SOULÉ'S

Primary Philosophic Arithmetic,

EMBRACING MENTAL AND WRITTEN

Arithmetical Exercises and Examples,

PERTAINING TO THE WORK OF HANDLING WHOLE AND FRACTIONAL NUMBERS AND A GREAT VARIETY OF IMPORTANT PRACTICAL PROBLEMS.

BEING A COMPLETE INTRODUCTORY WORK

TO THE

SCIENCE OF NUMBERS.

NEW ORLEANS:
CLARK & HOFELINE, PRINT, 112 GRAVIER ST.
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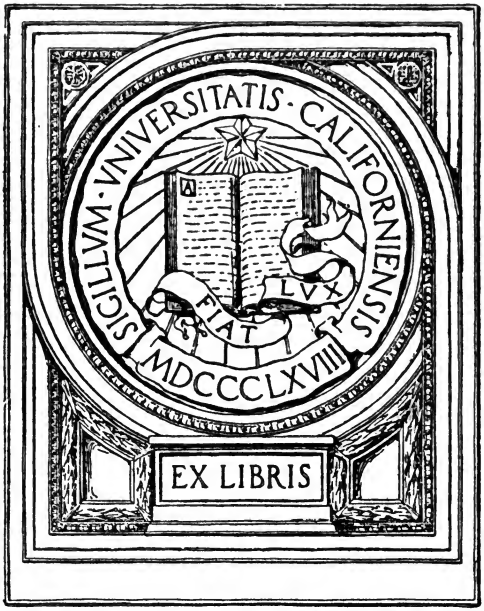

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Arithmetical Exercises and Examples,

PERTAINING TO THE WORK OF HANDLING NUMBERS AND THE APPLICATION THEREOF TO SIMPLE PRACTICAL QUESTIONS INVOLVING THE PRINCIPLES OF

Addition, Subtraction, Multiplication and Division

OF WHOLE AND FRACTIONAL NUMBERS.

Also a great variety and a large number of important practical problems, such as occur in the Countingroom, Factory, Workshop, on the Plantation, and throughout the various departments of business life.

Designed as a Supplement to

Soule's Philosophic, Commercial and Exchange Calculator

AND, AS AN

INTRODUCTORY WORK

TO THE

SCIENCE OF NUMBERS.

By GEO. SOULÉ,

Practical and Consulting Accountant, Commercial Lawyer, President of Soulé's Commercial College and Literary Institute, author of "Contractions in Numbers," and the "Analytic and Philosophic, Commercial and Exchange Calculator."

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PREFACE.

THE design of this work is twofold: 1. As a supplement to the author's large and advanced treatise, which does not contain sufficient primary work to meet the wants of young pupils. 2. As an introductory treatise to the science of numbers. It is especially designed to supply the requirements of primary and intermediate classes, and, at the same time, it presents much practical work of rare import to the advanced student. Bills and invoices of various forms for many departments of business constitute a special feature of the work. It is believed to possess superior merit on the following points: 1. In the arrangement and character of the mental exercises and the logical methods of mental training. 2. In the extent, variety, practical and scientific character of the problems. 3. In the elucidation of subjects. 4. In the philosophic solution of problems, by which system all the reasoning organs of the mind are expanded and the learner capacitated, not only to produce the results of problems, but to observe fine distinctions, reason logically, and deduce correctly; thus qualifying for a high plane of usefulness in various vocations of life.

The philosophic system is believed to be the most valuable improvement yet made to impart a thorough knowledge of the principles of numbers and capacitate the learner to utilize the same in the practical affairs of business life, and as it is the only natural system, it is destined at no distant day, to be adopted by all reasoning minds and efficient instructors. But, notwithstanding its superiority and the fact that its advocates include many

of the most profound mathematical minds, yet, like every other improvement or discovery in education, commerce, art, or science, it has opponents and is regarded with indifference by those who are satisfied with the non-progressive and non-reasoning methods of past ages.

With an experience extending through a period of twenty years with nearly four thousand students, the author has tested the advantages of the philosophic system, and from a full knowledge of its superior merits, he conscientiously assures his co-laborers in the mathematical field of education that a more thorough knowledge of the science of numbers can be imparted, and in far less time, by this system than by the usual methods and systems of work.

The science of numbers is the front door to the grand Temple of mathematics, in which are displayed some of the most beautiful principles of logic and profound syllogistic, analogical and axiomatical truths to be found in the vast fields of thought. And all who aspire to pre-eminence in brain power—all who hope to ascend to the highest planes of mathematical knowledge, must devote themselves earnestly to this subject.

In the selection of the material and the elements for it problems, this work does not present the toys and play things of the nursery, nor does it confine itself to the articles bought and sold on 'change. Instead of gyrating in the non-practical and non-progressive paths described by its hundreds of predecessors, it has diverged into new channels and derived the facts and elements of many of its problems from geography, history and chronology; from educational and commercial statistics; from Natural Philosophy, Astronomy, Geology and Chemistry; from Anatomy, Physiology and Hygiene; and from many other

departments of scientific knowledge. Through this means, the work is rendered far more interesting, and as it brings into use different organs of the mind from those which consider the computation of numbers only, it thereby imbues the mind of the learner with much valuable information without the cost of additional study or the expenditure of additional time.

The work has been prepared during such intervals of time as the author could command from his professional duties as teacher of Business Sciences, and Consulting and Practical Accountant; and, nowithstanding great care has been bestowed upon it, it is not improbable that some typographical or other errors may have escaped notice. Should any such be found, the author will esteem it a favor to be informed of them, in order that they may be expunged in future editions.

The author avails himself of this occasion to extend his thanks to his associate instructors and advanced students for their kindly aid in proof reading, and especially does he express his gratitude to his assistant instructor, Mr. B. D. Rowlee, for services cheerfully rendered in proof reading and the re-working of problems.

Soliciting for the work a thorough examination and a just measure of its merits, with the earnest hope that it may prove acceptable, and be of service in unfolding the principles of the beautiful science of numbers, and aid in advancing the interests of the rising generation, it is now submitted to the public.

THE AUTHOR.

NEW ORLEANS, Jan. 4, 1877.

TOPICAL INDEX.

DEFINITIONS.

1. **Definition** is the meaning or import of a word expressed by other words.

2. **Science** is classified knowledge.

3. **Quantity** is any thing that can be increased or diminished.

4. **A Unit** is a single thing of whatsoever denomination or nature.

5. **A Number** is a unit or a collection of units.

6. **An Abstract Number** is one in which the kind of unit or quantity is not designated, thus: three, four, five, etc.

7. **A Denominate or Concrete Number** is one in which the kind of unit is designated. Thus: two pounds, five yards, nine dollars, etc.

8. **A Compound Number** is a denominate number expressed in two or more denominations, thus: 5 years, 4 months and 8 days; 2 miles, 5 furlongs and 10 rods; 2 yards, 2 feet and 5 inches.

9. **An Arithmetical Complement of a Number** is the difference between the number and a unit of the next higher order, thus: 3 is the arithmetical complement of 7; 26 is the arithmetical complement of 74; 19 is the arithmetical complement of 981.

10. **A Problem** is a question proposed or given for solution.

11. **Philosophy**—the knowledge of phenomena as explained by and resolved into causes and reasons, powers and laws.

12. **Arithmetic** is the Science of Numbers: or to define it more extendedly, it is that branch of Mathematics which treats of the properties and relations of numbers when expressed by the aid of figures, either singly or combined. These principles and relations of numbers combined with the facts relating to problems, are applied, by the reasoning powers of man to the solution of all numerical problems of business affairs and practical life.

13. **Figures;** in Arithmetic *figures* are characters used to represent numbers. The ten Arabic figures which we use, are

Naught or Cipher	One	Two	Three	Four	Five	Six	Seven	Eight	Nine
0	1	2	3	4	5	6	7	8	9

By properly combining these ten figures all possible numbers may be represented.

The 1, 2, 3, 4, 5, 6, 7, 8, and 9 are sometimes called digits. They are also called the significant figures because each signifies a number when alone.

The 0 is so called because by itself it does not signify or represent any number. It expresses number only when used in connection with other figures.

14. **Value of Figures**—Figures have two values, a *simple* and a *local* value: thus when we write 2, independent of other figures, it has only a simple value, representing two units; but when we write it to the left of another figure or figures, thus, 23 or 241, it has a *local* value as well as a simple value. This local value depends on the scale or system of numbers employed and its location in the scale.

15. **Order of Figures**—The successive places occupied by figures are often called orders. Thus a figure in the first place is called a figure of the *first* order, or of the order of *units;* a figure in the second place is a figure of the second order, or of the order of *tens;* in the third place, of the third order, or of the order of *hundreds;* and so on, each figure next to the left belonging to a distinct order, the unit of which is *tenfold* the size or value of a unit of

the order of the figure on its right; and this increase in value from *right to left by ten* constitutes the *Decimal Scale* or *System* of numbers.

16. **Notation** is a method of *writing* numbers. There are two systems, the *Arabic* and *Roman*.

By the **Arabic** Notation numbers are expressed or written by *figures*.

By the **Roman** Notation numbers are expressed or written in letters.

17. **Numeration** is the method of reading written numbers.

There are two systems of numerating or reading numbers, the French and the English.

The *French* system is the one in general use in the United States and the Continent of Europe.

The *English* system is that generally used in England and the English Provinces.

FRENCH SYSTEM OF NUMERATION.

18. The French system separates figures into groups or periods of three figures each, and gives a different name to each period, thus:

Period of Octillions.	2	Hundreds of Octillions.	
	8	Tens of Octillions.	
	4,	Octillions.	
Period of Septillions.	5	Hundreds of Septillions.	
	6	Tens of Septillions.	
	1,	Septillions.	
Period of Sextillions.	3	Hundreds of Sextillions.	
	3	Tens of Sextillions.	
	8,	Sextillions.	
Period of Quintillions.	1	Hundreds of Quintillions.	
	7	Tens of Quintillions.	
	9,	Quintillions.	
Period of Quadrillions.	6	Hundreds of Quadrillions.	
	0	Tens of Quadrillions.	
	4,	Quadrillions.	
Period of Trillions.	9	Hundreds of Trillions.	
	3	Tens of Trillions.	
	2,	Trillions.	
Period of Billions.	4	Hundreds of Billions.	
	8	Tens of Billions.	
	7,	Billions.	
Period of Millions.	2	Hundreds of Millions.	
	6	Tens of Millions.	
	3,	Millions.	
Period of Thousands.	1	Hundreds of Thousands.	
	9	Tens of Thousands.	
	6,	Thousands.	
Period of Units.	5	Hundreds.	
	4	Tens.	
	8,	Units.	

The periods above Octillions, in regular order, are Nonillions, Decillions Undecillions, Duodecillions, Tredecillions, Quatuordecillions, Quindecillions Sexdecillions, Septendecillions, Octodecillions, Novemdecillions, Vigintillions, &c.

ENGLISH SYSTEM OF NUMERATION.

19. The English system of numeration separates the figures into groups or periods of six figures each, and designates each period by a distinct name, thus:

Period of Quadrillions.	2	Hundreds of Thousands of Quadrillions.
	8	Tens of Thousands of Quadrillions.
	4	Thousands of Quadrillions.
	5	Hundreds of Quadrillions.
	6	Tens of Quadrillions.
	1,	Quadrillions.
Period of Trillions.	3	Hundreds of Thousands of Trillions.
	3	Tens of Thousands of Trillions.
	8	Thousands of Trillions.
	1	Hundreds of Trillions.
	7	Tens of Trillions.
	9,	Trillions.
Period of Billions.	6	Hundreds of Thousands of Billions.
	0	Tens of Thousands of Billions.
	4	Thousands of Billions.
	9	Hundreds of Billions.
	3	Tens of Billions.
	2,	Billions.
Period of Millions.	4	Hundreds of Thousands of Millions.
	8	Tens of Thousands of Millions.
	1	Thousands of Millions.
	2	Hundreds of Millions.
	9	Tens of Millions.
	3,	Millions.
Period of Units.	1	Hundreds of Thousands.
	9	Tens of Thousands.
	6	Thousands.
	5	Hundreds.
	9	Tens.
	8,	Units.

By examining and comparing the two systems, it will be observed that they are the same to the ninth figure or the hundreds of millions, but at that figure a variation is made. Hence, if we wish to know the value of numbers higher than hundreds of millions, when we hear them spoken or see them in print, we must know whether they are expressed according to the French or the English system of numeration.

THE ROMAN SYSTEM OF NOTATION.

20. In the Roman system of notation the letter I represents *one;* V, *five;* X, *ten;* L, *fifty;* C, *one hundred;* D, *five hundred* and M, *one thousand.* The intermediate and succeeding numbers are expressed according to the following principles:

First.—Every time a letter is repeated, its value is repeated; thus II represents *two;* XX represents *twenty.*

Second.—When a letter of *lesser* value is placed before one of *greater* value, the lesser is taken from the greater; if placed after the greater, it is to be added to it. Thus, IV represents *four,* while VI represents *six;* XL represents *forty,* LX represents *sixty.*

Third.—A line or bar —, placed over a letter, increases its value a thousand *times.* Thus $\overline{X}$ represents *ten thousand;* $\overline{L}$ represents *fifty thousand.*

TABLE OF ROMAN CHARACTERS.

I	one.	XXV	twenty-five.
II	two.	XXVI	twenty-six.
III	three.	XXVII	twenty-seven.
IV	four.	XXVIII	twenty-eight.
V	five.	XXIX	twenty-nine.
VI	six.	XXX	thirty.
VII	seven.	XL	forty.
VIII	eight.	L	fifty.
IX	nine.	LX	sixty.
X	ten.	LXX	seventy.
XI	eleven.	LXXX	eighty.
XII	twelve.	XC	ninety.
XIII	thirteen.	C	one hundred.
XIV	fourteen.	CC	two hundred.
XV	fifteen.	CCC	three hundred.
XVI	sixteen.	CCCC	four hundred.
XVII	seventeen.	D	five hundred.
XVIII	eighteen.	DC	six hundred.
XIX	nineteen.	DCC	seven hundred.
XX	twenty.	DCCC	eight hundred.
XXI	twenty-one.	DCCCC	nine hundred.
XXII	twenty-two.	M	one thousand.
XXIII	twenty-three.	MM	two thousand.
XXIV	twenty-four	MDCCCLXXVI	1876.

EXERCISES IN NOTATION AND NUMERATION.

21. In **Writing Numbers** begin at the left hand with the highest order and write each period in regular order, separating them by commas.

Write in figures the following numbers and numerate them according to the French system of numeration.

1. One thousand, six hundred and ninety-four.
2. Eighteen hundred and seventy-seven.
3. Twenty-four hundred and six.
4. Three hundred forty-one thousand and twenty-two.
5. Sixty-five million, one hundred thirty-two thousand, three hundred and eighty-seven.
6. Twelve billion, sixteen million, forty-three thousand, one hundred and eleven.
7. Nine hundred thousand, three hundred and fifty.
8. Six million, one hundred and sixty-nine thousand, four hundred and thirty-seven.
9. Seventy-six million, four hundred thousand, one hundred.
10. Twenty-two billion, one hundred three million, five hundred seventy-six thousand, one hundred and two.
11. One hundred two trillion, one hundred twenty-five million, four hundred and three.
12. Eight trillion, seven billion and seventy-six.

22. Write in figures the following numbers and numerate them according to the English system of numeration:

1. Four hundred twenty-three thousand, five hundred and fourteen.
2. Six hundred nineteen thousand, one hundred fifty-two million, twenty-one thousand and forty-seven.
3. Fifty-three billion, two hundred twelve thousand, twenty-six million, seventy-five thousand three hundred and eighty-four.

23. Write in the Roman System of Notation the following numbers:

9, 12, 14, 37, 49, 83, 108, 519, 1519, 14704, 88976, 13140363.

ADDITION.

24. **Addition**—*Increasing*—is the process of uniting two or more numbers of the same name or kind, so as to make one equivalent number.

25. The number obtained by this process is called the **Sum** or **Amount.**

26. The **Sign of Addition** is a perpendicular cross, $+$, called plus; it means more; thus $7 + 9$ is read, 7 plus 9, and means that 7 and 9 are to be added. When used after a number, thus, $5 +$, which is read 5 plus, it means 5 and a small excess.

27. The **Sign of Equality** is $=$. It is read equals, or equal to, and denotes that the numbers between which it is placed are equal to each other; thus $7 + 9 = 16$ means that 7 and 9 added are equal to 16. The expression is read, 7 plus 9 equals 16.

28. A **Numerical Equation** is an equality between two numerical expressions, which though differing in form from each other are equivalent. Each expression is called a term of the equation. Thus $5 + 8 = 13$ is a numerical equation in which the $5 + 8$ is called the first member of the equation and 13 the second member, and both are called the terms of the equation.

29. *Principle of Addition.* Numbers of the same kind, order or character only, can be added. Thus we cannot add 2 apples and 3 oranges, nor 5 pounds of sugar and 6 boxes of peaches; nor 6 units and 5 hundreds; nor $\frac{1}{2}$ and $\frac{3}{4}$, etc. We can only add apples to apples, oranges to oranges, sugar to sugar, peaches to peaches, units to units, hundreds to hundreds, halves to halves, fourths to fourths, etc. We can *collect* together things of different kinds, apples, peaches, oranges, etc., but by collecting them together we do not increase the number or sum of either and hence there is no addition.

TABLES.

ADDITION TABLE.

30. No. I.

NOTE. In learning these tables and handling all numbers, all intermediate words and thoughts that occur between the numbers to be combined and the result of the desired combination should be omitted. Thus, instead of saying or thinking, that 2 and 2 are 4, 3 and 5 are 8, etc., say or think 4 ; 8; etc.

1	1
1 1	2
1 2	3
2.1 2.3	4
1.2 4.3	5
1.2.3 5.4.3	6
1.2.3 6.5.4	7
1.2.3.4 7.6.5.4	8
1.2.3.4 8.7.6.5	9

Explanation—In this table we show 20 different combinations of the 9 significant figures to produce results from 1 to 9. It may be said that three 1's make 3, three 2's make 6, etc., and that they are regular combinations; but we see by the table that two 1's are 2, and that two 2's are 4, etc. Hence, though the table does not contain all the possible combinations, it does contain all that are essential and of value in this connection.

ADDITION TABLE.

31. No. II.

1.2.3.4.5 9.8.7.6.5	10	*Explanation.*—In this table we show the 25 different combinations of the 9 significant figures the sum of which equals *ten* or more. To attain rapidity in adding, it is absolutely necessary that, the learner should be so familiar with these combinations that he can instantly see the result without adding, i. e. he must know the result by the combination, just as he knows the value of 4, or 5, by the combination of lines forming the figure, or as he knows the pronunciation of a word without spelling it.
2.3.4.5 9.8.7.6	11	
3.4.5.6 9.8.76.	12	
4.5.6 9.8.7	13	
5.6.7 9.8.7	14	
6.7 9.8	15	
7.8 9.8	16	
8 9	17	
9 9	18	

☞ The rapid increasing and decreasing operations in the science of numbers depend upon the capacity of the calculator to instantly apprehend and accurately apply, the result of two or more figures, no matter how they may be combined. And the object of these tables is to aid in acquiring the desired capacity.

ADDITION AND SUBTRACTION TABLE.

32. TABLE III.

1 & ?=9	1 & ?=8	1 & ?=7	1 & ?=6	1 & ?=5	1 & ?=4	1 & ?=3	1 & ?=2
2 9	2 8	2 7	2 6	2 5	2 4	2 3	
3 9	3 8	3 7	3 6	3 5	3 4		
4 9	4 8	4 7	4 6	4 5			
5 9	5 8	5 7	5 6				
6 9	6 8	6 7					
7 9	7 8						
8 9							

EXPLANATION.

In this table we present the 35 combinations of the significant figures in which the difference between each is to be supplied by the learner. This is a very important table for rapid work in subtraction, by the addition method, and should receive careful attention.

ADDITION AND SUBTRACTION TABLES.

33. TABLE IV.

1 & ?=100	26 & ?=100	51 & ?=100	76 & ?=100
2 100	27 100	52 100	77 100
3 100	28 100	53 100	78 100
4 100	29 100	54 100	79 100
5 100	30 100	55 100	80 100
6 100	31 100	56 100	81 100
7 100	32 100	57 100	82 100
8 100	33 100	58 100	83 100
9 100	34 100	59 100	84 100
10 100	35 100	60 100	85 100
11 100	36 100	61 100	86 100
12 100	37 100	62 100	87 100
13 100	38 100	63 100	88 100
14 100	39 100	64 100	89 100
15 100	40 100	65 100	90 100
16 100	41 100	66 100	91 100
17 100	42 100	67 100	92 100
18 100	43 100	68 100	93 100
19 100	44 100	69 100	94 100
20 100	45 100	70 100	95 100
21 100	46 100	71 100	96 100
22 100	47 100	72 100	97 100
23 100	48 100	73 100	98 100
24 100	49 100	74 100	99 100
25 100	50 100	75 100	

EXPLANATION.

We present this table to aid the learner in instantly seeing the difference between 100 and any number from 1 to 99. It is of special value in addition and subtraction, and all who expect to become rapid Calculators, must be proficient in this character of work.

SIGNS AND ABBREVIATIONS.

34. The following are the principal signs and abbreviations in general use among merchants and business men:

@ At.	Co. Company.
a/c Account.	Cr. Credit or creditor.
1^1 One and one-quarter.	Dr. Debit or Debitor.
1^2 One and one-half.	Gal. Gallons.
1^3 One and three-quarters.	Ps. Pieces.
℔ Per.	Yd. Yards.
lb Pound (weight).	Fr't. Freight.
$ Dollar or dollars.	Rec'd. Received.
¢ Cent. or cents.	Pay't. Payment.
% Per cent. or per centum.	Inst. This month.
Amt. Amount.	Prox. The next month.
Bbl. Barrel.	Ult. The last month.
Doz. Dozen.	£ Pound Sterling.
B. L. Bill of Lading.	O. K. All Right.
Blk. Black.	Fr. Franc, French coin.
Shipt. Shipment.	Fwd. Forward.
Sunds. Sundries.	Bal. Balance.
Dft. Draft.	Cons't. Consignment.
Com. Commission.	hhds, hogsheads.
Do. The same.	Mdse. Merchandise.

/ Shillings, thus $^2/_6$ two shillings and sixpence.

Mk. Marks, the German monetary unit.

✓ Check mark, correct, approved.

⊕ Cifrao, used to separate the milreis from the reis in Brazil money.

17 doz. $\$\frac{4}{10}$, $\$\frac{6}{12}$, $\$\frac{7}{15}$=17 doz., 4 of which are at $10 per doz., 6 @ $12 and 7 @ $15.

8 doz. $\frac{2}{4}$ @ 5 / $\frac{6}{5}$ @ $\frac{4}{6}$, 2 doz. No. 4 @ 5 shillings per doz., and 6 doz. No. 5 @ 4 shillings sixpence per dozen.

35. Name the *unit* result of the following numbers:

9	8	9	8	9	8	9	8	8	7	9	5	6
9	8	6	6	4	4	2	2	9	8	7	9	7
—	—	—	—	—	—	—	—	—	—	—	—	—
3	8	7	6	8	5	5	6	9	4	7	5	
9	3	3	4	4	7	5	6	1	7	7	8	
—	—	—	—	—	—	—	—	—	—	—	—	
3	4	5	1	6	9	8	2	3	6	5		
5	2	2	3	3	2	1	5	7	4	7		
7	9	8	7	8	2	4	6	8	9	3		
—	—	—	—	—	—	—	—	—	—	—		
7	1	8	4	6	5	4	9	7	9	7		
4	8	2	7	7	8	5	6	8	8	5		
5	8	9	9	8	9	8	8	5	6	9		
—	—	—	—	—	—	—	—	—	—	—		

For further explanation of Addition, the importance of it, and the most rapid processes of adding see Soule's Contractions in Numbers.

EXERCISES.

1. Write all the combinations of two figures that make 10, 11, 12, 13, 14, 15, 16, 17 and 18.
2. Commence with 1 and orally add thereto 2, and continue to add 2 to the successively occurring sums until you produce 21. Thus 3, 5, 7, 9, 11, 13, etc.
3. Commence with 1 and in like manner add 3 until you produce 31. Thus 4, 7, 10, 13, etc.
4. Commence with 1 and in like manner add 4 until you produce 41.
5. Commence with 1 and in like manner add 5 until you produce 51.
6. Commence with 1 and in like manner add 6 until you produce 61.

7. Commence with 1 and in like manner add 7 until you produce 71.

8. Commence with 1 and in like manner add 8 until you produce 81.

9. Commence with 1 and in like manner add 9 until you produce 91.

10. Orally add by 2's until you produce 20.

11. " " 3's " " 30.

12. " " 4's " " 40.

13. " " 5's " " 50.

14. " " 6's " " 60.

15. " " 7's " " 70.

16. " " 8's " " 80.

17. " " 9's " " 90.

18. " " 10's " " 100.

19. Commence at 1 and orally add by 3 and 5 alternately until you produce 100.

20. Commence at 1 and orally add by 4 and 7 alternately until you produce 100.

EXAMPLES IN ADDITION.

36. Add the following numbers: 6376, 564, 309, 485 and 5092.

OPERATION.

Thousands.	Hundreds.	Tens.	Units.
6	3	7	6
	5	6	4
	3	0	9
	4	8	5
5	0	9	2
Sum 12	8	2	6
	1	3	2

Explanation.—In all addition problems we first write the numbers so that units of the same order will stand in the same column, i. e., *units* in the units or first column; *tens* in the tens or second column; hundreds in the hundreds or third column and so on through the numbers. We then begin at the units or first column and add the columns separately. In adding the first column, we commence with the 2 and 5, and name only the successive results thus, 7, 16, 20, 26; which is 2 *tens* and 6 *units;* the 6 we write in the first place or column of units and place the 2 tens which is to be carried to the column of tens directly below the 6 in a small figure. Then adding the 2

tens to the tens column, we say, 11, 19, 25, 32; which is 3 *hundred* and 2 *tens:* the 2 tens we write in the column of tens and place the 3 hundreds, which is to be carried to the hundreds column directly under it. Then adding the 3 hundred to the hundreds column, we say, 7, 10, 15, 18, which is 1 *thousand* and 8 *hundred;* the 8 hundred we write in the hundreds column and the carrying figure, 1 thousand, directly under. Then adding the 1 thousand, to the fourth or thousands' column, we say 6, 12, which is 1 *ten thousand* and 2 *thousand*, and this being the last column to add we write the figures in their respective columns and produce 12826 as the *sum* of all the numbers.

When adding, set the result in pencil figures, being careful to place the carrying figure or figures directly beneath the unit figure of each column added as shown in the preceding problem.

PROOF OF ADDITION.

The best proof of the correctness of addition is to be proficient in your work, and then re-add the columns in the reverse direction.

What is the sums of the following groups of numbers?

(2)	(3)	(4)	(5)	(6)
4304	780	89	777	9040
291	1261	706	888	1288
643	537	73	999	9907
98	309	4009	666	6543
1400	6987	8888	645	2018

Add the following groups of numbers:

(7)	(8)	(9)	(10)	(11)	(12)
818	412	582	328	809	981
390	297	578	346	523	350
970	318	757	386	605	269
276	824	420	672	848	789
752	932	731	793	945	696
843	373	542	864	397	136
865	576	853	965	684	169
129	876	684	448	976	295
768	444	743	404	666	468
904	102	915	151	217	687
972	814	686	148	879	825
114	331	637	263	516	951
346	554	917	295	259	784
545	161	650	161	896	122
622	197	411	461	864	440
749	490	237	874	565	450
717	876	349	898	150	414
222	902	489	769	514	654
234	396	698	243	446	789
166	484	228	174	576	458
365	235	433	952	489	747
272	386	949	683	394	636
729	624	687	574	407	241
955	897	762	956	812	477
177	477	849	658	798	681

Add the following groups of numbers:

(13)	(14)	(15)	(16)	(17)	(18)
864	677	595	849	539	257
363	305	249	283	377	476
629	420	463	327	762	426
145	982	830	651	235	684
174	217	221	543	856	492
144	326	232	502	950	343
176	111	151	113	446	602
767	871	387	438	834	182
644	512	516	455	540	955
747	814	247	328	919	858
156	376	331	633	358	989
106	468	281	624	149	855
872	189	828	581	268	954
694	177	986	491	662	126
788	885	817	888	693	136
866	264	918	992	682	564
944	294	289	202	355	163
922	896	259	548	223	764
116	597	381	365	521	921
911	814	329	208	530	515
866	277	678	662	874	735
179	476	640	764	528	393
129	716	821	287	584	556
659	802	457	848	625	888
778	584	587	255	262	932

Add the following groups of numbers:

19	20	21	22	23	24
883	792	743	153	919	547
356	414	560	214	620	380
595	454	871	248	922	616
638	366	349	636	369	874
679	464	955	549	158	682
594	933	936	694	862	232
953	686	746	783	874	713
178	541	793	225	935	499
215	939	798	649	951	874
119	201	324	232	959	779
753	871	687	478	865	622
311	438	843	484	724	718
182	218	421	252	645	180
686	869	586	648	148	477
896	189	518	551	227	396
996	595	959	995	193	495
293	521	152	475	947	568
262	727	572	977	797	130
515	259	425	362	736	111
833	585	458	485	328	682
745	177	217	631	803	685
125	413	841	194	729	996
245	825	972	698	169	492
968	868	489	258	845	194
540	799	386	277	477	864

25	26	27	28
778999	979644	156563	1333182
115224	130466	994544	9979667
964892	898567	836869	7391573
578678	787543	234246	3517569
577594	964432	765183	8598674
668678	699678	345927	2513756
669657	978321	654678	3454210
539886	678789	456432	7656754
664756	564673	345718	5467856
795568	895437	765391	5645781
699689	569123	673123	7893344
689786	678982	437987	3216675
688968	869771	566789	4569911
935789	668339	544321	6543344
778896	956234	891389	9576677
659669	195842	219720	1539902
363769	957454	625221	6662234
351994	573367	431348	4235564

29. Add 6, 8, 9, 7, 6, 8, 5, 4, 9, 4, 8, 7, 6, 9, 14, 19, 18, 27, 38, 47, 59, 65, 74, 83, 92. Ans. 632.

30. Add 528, 791, 14389, 888, 91361, 587, 301, 7004, 52800, 7106, 42881. Ans. 218,636.

31. Add 476010, 51873, 98, 48932, 3581427, 67843, 21050, 3672. Ans. 4,250,905.

32. Add 63, 94, 85, 74, 63, 52, 41, 39, 48, 57, 66, 75, 84, 93, 27, 18, 60, 80, 19, 88, 99, 77, 66, 55, 44, 33, 22, 11, 98, 97, 96, 86, 76, 65, 54, 43. Ans. 2248.

33. Add seven million four thousand and ninety-six, and three hundred eighty-seven thousand five hundred and sixty-two. Ans. 7391658.

34. Find the sum of 4888765, 92238, 1600084, 8888888, 99999999999, 4100000808707 and 222222333-333444444. Ans. 22222653334972312 5.

35. Find the sum of 999999999, 88888888, 7777777, 666666, 55555, 4444, 333, 22, 1, and sixty-three millions. Ans. 1160393685.

36. Add 789, 679, 987, 140018, 191070, 871230432, 49706, 40000, 80000000 and eleven hundred and eleven.

Ans. 951,654,792.

37. Add five hundred thousand nine hundred thirty-nine, and eleven thousand eleven hundred and eleven.

Ans. 513050.

37. **Dollar and Cent signs.** The dollar sign is $, and the cent sign is ¢. When the dollar sign is placed before numbers they are read as dollars. Thus $45 is read 45 dollars. When the cent sign is placed after numbers they are read as cents. Thus 14¢ is read 14 cents. When dollars and cents are written together the cents are separated from the dollars by a point (.) and the sign of cents is omitted. Thus $16.45 is read 16 dollars and 45 cents.

Since there are 100 cents in 1 dollar, *cents* always occupy *two* places and only two in connection with dollars. When the number of cents is less than 10 a *naught* must be used to fill the *tens* column or the first place at the right of the point. Thus 8 dollars and 5 cents are written $8.05.

When cents only are written they are expressed as follows: 25 cents, or 25¢ or $.25.

When writing numbers representing dollars and cents for the purpose of addition, they must be set so that dollars will be under dollars and cents under cents in the regular order of units, tens, hundreds, etc., and the points (.) that separate dollars and cents must be in a vertical line.

The dollar sign ($) and the point (.) should never be omitted when writing dollars and cents.

		38	39	40	41	42
38.	Add	$14.50	$34.16	$75.	$ .88	$180.40
		8.	9.08	4.45	11.	48.08
		4.25	14.83	67.06	5.13	91.16
		12.15	8.	.35	7.02	7.05
		$38.90	$66.07	$146.86	$24.03	$326.69

	43	44	45	46	47
Add	$321.	$521.16	$ 9.45	$ 431.	$194.15
	640.80	83.25	80.	124.	8.05
	9.13	19.30	17.	381.	73.75
	75.20	8.	.65	569.	6.13
	100.05	4.07	6.10	827.	.95
	$1146.18	$635.78	$113.20	$2332.	$283.03

48. Add $8.12, $9, $.50, $3.40, $37.05, $.75 and $12.12. Ans. $70.94.

49. Add $43.10, $17, $5, 48¢, 75¢, $11, $24.14, $3. Ans. $104.47.

50. Add $108, $97.16, $84.12, $.75, $8, $6.40, 25¢, $18. Ans. $322.68.

51. Add $580.10, $671.23, $791.98, $88, 45¢, 5¢, $3.10. Ans. $2,137.91.

52. Add $999.99, $888.88, $777.77, $666.66, $555.55, $444.44, $333.33, $222.22, $111.11 and 1¢. Ans. $4,999.96.

53. Add $987.65, $876.54, $765.43, $654 32, $543.21, $123.45, $234.56, $345.67, $456.78, $567.89, $678.90 and $789. Ans. $7,123.40.

54. Middlemiss bought a hat for $2, a coat for $9.50, a pair of shoes for $2.75, a pair of pants for $4, a vest for $1.75, and had $41.05 left. How much money had he at first? Ans. $61.05.

55. Miss Smith paid for a broom 35¢, for soap $1.60, for starch 75¢, for matches 5¢, for salt 15¢, for sugar $1.50, for rice $2, for butter 80¢, Graham flour $1.25 and for a hygienic cook book $1. What was the sum paid for all? Ans. $9.45.

56. Prophet paid for a reader $1.35, for an arithmetic $1.50, for a history $2, for a set of drawing instruments $3.70, for paper $.60, for pens $.15, for ink $.05, for a pair of Indian clubs $3.50, and for the boy's own book $1. What did all cost? Ans. $13.85.

57. Conrad paid $1.75 for Chesterfield's letters; $1.80 for Cutter's Anatomy, Physiology and Hygiene; $1.75

for Comb's Constitution of Man; $1.25 for How to Read Character by Wells; $1.50 for Nordhoff's Politics for Young Americans; $1.75 for Physical Perfection by Jacques; $4 for Plutarch's Lives; $8 for Shakspeare's Works; $2 for the Literary Reader; $6 for Carey's Social Science; $5 for Parson's Laws of Business; $5 for Soule's Philosophic Work on Commercial and Exchange Calculations, and $1 for Cushing's Manual. How much did he pay for all?

Ans. $40.80.

58. If you should travel by rail 160 miles, by steamer 214, and walk 8, how far would you travel?

Ans. 382.

59. A planter raises 9842 pounds of sugar, 2351 pounds of cotton, 1827 pounds of rice, 3840 bushels of corn, 325 bushels of sweet potatoes and 194 bushels of beans. How many pounds and how many bushels does he raise in all? Ans. 14020 pounds, 4359 bushels.

60. Conrad loaned to Purcell $9; to Gresham $3.50; to Hanna 75¢; to Mitchell 85¢; to Sweeney 5¢; to Bothick $1; to Keen 25¢; to Abbott 75¢; to Prophet 50¢. What sum did he loan to all? Ans. $16.65.

61. Keen has $143.05; Couret $91; McCoard $18.30; Bush 90¢; Nevers 25¢; Fischer $5.05; Beck $9; Meyers $6; Levy $7; Brown $7; Rice $45; Shotwell $27; Wise $6.80; Moffett $5.50; Lindsey $88.70. How much have all? Ans. $460.55.

62. A merchant bought four adjacent lots of ground for $6850. He built a house thereon which cost $11875. Paid for fences $912; for flagging $1819.55; for furniture $3481.12. How much did the whole cost?

Ans. $24,937.67.

63. If you pay $175 for a horse, $450 for a carriage, $75 for a set of harness, $38 for a saddle and bridle and $6.50 for a whip. What will the whole cost?

Ans. $744.50.

64. A planter has 54 cows, 321 sheep, 174 mules, 23 horses, 42 oxen, 43 calves, 7 colts. How much live stock has he altogether? Ans. 664.

65. A merchant bought at one time 250 barrels Flour for $1500; at another 345 barrels for $2415; and at another 200 barrels for $1625. How many barrels did he buy and what was the total cost? Ans. 795 Bbls., $5540 Cost.

66. The weight of ten bales of cotton is as follows: 481, 503, 398, 462, 470, 479, 401, 397, 463, 511 pounds, what is the total weight? Ans. 4565.

67. Bought at one time 43 yards of calico and 32 yards of silk; at another 104 yards of calico and 24 yards of silk, and at another 96 yards of calico and 48 yards of silk. How many yards of each kind did I buy?

Ans. Calico 243, Silk 104.

68. Paid $425 for a lot of sugar, $120 for rice and $75 for potatoes. Sold the sugar at a profit of $41 and the rice and potatoes at cost. What did I get for the whole?

Ans. $661.

69. From New Orleans to the Rigolets is 31 miles; hence to Montgomery, 18; hence to Bay St. Louis, 3; hence to Pass Christian, 6; hence to Mississippi City, 13; hence to Biloxi, 9; hence to Ocean Springs, 4; hence to East Pascagoula, 16; hence to St. Elmo, 21; hence to Mobile, 20. How many miles to Mobile? Ans. 141.

70. From New Orleans to Kenner is 10 miles; hence to Manchac, 27; hence to Pouchatoula, 11; hence to Hammond, 4; hence to Amite, 16; hence to Tangipahoa, 10; hence to Osyka, 10; hence to Magnolia, 10; hence to Mc Comb City, 7; hence to Summit, 3; hence to Bogue Chitto, 10; hence to Brookhaven, 10; hence to Beauregard, 11; hence to Crystal Springs, 19; hence to Terry, 9; hence to Jackson, 15; hence to Madison, 13; hence to Canton, 11. How many miles is it to Canton? Ans. 206.

71. A young man paid $125 for a year's tuition at college, $22.50 for books, lost $40, and has $378.35 on hand. How much had he at first? Ans. $565.85.

72. A boy gave Jane 6 oranges, Kate 4, John 3, he ate 2, and had 5 remaining. How many had he at first?

Ans. 20.

73. Louisiana contains 41255 square miles; Mississippi, 47156; Texas, 237504; Arkansas, 52198; Tennessee, 45600; Kentucky, 37680; Alabama, 50722; Georgia, 52009; South Carolina, 29385; North Carolina, 50704; Missouri, 67380; Virginia, 61352; Maryland, 11124; Florida, 59268; California, 188982. How many square miles in the fifteen states? Ans. 1,032,319.

74. The population of London is 3311000; Paris, 1852000; St. Petersburg, 667000; Rio Janeiro, 275000; Constantinople, 400000; Vienna, 834000; Berlin, 825000; Lisbon, 224000; Pekin, 1648000; Tokio or Jeddo, 790000; Bombay, 647000; Madrid, 332000; Glasgow, 489000; Dublin, 311000; Amsterdam, 278000; Brussels, 176000; Stockholm, 139000; Copenhagen, 181000; Cairo, (Egypt) 354000; Tunis, 125000. What is the population of all? Ans. 13,858,000.

75. The length of the Mississippi River is 4200 miles; of the Nile, 4000; Amazon, 3750; Yenisei, 3400; Obi, 3000; Yang-tse-Kiang, 3320; Niger, 3000; Lena, 2700; Amoor, 2650; Volga, 2000; Ganges, 1600; Brahmapootra, 2300; La Plata, 2300; Mackenzie, 2300; St. Lawrence, 2000; Saskatchewan, 1900; Orinoco, 1550; Columbia, 1020; Colorado, 600; Yukon, 1600; Red River, 1500. What is the combined length of all?

Ans. 50,690.

76. Lake Superior is 400 miles in length; Lake Michigan, 320; Lake Huron, 240; Lake Erie, 240; Lake Ontario, 180; Lake Baikal, 375; Lake Pontchartrain, 40. What is the combined length of all? Ans. 1795.

77. There are in the world 394000000 Christians; 500000000 Buddhists; 145000000 Brahmins; 100000000 Confucians; 15000000 Shinloan; 199000000 Mohammedans; 7000000 Israelites. How many combined?

Ans. 1,360,000,000.

78. Mount Everest of the Himalaya chain in Asia the highest point on the globe, is 29062 feet high; Mt. St. Elias, the highest mountain in North America, is 17900 feet; Mt. Illampu, the highest mountain in South America,

is 24812 feet; Mt. Blanc, the highest mountain in Europe, is 15780 feet; Mt. Kilima Njaro, the highest mountain in Africa, is 20065 feet; Mt. Kosciusko, the highest mountain in Australia, is 7176 feet. What is the combined height of all? Ans. 114,795 feet.

79. By the census of 1870, the population of New York was 942992; Philadelphia, 674022; Brooklyn, 396099; St. Louis, 310864; Chicago, 298977; Baltimore, 267354; Boston, 250526; Cincinnati, 216239; New Orleans, 191418; San Francisco, 149473; Buffalo, 117714; Washington, 109199; Newark, 105059; Louisville, 100753; Mobile, 32034; Galveston, 13818; Memphis, 40226. What is the population of all combined?

Ans. 4216767.

80. On Monday 85482 persons entered the gates at the Centennial Exhibition, Philadelphia; on Tuesday, 108421; on Wednesday, 98792; on Thursday, 91953; on Friday, 103819, and on Saturday, 174587. How many entered in the six days? Ans. 663,054.

81. The standing army of the United States is 32000; of Great Britain and Ireland, 192000; of France, 454000; of the German Empire, 402000; of Russia, 766000; of Spain, 284000; of Switzerland, 201000; of Italy, 205000; of Brazil, 25000; of Mexico, 21000; of Turkish Empire, 93000; of Sweden, 150000; of Holland, 62000; of Portugal, 33000; of Belgium, 40000. How many men in all? Ans. 2,960,000.

82. Homer was born 733 years before the Christian Era. How many years from the birth of Homer to the year 1876? Ans. 2609.

83. During the fiscal year ending Sept. 1st 1876, the receipts of cotton at various points were as follows:

New Orleans, 1401563 bales; Galveston, 465529; Mobile, 371298; Savannah, 521437; Charleston, 389698; Wilmington, 78267; Norfolk, 469997; Baltimore, 18821; New York, 219609; Boston, 75065; Philadelphia, 58632; Various, 57976. How many bales were received during the year? Ans. 4127892.

84. From Aug. 31st 1875 to Sept. 1st 1876, the production of Sugar in Louisiana was as follows:

Parish of Livingston, 4 hogsheads; St. Tammany, 16; East Feliciana, 37; Lafayette, 187; West Feliciana, 339; Vermillion, 609; Avoyelles, 1582; St. Landry, 1768; St. Martin, 1884; Orleans, 1041; St. Bernard, 2097; East Baton Rouge, 2544; Rapides, 2453; Pointe Coupee, 2762; Iberia, 3632; Jefferson, 3671; West Baton Rouge, 4155; St. Charles, 5808; St. John, 8335; Plaquemines, 9068; Iberville, 9814; Lafourche, 11302; Terrebonne, 10888; St. James, 13437; Ascension, 14267; St. Mary, 14318; Assumption, 14712. How many hogsheads were produced during the year? Ans. 140730.

85. From July 1st 1875 to July 1st 1876, the monthly receipts of coffee in New Orleans, were as follows:

July, 9635 bags; August, 25987; September, 24851; October, 8832; November, 34452; December, 4800; January, 32219; February, 16042; March, 4000; April, 10512; May, 9000; June, 15120. How many bags were received during the year? Ans. 195450.

86. From New Orleans to Carrolton is 7 miles; hence to Donaldsonville, 71; hence to Plaquemines, 32; hence to Baton Rouge, 20; hence to Port Hudson, 23; hence to Bayou Sara, 12; hence to mouth Red River, 40; hence to Natchez, 72; hence to Rodney, 45; hence to Grand Gulf, 18; hence to Vicksburg, 61; hence to the Louisiana Line, 97; hence to Helena, 230; hence to Columbus, 329; hence to Cairo, 20; hence to Cape Girardeau, 50; hence to St. Louis, 151. How many miles to St. Louis by river?

Ans. 1278 miles.

87. From New Orleans to the mouth of Red River is 210 miles; hence to Black River, 40; hence to Alexandria, 110; hence to Grand Ecore, 120; hence to Grand Bayou, 95; hence to New Hope, 60; hence to Waterloo, 30; hence to Shreveport, 35. How many miles to Shreveport by the river? Ans. 700 miles.

88. From New Orleans to Algiers Depot is 1 mile. hence to Gretna, 3; hence to Jefferson, 9; hence to St'.

Charles, 6; hence to Boutte, 6; hence to Bayou des Alemedes, 8; hence to Raceland, 8; hence to Ewin,'s, 6; hence to Lafourche, 6; hence to Terrebonne, 3; hence to Chucahoula, 6; hence to Tigerville, 5; hence to L'Ourse, 4; hence to Bayou Beuf, 3; hence to Ramos, 3; hence to Morgan City, 4; hence to Galveston, 240. How many miles to Galveston? Ans. 321 miles.

89. 24 peaches were eaten, 5 being spoiled, were thrown away, and 32 remained in the basket. How many were there at first? Ans. 61.

90. A man was 26 years of age when he was married. How old will he be when he has been married 14 years? Ans. 40 years.

91. A young man graduated from college when he was 22 years of age. He married 6 years afterwards, 2 years after that he was presented with a son. What will be his age when the son is 21 years old? Ans. 51 years.

92. A lady paid $6.50 for a dress, $8 for a shawl, $4 for a bonnet and $3.75 for a pair of shoes. What was the total cost? Ans. $22.25.

93. A boy sold his pony for $45, and lost $15 by the sale. What did the pony cost him? Ans. $60.

94. A merchant paid for a lot of goods $580, he sold them and gained $190. How much did he receive for them? Ans. $770.

95. Henry is 16 years old, James is 3 years older, and William is 2 years older than James. How old are James and William? Ans. James 19, William 21.

96. The internal framework of the human body consists of bones, which united by strong ligaments constitute the *skeleton*. In the skull are 8 bones; in the face 14; in each ear 3; in the tongue 1; in the trunk and spinal column and pelvis 55; in each shoulder 2; in each arm 3; in each wrist 8; in the palm of each hand 5; in each thumb 2; in each finger 3; in each leg 4; in each ankle 7; in each foot 5; in each great toe 2; in each of the other toes 3; and there are 32 teeth. How many bones in the whole body? Ans. 240.

97. How many pupils in a school in which there are 6 grades, the first containing 63, the second 58, the third 27, the fourth 49, the fifth 35 and the sixth 24? Ans. 256.

98. Bothick has $420; Conrad has $130 more than Bothick, and Prophet has as much as Bothick and Conrad together. What sum have all three? Ans. $1940.

99. Keen, Soule and Abbott form a copartnership, Keen invests $3400, Soulé $4000, and Abbott $500 more than both Keen and Soulé. What is the capital of the firm? Ans. $15,300.

100. A father gave his son seven thousand eight hundred dollars; his daughter nineteen hundred and fifty dollars; and his wife three thousand five hundred more than he gave to both, the son and daughter. What sum did he give away? Ans. $23,000.

SUBTRACCION, (Decreasing.)

39. **Subtraction** is the process or operation of finding the difference between two numbers of the same kind.

40. The result obtained by subtraction is called the **Difference** or **Remainder.**

41. The greater number is called the **Minuend,** which means a number to be decreased.

42. The lesser number is called the **Subtrahend,** which means the number to be subtracted.

43. The **sign of subtraction** is a horizontal line, —. It is read *minus* and means less.

When this sign is placed between two numbers it means that the number *after* it, is to be subtracted from the number *before* it. Thus 8 — 3 is read 8 minus 3.

For Subtraction Tables and contracted methods of subtraction see Soule's Contractions in Numbers.

44. The sign, ( ), *parenthesis*, or ——, *vinculum*, indicates that the numbers included within the parenthesis, or below the vinculum, are to be considered as one, or together. Thus (9+3)—5=7, or with the vinculum thus $\overline{9+3}-5=7$.

45. ORAL EXERCISES.

1. Commence at 50 and orally count to 0 by continually subtracting 1, thus: 49, 48, 47, 46, 45, etc.
2. Commence at 50 and orally count to 0 by continually subtracting 2, thus: 48, 46, 44, 42, etc.
3. Commence at 50 and orally count to 0 by successively subtracting 3, thus 47, 44, 41, 38, etc.
4. In like manner commence at 50 and subtract respectively 4, 5, 6, 7, 8, 9 and 10 until you produce 0 or a number less than the subtracted number, thus 46, 41, 35, 28, etc.
5. Commence at 50 and subtract alternately 2 and 5 until you produce 1, thus, 48, 43, 41, 36, etc.
6. Commence at 50 and subtract alternately 8 and 3 until you produce 6, thus 42, 39, 31, etc.

46. *To subtract one number from another when any figure of the subtrahend is less than the corresponding figure of the minuend.*

1. From 897 subtract 641.

OPERATION.

897	or	641	*Explanation.*—First set the numbers with the less *under* or *over* the greater, so that units of the same order will stand in the same column. Then commence with the units figure and subtract each order separately; thus, 1 from 7 leaves 6; 4 from 9 leaves 5; 6 from 8 leaves 2. By this work we obtain the difference or remainder, 256.
641		897	
256		256	

Subtract the following:

(2)	(3)	(4)	(5)	(6)
843	384	978	425	9876
521	762	655	679	3456

47. *To subtract one number from another when any figure of the subtrahend is greater than the corresponding figure of the minuend.*

1. From 4173 subtract 2345.

FIRST OPERATION.

				Thousands. Hundreds. Tens. Units.
Minuend	4173	or	Subtrahend	2346
Subtrahend	2346		Minuend	4173
Difference	1827		Difference	1827

Explanation.—Having written the numbers as in the preceding problem, with the lesser number either above or below the greater, we observe that 6 units cannot be taken from 3 units; we therefore mentally add 10 to the 3 units making 13 units, and then say 6 from 13 leaves 7; then as we added 10 to the minnend we now mentally add its equivalent, 1 ten, to the tens figure of the subtrahend, and say 5 from 7 leaves 2; we next observe that 3 hundreds cannot be taken from 1 hundred, we therefore mentally add 10 hundreds to the 1 hundred making 11 hundreds, and then say 3 from 11 leaves 8; then having added 10 hundreds to the hundreds figure of the minuend we now mentally add 1 thousand, the equivalent of the 10 hundreds, to the thousands figure of the subtrahend and say 3 from 4 leaves 1. This completes the operation and gives 1827 as the difference of the two numbers.

The addition of 10 to the units and 10 hundreds to the hundreds of the minuend, and its equivalent 1 ten and 1 thousand to the tens and thousands columns of the subtrahend, is done upon the principle that the difference between two numbers is the same as the difference between the same two numbers *equally* increased.

In all problems of subtraction the operation of adding 10 to the minuend and its equivalent, 1, of the next higher order to the subtrahend is repeated as often as the subtrahend figure is greater than its corresponding minuend figure.

To *Prove* subtraction add the difference or remainder to the *subtrahend* and if the sum is equal to the *minuend* the work may be considered correct.

SECOND OPERATION.

4173	or	2346
2346		4173
1827		1827

Explanation. We will here perform the operation by addition which is a simpler and better method than the preceding, and consists simply in adding to the subtrahend such a number as will make it equal to the minuend. Thus commencing with the unit figure of the subtrahend or smaller number, we say 6 and 7 make 13; and set the 7 in the units place of the difference; then carrying 1 we say 5 and 2 make 7, and set the 2 in the tens column of the difference; then we say 3 and 8 make 11, and write the 8 in the third column or hundreds place of the difference; then carrying 1 we say 3 and 1 make 4, and write the 1 in the fourth place of the difference. This completes the operation.

2. From 73245 subtract 1228.

FIRST OPERATION.

73245
1228
72017

Explanation. Here we say 8 from 15 leaves 7; 3 from 4 leaves 1; 2 from 2 leaves 0; 1 from 3 leaves 2; 0 from 7 leaves 7.

SECOND OPERATION.

73245
1228
72017

Explanation. Here we say 8 and 7 make 15; 3 and 1 make 4; 2 and 0 make 2; 1 and 2 make 3; 0 and 7 make 7.

3. From 56802 subtract 50531.

FIRST OPERATION.

56802
50531
6271

Explanation.—Here we say 1 from 2 1; 3 from 10, 7; 6 from 8, 2; 0 from 6, 6; 5 from 5, 0, which being the last figure on the left has no value, and hence is not set.

SECOND OPERATION.

56802
50531
6271

Explanation.—Here we say 1 and 1 =2; 3 and 7=10; 6 and 2=8; 0 and 6=6; 5 and 0=5. The naught is not set for the reason given in the first solution.

EXAMPLES.

Write the following groups of numbers as they are here

written and subtract the lesser from the greater of each group:

1	2	3	4	5	6
467	1807	3842	607	3001	6879
342	4251	1291	8013	1009	9640

Subtract the following numbers:

7. From 5307 take 309. Ans. 4998.
8. From 1090 take 1009. Ans. 81.
9. From 7608 take 3705. Ans. 3903.
10. From 184240 take 39460. Ans. 144780.
11. From 41074089 take 1875429. Ans. 39198660.
12. From 9876543210 take 1234567890. Ans. 8641975320.

48. *To Subtract Dollars and Cents.*

1. What is the difference between \$483 and \$51.65. Ans. \$431.35.

OPERATION.

\$483.00
51.65
\$431.35

Explanation.—In all problems of this kind we first set the numbers in the same manner as when adding dollars and cents, with dollars under dollars and cents under cents, so that units of the same order will stand in the same column and the points in a vertical line.

When there are no cents in the minuend, we fill the place of cents with naughts.

The operation of subtraction is performed with dollars and cents, the same as with other numbers.

What is the difference between the numbers in each of the following groups?

1	2	3	4	5	6
\$16.25	\$8.00	\$.75	\$41.04	\$10.50	\$1.93
9.38	3.75	.59	6.61	4.78	.47
\$6.87	\$4.25				

7	8	9	10	11
$681.85	$127.05	$248.00	$49.11	$8527.09
90.38	105.50	181.15	9.89	2798.17

12	13	14	15
875.00	1971.50	1640.10	5184.62
43.50	880.00	1270.60	529.78

16. Paid for rice $5500, and for sugar $6875.40. How much more was paid for sugar than rice?

Ans. $1375.40

17. Bought a lot of flour for $2225, and sold the same for $2800, what was the gain? Ans. $575.

18. It is 700 miles to Shreveport and 320 to Galveston. How much farther is it to Shreveport than to Galveston?

Ans. 380 miles.

19. The ant has fifty eyes, the dragon fly 12000; how many more has the dragon fly than the ant?

Ans. 11950 eyes.

20. The total coinage of gold and silver at the different mints of the U. S. during the fiscal year ending June 30th 1875, was $43,854,708. Of this amount $33,553,965 was gold, what was the amount of silver coined.

Ans. $10,300,743.

21. A student had 40 problems to work, he worked 17, how many has he yet to work? Ans. 23.

22. Man has 26 bones in each foot, and 19 in each hand, how many more has he in the foot than in the hand?

Ans. 7.

23. Sound travels through the air at the rate of 1118 feet per second, and a bullet fired from a rifle travels 1750 feet per second; how much faster does the ball travel than sound? Ans. 632 feet per second.

24. Physiologists have determined with the aid of the microscope, that the lungs of man contain not less than 600,000,000 air cells; they have also determined that a single drop of human blood contains more than 4,000,000,000

of corpuscles; how many more corpuscles in one drop of blood than air cells in the lungs? Ans. 3,400,000,000.

25. Geologists have demonstrated that the formation of the stalactites and stalagmites in the Mammoth Cave of Kentucky, required not less than 75000 years of time; and that the wearing away of the rock of Niagara Falls by friction, from Queenstown where they first were after the glacial epoch, to their present location, 7 miles above, required at least 40,000 years; how much longer did it require to form the stalactites and stalagmites, than for the Falls of Niagara to recede to their present location?

Ans. 35000 years.

How many years from the date of each of the following events to the present year?

26. Quills first used for writing, 636 A. D.

27. Figures used by the Arabs, borrowed from the Indians, 813 A. D.

28. High towers first erected on churches, 1000 A. D.

29. Glass Windows first used in England, 1180 A. D.

30. Chimneys built in England, 1236 A. D.

31. Spectacles invented by Spina, 1299 A. D.

32. Woolen cloths first made in England, 1331 A. D.

33. Muskets used in England, 1421, A. D.

34. Printing invented, 1440 A. D.

35. Almanacs first published in Buda, 1460 A. D.

36. Tobacco discovered in St. Domingo, 1496 A. D.

37. Spinning-wheel invented at Brunswick, 1530 A. D.

38. Needles first made in England by an East Indian, 1545 A. D.

39. Decimal Arithmetic invented at Bruges, 1602 A. D.

40. Circulation of the blood discovered by Harvey, 1619 A. D.

41. Newspapers first published, 1630 A. D.

42. Coffee brought to England, 1641 A. D.

43. Steam engines invented by the Marquis of Worcester, 1649 A. D.

44. Cotton first planted in the United States, 1769 A. D.

45. Cotton first spun in America, 1787 A. D.

46. Steam first used to propel boats by Fulton, in America, 1807 A. D.

47. First Locomotive was made at Liverpool, 1829 A. D.

48. Electro-Magnetic Telegraphy invented by Morse, of America, 1832 A. D.

49. America was discovered in 1492 A. D.

50. The electric telegraphy was first used in the United States in 1844 A. D.

51. General George Washington was born in 1732 and died in 1799; General R. E. Lee was born in 1807 and died in 1870. How much older was General Washington than General Lee, when he died? Ans. 4 years.

52. What is the difference between 23222 and 11 thousand 11 hundred and 11? Ans. 11111.

53. What is the difference between 6 dozen dozen and half a dozen dozen? Ans. 792.

54. What number must be added to 68741 to make a million? Ans. 931259.

55. Philadelphia has 153151 buildings; New Orleans 35600. How many more has Philadelphia than New Orleans? Ans. 117551.

56. James, who is 23, is 7 years older than Henry; how old is Henry? Ans. 16 years.

57. William has $500 which is $150 more than I, and I have $75 more than Lewis, how much has Lewis, and how much have I? Ans. Lewis has $275.
I have $350.

58. There are two parties who owe me $8000, one of them owes $4250. The other wishes to pay me $1700 on account; how much will he then owe?
Ans. $2050.

59. A speculator bought a lot of apples for $215., and sold them for such a price, that if he had got $22.50 more he would have gained as much as they cost him. How much did he sell them for? Ans. $407.50.

60. From New Orleans to Vicksburg is 401 miles, and

to Natchez 277 miles; how far is it from Natchez to Vicksburg? Ans. 124 miles.

61. What is the difference between one million, seventeen thousand and seven, and one thousand sixteen hundred and sixteen? Ans. 1,014,391.

62. The sum of two numbers is 1463, one of the numbers is 628, what is the other? Ans. 835.

63. The velocity of our earth on its yearly voyage through space, around the sun, is 99733 feet per second; the velocity of a 12 pound cannon ball fired from a gun with an average charge of powder is 1734 feet per second, how many feet farther does the earth travel, in each second, than a cannon ball? Ans. 97999 feet, or 18 miles and 2959 feet.

64. What number is that to which if 17821 be added the sum will be 37907? Ans. 20086.

65. At an election the defeated candidate received 23742 votes; had he received 5112 votes more he would have been elected by 1000 majority; how many votes did the elected candidate receive? Ans. 27854.

66. A father divided his plantation consisting of 4500 acres between his five sons Albert, Edward, William, Frank and Robert. To Albert he gave 800 acres; to Edward he gave 150 acres more than he gave Albert; to William he gave 100 acres less than he gave Edward; to Frank he gave as much as he gave Edward, and the remainder he gave to Robert. How many acres did Robert receive? Ans. 950 acres.

MULTIPLICATION—(Increasing.)

49. **Multiplication** is the process or operation of increasing one of two numbers as many times as there are units in the other. Or, differently explained, it is a short method of performing addition.

The number to be multiplied is called the *multiplicand*.

The number which shows how many times the multiplicand is to be increased or repeated is called the *multiplier*.

The result obtained by the operation of multiplying is called the *Product*.

The multiplicand and multiplier are called *factors*. The meaning of the word factor is *maker* or *producer*.

50. The **Sign of Multiplication** is an oblique cross, $\times$. It is read *multiplied by* or *times*. Thus 8×3, is read 8 multiplied by 3, or 3 times 8.

51. **Principles of Multiplication.** In all cases of multiplication the multiplier must be regarded as an abstract number. Two denominate numbers cannot be multiplied together as denominate numbers.

In all multiplication operations the *product* is the *same* in *name* or *kind* as the multiplicand.

For extended work on contracted methods of multiplying see Soulé's Contractions in Numbers.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100
11	22	33	44	55	66	77	88	99	110
12	24	36	48	60	72	84	96	108	120
13	26	39	52	65	78	91	104	117	130
14	28	42	56	70	84	98	112	126	140
15	30	45	60	75	90	105	120	135	150
16	32	48	64	80	96	112	128	144	160
17	34	51	68	85	102	119	136	153	170
18	36	54	72	90	108	126	144	162	180
19	38	57	76	95	114	133	152	171	190
20	40	60	80	100	120	140	160	180	200

Explanation.—We recommend this table as being far superior to the one presented in the School and College Text Books of the country, and urge all who aspire to proficiency in computing numbers to learn it. In learning this table, or in the use of it, we caution the calculator against the use of all intermediate words, whether he speaks or thinks them; thus, instead of saying or thinking, 9 times 3 are 27; 17 times 6 are 102, &c., say or think, 9, 3, 27; 17, 6, 102, &c.

In reading we do not stop to spell orally or mentally the words that compose the sentences; from the combination of the letters we see what the words are without looking specially at each individual letter, and to read or operate with rapidity in the combination of numbers, we must omit all superfluous talk or thought.

52. ORAL EXERCISES.

1. At 8 cents a pound what will 9 pounds of rice cost?

Solution.—In all problems of this kind we reason thus: Since 1 pound costs 8¢, 9 pounds will cost 9 times as much, which is 72¢.

2. What will 7 yards cost at 12¢ per yard?
3. What will 4 books cost at 20¢ each?
4. At 13¢ per dozen what will 6 dozen cost?
5. If 1 box cost $3 what will 23 boxes cost?
6. Flour is worth $7 per barrel, what are 25 barrels worth?
7. Bought 14 pounds of sugar at 8¢ per pound, what did it cost?
8. At $6 per cord what will 34 cords of wood cost?
9. Paid $4 per barrel for potatoes and bought 47 barrels, what did they cost?
10. If you receive $2 per day for labor and work 17 days how much money will you receive?

Solution.—Since 1 day's labor is worth $2, 17 days' labor is worth 17 times as much, which is $34. Or thus, since I receive $2 for 1 day's work, for 17 days' work I will receive 17 times as much, which is $34.

11. Multiply from 0 times 8 to 15 times 8 and reverse.
12. Multiply from 0 times 9 to 16 times 9 and reverse.
13. Multiply from 0 times 11 to 16 times 11 and reverse.
14. 12 inches make a foot, how many inches in 16 feet?
15. 4 quarts make a gallon. How many quarts in a barrel that holds 42 gallons?
16. What will 6 dozen shirts cost @ $18 per dozen?
17. If you buy 15 boxes of peaches @ $2 per box, what will they cost?
18. Multiply from 0 times 12 to 19 times 12 and reverse.
19. How many are 9 times 12 plus 8?
20. How many are 12 times 7 minus 6?
21. If you buy 7 pencils at 5 cents each and hand to

the seller 50¢, how much change ought you to receive?

22. A merchant bought 23 barrels of apples at $4 per barrel and paid $65 on account. How much does he still owe?

53. *To multiply when the multiplier consists of only one figure.*

1. What is the product of 947 multiplied by 6?

OPERATION.

(Units. Tens. Hundreds.)

Multiplicand	947
Multiplier	6
Product	5682

Explanation.—In all problems of this kind we place the multiplier under the units' figure of the multiplicand and then commencing with the units figure we say, 6 times 7 are 42, which is 4 *tens* and 2 *units*; the 2 *units* we place in the units place of the product and retain in the mind the 4 tens to add to the column of tens; we next say 6 times 4 are 24 plus the 4 tens retained in the mind, are 28, which is 2 *hundreds* and 8 *tens*, the 8 *tens* we write in the tens column of the product, and retain in the mind the 2 hundreds to add to the column of hundreds. We then say 6 times 9 are 54, plus 2 hundreds are 56, which is 5 *thousand* and 6 *hundreds*, which we write respectively in the thousands and hundreds columns of the product. This completes the operation and gives a product of 5682.

In practice, instead of saying 6 times 7 are 42, 6 times 4 are 24, etc., we should only name the result of the combination, thus 42, 24, etc. *In handling figures we should always pronounce the results of the combinations without naming the figures that make the result, just as we pronounce words without spelling or naming the letters that make the words.*

54. To **Prove** the operations of multiplication, repeat the work or multiply the multiplier by the multiplicand. If the result is the same as the first, the work is probably correct.

EXAMPLES.

Perform the following multiplications:

	2	3	4	5
Multiplicand	543	983	2769	76895
Multiplier	7	8	5	9
Product	3801	7864	13845	692055

6	7	8	9
8764	2987	9876	85421
5	8	7	9
10	11	12	13
46532	58674	9861	81453
14	15	17	19

14. What will 4 pianos cost at $425 each?

Ans. $1700.

15. At $65 each what will 9 wagons cost?

Ans. $585.

16. What will 7 lots of ground cost at $1875 each?

Ans. $13125.

17. At $6 per barrel what will be the cost of 245 barrels of flour?

OPERATION.

Multiplier 245
Multiplicand 6

$1470 Ans.

Explanation.—The analysis for this problem is, since 1 barrel of flour cost $6, 245 barrels will cost 245 times as much. The $6 is the real multiplicand, but in the operation we used it as the multiplier. This we do for convenience in performing the operation, in all problems where the multiplicand is less than the multiplier. The result is the same whichever factor we use as a multiplier.

18. What will 42 dozen hats cost at $9 per dozen?

Ans. $378.

19. At $7 a piece what will 48 chairs cost?

Ans. $336.

55. *To multiply when the multiplier consists of more than one figure.*

1. What is the product of 397 multiplied by 653?

	Hunds. of thous.	Tens of thous.	Thousands.	Hundreds.	Tens.	Units.
Multiplicand			3	9	7	
Multiplier			6	5	3	
1st Partial product by 3 units			1	1	9	1 = 3 times the multiplicand.
2d Partial product by 5 tens			1	9	8	5 = 50 times "
3d " " 6 h'ds.		2	3	8	2	= 600 " "
Total product		2	5	9	2	4 1 = 653 " "

Explanation.—In all problems of this kind we first write the multiplier under the multiplicand so that units of the same order will stand in the same column, and then multiply by one figure at a time. We first multiply by the units figure, then the *tens, hundreds* and so on in regular order through the multiplier and add the several partial products together and thus obtain the required product.

In this problem we first multiply by 3, the units figure, in the same manner as explained in the first problem where there was but one figure in the multiplier, and obtain 1191 as the first partial product. This we write below the multiplier so that units of the same order will stand in the same column.

Next we multiply by the 5 tens; we say 5 times 7 are 35, which is 3 hundreds and 5 tens; we write the 5 tens in the tens column directly below the multiplying figure and reserve in the mind the 3 hundreds to add to the hundreds column. We then say 5 times 9 are 45 + 3 hundreds which were reserved are 48 hundreds which is 4 thousands and 8 hundreds; we write the 8 hundreds in the column of hundreds and reserve the 4 thousands to add to the thousands column. We then say 5 times 3 are 15 plus 4 thousands, reserved, are 19 thousands, which is 1 ten thousand and 9 thousands, which we write in their respective columns.

We then in like manner multiply by the 6 hundreds in the multiplier, being careful to write the first figure obtained (2) in the hundreds column, directly under the 6 of the multiplier, and the other figures in their respective columns, *thousands, ten*

thousands and *hundred thousands*. We then add the *partial* products together and obtain 259241 as the whole product of 397 multiplied by 653.

In practice remember to name or think only the results of the numerical combinations when adding or multiplying.

EXAMPLES.

2. Multiply 3426 by 457.

OPERATION.

Multiplicand 3426
Multiplier 457

1. Partial prod. by 7 units 23982 = 7 times the multiplicand
2. Partial prod. by 5 tens 17130 = 5 times the multiplicand
3. Partial prod. by 4 h'ds 13704 = 4 times the multiplicand

Whole product 1,565,682 = 457 times the multiplicand

3. Multiply 647 by 58.

OPERATION.

647
58

5176
3235

37,526 Ans.

4. Multiply 21794 by 2365

OPERATION.

21794
2365

108970
130764
65382
43588

51,542,810 Ans.

5. Multiply 28433 by 4172

6. Multiply 989769 by 248193

Multiply the following numbers.

7.	483	by 569	11.	1847	by 84
8.	924	by 237	12.	2346	by 127
9.	1683	by 328	13.	671	by 508
10.	581	by 76	14.	8765	by 2046

Operation of the 13th problem.

671
508
———
5368
3355
———
340868

Explanation.—In all problems where there are naughts in the multiplier we multiply by the significant figures only, for the reason that the product of any number by 0 is 0.

56. *To multiply when either the multiplicand or multiplier or both have naughts on the right.*

1. Multiply 463 by 200.

OPERATION.

463
200
———
92600

Explanation.—In all problems of this kind we write the significant figures so that units of the same order may stand in the same column and write the naughts on the right of the significant figures. We then multiply the significant figures and annex to the product as many naughts as there are in the multiplier or multiplicand, or both. The basis or reason of this is that the removal of a figure or number one place to the left increases its value ten fold, and the annexing of a naught removes the significant figures one place to the left, thereby increasing them ten fold, and hence annexing a naught is in effect multiplying by 10; and for the same reason annexing two naughts is multiplying by 100, the annexing of 3 naughts is multiplying by 1000, etc., for other powers of 10. In this problem we first use the multiples 2 *hundred* as 2 *units*, hence the first partial product, 926, was 100 times too small, we then by annexing the two naughts multiply it by 100 and obtained 92600 as the correct product.

2. Multiply 3400 by 26.

OPERATION.

3400
26
———
204
68
———
88400 Ans.

3. Multiply 940 by 4700

OPERATION.

940
4700
———
658
376
———
4418000 Ans.

Multiply 5020 by 420.

OPERATION.

5020	
420	
1004	
2008	
2108400	

Multiply 82000 by 483.

82000	or	483	
483		82000	
246		966	
656		3864	
328		39606000	
39606000			

4. Multiply 842 by 600.
5. Multiply 1208 by 1020.
6. Multiply 9900 by 707.
7. Multiply 23500 by 12030.
8. Multiply 1000 by 6208.
9. Multiply 81009 by 90200.
10. Multiply 45670 by 5780.
11. Multiply 987000 by 49.

57. *To multiply by the Factors of a number.*

NOTE.—**Factors** of a number are such numbers as will when multiplied together produce the number. Thus 6 and 6 are the factors of 36; 7 and 8 are the factors of 56, or it is a number that will exactly divide a number.

1. Multiply 2435 by 42.

OPERATION.

2435
7
17045
6
102270

Explanation.—In all problems of this kind we separate the multiplier into two or more factors and multiply the multiplicand by one of the factors and the resulting product by another factor and so on until we have used all the factors. The last product will be the correct product.

2. Multiply 781 by 63.
3. Multiply 3140 by 36.
4. Multiply 588 by 81.
5. Multiply 480 by 361.
6. Multiply 1756 by 125
7. Multiply 3281 by 128

58. *To multiply when the multiplicand or multiplier contains dollars and cents.*

1. Multiply $342.15 by 6.

OPERATION.

$342.15
6

Product $2052.90

Explanation.—In all problems of this kind we multiply in the regular manner and then prefix the dollar sign $ and place the point (.) two places from the right. Our answer is then in dollars and cents.

EXAMPLES.

2. What will 1682 pounds of sugar cost at 9¢ per pound? Ans. $151.38.

3. A merchant's monthly expenses are $1342.75. What are they for 12 months? Ans. $16113.00

4. It costs a family $2.30 a day for marketing, what will be the expense for 30 days? Ans. $69.00.

5. What will 37 boxes oranges cost at $3.75 per box? Ans. $138.75.

6. At 16 cents per pound what is the value of 23780 pounds Cotton? Ans. $3804.80.

7. If it costs $17500 to construct one mile of railroad what would be the cost to build 364 miles? Ans. $6370000.

8. What will 875 tons of railroad iron cost, at $55 per ton? Ans. $48125.

9. Multiply one million and twenty-six by nineteen thousand seven hundred and ten. Ans. 19710512460.

10. One cubic foot contains 1728 cubic inches. How many cubic inches in 324 cubic feet? Ans. 559872.

11. One square foot contains 144 square inches. How many square inches in 95 square feet? Ans. 13680.

12. One gallon contains 231 cubic inches. How many cubic inches in a cistern that holds 3500 gallons? Ans. 808500.

13. One bushel contains 2150.42 cubic inches. How many cubic inches in 20 bushels? Ans. 43008.40.

14. One mile contains 5280 feet. How many feet in 25 miles? Ans. 132000.

15. One year contains 365 days. How many days in 21 years? Ans. 7665.

16. The human heart beats 4200 times an hour. How many times does it beat in 10 years, there being 24 hours in one day and 365 days in each year? Ans. 367920000.

17. Sound travels 1118 feet per second. How far will it travel in 10 minutes, there being 60 seconds in a minute. Ans. 670800 feet.

18. Light travels 192500 miles per second. How many miles will it travel in 1 day, there being 24 hours in a day, 60 minutes in an hour, and 60 seconds in a minute. Ans. 16,632,000,000.

19. A railroad train runs 25 miles an hour. How far will it go in 3 days, allowing 3 hours for lost time in stoppages? Ans. 1725.

20. If a person respire 20 times in a minute, how many times will he breathe in a day? Ans. 28,800.

21. If a person inhales 1 gallon of air at each respiration, and respires 20 times per minute, how many gallons will he inhale in 24 hours. Ans. 28800.

22. At $17 per ounce what is the worth of 9 pounds of gold, there being 12 ounces in a pound Troy or Mint weight? Ans. $1836.

23. How many pounds of coffee in 180 bags if each bag contains 162 pounds? Ans. 29160.

24. How many pounds of cotton in 87 bales, if each bale weighs 475 pounds? Ans. 41325.

25. What will 27893 pounds of tobacco cost at 56 cents per pound? Ans. $15,620.08.

26. What will 1870 acres of land cost at $18 per acre? Ans. $33,660.

27. The Senate and House of Representatives of the State of Louisiana consists of 138 members who receive $8 per day. The regular session continues 60 days. What is the yearly expense for the salaries of the State's law makers? Ans. $66240.

27. A contractor has 865 men employed at $1.50 per day. What are the weekly wages of all for 6 days' labor? Ans. $7785.

28. What will it cost to build 37428 cubic yards of levee at 45 cents per cubic yard? Ans. $16842.60.

29. A steamboat arrives with 3840 bales of cotton; 1320 sacks cotton seed and 580 barrels molasses. Her freight charges are $2 per bale for cotton, 25¢ per sack for cotton seed and 50¢ per barrel for molasses. What is the amount of her freight bills? Ans. $8300.

30. A drayman charges 75 cents a load, and he has hauled 63 loads. How much is due him?

Ans. $47.25.

31. What will it cost to slate the roof of a house containing 52 squares at $13.25 per square? Ans. $689.

32. The walks around a dwelling contain 129 square yards. What will it cost to flag them with German flags at $3.10 per square yard? Ans. $399.90.

33. What will it cost to pave a street containing 20000 square yards, with stone at $4.75 per square yard?

Ans. $95000.

34. Bought 2180 barrels of coal at 48¢ per barrel. What was the cost? Ans. $1046.40.

35. Multiply 5 billions and 16 by 5 millions and 1 thousand. Ans. 25,005,000,080,016,000.

36. A Hogshead of sugar contains 1085 pounds; how many pounds in 107 hogsheads of equal weight?

Ans. 116,095

37. A planter produced 68 bales of cotton, if the average weight of the bales was 460 pounds, and the cotton sold for 13 cents per pound, how much money would it bring? Ans. $4,066.40

38. What will 3 cases containing 2 dozen pairs each of shoes cost @ $2.90 per pair? Ans. $208.80.

39. If it costs $1.50 a day to support one person, what will it cost to support a family of 13 for one year or 365 days? Ans. $7117.50.

40. There are 35600 dwellings in New Orleans, allowing 7 persons to each dwelling, what would be the population of the city? Ans. 249200.

41. A merchant sold *three dozen dozen* ladies' hose at

one quarter of a dozen dozen cents a pair. How much did he receive for them? Ans. $155.52

42. The pressure of the atmosphere is 15 pounds on every square inch of surface. The exterior surface of a man of average size is about 2500 square inches. How many pounds weight does he sustain?

Ans. 37500 pounds.

43. How many dollars are 375 $10 gold pieces worth?

Ans. $3750.

44. What is the value of 2146 dimes? Ans. $214.60

45. What is the value of 1010 quar. dol. Ans. $252.50

46. What is the value of 728 nickels? Ans. $36.40

47. What is the value of 1612 half. dol. Ans. $806.00

48. During the fiscal year ending Sept. 1, 1876, there was received 30181 hogsheads of Tobacco. If each hhd. contained 12 pounds of poison, how many pounds of poison were there in the whole? Ans. 362172.

49. The circumference of the earth is nearly 25000 miles, the distance to the sun is 3800 times as many miles. How far is it to the sun? Ans. 95000000.

50. 4875 is the thirteenth part of a number. What is the number? Ans. 63375.

51. The sun is 1384500 times as large as the earth; the earth is 45 times as large as the moon. How many times is the sun larger than the moon? Ans. 62302500.

52. Light travels 192500 miles a second and it requires 100000 years to travel to us from some of the fixed stars that are seen with the telescope. Allowing 365 days, 5 hours, 48 minutes and 49 seconds to a year and remembering that there are 24 hours in a day, 60 minutes in an hour and 60 seconds in a minute, how far distant are such stars?

Ans. 607470883250000000 miles.

53. A man's receipts are $1800 a year and his disbursements are $1125 a year. How much are his net receipts in 3 years? Ans. $2025.

54. It is estimated by Astronomers that 7500000 visible meteors fall upon the earth daily; it is also estimated that the average weight of each is 100 grains. From these

figures and allowing 365 days to the year, what is the annual growth of the earth in weight by the accession of the visible meteoric matter? Ans. 273750000000 grains.

DIVISION.—(Decreasing.)

59. **Division** is the process of finding how many times one number is equal to another. Or it is the process of finding one of the factors of a given product when the other factor is known.

60. The **Dividend** is the number to be *divided* or it is the number to be measured.

61. The **Divisor** is the number by which we *divide* or it is the number used as a unit of measure.

62. The **Quotient** is the result of the division, and shows how many times the dividend is equal to the divisor.

63. The **Remainder** is the number left after dividing dividends, which are not multiples of the divisor, or which are not an exact number of times equal to the divisor. It must always be less than the divisor.

64. The **Sign of Division** is a horizontal line with a point above and below, thus $\div$. It is read *divided by;* and it indicates that the number before it, is to be divided by the number after it; thus $25 \div 5$, is read 25 *divided by* 5.

The horizontal line, and the vertical or curved line when placed between two numbers also indicates *division.* Thus, $\frac{36}{4}$, 4|36 or 4)36, are all read 36 divided by 4.

65. PRINCIPLES OF DIVISION.

1. When the *divisor* and *dividend* are both denominate or both abstract numbers, the quotient will be an abstract number.

2. When the divisor is an abstract number and the dividend a denominate number, the quotient will be a denominate number.

3. When there is a remainder it is a part of the dividend and is therefore the same in name or kind.

4. Multiplying the dividend or dividing the divisor *multiplies* the quotient.

5. Dividing the dividend or multiplying the divisor divides the quotient.

6. Multiplying or dividing both the divisor and dividend by the same number does not change the quotient.

66. *Proof of Division.* Multiply the quotient by the divisor and if there is no remainder the product should be equal to the dividend; when there is a remainder add it to the product, and if the work is correct the sum will equal the dividend.

Division operations may be performed by the process of addition or subtraction. But as these processes are too lengthy for practical purposes, we will not give them place here.

For contracted methods in division, see Soulé's Contractions in Numbers.

67. ORAL EXERCISES.

1. How many times is 0 equal to 1? or $0 \div 1 = ?$
2. " " 1 " 0? or $1 \div 0 = ?$

Ans. An infinite number of times.

3. How many times is 1 equal to 1? or $1 \div 1 = ?$
4. " " 2 " 1? or $2 \div 1 = ?$
5. " " 3 " 1? or $3 \div 1 = ?$
6. " " 4 " 2? or $4 \div 2 = ?$
7. " " 8 " 2? or $8 \div 2 = ?$
8. " " 9 " 3? or $9 \div 3 = ?$
9. " " 12 " 4? or $12 \div 4 = ?$
10. " " 20 " 5? or $20 \div 5 = ?$
11. " " 24 " 6? or $24 \div 6 = ?$
12. " " 35 " 7? or $35 \div 7 = ?$
13. " " 56 " 8? or $56 \div 8 = ?$
14. " " 63 " 9? or $63 \div 9 = ?$
15. " " 72 " 9? or $72 \div 9 = ?$
16. " " 80 " 10? or $80 \div 10 = ?$
17. " " 88 " 11? or $88 \div 11 = ?$
18. " " 96 " 12? or $96 \div 12 = ?$

19. $\frac{36}{6}$=? 4)42=? 9)45=? 77÷ 7?
$\frac{64}{8}$=? 6)48=? 5)55=? 84÷12?

20. How many times is 24 equal to 3, to 4, to 6, to 8, to 12, to 24?
21. How many times is 36 equal to 3, to 4, to 6, to 9, to 12, to 36?
22. How many times is 42 equal to 2, to 6, to 7, to 42?
23. " " 64 " 2, to 4, to 8, to 64?
24. " " 72 " 2, to 8, to 9, to 72?

68. FRACTIONAL NUMBERS.

When we divide a unit or a number of units of any kind into equal parts, these parts are sometimes called fractions. The name of the equal parts varies according to the number of parts into which the thing or number was divided.

When the unit or number is divided into 2 equal parts 1 of the parts is called *one-half*, and is written thus $\frac{1}{2}$. If divided into 4 equal parts 1 of the parts is called *one-fourth*, and is written thus $\frac{1}{4}$; 3 of the parts are called *three-fourths* and are written thus $\frac{3}{4}$.

In like manner we obtain *fifths*, *sixths*, *sevenths*, *eighths*, *twelfths*, *sixteenths*, *twenty-firsts*, *etc.*

In writing fractional numbers in figures we place the number which shows the *name* of the parts below a horizontal line as a *divisor*, and the number which shows *how many* parts are taken or used, above the line as a *dividend.*

The following examples will fully elucidate this work:

Two-thirds are written $\frac{2}{3}$.	Seven-twelfths are written $\frac{7}{12}$
Three-fourths are written $\frac{3}{4}$.	Nine-tenths are written $\frac{9}{10}$
Five-eighths are written $\frac{5}{8}$	Fifteen-sixteenths written $\frac{15}{16}$
Seven-ninths are written $\frac{7}{9}$	Eleven-eightieths written $\frac{11}{80}$

How do you find $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, etc. of any number?
How do you find $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, etc. of any number?

What is $\frac{1}{2}$ of 4?	What is $\frac{1}{5}$ of 15?	What is $\frac{2}{3}$ of 9?
" " $\frac{1}{3}$ of 6?	" " $\frac{1}{6}$ of 18?	" " $\frac{3}{4}$ of 12?
" " $\frac{1}{4}$ of 8?	" " $\frac{1}{7}$ of 28?	" " $\frac{7}{8}$ of 40?

EXAMPLES.

1. If 3 hats cost $6 what will 1 hat cost?

Ans. $2.

Analytic solution.—Since 3 hats cost $6, 1 hat will cost $\frac{1}{3}$ part of $6, which is $2.

2. If 8 yards cost 56 cents what will 1 yard cost?

3. Paid $36 for 6 barrels of flour, what did 1 barrel cost?

4. 9 gallons of molasses cost $4.50, what did 1 gallon cost? Ans. 50¢.

5. Bought 12 shirts for $30, how much did 1 cost?

Ans. $2.50.

6. Paid $1.00 for 8 pounds of sugar, what was the price per pound? Ans. $.12½.

7. 7 dozen oranges cost $2.10, what was the price per dozen? Ans. $.30.

8. Bought 20 peaches for 60¢, how much did 1 peach cost? Ans. $.03.

9. At $2 a yard how many yards can you buy for $24?

Ans. 12 yards.

Analytic solution.—Since $2 buy 1 yard, $1 will buy $\frac{1}{2}$ of a yard, and $24 will buy 24 times $\frac{1}{2}$ a yard, which is, $\frac{24}{2}$ or 12 yards.

Or thus. Since $2 buy 1 yard, for $24 we can buy as many yards as $24 is equal to $2, which is 12 times.

10. At 9 cents per pound how many pounds can be bought for 45 cents? Ans. 5 pounds.

Analytic solution.—Since 9 cents buy 1 pound, 1 cent will buy $\frac{1}{9}$ of a pound, and 45 cents will buy 45 times $\frac{1}{9}$ of a pound, which are $\frac{45}{9}$ or 5 pounds.

11. Flour is worth $8 per barrel, how many barrels can be purchased for $56? Ans. 7 barrels.

Analytic solution.—Since $8 buy 1 barrel, $1 will buy $\frac{1}{8}$ of a barrel, and $56 will buy 56 times $\frac{1}{8}$ of a barrel, which are $\frac{56}{8}$ or 7 barrels.

12. For $.95 how many papers can you buy at 5 cents a paper? Ans. 19 papers.

13. If 25 cents buy 1 yard how many yards will 75 cents buy? Ans. 3 yards.

14. At $3 a piece how many chairs can be bought for $36? Ans. 12 chairs.

15. If the printer charges $1.50 to set 1 page of this book, how many pages can be set for $75?

Ans. 50 pages.

WRITTEN EXERCISES.

69. *To divide when the divisor does not exceed 12.*

1. Divide 3648 by 5.

OPERATION.

Divisor 5) 3648 dividend

Quotient 729 and 3 rem.

Explanation.—In all problems of this kind we write the numbers as shown in the operation, and then begin on the left of the dividend to divide. We begin on the left in order to carry the remainder, if any, of the higher order of units to the next lower order. In this problem we first take the 3 (thousands,) and as it is not equal to 5, we therefore unite it with the 6 hundreds, making 36 hundreds, which by trial multiplication and subtraction mentally performed, we find is equal to 5, 7 (hundreds) times and 1 remainder; the 7 we write in the hundreds column of the quotient line, directly under the 6 the last figure used of the dividend; then to the 1 remainder we mentally annex the 4 tens, making 14 tens, as the second partial dividend, and which by mental multiplication and subtraction, we find is equal to 5, 2 (tens) times and 4 remainder; the 2 we write in the tens column of the quotient line, and to the 4 we mentally annex the units figure of the dividend, making 48 units as the third and last partial dividend; this we find, by mental multiplication and subtraction to be equal to 5, 9 times and 3 remainder.

The remainder is usually expressed fractionally by writing it over the divisor, thus $\frac{3}{5}$, this expresses the part of a unit of times that the remainder is equal to the divisor.

SHORT DIVISION.

Operations in division according to the foregoing method are called *short division*, because the multiplication and subtraction work in finding the remainder of the partial dividends were mentally performed.

2. How many times is 846 equal to 6?

OPERATION.

Divisor 6)846 Dividend

Quotient 141

Explanation.—In the preceding problem we gave a full and explicit explanation of each step of the operation. In practice much of the explanation therein given is omitted, and the work performed thus:. Commencing with the left hand figure we say 8 is equal to 6, 1 time and 2 remainder; 24, is equal to 6, 4 times; 6 is equal to 6, 1 time.

Work the following indicated divisions.

3	4	5	6
7)847	8)12327	9)1085	11)2386
121	$1540\frac{7}{8}$	$120\frac{5}{9}$	$216\frac{10}{11}$
7	8	9	10
8)1471	4)11899	9)81018	12)10824
11	12	13	14
2344 / 3	7 | 93020	21345÷8	9)76451

Divide the following numbers:

15. 9872 by 4
16. 1483 " 7
17. 1691 " 9
18. 41070 " 8
19. 10286 by 6
20. 48710 " 7
21. 10008 " 9
22. 199999 " 8
23. What is $\frac{1}{4}$ of \$528?
24. " are $\frac{3}{5}$ of \$1005?
25. What are $\frac{3}{4}$ of \$448?
26. " " $\frac{7}{8}$ of \$6444?

Operation for the 24th problem.

5)\$1005

\$ 201 = $\frac{1}{5}$
3

\$603 Ans.

Operation for the 26th problem.

8)\$6444

\$ 805.50 = $\frac{1}{8}$
7

\$5638.50 Ans.

27. How many apples can be bought for \$2.25 at 5 cents a piece? Ans. 45 apples.

28. At 15 cents a pound, how many pounds can you buy for $3.15? Ans. 21 pounds.

29. Paid $90 for 10 volumes of Chambers' Cyclopedia, what was the price of one volume? Ans. $9.

30. If 8 men are to receive $5791 in equal parts, what will be each man's share? Ans. 723\frac{7}{8}$.

31. The dividend is 63, and the quotient is 9, what is the divisor? Ans. 7.

32. The quotient is 15, the divisor 3, and the remainder 2, what is the dividend? Ans. 47.

33. The quotient is 36, and the divisor 6, what is the dividend? Ans. 216.

34. The dividend is 72, and the divisor is 4, what is the quotient? Ans. 18.

35. How many pounds of cotton at 11 cents a pound will be required to pay for 33 pounds of sugar @ 8 cents a pound? Ans. 24.

70. *To divide when the divisor exceeds 12.*

1. Divide 7387 by 36.

OPERATION.

Divisor, Dividend, Quotient

36) 7387 (205$\frac{7}{36}$

72

187

180

7 remainder

Explanation.—We first write the numbers as shown in the operation and commence to divide as explained in the first written example. But as the divisor is too large to be conveniently used mentally, we therefore write tho operation of multiplying the divisor by the quotient figures, and subtracting the successive products from the several partial dividends.

In performing the division we first see that 7, (thousands) are not equal to 36, and hence there will be no thousands in the quotient. We then annex to the 7 thousands the 3 hundreds, making 73 hundreds as the first partial dividend; this is equal to 36, 2 times, and a remainder; we write the 2 in the hundreds column of the quotient, multiply the divisor by it, write the product under and subtract the same from the 73 hundreds of the dividend. This work gives us 1 hundred remainder, to which we annex the 8 tens, making 18 tens as the second partial dividend; this partial dividend not being equal to 36, we write

0 (no tens) in the tens column of the quotient, and annex to the 18 tens the 7 units, making 187 units as the third and last partial dividend. This is equal to 36, 5 times and a remainder, we write the 5 in the quotient, and multiply and subtract as we did with the first obtained figure of the quotient, and thus produce 7 remainder, which we write over the divisor as explained in short division.

LONG DIVISION.

Operations in division, according to the above method, are called *long division*, for the reason that the multiplication and subtraction work in finding the remainders of the partial dividends is written.

2. How many times is 66804 equal to 53?

Ans. $1260\frac{24}{53}$.

OPERATION.

Divisor	Dividend	Quotient.
53)	66804	($1260\frac{24}{53}$
	53	
	138	
	106	
	320	
	318	
	24	

Proof.

1260	Quotient.
53	Divisor.
3780	
6300	
24	Remainder.
66804	Dividend.

3. What is the quotient of $107941 \div 396$?

OPERATION.

396)	107941	(272 Quotient.
	792	
	2874	
	2772	
	1021	
	792	
	229	Remainder.

4. Divide 7167901 by 11267.

OPERATION.

11267)	7167901	(636 Quoti't.
	67602	
	40770	
	33801	
	69691	
	67602	
	2089	Remainder.

5. Divide 784 by 82.

STATEMENT.

$82)784(9\frac{6}{82}$ Ans.

6. Divide 91070 by 8761.

STATEMENT.

$8761)91070(10\frac{3460}{8761}$ Ans.

7. Divide 2461 by 74.
8. Divide 4809 by 91.
9. Divide 13872 by 263.
10. Divide 54123 by 1423.
11. Divide 628100 by 156.
12. Divide 10000 by 304.
13. Divide 37021 by 2002.
14. Divide 8888888 by 332211.
15. Divide $6805 equally between 5 men, and what will be the share of each? Ans. $1361.
16. What is the *sixty-fourth* part of $44800? Ans. $700.
17. 145 men picked 1305000 pounds of cotton, supposing they all picked an equal quantity, how much did one man pick? Ans. 9000 pounds.
18. A father gave his 7 sons a Christmas present of $353.50 to be shared equally, what was each one's share? Ans. $50.50.

71. *To divide when there are naughts on the right of the divisor.*

1. Divide 2843 by 200. Ans. $14\frac{43}{200}$

OPERATION.

2|00) 28|43 (

14 and 43 Rem.

Explanation.—Since by our scale of numbers they increase from right to left in a tenfold ratio, and decrease from left to right in a corresponding manner, it is clear that the removal of any order of figures from left to right diminishes its value ten times for each place of removal. And as previously shown, that the annexing of naughts multiplies numbers, by removing them to places of higher value, so in like manner cutting figures off from the right of a number removes the remaining orders to the right, and hence decreases them tenfold for every figure cut off. Hence to cut off *one* figure is dividing by 10; to cut off *two* figures divides by 100; to cut off three figures divides by 1000 and so on.

Considering these principles, in all cases of this kind we cut off the naughts from the right of the divisor and the same number of figures from the right of the dividend; and then divide the remaining figures of the dividend by the remaining figures of the divisor. When there is a remainder annex the figures cut off, and we obtain the true remainder.

2. Divide 87931 by 1000. Ans. $87\frac{931}{1000}$.

OPERATION.

1|000) 87|931

Quotient 87 and 931 Remainder.

3. Divide 178 by 10.

OPERATION.

1|0)17|8

Quotient 17—8 Remainder
Ans. $17\frac{8}{10}$

4. Divide 6581 by 300.

OPERATION.

300)6581

Quotient 21—281 Rem.
Ans. $21\frac{281}{300}$

5. Divide 71468071 by 341000.

OPERATION.

341|000)71468|071(209$\frac{290071}{341000}$ Ans.
682

3268
3069

199

6. Divide 8897600 by 8100. Ans. $1098\frac{38}{81}$.
7. Divide 1000000 " 10000. Ans. 100.
8. Divide 99999 by 9000. Ans. $11\frac{999}{9000}$.
9. Divide 33440 by 270. Ans. $123\frac{230}{270}$.
10. Divide 140817 by 6800. Ans. $20\frac{4817}{6800}$.

72. *To divide by the Factors of a number.*

1. Divide 936 by 24.

OPERATION.

4)936

6)234

39

Explanation.—In all problems where the divisor is a composite number we may divide by the factors and thus shorten the operation In this example the factors are 4 and 6, and we first divide by 4 which gives a quotient 6 times too large, for the reason that 4 is but $\frac{1}{6}$ of 24 the true divisor. We therefore divide this quotient by 6 and obtain the true quotient.

2. Divide 588 by 28. The factors are 4 and 7.
Ans. 21.

3. Divide 6976 by 32. The factors are 4 and 8.
Ans. 218.

4. Divide 2583 by 63. The factors are 7 and 9.
Ans. 41.

5. Divide 10206 by 81. The factors are 9 and 9.
Ans. 126.

6. Divide 11984 by 56. The factors are 8 and 7.
Ans. 214.

7. Divide 1607 by 72, using the factors 3, 4 and 6, and find the true remainder.

Ans. 22 quotient, and 23 remainder.

FIRST OPERATION.

3) 1607

4) 535 2, 1st remainder.

6) 133 3, 2d remainder.

22 1, 3d remainder.

Explanation—In this example using as divisors 3, 4 and 6, the factors of 72. we obtain for remainders 2, 3 and 1.

The first remainder 2, is clearly units of the given dividend, and hence a part of the true remainder.

The second remainder, 3 being *fourths* of the second dividend, 535 which are reciprocal *thirds* of the given dividend, it is hence $\frac{3}{4}$ of the reciprocal of $\frac{1}{4}$ of $\frac{1}{3} = 9$, of the given dividend and true remainder.

The third remainder, 1 being *sixths* of the third dividend 133, which are reciprocal *twelfths* of the given dividend, it is hence $\frac{1}{6}$ of the reciprocal of $\frac{1}{6}$ of $\frac{1}{4}$ of $\frac{1}{3} = 12$ of the given dividend

and true remainder. Therefore 2, the first remainder, plus 9, the unit value of the second remainder, plus 12, the unit value of the third remainder = 23, the true remainder. Or we may obtain the true remainder without considering the reciprocal relationship of the quotients and divisors, thus :

First remainder.	2
Plus 2d, remainder 3, × the preceding divisor 3, =	9
Plus 3d, remainder 1, × all the preceding divisors, 4 and 3 =	12
which added gives the true remainder	23

From the foregoing we see that the true remainder may be obtained by adding to the first remainder the product of the other remainders by all the divisors preceding the one which produced it.

8. Divide 7851 by 64, using the factors 8 and 8.

Ans. 122 quotient, 43 remainder.

OPERATION.

8)7851

8)981—3, 1st remainder.

122—5, 2d remainder.

Explanation.—Here the 1st remainder is 3, to which we add the product of the 2d remainder 5, multiplied by the preceding divisor 8, equals 40, making 43, the true remainder.

9. Divide 17803 by 96, using the factors 2, 3, 4 and 4.

Ans. $185\frac{43}{96}$.

OPERATION.

2)17803

3)8901—1

4)2967—0

4)741—3

185—1

Explanation.

1st remainder	1
2d remainder 3×3×2=	18
3d remainder 1×4×3×2=	24
True Remainder.	43

10. Divide 27865 by the factors of 81.

Ans. $344\frac{1}{81}$.

11. Divide 101041 by the factors of 84.

Ans. $1202\frac{73}{84}$

12. Divide 899 by the factors of 108. Ans. $8\frac{35}{108}$.

13. If $4691 are divided equally between 35 men, what will each one receive? Ans. $134\frac{1}{35}$.

14. There are 32 quarts in one bushel, how many bushels are there in 1536 quarts? Ans. 48 bushels.

15. A hogshead of wine contains 63 gallons. How many hogsheads in 2898 gallons? Ans. 46 hogsheads.

16. One of the factors of 10800 is 225; what is the other? Ans. 48.

17. What number multiplied by 137 will give 959137 for the product? Ans. 7001.

18. Multiplying 372 by an unknown number gives 44640; what is the number? Ans. 120.

19. What is the quotient of 9126 divided by 9? Ans. 1014.

20. Divide four million, eight thousand and sixteen by MMDCXLIV. Ans. $1515\frac{2356}{2644}$.

21. What number is that to which, if sixteen be added the sum multiplied by 8 and 13 substracted from the product the remainder will be 339? Ans. 28.

22. There is a number from which if you subtract 55, and divide the remainder by 12, your quotient will be 36. What is that number? Ans. 487.

23. A merchant owes a debt of $1875, which he agreed to pay by weekly installments of $25. He has made 55 payments, how many more payments has he to make. Ans. 20.

24. A merchant bought 350 barrels of flour at $6 a barrel, and sold it at $7.50 per barrel. The gain he gave in equal parts to 4 worthy boys to aid them in obtaining an education. What was the cost and selling price of the flour, and how much money did each boy receive.

Ans. $2100 cost, $2625 selling price, $131.25 each boy received.

25. An acre contains 160 square rods; how many acres in a plantation containing 123200 square rods?

Ans. 770 acres.

26. A boy sold 50 oranges at 5¢ each and thereby gained $1.50. At what rate did he buy the oranges?

Ans. 2¢ a piece.

27. How many times 136 will produce 1768?

Ans. 13.

28. Divide the product of 750 and 875 by their difference. Ans. 5250.

29. The diameter of the earth at the equator is 7925 miles; how long would it take a locomotive to travel that distance at the rate of 25 miles an hour?

Ans. 317 hours=13 days 5 hours.

30. The first Atlantic Telegraph Cable as originally made cost $1258250. 10 miles of deep sea cable was made at a cost of $1450 per mile, and 25 miles of shore ends was made at a cost of $1250 per mile. The remainder cost $485 per mile. How many miles of Cable were made?

Ans. 2535 miles.

31. A grocer wishes to put 3335 pounds of sugar in 3 kinds of boxes, containing respectively 20, 50 and 75 pounds, using the same number of boxes of each kind or size. How many boxes will he require?

Ans. 23 of each size.

32. The Northern Pacific Railroad from Lake Superior to Puget Sound, as located, is 2000 miles long. The estimated cost and equipment of the road, including interest is $85277000. What will be the average cost per mile?

Ans. $42638.50.

33. It is estimated that, by reason of intemperance the United States loses annually $98400000. How many School Houses costing $5000 each, and how many Libraries costing $3000 could be established with this amount of money? Ans. 12300 of each.

34. Ten freedmen agreed to pick 20000 pounds of cotton and receive for their labor $\frac{1}{5}$ of the cotton picked. After they had picked 7000 pounds 4 freedmen quit, leaving the other 6 to finish the work. How much cotton is each entitled to when the work is finished?

Ans. 140 pounds each for those who left, and $573\frac{2}{6}$ each for those who remained.

35. A merchant bought 800 gallons of molasses at 65¢ and sold ½ of it at 72¢ a gallon. From the profit he bought his children a set of Cutter's Anatomical and Physiological charts, and had $8.20 left. What did the charts cost? Ans. $19.80

36. The capacity of steam engines is measured by *horse power;* and 1 horse power is a force that will raise 33000 pounds 1 foot in 1 minute. How many horse power has a steam engine that possesses a capacity of 1188000 pounds? Ans. 36.

37. The average weight of man is 150 pounds. About ⅕ of this weight is blood. Allowing that the heart throws out 2 ounces of blood at each pulsation, and that it beats 72 times a minute, and that 16 ounces make a pound, how long will it take the heart to circulate all the blood in the body? Ans. 3⅓ minutes.

38. Prof. Wilson, a physician and physiologist, has counted in the skin of the palm of the hand 3528 perspiratory pores to the square inch; but as there are less to the square inch on some other parts of the body, he estimates that 2800 is a fair average to allow to the square inch for the whole surface of the body. The average size man has 2500 square inches of body surface, which would give 7000000 perspiratory pores. Through these pores fully 2 pounds of perspiration, water, refuse matter and worn out tissue pass every 24 hours. If a man weighs 150 pounds, how long will it take for matter equal to the weight of the body to pass through the perspiratory pores, if they are kept open as they should be by daily bathing? Ans. 75 days.

39. Our earth is about 95000000 miles from the sun, Neptune, the most distant member of our Solar System, is about 2850000000. How many times farther from the Sun is Neptune than our earth? Ans. 30.

40. The velocity of the earth on its yearly voyage around the Sun is 99733 feet per second. The velocity of a cannon ball fired from a gun with an average charge of

powder is 1750 feet a second. How many times faster is the velocity of the earth than a cannon ball?

Ans. $561\frac{733}{1750}$.

41. Geo. Peabody, of Mass., gave, while living, to 27 schools and colleges, library associations, benevolent societies, state public schools, etc., not including many private presents, $7875000. Of this amount $3300000 were given to the public schools of the South. What part of the whole specified donation did he give to the South?

Ans. $\frac{3300000}{7875000}=\frac{132}{315}$.

42. Stephen Girard, of Philadelphia, gave $60000000 for the founding and support of Girard College. Soule's College in New Orleans is worth $30,000. How many such colleges could be built with the amount of money given by Mr. Girard to establish one college?

Ans. 200.

43. The air which surrounds our earth, and of which we each inhale 600 gallons every hour, is composed of four parts of Nitrogen and 1 part of Oxygen. How many gallons of each are there in a room 22 feet long and 21 feet wide and 10 feet high, which contains 34560 gallons of air? Ans. 6912 Oxygen, 27648 Nitrogen.

44. A room contains 34560 gallons of air, a man inhales 600 gallons per hour, how long will it take for 10 men to inhale the air in the room. Ans. $5\frac{4560}{6000}$ hours.

45. A room 16 feet long, 10 feet wide, and 8 feet high, contains 1280 cubic feet of air. Every time a person breathes he throws out from his lungs a sufficient quantity of carbonic acid, or carbon di-oxide, (a most deadly gas,) to pollute or render poisonous and unfit for breathing 3 cubic feet of air, and he breathes 20 times a minute. How long will it take for the air of a room of the above dimensions to become poisonous if occupied by 5 persons, and no change of air is made by ventilation.

Ans. $4\frac{80}{300}$ minutes.

46. A man produces by breathing at least 6 gallons of carbonic acid gas every minute, a single burning gas jet, 10 gallons, an ordinary stove, 60 gallons. How many gallons

of carbonic acid gas will an audience of 1000 people, 2 heated stoves, and 50 burning gas jets produce in 3 hours, and how many times would the quantity fill a room 100 feet long, 50 feet wide and 30 feet high?

Ans. 1191600 gallons. $1\frac{160596\,00}{259\,200\,000}$ time.

NOTE.—There are 60 minutes in an hour, 231 cubic inches in a gallon, and 1728 cubic inches in a cubic foot.

47. Astronomers estimate that 7500000 visible meteors fall upon the earth daily, the average weight of which is estimated to be 100 grains. Allowing for an equal quantity of matter to be brought down by the invisible meteors and the ærolites, how many pounds a year does our earth increase in weight, there being 7000 grains in a pound, and 365 days in a year? Ans. $78214285\frac{5}{7}$ pounds.

73. PROBLEMS INVOLVING THE ENGLISH MONEY OF ACCOUNT.

1. What will 13840 pounds of cotton cost at 8 pence a pound. Ans. £461. 6s. 8d.

OPERATION.

13840
8d.

12) 110720d.

2ø) 9226 8d.

£ 461 6s.

Explanation.—In this problem the price is given in one of the subdivisions of the English monetary unit, and hence we must know what that unit and its subdivisions are, before we can solve the problem. The English monetary unit is the *Pound Sterling*, which is divided into 20 *Shillings*; each shilling is divided into 12 *Pennies*, and each penny into 4 *Farthings*. With this knowledge of English money we can work all problems of the above character. In this example we first multiply the price of one pound by the number of pounds and thus produce the value of the whole in pence. Then to reduce the pence to shillings, we divide them by 12, and obtain 9226 shillings and a remainder of 8, which being a part of the dividend is therefore 8d. Then to reduce the shillings to pounds we divide them by 20, and obtain 461 pounds and a remainder of 6, which being a part of the second dividend is therefore 6s. In the English monetary system the following abbreviations are used: £. represents pounds, s. represents shillings, d. represents pence, and f. represents farthings.

2. What is the value of 483 yards of cloth at 16 shilling per yard?

Ans. £386. 8s.

OPERATION.

483
16

20) 7728 shillings
£ 386 8s.

3. What will 241 boxes cheese cost at £3 per box?

Ans. £723.

OPERATION.

241
3

723 pounds.

4. Sold 486 yards of calico at 5 pence a yard, what did it amount to? Ans. £10 2s. 6d.

5. Bought 38495 pounds of good middling cotton at 7 pence a pound. How much did it cost.?

Ans. £1122 15s. 5d.

6. What is the value of 850 barrels of flour at 34 shillings a barrel? Ans. £1445.

7. How much will 1812 tons of iron cost at £52, 4s. per ton? Ans. £94586 8s.

8. Bought 38421 pounds of cotton at 9 pence per pound what did it cost? Ans. £1440 15s. 9d.

74. MISCELLANEOUS PROBLEMS INVOLVING THE PRINCIPLES OF ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION.

1. The subtrahend is 216, and the remainder 184, what is the minuend? Ans. 400.

2. A grocer paid $350 for some tea and coffee; for the tea he paid $50 more than for the coffee, what did he pay for each? Ans. tea $200, coffee $150.

3. John has 25 cents, and James has four times as many lacking 10 cents, how many cents has James?

Ans. 90 cents.

4. A slate cost 15 cents; an arithmetic four times as much as the slate, and a philosophy twice as much, lacking 25 cents, as the slate and arithmetic. What did they all cost? Ans. $2.00.

5. The sum of two numbers is 480, and their difference is 80, what are the numbers? Ans 200, 280.

6. A man purchased a horse and cow. For the horse he paid $175, and for the cow $110 less than for the horse, what did the cow cost? Ans. $65.

7. The less of two numbers is 224 and their difference 100, what is the greater? Ans. 324.

8. The product of two numbers is 6450, and one of the numbers is 150, what is the other? Ans. 43.

9. A merchant bought 415 yards calico at 10 cts. per yard and sold it for 13 cts. per yard. How much did he gain? Ans. $12.45.

10. The dividend is 37500 and the quotient 75, what is the divisor? Ans. 500.

11. A boy sold 5 chickens at 25¢ a piece; 8 ducks at 50¢ each; received in payment 3 pigeons at 30¢ each, and the balance in money; how much money did he receive? Ans. $4.35.

12. The divisor is 37; the quotient 21, and the remainder 23, what is the dividend? Ans. 800.

13. A news boy sold 20 papers at 5¢ each, and with the money bought oranges at 4¢ each, how many oranges did he get? Ans. 25.

14. The first battle of the Revolution was fought April 19, 1775, how many years, months and days have passed since then?

15. H. Zuberbier has an orange orchard consisting of 480 trees, and each tree produces 5 barrels of oranges which are worth in the market $4 a barrel; what is the value of his orange crop? Ans. $9600.

16. C. Quentell bought a barrel of sirop de batterie containing 43 gallons at 95¢ per gallon; 4 gallons having leaked out he sold the remainder at $1.05 a gallon. How much did he gain by the transactions? Ans. $.10 gain.

17. W. C. Martin bought 354 barrels of flour for $2478. He sold the same at $7.50 per barrel; how much did he gain? Ans. $177.

18. The Capital Stock of a Manufactory is $100000

which is divided into 200 shares. What are 5 shares worth?
Ans. $2500.

19. J. Muller sold to O. Braun 25 barrels of apples at $4 per barrel and 124 barrels of potatoes at $3.25 per barrel; he received in payment 1 hogshead of sugar containing 1143 pounds at 8¢ and the remainder in money; how much money did he receive? Ans. $411.56.

20. A speculator bought 528 cords of wood at $6.50 per cord. He re-corded the wood so that it measured 579 cords which he sold at $6.75 a cord; how much did he gain? Ans. $476.25.

CANCELLATION.

75. **Cancellation** is the process of shortening the operations of division, or of the indicated result of multiplication and division operations combined, by rejecting equal factors from both dividend and divisor or from both increasing and decreasing numbers.

The operation is performed by drawing a line across each factor *cancelled* or cut out.

76. The **Principles of Cancellation,** are, 1. Rejecting or Cancelling a factor from any number is in effect dividing the number by that factor. 2. Rejecting or cancelling equal factors from both dividend and divisor, or from both increasing and decreasing numbers in an indicated result, does not change the quotient or result.

EXAMPLES.

Divide $7\times3\times4$ by 7×4

Operation by Cancellation.

$$\begin{array}{r|l} \cancel{7} & \cancel{7} \\ \cancel{4} & 3 \\ & \cancel{4} \\ \hline & 3 \text{ Ans.} \end{array}$$

Explanation.—In all problems where we have both multiplication and division operations to perform, we use a vertical or perpendicular line which we call the statement line. This line is used to facilitate the work by separating the dividends and divisors, or the increasing and decreasing numbers. The dividends or increasing numbers are always placed upon the right hand side of the

line and the divisors or decreasing numbers are always placed upon the left hand side.

In this example having written the numbers that constitute the dividend and divisor, respectively upon the right and left hand side of the statement line, we *cut out* or *cancel* the equal factors 7's and 4's in the numbers constituting the dividend and divisor and thus obtain 3 the answer to the problem.

To perform the work without the aid of Cancellation we would be obliged to make the following figures: $7\times3=21$, which $\times4=84$ the dividend; then $7\times4=28$ the divisor; then

28)84(3 Ans.
84
—

2. Multiply 25, 48 and 88 together and divide the product by the product of 10, 36 and 8.

Operation by Cancellation.

2 10 25 5
3 36 48 4 2
8 88 11
———
110
—
$36\frac{2}{3}$ Ans.

Explanation.—In this example we write the numbers on the line as above directed and then cancel the 10, and 25 by 5; then the 36 and 48 by 12; then the 8 and 88 by 8; then the 4 and 2 by 2. This is all that can be cancelled and we then multiply together the 5, 2 and 11 and divide the product by 3, and thus obtain the true result $36\frac{2}{3}$.

Should the student experience any difficulty in this kind of work, he should be orally drilled on the factors of numbers and composite numbers.

3. Divide the product of 32×3 by $8\times9\times16$.

Operation by Cancellation.

8 32 4
3 9 3
4 16
———
12 | $1=\frac{1}{12}$ Ans.

Explanation.—Having written the numbers on the statement line, we first cancel the 8 and 32 by 8; then the 9 and 3 by 3; then the 16 and 4 by 4. Now having no more numbers on the increasing side of the line to cancel we multiply together the remaining numbers on the decreasing side of the line and thus produce the correct result $\frac{1}{12}$.

In all cases where, after cancelling, no factor appears on either

side of the statement line, the factor 1, is always understood, as being there. Its non-appearance is in consequence of not having written it when we cancelled a number by itself.

4. A merchant sold 25 boxes of candles containing 36 pounds each at 16¢ per pound and received in payment starch at 6 cents per pound. How many boxes each containing 30 pounds did he receive? Ans. 80 boxes.

Operation by Cancellation

$$\begin{array}{r|l} 6\,16 & \\ 5\ \cancel{30}\ 25\ 5 & \\ \cancel{36}\ 6 & \\ \hline & 80 \text{ boxes Ans.} \end{array}$$

Cancel and work the following line statements or results:

2 6	12 45	31 124	9 76
3 9	9 56	5 17	20 91
18 54	7 84	51 10	70 140
27	16 4	4	25
1 Ans.	70 Ans.	$\frac{2}{3}$ Ans.	$1921\frac{1}{9}$ Ans.

5. Divide the product of 6, 7, 12 and 22 by the product of 11, 3, 14 and 8. Ans. 3.

6. What is the quotient of $28\times65\times7\times78\div56\times130\times42\times13$? Ans. $\frac{1}{4}$.

7. Multiply 21, 55 and 128 together and divide the product by $14\times25\times64$. Ans. $6\frac{3}{5}$.

8. How many bushels of corn at 70¢ each will pay for 140 gallons molasses at 65 cents a gallon?

Ans. 130 bushels.

9. Bought 420 pounds of sugar at 6 cents a pound and gave in payment 360 pounds of rice. What was the price of the rice? Ans. 7 cents.

10. Sold a drayman 64 bushels of oats at 75 cents a bushel, for which he is to pay in drayage at 50 cents a load. How many loads must he haul? Ans. 96 loads.

11. How many pounds of butter at 35¢ per pound will

pay for 245 pounds of rice at 5 cents per pound?

Ans. 35 pounds.

12. Paid 65¢ for 5 yards of calico, what will 27 yards cost at the same rate?

Analytic Solution by Cancellation.

$$\begin{array}{r|l} 5 & 65 \quad 13 \\ & 27 \\ \hline & \$3.51 \text{ Ans.} \end{array}$$

Explanation.—In all practical problems of this kind we give a reason for each step of the operation, and make the whole statement to indicate the final result without performing any of the intermediate work. In this problem, we place the 65¢ on the increasing side of the statement line as our premise and reason thus: since 5 yards cost 65¢, 1 yard will cost $\frac{1}{5}$ part of it, and 27 yards will cost 27 times as much as 1 yard.

13. If 17 barrels of flour cost $110.50, what will 500 barrels cost at the same rate? Ans. $3250.

14. If $\frac{3}{4}$ of a dozen apples cost 40 cents, what will $13\frac{1}{2}$ dozen cost? Ans. $7.20.

PROPERTIES OF NUMBERS.

DEFINITIONS.

77. **An Integer** is a whole number; as 1, 5, 6, 18, etc. Whole numbers are divided into two classes, *Prime* and *Composite.*

78. **A Prime number** is one that can only be divided, without a remainder, by itself and 1; as 1, 2, 3, 5, 7, 11, 13, 17, etc.

79. **A Composite number** is one that can be divided without a remainder, by some other whole number than itself and 1; as 4, 9, 12, 15, 24, etc.

All composite numbers are the product of two or more other numbers.

Numbers are prime to each other when they have no common factor that will divide each without a remainder; as 6, 13, 20, etc.

80. An **Even** number is one that can be divided by 2, without a remainder; as 4, 8, 12, 56, etc.

81. **An Odd** number is one that cannot be divided by 2, without a remainder, as 1, 7, 19, 45, 133, etc.

82. **A Factor of a Number** is a number that will divide it without a remainder or by being taken an entire number of times, will produce it; as 4 is a factor of 16, and 5 a factor of 25.

Every factor of a number is a divisor of it.

83. **A Prime Factor** of a number is a prime number that will divide it without a remainder; thus, 1, 2, 3 and 5 are the prime factors of 30.

84. **A Composite Factor** of a number is a composite number that will divide it without a remainder; thus, 6 and 8 are composite factors of 48.

85. **An Aliquot** part of a number is such a part as will divide it without a remainder; thus, 1, 2, 3, 4, 6 and 8 are aliquot parts of 24.

86. **The Reciprocal** of a number is the quotient of 1 divided by the number; thus the reciprocal of 8 is $1 \div 8 = \frac{1}{8}$; and the reciprocal of $\frac{1}{4}$ is $1 \div \frac{1}{4} = 4$.

87. **The Power** of a number is the product obtained by multiplying the number by itself a certain number of times; thus, 36 is the second power of 6; 125 is the third power of 5.

88. **The Multiple** of a number is the product obtained by multiplying the number by any other number any number of times. Or it is a number divisible by a given number without a remainder; thus, 14 is a multiple of 7 and 2, and 54 is a multiple of 2, 3, 9, and 27.

89. **A Common Multiple** *of two or more numbers* is a number divisible by each of them without a remainder; thus, 24 is a common multiple of 2, 3, 4, 6 and 12.

90. **The Least Common Multiple** of two or more

numbers is the least number that is divisible by each of them without a remainder; thus, 12 is the *least* common multiple of 2, 3, 4, 6 and 12.

DIVISIBILITY OF NUMBERS.

91. **A Divisor**, or measure of a number, is any number that will divide it without a remainder; thus, 4 is a divisor or measure of 12, and 5 is a divisor or measure of 20.

One number is said to be **Divisible** by another when the remainder is 0.

92. **A Common Divisor** of two or more numbers is a number that will divide each of them without a remainder; thus, 2 is a common divisor of 12, 18 and 24.

93. **The Greatest Common Divisor** of two or more numbers is the greatest number that will divide each of them without a remainder; thus, 6 is the *greatest* common divisor of 12, 18 and 24.

94. Every number ending with 0, 2, 4, 6 or 8, is divisible by 2.

95. Every number is divisible by 4 when its units and tens figures are divisible by 4; thus, 156, 264, 34512, 561308, are each divisible by 4.

96. Every number is divisible by 8 when the units, tens and hundreds figures are divisible by 8; thus 3824, 12512, 190720 are each divisible by 8.

97. All numbers ending in 0 or 5 are divisible by 5; thus, 10, 15 and 35 are divisible by 5.

98. Every number, the sum of whose figures is divisible by 3 or 9 without a remainder, is divisible by 3 or 9; thus, 135, 3456, 12345912, etc., are each divisible by 3 and by 9.

99. Every *even* number, the sum of whose figures is

divisible by 3 without a remainder, is divisible by 6; thus 318, 12414, etc., are divisible by 6.

FRACTIONS.

100. A **Fraction** is one or more of the equal parts of a unit of any kind, or of a collection of units taken together. Or more briefly, a part of anything, or a numerical expression of a part of a unit.

101. A **Fractional Unit** is one of the equal parts into which any integral unit is divided. If the integral unit is divided into two equal parts, each is called *a half*; if into three, each is called *a third*; if into four, each is called *a fourth*; and so on according to the number of parts into which the integral unit is divided.

102. Fractions are divided into two kinds, *Common* or *Vulgar*, and *Decimal Fractions*.

Common Fractions are expressed by two numbers, one written above the other, with a horizontal line between them. The number below the line is called the *Denominator*, and the number above the line is called the *Numerator*. Thus $\frac{1}{2}$ (*one half*,) $\frac{3}{4}$ (*three fourths*,) $\frac{5}{6}$ (*five sixths*,) $\frac{7}{8}$ (*seven eights*,) and $\frac{13}{17}$ (*thirteen seventeenths*) are fractions, the denominators of which are, respectively, 2, 4, 6, 8, and 17. The Numerator and Denominator together, are called the *terms* of the fraction.

The *Denominator* of a fraction shows the number of equal parts into which the unit is divided.

Thus in the fraction $\frac{5}{8}$ the 8 is the denominator and shows that the unit is divided into 8 equal parts called *eighths*.

The *Numerator* of a fraction shows the number of equal parts taken to form the fraction.

Thus in $\frac{5}{8}$, the numerator is 5 and shows that 5 of the 8 equal parts are taken or expressed by the fraction.

All fractions arise from division and are expressions of unexecuted division in which the *numerator* is the *dividend*, the *denominator* the *divisor*, and the fraction itself the *quotient*.

Decimal Fractions are those in which the denominators are not generally expressed, but are always 10, or a power of ten; thus, .5, .75, .821, read respectively *five tenths*, *seventy-five hundredths*, and *eight hundred and twenty-one thousandths*, are decimal fractions. To write these fractions as *common fractions*, they would be written thus, $\frac{5}{10}$, $\frac{75}{100}$ and $\frac{821}{1000}$.

The point (.) placed before the 5, 7 and 8, in the above decimally expressed fractions, is called the **decimal point,** and is used to abbreviate the work.

103. CLASSIFICATION OF FRACTIONS.

For convenience fractions are classed under the following heads: *Proper Fractions; Improper Fractions; Simple Fractions; Mixed Numbers; Compound Fractions; Complex Fractions.*

104. **A Proper Fraction** is one in which the numerator is less than the denominator, as $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$.

105. **An Improper Fraction,** is one in which the numerator is equal to or greater than the denominator; as $\frac{5}{5}$, $\frac{7}{3}$, $\frac{9}{8}$ and $\frac{16}{2}$.

106. **A Simple Fraction** is one in which both terms are whole numbers, and may be either a *proper* or *improper* fraction; as $\frac{3}{5}$, $\frac{9}{7}$, $\frac{11}{16}$ or $\frac{24}{8}$.

107. **A Mixed Number,** is a number composed of a whole number and a fraction; as $2\frac{1}{2}$, $5\frac{3}{4}$ and $21\frac{5}{12}$.

108. **A Compound Fraction** is a fractional part of a fraction or mixed number; as $\frac{3}{4}$ of $\frac{5}{8}$ and $\frac{1}{2}$ of $\frac{7}{16}$ of $12\frac{3}{4}$.

109. **A Complex Fraction** is one that has one or more of its terms fractional; as:

$$\frac{\frac{3}{4}}{\frac{5}{8}} \text{ of } \frac{6\frac{1}{3}}{5\frac{3}{7}} \text{ and } \frac{3}{\frac{2}{3}} \text{ of } \frac{\frac{7}{8}}{1\frac{1}{2}} \text{ of } \frac{6\frac{1}{4}}{8}$$

110. **The Reciprocal of a Fraction** is the result of

1 divided by the fraction. Thus the reciprocal of $\frac{4}{5}$ is $1 \div \frac{4}{5} = \frac{5}{4} = 1\frac{1}{4}$.

111. **The Value of a Fraction** is the result of its numerator divided by its denominator. Thus $\frac{8}{2} = 4$, $\frac{21}{8} = 2\frac{5}{8}$.

112. GENERAL PRINCIPLES OF FRACTIONS.

1. *Multiplying* the numerator, or *dividing* the denominator, *multiplies* the fraction.

2. *Dividing* the numerator, or *multiplying* the denominator, *divides* the fraction.

3. *Multiplying* or *dividing* both numerator and denominator by the same number does not change the value of the fraction.

113 .REDUCTION OF FRACTIONS.

Reduction of Fractions is the process of changing their *form* without altering their *value*.

114. A fraction is reduced to **Higher Terms** when the numerator and denominator are expressed in larger numbers. Thus, $\frac{1}{2} = \frac{2}{4}$ or $\frac{4}{8}$ or $\frac{8}{16}$; $\frac{2}{3} = \frac{4}{6}$ or $\frac{8}{12}$ or $\frac{16}{24}$, etc.

115. A fraction is reduced to **Lower Terms** when the numerator and denominator are expressed in smaller numbers. Thus $\frac{8}{12} = \frac{4}{6}$ or $\frac{2}{3}$; $\frac{15}{45} = \frac{3}{9}$ or $\frac{1}{3}$.

116. A fraction is reduced to its **Lowest Terms** when its numerator and denominator cannot be divided by any number greater than 1, or, when its numerator and denominator are prime to each other.

117. **Whole numbers** may be reduced to fractions having any desired denominator.

118. ORAL EXERCISES.

1 unit, abstract, or denominate of any kind equals, how many halves? thirds? fourths? fifths? sixths? sevenths? eighths? ninths?

2=how many halves? thirds? fourths? fifths? sixths? sevenths? eighths? ninths?

3=how many halves? thirds? fourths? fifths? sixths? sevenths? eighths? ninths?

4=how many halves? thirds? fourths? fifths? sixths? sevenths? eighths? ninths?

5=how many halves? thirds? fourths? fifths? sixths? sevenths? eighths? ninths?

6=how many halves? thirds? fourths? fifths? sixths? sevenths? eighths? ninths?

119. What kind of numerical work is the above called?

$\frac{1}{2}$ = how many fourths? sixths? eighths? tenths? twelfths? fourteenths?

$\frac{1}{3}$ = how many sixths? ninths? twelfths? fifteenths? eighteenths? twenty-firsts?

$\frac{1}{4}$ = how many eighths? twelfths? sixteenths? twentieths? twenty-fourths?

$\frac{1}{5}$ = how many tenths? fifteenths? twentieths? twenty-fifths? thirtieths?

$\frac{1}{8}$ = how many sixteenths? twenty-fourths? thirty-seconds? fortieths? sixty-fourths?

$\frac{1}{16}$ = how many thirty-seconds? forty-eighths? sixty-fourths? eightieths?

What kind of numerical work is the above called?

Answer the following numerical questions:

$\frac{2}{4}=\frac{1}{2}$	$\frac{6}{9}=?$	$\frac{10}{15}=?$	$\frac{14}{28}=?$	$\frac{6}{36}=?$	$\frac{15}{75}=?$
$\frac{3}{6}=?$	$\frac{7}{21}=?$	$\frac{11}{22}=?$	$\frac{15}{45}=?$	$\frac{7}{49}=?$	$\frac{75}{100}=?$
$\frac{4}{12}=?$	$\frac{8}{24}=?$	$\frac{12}{48}=?$	$\frac{16}{64}=?$	$\frac{8}{96}=?$	$\frac{125}{750}=?$
$\frac{5}{10}=?$	$\frac{9}{27}=?$	$\frac{13}{26}=?$	$\frac{17}{51}=?$	$\frac{9}{108}=?$	$\frac{450}{900}=?$

120. What kind of numerical work is the above called?

How many halves = or make a unit?
" " thirds = " " "
" " fourths = " " "
" " fifths = " " "
" " sixths = " " "
" " sevenths = " " "
" " eighths = " " "

How many ninths = or make a unit?
" " tenths = " " "
" " elevenths = " " "
" " twelfths = " " "

121. 1 unit equals how many eighths? $\frac{8}{8} = ?$
1 " " " twelfths? $\frac{12}{12} = ?$
3 units equal " thirds? $\frac{9}{3} = ?$
4 " " " fourths? $\frac{16}{4} = ?$
5 " " " halves? $\frac{10}{2} = ?$
$1\frac{1}{2}$ " " " halves? $\frac{3}{2} = ?$
$1\frac{1}{2}$ " " " fourths? $\frac{6}{4} = ?$
$1\frac{1}{3}$ " " " thirds? $\frac{4}{3} = ?$
$1\frac{2}{3}$ " " " sixths? $\frac{10}{6} = ?$
$1\frac{3}{4}$ " " " eighths? $\frac{14}{8} = ?$
$3\frac{1}{4}$ " " " fourths? $\frac{13}{4} = ?$
$2\frac{3}{4}$ " " " sixteenths? $\frac{44}{16} = ?$
$4\frac{3}{8}$ " " " eighths? $\frac{35}{8} = ?$

122. What is the value of the following fractional expressions?

$\frac{1}{2}$ of $\frac{1}{2} = ?$
$\frac{1}{2}$ of $\frac{1}{4} = ?$
$\frac{1}{2}$ of $\frac{1}{3} = ?$
$\frac{1}{3}$ of $\frac{3}{4} = ?$
$\frac{3}{4}$ of $\frac{5}{8} = ?$
$\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4} = ?$

$\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{1}{8} = ?$
$\frac{5}{8}$ of $\frac{4}{5}$ of $\frac{3}{5} = ?$
$\frac{1}{4}$ of $\frac{5}{6}$ of $\frac{9}{10} = ?$
$\frac{3}{4}$ of $\frac{5}{8}$ of $\frac{4}{15} = ?$
$\frac{3}{8}$ of $\frac{8}{9}$ of $\frac{5}{16} = ?$
$\frac{9}{16}$ of $\frac{4}{9}$ of $\frac{7}{12} = ?$

123. What is the reciprocal of 1, of 2, of 3, of $\frac{1}{2}$, of $\frac{3}{4}$ of $2\frac{5}{8}$?

What is the value of $\frac{6}{2}$, of $\frac{12}{4}$, of $\frac{16}{3}$, of $\frac{24}{5}$, of $\frac{37}{8}$?

Analyse the fraction $\frac{3}{4}$.

Analysis:—$\frac{3}{4}$ is a proper fraction, since the numerator is less than the denominator; 4 is the denominator, and shows that the unit is divided into 4 equal parts; $\frac{1}{4}$ is the *fractional unit*, since it is ONE of the *four* equal parts into which the unit is divided; 3 is the *numerator* and shows that *three* of these equal parts are taken; 3 and 4 are the terms of the fraction, and its value is less than 1, or unity.

In like manner analyse the following fractions:

$\frac{3}{8}$, $\frac{5}{3}$, $\frac{9}{16}$, $\frac{12}{4}$, $\frac{15}{32}$, $\frac{9}{5}$, $\frac{41}{15}$, $\frac{24}{55}$.

124. Mentally add the following fractions:

$\frac{1}{2}+\frac{1}{2}=?$	$\frac{2}{3}+\frac{5}{6}=?$	$\frac{1}{2}+\frac{1}{4}+\frac{7}{8}=?$
$\frac{1}{4}+\frac{3}{4}=?$	$\frac{3}{4}+\frac{7}{8}=?$	$\frac{2}{3}+\frac{5}{6}+\frac{7}{12}=?$
$\frac{1}{3}+\frac{2}{3}=?$	$\frac{1}{3}+\frac{3}{4}=?$	$\frac{3}{4}+\frac{7}{8}+\frac{9}{16}=?$
$\frac{2}{3}+\frac{2}{3}=?$	$\frac{7}{8}+\frac{9}{16}=?$	$\frac{7}{8}+\frac{1}{2}+\frac{1}{6}=?$
$\frac{3}{4}+\frac{3}{4}=?$	$\frac{7}{12}+\frac{7}{12}=?$	$\frac{3}{5}+\frac{9}{10}+\frac{14}{15}=?$
$\frac{1}{2}+\frac{3}{4}=?$	$\frac{5}{9}+\frac{23}{27}=?$	$\frac{1}{4}+\frac{3}{5}+\frac{2}{3}=?$

125. Answer by mental work the following numerical questions:

$\frac{3}{4}-\frac{1}{4}=?$	$\frac{1}{2}-\frac{1}{4}=?$	$3\frac{1}{2}-1\frac{3}{8}=?$
$\frac{2}{3}-\frac{1}{3}=?$	$\frac{2}{3}-\frac{1}{6}=?$	$\frac{1}{2}+\frac{2}{3}-\frac{3}{8}=?$
$\frac{7}{8}-\frac{5}{8}=?$	$\frac{1}{2}-\frac{3}{8}=?$	$\frac{2}{3}+\frac{3}{4}-1\frac{7}{12}=?$
$\frac{5}{16}-\frac{1}{8}=?$	$1-\frac{5}{12}=?$	$2\frac{1}{2}-\frac{5}{8}+\frac{9}{16}=?$

126. ORAL EXERCISES.

1. What will $2\frac{1}{2}$ yards cost at $\$\frac{3}{4}$ per yard?

Ans. $\$1\frac{7}{8}$.

Analytic solution.—Since 1 yard cost $\$\frac{3}{4}$, $\frac{1}{2}$ a yard will cost $\frac{1}{2}$ as much, which is $\$\frac{3}{8}$; and $\frac{5}{2}$ yards ($2\frac{1}{2}$ yards reduced to halves) will cost 5 times as much, which is $\$\frac{15}{8}$ or $\$1\frac{7}{8}$.

2. What will $2\frac{3}{4}$ pounds cost at $7\frac{1}{2}$¢ per pound?

Analytic solution.—Since 1 pound cost $\frac{15}{2}$ cents, $\frac{1}{4}$ of a pound will cost $\frac{1}{4}$ part which is $\frac{15}{8}$ cents; and $\frac{11}{4}$ will cost 11 times $\frac{15}{8}$ cents, which is $\frac{165}{8}$ cents or $20\frac{5}{8}$ cents.

In like manner solve the following problems.

3. At $\$\frac{3}{8}$ a yard what will $5\frac{3}{4}$ yards cost?

Ans. $\$2\frac{5}{32}$.

4. A dozen is worth $\$3\frac{1}{3}$, what are $4\frac{1}{3}$ dozen worth?

Ans. $\$15\frac{1}{6}$.

5. What is the value of $6\frac{1}{4}$ dozen apples at $\$\frac{3}{8}$ per dozen? Ans. $\$2\frac{11}{32}$.

6. What cost $8\frac{1}{2}$ gross at $\$2\frac{3}{4}$ per gross?

Ans. $\$23\frac{3}{8}$.

1. If $\frac{3}{4}$ of a pound cost 12 cents, what is 1 pound worth? Ans. 16 cents.

Analytic solution.—Since $\frac{3}{4}$ of a pound cost 12 cents, $\frac{1}{4}$ will cost $\frac{1}{3}$ of 12 cents, which is 4 cents, and $\frac{4}{4}$ or 1 pound will cost 4 times 4 cents, which is 16 cents.

2. $\frac{2}{3}$ of a dozen cost \$10, what is the value of $2\frac{1}{2}$ dozen? Ans. \$37½.

Analytic solution.—Since $\frac{2}{3}$ of a dozen cost \$10, $\frac{1}{3}$ of a dozen will cost $\frac{1}{2}$ as much, which is \$5, and $\frac{3}{3}$ or a whole dozen will cost 3 times as much, which is \$15; and since 1 dozen cost \$15, $\frac{1}{2}$ dozen will cost $\frac{1}{2}$ as much, which is \$7½ and $\frac{5}{2}$ dozen will cost 5 times as much which is \$37½.

In like manner solve the following problems:

3. If $\frac{3}{8}$ of a yard cost $2\frac{1}{4}$, what will 1 yard cost? Ans. \$6.

4. If $\frac{3}{8}$ of a yard cost $2\frac{1}{4}$, what will $\frac{1}{8}$ of a yard cost? Ans. \$¾

5. $\frac{3}{4}$ of a number is 15, what is the number? Ans. 20.

6. If $\frac{2}{3}$ of a number is 8, what is $1\frac{3}{4}$ times the number? Ans. 21.

7. If $\frac{2}{3}$ of a dozen cost \$8, what will $\frac{3}{4}$ of a dozen cost at the same rate? Ans. \$9.

8. What part of 4 is 3? Ans. $\frac{3}{4}$.

Analytic solution.—Here by the terms of the question we have 3 to divide or measure by 4, and by the exercise of our reason we proceed thus: since 3 is equal to 1, 3 times, it is equal to 4 $\frac{1}{4}$ of 3 times, which is $\frac{3}{4}$. Or thus. Since 1 is $\frac{1}{4}$ of 4, 3 is 3 times $\frac{1}{4}$, which is $\frac{3}{4}$.

9. What part of 5 is $\frac{2}{3}$? Ans. $\frac{2}{15}$.

Analytic solution.—Since $\frac{3}{3}$ is equal to 1, $\frac{2}{3}$ of a time, it is equal to 5 the $\frac{1}{5}$ part of $\frac{2}{3}$ of a time, which is $\frac{2}{15}$.

10. What part of $\frac{4}{5}$ is 7? Ans. $8\frac{3}{4}$.

Analytic solution.—Since 7 is equal to one 7 times, it is equal to $\frac{1}{5}$, 5 times 7 which is 35, and to $\frac{4}{5}$ instead of $\frac{1}{5}$ to $\frac{1}{4}$ part of 35, which is $8\frac{3}{4}$.

What part of $\frac{3}{8}$ is $\frac{5}{9}$? Ans. $1\frac{13}{27}$.

Analytic solution.—Since $\frac{5}{5}$ is equal to 1, $\frac{5}{5}$ of a time, it is equal to $\frac{1}{8}$ 8 times $\frac{5}{5}$ which is $\frac{40}{9}$, and to $\frac{3}{8}$ instead of $\frac{1}{8}$, to $\frac{1}{8}$ part of $\frac{40}{9}$, which is $\frac{40}{27}$ or $1\frac{13}{27}$.

12. What part of $\frac{7}{8}$ is $\frac{4}{5}$? Ans. $\frac{32}{35}$.
13. What part of $3\frac{1}{2}$ is $2\frac{1}{4}$? Ans. $\frac{9}{14}$.
14. What part of 5 is $\frac{3}{4}$ of 2? Ans. $\frac{3}{10}$.
15. What part of $\frac{4}{5}$ is $\frac{3}{7}$ of $\frac{5}{8}$? Ans. $\frac{75}{224}$.
16. 9 is $\frac{1}{8}$ of what number? Ans. 72.

Analytic solution.—Since 9 is $\frac{1}{8}$ of a number, $\frac{8}{8}$ or the whole number is 8 times 9 or 72.

17. 13 is $\frac{1}{7}$ of what number? Ans. 91.
18. $21\frac{3}{10}$ is $\frac{1}{5}$ of what number? Ans. $106\frac{1}{2}$.
19. $\frac{3}{35}$ is $\frac{1}{7}$ of what number? Ans. $\frac{3}{5}$.
20. 24 is $\frac{4}{5}$ of how many times 3? Ans. 10.

Analytic solution.—Since 24 is $\frac{4}{5}$ of the number, $\frac{1}{5}$ is $\frac{1}{4}$ part of 24 which is 6, and $\frac{5}{5}$ or the whole number is 5 times 6, which is 30; and as 30 is equal to 3, 10 times, therefore 24 is $\frac{4}{5}$ of 10 times 3.

21. 32 is $\frac{4}{7}$ of how many times 8? Ans. 7.
22. 28 is $\frac{7}{15}$ of how many times 12? Ans. 5.
23. $\frac{3}{4}$ of 48 is $\frac{2}{3}$ of what number? Ans. 54.

Analytic solution.—Since 48 is the whole of a number, $\frac{1}{4}$ of the number is $\frac{1}{4}$ part of 48, which is 12, and $\frac{3}{4}$ is 3 times 12, which is 36; and since 36 is $\frac{2}{3}$ of an unknown number, $\frac{1}{3}$ of it is $\frac{1}{2}$ of 36, which is 18, and $\frac{3}{3}$ or the whole number is 3 times 18, which is 54.

24. $\frac{8}{9}$ of 63 is $\frac{4}{11}$ of what number? Ans. 154.
25. $\frac{3}{8}$ of $\frac{2}{3}$ of 64 is $\frac{2}{13}$ of what number? Ans. 104.
26. $\frac{1}{4}$ of $\frac{4}{7}$ of 42 is $\frac{5}{6}$ of what number? Ans. $7\frac{1}{5}$.
27. $\frac{3}{4}$ of 32 is $\frac{2}{3}$ of 4 times what number? Ans. 9.

Analytic solution.—Since 32 is the whole of a number, $\frac{1}{4}$ of the number is $\frac{1}{4}$ part of 32, which is 8, and $\frac{3}{4}$ is 3 times 8, which is 24; and since 24 is $\frac{2}{3}$ of 4 times an unknown number, $\frac{1}{3}$ of 4 times the number is $\frac{1}{2}$ of 24, which is 12, and $\frac{3}{3}$ or the whole of 4 times the number is 3 times 12, which is 36; and since 36 is 4 times the number, $\frac{1}{4}$ of 36, which is 9, is the required number.

28. $\frac{7}{8}$ of 40 is $\frac{5}{6}$ of 7 times what number? Ans. 6.
29. $\frac{4}{5}$ of 56 is $\frac{4}{9}$ of 6 times what number? Ans. 12.
30. $\frac{2}{3}$ of $\frac{3}{4}$ of 66 is $3\frac{3}{7}$ of 3 times what number? Ans. 3.

31. What is the $\frac{1}{3}$ and $\frac{1}{2}$ of a $\frac{1}{3}$, of $\frac{2}{3}$ of 15?

Ans. 5.

127. GREATEST COMMON DIVISOR.

For a definition of *a divisor*, *a common divisor*, and *the greatest common divisor*, see page 77.

1. What is the greatest common divisor of 42, 56 and 210?

OPERATION.

2	42. 56. 210
7	21. 28. 105
	3 4 15

$2 \times 7 = 14$ Ans.

Explanation.—In all problems of this kind we first divide by any factor that will divide all the numbers; then we divide in like manner the successive quotients thus obtained, until we obtain quotients that have no common factor; then we multiply all the divisors together and in the product we have the greatest common divisor.

When there is no number greater than 1 that will divide all the numbers without a remainder, then 1 is the greatest common divisor.

When there are two large numbers the operation may be more easily performed by first dividing the larger number by the smaller, and if there is a remainder divide the preceding divisor by it, and thus continue until there is no remainder. When there are more than two numbers, proceed as with two, and then with the greatest common divisor of the two and one of the other numbers, and thus continue until through all the numbers. The last divisor will be the greatest common divisor.

2. What is the greatest common divisor of 88 and 24? Ans. 8.

OPERATION.

```
24)88(3
   72
   —
  16)24(1
     16
     —
      8)16(2
        16
        —
```

3. What is the greatest common divisor of 195, 285, and 315? Ans. 15.

OPERATION.

```
285)315(1
    285
    —
     30)285(9
        270
        —
         15)30(2
            30
            —
15)195(13
   15
   —
   45
   45
   —
```

What is the greatest common divisor of the following numbers?

4. Of 441 and 567? Ans. 63.
5. Of 90, 315 and 810? Ans. 45.
6. Of 654, 216, and 108? Ans. 6.
7. Walker has 25 and Caruthers 45 dimes, how shall they arrange them in packages, so that each shall have the same number in each package? Ans. 5 in each package.
8. A planter has 697 bushels of corn and 204 bushels of rough rice, which he wishes to put in the least number of bins containing the same number of bushels, without mixing the two kinds. How many bushels must each bin hold? Ans. 17 bushels.
9. A Commission Merchant has 2490 bushels of wheat, 1886 bushels of corn and 8438 bushels of oats, which he wishes to ship in the least number of sacks of equal size that will exactly hold either kind of grain. How many sacks will he require? Ans. 6407.

128. LEAST COMMON MULTIPLE.

For a definition of a *Multiple*, a *Common Multiple*, and a *Least Common Multiple*, see page 76.

1. What is the least common multiple of 5, 6, 8, 21, 28.

OPERATION.

2) 5. 6. 8. 21 28
2) 5 3 4 21 14
3) 5 3 2 21 7
7) 5 1 2 7 7
5 1 2 1 1

$2\times2\times3\times7\times5\times2=840$ Ans.

Explanation.—In all problems of this kind we first arrange the numbers on a horizontal line, and then divide by the *smallest prime* number that will divide two or more without a remainder and write the quotient and undivided numbers in a line below; this process of dividing we continue until there are no two numbers that can be divided by the same number without a remainder; then we multiply the divisors and the numbers in the last line together, and the product is the least common multiple.

When there is any number that will divide any of the others without a remainder it may be cancelled before commencing to divide.

2. What is the least common multiple of 4, 9, 12, 15, and 24? Ans. 360.

What is the least common multiple of the following:

3. Of 8, 4, 9 and 30? Ans. 360.
4. Of 50, 27, 3, 45 and 63? Ans. 9450.
5. Of 21, 36, 11 and 22? Ans. 2772.
6. Of 800, 600, 10, 40 and 12? Ans. 2400.
7. Of 8, 18, 20 and 70? Ans. 2520.
8. A drayman has 2 drays and 2 floats; on 1 dray he can haul 9 barrels of flour, and on the other 12 barrels; on 1 float he can haul 18 barrels, and on the other 21 barrels; what is the least number of barrels that will make full loads for either of the drays or floats.

Ans. 252.

9. A fruit dealer desires to invest an equal amount of money in oranges, peaches and grapes, and to expend as small a sum as possible; the price of oranges is \$2.40 per box; peaches \$1.60, and grapes for a medium article, 90 ¢., and for first quality, \$1.20; of these two qualities the fruit dealer took the cheaper. How much more money did he invest than he would had he taken the grapes at \$1.20 per box? Ans. \$28.80.

WRITTEN EXERCISES.

REDUCTION OF FRACTIONS.

129. *To reduce fractions to their lowest terms.*

Reduce $\frac{56}{64}$ to its lowest terms.

FIRST OPERATION.

$2)\frac{56}{64} = \frac{28}{32}$; $4)\frac{28}{32} = \frac{7}{8}$ Ans.

SECOND OPEATION.

$8)\frac{56}{64} = \frac{7}{8}$ Ans.

Explanation.—In all problems of this kind we divide both the numerator and denominator successively by each of the common factors they contain. Or as shown in the second operation we may produce the same result with less figures, by dividing both terms of the fraction by their greatest common divisor.

By this reduction we change the form of the fraction $\frac{56}{64}$, but we do not alter or change its value, for the fractional unit of the resulting fraction ($\frac{1}{8}$) is 8 times as great while the number taken is $\frac{1}{8}$ as great.

When the t rms of the fraction have no common factor greater than 1, the fraction is in its lowest terms and is called an *irreducible* fraction.

The object of reducing fractions to their lowest terms is to enable us to more easily and readily understand their value.

Reduce the following fractions to their lowest terms:

2. $\frac{14}{28}$, $\frac{15}{85}$, $\frac{24}{72}$, $\frac{42}{80}$. Ans. $\frac{1}{2}$, $\frac{3}{7}$, $\frac{1}{3}$, $\frac{21}{40}$.

3. $\frac{324}{720}$, $\frac{288}{360}$, $\frac{231}{933}$. Ans. $\frac{9}{20}$, $\frac{4}{5}$, $\frac{77}{311}$.

4.	$\frac{1965}{26\,20}$	Ans. $\frac{3}{4}$.	8.	$\frac{139\,15}{200\,30}$	Ans. $\frac{2783}{4006}$.
5.	$\frac{6240}{156\,30}$	Ans. $\frac{208}{521}$.	9.	$\frac{259\,20}{518\,40}$	Ans. $\frac{1}{2}$.
6.	$\frac{224}{490}$	Ans $\frac{16}{35}$.	10.	$\frac{3575}{4719}$	Ans. $\frac{25}{33}$.
7	$\frac{972}{1368}$	Ans. $\frac{27}{38}$.	11.	$\frac{168\,48}{349\,92}$	Ans. $\frac{13}{27}$.

130. *To reduce whole or mixed numbers to improper fractions.*

1. Reduce $5\frac{3}{8}$ to an improper fraction or to *thirds*.

OPERATION.

$5\frac{3}{8}$

$\frac{17}{8}$ Ans.

Explanation.—In all problems of this kind we reason thus: Since there are **3** thirds in every unit or whole number, in 5 units there are **5** times as many, which is 15 + the $\frac{3}{8}$ make $\frac{17}{8}$.

2. Reduce 9 to a fraction whose denominator is 6.

OPERATION.

$9\times 6=\frac{54}{6}$ Ans.

131. *Reduce the following numerical expressions to improper fractions.*

3.	$8\frac{1}{2}$	Ans. $\frac{25}{3}$.	8	$71\frac{1}{2}$	Ans. $\frac{143}{2}$.
4.	$16\frac{1}{2}$	Ans. $\frac{33}{2}$.	9.	$68\frac{3}{7}$	Ans. $\frac{479}{7}$.
5.	$17\frac{3}{4}$	Ans. $\frac{71}{4}$.	10.	$2183\frac{3}{4}$	Ans. $\frac{8735}{4}$.
6.	$32\frac{5}{8}$	Ans. $\frac{261}{8}$.	11.	$23\frac{8}{27}$	Ans. $\frac{629}{27}$.
7.	$435\frac{3}{5}$	Ans. $\frac{2178}{5}$.	12.	$108\frac{1}{105}$	Ans. $\frac{11341}{105}$.

13. Reduce 14 to a fraction whose denominator is 9.
14. " 37 " " " " " " 24.
15. " $54\frac{3}{4}$ " " " " " " 16.

132. *To redu e improper fractions to whole or mixed numbers.*

Reduce $\frac{17}{4}$ to a mixed number.

OPERATION.

$\frac{17}{4}=4\frac{1}{4}$ Ans.

or

$17\div 4=4\frac{1}{4}$ Ans.

Explanation.—In all problems of this kind we reason thus. Since there are 4 *fourths* in 1 unit or whole number, in 17 *fourths* there are as many units as 17 is equal to 4, which is 4 times with 1 remainder, or altogether $4\frac{1}{4}$ as the proper quotient or answer.

Reduce the following improper fractions to whole or mixed numbers.

2.	$\frac{24}{6}$	Ans. 4.	6.	$\frac{29}{8}$	Ans. $3\frac{5}{8}$.
3.	$\frac{47}{9}$	Ans. $5\frac{2}{9}$.	7.	$\frac{72}{13}$	Ans. $5\frac{7}{13}$.
4.	$\frac{144}{3}$	Ans. 48.	8.	$\frac{483}{51}$	Ans. $9\frac{8}{17}$.
5.	$\frac{222}{12}$	Ans. $18\frac{1}{2}$.	9.	$\frac{279104}{809}$	Ans. $344\frac{48}{809}$.

133. *To reduce Compound Fractions to Simple Fractions.*

Reduce $\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{7}{8}$ to a simple fraction.

OPERATION.

$\frac{1}{2}\times\frac{3}{5}\times\frac{7}{8}=\frac{21}{80}$ Ans.

Explanation.—In all problems of this kind we multiply together all the numerators for a new numerator and all the denominator for a new denominator.

When a compound fraction contains whole or mixed numbers they must first be reduced to improper fractions.

When there are common factors in both terms of a compound fraction they should be cancelled before multiplying. By this cancelling the common factors the work is shortened, and the result unchanged for the reason that dividing both terms of a fraction by the same number does not alter its value.

2. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{8}$ to a simple fraction.

OPERATION.

$$\frac{\cancel{2}}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{4}}\times\frac{5}{8}=\frac{5}{16}\text{ Ans.}$$

3. Reduce $\frac{3}{5}$ of $7\frac{1}{2}$ of $\frac{2}{9}$ of 4 of $\frac{1}{12}$ to a simple fraction.

OPERATION.

$$\frac{\cancel{3}}{\cancel{5}}\times\frac{\cancel{15}}{\cancel{2}}\times\frac{\cancel{2}}{\cancel{9}}\times\frac{\cancel{4}}{1}\times\frac{1}{\cancel{12}}=\frac{1}{3}\text{ Ans.}$$

4. Reduce the following compound fractions to simple ones.

5. $\frac{2}{7}$ of $1\frac{4}{16}$ of $\frac{5}{11}$. Ans. $\frac{5}{44}$.
6. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{7}{16}$. Ans. $\frac{7}{48}$.
7. $\frac{4}{5}$ of $3\frac{1}{3}$ of $\frac{1}{2}$. Ans. $1\frac{1}{3}$.
8. $\frac{1}{4}$ of $8\frac{1}{4}$. Ans. $2\frac{1}{16}$.
9. $\frac{9}{16}$ of 96. Ans. 54.
10. $\frac{3}{8}$ of $\frac{1}{13}$ of $17\frac{1}{3}$. Ans. 6.
11. $\frac{2}{3}\times\frac{7}{9}\times\frac{5}{11}$. Ans. $\frac{70}{297}$.
12. $5\times2\times\frac{3}{57}$. Ans. $\frac{10}{19}$.

134. *To reduce fractions of different denominators to equivalent fractions of a common denominator or of the least common denominator.*

135. **Definitions and Principles,** pertaining to this kind of reduction of fractions.

136. A **Common Denominator** is a denominator common to two or more fractions.

137. **The Least Common Denominator** of two or more fractions is the least denominator to which all the fractions can be reduced.

138. A **Common Denominator** of two or more fractions is a common multiple of their denominators; and the *Least Common Denominator* of two or more fractions is the least common multiple of their denominators, for the reason that all higher terms of a fraction are multiples of its corresponding lower or lowest terms.

1. Reduce $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{7}{8}$, to equivalent fractions having a common denominator.

OPERATION.

$\frac{1}{3}$, $\frac{3}{4}$, $\frac{7}{8}$,

$3\times4\times8=96$ common denominator

$\frac{1}{3}$ of 96 $=32$ hence $\frac{32}{96}$, equivalent of $\frac{1}{3}$

$\frac{3}{4}$ of 96 $=72$ hence $\frac{72}{96}$, equivalent of $\frac{3}{4}$

$\frac{7}{8}$ of 96 $=84$ hence $\frac{84}{96}$, equivalent of $\frac{7}{8}$

Explanation.—In all problems of this kind we obtain the *common denominator* by multiplying together the denominators of all the fractions Then to find the respective numerators we take such a part of the

common denominator as the respective fractions are parts of a unit, as shown in the operation.

Reduce the following fractions to equivalent fractions, having a common denominator.

2. $\frac{3}{5}$, $\frac{2}{3}$, and $\frac{5}{7}$. Ans. $\frac{63}{105}$, $\frac{70}{105}$, and $\frac{75}{105}$.
3. $\frac{9}{10}$, $\frac{1}{4}$, and $\frac{8}{12}$. Ans. $\frac{216}{240}$, $\frac{120}{240}$, and $\frac{160}{240}$.
4. $\frac{8}{15}$, and $\frac{27}{32}$. Ans. $\frac{256}{480}$, and $\frac{405}{480}$.
5. $\frac{1}{2}$, $\frac{5}{16}$, $\frac{9}{10}$, $\frac{11}{12}$, and $\frac{1}{6}$.
Ans. $\frac{115200}{230400}$, $\frac{72000}{230400}$, $\frac{207360}{230400}$, $\frac{211120}{230400}$ and $\frac{38400}{230400}$.
6. $\frac{8}{17}$, $\frac{2}{3}$, $\frac{1}{4}$, and $3\frac{1}{6}$.
Ans. $\frac{576}{1224}$, $\frac{816}{1224}$, $\frac{306}{1224}$ and $\frac{3876}{1224}$.

7. Reduce $\frac{1}{3}$, $\frac{3}{4}$ and $\frac{7}{8}$ to equivalent fractions having the least common denominator.

OPERATION.

$$\begin{array}{r|l} 2 & 3.\ 4.\ 8 \\ \hline 2 & 3.\ 2.\ 4 \\ \hline & 3\ \ 1\ \ 2 \end{array}$$

$2\times2\times3\times2=24$ Least Common Denominator.

$\frac{1}{3}$ of $\frac{24}{24}=8$ hence $\frac{8}{24}$ is the equivalent of $\frac{1}{3}$.
$\frac{3}{4}$ of $\frac{24}{24}=18$ hence $\frac{18}{24}$ is the equivalent of $\frac{3}{4}$.
$\frac{7}{8}$ of $\frac{24}{24}=21$ hence $\frac{21}{24}$ is the equivalent of $\frac{7}{8}$.

Explanation.—In all problems of this kind we first find the *least Common Multiple* of the denominators of all the fractions as explained in article 128 page 88 which is the *least common denominator*. Then, having the least common denominator to find the respective numerators we take such a part of the least common denominator as the respective fractions are parts of a unit, as shown in the operation.

Reduce the following fractions to equivalent fractions having a least common denominator.

8. $\frac{2}{3}$, $\frac{5}{6}$ and $\frac{7}{8}$. Ans. $\frac{16}{24}$, $\frac{20}{24}$ and $\frac{21}{24}$.
9. $\frac{5}{12}$, $\frac{1}{4}$ and $\frac{9}{16}$. Ans. $\frac{20}{48}$, $\frac{12}{48}$ and $\frac{27}{48}$.
10. $\frac{8}{15}$, $\frac{14}{45}$ and $\frac{21}{60}$. Ans. $\frac{96}{180}$, $\frac{56}{180}$ and $\frac{63}{180}$.
11. $\frac{1}{10}$, $\frac{30}{65}$, $\frac{19}{52}$ and $\frac{3}{26}$. Ans. $\frac{26}{260}$, $\frac{120}{260}$, $\frac{95}{260}$ and $\frac{30}{260}$.
12. $5\frac{1}{2}$, $\frac{3}{4}$, $1\frac{5}{16}$, and $\frac{1}{3}$. Ans. $\frac{264}{48}$, $\frac{36}{48}$, $\frac{63}{48}$ and $\frac{16}{48}$.
13. $\frac{16}{21}$, 8, $\frac{20}{63}$ and 1. Ans. $\frac{48}{63}$, $\frac{504}{63}$, $\frac{20}{63}$ and $\frac{63}{63}$.

14. $3\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{3}$ and $\frac{5}{6}$. Ans. $\frac{42}{12}$, $\frac{9}{12}$, $\frac{8}{12}$ and $\frac{10}{12}$.

15. $\frac{80}{111}$, 3, $\frac{1}{4}$ and $\frac{15}{16}$.

Ans. $\frac{1280}{1776}$, $\frac{5328}{1776}$, $\frac{444}{1776}$ and $\frac{1665}{1776}$.

DENOMINATE FRACTIONS.

139. **A Denominate Fraction** is one whose unit is denominate. Thus $\frac{3}{4}$ of a yard is a denominate fraction.

140. *To reduce a denominate fraction from a greater unit to a less.*

1. Reduce $\frac{3}{4}$ of a yard to inches.

OPERATION.

4 | 3
3
12 3

27 inches. Ans.

Explanation.—In all problems of this kind we multiply the fraction by the units of the scale to which it belongs until we reach the unit required.

In this example we multiply the $\frac{3}{4}$ yard by 3 to reduce it to feet, then by 12 to reduce it to inches, the unit required.

2. In $\frac{7}{8}$ of a ton how many ounces? Ans. 28000.

141. *To reduce a denominate fraction from a less unit to a greater.*

1. Reduce $\frac{2}{3}$ of a pound to a fraction of a ton.

OPERATION.

3 | 2
25
2 4
20

$\frac{1}{3000}$ ton Ans.

Explanation.—In all problems of this kind we divide the fraction by the units of the scale to which it belongs until we reach the unit required. In this example we divide the $\frac{2}{3}$ pound by 25 to reduce it to quarters; then by 4 to reduce it to hundredweights; and then by 20 to reduce it to tons, the unit required.

2. Reduce $\frac{3}{8}$ of a penny to a fraction of a pound sterling. Ans. $\frac{1}{640}$.

ADDITION OF FRACTIONS.

142. Addition of fractions is the process of adding two or more fractional numbers of the same kind, or of the same denomination.

(1.) Add $\frac{1}{2}$ $\frac{3}{8}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{5}{6}$ and $\frac{7}{8}$ together.

OPERATION.

$$\frac{1\ 2\ 3\ 4\ 5\ 7}{2\ 3\ 4\ 5\ 6\ 8}$$

Whole numbers	Fractions obtained in adding
1	
1	$\frac{1\ 1\ 3\ 5\ 17}{4\ 2\ 4\ 8\ 40}$
1	
1	
$4\frac{17}{40}$ Ans.	

Explanation. In this example there are no two fractions alike, hence they cannot be added until we shall have reduced them to fractions of the same kind. To facilitate and simplify the operation, we here reduce and add but two fractions at a time, and we first select such two as may be the most easily reduced and added. Accordingly, by inspection, we select the $\frac{1}{2}$ and $\frac{3}{4}$ as the fractions to first reduce and add; and by the exercise of our reason we see that $\frac{1}{2}$ is equal to $\frac{2}{4}$ which, added to the $\frac{3}{4}$, make $\frac{5}{4}$, which, for the reason that $\frac{4}{4}$ make 1 is equal to 1 and $\frac{1}{4}$. We set the 1 in the column of whole numbers, and the $\frac{1}{4}$ in the column of fractions. We then cancel the $\frac{1}{2}$ and $\frac{3}{4}$, and select the $\frac{3}{8}$ and $\frac{5}{6}$ as the next two fractions to reduce and add. Again, we see, reasoning analytically, that $\frac{1}{3}$ is equal to $\frac{2}{6}$ and $\frac{3}{8}$ are equal to $\frac{4}{6}$, which, added to the $\frac{5}{6}$, make $\frac{9}{6}$, which is equal to 1 and $\frac{3}{6}$, which, reduced, equals $\frac{1}{2}$. The 1 we set in the column of whole numbers, and the $\frac{1}{2}$ in the column of fractions, and cancel the $\frac{3}{8}$ and $\frac{5}{6}$. We next add the $\frac{1}{4}$ and $\frac{1}{2}$, and by our reason we see that $\frac{1}{2}$ is equal to $\frac{2}{4}$, which, added to the $\frac{1}{4}$, make $\frac{3}{4}$, which we set in the column of fractions, and then cancel the $\frac{1}{4}$ and $\frac{1}{2}$. We then select the $\frac{3}{4}$ and $\frac{7}{8}$ as the next two fractions to add, and reducing the $\frac{3}{4}$ to 8ths, we see by our reason that $\frac{1}{4}$ is equal to $\frac{2}{8}$ and $\frac{3}{4}$ are equal to 3 times as many, which is $\frac{6}{8}$, which added to the $\frac{7}{8}$ make $\frac{13}{8}$ which is equal to 1 and $\frac{5}{8}$; we set the 1 in the column of whole numbers and the $\frac{5}{8}$ in the column of fractions, and cancel the $\frac{3}{4}$ and $\frac{7}{8}$. We then proceed to reduce and add the two remaining fractions $\frac{4}{5}$ and $\frac{5}{8}$. By inspection, the exercise of our reasoning faculties, and the use of our knowledge of the principles of numbers as contained in the preceding work, we see that the $\frac{4}{5}$ and $\frac{5}{8}$ are not only not alike, but that we can neither reduce the $\frac{4}{5}$ to 8ths nor the $\frac{5}{8}$ to 5ths, and, therefore, before we can add them we must reduce both the $\frac{4}{5}$ and $\frac{5}{8}$ to equivalent fractions of the same kind, or whose denominators are alike. To do this, we first observe that the denominators are not divisible by the same number, greater than 1, and hence the product of them, 40

is the least number that both of the fractions are reducible to, or, in other words their product 40 is the least common denominator of the two fractions. Having this, we next reduce the $\frac{4}{5}$ and $\frac{5}{8}$ to 40ths, and by our reason we see that $\frac{1}{5}$ is equal to $\frac{8}{40}$ and $\frac{4}{5}$ are equal to 4 times as many, which is $\frac{32}{40}$; then that $\frac{1}{8}$ is equal to $\frac{5}{40}$, and $\frac{5}{8}$ are equal to 5 times as many, which is $\frac{25}{40}$, w ich added to the $\frac{32}{40}$ make $\frac{57}{40}$, which for the reason that $\frac{40}{40}$ make a whole one, is equal to 1 and $\frac{17}{40}$, which we place in their respective columns and cancel the $\frac{4}{5}$ and $\frac{7}{8}$. The operation of adding the fractions is now completed, and by adding the whole numbers and annexing the remaining fractions, we have as the correct result $4\frac{17}{40}$.

The foregoing problem illustrates the most rational, easy and rapid system of adding fractions known, and as fractions are so indispensable and of so frequent occurrence in practical life, the principles involved in the system should be thoroughly understood.

In practical work, we would very much shorten the operation by adding several fractions at once, and mentally performing the most, if not all of the reduction and addition work, without stating the results. Thus, in the above problem, we would add the $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{7}{8}$ at once. We can instantly see that their sum is $1\frac{7}{8}$ or $2\frac{1}{8}$, and without naming or setting the $2\frac{1}{8}$, we add to it mentally the result of $\frac{2}{3}$ and $\frac{5}{6}$, which we mentally see is $\frac{9}{6}$ or $1\frac{1}{2}$, making $3\frac{5}{8}$, which are the only figures we set. Thus all the fractions, except $\frac{4}{5}$ and $\frac{5}{8}$, are added at one mental operation. Then we mentally add the sum of $\frac{4}{5}$ and $\frac{5}{8}$ by the same process of reasoning as given in the illustration of the above example, and obtain the correct result $4\frac{17}{40}$.

(2.) Add $\frac{3}{4}$, $\frac{7}{9}$ and $\frac{8}{11}$ together.

OPERATION.

Whole numbers.		Fractions obtained by adding.
	$\frac{3}{4}$ $\frac{7}{9}$ $\frac{8}{11}$	$\frac{19}{\quad}$
	$\frac{4}{4}$ $\frac{9}{9}$ $\frac{11}{11}$	$\frac{36}{\quad}$
1	27 28 288	269
1	$\frac{55}{36}$	$\frac{497}{396}$
$2\frac{101}{396}$	Ans.	

Explanation. By inspection and reason we see that there are no two fractions alike, and that we cannot reduce either of them to an equivalent denomination of any other; therefore we select the smallest two, $\frac{3}{4}$ and $\frac{7}{9}$, and reduce them to equivalent fractions of the same kind, or of the same denominator, which we find, by multiplying the denominators together, to be 36.

We now find by reasoning as in the preceding example, that $\frac{3}{4}$ and $\frac{7}{9}$ are equal respectively to $\frac{27}{36}$, and $\frac{28}{36}$, and collectively to $\frac{55}{36}$, which is equal to 1 and $\frac{19}{36}$. Then proceeding as in the first case, we add the $\frac{8}{11}$ and $\frac{19}{36}$.

(3.) Add $\frac{1}{2}$ of $\frac{2}{3}$ of $2\frac{1}{4}$, $\frac{3}{8}$ $\frac{5}{6}$ and $\frac{9}{23}$ together.

OPERATION.

Statement showing the reduction of the fractions.

$$\begin{array}{cccccc} & & 3 & & & \\ 1 & 2 & 9 & 3 & 5 & 9 \\ -\times- & -\times- & - & - & - & — \\ 2 & 3 & 4 & 8 & 6 & 23 \end{array}$$

Statement showing the result of the reduction and the addition of fractions.

Whole numbers.					Fractions obtained by adding.	
	3	3	5	9	1	23
	—	—	—	—	—	—
	4	8	6	23	8	24
1		20	216		3	529
1		$\frac{28}{24}$			$\frac{745}{552}$	
$2\frac{1923}{55\,2}$ Ans.						

Explanation. Here we have compound fractions and mixed numbers, and before adding, we reduce the mixed numbers to improper fractions, and the compound fractions to simple ones. Then we add the $\frac{3}{4}$ and $\frac{3}{8}$, which are equal to 1 and $\frac{1}{8}$; then the $\frac{5}{6}$ and $\frac{1}{8}$, which are equal to $\frac{23}{24}$; then the $\frac{23}{24}$ and $\frac{9}{23}$, which are equal to 1 and $\frac{19}{552}$. Then adding the whole numbers, and annexing the fractions, we have $2\frac{1923}{552}$ as the correct result.

EXAMPLES.

(4.) Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{9}{10}$. Ans. $4\frac{21}{40}$

(5.) Add $\frac{5}{8}$, $\frac{7}{9}$ and $\frac{13}{20}$. Ans. $2\frac{19}{360}$

(6.) Add $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{9}{16}$. Ans. $2\frac{7}{16}$

(7.) Add $\frac{7}{8}$, $\frac{11}{16}$, $\frac{23}{32}$ and $\frac{52}{64}$. Ans. $3\frac{6}{64}$

(8.) Add $\frac{3}{5}$, $\frac{9}{10}$ $\frac{14}{15}$ and $\frac{17}{20}$. Ans. $3\frac{17}{60}$

(9.) $\frac{3}{7}$ of $\frac{14}{23}$ and $2\frac{1}{2}$ of $\frac{3}{8}$ of $\frac{4}{15}$. Ans. $\frac{47}{92}$

(10.) Add $2\frac{1}{2}$, $6\frac{3}{4}$, $5\frac{5}{8}$ and $2\frac{1}{8}$. Ans. 17

(11.) $\frac{2}{3}$, $\frac{5}{6}$ and $\frac{7}{12}$. Ans. $2\frac{1}{12}$.

(12.) $\frac{3}{5}$, $\frac{2}{7}$ and $\frac{1}{3}$. Ans. $1\frac{23}{105}$.

(13.) $1\frac{1}{10}$, $6\frac{2}{5}$, $18\frac{19}{20}$ and $2\frac{7}{30}$. Ans. $28\frac{41}{60}$.

(14.) $\frac{7}{8}$, $\frac{9}{20}$ and $\frac{1}{25}$. Ans. $1\frac{161}{200}$.

(15.) $3\frac{1}{2}$, $1\frac{1}{3}$, $2\frac{1}{4}$ and $4\frac{5}{12}$. Ans. $11\frac{1}{2}$.

(16.) $\frac{13}{18}$, $\frac{8}{15}$, $\frac{11}{20}$ and $\frac{13}{30}$. Ans. $2\frac{43}{180}$.

(17.) $2\frac{1}{2}$, $3\frac{1}{3}$, $4\frac{1}{4}$ and 5. Ans. $15\frac{1}{12}$.

(18.) $\frac{3}{4}$, $\frac{1}{8}$, $\frac{2}{3}$ and $\frac{5}{12}$. Ans. $1\frac{97}{168}$.

(19.) $7\frac{1}{2}$, $5\frac{7}{8}$ and $10\frac{3}{4}$. Ans. $23\frac{11}{24}$.

(20.) $1\frac{4}{45}$, $\frac{27}{39}$ and $\frac{2}{21}$. Ans. $1\frac{404}{4095}$.

(21.) $1\frac{14}{15}$, $3\frac{9}{10}$, $1\frac{3}{5}$ and $\frac{19}{20}$. Ans. $21\frac{19}{60}$.

(22.) $\frac{7}{8}$, $1\frac{7}{12}$, $10\frac{5}{6}$ and 5. Ans. $18\frac{7}{24}$.

(23.) $125\frac{4}{7}$, $327\frac{5}{12}$ and $25\frac{1}{4}$. Ans. $478\frac{5}{21}$.

(24.) $\frac{140}{320}$, $\frac{57}{80}$, $1\frac{1}{10}$, $\frac{19}{20}$ and $\frac{1050}{1600}$. Ans. $3\frac{137}{160}$.

(25.) What is the weight of 10 sacks of wheat which weigh respectively, $154\frac{1}{2}$, $149\frac{1}{4}$, $160\frac{3}{4}$, $157\frac{3}{8}$, $152\frac{1}{2}$, $141\frac{5}{8}$, $163\frac{1}{4}$, $158\frac{1}{2}$, $139\frac{1}{4}$ and $161\frac{3}{4}$ pounds? Ans. $1539\frac{1}{2}$ lbs.

(26.) Add $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{4}{5}$ and $2\frac{1}{3}$ of $\frac{9}{14}$ of 1. Ans. $1\frac{4}{5}$.

(27.) Add $\frac{3}{5}$ of $\frac{8}{9}$ of 4 and $\frac{2}{3}$ of $\frac{1}{2}$ of $\frac{5}{2}$. Ans. $2\frac{29}{30}$.

(28.) How many yards in 8 bolts of domestic, measuring as follows: $40\frac{3}{4}$, $39\frac{1}{2}$, $43\frac{1}{4}$, $42\frac{1}{8}$, $43\frac{5}{8}$, $38\frac{1}{2}$, $39\frac{9}{16}$ and $41\frac{1}{2}$ yards? Ans. $328\frac{13}{16}$.

(29.) 14 bags of Coffee weigh as follows: $162\frac{7}{16}$, $163\frac{5}{8}$, $161\frac{5}{16}$, $164\frac{1}{2}$, $165\frac{1}{8}$, $164\frac{3}{4}$, $165\frac{1}{2}$, $162\frac{3}{4}$, $165\frac{3}{16}$, $164\frac{5}{8}$, $165\frac{3}{4}$, $165\frac{1}{4}$, $164\frac{11}{16}$ and $165\frac{1}{2}$ pounds; how many pounds in all? Ans. 2301.

(30.) A merchant bought $1153\frac{1}{2}$ pounds of rice for $\$92\frac{1}{4}$; $871\frac{1}{4}$ pounds of sugar for $\$87\frac{1}{2}$; $580\frac{3}{8}$ pounds of coffee for $\$115\frac{7}{8}$; $240\frac{1}{4}$ pounds of cheese for $\$43\frac{1}{2}$; and $408\frac{3}{4}$ pounds of graham flour for $\$18\frac{3}{8}$. What was the total number of pounds, and the total cost of all he purchased? Ans. $3254\frac{5}{8}$ pounds, $\$357\frac{1}{8}$ cost.

(31.) Add $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{10}$, and $3\frac{3}{4}$ of $\frac{7}{15}$ of $1\frac{1}{2}$. Ans. $5\frac{59}{60}$.

(32.) Add $\frac{5}{7}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{11}{13}$, $1\frac{1}{5}$ and $\frac{1}{2}$ of 3. Ans. $5\frac{5883}{13065}$.

SUBTRACTION OF FRACTIONS.

114. Subtraction of fractions is the operation in numbers of finding the difference between two fractional numbers that are of the same denomination; it is hence the converse of addition.

(1.) What is the difference between $\frac{3}{4}$ and $\frac{4}{5}$? Ans. $\frac{1}{20}$

OPERATION

$\frac{3}{4} = 15$ | or $\frac{3}{4}$ $\frac{4}{5}$
$\frac{4}{5} = 16$ | 15 16
$\frac{1}{20}$ Ans. | $\frac{1}{20}$ Ans

Explanation Here we see that the denominations are not the same, and therefore, before we can subtract, we must reduce the fractions to a common denominator. By inspection and in accordance with the principles as explained in the first problem of addition, we see that the least common denominator is 20; then, that $\frac{3}{4}$ are equal to $\frac{15}{20}$ and that $\frac{4}{5}$ are equal to $\frac{16}{20}$, and that the difference is $\frac{1}{20}$.

(2.) From $28\frac{5}{8}$ take $7\frac{1}{2}$. Ans. $21\frac{1}{8}$.

OPERATION.

$28\frac{5}{8}$
$7\frac{1}{2}$
$21\frac{1}{8}$ Ans.

Explanation In performing the operation of the question before us, we first observe that the fractions which constitute a part of the numbers to be substracted are not of the same kind or denomination, and hence, before we can perform the work, we must reduce them to equivalent fractions. We next observe that the $\frac{1}{2}$ may be reduced to 8ths, and by the exercise of our reason we see that it is equal to $\frac{4}{8}$, which taken from $\frac{5}{8}$ leaves $\frac{1}{8}$; this completes the work with the fractions, and we have but to find the difference between the whole numbers as in simple subtraction.

(3.) What is the difference between $37\frac{3}{8}$ and $12\frac{7}{9}$? Ans. $24\frac{43}{72}$.

OPERATION.

$37\frac{3}{8} = 27$ } 72 } 99
$12\frac{7}{9} = 56$
$24\frac{43}{72}$ Ans.

Explanation. By inspection we here see that the fractions belonging to the whole numbers are not of the same denomination, and that neither can be reduced to an equivalent fraction of the same term

as the other, and therefore we must reduce both to equivalent fractions having a common denominator, before we can subtract; and by the exercise of our reason we see that 72 is the smallest number to which both can be reduced, which, for convenience, we set below the fractions, and by the same reasoning as given in the preceding examples we see that $\frac{7}{9}$ are equal to $\frac{56}{72}$ and that $\frac{3}{8}$ are equal to $\frac{27}{72}$, which, for convenience, we carry to the right of the respective fractions, and to economise time we set only the numerators. We now observe that the upper fraction, belonging to the greater number, is less than the lower fraction, belonging to the lesser number. Therefore, before we can subtract the fractions, we must add 1, reduced to 72ds, to $\frac{27}{72}$, which gives us $\frac{28}{72}$. We now take $\frac{56}{72}$ from $\frac{28}{72}$ and have a remainder of $\frac{43}{72}$ as the fractional part of our answer. We now add 1 to the subtrahend, because we previously added 1 to the minuend, making it 13, which we subtract from 37 and have a remainder of 24, which we write below the line and complete the operation.

EXAMPLES.

4. What is the difference between $\frac{11}{16}$ and $\frac{3}{8}$? Ans. $\frac{5}{16}$.
5. What is the difference between $\frac{3}{5}$ and $\frac{7}{9}$? Ans. $\frac{8}{45}$.
6. What is the difference between $5\frac{1}{2}$ and $3\frac{1}{8}$? Ans. $2\frac{3}{8}$.
7. What is the difference between 7 and $3\frac{1}{16}$? Ans. $3\frac{15}{16}$.
8. What is the difference between $23\frac{3}{7}$ and 14? Ans. $9\frac{3}{7}$.

What is the difference between the following numbers.

9. $\frac{5}{7}$ and $\frac{4}{9}$. Ans. $\frac{17}{63}$.
10. $\frac{15}{24}$ and $\frac{2}{5}$. Ans. $\frac{9}{40}$.
11. $\frac{6}{8}$ and $\frac{3}{10}$. Ans. $\frac{9}{20}$.
12. $\frac{4}{17}$ and $\frac{3}{8}$. Ans. $\frac{19}{136}$.
13. $\frac{11}{12}$ and $\frac{5}{7}$. Ans. $\frac{17}{84}$.
14. $\frac{4}{35}$ and $\frac{84}{120}$. Ans. $\frac{41}{70}$.
15. $2\frac{3}{5}$ and $1\frac{1}{2}$. Ans. $1\frac{1}{10}$.
16. $9\frac{1}{3}$ and $2\frac{3}{4}$. Ans. $6\frac{7}{12}$.
17. $\frac{1}{2}$ of $\frac{4}{5}$ and $\frac{1}{6}$ of $\frac{3}{4}$. Ans. $\frac{11}{40}$.
18. $8\frac{1}{2}$ and $3\frac{7}{9}$. Ans. $4\frac{13}{18}$.
19. $12\frac{5}{9}$ and $9\frac{2}{7}$. Ans. $2\frac{53}{63}$.
20. $25\frac{3}{6}$ and $9\frac{7}{10}$. Ans. $16\frac{7}{15}$.
21. 9 and $3\frac{4}{25}$. Ans. $5\frac{21}{25}$.
22. $4\frac{2}{15}$ and $3\frac{7}{15}$. Ans. $3\frac{4}{15}$.
23. $7\frac{1}{18}$ and 4. Ans $3\frac{1}{18}$.

24. $31\frac{1}{2}$ and $17\frac{5}{8}$. Ans. $13\frac{7}{8}$.

25. From $6\frac{1}{2}$ of $\frac{4}{13}+13\frac{5}{6}$ take $\frac{1}{3}$ of $\frac{3}{4}$ of $15-\frac{2}{5}$ of $\frac{7}{2}$. Ans. $13\frac{29}{60}$.

26. From $8\frac{1}{2}+6\frac{3}{5}-\frac{9}{16}$ take $\frac{1}{2}$ of $\frac{2}{3}$ of 3 of $1\frac{3}{5}+2\frac{1}{5}$. Ans. $10\frac{91}{240}$.

27. From $17\frac{3}{4}+\frac{5}{8}$ take $6\frac{3}{8}-\frac{3}{4}$. Ans. $12\frac{11}{24}$.

28. G. K. Shotwell had $\$38\frac{3}{4}$; he gave $\$2\frac{1}{2}$ for a pair of Indian clubs, $\$5\frac{3}{4}$ for books, $\$1\frac{1}{4}$ for a drawing board, and $\$\frac{5}{8}$ for ink and pencils. How much had he left?
Ans. $\$28\frac{5}{8}$.

29. W. A. Weaver had $\$7\frac{1}{4}$ and his friend gave him $\$\frac{1}{2}$ more; A. Denis had $\$16\frac{1}{2}$ and he spent $\$5\frac{7}{8}$; How much more has A. Denis than W. A. Weaver? Ans. $\$2\frac{7}{8}$.

30. E. Schwartz bought 2 bags of coffee each weighing $163\frac{1}{2}$ pounds; he sold $27\frac{1}{4}$ pounds, $50\frac{3}{8}$ pounds, $87\frac{1}{4}$ pounds and $45\frac{3}{4}$ pounds; how many pounds has he left?
Ans. $116\frac{3}{8}$

31. S. Myers bought $75\frac{1}{2}$ gallons of molasses; he used $4\frac{1}{4}$, lost by leakage $2\frac{3}{8}$, and sold $22\frac{3}{4}$ gallons. How much has he left? Ans. $46\frac{1}{8}$ gallons.

32. What is the difference between a dozen times 6, plus $6\frac{1}{2}$, and 6 times a dozen minus one dozen and a half?
Ans. $24\frac{1}{2}$.

33. J. Farrell bought 6 chests of tea weighing $38\frac{1}{2}$, $42\frac{3}{4}$, $41\frac{5}{8}$, $44\frac{1}{4}$, $39\frac{1}{2}$ and $43\frac{3}{4}$ pounds; he sold $120\frac{7}{8}$ pounds and used $5\frac{3}{4}$ pounds. How many pounds has he on hand?
Ans. $123\frac{3}{4}$.

34. J. Barba owned the Steamer Isabel, he sold $\frac{3}{8}$; what is $\frac{1}{2}$ of his present interest? Ans. $\frac{5}{16}$.

35. From the sum of $6\frac{1}{4}$ and $8\frac{5}{8}$ take the difference of $14\frac{3}{7}$ and $9\frac{4}{5}$. Ans. $10\frac{69}{280}$.

36. What number is that to which if $16\frac{1}{2}$ be added the sum will be $44\frac{3}{8}$? Ans. $27\frac{5}{8}$.

37. C. E. McNeil bought $\frac{1}{2}$ of $\frac{2}{3}$ of a vessel and sold $\frac{2}{3}$ of $\frac{3}{4}$ of his share. How much of the whole vessel has he left? Ans. $\frac{1}{6}$.

38. E. Brinkman bought a barrel of molasses containing $41\frac{1}{4}$ gallons; he sold $9\frac{1}{2}$ gallons; how many gallons remain in the barrel? Ans. $31\frac{3}{4}$ gallons.

39. C. G. De Russy bought two sacks of coffee weighing respectively $161\frac{1}{2}$ and $163\frac{3}{4}$ pounds. He sold to J. Walter $186\frac{1}{2}$ pounds; how many has he left?

Ans. $138\frac{3}{4}$ pounds.

40. A. Buchanan sold to J. Bruns $\frac{1}{2}$ of $\frac{5}{8}$ of his plantation, what part has he left? Ans. $\frac{11}{16}$.

41. What is the difference between $\frac{1}{2}$ of $\frac{1}{2}$ plus $\frac{2}{3}$ and $\frac{2}{3}$ of $\frac{1}{2}$ plus $\frac{1}{2}$? Ans. $\frac{1}{12}$.

42. J. J. Schonekas and M. Shlenker were each $\frac{1}{2}$ owner of a broom and brush factory. Schonekas sold $\frac{1}{2}$ of his interest to E. F. Meyer, and then $\frac{1}{2}$ of his remaining interest to M. Shlenker who subsequently sold $\frac{1}{2}$ of $\frac{3}{4}$ of his whole interest to F. Kranz. What is the present interest of each owner? Ans. J. J. Shonekas $\frac{1}{8}$; E. F. Meyer $\frac{1}{4}$; M. Shlenker $\frac{25}{64}$ and F. Kranz $\frac{15}{64}$.

43. J. J. Manson owned $\frac{3}{4}$ of the Steamer Natchez. He sold to G. Lindsey $\frac{1}{4}$ interest in the Steamer, and to W. S. Keaghey $\frac{1}{4}$ of his remaining interest. What is the present interest of each in the boat? Ans. Manson $\frac{3}{8}$; Lindsey $\frac{1}{4}$ and Keaghey $\frac{1}{8}$.

MULTIPLICATION OF FRACTIONS.

115. Multiplication of fractions is the process of multiplying when one or both of the factors contain fractional numbers.

In the multiplication of simple numbers we saw that the result of multiplication operations was increasing, but in the multiplication of fractions, when the multiplier is less than a unit, the result is decreasing. This is evident from the fact that multiplication is the process of repeating the multiplicand as many times as there are units in the multiplier, and therefore, when the multiplier is less than a unit, the multiplicand will be repeated only a part of a time, or such a part of itself as the multiplier is part of a unit.

To elucidate the principles of the subject and render clear the reasoning we present our first questions in practical language; and to aid still farther in comprehending the work, we give the following *practical* definition of multiplication.

116. MULTIPLICATION is that operation in the practical business computation of numbers of finding the cost of either a part of one, or of many pounds, yards, barrels, etc., when we have the cost of one pound, yard, barrel, etc. On the principle or fact embraced in this definition, we *found* our reasoning for the solution of every question that can possibly be presented in multiplication, either of simple numbers or of fractions.

Considering the foregoing, we see that in all multiplication questions of a practical nature, we must necessarily *reason from one, or unity, to a part of one or many.* Thus, if 1 pound cost 50 ¢, $\frac{1}{4}$ of a pound will cost $\frac{1}{4}$th part of it; and if 1 yard costs \$2, 3 yards will cost *three* times as much, or 3 times \$2.

In the solution of abstract questions we apply the same system of reasoning without naming the factors, and thereby avoid all of the arbitrary rules given in the Arithmetics of the day

(1.) What will $41\frac{3}{4}$ pounds of coffee cost at $21\frac{1}{2}¢$ per pound? Ans. $\$8.97\frac{5}{8}$

OPERATION.

¢	
2	43
4	167
–	———
8	7181
	———
	$\$8.97\frac{5}{8}$ Ans.

Explanation. In this example by inspection and the exercise of our reason, we see that we have in the solution both increasing and decreasing work to perform, and hence to facilitate the operation of our work, we use a perpendicular or *statement line*, on the right hand side of which we place all increasing numbers, and on the left hand side all decreasing numbers. But, be it remembered, we never place a number on either side without giving a reason therefor, and in commencing the solution statement of any problem we always place at the top right hand side the number representing the article or thing to be increased or decreased, or that which the conditions of the question require the answer to be in.

By further inspection and reasoning we see that $21\frac{1}{2}¢$ are to be increased $41\frac{3}{4}$ times, and hence we will place the same at the top and on the increasing side of our statement line; but, before doing so, in order to facilitate the work, we first reduce the $21\frac{1}{2}$ to *half* cents, which equals $\frac{43}{2}$ cents, the denominator of which we place on the decreasing side and the numerator on the increasing side of the statement line. We then reason as follows: since 1 pound cost $\frac{43}{2}$ cents, $\frac{1}{4}$ of a pound will cost $\frac{1}{4}$ part of it, and as this conclusion is a decreasing one, we write the 4 on the decreasing side; then, since $\frac{1}{4}$ costs the result of the statement thus far made, $\frac{167}{4}$, $41\frac{3}{4}$ reduced, will cost 167 times as much, which, because the conclusion is an increasing one, we write on the increasing side, and thus complete the reason and statement. It may be asked how we know that if 1 pound costs $\frac{43}{2}¢$, $\frac{1}{4}$ of a pound will cost $\frac{1}{4}$ part of it, and that $\frac{167}{4}$ will cost 167 times as much. We answer, by the exercise of our reasoning faculties, our common sense, our judgment, which is the only way that mortal man knows anything.

In working out the statement, there being no common factors in the increasing and decreasing numbers that can be cancelled, we have but to multiply the increasing numbers together, which produce 7181, and the decreasing numbers together, which produce 8; then we divide the 7181 by 8 and obtain $\$8.97\frac{5}{8}$ as the result of the reasoning and operation.

In all simple statements the result is always of the same kind or character as represented by the number first placed on the statement line.

Multiply $\frac{4}{5}$ by $\frac{5}{8}$, or to express the problem in practical language:

(2.) What will $\frac{5}{8}$ of a yard cost at $\frac{4}{5}$ of a dollar per yard? Ans. $\$\frac{1}{2}$.

OPERATION.

$$\begin{array}{c|c} & \$. \\ \cancel{5} & \cancel{4} \\ \cancel{8} & \cancel{5} \\ 2 & \\ - & - \\ & \$\ \frac{1}{2}\ \text{Ans.} \end{array}$$

Explanation. As explained in the above example, by inspection and reason we see that the $\frac{4}{5}$ of a dollar is the number to be multiplied, and also the number representing the nature of the answer; hence we first place the same on our statement line, and then reason as follows: if, or since, 1 yard costs $\frac{4}{5}$ of a dollar, $\frac{1}{8}$ of a yard will cost $\frac{4}{5}$ part of it, and $\frac{5}{8}$ will cost 5 times as much as $\frac{1}{8}$. The 8 and 5 are placed respectively on the decreasing and increasing sides of the statement line, because the reasoning, when they were respectively used, was decreasing and increasing.

In working out the statement, we first cancel the 5's, and then the 8 by the 4, and thus obtain $\frac{1}{2}$ a dollar as the correct result.

The reasoning and operation of the foregoing problems will solve every question that can be presented in multiplication of fractions.

(3.) What will $58\frac{1}{2}$ pounds cost at $16\frac{2}{3}¢$ per pound?

OPERATION.

$$\begin{array}{c|c} & ¢ \\ \cancel{3} & \cancel{50}\ 25 \\ 2 & 117\ 39 \\ - & - \\ & \$9.75\ \text{Ans.} \end{array}$$

Explanation. The reasoning for the solution of this problem is the same as that given in the first example; hence we will very much abridge our explanation. We first reduce and place on the line the $16\frac{2}{3}¢$; then having the cost of 1 pound, we see by our reason that $\frac{1}{2}$ a pound will cost $\frac{1}{2}$ as much, and $1\frac{1}{2}^7$ pounds will cost 117 times as much as $\frac{1}{2}$.

This completes the statement, and in working the same we first cancel the 50 by 2, then the 117 by the 3. This cancels all of the decreasing figures, and we have but to multiply the 25 by 39 and produce the answer, $9.75.

(4.) What will $3\frac{3}{4}$ dozen cost at $\$3\frac{2}{5}$ per dozen?

OPERATION.

	$
5	17
4	15 3
–	———
	51
	———
	$12\frac{3}{4}$ Ans.

Explanation For reasons above given, we reduce and place the $\$3\frac{2}{5}$ on the line; then we see that since 1 dozen costs $\$\frac{1}{5}$, $\frac{1}{4}$ of a dozen will cost $\frac{1}{4}$ part of it, and $\frac{15}{4}$ will cost 15 times as much.

(5.) What will $5\frac{3}{8}$ bushels cost at $15\frac{1}{2}¢$ per pint?

OPERATION.

	¢
2	31
	2
	8
	4=124
8	43
–	———
	$53.32 Ans.

Explanation. By inspection and reason, we see that the $15\frac{1}{2}¢$ is the number to be increased; hence we reduce and place the same on the line and proceed to reason as follows: if 1 pint costs $\frac{31}{2}¢$, 2 pints or a quart will cost 2 times as much, and if 1 quart costs the result of the statement now made, 8 quarts or a peck will cost 8 times as much, and if a peck costs the result of this statement, that 4 pecks or a bushel will cost 4 times as much, and if a bushel costs the result of this statement, that $\frac{1}{8}$ of a bushel will cost $\frac{1}{8}$ part of it, and $\frac{43}{8}$ will cost 43 times as much.

(6.) What will $50\frac{1}{4}$ pounds of tea cost at $10\frac{1}{2}¢$ per ounce?

OPERATION.

	¢
2	21
	16 2
4	201
–	———
	$84.42 Ans.

Explanation. Here we reduce and place the $10\frac{1}{2}¢$ on the line, and reason thus: since 1 ounce costs $\frac{21}{2}¢$, 16 ounces or 1 pound will cost 16 times as much, and since 1 pound costs the result of this statement, $\frac{1}{4}$ of a pound will cost $\frac{1}{4}$ part of it, and $\frac{201}{4}$ will cost 201 times as much. This completes the reasoning and statement, which worked gives $84.42, answer.

(7.) What will 24 pounds cost at $9\frac{1}{2}¢$ per pound?

OPERATION.

¢
2 | 19
24
12
\$2.28 Ans.

Explanation. Reducing and placing the $9\frac{1}{2}¢$ on the line. we reason thus: if 1 pound costs $\frac{19}{2}¢$, 24 pounds will cost 24 times as much. This completes the statement, which worked gives \$2.28, the answer.

(8.) What will $14\frac{2}{3}$ dozen cost at \$5 per dozen?

OPERATION.

\$
5
3 | 44
220
\$73$\frac{1}{3}$ Ans.

Explanation. Placing the \$5 on the line, we reason thus: if 1 dozen costs \$5, $\frac{1}{3}$ of a dozen will cost $\frac{1}{3}$ of it, and $\frac{44}{3}$ will cost 44 times as much. This completes the statement, which worked out gives \$73$\frac{1}{3}$, the answer.

(10.) What will 6 dozen and 7 chickens cost at \$4.87$\frac{1}{2}$ per dozen? Ans. \$32.09$\frac{3}{8}$

OPERATION.

¢
2 | 9.75
12 | 79
\$32.09$\frac{3}{8}$ Ans.

Explanation. Reducing and placing the \$4.87$\frac{1}{2}$ on the line we reason thus: Since 1 dozen cost $\frac{975}{2}¢$ 1 chicken will cost the 12th part and 79 chickens 79 times as much as 1; or thus, since 1 dozen costs $\frac{975}{2}¢$, $6\frac{7}{12}$ dozen will cost $6\frac{7}{12}$ times as much.

(11.) What will 42 pounds and 11 ounces of butter cost at $22\frac{1}{2}¢$ per pound? Ans. \$9.60$\frac{15}{32}$

OPERATION.

¢
2 | 45
16 | 683
\$9.60$\frac{15}{32}$ Ans.

Explanation. As usual we here place the price of one pound on the statement line and reason as follows Since 1 pound cost $\frac{45}{2}¢$ 1 ounce will cost the 16th part and 683 ounces (which is 42 pounds and 11 ounces) will cost 683 times as much.

117. *To multiply Abstract Fractional numbers.*

1. Multiply $8\frac{1}{3}$ by $3\frac{3}{4}$.

OPERATION.

$$\begin{array}{r|l} 3 & 25 \\ 4 & 15\ 5 \\ \hline & 125 \\ \hline & 31\frac{1}{4}\ \text{Ans.} \end{array}$$

Explanation.—In this problem both factors are abstract numbers, hence we cannot give the same analogical reasoning as we gave in the foregoing problems where the factors were denominate numbers; although were we to do so, the result so far as the figures are concerned, would be correct. We therefore, reduce and place the $8\frac{1}{3}$, the number to be multiplied, on the statement line and reason as follows: Since 1 time $\frac{25}{3}$ is equal to $\frac{25}{3}$, $\frac{1}{4}$ time the same is $\frac{1}{4}$ part of $\frac{25}{3}$ and $\frac{15}{4}$ are 15 times as many.

MISCELLANEOUS EXAMPLES IN MULTIPLICATION OF FRACTIONS.

1. What will 16 yards cost at $14\frac{1}{4}¢$ per yard?
Ans. $2.36.
2. What will $23\frac{3}{4}$ pounds cost at $35¢$ per pound?
Ans. $8.31\frac{1}{4}$.
3. What will $\frac{3}{4}$ of a yard cost at $\$\frac{5}{8}$ per yard?
Ans. $\$\frac{15}{32}$.
4. What will $\frac{1}{2}$ of a yard cost at $\$\frac{1}{2}$ per yard?
Ans. $\$\frac{1}{4}$.
5. What will $8\frac{1}{2}$ pounds cost at $7\frac{1}{2}¢$ per pound?
Ans. $63\frac{3}{4}¢$.
6. What will $10\frac{3}{4}$ pounds cost at $9\frac{1}{4}¢$ per pound?
Ans. $99\frac{7}{16}¢$.
7. What will $19\frac{5}{8}$ pounds cost at $18\frac{5}{8}¢$ per pound?
Ans. $\$3.60\frac{55}{64}¢$.
8. What will $25\frac{3}{4}$ yards cost at $17\frac{3}{4}¢$ per yard?
Ans. $\$4.55\frac{7}{12}$.
9. What will $11\frac{3}{8}$ yards cost at $12\frac{1}{2}¢$ per yard?
Ans. $\$1.42\frac{3}{16}$.
10. What will $21\frac{3}{4}$ yards cost at $16\frac{1}{2}¢$ per yard?
Ans. $\$3.58\frac{7}{8}$.
11. What will $14\frac{1}{4}$ pounds cost at $12\frac{1}{4}¢$ per pound?
Ans. $\$1.74\frac{9}{16}$.

12. What will $31\frac{1}{8}$ pounds cost at $11\frac{1}{8}¢$ per pound?
Ans. $\$3.46\frac{17}{64}$.

13. Multiply $\frac{1}{2}\frac{1}{4}$ by 12. Ans. $5\frac{1}{2}$.

14. Multiply $\frac{1}{1}\frac{5}{9}$ by 13. Ans. $10\frac{5}{19}$.

15. Multiply $\frac{9}{16}$ by 19. Ans. $10\frac{11}{16}$.

16. Multiply 7 by $\frac{2}{5}$. Ans. $2\frac{4}{5}$.

17. Multiply 13 by $\frac{7}{10}$. Ans. $9\frac{1}{10}$.

18. Multiply 105 by $\frac{4}{35}$. Ans. 12.

19. Multiply 136 by $\frac{33}{100}$. Ans. $44\frac{22}{25}$.

20. Multiply 12 by $31\frac{5}{6}$. Ans. 382.

21. Multiply 25 by $3\frac{2}{5}$. Ans. 85.

22 Multiply 19 by $\frac{3}{11}$. Ans. $5\frac{2}{11}$.

23. Multiply $\frac{11}{12}$ by $\frac{18}{35}$. Ans. $\frac{33}{70}$.

24. Multiply $\frac{28}{29}$ by $\frac{1}{7}$. Ans. $\frac{16}{29}$.

25. Multiply $11\frac{1}{2}$ by $1\frac{3}{5}$. Ans. $18\frac{2}{5}$.

26. Multiply $2\frac{3}{8}$ by $21\frac{1}{3}$. Ans. $50\frac{2}{3}$.

27. Find the value of $\frac{3}{7}$ of $\frac{1}{3}$ of $\frac{5}{9}$ of $\frac{6}{64}$ of 4.
Ans. $\frac{5}{16}$.

28. Multiply $7\frac{7}{10}$ by $\frac{5}{6}$. Ans. $6\frac{5}{12}$.

29. What is the product of $\frac{9}{10}$, $\frac{2}{7}$, $\frac{5}{9}$ and $\frac{1}{4}$?
Ans. $\frac{1}{28}$.

30. What is the product of $1\frac{5}{6}$, $\frac{3}{5}$, 2 and $5\frac{1}{3}$?
Ans. $11\frac{11}{15}$.

31. What is the product of $\frac{8}{15}$ of $2\frac{1}{4}$ by $\frac{1}{5}$ of $7\frac{1}{3}$?
Ans. $1\frac{19}{25}$.

32. What is the product of $12\frac{1}{2}$ multiplied by $5\frac{1}{2}$ times $6\frac{3}{4}$? Ans. $464\frac{1}{16}$.

33. At $\frac{10}{11}$ of a dollar a pound, what will $\frac{9}{10}$ of a pound of tea cost? Ans. $\frac{9}{11}$ of a dollar.

34. What will $5\frac{3}{4}$ dozen buttons cost at $\frac{2}{46}$ of a dollar per dozen? Ans. $\frac{1}{4}$ of a dollar.

35. What will $4\frac{3}{4}$ yards cost at $4\frac{1}{4}¢$ per yard?
Ans. $20\frac{3}{16}¢$.

36. What will $9\frac{1}{2}$ yards cost at $9\frac{3}{4}¢$ per yard?
Ans. $92\frac{5}{8}¢$.

37. What will $12\frac{7}{8}$ yards cost at $12\frac{1}{2}¢$ per yard?
Ans. $\$1.60\frac{1}{16}$.

38. What will $12\frac{1}{2}$ pounds cost at $12\frac{1}{2}¢$ per pound?
Ans. $\$1.56\frac{1}{4}$.

39. What will $6\frac{1}{4}$ pounds cost at $6\frac{3}{4}¢$ per pound?
Ans. $42\frac{3}{16}¢$.

40. What will $8\frac{2}{3}$ pounds cost at $8\frac{1}{3}¢$ per pound?
Ans. $72\frac{2}{9}¢$.

41. What will $19\frac{5}{8}$ pounds cost at $19\frac{5}{8}¢$ per pound?
Ans. $\$3.80\frac{1}{64}$.

42. What will $9\frac{3}{4}$ pounds cost at $11\frac{3}{4}¢$ per pound?
Ans. $\$1.14\frac{9}{16}$.

43. What will $15\frac{1}{2}$ pounds cost at $10\frac{1}{2}¢$ per pound?
Ans. $\$1.62\frac{3}{4}$.

44. What will $40\frac{5}{8}$ pounds cost at $22\frac{5}{8}¢$ per pound?
Ans. $\$9.19\frac{9}{64}$.

45. What will $2812\frac{1}{2}$ gallons cost at \$4.50 per gallon?
Ans. \$12656.25.

46. What cost $471\frac{3}{4}$ gallons at $\$3\frac{3}{8}$ per gallon?
Ans. $\$1592\frac{5}{32}$.

47. Sold $937852\frac{1}{2}$ pounds of cotton at $14\frac{15}{32}¢$ per pound, what did it amount to? Ans. $\$135695.53\frac{23}{64}$.

48. If a man earns $\$2\frac{1}{2}$ in 1 day how much will he earn in $16\frac{1}{2}$ days? Ans. $\$41\frac{1}{4}$.

49. A Contractor pays $\$1\frac{1}{4}$ per day for labor and he has 370 men employed for six days. How much money will it take to pay them? Ans. \$2775.

50. E. J. Denis paid $\frac{9}{10}$ of a dollar for a book and for paper $\frac{2}{3}$ of the cost of the book. How much did he pay for paper? Ans. 60 cents.

51. Distillers of the essence of rose have determined by experience that it requires 48000 pounds of rose leaves to

make or distill one pound of the ottar of rose. How many pounds of rose leaves will it require to distill $50\frac{7}{8}$ pounds of the ottar of rose? Ans. 2442000 pounds.

52. If a pound and a half costs a cent and a half what will $25\frac{1}{2}$ pounds cost? Ans. $25\frac{1}{2}$ cents.

53. F. Querens Jr. owned $\frac{15}{16}$ of the Steamer Katie and sold $\frac{2}{3}$ of his share to G. M. Leahy, what part of the whole Steamer did he sell? Ans. $\frac{5}{8}$.

54. R. E. Terregrossa can work the problems in this book in $4\frac{3}{4}$ months, how many months would it take him to work $\frac{2}{3}$ of them? Ans. $3\frac{1}{6}$ months.

55. E. Schwartz paid $\$\frac{5}{6}$ for 1 gallon of molasses, what is $\frac{3}{4}$ of a gallon worth at the same rate? Ans. $\$\frac{5}{8}$.

56. What will $7\frac{1}{2}$ boxes of raisins cost at $\$2\frac{1}{4}$ per box? Ans. $\$16\frac{7}{8}$.

57. On one occasion at the New Orleans Opera $\frac{1}{2}$ of the ladies and gentlemen present were French; $\frac{1}{2}$ of the remainder American; $\frac{1}{3}$ of the remainder German, and the others were of different nationalities. What part were Americans, what part Germans and what part were of different nationalities? Ans. $\frac{1}{4}$ Americans, $\frac{1}{12}$ Germans and $\frac{1}{6}$ of different nationalities.

58. C. Reynolds owned $\frac{7}{8}$ of a plantation and sold $\frac{2}{3}$ of his share to D. C. Williams, who sold $\frac{1}{2}$ of what he purchased to E. Szymanowski, who sold $\frac{3}{4}$ of what he purchased to N. Forcheimer. What is Forcheimer's share in the plantation? Ans. $\frac{7}{32}$.

59. J. Byrnes owned $\frac{4}{5}$ of 2000 acres of land and sold $\frac{3}{4}$ of his share to E. H. Wells, who sold $\frac{5}{8}$ of what he purchased to H. Clark. How many acres have each?

Ans. J. Byrnes 400; E. H. Wells 450; and H. Clark 750 acres.

DIVISION OF FRACTIONS.

126. Division of fractions is the process of dividing when the divisor or dividend, or both, contain fractional numbers.

In the division of simple numbers we saw that the result of division operations was decreasing, but in the division of fractions, when the divisor is less than a unit, the result is increasing. This fact is plain, for the reason that the operation of division is the process of finding how many times the dividend is equal to the divisor, and, hence, when the divisor is less than 1, the dividend will be equal to the divisor as many times itself as the divisor is part of 1.

In practical operations we usually have the three following cases or questions in division of fractional numbers.

1st. To *find* the cost of *one* pound, yard, or article of of any kind, when we have the cost of *many* pounds, yards or articles of any kind given.

2d. To *find* the cost of *one* pound, yard or article of any kind, when we have the cost of *a part* of a pound, yard or article of any kind given.

3d. To *find* the *number* of pounds, yards or articles of any kind that can be bought with a specified sum, when we have the price of *one*, or *a part of one* pound, yard or article of any kind given.

From these questions we see that division is the converse of multiplication and that from the nature of the question, we must reason from *many* to *one* or from *a part of one* to *one*. Thus: 1st. if 5 pounds cost 50¢, 1 pound will cost the $\frac{1}{5}$th part of it; in the 2d. case, if $\frac{3}{4}$ of a yard cost \$2, $\frac{1}{4}$ of a yard will cost the $\frac{1}{3}$ part of it, and $\frac{4}{4}$ths, or a *whole* yard, will cost 4 times as much; and in the third case, if $\frac{2.5}{2}$¢ buy 1 yard, or any other thing, $\frac{1}{2}$¢ will buy the $\frac{1}{2.5}$th part of it, and $\frac{2}{2}$, or a whole cent, will buy 2 times as much.

For the full reasoning, for this, case, see the explanation of the 2d problem.

(1.) Bought $7\frac{1}{2}$ pounds of sugar for $78\frac{3}{4}¢$. What was the cost per pound?

OPERATION.

¢

2 4 | 3 1 5 21

1 5 | 2

$10\frac{1}{2}¢$ Ans.

Explanation. By inspection and the exercise of our reasoning faculties, we see that, as the $78\frac{3}{4}¢$ are the cost of $7\frac{1}{2}$ pounds. it must be divided by $7\frac{1}{2}$ in order to obtain the cost of 1 pound. We therefore, reduce and place the $78\frac{3}{4}$ on our statement line. In all division operations, in order to facilitate the work, we thus place the number to be divided.

We then reason from many to 1, as follows: since $\frac{15}{2}$ pounds, which is $7\frac{1}{2}$ reduced to *halves*, cost $\frac{315}{4}¢$, *one-half* a pound will cost the 15th part of it, and 2 halves, or a whole pound, will cost 2 times as much. This completes the reasoning and statement. The 15 and 2 are placed respectively on the decreasing and increasing sides of the line, for the reason that when they were used the conclusion arrived at were respectively decreasing and increasing.

(2.) At $10\frac{3}{4}¢$ per pound, how many pounds can be bought for $\$3.92\frac{3}{8}$?

OPERATION.

lb

1

4 3 | 4

2 8 | 3 1 3 9 73

$36\frac{1}{2}$ lb Ans.

Explanation. By inspection and reason, we see that the question requires pounds for the result or answer. Therefore, in all of the reasoning we must either increase or decrease pounds. To aid in rendering the solution easily understood, we first place the 1 pound that cost $10\frac{3}{4}¢$ on the right of our statement line, and reason as follows: since $\frac{43}{4}¢$, which is $10\frac{3}{4}¢$ reduced. buy 1 pound, $\frac{1}{4}$ of a cent will buy the 43d part, and 4 fourths or a whole cent will buy 4 times as much; then, since 1 cent will buy the result of the statement now made, $\frac{1}{8}$ of a cent will buy $\frac{1}{8}$ part, and $\frac{3139}{8}¢$ will buy 3139 times as much. This completes the reasoning and statement.

The placing of the 1 pound on the statement line may be omitted and the reasoning given in the same manner as when t is thus placed.

(3.) At $\$\frac{3}{4}$ per yard, how many yards can we buy for $\$\frac{7}{8}$?

OPERATION.

Y
1
3 | 4
2 8 | 7
6 | $1\frac{1}{6}$ yard, Ans.

Explanation. For reasons given in preceding examples, we first place 1 yard on our line, and then reason as follows: since $\frac{3}{4}$ of a dollar buy 1 yard, $\frac{1}{4}$ will buy the 3d part, and $\frac{4}{4}$ or a whole dollar, 4 times as much; then, since 1 dollar will buy the result of our statement, $\frac{1}{8}$ of a dollar will buy the 8th part, and $\frac{7}{8}$, 7 times as much. This completes the reasoning and statement.

(4.) At $\$1\frac{2}{5}$ per pound, how many pounds can we buy for $\$38$?

OPERATION.

lb
1
7 | 5
38
190
$27\frac{1}{7}$ lb Ans.

Explanation. Having placed 1 pound on our line, we reason thus: since $\frac{7}{5}$ dollars buy 1 pound, $\frac{1}{5}$ will buy the 7th part, and $\frac{5}{5}$, or a whole dollar, 5 times as much, and 38 dollars will buy 38 times as much as $1. This completes the reasoning and statement.

(5.) At $3 per dozen, how many dozen can we buy for $\$84\frac{3}{8}$?

OPERATION.

Doz.
1
3 | 675
8 | 225
$28\frac{1}{8}$ doz Ans.

Explanation. We place 1 dozen, the equivalent of $3, on the line, and reason thus; if $3 buy 1 dozen, $1 will buy the 3rd part; then, if $1 buys the result of the statement now made, $\frac{1}{8}$ of a dollar will buy the 8th part, and $\$67\frac{5}{8}$ will buy 675 times as many.

(6.) Bought $9\frac{3}{8}$ yards for $\$22\frac{1}{2}$. What was the price per yard?

OPERATION.

$$
\begin{array}{r|l}
 & \$ \\
2 & 45\ 3 \\
5\ 75 & 8\ \ 4 \\
\hline
 & 12 \\
\hline
 & \$2\frac{2}{5}\ \text{Ans.}
\end{array}
$$

Explanation. For reasons given in the first example of division, we reduce and place the $\$22\frac{1}{2}$ on our line, and then reason thus: if $\frac{75}{8}$ yards cost $\frac{45}{2}$ dollars, $\frac{1}{8}$ of a yard will cost the 75th part, and $\frac{8}{8}$ or a whole yard will cost 8 times as much. This completes the reasoning and statement. In working out the statement, we first cancel the 75 and 45 by 15; then the 8 by the 2; then we multiply the 3 and 4 together and divide the result by the 5.

(7.) Bought $\frac{3}{4}$ of a pound for $30\cent$. What was the price per pound?

OPERATION.

$$
\begin{array}{r|l}
 & 30\ 10 \\
3 & 4 \\
\hline
 & 40\cent\ \text{Ans.}
\end{array}
$$

Explanation. Placing the cost on the line, we reason thus; if $\frac{3}{4}$ of a pound cost $30\cent$, $\frac{1}{4}$ will cost the 3d part, and $\frac{4}{4}$ or a whole pound will cost 4 times as much; which worked out gives $40\cent$ or the cost of 1 pound.

(8.) Bought 5 boxes of mdse. for $\$81\frac{1}{4}$. What was the price per box?

OPERATION.

$$
\begin{array}{r|l}
 & \$ \\
4 & 325 \\
5 & 65 \\
\hline
 & \$16\frac{1}{4}\ \text{Ans.}
\end{array}
$$

Explanation. The $\$81\frac{1}{4}$ being the number to be divided, we reduce and place the same on the line; then reason thus: if 5 boxes cost $\frac{325}{4}$ dollars, 1 box will cost the 5th part.

EXAMPLES.

(9.) Bought $18\frac{3}{4}$ pounds for $37\frac{1}{2}\cent$. What was the price per pound? Ans. $2\cent$.

10. At $6\frac{1}{4}\cent$ per pound, how many pounds can be bought for $96\frac{5}{8}\cent$? Ans. $15\frac{2}{5}\frac{3}{0}$.

11. Bought $250\frac{1}{3}$ dozen for $\$1251\frac{2}{3}$. What was the price per dozen? Ans. $\$4.99\frac{4}{7}\frac{7}{5}\frac{1}{1}$.

12. At $79\frac{3}{11}$¢ per pound, how many pounds can be bought for 7287$\frac{1}{7}$¢? Ans. $91\frac{28}{30}\frac{23}{52}$.

13. At \$½ a piece, how many chickens can be bought for \$25½? Ans. 51.

127. DIVISION OF ABSTRACT NUMBERS.

(1.) Divide $22\frac{3}{4}$ by $5\frac{1}{2}$.

OPERATION.

$$\begin{array}{r|l} 2\ \ 4 & 91 \\ 11 & 2 \\ \hline 22 & \\ \hline & 4\frac{3}{2}\ \text{Ans.} \end{array}$$

Explanation. The real question to be determined in this example is, how many times is $22\frac{3}{4}$ equal to $5\frac{1}{2}$, and as both the dividend and divisor are abstract numbers, we cannot, therefore, logically reason as in the preceding problems, and accordingly proceed as follows: the $22\frac{3}{4}$ being the number to be divided, we first reduce and place the same on our statement line; then by inspection and the exercise of our reason, we see that $22\frac{3}{4}$ is equal to 1, $22\frac{3}{4}$ times, or reduced that $\frac{91}{4}$ are equal to 1, $\frac{91}{4}$ times; and if equal to 1, $\frac{91}{4}$ times, it is equal to $\frac{1}{2}$, twice as many times, and to $\frac{11}{2}$ instead of $\frac{1}{2}$, the $\frac{1}{11}$ part. This completes the reasoning and statement of the problem, and the same character of reasoning and statement will solve all division problems in abstract numbers.

(2.) Divide $\frac{2}{15}$ by $\frac{3}{4}$.

OPERATION.

$$\begin{array}{r|l} 15 & 2 \\ 3 & 4 \\ \hline & \\ & \frac{8}{45}\ \text{Ans.} \end{array}$$

Explanation. In this example the dividend being less than the divisor, the question is, what part of a time is the $\frac{2}{15}$ equal to $\frac{3}{4}$. Placing the $\frac{2}{15}$ on the line, we reason, as in the above example, thus: $\frac{2}{15}$ are equal to 1, $\frac{2}{15}$ of a time; and if equal to 1, $\frac{2}{15}$ times, it is equal to $\frac{1}{4}$, 4 times as many times, and to $\frac{3}{4}$ instead of $\frac{1}{4}$, the 3d part.

(3.) Divide 3 by $\frac{2}{3}$.

OPERATION.

$$\begin{array}{r|l} & 3 \\ 2 & 3 \\ \hline & 9 \\ \hline & 4\frac{1}{2} \text{ Ans.} \end{array}$$

Explanation. We first place the 3 on the line, and reason thus: 3 is equal to 1, 3 times, and to $\frac{1}{3}$ instead of 1, 3 times as many times; and to $\frac{2}{3}$ instead of $\frac{1}{3}$, the $\frac{1}{2}$ part.

(4.) Divide $14\frac{2}{5}$ by 9.

OPERATION.

$$\begin{array}{r|l} \cancel{5} & \cancel{72} \\ 9 & 8 \\ \hline & 1\frac{3}{5} \text{ Ans.} \end{array}$$

Explanation. Placing the number to be divided on the line, we reason thus: $\frac{1}{5}^2$ are equal to 1, $\frac{1}{5}^2$ times, and to 9 instead of 1, the 9th part.

The solution of the 4 preceding problems elucidates the only correct reasoning for dividing abstract fractional numbers. But for practical work we would not advise a change from the reasoning given where the numbers are denominate.

MISCELLANEOUS EXAMPLES IN DIVISION OF FRACTIONS.

1. Bought 4 yards for $\$14\frac{1}{2}$, what was the cost per yard? Ans. $\$3\frac{5}{8}$.
2. Sold $8\frac{1}{2}$ pounds for \$1.87, what was the price per pound? Ans. 22 cents.
3. Paid $37\frac{1}{2}$ cents for $6\frac{1}{4}$ yards of calico, what was the price per yard? Ans. 6 cents.
4. At $\$1\frac{3}{8}$ per gallon, how many gallons can be bought for $\$148\frac{1}{2}$? Ans. 108 gallons.
5. Divide $\frac{6}{7}$ by 2. Ans. $\frac{3}{7}$.
6. Divide $\frac{9}{21}$ by 3. Ans. $\frac{1}{7}$.
7. Divide $1\frac{4}{15}$ by 5. Ans. $1\frac{4}{75}$.
8. Divide $\frac{6}{7}$ by 5. Ans. $\frac{6}{35}$.
9. Divide $7\frac{1}{5}$ by 9. Ans. $\frac{4}{5}$.
10. Divide 2 by $\frac{6}{7}$. Ans. $2\frac{1}{3}$.

11. Divide 3 by $\frac{9}{21}$. Ans. 7.
12. Divide 5 by $\frac{14}{15}$. Ans. $5\frac{5}{14}$.
13. Divide 21 by $\frac{7}{11}$. Ans. 33.
14. Divide 105 by $\frac{15}{17}$. Ans. 119.
15. Divide $\frac{10}{11}$ by $\frac{5}{22}$. Ans. 4.
16. Divide $\frac{21}{23}$ by $\frac{7}{92}$. Ans. 12.
17. Divide $2\frac{1}{4}$ by $\frac{3}{4}$. Ans. 3.
18. Divide $\frac{13}{17}$ by $\frac{4}{11}$. Ans. $2\frac{7}{68}$.
19. Divide $3\frac{1}{2}$ by $2\frac{1}{5}$. Ans. $1\frac{13}{22}$.
20. Divide $\frac{6}{7}$ of $\frac{14}{81}$ by $\frac{8}{9}$. Ans. $\frac{1}{12}$.

21. If one pound of tea cost $\frac{5}{6}$ of a dollar, how many pounds can be bought for $25? Ans. 30 lbs.

22. Six barrels of flour were divided among some poor families in such a manner that each received $\frac{2}{3}$ of a barrel; how many families were there? Ans. 9.

23. If a boy can earn $\frac{7}{11}$ of a dollar in one day; how many days will it take him to earn $21? Ans. 33 days.

24. Henry walked 25 miles, which was $\frac{5}{6}$ of the distance Robert walked; how many miles did Robert walk? Ans. 30 miles.

25. At the battle of Germantown the British lost about 600 men; this was $\frac{3}{5}$ of the number lost by the Americans; and the number lost by the Americans was $\frac{2}{5}$ of the number they received as re-enforcements just before the battle. How many men did the Americans lose, and how many receive as re-enforcements?
Ans. 1000 men lost, 2500 re-enforcements.

26. A man had his store insured for $9000, which was $\frac{8}{9}$ of $\frac{9}{11}$ of its value; what was the store worth?
Ans. $12375.

27. Sulphur will fuse at 232° Fahrenheit. This is $7\frac{1}{4}$ times the temperature required to melt ice. At what temperature will ice melt? Ans. 32°.

28. A quantity of mercury weighed $32062\frac{1}{2}$ lbs., which is $13\frac{1}{2}$ times the weight of an equal bulk of water. What would an equal bulk of water weigh? Ans. 2375 lbs.

29. A pound of water at 212° F. was mixed with a

pound of powdered ice at 32°. The united temperature of the two was $4\frac{9}{13}$ times the temperature of the mixture when the ice became melted. What was the temperature of the two pounds after the ice became melted?

Ans. 52°.

30. When the air was at the freezing point, a cannon $2761\frac{3}{3}$ feet distant from New Orleans was discharged. $25\frac{1}{3}$ seconds elapsed after the discharge before the sound reached New Orleans. How many feet per second did the sound travel? Ans. 1090 feet.

31. Divide $287\frac{3}{4}$ by 5. Ans. $57\frac{11}{20}$.

Operation without the line statement.

$$5)287\frac{3}{4}$$

$$57\frac{11}{20} \text{ Ans.}$$

Explanation. We first divide the 287 by the process of short division and obtain a quotient of 57 and a remainder of 2; this remainder we reduce to a fraction whose denominator is the same as that of the fraction to be divided, add it to this fraction and then divide the sum by 5 and annex the result to the quotient 57. Thus $2=\frac{8}{4}+\frac{3}{4}=\frac{11}{4}$, and $\frac{11}{4}\div 5=\frac{11}{20}$.

32. Divide $1471\frac{3}{16}$ by 9. Ans. $163\frac{67}{144}$.

33. Divide $1044\frac{2}{3}$ by 12. Ans. $87\frac{1}{18}$.

34. E. T. Churchill divided $14\frac{7}{12}$ dozen apples among 3 boys and 2 girls; he gave each girl twice as many as each boy. How many did each boy and each girl receive?

Ans. $2\frac{1}{12}$ doz. each boy, $4\frac{1}{6}$ doz. each girl.

35. Divide 1 by $\frac{1}{5}$. Ans. 5.

36. Divide $\frac{1}{5}$ by 1. Ans. $\frac{1}{5}$.

37. Divide $\frac{8\frac{1}{3}}{6\frac{1}{4}}$ of $\frac{\frac{3}{16}}{\frac{11}{12}}$ by $\frac{9\frac{3}{4}}{4\frac{5}{7}}$ of $\frac{\frac{1}{2}}{3}$ Ans. $\frac{6}{7}$.

38. If $4\frac{1}{2}$ pounds of coffee cost 90 cents, what will $22\frac{3}{4}$ pounds cost? Ans. $4.55.

39. R. E. L. Fleming owns $\frac{3}{8}$ of the capital stock of a factory valued at $24000; he gives $\frac{1}{2}$ of $\frac{1}{3}$ to educational societies, and the remainder he divides equally between his

four children. How much does he give to educational societies and how much does each child receive?

Ans. $1500 to educational societies.
$1875 each child receives.

40. C. Craft has $65\frac{1}{2}$ yards of cloth, 2 yards wide, how many yards of lining $\frac{3}{8}$ of a yard wide will be required to line it? Ans. $196\frac{1}{2}$ yards.

41. Divide 18 oranges between A. and B. so that A. will have $\frac{1}{4}$ more than B. What number will each have?

Ans. A. 10; B. 8.

42. Divide 18 oranges between A. and B. so that A. will have $\frac{1}{4}$ less than B. What number will each have?

Ans. A. $7\frac{5}{7}$; B. $10\frac{2}{7}$.

43. A., B. and C. are to receive $26 in proportion to $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$. What will each receive?

Ans. A. $12; B. $8; C. $6.

44. A. and B. can do a piece of work in 10 days; A. alone can do it in 15 days. How many days will it take B. to do it? Ans. 30 days.

45. A. and B. can do a piece of work in 14 days. A. can do $\frac{3}{4}$ as much as B. How many days will it take each to do it, working alone? Ans. $24\frac{1}{2}$ days for B.
$32\frac{2}{3}$ days for A.

46. Three persons, A., B. and C., do a piece of work; A. and B. together do $\frac{7}{9}$ of it, and B. and C. do $\frac{7}{11}$ of it. What part of the work is done by B? Ans. $\frac{41}{99}$.

47. A planter remits his factor $500 to invest in rice and coffee, in equal sums. He pays $9\frac{1}{4}¢$ per pound for rice, and $23\frac{3}{8}¢$ per pound for coffee. How many pounds of each did he purchase?

Ans. $2702\frac{26}{37}$ lb rice. $1069\frac{97}{187}$ lb coffee.

48. W. Quintel has $100: he gives $\frac{2}{5}$ of it for five barrels of flour, and $\frac{1}{3}$ of the remainder for 4 barrels of potatoes, and with the remainder he buys coffee at $20¢$ per pound. How much coffee did he buy?

Ans. 200 lb coffee.

49. C. Wehrmann owned $\frac{9}{10}$ of a stock of goods: he

sold $\frac{1}{5}$ of his share for \$5000, and $\frac{1}{4}$ of the remainder for \$5000. and then the balance of his interst for \$15000. What part did he sell the last time, and what would the whole stock be worth at that rate?

Ans. $\frac{27}{50}$ sold last.
\$27777$\frac{7}{9}$ value of stock.

50. What quantity, from which if you subtract $\frac{3}{8}$ of itself, the remainder will be 15? Ans. 24.

148.—MISCELLANEOUS EXAMPLES, INVOLVING THE PRINCIPLES OF ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION OF FRACTIONS.

1. Find the difference between $\frac{5}{6}$ and $\frac{3}{5}$, $\frac{6}{7}$ and $\frac{5}{6}$, $\frac{2}{3}$ and $\frac{7}{11}$, $3\frac{7}{9}$ and $2\frac{3}{7}$, $4\frac{2}{3}$ and $\frac{1}{2}$ of $3\frac{1}{3}$? Ans. to last, 3.

2. Find the sum of $\frac{4}{7}$ of $\frac{9}{20}$ and $\frac{3}{4}$ of $\frac{8}{21}$. Ans. $\frac{19}{35}$.

3. To the quotient of $2\frac{3}{5}$ divided by $5\frac{1}{5}$, add the quotient of $3\frac{3}{4}$ divided by $\frac{1}{2}\frac{1}{1}$. Ans. $7\frac{1}{2}$.

4. A number was divided by $\frac{3}{4}$, and gave a quotient of 20, what was the number? Ans. 15.

5. What number is that, which being multiplied by $\frac{7}{11}$ gives as a product $\frac{14}{33}$? Ans. $\frac{2}{3}$.

6. What number is that, from which, if you take $\frac{3}{5}$ of itself, the remainder will be 12? Ans. 30.

7. What number is that, to which, if you add $\frac{3}{5}$ of itself, the sum will be 40? Ans. 25.

8. A. owns $\frac{3}{5}$ of a store which is worth \$25000. He sells $\frac{5}{6}$ of his share; what part does he still own, and what is it worth? Ans. owns $\frac{1}{10}$, worth \$2500.

9. \$mith owns $\frac{5}{11}$ of a cotton mill and sells $\frac{3}{10}$ of his share to Jones for \$33000; what is the mill worth at that rate? Ans. \$242000.

10. John has 5 cents, and James $\frac{3}{4}$ of 8 cents; what part of James' money is John's? Ans. $\frac{5}{6}$.

11. One planter raised 500 bales of cotton, another raised 250; what part of the first one's crop is the second? Ans. $\frac{1}{2}$.

12. The sum of four fractions is $1\frac{5}{6}$. Three of the fractions are $\frac{3}{7}$, $\frac{1}{2}$ and $\frac{2}{3}$; what is the fourth? Ans. $\frac{5}{21}$.

13. What number is that, to which if $\frac{3}{7}$ of $\frac{3}{3}\frac{5}{6}$ of $1\frac{1}{5}$ be added, the sum will be $1\frac{1}{2}$? Ans. 1.

14. Two boys bought a bushel of oranges, one paying $2\frac{1}{8}$ dollars and the other $4\frac{3}{8}$ dollars: what part of it should each have? Ans. first, $\frac{1}{4}\frac{3}{1}$; second, $\frac{2}{4}\frac{8}{1}$.

15. A farmer sold $\frac{5}{7}$ of his mules on Monday; on Tuesday he bought $\frac{3}{5}$ as many as he sold, and then had 40; how many mules had he at first? Ans. 56.

16. F. Gernon gave $\frac{1}{5}$, $\frac{1}{2}$ and $\frac{1}{4}$ of his money to different benevolent institutions, and had $1000 left. How much had he at first? Ans. $20000.

17. J. D. Bothick owning $\frac{6}{11}$ of a rice mill, sold $\frac{3}{3}$ of his share for $8800. What was the value of the mill?
Ans. $24200.

18. A book-keeper worked $91\frac{1}{2}$ days, and after paying $\frac{2}{3}$ of $\frac{3}{5}$ of his earnings for board and washing, had $438 remaining. How many dollars did he receive in all, and how many per day? Ans. $730 in all, $8 per day.

19. A planter gave 50 bales of cotton at $50 $1\frac{1}{10}$ per bale for flour at $7\frac{1}{2}$ per barrel. How many barrels of flour did he receive? Ans. 334.

20. Prophet can do a piece of work in 6, and Fisher can do the same in 8 days; how many days will it take both together to do the work? Ans. $3\frac{3}{7}$ days.

21. Myers, Levy and Hoffman can do a piece of work in 10 days; Myers and Levy can do it in 15 days; in what time can Hoffman do it, working alone? Ans. 30 days.

22. A man died and left his wife $14400, which was $\frac{3}{4}$ of $\frac{3}{3}\frac{2}{3}$ of his estate. At her death she left $\frac{5}{6}$ of her share to her daughter. How many dollars did the daughter receive, and what part was it of her father's estate?
Ans. $12000, $\frac{2}{3}\frac{0}{3}$ of her father's estate.

23. A mule and dray cost $240; the mule cost $1\frac{3}{5}$ times as much as the dray. What did each cost?
Ans. $150 mule, $90 dray.

24. A man engaging in trade lost $\frac{3}{7}$ of the money he invested, he then gained $1000, when he had $3800; what did he have at first, and what was his loss?

Ans. $4900 at first, $2100 loss.

25, Forcheimer lost $\frac{3}{8}$ of his fish-line, and then added $25\frac{1}{2}$ feet when it was just $\frac{3}{4}$ of its original length. What was its original length? Ans. 204 feet.

26. How many bushels of apples at $\$\frac{4}{5}$ a bushel, will pay for $\frac{9}{10}$ of a barrel oranges at $\$6\frac{2}{3}$ a barrel?

Ans. $7\frac{1}{2}$ bushels.

27. Sweeney paid $\frac{4}{7}$ of his year's wages for board, $\frac{3}{5}$ of the remainder for clothes, and had $80 left; how many dollars did he receive for labor? Ans. $560.

28. Purcell, having a certain number of cents, gave one-half of them and half a cent over to one beggar; one-half of what he had remaining and half a cent over to a second beggar; and to a third, one-half of what he then had and half a cent over, and had left 3 cents. How many cents had he at first? Ans. 31 cents

29. John lives with his parents, but works for Mr. Smith who pays him $210 per year. His parents board him, but he has his clothes to buy. He spends $\frac{2}{7}$ of his wages for cigars, $\frac{2}{5}$ of the remainder for theater tickets, $\frac{1}{3}$ of the remainder for wine, $\frac{1}{2}$ of what he then has for novels. How much has he remaining at the end of the year to pay for his clothes? Ans. $30.

30 Joseph worked on the same conditions as John. He gave $\frac{1}{14}$ of his wages to the cause of charity, $\frac{1}{3}$ of the remainder for useful books, $\frac{1}{4}$ of the remainder to be taught evenings, paid $100 for clothes, and deposited the balance in the bank. How many dollars did he put in the bank?

Ans. $50.

31. W. T. Harris and C. E. Jones have $1899, Jones has $3\frac{1}{2}$ times as much as Harris; how much has each?

Ans. Harris $422, and Jones $1477.

32. J. C. Beals can solve 25 problems in 50 minutes and

H. H. Barlow can solve them in 30 minutes. In what time can both solve them? Ans. $18\frac{3}{4}$ minutes.

33. E. Meyer purchased 200 barrels of flour for $1450 and sold $\frac{3}{4}$ of it at a profit of $\$\frac{1}{2}$ per barrel, and the remainder at $\$7\frac{1}{10}$ per barrel. How much did he gain?

Ans. $67.50.

34. What is the numerical value of

$$\frac{4\frac{1}{2}-\frac{3}{4}}{2\frac{1}{3}+1\frac{1}{3}}?$$

Ans. $1\frac{1}{4}$.

35. M. Ernst bought $3841\frac{1}{2}$ pounds of cotton at $7\frac{3}{4}$ pence per pound; what did it cost? Ans. £124, 11.6¼ d.

36. J. W. Anderson has 3 dozen oranges which he wishes to divide between Miss Kate and Miss Lucy, so that Miss Kate shall receive $\frac{1}{4}$ more than Miss Lucy. How many will each receive?

Ans. Miss K. 20 and Miss L. 16.

37. A tree 110 feet high, had $\frac{2}{5}$ of it broken off in a storm; how much of it was left standing? Ans. 44 feet.

38. What cost $22\frac{3}{4}$ pounds of coffee at $\$21\frac{3}{4}$ per pound?

Ans. $\$4.94\frac{13}{16}$.

39. If $18\frac{3}{4}$ yards cost $\$3.37\frac{1}{2}$ what will $3\frac{1}{3}$ yards cost?

Ans. 60 cents.

40. W. D. Maxwell has $600 of which he wishes to give to A. $\frac{1}{3}$, B. $\frac{1}{4}$, C. $\frac{1}{5}$ and D. $\frac{1}{6}$; how much will each receive? Ans. A. $200, B. $150, C. $120 and D. $100.

41. R. L. Paul has $600 which he wishes to give to A. B., C. and D. in the proportion of $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$; how much will each receive? Ans. A. $\$210\frac{10}{19}$, B. $\$157\frac{17}{19}$, C. $\$126\frac{6}{19}$ and D. $\$105\frac{5}{19}$.

42. C. M. Huber and A. J. Hohensee bought on speculation $800 worth of merchandise, of which Huber paid $500 and Hohensee $300; they sold to W. A. Tomlinson $\frac{1}{3}$ of the whole for $400. How much of the $400 must Huber and Hohensee receive respectively, in order to constitute each $\frac{1}{2}$ owner in the remainder of the goods?

Ans. Huber $350 and Hohensee $50.

43. If a yard and a half cost a dollar and a half what will twelve and a half yards cost? Ans. $12\frac{1}{2}$.

44. If 3 is the third of 6 what will the fourth of 20 be? Ans. $7\frac{1}{2}$.

45. Geo. Meyer owned a quantity of rice, of which he sold $\frac{4}{5}$ for $99.60; what is $\frac{3}{8}$ of the remainder worth at the same rate? Ans. $16.60.

46. F. Miller paid $60 for $\frac{3}{4}$ of an acre of land; what is the value of $\frac{5}{8}$ of an acre? Ans. $50.

47. S. Benavides bought $937852\frac{1}{2}$ pounds of cotton at $14\frac{5}{32}$¢ per pound; what was the cost? Ans. $135695.53\frac{23}{64}$.

48. J. Koch invested $\frac{1}{4}$ of his money in sugar, $\frac{1}{3}$ in rice, $\frac{5}{8}$ in coffee and deposited in bank $2645. How much money had he at first? Ans. $63480.

49. L. Meyer spends $\frac{1}{3}$ of his time in study, $\frac{1}{4}$ in labor, $\frac{1}{8}$ in rest and recreation, and the remainder in sleep. How many of the 24 hours of a day does he sleep? Ans. 7 hours.

50. A loafer spends 4 hours per day sauntering on street corners, 3 hours smoking and drinking, $\frac{1}{4}$ of the day in sleep, $\frac{1}{8}$ of the day in drunkenness, $\frac{1}{12}$ in eating, $\frac{1}{12}$ in quarreling and the remainder of the day in gaming. How many hours does he spend in gaming? Ans. 3 hours.

51. An industrious young lady spends $\frac{1}{4}$ of her time in the performance of household affairs, $\frac{1}{8}$ in reading good books, $\frac{1}{12}$ in physical exercise in the open air and sunlight, $\frac{1}{8}$ in the practice of music, singing and parlor amusements, or social intercourse, 2 hours per day in eating, and the remainder of the day in sleeping. How many hours per day does she devote to each?

Ans. 6 hours to household affairs; 4 hours to reading; 2 hours to exercise; 3 hours to music, etc.; 2 hours to eating, and 7 hours to sleep.

52. A fashionable young lady spends $\frac{1}{3}$ of her time in dressing, painting and making her toilet, $\frac{1}{8}$ in reading novels and papers of senseless fiction, $\frac{1}{4}$ in making calls and

gossiping, $\frac{1}{12}$ in street promenading, $\frac{1}{24}$ in criticising industrious young men, and speculating upon the qualities and fortune of an anticipated husband, $\frac{1}{24}$ in making remarks derogatory to the character of those who labor, while her own mother is perhaps cooking or washing, $\frac{1}{12}$ in entertaining young men, and the remainder in eating and sleeping. How many hours does she devote to useful service, and how many to eating and sleeping?

Ans. 0 hours to useful service; 8 hours to eating and sleeping.

53. A man willed $\frac{1}{4}$ of his property to his wife, $\frac{3}{4}$ of the remainder to his daughter, and the remainder to his son; the difference between his wife and daughter's share was $8000. How much did he give his son?

Ans. $4800.

54. R. W. Tyler owned a $\frac{3}{4}$ interest in a factory; he sold to C. Modinger $\frac{1}{2}$ of his interest for $15000. What interest does he still own, and how much is it worth at the rate received for the part sold?

Ans. he still owns $\frac{3}{8}$, worth $15000.

55. J. Cassidy owned $\frac{7}{8}$ of the Steamer R. E. Lee. He sold to G. Buesing $\frac{1}{8}$ interest in the Steamer for $20000; and to J. C. Beals $\frac{1}{4}$ of his remaining interest at the same rate. What did he receive for the last sale, and what is his remaining interest in the boat?

Ans. he received $30000; $\frac{9}{16}$ remaining interest.

56. N. Puech and A. Palacio bought on joint account each $\frac{1}{2}$ the New Orleans Cotton Factory. N. Puech sold $\frac{1}{2}$ of his interest to R. Krone, and subsequently $\frac{1}{2}$ of his remaining interest to A. Palacio, who subsequently sold $\frac{1}{2}$ of $\frac{3}{4}$ of his whole interest to R. Lynd for $7500. What is the factory worth at the same rate, and what is each owner's interest?

Ans. $32000 value of Factory; Puech owns $\frac{1}{8}$; Krone $\frac{1}{4}$; Palacio $\frac{25}{64}$, and Lynd $\frac{15}{64}$.

57. L. Kaiser bought $\frac{2}{3}$ of $\frac{3}{4}$ of $28\frac{1}{2}$ barrels of apples,

and sold to S. L. Crawford $\frac{3}{4}$ of 9 barrels for \$20$\frac{1}{4}$, which was \$1.50 more than the same cost. What was the cost of the whole, and how many barrels has he unsold?

Ans. \$39$\frac{7}{12}$ cost; 7$\frac{1}{2}$ barrels unsold.

58. W. D. Maxwell gives $\frac{1}{3}$ of his annual income to aid meritorious young men in obtaining an education; $\frac{1}{2}$ of the remainder for the publication and free distribution of books treating of the awful injury to the human race by the use of tobacco, tea, coffee and wine; $\frac{1}{3}$ of the second remainder for various benevolent purposes. The balance \$5490 he retains for his own personal use; how much does he give for each object named?

Ans. \$8235 for meritorious young men; \$8235 for the publication and distribution of books; and \$2745 for various benevolent purposes.

59. What is the smallest sum of money for which I could purchase a number of bushels of oats, at \$$\frac{5}{16}$ a bushel; a number of bushels of corn, at \$$\frac{5}{8}$ a bushel; a number of bushels of rye, at \$1$\frac{1}{2}$ a bushel; or a number of bushels of wheat, at \$2$\frac{1}{4}$ a bushel; and how many bushels of each could I purchase for that sum?

Ans. \$22$\frac{1}{2}$; 72 bushels of oats; 36 bushels of corn; 15 bushels of rye; 10 bushels of wheat.

60. There is an island 15 miles in circuit, around which A. can travel in $\frac{3}{4}$ of a day, B. in $\frac{1}{3}$ of a day, and a horse car in $\frac{3}{10}$ of a day. Supposing all to start together from the same point to travel around it in the same direction, how long must they travel before coming together again at the place of departure, and how many miles will each have traveled?

Ans. 10$\frac{1}{2}$ days; A. 210 miles; B. 180 miles; Horse Car 525 miles.

DECIMAL FRACTIONS.

150. **A Decimal Fraction** is one whose integral unit is divided according to the decimal scale; therefore the denomi-

nator is some power of ten; as 10, 100, 1000, etc. The word *decimal* is derived from the Latin word *decem*, which means ten.

151. **The Decimal Point** (.) is used to distinguish decimals from whole numbers. When there are mixed numbers, it also separates the whole numbers from the decimals.

The following are decimal fractions: $\frac{3}{10}$, $\frac{15}{100}$, $\frac{137}{1000}$, and $\frac{423}{10000}$; they are here written as common fractions, but generally the denominator of decimal fractions is omitted and the value is indicated by the location of the *decimal point* before the numerator.

To write these fractions according to the *decimal notation*, they would be written thus:

$\frac{7}{10}$ decimally expressed is .7.
$\frac{15}{100}$ decimally expressed is .15.
$\frac{137}{1000}$ decimally expressed is .137.
$\frac{423}{10000}$ decimally expressed is .0423.

152. **Notation of Decimals.** Whenever decimal fractions are expressed decimally, the numerator must have as many decimal places as there are naughts in the denominator. Thus $\frac{4}{10}$=.4; $\frac{16}{100}$=.16; $\frac{1456}{10000}$=.1456. When the number of naughts in the denominator is greater than the number of figures in the numerator, naughts must be prefixed to the numerator until the number of *places* is equal to the naughts in the demominator. Thus $\frac{4}{100}$=.04; $\frac{7}{1000}$=.007; $\frac{125}{100000}$=.00125, etc.

When the number of naughts in the denominator is less than the figures in the numerator, the result or value of the fraction will embrace a whole number and a fraction.

153. **A Pure or Simple Decimal** consists of a decimal fraction, decimally expressed or written. Thus .5, .42, .875 and .1256 are pure decimals, and are read respectively 5 tenths; 42 hundredths; 875 thousandths and 1256 ten thousandths.

154. **A Mixed Decimal** consists of a whole number and a decimal. Thus 24.5 and 41.25 are mixed decimals. They are read respectively, 24 and 5 tenths; 41 and 25 hundredths.

155. **A Complex Decimal** consists of a decimal with a common fraction annexed. Thus .15$\frac{3}{4}$ and .005$\frac{1}{8}$ are complex decimals. They are read respectively, 15$\frac{3}{4}$ hundredths; 5$\frac{1}{8}$ thousandths.

156. **A Circulating Decimal** is one in which a figure or set of figures constantly repeats itself. Thus $\frac{1}{3}$=.3333+, $\frac{1}{7}$=.142857+, $\frac{11}{15}$=.73333+. The figure or set of figures which is repeated is called a *Repetend*. If the repetend consists of only one figure, a dot is placed over it; if of a set of figures, a dot is placed over the first and last figures, as $\frac{1}{3}$=.$\dot{3}$, $\frac{2}{3}$=.$\dot{6}$, $\frac{1}{11}$=.$\dot{0}\dot{9}$, $\frac{1}{7}$=.$\dot{1}42857\dot{}$.

157. **A Pure Circulating Decimal** is one which contains only the repetend; as $\frac{2}{3}$=.$\dot{6}$, $\frac{1}{7}$=.$\dot{1}42857\dot{}$, $\frac{1}{9}$=.$\dot{1}$.

158. **A Mixed Circulating Decimal** is one which contains other figures than the repetend; as $\frac{1}{6}$=.1$\dot{6}$, $\frac{583}{900}$=.64$\dot{7}$.

There are still other kinds of circulating decimals, but as they are of very little practical importance, we will not here consider them.

159. Decimal fractions, like whole numbers, decrease towards the right and increase towards the left in a *ten-fold* ratio, and hence the prefixing of naughts between the decimal figures and the decimal point, or the removal of the decimal point towards the left diminishes their value *ten-fold*, or divides the decimal by ten for each order or place removed, and conversely the removal of the decimal point to the right, increases the value *ten-fold* or multiplies the decimal by ten for each place removed.

Annexing naughts to decimals does not change their

value, because the significant figures are not thereby removed nearer to nor farther from the decimal point.

Decimal *orders* are also called decimal *places*, each order being counted as one place. Thus in .0043 there are *four decimal places*, although the 3 is of the *fifth decimal place* from *unity*, the base of the system.

The following table will illustrate more fully the relation of whole numbers and decimals, with their increasing and decreasing orders to the left and right of the decimal point.

TABLE.

WHOLE NUMBERS.									DECIMALS.								
Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	**Units**	Decimal Point.	Tenths.	Hundredths.	Thousandths.	Ten-Thousandths.	Hundred-Thousandths	Millionths.	Ten-Millionths.	Hundred-Millionths.
9	8	7	6	5	4	3	2	1	.	2	3	4	5	6	7	8	9
Orders of ascending scale.									Orders of descending scale								

This number is read 987 *million* 654 *thousand* 321, and 23 million 456 thousand 789 *hundred-millionths*.

In order to clearly understand decimals, we must bear in mind that the *unit* one is the basis of all numbers, integral and fractional, abstract and denominate, and that all mathematical operations have this fundamental principle for their origin, and every number is but a multiple, either ascending or descending of unity or one.

The names of the decimal orders are derived from the names of the orders of whole numbers. Thus the names of the orders in the ascending scale, are, after units, *tens*,

hundreds etc., and the orders in the decending scale, are, after units, *tenths*, *hundredths* etc., the decimal orders being the reciprocal of the orders of whole numbers equally distant with themselves from the units.

Numeration of Decimals. In reading decimal fractions the entire decimal is regarded as reduced to units of the lowest order expressed, and the name of this order is given to the entire number of decimal units. Thus .25 is read *twenty-five hundredths.*

Before reading a decimal, we must determine 1st. How many units are expressed. To do this, we numerate and read the significant figures of the decimal as in whole numbers. 2nd. We must determine the name of the lowest order in the decimal. To do this, we numerate the number decimally. Thus to read .001073, we commence at the 3 and numerate to the 1 thousand, and thus find that 1073 units are expressed; then we commence at the decimal point and numerate decimally to the 3 and thus find that millionths is the lowest order, we then read 1073 millionths.

160. EXERCISES.

Read the following numbers:

1. 16.008; reads thus, sixteen units and eight thousandths.

2. .94⅜; reads thus, ninety-four and three-eights hundreths.

3. 5067.4005; reads thus, 5067 units and 4005 tenthousandths.

4. Write and read 197.8; 4.68907; .00073; 48.769146.

5. Write and read 2.491; 10.0101089167; 582.400410905.

6. Write and read 5841.291⅖; 8000.0000000217; 9876541.1000001.

161. **Writing Decimals.** In writing decimals we write down the given number as if it were a whole number; then, to facilitate the operation, we numerate from right to left, beginning the numeration with *tenths*, and continue until we come to the required place or order, always writing 0's to fill the places not occupied by significant figures. Thus, to write 25 ten thousandths we first write the 25; then we begin at the right and numerate thus, tenths, hundredths, thousandths, ten thousandths; by this we find that *four* places are required and as there are but two figures in the number we prefix two 0's and obtain the correct result .0025.

1. Write 104 hundred thousandths. Ans. .00104.

OPERATION.
.00104.

Explanation. According to the above directions we write the 104 and then commence on the right and numerate thus; tenths, hundredths, thousandths, ten thousandths, hundred thousandths. This numeration shows that *five* places are required and as we have but *three* we therefore prefix two 0's.

162. EXERCISES.

1. Write 10101 hundred billionths.
Ans. .00000010101.

Write decimally, numerate and read the following:

2. 314 millionths.
3. 12 thousandths.
4. 107 billionths.
5. 1 trillionth.
6. 1205 ten millionths.
7. 897 hundred billionths.
8. 1 sextillionth.
9. 21001 ten vigintillionths.

10. $\frac{5\frac{1}{2}}{10}$
11. $\frac{25}{100}$
12. $\frac{742}{1000}$
13. $\frac{1042\frac{1}{2}}{100000}$
14. $\frac{50409}{100000000}$
15. $\frac{748\frac{9}{3\,4}}{1000000}$
16. $\frac{1}{1000000000}$
17. $\frac{87}{10000}$
18. $\frac{1\frac{3}{4}}{100}$
19. $\frac{990099}{1000000000000}$

163. PRINCIPLES.

From the foregoing work we recapitulate the following principles:

1. Decimals are governed by the same laws of notation as whole numbers, hence the value of any decimal figure depends upon the place it occupies.

2. Each removal of the decimal point one place to the *right* is equivalent to multiplying the decimal by 10.

3. Each removal of the decimal point one place to the *left* is equivalent to dividing the decimal by 10.

4. Annexing or rejecting naughts at the right of any decimal does not change its value.

REDUCTION OF DECIMALS.

164. *To reduce Decimal Fractions to a common denominator.*

1. Reduce .7, .18, .2581 and .045 to a common denominator.

OPERATION.

.7000
.1800
.2581
.0450

Explanation. To reduce decimals to a common denominator we have but to annex a sufficient number of 0's to give each decimal the same number of places.

165. *To reduce a decimal to a common fraction.*

1. Reduce .25 to a common fraction.

OPERATION.

$\frac{25}{100}=\frac{1}{4}$ Ans.

Explanation. In all problems of this kind we simply write the decimal as a common fraction and then reduce it to its lowest terms.

2. Reduce .125 to a common fraction.

OPERATION.

$\frac{125}{1000}=\frac{1}{8}$ Ans.

3. Reduce .59$\frac{3}{8}$ to a common fraction.

FIRST OPERATION.

$\frac{59\frac{3}{8}}{100}=\frac{\frac{475}{8}}{\frac{800}{8}}$ and $\frac{475}{8}\div\frac{800}{8}=\frac{475}{800}=\frac{19}{32}$ Ans.

Explanation. To reduce complex decimals to simple fractions we first write the decimal as a common fraction; then we reduce both the numerator and denominator to the fractional unit of the denominator contained in the numerator term of the fraction, and thus obtain a complex fraction, which we reduce to a simple fraction.

SECOND OPERATION.

$$\frac{59\frac{3}{8}}{100} = \frac{475}{800} = \frac{19}{32} \text{ Ans.}$$

Explanation. Here, when reducing the fraction to the fractional unit of the denominator contained in the numerator term of the fraction, we shorten the work by omitting the denominator (8) in both terms of the complex fraction, and writing the result as a simple fraction. By this process we save the operation of division, the result of which, is the cancelling of the denominator in both terms of the complex fraction.

Reduce the following decimals to common fractions:

4.	.8.	Ans. $\frac{8}{10}$.	13.	.88.	Ans.
5.	.05	Ans. $\frac{5}{100}$.	14.	.909.	Ans.
6.	.25.	Ans.	15.	.00025.	Ans.
7.	.125.	Ans.	16.	$.48\frac{1}{2}$.	Ans. $\frac{97}{200}$.
8.	.675.	Ans.	17.	$.055\frac{3}{4}$.	Ans. $\frac{223}{4000}$.
9.	.105.	Ans.	18.	$.008\frac{2}{7}$.	Ans.
10.	.07.	Ans. $\frac{7}{100}$.	19.	$.00054\frac{7}{16}$.	Ans.
11.	.005.	Ans. $\frac{1}{200}$.	20.	.999.	Ans.
12.	.1045.	Ans.	21.	$4007\frac{1}{11}$.	Ans.

167. *To reduce common fractions to equivalent decimals.*

1. Reduce $\frac{3}{8}$ to a decimal.

OPERATION.

$$8)3.000$$

$$.375 \text{ Ans.}$$

Explanation. To reduce common fractions to decimals, we annex naughts to the numerator and divide by the denominator, then point off as many places for decimals as there were 0's annexed. When a remainder continues beyond four or six places, we discontinue dividing and write the sign + to the right of the last figure obtained, which indicates that the quotient is not complete. The annexing of 0's to the numerator is equivalent to multiplying it by 10 for each naught annexed, consequently the quotient obtained is as many times 10 too great as there were 0's annexed; and hence the reason for pointing off as

many places in the quotient as there were 0's annexed to the numerator.

2. Reduce $\frac{5}{7}$ to an equivalent decimal.

OPERATION.

7)5.000000

.714285+ Ans.

3. Reduce $\frac{3}{725}$ to an equivalent decimal.

FIRST OPERATION.

725)3.000000(4137+
2 900
.004137+ Ans.
1000
725
2750
2175
5750

SECOND OPERATION.

725)3. (.004137+ Ans.
30 tenths.
300 hundredths.
3000 thousandths.
2900
1000 ten-thousandths.
725
2750 hundred thousandths,
2175
5750 millionths.
5075
675 Remainder.

Explanation. Here, in the first operation, we annex six 0's and obtain but 4 figures in the quotient. Therefore in order to point off as many decimal places as we annexed 0's, we prefix two 0's and thus obtain the correct result. The reason for this will appear clear if we consider each step of the work as performed in the second operation. We are to divide or measure 3 by 725, and we first see that 3 is not equal to 725 any whole or unit number of times; we therefore write the decimal point in the quotient, annex a 0 to the 3 units and thus reduce it to 30 tenths, which we also see is not equal to 725 any tenth times, and hence we write 0 in the tenth's place of the quotient; we then annex another 0 and thereby reduce the 30 tenths to 300 hundredths, which we see is not equal to 725 any hundredths times, and hence we write 0 in the hundredths place of the quotient; we then annex another 0 and thereby reduce the 300 hundredths to 3000 thousandths, which we see

is equal to 725 4 times, with a remainder. We have now obtained the first significant figure of the decimal, and we continue the division in the usual manner to the sixth decimal place and annex the + sign to indicate that there is still a remainder.

4. Reduce $6\frac{3}{4}$ to a decimal.

FIRST OPERATION.

$6\frac{3}{4}=\frac{27}{4}$ and $\frac{27}{4}=4)27.00$

6.75 Ans.

SECOND OPERATION.

$6\frac{3}{4}=6$ and $\frac{3}{4}$; and

$\frac{3}{4}=4)3.00$

$.75+6=6.75$ Ans.

Reduce the following fractions to equivalent decimals not exceeding 6 places.

5. $\frac{23}{32}$ Ans. .71875
6. $\frac{210}{625}$ Ans. .336
7. $\frac{4}{125}$ Ans. .032
8. $\frac{3}{4}$ of $\frac{1}{7}$ Ans. .107142+
9. $\frac{5}{8}$ Ans. .625
10. $\frac{1}{13}$ Ans. .076923+
11. $.37\frac{1}{16}$ Ans. .370625
12. $47.18\frac{3}{4}$ Ans. 47.1875

Reduce $\frac{2}{3}$ to a complex decimal of 3 places.

OPERATION.

$3)2.000$

$.666\frac{2}{3}$ Ans.

13. Reduce $\frac{3}{7}$ to a complex decimal of 4 places.

Ans. $.4285\frac{5}{7}$.

14. Reduce $\frac{2}{9}$ to a complex decimal of 6 places.

Ans. $.222222\frac{2}{9}$.

168. ADDITION OF DECIMALS.

Since decimals increase from right to left, and decrease from left to right in a tenfold ratio as do simple whole numbers, they may be added, subtracted, multiplied and divided in the same manner.

1. Add .785, .93, 166.8, 72.5487 and 4.17.

OPERATION.

.785
.93
166.8
72.5487
4.17

245.2337 Ans.

Explanation. In all problems of this kind we write the numbers so that units of the same order will stand in the same column, and the decimal point be in a vertical line; then we add as in simple whole numbers.

When the addition is completed we point off in the sum, from the right hand, as many places for decimals as equal the greatest number of decimal places in any of the numbers added.

Add the following numbers.

2. 3.25, 42.348, 748.4 and 29.32. Ans. 823.318.
3. .0049, 47.0426, 37.041 and 360.0039. Ans. 444.0924.
4. 1121.6116, 61.87, 46.67, 165.13 and 676.167895. Ans. 2071.449495.
5. .8, .09, 34.275, 562.0785 and 1.01. Ans. 598.2535.
6. 81.61356, 6716.31, 413.1678956, 35.14671, 3.1671 and 314.6. Ans. 7564.0052656.
7. 1.01¾, 240.06¼, 999.9, 80.6051 and .17. Ans. 1321.7576.
8. What is the sum of the following numbers: twenty-five, and seven millionths; one hundred forty-five, and six hundred forty-three thousandths; one hundred seventy-five, and eighty-nine hundredths; seventeen, and three hundred forty-eight hundred-thousandths. Ans. 363.536487.
9. A farmer has sold at one time 3 tons and 75 hundredths of a ton of hay, at another time 11 tons and 7 tenths of a ton, and at a third time 16 tons and 125 thousandths of a ton. How much has he sold in all? Ans. 31.575.

169. SUBTRACTION OF DECIMALS.

1. From 345.3046 subtract 92.1435847.

OPERATION.

345.3046
92.1435847
253.1610153 Ans.

Explanation. In all problems of this kind we write the numbers so that units of the same order will stand in the same column, and the decimal points be in a vertical line; then we subtract as in simple whole numbers, and point off in the difference, from the right hand, as many places for decimals as equal the greatest number of decimal places in either the minuend or subtrahend.

When the decimal places in the subtrahend exceed those in the minuend, naughts are understood to occupy the vacant places, and may be filled in if it is desired.

EXAMPLES.

2.	3.	4.
81.04089	121.25	532.8
14.587	109.05438	9.00451681
66.45389 Ans.	12.19562 Ans.	523.79548319 Ans.

5. From 461.072 take 427.125. Ans. 33.947.
6. From 17.5 take 4.19. Ans. 13.31.
7. From 4000.0004 take 4.3. Ans. 3995 7004.
8. From three million take three millionths. Ans 2999999.999997.
9. From 11 take 1 and 9 thousand trillionths. Ans. 9.999999999991.
10. From 24000 subtract 2.078. Ans. 23997.922.
11. From 886.333 subtract 98.5427. Ans. 787.7903.

170. MULTIPLICATION OF DECIMALS.

1. Multiply 26.58 by 4.3.

OPERATION.

26.58
4.3
7974
10632
114.294 Ans.

Explanation. In all problems of this kind we multiply as in whole numbers, and point off on the right of the product as many places for decimals as there are decimal places in both the multiplicand and multiplier. The reason for thus pointing off the 3 decimal places in this problem is obvious from the fact that in the multiplicand we have 2 decimal places or hundredths, which we used as

whole numbers and thereby produced a product 100 times too great; and in the multiplier we have 1 decimal place or tenths which we also used as a whole number and thereby produced a product 10 times too great; and both together gives a product 1000 times too great; hence to obtain the correct product we divide or point off 3 decimal places.

SECOND OPERATION.

100 | 2658
10 | 43

114294 / 1000 Ans.

or decimally written
114.294 Ans.

Explanation. In this operation we reduce the factors to common fractions and then multiplying them together, we obtain a product of $\frac{114294}{1000}$ which written decimally is 114.294 This process shows in another way why we point off on the right of the product as many places for decimals as there are decimal places in both factors.

2. Multiply 4.024 by .0056.

OPERATION

4.024
.0056

24144
20120

.0225344 Ans.

Explanation. In all problems of this kind where the number of figures in the product is not equal to the number of decimal places in the two factors, we must prefix a sufficient number of 0's to supply the deficiency. In this example, we prefix one 0. The reason of this will appear evident by working the example as a common fraction as shown in the second operation of the first problem.

EXAMPLES.

3. Multiply 27 by .9. Ans. 24.3.
4. Multiply .38 by 8. Ans. 3.04.
5. Multiply .75 by .42. Ans. .3150.
6. Multiply .006 by .0103. Ans. .0000618.
7. Multiply 340.012 by 61.23. Ans 20818.93476.
8. Multiply .1234 by 1234. Ans. 152.2756.
9. Multiply 1500 by .00014. Ans. .21.
10. What is the product of one thousand and twenty-five multiplied by three hundred and twenty seven ten-thousandths? Ans. 33.5175.
11. What is the product of seventy-eight million two hundred five thousand and two, multiplied by fifty-three hundredths? Ans. 41448651.06.

12. Multiply one hundred and fifty-three thousandths by one hundred and twenty-nine millionths.

Ans. .000019737.

13. Multiply 1 thousand by 1 thousandth. Ans. 1.

14. Multiply 2 million by 2 billionths Ans. .004.

15. What will 37.23 tons of hay cost at $20.75 per ton?

Ans. $772.52+.

16. What will 428.431 bushels cost at $1.125 per bushel? Ans. $481.98+.

171. *To multiply a decimal or mixed number by 10, 100, 1000, etc.*

1. Multiply 428.375 by 100.

OPERATION.
42837.5 Ans.

Explanation In all problems where the multiplier is 10, 100, etc., we simply remove the decimal point as many places to the right as there are naughts in the multiplier, annexing naughts if required.

2. Multiply 271.32 by 1000. Ans. 271320.

3. Multiply .756 by 100. Ans. 75.6.

4. Multiply .025 by 10. Ans. .25.

5. Multiply 61.052 by 10000. Ans. 610520.

172. DIVISION OF DECIMALS.

1. Divide 17.094 by 8.14.

FIRST OPERATION.

8.14)17.094(2.1 Ans.
16 28

814
814

Explanation. In all problems of this kind, we divide as in whole numbers, and then *point off as many places for decimals from the right of the quotient as the number of decimal places in the dividend exceeds those in the divisor, observing to supply any deficiency by prefixing naughts.* In this problem the excess is one, and we therefore point off one decimal place in the quotient. The reason for thus pointing is obvious from the fact that in the dividend we had 3 decimals or thousandths, and in the divisor we had 1 decimal or tenths, and thousandths divided or decreased by tenths gives hundredths as a quotient. The reason will also appear plain if we observe that the dividend is the product of the divisor and

quotient multiplied together, and hence we point off enough decimal places in the quotient to make the number in the two factors equal to the number in the product or dividend, according to the principles shown in the first problem of multiplication of decimals

SECOND OPERATION.

$$\begin{array}{r|l} 1000 & 17094 \\ 814 & 100 \\ \hline & \hline \\ & 2\frac{1}{10} \text{ Ans.} \end{array}$$

Decimally written 2.1 Ans.

Explanation. In this operation we reduce the decimals to common fractions and then proceed as in the division of mixed numbers. The reduction of the dividend and divisor to common fractions and then the mixed numbers to improper fractions, is performed thus: the dividend $17.094 = 17\frac{094}{1000} = \frac{17094}{1000}$; the divisor $8.14 = 8\frac{14}{100} = \frac{814}{100}$. This method also shows the reason for pointing off and may be used for all problems in decimal fractions.

2. Divide 7898.56 by 2.4683.

OPERATION.

$$\begin{array}{l} 2.4683)7898.5600(3200. \text{ Ans.} \\ \quad 74049 \\ \quad \overline{} \\ \quad 49366 \\ \quad 49366 \\ \quad \overline{} \\ \quad 00 \end{array}$$

Explanation. Here we have an excess of decimals in the divisor, and in all cases of this kind we first make them equal by annexing naughts to the dividend, and the quotient will be in whole numbers. The reason for annexing the naughts will appear more obvious by solving the problem in the form of a common fraction.

3. Divide 7.0761 by 687.

OPERATION.

$$\begin{array}{l} 687)7.0761(103 \\ \quad 687 \quad .0103 \text{ Ans.} \\ \quad \overline{} \\ \quad 2061 \\ \quad 2061 \\ \quad \overline{} \end{array}$$

Explanation. In this problem there are 4 decimal places in the dividend and none in the divisor, hence according to the foregoing instruction we must point off 4 decimal places in the quotient, and as there are but 3 figures in the quotient we prefix 1 naught. In all problems of this kind, 0's are prefixed to supply any deficiency of figures that may occur.

4. Divide 47.789 by 39.27.

OPERATION.

39.27)47.789(1.2168+ Ans.
3927
8519
7854
6650
3927
27230
23562
36680
31416

Explanation. In this problem we have a remainder, after dividing the dividend, of 665; to this and the 2 successive remainders we annex 0's and continue the division until we have produced 4 decimal places. The annexing of 0's reduces the successive remainders to the next lower order of tenths and hence all quotient figures produced by annexing 0's are decimals. We therefore, point off from the right of the quotient as many places for decimals as the number of decimals in the dividend exceed those of the divisor, plus the number of 0's annexed. This is done in all division problems where 0's are annexed, and a sufficient number of 0's should be annexed to produce 4 or 6 decimal places. When there is a remainder after the last division the plus (+) sign should be annexed to the answer to indicate that the quotient is incomplete.

5. Divide 1.12233 by 12.

OPERATION.

12)1.12233
9352+=.09352+ Ans.

6. Divide 11.2233 by 12.

OPERATION.

12)11.2233
.9352+ Ans.

7. Divide .112233 by 12.

OPERATION.

12).112233
9352+=.009352+ Ans.

8. Divide 112.233 by 12.

OPERATION.

12)112.233
9.3627+ Ans.

9. Divide .0004869 by 396.

OPERATION.

396)4869(12+ Ans.
396 =.0000012+.
909
792
117

10. Divide .0004869 by 3.96.

OPERATION.

3.96).0004869(12+ Ans.
396 =.00012+
909
792
117

11. Divide .0004869 by .0396.

FIRST OPERATION.	SECOND OPERATION.
.0396).0004869(122+ Ans.	396)4869(122+=.0122+ Ans.
396 =.0122+	396
909	909
792	792
1170	1170
792	792
378	378

12. Divide 67.8632 by 32.8. Ans. 2.069.
13. Divide 983 by 6.6. Ans. 148.939+.
14. Divide 13192.2 by 10.47. Ans. 1260.
15. Divide 67.56785 by .035. Ans. 1930.51.
16. Divide .00125 by .5. Ans. .0025.
17. Divide 7.482 by .0006. Ans. 12470.
18. Divide 1 by 999. Ans. .001001+.
19. Divide 84375 by 3.75. Ans. 22500.
20. Divide 1081 by 39.56 Ans. 27.3255+.
21. Divide 35.7 by 485. Ans. .0736+.
22. If rice costs $.0775 per pound, how many pounds can be bought for $40.64875? Ans. 524:5 pounds.
23. Sold 14.75 acres of land for $191.75. What was the price per acre? Ans. $13.
24. Divide four thousand three hundred twenty-two, and four thousand five hundred seventy-three ten-thousandths by eight thousand and nine thousandths.

Ans. .5403+.

173. *To divide Decimal Fractions by 10, 100, 1000, etc.*

1. Divide 48.76 by 10.

OPERATION.
4.876 Ans.

Explanation. In all problems of this kind we simply remove the decimal point as many places to the left as there are 0's in the divisor. The reason for this was fully shown on page 133. When there are not a sufficient number of figures in the dividend to allow this to be done naughts must be prefixed to supply the deficiency.

2. Divide 875.25 by 100. Ans. 8.7525.
3. Divide .5231 by 1000. Ans. .0005231.
4. Divide 72 by 10000. Ans. .0072.
5. Divide 9.85 by 100. Ans. .0985.
6. Divide .025 by 200. Ans. .000125.
7. Divide 412.99 by 10. Ans. 41.299.

174. MISCELLANEOUS PRACTICAL PROBLEMS.

Such as occur in the counting room, factory, workshop, on the plantation, and in the various departments of business life.

1. What is the cost of 1465 pounds of corn at 84 cents per bushel, and how many bushels are there?

Ans. \$21.97½ cost.
26 Bush., 9 lbs.

2. Sold 5294 pounds of hay at \$23.75 per ton. How many tons were there, and what was the value of it?

Ans. 2 tons, 1294 lbs.
\$62.86⅝ value.

3. Bought $320\frac{42}{60}$ bushels of wheat at \$1.95 per bushel. What was the cost? Ans. \$625.36½.

4. Bought 1136½ pounds of dried peaches at \$5.80 per bushel. How many bushels were there and what did they cost? Ans. 34 bushels, 14½ lbs.
\$199.74$\frac{28}{33}$ cost.

5. Bought 15 bushels and 31 pounds of corn at 78½ cents per bushel. What was the cost?

Ans. \$12.20$\frac{107}{112}$.

6. Bought 3 coops of chickens containing 2 dozen and 7 chickens each, at \$4.35 per dozen. What did they cost? Ans. \$33.71¼.

7. What will 74 pounds and 11 ounces of butter cost at 42½ cents per pound? Ans. \$31.74$\frac{7}{32}$.

8. Bought 36 pounds and 7 ounces of tea at \$1.12½ per pound. What did it cost? Ans. \$40.99$\frac{7}{32}$.

9. Butter is worth 45 cents per pound. How much can be bought for 20 cents? Ans. 7$\frac{1}{9}$ ounces.

10. What is the cost of 31845 feet of lumber at $22.25 per M.? Ans. $708.55½.

11. What will 183 feet of lumber cost at $25.75 per M.? Ans. 4.71\frac{9}{40}$.

12. Bought 3 bales of hay weighing as follows: (1) 421 pounds (2) 394 pounds, (3) 487 pounds, at $22.50 per ton. What did it cost? Ans. $14.64¾.

13. Sold 3¼ dozen boxes Spencerian pens at $108 per gross. What did they amount to? Ans. $29.25.

14. How much coffee can I buy for 5 cents when a pound costs 28 cents? Ans. $2\frac{6}{7}$ ounces.

15. What is the cost of 400 T. 2 cwt. 3 qrs. 20 lbs. of iron at $60 per ton of 2240 pounds?

Ans. $24008.78¾.

16. A planter shipped 6 dozen dozen boxes of peaches to market, but being delayed on the way ½ a dozen dozen boxes spoiled; the remainder were sold at 70 cents per box. What did they amount to? Ans. $554.40.

17. Bought 12 dozen and 5 hats at $11 per dozen. What did they cost? Ans. $136.58¾.

18. What is the amount due for the freight of 40000 pounds of merchandise for 965 miles at 5¢ for 100 pounds for 100 miles? Ans. $193.

19. What is the cost of 2381¾ pounds of cotton at $17\frac{8}{16}$ cents per pound? Ans. 424.24\frac{59}{64}$.

20. What is the cost of a 14 carat gold chain that weighs 4 oz. 7 pwt. 15 gr. at $1.20 per pwt. for pure gold, allowing ½¢ per grain on full weight for manufacturing and the alloy? Ans. $71.85¼.

21. A hardware merchant received from Liverpool an invoice of iron weighing 2 T. 2 cwt. 3 qrs. 20 lbs., long ton weight; the invoice price was £12, 17s. 6d. per ton; What did the whole cost in sterling money?

Ans. £27, 12s. $8\frac{1}{2}\frac{3}{8}$d.

22. Bought 4692 pounds of barley at $.88 per bushel. How many bushels were there and what was the cost?

Ans. 97 bush. 36 lbs.
Cost $86.02.

23. Bought 2765 pounds of oats at 76¢ per bushel. What was the cost, and how many bushels were there?
Ans. $65.66⅔ cost.
86 bush. 13 lbs.

24. What is the cost of 4878 pounds of wheat at $2.45 per cental? Ans. $119.511.

25. What is the cost of 200 sacks of guano each weighing 162 pounds, at $52¼ per ton? Ans. $846.45.

26. What is the value of 5790 hoop-poles at $18 per M?
Ans. $104.22.

27. What is the value of 8750 shingles at $8.75 per M?
Ans. $76.56¼.

28. What is the value of 11428 fence pickets at $9 per M? Ans. $102.852.

29. What is the value of 1364 pine apples at $11½ per C.? Ans. $156.86.

30. What is the cost of 2417 cocoanuts at $8.25 per C.? Ans. $199.40¼.

31. What is the value of 78420 railroad ties at $75 per M.? Ans. $5881.50.

32. What is the freight on 540 bales cotton, weighing 243084 pounds, ⅝d. per pound from New Orleans to Liverpool? Ans. £633 0s. 7½d.

33. What is the freight in United States currency on 25000 bushels corn from New Orleans to Liverpool, at 24s. per imperial quarter of 480 pounds; allowing £1 to be equal to $4.87? Ans. $17045.

34. How many square feet in a pavement 120 feet 4 inches long and 10 feet wide? Ans. 1203⅓ sq. feet.

35. How many square yards in a plat of ground 140 feet 3 inches long and 64 feet 6 inches wide?
Ans. 1005⅓ sq. yards.

36. How many *squares* in the roof of a building 78 feet 6 inches long, and 48 feet 4 inches wide?
Ans. 37.94⅙ squares.

37. How many square yards of plastering in the walls and ceiling of a room which is 40 feet 6 inches long, 24 feet 8 inches wide, and 14 feet high, deducting 34 square

yards for doors, windows and base-board, and what will it cost at 35 cents per square yard?

Ans. $279\frac{20}{27}$ sq. yards.
$\$97.90\frac{25}{27}$ cost.

38. How many sq. feet in 8 boards, each measuring 16 feet long and 17 inches wide, and what will they cost at $2\frac{1}{2}¢$ per foot? Ans. $181\frac{1}{3}$ feet.
$\$4.53\frac{1}{3}$ cost.

39. How many square feet in 13 pieces of plank, each measuring 20 feet 6 inches long, 14 inches wide and 3 inches thick, and what is the cost at $23 per M.?

Ans. $932\frac{3}{4}$ feet.
$\$21.45\frac{3}{4}$ cost.

40. How many square feet in a circle, the diameter of which is 12 yards? Ans. 1017.8784 sq. feet.

41. How many shingles will it require to shingle a building, the roof of which measures 44 feet 7 inches from eave to eave, without allowances, by 50 feet 4 inches long, allowing a shingle to cover a space 4 inches wide and 5 inches long? Ans. 16157.

42. A yard is 24 feet 3 inches long by 11 feet 5 inches wide; how many brick, 4 by 8 inches will it take to pave it, no allowance to be made for the openings between the bricks? Ans. $1245\frac{27}{32}$.

43. How many square yards of paving in a sidewalk 64 feet long and 11 feet 8 inches wide?

Ans. $82\frac{26}{27}$ square yards.

44. How many flags, each 16 inches square, will it require to flag a walk 22 yards 1 foot 4 inches long and 6 feet 8 inches wide? Ans. $252\frac{1}{2}$.

45. How many yards of carpeting that is 27 inches wide, will it take to cover the floor of a room that is 25 feet 6 inches long, and 22 feet 9 inches wide, making no allowance for waste in matching or turning under?

Ans. $85\frac{17}{18}$ yards.

46. A water pipe is 50 feet 9 inches long, and its diameter is 30 inches; what is its concave surface?

Ans. 57397.032 inches.

47. How many cubic feet in a box 5 feet long, 3 feet wide and 4 feet deep? Ans. 60 cubic feet.

48. What is the freight on a box 6 feet 4 inches long, 4 feet wide and 3 feet 9 inches deep at 25 cents per cubic foot? Ans. $23\frac{3}{4}$.

49. What will be the freight on a box 9 feet 3 inches long, 4 feet 6 inches wide, 2 feet 10 inches deep, at 30 cents a cubic foot? Ans. $35.38\frac{1}{8}$.

50. How many bushels will a bin hold, that is 10 feet long, 8 feet 6 inches wide, and 5 feet 2 inches deep?

Ans. 352.90+ bushels.

51. How many cords of wood in two ranks, each 44 feet long and 6 feet 3 inches high? Ans. $17\frac{3}{16}$ cords.

52. How many barrels will a quadrilateral cistern hold, whose height is 12 feet and width of side 5 feet 8 inches?

Ans. $91\frac{821}{1617}$ barrels.

53. How many cubic yards in a levee 80 rods long, 60 feet wide at the base, $12\frac{1}{2}$ feet at the top, and 5 feet 4 inches average depth? Ans. $9451\frac{23}{27}$.

54. How many gallons will a box hold, that is 5 feet long, 2 feet 4 inches wide, and 3 feet deep?

Ans. 261.81+ gallons.

55. How many cubic feet in a cylinder 6 feet long 3 feet 4 inches in diameter? Ans. 52.36 cubic feet.

56. How many gallons in a cylindrical cistern, 9 feet 6 inches high, and 7 feet 2 inches in diameter?

Ans. 2866.6896 gals.

57. How many pints in a cylindrical vessel, whose height is 14 inches and diameter $12\frac{1}{2}$ inches?

Ans. 59.5 pints.

58. How many bushels in a cylinder shaped box, whose height is 10 feet, and diameter 10 feet?

Ans. 631.125 bu.

59. How many cubic feet in a frustum of a cone, whose height is 6 feet, diameter of the greater end is 4 feet and of the smaller end 3 feet? Ans. 58.1196 cubic feet.

60. How many gallons in a cistern which is in the form of a frustum of a cone, whose height is 9 feet 6 inches,

lower base 7 feet 2 inches, and upper base 6 feet 8 inches?
Ans. 2671.3392 gals.

61. A farmer has a heap of grain in a conical form, the diameter of which is 14 feet 4 inches, and the depth 5 feet 3 inches; how many bushels does it contain?
Ans. 226.906.

62. A barrel is 26 inches long, 17 inches in diameter at the head, and 20 inches in diameter at the bung or center. The staves have a medium curve. How many gallons will it hold? Ans. 31.244+ gallons.

For full information and a thorough elucidation of all questions pertaining to the mensuration of surfaces and solids, as contained in the foregoing miscellaneous problems, and in the following bills, see Soulé's Philosophic Commercial and Exchange Calculator, pages 741 to 796.

174. BILLS AND INVOICES.

Bills in a general sense, embrace all written statements of accounts and many legal instruments of writing; but in a more common and limited sense they are statements of goods sold or delivered, services rendered or work done, with the price or value, quality or grade of each article or item. They should state the place and date of each sale, the names of the buyer and seller, the extra charges or discount to be allowed, and the terms of the sale.

When goods are bought to sell again, or when bills are rendered to a jobber or retailer, or consigned to an agent, the bill is then called an *invoice*.

It is the custom of accountants and merchants, when making bills to commence the name of each article with a capital.

When a charge is made for the box, barrel, jar etc. containing goods, it is customary to write its price above and to the right of it and add the same to the cost of the goods it contains.

In making extensions, fractions of cents are not used in the

product; when they are ½ or more they are counted cents when they are less than ½ they are not counted.

In making the following bills, students should use pen and ink and give earnest attention to the proper form and spacing, to plain, neat and rapid penmanship of both words and figures, and above all to the accuracy of extentions and additions.

When notes or bills of exchange are given in payment, the student should draw the same and correctly mature them.

No. 1.

NEW ORLEANS, *Jan. 2, 1877.*

H. A. & R. C. Spencer,

Bot. of A. L. & E. Soule.

TERMS—Cash.

1876					
Dec.	16	2 bags Rio Coffee, 325 lbs. 50 c.	@	$ 23½ c.	$ 76 38
		1 bbl. Sugar, 234½ lbs.	"	9 c.	21 61
		½ Chest Black Tea, 35 lbs.	"	87½ c.	30 63
		1 bbl. Rice, 248–16 =227	"	8 c.	18 16
		40 gal. N. O. Molasses	"	75 c.	30 00
		6 doz. Brooms	"	4.15	24 90
		3 bbls. XXX Family Flour	"	8.12½	24 38
		25 lbs. Cream Crackers	"	16 c.	4 00
		50 lbs. Graham do	"	15 c.	7 50
		20 lbs. W. Butter	"	30 c.	6 00
		Rec'd pay't,			$243.56

A. L. & E. SOULÉ,
Per S. Richardson.

No. 2.

NEW ORLEANS, *Jan'y 31, 1877.*

S. S. Packard and E. G. Folsom,
Bot. of W. H. and Frank Soule.

TERMS—Note at 30 days.

1877					
Jan.	31	453½ lbs. Mocha Coffee,	@	$	25 c.
		241 " Rio Coffee,	"		18¾c.
		316¼ " C. Sugar,	"		12½c.
		72 " Duryea's Starch,	"		6¼c.
		64 " N. Y. C. Cheese,	"		17½c.
		52 " W. F. Cheese,	"		15 c.
		180 " B. Sugar,	"		7½c.
		80 doz. C. Eggs,	"		37½c.
		42 gals. N. O. Molasses,	"		62½c.
		320 lbs. G. Butter,	"		35 c.
		23 " Almonds,	"		27 c.
		76 " Y. H. Tea,	"		74 c.
		68 boxes Shrimp,	"		48 c.
		84 boxes Lobsters,	"		34 c.
		92 gals. N. O. G. Syrup,	"		96 c.
		114 " B. Whiskey,	"		1.08
		112 bags Salt,	"		93 c.
		320 Bbls. Sweet Potatoes,	"		1.25
		82 kits, No. 1 Mackerel,	"		2.50
		63 lbs. S. Crackers,	"		11 c.
		24¾ " P. L. Soap,	"		8½c.
		18½ " Codfish,	"		9¼c.
		Drayage $31.25, boxes $2.50.			

Rec'd pay't by note at 60 days, $1,492 09

W. H. & F. SOULÉ,
per J. J. Manson.

NOTE.—All of the extensions of this bill should be made mentally. For rapid mental work see Soulé's Contractions in Numbers.

No. 3.

NEW ORLEANS, *Jan'y 31, 1877.*

J. M. Butchee,

Bot. of J. B. Cundiff.

TERMS—60 days credit.

875	bbls. Nes. Potatoes	@ $ 4.25	
440	" P. B. Potatoes,	" 3.87½	
325	" Perfect'n Flour,	" 8.50	
1324	" St. L. XX "	" 6.62½	
112	" F'ily Clear Pork	" 17.50	
650	" Prime Pork,	" 13.75	
220	kegs Pig Feet,	" 7.50	
124	half bbls F. M. Beef,	" 11.	
1872	lbs. Choice Ham,	" 14 c.	
289	" B. Bacon,	" 9½c.	
106	Pig Tongues,	" 8 c.	

Rec'd pay't, $31167 27

NOTE—All the extensions of this bill shall be made mentally.

No. 4.

NEW YORK, *Dec. 8, 1876.*

H. C. Spencer & Co.,

Bot. of B. D. Rowlee & Co.

TERMS—Cash.

20	doz. Missionary Bibles,	@ $15.25	
108	" small New Testam't,	" 2.50	
65	" Prayer Books,	" 2.25	
65	" Hymn Books,	" 3.	
3	Bible Dictionaries,	" 4.	
½	doz. Webster's Dict'ry	" 50.	

Rec'd pay't, $ 953 25

B. D. ROWLEE & CO.
Per E. Conrad.

No. 5.

NEW ORLEANS, *Jan'y 31, 1877.*

Wm. Melchert & Co.,

TERMS—Cash. *Bot. of L. L. Williams & Co.*

	321	lbs.	Tobacco Low Lugs,	@	6 c.	
	1140	"	" Med. Lugs,	"	7½c.	
	509	"	" Low Leaf,	"	9¼c.	
	965	"	" Med. Leaf,	"	11½c.	
	398	"	" Good Leaf,	"	13¾c	
	2416	"	" Fine Leaf,	"	15 c	
	713	"	" Selections,	"	16⅝c.	

Rec'd pay't, $

L. L. WILLIAMS & CO.

No. 6.

NEW ORLEANS, *Dec. 17, 1876.*

Bryant and Nelson,

TERMS—Dft. 30 days. *Bot. of Wm. Horn & Co.*

1876						
	1420	lbs.	Sugar, Common,	@	5½c.	
	1927	"	" Good,	"	7¼c.	
	2810	"	" Fair,	"	7⅞c.	
	902	"	" Prime,	"	8½c.	
	813	"	" Choice,	"	9¼c.	
	2741	"	" Yellow Cent'al	"	10½c.	

Rec'd pay't, $

No. 7.

NEW ORLEANS, *Dec. 23, 1876.*

Montgomery & Lettellier,

TERMS—3 mos. *Bot. of Sadler & Smith.*

	7	Gross	Chewing Tobacco	@	$13.
	180	lbs.	Smoking do	"	1.40
	6	M.	Havana Cigars	"	70.
	2	M.	N. O. Manufacture do	"	30.

Rec'd pay't. $

No. 8.

NEW ORLEANS, *Jan'y 21, 1877.*

Jones and Carpenter,

Bot. of Stewart & Henderson.

TERMS—Note 60 days.

1877				
	34 bbls. La Orange, large,	@	$5.75	
	27 boxes Messina Lemons,	"	6.00	
	63 cases Malaga Grapes,	"	1.75	
	45 boxes California Pears,	"	4.50	
	5 mats Dates, 593 lbs.,	"	7½c.	

Rec'd pay't, by note at 60 days. $

STEWART & HENDERSON.

No. 9.

NEW ORLEANS, *Nov. 17, 1876.*

Geo. B. Brackett & Co.,

Bot. of R. Spencer Soule.

TERMS—Cash.

	1427 bu. No. 1 Winter Wheat	@	$1.55	
	856 " No. 2 Winter Wheat	"	1.47	
	420 " Ill. No. 1 White do	"	1.41	
	3145 " W. Corn	"	.70	
	1040 " B. Oats	"	.55	

Rec'd pay't, $

R. SPENCER SOULE.

No. 10.

NEW ORLEANS, *Feb. 4, 1877.*

F. L. & W. P. Richardson,

Bot. of P. W. Sherwood & Co.

TERMS—1 mo.

	30 box. Sperm Candles, 596 lbs.	@	.35½	
	24 do. Adam. Ex. do. 483	"	" .28	
	15 do. Sil. Gloss St'rch 360	"	" .10¾	

Rec'd pay't. $

No. 11.

NEW ORLEANS, *Jan. 19, 1877.*

Heald & Howe,

Bot. of Cole & Montague.

TERMS—Due Bill 1 mo.

342	lbs. La. Pecans	@	$	.13
289	" Taragona Almonds	"		.21
175	" Naples Walnuts	"		.17
196	" Brazil Nuts	"		.11
268	" Western Chestnuts	"		.18
160	Boxes Figs	"		.20
585	Cocoanuts @ $45 per M.			
61	Bunches Bananas	"		1.75
14	do. Plantains	"		.85
327	Pine Apples @ $80 per M.			

Rec'd pay't, $

COLE & MONTAGUE.

No. 12.

NEW ORLEANS, *Feb. 1, 1877.*

Hibbard & Gray,

Bot. of Odell & Faddis.

TERMS—Cash.

2714	lbs. Black Moss	@	4	¢
1829	" Gray do.	"	1¼	¢
913	" Wool	"	24	¢
74	" Live Geese Feathers	"	65	¢
1528	Packages Broom Corn	"	6	¢
752	lbs. Baling Twine	"	14	¢
800	yds. Indian Bagging	"	11	¢

Rec'd pay't, $

ODELL & FADDIS,

Per C. P. Meads.

No. 13.

NEW ORLEANS, *Jan. 7, 1877.*

Spaulding & Musselman,

Bot. of Warr & Bogardus.

TERMS—60 days.

78½	yds. Black Silk,	@	$2.90	
148	" Muslin,	"	16 c.	
62	" Cassimeres,	"	1.75	
38¼	" Blk. F. Broadcloth	"	5.25	
45	" " " Doeskin,	"	1.50	
324	" American Satinets,	"	95 c	
3 cases, each 40 ps. Amoskeag Sheetings, 1123¹ 1204 1096²	3423¾ yds.	"	17½c.	
142	yds. 6–4 Alpaca,	"	32 c.	
560	" Union Ginghams,	"	11¾c.	
491	" Am. Fancy Prints,	"	12 c.	
107	" Manch'ter Delains,	"	21¼c.	
10	doz. Handkerchiefs,	"	2.15	
16	" $\$\frac{8}{4}$., $\$\frac{4}{4.50}$, $\$\frac{4}{3.75}$ Ladies Hose.			
2 ps.	61½ yds. Can. Flannel	"	18 c.	

Rec'd pay't, $

No. 14.

NEW ORLEANS, *Feb. 8, 1877.*

Tasker & Felton,

Bot. of Allen & Shields.

TERMS—Cash.

50 lbs. Casing Nails,	@	$	7c.
1¾ doz. Mortice Locks,	"		7.50
½ " Porcelain Knobs,	"		4.75
50 pr. Butts,	"		25c.
3 Gross Screws,	"		75c.
8 bars, 1½ × ½ Bar Iron, 254 lbs.,	"		5c.
2 Rowland No. 2 Spades,	"		1.

Rec'd pay't, $

ALLEN & SHIELDS.

No. 15.

NEW ORLEANS, *Mar. 9, 1877.*

Lee and Ward,

Bot. of Harris & DeRussy.

TERMS—Due end mo.

3 reams Cap Paper,	@	$	3.25
2 doz. Ebony Rulers,	"		3.50
4 6 qr. Med. Ledgers 24 qrs.	"		1.75
3 3 " Demy Journals, 9	"	"	1.25
3 3 " " Cash Books, 9	"	"	1.25
3 6 " " Sales Books 18	"	"	1.15
4 gross Pen-holders,	"		2.10
¾ doz. Black Ink,	"		4.50
½ ream Blotting Paper,	"		2.50
2 doz. Mucilage,	"		2.75
3 " Carmine,	"		2.15
3 5/12 doz. Bill-books,	"		4.25

Rec'd pay't, $

No. 16.

NEW ORLEANS, *April 1, 1877.*

E. J. & R. Paul,

Bot. of Gresham & Harp.

TERMS—Due bill 30 days.

½	doz. Comb's Con. of Man,	@ $12.	
2	" Dana's Geological Story briefly told,	" 10.	
6	" Soule's Phi. Arithmetics	" 42.	
6	" " Con. in Numbers,	" 18	
6	" " Prim. Arithmetic	" 9.	
¾	" Webster's Acad. Dict.,	" 18.	
2	" Swinton's Lan. Lessons,	" 3.25	
1½	" Steel's Nat. Philosophy,	" 12.	
⅓	" Spencer's Science of Sociology,	" 10.50	
2	copies Wood's Byron,	" 3.	
4	" Dick's Shakspeare,	" 4.50	
drayage 75¢, box 50¢.			

Rec'd pay't. $

GRESHAM & HARP.

No. 17.

NEW ORLEANS, *Jan. 1, 1876.*

T. Janney,

To A. Laborde. *Dr.*

	For furnishing, making, and laying Bru sels carpeting 22 inches wide in 2 rooms measuring as follows:		
	No. 1. 24 ft. 3 in. × 20 ft. 9 in.		
	No. 2. 18 ft. 6 in. × 17 ft. 8 in.		
*	— yards,	@ $1.95	
68	" 6–4 Che. Mat.	" 90c.	

Rec'd pay't. $

* In this bill no allowance is made for waste in matching or otherwise.

No. 18.

NEW ORLEANS, *Jan. 25, 1877.*

Warner & Carnell,

Bot. of Goldsmith and Clark.

TERMS—Note at 60 days.

2143 lbs.———bush. Yellow Corn,	@ $	61c.
1241 lbs.——— " Texas Wheat,	"	1.70
852 lbs.——— " White Oats,	"	54c.
789 lbs.——— " Barley	"	83c.
1427 " ———Cwt. Bran,	"	75c.
3743 " ———Tons Timothy Hay,	"	18.50
1701 lbs.———Tons Clover Hay,	"	20.

Rec'd pay't, by note at 60 days. $

GOLDSMITH AND CLARK.

No. 19.

NEW ORLEANS, *Jan. 3, 1877.*

Geo. F. Bartley & Co.,

To Steamship Knickerbocker and Owners, Dr.

For Freight on —— cubic feet @ 25¢	
The same being contents of 8 boxes measuring as follows:	
Nos. 1, 2 & 3, 5 ft. 4 in. x 4 ft. 6 in. x 2 ft. 8 in. =	
Nos. 4, 5 & 6, 6 ft. 2 in. x 3 ft. 0 in. x 2 ft. 11 in. =	
Nos. 7 & 8, 12 ft. 3 in. x 2 ft. 1 in. x 1 ft. 6 in. =	

Rec'd pay't. $

No. 20.

NEW ORLEANS, *Jan. 4, 1877.*

O. Emmet & Co.,

To B. Criswell. *Dr.*

For rent of house No. 386 Dryades St., from Oct. 7, 1876, to Jan. 1, 1877, $2\frac{2}{3}\frac{3}{0}$ months, @ $35		
For services as collector from Sept. 19, 1876, to Jan. 4, 1877, both inclusive, $3\frac{1}{3}\frac{6}{0}$ months, @ $75		
Rec'd pay't,		$

No. 21.

NEW ORLEANS, *Jan. 9, 1877.*

The La. Levee Co.,

To James Selleck & Co.

For Constructing ——— cubic yards Levee @ 45¢ as per the following measurements:		
1st Section $890\frac{3}{1}\frac{}{2}$ ft. long, 70 ft. wide at the base and 30 ft. at the top, with an average depth of $8\frac{4}{1}\frac{}{2}$ ft.		
2nd Section 165 ft. long, 60 and 25 ft. respectively for the lower and upper widths, and 6, $7\frac{1}{2}$, $5\frac{1}{2}$, 8, 9, and $6\frac{1}{2}$ ft. in depth at different points.		
For Excavating ——— cubic yards Earth @ 45¢, the same being the contents of a cellar measuring as follows:		
92 ft. long and 50 ft. wide at the top, and 86 ft. long and 44 ft. wide at the bottom, average depth 8 ft. 4 in.		
Rec'd pay't,		

No. 22.

NEW ORLEANS, *Jan. 16, 1877.*

Geo. Soule,

To Clark & Hofeline. *Dr.*

For composition Arithmetical Exercises and examples 180 pp., 1600 ems a page, @ 75c. per m.		
For press work on 54 token, @ $ 50c.		
For 4 Reams Paper, " 6.00		
For Binding 500 sep., " 25c.		

Rec'd pay't, $

CLARK & HOFELINE.

No. 23.

NEW ORLEANS, *Jan. 18, 1877.*

Western Union Telegraph Co.,

To Jacob Simon & Co., Dr.

For ——— cubic feet Timber @ $24 per 100; the same being the contents of 50 Telegraph poles measuring as follows:		
40 Poles are 70 feet long, 16x16 inches at the larger end and so remain for a distance of 10 feet, at which point they begin and taper regularly to the smaller end, which is 6x6 inches.		
10 Poles are 60 feet long, 16x12 inches at the larger end, 6x4 inches at the smaller end, and taper regularly the whole length.		

Rec'd pay't.

No. 24.

NEW ORLEANS, *Jan'y 29, 1877.*

S. Dreyfuss,

To V. Keiffer. *Dr.*

Jan.	1	To old balance, as per bill rendered.		91	10
	6	12 cords Ash Wood	@ $7.	84	00
	6	4 cords Oak Wood	" $6.50	26	00
	14	50 bbls. Pittsburg Coal	" 60c.	30	00
				$231	10
		Cr.			
	8	By Cash	$50		
	29	" 6 days Labor, at $4,	$24	74	00
		Balance due Jan. 29, 1877.		$ 157	10

Settled by note at 60 days.

V. KEIFFER.

No. 25.

NEW ORLEANS, *Jan. 12, 1877.*

A. & S. H. Soule,

To Z. M. Pike & Co. *Dr.*

	To 4378 feet Com. Boards @ $21. per m.		
	" 1760 feet Dressed Flooring " $28.50 per m.		
	" 5125 Bricks " $14.25 per m.		
	" 9250 Cypress Shingles " $6.50 per m.		
	" Cartage and Labor	14	25
	Rec'd pay't,	$	

No. 26.

NEW ORLEANS, *Dec. 31, 1876.*

Levy & King,

To R. & C. Rice, Dr.

For ——— sq. yds. North River Flags @ $7.50 as per the following measurements:

Nos. 1, 2 & 3, are each 4 ft. 3 in. by 3 ft. 6 in. = sq. ft.

Nos. 4, 5 & 6 are each 4 ft. 8 in. by 3 ft. 4 in. = sq. ft.

Nos. 7 & 8 are each 4 ft. 0 in. by 3 ft. 0 in. = sq. ft.

Nos. 9, 10 & 11 are each 3 ft. 4 in. by 2 ft. 9 in. = sq. ft.

For ——— sq. yds. German Flags @ $2.25, comprising 152 Flags, each 22x16 inches.

For ——— sq. yds. Brick Pavement @ $1.15, contained in a sidewalk measuring 124 ft. 4 in. long by 11 ft. 9 in. wide.

For 124 ft. 4 in. Curbing @ $ 1.30

For ——— cu. yds. Granite " $16.00 contained in 23 blocks of stone measuring as follows:

Nos. 1 to 7 inclusive, are each 26x15x10 inches, =

Nos. 8 to 20 inclusive, are each 23x16x9¼, =

Nos. 21 to 23 inclusive, are each 42x35x21 inches, =

Rec'd pay't, $

No. 27.

NEW ORLEANS, *Jan. 28, 1877.*

Invoice of Sundries, purchased by J. Simmons & Co. and shipped per Steamer La Belle, for acc't. and risk of James Byrnes, Shreveport, La.

87 bbls. Molas's, 3498 gals. @	60c.		
20 hhds. Sugar, 23780 lbs. "	9c.		
10 bbls. Rice, 2150 lbs. "	5c.		
Charges.		4346	50
Drayage		17	50
Insurance on $4800.40 "	⅝ %	35	00
Commission on $4364.00 "	2½ %	109	10
		$ 4503	10

No. 28.

NEW ORLEANS, *Jan. 14, 1877.*

W. H. Carey,

To Geo. Jumonville, Dr.

For Slating a roof measuring 72 ft. 4 in. by 49 ft. 10 in. and containing —— squares	@ $14.50	
For 239 ft. Guttering	" .90	

Rec'd pay't.

No. 29.

NEW ORLEANS, *Feb. 1, 1877.*

Mississippi Valley Transportation Co.,

To Buck & Richardson, Dr.

For Servises rendered in cause No. 55472. "Steamer R. E. Lee and owners vs. Miss. V. T. Co."	

Rec'd pay't.

No. 30.

NEW ORLEANS, *Nov. 4, 1876.*

New Orleans, St. Louis and Chicago R. R.,
To W. L. & H. Hall, Dr.

	For 150 Cisterns holding ——— gals. @ 2½¢ per gal. The inside measurement of each cistern is as follows: 11 ft. 3 in. perpend cular height, lower base 9 ft. 2 in. and upper base 8 ft. 5 in.	
	Rec'd pay't.	

TABLES OF WEIGHTS AND MEASURES.

175. **Weight** is that property of bodies by virtue of which they tend toward the center of the earth, and the resistance required to overcome this centralizing pressure, or gravitating tendency of bodies, is what is named weight; Weight varies according to the quantity of matter a body contains, and its distance from the centre of the earth.

176. **A Measure** is a *standard unit* established by law or custom, by which *quantity*, such as extent, dimension, capacity, amount or value, is measured or estimated.

There are seven kinds of measure:

1st. Length. 2d. Surface or Area. 3d. Solidity or Capacity. 4th. Weight or Force of Gravity. 5th. Time. 6th. Angles. 7th. Money or Value.

COMPARISON OF STANDARD UNITS OF MEASURE.

1.	The Yard=3 feet=	36 inches.
2.	The Meter=3.2808 feet=	39.37 inches.
3.	The Vara=2.7778 feet=	33⅓ inches.
4.	The Troy and Apothecaries pound=12 oz.=	5760 grains.
5.	The Avoirdupois pound=16 oz.=	7000 grains.
6.	The Wine gallon=	231 cubic inches.
7.	The Beer gallon—(nearly obsolete)=	282 cubic inches.
8.	The Dry gallon=	268.8 cubic inches.

9. The Imperial gallon of England= 277.274 cubic inches.
10. The Bushel=4 pks.=32 qts.=64 pts.= 2150.42 cubic inches.
11. The Imperial Bushel of England=2218.192 cubic inches.
12. The Diamond grain is equal to .8 of a grain Troy.
13. The Gallon, wine measure, of distilled water weighs 8.3388 pounds Avoirdupois or 10.134 pounds Troy.
14. The *Civil Day* commences and ends at midnight, and the *Astronomical Day*, at noon of the Civil Day.
15. The *Solar Day* is the interval of time between two successive passages of the sun across the meridian of any place, and they are of unequal length on account of the unequal orbital motion of the earth and the obliquity of the ecliptic.

RATIOS.

When the diameter of a circle is 1, the circumference is 3.1416.
When the area of a square is 1, the area of a circle, the diameter of which is equal to one side of the square, is .7854.
When the solidity of a cube is 1, the solidity of a sphere, the diameter of which is equal to one side of the cube, is .5236.

WEIGHT OF COIN.

$10000 Gold=258000 gr.=44 lbs. 9 oz. 10 pwt. 0 gr. Troy.
$1000 Silver dollars old issue=412500 gr.=71 lbs. 7 oz. 7 pwt. 12 gr.
$1000000 Gold weigh 53750 ounces Troy or 3685.71 Avoirdupois pounds.
$1000000 Silver Trade dollars weigh 875000 ounces Troy or 60000 pounds Avoirdupois.
$1000000 Silver, half and quarter dollars, 20 cent pieces and dimes, weigh 803750 ounces Troy or 55114.28 Avoirdupois pounds.

VALUE OF COIN.

Gold coin=about 86 cents per pennyweight.
Silver coin=about $1.11 per ounce.

For more extended tables of weights and measures, a condensed history of time measure and the units of measure in use in the early ages of civilization, see Soulé's Philosophic, Commercial and Exchange Calculator, pages 121 to 152 and 497 to 519.

I.—UNITED STATES MONEY.

TABLE.

10 Mills (m.)	=	1 Cent,	¢
10 Cents	=	1 Dime,	d.
10 Dimes	=	1 Dollar,	$
10 Dollars	=	1 Eagle,	E.

II.—ENGLISH MONEY.

TABLE.

4 Farthings (far.)	=	1 Penny,	d.
12 Pence	=	1 Shilling,	s.
20 Shillings	=	1 Pound (Sovereign,)	£
£1	=	$4.8665	

III.—FRENCH MONEY.

TABLE.

10 Centimes	=	1 Decime.
10 Decimes or 100 Centimes	=	1 Franc.
Fr. 1	=	19 3 Cents.

TIME MEASURE.

60 Seconds (s.)	=	1 Minute,	m.
60 Minutes	=	1 Hour,	h.
24 Hours	=	1 Day,	d.
7 Days	=	1 Week,	wk.
365 Days	=	1 Common Year,	yr.
366 Days	=	1 Leap Year,	yr.
100 Years	=	1 Century,	c.

The names and orders of the months, and the number of days contained in each, are now as follows:

Names.	No.	No. days.	Names.	No.	No. days.
January,	1st,	31	July,	7th,	31
February	2d,	28	August,	8th,	31
March,	3d,	31	September,	9th,	30
April,	4th,	30	October,	10th,	31
May,	5th,	31	November,	11th,	30
June,	6th,	30	December,	12th,	31

The number of days in each, may be readily remembered by committing to memory the following lines:

"Thirty days hath September,
April, June, and November;
And all the rest have thirty-one,
Save February, which alone
Hath twenty-eight; and this, in fine,
One year in four hath twenty-nine."

(LINE) LINEAR OR LONG MEASURE.

TABLE.

12	Inches (in.)	= 1	Foot,	ft.
3	Feet	= 1	Yard,	yd.
$5\frac{1}{2}$	Yards or $16\frac{1}{2}$ feet	= 1	Rod or Pole,	rd. *or* po.
40	Rods	= 1	Furlong,	fur.
8	Furlongs	= 1	Mile (Statute Mile)	m.
3	Miles	= 1	League,	lea.

MARINERS' MEASURE.

TABLE.

6	Feet	= 1 Fathom.
120	Fathoms	= 1 Cable-length.
880	Fathoms or $7\frac{1}{3}$ Cable-lengths	= 1 Mile.

A *knot*, or geographical mile, is $\frac{1}{60}$ of a degree, and is equivalent to 1.15257 statute miles.

The length of a degree at the Equator is nearly equal to $69\frac{1}{6}$ statute miles. The length of an average degree on the meridian is 69.042 statute miles.

MISCELLANEOUS UNITS OF LINEAR MEASURE.

TABLE.

$\frac{1}{12}$	of an Inch	= A Line (American).
$\frac{1}{10}$	of an Inch	= A Line (French).
4	Inches	= A Hand.
3	Inches	= A Palm.
9	Inches	= A Span.
3	Feet	= A Pace.
$2\frac{1}{3}$	Feet (28 in.)	= A Military Pace.

CLOTH MEASURE.

TABLE.

2¼	Inches (in.)	=	1 Nail,	na.
4	Nails (9 inches)	=	1 Quarter,	qr.
4	Quarters	=	1 Yard,	yd.

This table formerly contained:

The Flemish Ell, which equaled 3 quarters or 27 inches;
The English Ell, which equaled 5 quarters or 45 inches;
The French Ell, which equaled 6 quarters or 54 inches.

These units of measure are nearly out of use.

SURVEYORS' AND ENGINEERS' MEASURE.

TABLE.

7.92	Inches	=	1 Link,	li.
25	Links	=	1 Rod or Pole,	rd. *or* po.
4	Poles or	=	1 Chain,	ch.
66	Feet			
80	Chains	=	1 Mile,	m.

Engineers use another chain which consists of 100 links, each 1 foot long.

SQUARE OR SURFACE MEASURE.

TABLE.

144	Square Inches (sq. in.)	=	1 Square Foot,	sq. ft.
9	Square Feet	=	1 Square Yard,	sq. yd.
30¼	Square Yards	=	1 Square Rod, *or* Perch,	sq. rd. p.
40	Square Rods or Perches	=	1 Rood,	r.
4	Roods	=	1 Acre,	a.
640	Acres	=	1 Square Mile, *or* Section,	sq. m. sec.
36	Square Miles (6 miles sq.)	=	1 Township,	T.
16	Perches	=	1 Square Chain,	sq. ch.
10	Square Chains	=	1 Acre,	

CUBIC OR SOLID MEASURE.

TABLE.

1728	Cubic Inches	=	1 Cubic Foot.
27	Cubic Feet	=	1 Cubic Yard.
16	Cubic Feet	=	1 Cord Foot
8	Cord Feet or 128 Cubic Feet	=	1 Cord of Wood.
24¾	Cubic Feet, or 16½ feet long, 1½ high and 1 foot wide	=	1 Perch.
40	Cubic Feet of round timber,	=	1 Ton or Load.
50	Cubic Feet of hewn timber		

A cubic foot contains 7 4805 Wine gallons.
A cubic foot contains .2374 barrels "
A cubic foot contains .8082 bushels.

LIQUID MEASURE.

TABLE.

4	Gills (gi.)	=	1 Pint,	pt.
2	Pints	=	1 Quart,	qt.
4	Quarts	=	1 Gallon,	gal. =231 cubic in.
31½	Gallons	=	1 Barrel,	bbl.
2	Barrels or 63 gallons	=	1 Hogshead,	hhd.
2	Hogsheads	=	1 Pipe,	P.
2	Pipes	=	1 Tun,	T.

DRY MEASURE.

TABLE.

2	Pints (pt.)	= 1 Quart,	qt.
8	Quarts	= 1 Peck,	pk.
4	Pecks	= 1 Bushel,	bu.
8	Bushels	= 1 Quarter,	qr.
36	Bushels	= 1 Chaldron,	ch.

TROY OR MINT WEIGHT.

TABLE.

24	Grains (gr.)	= 1 Pennyweight,	dwt. or pwt.
20	Pennyweights	= 1 Ounce,	oz.
12	Ounces	= 1 Pound,	℔.

AVOIRDUPOIS WEIGHT.

TABLE.

27$\frac{11}{32}$	Grains	= 1	Dram,	dr.
16	Drams	= 1	Ounce,	oz.
16	Ounces	= 1	Pound,	lb.
25	Pounds	= 1	Quarter,	qr.
4	Quarters or 100 pounds	= 1	Hundredweight,	cwt.
20	Hundredweight or 2000 pounds	= 1	Ton,	t.
480	Pounds	= 1	Imperial Quarter.	
100	Pounds is also called	1	Cental,	c.

The cwt. in England, and in some cases in the United States, is 112 pounds, or 4 quarters of 28 pounds. The ton English is 2240 pounds. This is called the *long ton*, and 2000 pounds, the *short ton*.

APOTHECARIES' WEIGHT.

TABLE.

20	Grains (gr.)	= 1	Scruple,	scr. or ℈
3	Scruples	= 1	Dram,	dr. " ʒ
8	Drams	= 1	Ounce,	oz. " ℥
12	Ounces	= 1	Pound,	lb. " ℔

MEDICAL DIVISIONS OF THE GALLON.

TABLE.

60	Minims (m.)	= 1	Fluidram,	f ʒ
8	Fluidrams	= 1	Fluidounce,	f ℥
16	Fluidounces	= 1	Pint,	O.
8	Pints	= 1	Gallon,	Cong.

O. is an abbreviation of *octans*, the Latin for one-eighth; *Cong.* for *congiarium*, the Latin for gallon.

DIAMOND WEIGHT.

TABLE.

16 Parts	= 1	Grain.
4 Grains	= 1	Carat.
1 Carat	= 3$\frac{1}{5}$	Grains Troy, nearly.

ASSAYERS' WEIGHT.

TABLE.

1 Carat	=	10 Pwts. Troy.
1 Carat grain	=	2 Pwts. 12 grains, or 60 grains Troy.
24 Carats	=	1 Pound Troy.

The term carat is also used to express the fineness of gold,—each carat meaning a twenty-fourth part.

SHOEMAKERS' MEASURE.

No. 1 small size is $4\frac{1}{8}$ inches, and every succeeding No. increases $\frac{1}{3}$ of an inch to 13.

No. 1 large size is $8\frac{1}{4}$ inches, and every succeeding No. increases $\frac{1}{2}$ of an inch to 15.

CIRCULAR MEASURE.

TABLE.

60 Seconds		= 1 Minute,	′
60 Minutes		= 1 Degree,	°
30 Degrees		= 1 Sign,	s.
12 Signs or 360°		= 1 Circle,	c.
90 degrees make	1 quadrant or right angle.		
60 " "	1 sextant or sixth of a circle.		
180 " "	1 semi-circle or half-circle.		

MISCELLANEOUS TABLES.

BOOKS AND PAPER.

SIZE OF PAPER.

	Inches.		Inches.
Demy	17 by 22	Letter	10 by 15
Medium	19 " 24	Folio post	16 " 21
Double medium	24 " 38	Foolscap	14 " 17
Super-royal	21 " 27	Crown	15 " 20
Imperial	22 " 32	Double Elephant	26 " 40

A sheet (medium) folded in 2 leaves is called folio.
" " " 4 " " quarto or 4to.
" " " 8 " " octavo or 8vo.
" " " 12 " " duodecimo or 12 mo.

A sheet (medium) folded in 16 leaves is called 16mo.
" " " 18 " " 18mo.
" " " 24 " " 24mo.
" " " 32 " " 32mo.

24 Sheets = 1 Quire.
480 Sheets = 20 Quires = 1 Ream.
2 Reams = 1 Bundle; 5 Bundles = 1 Bale.
12 units = 1 dozen.
144 units = 12 dozen = 1 gross.
12 gross = 1 great gross.
20 units = 1 score.
56 lb. = 1 firkin of butter.
100 lb. = 1 quintal of dried fish.
196 lb. = 1 barrel of flour.
200 lb. = 1 barrel of flour in California.
200 lb. = 1 barrel of beef, pork, or fish.
280 lb. = 1 barrel of salt.
100 lb. = 1 cask of raisins.
14 lb. iron or lead = 1 stone.
12 barrels of wheat = 7 English quarters.
31½ stone = 1 Pig; 8 pigs = 1 fother.
256 pounds of soap = 1 barrel.
25 pounds of powder = 1 keg.
18 Inches = 1 Cubit.

WEIGHT OF GRAIN AND PRODUCE PER BUSHEL,

AS USED IN NEW ORLEANS WHEN THERE IS NO AGREEMENT TO THE CONTRARY.

Wheat,	bush. 60 lb.		Flaxseed,	bush. 56 lb.	
Corn,	" 56 "		Hempseed,	" 44 "	
Rye,	" 56 "		Buckwheat,	" 52 "	
Oats,	" 32 "		Castor Beans,	" 46 "	
Barley,	" 48 "		Dried Peaches,	" 33 "	
Irish Potatoes,	" 60 "		Dried Apples,	" 24 "	
Sweet Potatoes,	" 60 "		Onions,	" 57 "	
Beans,	" 62 "		Coarse Salt,	" 50 "	
Bran,	" 24 "		Fine Salt,	" 50 "	
Clover seed,	" 60 "		Stone Coal,	" 80 "	
Timothy seed,	" 45 "		Corn Meal,	" 44 "	
Barley Malt,	" 34 "		Plastering Hair,	" 7 "	
Peas, split,	" 60 "		Blue Grass seed,	" 10 "	
Small Hominy,	" 50 "				

Soule's Commercial College and Literary Institute.

Cor. St. Charles and Lafayette Sts.,
NEW ORLEANS, LA.

Founded in 1856.
Chartered in 1861.

This Institution comprises the following Schools on the elective system:

A Full Course Practical Business College,

In which mercantile and banking officers are connected, daily lectures given, and business regularly transacted.

An Eclectic English Institute,

For boys thirteen years and upward. And an

Object Teaching School,

For boys from eight to fourteen years.

There are also departments for the ANCIENT and MODERN LANGUAGES, HIGHER MATHEMATICS, DRAWING, NATURAL SCIENCES, ETC.

This institution has a large, efficient and faithful corps of teachers, who have qualified for special service in the various fields of education; its curriculum embraces

THIRTY-TWO BRANCHES OF STUDY,

taught by the most improved, progressive and philosophic methods.

Lectures are given on the various subjects studied, and also on ANATOMY, PHYSIOLOGY, HYGIENE, PHRENOLOGY, ASTRONOMY, GEOLOGY, ETC.

All persons interested in the cause of progressive education are cordially invited to visit and inspect the special advantages of this institution. Parents of our students are solicited to often consult and advise us of their wishes in regard to the educational interests of their sons. For circulars, address,

GEO. SOULE,
President and Proprietor.