FACULTY WORKING PAPER NO. 824

Some Further Thoughts on the NTU Value and Related Matters: A Rejoinder to Aumann

Alvin E. Roth

College of Commerce and Business Administration
Bureau of Economic and Business Research
University of Illinois, Urbana-Champaign

BEBR

FACULTY WORKING PAPER NO. 824

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

November 1981

Some Further Thoughts on the NTU Value
and Related Matters: A Rejoinder to Aumann

Alvin E. Roth, Professor
Departments of Business Administration
and Economics

Abstract

A recent paper by Aumann [1981] replies to criticisms of the NTU value raised by Roth [1980] and Shafer [1980], and comments on the current state of research on the NTU value. This note presents the argument that the NTU value reflects phenomena which might better be viewed as belonging to the realm of transferable utility games, rather than non-transferable utility games. A new example is presented, and avenues for future research are discussed.

1. Introduction

In an important recent paper, R. J. Aumann [1981] replies in detail to criticisms of the NTU (non-transferable utility) value (Shapley [1969]) appearing in Roth [1980][1] and Shafer [1980]. Aumann's response is wide ranging: he begins by noting that other solution concepts (e.g., the core, competitive equilibrium, and Nash equilibrium) can give couterintuitive results, but still retain their analytical value. He argues that the examples used to criticize the NTU value are simply the same sort of counterintuitive results. The purpose of this paper is to explain why I find Aumann's response unpersuasive, and why the NTU value seems to have important weaknesses not shared by other solution concepts, and arising from different causes.[2]

Some of the difference between the NTU value and other solution concepts arises from the fact that, unlike the NTU value, most other commonly used solution concepts--e.g., the core, competitive equilibrium, Nash equilibrium, and even the Shapley [1953] value for transferable utility games—are defined, or can be rigorously interpreted, in terms of simple assumptions about the <u>behavior of individuals</u> in simple choice situations. Part of the analytic usefulness of these concepts derives from the fact that situations in which they yield counterintuitive results can be analysed to see why the related individual choice behavior might also be counterintuitive in those situations.[3] The NTU value, in contrast, has not as yet been given any rigorous interpretation in terms of individual behavior.

Before going on, it will be helpful to explain the point of view I take in discussing these matters. I agree with Aumann that economic theory isn't mathematics, solution concepts aren't theorems, and that one or two counterintuitive examples don't destroy a solution concept. Instead, I think that using any particular solution concept to explore some model of economic activity is akin in some respects to experimental science. The starting hypothesis is that the solution concept reliably reflects underlying economic phenomena, and each model "successfully" analysed is a data point lending support to that hypothesis. But experimenters also have to consider competing hypotheses consistent with the data. This paper will advance such a competing hypothesis about the NTU value.

The competing hypothesis is that the NTU value is a reflection, for games without transferable utility, of phenomena which occur in games which do have transferable utility, and which need not in general have economic meaning in the non-transferable utility case.

This paper is organized as follows. Section 2 defines the NTU value, and discusses why the formal behavioral interpretation of the Shapley value for transferable utility games (Roth [1977a,b]) does not carry over to the NTU value. Section 3 begins by briefly discussing Aumann's criticism of the analysis presented in Roth [1980], and goes on to present a new example to illuminate the issue, and to support the competing hypothesis discussed above. Section 4 concludes.

2. The relationship between the TU and NTU values

The NTU value for a game without transferable utility is defined to be equal to the TU value (Shapley [1953]) of a related game in which utility _is_ transferable. This section considers why we should expect to find difficulties in interpreting the NTU value along the customary lines for interpreting the TU value.

The "value" for games with transferable utility which appears most widely in the literature was introduced by Shapley [1953]. He proposed a set of axioms on "value functions" defined on the class of games with transferable utility, and demonstrated that there was a unique function satisfying these axioms. The interpretation suggested for this value function was that it summarized, for each player in each game, the value of playing the game.

Roth [1977a,b] gave this interpretation a rigorous foundation in a behavioral context, by showing that the Shapley value could be viewed as the von Neumann-Morgenstern [1953] utility function of a certain kind of risk-neutral individual, faced with the choice of positions in a game, and across games. That is, the Shapley value for a position in a game reflects its desirability relative to other positions to such a risk-neutral individual, in the well understood sense captured by von Neumann-Morgenstern utility functions.[4]

To understand how the NTU value is related to the TU value, we must first consider how games without transferable utility (NTU games) are related to games with transferable utility (TU games). For TU games, a single number is sufficient to summarize the worth of a coalition, since the coalition is free to distribute its utility payoffs in any way (so only the sum of these payoffs is required to represent the set of feasible payoffs). It has been recognized since quite early in the history of game theory that this assumption of unlimited transferability of utility is highly unrealistic (cf. Luce and Raiffa [1957], Aumann and Peleg [1960]).

In an NTU game, this assumption is not made; a coalition may have different potential activities which pay different members differently, and it cannot in general alter the distribution of payoffs. If we were to change an NTU game into a TU game by somehow allowing utility to be freely transferred, we would be enlarging the set of feasible payoffs which a coalition could achieve, to include all possible redistributions of its 'most profitable' activity.

The NTU value is defined as follows. Consider an n-person NTU game G, and the related TU game g which would result if we assumed that utility could in fact be freely transferred in the original game G. This TU game g has a TU value which may or may not be a feasible outcome in the original NTU game G. If it is, then the NTU value of G is defined to be equal to the TU value of g. If not, then the game G is transformed to another game G_λ, by multiplying the payoffs of each player i by some non-negative number λ_i, and the related TU game g_λ is considered. Shapley [1969] proved via a fixed-point argument that there exists some non-negative vector $\lambda = (\lambda_1,\ldots,\lambda_n)$ such that the TU value of g_λ is feasible in the NTU game G_λ. The corresponding outcome in G is its NTU value: i.e., $x = (x_1,\ldots,x_n)$ is the NTU value of G if x is feasible in G and if $(\lambda_1 x_1,\ldots,\lambda_n x_n)$ is the TU value of g_λ.[5]

The key assumption we must therefore make if we are to believe that the NTU value inherits the principal properties of the TU value is that, as far as the 'value' of a game is concerned, the NTU game G is equivalent to the TU game g_λ.

This assumption can be thought of as consisting of two parts. The first part is an assumption about the equivalence of games under the transformation involved in multiplying the payoffs by the corresponding components of the vector λ. The second part is an assumption about the equivalence of NTU games and TU games (without needing to modify the scale of the payoffs).

Concerning the first part of this assumption, it should be noted that multiplication of the payoffs by a vector λ whose components are unequal destroys the interpretation referred to earlier of the value as a von Neumann-Morgenstern utility for playing a game. The reason is that, in order for different positions to be comparable to one another, the payoffs available in each position must be defined in common units. So we cannot expect that the NTU value will represent the expected utility of playing each position in a game the way the TU value does.[6]

As to the second part of the assumption--the equivalence from the point of view of a 'value function' of NTU and TU games--the next section will address this issue. Some examples will be considered in which this assumption can be examined without the additional complication of a λ-transformation; i.e., examples in which the NTU value requires no re-scaling ($\lambda = (1,1,1)$).

3. Discussion of some examples

Let us first consider an example from Roth [1980], whose analysis there is challenged by Aumann. There are three players: each can get 0 by himself; players 1 and 2 acting together can achieve the payoff vector $(\frac{1}{2},\frac{1}{2},0)$; players 1 and 3 together can achieve $(p,0,1-p)$ where p is some fixed amount strictly less than $\frac{1}{2}$; players 2 and 3 together can similarly achieve $(0,p,1-p)$; and all three players acting together can achieve any division (x_1,x_2,x_3) whose components are nonnegative and sum to 1. Utility is not transferable. In fact, no player can make any sort of sidepayment to any other player: for each coalition, only the payoffs given above are possible. Aumann [1980] addresses his comments to the case $p = \frac{1}{3}$, and it will be sufficient to consider that case here.[7]

Note that the sum of the payoffs to each coalition is always 1. So if utility were transferable, the resulting TU game would be the symmetric game whose TU value is $(\frac{1}{3},\frac{1}{3},\frac{1}{3})$. Since this outcome is feasible in the NTU game actually being considered, it is therefore defined to be the NTU value for that game (with $\lambda = (1,1,1)$).

In Roth [1980] it was argued that, in the idealized cooperative game in which it is common knowledge[8] that all the players are perfectly rational, the outcome $(\frac{1}{2},\frac{1}{2},0)$ had a much better claim, since both players 1 and 2 prefer this to any other feasible outcome, and since the rules allow them to achieve it. Aumann [1980] argues that cooperative games can be thought of as modelling less idealized situations,[9] and that the outcome $(\frac{1}{2},\frac{1}{2},0)$, which gives nothing to player 3, is thus a much too unforgiving assessment of the game--the NTU value, which gives each player $\frac{1}{3}$, is more forgiving in the sense that it recognizes a possibility that each player will get something, which cannot be ignored when the game is not viewed as an idealized model.[10]

In summary, Aumann's defense of the NTU value in this example revolves around the argument that it gives to each player a payoff which reflects the possibility that he might or might not be in the winning coalition, which cannot be ignored if games are to be taken to represent non-ideal situations. By contrast, my position is that the NTU value is simply reflecting the symmetry of the related TU game, in a manner which is inappropriate in the (non-symmetric) NTU game. The following example, which is related to a game studied by Owen [1972], will help to illuminate these two conflicting points of view.

The new example

There are three players: each can assure himself of getting 0; players 1 and 2 acting together can achieve the payoff vector (2,-1,0); players 1 and 3 or players 2 and 3 acting together can achieve only the payoff vector (0,0,0), and players 1, 2, and 3 acting all together can achieve any payoff (x_1,x_2,x_3) whose components are not less than -1 and which sum to 1. Utility is not transferable--no sidepayments of any sort are feasible.

Note that, in this game, players 1 and 2 acting together can engage in an activity with a payoff of 2 to player 1, and -1 to player 2. Since player 2 can assure himself of a payoff of at least 0, no assumption other than the individual rationality of player 2 himself is needed to conclude that players 1 and 2 will not form a coalition to carry out this activity. No other one or two-player coalition has any productive activities available to it. Only the three-player coalition has any individually rational productive activities available to it, and its potential activities are perfectly symmetric in the payoffs they give to the players.

For this latter reason, a compelling case could be made that the outcome $(\frac{1}{3},\frac{1}{3},\frac{1}{3})$ is the most representative of the players' prospects in this game. At the very least, to argue that player 3 should have a positive payoff it is sufficient to note that no individually rational productive agreement can be reached without his cooperation. Nevertheless, the NTU value for this game is the outcome $(\frac{1}{2},\frac{1}{2},0)$ (with $\lambda = (1,1,1)$).

The technical reason for this is clear: if utility <u>were</u> transferable, player 3 would be a dummy in the corresponding TU game. In that game, the coalition of players 1 and 2 would have a worth of 1, player 3 would add nothing, and so he would get nothing at the TU value, which is $(\frac{1}{2},\frac{1}{2},0)$. Because this is the TU value in the TU game, it is <u>by definition</u> also the NTU value in the NTU game.

Lets look a little more closely at this sleight of hand. In the NTU game, players 1 and 2 can't get together on their own since a side-payment from 1 to 2 would be required before any joint activity would be rational for player 2, and such sidepayments are impossible. But player 3 can act as an intermediary: when he is a member of the coalition along with 1 and 2, any distribution of the gains from cooperation is possible. Of course, if it were suddenly possible (as in the TU game) for player 1 to make sidepayments to player 2, then the services of player 3 would have no value. But to conclude <u>for this reason</u> that player 3's services are of absolutely no value even when such sidepayments <u>cannot</u> be made strikes me as absurd. Yet this is what the NTU value does.

The similarity between the two examples discussed above is that both were chosen to emphasize the great difference which can exist between an NTU game and the related TU game. This difference is ignored by the NTU value. The dissimilarity between the two examples is that, loosely speaking, the roles of the payoff vectors $(\frac{1}{3},\frac{1}{3},\frac{1}{3})$ and $(\frac{1}{2},\frac{1}{2},0)$ are reversed. The arguments which Aumann [1980] made for $(\frac{1}{3},\frac{1}{3},\frac{1}{3})$ as the NTU value of the first example would have to be reversed in the case of the second example, where it is the NTU value which assigns 0 to the third player,[11] and which thus reflects no possibility that he will be part of the "winning coalition".

Even if some story can be constructed to simultaneously defend the NTU value in both games, it would still have to contend with what should by now be a very plausible competing hypothesis: the NTU value is a reflection, for NTU games, of the phenomena which occur in the corresponding TU games, whether or not these phenomena continue to have economic meaning in the NTU games.

4. Conclusions

We have considered two alternative hypotheses to account for the behavior of the NTU value. The "original hypothesis" is that the NTU value reflects the underlying economics of NTU games in something like the way the TU value reflects the economics of TU games. According to this hypothesis (cf. Aumann [1981]), the fact that examples of the kind considered here seem counterintuitive results from our present incomplete understanding of the NTU value; and when in the future we understand it better, these examples will no longer be disturbing. Viewed in this way, intuitive results obtained using the NTU value are seen as supporting evidence for the hypothesis.

The "competing hypothesis" is that, for each NTU game, the NTU value generates numbers based on some TU game which may not be sufficiently similar to the original NTU game to yield meaningful results. According to this hypothesis, the fact that many intuitive results can be obtained using the NTU value reflects the fact that TU games can often be used to illuminate economic phenomena (which is why there is such a large literature concerned with TU games). Viewed in this way, the counterintuitive examples discussed earlier are seen as supporting the hypothesis by demonstrating how dissimilar an NTU game can be from the TU game selected by the NTU value.

If we accept the competing hypothesis, we need to consider whether the NTU value might nevertheless continue to be a valuable tool capable of often providing some economic insight, even if that insight is into the economics of TU games. Here we need to be extremely careful. If we are studying phenomena which we believe can be adequately modelled by TU games, then it is surely more straightforward to study TU games directly. And if we are studying phenomena for which the assumption of transferable utility is sufficiently unrealistic so as to require modelling by NTU games, then we cannot rely on a solution concept which reinterprets an NTU game as a TU game.

So what can be said at this point about the status of the two alternative hypotheses? While I believe that the preponderance of the evidence at this point lies with the competing hypothesis, these are clearly the kinds of issues for which we cannot expect a completely "clean" resolution. And even someone who is persuaded by the arguments I have presented here would be foolish not to be impressed by the fact that it is no less a champion than Aumann who rises to the defense of the NTU value. So what does all this mean for future research on the NTU value?

It seems to me that two avenues of research present themselves as likely candidates to those hoping to find supporting evidence for the original hypothesis. The first of these is research into the foundations of the NTU value, aimed at finding some rigorous behavioral interpretation, which might help us understand counterintuitive examples for the NTU value the way we understand them for other solution concepts (cf. footnote 3). Of course, a likely outcome to this line of research is that it would lead to changes in the definition of the NTU value.[12] The second of these avenues is research aimed at using the NTU value to identify important economic phenomena which occur in NTU games but not in TU games. This would also tend to weaken the competing hypothesis. In the meantime, however, and as long as the competing hypothesis remains viable, there is reason to regard with some suspicion the results of using the NTU value to analyse models of economic interaction in which utility cannot be transferred, since these results may offer only an imperfect reflection of phenomena which occur in games with transferable utility.

In closing, I can't resist observing that "philosophical" discussions like Aumann [1980] and that attempted here occupy a set of almost measure zero in the open literature. This is perhaps a comment on an aspect of the sociology of our profession: since statements about the usefulness and plausibility of mathematical ideas aren't themselves mathematical statements, there isn't much space devoted to them in the mathematical literature; and since these ideas are of primary interest to mathematical economists, there isn't much attention to them in the non-mathematical literature either. There is thus a paucity of the kind of meta-theoretical discussion which might help to guide theoretical work.

Footnotes

[1]Roth [1980] criticized not only the NTU value, but also a closely related concept due to Harsanyi [1963], [1977], which Shapley [1969, p. 260] cites as the motivation for the NTU value. However, since Aumann directs his comments at the NTU value, and since Harsanyi [1980] has indicated substantial agreement with the conclusions of Roth [1980], the comments in this paper will be directed at the NTU value alone.

[2]Of course, this rejoinder expresses my own views only, and does not speak for other critics of the NTU value. And it goes without saying that the purpose of this kind of criticism is not to discourage research on the NTU value and related matters, but to suggest directions in which further research might be fruitful, and provide a guide to interpreting its results.

[3]Consider, for example, the famous example which Aumann draws to our attention of a market with one buyer and two sellers. Both the core and the competitive equilibrium assign all the benefits of trade in this game to the buyer, and none to the sellers. Even if we find this prediction counterintuitive, we can still note that <u>if</u> the sellers engaged in the kind of cutthroat competition presumed by the core (through the definition of what constitutes a dominated outcome) or the competitive equilibrium, then the buyer would indeed be able to play them off against one another to obtain as close to all of the benefits from trade as he liked. It is precisely this fact which gives the sellers some incentive to show restraint in their competition with one another, and which causes us to find the original prediction counterintuitive.

[4]Individuals who are not risk neutral will have different utility functions. See Roth [1977b].

[5]The NTU value is also called the <u>λ-transfer value</u>, in recognition of the fact that it comes from the TU game g_λ, in which utility is transferable at "exchange rates" given by the vector λ. A nice discussion of this can be found in Aumann [1975].

[6]The equivalence of games under the λ-transformation can also be questioned on empirical grounds, which is briefly referred to in footnote 10.

[7]However it is worth keeping in mind that the NTU value makes the same prediction for any value of p, and the criticism of the value discussed in Roth [1980] applies for values of p less than $\frac{1}{2}$. The case $p = 0$ provides a particularly clear test of the NTU value, and the comments of Aumann [1980] do not appear to address this case.

[8]In the sense of Aumann [1976] and Milgrom [1981].

[9]Aumann first models the game as a noncooperative game in which pairs of players have random access to each other, in secret. He also briefly considers an analogy to the coalition forming process which follows parliamentary elections.

[10]Let me clarify my point of view on the subject of when cooperative games of the kind considered are appropriately viewed as modelling "ideal" interactions among "perfectly rational" players. It seems to me that one of the principal goals of game theory is to study perfectly rational behavior in ideal conditions precisely in order to provide a benchmark against which theory and observation about actual economic behavior can be measured. (One of the most eloquent explanations of the use of idealized models in game theory is the discussion of how to model perfect competition given by Aumann [1964].) At the same time, it is obviously of enormous importance to study how real people interact in economic situations which approximate the idealized models. For this purpose, empirical observation seems to me to be an essential element. In this context it is worth noting that there is very strong experimental evidence against the assumption made in the definition of the NTU value, that games are equivalent under the λ-transfer procedure. For a survey of the experimental results on this subject for simple bargaining games, see Roth and Malouf [1979]; some more recent results are contained in Roth, Malouf, and Murnighan [1981], Roth and Malouf [1982], and Roth and Murnighan [1982].

[11]In general, if the game is taken to represent a variety of possible realizations, it seems very difficult to justify that the NTU value reflects no possibility that player 3 will get any benefit, even though no agreements can be reached without his cooperation.

[12]One promising effort in the direction of such an overhaul of the definition is the work of Hart and Kurz [1981], which directly addresses questions raised in Roth [1980].

Bibliography

Aumann, Robert J., "Markets with a Continuum of Traders," Econometrica, 32, pp. 39-50, 1964.

Aumann, Robert J., "Values of Markets with a Continuum of Traders," Econometrica, 43, pp. 611-646, 1975.

Aumann, Robert J., "Agreeing to Disagree," The Annals of Statistics, 4, pp. 1236-1239, 1976.

Aumann, Robert J., "On the NTU Value," Working Paper #105, Economics Series, Institute for Mathematical Studies in the Social Sciences, Stanford University, 1981.

Aumann, Robert J. and Bezalel Peleg, "Von Neumann-Morgenstern Solutions to Cooperative Games without Side Payments," Bulletin of the American Mathematical Society, 66, pp. 173-179, 1960.

Harsanyi, John C., "A Simplified Bargaining Model for the n-Person Cooperative Game," International Economic Review, vol. 4, pp. 194-220 1963.

Harsanyi, John C., Rational Behavior and Bargaining Equilibrium in Games and Social Situations, Cambridge University Press, Cambridge, 1977.

Harsanyi, John C., "Comments on Roth's Paper: "Values for Games Without Sidepayments"," Econometrica, 48, p. 477, 1980.

Hart, Sergiu and Mordecai Kurz, "On the Endogenous Formation of Coalitions," Technical Report #328, The Economics Series, Institute for Mathematical Studies in the Social Sciences, Stanford, 1981.

Luce, R. Duncan, and Howard Raiffa, Games and Decisions: Introduction and Critical Survey, John Wiley, New York, 1957.

Milgrom, Paul, "An Axiomatic Characterization of Common Knowledge," Econometrica, 49, pp. 219-221, 1981.

Roth, Alvin E., "The Shapley Value as a von Neumann-Morgenstern Utility," Econometrica, vol. 45, pp. 657-664, 1977a.

Roth, Alvin E., "Bargaining Ability, the Utility of Playing a Game, and Models of Coalition Formation," Journal of Mathematical Psychology, vol. 16, pp. 153-160, 1977b.

Roth, Alvin E., "Values for Games Without Sidepayments: Some Difficulties with Current Concepts," Econometrica, 48, pp. 457-465, 1980.

Roth, Alvin E. and Michael W. K. Malouf, "Game-Theoretic Models and the Role of Information in Bargaining," Psychological Review, vol. 86, pp. 574-594, 1979.

Roth, Alvin E. and Michael W. K. Malouf, "Scale Changes and Information in Bargaining: An Experimental Study," Mathematical Social Sciences, forthcoming, 1982.

Roth, Alvin E., Michael W. K. Malouf, and J. Keith Murnighan, "Sociological versus Strategic Factors in Bargaining," Journal of Economic Behavior and Organization, vol. 2, pp. 153-177, 1981.

Roth, Alvin E. and J. Keith Murnighan, "The Role of Information in Bargaining: An Experimental Study," Econometrica, forthcoming, 1982.

Shafer, Wayne J., "On the Existence and Interpretation of Value Allocations," Econometrica, 48, pp. 467-476, 1980.

Shapley, Lloyd S., "A Value for n-Person Games," in Contributions to the Theory of Games, II, (H. W. Kuhn and A. W. Tucker, eds.) Princeton University Press, Princeton, 1953.

Shapley, Lloyd S., "Utility Comparison and the Theory of Games," in La Decision: Aggregation et Dynamique des Ordres de Preference, Paris: Editions du Centre National de la Recherche Scientifique, pp. 251-263, 1969.

von Neumann, John and Oskar Morgenstern, Theory of Games and Economic Behavior, Princeton University Press, Princeton, 1944 (2nd edition 1947, 3rd edition 1953).