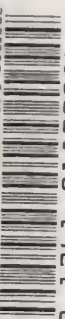


UNIVERSITY OF ST. MICHAEL'S COLLEGE



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Solutions to Exercises In Fundamentals of Logic

by

James D. Carney

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Solutions to Exercises in FUNDAMENTALS OF LOGIC

James D. Carney
Richard K. Scheer

The Macmillan Company, New York

INTRODUCTORY NOTE

This set of solutions to exercises in the textbook, Fundamentals of Logic, is presented for the convenience of instructors. It aims simply to dispose of the class preparation chore of solving the exercises presented to students in the text.

For all except the simplest exercises in Part II, "Formal Logic," (Chapters 7 - 15), we provide solutions. Limits of space in a booklet to be presented free of charge prevent including the solution to every exercise in Part I, "Informal Logic," and Part III, "The Logical Structure of Science." Some of these, of course, are so elementary they offer no problem to instructors; but others require such lengthy explanation that it is feasible only to give solutions to representative exercises of their kind.

In Part I (Chapters 1 - 6), most of the exercises have more than one defensible answer. Accordingly, correct answers may be found that do not appear here.

The instructor should notice that the Roman numerals designating groups of solutions in this manual correspond to numerals in the textbook that designate groups of exercises; these numerals do not refer to section numbers in the text.

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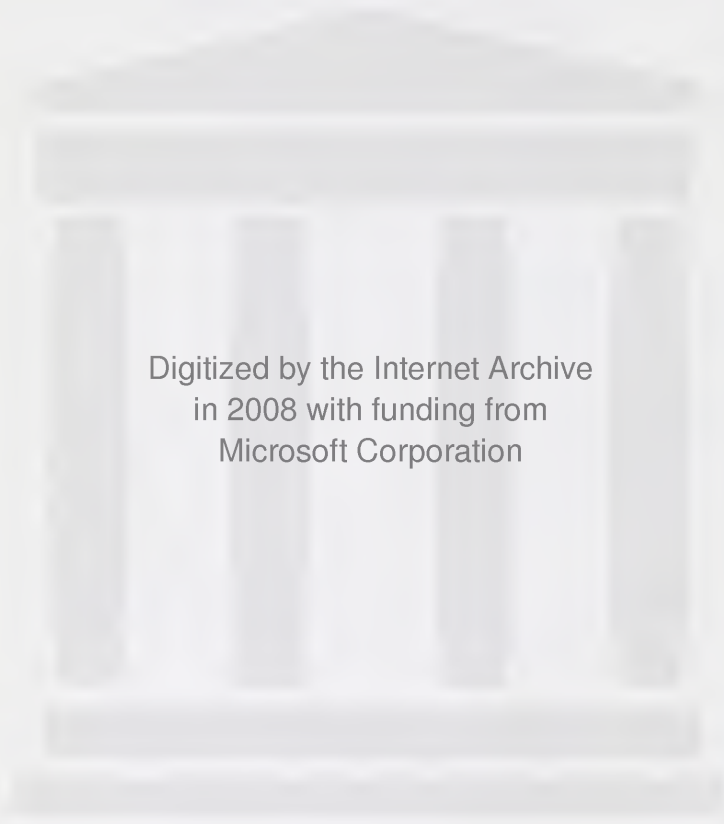
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CHAPTER ONE -- LOGICALLY APPRAISING ARGUMENTS

- I 1. Premiss: Advertising stimulates the economy by inducing the public to buy what they do not essentially need.
Premiss: Advertising creates mass production, employment, and greater physical well being by informing people of the availability of new or improved products.
Conclusion: Advertising is something we should have in America.
6. Hume's argument:
Premiss: It is more probable that the witnesses to the truth of a miracle are mistaken, than that the miracle should happen.
Premiss: Witnesses are the only evidence for miracles.
Conclusion: We do not have conclusive evidence for miracles.
- Boswell's argument:
Premiss: It is reasonable to think that God employs miracles in order to benefit mankind.
Premiss: Men who attest to miracles have no interest in deceiving us, and so on.
Premiss: Prophecies have been fulfilled.
Conclusion: We have as strong evidence for miracles as it is possible to have (supposing that miracles are possible).
9. Premiss: The invasion would not solve the problem in Latin America but would intensify it.
Premiss: If successful, the leaders of the invasion would establish a government which would fail or partially fail on the social and political front. In either case the U.S. would be blamed.
Premiss: If the invasion failed, the two possible reactions of people to the U.S. would be detrimental to the U.S.

Premiss: Cuba, if it is not a Soviet military base, can be tolerated.

Premiss: The U.S. should address itself to the economic needs of Latin America.

Conclusion: The planned invasion by Cuban refugees (March, 1961) should be abandoned.

II Five, six, seven, and nine are clearly correct. See answers below (Chapter Two, III).

III One, three, and four are correct.

CHAPTER TWO -- TRADITIONAL INFORMAL FALLACIES

I 1. Converse accident 2. Ad populum 3. Ad baculum 4. Ad hominem 5. Converse accident 6. Ad hominem 7. Ad ignorantiam 8. Ignoratio elenchi 9. Ignoratio elenchi 10. Petitio principii 11. Ad ignorantiam 12. Ad hominem 13. No fallacy 14. Ad verecundiam or ad hominem 15. Ad hominem, Ad populum 16. Ad populum, Genetic fallacy 17. A complex question but no fallacy. 18. Ad hominem 19. Genetic fallacy 20. Ad hominem 21. Ad hominem 22. Petitio principii

II 1. Hasty generalization 2. Post hoc 3. Post hoc 4. Special pleading 5. Opposition 6. Post hoc 7. Opposition 8. Hasty generalization 9. Special pleading 10. Opposition or post hoc

III 1. Division 2. Composition 3. Composition 4. Division 5. Equivocation 6. Equivocation 7. Equivocation 8. Equivocation 9. Equivocation 10. Equivocation 11. Equivocation 12. Quoting out of context

IV 1. Complex question 2. Tu quoque 3. Post hoc 4. Post hoc 5. Ad populum 6. Equiv-

- ocation or accident 7. False cause but not post hoc. 8. No fallacy--at least no ad verecundiam 9. Satire, but, if regarded as serious, special pleading 10. Ad populum, Post hoc, Equivocation, Ad hominem 11. Petitio principii 12. Tu quoque, Composition 13. Complex question 14. Ad verecundiam 15. Composition 16. Ignoratio elenchi 17. Special pleading 18. Ignoratio elenchi 19. Accident, Equivocation, Ad populum 20. Ad verecundiam, Hasty generalization 21. Equivocation 22. Ad hominem, Ignoratio elenchi, Post hoc, Special pleading, Ad populum 23. Ignoratio elenchi 24. Ignoratio elenchi 25. Ignoratio elenchi 26. Ignoratio elenchi
- V (I) 1. Special pleading (?), Post hoc 2. Special pleading (?) 3. Equivocation 4. Equivocation 5. --- 6. --- 7. --- 8. Each side gives only the reasons for, thus special pleading. 9. ---
- (II) 1. --- 2. --- (poor dilemma) 3. --- 4. Glendower commits a post hoc. 5. Ad populum, Ad ignorantiam, Special pleading, Ad hominem 6. Equivocation, Ad populum 7. Equivocation, Ad populum, (poor analogy)
- (III) (Supposing circumstances which would make each incorrect) 1. Petitio principii 2. Ad baculum 3. Ad populum 4. Division 5. Equivocation 6. Ad ignorantiam 7. Ad ignorantiam 8. False cause (not post hoc) 9. Ad hominem 10. --- (poor analogy)

CHAPTER THREE -- DEFINITIONS

- I A shoe is a covering for the foot which does not reach above the ankle, which is worn for everyday

activities, and which is normally made of leather and has a more or less stiff sole.

- II 1. In Chaucer's Miller's Tale there is a character whose mouth itched for a whole day ("My mouth hath icched al this longe day").
5. The animals called dachshunds at dog shows.
10. Statements which make up the Bill of Rights.
- III 1. "Itch": (C1) The character showed--by facial expressions--that he had an irritation in his mouth. (C2) He tried in various ways to relieve the irritation. (C3) He said, "My mouth hath icched al this longe day."
5. "Dachshund": (C1) A small dog (animal), (C2) with short legs, (C3) long body, (C4) with long drooping ears, and (C5) a short sleek coat.
10. "Laws": (C1) A body of rules, (C2) formally enacted by a governing body, (C3) and recognized as binding on the citizens of the country.
- IV 1. "Conservative": (1) a member of the Conservative party of a country (2) one who wishes to keep intact and unchanged the existing political institutions (3) one who is disposed to maintain existing views and habits (4) a preservative agent.
8. "Work": (1) someone's occupation (2) a literary or musical composition (3) to perform or do something related to one's occupation.
17. "Satellite": (1) a secondary planet which revolves round a larger one (2) a country which is subservient to another (3) a man made object put in orbit round the earth.
- V If X has these characteristics: (C1) weekly publication, (C2) paper cover, (C3) articles by various authors, then, in ordinary circumstances, we call it a magazine.
- If X has these characteristics: (C1) monthly publication, (C2) hard cover, (C3) articles by various authors, then, in ordinary circumstances, we call it a magazine (e.g. Horizon).

VI "Moses" (Old Testament figure--as the word is now used)
 "Work" (as used in connection with what someone does)

- | | | |
|-----|------------------------|----------------------|
| VII | 1. none of these | 8. d |
| | 2. none of these | 9. f |
| | 3. e | 10. d |
| | 4. c or, perhaps, b | 11. a thus c |
| | 5. none of these | 12. c or, perhaps, b |
| | 6. b (in some contexts | 13. d |
| | a, thus c) | 14. d |
| | 7. d | |

VIII 5. "War": The conflict beginning around 1914 between the Allies and Germans is a clear-cut example of a war. The conflict in 1962-1963 between the U.S. and France over trade and defense is a clear-cut example of something we would not hesitate to say is not a war between the U.S. and France.
 Should the relations between the U.S. and the Soviet Union following the Second World War be called a war?
 Is the activity of the U.N. army in Africa against certain forces to be called a war?

- IX 1. Vagueness, sense two, does not imply that there is not a sharp distinction between clear-cut cases of things which are X and those which are not X.
2. If "adequate" means "successful," then there are many adequate ways. If "adequate" means "the only way open so as to prevent misunderstandings," then exact definitions, in some circumstances, could be "inadequate." If "adequate" means "giving a description of the necessary and sufficient conditions," then the first sentence is a tautology and it becomes logically impossible to give an adequate definition for most of our ordinary words.
3. "Aggression" is a class b or d word, thus there is no exact definition to be found or there is no common meaning.

4. Words are picked up by children without being given verbal definitions. We also give ostensive definitions which children and adults properly understand (ordinarily). There is no infinite regress.
 5. A reportive definition can be spoken of as true or false, since such a definition is a statement of how a word is used (at some time by some group of persons).
 6. If such rules were set down no one would follow them, so such an activity would be idle and silly. If such rules were followed, certain undesirable consequences would follow. For example, new metaphors would be ruled out. There would also still be the possibility of misunderstanding since ostensive definitions can be misunderstood.
 7. If this means that things could turn up which we would hesitate to call X or not X, then this statement is true.
 8. See answer to 3.
 9. The fact that two things are called by the same name does not insure that they share a common set of characteristics.
 10. For most ordinary words we have more or less the same paradigm examples in mind. In some contexts two people can use words in accordance with analytic or exact definitions.
 11. There are certain contexts where stipulative definitions are appropriate. Humpty Dumpty's use of them, as is clear, creates misunderstanding and confusion and hence negates the value they have in appropriate circumstances.
- X 1. This appears to be a pseudo-dispute. That is, each party means something different by the word "conservative." A means someone who opposes the aims of the working class and probably, one who champions the aims of the wealthy. B means someone who wants to maintain the status quo. C does not seem to be serious here, but, perhaps, he means by "conservative" what A means.
2. Pseudo-dispute. A and B have different criteria for their use of "progress." A means by "prog-

- ress" decreasing human sin. B means technological improvements.
3. Factual dispute. What did the writers of the First Amendment intend the phrase "freedom of speech and the press" to cover?
 4. Factual dispute, though it looks like a pseudo-dispute. A and B mean the same by "negotiate." B thinks negotiation will lead to an agreement in which each side gains more than it gives up (he calls this "true negotiation"). A does not believe that there is anything that either side can give up which the other side wants. This dispute could be looked at as a definition dispute. B, for some reason which is not clear, thinks we ought to use the word "negotiation" to cover "interchange which leads to an agreement in which each side gains more than it gives up." A, it seems, would want the word to retain its common meaning--discussion aimed at settling differences between two parties.
 5. Pseudo-dispute. Goldwater, it appears, considers being able to dispose of one's profits in the way one wants to a necessary characteristic of being free as he uses the word "freedom." The disputer does not regard this characteristic as necessary in order to speak of someone as "free," though he does think having some property is a necessary characteristic in order to speak of someone as free. It is quite possible that there are some characteristics which both parties include in "being free." For example, belonging to the political party of one's choice and bringing up one's children as one chooses. If the disputer argued that the possession of these characteristics is incompatible, in our society as it is today, with one's being free to dispose of profits as one likes, and if Goldwater opposed this, then this would be in all likelihood a factual dispute.
 6. Word-extension dispute. A thinks that the absence of this characteristic: freedom from military control, is sufficient to prevent our speaking of what was done in the 19th century in the South as "ratification." B thinks all

that is necessary was present, so that it is proper to speak of the action in question as "ratification."

7. Word-extension dispute. Goldwater believes that the characteristics, few though they may be, found in common between federal matching funds and clear-cut cases of blackmail and bribery are sufficient to use these latter notions to cover federal matching funds.
8. Factual dispute. A appears to have misunderstood the Louisiana statute. The statute does not cover things which "might well have led to disturbing and alarming the public."
9. Pseudo-dispute. The lumberman and passer-by have, in these circumstances, different criteria for their use of the phrase "the same axe." The passer-by counts "being of the same material" among his criteria for saying X is the same axe. The lumberman employs "having been used over such-and-such a period of time" as his criteria for saying X is the same axe.
10. Not a verbal dispute of the kind described. Also it is significantly different from clear-cut cases of factual disputes.
11. Could be construed as a definition dispute (of the second kind discussed). Each has a theory of perception which he believes is true and is imposing on the situation in question.
12. Word extension dispute. An extraordinary case. Commonly when we speak of "same person" such characteristics as same appearance, same character, same body, and so on are present. In the Jekyll and Hyde case some are present and some are missing. Should we extend "same person" to cover Jekyll and Hyde?
13. Certainly in this case the man had some of the characteristics commonly present in those situations where we say of a person that he is dead--but, of course, his heart started beating again. There seems to be no basis for saying that one party wants to extend the word "dead" in this extraordinary case. But B seems to want this. He also, for some reason, is anxious to find an instance of a "resurrection."

If B does call this situation an instance of a resurrection, and A opposes this, then this would, it seems, be a pseudo-dispute.

14. A verbal dispute. Jesus makes the point that only in the context of earthly life can we speak of "being married" and "not being married." Depending on how the Sadducees react to this point, it could develop into a word extension dispute.
15. Factual dispute. It is false advertising. B is making a joke or is attempting to defend a false position by making it appear to be a verbal dispute of the pseudo type--i.e. in one sense of "economy" it is the economy size.
16. A good case can be made for this being a word extension dispute. The case in question has several of the characteristics of clear-cut cases of false advertising which come to mind. For example, most were deceived by what the salesmen said. The salesmen intended to deceive. They said just those things which in these circumstances would deceive. The story goes, however, that this case came to court, and the judge said that this was not a case of false advertising.
17. On the whole this can be regarded as a word extension dispute. The episode in question has some (and thus lacks some) of the characteristics of a "military expedition or enterprise from the U.S."

XI Since the distinction between real and nominal definition is not clear, we omit this distinction in the answer, though it is worthwhile raising this issue in class. A great deal of writing would be necessary to justify any answer we might here provide. For one thing, further stipulations are needed to employ the distinction in these exercises.

1. Definition by analysis. Persuasive definition.
2. Definition by analysis. Persuasive definition.
3. Definition by analysis. Stipulative definition.
4. Definition by analysis. He intends that this be a lexical definition, thus we have a false lexical definition.

5. Persuasive definition of "civilized man".
Definition by analysis.
6. Cites of a few characteristics which are present in those who are correctly called "cynics" and "hypocrites." There appears to be no attempt at a definition.
7. Persuasive definition.
8. Definition by example. Lexical definition.
9. Definition by analysis. Theoretical definition (or real definition).
10. Definition by analysis. Stipulative definition.
11. Definition by analysis. Theoretical definition.
12. Definition by analysis. All are intended to be lexical definitions (or, perhaps, real definitions).
13. No definition. "Everyone who commits sin is a slave" appears to be a factual claim.
14. The Century Dictionary definition gives a lexical definition for "philosophy" as the word was and is used in certain contexts. Definition by analysis. James is expressing dismay or some other emotion and is not giving a definition.
Wittgenstein is not giving a definition. Perhaps this can be regarded as a stipulative definition.
15. Lexical definition for "philosophy" as it is used in certain contexts. Definition by analysis.
16. Lexical definition. Definition by analysis.
17. Stipulative definition for the symbol "•".
18. Definition by example. Stipulative definition for "sense-data."

CHAPTER FOUR -- USE OF LANGUAGE

Uses of Language

- I The answers to the elementary examples are obvious. The others involve an analysis which is prohibitively long for this manual.

II Though it is easy enough to imagine or report examples the analysis of the examples is involved, so no attempt will be made to do these exercises.

- III
1. Confuses making a promise with making a prediction.
 2. Confuses contingent statements with logical truths.
 3. Confuses statements which express acts of consciousness with statements which report physical acts.
 4. Confuses expressions of intention with predictions.
 5. Confuses statements of intention with statements which assert a similarity between two things.
 6. Confuses dream reports with reports of past experiences.
 7. Confuses contingent statements with logical truths.
 8. Confuses logical truths (or grammatical statements) with statements which are justified by observation.
 9. Confuses expressions of wishes with predictions.
 10. Confuses the ceremonial use of language with the informative use.
 11. Confuses the directive use of language with the informative use.
 12. Confuses statements in fiction with the ordinary non-fictional informative use of language.
 13. Confuses aesthetic judgments with the informative use of language.
 14. Confuses statements which express how things appear with those which express how things are.

Nonsense

- I
- | | | | | | |
|-------|-----------------------|----------------------------|------------------------|-------|-------------|
| 1. 1 | 2. 2 or none of these | 3. 2, 10, or none of these | 4. 10 | 5. 10 | 6. 10 |
| 7. 10 | 8. 10 | 9. 10 | 10. 1 or none of these | 11. 1 | 12. 6 and 9 |
| | | | | | 13. none |

- II When we say "context-mixing" below, this does not necessarily exclude the possibility that the item might also involve what we are calling category-mixing (and vice-versa). We give what seems to us

to come in mind most readily.

1. Category-mixing. Confuses "nobody" with a proper name.
2. Category-mixing. Context-mixing. An infinite series is not a finite series. Uses the notion of "not being able" as it is used in contexts, say, of counting to the end of a finite series.
3. Context-mixing. Uses "looks like X" as it is used in context where "X" already has a meaning. In this context "horse" so far has not been given a meaning.
4. Context-mixing. It would only make sense to speak of "stingy right hands" in the context of a person's relationship with other persons.
5. Context-mixing. Uses notions of "ownership" as it is used in those contexts where there is agreement and established laws.
6. What X says is misleading and would be context mixing if he uses "walk such-and-such number of miles an hour" as it is used in ordinary contexts (such as walking down a road).
7. Perhaps heaven differs sufficiently from what is around us so that it would be context-mixing to speak of it as a "place."
8. Category-mixing? (Regards God as like a person in ways in which he is not.)
9. Context-mixing. Uses "hang downward" (and so on) as it is used in ordinary contexts. (Bats hang downward with their feet higher than their heads.)
10. Category-mixing. Regards colors and shapes as more like tables and cigars than they are.
11. Category-mixing. Regards the word "good" as a word which names something as do, for example, proper names.
12. Category-mixing. Confuses names of real things with names of fictitious things.
13. Category-mixing. Regards "thing" as a name like "table," "stone," etc.
14. Open invitation to context-mixing, it seems. Some concepts curdle into (context-mixing) nonsense when put together.
15. Context-mixing. Uses notions which need human contexts to have the sense they have.

CHAPTER FIVE -- ANALOGY

1. Explanatory analogy. It would seem that this is a poor explanatory analogy. It gives the impression that institutional authority is a bit more arbitrary, independent of popular support, and so on, than it is generally. The analogy can thus be criticized by calling into question the supposed similarities.
2. Whether or not this is an argumentative or explanatory analogy it is poor. If explanatory, then the supposed similar elements are not similar. If it is an argumentative analogy, then one has dissimilar elements.
3. Argumentative analogy. A poor argumentative analogy. There are a great number of relevant dissimilarities (in the context of Western democracies). In fact, what are the similarities?
4. Three successive arguments by logical analogy. All are good since what Alice says is not true and they bring this out.
5. Argumentative analogy. Actually two analogies are used to support the conclusion--the first statement: Equal armaments on both sides will not prevent war, since in the past it has not; and new weapons will not prevent war, since the development of new weapons did not prevent war in the past. A case can be made for these both being poor. There are several relevant differences. In the first argument, it can be argued that more or less equal armaments can be maintained--i.e., armaments necessary for deterrence. In the second argument it can be maintained that the old weapons were not able to destroy whole hemispheres.
6. Russell is giving two examples of prohibition. Prohibition in America and the prohibition against eating laurel leaves. Apparently the force of this is: what is true in these cases--if a person(s) is not prohibited from doing something, then he will not do it or will not do it in the degree he would do it if he were not prohibited--is true of prohibiting pornography. If this is correct, this is an argumentative analogy. What makes the laurel leaves analogy poor is the fact that some persons

are interested in pornography even in those circumstances where it is available and nothing prevents them from indulging in it. The Prohibition argument looks considerably stronger.

7. A good argument by logical analogy.
8. This is an explanatory analogy. Given what Jesus had in mind, it is a good one.
9. A good explanatory analogy.
10. An explanatory analogy. If there exists the possibility of either capitalism or communism gaining the world, and one can make a good case for this being true, then, obviously, this argumentative analogy is poor. The element said to be similar would not be similar, and this element must be similar for the argument to stand up.
11. Explanatory analogy. A good explanatory analogy, though the candidate might be offended.
12. No analogy. Don B. is making a distinction between official Christians and real Christians. The force of the distinction shows that Don G. is correct if he has official Christians in mind, whereas Don B.'s first statement (lesser evils are not valid in a religious society) is true if real Christians are kept in mind.
13. A good argumentative analogy.
14. A good refutation by logical analogy of "medical materialism." The argument of medical materialism implies not only that religious opinions are mistaken but all beliefs are mistaken.
15. A poor analogical argument. The conditions which made the few communists a danger in the other countries (supposing this is true) were missing in the U.S. when this was written.
16. With a little effort one can find a fairly good argument by logical analogy in this item.
17. An implicit argumentative analogy. We would not think highly of individuals who act in such-and-such a way, so, similarly, we should not think highly of nations when they behave in similar ways. One point against this analogy is that ordinarily an individual would know that he was acting in this way if he did act in this way, whereas individuals which make up countries generally feel they are doing what is right and are ignorant that

they are ignoring all interests except their own. They also often do not have the motives that Russell's individuals have (thinking they are morally and intellectually superior). These two differences certainly tend to lessen our ill feelings towards the actions of the people of a country.

18. An explanatory analogy. What makes this poor is that the statement in the analogy: "the flame is not a distinct entity," seems to involve some kind of equivocation. There are also grounds for arguing that even if we assume that the flame is not a "distinct entity," the elements thought to be similar are not similar.
19. An explanatory analogy. It is seriously doubted by many that explanations, aims, and methods in history are the same as those in the physical sciences, as Hume says. This makes the analogy poor. Perhaps there was a closer connection between the two in Hume's time than there is today.
20. Analogy used to suggest hypotheses.
21. An explanatory analogy. Criticism of this analogy would most likely be directed to Descartes' belief that our beliefs are just a matter of opinion, as is suggested by this analogy.
22. A good argument by logical analogy.

CHAPTER SIX -- DILEMMAS AND PARADOXES

Dilemmas

1. Take the dilemma by the horns. The second if--then premiss is weak.
2. Much depends on what is meant by "fated." In one sense of the word the argument is invalid--the conclusion does not follow from the premisses.
3. This does not seem to be a dilemma, though with great effort it might be reworked into a dilemma. On the face of it the argument is an instance of modus tollens:
If God desires to prevent helpless human beings

from suffering (is benevolent) and has the power (is omnipotent), then no helpless human being would suffer.

Helpless human beings do suffer.

Therefore either God does not desire to prevent helpless human beings from suffering or has not the power.

Understood in this way, the first premiss is open to criticism. One can desire something and be able to do it and still not do it (because of other considerations).

4. Slip between the horns.
5. Take the dilemma by the horns. Is it true that if one does not know a subject, then he cannot inquire about it?
6. In the circumstances this appears to be a realistic dilemma.
7. (Perhaps) take it by the horns. There is a problem about what is meant by "pushing too far" and "carried to its fullness." On an interpretation which does not make the 'if--then' premisses tautologies, it would seem that a good case can be made for doubting both.
8. Either we have the will to use nuclear weapons or we do not.
 If we do not, then there is no deterrent.
 If we do, then "having the will" implies "being willing to exercise it," and thus destroying what we are trying to save by having the deterrent.

This seems realistic (supposing that this formulation is correct).

9. One can, it seems, question whether the only way to stop Caesar's "potential tyranny" is by murdering him. Depending on how the dilemma is explicitly formulated, one would slip between the horns or take it by the horns.
10. Either we should or should not pay taxes.
 If we should pay taxes, then we should support or go along with a tyrant and with unjust government.
 If we should not pay taxes to Caesar, then we should disobey the existing laws and ruling body.

The dilemma appears to be realistic (considering the circumstances). Jesus' reply stresses, it seems, that for the Christian everything is "due to God." What would a Christian conscience demand here?

11. Realistic dilemma.
12. Hume is confronting certain theologians with this dilemma. Hume himself does support all aspects of it, but is starting from what he believes certain theologians hold. From these assumptions, he generates what seems to be a realistic dilemma.

Paradoxes

As was mentioned in the text, there is no commonly accepted solution or resolution of any of these paradoxes. The instructor may find suggested solutions for some in

G. Ryle; Dilemmas.

Whitehead and Russell; Principia Mathematica.

W. V. Quine, "Paradox," Scientific American, April, 1962.

and in the various journals, especially Mind.

For a discussion of 5 see Martin Gardner, "Mathematical Games," Scientific American, March, 1963.

CHAPTER SEVEN -- VALIDITY

- I
1. $(C \cdot Ci) \supset Ci$; valid
 2. Not deductive
 3. $((E \vee I) \cdot \sim I) \supset \sim E$; invalid
 4. $((P \supset E) \cdot \sim E) \supset \sim P$; valid
 5. Not deductive
 6. $((M \vee \sim M) \cdot \sim M) \supset T$; invalid
 7. $\sim(\sim R \cdot H) \supset R$; invalid
 8. $((N \supset B) \cdot (B \supset S)) \supset (\sim S \supset \sim N)$; valid
 9. $((C \vee B) \supset \sim T) \cdot \sim C \supset \sim T$; invalid
 10. $((P \cdot \sim R) \supset \sim B) \cdot B \supset (R \vee \sim P)$; invalid
- II
1. C (supposing the father was not adopted)
 2. LT (supposing the father was not adopted)
 3. CT (if true)
 4. LT
 5. CT (if true)
 6. CT (if true)
 7. CT
 8. LT
 9. CT
 10. LT
- III In normal circumstances, allowing for the usual meanings of the words, none of these is necessarily contradictory.

CHAPTER EIGHT -- STATEMENT CONNECTIVES

- | | | |
|---|-----------------------|------------------------|
| I | 1. TF | 11. N |
| | 2. N (counterfactual) | 12. N |
| | 3. TF | 13. N (counterfactual) |
| | 4. TF | 14. N |
| | 5. GC | 15. GC, LT |
| | 6. N (counterfactual) | 16. N |
| | 7. N | 17. N |
| | 8. LT | 18. TC |
| | 9. TF | 19. N (counterfactual) |
| | 10. LT | 20. TF |

- | | | | |
|----|------|------|-------|
| II | 1. b | 5. b | 8. b |
| | 2. a | 6. b | 9. b |
| | 3. b | 7. a | 10. b |
| | 4. b | | |

- III
1. NP
 2. NP
 3. $S \supset T$
 4. $M \cdot F$
 5. $H \supset B$
 6. $H \supset B$
 7. (generalized conditional) NP
 8. $B \vee \sim H$
 9. $L \cdot \sim B$
 10. NP
 11. $\sim F \cdot J$
 12. $S \vee \sim S$
 13. $Y \supset I$
 14. $Ja \cdot Ji$
 15. $B_1 \cdot B_2$ or just B (ambiguous)

- IV
1. F
 2. F
 3. $T \supset (B \vee J)$ or $T \supset ((B \vee J) \cdot \sim (B \cdot J))$; T
 4. F
 5. F
 6. $(T \cdot B) \supset \sim (J \vee H)$; T
 7. T
 8. F
 9. $T \equiv \sim (B \vee J)$; T
 10. F
 11. $((T \cdot J) \cdot B) \supset (H \cdot D)$; T
 12. T
 13. T
 14. $(J \cdot H) \cdot (T \supset (Ja \cdot Ha))$; F

V Assuming, where no parentheses exist, that ' \sim ' is the weakest connective.

1. \sim, \cdot
2. \supset, \sim
3. \cdot (2nd), \sim (1st), \cdot, \sim and \supset .
4. \cdot (1st), \supset and \sim (2nd), \sim and \cdot .
5. \supset, \equiv, \sim .
6. \supset (1st), \sim (3rd), \supset (2nd), \sim (1st, 3rd), \equiv, \vee and \cdot, \sim .

7. $\cdot, \sim (1st), \supset (1st), \sim (2nd), \vee$ and \supset, \sim
8. $\cdot (2nd), \sim (1st), \equiv, \cdot, \vee, \sim$.
9. $\vee (1st), \vee (2nd), \sim (1st), \cdot (1st), \cdot, \vee, \sim$.
10. $\equiv, \sim (2nd), \cdot (2nd), \cdot, \sim$

VI	1. F	5. F	8. F
	2. T	6. T	9. F
	3. F	7. F	10. F
	4. T		

CHAPTER NINE -- TRUTH TABLES

I 2, 3, 4, 6, 7, 12, 13, 15, 16, 17 are equivalent.

II	1. T	8. T	15. N
	2. N	9. T	16. T
	3. N	10. C	17. T
	4. N	11. T	18. C
	5. C	12. N	19. C
	6. T	13. N	20. C
	7. N	14. T	

III	1. $(\sim(G \cdot S) \cdot S) \supset \sim G$	Valid
	2. $((R \vee N) \cdot \sim N) \supset R$	Valid
	3. $((A \vee P) \cdot \sim A) \supset \sim P$	Invalid
	4. $((B \supset T) \cdot \sim B) \supset \sim T$	Invalid
	5. $(\sim(C \cdot \sim S) \cdot \sim S) \supset C$	Invalid
	6. $((L \vee B) \cdot \sim B) \supset L$	Valid
	7. $((C \supset \sim P) \cdot (\sim P \supset M)) \supset (C \supset M)$	Valid
	8. $((T \vee I) \cdot \sim(T \cdot I)) \supset (T \supset \sim I)$	Valid
	9. $(P \supset ((S \vee E) \cdot \sim S)) \supset E$	Invalid
	10. $((D \vee S) \cdot \sim S) \supset D$	Valid
	11. $((H \supset W) \cdot \sim H) \supset \sim W$	Invalid
	12. $((H \supset C) \cdot (W \supset H) \cdot \sim C) \supset \sim W$	Valid
	13. $(L \supset (M \vee (P \cdot S))) \cdot \sim(M \vee (P \cdot S)) \supset \sim L$	Valid
	14. $((N \supset (D \vee G)) \cdot (\sim N \cdot \sim D)) \supset G$	Invalid
	15. $((\sim Fi \supset \sim Fr) \cdot (\sim T \supset \sim Fi)) \supset ((T \vee Fr) \supset Fi)$	Valid
	16. $(\sim(M \vee E) \supset \sim A) \supset ((\sim E \cdot A) \supset M)$	Valid
	17. Invalid	

18. Invalid
19. Valid
20. Valid

IV 1, 2, 3, 5, 6, 7, 9, 11, 12, 14, 15, 16, 18, 20
are valid.

CHAPTER TEN -- ELEMENTARY INFERENCES

- | | | | | | |
|---|----|----|-----------------------------|-----|------------|
| I | A. | 1. | 3 (1,2) HS | 8. | 2 (1) Add |
| | | 2. | 4 (1,2) Conj | | 3 (2) Add |
| | | | 5 (3,4) CD | | 4 (3) Add |
| | | 3. | 2 (1) Add | | 5 (4) Add |
| | | 4. | 3 (1,2) HS | 9. | 3 (1,2) MP |
| | | 5. | 3 (1,2) DS | | 4 (3) Simp |
| | | 6. | 3 (2) DM | | 5 (4) Simp |
| | | | 4 (3) DN | | 6 (5) Add |
| | | | 5 (4) DN | 10. | 2 (1) Simp |
| | | | 6 (1,5) MP | | 4 (2,3) DS |
| | | 7. | 5 (1,2) Conj | | 6 (4,5) DS |
| | | | 6 (3,5) CD | | |
| | | | 7 (4,6) DS | | |
| | | | 8 (7) Add | | |
| | B. | 1. | 1 $G \supset (S \supset U)$ | | P |
| | | | 2 G | | P |
| | | | 3 $S \supset U$ | | (1,2) MP |
| | | | 4 $\sim U \supset \sim S$ | | (3) Trans |
| | | | 5 $\sim U$ | | P |
| | | | 6 $\sim S$ | | (4,5) MP |
| | | 2. | 1 $N \supset M$ | | P |
| | | | 2 $M \supset D$ | | P |
| | | | 3 $M \supset P$ | | P |
| | | | 4 $\sim P$ | | P |
| | | | 5 $M \vee N$ | | P |
| | | | 6 $\sim P \supset \sim M$ | | (3) Trans |
| | | | 7 $\sim M$ | | (4,6) MP |
| | | | 8 $N \supset D$ | | (1,2) HS |
| | | | 9 N | | (5,7) DS |
| | | 10 | D | | (8,9) MP |

3.	1	$B \supset J$	P
	2	$H \supset D$	P
	3	$\sim(\sim J \vee \sim D) \supset U$	P
	4	$\sim U$	P
	5	$\sim U \supset \sim \sim(\sim J \vee \sim D)$	(3) Trans
	6	$\sim \sim(\sim J \vee \sim D)$	(4,5) MP
	7	$\sim J \vee \sim D$	(6) DN
	8	$J \supset \sim D$	(7) Imp
	9	$B \supset \sim D$	(1,8) HS
	10	$\sim D \supset \sim H$	(2) Trans
	11	$B \supset \sim H$	(9,10) HS
	12	$\sim B \vee \sim H$	(11) Imp
4.	1	P	P
	2	$(P \vee R) \supset D$	P
	3	$P \vee R$	(1) Add
	4	D	(2,3) MP
	5	$P \cdot D$	(1,4) Conj
5.	1	$\sim((A \cdot A) \vee D) \supset Z$	P
	2	$\sim Z$	P
	3	$\sim Z \supset \sim D$	P
	4	$\sim Z \supset \sim \sim((A \cdot A) \vee D)$	(1) Trans
	5	$\sim \sim((A \cdot A) \vee D)$	(2,4) MP
	6	$(A \cdot A) \vee D$	(5) DN
	7	$\sim D$	(2,3) MP
	8	$A \cdot A$	(6,7) DS
	9	A	(8) Taut
6.	1	$E \supset F$	P
	2	$(F \supset D) \cdot (F \supset C)$	P
	3	$\sim D \vee \sim C$	P
	4	$F \supset D$	(2) Simp
	5	$\sim D \supset \sim F$	(4) Trans
	6	$F \supset C$	(2) Simp
	7	$\sim C \supset \sim F$	(6) Trans
	8	$(\sim D \supset \sim F) \cdot (\sim C \supset \sim F)$	(5,7) Conj
	9	$\sim F \vee \sim F$	(3,8) CD
	10	$\sim(F \cdot F)$	(9) DM
	11	$\sim F$	(10) Taut
	12	$\sim F \supset \sim E$	(1) Trans
	13	$\sim E$	(11,12) MP
7.	1	$T \supset (C \cdot O)$	P
	2	$T \cdot B$	P
	3	$(F \cdot F) \vee \sim(\sim W \cdot B)$	P
	4	$W \supset \sim(C \vee D)$	P
	5	T	(2) Simp

6	$C \cdot O$	(1,5) MP
7	C	(6) Simp
8	B	(2) Simp
9	$(\sim W \cdot B) \supset (F \cdot F)$	(3) Imp
10	$(\sim W \cdot B) \supset F$	(9) Taut
11	$\sim\sim(C \vee D) \supset \sim W$	(4) Trans
12	$(C \vee D) \supset \sim W$	(11) DN
13	$(C \vee D)$	(7) Add
14	$\sim W$	(12,13) MP
15	$\sim W \cdot B$	(8,14) Conj
16	F	(10,15) MP

II A.	1.	2	(1) DN	6.	3	(1,2) Conj
		3	(2) DM		4	(3) Imp
		4	(3) DM		5	(4) DN
	2.	2	(1) Assoc		6	(5) Dist
		3	(2) Comm	7.	2	(1) Equiv
		4	(3) Assoc		3	(2) Trans (twice)
	3.	3	(2) Trans		4	(3) DN
		4	(3) DM		5	(4) DM
		5	(4) Imp		6	(5) Imp
		6	(1,5) MP		7	(6) Trans
	4.	4	(2) Exp		8	(7) DM
		5	(3) Comm	8.	2	(1) Equiv
		6	(5) Comm		3	(2) DM
		7	(4,6) MP		4	(3) Comm
		8	(7) DN		5	(4) Imp
		9	(8) Imp		6	(5) Comm
		10	(1,9) MP		7	(6) Dist
	5.	2	(1) Add			
		3	(2) DM			
		4	(3) Equiv			

B.	1.	1	$M \supset (\sim R \supset U)$	P
		2	$M \cdot \sim R$	P
		3	M	(2) Simp
		4	$\sim R \supset U$	(1,3) MP
		5	$\sim R$	(2) Simp
		6	U	(4,5) MP
	2.	1	$W \equiv F$	P
		2	$\sim(W \vee D)$	P
		3	$(W \supset F) \cdot (F \supset W)$	(1) Equiv
		4	$W \supset F$	(3) Simp
		5	$\sim W \cdot \sim D$	(2) DM

	6	$\sim D$	(5) Simp
	7	$\sim D \vee \sim F$	(6) Add
3.	1	$C \supset (K \supset W)$	P
	2	$K \cdot \sim W$	P
	3	$\sim (K \supset W) \supset \sim C$	(1) Trans
	4	$\sim \sim (K \cdot \sim W)$	(2) DN
	5	$\sim (\sim K \vee \sim \sim W)$	(4) DM
	6	$\sim (\sim K \vee W)$	(5) DN
	7	$\sim (K \supset W)$	(6) Imp
	8	$\sim C$	(3,7) MP
4.	1	$G \supset (T \supset U)$	P
	2	$G \cdot \sim U$	P
	3	G	(2) Simp
	4	$T \supset U$	(1,3) MP
	5	$\sim U \supset \sim T$	(4) Trans
	6	$\sim U$	(2) Simp
	7	$\sim T$	(5,6) MP
5.	1	$G \vee (L \cdot T)$	P
	2	$G \supset \sim T$	P
	3	T	P
	4	$\sim \sim T \supset \sim G$	(2) Trans
	5	$T \supset \sim G$	(4) DN
	6	$\sim \sim G \vee (L \cdot T)$	(1) DN
	7	$\sim G \supset (L \cdot T)$	(6) Imp
	8	$T \supset (L \cdot T)$	(5,7) HS
	9	$L \cdot T$	(3,8) MP
	10	L	(9) Simp
6.	1	$(F \supset M) \cdot (E \supset F)$	P
	2	E	P
	3	$E \supset F$	(1) Simp
	4	F	(2,3) MP
	5	$F \supset M$	(1) Simp
	6	M	(4,5) MP
7.	1	$P \vee D$	P (unnecessary)
	2	S	P
	3	$S \supset C$	P
	4	C	(2,3) MP
8.	1	$P \supset D$	P
	2	$D \supset U$	P
	3	$U \supset I$	P
	4	$P \supset U$	(1,2) HS
	5	$P \supset I$	(3,4) HS
9.	1	A	P
	2	N	P

	3	$(A \cdot N \cdot F) \supset \sim S$	P
	4	$(Fr \supset F) \cdot Fr$	P
	5	$Fr \supset F$	(4) Simp
	6	Fr	(4) Simp
	7	F	(5,6) MP
	8	$A \cdot N \cdot F$	(1,2,7) Conj (twice)
	9	$\sim S$	(3,8) MP
10.	1	$P \supset C$	P
	2	$C \supset \sim (F \cdot A)$	P
	3	F	P
	4	$P \supset \sim (F \cdot A)$	(1,2) HS
	5	$\sim \sim (F \cdot A) \supset \sim P$	(4) Trans
	6	$(F \cdot A) \supset \sim P$	(5) DN
	7	$F \supset (A \supset \sim P)$	(6) Exp
	8	$A \supset \sim P$	(3,7) MP
	9	$\sim \sim P \supset \sim A$	(8) Trans
	10	$P \supset \sim A$	(9) DN
11.	1	$C \supset (H \vee D)$	P
	2	$\sim H$	P
	3	$C \supset (\sim \sim H \vee D)$	(2) DN
	4	$C \supset (\sim H \supset D)$	(3) Imp
	5	$(C \cdot \sim H) \supset D$	(4) Exp
	6	$\sim H \supset (C \supset D)$	(5) Exp
	7	$C \supset D$	(2,6) MP
12.	1	$((R \cdot D) \vee W) \cdot \sim ((R \cdot D) \cdot W)$	P
	2	W	P
	3	$\sim \sim W$	(2) DN
	4	$\sim ((R \cdot D) \cdot W)$	(1) Simp
	5	$\sim (R \cdot D) \vee \sim W$	(4) DM
	6	$\sim (R \cdot D)$	(3,5) DS
13.	1	$D \supset T$	P
	2	$\sim D \supset H$	P
	3	$H \supset F$	P
	4	$(D \supset T) \cdot (H \supset F)$	(1,3) Conj
	5	$\sim \sim D \vee H$	(2) Imp
	6	$D \vee H$	(5) DN
	7	$T \vee F$	(4,6) CD
	8	$\sim \sim T \vee F$	(7) DN
	9	$\sim T \supset F$	(8) Imp
14.	1	$(D \cdot \sim (A \vee B)) \supset C$	P
	2	$\sim (C \vee A)$	P
	3	$D \cdot (C \vee \sim B)$	P
	4	D	(3) Simp

5	$D \supset (\sim(A \vee B) \supset C)$	(1) Exp
6	$\sim(A \vee B) \supset C$	(4,5) MP
7	$\sim C \supset \sim\sim(A \vee B)$	(6) Trans
8	$\sim C \supset (A \vee B)$	(7) DN
9	$\sim C \supset (\sim\sim A \vee B)$	(8) DN
10	$\sim C \supset (\sim A \supset B)$	(9) Imp
11	$(\sim C \cdot \sim A) \supset B$	(10) Exp
12	$\sim C \cdot \sim A$	(2) DM
13	B	(11,12) MP
15.	1 $(D \cdot (M \vee F)) \vee (D \cdot (G \vee A))$	P
	2 $D \cdot ((M \vee F) \vee (G \vee A))$	(1) Dist
	3 D	(2) Simp

III	1.	1	$A \supset B$	P
		2	A	Hyp. RCP
		3	B	(1,2) MP
		4	$A \cdot B$	(2,3) Conj
		5	$A \supset (A \cdot B)$	(2,4) RCP
	2.	1	$L \supset F$	P
		2	$L \supset (F \supset B)$	P
		3	$F \supset (B \supset S)$	P
		4	L	Hyp. RCP
		5	F	(1,4) MP
		6	$F \supset B$	(2,4) MP
		7	B	(5,6) MP
		8	$B \supset S$	(3,5) MP
		9	S	(7,8) MP
		10	$L \supset S$	(4,9) RCP
	3.	1	$\sim F \supset (R \supset L)$	P
		2	$F \vee H$	P
		3	R	P
		4	$\sim F$	Hyp. RCP
		5	$R \supset L$	(1,4) MP
		6	L	(3,5) MP
		7	$\sim F \supset L$	(4,6) RCP
	4.	1	$A \supset ((B \cdot C) \vee E)$	P
		2	$(B \cdot C) \supset \sim A$	P
		3	$D \supset \sim E$	P
		4	A	Hyp. RCP
		5	$(B \cdot C) \vee E$	(1,4) MP
		6	$\sim\sim A \supset \sim(B \cdot C)$	(2) Trans
		7	$A \supset \sim(B \cdot C)$	(6) DN
		8	$\sim(B \cdot C)$	(4,7) MP
		9	E	(5,8) DS

	10	$\sim\sim E \supset \sim D$	(3) Trans
	11	$E \supset \sim D$	(10) DN
	12	$\sim D$	(9,11) MP
	13	$A \supset \sim D$	(4,12) RCP
5.	1	$(P \vee Q) \supset R$	P
	2	$(S \vee T) \supset ((A \vee B) \supset P)$	P
	3	S	Hyp. RCP
	4	A	Hyp. RCP
	5	$S \vee T$	(3) Add
	6	$(A \vee B) \supset P$	(2,5) MP
	7	$A \vee B$	(4) Add
	8	P	(6,7) MP
	9	$P \vee Q$	(8) Add
	10	R	(1,9) MP
	11	$A \supset R$	(4,10) RCP
	12	$S \supset (A \supset R)$	(3,11) RCP
6.	1	$(S \supset W) \cdot (E \supset F)$	P
	2	$S \cdot E$	Hyp. RCP
	3	$S \supset W$	(1) Simp
	4	S	(2) Simp
	5	W	(3,4) MP
	6	$E \supset F$	(1) Simp
	7	E	(2) Simp
	8	F	(6,7) MP
	9	$W \cdot F$	(5,8) Conj
	10	$(S \cdot E) \supset (W \cdot F)$	(2,9) RCP
IV	1.	1 B $\vee \sim C$	P
		2 C	P
		3 $\sim B$	Hyp. RAA
		4 $\sim C$	(1,3) DS
		5 $C \cdot \sim C$	(2,4) Conj
		6 B	(3,5) RAA
	2.	1 $B \supset A$	P
		2 $\sim(A \cdot \sim C) \supset B$	P
		3 $\sim A$	Hyp. RAA
		4 $\sim A \supset \sim B$	(1) Trans
		5 $\sim B$	(3,4) MP
		6 $(\sim A \vee \sim \sim C) \supset B$	(2) DM
		7 $(\sim A \vee C) \supset B$	(6) DN
		8 $\sim A \vee C$	(3) Add
		9 B	(7,8) MP
		10 $B \cdot \sim B$	(5,9) Conj
		11 A	(3,10) RAA

3.	1	$\sim B \supset E$	P
	2	$D \supset \sim E$	P
	3	$\sim (\sim D \cdot \sim B)$	P
	4	$\sim B$	Hyp. RAA
	5	E	(1,4) MP
	6	$\sim \sim E \supset \sim D$	(2) Trans
	7	$E \supset \sim D$	(6) DN
	8	$\sim D$	(5,7) MP
	9	$\sim \sim D \vee \sim \sim B$	(3) DM
	10	$D \vee B$	(9) DN (twice)
	11	B	(8,10) DS
	12	$B \cdot \sim B$	(4,11) Conj
	13	B	(4,12) RAA
4.	1	$B \supset W$	P
	2	$W \supset \sim P$	P
	3	$\sim B \vee P$	P
	4	$\sim (B \supset \sim W)$	Hyp. RAA
	5	$\sim (\sim B \vee \sim W)$	(4) Imp
	6	$B \cdot W$	(5) DM
	7	B	(6) Simp
	8	$B \supset \sim P$	(1,2) HS
	9	$\sim P$	(7,8) MP
	10	$\sim \sim B$	(7) DN
	11	P	(3,10) DS
	12	$P \cdot \sim P$	(9,11) Conj
	13	$B \supset \sim W$	(4,12) RAA
5.	1	$F \vee W$	P
	2	$F \supset D$	P
	3	$W \supset K$	P
	4	$\sim (D \vee K)$	Hyp. RAA
	5	$\sim D \cdot \sim K$	(4) DM
	6	$\sim D$	(5) Simp
	7	$\sim K$	(5) Simp
	8	$\sim K \supset \sim W$	(3) Trans
	9	$\sim W$	(7,8) MP
	10	$\sim \sim F \vee W$	(1) DN
	11	$\sim F \supset W$	(10) Imp
	12	$\sim W \supset \sim \sim F$	(11) DN
	13	$\sim W \supset F$	(12) DN
	14	$\sim D \supset \sim F$	(2) Trans
	15	F	(9,13) MP
	16	$\sim F$	(6,14) MP
	17	$F \cdot \sim F$	(15,16) Conj
	18	$D \vee K$	(4,17) RAA

- V
1. Inconsistent: $P - F, Q - T$
 2. Consistent
 3. Inconsistent: $A - F, B - T$
 4. Consistent
 5. Consistent
 6. Consistent
 7. Inconsistent: $P - F, Q - T, R - F$
 8. Consistent

- VI
1. $\sim (P \vee Q) \vee R$ P
 $P \cdot S$ P
 $(P \vee Q) \supset R$ (1) Imp
 P (2) Simp
 $P \vee Q$ (4) Add
 R (3,5) MP
 $P \cdot R$ (4,6) Conj
 1. $(S \supset Q) \supset R$ P
 $(P \cdot S) \supset Q$ P
 $P \supset (S \supset Q)$ (2) Exp
 $P \supset R$ (1,3) HS
 1. $S \supset P$
 $P \supset \sim (V \cdot N)$
 $\sim V \supset \sim P$
 $\sim P \cdot N$
 P Invalid
 1. $\sim (P \vee M) \vee (S \cdot R)$ P
 $\sim S$ P
 $(P \vee M) \supset (S \cdot R)$ (1) Imp
 $\sim (S \cdot R) \supset \sim (P \vee M)$ (3) Trans
 $(\sim S \vee \sim R) \supset \sim (P \vee M)$ (4) DM
 $\sim S \vee \sim R$ (2) Add
 $\sim (P \vee M)$ (5,6) MP
 $\sim P \cdot \sim M$ (7) DM
 $\sim M$ (8) Simp
 - Inconsistent Premises
 1. $(A \vee B) \supset (C \cdot D)$ P
 $(D \vee E) \supset F$ P
 A P
 $A \vee B$ (3) Add
 $C \cdot D$ (1,3) MP
 D (5) Simp
 $D \vee E$ (6) Add
 F (2,7) MP

7.	1	$(S \vee W) \supset (B \cdot T)$	P
	2	$(T \vee H) \supset B$	P
	3	S	P
	4	$S \vee W$	(3) Add
	5	$B \cdot T$	(1,4) MP
	6	T	(5) Simp
	7	$T \vee H$	(6) Add
	8	B	(2,7) MP
8.	1	$P \supset R$	P
	2	$(\sim P \vee R) \supset (S \supset Q)$	P
	3	$(P \cdot P) \supset R$	(1) Taut
	4	$P \supset (P \supset R)$	(3) Exp
	5	$(P \supset R) \supset (S \supset Q)$	(2) Imp
	6	$P \supset (S \supset Q)$	(4,5) HS
9.		Invalid Argument	
10.	1	$(Q \cdot (R \vee S)) \supset \sim P$	P
	2	P	P
	3	S	P
	4	$P \supset \sim (Q \cdot (R \vee S))$	(1) Trans, DN
	5	$\sim (Q \cdot (R \vee S))$	(2,4) MP
	6	$\sim ((Q \cdot R) \vee (Q \cdot S))$	(5) Dist
	7	$\sim (Q \cdot R) \cdot \sim (Q \cdot S)$	(6) DM
	8	$\sim (Q \cdot S)$	(7) Simp
	9	$\sim Q \vee \sim S$	(8) DM
	10	$S \supset \sim Q$	(9) Imp
	11	$\sim Q$	(3,10) MP
11.	1	$\sim A \supset \sim B$	P
	2	$A \supset C$	P
	3	$B \vee D$	P
	4	$D \supset E$	P
	5	$B \supset A$	(1) Trans, DN (twice)
	6	$B \supset C$	(2,5) HS
	7	$\sim \sim B \vee D$	(3) DN
	8	$\sim B \supset D$	(7) Imp
	9	$\sim B \supset E$	(4,8) HS
	10	$\sim E \supset B$	(9) Trans, DN
	11	$\sim E \supset C$	(6,10) HS
	12	$\sim \sim E \vee C$	(11) Imp
	13	$E \vee C$	DN (13)
12.	1	$\sim A \vee \sim B$	P
	2	$(A \supset C) \cdot ((A \cdot C) \supset B)$	P
	3	$A \supset C$	(2) Simp
	4	$A \supset \sim B$	(1) Imp

	5	$(A \cdot C) \supset B$	(2) Simp
	6	$B \supset \sim A$	(4) Trans, DN
	7	$(A \cdot C) \supset \sim A$	(5,7) HS
13.	1	$P \supset (Q \vee (R \cdot S))$	P
	2	$\sim R \vee \sim S$	P
	3	$\sim Q$	P
	4	$\sim (R \cdot S)$	(2) DM
	5	$\sim (Q \vee (R \cdot S)) \supset \sim P$	(1) Trans
	6	$(\sim Q \cdot \sim (R \cdot S)) \supset \sim P$	(5) DM
	7	$\sim Q \cdot \sim (R \cdot S)$	(3,4) Conj
	8	$\sim P$	(6,7) MP
14.		Invalid argument	
15.		Invalid argument	
16.	1	$(P \supset Q) \cdot (R \supset S)$	P
	2	$\sim (Q \equiv R)$	P
	3	$(\sim P \supset R) \vee (Q \equiv R)$	P
	4	$\sim P \supset R$	(2,3) DS
	5	$\sim \sim P \vee R$	(4) Imp
	6	$P \vee Q$	(5) DN
	7	$Q \vee S$	(1,6) CD
17.		Invalid	
18.	1	$C \equiv J$	P
	2	$\sim J$	P
	3	$(C \supset J) \cdot (J \supset C)$	(1) Equiv
	4	$C \supset J$	(3) Simp
	5	$\sim J \supset \sim C$	(4) Trans
	6	$\sim C$	(2,5) MP
	7	$\sim C \vee (E \vee \sim K)$	(6) Add
	8	$C \supset (E \vee \sim K)$	(7) Imp
19.		Invalid argument	
20.		Inconsistent premisses	
21.		Invalid	
22.		Not truth-functional	
23.		Inconsistent premisses	
24.	1	$(J \vee R) \supset (D \cdot V)$	P
	2	J	Hyp. RCP
	3	$J \vee R$	(2) Add
	4	$D \cdot V$	(1,3) MP
	5	D	(4) Simp
	6	$J \supset D$	(2,5) RCP
	7	$\sim J \vee D$	(6) Imp
25.	1	$U \supset (V \vee W)$	P
	2	$(W \cdot X) \supset Y$	P
	3	$\sim Z \supset (X \cdot \sim Y)$	P

4	U	Hyp. RCP
5	$V \vee W$	(1,4) MP
6	$W \supset (X \supset Y)$	(2) Exp
7	$\sim (X \cdot \sim Y) \supset Z$	(6) Trans, DN
8	$\sim X \vee \sim \sim Y \supset Z$	(7) DM
9	$\sim X \vee Y \supset Z$	(8) DN
10	$(X \supset Y) \supset Z$	(9) Imp
11	$W \supset Z$	(6,10) HS
12	$\sim \sim V \vee W$	(5) DN
13	$\sim V \supset W$	(12) Imp
14	$\sim V \supset Z$	(11,13) HS
15	$U \supset (\sim V \supset Z)$	(4,14) RCP
16	$U \supset (\sim \sim V \vee Z)$	(15) Imp
17	$V \supset (V \vee Z)$	(16) DN
26.	1 $H \supset (L \cdot R)$	P
	2 $(L \vee W) \supset P$	P
	3 $W \vee H$	P
	4 $\sim \sim W \vee H$	(3) DN
	5 $\sim W \supset H$	(4) Imp
	6 $\sim W \supset (L \cdot R)$	(1,5) HS
	7 $\sim \sim W \vee (L \cdot R)$	(6) Imp
	8 $\sim (\sim \sim W \cdot \sim (L \cdot R))$	(7) DM, DN (twice)
	9 $\sim (\sim W \cdot \sim (L \cdot R))$	(8) DN
	10 $\sim (\sim W \cdot (\sim L \vee \sim R))$	(9) DM
	11 $\sim ((\sim W \cdot \sim L) \vee (\sim W \cdot \sim R))$	(10) Dist
	12 $\sim (\sim W \cdot \sim L) \cdot \sim (\sim W \cdot \sim R)$	(11) DM
	13 $\sim (\sim W \cdot \sim L)$	(12) Simp
	14 $W \vee L$	(13) DM, DN (twice)
	15 P	(2,14) MP
27.	Invalid	
28.	Invalid	
29.	Not truth-functional	
30.	1 $(R \cdot F) \vee D$	P
	2 $\sim D \vee F$	P
	3 $\sim F$	Hyp. RAA
	4 $D \supset F$	(2) Imp
	5 $\sim F \supset \sim D$	(4) Trans
	6 $\sim D$	(3,5) MP
	7 $\sim (\sim (R \cdot F) \cdot \sim D)$	(1) DM, DN (twice)
	8 $\sim ((\sim R \vee \sim F) \cdot \sim D)$	(7) DM
	9 $\sim ((\sim D \cdot \sim R) \vee (\sim D \cdot \sim F))$	(8) Dist
	10 $\sim (\sim D \cdot \sim R) \cdot \sim (\sim D \cdot \sim F)$	(9) DM
	11 $\sim (\sim D \cdot \sim F)$	(10) Simp
	12 $D \vee F$	(11) DM, DN (twice)

13	$\sim \sim F \vee D$	(12) DN
14	$\sim F \supset D$	(13) Imp
15	D	(3,14) MP
16	$D \cdot \sim D$	(6,15) Conj
17	F	(3,16) RAA

CHAPTER ELEVEN -- QUANTIFICATION

- I
- | | |
|------------------------------------|--|
| 1. $(\exists x)(Px \cdot Nx)$ | 12. $(\exists x)(Lx \cdot \sim Ax)$ |
| 2. $(\exists x)(Bx \cdot \sim Sx)$ | 13. $(\exists x)(Fx \cdot Lx)$ |
| 3. $(\exists x)(Tx \cdot \sim Fx)$ | 14. $(\exists x)(Ex \cdot Sx)$ |
| 4. $(\exists x)(Bx \cdot Mx)$ | 15. $(\exists x)(Ex \cdot Fx)$ |
| 5. $(\exists x)(Px \cdot Sx)$ | 16. $(\exists x)(Bx \cdot Fx)$ |
| 6. $(\exists x)(Bx \cdot \sim Yx)$ | 17. $(\exists x)(Hx \cdot Sx)$ or/and |
| 7. $(\exists x)(Px \cdot Fx)$ | $(\exists x)(Hx \cdot \sim Sx)$ |
| 8. $(\exists x)(Sx \cdot Fx)$ | 18. $(\exists x)(Fx \cdot \sim Hx)$ |
| 9. $(\exists x)(Px \cdot Sx)$ | 19. $(\exists x)(Sx \cdot Dx)$ |
| 10. $(\exists x)(Bx \cdot Wx)$ | 20. $(\exists x)(Px \cdot \sim Ax)$ or/and |
| 11. $(\exists x)(Vx \cdot Ax)$ | $(\exists x)(Px \cdot Ax)$ |
- II
- | | |
|--|--|
| 1. $(x)(Px \supset Nx)$ | 11. $(x)(Wx \supset Vx)$ |
| 2. $(x)(Px \supset Nx)$ | 12. $(\exists x)(Hx \cdot \sim Tx)$ |
| 3. $\sim (x)(Px \supset \sim Nx)$
or $(\exists x)(Px \cdot Nx)$ | 13. $(x)(Hx \supset \sim Tx)$ |
| 4. $(x)(Nx \supset Px)$ | 14. $(\exists x)(Bx \supset Lx)$ |
| 5. $\sim (x)(Px \supset Nx)$
or $(\exists x)(Px \cdot Nx)$ | 15. $(x)(Px \supset \sim Lx)$ |
| 6. $(x)(Px \supset \sim Nx)$ | 16. $(\exists x)(Px \cdot \sim Rx)$ or/and |
| 7. $(\exists x)(Fx \cdot Ox)$ | $(\exists x)(Px \cdot Rx)$ |
| 8. $(x)(Cx \supset Ox)$ | 17. $(x)(Gx \supset Lx)$ |
| 9. $(x)(Bx \supset Fx)$ | Lx = nothing can save x. |
| 10. $(x)(Dx \supset \sim Bx)$ | 18. $(x)(Gx \supset Lx)$ |
| | 19. $(x)(Lx \supset Gx)$ |
| | 20. $(x)(Gx \supset Lx)$ |
- III
- | |
|---|
| 1. $(x)(Px \supset (\exists y)(Cy \cdot Exy))$ |
| 2. $(\exists x)(Dx \cdot (y)(Py \supset Fxy))$ |
| 3. $(\exists x)(Ax \cdot (\exists y)(Py \cdot Bxy))$ |
| 4. $(\exists x)(Sx \cdot (\exists y)(Ty \cdot Exy))$ |
| 5. $(x)(Px \supset (y)(Cy \supset Hxy))$ |
| 6. $(x)(Px \supset (y)(Ly \supset \text{Loves } xy))$ |
| 7. $(x)(Lx \supset (y)(Py \supset Wyx))$ |

8. $(x)(Cx \supset (y)(Py \supset \sim Cyx))$
9. $(x)(Cx \supset (\exists y)(Py \cdot \sim Vyx))$
10. $(\exists x)(Nx \cdot (y)(Oy \supset Lxy))$
 $Oy = y$ is a number other than 0.
11. $(\exists x)(Px \cdot (y)(Cy \supset \sim Vxy))$
12. $(x)(y)(Nx \cdot Ny \supset (Lxy \supset \sim Exy))$
13. $(x)(Rx \supset (y)(Fy \supset Lxy))$
14. $(x)(Cx \supset (\exists y)(Hy \cdot Wxy))$
15. $(x)(Px \supset (y)(Ty \supset \sim Lxy))$
16. $(x)(Mx \supset (\exists y)(Wy \cdot Fyx))$
17. $(x)(Px \supset (\exists y)(Sy \cdot \sim Jxy))$
18. $(\exists x)(Bx \cdot (\exists y)(Py \cdot \sim Iyx))$
19. $(\exists x)(Sx \cdot (y)(Ey \supset Wxy))$
20. $(x)(Nx \supset (\exists y)(Ny \cdot Gyx))$

- IV
- | | |
|------------------------|------------------------|
| 1. y free; x bound | 2. x,y free; x,y bound |
| 3. y free; x,y,z bound | 4. x,y,z bound |
| 5. x free; x,y bound | 6. x free; x,y,z bound |

CHAPTER TWELVE -- ARISTOTELIAN LOGIC

- I
- | | |
|--------------------|----------|
| 1. Not a syllogism | 7. R2 |
| 2. Not a syllogism | 8. R3,4 |
| 3. Valid | 9. R6 |
| 4. R1 | 10. R5,6 |
| 5. Valid | 11. R6 |
| 6. R2 | |
- II
1. Premises 1 and 3 yield "All B are D" which, with premiss 2, yields "No B are M".
 2. Premises 1 and 3 yield "No CT are W" which, with premiss 2, yields "No CT are C".
 3. Premises 1 and 4 yield "All S are C". Premises 3 and 5 yield "All SH are T". "All SH are T" and 2 yield "All SH are S", which, with "All S are C" yield "All SH are C", i.e. "Shakespeare was clever".
 4. Premises 1 and 3 yield "All T are R", which, with premiss 2, yields "No T are H", i.e. "No Hedgehogs take in the Times".

5. Premises 1 and 4 yield "All L are RO", which with 2, yields "No L are S", which, with 5, yields "No HA are S", which, with 3, yields "No EE are S", i.e. "These Sorites are not easy examples."

- III
1. All Athenians are men.
 2. All rare people should be honored.
 3. All human beings make mistakes.
 4. You are English.
 5. Whatever the critics say is best is best.
 6. Some major nations are in Europe.
 7. Whenever Russia increases hers, we need to increase ours.
 8. ____.

CHAPTER THIRTEEN -- INFERENCES INVOLVING QUANTIFIERS

- | | | | | | |
|---|----|----|---|---------------------------------|------------|
| I | A. | 1. | 1 | $(x)(Cx \supset Yx)$ | P |
| | | | 2 | $(\exists x)(Ox \cdot Cx)$ | P |
| | | | 3 | $Oa \cdot Ca$ | (2) El |
| | | | 4 | $Ca \supset Ya$ | (1) UG |
| | | | 5 | Oa | (3) Simp |
| | | | 6 | Ca | (3) Simp |
| | | | 7 | Ya | (4,6) MP |
| | | | 8 | $Oa \cdot Ya$ | (5,7) Conj |
| | | | 9 | $(\exists x)(Ox \cdot Yx)$ | (8) EG |
| | 2. | 1 | 1 | $(x)(Cx \supset \sim Ux)$ | P |
| | | 2 | 2 | $(\exists x)(Ax \cdot Cx)$ | P |
| | | 3 | 3 | $Aa \cdot Ca$ | (2) El |
| | | 4 | 4 | $Ca \supset \sim Ua$ | (1) U1 |
| | | 5 | 5 | Aa | (3) Simp |
| | | 6 | 6 | Ca | (3) Simp |
| | | 7 | 7 | $\sim Ua$ | (4,6) MP |
| | | 8 | 8 | $Aa \cdot \sim Ua$ | (5,7) Conj |
| | | 9 | 9 | $(\exists x)(Ax \cdot \sim Ux)$ | (8) EG |
| | 3. | 1 | 1 | $(x)(Ax \supset Bx)$ | P |
| | | 2 | 2 | $(x)(Px \supset \sim Bx)$ | P |
| | | 3 | 3 | $Aa \supset Ba$ | (1) U1 |
| | | 4 | 4 | $Pa \supset \sim Ba$ | (2) U1 |

	5	$\sim Ba \supset \sim Aa$	(3) Trans
	6	$Pa \supset \sim Aa$	(4,5) HS
	7	$(x)(Px \supset \sim Ax)$	(6) UG
4.	1	$(\exists x)(Px \cdot Bx)$	P
	2	$(x)(Px \supset Hx)$	P
	3	$Pa \cdot Ba$	(1) E1
	4	$Pa \supset Ha$	(2) U1
	5	Pa	(3) Simp
	6	Ba	(3) Simp
	7	Ha	(4,5) MP
	8	$Ha \cdot Ba$	(6,7) Conj
	9	$(\exists x)(Hx \cdot Bx)$	(8) EG
5.	1	$(x)(Rx \supset Px)$	P
	2	$(\exists x)(Rx \cdot \sim Dx)$	P
	3	$Ra \cdot \sim Da$	(2) E1
	4	$Ra \supset Pa$	(1) U1
	5	Ra	(3) Simp
	6	$\sim Da$	(3) Simp
	7	Pa	(4,5) MP
	8	$Pa \cdot \sim Da$	(6,7) Conj
	9	$(\exists x)(Px \cdot \sim Dx)$	(8) EG
6.	1	$(x)(Wx \supset Ax)$	P
	2	$(x)(Ax \supset \sim Gx)$	P
	3	$Wa \supset Aa$	(1) U1
	4	$Aa \supset \sim Ga$	(2) U1
	5	$Wa \supset \sim Ga$	(3,4) HS
	6	$\sim \sim Ga \supset \sim Wa$	(5) Trans
	7	$Ga \supset \sim Wa$	(6) DN
	8	$(x)(Gx \supset \sim Wx)$	(7) UG
7.	1	$(x)(Wx \supset Ax)$	P
	2	$(x)(Ax \supset \sim Gx)$	P
	3	$(\exists x)Gx$	P
	4	Ga	(3) E1
	5	$Wa \supset Aa$	(1) U1
	6	$Aa \supset \sim Ga$	(2) U1
	7	$Ua \supset \sim Ga$	(5,6) HS
	8	$\sim \sim Ga \supset \sim Wa$	(7) Trans
	9	$Ga \supset \sim Wa$	(8) DN
	10	$\sim Wa$	(4,9) MP
	11	$Ga \cdot \sim Wa$	(4,10) Conj
	12	$(\exists x)(Gx \cdot \sim Wx)$	(11) EG
8.	1	$(\exists x)(Px \cdot \sim Lx)$	P
	2	$(x)(Px \supset Gx)$	P
	3	$Pa \cdot \sim La$	(1) E1

	4	$Pa \supset Ga$	(2) U1
	5	Pa	(3) Simp
	6	$\sim La$	(3) Simp
	7	Ga	(4,5) MP
	8	$Ga \cdot \sim La$	(6,7) Conj
	9	$(\exists x)(Gx \cdot \sim Lx)$	(8) EG
I C. 1.	1	$(x)(Nx \supset \sim Gx)$	P
	2	$(\exists x)(Nx \cdot Bx)$	P
	3	$Na \cdot Ba$	(2) El
	4	$Na \supset \sim Ga$	(1) U1
	5	Na	(3) Simp
	6	Ba	(3) Simp
	7	$\sim Ga$	(4,5) MP
	8	$Na \cdot Ba \cdot \sim Ga$	(5,6,7) Conj (twice)
	9	$(\exists x)(Nx \cdot Bx \cdot \sim Gx)$	EG (8)
2.	1	$(x)(Gx \supset Bx)$	P
	2	$\sim B(e)$	P
	3	$G(e) \supset B(e)$	(1) U1
	4	$\sim B(e) \supset \sim G(e)$	(3) Trans
	5	$\sim G(e)$	(2,4) MP
3.	1	$(x)((\sim Mx \cdot \sim Cx) \supset Dx)$	P
	2	$\sim C(e)$	P
	3	$\sim M(e)$	P
	4	$(\sim M(e) \cdot \sim C(e)) \supset D(e)$	(1) U1
	5	$\sim M(e) \cdot \sim C(e)$	(2,3) Conj
	6	$D(e)$	(4,5) MP
4.	1	$(\exists x)(Sx \cdot Ix \cdot \sim Cx)$	P (unnecessary)
	2	$(x)(Sx \supset Hx)$	P
	3	$(\exists x)(Sx \cdot \sim Ix)$	P
	4	$(x)(Gx \supset Ix)$	P
	5	$Sa \cdot \sim Ia$	(3) El
	6	$Sa \supset Ha$	(2) U1
	7	$Ga \supset Ia$	(4) U1
	8	Sa	(5) Simp
	9	Ha	(6,8) MP
	10	$\sim Ia \supset \sim Ga$	(7) Trans
	11	$\sim Ia$	(5) Simp
	12	$\sim Ga$	(10,11) MP
	13	$Ha \cdot \sim Ga$	(9,12) Conj
	14	$(\exists x)(Hx \cdot \sim Gx)$	(13) EG
5.	1	$(x)(Mx \supset (Sx \vee Ax))$	P
	2	$(\exists x)(Mx \cdot \sim Ax)$	P

3	$Ma \cdot \sim Aa$	(2) El
4	$Ma \supset (\bar{S}a \vee Aa)$	(1) Ul
5	Ma	(3) Simp
6	$Sa \vee Aa$	(4,5) MP
7	$\sim\sim Aa \vee Sa$	(6) DN
8	$\sim Aa \supset Sa$	(7) Imp
9	$\sim Aa$	(3) Simp
10	Sa	(8,9) MP
11	$Ma \cdot Sa$	(5,10) Conj
12	$(\exists x)(Mx \cdot Sx)$	(11) EG
6.	1 $(x)(Bx \equiv (Fx \vee Lx))$	P
	2 Bs	P
	3 $Bs \equiv (Fs \vee Ls)$	(1) Ul
	4 $(Bs \supset (Fs \vee Ls)) \cdot ((Fs \vee Ls) \supset Bs)$	(3) Equiv
	5 $Bs \supset (Fs \vee Ls)$	(4) Simp
	6 $Fs \vee Ls$	(2,5) MP
7.	1 $(x)((Dx \cdot Ix) \supset \sim Cx)$	P
	2 $(x)(Tx \supset Dx)$	P
	3 $(x)(\sim Cx \supset Ax)$	P
	4 $(Da \cdot Ia) \supset \sim Ca$	(1) Ul
	5 $Ta \supset Da$	(2) Ul
	6 $\sim Ca \supset Aa$	(3) Ul
	7 $Da \supset (Ia \supset \sim Ca)$	(4) Exp
	8 $Ta \supset (Ia \supset \sim Ca)$	(5,7) HS
	9 $(Ta \cdot Ia) \supset Aa$	(8) Exp
	10 $(x)((Tx \cdot Ix) \supset Ax)$	UG (9)
8.	1 $(x)(Ax \supset \sim Rx)$	P
	2 $(x)(Ax \supset Gx)$	P
	3 $(\exists x)Ax$	P
	4 Aa	(3) El
	5 $Aa \supset \sim Ra$	(1) Ul
	6 $Aa \supset Ga$	(2) Ul
	7 $\sim Ra$	(4,5) MP
	8 Ga	(4,6) MP
	9 $Ga \cdot \sim Ra$	(7,8) Conj
	10 $(\exists x)(Gx \cdot \sim Rx)$	(9) EG
9.	1 $(x)(Bx \supset (Sx \cdot Fx))$	P
	2 $(x)((Bx \cdot Cx) \supset Jx)$	P
	3 $(x)((Fx \cdot \sim Rx) \supset Cx)$	P
	4 $(\exists x)(Bx \cdot \sim Rx)$	P
	5 $Ba \cdot \sim Ra$	(4) El
	6 Ba	(5) Simp
	7 $Ba \supset (Sa \cdot Fa)$	(1) Ul

- | | | |
|----|-------------------------------------|---------------------------|
| 8 | $Sa \cdot Fa$ | (6,7) MP |
| 9 | $(Ba \cdot Ca) \supset Ja$ | (2) U1 |
| 10 | $(Fa \cdot \sim Ra) \supset Ca$ | (3) U1 |
| 11 | $Fa \supset (\sim Ra \supset Ca)$ | (10) Exp |
| 12 | Fa | (8) Simp |
| 13 | $\sim Ra \supset Ca$ | (11,12) MP |
| 14 | $\sim Ra$ | (5) Simp |
| 15 | Ca | (13,14) MP |
| 16 | $Ba \cdot Fa \cdot Ca$ | (6,12,15) Conj
(twice) |
| 17 | $(\exists x)(Bx \cdot Fx \cdot Cx)$ | (16) EG |

10. The syllogisms are as follows:

1. (1), (5) \therefore (11)
2. (8), (11) \therefore (12)
3. (4), (12) \therefore (13)
4. (6), (13) \therefore (14)
5. (10), (14) \therefore (15)
6. (2), (15) \therefore (16)
7. (9), (16) \therefore (17)
8. (7), (17) \therefore (18)
9. (3), (18) \therefore (19)

- | | | | | |
|----|----|----|---|-------------|
| II | 1. | 1 | $(x)(y)(\exists z)((Px \cdot Py \cdot Lxy) \supset Nxyz)$ | P |
| | | 2 | $Ps \cdot Pa \cdot Lsa$ | P |
| | | 3 | $(y)(\exists z)((Ps \cdot Py \cdot Lsy) \supset Nsy z)$ | (1) U1 |
| | | 4 | $(\exists z)(Ps \cdot Pa \cdot Lsa) \supset Nsaz$ | (3) U1 |
| | | 5 | $(Ps \cdot Pa \cdot Lsa) \supset Nsab$ | (4) E1 |
| | | 6 | $Nsab$ | (2,5) MP |
| | | 7 | $(\exists z)Nsaz$ | (6) EG |
| | 2. | 1 | $(x)(y)((Fx \supset Gx) \supset (Ay \supset By))$ | P |
| | | 2 | $(\exists x)\sim Fx$ | P |
| | | 3 | $(\exists y)Ay$ | P |
| | | 4 | $\sim Fa$ | (2) E1 |
| | | 5 | Ab | (3) E1 |
| | | 6 | $(y)((Fa \supset Ga) \supset (Ay \supset By))$ | (1) U1 |
| | | 7 | $(Fa \supset Ga) \supset (Ab \supset Bb)$ | (6) U1 |
| | | 8 | $((Fa \supset Ga) \cdot Ab) \supset Bb$ | (7) Exp |
| | | 9 | $\sim Fa \vee Ga$ | (4) Add |
| | | 10 | $Fa \supset Ga$ | (9) Imp |
| | | 11 | $(Fa \supset Ga) \cdot Ab$ | (5,10) Conj |
| | | 12 | Bb | (8,11) MP |
| | | 13 | $(\exists x)Bx$ | (12) EG |
| | 3. | 1 | $(x)(y)((Fx \cdot By) \supset Exy)$ | P |
| | | 2 | $(x)(y)((Fx \cdot \sim Fry) \supset \sim Exy)$ | P |

	3	$(\exists x)Fx$	P
	4	$(\exists x)Bx$	P
	5	Fa	(3) E1
	6	Bb	(4) E1
	7	$(y)((Fa \cdot By) \supset Eay)$	(1) U1
	8	$(Fa \cdot Bb) \supset Eab$	(7) U1
	9	$Fa \supset (Bb \supset Eab)$	(8) Exp
	10	$(y)((Fa \cdot \sim Fry) \supset \sim Eay)$	(2) U1
	11	$(Fa \cdot \sim Frb) \supset \sim Eab$	(10) U1
	12	$Fa \supset (\sim Frb \supset \sim Eab)$	(11) Exp
	13	$Bb \supset Eab$	(5,9) MP
	14	Eab	(6,13) MP
	15	$\sim Frb \supset \sim Eab$	(5,12) MP
	16	$Eab \supset Frb$	(15) Trans, DN (twice)
	17	Frbb	(14,16) MP
	18	$(\exists x)(Frx)$	(17) EG
4.	1	$(x)(y)(Bx \supset (Gy \supset \sim Fxy))$	P
	2	$(\exists x)(\exists y)(\sim (Bx \cdot Gy) \supset Hxy)$	P
	3	$(x)(y)(Hxy \supset Ix)$	P
	4	$(x)(y)Fxy$	P
	5	$(\exists y)(\sim (Ba \cdot Gy) \supset Hay)$	(2) E1
	6	$\sim (Ba \cdot Gb) \supset Hab$	(5) E1
	7	$(y)(Hay \supset Ia)$	(3) U1
	8	$Hab \supset Ia$	(7) U1
	9	$(y)(Ba \supset (Gy \supset \sim Fay))$	(1) U1
	10	$Ba \supset (Gb \supset \sim Fab)$	(9) U1
	11	$(y)Fay$	(4) U1
	12	Fab	(11) U1
	13	$(Ba \cdot Gb) \supset \sim Fab$	(10) Exp
	14	$\sim \sim Fab \supset \sim (Ba \cdot Gb)$	(13) Trans
	15	$Fab \supset \sim (Ba \cdot Gb)$	(14) DN
	16	$\sim (Ba \cdot Gb)$	(12,15) MP
	17	Hab	(6,16) MP
	18	Ia	(8,17) MP
	19	$(\exists x)Ix$	(18) EG
5.	1	$(y)(x)((Gy \cdot Dx) \supset (Fxy \supset Yy))$	P
	2	$(x)(y)((Gy \cdot Dx) \supset (Cxy \supset Fxy))$	P
	3	$(\exists x)(\exists y)(Dx \cdot Gy \cdot Cxy)$	P
	4	$(\exists y)(Da \cdot Gy \cdot Cay)$	(3) E1
	5	$Da \cdot Gb \cdot Cab$	(4) E1
	6	$(x)((Gb \cdot Dx) \supset (Fxb \supset Yb))$	(1) U1
	7	$(Gb \cdot Da) \supset (Fab \supset Yb)$	(6) U1
	8	$Gb \supset (Da \cdot (Fab \supset Yb))$	(7) Exp

9	$(y)((Da \cdot Gy) \supset (Cay \supset Fay))$	(2) U1
10	$(Da \cdot Gb) \supset (Cab \supset Fab)$	(9) U1
11	Da	(5) Simp
12	Gb	(5) Simp
13	$Da \cdot Gb$	(11,12) Conj
14	$Cab \supset Fab$	(10,13) MP
15	Cab	(5) Simp
16	Fab	(14,15) MP
17	$Da \cdot (Fab \supset Yb)$	(8,12) MP
18	$Fab \supset Yb$	(17) Simp
19	Yb	(16,18) MP
20	$Gb \cdot Yb$	(12,19) Conj
21	$(\exists x)(Gx \cdot Yx)$	(20) EG
6.	1 $(\exists x)Rx$	P
	2 $(x)(Rx \supset (\exists y)(By \cdot Vxy))$	P
	3 $(x)(Rx \supset (y)(Ly \supset \sim Vxy))$	P
	4 Ra	(1) E1
	5 $Ra \supset (\exists y)(By \cdot Vay)$	(2) U1
	6 $(\exists y)(By \cdot Vay)$	(4,5) MP
	7 $Bb \cdot Vab$	(6) E1
	8 $Ra \supset (y)(Ly \supset \sim Vay)$	(3) U1
	9 $(y)(Ly \supset \sim Vay)$	(4,8) MP
	10 $Lb \supset \sim Vab$	(9) U1
	11 Vab	(7) Simp
	12 $\sim \sim Vab \supset \sim Lb$	(10) Trans
	13 $Vab \supset \sim Lb$	(12) DN
	14 $\sim Lb$	(11,13) MP
	15 Bb	(7) Simp
	16 $Bb \cdot \sim Lb$	(14,15) Conj
	17 $(\exists x)(Bx \cdot \sim Lx)$	(16) EG
7.	1 $(x)((Px \cdot Cx) \supset Sx)$	P (unnecessary)
	2 $(\exists x)((Px \cdot Cx) \cdot (y)(Vy \supset \sim Txy))$	P
	3 $(\exists x)Vx$	P
	4 $(x)(y)((Px \cdot Vy) \supset (\sim Txy \supset \sim Wx))$	P
	5 Va	(3) E1
	6 $(Pb \cdot Cb) \cdot (y)(Vy \supset \sim Tyb)$	(2) E1
	7 $(y)((Pb \cdot Vb) \supset \sim Tby) \supset \sim Wb$	(4) U1
	8 $(Pb \cdot Va) \supset (\sim Tba \supset \sim Wb)$	(7) U1
	9 $((Pb \cdot Va) \cdot \sim Tba) \supset \sim Wb$	(8) Exp
	10 $(Pb \cdot Cb) \supset Sb$	(1) U1
		(unnecessary)
	11 $(y)(Vy \supset \sim Tyb)$	(6) Simp
	12 $Va \supset \sim Tab$	(11) U1

13	$\sim \text{Tab}$	(5,12) MP
14	$(x)(y)(\sim \text{Txy} \supset \sim \text{Tyx})$	P
15	$(y)(\sim \text{Tay} \supset \sim \text{Tya})$	(14) U1
16	$\sim \text{Tab} \supset \sim \text{Tba}$	(15) U1
17	$\sim \text{Tba}$	(13,16) MP
18	$\text{Va} \cdot \sim \text{Tba}$	(5,17) Conj
19	Pb	(6) Simp, Assoc.
20	$\text{Pb} \cdot \text{Va} \cdot \sim \text{Tba}$	(18,19) Conj
21	$\sim \text{Wb}$	(9,20) MP
22	Cb	(6) Simp, Assoc.
23	$(\text{Pb} \cdot \text{Cb}) \cdot \sim \text{Wb}$	(19,21,22) Conj (twice)
24	$(\exists x)((\text{Px} \cdot \text{Cx}) \cdot \sim \text{Wx})$	(23) EG
8.	1 $(\exists x)(y)(\sim \text{Dx} \supset (\sim \text{Gy} \cdot \text{Dxy}))$	P
	2 $(y)(\sim \text{Da} \supset (\sim \text{Gy} \cdot \text{Day}))$	(1) E1
	3 $\sim \text{Da} \supset (\sim \text{Gb} \cdot \text{Dab})$	(2) U1
	4 Gb	Hyp. RCP
	5 $\sim (\sim \text{Gb} \cdot \text{Dab}) \supset \sim \sim \text{Da}$	(3) Trans
	6 $\sim (\sim \text{Gb} \cdot \text{Dab}) \supset \text{Da}$	(5) DN
	7 $(\sim \sim \text{Gb} \vee \sim \text{Dab}) \supset \text{Da}$	(6) DM
	8 $(\text{Gb} \vee \sim \text{Dab}) \supset \text{Da}$	(7) DN
	9 $\text{Gb} \vee \sim \text{Dab}$	(4) Add
	10 Da	(8,9) MP
	11 $\text{Gb} \supset \text{Da}$	(4,10) RCP
	12 $(y)(\text{Gy} \supset \text{Da})$	(11) UG
	13 $(\exists x)(y)(\text{Gy} \supset \text{Dx})$	(12) EG

III 1. In a universe of one individual

$\text{Sa} \quad \text{T}$

$\text{Na} \quad \text{T}$

$\therefore \text{Sa} \cdot \sim \text{Na} \quad \text{F}$

2. Valid

3. In a universe of one individual

$\text{Aa} \supset \text{Ba}$

$\text{Ca} \supset \text{Aa}$

$\therefore \text{Ca} \cdot \text{Ba} \quad \text{Take Ca, Ba, and Aa as false}$

4. In a universe of one individual

$\text{Ca} \supset (\text{Pa} \vee \text{Ta})$

$\text{Ha} \supset \text{Pa}$

$\text{Ca} \cdot \text{Ha}$

$\therefore \text{Ca} \cdot \text{Ta} \quad \text{Take Ta as false and the rest true.}$

5. In a universe of two individuals

$$\frac{(Ba \cdot \sim Ca) \vee (Bb \cdot \sim Cb)}{(Ca \cdot Da) \vee (Cb \cdot Db)} \\ \frac{(Ba \cdot \sim Da) \vee (Bb \cdot \sim Db)}{(Ba \cdot \sim Da) \vee (Bb \cdot \sim Db)}$$

Take Ba, Bb, Da, Cb, Db as true--the rest false.

- IV 1. 1 $(x)(Bx \supset Hex)$ P
 2 Bh P
 3 $Bh \supset Heh$ (1) U1
 4 Heh (2,3) MP
 2. 1 $H \vee E$ P
 2 $G \supset \sim H$ P
 3 $\sim E$ Hyp. RCP
 4 $\sim \sim E \vee H$ (1) DN
 5 $\sim E \supset H$ 4 Imp
 6 H (3,5) MP
 7 $\sim \sim H \supset \sim G$ (2) Trans
 8 $H \supset \sim G$ (7) DN
 9 $\sim G$ (6,8) MP
 10 $\sim E \supset \sim G$ (3,9) RCP
 3. 1 $(x)(Fx \supset Hx)$ P
 2 $(\exists x)(Nx \cdot Fx)$ P
 3 $Na \cdot Fa$ (2) E1
 4 $Fa \supset Ha$ (1) U1
 5 Fa (3) Simp
 6 Ha (4,5) MP
 7 Na (3) Simp
 8 $Na \cdot Ha$ (6,7) Conj
 9 $(\exists x)(Nx \cdot Hx)$ (8) EG
 4. Invalid: " $((T \supset (N \supset C)) \cdot (\sim N \supset E) \cdot \sim E) \supset \sim T$ "
 is not a tautology
 5. 1 $(x)(Fx \supset (Hx \vee Px))$ P
 2 Fh P
 3 $Fh \supset (Hh \vee Ph)$ (1) U1
 4 $Hh \vee Ph$ (2,3) MP
 5 $\sim \sim Hh \vee Ph$ (4) DN
 6 $\sim Hh \supset Ph$ (5) Imp
 6. Invalid in a universe of two individuals.

$$\frac{(Ba \supset Ta) \cdot (Bb \supset Tb)}{(Ba \cdot \sim Ca) \vee (Bb \cdot \sim Cb)} \\ \therefore (Ta \supset \sim Ca) \cdot (Tb \supset \sim Cb)$$

 Take Ba, Ta, Tb, Ca as true and the others false.
 7. 1 $P \supset \sim T$ P
 2 $H \vee P$ P

3	$H \supset N$	P
4	$\sim N$	P
5	$\sim N \supset \sim H$	(3) Trans
6	$\sim H$	(4,5) MP
7	$\sim \sim H \vee P$	(2) DN
8	$\sim H \supset P$	(7) Imp
9	P	(6,8) MP
10	$\sim T$	(1,9) MP

8. Invalid in a universe with one individual

$Fa \supset Na$
 $Ba \supset (Fa \vee \sim Na)$

Fa

$\therefore \sim Na$		Take Fa, Na as true.
9.	1 $(x)(\sim Fx \supset (\sim Wx \cdot \sim Bx))$	P
	2 $(x)(\sim Tx \supset \sim Fx)$	P
	3 $\sim Fa \supset (\sim Wa \cdot \sim Ba)$	(1) U1
	4 $\sim Ta \supset \sim Fa$	(2) U1
	5 $\sim Ta \supset (\sim Wa \cdot \sim Ba)$	(3,4) HS
	6 $\sim (\sim Wa \cdot \sim Ba) \supset \sim \sim Ta$	(5) Trans
	7 $\sim (\sim Wa \cdot \sim Ba) \supset Ta$	(6) DN
	8 $(Wa \vee Ba) \supset Ta$	(7) DM
	9 $(x)((Wx \vee Bx) \supset Tx)$	(8) UG
10.	1 $(x)(Tx \supset Vx)$	P
	2 $(\exists x)(Rx \cdot (y)(Vy \supset \sim Lxy))$	P
	3 $Ra \cdot (y)(Vy \supset \sim Lay)$	(2) E1
	4 $(y)(Vy \supset \sim Lay)$	(3) Simp
	5 $Vb \supset \sim Lab$	(4) U1
	6 $Tb \supset Vb$	(1) U1
	7 Ra	(3) Simp
	8 $Tb \supset \sim Lab$	(5,6) HS
	9 $(y)(Ty \supset \sim Lay)$	(8) UG
	10 $Ra \cdot (y)(Ty \supset \sim Lay)$	(7,9) Conj
	11 $(\exists x)(Rx \cdot (y)(Ty \supset \sim Lxy))$	(10) EG

CHAPTER FOURTEEN -- AXIOM SYSTEMS

- I 1. Replacing 'p' by ' $\sim p$ ', we get ' $\sim p \supset (q \vee \sim \sim p)$ ' or ' $\sim \sim p \vee (q \vee \sim \sim p)$ ' or ' $p \vee (q \vee p)$ '. The value of ' $p \supset (q \vee \sim p)$ ' or ' $\sim p \vee (q \vee \sim p)$ ' is the same as that of ' $p \vee (q \vee p)$ '.

II 3. For 'v', use the table

v	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	2	0
3	0	3	0	3

- III 1. $p \supset (p \supset q)$
 $\sim p \vee (\sim p \vee q)$; not a tautology
2. $p \equiv \sim p$
 $(p \supset \sim p) \cdot (\sim p \supset p)$
 $(\sim p \vee \sim p) \cdot (p \vee p)$; not a tautology
3. $(p \supset q) \cdot \sim q \supset \sim p$
 $(\sim p \vee q) \cdot \sim q \supset \sim p$
 $\sim((\sim p \vee q) \cdot \sim q) \vee \sim p$
 $\sim \sim(\sim(\sim p \vee q) \vee \sim \sim q) \vee \sim p$
 $((p \cdot \sim q) \vee q) \vee \sim p$
 $((q \vee p) \cdot (q \vee \sim q)) \vee \sim p$
 $(\sim p \vee (q \vee p)) \cdot (\sim p \vee (q \vee \sim q))$
 $(\sim p \vee q \vee p) \cdot (\sim p \vee q \vee \sim q)$; a tautology
4. $(p \supset q) \cdot (q \supset r) \supset (p \supset r)$
 $(\sim p \vee q) \cdot (\sim q \vee r) \supset (\sim p \vee r)$
 $\sim((\sim p \vee q) \cdot (\sim q \vee r)) \vee (\sim p \vee r)$
 $(\sim(\sim p \vee q) \vee \sim(\sim q \vee r)) \vee (\sim p \vee r)$
 $((p \cdot \sim q) \vee (q \cdot \sim r)) \vee (\sim p \vee r)$
 $((p \cdot \sim q) \vee q) \cdot ((p \cdot \sim q) \vee \sim r) \vee (\sim p \vee r)$
 $((q \vee p) \cdot (q \vee \sim q)) \cdot ((\sim r \vee p) \cdot (\sim r \vee \sim q))$
 $\vee (\sim p \vee r)$
 $(\sim p \vee r \vee q \vee p) \cdot (\sim p \vee r \vee q \vee \sim q) \cdot$
 $(\sim p \vee r \vee \sim r \vee p) \cdot (\sim p \vee r \vee \sim r \vee \sim q)$;
a tautology
5. $p \cdot q \equiv \sim(\sim p \vee \sim q)$
 $((p \cdot q) \supset \sim(\sim p \vee \sim q)) \cdot (\sim(\sim p \vee \sim q) \supset (p \cdot q))$
 $(\sim \sim(\sim p \vee \sim q) \vee \sim \sim(\sim p \vee \sim q)) \cdot$
 $(\sim \sim(\sim p \vee \sim q) \vee (p \cdot q))$
 $((\sim p \vee \sim q) \vee \sim(\sim p \vee \sim q)) \cdot ((\sim p \vee \sim q) \vee (p \cdot q))$
 $((\sim p \vee \sim q) \vee (p \cdot q)) \cdot ((\sim p \vee \sim q) \vee (p \cdot q))$
 $(\sim p \vee \sim q \vee p) \cdot (\sim p \vee \sim q \vee q) \cdot (\sim p \vee \sim q \vee p) \cdot$
 $(\sim p \vee \sim q \vee q)$; a tautology

IV Because the transformations used in obtaining the CNF from 'P' are themselves equivalences. The CNF

of 'P' is equivalent to 'P' and is therefore derivable if 'P' is.

V If each conjunct does contain an individual variable and its negation then it is of the form ' $(\dots v p \vee \dots v \neg p \vee \dots)$ ' which is clearly a tautology. Thus each conjunct of a derivable CNF must contain a variable and its negation.

- VI
1. Take 'e' as '1' if 'o' is multiplication and 'x-1' as the inverse of 'x'. Take 'e' as '0' if 'o' is addition and 'x-1' as '-x'.
 2. Take 'e' as '1' and 'x-1' as the inverse of 'x'.
 3. Take the variables to be integers; for example, 'x-1' as '-x' and 'e' as '0'.
 4. No

VII T6 $(p \vee q) \supset ((r \vee q) \vee p)$

Proof 1 $q \supset r \vee q$ A2 p/r
 2 $(q \supset r \vee q) \supset (p \vee q \supset p \vee (r \vee q))$ A4 r/r $\vee q$
 3 $p \vee q \supset p \vee (r \vee q)$ (1,2) R1
 4 $p \vee (r \vee q) \supset (r \vee q) \vee p$ A3 q/r $\vee q$
 5 $p \vee q \supset (r \vee q) \vee p$ (3,4) DT2

T7 $r \vee (p \vee q) \supset r \vee (p \vee (q \vee s))$

Proof 1 $q \supset q \vee p$ A2, A3 DT2
 2 $q \supset q \vee s$ (1) p/s
 3 $(q \supset q \vee s) \supset (p \vee q \supset p \vee (q \vee s))$ A4 r/q $\vee s$
 4 $p \vee q \supset p \vee (q \vee s)$ (2,3) R1
 5 $(p \vee q \supset p \vee (q \vee s)) \supset (r \vee (p \vee q) \supset r \vee (p \vee (q \vee s)))$ A4 p/r, q/p $\vee q$, r/p $\vee (q \vee s)$
 6 $r \vee (p \vee q) \supset r \vee (p \vee (q \vee s))$ (4,5) R1

T8 $(p \vee q) \vee (q \vee r) \supset p \vee (q \vee r)$

Proof 1 $(p \vee q) \vee (q \vee r) \supset (p \vee (q \vee r)) \vee (p \vee q)$ T6 p/p $\vee q$, q/q $\vee r$, r/p
 2 $(p \vee (q \vee r)) \vee (p \vee q) \supset (p \vee (q \vee r)) \vee (p \vee (q \vee r))$ T7 r/p $\vee (q \vee r)$, s/r
 $\vee (p \vee (q \vee r))$

- | | | |
|---|---|--------------------------|
| 3 | $(p \vee (q \vee r))$
$\vee (p \vee (q \vee r))$
$\supset p \vee (q \vee r)$ | A1 $p/p \vee (q \vee r)$ |
| 4 | $(p \vee q) \vee (q \vee r)$
$\supset (p \vee (q \vee r))$
$\vee (p \vee (q \vee r))$ | (1,2) DT2 |
| 5 | $(p \vee q) \vee (q \vee r)$
$\supset p \vee (q \vee r)$ | (3,4) DT2 |

T9 $(p \vee q) \vee r \supset p \vee (q \vee r)$
Proof Use T3, T8, DT 2

T10 $\underline{\quad}$

T11 $\underline{\quad}$

- | | | |
|---------|--|---|
| T12 | $p \supset (q \supset p \cdot q)$ | |
| Proof 1 | $(\sim p \vee \sim q)$
$\vee \sim (\sim p \vee \sim q)$ | T4 $p/\sim p \vee \sim q$ |
| 2 | $(\sim p \vee \sim q) \vee p \cdot q$ | (1) R3 |
| 3 | $(\sim p \vee \sim q) \vee p \cdot q$
$\supset \sim p \vee (\sim q \vee p \cdot q)$ | T9 $p/\sim p, q/\sim q,$
$r/p \cdot q$ |
| 4 | $\sim p \vee (\sim q \vee p \cdot q)$ | (2,3) R1 |
| 5 | $p \supset (q \supset p \cdot q)$ | (4) R3 (twice) |

T13 $(p \vee q) \vee r \equiv p \vee (q \vee r)$
Proof Use T9, T10, DT 12

T14-T17

- | | | |
|---------|--|-----------------------------------|
| T18 | $(p \supset q) \supset (\sim q \supset \sim p)$ | |
| Proof 1 | $q \supset \sim \sim q$ | T15 p/q |
| 2 | $(q \supset \sim \sim q)$
$\supset (\sim p \vee q \supset \sim p \vee \sim \sim q)$ | A4 $r/\sim \sim q,$
$p/\sim p$ |
| 3 | $\sim p \vee q \supset \sim p \vee \sim \sim q$ | (1)(2) R1 |
| 4 | $\sim p \vee \sim \sim q$
$\supset \sim \sim q \vee \sim p$ | (3) |
| 5 | $\sim p \vee q \supset \sim \sim q \vee \sim p$ | (3)(4) DT2 |
| 6 | $(p \supset q) \supset (\sim q \supset \sim p)$ | (5) R3 (twice) |

T19-T27

- | | | |
|---------|-------------------------------------|-------------------------|
| T28 | $p \supset q \supset p$ | |
| Proof 1 | $\sim p \supset \sim q \vee \sim p$ | A2 $q/\sim p, p/\sim q$ |

2	$\sim q \vee \sim p \supset \sim p \vee \sim q$	A3 $p/\sim q, q/\sim p$
3	$\sim p \supset \sim p \vee \sim q$	(1) (3) DT2
4	$\sim (\sim p \vee \sim q) \supset \sim \sim p$	(3) DT18
5	$\sim \sim p \supset p$	T16
6	$\sim (\sim p \vee \sim q) \supset p$	(4) (5) DT2
7	$p \cdot q \supset p$	(6) R3

CHAPTER FIFTEEN -- CLASSES

I (a) 2, 3, 5, 7, 8, 9, 10 (b) 1 (c) 4, 6

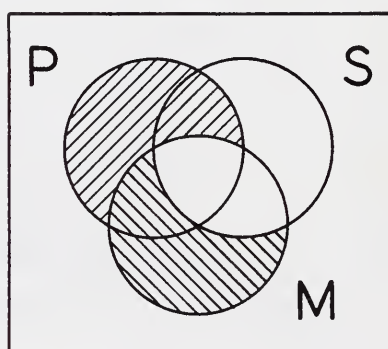
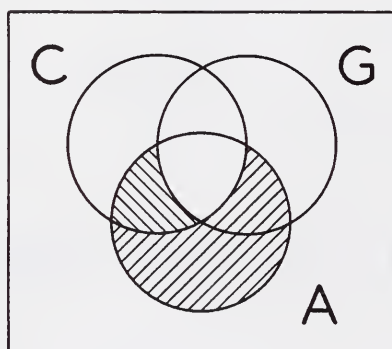
II (a) 1, 4

III 1. 5 2. yes 3. yes; no 4. no; yes
5. no 6. yes

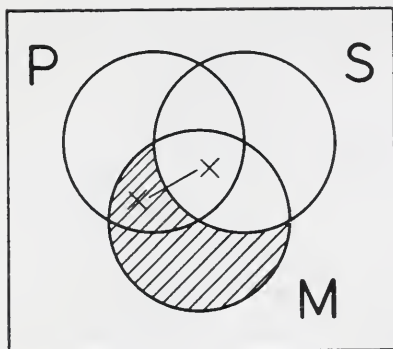
IV (a) 1. $\{3\}$ 2. $\{6,7\}$ 3. $\{1,2,3\}$
4. null 5. $\{1,3,4,5, \{3,4,5\}\} -$
 $\{1,2,3,6,7, \{1,2,3\}\} = \{4,5, \{3,4,5\}\}$
6. $\{1,2,3,4,5\}$ 7. $\{6,7, \{1,2,3\}, 1,2\}$
8. null 9. $\{1,2,3,4,5, \{6,7, \{1,2,3\}\}\}$
10. null

(b) 2, 6, 8 are true.

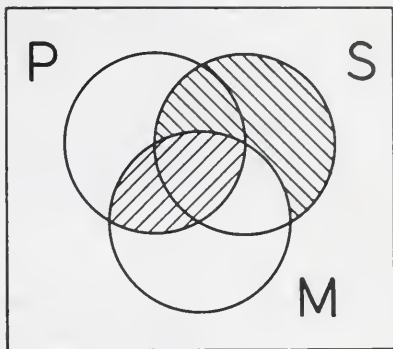
V 1. 2.



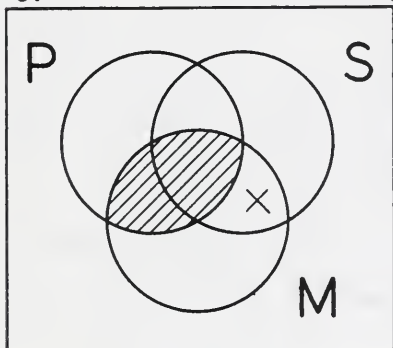
3.



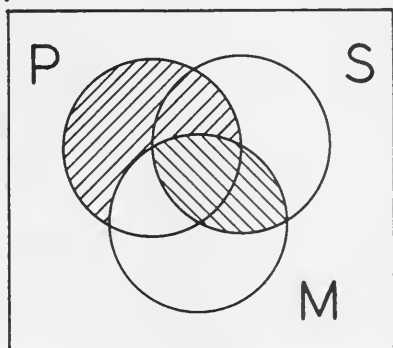
4.



5.



6.



4 and 5 are valid

7. 2. There are P's.

6. There are S's.

VI A

$$(1) A \cup \Lambda = A$$

$$x \in A \cup \Lambda \equiv x \in A \vee x \in \Lambda$$

But $\sim (x \in \Lambda)$. So

$$(x \in A \vee x \in \Lambda) \cdot \sim (x \in \Lambda) \supset x \in A.$$

And, since both

$$x \in A \supset x \in A \vee x \in \Lambda$$

$$x \in A \vee x \in \Lambda \supset x \in A$$

we have

$$x \in A \vee x \in \Lambda \equiv x \in A$$

$$(7) A \cup \bar{A} = V$$

$$x \in A \cup \bar{A} \equiv x \in A \vee \sim (x \in A)$$

And since $x \in V$ and

$x \in A \vee \sim (x \in A)$
 are both logical truths,
 $x \in A \vee \sim (x \in A) \equiv x \in V$.

$$\begin{aligned}
 (14) \quad \bar{\bar{A}} &= A \\
 x \in A &\equiv \sim \sim (x \in A) \\
 &\equiv \sim (x \in \bar{A}) \\
 &\equiv x \in \bar{\bar{A}}
 \end{aligned}$$

$$\begin{aligned}
 (16) \quad A \vee B &\neq \bar{A} \rightarrow A \neq \bar{A} \vee B \neq \bar{A} \\
 &\text{Since there is an element } x \text{ such that} \\
 &x \in A \vee x \in B, \text{ it follows that} \\
 &\text{either } x \in A \text{ or } x \in B.
 \end{aligned}$$

$$\begin{aligned}
 (21) \quad A \cap (A \vee B) &= A \\
 x \in A \cap (A \vee B) &\equiv x \in A \cdot (x \in A \vee x \in B) \\
 &\equiv (x \in A \cdot x \in A) \vee (x \in A \cdot x \in B)
 \end{aligned}$$

Now

$$x \in A \cdot (x \in A \vee x \in B) \supset x \in A$$

and

$$x \in A \supset (x \in A \cdot x \in A) \vee (x \in A \cdot x \in B)$$

so

$$x \in A \cap (A \cup B) \equiv x \in A.$$

$$\begin{aligned}
 (25) \quad A - A &= \bar{A} \\
 x \in A - A &\equiv x \in A \cdot \sim (x \in A) \\
 &\text{But } x \in \bar{A} \text{ and } x \in A \cdot \sim (x \in A) \\
 &\text{are logical contradictions, so} \\
 x \in A - A &\equiv x \in \bar{A}
 \end{aligned}$$

$$\begin{aligned}
 (30) \quad (A - B) \cup B &= A \vee B \\
 x \in (A - B) \cup B &\equiv (x \in A \cdot \sim (x \in B)) \vee x \in B \\
 &\equiv (x \in A \vee x \in B) \cdot (\sim (x \in B) \vee x \in B) \\
 &\text{And, since } (\sim (x \in B) \vee x \in B) \text{ is a} \\
 &\text{logical truth} \\
 x \in (A - B) \cup B &\equiv (x \in A \vee x \in B)
 \end{aligned}$$

VI The proofs are condensed.

- (9) $A \cup A = A$
 1 $A \cup (A \cap \bar{A}) =$ (5)
 $(A \cup A) \cap (A \cup \bar{A})$
 2 $A \cup \bar{A} = (A \cup A) \cap (A \cup \bar{A})$ (8)
 3 $A \cup \bar{A} = (A \cup A) \cap V$ (7)
 4 $A = A \cup A$ (1), (2)
- (11) $A \cup V = V$
 1 $A \cup (\bar{A} \cap V) =$ (5)
 $(A \cup \bar{A}) \cap (A \cup V)$
 2 $A \cup (\bar{A} \cap V) =$ (7)
 $V \cap (A \cup V)$
 3 $A \cup \bar{A} = V \cap (A \cup V)$ (2)
 4 $A \cup \bar{A} = A \cup V$ (2)
 5 $V = A \cup V$ (7)
- (12) $A \cap \bar{A} = \bar{A}$
 1 $\bar{A} \cap (A \cup V) =$ (6)
 $(\bar{A} \cap A) \cup (\bar{A} \cap V)$
 2 $\bar{A} \cap (A \cup V) = (\bar{A} \cap A) \cup \bar{A}$ (2)
 3 $\bar{A} \cap V = (\bar{A} \cap A) \cup \bar{A}$ (11)
 4 $\bar{A} = \bar{A} \cap A$ (1), (2)
 5 $\bar{A} = A \cap \bar{A}$ (4)
- (15) $A = \bar{\bar{B}} \supset B = \bar{A}$
 1 $A = \bar{\bar{B}}$ Hyp.
 2 $\bar{A} = \bar{B}$ line 2
 3 $A = B$ (14)
 4 $B = \bar{A}$ line 3
 5 $A = \bar{\bar{B}} \supset B = \bar{A}$ (1), (4)
- (16) $A \cup B \neq \bar{A} \supset A \neq \bar{A} \vee B \neq \bar{A}$
 1 $A \cup B \neq \bar{A}$ Hyp.
 2 $A = \bar{A}$ Hyp.
 3 $\bar{A} \cup B \neq \bar{A}$ line 1,2
 4 $B \neq \bar{A}$ (1)
 5 $A \cup B \neq \bar{A} \supset A =$ line 1,2,4
 $\bar{A} \supset B \neq \bar{A}$
 6 $A \cup B \neq \bar{A} \supset A \neq \bar{A} \vee B \neq \bar{A}$ Def.
- (17) $A \cap B \neq \bar{A} \supset A \neq \bar{A}$
 1 $A \cap B \neq \bar{A}$ Hyp.
 2 $A = \bar{A}$ Hyp. (for reductio
 ad absurdum proof)
 3 $\bar{A} \cap B \neq \bar{A}$ line 1,2
 4 $\bar{A} \neq \bar{A}$ (12) contradiction
 5 $A \neq \bar{A}$ line 2,4
 6 $A \cap B \neq \bar{A} \supset A \neq \bar{A}$ line 1,5

$$(19) \quad A \cap (B \cap C) = (A \cap B) \cap C$$

$$\begin{array}{ll} 1 & x \in (A \cap (B \cap C)) \equiv x \in A \cdot (x \in B \cdot x \in C) \quad \text{Def.} \\ 2 & \equiv (x \in A \cdot x \in B) \cdot x \in C \quad \text{Taut.} \\ 3 & \equiv x \in (A \cap B) \cap C \end{array}$$

$$(24) \quad \overline{(A \cap B)} = \bar{A} \cup \bar{B}$$

$$1 \quad \overline{\bar{A} \cup \bar{B}} = \overline{\bar{A}} \cap \overline{\bar{B}} \quad (23)$$

$$2 \quad \overline{\bar{A} \cup \bar{B}} = \overline{\bar{A}} \cap \overline{\bar{B}} \quad (14), (15)$$

$$3 \quad \overline{\bar{A} \cap \bar{B}} = \overline{\bar{A}} \cup \overline{\bar{B}} \quad (23)$$

$$4 \quad \overline{\bar{A} \cap \bar{B}} = \overline{\bar{A}} \cup \overline{\bar{B}} \quad (14)$$

$$(26) \quad A - (A \cap B) = A - B$$

$$1 \quad A \cap (\overline{A \cap B}) = A \cap (\bar{A} \cup \bar{B}) \quad (24)$$

$$2 \quad A \cap (\bar{A} \cup \bar{B}) = \quad (6)$$

$$(A \cap \bar{A}) \cup (A \cap \bar{B})$$

$$3 \quad (A \cap \bar{A}) \cup (A \cap \bar{B}) = \quad (8)$$

$$\Lambda \cup (A \cap \bar{B})$$

$$4 \quad \Lambda \cup (A \cap \bar{B}) = A \cap \bar{B} \quad (1)$$

$$5 \quad A \cap \bar{B} = A - B \quad \text{Def.}$$

$$6 \quad A - (A \cap B) = A - B$$

$$(29) \quad (A - B) - A = \Lambda$$

$$1 \quad (A \cap B) \cap \bar{A} = \bar{A} \cap (A \cap \bar{B}) \quad (4)$$

$$2 \quad \bar{A} \cap (A \cap \bar{B}) = (\bar{A} \cap A) \cap \bar{B} \quad (19)$$

$$3 \quad (\bar{A} \cap A) \cap \bar{B} = \Lambda \cap \bar{B} \quad (8)$$

$$4 \quad \Lambda \cap \bar{B} = \Lambda \quad (12)$$

$$5 \quad (A - B) - A = \Lambda$$

$$(31) \quad (A \cup B) - B = A - B$$

$$1 \quad (A \cup B) \cap \bar{B} = \bar{B} \cap (A \cup B) \quad (4)$$

$$2 \quad (\bar{B} \cap A) \cup (\bar{B} \cap B) = (\bar{B} \cap A) \cup \Lambda \quad (6)$$

$$3 \quad (\bar{B} \cap A) \cup \Lambda = \bar{B} \cap A \quad (8)$$

$$4 \quad \bar{B} \cap A = A \cap \bar{B} = A - B \quad (1)$$

$$5 \quad (A \cup B) - B = A - B \quad (4), \text{Def.}$$

$$B \quad 1. \quad \bar{A} \cup (B \cap C \cap D) = (\bar{A} \cup B) \cap (\bar{A} \cup C) \cap (\bar{A} \cup D)$$

$$1 \quad \bar{A} \cup (B \cap C \cap D) = \quad (5)$$

$$(\bar{A} \cup B) \cap (\bar{A} \cup (C \cap D))$$

$$2 \quad (\bar{A} \cup B) \cap (\bar{A} \cup (C \cap D)) = \quad (5)$$

$$(\bar{A} \cup B) \cap (\bar{A} \cup C) \cap (\bar{A} \cup D)$$

$$2. \quad (\overline{A \cap B}) \cap (\overline{A \cap C}) \cup \bar{C} \cup (D \cap E) \cup$$

$$(A \cap (B \cup C)) = \vee$$

$$1 \quad (\text{Left side 2.}) = C \cup (D \cap E) \quad (18)$$

$$\cup ((A \cap B) \cap (A \cap C))$$

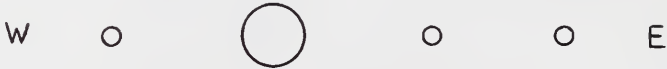
$$\cup (A \cap (B \cup C))$$

- 2 $RS\ 1 = \bar{C} \cup (D \cap E) \cup ((\overline{A \cap B}) \cap (\overline{A \cap C})) \cup ((A \cap B) \cup (A \cap C))$ (6)
- 3 $RS\ 2 = \bar{C} \cup (D \cap E) \cup (\overline{A \cap B}) \cup (\overline{A \cap C})$ (23)
- 4 $RS\ 3 = \bar{C} \cup (D \cap E) \cup V$ (7)
- 5 $RS\ 4 = V$
3. $(A \cap \bar{B}) \cap ((B \cap C) \cup D) = A \cap C$
 - 1 $LS = ((A \cap \bar{B}) \cap (B \cap C)) \cup ((A \cup \bar{B}) \cap D)$ (6)
 - 2 $RS\ 1 = (A \cap (\bar{B} \cap B) \cap C) \cup ((A \cap \bar{B}) \cap D)$ (18)
 - 3 $RS\ 2 = (A \cap C) \cup ((A \cap \bar{B}) \cap D)$ (8), (1)
 - 4 $RS\ 3 = (A \cap C) \cup (A \cap \bar{B}) \cap (A \cap C) \cup D$ (5)
 - 5 $RS\ 4 = ((A \cap C) \cup A) \cap ((A \cap C) \cup \bar{B}) \cap ((A \cap C) \cup D)$ (5)
 - 6 $RS\ 5 = (A \cap C) \cap ((A \cap C) \cup \bar{B}) \cap ((A \cap C) \cup D)$ (5), (9)
 - 7 $RS\ 6 = (A \cap C) \cap ((A \cap C) \cup D)$ (21)
 - 8 $RS\ 7 = A \cap C$ (21)
4. $((A \cap \bar{B}) \cap (\overline{A \cup D})) \cup ((A \cap \bar{B}) \cap (A \cup D)) = A \cap \bar{B}$
 - 1 $LS = (((A \cap \bar{B}) \cap (\overline{A \cup D})) \cup (A \cap \bar{B})) \cap (((A \cap \bar{B}) \cap (\overline{A \cup D})) \cup (A \cup D))$ (5)
 - 2 $RS\ 1 = ((A \cap \bar{B}) \cup (A \cap \bar{B})) \cap ((A \cap \bar{B}) \cup (A \cup D)) \cap ((A \cup D) \cup (A \cup D))$ (5)
 - 3 $RS\ 2 = (A \cap \bar{B}) \cap ((A \cap \bar{B}) \cup (\overline{A \cup D})) \cap ((A \cup D) \cup (A \cap B)) \cap V$ (9), (7)
 - 4 $RS\ 3 = (A \cap \bar{B}) \cap ((A \cap \bar{B}) \cup (A \cup D))$ (2), (21)
 - 5 $RS\ 4 = A \cap \bar{B}$
5. $(A \cup (B \cap C)) \cup (D \cap E) \cup \bar{C} \cup (A \cup B) \cap (A \cup C) = V$
 - 1 $LS = ((A \cup B) \cap (A \cup C)) \cup (A \cup B) \cap (A \cup C) \cup \bar{C} \cup (D \cap E)$ (5)
 - 2 $RS\ 1 = V \cup \bar{C} \cup (D \cap E)$ (7)
 - 3 $RS\ 2 = V$

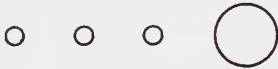
PART III

CHAPTER SIXTEEN -- SCIENCE AND HYPOTHESES

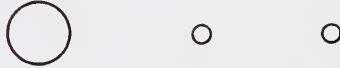
2. Galileo's first hypothesis is introduced in connection with this phenomenon observed on the 7th:



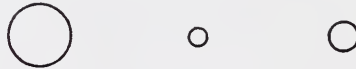
and the hypothesis is: All three bodies near Jupiter are fixed. On the 8th he observed this phenomenon:



By assuming that Jupiter moved east he was able to retain his hypothesis; however, even with this assumption the hypothesis does not explain why the bodies were "nearer one another than before." On the 10th he observed:



Now to retain the first hypothesis would be to go against known astronomical regularities, for if the hypothesis were true, then Jupiter would have to move east then west. The explanation now called for is that the bodies move. On the 11th he observed:



All of these phenomena naturally suggest Galileo's second hypothesis: ". . . there were in the heaven three stars which revolved round Jupiter." There are no competing hypotheses since the second was entertained after the first was abandoned. Both hypotheses are empirical, and it is apparent that further observations would verify the second hypothesis.

CHAPTER SEVENTEEN -- CRUCIAL EXPERIMENTS AND INDUCTIVE TECHNIQUES

- I 3. The hypothesis which competed with the phlogiston theory--in combustion things combine--is not mentioned here. The phlogiston theory is naturally suggested by the appearance of something being released when, say, a piece of paper burns. This hypothesis was theoretical--for as the word "phlogiston" was used in the theory it was conceptually impossible at the time to observe phlogiston.

In the passage an experimental result is described which, in part, led to the downfall of the theory. If the phlogiston theory were true, then it would seem that when metal is calcinated, and thus loses phlogiston, the calx should weigh less than the metal. The results of calcination, however, were the opposite. This did not result in the abandonment of the theory, since defenders supposed that phlogiston had a "negative weight." Perhaps if this had been the only way to save the theory, the theory would have been given up, but there were other ways to make it compatible with the known P's (see, e.g., the reaction to Lavoisier's experiment).

- III 1. Since he supposed that the nervous paralysis of the hens had the same cause as the similar paralysis of the prisoners (called "beri-beri") Eijkman looked for what the hens and prisoners had in common and discovered that they both fed almost entirely on polished rice. Here the method of agreement is employed. It supports the hypothesis that the disease is caused by the exclusive diet of polished rice (or by the lack of something). He then noticed that other prisoners who had beri-beri ate polished rice while those who ate unpolished rice did not have the disease. Eijkman employed the method of

difference here. The results provided additional support for his hypothesis. Eijkman's controlled experiments (with the hens) in which all the conditions were the same except that one group was fed on polished rice while the other was fed on unpolished rice (method of difference) gave strong support that it was the lack of what was in the husks which caused beriberi in those fed exclusively on rice.

CHAPTER EIGHTEEN -- PATTERNS OF SCIENTIFIC EXPLANATIONS

I 3. Here, in brief, is the explanation:

- (1) The American rich have a fear of expropriation.
- (2) Many people in America, other than the rich, can display the traditional signs of wealth (luxury cars, etc.). Therefore the American rich no longer display their wealth.

There is implicit in the premisses the notion that fear and the lack of success in displaying wealth generally lead the wealthy to avoid displaying their wealth (given the existing conditions in some countries today). Since this probabilistic generality is part of the explicans, the explicandum would not be a deductive consequence of the explicans. The explanation is a probabilistic one. It is not a historical explanation (though a similar explanation in a different context could be) nor a teleological explanation. It deals with the reasons the American rich have (or had) for not displaying their wealth. It seems to be an instance of a fairly common kind of sociological explanation (the reasons such-and-such people have for behaving in such-and-such a way). Galbraith cites some of the events which are connected

with and, in part, produce the fear considered in (1). If these had been explained in more detail, we would have the ingredients of an empathetic explanation.

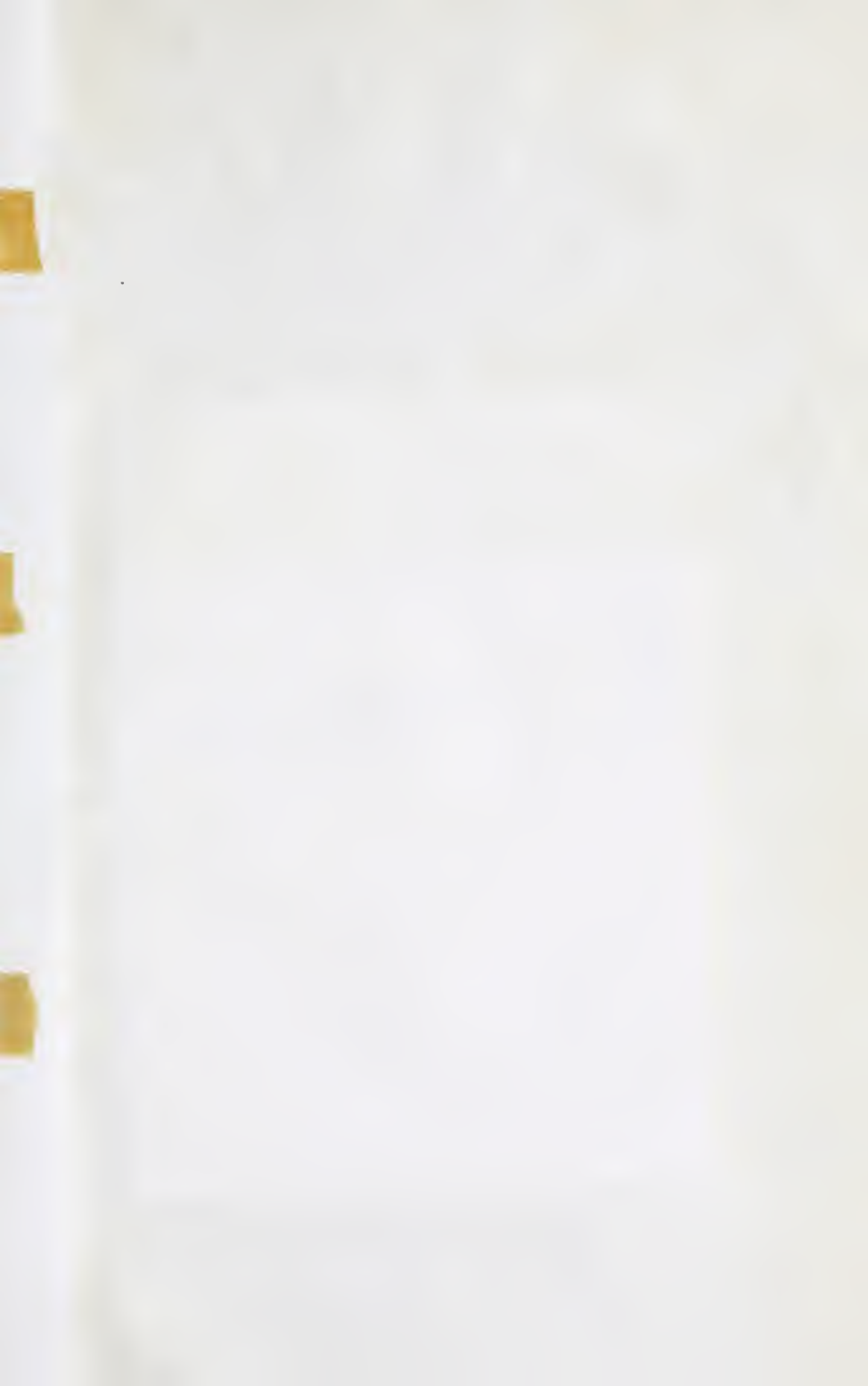
- II 1. Psuedo-explanation (not testable).
- 2. Psuedo-explanation (explicans follows from the explicandum).
- 3. Psuedo-explanation (not testable).
- 4. Not testable in light of the fact that we would not know how to test it. If it is regarded as scientific, then it is a psuedo-explanation.
- 5. Perhaps a non-scientific explanation. If it is regarded as scientific, it is not testable.
- 6. Psuedo-explanation (not testable).

CHAPTER NINETEEN -- SOME LOGICAL FEATURES OF SCIENCE

- II 1. No conflict. Even if saying (long ago) that the sun sets in the west entailed that the sun revolves around the earth (which is doubtful), we do not mean this today.
 - 2. No conflict. He was not "conscious" according to his definition of the word.
 - 3. No conflict. In this passage what is being called a "cyclone" is different from what we ordinarily call a "cyclone."
 - 4. No conflict. "Structure" is being used in such a way that liquids having an internal architecture is compatible with the statement "Liquids are fluid," as we would ordinarily understand this.
- III 4. Though the vastness of the universe seems to produce a feeling of humility and terror in Russell, Eddington, and Jeans, it does not produce this feeling in everyone. To paraphrase a remark by Ramsey, in connection with "humility": the stars cannot think, love, or

lead virtuous lives, and these are the qualities which impress me, not size or distance. It is those qualities which I myself can take credit for and which others have to a greater degree than I which bring on feelings of humbleness. I can take no credit for weighing 160 lbs.

Do such astronomical facts show that man is unimportant or insignificant? If it were true that those who believe that man is important and not insignificant based their belief on the assumption that the limits of the universe are a few thousand feet or that men are to be found all over the universe, then such facts would certainly upset their belief. But do those who believe that man is significant have this as their reason? Such beliefs come up in religious and certain philosophical contexts. For example, that man is important and not insignificant follows from the Christian notion that man is created in the image of God. And no matter what the relative size of man to the universe may be, such facts are irrelevant to such beliefs.





Carney

Solutions to exercises in
Fundamentals of logic

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S6

