut, which is thereby forced at this spot to make a curve westward. The section No. 25 passes across the Ashuelot near its mouth, directly through the village, northwesterly over the hill, and then descends towards the Connecticut; so that all the terraces on it to the right of this hill belong to the Ashuelot; while those to the left belong to the Connecticut. The difference in their height and size on the two rivers affords a good illustration of the fact that the larger the river the higher the terraces. The character of the materials, too, illustrates another fact, viz., that they are coarser on small and rapid streams than on larger and more tranquil ones. Excepting the lowest, which are narrow, the terraces on the Ashuelot are all gravel, mixed with sand, and often the fragments are quite large; while on the Connecticut are no pebbles of consequence, but sand underlaid by a thick bed of clay. A third circumstance deserves notice: On the Ashuelot the terraces have a rapid slope towards its mouth, corresponding to that of the river, which here falls so much as to afford a good site for manufactories; whereas, on the Connecticut, the eye cannot perceive that the terraces are not strictly horizontal. Indeed, they probably decline but little from Brattleborough to this place, and the two higher ones are nearly continuous between the two places. The higher terrace along the Connecticut, not measured, is sandy and irregular, and more properly deserves the name of a beach.

26. This section (Plate II) is on the west side of Connecticut river, in the north part of Vernon, and differs but little from that already described on the same river in Hinsdale. The height of the fourth terrace, however, is greater; but the spot is not a great distance south of the gorge in the river at Brattleborough, and hence we should expect a greater amount of terrace materials.

5. *In the narrow Basin from Brattleborough to Bellows Falls.*

So narrow is the valley between Brattleborough and Westminster, that it deserves the name of a defile rather than a basin. And yet terraces are found nearly the whole distance, though usually quite narrow. Opposite Brattleborough, on the east side of Connecticut river, West River Mountain rises very precipitously to the height, above the river, of 1050 feet, as I ascertained by a not very accurate mode of observation. On the west side of the river, the hills rise more gradually, yet the rocks press closely upon the bank. Within a distance of not over half a mile, two tributary streams empty into the Connecticut; the most northerly called West river, of considerable size; and the one at the south end of the village, small, and called Whetstone brook. Such streams, debouching in such a spot, and at right angles to the course of the Connecticut, are sure to produce numerous terraces. So numerous are they, and so complicated, that I judge it necessary to devote a map to them alone, so far as I have traced them out (see Plate V;) for I have not obtained quite all the facts in respect to the sections that would have been desirable, yet I have enough to be very instructive as to river terraces.

27. This section (Plate II) commences on the west bank of the Connecticut and

the south bank of Whetstone brook, and runs southwesterly to the top of the elevated sandy plain that passes into the Basin No. 4, just considered. (See the line of the section on Plate V.) The terraces appear to be the joint result of Whetstone brook and of Connecticut river. They are, therefore, more numerous than is usual on the Connecticut, and less so than on this same Whetstone brook, a mile from its mouth, as the next section will show.

The Connecticut valley was probably occupied originally by terrace materials as high as the uppermost of the above terraces on this section, and when the waters gradually subsided, both the Connecticut and Whetstone brook formed channels through these materials, and produced the successive terraces. Why terraces, rather than a continuous slope, were formed, I shall endeavor to show in another place.

28. This is a quite instructive section, commencing on the south bank of West river at its point of junction with the Connecticut, then extending southwesterly across the village of Brattleborough to the high bank of Whetstone brook, a little west of the village, opposite Burge's factories; thence across the brook, and up the opposite bank, so as to cross the successive terraces, ten in number. The upper one was not measured, on account of the rain. Nor did I ascertain the height of the brook, where the section crosses it, above Connecticut river.

It will be seen that No. 5, on the left hand part of this section, consists in part of an insulated hillock, crossed a little north of the village; and in the main part of a broad terrace, on which stands the upper and northwest portion of the village. This terrace, as I found by levelling, slopes towards Connecticut river at the rate of 20 feet in 50 rods. Possibly this might have been in part the result of rains for a long period, bringing down from the hill by which the terrace is bounded, deposits of sand. More probably the terrace was formed by the conjoint action of West river and Whetstone brook as a delta terrace, and that its slope was produced by the rapidity of the currents.

All these terraces are underlaid by argillaceous slate, which shows itself all along the banks of the streams. It is doubtless this solid rock that has determined the present channels of the tributaries to the Connecticut, and caused them to enter that river nearly at right angles. The mere sand and loam of the terraces would soon be washed away in time of freshets, were it not for this rocky foundation.

In this section we see a good exemplification of the statement made on a preceding page, that the smaller the stream the smaller are the terraces, and often more numerous too. Here we have ten on Whetstone brook, and nine on West river, yet they do not rise so high as the fourth, on the Connecticut, in Vernon.

Had I explored the hills by which the valley at Brattleborough is bounded on the west, I might have found beaches, or imperfect terraces, at a much higher level. But when I examined that region my attention had not been called, as it was subsequently, to the subject of beaches. The same remark will apply to nearly all the terraces of which I have given sections on the Connecticut.

I regret that I did not measure a section across Whetstone brook through the middle of the village of Brattleborough, along the track marked by the figures 1, 2, 2, 3, 4, on Plate V. Here it would seem are fewer terraces than at the mouth of the stream. Possibly more careful examination might have detected others, and probably also the original surface has been here somewhat altered by the grading of the streets.

29. This section commences with the highest distinct terrace in Westminster, a little south of the village (which stands upon the second terrace, reckoning upwards), and crosses Connecticut river into Walpole. But unfortunately I was unable to measure the terraces on the east side of the river, and have marked them only as they appeared from the west side. They are very distinct on both sides, and perhaps they correspond in height, though I usually found in such cases, that actual measurement showed considerable difference in elevation where the eye could discern none.

30. At the upper end of the basin under consideration, the terraces are numerous and distinct, just below, as well as above Bellows Falls in the next basin. No. 30 crosses Connecticut river at the mouth of Saxon's river on the west side, and of Cold river on the east side. Of course the terraces are compounded of the effects of the three rivers. It will be seen that there is no correspondence in their height on opposite sides of Connecticut river, except that the upper terrace very probably once filled the valley; for the difference in height between the opposite terraces (17 feet) is not greater than we might expect on the supposition that the materials were drifted into a former lake, or estuary, by the adjacent streams. These materials are, for the most part, coarse sand, sometimes mixed with gravel. On the east side ledges of rocks appear on the slope of the third terrace.

As an illustration of this paper, I have given a sketch (taken by Mrs. Hitchcock) of the general aspect of the terraces of the above section, as they appear about a mile south of where it crosses Connecticut river, on the road to Walpole. (See Plate IX, Fig. 1.) The view from this spot of the gorge with its terraces, and of some of the principal buildings in the romantic village of Bellows Falls, is very fine, and deserves the attention of the artist for its scenographic beauties. My object in giving its outlines was to exhibit the terraces as a good example of the very artificial appearance of many spots along the rivers of New England. Certainly it does seem, as we look at these terraces, as if they were the work of man.

31. On the preceding section, on the west side of Connecticut river, I have represented two *glacis terraces*. On No. 31 I have shown them on a large scale, and laid them down accurately, so as to give a good idea of this kind of terrace. It will be seen that they constitute merely undulating portions of the lowest terrace, and perhaps ought not to be reckoned as distinct terraces. Yet they are sometimes of considerable height, and certainly deserve notice, because they show us one of the modes in which water accumulates terrace materials. How they are formed I will consider in another place. But there are certain laws concerned in their production. Thus, the depression between them always corresponds in its longest direction with the course of the current that produced them. One side, also, and I believe always that next the stream, is steeper than the other.

In almost all extensive meadows this sort of terraces may be seen more or less distinct. Excellent examples occur in Hatfield and Hadley, not merely in the meadows, but they are seen in crossing the villages, from street to street, in an east and west direction, or at right angles to the course of the stream that made the deposits.

It was from such examples as this section exhibits that I first got the type of a glacis terrace; but in passing subsequently through some of the higher valleys of the Alps, I sometimes observed the terrace materials arranged so as to form one continuous slope from the rocky side of the valley to the stream. I noticed this most distinctly on the Eau Noire, in the pass of Tête Noire. Here the materials were quite coarse, the fragments often large enough to be called boulders, though I fancy most geologists would be puzzled to say just how large a *pebble* may be, or how small a *boulder*. The same sort of terrace I saw in other places in the Alps, and I have observed them in the mountainous parts of our own country, though but seldom, and they were imperfect. They perhaps furnish a better type for the glacis terrace than that already described. If, however, we regard the gentle slope on one side as a characteristic of this terrace, then both the above descriptions of terrace will belong to it.

6. *In the Basins extending from Bellows Falls to Wells River.*

The mountains at Bellows Falls crowd closer upon the river than at any place south of this spot, except perhaps at Holyoke and Tom. Kilburn Peak, on the east bank, rises almost perpendicularly, over 800 feet. On the west side, as at Brattleborough, the mountains recede further, and have an escarpment less steep; yet the rocks show themselves almost everywhere in the gorge, and form a ridge which produces the falls. All the circumstances here are favorable to the formation of terraces. Sections 30 and 31 are only a mile and a half south of the village of Bellows Falls, and the highest terraces extend through the village into the sixth basin. So remarkably are they grouped together here, that a distinct and separate map seemed indispensable. (Plate VI., No. 1.)

32. This section crosses Connecticut river directly through the village of Bellows Falls and a few rods above the principal cataract. The heights are given from the foot of the falls. The depression on the left was evidently once occupied by the river when at a higher level. I regret that I was not able to measure all the terraces—none, indeed, on the east side of the river; but I am not aware that they are peculiarly instructive.

It was my intention for a long time to continue to get the heights of terraces through the whole course of Connecticut river, at least as frequently as they are given above. But I began to be convinced that I had already measured enough for all important purposes in relation to river terraces. The phenomena of beaches arrested my attention more and more, and it seemed a very important point to ascertain how high they could be found upon the sides of our mountains. To this problem I addressed myself, both in this country and in Europe, and shall briefly give the results. But something more needs first to be said concerning the terraces.

As to those above Bellows Falls on Connecticut river, I have but little to state; for although I have passed over the region several times, it has been rapidly, and I can only say that at least three terraces may be traced nearly all the way to

Wells river. Sometimes I noticed four, or even more. But with one or two exceptions, I have marked only three on the map, and I fear that I have but very inaccurately represented the position and relative width of these. Neither do I suppose that the basins above Charleston, are accurately laid down. In some places, as at Wethersfield, and above Haverhill, the terraces are very perfect and beautiful.

33. My son, Charles Henry Hitchcock, measured this section at White river junction, with the aneroid barometer, and I have thought it worthy to be added in this place, especially as I know from my own observations that its outlines are correct. It commences at Connecticut river, and passes west, near the railroad station. The old river bed, on its west part, was probably formerly occupied by White river, which entered the Connecticut, a little below its present junction. I am not certain, however, that this was the case.

Terraces chosen as the Sites of Towns.

It is a curious fact that the most attractive villages in the valley of Connecticut river, owe their chief beauty to being placed upon terraces. Among these towns we may mention Wethersfield, Ct.; Hartford, East Hartford, Windsor, East Windsor, Springfield, West Springfield, Northampton, Hatfield, Deerfield, Greenfield, Northfield, Hinsdale, Brattleborough, Westminster, Walpole, Bellows Falls, Charleston, Wethersfield, Vt.; Windsor, Hanover, Oxford, Haverhill, and Newbury. Probably but few of the inhabitants have ever thought as to what they are indebted for the beauty of their towns.

Terraces and Beaches out of the Connecticut Valley, but in New England or New York.

I have already described the terraces on Westfield river, among the mountains west of the Connecticut valley. But they occur on almost all the rivers of New England, and I have not attempted the Herculean task of measuring or even mapping but a small part of those which I have visited since engaged in these researches. After finding the features of them to be essentially alike on all rivers, I became convinced that the measurement of great numbers was not important. I will only refer to those on a few rivers, which I have observed with special interest, as well as to beaches, which I have noticed on the adjoining hills.

Merrimack river abounds with terraces, the most perfect of which are in New Hampshire. They give great beauty to many of the towns along that river. From the south line of the State to Franklin I have traced them, and with some interruptions, two or three of moderate height may be seen on one side or the other, or both sides, nearly the whole distance, as I have shown without much accuracy on Plate III. Near the mouth of the river I found terraces, but could rarely find more than one well defined, and so have I represented them on the same map.

Plum Island, stretching along south of the mouth of the Merrimack, is a good example of a modern beach. (See Plate III.) Some other features of the surface geology of that region I have delineated, and shall notice further on.

A slight examination led me to the conclusion that the terraces are of unusual interest upon Ammonoosuck river, which comes from the White Mountains and empties into the Connecticut.

I have followed up the Waterquechee river in Vermont to a considerable distance, and find some interesting terraces a little below the village of Quechee, where is a wild gorge. Above this not less than seven terraces occur on the southwest side of the stream, and four on the opposite side, as I have indicated simply by lines upon Plate III., connected with my paper on the marks of drift and glaciers.

On the same map I have sketched most of the terraces on Deerfield river above Shelburne Falls, where the Ghor terminates. Generally, we have along this stream only two terraces, as represented, though sometimes more exist, as section 34 shows, to be described below. But where small streams enter Deerfield river, I have noticed fine examples of the Delta Terrace, and several of these are marked upon the map, and will be more particularly described further on.

I now proceed to describe Section No. 34, just referred to, as well as several others, mostly of beaches out of the Connecticut valley.

34. Beyond the barrier across Deerfield river a little west of Shelburne Falls, commences a rather broad valley, which must have been once a lake, extending perhaps fifteen miles, to the foot of Hoosac mountain. Here, as we might expect, we find good examples of terraces. I have measured, however, only a single series, lying on the south side of Deerfield river, and at the mouth of a small tributary coming in from the south through Buckland. It will be seen from the section, No. 34, that the terraces are all of them low. They seem to be the result of the joint action of both rivers.

35. In the southeast part of Heath is a mountain, to which the Indian name of *Pocumtuck* was formally given in 1855, by the Senior Class in Amherst College who graduated in 1856. It was used as a station in the trigonometric survey of Massachusetts, and consequently its height above tidewater is known to be 1888 feet. From this point I levelled northwesterly about two miles, till I struck a deposit of water-worn sand and gravel, of limited extent, but which I must regard as an ancient beach; for I know not how else to explain the occurrence of comminuted and sorted materials in a spot so elevated and open to the surrounding country. The section will give an idea of its position.

36. The summit-level of the Western Railroad, in Washington, is 1456 feet above the ocean; the cut in the rock being 60 feet. On all sides of this cut I find deposits of sand and sometimes gravel, at least to the height of 134 feet above the original rock. This would give 1590 feet above the ocean as the highest mark of distinct sea action at this place, although very probably similar deposits can be found in the vicinity at a higher level. But I am a little doubtful as to some of these banks of sand; for the rock here is a variety of gneiss easily disintegrated, and the result of the disintegration is coarse sand. I cannot thus explain, however, the thicker deposits, certainly not those with pebbles, and these are seen nearly at the height above named.

This spot was doubtless one of the lowest, if not the lowest, pass through the dividing ridge between the Hudson and Connecticut rivers, and therefore we should expect marks of sea action here, if the ocean once stood above the mountains of New England.

37. French's Hill, in Peru, on Hoosac Mountain, is one of the highest peaks in Massachusetts, and as its height was ascertained in the trigonometric survey, I visited the spot in the hope of finding beaches or terraces in the vicinity, whose height, also, above the ocean, could be easily determined. The section No. 37 exhibits the result. By carrying a level downward from the top of French's Hill we strike what I conceive to be an ancient beach, 217 feet below the summit, or 2022 feet above the ocean. It is level like a terrace, but the materials are not very thoroughly rounded, like those of the lower beaches and terraces; yet they are more worn than drift usually is, and I can impute the level top of the deposit to water only.

Passing eastward from this beach, we cross a brook, which rises in a pond, and then go over a hill of considerable height. In descending it easterly, I fancied the existence of another beach; but, going onward, nearly three miles from French's Hill, and descending about 470 feet, we reach a small stream, where are at least three terraces, made up of coarse materials, sand, gravel, and bowlders, the highest on the west bank being 85 feet above the stream, and 1852 feet above the ocean. This is the highest river terrace I have yet met with in New England; but I see no reason why they may not be found at a higher level in some of our mountains, since, as I conceive, they are mainly the result of the action of the stream itself. In this instance, however, it is rather difficult to imagine the former existence of any barrier high enough to shut in the water, so that it would overflow these terraces: so that probably the sinking of the waters of the ocean may have had an important influence on their production. On the east side of the stream are three terraces of about a corresponding height, but I did not measure them.

Proceeding eastward from this elevated region, I met with other deposits at a lower level, more obviously once constituting the shores of an ocean; but not then having barometers with me, I could not measure their height.

In going westward, also, from Peru, or any other culminating point of Hoosac Mountain, into the valleys of Berkshire County, we meet with many examples of comminuted and rearranged drift, in the form of beaches, and in the valleys of terraces. But I have not measured the height of these, save a single example on the Western Railroad in Dalton, which I find by the aneroid barometer to be 1228 feet above Hudson river.

In the west part of Whately, on the ridge between that town and Conway, I found a distinct beach of sand and gravel, which by the aneroid and siphon barometers I ascertained to be 697 feet above Connecticut river, and 802 feet above the ocean. In the northwest part of Conway, called Shirkshire, I found another, of coarse gravel and sand, 935 feet above the river, and 1040 above the ocean. Two miles further west, in Ashfield, is another, mostly of sand, 976 feet above Connecticut river, and 1081 above the ocean. A mile further north, an imperfect beach shows itself, 1216 feet above the river, and 1321 above the ocean.

Still further northwest, on the opposite side of the ridge, is another sandy beach, nearly as high, but I did not measure its elevation.

In all the above cases, and, indeed, wherever I have discovered the most distinct beaches, they occupy such a position among the hills, that if the country were covered by water a few feet above the beaches, they would become inlets or harbors, and I fancy that if our present harbors, either along the ocean, or the shores of our larger lakes, were to be left by the waters, the surface would be no imperfect counterpart to these ancient beaches. Indeed, when standing on these beaches, and looking in the direction which must have been *seaward*, if my suppositions are correct, I have often felt that it required no great stretch of imagination to see the ancient waves rolling in upon the beach, and silting up the harbor.

Upon Map No. 1, I have marked beaches at Franconia Notch and the White Mountain Notch, which are two passes through that gigantic range of mountains. In those passes, a little west from their narrowest part, we find accumulations of water-worn detritus, stratified and laminated, which I doubt not were left there by the breakers of an ancient ocean. At least it is certain that no existing streams could have formed them, and yet water must have been concerned in their production. By my aneroid barometer, I found the highest point in the road, which passes westerly from the Franconia Notch house, to be 2665 feet above the ocean, and 2259 above Connecticut river. This is not so distinct a beach, however, as is shown at the height of 2449 feet above the ocean. Gibbs' hotel, at the White Mountain Notch, which occupies the top of a beach, in my opinion, is 2018 feet above the ocean by a mean of the two barometers, and 1612 above Connecticut river. But I fear this measurement may vary somewhat from the truth.

38. This is a very imperfect section, from the mouth of Connecticut river to that of the Thames, at New London, or a little north of the city. I had no intention of making such a section when I crossed that district in the road nearest to the coast, not far from the route of the New London and New Haven railroad. But having taken a few barometrical observations, and finding the two barometers to agree unusually well, I thought it best to put down the different terraces and beaches which I observed, although I have given the heights of only a few; and probably some terraces, at least, are omitted. Perhaps all should be called beaches, as they lie open entirely to the ocean. But the rivers seem to me to have had more to do in their formation than the ocean. The beach marked 17 feet high, on the west bank of Connecticut river, seems to me of the same height, as the very distinct one, commencing on both sides of the Thames, and extending as far as Norwich. This, however, is in fact a terrace, and at New London there is a rocky barrier, which doubtless had something to do with its formation. I regret that I could not spend a longer time along this section, and make more measurements. At the time, I thought the terraces and beaches too low to be measured accurately by the barometer, and I had no level with me. I think it would be instructive to run such a section along much of the coast of New England; yet I think the one given is an epitome of what we should find in the whole distance.

39. In passing from Schenectady to Albany and Springfield, I took observations with the aneroid barometer at certain places, which I had often observed to be

the tops of terraces and beaches, and have given the result on this section, which commences at the highest part of the sandy plain lying between Albany and Schenectady, and, following the railroad, terminates at the highest point on the road of the Hoosac range. The horizontal scale is so small compared with the vertical, that the section is very much distorted, and gives but a poor idea of the country passed over. On the east side of the Hudson, after rising to the third broad terrace, the ascent is gradual most of the way to the State line between New York and Massachusetts, a distance of 38 miles. Between that point and Pittsfield, eleven miles, the surface is chiefly covered with unmodified drift. Thence eight miles to Hinsdale, the drift is frequently covered by re-arranged drift, which I suppose to have been modified by the ocean, beating against the side of Hoosac Mountain. The same is true of the remaining five miles, which brings us to Washington, on the summit level, and, as already explained, I have regarded the sea action there as extending upwards above the railroad 200 feet.

Though at each of the railroad stations where I took observations, I have represented a distinct beach on the section, it must not be supposed that such is the fact at those places, while between them no beaches exist. I mean only to indicate that beach materials exist at those places, but exactly how many distinct beaches exist along the route, I am unable to say. That the whole of this inclined plane once constituted the shore of a retiring ocean, I cannot doubt; but how many pauses there might have been in the vertical movement, so as to form marked beaches, is a point I have not determined.

At some of the stations of medium height, say at Chatham and East Chatham, I noticed those irregular elevations and depressions of sand and gravel, which I have already described as occurring among the highest of the perfect terraces, and below the most distinct beaches. From this fact we must infer that at that particular level of the waters some peculiar action must have taken place, necessary to produce these modified effects. I refer to those accumulations which I have denominated Moraine Terraces.

40. This section was taken by the aneroid barometer, on the west side of Genesee river, in Mount Morris, which lies at the lower end of that remarkable gorge cut by the river from Portage to that place. There is nothing very instructive in the section. We see, however, that the terraces here are of great height, and they are, also, in general quite broad. An enormous quantity of detrital matter has in past ages been brought into the Genesee valley, and there are some quite instructive facts in relation to former changes of river beds. But this subject I shall reserve for my paper on Erosions.

Terraces on Rivers and Lakes at the West in our Country.

I have not had much opportunity to examine our western rivers and lakes with reference to the surface geology of their banks. The Ohio did not seem to me remarkable for its terraces, nor did the Great Kanawha. On them both we meet occasionally with two terraces, sometimes three. The horizontal position of the sandstone and limestone strata in the Western States, exposes one to error in this matter, by mistaking a terrace of rock for one of sand or gravel. There is no danger of such a mistake in New England.

The terraces and beaches around a considerable part of Lake Superior have been described with great scientific accuracy by Professor Agassiz, in his work on Lake Superior, and by Messrs. Desor and Whittlesey in the Reports of Foster and Whitney on the Lake Superior Land District. The latter gentlemen have, also, included a considerable part of the shores of Lake Michigan.

From the details given by these gentlemen, I judge that surface geology in the regions of these great lakes corresponds essentially to that of New England. Though the different forms assumed by the materials may in their writings often be given under names different from those I have used in this paper, the things described appear to be identical. There is a coarse drift underlying all the other forms of detritus, and above this lie deposits of clay, sand, and loam, overspread in many places and mingled with blocks of various sizes, generally more or less rounded. M. Desor considers the lowest deposit of the clay some 60 feet thick, and those of the sand and gravel above, some 360 feet thick, to belong to the drift, because mixed with, and covered over, with boulders. He divides all the superficial deposits into three parts. 1. Drift proper, with the above four subdivisions. 2. Terraces belonging to a later epoch—a part of the terraces he includes in the drift. 3. Alluvial deposits, embracing all those forms of detritus that have accumulated since the continent began to rise from the ocean, such as beaches, terraces, nooks, belts, bars, marshes, flats, and subaqueous ridges.

As to the number of terraces, M. Desor speaks of as many as seven in some places, and Professor Agassiz says that "six, ten, even fifteen, may be distinguished on one spot." The number, all agree, varies very much in different parts of the same lake. Professor Agassiz thinks that "these various terraces mark the successive paroxysms or periods of re-elevation" of the shores of the lake. Desor adopts the same view, certainly so far as to say that the terraces indicate pauses in the vertical movement, which, however, he would make general over the continent; for he finds the drift deposit at the top of the highest parts of the country around these lakes, not less than 1000 feet above their present level.

Now it will be seen that while I agree with these gentlemen in regard to the essential facts of surface geology, we differ as to the mode of stating them, and somewhat in the theory of the whole subject. We all agree in supposing the phenomena to require vertical movements in our continent, or its submergence and emergence since the tertiary epoch. But while they suppose that there were pauses in the vertical movement, long enough to form the different terraces, I have been led to suppose that most of them, certainly river terraces, must have been formed without such pauses, and simply by uninterrupted emergence or drainage of the country. We agree as to the occurrence of a deposit of coarse drift at the bottom of the series; but while they regard the superimposed clay and sand as true drift, I suppose them modified drift, and produced almost entirely by water, save that floating icebergs have dropped the large boulders. They, certainly M. Desor, suppose the drift period to have terminated when the continent began to emerge, and the alluvial to have then commenced; but I regard drift proper as the result of several agencies—icebergs, glaciers, landslips, and waves of translation—which, indeed, operated most powerfully in the earliest periods, but have ever since continued to act and are still acting. And so of alluvial agencies: we find evidence of their operation from the close of the tertiary period; nay, much further back; but they have gone on increasing in power to this time. Thus the drift and alluvial agencies have had *a parallel operation* from the first, and hence the difficulty of separating drift and alluvium, and the propriety of regarding the whole as one prolonged period, with synchronous deposits. These views will be more fully developed in the subsequent parts of this paper, and I mention them now to avoid misapprehension.

In Professor Owen's Report on the Geology of Wisconsin, Iowa, &c., many interesting facts in surface geology are mentioned; such as terraces and old river beds. On the St. Peter's river he describes two terraces above the meadows, one 130 feet, and the other 230 feet high—the latter of coarse materials. On Red river, according to Captain Marcy (*Report*, p. 35), are three, the lowest from 2 to 6 feet above the stream; the second from 10 to 20 feet; and the third, from 50 to 100 feet; forming the most elevated bluffs along the river.

On the River Jordan, in Palestine. Dr. Anderson, geologist to the exploring expedition sent out by the United States Government to the Dead Sea, has given us an account of the terraces in the valley of the Jordan, a river so remarkable for its tortuosities and rapid descent. He says: "There are almost everywhere in the Jordan valley, distinct traces of two independent terraces. The upper terrace extends to the basis of the rocky barriers of the Ghor, both on the east side and the west, and appears to have been due to a geological condition long preceding the existence of the actual river, yet subsequent to the removal of the material which once occupied the space between the two opposing cliffs." We understand him to mean that there are two terraces besides the meadows, or lower bank of the river; so that I should speak of the river, according to the views presented in this paper, as having three terraces. Near Beìsan, or Scythopolis, Dr. Anderson says there are three terraces—four I suppose by my nomenclature. He does not make an estimate of the height of the two great terraces of the Jordan, though in one place he speaks of banks of stratified gravel rising sometimes 100 feet. Dr. Robinson, in his *Biblical Researches in Palestine, &c.*, describes the valley of the Jordan near the Dead Sea, and says that the immediate valley, which is usually nearly a mile wide, is bounded by a terrace (the first or lowest of Dr. Anderson I suppose) 50 to 60 feet high at its southern part, but not more than 40 feet further north. He also describes a small terrace near the Dead Sea, only 5 or 6 feet above the meadow, which does not extend far up the stream. The width of the whole ghor or valley to its rocky sides varies from 5 to 10 or 12 miles.

Delta and Moraine Terraces.

Very distinct delta terraces may be seen near the mouths of most of the tributaries of the Connecticut and on the branches of those tributaries; but they do not occur usually at the present mouths of the streams, but rather at the point where they formerly emerged from the mountains, into what is now a valley with terraces, but was then an estuary, or lake, or broad river. The materials, brought down from the mountains by the tributaries, were pushed forward into these expansions of water, and spread, in part at least, over the bottom. As the drainage went on, these subaqueous deposits gradually emerged in the form of deltas, and were subsequently cut through by the streams. The result would be, as I shall shortly attempt to show, that a new set of lateral terraces would be formed in the delta terraces. Hence at present, several of the sections of terraces that have been described on the preceding pages, cross from one side of an eroded delta terrace to the other. This is the case in No. 13, in which the right hand terraces were all formed upon a delta terrace of Deerfield river. The same is true of the left hand portion of No. 25, which crosses the Ashuelot river in Hinsdale, New Hampshire, as also of No. 28 in Brattleborough, which crosses the original delta terraces of West river and Whetstone brook. On Deerfield river, in Charlemont, I noticed good examples of delta terraces on at least three small streams, that come in from the mountains of Coleraine, Heath, and Rowe, on North river, Mill brook, and Pelham brook, as is shown on Plate IV.

The Moraine Terrace is certainly one of the most remarkable of all the forms of surface geology, as it is one of the most difficult to explain. It is now more than twenty years since I first attempted to describe this phenomenon, and though I have called in the aid of drawings, I feel that I have yet given but an imperfect idea of it to those who have not seen it in nature. Wherever I have travelled, however, these singular elevations and depressions of sand and gravel have awakened my attention, and the localities have multiplied beyond the power of memory to recall. I do not, however, recollect to have met with them anywhere, save in such circumstances that in the drainage of a country the spot must have formed a shore sufficiently steep to have arrested and stranded floating icebergs. I will refer to a few localities.

To begin with the eastern part of Massachusetts, we find these terraces near the extremity of Cape Cod, in Truro, of sand, very strikingly piled up and gouged out. At Plymouth they are more gravelly. In passing west from the coast, we meet with the first general rise of the country. In about twenty miles, and all along this ancient coast line in Connecticut, Massachusetts, and New Hampshire, we find these terraces, not quite so high, however, as in more mountainous regions. In the valley of Connecticut river, all along its eastern side, where the alluvial plain abuts against the bounding hills, they are very common. Still more striking are they along the western foot of Hoosac and Green Mountains, in Massachusetts and Vermont. Let any one pass from Dalton, in Berkshire county, to Cheshire, along the Gulf Road, and he will be a witness of this phenomenon in its grandest form. It is very striking, also, in the east part of Granville, in Hampden county, at the west foot of Sodom Mountain, in a region scarcely penetrated by roads.

These singular forms of the surface do not occur in the lowest and most perfect terraces, but generally as a part of the highest in a district. The materials are always rounded and sorted, and water has most unquestionably played an important part in their production. But I am sure that no logical mind, accustomed to geological reasoning, will doubt that some other agent must be called in to explain their formation; and their position and relative elevation, as stated above, are important elements in forming a theory of their origin. But more on this subject in the sequel.

2. Surface Geology in Europe.

It is not my intention to give even a summary of the facts collected by Mr. Chambers and others upon surface geology in Europe, except perhaps to refer to a few of them; but, having travelled through several European countries since my attention was turned to this subject, I could not but have my eyes open to it as I passed over the surface. The results of my hasty observations I will now give, though aware that they may be comparatively of little importance.

Wales.

It happened that the first country which I visited after landing at Liverpool, was North Wales, and not expecting to go there when I left home, I had not refreshed my memory with the statements of English geologists respecting its surface geology, and, therefore, I passed over its lofty mountains and through its rugged passes, with no hypothesis in mind, or expectation of what I should meet. In going from Carnarvon to Llanberris, I thought the detritus, to the height perhaps of 300 feet above the sea, indicated sea action; that is, the detritus was not coarse drift, but had been worked over by the action of water. Above that height I found occasionally small accumulations of rounded and comminuted materials, in some partially sheltered spots on the sides of the steep mountains. The highest spot of this kind on Snowdon, my barometer made 2547 feet above the ocean; but in the higher, or rather midway heights of Snowdonia, my attention was arrested by the marks of ancient glaciers. I had not then seen a glacier, but the marks were so obvious that I could not hesitate to refer them to that agency, and the conviction is still stronger since I have been among the Alps of Switzerland, and especially since I have learnt the opinion of Professor Ramsey, who has charge of the geological survey of Great Britain. He finds drift in those mountains 2300 feet high, and thinks that there have been two periods of glaciers there, one before and the other since the drift period. But I will give more details on this point in my paper on the Ancient Glaciers of Hoosac and Green Mountains.

I ought to add, that I saw scarcely any terraces in Wales, nor were the ancient beaches at all striking. On Cader Idris I saw none; but east of that mountain, on the road to Machynleth, is a pass, 762 feet above the ocean by my barometer, and there I saw some evidence of a beach. Although there is proof enough that Wales has been again and again, and for vast periods beneath the ocean, and experienced deep denudation, I did not see there as much evidence of its last drainage as in Scotland or New England.

England.

I traversed England in various directions, and yet generally over its more level parts, and did not see much evidence of drift agency, nor many terraces. The latter I did not expect to find well developed, save in regions where rivers are bordered by hills of considerable elevation, so arranged as to form basins. Yet I did expect to see them on the romantic banks of the Wye, but was disappointed; though materials exist, they are not well formed into terraces. And the same is true of all the streams of England where I passed them. Beds of gravel and sand do, indeed, occur extensively, but they seemed to me to be beaches, or rather old sea bottoms, and not terraces, and many of them sandy and gravelly bottoms of former seas, belonging to a period anterior to the drift, being the beds of tertiary strata.

Very probably good examples of terraces may be found in the more hilly parts of England; and geologists describe deposits of drift derived from Scandinavia and Scotland. But they generally make no distinction between drift and remodelled and comminuted drift, which last forms deposits of far posterior date. I think I see in their descriptions, however, marks of what I call ancient beaches and sea-bottoms of postdiluvian date.

Ireland.

I visited only the northeast of Ireland, passing from Dublin to Belfast, through Dundalk, Castleblayney and Armagh; from Belfast, along the coast, to Fair Head, and the Giant's Causeway, and from thence back to Belfast, through Ballymoney, Ballymena, Antrim, and Carrickfergus. A little south of Castleblayney, I met with genuine unmodified drift, scattered over the slate and silurian rocks, and I saw striæ and embossed rocks; the direction of the striæ being from northwest to southeast nearly. Here, also, were frequent examples of what I suppose to be the Swedish *Osar*, viz., ridges of sand and gravel running northwest and southeast, the rounded summit sloping very gradually, especially at its southeast extremity. At the other end the slope is not so distinct, and indeed the ridge is sometimes terminated by some obstruction.

In Col. Portlock's "Report of the Geology of Londonderry, Tyrone, and Fermanagh," which lie north and northwest from the region I am describing, he states "that these trains of sand and gravel are found at an elevation of nearly 1000 feet." He says, also, that "in the eastern parishes of Derry the form of detritus is peculiar and beautiful. It appears like so many streamers attached to each basaltic knoll, and directed from north to south." These ridges are somewhat different from any that I have observed in the United States; or rather, they seem more distinctly to be the result of a current heaping up materials behind some obstruction; precisely, in fact, what we see in the beds of our large rivers, or smaller lakes. Whereas, with us, similar ridges, which I denominate Moraine Terraces, are often curved, have steeper escarpments, and do not seem connected with obstructions.

They do not seem to correspond to the descriptions given by authors of Osars. Yet M. Desor, who is familiar with such deposits in Scandinavia, describes them as occurring around our western lakes; and he refers to the gravelly ridges at Andover, Mass., as of the same kind. As to the latter, taking Sir R. Murchison's description of Osars (Russia, vol. i. p. 547) as a guide, I have doubted very much whether they could be Osars, since they are too crooked, too narrow, and to long, to be produced by a current sweeping past some obstruction, either a rock or an iceberg. Seffstrom regarded the Osar as peculiar to Sweden, though probably wrong in such a view. But I ought perhaps hardly to give an opinion adverse to such authority on the subject. As remarked on another page, I have represented Osars in four places in New Hampshire on Map No. 1, Plate III, viz., at the *Pot-hole Gorge* in Union, near Fabyan's tavern, in the White Mountains, and a few miles south of Conway, on the road to Centre Harbor, and just within the bounds of the town of Eaton. These may be Osars, yet my doubts as to the fact are not all cleared up.

Along the northeast coast of Ireland the streams are little more than brooks, yet the glens are numerous, and I looked into them with interest, expecting to see perfect terraces. But they are infrequent and imperfect. So in the gently undulating region from the Giant's Causeway, through Ballymoney and Ballymena to Belfast, although rocks seldom appear in place, and a coarse detritus covers the surface; yet it does not assume the form of distinct beaches or terraces. They doubtless exist, however, in other parts of the island; and yet, although in the able papers and volumes of Berger, Weaver, and Portlock, on the Geology of Ireland, I find decided evidence of ancient beaches, I have not met with any description of distinct terraces.

Scotland.

I entered Scotland by the way of the Frith of the Clyde, and soon noticed the general resemblance of its banks to those of American rivers. A few miles below Glasgow, two, and sometimes three, terraces were obvious from the steamboat. They were of small elevation, however, not more, I judged, than 20 or 30 feet, and there is no barrier between them and the ocean. Subsequently I passed through the Highlands by the way of Loch Lomond and Glencoe, and thence to the Parallel Roads of Glen Roy. On this route the surface geology bears a strong resemblance to that of New England. At the foot of hills great quantities of modified drift appear in the form of beaches, rather than of terraces. Sometimes, as in the valley near the lead mines of the Marquis of Breadalbane, the coarse gravel is piled up in those irregular masses with deep depressions, which I have called Moraine Terraces. These, both in Scotland and America, have been regarded as the moraines of ancient glaciers. Once I was prepared to adopt this opinion, but since I have seen undoubted moraines in the Alps, I feel compelled to dissent from it. The fatal objection to such an opinion is, that the materials, composing these supposed moraines, have been modified and in a measure sorted by water—a condition never seen in genuine moraines, at least to any great extent. Fragments of all sizes and shapes are crowded along promiscuously by glaciers, and though some of them are rounded and others ground to powder, there is no separation of one sort and size from another. Wherever we find such separation, however imperfect, we may be sure that the materials, even though originally produced by glaciers, have been remodelled by water. And such are most of those cases which I have seen of supposed moraines in the United States, which I thought strongly to resemble those above alluded to in Scotland. In passing from Fort William to Glen Roy, along the northwest side of Ben Nevis, vast accumulations of such materials occur, which appear to me to have once been sea-beaches or sea-bottoms. In descending towards the Spean on that road, we meet with very fine terraces, sometimes three or four tiers of them. They are, also, seen along the Roy, even beneath the Parallel Roads, where they have been long since figured by Dr. Macculloch, in the Transactions of the London Geological Society.

These Parallel Roads are certainly the most remarkable terraces I have ever set my eyes upon: peculiar from their narrowness and from their perfect horizontality and parallelism. The first fact may perhaps be explained from their occurrence on hills so steep that they could not retain wide platforms of loose materials. The other facts lead the mind almost irresistibly to the conclusion that the body of water, which once filled these glens, must have paused for a time at the successive roads, as it was drained off. But was it the sea, or a lake, whose barrier towards the ocean has disappeared? Did not the markings extend towards the ocean below that point on the Roy river, a mile or two above its mouth, where such large quantities of detritus lie upon its west side, we might say that the valley at that spot was once choked up with detrital matter to the height of the roads, and subsequently eroded. But since the terraces can be traced far down the valley of the Spean, where it becomes quite broad, such erosion never could be accomplished by the river. Nothing but the ocean could have opened such a broad valley. To suppose the space to have been choked up by a glacier, descending from Ben Nevis, does not relieve the matter, because the materials now occupying the valley have been most evidently worn and comminuted by water, and are not the simple moraine of a glacier. Moreover, I noticed that in some places at least, the side of the mountains above the highest road, was covered by such sand and pebbles as constitute the terraces. I did not ascertain whether the same is true to the very top of the hills; yet such was my impression, and if correct, it destroys the idea of lakes and obliges us to admit the presence of the ocean.

But I fear that I am affording ground for the charge of vanity in venturing an opinion on questions which have divided the judgment of so many able men, who have devoted much more of attention to the phenomena than I have. The new suggestions I have made, in respect to the nature of the materials forming the parallel roads and spread over the sides of the valleys, is my only apology.

In ascending towards the higher parts of the Highlands, especially on approaching Glencoe—that most romantic of all the Highland glens—I found the detritus becoming coarser, and the fragments more angular, with slight evidence of being sorted, very similar, indeed, to the unmodified drift of New England. Just at the entrance of Glencoe, I noticed striæ upon the ledges running nearly N. W. and S. E. At Oban, on the western shore of Scotland, I observed similar markings, having a direction N. 50° to 60° W., and S. 50° to 60° E. A good example, also, I observed upon the railway track at the Rath station, between Glasgow and Edinburgh—to say nothing of the examples pointed out around the latter city by her eminent geologists.

Perhaps it may be superfluous to mention that, on the hill lying directly east of the village of Oban (where I was detained by ill health), Mrs. Hitchcock found detrital accumulations of recent shells from 200 to 250 feet above the ocean. Prof. James Nicol, in his *Guide to the Geology of Scotland*, mentions that a raised beach occurs not far from Oban, but only some 30 feet above the sea. Others, however, may have described the higher beach to which I allude. I noticed among the shells those of Ostrea, Mytilus, Mya, &c.

Valley of the Rhine.

In travelling through Belgium, the most of which appears as if recently reclaimed from the sea, and is, in fact, probably a not very ancient sea-bottom, I saw no terraces nor beaches till I reached its northeast part. In the vicinity of Liege, beds of gravel appear which I regard as beaches; and, as we approach the Rhine, the railroad is tunnelled through a high deposit of this material. Emerging into the broad valley of the Rhine, we find distinct, though not high, terraces. They are such as are sometimes produced by the slow alteration of a river's bed, by the wearing away of one of its banks and depositing a lower bank on the opposite side. Such a terrace, some miles long and 15 to 20 feet high, I saw on the right side of the railway between Cologne and Bonn, near the latter city. The beaches are composed of sorted gravel and sand, but I observed no genuine drift in passing through the Ardennes mountains. A little above Bonn, is one very distinct terrace, on the south side of the river, above the meadow, with deposits like beaches above. Before reaching the Siebengebirge, or seven mountains, are remains of terraces, some of which have a rapid slope down the stream. But possibly these are rocky platforms covered by detritus. Between the Siebengebirge and Aldernach, we pass occasionally narrow meadows, on one side or the other, with terraces, and sometimes beaches, higher up. Generally there are only two terraces besides the meadow. The lower ones at least are composed, as I was told, mainly of *Löess.* One of these terraced basins I noticed opposite Linz, at the mouth of the Ahr; another opposite Niederbreisig. But I think it useless to particularize, as the terraces all have the same general characters. They are usually of rather moderate height and not wide.

Above Aldernach the valley expands, with at least one terrace above the meadows. From Coblentz to Bingen, the river is crooked, and the banks crowd so closely upon it that terraces hardly exist. Above Bingen, terraces appear, especially on the north side. The Chateau of Johannisberg, the property, as I was told, of Prince Metternich, stands upon one of these, not less than 100 feet above the river. Above this place, the mountains recede far from the river, and the country is undulating, seeming like the bottom and shores of an ancient sea. But the river terraces are few and imperfect. Near Heidelberg, on the north bank, a few are placed along the foot of the hills. At Wiesbaden and Frankfort, the detrital matter appeared to me like old sea-bottoms, and the long sandy plain passed over between Frankfort and Heidelberg, is probably a terrace of similar origin.

For the next 200 miles, between Heidelberg and Basle, I can only say that the valley of the Rhine is broad most of the way, and I saw but a few well-marked terraces, with now and then a beach above them. But I doubt not that examination would show them both to be numerous, though probably not so distinct as in narrow valleys. Upon the whole, I may say that the phenomena of surface geology on the Rhine, as far as I observed them, correspond entirely with those upon the larger rivers of our country.

Switzerland.

We next reached Switzerland, but in passing towards Zurich, through Bruges and Baden, we continued for a time along the south bank of the Rhine. A little beyond Basle, near the mouth of the Birs, terraces are very fine; and, in fact, they continue to be exhibited along the Rhine as far as I followed it, viz., to Mumpy. The two lowest are very distinct, and then we frequently have irregular ones still higher, which I should call beaches. Near Basle, I measured a terrace, the third in height—and, so far as I saw, the highest—which I found, by the aneroid barometer, to be 228 feet above the Rhine, and 983 feet above the ocean. At Rhinefelder, I took the approximate heights of three successive terraces, and observed at least one other below the lowest of these, and also what seemed to me to be beaches above the highest; these are represented on section No. 41. The highest, it will be seen, is 306 feet above the Rhine, and 1226 above the ocean. Further up the Rhine, near Mumpy, I measured what seemed to me a beach, 696 above the river; and found the highest part of the road between Mumpy and Bruges to be 941 feet above the Rhine at the former place, and 1915 feet above the ocean. At this summit, the detritus was perhaps drift, though I thought it had been modified by water subsequently. After leaving the Rhine at Mumpy, we followed up a small stream with terraces, but they slope rapidly towards the stream, and are, properly speaking, glacis terraces.

Around Bruges, where the Reuse and Limmat join the Aar, the terraces are very fine, and may be seen extending down the river several miles. Between Bruges and Zurich, through Baden, we see some terraces on the small streams, but they are not striking. Most of the detritus seemed to be drift, yet somewhat modified.

The northern part of Lake Zurich I found to be fringed by three or four terraces, which are often chosen as the sites of villages and scattered houses. Leaving the lake at Horgen, on the west shore, we ascended the ridge separating Lake Zurich from Lake Zug. Section No. 42 will give some idea of the terraces on that route. Two of the terraces I measured; and the beach represented as 843 feet above the lake may be only drift, yet it seemed to me to have been modified by water. If so, this gives us a beach 2185 feet above the ocean.

Crossing Righi, I went to Lucerne; next to Bern, and from thence to Vevay, on Lake Leman; thus passing lengthwise through the greater part of the great valley of Switzerland between the Alps and Jura.

North of Lake Zug is a wide plain, but little above the lake, and appearing like an ancient bottom of the lake, as I doubt not it was. On the east shore of the lake, I thought I saw one or two imperfect terraces. Around the western part of Lucerne lake, I saw none that I recollect.

In going from Lucerne to Bern, we ascended the Little Emmen as far as Scupsheim, and then passed over to a branch of the Great Emmen, which, however, we left ere many miles, and passed over an undulating country, where are numerous accumulations of water-worn materials which constitute what I call beaches, or perhaps, more properly, ancient sea-bottoms. Along all the rivers on this route, terraces are common and often quite perfect; for example, a little south of the village of Langnan, in the Emmenthal. I ought, however, to mention that the sandstone along this route sometimes assumes a terrace form, and, where covered by soil, I might have mistaken such a terrace for one composed of detritus. Yet I am sure that many unconsolidated postdiluvian terraces exist on these rivers. On the Reuss, a little out of Lucerne, I measured one that is 267 feet above the lake, and still further on another that is 325 feet above the same. Towards the summit level of the route, near Scupsheim, I measured a detrital accumulation—which, with some doubt, I call a beach—894 feet above the lake, and 2274 feet above the ocean. The summit I found to be 1287 feet above the lake, and 2667 feet above the ocean.

Around Bern, and wherever I travelled on the banks of the Aar, the terraces are well characterized. They consist mainly of gravel and sand; but as we recede from the river, and come to the beaches, the materials are coarser and pass into drift, the boulders rarely exceeding two feet in diameter; yet they are mainly of the older crystalline rocks, while those in place are sandstone.

From Bern towards Vevay, the detritus, till we reach Bulle, beyond Freyburg, is evidently water worn and sorted into terraces and beaches. Some distance beyond Bulle, genuine drift (perhaps the old moraine of the Rhone glacier) began to appear, and continued, so far as I could judge in the fading twilight, nearly to Vevay, where we strike some lake terraces—which Robert Chambers has described as delta terraces—at the heights of 108, 165, and 442 feet above the lake. The highest terrace, or beach more probably, which I passed on this route, my barometer indicated to be 981 feet above Bern, and 2640 above the ocean.

From Lausanne to Geneva, the west shore of Lake Leman is fringed with terraces. In some places I noticed three or four, though not so many are continuous; probably none of them are all the way. Back several miles from the lake, the country appeared to me to be covered with such materials as terraces or beaches are usually composed of. Some of the terraces near the lake I could see, from the steamboat, to be composed of laminated sand and fine gravel. In entering the harbor of Geneva, I noticed several large Alpine boulders.

From Geneva I turned eastward and followed the Arve nearly to its source, on the route usually taken to Chamouny. On looking over Mr. R. Chambers' paper on the valleys of the Rhine and the Rhone, since my return home, I find that he took the same route, and has anticipated some of my observations. I shall, however, give the few which I collected as they appeared to me.

At the south end of Lake Leman, where the Arve from the region of Mont Blanc unites with the Rhone, a mile below Geneva, as it comes from the lake, is a deep accumulation of detritus, through which both the rivers have worn a passage. It was mainly brought down by the Arve, a rapid and tumultuous stream, almost always loaded with matter mechanically suspended. It is in fact the delta terrace of that river and the Rhone, and extends back to the Saleve Mountains. A mile or two east of the city we reach its highest part in that direction, before crossing the Sardinian frontier. I found the terrace there to be 137 feet above the lake. Passing from this level towards the Arve, we find one or two lower terraces: which are composed of pebbles and boulders mixed with clay, not unlike the "boulder clay" of Scotland.

There can hardly be a doubt that Lake Leman owes its existence to this delta terrace of the Arve. But this point will be better understood when I have treated of analogous cases in my paper on Erosions.

As we proceed along the Arve towards its source, we find terraces more or less distinct most of the way to Sallenches, which is 36 miles from Geneva. These terraces for the most part slope with the stream, and they, also, usually slope towards the river, often rapidly, so as to form the Glacis Terrace. In some places, especially where the valley is narrow, there is only a single slope, as is very common in the higher Alpine valleys, where the river runs at the foot of one of the hills.

The materials of the terraces are usually coarse, though sometimes we pass alluvial meadows. But the higher terraces are very coarse, often like unmodified drift. A few miles below Bonneville, I measured a terrace, not the highest, and found it 314 feet above Leman, 1544 above the ocean, and about 134 above the Arve. A little beyond Bonneville, I measured another, which was 372 feet above Leman, and 1603 above the ocean. At Sallenches, I found one which is 581 feet above Leman, 1811 above the ocean, and about 120 above the Arve. Around Sallenche the terraces are fine, but on the north side of the river I suspect the existence of a portion of a former terminal moraine. Still lower on the stream I thought I discovered another, and when we had gone a league or so beyond St. Martins, and began to ascend the enormous masses of coarse detritus unmodified, I could not doubt that we had reached the terminal moraines of former glaciers.

Four or five miles before reaching Chamouny, we pass a long and narrow defile, and as we might have expected, found terraces above where the valley opens, in which Chamouny is situated. A few of them may be level-topped; but they mostly slope rapidly towards the Arve. Chamouny (Union Hotel), according to my barometer, is 3270 feet above the ocean: 3425, according to Johnston's Dictionary of Geography, and 3190 French feet, according to Keller's map.

Some distance above Chamouny, and just beyond the termination of the Mer de

Glace, the Arve valley is blocked up by an enormous mass of coarse detritus, which was probably the right hand lateral moraine of the glacier, when it formerly extended across the valley. It is nearly 200 feet high, as I judged, save that on the north side of the valley the Arve has worn a passage to its bottom through the moraine. Above this barrier was once a lake, and the result has been, that at least three terraces have been formed on both sides of the river. The highest of these terraces I found to be 670 feet above Chamouny, and 4100 above the ocean.

Still higher up the stream, just beyond the glacier, called Argentiere, is another similar moraine, which produced some terraces, less distinct, however, than in the lower basin. I did not pass into the bed of this ancient lake, but took an observation, towards the hamlet of le Tour, on a level with the terraces, as near as I could judge with the eye, and found the height to be 926 feet above Chamouny, and 4351 feet above the ocean; the highest point where I have ever seen terraces.

After passing the summit between the valleys of the Arve and the Rhone, on the Tête Noire Route, we came upon the Eau Noire, which descends into the Rhone. The highest terraces I noticed on the Eau Noire, which are small and of quite coarse materials, are 793 feet above Chamouny, and 4218 feet above the ocean. But the valley of this stream, for several miles on the Sardinian side of its course, affords a fine example of that sort of glacis terrace, which consists of one broad slope towards the stream from the mountain side. The materials are quite coarse, yet rounded, and evidently the result more or less of aqueous agency. Yet along this stream the erosions of former glaciers are quite manifest, high up on the precipices that bound the gorge.

As we descend towards Martigny, on the Rhone, we have a view of the valley of that river some dozen miles up the stream, traversed by the Simplon road. It looks very much like an estuary recently abandoned, and I could see no terraces. The detritus is spread, with nearly an even surface from one steep side of the valley to the other, having a downward slope equal to that of the rapid stream. Such, for the most part, is the character of the Rhone valley half way from Martigny to Lake Leman. Frequently, however, the alluvial sides of the valley slope towards the river in the glacis form, and sometimes I noticed more than one of this kind of terrace, arranged in successive steps, like the level topped terraces. At St. Maurice, where the river goes through a narrow gorge, and the road passes from the Valais into the Vaud, we meet with terraces of the common form, which I found to be 250 feet above Leman, and 1480 feet above the ocean; or on a level with the surface at Martigny. The Rhone, however, at St. Maurice, is 70 feet lower than at Martigny, according to Keller's map.[1]

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[1] In several of the valleys of the Alps, I was struck with a singular optical deception, which I have not seen noticed by travellers. In ascending valleys with steep and lofty sides, the road sometimes descends slightly for some distance, in consequence of the detritus, which spreads out over the whole valley. In some cases of this sort, I felt a little anxiety to see the postilion urging on his horses at so furious a rate, down what appeared to me a quite steep hill. But on looking back, I found that we were scarcely descending at all. And, indeed, I found that a great part of the way we seemed to be

France.

I passed from Geneva to Paris, through Dijon and Tonnerre, and from Paris to Boulogne: also from Calais to Lille, in the north of France, but I have not much to say of the terraces. The country generally is too flat and free from mountains to make their occurrence probable. On the route from Geneva to Paris, terraces are uncommon, though the limestone, which I believe underlies the whole country, sometimes assumes this form: as for example, in the hills surrounding Poligny. Around Campanogle river, terraces are well characterized, and at a higher level I saw some beaches. As to drift derived from a distance, I saw no good example; however, I crossed the Jura mountains in the night. In many places the limestone is worn into a thousand fantastic shapes at the surface, and appears extremely jagged; showing that drift agency has not smoothed it down.

Scandinavia.

This country I did not visit, and I allude to it here for the purpose of quoting a remarkable fact, mentioned by Mr. Robert Chambers, in his description of some of its terraces. These he traced at least to the height of 2162 feet above the ocean, and found the highest bearing a strong resemblance to the Parallel Roads of Lochaber, in Scotland. But the fact to which I allude, is this: "that there is a district in Finmark, of 40 geographical miles in extent, which has sunk 58 feet at one extremity and risen 96 at the other." (Ed. New Phil. Journ., Jan. 1850.) If the terraces there are as irregular as in this country, and as much wanting in continuity over wide districts, this would be a very difficult fact to determine. But I cannot doubt that one so familiar with this subject as Mr. Chambers, would be on his guard against confounding different terraces.

3. Terraced Island in the East Indian Archipelago.

Rev. Charles Hartwell, American missionary in China, on his passage thither, took a sketch of Sandalwood Island, on account of its terraced appearance. Formerly my pupil, and knowing the deep interest I felt in terraces, he sent the sketch to me; and in the dearth of information respecting terrace phenomena in that part of the world, I have thought it ought to be preserved. I have accordingly added it to the illustrations of this paper in Plate XII, Fig. 6. It was taken at the distance of eight miles. *a*, is a projecting point of terraces; *b*, the S. E. point of the island; *c*, detached isle west of the point *b*, and near the southern

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descending, when in fact there was a slight slope upward. I observed, also, that when we were on one side of a valley, say 80 or 100 rods wide, and where in fact the two sides sloped somewhat rapidly towards the river in the centre, it seemed as if there was a continuous slope to the opposite side, where the steep rocky mountains rose. I shall not attempt to explain these phenomena, though confident that they are not the result of anything peculiar in my own perceptions.

shore. This was the only terraced island seen by Mr. Hartwell during his voyage. He says it is volcanic in part, and the terraced margin may be coral reefs. It is covered with vegetation, sandalwood being abundant.

Other Forms of Surface Geology.

In the commencement of this paper I have enumerated other distinct forms of detrital matter coming within the province of surface geology. I have not studied these so carefully as the terraces and beaches, and, therefore, my descriptions will be short, though I trust they may deserve the attention of observers.

1. *Sea-Bottoms.*

If we find evidence of the existence of shores of ancient seas, we should expect to discover the remains of their bottoms; and if I mistake not, New England, especially in its less elevated portions, does present the gravelly and sandy plains and low ridges, which can be explained only by the former presence of the ocean above them, with its waves, tides, and currents. In the vicinity of Connecticut river, they are less obvious, because in the lower parts of the valley drainage has, in a measure, obliterated the marks of oceanic action, and the materials have been converted into terraces and beaches. The sides of the valley also rise too rapidly to expect many such accumulations of detritus as form sea-bottoms. But when we get into the comparatively low region, within twenty or thirty miles of the coast, in Massachusetts, Rhode Island, and Connecticut, the surface is in a great measure covered with such materials, and in such forms as the ocean must have produced. Though I am not prepared to mark these definitely upon a map, yet I have ventured to define a few of them near the mouth of the Merrimack, and also in Berkshire valleys, in Plate III; I have likewise marked a strip as of this character, along the route of the New London, Palmer, and Amherst Railroads, and from Merrimack river to Saco river, along the northwest side of Lake Memphremagog.

2. *Submarine Ridges.*

I agree with Mr. Whittlesey in the opinion that the ridges which encircle Lakes Ontario and Erie, were probably formed beneath the waters. These lake ridges—the lowest certainly—may not have been *submarine* in the strict sense of the term, though as it is certain the ocean once stood above the western lakes, it is not easy to say at what altitude the waters became first brackish, perhaps, and then fresh.

I have ventured to mark one submarine ridge near the mouth of the Merrimack, on its south side, and to extend it southerly along the coast at least to Ipswich; beyond which I have not attempted to trace it. The highest part of the city of Newburyport occupies the summit of this ridge, which has a slope both towards the ocean and towards the country. This ridge preserves a pretty uniform height, nearly to Ipswich.

This ridge may prove to have been an ancient beach, but its slope towards the interior and its singularity have led me to refer it to a submarine origin. Others, doubtless, will be found along the coast.

3. *Osars.*

I have perhaps already said all that is necessary as to the existence of Osars in this country. I cannot see why they should not occur here as well as in Europe, since all the other forms of modified and unmodified drift are so similar on cis-atlantic and transatlantic shores. But what I call Moraine Terraces cannot be referred to accumulations of detritus by a current sweeping past an obstruction; and, therefore, they are not osars, if such a mode of formation be essential. I should be inclined to refer to osars those remarkable trains of blocks, starting in Richmond in Berkshire county, and extending southeasterly several miles, described by me in the American Journal of Science, XLIX, 253; but they are too long to answer Murchison's description. I will mention one or two cases, however, in the vicinity of the White Mountains, which seem to me more like osars than any examples I have met with in this country, though not satisfied that they are so; but I have placed them on Plate III, in order to call the attention of geologists to those spots. One is a remarkable mound of gravel, near Fabyan's Tavern, five miles from the notch in the White Mountains. Its height cannot be less than twenty or thirty feet above the surrounding surface; and its top (measured with the aneroid barometer) is 1537 feet above the ocean. (See a sketch and description of it in Vol. I of the Transactions of the Association of American Geologists and Naturalists, Plate viii, Fig. 10.)

The other case presents us with several ridges of sand, nearly straight, in a valley lying southwest from Adams' Tavern, in Conway, New Hampshire, towards Eaton. The principal ridge may be a half a mile long, terminated on the north by a pond. These ridges seem to me to differ from those in Andover, in being nearly straight; but they need further examination.

4. *Deltas and Dunes.*

Connecticut river has, of course, made some delta-like accumulations at its mouth, but they are not extensive, being probably swept away by tides and currents. The same is true of the smaller rivers of New England, but as I have not studied any of these with care, I pass them by.

The dunes of southeastern Massachusetts have long since been described. They are sometimes quite high and large, requiring vigorous and expensive efforts to arrest their progress. Along the Connecticut valley small ones exist in Hadley, Granby, Montague, and Enfield, Ct., which are slowly advancing southeasterly, in consequence of the predominance of northwesterly winds. These dunes are derived from the sands of one of the higher terraces of the valley.

5. *Changes in the Beds of Rivers.*

1. *On Connecticut River.*

On the maps which I have given of Connecticut and Deerfield rivers, I have marked numerous ancient river beds. These are of two kinds, the most ancient, showing a deserted rocky gorge, where once the stream flowed at a higher level than at present; and the other, a depression in alluvial meadows, once the bed of the stream, and from which it has been generally slowly deflected by the wearing away of a bank. Changes of the first sort probably present us with old river beds of the antediluvian period, as I shall attempt to show in my paper on Erosions; but the latter class are postdiluvian, and sometimes occur in our own times; one good example of which change may be seen along the Connecticut, at the foot of Mount Holyoke. I was surprised to find how numerous these ancient river beds are, and I doubt not that time and further research would bring to light many more than I have exhibited on the maps. I will briefly describe such as I have found. Of some of them I am not quite sure, but generally they are distinct. The antediluvian river courses it is sometimes difficult to distinguish from erosions by the ocean; and the alluvial ones may be confounded with the troughs between glacis terraces.

The most southerly deserted beds of Connecticut river are in Portland, opposite Middletown. The present bed of that stream through the first range of mountains, appears to me to be in a measure postdiluvian. It curves around two hills of considerable height, between which, as it appeared to me, is a former bed of the stream. I feel quite confident, also, that it once ran on the east side of both the hills, at a considerable elevation above the present level.

On the west side of the river, I think I can trace an ancient, though postdiluvial bed of the river, through Wethersfield, passing a little west of the village, and also through the west part of Hartford, so as to unite with the present bed a little above the city, bringing the city upon the east bank, had it then existed.

Along the east bank a depression commences on the east margin of the meadows, in East Hartford, and continues as far as the south part of Enfield. I have not, however, followed the old bed through the whole of this distance, and may be in error.

In the town of Springfield, a similar appearance is exhibited along the foot of the highest terrace on which the United States Armory stands, from a small stream at the south end of the town, to near the mouth of Chicopee river. I think the river once ran where we now find the principal street of the place. An isolated terrace, a little north of the town, marks the west bank of the former stream, as shown on Section No. 3, Plate I.

Commencing in the west part of South Hadley, an ancient bed is marked on the map, passing to the east part of Granby, through a part of Ludlow, thence into the west part of Belchertown, where it passes through the gorge between the east end of Mount Holyoke (or Norwottuck, as the eastern part is called), and the gneiss hills of Pelham; thence through the east part of Amherst, into Leverett, where it runs along the east base of Mettawampe (Toby), and thence along the east part of Montague, to the mouth of Miller's river. In the south part of Amherst, at a later period, when the waters had sunk below the gorge at the east end of Norwottuck, I suspect that the current ran along the north base of Holyoke, and entered the present bed of the Connecticut, opposite Northampton. At a period somewhat later, I think another bed ran from this same place along the west side of Amherst; thence to Sunderland, at the foot of the higher terraces, where, at the north of the village, it coalesced with the present bed.

In Hatfield, somewhat north of the village, is a distinct ancient bed of postdiluvian date, but of no great extent.

In the east part of Leverett, is a valley which was probably once the bed of Connecticut river, earlier, without doubt, than the bed so distinct along the foot of Mettawampe. The two unite at Amherst on the south, and in the north part of Leverett on the north.

In Vermont and New Hampshire, I have not examined the Connecticut with care enough to discover its ancient beds, save in two places. In Claremont, I think it formerly ran about two miles east of its present bed, from which the old bed is separated by a hill of considerable height. In the southwest part of Piermont, also, I thought I discovered an abandoned bed, but had not time to explore it carefully.

Opposite Mount Holyoke, in Hadley, is an example (referred to above), of a recent change in the bed of the Connecticut, of considerable extent. Formerly the river made a curve here of three or four miles long, while its actual advance towards the ocean was only about 100 rods. Ten or twelve years ago, during a freshet, a passage was cut through this neck, and since that time, the stream has left its old channel, which is fast filling up, and across which Connecticut River Railroad now runs.

2. *In Orange, New Hampshire.*

On Plate III, a distinct ridge of mountains is represented as running from Bellows Falls, in New Hampshire, to the White mountains. It is not intended to convey the idea, however, that such a continuous ridge exists: but only that it is the summit between Connecticut and Merrimack rivers, from which tributaries of those rivers run in opposite directions. In that summit, in the town of Orange, is a depression in the range, through which the Northern Railroad passes, at an elevation above Connecticut river, at West Lebanon, of 682 feet, and of 830 above the Merrimack. Here pot-holes of great size indicate the former passage of a stream of water for a long time, from the Connecticut into the Merrimack valley. In other words, it seems to have been one of the outlets of the waters of the Connecticut valley, where they stood at that height. But this is hardly a case of the change of a river's bed, since no correspondent stream now exists. Two small brooks, commencing in the peat swamps lying on each side of the ridge, and running, one easterly and the other westerly, are all the representatives remaining of the powerful current that once crossed this spot. It will be more particularly described in my paper on Erosions.

3. *In Cavendish, Vermont.*

On Plate III, Black river and William's river, in Vermont, are seen to run nearly parallel courses. It appears that they were once united: at least the principal branch of Black river formerly ran southerly into the present bed of William's river. Whoever will pass through "Proctorsville Gulf," in Cavendish, shown on Plate III, as an old river bed, will be satisfied that it was indeed once the channel of Black river. Its present summit, raised considerably by detritus, is (by the aneroid barometer) 792 feet above Connecticut river, at the top of Bellows Falls, and about 100 feet above the Black river at Proctorsville; so that if this river were 100 feet higher at that spot, it might run through the gulf. The sides of the "gulf" are quite steep and high, resembling the banks of many of our mountain streams that have been worn deeply by water.

At Duttonville, in Cavendish, two miles lower down the stream than Proctorsville, is another more obvious ancient bed of the Black river. This, also, is filled with detritus where it branches off from the present bed, but within 100 rods of that spot, on the route of the Rutland and Burlington Railroad, we find large and distinct pot-holes; the infallible mementos of a former rapid current. This old bed may be traced some six or eight miles towards Connecticut river, where it unites again with the present channel of Black river. By the detritus which chokes up the old bed, at Duttonville, that river was compelled to turn to the left, where it has worn out a gorge through the rocks nearly 100 feet deep, producing a romantic cataract, called Great Falls; the foot of which is 183 feet below the old river bed. These two cases, belonging as they do in part to antediluvian agencies, will be described again in my paper on Erosions.

4. *On Deerfield River.*

One of these occurs near Shelburne Falls, in Buckland, where pot-holes exist in the sides of the old channel, 80 feet above the present stream, as may be seen on Plate IV. But a description of the spot is reserved for my paper on Erosions.

Where Deerfield river debouches into the valley of Connecticut river, from its mountain gorge, it has formed an alluvial plat of unrivalled fertility. And here is displayed the best example of changes in the bed of a river by alluvial action that I have ever seen. As all the early part of my life was spent in that valley, I became familiar with these ancient river beds, and I have sketched them in Plate IV. Some others, less obvious, perhaps, might have been added: but it will be seen that not less than fourteen are put down on this spot, only four miles long and one mile broad. Nay, from the manner in which rivers in alluvial spots change their courses, viz., by the gradual wearing away of one of their banks, I cannot doubt that every part of these four square miles, save Pine Hill, in the northern part, and perhaps some limited spots where the village stands, has once constituted a part of the channel of the stream.

In the extreme northern part of Deerfield, only a mile south of the village of Greenfield, occurs an old rocky bed of Green river, a tributary of Deerfield river. Here are pot-holes in the red sandstone, and a gorge in the same, while the present river runs in a channel worn in sand and clay, several rods further west, and at a considerably lower level. (See Plate IV.)

5. *On Agawam River.*

I have traced out three examples on this river of antediluvian date. One is in Russell, on the west side of the present stream. The old bed is filled to a considerable height with sand and gravel, compelling the river to find its way through a rocky barrier.

A second of these beds may be seen to the east of Chester village, at the junction of its east and west or principal branches. A third is some three miles above this point on the east branch. (See my paper on Erosions.)

I might refer to many other examples of ancient beds of rivers, not connected with the Connecticut. But since most of these are older than the alluvial period, they will more properly be noticed in my paper on Erosions. They would not be mentioned here at all, were it not that the accumulation of detrital matter during the last sojourn of the continent below the waters, seems to have been the means of commencing many of those defiles in which rivers now run.

Results, or Conclusions from the Facts.

I shall now proceed to state the conclusions at which my own mind has arrived from the facts which I have observed respecting surface geology, especially terraces, beaches, and drift. And as these conclusions are not based altogether upon the details above given, I shall present a summary of the arguments by which they are sustained, and the collateral facts and considerations on which they rest.

1. Postdiluvian terraces and beaches all lie above the coarse unstratified and unmodified drift, as well as above the striæ, furrows, and *roches moutonnes*, connected with drift. Hence the terraces and beaches are the result of operations subsequent to the drift period.

I wish not to be understood as maintaining that no genuine drift shows evidence of stratification and other modifying effects of water. Such effects do present themselves sometimes in the midst of detritus, which generally, in position and character, affords unequivocal evidence of being true drift. Limited beds of sand and clay are met with sometimes in the midst of such materials, and sometimes we find masses of coarse irregular detritus and scattered blocks above deposits that are distinctly sorted and stratified. But, as a general fact, the sorted and stratified materials lie above the drift.

I wish, also, to add, that it is no easy matter always to draw a line between unmodified drift and the modified materials of beaches and terraces. The graduation of one into the other is often so insensible, that we cannot tell where the one ends and the other begins. But I shall refer to this again in another place.

2. The successive beaches and terraces, as we descend from the highest to the lowest, in any valley, seem to have been produced by the continued repetition of essentially the same agencies by which the materials—originally coarse drift—have been made finer and finer, and have been more carefully sorted and arranged into more and more perfect beaches and terraces.

This seems to be the general law: at least such is the conviction produced in my own mind. Yet occasionally we meet with limited deposits, as already remarked, of fine materials in the midst of, or beneath those very coarse. This only shows that in certain places the comminuting and sorting processes were carried on at an early date as perfectly as afterwards when they were extended to large areas.

3. By far the largest part of the materials constituting the beaches and terraces is modified drift, in other words, fragments torn from the rocks in place by all the eroding agencies down to the close of the drift period.

This position is proved by the occurrence of drift scratches and furrows over most of the rocks in place, in the valleys as well as on the hills. Indeed, I expect to show in my paper on Erosions, that some rivers have made deep and long cuts through the rocks since that period: for instance, the gorge of Niagara river, from the Falls to Ontario, and the still deeper cut between Portage and Mount Morris, on Genesee river. But in the valley of Connecticut river no such gorges have been worn, since we find the drift striæ in many places almost as low as the surface of the present stream, even at those points where once gorges were worn out. Thus, at Bellows falls, the rocks at the top of the falls, even to the water's edge, exhibit distinct and beautiful examples of furrows and protuberances produced by the drift agency, although the cataract has undoubtedly receded considerably at this spot since that force acted. At Brattleborough the slate on the west side of the river shows drift furrows only a few feet above the river. Here too was once a gorge: but it was worn out earlier than the drift period. At Sunderland, where Mettawampe and Sugar Loaf, between which the Connecticut now runs, were doubtless once united, drift scratches now show themselves almost on a level with the stream. The same is true on the trap rocks at Titan's Pier, in the gorge between Holyoke and Tom, which were once still more certainly united. As to the gorge a little below Middletown, I am not able to speak certainly, yet so far as I could judge, in passing upon a steamboat, I do not doubt the occurrence of drift erosions at a low level.

4. Hence on such rivers as the Connecticut, wherever, indeed, we can find marks of drift agency low down on the rocks at gorges, we cannot suppose that rocky barriers closed those gorges during the period when the terraces were forming; and, therefore, we cannot call in their aid to explain the formation of the terraces.

5. The highest distinct terraces which I have measured above the rivers on which they occur, are as follows: On Connecticut river, at Bellows Falls, 226 feet; on Deerfield river, 236 feet; on Genesee river, at Mount Morris, 348 feet; on the

Rhine, near Rhinefelder, 306 feet. Some of the accumulations of gravel and sand above these might perhaps be called terraces; but I think they are more appropriately called beaches. So far up the sides of the valleys as these banks appear to have been formed mainly by the rivers that now run through them, when at a higher level, and forming a chain of narrow lakes, thus high should I denominate them terraces. But when we reach such a height that the waters producing the banks must have overtopped most of the hills and communicated with the ocean, or constituted a part of it, then they ought to be called, as they undoubtedly were, beaches—it may be the shores of a bay, or estuary, or frith; but still produced more by breakers than by currents, and, therefore, have not a level top.

6. The most perfect beaches in New England vary in height from 800 to 1200 feet above the ocean. (In Pelham, Shutesbury, Whately, Conway, Ashfield, &c.) Others occur less distinct, as we might expect they would be, from 1200 to 2600 feet above the ocean (at Dalton, Hinsdale, Washington, Peru, White Mountain Notch, and Franconia Notch). I can hardly doubt that further examination will discover others at a still greater altitude.

On Snowden, in Wales, I found a few traces of sea-beaches at several altitudes, the highest 2547 feet above the ocean. Still more distinct marks of a beach occur a little east of Cader Idris, 762 feet above the same level.

In the north part of Switzerland, near Mumpy, I measured what I called a beach, 1670 feet above the ocean; on the west side of Lake Zurich, another, a little doubtful, perhaps, 2105 feet; between Lucerne and Bern, near Scupsheim, another, 2274 feet; and between Bern and Vevay another, 2640 feet, above the present ocean level.

7. The number, height, and breadth, of the river terraces, vary with the size of the river, the width of the valley, and the velocity of the current above the place where the deposits are made. Generally the number is greater upon small than large streams, while the height is less. This may be seen upon the subjoined sections. Thus the terraces on the Connecticut rarely exceed three or four; but on its tributaries, where they enter the Connecticut especially, the number rises sometimes as high as ten, as on the Ashuelot, in Hinsdale, Whetstone brook, and West river in Brattleborough, and Saxon's river, at Bellows Falls. In these cases the terraces on the tributaries are formed in the terraces of the principal stream; yet though the former are more numerous they rise no higher than the latter.

8. The river terraces, excepting the delta terraces, rarely correspond in number or in height on opposite sides of the stream. The delta terrace, whenever worn through by a stream, will, of course be of equal height on both sides of the river. When the valley is wide, and several terraces exist on opposite sides, by the eye alone we are apt to imagine an exact correspondence in height. But the application of the level usually dissipates such an impression, as nearly all the subjoined sections, which extend across the stream, will show. Had I carried these sections across the river more frequently, it would have appeared that sometimes no terraces exist on one side, while there are many on the other; or that the number differs much on opposite sides.

9. River terraces usually slope toward the mouth of the stream, to the same

amount as the current descends, and sometimes more. It is on the smaller and more rapid streams that we see this slope most conspicuously; indeed, on these it is so obvious that I deemed measurements unnecessary. I have made only a single one, and that shows the slope in a delta terrace in the west part of Deerfield, which terrace was produced by a small stream called Mill river, which, as it entered the former estuary, thrust forward a quantity of sand marked as a terrace on Plate IV. This deposit would of course be thickest nearest the shore and diminish outwardly. The amount as I measured it by the aneroid barometer, is thirty-nine feet, in less than half a mile, a slope which of course had no reference to that of the current.

I have said that the slope in some cases is greater than that of the stream. To illustrate this, let us refer to the wide and long basin from Mt. Holyoke to Middletown, in which the current of the Connecticut must have been gentle, nor could the tributaries have brought in materials sufficient to fill up the broad valley as high as where it is much narrower. Hence we should expect, that as we pass south from Holyoke, the upper terrace would become thinner and thinner. Such I suppose to be the fact, as stated in my description of the sections in the part of Connecticut valley above alluded to. In a distance of forty or fifty miles, I have thought we have evidence of a descent of more than 140 feet, besides the descent of the river. The only doubt I have in the case, arises from the difficulty of determining whether the upper terrace, to which my sections extend, is continuous throughout this whole distance.

10. Terraces are usually the highest about gorges in river courses. Such is the fact at Bellows Falls, at Brattleborough, at Montague, at South Hadley, and a little above Middletown, where Rocky Hill on the west side of the river produces a narrow gulf for the river. Also between Tekoa and Middle Tekoa, on Agawam river (Section No. 19.) The materials are not accumulated around these narrow passes because they were then closed, for we have shown that since the drift period most of them have not been closed. But the narrowness of the valley at these spots would, to some extent, retard the streams when swollen, and cause it to deposit more of its suspended matter than in the middle part of the basin. In general it is on the lower side of the gorge that the accumulation is the greatest, because there the waters would spread out laterally and produce eddies or ponds. But sometimes it is above the gorge where the terrace is highest, as on Tekoa.

11. The chief agent in the formation of terraces and beaches appears to have been water. The following facts establish this conclusion beyond all reasonable doubt. 1. The materials have been so comminuted and rounded as no other agent but water can do. Glaciers and stranded icebergs may, indeed, crush and sometimes partially round abraded fragments of rock, but they do not produce deposits of rounded and smoothed pebbles, such as form most of the terraces and beaches. 2. The materials are sorted, so that those of different sizes occupy distinct layers. This effect water alone, of all natural agencies, in the form of waves or currents, can produce. The size of the fragments indicates the strength of the breaker, or the current. 3. The deposition of the layers in horizontal or nearly horizontal position, can be effected only by water. In order to produce the level tops of the terraces water must have once stood above them, while currents strewed the mate-

rials along the bottom. So too, though we find more irregularity in the beaches, yet along what was by the supposition once the line of a coast, they are level, while seaward they are rounded and sloping, like beaches now forming.

In the case of moraine terraces, however, I think it unquestionable that some other agent, besides water, must be called in to explain their formation. If masses of ice were stranded for a long time on the spot where they occur, and currents of water had accumulated the sand and gravel around them, and afterwards the waters had retired and the ice melted, it seems to me that the surface would be left in that peculiar condition which the phenomena under consideration present. I can, however, conceive how strong eddying currents alone might pile up sand and gravel to some extent in a similar manner. But when I meet with these ridges, knolls, and depressions, over wide surfaces, and a hundred feet in height and depth, I have strong doubts whether we must not call in the aid of stranded ice. Water, however, even in this case, must have been the principal agent. But more on this subject in a subsequent paragraph.

12. If the preceding conclusions be admitted, it will follow, that at as high a level as we can find accumulations of rounded and sorted materials, we may be sure of the long continued presence of water, since the drift period, or during the alluvial period. Hence I feel sure from the facts which I have stated, that over the northern parts of this country, this body of water must have stood at least 2000 feet above the present sea level; and I might safely put it at 2500 feet: for up to that height I have found drift modified by water. At an equal height have I observed it on the continent of Europe.

13. The water that stood at such a height on the continents, must have been the ocean. For most of the mountains in the United States are below that level, and consequently must have been enveloped by the waters. Not a few instances occur, indeed, nearly all the examples of beaches which I have described are of this character, in which Plate XII, Fig. 4 represents their situation. Between the old beach and the present ocean there are no barriers high enough to prevent the water that covered the beach from communicating with the ocean: and the fact that the surface, almost everywhere, is smoothed, rounded, and striated by the drift agency, even to the bottom of the valleys, precludes the idea that rocky barriers existed when the beaches were formed high enough to shut out the ocean: for those beaches were formed since the drift period.

I know of no way of avoiding the conclusion that these waters were oceanic, unless it be by supposing barriers to have been formed by vast accumulations of detritus and ice, which subsequently disappeared, after having formed and sustained lakes and inland seas long enough to form the beaches. But this must have required barriers, sometimes perhaps a hundred miles long, and in some places at least 1000 feet high. If they once existed, and were formed of detritus, what can have become of it? Was it carried into the ocean? This would have been impossible by the breaking away of the barrier, even though ruptured in several places; and we may not, by the very supposition, call in the breakers of an ocean to wear it away. Was it an icy barrier? Is it not incredible that an embankment of this material, so many miles long, and so many hundred feet thick, should have been able to sustain for centuries vast bodies of water, while it was comminuting and depositing extensive beaches. I am fully satisfied, that even though the geologist may, in his study, conceive of such icy or detrital barriers, he could not maintain his opinion, were he to stand upon these beaches, and turn his eyes towards the present ocean, and see what an immense mass of materials must be required to fill up the country to the level of his eye, so as to cut off all communication with the ocean. Certainly nothing like such piles have been witnessed in any place on earth. It is true of some Alpine valleys, that their lower ends have been choked with ice and detritus, so as to form ponds above; but where do we find an example, in which the sides of such valleys, many miles long, are formed by the same materials?

Some, I know, consider no evidence of the presence of the ocean decisive, unless it have left marine remains, and such we find in the United States only among the more recent beaches and terraces, for example the clays around lake Champlain, and along the St. Lawrence, at Montreal, &c., which are only a few hundred feet above the sea. Why they do not occur among the more ancient pleistocene strata, I mean the terraces and beaches, I know of no more probable reason, than that animals and plants were not then living in the waters that made these deposits. But that the beaches and terraces were formed by water, no one, who will examine them, can doubt. This being admitted, I am forced irresistibly to the conclusion that this body of water must have been oceanic, for the simple reason that a sheet of water thick enough to reach such spots, must have spread on all sides far enough to form a sea.

It is possible that some may resort to the supposition, that though no high rocky barriers have been worn down since the formation of the beaches and terraces, yet there may have been great changes of relative level since that time, so that places, which are now lifted high above the general surface, may then have occupied depressions where lakes existed. I can hardly believe that any one practised in surface geology would adopt such an opinion, for he will see that nowhere have terraces or beaches been disturbed by any such movements, but retain exactly the contour and levels which they had when deposited. This they could not have done if there had been any appreciable changes of relative level: and to meet the case, such changes must have been very great. The hills, too, that were rounded by the drift agency, present their *stoss*, or abraded sides, to the north, just as they must have done when struck by such agency: and at the foot of other hills, boulders are accumulated, just as they would be, if those hills stood there during the drift period. In short, though there be evidence that the land as a whole has either risen, or the water has retired from it, since the drift period, in thirty years' examination I have never met with a single example of any change of relative level in different parts of the surface by vertical movements since that time; nor have I seen any such changes described, save that sort of see-saw movement which Mr. Chambers found in Scandinavia, and which may have happened, also, in our own country, but which has never disturbed the relative levels in the sense above supposed.

14. It is hardly venturing beyond a legitimate conclusion, in view of the pre-

ceding facts, to say, that all the northern part of this continent, at least all east of the Mississippi, has been covered by the ocean since the drift period. For admit that these waters rose 2000 feet above the present ocean, and how few mountains even, would project above the surface. A few rocky islands only would be seen, the largest around the White mountains and in the northern part of New York, while the chief portions of the land would have disappeared: nor in the opinion of many geologists is the evidence wanting, in the marks of drift agency everywhere, save at the very top of Mount Washington, that all the hills, higher than 2000 feet, save that single peak, were at that period beneath the waters.

15. Admitting the existence of the ocean over the whole, or the greater part of North America (and the same may be said of other continents, with similar phenomena), and a gradual elevation of the land, or a depression of the ocean to commence and continue to the present time, we can see how, by the drainage of the uneven surface, and the action of waves, tides, and oceanic and fluviatile currents, the whole system of beaches and terraces, as well as other forms of surface geology, were produced.

16. Let us begin with the beaches, which must have been formed the earliest. As the elevated portions of the surface began to emerge from the waters, covered probably to a considerable extent by drift detritus, the waves would act upon the shores and comminute the materials, causing them to accumulate in bays and friths. Yet at first the quantity must have been small, both from the limited extent of coast, and deficiency of materials; and if the elevatory movement was rather rapid, the fragments would not be reduced very small, nor thoroughly rounded. Hence the highest beaches might be difficult to distinguish from the drift, especially as the drift, while beneath the waters (I say nothing here of the time or mode of its origination, save that the period was earlier than the rise of the land), would most probably be made to assume a beach-form in some places. If the elevation proceeded equably, the wave-worn detritus might be strewed somewhat evenly over the sloping surface, and not form distinct beaches. But if there were pauses in the movement, we might look for beaches at successive levels. Yet there would doubtless be great inequality in their position and character, nor should we expect, unless the pauses were long, and the quantity of detritus great, that they would form regular fringes around the islands: but rather that they would be found in the successive bays that would be formed in different places, as the irregular bottom of the sea emerged.

I have supposed pauses in the vertical movement: and these doubtless would produce beach deposits at successive levels. But when enough of land had emerged to give rise to rivers, I think we can see how similar beaches might be formed without paroxysmal movements. A river would carry detritus into the sea, which might be spread along the coast by oceanic currents, and form a bank beneath the waters. Gradually would this be raised by new depositions, and by the uniform rise of the shore, until it would reach the surface, forming a marsh at first; and as the process of elevation went on, a dry and raised beach, modified by the breakers while within their reach. But when the river could no longer deposit its sediment upon this bank, it would be carried forward into the water beyond, and there begin to form a new bank, which in like manner, would at length reach the surface; and then a third bank would be formed, all the while the vertical movement proceeding without pause or paroxysm.

It may be thought that in such a case the sediment would be deposited in one continuous slope or talus: and it would be without a current along the coast to wear away the successive banks on the outer margin; and thus, it seems to me, the result might be terraces, or rather successive beaches, at different levels. And thus might the lower beaches, that now fringe the coasts of North America, have been formed by a secular and perfectly uniform elevation of the continent. Until rivers existed, however, I should expect the beaches to be very irregular and indistinct, unless there were pauses in the upward movement: and so I do find them near their upper limit, while the lowest beaches on our present shores, are almost as perfect as river terraces, especially at the mouths of rivers, where perhaps, they should be called terraces.

17. Let us now take a bird's-eye view of the continent, raised high enough to bring nearly all the surface above the waters, which is now above the level of the highest terraces. We see the valleys occupied as arms of the sea, in the forms of friths, estuaries, and bays, and in some places, bodies of water exist, cut off entirely from the ocean. Some of the estuaries, too, are so narrowed in particular places, by the approach of barriers on opposite sides of the estuary, as to form, as it were, a chain of lakes, connected by straits. Such would be the aspect at the time supposed of the Connecticut valley. Along the shores, we see on a diminished scale, those rivers which are now its tributaries, emptying into the lake-like estuary, and thus producing a current towards the ocean. Their waters, acting on the drift over which they run, would comminute and carry into the estuary the smaller particles, and thus form shoals, or banks, along their mouths. Meanwhile the ocean is sinking, and at length these banks will come to the surface, and constitute small deltas to the rivers. The streams, too, will wear down their beds, as the estuary sinks, and hence they must cut passages through their deltas, and urge forward a new mass of sorted materials into the now diminished estuary. Thus another delta may be formed, and even a third, or fourth, in the same manner; and even though the vertical movement be perfectly uniform, the current towards the ocean, produced by the tributaries, will so act upon the outer margin of the embankments, as to form terraces, rather than a simple talus.

In this manner, it seems to me, may the delta terraces have been formed by the slow drainage of the country, and without supposing pauses in the vertical movement. These are in fact, among the most usual and striking of the terraces.

Though formed in essentially the same manner as the beaches above described, they would be more regular on their tops, because not exposed as the beaches were, during their emergence, to the action of the breakers.

Mr. Charles Darwin, I believe, first suggested the mode in which delta terraces were formed, as described above, in his paper on the Parallel Roads of Glen Roy. Mr. Robert Chambers, however, has pointed out a case in Switzerland, which fully confirms these views. In the canton of Unterwalden, the lake of Lungern has been artificially lowered within the last sixty years. Where the head of the lake formerly was, and into which a number of small streams formerly emptied, several deltas are laid bare by the draining off of the water, and they are cut through by the streams, which have worn deep chasms through the loose materials, and are still wearing them backwards towards the Alps.

18. We will now inquire, how, in like circumstances, lateral terraces may have been formed. As the comminuted and sorted materials are projected into the main valley, now an estuary, which, as it sinks, is putting on the characters of a river, they will be swept towards the ocean by the current, a greater or less distance, according to the velocity of the stream. Thus will the delta terraces of the tributary, become in part lateral terraces to the principal valley.

19. There is another mode in which lateral terraces may be formed, as suggested by Robert Chambers, in his paper on the Valleys of the Rhine and the Rhone. In the successive basins that form the chain of lakes produced by the drainage of a country, the detritus brought into the basins by their affluents, will more or less be spread over their entire bottoms, although, as above suggested, banks may be formed, also, along the shores. The materials there spread over the bottom, may accumulate to a great depth, if the straits connecting the several expansions of water are narrow, and the water not so deep as in the basins. At length, however, as the drainage goes on, the bed of the basins will be brought to the surface, and the waters, narrowed into a river, will cut a passage through the detritus, leaving probably on each shore a terrace of the same height. The current, however, might crowd so closely upon one side of the valley as to sweep away all the detritus there, and leave a terrace on one side only.

20. There is a third mode in which lateral terraces might be, and doubtless have been formed. In the case last supposed, the river is represented as simply cutting a chasm through its sandy, clayey, or gravelly bottom. But powerful freshets occur not unfrequently on all rivers: and in their swollen condition, and with increased velocity, they act powerfully upon their banks, especially if of alluvial materials. And if the course of the stream be tortuous, as is always the case, one bank will be acted upon more powerfully than the other. This action will produce a meadow on one side of the stream, but little raised, it may be, above the river in its ordinary state. Successive inundations will eat away the bank more and more, and thus widen the alluvial flat. The stream will thus be spread out over a wide surface during its floods, and of course its velocity will be lessened. This will cause a deposition of suspended matter to take place, whereby the meadows will increase in height. Meanwhile the stream will continue to wear its channel deeper, the supposition being that the drainage is still going on. At length the channel will become so deep, and the meadows so high, that even in freshets the waters will not spread over the meadows. They have now become a permanent terrace, bounded by the river on one side, and by a steep escarpment on the other, that leads to the higher terrace.

As the river no longer rises over the meadows in time of floods, the process already described is repeated, and a third terrace is the result; and so a fourth, a fifth, &c., may be formed, if the river sink deep enough and time be given.

21. A modification of the above process may in some cases be witnessed. The

stream sometimes wears away one of its banks to such a depth, that the channel gradually changes towards that side, while the back water produced on the other side causes a deposit, which is increased by freshets, and although its upper surface becomes nearly level, it yet forms a terrace which properly deserves the name of a glacis terrace. After this process of lateral change has gone on for some years, it not unfrequently happens that the river suddenly deserts its old bed, in consequence of having found a new channel. Successive floods fill up the deserted bed, sometimes so as to make a level-topped terrace: but in other cases, it is only partially filled, and exhibits, at least for centuries, evidence of the former presence of the stream. Such are the old river beds in Deerfield meadows, shown on Plate IV. In the short one directly west of the village, the whole process has been gone through since my boyish days, and I have watched its progress with interest from year to year.

22. It is I apprehend, by modifications of this process, that that variety of glacis terrace exhibited on section No. 31 was produced. Sometimes they may also have resulted from the accumulation of sand and loam on one shore, by the lateral influence of a strong current. I am not prepared to say exactly how that variety of glacis terrace, found in the Alps and other mountainous districts, consisting of rather rapid slopes of the whole alluvial formation of a valley towards the stream, was produced. It may, however, have resulted from the sliding down of detrital matter towards the stream from the steep adjoining hill-sides, during the semi-fluid condition of the surface in the spring, or after powerful rains.

23. On the supposition above made, that during the drainage of a valley like the Connecticut, it assumed the condition of a chain of small lakes, we can see how it is, that around the gorges or straits between them, the terraces should be higher than in the wider parts of the valleys. For the contraction of the stream at the gorges, would check the current there, and thus cause more of the suspended matter to be deposited. Very probably it might so fill up the gorges, that, as the continent rose, it would require a great length of time to wear them down to their present depth.

24. We see then that the various forms of river terraces, whether called delta, lateral, gorge, or glacis terraces, may be formed by the simple drainage of the country, as the surface emerges from the ocean. Nor need we, as has generally been thought necessary, suppose that there were pauses in the vertical movement. That such pauses may have occurred I admit, and that in this way some terraces and beaches may have been produced; but to form the river terraces we need not call in their aid.

25. I now proceed a step further, and will state certain facts, which prove that river terraces in general could not have been produced by pauses in the vertical movement of the land.

1. If thus produced, they ought to be the same in number and height in the different basins of the same river, and on different rivers not very remote from one another. For, even though we might admit some small difference in their height if thus produced, their number must correspond, since the water would sink equally in the different basins. But a reference to the sections attached to this

paper, and to the tabular heights of the terraces, will show that the facts are widely diverse from this supposition. Along the Connecticut, indeed, the most usual number is three or four: but on some of its tributaries they rise as high as eight or ten. Which number, in such a case, shall we assume as indicating the pauses in the vertical movement? If the smallest, then how are we to explain the excess? If the larger number, then why did not the waters leave traces of their influence alike numerous wherever they acted an equal length of time.

2. On this supposition, the terraces ought to agree essentially, at least in height, on opposite sides of a valley. Circumstances might, indeed, erase all traces of their action in particular spots, but such great irregularity as exists in this respect, cannot be thus explained. Terraces thus formed would leave evidence of their existence, as the Parallel Roads of Lochaber have done, on the steep flanks of the Scottish Highlands; which I am willing to admit were produced by successive uplifts of the land, or subsidence of the waters.

3. The difference in the number and height of the terraces upon the principal stream and its tributaries at their debouchure, affords decisive proof that said terraces were not the result of the paroxysmal elevation of the land. Here we find two sets of terraces formed in the same bank of detritus; one set, usually the smallest in number, on the main river, and the other set, formed by the erosion of the tributary through the first. Of these, the maps and sections appended, afford numerous examples. Thus, at the mouth of the Ashuelot river, in Hinsdale (No. 25), we have five terraces on that river, and three, or perhaps four, on the Connecticut. Just below Bellows falls, we find at the mouth of Saxon's river (No. 30), as many as six terraces, while on the Connecticut, a little to the south, in Westminster (No. 29), are only four. In the north part of Vernon (No. 26), are only four on the Connecticut: but on West river, in Brattleborough, perhaps two miles north, we find nine, and on Whetstone brook, ten (No. 28 and Plate III). Moreover, the latter rise no higher than, if as high, as the former. And since both sets are found in the same bank of sand and gravel, it is certain, that if one set were produced by pauses in the retiring waters, the other set could not be: since no possible reason can be assigned, why in the same bank of materials the terraces on one stream should be twice as numerous as those on the other, if produced by pauses in the retiring waters.

26. These facts, especially the last named, afford almost equally strong evidence that river terraces could not have been produced by the sudden bursting of barriers. In the valley of the Connecticut, if such barriers existed, they must have consisted of sand and gravel, choking up the gorges, and not of solid rock, since the traces of drift agency occur so low down at those gorges. That detrital barriers may have existed to some extent, perhaps with the addition of ice, I will admit. But that they had little to do with the formation of terraces, is clear from the above facts; since if suddenly lowered they could not have produced a different number of terraces on the principal stream from those on the tributaries, nor such irregularity as we find in their height and number upon opposite sides of the river, although they might have formed more in one basin than in another.

27. In a former paragraph (11) I have given an intimation of the views which

I have been finally led to adopt, as to the formation of moraine terraces. I regard them as mainly deposits by water, urged in currents through the sinuosities of stranded icebergs. The subsequent melting of the ice, as the surface was drained, would leave it with those convolutions and anfractuosities, so like those upon the human brain. That powerful currents occur among stranded icebergs, we have the testimony of Sir James Ross, who "mentions that the streams of tide were so strong amid grounded icebergs at the south Shetlands, that eddies were produced behind them; so that as far as such streams were concerned, they acted as rocks. Navigators have observed icebergs sufficiently long aground in some situations, that even mineral matter might be accumulated at their bases in favorable situations, while tide currents may run so strongly between others, that channels might be cut by them in bottoms sufficiently yielding, and at depths where the friction of these streams would be experienced. Much modification of sea bottoms might thus be produced by grounded icebergs, &c." (*De la Beche's Geological Observer*, p. 254.)

Such masses of ice are liable, at some seasons of the year, to be crowded forward by other ice, so as to plough furrows in the loose materials, and grind down and striate the rocks in place. Sir Charles Lyell quotes an interesting case, in which mounds analogous to moraine terraces were produced "by the pressure of ice." From the account given by Messrs. Dease and Simpson, of their recent Arctic discoveries, we learn, that in lat. 71° N. long. 156° W., they found "a long low spit, named Point Barrow, composed of gravel and coarse sand, in some parts more than a quarter of a mile broad, which the pressure of the ice had forced up into numerous mounds, that viewed from a distance assumed the appearance of large boulder rocks." (*Lyell's Principles of Geology*, p. 230.)

Such statements, especially the last one, give great plausibility to the theory which I have adopted. It is still further strengthened by the fact, that these moraine terraces occur in spots, which must have been the shores of the ocean, or of estuaries, or of lakes, as the waters were retiring; and, therefore, just the spots where icebergs might be expected to get stranded. They are found, also, as a part of the earlier terraces, not long posterior to the drift, while as yet we may presume the temperature was low enough to allow of the long continued presence of ice along the shores.

But though the preceding views may explain the rounded hillocks and intervening depressions of the moraine terraces, something more seems necessary to account for those remarkable ridges of sand and gravel, usually more or less serpentine, that accompany the mounds in some instances, as at Andover. Now in high latitudes the shores are found sometimes to be composed of layers of sand, gravel, and ice, more or less interstratified; that is, the waters throw up gravel and sand upon and among the ice along the shores. As the ice melts away, we might expect ridges of sand and gravel to remain, being crooked or straight as the shores were. It seems to me that this may have been the origin of such ridges of this kind, as have fallen under my observation, the most striking of which are in Andover, Mass.

I have seldom been so much perplexed to find a name for any natural object as for these moraine terraces. Without some new term they cannot be referred to, without much circumlocution. In my Reports on the Geology of Massachusetts, and in a paper on the subject, in the first volume of the Transactions of the American Association of Geologists and Naturalists, I called them, in the first work, *Diluvial Elevations and Depressions;* and in the other, *Iceberg Moraines,* but these terms are quite unsatisfactory; and after having ascertained that these objects are connected with, and frequently form a part of, one of the higher terraces, I have named them, merely on the ground of some external resemblances, *Moraine Terraces,* which I shall use only until I can find a better term.

I have not gone into minute details respecting these curious forms of modified drift, because they are given in the works above referred to, and in my Elementary Geology. By recurring to those details, the reasons will be obvious why we cannot explain the phenomena by water alone, nor by ice alone. Their conjoint agency, it seems to me, may do it.

I ought to add, perhaps, that I have sometimes seen appearances in the bottom of an old river bed, somewhat analogous to the moraine terraces. As such a bed was being filled, when beneath the waters, with sand and gravel, spots were left here and there, several feet deep, which were not filled for want of materials, or from the direction of the currents. But I cannot believe that depressions so deep and numerous, and separated by ridges so narrow and steep, as some of the moraine terraces exhibit, could be the result of mere currents of water.

28. As to lake terraces I can say but little with much confidence. I cannot doubt, however, that those around most of the small and narrow lakes, such as those of New York and of Switzerland, fall into the same category as the river terraces, while yet the water was high enough to form chains of small lakes. For the drainage of the modern lakes appears to have been going on in the same manner as the estuaries, that become ultimately converted into rivers. Such seemed to me to be the case with Lake Zurich and Leman; and such, I am told, is the fact in respect to the smaller lakes of New York, so that they do not seem to require us to call in any new principle besides those already applied to river terraces.

As to the larger lakes, I have had no opportunity to examine any of them, save the one called the Ridge Road, of Ontario, which has more the appearance of a beach, or rather a submarine ridge, than a terrace. Professor Agassiz describes those around Lake Superior, as varying very much in number in different places, "six, and rising from the height of a few feet, to several hundred. He says, that ten, even fifteen such terraces may be distinguished on one spot, forming, as it were, the steps of a gigantic amphitheatre." He distinguishes between these lake terraces and the delta terraces, at the mouths of rivers, which he also describes: and he states also, that the lake terraces "present everywhere undoubted evidence, that they were formed by the waters of the lake itself." He supposes that the shores of the lake have experienced vertical movements; first a depression and then a rise, and that "these various terraces mark the successive paroxysms or periods of re-elevation" (p. 104, *Lake Superior*, &c.). He supposes the terraces to have been formed, and of course the last elevation of the land to have taken place, subsequent to the drift period: for he remarks, "It is clear that the formation of the terraces was subsequent. They overlie the grooved and rounded rocks" (p. 103). Yet, if I understand Prof. Agassiz, he ascribes these vertical movements to the injections of trap veins, so common along the shores. "This process of intersection, these successive injections of different materials (in the veins), have evidently modified at various epochs, the relative level of the lake and land, and probably also occasioned the modification which we notice in the deposition of the shore drift, and the successive amphitheatric terraces, which border, at various heights, its shores" (p. 424).

Now, with so little personal knowledge of lake terraces, it may be presumption in me to call in question any of these conclusions. But a few suggestions may not be improper.

Were Lake Superior, itself an ocean, alone concerned, we might have less difficulty in admitting these views, and in supposing that its terraces mark the pauses in the uplifts of its shores. But I apprehend that scarcely a lake exists in our country that does not show distinct terraces, nay even ponds, covering only a few hundred acres, exhibit them distinctly. I know of some such in New England. Now surely we cannot suppose that the shores of each of these smaller lakes and ponds have undergone any *such* elevation since the drift period: I mean to say that they have been elevated only as a part of the continent, and not by a local movement, as must have been the case if the shores are raised above the waters. So that if we could dispose of the Lake Superior terraces in this manner, those of other lakes would still remain unaccounted for. Moreover, as to the cause assigned for this rise of the shores, viz., trap dykes, I do not see how these could have been concerned in the last movement which produced the terraces. For the surface of these dykes is smoothed and striated by the drift agency, which shows them to have been injected long before the drift period, whereas the terraces have all been formed since.

I agree with Professor Agassiz in the opinion, that subsequent to the drift period, our continent has been beneath the ocean, and has subsequently risen. But it seems to me that it came up bodily, or as a whole; at any rate, I have not met with any evidence of local elevations. Supposing it was the ocean that spread over all our continent; as that was gradually raised, the waters might have left evidence of their recession, and of their successive pauses (if any prefer that view), in the form of terraces around all our lakes. I think that a rise of the land, unattended by paroxysms and pauses, may more easily show us why the number and height of terraces differ so much on different bodies of water, and that the unequal number which we find on the same lake, or river, may thus be more satisfactorily explained. For if there were such pauses to any great extent, I do not see why the number and height of the principal terraces should not correspond everywhere, even though we leave out of the account the irregularities of the minor terraces. Yet I admit the occurrence, occasionally, of such pauses. I could not, for instance, look on the Parallel Roads of Lochaber, in Scotland, without feeling that probably they mark paroxysmal movements of the waters. But it cannot be denied that men, even geologists, are too prone to resort to paroxysms and irregular action to explain phenomena; and I look upon the labors of Sir

Charles Lyell as of great value in this respect; although I might suppose that his views of uniformity are sometimes carried too far. The rule which I theoretically adopt, is, to admit paroxysms wherever there is evidence of their action, but not introduce them for the sake of eking out an hypothesis. For we ought ever to remember, that in nature, uniformity is the law, and paroxysm the exception.

I will only add, that if it be admitted that the facts adduced in this paper prove the presence, since the drift period, of the ocean at the height of 2000, or even 1200 feet, above its present level, then it must have extended over nearly all of our western country; and unless Professor Agassiz says that he had his eye upon this matter along the shores of Superior, I cannot avoid entertaining the expectation, that what I call beaches will yet be found at a much higher level there, than the 331 feet terrace, measured by Mr. Logan.

29. The period when the formation of beaches and terraces commenced was immensely remote. The proof of this position will more appropriately be given in my paper on Erosions. I trust there to prove, that the whole of the gulf between Niagara falls and lake Ontario has been worn out by the river since the drift period: as well as the gorge between Portage and Mount Morris, on Genesee river, and several analogous gulfs in other parts of the country. I expect also to show, that some of the old river beds, pointed out in this paper, were beds through which rivers ran before the continent went down beneath the ocean the last time. Such facts, if admitted, give an antiquity to the drift period little imagined heretofore; and may excite astonishment that the drift striæ should be so fresh and distinct

30. The facts and reasonings that have been presented, exhibit to us one simple, grand, and uninterrupted series of operations, by which all the changes in the superficial deposits since the drift, have been produced. We see the continent slowly emerging from the ocean; rivers commencing their wearing action on the islands; waves and oceanic currents meeting the detritus of rivers and comminuting, sorting, and arranging the same, in the shape of beaches and terraces, while it may be that icebergs and glaciers modified the whole. It may be, too, that paroxysmal movements occasionally accelerated, retarded, or modified, the effects. The period over which the uninterrupted operation of these agencies can be traced, may be regarded as the alluvial, and we can refer them back at least to the tertiary epoch.

31. It is obvious, however, that it is only the present form and admixture of the loose materials on the earth's surface, that can be referred to the post-tertiary period. We infer that their present arrangement is post-tertiary because they lie in some places above the tertiary. In others, however, they lie upon older rocks—sometimes upon the oldest known. And in such case, though the presumption is strong that their present disposition and mixture are not older than the tertiary, yet the time of the abrasion, comminution, and rounding of the fragments, may have been vastly earlier—as early, indeed, as the consolidation of the rocks on which they now repose. They may have formed other terraces and beaches on other continents; and it is quite possible that in some cases those old terraces and beaches may still remain, not having been remodelled by the last vertical movement of the continents. In an important sense, therefore, the alluvial period may have been

contemporaneous with all other periods; or rather, each period had its alluvium, and sometimes the same alluvium may have belonged to successive periods. These facts give a peculiarity to the alluvial formation possessed by no other.

32. It appears that the time since man came upon the globe, has been only a small part of the alluvial period. For we find none of his remains, nor works, except in the superficial portions of the terraces. The lowest of these, save alluvial meadows, are often the seat of his most ancient works—his habitations and forts. The remotest epochs of history rarely, if ever, reach back to the time when the most recent terrace, save overflowed meadows, was formed. Even if it be admitted, which yet requires proof, that his remains are found with those of extinct animals, this by no means throws back his origin, as has been supposed, to what is usually understood by the drift period, for many races of animals have disappeared since alluvial agencies have been at work.

33. A large proportion of those superficial deposits in high latitudes, that have been usually included in drift, appear from the views that have been presented, to have been the work of agencies greatly posterior; analogous probably to those that produced the lowest and coarsest drift, but still greatly modified. These agencies have taken the drift and worked it over, and though the same kind of drift as the oldest is still produced in some parts of the globe, yet it is undesirable to confound modified with unmodified drift, since it embarrasses our reasonings as to the origin of that coarse deposit which usually lies beneath all others that are unconsolidated, and which all geologists agree in regarding as drift. The superimposed beds of gravel, sand and clay, demand only water to explain their origin; whereas all geologists at this day would agree that the coarse drift must have been the result, in part at least, of glacial action. Besides to blend drift proper and modified drift is almost as much of an anachronism as to regard the conglomerates of the triassic or carboniferous period, as contemporaneous with the fragments of which they are composed.

34. But after all, the idea so long and generally maintained, that the drift agency operated for a certain length of time after the tertiary epoch, and then ceased, and was succeeded by alluvial action, which did not operate during the drift period, I find myself compelled to abandon. For I find evidence that both these agencies have been in parallel operation from the close of the tertiary epoch, to say nothing of earlier periods. They have varied only in the amount of their action. During the earlier part of the period, drift agency largely predominated, as the alluvial agency has since done. Hence the attempt to fix upon a certain definite time when drift agency ceased and alluvial agency commenced, has so signally failed, and scarcely no two geologists have drawn the line in the same place. But I shall recur to this point again after laying down a few more positions.

35. It appears that the organic remains which have been referred to the drift, do, in fact, belong to modified drift, and generally to a very late stage of the alluvial period. The marine remains are the oldest, such as are found on the shores of Lake Champlain, and on the banks of the St. Lawrence, at Montreal; on Long Island, at Brooklyn; at Portsmouth, New Hampshire, and at Portland and other

places in Maine, only some four or five hundred feet above the present ocean; and they occur in clay or gravel that has been thoroughly rounded. These remains (along our coast) belong altogether, I believe, to existing species, and the molluscs even yet retain the epidermis. They must, therefore, have been deposited at a period vastly posterior to the drift. The *Delphinus Vermontanus*, described by Professor Thompson, from the clays near Lake Champlain, was found only one hundred and fifty feet above the present sea level, and hence we should not think it strange that he found it difficult to distinguish it from an existing species.

Still more recent are the remains of extinct land animals, which have often in a general way been referred to the drift. I mean the mastodon, elephant, horse, &c., for they occur most usually in peat and marl swamps, and these may have been quite recent. Such is the case at Newburg and Geneseo, New York, and at the summit-level of the Burlington and Rutland railroad in Mt. Holly, Vermont.

In Wales, marine shells were found nearly 1400 feet above the sea, in what, though called drift, was most probably modified drift, which I saw at even a greater elevation in that country.

36. So far as this continent is concerned, I think we may as yet safely say, that there is no evidence of the existence of life in the seas that covered it during the period of unmodified drift; and, indeed, we might say the same of a considerable part of modified drift and alluvium. I mean that the lowest drift and most of the terraces have not furnished any example of fossil animal or plant. And when we find such proof of glacial agency, especially in the oceans, during those periods, we do not wonder that life was mostly absent. Sir Charles Lyell has also assigned some other reasons for this paucity of organic remains, in the pleistocene deposits, which are probable. (*Manual of El. Geol.*, p. 136.)

37. With such views of the climate in regions now temperate, we should expect, that as mountains emerged from the ocean, glaciers would be formed upon their crests and slopes. Those descending towards the ocean, would produce striæ upon the rocks, radiating from the highest points, or directed outwardly from the axes of ridges, and more or less obliterating the traces of the drift agency, where, as in our country, the striæ that have resulted from it, run nearly in a north and south direction over the whole continent. As far down the mountains as the glaciers extended, they would obliterate, also, the beaches and terraces that may have been formed by the retiring waters.

In Wales, as I have already stated, the marks of ancient glaciers seem to me most manifest, and they have erased most of the marks of the former presence of the ocean, though they do not prove that the country was not all once beneath the ocean, but only that the glaciers have since occupied its higher parts and so changed the surface that the proofs of oceanic agency are less obvious. And, moreover, the ragged aspect of the highest peaks, makes it probable that they never were rounded, as nearly all the mountains in our country are, by drift agency.

In Switzerland, I think we can easily find proofs of the action of water from 2000 to 3000 feet high: but all the regions more elevated, show marks of ancient or of existing glaciers. And here, also, the lofty summits have not been truncated by glaciers or drift agency.

In America the evidence of ancient glaciers is less striking. I think, however, that I have discovered them upon some of our mountains, and the subject is of such importance, that I have devoted a separate paper accompanying this, to the details of my observations relative to them.

38. Countries corresponding in their modified drift, or rather their beaches and terraces, may be regarded as having occupied about the same length of time in their last emergence from the ocean, and consequently are of nearly the same subaerial age. Perhaps I ought to add, that this principle would require that there should be a general correspondence, also, in the outlines of the surface, and the nature of the rocks, as well as in the rapidity with which the waters withdrew. For, since in my view the terraces and beaches were produced by the drainage of the country, the length of time occupied would depend very much upon the contour of the surface, and the character of the rocks. All these circumstances being the same, I do not see why the time occupied by the drainage should not be the same. In the northern parts of the United States, in Scotland, and Scandinavia, so far as my observation in the two first countries, and information concerning the third, extend, all the above circumstances are essentially alike, and hence I should regard their postdiluvial ages as nearly equal. The facts mentioned elsewhere as to the terraces of the river Jordan, would lead to the conclusion that Palestine and Syria, regarded by so many writers as having experienced great vertical movements, have remained essentially unchanged nearly as long as New England; and the facts respecting the Arabah and its Wadys, south of the Dead Sea, confirm this opinion. This point I have discussed more fully in the first volume of the Transactions of the American Association of Geologists and Naturalists.

39. It is a well known question of great interest, whether the drainage of continents, since the drift period, has been effected by the elevation of the land, or the depression of the oceans. The able expositions of the latter hypothesis, by Professor James D. Dana, in the American Journal of Science, incline me to adopt it, at least partially, some of the facts, concerning beaches and terraces, affording a presumption in its favor. It is not very easy to conceive how a broad continent can be lifted up, and permanently sustained, to the average height of nearly a thousand feet. Still more difficult is it to imagine how this can be done so as not to rupture or disturb the superficial deposits upon it. We should expect that in some places the elevation would be much greater than in others, and consequently the lines of level of the beaches and terraces would be changed, and the materials in some places be disturbed, as they are in regions subject to earthquakes. But I have never met with a single example of such disturbance. And the only case I know of, is the one described by Mr. Chambers, in Finmark, where a seesaw movement, of more than two feet in a mile, has been traced over an extent of 40 miles. Such cases may be discovered in our country; but, so far as I can judge, the change of level has been effected here in the most quiet manner, and the surface has risen in every part alike, and its whole contour remains as when

the waters left it. Such a fact corresponds better with the idea of a retiring ocean than of a rising continent. And upon the whole, though I cannot doubt that lateral pressure and internal volcanic force have produced limited vertical movements; I am more and more inclined to believe that the waters have in a great measure withdrawn in the manner suggested by Prevost and Dana.

40. The phenomena of drift, in distinction from terraces and beaches, although an important part of surface geology, I have not dwelt upon in this paper, because they are now generally known, and, so far as North America is concerned, I have published them elsewhere. But some suggestions upon the theory of drift seem important in this place, in order to bring out my views fully upon surface geology. I have endeavored to show that a large proportion of what has been usually regarded as drift, has been the result of subsequent alluvial agencies. There still, however, remains an irregular coarse deposit beneath the modified beaches and terraces, whose origin is a matter of great interest. The subject is narrowed, but not disposed of. There yet remain the great boulders, mixed with rounded fragments and sand and clay, as well as the striated and embossed surfaces, to be explained. And in respect to the agency by which the phenomena have been produced, the following positions, which are most of them essentially those taken by Professor Naumann, appear to me most unquestionably true:—

1. The eroding materials must have been comminuted stone.
2. They must have been borne along under heavy pressure.
3. The moving force must have operated slowly and with prodigious energy.
4. It must have been nearly uniform in direction, yet capable of conforming somewhat to an uneven surface, and of some divergence when meeting with obstacles.
5. The vehicle of the eroding materials cannot have been water alone.
6. It must have been a firm and heavy mass, yet somewhat plastic.
7. The grinding and crushing mass must have been impelled by such a *vis a tergo*, as would urge it over hills of considerable height.
8. A part of the phenomena can be explained only by the presence and agency of water in some places, at least to sort out, arrange, and deposit layers of sand, clay, and gravel, which are sometimes found beneath the large boulders that are scattered over the surface, or sometimes mixed with the finer stratified deposits.

Were this the proper place, I would quote a multitude of facts to sustain these positions. But since to do this would be less original than the other parts of this paper, I will refer only to a single observation, made by me in the White Mountains, in 1851, and which I have described in the 14th volume, 2d Series, of the American Journal of Science, p. 73, to illustrate the fifth of the above positions. On the southwest side of La Fayette Mountain, near the Franconia Notch, I followed the track of a recent summer slide, which had never been explored. The perils which I encountered in this attempt, greater than I have ever met in a mountain excursion, are detailed in the Journal of Science, but will here be omitted, and I shall give only a part of the facts.

I found a path several rods wide ploughed out by an immense mass of coarse drift, some of the boulders being from 10 to 20 feet in diameter. They still lie along the borders of the gulf in ridges that correspond exactly to the lateral moraines of Alpine glaciers, and at the end of the slide we have a terminal moraine. The rock in place is laid bare most of the way, and although considerably smoothed, it is not striated to any extent. I cannot conceive of a fairer opportunity to test this matter than on this spot. The size and quantity of the moving mass of detritus, and the rapidity with which it must have descended on a slope of 10° to 38°, were all favorable to the production of an exact counterpart of drift action, if water only was the transporting agent. But it failed just where we should expect it to fail, viz.: in the formation of striæ and furrows.

Where now, save in glaciers, icebergs, and ice-islands, can we find agencies that meet the conditions of the above principles respecting drift? Glaciers, as every one knows, who has observed their effects in the Alps, do produce phenomena corresponding to those of drift in northern regions, in almost every respect. Nor can we doubt that icebergs and ice-floes, large enough to grate along the bottom of the sea, would do the same, although the proof is more difficult to obtain, because the scene of the operation is beneath the ocean. But such icebergs and floes as I suppose, would, it seems to me, operate almost precisely like glaciers. For I assume that they are so large and thick that they reach and press heavily upon the bottom: such icebergs and icefloes in fact, as northern voyagers have described, whose surface was so large that they travelled for days upon them, or their vessel was frozen into them, without their suspecting that they were in motion, till an observation for latitude and longitude showed them that they were upon a drifting mass. Let such masses be put into motion by currents and winds, ever so slowly, and how powerfully would they scour the rocky bottom, wherever they reached it, especially if their under side were armed with fragments of stone.

To which phase of this glacial agency, then, shall we refer the phenomena of drift? Before attempting to answer this question, I shall make a few remarks upon another point, viz.: whether in such a country as the United States and Canada, we can fix upon the geological period when the drift agency operated? Was it previous to the last submergence of the surface, or during its subsidence, or while it was emerging?

There is one fact that leads to the conclusion that the greater part of this work was done before the continent had emerged very considerably from the waters. In my paper on Erosions, I point out several instances in which the beds of rivers, that existed before the submergence of the continent, apparently became so filled with detritus, while beneath the ocean, that the postdiluvian rivers were forced to leave these old channels and wear out new beds, sometimes through solid rocks. True, this detritus is often made up of materials much comminuted, and formed into terraces, and, therefore, may not have accumulated till the continent had been lifted considerably from the ocean. But since these old beds of rivers often show drift scratches beneath the detritus, they must have been made previous to its accumulation, and, therefore, before the drainage had proceeded very far.

On the other hand, there is a fact that leads the mind to the conclusion that the work of erosion went on for some time after the continent began to emerge. A careful examination of the rounded and striated rocks at different altitudes, will satisfy any one that in the valleys the work is considerably more fresh and less affected by decomposing agencies than on high mountains. The erosions are also deeper in the valleys. Sometimes, as on Holyoke, in Massachusetts, a succession of valleys crossing a mountain ridge, have been excavated, to a considerable depth; but I never saw any such drift valleys on the tops of high mountains. All this looks as if the work at high altitudes was completed first, and continued in the valleys after the emergence of the mountains. Yet, in this country, such anachronism could not have been long continued, for in that case, the emergence of the high mountains would have changed the direction of the abrading force into the valleys, from a north and south direction, and this appears to have been the case only to a limited extent. While only the higher parts of the mountains were above the waters, as islands, they would not very much affect the direction of the force, if it consisted of large icebergs.

Some may imagine that rocks much elevated are more liable to surface disintegration than when in valleys. This may sometimes be true, but I doubt whether, with most rocks the reverse is not the fact. The best example of freshness in rocks rounded and striated at high levels, that I have met with, may be seen on the top of Monadnoc, in New Hampshire, 3000 feet above tide water. Yet apparently it is not as recent there as in the bottoms of some of the valleys.

Upon the whole, I think that we must throw back the drift period with the exception above named, at least as far as the time when this and other countries were sinking beneath the ocean. But did the work take place during that subsidence, or previous to it? My own conviction is, that we have evidence that the work extended into both those periods. If before the time of subsidence, it was accomplished by glaciers on a former continent. If we find evidence, as I think we do, in Wales, in Scotland, in some parts of Switzerland, and in New England, that glaciers existed before the last submergence, the detritus accumulated by them, although modified somewhat by oceanic action, ought to be regarded as a part of the drift deposit. We know, also, that since the emergence of the land, glaciers, in some countries, have been producing genuine drift. It is well known that eminent men have referred the whole of the drift to glaciers, and they seem to me to have proved uncontrovertibly, that the smoothing, rounding, and striating of the rocks in northern regions, have been the result of large heavy bodies of ice, forced along the surface by a *vis a tergo*. Now did the glacier theory apply to other countries as well as to Switzerland, so far as my slight examination of that country enables me to judge, I could not well resist its adoption. But in Great Britain, and especially in this country, there are peculiarities in the drift phenomena, that lead me to hesitate, and inquire whether they are not better explained by the passage over the surface of large icebergs and ice-floes, whose effects scarcely differ from those of glaciers. Some of the reasons for such an opinion are the following:—

1. The occurrence of striæ upon the northern slopes of mountains, even to a

considerable height, is better explained by icebergs than by glaciers. In some instances the grinding body must have been forced upward, above the general surface, which is also striated hundreds if not thousands of feet, as on Mt. Monadnoc and the White Mountains. Now a glacier, descending as a whole in every known instance, is able to force portions of its mass over obstacles a few feet only in height. But here we must suppose one not on a slope, but moving over a level surface for hundreds of miles, to be able to crowd large portions of its mass hundreds of feet over opposing mountains. If we could suppose a huge iceberg, suspended in an ocean rising above the mountains, to impinge against its top, with an immense momentum, it might force a portion of its mass over the top; especially if at the same time the mountain were sinking; though perhaps this descent would be too slow to meet the case.

2. Iceberg action explains better than that of glaciers, that sorting of materials and of laminations, which we sometimes find in the drift. I know it is customary to speak of drift, (I mean the lowest and coarsest variety,) as a mass mingled in perfect confusion. But I have rarely seen a section in it, of very considerable extent, in which I could not discover some marks of the action of water in the parallel arrangement and separation of the materials into finer and coarser. I have often been struck with this evidence of a tumultuous and quiet action in close juxtaposition; and we know that not unfrequently the aqueous action appears to have predominated. But if huge icebergs tore off and accumulated the detritus, we might expect that the currents which bore them onward would, to some extent, separate and arrange the materials, especially where masses of ice were stranded; and that sometimes the icebergs would be absent altogether. Glaciers, however, have no such power, save that the stream which usually issues from them, will cause some alluvial accumulations in the valley below the terminal moraines, but not in the midst of the moraines.

3. The facts concerning the dispersion of boulders can be more satisfactorily explained by icebergs than by glaciers. It appears that the work of scattering these boulders continued till after the time when a large part of the beaches and terraces were formed, for they are scattered over the surface of these sandy deposits. (See Mr. Desor's account of the Drift of the Lake Superior land District, in *Foster and Whitney's Report*, p. 190.) Now glaciers could not have done this; for they would have ploughed a track through the stratified deposits of sand and clay beneath, if they had transported these boulders; and so would such icebergs as I have supposed might have produced the drift below the terraces and beaches. But such icebergs as now traverse the Atlantic might have carried boulders over the beaches and terraces and dropped them from time to time, as we now find them scattered over the western prairies. By the same agency, also, we can explain the intermixture of coarse angular blocks in any of the beach and terrace deposits.

4. The supposition that a glacier once existed on this continent, wide enough to reach from Newfoundland to the Rocky Mountains, is the grand difficulty in the way of the glacier theory. All known glaciers occur in valleys, not many miles

wide, and so did the supposed ancient glaciers, of which traces now exist. But the North American glacier must have extended uninterruptedly almost over hill and valley, for at least 2,000 miles; nor even with that width could it have found higher ground on its borders, unless it were the Rocky Mountains on the west, concerning whose drift phenomena we know but little.

Again, all known glaciers are situated upon slopes, greater or less. Indeed, how could they advance, if not upon slopes? For though expansion by freezing might have some influence in urging them forward, as maintained by authors, yet the facts and reasonings of Prof. Forbes seem to show very conclusively, that gravity is the principal cause of their onward march. At any rate, I know of no example where a glacier does advance upon a level surface—certainly where hills oppose its progress. It is surely, then, a great demand upon our faith, to ask us to believe that the broad North American glacier has crowded southerly 500 or 600 miles, over a highly uneven but not sloping surface, and that simply by expansion. Even should it be proved that we have centres of dispersion in the White Mountains, or the mountains of northern New York, we must still admit a great movement from the north sweeping the whole country, save a few peaks. Nor does it relieve the difficulty to suppose an enormous thickness of the sheet of ice in the arctic regions, from which the great glacier proceeded; for its movement was on the surface of the earth; and this had no greater average height to the north than in the United States.

As to those supposed traces of ancient glaciers, to be described in my paper on that subject, as occurring in New England, the probability is, that they were made earlier than the drift scratches. At any rate, the latter are altogether the most obvious phenomenon, and the principal thing to be accounted for; and it is their characteristics that are reconciled with so much difficulty with the effects of glaciers.

5. I find some difficulty in reconciling to the glacier theory, the diversity of direction taken by the drift agency in different parts of the country. Over the mountains of New England the course was south and southeasterly. But in the valley of Lake Superior, it was nearly southwest. What could have determined different glaciers in directions so diverse, especially as they must have ascended rather than descended, both in New England and to the southwest of Superior, I am unable to conceive. But supposing icebergs to be driven forward by currents in the ocean, and there is no difficulty in accounting for such diversity of direction in the striae and boulders.

Upon the whole, those difficulties seem too formidable to admit of the adoption of the unmodified glacier hypothesis. I lean, therefore, at present, toward that which imputes most of the work on this continent to immense icebergs, icefloes, and shore ice; not because that view is free from difficulties; for I acknowledge them to be many; but they appear less to me now than in the other hypothesis. Perhaps, however, the iceberg hypothesis, as I have stated it, falls but little short of that of the glaciers. For I agree with Professor Agassiz, that to sustain the former, " we must assume an ice period—nothing less than an extensive cap of ice upon both poles." "This," says he, "is the very theory which I advocate; and unless the advocates of that theory go to that length in their premises, I venture to say, without fear of contradiction, that they will find the source of their icebergs fall short of the requisite conditions which they must assume upon due consideration, to account for the whole phenomena as they have really been observed." (*Lake Superior*, p. 406.) I think that could we get access to the floor of the Arctic Ocean, where the icefloes probably occupy more space than the water, that partially bears them up, we should find a work going on very similar to that which produced the drift. On such icebergs and icefloes, for the present I take my stand. But as I look toward the shore, and see my neighbor standing upon a glacier, I can hardly tell the difference between the two foundations; and whenever he will show me that his glacier is advancing southerly over a level surface, as does my iceberg, I will gladly place myself by his side.

As to the origin of that more intense cold which once prevailed over New England and other countries much farther from the pole than at present, I have no hypothesis to offer. But as to the fact it seems to me that the undeniable former great extension and thickness of glaciers in Switzerland, Scandinavia, Wales, and perhaps Scotland, and the absence of organic remains from drift, in general, make it certain. I have sometimes imagined that the upheaval of the bed of the northern ocean, according to De la Beche, or the earthquake waves of the Professors Rogers, sweeping southerly from the same region, might afford an explanation. But such forces would produce only a temporary submersion and icy deposit; whereas the evidence of the long continued presence and action of water and ice, and of the slow emergence of the land from the ocean, evince its permanent submergence.

41. Let me now present a summary of my present views of the origin of that deposit, properly called drift, excluding all which I have described as modified drift.

1. *Glaciers.*—It seems to me that the moraines of glaciers affords a good type of drift, viz.: a confused mixture of abraded materials of almost every size, driven mechanically forward. I cannot see why we should limit the impelling force to water as does the ordinary definition of the term drift.

If these views are correct (and I presume no geologist will dissent), then we have one agency in this work in which all are agreed, and which is still in operation before our eyes. Moreover it has been at work from the earliest times in which we have any evidence of drift action. Certainly it goes back as far as the tertiary period, perhaps further. Before the last submersion of our continents it may have operated long and powerfully; and if the views of some geologists are true, it then accumulated the great body of the drift now before our eyes. And still in northern regions, and even in central Europe, it is adding to the mass daily.

2. *Icebergs.*—Wherever these reach the bottom and are urged forward, it cannot be doubted that they must produce essentially the same effects as glaciers. And for the reasons already given, I must suppose that in some countries—our own for

instance—most of the drift has been thus produced, and most of the erratic blocks thus scattered.

This agency, too, we can trace back to the dawn of the drift period, and it is still in operation on a stupendous scale in arctic and antarctic regions. What we witness of its effects in temperate seas shows only its power in transporting afar blocks of stone which it has torn from the shore.

3. *Mountain Slides produced by Aqueous Agency.*—If any one doubts whether this should be reckoned among the causes of drift, let him visit the case described in this paper, on what I call *Moraine Brook,* in Mount La Fayette, at Franconia, and he will see first, that the materials torn off from the ledges and strewed along for two miles, cannot be distinguished from coarse drift; and secondly, that they are so arranged as not to be distinguishable from the lateral and terminal moraines of a glacier. Why then should they not be regarded as drift?

4. *Waves of Translation, produced by the Paroxysmal Upheaval of Continental Masses, or Earthquake Undulations.*—Whether any certain example of such a movement can be pointed out—unless we admit drift generally to have been thus produced—I exceedingly doubt. But hypothetically we can realize that such waves would tear off fragments of rocks and roll them along and smooth, if they did not groove, the rocks. This action would, indeed, be too short, violent, and irregular, to explain all the features of drift, which were the work of agents acting ages upon ages: yet, from the phenomena occasionally exhibited by earthquake waves along the coast, we may reasonably include this force among those concerned in the production of drift. And in some countries it may have done much of the drift work.

5. Perhaps I ought to add, as a fifth cause of drift, those ice floods that occur almost every winter in the rivers of northern and mountainous countries. Often in these cases, the river is choked with fragments of ice, so that its banks are full. Yet there is water enough to keep it slowly in motion. It differs, in fact, from a glacier, only in being more fluid, so that its motion is more rapid. But it grates powerfully upon the sides and bottom of the stream, and produces miniature moraines. I see not why such detritus should not be regarded as drift as much as the moraines of glaciers or icebergs.

42. According to these views, drift is the result of several agencies that have been in operation upon the earth's surface, certainly since the tertiary period, and in some countries, from a much earlier date. They have varied in intensity at different times, and in different circumstances, and each one has had a predominance at certain times. But all of them are still in action in some parts of the globe, and perhaps with as much power as ever.

43. In like manner, alluvial agencies have had an operation parallel to those producing drift, and as far back, though the present forms of alluvium are chiefly posterior to the tertiary epoch. But perhaps the whole formation is not so.

44. Drift and alluvium ought to be regarded as only varieties of the same formation. And since water has always been present and essential in the operation of the other agencies, the whole formation should take the name of alluvium. Chro-

nologically, we might divide this formation into the following periods; which, however, must not be understood as completely isolated from one another, but only as marking the times when certain phenomena predominated.

1. The Period of unmodified Drift.
2. The Period of Beaches, Osars, Submarine Ridges, and Sea Bottoms.
3. The Period of Terraces.
4. The Historic Period, or the Period of Deltas and Dunes.

Lithologically, Alluvium may be subdivided as follows:

1. Drift unmodified, embracing angular and rounded boulders, gravel, sand, and clay.
2. Modified Drift, embracing the following forms:
    1. Beaches, ancient and modern.
    2. Osars.
    3. Submarine Ridges.
    4. Sea Bottoms and Lake and River Bottoms.
    5. Terraces.
    6. Deltas.
    7. Dunes.
    8. Moraines.

Such views, essentially, have been advanced by previous writers of great ability. Thus, Sir Charles Lyell groups together all the strata above the tertiary, under the name of Post-Pliocene, of which the Recent embraces the deposits coeval with man, and Drift, those anterior to man. We find, also, that the eminent palæontologist and geologist, M. Alcide D'Orbigny, in his *Cours Elementaire de Palæontologie et Geologie,* comprehends in his *Terrains Contemporains, ou Epoque actuelle,* every thing above the tertiary. Still more specifically like my own, are the views expressed by William C. Redfield, Esq., at the meeting of the American Association for the advancement of Science, at Cambridge, in 1850. He remarked, "that the phenomena of the boulders and drift should be attributed to mixed causes, and that the theories which refer these phenomena to the several agencies of glaciers, icebergs, and packed ice, are, in truth, more nearly coincident than is commonly imagined." I understand M. Desor, also, who has had opportunities for examining drift phenomena, not inferior to those of any man living, as inclined to similar views. He supposes that "the surface boulders, like many of those buried in the drift, clay, and sand, have been transported by the floating ice:" and he says that since "glaciers in our days occur chiefly in the valleys of the highest mountain chains, it is difficult to conceive how they could exist and move in a wide and level country like the northern parts of the United States and Canada." (*Foster and Whitney's Report,* p. 215.)

45. Such are the results to which I have been conducted by the facts respecting surface geology which have fallen under my notice. I am aware that these are subjects of great difficulty, and that I am in conflict with the views of eminent geologists on several points; as I am, indeed, with my own opinions, as held several years ago. And yet for a long time I have stood chiefly aloof from the

various hypotheses that have been broached respecting surface geology. But I could not refuse to follow where facts seemed to lead the way. It becomes me, however, to be very modest in urging my conclusions upon others. If they cannot adopt my explications, I hope they will at least find my facts to be of some little service in reaching better conclusions.

Heights of River Terraces and Ancient Beaches above the Rivers to which they are contiguous, and above the Ocean (mean height), as measured by EDWARD HITCHCOCK *in* 1849, 1850, *and* 1851. *Heights in English feet.*

S. B. prefixed indicates that the heights were obtained by the Syphon Barometer; A. B., by the Aneroid Barometer. In all other cases, they were measured by the common levelling instrument. The figures inclosed in parentheses refer to the sections where the terraces and beaches are represented.

	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10	10	11	11
	Above the river.	Above the ocean.	River.	Ocean.	River.	Ocean.	River.	Ocean.	River.	Ocean.	River.	Ocean.	River.	Ocean.	River.	Ocean.	River.	Ocean.	River.	Ocean.	River.	Ocean.
ON THE SHORE OF THE OCEAN.																						
Lyme, Ct., beach (33) S. B.		14																				
Lyme, Ct., beach (33) A. B.		19																				
East Lyme beach (ancient) (33) S. B.		31																				
East Lyme beach (ancient) (33) A. B.		31																				
East Lyme River terrace (highest) (33) S. B.		119																				
East Lyme River terrace (highest) (33) A. B.		102																				
Flanders (New London) beach (33) S. B.		34																				
Flanders (New London) beach (33) A. B.		39																				
ON CONNECTICUT RIVER.																						
Glastenbury, Ct., terraces (7)	20	36	50	66	174	190																
East Hartford terraces (6)	15	36	40	61																		
Wethersfield terraces (8)			57	76																		
Windsor terraces (9)	19	44	49	74	114	139																
East Windsor terraces (5)	22	47	71	96																		
Longmeadow terraces (4)	24	88	136	200																		
Springfield terraces (3)	21	85	36	100	67	131	136 (?)	200														
Springfield Willimansett do. (2)	40	114	194	268																		
South Hadley terraces (1) S. B.	182	287																				
South Hadley terraces (1) A. B.	193	298																				
Northampton terraces (10)	57	162	97	202																		
Hatfield terraces (10)	22	130	28	136	38	146	53	161														
Hadley terraces (10)	27	135	54	162	181	289																
Whately terraces (12)	32	146	46	160	92	206																
Turner's Falls, Montague terraces (18)	18	212	97	291	175	369																
Northfield (south part) terraces (19)	24	224	90	290	131	331																
Northfield (north part) terraces (20)	24	224	87	287	112	312																
Hinsdale, on south side of Ashuelot River, near its mouth (21)	7	215	14	222	36	244	66	274	150	358												
Hinsdale, south side of the Ashuelot (21)	7	215	14	222	37	245	73	281	162	370												
Hinsdale, on Connecticut River, beyond the village (21)	21	231	96	306	159	369	Unk.															
Vernon (22)	21	234	38	251	107	320	237	450														
Brattleborough, mouth of Whetstone Brook, towards the S. W. (23)	20	234	74	288	86	300	118	332	165	379	200	414										
Brattleborough, mouth of West River, across the village S. W. (24)	25	239	47	261	58	272	70	284	80	294	100	314	114	328	132	246	221	435				
Brattleborough, on Whetstone Brook (24)	8		14		23		44		56		83		102		138		153		Unk.			
Westminster (25)	24	255	94	325	139	370	171	402														
Langdon, mouth of Cold River, south side (24)	16	251	94	329	243	478																
Westminster, near Bellows Falls, opposite Cold River (24)	26	261	34	269	38	273	83	318	117	352	138	373	161	396	226	461						
Bellows Falls, above the falls (26)	41	327	122	408																		
Pelham, ancient beaches (10) S. B.	583	688	921	1029	1049	1151																
Pelham, ancient beaches (10) A. B.	567	672																				
Shutesbury, ancient beaches	1053	1167	1103 (?)	1217 (?)																		
Amherst, near Mount Pleasant, ancient beach (10)	329	437																				
Whately, north part, beach S. B.	704	809																				
Whately, north part, beach A. B.	690	795																				
Conway (Shirkshire) beach S. B.	942	1047																				
Conway (Shirkshire) beach A. B.	928	1033																				
Ashfield, N. E. part, beach, A. B.	976	1031	1216	1321																		
Heath beach (30)	1438	1561																				

Heights of River Terraces and Ancient Beaches—Continued.

	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10	10	11	11
	Above the river.	Above the ocean.	River.	Ocean.	River.	Ocean.	River.	Ocean.	River.	Ocean.	River.	Ocean.	River.	Ocean.	River.	Ocean.	River.	Ocean.	River.	Ocean.	River.	Ocean.
ON AGAWAM RIVER.																						
Westfield terraces (17)	11	136	25	150	83	208	114	239	134 (?)	259 (?)												
ON FORT RIVER.																						
Amherst to Pelham, south side of the river, terraces (11)[1]	7	172	14	179	31	196	80	185	130	235	180	285	235	340	268	373	323	428				
Amherst to Pelham, north side of Fort River, delta terraces (11)[1] { S. B. A. B. }	7	172	14	179	131	236	155	260	176	281	210	305	248	353	318	423	376	481	387	492	429	534
ON DEERFIELD RIVER.																						
Deerfield, Foot's Ferry, south side of the river, terraces (27)	15	155	24	164	Unk.		Unk.		178	318												
Same place, north side of river (27)	15	155	29	169	36	176	94	234	103	243												
Deerfield, south end of village, towards mountain, terraces (14)	10	140	38	168	122	250	196	326	S. B. A. B. 235	S. B. A. B. 315												
Deerfield, Carter's land terraces (14)	20	146	64	190	183	309																
Deerfield, Pettee's plain terraces (15)	108	232	159	283																		
Deerfield to top of Pine Hill, terraces (16)	20	143	57	180	95	218																
Buckland, south side of river, terraces (29)	11		21		32		45		75													
Deerfield Mountain, beach (14)	407	537																				
TERRACES AND BEACHES IN OTHER PARTS OF NEW ENGLAND.																						
Peru, Mass., terraces on a brook (32)	46	1813	65	1832	84	1851																
Peru, Mass., ancient beach (32)		2022																				
Washington, Mass., ancient beach summit level of Western Railroad (31)	46	1590																				
Bath, N. H., terrace near the mouth of Ammonoosuck River, A. B.	523	934	[2]																			
Notch House, Franconia beach { S. B.	1524	1930	[2]																			
Notch House, Franconia beach { A. B.	1463	1869	[2]																			
Notch House, beaches west on the road, A. B.	2043	2449	2259	2665	[2]																	
Osar near Fabyan's, White Mts., A. B.	1131	1537	[2]																			
Notch of White Mts., Gibbs's Hotel, beach { S. B.	1667	2073	[2]																			
Notch of White Mts., Gibbs's Hotel, beach { A. B.	1557	1963	[2]																			
ON HUDSON RIVER.																						
Sandy terrace, or beach between Albany and Schenectady (34), A. B.	335																					
Greenbush, opposite Albany, terraces (34), A. B.	64		135		179																	
Beaches on the Western Railroad, east of Hudson River (34), A. B.	330		554		642		890		1111		1378			1590								
ON GENESEE RIVER.																						
Highest terrace east of portage towards Nunda, A. B.	235	1410																				
Mount Morris terraces, west of the river (35), A. B.	73	616	116	659	229	772	348	891														
ON THE RHINE, GERMANY.																						
Near Basle, 3d terrace, A. B.					228	983																
Near Rhinefelder (36), A. B.			102	1022	200	1120	306	1226														
Between Mumpy and Brugg, beaches, A. B.	696	1669	941	1915																		

[1] The numbers in these lists represent the mean height of a series of delta terraces on both sides of Fort river, above that river in Amherst, and above the ocean on the south side. They were obtained by levelling on the north side by both kinds of barometers, except the two lowest. It would, perhaps, have been better to have given the heights above Connecticut river, as is done in the section No. 11.

[2] Above Connecticut river and the ocean.

Heights of River Terraces and Ancient Beaches—Continued.

	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10	10	11	11
	Above the river.	Above the ocean.	River.	Ocean.	River.	Ocean.	River.	Ocean.	River.	Ocean.	River.	Ocean.	River.	Ocean.	River.	Ocean.	River.	Ocean.	River.	Ocean.	River.	Ocean.
ON LAKE ZURICH.																						
From Horgen S. W., terraces and beach (37), A. B.	263	1605	392	1734	843	2185 [1]																
FROM LUCERNE TO LAKE LEMAN, THROUGH BERN.																						
On the Reuss, between Lucerne and Bern, terraces or beaches, A. B.	267	1647	325	1705	894	2274 [2]																
Between Bern and Leman, terrace, the highest on the road, A. B. .	981	2641 [3]																				
ON THE ARVE.																						
Highest terrace between the Loire and Lake Leman, A. B. . .	137	1367 [4]																				
Near Bonneville, terraces, A. B. .	314	1544	372	1603 [4]																		
Near Sallenches, A. B.	581	1811 [1]																				
Terrace highest at Argentiere, opposite the glacier, A. B. . .	675	4100 [5]																				
Terrace at Le Tour hamlet, A. B. .	926	4351 [5]																				
Terrace on the Eau Noire (highest), A. B.	793	4218 [5]																				
Terrace at St. Maurice on the Rhone, A. B.	250	1480 [6]																				
IN WALES, GREAT BRITAIN.																						
Beach east of Cader-Idris, between Dolgelly and Machynlleth, A. B.		969																				
Highest beach (?) on Snowdon, A. B.		2547																				

[1] Above the lake and the ocean.
[2] Above Lake Lucerne and the ocean.
[3] Above Bern and the ocean.
[4] Above the lake and the ocean.
[5] Above Chamouny and the ocean.
[6] Above Leman and the ocean.

ILLUSTRATIONS OF SURFACE GEOLOGY.

PART II.

ON THE EROSIONS OF THE EARTH'S SURFACE,

ESPECIALLY BY RIVERS.

ON THE EROSIONS OF THE EARTH'S SURFACE.

General Remarks.

The vast amount of denudation which the earth's surface has experienced is shown by the following facts:—

1. The great amount of boulders, gravel, sand, clay, and loam, that is spread over the solid rocks.

That these materials once constituted a part of the solid strata, it would seem, cannot be doubted by any one who has observed natural operations at all. For he must have seen the process of abrasion and comminution going on everywhere. Let him go to the shores of any river, and he will see the work in progress. Where the stream is rapid, the materials at its bottom and along its shores will be coarse and not thoroughly rounded; where less violent in its movement, well-worn pebbles will be seen mingled with coarse sand; that is, such materials as that amount of current would urge along. Fine sand, clay, and loam, will appear where the stream is very slow; because such a current can separate and sweep along only the minute fragments of which such deposits are composed. But in all these cases the fragments, if examined, will be found to be portions of the rocks over which the stream passes; and, moreover, we find in many places that the river, sometimes in the form of ice, has power to break off and grind down portions of the rocks.

Now these detrital materials are spread over perhaps nine-tenths of the surface, even in mountainous regions, save in some very precipitous and elevated parts. Their thickness, also, often amounts to hundreds of feet. In short, the loose materials spread over four-fifths of the surface, amount to a thick rock formation; and all accumulated by the slow processes of erosion now going on before our eyes. How vast the period requisite to accomplish the work!

2. By the deep troughs worn out of the loose materials by rivers. After the detritus has been deposited the stream sinks by wearing away a portion of the mass. This process has sometimes gone on to a depth of 100 or 200 feet; and though this is a small erosion in comparison with that already named, it deserves notice in this connection.

3. Nearly all the fossiliferous rocks are composed of materials abraded from previously existing rocks, and subsequently consolidated. The former, it is well known, are several miles thick in all the countries where they have been measured.

They thus prove an amount of erosion previous to the alluvial period, immensely greater than during the deposition of drift and alluvium.

According to the views of many eminent geologists, I might consider nearly all the crystalline stratified rocks among those originally detrital. But as this is debatable ground, I leave them out, although all geologists, I believe, admit that there are metamorphic rocks of considerable thickness having such an origin.

4. By the rounded, smoothed, and striated appearance of most of our hills and mountains in the northern portions of continents. These phenomena indicate an erosive agency that has operated long and powerfully, especially on the northern slopes of mountains, to wear them down. To this force we might add that of glaciers, which produce similar effects; and as all know, the two agencies are by some regarded as identical.

5. The marks of erosions in gorges and on the steep sides of valleys, teach the same lesson. These, I apprehend, are more common than has been supposed; and it is the chief object of this paper to describe and elucidate them. It is an agency distinct from that producing drift, being referable to two sources, viz: rivers, and the ocean acting upon its shores.

6. But perhaps the vast amount of materials that must be supplied to fill up deficiencies in the strata, shows most strikingly the enormous erosions that have taken place. Facts on this subject have not, indeed, been accurately determined in many countries. Yet we know enough to be satisfied that miles in depth have often been taken away; as indeed we might presume must have been the case to supply materials enough for the sedimentary rocks.

All these facts speak the same language, and impress the careful observer with the magnitude of the work of erosion that has been going on from the earliest times. Yet it is only the careful observer who will be impressed with these proofs. Those who take only general views of the rocks and the surface geology, can easily persuade themselves that even the fragmentary rocks were created just as we now find them; and some extend such an hypothesis even to the water-worn pebbles and banks of sand.

Agents of Erosion.

It may be well briefly to enumerate the agents of erosion upon the earth's surface before detailing their effects. They may all be grouped under Atmospheric Air and Water.

1. *Atmospheric Air.*

The four constituents of atmospheric air, oxygen, nitrogen, carbonic acid, and aqueous vapor, are all concerned in the disintegration of the solid rocks, which thus become prepared to be easily acted on by mechanical agencies. Probably oxygen, of which so great a quantity exists in the air, is the most efficient of these agencies, since it has so strong an affinity for most other substances that they will quit their weaker combinations to unite with it. Peroxidation, also, from the same cause, is very common; as in the case of iron and manganese, which are almost universally present in the rocks and the soils.

Nitrogen seems rarely to operate directly upon the rocks: but when converted into nitric acid and ammonia, as it sometimes is, these compounds act with much energy as disintegrating agents.

Perhaps carbonic acid is the most efficient of all agents in the work of erosion. But as it acts chiefly when dissolved in water, I reserve details to the next head. And as to aqueous vapor in the atmosphere, its effects upon the rocks, are not so marked as to be easily described, although doubtless it assists the other constituents of the air in this work.

2. *Water.*

Water acts upon rocks and minerals in three modes, in all of which it is energetic.

1. *As a medium for other decomposing agents.*—These it dissolves, and thus enables them to act upon the rocks. Carbonic acid should stand at the head of this list. It seems to be the only acid, with a few rare exceptions, that exists in the water, which penetrates the rocks, and is able to decompose the silicates of alkalies, the alkaline earths, and protoxides of iron and manganese, at ordinary temperatures. The alkaline carbonates when formed, will decompose solutions of sulphate of lime, or manganese, and the chlorides of calcium and magnesium. Chemical changes thus begun, others will follow in a wider range, all commencing with carbonic acid.

Bicarbonate of lime is another agent widely diffused and productive of extensive changes: such, for instance, as the formation of carbonate of lime, the most abundant of the salts formed in the earth's crust.

The alkaline carbonates are not so generally found in natural waters: but whenever present, as the result of other agents, they effect important changes, such as prepare the way often for erosions by mechanical agencies.

2. *Water, alone, dissolves not a few of the ingredients of rocks.*—The process is much slower generally than when aided by carbonic acid. Nevertheless, pure water will dissolve most of the refractory minerals. Hot water will do it most rapidly: but cold water will do essentially the same, if sufficient time be given. Professors W. B. and R. E. Rogers, in this way dissolved portions of more than thirty rocks and minerals, seemingly the most unyielding, such as feldspar, mica, augite, tourmaline, hornblende, chalcedony, epidote, talc, serpentine, obsidian, lava, greenstone, gneiss, and hornblende slate. It has been probably by means of water chiefly, that the various pseudomorphous processes, which we find to have gone on so extensively in the mineral kingdom, have been accomplished. So great have these changes been, that an able writer (Bischof) says, that "strictly speaking, we do not know with regard to any single mineral, whether it is still in its original condition, or has been more or less altered."

The power of water to penetrate rocks and minerals should be stated in this connection. It not only makes its way into the cracks, fissures, and planes of stratification and lamination, but also through the mass of most rocks. This it does chiefly through capillary attraction: and few rocks or minerals, long immersed in water, escape its all pervading influence.

By means of these agents of chemical change in the atmosphere and the waters (some others of minor influence might be mentioned), we sometimes find the surface of rocks, to the depth of 10 to 15 feet, so thoroughly disintegrated that they can easily be removed by the shovel. This fact may be observed in many rocky regions south of Pennsylvania. The drift agency further north, has swept off the disintegrated mass, so that in New England we get but a feeble idea of its extent.

In this way are the rocks prepared to be acted upon mechanically by erosive agencies. These too are chiefly water in some form.

3. *Water acts mechanically*, first, as breakers or waves, tides, and oceanic currents. These all act conjointly, for the most part, and it cannot be doubted that nearly all the materials of which the sedimentary rocks, consolidated and unconsolidated, are composed, have been accumulated and deposited by this joint action. To be sure, waves, tides, and currents act with the most important results upon loose detritus: but if we suppose a continent gradually rising or falling, every part of its surface will be brought under the denuding agency; and the projecting naked rocks, subject to the ceaseless action, cannot but yield to its force. Indeed, of all the causes operating to wear down the surface, waves, tides, and currents, have been the most efficient, and have done most to give our present continents their form and outline.

Secondly, as fresh water currents, chiefly in the form of rivers, which drain the land. They would have but little effect upon the rocks were not the latter softened and disintegrated. But loosened materials it can sweep off, and distribute, according to its velocity. And when once it has set detritus in motion, that will tear away projecting fragments, which the water alone could not remove. In some mountain slides, such as that described in my paper on Terraces, as produced on Mount La Fayette, by a powerful shower, the work of erosion accomplished by the water and detritus is almost equal to that of glaciers.

Thirdly, the expansive force of water, when freezing, is one of the mightiest of all known agencies for lifting rocks out of their beds. If water finds its way into cracks and cavities in the rock, and then freezes as solid as we know it may do in northern regions, it will exert a power which even gunpowder could not equal. Thus would the fissures be widened, giving an opportunity for a larger quantity of water to freeze in them the subsequent winter, with a still stronger force and wider effect. Thus, in time, are the most solid and deep-seated masses so heaved out of their original beds, that ice floods, or other agencies convert them into boulders and roll them along the surface.

Fourthly, Glaciers, Icebergs, Ice Floes, and Ice Floods, form agents of erosion of tremendous power. In these cases, blocks of stone, gravel, and sand are frequently frozen into the bottom of the ice, so as to act like enormous rasps upon the surface, over which they move with almost irresistible power. I need not go into details on this subject. For any one who has read the works of arctic and antarctic voyagers in latter times, and the histories of Alpine glaciers, must be impressed with the energy of these agencies. And whoever has examined the surface of the northern parts of this continent with a geological eye, cannot doubt that he has before him examples of their former operation. If the glacial phenomena that now exist in the northern part of Greenland, as they are described in the works of Dr. Kane, once existed here, they would satisfactorily explain the drift exhibitions of North America.

The ice floods in mountain torrents, above alluded to, possess a power in the removal of detritus, second only to glaciers and ice floes. These several agencies are indeed very similar. For a glacier seems to be only a river of ice urged forward mainly by the force of gravity, aided slightly, perhaps, by the freezing of water in the crevices. I have sometimes seen a mountain stream in New England, crowded with blocks of ice so wedged together, that I have safely walked over its surface; and yet the mass was slowly in motion, and it closely resembled a glacier, even in its erosive power. That of glaciers we know to be still greater; nor can that of large icebergs, when they plough upon the bottom of the ocean, be less, but in some cases it must be greater.

To these agents of erosion, perhaps, I should add those of heat and gravity, and the action of plants and animals upon the rocks. But the first two are implied in the agencies already named; and the two latter are so limited in their action, as hardly to need description in this place.

Conjoined Results of these Agencies.

They have sometimes acted together and sometimes successively upon the same surface: and sometimes the latest action has obliterated the previous ones. But the final results we can trace in the following phenomena.

1. *In the Character of the present Shores of the Ocean.*

This presents two phases. The first consists of the beaches, bars, hooks, and shoals of loose materials which the breakers, tides, and currents, have worn off, sorted, and deposited. In some places the projecting shores of unconsolidated materials have wasted away over a wide surface, while in others the sand-banks have been extended a great distance. The encroachments upon the solid rocks, that project into the waters, is less obvious during the life of man; but in many places it is constant, and, therefore, in the course of ages must be very great.

The other phase of oceanic action, is exhibited in the fiords that are found frequently along the coast. These consist of narrow friths that run up between narrow headlands, as in Sweden and Norway, and along the coast of Maine, in this country. It is easy to see that they have resulted from an alternation of harder and softer strata, on the latter of which the sea has operated more effectually than upon the former, aided sometimes by the drift agency. The extent to which this action has been carried on in many places, is truly surprising, and indicates a vast period of time for its accomplishment. Along the coast of Massachusetts, for instance, where we see that Cape Ann and the rocks of Cohasset consist of unyielding syenite, while around Boston, they are softer metamorphic slates, we cannot doubt why Boston Harbor has been scooped out by the action of tides and breakers; and probably we would extend the same conclusion to the whole of Massachusetts Bay; although at present Cape Cod is extending in a northeasterly direction, because a current sets in that direction along the coast. In passing from Cape Ann to the Bay of Fundy, some 300 or 400 miles, we find almost the whole coast serrated by fiords, some of them 20 miles long, including the many islands that once constituted continuous ridges.

2. *In the Extensive Denudations of the Strata by Oceanic Agency, when the Surface of Continents sunk beneath, and emerged from, the Waters.*

This has doubtless been the most powerful of all the agencies of erosion which the surface has undergone. In South Wales, where the geology has been examined with almost unequalled thoroughness and accuracy, by the Ordnance Geological Surveyors, Professor Ramsay has made it almost certain, that as much as 10,000 feet in perpendicular thickness has disappeared. I do not think Geological observations in this country have been prosecuted with the minute accuracy requisite to determine the denudation here. From what I have observed, however, it would not seem extravagant to assert, that an equal amount of strata have disappeared from some parts of our country. In my paper on Terraces, I have endeavored to show, that since the tertiary period, the continent has once sunk below the ocean, and once emerged from it. Furthermore, I intend in this paper to point out certain valleys that must have been occupied by rivers before the continent's last submergence. Tracing back its history still further, we may be sure that during the deposition of the coal measures, a large portion of it at least must have been below the waters. Yet previously, or perhaps contemporaneously, large portions of the surface must have been dry land, to nourish the prolific flora which produced the coal. During the Devonian and Silurian periods, we have still clearer evidence that almost the entire continent was covered by the ocean.

We may then be nearly or quite sure of at least three depressions of the North American continent, beneath, and an equal number of elevations above the ocean, since the fossiliferous rocks began to be formed. And, in general, it is clear that these vertical movements were but slightly paroxysmal; so that every part of the surface has been again and again exposed to the long-continued action of waves, tides, and currents. The amount of erosion must have been prodigiously great; and, in my opinion, we find the evidence of it almost everywhere in the mountainous districts of our country. Even where our valleys are so narrow at their lower part, that rivers may well have worn them out, their upper part is so widened that only waves and tides, rushing back and forth between rocky islands, could have caused it. Indeed, the ragged isolated appearance of a large part of our mountains, save on their northern sides, can be explained only on the supposition of having been subject to powerful oceanic action.

Sir Charles Lyell, in connection with Professor John Locke, has made an estimate of the amount of denudation of the rocks above the lower Silurian at Cincinnati. It proceeds on the supposition that the Appalachian and Illinois coal fields were once continuous. In that case, the strata at Cincinnati must have been swept off to the depth of about 2000 feet. Plate XII, Fig. 3, copied mainly from Lyell's Travels in the United States, will give an idea of this example. Prof. Locke had promised me a more accurate statement of this case, but his recent death has deprived me of this expected aid. I regret it, because I am aware that some geologists do not place much confidence in this example, as indicating the amount of erosion, and I feel myself unable to form a very decided opinion concerning it. It has every appearance of a plain case, if we can judge from the figure, and Prof. Locke felt quite confident of its reliability.

I will venture to refer to one other example, which, in my estimation, will give some approximate idea of the amount of this work of erosion. It is the part of the Connecticut valley occupied by secondary rocks, especially the part reaching from New Haven to Mettawampe, in Sunderland. It will be seen by Plate III, that this valley is traversed by a few narrow and precipitous ridges, which consist of trap based on sandstone, or rather interposed between its strata, as enormous dykes or beds. Mettawampe and Sugar Loaf, however, are sandstone, and the former rises higher than any mountain of secondary rock in the valley, being 1175 feet above Connecticut river, and 1295 above the ocean. Sugar Loaf is about 500 feet above the river. Plate XII, Fig. 2 is a section crossing the whole valley at this place, from the gneissoid rock of Leverett on the east, to the highly inclined strata of mica slate in the west part of Deerfield. The sandstone, dipping easterly from 5° to 30°, has an unknown depth, and rises to the top of Mettawampe. In passing up the valley from Long Island Sound, this mountain and Sugar Loaf, which is the southern termination of a ridge running north through Deerfield, Greenfield, and Gill, are the first high bluffs of sandstone without trap, with which we meet, and they stand up with almost perpendicular walls, having every appearance of being merely the remnants of a formation that once filled the valley. There is no appearance of any dislocation of the strata, although there does exist, near the base of Mettawampe, a narrow bed of trap, as shown on the section. The valleys east and west (right and left on the section) of Sugar Loaf, exhibit every appearance of having been worn out by water, and it is difficult to avoid the conclusion that the sandstone, at least to the height of Mettawampe, once filled the valley of the Connecticut to Long Island Sound; a distance of nearly 100 miles; and that long exposure to oceanic action has worn the whole away as far as Mettawampe, except where protected by the overlying trap. This latter rock, being excessively hard, has in a great measure resisted these agencies, and now stands out in ridges, whose rounded and furrowed surface indicates long continued powerful erosion. I can assign no possible reason why these trap ridges thus remain, except that the softer sandstone has been worn away, nor can I imagine any other agency which could have accomplished the work, save the ocean. Judging from its effects in other parts of the world, it is not extravagant to conclude that this is sufficient for the mighty work. Nay, the probability is, that even Mettawampe shows us by no means the original elevation of the sandstone. It may have been far greater;

for the top of that mountain shows as distinct marks of erosion as any other portion of the valley.

Under the quaint term of Purgatories, I introduce another evidence of oceanic denudation, which we can connect with operations now going on in some places upon the coast. In several works on geology I have given examples of this peculiar sort of erosion; yet they do not seem to have arrested the attention of geologists, at least under this name. Of the origin of the name I know nothing, but I take it as I find it among the people. Along the coast we find sometimes long and deep chasms in the rocks, into which the waves still rush during storms, and by their concussion wear away the strata and lengthen the gorge. Sometimes this work is aided by the jointed structure of the rock, which produces parallel fissures, and when these are only a few feet apart, they enable the waves to carry on the work of erosion more rapidly. In my Report on the Geology of Massachusetts, I have pointed out two striking examples of these Purgatories, where we see the process actually going on, on the southern shores of Rhode Island. It is no wonder that those who never thought of the manner in which they were produced, should have given the same name to a much larger and longer chasm in the town of Sutton, Mass., far removed from the ocean. I have discovered another at a still greater altitude and further from the ocean, in Great Barrington, Mass. But I have given so full an account of these cases in my report above referred to, that I need only refer to that work. I can imagine no other explanation of their origin that will at all meet the facts, although this has its difficulties.

I will here refer to another example, which, although I have not seen it, I have been led from Dr. Charles Jackson's description to regard as a Purgatory. I refer to the Dixville Notch, in the north part of New Hampshire. Here, through the summit of a mountain of mica slate, which forms the dividing ridge between the Connecticut and Androscoggin rivers, we find a fissure from 600 to 800 feet deep, with nearly perpendicular sides. Its situation forbids the supposition that the gulf can have been produced by fluviatile action, since the streams here run in opposite directions into the Connecticut and Androscoggin. But it is just the situation where the waves of an increasing or retiring ocean would act most powerfully. The chasm may have been once occupied by a trap dyke, as supposed by Prof. Hubbard (*American Journal of Science*, vol. ix. p. 160, N. S.). But my inquiry simply is, how the trap, or other material which once filled it, has been removed. And it seems to me that we must resort to oceanic agency.

I have little doubt that careful examination would discover many more of these "Purgatories" in the mountainous parts of New England. Indeed, what are the famous "Notches" at Franconia and the White Mountains, but examples somewhat modified of the same kind?

3. *Erosions by Drift.*

This agency, as I have endeavored to show in another place, may be regarded as chiefly or entirely ice and water. Yet these causes have operated under such circumstances as to demand a notice distinct from that of the ocean, or of glaciers.

In our country, as well as in northern Europe, the force appears to have had a southern direction. Consequently, the northern sides of our hills and mountains, as well as their tops, are rounded and striated, while their lee side is rough and precipitous.

If any one wished to be impressed with the extent and power of this agency, let him compare the rounded appearance of the mountains and hills of New England, with the pointed and jagged aspect of those of Wales, or the Alps, especially above the line where this agency has operated. He will see that the amount swept away must have been immense; nor will his conviction of the quantity be lessened when he examines the loose detritus covering the surface of northern countries, most of which was originally drift.

4. *Erosions by Rivers.*

To this action I have given more attention than to any other form of erosion. At the outset, however, I found myself embarrassed by the difficulty of distinguishing between river action and the other denuding agencies above described. So far as I know, very few definite rules on this subject have been proposed by geologists. I give the following as the result of my own examinations. They are not in all cases as decisive as I could desire. But they seem to me sufficiently so in general, to enable one to discriminate between the different agencies, and they have led my mind to some interesting conclusions from phenomena which I had hitherto overlooked.

Marks by which River Action can be distinguished from Drift Agency.—1. By the direction in which the denuding force has acted. Since the drift agency in several countries took a southerly course, we may conclude erosions made in other directions to have resulted from rivers. Or more specifically, having determined the course taken by the drift in a given district, we may refer other marks of aqueous action to rivers or the ocean. I ought to make an exception of those few cases where the marks of ancient glaciers have been found in the same countries as the drift phenomena, with which they agree, except that the striæ made by glaciers follow down the valleys in whatever direction they run. In every other respect the glacier marks correspond to those of drift, and can be distinguished from fluviatile action by the marks that follow.

2. Drift agency has eroded the northern slopes of mountains; but rivers have cut channels through the rocks in every direction, or where they have pressed against the sides of hills, they have formed steep precipices.

3. The drift agency has but slightly conformed to the minor irregularities of surface, but has operated like a huge plane, or rasp, to remove protuberances. Whereas rivers have insinuated themselves into the minutest anfractures and cavities, smoothing the uneven surface—the depressions almost as much as the protuberances.

4. Drift agency has covered the eroded surfaces with striæ and furrows of various sizes, from the finest scratches, to troughs several inches deep. But save in a few spots, where ice with gravel frozen into it has been crowded over the surface,

no such markings are the result of river action. The rock is smoothed sometimes almost to a polish, but not distinctly scratched, unless something more than water, or gravel and sand driven by water, has acted upon it.

5. The drift agency sometimes operated in an up-hill direction, even to the height of some hundreds of feet, whereas, rivers can operate only upon a level or upon a descent.

6. Pot-holes in the rocks are produced by rivers where they form cataracts, but never by the drift agency.

7. Where successive layers of rock are superimposed upon one another, some are more easily worn away than others, and usually the central parts of a stratum are the hardest. When currents of water act on such ledges, the edges of the layers will be rounded and interspaces or grooves be produced: not regular, indeed, but more or less deep and wide, according to the greater or less ease with which the strata are disintegrated. But no such effects are produced by drift agency. All parts of the surface, whether harder or softer, are swept down to the same level, or nearly so.

Marks by which to distinguish between Fluviatile and Oceanic Agencies.—This is a much more difficult case than the last, and in some instances, I despair of determining by which of these agencies erosions were produced. In most cases, however, I think the distinguishing marks are clear to a practised eye.

1. Fluviatile action produces pot-holes, when rivers have cataracts, but they never result from the action of oceanic waves, tides, or currents.

2. When chasms or gorges are worn in the rocks by waves and tides, they are usually almost straight, and generally follow the jointed structure of the rock, producing purgatories. But when rivers wear out long chasms, they are usually more or less crooked, as that of the Niagara river, for instance.

3. Rivers have little power to form wide valleys. Sometimes, however, as a stream cuts its bed deeper and deeper, either in consequence of the strata being softer on one of its banks, or of a curvature in its course, it moves laterally so as to leave a sloping bank on one side, perhaps to a great height. In that case, however, the opposite bank will be steep. If both banks slope nearly alike, so much as to make the upper part of the valley quite broad, we must impute much of the erosion to oceanic action; to the flux and reflux of the waters through the opening for ages. The lower and narrower part of the same valley, in such case, may be the result of river action. Or the river may have begun the work at a high level, and it has been subsequently modified by the ocean. I apprehend that most of the valleys in mountainous regions have been produced by this joint agency.

4. If the crest of a mountain is crossed by parallel valleys of different heights, evidently eroded, the presumption is, that the denudation was accomplished mainly by oceanic action; by the flux and reflux waves and tides, aided, perhaps, by icebergs, during the upheaval of the land. For though a river, in such a case, might sometimes change its bed, so as to wear each successive one deeper and deeper, the supposition would imply that the successive beds had originally nearly the same relative depth, and it is not easy to see why a lake should have so many outlets at the same time. Lakes do, indeed, sometimes have two outlets, at oppo-

site extremities; but I do not recollect a case in which their drainage is effected by several parallel outlets.

A case to illustrate this principle occurs in the valley of Connecticut river. The trap range, called Holyoke and Tom (see Plate III), crosses that valley, or nearly so, obliquely; its northern extremity (Holyoke and Norwottuck), turning across the valley almost at right angles. Its crest is crossed by numerous valleys of erosion, of very unequal depth, in one of which Connecticut river now runs. I have no doubt that the drift agency has had a good deal to do with their erosion, yet such is the situation of these valleys, that when the ocean gradually receded from the surface, the waves and tides must have acted with great force in the manner above described. During the last depression of this region below the ocean, the drift agency probably swept over the ridge and modified the small valleys; but probably the ocean did most of the work long before. I should go more into detail in respect to this case, had I not already done so in my Report on the Geology of Massachusetts, and in my Elementary Geology. I have not, however, in those works advanced the above hypotheses to account for the denudation, but have merely inferred that water and ice must have been the agents.

5. If the face of a mountain be steep and show marks of denudation; if it be an outlier; that is, have no corresponding eminence opposite, so as to form a valley, and if there be no evidence of a dislocation of the strata, we must impute the erosion to oceanic agency; since fluviatile agency is out of the question. But if, while the continent was sinking or rising, its waves and currents beat against the mountain, it might so wear away the strata as to leave a mural face. In this case, however, we must suppose a previous inequality of surface, so as to enable the waves to act upon the shore.

6. The main force of the ocean is directed towards the axis of mountain chains, although tides and currents will be parallel thereto. Hence the eroded valleys will run towards the crest of the mountain chain. If, therefore, we find valleys running very much oblique to the axis, we may presume them to be formed by rivers. It must be remembered, however, that the direction of the strata will greatly modify the direction of erosions, as in the case of fiords. The unequal hardness of the strata, also, will operate in the same way: so that the application of this distinctive mark will require caution.

7. In a few cases, where a river has worn a passage through a mountain ridge, and at the same time has, from time to time, made lateral changes in its bed, it might leave a succession of precipices, which were its former banks, on one side, while on the other, they might be worn away. In such a case, however, we must suppose that the stream, after excavating a bed, should suddenly desert it; else, if the lateral change were slow and equable, it would leave on the deserted side, only a uniform slope.

In suggesting this distinction, I have had a particular case in view, which I will shortly describe; but about which I am in doubt, whether to refer it to oceanic or fluviatile action, or to both united.

Modes and Extent of Erosion by Rivers.—1. The manner in which rivers are formed, as a continent rises from the ocean, has been described in my paper on

Terraces. During the rise of the land, the water would remain only over its depressed portions, and it might be that the lakes or ponds thus formed, if ranged along some extended depression of surface, would constitute a chain of lakes. The water poured into them from the neighboring hills, would produce an oceanward current, and this, passing through the barriers of the lakes, would begin to wear them away. This would be the first step towards a river.

2. The above processes continuing to go on, the lakes would become narrower and the barriers be more deeply eroded, so that what we call a river would be the result. The matter, however, which was worn away at the barriers would in part be deposited in the deeper and more quiet water between them; and in part be carried forward to the ocean. Hence the process would be one both of erosion and of filling up.

3. That, upon the whole, the process of excavation exceeds that of filling up, will be evident from the following facts:—

1. The increase in the deltas of rivers.

The Merrimack sends forward, annually, about 839,171 tons of sediment to increase its delta at Newburyport.

The Ganges pours into the ocean, each year, 355,361,464 tons of mud.

The Mississippi carries forward 28,188,383,892 cubic feet, or one cubic mile in five years and eighty-one days. Its whole delta contains 2720 cubic miles: and, therefore, at the rate above indicated, 14,204 years would have been requisite to form it.

But such examples need not be multiplied, for every river tells the same story. The amount of sediment at its debouchure is ever increasing, and, therefore, its bed must be continually widening and deepening.

2. Some portions of the banks of most rivers are composed of loose materials, which form precipitous walls, and thus make it almost certain that the depression now occupied by the river, was once occupied by the same sort of materials as the banks, which the waters have carried away.

3. Wherever rivers run through rocky gorges, especially if cataracts exist, we find distinct evidence, in the worn appearance of the rocky banks, and sometimes by pot-holes, that the stream once ran at a higher level than at present, as at the Great Falls, on the Potomac, near Washington: at the Falls on Genesee river, at Portage, and on the Mohawk, at Trenton, New York: on the Connecticut river, at Bellows Falls, New Hampshire. If the surface of the rock, however, has been exposed for a very long time, the atmosphere and frost are very apt to cause it to scale off so as to obliterate traces of river action.

4. The modes in which rivers excavate their beds has been already given, essentially, in describing the effects of water and ice upon the rocks. They are briefly as follows:—

1. By solution of the agents of chemical change.

2. By direct solution of the constituents of rocks.

3. By urging forward loose materials, such as sand, gravel, and boulders, over the surface. When a gyratory motion is produced in the water, the eroding materials produce pot-holes.

4. By entering the fissures of rocks and freezing, so as to separate them by expansion. This is one of the most powerful modes in which the work of excavations is carried forward.

5. By ice floods. In these cases the stream becomes choked with ice, with only water enough to make it plastic, and enable gravity to urge it forward. The moving mass does, indeed, very strikingly resemble a glacier, and it moves forward with a similar immense power; ploughing up the loose surface, tearing off the projecting rocks, and sometimes forming new channels for the river.

6. Where there are cataracts in rivers, all these modes of erosion usually act with a maximum intensity: and at this day probably the principal amount of erosion by rivers takes place where there are cataracts. These cataracts are constantly receding, although when measured by the life of man, the rate of retrocession is scarcely perceptible; but measured by geological periods, it becomes very manifest, and we find evidence, that in this manner long and deep gorges have been produced, and lofty barriers removed. The consequence of this latter process is, that the river below and above the barrier, thus partially or wholly removed, will excavate a deeper bed in the loose materials there accumulated.

5. Without attempting to determine the precise amount of erosion by rivers, I wish to state distinctly that I do not impute to this agency the whole, or even the larger part, of the formation of the valleys through which rivers now run. Much less do I maintain that present rivers have produced these valleys: for there is proof in some cases, that other streams once flowed through valleys now occupied, perhaps, by rivers totally unable to have eroded them. For the formation of most of our present valleys we may assign the following agencies:—

1. The original upheaval and dislocation of the strata.
2. Long continued oceanic action.
3. The drift agency.
4. Rivers on former continents.
5. Existing rivers.

I impute to rivers only such a part in the work of erosion as can be proved by an application of the preceding principles.

Caution in the application of the preceding Rules.—1. The older the rock through which rivers have cut their way (*cœteris paribus*), the greater should we expect the amount of erosion.

2. But, secondly, the position of the strata, if the rock be stratified, and the amount of water acting upon them, or the number and direction of the fissures, if the rock be unstratified, will greatly modify the amount of erosion. If the strata cross the stream and dip in the same direction as the slope of the river, the action of the water will be much more powerful than if the dip is in the opposite direction. Or if the inclination of the strata corresponds with that of the stream, the erosion will obviously be slower. Again, some unstratified rocks present but few fissures, while others are full of them, and this fact will make a great difference in the erosion.

3. Rocks, essentially alike in chemical composition, may yet vary very much in hardness, and in the ease with which they might be disintegrated. How great

the difference is, for example, between chalk and Silurian limestone, especially when a small proportion of silex enters into the composition of the latter. Certain kinds of syenite disintegrate with great ease, compared with common trap: yet both are composed of feldspar and hornblende.

4. Rocks, essentially alike, may yet be decomposed with very different degrees of facility, on account of the presence in some of them of such minerals as carbonate or sulphuret of iron or manganese in some form. It is surprising sometimes to see to what depth the whole character of the rock will be changed, and how it will be disaggregated, so that aqueous agency can easily denude its surface.

Detail of Facts.

Guided by the preceding principles, I have made a collection of examples, which I suppose to be cases of erosion mainly by rivers. In some of them, however, other agencies have been largely concerned, perhaps more largely than the rivers. I have not confined myself to examples founded on personal observation, and of course, in those cases which I have not seen, I feel less confidence than in the others; for careful examination is sometimes necessary to decide certainly whether river agency has produced the gorges. Yet by observing the characters of those erosions, which personal examination refers with great confidence to river action, we can with great probability refer other cases to the same cause, which we have only seen described by travellers or geographers.

The first example below, is not one of the most satisfactory; yet as it is the case which first called my attention to the subject, I shall describe it with more detail than usual.

I cannot doubt that a more extensive examination than my time will allow of the works of travellers and geographers would enable me easily to double the following list. Still, as travellers usually describe such scenery only in general terms, the geologist can but seldom decide certainly what cases are examples of erosion by rivers.

So far as it is in my power, I shall describe these erosions under the head of the different rocks in which they exist.

1. *Erosions in the Hypozoic or older Crystalline Rocks, such as Gneiss, Mica Slate, Talcose Slate, &c.*

a. In Buckland, on Deerfield River, a little west of Shelburne Falls.

A ridge of gneiss and hornblende slate lies west of the village of Shelburne Falls, through which Deerfield river has cut a passage. On the road from that village to Charlemont, where it crosses this ridge, we meet with pot-holes in the ledges of gneiss; and, indeed, the road occupies an old bed of the river. These pot-holes are 80 feet above the present bed of the stream, and the terrace materials rise to that height on the north side of the river. This proves that the stream was once dammed up to that height, else the pebbles and sand could not have been sorted and deposited. Such a rise must have thrown the waters over a basin which extends several miles into Charlemont, as shown on Plate IV. The old river bed is marked on that map, as well as the present course of the stream. It is clear, then, that since the deposition of these terrace materials, the river has not only changed its course a considerable distance to the north, but has cut a new channel 80 feet deep, through very hard gneiss rock. It was probably the blocking up of the old river bed by the gravel deposited while the waters stood over the spot, that caused the river to change its course. The evidence on which such an explanation rests, is not quite as striking at this spot as at some others of a similar character to be subsequently described, and, therefore, I will not dwell upon it. But if admitted, it shows us *the amount of erosion by the river in very hard rock, since the deposition of the gorge terrace on its bank.* And since the terrace lies above the drift, we are sure that so much work at least has been done by the river since the drift period. Nay, after that period, the materials of the terrace at the gorge must have been very slowly accumulated, so that this erosion of 80 feet may not carry us more than half way to the period of the drift.

At the top of Plate IV, is a section of the mountain through which Deerfield river has cut a passage, as above described: it runs in the direction of the axis of the mountain; that is, nearly north and south. On the north side of the river the mountain rises to the height of more than 1800 feet above the ocean, and forms Mount Pocomtuck (formerly Walnut Hill). On the south the ridge ascends rather rapidly, till within half a mile it has reached the height of 545 feet above the present river bed. Then it descends 218 feet, into a valley now covered with gravel and boulders, looking like a former river bed. Then the ridge rises for several miles and attains a height nearly equal to Pocomtuck.

Having ascertained the action of the river 86 feet above its present bed, as proved by the pot-holes, I was led to inquire whether any marks of its erosions existed at a higher level, towards the south. I found that the north slope of the hill exhibited a succession of ragged walls for a considerable height, as shown on the section connected with Plate IV. These have the appearance of successive banks of the river, as it stood at different elevations. These walls present a curve horizontally, whose convex side is towards the northeast, which would be exactly the effect of the river sweeping around towards the southeast in a curve of that description, as it must do to correspond with its present course. (See Map.)

At first I felt very little doubt that these facts were decisive proof of the former action of this river, at least to the height of 545 feet above its present bed. But some doubts as to this point have been subsequently excited. If the river wore down the whole of this gulf by a slow and uniform action, I can hardly see why the south bank should not have a uniform slope instead of several steps. Nor do I see any reason why it should have changed its bed so many times suddenly, unless we suppose such a state of things to have existed at each lateral movement, as at the last—that is, a filling up of the old bed by loose materials, because the region had subsided beneath the ocean. This would suppose more vertical movements than have generally been admitted. Again, if the sea once, or more than once, stood over this spot, we should expect that the flux and reflux of the waves through the depression in the crest of the mountain, which may have existed, would wear it away, as we now find it. After all, however, water flowing in the same direction as the present river, affords a more natural explanation of the erosions at this spot than any other supposition; and I apprehend that they may have been the result of both kinds of agency: for when a mountainous region, like the one under consideration, is either gradually sinking beneath, or rising above, the ocean, what is at first an ocean, becomes an estuary, and then a river.

The Ghor[1] is a deep narrow valley, extending from Shelburne Falls to Deerfield Meadows, about eight miles. Throughout most of this distance the stream flows obliquely across the hard strata of mica slate and gneiss, which have a high dip in the same direction as the slope of the stream. The rocks crowd so closely upon the river, and rise so precipitously for several hundred feet, that no attempt has ever been made to make a road parallel to the stream, and only in one place is it crossed by a road, and there with difficulty. Near the upper extremity of this valley we find Shelburne Falls, whose height I know not: but there we see the effect of the cataract upon the hard and almost unstratified gneiss rock, in the formation of pot-holes of enormous size, some of them being as much as twenty feet deep, and eight or ten in diameter. There, too, we see the effect of the expansion of freezing water in the fissures, in the removal of huge blocks from their native beds; so that upon the whole we cannot doubt that the cataract is receding. Nor can the geologist doubt that it may have receded the whole distance of eight miles from Deerfield Meadows. Nay, perhaps previous cataracts at higher levels, may, in like manner, have worn backwards, so as to form the whole of this Ghor. Its situation is such, and it is so crooked, that it seems difficult to suppose the sea to have had much to do with its excavation, except, perhaps, to widen its upper part.

b. Ancient River Bed at the Summit Level of the Northern Railroad in New Hampshire.

On Map No. 1, a mountain ridge is represented as running from Connecticut river, at Bellows Falls, northeasterly to the White Mountains. No such distinct mountain exists there: but it marks the dividing ridge between Connecticut and Merrimack rivers. The valley of the latter is about 150 feet lower than that of the former. Through this dividing ridge I know of not more than four depressions of considerable depth. One of them is in the town of Orange, on the Northern Railroad, and is 682 feet above the Connecticut, at Lebanon, and 830 feet above the Merrimack. Another is at Whitfield, on the White Mountain Rail-

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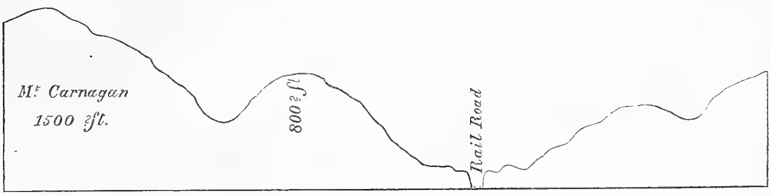
[1] The Geological Class in Amherst College, a few years ago, having forced their way on foot through this wild and difficult gully, seldom trodden by man, felt at liberty to propose for it the Arabic name of *Ghor;* which may be used till a better one is suggested. In like manner, the class that graduated in 1856, visited Walnut Hill, referred to in the text, and imposed upon it by ceremonies, the name of Mount *Pocomtuck;* the Indian name for Deerfield river, which washes its southern base.

road, and is 650 feet above the Connecticut. The third is the Franconia Notch, which is about 2295 feet above the same river. And a fourth is the White Mountain Notch, which is 1557 feet above.

On the west side of Connecticut river we find the lofty Green Mountain range, running parallel to the river, its culmination being some 30 miles distant. Through this ridge there are two depressions occupied by railroads. The Rutland and Burlington road crosses at the Mount Holly Gap, 1350 feet above Connecticut river, at Bellows Falls. The Central Railroad passes the summit, near Montpelier, 930 feet above the Connecticut, at Lebanon. Still further north, the Passumpsic and Connecticut Railroad (not yet finished) passes over the summit at about 900 feet above the Connecticut at West Lebanon.

At Bellows Falls, the hills extending easterly and westerly to these two lofty dividing ridges, crowd so closely upon the Connecticut as to leave only a narrow gorge, although the mountain on the east side of the river (Kilburn Peak, formerly Fall Mountain,[1] 828 feet above the top of the falls and 1114 feet above the ocean) is the highest. Yet if this gap were closed, it would raise the waters high enough to flow out laterally through some of the passes above mentioned as the location of railroads. And when we see the evidence of erosion on the west face of Kilburn Peak, to the height of 900 feet, we cannot but suspect that this gap was once closed, and that the waters did spread out so as to form a lake, extending to the dividing ridges east and west, and northwards perhaps even to Canada. If, therefore, we could find evidence of the former passage of water through some of the above named gaps, it would make such a conjecture almost certain. Such evidence we do find at the Summit level of the Northern railroad, in Union, two and a half miles from the station house in Canaan.

In approaching this spot by railroad from Connecticut river, we ascend a small tributary to Canaan. There we have before us a mountain ridge, running nearly N. E. and S. W., with a deep depression in Union. To the north, as a part of the ridge, lies Mount Carnagan, which I judged to be at least 1500 feet above the railroad. The cut below will give some idea of the appearance of the range as we approach it from the west. The stream has diminished to a small brook, which



[1] In 1856 the Class in Amherst College, that will graduate in 1857, visited Fall Mountain, and formally imposed on it the name of *Kilburn Peak;* to commemorate the memory of two men by the name of Kilburn and Peak, who, with their families, the earliest settlers of Walpole, performed a feat of courage and self-defence at the foot of this mountain, perhaps more daring and extraordinary than the whole history of Indian warfare in this country can present, and rivalling that of Leonidas and his Spartans, at Thermopylæ.—See *New Hampshire Historical Collections.*

has its origin in a peat swamp, that lies immediately to the west of the highest part of the gorge. Not long before we reach the cut for the railroad, we pass one or two long ridges of sand and gravel, running N. W. and S. E., and resembling very much genuine *osars,* and I have marked them as such on the map. The rock is gneiss, traversed by large veins of coarse granite and feldspar, trap and quartz. The artificial cut is 30 feet deep and 1200 feet long; and along it, near the east part of the ridge, are seen the remains of several pot-holes. In short, there is the most conclusive proof that a cataract once existed here, and that the waters ran from the Connecticut into the Merrimack valley. For on the west side of the ridge occur very distinct marks of drift agency from N. W. to S. E., and this would have obliterated the river action had it been on that side; as it would have been, had the current passed from S. E. to N. W. At present, on the Merrimack side of the ridge, is a peat swamp, from which a small brook issues towards the east.

The conclusion from these facts seems irresistible, that the valley of the Connecticut was once filled with water to the height of 682 feet above its present bed, and that here was one of its outlets. Over the whole of the valley, for many rods to the right and left of the railroad, we see marks of very powerful aqueous action, nearly obliterated, indeed, in some places by the drift agency, but still manifest to a practised eye. But it is clear that the water poured through this outlet before the drift period; consequently it was on a former continent, the one that was submerged at the drift period. During that last submergence, the pebbles and sand, still found so abundantly on either side of the ridge, and even beneath the peat in the very gorge, were deposited.

In looking at this outlet of a lake of a former continent, one cannot doubt that a great amount of erosions has here taken place. The great width of the openings in the ridge, would indicate the action of waves and oceanic currents, but that the waters of the lake itself did much of the work, can hardly be doubted.

2. *Gorge at Bellows Falls.*—The preceding facts and reasoning make the conclusion almost irresistible, that the gap at Bellows Falls, through which Connecticut river now runs, was once closed, at least to the height of 682 feet, with the addition of the fall in the river between Lebanon and that place, say 40 feet, which gives 722 feet. There was probably another outlet to the lake at that height, and perhaps the reason why the one at Bellows Falls sunk faster than that in Union lies in the character of the rock in the two localities; that at Bellows Falls being more slaty and more full of fissures.

At Bellows Falls, as well as at Union, we have evidence that nearly the whole of the erosions was accomplished previous to the drift period. For at the top of the falls, that is, in the bottom of the valley, we find very beautiful examples of striæ and *roches moutonnees,* while only a few rods or feet below them, are fine illustrations of river action upon the rocky banks of the stream about the falls. We are certain, then, that the gorge was mostly excavated previous to the drift period, and we may put the work at least as far back as the period of dry land, which preceded the last submergence of the continent.

I once scaled the almost perpendicular face of Kilburn Peak, on the east bank of the stream, to see if I could not discover marks of former fluviatile action on its face. Perhaps I ought to have concluded that drift agency had obliterated all traces of river action. But I had noticed that bottoms of valleys have been more affected by that agency than high mural points. Accordingly, on the face of this mountain, I found much less of drift action than at its base: and I fancied that in many places, especially in depressions of the surface, I could see the smoothing and rounding effects so peculiar to running water. In general, however, long exposure to atmospheric agencies has caused a scale to fall off from the surface, and thus nearly destroyed its original character. But even in that case, the general contour might not be destroyed, and thus we may sometimes detect river action where the surface has become rough.

But it is at the top of Kilburn Peak I think the marks of ancient currents of water are most obvious. Here we sometimes see what seem to have been the shores of ancient currents: namely, ragged walls running out in the direction of the valley, that is north and south, but inclining to N. E. and S. W., as if the outlet of the lake, in those early times, had that direction, because certain joints in the rocks have the same, and thus made the erosion easier. But though the marks of ancient fluviatile action on the west side and top of this mountain seemed to me quite distinct, I do not forget how difficult it is to distinguish such action from that of the ocean. But that the river itself was the chief agent in forming this gorge of 800 feet high, I cannot doubt.

3. *Gorge at Brattleborough.*—Wantastoguit mountain, at Brattleborough, 1050 feet above the river, according to my measurements, corresponds in position and shape so nearly to Kilburn Peak, and there is such a general resemblance between the narrow valley of Brattleborough and that of Bellows Falls, that we can hardly doubt that the agencies which operated in the one place acted in the other. I found on the west face and top of Wantastoguit mountain, quite as distinct marks of erosion by water as on Kilburn Peak, on the top perhaps a little more distinct; especially if we admit that the gulfs running N. N. E. and S. S. W., with ragged mural faces, were caused by water from the ancient lake. Perhaps, however, in the less elevation of the hills on the west side of Connecticut river, at Brattleborough, for several miles, we have stronger evidence of oceanic action. But I cannot doubt, that though we have no cataract in the river at Brattleborough, as at Bellows Falls, the stream has had an important agency in past ages and on a former continent, in the removal of a barrier, which once dammed up the Connecticut at this place, and formed a lake reaching to Bellows Falls. That barrier may, indeed, have extended a considerable part of the distance to Bellows Falls, as the narrow and deep gulf reaching even beyond Putney testifies.

4. *Gorge between Mettawampe and Sugar Loaf at Sunderland.*—In another part of this paper I have referred the erosions of most of this valley to oceanic action, though I cannot doubt that the river exercised an important agency. How much, it is impossible now to say: since like the gorge at Bellows Falls and Brattleborough, the erosion was previous to the drift period.

5. *Gorge between Holyoke and Tom, at South Hadley.*—The same remarks will

apply to the passage cut through the trap at this place, as were made in relation to that at Sunderland.

The two last examples, being the one in sandstone and the other in trap, would more logically be described in another part of this paper, but it seemed most natural in passing down the Connecticut to notice the gorges consecutively.

6. *Gulf between Middletown and the mouth of Connecticut river.*—As you pass through this gulf in a steamboat, and see how, in many places, the high and rocky banks crowd down upon the river, and even jut into it, you cannot resist the conviction that the stream itself, or one of a similar character on a former continent, must have had much to do with its erosion. You cannot believe either that it is a gorge produced solely by original folding of the strata, or by oceanic action. It is too long, say about 20 miles, and probably I might add, too crooked, to admit a sufficient force of waves and tides to accomplish the work. However it is not one of the most decided and certain examples of fluviatile erosion.

7. *Ravine through which Agawam river flows, extending from Mount Tekoa, where the river debouches into the valley of Connecticut river, nearly to the summit level of the Western railroad, along the main branch of the river, about twenty-five miles.*—If we were to follow up any other branch of this river, we should find similar ravines. The main one under consideration crosses the strata often at right angles, and there is no evidence of their dislocation on either side; hence its erosion may reasonably be imputed to the river, or the ocean. It is deeper in many places than the Ghor, on Deerfield river, and there are at least two cataracts along its course of considerable height, where the work of erosion is going on. In most places, however, it is wider than the Ghor, admitting of farms and villages. There is scarcely any part of it that presents walls of rock so obviously eroded as at Tekoa, where the river emerges into the alluvial plain of Westfield.

From the fact that an enormous vein of granite is seen in the bed of Agawam river in several places, as at Salmon Falls, I have suggested in my Final Report on the geology of Massachusetts, p. 691, that it might once have extended through a great portion of this ravine; and if so, that it gives the reason why the river chose this track: because such a vein would be more easily worn away than the mica slate. I still think that in this way we may account for a part of the erosion: but I have not found the evidence that the vein occurs through any considerable portion of the river's course.

8. *Ancient bed of Agawam river in Russell.*—This is a well marked example, lying immediately north of the railroad station in Russell. Standing at that spot, and looking north, you have before you a rocky hill, several hundred feet high, on the right or east side of which the river and the railroad now run. But on the left side, the common road passes through a valley about as wide as the river, and filled to a considerable height with terrace materials, gravel and coarse sand, at the north end, but finer towards the south. Near the north end the road attains an elevation above the present river a few rods further north of 74 feet. This is the present height, or nearly so, of the old bed of the river above the existing stream. But upon both sides of the old bed, the steep hills are fringed with the remnants of a former terrace, rising 208 feet above the river; and this doubtless filled the

old bed entirely, but was subsequently worn away, not by Westfield river, but by less powerful agencies. During the last submergence of the continent, doubtless the former bed of the river became filled to the height of 208 feet, so that upon its emergence, the river found a lower channel on the east side of the hill, where it has cut the deep rocky channel in which it now runs. At the north end of this gorge (which is not far from a mile long), and only a few rods north of the Russell depot, we find pot-holes on the west bank, nearly 70 feet above the stream. The rock is mica slate, traversed by huge granite veins. The greatest sceptic could not doubt, after visiting this spot, that the river has lowered its channel at least 70 feet, and admitting this, what reasonable man can suppose that the work has not been carried on at least to the height of 208 feet. The evidence that the river once ran in the old channel is so strong, that the farmers who live in the vicinity, have no doubt of the fact, though unconscious of the interesting geological conclusions resulting from it. For they see the proof in the water-worn appearance of the rocky sides of the old bed, and in the fact that they find logs in the alluvial deposit to the depth of nearly 30 feet.

This then is a case of postdiluvian gorge, in a convenient situation for examination, since the Western railroad passes over it, and a delay from one train to another, would afford time for the exploration. The length of the gorge is not, indeed, as long as from Niagara Falls to Ontario; but the rock here is much more difficult to wear away. A tolerable idea of this case may be obtained from Plate III.

9. *Another old bed occurs on this same river*, or perhaps I should say on its principal or eastern branch, where it unites with the western branch, at Chester village. It lies a little east of the village, is perhaps a mile long, and is separated from the present bed of the river by a hill, perhaps 500 feet high.

10. Still further up this east branch, say about four miles above Chester village, in Norwich, on the east side of the present stream, and separated from it by a hill of some height, is a deserted bed, which may be half a mile long. A small village occurs at the spot, and though I have not made accurate measurements either at this old bed, or at that described in the last paragraph, they both appeared to me to be examples of antediluvial channels through which the river ran on the last continent.

11. *Gorge on Little river, in Russell and Blanford.*—Little river is a tributary of Westfield or Agawam river, into which it empties a little east of the village in Westfield, after having pursued a nearly parallel course through Blanford, Russell, and Westfield. Five miles west of Westfield village, it emerges from the mountains, that bound the west side of Connecticut valley. From this point, for six or seven miles up the river, we find it with occasional interruptions, occupying the bottom of a deep and crooked gorge, so difficult to be crossed that rarely do we find a road over it, nor do any roads lead along the banks near the gorge.

The road to Russell from Westfield ascends the mountain on the north side of the gorge, and here I observed two or three quite interesting facts. By the roadside, perhaps 150 feet above the river, are most distinct marks upon the rocks of the former action of the river. The surface is rounded and smoothed, just as we often see near falls. I was interested to see how high this fluviatile action might be traced, and found it to grow fainter and fainter as I ascended, but I thought it quite distinct 300 feet above the river (aneroid). It is I think the best example that I ever saw of the gradual disappearance of these marks upwards.

At this spot as we rise above the marks of river action, we meet with what I have regarded as traces of ancient glaciers. These have already been noticed in Part I, on Surface Geology, and will be fully described in Part III, on the Marks of Ancient Glaciers.

I have not ascertained the precise length of this gorge on Little river, though I presume it is six or seven miles, with some interruptions, and three or four miles in its lower part without interruptions, by wider openings. Though the hills that bound the gulf are of very unequal height, yet I think we cannot regard their average height as more than 600 to 800 feet. It reminded me of the Ghor in Deerfield river. It is too crooked to impute much of its erosion to the ocean, though doubtless its upper part may have been widened by that agency.

12. *Ancient river beds in Cavendish, Vermont.*—Williams river and Black river, streams of nearly the same size, rise in the Green Mountains, and running nearly parallel, empty into Connecticut river; the former, two or three miles north of Bellows Falls, and the latter, ten or eleven miles further north. Through most of their course they are separated by mountains, rising sometimes, to near a thousand feet in height. Yet there are at least two gulfs, the Duttonsville one and that at Proctorsville, in Cavendish, connecting the valleys of the two streams, and through which Black river once flowed into Williams river: in other words, it is probable that Black river was once a tributary of Williams river. The evidence of the position I shall now present.[1]

The Duttonsville Gulf.

The Rutland and Burlington railroad passes up Williams river from Bellows Falls 18 miles to Gassett's station. There it turns to the right and crosses to Black river, through the Duttonsville gulf. Through its whole course that gulf bears evidence, to a practised eye, of being the former bed of a river, but just before we reach Duttonsville, we find deep pot-holes in the gneiss rock, perhaps 50 feet above Black river. This old river bed, especially near Duttonsville, is choked up to the depth of several feet by terrace materials, which must have been deposited during the last submergence of the continent beneath the ocean. These formed a bank so high, that as the surface emerged, and a river began to run down

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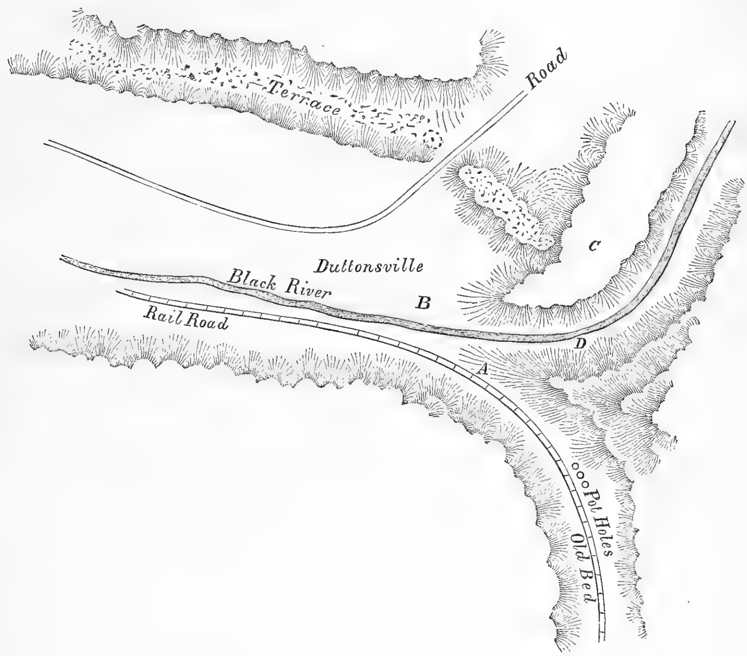
[1] I am much indebted to William F. Hall, Esq., now of Washington city, and to Hon. William Henry, of Bellows Falls, for calling my attention to these cases. To the latter gentleman I am, also, indebted for a free ticket on the Rutland and Burlington railroad on a visit to the spot. Nor is this the only time in which I have been thus liberally treated by gentlemen connected with that railroad. Indeed, it is but justice to say, that in no other part of this country have I found all classes of the community so ready to appreciate the connection between scientific researches and the public welfare, and so ready to help them forward, as in Vermont.

the valley, it was turned to the left and found a new channel to the left of the mountain lying east of Duttonsville. On Plate III a sketch of the region is laid down, which will give some idea of the ancient and present courses of Black river. But the published map of that region is so imperfect, that the following sketch, taken by the eye, will probably present a better outline in part.



During the drainage of the country a pond would occupy the basin B, at Duttonsville, extending up the river as far as Proctorsville and perhaps even to Ludlow, and the water would find an outlet at the lowest point. On the north side it was kept in by a gravel terrace, extending to the rocky hill C, and as stated above, the old bed at A was raised by a similar deposit. The result was, that the rocky ridge at D, was the lowest point, and there the stream flowed over and commenced its erosion of the strata. That work has gone on till a gorge has been worn back half or three-quarters of a mile, and the work is now progressing in the hard gneiss rock. According to my barometrical measurements the river falls in this whole distance as many as 183 feet.

As may be seen in Plate III, the old river bed, after continuing, about three miles, towards Gassett's railroad station, forsakes the railroad track, and finds its way to the present bed of Black river some seven or eight miles below Duttonsville. But a similar bed is represented as continuing as far as the Gassett station. No pot-holes, indeed, occur along this ravine, but we cannot doubt that a stream once flowed through it, and joined Williams river. Indeed, its bottom is only a few feet higher than that of the ravine just described, which branches from it to the left. Yet since the stream must have flowed through the lowest valley at the latest period, we must regard the valley running to Gassett's as the bed of a river at an earlier date. But this subject will be referred to more at length in a subsequent paragraph.

The Proctorsville Gulf.

The bed of the ancient river at Duttonsville is 675 feet above the top of Bellows Falls. Passing from this place two miles up the Black river, we find a rather broad valley almost level, as far as Proctorsville, another flourishing village. Running nearly south from this village, we find a deep narrow ravine, cutting through the high mountain and opening at its southern extremity into the valley of a tributary of Williams river. I found no pot-holes in the sides of this ravine, but every other mark of a former current of water, which wore out the gorge in fact, is seen on the surface. The highest point in the gulf, perhaps a mile south of Proctorsville, is 117 feet above the old river bed at Duttonsville, or 792 feet above the top of Bellows Falls. At the summit the gorge shows a deposit of terrace materials, how deep I cannot say. But the fact is sufficient to show that no stream has passed through the gorge since the last emergence of the continent. But that Black river—or rather the *progenitor* of that river, on a former continent—once passed through this gorge, and was in fact a part of Williams river, will be obvious by an inspection of the rough outline on Plate III. But at what period of antediluvian history did this take place?

If the principle above alluded to be true, viz., that where more than one lateral ravine, once the beds of rivers, open from a common valley, that which is the lowest was last occupied by the stream, then the Duttonsville gulf is more recent than the Proctorsville gulf. I have inferred that the former was the bed of a stream on the continent which immediately preceded the present. Was the latter worn out during the same period; or might it have been the work of a stream on a still earlier continent, that is, the second one anterior to the present? If we knew the depth of the detritus at the summit of the Proctorsville gulf, it might aid in deciding this point. But I can hardly believe that its depth equals the difference of level between the two gulfs. If not, then the Proctorsville gulf must have been higher than the other, during the period of emergence previous to the present. The country below Proctorsville, also, must have been blocked up high enough to throw the waters through the Proctorsville outlet. The amount of erosion since that time, on such a supposition, must have been enormous to bring the region below Duttonsville into its present state. And it would not be an improbable supposition, that the Proctorsville gulf, as well as the right hand branch of the Duttonsville gulf, already described, may have been the bed of a stream on a continent earlier than the last. But I despair of being able to prove this decidedly by any facts within the reach of present observation. And yet those detailed above, do appear to me to prove at least a great difference in the ages of these two gulfs. But whether the period between them embraced a submergence of the continent, is another question. To be able to trace back with clearness erosions accomplished on even the last continent, is more than I ever expected to be able to do. The above facts come nearer to extending our vision across another mighty chasm, and witnessing events in surface geology upon a still earlier continent, than any I have ever met with. But whether this be a problem resolvable by the geologist I am in doubt.

It is only recently that this subject of distinguishing between postdiluvian and antediluvian river beds has arrested my attention. And from the number of cases that have already fallen under my observation, I cannot doubt that they are quite frequent. I have some other examples to which I shall refer on a subsequent page.

13. *Gorge at Great Falls on the Potomac, twelve miles west of Washington city.*— The top of these falls is 112 feet above tide water; and the water at the cataract descends 82 feet. The rock is a hard mica slate, whose strike is N. a little W. by the needle, and whose dip is about 70° easterly. Consequently the water has acted upon the edges of the strata, and in circumstances poorly adapted to erosion. Yet as you stand upon the high bank near the falls, and look to the south, you see a gulf, from 60 to 65 feet high, with almost perpendicular walls of naked rock, extending nearly four miles. One cannot stand there and not be satisfied that the river must have worn out that gulf. Indeed, in going towards Georgetown, he will see that in many other places the work of erosion has been going on. And when we see the unfavorable position of the rock for being acted upon at this place, and the great amount of erosion, we can hardly avoid the conviction that a greater work has been done here than at Niagara; as indeed we might expect, when we remember that the rock over which the Potomac flows is probably much the oldest.[1]

14. *Passage of the Hudson through the Highlands.*—This celebrated gorge is nearly twenty miles long, and is remarkable for being worn out so that its bottom is below mean tide water. The hills on its sides rise in some instances as much as 2600 feet, and in many places the walls are very precipitous. The rock is gneiss, of a kind not easily disintegrated or eroded. Nor is there any evidence of any convulsive movement in the strata.

While, therefore, this is clearly a case of erosion, it seems almost equally obvious that the waters of the present river could not have done it: for they are too quiet, and have so little descent that tide water extends nearly 100 miles up the river beyond the Highlands: and, moreover, the low level of the bed of the gorge precludes the idea of a former cataract, whose recess might have accomplished the erosion. This, therefore, was probably a work mainly performed in some past period, when the continent was at a higher level. It was doubtless the joint result of oceanic and fluviatile action: for it is too crooked to allow us to impute it all to the ocean. Very probably the whole process was gone through at different periods, with long intervals, it may be, of rest. There is no evidence

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[1] I visited this spot in 1849, in company with Professors Henry and Guyot, Count Pourtales, and Mr. Saxton; and from these gentlemen I obtained several of the facts mentioned in the text. This same gorge is given as an example of the erosion of a river in Hutton's Theory of the Earth, by Playfair.

that much of it was effected since the drift period. Most likely it is a valley of very great antiquity.

15. *High Falls on the Hudson, in Luzerne, Warren county, New York.*—I depend entirely upon Professor Emmons' account of this gorge, in his Report on the Second Geological District, p. 188. It lies at the junction of gneiss and Potsdam sandstone. It is a mile long, and the wall of gneiss rises in some parts of this distance to the height of 100 feet. From Professor Emmons' description, I should judge this to be a genuine example of river erosion.

16. *Little Falls on the Mohawk, Oneida county, New York.*—The rock here is gneiss, through which the river has cut its way. Professor Vanuxem says that on its east side the walls of rock are 100 feet high, and that westward it gradually declines in height. The length of the gorge I am unable to state. It is an unequivocal example of river erosion: for pot-holes are found at various heights in its walls.—*Vanuxem's Report on the Geology of the Third District*, p. 208.

17. *Gorge on the Ottaqueechy river, at Hartford, in Vermont.*—The river here passes through a gulf a mile long, one side of which is 100 feet high and continuous: the opposite side being more irregular. Falls exist at Queechy village, 20 feet high, a mile above the gulf, with pot-holes on the sides and the bottom. Between these and the gulf are meadows, with seven terraces on one side of the river, and four on the other, as given in my paper on Terraces. Probably to determine the amount of erosion here, we must add the length of the gulf to its distance below the falls. But my examination of the spot was so hasty that I could not give a sketch of its features. The walls forming the gulf are mica slate, with trap, which Professor Hubbard supposes to have once occupied the gorge.—*American Journal of Science*, vol. IX., *New Series*, p. 160.

18. *Grandfather Bull's Falls, on Wisconsin river, in gneiss, mica slate, and trap.*—The cut is one and a half miles long, and 150 feet deep. Professor Owen describes another cataract a mile further up the stream, in trap; and the two may perhaps have formed parts of one continuous erosion, though more probably the work may have been going on contemporaneously at both places. *Owen's Report to the Government, in* 1848, p. 97.

19. *Gates of the Rocky Mountains.*—This remarkable chasm lies near the head waters of Missouri river, where it emerges from the Rocky Mountains. The average height of the walls is 1200 feet, and the chasm nearly six miles long. I am not sure that the rocks are crystalline, or hypozoic.—*Encyclopedia of Geography*, vol. III., p. 373.

20. *Sixty miles easterly from the Gates of the Rocky Mountains the Missouri forms a succession of cataracts, second only to Niagara.*—In the space of seventeen miles, the river falls 360 feet, beside the great fall of 90 feet. I refer to this place as probably affording, like the last, a striking example of river erosion.—*Encyc. Geog.*, vol. III., p. 373.

21. Robert Maclagan, Esq., of the Bengal Engineers, who has resided ten years near the Himalaya Mountains, informs me that on the Sutlej river, there is a gorge through gneiss, as much as 1500 feet deep and a mile long. Since his return to Europe he has sent me the following letter on this subject:—

EDINBURGH, 129 GEORGE STREET, *February* 10, 1854.

MY DEAR SIR: In course of conversation with you at Amherst, when I enjoyed the pleasure of a visit to your house, I communicated to you my impression of the height of certain precipitous cliffs on the banks of the Sutlej river, in the Himalaya, mentioning, that to the best of my recollection, I had estimated them at the time to reach the height of 1500 feet. I find on referring to my notes that I was correct in this recollection. I had set them down as of that height and possibly higher. The place is on the Sutlej, about seven miles above the confluence of the Buspa, in the district called Koonáwur, the great grape-growing country of the Himalaya. In addition to the note of the estimated height of the cliffs, I had observed in my note-book that they were very precipitous, almost and sometimes quite vertical. The path was all along the face of the cliff, now mounting high up to avoid some impracticable projection, and again similarly descending; the ascents managed by rude steps, at times very high and perpendicular. The path throughout a mere ledge, often extremely narrow, and occasionally supplemented by a trunk of a tree thrown across a chasm and in contact with the vertical face of rock, its ends resting on the projecting ledges forming the path.

The above is the description of the place as obtained from my note-book. At the base of this cliff flowed the Sutlej, here a very full and impetuous river. The rocks are gneiss and clay slate.

I am ashamed to have so long omitted to write to you and give the above information, which may be interesting, as confirming what I stated to you, with only half confidence, at Amherst.

22. The famous Cow's Mouth, in the Himalaya mountains, appears to be an enormous gorge cut by the Ganges, through a part of that chain. Some other similar cuts are described on that river; but I have not the authorities at hand for a minute description.

23. *Ravines on the west side of the Sierra Nevada Mountains, in California.*—The general character of the western slope of these mountains is thus stated by Philip T. Tyson, Esq., in his Report to the Government, on the Geology of California (p. 7): "The western flanks of the Sierra, as far as observed, consist of a vast mass of metamorphic and hypogene rocks, stretching from the Sacramento valley to the axis of the mountain. This mass of matter has an average slope from the valley upwards of 180 feet to the mile, thus giving a great rate of fall to the streams which rise in the vicinity of the snow peaks: these, aided by the decomposing energies of atmospheric agents, have excavated ravines of enormous depths, reaching along some branches of the American river at least 3500 feet. Into these, other ravines open with their innumerable tributaries, which, by intersecting the country in every direction, give it the appearance of a group of rounded and conical mountains." The following are examples of these ravines:—

1. South Fork of Yuba river: about 3000 feet deep.
2. North Fork of the American river: 3000 feet deep.
3. Middle Fork, not quite so deep.
4. South Fork, of a similar character.
5. Mokelumne river: 2000 feet.

The following extract from a private letter, from Mr. J. S. Daggett, principal of the Academy, in Americus, Georgia, gives so clear an idea of some of the features of the western slope of the Sierras of California, that I take the liberty, without consulting him, to insert it:—

"In the passage from San Francisco to the mining or mountainous region of the interior of the State of California, one cannot but be sensibly impressed with the geological features of the country, which give indication, amounting almost to positive proof, that the whole of that part of the State has (recently) been submerged in the waters of the ocean. The very entrance to San Francisco bay, has evidently been made by the erosive power of the sea; it being not more than a quarter of a mile in width, with cliffs rising nearly perpendicularly several hundred feet on either side. And the mouth of the Sacramento river is still less in width, and having similar cliffs of solid stone, through which in time it has worn its way down to its present level. This is where it forms its passage through the coast range of the Sierra Nevada mountains, between which and the interior range lies the valley of the Sacramento. This valley is remarkable for its appearance of having once been the bed of an inland sea. You ascend the river some eighty miles from its entrance into the bay, by steamer, and after having passed the coast range, its banks stretch away into the vast level of the plain, with nothing to intercept the view but the tall waving grass and weeds. You then proceed about the same distance by stage to the mountains, and as you are whirled along over an almost level surface of sand, gravel, and marl, occasionally crossing the beds of rivers now dry, you are led to think that where you are now riding leisurely along, the mighty giants of ocean once sported and played. On leaving the valley to ascend the mountains, you see on either hand, stretching away to the north and south, perpendicular cliffs of basaltic rock, and vast columnar palisades, which present every appearance of having once withstood the action of the waves. On arriving among the mountains, the rocks indicate their volcanic origin, and present a varied and interesting aspect. I never witnessed scenery more grand and terrific than that on the rivers that flow down the western slope of these mountains. You are walking along over gentle eminences and little vales sprinkled with a thousand various flowers, and crowned with giant pines and cedars, when suddenly your ear catches the faint roar of distant waters, and immediately you are standing upon the brink of a precipice more than two thousand feet in perpendicular height, at the base of which you see the river foaming and dashing along, over rocks and cliffs, and madly seeking its way to the far off valley of the Sacramento, the opening to which you can just discern in the distance. Turning your eyes towards the source of the river, you behold the eternally snow-clad summits of the Sierras, peering high amid the clouds, and reflecting the beams of the sun. Opposite to you you see the various rock formations through which the river for numberless years has been cutting its way. Occasionally veins of quartz appear like banks of snow amid augitic and feldspathic granite, awakening interesting conjectures in the scientific mind. These manifestations of power have an effect of awe and sublimity upon the mind of the beholder, and lead him to wonder and adore the Omnipotent Creator."

24. *Passage of the river Zaire through the mountains, a distance of* 40 *miles, in Central Africa.*—The width of the stream is from 300 to 500 yards. The channel, everywhere "bristled with rocks" of mica slate, quartz rock, and syenite: in many places they were "stupendous overhanging rocks."—*Tuckey's Narrative*, pp. 176 and 349.

25. *Valley of erosion in the western part of New Fane, in Vermont.*—Upon a lofty hill in the west part of New Fane, is an extensive bed of serpentine, associated with soapstone, running nearly north and south. On the west side the hill slopes rapidly towards a small stream, which lies a little over 300 feet below the summit. A similar slope rises on the west side of the brook, extending into Dover. In the soapstone bed, near the top of the hill, are distinct pot-holes, which I regard as decisive evidence that a stream once ran there and formed a cataract. The conclusion is irresistible that the present stream, or its progenitor, once ran over this spot, and consequently that broad valley has been subsequently worn out. On Plate XII., Fig. 7, I have exhibited this valley with its sides having the slope which was determined by the clinometer.

This is an instructive case. For if this valley has been the result of river action, one could easily be made to believe that almost any other valley in the mountainous parts of our country had a similar origin.

2. *In Metamorphic and Silurian Rocks and newer Sandstones.*

1. *Gulf between Lake Ontario and Niagara Falls.*—These falls are at present six and a half miles from Lake Ontario, at Lewiston: and the whole distance the river runs in a gulf, which at the falls is 160 feet, and at Lewiston, 300 feet deep, and generally about twice as wide at the top as at the bottom. The rocks passed through by the receding falls are the Medina sandstone, the Clinton group of limestone and shale, and the Niagara limestone and shale. All these rocks, except the Niagara group, having a slight southerly dip, have disappeared beneath the bed of the river, and the falls are now in the Niagara group entirely, the shale lying beneath the limestone.

At the Whirlpool, a little more than three miles below the falls, on the west bank of the river, the continuity of the rock forming the bank is interrupted by a deep ravine filled with drift materials. This ravine may be traced two miles in a northwest direction, and from thence another depression can be followed to Ontario, at St. Davids, four miles west of Queenston.

It appears probable, as Professor James Hall has shown in his Report on the fourth District of New York, p. 389, that this ancient ravine may have been formed by oceanic rather than fluviatile action. For its opening on the lake at St. Davids is two miles wide: while that of Niagara river is about a third of one mile. And width of opening is one of the peculiarities of oceanic action, when it forms indentations along a coast, save in the case of purgatories, which are dependent upon a peculiar structure of the rocks. Although, therefore, it be not certain that Niagara river, or the river on a former continent that corresponded to the present Niagara, emptied into Ontario through this ancient ravine, yet since the ravine can be traced no further than the present river, this probably was the lowest part of the country between the falls and Ontario, and not improbably, therefore, the water of lake Erie would find this outlet into Ontario. It is clear, however, that the present channel of the river from Ontario to the falls, has been excavated since the drift period. For when the ravine to St. Davids was blocked up by drift materials, the stream would be forced to find its present rocky channel. Even though the drift rose only a foot higher than the rocks, it would as effectually force the waters over the rocks as if it formed a mountain. Could the river have once surmounted the drift, its work would have been comparatively easy in wearing out a bed through the old ravine. But till it was able to flow over the barrier, it would have no power over it, and must commence its slow work of wearing away the solid rock. The present gulf shows us what it has done since the drift period.

The above case, as well as the three following examples, are treated in much greater detail, and with much ability, by Professor James Hall, in his Report on the fourth District of New York.

2. *Gulf of Genesee River between Rochester and Lake Ontario.*—The Genesee river is remarkable for the striking examples of erosion which it exhibits. Beginning at its mouth, on the south shore of Ontario, we find three cataracts between that point and Rochester, which is about seven miles. Three distinct groups of

strata are crossed, viz: the Medina sandstone (lowest), the Clinton group next, and the Niagara group highest. It is evidently the different hardness of these groups, or varying facility of decomposition, that have produced these falls. In such a case we have indubitable proof that the river has done the work. These falls at first were but one, and at this time the lower ones are gaining probably upon the upper one, and the time may come when they will unite again.

A few miles east of the mouth of Genesee river, the Irondequoit creek empties into the lake, flowing in a deeper channel than the Genesee. But it passes through deposites of sand and gravel, and Professor Hall suggests, with much probability, that the Genesee once ran in the channel of the Irondequoit. But when that was filled with deposits of sand and gravel, and the region elevated, the Genesee was turned westward and compelled to cut out its present rocky bed, like the Niagara, of about seven miles in length. I am not able to state the amount of descent in the three falls: my aneroid gave 107 feet for the height of the largest at Rochester.

3. *Gulf of the Genesee River between Mount Morris and Portage.*—We have at this place a still more remarkable example of a postdiluvian cut in the rock in consequence of the filling up of the old channel. From Rochester to Mount Morris the Genesee river occupies for the most part a broad valley with no gorges of importance. But at Mount Morris it issues from high walls of Devonian rocks (the Portage and Chemung groups), and if we follow its course upwards to Portage, fourteen miles, we shall find its bed to be a deep cut in solid rock much of the way, with nearly perpendicular walls, but sometimes sloping so as to admit narrow meadows. It is not till you get considerably above St. Helena that you come to cataracts. In Portage, within a distance of less than two miles, are three falls, whose whole amount, with the intervening rapids, by my aneroid barometer, is 370 feet. The falls are said to be 60, 90, and 110 feet. *Am. Journ. Sci.*, vol. XVIII., p. 209. The depth of the gorge in some places is not less than 350 feet, and its width only about 600 feet, the banks being nearly perpendicular. Were the quantity of water in Genesee river as great as in Niagara river, the scenery on the former at Portage would be more imposing, on account of the greater depth of the gulf. As it is, it is well worth a visit, now easily made, as the railroad from Hornellsville to Attica crosses the river a little below the middle falls, if I rightly recollect.

In passing from Portage at the south end of this gorge, and near the upper falls, towards Nunda, we rise upon a deposit of sand and gravel of great depth, according to my measurements, 235 feet thick at the head of this upper fall. This deposit extends to Nunda, which place, by the aneroid barometer, lies 135 feet below the Genesee at Portage. From Portage a canal extends through Nunda to Mount Morris, following down from the former place a tributary of the Genesee. I fully agree with Professor Hall in his suggestion, that this was once the bed of the Genesee river: which being filled with drift and terrace materials, while the country was yet beneath the ocean, was compelled, upon the emergence of the land, to find a new tortuous channel more to the left. The result has been that it has cut out its present channel; that is, the deep gorge between Portage and

Mount Morris, since the drift period. I copy on Plate XII., Fig. 3, Mr. Hall's sketch illustrative of this view. On the right is shown the present bed of the river, and on the left, the ancient valley, now filled with sand, gravel, and clay.

4. *Bed of Oak Orchard Creek in Orleans county, New York.*—This is a small stream that empties into lake Ontario, passing across the same strata as the Genesee, viz: the Medina sandstone and the Clinton and Niagara groups. As we might expect, we find a similar series of cataracts and rapids: but I am unable to give any details as to their height, distance from one another, &c. The case is interesting, however, as lending additional confirmation to the views already presented, as to the fluviatile origin of the erosions in such streams as the Niagara and Genesee.

5. *Gorge on the Au Sable River in Essex county, New York.*—On the west side of lake Champlain, not far from Keesville, this river has cut a passage for a great distance through the Potsdam sandstone, which shows strikingly the excavating power of water. At Birmingham is a gorge two miles long and 100 feet deep. The best place for visiting it, is at a spot called High Bridge, where stairs have been cut in the walls to the bottom of the gulf, and as you stand there, the frowning and even overhanging walls almost shut out the light of day. No man at that spot could imagine any other agency but the stream itself to produce such a gulf. I mean no man accustomed to reason upon this class of geological phenomena. The average width of the gorge is only from 20 to 40 feet.

This spot may be reached by steamboat and two or three miles land travel, from Burlington, Vermont, and well repays the visitor.—*Emmons' Geological Report on the Second District*, p. 266.

6. *Water Gap on Delaware River, in New Jersey.*—Macculloch, in his Geographical Dictionary, states this gap to be 1200 feet deep and two miles long. I have not visited the spot, nor have I been able to ascertain whether the rocks be crystalline or Silurian.

In examining the valley in which Port Jervis and Delaware are situated, 40 miles above the gap, on Delaware river, I became satisfied that this river once ran northeasterly towards the Hudson river. And I am informed by H. N. Farnum, Esq., of Port Jervis, that the summit level of the Delaware and Hudson canal is only 115 feet above the Delaware opposite Port Jervis. If, therefore, the Water Gap were closed to the height of 115 feet, plus the descent of the river between the gap and Port Jervis, the Delaware would be turned into the Hudson. Such I can hardly doubt was the course of its predecessor on a former continent. But during the last submergence of that region, the old bed was filled with gravel and sand, so as to turn the Delaware towards the Water Gap, and probably some of the erosion there has been effected since the last emergence of our continent. The valley of the Delaware and Hudson canal, therefore, adds another example of an antediluvian river bed. I do not, however, feel so confident in this conclusion as I should if I had examined the whole ground.

7. *Gorge on Delaware River from Port Jervis to Narrowsburg.*—This is a deep and crooked gorge about 25 miles long, exhibiting some of the wildest scenery in our country, yet distinguished by two works of art of great magnitude and import-

ance: one is the canal leading from the coal mines of northern Pennsylvania; the other, the Erie railroad: both cut out of rock in many places, and overhung as it were by ragged precipices. It is impossible to ascertain the depth eroded by the river, because the banks are so irregular. Near the lower end, however, it is obvious that Mount Butler, on the New York side of the river, once constituted the barrier that has been cut through. It is 750 feet above the river at its base, and I thought I discovered traces of river action nearly all the way upwards on its steep face, and in some places on the top, although drift striæ are found in some prominent places. From Narrowsburg to Port Jervis the river descends, according to the aneroid barometer, 215 feet; so that if the barrier was once closed as high as Mount Butler, a narrow lake must have reached much further than Narrowsburg.

The course of the stream through this gorge is quite crooked. Of course it has been thrown with great power against particular spots and worn them away more rapidly, so as to form flats on the opposite side. In such cases these flats are almost invariably occupied by terraces of rather coarse gravel and considerable elevation. The serpentine course of the stream precludes the idea of the ocean's having worn out the gorge to any great extent.

8. *The Grand Cañon on Canadian River, in the country of the Camanche Indians.*—The southwestern portion of the United States, this side of the Rocky Mountains, is remarkable for the numerous deep gulfs through which the rivers run. These are called *Cañons.* Often they occur in a level region, where the strata, usually sandstone, lie nearly horizontal. In such a case the traveller, as he passes over the plain, sees no signs of a river till he finds himself suddenly stopped by a gulf, it may be several hundred feet deep, with walls nearly perpendicular, and sometimes for a day or two may he travel along the stream, unable to find a spot where he or his animals can get to the water. He meets with another difficulty, also, if he passes along the stream in the hope of finding a crossing place. When he comes to a tributary stream, he will find most likely a cañon, nearly as deep as on the main river, and he will be forced to diverge along the tributary, till he can find a passage over the gulf. Thus will he be compelled to deviate almost continually from his direct course, and moreover be tantalized by the sight of water in the inaccessible gulf below him, while his animals are nearly perishing with thirst.

I apprehend that travellers apply the term *cañon* to mountain gorges as well as the gulfs above described, and doubtless it would be proper to use the term in describing eroded gorges in the northern parts of our country; certainly to such as exist on the Niagara and Genesee rivers. But some of those described by officers connected with the United States army, are of a depth and extent much greater than any that have been mentioned. I shall give only a few examples, partly because out of the many that have been described by travellers, the facts respecting them are not given with sufficient definiteness to answer my purpose.

Some writers do, indeed, speak of convulsions as the cause of these cañons, just as they do concerning the gulfs at Niagara and Portage. But the fact that they exist along the tributaries, as well as the main stream, shows that they are the work of erosion. For when have faults been known to take the ramified form of the tributaries to a river?

Lt. J. W. Abert, in his Report to the Government (p. 22), describes the Grand Cañon on the Canadian as an immense gulf, several hundred feet deep, with almost perpendicular walls. Mr. Stanley says, "we travelled fifty miles, the whole of which distance is bounded in by cliffs several hundred feet high, in many places perpendicular." Lt. Peck found the walls to be about 250 feet high, but he does not mention the length of the cut. The rock is described as shale.

9. *Cañons on the Pecos River, in New Mexico.*—These are thus described by Capt. S. G. French, in his Report to the Government, of a route over which he passed from San Antonio, in Texas, to El Paso del Norte, p. 45. "The Pecos is a remarkable stream, narrow and deep, extremely crooked in its course, and rapid in its current. Its banks are steep, and in a course of 240 miles, there are but few places where an animal can approach them for water in safety. Not a tree or bush marks its course; and one may stand on its banks and not know that the stream is near."

10. *Cañon of Chelly, in New Mexico.*—On the Map of Lt. James H. Simpson, in his Report to the Government, of an expedition among the Navajos Indians, west of the Rio Grande, we find no less than four Cañons laid down and noticed. But the most remarkable is that of Chelly, on the Rio de Chelly of Simpson, but the Red River of Monk's Map, in long. 109½° and N. lat. 36°. It is cut through red sandstone: its width at bottom varies from 150 to 300 or 400 feet: the height of its perpendicular wall is from 200 to 800 feet: and its whole length not less than 25 miles. This is certainly one of the most remarkable defiles that have ever been described. A view of this cañon, eight miles from its mouth, as given by Lt. Simpson, has been copied and accompanies this paper. See Plate XII. fig. 9.

11. A cañon still more remarkable, certainly for length, has been described by Capt. R. B. Marcy, of the United States Army, in a lecture before the American Geographical and Statistical Society, in New York, March 22, 1853, giving an account of his exploration of the head branches of Red river, in Texas. This river takes its rise in the desert table land, called Llano Estacado, which is elevated above the sea 3650 feet, and which extends from the Canadian river southerly for 400 miles, between 101° and 104° W. long., and 32° 30′ N. lat. to 36° 20′. The gorge on Red river, as it comes out from the sandstone of this *mesa*, says Capt. Marcy, "is 70 miles long, and the escarpments from 500 to 800 feet high on each side, and in many places they approach so near the water's edge, that there is not room for a man to pass; and occasionally it is necessary to travel for miles in the bed of the river, before a spot is found where a horse can clamber up the precipitous sides of the chasm." Near the upper part of the chasm, he says, "the gigantic walls of sandstone, rising to the enormous height of 800 feet on each side, gradually closed in, until they were only a few yards apart, and at last united above us, leaving a long narrow corridor beneath, at the base of which the head spring of the principal branch of Red river takes its rise." "The magnificence of the views that presented themselves, as we approached the head of the river, exceeded anything I had ever beheld. It is impossible for me to describe

the sensations of intense pleasure I experienced, as I gazed on these grand and novel displays of nature."—*See also Capt. Marcy's Report*, p. 55, *et seq.*

12. *Hot Spring Gate, on the River Platte, in about* 107° *W. longitude.*—The river here passes through a hill of white calcareous sandstone, a distance of about 1200 feet, with a depth of about 360 feet. At both extremities is a smooth green prairie. Col. J. C. Fremont has named, described, and given a sketch of this gorge in his first Report, p. 55.

13. *Rapids in St. Louis River, west of Lake Superior, towards the Portage aux Coteaux.*—The gorge is 36 miles long at least, and the walls from 30 to 40 feet high, in argillaceous slate. In that distance are four distinct falls, each made up of several distinct cascades. Here doubtless the work of erosion and retrocession is going on at every cascade.—*Owen's Report on Wisconsin and Iowa, in* 1848, p. 79.

14. *Cañon in the Rocky Mountains, on one of the branches of Snake or Lewis River.*—The distance through it occupied a half day's travel, the walls are very precipitous and high, and the rocks are sandstone, limestone, and gypsum.—*Parker's Exploration*, 3d edition, p. 87.

15. *Gulfs of Loraine and Redmond, in Jefferson county, New York.*—These appear to be genuine cañons upon the small streams flowing through the Trenton limestone, Utica slate, and Loraine shales of those towns. Those are the most striking upon South Sandy creek. The walls are perpendicular, and vary in height from 100 to 300 feet. The width of the gulf varies of course, and is sometimes as much as sixteen rods. The length of some of them is over twelve miles, reaching to the very starting-point of the streams.—*Emmons' Report on the Second Geological District of New York*, p. 408.

16. *Gorge on Cox River, in New South Wales, in Australia.*—This is 2200 yards wide and 800 feet deep, cut in horizontally stratified sandstone. From this valley Major Mitchell estimates that 134 cubic miles of stone have been removed.—*Am. Journal Science*, vol. IX., New Series, p. 290.

17. *Kangaroo Valley* is another example of erosion in the same country. It is two or three miles wide, and from 1000 to 1800 feet deep, opening outward through a comparatively narrow gorge. Professor Dana estimates the amount of rock necessary to fill the valley, and which has been removed, to be equal to "a rectangular ridge 12 miles long, two miles wide, and 2000 feet high."

The above are only two out of a multitude of valleys in New South Wales, which have been excavated in horizontal strata of sandstone. They are usually narrowest and deepest towards the sea, resembling a harbor with a narrow entrance. Professor Dana has shown in a conclusive manner, that these valleys are the work of running water, and not of convulsions or of original creation.—*Am. Journal of Science*, vol. IX., New Series, p. 289.

18. *Gorge of the Rhine, between Coblentz and Bingen.*—All that distance, nearly 50 miles, the river has cut across ranges of mountains of the older fossiliferous rocks, to a depth sometimes as great as 1000 feet. The gulf is a true mountain gorge, and the banks are so precipitous as scarcely to afford room for a narrow terrace. The idea that the waves of the ocean, or a rent by internal forces, pro-

duced this gulf, is made exceedingly improbable by the tortuousness of its course, which would prevent the action of waves, and would be followed by no volcanic rent. It corresponds, however, to the known effects of currents of water, when a country is undergoing drainage.

I might perhaps consider the gorge of the Rhine as commencing as far down as the Drachenfels, and extending even to Mayence. But the valley is a good deal wider at these extremities, and I prefer to confine this example to that portion of it which seems unequivocally the work of river action.

The strata cross the Rhine nearly at right angles, and appeared to me, from the steamboat, to dip 60° to 70° S. easterly.

19. *Valley of erosion in Dorset, Vermont.*—Those who have passed from Manchester to Rutland, in Vermont, on the Western Vermont railroad, will not forget how narrow the valley is, especially in Dorset and Danby. Its east side is formed by the Green Mountains, and its west side by a ridge not so high, which at its southern extremity has received the name of Dorset Mountain. Near the base of Dorset mountain the Otter creek takes its rise, and runs northerly into Lake Champlain, at Vergennes. Near the same spot rises the Battenkill, which runs southwesterly and empties into the Hudson at Greenwich. Both these streams are mere brooks at the base of Dorset mountain, and the idea that they ever wore out the valley in which they run, is quite absurd, especially as they flow in opposite directions. Dorset mountain, according to the careful measurements of Mr. W. A. Burnham, teacher in Burr Seminary, in Manchester, is 1627 feet above the valley, whose summit-level must be near the base of the mountain. This is, however, a valley of erosion; for near the top of Dorset mountain is a thick bed of white limestone, which is interstratified with a metamorphic talcose slate, sometimes called the Taconic slate. The mountain rises very precipitously from the valley, being almost perpendicular on its east side, and in the limestone, not far from 1600 feet above the valley, is a cavern opening towards the valley, and sloping towards the west, as represented in Plate XII. fig. 5. On exploring this cavern for several rods, I met with unequivocal evidence that it had been formed by running water. I traced it several rods into the mountain, and think it may be followed much further.

Now the conclusion is a legitimate one that a stream of water of considerable size once, and for a long time, ran through this opening. Consequently the valley east of it must have been filled to the height of the stream, in order to form a surface for a river bed. Consequently the valley, 1600 feet deep, and many miles long, must have been excavated since that period; for I saw no evidence of any upheaval of this mountain at a subsequent date.

What agencies were concerned in this work, it may be difficult fully to understand. It is certain that existing streams have not produced it. Drift agency, while the continent was beneath the ocean, may have had some effect; as, also, the slow action of the waves during the vertical movements of the land. But the length, narrowness, and depth of the valley, and the steepness of its sides, agree better with river action, and I cannot doubt that the work was mainly accomplished by that agency on some continent long, long anterior to the present.

Further north, on the same continuous ridge, is at least one other cavern in limestone, which is said to have been penetrated 150 feet in depth, without reaching its bottom. But I have not visited it, and know neither its height above the valley, nor whether it was an ancient river bed: though every such cavern, which I have visited in the limestone of New England, has seemed to have been thus produced.

If this valley in Dorset was formed by aqueous erosion, it is highly probable that the many other deep and narrow valleys in the same metamorphic rocks in Vermont and Massachusetts, especially in Berkshire county, were formed in a similar manner. On no other theory could I explain their existence, even had we not this striking fact of the eroded cavern on the top of Dorset mountain.

I might extend this inference to a large part of the deep valleys of our country. At least, such facts afford a presumption that many of them were probably the beds of rivers on former continents. Here, then, it appears to me, is an interesting field of geological inquiry, rarely entered, yet capable of exploration. I mean the determination of the period and manner in which our ancient valleys have been formed.

20. *Gorge on New River, a tributary of the Kenawha, in western Virginia.*—Dr. Hildreth describes this gorge as having nearly perpendicular walls of 800 feet in height, and its whole length is 50 or 60 miles. Indeed the entire valley of the Kenawha river, so far as I have ascended it, appears manifestly to have been worn out in the nearly horizontal sandstone, shale, and fire-clay, of the coal formation. The hills along its lower part, however, rarely rise higher than 400 or 500 feet.—*American Journal of Science*, vol. XXIX. p. 91.

21. *The Valley of the Mississippi, for two hundred miles above the mouth of the Missouri.*—I select this part of this valley, because the proof of its erosion seems quite obvious, by looking at Professor Owen's geological map, appended to his Report upon Wisconsin, Iowa, and Minnesota; or upon a similar map in Taylor's Statistics of Coal. On the east side of the river and at some distance, is exhibited the Illinois coal field; and on the west side, that of Missouri and Iowa. Approaching the river from either side, we find the coal measures swept away, and the carboniferous limestone, the next rock beneath, brought to light. Still nearer the river, we find rocks of a still older date, because the valley is deeper. How obvious that these coal fields were once united, and that the coal measures have been swept away by the action of water! What portion is gone I am unable to state: but the fact that powerful erosion has taken place seems too evident to be doubted. Most other river beds present similar facts: but they do not usually stand out so distinctly.

22. *Big Cañon on the Rio Colorado of the West.*—This occurs in W. long. 115° and N. lat. 36°; but I have not been able to find any detailed account of its extent. Where Capt. Sitgreaves struck a cañon on the Zuñi, or Little Colorado, which he was assured extended to the Rio Colorado, its depth was 120 feet, less probably than that of the Big cañon.—*Sitgreaves' Report to Government*, p. 8.

23. *Dalles of the Wisconsin River, in Wisconsin.*—The length of this gorge in sandstone, is five or six miles, and the height of the wall from 40 to 120 feet.—*Owen's Report on a Survey of Wisconsin, &c.*, p. 517.

3. *In Limestone chiefly.*

Some of the cases already described are partly in limestone and some of those now to be presented are partly in other rock; but I shall bring those only under the present division that are chiefly in limestone.

1. *Gulf at the Natural Bridge, in Rockbridge county, Virginia.*—The width of this gorge is 50 feet at bottom, and 90 feet at top; and the height of the bridge is 215 feet above the stream; its length I have not been able to ascertain.

2. *Gulf at the Natural Bridge, in Lee county, Virginia.*—The walls here are 339 feet high, and the width of the stream from 35 to 55 feet. The length of the gulf is not given, but the stream itself (Stock creek) is only a few miles long.

3. *Glenn's Falls, on Hudson River, Warren county, New York.*—These are in black limestone, and the gorge is of considerable depth and length, but though I have visited the spot I have made no measurements. The height of the falls is about 50 feet.

4. *Trenton Falls, on West Canada Creek, in Oneida county, New York.*—These are also in the black Trenton limestone. The gorge is very deep and extends for at least two miles; in which space are six cataracts. In passing through this gorge I was much impressed with the power of water to wear away unyielding rock.

5. *St. Anthony's Falls, on the Mississippi.*—The surface rock, over which this large river, 1800 feet wide, is precipitated, is limestone, underlaid by friable sandstone. The latter easily disintegrates and undermines the limestone, which falls at length by the force of gravity, piece after piece. In this manner have these falls receded seven miles from the mouth of St. Peter's river. The fall of water at present is only about 17 feet. From these falls to the mouth of the Wisconsin, some 130 miles, the river passes through limestone, and has walls of rock, but I have not met with any description definite enough to decide whether its bed has been eroded all the way.

6. *Cañada (little Cañon), of Santa Domingo, in Oaxaca, a province of Mexico.*—This gorge is from 10 to 30 feet wide, 25 miles long, and the immediate walls 300 feet high. Back from the river a mile, the mountains rise to the height of 2000 feet. This case was described to me by the late Mr. George R. Ferguson, who was employed as an engineer upon the Tehuantepec railroad. The rock is limestone.

7. The same gentleman mentioned another gorge on the river Tobasco, in the province of Chiapas, in Mexico. It is 300 feet deep, but its length he could not give. This also is in limestone.

8. *Defile of Karzan, on the Danube, on the borders of Hungary and Turkey, a little above Orsova.*—The river here is only about 600 feet wide, and the perpendicular walls of limestone and slate, are 2000 feet high; and the water is 170 feet deep. For many miles above this, a similar defile exists, and it is one of the most remarkable gorges, or rather succession of gorges, between successive basins, on the globe.—*Murray's Handbook for Southern Germany*, 5th edition, p. 511.

9. *The Via Mala, on the Rhine, near Thusis, in Switzerland.*—The rocks are slate and limestone, and the river is here compressed for the distance of four miles, into

a gorge often not more than 30 feet wide, but 1600 feet deep; said to be the most remarkable defile in Switzerland. A road has been blasted through the overhanging rocks, high above the river, the Middle Bridge being not less than 400 feet high. Yet, in 1834, the river rose nearly to this bridge.—*Handbook for Switzerland.* Paris, 1849, p. 222.

10. *Wady Barida, in Anti-Lebanon, in Syria.*—This is a long gorge (length not given), in limestone, with walls from 600 to 800 feet high.—*Described by Rev. Mr. Thompson, American Missionary, in the Bibliotheca Sacra,* vol. V. p. 762.

11. *Gorge and Natural Bridge on Dog River, the Lycus of the Ancients, in Mount Lebanon.*—The width of this gorge is from 120 to 160 feet; its length six miles, and the height of the bridge, 70 to 80 feet. Span of the arch, 163 feet.—*Described by Mr. Thompson, in the Bibliotheca,* vol. V. p. 2.

12. *Gorge in limestone and a Natural Bridge, on the River Litany, in Mount Lebanon.*—This stupendous gorge is many miles long: and so narrow in many places that persons standing on the opposite sides can converse. The walls are in the deepest part a thousand feet high. The bridge is formed by large rocks falling from the cliffs. This spot deserves more minuteness of detail. It is described by Rev. Eli Smith, of Beirut.—*Bibliotheca Sacra,* vol. VI. p. 373.

13. *On the Euphrates, near Diadeen, in Armenia.*—A natural bridge occurs here, 100 feet wide, 150 feet high, and more than 100 feet long. Another bridge occurs 50 rods lower down the stream, 40 feet high, and 100 feet wide. The banks of the river, for miles above and below these bridges, are 100 feet high. No less than eight hot sulphur springs occur on the banks of the river at the bridges, which reach down even to high water.—*Letter from Rev. Justin Perkins, D. D., American Missionary, dated at Ooroomiah, July* 20, 1848.

14. *On the River Raveondooz, near a town of that name in the Koordish Mountains.*—This river, says Dr. Perkins, is "about as large as Chicopee river (in Massachusetts), and is engulfed between perpendicular limestone banks, that rival in awful grandeur those of the Euphrates, above Diadeen, and are indeed quite unparalleled by anything of the kind I have ever seen, even the banks of the Niagara below the falls; except that the river itself is small. These perpendicular rocky banks are in some places nearly a thousand feet high."—*Letter from Dr. Perkins, dated July* 9, 1849.

15. "There is a similar gorge, on our return route (from Mosul to Ooroomiah), on the river Sheen, in Jeloo."—*Same letter.*

16. *Wady el Jeib, at the south end of the Dead Sea, in Palestine.*—This is a gorge lying at the bottom of Wady Arabah, a wady within a wady, and has been apparently excavated by the winter streams that flow northward into the Dead Sea. It commences 40 miles south of that sea, and terminates a few miles south of it, where a limestone terrace stretches across the wady Arabah. The width of the defile, at its lower part, is half a mile, and the height of its walls 150 feet. It is in soft limestone, belonging probably to the cretaceous formation.—*See Robinson and Smith's Bib. Researches in Palestine, &c.*

4. *In Unstratified Rocks chiefly.*

1. *Devil's Gate.*—Near the rock Independence, on the Sweet Water, in the Rocky Mountains. Length of the gorge, 900 feet. Height of the walls, 400 feet. Width of the gorge, 105 feet. In granite.—*Fremont's First Tour,* p. 67, with a drawing; also *Fremont's Second Tour,* p. 164, with a plate.

2. *The American Falls, on Lewis' Fork of Columbia River.*—Width of the river, which is contracted at the falls, 870 feet. From these falls the river runs between walls of trap, with occasional interruptions, to the Dalles, or "trough," of the lower Columbia, 800 miles.

3. *The Dalles, or "trough," and rapids, near the mouth of Columbia River.*—The basaltic walls here, although not of great height, are continuous for six miles. Perhaps I ought to consider this example as embraced in the last.—*Parker's Exploring Tour,* pp. 142 and 318.

4. *The Cascades on the Columbia,* 50 *miles below the Dalles, or falls.*—The walls are trap, from 100 to 400 feet high, and five miles long.—*Parker's Ex. Tour,* p. 142 and 318.

5. *Gorge on Columbia River, a little below Fort Wallah Wallah.*—This gorge in trap, is from two to three miles long, and 300 feet deep.—*Parker's Ex. Tour,* p. 132.

6. *Pavilion River,* which empties into the Columbia a little above Wallah Wallah, is walled up with trap some 15 or 20 miles.—*Same work,* p. 289.

It seems that the Columbia river and many of its tributaries pass through deep and almost continuous cuts in the hard trap for several hundred miles. The above cases are merely some of the most striking spots.

7. *The Dalles of St. Croix River, in Wisconsin,* 30 *miles above its mouth.*—This gorge in trap, is at least half a mile long, and from 100 to 170 feet deep.—*Owen's Report on a Survey of Wisconsin, &c.,* p. 164, *and a beautiful sketch on* p. 142.

8. *Gorge and Falls, on Pigeon river, in Wisconsin.*—This is near the mouth of the stream, which is 75 feet wide, falls 60 feet, and then pursues its way for 600 feet, in a deep trough in trap.—*Owen's Rep.,* p. 405, *with a sketch.*

9. *Adirondac Pass, in the Mountains of Essex county, New York.*—This, as described by Professor Emmons, appears to be an immense gulf in the peculiar granite, or hypersthene rock of that region, whose bottom is filled to a great depth with fragments of rock broken from the walls. Those walls on one side present a perpendicular front 1000 feet high, and three-quarters of a mile long. Professor Emmons thinks that the detritus is 500 feet deep, making the original gulf 1500 feet. Whether it was excavated by a river, or by the ocean, producing a purgatory, his description does not enable us to determine.—*Emmons' Report on the Geology of New York,* p. 216.

10. *Erosions in trap, in the Ghaut mountains of southern India.*—Probably the largest outburst or overflow of trap in the world exists in southern India, extending from latitude 16° to 25°, at least, or nearly 600 miles, and some hundreds of

miles east and west. The Ghaut mountains lie near the western coast, rising from 2000 to 7000 feet, with high table lands stretching away from their east side. This region is penetrated by numerous valleys, sometimes 600, 800, or even 1000 feet deep, whose precipitous sides exhibit numerous alternations of compact, amygdaloidal, and tufaceous trap, capped by laterite and red clay, in layers apparently horizontal. The same layers appear on both sides of the valleys undisturbed; showing, beyond question, that the depressions have been the work of erosion rather than of internal upheaving forces. These valleys are numerous, especially along the western face of the Ghauts, and the eye can often take in a distance of 10 to 15 miles; the layers of trap showing continuous stripes all the way; nay, much further, if the observer travels along the valley.

These are certainly striking examples of erosion by streams, in a country where we cannot suppose ice to have aided the work. But tropical rains are very powerful. I am indebted for these facts to Rev. Ebenezer Burgess, missionary at Satara, in southern India, and who was formerly a resident for years at Ahmednugger, which lies in the same great trap region. To him, also, I am indebted for the facts stated in the next example. He visited Table Mountain, on his return recently, and obtained specimens of the different rocks composing it.

11. *Table Mountain, at the Cape of Good Hope, in Southern Africa.*—This is a vast mass of horizontal strata of sandstone, some 600 or 800 feet thick, superimposed upon granite and older inclined sandstone and metamorphic slate. The height of the mountain is stated at 3800 feet; which makes it visible 30 to 40 miles at sea. That this outlier of sandstone, capping Table mountain, must once have had a wide extent, no geologist will doubt, nor can it be reasonably questioned that it has been brought into its present shape by the action of the ocean, when this was at a higher level, or the mountain at a lower level. The slate and lower sandstone, that are inclined at a large angle, must have been tilted up by a force beneath, or acting laterally. But if the mountain has been raised since the deposition of the horizontal sandstone, it must have been a secular elevation, bringing up the continent bodily and equably.

12. *Table lands and intervening Valleys in the vicinity of Natal, in South Africa.*—Accident has put into my hands two sketches of the scenery in that region, with a description, from the pencil and pen of Mrs. Lydia B. Grout, wife of Rev. Mr. Grout, American missionary among the Zulus; and these are too appropriate to the object of this paper, and too well executed to be lost. I therefore have taken the liberty to append these drawings (Plate XI. figs. 1 and 2), and copy the accompanying descriptions, from a letter written by Mrs. Grout.

A glance at these drawings will satisfy the geologist that they represent a region analogous to Cape Town, and this makes it probable that these table lands are very extensive in Southern Africa, since Natal is some 800 miles north of the Cape. And we see enough in the drawing and description to satisfy any one that the erosions in Southern Africa have been on the same great scale as on other continents.

"The scene," says Mrs. Grout, "which it (Plate XI. fig. 2) portrays, is about three miles from our station. In going to it (the station), or rather to the brow of the hill or precipice on this side of it, we cross a table land like the one shown in the drawing. These table lands, with ten thousand little hills, are distinguishing natural features extending the whole length of the colony. Table lands are on each side of the valley of little hills. They are, however, broken at intervals, of perhaps about six miles; thus affording a passage to the large rivers that flow into the Indian ocean. Their terminations towards the valley are sometimes perpendicular precipices, several hundred feet in height, covered with bushes near the top, and breaking into numerous hills below. There is seldom a descent to the valley sufficiently gradual to allow a wagon to be driven down. I think there is not more than one such declivity from each table land."

"In some places these platforms are perpendicular for 20 feet or more at the top, and expose a face of sandstone, broken into a thousand fragments, which to a great extent retain their places. Sometimes, however, these fragments are strewn over the whole of the slopes of which I have spoken. There is an example in the foreground of the drawing (Plate XI. Fig. 2) on the right side. Some of the fragments, as exhibited, are very irregular, while others are rectangular. The width of the scene presented is perhaps four miles; but in most places the great valley is more extended. The widest part we have travelled over is about ten miles."

"Sometimes in the midst of these little hills single mountains rise, which seem to correspond in height with the table lands, and their sides present the same variety in appearance as do the latter. Examples of these mountains are given in the outline (Plate XI. Fig. 1). The tops are not more than 5 or 6 feet wide, and with the sides are covered with grass."

"It seems to an observer of this scenery, that the whole region, including table lands, mountains, peaks, and small hills, was once an immense plain, and that some mighty convulsion of nature brought them into their present state. Whether this great change was produced by fire or water, we are not geologists enough to decide."

These views of Mrs. Grout are doubtless correct, except "the mighty convulsions of nature," which were probably little more than the quiet and slow action of the present rivers, aided, perhaps, by the waves of a former ocean. But that no violent convulsion of nature has been concerned, is obvious from the horizontal position of the sandstone, forming the upper part of the table lands and the caps of the mountains. The case seems analogous to the *cañons* of our southwestern states.

This case might perhaps have been more properly introduced under the examples in sandstone. But I place it here in connection with the example from Cape Town, as it seems to belong to the same class of phenomena.

13. *Pass of Dariel Caucasus, on the River Terek, in Asia.*—Maculloch's Geographical Dictionary represents this pass as occurring in porphyry and schist, as being 120 miles long, and having walls 3000 feet high. Sir R. Ker Porter speaks of the walls as only 1000 feet high.—*Travels*, vol. I. p. 75.

14. *Source of the River Jordan, above Lake Huleh, in the mountains of Lebanon.*—Mr. Thompson, American missionary, describes it as a constantly deepening gorge, in basalt, six miles long.—*Bibliotheca Sacra*, vol. III. p. 135.

15. *Valleys in the volcanic islands of the Pacific Ocean.*—These valleys have been described, and their origin discussed with great ability, by Professor J. D. Dana, in his Geological Report of the United States Exploring Expedition. He divides them into three kinds: 1. "A narrow gorge with barely a pathway for a streamlet at bottom, the enclosing sides diverging upwards at an angle of 30° to 60°." 2. "A narrow gorge, having the walls vertical, or nearly so, and a flat strip of land at bottom, more or less uneven, with a streamlet." 3. "Valleys of the third kind have an extensive plain at bottom, quite unlike the strip of land just described." The valleys are one, two, or even three thousand feet deep, and the dividing ridges often so narrow as to be knife-like and tortuous. Professor Dana imputes their origin mainly to two causes: first, volcanic agency, which lifted up the mountains and produced inequalities and gulfs. Secondly, the action of rains producing brooks and rivers. The latter cause he thinks the chief one, though the ocean, especially when the islands were nearly submerged, must have produced some effect.

The phenomena of valleys in some parts of the great Appalachian coal field, as along the Ohio and Great Kenawha rivers, appear to me to sustain the view taken by Professor Dana, that streams of water chiefly have eroded the valleys of the Pacific islands. For along those rivers the coal measures lie nearly horizontal, and the rivers have evidently worn out their beds to the depth of some hundred feet, leaving bluffs of sandstone along their margin. And wherever brooks and streamlets have found their way to the main river we observe that they have worn out channels having the same steep sides as those of the Pacific islands. The ridges too, intervening between the brooks, are sharp, narrow, and tortuous, though not extremely so, like those of the Pacific islands. Now, in the horizontal coal strata, which have never been disturbed, we can impute the valleys and ridges to nothing but running water, and it is reasonable to refer the analogous phenomena of the Pacific islands to the same cause.

Conclusions.

From the facts that have been detailed, we may derive several inferences of considerable geological importance. With these I shall conclude this paper.

1. Some of the erosions that have been described, must have been commenced as early as the oldest rocks were consolidated.

They occur in the oldest hypozoic rocks, and were begun probably by the drainage of land at its first emergence from the waters. The hypozoic rocks are not indeed necessarily older than the fossiliferous. But sometimes they lie below the fossiliferous, and are too thick to be regarded as their lower metamorphic beds. I should place the following cases as among the earliest described in this paper:—

1. Valley of Connecticut river, which is for the most part formed in hypozoic rocks.

2. The Ghor, on Deerfield river, a tributary of the Connecticut: or rather the whole valley of Deerfield river west of Deerfield.

3. The valley of Hudson river, for the most part in hypozoic and the oldest Silurian rocks.

4. The valley of Agawam river, from Mount Tekoa, in Westfield, to the summit level of the Western railroad.

5. The cut at the summit level of the Northern railroad, in New Hampshire.

6. Dorset valley, on the west side of the Green Mountains, through which the small streams called Otter creek and the Batten kill now run. The rocks are hypozoic, or very old metamorphic.

7. Gorge on Little river, in Russell and Blandford, No. 13, in hypozoic rocks.

8. Gorge on the Potomac, below Great Falls, in Virginia, No. 15, in hypozoic rocks.

To these cases I might add probably nearly every valley through which rivers of considerable size run in the hypozoic regions of our country, especially of New England. But my object in this paper is not to describe all cases of erosions, but only to give some good examples, in order to call the attention of geologists to the subject.

As, however, it does not follow because a gorge is found in hypozoic rocks, that it is very ancient, I have thought that the following principles may enable us to decide with much probability whether a valley is of the most ancient class.

1. Such a valley must occur in the oldest rocks, viz: the hypozoic, early metamorphic, or Silurian.

2. It will have great width in its upper parts, its slopes will be gentle, its sides rounded, and with few precipitous gorges. Such effects could be produced only by oceanic agency, as the continent was repeatedly submerged and raised from the deep. The waves and currents, rushing back and forth through the gorges produced by streams, would give this breadth and rounded outline of the sides, and I know of no other cause that could have produced the effect.

3. The rivers in the oldest valleys have nearly ceased to deepen their beds, except perhaps where cataracts occur, and these are not usually of the most striking character.

4. The drift agency in such valleys has smoothed and striated the rocks nearly to the present level of the streams, and thus afforded proof that the beds have not been much deepened since the drift period.

2. The work of erosion in these oldest valleys must have been repeatedly interrupted, varied, and renewed, by vertical movements. Some of the continents, perhaps all of them, have been subject to such movements, as is obvious from the character of the strata and their embedded organic remains. At one period, for instance, dry land must have existed over wide areas, and then these must have been submerged to receive a marine deposit now covering them.

3. Some of the erosions that have been described in this paper are clearly the beds of antediluvial rivers: that is, of rivers existing upon this continent before its last submergence beneath the ocean; which beds were deserted when the sur-

face emerged from the waters: although essentially the same rivers as existed previously, must have been the result of drainage.

The grounds on which I refer the cases mentioned below and described in detail in this paper to the latest of former continents are the following:—

1. The occurrence of pot-holes in the walls of gorges, which are either dry, or the bed of a brook too small to have produced them.

2. The outlet of such gorges in one direction into valleys now containing streams large enough to have formed the gorges, and in the other direction, into valleys leading at a gentle descent to some rivers.

These two facts make it certain that the gorges were once the beds of rivers.

3. An accumulation of water-worn and, perhaps sorted materials, viz: gravel and sand, to a considerable depth. This accumulation appears to me to have been made during the last submergence of the land, and to be the cause that prevented the ancient rivers from occupying their old channels upon the drainage of the country, and compelled them, at least for a considerable distance, to find a new channel. I consider the following as examples of this phenomenon, most of them very decided; that is, of these antediluvial river beds.

1. An old bed of Niagara river, commencing on the Canada shore, near the Whirlpool, and passing circuitously to St. Davids, four miles west of Queenstown.

2. An old bed of Genesee river, extending from the mouth of Irondequoit creek, nearly to Rochester.

3. An old bed of the same river, extending from Portage to Mount Morris, some twelve or fourteen miles, now filled with sand and gravel.

4. Proctorsville Gulf, an ancient bed of Black river, in Cavendish, Vermont.

5. A former bed of Connecticut river, in Portland, Connecticut. Indeed there are two such beds: but I have examined the most easterly one with most attention. It is near the junction of the sandstone and hypozoic rocks, and is filled with gravel to the height of about 200 feet above the river at present. My impression is that the Connecticut ran in this channel on the last continent before the present, and that the detritus which was thrown into it during the last subaqueous sojourn of the continent, turned the river into its present channel upon the emergence of the land. But I am not sure that this old channel was occupied by the river at so recent a date. It might have been at a still earlier date. My doubts spring from the great height of the old bed above the present river.

6. Former bed of Delaware river, along the valley now occupied by the Delaware and Hudson canal, from Port Jervis, or Delaware, to Hudson river.

7. Bed of an ancient river in Antwerp, Jefferson county, New York. I venture to place this example among the river beds of the last continent, although I have never examined it. But taking Professor Emmons' description of the deserted river bed in that place, and looking at a map of the region, I venture to predict that it will be found that the Oswegatchie river once ran into what is now the Indian river, and was forced by the filling up of its channel when beneath the ocean, to take another very circuitous route to Ogdensburg.

8. An ancient bed of Agawam river, in Russell. See No. 10 of erosions in hypozoic rocks.

9. A similar bed on the same river, at Chester village (now called Huntington). See No. 11 of this paper, in hypozoic rocks.

10. A similar bed on the east branch of this same river, four miles above Chester village. See No. 12, in hypozoic rocks, of this paper.

4. Some cases of erosion described in this paper appear to have been mainly intermediate, as to the time of their formation, between the antediluvial river beds just enumerated, and the earliest formed in the hypozoic rocks.

I would not undertake to decide positively how many times this continent, or large portions of it, may have been beneath and above the ocean. But I do not see how we can escape the conclusion that it must have been submerged at least three times. During the Silurian period we must admit its submergence; and during the carboniferous period, certainly large portions of the surface must have been above the waters to allow a gigantic growth of plants. The triasic, oolitic, and cretaceous deposits, must have been made on surfaces beneath the ocean. The tertiary strata seem to have been formed chiefly in estuaries, and with dry land in the vicinity, which indicates a second emergence. The evidence that the same surface was beneath the waves during the deposition of drift, has been presented in the paper on Surface Geology: and we have the proof beneath our feet of the third emergence of our country during the alluvial period.

During all these vertical movements, erosions of the surface must have been going on. I have referred to some examples of this work, commencing at the earliest period, or during the first emergence and drainage of land: and also some cases referrible to the last upward movement. The following cases seem most probably to have been produced at an intermediate period, but precisely when (as to geological sequence rather than chronological dates), I am unable to determine.

1. The Proctorsville gulf, in Cavendish, Vermont. See the description in this paper, No. 14, in hypozoic rocks.

2. Gorge on Delaware river, from Port Jervis to Narrowsburg, in Pennsylvania. A very old erosion, perhaps among the oldest.

3. The cañons of the southwest, described in this paper. Very old.

4. Gorge on New river and the Kenawha, in western Virginia, in coal sandstone. No. 20, in fossiliferous rocks, of this paper.

5. Gorges in New South Wales, Australia, in sandstone. Nos. 16 and 17, in fossiliferous rocks.

6. Natural bridges in Virginia.

7. Do. on the Euphrates, near Diadeen, in Armenia.

8. Do. on Dog river, in Mount Lebanon.

9. Gorge on the river Ravendooz, in Kurdistan.

10. Wady el Jeib, in Palestine.

11. Via Mala, in Switzerland.

12. Defile of Karzan, on the Danube.

13. Old river bed, east side of Mettawampe, in Massachusetts.

14. Gorges through trap, on the Columbia river and its tributaries.

The grounds on which I refer these cases to a period intermediate between the earliest and the antediluvial beds (and I might have added more for nearly equally good reasons), are one or more of the following:—

1. Some of these gorges are in rocks more modern than the oldest.

2. Some of them have sides too precipitous for erosions of the earliest date.

3. Yet some of them are situated too high above the present contiguous streams to have been worn out so recently as the last sojourn of this continent above the waters, previous to the present.

5. The most numerous cases of erosion, which I have described, appear to be postdiluvial, or produced during the alluvial period.

1. All erosions in unconsolidated strata, lying above the tertiary strata, must, from the nature of the case, be of this description; since such deposits did not exist certainly in their present position previously. Hence all those examples of old river beds in alluvium, along the Connecticut and its tributaries, exhibited on Plates III. and IV., and described in my paper on Surface Geology, because connected with the subject of terraces, belong to this class of erosions. But they are not limited to the unconsolidated strata.

2. The gulf from Niagara Falls to Ontario through which the river now runs.

3. The present bed of Genesee river, below Rochester.

4. The same, between Portage and Mount Morris.

5. The present bed of Black river, in Vermont, below Proctorsville, at least for several miles.

6. The present bed of Connecticut river, for some miles below Middletown.

7. The present bed of Westfield river, in Russell, parallel to where an old bed appears.

8. A similar case, perhaps a mile long, on the same river, at Chester village.

9. A similar case, four miles above Chester village, on the east branch of the same river.

10. Present bed of Delaware river, through the Gap.

I am satisfied that a multitude of similar cases exist in our country as well as on other continents, if care were only taken to trace them out. I judge so from the ease with which I have found those above enumerated.

6. The character of the rock, the position of the strata, their chemical character, and the nature of the climate, as to heat and cold and moisture, are circumstances affecting the amount of erosion, to be taken into account in comparing the work in different places.

7. Hence we need a number of cases of erosion in different rocks, in countries which we wish to compare together in this respect.

8. Taking such an average as our guide, as far as we can do from the cases that have been described, we infer that this work has not differed much in amount on different continents. It has been great and long continued on them all.

9. In rivers without cataracts or rapids, the work of erosion has nearly ceased, and the marks of drift agency extend nearly or quite down to their present level.

10. In some places, especially between cataracts, and in low alluvions, rivers are filling up their beds. Ex. gr. The Mississippi near its mouth and the Po, whose bed in some places is above the houses on its banks.

11. It is mainly at rapids and cataracts that rivers are now deepening their beds.

12. The details that have been given enable us to form some idea of the length of time that has elapsed since the close of the drift period.

The evidence on this point, rests on the assumption I have made in the preceding details, that certain old river beds that existed on the last continent, became so filled with modified drift during the sojourn of the surface beneath the ocean, that when it rose, the old rivers were compelled to seek new channels, and in some cases we have the amount of their erosion in the solid rock since that period. If this explanation be admitted, it follows that probably such cuts as the Niagara has made in the rocks below the cataract, in Genesee river, below Rochester, and between Mount Morris and Portage; in fact, all the ten cases referred to under the third inference, have been formed during the alluvial period: or since the close of the drift period. Nay, these old beds seem to have been filled with modified drift, and, therefore, the gulfs eroded since the last emergence of our continent from the waters, do by no means reach back to the drift period: that is, if we suppose the coarser and legitimate drift to have been produced while the continent was sinking. But since it is so difficult to fix the limits between drift and modified drift, we will regard the drift period as not closing till the work of erosion had commenced upon the rising continent. And even with such limits, what an immense period has elapsed since the period of the striation of rocks and the dispersion of the erratics closed, and the alluvial commenced.

But other facts in the history of alluvium correspond to the evidence which erosions present of the great antiquity even of the drift period. I refer specially to the vast deltas that have been pushed forward at the mouths of the large rivers of the globe, and the enormous accumulation of debris on the face of steep mountains. As mentioned in another part of this paper, the delta of the Mississippi, at its present rate of increase, must have required over 14,000 years to accumulate.

The growth and extent of coral reefs lead us to the same conclusion as to the length of the alluvial period. But perhaps the erosions of the surface form an argument for the earth's great antiquity more readily apprehended by men generally than any other.

ILLUSTRATIONS OF SURFACE GEOLOGY.

PART III.

TRACES OF ANCIENT GLACIERS

IN

MASSACHUSETTS AND VERMONT.

17

TRACES OF ANCIENT GLACIERS IN MASSACHUSETTS AND VERMONT.

Whoever is familiar with the phenomena of drift in this country, and has examined the effects of glaciers in the Alps, will be struck with the resemblance in most respects, and may perhaps infer a complete identity. I cannot, for the reasons already assigned in my paper on Surface Geology, adopt this opinion, but suppose it possible to distinguish between the two agencies by the following marks:

1. By the direction of the striæ and the position of the *stoss side* of the *roches moutonnes.* There is great uniformity and almost parallelism in the drift striæ in our country over wide surfaces. If, therefore, we find other striæ differing in direction very much from these, and the marks also having their stoss side very different as to the cardinal points, the presumption is strong that the more limited striæ were produced by glaciers.

2. Glacier striæ are limited to valleys, and proceed from the crests of the mountains outwardly, and the stoss side of the embossed ledges is always the upper side, that is, it faces up the valley, showing that the abrading body descended the valley. But drift striæ, although frequently found in the valleys, are also common upon the tops of the mountains; in this country with only one exception that I know of, viz: Mount Washington, in New Hampshire, which seems to have been above the agency.

3. The striæ of glaciers always descend from higher to lower levels, except in limited spots, where they may be horizontal. But drift striæ frequently ascend, the stoss side of hills and mountains, hundreds of feet high, being the lower side.

4. Drift is spread more or less promiscuously over most of the surface: but the detrital matter swept along by glaciers, occurs, either as lateral moraines along the sides of valleys, or accumulated in greater quantity where the valley makes a curve, or blocking up the valley as terminal moraines. In the latter case, however, the modern river occupying the valley, has usually worn away a part of the moraine, and during that process, it may be, has partially covered the other part with modified drift in the form of terraces.

Within the last five years I have had an opportunity to apply these principles in three widely separated countries, viz: Wales, Switzerland, and New England. I made a practical application of them in Scotland, but not with so satisfactory results.

As already stated in my paper on Surface Geology, when I went among the mountains of Wales, I had no recollection of the statements of the eminent geologists of Great Britain respecting its superficial deposits and markings: nor had I then been in a country of glaciers. I soon recognized erosions on the sides and bottoms of the valleys, quite similar to the drift markings with which I had been familiar in New England. But I found several differences. In Wales the grooves and striæ followed the valleys, I thought almost exclusively, radiating from the higher peaks of the Snowdonian range; nor did they reach to the top of the sides of the valleys, but the mountains above were ragged, not embossed as in the United States. I could not doubt that the erosions were produced by some force proceeding from the central and elevated parts of the country, following down the valleys, and in some spots I found that the slate rocks on the sides of the valley, had not merely been smoothed and scored, but knocked over, as if by a heavy body crowding against their upturned edges, and urging its way downwards. In short, I could not doubt that I had before me the marks of ancient glaciers. And I stated my convictions on the subject before the British Association for the Advancement of Science, where I was happy to have them confirmed by Professor Ramsay. That gentleman, I find, considers a glacier period to have preceded the drift period in Wales, and a second period of glaciers to have followed.

I make these statements to show how this subject has opened upon my mind. And for the same reason I will subjoin some details of the facts that fell under my observation in a sojourn of only a fortnight in Switzerland, respecting the former greater extent of its glaciers. The facts which I shall state can add nothing of importance to the more important ones adduced by Agassiz, Guyot, and others, and I suppose they have all been described. But I give them, both as a testimony in favor of the views of those gentlemen, and because they prepare the way for facts somewhat analogous, in New England.

As I ascended Mount Righi from the side of Lake Zug, far above the ruins of the famous Rossberg slide, certainly as high as the Stafflehaus, which, according to my barometer, is 4854 feet above the ocean, we find strewed along the steep side of the mountain, blocks of granite and gneiss, mixed with the Nagleflue, of which the mountain is composed. These crystalline boulders must have come from the higher parts of the Bernese Alps, and have constituted a lateral moraine. At least I can in no other way explain their occurrence in such a situation.

In ascending the Arve, from Geneva, we meet with remnants of former moraines far below existing glaciers. Some four or five miles before reaching Chamouny, we pass a defile, one or two miles long, where striæ and *roches moutonnes* are very distinct; the former conforming to the direction of the valley, and corresponding exactly to the effects of existing glaciers. How could I doubt that they originated in glaciers? If in North America I might strive to explain them by the action of huge icebergs, yet how useless to talk of icebergs in a narrow and retired valley of the Alps?

Most travellers who visit Chamouny ascend to the Flegere, on the northwest side of the valley. Everywhere in the vicinity of the Chalet there, the rocks are striated and rounded; and as well as I could judge, the same is the case several hundred feet higher than the Chalet, which is 3500 feet above Chamouny, and 6925 feet above the ocean. This is much above existing glaciers in that vicinity. The striæ appeared to be directed down the valley of the Arve, and I could not doubt that this valley was once filled by a glacier to the height of nearly 4000 feet, which has entirely disappeared.

In passing from Chamouny to Martigny, through the Pass of Tete Noire, in the wild gorge that crosses the dividing ridge between the Arve and the Rhone, I noticed, several hundred feet above the gorge, which is 4200 feet above the ocean, distinct marks of a glacier that once descended towards the Rhone. The smoothed and striated wall must be over 5000 feet above the ocean.

On the way from Martigny to lake Leman, down the valley of the Rhone, although the mountains on either side are bold and rocky, I did not notice such distinct traces of glacial action as in the higher Alps. Yet in several places, especially where the ledges crowd into the valley so as to form gorges, they are rounded and furrowed. Some distance before reaching St. Maurice, I never saw so distinct examples of embossed rocks, and on them we can see distinctly that the abrading force was directed down the valley, since the most distinctly rounded side—*the stoss side*—of the embossed masses, faces up the valley. It seems as if we hardly needed stronger proof of an ancient glacier descending this valley.

I had no opportunity to trace the ancient glaciers of the Alps across the great valley of Switzerland to the Jura chain, as Professor Guyot has done. It did, however, appear to me, that for the most part the drift in that valley is modified drift; that is, has been comminuted and rearranged since it was originally produced by the glaciers. I feel quite sure that the terraces around lake Zurich and Lucerne, and along the Rhine, the Aar, and the Arve, lie above the drift and have been formed by the drainage of the country. Hence I infer that this valley, certainly as high as 2000 feet above the ocean, has been under water since the period of some of these ancient glaciers. If so, what else could such a body of water be, but the ocean?

Marks of ancient glaciers have been looked for in this country for a long period with deep interest: I mean, marks in distinction from those of drift, waiving the question whether the latter has originated from glaciers. I have never visited the culminating points of our country without an eye open for such phenomena. But until lately without success. I had supposed, however, and perhaps others have done the same, that the most probable place for such marks was among the White mountains of New Hampshire. Nor can I doubt that glaciers once existed there. But the nature of the rock is not well adapted to retain the traces either of these or of drift agency. It seems probable, moreover, that the ocean has stood above our continent since the glacier period, and the drainage has obscured the traces of glaciers, not merely by erosion, but by modifying the moraines. I apprehend, indeed, that this has been a chief reason, all over our country, why it has been so difficult to trace out the marks of glacier agency. I would not be absolutely certain that I have overcome this difficulty. Yet I have now discovered so many examples, not only of embossed and striated rocks, but of detrital accumulations, which I cannot refer to the drift agency, that I cannot resist the conviction that they did originate in glaciers. The marks are not as striking here as in Wales, or Switzerland; but they are too numerous and obvious to be set aside as of no weight. I shall now proceed to give the details.

I have found all these markings upon the eastern slope of that broad range of mountains extending along the whole western side of New England, the one in Vermont, as the Green Mountains, and in Massachusetts, as Hoosac Mountain. This range in Vermont rises more than 4000 feet above the ocean: but in Massachusetts not over 2500 feet. My examinations have been mostly confined to Massachusetts, though it is obvious that Vermont promises to be a better field, because its mountains are higher. The west slope of this range of mountains is much the steepest, and the streams few and short. I have explored but a few of them, and have discovered no certain traces of glaciers, but I expect they will be found, especially in Vermont.

The annexed map, Plate VIII, extending as far as I have made any explorations, will give at a glance the principal facts which I refer to the action of former glaciers, and will make great minuteness of detail unnecessary.

My first discovery on this subject was quite accidental. I was exploring the gorge through which Little river debouches from the mountains, near the line between Westfield and Russell, into the valley of Connecticut river. As I passed along the north branch on the steep southerly face of Middle Tekoa, most distinct striæ and embossed rocks, arrested my attention, on a belt at least 140 feet wide vertically. As I knew the drift striæ in this region to run between north and south and S. 30° E., I was struck with this remarkable exception, and finding that the direction of the striæ corresponded with the course of the gorge through which Little river had cut its way, I was led to inquire whether the whole was not the effect of a glacier once descending through the valley of that river.

In 1853, in a Report to the Government of Massachusetts, I gave an account of this case, so far as it had then been explored, and of some other cases in the vicinity. I have continued, since that time, to follow up these inquiries and to extend them into other valleys in the same mountain range. The result is a still stronger conviction that the traces of ancient glaciers can be identified, though obscured by the subsequent operation of the drift and alluvial agencies. I say *subsequent* operation, and yet I confess that some of the striæ which I refer to glaciers, seem quite as recent as any found by the drift agency that I have ever seen; and really I do not feel quite satisfied which of these agencies was the earliest. Perhaps there were two periods of glaciers, one before, and the other subsequent to the drift.

The road from Westfield to Russell, just after crossing the line between the towns, rises rapidly along the south side of Little river, over ledges of mica slate, which have a dip almost 90°, and a strike not far from north to south. Till we reach the height of about 300 feet, these rocks exhibit that irregular yet smoothed surface characteristic of river action, in distinction from that of the drift agency, the ocean, or glaciers. And when we look down into the deep gorge of the river between Middle and South Tekoa, we infer at once that subsequent to the drift or glacier period, Little river has worn out its bed to that depth. But when we rise higher and get a little beyond the farm house of Ichabod Blakesley, we meet, on the north side of the road, and close to it, very distinct striæ running almost exactly east and west, on a surface sloping easterly 10° or 12°. On the right the mountain, partly wooded and partly pasture ground, is very steep, and for 150 feet at least, the frequently uncovered rocks are striated, and much higher they exhibit evidence of having been abraded and embossed, though most of the *striæ* have disappeared. This evidence of a greater antiquity to the work of erosion as we ascend, is quite manifest. The highest part of the mountain, 314 feet above the striæ first named, is covered with forest, and the rock is rarely visible. Here we find several interesting boulders, of which I shall speak subsequently. But if we return to the striæ by the road side, and follow the road upwards no great distance, we shall reach the summit of the ridge, which runs southerly towards the river. Here we see at once, would be the spot where a glacier descending this valley, must have been most crowded, because on the opposite side of the river, South Tekoa rises up in the same manner as middle Tekoa, and the ridge was no doubt continuous across the river. Accordingly, in the road where it crosses this ridge and slopes somewhat towards the west, we find the abrasion to have been powerful, and the striæ numerous. We see, also, that the west side of the ridge is the *stoss* side, and if we follow the ridge upward above the road, we shall find almost to the summit, that the west or northwest side has been struck and smoothed, while the east is the lea side.

Returning to the point in the road where it crosses the ridge, and looking up the valley, we see that Little river comes in from the southwest, and a small stream from the northwest; and if a glacier once descended the former, a smaller one probably came down the latter, both uniting at this place, and of course this would be a point of severe pressure. If we turn easterly and look into the valley of the Connecticut, we shall see that South Tekoa extends easterly but a little way, so that the glacier, after passing this gorge, would find ample room to expand southerly, so that it would no longer crowd and striate Middle Tekoa. Accordingly, I have not found much evidence on the face of that mountain of glacier action more than half a mile or so east of the summit of this ridge.

I ought to mention, that the mountain known in the region as Tekoa, is a prominent peak of mica slate, on the north side of Westfield river, in the town of Montgomery. For convenience I call the mountain south of Westfield river, between that and Little river, in Russell, Middle Tekoa, and that south of Little river, in Granville, South Tekoa; although those names are not used in the vicinity. South Tekoa, by my barometer, is 1054 feet above the ocean, and Middle Tekoa about the same height. They all originally belonged to a continuous ridge, subsequently cut across by the rivers.

The north slope of South Tekoa lying directly opposite the striated ridge above described, in Middle Tekoa, is covered with a dense forest which prevents the rock from being seen to much extent. But though I saw no striæ, it was obvious that the west was the stoss side, as on the north side of the river.

All the facts in the vicinity, therefore, force the conclusion upon the geologist, that a glacier once descended the valley of Little river into the Connecticut valley.

But how is the region to the west, from which the glacier must have come? We see clearly that it is a mountainous region, and on consulting the Map of Massachusetts, based upon trigonometrical surveys, we find several mountains in a southwest, west, and northwest direction, high enough to have formed starting points for a glacier. Winchell's Hill, in Granville, lies in a southwest direction, a little over six miles distant, rising to the height of 1362 feet above the ocean, nearly 800 feet above the lowest of the glacier striæ, and 500 above the highest; giving a descent of nearly 100 feet in a mile. The same mountain extends nearly through Granville of nearly the same height, and its northern extremity is distant from the gorge in Russell only four or five miles. To the northwest of the spot, near the middle of Blanford, six and a half miles distant, we find Dug Hill, 1622 feet above the ocean. More to the west, and eight and a half miles from the gorge, Jackson's Hill, 1717 feet high: in the same direction, nearly 20 miles from the gorge, we find the Becket Station of the Trigonometrical Survey, which is 2193 feet high. Still more to the right, 22 miles distant, is French's Hill, in Peru, 2339 feet high. Indeed, the country rises to the west over a space of 90°, for nearly 20 miles: Hoosac mountain forming the culminating ridge; and Little river is one of the outlets through which glaciers, if they pressed downward from these mountains, would find their way to the Connecticut valley.

The inquiry, however, arose in my mind, whether these striæ, on the south slope of Middle Tekoa, were not the result of some modified form of the drift agency. And on examination, I did find on the west side of the Connecticut valley, that what I call drift striæ, instead of running north and south, as they usually do, turn southwesterly, south of Southampton, as much in some places, as S. 65° W. I suspected at first, either that these markings were produced by the glacier after it reached the Connecticut valley, or that the supposed glacier scratches were the result of drift agency operating up hill. But when I found that the stoss side of the glacier striæ was the west side, and that of the drift striæ was the northeast, both these suppositions were shown to be untenable, and I accounted for the southwest direction of the drift striæ by the expansion to the right, of the Connecticut valley south of Southampton. I think this the right interpretation of the facts: but I could wish to give them further examination. However they should be explained, it seems to me that they cannot invalidate the conclusion respecting the former descent of a glacier down the valley of Little river.

Still further to settle this question, I determined to visit the tops of the mountains north and west of the striated gorge, to ascertain the direction there of the drift agency. The country is very wild for the heart of New England, and excursions on foot can alone, in most cases, carry us to the summits. I first visited the hills in the southwest part of Russell, forming the north side of Little river; and there, about 1100 feet above the ocean, I found the rocks distinctly abraded and embossed by a force from the north: yet the striæ were mostly obliterated by the disintegration of the surface of the coarse mica slate. This was obviously a case of drift agency, and is so represented on the map.

The next locality to which I would refer, is two miles northwest of East Granville village, on the road to Blanford. Near the top of the hill, 1176 feet above Connecticut river, and 1240 above the ocean, the rocks are smoothed; and striæ, though almost obliterated, can be traced, running S. 10° E. and N. 10° W. On the same surface, also, especially the northern slope, I think I could discern striæ having a direction S. 60° W. and N. 60° E. Which set of striæ were made first, I found it difficult to ascertain. A little further north, 65 feet above the first named point, I found striæ running S. 20° E. and N. 20° W. But the cross striæ were not visible. That the stoss side in both cases, where the striæ approach nearest to the meridian, I could not doubt was the north side: but in the other case, I could not satisfy myself which side had been struck. I suspect that the latter were produced by the glacier that descended through the gorge on Little river, already described, which probably commenced much further to the west. The former striæ appeared to belong to the drift.

Returning now to the spot on the north side of Little river where the supposed glacier striæ exist, and ascending the steep face of the mountain northerly, we find, as already described, the striation evidently less and less distinct, though the abrasion is obvious enough, especially if we follow up the crest of the ridge. At the top of the first summit, 314 feet above the lowest striæ, that is, about 785 feet above the village of Westfield, and 956 above the ocean, we meet with several quite large and striking boulders, one of which measured 55 feet in circumference. One of our party, Mr. Henry B. Nason, of the Scientific Department in Amherst college, took a sketch of two of the most remarkable of these, which forms Plate X, Fig. 2. They are partially enveloped by shrubs and trees, and access to them is rather difficult; but they are well worth the trouble of visiting. I regard them as the result of drift agency rather than of glaciers, although it is possible that the glacier might once have overtopped this hill.

This eminence is one of the summits of Middle Tekoa, and it overlooks a wide extent of country to the northwest, from whence the drift agency came. To the northeast, however, other ridges of the mountain rise considerably higher. We passed to the north end of what may be called a middle ridge of the mountain, perhaps half a mile from the boulders, and where it begins to slope northerly towards Westfield river. Here the marks of drift action are very manifest in the rounding and abrasion of the rocks, and the north side was the stoss side. Generally the small striæ have disappeared; but in a few places I found grooves running S. 20° E. and N. 20° W. Towards the south end of the hill is quite an accumulation of boulders. The smoothed rocks show themselves occasionally as we descend the hill southerly, very nearly as low down as the highest of the rocks striated at right angles by the glacier. Indeed, the two agencies can be traced very near to each other in several places.

All the circumstances then at this spot seem to conspire to sustain the opinion, that either before or after the drift period, a glacier descended through the gorge of Little river, which has subsequently deepened its bed nearly 300 feet. The results of the three kinds of action, the fluviatile, the glacial, and the drift, are here in so close juxtaposition that one or two hours' walk will bring distinct examples of each under the eye; and although I have found analogous phenomena in other places, I know of no other spot where the *ensemble* of the facts (except the moraines) are so distinct and near together.

But if a glacier once descended this valley, doubtless other valleys opening eastward from the same mountain range, must have been subjected to similar action. Guided by this inference, I have examined other spots where the outline of the surface seemed to promise most in this respect.

In the east part of Granville is a depression of the surface, forming a valley north and south, and bounded easterly by an elevation of considerable height, called Sodom mountain. This ridge is cut in two by a small stream, and a deep valley is formed, opening easterly into Southwick. It occurred to me that this might be such a gorge as a glacier might pass through. I accordingly found at its entrance, near the termination of the road, at a point 565 feet above Connecticut river, and 630 feet above the ocean, near the house of Mrs. Jones, that very distinct striæ on the mica slate run E. 20° S. and W. 20° N., pointing easterly directly into the gorge, and westerly to the high region from whence a glacier might have come. Unfavorable weather prevented me from penetrating far into the gorge, where no road exists; but I cannot doubt that we have here another example of glacier action.

The next region to which I directed my explorations, was on the north slope of the range of mountains lying between Little river and Westfield river, presuming from the course and lofty sides of the latter, that a glacier or glaciers may have descended that valley also. I followed an old turnpike road from Blanford to Westfield, through the north part of Russell, and found that it follows what looks much like an old abandoned river bed, or that of a glacier, perhaps both. At any rate, some agency had acted upon the up stream side of the ledges and rounded them; though the striæ are mostly obliterated by disintegration. The direction of this valley is nearly east and west, and where it joins the present bed of Westfield river, the south bank is distinctly striated. This would be near the spot where a glacier, descending this valley, would unite with one descending the Westfield river valley, and of course the pressure would be here at a maximum.

The east branch of Westfield river, which I believe is rather larger than the west branch, runs so nearly south, and consequently so nearly coincides with the course taken by the drift, that I apprehend, had a glacier once descended this branch, it would now be impossible to distinguish between the effects of the two agencies. That the up stream side is the stoss side, is quite obvious; but the work may have been done by drift as probably as by a glacier.

The west branch of this river, however, which has a direction from 30° to 40° S. of E. and N. of W., is more favorably situated for distinguishing between the agency of drift and a glacier. And yet it must be confessed that the direction of the drift agency on the mountains to the west of this river, if I have not confounded the effects with those produced by glaciers, is quite irregular, and sometimes gets round towards east and west, as far as 45°. But in the valley of Westfield river, I think I have found some other evidence of a descent of a glacier besides striæ.

The part of this river that I have examined with the most care, lies between the junction of the east and west branches, at Chester village and Chester Factories, which are a little more than six miles higher up the stream. It passes over this distance almost at right angles across nearly perpendicular strata of mica slate, portions of which project occasionally, so as to form prominent objects against which a force pressing down the valley must have struck; and in fact most of the distance, especially its upper part, these exposed ledges appear to have been much abraded and by a force directed down the valley. I found, also, in at least three places, such accumulations of boulders, as could not be accounted for in any other way but by supposing them the moraines of glaciers. These are shown upon the map, Plate VIII. The first one, after leaving Chester Factories and going eastward, occurs a mile distant, on the south side of the river, lodged at the foot of a projecting hill, as indeed I have always found them. It would seem, that as the glacier passed such projections, which would form gorges, or at least obstructions on one side, the fragments borne along by it would be shaken off.

The second example is on the same side of the river, three and a half miles below the Factories. The third is on the north side of the river, near the house, occupied when I visited the spot, by Ethel Osborne. This I think is the best example. The large and for the most part angular fragments lie along the side of a hill that rises 125 feet above them, and they rise above the river perhaps 100 or 200 feet. The great size, angular form, and large amount of these fragments, struck me as rendering their glacier origin extremely probable, taken in connection with the rounded aspect of the projecting bluffs. The descent from the Factories to Chester village, however, by my aneroid barometer, is only 246 feet, or 41 feet in a mile, which gives a slope of only 0° 27′. This is more than double some of the slopes of ancient glacier action in the Alps. (*De la Beche's Geological Observer*, p. 269. Philadelphia, 1851.) It ought to be stated, that in several places along this valley, I found river action 150 feet above the present stream, so that the original slope may have been much modified. Besides, immediately west of Chester Factories, the mountains, whence the glacier must have come, rise much more rapidly, and the grade of the crooked valley, along which the Western railroad is carried, is sometimes as high as 90 feet to the mile. This upper part of the valley I have not examined carefully with reference to glacier action. But if a thick glacier came down from the high region west of the Factories, we can easily conceive its lower extremity to be pushed forward four miles upon a more moderate slope. For Becket, which lies at the summit of this elevated region, is more than 1200 feet above the Factories, and only five or six miles distant.

These were the first accumulations of detritus that I had met in our country that bore any satisfactory resemblance to the moraines of Alpine glaciers. Those that had been pointed out to me were composed of materials that had been extensively modified by water. And I ought to add, that those which I now refer to glaciers on Westfield and Deerfield rivers, have undergone some changes from the action of water, subsequent to that of glaciers. In some cases it is obvious that water has stood entirely or partially above the moraine and covered its surface at least, with rounded and sorted materials; so that terraces may frequently be seen partially resting upon the moraines. The same thing may be seen in the Alps. In ascending the valley of the Arve, beyond Chamouny, we pass over an enormous moraine, probably once produced by the Mer de Glace, which moraine once blocked up the whole valley to the height of 150 or 200 feet, but the Arve has cut a passage through it on the north side, and while eroding its present bed, it formed several terraces on both banks to the height of 50 feet, which extend to the village of Argentiere. Beyond that place, another moraine blocked up the valley, and has been in like manner cut through by the river, and I thought I could see terraces above the barrier in the hamlet of le Tour, which I did not enter. These effects were produced in the Alps without any general submergence of the country, and therefore the moraines are but little obscured in their characters. In the great valley of Switzerland, which appears to have been beneath the ocean for a long time subsequent to the ancient widely extended glaciers, the masses of detritus, once probably moraines, have been much modified on their surface, but within retain more nearly the character of unchanged moraines.

In the same manner do the moraines which I am describing in Massachusetts appear to me to have been modified and obscured by the long-continued presence and the action of water, as the surface emerged from the deep. It is this fact that seems to me to have obscured the phenomena so much that I have long hesitated to admit the existence of genuine moraines among our mountains. But the cases which I describe in this paper, taking into account this subsequent modifying influence of water, I cannot but hope will bear the test of examination. I shall refer to others besides those on Westfield river; and I have reason to suppose that if it had been in my power to examine the valleys, I might multiply examples. I think, for instance, that they exist in the valley of Saco river among the White mountains; but they are not numerous or striking.

Deerfield river, between Florida and Deerfield, crosses the ridges of mica slate, talcose slate, and gneiss, more nearly in an easterly direction than does the Westfield river. From Shelburne falls to where it debouches into Deerfield meadows, the river occupies a deep and wild gulf, which is called the Ghor. Above Shelburne falls, through most of Charlemont, nearly ten miles, the valley is broader and is occupied by terraces a considerable portion of the way, as represented upon the map. The descent of the river thus far is moderate: but for four or five miles beyond, the lofty hills crowd closer upon the river and the descent is greater. This brings us to the Tunnel which is commenced for penetrating Hoosac mountain, and above this point, Deerfield river, which through Charlemont runs E. S. E., takes a nearly southwest direction. It comes down from Vermont, through one of the wildest gorges in New England, scarcely admitting of roads or cultivation. From the Tunnel the road passes northwesterly over Hoosac mountain, rising 1415 feet above the Tunnel, or 1860 feet above Shelburne falls, or about 2480 feet above the ocean. From this high ridge must a glacier have come, if one ever descended Deerfield river, in Massachusetts.

Accordingly on the east face of Hoosac mountain, I found striæ running N. W. and S. E. on a steep easterly slope, the mountain itself running nearly north and south. They may be very distinctly seen, passing of course obliquely down the mountain, at least 800 feet above the Tunnel, and although the drift striæ on the top and west edge of the mountain have nearly the same direction, I have never seen any such as far below the summit of a steep hill as 600 feet on its lee side in any other place, and as I find other proofs of a glacier descending Deerfield river from this point, I have connected these striæ on the east face of the mountain with such a glacier.

As the Tunnel, whether ever completed or not, will always be a spot easily found, I make it a starting point in my description of glacier action. A mile or two north of the Tunnel, up Deerfield river, a projecting hill on the west side shows fluviatile action from 100 to 200 feet high, above which line the rocks are embossed, as they are all along the high hill on that side of the river. To do this, the force must have come from the N. N. E. down the valley of Deerfield river, or about at right angles to the direction of the drift striæ on the top of Hoosac mountain. Above this point I have never been able to force my way but once, and then I was unable to examine the hills to much height for want of time and the great difficulty of getting along. But below the Tunnel the river turns suddenly towards the east, and the hill around which it curves, is distinctly embossed at its top, and so indeed, more or less, are all the projecting points on each side of the river below the Tunnel to Charlemont, and perhaps, also, I might say, to Shelburne falls. For several miles east of the Tunnel the river is quite crooked and the adjoining hills very high and precipitous, so that a good opportunity is presented for observing the outlines of the projecting cliffs above the line where the river has acted, which, in some places, I find as high as 100 or even 200 feet; but in other places, the erosion seems to have been nothing since the striating and embossing period. Thus, one mile below the village of Charlemont (West Charlemont, which I believe is called the centre), the north bank to the very water's edge, and even beneath the stream, is finely striated: the striæ pointing directly down the stream, or a little south of east. According to my views, at such a spot the river has not deepened its bed at all since the glacier period. But where the current is rapid in Zoar and Florida, it is quite obvious that the bed has been deepened a good deal, and we do not find glacier or drift action within 100 or even 200 feet of the stream perpendicularly.

Cold river is a smaller branch of Deerfield river than that which comes in from Vermont, as just described. It starts in the northwest part of Florida, on the top and near the west side of Hoosac mountain, and runs diagonally, in a southeast direction, nearly across the town, and near the eastern slope of the mountain it turns more easterly, and empties into Deerfield river in the west part of Charlemont, some miles below the Tunnel. I followed this river through Florida, as nearly as the roads would permit. On its west side, not far from the middle of the town, I found striæ running S. 22° E. Further down the stream, where it turns more easterly, the striæ point S. 45° E. Still further down, and where the eastern slope of the mountain commences, the striæ are directed still more to the east, pointing in fact almost directly down Deerfield river, viz: S. 55° E. These facts certainly sustain the presumption that a glacier once descended this valley. They are shown, as well as may be, on the Map of Drift and Glacier Striæ.

In three places below the Tunnel, and within five miles of it, on Deerfield river, I have found accumulations of boulders and detritus, which I venture to denominate moraines. The first is not far from a mile and a half below the Tunnel, and rises on the north side of the river to the height of 60 or 70 feet. A portion of the same materials may be seen south of the river, but less striking. The second case occurs on the north side of the river, just below the soapstone bed, or quarry, for it has scarcely been quarried. The third, of more doubtful character, is a little below Zoar bridge, and is, also, on the north side of the river. They have all received considerable modification from water in the manner already described, but are inexplicable without calling in some other agency, and that agency, if a glacier, affords a reasonable explanation.

If we take the whole distance from Shelburne falls, about 14 miles, the descent is but small, only as I made it, 445 feet, equal to 30 feet in the mile, and 0° 20′ *en arc.* As far as we find moraines, however, the slope is great enough I judge, for the advance of a modern glacier. But I do find some evidence in the striæ and rounded rocks, even to the falls, that the glacier extended the whole distance, and if thus far then doubtless through the Ghor, whose slope is greater. At any rate, it seems to me, that as far as the moraines occur we may presume upon the former existence of a glacier, although the direction of the striæ does not differ much from that of the drift agency where it swept over Hoosac mountain. But it ought also to be stated, that on the lofty hills of Rowe, Heath, Shelburne, Conway, and Hawley lying north and south of Deerfield river, the course of the drift striæ rarely varies more than 10° from the meridian, and this is almost at right angles to the force that striated and embossed the valley of the river.

Passing now to the vicinity of Shelburne falls, we find a tributary of Deerfield river coming in from the northeast, and called North Branch. The junction lies on the east foot of a lofty ridge, through which Deerfield river has cut its way. I felt a peculiar interest in examining the valley of North Branch, because a glacier might once have descended it, and if so, its course for some distance must have been from the N. E. to the S. W. I found such to be the fact most decidedly, as far as I have explored the valley. The striæ on both sides of the stream are very manifest in several places, and to the height on the west side where alone I measured them, of 400 feet at least. The rounded points of the ledges show conclusively that the abrading force struck the northeast side. This abrasion may be traced downward to the point of the mountain, where the tributary enters Deerfield river, and on the north side of the same mountain, we find marks of the force that swept down Deerfield river; the two forces having met at an angle greater than a right angle, as the map will show.

The high land that rises between these two rivers forms Mount Pocomtuck, which is almost 1800 feet above the ocean. And I find that the striæ, almost to the top of this mountain, run nearly N. E. and S. W., as along the North Branch. Hence, if produced by a glacier, it must have risen very high; so high in fact, as to have swept over most of the region east of Hoosac mountain. Indeed, I found the striæ on the high regions of Rowe and Heath to run generally from 5° to 20° W. of south. These facts, I confess, excite a doubt whether this force from the northeast was a glacier or an iceberg. But that is a very unusual direction in

New England for drift striæ: and I should be glad to study the phenomena further.

There are a few other tributary streams on the eastern slope of Hoosac mountain, which I should be glad to examine more carefully, with reference to this question of ancient glaciers. I cannot but feel, however, that I have pointed out facts enough to induce others to make further explorations; enough, also, I trust, to produce the belief that glaciers did once exist in these regions.

Gladly would I have carried these researches into regions beyond Massachusetts, where the probability is still stronger that traces of glaciers might be found. A single excursion into Vermont, however, is all I have been able to make. I have spent a little time upon the branches of the Queechy or Waterqueechee river, in Windsor county, Vermont. The branches of that river, that pass through the gold region of Vermont, run east and northeast, and I was anxious to determine if marks of a glacier could be found descending those valleys, since the direction is nearly 180° different from that of the striæ on North Branch, just described. The result of my examinations I have given at the top of the map, where I have added a sketch of the Queechee region on a scale larger than that of the map below. The intervening space in Vermont is well worthy of examination, though the direction of the streams is almost coincident with that of the drift agency.

The marks of ancient glaciers on the branches of the Queechee are not so decided, I think, as upon the rivers already described in Massachusetts. Yet taken together, they have produced the conviction in my mind that such a glacier once descended that river as far as Woodstock at least. In ascending that stream above that village, I found within a mile or so, accumulations of detritus on the north bank, such as I have referred to moraines. Several miles further west, just before entering the village of Bridgewater, I found a still more decided example. The detritus here once extended across the entire valley, but has been worn away by the river on one side, just as I have described in the vicinity of Chamouny, in Savoy. Water has in this case considerably modified the materials at the surface of the heap.

Beyond Bridgewater I followed for several miles, a branch of the river that comes in from Plymouth, in a northeast direction. The mountains along this stream are high, and in several places it was obvious that the southwest side was the stoss side: though the striæ are mostly obliterated. I afterwards followed up another branch of the river to the gold mine, in Bridgewater, and I thought I saw evidence here, also, of glacial action on the west side of the ledges, but the evidence was not very striking.

The highest point which I reached on the road to Plymouth was 450 feet above Woodstock, distant about 10 miles, and the gold mine is 820 feet above that place. These heights would give a moderate slope, but great enough for a glacier.

It would be desirable to follow the road beyond the gold mine to the top of the Green mountains, as Killington Peak lies in that direction, one of the highest points of these mountains, as much as 4000 feet above the sea. The highest point which I have mentioned above, viz: the gold mine, is only 1580 feet above the ocean.

I am aware that the details which I have given in this paper will impress the reader with the limited extent of my researches compared with the field that lies yet unexplored. I have endeavored, however, to visit those points most likely to afford satisfactory results. If I have done enough in so difficult a matter to stimulate younger and more vigorous explorers to push these investigations into all the Alpine districts of our country, my deficiencies will ere long be supplied, and what I now grope after in the twilight may be made to stand out in the clearness of day, and with the stability of established truth.

NOTE.—In looking over the preceding pages, as they have passed through the press, it has occurred to me that the few references which I have made to the many eminent men on both sides of the Atlantic, who have written upon Surface Geology, might possibly be imputed to an overweening opinion of the superior value of my own observations. I can hardly believe, however, after what I have said on page 34, that any will think me guilty of such folly; certainly not in respect to my few and unimportant observations upon Europe. The fact is, I had been much interested in New England with surface geology under the form of terraces and beaches, or in more general terms, as unmodified and modified drift, and I was anxious to see with my own eyes how nearly these phases of the phenomena in Europe agreed with those at home. But the thought never entered my mind, that I should seem to be exalting my own few and defective observations above those of the scores of eminent men, who have been studying similar phenomena. I referred to the labors of Mr. Chambers and Professor Ramsay, because I had followed so closely in their track. If others have looked at the subject from the same point of view, I am not aware of it. I hesitated much whether it were best to give these European facts, as well as those in our country out of New England, because they are so few and scattered, but not because I imagined I was ignoring or neglecting the labors of others. And they do seem to me sufficient to show an identity between certain phenomena of surface geology in widely separated regions.

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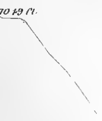

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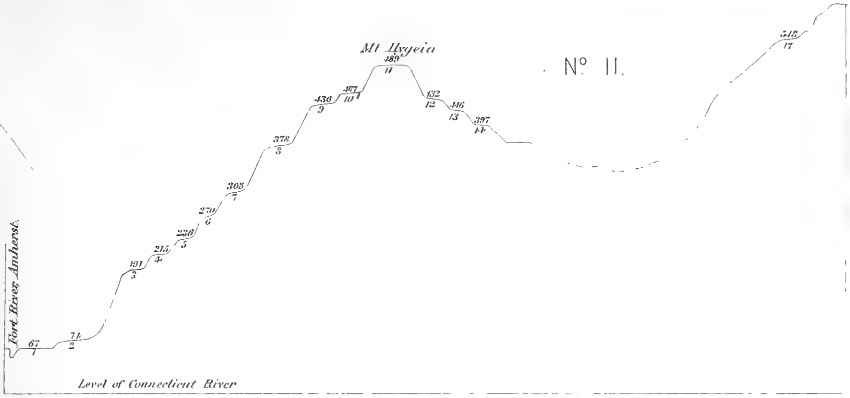

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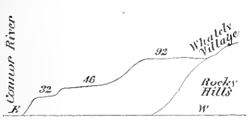

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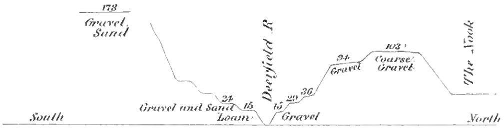

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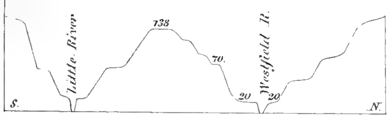
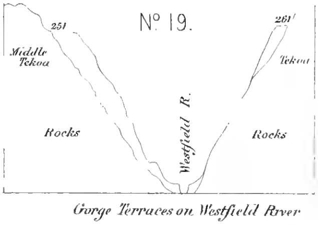


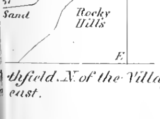
Section across a tongue of Terraces in Westfield.

No. 19.



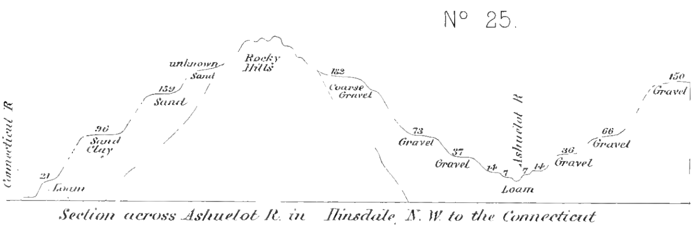
Gorge Terraces on Westfield River

24.

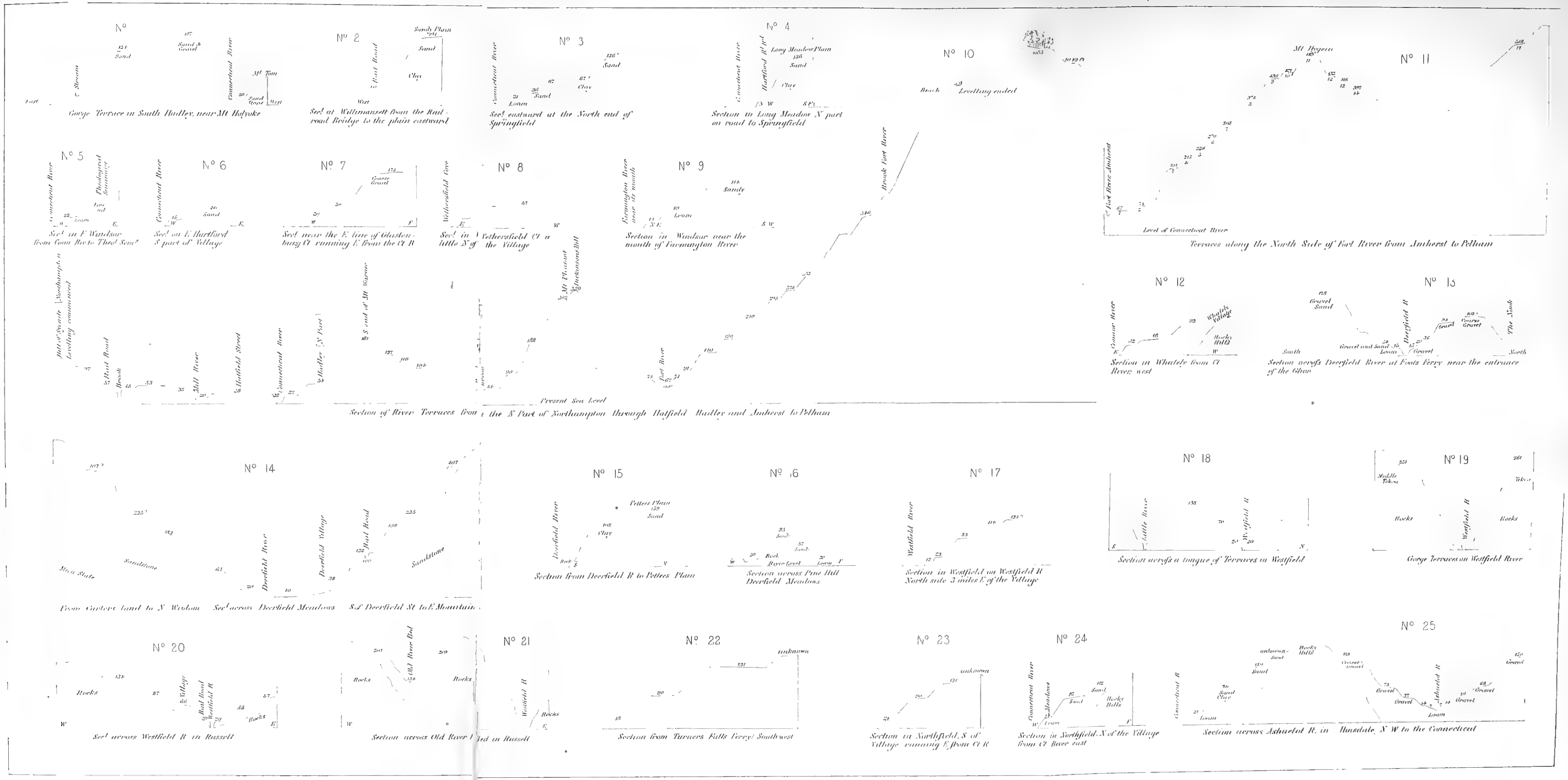


...thfield N. of the Village ...east.

No. 25.



Section across Ashuelot R. in Hinsdale, N. W. to the Connecticut

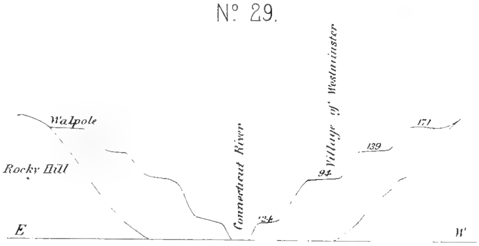


Gorge Terrace in South Hadley, near Mt Holyoke
No 2
Secn at Willimansett from the Rail-road Bridge to the plain eastward
No 3
Secn eastward at the North end of Springfield
No 4
Section in Long Meadow N part on road to Springfield
No 10
Levelling ended
No 11
Terraces along the North Side of Fort River from Amherst to Pelham
No 5
Secn in E Windsor from Conn Rr to Theol Semy
No 6
Secn in E Hartford S part of Village
No 7
Secn near the E line of Glastenbury Ct running E from the Ct R
No 8
Secn in Wethersfield Ct a little N of the Village
No 9
Section in Windsor near the mouth of Farmington River
No 12
Section in Whately from Ct River west
No 13
Section across Deerfield River at Foots Ferry near the entrance of the Glen
Present Sea Level
Section of River Terraces from the N Part of Northampton through Hatfield Hadley and Amherst to Pelham
No 14
From Carters land to N Windsor
Secn across Deerfield Meadows
S of Deerfield St to E Mountain
No 15
Section from Deerfield R to Petters Plain
No 16
Section across Pine Hill Deerfield Meadows
No 17
Section in Westfield on Westfield R North side 3 miles E of the Village
No 18
Section across a tongue of Terraces in Westfield
No 19
Gorge Terraces on Westfield River
No 20
Secn across Westfield R in Russell
Section across Old River Bed in Russell
No 21
No 22
Section from Turners Falls Ferry Southwest
No 23
Section in Northfield, S of Village running E from Ct R
No 24
Section in Northfield N of the Village from Ct River east
No 25
Section across Ashuelot R. in Hinsdale N W to the Connecticut

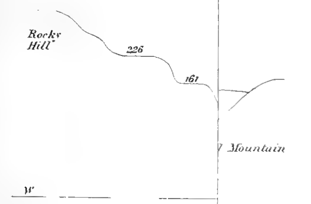
N

Section from the Connecti age and part of Vernon.

N°. 29.

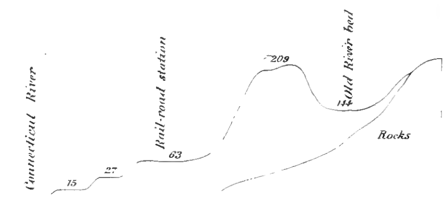


Section acrofs Connecticut River from N W to S E from Westminster Vt. to Walpole N. H.



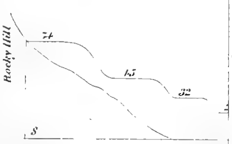
Section across Connecticut s [above the Falls.

N°. 33.



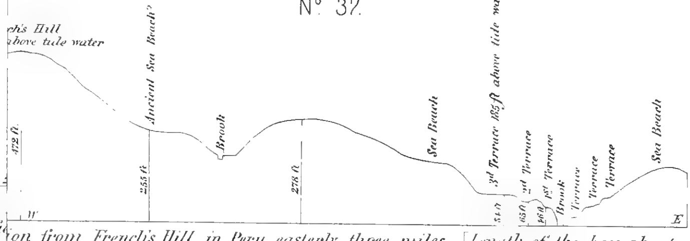
Section westward from Connecticut River at White River Junction.

N° 34.



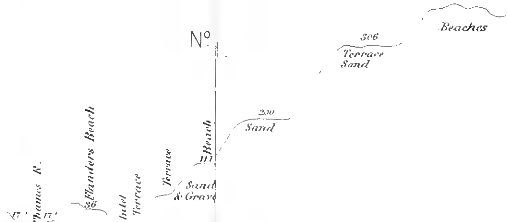
Section on S. side of Deerfi Buckland.

N°. 37.



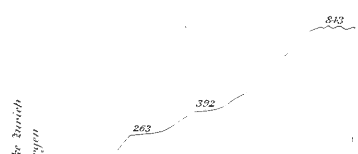
on from French's Hill in Peru easterly three miles. [Length of the base about e miles.]

N°



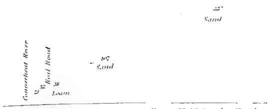
Section from the mouth of mouth of the Thames hine from Rhinefelder towards

N° 42.



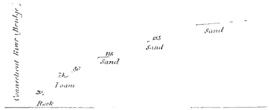
Section on the Western shore of Lake Zurich.

Nº 26



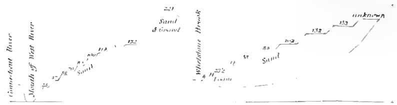
Section from the Connecticut River N W in the North part of Vernon

Nº 27



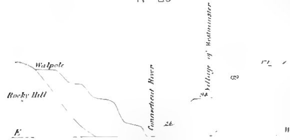
Section from the Bridge over the Connecticut River in Brattleboro to the Southwest

Nº 28



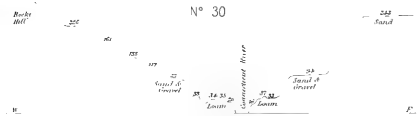
Section in Brattleboro from the mouth of West River across the Village and Whetstone Brook

Nº 29



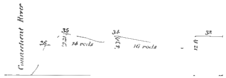
Section across Connecticut River from N W to S E from Westminster Vt. to Walpole N. H

Nº 30



Section across Connecticut River in Walpole near the mouth of Cold and Saxon Rivers.

Nº 31



Section of two Glacis Terraces near the mouth of Saxons River

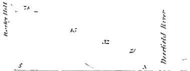
Nº 32

Section across Connecticut River at Bellows Falls [above the old Barrier] Height given from the foot of the Falls.

Nº 33

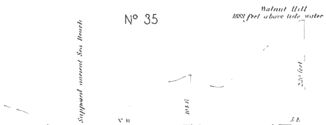
Section westward from Connecticut River at White River Junction

Nº 34



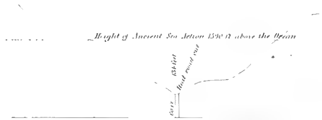
Section on S side of Deerfield River in Buckland.

Nº 35



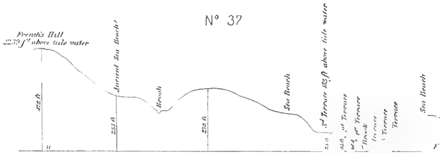
Section in Heath from Walnut Hill N Easterly about two miles.

Nº 36



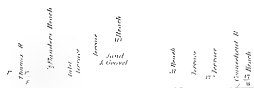
Section across the deep cut at the summit level of the Western Rl Rd in Washington Mass

Nº 37



Section from French's Hill in Peru easterly three miles (Length of the base about three miles)

Nº 38

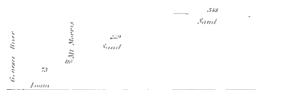


Section from the mouth of Connecticut River to the mouth of the Thames River

Nº 39

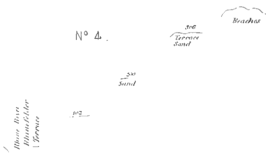
Section across Hudson River to the Summit level of the Western Rail road

Nº 40



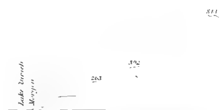
Section at Mt Morris N York

Nº 41



Terraces on the Rhine from Rhinefelder towards Bruges

Nº 42



Section on the Western shore of Lake Zurich

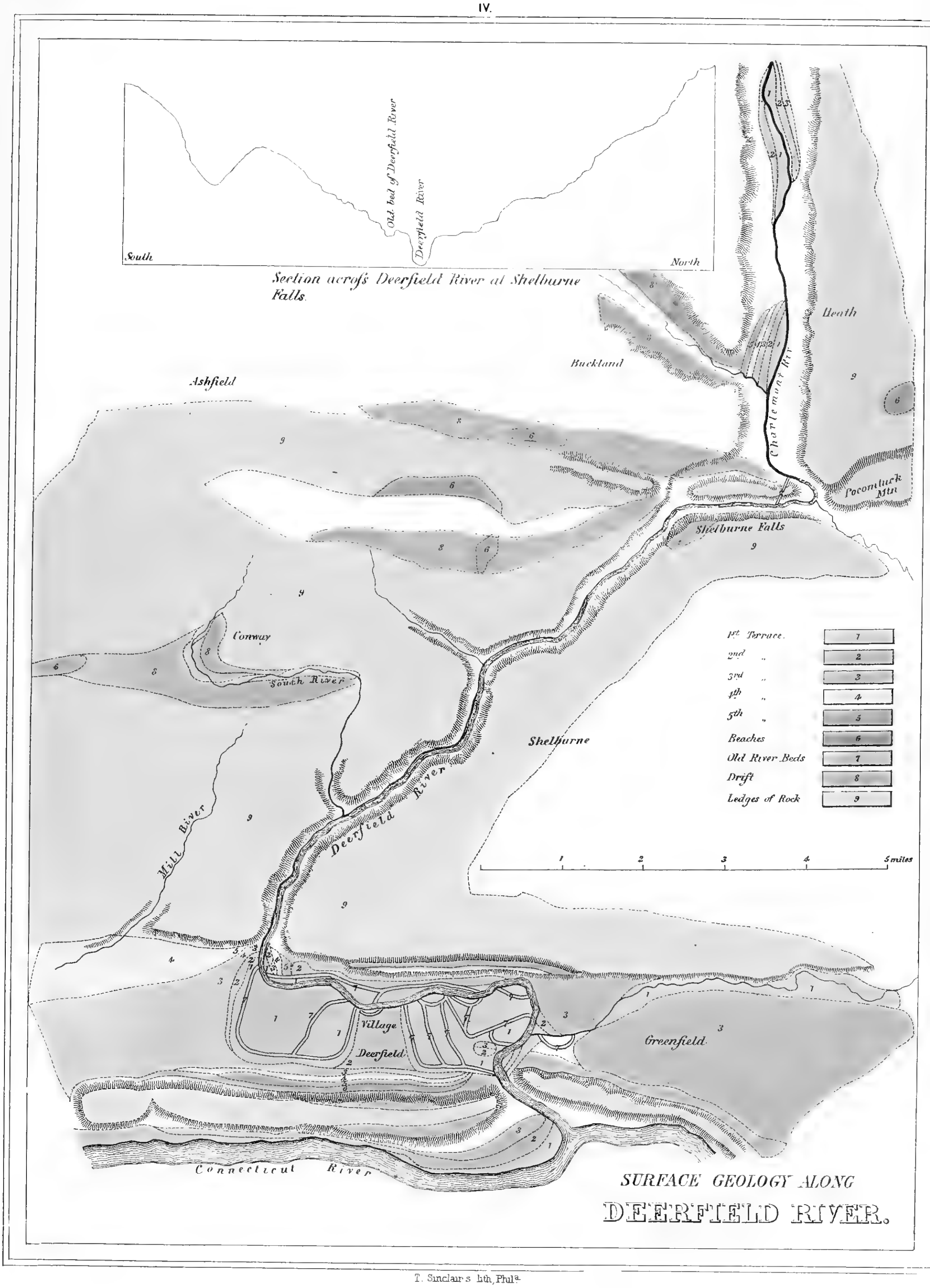
THE

SURFACE GEOLOGY

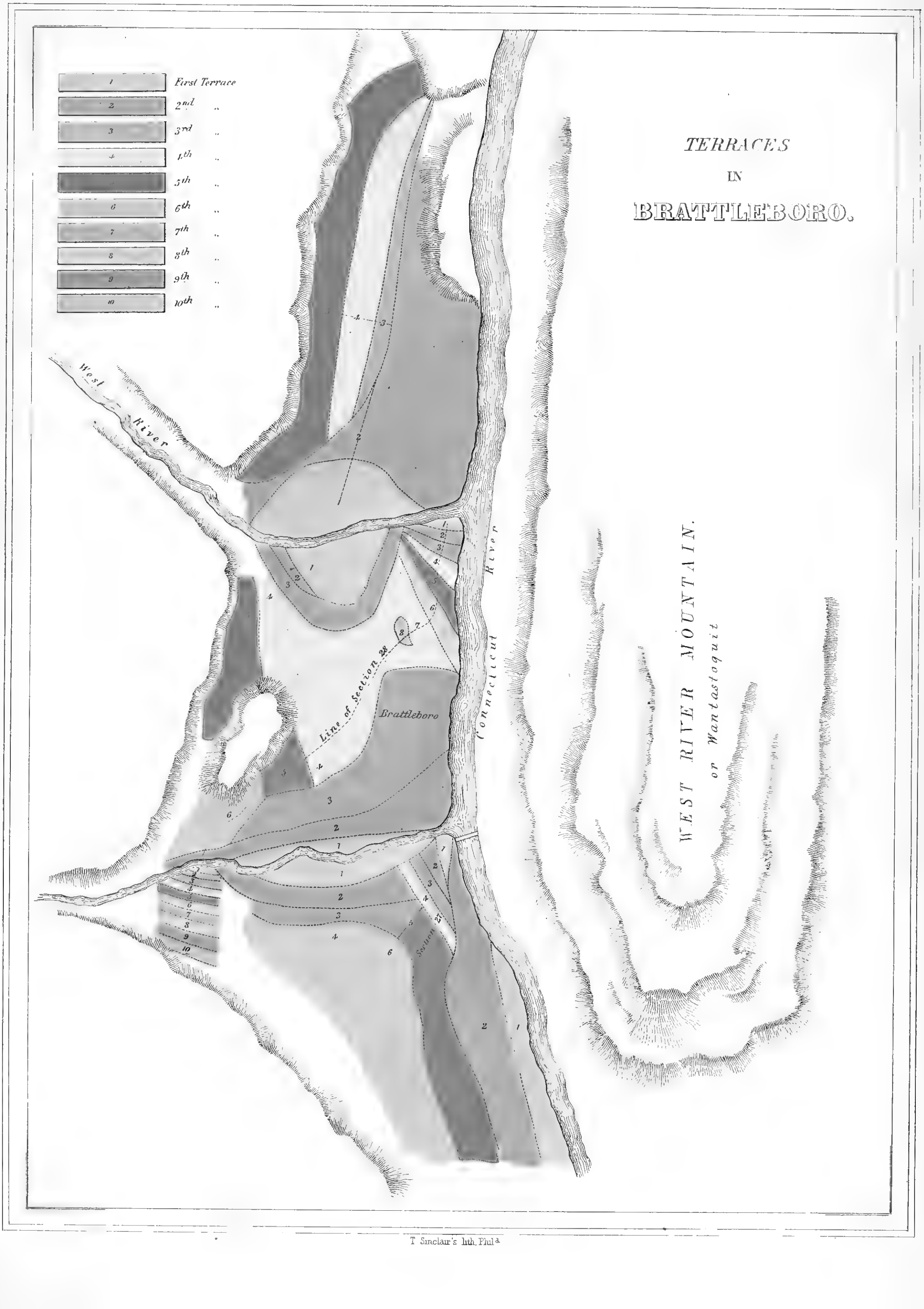
CHIEFLY OF THE

CONNECTICUT VALLEY.

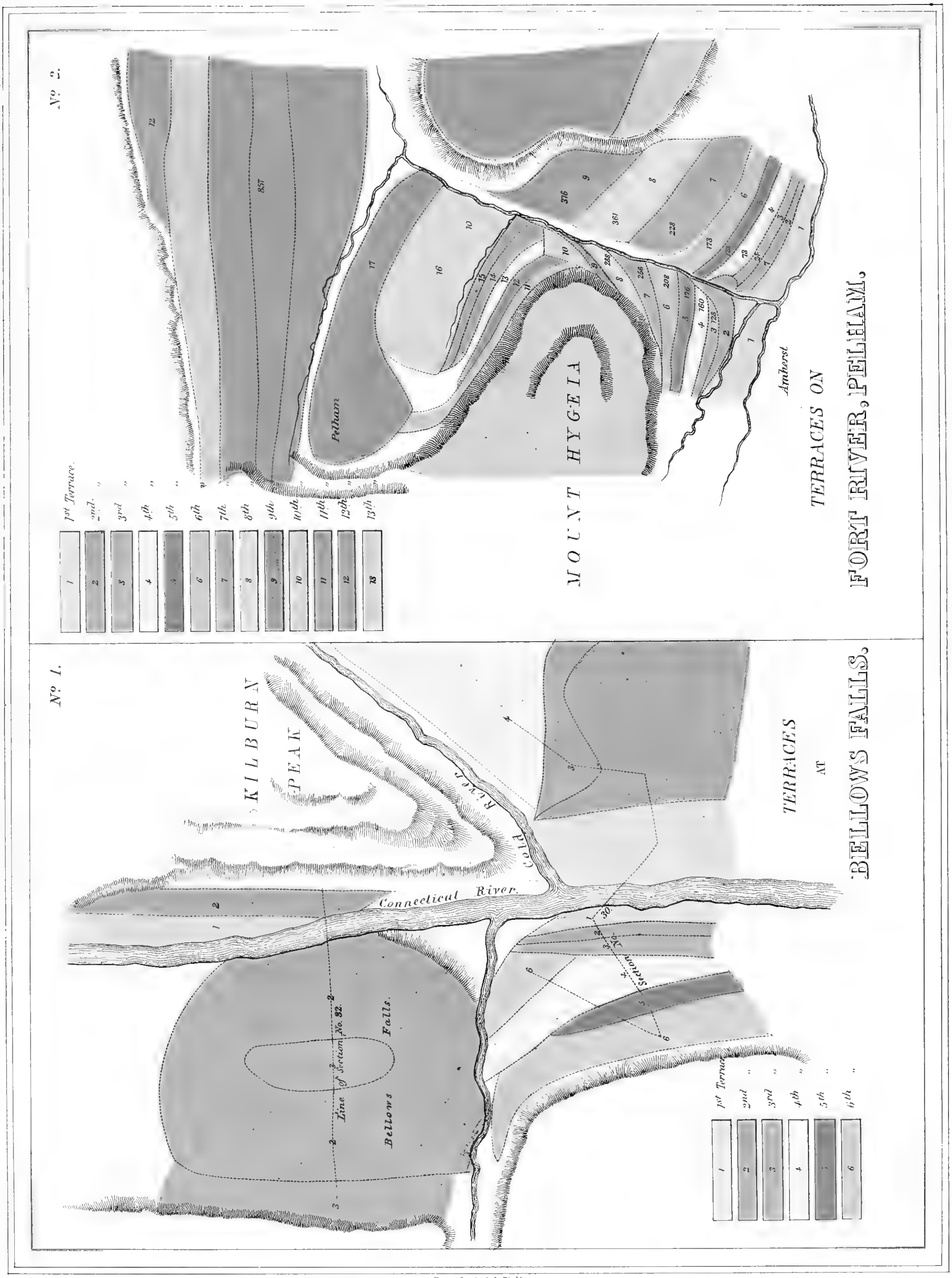
T. Sinclair's lith Phila



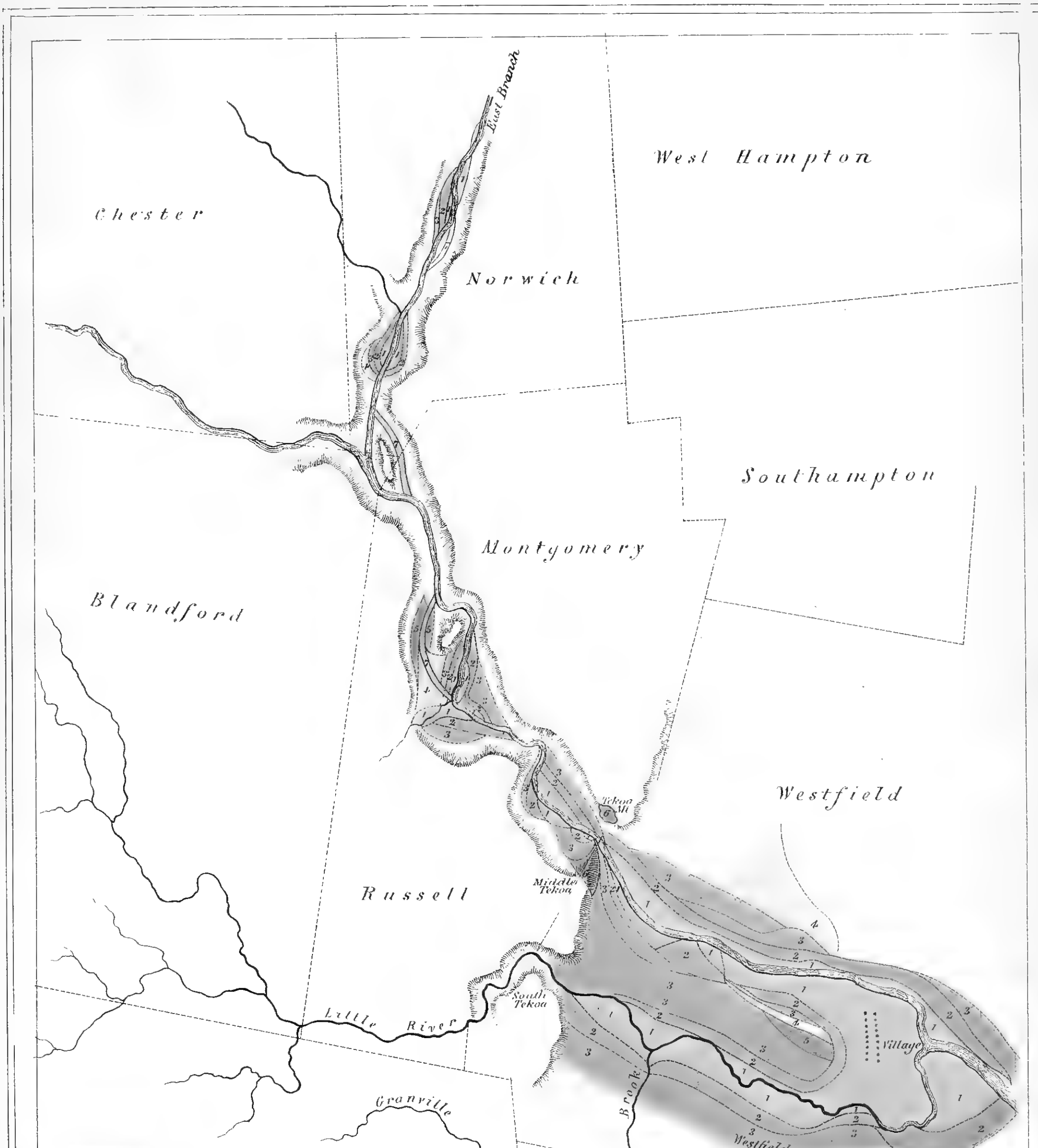
T. Sinclair's lith, Phil[a].



T. Sinclair's lith. Phil^a.

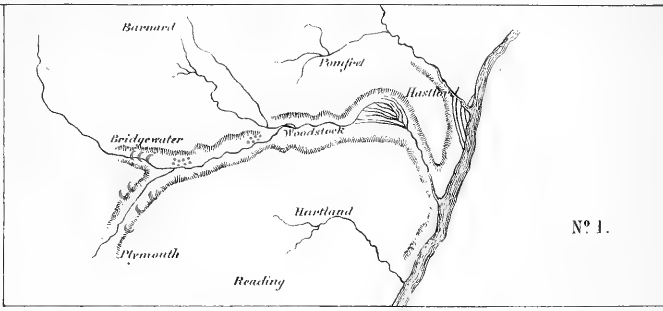


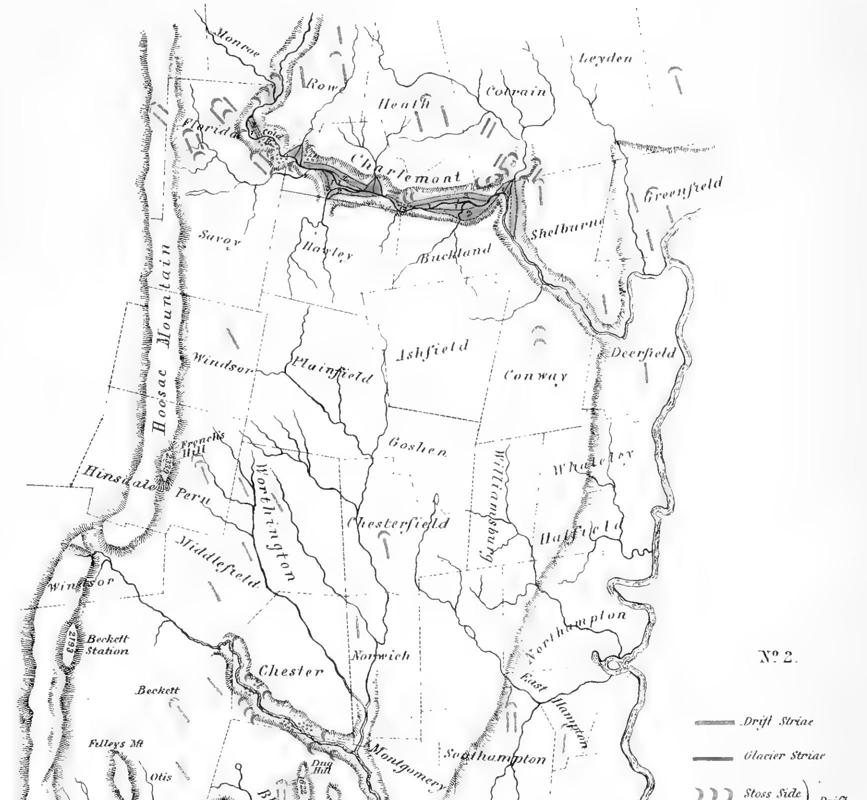
T. Sinclair's lith. Phila.



T. Sinclair's lith, Phila.

MAP OF DRIFT & GLACIER STRIAE & MORAINES IN MASSACHUSETTS 1856.





T. Sinclair's lith, Phila

Fig 1 Terraces in the gorge at Bellows Falls

Fig. 2 Terraces in Pelham

T. Sinclair's lith, Phil^a.

Fig. 1. Terraces on Westfield River.
[at the Russell Station]

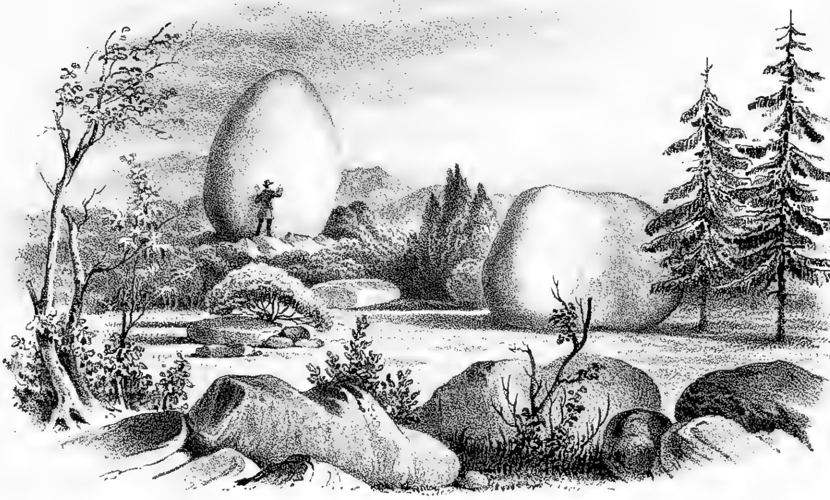


Fig. 2. Boulders on Mount Tekoa
[viewed from the southwest]

T. Sinclair's lith. Phil^a.

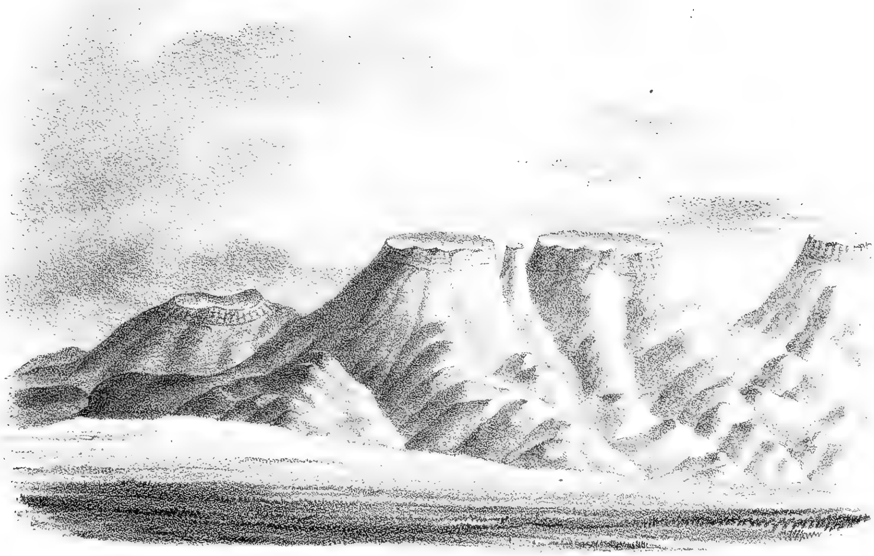


Fig. 1. View of Eroded hills near Natal, S. Africa

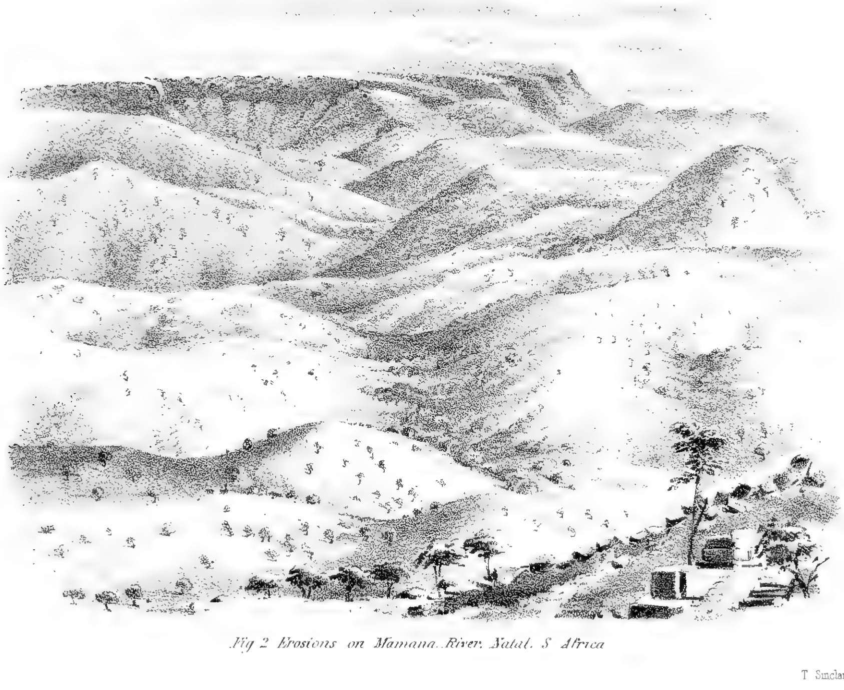
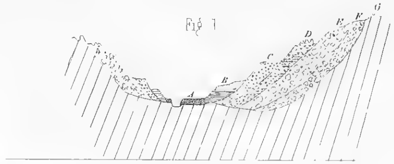


Fig 2 Erosions on Mamana River, Natal, S. Africa

T. Sinclair's lith. Phil^a.

Fig 1



Ideal Section of a Terraced Valley

Fig 2

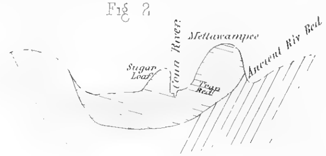


Fig 3

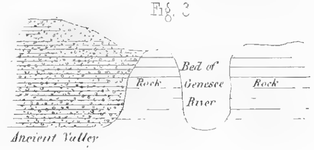


Fig 4

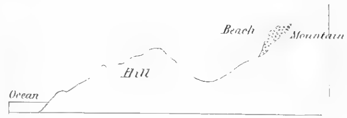


Fig 5

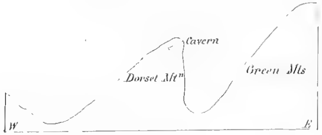


Fig 6

View of Sandalwood Island in the East Indian Archipelago

Fig 7

Fig 8

Fig 9

Cañon of Chelly.

SMITHSONIAN CONTRIBUTIONS TO KNOWLEDGE.

OBSERVATIONS

ON

MEXICAN HISTORY AND ARCHÆOLOGY,

WITH A SPECIAL NOTICE OF

ZAPOTEC REMAINS,

AS

DELINEATED IN MR. J. G. SAWKINS'S DRAWINGS OF MITLA, ETC.

BY

BRANTZ MAYER.

[ACCEPTED FOR PUBLICATION, JUNE, 1856.]

COMMISSION

TO WHICH THIS PAPER HAS BEEN REFERRED.

SAMUEL F. HAVEN,
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PHILADELPHIA.

MEXICAN HISTORY AND ARCHÆOLOGY;

ZAPOTEC ARCHITECTURE, ETC., AT MITLA.

CHAPTER I.

During the last twenty years, the attention of students has been directed with much zeal to the investigation of American Archæology. The peopling of our continent, the romantic ideas attached to the remnants of our Indian race, the strangeness of their architectural remains, and sometimes mere curiosity, have been the motives for this labor; yet it is to be regretted that no very definite *historical* results have been obtained from these studies, and that it is probable the future will be equally barren of scientific certainty. The works of McCulloh, Schoolcraft, Gallatin, Rafinesque, Bradford, Squier, Davis, Lapham, Whittlesey, and others, in regard to the aboriginal remains within the limits of the United States; and the publications of the American Ethnological Society; the vast repository of Lord Kingsborough's volumes relative to Mexican antiquities; the admirable work of Antonio Gama; the illustrated publications of Stephens, Catherwood, Norman, and Squier, on Yucatan, Central America, Nicaragua, and Honduras; the Crania Americana of Morton, and the Peruvian Antiquities of Von Tschudi,—have presented us, mainly, the physical remains of our ancient continent; but, while they serve to stimulate our curiosity and wonder, they have done very little in elucidating the national antiquity or personal story of our aborigines. After a careful study of all these books, the question may still be properly asked: Who were the Indians of this northern continent and whence did they come? Who were the Toltecs, Chichimecs, and Aztecs of Mexico? What was their origin, and what are the facts and exact chronology of their history? Who built and dwelt in the civilized cities of Yucatan? What was the origin of the wealth, refinement, and polity of Peru? Who were the Araucanians? In fact, excepting the fanciful traditions of the northern tribes at the period of European occupation, and the few scattered "picture writings" and legends of Mexico, we have very little but architectural, image, and utensil remains, to inform us how far the inhabitants of the western world had advanced beyond the mere supply of animal wants, towards those higher degrees of intellectual and social progress, in which taste, sensibility, and moral feeling expand into civilization.

This *progress* is shown by the traditions or written history of all people who have emerged from barbarism. They hunger, and, at first, allay the cravings of appetite by the fruits of the earth, or invent the simplest instruments to pursue the chase. They suffer from cold, and clothe themselves in the skins of beasts they have slain. They are exposed to the rain and frost of winter, or the heat of summer, and, after finding the forest boughs inadequate for protection, they learn to build for temporary or permanent comfort. As the family grows into a tribe, and the tribe multiplies its numbers, they congregate in villages or towns, which, through fear or affection, become affiliated by the bond of nationality. During this process, which often requires centuries, according to the grade of aggregate intellect, all the wants and passions of society make themselves gradually known. They develop gradually in the natural growth of a people. Municipalities and states beget police, law, government. The changes of day and night are beheld; the regular motion of sun, moon, and stars is noted; seasons are marked; and the simpler portions of astronomy are developed in the scientific division of time, as chronicled in the dial of the sky. The rivalry of neighboring states begets wars; and thence, protection ensues in the shape of arms, soldiery, arsenals, military experience, and fortifications. The inevitable conviction of a creative and preservative Power impresses the minds of all with a religious sentiment, which begets worship and builds temples, either for adoration or propitiation, according as the national mind is exalted or grovelling. And, finally, as the people observe the necessity of recurring to the past for facts and principles, they advance from oral tradition to written and monumental records, which modern civilization endeavors to ripen into history.

Now, in the absence of explicit records in regard to American nations, the object of antiquarian research, at present, is not so much to penetrate, by fanciful guesses or resemblances, the periods antecedent to the European occupation of our continent, as to fix the world's attention on the *actual* condition of the aboriginal nations at the period of the conquest, and to endeavor, from their remains, to form a fair estimate of their relative *status* at that time. I consider this the true and best object to propose; because, most of the records—legendary, hieroglyphic, or monumental—concerning the antiquity of the chief centres of civilization on this continent, which were rescued from destruction, have been deciphered as far as practicable, and their valuable facts detailed by investigators. Of all things, the American antiquarian should, as yet, avoid the peril of starting in his investigations *with an hypothesis*, for the chances are very great that, in the mythic confusion of our aboriginal past, he will find abundant hints to justify any ideas excited by his credulity or hopes. In the present state of our archæology, all labors should be *contributions to that store of facts*, which, in time, may form a mass of testimony whence future historians shall either draw a rational picture of ante-Columbian civilization, or be justified in declaring that there is nothing more to be disclosed.

The ancient history of our own tribes, it is well known, is a matter of tradition alone, for they had no written language; or, if they had, their story was not engraved on monuments or transmitted on imperishable materials. Their wampum and pictographs may scarcely be entitled to consideration for permanent or historical purposes. Among the Peruvians, the *quipo* was only a species of *memoria technica*, and served rather to aid arithmeticians and financiers, than to establish an independence of individual recollection. The Aztecs, and perhaps their predecessors in the valley of Mexico, possessed a "picture writing," which was chiefly used for the recording of facts apart from abstract ideas; but the Spaniards who seized Peru and Mexico did not protect these simple archives, flimsy as they were, from destruction by an ignorant soldiery and their superstitious companions. The Mexican "picture writing" consisted of several elements: an arbitrary system of symbols to denote years, months, days, seasons, the elements, and events of frequent occurrence; an effort to delineate persons and their acts by rude drawings; and a phonetic system, which, through objects, conveyed sounds that, singly or in combination, expressed the facts they were designed to record. This imperfect and mixed process of painting and symbolizing thought, was stopped at this stage, for it was the extent of Aztec invention at the period of the conquest, and it is difficult to judge, from the known character of the people, whether further progress would have been made. But this inquiry is of comparatively small importance, as the archives of Mexico and Tezcoco, containing "picture writings" which were regarded by the Spaniards as the "symbols of a pestilent superstition," were piled in a heap by order of Zumarraga, the first archbishop of Mexico, and reduced to ashes.[1] This species of literary *auto da fé* was imitated by other Spanish authorities, so that every painted paper or graven image they found was soon annihilated by the invaders. Still, a few of these relics escaped the general wreck, and were deposited in the Royal Libraries of Paris, Berlin, and Dresden; the Imperial Library of Vienna; the Museum and Vatican at Rome; the library of the Institute at Bologna; and in the Bodleian Library at Oxford.

In summing up the character of the most important of these relics, Mr. Gallatin observes that, "whatever may have been the value of the Mexican paintings destroyed by the Spanish clergy, it has now been shown that those which have been preserved contain but a meagre account of the Mexican history for the one hundred years preceding the conquest, and hardly anything that relates to prior events."[2] The consequence of this is, that the antecedent history of the aboriginal nations inhabiting the territory of modern Mexico must rest upon the reports of early Spanish writers, their monumental remains, and, perhaps mainly, on the questionable authority of Ixtlilxochitl.[3]

---

[1] Prescott, Conquest of Mexico, I, 101. See his authorities.

[2] Am. Ethnological Soc. Trans., I, 145.

[3] The sources of information in regard to early Mexican *history* and *antiquity* are the following:—

No. 1. The Mexican Paintings, &c. These are engraved in Lord Kingsborough's 1st, 2d, and 3d volumes of Mexican Antiquities.

- The Codex Vaticanus, No. 3776.
- " Vaticanus, No. 3738.
- " Borgianus, of Veletri.
- " Bologna.
- " Pess Hungary, of Mr. Fejervari.
- " Oxford, Arbp: Laud.
- " Vienna.
- " Oxford, Bodleian.
- " Oxford, Selden.
- " Berlin, of Humboldt.
- " Dresden.
- " Boturini.
- " Paris, Tell:
- " Tellurianus Remensis.
- " Oxford, Mendoza Collection.

(Continued over Page.)

"Clavigero," says Prescott,[1] "talks of Boturini's having written 'on the faith of *Toltec* historians.'[2] But that scholar does *not pretend to have ever met a Toltec MS. himself*, and had heard of *only one* in the possession of Ixtlilxochitl.[3] The latter writer tells us that his account of the Toltec and Chichimec nations was 'derived from *interpretation*' (probably of the Tezcocan paintings), 'and from the traditions of old men;'[4] poor authority for events which had passed centuries before." This depreciation of the sources of recorded and traditionary information in regard to Mexico by Mr. Prescott, has drawn a critical notice from Don José F. Ramirez, in his notes on the Spanish translation of the history of the conquest, published in Mexico.[5] The criticism, though earnest and ingenious, does not seem to improve our sources of knowledge and their authoritative value. Señor Ramirez was naturally anxious to sustain the idea of an extremely ancient civilization, and to destroy as much as possible the fabulous air which some of the Spanish narratives were

---

No. 2. Torquemada's "Monarchia Indiana."
3. Bernardino de Sahaguns's "Historia Universal de Nueva Espania."
4. Boturini's "Idea da una Nueva Historia General de la America Septentrional."
5. Fernando de Alva Ixtlilxochitl's "Relaciones, Historia Chichimeca."
6. Castañeda's "Viaje a Cibola," 1540.
7. Fray Bartolomé de las Casas, "Historia General de las Indias," &c. &c.
8. Antonio de Herrera's "Historia General de las Indias Occidentales."
9. Torebio de Benavente, "Historia General de los Indios de Nueva Espania."
10. Pietro Martire de Anglera, "Decades de Orbe Novo." 1587.
11. Gonzalo de Oviedo y Valdes, "Historia General de las Indias."
12. Diego Muños Camargo's "Historia de Tlascala—pedazo de historia verdadera."
13. Francisco Lopez de Gomara's "Cronica de la Nueva Espania."
14. Bernal Diaz del Castillo's "Historia Verdadera de la Conquesta de la Nueva Espania."
15. Pesquisia contra "Pedro de Alvarado y Nuño de Guzman."
16. Don Martin Veytia's "Historia Antigua de Mejico."
17. Clavigero's "Storia Antica de Messico."
18 Antonio Leon y Gama's "Descripcion de las dos Piedras," &c. &c. &c. 1832.
19. Lord Kingsborough's "Mexican Antiquities." London, 1830.
20. Cavo y Bustamante's "Tres Siglos de Mejico."
21. Alaman's works on Mexican History, &c. &c.
22. Nebel, "Voyage Pittoresque et Archæologique à Mexique."
23. Stephens's works on Central America, Yucatan, and Chiapas.
24. Norman's works on Yucatan and Mexico.
25. Catherwood's illustrations of Stephens's works.
26. Bartlett's "Personal Narrative."
27. Mexico: Aztec, Spanish, and Republican.
28. De Solis, "Historia de la Conquista de Mejico."
29. Robertson's "History of America."
30. Prescott's "History of the Conquest of Mexico."
31. Ramirez, Notes on the Spanish translation of the last work;—Mexico, 1844.
32. The vols. of the American Ethnological Society's Transactions."

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[1] Prescott, Conq. Mex., I, 12, *note*.
[2] Storia de Messico, I, 128.
[3] Nueva Historia General, p. 110.
[4] Ixtlilxochitl, Rel.
[5] Prescott, Conquista de Mejico, vol. II; *notes*, p. 1.

calculated to throw around it. He admits, I think with great justice, that Antonio de Leon y Gama "has achieved the first and *only* rigorously archæological investigation in his country;"[1] and he very properly adds, in regard to these mythic periods, that "historical criticism, notwithstanding the quantity written on the subject, is probably the most difficult and least advanced portion of Mexican literature; for, while some of our writers incur imminent risk from excessive credulity, others are governed by a scepticism which is radically destructive of all scientific investigation. A history may be true and highly instructive, though it contains the most incredible absurdities; for while it states what may be absolutely false, either through invention or insufficient proof, *it may faithfully transmit the traditions, beliefs, and customs of the people it describes.* * * * * Mexican history, like that of all nations, is made up of two classes of narratives; the usages, customs, and ruling beliefs *which present the type of the people,* and of the public and private life of its eminent men, together with facts which concern *the mass* of the community, and constitute the very life and essence of a people."[2]

Thus, it may be said that the deciphered picture writings found among the Mexicans by the Spaniards, together with the traditions recorded by Ixtlilxochitl, Sahagun, and others, will, in all likelihood, be found to present a typical idea of the individual, tribal, and national character. Some great historical facts may stand out in bold relief; some persons, and certain biographical incidents may appear in shadowy outline through the veil of the past; but the whole antiquity, blurred by dilapidation, looms up dimly, like a noble ruin in the gloom of twilight.

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[1] Gama's "Descripcion historica y chronologica de las dos piedras descubiertas en la plaza principal de esta ciudad." Mexico, 1832. 2d edition.

[2] Ramirez; Notes to the Spanish translation of Prescott's Conq. Mex., II, p. 8 (*of notes*).

CHAPTER II.

THE letters of Cortez to the Emperor Charles V., and the writings of Bernal Diaz del Castillo, Sahagun, Torquemada, Las Casas, Oviedo, Boturini, Veytia, and Clavigero, digested as they have been in the valuable work of Mr. Prescott, display a picture of the Aztec people as they existed at the period of European occupation. We are informed, no doubt accurately, as to much of the religion, laws, science, and social life of the conquered. The Spanish exaggerations were thoroughly examined, and the essential, characteristic facts have been preserved for our acceptance.

The ancient history of the foundation of the Aztec empire, stripped of most of its myths, may be comprised in a few paragraphs.

At the period of the conquest by Cortez, the Vale of Anahuac, with its assemblage of lakes, levels, and mountains, seems to have been the conceded seat and centre of greatest civilization on the northern continent. Yucatan and the territory of the Zapotecs were doubtless inhabited by a refined people; but they were probably subordinate to the Aztecs by conquest. The received traditions as to the Vale of Anahuac declare that the original inhabitants came from some unknown place "at the north," and, in the fifth or eighth century, settled at Tollan or Tula, in the neighborhood of the Mexican Valley. This spot became the parent hive of an industrious and progressive people, whose northern frames and characters were civilized and not emasculated by the more genial climate to which they migrated. They cultivated the soil, built extensive cities, conquered their neighbors, and, after performing their allotted task in the development of our continent, wasted away in the tenth or eleventh century, under the desolation of famine and unsuccessful wars. The Toltec remnant emigrated southward; and, during the next hundred years, the valleys and mountains of this beautiful region were nearly abandoned, until a rude tribe, known as the Chichimecas, came "from the north," and settled among the ruins abandoned by the Toltecs. Some years afterwards, six tribes of the Nahuatlacs reached the valley, announcing the approach of another band "from the north," known as the Aztecs. About this period, the Acolhuans, who bordered on the Chichimecas before their southward emigration, entered the Valley of Anahuac, and allied themselves with their ancient neighbors. These tribes appear to have been the founders of the Tezcocan government, which, in the fifteenth century was consolidated by the courage and talents of Nezahualcoyotl.

Thus it was that wave after wave of population poured "*from the north*" into the valley, till it was reached by the Aztecs, who, about the year 1160, left their mysterious and unknown "northern" site at Aztlan. Their wanderings were slow. It is alleged that one hundred and sixty-five years elapsed before they descried "an eagle grasping in his claw a writhing serpent, and resting on a cactus which sprang from a rock in the Lake of Tezcoco. This had been designated by the Aztec oracles as the spot where the tribe should settle, after its long and weary migration; and, accordingly, the city of Tenochtitlan was founded on the sacred rock, and, like another Venice, rose from the bosom of the placid waters.

"It was nearly a hundred years after the founding of the city, and in the beginning of the fifteenth century, that the Tepanecs attacked the Tezcocan monarchy. The Tezcocans and the Aztecs united to put down the spoiler, and, as a recompense for the important services of the allies, the supreme dominion of the territory of the Tezcocans was transferred to the Aztecs. The Tezcocan sovereigns thus became, in a measure, mediatized princes of the Mexican throne; and the two states, together with the neighboring small state of Tlacopan, south of Lake Chalco, formed an offensive and defensive league, which was sustained with unwavering fidelity throughout the wars of the succeeding century. The bold allies united in the spirit of conquest and plunder which characterizes a rude, martial people, as soon as they are surrounded by the necessaries and comforts of life in their own country, or whenever the increase of population begins to require a vent through which it may expend those energies which would explode in civil war, if pent up within so small a realm as the Valley of Mexico. Accordingly, we find that the sway of these tribes, which had but just nestled among the rocks and marshes of the lakes, was quickly spread beyond the mountains that hemmed in the valley. The Aztec arms were triumphant throughout all the plains that swept down towards the Atlantic and Pacific, and penetrated, as is alleged by some authorities, even to Guatemala and Nicaragua."[1]

Large, however, as was this dominion of the Aztecs and their allies, it must be recollected that their territorial power did not cover the entire region which was known subsequently as New Spain or Mexico. In addition to the tribes or states I have mentioned in this notice, as constituting the nucleus of the empire at the period of the conquest, there were numerous other aboriginal powers, among which the Cholulans and Tlascalans were the most eminent. Besides these, there were, on territory now comprehended within the Mexican Republic, the Tarascos, who inhabited Michoacan, an independent sovereignty; the barbarous Ottomies; the Olmecs; the Xilancas; the Mistecas; and the Zapotecs. The Aztec arms had recently subdued the region of Oajaca, and the last-named tribe, with all its civilization, had submitted to Ahuitzotl.[2]

There was something, doubtless, in the geographical position and geological structure of this remarkable region, that assisted in making it the seat of empire. History shows that colonial offshoots are modified by climatic change. The great

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[1] Mexico: Aztec, Spanish, and Republican, I, 96.

[2] As an illustration of the uncertainty of the early aboriginal history of Mexican tribes and nations, and especially of their chronology, I annex the following tables of their emigrations from the north, and of the duration of the reigns of Mexican sovereigns. They were compiled by Mr. Gallatin from a comparison of Ixtlilxochitl, Sahagun, Veytia, Clavigero, the Mendoza collection of ancient picture writings, the Codex Tellurianus, and Acosta, and inserted in the 1st vol. of our Ethnological Society's Transactions, p. 162. The tables will be found on the next page.

features of Mexico are the same now that they were in the tenth and sixteenth centuries. The waters of the Atlantic, sweeping along the central parts of our continent, and compressed within the gulf by the curving shores of Florida and

	Ixtlilxochitl.	Sahagun.	Veytia.	Clavigero.
TOLTEC EMIGRATION, &c.				
Arrived at Huehuetlalpallan	387	...	...	...
Departed from Huehuetlalpallan	...	...	596	544
They found Tula	498	...	713	720
Monarchy begins	510	...	...	667
Monarchy ends	959	...	1116	1051
CHICHIMECAS AND ACOLHUANS OR TEZCOCANS.				
Xolotl, 1st king, occupies the valley of Mexico	963	...	1120	about 1170
Napoltzin, 2d king, ascends the throne	1075	...	1232	13 cen.
Huetzin } Tlotzin } 3d king, so called erroneously, ascends the throne	1107	...	1263	14 cen.
Quinantzin, 4th king, ascends the throne	1141	...	1298	14 cen.
Tlaltecatzin, 1st king according to Sahagun, ascends the throne	...	1246	...	...
Techotlalatzin 5th (2d Sahagun) ascends the throne	1253	1271	1357	14 cen.
Ixtlilxochitl 6th (3d Sahagun) ascends the throne	1357	1331	1409	1406
Netzahual-Coyotzin 7th (4th, Sahagun) ascends the throne	1418	1392	1418	1426
Netzahual-Pilzintli 8th (5th, Sahagun) ascends the throne	1462	1463	...	1470
Netzahual-Pilzintli dies	1515	1516	...	1516
TEPANECS, OR TECPANECS OF ACAPULCO.				
Acolhua arrives	1011	...	1158	...
Acolhua, 2d son of Acolhua 1st, arrives	...	...	1239	...
Tezozomac, son according to D'Alva, grandson according to Veytia, of the 1st Acolhua, arrives	1299	1348	1343	...
Maxtlan, son of Tezozomac, arrives	1427	...	1427	1422
MEXICAN OR AZTEC EMIGRATION.				
Mexicans leave Aztlan	...	...	1064	1160
" arrive at Huelcolhuacan	...	...	...	1168
" " at Chicomotzoc	...	...	1168	...
" " at valley of Mexico	1141	...	1227	1216
" " at Chapultepec	...	...	{ 1248 1276	1245

	Mendoza's Collection.	Codex Tellurianus.	Acosta.	Siguenza.	Ixtlilxochitl.	Sahagun.	Veytia.	Clavigero.
MEXICAN OR AZTEC POWER.								
Foundation of Mexico or Tenochtitlan	1324	...	...	1325	1220	...	1325	1325
Acamapichtli, elected king	1375	1399	1384	1361	1141	1384	1361	1352
Huitzilihuitl, accession	1396	1406	1424	1403	1353	...	1402	1389
Chimalpopoca	1417	1414	1427	1414	1357	...	1414	1409
Ytzcoatl	1427	1426	1437	1427	1427	...	1427	1423
Montezuma 1st	1440	1440	1449	1440	1440	...	...	1436
Acayacatl	1469	1469	1481	1468	1469	...	...	1464
Tizoc	1482	1483	1487	1481	1483	...	...	1477
Ahuitzol	1486	1486	1492	1486	1486	...	...	1482
Montezuma 2d	1502	1502	1503	1502	1503	...	...	1502
DURATION OF REIGNS OF MEXICAN KINGS.								
Acamapichtli	21	7	40	42	150	21	41	37
Huitzilihuitl	21	8	3	11	50	21	12	20
Chimalpopoca	10	12	10	13	70	10	13	14
Ytzcoatl	13	14	12	13	13	14	...	13
Montezuma 1st	29	29	32	28	29	30	...	28
Acayacatl	13	14	6	13	14	14	...	13
Tizoc	4	3	5	5	3	4	...	5
Ahuitzol	16	16	11	16	17	8	...	16
Montezuma 2d	17	17	16	17	17	19	...	17

The discrepancies between these authorities, amounting, in many cases, not only to years but centuries, show the extremely unreliable and mythic character of the records and traditions of the ante-Columbian period.

Yucatan, whirl the shifting bed of the sea in continual eddies at the mouths of the few rivers that pour into it, and create the formidable bars and shoals which make the eastern coast so dangerous an anchorage. But on the west, the shores of the Pacific are favored with tranquil and commodious havens, while numerous indentations break the rugged outline of the coast with landlocked bays.

The voyager may sail from the extreme eastern shores of our continent to the very centre of the Mexican Gulf-coast, along a low sandy beach, visible only at a short distance from the sea; but as he advances to that point, the snowy peak of Orizaba, towering seventeen thousand feet above the ocean, looms up in the distance like an outpost sentinel of Mexico, indicating his approach to the dividing ridge of lofty mountains. The vast Cordillera which rises near the Frozen Sea, descends southward in a series of mighty waves through the whole of this continent, until it is lost in the ocean at Cape Horn; while at the Isthmus which links the great body of North to South America, it parts the two seas that strive to meet across this narrowest portion of the Western World. Between the 16th and 33d degrees of north latitude, this mountain range sends forth a multitude of spurs and branches, and, within that confined space, piled on a massive base of *sierras*, rising from the Atlantic till they reach the height of nearly eighteen thousand feet, and thence plunging westward into the Pacific, is the territory of Mexico, hung upon these sloping cliffs, and resting among the sheltered recesses of their upland valleys.

Two important rivers may be said to form the *natural* northern boundary of this region. The snow that melts on the Sierra Nevada, descends, one-half to feed the fountains of the Rio Grande, which winds through an immense extent of country before it falls into the Gulf of Mexico—and one-half to swell the Colorado of California, before it reaches the Pacific through the Sea of Cortez. The sources of these two streams nearly meet at the same mountain, in the neighborhood of the fortieth degree; but the configuration of the earth essentially varies between the northern and southern sides of these rivers. From their northern banks the land recedes in comparative levels, interspersed with arid wastes and prairies, sloping gradually to the Pacific and Atlantic; while from their southern banks the country almost directly breaks into the steeps of the Sierra Nevada, whose multiplied veins enlace the whole of Mexico with a massive network. Uncertain streams—none of which are navigable, and all dependent on rain for their floods—pour down the precipitous defiles, on their way to the seas. As the centre of this territory is approached, the naked Cordilleras become loftier and loftier, as if to guard, with double security, the heart of the nation; while, in the midst of this sublime congregation of mountains, rise still more majestic peaks crowned with eternal snow, presiding over the beautiful valley of Anahuac, wherein the ancient Aztec capital nestled on the border of its crystal lake. Flanked by two oceans, and rising from both to the rich plateaus of the table-land, Mexico possesses, on both acclivities, all the temperatures of the world, and ranges from the orange and plantain on the seashore, to eternal ice on the precipices that overhang the higher valleys. Change of climate is attained merely by ascending, and, in a region where the country rises steeply, the broad-leaved aloe and feathery palm may be seen relieved against the

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everlasting snow of Popocateptl. All these delightful climates produce the fruits and flowers of the tropics on the same parallel of latitude that crosses continual frost, while, over all, a never ending spring bends its cloudless arch. Nor are these the only allurements of this wonderful land, for nature, as if unsatisfied with pampering the tastes of man by crowding the surface of the earth with everything that might please his appetite or delight his eye, has veined its sterile mountains with precious ores in exhaustless quantity.

It is not surprising that hardy races *from the northern hive,* where vigor is gained from toil and where toil wrests existence from an ungenerous soil, abandoned their savage habits and were subdued into a masculine civilization by a country and climate like these. It was a tropical Switzerland. Such a people, by migration, may lose nothing of their energy except its barbarism, and gain nothing from the softer skies but their genial blandness.

CHAPTER III.

It is conceded that, at the period of the first European occupation, all parts of North and South America were peopled; and Dr. Morton, in his elaborate "Inquiry into the Distinctive Characteristics of the Aboriginal Race of America,"[1] says, "That the study of *physical conformation alone*, excludes every branch of the Caucasian race from any obvious participation in the peopling of this continent." * * * * "Our conclusion," he continues, "long ago deduced from a patient examination of the facts thus briefly and inadequately stated, is that the American race is essentially separate and peculiar, whether we regard it in its physical, its moral, or its intellectual relations. * * * * I maintain that the organic characters of the people themselves, through all the endless ramifications of tribes or nations, prove them to belong to one and the same race, and that this race is distinct from all others."

Without stopping to discuss Dr. Morton's opinion, let us now consider the general characteristics of the remains still visible on this continent, and especially of the architectural antiquities of Mexico.

"Architecture is one of those massive records, either of intelligence or absurdity, which require too much labor in order to perpetuate a falsehood. It shows what the men could do, be it good or bad, elegant or hideous, civilized or barbaric. The men who built the edifices of Uxmal, Palenque, Copan, and Chichen-Itza, were far removed from the condition of nomadic tribes. Taste and luxury had long been grafted on the mere wants of the natives. They had learned to build, not only for protection against weather, but for permanent residences whose internal arrangements afforded comfort, and whose external embellishment might gratify public taste. Order, symmetry, elegance, beauty of ornament, gracefulness of symbolic imagery, had all combined for the manifestations which are always beheld among people who are not only anxious to gratify others as well as themselves, but to vie with each other in the exhibition of individual tastes. Here, however, as in Egypt, the remains are chiefly of temples, palaces, and tombs. The worship of God, the safety of the body after death, and obedience to authority, are demonstrated by the temple, tomb, and rock-built palace. The masses who felt or imagined they had no constant abiding place on earth, and that posterity had little interest in them as individuals, did not, in all likelihood, build those numerous and comfortable dwellings, under whose influence modern civilization has so far surpassed the barren *humanism* of the valley of the Nile."[2]

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[1] Pp. 35, 36, 2d edition, Philadelphia, 1844.

[2] Mexico; Aztec, Spanish, and Republican, Vol. I.

"If the far-off past has not always been able to write its name, it has left its mark," says Robert Cary Long, in his ingenious discourse on the ancient architecture of America, delivered before the New York Historical Society, in 1849.[1] "Its stony autographs loom out largely from the page of time. Egypt has piled hers in Pyramids; India has quaintly carved hers in the Rocks of Ellora; Greece has delicately shaped hers, in a form of ever living beauty, upon the Acropolis; Rome has rounded hers in magnificent proportions in the dome of the Pantheon; and the Middle Ages have 'illuminated' their signature with those heaven-reaching coruscations, the Gothic cathedrals." * * * * "In the monuments of the past we have the human deposit of the ages—*the truth of the historical past.* Architecture, in this view, is the geology of humanity. Ceasing its testimony at the present surface of the globe, geology tells nothing of that subsequent history which commences with the existence of men. Here, architecture resumes the thread of the narrative, and bears witness of that compound existence to which it owes its origin. * * * * * That consecutiveness which is dimly descried in documents, in architecture is apparent; that human progress, in which all believe, but which so few show forth distinctly, is beautifully narrated in the monumental series."

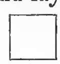
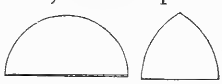
In the absence of unquestionable historic and recorded evidence, I have always considered *architectural forms,* disclosed in the remains of antiquity, as the most valuable hints for detecting the relative stages of the human family in the process of civilization. Craniology and osteologic science may show the relative capacity of races for civilization, but they do not demonstrate the degree attained; while the Druidical stonehenge, the Indian mound, the Egyptian tomb and palace, the Greek temple, and the Roman Coliseum, are types of the progressive intellectual grades of their respective builders.

It is true that, where there are intertribal or international communications between people, the arts of the most advanced may be adopted by those who are in the rear; but it is dangerous, and I think unscientific, to start with the theory that resemblances, or even identities, in any of the arts, indicate either international connection or imitation. The basis of all action is the mind, and we know that it originates similar inventions,—according to individual capacity,—throughout the most widely separated conditions of the human family.

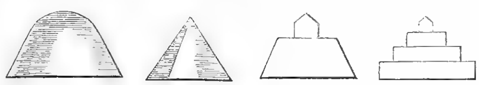
"Analogies of this kind," says Baron Humboldt, in his Voyage Pittoresque, "prove very little in favor of the ancient intercommunication between people, for, under all the zones, men have indulged in a *rhythmic repetition* of the same forms."

To understand the force of this and its sensible value, let us recur to the simple and natural process in the law of inventive progress. A hunter or shepherd will content himself by leaning the branches of trees against each other to shield himself from sun or rain in his temporary bivouac, and, hence the first form is that of the tent:  . If he is a wanderer, and inhabits, at times, the plains as well as the forest, he will construct a permanent and portable covering of skins and poles,

[1] Long's Ancient Architecture of America, pp. 5 and 6.

so as to constitute the Indian lodge, which preserves the same shape as the tent. As he becomes less nomadic, begins to possess property, family, flocks, and herds, and requires more covered space for protection as well as comfort, he discovers that a square affords more commodious room than an angle, and his edifice assumes a new shape by the use of several of his simple architectural elements, instead of two. Accordingly, he plants his stouter timbers upright in the ground, and lays across them a covering of branches and leaves, so as to form a square: . But this, in the course of time, admits of improvement—especially as the flat covering is not as sure a protection against rain as his original tent; and, accordingly, on the last of his inventions he elevates the first, so as to preserve his space and insure additional comfort: . Perhaps, instead of forming his tent by simple boughs or poles, lodged against each other, he has contented himself with bending the saplings together, and thus produces the elemental shapes of the Roman and Gothic arches: . As wandering families unite in tribes, and tribes grow into communities, and communities associate in municipalities or nations, their most skilful builders discover that mechanical genius has no more elements for architectural progress in forms than a straight line and a curve; so that all invention is limited, by an irreversible law, to their wise and tasteful combination.

Is it hazarding too much, then, to assert that, in early stages of civilization, we must naturally expect to see much of the type of national *status* in architectural combinations of the mound and pyramid?



Again; is it venturing too far to suggest that, when people emerge from early stages of civilization, and rise to vigorous, masculine, and refined nationality, they abandon the propped weakness of leaning pyramidal shapes, and seek the massive, self-sustaining independence of upright, perpendicular forms?[1]

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[1] These are general suggestions upon the world's progress in mechanics and taste, and altogether independent of art as controlled by climatic or geological necessities. A perfectly flat roof in Switzerland would cave in under accumulated snows, and an unsupported edifice in a volcanic region would be destroyed wherever earthquakes were frequent and violent.

CHAPTER IV.

THE aborigines of our country at the period of the Discovery, or their ancestors, were all more or less engaged in building for defence or worship. The elaborate works of Squier, Davis, Whittlesey, and Lapham, published by the Smithsonian Institution, have described, perhaps everything of value among the Indian remains within our territory.[1]

These aboriginal relics—chiefly earthworks—may be comprised in two classes: simple Mounds, and Enclosures bounded by parapets and circumvallations or walls. The mounds are asserted to have been places of sepulture, sacrifice, and worship, or sometimes devoted to various mixed uses; while the enclosures were intended either for defence, or for sacred or superstitious purposes. The rude pyramidal mounds were frequently of great and massive dimensions, while the *bird and beast shapes* of their ground plans, in Wisconsin, as described in the work of Mr. Lapham, are as singular as they are inexplicable.[2]

The mound, or *heap-shape*—derived, perhaps originally, from the earth that was piled over a body in burial—seems to have been the most common throughout our entire territory as far as the northern shores of the Gulf of Mexico and the Rio Grande. It indicates the early condition of art or the unprogressive character of the builders, who either disappeared from the land, degenerated into the modern Indian, or passed southward to become the progenitors of semi-civilization in more genial regions.

In the mounds have been found ornaments, carvings, pipes, skeletons, shells, spear and arrow-heads, hornstone knives, axes, copper chisels and gravers, silver, galena, and various utensils of pottery; but all the *forms* of these implements, and especially those of the domestic vessels and images, indicate a rude state of art, taste, invention, and wants. No discoveries have yet been made to show that the mound-builders communicated or preserved facts by permanent records or monuments; and their nearest approach to *printing* is a figured stamp, found, some years since, in a mound at Cincinnati, which resembles the stamps I have seen in Mexico, used by the ancient people of that region, either to impress marks upon paper or patterns on their stuffs.[3]

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[1] See Squier's Paper in the 2d Vol. Trans. Am. Eth. Soc., pp. 136, 137, 138, and his Ancient Mon. Vall. Miss., and of N. York, &c. &c.; Whittlesey's Descrip. of Ancient Works in Ohio; Lapham's Antiq. of Wisconsin.

[2] See Lapham's Antiquities of Wisconsin in the Smithsonian Contributions.

[3] This stamp, of which I possess a cast, is very accurately represented in Squier and Davis's Ancient Mon. Val. Mississippi, p. 275. The inscribed stones and rocks that have been found are very apocryphal as to period and purpose; nor are they numerous enough to indicate an ancient system.

Quitting the shores of the Gulf of Mexico, and penetrating the old northern territories of New Spain, we find, for the first time in our southern progress, the remains which have become so generally known in Spanish, as the "CASAS GRANDES," or Large Houses; all of which are probably ruins of villages and towns occupied by the aboriginal tribes described by Castañeda, in the expedition of Francisco Vasquez de Coronado, in 1541, in search of the rich cities which had been reported to exist in those northern regions. The accounts of Castañeda and of modern travellers, coincide as to the character of architecture, ground-plans, and general purposes of the remains; and it is here that we see *perpendicular walls*, another evidence of an improved degree of civilization. The houses were not built of stones, but of *adobés*, or sun-dried bricks; and, as the natives had no lime, they substituted for it a mixture of earth, coals, and ashes. Some of these houses were four stories high, while their interiors were reached by ladders from the outside, so as to render the external, *doorless* walls, protections against enemies in the wars which seem to have been almost constantly occurring. The village of Acuco, described by the Spanish writers as lying between Cibola and Tiguex, was built on top of a perpendicular rock, which could only be ascended by three hundred steep steps cut in the stone, and clambering eighteen feet more by the aid of simple holes or grooves in the precipice. The tribes are spoken of as agricultural and warlike, nor does it seem that they had advanced further in social progress than by constructions for defence and comfort, of a superior character to those of the tribes beyond the waters of the Rio Grande. The fact is established, by Coronado's expedition, says Mr. Gallatin, that "at the time of the conquest by Cortez, there was, northwardly, at the distance of eight hundred or one thousand miles from the city of Mexico, a collection of Indian tribes in a state of semi-civilization, *intermediary* between that of the Mexicans and the social state of any other aborigines."[1]

Moving southward, we enter the present actual territory of the Mexican Republic, and encounter the first remarkable architectural remains of antiquity in the State of Zacatecas, on an eminence called the "Cerro de los Edificios," or Hill of the Buildings, situated about twelve leagues southwest from the city of Zacatecas, about one league north of La Quemada, and in the neighborhood of $22\frac{1}{2}°$ north latitude, at an elevation of 7,406 feet above the sea. Clavigero speaks of Chico-mozoc, or Chico-comoc, a sojourning place of the Aztecs in their southward emigration, and inclines to the belief that these remains are the relics of their provisional architecture. A very full account of the ruins is given in Captain Lyons's travels in Mexico, and another in Nebel's "Voyage Pittoresque et Archæologique," in which the walls, squares, pyramids, terraces, roads, pavements, &c., are described and partially delineated. The site of the remains seems to have been the citadel, fortress, or defensive portion of a settlement which was spread out extensively over the adjacent plain. The northern side of the hill rises by an easy slope from the plain, and is guarded by a double wall and a kind of bastion; while on the other sides,

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[1] See Castañeda, Voyage à Cibola, Paris, 1838. Am. Eth. Soc. Trans., Vol. II, p. lxxxiii of *introduction.* Mr. Gallatin of course means the "social state of any other" *northern* "aborigines." See, also, Mr. "Bartlett's Personal Narrative," in relation to the *North Mexican* remains.

the precipitous rocks of the hill itself form natural defences. The whole elevation is covered with fragments; the rock-built walls (many of which are twenty-two feet in thickness) are sometimes joined by mortar of no great tenacity, but are retained in their positions mainly by their massiveness.[1]

If we leave these loftier regions of the table-lands of Mexico, and descend towards the eastern coast of Mexico, through the State of San Louis Potosi, we find the architectural remains, sculpture, &c., visited by Mr. Norman, in 1844.[2] The relics discovered by this intelligent traveller were of mounds, pyramids, edifices, tombs, images, fragments of *obsidian* knives or arrows, and pottery. Hewn blocks of concrete sandstone were, in many instances, the materials used for building; and, besides the images of clay, he found others rudely cut in stone in bold relief. The most significant of these remains, as well as the most extensive evidences of civic civilization, were placed, by Mr. Norman, at about 22° 9′ of north latitude, and 98° 31′ of west longitude.

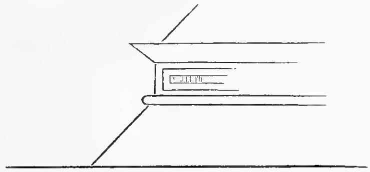
The State of Vera Cruz, in Mexico, adjoins Tamaulipas on the south, and here, in the vicinity of Panuco, an old town of the Huestecos, Mr. Norman found remains of architecture and sculpture scattered over an area of many miles, the history and traditions of which are altogether unknown among the present indolent inhabitants of the region. Three leagues south of Panuco are more ruins, known as those of Chacuaco, represented as covering about three square leagues, all of which seem to have been comprised within the bounds of a large city. Five leagues southwest of these are some remains at San Nicolas; and six leagues, in nearly the same direction, are others, at La Trinidad. More relics of the same character, together with quantities of pottery, vessels, clay images, &c. &c., are found in the same district; and it is to be regretted that the character of the inhabitants, as well as the health of the region, do not invite a more thorough scientific examination of the State.

Sixteen leagues from the sea, and fifty-two north of the city of Vera Cruz, on the eastern slope of the Cordillera, and two leagues from the Indian hamlet of Papantla, lie, spread over the plain, the massive ruins of an ancient city, which, in its palmy days, was perhaps more than a mile and a half in circuit. The best account we have of this spot is to be found in Nebel's work, and, if we can rely on the accuracy of his drawing of the Pyramid—called by the neighboring Indians "El Tajin"—it is unquestionably one of the most perfect and symmetrical relics of antiquity within the present limits of the Mexican republic. Time has done its work upon the edifice; but, according to Nebel, the whole form and character of the architecture are still discernible beneath the trees and vines that have sprung up among its loosened joints. The pyramid is represented by this artist as being built of sandstone, nicely squared and united, and covered with a hard stucco, which seems to have been painted. Its base, on all sides, is one hundred and twenty feet; and as it is ascended by a stair, composed of fifty-seven steps, each

[1] See Lyons's Travels in Mexico; Nebel's Voyage, &c. &c.; Mexico; Aztec, Spanish, and Republican; Clavigero, 'Storia de Messico.'

[2] Norman's Rambles by Land and Water, and Notes of Travel in Cuba and Mexico.

measuring a foot in height, it may be calculated that the summit was at least sixty feet from the ground. It consists of seven stories or bodies, each decreasing in size as it ascends from the base, and all of the form shown by the annexed profile of the lower story:—



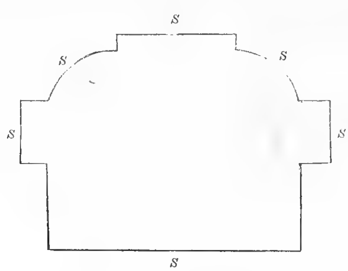
A few miles from Papantla, near an Indian *rancho,* called Mapilca, Mr. Nebel discovered more *pyramids,* carved stones, and the ruins of an extensive town, but everything was so overgrown with the tropical vegetation, that he found it impossible to penetrate the district, and examine the relics. The artist has preserved the drawing of only a single sculpured stone, which he describes as twenty-one feet long and of close-grained granite. The figures carved on the fragment differ from the ancient sculptures found east of the main Cordillera, and somewhat resemble those in Oajaca. By excavating in front of the stone, Mr. Nebel discovered a road formed of irregular blocks, not unlike the old Roman pavements.

About fifteen leagues west of Papantla, and still in the State of Vera Cruz, in a small plain at the foot of the eastern Cordillera, are the remains known as those of Tusapan, which is supposed to have been a settlement of Totonacs. The vestiges of this small aboriginal establishment are nearly obliterated, and the only striking objects at present are a fountain—in human shape,—and a pyramid of four stories or bodies, in which the pyramidal and *vertical* lines are again united—the second story being reached, at a door, by a flight of steps. This pyramid is built of *stones,* of unequal sizes, and has a base of thirty feet on each of its four sides. In front of the door stands a pedestal, but the idol it probably supported has been destroyed. Around the pyramid are scattered masses of stone, rudely carved, to represent men and various animals; yet, from the inferior manner in which the work is executed, we may judge that the art of *ornamentation* was just beginning to be engrafted on the pyramidal and vertical architecture of the builders. The fountain to which I have alluded, is cut from solid rock; is nineteen feet high, and represents a female in an indecent, squatting attitude. The remains of a pipe which conveyed water to the image, is still seen in the back of the head, and the liquid passed through the body of the gigantic work, till it was discharged below the figure into a basin and canal, which carried it to the neighboring town.

On the Island of Sacrificios, just south of the present city of Vera Cruz, there are no longer any architectural remains of edifices used for those brutal rites which made the spot so celebrated at the period of the conquest; but the soil has yielded many relics in the shape of vases, images, carvings, sepulchres, and skeletons; and it is said that fragments of pottery and obsidian are still found in considerable quantities.

If we go westward from this spot, and penetrate the State of Vera Cruz until we strike a ridge of mountains in the district of Misantla, about thirty miles from the well known and beautiful town of Jalapa, we encounter a precipitous elevation, near the Cerro of Estillero, on whose narrow strip of table-land the remains of an extensive town were discovered in 1835. It is described as perfectly isolated. Steep rocks and ravines surround the mountain, and beyond these precipices there is a lofty wall of hills from the summit of which the sea is visible. As the mountain plain is approached, the traveller discovers a broken wall of massive stones united by a weak cement, which seems to have constituted the boundary or fortification of a circular area or open space, in whose centre a pyramid, with three stages (but without any mixture of vertical lines in the shape), rises to a height of eighty feet, having a base of forty feet, on two sides, by forty-nine on the two others. Beyond the encircling wall are the remains of the town, extending northward for nearly three miles along the table-land. The stone foundations—large, square, and massive—are still distinguishable, and the lines of the streets may be traced in blocks, about 300 yards from each other. Some of the walls of these edifices are still standing, in broken masses, at a height of three or four feet from the ground. South of the town are the fragments of a low wall, evidently intended for defence in that quarter; while, north of it, there is a tongue of land, jutting out towards the precipitous edge of the mountain, the centre of which is occupied by a mound, supposed by explorers to have been the cemetery of the ancient inhabitants. Twelve tombs, built of stone, and a number of carved figures, vases, and utensils were exhumed; but the images and minor objects were taken to Vera Cruz, and all trace of them has unfortunately been lost.[1]

In November, 1843, further east of these remains, Don José Maria Esteva found in a thick forest, about three miles and a half from the Puente Nacional or national bridge, the interesting remains of architecture which had been first visited in 1819 or '20 by a clergyman named Cabeça de Vaca. The temple or teocalli *seems to be an exceedingly steep pyramid of steps*, the *base* of which is shaped as follows:



It is elevated on a mount about one hundred and fifty feet above the level of a stream which flows at its feet; and, in consequence of the inequality of the ground, is thirty-three Spanish feet high on some of its sides and forty-two on others. It fronts eastwardly, and the platform of its top is reached by thirty-four

[1] Mosaico Mejicano.

steps, so as to be almost perpendicular to the base. This platform is forty-eight Spanish feet broad and seventy long, and the steps rise on all the sides indicated on the above ground-plan by the letter *S*. The entire structure is of sand, lime, and large stones taken from the bed of the stream; and though very old and of course covered with a thick mantle of tropical plants and trees, its form is declared to be almost perfect. At first it was supposed to be solid, but an entrance was discovered from the west, but so small and clogged that the explorers were not disposed to venture within for fear of venomous insects and serpents with which the interior in all likelihood is swarming.[1]

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[1] See Museo Mejicano, II, 465, for plate and description.

CHAPTER V.

East of the State of Vera Cruz, but separated from it by Tobasco and the southern bend of the Gulf of Mexico, lies the State of Yucatan; and, southeast of it, the State of Chiapas.

The physical character of these States demonstrates the prolific and agreeable climate that probably attracted the large population with which the region must have been filled before the Spanish conquest. Since 1840, three important works have been issued by the American press relative to the architectural remains in these States. Two of these are from the pen and pencil of the late Messrs. John L. Stephens and Catherwood, while the third is the result of a visit paid to Yucatan in 1841–2, by Mr. B. M. Norman.[1] In the "long, irregular route" pursued by Stephens and Catherwood, "they discovered the remains of *fifty-four ancient cities,* most of them but a short distance apart, though, from the great change that has taken place in the country and the breaking up of old roads, having no direct communication with each other. With but few exceptions, all were lost, buried, and unknown, never before visited by a stranger, and some of them, perhaps, never looked upon by the eyes of a white man." In Chiapas, the travellers encountered remarkable architectural remains at Ocozingo and Palenque, between 16° and 18° of N. latitude; and passing thence to Yucatan, they found the more northern peninsular region crowded with monumental ruins at Maxcanu, Uxmal, Sacbey, Xampon, Sanacte, Chun-hu-hu, Labpahk, Iturbide, Mayapan, San Francisco, Ticul, Nochacab, Xoch, Kabah, Sabatsche, Labna, Kenick, Izamal, Saccacal, Tecax, Akil, Mani, Macoba, Becanchen, Peto, Chichen, in the interior of the State; and at Tuloom, Tancar, and on the island of Cozumel, on its eastern coast. All these architectural remnants of the past, lie between the 18° and 21½° of N. latitude. Of all this numerous catalogue, the remains at Palenque in Chiapas, and of Uxmal and Chichen in Yucatan, are certainly the most remarkable for their architectural forms as well as embellishments; but they have been made known so popularly throughout the world by the books of our countrymen, that it is unnecessary to dwell upon their characteristics in this summary sketch. Mr. Stephens believed, after full investigation, that most of these cities and towns were occupied by the original builders and their descendants, at the time of the conquest.[2] If any reliance is to be placed

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[1] Rambles in Yucatan, by B. M. Norman, 1 vol.; Stephens' Incidents of Travel in Central America, Chiapa, and Yucatan, 2 vols.; and Stephens' Incidents of Travels in Yucatan, 2 vols., both of the latter works being illustrated by Mr. Catherwood, who has since published many of his drawings in a separate folio.

[2] See his first work, Vol. II, Chapter XXVI; and his second, Vol. II, p. 444. See, also, Trans. Am. Eth. Soc., Vol. I, and Stephens' Yucatan, for an account of the calendar and language of the people, and some other ethnographic facts.

on the theory of progressive architectural forms, the drawings of Catherwood show that these tribes or nations of the aborigines had advanced to a very important stage, though their style of "ornamentation" indicates that they had not entirely abandoned the barbaric for the beautiful.

Returning again, northward, from the extreme southern limits of Mexico, we find, in the State of Puebla—which lies directly west of the northern part of the State of Vera Cruz—at about 19° of north latitude, the well known remains of the Pyramid of Cholula. It was originally constructed of *adobés,* or sun-dried bricks, and may therefore be considered a sort of earthwork. The huge pyramidal mass rises abruptly from the plain of Puebla to a height of 204 feet,[1] and was composed of four stages or stories connected by terraces; but the materials of the mound have been so worn by the attrition of time and seasons, that at present it resembles one of those Indian heaps of our own West, with which the reader has been made acquainted in the volumes of Squier and Davis. The most striking and valuable facts in regard to it—as its shape was simply pyramidal—are to be found in the labor and materials which were expended on a work whose base line measures 1,060 feet, and whose present elevation reaches 204.

Adjoining the State of Puebla, immediately west of it, and, of course, in the neighborhood of the same latitude, we enter the State of Mexico, the seat and centre of the Aztec population which submitted to Cortez. The Spanish settlement which occupied the site of the ancient capital, very soon obliterated every *architectural* vestige of the aborigines, so that I am not aware, either from my own personal examinations, or from the reports of travellers, that any remains of temples, palaces, pyramids, or other edifices, are preserved in or very near the city of Mexico. The National Museum, and a few private collections, are full of small relics of various characters, which have been found on the surface or disinterred in the neighborhood. These relics are either of stone, carved with skill or roughly; or of clay burnt to the requisite hardness for utensils. To the images or objects, connected, as is supposed, with the religion and science of the Aztecs, various and perhaps arbitrary names have often been affixed by antiquarians, but their description belongs to another branch of archæology than that which now engages our attention.[2]

But, if the city of Mexico and its *immediate* neighborhood are destitute of ancient architecture, the present limits of the State are not without some valuable remains of that character. Across the Lake of Tezcoco, at a distance of about twelve miles from the capital, and in the northwestern part of the modern town of Tezcoco, the

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[1] According to the accurate *scientific* measurements of Lieut. Semmes, of the U. S. Navy, and Lieut. Beauregard, of the U. S. Engineers, thus differing from Humboldt, whose work states the elevation to be 162 feet. See Mexico, Aztec, Spanish, and Republican, II, 230.

[2] The reader will find a full account of these lesser remains in my first and second volumes of "Mexico, Aztec, Spanish, and Republican;" and, of two or three of the most important, in Gama's "Descripcion de las dos Piedras, &c." The *size and sculpture* of some of the larger stones are quite wonderful; the image called "Teoyaomiqui," is cut from *a single block of basalt,* nine feet high and five and a half broad; the "Sacrificial stone," also of basalt, is cylindrical, nine feet in diameter and three high; while the "Calendar stone," of the same material, is eleven feet eight inches in diameter, and about two feet in thickness.

explorer will find a shapeless mass of burnt bricks, mortar, and earth, thickly overgrown with shrubbery and aloes, among which there are several slabs of basalt neatly squared, and laid due north and south, forming, in all likelihood, the only fragments of one of those royal residences for which the Tezcocan princes were celebrated by the conquerors. When Mr. Poinsett visited Tezcoco, in 1825, this heap had not been pillaged, for architectural purposes, as much as it has been since; and, among the ruins, he found a *regularly arched* and well-built passage, sewer, or aqueduct, formed of cut stones of the size of bricks, cemented with the strong mortar used by the aborigines of the Valley in all their works. In the door of a room, he noticed the remains of *a very flat arch*, the stones of which were of prodigious bulk.

In the southern portion of Tezcoco, are the extensive remains of *three* pyramidal masses, whose forms were still tolerably perfect in 1842. They adjoin each other in a direct line from north to south; and, according to a rough measurement by myself, are about 400 feet in extent on each front of their bases. These erections were constructed partly of burnt and partly of sun-dried bricks, mixed with fragments of pottery and thick coverings of cement, through which small canals had been grooved to carry off the water from the upper terrace. Bernal Diaz del Castillo says that the chief *teocalli* of Tezcoco was ascended by 117 steps; and, from the quantity of obsidian fragments, vessels, and images, found on the sides of these structures, it may be surmised that, like the teocallis of the capital, they were devoted to the same bloody rites that are described in the writings of the Spanish chroniclers and of Mr. Prescott.

About three miles east of Tezcoco, across the gently sloping levels, a sharp, conical mountain rises precipitously from the plain, and though now covered with a thick growth of nopals, agaves, and bushes, seems to have been the site of some Aztec or Tezcocan works of considerable importance. The hill is full of the *debris* of ancient pottery and *obsidian;* and, about fifty feet below the top, facing the north, the mountain rock has been cut into seats surrounding a sort of grotto or recess in a steep wall, which tradition says was once covered with a calendar. The sculptures have been entirely destroyed by modern Indians, who cut them to pieces in search for treasure, as soon as they found the spot became an object of interest to foreigners.

Winding downwards by the remains of ancient terraces cut in the hill, we find the path suddenly terminated by an abrupt wall which plunges down the mountain precipitously for two hundred feet. Here, another recess has been cut in the solid rock, also surrounded by seats, while in the centre of the area is a basin, into which the water was conveyed by a system of ingenious engineering. East of this hill, and filling a ravine, are the remains of the stone, masonry, earthwork, and aqueduct pipes, by which the ancients brought the mountain streams to the Hill of Tezcocingo, from the more eastern and loftier elevations.[1]

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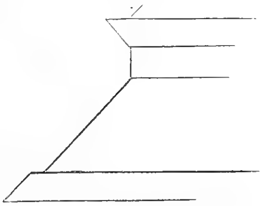
[1] There is an account, in Spanish, of the palace and gardens of Nezahualcoyotl, at Tezcocingo, extracted from Ixtlilxochil's History of the Chichimecas, in the third volume of Prescott's History of the Conquest of Mexico, p. 430. The hill referred to by the Indian historian is, probably, the one whose remains I have noticed.

A ride on horseback of three hours will bring a traveller from Tezcoco, north-eastwardly, to the village of San Juan, lying in a plain hemmed in by mountain spurs and ridges on all sides except towards the east, where a depression in the chain leads into the plain of Otumba. In the centre of this valley of San Juan are the two pyramids known as the Tonatiuh-Ytzagual, or House of the Sun, and the Meztli-Ytzagual, or House of the Moon, and generally denominated the Pyramids of Teotihuacan. At the distance from which they are first beheld in crossing the hills, the foliage and bushes that cover them are not easily discerned; but as they are approached, the work of nature appears to have encroached on that of art to such a degree, that all the sharp outlines of the pyramid are blurred and broken. In advancing towards these works, the evident traces of an old road, covered for several inches with hard cement, may still be observed; and, at their feet, smaller mounds and stone heaps extend in long lines from the southern side of the "House of the Moon." Earth and perhaps *adobes*, seem to have been the chief materials used in the erection of these pyramids; but, in many places, the remains of a thick coating of cement with which they were incrusted in the days of their perfection, were still to be found in the year 1842. The base line of the House of the Sun is stated, by Mr. Glennie, to be 682 feet, and its perpendicular height 121.

Returning again to the city of Mexico, and going thence southward over the mountain barrier that surrounds the valley of Mexico, we descend into the warmer regions of the valley of Cuernavaca; and, about eighteen miles south of the town of that name, near the latitude north of 18½ degrees, but still in the State of Mexico, we encounter the *Cerro* of Xochicalco, or "hill of flowers," which, a few years back, was still crested by the remains of a *stone pyramid*. The base of the hill is reached across a wide plain intersected by ravines, and is surrounded by the remains of a deep wide ditch. The summit is gained by winding along five spiral terraces, supported with stones joined by cement. Along the edge of this winding path are the remains of bulwarks fashioned like the bastions of a fortification. On the top of the hill there is a broad level, the eastern portion of which is occupied by three truncated cones, while on the three other sides of the esplanade there are masses of stones, (which may have formed parts of similar tumuli), all of which were evidently carefully cut and covered with stucco. In the centre of the area are the remains of the first story or body of the pyramid, which, before its destruction by the neighboring planters, who used the carved and squared stones for building, is said to have consisted of five pyramidal masses placed on each other, somewhat in the style of the pyramid of Papantla. The story that has been spared is rectan-



Outline of part of Xochicalco.

gular, faces due north and south, and measures sixty-four feet on the northern front above the plinth, and fifty-eight on the western. The distance between the plinth and frieze is about ten feet, the breadth of the frieze three and a half feet, and the height of the cornice one foot five inches.

The most perfect portion is the northern front, and here the sculpture *in relief on the pyramid* is between three and four inches deep and distinctly perfect. The massive stones, some of which are seven feet long and two feet six inches broad, are all laid upon each other without cement, and kept together simply by the weight of the incumbent mass.

The dimensions of the fragments of so fine a structure will give the reader an idea of the ingenuity as well as the labor employed in its building; for it must be recollected that the aboriginal skill was not taxed in the shaping or adornment of the stones in a neighboring quarry, but that the weighty materials were drawn from a considerable distance and carried up a hill 300 feet high, without the use of horses. The sculptures on this monument are somewhat rude and grotesque, but they appear to resemble the images delineated in the works of Stephens and Catherwood, as found by them in Yucatan and Chiapas. There seems to be no doubt, from the lines and irregularity of the stones, that the reliefs were cut after the pyramid was erected.

Besides the *external* works of pyramid and terraces, it is said that the *interior* of the hill was hollowed into chambers. Some years since a party of gentlemen, under orders from the Mexican government, explored the subterranean portions, and, after groping through narrow passages, whose walls were covered with a hard glistening gray cement, they came to three entrances between two huge pillars cut in the mountain rock. Through these portals they entered a chamber, whose roof was a regular cupola built of stones ranged in diminishing circles, while, at the top of the dome was an aperture which probably led to the surface of the earth or to the summit of the pyramid. Nebel, who visited the ruins some years ago, relates, as an Indian tradition, that this aperture was immediately above an altar placed in the centre of the chamber, and that the sun's rays fell directly on the centre of the shrine when the luminary was vertical! This idea is perhaps a fair specimen of the traditions and guesses with which ingenious archæologists bewilder themselves and their readers.[1]

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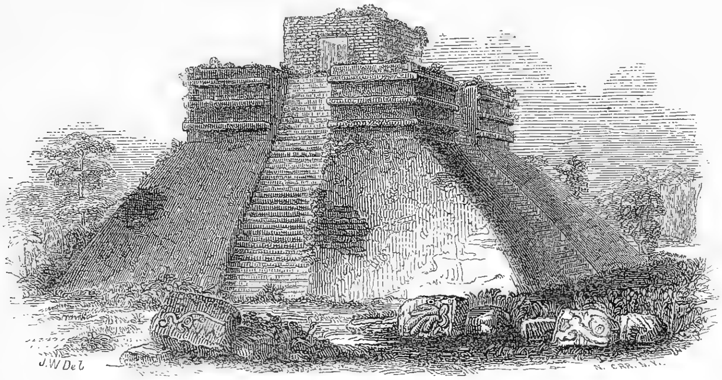
[1] See Revesta Mejicana, I, 539. Mexico; Aztec, Spanish, and Republican, II, 284. Nebel, Voyage Archæologique et Pittoresque: Plate—Xochicalco.

CHAPTER VI.

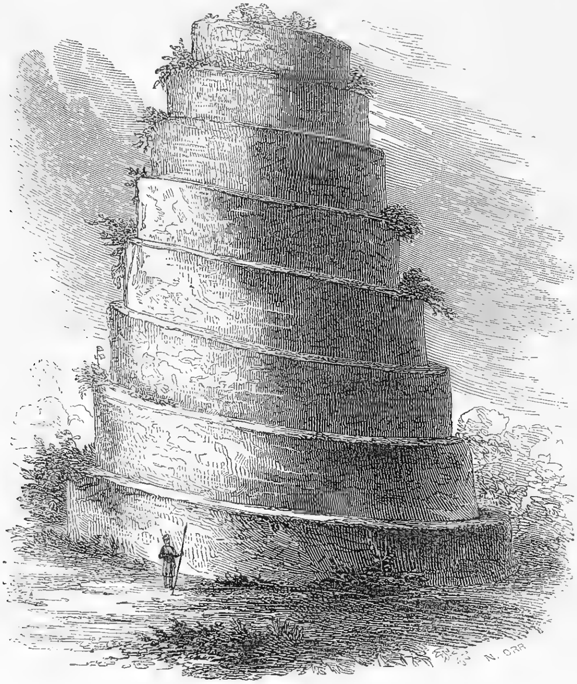
South of the State of Vera Cruz, adjoining the State of Chiapas, and on the western slopes of the Cordillera, bounded by the Pacific, lies the State of Oajaca. This region, from the great quantity of architectural and image-remains found throughout it, seems to have been the seat of an advanced civilization, though its history is much less known than that of the central portions of Mexico. The State has been by no means thoroughly explored, either for its resources or antiquities; but most interesting remains are known to exist at Tachila, where there are tumuli; at Monte Alban, two leagues S. W. from the town of Oajaca, where there are tumuli and pyramids; at Coyúla; at San Juan de los Cúes; at Guengola; at Quiotepec, and at Mitla. Most of the relics present pyramidal shapes, in combination with the vertical; a specimen of which is here copied from Lord Kingsborough's plates of Dupaix's expedition.



Remains near Tehuantepec, Oajaca.

In 1844, an examination was made, by order of the Governor of Oajaca, of the remains near Quiotepec, a village about thirty-two leagues northwardly from the capital of the State. These ruins, originally constructed of cut stone, are found on the Cerro de las Juntas, or Union Hill, so called from its neighborhood to the *junction* of the Rivers Salado and Quiotepec.

The eminence is said to be covered, in every direction, with remains of works of a *defensive* character, designed, as it appears, to protect the dwellings erected on the hill, and the large temple and palace, whose massive ruins still crown the summit. These fragments of the past are represented to be somewhat similar to those of Chicocomoc or Quemada, in the northern part of Mexico, which I have already described in the notice of architectural antiquities in Zacatecas. The resemblance consists in the style of building, and the mingling of worship and civic defences. There does not appear, however, to be any similarity between these ruins and the remains found in Yucatan and Chiapas, where the designs are much carved and ornamented, denoting, perhaps, a higher degree of luxury, taste, and civilization. The temples of Quiotepec, and that of Chicocomoc, or Quemada, are both pyramidal, like most of the Mexican structures; but the architectural style generally, at the former place, is rather more sumptuous than that at Quemada.[1]



Remains near Tehuantepec, Oajaca.

The most interesting, perhaps, of the architectural remains within the *present* bounds of Mexico, in Oajaca, are those of MITLA; and, as it was not until the year 1494 that the Aztecs *finally* subdued the people of MICTLA, in the province of Huaxaca,[2] it is not likely that the constructive talent or tastes of that region were modified or controlled by the inhabitants of the Valley of Anahuac. The same remark applies to all the other districts, in every quarter outside the valley, where the aborigines became subject to the Aztecs, either by alliance or conquest. It is

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[1] See Museo Mejicano, Vol. III, p. 329, for drawings of these monuments. See, also, Vol. I, p. 401, of the same work, and Vol. III, p. 135, for accounts of Zapotec remains; and Vol. I, p. 246, for an imperfect notice of military fortifications, &c. &c., near Guengola, Tehuantepec.

[2] Gama; Gallatin, Eth. Soc. Trans., Vol. I, 137. Mexican Chronology. Clavigero, Lond. ed., Vol. I, p. 185.

very probable that hundreds of the unfortunate Zapotec inhabitants of Mitla and Huaxaca, or Oajaca, who had become prisoners to Aheutzotl, in *previous* wars, swelled the splendid but brutal sacrifice of human victims, with which the great temple of Mexico was dedicated in 1487.[1]

Very soon after the *first* success of Cortez in the city of Mexico, the people of Oajaca sent embassies to claim his protection; and, as soon as the country was absolutely conquered, and the victor had learned the value of the region from the reports of Alvarado and the Spaniards who began to settle there, he seems to have selected it as his own particular domain. When the crown raised him to the dignity of "Marquess of the Valley of Oajaca," he was endowed with a vast tract of land in the province, and there is no doubt that his twenty large towns, and twenty-three thousand vassals, were to be found mainly within the boundary of his Zapotec territory. These facts are mentioned to show that the acts of Cortez himself indicate the value of the region in which Mitla lies; and, in all likelihood, illustrate the degree of civilization it possessed prior to the Aztec conquest. It is to be regretted that there are so few traces of the ancient Zapotec tribes, and that we are left to grope in the dark, with scarcely a cobweb to guide us through the ruined labyrinth of their history. The great natural features and characteristics of the region remain of course the same; and from its general salubrity, its fertility of soil, the nature of its productions, its geological structure, and beauty of natural scenery, we may fairly suppose that its famous "valley" possessed many attractions similar to those which induced the Aztecs to make their lodgement in the Vale of Anahuac. Zachila, which is a corruption of the word Záachillattóô, as written in an ancient MS. seen by Dupaix, is situated in the midst of the great Valley of Oajaca, and, in former times, is said to have been the seat and court of the Zapotec kings. Ten or twelve leagues southeastwardly from the town of Oajaca, engulfed in a deep valley, crested with *cerros* whose dry, sterile, and poorly watered soil is probably more prolific of snakes and poisonous insects than of anything else, lies the modern village of San Pablo-Mitlan. Its name was derived from Mictlan, or Miquitlan, "a place of sadness," which it probably received from the Aztecs, while the Zapotec appellation seems to have been Liuba or Leoba, "the tomb." It is here that we

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[1] The cruelty of the Mexican sacrifices of *human beings* has always been one of the principal arguments against the civilization, and in favor of the barbarism of the Aztecs. All religion includes the idea of sacrifice—spiritual or physical—actual or symbolical. The Christian sacrifices his selfish nature; the Idolater propitiates by victims. The Aztec sacrifice arose, probably, from a blended motive of propitiation and *policy*. The human sacrifice by that people was, perhaps, founded on the idea that the best way of getting rid of culprits, dangerous people, and prisoners of war taken in immense numbers, and whom it was impossible to support or retain in subjection without converting a large portion of their small kingdom into a jail—was to offer them to their gods. It is true, that *savage* nations, such as the Africans of Dahomé, &c., admit the purest *barbaric* notions of human sacrifice; but can such cruel contradictions be attributed—with their more brutal motives—to the Aztecs, who, in other respects, possessed so many titles to civilization? Still, it must be admitted, that if we regard the grossness of the Aztec idolatry alone, at the time of the conquest, we could form no idea of that people's intellectual progress in other respects. Yet their architecture, laws, government, private life, and astronomical knowledge, show that their social condition was much more refined than their faith, so that we must suppose the Valley of Anahuac was full of priestcraft and superstition, and that its cultivated society was in advance of its religion.

find the architectural remains which were first made known, partially, by the drawings of Don Luis Martin, in 1802, of Dupaix, in 1806, and are now shown in the accompanying pictures, drawn on the spot, in 1837, by Mr. J. G. Sawkins.

According to the traditions reported by the earlier explorers, the chief object designed in the erection of these edifices was to preserve the remains of Zapotec princes; and it is alleged, that at the death of a son or brother, the sovereign retired to this place, and taking up his residence in a portion of the building which was calculated for habitation, performed religious services and gave vent to ceremonious sorrow. Other reports, of the same period, say that these solitary and dreary abodes were inhabited by an association of priests who devoted their lives to expiatory services for the dead. It must be confessed that the site is admirably calculated for any one, or all, of these gloomy purposes; for, according to the accounts of travellers, the silence of the lonely valley, which is reached conveniently but by one approach, is unbroken even by the songs of birds. Perhaps it was—not only in location, but destination—an aboriginal Escorial, where life, death, and religion mingled their austere but courtly pageants.

Plate No. 1 presents a general picture of the ruins; while the following cut, A, taken from a drawing by Martin, in 1802 (and, perhaps, *not* strictly accurate, except as to *parts* of the main edifice), shows a ground-plan or sketch of the whole group, so as to make the scene intelligible to the reader.[1]

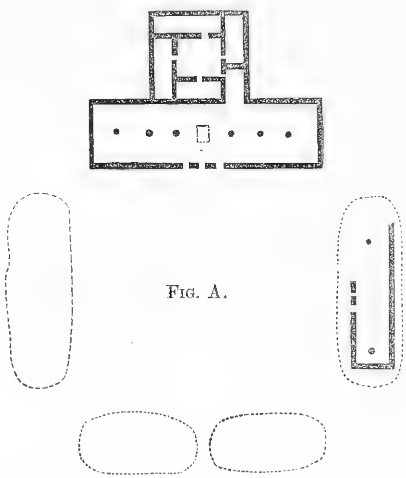
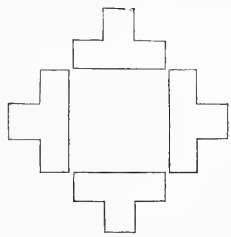


FIG. A.

A large portion of the valley in the neighborhood of the three mountains, seen in Plate 1, is said to be still covered with heaps indicating the sites of ancient architecture; but, as most of the ground is under cultivation, every relic of the architecture itself is destroyed, and even the ground-plans have become so indistinct

[1] Martin, for instance, seems to indicate five remains, while there are only four; and gives two columns at the entrance of the remaining building, while there are three.

as to make all researches useless. But the group which at present interests us, seems, from Mr. Sawkins's observations, to have consisted originally of four connected, or nearly connected, buildings, each one fronting a cardinal point, the whole inclosing a square court. The original erections *may*, in all likelihood, have resembled the following sketch, in their ground plan:—



Of the southernmost of these edifices, Mr. Sawkins found five upright columns still standing—four supporting portions of a wall, while the fifth, which was taller than the rest, stood alone. These fragments are seen in Plate No. 1, immediately in front of the spectator. On the west of the square, there are the remains of crumbling and indistinct walls; on the north, everything seems to be obliterated; while, on the east of the quadrangle, is the edifice forming the main feature of Plate No. 1, and which is represented, at large, *from the rear*, in Plate No. 2.

Passing over the court-yard, or quadrangle—still floored with a hard cement and slabs of sandstone—we approach the entrance of this building, which consists of four apertures between three low, square columns, or door jambs, through which the interior can only be reached in a crouching posture. These four apertures admit the passage, through each, of but one person at a time. On either side of this portal, as seen in No. 1, there are niches or recesses, on the front, which were probably filled by images. This portion of the exterior wall, or *façade*, is said by Mr. Sawkins to be, at present, without any adornments; but whether such was its original state, or whether it has been stripped of its coverings by the neighboring Mexicans, we are not distinctly informed. The large stones forming the cornice over the entrance, were especially remarked by our traveller, as indicating—both by *size* and neatness of workmanship—the ingenuity and power of the builders.[1]

Upon entering through one of the low and narrow adits, just described, Mr. Sawkins found himself in an oblong court or apartment, of very considerable size. Its walls were covered with a rich, highly polished, red plaster, so hard as to resist the knife. At the two ends of this court there were niches, as well as one directly in front of the entrance; but the images or utensils they were intended for by the aborigines, had long disappeared. It was in a line along the centre of this

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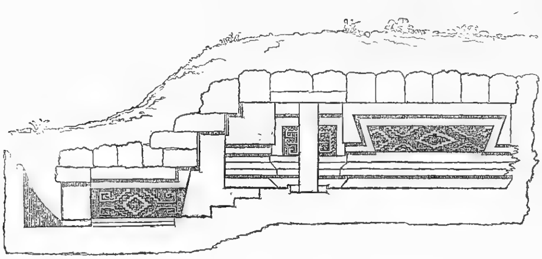
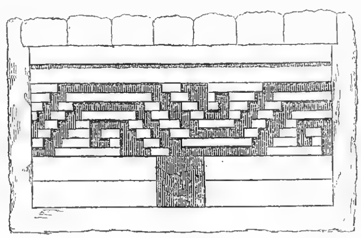
[1] Mr. Glennie, a British traveller, states the dimensions of some of the stones above the entrances of these buildings to be: eighteen feet long, four feet ten inches broad, three feet six inches thick; another is nineteen feet four inches long, four feet ten and a half inches broad, and three feet nine inches thick; a third is nineteen feet six inches long, four feet ten inches broad, and three feet four inches thick!

apartment that Martin, in 1802, and Dupaix, four years after, found the six *cylindrical stone columns,* without bases or capitals and of a single shaft, the position of which is shown in the ground-plan I have given, on another page, from Martin's drawing.[1] But when Mr. Sawkins visited Mitla, in 1837, the columns had been removed, probably by the present villagers, for their domestic purposes. These columns had evidently been intended to support the roof which formerly covered this portion of the edifice, and are represented by Dupaix to have been one *vara* in diameter and five and a half *varas* high; or *near* three feet in diameter by about fifteen in altitude!

The large court, or saloon, just described, communicated at its rear, by a narrow passage (as will be seen in Martin's plan), with another body of the edifice, which that artist represents to have been a sort of *interior court,* surrounded by four rooms without windows, each of which was entered by a single door. Don Luis Martin represents it, evidently, as a structure resembling the modern edifices of the Mexicans, which are similarly constructed around a *patio,* or court, without external windows. It is probable that such may have been the state of the ruins in 1802, but when they were seen by Mr. Sawkins, in 1837, he found the whole interior quadrangle an unoccupied area, while three of its walls were covered with nine long recesses on each side, in three tiers, each recess being large enough for the reception of a human body. These vaults were plastered with the same kind of cement that was found in the first apartment, but they were all empty.

In the centre of the *main* court-yard of the whole group, there are said to be subterranean apartments similar to those which have been found elsewhere in this valley, and which have been represented as adorned in the following cuts.





[1] See cut on page 28.

If we leave the interior of this building, we may now obtain an accurate and excellent idea of its outside from the minute drawings of Mr. Sawkins, in Plate No. 2. It is a monument which cannot fail to strike the student of American architectural archæology as being the first effort of the aborigines that not only abandons the vertical and pyramidal, but absolutely *reverses* the latter, and, at the same time, indulges in a style of elaborate and *regular* adornment which far surpasses many remains of Etruscan art, and may almost be said to resemble the Greek. These exteriors have been constructed with great labor as well as ingenuity. Above the ground, the building,—whose *interior* wall is formed of *adobés*, or sun-dried bricks,—is cased, for about five feet, with a pyramidal base of stone slabs about two inches thick; and, from this point, the walls, still of stone, and sharply cut, begin to *incline outwards* till they reach a height of near twenty-five feet. Each of the seven exterior walls, as seen in Plate 2, is divided into nine compartments, corresponding with the sepulchral recesses or vaults we noticed on the interior. From the point where the walls strike outwards from the perpendicular, all the *corners* and *divisions* appear to be formed by stouter stones than the slabs which encase the base. The bands, which are the frames, as it were, of each of these sixty-three divisions, are all of solid stone, cleanly and sharply chiselled; while the ornamental figures contained in the squares are formed by a Mosaic work of small square stones, artistically placed beside each other, in high relief, and imbedded in a mass of adamantine cement, similar to that which covers the interior walls. The spectator who looks at one end of this singular building, with its basket-like outline and *beautiful* adornments, might almost fancy that he stood in front of a gigantic *sarcophagus*, designed and sculptured in advanced periods of Grecian and Roman art.[1]

About half a mile west of these ruins, Mr. Sawkins found a large, dark red, porphyritic column, which, for the sake of illustration, he has taken the liberty to represent in Plate No. 2, as lying near the edifice. It had probably been carried off from the building by some vandals, and abandoned before they could devote it to their private uses. The artist states that the marks of the chisel or chipping tool are still visible on this column, and remarks that many blocks, from these and other edifices of the valley, were employed in building the church which is seen in Plate No. 1. To the southwest, near the point indicated in the picture by the union of the three hills with the plain, Mr. Sawkins saw the ruins of many other edifices, but all were so dilapidated that nothing could be made out. Wherever he detected the remains of *cement* or *mortar*, either on the roads, in the open air, or on walls, he found it still perfectly hard and serviceable, and but little injured either by time or attrition. There seems to have been a great fondness among the Zapotecs for *red*, and it is alleged that a color, which is so unpicturesque in architecture, seems to have been plentifully distributed over the exterior as well as the interior of the remarkable edifice we have been considering.

Plate No. 3 exhibits the characteristics of the image-remains of the Zapotecs.

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[1] Humboldt says that the walls extend, on *a line*, about forty metres, and are five or six high: a metre, in round numbers, is $39\frac{2}{9}$ English inches.

No. 1 was drawn by me from the original in sandstone, which I found in Mexico in 1842, in the fine collection of the Conde del Peñasco. Archæologists who are familiar with the style of images found among Aztec remains, in the Valley of Mexico, as well as with the same class of objects from Yucatan, Tabasco, and elsewhere in that quarter, will at once observe their difference from the images represented in the plate. Grotesque and hideous as they are, they seem to possess, in the symmetrical arrangement of the designs, and in their *originality*, many more elements of *art* than are found in the images of the Aztec or Maya tribes. I have introduced them here for the purpose of hinting that, in all the Zapotec remains of architecture and ornament that have come down to us, we find traces of rather more inventive talent and taste than among the other aboriginal tribes with which we are acquainted.[1]

About a league northeasterly from the ruins of Mitla, Mr. Sawkins visited the remains of the Zapotec fortification which he has represented in Plate No. 4. A steep, isolated hill, about three hundred feet high, with a base nearly a league in extent, rises in this spot and commands the whole plain. The broad, oval summit, whose greatest diameter is about six hundred feet, is reached with difficulty from all sides except the southern. By this approach, the entrance or gateway is attained in a wall about six feet thick and eighteen high. The plate shows the character of the works, which contain a second or inner wall, as is seen in the rear of the first behind the gateway; while in the interior, are the remains of three edifices, which were probably intended for the barracks of the defenders. Two of these buildings are on the southern side, overlooking the approach by the gateway, while the remaining one is placed towards the east. It seems from the heaps of *piled* stones, still to be seen by modern travellers, and from the huge masses of isolated rock found by Mr. Sawkins and represented in his sketch, that these were the principal weapons with which the defenders protected themselves against assailants. How the possessors of this ancient fortress supplied themselves with water, on the top of an abrupt, isolated hill of 300 feet elevation, we are not yet informed by any explorers. It is stated by some travellers that several thousand men might have gathered for protection within these walls; but it may well be doubted whether the structure was ever designed for anything but a temporary refuge in times of extreme danger, when the plain had been invaded and ravaged.

I have now completed a catalogue of such architectural remains in Mexico as have become known to us, either by personal observation or the reports of travellers. If we proceed southward, beyond Yucatan and Chiapas, and pass throughout the various states of what is geographically known as "Central America," we find, in all of them, innumerable images and vessels, and fewer monumental or architectural

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[1] The only other *ornamental* remains possessing nearly equal claims to symmetrical design, are represented in some Peruvian ruins near Truxillo, South America. See Rivero and Von Tschudi.

remains of importance than we encountered in Mexico. The taste, too, as well as the design and sculpture, is inferior; nor shall we again meet with traces of evident superiority, until we pass the broad belts of the equatorial forests and rivers, and descend beyond the Amazon to the ancient realm of the Incas in Peru.

I will not close this paper by offering any theory in regard to climatic influences on the degrees of civilization found among the aboriginal races of our continent at the period of the Spanish conquest. Still, I hope it may not be considered improper to remark that, while the hot regions in the neighborhood of the equatorial part of our hemisphere appear nearly destitute of monumental, traditional, or recorded remains of their inhabitants, we find, according to all these sources of knowledge, that the best samples of aboriginal civilization have apparently originated and ripened, between 10 and 25 degrees of north latitude, and between 10 and 25 degrees of south latitude. While the equatorial heat degenerated man into an indolent vegetation, the northern and southern portions of the tropics rendered him progressive and fostered his social instincts. From these points, the marks of civilization seem gradually to fade away towards both poles, till they merge, through the nomadic warrior, into the squalid Esquimaux of the north, and, through the Araucanian, into the barbarous Fuegan of the south.

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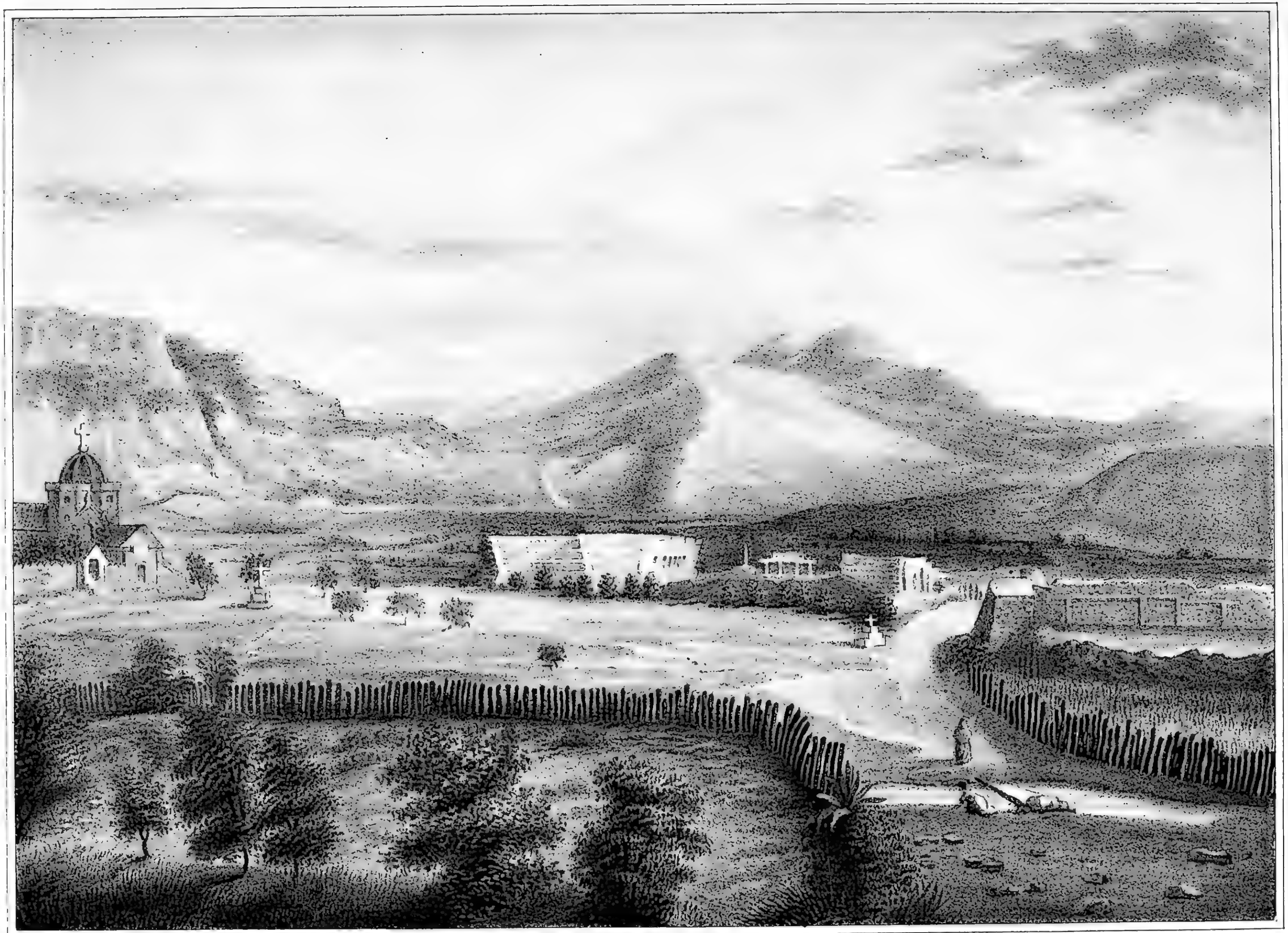
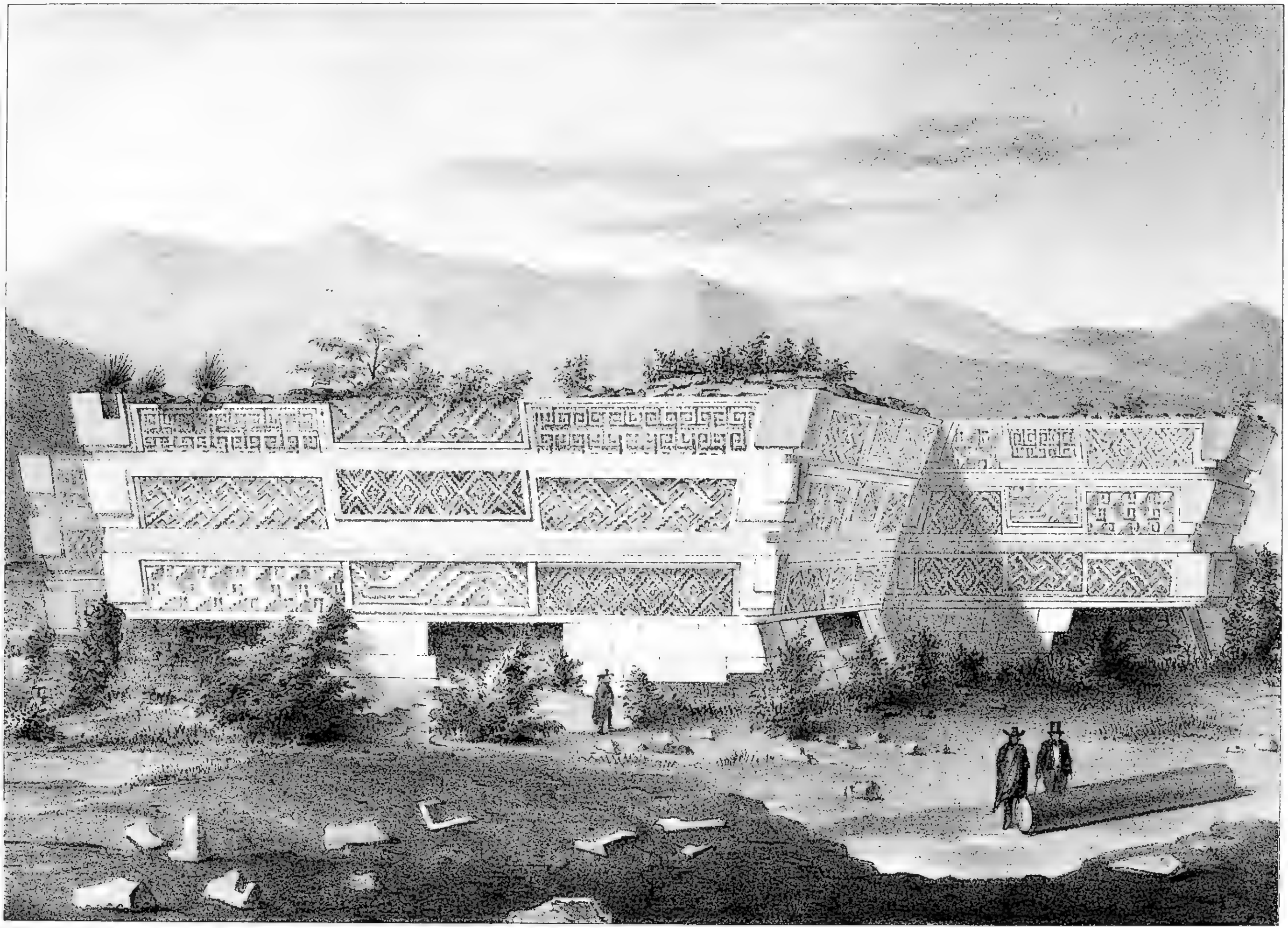
I.



T. Sinclair's lith. Phil^a.

A GENERAL VIEW OF THE ARCHITECTURAL REMAINS AT MITLA.



J. G. Sawkins Delᵗ

II.

ARCHITECTURAL RUINS NEAR MITLA.

Fig. 1

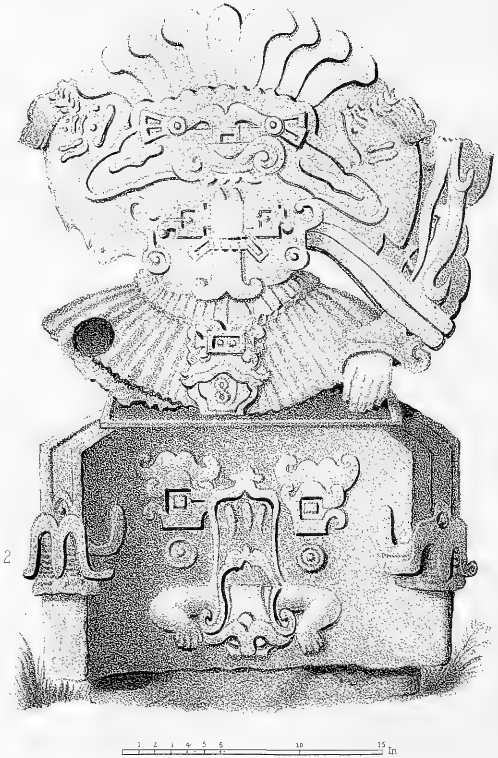


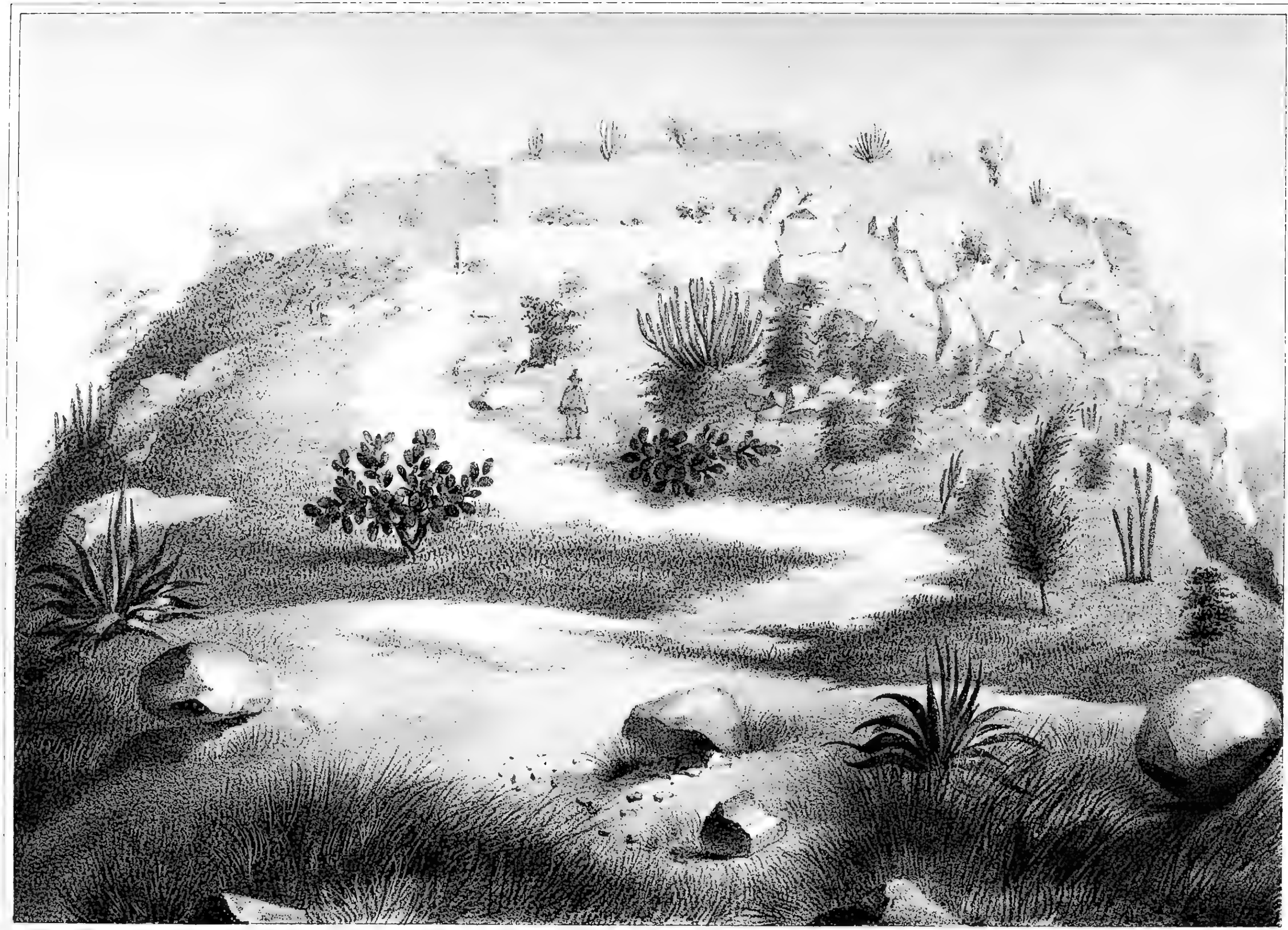
Fig. 2

Fig. 3.

J. G. Sawkins, Delt. T. Sinclair's lith, Phil.

IDOLS FOUND AT MITLA.

IV.



ANCIENT FORTIFICATION NEAR MITLA.

SMITHSONIAN CONTRIBUTIONS TO KNOWLEDGE.

RESEARCHES

ON THE

AMMONIA-COBALT BASES.

BY

WOLCOTT GIBBS AND FREDERICK AUG. GENTH.

[ACCEPTED FOR PUBLICATION, JULY, 1856.]

COMMISSION

TO WHICH THIS PAPER HAS BEEN REFERRED.

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ANALYTICAL INDEX.

RESEARCHES ON THE AMMONIA-COBALT BASES.

PART I.

The facility with which alkaline solutions of many metallic protoxides absorb oxygen from the air, attracted the attention of chemists at an early period. The protosalts of iron, manganese and cobalt, are particularly remarkable in this respect. In the presence of an excess of the fixed caustic alkalies and their carbonates, salts of the protoxides of these metals are more or less rapidly converted into basic salts of their higher oxides. A similar effect appears to be produced by all of the more powerful fixed bases, while it is remarkable that neutral or acid solutions of the same salts are oxidized much more slowly, an effect which is perhaps owing to the tendency which per-salts in general exhibit to become basic, and to the influence which an excess of acid exerts in producing neutral or acid compounds.

Ammonia acts like potash and soda in causing the oxidation of solutions of iron and manganese. In the case of these two metals either basic salts or hydrates of the peroxides are formed, which contain no ammonia, at least in chemical combination. With salts of protoxide of cobalt the result of the oxidation is very different. The sesquioxide of cobalt at the instant of its formation unites with a certain number of equivalents of ammonia so as to produce a conjugate base of which ammonia forms an integral portion. The new base partakes in some measure of the properties of the alkalies, the peculiar character of the salts of cobalt being wanting. It is with this class of bases that we have at present to deal.

The earliest observations which we possess upon the oxidation of the salts of cobalt are due to Leopold Gmelin, who, in a memoir, published in 1822,[1] described the changes of color which are produced when ammoniacal solutions of the chloride, sulphate, and nitrate of cobalt are exposed to the air. The solutions under these circumstances absorbed oxygen and became brown, and Gmelin considered it probable that they contained a cobaltic acid. Dingler,[2] who subsequently endeavored to determine the amount of oxygen absorbed, inferred that the cobaltic acid consisted of one equivalent of cobalt and two equivalents of oxygen, since the brown solution gave with sulphide of ammonium a black precipitate of bisulphide of cobalt. Winkelblech[3] denied the existence of a metallic acid in the solution, but

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[1] Neues Journal der Chemie und Physik. Neue Reihe, V, 235.
[2] Kastner's Archiv, XVIII, 249.
Annalen der Pharmacie, XIII, 148, 253.

though his memoir contained many interesting and valuable contributions to our knowledge of the oxides of cobalt, it threw no light upon the true nature of the ammoniacal compounds, except by establishing in them the existence of sesquioxide of cobalt. The subject was next investigated by Beetz,[1] who analyzed an ammoniacal sulphate and nitrate of sesquioxide of cobalt formed during the direct oxidation of ammoniacal solutions. These analyses led to the formulas $Co_2O_3.3SO_3+3NH_3+NH_4O$, and $Co_2O_3.3NO_5+3NH_3+NH_4O$, but as the substances employed were not crystallized, and as the analytical methods were difficult to execute, but little reliance could be placed in the results. Beetz, however, considered the sesquioxide of cobalt in these compounds as playing the part of an acid, the ammonia being present as a salt of ammonium.

The oxidation of ammoniacal solutions of various salts of cobalt was also observed by Rammelsberg,[2] and the products of the action in several cases analyzed. None of the formulas obtained, however, appear to belong to well defined and distinct compounds.

A memoir published by one of ourselves, in 1851,[3] contained the first distinct recognition of the existence of perfectly well defined and crystallized salts of ammonia-cobalt bases; in fact we have not been able to trace in any earlier paper even the idea of the existence of such a class of compounds. The results made public in this paper had been obtained by the author, in Marburg, in 1847, had been at that time freely though verbally communicated, and a suite of the salts obtained had been left in the laboratory at Giessen. Want of opportunity prevented a complete and systematic investigation, particularly from the analytical point of view. The memoir in question contained, however, besides several analyses, an accurate description of the two bases now to be described under the names of Roseocobalt and Luteocobalt. Though the analyses were from necessity not sufficiently complete and extended to fix the constitution of the bases in question, yet the fact is indisputable that this memoir contained, not merely the first announcement of the existence of ammonia-cobalt bases, but also a scarcely less accurate and complete description of two of these bases than any which has since appeared.

As its title states, the memoir in question was intended simply as a preliminary notice; circumstances, however, prevented a speedy resumption and continuation of the subject. In a paper published in 1851,[4] Claudet described with some detail the properties of the chloride of Purpureocobalt, and the mode of obtaining it, as well as a few other ammonia-cobalt salts. With the exception, however, of more complete analyses, the memoir in question contained nothing which is not to be found in the previously published paper above alluded to. In two notices communicated to the Academy of Sciences[5] in the same year, Frémy announced as his

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[1] Pogg. Ann., LXI, 494, 480, 490.

[2] Pogg. Ann., XLVIII, 208. XLIV, 268.

[3] Nordamerikanischer Monatsbericht für Natur. und Heilkunde, 1. Januar. 1851. Vorläufige Notiz über gepaarte Kobaltverbindungen von Dr. Friedrich August Genth.

[4] Phil. Mag., II, 253, and Ann. de Chimie et de Physique, XXIII, 483.

[5] Comptes Rendus, XXXII, 509, 808.

own, the discovery of a class of compounds containing cobalt and ammonia, and produced by the oxidation of ammoniacal solutions of protosalts of cobalt. In the following year his complete memoir appeared.[1] In this Frémy describes anew the ammonia-salts of protoxide of cobalt, first obtained by H. Rose, passes then to the description of two new classes of compounds discovered by himself, and named by him Oxy-cobaltiaque and Fusco-cobaltiaque, and finally describes at some length the principal salts of Genth's two bases, the constitution of which he correctly determines. Frémy appears not to have been aware that these two bases had been described in a manner little less complete than his own two years before the appearance of his memoir. The chloride of Luteocobalt and its platinum salt have also been described and analyzed by Rogojski,[2] and what we now term the chloride of Purpureocobalt, by Gregory,[3] who corrected the analyses of Frémy.

The researches of Claus[4] on the ammonia-iridium and ammonia-rhodium bases established the existence of compounds of these metals exactly analogous to Roseocobalt and its salts, and chemists will look with impatience for the publication of his results in detail. Recently Weltzien has published some theoretical views on the constitution of the ammonia-cobalt bases which possess much interest.

The salts of Xanthocobalt were discovered in November, 1852, by W. G., and the principal results which are contained in the present memoir were communicated to the American Association for the Advancement of Science, at its meeting in Cleveland in August, 1853. The formulas of several of the more remarkable bases are also given in a Report on the recent progress of organic chemistry, read before the same association, at its Providence meeting in August, 1855. The nomenclature of the ammonia-cobalt bases proposed by Frémy is so simple and convenient that we have adopted and extended it to meet every case. We have, however, considered it desirable to drop the terminal syllable "iaque," employed by Frémy, not merely because it is not an English termination, but because by omitting it we obtain shorter and more convenient words. Thus, we say Roseocobalt and Luteocobalt, instead of Roseo-cobaltiaque and Luteo-cobaltiaque, or Roseo-cobaltia and Luteo-cobaltia, which are the English equivalents. The shorter names, as will hereafter appear, also agree better with our own theoretical views, since we consider the compounds in question conjugate metals and not ammonias.

With the view of making the description of our salts as complete as possible, we have followed the excellent example of Frémy, and referred the colors of these substances to Chevreul's chromatic scale. Frémy had the advantage of Chevreul's own determinations. We have employed, for the purpose, the chromatic scales recently published in Paris by Digeon, and which appear to be reliable; in any event they give *some* precision to determinations of color. As we have found that very many of the salts of the ammonia-cobalt bases exhibit a well marked dichro-

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[1] Ann. de Chimie et de Physique, XXXV, 257.

[2] Journal für praktische Chemie.

[3] Ann. der Chemie und Pharmacie, LXXXVII, 125.

[4] Bulletin de l'Académie de St. Petersburg, 1855, XIII, 97, quoted in Handwörterbuch der reinen und angewandten Chemie, VI, 843.

ism, we have in most cases examined the light reflected from layers of crystals, by Haidinger's dichroscopic lens, and have given the colors of the ordinary and extraordinary images as obtained in this way. As a curious physical result, we may here mention that, in general, the cobalt color predominates in the ordinary image.

We are indebted to Prof. Dana for the determination of the systems to which many of our crystals belong, and of their principal forms, as well as for our figures, and embrace this opportunity of expressing our grateful acknowledgment of his valuable assistance.

METHODS OF ANALYSIS.

The accurate quantitative determination of the different elements which enter into the constitution of the ammonia-cobalt bases and their salts, is attended with great difficulties. We have in general found it necessary to study out with much labor the methods of analysis proper to be used in each particular case; and it has been only after many trials that we have at length been able to obtain accurate results. Before proceeding to the description of the compounds in question, it may therefore be proper to state the analytical methods employed.

Cobalt. The determination of the cobalt in these salts may, in most cases, be very easily and accurately effected by the following process. A weighed portion of the salt is gently heated in a deep platinum crucible, with a quantity of pure and strong sulphuric acid sufficient to moisten the whole mass. Some effervescence is generally produced by the addition of the acid, but there is no danger of loss if the crucible be sufficiently large, and if the heat be applied only after the first action of the acid is over. The mixture is to be gently heated over a spirit lamp, until the excess of the acid, sulphate of ammonia, and other volatile matters have been expelled. During the whole time of heating, the cover of the crucible must be so placed as to prevent the possibility of loss by spattering, and at the same time to permit the escape of volatile matters. When, however, the quantity of acid has not been too great, the whole process goes on very quietly to the end, when the mass becomes dry. The heat is finally to be raised, for an instant, to low redness, the cover of the crucible being quickly lifted off and then replaced. The crucible is then to be allowed to cool and weighed, when the quantity of cobalt may easily be calculated from the weight of the dry and pure sulphate. After the weighing, the mass in the crucible must be carefully examined. It should have a fine rose color, and be perfectly soluble in warm water, leaving no black residue. In case this is observed, which happens only when the heat has been too high, a drop of sulphuric acid and a few drops of oxalic acid may be added, and the whole evaporated to dryness, and again ignited. When, however, there is much oxide of cobalt present, it is better to reject the analysis at once. With a little care and practice the operation succeeds almost invariably, and the result, as we shall hereafter show, leaves nothing to be desired in point of accuracy. When chlorine is present in the salt to be analyzed, a little free chlorine is sometimes found among the products of the action of the sulphuric acid, and the platinum crucible is slightly acted upon. In such cases we usually add a little oxalate of ammonia to the salt before dropping the acid upon it. The quantity of salt to be taken for analysis may vary from three to five decigrammes; when more is used, there is apt to be some loss from effervescence. In consequence of the small quantities of substance employed, the weighings must be as accurate as possible. In calculating the weight of the cobalt from that of the sulphate, we have the advantage of determining one substance from another with an equivalent more than twice as high.

In certain cases, as, for example, when phosphoric acid, chromic acid, &c., are present, the above method cannot be employed. In such compounds we have found it advantageous to separate the cobalt as a hydrate of the sesquioxide, by boiling the salt with a solution of caustic potash, washing the precipitate thoroughly, and estimating the ignited precipitate as Co_6O_7, or as metallic cobalt after reduction by hydrogen. Frémy justly observes that this ignited oxide usually contains potash; but an accurate result may always be obtained by washing it well with boiling water after the ignition, and weighing a second time. It is remarkable that Frémy asserts that cobalt may be accurately estimated in the form of sulphate, in consequence of the stability of this salt, while the direct application of the method, as we have described it above, appears to have escaped him entirely.

Hydrogen. We have in almost all cases determined hydrogen directly by combustion with chromate of lead, metallic copper being placed in the anterior part of the tube. In the case of the nitrates, however, an excess of hydrogen in the result is almost unavoidable, because it is impossible, even with freshly reduced copper, to decompose completely the great quantity of oxides of nitrogen formed during the combustion. In other cases this effect is much less marked, and the hydrogen determinations are at least as accurate as in ordinary organic analyses.

Chlorine. The accurate determination of the chlorine in the ammonia-cobalt salts is very difficult. Nitrate of silver, it is true, precipitates chlorine from most of its combinations in these salts, but the precipitation is never complete, because the chloride of silver is somewhat soluble in the ammonia-cobalt chlorides, forming with them peculiar double salts. By long boiling with free nitric acid in the solution, nearly all the chlorine may be determined as chloride of silver, but very accurate results cannot be obtained in this manner. The best method consists in igniting the chloride with lime in a combustion tube, in the manner usually practised with organic bodies. In some cases, however, we have obtained very good results by decomposing the solution of the chloride by sulphurous acid, or by boiling the solution of the salt until it is completely decomposed, adding sulphurous or nitrous-nitric acid to reduce the sesquioxide of cobalt, and then precipitating with silver. The process is, however, always troublesome, and requires much time and great care.

Carbon. This element is best determined by the usual processes of organic analysis. In consequence, however, of the very large quantity of oxides of nitrogen, which are always produced during the combustion of these salts, we have found it very advantageous to employ a method first suggested, we believe, by Winkelblech, and which consists in mixing with the oxide of copper a quantity of finely divided metallic copper, in the form in which it is obtained by reducing the oxide by hydrogen. In this manner the formation of the oxides of nitrogen may be completely prevented. Great care must, however, be taken when it is wished to determine hydrogen at the same time with carbon, because, copper reduced from the oxide by hydrogen, always contains water, which it is difficult to separate.

Nitrogen. No element has presented such difficulties as nitrogen. We have found it impossible to obtain results within two or three per cent. of the truth by employing the old methods of analysis, that of Dumas for instance. The quantity of nitric oxide formed during the combustion is surprising, and it is absolutely impossible to get rid of it by means of ignited metallic copper, placed in front of the combustion tube. Will and Varrentrapp's method with soda lime is inapplicable, because one equivalent of ammonia is always decomposed by the equivalent of oxygen set free in the reduction of sesquioxide to protoxide of cobalt. Good results could not be obtained by boiling the salts with caustic alkalies, collecting the ammonia in chlorhydric acid, and determining it by bichloride of platinum. Even after the reduction of the sesquioxide of cobalt to protoxide by means of sulphurous acid, this method was found unreliable. The improvements made by Simpson in the absolute determination of nitrogen by volume at last furnished us with a reliable process; and nearly all the analyses in this memoir were executed by his method. The improvement introduced by Simpson consists essentially in mixing oxide of mercury with the oxide of copper employed to effect the combustion. The vapor of metallic mercury completely decomposes the oxide of nitrogen, and any excess of free oxygen is absorbed by means of metallic copper. By this method we have analyzed most of our compounds without special difficulty, though we have often found it necessary to employ a much larger proportion of oxide of mercury than is recommended by Simpson. One class of ammonia-cobalt bases have, however, been the source of frequent analytical failures, and of great loss of time and material. We refer to the salts of Xanthocobalt, a base containing deutoxide of nitrogen, and giving off this gas at a gentle heat, below that at which oxide of mercury is decomposed. Simpson's method has not always been found accurate, since even when a very large amount of oxide of mercury is employed there is frequently much nitric oxide in the nitrogen collected for measurement. In many cases the simple admixture of a large proportion of metallic copper with the oxide, as recommended by Winkelblech, has been found to give most excellent results. It is proper also to state here that, in consequence of difficulties in obtaining proper apparatus with which European chemists do not have to contend, we have, in the majority of cases, measured the volume of nitrogen in the old way, using, however, very accurately graduated tubes for collection, and correcting with great care for temperature and pressure. We have also found it advantageous to operate upon quantities of substance sufficient to yield at least two hundred cubic centimetres of gas, since in this way the error of reading becomes extremely small.

Sulphuric Acid. This acid cannot be accurately determined in the ammonia-cobalt salts by direct precipitation with chloride of barium. In almost all cases, a great apparent excess of acid is obtained, and this may amount to five per cent., even when the sulphate of baryta appears to have been completely washed. We have, therefore, in all cases preferred to decompose the salt to be analyzed, by boiling it with a little ammonia. After complete precipitation of the sesquioxide of cobalt, chlorhydric acid is to be added to reduce and dissolve the oxide, when the sulphuric acid may be directly thrown down by chloride of barium. Even with these precautions, our results are not unfrequently two-tenths or three-tenths of one per cent. too high, almost never too low.

Oxalic Acid. The ordinary methods for the quantitative estimation of this acid fail entirely with the class of salts under consideration. A solution of terchloride of gold is reduced only after very long and tedious boiling, and then incompletely. Even after previous reduction of the cobalt to the form of protoxide, the method is found to be very inconvenient and inaccurate. The conversion of the oxalic into carbonic acid by oxidation, and its determination from the weight of this last, gave no better results, inasmuch as the oxidation is effected with difficulty. We have therefore in all cases had recourse to the ultimate organic analysis, which alone gives reliable results.

The methods employed in the determination of other substances will be described, when necessary, in treating of particular compounds.

ROSEOCOBALT.

The description of the salts of Roseocobalt forms, upon the whole, the most convenient starting point in a statement of the results of our investigation. These salts are in general easily obtained, and the products of their decomposition include several of the other bases, which we shall have occasion to describe. They are almost all well crystallized, and are in general nearly insoluble in cold water, soluble without decomposition in warm water slightly acidulated, but easily decomposed when the neutral solutions are boiled, a hydrated hyper-oxide of cobalt being thrown down, while free ammonia is given off. The salts of Roseocobalt have a purely saline, not metallic taste; their color varies, being sometimes dull or brick-red, and sometimes cherry-red. They are usually dichrous, though a few of them do not exhibit this property in a marked degree. Heat decomposes the dry salts readily, the final products of the decomposition being usually ammonia, a salt of ammonium and a salt of protoxide of cobalt. Intermediate products are, however, sometimes formed, as we shall hereafter see. Thus in many cases the salts of Roseocobalt on boiling yield salts of Luteocobalt, which then, by continued boiling, are completely decomposed. The salts of Roseocobalt may almost always be prepared by the direct oxidation of ammoniacal solutions of salts of protoxide of cobalt, but the particular circumstances, which accompany the formation of each one, will be best considered in treating of the separate compounds. Roseocobalt is a triacid base.

CHLORIDE OF ROSEOCOBALT.

An ammoniacal solution of chloride of cobalt absorbs oxygen readily from the air, becomes at first brown and then gradually passes through various shades of color to a deep red. The red solution leaves upon a filter a quantity of hydrate of sesquioxide of cobalt, which is sometimes almost inappreciable, sometimes in comparatively large amount. In one experiment, in which we employed perfectly pure chloride of cobalt and pure ammonia, there was no deposit whatever of oxide. In this case, however, no chloride of Roseocobalt, but only chloride of Purpureocobalt was formed. When impure materials are used the precipitate is abundant, and contains many of the impurities of the substances employed, as well as much sesquioxide of cobalt. The rate at which oxygen is absorbed varies much with the degree of concentration of the solution, with the temperature, with the quantity of ammonia present, and with the extent of liquid surface exposed to the air. Frequent agitation of the solution materially shortens the time required for complete oxidation, and the same effect is produced by passing a current of oxygen directly through the liquid, which soon becomes brown and subsequently red. As a general rule, the first effect of the oxidizing action is to give the liquid a brown color, the layer next the surface being the first to change its tint. The brown color then passes gradually into a deep red, and the oxidation is complete, when the whole mass of liquid has the color of red Burgundy wine.

The presence of chloride of ammonium is not necessary in this process; a large quantity of this salt in the solution often gives a lilac or purple precipitate as the oxidation advances, but this is composed principally of the chloride of Purpureocobalt. As will be seen from the above, the chloride of Roseocobalt is not always formed during the oxidation of an ammoniacal solution of chloride of cobalt. On the contrary, it often happens that not a trace of this salt can be obtained from the oxidized solution, which contains only the chloride of Purpureocobalt. We have observed the absence of the chloride of Roseocobalt only in solutions which had been oxidized in a warm room, or during the summer season. This fact, taken in connection with the facility with which heat transforms solutions of Roseocobalt into those of Purpureocobalt, renders it, to say the least, extremely probable, either that a comparatively high temperature prevents the formation of the chloride of Roseocobalt entirely, or else that this salt is converted into chloride of Purpureocobalt as fast as it is formed in the solution.

To obtain the chloride of Roseocobalt from the oxidized solution, cold and strong chlorhydric acid is to be added to it, the slightest elevation of temperature being carefully avoided. A brick-red precipitate is thrown down, which is to be washed with strong chlorhydric acid and then with ice-cold water, thrown upon a filter, and dried by pressure, great care being taken to operate at as low a temperature as possible.

As the formula of the chloride of Roseocobalt is, $5NH_3.Co_2Cl_3+2HO$, its formation by the oxidation of the ammoniacal solution of chloride of cobalt may be explained by the equation

$$6CoCl+10NH_3+3O=2(5NH_3.Co_2Cl_3)+Co_2O_3.$$

In those cases in which no sesquioxide of cobalt is precipitated, we may suppose that the sesquioxide unites directly with ammonia, as represented by the equation

$$6CoCl+15NH_3+3O=2(5NH_3.Co_2Cl_3)+5NH_3.Co_2O_3.$$

On adding an excess of chlorhydric acid to such an oxidized solution, $3(5NH_3.$

Co_2Cl_3) must be formed, which satisfactorily explains the precipitation of the brick-red chloride by the acid.

Frémy assigns to the brown substance, which is the first product of the oxidation, the formula $4NH_3.Co_2O_3$. We have not yet been able to obtain this substance in a condition fit for analysis, and Frémy does not consider the formula, which he proposes, as by any means established.

It will be seen from the above that, in the formation of the chloride of Roseocobalt, the elements of ammonia unite with sesquioxide or sesquichloride of cobalt at the instant that these are formed by the absorption of oxygen from the air. Claus has recently shown that the sesquichloride of rhodium[1] unites directly with five equivalents of ammonia to form a chloride exactly analogous to the chloride of Roseocobalt, and having the formula $5NH_3.Rh_2Cl_3$. We have made various experiments to determine whether sesquioxide of cobalt once formed could unite directly with ammonia. A solution of chloride of ammonium was poured upon freshly prepared sesquioxide of cobalt, strong ammonia-water added, and the whole allowed to stand for some time in a closed bottle and in a rather dark closet. Even after many weeks, however, only traces of chloride of Roseocobalt could be detected. A quantity of sesquioxide of cobalt was dissolved in strong acetic acid, and to the solution chloride of ammonium and ammonia-water added. In this case chloride of Roseocobalt was formed after a few days, but it is doubtful whether its formation was not due to the oxidation of a small quantity of protoxide of cobalt in the sesquioxide employed. In another experiment, strong ammonia was added to freshly prepared sesquioxide of cobalt, and the whole allowed to stand for several weeks, after which time it was boiled with chlorhydric acid, and considerable quantities of chloride of Purpureocobalt, Luteocobalt, and Praseocobalt, were obtained. This experiment leaves no doubt that the Ammonia-cobalt bases can be prepared by the direct action of ammonia upon sesquioxide of cobalt, though this mode of preparation is not economical.

The chloride of Roseocobalt may also be prepared by adding cold and strong chlorhydric acid to a completely oxidized solution of the ammoniacal nitrate or sulphate of cobalt. A brick-red precipitate is formed in either case, which must be purified by repeated washing with chlorhydric acid. Strong chlorhydric acid also precipitates the chloride from solutions of the sulphate and nitrate of Roseocobalt. In all these cases, however, it is difficult to obtain the chloride in a perfectly pure state.

The chloride of Roseocobalt is usually precipitated as a brick-red powder, which, under the microscope, appears to be composed of indistinct granular crystals. It may be purified, though with difficulty, by solution in ice-cold water and spontaneous evaporation in the cold. The salt is soluble in cold, as well as in hot water, with a dark-red but not violet-red color, the portions still undissolved becoming lilac or purple before dissolving. The most remarkable property of this salt is the facility with which it is converted into chloride of Purpureocobalt. The hot

[1] Since the above was written, Claus has extended his observation to the sesquichloride of iridium, which forms a similar base with five equivalents of ammonia.

solution yields, on cooling, small but brilliant crystals of the latter chloride; in fact even solution in warm water converts a portion of chloride of Roseocobalt into chloride of Purpureocobalt, as may easily be observed by the change of color. This transformation is, however, far more striking when a solution of chloride of Roseocobalt is boiled with a little chlorhydric acid: the solution speedily changes its color from a dull-red to a beautiful violet-red, and on cooling deposits an abundant crystallization of chloride of Purpureocobalt. There is in this case a direct conversion of Roseocobalt into Purpureocobalt, an isomeric radical; the reactions of the violet-red solution being entirely different from those of the same solution previous to boiling with acid, except in one or two particulars, to be pointed out hereafter. The dry chloride of Roseocobalt is also slowly converted into chloride of Purpureocobalt by keeping, changing its color to violet-red; in this case, however, the change is not complete, even after a long time, unless heat be applied. The formula of the chloride of Purpureocobalt, as we shall hereafter show, is

$$5NH_3.Co_2Cl_3.$$

This differs from that of the chloride of Roseocobalt only by containing no water of crystallization. The change which takes place in the conversion of one chloride into the other does not, however, consist in the mere loss of water. As we shall show, the chloride of Roseocobalt corresponds to a triacid oxide, while that of Purpureocobalt yields a biacid oxide. It is to be carefully borne in mind, that the substance which we have called chloride of Roseocobalt is not the chloride of Roseocobaltiaque of Frémy, Claudet, and other chemists who have studied the subject. To the chloride described by Frémy under the name of chloride of Roseocobaltiaque we have given the name of chloride of Purpureocobalt. The necessity of this change of name has arisen from the fact that hitherto two different bases have been confounded, the chloride of Purpureocobalt having been considered as the chloride corresponding to the sulphate and nitrate of Roseocobalt.

The chloride of Roseocobalt is dichrous, the ordinary being paler than the extraordinary image; both are rose-red, with a faint brownish orange tint.

Chloride of Roseocobalt, as already mentioned, has the formula

$$5NH_3.Co_2Cl_3+2HO$$

as the following analyses show:

0.7291 grs. gave 0.4235 grs. of sulphate of cobalt = 22.10 per cent. of cobalt.
0.6247 grs. gave 0.3619 grs. " " = 21.99 " "
0.9834 grs. gave 0.5742 grs. chloride of silver = 39.57 " chlorine.
1.8112 grs. gave 2.9095 grs. " " = 39.71 " "
2.2390 grs. gave 1.2829 grs. water = 6.37 " hydrogen.
1.4993 grs. gave 0.8784 grs. " = 6.50 " "
1.2235 grs. gave 282 c. c. nitrogen at 22.°5 C. and $766^{mm}.82$ (at 23° C.) = 254.91 c. c. at 0° and 760^{mm} = 26.16 per cent.

The formula requires—

	Calculated.	Found.	
Cobalt	21.97	21.99	22.10
Chlorine	39.66	39.57	39.71
Hydrogen . . .	6.33	6 37	6.50
Nitrogen . . .	26.08	26.16	—

With respect to this formula, it must be remarked that it is extremely difficult to obtain this chloride perfectly free from chloride of Purpureocobalt, into which it is so easily converted. The uncertainty, however, will concern only the number of equivalents of water. The chloride of Roseocobalt combines with the chlorides of the electro-negative metals to form well defined salts. The platinum salt, which we have not yet fully examined, appears to have the formula

$$5NH_3.Co_2Cl_3+3PtCl_2+8HO.$$

A neutral solution of the chloride of Roseocobalt is easily decomposed by boiling, with evolution of ammonia, and precipitation of a black powder. This powder is probably a hydrate of the magnetic oxide, Co_3O_4+xHO, but we have deferred its examination to the second part of our memoir. The reactions of the chloride of Roseocobalt are as follows:

Terchloride of gold gives no precipitate at first, but after standing a lilac or purple precipitate, which is probably merely the chloride of Roseocobalt.

Bichloride of platinum gives a pale orange red precipitate.

Chloride of mercury gives a pale rose or flesh-colored flocky precipitate.

Ferridcyanide of potassium gives beautiful orange-red oblique rhombic crystals.

Cobaltidcyanide of potassium gives fine red crystals.

Ferrocyanide of potassium gives a cinnamon, passing to a chocolate brown precipitate.

Oxalate of ammonia gives a brick-red precipitate of small granular crystals.

Neutral chromate of potash gives no precipitate.

Bichromate of potash gives a dark brick-red precipitate.

The following reactions, which were obtained with a solution of the hydrated nitrate of Roseocobalt may also be introduced in this place.

Pyrophosphate of soda gives a dull rose-red precipitate soluble in an excess of the precipitant to a clear red liquid, which in a few minutes solidifies to a mass of fine rose-red needles.

Picrate of ammonia gives a fine bright orange-red precipitate soluble in hot water.

Iodide of potassium gives no precipitate either with the chloride or nitrate.

The precipitate with chloride of mercury is readily soluble in chlorhydric acid, and the solution after standing gives beautiful small granular crystals of a brownish red color.

The reactions which are peculiar to the sulphate of Roseocobalt will be described when speaking of that salt.

SULPHATE OF ROSEOCOBALT.

An ammoniacal solution of sulphate of cobalt absorbs oxygen readily from the air, becoming at first brown and then dark red. The time required for complete oxidation varies remarkably. The process is sometimes complete in a few days, but often requires many weeks. From the perfectly oxidized solution, sulphuric acid cautiously added usually throws down the sulphate of Roseocobalt as a bright red crystalline powder, which, after washing with cold water, is readily purified by solution and crystallization, a very small quantity of acid being added to prevent decomposition.

The sulphate of Roseocobalt is, however, not always the only salt formed under these circumstances. In some cases in which the ammoniacal liquid was allowed to stand several months until there remained only a dry mass of red crystals, warm water dissolved out a red salt in small quantity, much more soluble than the sulphate of Roseocobalt, and giving different and very characteristic reactions. In other cases, and especially when a little chloride of cobalt was originally present, warm water dissolved out another sulphate, crystallizing in octahedra of an orange-red color, the examination of which is not yet complete.

We are unable to confirm Frémy's assertion that sulphuric acid precipitates from oxidized ammoniacal solutions of sulphate of cobalt, an *acid* sulphate of Roseocobalt, having the formula

$$5NH_3.Co_2O_3,5SO_3+5HO.$$

The salt precipitated under these circumstances is merely the neutral sulphate, as repeated analyses have shown, and as the crystalline form at once proves. The formula of the neutral sulphate is

$$5NH_3.Co_2O_3,3SO_3+5HO$$

as the following analyses show:

0.8160 grs. gave 0.3800 grs. sulphate of cobalt = 17.72 per cent. cobalt.
0.8272 grs. gave 0.3869 grs. " " = 17.79 " "
0.2760 grs. gave 0.2908 grs. sulphate of baryta = 36.11 per cent. sulphuric acid.
0.5731 grs. gave 0.6074 grs. " " = 36.38 " " "
1.4925 grs. gave 0.8250 grs. water = 6.14 " hydrogen.
1.2840 grs. gave 0.7109 grs. " = 6.15 " "
1.2108 grs. gave 220.5 c. c. nitrogen at 18°.4 C. and 761^{mm}.23 (at 18°.9) = 201.97 c. c. at 0° and 760^{mm} = 20.95 per cent. nitrogen.
1.0404 grs. gave 194.2 c. c. nitrogen at 20°.5 C. and 754^{mm}.37 (at 20°.5) = 174.67 c. c. at 0° and 760^{mm} = 21.08 per cent. nitrogen.

The formula as above stated requires

	Eqs.	Calculated.		Found.		Mean.
Cobalt	2	59.0	17.71	17.72	17.79	17.75
Sulphuric acid	3	120.0	36.03	36.11	36.38	36.24
Hydrogen	20	20.0	6.00	6.14	6.15	6.14
Nitrogen	5	70.0	21.02	20.95	21.08	21.01
Oxygen	8	64.0	19.24	—	—	18.86
		333.0	100.00			100.00

The sulphate of Roseocobalt has a fine cherry-red color. The light reflected from a layer of the crystals when analyzed by the dichroscopic lens, gives a rose-red ordinary, and an orange-red extraordinary image. The dichroism is very distinct. In this, as in our other observations upon dichroism, the reflected rays examined made an angle of about 60° with the normal, but no material variations of color could be observed by changing the angle of incidence. The exact color of the crystals is, according to Chevreul, the second red 2-10ths. The crystals of sulphate of Roseocobalt belong to the dimetric or square prismatic system, as determined by Prof. Dana. The observed forms are represented in Figs. 1, 2, and 3. The measured angles are as follows:

$1 : 1 \ = 107° 20'$

$1 : ii \ = 126° 20'$

$ii : 1i = 137° 21'$

$O : 1i = 132° 35'$ (calc. $132° 39'$)

$a = 1.0866.$

FIG. 1.

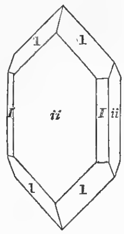


FIG. 2.

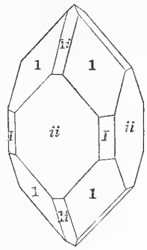
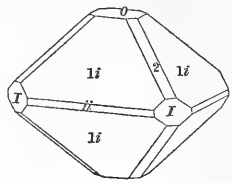


FIG. 3.



Prof. Dana remarks that the angles are very close to those of Cerasine; and also to those of Scheeletine or tungstate of lead, if $1i$ be $\frac{1}{2}$ and 1 be $1i$.

The sulphate of Roseocobalt is nearly insoluble in cold water, but is soluble in much boiling water, and crystallizes readily as the solution cools. By slow evaporation, it may be obtained in large crystals, which, however, seldom exhibit very perfect faces. Ammonia in dilute solution dissolves the sulphate, giving a fine purple solution, from which the salt crystallizes unchanged. The neutral solution is readily decomposed by boiling, ammonia being evolved, and a dark brown precipitate of the hydrated magnetic oxide of cobalt, Co_3O_4+3HO, thrown down, while sulphate of Luteocobalt remains in solution. The decomposition in this case extends to the sulphate of Luteocobalt also, so that much less than one equivalent of this salt is obtained for two equivalents of the sulphate of Roseocobalt decomposed.

Strong ammonia poured upon dry sulphate of Roseocobalt usually changes its color almost immediately from a red to a buff yellow, while the liquid itself becomes red. The buff colored substance formed in this case, is the sulphate of Luteocobalt; the red solution contains sulphate of Roseocobalt.

When dry sulphate of Roseocobalt is carefully heated in a porcelain or platinum crucible, ammonia is evolved, and there remains a lilac-red mass, which contains sulphate of Luteocobalt, sulphate of Purpureocobalt, and a leek-green crystalline substance which we have called provisionally Praseocobalt.

We shall speak of all these reactions more fully when treating of the sulphate of Luteocobalt.

A current of the red gas which arises from the action of nitric acid upon starch, and which probably consists chiefly of NO_4, converts an acid, neutral, or ammoniacal solution of sulphate of Roseocobalt into one of the nitrate of Xanthocobalt.

A solution of sulphurous acid gently heated with sulphate of Roseocobalt gives, in a few minutes, an orange precipitate of a substance containing ammonia, sesquioxide of cobalt, sulphurous and sulphuric acid, and which we shall describe fully in the second part of our memoir.

Strong sulphuric acid digested with sulphate of Roseocobalt yields, under some circumstances, sulphate of ammonia and sulphate of Luteocobalt. In other cases it yields the acid sulphate of Purpureocobalt. By double decomposition with salts of barium the sulphate of Roseocobalt yields the other salts of this base.

The reactions of the sulphate are somewhat different from those of the chloride, as will be seen from the following statement.

Ferridcyanide of potassium gives no precipitate at first, but after two hours very distinct and well defined small augitic crystals.

Cobaltidcyanide of potassium behaves in a precisely similar manner, giving red crystals.

Neutral chromate of potash gives no precipitate. The bichromate gives none at first, but after two or three hours, groups of reddish brown needles.

We shall hereafter state our reasons for believing that in certain cases there is a conversion of the triacid Roseocobalt in the sulphate of this base, into the biacid Purpureocobalt.

ANHYDROUS NITRATE OF ROSEOCOBALT.

The ammoniacal solution of nitrate of cobalt absorbs oxygen very readily from the air, and the oxidation is usually complete after a few days. As a general rule, a considerable quantity of nitrate of Luteocobalt is formed under these circumstances, and being insoluble in the ammoniacal liquid, forms a bright yellow crystalline precipitate upon the bottom and sides of the vessel. During the process of the oxidation, crystals of the compound described by Frémy as the nitrate of Oxycobaltiaque are frequently formed in some quantity, but these disappear at a later stage of the oxidation, when the liquid takes a deep wine-red color. The crystals of nitrate of Oxycobaltiaque were first observed by Leopold Gmelin. We have not particularly examined or analyzed them, though Frémy's analyses do not appear to us satisfactory.

The dark-red liquid formed under these circumstances contains nitrate of Roseocobalt. When nitric acid is added to this solution, a brick-red precipitate is thrown down, which is the hydrated nitrate of Roseocobalt. This nitrate is readily soluble in water, and exists unchanged in the solution, but by boiling with nitric acid, the solution yields a fine violet-red crystalline precipitate of the anhydrous nitrate of Roseocobalt. The presence of nitrate of ammonia facilitates the oxidation and formation of nitrate of Roseocobalt, but is not indispensable.

The preparation of pure nitrate of Roseocobalt is attended with difficulty, as the precipitated crystalline nitrate almost always contains a little nitrate of Luteocobalt. It is best to dissolve the crude nitrate in water, to which a little ammonia has been added, to filter and allow the solution to evaporate spontaneously. After some days, large and well defined crystals of the nitrate are formed, while the bottom of the evaporating vessel is covered with minute red crystals of the same salt. The difference between the appearance of the large and small crystals is so great that we suspected a difference in their constitution. Analysis and the behavior of the two kinds towards reagents, showed, however, no difference. A marked variation in the color of the large and small crystals of the same substance is very commonly observed in the ammonia-cobalt compounds, and might easily lead to erroneous conclusions. The nitrate of Roseocobalt is readily prepared by decomposing a solution of the chloride with nitrate of silver, but the solubility of the chloride of silver in chloride of Roseocobalt renders it somewhat difficult to obtain a pure salt in this manner. Nitrate of copper also gives nitrate of Roseocobalt with chloride of copper, when mixed with an equivalent proportion of chloride of Roseocobalt; but the purification is difficult. Finally, a pure nitrate may be prepared by double decomposition of nitrate of baryta and sulphate of Roseocobalt. The anhydrous nitrate of Roseocobalt, when in large crystals, has a fine red color, which, according to Chevreul's determination, as given by Frémy, is the first red $\frac{2}{10}$. The crystals are dichrous; the ordinary image is clear rose red, the extraordinary image bright red. According to Prof. Dana, this salt, like the sulphate, crystallizes in forms belonging to the dimetric system. Figs. 4 and 6 represent some of the more usual combinations, Fig. 5 is a very rare form, which was obtained only once.

1 : 1 (over the base) = 82° 40′.

Whence

0 : 1 (not observed) = 138° 20′.

I : 1 = 131° 40.

Fig. 4.

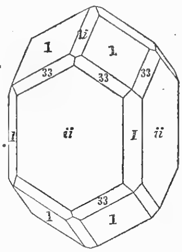


Fig. 5.

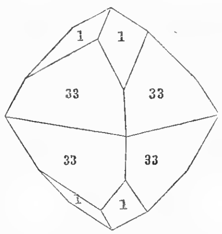
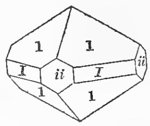


Fig. 6.



Ammonia dissolves the nitrate with a fine purple red tint, and the salt usually crystallizes unchanged from the solution, though sometimes the hydrous nitrate is obtained. In cold water the nitrate is rather insoluble, though more soluble than the sulphate. Hot water dissolves it rather more easily; but the solution, unless it be acid, is quickly decomposed, and this effect is very speedily produced by boiling. The products of the decomposition in this case are a dark-brown oxide of cobalt, and a solution containing the nitrate of Luteocobalt and nitrate of ammonia. The quantity of Luteocobalt is small in comparison with that of the nitrate of Roseocobalt employed.

When heated, the nitrate of Roseocobalt explodes, though not with violence. A black anhydrous oxide remains, which is probably Co_2O_3. The reaction in this case is easily explained, if we remark that the oxygen in the nitric acid is exactly sufficient to form water with the hydrogen of the ammonia. The simplest equation representing the reaction is

$$5NH_3.Co_2O_3,3NO_5=Co_2O_3+8N+15HO.$$

In point of fact, however, the decomposition is less simple, as red vapors are always evolved.

When a current of NO_4 is passed through a solution of nitrate of Roseocobalt a rapid absorption takes place, and after a short time crystals of nitrate of Xanthocobalt are deposited.

A solution of sulphurous acid converts the nitrate of Roseocobalt, at first into an orange-colored compound containing SO_2, and afterward reduces this completely to nitrate and sulphate of cobalt and nitrate of ammonia.

Nitrate of Roseocobalt has the formula

$$5NH_3.Co_2O_3,3NO_5$$

as the following analyses show:

0.2308 grs. gave 0.1081 grs. sulphate of cobalt = 17.82 per cent. cobalt.
0.1272 grs. gave 0.0599 grs. " " = 17.92 " "
0.1448 grs. gave 0.0681 grs. " " = 17.90 " "
0.9370 grs. gave 0.3915 grs. water = 4.64 per cent. hydrogen.
0.6632 grs. gave 0.2862 grs. " = 4.79 " "
2.7312 grs. gave 1.1258 grs. " = 4.57 " "
0.5564 grs. gave 168 c. c. nitrogen at 24° C. and $766^{mm}.31$ (at 24°.5) = 150.54 c. c. at 0° and 760^{mm} = 33.98 per cent.
0.7400 grs. gave 213 c. c. nitrogen at 13°.5 C. and $764^{mm}.28$ at (13.°8) = 200.55 c. c. at 0° and 760^{mm} = 34.03 per cent.

The formula above mentioned requires

	Eqs.	Calculated.		Mean.	Found.		
Cobalt	2	59.0	17.87	17.88	17.82	17.92	17.90
Hydrogen	15	15.0	4.55	4.60	4.64	4.79	4.57
Nitrogen	8	112.0	33.93	34.01	33.98	34.03	—
Oxygen	18	144.0	43.65	43.51	—	—	—
		330.0	100.00	100.00			

When nitrate of Roseocobalt is dissolved in water containing much nitrate of ammonia and a little ammonia, and the solution is allowed to evaporate spontaneously, beautiful purple-red scaly crystals separate. These crystals cannot be purified by recrystallization, as they are decomposed by solution in water. When boiled with chlorhydric acid there is copious effervescence and a purple-red solution is obtained, which appears to contain the chloride of Purpureocobalt. The empirical formula of the scaly nitrate appears to be $5NH_3.Co_2O_3,2NO_5+7HO$. From the effervescence with muriatic acid we are disposed to consider it $4NH_3.Co_2O_3,NO_5+NH_4O,NO_5+6HO$, but further investigation is required before we can pronounce with certainty on this point.

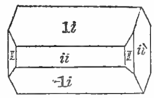
HYDROUS NITRATE OF ROSEOCOBALT.

When ammonia is added in excess to a solution of the nitrates of cobalt and of ammonia and the solution is exposed to the air, oxidation takes place with considerable rapidity, and as we have already stated when speaking of the anhydrous nitrate, the solution becomes dark purple-red, while yellow scales of the nitrate of Luteocobalt are more or less abundantly deposited upon the bottom of the vessel. When the red liquid is boiled with nitric acid in excess, a dark crimson precipitate of nitrate of Roseocobalt is formed, while a portion of the same salt remains in solution. It has hitherto been supposed from these facts that the anhydrous nitrate of Roseocobalt is a direct product of the oxidation of the ammoniacal liquid. This, however, is not the case. If the oxidized liquid be filtered from the nitrate of Luteocobalt and allowed to evaporate spontaneously, very fine large oblique rhombic crystals are formed, which are the hydrous nitrate of Roseocobalt.

The crystals of this nitrate belong to the monoclinic or oblique rhombic system, according to Prof. Dana's determination. The observed forms are *I*, 1*i*, *ii*, –1*i*, *iì*, or in other symbols, ∞, 1-∞, ∞-∞, —1-∞, ∞-∞̆. Fig. 7. The angles are

FIG. 7.

$I : I = 103°$
$1i : ii = 136°$
$-1i : ii = 140° 30'$
$1i : -1i = 96° 30'$ and $83° 30'$



The hydrous nitrate of Roseocobalt is readily soluble even in cold water; the hot neutral solution is very easily decomposed with evolution of ammonia and precipitation of a black powder. The addition of a few drops of nitric acid prevents the decomposition. An excess of nitric acid added to a cold solution of the nitrate produces a brick-red precipitate, which is readily soluble in cold water, and which is the unchanged salt. The solution has a dark brick-red color, and exhibits all the reactions of the chloride of Roseocobalt. When boiled with an excess of nitric acid for some time, the brick-red color gradually becomes violet-red, and there remains at last a beautiful violet precipitate, which is the anhydrous nitrate of Roseocobalt. From this it appears that, in some cases at least, and particularly when nitrate of ammonia is present during the oxidation, the hydrous nitrate of Roseocobalt is the first product of the oxidation of an ammoniacal solution of nitrate of cobalt, and that it is the action of nitric acid upon this salt which converts it into the anhydrous nitrate. The nitrate of Roseocobalt obtained by direct oxidation may be recrystallized by adding to its solution a few drops of nitric acid and allowing it to stand a few days for spontaneous evaporation. In this manner beautiful crystals are obtained, adhering to the bottom of the evaporating vessel, and mixed with a dull-red matter in crystalline crusts, which exhibits the same reactions with the large and clear crystals, and appears to have the same constitution, though upon this point we cannot speak with certainty at present.

We consider the formula of the hydrous nitrate of Roseocobalt as most probably

$$5NH_3.Co_2O_3,3NO_5+2HO$$

as the following analyses indicate:

0.8265 grs. gave 0.3703 grs. sulphate of cobalt = 17.05 per cent. cobalt.
0.5275 grs. gave 0.2370 grs. " " = 17.09 " "
0.8012 grs. gave 217.8 c. c. nitrogen at 11°.5 C. and $761^{mm}.48$ (at 11°.4 C.) = 206.05 c. c. at 0° and 760^{mm} = 32.30 per cent.
0.7667 grs. gave 207 c. c. of nitrogen at 14° C. and $766^{mm}.56$ (at 14°.2 C.) = 194.91 c. c. at 0° and 760^{mm} = 31.92 per cent.

The formula requires

	Eqs.	Calculated.	Found.	
Cobalt . . .	2	16.95	17.05	17.09
Nitrogen . .	8	32.18	32.30	31.92

It is true that the analyses here agree with the formula as well as can be reasonably expected. We have, however, found in other crystals from the same mass 17.82, 17.86, 17.92, 18.06 per cent. cobalt, numbers which agree much better with the formula of an anhydrous nitrate, having the same formula as the nitrate of Roseocobalt already described, which contains 17.87 per cent. In any event, the doubt appears to be simply with respect to the quantity of water in the salt, the ratio of the equivalents of cobalt and nitrogen being as 1 to 4, or 2 to 8. We shall return to this point at another time.

OXALATE OF ROSEOCOBALT.

The oxalate is precipitated from the chloride almost immediately by the addition of a solution of oxalate of ammonia. From a solution of the nitrate it is deposited much more slowly, often only after some hours, and sometimes in remarkably distinct and well formed crystals. The oxalate as first thrown down may be purified by solution in ammonia-water and recrystallization by spontaneous evaporation. The salt then forms beautiful prismatic crystals, which are nearly insoluble in water, and which have a fine cherry-red color, resembling the crystals of sulphate of Roseocobalt. The crystals are dichrous, the ordinary image being pale violet, while the extraordinary image is dark rose-red. The precipitated oxalate has a dull brick-red color. According to Prof. Dana, the crystals of oxalate of Roseocobalt belong to the right rhombic or trimetric system, the observed forms being a rhombic prism of about 101° 48′, with a brachydome of 108° 54′.

The constitution of the oxalate of Roseocobalt is represented by the formula,

$$5NH_3.Co_2O_3,3C_2O_3+6HO$$

as the following analyses show:

0.5504 grs. gave 0.2625 grs. sulphate of cobalt = 18.15 per cent.
0.3325 grs. gave 0.1585 grs. " " = 18.14 "
1.5381 grs. gave 0.6170 grs. carbonic acid = 32.82 per cent. oxalic acid.

The formula requires

	Eqs.	Calculated.	Found.	
Cobalt . . .	2	17.87	18.15	18.14
Oxalic acid . .	3	32.73	32.82	—

COBALTIDCYANIDE OF ROSEOCOBALT.

This beautiful salt is precipitated from a solution of the chloride or hydrous nitrate of Roseocobalt, by a solution of cobaltidcyanide of potassium. It may be prepared with equal facility from a solution of the chloride of Purpureocobalt, which under these circumstances, as we conceive, undergoes a direct change into a salt of the triacid Roseocobalt. The cobaltidcyanide is usually precipitated at once in the form of cherry-red prismatic crystals, which, so far as it is possible to judge from their appearance under the microscope, belong to the oblique rhombic or monoclinic system, much resembling some of the simpler forms of augite. The salt is very insoluble in cold water; hot water readily decomposes it. It forms an extremely characteristic test for the salts of Roseocobalt in general, as well as for the chloride of Purpureocobalt, but, as already remarked, it is precipitated from the sulphate of Roseocobalt only after some hours. The crystals are usually remarkably large when compared with the mass of liquid from which they are thrown down. They are more distinct in form the more slowly the precipitation takes place. The salt has the formula

$$5NH_3.Co_2Cy_3+Co_2Cy_3+3HO$$

as the following analyses show:

0.1924 grs. (from chloride of Purpureocobalt) gave 0.1540 grs. sulphate of cobalt = 30.46 per cent. cobalt.

0.7150 grs. (from chloride of Roseocobalt) gave 0.5755 grs. sulphate of cobalt = 30.63 per cent. cobalt.

0.8111 grs. (from chloride of Roseocobalt) gave 272 c. c. of nitrogen at 10° C. and $761^{mm}.99$ (at 10° C.) = 259.48 c. c. at 0° and 760^{mm} = 40.18 per cent.

The formula requires

	Eqs.	Calculated.	Found.	
Cobalt . . .	4	30.57	30.63	30.46
Nitrogen . .	11	39.89	40.18	

The analyses of the ferridcyanide of Roseocobalt give additional evidence of the correctness of the formula adopted for this salt.

FERRIDCYANIDE OF ROSEOCOBALT.

The ferridcyanide is formed like the cobaltidcyanide by adding a solution of ferridcyanide of potassium to one of chloride or nitrate of Roseocobalt or chloride of Purpureocobalt. A very beautiful orange-red precipitate is thrown down in distinct and usually extremely well-defined crystals, which under the microscope, exactly resemble those of the corresponding cobalt salt. The crystals exhibit a remarkable dichroism, the ordinary image being of a fine purple rose color, while the extraordinary image is bright orange red. The crystals are insoluble in cold water; hot water easily decomposes them, ammonia being evolved, while a dark-brown precipitate is thrown down. Heat decomposes the dry salt very gradually and uniformly, and leaves a black residue which contains much carbon. We availed ourselves of this fact to determine accurately the sum of the cobalt and iron in the salt, by first decomposing it by heat, then burning off the carbon in a gentle current of oxygen, and finally reducing the mixed oxides of cobalt and iron in a current of hydrogen. The formula of this salt is

$$5NH_3.Co_2Cy_3+Fe_2Cy_3+3HO$$

as the following analyses show:

0.3613 grs. gave 0.1086 grs. metallic iron and cobalt = 30.05 per cent.

1.5028 grs., burnt with oxide of copper and oxygen, gave 1.0302 grs. carbonic acid = 18.69 per cent. carbon.

The formula requires

	Eqs.	Calculated.	Found.
Cobalt and iron . .	4	30.02	30.05
Carbon . . .	12	18.79	18.69

There can be no reasonable doubt that this salt is isomorphous with the corresponding cobalt salt. Like the latter it is an extremely valuable test for the salts of Roseocobalt and for certain salts of Purpureocobalt.

OXIDE OF ROSEOCOBALT.

The oxide of Roseocobalt exists only in solution. It is obtained either by decomposing the chloride by oxide of silver, or by adding baryta-water to a cold solution of the sulphate; the latter method is the better one, because the chloride of silver is soluble in solutions of the chloride of Roseocobalt. The solution as thus obtained is red, has an alkaline non-metallic taste and reaction, and is very easily decomposed. By standing in the air it absorbs carbonic acid and forms a carbonate.

MAGNETIC OXIDE OF COBALT.

In connection with the salts of Roseocobalt we may perhaps with propriety describe the peculiar oxide of cobalt, which is, in some cases at least, one of the products of their decomposition. The hydrated oxide which is precipitated by boiling the neutral salts of Roseocobalt with an alkaline solution is considered by Frémy as a hydrate of the sesquioxide, and he attributes to it the formula Co_2O_3,HO. According to the same chemist, all the other ammonia-cobalt bases give this hydrate by boiling with solutions of the alkalies. It does not appear probable that the oxide obtained by boiling the neutral solution should have a different constitution. We have however found that the dark-brown oxide obtained by boiling a solution of sulphate of Roseocobalt, and afterward washing and drying the precipitate in the air, has the formula

$$Co_3O_4 + 3HO$$

as the following analyses show:

I. 0.4430 grs. gave 0.6990 grs. sulphate of cobalt = 60.05 per cent. of cobalt.

II. 0.9269 grs. gave 0.1672 grs. water (ignited with chromate of lead) = 18.03 per cent.

III. 0.7150 grs. ignited in hydrogen gas gave 0.3010 grs. water, which in connection with the 2d analysis gives 21.38 per cent. oxygen in the oxide of cobalt.

The formula requires

	Eqs.		Calculated.	Found.
Cobalt . . .	3	88.5	60.00	60.05
Oxygen . . .	4	32.0	21.69	21.38
Water . . .	3	27.0	18.30	18.03
		147.5	100.00	99.46

Frémy's formula requires 64 per cent. of cobalt. Claudet gives also the formula $Co_3O_4 + 3HO$ as probable, but without analyses. On the other hand, it is possible that the different salts, not only of Roseocobalt but of the other similar bases, may give different oxides by decomposition. We propose to examine this point more fully hereafter.

The hydrate above mentioned and analyzed is a very dark-brown powder, which dries to a black mass with a gummy lustre. The powder is dark brown. Oxalic acid dissolves it to a green solution, without evolution of gas, but this is decomposed by heating. Chlorhydric acid also dissolves the oxide, with evolution of chlorine and formation of the protochloride.

We have already mentioned that the anhydrous magnetic oxide of cobalt is sometimes obtained during the decomposition of the chloride of Roseocobalt by heat. We are, however, not able to state precisely under what circumstances this occurs; either water or the oxygen of the air must take part in the decomposition, since the chloride contains no oxygen. The anhydrous oxide occurs in the form of small steel-gray octahedra, which are very hard, and which can only be dissolved by long heating with sulphuric acid, or by fusion with sulphate of potash. Nitric, chlorhydric, and nitro-muriatic acids have no decided action upon them.

1.6425 grs. of this oxide gave 1.2059 grs. of metallic cobalt = 73.41 per cent.
1.6425 grs. ignited in hydrogen gave 0.4879 grs. water = 25.91 per cent. oxygen.

The formula Co_3O_4 requires

	Eqs.		Calculated.	Found.
Cobalt . . .	3	88.5	73.44	73.41
Oxygen . . .	4	32.0	26.56	25.91
		120.5	100.00	99.32

This oxide has recently been described by Schwarzenberg, who obtained it by igniting chloride of cobalt with free access of air, until the chlorine is expelled. It is, therefore, very probable that in the decomposition of chloride of Roseocobalt by heat, the chloride of cobalt is first produced, and then decomposed in the manner observed by Schwarzenberg.

With respect to the black sulphide which is thrown down from solutions of the ammonia-cobalt bases by sulphide of ammonium, it can scarcely be doubted that this is the bisulphide, as Claudet's analyses lead directly to the formula CoS_2, corroborating the results obtained by Dingler already alluded to.

PURPUREOCOBALT.

The salts of Purpureocobalt are often found among the direct products of the oxidation of ammoniacal solutions of cobalt. They are often formed from the salts of Roseocobalt by heating or by boiling with strong acids, the cobalt passing, as we conceive, from one modification to another. The salts of Purpureocobalt are also formed in great abundance by the action of acids upon salts of Xanthocobalt, and we are disposed to think that they may also occur, though rarely, among the products of the decomposition of salts of Luteocobalt.

The salts of Purpureocobalt are distinguished by a fine violet-red or purple color, which is common to nearly all of them, and which is very different from the comparatively dull red of the salts of Roseocobalt. They are in general somewhat less soluble than the compounds of Roseocobalt, and crystallize, for the most part, in well defined crystals. When neutral they have a purely saline, non-metallic taste.

Heat readily decomposes the salts of this base; the final products of the decomposition are the same as in the case of the salts of Roseocobalt, but intermediate products are often formed. The neutral solutions are readily decomposed by boiling, the products of the decomposition being a black or dark-brown oxide of cobalt and a salt of ammonium, free ammonia being given off. In some cases, however, salts of Luteocobalt are intermediate products of this decomposition.

All the salts of Purpureocobalt by long boiling with an excess of chlorhydric acid yield the chloride.

CHLORIDE OF PURPUREOCOBALT.

The substance which we shall describe under the name of chloride of Purpureocobalt is the same as that to which Frémy gave the name of chloride of Roseocobaltiaque. In the course of our investigations it at length became clear that, under the name of salts of Roseocobalt, the compounds of two perfectly distinct bases have hitherto been confounded. It became, therefore, necessary to devise a new name. The purple color of the salts which correspond to the chloride now to be described, led us to adopt the name of Purpureocobalt for the radical of these salts, as more appropriate than Roseocobalt, which we have retained for most of the salts to which it was originally applied. Such a change is to be regretted; it could not, however, have been avoided, without an introduction of two entirely new names.

We have already stated that the chloride of Purpureocobalt is often a product of the direct oxidation of an ammoniacal solution of the chloride of cobalt exposed to the air. In these cases it is sometimes mixed with chloride of Roseocobalt, and sometimes forms the entire product of the oxidation. We believe that the temperature at which the process of oxidation goes on is the condition which determines the character and the amount of the chloride which is formed during the oxidation. The chloride of Roseocobalt is, in our view, the first product of the oxidation under all circumstances. At a moderately high temperature, however, the chloride passes as fast as it is formed into chloride of Purpureocobalt, which may thus be the only final product of the oxidation, or may be mixed with variable proportions of the chloride of Roseocobalt.

In one experiment made during the summer season, and in which chemically pure chloride of cobalt and ammonia were employed, the process of oxidation went on very slowly, without the precipitation of any trace of sesquioxide of cobalt. The liquid had a dull purple color, and gave with reagents no precipitates or reactions to indicate the presence of chloride or oxide of Purpureocobalt or Roseocobalt. On boiling with chlorhydric acid, however, the chloride of Purpureocobalt was thrown down in abundance, and no other substance could be detected in the supernatant liquid. In this case the oxidized liquid gave no precipitate with chlorhydric acid in the cold, but the cold solution, after some hours standing, deposited distinct crystals of chloride of Purpureocobalt. We believe that in this case a combination of the oxide and chloride existed in the solution, so that, as we have already suggested in speaking of the chloride of Roseocobalt, the oxidation itself would be expressed by the equation

$$6CoCl+15NH_3+3O=5NH_3.Co_2O_3+2(5NH_3.Co_2Cl_3).$$

If we admit that the oxide and chloride of Purpureocobalt as thus formed are actually in combination so as to form a sort of oxychloride, and are not merely mechanically mixed, we may perhaps explain, why no precipitates of salts of Purpureocobalt are obtained in the oxidized liquid, since the reagents added might not be able to overcome the affinity between the oxide and chloride. It is easy to see that by boiling with chlorhydric acid, the combination of oxide and chloride will give the chloride alone, since we may have the equation

$$5NH_3.Co_2O_3+_2(5NH_3.Co_2Cl_3)+3HCl=3(5NH_3.Co_2Cl_3)+3HO.$$

We have already mentioned that the chloride of Roseocobalt is readily converted into chloride of Purpureocobalt, by boiling with chlorhydric acid, or even by gently heating its solution. This circumstance explains why only the chloride of Purpureocobalt is obtained by boiling a completely oxidized ammoniacal solution of chloride of cobalt, even when this solution contains only Roseocobalt, or a mixture of Roseocobalt and Purpureocobalt. It also enables us to understand why all writers upon the ammonia-cobalt bases, up to the period of the present investigation, entirely overlooked the chloride of Roseocobalt, and consequently described the salts of that base, as if they corresponded to, and contained the same radical as chloride of Purpureocobalt.

It is not necessary to add chloride of ammonium to the ammoniacal solution of chloride of cobalt, in preparing the chloride of Purpureocobalt. It is, however, advantageous to do so, because, the oxide of Purpureocobalt is converted by it as fast as formed into the chloride, and the formation of the oxychloride is prevented. To prepare the chloride of Purpureocobalt in a pure state from the oxidized liquid, it is only necessary to boil this with an excess of chlorhydric acid. A crimson powder is thrown down, while the supernatant liquid becomes nearly colorless, provided, at least, that a pure salt of cobalt was employed, and that the oxidation was complete. The mother liquor is to be poured off and the precipitate dissolved in a large quantity of boiling water, to which enough chlorhydric acid has been added to render the solution distinctly acid. On cooling, the solution gives small but beautiful crystals of the chloride, almost perfectly pure. A second crystallization usually removes all traces of impurity. It is not, however, necessary to use a pure chloride of cobalt in preparing the chloride of Purpureocobalt. Any commercial oxide will answer, even when arsenic, nickel, iron, &c., are present. On the other hand, it is easy to prepare a perfectly pure chloride of cobalt, by heating the chloride of Purpureocobalt in a porcelain crucible until vapors of ammonia and chloride of ammonium cease to be given off. The pure chloride of cobalt thus obtained, is remarkable for its beauty of color, the anhydrous chloride forming pale blue talcose scales, while the solution and the crystals obtained from this have a very fine violet-red tint.

The chloride of Purpureocobalt may be prepared by other methods. One of the most interesting of these is, by the action of strong chlorhydric acid upon a salt of Xanthocobalt. It is almost a matter of indifference which salt of Xanthocobalt is employed. As, however, the nitrate is, perhaps, most easily obtained in a pure state, it is usually most advantageous to employ this. The nitrate has the formula

$$NO_2.5NH_3.Co_2O_3,2NO_5+HO.$$

On boiling with chlorhydric acid, the salt is slowly converted from a brown-yellow to a lilac-purple powder, which is insoluble in the supernatant acid liquid. After boiling strongly for an hour or two, almost all the original salt is decomposed, NO_2 is given off in abundance during the boiling, while a lilac colored uncrystallized mass remains at the bottom of the flask. The supernatant liquid is to be poured off, and boiling water added to the insoluble portion. A brown-yellow or dark sherry wine colored solution is usually formed, which is again to be poured off, and the washing repeated till the liquid has a clear purple color. The red mass is then to be dissolved in boiling water, to which a little chlorhydric acid has been added, and filtered. On cooling, the chloride of Purpureocobalt crystallizes in small brilliant crystals, which must be repeatedly recrystallized to separate all traces of impurity. The washings, on boiling with chlorhydric acid, yield a fresh portion of the chloride. The reaction which takes place under these circumstances may be expressed by the equation

$$NO_2.5NH_3.Co_2O_3,2NO_5+3HCl=NO_2+2NO_5,HO+5NH_3.Co_2Cl_3+HO.$$

As already remarked, the chloride or sulphate of Xanthocobalt may be employed in a precisely similar manner, and also yield the chloride of Purpureocobalt. When the sulphate is used, however, the resulting chloride is apt to retain sulphuric acid with much obstinacy, and can with difficulty be freed from it.

Another method of preparing the chloride of Purpureocobalt, consists in boiling the chloride or nitrate of Roseocobalt with chlorhydric acid. This method is very convenient, and yields a very pure chloride.

The chloride of Purpureocobalt may also be prepared by boiling the acid sulphate of this base with chlorhydric acid. In this case, however, as in all others in which sulphuric acid is present in the solution, the chloride should be boiled with a little chloride of barium, and repeatedly recrystallized, to separate traces of the isomorphous sulphate of Roseocobalt formed at the same time.

The chloride of Purpureocobalt has a beautiful violet-red or purple color, and is dichrous, the ordinary ray being colorless, while the extraordinary ray has a rich violet-red tint. Its solution is violet-red. The salt is nearly insoluble in cold water, but is soluble without decomposition in boiling water, to which a few drops of chlorhydric acid have been added. From this solution it separates, on cooling, in very brilliant small crystals, which are simpler in form, the purer the solution from which they have crystallized. The crystals belong to the square prismatic or dimetric system, according to Prof. Dana, and not to the regular system, as stated in previous memoirs. The observed forms are the octahedron and first and second prism. P . ∞ P ∞ . P ∞ . The angles in Dana's notation are (see Fig 1):

1 : 1 over the base = 114° 8′; over the top = 65° 52′.
1 : 1 over the terminal edge = 107°—107° 20′ (calculated 107° 12′).
O : 1 (calculated) = 122° 56′.
O : 1*i* (calculated) = 132° 30′.
1*i* : 1*i* over the base = 95°; over the top = 85°.
a = 1.0916.

The crystals are usually small but extremely well formed; those which are obtained from solutions containing a little chloride of mercury most frequently exhibit the planes of the first and second prism, and are larger than those which separate from pure solutions. From these measurements, it appears that the chloride of Purpureocobalt is isomorphous with the sulphate of Roseocobalt. This isomorphism is the more remarkable, inasmuch as a precisely similar case occurs with the chloride and sulphate of Luteocobalt, between which there is a similar difference of constitution. Thus we have

$$5NH_3.Co_2Cl_3=5NH_3.Co_2O_3,3SO_3+5HO$$
$$6NH_3.Co_2Cl_3=6NH_3.Co_2O_3,3SO_3+5HO.$$

From this it appears that in both cases we have the crystallographic equality

$$3Cl=O_3,3SO_3+5HO.$$

The density of the crystals of chloride of Purpureocobalt, as taken in alcohol, is 1.802 at 23° C.; the atomic volume of the chloride is consequently 139.0.

The chloride of Purpureocobalt has the formula

$$5NH_3.Co_2Cl_3$$

as appears from the following analyses:

0.5215 grs. gave 0.3230 grs. of sulphate of cobalt = 23.58 per cent. cobalt.
0.4962 grs. gave 0.3073 grs. " " = 23.57 " "
0.8514 grs. gave 0.5269 grs. " " = 23.55 " "
1.4116 grs. gave 0.8732 grs. " " = 23.55 " "
0.7550 grs. gave 0.4105 grs. water . . . . = 6.04 per cent. hydrogen.
0.6116 grs. gave 0.3412 grs. " = 6.19 " "
0.6124 grs. gave 0.3365 grs. " = 6.10 " "
1.4184 grs. gave 0.7800 grs. " = 6.11 " "
1.6636 grs. gave 2.8540 grs. chloride of silver = 42.40 per cent. chlorine.
0.4754 grs. gave 0.8200 grs. " " = 42.49 " "
0.1966 grs. gave 0.3365 grs. " " = 42.31 " "
0.4972 grs. gave 0.8553 grs. " " = 42.52 " "
0.5963 grs. gave 143 c. c. nitrogen at 18° C. and $772^{mm}.4$ (at 18°.4 C.) = 133.19 c. c. at 0° and 760^{mm} = 28.05 per cent. nitrogen.
0.5542 grs. gave 134 c. c. nitrogen at 19° C. and $766^{mm}.56$ (at 19°.4 C.) = 123.26 c. c. at 0° and 760^{mm} = 27.93 per cent. nitrogen.
0.6036 grs. gave 168.16 c. c. nitrogen at $752^{mm}.60$ and 15° C, h = $93^{mm}.5$, t = 15°.6 C, = 135.08 c. c. at 0° and 760^{mm} = 28.11 per cent. nitrogen.
0.5755 grs. gave 160.39 c. c. nitrogen at $771^{mm}.39$ and 15° C, h = $112^{mm}.7$, t = 15° C, = 128.86 c. c. at 0° and 760^{mm} = 28.12 per cent. nitrogen.

The nitrogen in these as in all our analyses was moist.

Hence we have

	Eqs.		Theory.	Mean.		Found.		
Cobalt .	2	59	23.55	23.56	23.58	23.57	23.55	23.55
Chlorine .	3	106.5	42.50	42.43	42.49	42.31	42.52	42.40
Hydrogen .	15	15	5.98	6.11	6.04	6.19	6.10	6.11
Nitrogen .	5	70	27.97	28.01	28.05	27.93	28.12	28.11
		250.5	100.00	100.11				

The agreement of these analyses leaves no reasonable doubt that the true formula of the chloride of Purpureocobalt is $5NH_3.Co_2Cl_3$, as first correctly determined by Rogojski, and subsequently by Gregory. Frémy gives in addition one equivalent of water, while Claudet makes 16 in place of 15 equivalents of hydrogen. That the salt, however, contains but 15 equivalents is clear, from the fact that free nitrogen and hydrogen are found among the products of its decomposition by heat in an atmosphere of carbonic acid gas, which could not be the case upon Claudet's view, since we should then have the equation

$$N_5H_{16}Co_2Cl_3=5NH_3+2CoCl+HCl,$$

while the presence of free nitrogen and hydrogen renders it probable that the decomposition is in reality expressed by the equation

$$5NH_3.Co_2Cl_3=2CoCl+NH_4Cl+3NH_3+N+H_2.$$

We have more than once endeavored to determine the amount of gas actually given off during this decomposition, with the view of verifying the equation just given. In every case, however, a portion of the chloride of cobalt was reduced, either by the free ammonia or by the hydrogen, so that much metallic cobalt was found mixed with the chloride. A neutral solution of the chloride of Purpureocobalt is readily decomposed by boiling, a dark-brown precipitate, probably of the hydrated magnetic oxide, being thrown down, while the solution becomes brown-yellow, and contains chloride of ammonium and chloride of Luteocobalt, ammonia being at the same time given off. The quantity of chloride of Luteocobalt which is thus formed is always very small, being very much less than one equivalent for two equivalents of the chloride of Purpureocobalt.

On the other hand, a solution of chloride of Purpureocobalt may be boiled for a very long time with concentrated chlorhydric acid without decomposition, and this stability in the presence of acids is one of the most remarkable peculiarities of the whole class of ammonia-cobalt salts.

Chlorhydric acid and the alkaline chlorides precipitate chloride of Purpureocobalt from its solutions almost completely, slowly in the cold, but instantly on boiling. Ignited in a current of hydrogen, the salt yields metallic cobalt as a gray spongy mass. Heated in an open crucible, the salt fuses and swells up, giving off abundant vapors of chloride of ammonium and ammonia, while pure chloride of cobalt remains in lavender-blue scales. In some cases, however, this is mixed with metallic cobalt, while in others, in which the ignition takes place with free access of air, brilliant iron-black octahedra are formed, which are the anhydrous magnetic oxide of cobalt, Co_3O_4. The red gas arising from the action of nitric acid upon starch or sawdust exerts a very remarkable influence upon the chloride of Purpureocobalt, converting it into the nitrate of a base, which will be described further on under the name of Xanthocobalt.

Sulphurous acid solution throws down from solutions of the chloride a dull orange-brown precipitate, which appears to be a sulphite. By boiling with an excess of the acid this is reduced, and there remains a solution of a protosalt of cobalt.

Sulphuric acid, under certain conditions, converts chloride of Purpureocobalt into the acid sulphate of the same base.

Zinc may be boiled a long time with an acid solution of the chloride without producing decomposition or reduction. Formic and oxalic acids have no reducing action. Protochloride of tin simply unites with the chloride of Purpureocobalt so as to form a chloro-salt.

The chloride of Purpureocobalt exhibits a remarkable tendency to unite with metallic chlorides to form chloro-salts. Such compounds are formed with the chlorides of Platinum, Palladium, Mercury, Tin, Zinc, and various other metals. The chloride of Purpureocobalt even dissolves chloride of silver in large quantity, doubtless forming with it a double chloride. It is for this reason, that it is not generally advantageous to prepare the salts of Purpureocobalt by double decomposition between the chloride and salts of silver.

The reactions of a pure solution of the chloride of Purpureocobalt are as follows:

Ferrocyanide of potassium gives a yellowish precipitate which quickly becomes chocolate-brown.

Ferridcyanide of potassium gives a beautiful bright orange-red crystalline precipitate.

Cobaltidcyanide of potassium gives a fine red crystalline precipitate.

Oxalate of ammonia gives a beautiful purple-red precipitate of fine needles.

Pyrophosphate of soda gives a lilac precipitate easily soluble in an excess of the precipitant.

Neutral chromate of potash gives a brick-red precipitate.

Bichromate of potash gives orange-yellow scales.

Picrate of ammonia gives a beautiful yellow precipitate.

Terchloride of gold precipitates the chloride unchanged.

Bichloride of platinum gives a fine cinnamon-brown precipitate of crystalline scales.

Sulphide of ammonium gives a black precipitate.

Chloride of mercury gives fine rose-red needles, easily decomposed.

Bichloride of tin gives pale peachblossom-red silky needles.

Molybdate of ammonia gives a pale peachblossom-red precipitate.

Alkalies and their carbonates give no precipitate.

Iodide and bromide of potassium give no precipitate.

Chlorhydric acid and the alkaline chlorides throw down the chloride of Purpureocobalt from its solutions as a violet-red powder.

The reactions of the chloride of Purpureocobalt with the ferridcyanide and cobaltidcyanide of potassium and with oxalate of ammonia are not sufficient to distinguish it from the chloride of Roseocobalt, which, when pure and freshly prepared, gives also precipitates with these reagents. The character of the precipitates with oxalate of ammonia, bichloride of platinum, and bichromate of potash enable us, however, readily to distinguish the chloride of Purpureocobalt from chloride of Roseocobalt, with which, however, it is not likely to be confounded.

CHLORPLATINATE OF PURPUREOCOBALT.

When a solution of bichloride of platinum is added to one of the chloride of Purpureocobalt, a brown-red precipitate is thrown down, which is a combination of the two chlorides. When dried it has a fine rich brown-red color and high lustre. The crystals seen under the microscope are usually aggregations of flat pale reddish-brown needles. They are very distinctly dichrous, the ordinary image being pale violet-rose, while the extraordinary image is rich orange-red.

The chlorplatinate is nearly insoluble in cold, and with great difficulty soluble in hot water. It resists the action of reducing agents much more powerfully than the chlorplatinates of the alkaline metals. Thus it must be boiled for a very long time with zinc and chlorhydric acid before a complete reduction of the platinum is effected. If the process be interrupted before the reduction is complete, brilliant yellow granular crystals are often formed in the liquid. We have not determined the constitution of these crystals, but they are not chlorplatinate of ammonium. Sulphurous acid reduces this double chloride readily, and yields a red solution containing the protochlorides of platinum and of cobalt. We may here remark, that so far as our observation has hitherto extended, the action of a reducing agent upon any constituent of a compound containing an ammonia-cobalt base extends invariably to the ammonia-cobalt base itself.

The chlorplatinate of Purpureocobalt has the formula

$$5NH_3.Co_2Cl_3+2PtCl_2$$

as the following analyses show:

0.6765 grs. (reduced by boiling with SO_2 and the platinum precipitated as sulphide by $NaO.S_2O_2$ after adding HCl) gave 0.2267 grs. of platinum = 33.51 per cent.

0.9521 grs. gave 0.3169 grs. of platinum and 0.2483 grs. sulphate of cobalt = 9.93 per cent. cobalt.

—— grs. gave —— grs. of chloride of silver = 41.80 per cent. chlorine.

The formula requires

	Eqs.	Calculated.	Found.
Cobalt	2	10.10	9.93
Platinum . . .	2	33.50	33.51
Chlorine	7	42.01	41.80

This salt is identical with the chlorplatinate described and analyzed by Claudet, and for which that chemist found the same formula, with the exception of the hydrogen, which he makes 16 in place of 15 equivalents. We have also obtained it from a chloride which gave the reactions of chloride of Roseocobalt, but we must leave it for the present undecided whether in this case there was a conversion of Roseocobalt into the isomeric Purpureocobalt, by the action of the chloride of platinum, or whether the chloride of Roseocobalt had already undergone the change. We consider it certain that the salt in question is a salt of Purpureocobalt, because it contains *two* in place of *three* equivalents of bichloride of platinum. We shall show further on, that the oxygen salts of this base contain either *two* or *four* equivalents of acid, and it is well known that in the chlorplatinates there is—we believe invariably—but one equivalent of bichloride of platinum for each equivalent of chlorine in the chloride with which it is united. Since there are three equivalents of chlorine in this chloride of Purpureocobalt, we infer that two of them are differently combined from the other two, so that the rational formula of the chlorplatinate is

$$5NH_3.Co_2Cl.Cl_2+2PtCl_2.$$

We shall develop this view more fully when speaking of the oxygen salts of Purpureocobalt.

OXALATE OF PURPUREOCOBALT.

This most beautiful salt is readily prepared by adding a solution of oxalate of ammonia to one of chloride of Purpureocobalt. After a short time violet-red needles are thrown down, which may be washed with cold water. As thus prepared, the salt is almost chemically pure. The color of the oxalate of Purpureocobalt is the violet $\frac{6}{10}$ of the first circle of Chevreul's scale; the crystals are not sensibly dichrous. We have not, as yet, obtained measurable crystals of this salt. Under the microscope four- and six-sided acicular prisms are distinguishable, but without characterizing terminal planes.

The oxalate of Purpureocobalt has the formula

$$5NH_3.Co_2O_3,2C_2O_3+3HO$$

as the following analyses show:

0.2723 grs. gave 0.1574 grs. sulphate of cobalt = 22.00 per cent. of cobalt.
0.3545 grs. gave 0.2045 grs. " " = 21.95 " "
0.8970 grs. burnt with oxide of copper gave 0.2973 grs. carbonic acid = 27.11 per cent. of oxalic acid.

The formula requires

	Eqs.	Calculated.	Found.	
Cobalt . . .	2	22.09	22.00	21.95.
Oxalic acid . .	2	26.96	27.11	—

The oxalate is nearly insoluble in cold water, and not very soluble in boiling water, even after addition of free oxalic acid. The salt does not crystallize well from its solutions, and we have always obtained it in the most beautiful form by direct precipitation from the chloride. The salt is neutral to test paper, and is the only neutral oxysalt of Purpureocobalt which we have yet obtained. It will appear from what follows, extremely probable that there is an acid oxalate of Purpureocobalt containing four equivalents of oxalic acid. We have not obtained such a salt, however, in one or two experiments made for the purpose.

ACID SULPHATE OF PURPUREOCOBALT.

Our efforts to obtain a neutral sulphate of Purpureocobalt containing two equivalents only of sulphuric acid have hitherto been fruitless. When a solution of chloride of Purpureocobalt is treated with sulphate of silver, chloride of silver is formed, and the red supernatant liquid yields, on evaporation, crystals of sulphate of Roseocobalt. Precisely the same result is obtained with the chloride and nitrate of silver; the red solution yielding crystals of nitrate of Roseocobalt. We consider it probable that in these cases the sulphate and nitrate of Purpureocobalt, $5NH_3.Co_2O_3,2SO_3$, and $5NH_3.Co_2O_3,2NO_5$, are really formed by double decomposition, but that during evaporation the equivalent of free sulphuric or nitric acid formed at the same time with the sulphate or nitrate, reacts upon this so as to convert it into a salt of Roseocobalt with three equivalents of acid. In equations we should have for the sulphate

$$5NH_3.Co_2Cl_3+3AgO,SO_3+HO=5NH_3.Co_2O_3,2SO_3+HO,SO_3+3AgCl.$$

$$5NH_3.Co_2O_3,2SO_3+HO,SO_3=5NH_3.Co_2O_3,3SO_3+HO.$$

When oil of vitriol is poured upon chloride of Purpureocobalt in quantity sufficient to make a thick paste, the mass assumes a fine purple color, and swells up very much at first, so that a large vessel is necessary. If the solution, after the evolution of chlorhydric acid has ceased, be diluted with about twice its volume of water, and allowed to stand for a few hours, a large mass of beautiful violet-red needles is deposited. The mother liquor, after standing for a longer time, deposits more crystals. These crystals are to be quickly washed with a little cold water, drained and dried by pressure between folds of bibulous paper. They are usually free from chlorine, and are very nearly pure acid sulphate of Purpureocobalt. The mother liquor contains more of the acid sulphate together with small quantities of another sulphate which we shall describe more fully hereafter, and frequently a little undecomposed chloride. By boiling this mixture with chlorhydric acid chloride of Purpureocobalt is formed, which may be employed in preparing a fresh portion of the acid sulphate.

The acid sulphate of Purpureocobalt may also be prepared by the action of strong sulphuric acid upon the sulphate of Roseocobalt. For this purpose oil of vitriol is to be poured upon the sulphate in quantity sufficient to produce an oily liquid on heating in a water bath. The digestion is to be continued for one or two hours, according to the quantity of salt employed, care being taken that no oxygen is evolved. The dark purple liquid is to be suffered to cool, diluted with an equal bulk of water, and allowed to crystallize.

The acid sulphate as thus obtained is difficult to purify. By dissolving it in a small quantity of hot water, and evaporating it quickly, fine crystals may sometimes be obtained. When, however, the solution is evaporated slowly in the air, crystals of sulphate of Roseocobalt are formed in abundance, while the mother liquor contains free sulphuric acid. When a solution of the acid sulphate is neutralized with ammonia, and allowed to crystallize by slow evaporation, the sulphate of Roseocobalt is also obtained, but by rapid evaporation dark-red, prismatic crystals are sometimes formed, which we have not yet obtained in sufficient quantity for a complete analysis. They may prove to be the neutral sulphate of Purpureocobalt.

The acid sulphate of Purpureocobalt crystallizes in fine, red, prismatic crystals, which, according to Prof. Dana, belong to the trimetric system, and are hemihedral. The observed forms are I, $i\breve{i}$, $\frac{1}{2}\breve{i}$, $\frac{1\bar{2}}{2}$, $i\bar{2}$ (?) or in other symbols ∞, $\infty\text{-}\breve{\infty}$, $\frac{1}{2}\text{-}\breve{\infty}$, $\frac{1\text{-}\bar{2}}{2}$, $\infty\text{-}\bar{2}$ (?):

$I : I = 106°$.

$I : i\breve{i} = 127°$ (126° 50′—127° 10′)

$\frac{1}{2}\breve{i} : \frac{1}{2}\breve{i} = 122° 42′$.

$1\bar{2} : 1\bar{2} = 67° 54$ $a : b : c = 1.0927 : 1 : 1.3271$

Fig. 8.

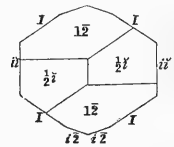


Fig. 8 represents an end view of a crystal of this salt; $1\bar{2}$ is hemihedral and $i\bar{2}$ usually so: the symbol $i\bar{2}$ is probably correct, though the observed angle varies much.

The acid sulphate is very soluble in water, and has a distinct though not strongly acid taste. It reddens litmus, and expels carbonic acid from the carbonates.

The formula of this salt is

$$5NH_3.Co_2O_3,4SO_3+5HO$$

as the following analyses show:

0.620 grs. gave 0.2577 grs. sulphate of cobalt = 15.82 per cent. cobalt.
1.1402 grs. gave 0.4756 grs. " " = 15.86 " "
1.5317 grs. gave 1.9270 grs. sulphate of baryta = 43.19 " sulphuric acid.
1.5843 grs. gave 0.7570 grs. water = 5.31 per cent. hydrogen.
1.1869 grs. gave 189.5 c. c. of nitrogen at 15° C. and $775^{mm}.2$ (at 15°.3 C.) = 179.6 c. c. at 0° and 760^{mm} = 19.00 per cent. nitrogen.

The formula requires

	Eqs.	Calculated.	Found.	
Cobalt . .	2	15.81	15.82	15.86
Sulphuric acid .	2	42.89	43.19	
Hydrogen . .	20	5.36	5.31	
Nitrogen . .	5	18.76	19.00	

The acid sulphate gives no precipitate with $3KCy,Co_2Cy_3$, but only a fine red liquid, which, on evaporation, yields a red mass. Boiled with chlorhydric acid the sulphate yields the chloride of Purpureocobalt, easily recognized by oxalate of ammonia, with which, however, the acid sulphate itself gives no precipitate. When precipitated with nitrate of baryta the acid sulphate yields a red liquid which probably contains a nitrate of Purpureocobalt, but which on evaporation gives crystals of nitrate of Roseocobalt. It is well worthy of notice, that this red liquid contains a large quantity of sulphate of baryta in solution, which it deposits during evaporation.

The products of the decomposition of the acid sulphate are similar to those of the other salts of Purpureocobalt. A rapid current of NO_x passed into the solution gives, after a short time, an abundant precipitate of the nitrate of Xanthocobalt.

The constitution of the acid sulphate might be represented by either of the following formulæ, besides that already given:

$$5NH_3.Co_2O_3,3SO_3+HO,SO_3+4HO$$
$$5NH_3.Co_2O_3,2SO_3+2HO,SO_3+3HO.$$

We reject the first of these formulæ because Purpureocobalt is a biacid and not a triacid base. The second formula appears to us less probable than that which we have adopted, in the first place, because a salt so constituted ought to be strongly acid, and in the second place, because we shall presently show that there exists an oxalo-sulphate of Purpureocobalt, in which *two* equivalents of sulphuric are replaced by *two* of oxalic acid, and another and neutral oxalo-sulphate in which one equivalent of oxalic acid replaces one equivalent of sulphuric acid.

ACID OXALO-SULPHATE OF PURPUREOCOBALT.

When sulphate of Roseocobalt is boiled for several hours with an excess of a solution of oxalic acid, a clear red solution is formed, which on evaporation deposits an abundance of crystals of a bright brick-red color, and indistinct acicular form. These crystals are soluble in hot water without decomposition, and may be purified, though with difficulty, by recrystallization. Their constitution is represented by the formula

$$5NH_3.Co_2O_3,2SO_3,2C_2O_3+3HO$$

as appears from the following analyses:

0.7433 grs. gave 0.3315 grs. sulphate of cobalt = 16.97 per cent. cobalt.
1.3912 grs. gave 0.9535 grs. sulphate of baryta = 23.50 " sulphuric acid.
1.6895 grs. gave 1.1564 grs. " " = 23.49 " sulphuric acid.
2.7702 grs. gave 0.7070 grs. carbonic acid = 20.88 " oxalic acid.
2.0198 grs. gave 340 c. c. of nitrogen at 14°.5 C. and $763^{mm}.01$ (at 15° C.) = 318.1 c. c. at 0° and 760^{mm} = 19.78 per cent. nitrogen.

The formula requires

	Eqs.	Calculated.	Found.	
Cobalt . . .	2	17.00	16.97	
Sulphuric acid .	2	23.05	23.49	23.50
Oxalic acid . .	2	20.74	20.88	
Nitrogen . .	5	20.17	19.78	

The reactions of this remarkable salt resemble closely those of the acid sulphate. It has an acid taste and reaction, gives no precipitate with oxalate of ammonia, or cobaltidcyanide of potassium, and yields chloride of Purpureocobalt by boiling with an excess of chlorhydric acid. The formula of this salt may be written in various ways. In the first place, we may consider it as a double salt represented by the formula

$$5NH_3.Co_2O_3,4SO_3+5NH_3.Co_2O_3,4C_2O_3+6HO.$$

The advantage of simplicity is evidently in favor of the formula we have adopted. We may also consider it as represented by

$$5NH_3.Co_2O_3,2SO_3+2C_2O_3,HO+HO.$$

In this case the salt should have a strongly acid taste which it has not. On the whole the formula

$$5NH_3.Co_2O_3\left\{\begin{matrix}2SO_3\\2C_2O_3\end{matrix}\right.+3HO$$

appears to deserve the preference.

NEUTRAL OXALO-SULPHATE OF PURPUREOCOBALT.

When ammonia is added to a solution of the acid oxalo-sulphate just described, a fine violet-red color is produced, and if no more ammonia be added than is sufficient to completely neutralize the acid reaction, the liquid yields, on evaporation, beautiful red prismatic crystals of a neutral salt. The neutral oxalo-sulphate is much less soluble in water than the acid salt, and has a purely saline taste: it is easily decomposed by boiling. The formation of this salt is represented by the equation

$$5NH_3.Co_2O_3,2SO_3,2C_2O_3+2NH_4O=5NH_3.Co_2O_3,SO_3,C_2O_3+NH_4O,SO_3+NH_4O,C_2O_3.$$

The fact that the ammonia unites with both sulphuric and oxalic acid, and not simply with *two* equivalents of oxalic acid, throws much light on the constitution of the acid oxalo-sulphate, and shows, we think, clearly that the formula of this salt cannot be

$$5NH_3.Co_2O_3,2SO_3+2C_2O_3,HO+HO.$$

The constitution of the neutral oxalo-sulphate is represented by the formula

$$5NH_3.Co_2O_3,\left\{\begin{matrix}SO_3\\C_2O_3\end{matrix}\right.+7HO$$

as appears from the following analyses:

0.6367 grs. gave 0.3217 grs. sulphate of cobalt = 19.23 per cent. cobalt.

0.6721 grs. gave 0.2569 grs. sulphate of baryta = 13.12 per cent. sulphuric acid.

0.9760 grs. gave 191 c. c. nitrogen at 17°.25 C. and $767^{mm}.58$ (at 17°.8) = 177.43 c. c. at 0° and 760^{mm} = 22.83 per cent.

The formula requires

	Eqs.	Calculated.	Found.
Cobalt . . .	2	19.21	19.23
Sulphuric acid .	1	13.02	13.12
Oxalic acid . .	1	11.72	
Nitrogen . .	5	22.80	22.83

We may further remark that the character of the action of ammonia upon the acid oxalo-sulphate leads us to hope that the neutral sulphate of Purpureocobalt may be obtained by the action of this agent upon the acid sulphate, and that in fact, this is the salt already mentioned as so obtained, but not yet analyzed. The two oxalo-sulphates described constitute, we believe, the types of an entirely new class of salts, and lead to the idea that sulphuric and oxalic acids may possibly be capable of replacing each other in other combinations.

OXIDE OF PURPUREOCOBALT.

The oxide of Purpureocobalt, like that of Roseocobalt, appears to exist only in solution. It may be prepared, either by decomposing the acid sulphate by baryta water, or by digesting a solution of the chloride with oxide of silver in the cold. The solution is not pure in either case, containing either sulphate of baryta or chloride of silver in solution. The oxide, as thus prepared in solution, forms a violet-red liquid, which absorbs carbonic acid readily from the air, and which is decomposed by concentration.

The constitution of the oxygen salts of Purpureocobalt, which we have described, as well as that of the chlorplatinate of this radical, appears to us to leave no reasonable doubt that the oxide is essentially *biacid*. According to the rule that the number of equivalents of acid in a salt is equal to the number of equivalents of oxygen in the base, the rational constitution of the oxide of Purpureocobalt will be expressed by the formula

$$5NH_3.Co_2O.O_2.$$

We shall develop this view more fully when occupied with the purely theoretical portion of the subject, and in the second part of our memoir we shall endeavor, by the analysis and description of other salts of Purpureocobalt, to throw more light upon the nature of this remarkable radical. The chromates, pyrophosphate, and picrate of Purpureocobalt have, in particular, occupied our attention.

We have mentioned, in speaking of the reactions of chloride of Purpureocobalt, that both the cobaltidcyanide and the ferridcyanide of potassium give precipitates in its solution. The constitution, crystalline form and physical appearance of these two precipitates exactly agree with those of the cobaltidcyanide and ferridcyanide of Roseocobalt, and we have, therefore, not hesitated to identify them with these last. We believe that in this case there is a conversion of Purpureocobalt into Roseocobalt, since in the salts in question there are three equivalents of cyanogen in the electropositive for three in the electronegative cyanide, the formulæ being as mentioned above

$$5NH_3.Co_2Cy_3+Co_2Cy_3+3HO, \text{ and } 5NH_3.Co_2Cy_3+Fe_2Cy_3+3HO.$$

As Purpureocobalt is certainly biacid, its cobaltidcyanide and ferridcyanide should have the formulæ

$$3(5NH_3.Co_2Cy_3)+2Co_2Cy_3, \text{ and } 3(5NH_3.Co_2Cy_3)+2Fe_2Cy_3,$$

although the frequent occurrence of basic double cyanides may render this point less clear than the others which also involve the biacid character of the radical.

LUTEOCOBALT.

The salts of Luteocobalt have a yellow or brown-yellow color, and are almost always well crystallized. They are in general more soluble in water than the corresponding salts of Roseocobalt; the solutions have a brown-yellow color. The salts of Luteocobalt are very stable in the presence of acids in general, but are decomposed by long heating with sulphuric acid. The neutral and alkaline solutions are readily decomposed by boiling, like the salts of the other cobalt bases. Nearly all of them have a purely saline taste. When hydrated, these salts generally effloresce in dry air or in vacuo, and become opaque, with a peculiar porcelain-like lustre and reddish-buff color. The salts of Luteocobalt may be formed, like those of the other bases described, by direct oxidation: it is well worthy of notice, however, that they are often found among the products of the decomposition of the salts of Roseocobalt and Purpureocobalt. This is especially remarkable, because the constitution of Roseocobalt is simpler than that of Luteocobalt, the former base being $5NH_3.Co_2O_3$, while the latter is $6NH_3.Co_2O_3$. We have here a singular inversion of the usual law, that the products of the decomposition of a complex molecule are more simple in constitution than the body decomposed.

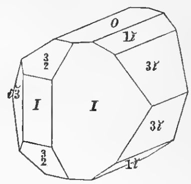
Luteocobalt, like Roseocobalt, is a triacid base.

CHLORIDE OF LUTEOCOBALT.

When an ammoniacal solution of chloride of cobalt, to which a large quantity of coarsely powdered chloride of ammonium has been added, is exposed to the air for some days, it often happens that no traces of chloride of Roseocobalt or Purpureocobalt are found, but the bottom of the vessel becomes covered with orange-yellow crystals, which are the chloride of Luteocobalt. Chlorhydric acid precipitates an additional quantity of the salt from the supernatant liquid. The raw chloride, as thus obtained, is easily purified by solution in hot water, filtration, and repeated crystallization. This method of preparing the salt is by no means always successful, and very frequently results only in the formation of chloride of Roseocobalt and Purpureocobalt, with scarcely a trace of the chloride of Luteocobalt. We have, however, almost invariably succeeded in preparing, by this process, a mixture of the sulphate and chloride of Luteocobalt, by employing a solution containing both the chloride and sulphate of cobalt. The sulphato-chloride resulting, by boiling with chlorhydric acid and chloride of barium, yields a solution from which the pure chloride may be obtained by repeated crystallization. The chloride of Luteocobalt crystallizes by slow evaporation, in remarkably beautiful brownish-orange colored crystals, which belong to the trimetric or right rhombic system, and which are isomorphous with the sulphate of Luteocobalt. According to Prof. Dana, the usual forms are, in his modification of Naumann's notation, $O, \infty, \frac{\frac{3}{2}}{2}, \frac{\infty - \breve{3}}{2}, 1 - \breve{\infty}, 8 - \breve{\infty},$

with the angle I : I = 113°. 16′. Fig. 9 represents a crystal of this salt with Dana's notation for the faces:

Fig. 9.



I : I = 113° 16′

O : 1ĭ = 145° 55′

O : 3ĭ = 116° 13′

1ĭ : 1ĭ = 112° 2′ (over O)

3ĭ : 3ĭ = 52° 26′ (over O)

3ĭ : 3ĭ = 127° 34′ (adjacent)

O : $\frac{3}{2}$ = 118° 35′ (by observation)

Frémy states that this salt crystallizes in regular octahedrons; in this case it must be dimorphous, but we have never observed any forms belonging to the regular system.

The chloride of Luteocobalt is readily soluble in boiling water, and crystallizes in a great measure from the solution on cooling. Chlorhydric acid and alkaline chlorides precipitate it unchanged. When boiled with sulphuric acid, the salt gives off abundance of chlorhydric acid gas, but it is difficult to drive off all the acid without decomposing a portion of the resulting sulphate. The salt is slowly decomposed by boiling ammonia, chloride of ammonium, and a dark brown oxide of cobalt being the only products of the decomposition which we have been able to detect. Reducing agents in general act upon this salt as upon chloride of Roseocobalt and Purpureocobalt. We have not yet, however, been able to obtain with the chloride of Luteocobalt compounds analogous to those which are produced by the action of sulphurous acid and deutoxide of nitrogen upon the chlorides of Roseocobalt and Purpureocobalt, although we have repeatedly made the attempt.

The chloride of Luteocobalt is dichrous. In the dichroscopic lens the ordinary image is pale violet, while the extraordinary image is orange-violet. The color of the salt, in coarse powder, approaches the orange-yellow of the first circle, but the color of the mass of crystals could not be defined by the chromatic scale, which we employed. Chloride of Luteocobalt exhibits a remarkable tendency to form chloro-salts with metallic chlorides. These salts are formed with great ease, by the direct union of the two chlorides, and are worthy of notice for their stability and capacity of crystallization. Of these salts, which are very numerous, we have examined only the compounds with gold and platinum.

The analyses of chloride of Luteocobalt lead to the formula

$$6NH_3.Co_2Cl_3,$$

0.2036 grs. gave 0.1180 grs.	sulphate of cobalt	=	22.05	per cent.	cobalt.
0.3350 grs. gave 0.1938 grs.	" "	=	22.01	"	"
0.5110 grs. gave 0.2970 grs.	" "	=	22.11	"	"
0.3335 grs. gave 0.1930 grs.	" "	=	22.02	"	"
0.2942 grs. gave 0.4723 grs.	chloride of silver	=	39.67	"	chlorine.
0.4886 grs. gave 0.7846 grs.	" "	=	39.78	"	"
0.3902 grs. gave 0.2346 grs.	water	=	6.68	"	hydrogen.
0.4617 grs. gave 0.2800 grs.	"	=	6.73	"	"

0.7305 grs. gave 200.5 c. c. of nitrogen at 21°.5 C. and $765^{mm}.80$ (t = 22°.2 C.) = 182.3 c. c. at 0° and 760^{mm} = 31.34 per cent. nitrogen.

0.7773 grs. gave 212 c. c. of nitrogen at 18° C. and $762^{mm}.75$ (t = 18°.6 C.) = 194.94 c. c. at 0° and 760^{mm} = 31.49 per cent. nitrogen.

Comparing these with the calculated results, we have

	Eqs.		Theory.	Mean.	Found.			
Cobalt .	2	59.0	22.06	22 05	22.05	22.01	22.11	22.02
Chlorine .	3	106.5	39.79	39.73	39.68	39.78	—	—
Hydrogen .	18	18.0	6.73	6.70	6.68	6.73	—	—
Nitrogen .	6	84.0	31.42	31.41	31.49	31.34	—	—
		267.5	100.00	99.89				

The formula $6NH_3.Co_2Cl_3$ is given, by both Frémy and Rogojski, and no reasonable doubt can be entertained of its accuracy. The density of the chloride of Luteocobalt, as taken in alcohol, is 1.7016 at 20° C., its atomic volume is consequently 157.2.

The reactions of the chloride of Luteocobalt are as follows:

Iodide of potassium gives a bright yellow precipitate.

Bromide of potassium gives a less brilliant yellow precipitate.

Ferrocyanide of potassium gives a chamois colored precipitate, which becomes black on boiling.

Ferridcyanide of potassium gives beautiful yellow needles, which are nearly insoluble.

Cobaltidcyanide of potassium gives a pale fawn colored precipitate of fine needles.

Terchloride of gold gives bright yellow granular crystals of the chloraurate.

Bichloride of platinum gives yellow or orange-yellow needles of the chlorplatinate.

Chromate of potash gives a bright yellow precipitate of the chromate.

Oxalate of ammonia gives a buff yellow precipitate, soluble in oxalic acid.

Tribasic phosphate of soda gives, after a short time, a yellow precipitate.

Pyrophosphate of soda gives a pale buff colored precipitate.

Picrate of ammonia gives a beautiful yellow precipitate of very fine silky needles.

Alkalies and their carbonates produce no precipitate in the cold.

Sulphide of ammonium gives a black precipitate.

CHLORPLATINATE OF LUTEOCOBALT.

Chloride of platinum produces, immediately, in a solution of the chloride of Luteocobalt, a beautiful orange or yellow precipitate of the chlorplatinate. When the solutions employed are concentrated, the precipitate is orange colored; when the solutions are dilute, yellow needles are thrown down. The difference is here only in the quantity of water of crystallization, and the orange granular crystals may be converted into the pale yellow needles by solution in a large quantity of hot water and recrystallization.

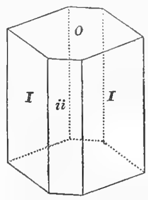
According to Prof. Dana's measurements, the acicular crystals belong to the monoclinic system, so far as it is possible to determine. The crystals are usually hollow and much striated longitudinally. The observed forms are I, *ii* and O, and the angles

Fig. 10.

I : I = 107° 10′

I : *ii* = 143° 50′

O : *ii* = 114° 15′



Twin crystals are frequent, the composition being parallel to the plane *O*. The salt is very slightly soluble in cold water, but dissolves in much boiling water, from which it separates on cooling. When gently heated in a porcelain crucible it gives off ammonia and chloride of ammonium, and becomes green. The green mass, on solution in water, gives globular aggregations of minute crystals of a buff color, which may be a new salt, but which we have not specially examined. Zinc decomposes the chlorplatinate of Luteocobalt only by very long boiling in an acid solution, metallic platinum being separated as a black powder, while chlorides of cobalt and ammonium are formed.

The formula of the orange salt is

$$6NH_3.Co_2Cl_3+3PtCl_2+6HO$$

as the following analyses show:

1.220 grs. gave 0.4321 grs. metallic platinum = 35.41 per cent.
1.220 grs. gave 0.2261 grs. sulphate of cobalt = 7.05 per cent. cobalt.

	Eqs.	Calculated.	Found.
Cobalt . . .	2	7.10	7.05
Platinum . .	2	35.64	35.41

The formula of the yellow salt is

$$6NH_3.Co_2Cl_3+3PtCl_2+21HO$$

as appears from the analyses:

0.2638 grs. gave 0.0822 grs. metallic platinum = 31.16 per cent.
0.4449 grs. gave 0.6037 grs. chloride of silver = 33.54 per cent. chlorine.

	Eqs.	Calculated.	Found.
Platinum . .	3	30.99	31.16
Chlorine . . .	9	33.42	33.54

Rogojski found in this salt but one and a half equivalents of water, but his analyses are not very satisfactory, giving a large excess of platinum, hydrogen, and cobalt.

CHLORAURATE OF LUTEOCOBALT.

A solution of terchloride of gold produces immediately in solutions of the chloride of Luteocobalt a beautiful yellow precipitate of small granular crystals. These crystals are very insoluble in cold water, but more readily soluble in boiling water acidulated with chlorhydric acid. Reducing agents separate gold with full metallic lustre. The formula of this salt is

$$6NH_3.Co_2Cl_3+AuCl_3$$

as the analyses satisfactorily show:

0.7308 grs. gave 0.1025 grs. Co_6O_7 = 10.53 per cent. cobalt.
0.7308 grs. gave 0.2530 grs. gold = 34.62 per cent.
0.6457 grs. gave 0.9714 grs. chloride of silver = 37.36 per cent of chlorine.

	Eqs.	Calculated.	Found.
Cobalt . . .	2	10.33	10.53
Gold . . .	1	34.50	34.62
Chlorine . . .	6	37.30	37.36

IODIDE OF LUTEOCOBALT.

Iodide of potassium produces immediately in solutions of the chloride, sulphate, or nitrate of Luteocobalt, a remarkably beautiful bright yellow precipitate of the iodide of Luteocobalt. This precipitate is rather insoluble in cold water, but readily soluble in hot water. The solution yields by spontaneous evaporation brown-yellow crystals, which appear to have the same form as the chloride.

0.2224 grs. of this salt gave 0.06308 grs. sulphate of cobalt, corresponding to 10.79 per cent. cobalt.

The formula $6NH_3.Co_2I_3$ requires 10.88 per cent. cobalt.

The color of the precipitated and dried iodide is very fine, and its brilliancy led us to hope that it might be advantageously employed as a pigment. On trial, however, the color was found wanting in body; the yellow, moreover, changes to a brown-yellow when the powder is ground in oil or water.

BROMIDE OF LUTEOCOBALT.

Bromide of potassium gives a rather dull yellow precipitate in solutions of Luteocobalt. The precipitate, re-dissolved in hot water, gives, on slow evaporation, wine-yellow crystals of the bromide. These crystals have the same form as those of the chloride, and their formula is therefore

$$6NH_3.Co_2Br_3.$$

COBALTIDCYANIDE OF LUTEOCOBALT.

Cobaltidcyanide of potassium produces in solutions of Luteocobalt a pale yellowish flesh-colored precipitate of the double cyanide of cobalt and Luteocobalt. The salt is insoluble in cold water, and easily decomposed by boiling water. It cannot, therefore, be re-crystallized for analysis. Under the microscope the crystals are seen to belong to the oblique rhombic system; they are too small to admit of accurate measurement. The formula of this salt is

$$6NH_3.Co_2Cy_3+Co_2Cy_3+HO.$$

0.4835 grs. gave 0.3923 grs. sulphate of cobalt = 30.88 per cent. cobalt.
0.7652 grs. gave 0.3580 grs. water = 5.19 per cent. hydrogen.
2.1845 grs. gave 0.6800 grs. carbonic acid = 18.82 per cent. carbon.

	Eqs.	Calculated.	Found.
Cobalt . . .	4	30.57	30.88
Carbon . . .	12	18.70	18.82
Hydrogen . .	19	4.93	5.19

A solution of ferridcyanide of potassium produces a most beautiful precipitate of orange-yellow needles in solutions of Luteocobalt. These, under the microscope, have the same form as the corresponding cobalt salt, and their formula is therefore

$$6NH_3.Co_2Cy_3+Fe_2Cy_3+HO.$$

SULPHATE OF LUTEOCOBALT.

The sulphate of Luteocobalt is easily procured mixed with the chloride, when solutions of both chloride and sulphate of cobalt are rendered ammoniacal and exposed to the air after the addition of coarsely powdered chloride of ammonium in large excess. The mass of yellow crystals formed upon the bottom of the vessel, after a few days, is a mixture of the two salts. To obtain the sulphate from this mass, the solution in hot water is to be filtered and digested with sulphate of silver, after addition of a few drops of sulphuric acid. In this manner the whole of the chloride may be decomposed, and the filtered solution on evaporation will yield fine crystals of the sulphate. We have frequently prepared large quantities of the sulphate by this method. Another mode of preparing the sulphate of Luteocobalt, which is often very convenient, consists in pouring ammonia upon the sulphate of Roseocobalt, thrown down by cautious addition of sulphuric acid to perfectly oxidized solutions of the ammoniacal sulphate of cobalt. When this sulphate is powdered, and strong ammonia poured upon it, its color frequently changes from red to a dull buff, while the supernatant liquid takes a fine red color. The buff powder on solution in hot water and evaporation yields crystals of sulphate of Luteocobalt. The red liquid is merely a solution of sulphate of

Roseocobalt in ammonia. The reaction which takes place in this case may be represented by the equation

$$5NH_3.C_2O_3,3SO_3+NH_3=6NH_3.Co_2O_3,3SO_3,$$

the sulphate of Roseocobalt simply absorbing one equivalent of ammonia. The quantity of sulphate of Roseocobalt dissolved in the ammonia is very variable, being sometimes extremely small. In other cases, however, no sulphate of Luteocobalt is formed, but only a solution of sulphate of Roseocobalt in ammonia, from which, by evaporation, the sulphate crystallizes unchanged in large dark-red crystals, frequently of the form represented in Fig. 3. We are unable to assign a satisfactory reason for the capriciousness of the behavior of the red sulphate towards ammonia.

Frémy asserts that sulphuric acid, continuously added to a completely oxidized ammoniacal solution of sulphate of cobalt, throws down an acid sulphate of Roseocobalt to which he assigns the formula $5NH_3.Co_2O_3,5SO_3+5HO$. When this acid sulphate is boiled for a few minutes with ammonia, a yellow precipitate of sulphate of Luteocobalt is thrown down. The author does not attempt to explain the reaction which takes place in this case, but states that the red mother liquor from which the sulphate of Luteocobalt has separated, yields on evaporation crystals of the neutral sulphate of Roseocobalt. We have never succeeded in preparing an acid sulphate of Roseocobalt by the process above mentioned, nor by any other. On the contrary, we have uniformly found that sulphuric acid precipitates from the oxidized solution only the neutral sulphate of Roseocobalt in small bright-red crystals, easily recognized by their form. Frémy's salt must have contained free sulphuric acid, in consequence of imperfect washing.

When a pure solution of the sulphate of Roseocobalt is boiled, ammonia is evolved, while sulphate of ammonia and sulphate of Luteocobalt remain in solution, and a dark-colored oxide of cobalt is precipitated. From the solution the sulphate of Luteocobalt may be obtained by evaporation and crystallization. This method yields but little, and is not to be recommended.

Sulphate of Luteocobalt is also sometimes obtained, with other products, by digesting sulphate of Roseocobalt with sulphuric acid, before the period of complete decomposition sets in.

A very simple and easy method of preparing the sulphate of Luteocobalt consists in decomposing the dry sulphate of Roseocobalt by heat. When the latter salt is gently heated in a porcelain crucible over a spirit lamp, or better still, in a glass flask in a bath of rosin oil to about the temperature of melting lead, ammonia is given off in abundance, and the mass, which should be constantly stirred, assumes a fine purple-lilac hue. The heat, when the lamp alone is used, must never rise to low redness, and no vapors of sulphate of ammonia should be given off. The resulting mass is then to be dissolved in hot water, which gives a fine purple red solution, and chlorhydric acid added in excess. An orange precipitate of sulphatochloride of Luteocobalt is immediately thrown down, which is easily purified, as above, by sulphate of silver and recrystallization. The acid mother liquor sometimes deposits more sulphate on cooling. The supernatant liquid contains chloride of Luteocobalt, chloride of Purpureocobalt, and a leek-green crystalline body, which we have called provisionally Praseocobalt, but which we have not yet carefully studied.

The sulphate of Luteocobalt, like the chloride, has a fine wine-yellow color, and crystallizes readily. The crystals belong to the right rhombic or trimetric system; they are hemihedral and isomorphous with the chloride of Luteocobalt. According to Prof. Dana's determinations, the more usual forms are represented in Figs. 11, 12, 13, 14. In Figs. 13 and 14 the sulphate is mixed with the chloride.

$I : I = 113° 38'$	$O : \frac{3}{4} = 137° 19'$	$i\breve{\tfrac{3}{2}} : i\breve{\tfrac{3}{2}} = 88° 44'$ and $91° 16'$
$O : 1\breve{i} = 146° 4'$	$O : \frac{3}{2} = 118° 28'$	$1\bar{i} : 1\bar{i}$ (over O) $= 88° 22'$
$1\breve{i} : 1\breve{i} = 112° 8'$ (over O)	$O : 3\bar{i} = 107° 57'$	
$3\breve{i} : 3\breve{i} = 127° 18'$ (adjacent)	$O : 1\bar{i} = 134° 11'$	

These forms in other symbols are $O, \infty, 3\text{-}\bar{\infty}, \frac{\frac{3}{2}}{2}, \frac{\frac{3}{4}}{2}, 1\text{-}\breve{\infty}$, (Fig. 11). $O, \infty, \frac{\frac{3}{2}}{2}, \frac{\frac{3}{4}}{2}, 1\text{-}\breve{\infty}, 3\text{-}\breve{\infty}$ (Fig. 12). $1\text{-}\bar{\infty}, \infty\text{-}\breve{\tfrac{3}{2}}, 3\text{-}\breve{\infty}$, (Fig. 13). $1\text{-}\bar{\infty}, \infty\text{-}\breve{\tfrac{3}{2}}, 3\text{-}\breve{\infty}, 1\text{-}\infty$ (Fig. 14).

$$a : b : c = 1{,}039 : 1 : 1{,}539.$$

Fig. 11.

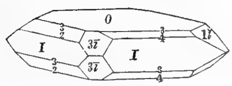


Fig. 12.

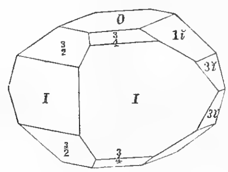


Fig. 13.

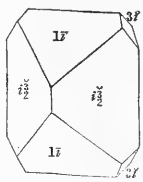
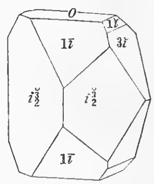


Fig. 14.



Figs. 15, 16, 17, are different in habit from the preceding, and do not agree precisely in angles. The forms as lettered are referred to a different fundamental form. Adopting the same fundamental form as in the above figures, the lettering would be as follows:

Lettering on figures,	O	$i\breve{2}$	$1\breve{2}$	$\frac{1}{2}$	$1\bar{i}$	I	$i\bar{i}$	$\frac{1}{2}\bar{i}$	$\frac{1}{4}$	$\frac{3}{4}$.
New lettering,	O	I	$\frac{3}{2}$	$\frac{3}{2}\breve{2}$	$3\breve{i}$	$i\breve{2}$	$i\breve{i}$	$\frac{3}{2}\breve{i}$	$\frac{3}{4}\breve{2}$	$\frac{9}{4}\breve{2}$.

Angles obtained and calculated for Fig. 13 (putting the lettering on the figure in brackets):

$I : I$ $(i\breve{2} : i\breve{2}) = 64° 28'$ and $115° 32'$	$\frac{3}{2}\breve{2} : \frac{3}{2}\breve{2}$, $(\frac{1}{2} : \frac{1}{2}) = 124° 3'$ (adjacent).
$O : \frac{3}{2}$ $(O : 1\breve{2}) = 120° 36'$	$O : 3\breve{i}$ $(O : 1\bar{i}) = 119°$.
$O : \frac{3}{2}\breve{2}$ $(O : \frac{1}{2}) = 130° 59'$	$i\breve{2} : i\breve{2} = 103° 10'$ (by calculation).

Angles obtained and calculated for Figs. 14, 15:

$i\breve{2} : i\breve{2}$ $(I : I) = 103° 30'$	$O : \frac{3}{4}\breve{2}$ $(O : \frac{1}{4}) = 150° 16'$.
$O : \frac{8}{2}$ $(O : 1\breve{2}) = 120° 40'$	$O : \frac{9}{4}\breve{2}$ $(O : \frac{3}{4}) = 120° 16'$.
$O : \frac{3}{2}\breve{i}$ $(O : \frac{1}{2}\bar{i}) = 138° 6'$.	

Fig. 18 has still a different habit. The occurring vertical prism, lettered I, gave the angle (approximately) 101° 30′, and the dome $1\breve{i}$ has the angle 109° 36′, giving $O : 1\breve{i} = 144° 48'$, near the angle in Figs. 11, 12.

Fig. 15.

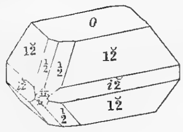


Fig. 16.

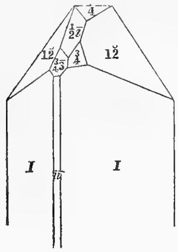


Fig. 17.

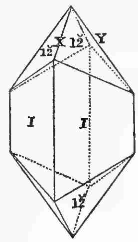
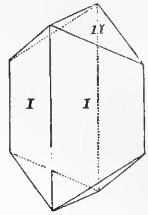


Fig. 18.



The sulphate is rather insoluble in cold but is freely soluble in hot water; the dilute solution is yellow, the concentrated solution dark sherry wine colored. By double decomposition with salts of barium it yields the other salts of Luteocobalt. The sulphate like the chloride is dichrous, the ordinary image being pale rose-red while the extraordinary image is bright orange. The color of the salt in coarse powder approaches the orange No. 5 of the first circle.

Sulphuric acid does not precipitate this salt from its solution, but chlorhydric and nitric acids throw down in the cold mixtures of the chloride with the sulphate and nitrate. The salt is decomposed with very great difficulty by long boiling, even after the addition of a little ammonia. No new base is formed during the decomposition. When, however, the dry salt is gently heated in a porcelain crucible, ammonia is evolved, and if the heat be regulated so that no sulphate of ammonia is given off, while the mass is constantly stirred, there remains after a few minutes a red mass, which on solution in water gives a fine red liquid containing a sulphate of a red base, which is probably Purpureocobalt. The reaction is, however, a very uncertain one, and has succeeded in our hands but once. We have in most cases obtained by the process described only a mixture of sulphate of Luteocobalt, sulphate of cobalt, and sulphate of ammonia. We shall consider this subject more fully hereafter. Sulphuric acid, if not too dilute, readily decomposes the sulphate of Luteocobalt when the solution is heated. It appears probable that there exists an acid sulphate of this base, as there is an acid carbonate, but we have not been able to obtain it as yet.

Sulphate of Luteocobalt has the formula

$$6NH_3.Co_2O_3,3SO_3+5HO$$

as the following analyses satisfactorily show. The salt analyzed was dried by pressure between folds of bibulous paper only.

0.3618 grs. gave 0.1600 grs. sulphate of cobalt = 16.83 per cent. cobalt.
0.4993 grs. " 0.2205 grs. " " = 16.80 " "
0.4790 grs. " 0.2122 grs. " " = 16.85 " "
1.2023 grs. " 1.2020 grs. sulphate of baryta = 34.32 per cent. sulphuric acid.
0.8203 grs. " 0.8250 grs. " " = 34.52 " " "
0.9355 grs. " 0.5650 grs. water = 6.71 per cent. hydrogen.
1.1194 grs. " 0.6722 grs. " = 6.67 " "

1.0005 grs. gave 205 c. c. nitrogen at 12°.5 C. and 753^{mm}.61 (at 12°.7) = 191.16 c. c. at 0° and 760^{mm} = 24.00 per cent. nitrogen.

0.9018 grs. gave 185.5 c. c. nitrogen at 19° C. and 763^{mm}.36 (at 19°.5) = 171.26 c. c. at 0° and 760^{mm} = 23.85 per cent. nitrogen.

Our formula requires

	Eqs.		Calculated.	Mean.	Found.		
Cobalt . .	2	59.0	16.85	16.83	16.80	16.85	16.83
Sulph. acid .	3	120.0	34.28	34.42	34.52	34.32	
Hydrogen .	23	23.0	6.57	6.69	6.71	6.67	
Nitrogen .	6	84.0	24.00	23.97	24.00	23.85	
Oxygen . .	8	64.0	18.28	18.09	—	—	
		350.0	100.00				

According to Frémy, the sulphate contains but four equivalents of water of crystallization. In vacuo or in dry air the sulphate of Luteocobalt effloresces, becomes opaque and reddish buff colored, and loses 4 eqs. or 10.13 per cent. of water.

Rogojski did not succeed in obtaining the sulphate of Luteocobalt by decomposing the chloride with sulphate of silver. According to this chemist, there is produced under these circumstances a sulphato-chloride which has the formula

$$6NH_3.Co_2O_3,3SO_3+6NH_3.Co_2Cl_3.$$

We have already mentioned, however, that the chloride and sulphate of Luteocobalt are isomorphous, and we have accordingly found, as might be expected, that these two salts are capable of crystallizing together in all proportions, and cannot be separated by crystallization alone. To show the variation in the constitution of the mixed chloride and sulphate, it will be sufficient to give a few cobalt determinations made with the salt as prepared at different times.

0.1510 grs. gave 0.0673 grs. sulphate of cobalt = 16.96 per cent. cobalt.
0.7075 grs. " 0.3210 grs. " " = 17.26 " "
0.1205 grs. " 0.0680 grs. " " = 21.47 " "

The parallel which Rogojski draws between the salt which he analyzed and the sulphate of Gros's base, which Gerhardt considers as a sulphato-chloride of Diplatinamin, must therefore be considered as illusory.

CHROMATE OF LUTEOCOBALT.

A solution of the neutral chromate of potash gives a fine yellow precipitate in solutions of the chloride, nitrate, and sulphate of Luteocobalt. The precipitate is soluble in hot water, and crystallizes readily from the solution in brown-yellow crystals, which resemble those of the sulphate. We have not analyzed this salt, but it is almost certain that its true formula is

$$6NH_3.Co_2O_3,3CrO_3+5HO,$$

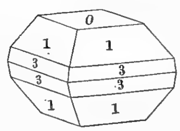
since it forms with chloride of Luteocobalt crystallizable mixtures in various proportions, which exhibit in the greatest beauty and distinctness the characteristic forms of the crystals of the sulphato-chlorides above alluded to. The pure chromate can only be obtained by precipitating the nitrate of Luteocobalt by chromate of potash, as the precipitate from the chloride always contains chlorine, and that from the sulphate, sulphuric acid.

NITRATE OF LUTEOCOBALT.

This beautiful salt is almost invariably obtained during the oxidation of an ammoniacal solution of nitrate of cobalt, and is deposited upon the bottom of the vessel in bright orange crystalline scales. The supernatant liquid is usually red, and contains nitrate of Roseocobalt. The orange-yellow salt is easily purified by re-crystallization. The salt may also be easily prepared from the chloride or sulphate by double decomposition with nitrate of silver or of baryta. The nitrate of Luteocobalt crystallizes readily in forms which belong to the square prismatic or dimetric system. According to Professor Dana, the dimensions and angles of the crystals are as follows:

1 : 1 (over the base) = 110° 20′
O : 1 = 124° 50′
O : 3 = 103° 4′
3 : 3 (over the base) = 153° 52′
a = 1.0161
O : i (not observed) = 134° 33′

FIG. 19.



The crystals are usually small and often very brilliant. The salt is readily soluble in hot water, and separates in small crystals on cooling. Chlorhydric acid throws it down from its solution as a yellow crystalline powder; nitric acid also precipitates it, but sulphuric acid converts it into sulphate with more or less complete decomposition. The nitrate of Luteocobalt is anhydrous, and has the formula

$$6NH_3.Co_2O_3,3NO_5$$

as the following analyses show:

0.1972 grs. gave 0.0880 grs. sulphate of cobalt = 16.98 per cent. cobalt.
0.2090 grs. " 0.0928 grs. " " = 16.87 " "
1.0859 grs. " 0.5151 grs. water = 5.27 per cent. hydrogen.
0.9126 grs. " 0.4337 grs. " = 5.28 " "
0.6242 grs. gave 188 c. c. at 11°.5 C. and 772^{mm}.40 at 11°.94 = 180.56 c. c. at 0° and 760^{mm} = 36.33 per cent. nitrogen.
0.7394 grs. gave 226 c. c. at 13°.5 C. and 766^{mm}.05 at 14° = 213.29 c. c. at 0° and 760^{mm} = 36.23 per cent. nitrogen.

The formula requires

	Eqs.	Calculated.	Mean.	Found.	
Cobalt . . .	2	17.00	16.93	16.98	16.89
Hydrogen . .	18	5.18	5.27	5.27	5.28
Nitrogen . .	9	36.31	37.28	36.23	36.33

Frémy and Rogojski deduce the same formula from very imperfect analyses. Heat decomposes the dry nitrate of Luteocobalt with a slight explosion, a black powder of an oxide of cobalt remaining. It may be remarked that the oxygen and hydrogen in this salt are exactly in the ratio to form water.

OXALATE OF LUTEOCOBALT.

When a solution of oxalate of ammonia is added to one of a soluble salt of Luteocobalt, a buff colored precipitate of fine needles is thrown down, which is insoluble both in hot and cold water, but which readily dissolves in a solution of oxalic acid. From this solution the neutral oxalate crystallizes in beautiful prismatic crystals, having the color of the sulphate and chloride. In dry air the crystals lose water like those of the other hydrated salts of Luteocobalt. The oxalate has the formula

$$6NH_3.Co_2O_3,3C_2O_3+4HO$$

as the following analyses show:

0.4330 grs. gave 0.2040 grs. sulphate of cobalt	= 17.99 per cent. cobalt.		
0.4228 grs. gave 0.2000 grs. " "	= 18.00 " "		
0.5345 grs. gave 0.2529 grs. " "	= 18.01 " "		
2.0805 grs. gave 0.8380 grs. carbonic acid	= 32.95 per cent. oxalic acid.		

The formula requires

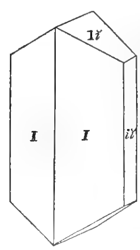
	Eqs.	Calculated.	Found.		
Cobalt . .	2	17.93	17.99	18.00	18.01
Oxalic Acid .	3	32.82	32.95	—	—

It would *à priori* appear probable that there exists an acid oxalate of Luteocobalt corresponding to the acid carbonate, but we have not yet been able to obtain such a salt. The oxalic acid in this compound cannot be easily reduced by a solution of terchloride of gold, nor can it be completely separated from the base by means of a solution of chloride of calcium.

CARBONATES OF LUTEOCOBALT.

The neutral carbonate of Luteocobalt is readily formed by decomposing a solution of chloride of Luteocobalt by carbonate of silver. The yellow solution, by evaporation, yields sherry-wine colored crystals of the carbonate. The salt closely resembles the other soluble salts of Luteocobalt; is easily soluble in hot water, and crystallizes well by slow evaporation. During evaporation, however, the solution absorbs carbonic acid from the air, and crystals of the acid carbonate are found mixed with those of the neutral salt. According to Prof. Dana's measurement, the crystals of the neutral carbonate belong to the trimetric system, and approach Aragonite in form. Fig. 20 represents a crystal of this salt:

Fig. 20.



I : I = 116° 50′

I : iĭ = 121° 35′

1ĭ : 1ĭ (top) = 114° 16′

1ĭ : 1ĭ over iĭ = 65° 44′

$a : b : c = 1.0509 : 1 : 1.6265$

The constitution of the neutral carbonate appears to be represented by the formula

$$6NH_3.Co_2O_3,3CO_2+7HO.$$

0 2495 grs. gave 0.1220 grs. sulphate of cobalt = 18.61 per cent. cobalt.
0.3518 grs. gave 0.0786 grs. carbonic acid = 22.34 per cent.

The formula requires

	Eqs.		Found.
Cobalt . . .	2	18.79	18.61
Carbonic acid . .	3	21.01	22.34

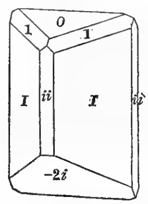
The excess of carbonic acid and the deficiency in cobalt, are doubtless due to the presence of a portion of the acid carbonate. The neutral carbonate loses its water of crystallization in dry air, and becomes opaque, with the lustre of porcelain, like many other hydrated salts of this base.

The acid carbonate of Luteocobalt is most readily prepared by passing a current of carbonic acid gas into a solution of the neutral salt. The acid carbonate usually separates, after a very short time, in the form of large brown-red or sherry-wine colored crystals, which are less soluble than those of the neutral carbonate. According to Prof. Dana, the crystals of this salt belong to the monoclinic system, and closely approach Barytocalcite in form. Fig. 21 represents a crystal of this salt.

$O : ii = 108° 16'$
$O : I = 77° 40'$ and $102° 20'$
$I : I = 85° 54'$
$O : 1 = 139° 50'$
$I : 1 = 142° 30'$
$O : -2i = 111° 46'$

$a : b : c = 0.7219 : 1 : 0.8398$
$C = 71° 44'.$

Fig. 21.



In Barytocalcite the angle corresponding to $O : ii = 106° 54'$ and that corresponding to $I : I = 84° 52'$.

The acid carbonate of Luteocobalt retains its water of crystallization in the air, but loses it under the air-pump. The salt is particularly interesting as being the only acid salt of Luteocobalt which we have as yet been able to obtain. The formula of this salt is

$$6NH_3.Co_2O_3,3Co_2+HO,CO_2+5HO$$

as the following analyses show:

0.4715 grs. gave 0.2246 grs. sulphate of cobalt = 18.12 per cent. cobalt.
1.0506 grs. gave 0.2830 grs. carbonic acid = 26.93 per cent.

The formula requires

	Eqs.		Found.
Cobalt . . .	2	18.04	18.12
Carbonic acid . .	4	26.91	26.93

The very distinctly marked triacid character of Luteocobalt, considered as a base, renders it, to say the least, improbable that the formula of this salt should be written

$$6NH_3.Co_2O_3.4CO_2+6HO.$$

OXIDE OF LUTEOCOBALT.

The oxide of Luteocobalt may be obtained by decomposing a solution of the sulphate with baryta water. The solution is brown-yellow, and has an alkaline taste and reaction. It cannot be evaporated without decomposition, ammonia being evolved and a black powder separated. The solution absorbs carbonic acid from the air, and on evaporation, yields crystals of the carbonates; with acids it yields the salts of the base. The oxide of Luteocobalt appears to form compounds with salts of copper, which may be analogous to the ammonia-salts of that metal, or which again may be only double salts of copper and Luteocobalt. A solution of the oxide added to one of sulphate of copper gives, after standing, beautiful chrome-green crystals of a new salt, which we have not yet had an opportunity of examining.

XANTHOCOBALT.

The salts of Xanthocobalt may be prepared by two distinct processes, either directly from ammoniacal solutions of salts of cobalt, or from neutral, acid, or ammoniacal solutions of the salts of Roseocobalt or Purpureocobalt. In any case, the gas arising from the action of nitric acid upon starch or sawdust, and which may be considered as a mixture of CO_2, NO_2, NO_3, and NO_4—the last probably in excess—is to be passed into the solution. A more or less rapid absorption accompanied by copious fumes of carbonate of ammonia takes place. The color of the liquid gradually changes to a dark reddish-brown or dark sherry-wine color, and the liquid on cooling generally deposits an abundance of a salt of Xanthocobalt in brown-yellow crystals. When a current of the red gas above mentioned, and which we shall denote by the symbol NO_x, is passed into an acid solution of cobalt and ammonia, an absorption also usually takes place, but in this case no Xanthocobalt is produced, but only a beautiful yellow crystalline salt, which is the ammonia nitrite of cobalt, corresponding to the yellow potash salt, first described by Fischer,[1] some years afterward re-discovered and analyzed by St. Evre,[2] and finally examined by Stromeyer.[3] The formation of this compound is easily prevented by keeping the solution ammoniacal by constant additions of ammonia.[4]

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[1] Pogg. Ann., LXXIV, 115, and LXXXVIII, 496.

[2] Compt. Rend., XXXIV, 479, and Ann. de Chimie et de Physique, XXXVIII, 177.

[3] Journ. für Pract. Chemie, LXI, 41, and Annalen der Chemie und Pharmacie, XCVI, 218.

[4] The employment of the double nitrite of cobalt and potash as a means of separating cobalt from other metals was first suggested by Fischer, and afterward by St. Evre and Stroméyer. As the salt is insoluble in a solution of acetate of potash and in alcohol, it is easy to wash it completely, but after washing it is difficult to estimate the cobalt accurately by drying and weighing upon a filter. We have found that a very satisfactory result may be obtained by weighing the cobalt in the form of mixed sulphates of cobalt and potash. According to Stromeyer, Fischer's salt has the formula

$$Co_2O_3.2NO_3+3KO,NO_3+2HO.$$

The salts of Xanthocobalt have a dark sherry-wine or brown-yellow color. They are rather more soluble, both in hot and cold water than the salts of Roseocobalt, Purpureocobalt, or Luteocobalt, the solutions when dilute have a yellow color; when concentrated they are dark-brown yellow. Solutions of these salts are decomposed by boiling, though sometimes with difficulty; in some cases, however, a temperature much below the boiling point produces an incipient decomposition after a short time, so that great care must be taken in evaporating for the purpose of crystallization. During the decomposition by boiling, ammonia is given off, while a heavy black or dark-brown powder is thrown down. The addition of a mineral acid prevents the decomposition, but produces, even when in very small quantity, a chemical change of a different kind, deutoxide of nitrogen being given off, while a salt of Purpureocobalt is formed, which remains mixed with the undecomposed salt of Xanthocobalt, and which it is difficult to separate. A few drops of acetic acid, however, added to a solution of a salt of Xanthocobalt so as to produce a slightly acid reaction, will usually suffice to prevent decomposition, if the solution be evaporated at a gentle heat. The dry salts of Xanthocobalt heated in a porcelain crucible are easily decomposed, with copious evolution of red vapors, and afterwards of ammonia, while a black powder remains. We have not been able to detect any new ammonia-cobalt bases among the products of the decomposition of solutions of these salts, by simple boiling or heating in the dry way.

Xanthocobalt differs from the other ammonia-cobalt bases by containing deutoxide of nitrogen as a couplet in addition to ammonia. Considering it as a primary radical we attribute to it the formula $NO_2.5NH_3.Co_2$, and it is evident that it differs from Roseocobalt and Purpureocobalt only by one equivalent of deutoxide of nitrogen. The passage of the salts of these two bases into salts of Xanthocobalt by simple absorption of NO_2 is thus easily explained, as is the decomposition of the salts of Xanthocobalt into NO_2 and salts of Purpureocobalt.

It is, however, proper to remark that, inasmuch as the salts of Xanthocobalt which we have examined in all cases appear to contain water in the proportion of at least one equivalent, it is possible that the view we have taken of their constitution may not be precisely accurate, and that they may contain NO_3 as a couplet in place of NO_2. This will be evident from a simple inspection of the formulæ of the

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When this is heated with sulphuric acid till the excess of acid is expelled, there remains a mixture of sulphates having the formula

$$2CoO,SO_3+3KO,SO_3.$$

We, therefore, dry the salt carefully, burn the filter upon the cover of a deep platinum crucible, and bring the ashes and salt into the crucible itself. A few drops of sulphuric acid are then added, so as to cover the mass, and the whole is then treated precisely as we described on page 4, though in this case a higher heat is to be used. From the weight of the mixed sulphates the quantity of cobalt is easily calculated. This method gives very accurate results, because the cobalt forms only about 14 per cent. of the substance weighed. In an experiment made to test the accuracy of Stromeyer's formula

0.6671 grs. gave 0.6437 grs. mixed sulphates of cobalt and potash = 13.66 per cent. cobalt.

The formula requires 13.63 per cent.

chloride, nitrate, and sulphate of Xanthocobalt, which are, as we write them at present

$$NO_2.5NH_3.Co_2O,Cl_2+HO.$$
$$NO_2.5NH_3.Co_2O_3,2NO_5+HO.$$
$$NO_2.5NH_3.Co_2O_3,2SO_3+HO.$$

No chemical analyses could, probably, in substances of such high equivalents, enable us to say with certainty that there may not be in these formulæ an equivalent of hydrogen too much. In this case the three formulæ would read

$$NO_3.5NH_3.Co_2O,Cl_2.$$
$$NO_3.5NH_3.Co_2O_3,2NO_5.$$
$$NO_3.5NH_3.Co_2O_3,2SO_3.$$

The difficulty of analyzing these salts must also be fairly considered in forming an opinion as to their constitution. We adopt provisionally the formulæ first given in the hope that a more extended study of the products of decomposition will enable us hereafter to fix the constitution with certainty. In the mean time we will only remark that the analyses of the nitrate of Xanthocobalt, which is of all salts of the base the easiest to prepare in a state of absolute purity, correspond exactly to the formula

$$NO_2.5NH_3.Co_2O_3,2NO_5+HO.$$

CHLORIDE OF XANTHOCOBALT.

The chloride of Xanthocobalt can be most readily prepared by decomposing a solution of the sulphate by chloride of barium. After separation of the sulphate of baryta by filtration, a few drops of acetic acid may be added to the filtrate, which, after evaporation by a gentle heat, will yield on standing large crystals of the chloride. This process is in fact the only one by which we have been able as yet to prepare this salt.

The chloride of Xanthocobalt cannot be prepared by the direct action of a current of NO_x upon an ammoniacal solution of the chloride of cobalt. It is true that in this case an abundance of a brown-yellow crystalline salt is obtained, which might easily be mistaken for the chloride. This salt is, however, a mixture of a large quantity of the nitrate of Xanthocobalt with a much smaller quantity of another salt, which appears to be the chloride of a distinct base, but which is not yet fully studied.[1]

The equation which represents the formation of the chloride of Xanthocobalt by double decomposition, is simply

$$NO_2.5NH_3.Co_2O_3,2SO_3+2BaCl=2BaO,SO_3+NO_2.5NH_3.Co_2O,Cl_2.$$

The chloride of Xanthocobalt crystallizes in brown-yellow crystals, which when

[1] It is this chloride in an impure state to which, in my Report on the Recent Progress of Organic Chemistry, the formula $2NO_2.5NH_3.Co_2O_2Cl$, is attributed, and which is there called Dixanthocobalt. W. G.

large, have a faint reddish tinge. They are usually very well defined, and exhibit a beautiful iridescence upon their surfaces, but we cannot at present describe their form. The salt is quite soluble in hot water, but the solution is easily, though only partially, decomposed at a boiling heat, and even, when quite neutral, at a lower temperature. On boiling, ammonia is evolved in abundance, while a heavy black precipitate is thrown down; it requires very long boiling to produce a complete decomposition. The salt is rather insoluble in cold water; chlorhydric acid precipitates it unchanged in the cold, and the same effect is produced by the solutions of the alkaline chlorides. Acids, even when dilute, easily and completely decompose the chloride of Xanthocobalt by boiling. Chlorhydric acid in large excess, by long boiling, converts the salt completely into chloride of Purpureocobalt, deutoxide of nitrogen, and occasionally chlorine, being given off. Sulphuric and nitric acid also decompose the chloride, but with these acids the decomposition generally goes so far as to produce salts of cobalt, or double salts of cobalt and ammonia. Oxalic acid also produces complete decomposition by long boiling, oxalate of cobalt and oxalate of ammonia being formed. Even strong acetic acid produces, on boiling with the salt, a partial decomposition. Alkalies readily decompose a solution of the chloride when heated, ammonia being evolved, while a black powder is thrown down. The dry chloride is easily decomposed below a red heat. An abundance of red vapors is at first given off, afterward ammonia is evolved, and finally a black superoxyd of cobalt remains.

Reducing agents, in certain cases, completely decompose the solution of the chloride of Xanthocobalt, giving salts of cobalt and ammonium. Thus a current of sulphydric acid gas passed for some time through the solution gives a precipitate of sulphur and sulphide of cobalt, while nitrogen gas escapes, and chloride and sulphide of ammonium remain in solution. The decomposition is similar in an acid solution, but of course no sulphide of cobalt is precipitated.

We made this experiment in the hope of decomposing the NO_2 in the chloride in a manner similar to the well known reaction in the case of organic compounds containing NO_4. For the same reason we also boiled the chloride with acetic acid and iron filings, in the hope of obtaining a reaction analogous to that which is produced under the same circumstances with nitro-benzol, nitro-naphtalin, &c., as observed by Béchamp. The experiments, however, led to no interesting result.

Chloride of Xanthocobalt has the formula

$$NO_2.5NH_3.Co_2O,Cl_2+HO,$$

as the following analyses show:

0.2652 grs. gave 0.1557 grs. sulphate of cobalt = 22.34 per cent. cobalt.
0.5865 grs. gave 0.3435 grs. " " = 22.29 " "
0.8240 grs. gave 0.8980 grs. chloride of silver = 26.94 per cent. chlorine.
1.5200 grs. gave 1.6740 grs. " " = 27.22 " "
0.4983 grs. gave 0.2773 grs. water = 6.18 " hydrogen.
0.5897 grs. gave 0.3272 grs. " = 6.16 " "
1.0768 grs. gave 297 c. c. nitrogen at 16°.9 C. and $758^{mm}.43$ (at 17°.2 C.) = 273.02 c. c. at 0° and 760^{mm} = 31.84 per cent.
1.0657 grs. gave 297 c. c. nitrogen at 19°.5 C. and $763^{mm}.26$ (at 20° C.) = 271.30 c. c. at 0° and 760^{mm} = 31.97 per cent.

The formula requires

	Eqs.		Calculated.	Mean.	Found.	
Cobalt . .	2	59	22.52	22.32	22.34	22.29
Chlorine . .	2	71	27.09	27.08	26.94	27.22
Hydrogen .	16	16	6.10	6.17	6.18	6.16
Nitrogen . .	6	84	32.06	31.90	31.84	31.97
Oxygen . .	4	32	12.23	12.53	—	—
		262	100.00			

CHLORAURATE OF XANTHOCOBALT.

This salt is readily formed by adding a solution of terchloride of gold to one of the chloride of Xanthocobalt. Beautiful yellow needles are thrown down, which may easily be purified by re-solution in hot water and crystallization. The chloraurate of Xanthocobalt crystallizes in beautiful brown-yellow iridescent prisms, which belong to the trimetric or right rhombic system, according to Prof. Dana's measurements. The observed forms are ∞, $1\text{-}\bar{\infty}$, the only observed angle is $1\bar{i} : 1\bar{i} = 130° 24'$. I : I varies much and could not be accurately determined.

The formula of this salt is

$$NO_2.5NH_3.Co_2O,Cl_2+AuCl_3+2HO.$$

The data of the analysis are as follows:

0.2687 grs. gave 0.1645 grs. metallic gold and sulphate of cobalt = 10.28 per cent. cobalt.
0.3028 grs. gave 0.1035 grs. metallic gold (reduced by SO_2) = 34.18 per cent.
0.2749 grs. gave 0.3461 grs. chloride of silver = 31.12 per cent. chlorine.

The formula requires

	Eqs.	Calculated.	Found.
Cobalt . .	2	10.26	10.28
Gold . . .	1	34.28	34.18
Chlorine . .	5	30.89	31.12

CHLORPLATINATE OF XANTHOCOBALT.

This salt is easily prepared by direct combination. It has a fine orange-yellow color, and is with difficulty soluble both in hot and cold water, but it may be recrystallized from a hot solution in dilute chlorhydric acid. It is represented by the formula

$$NO_2.5NH_3.Co_2O,Cl_2+2PtCl_2+2HO$$

as the following analyses show:

0.6830 grs. gave 0.1421 grs. chloride of cobalt = 9.45 per cent. cobalt.
0.6830 grs. gave 0.2187 grs. platinum = 32.02 per cent.

The formula requires

	Eqs.	Calculated.	Found.
Cobalt . .	2	9.52	9.45
Platinum . .	2	31.87	32.02

The analysis was made in this case by gently igniting the salt in a platinum crucible, and afterwards dissolving out the chloride of cobalt by long boiling with chlorhydric acid.

CHLORHYDRARGYRATE OF XANTHOCOBALT.

This salt is formed by adding a cold solution of chloride of mercury to a solution of the chloride of Xanthocobalt. It is usually thrown down in the form of brilliant pale brownish-yellow talcose scales, which on re-solution in hot water containing a little free chlorhydric acid, separate as the solution cools in brown-yellow needles. The two forms appear to have the same chemical constitution. The salt is quite insoluble in cold, and with difficulty soluble in hot water, but the solution occurs without sensible decomposition. The chlorhydrargyrate of Xanthocobalt has the formula

$$NO_2.5NH_3.\dot{C}o_2O,Cl_2+4HgCl+2HO.$$

as the analyses satisfactorily show:

0.7025 grs. gave 0.1375 grs. sulphate of cobalt = 7.44 per cent. cobalt.
0.8898 grs. gave 0.9377 grs. chloride of silver = 26.05 per cent. chlorine.

The formula requires

	Eqs.	Calculated.	Found.
Cobalt . .	2	7.25	7.44
Chlorine . .	6	26.19	26.05

FERROCYANIDE OF XANTHOCOBALT.

This salt is precipitated almost immediately when a solution of ferrocyanide of potassium is added to one of the nitrate of Xanthocobalt. We have not been able to obtain it, however, either from the chloride or the sulphate, with which the ferrocyanide of potassium gives only turbid solutions. The salt is precipitated in prismatic crystals which appear to belong to the oblique rhombic system. Its color is a very beautiful bright orange-red, corresponding nearly with the red-orange No. 5, of the 2d circle of Chevreul's scale. When freshly prepared, it is one of the most beautiful salts which chemistry can exhibit, but it loses some of its brilliancy of tint by keeping, and becomes a little duller and darker, probably from a slight decomposition upon the surface. The crystals exhibit a fine dichroism by reflection, the ordinary image being pale reddish orange, while the extraordinary image is bright orange. The ferrocyanide of Xanthocobalt is almost insoluble in cold, and is immediately decomposed by hot or even by warm water. The crystals lose water and are partially decomposed in vacuo, or even in pleno over sulphuric acid. They can, therefore, only be dried by pressure between folds of bibulous paper. The impossibility of purifying this salt by recrystallization, and the facility with which it is decomposed, render it difficult to obtain it in a perfectly pure state.

The formula of the ferrocyanide appears to be

$$NO_2.5NH_3.Co_2O,Cy_2+FeCy+7HO$$

as the following analyses show:

0.6288 grs. gave 0.1557 grs. metallic cobalt and iron = 24.76 per cent.
0.9047 grs. gave 0.2239 grs. " " " = 24.74 "
0.5896 grs. gave 0.0690 grs. sesquioxide of iron = 8.19 " iron.
0.6006 grs. gave 0.1311 grs. Co_2O_3+4CoO = 16.39 " cobalt.
1.0820 grs. gave 0.4163 grs. carbonic acid = 10.49 " carbon.
1.1645 grs. gave 0.6439 grs. water = 6.14 per cent. hydrogen.
0.8718 grs. gave 271 c. c. nitrogen at 20°.8 C. and $765^{mm}.04$ (at 21°.3 C.) = 246.51 c. c. at 0° and 760^{mm} = 35.51 per cent.
0.9471 grs. gave 292 c. c. nitrogen at 17°.7 C. and $763^{mm}.01$ (at 18° C.) = 268.96 c. c. at 0° and 760^{mm} = 35.66 per cent.

The formula requires

	Eqs.	Calculated.		Found. 1	2	3
Cobalt	2	16.80	24.77	16.39	24.76	24.74
Iron	1	7.97		8.19		
Carbon	6	10.25		10.49		
Hydrogen	22	6.26		6.14		
Nitrogen	9	35.89		35.66	35.51	

These analyses agree with the formula as well as can be reasonably expected, when the difficulty of obtaining a perfectly pure salt is taken into consideration. It is interesting to remark that in this salt, in which there is but one equivalent of basic cyanide, there are still two equivalents of cyanogen in the basic for one in the acid or electronegative cyanide.

SULPHATE OF XANTHOCOBALT.

When pure sulphate of cobalt is dissolved in water, an excess of ammonia added, and a current of NO_x passed through the liquid, an absorption takes place, and copious fumes of carbonate of ammonia are given off. The liquid gradually assumes a dark brown-yellow color, and frequently deposits an abundance of crystals during the passage of the current of gas. Ammonia must be added from time to time, so as to keep the solution strongly alkaline, and prevent the appearance of red vapors at the surface of the liquid. It is not necessary to apply heat, as the temperature rises from the absorption of the gas. When the operation is over, which is indicated by the color of the liquid, the solution may be filtered and allowed to evaporate spontaneously, when a mass of brown-yellow crystals is obtained. These may be purified by re-solution in hot water with addition of a few drops of acetic acid and re-crystallization.

This method of preparing the salt is a convenient one, and large quantities may be prepared in a few hours. The time required for the completion of the oxidation depends upon the quantity of sulphate of cobalt employed, and may last from one to twelve hours. It is best to employ a rapid current of the gas. The formation of the sulphate of Xanthocobalt, under these circumstances, may be represented by the equation

$$2CoO,SO_3+5NH_3+NO_3=NO_2.5NH_3.Co_2O_3,2SO_3$$

if we suppose the oxidizing agent to be NO_3. If, however, we consider the active agent to be NO_4, we may represent the action by the equation

$$2CoO,SO_3+6NH_3+2NO_4+HO=NO_2.5NH_3.Co_2O_3,2SO_3+NH_4O,NO_5.$$

A large quantity of free nitrite of ammonia is always formed during the process, as we believe, by the direct action of NO_3 upon ammonia and water. The addition of an acid to the mother liquor from which the sulphate has crystallized, produces an active effervescence arising from the decomposition of the nitrite of ammonia.

The process which we have indicated has generally proved perfectly satisfactory in preparing the sulphate. It is proper to state, however, that in some experiments, made for the purpose of revision, we obtained by the process above given no sulphate of Xanthocobalt, but only the nitrate. The cause of this we are at present unable to assign, and further experiments are wanting to explain so unexpected a result.

The sulphate of Xanthocobalt crystallizes in thin plates, the form of which we have not been able to determine; they appear to belong to the right rhombic system. The crystals have a fine brown-yellow color, which, however, we cannot define by means of the chromatic scale. The salt is rather soluble in hot, but much less soluble in cold water. Heat readily decomposes the neutral solution, a black powder being thrown down, while ammonia is given off. The dry salt is decomposed like the chloride.

Strong sulphuric acid dissolves the sulphate to a red oily liquid, but little deutoxide of nitrogen being given off. The addition of water to this solution causes a violent effervescence from the escape of deutoxide of nitrogen, and probably nitrous acid. There remains a red liquid consisting chiefly of the double sulphate of cobalt and ammonia, but almost always containing a little acid sulphate of Purpureocobalt. Even very dilute sulphuric acid readily decomposes sulphate of Xanthocobalt by boiling. Long boiling with chlorhydric acid also decomposes this salt, the products of the decomposition being, as already stated, chloride of Purpureocobalt, free sulphuric acid, and deutoxide of nitrogen.

Sulphate of Xanthocobalt has the formula

$$NO_2.5NH_3.Co_2O_3,2SO_3+HO$$

as the following analyses show:

0.5776 grs. gave 0.3142 grs. sulphate of cobalt	= 20.69	per cent.	cobalt.	
0.5630 grs. gave 0.3060 grs. " "	= 20.68	"	"	
1.2370 grs. gave 1.0150 grs. sulphate of baryta	= 28.16	"	sulphuric acid.	
1.1472 grs. gave 0.9242 grs. " "	= 27.65	"	"	"
0.7713 grs. gave 0.3868 grs. water	= 5.56	"	hydrogen.	
0.4468 grs. gave 0.2285 grs. "	= 5.68	"	"	

0.6020 grs. gave 169.48 c. c. nitrogen at 14° C., and $769^{mm}.61$ ($h = 91^{mm}$) = 140.87 c. c., at 0° and 760^{mm} = 29.37 per cent.

1.2624 grs. gave 300 c. c. nitrogen, at 5° C., and $775^{mm}.20$ (at 5°.3 C.) = 296.68 c. c., at 0° and 760^{mm} = 29.51 per cent.

The formula requires

	Eqs.		Calculated.	Mean.	Found.	
Cobalt . .	2	59	20.55	20.68	20.69	20.68
Sulphuric acid .	2	80	27.94	27.90	27.65	28.16
Hydrogen . .	16	16	5.57	5.62	5.56	5.68
Nitrogen . .	6	84	29.26	29.44	29.37	29.51
Oxygen . .	12	96	16.68	16.30	—	—
		335	100.00			

A solution of sulphurous acid dissolves sulphate of Xanthocobalt without decomposition. On boiling, however, a complete decomposition takes place, bubbles of gas are given off, and there remains a red solution of the double sulphate of cobalt and ammonia. The gas which is evolved produces no red vapors on contact with the air, and is probably protoxide of nitrogen. The sulphate of Xanthocobalt is also decomposed by boiling with urea, an abundance of a colorless gas without odor being given off. The reactions of the sulphate are similar to those of the chloride.

NITRATE OF XANTHOCOBALT.

The nitrate of Xanthocobalt may be prepared like the sulphate, by passing a current of NO_x into an ammoniacal solution of nitrate of cobalt. The formation of the nitrate goes on very rapidly, and crystals are usually deposited in abundance long before the oxidation is complete. It is best in this case, as in the preparation of the sulphate, to employ a pure salt of cobalt and pure ammonia, as the subsequent purification of the nitrate of Xanthocobalt becomes much more easy. The equations representing the formation of the nitrate are similar to those which we have given for the sulphate.

The nitrate of Xanthocobalt may also be easily prepared by the action of NO_x upon neutral, acid, or alkaline solutions of the chlorides, sulphates and nitrates of Roseocobalt and Purpureocobalt. We have alluded to the formation of the nitrate of Xanthocobalt by this process already, and will here enter more into detail.

It is almost a matter of indifference which salt is selected, as the nitrate is prepared with nearly equal facility from all. The salt is to be dissolved in water, and, to hasten the process, ammonia added; a large excess of ammonia is not necessary, but the process always goes on more rapidly in an ammoniacal than in an acid or neutral solution. The current of gas resulting from the action of nitric acid upon starch or sawdust is then to be passed into the liquid, which speedily becomes hot and gradually changes its color from violet to orange-red, and at last to orange, while orange-yellow crystals of the nitrate of Xanthocobalt are precipitated. The liquid on cooling gives more crystals; the process should not be continued after the whole mass has assumed a clear orange-yellow color. The mother-liquor from which the crystals of the nitrate of Xanthocobalt have separated contains only salts of ammonia, with a little nitrate in solution. The whole process is very easy to execute, and yields a very pure nitrate. The reactions which occur in this process are remarkably beautiful, and may be expressed, in the case of the action of the gas upon the chloride and sulphate of Roseocobalt, by the equations

$$5NH_3.Co_2Cl_3+3NO_4+3HO=NO_2.5NH_3.Co_2O_3,2NO_5+3HCl,$$
$$5NH_3.Co_2O_3,3SO_3+3NO_4+3HO=NO_2.5NH_3.Co_2O_3,2NO_5+3HO,SO_3.$$

It will be seen from these equations that when neutral solutions of the salts of Roseocobalt or Purpureocobalt are employed, chlorhydric and sulphuric acids are set free. The influence of an excess of ammonia in facilitating the process is thus easily understood. The nitrate of Xanthocobalt, being much less soluble in cold water than the ammonia salts, is easily purified by re-crystallization. In treating of the sulphate and chloride of Xanthocobalt, we have mentioned that the nitrate of Xanthocobalt is often formed while preparing these salts directly. As a mode of preparation, this method is not to be recommended, but it possesses much theoretical interest. The equation which represents the action when the nitrate of Xanthocobalt is formed directly by the action of NO_x upon an ammoniacal solution of the sulphate of cobalt, is possibly the following

$$2CoO,SO_3+7NH_3+NO_3+3NO_4+2HO=NO_2.5NH_3.Co_2O_3,2NO_5+2NH_4O,SO_3+NO_2.$$

When the chloride of cobalt is employed instead of the sulphate, other products are always formed simultaneously with the nitrate of Xanthocobalt, and the reaction becomes therefore more complicated.

The nitrate of Xanthocobalt crystallizes in small brilliant crystals, which, according to Prof. Dana's measurement, belong to the dimetric system. The only form observed as yet is an octahedron, the angle at the base being 100° 45′—101° 15′.

The salt is dichrous, the ordinary image being pale orange, while the extraordinary is bright orange-yellow.

The salt has a clear brown-yellow color, and the mass of crystals is usually very brilliant. It is quite soluble in hot, but rather insoluble in cold water; the solution is readily decomposed by boiling, with evolution of ammonia and precipitation of a heavy black powder. The dry salt is readily decomposed by heating, abundance of red vapors being given off, while a black oxide remains. The nitrate is completely decomposed by boiling with chlorhydric acid, red vapors mixed with chlorine being given off, while there remains a solution of chloride of Purpureocobalt, from which crystals of this salt are deposited on cooling. When boiled with nitric acid, a similar decomposition is produced, and crystals of nitrate of Roseocobalt are obtained in small quantity. It is probable that nitrate of Purpureocobalt is the first product of this reaction, and that this by boiling with excess of acid, passes into the nitrate of Roseocobalt. As a general rule, the quantity of nitrate of Roseocobalt produced is small, and the decomposition results in the conversion of the nitrate of Xanthocobalt into the nitrates of cobalt and ammonia.

Nitric acid precipitates nitrate of Xanthocobalt from its solution without sensible decomposition in the cold. By long boiling acetic acid completely reduces nitrate of Xanthocobalt, and a solution of cobalt is obtained which is free from ammonia-cobalt bases. Oxalic acid also reduces the nitrate by boiling, oxalate of cobalt being thrown down.

Nitrate of Xanthocobalt has the formula

$$NO_2.5NH_3.Co_2O_3,2NO_5+HO$$

as the following analyses appear to show:

0.3917 grs. gave 0.1927 grs. sulphate of cobalt = 18.72 per cent. cobalt.
0.6960 grs. gave 0.3405 grs. " " = 18.76 " "
0.5935 grs. gave 0.2925 grs. " " = 18.75 " "
0.5585 grs. gave 0.2535 grs. water = 5.04 " hydrogen.
0.4958 grs. gave 0.2330 grs. " = 5.21 " "
0.9483 grs. gave 0.4270 grs. " = 5.09 " "
0.6145 grs. gave 187.45 c. c. nitrogen at 6°.5 C., and $769^{mm}.5$ (at 7° C.), (h = $35^{mm}.0$) = 174.55 c. c. at 0° and 760^{mm} = 35.73 per cent.
0.6024 grs. gave 183.30 c. c. of nitrogen at 5° C., and $759^{mm}.0$ (at 6° C.), (h = $37^{mm}.0$) = 169.20 c. c. at 0° and 760^{mm} = 35.27 per cent.
0.5912 grs. gave 184.66 c. c. nitrogen at 11°.5 C., and $761^{mm}.0$ (at 12° C.), (h = $32^{mm}.5$) = 167.13 c. c. at 0° and 760^{mm} = 35.50 per cent.

Hence we have

	Eqs.	Calculated.		Mean.	Found.		
Cobalt .	2	59.0	18.73	18.74	18.76	18.72	18.75
Hydrogen	16	16.0	5.08	5.11	5.04	5.21	5.09
Nitrogen .	8	112.0	35.55	35.50	35.73	35.50	35.27
Oxygen .	16	128.0	40.64	40.65	—	—	—
		315.0	100.00	100.00			

As the nitrate of Xanthocobalt is, of all the salts of this base, that which is most easily prepared in a state of purity, and as its reactions are most characteristic of the base, we shall give them in this place.

Chlorhydric acid in excess gives a buff-yellow precipitate.

Alkaline carbonates give no precipitate.

Ferrocyanide of potassium precipitates beautiful orange-red crystals.

Ferridcyanide of potassium gives no precipitate.

Cobaltidcyanide of potassium gives no precipitate.

Chromate of potash gives a fine clear yellow precipitate.

Bichromate of potash gives beautiful orange-red needles.

Oxalate of ammonia gives a voluminous precipitate of pale yellow needles.

Picrate of ammonia gives beautiful clear yellow needles.

Phosphate of soda gives no precipitate.

Pyrophosphate of soda gives no precipitate.

Chloride of mercury gives a buff-colored scaly precipitate.

Protochloride of tin gives, after a short time, granular yellow crystals.

Bichloride of platinum gives an orange-yellow precipitate.

Terchloride of gold gives, after addition of chlorhydric acid and standing, yellow needles.

Iodide and bromide of potassium give no precipitates.

OXALATE OF XANTHOCOBALT.

The oxalate of Xanthocobalt is precipitated when a solution of oxalate of ammonia is added to one of the chloride, nitrate or sulphate of the base. After a very short time yellow acicular crystals make their appearance, the separation being greatly facilitated by strongly agitating the solution. The precipitate is to be thrown on a filter, well washed with cold water and dried, first by pressure and afterwards in pleno over sulphuric acid. As thus prepared, the salt has a pale yellow color, and consists of fine needles, the form of which cannot be determined, even under the microscope. It is nearly insoluble in cold and but very slightly soluble in hot water. The solution is readily decomposed by boiling. The insolubility of this oxalate and its characteristic appearance render it of great value in detecting the presence of salts of Xanthocobalt.

Oxalate of Xanthocobalt has the formula

$$NO_2.5NH_3.Co_2O_3,2C_2O_3+5HO$$

as appears from the following analyses:

0.3191 grs. gave 0.1762 grs. sulphate of cobalt	= 21.01	per cent.	cobalt.	
0.2980 grs. gave 0.1650 grs. " "	= 21.06	"	"	
2.2520 grs. gave 0.7075 grs. carbonic acid	= 25.70	"	oxalic acid.	
2.2850 grs. gave 0.7260 grs. " "	= 25.99	"	"	"

The calculated results are

	Eqs.	Theory.	Found.	
Cobalt . . .	2	21.14	21.01	21.06
Oxalic acid . .	2	25.81	25.70	25.99

THEORETICAL CONSIDERATIONS.

The empirical constitution of the ammonia-cobalt bases being, as we believe, established, it remains to offer an exposition of our views of their theoretical structure. Claudet[1] and Weltzien[2] have endeavored to reduce the salts of Roseocobalt and Luteocobalt to the type of ammonium, while Frémy has abstained from adopting any particular theory, and gives, without comment, the results of his analyses in the shape of empirical formulæ. Claudet's view is necessarily erroneous, from the fact that his formula for what we term the chloride of Purpureocobalt is incorrect, inasmuch as he assigns to it 16 in place of 15 equivalents of hydrogen. Our own numerous analyses, as well as those of Rogojski and Gregory, have clearly shown that the number of equivalents of hydrogen is fifteen.

For an exposition of Weltzien's views we must refer to his paper; they appear to us wanting in simplicity, since they require us to admit, not merely an equiva-

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[1] Phil. Mag. (4) II, 253.

[2] Annalen der Chemie und Pharmacie, XCVII, 19.

lent replacement of hydrogen by ammonium and cobalt, but even that the compound ammonium thus formed may replace cobalt in its sesqui-combinations. Thus, according to Weltzien, the formula of nitrate of Luteocobalt is

$$\left.\begin{array}{l} N\left\{\begin{array}{l} H \\ NH_4 \\ NH_4 \\ Co \end{array}\right. \\ N\left\{\begin{array}{l} H \\ NH_4 \\ NH_4 \\ Co \end{array}\right. \end{array}\right\} O_3+3NO_5,$$

which is obviously reducible to the type $R_2O_3+3NO_5$.

By adopting Gerhardt's view of the constitution of the sesquioxides, Rogojski reduces the formula of chloride of Luteocobalt, $6NH_3.Co_2Cl_3$, to the form $2NH_3.coCl$ in which co represents cobalt with $\frac{2}{3}$ of its usually received equivalent. To this body he gives the name Dicobaltinamin, and considers it analogous to the chlorides of Diplatosamin and Palladiamin $2NH_3.PtCl$ and $2NH_3.PdCl$. This view applies very well to several other salts of Luteocobalt, as for example, the bromide, iodide, nitrate, and chlorplatinate, the formulæ of which become

$$\begin{array}{ll} 2NH_3.coBr & \text{or } N_2H_5co,HBr \\ 2NH_3.coI & \text{or } N_2H_5co,HI \\ 2NH_3.coO,NO_5 & \text{or } N_2H_5co,HO,NO_5 \\ 2NH_3.coCl+PtCl_2+2HO & \text{or } N_2H_5co,HCl+PtCl_2+2HO. \end{array}$$

We have remarked already that the compound $6NH_3.Co_2Cl_3+6NH_3.Co_2O_3,3SO_3$, which Rogojski describes, and to which he attributes the formula

$$2NH_3.coCl+2NH_3.coO,SO_3, \text{ or } N_2H_5co,HCl+SO_4H_2,2N_2H_5co.$$

has no real existence, but is merely a mixture of the chloride and sulphate which are isomorphous salts. Rogojski's parallel between this and the sulphate of Gros's base, which according to Gerhardt's view has the formula

$$N_2H_4pt_2,2HCl+N_2H_4pt_2,SO_4H_2$$

is consequently illusory.

On the other hand, moreover, it must be remembered that Rogojski's view applies only to the compounds of Luteocobalt, and fails entirely to reduce the formulæ of the other cobalt bases to more simple expressions, since it requires us, in these cases, to admit fractions of equivalents. Thus the formula of the chloride of Purpureocobalt becomes on this view

$$N_{\frac{5}{3}}H_5coCl,$$

while that of its chlorplatinate must be written

$$3N_{\frac{5}{3}}H_5coCl+2PtCl_2.$$

Even in the case of those compounds of Luteocobalt which contain water, Rogojski's view ceases to give simple expressions, since in the majority of these the number of equivalents of water is not divisible by three. The difficulty becomes still greater in the case of the compounds of Xanthocobalt. We have, therefore, no hesitation in rejecting Rogojski's theory as too limited in its application.

We consider the ammonia-cobalt bases as conjugate compounds of sesquioxide, sesquichloride, &c., of cobalt, the five or six equivalents of ammonia, or of ammonia and deutoxide of nitrogen, forming the conjunct, and serving to give to the sesqui-compound of cobalt the degree of stability which it possesses in this class of bodies. Accordingly we should prefer to write the formula of chloride of Luteocobalt

$$6NH_3{\frown}Co_2.Cl_3$$

employing the connecting circumflex, as Kolbe has suggested, as a symbol of conjugation. Adopting this view, we have the following conjugate radicals, which we assume as existing in the ammonia-cobalt bases precisely in the same sense in which we assume Co_2 as existing in the sesquichloride and sesquioxide of cobalt.

Roseocobalt	$5NH_3{\frown}Co_2$
Purpureocobalt	$5NH_3{\frown}Co_2$
Luteocobalt	$6NH_3{\frown}Co_2$
Xanthocobalt	$NO_2{\frown}5NH_3{\frown}Co_2$.

We may, however, remark that while this view offers a satisfactory explanation of the fact that two of the ammonia-cobalt bases are triacid, forming in all cases neutral compounds with three equivalents of acid, two are biacid bases, and of these latter, one, namely Purpureocobalt, forms acid compounds with four equivalents of acid.

In these cases we meet with the same difficulty which occurs with the ordinary salts of sesquioxides; thus sesquioxide of iron may unite with one, two, or three equivalents of acid, though there appear to be three in the neutral salts. While some chemists assume that salts of sesquioxide of iron containing one or two equivalents of acid are basic, others consider the different equivalents of oxygen as united in different modes, so that the oxide may be, according to circumstances,

$$Fe_2O_3$$
$$Fe_2O_2.O$$
$$Fe_2O.O_2.$$

Peligot's theory of the constitution of sesquioxide of uranium is another case in point; as this oxide unites with but a single equivalent of acid, it may be considered, as Peligot has shown, as the oxide of a radical, $U_2O_2.O$, so that in this case also, the rule that a base unites with as many equivalents of acid as the base itself contains equivalents of oxygen is satisfied.

To explain the biacid character of two of the ammonia-cobalt bases, we may therefore assume in them the existence of secondary radicals containing oxygen. Thus in Purpureocobalt and Xanthocobalt the primary radicals are

$$5NH_3{\frown}Co_2$$
$$NO_2{\frown}5NH_3{\frown}Co_2;$$

while the secondary radicals are

$$5NH_3.Co_2O$$
$$NO_2.5NH_3.Co_2O.$$

The oxides on this view are

$$5NH_3.Co_2O.O_2$$
$$NO_2\,5NH_3.Co_2O.O_2$$

and are consequently of the form RO_2, so that their biacid character is explained. The doctrine of polyacid bases is by no means new; it is in fact contained in the empirical law above referred to, that there are in neutral salts as many equivalents of acid as there are of oxygen in the base, bearing in mind, however, that the oxygen in the base must be outside of the radical. The ammonia-cobalt bases like the conjugate metals produced by the union of ethyl, methyl, &c., with antimony, arsenic, and bismuth, serve, however, to place the doctrine of polyacid bases upon the same footing as that of the polybasic acids, so that the two theories are in this way complementary to each other. From this point of view it is interesting to remark, that the chlorplatinates and double cyanides of the ammonia-cobalt bases follow the same law as the oxygen salts, thus we have

$$6NH_3.Co_2Cl_3+3PtCl_2$$
$$5NH_3.Co_2Cl.Cl_2+2PtCl_2$$
$$NO_2.5NH_3.Co_2O.Cl_2+2PtCl_2$$
$$6NH_3.Co_2Cy_3+Co_2Cy_3$$
$$NO_2.5NH_3.Co_2O.Cy_2+FeCy.$$

In point of fact, the presence of but two equivalents of bichloride of platinum in the chlorplatinate of Purpureocobalt first led us to suspect that the true oxygen salts of this base would be found to contain but two equivalents of acid.

Two other points require special notice in this connection. We have already shown that the oxide of Purpureocobalt, in at least two cases, is capable of uniting with *four* equivalents of acid so as to form feebly acid salts. We consider these salts the true bi-salts of the base, and not as double salts of Purpureocobalt and water. In other words, we hold that they bear the same relation to the neutral salts of the base which bichromate of potash does to the neutral chromate. If this view be correct, we may perhaps expect to find salts of Roseocobalt or Luteocobalt containing *six* equivalents of acid. The only acid salt of Luteocobalt hitherto discovered is the carbonate, but this in reality, in our view, is a double salt of Luteocobalt and water, and has the formula

$$6NH_3.Co_2O_3,3CO_2+HO,CO_2.$$

The fact, that both the acid and neutral oxalo-sulphate of Purpureocobalt contain two distinct acids, is also a very instructive one, since it completes the analogy between the polyacid bases and the polybasic acids. A polybasic acid, as, for example, tartaric acid, may unite with two different bases at once, and we now learn that a polyacid base may in like manner unite with two different acids. Thus we have

$$C_8H_4O_{10}\left\{\begin{matrix}KO\\NaO\end{matrix}\right. \quad - \quad 5NH_3.Co_2O_3\left\{\begin{matrix}SO_3\\C_2O_3\end{matrix}\right. \quad - \quad 5NH_3.Co_2O_3\left\{\begin{matrix}2SO_3\\2C_2O_3.\end{matrix}\right.$$

The other point to which we refer, is the peculiarity in the constitution of Xanthocobalt, in which one equivalent of oxygen in the secondary radical is not capable of replacement by chlorine, so that we have for the chloride of this base the formula

$$\widehat{NO_2.5NH_3.}Co_2O.Cl_2,$$

while for the chloride of Purpureocobalt, we have

$$5\widehat{NH_3.Co_2}Cl.Cl_2$$

and not, as we might expect from the analogy of Xanthocobalt,

$$5\widehat{NH_3.Co_2}O.Cl_2.$$

The appearance of deutoxide of nitrogen as a conjunct (Paarling) is in itself well worthy of attention, and Xanthocobalt forms, we believe, the only known instance in which this occurs. It seems *à priori* probable, that iridium and rhodium bases corresponding to Xanthocobalt may be prepared by passing a current of NO_x into ammoniacal solutions of protosalts of those metals, or into solutions of Claus' bases, and we have already instituted experiments with these metals, the results of which we hope hereafter to communicate.

The theory which we have proposed for the ammonia-cobalt bases has also been brought forward by Claus, and applied to his rhodium and iridium compounds. Claus has extended the view in question to the ammonia compounds of metallic protoxides, and we conceive with advantage, in the case of those bases which contain more than one equivalent of ammonia, as for instance, the platinum, palladium, and iridium bases, having the formulæ

$$2NH_3.PtO \qquad 2NH_3.PdO \qquad 2NH_3.IrO.$$

The discovery of the biacid character of Purpureocobalt and Xanthocobalt, in connection with the views which we have expressed with respect to the molecular structure of these bases, has led us to extend the theory of conjugation to the ammonia-platinum compounds. We consider these also as conjugate bases, and are of opinion that their constitution may be more simply expressed upon this than upon any other view yet proposed. The ammonia-platinum bases at present known are eight in number, of which two were discovered by Reiset, one by Gros, two by Raewsky, and three by Gerhardt. The empirical formulæ of these bases are as follows:

Reiset's first base		$= N_2H_6PtO$	uniacid
"	second base	$= NH_3PtO$	"
Gros's base		$= N_2H_6PtClO$	"
Gerhardt's	first base	$= NH_3PtO_2$	"
"	second base	$= N_2H_6PtO_2$	"
"	third "	$= NH_3PtClO$	"
Raewsky's	first base	$= N_4H_{12}Pt_2ClO_5$	biacid
"	second "	$= N_4H_{12}Pt_2Cl_2O_4$	"

If we apply to these bases the results which we have obtained in the case of the ammonia-cobalt compounds, we may consider them as conjugates of platinum, chloride of platinum, and oxide of platinum, with one or two equivalents of ammonia, excepting Raewsky's bases, which may be regarded as containing a deutoxide of chlorine with four equivalents of ammonia. The formulæ of these bases become on this view

Reiset's second base	=	$NH_3.Pt.O$	uniacid
Gerhardt's first "	=	$NH_3.PtO.O$	"
" third "	=	$NH_3.PtCl.O$	"
Reiset's first base	=	$2NH_3.Pt.O$	"
Gerhardt's second base	=	$2NH_3.PtO.O$	"
Gros's second base	=	$2NH_3.PtCl.O$	"
Raewsky's first "	=	$ClO_2.4NH_3.Pt_2O.O_2$	biacid
" second base	=	$ClO_2.4NH_3.Pt_2Cl.O_2$	"

In comparing the formulæ of these bases with those of the ammonia-cobalt compounds, we remark several points of analogy. These are most striking in the case of Raewsky's two bases, which we consider analogous to Xanthocobalt. Thus we have

Oxide of Xanthocobalt . . . $NO_2.5NH_3.Co_2O.O_2$

Oxide of Raewsky's first base . . $ClO_2.4NH_3.Pt_2O.O_2$.

Raewsky's second base contains chlorine in the radical in place of oxygen. We consider it, to say the least, very probable that there exists an analogous cobalt base having the formula

$$NO_2.5NH_3.Co_2Cl.O_2$$

like

$$ClO_2.4NH_3.Pt_2Cl.O_2$$

and we may here remark that the compound which we have mentioned as one of the products of the action of a current of NO_x upon an ammoniacal solution of chloride of cobalt appears to contain chlorine in the radical, since we have found this element in the dark brown-yellow oxalate which is thrown down by oxalate of ammonia from the solution.

The constitution of Gros's base becomes perfectly intelligible upon this view, as does that of the analogous base containing but one equivalent of ammonia. It can scarcely fail to escape notice that the theory of conjugates brings all the platinum bases under one point of view, and exhibits the analogy in their constitution in a very striking manner, by arranging them at once in three groups, of which the first two are exactly parallel. Thus we have for the radicals of the first six bases mentioned the formulæ

$NH_3.Pt$	$2NH_3.Pt$
$NH_3.PtO$	$2NH_3.PtO$
$NH_3.PtCl$	$2NH_3.PtCl$,

each uniting with a single equivalent of oxygen to form a uniacid base. The occurrence of a deutoxide of chlorine as a couplet is not more remarkable or more improbable than that of deutoxide of nitrogen; but experimental evidence is still wanting to support the view which we have taken of the constitution of Raewsky's bases which have as yet been very imperfectly examined. Claus[1] has also applied the theory of conjugates to several of the ammonia-platinum bases, but has not considered the subject from precisely the point of view which we have taken, though his ideas, in the main, are the same. It is in the explanation of the difference in the saturating capacity of the various bases that Claus' view appears to us less satisfactory than that which we have proposed. It is scarcely necessary to remark that our theory applies to all bases containing ammonia and a metallic oxide. We may, however, observe that it also harmonizes perfectly with the ammonium theory if we consider ammonium as a conjugate hydrogen, or as represented by the formula $\widehat{NH_3.H}$.

We will here remark that our view of the theoretical constitution of the ammonia-cobalt bases was distinctly enunciated in the paper already referred to, as published by one of ourselves, in 1851.

A glance at the formulæ of the ammonia-cobalt bases, suggests the possibility of generalizing the results which we have obtained, by two distinct methods. In the first place, it is evidently possible, theoretically at least, to replace one or more equivalents of ammonia in these compounds by an equal number of equivalents of a compound ammonia, as for example, by methylamin, ethylamin, &c. Thus there may be, for example, a species of Roseocobalt, having the formula

$$5N(C_2H_3)\widehat{_3.Co_2},$$

which would represent the base which we have described under that name, in which methylamin replaces ammonia. We have not yet been able to investigate this point with any degree of thoroughness, and will here only mention, that an experiment made by adding a solution of piperidin to one of chloride of cobalt and allowing the solution to stand for some time in a half-filled flask, with frequent agitation, led to no decisive result. We selected piperidin for this experiment, because this alkaloid is comparatively easy to prepare, and does not itself oxidize by exposure to the air.

In the next place, the results of our investigation may be generalized by replacing cobalt by other metals. This, as we have already remarked, has been done by Claus, who obtained ammonia-rhodium and ammonia-iridium bases corresponding to Roseocobalt, and like this, triacid bases. To judge, however, from the imperfect notices of Claus' papers which have hitherto reached us, his compounds are not isomorphous with those of Roseocobalt.

We have ourselves made many experiments in this direction, though as yet without interesting results. Iron and manganese promised to afford similar classes of compounds, yet in their behavior toward ammonia and oxygen the proto-salts

[1] Chemisch-Pharmaceutisches Central Blatt No. 25, p. 189, Oct. 1854.

of these metals exhibit no analogy to those of cobalt. With chromium the case may be different, but we cannot as yet pronounce with certainty on this point. Experiments with nickel failed entirely, and yielded only ammonia-salts of the protoxide.

The behavior of ammoniacal solutions of proto-salts of iron and manganese toward the mixture of gases which we have denoted by the formula NO_x is worthy of notice. When tartaric acid is added to a solution of proto-chloride of iron or manganese, and after the addition of a large excess of ammonia, a rapid current of NO_x is passed through the solution, the liquid soon becomes dark colored, and after a time the whole of the iron or manganese is precipitated as a dark brown or nearly black flocky substance. The filtrate is free from the metal employed. No ammonia-iron or ammonia-manganese base is, however, formed under these circumstances.

In order to afford a general view of the results of our investigation, we will here give a table of the formulæ of the bodies which we have analyzed, or whose constitution has been inferred from analogy of crystalline form.

Roseocobalt.

Chloride	$5NH_3.Co_2.Cl_3+2HO$.
Chlorplatinate . . .	$5NH_3.Co_2.Cl_3+3PtCl_2+8HO$. (?)
Sulphate	$5NH_3.Co_2.O_3,3SO_3+5HO$.
Nitrate	$5NH_3.Co_2.O_3,3NO_5$.
Hydrous nitrate . . .	$5NH_3.Co_2.O_3,3NO_5+2HO$.
Oxalate	$5NH_3.Co_2.O_3,3C_2O_3+6HO$.
Ferridcyanide . . .	$5NH_3.Co_2.Cy_3+Fe_2Cy_3+3HO$.
Cobaltidcyanide . . .	$5NH_3.Co_2.Cy_3+Co_2Cy_3+3HO$.

Purpureocobalt.

Chloride	$5NH_3.Co_2Cl.Cl_2$.
Chlorplatinate . . .	$5NH_3.Co_2Cl.Cl_2+2PtCl_2$.
Acid sulphate . . .	$5NH_3.Co_2O.O_2,4SO+5HO$.
Oxalate	$5NH_3.Co_2O.O_2,2C_2O_3+3HO$.
Acid oxalo-sulphate . .	$5NH_3.Co_2O.O_2,2C_2O_3,2SO_3+3HO$.
Neutral oxalo-sulphate .	$5NH_3.Co_2O.O_2,C_2O_3,SO_3+7HO$.

Luteocobalt.

Chloride	$6NH_3.Co_2.Cl_3$.
Iodide	$6NH_3.Co_2.I_3$.
Bromide	$6NH_3.Co_2.Br_3$.
Chlorplatinate . . .	$6NH_3.Co_2.Cl_3+3PtCl_2+21HO$.
" . . .	" " " $+6HO$.

Chloraurate	$6NH_3.Co_2.Cl_3+AuCl_3$.
Cobaltidcyanide . . .	$6NH_3.Co_2.Cy_3+Co_2Cy_3+HO$.
Sulphate	$6NH_3.Co_2.O_3,3SO_3+5HO$.
Nitrate	$6NH_3.Co_2.O_3,3NO_5$.
Oxalate	$6NH_3Co_2.O_3,3C_2O_3+4HO$.
Carbonate	$6NH_3.Co_2.O_3,3CO_2+7HO$.
Acid carbonate . . .	$6NH_3.Co_2.O_3,3CO_2+HO,CO_2+5HO$.
Chromate	$6NH_3.Co_2.O_3,3CrO_3+5HO$.

Xanthocobalt.

Chloride	$NO_2.5NH_3.Co_2O.Cl_2$.
Chlorplatinate . . .	$NO_2.5NH_3.Co_2O.Cl_2+2PtCl_2+2HO$.
Chloraurate	$NO_2.5NH_3.Co_2O.Cl_2+AuCl_3+2HO$.
Chlorhydrargyrate . .	$NO_2.5NH_3.Co_2O.Cl_2+4HgCl+2HO$.
Ferrocyanide	$NO_2.5NH_3.Co_2O.Cy_2+FeCy+7HO$.
Sulphate	$NO_2.5NH_3.Co_2O.O_2,2SO_3+HO$.
Nitrate	$NO_2.5NH_3.Co_2O.O_2,2NO_5+HO$.
Oxalate	$NO_2.5NH_3.Co_2O.O_2,2C_2O_3+5HO$.

In concluding[1] for the present an investigation to which we have devoted our leisure for several years, and which has been one of extraordinary difficulty, we desire to state our conviction that the subject is by no means exhausted, but that on the contrary there is scarcely a single point which will not amply repay a more extended study. The number of bases which the sesquioxide of cobalt is capable of forming with ammonia is perhaps very large, and a careful study of the products of the decomposition of the salts of each base promises to yield an abundant harvest of interesting combinations. It is our hope to be able to return to the subject hereafter, and in a second part of our memoir to clear up some points which we have not as yet had time and opportunity fully to consider. In the mean time, we invite the attention of chemists to a class of salts which for beauty of form and color, and for abstract theoretical interest, are almost unequalled either among organic or inorganic compounds.

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[1] I should do no justice to my own feelings if I did not in this place gratefully acknowledge the assistance which I have received in the analytical part of the labor from my friend and pupil, Mr. James R. Brant, whose zeal and skill have alone rendered it possible for me, amid the duties of a laborious professorship, to bring my own share of the work to a conclusion. W. G.

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NEW TABLES

FOR DETERMINING THE

VALUES OF THE COEFFICIENTS,

IN THE

PERTURBATIVE FUNCTION OF PLANETARY MOTION,

WHICH DEPEND UPON THE

RATIO OF THE MEAN DISTANCES.

BY

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ACCEPTED FOR PUBLICATION

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JOSEPH HENRY,
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INTRODUCTION.

§ 1. The first important step, in the reduction of the planetary perturbations to numbers, is the determination of those coefficients in the perturbative function which depend upon the ratios of the mean distances.

This work has been done by different astronomers, but lastly and most completely by Leverrier, whose results were published in *Liouville's Journal* for 1841. Since that time Neptune and thirty-eight Asteroids have been added to the catalogue of planets.

If, through the ordinary forms of development, we wish to determine the secular and periodic inequalities of the elements of the orbits of the fifty planets, it will be necessary to find the coefficients which correspond to three hundred and sixty-four values of the ratio of the mean distances.

This estimate is upon the supposition that the mutual action of the Asteroids is neglected.

2. At Professor Peirce's suggestion, and with the approval of Commander Davis, the Superintendent of the American Ephemeris and Nautical Almanac, I have undertaken this work, as a part of the systematic labor of a thorough revision of most of the planetary theories, now being carried on, under the superintendence of Commander Davis, as fast as can be done consistently with the demands which the regular issues of the Almanac make upon the annual appropriations made by Congress for its support.

3. The following notation and forms are found in any work upon the theory of the planetary perturbations.

$$(1 - 2\,a \cos l + a^2)^{-s} = \tfrac{1}{2}\, b_s^{(0)} + b_s^{(1)} \cos l + b_s^{(2)} \cos 2\,l + b_s^{(3)} \cos 3\,l + \cdots\; b_s^{(i)} \cos i\,l.$$

$$(1)\quad b_s^{(i)} = 2\,\frac{s\,(s+1)\,(s+2)\cdots(s+i-1)}{1\cdot 2\cdot 3\cdot 4\cdots i}\, a^i \left(1 + \frac{s\,(s+i)}{1\,(i+1)}\, a^2 + \frac{s\,(s+1)\,(s+i)\,(s+i+1)}{1\cdot 2\cdot (i+1)\,(i+2)}\, a^4 + \cdots\right).$$

In these formulas a denotes the ratio, taken less than unity, of the mean distances of two planets; and $s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$, &c.

The series for computing the values of $b_{\frac{1}{2}}^{(0)}$, $b_{\frac{1}{2}}^{(1)}$, $b_{\frac{1}{2}}^{(2)}$, &c., $b_{\frac{3}{2}}^{(0)}$, $b_{\frac{3}{2}}^{(1)}$, $b_{\frac{3}{2}}^{(2)}$, &c., $b_{\frac{5}{2}}^{(0)}$, $b_{\frac{5}{2}}^{(1)}$, $b_{\frac{5}{2}}^{(2)}$, &c., or as many of them as are needed, are readily obtained from (1) by substituting the proper values of s and i.

The values of $a\, D_\alpha\, b_s^{(i)}$, $a^2\, D_\alpha^2\, b_s^{(i)}$, $a^3\, D_\alpha^3\, b_s^{(i)}$, &c., are also needed; and the series for computing them are found by taking the successive derivatives of (1) and multiplying them by a, a^2, a^3, &c.

4. We see from this, that all values of $b_s^{(i)}$ and its derivatives, any one of which may be denoted by $f(a)$, will depend upon a series of the form

$$(2) \qquad f(a) = A_0\, a^i + A_1\, a^{i+2} + A_2\, a^{i+4} + A_3\, a^{i+6} + A_4\, a^{i+8} + \cdots,$$

in which A_0, A_1, A_2, &c. are known functions of s and i; and to obtain the values of $b_s^{(i)}$ and its derivatives in the mutual action of any two planets, it is only necessary to substitute the proper value of a, and sum the series to as many terms as will give them with the requisite degree of approximation.

5. It is obvious, that not only must these series converge, which they all do, though in very different degrees, but the facility with which they can be used must depend very much upon the degree of convergency. Now it is found, that, although all the values of a fall between 0 and 0.75, these series, with but few exceptions, converge so slowly that they are nearly useless in their present form; and the great problem has been to transform them into others converging more rapidly.

This problem Leverrier has treated with ability and success, in the paper to which reference has been made. The coefficients in his transformed series have received the name of the *Leverrier Coefficients.*

6. At the request of the Superintendent of the American Ephemeris and Nautical Almanac, the values of the Leverrier Coefficients were computed by the late Sears C. Walker, assisted by Mr. Pourtalès, and are published in an Appendix to that work for 1857. This carefully prepared paper has been of great aid to me, especially the manuscript sheets containing the numerical values of the coefficients carried to a high degree of accuracy, which were better adapted to the changes demanded by the form of the following Tables.

7. Upon undertaking this work, the first question which presented itself was, How do the series giving the values of $b_s^{(i)}$, $a\,D_a\,b_s^{(i)}$, $a^2\,D_a^2\,b_s^{(i)}$, &c., vary with a? Now, for special values of s and i, these coefficients are simply functions of a; and, if their variation with reference to a is slow, terminating in low orders of differences, we may not only make this circumstance a check upon the accuracy of the work, if we compute them for equidistant values of a, but we may tabulate them with reference to a as an argument, and afterwards enter these tables with the special values of a for the system, and take out their corresponding values. If these Tables were extended from $a = 0$ to $a = 0.75$, we should include all the three hundred and sixty-four values of a; and, if the argument intervals were sufficiently small, all the coefficients could be taken from them with trifling labor. Besides, they would undoubtedly include all the planets hereafter to be discovered.

It was soon found, however, that these variations, instead of being slow, were in most cases so rapid as to make them entirely useless, with any reasonable amount of labor, for the purposes indicated.

8. But, resuming the equation

$$f(a) = A_0\, a^i + A_1\, a^{i+2} + A_2\, a^{i+4} + A_3\, a^{i+6} + A_4\, a^{i+8} + \cdots,$$

may we not find

$$f(a) = f'(a) \text{ (a transformed series)},$$

in which $f'(a)$ is an exact function of a, involving nearly the whole variation of

$f(a)$, while the transformed series shall vary so slowly with reference to a, as to be perfectly adapted to the ends already specified?

We should then tabulate $\frac{f(a)}{f'(a)}$, and, having taken a value of it from the Tables for some value of a, it would only be necessary to multiply this tabular value by the corresponding value of $f'(a)$ to obtain the required value of $f(a)$.

And if we could tabulate the logarithmic, instead of the numerical, values of $\frac{f(a)}{f'(a)}$, we should find $\log f(a)$ at once, which is always needed.

On these considerations the following Tables are based; and, fortunately, only slight and quite obvious changes in Leverrier's series were needed, thus, with corresponding modifications, making the valuable labor of Mr. Walker entirely available.

9. Since we wish to tabulate the logarithmic values of $\frac{f(a)}{f'(a)}$, it must be finite for $a = 0$.

But equation (2) fulfils this condition at once if we give it the form

$$\text{(3)} \qquad \frac{f(a)}{a^i} = A_0 + A_1 a^2 + A_2 a^4 + A_3 a^6 + A_4 a^8 + \cdots;$$

for when $a = 0$,

$$\frac{f(a)}{a^i} = A_0.$$

Now, the rate at which $\log \frac{f(a)}{a^i}$ is changing relatively to a, at any point between the limits of $a = 0$ and $a = 0.75$, is readily determined; and, if it be sufficiently slow for practical use, then the series (3) needs no further change.

Such was found to be the case with $\log \frac{b_{\frac{1}{2}}^{(i)}}{a^i}$, which is tabulated on pages 1 to 7.

10. When, however, as in most cases, the variation of the series (3) is too rapid to be practical, it may be transformed in the following manner. If we both multiply and divide it by $1 - a^2$ we shall obtain

$$\frac{f(a)}{a^i} = \frac{1}{1-a^2}\left\{A_0 + (A_1 - A_0)\, a^2 + (A_2 - A_1)\, a^4 + (A_3 - A_2)\, a^6 + \cdots\right\};$$

and by putting

$$\beta^2 = \frac{a^2}{1-a^2}, \quad A_1 - A_0 = \delta A_0, \quad A_2 - A_1 = \delta A_1, \quad A_3 - A_2 = \delta A_2, \quad \text{\&c.},$$

it becomes

$$\text{(4)} \qquad \frac{f(a)}{a^{i-2}\beta^2} = A_0 + \delta A_0\, a^2 + \delta A_1\, a^4 + \delta A_2\, a^6 + \delta A_3\, a^8 + \cdots, \text{ \&c.}$$

The whole variation of (3) from $a = 0$ to $a = 0.75$ is

$$(a) \qquad A_1 (\tfrac{3}{4})^2 + A_2 (\tfrac{3}{4})^4 + A_3 (\tfrac{3}{4})^6 + \cdots;$$

the whole variation of (4) between the same limits is

$$(b) \qquad \delta A_0 (\tfrac{3}{4})^2 + \delta A_1 (\tfrac{3}{4})^4 + \delta A_2 (\tfrac{3}{4})^6 + \cdots;$$

the difference of these variations is

$$(c) \qquad A_0 (\tfrac{3}{4})^2 + A_1 (\tfrac{3}{4})^4 + A_2 (\tfrac{3}{4})^6 + \cdots.$$

The coefficients in (3) are positive for all admissible values of s and i; and if they are all increasing, (b) will be positive, and (4) will vary more slowly than (3); but if they are all decreasing, (b) will be negative, and (4) will vary more slowly than (3) only when (b) is numerically less than (a). Otherwise (3) will be preferable to (4). But the reverse may, at the same time, be true with respect to the convergency of these series.

11. By a similar transformation of (4), putting

$$\delta A_0 - A_0 = \delta^2 A_0, \quad \delta A_1 - \delta A_0 = \delta^2 A_1, \quad \delta A_2 - \delta A_1 = \delta^2 A_2, \quad \&c.,$$

we obtain

(5) $$\frac{f(a)}{a^{i-4}\beta^4} = A_0 + \delta^2 A_0\, a^2 + \delta^2 A_1\, a^4 + \delta^2 A_2\, a^6 + \cdots.$$

Continuing the same transformations and notation, we find

(6) $$\frac{f(a)}{a^{i-6}\beta^6} = A_0 + \delta^3 A_0\, a^2 + \delta^3 A_1\, a^4 + \delta^3 A_2\, a^6 + \cdots,$$

(7) $$\frac{f(a)}{a^{i-8}\beta^8} = A_0 + \delta^4 A_0\, a^2 + \delta^4 A_1\, a^4 + \delta^4 A_2\, a^6 + \cdots,$$

(8) $$\frac{f(a)}{a^{i-10}\beta^{10}} = A_0 + \delta^5 A_0\, a^2 + \delta^5 A_1\, a^4 + \delta^5 A_2\, a^6 + \cdots,$$

&c., &c.

When we have a series corresponding to special values of s and i, that is, when the coefficients in our series have been reduced to numbers, the number of such transformations needed to obtain the series most desirable, both with respect to convergency and the amount of variation relatively to a, will be a practical question which can, in most cases, be settled by simple inspection.

It has been found practicable to tabulate the following functions,

$$\log \frac{b_{\frac{1}{2}}^{(i)}}{a^i}, \quad \log \frac{a\, D_\alpha\, b_{\frac{1}{2}}^{(i)}}{a^{i-2}\beta^2}, \quad \log \frac{a^2 D_\alpha^2\, b_{\frac{1}{2}}^{(i)}}{a^{i-4}\beta^4}, \quad \log \frac{a^3 D_\alpha^3\, b_{\frac{1}{2}}^{(i)}}{a^{i-6}\beta^6}, \quad \log \frac{a^4 D_\alpha^4\, b_{\frac{1}{2}}^{(i)}}{a^{i-8}\beta^8},$$

$$\log \frac{a\, b_{\frac{3}{2}}^{(i)}}{a^{i-3}\beta^4}, \quad \log \frac{a^2 D_\alpha\, b_{\frac{3}{2}}^{(i)}}{a^{i-5}\beta^6}, \quad \log \frac{a^3 D_\alpha^2\, b_{\frac{3}{2}}^{(i)}}{a^{i-7}\beta^8}, \quad \log \frac{a^2\, b_{\frac{5}{2}}^{(i)}}{a^{i-6}\beta^8},$$

for all values of i generally needed in the perturbative theories of the planets. These tabular values have been limited to $i = 9$; since, for values of i larger than 9, only few values of these, or any other functions, are needed.

12. The following special case will show the effect of the above transformations. In order to take the differences more readily, the series may be written vertically, with the corresponding function at the top of the column.

	$\frac{\alpha^3 D_\alpha b_{\frac{5}{2}}^{(10)}}{\alpha^{12}}$	$\frac{\alpha^3 D_\alpha b_{\frac{5}{2}}^{(10)}}{\alpha^{10}\beta^2}$	$\frac{\alpha^3 D_\alpha b_{\frac{5}{2}}^{(10)}}{\alpha^8\beta^4}$	$\frac{\alpha^3 D_\alpha b_{\frac{5}{2}}^{(10)}}{\alpha^6\beta^6}$	$\frac{\alpha^3 D_\alpha b^{(10)}}{\alpha^4\beta^8}$	$\frac{\alpha^3 D_\alpha b_{\frac{5}{2}}^{(10)}}{\alpha^2\beta^{10}}$
a^0	+ 567.35451	+ 567.35451	+ 567.35451	+576.35451	+576.35451	+576.35451
a^2	+ 1934.16309	+ 1366.80858	+ 799.45407	+232.09956	—335.25495	—902.60946
a^4	+ 4442.53086	+ 2508.36777	+1141.55919	+342.10512	+110.00556	+445.26051
a^6	+ 8494.50955	+ 4051.97869	+1543.61092	+402.05173	+ 59.94661	— 50.05895
a^8	+ 14547.79565	+ 6053.28610	+2001.30741	+457.69649	+ 55.64476	— 4.30185
a^{10}	+ 23114.83086	+ 8567.03521	+2513.74911	+512.44170	+ 54.74521	— 0.89955
a^{12}	+ 34762.53860	+11647.70774	+3080.67253	+566.92342	+ 54.48172	— 0.26349
a^{14}	+ 50112.23098	+15349.69238	+3701.98464	+621.31211	+ 54.38869	— 0.09303
a^{16}	+ 69839.57190	+19727.34092	+4377.64854	+675.66390	+ 54.35179	— 0.03690
a^{18}	+ 94674.56137	+24834.98947	+5107.64855	+730.00001	+ 54.33611	— 0.01568
a^{20}	+125401.52839	+30726.96702	+5891.97755	+784.32900	+ 54.32899	— 0.00712
a^{22}	+162859.12777	+37457.59938	+6730.63236	+838.65481	+ 54.32581	— 0.00318

Now it is evident that the series which gives the value of $\frac{a^3 D_\alpha b^{(10)}_{\frac{5}{2}}}{a^2 \beta^{10}}$ converges more rapidly than any of the preceding ones; and a little examination will show that it also converges more rapidly than the one which will result from taking the next order of differences. It is therefore the one best adapted to finding special values of $a^3 D_\alpha b^{(10)}_{\frac{5}{2}}$; but the function $\frac{a^3 D_\alpha b^{(10)}_{\frac{5}{2}}}{a^4 \beta^8}$ is much the best one of the set to tabulate, since its whole variation, between the limits of $\alpha = 0$ and $\alpha = 0.75$, is only about half of that of the two functions which immediately precede and follow it.

13. The series for computing those coefficients of which but few values can ever be needed, are given on pages 42, 43, and 44. The coefficients corresponding to such high values of i are mostly needed in the long-period terms in the theory of Venus and the Earth.

The few values of $a^4 D^3_\alpha b^{(i)}_{\frac{1}{2}}$ needed in the theory of Jupiter and Saturn, were computed from the series given by Mr. Walker.

14. Resume (4), and write it

$$(9) \qquad f(\alpha) = A_0 \alpha^{i-2} \beta^2 + \beta^2 (\delta A_0 \alpha^i + \delta A_1 \alpha^{i+2} + \delta A_2 \alpha^{i+4} + \delta A_3 \alpha^{i+6} + \cdots).$$

Now, since the series in this parenthesis has precisely the same form as (2), it may, by putting

$$\delta A_1 - \delta A_0 = \delta^2 A_0, \quad \delta A_2 - \delta A_1 = \delta^2 A_1, \quad \delta A_3 - \delta A_2 = \delta^2 A_2, \quad \&c.,$$

be written

$$\delta A_0 \alpha^{i-2} \beta^2 + \beta^2 (\delta^2 A_0 \alpha^i + \delta^2 A_1 \alpha^{i+2} + \delta^2 A_2 \alpha^{i+4} + \delta^2 A_3 \alpha^{i+6} + \cdots);$$

and, by substituting this form in (9), it becomes

$$(10) \qquad f(\alpha) = A_0 \alpha^{i-2} \beta^2 + \delta A_0 \alpha^{i-2} \beta^4 + \beta^4 (\delta^2 A_0 \alpha^i + \delta^2 A_1 \alpha^{i+2} + \delta^2 A_2 \alpha^{i+4} + \cdots).$$

By a similar transformation

$$(11) \quad f(\alpha) = A_0 \alpha^{i-2} \beta^2 + \delta A_0 \alpha^{i-2} \beta^4 + \delta^2 A_0 \alpha^{i-2} \beta^6 + \beta^6 (\delta^3 A_0 \alpha^i + \delta^3 A_1 \alpha^{i+2} + \delta^3 A_2 \alpha^{i+4} + \cdots);$$

and, continuing this process indefinitely, we finally get

$$(12) \qquad f(\alpha) = \alpha^{i-2} \beta^2 (A_0 + \delta A_0 \beta^2 + \delta^2 A_0 \beta^4 + \delta^3 A_0 \beta^6 + \delta^4 A_0 \beta^8 + \cdots \delta^n A_0 \beta^{2n}).$$

In practice, the value of $f(\alpha)$ is obtained from the most convergent of these transformed series, which may usually be determined by simple inspection.

These are the forms adopted by Leverrier; but we have derived them by a much simpler process.

15. Professor Peirce has also suggested transformations, which, as far as convergency is concerned, leave nothing to be desired; but, as most of his series are functions of differences of the required quantities, tables constructed upon this basis would not have been so practical as those we have given. Besides, it was very desirable to make the whole subject rest upon a numerical basis prepared with such great care as Mr. Walker's paper.

The coefficients A_0, δA_0, $\delta^2 A_0$, $\delta^3 A_0$, &c., are the *Leverrier Coefficients* computed by Mr. Walker.

It will be seen that these coefficients are readily converted into those necessary to compute the functions we have tabulated. For this reason, we have adopted the series involving the same order of differences as those adopted by Leverrier; although in some cases the series expressed in terms of the order preceding or following vary less rapidly relatively to a.

In constructing the following Tables, each term was computed for every .03 of a between the necessary limits, and the accuracy of the computations for all the larger terms was tested by differences. The series were then summed to as many terms as were necessary to give the last figure in the tabular logarithms accurate, and the numerical values of the function checked by differences. Logarithms of these values were then taken and checked in the same way. Next, these logarithms were interpolated to every .01 of a, and those functions of which but few values are ever needed are printed in this form; the remaining ones interpolated to every .005, are practically without third differences, and in this form they are printed. Nearly all the functions given to every .005 of a are accompanied by the logarithms of their variations for every .001 of a. These variations correspond to the value of the argument opposite to which they are written, and their differences are so small that the second differences of the functions may be taken into account without any difficulty.

Since $f'(a)$ is usually a small fraction, $f(a)$ will always be found with a high degree of accuracy.

16. In printing these Tables, the proof was read by copy, and then the page was stereotyped. Plate-proofs were then tested by taking a new set of differences, and all the errors thus found were corrected in the plates. Proofs from the corrected plates were used in preparing the Supplement. As fast as the pages of the Supplement were stereotyped, they were tested by a duplicate computation entirely independent of the first. The series printed on pages 42, 43, 44, were also computed in duplicate.

In conclusion, I must express my obligations to my friend and colleague, Mr. Isaac Bradford, for the valuable aid he has rendered me in preparing the Tables for the press, and in computing a duplicate of the Supplement.

LOGARITHMIC VALUES OF $\frac{b_{\frac{1}{2}}^{(i)}}{\alpha^i}$

α	$\log b_{\frac{1}{2}}^{(0)}$	log var. for .001 of α.	$\log \frac{b_{\frac{1}{2}}^{(1)}}{\alpha}$	log var. for .001 of α.	$\log \frac{b_{\frac{1}{2}}^{(2)}}{\alpha^2}$	log var. for .001 of α.	$\log \frac{b_{\frac{1}{2}}^{(3)}}{\alpha^3}$	log var. for .001 of α.
.000	0.3010300	— ∞	0.0000000	— ∞	9.8750613	— ∞	9.7958800	— ∞
.005	.3010328	1.0414	.0000040	1.2068	.8750655	1.2480	.7958847	1.2765
.010	.3010410	.3385	.0000161	.5106	.8750790	.5587	.7958989	.5786
.015	.3010546	.5145	.0000364	.6884	.8751017	.7364	.7959226	.7559
.020	.3010737	.6395	.0000650	.8143	.8751335	.8603	.7959559	.8814
.025	0.3010982	1.7356	0.0001017	1.9112	9.8751744	1.9571	9.7959987	1.9782
.030	.3011281	.8142	.0001466	1.9903	.8752243	2.0366	.7960511	2.0573
.035	.3011634	.8814	.0001996	2.0577	.8752832	.1031	.7961129	.1245
.040	.3012042	.9395	.0002608	.1153	.8753511	.1614	.7961843	.1824
.045	.3012504	1.9903	.0003301	.1667	.8754281	.2125	.7962652	.2338
.050	0.3013020	2.0362	0.0004076	2.2127	9.8755142	2.2586	9.7963556	2.2797
.055	.3013591	.0777	.0004933	.2543	.8756094	.3002	.7964556	.3214
.060	.3014217	.1159	.0005872	.2922	.8757138	.3381	.7965652	.3595
.065	.3014897	.1508	.0006893	.3274	.8758273	.3731	.7966844	.3945
.070	.3015632	.1833	.0007997	.3598	.8759499	.4055	.7968132	.4267
.075	0.3016422	2.2138	0.0009183	2.3901	9.8760817	2.4358	9.7969516	2.4570
.080	.3017269	.2422	.0010452	.4183	.8762227	.4642	.7970996	.4853
.085	.3018170	.2688	.0011804	.4450	.8763729	.4908	.7972573	.5120
.090	.3019127	.2940	.0013239	.4702	.8765324	.5161	.7974247	.5371
.095	.3020139	.3178	.0014757	.4940	.8767011	.5398	.7976018	.5611
.100	0.3021205	2.3405	0.0016358	2.5166	9.8768790	2.5625	9.7977887	2.5839
.105	.3022328	.3621	.0018042	.5382	.8770662	.5841	.7979854	.6054
.110	.3023507	.3827	.0019811	.5588	.8772628	.6049	.7981918	.6258
.115	.3024742	.4024	.0021664	.5785	.8774687	.6245	.7984080	.6458
.120	.3026033	.4214	.0023601	.5974	.8776841	.6434	.7986342	.6649
.125	0.3027380	2.4396	0.0025622	2.6156	9.8779088	2.6616	9.7988703	2.6831
.130	.3028785	.4572	.0027728	.6331	.8781429	.6791	.7991162	.7005
.135	.3030246	.4741	.0029919	.6501	.8783865	.6961	.7993721	.7176
.140	.3031763	.4904	.0032196	.6665	.8786395	.7125	.7996382	.7339
.145	.3033338	.5062	.0034559	.6822	.8789021	.7283	.7999142	.7497
.150	0.3034971	2.5215	0.0037008	2.6975	9.8791744	2.7436	9.8002002	2.7648
.155	.3036662	.5364	.0039543	.7124	.8794563	.7585	.8004962	.7797
.160	.3038410	.5508	.0042165	.7268	.8797478	.7729	.8008024	.7941
.165	.3040216	.5648	.0044874	.7408	.8800490	.7868	.8011187	.8081
.170	.3042081	.5784	.0047671	.7545	.8803599	.8004	.8014452	.8217
.175	0.3044005	2.5917	0.0050556	2.7678	9.8806806	2.8137	9.8017820	2.8350
.180	.3045986	.6047	.0053529	.7807	.8810111	.8266	.8021291	.8479
.185	.3048028	.6173	.0056591	.7933	.8813515	.8393	.8024866	.8606
.190	.3050129	.6296	.0059742	.8056	.8817018	.8516	.8028546	.8730
.195	.3052290	.6416	.0062982	.8176	.8820621	.8637	.8032331	.8851
.200	0.3054510	2.6533	0.0066313	2.8294	9.8824325	2.8755	9.8036223	2.8969
.205	.3056790	.6648	.0069734	.8409	.8828129	.8871	.8040220	.9085
.210	.3059131	.6761	.0073246	.8522	.8832035	.8984	.8044325	.9199
.215	.3061533	.6872	.0076849	.8633	.8836043	.9095	.8048536	.9310
.220	.3063997	.6981	.0080545	.8741	.8840153	.9204	.8052856	.9419
.225	0.3066523	2.7087	0.0084333	2.8847	9.8844366	2.9310	9.8057285	2.9526
.230	.3069110	.7191	.0088214	.8952	.8848682	.9414	.8061822	.9631
.235	.3071760	.7294	.0092189	.9055	.8853104	.9517	.8066470	.9734
.240	.3074473	.7395	.0096258	.9156	.8857629	.9618	.8071227	.9835
.245	.3077249	.7494	.0100422	.9255	.8862262	.9718	.8076096	2.9935
.250	0.3080089	2.7592	0.0104682	2.9353	9.8867001	2.9817	9.8081078	3.0035

LOGARITHMIC VALUES OF $\dfrac{b_{\frac{1}{2}}^{(i)}}{\alpha^i}$

α	$\log b_{\frac{1}{2}}^{(0)}$	log var. for .001 of α.	$\log \dfrac{b_{\frac{1}{2}}^{(1)}}{\alpha}$	log var. for .001 of α.	$\log \dfrac{b_{\frac{1}{2}}^{(2)}}{\alpha^2}$	log var. for .001 of α.	$\log \dfrac{b_{\frac{1}{2}}^{(3)}}{\alpha^3}$	log var. for .001 of α.
.250	0.3080089	2.7592	0.0104682	2.93533	9.8867001	2.98162	9.8081078	3.00328
.255	.3082993	.7689	.0109038	.94493	.8871848	2.99131	.8086172	.01292
.260	.3085962	.7784	.0113491	.95440	.8876803	3.00084	.8091380	.02243
.265	.3088996	.7877	.0118041	.96374	.8881867	.01022	.8096702	.03181
.270	.3092095	.7969	.0122689	.97296	.8887042	.01945	.8102141	.04107
.275	0.3095260	2.8060	0.0127437	2.98206	9.8892326	3.02853	9.8107695	3.05022
.280	.3098492	.8150	.0132284	.99104	.8897721	.03749	.8113367	.05925
.285	.3101791	.8238	.0137232	2.99991	.8903228	.04634	.8119157	.06816
.290	.3105157	.8325	.0142282	3.00866	.8908847	.05509	.8125066	.07696
.295	.3108591	.8412	.0147434	.01729	.8914580	.06375	.8131095	.08565
.300	0.3112093	2.8498	0.0152688	3.02581	9.8920428	3.07231	9.8137246	3.09424
.305	.3115665	.8582	.0158046	.03423	.8926392	.08077	.8143519	.10273
.310	.3119307	.8665	.0163508	.04255	.8932472	.08915	.8149914	.11113
.315	.3123019	.8748	.0169075	.05079	.8938670	.09745	.8156434	.11945
.320	.3126801	.8830	.0174748	.05895	.8944989	.10567	.8163079	.12768
.325	0.3130655	2.8910	0.0180528	3.06703	9.8951427	3.11381	9.8169851	3.13583
.330	.3134581	.8990	.0186417	.07505	.8957986	.12187	.8176751	.14390
.335	.3138580	.9070	.0192415	.08300	.8964667	.12985	.8183780	.15190
.340	.3142653	.9149	.0198523	.09088	.8971471	.13776	.8190938	.15983
.345	.3146800	.9227	.0204743	.09870	.8978400	.14562	.8198228	.16770
.350	0.3151022	2.9304	0.0211075	3.10645	9.8985454	3.15341	9.8205651	3.17550
.355	.3155320	.9381	.0217521	.11414	.8992636	.16113	.8213208	.18324
.360	.3159694	.9458	.0224081	.12177	.8999946	.16879	.8220901	.19092
.365	.3164145	.9533	.0230757	.12934	.9007386	.17639	.8228730	.19855
.370	.3168675	.9608	.0237550	.13686	.9014957	.18393	.8236697	.20613
.375	0.3173284	2.9683	0.0244461	3.14432	9.9022660	3.19142	9.8244804	3.21366
.380	.3177972	.9757	.0251492	.15174	.9030496	.19886	.8253051	.22115
.385	.3182741	.9831	.0258644	.15911	.9038467	.20626	.8261442	.22861
.390	.3187591	.9904	.0265918	.16643	.9046574	.21361	.8269978	.23603
.395	.3192523	2.9977	.0273315	.17371	.9054820	.22092	.8278662	.24340
.400	0.3197539	3.0050	0.0280836	3.18095	9.9063205	3.22818	9.8287493	3.25073
.405	.3202639	.0122	.0288484	.18816	.9071731	.23543	.8296475	.25804
.410	.3207824	.0194	.0296259	.19533	.9080401	.24265	.8305608	.26530
.415	.3213095	.0265	.0304163	.20246	.9089216	.24985	.8314895	.27252
.420	.3218454	.0336	.0312199	.20957	.9098177	.25703	.8324336	.27970
.425	0.3223901	3.0407	0.0320366	3.21665	9.9107288	3.26418	9.8333935	3.28684
.430	.3229438	.0478	.0328667	.22371	.9116551	.27130	.8343693	.29395
.435	.3235065	.0548	.0337104	.23075	.9125966	.27839	.8353612	.30104
.440	.3240783	.0619	.0345680	.24776	.9135537	.28545	.8363693	.30810
.445	.3246595	.0689	.0354394	.24475	.9145263	.29248	.8373940	.31515
.450	0.3252500	3.0759	0.0363250	3.25172	9.9155148	3.29948	9.8384353	3.32219
.455	.3258502	.0828	.0372248	.25866	.9165192	.30645	.8394938	.32921
.460	.3264600	.0897	.0381391	.26559	.9175398	.31341	.8405695	.33623
.465	.3270796	.0966	.0390681	.27252	.9185770	.32037	.8416628	.34325
.470	.3277091	.1035	.0400120	.27943	.9196308	.32732	.8427738	.35025
.475	0.3283488	3.1104	0.0409710	3.28632	9.9207017	3.33125	9.8439029	3.35724
.480	.3289986	.1173	.0419454	.29320	.9217898	.34117	.8450504	.36422
.485	.3296589	.1242	.0429354	.30007	.9228954	.34809	.8462164	.37118
.490	.3303297	.1311	.0439411	.30693	.9240187	.35501	.8474011	.37814
.495	.3310112	.1379	.0449628	.31377	.9251600	.36192	.8486050	.38509
.500	0.3317034	3.1447	0.0460006	3.32061	9.9263197	3.36883	9.8498283	3.39205

LOGARITHMIC VALUES OF $\frac{b_{\frac{1}{2}}^{(i)}}{a^i}$

a	$\log b_{\frac{1}{2}}^{(0)}$	log var. for .001 of a.	$\log \frac{b_{\frac{1}{2}}^{(1)}}{a}$	log var. for .001 of a.	$\log \frac{b_{\frac{1}{2}}^{(2)}}{a^2}$	log var. for .001 of a.	$\log \frac{b_{\frac{1}{2}}^{(3)}}{a^3}$	log var. for .001 of a.
.500	0.3317034	3.14472	0.0460006	3.32061	9.9263197	3.36883	9.8498283	3.39205
.505	.3324067	.15159	.0470550	.32746	.9274980	.37574	.8510714	.39900
.510	.3331213	.15846	.0481260	.33431	.9286951	.38265	.8523345	.40595
.515	.3338472	.16533	.0492141	.34116	.9299114	.38955	.8536180	.41290
.520	.3345847	.17221	.0503196	.34801	.9311472	.39645	.8549222	.41985
.525	0.3353340	3.17909	0.0514426	3.35486	9.9324028	3.40336	9.8562475	3.42681
.530	.3360952	.18598	.0525835	.36171	.9336786	.41028	.8575942	.43378
.535	.3368686	.19288	.0537426	.36858	.9349749	.41721	.8589627	.44075
.540	.3376545	.19979	.0549200	.37546	.9362921	.42415	.8603533	.44774
.545	.3384530	.20670	.0561163	.38236	.9376305	.43110	.8617665	.45475
.550	0.3392641	3.21361	0.0573318	3.38926	9.9389905	3.43806	9.8632028	3.46177
.555	.3400883	.22053	.0585668	.39616	.9403725	.44504	.8646625	.46880
.560	.3409258	.22747	.0598216	.40308	.9417769	.45204	.8661460	.47585
.565	.3417768	.23444	.0610966	.41003	.9432042	.45905	.8676538	.48292
.570	.3426415	.24143	.0623923	.41700	.9446547	.46608	.8691865	.49001
.575	0.3435203	3.24844	0.0637089	3.42400	9.9461289	3.47313	9.8707443	3.49711
.580	.3444135	.25547	.0650469	.43102	.9476273	.48020	.8723279	.50424
.585	.3453213	.26252	.0664068	.43805	.9491503	.48728	.8739378	.51140
.590	.3462439	.26960	.0677889	.44511	.9506984	.49440	.8755746	.51859
.595	.3471817	.27671	.0691938	.45220	.9522722	.50155	.8772387	.52581
.600	0.3481352	3.28385	0.0706219	3.45931	9.9538720	3.50873	9.8789308	3.53306
.605	.3491044	.29104	.0720735	.46645	.9554987	.51594	.8806513	.54033
.610	.3500898	.29826	.0735493	.47362	.9571527	.52319	.8824010	.54763
.615	.3510917	.30550	.0750497	.48081	.9588346	.53047	.8841803	.55498
.620	.3521105	.31278	.0765751	.48805	.9605450	.53779	.8859901	.56236
.625	0.3531466	3.32010	0.0781263	3.49534	9.9622846	3.54516	9.8878310	3.56979
.630	.3542004	.32746	.0797036	.50266	.9640540	.55257	.8897038	.57726
.635	.3552723	.33486	.0813080	.51002	.9658539	.56001	.8916092	.58478
.640	.3563626	.34230	.0829399	.51742	.9676851	.56750	.8935480	.59234
.645	.3574719	.34980	.0845999	.52488	.9695482	.57505	.8955210	.59995
.650	0.3586005	3.35735	0.0862889	3.53239	9.9714441	3.58264	9.8975290	3.60761
.655	.3597490	.36495	.0880074	.53995	.9733735	.59028	.8995728	.61532
.660	.3609178	.37261	.0897560	.54757	.9753374	.59798	.9016534	.62309
.665	.3621075	.38034	.0915358	.55526	.9773364	.60573	.9037717	.63093
.670	.3633187	.38813	.0933476	.56301	.9793717	.61354	.9059286	.63883
.675	0.3645519	3.39599	0.0951921	3.57081	9.9814440	3.62142	9.9081254	3.64680
.680	.3658077	.40392	.0970703	.57869	.9835545	.62936	.9103631	.65484
.685	.3670868	.41191	.0989830	.58665	.9857040	.63738	.9126428	.66294
.690	.3683897	.41997	.1009312	.59467	.9878939	.64547	.9149655	.67111
.695	.3697172	.42811	.1029158	.60277	.9901251	.65364	.9173325	.67936
.700	0.3710699	3.43633	0.1049381	3.61095	9.9923988	3.66190	9.9197451	3.68770
.705	.3724486	.44464	.1069990	.61921	.9947164	.67025	.9222048	.69613
.710	.3738540	.45304	.1090996	.62756	.9970793	.67869	.9247131	.70465
.715	.3752870	.46153	.1112412	.63599	9.9994889	.68724	.9272713	.71327
.720	.3767484	.47011	.1134250	.64450	0.0019466	.69589	.9298809	.72198
.725	0.3782393	3.47880	0.1156523	3.65310	0.0044542	3.70465	9.9325436	3.73078
.730	.3797604	.48758	.1179244	.66179	.0070132	.71352	.9352612	.73967
.735	.3813128	.49647	.1202426	.67056	.0096253	.72251	.9380354	.74867
.740	.3828975	.50546	.1226084	.67942	.0122922	.73161	.9408680	.75777
.745	.3845155	.51455	.1250231	.68837	.0150159	.74082	.9437609	.76697
.750	0.3861679	3.52374	0.1274884	3.69741	0.0177985	3.75015	9.9467161	3.77627

LOGARITHMIC VALUES OF $\frac{b_{\frac{1}{2}}^{(i)}}{\alpha^i}$

α	$\log \frac{b_{\frac{1}{2}}^{(4)}}{\alpha^4}$	log var. for .001 of α.	$\log \frac{b_{\frac{1}{2}}^{(5)}}{\alpha^5}$	log var. for .001 of α.	$\log \frac{b_{\frac{1}{2}}^{(6)}}{\alpha^6}$	$\log \frac{b_{\frac{1}{2}}^{(7)}}{\alpha^7}$	$\log \frac{b_{\frac{1}{2}}^{(8)}}{\alpha^8}$	$\log \frac{b_{\frac{1}{2}}^{(9)}}{\alpha^9}$
.000	9.737888	$-\infty$	9.692131	$-\infty$	9.65434	9.62216	9.59413	9.56931
.005	.737894	0.3222	.692137	0.3222	.65435	.62217	.59414	.56932
.010	.737909	.5911	.692151	.5911	.65436	.62218	.59415	.56933
.015	.737934	.7643	.692176	.7782	.65439	.62220	.59417	.56936
.020	.737967	.8921	.692211	.9031	.65442	.62224	.59421	.56939
.025	9.738011	0.9868	9.692256	0.9956	9.65447	9.62229	9.59426	9.56944
.030	.738064	1.0682	.692310	0.0719	.65452	.62234	.59431	.56949
.035	.738128	.1367	.692374	1.1399	.65458	.62240	.59437	.56955
.040	.738201	.1931	.692448	.1987	.65466	.62248	.59445	.56963
.045	.738284	.2455	.692532	.2504	.65474	.62257	.59454	.56972
.050	9.738377	1.2923	9.692626	1.2967	9.65484	9.62266	9.59464	9.56982
.055	.738480	.3344	.692730	.3404	.65495	.62277	.59475	.56993
.060	.738593	.3711	.692845	.3784	.65506	.62289	.59487	.57005
.065	.738715	.4065	.692969	.4133	.65518	.62302	.59500	.57018
.070	.738848	.4393	.693104	.4472	.65532	.62315	.59514	.57032
.075	9.738991	1.4698	9.693249	1.4771	9.65547	9.62330	9.59529	9.57047
.080	.739143	.4983	.693404	.5065	.65562	.62346	.59545	.57063
.085	.739306	.5238	.693569	.5315	.65579	.62363	.59562	.57080
.090	.739477	.5478	.693744	.5563	.65597	.62382	.59580	.57098
.095	.739659	.5740	.693930	.5821	.65616	.62401	.59599	.57117
.100	9.739852	1.5977	9.694126	1.6043	9.65635	9.62421	9.59619	9.57138
.105	.740055	.6181	.694332	.6263	.65656	.62442	.59640	.57160
.110	.740267	.6375	.694549	.6474	.65678	.62464	.59663	.57182
.115	.740489	.6571	.694776	.6665	.65701	.62487	.59687	.57205
.120	.740721	.6766	.695013	.6857	.65726	.62511	.59711	.57229
.125	9.740964	1.6955	9.695261	1.7042	9.65751	9.62536	9.59736	9.57255
.130	.741217	.7127	.695519	.7217	.65777	.62562	.59763	.57281
.135	.741480	.7300	.695788	.7388	.65804	.62589	.59791	.57309
.140	.741754	.7459	.696067	.7544	.65833	.62618	.59819	.57338
.145	.742038	.7619	.696356	.7701	.65863	.62648	.59848	.57368
.150	9.742332	1.7774	9.696656	1.7853	9.65893	9.62678	9.59879	9.57400
.155	.742637	.7924	.696966	.8000	.65924	.62710	.59911	.57432
.160	.742952	.8069	.697287	.8143	.65957	.62742	.59943	.57465
.165	.743278	.8209	.697618	.8280	.65991	.62776	.59977	.57499
.170	.743614	.8344	.697960	.8420	.66025	.62811	.60012	.57535
.175	9.743961	1.8476	9.698313	1.8549	9.66061	9.62847	9.60049	9.57572
.180	.744318	.8609	.698676	.8675	.66098	.62885	.60087	.57609
.185	.744686	.8733	.699050	.8808	.66136	.62923	.60126	.57648
.190	.745065	.8854	.699436	.8943	.66175	.62962	.60165	.57688
.195	.745455	.8976	.699833	.9063	.66215	.63003	.60206	.57729
.200	9.745855	1.9096	9.700243	1.9185	9.66257	9.63045	9.60249	9.57772
.205	.746266	.9212	.700663	.9299	.66300	.63088	.60292	.57816
.210	.746688	.9325	.701095	.9415	.66343	.63132	.60337	.57860
.215	.747122	.9435	.701537	.9523	.66387	.63177	.60383	.57905
.220	.747566	.9547	.701991	.9633	.66433	.63223	.60430	.57952
.225	9.748022	1.9652	9.702456	1.9731	9.66480	9.63270	9.60478	9.58000
.230	.748489	.9745	.702931	.9832	.66529	.63319	.60527	.58049
.235	.748967	.9854	.703418	1.9934	.66579	.63369	.60577	.58100
.240	.749456	1.9956	.703915	2.0030	.66629	.63419	.60628	.58152
.245	.749957	2.0060	.704425	.0137	.66680	.63471	.60680	.58205
.250	9.750470	2.0162	9.704947	2.0237	9.66733	9.63524	9.60734	9.58259

LOGARITHMIC VALUES OF $\frac{b_{\frac{1}{2}}^{(i)}}{a^i}$

α	$\log \frac{b_{\frac{1}{2}}^{(4)}}{a^4}$	log var. for .001 of α.	$\log \frac{b_{\frac{1}{2}}^{(5)}}{a^5}$	log var. for .001 of α.	$\log \frac{b_{\frac{1}{2}}^{(6)}}{a^6}$	$\log \frac{b_{\frac{1}{2}}^{(7)}}{a^7}$	$\log \frac{b_{\frac{1}{2}}^{(8)}}{a^8}$	$\log \frac{b_{\frac{1}{2}}^{(9)}}{a^9}$
.250	9.750470	2.0154	9.704947	2.0237	9.66733	9.63524	9.60734	9.58259
.255	.750994	.0253	.705481	.0338	.66787	.63579	.60789	.58315
.260	.751530	.0350	.706028	.0434	.66842	.63634	.60845	.58372
.265	.752078	.0445	.706587	.0531	.66898	.63691	.60902	.58430
.270	.752638	.0539	.707158	.0622	.66957	.63750	.60962	.58490
.275	9.753210	2.0630	9.707741	2.0715	9.67016	9.63809	9.61022	9.58551
.280	.753794	.0719	.708337	.0803	.67076	.63870	.61083	.58612
.285	.754390	.0810	.708945	.0892	.67137	.63932	.61146	.58675
.290	.754999	.0899	.709564	.0980	.67200	.63996	.61210	.58739
.295	.755620	.0986	.710197	.1065	.67264	.64061	.61275	.58805
.300	9.756253	2.1072	9.710842	2.1153	9.67330	9.64128	9.61342	9.58872
.305	.756900	.1156	.711500	.1235	.67396	.64196	.61410	.58940
.310	.757558	.1239	.712171	.1319	.67464	.64265	.61479	.59010
.315	.758230	.1323	.712855	.1405	.67533	.64335	.61550	.59081
.320	.758914	.1406	.713552	.1489	.67604	.64407	.61622	.59154
.325	9.759611	2.1489	9.714263	2.1572	9.67676	9.64480	9.61695	9.59228
.330	.760322	.1569	.714988	.1655	.67750	.64554	.61770	.59303
.335	.761047	.1650	.715727	.1735	.67825	.64630	.61846	.59380
.340	.761784	.1732	.716479	.1813	.67901	.64707	.61924	.59458
.345	.762535	.1807	.717245	.1892	.67979	.64786	.62003	.59538
.350	9.763300	2.1884	9.718025	2.1971	9.68058	9.64866	9.62084	9.59618
.355	.764079	.1962	.718819	.2049	.68139	.64947	.62166	.59700
.360	.764871	.2039	.719628	.2127	.68221	.65030	.62249	.59784
.365	.765678	.2117	.720452	.2204	.68305	.65114	.62334	.59869
.370	.766499	.2191	.721290	.2281	.68390	.65200	.62421	.59956
.375	9.767334	2.2269	9.722143	2.2358	9.68477	9.65287	9.62509	9.60045
.380	.768184	.2345	.723011	.2433	.68565	.65376	.62599	.60136
.385	.769050	.2420	.723894	.2507	.68655	.65466	.62690	.60228
.390	.769929	.2494	.724792	.2581	.68745	.65559	.62783	.60322
.395	.770824	.2567	.725705	.2653	.68838	.65652	.62877	.60417
.400	9.771735	2.2639	9.726634	2.2725	9.68932	9.65747	9.62973	9.60513
.405	.772661	.2711	.727578	.2797	.69028	.65844	.63070	.60611
.410	.773602	.2783	.728538	.2869	.69125	.65942	.63169	.60711
.415	.774559	.2856	.729514	.2940	.69224	.66042	.63270	.60812
.420	.775532	.2927	.730505	.3013	.69325	.66144	.63372	.60917
.425	9.776521	2.2999	9.731514	2.3086	9.69428	9.66248	9.63475	9.61022
.430	.777527	.3071	732540	.3160	.69532	.66353	.63580	.61129
.435	.778550	.3143	.733583	.3233	.69638	.66460	.63688	.61238
.440	.779589	.3214	.734645	.3304	.69746	.66569	.63797	.61348
.445	.780646	.3286	.735724	.3375	.69856	.66680	.63908	.61461
.450	9.781721	2.3357	9.736821	2.3446	9.69966	9.66791	9.64022	9.61574
.455	.782814	.3428	.737936	.3517	.70079	.66905	.64137	.61692
.460	.783923	.3496	.739069	.3587	.70194	.67022	.64254	.61812
.465	.785050	.3568	.740221	.3657	.70311	.67141	.64373	.61935
.470	.786197	.3638	.741391	.3727	.70429	.67261	.64494	.62058
.475	9.787362	2.3709	9.742580	2.3797	9.70549	9.67383	9.64617	9.62183
.480	.788546	.3777	.743789	.3867	.70672	.67508	.64743	.62310
.485	.789749	.3849	.745018	.3939	.70797	.67635	.64870	.62440
.490	.790972	.3920	.746266	.4009	.70923	.67763	.65000	.62571
.495	.792215	.3990	.747535	.4081	.71051	.67893	.65132	.62704
.500	9.793478	2.4057	9.748825	2.4150	9.71182	9.68026	9.65266	9.62840

LOGARITHMIC VALUES OF $\frac{b_{\frac{1}{2}}^{(i)}}{\alpha^i}$

α	$\log \frac{b_{\frac{1}{2}}^{(4)}}{\alpha^4}$	log var. for .001 of α.	$\log \frac{b_{\frac{1}{2}}^{(5)}}{\alpha^5}$	log var. for .001 of α.	$\log \frac{b_{\frac{1}{2}}^{(6)}}{\alpha^6}$	$\log \frac{b_{\frac{1}{2}}^{(7)}}{\alpha^7}$	$\log \frac{b_{\frac{1}{2}}^{(8)}}{\alpha^8}$	$\log \frac{b_{\frac{1}{2}}^{(9)}}{\alpha^9}$
.500	9.793478	2.4057	9.748825	2.4150	9.71182	9.68026	9.65266	9.62840
.505	.794761	.4128	.750136	.4223	.71315	.68161	.65402	.62977
.510	.796065	.4196	.751469	.4292	.71450	.68298	.65541	.63117
.515	.797390	.4267	.752823	.4362	.71587	.68437	.65682	.63259
.520	.798736	.4338	.754199	.4433	.71727	.68578	.65825	.63404
.525	9.800104	2.4408	9.755598	2.4503	9.71869	9.68722	9.65970	9.63551
.530	.801495	.4478	.757019	.4572	.72014	.68868	.66118	.63699
.535	.802909	.4547	.758463	.4643	.72161	.69016	.66268	.63850
.540	.804346	.4616	.759932	.4716	.72311	.69167	.66421	.64002
.545	.805806	.4688	.761425	.4784	.72463	.69320	.66576	.64157
.550	9.807289	2.4758	9.762941	2.4852	9.72617	9.69476	9.66734	9.64314
.555	.808797	.4830	.764482	.4925	.72774	.69634	.66895	.64474
.560	.810330	.4900	.766049	.4997	.72933	.69795	.67058	.64637
.565	.811889	.4972	.767642	.5068	.73095	.69959	.67224	.64803
.570	.813474	.5042	.769260	.5138	.73259	.70125	.67392	.64970
.575	9.815084	2.5115	9.770906	2.5211	9.73427	9.70294	9.67563	9.65140
.580	.816721	.5185	.772580	.5282	.73597	.70466	.67737	.65314
.585	.818386	.5257	.774282	.5357	.73770	.70641	.67914	.65491
.590	.820078	.5329	.776014	.5428	.73946	.70819	.68093	.65670
.595	.821799	.5402	.777773	.5504	.74125	.71000	.68275	.65851
.600	9.823549	2.5475	9.779563	2.5575	9.74306	9.71185	9.68460	9.66037
.605	.825329	.5551	.781385	.5651	.74491	.71372	.68649	.66227
.610	.827139	.5624	.783237	.5722	.74679	.71563	.68841	.66420
.615	.828980	.5698	.785121	.5799	.74870	.71757	.69036	.66617
.620	.830853	.5773	.787038	.5872	.75065	.71954	.69235	.66817
.625	9.832758	2.5848	9.788988	2.5949	9.75263	9.72154	9.69438	9.67021
.630	.834696	.5923	.790972	.6022	.75465	.72359	.69644	.67228
.635	.836669	.5999	.792991	.6100	.75671	.72567	.69854	.67440
.640	.838676	.6075	.795046	.6172	.75880	.72779	.70068	.67655
.645	.840719	.6151	.797137	.6253	.76093	.72995	.70286	.67874
.650	9.842798	2.6228	9.799266	2.6328	9.76310	9.73214	9.70508	9.68097
.655	.844915	.6306	.801433	.6409	.76531	.73437	.70734	.68325
.660	.847070	.6385	.803641	.6485	.76755	.73665	.70964	.68556
.665	.849265	.6464	.805889	.6568	.76984	.73897	.71198	.68793
.670	.851500	.6544	.808178	.6645	.77217	.74133	.71437	.69034
.675	9.853777	2.6624	9.810510	2.6729	9.77454	9.74374	9.71680	9.69280
.680	.856096	.6705	.812887	.6808	.77696	.74619	.71928	.69530
.685	.858458	.6787	.815309	.6894	.77943	.74869	.72181	.69786
.690	.860866	.6869	.817779	.6974	.78194	.75124	.72438	.70046
.695	.863321	.6952	.820296	.7062	.78450	.75384	.72700	.70312
.700	9.865823	2.7036	9.822863	2.7143	9.78712	9.75649	9.72968	9.70583
.705	.868374	.7121	.825481	.7232	.78979	.75919	.73241	.70860
.710	.870976	.7206	.828149	.7315	.79249	.76195	.73520	.71141
.715	.873630	.7291	.830873	.7406	.79526	.76477	.73805	.71429
.720	.876338	.7377	.833651	.7490	.79809	.76764	.74095	.71722
.725	9.879102	2.7465	9.836487	2.7582	9.80099	9.77057	9.74391	9.72021
.730	.881923	.7556	.839382	.7668	.80395	.77356	.74694	.72327
.735	.884803	.7647	.842337	.7761	.80698	.77661	.75003	.72639
.740	887745	.7739	.845353	.7847	.81008	.77973	.75319	.72957
.745	.890750	.7835	.848433	.7941	.81326	.78292	.75642	.73282
.750	9.893819	2.7935	9.851577	2.8037	9.81650	9.78617	9.75972	9.73613

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$

α	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(0)}}{\beta^2} = \log \frac{\alpha^2 D_\alpha b_{\frac{1}{2}}^{(1)}}{\beta^2}$	log var. for .001 of α.	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(2)}}{\beta^2}$	log var. for .001 of α.	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(3)}}{\alpha \beta^2}$	log var. for .001 of α.	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(4)}}{\alpha^2 \beta^2}$	log var. for .001 of α.
.000	0.0000000	$-\infty$	0.1760913	$-\infty$	0.2730013	$-\infty$	0.3399481	$-\infty$
.005	.0000012	0.7160	.1760895	0.8573	.2729984	1.0719	.3399445	1.1523
.010	.0000052	1.0253	.1760841	1.1583	.2729896	.3749	.3399338	.4533
.015	.0000119	.2041	.1760751	.3384	.2729749	.5490	.3399161	.6294
.020	.0000213	.3304	.1760624	.4624	.2729543	.6739	.3398913	.7528
.025	0.0000334	1.4281	0.1760461	1.5611	0.2729278	1.7708	0.3398595	1.8488
.030	.0000482	.5078	.1760261	.6375	.2728954	.8500	.3398207	.9284
.035	.0000657	.5798	.1760026	.7041	.2728571	.9170	.3397748	1.9956
.040	.0000861	.6355	.1759755	.7634	.2728129	1.9740	.3397218	2.0530
.045	.0001091	.6902	.1759447	.8142	.2727629	2.0253	.3396618	.1045
.050	0.0001349	1.7340	0.1759103	1.8597	0.2727069	2.0704	0.3395947	2.1498
.055	.0001635	.7782	.1758723	.9020	.2726451	.1119	.3395206	.1920
.060	.0001949	.8142	.1758306	.9395	.2725774	.1504	.3394393	.2294
.065	.0002290	.8512	.1757853	1.9740	.2725037	.1852	.3393510	.2648
.070	.0002659	.8819	.1757364	2.0069	.2724241	.2175	.3392555	.2971
.075	0.0003054	1.9138	0.1756838	2.0366	0.2723386	2.2479	0.3391529	2.3271
.080	.0003478	.9405	.1756276	.0652	.2722471	.2760	.3390431	.3552
.085	.0003928	.9675	.1755677	.0906	.2721497	.3027	.3389262	.3824
.090	.0004406	1.9921	.1755043	.1166	.2720463	.3275	.3388020	.4069
.095	.0004911	2.0170	.1754371	.1398	.2719370	.3514	.3386708	.4310
.100	0.0005445	2.0390	0.1753663	2.1613	0.2718217	2.3740	0.3385323	2.4536
.105	.0006005	.0599	.1752920	.1835	.2717004	.3955	.3383866	.4751
.110	.0006593	.0806	.1752139	.2035	.2715731	.4159	.3382337	.4955
.115	.0007209	.1003	.1751322	.2232	.2714398	.4355	.3380736	.5156
.120	.0007853	.1186	.1750468	.2415	.2713005	.4542	.3379061	.5340
.125	0.0008524	2.1373	0.1749578	2.2600	0.2711552	2.4724	0.3377315	2.5524
.130	.0009224	.1541	.1748650	.2769	.2710038	.4897	.3375495	.5696
.135	.0009951	.1714	.1747686	.2940	.2708464	.5064	.3373603	.5868
.140	.0010707	.1869	.1746684	.3096	.2706829	.5224	.3371637	.6027
.145	.0011490	.2030	.1745646	.3255	.2705134	.5380	.3369597	.6189
.150	0.0012302	2.2180	0.1744570	2.3400	0.2703378	2.5531	0.3367483	2.6336
.155	.0013142	.2325	.1743458	.3549	.2701561	.5677	.3365295	.6483
.160	.0014010	.2465	.1742308	.3684	.2699682	.5818	.3363033	.6624
.165	.0014906	.2610	.1741122	.3824	.2697742	.5957	.3360698	.6763
.170	.0015832	.2741	.1739898	.3955	.2695740	.6091	.3358287	.6896
.175	0.0016786	2.2883	0.1738637	2.4082	0.2693676	2.6222	0.3355802	2.7026
.180	.0017770	.3001	.1737339	.4206	.2691550	.6349	.3353243	.7155
.185	.0018782	.3126	.1736004	.4329	.2689362	.6474	.3350608	.7280
.190	.0019823	.3247	.1734631	.4446	.2687111	.6593	.3347897	.7400
.195	.0020893	.3364	.1733221	.4563	.2684798	.6711	.3345111	.7520
.200	0.0021992	2.3475	0.1731773	2.4674	0.2682422	2.6825	0.3342248	2.7636
.205	.0023120	.3587	.1730288	.4788	.2679984	.6939	.3339309	.7748
.210	.0024277	.3699	.1728764	.4894	.2677482	.7048	.3336294	.7859
.215	.0025464	.3809	.1727203	.4999	.2674917	.7155	.3333202	.7970
.220	.0026681	.3916	.1725604	.5100	.2672288	.7260	.3330031	.8075
.225	0.0027928	2.4021	0.1723968	2.5203	0.2669595	2.7364	0.3326783	2.8179
.230	.0029205	.4123	.1722293	.5299	.2666838	.7465	.3323456	.8281
.235	.0030512	.4222	.1720581	.5398	.2664017	.7565	.3320051	.8382
.240	.0031849	.4320	.1718830	.5490	.2661131	.7662	.3316567	.8480
.245	.0033216	.4418	.1717042	.5582	.2658180	.7758	.3313004	.8578
.250	0.0034614	2.4511	0.1715214	2.5672	0.2655164	2.7852	0.3309360	2.8673

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$

α	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(0)}}{\beta^2} = \log \frac{\alpha^2 D_\alpha b_{\frac{1}{2}}^{(1)}}{\beta^2}$	log var. for .001 of α.	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(2)}}{\beta^2}$	log var. for .001 of α.	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(3)}}{\alpha \beta^2}$	log var. for .001 of α.	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(4)}}{\alpha^2 \beta^2}$	log var. for .001 of α.
.250	0.0034614	2.4511	0.1715214	2.5672	0.2655164	2.7852	0.3309360	2.8673
.255	.0036042	.4604	.1713349	.5762	.2652083	.7943	.3305637	.8766
.260	.0037501	.4696	.1711444	.5851	.2648935	.8033	.3301833	.8858
.265	.0038991	.4787	.1709501	.5938	.2645722	.8122	.3297949	.8949
.270	.0040512	.4876	.1707519	.6024	.2642443	.8211	.3293982	.9039
.275	0.0042064	2.4964	0.1705499	2.6108	0.2639097	2.8299	0.3289934	2.9127
.280	.0043648	.5050	.1703439	.6190	.2635684	.8385	.3285803	.9214
.285	.0045262	.5134	.1701341	.6271	.2632204	.8469	.3281590	.9300
.290	.0046908	.5217	.1699203	.6350	2628656	.8552	.3277293	.9384
.295	.0048585	.5299	.1697026	.6428	.2625040	.8634	.3272913	.9467
.300	0.0050295	2.5380	0.1694811	2.6505	0.2621355	2.8714	0.3268448	2.9549
.305	.0052037	.5461	.1692555	.6581	.2617603	.8794	.3263899	.9630
.310	.0053813	.5541	.1690260	.6656	.2613780	.8873	.3259264	.9710
.315	.0055621	.5620	.1687926	.6730	.2609889	.8951	.3254543	.9789
.320	.0057462	.5699	.1685551	.6802	.2605927	.9027	.3249736	.9868
.325	0.0059336	2.5777	0.1683137	2.6873	0.2601896	2.9102	0.3244842	2.9946
.330	.0061244	.5853	.1680683	.6944	.2597794	.9177	.3239860	3.0023
.335	.0063186	.5928	.1678189	.7014	.2593621	.9251	.3234790	.0099
.340	.0065161	.6002	.1675654	.7083	.2589376	.9324	.3229631	0174
.345	.0067170	.6076	.1673079	.7152	.2585060	.9397	.3224383	.0248
.350	0.0069213	2.6149	0.1670463	2.7220	0.2580671	2.9469	0.3219044	3.0321
.355	.0071290	.6222	.1667807	.7287	.2576210	.9540	.3213615	.0393
.360	.0073403	.6294	.1665109	.7353	.2571676	.9610	.3208095	.0465
.365	.0075550	.6365	.1662371	.7418	.2567068	.9679	.3202482	.0537
.370	.0077732	.6435	.1659591	.7482	.2562386	.9748	.3196776	.0608
.375	0.0079949	2.6505	0.1656771	2.7545	0.2557630	2.9817	0.3190976	3.0679
.380	.0082203	.6574	.1653909	.7608	.2552798	.9885	.3185083	.0849
.385	.0084492	.6643	.1651007	.7670	.2547891	2.9952	.3179094	.0919
.390	.0086818	.6711	.1648062	.7731	.2542908	3.0019	.3173010	.0988
.395	.0089180	.6778	.1645076	.7792	.2537849	.0085	.3166828	.1056
.400	0.0091580	2.6845	0.1642048	2.7852	0.2532712	3.0150	0.3160549	3.1023
.405	.0094016	.6911	.1638978	.7912	.2527498	.0215	.3154172	.1090
.410	.0096491	.6977	.1635865	.7971	.2522205	.0279	.3147696	.1157
.415	.0099004	.7043	.1632711	.8029	.2516834	.0343	.3141120	.1223
.420	.0101554	.7108	.1629513	.8087	.2511382	.0406	.3134444	.1289
.425	0.0104142	2.7173	0.1626273	2.8144	0.2505852	3.0469	0.3127666	3.1354
.430	.0106770	.7237	.1622990	.8200	.2500240	.0531	.3120785	.1419
.435	.0109436	.7301	.1619664	.8256	.2494548	.0593	.3113801	.1483
.440	.0112143	.7365	.1616296	.8312	.2488773	.0655	.3106712	.1547
.445	.0114889	.7429	.1612884	.8368	.2482916	.0717	.3099518	.1611
.450	0.0117677	2.7492	0.1609428	2.8423	0.2476976	3.0779	0.3092218	3.1675
.455	.0120504	.7555	.1605929	.8478	.2470953	.0840	.3084810	.1738
.460	.0123373	.7618	.1602386	.8532	.2464844	.0900	.3077293	.1801
.465	.0126283	.7681	.1598799	.8585	.2458651	.0960	.3069666	.1864
.470	.0129236	.7744	.1595167	.8638	.2452372	.1020	3061928	.1927
.475	0.0132230	2.7806	0.1591492	2.8690	0.2446005	3.1079	0.3054078	3.1990
.480	.0135269	.7867	.1587771	.8742	.2439550	.1138	.3046114	.2052
.485	.0138349	.7928	.1584006	.8793	.2433008	.1197	.3038036	.2114
.490	.0141474	.7989	.1580197	.8844	.2426378	.1255	.3029843	.2176
.495	.0144642	.8050	.1576342	.8895	.2419657	.1313	.3021533	.2238
.500	0.0147856	2.8111	0.1572442	2.8946	0.2412846	3.1371	0.3013104	3.2299

LOGARITHMIC VALUES OF $f(\alpha)\,.\,\alpha\, D_\alpha\, b_{\frac{1}{2}}^{(i)}$

α	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(0)}}{\beta^2} = \log \frac{\alpha^2 D_\alpha b_{\frac{1}{2}}^{(1)}}{\beta^2}$	log var. for .001 of α.	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(2)}}{\beta^2}$	log var. for .001 of α.	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(3)}}{\alpha \beta^2}$	log var. for .001 of α.	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(4)}}{\alpha^2 \beta^2}$	log var. for .001 of α.
.500	0.0147856	2.8111	0.1572442	2.8946	0.2412846	3.13716	0.3013104	3.22984
.505	.0151114	.8172	.1568496	.8996	.2405943	.14290	.3004556	.23598
.510	.0154419	.8233	.1564504	.9046	.2398948	.14866	.2995886	.24204
.515	.0157770	.8293	.1560467	.9096	.2391860	.15442	.2987095	.24812
.520	.0161169	.8353	.1556384	.9145	.2384678	.16011	.2978180	.25416
.525	0.0164615	2.8413	0.1552254	2.9194	0.2377401	3.16584	0.2969140	3.26021
.530	.0168108	.8473	.1548079	.9242	.2370028	.17149	.2959973	.26628
.535	.0171650	.8533	.1543857	.9290	.2362559	.17713	.2950678	.27231
.540	.0175242	.8593	.1539588	.9337	.2354991	.18281	.2941253	.27830
.545	.0178883	.8653	.1535272	.9384	.2347325	.18842	.2931698	.28430
.550	0.0182574	2.8712	0.1530910	2.9431	0.2339559	3.19401	0.2922009	3.29030
.555	.0186316	.8772	.1526500	.9477	.2331692	.19959	.2912186	.29627
.560	.0190110	.8831	.1522042	.9523	.2323724	.20515	.2902227	.30224
.565	.0193957	.8891	.1517537	.9569	.2315653	.21075	.2892130	.30822
.570	.0197858	.8950	.1512984	.9615	.2307478	.21627	.2881893	.31416
.575	0.0201812	2.9010	0.1508383	2.9661	0.2299198	3.22182	0.2871515	3.32015
.580	.0205821	.9069	.1503734	.9706	.2290811	.22737	.2860993	.32609
.585	.0209885	.9129	.1499037	.9751	.2282317	.23289	.2850326	.33207
.590	.0214005	.9189	.1494290	.9796	.2273714	.23845	.2839511	.33806
.595	.0218183	.9249	.1489495	.9840	.2265001	.24398	.2828546	.34402
.600	0.0222419	2.9309	0.1484651	2.9884	0.2256176	3.24949	0.2817429	3.34998
.605	.0226713	.9369	.1479758	.9927	.2247239	.25493	.2806159	.35595
.610	.0231067	.9429	.1474816	2.9970	.2238189	.26041	.2794733	.36192
.615	.0235481	.9489	.1469824	3.0013	.2229024	.26590	.2783149	.36788
.620	.0239958	.9549	.1464783	.0056	.2219742	.27142	.2771404	.37390
.625	0.0244497	2.9610	0.1459691	3.0099	0.2210342	3.27688	0.2759495	3.37989
.630	.0249100	.9671	.1454550	.0141	.2200823	.28235	.2747421	.38589
.635	.0253768	.9732	.1449358	.0183	.2191184	.28780	.2735179	.39189
.640	.0258501	.9793	.1444116	.0225	.2181423	.29327	.2722766	.39794
.645	.0263301	.9854	.1438824	.0267	.2171538	.29877	.2710179	.40398
.650	0.0268170	2.9915	0.1433481	3.0308	0.2161527	3.30423	0.2697416	3.41000
.655	.0273108	2.9977	.1428088	.0349	.2151389	.30972	.2684475	.41604
.660	.0278117	3.0039	.1422643	.0390	.2141122	.31521	.2671352	.42210
.665	.0283197	.0101	.1417148	.0430	.2130725	.32069	.2658044	.42820
.670	.0288350	.0163	.1411602	.0470	.2120196	.32618	.2644548	.43431
.675	0.0293578	3.0225	0.1406005	3.0510	0.2109533	3.33167	0.2630860	3.44044
.680	.0298882	.0288	.1400356	.0549	.2098734	.33713	.2616977	.44657
.685	.0304264	.0351	.1394657	.0588	.2087798	.34265	.2602897	.45274
.690	.0309724	.0414	.1388907	.0627	.2076722	.34815	.2588615	.45891
.695	.0315265	.0478	.1383106	.0665	.2065505	.35368	.2574128	.46512
.700	0.0320889	3.0542	0.1377253	3.0703	0.2054144	3.35919	0.2559433	3.47132
.705	.0326597	.0607	.1371349	.0741	.2042639	.36470	.2544526	.47758
.710	.0332390	.0672	.1365392	.0778	.2030985	.37029	.2529401	.48387
.715	.0338271	.0738	.1359384	.0815	.2019182	.37581	.2514056	.49016
.720	.0344242	.0804	.1353325	.0852	.2007226	.38141	.2498487	.49649
.725	0.0350304	3.0870	0.1347215	3.0889	0.1995116	3.38696	0.2482688	3.50284
.730	.0356461	.0937	.1341054	.0924	.1982850	.39256	.2466656	.50923
.735	.0362714	.1005	.1334843	.0959	.1970425	.39812	.2450386	.51566
.740	.0369065	.1073	.1328581	.0994	.1957840	.40367	.2433872	.52210
.745	.0375516	.1141	.1322271	.1029	.1945092	.40929	.2417111	.52861
.750	0.0382071	3.1210	0.1315910	3.1063	0.1932178	3.41490	0.2400096	3.53511

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$

α	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(5)}}{\alpha^3 \beta^2}$	log var. for .001 of α.	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(6)}}{\alpha^4 \beta^2}$	log var. for .001 of α.	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(7)}}{\alpha^5 \beta^2}$	log var. for .001 of α.	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(8)}}{\alpha^6 \beta^2}$	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(9)}}{\alpha^7 \beta^2}$
.000	0.3911007	$-\infty$	0.432493	$-\infty$	0.467257	$-\infty$	0.49722	0.52355
.005	.3910968	1.1931	.432489	0.2304	.467252	0.2552	.49721	.52354
.010	.3910851	.4941	.432476	.5315	.467239	.5315	.49720	.52353
.015	.3910656	.6702	.432455	.6989	.467218	.7060	.49718	.52351
.020	.3910383	.7938	.432426	.8195	.467188	.8400	.49715	.52348
.025	0.3910033	1.8909	0.432389	0.9191	0.467149	0.9395	0.49711	0.52344
.030	.3909605	1.9712	.432343	1.0000	.467101	1.0213	.49706	.52339
.035	.3909098	2.0382	.432289	.0644	.467044	.0863	.49700	.52333
.040	.3908513	.0952	.432227	.1239	.466979	.1431	.49694	.52326
.045	.3907851	.1467	.432156	.1732	.466905	.1931	.49686	.52318
.050	0.3907111	2.1925	0.432078	1.1931	0.466823	1.2355	0.49677	0.52309
.055	.3906293	.2345	.431991	.2600	.466733	.2767	.49667	.52299
.060	.3905396	.2723	.431896	.2966	.466634	.3180	.49657	.52289
.065	.3904421	.3075	.431793	.3304	.466525	.3541	.49646	.52277
.070	.3903367	.3396	.431682	.3635	.466408	.3858	.49634	.52265
.075	0.3902235	2.3699	0.431562	1.3979	0.466282	1.4149	0.49621	0.52251
.080	.3901024	.3983	.431433	.4281	.466148	.4440	.49607	.52237
.085	.3899734	.4245	.431295	.4533	.466005	.4713	.49592	.52222
.090	.3898365	.4499	.431149	.4771	.465853	.4969	.49577	.52206
.095	.3896916	.4736	.430995	.5004	.465692	.5185	.49560	.52189
.100	0.3895388	2.4964	0.430833	1.5237	0.465522	1.5415	0.49543	0.52171
.105	.3893780	.5180	.430662	.5465	.465344	.5635	.49524	.52152
.110	.3892092	.5383	.430482	.5655	.465157	.5843	.49505	.52132
.115	.3890325	.5580	.430294	.5843	.464961	.6042	.49485	.52111
.120	.3888478	.5770	.430097	.6042	.464756	.6232	.49464	.52090
.125	0.3886550	2.5953	0.429892	1.6232	0.464541	1.6414	0.49441	0.52067
.130	.3884541	.6125	.429679	.6395	.464318	.6589	.49418	.52044
.135	.3882452	.6294	.429457	.6570	.464086	.6757	.49394	.52019
.140	.3880282	.6456	.429225	.6739	.463845	.6919	.49369	.51994
.145	.3878031	.6612	.428984	.6902	.463595	.7075	.49343	.51967
.150	0.3875699	2.6765	0.428735	1.7058	0.463337	1.7226	0.49317	0.51940
.155	.3873285	.6912	.428478	.7193	.463069	.7372	.49290	.51911
.160	.3870788	.7052	.428213	.7324	.462792	.7512	.49261	.51882
.165	.3868209	.7195	.427939	.7466	.462506	.7649	.49232	.51851
.170	.3865547	.7329	.427656	.7604	.462210	.7781	.49201	.51820
.175	0.3862802	2.7460	0.427364	1.7738	0.461905	1.7917	0.49169	0.51788
.180	.3859974	.7589	.427063	.7867	.461591	.8048	.49137	.51755
.185	.3857062	.7714	.426753	.7993	.461267	.8169	.49103	.51721
.190	.3854067	.7836	.426434	.8115	.460934	.8292	.49069	.51686
.195	.3850988	.7954	.426106	.8234	.460592	.8413	.49033	.51650
.200	0.3847825	2.8071	0.425769	1.8350	0.460240	1.8537	0.48997	0.51613
.205	.3844576	.8185	.425424	.8463	.459878	.8657	.48960	.51574
.210	.3841241	.8297	.425069	.8573	.459507	.8762	.48922	.51535
.215	.3837820	.8406	.424705	.8680	.459127	.8870	.48882	.51495
.220	.3834314	.8514	.424331	.8785	.458737	.8976	.48842	.51454
.225	0.3830721	2.8617	0.423948	1.8898	0.458337	1.9085	0.48801	0.51412
.230	.3827041	.8720	.423556	.8998	.457927	.9190	.48759	.51369
.235	.3823274	.8821	.423155	.9095	.457508	.9294	.48716	.51324
.240	.3819419	.8921	.422744	.9190	.457079	.9390	.48671	.51279
.245	.3815475	.9019	.422324	.9294	.456640	.9484	.48626	.51232
.250	0.3811442	2.9113	0.421894	1.9385	0.456191	1.9581	0.48580	0.51185

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$

α	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(5)}}{\alpha^3 \beta^2}$	log var. for .001 of α.	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(6)}}{\alpha^4 \beta^2}$	log var. for .001 of α.	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(7)}}{\alpha^5 \beta^2}$	log var. for .001 of α.	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(8)}}{\alpha^6 \beta^2}$	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(9)}}{\alpha^7 \beta^2}$
.250	0.3811442	2.9113	0.421894	1.9385	0.456191	1.9581	0.48580	0.51185
.255	.3807320	.9207	.421455	.9482	.455732	.9675	.48532	.51136
.260	.3803108	.9300	.421006	.9578	.455263	.9768	.48484	.51086
.265	.3798805	.9392	.420547	.9673	.454783	.9860	.48434	.51035
.270	.3794411	.9483	.420078	.9766	.454293	1.9952	.48384	.50984
.275	0.3789926	2.9572	0.419599	1.9858	0.453793	2.0043	0.48332	0.50931
.280	.3785348	.9660	.419110	1.9948	.453283	.0133	.48280	.50877
.285	.3780678	.9747	.418612	2.0037	.452762	.0222	.48226	.50821
.290	.3775915	.9832	.418104	.0124	.452231	.0310	.48171	.50765
.295	.3771059	.9916	.417586	.0210	.451688	.0398	.48115	.50707
.300	0.3766108	2.9999	0.417058	2.0294	0.451135	2.0484	0.48058	0.50649
.305	.3761062	3.0081	.416519	.0376	.450571	.0567	.48000	.50589
.310	.3755920	.0162	.415970	.0456	.449997	.0648	.47941	.50529
.315	.3750682	.0242	.415411	.0534	.449411	.0728	.47881	.50467
.320	.3745347	.0321	.414841	.0612	.448815	.0807	.47819	.50404
.325	0.3739914	3.0400	0.414260	2.0689	0.448208	2.0885	0.47756	0.50339
.330	.3734381	.0478	.413669	.0766	.447589	.0963	.47692	.50274
.335	.3728749	.0555	.413067	.0842	.446959	.1041	.47627	.50207
.340	.3723017	.0631	.412454	.0918	.446317	.1118	.47560	.50139
.345	.3717185	.0706	.411830	.0993	.445664	.1195	.47493	.50069
.350	0.3711251	3.0781	0.411196	2.1068	0.445000	2.1271	0.47424	0.49999
.355	.3705214	.0855	.410551	.1143	.444324	.1347	.47354	.49927
.360	.3699074	.0928	.409895	.1218	.443637	.1422	.47283	.49854
.365	.3692830	.1001	.409227	.1292	.442938	.1496	.47211	.49779
.370	.3686481	.1073	.408548	.1366	.442227	.1570	.47137	.49704
.375	0.3680027	3.1145	0.407857	2.1439	0.441503	2.1643	0.47062	0.49627
.380	.3673465	.1216	.407155	.1511	.440767	.1715	.46986	.49549
.385	.3666795	.1286	.406441	.1581	.440018	.1786	.46908	.49469
.390	.3660017	.1356	.405715	.1651	.439257	.1857	.46829	.49389
.395	.3653130	.1425	.404978	.1720	.438483	.1927	.46749	.49307
.400	0.3646132	3.1494	0.404229	2.1789	0.437697	2.1997	0.46668	0.49224
.405	.3639023	.1563	.403468	.1859	.436898	.2068	.46585	.49139
.410	.3631801	.1631	.402694	.1928	.436086	.2139	.46501	.49053
.415	.3624465	.1699	.401908	.1987	.435262	.2210	.46416	.48965
.420	.3617013	.1767	.401109	.2056	.434424	.2280	.46329	.48876
.425	0.3609445	3.1834	0.400298	2.2135	0.433572	2.2350	0.46241	0.48785
.430	.3601760	.1900	.399474	.2203	.432707	.2419	.46151	.48693
.435	.3593956	.1965	.398637	.2271	.431828	.2488	.46060	.48599
.440	.3586033	.2030	.397787	.2339	.430936	.2556	.45967	.48504
.445	.3577989	.2095	.396923	.2407	.430030	.2623	.45873	.48407
.450	0.3569822	3.2160	0.396046	2.2475	0.429109	2.2690	0.45778	0.48309
.455	.3561533	.2226	.395155	.2542	.428174	.2757	.45681	.48209
.460	.3553118	.2291	.394252	.2608	.427225	.2824	.45582	.48108
.465	.3544577	.2356	.393335	.2673	.426261	.2891	.45482	.48005
.470	.3535908	.2421	.392403	.2737	.425282	.2957	.45380	.47900
.475	0.3527110	3.2486	0.391457	2.2801	0.424287	2.3023	0.45277	0.47794
.480	.3518181	.2550	.390497	.2865	.423277	.3089	.45173	.47686
.485	.3509121	.2614	.389522	.2929	.422252	.3154	.45067	.47576
.490	.3499927	.2678	.388533	.2994	.421212	.3219	.44959	.47465
.495	.3490599	.2741	.387529	.3059	.420155	.3284	.44849	.47352
.500	0.3481133	3.2804	0.386510	2.3124	0.419082	2.3348	0.44738	0.47237

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$

α	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(5)}}{\alpha^3 \beta^2}$	log var. for .001 of α.	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(6)}}{\alpha^4 \beta^2}$	log var. for .001 of α.	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(7)}}{\alpha^5 \beta^2}$	log var. for .001 of α.	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(8)}}{\alpha^6 \beta^2}$	$\log \frac{\alpha D_\alpha b_{\frac{1}{2}}^{(9)}}{\alpha^7 \beta^2}$
.500	0.3481133	3.28044	0.386510	2.3124	0.419082	2.3348	0.44738	0.47237
.505	.3471527	.28673	.385476	.3189	.417993	.3413	.44624	.47121
.510	.3461781	.29301	.384426	.3254	.416887	.3478	.44509	.47003
.515	.3451893	.29925	.383361	.3319	.415765	.3543	.44392	.46883
.520	.3441862	.30548	.382280	.3384	.414626	.3608	.44274	.46761
.525	0 3431686	3.31175	0.381182	2.3448	0.413469	2.3673	0.44154	0.46637
.530	.3421362	.31802	.380068	.3512	.412295	.3738	.44032	.46512
.535	.3410889	.32424	.378938	.3575	.411104	.3803	.43908	.46384
.540	.3400264	.33049	.377792	.3638	.409894	.3868	.43782	.46255
.545	.3389486	.33668	.376628	.3701	.408666	.3933	.43654	.46123
.550	0.3378551	3.34298	0.375447	2.3764	0.407420	2.3998	0.43525	0.45990
.555	.3367458	.34920	.374249	.3828	.406155	.4063	.43394	.45854
.560	.3356206	.35537	.373033	.3892	.404871	.4128	.43260	.45716
.565	.3344792	.36162	.371798	.3956	.403567	.4193	.43124	.45575
.570	.3333212	.36788	.370545	.4020	.402244	.4258	.42986	.45433
.575	0.3321464	3.37408	0.369274	2.4084	0.400902	2.4323	0.42846	0.45288
.580	.3309547	.38028	.367984	.4148	.399539	.4389	.42704	.45142
.585	.3297460	.38650	.366674	.4212	.398154	.4455	.42560	.44993
.590	.3285197	.39277	.365344	.4276	.396747	.4521	.42413	.44842
.595	.3272756	.39905	.363995	.4341	.395319	.4587	.42264	.44688
.600	0.3260133	3.40532	0.362626	2.4406	0.393870	2.4654	0.42113	0.44532
.605	.3247328	.41152	.361237	.4471	.392399	.4720	.41960	.44373
.610	.3234338	.41777	.359827	.4536	.390905	.4786	.41804	.44212
.615	.3221159	.42407	.358395	.4601	.389388	.4852	.41646	.44048
.620	.3207788	.43037	.356941	.4666	.387848	.4919	.41485	.43882
.625	0.3194221	3.43670	0.355466	2.4732	0.386284	2.4986	0.41321	0.43713
.630	.3180454	.44305	.353968	.4798	.384696	.5053	.41155	.43542
.635	.3166485	.44935	.352448	.4864	.383083	.5121	.40986	.43368
.640	.3152312	.45567	.350904	.4930	.381445	.5189	.40815	.43190
.645	.3137931	.46202	.349338	.4996	.379781	.5257	.40641	.43010
.650	0.3123337	3.46845	0.347747	2.5062	0.378090	2.5325	0.40464	0.42827
.655	.3108524	.47492	.346132	.5129	.376372	.5394	.40284	.42641
.660	.3093489	.48136	.344491	.5196	.374627	.5463	.40102	.42452
.665	.3078230	.48779	.342825	.5263	.372854	.5532	.39917	.42260
.670	.3062742	.49426	.341133	.5330	.371053	.5601	.39728	.42064
.675	0.3047022	3.50079	0.339414	2.5397	0.369223	2.5670	0.39536	0.41865
.680	.3031063	.50729	.337668	.5465	.367363	.5740	.39341	.41663
.685	.3014863	.51383	.335894	.5533	.365473	.5811	.39143	.41457
.690	.2998416	.52013	.334092	.5602	.363552	.5882	.38941	.41248
.695	.2981716	.52706	.332261	.5672	.361599	.5953	.38736	.41035
.700	0.2964760	3.53370	0.330400	2.5742	0.359613	2.6025	0.38528	0.40818
.705	.2947542	.54038	.328509	.5812	.357594	.6098	.38316	.40598
.710	.2930056	.54711	.326588	.5883	.355541	.6172	.38101	.40374
.715	.2912296	.55388	.324637	.5954	.353453	.6246	.37882	.40146
.720	.2894256	.56059	.322652	.6026	.351328	.6320	.37659	.39913
.725	0.2875930	3.56755	0.320633	2.6098	0.349167	2.6395	0.37432	0.39676
.730	.2857311	.57447	.318580	.6170	.346968	.6470	.37201	.39435
.735	.2838391	.58147	.316493	.6243	.344731	.6545	.36966	.39190
.740	.2819163	.58852	.314370	.6316	.342455	.6621	.36727	.38940
.745	.2799618	.59563	.312211	.6390	.340139	.6697	.36484	.38686
.750	0.2779750	3.60278	0.310015	2.6464	0.337781	2.6774	0.36236	0.38427

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$

α	$\log \frac{\alpha^4 D_\alpha^2 b_{\frac{1}{2}}^{(0)}}{\beta^4}$	log var. for .001 of α.	$\log \frac{\alpha^3 D_\alpha^2 b_{\frac{1}{2}}^{(1)}}{\beta^4}$	log var. for .001 of α.	$\log \frac{\alpha^4 D_\alpha^2 b_{\frac{1}{2}}^{(2)}}{\beta^4}$	log var. for .001 of α.	$\log \frac{\alpha^3 D_\alpha^2 b_{\frac{1}{2}}^{(3)}}{\beta^4}$	log var. for .001 of α.
.000	0.0000000	$-\infty$	0.3521825	$-\infty$	0.1760913	$-\infty$	0.5740313	$-\infty$
.005	.0000148	1.77525	.3521835	0.5688	.1760967	1.3344	.5740254	1.3729
.010	.0000596	2.07737	.3521862	0.8513	.1761130	.6375	.5740077	.6739
.015	.0001343	.25310	.3521907	1.0334	.1761402	.8142	.5739783	.8488
.020	.0002387	.37785	.3521970	.1643	.1761781	1.9395	.5739371	1.9740
.025	0.0003730	2.47480	0.3522052	1.2600	0.1762268	2.0358	0.5738842	2.0704
.030	.0005371	.55376	.3522151	.3384	.1762863	.1152	.5738196	.1492
.035	.0007309	.62055	.3522269	.4048	.1763569	.1818	.5737432	.2164
.040	.0009545	.67834	.3522404	.4624	.1764385	.2400	.5736550	.2746
.045	.0012078	.72933	.3522558	.5132	.1765309	.2909	.5735550	.3259
.050	0.0014906	2.77481	0.3522730	1.5587	0.1766340	2.3364	0.5734433	2.3714
.055	.0018030	.81584	.3522921	.6020	.1767480	.3780	.5733198	.4129
.060	.0021451	.85345	.3523129	.6395	.1768729	.4159	.5731845	.4505
.065	.0025166	.88784	.3523356	.6739	.1770086	.4505	.5730375	.4854
.070	.0029174	.91960	.3523601	.7058	.1771551	.4828	.5728787	.5174
.075	0.0033476	2.94919	0.3523865	1.7372	0.1773125	2.5127	0.5727082	2.5475
.080	.0038070	2.97681	.3524146	.7649	.1774806	.5403	.5725259	.5756
.085	.0042956	3.00277	.3524446	.7910	.1776596	.5664	.5723318	.6018
.090	.0048133	.02710	.3524764	.8155	.1778495	.5913	.5721260	.6265
.095	.0053600	.05015	.3525100	.8401	.1780502	.6151	.5719085	.6501
.100	0.0059356	3.07188	0.3525454	1.8621	0.1782617	2.6373	0.5716791	2.6724
.105	.0065400	.09251	.3525827	.8831	.1784840	.6586	.5714380	.6937
.110	.0071730	.11214	.3526218	.9031	.1787172	.6787	.5711851	.7140
.115	.0078346	.13085	.3526628	.9232	.1789613	.6979	.5709205	.7332
.120	.0085247	.14873	.3527056	.9415	.1792162	.7165	.5706442	.7516
.125	0.0092431	3.16584	0.3527502	1.9600	0.1794819	2.7341	0.5703561	2.7694
.130	.0099897	.18219	.3527967	.9777	.1797583	.7511	.5700563	.7864
.135	.0107644	.19794	.3528451	1.9938	.1800455	.7674	.5697447	.8027
.140	.0115672	.21309	.3528953	2.0103	.1803435	.7832	.5694214	.8185
.145	.0123977	.22753	.3529474	.0253	.1806524	.7983	.5690863	.8338
.150	0.0132558	3.24150	0.3530013	2.0406	0.1809721	2.8130	0.5687395	2.8485
.155	.0141415	.25498	.3530571	.0553	.1813025	.8272	.5683809	.8627
.160	.0150545	.26797	.3531147	.0696	.1816437	.8410	.5680106	.8764
.165	.0159948	.28049	.3531743	.0828	.1819958	.8543	.5676286	.8897
.170	.0169621	.29257	.3532357	.0962	.1823586	.8671	.5672348	.9026
.175	0.0179563	3.30421	0.3532991	2.1092	0.1827322	2.8796	0.5668293	2.9153
.180	.0189774	.31563	.3533643	.1219	.1831166	.8918	.5664121	.9274
.185	.0200250	.32667	.3534315	.1341	.1835118	.9038	.5659832	.9393
.190	.0210989	.33730	.3535005	.1461	.1839177	.9153	.5655425	.9505
.195	.0221991	.34761	.3535715	.1577	.1843344	.9264	.5650901	.9622
.200	0.0233253	3.35759	0.3536443	2.1696	0.1847618	2.9374	0.5646260	2.9731
.205	.0244774	.36732	.3537190	.1807	.1852000	.9480	.5641501	.9838
.210	.0256552	.37673	.3537957	.1914	.1856489	.9583	.5636626	2.9942
.215	.0268584	.38592	.3538744	.2019	.1861085	.9686	.5631633	3.0044
.220	.0280869	.39484	.3539550	.2127	.1865789	.9784	.5626524	.0143
.225	0.0293405	3.40350	0.3540375	2.2227	0.1870600	2.9881	0.5621298	3.0241
.230	.0306191	.41192	.3541219	.2325	.1875519	2.9975	.5615955	.0336
.235	.0319224	.42018	.3542083	.2425	.1880545	3.0069	.5610495	.0429
.240	.0332503	.42813	.3542967	.2519	.1885678	.0159	.5604919	.0520
.245	.0346024	.43591	.3543871	.2615	.1890918	.0248	.5599226	.0609
.250	0.0359786	3.44349	0.3544794	2.2709	0.1896265	3.0335	0.5593416	3.0696

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$

α	$\log \frac{\alpha^4 D_\alpha^2 b_{\frac{1}{2}}^{(0)}}{\beta^4}$	log var. for .001 of α.	$\log \frac{\alpha^3 D_\alpha^2 b_{\frac{1}{2}}^{(1)}}{\beta^4}$	log var. for .001 of α.	$\log \frac{\alpha^4 D_\alpha^2 b_{\frac{1}{2}}^{(2)}}{\beta^4}$	log var. for .001 of α.	$\log \frac{\alpha^3 D_\alpha^2 b_{\frac{1}{2}}^{(3)}}{\beta^4}$	log var. for .001 of α.
.250	0.0359786	3.44349	0.3544794	2.2709	0.1896265	3.0335	0.5593416	3.0696
.255	.0373787	.45086	.3545737	.2801	.1901721	.0420	.5587489	.0781
.260	.0388026	.45808	.3546699	.2891	.1907281	.0504	.5581446	.0865
.265	.0402500	.46509	.3547681	.2979	.1912950	.0586	.5575288	.0947
.270	.0417206	.47191	.3548683	.3066	.1918725	.0666	.5569014	.1027
.275	0.0432142	3.47860	0.3549706	2.3155	0.1924607	3.0745	0.5562623	3.1105
.280	.0447307	.48509	.3550750	.3238	.1930595	.0823	.5556116	.1182
.285	.0462698	.49145	.3551813	.3320	.1936691	.0898	.5549493	.1257
.290	.0478313	.49763	.3552897	.3402	.1942893	.0973	.5542754	.1332
.295	.0494150	.50369	.3554002	.3483	.1949202	.1047	.5535900	.1406
.300	0.0510207	3.50962	0.3555127	2.3564	0.1955618	3.1119	0.5528930	3.1478
.305	.0526481	.51540	.3556273	.3643	.1962141	.1190	.5521845	.1549
.310	.0542970	.52106	.3557439	.3721	.1968770	.1260	.5514644	.1618
.315	.0559672	.52654	.3558627	.3797	.1975505	.1328	.5507328	.1686
.320	.0576586	.53193	.3559835	.3871	.1982346	.1395	.5499897	.1753
.325	0.0593709	3.53721	0.3561064	2.3944	0.1989295	3.1462	0.5492351	3.1820
.330	.0611038	.54236	.3562314	.4017	.1996350	.1528	.5484690	.1886
.335	.0628571	.54738	.3563586	.4090	.2003512	.1593	.5476915	.1950
.340	.0646305	.55226	.3564879	.4162	.2010780	.1657	.5469026	.2012
.345	.0664238	.55708	.3566194	.4234	.2018154	.1720	.5461023	.2074
.350	0.0682369	3.56177	0.3567530	2.4305	0.2025634	3.1782	0.5452907	3.2135
.355	.0700695	.56637	.3568888	.4374	.2033221	.1842	.5444677	.2194
.360	.0719213	.57085	.3570267	.4443	.2040915	.1902	.5436334	.2253
.365	.0737922	.57526	.3571669	.4511	.2048715	.1961	.5427877	.2311
.370	.0756819	.57955	.3573093	.4578	.2056621	.2019	.5419307	.2368
.375	0.0775902	3.58377	0.3574539	2.4644	0.2064634	3.2076	0.5410625	3.2424
.380	.0795169	.58787	.3576007	.4710	.2072753	.2133	.5401830	.2479
.385	.0814617	.59190	.3577497	.4776	.2080978	.2189	.5392924	.2534
.390	.0834245	.59581	.3579009	.4842	.2089309	.2244	.5383907	.2588
.395	.0854049	.59970	.3580545	.4907	.2097747	.2299	.5374776	.2642
.400	0.0874028	3.60347	0.3582104	2.4971	0.2106291	3.2353	0.5365537	3.2694
.405	.0894179	.60715	.3583686	.5035	.2114941	.2407	.5356185	.2745
.410	.0914500	.61077	.3585291	.5098	.2123698	.2460	.5346723	.2796
.415	.0934989	.61430	.3586920	.5161	.2132561	.2512	.5337152	.2846
.420	.0955644	.61778	.3588572	.5223	.2141530	.2564	.5327472	.2895
.425	0.0976463	3.62118	0.3590248	2.5284	0.2150606	3.2615	0.5317682	3.2943
.430	.0997443	.62449	.3591948	.5345	.2159789	.2665	.5307783	.2990
.435	.1018582	.62774	.3593672	.5406	.2169079	.2715	.5297776	.3036
.440	.1039878	.63091	.3595420	.5466	.2178475	.2764	.5287662	.3082
.445	.1061330	.63405	.3597193	.5526	.2187977	.2813	.5277440	.3128
.450	0.1082935	3.63711	0.3598990	2.5585	0.2197585	3.2861	0.5267112	3.3173
.455	.1104690	.64008	.3600812	.5644	.2207300	.2909	.5256678	.3217
.460	.1126594	.64302	.3602658	.5703	.2217122	.2956	.5246137	.3260
.465	.1148645	.64588	.3604529	.5761	.2227051	.3003	.5235491	.3303
.470	.1170839	.64866	.3606426	.5819	.2237087	.3049	.5224741	.3345
.475	0.1193175	3.65141	0.3608318	2.5877	0.2247230	3.3095	0.5213888	3.3387
.480	.1215652	.65412	.3610295	.5935	.2257481	.3140	.5202932	.3428
.485	.1238268	.65675	.3612268	.5992	.2267838	.3185	.5191873	.3468
.490	.1261019	.65933	.3614267	.6049	.2278303	.3230	.5180713	.3507
.495	.1283905	.66185	.3616293	.6106	.2288875	.3274	.5169452	.3546
.500	0.1306923	3.66433	0.3618345	2.6162	0.2299554	3.3318	0.5158090	3.3584

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$

α	$\log \frac{\alpha^4 D_\alpha^2 b_{\frac{1}{2}}^{(0)}}{\beta^4}$	log var. for .001 of α.	$\log \frac{\alpha^3 D_\alpha^2 b_{\frac{1}{2}}^{(1)}}{\beta^4}$	log var. for .001 of α.	$\log \frac{\alpha^4 D_\alpha^2 b_{\frac{1}{2}}^{(2)}}{\beta^4}$	log var. for .001 of α.	$\log \frac{\alpha^3 D_\alpha^2 b_{\frac{1}{2}}^{(3)}}{\beta^4}$	log var. for .001 of α.
.500	0.1306923	3.66433	0.3618345	2.6162	0.2299554	3.3318	0.5158090	3.3584
.505	.1330072	.66677	.3620424	.6218	.2310341	.3361	.5146628	.3621
.510	.1353349	.66913	.3622531	.6274	.2321236	.3404	.5135068	.3658
.515	.1376753	.67146	.3624664	.6330	.2332239	.3447	.5123410	.3694
.520	.1400280	.67374	.3626825	.6385	.2343350	.3489	.5111656	.3729
.525	0.1423930	3.67598	0.3629013	2.6440	0.2354569	3.3531	0.5099807	3.3764
.530	.1447701	.67817	.3631229	.6495	.2365898	.3573	.5087863	.3798
.535	.1471592	.68032	.3633474	.6550	.2377335	.3614	.5075826	.3832
.540	.1495601	.68242	.3635746	.6605	.2388882	.3655	.5063696	.3865
.545	.1519724	.68449	.3638047	.6659	.2400538	.3696	.5051474	.3898
.550	0.1543961	3.68651	0.3640378	2.6713	0.2412303	3.3736	0.5039161	3.3930
.555	.1568310	.68849	.3642738	.6767	.2424178	.3776	.5026759	.3961
.560	.1592768	.69042	.3645128	.6821	.2436162	.3816	.5014270	.3991
.565	.1617335	.69234	.3647548	.6875	.2448256	.3856	.5001694	.4021
.570	.1642009	.69420	.3649998	.6929	.2460460	.3895	.4989032	.4050
.575	0.1666788	3.69603	0.3652478	2.6983	0.2472775	3.3934	0.4976287	3.4078
.580	.1691671	.69782	.3654989	.7036	.2485201	.3973	.4963459	.4105
.585	.1716656	.69958	.3657531	.7090	.2497738	.4011	.4950550	.4132
.590	.1741740	.70129	.3660105	.7144	.2510387	.4049	.4937562	.4158
.595	.1766923	.70298	.3662711	.7197	.2523148	.4087	.4924496	.4184
.600	0.1792203	3.70465	0.3665348	2.7250	0.2536021	3.4125	0.4911355	3.4209
.605	.1817580	.70628	.3668018	.7303	.2549007	.4164	.4898139	.4233
.610	.1843050	.70786	.3670721	.7357	.2562106	.4202	.4884850	.4257
.615	.1868613	.70942	.3673458	.7410	.2575319	.4239	.4871491	.4280
.620	.1894267	.71094	.3676228	.7463	.2588647	.4276	.4858064	.4302
.625	0.1920011	3.71246	0.3679033	2.7516	0.2602089	3.4313	0.4844570	3.4323
.630	.1945844	.71394	.3681872	.7569	.2615646	.4350	.4831010	.4343
.635	.1971764	.71538	.3684746	.7622	.2629318	.4387	.4817388	.4363
.640	.1997770	.71681	.3687655	.7675	.2643106	.4424	.4803707	.4382
.645	.2023860	.71820	.3690600	.7728	.2657011	.4461	.4789968	.4400
.650	0.2050034	3.71958	0.3693581	2.7781	0.2671033	3.4497	0.4776174	3.4417
.655	.2076289	.72093	.3696599	.7834	.2685173	.4533	.4762327	.4432
.660	.2102625	.72225	.3699655	.7887	.2699431	.4569	.4748431	.4447
.665	.2129041	.72354	.3702748	.7940	.2713808	.4605	.4734488	.4461
.670	.2155535	.72482	.3705879	.7994	.2728305	.4641	.4720501	.4474
.675	0.2182107	3.72607	0.3709049	2.8048	0.2742922	3.4777	0.4706472	3.4486
.680	.2208755	.72732	.3712259	.8102	.2757661	.4713	.4692405	.4497
.685	.2235479	.72854	.3715509	.8156	.2772522	.4749	.4678303	.4508
.690	.2262277	.72971	.3718800	.8210	.2787505	.4785	.4664168	.4517
.695	.2289148	.73089	.3722132	.8264	.2802612	.4821	.4650005	.4525
.700	0.2316091	3.73204	0.3725505	2.8318	0.2817844	3.4856	0.4635819	3.4532
.705	.2343105	.73320	.3728921	.8373	.2833202	.4891	.4621612	.4538
.710	.2370190	.73432	.3732380	.8427	.2848686	.4927	.4607387	.4541
.715	.2397345	.73542	.3735882	.8482	.2864298	.4963	.4593149	.4544
.720	.2424568	.73651	.3739429	.8537	.2880039	.4999	.4578904	.4546
.725	0.2451859	3.73759	0.3743022	2.8592	0.2895911	3.5035	0.4564654	3.4548
.730	.2479217	.73865	.3746660	.8647	.2911913	.5070	.4550404	.4548
.735	.2506642	.73970	.3750345	.8702	.2928047	.5105	.4536158	.4545
.740	.2534133	.74073	.3754077	.8757	.2944315	.5141	.4521922	.4541
.745	.2561688	.74175	.3757858	.8812	.2960718	.5177	.4507701	.4537
.750	0.2589307	3.74276	0.3761687	2.8867	0.2977257	3.5213	0.4493497	3.4532

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$

α	$\log \frac{\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(4)}}{\beta^4}$	log var. for .001 of α.	α	$\log \frac{\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(4)}}{\beta^4}$	log var. for .001 of α.	$\log \frac{\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(5)}}{\alpha \beta^4}$	log var. for .001 of α.	$\log \frac{\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(6)}}{\alpha^2 \beta^4}$
.000	0.8170693	— ∞	.250	0.7929961	3.29044	0.9644552	3.36940	1.1000106
.005	.8170598	1.57978	.255	.7920099	.29951	.9632721	.37867	.0987100
.010	.8170313	1.88081	.260	.7910030	.30843	.9620637	.38778	.0973812
.015	.8169838	2.05690	.265	.7899755	.31719	.9608299	.39676	.0960240
.020	.8169173	.18213	.270	.7889272	.32580	.9595705	.40559	.0946382
.025	0.8168317	2.27898	.275	0.7878581	3.33429	0.9582855	3.41428	1.0932238
.030	.8167272	.35813	.280	.7867681	.34262	.9569746	.42287	.0917806
.035	.8166036	.42521	.285	.7856571	.35084	.9556378	.43133	.0903084
.040	.8164610	.48316	.290	.7845251	.35892	.9542748	.43968	.0888071
.045	.8162993	.53453	.295	.7833720	.36686	.9528856	.44789	.0872763
.050	0.8161187	2.58024	.300	0.7821977	3.37471	0.9514701	3.45596	1.0857161
.055	.8159190	.62180	.305	.7810022	.38245	.9500283	.46392	.0841263
.060	.8157002	.65973	.310	.7797853	.39009	.9485599	.47185	.0825065
.065	.8154623	.69461	.315	.7785470	.39763	.9470646	.47969	.0808565
.070	.8152052	.72689	.320	.7772872	.40504	.9455422	.48742	.0791762
.075	0.8149291	2.75694	.325	0.7760058	3.41236	0.9439926	3.49505	1.0774653
.080	.8146338	.78504	.330	.7747028	.41959	.9424158	.50259	.0757236
.085	.8143194	.81158	.335	.7733780	.42673	.9408116	.51004	.0739510
.090	.8139858	.83658	.340	.7720314	.43381	.9391798	.51740	.0721473
.095	.8136330	.86022	.345	.7706629	.44077	.9375202	.52467	.0703122
.100	0.8132610	2.88264	.350	0.7692724	3.44764	0.9358327	3.53190	1.0684455
.105	.8128698	.90401	.355	.7678598	.45444	.9341171	.53900	.0665469
.110	.8124593	.92445	.360	.7664251	.46117	.9323733	.54605	.0646163
.115	.8120294	.94409	.365	.7649681	.46782	.9306011	.55304	.0626534
.120	.8115801	.96275	.370	.7634887	.47442	.9288003	.55996	.0606579
.125	0.8111116	2.98064	.375	0.7619869	3.48090	0.9269707	3.56682	1.0586296
.130	.8106237	2.99791	.380	.7604625	.48732	.9251122	.57360	.0565683
.135	.8101164	3.01460	.385	.7589155	.49373	.9232246	.58031	.0544736
.140	.8095896	.03068	.390	.7573458	.50002	.9213076	.58699	.0523453
.145	.8090433	.04618	.395	.7557532	.50630	.9193610	.59362	.0501831
.150	0.8084775	3.06115	.400	0.7541377	3.51245	0.9173847	3.60021	1.0479867
.155	.8078921	.07569	.405	.7524992	.51854	.9153783	.60670	.0457559
.160	.8072872	.08976	.410	.7508375	.52462	.9133418	.61317	.0434903
.165	.8066626	.10346	.415	.7491526	.53063	.9112748	.61957	.0411897
.170	.8060184	.11674	.420	.7474445	.53656	.9091772	.62595	.0388538
.175	0.8053544	3.12963	.425	0.7457129	3.54244	0.9070487	3.63227	1.0364822
.180	.8046706	.14220	.430	.7439577	.54827	.9048891	.63855	.0340746
.185	.8039670	.15442	.435	.7421789	.55405	.9026982	.64477	.0316306
.190	.8032436	.16637	.440	.7403763	.55980	.9004757	.65097	.0291500
.195	.8025003	.17800	.445	.7385498	.56548	.8982214	.65714	.0266325
.200	0.8017370	3.18938	.450	0.7366994	3.57111	0.8959350	3.66324	1.0240775
.205	.8009537	.20047	.455	.7348249	.57670	.8936163	.66932	.0214848
.210	.8001505	.21133	.460	.7329263	.58224	.8912651	.67536	.0188542
.215	.7993271	.22194	.465	.7310034	.58776	.8888811	.68135	.0161851
.220	.7984835	.23234	.470	.7290560	.59322	.8864640	.68731	.0134772
.225	0.7976198	3.24249	.475	0.7270841	3.59861	0.8840136	3.69323	1.0107301
.230	.7967358	.25246	.480	.7250876	.60399	.8815296	.69914	.0079435
.235	.7958315	.26221	.485	.7230664	.60929	.8790117	.70501	.0051169
.240	.7949068	.27180	.490	.7210204	.61459	.8764597	.71084	1.0022499
.245	.7939617	.28122	.495	.7189494	.61984	.8738732	.71665	0.9993421
.250	0.7929961	3.29044	.500	0.7168533	3.62505	0.8712520	3.72244	0.9963932

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$

α	$\log \frac{\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(4)}}{\beta^4}$	log var. for .001 of α.	$\log \frac{\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(5)}}{\alpha \beta^4}$	log var. for .001 of α.	$\log \frac{\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(6)}}{\alpha^2 \beta^4}$	$\log \frac{\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(7)}}{\alpha^3 \beta^4}$	$\log \frac{\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(8)}}{\alpha^4 \beta^4}$	$\log \frac{\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(9)}}{\alpha^5 \beta^4}$
.500	0.7168533	3.62505	0.8712520	3.72244	0.9963932	1.1015301	1.1921458	1.2717495
.505	.7147320	.63020	.8685958	.72817	.9934026	.0983116	.1887627	.2682421
.510	.7125854	.63534	.8659043	.73390	.9903699	.0950461	.1853291	.2646814
.515	.7104134	.64042	.8631771	.73959	.9872945	.0917332	.1818442	.2610667
.520	.7082160	.64548	.8604141	.74525	.9841761	.0883721	.1783075	.2573972
.525	0.7059929	3.65050	0.8576149	3.75090	0.9810143	1.0849624	1.1747183	1.2536721
.530	.7037441	.65548	.8547791	.75651	.9778085	.0815034	.1710757	.2498906
.535	.7014695	.66041	.8519065	.76211	.9745582	.0779945	.1673790	.2460519
.540	.6991689	.66531	.8489966	.76769	.9712628	.0744350	.1636277	.2421552
.545	.6968422	.67019	.8460492	.77325	.9679220	.0708243	.1598208	.2381997
.550	0.6944893	3.67502	0.8430640	3.77877	0.9645352	1.0671616	1.1559575	1.2341844
.555	.6921102	.67984	.8400406	.78429	.9611019	.0634464	.1520371	.2301084
.560	.6897047	.68462	.8369786	.78979	.9576215	.0596779	.1480588	.2259709
.565	.6872728	.69935	.8338777	.79525	.9540934	.0558554	.1440216	.2217708
.570	.6848143	.69406	.8307375	.80071	.9505173	.0519782	.1399248	.2175071
.575	0.6823291	3.69872	0.8275578	3.80614	0.9468923	1.0480455	1.1357673	1.2131789
.580	.6798172	.70336	.8243381	.81157	.9432180	.0440565	.1315485	.2087852
.585	.6772784	.70796	.8210781	.81694	.9394937	.0400105	.1272672	.2043248
.590	.6747127	.71253	.8177775	.82231	.9357189	.0359065	.1229225	.1997967
.595	.6721199	.71706	.8144358	.82768	.9318929	.0317438	.1185134	.1951996
.600	0.6695000	3.72156	0.8110526	3.83305	0.9280152	1.0275216	1.1140389	1.1905326
.605	.6668530	.72602	.8076275	.83838	.9240849	.0232390	.1094979	.1857943
.610	.6641786	.73046	.8041603	.84368	.9201015	.0188951	.1048894	.1809835
.615	.6614769	.73486	.8006504	.84898	.9160643	.0144890	.1002122	.1760989
.620	.6587479	.73923	.7970976	.85426	.9119726	.0100198	.0954653	.1711393
.625	0.6559915	3.74354	0.7935013	3.85952	0.9078257	1.0054864	1.0906475	1.1661034
.630	.6532075	.74783	.7898612	.86477	.9036228	1.0008881	.0857574	.1609896
.635	.6503960	.75210	.7861769	.87002	.8993633	0.9962237	.0807939	.1557968
.640	.6475569	.75631	.7824479	.87524	.8950464	.9914922	.0757559	.1505235
.645	.6446901	.76051	.7786739	.88044	.8906713	.9866926	.0706419	.1451681
.650	0.6417957	3.76467	0.7748545	3.88562	0.8862373	0.9818240	1.0654506	1.1397290
.655	.6388735	.76879	.7709893	.89080	.8817436	.9768850	.0601806	.1342047
.660	.6359236	.77286	.7670779	.89596	.8771894	.9718745	.0548306	.1285935
.665	.6329460	.77691	.7631198	.90109	.8725739	.9667913	.0493991	.1228937
.670	.6299407	.78090	.7591148	.90621	.8678963	.9616344	.0438847	.1171036
.675	0.6269078	3.78487	0.7550624	3.91132	0.8631558	0.9564024	1.0382857	1.1112215
.680	.6238473	.78879	.7509620	.91642	.8583515	.9510942	.0326008	.1052454
.685	.6207591	.79266	.7468133	.92148	.8534826	.9457085	.0268282	.0991735
.690	.6176434	.79650	.7426160	.92654	.8485481	.9402440	.0209663	.0930037
.695	.6145003	.80027	.7383695	.93158	.8435473	.9346995	.0150135	.0867340
.700	0.6113298	3.80401	0.7340736	3.93659	0.8384792	0.9290734	1.0089679	1.0803623
.705	.6081320	.80772	.7297279	.94159	.8333430	.9233646	1.0028279	.0738864
.710	.6049071	.81136	.7253320	.94657	.8281376	.9175715	0.9965915	.0673040
.715	.6016552	.81495	.7208856	.95152	.8228622	.9116927	.9902568	.0606128
.720	.5983766	.81849	.7163882	.95647	.8175158	.9057268	.9838220	.0538104
.725	0.5950713	3.82197	0.7118395	3.96138	0.8120975	0.8996722	0.9772849	1.0468943
.730	.5917396	.82542	.7072391	.96630	.8066063	.8935272	.9706436	.0398620
.735	.5883816	.82879	.7025865	.97117	.8010411	.8872906	.9638960	.0327109
.740	.5849976	.83212	.6978814	.97603	.7954010	.8809605	.9570400	.0254382
.745	.5815878	.83539	.6931234	.98089	.7896850	.8745352	.9500731	.0180410
.750	0.5781524	3.83861	0.6883121	3.98571	0.7838921	0.8680134	0.9429928	1.0105166

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$

α	$\log \frac{\alpha^5 D^3_\alpha b^{(0)}_{\frac12}}{\beta^6}$	log var. for .001 of α.	$\log \frac{\alpha^6 D^3_\alpha b^{(1)}_{\frac12}}{\beta^6}$	log var. for .001 of α.	$\log \frac{\alpha^5 D^3_\alpha b^{(2)}_{\frac12}}{\beta^6}$	log var. for .001 of α.	$\log \frac{\alpha^6 D^3_\alpha b^{(3)}_{\frac12}}{\beta^6}$	log var. for .001 of α.
.000	0.8293038	— ∞	0.3521825	— ∞	0.8750613	— ∞	0.5740313	— ∞
.005	.8293090	1.3118	.3522177	2.14890	.8750643	1.0863	.5740463	1.77743
.010	.8293243	.6096	.3523234	.45025	.8750735	.3874	.5740912	2.07773
.015	.8293497	.7868	.3524997	.62645	.8750887	.5623	.5741659	.25334
.020	.8293855	1.9138	.3527465	.75128	.8751100	.6884	.5742704	.37803
.025	0.8294317	2.0094	0.3530637	2.84794	0.8751375	1.7853	0.5744047	2.47480
.030	.8294883	.0909	.3534511	.92665	.8751711	.8645	.5745688	.55388
.035	.8295550	.1571	.3539084	2.99317	.8752108	.9314	.5747627	.62055
.040	.8296318	.2148	.3544354	3.05061	.8752566	1.9894	.5749863	.67834
.045	.8297189	.2657	.3550319	.10113	.8753085	2.0414	.5752396	.72933
.050	0.8298162	2.3113	0.3556977	3.14619	0.8753666	2.0870	0.5755225	2.77481
.055	.8299238	.3529	.3564323	.18696	.8754308	.1284	.5758349	.81598
.060	.8300416	.3906	.3572354	.22396	.8755010	.1661	.5761769	.85358
.065	.8301696	.4252	.3581069	.25787	.8755773	.2008	.5765485	.88807
.070	.8303078	.4573	.3590462	.28914	.8756598	.2330	.5769496	.91991
.075	0.8304562	2.4871	0.3600529	3.31814	0.8757484	2.2634	0.5773801	2.94959
.080	.8306148	.5151	.3611266	.34514	.8758431	.2913	.5778400	2.97727
.085	.8307836	.5413	.3622668	.37041	.8759439	.3176	.5783292	3.00320
.090	.8309626	.5661	.3634730	.39406	.8760508	.3424	.5788475	.02759
.095	.8311517	.5893	.3647446	.41635	.8761638	.3658	.5793949	.05069
.100	0.8313510	2.6113	0.3660812	3.43736	0.8762829	2.3881	0.5799712	3.07254
.105	.8315604	.6324	.3674821	.45718	.8764081	.4092	.5805765	.09321
.110	.8317800	.6526	.3689466	.47606	.8765395	.4294	.5812106	.11294
.115	.8320097	.6717	.3704741	.49382	.8766771	.4490	.5818735	.13175
.120	.8322495	.6900	.3720640	.51078	.8768208	.4674	.5825650	.14965
.125	0.8324994	2.7075	0.3737157	3.52692	0.8769705	2.4851	0.5832850	3.16684
.130	.8327594	.7244	.3754283	.54227	.8771264	.5024	.5840334	.18327
.135	.8330294	.7405	.3772013	.55698	.8772884	.5188	.5848101	.19910
.140	.8333094	.7561	.3790338	.57101	.8774565	.5345	.5856150	.21426
.145	.8335995	.7711	.3809252	.58443	.8776308	.5497	.5864480	.22886
.150	0.8338997	2.7856	0.3828745	3.59726	0.8778112	2.5644	0.5873089	3.24289
.155	.8342099	.7996	.3848812	.60959	.8779977	.5788	.5881975	.25648
.160	.8345300	.8132	.3869444	.62140	.8781903	.5926	.5891138	.26961
.165	.8348601	.8263	.3890633	.63274	.8783891	.6061	.5900578	.28226
.170	.8352002	.8390	.3912371	.64363	.8785940	.6191	.5910291	.29447
.175	0.8355503	2.8513	0.3934650	3.65408	0.8788050	2.6316	0.5920277	3.30629
.180	.8359103	.8632	.3957460	.66414	.8790223	.6438	.5930535	.31778
.185	.8362802	.8748	.3980795	.67381	.8792456	.6559	.5941062	.32880
.190	.8366600	.8861	.4004645	.68310	.8794751	.6676	.5951857	.33957
.195	.8370496	.8971	.4029001	.69206	.8797107	.6789	.5962918	.35002
.200	0.8374490	2.9078	0.4053855	3.70067	0.8799525	2.6898	0.5974244	3.36013
.205	.8378583	.9182	.4079197	.70898	.8802004	.7007	.5985834	.37000
.210	.8382775	.9283	.4105019	.71698	.8804544	.7111	.5997686	.37957
.215	.8387064	.9382	.4131313	.72468	.8807146	.7214	.6009798	.38885
.220	.8391451	.9479	.4158069	.73212	.8809810	.7315	.6022168	.39787
.225	0.8395935	2.9574	0.4185278	3.73928	0.8812536	2.7414	0.6034794	3.40672
.230	.8400517	.9666	.4212930	.74617	.8815323	.7511	.6047676	.41531
.235	.8405196	.9756	.4241017	.75284	.8818172	.7605	.6060812	.42361
.240	.8409971	.9844	.4269530	.75924	.8821083	.7697	.6074200	.43173
.245	.8414842	2.9930	.4298461	.76545	.8824055	.7787	.6087837	.43968
.250	0.8419810	3.0014	0.4327800	3.77142	0.8827089	2.7875	0.6101722	3.44742

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$

α	$\log \frac{\alpha^5 D_\alpha^3 b_{\frac{1}{2}}^{(0)}}{\beta^6}$	log var. for .001 of α.	$\log \frac{\alpha^6 D_\alpha^3 b_{\frac{1}{2}}^{(1)}}{\beta^6}$	log var. for .001 of α.	$\log \frac{\alpha^5 D_\alpha^3 b_{\frac{1}{2}}^{(2)}}{\beta^6}$	log var. for .001 of α.	$\log \frac{\alpha^6 D_\alpha^3 b_{\frac{1}{2}}^{(3)}}{\beta^6}$	log var. for .001 of α.
.250	0.8419810	3.0014	0.4327800	3.77142	0.8827089	2.7875	0.6101722	3.44742
.255	.8424874	.0096	.4357539	.77720	.8830185	.7961	.6115853	.45497
.260	.8430033	.0177	.4387668	.78276	.8833343	.8046	.6130228	.46230
.265	.8435288	.0256	.4418179	.78813	.8836563	.8130	.6144846	.46950
.270	.8440638	.0333	.4449062	.79330	.8839844	.8212	.6159705	.47651
.275	0.8446083	3.0409	0.4480309	3.79831	0.8843187	2.8293	0.6174802	3.48461
.280	.8451623	.0483	.4511911	.80313	.8846592	.8372	.6190137	.49002
.285	.8457258	.0555	.4543860	.80779	.8850059	.8450	.6205706	.49653
.290	.8462987	.0626	.4576147	.81226	.8853589	.8526	.6221508	.50291
.295	.8468809	.0696	.4608763	.81658	.8857181	.8601	.6237541	.50918
.300	0.8474724	3.0765	0.4641699	3.82077	0.8860834	2.8675	0.6253803	3.51522
.305	.8480733	.0832	.4674948	.82479	.8864550	.8747	.6270292	.52120
.310	.8486836	.0898	.4708500	.82867	.8868328	.8818	.6287007	.52703
.315	.8493032	.0963	.4742348	.83240	.8872169	.8888	.6303945	.53272
.320	.8499319	.1027	.4776484	.83602	.8876072	.8957	.6321104	.53829
.325	0.8505699	3.1090	0.4810899	3.83949	0.8880037	2.9026	0.6338482	3.54376
.330	.8512171	.1151	.4845585	.84285	.8884064	.9094	.6356077	.54913
.335	.8518734	.1211	.4880535	.84607	.8888154	.9161	.6373889	.55434
.340	.8525389	.1270	.4915741	.84917	.8892306	.9227	.6391914	.55944
.345	.8532135	.1329	.4951195	.85216	.8896521	.9292	.6410150	.56444
.350	0.8538971	3.1387	0.4986890	3.85503	0.8900799	2.9355	0.6428595	3.56935
.355	.8545897	.1444	.5022817	.85781	.8905139	.9417	.6447248	.57416
.360	.8552914	.1500	.5058970	.86047	.8909542	.9479	.6466106	.57887
.365	.8560021	.1555	.5095340	.86304	.8914008	.9540	.6485168	.58348
.370	.8567217	.1609	.5131922	.86550	.8918537	.9600	.6504431	.58799
.375	0.8574502	3.1661	0.5168708	3.86788	0.8923129	2.9660	0.6523893	3.59241
.380	.8581876	.1713	.5205691	.87015	.8927784	.9719	.6543552	.59675
.385	.8589338	.1764	.5242864	.87233	.8932503	.9777	.6563407	.60100
.390	.8596887	.1815	.5280222	.87443	.8937285	.9834	.6583455	.60517
.395	.8604524	.1865	.5317756	.87644	.8942130	.9891	.6603695	.60927
.400	0.8612249	3.1914	0.5355459	3.87836	0.8947038	2.9947	0.6624125	3.61329
.405	.8620061	.1962	.5393327	.88021	.8952010	3.0003	.6644743	.61723
.410	.8627960	.2010	.5431352	.88197	.8957045	.0058	.6665546	.62109
.415	.8635946	.2057	.5469530	.88367	.8962144	.0113	.6686533	.62487
.420	.8644018	.2103	.5507853	.88528	.8967307	.0167	.6707702	.62858
.425	0.8652175	3.2148	0.5546316	3.88683	0.8972534	3.0220	0.6729051	3.63221
.430	.8660418	.2193	.5584913	.88830	.8977825	.0272	.6750577	.63578
.435	.8668746	.2237	.5623638	.88972	.8983179	.0324	.6772280	.63929
.440	.8677158	.2281	.5662486	.89106	.8988597	.0375	.6794157	.64273
.445	.8685655	.2324	.5701452	.89234	.8994080	.0426	.6816207	.64611
.450	0.8694236	3.2367	0.5740528	3.89355	0.8999628	3.0476	0.6838428	3.64943
.455	.8702901	.2409	.5779712	.89470	.9005240	.0526	.6860818	.65269
.460	.8711649	.2451	.5818998	.89579	.9010916	.0575	.6883374	.65589
.465	.8720480	.2492	.5858380	.89683	.9016657	.0624	.6906094	.65903
.470	.8729393	.2532	.5897855	.89781	.9022463	.0673	.6928978	.66211
.475	0.8738387	3.2571	0.5937417	3.89874	0.9028334	3.0721	0.6952024	3.66513
.480	.8747464	.2610	.5977059	.89962	.9034270	.0769	.6975229	.66811
.485	.8756623	.2648	.6016780	.90045	.9040271	.0816	.6998593	.67104
.490	.8765865	.2686	.6056575	.90123	.9046338	.0863	.7022113	.67391
.495	.8775187	.2723	.6096439	.90196	.9052470	.0909	.7045787	.67673
.500	0.8784588	3.2760	0.6136367	3.90264	0.9058667	3.0955	0.7069614	3.67950

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$

α	$\log \frac{\alpha^5 D_\alpha^3 b_{\frac{1}{2}}^{(0)}}{\beta^6}$	log var. for .001 of α.	$\log \frac{\alpha^6 D_\alpha^3 b_{\frac{1}{2}}^{(1)}}{\beta^6}$	log var. for .001 of α.	$\log \frac{\alpha^5 D_\alpha^3 b_{\frac{1}{2}}^{(2)}}{\beta^6}$	log var. for .001 of α.	$\log \frac{\alpha^6 D_\alpha^3 b_{\frac{1}{2}}^{(3)}}{\beta^8}$	log var. for .001 of α.
.500	0.8784588	3.2760	0.6136367	3.90264	0.9058667	3.0955	0.7069614	3.67950
.505	.8794070	.2797	.6176356	.90328	.9064930	.1001	.7093593	.68222
.510	.8803632	.2833	.6216400	.90387	.9071259	.1046	.7117721	.68489
.515	.8813274	.2869	.6256498	.90442	.9077654	.1091	.7141998	.68752
.520	.8822994	.2904	.6296644	.90493	.9084115	.1136	.7166421	.69011
.525	0.8832793	3.2939	0.6336836	3.90540	0.9090643	3.1180	0.7190988	3.69266
.530	.8842670	.2973	.6377069	.90583	.9097237	.1224	.7215699	.69516
.535	.8852625	.3007	.6417341	.90622	.9103898	.1268	.7240552	.69762
.540	.8862658	.3041	.6457646	.90658	.9110626	.1311	.7265545	.70004
.545	.8872768	.3074	.6497983	.90690	.9117421	.1354	.7290676	.70242
.550	0.8882955	3.3107	0.6538348	3.90718	0.9124283	3.1397	0.7315945	3.70477
.555	.8893219	.3139	.6578738	.90742	.9131213	.1439	.7341349	.70708
.560	.8903559	.3171	.6619149	.90763	.9138211	.1481	.7366888	.70935
.565	.8913975	.3203	.6659580	.90781	.9145276	.1523	.7392560	.71159
.570	.8924466	.3234	.6700026	.90796	.9152409	.1564	.7418363	.71380
.575	0.8935033	3.3265	0.6740486	3.90809	0.9159611	3.1605	0.7444296	3.71597
.580	.8945674	.3295	.6780955	.90819	.9166881	.1646	.7470358	.71812
.585	.8956390	.3325	.6821432	.90826	.9174220	.1687	.7496548	.72023
.590	.8967180	.3355	.6861914	.90831	.9181628	.1728	.7522865	.72231
.595	.8978044	.3384	.6902399	.90833	.9189106	.1768	.7549308	.72436
.600	0.8988982	3.3413	0.6942883	3.90832	0.9196653	3.1808	0.7575875	3.72637
.605	.8999993	.3442	.6983366	.90828	.9204270	.1848	.7602564	.72835
.610	.9011076	.3471	.7023844	.90823	.9211957	.1888	.7629375	.73032
.615	.9022232	.3499	.7064315	.90815	.9219714	.1927	.7656306	.73226
.620	.9033460	.3527	.7104777	.90804	.9227542	.1966	.7683357	.73417
.625	0.9044760	3.3555	0.7145228	3.90790	0.9235441	3.2005	0.7710526	3.73605
.630	.9056131	.3582	.7185666	.90774	.9243411	.2044	.7737812	.73791
.635	.9067574	.3609	.7226090	.90757	.9251452	.2083	.7765215	.73974
.640	.9079087	.3636	.7266497	.90738	.9259564	.2122	.7792734	.74155
.545	.9090671	.3662	.7306886	.90718	.9267749	.2160	.7820367	.74334
.650	0.9102325	3.3688	0.7347255	3.90696	0.9276007	3.2198	0.7848114	3.74512
.655	.9114049	.3714	.7387602	.90672	.9284338	.2236	.7875974	.74688
.660	.9125842	.3739	.7427926	.90647	.9292743	.2274	.7903945	.74862
.665	.9137705	.3764	.7468224	.90620	.9301222	.2312	.7932027	.75034
.670	.9149636	.3789	.7508496	.90591	.9309774	.2350	.7960220	.75204
.675	0.9161636	3.3814	0.7548741	3.90560	0.9318400	3.2388	0.7988523	3.75372
.680	.9173704	.3839	.7588957	.90527	.9327100	.2425	.8016934	.75537
.685	.9185840	.3863	.7629142	.90493	.9335876	.2462	.8045455	.75701
.690	.9198044	.3887	.7669295	.90458	.9344727	.2499	.8074084	.75864
.695	.9210316	.3911	.7709416	.90422	.9353655	.2536	.8102820	.76026
.700	0.9222655	3.3935	0.7749503	3.90385	0.9362660	3.2573	0.8131664	3.76188
.705	.9235061	.3958	.7789555	.90346	.9371742	.2610	.8160614	.76348
.710	.9247533	.3981	.7829571	.90306	.9380901	.2647	.8189669	.76506
.715	.9260072	.4004	.7869550	.90265	.9390139	.2684	.8218829	.76662
.720	.9272677	.4027	.7909492	.90223	.9399456	.2721	.8248095	.76817
.725	0.9285348	3.4050	0.7949395	3.90181	0.9408852	3.2758	0.8277466	3.76971
.730	.9298084	.4072	.7989258	.90138	.9418327	.2795	.8306941	.77124
.735	.9310885	.4094	.8029081	.90094	.9427882	.2832	.8336519	.77274
.740	.9323751	.4116	.8068863	.90049	.9437518	.2868	.8366199	.77424
.745	.9336682	.4138	.8108603	.90003	.9447234	.2904	.8395983	.77573
.750	0.9349677	3.4159	0.8148302	3.89957	0.9457032	3.2940	0.8425871	3.77722

LOGARITHMIC VALUES OF $f(\alpha)\cdot\alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$

α	$\log \frac{\alpha^5 D^3_\alpha b^{(4)}_{\frac{1}{2}}}{\beta^6}$	log var. for .001 of α.	α	$\log \frac{\alpha^5 D^3_\alpha b^{(4)}_{\frac{1}{2}}}{\beta^6}$	log var. for .001 of α.	α	$\log \frac{\alpha^5 D^3_\alpha b^{(4)}_{\frac{1}{2}}}{\beta^6}$	log var. for .001 of α.
.000	1.1180993	$-\infty$	.250	1.0982753	3.1879	.500	1.0464236	3.3824
.005	.1180912	1.5105	.255	.0974979	.1954	.505	.0452164	.3830
.010	.1180669	.8129	.260	.0967070	.2028	.510	.0440079	.3834
.015	.1180263	1.9894	.265	.0959029	.2099	.515	.0427985	.3836
.020	.1179693	2.1152	.270	.0950856	.2168	.520	.0415886	.3837
.025	1.1178960	2.2116	.275	1.0942553	3.2235	.525	1.0403786	3.3837
.030	.1178064	.2909	.280	.0934122	.2301	.530	.0391690	.3835
.035	.1177005	.3575	.285	.0925563	.2365	.535	.0379601	.3832
.040	.1175784	.4156	.290	.0916879	.2427	.540	.0367523	.3827
.045	.1174402	.4665	.295	.0908070	.2488	.545	.0355461	.3821
.050	1.1172858	2.5148	.300	1.0899139	3.2548	.550	1.0343419	3.3813
.055	.1171152	.5531	.305	.0890086	.2607	.555	.0331401	.3803
.060	.1169284	.5906	.310	.0880914	.2663	.560	.0319413	.3792
.065	.1167255	.6253	.315	.0871623	.2718	.565	.0307458	.3779
.070	.1165065	.6572	.320	.0862216	.2771	.570	.0295542	.3764
.075	1.1162713	2.6870	.325	1.0852694	3.2823	.575	1.0283669	3.3747
.080	.1160201	.7146	.330	.0843059	.2874	.580	.0271843	.3728
.085	.1157529	.7407	.335	.0833313	.2923	.585	.0260071	.3708
.090	.1154696	.7652	.340	.0823457	.2971	.590	.0248359	.3685
.095	.1151704	.7884	.345	.0813494	.3017	.595	.0236711	.3660
.100	1.1148553	2.8103	.350	1.0803425	3.3062	.600	1.0225131	3.3633
.105	.1145243	.8312	.355	.0793253	.3106	.605	.0213624	.3604
.110	.1141774	.8510	.360	.0782979	.3148	.610	.0202198	.3573
.115	.1138147	.8699	.365	.0772606	.3189	.615	.0190855	.3539
.120	.1134363	.8879	.370	.0762137	.3229	.620	.0179601	.3503
.125	1.1130422	2.9052	.375	1.0751572	3.3268	.625	1.0168443	3.3465
.130	.1126324	.9218	.380	.0740915	.3305	.630	.0157386	.3425
.135	.1122070	.9377	.385	.0730168	.3341	.635	.0146436	.3382
.140	.1117660	.9530	.390	.0719333	.3376	.640	.0135599	.3336
.145	.1113095	.9677	.395	.0708413	.3409	.645	.0124879	.3287
.150	1.1108377	2.9819	.400	1.0697410	3.3441	.650	1.0114284	3.3235
.155	.1103505	2.9955	.405	.0686327	.3472	.655	.0103819	.3179
.160	.1098480	3.0088	.410	.0675164	.3502	.660	.0093490	.3121
.165	.1093302	.0216	.415	.0663926	.3531	.665	.0083303	.3059
.170	.1087973	.0340	.420	.0652616	.3558	.670	.0073264	.2993
.175	1.1082493	3.0459	.425	1.0641236	3.3584	.675	1.0063379	3.2924
.180	.1076862	.0573	.430	.0629790	.3609	.680	.0053655	.2851
.185	.1071082	.0684	.435	.0618280	.3632	.685	.0044098	.2774
.190	.1065153	.0792	.440	.0606708	.3654	.690	.0034716	.2693
.195	.1059077	.0897	.445	.0595078	.3675	.695	.0025512	.2607
.200	1.1052854	3.1000	.450	1.0583394	3.3695	.700	1.0016494	3.2516
.205	.1046486	.1100	.455	.0571658	.3714	.705	1.0007667	.2419
.210	.1039972	.1197	.460	.0559874	.3732	.710	0.9999041	.2317
.215	.1033314	.1291	.465	.0548044	.3749	.715	.9990620	.2210
.220	.1026513	.1382	.470	.0536171	.3764	.720	.9982411	.2096
.225	1.1019570	3.1470	.475	1.0524257	3.3777	.725	0.9974421	3.1974
.230	.1012485	.1556	.480	.0512307	.3788	.730	.9966657	.1846
.235	.1005260	.1640	.485	.0500328	.3799	.735	.9959125	.1710
.240	.0997895	.1722	.490	.0488322	.3809	.740	.9951832	.1566
.245	.0990392	.1802	.495	.0476290	.3817	.745	.9944784	.1412
.250	1.0982753	3.1879	.500	1.0464236	3.3824	.750	0.9937989	3.1247

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$

α	$\log \frac{\alpha^4 D_\alpha^3 b_{\frac{1}{2}}^{(5)}}{\beta^6}$	log var. for .001 of α.	$\log \frac{\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(6)}}{\beta^6}$	log var. for .001 of α.	$\log \frac{\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(7)}}{\alpha \beta^6}$	log var. for .001 of α.	$\log \frac{\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(8)}}{\alpha^2 \beta^6}$	$\log \frac{\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(9)}}{\alpha^3 \beta^6}$
.500	1.3175300	3.78351	1.5388628	3.91156	1.7245949	3.97603	1.8841185	2.0237101
.505	.3144788	.78742	.5347605	.91656	.7198329	.98161	.8789111	.0181845
.510	.3114002	.79127	.5306109	.92151	.7150096	.98716	.8736321	.0125799
.515	.3082944	.79505	.5264140	.92641	.7101246	.99266	.8682810	.0068954
.520	.3051617	.79877	.5221698	.93126	.7051775	3.99813	.8628572	2.0011302
.525	1.3020024	3.80243	1.5178781	3.93606	1.7001679	4.00356	1.8573598	1.9952832
.530	.2988168	.80600	.5135388	.94082	.6950953	.00897	.8517883	.9893537
.535	.2956051	.80950	.5091519	.94553	.6899594	.01434	.8461418	.9833406
.540	.2923677	.81293	.5047174	.95020	.6847597	.01968	.8404198	.9772429
.545	.2891048	.81628	.5002353	.95482	.6794959	.02499	.8346214	.9710598
.550	1.2858169	3.81956	1.4957055	3.95939	1.6741676	4.03027	1.8287460	1.9647903
.555	.2825044	.82278	.4911280	.96391	.6687743	.03551	.8227928	.9584332
.560	.2791678	.82592	.4865029	.96838	.6633157	.04072	.8167612	.9519878
.565	.2758073	.82899	.4818301	.97280	.6577914	.04590	.8106504	.9454529
.570	.2724230	.83198	.4771098	.97718	.6522010	.05104	.8044598	.9388273
.575	1.2690157	3.83488	1.4723419	3.98151	1.6465443	4.05615	1.7981886	1.9321103
.580	.2655859	.83771	.4675266	.98578	.6408209	.06124	.7918362	.9253007
.585	.2621339	.84046	.4626639	.99000	.6350304	.06629	.7854016	.9183974
.590	.2586605	.84313	.4577540	.99417	.6291725	.07130	.7788841	.9113994
.595	.2551658	.84572	.4527970	3.99829	.6232468	.07627	.7722828	.9043054
.600	1.2516506	3.84823	1.4477930	4.00237	1.6172530	4.08121	1.7655971	1.8971144
.605	.2481152	.85065	.4427422	.00639	.6111908	.08612	.7588260	.8898252
.610	.2445605	.85298	.4376448	.01035	.6050598	.09099	.7519689	.8824366
.615	.2409868	.85524	.4325010	.01426	.5988598	.09583	.7450250	.8749475
.620	.2373950	.85741	.4273111	.01811	.5925906	.10064	.7379935	.8673566
.625	1.2337856	3.85948	1.4220753	4.02190	1.5862519	4.10541	1.7308738	1.8596627
.630	.2301594	.86145	.4167940	.02563	.5798135	.11014	.7236650	.8518647
.635	.2265170	.86332	.4114675	.02930	.5733651	.11484	.7163665	.8439614
.640	.2228592	.86509	.4060961	.03291	.5668166	.11950	.7089777	.8359514
.645	.2191867	.86678	.4006803	.03646	.5601979	.12411	.7014976	.8278335
.650	1.2155004	3.86839	1.3952205	4.03994	1.5535089	4.12868	1.6939258	1.8196064
.655	.2118010	.86989	.3897172	.04335	.5467494	.13320	.6862614	.8112688
.660	.2080894	.87128	.3841710	.04668	.5399194	.13768	.6785038	.8028196
.665	.2043664	.87255	.3785824	.04995	.5330189	.14212	.6706524	.7942574
.670	.2006330	.87370	.3729520	.05315	.5260480	.14651	.6627066	.7855810
.675	1.1968901	3.87474	1.3672805	4.05627	1.5190067	4.15085	1.6546657	1.7767891
.680	.1931386	.87567	.3615686	.05931	.5118952	.15513	.6465292	.7678804
.685	.1893795	.87649	.3558170	.06227	.5047136	.15936	.6382964	.7588536
.690	.1856139	.87719	.3500266	.06515	.4974621	.16355	.6299668	.7497077
.695	.1818428	.87775	.3441982	.06795	.4901410	.16768	.6215400	.7404412
.700	1.1780676	3.87817	1.3383328	4.07066	1.4827506	4.17174	1.6130156	1.7310529
.705	.1742892	.87846	.3324313	.07328	.4752911	.17573	.6043930	.7215417
.710	.1705090	.87861	.3264947	.07581	.4677632	.17965	.5956720	.7119063
.715	.1667282	.87862	.3205243	.07824	.4601673	.18351	.5868523	.7021457
.720	.1629480	.87847	.3145211	.08056	.4525042	.18731	.5779336	.6922589
.725	1.1591695	3.87815	1.3084864	4.08278	1.4447745	4.19103	1.5689157	1.6822445
.730	.1553946	.87766	.3024215	.08489	.4369791	.19467	.5597984	.6721019
.735	.1516245	.87700	.2963277	.08688	.4291185	.19824	.5505817	.6618301
.740	.1478611	.87616	.2902067	.08876	.4211937	.20174	.5412654	.6514282
.745	.1441056	.87514	.2840600	.09053	.4132056	.20516	.5318497	.6408954
.750	1.1403597	3.87394	1.2778890	4.09219	1.4051551	4.20851	1.5223347	1.6302305

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$

α	$\log \frac{\alpha^8 D_\alpha^4 b_{\frac{1}{2}}^{(0)}}{\beta^8}$	log var. for .001 of α.	$\log \frac{\alpha^7 D_\alpha^4 b_{\frac{1}{2}}^{(1)}}{\beta^8}$	log var. for .001 of α.	$\log \frac{\alpha^8 D_\alpha^4 b_{\frac{1}{2}}^{(2)}}{\beta^8}$	log var. for .001 of α.	$\log \frac{\alpha^7 D_\alpha^4 b_{\frac{1}{2}}^{(3)}}{\beta^8}$	log var. for .001 of α.
.000	0.8293038	$-\infty$	1.4490925	$-\infty$	0.8750613	$-\infty$	1.5160393	$-\infty$
.005	.8293730	2.44404	.4491045	1.68034	.8751247	2.40449	.5160471	1.49693
.010	.8295817	.74574	.4491404	1.98091	.8753151	.70518	.5160707	.79934
.015	.8299295	2.92148	.4492002	2.15746	.8756319	2.88093	.5161101	1.97589
.020	.8304163	3.04626	.4492841	.28330	.8760754	3.00544	.5161652	2.10037
.025	0.8310418	3.14270	1.4493921	2.38049	0.8766447	3.10188	1.5162361	2.19756
.030	.8318051	.22121	.4495240	.45879	.8773398	.18041	.5163227	.27645
.035	.8327055	.28713	.4496798	.52556	.8781598	.24660	.5164250	.34321
.040	.8337420	.34404	.4498594	.58343	.8791042	.30371	.5165431	.40140
.045	.8349136	.39406	.4500628	.63428	.8801722	.35388	.5166769	.45209
.050	0.8362197	3.43863	1.4502900	2.67979	0.8813630	3.39848	1.5168263	2.49748
.055	.8376591	.47870	.4505410	.72082	.8826755	.43866	.5169913	.53882
.060	.8392304	.51528	.4508158	.75846	.8841089	.47506	.5171719	.57634
.065	.8409326	.54824	.4511143	.79295	.8856617	.50843	.5173683	.61109
.070	.8427642	.57874	.4514365	.82478	.8873332	.53905	.5175803	.64306
.075	0.8447235	3.60690	1.4517823	2.85443	0.8891217	3.56736	1.5178080	2.67302
.080	.8468088	.63298	.4521517	.88207	.8910260	.59361	.5180513	.70088
.085	.8490185	.65727	.4525446	.90806	.8930446	.61805	.5183102	.72705
.090	.8513510	.67995	.4529610	.93247	.8951760	.64088	.5185847	.75174
.095	.8538043	.70115	.4534008	95559	.8974186	.66227	.5188748	.77510
.100	0.8563765	3.72105	1.4538639	2.97745	0.8997707	3.68235	1.5191805	2.79727
.105	.8590655	.73981	.4543502	2.99826	.9022307	.70124	.5195017	.81823
.110	.8618694	.75745	.4548597	3.01795	.9047969	.71905	.5198383	.83809
.115	.8647861	.77406	.4553924	.03679	.9074674	.73587	.5201905	.85721
.120	.8678134	.78975	.4559482	.05477	.9102404	.75177	.5205582	.87552
.125	0.8709488	3.80462	1.4565270	3.07203	0.9131139	3.76684	1.5209413	2.89298
.130	.8741904	.81861	.4571286	.08849	.9160859	.78109	.5213397	.90966
.135	.8775357	.83200	.4577530	.10435	.9191546	.79463	.5217535	.92583
.140	.8809823	.84463	.4584001	.11952	.9223178	.80740	.5221827	.94141
.145	.8845280	.85662	.4590698	.13418	.9255735	.81969	.5226273	.95636
.150	0.8881704	3.86801	1.4597620	3.14823	0.9289199	3.83124	1.5230871	2.97072
.155	.8919071	.87884	.4604766	.16185	.9323546	.84235	.5235621	.98471
.160	.8957357	.88912	.4612134	.17493	.9358757	.85286	.5240523	2.99817
.165	.8996538	.89890	.4619724	.18758	.9394809	.86288	.5245578	3.01123
.170	.9036589	.90822	.4627536	.19981	.9431683	.87243	.5250784	.02401
.175	0.9077487	3.91708	1.4635568	3.21171	0.9469357	3.88153	1.5256142	3.03607
.180	.9119207	.92551	.4643818	.22318	.9507808	.89020	.5261651	.04798
.185	.9161724	.93353	.4652285	.23431	.9547017	.89848	.5267310	.05949
.190	.9205014	.94117	.4660969	.24512	.9586962	.90637	.5273118	.07070
.195	.9249055	.94845	.4669869	.25561	.9627623	.91390	.5279077	.08164
.200	0.9293821	3.95539	1.4678983	3.26576	0.9668977	3.92113	1.5285186	3.09230
.205	.9339291	.96198	.4688309	.27563	.9711008	.92792	.5291445	.10257
.210	.9385439	.96826	.4697847	.28529	.9753685	.93444	.5297852	.11261
.215	.9432243	.97424	.4707596	.29460	.9796996	.94068	.5304407	.12248
.220	.9479679	.97993	.4717553	.30367	.9840917	.94662	.5311109	.13207
.225	0.9527726	3.98535	1.4727717	3.31252	0.9885429	3.95229	1.5317959	3.14132
.230	.9576360	.99050	.4738088	.32115	.9930511	.95769	.5324955	.15045
.235	.9625561	3.99540	.4748665	.32952	0.9976145	.96285	.5332098	.15939
.240	.9675306	4.00007	.4759444	.33766	1.0022311	.96778	.5339387	.16808
.245	.9725575	.00449	.4770425	.34565	.0068991	.97245	.5346822	.17655
.250	0.9776346	4.00870	1.4781606	3.35342	1.0116164	3.97691	1 5354403	3.18486

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$

α	$\log \frac{\alpha^8 D_\alpha^4 b_{\frac{1}{2}}^{(0)}}{\beta^8}$	log var. for .001 of α.	$\log \frac{\alpha^7 D_\alpha^4 b_{\frac{1}{2}}^{(1)}}{\beta^8}$	log var. for .001 of α.	$\log \frac{\alpha^8 D_\alpha^4 b_{\frac{1}{2}}^{(2)}}{\beta^8}$	log var. for .001 of α.	$\log \frac{\alpha^7 D_\alpha^4 b_{\frac{1}{2}}^{(3)}}{\beta^8}$	log var for .001 of α.
.250	0.9776346	4.00870	1.4781606	3.35342	1.0116164	3.97691	1.5354403	3.18486
.255	.9827598	.01269	.4792987	.36097	.0163813	.98115	.5362128	.19296
.260	.9879313	.01648	.4804565	.36835	.0211918	.98520	.5369997	.20091
.265	.9931467	.02008	.4816340	.37556	.0260462	.98904	.5378010	.20871
.270	0.9984041	.02356	.4828310	.38259	.0309427	.99269	.5386165	.21632
.275	1.0037017	4.02673	1.4840473	3.38948	1.0358794	3.99616	1.5394463	3.22376
.280	.0090380	.02980	.4852827	.39620	.0408547	3.99946	.5402904	.23106
.285	.0144111	.03266	.4865371	.40275	.0458670	4.00259	.5411487	.23825
.290	.0198190	.03539	.4878103	.40915	.0509144	.00556	.5420211	.24527
.295	.0252601	.03796	.4891023	.41542	.0559957	.00836	.5429076	.25212
.300	1.0307329	4.04039	1.4904129	3.42150	1.0611088	4.01103	1.5438081	3.25888
.305	.0362352	.04271	.4917419	.42753	.0662528	.01355	.5447226	.26553
.310	.0417655	.04487	.4930891	.43336	.0714257	.01593	.5456510	.27203
.315	.0473228	.04688	.4944544	.43909	.0766263	.01818	.5465933	.27839
.320	.0529053	.04878	.4958376	.44467	.0818532	.02032	.5475494	.28461
.325	1.0585116	4.05056	1.4972385	3.45015	1.0871050	4.02231	1.5485192	3.29079
.330	.0641400	.05223	.4986569	.45552	.0923803	.02421	.5495027	.29684
.335	.0697895	.05379	.5000928	.46077	.0976779	.02597	.5504999	.30276
.340	.0754585	.05524	.5015459	.46588	.1029965	.02762	.5515107	.30857
.345	.0811458	.05653	.5030161	.47090	.1083348	.02919	.5525350	.31433
.350	1.0868500	4.05783	1.5045032	3.47583	1.1136916	4.03065	1.5535727	3.31994
.355	.0925700	.05898	.5060071	.48064	.1190658	.03199	.5546239	.32548
.360	.0983044	.06004	.5075275	.48534	.1244562	.03326	.5556884	.33090
.365	.1040524	.06101	.5090644	.48996	.1298618	.03444	.5567663	.33626
.370	.1098125	.06189	.5106175	.49449	.1352813	.03552	.5578574	.34151
.375	1.1155839	4.06270	1.5121868	3.49892	1.1407139	4.03652	1.5589617	3.34673
.380	.1213654	.06342	.5137719	.50325	.1461585	.03745	.5600792	.35184
.385	.1271561	.06407	.5153728	.50750	.1516142	.03828	.5612098	.35683
.390	.1329550	.06466	.5169893	.51168	.1570798	.03904	.5623534	.36177
.395	.1387612	.06516	.5186212	.51576	.1625547	.03974	.5635100	.36665
.400	1.1445737	4.06561	1.5202683	3.51975	1.1680380	4.04037	1.5646795	3.37140
.405	.1503918	.06598	.5219305	.52366	.1735288	.04093	.5658618	.37613
.410	.1562144	.06630	.5236076	.52750	.1790261	.04142	.5670569	.38077
.415	.1620410	.06656	.5252996	.53127	.1845294	.04185	.5682648	.38534
.420	.1678706	.06676	.5270059	.53497	.1900377	.04222	.5694852	.38984
.425	1.1737025	4.06690	1.5287267	3.53859	1.1955504	4.04253	1.5707183	3.39427
.430	.1795360	.06699	.5304618	.54214	.2010666	.04279	.5719640	.39863
.435	.1853704	.06704	.5322110	.54561	.2065859	.04300	.5732222	.40293
.440	.1912050	.06704	.5339740	.54901	.2121074	.04316	.5744928	.40717
.445	.1970392	.06699	.5357508	.55234	.2176306	.04327	.5757758	.41135
.450	1.2028723	4.06688	1.5375410	3.55561	1.2231549	4.04332	1.5770712	3.41547
.455	.2087038	.06673	.5393447	.55881	.2286796	.04333	.5783788	.41952
.460	.2145330	.06654	.5411617	.56195	.2342041	.04330	.5796987	.42352
.465	.2203596	.06631	.5429918	.56503	.2397280	.04323	.5810307	.42748
.470	.2261827	.06605	.5448348	.56805	.2452507	.04312	.5823747	.43139
.475	1.2320021	4.06575	1.5466905	3.57102	1.2507718	4.04297	1.5837308	3.43525
.480	.2378174	.06541	.5485587	.57393	.2562905	.04278	.5850990	.43906
.485	.2436279	.06504	.5504394	.57677	.2618068	.04256	.5864791	.44281
.490	.2494331	.06464	.5523323	.57956	.2673203	.04230	.5878709	.44651
.495	.2552327	.06421	.5542373	.58229	.2728299	.04200	.5892746	.45015
.500	1.2610264	4.06375	1.5561542	3.58497	1.2783353	4.04167	1.5906901	3.45374

LOGARITHMIC VALUES OF $f(\alpha) . \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$

α	$\log \frac{\alpha^8 D_\alpha^4 b_{\frac{1}{2}}^{(0)}}{\beta^8}$	log var. for .001 of α.	$\log \frac{\alpha^7 D_\alpha^4 b_{\frac{1}{2}}^{(1)}}{\beta^8}$	log var. for .001 of α.	$\log \frac{\alpha^8 D_\alpha^4 b_{\frac{1}{2}}^{(2)}}{\beta^8}$	log var. for .001 of α.	$\log \frac{\alpha^7 D_\alpha^4 b_{\frac{1}{2}}^{(3)}}{\beta^8}$	log var. for .001 of α.
.500	1.2610264	4.06375	1.5561542	3.58497	1.2783353	4.04167	1.5906901	3.45374
.505	.2668138	.06326	.5580829	.58761	.2838364	.04131	.5921173	.45728
.510	.2725944	.06275	.5600232	.59019	.2893329	.04093	.5935561	.46078
.515	.2783679	.06221	.5619750	.59271	.2948245	.04052	.5950065	.46424
.520	.2841339	.06163	.5639381	.59519	.3003107	.04008	.5964685	.46766
.525	1.2898921	4.06102	1.5659124	3.59762	1.3057912	4.03962	1.5979420	3.47104
.530	.2956421	.06040	.5678976	.60000	.3112657	.03913	.5994269	.47438
.535	.3013838	.05976	.5698936	.60234	.3167340	.03861	.6009232	.47768
.540	.3071169	.05909	.5719004	.60464	.3221956	.03807	.6024308	.48094
.545	.3128412	.05840	.5739178	.60690	.3276505	.03751	.6039496	.48416
.550	1.3185563	4.05770	1.5759454	3.60912	1.3330981	4.03694	1.6054797	3.48733
.555	.3242621	.05698	.5779833	.61130	.3385384	.03635	.6070210	.49046
.560	.3299582	.05625	.5800313	.61342	.3439711	.03574	.6085734	.49356
.565	.3356445	.05549	.5820893	.61550	.3493960	.03511	.6101368	.49663
.570	.3413209	.05471	.5841570	.61754	.3548129	.03446	.6117112	.49966
.575	1.3469872	4.05392	1.5862344	3.61954	1.3602216	4.03380	1.6132966	3.50266
.580	.3526431	.05312	.5883213	.62149	.3656219	.03312	.6148929	.50563
.585	.3582884	.05230	.5904175	.62342	.3710138	.03242	.6165001	.50857
.590	.3639230	.05147	.5925229	.62532	.3763969	.03171	.6181181	.51148
.595	.3695468	.05063	.5946375	.62718	.3817712	.03099	.6197469	.51436
.600	1.3751596	4.04978	1.5967610	3.62899	1.3871364	4.03025	1.6213864	3.51720
.605	.3807612	.04892	.5988933	.63077	.3924925	.02950	.6230367	.52001
.610	.3863516	.04804	.6010344	.63252	.3978394	.02874	.6246976	.52279
.615	.3919307	.04715	.6031840	.63423	.4031769	.02797	.6263690	.52554
.620	.3974983	.04625	.6053419	.63590	.4085049	.02720	.6280510	.52826
.625	1.4030544	4.04534	1.6075082	3.63754	1.4138234	4.02642	1.6297436	3.53094
.630	.4085989	.04443	.6096826	.63915	.4191322	.02563	.6314467	.53360
.635	.4141316	.04351	.6118650	.64073	.4244313	.02483	.6331602	.53624
.640	.4196525	.04258	.6140552	.64228	.4297206	.02402	.6348841	.53886
.645	.4251616	.04164	.6162532	.64380	.4350000	.02321	.6366184	.54145
.650	1.4306587	4.04070	1.6184588	3.64529	1.4402694	4.02239	1.6383630	3.54402
.655	.4361438	.03975	.6206719	.64676	.4455287	.02156	.6401179	.54657
.660	.4416168	.03879	.6228923	.64820	.4507780	.02073	.6418831	.54909
.665	.4470778	.03783	.6251200	.64961	.4560173	.01989	.6436585	.55159
.670	.4525267	.03686	.6273549	.65099	.4612465	.01905	.6454441	.55406
.675	1.4579634	4.03588	1.6295968	3.65234	1.4664656	4.01820	1.6472399	3.55651
.680	.4633879	.03490	.6318456	.65366	.4716745	.01735	.6490458	.55895
.685	.4688002	.03392	.6341013	.65496	.4768732	.01650	.6508619	.56137
.690	.4742002	.03293	.6363637	.65624	.4820617	.01565	.6526880	.56377
.695	.4795879	.03194	.6386327	.65750	.4872400	.01479	.6545242	.56615
.700	1.4849634	4.03095	1.6409081	3.65873	1.4924081	4.01393	1.6563704	3.56851
.705	.4903266	.02996	.6431899	.65992	.4975660	.01307	.6582267	.57085
.710	.4956775	.02896	.6454779	.66109	.5027137	.01222	.6600929	.57318
.715	.5010161	.02796	.6477721	.66224	.5078512	.01137	.6619691	.57549
.720	.5063425	.02696	.6500724	.66337	.5129786	.01051	.6638553	.57778
.725	1.5116566	4.02596	1.6523786	3.66449	1.5180959	4.00965	1.6657514	3.58005
.730	.5169585	.02496	.6546907	.66558	.5232030	.00879	.6676575	.58231
.735	.5222481	.02395	.6570085	.66663	.5283001	.00793	.6695735	.58456
.740	.5275254	.02294	.6593319	.66768	.5333871	.00707	.6714994	.58679
.745	.5327905	.02193	.6616609	.66870	.5384641	.00621	.6734353	.58901
.750	1.5380434	4.02092	1.6639951	3.66963	1.5435311	4.00535	1.6753810	3.59120

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$

α	$\log \frac{\alpha^8 D_\alpha^4 b_{\frac{1}{2}}^{(4)}}{\beta^8}$	log var. for .001 of α.	$\log \frac{\alpha^7 D_\alpha^4 b_{\frac{1}{2}}^{(5)}}{\beta^8}$	log var. for .001 of α.
.450	1.3181597	3.87431	1.7185300	3.19681
.455	.3219119	.87631	.7177476	.19193
.460	.3256813	.87827	.7169743	.18667
.465	.3294677	.88018	.7162106	.18102
.470	.3332704	.88204	.7154572	.17464
.475	1.3370893	3.88385	1.7147146	3.16843
.480	.3409238	.88562	.7139834	.16144
.485	.3447738	.88734	.7132642	.15409
.490	.3486387	.88901	.7125575	.14622
.495	.3525183	.89063	.7118639	.13777
.500	1.3564121	3.89220	1.7111842	3.12875
.505	.3603199	.89372	.7105188	.11923
.510	.3642412	.89520	.7098683	.10907
.515	.3681757	.89664	.7092333	.09827
.520	.3721232	.89804	.7086144	.08676
.525	1.3760832	3.89940	1.7080122	3.07452
.530	.3800555	.90072	.7074272	.06149
.535	.3840398	.90200	.7068601	.04758
.540	.3880356	.90325	.7063114	.03274
.545	.3920428	.90447	.7057818	3.01687
.550	1.3960610	3.90565	1.7052718	2.99991
.555	.4000900	.90680	.7047820	.98182
.560	.4041295	.90792	.7043128	.96242
.565	.4081792	.90901	.7038649	.94156
.570	.4122389	.91006	.7034387	.91913
.575	1.4163084	3.91108	1.7030348	2.89476
.580	.4203872	.91207	.7026539	.86823
.585	.4244753	.91303	.7022965	.83929
.590	.4285723	.91397	.7019632	.80760
.595	.4326780	.91488	.7016544	.77276
.600	1.4367924	3.91577	1.7013706	2.73400
.605	.4409151	.91664	.7011124	.69055
.610	.4450459	.91749	.7008802	.64128
.615	.4491846	.91832	.7006746	.58444
.620	.4533311	.91912	.7004961	.51772
.625	1.4574852	3.91990	1.7003452	2.43759
.630	.4616466	.92066	.7002222	.33766
.635	.4658153	.92141	.7001276	2.20439
.640	.4699911	.92214	.7000621	1.77232
.645	.4741738	.92285	.7000259	.62839
.650	1.4783632	3.92354	1.7000196	1.24797
.655	.4825592	.92421	.7000436	1.89542
.660	.4867616	.92486	.7000980	2.14706
.665	.4909704	.92550	.7001837	.30707
.670	.4951853	.92613	.7003008	.42488
.675	1.4994063	3.92675	1.7004497	2.51838
.680	.5036332	.92735	.7006307	.59616
.685	.5078659	.92794	.7008443	.66266
.690	.5121043	.92852	.7010906	.72090
.695	.5163483	.92909	.7013702	.77276
.700	1.5205979	3.92965	1.7016832	2.81935
.705	.5248529	.93021	.7020299	.86183
.710	.5291133	.93075	.7024107	.90080
.715	.5333789	.93128	.7028257	.93697
.720	.5376497	.93180	.7032752	2.97025
.725	1.5419256	3.93232	1.7037595	3.00156
.730	.5462067	.93283	.7042788	.03089
.735	.5501928	.93334	.7048332	.05854
.740	.5547839	.93385	.7054231	.08472
.745	.5590800	.93436	.7060486	.10951
.750	1.5633810	3.93485	1.7067099	3.13309

α	$\log \frac{\alpha^6 D_\alpha^4 b_{\frac{1}{2}}^{(6)}}{\beta^8}$	$\log \frac{\alpha^5 D_\alpha^4 b_{\frac{1}{2}}^{(7)}}{\beta^8}$
.45	2.0559298	2.3395318
.46	.0494219	.3301744
.47	.0428223	.3206186
.48	.0361395	.3108669
.49	.0293763	.3009219
.50	2.0225397	2.2907869
.51	.0156361	.2804656
.52	.0086722	.2699618
.53	2.0016554	.2592802
.54	1.9945928	.2484258
.55	1.9874921	2.2374041
.56	.9803614	.2262212
.57	.9732091	.2148838
.58	.9660442	.2033993
.59	.9588757	.1917757
.60	1.9517132	2.1800219
.61	.9445666	.1681475
.62	.9374461	.1561629
.63	.9303622	.1440795
.64	.9233258	.1319092
.65	1.9163479	2.1196650
.66	.9094398	.1073608
.67	.9026129	.0950115
.68	.8958789	.0821326
.69	.8892496	.0702409
.70	1.8827367	2.0578541
.71	.8763520	.0454908
.72	.8701072	.0331704
.73	.8640137	.0209129
.74	.8580831	2.0087396
.75	1.8523269	1.9966725

α	$\log \frac{\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(8)}}{\beta^8}$	$\log \frac{\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(9)}}{\alpha \beta^8}$
.50	2.5232907	2.7272179
.51	.5107714	.7132065
.52	.4979736	.6988446
.53	.4848980	.6841291
.54	.4715458	.6690572
.55	2.4579187	2.6536266
.56	.4440185	.6378349
.57	.4298478	.6216804
.58	.4154099	.6051618
.59	.4007086	.5882782
.60	2.3857489	2.5710295
.61	.3705365	.5534160
.62	.3550781	.5354392
.63	.3393811	.5171014
.64	.3234541	.4984055
.65	2.3073069	2.4793559
.66	.2909508	.4599584
.67	.2743988	.4402204
.68	.2576651	.4201507
.69	.2407654	.3997600
.70	2.2237169	2.3790608
.71	.2065365	.3580683
.72	.1892506	.3368002
.73	.1718749	.3152766
.74	.1544351	.2935208
.75	2.1369568	2.2715594

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha^5 D_\alpha^5 b_{\frac{1}{2}}^{(i)}$

α	$\log \frac{\alpha^9 D_\alpha^5 b_{\frac{1}{2}}^{(0)}}{\beta^{10}}$	$\log \frac{\alpha^{10} D_\alpha^5 b_{\frac{1}{2}}^{(1)}}{\beta^{10}}$	$\log \frac{\alpha^9 D_\alpha^5 b_{\frac{1}{2}}^{(2)}}{\beta^{10}}$	$\log \frac{\alpha^{10} D_\alpha^5 b_{\frac{1}{2}}^{(3)}}{\beta^{10}}$	$\log \frac{\alpha^9 D_\alpha^5 b_{\frac{1}{2}}^{(4)}}{\beta^{10}}$
.45	2.3087238	1.9712301	2.3203701	1.997455	2.361716
.46	.3149009	1.9861043	.3262313	2.011489	.366355
.47	.3211404	2.0009204	.3321554	.025478	.371063
.48	.3274395	.0156758	.3381399	.039419	.375837
.49	.3337955	.0303685	.3441822	.053312	.380675
.50	2.3402055	2.0449963	2.3502799	2.067153	2.385577
.51	.3466671	.0595570	.3564305	.080941	.390543
.52	.3531778	.0740492	.3626318	.094674	.395569
.53	.3597350	.0884719	.3688814	.108349	.400653
.54	.3663365	.1028239	.3751770	.121967	.405796
.55	2.3729797	2.1171001	2.3815167	2.135525	2.410995
.56	.3796623	.1313080	.3878982	.149024	.416249
.57	.3863821	.1454472	.3943196	.162463	.421559
.58	.3931371	.1595092	.4007789	.175840	.426920
.59	.3999253	.1734979	.4072741	.189156	.432333
.60	2.4067448	2.1874132	2.4138035	2.202412	2.437796
.61	.4135934	.2012551	.4203654	.215606	443307
.62	.4204692	.2150236	.4269580	.228739	.448867
.63	.4273705	.2287189	.4335798	.241810	.454475
.64	.4342956	.2423414	.4402291	.254820	.460128
.65	2.4412429	2.2558912	2.4469044	2.267770	2.465827
.66	.4482108	.2693687	.4536044	.280660	.471568
.67	.4551976	.2827746	.4603275	.293490	.477352
.68	.4622018	.2961092	.4670725	.306262	.483180
.69	.4692221	.3093731	.4738380	.318976	.489050
.70	2.4762571	2.3225668	2.4806230	2.331632	2.494960
.71	.4833055	.3356909	.4874260	.344230	.500909
.72	.4903661	.3487461	.4942461	.356772	.506897
.73	.4974375	.3617329	.5010823	.369259	.512923
.74	.5045186	.3746522	.5079336	.381691	.518987
.75	2.5116083	2.3875046	2.5147989	2.394070	2.525089

α	$\log \frac{\alpha^{10} D_\alpha^5 b_{\frac{1}{2}}^{(5)}}{\beta^{10}}$	$\log \frac{\alpha^9 D_\alpha^5 b_{\frac{1}{2}}^{(6)}}{\beta^{10}}$	$\log \frac{\alpha^8 D_\alpha^5 b_{\frac{1}{2}}^{(7)}}{\beta^{10}}$	$\log \frac{\alpha^7 D_\alpha^5 b_{\frac{1}{2}}^{(8)}}{\beta^{10}}$	$\log \frac{\alpha^6 D_\alpha^5 b_{\frac{1}{2}}^{(9)}}{\beta^{10}}$
.50	2.131775	2.485940	2.818001	3.1177960	3.3857251
.51	.142678	.486643	.811397	.1063243	.3709978
.52	.153614	.487475	.804823	.0947314	.3560121
.53	.164581	.488439	.798289	.0830288	.3407758
.54	.175575	.489539	.791807	.0712287	.3252975
.55	2.186593	2.490780	2.785389	3.0593442	3.3095870
.56	.197632	.492163	.779046	.0473895	.2936549
.57	.208690	.493690	.772791	.0353790	.2775132
.58	.219765	.495363	.766635	.0233280	.2611750
.59	.230855	.497183	.760588	3.0112528	.2446543
.60	2.241958	2.499152	2.754664	2.9991701	3.2279671
.61	.253072	.501271	.748873	.9870972	.2111297
.62	.264195	.503541	.743228	.9750521	.1941606
.63	.275326	.505962	.737738	.9630537	.1770795
.64	.286464	.508535	.732415	.9511211	.1599076
.65	2.297607	2.511261	2.727268	2.9392737	3.1426674
.66	.308754	.514139	.722308	.9275314	.1253827
.67	.319905	.517167	.717544	.9159143	.1080790
.68	.331058	.520346	.712983	.9044423	.0907829
.69	.342214	.523674	.708632	.8931357	.0735222
.70	2.353370	2.527149	2.704502	2.8820142	3.0563260
.71	.364527	.530770	.700598	.8710971	.0392241
.72	.375684	.534532	.696926	.8604034	.0222470
.73	.386840	.538433	.693491	.8499514	3.0054257
.74	.397996	.542471	.690296	.8397583	2.9887915
.75	2.409151	2.546640	2.687339	2.8298407	2.9723758

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha\, b_{\frac{3}{2}}^{(i)}$

α	$\log \frac{\alpha^4 b_{\frac{3}{2}}^{(0)}}{\beta^4}$	log var. for .001 of α.	$\log \frac{\alpha^3 b_{\frac{3}{2}}^{(1)}}{\beta^4}$	log var. for .001 of α.	$\log \frac{\alpha^2 b_{\frac{3}{2}}^{(2)}}{\beta^4}$	log var. for .001 of α.	$\log \frac{\alpha\, b_{\frac{3}{2}}^{(3)}}{\beta^4}$	log var. for .001 of α.
.000	0.3010300	— ∞	0.4771213	— ∞	0.5740313	— ∞	0.6409781	— ∞
.005	.3010328	1.0374	.4771200	0.7324	.5740285	1.0492	.6409749	1.1206
.010	.3010409	.3365	.4771159	1.0374	.5740201	.3423	.6409649	.4281
.015	.3010544	.5132	.4771091	.2122	.5740065	.5105	.6409481	.6064
.020	.3010735	.6375	.4770996	.3365	.5739877	.6355	.6409245	.7324
.025	0.3010979	1.7348	0.4770874	1.4362	0.5739633	1.7340	0.6408941	1.8306
.030	.3011277	.8136	.4770723	.5159	.5739335	.8129	.6408568	.9154
.035	.3011630	.8808	.4770546	.5809	.5738983	.8802	.6408118	1.9818
.040	.3012037	.9385	.4770342	.6385	.5738576	.9390	.6407609	2.0363
.045	.3012498	1.9899	.4770111	.6902	.5738114	1.9908	.6407031	.0874
.050	0.3013014	2.0359	0.4769852	1.7364	0.5737597	2.0363	0.6406386	2.1332
.055	.3013584	2.0770	.4769566	.7774	.5737026	.0781	.6405672	.1753
.060	.3014208	.1145	.4769253	.8156	.5736400	.1162	.6404889	.2130
.065	.3014886	.1495	.4768912	.8506	.5735720	.1507	.6404039	.2477
.070	.3015619	.1815	.4768544	.8825	.5734985	.1832	.6403120	.2801
.075	0.3016406	2.2113	0.4768149	1.9122	0.5734195	2.2133	0.6402133	2.3102
.080	.3017246	.2392	.4767727	.9400	.5733351	.2412	.6401077	.3387
.085	.3018141	.2655	.4767278	.9661	.5732452	.2679	.6399952	.3653
.090	.3019090	.2902	.4766802	1.9912	.5731498	.2930	.6398758	.3903
.095	.3020093	.3139	.4766298	2.0149	.5730489	.3168	.6397496	.4140
.100	0.3021150	2.3361	0.4765767	2.0371	0.5729424	2.3395	0.6396164	2.4367
.105	.3022261	.3574	.4765209	.0584	.5728304	.3608	.6394763	.4581
.110	.3023427	.3774	.4764623	.0789	.5727129	.3809	.6393293	.4786
.115	.3024647	.3967	.4764010	.0983	.5725900	.4004	.6391753	.4981
.120	.3025920	.4151	.4763369	.1168	.5724615	.4193	.6390144	.5171
.125	0.3027247	2.4328	0.4762701	2.1348	0.5723274	2.4375	0.6388464	2.5351
.130	.3028629	.4497	.4762005	.1523	.5721877	.4548	.6386715	.5525
.135	.3030065	.4661	.4761281	.1688	.5720424	.4716	.6384895	.5694
.140	.3031555	.4818	.4760530	.1847	.5718915	.4877	.6383005	.5855
.145	.3033098	.4970	.4759751	.2004	.5717350	.5034	.6381045	.6011
.150	0.3034695	2.5116	0.4758944	2.2153	0.5715728	2.5185	0.6379014	2.6163
.155	.3036346	.5258	.4758109	.2300	.5714050	.5332	.6376912	.6309
.160	.3038051	.5395	.4757246	.2440	.5712315	.5472	.6374739	.6452
.165	.3039810	.5528	.4756355	.2577	.5710524	.5610	.6372494	.6592
.170	.3041622	.5657	.4755436	.2709	.5708676	.5743	.6370177	.6726
.175	0.3043488	2.5782	0.4754489	2.2835	0.5706771	2.5873	0.6367789	2.6855
.180	.3045408	.5904	.4753515	.2961	.5704810	.5999	.6365329	.6984
.185	.3047381	.6021	.4752512	.3086	.5702791	.6122	.6362796	.7109
.190	.3049408	.6135	.4751480	.3205	.5700715	.6243	.6360190	.7229
.195	.3051488	.6247	.4750420	.3320	.5698581	.6362	.6357512	.7348
.200	0.3053622	2.6356	0.4749332	2.3434	0.5696388	2.6478	0.6354760	2.7464
.205	.3055810	.6462	.4748215	.3543	.5694137	.6591	.6351935	.7576
.210	.3058051	.6565	.4747071	.3649	.5691827	.6700	.6349037	.7687
.215	.3060345	.6667	.4745898	.3756	.5689460	.6806	.6346064	.7797
.220	.3062693	.6766	.4744696	.3860	.5687034	.6912	.6343016	.7904
.225	0.3065094	2.6863	0.4743466	2.3962	0.5684549	2.7015	0.6339893	2.8008
.230	.3067548	.6957	.4742206	.4062	.5682005	.7117	.6336695	.8110
.235	.3070056	.7050	.4740918	.4160	.5679401	.7217	.6333422	.8210
.240	.3072617	.7141	.4739600	.4257	.5676737	.7314	.6330073	.8309
.245	.3075232	.7229	.4738253	.4349	.5674014	.7409	.6326648	.8405
.250	0.3077899	2.7315	0.4736878	2.4440	0.5671230	2.7503	0.6323147	2.8500

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha\, b_{\frac{3}{2}}^{(i)}$

α	$\log \frac{\alpha^4 b_{\frac{3}{2}}^{(0)}}{\beta^4}$	log var. for .001 of α.	$\log \frac{\alpha^3 b_{\frac{3}{2}}^{(1)}}{\beta^4}$	log var. for .001 of α.	$\log \frac{\alpha^2 b_{\frac{3}{2}}^{(2)}}{\beta^4}$	log var. for .001 of α.	$\log \frac{\alpha\, b_{\frac{3}{2}}^{(3)}}{\beta^4}$	log var. for .001 of α.
.250	0.3077899	2.7315	0.4736878	2.4440	0.5671230	2.7503	0.6323147	2.8500
.255	.3080620	.7400	.4735473	.4532	.5668386	.7596	.6319569	.8593
.260	.3083394	.7483	.4734039	.4620	.5665481	.7687	.6315914	.8686
.265	.3086221	.7564	.4732576	.4705	.5662515	.7775	.6312181	.8776
.270	.3089101	.7644	.4731084	.4792	.5659489	.7864	.6308370	.8866
.275	0.3092034	2.7723	0.4729562	2.4877	0.5656400	2.7951	0.6304480	2.8954
.280	.3095020	.7800	.4728010	.4959	.5653250	.8036	.6300511	.9040
.285	.3098059	.7875	.4726429	.5042	.5650038	.8120	.6296463	.9126
.290	.3101150	.7949	.4724817	.5123	.5646764	.8202	.6292334	.9210
.295	.3104294	.8022	.4723176	.5199	.5643428	.8284	.6288125	.9294
.300	0.3107491	2.8093	0.4721505	2.5279	0.5640028	2.8366	0.6283835	2.9376
.305	.3110740	.8163	.4719804	.5356	.5636564	.8446	.6279463	.9457
.310	.3114041	.8232	.4718073	.5432	.5633037	.8524	.6275010	.9537
.315	.3117395	.8300	.4716311	.5507	.5629446	.8601	.6270475	.9616
.320	.3120801	.8366	.4714519	.5580	.5625790	.8679	.6265856	.9694
.325	0.3124259	2.8432	0.4712697	2.5653	0.5622070	2.8754	0.6261154	2.9772
.330	.3127769	.8496	.4710844	.5725	.5618285	.8828	.6256368	.9848
.335	.3131331	.8560	.4708960	.5796	.5614434	.8903	.6251498	.9921
.340	.3134946	.8623	.4707045	.5867	.5610517	.8976	.6246542	2.9999
.345	.3138613	.8685	.4705099	.5936	.5606534	.9049	.6241500	3.0073
.350	0.3142331	2.8746	0.4703122	2.6006	0.5602484	2.9121	0.6236372	3.0146
.355	.3146102	.8806	.4701113	.6074	.5598367	.9192	.6231157	.0219
.360	.3149925	.8865	.4699073	.6139	.5594183	.9261	.6225854	.0291
.365	.3153800	.8923	.4697002	.6205	.5589931	.9332	.6220463	.0363
.370	.3157727	.8980	.4694899	.6271	.5585610	.9395	.6214983	.0434
.375	0.3161706	2.9036	0.4692764	2.6338	0.5581221	2.9469	0.6209413	3.0504
.380	.3165737	.9091	.4690596	.6402	.5576762	.9536	.6203753	.0573
.385	.3169819	.9146	.4688397	.6464	.5572234	.9603	.6198002	.0642
.390	.3173952	.9200	.4686166	.6527	.5567635	.9670	.6192159	.0711
.395	.3178137	.9254	.4683902	.6591	.5562966	.9736	.6186223	.0779
.400	0.3182374	2.9307	0.4681605	2.6652	0.5558225	2.9802	0.6180194	3.0847
.405	.3186662	.9359	.4679276	.6713	.5553412	.9867	.6174070	.0915
.410	.3191002	.9410	.4676914	.6773	.5548527	.9931	.6167851	.0981
.415	.3195393	.9461	.4674519	.6834	.5543569	2.9995	.6161537	.1046
.420	.3199835	.9511	.4672090	.6894	.5538538	3.0059	.6155127	.1112
.425	0.3204328	2.9561	0.4669628	2.6952	0.5533433	3.0122	0.6148619	3.1178
.430	.3208873	.9610	.4667133	.7010	.5528254	.0184	.6142012	.1243
.435	.3213469	.9658	.4664604	.7070	.5523000	.0247	.6135306	.1307
.440	.3218116	.9706	.4662041	.7126	.5517670	.0308	.6128500	.1372
.445	.3222813	.9753	.4659444	.7182	.5512264	.0370	.6121592	.1436
.450	0.3227571	2.9799	0.4656813	2.7241	0.5506781	3.0431	0.6114583	3.1499
.455	.3232371	.9845	.4654147	.7298	.5501220	.0492	.6107470	.1563
.460	.3237222	.9891	.4651446	.7354	.5495581	.0552	.6100253	.1625
.465	.3242124	.9936	.4648710	.7409	.5489864	.0612	.6092931	.1688
.470	.3247077	2.9981	.4645939	.7464	.5484067	.0672	.6085502	.1751
.475	0.3252081	3.0025	0.4643133	2.7519	0.5478189	3.0732	0.6077965	3.1813
.480	.3257135	.0069	.4640291	.7573	.5472230	.0791	.6070320	.1875
.485	.3262240	.0112	.4637414	.7629	.5466190	.0850	.6062566	.1936
.490	.3267395	.0154	.4634501	.7681	.5460067	.0909	.6054702	.1998
.495	.3272600	.0196	.4631551	.7735	.5453862	.0967	.6046726	.2059
.500	0.3277856	3.0238	0.4628565	2.7788	0.5447573	3.1025	0.6038636	3.2121

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha\, b_{\frac{3}{2}}^{(i)}$

α	$\log \frac{\alpha^4 b_{\frac{3}{2}}^{(0)}}{\beta^4}$	log var. for .001 of α.	$\log \frac{\alpha^3 b_{\frac{3}{2}}^{(1)}}{\beta^4}$	log var. for .001 of α.	$\log \frac{\alpha^2 b_{\frac{3}{2}}^{(2)}}{\beta^4}$	log var. for .001 of α.	$\log \frac{\alpha\, b_{\frac{3}{2}}^{(3)}}{\beta^4}$	log var. for .001 of α.
.500	0.3277856	3.0238	0.4628565	2.7788	0.5447573	3.1025	0.6038636	3.2121
.505	.3283162	.0279	.4625542	.7841	.5441199	.1083	.6030431	.2182
.510	.3288519	.0320	.4622482	.7895	.5434740	.1141	.6022111	.2242
.515	.3293926	.0360	.4619384	.7947	.5428194	.1199	.6013674	.2304
.520	.3299383	.0400	.4616249	.7998	.5421560	.1257	.6005118	.2363
.525	0.3304891	3.0440	0.4613077	2.8049	0.5414838	3.1314	0.5996442	3.2424
.530	.3310448	.0479	.4609867	.8102	.5408028	.1370	.5987645	.2484
.535	.3316055	.0517	.4606619	.8152	.5401128	.1428	.5978725	.2544
.540	.3321713	.0555	.4603333	.8203	.5394136	.1484	.5969681	.2604
.545	.3327421	.0593	.4600008	.8253	.5387053	.1541	.5960511	.2664
.550	0.3333178	3.0631	0.4596644	2.8304	0.5379877	3.1598	0.5951214	3.2724
.555	.3338984	.0668	.4593240	.8356	.5372607	.1654	.5941788	.2783
.560	.3344840	.0705	.4589796	.8405	.5365243	.1710	.5932232	.2843
.565	.3350746	.0742	.4586313	.8455	.5357783	.1766	.5922543	.2903
.570	.3356701	.0778	.4582790	.8504	.5350227	.1822	.5912720	.2962
.575	0.3362705	3.0814	0.4579227	2.8554	0.5342572	3.1878	0.5902761	3.3022
.580	.3368760	.0850	.4575622	.8604	.5334818	.1933	.5892664	.3082
.585	.3374865	.0885	.4571976	.8653	.5326964	.1989	.5882428	.3142
.590	.3381020	.0920	.4568288	.8703	.5319008	.2045	.5872050	.3204
.595	.3387224	.0954	.4564558	.8751	.5310950	.2100	.5861529	.3261
.600	0.3393478	3.0988	0.4560786	2.8800	0.5302788	3.2156	0.5850862	3.3320
.605	.3399780	.1022	.4556972	.8849	.5294520	.2212	.5840048	.3380
.610	.3406132	.1056	.4553115	.8897	.5286146	.2268	.5829085	.3440
.615	.3412533	.1089	.4549215	.8945	.5277664	.2323	.5817970	.3500
.620	.3418983	.1122	.4545271	.8994	.5269073	.2379	.5806700	.3560
.625	0.3425482	3.1155	0.4541283	2.9043	0.5260371	3.2434	0.5795273	3.3620
.630	.3432030	.1188	.4537250	.9090	.5251557	.2490	.5783687	.3680
.635	.3438628	.1221	.4533173	.9138	.5242629	.2546	.5771940	.3740
.640	.3445276	.1253	.4529050	.9187	.5233586	.2601	.5760028	.3800
.645	.3451973	.1285	.4524881	.9234	.5224427	.2657	.5747950	.3860
.650	0.3458717	3.1317	0.4520666	2.9283	0.5215150	3.2713	0.5735702	3.3921
.655	.3465512	.1348	.4516404	.9331	.5205752	.2768	.5723282	.3982
.660	.3472356	.1379	.4512094	.9378	.5196233	.2824	.5710688	.4043
.665	.3479249	.1410	.4507738	.9425	.5186591	.2880	.5697916	.4104
.670	.3486191	.1441	.4503334	.9472	.5176823	.2936	.5684962	.4165
.675	0.3493183	3.1471	0.4498881	2.9520	0.5166929	3.2992	0.5671824	3.4226
.680	.3500224	.1502	.4494379	.9568	.5156907	.3048	.5658500	.4288
.685	.3507314	.1532	.4489827	.9616	.5146754	.3104	.5644985	.4350
.690	.3514453	.1562	.4485224	.9663	.5136468	.3161	.5631274	.4412
.695	.3521641	.1592	.4480572	.9710	.5126047	.3218	.5617366	.4474
.700	0.3528879	3.1621	0.4475869	2.9758	0.5115489	3.3275	0.5603257	3.4537
.705	.3536166	.1650	.4471114	.9806	.5104791	.3332	.5588942	.4600
.710	.3543502	.1679	.4466307	.9853	.5093952	.3389	.5574417	.4663
.715	.3550887	.1708	.4461447	.9901	.5082968	.3447	.5559678	.4727
.720	.3558321	.1737	.4456533	.9949	.5071838	.3504	.5544723	.4791
.725	0.3565804	3.1765	0.4451566	2.9994	0.5060558	3.3562	0.5529545	3.4855
.730	.3573336	.1793	.4446545	3.0042	.5049127	.3620	.5514141	.4919
.735	.3580917	.1821	.4441468	.0091	.5037543	.3678	.5498506	.4984
.740	.3588547	.1849	.4436334	.0138	.5025803	.3736	.5482635	.5049
.745	.3596226	.1877	.4431144	.0186	.5013904	.3795	.5466525	.5114
.750	0.3603953	3.1905	0.4425897	3.0233	0.5001844	3.3854	0.5450170	3.5179

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha\, b_{\frac{3}{2}}^{(i)}$

α	$\log \frac{\alpha\, b_{\frac{3}{2}}^{(4)}}{\alpha\, \beta^4}$	log var. for .001 of α.	$\log \frac{\alpha\, b_{\frac{3}{2}}^{(5)}}{\alpha^2\, \beta^4}$	log var. for .001 of α.	α	$\log \frac{\alpha\, b_{\frac{3}{2}}^{(4)}}{\alpha\, \beta^4}$	log var. for .001 of α.	$\log \frac{\alpha\, b_{\frac{3}{2}}^{(5)}}{\alpha^2\, \beta^4}$	log var. for .001 of α.
.000	0.6921306	$-\infty$	0.7335233	$-\infty$	.250	0.6824083	2.9010	0.7230919	2.9322
.005	.6921268	1.1818	.7335191	1.2175	.255	.6820058	.9105	.7226595	.9417
.010	.6921154	.4829	.7335068	.5145	.260	.6815946	.9197	.7222176	.9510
.015	.6920964	.6590	.7334864	.6893	.265	.6811746	.9289	.7217662	.9602
.020	.6920698	.7839	.7334579	.8136	.270	.6807457	.9379	.7213052	.9692
.025	0.6920356	1.8808	0.7334213	1.9101	.275	0.6803078	2.9468	0.7208346	2.9782
.030	.6919938	1.9600	.7333766	1.9894	.280	.6798609	.9555	.7203542	.9870
.035	.6919444	2.0274	.7333237	2.0569	.285	.6794051	.9641	.7198640	2.9957
.040	.6918873	.0856	.7332626	.1151	.290	.6789402	.9726	.7193640	3.0043
.045	.6918226	.1367	.7331933	.1668	.295	.6784662	.9810	.7188541	.0128
.050	0.6917503	2.1826	0.7331158	2.2127	.300	0.6779829	2.9894	0.7183342	3.0212
.055	.6916703	.2245	.7330301	.2544	.305	.6774904	2.9976	.7178042	.0294
.060	.6915826	.2622	.7329362	.2925	.310	.6769885	3.0057	.7172642	.0375
.065	.6914874	.2971	.7328340	.3273	.315	.6764772	.0137	.7167140	.0456
.070	.6913844	.3296	.7327237	.3594	.320	.6759564	.0216	.7161534	.0537
.075	0.6912738	2.3596	0.7326052	2.3897	.325	0.6754261	3.0295	0.7155825	3.0616
.080	.6911555	.3879	.7324784	.4181	.330	.6748862	.0372	.7150011	.0694
.085	.6910295	.4147	.7323433	.4446	.335	.6743366	.0449	.7144093	.0771
.090	.6908957	.4398	.7322000	.4698	.340	.6737773	.0524	.7138068	.0848
.095	.6907542	.4634	.7320483	.4936	.345	.6732082	.0600	.7131937	.0924
.100	0.6906050	2.4860	0.7318884	2.5162	.350	0.6726291	3.0676	0.7125698	3.0999
.105	.6904480	.5076	.7317201	.5377	.355	.6720401	.0748	.7119350	.1074
.110	.6902832	.5281	.7315435	.5582	.360	.6714411	.0821	.7112893	.1148
.115	.6901106	.5478	.7313585	.5781	.365	.6708319	.0895	.7106325	.1221
.120	.6899302	.5665	.7311650	.5969	.370	.6702124	.0967	.7099646	.1294
.125	0.6897420	2.5848	0.7309632	2.6150	.375	0.6695826	3.1038	0.7092854	3.1366
.130	.6895458	.6023	.7307529	.6222	.380	.6689424	.1109	.7085949	.1438
.135	.6893418	.6189	.7305342	.6492	.385	.6682917	.1179	.7078929	.1509
.140	.6891299	.6352	.7303070	.6655	.390	.6676304	.1249	.7071794	.1580
.145	.6889101	.6509	.7300713	.6813	.395	.6669585	.1318	.7064542	.1650
.150	0.6886823	2.6662	0.7298269	2.6966	.400	0.6662758	3.1387	0.7057172	3.1720
.155	.6884465	.6807	.7295740	.7113	.405	.6655822	.1455	.7049683	.1789
.160	.6882028	.6950	.7293125	.7256	.410	.6648777	.1523	.7042073	.1859
.165	.6879510	.7091	.7290424	.7395	.415	.6641621	.1591	.7034341	.1927
.170	.6876910	.7226	.7287636	.7530	.420	.6634353	.1658	.7026487	.1995
.175	0.6874230	2.7357	0.7284762	2.7662	.425	0 6626973	3.1725	0.7018509	3.2063
.180	.6871469	.7484	.7281799	.7791	.430	.6619478	.1791	.7010406	.2131
.185	.6868627	.7609	.7278749	.7916	.435	.6611868	.1857	.7002176	.2198
.190	.6865703	.7732	.7275611	.8038	.440	.6604142	.1923	.6993819	.2264
.195	.6862696	.7851	.7272384	.8158	.445	.6596298	.1989	.6985333	.2331
.200	0.6859606	2.7968	0.7269068	2.8275	.450	0.6588334	3.2054	0.6976715	3.2398
.205	.6856433	.8081	.7265662	.8390	.455	.6580250	.2119	.6967963	.2464
.210	.6853177	.8193	.7262166	.8501	.460	.6572044	.2184	.6959078	.2529
.215	.6849837	.8301	.7258581	.8609	.465	.6563715	.2248	.6950060	.2594
.220	.6846414	.8408	.7254906	.8717	.470	.6555262	.2313	.6940906	.2660
.225	0.6842906	2.8514	0.7251139	2.8823	.475	0.6546683	3.2377	0.6931611	3.2726
.230	.6839313	.8616	.7247280	.8927	.480	.6537977	.2441	.6922175	.2791
.235	.6835634	.8718	.7243329	.9028	.485	.6529142	.2504	.6912598	.2855
.240	.6831870	.8816	.7239286	.9128	.490	.6520178	.2567	.6902876	.2920
.245	.6828020	.8914	.7235149	.9226	.495	.6511083	.2630	.6893009	.2984
.250	0.6824083	2.9010	0.7230919	2.9322	.500	0.6501855	3.2693	0.6882995	3.3049

LOGARITHMIC VALUES OF $f(\alpha) \, . \, \alpha \, b_{\frac{3}{2}}^{(i)}$

α	$\log \frac{\alpha \, b_{\frac{3}{2}}^{(4)}}{\alpha \, \beta^4}$	log var for .001 of α.	$\log \frac{\alpha \, b_{\frac{3}{2}}^{(5)}}{\alpha^2 \, \beta^4}$	log var. for .001 of α.
.500	0.6501855	3.26930	0.6882995	3.30486
.505	.6492492	.27559	.6872832	.31129
.510	.6482993	.28185	.6862517	.31771
.515	.6473356	.28812	.6852049	.32410
.520	.6463579	.29436	.6841426	.33047
.525	0.6453661	3.30060	0.6830646	3.33688
.530	.6443599	.30683	.6819705	.34327
.535	.6433392	.31304	.6808603	.34963
.540	.6423038	.31927	.6797337	.35601
.545	.6412534	.32552	.6785904	.36239
.550	0.6401878	3.33173	0.6774302	3.36875
.555	.6391069	.33794	.6762529	.37513
.560	.6380104	.34416	.6750581	.38150
.565	.6368981	.35038	.6738456	.38790
.570	.6357697	.35660	.6726152	.39428
.575	0.6346251	3.36278	0.6713666	3.40067
.580	.6334641	.36897	.6700995	.40707
.585	.6322864	.37519	.6688135	.41347
.590	.6310917	.38143	.6675085	.41988
.595	.6298797	.38766	.6661840	.42632
.600	0.6286502	3.39388	0.6648397	3.43276
.605	.6274030	.40011	.6634753	.43919
.610	.6261377	.40637	.6620906	.44563
.615	.6248540	.41265	.6606851	.45211
.620	.6235516	.41896	.6592585	.45860
.625	0.6222301	3.42524	0.6578104	3.46509
.630	.6208894	.43154	.6563405	.47162
.635	.6195290	.43788	.6548482	.47819
.640	.6181486	.44422	.6533331	.48477
.645	.6167479	.45058	.6517949	.49135
.650	0.6153265	3.45696	0.6502332	3.49797
.655	.6138840	.46336	.6486474	.50462
.660	.6124201	.46978	.6470371	.51129
.665	.6109343	.47624	.6454018	.51800
.670	.6094262	.48272	.6437410	.52474
.675	0.6078954	3.48923	0.6420542	3.53149
.680	.6063414	.49576	.6403409	.53828
.685	.6047638	.50233	.6386005	.54512
.690	.6031621	.50893	.6368324	.55200
.695	.6015358	.51556	.6350360	.55893
.700	0.5998845	3.52219	0.6332106	3.56590
.705	.5982077	.52889	.6313556	.57289
.710	.5965047	.53563	.6294704	.57994
.715	.5947750	.54240	.6275542	.58705
.720	.5930181	.54921	.6256063	.59419
.725	0.5912333	3.55608	0.6236260	3.60139
.730	.5894199	.56301	.6216125	.60863
.735	.5875773	.56996	.6195650	.61592
.740	.5857049	.57696	.6174828	.62325
.745	.5838019	.58403	.6153650	.63062
.750	0.5818675	3.59118	0.6132109	3.63801

α	$\log \frac{\alpha \, b_{\frac{3}{2}}^{(6)}}{\alpha^3 \, \beta^4}$	$\log \frac{\alpha \, b_{\frac{3}{2}}^{(7)}}{\alpha^4 \, \beta^4}$
.45	0.7305903	0.7591606
.46	.7287287	.7572243
.47	.7268093	.7552273
.48	.7248307	.7531679
.49	.7227912	.7510443
.50	0.7206891	0.7488549
.51	.7185229	.7465979
.52	.7162907	.7442712
.53	.7139905	.7418731
.54	.7116203	.7394012
.55	0.7091783	0.7368531
.56	.7066624	.7342264
.57	.7040700	.7315184
.58	.7013985	.7287267
.59	.6986453	.7258485
.60	0.6958077	0.7228805
.61	.6928828	.7198194
.62	.6898674	.7166619
.63	.6867581	.7134040
.64	.6835512	.7100416
.65	0.6802427	0.7065705
.66	.6768287	.7029864
.67	.6733049	.6992845
.68	.6696665	.6954596
.69	.6659084	.6915058
.70	0.6620250	0.6874169
.71	.6580104	.6831863
.72	.6538583	.6788068
.73	.6495618	.6742708
.74	.6451138	.6695702
.75	0.6405065	0.6646962

α	$\log \frac{\alpha \, b_{\frac{3}{2}}^{(8)}}{\alpha^5 \, \beta^4}$	$\log \frac{\alpha \, b_{\frac{3}{2}}^{(9)}}{\alpha^6 \, \beta^4}$
.50	0.7737748	0.7961217
.51	.7714460	.7937349
.52	.7690447	.7912731
.53	.7665688	.7887341
.54	.7640161	.7861154
.55	0.7613838	0.7834145
.56	.7586692	.7806285
.57	.7558697	.7777547
.58	.7529825	.7747899
.59	.7500045	.7717306
.60	0.7469324	0.7685735
.61	.7437625	.7653149
.62	.7404912	.7619507
.63	.7371144	.7584766
.64	.7336279	.7548882
.65	0.7300270	0.7511806
.66	.7263066	.7473484
.67	.7224617	.7433859
.68	.7184864	.7392869
.69	.7143744	.7350449
.70	0.7101195	0.7306530
.71	.7057142	.7261033
.72	.7011504	.7213873
.73	.6964196	.7164961
.74	.6915128	.7114199
.75	0.6864204	0.7061471

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$

α	$\log \frac{\alpha^5 D_\alpha b_{\frac{3}{2}}^{(0)}}{\beta^6}$	log var. for .001 of α.	$\log \frac{\alpha^6 D_\alpha b_{\frac{3}{2}}^{(1)}}{\beta^6}$	log var. for .001 of α.	$\log \frac{\alpha^5 D_\alpha b_{\frac{3}{2}}^{(2)}}{\beta^6}$	log var. for .001 of α.	$\log \frac{\alpha^4 D_\alpha b_{\frac{3}{2}}^{(3)}}{\beta^6}$	log var. for .001 of α.
.000	0.9542425	$-\infty$	0.4771213	$-\infty$	0.8750613	$-\infty$	1.1180993	$-\infty$
.005	.9542438	0.7324	.4771498	2.05690	.8750665	1.3304	.1180972	0.9138
.010	.9542479	1.0374	.4772353	.35774	.8750827	.6365	.1180911	1.2122
.015	.9542547	.2122	.4773777	.53364	.8751098	.8136	.1180809	.3874
.020	.9542642	.3365	.4775770	.65839	.8751478	1.9385	.1180667	.5119
.025	0.9542764	1.4346	0.4778331	2.75511	0.8751966	2.0354	1.1180484	1.6096
.030	.9542914	.5145	.4781460	.83398	.8752563	.1149	.1180260	.6893
.035	.9543091	.5809	.4785154	.90048	.8753269	.1818	.1179995	.7559
.040	.9543295	.6385	.4789412	2.95804	.8754083	.2393	.1179690	.8136
.045	.9543526	.6884	.4794233	3.00869	.8755004	.2900	.1179344	.8657
.050	0.9543783	1.7340	0.4799614	3.05389	0.8756033	2.3359	1.1178956	1.9117
.055	.9544068	.7760	.4805554	.09468	.8757171	.3771	.1178528	.9528
.060	.9544380	.8136	.4812050	.13176	.8758416	.4146	.1178059	1.9908
.065	.9544719	.8482	.4819098	.16572	.8759769	.4492	.1177549	2.0261
.070	.9545085	.8808	.4826696	.19711	.8761229	.4810	.1176997	.0584
.075	0.9545479	1.9112	0.4834842	3.22624	0.8762796	2.5106	1.1176405	2.0878
.080	.9545900	.9390	.4843532	.25337	.8764469	.5381	.1175773	.1159
.085	.9546348	.9647	.4852763	.27870	.8766248	.5640	.1175099	.1411
.090	.9546822	1.9894	.4862530	.30248	.8768133	.5884	.1174385	.1673
.095	.9547324	2.0133	.4872830	.32486	.8770124	.6116	.1173629	.1915
.100	0.9547853	2.0354	0.4883658	3.34598	0.8772222	2.6337	1.1172831	2.2141
.105	.9548409	.0565	.4895011	.36597	.8774426	.6545	.1171992	.2353
.110	.9548992	.0766	.4906884	.38493	.8776735	.6741	.1171112	.2555
.115	.9549602	.0962	.4919273	.40293	.8779148	.6929	.1170191	.2749
.120	.9550240	.1149	.4932173	.42006	.8781665	.7109	.1169229	.2936
.125	0.9550905	2.1326	0.4945579	3.43633	0.8784287	2.7281	1.1168225	2.3118
.130	.9551597	.1495	.4959484	.45188	.8787012	.7444	.1167179	.3289
.135	.9552316	.1658	.4973885	.46675	.8789841	.7601	.1166092	.3455
.140	.9553062	.1816	.4988776	.48095	.8792772	.7753	.1164963	.3616
.145	.9553835	.1965	.5004152	.49457	.8795806	.7901	.1163792	.3771
.150	0.9554634	2.2111	0.5020006	3.50760	0.8798942	2.8044	1.1162579	2.3922
.155	.9555461	.2256	.5036333	.52011	.8802180	.8181	.1161325	.4068
.160	.9556315	.2393	.5053128	.53213	.8805519	.8312	.1160029	.4210
.165	.9557196	.2526	.5070384	.54366	.8808959	.8438	.1158691	.4347
.170	.9558104	.2655	.5088095	.55473	.8812499	.8561	.1157310	.4480
.175	0.9559039	2.2781	0.5106254	3.56539	0.8816139	2.8680	1.1155886	2.4609
.180	.9560001	.2904	.5124856	.57564	.8819878	.8795	.1154420	.4734
.185	.9560991	.3023	.5143893	.58550	.8823715	.8906	.1152912	.4856
.190	.9562008	.3138	.5163360	.59500	.8827650	.9013	.1151362	.4975
.195	.9563052	.3250	.5183249	.60418	.8831683	.9118	.1149769	.5092
.200	0.9564122	2.3359	0.5203555	3.61301	0.8835813	2.9220	1.1148132	2.5206
.205	.9565219	.3466	.5224270	.62150	.8840039	.9319	.1146453	.5317
.210	.9566343	.3571	.5245386	.62971	.8844360	.9415	.1144730	.5425
.215	.9567495	.3673	.5266899	.63765	.8848777	.9508	.1142965	.5531
.220	.9568674	.3773	.5288802	.64529	.8853288	.9599	.1141156	.5635
.225	0.9569880	2.3870	0.5311086	3.65267	0.8857894	2.9687	1.1139304	2.5737
.230	.9571112	.3965	.5333746	.65980	.8862593	.9773	.1137409	.5837
.235	.9572371	.4058	.5356774	.66668	.8867384	.9857	.1135470	.5935
.240	.9573657	.4149	.5380163	.67332	.8872268	.9939	.1133487	.6031
.245	.9574970	.4239	.5403906	.67974	.8877243	3.0018	.1131460	.6125
.250	0.9576311	2.4327	0.5427997	3.68593	0.8882308	3.0095	1.1129389	2.6218

LOGARITHMIC VALUES OF $f(\alpha) . \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$

α	$\log \frac{\alpha^5 D_\alpha b_{\frac{3}{2}}^{(0)}}{\beta^6}$	log var. for .001 of α.	$\log \frac{\alpha^6 D_\alpha b_{\frac{3}{2}}^{(1)}}{\beta^6}$	log var. for .001 of α.	$\log \frac{\alpha^5 D_\alpha b_{\frac{3}{2}}^{(2)}}{\beta^6}$	log var. for .001 of α.	$\log \frac{\alpha^4 D_\alpha b_{\frac{3}{2}}^{(3)}}{\beta^6}$	log var. for .001 of α.
.250	0.9576311	2.4327	0.5427997	3.68593	0.8882308	3.0095	1.1129389	2.6218
.255	.9577678	.4412	.5452427	.69191	.8887463	.0171	.1127274	.6309
.260	.9579072	.4495	.5477190	.69770	.8892706	.0245	.1125114	.6398
.265	.9580493	.4577	.5502280	.70327	.8898039	.0316	.1122910	.6486
.270	.9581941	.4657	.5527688	.70866	.8903460	.0386	.1120661	.6573
.275	0.9583416	2.4736	0.5553408	3.71389	0.8908968	3.0454	1.1118367	2.6659
.280	.9584917	.4814	.5579434	.71891	.8914561	.0521	.1116028	.6743
.285	.9586445	.4891	.5605757	.72375	.8920240	.0586	.1113644	.6825
.290	.9588001	.4967	.5632371	.72845	.8926004	.0649	.1111214	.6906
.295	.9589584	.5041	.5659269	.73298	.8931851	.0711	.1108739	.6986
.300	0.9591194	2.5114	0.5686443	3.73734	0.8937781	3.0771	1.1106219	2.7065
.305	.9592830	.5185	.5713888	.74158	.8943793	.0829	.1103652	.7143
.310	.9594493	.5255	.5741597	.74565	.8949887	.0886	.1101040	.7220
.315	.9596183	.5324	.5769562	.74958	.8956061	.0943	.1098381	.7296
.320	.9597900	.5392	.5797777	.75338	.8962315	.0999	.1095675	.7370
.325	0.9599644	2.5459	0.5826236	3.75705	0.8968648	3.1053	1.1092923	2.7444
.330	.9601415	.5524	.5854931	.76057	.8975058	.1106	.1090124	.7517
.335	.9603213	.5588	.5883856	.76397	.8981546	.1157	.1087278	.7589
.340	.9605038	.5651	.5913004	.76725	.8988110	.1207	.1084385	.7660
.345	.9606889	.5714	.5942369	.77042	.8994750	.1256	.1081444	.7730
.350	0.9608766	2.5776	0.5971945	3.77346	0.9001464	3.1304	1.1078455	2.7800
.355	.9610670	.5837	.6001725	.77640	.9008252	.1351	.1075418	.7869
.360	.9612601	.5898	.6031704	.77923	.9015113	.1397	.1072334	.7937
.365	.9614559	.5958	.6061874	.78193	.9022046	.1442	.1069201	.8004
.370	.9616544	.6017	.6092229	.78455	.9029048	.1466	.1066018	.8071
.375	0.9618556	2.6075	0.6122764	3.78706	0.9036121	3.1528	1.1062787	2.8137
.380	.9620594	.6132	.6153472	.78947	.9043263	.1569	.1059507	.8202
.385	.9622658	.6188	.6181348	.79178	.9050472	.1609	.1056177	.8267
.390	.9624749	.6243	.6215385	.79400	.9057748	.1649	.1052797	.8331
.395	.9626867	.6298	.6246578	.79613	.9065090	.1688	.1049368	.8394
.400	0.9629012	2.6352	0.6277921	3.79818	0.9072498	3.1726	1.1045889	2.8457
.405	.9631184	.6405	.6309409	.80013	.9079971	.1763	.1042359	.8519
.410	.9633382	.6457	.6341035	.80200	.9087506	.1799	.1038778	.8580
.415	.9635607	.6509	.6372795	.80379	.9095103	.1835	.1035146	.8641
.420	.9637859	.6561	.6404684	.80551	.9102760	.1869	.1031462	.8702
.425	0.9640138	2.6612	0.6436696	3.80714	0.9110477	3.1902	1.1027727	2.8763
.430	.9642443	.6662	.6468825	.80870	.9118255	.1935	.1023940	.8823
.435	.9644774	.6712	.6501066	.81019	.9126091	.1967	.1020101	.8883
.440	.9647132	.6761	.6533415	.81159	.9133983	.1998	.1016209	.8942
.445	.9649517	.6810	.6565866	.81292	.9141931	.2028	.1012264	.9001
.450	0.9651929	2.6858	0.6598415	3.81418	0.9149933	3.2057	1.1008265	2.9059
.455	.9654368	.6905	.6631056	.81538	.9157989	.2086	.1004213	.9116
.460	.9656833	.6952	.6663784	.81652	.9166098	.2114	.1000107	.9173
.465	.9659324	.6998	.6696596	.81759	.9174259	.2141	.0995947	.9230
.470	.9661841	.7044	.6729486	.81859	.9182470	.2167	.0991732	.9287
.475	0.9664385	2.7089	0.6762451	3.81954	0.9190731	3.2193	1.0987462	2.9343
.480	.9666956	.7134	.6795486	.82042	.9199040	.2218	.0983136	.9399
.485	.9669554	.7178	.6828586	.82125	.9207397	.2243	.0978754	.9454
.490	.9672178	.7222	.6861747	.82203	.9215801	.2267	.0974316	.9509
.495	.9674828	.7266	.6894965	.82275	.9224250	.2290	.0969821	.9564
.500	0.9677505	2.7309	0.6928236	3.82342	0.9232743	3.2311	1.0965269	2.9619

LOGARITHMIC VALUES OF $f(\alpha) \,.\, \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$

α	$\log \frac{\alpha^5 D_\alpha b^{(0)}_{\frac{3}{2}}}{\beta^6}$	log var. for .001 of α.	$\log \frac{\alpha^6 D_\alpha b^{(1)}_{\frac{3}{2}}}{\beta^6}$	log var. for .001 of α.	$\log \frac{\alpha^5 D_\alpha b^{(2)}_{\frac{3}{2}}}{\beta^6}$	log var. for .001 of α.	$\log \frac{\alpha^4 D_\alpha b^{(3)}_{\frac{3}{2}}}{\beta^6}$	log var. for .001 of α.
.500	0.9677505	2.7309	0.6928236	3.82342	0.9232743	3.2312	1.0965269	2.9619
.505	.9680209	.7351	.6961556	.82404	.9241280	.2333	.0960660	.9673
.510	.9682939	.7393	.6994921	.82461	.9249858	.2353	.0955993	.9727
.515	.9685696	.7435	.7028326	.82511	.9258477	.2373	.0951268	.9781
.520	.9688479	.7476	.7061769	.82556	.9267135	.2393	.0946484	.9835
.525	0.9691289	2.7517	0.7095245	3.82596	0.9275831	3.2413	1.0941641	2.9888
.530	.9694125	.7558	.7128750	.82632	.9284564	.2432	.0936739	.9941
.535	.9696987	.7599	.7162282	.82664	.9293333	.2449	.0931777	2.9994
.540	.9699876	.7639	.7195836	.82691	.9302138	.2466	.0926753	3.0047
.545	.9702792	.7679	.7229409	.82713	.9310977	.2482	.0921668	.0099
.550	0.9705734	2.7718	0.7262998	3.82731	0.9319849	3.2498	1.0916522	3.0151
.555	.9708703	.7756	.7296600	.82745	.9328752	.2514	.0911314	.0203
.560	.9711698	.7794	.7330211	.82755	.9337685	.2529	.0906042	.0255
.565	.9714719	.7832	.7363882	.82761	.9346648	.2542	.0900708	.0307
.570	.9717766	.7869	.7397448	.82763	.9355639	.2555	.0895311	.0359
.575	0.9720840	2.7906	0.7431067	3.82761	0.9364656	3.2567	1.0889849	3.0410
.580	.9723940	.7943	.7464685	.82755	.9373699	.2579	.0884322	.0461
.585	.9727067	.7980	.7498297	.82745	.9382767	.2591	.0878730	.0512
.590	.9730220	.8016	.7531900	.82732	.9391858	.2602	.0873073	.0563
.595	.9733400	.8052	.7565492	.82716	.9400972	.2612	.0867349	.0613
.600	0.9736606	2.8088	0.7599070	3.82696	0.9410107	3.2622	1.0861559	3.0663
.605	.9739839	.8124	.7632630	.82673	.9419261	.2631	.0855701	.0714
.610	.9743098	.8159	.7666170	.82646	.9428434	.2640	.0849773	.0764
.615	.9746384	.8194	.7699688	.82615	.9437624	.2648	.0843777	.0814
.620	.9749696	.8229	.7733183	.82581	.9446829	.2655	.0837711	.0864
.625	0.9753035	2.8263	0.7766650	3.82545	0.9456049	3.2661	1.0831575	3.0914
.630	.9756400	.8297	.7800087	.82506	.9465282	.2667	.0825368	.0964
.635	.9759791	.8331	.7833493	.82464	.9474528	.2673	.0819083	.1014
.640	.9763208	.8365	.7866865	.82419	.9483784	.2678	.0812737	.1064
.645	.9766652	.8398	.7900202	.82370	.9493050	.2682	.0806312	.1114
.650	0.9770122	2.8431	0.7933500	3.82319	0.9502324	3.2685	1.0799813	3.1164
.655	.9773619	.8463	.7966758	.82266	.9511605	.2688	.0793239	.1214
.660	.9777141	.8495	.7999974	.82210	.9520891	.2690	.0786589	.1264
.665	.9780691	.8527	.8033146	.82151	.9530182	.2692	.0779863	.1314
.670	.9784267	.8559	.8066271	.82088	.9539477	.2693	.0773060	.1363
.675	0.9787869	2.8591	0.8099348	3.82022	0.9548774	3.2694	1.0766179	3.1412
.680	.9791497	.8623	.8132374	.81954	.9558071	.2694	.0759219	.1461
.685	.9795152	.8655	.8165349	.81884	.9567368	.2693	.0752179	.1510
.690	.9798834	.8686	.8198270	.81812	.9576662	.2692	.0745059	.1560
.695	.9802542	.8717	.8231135	.81738	.9585953	.2690	.0737857	.1610
.700	0.9806277	2.8748	0.8263943	3.81662	0.9595239	3.2687	1.0730572	3.1660
.705	.9810038	.8779	.8296692	.81583	.9604518	.2684	.0723203	.1710
.710	.9813826	.8809	.8329380	.81501	.9613790	.2680	.0715750	.1759
.715	.9817640	.8839	.8362006	.81417	.9623053	.2675	.0708212	.1808
.720	.9821480	.8869	.8394568	.81331	.9632305	.2670	.0700587	.1857
.725	0.9825347	2.8899	0.8427065	3.81243	0.9641546	3.2664	1.0692874	3.1907
.730	.9829241	.8928	.8459495	.81153	.9650774	.2657	.0685073	.1957
.735	.9833162	.8957	.8491857	.81061	.9659987	.2650	.0677182	.2007
.740	.9837109	.8986	.8524149	.80967	.9669183	.2642	.0669200	.2057
.745	.9841082	.9015	.8556371	.80872	.9678361	.2633	.0661125	.2107
.750	0.9845081	2.9044	0.8588521	3.80775	0.9687520	3.2624	1.0652956	3.2157

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$ AND $f(\alpha) \cdot \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$

α	$\log \frac{\alpha^3 D_\alpha b_{\frac{3}{2}}^{(4)}}{\beta^6}$	$\log \frac{\alpha^2 D_\alpha b_{\frac{3}{2}}^{(5)}}{\beta^6}$	$\log \frac{\alpha^2 D_\alpha b_{\frac{3}{2}}^{(6)}}{\alpha \beta^6}$	$\log \frac{\alpha^6 D_\alpha^2 b_{\frac{3}{2}}^{(4)}}{\beta^8}$	$\log \frac{\alpha^5 D_\alpha^2 b_{\frac{3}{2}}^{(5)}}{\beta^8}$	$\log \frac{\alpha^4 D_\alpha^2 b_{\frac{3}{2}}^{(6)}}{\beta^8}$
.45	1.2462513	1.3654966	1.4664714	1.7808579	1.9831531	2.1558365
.46	.2440090	.3623232	.4626444	.7812184	.9808581	.1517403
.47	.2417091	.3590645	.4587109	.7815805	.9785141	.1475485
.48	.2393511	.3557193	.4546692	.7819439	.9761213	.1432607
.49	.2369342	.3522865	.4505171	.7823082	.9736798	.1388765
.50	1.2344578	1.3487648	1.4462528	1.7826729	1.9711897	2.1343956
.51	.2319212	.3451528	.4418744	.7830375	.9686512	.1298178
.52	.2293237	.3414494	.4373799	.7834017	.9660644	.1251427
.53	.2266646	.3376530	.4327671	.7837651	.9634295	.1203703
.54	.2239432	.3337620	.4280339	.7841270	.9607465	.1155004
.55	1.2211585	1.3297751	1.4231780	1.7844868	1.9580157	2.1105325
.56	.2183099	.3256907	.4181969	.7848442	.9552371	.1054665
.57	.2153965	.3215073	.4130882	.7851986	.9524109	.1003021
.58	.2124174	.3172230	.4078493	.7855494	.9495373	.0950392
.59	.2093718	.3128360	.4024774	.7858961	.9466164	.0896774
.60	1.2062586	1.3083447	1.3969696	1.7862382	1.9436484	2.0842166
.61	.2030768	.3037472	.3913231	.7865749	.9406334	.0786568
.62	.1998255	.2990413	.3855348	.7869057	.9375716	.0729978
.63	.1965037	.2942250	.3796015	.7872300	.9344632	.0672395
.64	.1931102	.2892962	.3735198	.7875471	.9313083	.0613816
.65	1.1896440	1.2842528	1.3672864	1.7878564	1.9281071	2.0554241
.66	.1861037	.2790926	.3608975	.7881572	.9248596	.0493668
.67	.1824883	.2738130	.3543493	.7884488	.9215661	.0432098
.68	.1787965	.2684115	.3476379	.7887305	.9182267	.0369529
.69	.1750270	.2628856	.3407592	.7890016	.9148414	.0305960
.70	1.1711784	1.2572325	1.3337088	1.7892609	1.9114105	2.0241391
.71	.1672490	.2514496	.3264822	.7895079	.9079340	.0175823
.72	.1632375	.2455339	.3190746	.7897417	.9044121	.0109254
.73	.1591424	.2394823	.3114813	.7899614	.9008448	2.0041683
.74	.1549621	.2332916	.3036970	.7901660	.8972323	1.9973111
.75	1.1506950	1.2269581	1.2957158	1.7903546	1.8935747	1.9903539

α	$\log \frac{\alpha^2 D_\alpha b_{\frac{3}{2}}^{(7)}}{\alpha^2 \beta^6}$	$\log \frac{\alpha^2 D_\alpha b_{\frac{3}{2}}^{(8)}}{\alpha^3 \beta^6}$	$\log \frac{\alpha^2 D_\alpha b_{\frac{3}{2}}^{(9)}}{\alpha^4 \beta^6}$	$\log \frac{\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(7)}}{\beta^8}$	$\log \frac{\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(8)}}{\alpha \beta^8}$	$\log \frac{\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(9)}}{\alpha^2 \beta^8}$
.50	1.5311870	1.6064110	1.6739004	2.2776887	2.4051069	2.5196620
.51	.5262367	.6010163	.6681523	.2716203	.3979105	.5115865
.52	.5211495	.5954695	.6622389	.2654115	.3905365	.5033021
.53	.5159237	.5897673	.6561560	.2590609	.3829828	.4948056
.54	.5105563	.5839062	.6498995	.2525676	.3752471	.4860938
.55	1.5050442	1.5778822	1.6434651	2.2459305	2.3673271	2.4771630
.56	.4993841	.5716914	.6368482	.2391483	.3592207	.4680098
.57	.4935725	.5653297	.6300439	.2322199	.3509254	.4586306
.58	.4876061	.5587928	.6230473	.2251442	.3424387	.4490216
.59	.4814814	.5520763	.6158529	.2179200	.3337583	.4391790
.60	1.4751944	1.5451754	1.6084552	2.2105462	2.3248817	2.4290989
.61	.4687410	.5380850	.6008483	.2030217	.3158066	.4187774
.62	.4621172	.5307999	.5930259	.1953454	.3065303	.4082104
.63	.4553184	.5233146	.5849814	.1875161	.2970502	.3973937
.64	.4483403	.5156234	.5767081	.1795328	.2873639	.3863228
.65	1.4411778	1.5077199	1.5681986	2.1713944	2.2774688	2.3749934
.66	.4338258	.4995979	.5594452	.1631000	.2673624	.3634012
.67	.4262790	.4912507	.5501398	.1546487	.2570422	.3515417
.68	.4185316	.4826711	.5411738	.1460396	.2465056	.3394104
.69	.4105779	.4738516	.5316381	.1372719	.2357501	.3270027
.70	1.4024117	1.4647843	1.5218231	2.1283146	2.2247733	2.3143143
.71	.3940264	.4551609	.5117185	.1192570	.2135725	.3013405
.72	.3854153	.4458727	.5013135	.1100083	.2021454	.2880766
.73	.3765711	.4360103	.4905965	.1005981	.1904898	.2745180
.74	.3674863	.4258634	.4795555	.0910258	.1786035	.2606594
.75	1.3581529	1.4154214	1.4681777	2.0812911	2.1664846	2.2464957

LOGARITHMIC VALUES OF $f(\alpha) . \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$

α	$\log \frac{\alpha^8 D_\alpha^2 b_{\frac{3}{2}}^{(0)}}{\beta^8}$	log var. for .001 of α.	$\log \frac{\alpha^7 D_\alpha^2 b_{\frac{3}{2}}^{(1)}}{\beta^8}$	log var. for .001 of α.	$\log \frac{\alpha^8 D_\alpha^2 b_{\frac{3}{2}}^{(2)}}{\beta^8}$	log var. for .001 of α.	$\log \frac{\alpha^7 D_\alpha^2 b_{\frac{3}{2}}^{(3)}}{\beta^8}$	log var. for .001 of α.
.000	0.9542425	$-\infty$	1.5282738	$-\infty$	0.8750613	$-\infty$	1.4191293	$-\infty$
.005	.9543008	2.36810	.5282830	1.57054	.8751318	2.45056	.4191467	1.84634
.010	.9544759	.66894	.5283110	1.87332	.8753435	.75143	.4191995	2.14922
.015	.9547674	.84460	.5283577	2.04999	.8756960	2.92711	.4192877	.32572
.020	.9551751	2.96914	.5284232	.17522	.8761890	3.05158	.4194112	.45040
.025	0.9556988	3.06562	1.5285074	2.27184	0.8768221	3.14790	1.4195698	2.54704
.030	.9563382	.14423	.5286102	.35083	.8775947	.22637	.4197636	.62613
.035	.9570927	.21046	.5287317	.41764	.8785062	.29248	.4199926	.69285
.040	.9579617	.26759	.5288718	.47538	.8795557	.34945	.4202566	.75051
.045	.9589445	.31779	.5290305	.52621	.8807421	.39943	.4205556	.80127
.050	0.9600404	3.36248	1.5292077	2.57159	0.8820643	3.44392	1.4208894	2.84652
.055	.9612485	.40271	.5294034	.61278	.8835213	.48393	.4212579	.88745
.060	.9625680	.43922	.5296177	.65040	.8851117	.52019	.4216611	.92474
.065	.9639978	.47275	.5298505	.68494	.8868341	.55332	.4220988	.85899
.070	.9655369	.50328	.5301018	.71692	.8886870	.58372	.4225710	2.99065
.075	0.9671840	3.53162	1.5303716	2.74663	0.8906687	3.61180	1.4230775	3.01999
.080	.9689380	.55794	.5306598	.77430	.8927777	.63783	.4236181	.04735
.085	.9707976	.58246	.5309663	.80030	.8950121	.66202	.4241927	.07298
.090	.9727615	.60538	.5312912	.82478	.8973699	.68459	.4248011	.09705
.095	.9748283	.62685	.5316343	.84782	.8998492	.70570	.4254431	.11979
.100	0.9769965	3.64703	1.5319956	2.86970	0.9024480	3.72549	1.4261187	3.14130
.105	.9792647	.66603	.5323751	.89048	.9051641	.74409	.4268276	.16164
.110	.9816313	.68394	.5327727	.91020	.9079954	.76160	.4275696	.18093
.115	.9840946	.70085	.5331883	.92901	.9109397	.77810	.4283444	.19932
.120	.9866530	.71686	.5336219	.94699	.9139947	.79367	.4291520	.21684
.125	0.9893048	3.73201	1.5340734	2.96426	0.9171580	3.80838	1.4299919	3.23350
.130	.9920482	.74638	.5345429	.98078	.9204272	.82229	.4308640	.24952
.135	.9948815	.76003	.5350301	2.99656	.9237999	.83548	.4317682	.26482
.140	0.9978029	.77298	.5355350	3.01178	.9272738	.84796	.4327040	.27946
.145	1.0008105	.78531	.5360576	.02645	.9308463	.85981	.4336713	.29358
.150	1.0039026	3.79705	1.5365978	3.04056	0.9345149	3.87103	1.4346700	3.30711
.155	.0070773	.80822	.5371555	.05381	.9382770	.88169	.4356995	.32007
.160	.0103327	.81885	.5377307	.06696	.9421302	.89182	.4367596	.33258
.165	.0136667	.82897	.5383232	.08005	.9460720	.90144	.4378502	.34463
.170	.0170775	.83863	.5389331	.09234	.9500998	.91057	.4389708	.35622
.175	1.0205632	3.84784	1.5395601	3.10418	0.9542110	3.91926	1.4401212	3.36745
.180	.0241219	.85666	.5402042	.11571	.9584032	.92751	.4413013	.37827
.185	.0277517	.86503	.5408654	.12688	.9626737	.93536	.4425105	.38867
.190	.0314506	.87303	.5415435	.13767	.9670201	.94280	.4437485	.39874
.195	.0352167	.88068	.5422384	.14814	.9714398	.94990	.4450151	.40850
.200	1.0390482	3.88797	1.5429500	3.15833	0.9759305	3.95665	1.4463100	3.41792
.205	.0429431	.89494	.5436783	.16823	.9804897	.96305	.4476328	.42703
.210	.0468995	.90159	.5444231	.17785	.9851148	.96913	.4489832	.43584
.215	.0509155	.90792	.5451844	.18724	.9898034	.97490	.4503608	.44439
.220	.0549890	.91397	.5459621	.19635	.9945532	.98039	.4517654	.45266
.225	1.0591184	3.91975	1.5467560	3.20517	0.9993619	3.98561	1.4531965	3.46068
.230	.0633017	.92525	.5475660	.21381	1.0042271	.99054	.4546539	.46846
.235	.0675372	.93051	.5483921	.22223	.0091465	.99523	.4561372	.47596
.240	.0718230	.93551	.5492341	.23042	.0141179	.99968	.4576459	.48323
.245	.0761574	.94030	.5500919	.23838	.0191391	4.00389	.4591797	.49032
.250	1.0805386	3.94486	1.5509654	3.24618	1.0242079	4.00788	1.4607385	3.49721

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$

α	$\log \frac{\alpha^8 D_\alpha^2 b_{\frac{3}{2}}^{(0)}}{\beta^8}$	log var. for .001 of α.	$\log \frac{\alpha^7 D_\alpha^2 b_{\frac{3}{2}}^{(1)}}{\beta^8}$	log var. for .001 of α.	$\log \frac{\alpha^8 D_\alpha^2 b_{\frac{3}{2}}^{(2)}}{\beta^8}$	log var. for .001 of α.	$\log \frac{\alpha^7 D_\alpha^2 b_{\frac{3}{2}}^{(3)}}{\beta^8}$	log var. for .001 of α.
.250	1.0805386	3.94486	1.5509654	3.24618	1.0242079	4.00788	1.4607385	3.49721
.255	.0849649	.94919	.5518546	.25370	.0293223	.01165	.4623217	.50386
.260	.0894345	.95332	.5527593	.26121	.0344800	.01522	.4639290	.51032
.265	.0939458	.95725	.5536794	.26844	.0396791	.01860	.4655600	.51657
.270	.0984971	.96099	.5546147	.27547	.0449176	.02179	.4672142	.52262
.275	1.1030868	3.96454	1.5555651	3.28235	1.0501935	4.02479	1.4688914	3.52854
.280	.1077132	.96792	.5565305	.28907	.0555049	.02762	.4705913	.53428
.285	.1123749	.97114	.5575108	.29566	.0608500	.03027	.4723134	.53983
.290	.1170701	.97418	.5585059	.30207	.0662269	.03278	.4740573	.54521
.295	.1217976	.97706	.5595156	.30833	.0716339	.03512	.4758226	.55043
.300	1.1265557	3.97980	1.5605398	3.31448	1.0770692	4.03731	1.4776090	3.55552
.305	.1313430	.98238	.5615785	.32050	.0825310	.03936	.4794161	.56044
.310	.1361582	.98483	.5626315	.32636	.0880179	.04127	.4812435	.56522
.315	.1409998	.98713	.5636986	.33207	.0935281	.04306	.4830908	.56988
.320	.1458664	.98932	.5647797	.33768	.0990602	.04472	.4849578	.57439
.325	1.1507568	3.99137	1.5658747	3.34315	1.1046126	4.04624	1.4868439	3.57875
.330	.1556696	.99329	.5669834	.34852	.1101838	.04766	.4887488	.58299
.335	.1606036	.99511	.5681058	.35378	.1157726	.04898	.4906721	.58712
.340	.1655576	.99681	.5692417	.35891	.1213775	.05017	.4926135	.59111
.345	.1705303	.99839	.5703909	.36393	.1269972	.05126	.4945725	.59498
.350	1.1755207	3.99989	1.5715534	3.36886	1.1326303	4.05225	1.4965488	3.59874
.355	.1805275	4.00169	.5727290	.37367	.1382757	.05315	.4985420	.60237
.360	.1855497	.00254	.5739175	.37836	.1439321	.05395	.5005516	.60588
.365	.1905862	.00373	.5751188	.38299	.1495983	.05465	.5025774	.60931
.370	.1956359	.00482	.5763329	.38752	.1552731	.05528	.5046190	.61264
.375	1.2006979	4.00584	1.5775595	3.39194	1.1609555	4.05581	1.5066760	3.61583
.380	.2057712	.00680	.5787986	.39627	.1666444	.05627	.5087479	.61894
.385	.2108548	.00760	.5800499	.40051	.1723389	.05669	.5108345	.62194
.390	.2159477	.00835	.5813134	.40466	.1780378	.05696	.5129353	.62487
.395	.2210491	.00904	.5825889	.40872	.1837403	.05720	.5150501	.62769
.400	1.2261580	4.00965	1.5838762	3.41268	1.1894455	4 05733	1.5171785	3.63042
.405	.2312737	.01018	.5851752	.41657	.1951524	.05742	.5193200	.63304
.410	.2363952	.01065	.5864858	.42042	.2008602	.05747	.5214743	.63559
.415	.2415219	.01105	.5878080	.42418	.2065681	.05748	.5236411	.63806
.420	.2466530	.01139	.5891415	.42783	.2122752	.05741	.5258200	.64043
.425	1.2517877	4.01167	1.5904861	3.43139	1.2179809	4.05727	1.5280106	3.64273
.430	.2569254	.01189	.5918417	.43491	.2236844	.05706	.5302127	.64494
.435	.2620653	.01204	.5932082	.43837	.2293850	.05680	.5324257	.64706
.440	.2672067	.01215	.5945856	.44176	.2350819	.05649	.5346494	.64911
.445	.2723491	.01221	.5959736	.44504	.2407746	.05614	.5368834	.65108
.450	1.2774917	4.01220	1.5973720	3.44826	1.2464624	4.05575	1.5391274	3.65298
.455	.2826340	.01215	.5987807	.45144	.2521446	.05532	.5413810	.65481
.460	.2877754	.01204	.6001997	.45454	.2578208	.05484	.5436440	.65658
.465	.2929152	.01189	.6016287	.45757	.2634903	.05432	.5459160	.65827
.470	.2980531	.01171	.6030676	.46054	.2691527	.05375	.5481967	.65989
.475	1.3031884	4.01146	1.6045163	3.46344	1.2748074	4.05313	1.5504857	3.66143
.480	.3083206	.01119	.6059745	.46667	.2804539	.05248	.5527827	.66291
.485	.3134493	.01087	.6074423	.46909	.2860917	.05179	.5550873	.66432
.490	.3185740	.01051	.6089195	.47182	.2917205	.05107	.5573993	.66567
.495	.3236943	.01012	.6104059	.47450	.2973397	.05032	.5597183	.66697
.500	1.3288097	4.00969	1.6119014	3.47711	1.3029490	4.04955	1.5620441	3.66820

LOGARITHMIC VALUES OF $f(\alpha) \, . \, \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$

α	$\log \dfrac{\alpha^8 D_\alpha^2 b_{\frac{3}{2}}^{(0)}}{\beta^8}$	log var. for .001 of α.	$\log \dfrac{\alpha^7 D_\alpha^2 b_{\frac{3}{2}}^{(1)}}{\beta^8}$	log var. for .001 of α.	$\log \dfrac{\alpha^8 D_\alpha^2 b_{\frac{3}{2}}^{(2)}}{\beta^8}$	log var. for .001 of α.	$\log \dfrac{\alpha^7 D_\alpha^2 b_{\frac{3}{2}}^{(3)}}{\beta^8}$	log var. for .001 of α.
.500	1.3288097	4.00969	1.6119014	3.47711	1.3029491	4.04955	1.5620441	3.66820
.505	.3339198	.00922	.6134058	.47966	.3085482	.04872	.5643763	.66936
.510	.3390242	.00871	.6149190	.48217	.3141365	.04787	.5667145	.67045
.515	.3441225	.00818	.6164409	.48464	.3197137	.04700	.5690585	.67150
.520	.3492144	.00762	.6179714	.48706	.3252795	.04610	.5714081	.67250
.525	1.3542995	4.00703	1.6195103	3.48940	1.3308336	4.04517	1.5737628	3.67342
.530	.3593774	.00640	.6210514	.49168	.3363756	.04422	.5761224	.67429
.535	.3644478	.00574	.6226126	.49394	.3419053	.04324	.5784866	.67509
.540	.3695104	.00506	.6241758	.49614	.3474222	.04224	.5808550	.67586
.545	.3745650	.00435	.6257469	.49830	.3529263	.04120	.5832275	.67656
.550	1.3796112	4.00362	1.6273257	3.50040	1.3584172	4.04014	1.5856036	3.67722
.555	.3846488	.00286	.6289121	.50246	.3638947	.03906	.5879832	.67784
.560	.3896774	.00208	.6305059	.50446	.3693585	.03797	.5903662	.67840
.565	.3946969	.00129	.6321070	.50643	.3748084	.03686	.5927519	.67890
.570	.3997071	4.00047	.6337153	.50836	.3802441	.03573	.5951402	.67935
.575	1.4047077	3.99963	1.6353307	3.51024	1.3856656	4.03457	1.5975309	3.67975
.580	.4096986	.99878	.6369530	.51207	.3910727	.03340	.5999238	.68010
.585	.4146796	.99790	.6385821	.51387	.3964650	.03221	.6023185	.68042
.590	.4196503	.99700	.6402179	.51562	.4018425	.03100	.6047148	.68069
.595	.4246107	.99609	.6418603	.51736	.4072050	.02978	.6071124	.68090
.600	1.4295606	3.99515	1.6435091	3.51901	1.4125522	4.02856	1.6095110	3.68107
.605	.4344997	.99420	.6451641	.52062	.4178842	.02732	.6119104	.68119
.610	.4394278	.99323	.6468252	.52222	.4232008	.02605	.6143104	.68126
.615	.4443442	.99224	.6484924	.52379	.4285018	.02475	.6167107	.68129
.620	.4492507	.99124	.6501655	.52530	.4337871	.02344	.6191111	.68129
.625	1.4541451	3.99023	1.6518444	3.52678	1.4390566	4.02213	1.6215112	3.68124
.630	.4590281	.98920	.6535289	.52821	.4443102	.02081	.6239110	.68114
.635	.4638995	.98816	.6552189	.52963	.4495478	.01949	.6263101	.68100
.640	.4687593	.98712	.6569144	.53100	.4547693	.01815	.6287083	.68082
.645	.4736073	.98606	.6586151	.53232	.4599746	.01679	.6311054	.68060
.650	1.4784436	3.98499	1.6603210	3.53362	1.4651635	4.01542	1.6335012	3.68034
.655	.4832678	.98391	.6620319	.53486	.4703360	.01404	.6358954	.68004
.660	.4880799	.98282	.6637477	.53610	.4754921	.01265	.6382879	.67970
.665	.4928799	.98172	.6654683	.53732	.4806316	.01125	.6406783	.67931
.670	.4976677	.98061	.6671937	.53850	.4857546	.00984	.6430666	.67889
.675	1.5024432	3.97949	1.6689237	3.53963	1.4908610	4.00843	1.6454524	3.67843
.680	.5072064	.97836	.6706581	.54070	.4959507	.00702	.6478356	.67793
.685	.5119572	.97723	.6723967	.54175	.5010237	.00558	.6502159	.67739
.690	.5166955	.97609	.6741395	.54280	.5060798	.00413	.6525932	.67681
.695	.5214213	.97494	.6758865	.54382	.5111191	.00267	.6549672	.67619
.700	1.5261346	3.97378	1.6776375	3.54479	1.5161415	4.00121	1.6573377	3.67553
.705	.5308354	.97262	.6793923	.54574	.5211470	3.99974	.6597045	.67484
.710	.5355236	.97145	.6811510	.54667	.5261356	.99827	.6620675	.67412
.715	.5401991	.97026	.6829133	.54754	.5311072	.99679	.6644264	.67335
.720	.5448618	.96907	.6846791	.54840	.5360619	.99529	.6667809	.67254
.725	1.5495118	3.96788	1.6864484	3.54924	1.5409994	3.99379	1.6691309	3.67168
.730	.5541490	.96669	.6882210	.55003	.5459200	.99229	.6714763	.67079
.735	.5587734	.96550	.6899968	.55080	.5508235	.99078	.6738168	.66986
.740	.5633852	.96430	.6917757	.55155	.5557099	.98926	.6761522	.66891
.745	.5679842	.96309	.6935576	.55226	.5605792	.98775	.6784824	.66792
.750	1.5725704	3.96189	1.6953424	3.55296	1.5654316	3.98623	1.6808072	3.66691

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha^2 b^{(i)}_{\frac{5}{2}}$

α	$\log \frac{\alpha^8 b^{(0)}_{\frac{5}{2}}}{\beta^8}$	log var. for .001 of α.	α	$\log \frac{\alpha^8 b^{(0)}_{\frac{5}{2}}}{\beta^8}$	log var. for .001 of α.	α	$\log \frac{\alpha^8 b^{(0)}_{\frac{5}{2}}}{\beta^8}$	log var. for .001 of α.
.000	0.3010300	— ∞	.250	0.3583824	3.63495	.500	0.4973035	3.80720
.005	.3010543	1.98900	.255	.3605563	.64151	.505	.5005162	.80858
.010	.3011275	2.29070	.260	.3627628	.64788	.510	.5037389	.80990
.015	.3012496	.46687	.265	.3650014	.65406	.515	.5069713	.81118
.020	.3014205	.59173	.270	.3672716	.66005	.520	.5102131	.81241
.025	0.3016402	2.68851	.275	0.3695728	3.66587	.525	0.5134639	3.81360
.030	.3019086	.76745	.280	.3719047	.67152	.530	.5167235	.81475
.035	.3022256	.83404	.285	.3742666	.67700	.535	.5199915	.81585
.040	.3025910	.89159	.290	.3766580	.68231	.540	.5232674	.81690
.045	.3030047	.94231	.295	.3790784	.68747	.545	.5265512	.81791
.050	0.3034666	2.98762	.300	0.3815273	3.69247	.550	0.5298425	3.81888
.055	.3039766	3.02853	.305	.3840042	.69728	.555	.5331410	.81981
.060	.3045345	.06577	.310	.3865087	.70208	.560	.5364464	.82070
.065	.3051401	.09996	.315	.3890401	.70666	.565	.5397584	.82155
.070	.3057933	.13156	.320	.3915980	.71111	.570	.5430768	.82236
.075	0.3064939	3.16089	.325	0.3941819	3.71528	.575	0.5464013	3.82314
.080	.3072417	.18823	.330	.3967913	.71964	.580	.5497316	.82387
.085	.3080364	.21378	.335	.3994256	.72371	.585	.5530674	.82457
.090	.3088777	.23779	.340	.4020844	.72766	.590	.5564085	.82525
.095	.3097654	.26045	.345	.4047671	.73151	.595	.5597546	.82589
.100	0.3106993	3.28185	.350	0.4074734	3.73524	.600	0.5631055	3.82649
.105	.3116790	.30211	.355	.4102026	.73886	.605	.5664609	.82706
.110	.3127043	.32135	.360	.4129544	.74238	.610	.5698206	.82760
.115	.3137748	.33963	.365	.4157282	.74580	.615	.5731844	.82811
.120	.3148902	.35706	.370	.4185236	.74911	.620	.5765520	.82859
.125	0.3160502	3.37372	.375	0.4213401	3.75233	.625	0.5799233	3.82904
.130	.3172546	.38963	.380	.4241772	.75544	.630	.5832979	.82946
.135	.3185028	.40481	.385	.4270344	.75847	.635	.5866757	.82986
.140	.3197945	.41939	.390	.4299113	.76141	.640	.5900565	.83022
.145	.3211294	.43339	.395	.4328075	.76425	.645	.5934401	.83056
.150	0.3225071	3.44683	.400	0.4357223	3.76699	.650	0.5968262	3.83088
.155	.3239272	.45972	.405	.4386554	.76968	.655	.6002146	.83116
.160	.3253893	.47213	.410	.4416065	.77229	.660	.6036052	.83142
.165	.3268929	.48407	.415	.4445750	.77481	.665	.6069977	.83167
.170	.3284377	.49560	.420	.4475605	.77725	.670	.6103920	.83189
.175	0.3300233	3.50670	.425	0.4505626	3.77962	.675	0.6137879	3.83209
.180	.3316491	.51740	.430	.4535808	.78191	.680	.6171853	.83226
.185	.3333148	.52773	.435	.4566147	.78413	.685	.6205840	.83241
.190	.3350199	.53770	.440	.4596639	.78627	.690	.6239838	.83253
.195	.3367639	.54733	.445	.4627280	.78837	.695	.6273844	.83264
.200	0.3385463	3.55661	.450	0.4658066	3.79038	.700	0.6307858	3.83273
.205	.3403667	.56565	.455	.4688993	.79232	.705	.6341877	.83280
.210	.3422247	.57438	.460	.4720056	.79420	.710	.6375901	.83284
.215	.3441197	.58282	.465	.4751252	.79603	.715	.6409928	.83287
.220	.3460514	.59101	.470	.4782578	.79782	.720	.6443956	.83288
.225	0.3480192	3.59893	.475	0.4814029	3.79951	.725	0.6477984	3.83287
.230	.3500227	.60660	.480	.4845602	.80115	.730	.6512011	.83284
.235	.3520612	.61402	.485	.4877292	.80275	.735	.6546035	.83279
.240	.3541344	.62122	.490	.4909097	.80428	.740	.6580055	.83273
.245	.3562416	.62818	.495	.4941012	.80576	.745	.6614069	.83264
.250	0.3583824	3.63495	.500	0.4973035	3.80720	.750	0.6648075	3.83254

LOGARITHMIC VALUES OF $f(\alpha) \cdot \alpha^2 b_{\frac{5}{2}}^{(i)}$

α	$\log \frac{\alpha^6 b_{\frac{5}{2}}^{(2)}}{\beta^8}$	log var. for .001 of α.	α	$\log \frac{\alpha^6 b_{\frac{5}{2}}^{(2)}}{\beta^8}$	log var. for .001 of α.	α	$\log \frac{\alpha^6 b_{\frac{5}{2}}^{(2)}}{\beta^8}$	log var. for .001 of α.
.000	0.9420081	$-\infty$	.250	0.9352093	2.7363	.500	0.9146724	3.0415
.005	.9410054	1.0374	.255	.9349341	.7450	.505	.9141195	.0459
.010	.9419972	.3385	.260	.9346534	.7535	.510	.9135608	.0503
.015	.9419836	.5132	.265	.9343672	.7618	.515	.9129966	.0546
.020	.9419646	.6375	.270	.9340754	.7700	.520	.9124269	.0589
.025	0.9419402	1.7340	.275	0.9337782	2.7780	.525	0.9118516	3.0631
.030	.9419104	.8136	.280	.9334755	.7859	.530	.9112706	.0673
.035	.9418751	.8808	.285	.9331673	.7936	.535	.9106840	.0715
.040	.9418344	.9385	.290	.9328536	.8012	.540	.9100917	.0756
.045	.9417883	1.9899	.295	.9325344	.8088	.545	.9094938	.0797
.050	0.9417367	2.0359	.300	0.9322097	2.8162	.550	0.9088902	3.0838
.055	.9416797	.0770	.305	.9318795	.8235	.555	.9082810	.0878
.060	.9416173	.1149	.310	.9315438	.8306	.560	.9076663	.0917
.065	.9415494	.1501	.315	.9312026	.8376	.565	.9070459	.0956
.070	.9414760	.1824	.320	.9308558	.8445	.570	.9064198	.0995
.075	0.9413972	2.2122	.325	0.9305035	2.8514	.575	0.9057881	3.1035
.080	.9413130	.2400	.330	.9301457	.8581	.580	.9051507	.1073
.085	.9412234	.2662	.335	.9297823	.8647	.585	.9045077	.1111
.090	.9411284	.2909	.340	.9294134	.8712	.590	.9038590	.1149
.095	.9410280	.3145	.345	.9290390	.8776	.595	.9032047	.1187
.100	0.9409221	2.3371	.350	0.9286590	2.8839	.600	0.9025447	3.1224
.105	.9408107	.3583	.355	.9282735	.8902	.605	.9018790	.1261
.110	.9406939	.3786	.360	.9278825	.8964	.610	.9012077	.1298
.115	.9405716	.3979	.365	.9274860	.9025	.615	.9005307	.1335
.120	.9404439	.4163	.370	.9270839	.9084	.620	.8998480	.1371
.125	0.9403108	2.4341	.375	0.9266762	2.9143	.625	0.8991596	3.1407
.130	.9401722	.4511	.380	.9262629	.9201	.630	.8984656	.1442
.135	.9400282	.4676	.385	.9258441	.9259	.635	.8977660	.1477
.140	.9398787	.4835	.390	.9254198	.9316	.640	.8970607	.1512
.145	.9397238	.4987	.395	.9249899	.9372	.645	.8963498	.1546
.150	0.9395634	2.5135	.400	0.9245545	2.9427	.650	0.8956333	3.1580
.155	.9393976	.5277	.405	.9241135	.9482	.655	.8949111	.1614
.160	.9392263	.5416	.410	.9236669	.9536	.660	.8941834	.1647
.165	.9390496	.5550	.415	.9232147	.9590	.665	.8934500	.1680
.170	.9388674	.5680	.420	.9227569	.9643	.670	.8927110	.1713
.175	0.9386798	2.5806	.425	0.9222936	2.9695	.675	0.8919664	3.1746
.180	.9384867	.5928	.430	.9218247	.9747	.680	.8912161	.1779
.185	.9382882	.6047	.435	.9213502	.9798	.685	.8904602	.1811
.190	.9380842	.6163	.440	.9208701	.9849	.690	.8896987	.1843
.195	.9378747	.6277	.445	.9203844	.9899	.695	.8889316	.1875
.200	0.9376598	2.6388	.450	0.9198931	2.9948	.700	0.8881588	3.1907
.205	.9374394	.6496	.455	.9193962	2.9997	.705	.8873804	.1938
.210	.9372135	.6601	.460	.9188938	3.0045	.710	.8865964	.1969
.215	.9369821	.6704	.465	.9183858	.0093	.715	.8858068	.2000
.220	.9367453	.6804	.470	.9178722	.0141	.720	.8850116	.2031
.225	0.9365030	2.6903	.475	0.9173529	3.0188	.725	0.8842108	3.2061
.230	.9362552	.6999	.480	.9168280	.0235	.730	.8834044	.2091
.235	.9360019	.7093	.485	.9162975	.0281	.735	.8825924	.2121
.240	.9357432	.7184	.490	.9157614	.0325	.740	.8817748	.2151
.245	.9354790	.7274	.495	.9152197	.0370	.745	.8809516	.2181
.250	0.9352093	2.7363	.500	0.9146724	3.0415	.750	0.8801228	3.2211

LOG COEFFICIENTS OF THE POWERS OF α.

Powers of α	$\log \frac{b^{(10)}_{\frac{1}{2}}}{\alpha^{10}}$	$\log \frac{\alpha D_\alpha b^{(10)}_{\frac{1}{2}}}{\alpha^8 \beta^2}$	$\log \frac{\alpha^2 D^2_\alpha b^{(10)}_{\frac{1}{2}}}{\alpha^6 \beta^4}$	$\log \frac{\alpha^3 D^3_\alpha b^{(10)}_{\frac{1}{2}}}{\alpha^4 \beta^6}$	$\log \frac{\alpha^4 D^4_\alpha b^{(10)}_{\frac{1}{2}}}{\alpha^2 \beta^8}$	$\log \frac{\alpha^5 D^5_\alpha b^{(10)}_{\frac{1}{2}}}{\beta^{10}}$
α^0	+9.5470286	+0.5470286	+1.5012711	+2.4043611	+3.2494592	+4.0276105
α^2	+9.2257952	−0.1777338	−1.6152145	−2.7317201	−3.7080970	−4.5716785
α^4	+9.0823713	−9.5130370	+0.9691790	+2.5552876	+3.7457183	+4.7457183
α^6	+8.9861585	−9.1540663	+0.1600994	−1.7346282	−3.3981864	−4.6144071
α^8	+8.9123724	−8.8959819	+9.7123558	−0.7929022	+2.4576404	+4.1467928
α^{10}	+8.8518916	−8.6905235	+9.3958804	−0.2327679	+1.4283115	−3.1112854
α^{12}	+8.8003147	−8.5179803	+9.1506929	−9.8154811	+0.8046083	−1.9987102
α^{14}	+8.7551650	−8.3682204	+8.9506679	−9.4769745	+0.3428636	−1.2949972
α^{16}	+8.7149019	−8.2352918	+8.7818214	−9.1889289	+9.9754219	−0.7510711
α^{18}	+8.6784965	−8.1153744	+8.6357467	−8.9360287	+9.6710927	−0.2957320
α^{20}	+8.6452246	−8.0058554	+8.5069939	−8.7089594	+9.4122184	−9.8950964
α^{22}	+8.6145558	−7.9048619	+8.3918341	−8.5015836	+9.1875793	−9.5286211
α^{24}	+8.5860882	−7.8110000	+8.2876105	−8.3096049	+8.9895282	−9.1816721
α^{26}	+8.5595095	−7.7232009	+8.1923669	−8.1298642	+8.8125906	−8.8981216

Powers of α	$\log \frac{b^{(11)}_{\frac{1}{2}}}{\alpha^{11}}$	$\log \frac{\alpha D_\alpha b^{(11)}_{\frac{1}{2}}}{\alpha^9 \beta^2}$	$\log \frac{\alpha^2 D^2_\alpha b^{(11)}_{\frac{1}{2}}}{\alpha^7 \beta^4}$	$\log \frac{\alpha^3 D^3_\alpha b^{(11)}_{\frac{1}{2}}}{\alpha^5 \beta^6}$	$\log \frac{\alpha^4 D^4_\alpha b^{(11)}_{\frac{1}{2}}}{\alpha^3 \beta^8}$	$\log \frac{\alpha^5 D^5_\alpha b^{(11)}_{\frac{1}{2}}}{\alpha \beta^{10}}$
α^0	+9.5268252	+0.5682179	+1.5682180	+2.5224605	+3.4255504	+4.2706484
α^2	+9.2073119	−0.2054196	−1.6889414	−2.8588090	−3.8971069	−4.8347436
α^4	+9.0653399	−9.5463043	+1.0462084	+2.6871765	+3.9409679	+5.0166585
α^6	+8.9703643	−9.1922130	+0.2382761	−1.8700532	−3.5974309	−4.8900626
α^8	+8.8976491	−8.9384579	+9.7904561	−0.9318112	+2.6593168	+4.4254283
α^{10}	+8.8381032	−8.7368913	+9.4732818	−0.3755407	+1.6307492	−3.3923716
α^{12}	+8.7873497	−8.5678872	+9.2271350	−9.9626214	+1.0060926	−2.2830750
α^{14}	+8.7429307	−8.4213789	+9.0260938	−9.6290049	+0.5418519	−1.5846506
α^{16}	+8.7033201	−8.2914656	+8.8562818	−9.3463657	+0.1707002	−1.0489863
α^{18}	+8.6675010	−8.1743672	+8.7093425	−9.0993890	+9.8618412	−0.6057986
α^{20}	+8.6347593	−8.0675042	+8.5798437	−8.8787761	+9.5979913	−0.2223090
α^{22}	+8.6045716	−7.9690301	+8.4640573	−8.6784273	+9.3682395	−9.8798852
α^{24}	+8.5765428	−7.8775729	+8.3593185	−8.4941026	+9.1651830	−9.5662041
α^{26}	+8.5503661	−7.7920822	+8.2636600	−8.3227280	+8.9834735	−9.2733277

Powers of α	$\log \frac{b^{(12)}_{\frac{1}{2}}}{\alpha^{12}}$	$\log \frac{\alpha D_\alpha b^{(12)}_{\frac{1}{2}}}{\alpha^{10} \beta^2}$	$\log \frac{\alpha^2 D^2_\alpha b^{(12)}_{\frac{1}{2}}}{\alpha^8 \beta^4}$	$\log \frac{\alpha^3 D^3_\alpha b^{(12)}_{\frac{1}{2}}}{\alpha^6 \beta^6}$	$\log \frac{\alpha^4 D^4_\alpha b^{(12)}_{\frac{1}{2}}}{\alpha^4 \beta^8}$	$\log \frac{\alpha^5 D^5_\alpha b^{(12)}_{\frac{1}{2}}}{\alpha^2 \beta^{10}}$
α^0	+9.5083418	+0.5875231	+1.6289158	+2.6289158	+3.5831583	+4.4862482
α^2	+9.1902787	−0.2300891	−1.7550865	−2.9722348	−4.0642824	−5.0642824
α^4	+9.0495455	−9.5756293	+1.1152368	+2.8046272	+4.1132964	+5.2527521
α^6	+8.9556411	−9.2256368	+0.3085255	−1.9904750	−3.7731624	−5.1302153
α^8	+8.8838608	−8.9755358	+9.8609192	−1.0550089	+2.8372855	+4.6682848
α^{10}	+8.8251385	−8.7772635	+9.5433948	−0.5016665	+1.8098247	−3.6373347
α^{12}	+8.7751153	−8.6112584	+9.2966176	−0.0919487	+1.1851409	−2.5302996
α^{14}	+8.7313488	−8.4675049	+9.0948396	−9.7618367	+0.7198127	−1.8349419
α^{16}	+8.6923246	−8.3401421	+8.9242830	−9.4830034	+0.3466786	−1.3036584
α^{18}	+8.6570356	−8.2254225	+8.7766454	−9.2401260	+0.0351507	−0.8665692
α^{20}	+8.6247750	−8.1207929	+8.6465207	−9.0239023	+9.7681640	−0.4912748
α^{22}	+8.5950262	−8.0244282	+8.5301901	−8.8282356	+9.5350080	−0.1595579
α^{24}	+8.5673995	−7.9349743	+8.4249892	−8.6488982	+9.3284478	−9.8597170
α^{26}	+8.5415923	−7.8519162	+8.3289457	−8.4828361	+9,1432694	−9.5843549

LOG COEFFICIENTS OF THE POWERS OF α.

Powers of α	$\log \frac{b^{(13)}_{\frac{1}{2}}}{\alpha^{13}}$	$\log \frac{\alpha D_\alpha b^{(13)}_{\frac{1}{2}}}{\alpha^{11}\beta^2}$	$\log \frac{\alpha^2 D^2_\alpha b^{(13)}_{\frac{1}{2}}}{\alpha^9\beta^4}$	$\log \frac{\alpha^3 D^3_\alpha b^{(13)}_{\frac{1}{2}}}{\alpha^7\beta^6}$	$\log \frac{\alpha^4 D^4_\alpha b^{(13)}_{\frac{1}{2}}}{\alpha^5\beta^8}$	$\log \frac{\alpha^5 D^5_\alpha b^{(13)}_{\frac{1}{2}}}{\alpha^3\beta^{10}}$
α^0	+9.4913085	+0.6052519	+1.6844331	+2.7258258	+3.7258258	+4.6800683
α^2	+9.1744843	−0.2523084	−1.8150705	−3.0747017	−4.2143089	−5.2683366
α^4	+9.0348223	−9.6018732	+1.1777047	+2.9104912	+4.2676086	+5.4622206
α^6	+8.9418527	−9.2553117	+0.3724148	−2.0987823	−3.9303704	−5.3431579
α^8	+8.8708958	−9.0083430	+9.9250372	−1.1660456	+2.9963254	+4.8835891
α^{10}	+8.8129038	−8.8129039	+9.6073958	−0.6147041	+1.9709083	−3.8541404
α^{12}	+8.7635334	−8.6494833	+9.3602251	−0.2074551	+1.3460206	−2.7504203
α^{14}	+8.7203533	−8.5081058	+9.1579208	−9.8799920	+0.8803458	−2.0560178
α^{16}	+8.6818593	−8.3829425	+8.9867952	−9.6039936	+0.5061988	−1.5274218
α^{18}	+8.6470514	−8.2702724	+8.8385981	−9.3641317	+0.1931121	−1.0938681
α^{20}	+8.6152297	−8.1675634	+8.7079525	−9.1511073	+9.9241505	−0.7231561
α^{22}	+8.5858829	−8.0730084	+8.5911541	−8.9587904	+9.6887312	−0.3972342
α^{24}	+8.5586256	−7.9852682	+8.4855439	−8.7829812	+9.4797595	−0.1044932
α^{26}	+8.5331590	−7.9033201	+8.3891496	−8.6206213	+9.2920344	−9.8379853

Powers of α	$\log \frac{\alpha^4 D^3_\alpha b^{(9)}_{\frac{3}{2}}}{\beta^{10}}$	$\log \frac{\alpha^7 D^4_\alpha b^{(9)}_{\frac{3}{2}}}{\beta^{12}}$	$\log \frac{\alpha^{10} D^5_\alpha b^{(9)}_{\frac{3}{2}}}{\beta^{14}}$	$\log \frac{\alpha\, b^{(10)}_{\frac{3}{2}}}{\alpha^7\beta^4}$	$\log \frac{\alpha^2 D_\alpha b^{(10)}_{\frac{3}{2}}}{\alpha^5\beta^6}$	$\log \frac{\alpha^3 D^2_\alpha b^{(10)}_{\frac{3}{2}}}{\alpha^3\beta^8}$
α^0	+3.5504892	+4.3286404	+5.0276105	+0.8692480	+1.8692480	+2.8235105
α^2	−3.8306691	−4.6016417	−5.1157465	−0.5045488	−1.9177604	−3.0539393
α^4	+3.7377454	+4.6141274	+5.2313061	−9.8444969	+1.1980561	+2.7915914
α^6	−3.2929649	−4.3721462	−5.1327767	−9.4902214	+0.3103372	−1.9019875
α^8	+2.2695187	+3.8149424	+4.7877245	−9.2368834	+9.7814475	−0.9091387
α^{10}	+1.1580679	−2.7107658	−4.1534165	−9.0362239	+9.3830115	−0.3140332
α^{12}	+0.4446582	−1.5590758	+2.9663195	−8.8685333	+9.0561187	−9.8753576
α^{14}	+9.8800815	−0.8510769	+1.6656694	−8.7236808	+8.7752576	−9.5264328
α^{16}	+9.3916055	−0.3358277	+0.6207094	−8.5957154	+8.5267393	−9.2366687
α^{18}	+8.9407888	−9.9357315	−9.5245714	−8.4808186	+8.3022385	−8.9889307
α^{20}	+8.4955194	−9.6104595	−9.8502736	−8.3763796	+8.0962429	−8.7725202
α^{22}	+8.0062649	−9.3358673	−9.7399401	−8.2805253	+7.9048621	−8.5802930
α^{24}	+7.2834143	−9.0969135	−9.5734112	−8.1918647	+7.7252090	−8.4072467

Powers of α	$\log \frac{\alpha^4 D^3_\alpha b^{(10)}_{\frac{3}{2}}}{\alpha\beta^{10}}$	$\log \frac{\alpha^6 D^4_\alpha b^{(10)}_{\frac{3}{2}}}{\beta^{12}}$	$\log \frac{\alpha^9 D^5_\alpha b^{(10)}_{\frac{3}{2}}}{\beta^{14}}$	$\log \frac{\alpha\, b^{(11)}_{\frac{3}{2}}}{\alpha^8\beta^4}$	$\log \frac{\alpha^2 D_\alpha b^{(11)}_{\frac{3}{2}}}{\alpha^6\beta^6}$	$\log \frac{\alpha^3 D^2_\alpha b^{(11)}_{\frac{3}{2}}}{\alpha^4\beta^8}$
α^0	+3.7265804	+4.5716785	+5.3498298	+0.8885531	+1.9299458	+2.2929458
α^2	−4.0539393	−4.9340801	−5.6660997	−0.5295311	−1.9919291	−3.1813628
α^4	+3.9863590	+4.9789846	+5.7846840	−9.8742256	+1.2840853	+2.9336080
α^6	−3.5554083	−4.7512369	−5.6867987	−9.5239776	+0.4070709	−2.0531846
α^8	+2.5428102	+4.2044619	+5.3505899	−9.2741001	+9.8880594	−1.0652448
α^{10}	+1.4440987	−3.1045829	−4.7191795	−9.0764462	+9.4989245	−0.4719546
α^{12}	+0.7480065	−1.9458065	+3.5441036	−8.9113903	+9.1809532	−0.0330568
α^{14}	+0.2075078	−1.2178904	+2.2995123	−8.7688663	+8.9087975	−9.6827036
α^{16}	+9.7529392	−0.6736069	+1.4527647	−8.6429742	+8.6689098	−9.3909122
α^{18}	+9.3515837	−0.2413970	+0.7262120	−8.5299345	+8.4530947	−9.1409322
α^{20}	+8.9837165	−9.8857399	+9.9690896	−8.4271690	+8.2559630	−8.9222998
α^{22}	+8.6353832	−9.5855171	−8.1495270	−8.3328308	+8.0737506	−8.7280452
α^{24}	+8.2942457	−9.3252488	−9.2796669	−8.2455500	+7.9037041	−8.5530938

LOG COEFFICIENTS OF THE POWERS OF α.

Powers of α	$\log \frac{\alpha^4 D_\alpha^3 b^{(11)}_{\frac{3}{2}}}{\alpha^2 \beta^{10}}$	$\log \frac{\alpha^5 D_\alpha^4 b^{(11)}_{\frac{3}{2}}}{\beta^{12}}$	$\log \frac{\alpha^8 D_\alpha^5 b^{(11)}_{\frac{3}{2}}}{\beta^{14}}$	$\log \frac{\alpha b^{(12)}_{\frac{3}{2}}}{\alpha^9 \beta^4}$	$\log \frac{\alpha^2 D_\alpha b^{(12)}_{\frac{3}{2}}}{\alpha^7 \beta^6}$	$\log \frac{\alpha^3 D_\alpha^2 b^{(12)}_{\frac{3}{2}}}{\alpha^5 \beta^8}$
α^0	+3.8841883	+4.7872782	+5.6323763	+0.9062819	+1.9854631	+3.0268558
α^2	−4.2443399	−5.2046788	−6.0551829	−0.5520063	−2.0583348	−3.2945607
α^4	+4.1962330	+5.2754141	+6.2023168	−9.9007284	+1.3601209	+3.0585618
α^6	−3.7771807	−5.0615905	−6.1171159	−9.5539408	+0.5055677	−2.1860606
α^8	+2.7735204	+4.5243260	+5.7902188	−9.3070689	+9.9328078	−1.2029610
α^{10}	+1.6836121	−3.4302486	−5.1646981	−9.1120497	+9.5990642	−0.6121807
α^{12}	+0.9978052	−2.2721385	+3.9960709	−8.9493226	+9.2881409	−0.1741381
α^{14}	+0.4699147	−1.5837168	+2.7663615	−8.8088714	+9.0227514	−9.8235499
α^{16}	+0.0312468	−0.9830938	+1.9535877	−8.6848364	+8.7894428	−9.5308603
α^{18}	+9.6501229	−0.5353043	+1.3033967	−8.5734708	+8.5801015	−9.2796327
α^{20}	+9.3081801	−0.1620742	+0.7274119	−8.4722208	+8.3894100	−9.0596089
α^{22}	+8.9949767	−9.8439239	+0.1919566	−8.3792624	+8.2136752	−8.8638996
α^{24}	+8.6985355	−9.5675322	+9.6145070	−8.2932417	+8.0502026	−8.6877340

Powers of α	$\log \frac{\alpha^4 D_\alpha^3 b^{(12)}_{\frac{3}{2}}}{\alpha^3 \beta^{10}}$	$\log \frac{\alpha^5 D_\alpha^4 b^{(12)}_{\frac{3}{2}}}{\alpha \beta^{12}}$	$\log \frac{\alpha^7 D_\alpha^5 b^{(12)}_{\frac{3}{2}}}{\beta^{14}}$	$\log \frac{\alpha^2 b^{(10)}_{\frac{5}{2}}}{\alpha^4 \beta^8}$	$\log \frac{\alpha^3 D_\alpha b^{(10)}_{\frac{5}{2}}}{\alpha^2 \beta^{10}}$	$\log \frac{\alpha^2 b^{(11)}_{\frac{5}{2}}}{\alpha^5 \beta^8}$
α^0	+4.0268558	+4.9810983	+5.8841883	+1.7538545	+2.7538545	+1.8093718
α^2	−4.4111603	−5.4359432	−6.3702645	−1.8179720	−2.9554998	−1.8840054
α^4	+4.3784473	+5.5269955	+6.5431207	+1.1144544	+2.6486141	+1.1891914
α^6	−3.9695033	−5.3253849	−6.4715129	+0.2435490	−1.6994811	+0.3256103
α^8	+2.9734198	+4.7965683	+6.1537157	+9.7321501	−0.6336635	+9.8204600
α^{10}	+1.8903509	−3.7078557	−5.5344717	+9.3519389	−9.9540007	+9.4456437
α^{12}	+1.2116515	−2.5650467	+4.3711407	+9.0440696	−9.4208136	+9.1424797
α^{14}	+0.6918574	−1.7477473	+2.1485242	+8.7831029	−8.9685343	+8.8856534
α^{16}	+0.2625935	−1.3651767	+1.3483653	+8.5554353	−8.5667556	+8.6616562
α^{18}	+9.8923506	−0.7309881	+0.7181354	+8.3528354	−8.1974184	+8.4623356
α^{20}	+9.5639199	−0.4226375	+0.1834208	+8.1699061	−7.8478193	+8.2823486
α^{22}	+9.2657845	−0.0940097	+9.7072379	+8.0028837	−7.5052857	+8.1179885
α^{24}	+8.9913590	−9.8066683	+9.2650538			

Powers of α	$\log \frac{\alpha^3 D_\alpha b^{(11)}_{\frac{5}{2}}}{\alpha^3 \beta^{10}}$	$\log \frac{\alpha^2 b^{(12)}_{\frac{5}{2}}}{\alpha^6 \beta^8}$	$\log \frac{\alpha^3 D_\alpha b^{(12)}_{\frac{5}{2}}}{\alpha^4 \beta^{10}}$
α^0	+2.8507745	+1.8605244	+2.9397056
α^2	−3.0750739	−1.9438616	−3.1819476
α^4	+2.7886585	+1.2563714	+2.9116769
α^6	−1.8587518	+0.3990389	−1.9967340
α^8	−0.8116862	+9.8992840	−0.9639272
α^{10}	−0.1509997	+9.5291729	−0.3171937
α^{12}	−9.6377598	+9.2301490	−9.8179724
α^{14}	−9.2071979	+8.9769939	−9.4018570
α^{16}	−8.8301137	+8.7562746	−9.0400482
α^{18}	−8.4900709	+8.5598978	−8.7166293
α^{20}	−8.1764964	+8.3825710	−8.4215217
α^{22}	−7.8820120	+8.2206235	−8.1483558

SUPPLEMENT.

It now remains to take from these Tables the especial values of $b_s^{(i)}$ and its derivatives, which are needed in the perturbative theories of the planetary motions. The present Supplement will only include those coefficients which pertain to the principal Planets; those for the Asteroids being reserved for a future occasion.

In determining the values of α, the following values of the masses and mean motions, which are those adopted in the American Ephemeris and Nautical Almanac, will be used.

Mercury,	m^{I}	$= \frac{1}{4865751}$	n^{I}	$= 5381016''.218$
Venus,	m^{II}	$= \frac{1}{390000}$	n^{II}	$= 2106641.438$
The Earth,	m^{III}	$= \frac{1}{354936}$	n^{III}	$= 1295977.440$
Mars,	m^{IV}	$= \frac{1}{2680637}$	n^{IV}	$= 689051.030$
Jupiter,	m^{V}	$= \frac{1}{1047.879}$	n^{V}	$= 109256.719$
Saturn,	m^{VI}	$= \frac{1}{3501.6}$	n^{VI}	$= 43996.127$
Uranus,	m^{VII}	$= \frac{1}{24905}$	n^{VII}	$= 15424.5094$
Neptune,	m^{VIII}	$= \frac{1}{18780}$	n^{VIII}	$= 7872.77382$

We have, then, for the value of α, the ratio of the mean distances of any two planets, as Mercury and Venus, the formula

$$\alpha = \left(\frac{n^{\mathrm{II}}}{n^{\mathrm{I}}}\right)^{\frac{2}{3}} \left(1 + \frac{m^{\mathrm{I}} - m^{\mathrm{II}}}{3}\right);$$

which takes into account the correction, necessary in the perturbative theory, due to the masses.

But if each planet is considered with reference to the Earth, the values of the mean distances a^{I}, a^{II}, a^{III}, &c., may first be found, since $a^{\mathrm{III}} = 1$, and then the values of α.

We find for

Mercury,	a^{I}	$= 0.38709870$	$\log a^{\mathrm{I}}$	$= 9.58782172$
Venus,	a^{II}	$= 0.72333227$	$\log a^{\mathrm{II}}$	$= 9.85933784$
The Earth,	a^{III}	$= 1.00000000$	$\log a^{\mathrm{III}}$	$= 0.00000000$
Mars,	a^{IV}	$= 1.52369140$	$\log a^{\mathrm{IV}}$	$= 0.18289702$
Jupiter,	a^{V}	$= 5.20280136$	$\log a^{\mathrm{V}}$	$= 0.71623725$
Saturn,	a^{VI}	$= 9.53885533$	$\log a^{\mathrm{VI}}$	$= 0.97949626$
Uranus,	a^{VII}	$= 19.18357126$	$\log a^{\mathrm{VII}}$	$= 1.28292946$
Neptune,	a^{VIII}	$= 30.03680569$	$\log a^{\mathrm{VIII}}$	$= 1.47765375$

Hence, for

Mercury and Venus,	$\alpha = 0.5351603$	$\log \alpha = 9.7284839$	$\log \beta^2 = 9.6035107$
Mercury and The Earth,	$\alpha = 0.3870987$	$\log \alpha = 9.5878217$	$\log \beta^2 = 9.2461455$
Mercury and Mars,	$\alpha = 0.2540532$	$\log \alpha = 9.4049247$	$\log \beta^2 = 8.8388256$
Mercury and Jupiter,	$\alpha = 0.0744020$	$\log \alpha = 8.8715845$	$\log \beta^2 = 7.7455797$
Mercury and Saturn,	$\alpha = 0.0405813$	$\log \alpha = 8.6083254$	$\log \beta^2 = 7.2173667$
Mercury and Uranus,	$\alpha = 0.0201787$	$\log \alpha = 8.3048923$	$\log \beta^2 = 6.6099614$
Mercury and Neptune,	$\alpha = 0.0128875$	$\log \alpha = 8.1101680$	$\log \beta^2 = 6.2204080$
Venus and The Earth,	$\alpha = 0.7233323$	$\log \alpha = 9.8593378$	$\log \beta^2 = 0.0403481$
Venus and Mars,	$\alpha = 0.4747236$	$\log \alpha = 9.6764408$	$\log \beta^2 = 9.4637832$
Venus and Jupiter,	$\alpha = 0.1390275$	$\log \alpha = 9.1431006$	$\log \beta^2 = 8.2946777$
Venus and Saturn,	$\alpha = 0.0758301$	$\log \alpha = 8.8798416$	$\log \beta^2 = 7.7621877$
Venus and Uranus,	$\alpha = 0.0377058$	$\log \alpha = 8.5764084$	$\log \beta^2 = 7.1534347$
Venus and Neptune,	$\alpha = 0.0240815$	$\log \alpha = 8.3816841$	$\log \beta^2 = 6.7636201$
The Earth and Mars,	$\alpha = 0.6563009$	$\log \alpha = 9.8171030$	$\log \beta^2 = 9.8788884$
The Earth and Jupiter,	$\alpha = 0.1922042$	$\log \alpha = 9.2837628$	$\log \beta^2 = 8.5838733$
The Earth and Saturn,	$\alpha = 0.1048344$	$\log \alpha = 9.0205037$	$\log \beta^2 = 8.0458069$
The Earth and Uranus,	$\alpha = 0.0521279$	$\log \alpha = 8.7170705$	$\log \beta^2 = 7.4353228$
The Earth and Neptune,	$\alpha = 0.0332925$	$\log \alpha = 8.5223463$	$\log \beta^2 = 7.0451741$
Mars and Jupiter,	$\alpha = 0.2928598$	$\log \alpha = 9.4666598$	$\log \beta^2 = 8.9722626$
Mars and Saturn,	$\alpha = 0.1597353$	$\log \alpha = 9.2034008$	$\log \beta^2 = 8.4180265$
Mars and Uranus,	$\alpha = 0.0794269$	$\log \alpha = 8.8999676$	$\log \beta^2 = 7.8026836$
Mars and Neptune,	$\alpha = 0.0507275$	$\log \alpha = 8.7052433$	$\log \beta^2 = 7.4116055$
Jupiter and Saturn,	$\alpha = 0.5454325$	$\log \alpha = 9.7367410$	$\log \beta^2 = 9.6268336$
Jupiter and Uranus,	$\alpha = 0.2712113$	$\log \alpha = 9.4333078$	$\log \beta^2 = 8.8997962$
Jupiter and Neptune,	$\alpha = 0.1732142$	$\log \alpha = 9.2385835$	$\log \beta^2 = 8.4903967$
Saturn and Uranus,	$\alpha = 0.4972408$	$\log \alpha = 9.6965668$	$\log \beta^2 = 9.5164820$
Saturn and Neptune,	$\alpha = 0.3175722$	$\log \alpha = 9.5018425$	$\log \beta^2 = 9.0498539$
Uranus and Neptune,	$\alpha = 0.6386689$	$\log \alpha = 9.8052757$	$\log \beta^2 = 9.8381548$

With these values of α and β^2 we enter the Tables and take out the required coefficients, which are found on the following pages.

MERCURY AND VENUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.3368936	99.6206871	99.8972897	0.2388589	0.8016751
1	99.7822639	99.8922032	99.8418921	0.2669438	0.7985490
2	99.3919845	99.7578828	99.9878240	0.2639757	0.8170165
3	99.0444586	99.5682264	99.9860814	0.3492156	0.8295626
4	98.7168905	99.3555162	99.9084175	0.3914858	0.8842750
5	98.4009294	99.1300174	99.7873191	0.3775499	0.9354334
6	98.092563	98.896347	99.638442	0.3195425	0.9550919
7	97.789597	98.656996	99.470354	0.228810	
8	97.49060	98.413474	99.288216		
9	97.19490	98.166718			

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.3531933	0.5947884	1.1362336	0.5632360
1	0.2107051	0.6129323	1.1227699	
2	0.0186280	0.5544420	1.1137060	0.4107724
3	99.8048651	0.4467261	1.0786698	
4	99.5788115	0.3073166	1.0125432	
5	99.3448135	0.1467052	0.9191260	
6	99.1052495	99.9693565		
7	98.8615637			

MERCURY AND THE EARTH.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.3184766	99.2546919	99.3989310	99.4220427	99.7628838
1	99.6139899	99.6668702	99.2622822	99.5008233	99.7371661
2	99.0798287	99.4111232	99.5250938	99.4562433	99.7872022
3	99.5899659	99.0885481	99.4433846	99.6321510	99.7828049
4	98.1207040	98.7394382	99.2505507	99.6353560	
5	97.6633767	98.3760072	99.0025367		
6	97.213840	98.0035704	98.7215190		
7	96.76980	97.6249551			
8	96.32984				

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.0459805	99.9373245	0.2584646	99.8858911
1	99.7853941	0.0068851	0.2138736	
2	99.4615006	99.8803230	0.2202039	99.5589618
3	99.1118471	99.6682696	0.1450095	
4	98.7481282			
5	98.3755292			

MERCURY AND MARS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.3082438	98.8424025	98.9049136	98.5490181	98.7173894
1	99.4157453	99.4374778	98.6272822	98.7368905	98.6196100
2	98.6969416	99.0101961	99.0578698	98.5895868	98.7510792
3	98.0232940	98.5090174	98.8315886	98.9130185	98.6765937
4	97.3705928	97.9793098	98.4698494	98.8042736	
5	96.7300025	97.4344104	98.0460740		

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.7708872	99.2594445	99.4148023	99.2858964
1	99.3413761	99.4415556	99.2872887	
2	98.8396194	99.1903507	99.3590295	98.6705902
3	98.3096765	98.8193952	99.1976237	

MERCURY AND JUPITER.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.3016325	97.7458802	97.7512851	96.3240081	96.3404635
1	98.8724882	98.8742957	96.9719581	96.9819147	95.8193051
2	97.6192343	97.9212700	97.9252836	96.3693077	96.3848826
3	96.4116879	96.8895133	97.1923042	97.1993128	95.8853453
4	95.225311	95.827914			

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.1780369	97.5765287	97.5913778	98.0592196
1	98.2248103	98.2337851	97.0263193	
2	97.1930041	97.4982470	97.5148215	96.4373878
3	96.1313849	96.6112183	96.9189960	

MERCURY AND SATURN.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.3012093	97.2174553	97.2190649	95.2650906	95.2700362
1	98.6085938	98.6091299	96.1786498	96.1816249	94.4943723
2	97.0920105	97.3933388	97.3945312	95.3107115	95.3153870
3	95.6211696	96.0984995	96.4000520	96.4021379	94.5610483
4	94.1715118	94.7737328			

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	98.9109658	96.7814557	96.7859097	97.5221507
1	97.6951141	97.6977926	95.9650543	
2	96.4002605	96.7025422	96.7075262	95.3779943
3	95.0754879	95.5534144	95.8564546	

MERCURY AND URANUS.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.3010745	96.6099831	96.6103814	94.0494867	94.0507129
1	98.3049585	98.3050908	95.2672278	95.2679644	92.9744565
2	96.4849193	96.7860233	96.7863180	94.0952106	94.0963708
3	94.7106341	95.1878071	95.4889658	95.4894822	93.0413363

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	98.6063203	95.8694720	95.8705767	96.9119197
1	97.0872375	97.0879007	94.7487026	
2	95.4890174	95.7903568	95.7915942	94.1622404

MERCURY AND NEPTUNE.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log a\, D_a\, b_{\frac{1}{2}}^{(i)}$	$\log a^2\, D_a^2\, b_{\frac{1}{2}}^{(i)}$	$\log a^3\, D_a^3\, b_{\frac{1}{2}}^{(i)}$	$\log a^4\, D_a^4\, b_{\frac{1}{2}}^{(i)}$
0	0.3010482	96.2204167	96.2205795	93.2702259	93.2707270
1	98.1101949	98.1102487	94.6828366	94.6831374	92.0003003
2	96.0954270	96.3964873	96.3966076	93.3159697	93.3164439

i	$\log a\, b_{\frac{3}{2}}^{(i)}$	$\log a^2\, D_a\, b_{\frac{3}{2}}^{(i)}$	$\log a^3\, D_a^2\, b_{\frac{3}{2}}^{(i)}$	$\log a^2\, b_{\frac{5}{2}}^{(i)}$
0	98.4113602	95.2849716	95.2854233	96.5218167
1	96.6975924	96.6978632	93.9692959	
2	94.9046608	95.2058173	95.2063234	93.3829501

VENUS AND THE EARTH.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log a\, D_a\, b_{\frac{1}{2}}^{(i)}$	$\log a^2\, D_a^2\, b_{\frac{1}{2}}^{(i)}$	$\log a^3\, D_a^3\, b_{\frac{1}{2}}^{(i)}$	$\log a^4\, D_a^4\, b_{\frac{1}{2}}^{(i)}$	$\log a^5\, D_a^5\, b_{\frac{1}{2}}^{(i)}$
0	0.3777387	0.0751753	0.6062955	1.3304800	2.2339265	3.2571104
1	99.9742423	0.2158375	0.5955403	1.3366398	2.2349877	3.2581326
2	99.7222879	0.1752740	0.6510807	1.3429395	2.2404308	3.2609114
3	99.5096630	0.0996032	0.6782990	1.3697966	2.2484968	3.2659903
4	99.3155264	0.0078221	0.6768729	1.4000746	2.2645398	3.2732901
5	99.1322252	99.9065691	0.6533964	1.4221359	2.2869730	3.2844533
6	98.956047	99.7990095	0.6132848	1.4315468	2.3107746	3.3002061
7	98.784955	99.686930	0.5604115	1.4277420	2.3311298	3.3194820
8	98.617623	99.571455	0.4975245	1.4116546	2.3448582	3.3399585
9	98.453251	99.453263	0.4265994	1.3846568	2.3503817	3.3590269
10	98.291240	99.332957	0.3491038	1.3481201	2.3472192	3.3744826
11	98.131160	99.210872	0.2661455	1.3033678	2.3355127	3.3847833
12	97.972728	99.087305	0.1785773	1.2514069	2.3157346	3.3890323
13	97.809310	98.962477	0.0870730	1.1931751	2.2885064	3.3868287

i	$\log a\, b_{\frac{3}{2}}^{(i)}$	$\log a^2\, D_a\, b_{\frac{3}{2}}^{(i)}$	$\log a^3\, D_a^2\, b_{\frac{3}{2}}^{(i)}$	$\log a^4\, D_a^3\, b_{\frac{3}{2}}^{(i)}$	$\log a^5\, D_a^4\, b_{\frac{3}{2}}^{(i)}$	$\log a^6\, D_a^5\, b_{\frac{3}{2}}^{(i)}$
0	0.8590131	1.5254362	2.4126655			
1	0.8073433	1.5253162	2.4098989			
2	0.7277921	1.5068774	2.4040576			
3	0.6341595	1.4719142	2.3923886			
4	0.5318658	1.4235885	2.3731955			
5	0.4236621	1.3645764	2.3459447			
6	0.3111523	1.2969468	2.3107387			
7	0.1953592	1.2222139	2.2682820			
8	0.0769784	1.1416746	2.2190160			
9	99.9565006	1.0561729	2.1636585	3.2949178	4.4657772	5.6871063
10	99.8342894	0.9664059	2.1028268	3.2569379	4.4433426	5.6737091
11	99.7106216	0.8731223	2.0370794	3.2139903	4.4169447	5.6577201
12	99.5857112	0.7766490	1.9669190	3.1663382	4.3865409	5.6388507

i	$\log a^2\, b_{\frac{5}{2}}^{(i)}$	$\log a^3\, D_a\, b_{\frac{5}{2}}^{(i)}$
0	1.6520288	
2	1.6085195	
10	1.0169577	2.2415088
11	0.9206974	2.1682542
12	0.8216328	2.0912966

VENUS AND MARS.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.3283132	99.4769896	99.6938783	99.9122567	0.3810500
1	99.7173584	99.8005488	99.6119497	99.9555499	0.3723979
2	99.2735238	99.6229528	99.7993513	99.9412687	0.3998361
3	98.8731656	99.3848599	99.7725747	0.0571017	0.4094658
4	98.4930603	99.1221164	99.6547601	0.0909596	0.4862474
5	98.1247182	98.8458657	99.4881571		
6	97.764075	98.561057	99.2913313		
7	97.408846	98.270329			

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	0.2234242	0.3284515	0.7758332	0.2777165
1	0.0390136	0.3616490	0.7538055	
2	99.7989772	0.2810544	0.7474236	0.0667512
3	99.5354049	0.1372379	0.6997284	
4	99.2587233			
5	98.9736609			

VENUS AND JUPITER.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.3031463	98.2957335	98.3145632	97.4310860	97.4866127
1	99.1462753	99.1526329	97.7991402	97.8334042	97.2076815
2	98.1647907	98.4693659	98.4834389	97.4752552	97.5280039
3	97.2288875	97.7084935	98.0157400	98.0401876	97.2715070
4	96.314102	96.9180814	97.3990491	97.7096849	
5	95.41152	96.112051			

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.4631797	98.4100228	98.4604358	98.6396471
1	98.7792220	98.8102149	98.1417438	
2	98.0181761	98.3339507	98.3897984	97.5462166
3	97.2276932	97.7143505	98.0388280	

VENUS AND SATURN.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.3016559	97.7624999	97.7681145	96.3573617	96.3744455
1	98.8807804	98.8826583	96.9969247	97.0072649	95.8610681
2	97.6357877	97.9378624	97.9420319	96.4026436	96.4188144
3	96.4365002	96.9143532	97.2172126	97.2244928	95.9270734
4	95.2583823	95.8610061	96.3392568	96.6431106	

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.1865049	97.6015931	97.6170106	98.0763161
1	98.2415003	98.2508215	97.0598027	
2	97.2179397	97.5233450	97.5405529	96.4707681
3	96.1645717	96.6445102	96.9525494	

VENUS AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.3011847	97.1535110	97.1549003	95.1370825	95.1413554
1	98.5766400	98.5771026	96.0826950	96.0852642	94.3342873
2	97.0281355	97.3294231	97.3304520	95.1827220	95.1867614
3	95.5253753	96.0026771	96.3041581	96.3059585	94.4010004
4	94.0437995	94.6459986			

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	98.8788285	96.6853988	96.6892460	97.4577049
1	97.6310965	97.6334096	95.8369102	
2	96.3043381	96.6064484	96.6107545	95.2499591

VENUS AND NEPTUNE.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.3010933	96.7636511	96.7642173	94.3569146	94.3586608
1	98.3817785	98.3819670	95.4977600	95.4988080	93.3587986
2	96.6385344	96.9396695	96.9400890	94.4026241	94.4042749
3	94.9410424	95.4182373	95.7194509	95.7201858	93.4256500

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	98.6832809	96.1000820	96.1016540	97.0659721
1	97.2409619	97.2419058	95.0562346	
2	95.7195243	96.0209948	96.0227555	94.4696891

THE EARTH AND MARS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.3600512	99.9063288	0.3318839	0.9141702	1.6847103
1	99.9055625	0.0892258	0.3104128	0.9251659	1.6854936
2	99.6080871	0.0215561	0.3924579	0.9311108	1.6940369
3	99.3514193	99.9108645	0.4165453	0.9736804	1.7048208
4	99.1138839	99.7812022	0.3958855	1.0125710	1.7307937
5	98.887518	99.6406607	0.3448559	1.0303987	1.7642993
6	98.668488	99.4930083	0.2725474	1.0249435	
7	98.454661	99.340324	0.1846741	0.9987475	
8	98.244734	99.183856			
9	98.037747	99.024519			

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.6531967	1.1628096	1.9145598	1.2140322
1	0.5750996	1.1657938	1.9096196	
2	0.4610026	1.1367583	1.9017179	1.1418640
3	0.3297791	1.0816108	1.8836598	
4	0.1883851	1.0069848	1.8522916	
5	0.0402136	0.9176828	1.8074144	
6	99.8871919	0.8170486		

THE EARTH AND JUPITER.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.3051074	98.5859021	98.6218017	98.0209251	98.1228757
1	99.2898787	99.3021396	98.2375153	98.3018635	97.9506914
2	98.4493848	98.7572747	98.7843211	98.0636724	98.1609219
3	97.6543084	98.1362460	98.4493285	98.4960014	98.0117776
4	96.880286	97.4860667	97.9706649	98.2903438	
5	96.118424	96.8204336			
6	95.364507	96.145214			
7	94.616139	95.463471			

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.6214901	98.8565780	98.9497818	98.9687006
1	99.0753228	99.1337764	98.7422899	
2	98.4539620	98.7832731	97.8856386	98.1384352
3	97.7936485	98.2991609	98.6447454	
4	97.1379483			

THE EARTH AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.3022290	98.0464055	98.0571258	96.9279665	96.9601872
1	99.0223022	99.0259018	97.4236915	97.4433442	96.5760501
2	97.9180673	98.2211014	98.2290828	96.9728170	97.0033601
3	96.8594899	97.3380151	97.6425563	97.6564655	96.6412072
4	95.822063	96.4252059	96.9044969	97.2109487	
5	94.796844	95.4967015			

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.3323249	98.0307485	98.0598969	98.3718510
1	98.5271291	98.5448684	97.6335748	
2	97.6439443	97.9533446	97.9857812	97.0420271
3	96.7310949	97.2136156	97.5280162	

THE EARTH AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.3013256	97.4354695	97.4381242	95.7016881	95.7098238
1	98.7175135	98.7183990	96.5058565	96.5107585	95.0404733
2	97.3096947	97.6112174	97.6131856	95.7472204	95.7549146
3	95.9476085	96.4250747	96.7269674	96.7304084	95.1069740
4	94.6067012	95.2090280	95.6866818		

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.0207590	97.1091469	97.1164784	97.7425453
1	97.9134779	97.9178933	96.4022975	
2	96.7273112	97.0304071	97.0386052	95.8147221
3	95.5112547	95.9897052	96.2940507	

THE EARTH AND NEPTUNE.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.3011507	97.0452335	97.0463171	94.9203609	94.9236944
1	98.5225269	98.5228872	95.9202246	95.9222280	94.0632816
2	96.9199546	97.2211851	97.2219871	94.9660264	94.9691775
3	95.3631295	95.8403912	96.1417725	96.1431767	94.1300459

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	98.8244597	96.5227863	96.5257875	97.3487309
1	97.5227166	97.5245204	95.6199996	
2	96.1419128	96.4437851	96.4471449	95.0331992
3	94.7311760	95.2088388	95.5112218	

MARS AND JUPITER.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.3107113	98.9770487	99.0599399	98.8300984	99.0453381
1	99.4811814	99.5103889	98.8332179	98.9762844	98.9776184
2	98.8245306	99.1420587	99.2058544	98.8690316	99.0762273
3	98.2128292	98.7015819	99.0317499	99.1398730	99.0315969
4	97.6219911	98.2330619	98.7283933	99.0746535	
5	97.043213	97.7495567	98.3646694		
6	96.472329	97.2567116	97.9657798		
7	95.90695	96.7574834			
8	95.34575				

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.8548398	99.4756985	99.6755213	99.4671301
1	99.4835937	99.6149205	99.5814924	
2	99.0423516	99.4097419	99.6250671	98.9550827
3	98.5735187	99.0944484	99.4974752	
4	98.0898551			

MARS AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.3038316	98.4194228	98.4442570	97.6817908	97.7540338
1	99.2076032	99.2160220	97.9857638	98.0307110	97.5230775
2	98.2865336	98.5922635	98.6108769	97.7254579	97.7941901
3	97.4109882	97.8914051	98.2006829	98.2329418	97.5859297
4	96.5565380	97.1611435	97.6433727	97.9571530	
5	95.714274	96.415321			
6	94.879965	95.659857			

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.5296467	98.5995041	98.6652606	98.7770123
1	98.9049807	98.9456991	98.3962028	
2	98.2038928	98.5244111	98.5970262	97.7977385
3	97.4735386	97.9632879	98.2952057	

MARS AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.3017169	97.8030264	97.8091849	96.4387118	96.4574277
1	98.9009978	98.9030588	97.0578109	97.0691483	95.9629399
2	97.6761412	97.9783178	97.9828929	96.4839476	96.5016662
3	96.4969847	96.9749090	97.2779470	97.2859339	96.0288542
4	95.3389952	95.9416747	96.4200358		

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.2071792	97.6627331	97.6796282	98.1180826
1	98.2822098	98.2924315	97.1414900	
2	97.2787446	97.5845753	97.6034263	96.5521871
3	96.2454874	96.7257004	97.0344186	

MARS AND NEPTUNE.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.3013100	97.4117444	97.4142587	95.6541612	95.6618697
1	98.7056629	98.7065011	96.4702434	96.4748870	94.9810173
2	97.2860140	97.5875105	97.5893744	95.6997056	95.7069953
3	95.9120994	96.3895471	96.6913938	96.6946528	95.0475415
4	94.5593641	95.1616763	95.6393018		

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.0087906	97.0734690	97.0804149	97.7185002
1	97.8897057	97.8938878	96.3546840	
2	96.6917194	96.9947059	97.0024735	95.7671776
3	95.4638396	95.9422196	96.2463897	

JUPITER AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^5 D_\alpha^5 b_{\frac{1}{2}}^{(i)}$
0	0.3385227	99.6447536	99.9323668	0.2943834	0.8737065	1.5571443
1	99.7929617	99.9080126	99.8807510	0.3204252	0.8712043	1.5610525
2	99.4112293	99.7803232	0.0203404	0.3188200	0.8884925	1.5658199
3	99.0721127	99.5982403	0.0219675	0.3995635	0.9011930	1.5798031
4	98.7528969	99.3934022	99.9503069	0.4424607	0.9527604	1.5958171
5	98.445260	99.1759112	99.8362007	0.4325812	1.0028484	1.6320201
6	98.145206	98.9503245	99.6947800	0.3803462	1.0245924	1.6773991
7	97.850517	98.7190975	99.5343998	0.2962794	1.0130524	1.7122581
8	97.559818	98.483710	99.3601200	0.1880990	0.9715115	1.7254699
9	97.272369	98.245141	99.1752269	0.0612446		
10	96.98714	98.00416				
11	96.70391	97.76098				

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^3 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.3762360	0.6405825	1.1986313	1.8182258	0.6137241
1	0.2401571	0.6567683	1.1862536	1.8459979	
2	0.0555698	0.6014522	1.1770313	1.8554065	0.4698123
3	99.8496384	0.4991413	1.1438034	1.8414177	
4	99.6315700	0.3661981	1.0814342	1.8164763	
5	99.4056399	0.2121088	0.9931214	1.7744894	
6	99.1741927	0.0426529	0.8834073	1.7135764	
7	98.9386574	99.8615625	0.7563142	1.6344562	
8	98.6999680	99.6713775	0.6150429		
9	98.4587713	99.4738784	0.4620858		

JUPITER AND URANUS.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.3092856	98.9038847	98.9750571	98.6769679	98.8656377
1	99.4456908	99.4705769	98.7211775	98.8451251	98.7823853
2	98.7554468	99.0704995	99.1249909	98.7168378	98.8980896
3	98.1102711	98.5972679	98.9230322	99.0157985	98.8380776
4	97.486006	98.0957127	98.5882625	98.9276587	
5	96.873837	97.5790530	98.1921618		
6	96.269539	97.0529904	97.7605062		
7	95.670777	96.520507			

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.8086496	99.3576948	99.5322513	99.3671643
1	99.4060486	99.5195467	99.4207972	
2	98.9321593	99.2899439	99.4788381	98.7999575
3	98.4303359	98.9447839	99.3335720	
4	97.9135406			
5	97.3874000			

JUPITER AND NEPTUNE.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.3043311	98.4920408	98.5212246	97.8294469	97.9135311
1	99.2435350	99.2534573	98.0954862	98.1481025	97.7091037
2	98.3577319	98.6643058	98.6862239	97.8727521	97.9528337
3	97.5174107	97.9984222	98.3091854	98.3471076	97.7712573
4	96.698170	97.3032334	97.7863873	98.1024699	
5	95.891104	96.5925262			
6	95.091992	95.872200			

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.5693244	98.7113098	98.7879789	98.8595382
1	98.9791094	99.0268276	98.5465870	
2	98.3129556	98.6369224	98.7214024	97.9460003
3	97.6176584	98.1096632	98.4469597	
4	96.9068966			

SATURN AND URANUS.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.3313200	99.5310896	99.7692508	0.0342514	0.5374908
1	99.7419927	99.8345228	99.6981179	0.0711778	0.5313225
2	99.3188110	99.6739420	99.8691952	0.0618363	0.5549586
3	98.9388512	99.4547104	99.8528345	0.1653903	0.5661350
4	98.5790462	99.2113929	99.7509771	0.2034016	0.6339224
5	98.230944	98.9548198	99.6022336		
6	97.890500	98.6898232	99.4241234		
7	97.555494	98.418992			

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	0.2707586	0.4273481	0.9090815	0.3820620
1	0.1028521	0.4541657	0.8907357	
2	99.8815026	0.3825507	0.8829489	0.1946360
3	99.6372755	0.2530913	0.8404206	
4	99.3802273			
5	99.1149516			

SATURN AND NEPTUNE.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.3124955	99.0555102	99.1528574	98.9955021	99.2422368
1	99.5190405	99.5536677	98.9537897	99.1200212	99.1890516
2	98.8978754	99.2185249	99.2939236	99.0332935	99.2713572
3	98.3215112	98.8124824	99.1482174	99.2753084	99.2409713
4	97.765950	98.3787471	98.8776094	99.2325567	
5	97.222425	97.9301765	98.5478353		
6	96.686750	97.472343			

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.9060934	99.6037404	99.8337030	99.5787130
1	99.5675621	99.7205960	99.7562985	
2	99.1606227	99.5399608	99.7865741	99.1230705
3	98.7265189	99.2555761	99.6760942	
4	98.2777608			
5	97.8198198			

URANUS AND NEPTUNE.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.3560705	99.8638782	0.2648420	0.8115144	1.5497002
1	99.8877784	0.0586025	0.2397216	0.8242115	1.5502635
2	99.5777458	99.9827065	0.3297006	0.8296527	1.5598299
3	99.3088556	99.8618339	0.3517696	0.8771771	1.5712163
4	99.0592408	99.7213150	0.3246251	0.9177603	1.6003951
5	98.8208746	99.5695925	0.2650304	0.9330233	1.6368631
6	98.589898	99.4105746	0.1830624	0.9219949	1.6663271
7	98.364156	99.2464174	0.0848955	0.8883071	1.6808770
8	98.142317	99.078414	99.9745172	0.8359697	1.6782061
9	97.923459	98.907462	99.8546237	0.7683861	
10	97.70712	98.731127	99.7271213	0.6882557	
11	97.49272	98.558858	99.5934081		
12	97.27994	98.381556	99.4544996		

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	0.6048327	1.0748668	1.7937073	1.1101211
1	0.5187735	1.0791600	1.7879788	
2	0.3946344	1.0467691	1.7796214	1.0287654
3	0.2526312	0.9853565	1.7595863	
4	0.1001034	0.9027548	1.7242974	
5	99.9405998	0.8044232	1.6737988	
6	99.7761207	0.6940781	1.6095105	
7	99.6079077	0.5742955	1.5332226	
8	99.4367866	0.4469509	1.4465602	

SMITHSONIAN CONTRIBUTIONS TO KNOWLEDGE.

ASTEROID SUPPLEMENT

TO

NEW TABLES

FOR DETERMINING THE

VALUES OF $b_s^{(i)}$ AND ITS DERIVATIVES.

BY

JOHN D. RUNKLE,

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ACCEPTED FOR PUBLICATION

BY THE SMITHSONIAN INSTITUTION,

NOVEMBER, 1855.

COMMISSION

TO WHICH THIS PAPER HAS BEEN REFERRED.

BENJAMIN PEIRCE,
CHARLES H. DAVIS.

JOSEPH HENRY,
Secretary.

THE following pages contain the values of $b_s^{(i)}$ and its derivatives for all the Asteroids except those having large inclinations and eccentricities; and is the completion of the paper lately published by the Smithsonian Institution, entitled "New Tables for determining the Values of the Coefficients, in the Perturbative Function of Planetary Motion, which depend upon the Ratio of the Mean Distances."

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ASTEROID SUPPLEMENT.

THE large, and still increasing, number of the Asteroids has served to augment the labors of astronomers in a rapid ratio; so much so, that nothing short of general tables, by means of which their ephemerides can be rapidly computed, will ever reduce this department of astronomical labor to anything like its just proportion. The Asteroid problem must always remain one of a purely scientific character; and such results, based upon a solid foundation, can hardly be expected, so long as so much mechanical labor is required merely to prevent these bodies from being lost. The great work of Gauss, *Theoria Motus Corporum Cœlestium*, and the subsequent labors of many other distinguished astronomers, leave little to be desired so far as the determination of their orbits is concerned. Such, however, is by no means the case with regard to their perturbations by the larger planets. While the variations of the elements of those having small eccentricities and inclinations may be obtained through the development of the usual form of the perturbative function, there are others for which this method is practically nearly impossible for any close approximation. This fact has led to the suggestion of new theories of their general perturbations; and the study which is now bestowed upon this problem, in this country as well as in Europe, warrants the confident expectation that it will erelong be presented in its simplest and most practical form.

In order to facilitate as much as possible the computation of the perturbations of those to which the usual theory of development is applicable, I have thought it best to take from the Tables the necessary constants, $b_s^{(i)}$ and its derivatives, depending upon the ratio of the mean distances, giving at the same time a simple table for computing the variation of these constants relatively to this ratio, in order that they may readily be corrected for any change in the Asteroid's mean distance. This correction will be necessary for those Asteroids more recently discovered, and whose mean distances are not therefore very well known.

If, then, n_0 and m_0 are the mean motion and mass of the Earth, and n is the mean motion of the Asteroid in a Julian year, its mean distance a will be

$$a = \left(\frac{n_0}{n}\right)^{\frac{2}{3}} (1 + m_0)^{-\frac{1}{3}}.$$

Adopting the following values of n, the corresponding values of a are found to be for

①	Ceres,	$n = 281110''.5$	$\log a = 0.4424798$	$a = 2.770000$
②	Pallas,	$n = 281174.6$	$\log a = 0.4424138$	$a = 2.769579$
③	Juno,	$n = 297282.4$	$\log a = 0.4262850$	$a = 2.668609$
④	Vesta,	$n = 357318.7$	$\log a = 0.3730274$	$a = 2.360627$
⑤	Astræa,	$n = 313428.6$	$\log a = 0.4109720$	$a = 2.576155$
⑥	Hebe,	$n = 342978.4$	$\log a = 0.3848866$	$a = 2.425977$
⑦	Iris,	$n = 351620.0$	$\log a = 0.3776818$	$a = 2.386063$
⑧	Flora,	$n = 396782.4$	$\log a = 0.3426962$	$a = 2.201386$
⑨	Metis,	$n = 351546.5$	$\log a = 0.3777426$	$a = 2.386397$
⑩	Hygeia,	$n = 231877.3$	$\log a = 0.4982256$	$a = 3.149385$
⑪	Parthenope,	$n = 337564.7$	$\log a = 0.3894930$	$a = 2.451845$
⑫	Clio,	$n = 363291.7$	$\log a = 0.3682274$	$a = 2.334680$
⑬	Egeria,	$n = 313296.1$	$\log a = 0.4110944$	$a = 2.576881$
⑭	Irene,	$n = 311778.8$	$\log a = 0.4125000$	$a = 2.585235$
⑮	Eunomia,	$n = 301543.6$	$\log a = 0.4221644$	$a = 2.643409$
⑯	Psyche,	$n = 259387.5$	$\log a = 0.4657652$	$a = 2.922572$
⑰	Thetis,	$n = 333324.5$	$\log a = 0.3931530$	$a = 2.472595$
⑱	Melpomene,	$n = 372571.0$	$\log a = 0.3609250$	$a = 2.295752$
⑲	Fortuna,	$n = 346696.3$	$\log a = 0.3817650$	$a = 2.408602$
⑳	Massalia,	$n = 346935.0$	$\log a = 0.3815646$	$a = 2.407490$
㉑	Lutetia,	$n = 341251.3$	$\log a = 0.3863482$	$a = 2.434155$
㉒	Calliope,	$n = 260561.4$	$\log a = 0.4644578$	$a = 2.913788$
㉓	Thalia,	$n = 304604.2$	$\log a = 0.4192406$	$a = 2.625672$
㉔	Themis,	$n = 230157.6$	$\log a = 0.5003810$	$a = 3.165053$
㉕	Phocæa,	$n = 348415.7$	$\log a = 0.3803328$	$a = 2.400672$
㉖	Proserpine,	$n = 299449.4$	$\log a = 0.4241822$	$a = 2.655720$
㉗	Euterpe,	$n = 360561.4$	$\log a = 0.3770784$	$a = 2.382750$
㉘	Bellona,	$n = 279486.4$	$\log a = 0.4441574$	$a = 2.780721$
㉙	Amphitrite,	$n = 317485.8$	$\log a = 0.4072482$	$a = 2.554160$
㉚	Urania,	$n = 356829.1$	$\log a = 0.3734242$	$a = 2.362785$
㉛	Euphrosyne,	$n = 227218.7$	$\log a = 0.5041018$	$a = 3.192286$
㉜	Pomona,	$n = 312038.9$	$\log a = 0.4122586$	$a = 2.583799$
㉝	Polymnia,	$n = 353282.6$	$\log a = 0.3763164$	$a = 2.378573$
㉞	Circe,	$n = 294022.0$	$\log a = 0.4294780$	$a = 2.688302$
㉟	Leucothea,	$n = 252728.1$	$\log a = 0.4732954$	$a = 2.973688$
㊱	Atalanta,	$n = 284199.7$	$\log a = 0.4393088$	$a = 2.749849$
㊲	Fides,	$n = 301801.1$	$\log a = 0.4219174$	$a = 2.641906$
㊳	Leda,	$n = 285726.3$	$\log a = 0.4377644$	$a = 2.740088$
㊴	Lætitia,	$n = 281889.5$	$\log a = 0.4416786$	$a = 2.764895$
㊵	Harmonia,	$n = 379644.5$	$\log a = 0.3554798$	$a = 2.267148$
㊶	Daphne,	$n = 353182.1$	$\log a = 0.3763988$	$a = 2.379024$
㊷	Isis,	$n = 340776.8$	$\log a = 0.3867512$	$a = 2.436415$

Hence, since

$$\beta^2 = \frac{\alpha^2}{1-\alpha^2},$$

or, if $\alpha = \sin \delta$,

$$\beta^2 = \tan^2 \delta,$$

we have for

	MERCURY AND			VENUS AND		
ASTEROID	α	$\log\alpha$	$\log\beta^2$	α	$\log\alpha$	$\log\beta^2$
(1) Ceres,	0.1397469	9.1453419	8.2992490	0.2611307	9.4168580	8.8643882
(2) Pallas,	0.1397681	9.1454079	8.2993834	0.2611704	9.4169240	8.8645298
(3) Juno,	0.1450564	9.1615367	8.3323090	0.2710521	9.4330528	8.8992460
(4) Vesta,	0.1639812	9.2147943	8.4414266	0.3064153	9.4863104	9.0154400
(5) Astræa,	0.1502622	9.1768497	8.3636172	0.2807797	9.4483658	8.9323950
(6) Hebe,	0.1595641	9.2029351	8.4170708	0.2981612	9.4744512	8.9893366
(7) Iris,	0.1622333	9.2101399	8.4318632	0.3031489	9.4816560	9.0051782
(8) Flora,	0.1758432	9.2451255	8.5038916	0.3285804	9.5166416	9.0829018
(9) Metis,	0.1622106	9.2100791	8.4317386	0.3031065	9.4815952	9.0050436
(10) Hygeia,	0.1229125	9.0895961	8.1858034	0.2296742	9.3611122	8.7457598
(11) Parthenope,	0.1578802	9.1983287	8.4076200	0.2950155	9.4698448	8.9792348
(12) Clio,	0.1658037	9.2195943	8.4512948	0.3098207	9.4911104	9.0260468
(13) Egeria,	0.1502191	9.1767273	8.3633672	0.2807006	9.4482434	8.9321294
(14) Irene,	0.1497344	9.1753217	8.3604912	0.2797936	9.4468378	8.9290786
(15) Eunomia,	0.1464392	9.1656573	8.3407292	0.2736361	9.4371734	8.9081472
(16) Psyche,	0.1324514	9.1220565	8.2518000	0.2474985	9.3935726	8.8145976
(17) Thetis,	0.1565556	9.1946687	8.4001144	0.2925397	9.4661848	8.9712234
(18) Melpomene,	0.1686152	9.2268967	8.4663196	0.3150742	9.4984128	9.0422318
(19) Fortuna,	0.1607151	9.2060567	8.4234782	0.3003121	9.4775728	8.9961938
(20) Massalia,	0.1607893	9.2062571	8.4238896	0.3004507	9.4777732	8.9966344
(21) Lutetia,	0.1590280	9.2014735	8.4140716	0.2971596	9.4729896	8.9861286
(22) Calliope,	0.1328507	9.1233639	8.2544612	0.2482447	9.3948800	8.8173836
(23) Thalia,	0.1474284	9.1685811	8.3467060	0.2754845	9.4400972	8.9144712
(24) Themis,	0.1223040	9.0874407	8.1814266	0.2285372	9.3589568	8.7412102
(25) Phocæa,	0.1612460	9.2074889	8.4264188	0.3013041	9.4790050	8.9993426
(26) Proserpine,	0.1457604	9.1636395	8.3366054	0.2723677	9.4351556	8.9037866
(27) Euterpe,	0.1624588	9.2107433	8.4331028	0.3035704	9.4822594	9.0065074
(28) Bellona,	0.1392080	9.1436643	8.2958274	0.2601240	9.4151804	8.8607884
(29) Amphitrite,	0.1515562	9.1805735	8.3712386	0.2831977	9.4520896	8.9404862
(30) Urania,	0.1638316	9.2143975	8.4406112	0.3061354	9.4859136	9.0145640
(31) Euphrosyne,	0.1212607	9.0837199	8.1738730	0.2265876	9.3552360	8.7333624
(32) Pomona,	0.1498178	9.1755631	8.3609852	0.2799491	9.4470792	8.9296028
(33) Polymnia,	0.1627441	9.2115053	8.4346682	0.3041035	9.4830214	9.0081858
(34) Circe,	0.1439938	9.1583437	8.3257866	0.2690666	9.4298598	8.8923570
(35) Leucothea,	0.1301746	9.1145263	8.2364750	0.2432441	9.3860424	8.7985724
(36) Atalanta,	0.1407709	9.1485129	8.3057184	0.2630444	9.4200290	8.8711982
(37) Fides,	0.1465225	9.1659043	8.3412542	0.2737918	9.4374204	8.9086812
(38) Leda,	0.1412724	9.1500573	8.3088698	0.2639814	9.4215734	8.8745172
(39) Lætitia,	0.1400049	9.1461431	8.3008832	0.2616130	9.4176594	8.8661084
(40) Harmonia,	0.1707426	9.2323419	8 4775334	0.3190422	9.5038580	9.0543398
(41) Daphne,	0.1627133	9.2114229	8.4344990	0.3040458	9.4829390	9.0080048
(42) Isis,	0.1588805	9.2010705	8.4132448	0.2968840	9.4725866	8.9852450

	THE EARTH AND			MARS AND		
ASTEROID	α	$\log \alpha$	$\log \beta^2$	α	$\log \alpha$	$\log \beta^2$
(1) Ceres,	0.3610108	9.5575202	9.1756850	0.5499526	9.7403172	9.6370508
(2) Pallas,	0.3610657	9.5575862	9.1758370	0.5501526	9.7404832	9.6375268
(3) Juno,	0.3747270	9.5737150	9.2131442	0.5709682	9.7566120	9.6845672
(4) Vesta,	0.4236162	9.6269726	9.3398400	0.6454604	9.8098696	9.8537870
(5) Astræa,	0.3881754	9.5890280	9.2489844	0.5914594	9.7719250	9.7308190
(6) Hebe,	0.4122051	9.6151134	9.3111036	0.6280734	9.7980104	9.8138890
(7) Iris,	0.4191005	9.6223182	9.3285220	0.6385800	9.8052152	9.8379504
(8) Flora,	0.4542593	9.6573038	9.4149788	0.6921510	9.8402008	9.9636242
(9) Metis,	0.4190419	9.6222574	9.3283752	0.6384949	9.8051574	9.8377554
(10) Hygeia,	0.3175224	9.5017744	9.0497024	0.4838060	9.6846714	9.4851526
(11) Parthenope,	0.4078561	9.6105070	9.3000288	0.6214469	9.7934040	9.7987786
(12) Clio,	0.4283242	9.6317726	9.3515690	0.6526339	9.8146696	9.8703756
(13) Egeria,	0.3880660	9.5889056	9.2486962	0.5912928	9.7718026	9.7304432
(14) Irene,	0.3868121	9.5875000	9.2453892	0.5893821	9.7703970	9.7261280
(15) Eunomia,	0.3782993	9.5778356	9.2227462	0.5764114	9.7607326	9.6968516
(16) Psyche,	0.3421644	9.5342348	9.1225462	0.5213528	9.7171318	9.5720184
(17) Thetis,	0.4044334	9.6068470	9.2912624	0.6162317	9.7897440	9.7869150
(18) Melpomene,	0.4355872	9.6390750	9.3695238	0.6637003	9.8219720	9.8961430
(19) Fortuna,	0.4151787	9.6182350	9.3186354	0.6326040	9.8011320	9.8242486
(20) Massalia,	0.4153703	9.6184354	9.3191196	0.6328961	9.8013324	9.8245836
(21) Lutetia,	0.4108202	9.6136518	9.3075842	0.6259631	9.7965488	9.8090718
(22) Calliope,	0.3431960	9.5355422	9.1255090	0.5229247	9.7184392	9.5756132
(23) Thalia,	0.3808547	9.5807594	9.2295784	0.5803050	9.7636564	9.7056382
(24) Themis,	0.3159504	9.4996190	9.0449108	0.4814110	9.6825160	9.4795332
(25) Phocæa,	0.4165500	9.6196672	9.3220986	0.6346937	9.8025642	9.8290350
(26) Proserpine,	0.3765458	9.5758178	9.2180402	0.5737395	9.7587148	9.6908214
(27) Euterpe,	0.4196832	9.6229216	9.3299862	0.6394677	9.8058186	9.8399906
(28) Bellona,	0.3596190	9.5558426	9.1718296	0.5479441	9.7387396	9.6325348
(29) Amphitrite,	0.3915180	9.5927518	9.2577668	0.5965527	9.7756488	9.7423274
(30) Urania,	0.4232294	9.6265758	9.3388734	0.6448709	9.8094728	9.8524280
(31) Euphrosyne,	0.3132551	9.4958982	9.1366518	0.4773042	9.6787952	9.4698718
(32) Pomona,	0.3870272	9.5877414	9.2459566	0.5897100	9.7706384	9.7268680
(33) Polymnia,	0.4204209	9.6236835	9.3318370	0.6405906	9.8065806	9.8425712
(34) Circe,	0.3719820	9.5705220	9.2057244	0.5667858	9.7534190	9.6751256
(35) Leucothea,	0.3362828	9.5267046	9.1055276	0.5123911	9.7096016	9.5514684
(36) Atalanta,	0.3636564	9.5606912	9.1829862	0.5541000	9.7435882	9.6464614
(37) Fides,	0.3785145	9.5780826	9.2233228	0.5767394	9.7609796	9.6975898
(38) Leda,	0.3649519	9.5622356	9.1865478	0.5560740	9.7451326	9.6509244
(39) Lætitia,	0.3616774	9.5583214	9.1775288	0.5510848	9.7412184	9.6396362
(40) Harmonia,	0.4410828	9.6445202	9.3830044	0.6720742	9.8274172	9.9158034
(41) Daphne,	0.4203405	9.6236012	9.3316372	0.6404691	9.8064982	9.8422938
(42) Isis,	0.4104392	9.6132488	9.3066132	0.6253682	9.7961358	9.8077146

ASTEROID	JUPITER AND α	$\log \alpha$	$\log \beta^2$	SATURN AND α	$\log \alpha$	$\log \beta^2$
(1) Ceres,	0.5324055	9.7262425	9.5972418	0.2903912	9.4629835	8.9642268
(2) Pallas,	0.5323246	9.7261765	9.5970576	0.2903471	9.4629175	8.9640826
(3) Juno,	0.5129176	9.7100477	9.5526778	0.2797620	9.4467887	8.9289720
(4) Vesta,	0.4537223	9.6567901	9.4136850	0.2474749	9.3935311	8.8145096
(5) Astræa,	0.4951477	9.6947347	9.5116208	0.2700696	9.4314757	8.8958426
(6) Hebe,	0.4662826	9.6686493	9.4437696	0.2543257	9.4053903	8.8398212
(7) Iris,	0.4586110	9.6614445	9.4254400	0.2501414	9.3981855	8.8244328
(8) Flora,	0.4231154	9.6264589	9.3385890	0.2307809	9.3631999	8.7501690
(9) Metis,	0.4586752	9.6615053	9.4255938	0.2501764	9.3982463	8.8245618
(10) Hygeia,	0.6053246	9.7819883	9.7621738	0.3301637	9.5187293	9.0875848
(11) Parthenope,	0.4712547	9.6732557	9.4555768	0.2570376	9.4099967	8.8496782
(12) Clio,	0.4487351	9.6519901	9.4016310	0.2447547	9.3887311	8.8042904
(13) Egeria,	0.4952871	9.6948571	9.5119454	0.2701457	9.4315981	8.8961064
(14) Irene,	0.4968928	9.6962627	9.5156748	0.2710215	9.4330037	8.8991400
(15) Eunomia,	0.5080741	9.7059271	9.5415322	0.2771201	9.4426681	8.9200382
(16) Psyche,	0.5617304	9.7495279	9.6637084	0.3063860	9.4862689	9.0153486
(17) Thetis,	0.4752430	9.6769157	9.4650096	0.2592130	9.4136567	8.8575210
(18) Melpomene,	0.4412530	9.6446877	9.3834198	0.2406737	9.3814287	8.7887714
(19) Fortuna,	0.4629432	9.6655277	9.4358076	0.2525042	9.4022687	8.8331494
(20) Massalia,	0.4627295	9.6653273	9.4352980	0.2523878	9.4020683	8.8327214
(21) Lutetia,	0.4678547	9.6701109	9.4475086	0.2551831	9.4068519	8.8429470
(22) Calliope,	0.5600419	9.7482205	9.6598932	0.3054651	9.4849615	9.0124638
(23) Thalia,	0.5046651	9.7030033	9.5336678	0.2752608	9.4397443	8.9137078
(24) Themis,	0.6083363	9.7841437	9.7689974	0.3318063	9.5208847	9.0924256
(25) Phocæa,	0.4614190	9.6640955	9.4321654	0.2516730	9.4008365	8.8300908
(26) Proserpine,	0.5104402	9.7079449	9.5469810	0.2784107	9.4446859	8.9244112
(27) Euterpe,	0.4579743	9.6608411	9.4239124	0.2497941	9.3975821	8.8231452
(28) Bellona,	0.5344660	9.7279201	9.6019316	0.2915151	9.4646611	8.9678920
(29) Amphitrite,	0.4909202	9.6910109	9.5017814	0.2677638	9.4277519	8.8878142
(30) Urania,	0.4541370	9.6571869	9.4146848	0.2477010	9.3939279	8.8153552
(31) Euphrosyne,	0.6135706	9.7878645	9.7808704	0.3346613	9.5246055	9.1007972
(32) Pomona,	0.4966167	9.6960213	9.5150332	0.2708709	9.4327623	8.8986186
(33) Polymnia,	0.4571714	9.6600791	9.4219850	0.2493562	9.3968201	8.8215198
(34) Circe,	0.5167026	9.7132407	9.5613672	0.2818265	9.4499817	8.9359044
(35) Leucothea,	0.5715551	9.7570581	9.6858902	0.3117448	9.4937991	9.0319994
(36) Atalanta,	0.5285322	9.7230715	9.5884168	0.2882793	9.4598125	8.9573054
(37) Fides,	0.5077852	9.7056801	9.5408658	0.2769626	9.4424211	8.9195028
(38) Leda,	0.5266561	9.7215271	9.5841354	0.2872553	9.4582681	8.9539372
(39) Lætitia,	0.5314241	9.7254413	9.5950066	0.2898560	9.4621823	8.9624776
(40) Harmonia,	0.4357551	9.6392425	9.3699368	0.2376750	9.3759835	8.7772206
(41) Daphne,	0.4572582	9.6601615	9.4221928	0.2494034	9.3969025	8.8216958
(42) Isis,	0.4682889	9.6705139	9.4485410	0.2554200	9.4072549	8.8438088

	URANUS AND			NEPTUNE AND		
ASTEROID	α	log α	log β^2	α	log α	log β^2
① Ceres,	0.1443944	9.1595503	8.3282512	0.0922202	8.9648260	7.9333614
② Pallas,	0.1443724	9.1594843	8.3281162	0.0922062	8.9647600	7.9332282
③ Juno,	0.1391091	9.1433555	8.2951978	0.0888446	8.9486312	7.9007042
④ Vesta,	0.1230546	9.0900979	8.1868222	0.0785911	8.8953736	7.7934380
⑤ Astræa,	0.1342896	9.1280425	8.2639882	0.0857666	8.9333182	7.8698428
⑥ Hebe,	0.1264611	9.1019571	8.2109160	0.0807668	8.9072328	7.8173020
⑦ Iris,	0.1243805	9.0947523	8.1962758	0.0794379	8.9000280	7.8028050
⑧ Flora,	0.1147537	9.0597667	8.1252904	0.0732896	8.8650424	7.7324240
⑨ Metis,	0.1243979	9.0948131	8.1963996	0.0794491	8.9000888	7.8029278
⑩ Hygeia,	0.1641709	9.2152961	8.4424582	0.1048508	9.0205718	8.0459446
⑪ Parthenope,	0.1278096	9.1065635	8.2202800	0.0816280	8.9118392	7.8265820
⑫ Clio,	0.1217021	9.0852979	8.1770762	0.0777273	8.8905736	7.7837792
⑬ Egeria,	0.1343275	9.1281649	8.2642376	0.0857908	8.9334406	7.8700892
⑭ Irene,	0.1347629	9.1295705	8.2671008	0.0860689	8.9348462	7.8729216
⑮ Eunomia,	0.1377955	9.1392349	8.2867952	0.0880057	8.9445106	7.8923978
⑯ Psyche,	0.1523476	9.1828357	8.3758704	0.0972997	8.9881114	7.9803538
⑰ Thetis,	0.1288912	9.1102235	8.2277226	0.0823188	8.9154992	7.8339506
⑱ Melpomene,	0.1196729	9.0779955	8.1622562	0.0764313	8.8832712	7.7690870
⑲ Fortuna,	0.1255554	9.0988355	8.2045718	0.0801883	8.9041112	7.8110240
⑳ Massalia,	0.1254978	9.0986351	8.2041646	0.0801513	8.9039108	7.8106206
㉑ Lutetia,	0.1268875	9.1034187	8.2138866	0.0810391	8.9086944	7.8202504
㉒ Calliope,	0.1518891	9.1815283	8.3731932	0.0970072	8.9868040	7.9777144
㉓ Thalia,	0.1368709	9.1363111	8.2789702	0.0874151	8.9415868	7.8865052
㉔ Themis,	0.1649877	9.2174515	8.4468886	0.1053726	9.0227272	8.0503034
㉕ Phocæa,	0.1251421	9.0974033	8.2016610	0.0799243	8.9026790	7.8081412
㉖ Proserpine,	0.1384371	9.1412527	8.2909096	0.0884155	8.9465284	7.8964652
㉗ Euterpe,	0.1242078	9.0941489	8.1950500	0.0793276	8.8994246	7.8015908
㉘ Bellona,	0.1449532	9.1612279	8.3316786	0.0925771	8.9665036	7.9367454
㉙ Amphitrite,	0.1331431	9.1243187	8.2564048	0.0850344	8.9295944	7.8623406
㉚ Urania,	0.1231671	9.0904947	8.1876282	0.0786630	8.8957704	7.7942364
㉛ Euphrosyne,	0.1664073	9.2211723	8.4545402	0.1062742	9.0264480	8.0578292
㉜ Pomona,	0.1346880	9.1293291	8.2666090	0.0860211	8.9346048	7.8724352
㉝ Polymnia,	0.1239906	9.0933869	8.1935024	0.0791886	8.8986626	7.8000572
㉞ Circe,	0.1401356	9.1465485	8.3017102	0.0895002	8.9518242	7.9071412
㉟ Leucothea,	0.1550122	9.1903659	8.3912946	0.0990014	8.9956416	7.9955608
㊱ Atalanta,	0.1433439	9.1563793	8.3217752	0.0915493	8.9616550	7.9269652
㊲ Fides,	0.1377171	9.1389879	8.2862920	0.0879557	8.9442636	7.8919002
㊳ Leda,	0.1428351	9.1548349	8.3186218	0.0912243	8.9601106	7.9238506
㊴ Lætitia,	0.1441281	9.1587491	8.3266150	0.0920502	8.9640248	7.9317454
㊵ Harmonia,	0.1181817	9.0725503	8.1512096	0.0754790	8.8778260	7.7581332
㊶ Daphne,	0.1240136	9.0934693	8.1936694	0.0792036	8.8987450	7.8002230
㊷ Isis,	0.1270053	9.1038217	8.2147060	0.0811143	8.9090974	7.8210616

The want of accuracy in the mean distances of many of the Asteroids renders some simple and expeditious means desirable for correcting the coefficients for changes in the value of the argument a. The following formulas and tables enable us to compute these changes with every possible facility and accuracy, and leave nothing to be desired in the numerical solution of the problem.

Denote by A_i, B_i, C_i, D_i, &c. the variations of the tabulated functions for a change of .001 in the argument a, the logarithms of which at any value of a are found in the general Tables. If Δ denotes a difference of the function corresponding to a change $\Delta a = .001$ in the argument, and M is the modulus of the common system of logarithms, we readily find the following formulas.

VARIATIONS OF LOG $b_{\frac{1}{2}}^{(i)}$.

$$\Delta \log b_{\frac{1}{2}}^{(i)} = A_i + \frac{i M \Delta a}{a}$$

for all values of i.

VARIATIONS OF FIRST DERIVATIVES.

$$\Delta \log a D_a b_{\frac{1}{2}}^{(i)} = B_i + (i + 2\beta^2) \frac{M \Delta a}{a}$$

for all values of i except $i = 0$, for which the formula is

$$\Delta \log a D_a b_{\frac{1}{2}}^{(0)} = B_0 + (2 + 2\beta^2) \frac{M \Delta a}{a}.$$

VARIATIONS OF SECOND DERIVATIVES.

$$\Delta \log a^2 D_a^2 b_{\frac{1}{2}}^{(i)} = C_i + (i + 4\beta^2) \frac{M \Delta a}{a}$$

for all values of i except $i = 0$, and 1, for which the formulas are

$$\Delta \log a^2 D_a^2 b_{\frac{1}{2}}^{(0)} = C_0 + (2 + 4\beta^2) \frac{M \Delta a}{a}, \qquad \Delta \log a^2 D_a^2 b_{\frac{1}{2}}^{(1)} = C_1 + (3 + 4\beta^2) \frac{M \Delta a}{a}.$$

VARIATIONS OF THIRD DERIVATIVES.

$$\Delta \log a^3 D_a^3 b_{\frac{1}{2}}^{(i)} = D_i + (i + 6\beta^2) \frac{M \Delta a}{a}$$

for all values of i except $i = 0$, 1, and 2, for which the formulas are

$$\Delta \log a^3 D_a^3 b_{\frac{1}{2}}^{(0)} = D_0 + (4 + 6\beta^2) \frac{M \Delta a}{a}, \qquad \Delta \log a^3 D_a^3 b_{\frac{1}{2}}^{(1)} = D_1 + (3 + 6\beta^2) \frac{M \Delta a}{a},$$

$$\Delta \log a^3 D_a^3 b_{\frac{1}{2}}^{(2)} = D_2 + (4 + 6\beta^2) \frac{M \Delta a}{a}.$$

VARIATIONS OF FOURTH DERIVATIVES.

$$\Delta \log a^4 D_a^4 b_{\frac{1}{2}}^{(i)} = E_i + (i + 8\beta^2) \frac{M \Delta a}{a}$$

for all values of i except $i = 0$, 1, 2, and 3, for which the formulas are

$$\Delta \log a^4 D_a^4 b_{\frac{1}{2}}^{(0)} = E_0 + (4 + 8\beta^2) \frac{M \Delta a}{a}, \qquad \Delta \log a^4 D_a^4 b_{\frac{1}{2}}^{(2)} = E_2 + (4 + 8\beta^2) \frac{M \Delta a}{a},$$

$$\Delta \log a^4 D_a^4 b_{\frac{1}{2}}^{(1)} = E_1 + (5 + 8\beta^2) \frac{M \Delta a}{a}, \qquad \Delta \log a^4 D_a^4 b_{\frac{1}{2}}^{(3)} = E_3 + (5 + 8\beta^2) \frac{M \Delta a}{a}.$$

VARIATIONS OF FIFTH DERIVATIVES.

$$\Delta \log a^5 D_a^5 b_{\frac{1}{2}}^{(i)} = F_i + (i + 10\beta^2) \frac{M \Delta a}{a}$$

for all values of i except $i = 0$, 1, 2, 3, and 4, for which the formulas are

$$\Delta \log a^5 D_a^5 b_{\frac{1}{2}}^{(0)} = F_0 + (6 + 10\,\beta^2)\,\frac{M \Delta a}{a}, \qquad \Delta \log a^5 D_a^5 b_{\frac{1}{2}}^{(2)} = F_2 + (6 + 10\,\beta^2)\,\frac{M \Delta a}{a},$$

$$\Delta \log a^5 D_a^5 b_{\frac{1}{2}}^{(1)} = F_1 + (5 + 10\,\beta^2)\,\frac{M \Delta a}{a}, \qquad \Delta \log a^5 D_a^5 b_{\frac{1}{2}}^{(3)} = F_3 + (5 + 10\,\beta^2)\,\frac{M \Delta a}{a},$$

$$\Delta \log a^5 D_a^5 b_{\frac{1}{2}}^{(4)} = F_4 + (6 + 10\,\beta^2)\,\frac{M \Delta a}{a}.$$

VARIATIONS OF LOG $a\,b_{\frac{3}{2}}^{(i)}$.

$$\Delta \log a\, b_{\frac{3}{2}}^{(i)} = G_i + (i + 1 + 4\,\beta^2)\,\frac{M \Delta a}{a}$$

for all values of i.

VARIATIONS OF FIRST DERIVATIVES.

$$\Delta \log a^2 D_a\, b_{\frac{3}{2}}^{(i)} = H_i + (i + 1 + 6\,\beta^2)\,\frac{M \Delta a}{a}$$

for all values of i except $i = 0$, for which the formula is

$$\Delta \log a^2 D_a\, b_{\frac{3}{2}}^{(0)} = H_0 + (3 + 6\,\beta^2)\,\frac{M \Delta a}{a}.$$

VARIATIONS OF SECOND DERIVATIVES.

$$\Delta \log a^3 D_a^2\, b_{\frac{3}{2}}^{(i)} = K_i + (i + 1 + 8\,\beta^2)\,\frac{M \Delta a}{a}$$

for all values of i except $i = 0$, and 1, for which the formulas are

$$\Delta \log a^3 D_a^2\, b_{\frac{3}{2}}^{(0)} = K_0 + (3 + 8\,\beta^2)\,\frac{M \Delta a}{a}, \qquad \Delta \log a^3 D_a^2\, b_{\frac{3}{2}}^{(1)} = K_1 + (4 + 8\,\beta^2)\,\frac{M \Delta a}{a}.$$

VARIATIONS OF LOG $a^2\,b_{\frac{5}{2}}^{(i)}$.

$$\Delta \log a^2\, b_{\frac{5}{2}}^{(i)} = L_i + (i + 2 + 8\,\beta^2)\,\frac{M \Delta a}{a}$$

for all values of i.

It will be observed, that, in all these variations, simple multiples of $\frac{M \Delta a}{a}$ and $2\,\beta^2\,\frac{M \Delta a}{a}$ occur. If then for a change of δa in the argument we compute the terms $\frac{M \Delta a}{a}\,\delta a$ and $2\,\beta^2\,\frac{M \Delta a}{a}\,\delta a$, it will only be necessary to multiply them by 1, 2, 3, 4, &c. to have the variations of all the coefficients arising from these terms. The other terms, $A_i\,\delta a$, $B_i\,\delta a$, $C_i\,\delta a$, &c., of the variations are as readily found, since the logarithms of A_i, B_i, C_i, &c., which are given in the general Tables, are all to be increased by the same constant $\log \delta a$. If, besides, we have tables from which $\frac{M \Delta a}{a}$ and $2\,\beta^2\,\frac{M \Delta a}{a}$ can be taken for any value of a by simple inspection, we shall be able to compute the variations of all the coefficients needed for any Asteroid in a few minutes.

Such are the Tables given on the following pages. The unit place corresponds to the fifth decimal place of the logarithm to be corrected, while the unit place of A_i, B_i, C_i, &c. corresponds to the last decimal in the tabulated function, which is usually the seventh.

The logarithms of $b_s^{(i)}$ and its derivatives, found in the following pages, have been computed in duplicate to seven decimals; but only five are printed, which is ample to give all desired accuracy in the final result.

α	$\frac{M\Delta a}{a}$	$2\beta^2\frac{M\Delta a}{a}$	α	$\frac{M\Delta a}{a}$	$2\beta^2\frac{M\Delta a}{a}$	α	$\frac{M\Delta a}{a}$	$2\beta^2\frac{M\Delta a}{a}$	α	$\frac{M\Delta a}{a}$	$2\beta^2\frac{M\Delta a}{a}$
.030	1447.6	2.6	.090	482.5	7.8	.150	289.5	13.3	.210	206.7	19.1
.031	1400.9	2.7	.091	477.2	7.9	.151	287.6	13.4	.211	205.8	19.2
.032	1357.2	2.8	.092	472.0	8.0	.152	285.7	13.5	.212	204.8	19.3
.033	1316.0	2.8	.093	467.0	8.1	.153	283.8	13.6	.213	203.8	19.4
.034	1277.3	2.9	.094	462.0	8.2	.154	282.0	13.7	.214	202.9	19.5
.035	1240.8	3.0	.095	457.2	8.3	.155	280.2	13.8	.215	201.9	19.6
.036	1206.4	3.2	.096	452.4	8.4	.156	278.4	13.9	.216	201.0	19.7
.037	1173.7	3.3	.097	447.7	8.5	.157	276.6	14.0	.217	200.1	19.8
.038	1142.9	3.4	.098	443.2	8.6	.158	274.8	14.1	.218	199.2	19.9
.039	1113.6	3.4	.099	438.7	8.7	.159	273.1	14.1	.219	198.3	20.0
.040	1085.8	3.5	.100	434.3	8.8	.160	271.4	14.2	.220	197.4	20.1
.041	1059.3	3.6	.101	430.0	8.9	.161	269.7	14.3	.221	196.5	20.2
.042	1034.1	3.6	.102	425.8	9.0	.162	268.0	14.4	.222	195.6	20.3
.043	1010.0	3.7	.103	421.6	9.1	.163	266.4	14.5	.223	194.7	20.4
.044	987.0	3.8	.104	417.6	9.2	.164	264.8	14.6	.224	193.8	20.5
.045	965.1	3.9	.105	413.6	9.2	.165	263.2	14.7	.225	192.9	20.6
.046	944.1	4.0	.106	409.7	9.3	.166	261.6	14.8	.226	192.1	20.7
.047	924.0	4.1	.107	405.9	9.4	.167	260.0	14.9	.227	191.2	20.8
.048	904.8	4.2	.108	402.1	9.5	.168	258.5	15.0	.228	190.4	20.9
.049	886.3	4.3	.109	398.4	9.6	.169	257.0	15.1	.229	189.6	21.0
.050	868.7	4.4	.110	394.8	9.7	.170	255.5	15.2	.230	188.7	21.1
.051	851.5	4.4	.111	391.3	9.8	.171	254.0	15.3	.231	187.9	21.2
.052	835.2	4.5	.112	387.7	9.9	.172	252.5	15.4	.232	187.1	21.3
.053	819.4	4.6	.113	384.3	10.0	.173	251.0	15.5	.233	186.3	21.4
.054	804.2	4.7	.114	380.9	10.0	.174	249.6	15.6	.234	185.5	21.5
.055	789.6	4.8	.115	377.6	10.1	.175	248.2	15.7	.235	184.7	21.6
.056	775.5	4.9	.116	374.4	10.2	.176	246.8	15.8	.236	183.9	21.7
.057	761.9	5.0	.117	371.2	10.3	.177	245.4	15.8	.237	183.1	21.8
.058	748.8	5.1	.118	368.0	10.4	.178	244.0	15.9	.238	182.4	21.9
.059	736.1	5.2	.119	364.9	10.5	.179	242.6	16.0	.239	181.6	22.0
.060	723.8	5.2	.120	361.9	10.6	.180	241.2	16.1	.240	180.8	22.1
.061	711.9	5.3	.121	358.9	10.7	.181	239.9	16.2	.241	180.1	22.2
.062	700.5	5.4	.122	356.0	10.8	.182	238.6	16.3	.242	179.3	22.3
.063	689.4	5.5	.123	353.1	10.8	.183	237.3	16.4	.243	178.6	22.4
.064	678.6	5.6	.124	350.2	10.9	.184	236.0	16.5	.244	177.9	22.5
.065	668.1	5.7	.125	347.4	11.0	.185	234.7	16.6	.245	177.2	22.6
.066	658.0	5.8	.126	344.7	11.1	.186	233.4	16.7	.246	176.5	22.8
.067	648.2	5.9	.127	342.0	11.2	.187	232.2	16.8	.247	175.8	22.9
.068	638.7	6.0	.128	339.3	11.3	.188	230.9	16.9	.248	175.1	23.0
.069	629.4	6.0	.129	336.7	11.4	.189	229.7	17.0	.249	174.4	23.1
.070	620.4	6.1	.130	334.1	11.5	.190	228.5	17.1	.250	173.7	23.2
.071	611.7	6.2	.131	331.5	11.6	.191	227.3	17.2	.251	173.0	23.3
.072	603.2	6.3	.132	329.0	11.6	.192	226.1	17.3	.252	172.3	23.4
.073	594.9	6.4	.133	326.5	11.7	.193	224.9	17.4	.253	171.6	23.5
.074	586.9	6.5	.134	324.1	11.8	.194	223.8	17.5	.254	170.9	23.6
.075	579.1	6.6	.135	321.7	11.9	.195	222.7	17.6	.255	170.2	23.8
.076	571.4	6.7	.136	319.3	12.0	.196	221.6	17.7	.256	169.6	23.9
.077	564.0	6.8	.137	317.0	12.1	.197	220.5	17.8	.257	168.9	24.0
.078	556.8	6.8	.138	314.7	12.2	.198	219.4	17.9	.258	168.2	24.1
.079	549.7	6.9	.139	312.4	12.3	.199	218.3	18.0	.259	167.6	24.2
.080	542.9	7.0	.140	310.2	12.4	.200	217.2	18.1	.260	166.9	24.3
.081	536.1	7.0	.141	308.0	12.4	.201	216.1	18.2	.261	166.3	24.4
.082	529.6	7.1	.142	305.8	12.5	.202	215.0	18.3	.262	165.7	24.5
.083	523.2	7.2	.143	303.7	12.6	.203	213.9	18.4	.263	165.1	24.6
.084	517.0	7.3	.144	301.6	12.7	.204	212.8	18.5	.264	164.5	24.8
.085	510.9	7.4	.145	299.5	12.8	.205	211.8	18.6	.265	163.9	24.9
.086	505.0	7.5	.146	297.4	12.9	.206	210.7	18.7	.266	163.3	25.0
.087	499.2	7.5	.147	295.4	13.0	.207	209.7	18.8	.267	162.7	25.1
.088	493.5	7.6	.148	293.4	13.1	.208	208.7	18.9	.268	162.1	25.2
.089	488.0	7.7	.149	291.4	13.2	.209	207.7	19.0	.269	161.5	25.3
.090	482.5	7.8	.150	289.5	13.3	.210	206.7	19.1	.270	160.9	25.4

α	$\frac{M\Delta a}{a}$	$2\beta^2\frac{M\Delta a}{a}$	α	$\frac{M\Delta a}{a}$	$2\beta^2\frac{M\Delta a}{a}$	α	$\frac{M\Delta a}{a}$	$2\beta^2\frac{M\Delta a}{a}$	α	$\frac{M\Delta a}{a}$	$2\beta^2\frac{M\Delta a}{a}$
.270	160.9	25.4	.330	131.6	32.1	.390	111.3	39.9	.450	96.4	49.0
.271	160.3	25.5	.331	131.2	32.2	.391	111.1	40.0	.451	96.2	49.1
.272	159.7	25.6	.332	130.8	32.3	.392	110.8	40.1	.452	96.0	49.3
.273	159.1	25.7	.333	130.4	32.5	.393	110.5	40.3	.453	95.8	49.5
.274	158.5	25.8	.334	130.0	32.6	.394	110.2	40.4	.454	95.6	49.6
.275	157.9	25.9	.335	129.6	32.8	.395	109.9	40.6	.455	95.4	49.8
.276	157.3	26.0	.336	129.2	32.9	.396	109.6	40.7	.456	95.2	50.0
.277	156.8	26.1	.337	128.9	33.0	.397	109.4	40.8	.457	95.0	50.1
.278	156.2	26.2	.338	128.5	33.1	.398	109.1	41.0	.458	94.8	50.3
.279	155.6	26.3	.339	128.1	33.2	.399	108.8	41.2	.459	94.6	50.5
.280	155.1	26.4	.340	127.7	33.3	.400	108.6	41.3	.460	94.4	50.6
.281	154.5	26.5	.341	127.3	33.4	.401	108.3	41.4	.461	94.2	50.8
.282	154.0	26.6	.342	126.9	33.6	.402	108.0	41.6	.462	94.0	51.0
.283	153.5	26.7	.343	126.6	33.7	.403	107.8	41.7	.463	93.8	51.1
.284	152.9	26.8	.344	126.2	33.9	.404	107.5	41.9	.464	93.6	51.3
.285	152.4	26.9	.345	125.8	34.0	.405	107.2	41.0	.465	93.4	51.5
.286	151.9	27.0	.346	125.5	34.1	.406	107.0	42.1	.466	93.2	51.6
.287	151.3	27.1	.347	125.1	34.2	.407	106.7	42.3	.467	93.0	51.8
.288	150.8	27.2	.348	124.7	34.3	.408	106.4	42.5	.468	92.8	52.0
.289	150.3	27.3	.349	124.4	34.4	.409	106.2	42.6	.469	92.6	52.2
.290	149.7	27.4	.350	124.0	34.5	.410	105.9	42.7	.470	92.4	52.4
.291	149.2	27.6	.351	123.6	34.7	.411	105.6	42.9	.471	92.2	52.6
.292	148.7	27.7	.352	123.3	34.8	.412	105.4	43.0	.472	92.0	52.7
.293	148.2	27.8	.353	122.9	35.0	.413	105.1	43.1	.473	91.8	52.9
.294	147.7	27.9	.354	122.6	35.1	.414	104.8	43.3	.474	91.6	53.1
.295	147.2	28.0	.355	122.3	35.2	.415	104.6	43.4	.475	91.4	53.2
.296	146.7	28.1	.356	121.9	35.3	.416	104.3	43.6	.476	91.2	53.4
.297	146.2	28.2	.357	121.6	35.5	.417	104.1	43.8	.477	91.0	53.6
.298	145.7	28.3	.358	121.3	35.6	.418	103.9	43.9	.478	90.8	53.8
.299	145.2	28.4	.359	120.9	35.8	.419	103.6	44.0	.479	90.6	54.0
.300	144.7	28.6	.360	120.6	35.9	.420	103.3	44.2	.480	90.4	54.2
.301	144.3	28.7	.361	120.3	36.0	.421	103.1	44.3	.481	90.3	54.3
.302	143.8	28.8	.362	119.9	36.1	.422	102.8	44.5	.482	90.1	54.5
.303	143.3	28.9	.363	119.6	36.3	.423	102.6	44.7	.483	89.9	54.7
.304	142.9	29.0	.364	119.3	36.5	.424	102.4	44.8	.484	89.7	54.8
.305	142.4	29.1	.365	118.9	36.6	.425	102.1	45.0	.485	89.5	55.0
.306	141.9	29.2	.366	118.6	36.7	.426	101.9	45.2	.486	89.3	55.2
.307	141.5	29.3	.367	118.3	36.8	.427	101.7	45.3	.487	89.1	55.4
.308	141.0	29.4	.368	117.9	36.9	.428	101.4	45.5	.488	88.9	55.6
.309	140.5	29.6	.369	117.6	37.1	.429	101.2	45.6	.489	88.7	55.8
.310	140.1	29.7	.370	117.3	37.2	.430	101.0	45.7	.490	88.6	55.9
.311	139.6	29.8	.371	117.0	37.4	.431	100.7	45.9	.491	88.4	56.1
.312	139.1	29.9	.372	116.7	37.5	.432	100.5	46.1	.492	88.2	56.3
.313	138.7	30.0	.373	116.4	37.6	.433	100.3	46.2	.493	88.0	56.5
.314	138.2	30.1	.374	116.1	37.7	.434	100.0	46.4	.494	87.8	56.7
.315	137.8	30.3	.375	115.8	37.9	.435	99.8	46.6	.495	87.6	56.9
.316	137.4	30.4	.376	115.5	38.0	.436	99.6	46.7	.496	87.5	57.1
.317	136.9	30.6	.377	115.2	38.2	.437	99.3	46.9	.497	87.3	57.3
.318	136.5	30.7	.378	114.9	38.3	.438	99.1	47.0	.498	87.1	57.5
.319	136.1	30.8	.379	114.6	38.4	.439	98.9	47.1	.499	87.0	57.7
.320	135.6	30 9	.380	114.3	38.5	.440	98.6	47.3	.500	86.8	57.9
.321	135.2	31.0	.381	114.0	38.7	.441	98.4	47.5	.501	86.6	58.1
.322	134.8	31.1	.382	113.7	38.8	.442	98.2	47.6	.502	86.5	58.3
.323	134.4	31.2	.383	113.4	39.0	.443	97.9	47.8	.503	86.3	58.5
.324	134.0	31.4	.384	113.1	39.1	.444	97.7	48.0	.504	86.1	58.6
.325	133.6	31.5	.385	112.8	39.2	.445	97.5	48.1	.505	86.0	58.8
.326	133.2	31.6	.386	112.5	39.4	.446	97.2	48.3	.506	85.8	59.1
.327	132.8	31.8	.387	112.2	39.5	.447	97.0	48.5	.507	85.6	59.3
.328	132.4	31.9	.388	111.9	39.6	.448	96.8	48.6	.508	85.5	59.5
.329	132.0	32.0	.389	111.6	39.8	.449	96.6	48.8	.509	85.3	59.7
.330	131.6	32.1	.390	111.3	39.9	.450	96.4	49.0	.510	85.1	59.9

α	$\frac{M\Delta a}{a}$	$2\beta^2\frac{M\Delta a}{a}$	α	$\frac{M\Delta a}{a}$	$2\beta^2\frac{M\Delta a}{a}$	α	$\frac{M\Delta a}{a}$	$2\beta^2\frac{M\Delta a}{a}$	α	$\frac{M\Delta a}{a}$	$2\beta^2\frac{M\Delta a}{a}$
.510	85.1	59.9	.570	76.2	73.3	.630	68.9	90.8	.690	63.0	114.5
.511	85.0	60.1	.571	76.1	73.5	.631	68.8	91.1	.691	62.9	114.9
.512	84.8	60.3	.572	75.9	73.8	.632	68.7	91.4	.692	62.8	115.4
.513	84.6	60.5	.573	75.8	74.1	.633	68.6	91.8	.693	62.7	115.9
.514	84.5	60.7	.574	75.7	74.3	.634	68.5	92.6	.694	62.6	116.3
.515	84.3	60.9	.575	75.5	74.6	.635	68.3	92.4	.695	62.5	116.8
.516	84.1	61.1	.576	75.4	74.9	.636	68.2	92.8	.696	62.4	117.3
.517	84.0	61.3	.577	75.3	75.1	.637	68.1	93.1	.697	62.4	117.7
.518	83.8	61.5	.578	75.1	75.4	.638	68.0	93.4	.698	62.3	118.2
.519	83.6	61.7	.579	75.0	75.7	.639	67.9	93.8	.699	62.2	118.7
.520	83.5	61.9	.580	74.9	75.9	.640	67.8	94.1	.700	62.1	119.2
.521	83.3	62.1	.581	74.7	76.2	.641	67.7	94.5	.701	62.0	119.7
.522	83.1	62.3	.582	74.6	76.5	.642	67.6	94.9	.702	61.9	120.2
.523	83.0	62.5	.583	74.5	76.7	.643	67.5	95.2	.703	61.8	120.7
.524	82.8	62.7	.584	74.3	77.0	.644	67.4	95.5	.704	61 7	121.2
.525	82.7	62.9	.585	74.2	77.3	.645	67.3	95.9	.705	61.6	121.7
.526	82.6	63.1	.586	74.1	77.5	.646	67.2	96.2	.706	61.6	122.2
.527	82.4	63.3	.587	73.9	77.8	.647	67.1	96.6	.707	61.5	122.7
.528	82.2	63.5	.588	73.8	78.1	.648	67.0	97.0	.708	61.4	123.3
.529	82.1	63.7	.589	73.7	78.3	.649	66.9	97.3	.709	61.3	123.8
.530	81.9	63.9	.590	73.5	78.6	.650	66.8	97.7	.710	61.2	124.3
.531	81.7	64.2	.591	73.4	78.9	.651	66.7	98.1	.711	61.1	124.9
.532	81.6	64.4	.592	73.3	79.1	.652	66.6	98.4	.712	61.0	125.4
.533	81.4	64.6	.593	73.1	79.4	.653	66.5	98.8	.713	60.9	125.9
.534	81.3	64.9	.594	73.0	79.7	.654	66.4	99.2	.714	60.8	126.5
.535	81.2	65.1	.595	72.9	80.0	.655	66.3	99.6	.715	60.8	127.0
.536	81.0	65.3	.596	72.8	80.3	.656	66.2	100.0	.716	60.7	127.6
.537	80.8	65.5	.597	72.7	80.6	.657	66.1	100.4	.717	60.6	128.2
.538	80.7	65.7	.598	72.6	80.9	.658	66.0	100.8	.718	60.5	128.7
.539	80.5	66.0	.599	72.4	81.2	.659	65.9	101.2	.719	60.4	129.3
.540	80.3	66.2	.600	72.3	81.4	.660	65.8	101.6	.720	60.3	129.9
.541	80.2	66.5	.601	72.2	81.7	.661	65.7	102.0	.721	60.2	130.4
.542	80.0	66.7	.602	72.0	82.0	.662	65.6	102.4	.722	60.1	131.0
.543	79.9	66.9	.603	71.9	82.3	.663	65.5	102.8	.723	60.0	131.6
.544	79.8	67.1	.604	71.8	82.6	.664	65.4	103.2	.724	60.0	132.2
.545	79.6	67.3	.605	71.7	82.9	.665	65.3	103.6	.725	59.9	132.8
.546	79.4	67.6	.606	71.6	83.2	.666	65.2	104.0	.726	59.8	133.4
.547	79.3	67 8	.607	71.5	83.5	.667	65.1	104.4	.727	59.7	134.0
.548	79.1	68.0	.608	71.3	83.8	.668	65.0	104.8	.728	59.6	134.6
.549	79.0	68.3	.609	71.2	84.0	.669	64.9	105.3	.729	59.5	135.2
.550	78.9	68.5	.610	71.1	84.2	.670	64.8	105.7	.730	59.4	135.8
.551	78.7	68.7	.611	71.0	84.5	.671	64.7	106.1	.731	59.3	136.4
.552	78.6	69.0	.612	70.9	84.8	.672	64.6	106.5	.732	59.2	137.0
.553	78.5	69.2	.613	70.8	85.1	.673	64.5	106.9	.733	59.2	137.6
.554	78.3	69.5	.614	70.7	85.4	.674	64.4	107.3	.734	59.1	138.2
.555	78.2	69.7	.615	70.6	85.7	.675	64.3	107.8	.735	59.0	138.9
.556	78.1	69.9	.616	70.5	86.0	.676	64.2	108.2	.736	58.9	139.5
.557	77.9	70.2	.617	70.3	86.3	.677	64.1	108.6	.737	58.8	140.1
.558	77.8	70 4	.618	70.2	86 7	.678	64.0	109.1	.738	58.7	140.8
.559	77.7	70.6	.619	70.1	87.0	.679	64.0	109.5	.739	58.6	141.4
.560	77.5	70.8	.620	70.0	87.3	.680	63.9	109.9	.740	58.5	142.1
.561	77.4	71.1	.621	69.9	87.7	.681	63.8	110.4	.741	58.5	142.8
.562	77.3	71 3	.622	69.8	88.0	.682	63.7	110.8	.742	58.5	143.4
.563	77.1	71.6	.623	69.7	88.4	.683	63.6	111.2	.743	58.4	144.1
.564	77.0	71.8	.624	69.6	88.8	.684	63.5	111.7	.744	58.3	144.8
.565	76.9	72.0	.625	69.5	89.1	.685	63.4	112.1	.745	58.2	145.4
.566	76.7	72.2	.626	69.3	89.4	.686	63.3	112.6	.746	58.1	146.1
.567	76.6	72.5	.627	69.2	89.8	.687	63.2	113.1	.747	58.0	146.8
.568	76.5	72.7	.628	69.1	90.1	.688	63.2	113.5	.748	57.9	147.4
.569	76.3	73.0	.629	69.0	90.4	.689	63.1	114.0	.749	57.8	148.1
.570	76.2	73.3	.630	68.9	90.8	.690	63.0	114.5	.750	57.8	148.9

CERES AND MERCURY.

i	log $b_{\frac{1}{2}}^{(i)}$	log $\alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	log $\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	log $\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	log $\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30317	98.30032	98.31934	97.44036	97.49643
1	99.14855	99.15497	97.80605	97.84066	97.21934
2	98.16931	98.47392	98.48814	97.48451	97.53778
3	97.23565	97.71528	98.02259	98.04729	97.28313

i	log $\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 b_{\frac{5}{2}}^{(i)}$
0	98.46562	98.41702	98.46794	98.64467
1	98.78387	98.81518	98.15114	
2	98.02506	98.34098	98.39738	97.55551

CERES AND VENUS.

i	log $b_{\frac{1}{2}}^{(i)}$	log $\alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	log $\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	log $\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	log $\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30866	98.86817	98.93419	98.60257	98.77923
1	99.42831	99.45131	98.66661	98.78204	98.68770
2	98.72151	99.03548	99.08592	98.64286	98.81241
3	98.05983	98.54607	98.86992	98.95594	98.74416

i	log $\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.78661	99.30053	99.46371	99.31967
1	99.36843	99.47402	99.34309	
2	98.87840	99.23198	99.40891	98.72471

CERES AND THE EARTH.

i	log $b_{\frac{1}{2}}^{(i)}$	log $\alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	log $\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	log $\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	log $\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.31606	99.18307	99.30863	99.26745	99.57212
1	99.58006	99.62555	99.15090	99.36112	99.53802
2	99.01518	99.34214	99.44058	99.30306	99.59821
3	98.49481	98.99028	99.33731	99.50149	99.58608

i	log $\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.99388	99.81579	0.10171	99.77113
1	99.70620	99.90075	0.04682	
2	99.35318	99.75615	0.06022	99.40046

CERES AND MARS.

i	log $b_{\frac{1}{2}}^{(i)}$	log $\alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	log $\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	log $\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	log $\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.33926	99.65530	99.94784	0.31880	0.90544
1	99.79764	99.91499	99.89782	0.34400	0.90318
2	99.41961	99.79015	0.03469	0.34294	0.91998
3	99.08414	99.61133	0.03771	0.42177	0.93272

i	log $\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.38646	0.66077	1.22618	0.63611
1	0.25313	0.67615	1.21425	
2	0.07178	0.62218	1.20498	0.49583

CERES AND JUPITER.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.33647	99.61422	99.88792	0.22399	0.78240
1	99.77938	99.88798	99.83147	0.25264	0.77910
2	99.38678	99.75185	99.97913	0.24928	0.79789
3	99.03698	99.56013	99.97645	0.33576	0.81039
4	98.70714	99.34528	99.89714	0.37783	0.86597
5	98.38892	99.11660	99.77413	0.36276	0.91738

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.34707	0.58255	1.11957	0.54981
1	0.20283	0.60124	1.10580	
2	0.00871	0.54188	1.09679	0.39499
3	99.79282	0.43268	1.06126	

CERES AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.31054	98.96893	99.05044	98.81306	99.02522
1	99.47725	99.50595	98.82077	98.96160	98.95587
2	98.81690	99.13413	99.19682	98.85210	99.05628
3	98.20150	98.69005	99.01969	99.12600	99.01005
4	97.59598	97.89490	98.71290	99.05833	

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.84964	99.46254	99.65943	99.45585
1	99.47496	99.60419	99.56356	
2	99.03012	99.39638	99.60864	98.93780
3	98.55765	98.92597	99.47918	

CERES AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30331	98.32939	98.34970	97.49922	97.55890
1	99.16298	99.16984	97.84989	97.88680	97.29334
2	98.19797	98.50283	98.51802	97.54326	97.59998
3	97.27853	97.75834	98.06608	98.09245	97.35693

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.48114	98.46148	98.51570	98.67667
1	98.81339	98.84678	98.21080	
2	98.06871	98.38565	98.44566	97.61455

CERES AND NEPTUNE.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30196	97.93382	97.94212	96.70148	96.72657
1	98.96622	98.96900	97.25439	97.26964	96.29212
2	97.80626	98.10884	98.11501	97.74653	96.77030
3	96.69198	97.17019	97.47393	97.48469	96.35768

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.27420	97.86031	97.88298	98.25376
1	98.41373	98.42748	97.40558	
2	97.47500	97.78251	97.80777	96.81523

PALLAS AND MERCURY.

The values of a for Pallas and the principal Planets are so near the corresponding ones for Ceres, that the coefficients for Pallas are readily obtained from those for Ceres by means of the formulas and tables on pages 63 – 67.

The inclination and eccentricity of Pallas are so great, however, that its perturbations cannot be obtained to any great degree of approximation through the usual development without a disproportionate amount of labor. The values of $b_s^{(i)}$ and its derivatives for Pallas will, therefore, probably never be needed for the computation of its perturbations; and especially for the action of Jupiter and Saturn.

JUNO AND MERCURY.

The inclination of Juno is 13°, and the eccentricity is 0.27. Both of these elements are large; and the eccentricity especially is too great to render it necessary to give the coefficients for this Planet. If, however, all or any of them shall ever be needed, they may readily be found from the corresponding ones for Proserpine given hereafter.

To illustrate the use of the formulas for computing the variations of the coefficients corresponding to a change in the argument a, we will find the value of $\log a^4 D_\alpha^4 b_{\frac{1}{2}}^{(2)}$ for Juno and Jupiter from the value of the same coefficient for Proserpine and Jupiter.

The formula is

$$\Delta \log a^4 D_\alpha^4 b_{\frac{1}{2}}^{(2)} = E_2 + (4 + 8\,\beta^2)\,\frac{M \Delta a}{a}.$$

For Proserpine and Jupiter,

$$a = 0.5104402.$$

For Juno and Jupiter,

$$a = 0.5129176.$$

$$\delta a = \quad 2.4774.$$

With $a = 0.5116790$, the mean of the above values, which must be used when we wish to take second differences into account, find from the Tables, pp. 25 and 67,

$$\log E_2 = 2.0408, \qquad \frac{M \Delta a}{a} = 84.9, \qquad 2\,\beta^2\,\frac{M \Delta a}{a} = 60.2.$$

Then,

$$E_2\,\delta a = + 272.1, \qquad \frac{M \Delta a}{a}\,\delta a = 210.3, \qquad 2\,\beta^2\,\frac{M \Delta a}{a}\,\delta a = 149.1.$$

Hence,

$$\Delta \log a^4 D_\alpha^4 b_{\frac{1}{2}}^{(2)} = + 272.1 + 4 \times 210.3 + 4 \times 149.1 = 1709.7.$$

From the general Tables we find for Proserpine and Jupiter,

$$\log a^4 D_\alpha^4 b_{\frac{1}{2}}^{(2)} = 0.6459610,$$

For Juno and Jupiter,

$$\log a^4 D_\alpha^4 b_{\frac{1}{2}}^{(2)} = 0.6630584,$$

$$\Delta \log a^4 D_\alpha^4 b_{\frac{1}{2}}^{(2)} = \quad 1709.74.$$

Products of 210.3 and 149.1 by 1, 2, 3, 4, &c. give the corrections corresponding to these terms for all the coefficients.

VESTA AND MERCURY.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.30398	98.44290	98.46907	97.72948	97.80538
1	99.21922	99.22810	98.02122	98.06852	97.58314
2	98.30958	98.61556	98.63519	97.77304	97.84527
3	97.44544	97.92604	98.23577	98.26976	97.64578

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.54242	98.65560	98.70472	98.80352
1	98.92892	98.97179	98.44473	
2	98.23915	98.56072	98.63700	97.84562

VESTA AND VENUS.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.31167	99.02069	99.11137	98.92194	99.15432
1	99.50227	99.53438	98.90023	99.05583	99.09495
2	98.86513	99.18463	99.25466	98.96026	99.18423
3	98.27346	98.76341	99.09655	99.21489	99.14781

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.88312	99.54672	99.76291	99.52861
1	99.53019	99.67325	99.67839	
2	99.10813	99.48194	99.71429	99.04830

VESTA AND THE EARTH.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.32224	99.35018	99.52280	99.63057	0.02356
1	99.65878	99.72321	99.41169	99.69217	0.00669
2	99.16442	99.50256	99.64054	99.66268	0.04549
3	98.71404	99.21755	99.58475	99.81091	0.04882

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	0.11907	0.10255	0.47486	0.04725
1	99.89277	0.15441	0.44158	
2	99.60619	0.04944	0.44090	99.77389

VESTA AND MARS.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.35757	99.88016	0.29046	0.85080	1.60134
1	99.89462	0.07029	0.26679	0.86281	1.60200
2	99.58946	99.99762	0.35366	0.86847	1.61116
3	99.32531	99.88072	0.37657	0.91340	1.62232

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	0.62323	1.10845	1.83985	1.14968
1	0.54028	1.11221	1.83444	
2	0.42006	1.08114	1.82625	1.07195

VESTA AND JUPITER.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.32570	99.42566	99.62370	99.79754	0.23479
1	99.69378	99.76887	99.53061	99.84765	0.22325
2	99.22984	99.57437	99.73427	99.82785	0.25485
3	98.80959	99.31773	99.69651	99.95619	0.26241
4	98.40969	99.03594	99.56268	99.98494	0.34853
5	98.02159	98.74042	99.37837		

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.18011	0.23606	0.65211	0.18211
1	99.97927	0.27617	0.62600	
2	99.72084	0.18628	0.62148	99.94711
3	99.43830	0.02802	0.56838	

VESTA AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30786	98.81790	98.87724	98.49819	98.65898
1	99.40378	99.42437	98.58992	98.69423	98.55504
2	98.67352	98.98612	99.03131	98.53902	98.69314
3	97.98845	98.47371	98.79512	98.87240	98.61250
4	97.32433	97.93269	98.42250	98.75513	

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.75608	99.22050	99.36870	99.25415
1	99.31571	99.41098	99.23443	
2	98.80275	99.15091	99.31202	98.61926
3	98.26151	99.98244	99.14386	

VESTA AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30268	98.18765	98.20241	97.21267	97.25611
1	99.09258	99.09753	97.63628	97.66324	96.93329
2	98.05802	98.36182	98.37283	97.25718	97.29888
3	97.06907	99.54813	97.85402	97.87317	96.99779

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.40602	98.21525	98.28506	98.52230
1	98.66975	98.69411	97.92079	
2	97.85593	98.16850	98.21271	97.32726

VESTA AND NEPTUNE.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30170	97.79377	97.79980	96.42014	96.43847
1	98.89638	98.89840	97.04391	97.05501	95.93968
2	97.66693	97.96908	97.97356	96.46538	96.48274
3	96.48318	96.96109	97.26408	97.27190	96.00561

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.20246	97.64877	97.66532	98.10854
1	98.27291	98.28292	97.12283	
2	97.26486	97.57059	97.58906	96.53359

ASTRÆA AND MERCURY.

i	log $b^{(i)}_{\frac{1}{2}}$	log $\alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.30351	98.36485	98.38684	97.57107	97.63543
1	99.18056	99.18800	97.90339	97.94328	97.38372
2	98.23289	98.53807	98.55452	97.61497	97.67617
3	97.33076	97.81080	98.11911	98.14766	97.44703

i	log $\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.50016	98.51577	98.57429	98.71595
1	98.84943	98.88546	98.28370	
2	98.12195	98.44021	98.50493	97.68663

ASTRÆA AND VENUS.

i	log $b^{(i)}_{\frac{1}{2}}$	log $\alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.30990	98.93678	99.01303	98.74570	98.94599
1	99.46167	99.48842	98.77152	98.90377	98.86996
2	98.78659	99.10271	99.16121	98.78517	98.97775
3	98.15652	98.64428	98.97193	99.07134	98.92491

i	log $\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.82924	99.41060	99.59619	99.41166
1	99.44084	99.56207	99.49280	
2	98.98170	99.34361	99.54409	98.86954

ASTRÆA AND THE EARTH.

i	log $b^{(i)}_{\frac{1}{2}}$	log $\alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.31857	99.25758	99.40262	99.42831	99.77067
1	99.61535	99.66855	99.26679	99.50653	99.74525
2	99.08242	99.41391	99.52854	99.46245	99.79491
3	98.59377	99.09249	99.44766	99.63748	99.79079

i	log $\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^2 b^{(i)}_{\frac{5}{2}}$
0	0.04813	99.94227	0.26489	99.89063
1	99.78861	0.01125	0.22068	
2	99.46587	99.88538	0.22676	99.56540

ASTRÆA AND MARS.

i	log $b^{(i)}_{\frac{1}{2}}$	log $\alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.34652	99.75234	0.09270	0.54564	1.20115
1	99.84012	99.98042	0.05580	0.56406	1.20064
2	99.49501	99.88011	0.16920	0.56699	1.21354
3	99.19183	99.72986	0.18309	0.62974	1.22609

i	log $\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^2 b^{(i)}_{\frac{5}{2}}$
0	0.48415	0.84980	1.48475	0.84911
1	0.37451	0.85893	1.47627	
2	0.22138	0.81613	1.46706	0.73925

ASTRÆA AND JUPITER.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.33103	99.52609	99.76223	0.02294	0.52295
1	99.73973	99.83136	99.69014	0.06042	0.51657
2	99.31466	99.66924	99.86269	0.05066	0.54054
3	98.93285	99.44812	99.84542	0.15531	0.55160
4	98.57116	99.20322	99.74221	0.19299	0.62018
5	98.22121	98.94486	99.59177		

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.26631	0.41815	0.89666	0.37227
1	0.09692	0.44552	0.87799	
2	99.87387	0.37311	0.87032	0.18275
3	99.62789	0.24236	0.82733	

ASTRÆA AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30921	98.89990	98.97047	98.66865	98.85595
1	99.44375	99.46842	98.71508	98.83805	98.77179
2	98.75166	99.06659	99.12061	98.70856	98.88848
3	98.10465	98.59156	98.91711	99.00910	98.82757
4	97.47855	98.08819	98.58060	98.91965	

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.80617	99.35130	99.52455	99.36182
1	99.40184	99.51443	99.41209	
2	98.92615	99.28345	99.47098	98.79154
3	98.42252	98.93664	99.32470	

ASTRÆA AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30300	98.26497	98.28254	97.36887	97.42082
1	99.13100	99.13694	97.75277	97.78178	97.12949
2	98.13444	98.43877	98.45190	97.41314	97.46250
3	97.18346	97.66290	97.96972	97.99253	97.19352

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.44683	98.36306	98.41021	98.60602
1	98.74803	98.77698	98.07874	
2	97.97200	98.28678	98.33906	97.48383

ASTRÆA AND NEPTUNE.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30183	97.87024	97.87742	96.57370	96.59547
1	98.93452	98.93692	97.15882	97.17202	96.13202
2	97.74303	98.04540	98.05074	96.61885	96.63946
3	96.59724	97.07530	97.37867	97.38798	96.19777

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.24156	97.76422	97.78387	98.18762
1	98.34978	98.36168	97.27711	
2	97.37960	97.68623	97.70814	96.68731

HEBE AND MERCURY.

The inclination of Hebe is 14° 47′, and its eccentricity is 0.202. Both of these elements are so large that the coefficients for this Planet need not be given. They may, however, if needed, be readily found from the corresponding ones for Lutetia and Mercury, Lutetia and Venus, &c., by means of the variation formulas, as the values of δa are all less than 0.0025.

IRIS AND MERCURY.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log a\, D_a\, b^{(i)}_{\frac{1}{2}}$	$\log a^2\, D^2_a\, b^{(i)}_{\frac{1}{2}}$	$\log a^3\, D^3_a\, b^{(i)}_{\frac{1}{2}}$	$\log a^4\, D^4_a\, b^{(i)}_{\frac{1}{2}}$
0	0.30392	98.43330	98.45892	97.70999	97.78437
1	99.21448	99.22316	98.00673	98.05306	97.55858
2	98.30016	98.60604	98.62525	97.75359	97.82437
3	97.43136	97.91189	98.21143	98.25470	97.62131

i	$\log a\, b^{(i)}_{\frac{3}{2}}$	$\log a^2\, D_a\, b^{(i)}_{\frac{3}{2}}$	$\log a^3\, D^2_a\, b^{(i)}_{\frac{3}{2}}$	$\log a^2\, b^{(i)}_{\frac{5}{2}}$
0	99.53719	98.62084	98.68857	98.79267
1	98.91913	98.96111	98.42489	
2	98.22474	98.54588	98.62063	97.82604

IRIS AND VENUS.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log a\, D_a\, b^{(i)}_{\frac{1}{2}}$	$\log a^2\, D^2_a\, b^{(i)}_{\frac{1}{2}}$	$\log a^3\, D^3_a\, b^{(i)}_{\frac{1}{2}}$	$\log a^4\, D^4_a\, b^{(i)}_{\frac{1}{2}}$
0	0.31143	99.01032	99.09909	98.90007	99.12828
1	99.49726	99.52866	98.88428	99.03683	99.06699
2	98.85573	99.17452	99.24302	98.93854	99.15843
3	98.25909	98.74873	99.08115	99.19698	99.12013

i	$\log a\, b^{(i)}_{\frac{3}{2}}$	$\log a^2\, D_a\, b^{(i)}_{\frac{3}{2}}$	$\log a^3\, D^2_a\, b^{(i)}_{\frac{3}{2}}$	$\log a^2\, b^{(i)}_{\frac{5}{2}}$
0	99.87634	99.52979	99.74200	99.51386
1	99.51909	99.65928	99.65528	
2	99.09249	99.46472	99.69294	98.02609

IRIS AND THE EARTH.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log a\, D_a\, b^{(i)}_{\frac{1}{2}}$	$\log a^2\, D^2_a\, b^{(i)}_{\frac{1}{2}}$	$\log a^3\, D^3_a\, b^{(i)}_{\frac{1}{2}}$	$\log a^4\, D^4_a\, b^{(i)}_{\frac{1}{2}}$
0	0.32175	99.33863	99.50760	99.60519	99.99164
1	99.65339	99.71631	99.39355	99.66871	99.97383
2	99.15429	99.49153	99.62640	99.63757	0.01386
3	98.69922	99.20208	99.56765	99.78900	0.01640

i	$\log a\, b^{(i)}_{\frac{3}{2}}$	$\log a^2\, D_a\, b^{(i)}_{\frac{3}{2}}$	$\log a^3\, D^2_a\, b^{(i)}_{\frac{3}{2}}$	$\log a^2\, b^{(i)}_{\frac{5}{2}}$
0	0.10999	0.08236	0.44823	0.02720
1	99.87969	0.13619	0.41372	
2	99.58867	0.02875	0.41375	99.74765

IRIS AND MARS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.35605	99.86367	0.26451	0.81100	1.54903
1	99.88769	0.05845	0.23937	0.82371	1.54959
2	99.57759	99.98251	0.32939	0.82915	1.55916
3	99.30864	99.86159	0.35145	0.87670	1.57055

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.60459	1.07443	1.79311	1.10961
1	0.51849	1.07873	1.78737	
2	0.39430	1.04632	1.77901	1.02820

IRIS AND JUPITER.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.32629	99.43770	99.64004	99.82435	0.26890
1	99.69933	99.77625	99.54965	99.87279	0.25808
2	99.24014	99.58578	99.74943	99.85436	0.28865
3	98.82460	99.33354	99.71434	99.97970	0.29676
4	98.42939	99.05627	99.58434	0.00975	0.38061
5	98.04575	98.76532	99.40425		

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.19013	0.25760	0.68088	0.20423
1	99.99321	0.29601	0.65579	
2	99.73915	0.20837	0.65078	99.97502
3	99.46111	0.05356	0.59900	

IRIS AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30802	98.82790	98.88851	98.51892	98.68277
1	99.40867	99.42971	98.60516	98.71161	98.58137
2	98.68308	98.99595	99.04214	98.55965	98.71674
3	98.00268	98.48813	98.81001	98.88895	98.63864
4	97.34323	97.95173	98.44183	98.77518	

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.76211	99.23638	99.38747	99.26706
1	99.32618	99.42342	99.16598	
2	98.81780	99.16699	99.33116	98.64019
3	98.28117	98.78986	99.16577	

IRIS AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30272	98.19712	98.21220	97.23179	97.27665
1	99.09729	99.10237	97.65054	97.67808	96.95730
2	98.06739	98.37124	98.38250	97.27627	97.31885
3	97.08310	97.56220	97.86819	97.88776	97.02174

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.41100	98.25965	98.30031	98.53249
1	98.67933	98.70421	97.94011	
2	97.87014	98.18297	98.22810	97.34642

IRIS AND NEPTUNE.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30172	97.80315	97.80931	96.43896	96.45768
1	98.90106	98.90312	97.05799	97.06933	95.96324
2	97.67627	97.97844	97.98302	96.48419	96.50191
3	96.49717	96.97509	97.27813	97.28612	96.02916

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.20724	97.66292	97.67981	98.11821
1	98.28233	98.29256	97.14173	
2	97.27893	97.58476	97.60361	96.55243

FLORA AND MERCURY.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$
0	0.30443	98.50559	98.53565	97.85703	97.94351
1	99.25023	99.26046	98.11597	98.17014	97.74388
2	98.37099	98.67773	98.70033	97.90026	97.98264
3	97.53722	98.01835	98.32941	98.36850	97.80589

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.57679	98.73222	98.81110	98.87511
1	98.99296	99.04211	98.57473	
2	98.33330	98.65797	98.74485	97.97371

FLORA AND VENUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.31335	99.08897	99.19313	99.06645	99.32758
1	99.53512	99.57233	99.00536	99.18235	99.27993
2	98.92889	99.25104	99.33195	99.10371	99.35592
3	98.36740	98.85944	99.19785	99.33388	99.33090

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.92855	99.65888	99.90267	99.62780
1	99.60366	99.76681	99.83271	
2	99.21110	99.59610	99.85700	98.19529

FLORA AND THE EARTH.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.32576	99.42699	99.62550	99.80049	0.23854
1	99.69439	99.76968	99.53274	99.85041	0.22708
2	99.23098	99.57563	99.73594	99.83077	0.25856
3	98.81125	99.31947	99.69848	99.95877	0.26619

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.18121	0.23843	0.65527	0.18453
1	99.98080	0.27834	0.62927	
2	99.72286	0.18870	0.62470	99.95017

FLORA AND MARS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.36896	0.03121	0.54698	1.23993	2.11572
1	99.94198	0.19101	0.53182	1.24805	2.11674
2	99.66925	0.13864	0.59900	1.25445	2.12349
3	99.43658	0.04739	0.62561	1.28804	2.13287
i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$	
0	0.83115	1.45944	2.31773	1.58424	
1	0.76792	1.46044	2.31409		
2	0.67300	1.43746	2.30725	1.52857	

FLORA AND JUPITER.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.32218	99.34891	99.52112	99.62776	0.02002
1	99.65819	99.72245	99.40968	99.68957	0.00306
2	99.16331	99.50134	99.63898	99.65990	0.04386
3	98.71214	99.21584	99.58286	99.80849	0.04523
4	98.28198	98.90453	99.42355	99.82740	
5	97.86343	98.57920	99.21149		
i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$	
0	0.11806	0.10032	0.47191	0.04503	
1	99.89132	0.15239	0.43850		
2	99.60426	0.04715	0.43983	99.77099	
3	99.29229	99.86576	0.37570		

FLORA AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30695	98.75311	98.80476	98.36423	98.50628
1	99.37208	99.38991	98.49127	98.58264	98.38505
2	98.61134	98.92237	98.96157	98.40568	98.54164
3	97.89585	98.38001	98.69865	98.76588	98.44368
4	97.20136	97.80886	98.29693	98.62524	
i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$	
0	99.71753	99.11804	99.24864	99.17182	
1	99.24814	99.33144	99.09557		
2	98.70530	99.04724	99.18967	98.48409	
3	98.13396	98.63782	99.00276		

FLORA AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30247	98.12601	98.13885	97.08834	97.12673
1	99.06192	99.06624	97.54347	97.56697	96.77723
2	97.99699	98.30043	98.31000	97.13301	97.16943
3	96.97770	97.45650	97.76175	97.77841	96.84203
i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$	
0	99.37374	98.15153	98.18630	98.45628	
1	98.60745	98.62867	97.79526		
2	97.76341	98.07447	98.11312	97.20267	

FLORA AND NEPTUNE.

i	log $b_{\frac12}^{(i)}$	log $a D_a b_{\frac12}^{(i)}$	log $a^2 D_a^2 b_{\frac12}^{(i)}$	log $a^3 D_a^3 b_{\frac12}^{(i)}$	log $a^4 D_a^4 b_{\frac12}^{(i)}$
0	0.30161	97.73272	97.73796	96.29759	96.31357
1	98.86592	98.86767	96.95218	96.96176	95.78623
2	97.60612	97.90813	97.91202	96.34290	96.35802
3	96.39203	96.86963	97.17256	97.17937	95.85230

i	log $a b_{\frac32}^{(i)}$	log $a^2 D_a b_{\frac32}^{(i)}$	log $a^3 D_a^2 b_{\frac32}^{(i)}$	log $a^2 b_{\frac52}^{(i)}$
0	99.17133	97.55668	97.57110	98.04569
1	98.21160	98.22030	96.99980	
2	97.17325	97.47837	97.49446	96.41095

METIS AND MERCURY.

i	log $b_{\frac12}^{(i)}$	log $a D_a b_{\frac12}^{(i)}$	log $a^2 D_a^2 b_{\frac12}^{(i)}$	log $a^3 D_a^3 b_{\frac12}^{(i)}$	log $a^4 D_a^4 b_{\frac12}^{(i)}$
0	0.30392	98.43318	98.45879	97.70973	97.78410
1	99.21441	99.22310	98.00653	98.05285	97.55826
2	98.30004	98.60592	98.62512	97.75334	97.82410
3	97.43118	97.91187	98.22124	98.25451	97.62099

i	log $a b_{\frac32}^{(i)}$	log $a^2 D_a b_{\frac32}^{(i)}$	log $a^3 D_a^2 b_{\frac32}^{(i)}$	log $a^2 b_{\frac52}^{(i)}$
0	99.53712	98.62065	98.68836	98.79253
1	98.91900	98.96097	98.42463	
2	98.22455	98.54568	98.62042	97.82578

METIS AND VENUS.

i	log $b_{\frac12}^{(i)}$	log $a D_a b_{\frac12}^{(i)}$	log $a^2 D_a^2 b_{\frac12}^{(i)}$	log $a^3 D_a^3 b_{\frac12}^{(i)}$	log $a^4 D_a^4 b_{\frac12}^{(i)}$
0	0.31143	99.01018	99.09894	98.89979	99.12794
1	99.49720	99.52859	98.88408	99.03658	99.06663
2	98.85560	99.17439	99.24286	98.93826	99.15810
3	98.25890	98.74854	99.08095	99.19675	99.11977

i	log $a b_{\frac32}^{(i)}$	log $a^2 D_a b_{\frac32}^{(i)}$	log $a^3 D_a^2 b_{\frac32}^{(i)}$	log $a^2 b_{\frac52}^{(i)}$
0	99.87625	99.52957	99.74173	99.51367
1	99.51894	99.65910	99.65498	
2	99.09228	99.46450	96.69266	99.02580

METIS AND THE EARTH.

i	log $b_{\frac12}^{(i)}$	log $a D_a b_{\frac12}^{(i)}$	log $a^2 D_a^2 b_{\frac12}^{(i)}$	log $a^3 D_a^3 b_{\frac12}^{(i)}$	log $a^4 D_a^4 b_{\frac12}^{(i)}$
0	0.32174	99.33848	99.50740	99.60486	99.99122
1	99.65332	99.71622	99.39332	99.66840	99.97341
2	99.15416	99.49139	99.62622	99.63724	0.01345
3	98.69902	99.20188	99.56743	99.78872	0.01598

i	log $a b_{\frac32}^{(i)}$	log $a^2 D_a b_{\frac32}^{(i)}$	log $a^3 D_a^2 b_{\frac32}^{(i)}$	log $a^2 b_{\frac52}^{(i)}$
0	0.10988	0.08210	0.44788	0.02694
1	99.87991	0.13595	0.41336	
2	99.58844	0.02848	0.41340	99.74732

METIS AND MARS.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.35603	99.86346	0.26419	0.81051	1.54838
1	99.88760	0.05830	0.23903	0.82332	1.54894
2	99.57745	99.98233	0.32909	0.82866	1.55852
3	99.30843	99.86135	0.35114	0.87624	1.56991

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	0.60437	1.07401	1.79253	1.10912
1	0.51823	1.07832	1.78680	
2	0.39399	1.04589	1.77843	1.02767

METIS AND JUPITER.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.32630	99.43785	99.64025	99.82470	0.26934
1	99.69440	99.77635	99.54990	99.87312	0.25854
2	99.24028	99.58588	99.74963	99.85472	0.28910
3	98.82480	99.33375	99.71458	99.98000	0.29721
4	98.42965	99.05653	99.58462	0.01047	0.38104
5	98.04629	98.76565	99.40458		

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	0.19026	0.25788	0.68127	0.20452
1	99.99339	0.29627	0.65618	
2	99.73939	0.20866	0.65117	99.97538
3	99.46141	0.05389	0.59940	

METIS AND SATURN.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.30811	98.82803	98.88866	98.51919	98.68308
1	99.40873	99.42978	98.60536	98.71183	98.58171
2	98.68318	98.99608	99.04228	98.55991	98.72305
3	98.00286	98.48831	98.81020	98.88817	98.63898
4	97.34347	97.95198	98.44209	98.77544	

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.76218	99.23662	99.38771	99.26723
1	99.32631	99.42359	99.25626	
2	98.81799	99.16720	99.33140	98.64046
3	98.28143	98.79012	99.16606	

METIS AND URANUS.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.30272	98.19724	98.21233	97.23204	97.27691
1	99.09735	99.10243	97.65073	97.67827	96.95762
2	98.06751	98.37137	98.38262	97.27652	97.31911
3	97.08328	97.56239	97.86838	97.88796	97.02205

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.41107	98.25984	98.30051	98.53263
1	98.67945	98.70434	97.91036	
2	97.87033	98.18316	98.22830	97.34667

METIS AND NEPTUNE.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30172	97.80327	97.80943	96.43920	96.45793
1	98.90112	98.90318	97.05818	97.06952	95.96355
2	97.67638	97.97856	97.98314	96.48444	96.50217
3	96.49735	96.97527	97.27831	97.28630	96.02947

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.20730	97.66310	97.68001	98.11834
1	98.28246	98.29268	97.14198	
2	97.27911	97.58494	97.60381	96.55268

HYGEIA AND MERCURY.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30268	98.18663	98.20135	97.21061	97.25445
1	99.09207	99.09703	97.63474	97.66164	96.93071
2	98.05701	98.36080	98.37178	97.25513	97.29673
3	97.06756	97.54662	97.85249	97.87160	96.99520

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.40549	98.24368	98.28342	98.52120
1	98.66871	98.69302	97.91871	
2	97.85439	98.16694	98.21106	97.32520

HYGEIA AND VENUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30690	98.74868	98.79987	98.35509	98.49607
1	99.36993	99.38756	98.48453	98.57523	98.37348
2	98.60708	98.91799	98.95683	98.39658	98.53149
3	97.88950	98.37356	98.69202	98.75867	98.43218

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.71493	99.11105	99.24064	99.16632
1	99.24352	99.32613	99.08613	
2	98.69862	99.04019	99.18155	98.47485

HYGEIA AND THE EARTH.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.31249	99.05536	99.15267	98.99518	99.24185
1	99.51897	99.55358	98.95355	99.11974	99.18864
2	98.89773	99.21838	99.29375	99.03297	99.27097
3	98.32130	98.81227	99.14799	99.27504	99.24056

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.90599	99.60349	99.83339	99.57849
1	99.56740	99.72039	99.75595	
2	99.16039	99.53970	99.78625	99.12274

HYGEIA AND MARS.

i	log $b^{(i)}_{\frac{1}{2}}$	log $\alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.32950	99.49891	99.72425	99.96156	0.44417
1	99.72737	99.81424	99.64681	0.00217	0.43659
2	99.29197	99.64364	99.82750	99.99000	0.46242
3	98.89995	99.41328	99.80509	0.10074	0.47274

i	log $\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^2 b^{(i)}_{\frac{5}{2}}$
0	0.24239	0.36837	0.82948	0.31955
1	0.06477	0.39884	0.80902	
2	99.83240	0.32198	0.80200	0.11835

HYGEIA AND JUPITER.

i	log $b^{(i)}_{\frac{1}{2}}$	log $\alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.34917	99.78487	0.14229	0.62262	1.30187
1	99.85416	0.00288	0.10918	0.63916	1.30176
2	99.51958	99.91012	0.21536	0.64302	1.31358
3	99.22673	99.76883	0.23209	0.70099	1.32587
4	98.95340	99.60669	0.19103	0.74383	1.36192
5	98.69145	99.43279	0.11374	0.75242	1.40383

i	log $\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^2 b^{(i)}_{\frac{5}{2}}$
0	0.51840	0.91456	1.57357	1.92344
1	0.41604	0.92205	1.56601	
2	0.27176	0.88254	1.55698	0.82258
3	0.10828	0.80808	1.53281	

HYGEIA AND SATURN.

i	log $b^{(i)}_{\frac{1}{2}}$	log $\alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.31359	99.09372	99.19887	99.07653	99.33975
1	99.53857	99.57499	99.01268	99.19124	99.29285
2	98.93347	99.25565	99.33737	99.11372	99.36798
3	98.37409	98.86608	99.20488	99.34223	99.34369
4	97.83526	98.44901	98.94983	99.30957	

i	log $\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.93177	99.66671	99.91252	99.63484
1	99.58879	99.77342	99.84244	
2	99.21826	99.60409	99.86706	99.20556
3	98.80079	99.33430	99.76423	

HYGEIA AND URANUS.

i	log $b^{(i)}_{\frac{1}{2}}$	log $\alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.30399	98.44393	98.47016	97.73159	97.80764
1	99.21974	99.22864	98.02278	98.07019	97.58579
2	98.31059	98.61659	98.63626	99.77514	97.84752
3	97.44695	97.92756	98.23731	98.27138	97.64842

i	log $\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.54298	98.63719	98.70646	98.80470
1	98.92997	98.97294	98.44687	
2	98.24070	98.56232	98.63876	97.84773

HYGEIA AND NEPTUNE.

i	log $b^{(i)}_{\frac{1}{2}}$	log $\alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.30223	98.04654	98.05727	96.92824	96.96047
1	99.02237	99.02597	97.42390	97.44356	96.57640
2	97.91820	98.22124	98.22922	96.97309	97.00365
3	96.85969	97.33822	97.64276	97.65668	96.64155

i	log $\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.33240	98.03096	98.06011	98.37200
1	98.52727	98.54501	97.63385	
2	97.64415	97.95355	97.98600	97.04231

PARTHENOPE AND MERCURY.

i	log $b^{(i)}_{\frac{1}{2}}$	log $\alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.30377	98.40898	98.43225	97.66060	97.73127
1	99.20243	99.21066	97.97000	98.01394	97.49639
2	98.27628	98.58190	98.60008	97.70431	97.77154
3	97.39566	97.87600	98.18508	98.21660	97.55934

i	log $\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.52399	98.58347	98.64778	98.76527
1	98.89434	98.93414	98.37465	
2	98.18822	98.50828	98.57932	97.77646

PARTHENOPE AND VENUS.

i	log $b^{(i)}_{\frac{1}{2}}$	log $\alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.31086	98.98410	99.06820	98.84490	99.06284
1	99.48459	99.51425	98.84403	98.98906	98.99651
2	98.83115	99.14894	99.21370	98.88373	99.09357
3	98.22265	98.71158	99.04221	99.15193	99.05032

i	log $\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.85937	99.48713	99.68953	99.47696
1	99.49110	99.62426	99.59708	
2	99.05297	99.42136	99.63937	98.97009

PARTHENOPE AND THE EARTH.

i	log $b^{(i)}_{\frac{1}{2}}$	log $\alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	log $\alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.32056	99.30957	99.39762	99.54153	99.91180
1	99.63980	99.69906	99.34739	99.61007	99.89148
2	99.12868	99.46375	99.59104	99.57456	99.93476
3	98.66169	99.16298	99.52463	99.73423	99.93514

i	log $\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	log $\alpha^2 b^{(i)}_{\frac{5}{2}}$
0	0.08745	0.03181	0.38178	99.97741
1	99.84684	0.09080	0.32600	
2	99.54461	99.97699	0.34599	99.68195

PARTHENOPE AND MARS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.35241	99.82990	0.20092	0.71320	1.42061
1	99.87043	0.02950	0.17186	0.72777	1.42087
2	99.54785	99.94511	0.27000	0.73251	1.43154
3	99.26673	99.81389	0.28957	0.78524	1.44344

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.55943	0.99119	1.67876	1.01222
1	0.46516	0.99701	1.67215	
2	0.33081	0.96107	1.66341	0.92115

PARTHENOPE AND JUPITER.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.32787	99.46858	99.68228	99.89338	0.35693
1	99.71351	99.79532	99.59859	99.93774	0.34784
2	99.26641	99.61500	99.78860	99.92261	0.37592
3	98.86282	99.37391	99.76010	0.04044	0.38525
4	98.47951	99.10809	99.63972	0.07354	0.46351
5	98.10797	98.82872	99.47016		

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.21622	0.31321	0.75537	0.26181
1	0.02917	0.34748	0.73271	
2	99.78616	0.26542	0.72659	0.04703
3	99.51952	0.11929	0.67805	

PARTHENOPE AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30842	98.85334	98.91732	98.57210	98.74359
1	99.42108	99.44334	98.64397	98.75602	98.64849
2	98.70738	99.02094	99.06976	98.61219	98.77706
3	98.03882	98.52476	98.84786	98.93121	98.70525
4	97.39120	98.00008	98.49096	98.82622	

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.77754	99.27687	98.43551	99.30018
1	99.35285	99.45530	99.31095	
2	98.85608	99.20800	99.38015	98.69355
3	98.33117	98.84168	99.22170	

PARTHENOPE AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30282	98.22117	98.23709	97.28036	97.32762
1	99.10924	99.11461	97.28677	97.71582	97.01827
2	98.09117	98.39519	98.40706	97.32477	97.36964
3	97.11870	97.59791	97.90419	97.92485	97.08259

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.42367	98.29628	98.33914	98.55845
1	98.70366	98.72992	97.98920	
2	97.90625	98.21973	98.26728	97.39510

PARTHENOPE AND NEPTUNE.

i	log $b_{\frac{1}{2}}^{(i)}$	log $\alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	log $\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	log $\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	log $\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30176	97.82694	97.83345	96.48674	96.50650
1	98.91293	98.91511	97.09375	97.10572	96.02309
2	97.69995	98.00219	98.00702	96.53194	96.55064
3	96.53267	97.01064	97.31379	97.32222	96.08894

i	log $\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.21940	97.69883	97.65666	98.14279
1	98.30624	98.31704	97.18973	
2	97.31463	97.62073	97.64062	96.60026

CLIO AND MERCURY.

i	log $b_{\frac{1}{2}}^{(i)}$	log $\alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	log $\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	log $\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	log $\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30405	98.45280	98.47955	97.74960	97.82709
1	99.22413	99.23320	98.03618	98.08451	97.60849
2	98.31928	98.62539	98.64545	97.79311	97.86687
3	97.45995	97.94064	98.25057	98.28531	97.67104

i	log $\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.54781	98.65084	98.72142	98.81475
1	98.93902	98.98283	98.46522	
2	98.25402	98.57605	98.65392	97.86582

CLIO AND VENUS.

i	log $b_{\frac{1}{2}}^{(i)}$	log $\alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	log $\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	log $\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	log $\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.31192	99.03142	99.12411	98.94458	99.18131
1	99.50744	99.54031	98.91672	99.07554	99.12390
2	98.87545	99.19508	99.26673	98.98274	99.21098
3	98.28830	98.77855	99.11247	99.23345	99.17646

i	log $\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.89016	99.56425	99.78462	99.54394
1	99.54169	99.68785	99.70234	
2	99.12430	99.49978	99.73645	99.07130

CLIO AND THE EARTH.

i	log $b_{\frac{1}{2}}^{(i)}$	log $\alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	log $\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	log $\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	log $\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.32276	99.36216	99.53862	99.65693	0.05677
1	99.66436	99.73083	99.43050	99.71658	0.04084
2	99.17489	99.51398	99.65526	99.68877	0.07840
3	98.72936	99.23355	99.60248	99.83372	0.08250

i	log $\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.12855	0.12356	0.50262	0.06821
1	99.90639	0.17342	0.47057	
2	99.62436	0.07095	0.46919	99.80117

CLIO AND MARS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.35920	99.89764	0.31780	0.89264	1.65637
1	99.90185	0.08297	0.29560	0.90397	1.65712
2	99.60175	0.01344	0.37926	0.90983	1.66587
3	99.34261	99.90067	0.40297	0.95340	1.67678

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.64297	1.14431	1.88914	1.19210
1	0.56325	1.14755	1.88405	
2	0.44710	1.11784	1.87604	1.11808

CLIO AND JUPITER.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.32510	99.41333	99.60703	99.77012	0.19996
1	99.68809	99.76134	99.51113	99.82199	0.18764
2	99.21925	99.56264	99.71880	99.80074	0.22032
3	98.79414	99.30147	99.67824	99.93219	0.22730
4	98.38941	99.01502	99.54043	99.95955	0.31578
5	97.99650	98.71479	99.35177		

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.16993	0.21405	0.62276	0.15961
1	99.96503	0.25595	0.59558	
2	99.70209	0.16377	0.59160	99.91858
3	99.41490	0.00184	0.53712	

CLIO AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30771	98.80761	98.86565	98.47687	98.63455
1	99.39875	99.41887	98.57423	98.67638	98.52796
2	98.66367	98.97600	99.02018	98.51780	98.66890
3	97.97378	98.45885	98.77980	98.85539	98.58561
4	97.30486	97.91307	98.40259	98.73448	

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.74990	99.20417	99.34945	99.24090
1	99.30495	99.39822	99.21229	
2	98.78727	99.13438	99.29240	98.59773
3	98.24126	98.74857	99.12134	

CLIO AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30265	98.17788	98.19232	97.19297	97.24498
1	99.08773	99.09259	97.62158	97.64795	96.90855
2	98.04835	98.35209	98.36286	97.23750	97.27832
3	97.05461	97.55363	97.83940	97.85814	96.97310

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.40090	98.23038	98.26856	98.51180
1	98.65987	98.68360	97.90089	
2	97.84127	98.15359	98.19687	97.30751

CLIO AND NEPTUNE.

i	log $b_{\frac{1}{2}}^{(i)}$	log $\alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	log $\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	log $\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	log $\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30169	97.78411	97.79001	96.40073	96.41867
1	98.89156	98.89353	97.02939	97.04025	95.91537
2	97.65730	97.95943	97.96381	96.44599	96.46296
3	96.46875	96.94664	97.24959	97.25724	95.98134

i	log $\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.15752	97.63419	97.65038	98.09857
1	98.26320	98.27300	97.10335	
2	97.25036	97.55599	97.57405	96.51417

EGERIA AND MERCURY.

i	log $b_{\frac{1}{2}}^{(i)}$	log $\alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	log $\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	log $\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	log $\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30350	98.36460	98.38657	97.57056	97.63489
1	99.18044	99.18787	97.90301	97.94288	97.38308
2	98.23264	98.53782	98.55427	97.61447	97.67563
3	97.33039	98.81043	98.11873	98.14726	97.44639

i	log $\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.50003	98.51539	98.57387	98.71567
1	98.84917	98.88526	98.28318	
2	98.12157	98.43983	98.50451	97.68612

EGERIA AND VENUS.

i	log $b_{\frac{1}{2}}^{(i)}$	log $\alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	log $\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	log $\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	log $\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30989	98.93652	99.01272	98.74514	98.94532
1	99.46154	99.48827	98.77110	98.90329	98.86924
2	98.78633	99.10244	99.16092	98.78461	98.97711
3	98.15615	98.64389	98.97154	99.07089	98.92420

i	log $\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.82907	99.41017	99.59566	99.41129
1	99.44055	99.56172	99.49221	
2	98.98130	99.34319	99.54355	98.86898

EGERIA AND THE EARTH.

i	log $b_{\frac{1}{2}}^{(i)}$	log $\alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	log $\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	log $\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	log $\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.31874	99.25729	99.40224	99.42767	99.76988
1	99.61521	99.66838	99.26633	99.50595	99.74443
2	99.08215	99.41362	99.52819	99.46182	99.79413
3	98.59338	99.09209	99.44723	99.63694	99.78998

i	log $\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.04791	99.94177	0.26423	99.89015
1	99.78828	0.01080	0.21999	
2	99.46543	99.88486	0.22609	99.56475

EGERIA AND MARS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.34648	99.75195	0.09211	0.54472	1.19994
1	99.83995	99.98015	0.05516	0.56316	1.19943
2	99.49471	99.87975	0.16865	0.56608	1.21235
3	99.19141	99.72939	0.18250	0.62889	1.22490

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.48374	0.84903	1.48369	0.84823
1	0.37401	0.85818	1.47520	
2	0.22078	0.81534	1.46599	0.73825

EGERIA AND JUPITER.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.33105	99.52643	99.76270	0.02369	0.52392
1	99.73988	99.83157	99.69068	0.06114	0.51756
2	99.31494	99.66956	99.86313	0.05140	0.54150
3	98.93325	99.44873	99.84591	0.15598	0.55257
4	98.57172	99.20376	99.74272	0.19368	0.62109
5	98.22189	98.94552	99.59247		

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.26661	0.41876	0.89748	0.37292
1	0.09731	0.44610	0.87884	
2	99.87438	0.37374	0.87116	0.18354
3	99.62852	0.24308	0.82820	

EGERIA AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30922	98.90016	98.97078	98.66920	98.85659
1	99.44388	99.46856	98.71549	98.83852	98.77250
2	98.75192	99.06685	99.12091	98.70912	98.88912
3	98.10502	98.59194	98.91753	99.00954	98.82827
4	97.47905	98.08869	98.58111	98.92018	

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.80634	99.35171	99.52507	99.36217
1	99.40212	99.51477	99.41267	
2	98.92656	99.28389	99.47151	98.79210
3	98.42304	98.93718	99.32530	

EGERIA AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30300	98.26522	98.28280	97.36937	97.42137
1	99.13113	99.13706	97.75315	97.78516	97.13012
2	98.13468	98.43902	98.45215	97.41344	97.46303
3	97.18383	97.66327	97.97010	97.99292	97.19415

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.44697	98.36344	98.41062	98.60629
1	98.74828	98.77724	98.07925	
2	97.97237	98.28716	98.33947	97.48434

EGERIA AND NEPTUNE.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30183	97.87049	97.87767	96.57420	96.59597
1	98.93464	98.93705	97.15919	97.17240	96.13264
2	97.74328	98.04565	98.05099	96.61935	96.63997
3	96.59760	97.07566	97.37904	97.38835	96.19839

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.24168	97.76459	97.78425	98.18788
1	98.35002	98.36193	97.27761	
2	97.37997	97.68660	97.70853	96.68780

IRENE AND MERCURY.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30349	98.36172	98.38355	97.56471	97.62865
1	99.17901	99.18640	97.89866	97.93829	97.37539
2	98.22980	98.53495	98.55130	97.60863	97.66942
3	97.32615	97.80615	98.11442	98.14277	97.43906

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.49848	98.51097	98.56909	98.71247
1	98.84624	98.88210	98.27725	
2	98.11724	98.43539	98.49968	97.68025

IRENE AND VENUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30984	98.93344	99.00915	98.73870	98.93778
1	99.46005	99.48660	98.76639	98.89778	98.86103
2	98.78343	99.09943	99.15752	98.77820	98.96962
3	98.15183	98.63950	98.96696	99.06567	98.91606

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.82713	99.40521	99.58965	99.40710
1	99.43729	99.55773	99.48545	
2	98.97666	99.33815	99.53741	98.86245

IRENE AND THE EARTH.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.31845	99.25392	99.39795	99.42037	99.76081
1	99.61363	99.66642	99.26108	99.49930	99.73501
2	99.07914	99.41038	99.52418	99.45459	99.78515
3	98.58895	99.08750	99.44224	99.63073	99.78068

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.04554	99.93601	0.25676	99.88463
1	99.78454	0.00572	0.21206	
2	99.46034	99.87898	0.21846	99.55725

IRENE AND MARS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.34611	99.74747	0.08528	0.53411	1.18604
1	99.83799	99.97707	0.04783	0.55280	1.18554
2	99.49127	99.87563	0.16232	0.55565	1.19855
3	99.18653	99.72402	0.17580	0.61910	1.21121

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.47908	0.84017	1.47147	0.83806
1	0.36834	0.84950	1.46291	
2	0.21387	0.80625	1.45360	0.72688

IRENE AND JUPITER.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.33127	99.53026	99.76808	0.03237	0.53507
1	99.74162	99.83400	99.69679	0.06939	0.52887
2	99.31812	99.67316	99.86811	0.05998	0.55256
3	98.93785	99.45365	99.85160	0.16371	0.56372
4	98.57774	99.21003	99.74951	0.20167	0.63164
5	98.22933	98.95316	99.60049		

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.27002	0.42582	0.90701	0.37801
1	0.10187	0.45273	0.88862	
2	99.88013	0.38098	0.88085	0.19307
3	99.63572	0.25165	0.83824	

IRENE AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30927	98.90322	98.97430	98.67559	98.86403
1	99.44537	99.47022	98.72017	98.84395	98.78063
2	98.75482	99.06985	99.12426	98.71546	98.89649
3	98.10934	98.59632	98.92205	99.01469	98.83633
4	97.48477	98.09446	98.58699	98.92633	

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.80824	99.35663	99.53097	99.36628
1	99.40535	99.51870	99.41935	
2	98.93116	99.28887	99.47753	98.79856
3	98.42904	98.94343	99.33210	

IRENE AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30302	98.26809	98.28579	97.37518	97.42749
1	99.13255	99.13852	97.75747	97.78971	97.13741
2	98.13752	98.44187	98.45509	97.41944	97.46913
3	97.18807	97.66752	97.97439	97.99736	97.20145

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.44849	98.36782	98.41530	98.60942
1	98.75119	98.78034	98.08513	
2	97.97668	98.29156	98.34419	97.49015

IRENE AND NEPTUNE.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30184	97.87332	97.88055	96.57989	96.60181
1	98.93606	98.93848	97.16345	97.17674	96.13978
2	97.74610	98.04848	98.05385	96.62504	96.64579
3	96.60183	97.07990	97.38329	97.39266	96.20552

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.24314	97.76887	97.78866	98.19082
1	98.35287	98.36386	97.28333	
2	97.38422	97.69089	97.71296	96.69354

EUNOMIA AND MERCURY.

The inclination of Eunomia is 11° 44′, and its eccentricity 0.188; both too large to make it necessary to give the coefficients for this Asteroid. They may, however, be readily found from the corresponding ones for Fides, if needed.

PSYCHE AND MERCURY.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30295	98.25276	98.26984	97.34418	97.39479
1	99.12493	99.13070	97.73436	97.76552	97.09846
2	98.12237	98.42662	98.43936	97.38849	97.43655
3	97.16541	97.64478	97.95175	97.97364	96.16257

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.44036	98.34443	98.39034	98.59272
1	98.73565	98.76382	98.05375	
2	97.95366	98.26807	98.31899	97.45908

PSYCHE AND VENUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30787	98.81799	98.87734	98.49838	98.65929
1	99.40383	99.42441	98.59005	98.69438	98.55527
2	98.67361	98.98621	99.03141	98.53920	98.69335
3	97.98857	98.47384	98.79526	98.87255	98.61273

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.75613	99.22064	99.36887	99.25426
1	99.31581	99.41109	99.23463	
2	98.80289	99.15105	99.31219	98.61945

PSYCHE AND THE EARTH.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.31444	99.12915	99.24203	99.15200	99.43116
1	99.55435	99.59491	99.06740	99.25804	99.38966
2	98.96591	99.29000	99.37802	99.18858	99.45855
3	98.42211	98.91553	99.25742	99.40491	99.43943

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.95604	99.72552	99.98672	99.68802
1	99.64724	99.82327	99.92298	
2	99.27174	99.66403	99.94282	99.28248

PSYCHE AND MARS.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.33479	99.58823	99.85044	0.16436	0.70524
1	99.76775	99.87110	99.78965	0.19541	0.70115
2	99.36575	99.72755	99.94441	0.19038	0.72134
3	99.00677	99.52742	99.93775	0.28196	0.73712

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	0.32273	0.53358	1.05301	0.49637
1	0.17131	0.55461	1.03793	
2	99.96888	0.49161	1.02920	0.33182

PSYCHE AND JUPITER.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.34122	99.68285	99.98849	0.38278	0.98865
1	99.80979	99.93332	99.94248	0.40585	0.98699
2	99.44132	99.81576	0.07239	0.40613	1.00257
3	99.11525	99.64533	0.07888	0.48012	1.01536
4	98.80890	99.45264	0.01628	0.52360	1.06225
5	98.51342	99.24752	99.91285	0.51961	1.11040

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	0.41352	0.71382	1.29801	0.69526
1	0.28722	0.72720	1.28828	
2	0.11416	0.67662	1.27844	0.56417
3	99.92031	0.58249	1.24791	

PSYCHE AND SATURN.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.31167	99.02060	99.11126	98.92175	99.15408
1	99.50222	99.53433	98.90009	99.05566	99.09470
2	98.86534	99.18454	99.25456	98.96007	99.18400
3	98.27333	98.76327	99.09642	99.21473	99.14757
4	97.70216	98.31415	98.81136	99.16226	

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.88306	99.54657	99.76272	99.52848
1	99.53011	99.67313	99.67819	
2	99.10799	99.48179	99.71410	99.04811
3	98.65852	99.18380	99.59624	

PSYCHE AND URANUS.

i	log $b_{\frac{1}{2}}^{(i)}$	log $\alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	log $\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	log $\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	log $\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30358	98.37714	98.39973	97.59598	97.66205
1	99.18665	99.19430	97 92193	97.96291	97.41507
2	98.24498	98.55028	98.56720	97.63984	97.70266
3	97.34884	97.82896	98.13748	98.16683	97.47828

i	log $\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.50678	98.53461	98.59469	98.72964
1	98.86192	98.89903	98.30900	
2	98.14040	98.45915	98.52557	97.71162

PSYCHE AND NEPTUNE.

i	log $b_{\frac{1}{2}}^{(i)}$	log $\alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	log $\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	log $\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	log $\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30206	97.98087	97.99011	96.79608	96.82394
1	98.98966	98.99276	97.32512	97.34208	96.41069
2	97.85300	98.15576	98.16263	96.84106	96.86746
3	96.76202	97.24035	97.54440	97.55638	96.47609

i	log $\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.29842	97.93148	97.95667	98.30294
1	98.46109	98.47639	97.50077	
2	97.54559	97.85383	97.88189	96.90995

THETIS AND MERCURY.

i	log $b_{\frac{1}{2}}^{(i)}$	log $\alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	log $\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	log $\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	log $\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30372	98.40145	98.42531	97.64531	97.71487
1	99.19870	99.20678	97.95863	98.00185	97.47715
2	98.26888	98.57442	98.59230	97.68906	97.75522
3	97.38460	97.86488	98.17383	98.20482	97.54016

i	log $\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.51991	98.57191	98.63520	98.75682
1	98.88673	97.92582	98.35811	
2	98.17691	98.49666	98.56658	97.76053

THETIS AND VENUS.

i	log $b_{\frac{1}{2}}^{(i)}$	log $\alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	log $\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	log $\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	log $\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.31069	98.97601	99.05871	98.82788	99.04273
1	99.48067	99.50982	98.83161	98.97439	98.97480
2	98.82354	99.14103	99.20469	98.86683	99.07365
3	98.21137	98.70009	99.03019	99.13808	99.02881

i	log $\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.85417	99.47400	99.67344	99.46567
1	99.48248	99.61353	99.57917	
2	99.04077	99.40801	99.62294	98.95285

THETIS AND THE EARTH.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.32021	99.30064	99.45802	99.52200	99.88739
1	99.63561	99.69379	99.33403	99.59215	99.86625
2	99.12077	99.45520	99.58023	99.55524	99.91057
3	98.65009	99.15092	99.51140	99.71749	99.91024

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.08060	0.01634	0.36151	99.96229
1	99.83678	0.07699	0.32269	
2	99.53107	99.96116	0.32532	99.66183

THETIS AND MARS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.35134	99.81057	0.18183	0.68376	1.38199
1	99.86517	0.02183	0.15150	0.69894	1.38214
2	99.53874	99.93377	0.25220	0.70342	1.39317
3	99.25382	99.80933	0.27091	0.75781	1.40521

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.54601	0.96623	1.64449	0.98321
1	0.44917	0.97256	1.63759	
2	0.31164	0.93550	1.62875	0.88905

THETIS AND JUPITER.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.32838	99.47825	99.69561	99.91508	0.38466
1	99.71793	99.80133	99.61395	99.95821	0.37607
2	99.27459	99.62414	99.80096	99.94406	0.40342
3	98.87471	99.38649	99.77444	0.05960	0.41309
4	98.49535	99.12421	99.65701	0.09357	0.48965
5	98.12722	98.84841	99.49083		

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.22450	0.33073	0.77890	0.28010
1	0.04049	0.36377	0.75696	
2	99.80089	0.28339	0.75054	0.06970
3	99.53778	0.13992	0.70297	

THETIS AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30855	98.86125	98.92631	98.58817	98.76257
1	99.42493	98.44759	98.65604	98.76988	98.66939
2	98.71491	99.02870	99.07837	98.62853	98.79589
3	98.05003	98.53612	98.85962	98.94428	98.72599
4	97.40607	98.01508	99.67889	97.84208	

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.78237	99.28948	99.45053	99.31056
1	99.36116	99.46526	99.32807	
2	98.86798	99.22078	99.39547	98.71015
3	98.34669	98.85780	99.23913	

THETIS AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log a\, D_a\, b_{\frac{1}{2}}^{(i)}$	$\log a^2\, D_a^2\, b_{\frac{1}{2}}^{(i)}$	$\log a^3\, D_a^3\, b_{\frac{1}{2}}^{(i)}$	$\log a^4\, D_a^4\, b_{\frac{1}{2}}^{(i)}$
0	0.30285	98.22863	98.24482	97.29542	97.34346
1	99.11235	99.11841	97.69801	97.72754	97.03721
2	98.09854	98.40261	98.41472	97.33981	97.38542
3	97.12973	97.60898	97.91535	97.93636	97.10147
i	$\log a\, b_{\frac{3}{2}}^{(i)}$	$\log a^2\, D_a\, b_{\frac{3}{2}}^{(i)}$	$\log a^3\, D_a^2\, b_{\frac{3}{2}}^{(i)}$	$\log a^2\, b_{\frac{5}{2}}^{(i)}$	
0	99.42720	98.30786	98.35121	98.56653	
1	98.71121	98.73791	98.00443		
2	97.91744	98.23136	98.27947	97.41020	

THETIS AND NEPTUNE.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log a\, D_a\, b_{\frac{1}{2}}^{(i)}$	$\log a^2\, D_a^2\, b_{\frac{1}{2}}^{(i)}$	$\log a^3\, D_a^3\, b_{\frac{1}{2}}^{(i)}$	$\log a^4\, D_a^4\, b_{\frac{1}{2}}^{(i)}$
0	0.30177	97.83428	97.84093	96.50155	96.52162
1	98.91661	98.91882	97.10483	97.11700	96.04164
2	97.70729	98.00955	98.01446	96.54674	96.56573
3	96.54367	97.02165	97.32484	97.33342	96.10747
i	$\log a\, b_{\frac{3}{2}}^{(i)}$	$\log a^2\, D_a\, b_{\frac{3}{2}}^{(i)}$	$\log a^3\, D_a^2\, b_{\frac{3}{2}}^{(i)}$	$\log a^2\, b_{\frac{5}{2}}^{(i)}$	
0	99.22317	97.71096	97.72809	98.15041	
1	98.31366	98.32461	97.20460		
2	97.32570	97.63188	97.65211	96.61516	

MELPOMENE AND MERCURY.

The inclination of Melpomene is 10°, and its eccentricity is 0.215; both large. We can readily find, however, the coefficients for

Melpomene and Mercury, from those for Clio and Mercury;
" " Venus, " " Themis and The Earth;
" " The Earth, " " Harmonia and Jupiter;
" " Mars, " " " " Mars;
" " Jupiter, " " " " The Earth;
" " Saturn, " " " " Saturn;
" " Uranus, " " " " Uranus;
" " Neptune, " " " " Neptune.

FORTUNA AND MERCURY.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log a\, D_a\, b_{\frac{1}{2}}^{(i)}$	$\log a^2\, D_a^2\, b_{\frac{1}{2}}^{(i)}$	$\log a^3\, D_a^3\, b_{\frac{1}{2}}^{(i)}$	$\log a^4\, D_a^4\, b_{\frac{1}{2}}^{(i)}$
0	0.30387	98.42489	98.45003	97.69290	97.76598
1	99.21031	99.21885	97.99402	98.03951	97.53706
2	98.29190	98.59769	98.61653	97.73654	97.80607
3	97.41902	97.89948	98.20886	98.24151	97.59987
i	$\log a\, b_{\frac{3}{2}}^{(i)}$	$\log a^2\, D_a\, b_{\frac{3}{2}}^{(i)}$	$\log a^3\, D_a^2\, b_{\frac{3}{2}}^{(i)}$	$\log a^2\, b_{\frac{5}{2}}^{(i)}$	
0	99.53262	98.60791	98.67443	98.78318	
1	98.91055	98.95176	98.40750		
2	98.21211	98.53286	98.60632	97.80889	

FORTUNA AND VENUS.

i	log $b_{\frac{1}{2}}^{(i)}$	log $\alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	log $\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	log $\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	log $\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.31123	99.00123	99.08836	98.88095	99.10556
1	99.49288	99.52366	98.87034	99.02024	99.04255
2	98.84723	99.16566	99.23285	98.91954	99.13591
3	98.24648	98.73588	99.06766	99.18135	99.09592

i	log $\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.87044	99.51499	99.72377	99.50102
1	99.50938	99.74711	99.63509	
2	99.07880	99.44968	99.67432	99.00667

FORTUNA AND THE EARTH.

i	log $b_{\frac{1}{2}}^{(i)}$	log $\alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	log $\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	log $\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	log $\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.32133	99.32855	99.49437	99.58305	99.96385
1	99.64868	99.71031	99.37773	99.64829	99.94520
2	99.14542	99.48190	99.61409	99.61567	99.98633
3	98.68623	99.18854	99.55273	99.76993	99.98815

i	log $\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.10212	0.06477	0.42507	0.00981
1	99.86824	0.12036	0.38946	
2	99.57338	0.01074	0.39014	99.72480

FORTUNA AND MARS.

i	log $b_{\frac{1}{2}}^{(i)}$	log $\alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	log $\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	log $\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	log $\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.35476	99.84940	0.24217	0.77669	1.50395
1	99.88167	0.04827	0.21570	0.79002	1.50442
2	99.56725	99.96944	0.30851	0.79524	1.51436
3	99.29409	99.84496	0.32976	0.84456	1.52594

i	log $\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.58865	1.04517	1.75290	1.07526
1	0.49975	1.05000	1.74688	
2	0.37206	1.01636	1.73837	0.99057

FORTUNA AND JUPITER.

i	log $b_{\frac{1}{2}}^{(i)}$	log $\alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	log $\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	log $\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	log $\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.32682	99.44832	99.65452	99.84805	0.29908
1	99.70421	99.78279	99.56646	99.89506	0.28889
2	99.24920	99.59584	99.76286	99.87780	0.31858
3	98.83779	99.34746	99.73008	0.00051	0.32713
4	98.44669	99.07415	99.60341	0.03166	0.40903
5	98.06738	98.78720	99.42701		

i	log $\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.19904	0.27667	0.70639	0.22391
1	0.00554	0.31362	0.68216	
2	99.75531	0.22793	0.67675	99.99972
3	99.48121	0.07613	0.62610	

FORTUNA AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30815	98.83668	98.89844	98.53714	98.70372
1	99.41295	99.43442	98.61856	98.72691	98.60452
2	98.69148	99.00457	99.05166	98.57777	98.73752
3	98.01517	98.50078	98.82308	98.90352	98.66162
4	97.35981	97.96844	98.45880	98.79280	

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.76742	99.25034	99.40400	99.27845
1	99.33538	99.43439	99.27493	
2	98.83101	99.18113	99.34802	98.65860
3	98.29844	98.80774	99.18505	

FORTUNA AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30275	98.20543	98.22080	97.24857	97.29125
1	99.10142	99.10660	97.66306	97.69111	96.97837
2	98.07561	98.37952	98.39099	97.29303	97.33638
3	97.09540	97.57455	97.88063	97.90058	97.04277

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.41538	98.27231	98.31372	98.54146
1	98.68774	98.71312	97.95707	
2	97.88262	98.19567	98.24163	97.36324

FORTUNA AND NEPTUNE.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30173	97.81137	97.81765	96.45547	96.47454
1	98.90516	98.90726	97.07035	97.08191	95.98393
2	97.68445	97.98665	97.99131	96.50070	96.51876
3	96.50944	96.98738	97.29046	97.29860	96.04982

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.21144	97.67533	97.69255	98.12670
1	98.29060	98.30101	97.15832	
2	97.29127	97.59719	97.61640	96.56896

MASSALIA AND MERCURY.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30387	98.42530	98.45047	97.69374	97.76688
1	99.21052	99.21905	97.99465	98.04017	97.53812
2	98.29231	98.59810	98.61696	97.73738	97.80697
3	97.41962	97.90008	98.20947	98.24216	97.60092

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.53284	98.60854	98.67513	98.78364
1	98.91098	98.95222	98.40835	
2	98.21273	98.53350	98.60702	97.80973

MASSALIA AND VENUS.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.31124	99.00168	99.08889	98.88188	99.10667
1	99.49309	99.52391	98.87103	99.02103	99.04374
2	98.84764	99.16610	99.23334	98.92047	99.13702
3	98.24709	98.73651	99.06833	99.18211	99.09711

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.87072	99.51572	99.72468	99.50165
1	99.50986	99.64770	99.63691	
2	99.07947	99.45041	99.67523	99.00762

MASSALIA AND THE EARTH.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.32135	99.32904	99.49502	99.96571	99.96521
1	99.64891	99.71060	99.37851	99.26772	99.94660
2	99.14586	99.48237	99.61469	99.99831	99.98767
3	98.68686	99.18920	99.55345	99.38930	99.98953

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	0.10251	0.06563	0.42620	0.01066
1	99.86880	0.12113	0.39064	
2	99.57412	0.01162	0.39129	99.72592

MASSALIA AND MARS.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.35482	99.84976	0.24258	0.77736	1.50481
1	99.88196	0.04843	0.21619	0.79066	1.50528
2	99.56776	99.96974	0.30886	0.79589	1.51519
3	99.29480	99.84544	0.33015	0.84512	1.52678

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	0.58875	1.04559	1.75352	1.07560
1	0.49999	1.05036	1.74751	
2	0.37247	1.01682	1.73902	0.99107

MASSALIA AND JUPITER.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.32680	99.44779	99.65380	99.84689	0.29754
1	99.70397	99.78247	99.56564	99.89396	0.28737
2	99.24876	99.59534	99.76219	99.87664	0.31716
3	98.83715	99.34677	99.72930	99.99949	0.32563
4	98.44585	99.07327	99.60248	0.03058	0.40763
5	98.06633	98.78613	99.42589		

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	0.19860	0.27573	0.70514	0.22293
1	0.00494	0.31279	0.68086	
2	99.75452	0.22696	0.67547	99.99850
3	99.48022	0.07502	0.62477	

MASSALIA AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30815	98.83625	98.89797	98.53625	98.70269
1	99.41275	99.43418	98.61790	98.72615	98.60340
2	98.69107	99.00415	99.05120	98.57688	98.73650
3	98.01456	98.50016	98.82243	98.90280	98.66050
4	97.35900	97.96761	98.45796	97.79192	

i	$\log \alpha\, b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.76716	99.24966	99.40319	99.27792
1	99.33493	99.43388	99.27401	
2	98.83063	99.18044	99.34719	98.65769
3	98.29758	98.80686	99.18412	

MASSALIA AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30275	98.20502	98.22037	97.24775	97.29338
1	99.10122	99.10639	97.66245	97.69047	96.97734
2	98.07522	98.37913	98.39057	97.29221	97.33552
3	97.09480	97.57394	97.88002	97.89995	97.04173

i	$\log \alpha\, b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.41516	98.27167	98.31306	98.54102
1	98.68732	98.71265	97.95624	
2	97.88201	98.19504	98.24096	97.36242

MASSALIA AND NEPTUNE.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30173	97.81097	97.81724	96.45465	96.47371
1	98.90496	98.90706	97.06975	97.08159	95.98291
2	97.68405	97.98625	97.99091	96.49989	96.51792
3	96.50884	96.98678	97.28985	97.29798	96.04881

i	$\log \alpha\, b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.21124	97.67472	97.69192	98.12624
1	98.29019	98.30060	97.15751	
2	97.29066	97.59658	97.61577	96.56815

LUTETIA AND MERCURY.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30381	98.41546	98.44007	97.67373	97.74538
1	99.20564	99.21398	97.97977	98.02433	97.51293
2	98.28264	98.58832	98.60677	97.71742	97.78558
3	97.40516	97.88555	98.19475	98.22673	97.57582

i	$\log \alpha\, b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.52749	98.59341	98.65861	98.77255
1	98.90094	98.94130	98.38801	
2	98.19794	98.51828	98.59029	97.78965

LUTETIA AND VENUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.31101	98.99107	99.07638	98.85954	99.08017
1	99.48796	99.51807	98.85472	99.00171	99.01521
2	98.83769	99.15574	99.22147	98.89828	99.10846
3	98.23234	98.72146	99.05256	99.16387	99.06883

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.86386	99.49845	99.70341	99.48671
1	99.49852	99.63353	99.61251	
2	99.06347	99.43286	99.65354	98.98495

LUTETIA AND THE EARTH.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.32087	99.31727	99.47965	99.55838	99.93290
1	99.64341	99.70362	99.36007	99.62556	99.91327
2	99.13549	99.47112	98.60025	99.59124	99.95566
3	98.67167	99.17337	99.53303	99.74869	99.95663

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.09338	0.04517	0.39931	99.99052
1	99.85551	0.10177	0.36243	
2	99.55629	99.99067	0.36387	99.69932

LUTETIA AND MARS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.35335	99.83361	0.21754	0.73881	1.45422
1	99.87498	0.03706	0.18955	0.75287	1.45457
2	99.55572	99.95494	0.28551	0.75781	1.46494
3	99.27783	99.82647	0.30579	0.80915	1.47671

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.57117	1.01294	1.70863	1.03757
1	0.47909	1.01833	1.70226	
2	0.37446	0.98335	1.69361	0.94912

LUTETIA AND JUPITER.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.32744	99.46030	99.67093	99.87486	0.33328
1	99.70971	99.79019	99.58547	99.92028	0.32375
2	99.25940	99.60718	99.77807	99.90430	0.35247
3	98.85263	99.36313	99.74784	0.02411	0.36150
4	98.46614	99.09426	99.62491	0.05643	0.44123
5	98.09144	98.81181	99.45263		

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.20918	0.29827	0.73533	0.24628
1	0.01951	0.33362	0.71204	
2	99.77381	0.25009	0.70621	0.02768
3	99.50389	0.10166	0.65681	

LUTETIA AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30831	98.84656	98.90957	98.55764	98.72733
1	99.41777	99.43970	98.63362	98.74415	98.63057
2	98.70091	99.01429	99.06238	98.59817	98.76094
3	98.02919	98.51500	98.83777	98.91992	98.68747
4	97.37842	97.98720	98.47787	98.81261	

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.77341	99.26606	99.42266	99.29131
1	99.34573	99.44677	99.29627	
2	98.84587	99.19705	99.36704	98.67930
3	98.31784	98.82786	99.20676	

LUTETIA AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30279	98.21476	98.23046	97.26742	97.31403
1	99.10606	99.11135	97.67712	97.70576	97.00204
2	98.08483	98.38881	98.40052	97.31185	97.35610
3	97.10922	97.58840	97.89460	97.91497	97.06638

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.42029	98.28652	98.32878	98.55153
1	98.69718	98.72306	97.97612	
2	97.89663	98.20993	98.25683	97.38213

LUTETIA AND NEPTUNE.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30174	97 82061	97.82702	96.47401	96.49348
1	98.90977	98.91191	97.08423	97.09603	96.00715
2	97.69364	97.99587	98.00063	96.51923	96.53766
3	96.52321	97.00117	97.39560	97.31261	96.07302

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.21610	97.68927	97.70684	98.13624
1	98.29987	98.31051	97.17695	
2	97.30512	97.61115	97.63076	96.58752

CALLIOPE AND MERCURY.

The coefficients for this Asteroid may readily be obtained from the corresponding ones for Psyche, if needed.

THALIA AND MERCURY.

The coefficients for this Asteroid may be obtained from the corresponding ones for Fides.

THEMIS AND MERCURY.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30266	98.18224	98.19702	97.20176	97.24519
1	99.08989	99.09480	97.62814	97.65478	96.91960
2	98.05267	98.35643	98.36731	97.24629	97.28750
3	97.06106	97.54010	97.84593	97.86485	96.98412

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.40318	98.23701	98.27637	98.51648
1	98.66428	98.68835	97.90977	
2	97.84781	98.16024	98.20394	97.31633

THEMIS AND VENUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30683	98.74407	98.79475	98.34563	98.48522
1	99.36760	99.38512	98.47756	98.56724	98.36147
2	98.60265	98.91349	98.95191	98.38717	98.52074
3	97.88292	98.36693	99.68522	98.75115	98.42026

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.71223	99.10383	99.23213	99.16053
1	99.23876	99.32044	99.07634	
2	98.69174	99.03288	98.17289	98.46534

THEMIS AND THE EARTH.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.31237	99.05051	99.14687	98.97592	99.22955
1	99.51663	99.55089	98.94609	99.11076	99.17551
2	98.89322	99.21366	99.28826	99.02279	99.25878
3	98.31463	98.80544	99.14079	99.26659	99.22756

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.90277	99.59552	99.82347	99.57145
1	99.56218	99.71375	99.74507	
2	99.15308	99.53160	99.77612	99.11268

THEMIS AND MARS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.32918	99.49315	99.71623	99.94857	0.42753
1	99.72474	99.81063	99.63763	99.98988	0.41967
2	99.28715	99.63820	99.82007	99.97716	0.44571
3	98.89293	99.40582	99.79651	0.08923	0.45607

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.75784	0.35782	0.81532	0.30849
1	0.57845	0.38902	0.79446	
2	0.34407	0.31119	0.78760	0.10475

THEMIS AND JUPITER.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.34976	99.79196	0.15316	0.63944	1.32391
1	99.85720	0.00781	0.12083	0.65560	1.32388
2	99.52489	99.91664	0.22547	0.65964	1.33548
3	99.23425	99.77726	0.24278	0.71661	1.34770
4	98.96310	99.61714	0.20307	0.75930	1.38308
5	98.70334	99.44530	0.12746	0.76859	1.42451

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	0.52597	0.92876	1.59306	1.93983
1	0.42514	0.93592	1.58569	
2	0.28275	0.89710	1.57670	0.84085
3	0.12127	0.82388	1.55293	

THEMIS AND SATURN.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.31360	99.09862	99.20482	99.08696	99.35234
1	99.53974	99.57773	99.02024	99.20044	99.30623
2	98.93781	99.26050	99.34297	99.12406	99.38045
3	98.38058	98.87294	99.21216	99.25087	99.35691
4	97.84412	98.45810	98.95908	99.31946	

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.93510	99.67483	99.92273	99.64213
1	99.61410	99.78027	99.85355	
2	99.22566	99.61250	99.87748	99.21618
3	98.81031	99.34442	99.77560	

THEMIS AND URANUS.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.30398	98.44835	98.47468	97.74054	97.81643
1	99.22187	99.23090	98.02949	98.07685	97.59698
2	98.31487	98.62103	98.64078	97.78410	97.85633
3	97.45339	97.93416	98.24405	98.27813	97.65963

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.54536	98.64401	98.71313	98.80936
1	98.93453	98.97747	98.45592	
2	98.24741	98.56912	98.64539	97.85684

THEMIS AND NEPTUNE.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.30224	98.05009	98.06173	96.93703	96.96957
1	99.02454	99.02818	97.43046	97.45032	96.58642
2	97.92253	98.22559	98.23365	96.98187	97.01273
3	96.86618	97.34474	97.64920	97.66335	96.65256

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.33466	98.03757	98.06701	98.37653
1	98.53167	98.54951	97.64268	
2	97.65070	97.96017	97.99295	97.05111

PHOCÆA AND MERCURY.

The inclination and eccentricity of this Asteroid are both very large; but the coefficients may be obtained from the corresponding ones for Massalia.

PROSERPINE AND MERCURY.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30336	98.33777	98.35846	97.51618	97.57694
1	99.16713	99.17413	97.86253	97.90012	97.31468
2	98.20622	98.51115	98.52663	97.56020	97.61794
3	97.29088	97.77073	98.07860	98.10547	97.37820
i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$	
0	99.48563	98.47429	98.52950	98.68592	
1	98.82189	98.85591	98.22801		
2	98.08128	98.39853	98.45962	97.63156	

PROSERPINE AND VENUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30936	98.90791	98 97969	98.68537	98.87543
1	99.44765	99.47276	98.72733	98.85229	98.79308
2	98.75926	99.07444	99.12941	98.72519	98.90780
3	98.11594	98.60303	98.92902	99.02258	98.84869
i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$	
0	99.81115	99.36416	99.54003	99.37257	
1	99.41030	99.52472	99.42959		
2	98.93822	99.29649	99.48678	98.80846	

PROSERPINE AND THE EARTH.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.31747	99.22610	99.36263	99.36016	99 68626
1	99.60048	99.65029	99.21767	99.44468	99 65738
2	99.05414	99.38363	99.49116	99.39494	99.71129
3	99.55219	99.04947	99.40106	99.57966	99.70401
i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$	
0	0.02492	99.88859	0.19534	99.83947	
1	99.75365	99.96407	0.14683		
2	99.41824	99.83050	0.15579	99.49544	

PROSERPINE AND MARS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.34330	99.71090	0.03027	0.44827	1.07398
1	99.82209	99.95219	99.98811	0.46935	1.07285
2	99.46318	99.84178	0.11118	0.47081	1.08729
3	99.14649	99.67967	0.12088	0.54009	1.10004

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.44162	0.76832	1.37316	0.75656
1	0.32223	0.77986	1.36335	
2	0.15738	0.73256	1.35401	0.63437

PROSERPINE AND JUPITER.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.33318	99.56245	99.81361	0.10550	0.62934
1	99.75617	99.85451	99.74829	0.13910	0.62428
2	99.34469	99.70339	99.91029	0.13223	0.64596
3	98.97638	99.49476	99.89942	0.22909	0.65777
4	98.62796	99.26238	99.80636	0.26895	0.72073
5	98.29131	99.01691	99.66757	0.24412	0.77390

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.29903	0.48542	0.98767	0.44428
1	0.14029	0.50893	0.97119	
2	99.92943	0.44217	0.96283	0.26965
3	99.69610	0.32061	0.92306	

PROSERPINE AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30975	98.92873	99.00370	98.72885	98.92624
1	99.45776	99.48404	98.75918	98.88936	98.84848
2	98.77897	99.09482	99.15232	98.76841	98.95817
3	98.14521	98.63278	98.95996	99.05770	98.90361
4	97.53235	98.14250	98.63594	98.97754	

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.82417	99.39762	99.58046	99.40069
1	99.43230	99.55160	99.47512	
2	98.96956	99.33045	99.52803	98.85247
3	98.47900	98.99554	99.38895	

PROSERPINE AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30313	98.29196	98.31063	97.42344	97.47852
1	99.14440	99.15070	97.79344	97.82743	97.19808
2	98.16106	98.46561	98.47956	97.46763	97.51995
3	97.22331	97.70290	98.01009	98.03433	97.26193

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.46117	98.40425	98.45426	98.63551
1	98.77539	98.80613	98.13400	
2	98.01251	98.32816	98.38355	97.53855

PROSERPINE AND NEPTUNE.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.30189	97.89689	97.90452	96.62724	96.65035
1	98.94790	98.95036	97.19887	97.21289	96.19910
2	97.76964	98.07199	98.07766	96.67236	96.69423
3	96.63706	97.11507	97.41859	97.42849	96.26477

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.25522	97.80447	97.82535	98.21530
1	98.37657	98.38922	97.33093	
2	97.41958	97.72656	97.74983	96.74091

EUTERPE AND MERCURY.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.30393	98.43455	98.46023	97.71251	97.78709
1	99.21508	99.22381	98.00860	98.05505	97.56177
2	98.30140	98.60728	98.62653	97.75611	97.82707
3	97.43322	97.91372	98.22329	98.25665	97.62448

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.53787	98.62275	98.69065	98.79407
1	98.92040	98.96249	98.42745	
2	98.22661	98.54779	98.62279	97.82858

EUTERPE AND VENUS.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.31145	99.01166	99.10068	98.90290	99.13165
1	99.49791	99.52940	98.88635	99.03927	99.07061
2	98.85697	99.17583	99.24452	98.94135	99.16187
3	98.26095	98.75063	99.08314	99.19930	99.12371

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	99.87722	99.53198	99.74470	99.51575
1	99.52052	99.66109	99.65827	
2	99.09451	99.46695	99.69570	99.02897

EUTERPE AND THE EARTH.

i	$\log b^{(i)}_{\frac{1}{2}}$	$\log \alpha D_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^2 D^2_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^3 D^3_\alpha b^{(i)}_{\frac{1}{2}}$	$\log \alpha^4 D^4_\alpha b^{(i)}_{\frac{1}{2}}$
0	0.32181	99.34012	98.41956	99.60847	99.99576
1	99.65409	99.71720	99.30590	99.67174	99.97908
2	99.15560	99.49296	99.53823	99.64081	0.01795
3	98.70114	99.20408	99.47986	99.79183	0.02059

i	$\log \alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 D_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^3 D^2_\alpha b^{(i)}_{\frac{3}{2}}$	$\log \alpha^2 b^{(i)}_{\frac{5}{2}}$
0	0.02116	0.08497	0.45167	0.02979
1	99.79135	0.13854	0.41732	
2	99.50094	0.03142	0.41725	99.75104

EUTERPE AND MARS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.35624	99.86581	0.26840	0.81612	1.55585
1	99.88858	0.05999	0.24290	0.82874	1.55633
2	99.57913	99.98446	0.33251	0.83420	1.56584
3	99.31079	99.86404	0.35468	0.88150	1.57721
i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$	
0	0.60698	1.07881	1.79911	1.11474	
1	0.52129	1.08303	1.79342		
2	0.39762	1.05080	1.78508	1.03382	

EUTERPE AND JUPITER.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.32621	99.43613	99.63791	99.82086	0.26446
1	99.69861	99.77529	99.54649	99.86952	0.25355
2	99.23881	99.58429	99.47446	99.85091	0.28424
3	98.82265	99.33148	99.71202	99.77464	0.29229
4	98.42683	99.05363	99.58152	0.00652	0.37644
5	98.04281	98.76209	99.40089		
i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$	
0	0.18705	0.25480	0.67714	0.20035	
1	99.99018	0.29462	0.65191		
2	99.[illegible]3676	0.20548	0.64696	99.97138	
3	99.45815	0.05023	0.59501		

EUTERPE AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30800	98.82660	98.88705	98.51623	98 67968
1	99.40803	99.42902	98.60318	98.70935	98.57795
2	98.68185	98.99467	99.04073	98.55697	98 71367
3	98.00083	98.48626	98.80807	98.88680	98.63524
4	97.34078	97.94926	98.43933	98.77258	
i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$	
0	99.76132	99.23431	99.38501	99.26538	
1	99.32482	99.42181	99.25318		
2	98.81584	99.16490	99.32867	98.63747	
3	98.27862	98.78722	99.16293		

EUTERPE AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30270	98.19589	98.20193	97.22931	97.27405
1	99.09668	99.10174	97.64863	97.67615	96.95419
2	98.06617	98.37001	98.38124	97.27380	97.31625
3	97.08128	97.56038	97.86626	97.88587	97.01863
i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$	
0	99.41036	98.25778	98.29833	98.53117	
1	98.67808	98.70290	97.93760		
2	97.86830	98.18114	98.22610	97.34394	

EUTERPE AND NEPTUNE.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30171	97.80193	97.80807	96.43651	96.45518
1	98.90045	98.90251	97.05617	97.06748	95.96019
2	97.67505	97.97723	97.98179	96.48175	96.49943
3	96.49535	96.97327	97.27631	97.28427	96.02611

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.20662	97.66108	97.67793	98.11695
1	98.28111	98.29130	97.13928	
2	97.27710	97.58292	97.60173	96.54999

BELLONA AND MERCURY.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30314	98.29689	98.31576	97.43342	97.48908
1	99.14685	99.15322	97.80080	97.83523	97.21061
2	98.16593	98.42061	98.48462	97.47759	97.53046
3	97.23059	97.71020	98.01746	98.04187	97.27443

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.46379	98.41178	98.46232	98.64091
1	98.78039	99.81146	98.14411	
2	98.01991	98.33572	98.39170	97.54855

BELLONA AND VENUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30860	98.86354	98.93005	98.59501	98.77048
1	99.42654	99.44936	98.66106	98.77565	98.67808
2	98.71805	99.03189	99.08195	98.63534	98.80373
3	98.05469	98.54085	98.86452	98.94987	98.73462

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.78437	99.29472	99.45678	99.31487
1	99.36461	99.46941	99.33520	
2	98.87293	99.22610	99.40184	98.71706

BELLONA AND THE EARTH.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.31594	99.17915	99.30376	99.25904	99.56182
1	99.57820	99.62331	99.14483	99.35358	99.52720
2	99.01162	99.33836	99.43601	99.29472	99.58799
3	98.48956	98.98487	99.33151	99.49443	99.57540

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.99109	99.80921	0.09327	99.76501
1	99.70190	99.89526	0.03778	
2	99.34727	99.74942	0.05161	99.39186

BELLONA AND MARS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.33893	99.65064	99.94099	0.30800	0.89139
1	99.79557	99.91190	99.89027	0.33356	0.88903
2	99.41591	99.78564	0.02834	0.33227	0.90604
3	99.07883	99.60556	0.03075	0.41194	0.91877

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.38193	0.65184	1.21398	0.62619
1	0.24739	0.66756	1.20168	
2	0.06461	0.61301	1.19260	0.48432

BELLONA AND JUPITER.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.33679	99.89114	99.89493	0.23511	0.79682
1	99.78154	99.61906	99.83927	0.26334	0.79364
2	99.39068	99.75636	99.98563	0.26027	0.81220
3	99.04258	99.56619	99.98365	0.34583	0.88472
4	98.71444	99.35294	99.90558	0.38804	0.87966
5	98.39791	99.12689	99.78400	0.37382	0.93088

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.35165	0.59170	1.13203	0.55984
1	0.20872	0.60998	1.11849	
2	0.01613	0.55127	1.10944	0.40679
3	99.80183	0.44319	1.07428	

BELLONA AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.31062	98.97263	99.05477	98.82083	99.03439
1	99.47904	99.50797	98.82645	98.96829	98.96578
2	98.82038	99.13775	99.20094	98.85982	99.06537
3	98.20667	98.69528	99.02519	99.13233	99.01987
4	97.61383	98.22481	98.71996	99.06587	

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.85201	99.46854	99.66676	99.46099
1	99.47889	99.60908	99.57173	
2	99.03570	99.40249	99.61613	98.94568
3	98.56499	99.08540	99.48751	

BELLONA AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30333	98.33283	98.35329	97.50618	97.56630
1	99.16468	99.17160	97.85508	97.89226	97.30209
2	98.20136	98.50624	98.52155	97.55021	97.60742
3	97.28360	97.76346	98.07122	98.09779	97.36565

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.48298	98.46673	98.52136	98.68046
1	98.81688	98.85052	98.21786	
2	98.07387	98.39093	98.45139	97.62152

BELLONA AND NEPTUNE.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30196	97.93721	97.94557	96.70829	96.73357
1	98.96790	98.97069	97.25948	97.27485	96.30066
2	97.80963	98.11220	98.11844	96.75334	96.77728
3	96.69703	97.17524	97.47900	97.48985	96.36620

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.27594	97.86543	97.88828	98.25729
1	98.41714	98.43100	97.41243	
2	97.48009	97.78764	97.81215	96.82204

AMPHITRITE AND MERCURY.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30355	98.37249	98.39486	97.58656	97.65198
1	99.18435	99.19192	98.11492	97.95549	97.40322
2	98.24041	98.54566	98.56240	97.63044	97.69264
3	97.34201	97.82209	98.13054	98.15958	97.46647

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.50428	98.52748	98.58697	98.72445
1	98.85720	98.89393	98.29943	
2	98.13342	98.45199	98.51779	97.70217

AMPHITRITE AND VENUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.31006	98.94495	99.02251	98.76280	98.96606
1	99.46563	99.49286	98.78403	98.91842	98.89176
2	98.79430	99.11070	99.17024	98.80216	98.99764
3	98.16797	98.65592	98.98407	99.08520	98.94651

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.83440	99.42378	99.61219	99.42282
1	99.44949	99.57272	99.51074	
2	98.99400	99.35701	99.56042	98.88687

AMPHITRITE AND THE EARTH.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.31891	99.26652	99.41405	99.44772	99.79478
1	99.61957	99.67377	99.28073	99.52420	99.77029
2	99.09041	99.42248	99.53922	99.48167	99.81880
3	98.60552	99.10465	99.46090	99.65401	99.81551

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.05480	99.95758	0.28480	99.90535
1	99.79858	0.02478	0.24176	
2	99.47940	99.90104	0.24708	99.58535

AMPHITRITE AND MARS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.34748	99.76428	0.11083	0.57383	1.23800
1	99.84528	99.98863	0.07536	0.59153	1.23766
2	99.50406	99.89113	0.18607	0.59483	1.25015
3	99.20471	99.74420	0.20105	0.65579	1.26262

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.49662	0.87347	1.51721	0.87621
1	0.38970	0.88198	1.50908	
2	0.23985	0.84042	1.49993	0.76972

AMPHITRITE AND JUPITER.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.33045	99.51599	99.74806	0.00008	0.49358
1	99.73515	99.82498	99.67402	0.03807	0.48677
2	99.30625	99.65973	99.84957	0.02807	0.51142
3	98.92065	99.43531	99.83041	0.13497	0.52222
4	98.55524	99.18664	99.72420	0.17240	0.59243
5	98.20156	98.92464	99.57056		

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.25737	0.39957	0.87159	0.35255
1	0.08494	0.42809	0.85227	
2	99.85844	0.35405	0.84483	0.15874
3	99.60889	0.22067	0.80091	

AMPHITRITE AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30907	98.89180	98.96119	98.65176	98.83630
1	99.43981	99.46405	98.70270	98.82371	98.75029
2	98.74398	99.05866	99.11174	98.69178	98.86900
3	98.09323	98.57996	98.90506	98.99549	98.80625
4	97.49668	98.07289	98.56503	98.90339	

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.80115	99.33832	99.50895	99.35100
1	99.39330	99.50406	99.39444	
2	98.91396	99.27029	99.45506	98.77446
3	98.40663	98.92010	99.30672	

AMPHITRITE AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30297	98.25737	98.27465	97.35351	97.40453
1	99.12723	99.13305	97.74132	97.77279	97.11018
2	98.12693	98.43121	98.44411	97.39780	97.44635
3	97.17223	97.65163	97.95835	97.98078	97.17426

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.44281	98.35146	98.39784	98.59774
1	98.74033	98.76879	98.06319	
2	97.96059	98.27513	98.32656	97.46843

AMPHITRITE AND NEPTUNE.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30182	97.86273	97.86979	96.55862	96.58002
1	98.93078	98.93314	97.14753	97.16051	96.11315
2	97.73556	98.03791	98.04315	96.60378	96.62404
3	96.58604	97.06408	97.36742	97.37657	96.17891

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.23771	97.75287	97.77220	98.17984
1	98.34222	98.35393	97.26195	
2	97.36833	97,67486	97.69640	96.67221

URANIA AND MERCURY.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30398	98.44208	98.46820	97.72782	97 80357
1	99.21882	99.22768	98.01999	98.06720	97.58105
2	98.30877	98.61475	98.63434	97.77138	97.84348
3	97.44424	97.92483	98.23454	98.26848	97.64369

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.54197	98.63434	98.70334	98.80260
1	98.92808	98.97087	98.44304	
2	98.23792	98.55946	98.63560	97.84395

URANIA AND VENUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.31165	99.01981	99.11032	98.92008	99.15209
1	99.50184	99.53389	98.89887	99.05420	99.09256
2	98.86460	99.18377	99.25366	98.95840	99.18202
3	98.27224	98.76215	99.09324	99.21336	99.14545

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.88254	99.54527	99.76112	99.52734
1	99.52924	99.67205	99.67642	
2	99.10679	99.48047	99.71246	99.04641

URANIA AND THE EARTH.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.32220	99.34919	99.52150	99.62840	0.02083
1	99.65832	99.72262	99.41014	99.69006	0.00388
2	99.16356	99.50162	99.63933	99.66054	0.04279
3	98.71278	99.21623	99.58329	99.80904	0.04605

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.11829	0.10083	0.47258	0.04554
1	99.89165	0.15285	0.43920	
2	99.60470	0.04767	0.43858	99.77165

URANIA AND MARS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.35744	99.87875	0.28823	0.84736	1.59684
1	99.89403	0.06927	0.26444	0.85945	1.59749
2	99.58845	99.99632	0.35158	0 86509	1.60669
3	99.32389	99.87907	0.37441	0.91083	1.61787

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.62162	1.10552	1.83583	1.14623
1	0.53841	1.10933	1.83039	
2	0.41785	1.07815	1.82219	1.06819

URANIA AND JUPITER.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.32575	99.42669	99.62509	99.79982	0.24769
1	99.69426	99.76950	99.53223	99.84979	0.22621
2	99.23072	99.57534	99.73556	99.83011	0.25772
3	98.81087	99.31907	99.69803	99.95819	0.26533
4	98.41137	99.03767	99.56452	99.98705	0.35127
5	98.02368	98.74254	99.38057		

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.18096	0.23789	0.65455	0.18398
1	99.98046	0.27785	0.62853	
2	99.72240	0 18815	0.62397	99.94947
3	99.44024	0.03017	0.57098	

URANIA AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30788	98.81875	98.87820	98.49996	98.66100
1	99.40420	99.42482	98.59083	98.69571	98.55728
2	98.67434	98.98696	99.03223	98.54078	98.69515
3	97.98966	98.47494	98.79639	98.87381	98.61473
4	97.32595	97.93431	98.42415	98.76684	

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.75659	99.22185	99.37030	99.25525
1	99.31661	99.41204	99.23627	
2	98.80403	99.15228	99.31365	98.62104
3	98.26319	98.77105	99.14573	

URANIA AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30269	98.18846	98.20324	97.21430	97.25832
1	99.09298	99.09796	97 63750	97.66450	96.93534
2	98 05881	98.36273	98.37365	97.25881	97.30058
3	97.07027	97.54933	97.85522	97.87442	96.99983

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99 40645	98.24647	98.28636	98.52316
1	98.67056	98.69497	97.92244	
2	97.85714	98.16973	98.21402	97.32889

URANIA AND NEPTUNE.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30170	97.79457	97.80061	96.42174	96.44010
1	98.89678	98.89886	97.04511	97.05623	95.94668
2	98.66773	97.96988	97.97437	96.46699	96.48386
3	97.48437	96.96228	97.26528	97.27311	96.06762

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.20286	97.64998	97.66655	98.10936
1	98.27372	98.28374	97.12444	
2	97.26606	97.57180	97.59029	96.53520

EUPHROSYNE AND MERCURY.

Both the inclination and eccentricity of this Asteroid are large; but if the coefficients are needed, we may find those for

Euphrosyne and Mercury, from those for Hygeia and Mercury;
" " Venus, " " " " Venus;
" " The Earth, " " Themis and The Earth;
" " Mars, " " Thetis and Jupiter;
" " Jupiter, " " " " Mars;
" " Saturn, " " Themis and Saturn;
" " Uranus, " " Clio and Mercury;
" " Neptune, " " Themis and Neptune.

POMONA AND MERCURY.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30349	98.36221	98.38407	97.56572	97.62972
1	99.17926	99.18665	97.89941	97.93907	97.37699
2	98.23029	98.53545	98.55180	97.60963	97.67049
3	97.32688	97.80689	98.11516	98.14354	97.44032

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.49874	98.51173	98.56991	98.71302
1	98.84674	98.88264	98.27827	
2	98.11799	98.43615	98.50050	97.68126

POMONA AND VENUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30985	98.93397	99.00976	98.73981	98.93908
1	99.46030	99.48688	98.76720	98.89873	98.86244
2	98.78392	99.09995	99.15810	98.77931	98.97090
3	98.15257	98.64025	98.96774	99.06658	98.91746

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.82747	99.40606	99.59068	99.40782
1	99.43785	99.55841	99.48661	
2	98.97745	99.33902	99.53847	98.86357

POMONA AND THE EARTH.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.31847	99.25450	99.39869	99.42163	99.76236
1	99 61390	99.66676	99.26198	99.50044	99.73663
2	99.07966	99.41094	99.52486	99.45583	99.78669
3	98.58971	99.08829	99.44310	99.63180	99.78227

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0 04584	99.93700	0.25804	99.88558
1	99.78518	0.00670	0.21342	
2	99.46121	99.87999	0.21976	99.55853

POMONA AND MARS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.34619	99.74824	0.08649	0.53598	1.18851
1	99.83834	99.97761	0.04909	0.55465	1.18795
2	99.49188	99.87632	0.16342	0.55745	1.20100
3	99.18739	99.72493	0.17693	0.62082	1.21358

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.47989	0.84169	1.47364	0.83986
1	0.36931	0.85105	1.46504	
2	0.21504	0.80782	1.45581	0.72846

POMONA AND JUPITER.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.33123	99.52960	99.76716	0.03088	0.53315
1	99.74132	99.83358	99.69574	0.06797	0.52692
2	99.31758	99.67254	99.86726	0.05850	0.55066
3	98.93706	99.45280	99.85062	0.16238	0.56180
4	98.57671	99.20896	99.74834	0 20030	0.62982
5	98.22806	99.01335	99.59912		

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.26943	0.42460	0.90538	0.37914
1	0.10108	0.45159	0.88694	
2	99.87923	0.37973	0.87918	0.19109
3	99.63448	0.24989	0.83652	

POMONA AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30926	98.90269	98.97369	98.67449	98.86275
1	99.44511	99.46993	98.71936	98.84302	98.77923
2	98.75432	99.06934	99.12370	98.71473	98.89522
3	98.10860	98.59557	98.92127	99.01380	98.83495
4	97.48379	98.09347	98.58598	98.92527	

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.80791	99.35579	99.52996	99.36557
1	99.40479	99.51802	99.41820	
2	98.93037	99.28801	99.47650	98.79745
3	98.42801	98.94236	99.33093	

POMONA AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30301	98.26760	98.28527	97.37418	97.42644
1	99.13231	99.13827	97.75673	97.78893	97.13616
2	98.13703	98.44138	98.45458	97.41844	97.46808
3	97.18734	97.66679	97.97365	97.99660	97.20017
i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$	
0	99.44823	98.36707	98.41449	98.60888	
1	98.75068	98.77980	98.08412		
2	97.97594	98.29081	98.34338	97.48916	

POMONA AND NEPTUNE.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30184	97.87284	97.88006	96.57891	96.60080
1	98.93581	98.93823	97.16272	97.17600	96.13855
2	97.74561	98.04799	98.05336	96.62406	96.64479
3	96.60110	97.07917	97.38256	97.39192	96.20429
i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$	
0	99.24289	97.76814	97.78790	98.19032	
1	98.35238	98.36436	97.28235		
2	97.38349	97.69015	97.71220	96.69253	

POLYMNIA AND MERCURY.

This Asteroid has a large eccentricity; but if the coefficients are needed, they may be found from the corresponding ones for Euterpe.

CIRCE AND MERCURY.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30331	98.32692	98.34711	97.49421	97.55358
1	98.32009	99.16857	97.84608	97.88321	97.28705
2	97.35388	98.50037	98.51547	97.53827	97.59468
3	96.43323	97.75468	98.06208	98.08861	97.35065
i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$	
0	99.47982	98.45770	98.51163	98.67394	
1	98.81088	98.84409	98.20572		
2	98.06499	98.38185	98.44155	97.60953	

CIRCE AND VENUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30926	98.89638	98.96644	98.66132	98.84741
1	99.44204	99.46652	98.70970	98.83182	98.76246
2	98.74833	99.06315	99.11676	98.70127	98.88001
3	98.09969	98.58652	98.91187	99.00318	98.81831

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.80399	99.34566	99.51778	99.35709
1	99.39823	99.50992	99.40443	
2	98.92086	99.27774	99.46407	98.87412

CIRCE AND THE EARTH.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.31705	99.21358	99.34684	99.33314	99.65290
1	99.59455	99.64306	99.19829	99.42025	99.62257
2	99.04284	99.37157	99.47638	99.36816	99.67824
3	98.53555	99.03230	99.38251	99.55682	99.66962

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.01550	99.86734	0.06792	99.81940
1	99.73981	99.94551	0.11762	
2	99.39931	99.80879	0.12781	99.46773

CIRCE AND MARS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.34208	99.69466	0.00602	0.41031	1.02450
1	99.81498	99.94124	99.96167	0.43252	1.02307
2	99.45056	99.82672	0.08867	0.43332	1.03816
3	99.12845	99.65982	0.09655	0.50530	1.05094

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.42528	0.73670	1.32990	0.72093
1	0.30192	0.74929	1.31951	
2	0.13234	0.70010	1.31016	0.59365

CIRCE AND JUPITER.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.33410	99.57726	99.83473	0.13928	0.67284
1	99.76283	99.86402	99.77203	0.17139	0.66839
2	99.35681	99.71727	99.92985	0.16560	0.68918
3	98.99378	99.51355	99.92143	0.25940	0.70125
4	98.65082	99.28626	99.83240	0 30001	0.76202
5	98.31949	99.04594	99.69822	0.27809	0.81477

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.31259	0.51304	1.02512	0.47410
1	0.15806	0.53511	1.00947	
2	99.95209	0.47052	1.00088	0.30531
3	99.72381	0.35258	0.96236	

CIRCE AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30997	98.94033	99.01713	98.75312	98.95469
1	99.46239	99.49034	98.77694	98.91012	98.87941
2	98.78993	99.10617	99.16513	98.79253	98.98637
3	98.16149	98.64933	98.97720	99.07735	98.93428
4	97.55394	98.16430	98.65817	99.00085	

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.83148	99.41631	99.60324	99.41649
1	99.44459	99.56669	99.50058	
2	98.98704	99.34951	99.51116	98.87706
3	98.50171	99.01927	99.41491	

CIRCE AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30318	98.30278	98.32191	97.44535	97.50172
1	99.14977	99.15623	97.80977	97.84457	97.22561
2	98.17174	98.47638	98.49067	97.48949	97.54305
3	97.23929	97.71894	98.02628	98.05112	97.28939

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.46693	98.42079	98.47198	98.64738
1	98.78637	98.81786	98.15619	
2	98.02876	98.34477	98.40147	97.56052

CIRCE AND NEPTUNE.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30192	97.90758	97.91539	96.64872	96.67238
1	98.95313	98.95575	97.21493	97.22930	96.22601
2	97.78016	98.08265	98.08846	96.69381	96.61622
3	96.65288	97.13102	97.43461	97.44475	96.29165

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.26071	97.82063	97.84201	98.22641
1	98.38732	98.40028	97.35253	
2	97.43562	97.74274	97.76655	96.76240

LEUCOTHEA AND MERCURY.

If the coefficients for this Asteroid are ever needed, they may be found for

Leucothea and Mercury, from those for Psyche and Mercury;
" " Venus, " " Clio and Saturn;
" " The Earth, " " Themis and Saturn;
" " Mars, " " Proserpine and Jupiter;
" " Jupiter, " " " " Mars;
" " Saturn, " " Clio and Venus;
" " Uranus, " " Psyche and Uranus;
" " Neptune, " " " " Neptune.

ATALANTA AND MERCURY.

Both the inclination and eccentricity of this Asteroid are large. The coefficients may, however, be found for

Atalanta	and	Mercury,	from	those	for Ceres	and	Mercury ;
"	"	Venus,	"	"	"	"	Venus ;
"	"	The Earth,	"	"	"	"	The Earth
"	"	Mars,	"	"	Leda	and	Mars ;
"	"	Jupiter,	"	"	"	"	Jupiter ;
"	"	Saturn,	"	"	"	"	Saturn ;
"	"	Uranus,	"	"	"	"	Uranus ;
"	"	Neptune,	"	"	"	"	Neptune.

FIDES AND MERCURY.

i	log $b_{\frac{1}{2}}^{(i)}$	log $\alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	log $\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	log $\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	log $\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30334	98.34233	98.36336	97 52564	97.58703
1	99.16943	99.17652	97.86957	97.90756	97.32659
2	98.21079	98.51579	98.53145	97.56964	97.62798
3	97.29771	97.77762	98.08559	98.11276	97.39007

i	log $\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.48815	98.48146	98.53724	98.69114
1	98.82665	98.86104	98.23762	
2	98.08829	98.40572	98.46745	97.64108

FIDES AND VENUS.

i	log $b_{\frac{1}{2}}^{(i)}$	log $\alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	log $\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	log $\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	log $\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.31112	98.91285	98.98537	98.69568	98.88746
1	99.45256	99.47543	98.73489	98.86106	98.80622
2	98.76674	99.07928	99.13484	98.73544	98.91973
3	98.12584	98.61009	98.93636	99.03090	98.86171

i	log $\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.81423	99.37209	99.54960	99.37922
1	99.41551	99.53108	99.44038	
2	98.94566	99.30455	99.49654	98.81889

FIDES AND THE EARTH.

i	log $b_{\frac{1}{2}}^{(i)}$	log $\alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	log $\alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	log $\alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	log $\alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.31766	99.23148	99.36942	99.37177	99.70061
1	99.60302	99.65339	99.22612	99.45519	99.67234
2	99.05898	99.38880	99.49751	99.40644	99.72550
3	98.55931	99.07683	99.40905	99.58949	99.71879

i	log $\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	log $\alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	log $\alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.02885	99.89772	0.20714	99.84813
1	99.75960	99.97207	0.15939	
2	99.42637	99.83983	0.16783	99.50735

FIDES AND MARS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.34383	99.71781	0.04076	0.46468	1.09540
1	99.82515	99.95693	99.99953	0.48529	1.09438
2	99.46861	99.84827	0.12093	0.48702	1.10854
3	99.15423	99.67820	0.13138	0.55517	1.12127

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.44879	0.78202	1.39191	0.77204
1	0.33102	0.79313	1.38234	
2	0.16818	0.74551	1.37301	0.65201

FIDES AND JUPITER.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.33280	99.55616	99.80467	0.09118	0.61078
1	99.75333	99.85048	99.73821	0.12542	0.60559
2	99.33952	99 69749	99.90201	0.11808	0 62784
3	98.96881	99.48675	99.89007	0.21629	0.63934
4	98.61821	99.25220	99.79527	0.25578	0.70325
5	98.27928	99.00452	99.65455	0.22968	0.75658

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.29331	0.47363	0.97183	0.43169
1	0.13276	0.59789	0.95499	
2	99.91981	0.43016	0.94673	0.25455
3	99.68431	0.30704	0.90642	

FIDES AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30965	98.92377	98.99797	98.41847	98.91412
1	99.45535	99.48135	98.75160	98.58051	98.83528
2	98.77429	99.08997	99.14686	98.45802	98.94615
3	98.13859	98.62570	98.95259	98.74932	98.89052
4	97.52312	98.13318	98.62644	98.66759	

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.82106	99.08964	99.57082	99.39397
1	99.42706	99.54518	99.46429	
2	98.96210	99.02236	99.51818	98.84199
3	98.46930	98.68541	99.37805	

FIDES AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30311	98.28733	98.30581	97.41408	97.46861
1	99.14210	99.14834	97.78647	97.82010	97.18630
2	98.15650	98.46101	98.47481	97.45828	97.51008
3	97.21648	97.69604	98.00317	98.02676	97.25019

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.45755	98.39718	98.44669	98.63044
1	98.77060	98.80112	98.12452	
2	98.00556	98.32105	98.37590	97.52916

FIDES AND NEPTUNE.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30187	97.89232	97.89987	96.61807	96.64093
1	98.94553	98.94806	97.19200	97.20588	96.18760
2	97.76499	98.06743	98.07304	96.66318	96.68484
3	96.63015	97.10825	97.41175	97.42154	96.25328

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.25288	97.79757	97.81823	98.21055
1	98.37197	98.38449	97.32170	
2	97.41273	97.71964	97.74267	96.73171

LEDA AND MERCURY.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30321	98.30996	98.32940	97.54988	97.51713
1	99.15334	99.15990	97.82059	97.85595	97.24387
2	98.17882	98.48351	98.49804	97.50399	97.55839
3	97.24988	97.72957	98.03702	98.06226	97.30760

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99 47076	98.43176	98.48375	98.65526
1	98.79366	98.82564	98.17092	
2	98.03953	98.35579	98.41337	97.57509

LEDA AND VENUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30884	98.87839	98.94584	98.62383	98.80385
1	99.43328	99.45681	98.68221	98.80002	98.71474
2	98.73123	99.04551	99.09706	98.66399	98.83683
3	98.07428	98.56073	98.88512	98.97302	98.77098

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.79288	99.31685	99.48322	99.33317
1	99.37918	98.48697	99.36527	
2	98.89377	99.24853	99.42882	98.74620

LEDA AND THE EARTH.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.31641	99.19410	99.32240	99.29117	99.60125
1	99.58531	99.63186	99.16803	99.38243	99.56853
2	99.02520	99.35279	99.45349	99.32657	99.62706
3	98.50957	99.00549	99.35366	99.52144	99.61624

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.00176	99.83439	0.12555	99.78848
1	99.71833	99.04133	0.07236	
2	99.36986	99.77514	0.04856	99.42474

LEDA AND MARS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.34027	99.66964	99 96894	0.35205	0.94865
1	99.80397	99.92450	99.92104	0.37612	0.94670
2	99.33094	99.80348	0.05426	0.37578	0.96287
3	99.10038	99 62906	0.05913	0.45206	0.97565

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.40047	0.68831	1.26376	0.66675
1	0.27017	0.70263	1.25242	
2	0.09382	0.65044	1.24310	0.53132

LEDA AND JUPITER.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.33558	99.60071	99.86839	0.19296	0.74223
1	99.77334	99.87918	99.80972	0.22257	0.73855
2	99.37588	99.73922	99.96105	0.21863	0.75804
3	99.02127	99.54316	99.95623	0.30774	0.77039
4	98.68667	99.32380	99.87352	0.34933	0.82783
5	98.36370	99.09154	99.74648	0.33183	0.87978

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.33437	0.55705	1.08489	0.52192
1	0.18642	0.57693	1.07046	
2	99.98800	0.51570	1.06158	0.36209
3	99.76762	0.40336	1.02498	

LEDA AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.31033	98.95854	99.03831	98.79126	98.99952
1	99.47222	99.50027	98.80483	98.94285	98.92805
2	98.80711	99.12397	99.18529	98.83044	99.03082
3	98.18698	98.67527	99.00425	99.10829	98.98248
4	97.58773	98.19844	98.69302	99.03744	

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.84301	99.44572	99.63890	99.44148
1	99.46391	99.59151	99.54063	
2	99.01446	99.37929	99.58768	98.91570
3	98.53734	99.05653	99.45577	

LEDA AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30326	98.31974	98.33981	97.47967	97.53813
1	99.15819	99.16490	97.83533	97.87146	97 26876
2	98.18846	98.49323	98.50809	97.52375	97.57930
3	97.26430	97.74404	98.05154	98.07744	97.33242

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.47598	98.44671	98.49981	98.65602
1	98.80358	98.83627	98.19098	
2	98.05121	98.37081	98.42960	97.59494

LEDA AND NEPTUNE.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30194	97.92430	97.93242	96.68234	96.70690
1	98.96147	98.96419	97.24007	97.25500	96.26814
2	97.79679	98.09934	98.10538	96.72765	96.75067
3	96.67780	97.15598	97.45966	97.47020	96.33372

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.26930	97.84591	97.86811	98.24383
1	98.40415	98.81761	97.38633	
2	97.46072	97.76808	97.79281	96.79606

LÆTITIA AND MERCURY.

The coefficients for this Asteroid may be found from the corresponding ones for Ceres.

HARMONIA AND MERCURY.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30423	98.47913	98.50749	97.80317	97.88502
1	99.23715	99.24679	98.07597	98.12714	97.67598
2	98.34509	98.65151	98.67280	97.84654	97.92449
3	97.49852	97.97942	98.28990	98.32675	97.73826

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.56223	98.69140	98.76601	98.84475
1	98.96591	99.01231	98.51979	
2	98.29357	98.61688	98.69913	97.91961

HARMONIA AND VENUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.31261	99.06005	99.15830	99.00511	99.25466
1	99.52122	99.55619	98.96078	99.12844	99.20136
2	98.90209	99.22294	99.29906	99.04283	99.28278
3	98.32775	98.81887	99.15495	99.28322	99.25315

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.90912	99.61120	99.84300	99.58530
1	99.57245	99.72682	99.76650	
2	99.16747	99.54756	99.79606	99.13285

HARMONIA AND THE EARTH.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.32420	99.39428	99.58142	99.72787	0.14640
1	99.67928	99.74976	99.48116	99.78254	0.13282
2	99.20280	99.54456	99.69502	99.75895	0.16724
3	98.77015	99.27627	99.65003	99.89534	0.17322

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.15436	0.18022	0.57773	0.12521
1	99.94312	0.22497	0.54882	
2	99.67314	0.12902	0.54573	99.87470

HARMONIA AND MARS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.36382	99.94485	0.40343	1.00803	1.80630
1	99.92153	0.11744	0.37491	1.01768	1.80924
2	99.63506	0.05673	0.45020	1.02391	1.81695
3	99.38909	99.95480	0.47565	1.06235	1.82715

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.69826	1.24373	2.02578	1.31181
1	0.62709	1.24574	2.02145	
2	0.52146	1.21949	2.01400	1.24595

HARMONIA AND JUPITER.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.32436	99.38092	99.56357	99.69837	0.10903
1	99.67308	99.74168	99.46002	99.75503	0.09450
2	99.19122	99.53185	99.67844	99.72972	0.13020
3	98.75324	99.25855	99.63024	99.86964	0.13543
4	98.33597	98.95969	99.48173	99.89298	
5	97.92995	98.64694	99.64224		

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	0.14354	0.15659	0.54638	0.10136
1	99.92781	0.20343	0.51619	
2	99.65285	0.10481	0.51378	99.84405
3	99.35330	99.93328	0.45554	

HARMONIA AND SATURN.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30732	98.78034	98.83510	98.42047	98.57016
1	99.38540	99.40436	98.53271	98.62933	98.45637
2	98.63750	98.94918	98.99080	98.46167	98.60501
3	97.93485	98.41945	98.73921	98.81050	98.52453
4	97.25316	97 86101	98.34978	98.67983	

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.74363	99.16100	99.29879	99.20615
1	99.27650	99.36465	99.15379	
2	98.74626	99.09071	99.24077	98.54081
3	98.18760	98.69314	99.06189	

HARMONIA AND URANUS.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30256	98.15197	98.16559	97.14069	97.18134
1	99.07420	99.07942	97.58256	96.60746	96.84293
2	98.02199	98.32629	98.33644	97.18528	97.22385
3	97.01620	97.49511	97.80061	97.81829	96.90761

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.38731	98.19098	98.22863	98.48400
1	98.63368	98.65617	97.84810	
2	97.80237	98.11405	98.15496	97.25513

HARMONIA AND NEPTUNE.

i	$\log b_{\frac{1}{2}}^{(i)}$	$\log \alpha D_\alpha b_{\frac{1}{2}}^{(i)}$	$\log \alpha^2 D_\alpha^2 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^3 b_{\frac{1}{2}}^{(i)}$	$\log \alpha^4 D_\alpha^4 b_{\frac{1}{2}}^{(i)}$
0	0.30165	97.75844	97.76400	96.34922	96.36615
1	98.87876	98.88062	96.99083	97.00107	95.85087
2	97.63175	97.93381	97.93794	96.39450	96.41053
3	96.43044	96.90829	97.21113	97.21834	95.91688

i	$\log \alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 D_\alpha b_{\frac{3}{2}}^{(i)}$	$\log \alpha^3 D_\alpha^2 b_{\frac{3}{2}}^{(i)}$	$\log \alpha^2 b_{\frac{5}{2}}^{(i)}$
0	99.18443	97.59547	97.61075	98.07214
1	98.23744	98.24666	97.05163	
2	97.21187	97.51722	97.53427	96.46262

DAPHNE AND MERCURY.

The coefficients for Daphne may be obtained from the corresponding ones for Iris, if needed.

ISIS AND MERCURY.

The coefficients for this Asteroid may be obtained from the corresponding ones for Lutetia.