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Spatially-Limited Altruism, Mixed Clubs, and Local Income Redistribution

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Abstract

This paper extends the work of Pauly (1973) by analysing the optimal organization of an economy in which individuals experience spatially-limited altruism. With such altruism, the nonpoor members of society care more about the poor living near them than about those living farther away. The main theme of the paper is that while the proximity of the poor gives mixed communities an altruistic advantage over homogeneous communities, the intermixing of rich and poor generates an efficiency loss in that public consumption in mixed communities cannot be tailored to suit individual preferences. As a result, a mixed community configuration (where income redistribution proceeds through local transfers) may or may not be superior to a homogeneous configuration (in which redistribution is conducted by the federal government). In addition to analysing this altruism/efficiency loss trade-off, the paper characterizes equilibrium outcomes when communities are organized by competitive developers.

Spatially-Limited Altruism, Mixed Clubs, and Local Income Redistribution

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Jan K. Brueckner and Kangoh Lee*

1. Introduction

In an important paper, Pauly (1973) introduced the notion of spatially-limited altruism into the income redistribution literature. With this type of altruism, the nonpoor members of society care more about the poor living near them than about those living far away. Using this assumption, Pauly argued that income redistribution should be locally controlled and tailored to suit the features of each community. He argued that national redistribution, which imposes a nationally uniform policy, generates an efficiency loss by preventing such local discretion.

The purpose of the present paper is to give a more complete treatment of Pauly's idea by analysing the optimal organization of an economy in which people experience spatially-limited altruism. In contrast to Pauly's normative analysis, which assumed a fixed spatial distribution of the population, the present paper attempts to determine the spatial grouping of rich and poor that is best for society. Although a direct application of Pauly's model would suggest that the rich and poor should live together so that the rich can reap the greatest benefit from their generosity, the issue is more complex in the present framework. This is a consequence of the additional assumption that individuals in the economy consume public goods. Since public consumption in mixed communities cannot be tailored to suit individual preferences, enjoyment of the altruistic benefits of such communities entails an efficiency loss on the consumption side. This loss is avoided in homogeneous communities but enjoyment of altruism is sacrificed. Much of the analysis in the paper is devoted to analysing this altruism/efficiency loss trade-off and to identifying the different conditions under which mixed and homogeneous communities are desirable. The paper also analyses equilibrium community configurations under the assumption that communities are organized by competitive, profit-maximizing developers.[1] It is shown (subject to certain qualifications) that mixed communities emerge in equilibrium whenever such a configuration is desirable from an efficiency standpoint.

The lessons of the analysis regarding which level of government should carry out income redistribution are somewhat different than in Pauly's paper. It is clear that if the optimal organization of the economy entails mixed communities, then there is no need for a national redistribution program since local transfers can achieve identical results. While local redistribution can thus be optimal in that it may be no worse than a national policy, a national system is clearly required if the economy is to be organized into homogeneous communities. For this reason, reliance on local redistribution can be inefficient even in the presence of spatially-limited altruism, in contrast to Pauly's conclusion.

The paper's analytical framework is based on the standard economic model of clubs, as developed by Buchanan (1965), Berglas (1976b), and Berglas and Pines (1981).[2] Although the connection to Pauly makes the local redistribution question an important focus of the analysis, the paper's main contribution is to extend the theory of clubs by stating new conditions under which mixed clubs might be optimal. There has been considerable interest in this issue in the literature. Berglas (1976a), for example, showed that mixing is desirable when different types of people are complementary in production. More recently, Berglas (1984) proved the much less obvious result that mixing may be optimal in the presence of multiple public goods. In analysing spatially-limited altruism, this paper identifies a new force favoring the formation of mixed clubs.

2. Normative Analysis

The model has two types of individuals, a and b, with the a's feeling altruism for the b's, as specified further below. The b's comprise a fraction θ of the economy's total population N, with the a's accounting for $1-\theta$ of the total. Exogenous incomes for the two groups are I^a and I^b. Given the a's altruism, it is natural to suppose that the b's are relatively poor ($I^b < I^a$), although this assumption plays no role in the analysis. Consumption goods in the economy include a private good x and a congested public good z. The cost in terms of x of providing public consumption z to a community (hereafter "club") of n people is $C(z,n)$. C is increasing and convex in z, and congestion implies that the partial derivative C_n is positive. A further assumption is that for any $z > 0$, per capita cost $C(z,n)/n$ is a U-shaped function of n, which guarantees the existence of a positive finite optimal club size.

The (well-behaved) type-a and type-b utility functions are $U(x,z,k)$ and $V(x,z)$, where the k argument captures the altruism felt by the a's. A fundamental assumption is that this altruism is spatially limited, which means that an a-type cares more about the b's living in his own club than those living in other clubs. Moreover, the possibility of joint consumption of public goods means that k does not depend simply on the post-transfer income of the b's (as in the usual formulation of altruism) but instead reflects their achieved utility level. There are various ways of modeling the spatially-limited utility interdependence implied by these assumptions. One possibility would be to assume that k equals a population-weighted average of the utility levels of type-b consumers, with a higher weight applied to b's in the home club of a representative a-type. In other words, for an a-type living in a club containing n^b b-types, k would equal $[(\alpha + \beta)n^b v^{home} + \beta(\theta N - n^b)v^{away}]/\theta N$, where v^{home} is the type-b utility in the home club, v^{away} is the (average) type-b utility in other clubs, $\beta \geq 0$ is a parameter measuring "generalized" altruism (which is felt regardless of the location of the b's), and $\alpha \geq 0$ is parameter measuring "local" altruism (which is felt only toward b's in the same club). Recall from above that θN equals the number of b-types in the economy. While this is in some ways a natural formulation, it has the peculiar implication that for a given uniform b-type utility ($v^{home}=v^{away}$=constant), k is increasing in n^b, implying that an a-type is happier in a club with a larger type-b population. This seems inconsistent with typical behavior, under which the nonpoor care about the welfare of the poor but do not wish to live surrounded by them. A modification that addresses this objection would be to write the v^{home} term above as $[\alpha g(n^b) + \beta n^b]v^{home}/\theta N$, where g is a function satisfying $g(n^b) \geq 0$, $g(0) = 0$, and $g'(0) > 0$. This formulation allows k to rise initially with the type-b population when $v^{home}=v^{away}$, but the possibility that g' could turn negative means that further increases in n^b may ultimately depress k in a realistic fashion.

Since the appearance of the absolute population size n^b in the modified k formula is inconvenient in the later analysis, the formula is further altered to read $k = [(\alpha f(\sigma) + \beta)v^{home} + \beta v^{away}]$, where σ is the proportion of b-types in the club population and f is a function satisfying $f(\sigma) \geq 0$, $f(0) = 0$, and $f'(0) > 0$. This formulation eliminates the population weights on generalized altruism, so that for given home and away utilities, generalized altruism is independent of the distribution of the type-b population. This is plausible given that altruism may reflect an implicit division of the poor population into local and nonlocal components that ignores the relative sizes of these groups. An additional change is that local altruism now depends on the proportion of b's in the local population rather than their absolute number (the factor $1/\theta N$ is also suppressed). In the spirit of the earlier formulation, k initially rises with σ but further increases in the type-b proportion may be make the a's worse off. While this formulation is somewhat arbitrary, it captures the essential aspect of spatially-limited altruism and it proves to be convenient in the analysis.

The normative problem is to characterize Pareto-efficient club configurations in the model. For purposes of clarity, it is desirable to focus first on two simple configurations: one consisting of homogeneous type-a and type-b clubs and another consisting of identical mixed clubs. The first part of this section analyses these two configurations and proves a number of results regarding their relative merits. Once this analysis is complete, the discussion considers more complex configurations containing both mixed and homogeneous clubs. The ultimate goal is to state conditions which allow the optimal club configuration among all those considered to be identified.

Efficient configurations are analysed under the standard requirement of horizontal equity (identical utilities for identical people). Moreover, all clubs of a given type (mixed, homogeneous type-a, homogeneous type-b) are constrained to be identical (configurations violating this requirement are inefficient).[a] To analyse a homogeneous club configuration (denoted H hereafter), the first step is to note that since type-b utilities are uniform under the horizontal equity requirement, the altruism expression k is evaluated with $v^{home}=v^{away}=v$. Furthermore, since clubs are homogeneous, the type-b proportion σ equals zero in each club where the a-types live. Recalling that $f(0) = 0$, these facts mean that the altruism argument under H satisfies $k = 2\beta v \equiv \delta v$, where v is the uniform type-b utility and $\delta \equiv 2\beta$. A Pareto-efficient H configuration then solves the following problem:

$$\max \quad U(x^a, z^a, \delta v)$$

$$\text{s.t.} \quad V(x^b, z^b) = v \tag{1}$$

$$(1-\theta)Nx^a + \theta Nx^b + [(1-\theta)N/n^a]C(z^a, n^a)$$
$$+ (\theta N/n^b)C(z^b, n^b) = (1-\theta)NI^a + \theta NI^b. \tag{2}$$

Eq. (2) above is the resource constraint for an economy with homogeneous clubs. The RHS is total income in the economy, and the first two terms on the LHS give total consumption of the private good x. The remaining terms give the cost of public good provision in all the economy's clubs. Note that the number of clubs of each type equals group population [$(1-\theta)N$ for the a's, θN for the b's] divided by the relevant club population (n^a or n^b). As is standard in club theory, we ignore the fact that these expressions need not be integer-valued (the problem is inconsequential if N is large relative to optimal club sizes). The necessary conditions for an optimum in the above problem are the two constraints together with

$$n^a U_z / U_x = C_z^a \tag{3}$$

$$n^b V_z / V_x = C_z^b \tag{4}$$

$$C_n^i = C^i / n^i, \quad i=a,b, \tag{5}$$

where subscripts denote partial derivatives and where the i superscripts on C and C_n indicate that the functions are evaluated at (z^i, n^i), i=a,b. Eqs.

(3) and (4) are the Samuelson conditions for the two types of clubs, and (5) indicates that club populations are chosen to minimize the per capita cost of (optimal) public consumption.[4]

Consider now a configuration of identical mixed clubs, each of which mirrors the overall composition of the population (having a type-b proportion equal to θ). Let this configuration be denoted by CM, for "completely mixed." A key feature of the CM configuration is that, because of the proximity of the b's, the altruistic benefits enjoyed by the a's are greater than in the homogeneous clubs formed under H. This can be seen by computing the value of k in mixed clubs. Since $v^{home}=v^{away}=v$ and f is now evaluated at θ rather than zero, $k = [\alpha f(\theta) + \beta]v + \beta v = [\alpha f(\theta) + \delta]v$, which exceeds the previous value of δv. A disadvantage of CM, however, is that in contrast to the H configuration, the public good level in its mixed clubs cannot be tailored to suit individual preferences. Substituting the new value of k, a Pareto-efficient CM configuration solves the following problem:

$$\max \quad U(x^a, z, [\alpha f(\theta) + \delta]v)$$

$$\text{s.t.} \quad V(x^b, z) = v \tag{6}$$

$$(1-\theta)nx^a + \theta nx^b + C(z,n) = (1-\theta)nI^a + \theta nI^b. \tag{7}$$

Eq. (7) is the resource constraint for a representative club, with n giving the club's population and z the common public good level consumed by its residents. The necessary conditions for an (interior) optimum are the two constraints along with[5]

$$(1-\theta)nU_z/U_x + \theta nV_z/V_x = C_z \tag{8}$$

$$C_n = C/n. \tag{9}$$

Eq. (8) is the Samuelson condition for the mixed club, which reflects the compromise of tastes imposed by heterogeneity, and (9) is the per capita cost-minimization condition. The number of clubs N/n need not be integer-valued, but this problem is again ignored.[6]

It should be noted that although the formulation of the H and CM problems uses a common parametric v, achievement of a given type-b utility is not always feasible in both problems. It is easy to see, for example, that the lowest possible v value under H (which equals V(0,0), reflecting complete expropriation of the b's) is lower than the lowest v under CM, which is based on a positive value of z. Similarly, it may be shown that the highest feasible value of v is higher under H than under CM.[7] As a result, the feasible set of v's under CM is a subset of feasible set under H.

The key to comparing H and CM configurations for a common value of v is to note that the CM constraint (7) is equivalent to the H constraint (2) together with the side conditions

$$z^a = z^b, \quad n^a = n^b. \tag{10}$$

These "mixing constraints," which are necessary for common type-a and type-b consumption of public goods, reduce the size of the CM opportunity set relative to that of the H problem. Ordinarily, this would lead to a lower value of the objective function (type-a utility) under CM. However, since the altruistic advantage of mixed clubs means that, for given values of the choice variables, the CM objective function achieves a higher value than the H function, the effect of the smaller CM opportunity set may be reversed. Together, these considerations imply that the preferred configuration in a choice between H and CM cannot be determined in general.

Intuitively, this indeterminacy arises because there is a trade-off between the altruistic advantage of mixed clubs and the efficiency loss resulting from common consumption of public goods by people of different types. This trade-off will be analysed in detail below.

A critical difference between the H and the CM configurations is that the minimal institutional structure required to carry out income redistribution is different under the two regimes. Since income is redistributed via transfers between type-a and type-b clubs under H, this configuration cannot be implemented in the absence of a national redistribution system run by the federal government. While the federal government could also handle income redistribution in the CM configuration, the intermixture of the a's and b's means that local governments can achieve identical results. Since the CM configuration is potentially efficient, it follows that local redistribution may be compatible with efficiency in some circumstances. However, since an economy that relies on local redistribution cannot attain the H configuration, local redistribution may be inconsistent with the achievement of efficiency in other situations.

The choice between H and CM can be resolved in favor of CM provided that the the altruistic gain from mixed clubs dominates the associated efficiency loss. This outcome will obtain when the altruistic advantage from mixing is "large" or the efficiency loss from mixing is "small." Focusing on the second of these conditions, the above discussion suggests that the efficiency loss will be zero when the mixing constraints (10) hold at the H solution and small when these constraints are approximately satisfied. In other words, if the optimal homogeneous clubs of the groups are identical or nearly so, then the loss from the taste compromise in mixed clubs will not be significant. This notion is made precise as follows:

> **Proposition 1.** Suppose that for some $v = v'$, the mixing constraints (10) hold at the solution to the H problem, and suppose that v' is feasible in the CM problem. Then CM is preferred to H for all v in a neighborhood of v' when $\alpha > 0$.

This result can be established by first proving that when $\alpha = 0$ (when local altruism is absent), the CM and H solutions are identical whenever (10) holds at the H solution. This can be seen by multiplying (3) and (4) by $(1-\theta)$ and θ respectively and adding, which yields the Samuelson condition (8) for a mixed club with an α value of zero (note that the RHS's of (3) and (4) are identical by (10)). Since (5) and (9) are the same and (2) reduces to (7), it follows that the H solution satisfies the CM optimality conditions when $\alpha = 0$, implying that the two solutions are the same. Since the H and CM objective functions are also identical when $\alpha = 0$, the values of these functions are then equal at the respective solutions (in other words, $U^{CM} = U^{H}$). But since U^{CM} is increasing in α by the envelope theorem (the derivative is $f(\theta)vU_k^{CM} > 0$), it follows that U^{CM} exceeds U^{H} for $\alpha > 0$ when $v = v'$. By continuity, it then follows that CM is superior to H for all v lying in some neighborhood of v'.[8]

While it is possible to construct pathological examples where (10) holds at the H solution, this outcome arises naturally when preferences for x and z are the same. Suppose, for example, that $U(x,z,k) \equiv V(x,z) + W(k)$, so that k enters the type-a utility function in an additively separable manner and the (x,z) portion of the function is identical to the type-b utility function. Then it is easy to see that CM is superior for all values of v in a neighborhood of v'', where v'' is the type-b utility achieved in a homogeneous club with post-transfer income of $Y^a = (1-\theta)I^a +$

θI^b. The reason is that if the a's and b's enjoy identical post-transfer incomes under H ($Y^a = Y^b$), then (10) holds and CM is preferred. But since equality of the Y's means that they both must equal the average pre-transfer income in the economy $[(1-\theta)I^a + \theta I^b]$, it follows from Proposition 1 that when $\alpha > 0$, CM is preferred to H in a neighborhood of the v value associated with this Y^b. Suppose further that U satisfies the above assumptions and in addition $V(x,z) \equiv x + S(z)$, where $S' > 0$ and $S'' < 0$. Then CM is superior to H for any common value of v when $\alpha > 0$. This follows because when utility has the given transferable form, homogeneous clubs are identical regardless of the value of v (conditions (3)-(5) do not involve x, so that they yield the same common (z,n) solution for all values of v). Intuitively, the absence of income effects with transferable utility makes the earlier result hold regardless of the distribution of income.

In addition to the above results, it is intuitively clear that the efficiency loss from mixed clubs will be small when preferences are "similar" since this reduces the required compromise of tastes from joint consumption of public goods. In the first case above, for example, suppose that the (x,z) component of U is a function that is "similar" to V rather than identically equal to it. Then the efficiency loss from mixing will be low for v values near v", and CM will be superior in this range. This argument can be made precise in the transferable utility case, as follows:

Proposition 2. Suppose that $U(x,z,k) \equiv x + \Omega S(z) + W(k)$ and $V(x,z) \equiv x + S(z)$, with $S' > 0$ and $S'' < 0$ and $\Omega > 0$. Then when $\alpha > 0$, there exist positive numbers $\mu_1(v)$ and $\mu_2(v)$ such that CM is preferred to H at a given v when Ω satisfies $1 - \mu_1(v) < \Omega < 1 + \mu_2(v)$ and H is preferred to CM when $\Omega > 1 + \mu_2(v)$ or $\Omega < 1 - \mu_1(v)$.

This result says that when the groups' tastes are sufficiently similar, CM is preferred to H. To prove the proposition, first note that the derivative of $U^{CM} - U^{H}$ with respect to Ω equals $S(z^{CM}) - S(z^{aH})$ (the z's in this expression are optimal values under the two configurations). This expression can be signed by comparing the Samuelson conditions (4) and (8), which reduce to $n^{a}\Omega S'^{a} = C_{z}^{a}$ and $(1-\theta)n\Omega S' + \theta nS' = C_{z}$ respectively under the present assumptions. The key is to note that the first condition comes from setting θ equal to zero in the second condition. The optimal z values can be therefore be compared by computing the derivative of z in a mixed club with respect to θ. Totally differentiating the mixed-club Samuelson condition along with the cost-minimization condition (9) and using the second-order conditions for the CM problem, it is easy to show that z is an decreasing (increasing) function of θ when Ω is greater than (less than) unity.[9] It follows therefore that the z value corresponding to $\theta = 0$ (z^{aH}) is greater than (less than) the mixed-club value (z^{CM}) as Ω is greater than (less than) unity. Referring to the above derivative expression, it is then clear that $U^{CM} - U^{H}$ is decreasing (increasing) in Ω when Ω is greater than (less than) unity. Recalling from the discussion following Proposition 1 that $U^{CM} - U^{H} > 0$ when $\Omega = 1$, the existence of the $\mu_1(v)$ and $\mu_2(v)$ values in Proposition 2 then follows by continuity.[10]

With the superiority of CM demonstrated under a "small" efficiency loss from mixing, consider now the first of the earlier conditions: a "large" altruistic gain from CM. The altruistic advantage of CM depends on the magnitude of the parameter α, with larger values indicating a larger gain. Clearly, if α is large enough, this gain will be sufficient to offset any efficiency loss, making CM preferred to H. This notion is made precise as follows:

Proposition 3. For any common v in the H and CM problems, there exists a critical value of α, denoted $\alpha^*(v) \geq 0$, such that CM is preferred to H for all α satisfying $\alpha > \alpha^*(v)$ and H is preferred to CM for all α satisfying $\alpha^*(v) > \alpha \geq 0$. If the mixing constraints (10) are satisfied at the H solution, then $\alpha^*(v) = 0$. Otherwise, $\alpha^*(v) > 0$.

This result is established by first recalling from the proof of Proposition 1 that if (10) holds at the H solution, then $U^{CM} - U^{H}$ is zero for $\alpha = 0$ and positive for $\alpha > 0$. This shows that $\alpha^*(v) = 0$ for any v where the mixing constraints hold. If, on the other hand, (10) is not satisfied at the H solution for a given v, then the smaller size of the CM opportunity set matters and $U^{CM} - U^{H} < 0$ holds when $\alpha = 0$ (recall that the objective functions are the same when $\alpha = 0$). But since the utility difference is increasing in α, there exists some positive $\alpha^*(v)$ where the difference changes sign from negative to positive.

The choice between CM and H that has been the subject of the preceding analysis can be illustrated diagrammatically by drawing the utility frontiers for the economy under the two policies. These curves show the maximal value of U as a function of v under CM and H (they are graphs of U^{CM} and U^{H}). Examples of such curves are illustrated in Figure 1 (u denotes the type-a utility level).[11] The policy whose curve is higher at given v is, of course, preferred. Referring back to the respective optimization problems, the slopes of the H and CM frontiers equal $\delta U_k^{H} - \theta U_x^{H}/(1-\theta)V_x^{H}$ and $[\alpha f(\theta) + \delta]U_k^{CM} - \theta U_x^{CM}/(1-\theta)V_x^{CM}$ respectively. While these expressions are negative when U_k is zero, the slopes can be positive in the presence of altruism, implying that redistribution can raise both utility levels simultaneously over some range of v. This is seen in Figure 1, where the CM frontier has an upward sloping range (for later convenience, the H frontier is shown as downward sloping).

The H and CM club configurations discussed so far are in fact special cases in a more general choice problem. The analysis now focuses on this general problem with the goal of identifying the optimal club structure from among all those that are feasible. As will become clear, the previous results comparing H and CM are useful in this more comprehensive analysis. In the most complex possible club configuration, mixed clubs coexist with homogeneous clubs of each type. Clubs of a given type as before are identical, and horizontal equity requires that members of each group enjoy the same utility level regardless of whether they live in a mixed or homogeneous club. Letting Q denote the number of mixed clubs, σ denote the type-b proportion in their populations, x^{ah} and x^{bh} denote x consumption in homogeneous clubs, and $I \equiv (1-\theta)I^a + \theta I^b$, the general Pareto-optimality problem can be written

$$\max \quad U(x^a, z, [\alpha f(\sigma) + \delta]v)$$

$$\text{s.t.} \quad V(x^b, z) = v$$

$$U(x^a, z, [\alpha f(\sigma) + \delta]v) = U(x^{ah}, z^a, \delta v) \tag{11}$$

$$V(x^b, z) = V(x^{bh}, z^b) \tag{12}$$

$$\begin{aligned} &Q[(1-\sigma)nx^a + \sigma nx^b + C(z,n)] \\ &\quad + [(1-\theta)N - Q(1-\sigma)n][n^a x^{ah} + C(z^a, n^a)]/n^a \\ &\qquad + [\theta N - Q\sigma n][n^b x^{bh} + C(z^b, n^b)]/n^b = NI. \end{aligned} \tag{13}$$

Note that (11) and (12) are the horizontal equity constraints and that $[(1-\theta)N - Q(1-\sigma)n]/n^a$ and $[\theta N - Q\sigma n]/n^b$ are the numbers of homogenous type-a and type-b clubs (total group size minus the population in mixed clubs divided by n^i, i=a,b).[12] Implicit constraints in the problem are $0 \leq \sigma \leq 1$ and $0 \leq Q \leq \min\{(1-\theta)N/(1-\sigma)n, \theta N/\sigma n\}$, with the last inequality saying that mixed clubs cannot contain more than the total population of either group.

As before, the first-order conditions for choice of the x, z, and n variables reduce to the mixed- and homogeneous-club Samuelson and per capita cost-minimization conditions. The interesting questions in the general problem, however, concern the variables Q and σ. The following result is immediate:

> Proposition 4. In an optimal configuration, mixed clubs can coexist with at most one type of homogeneous club (either a or b).

This result follows directly from the fact that the Lagrangean expression L for the problem is linear in Q. This means that a corner solution is optimal, with Q either equal to zero or $\min\{(1-\theta)N/(1-\sigma)n, \theta N/\sigma n\}$. Since at least one of the equalities $(1-\sigma)nS = (1-\theta)N$, $\sigma nS = \theta N$ must therefore hold when Q is positive, it follows that the entire population of one or both groups fits into mixed clubs, as claimed. When $\sigma = \theta$, both equalities hold together and everyone lives in mixed clubs (this is the CM configuration).

A club structure containing both mixed and homogeneous type-a or type-b clubs will be referred to as a PM configuration (for "partially mixed"). A key question is whether some PM configuration is superior to the CM configuration analysed above. To answer this question, consider the gains and losses from allowing σ to deviate from θ. First, some change in σ will typically make each mixed club's population makeup more advantageous from an altruistic point of view. Recalling that local altruism depends on the type-b proportion through the function f, the a's will gain from increasing (decreasing) the type-b proportion relative to θ as $f'(\theta)$ is positive (negative). This altruistic gain has a cost, however, in that the individuals displaced from the mixed club must be guaranteed the same utility as those that remain. If the amount of extra resources required to achieve this equality is less than the (appropriately measured) altruistic gain, then some deviation of σ away from θ is desirable. The following result formalizes these considerations:

Proposition 5. CM cannot be optimal if one or both of the following inequalities holds at the CM solution:

$$\alpha U_k f'(\theta)v/U_x > [(x^{ah} + C^a/n^a) - (x^a + C/n)]/\theta(1-\theta) \quad (14)$$

$$-\alpha U_k f'(\theta)v/U_x > [(x^{bh} + C^b/n^b) - (x^b + C/n)]/(1-\theta)^2 \quad (15)$$

This result[13] is established by substituting $Q = \min\{(1-\theta)N/(1-\sigma)n,\ \theta N/\sigma n\}$ into the Lagrangean L and differentiating with respect to σ in the separate cases where $\sigma \geq \theta$ and $\sigma \leq \theta$. It is easily seen that $L_\sigma|_{\sigma\geq\theta}$ is positive at $\sigma = \theta$ when (14) holds and that $L_\sigma|_{\sigma\leq\theta}$ is negative at $\sigma = \theta$ when (15) holds. In either case, there exist PM configurations close to CM that yield higher values of the objective function than CM itself, establishing the Proposition. To relate the above inequalities to the previous intuitive discussion, note that the altruistic gain (loss) in terms of x from increasing σ above θ is captured by the first term in (14) while the loss (gain) from decreasing σ below θ is measured by the first term in (15). The resource costs discussed above appear on the RHS of these inequalities. The cost of displacing an a-type into a homogeneous club (the cost of a higher σ) equals the difference between per capita consumption of the a's in homogenous and mixed clubs, while the cost of displacing a b-type (of reducing σ) is the analogous difference in per capita type-b consumption. The factors $1/\theta(1-\theta)$ and $1/(1-\theta)^2$ are needed to apportion these costs among the a-types remaining in the mixed club.[14] It is easy to see that the type-b displacement cost is negative while the type-a cost is ambiguous in sign. The first claim ($x^{bh} + C^b/n^b < x^b + C/n$) follows because public consumption inefficiency in a mixed club means that the b's must consume more resources than in a homogenous club to achieve a given utility level. While this effect is also present for the a's, the countervailing altruistic advantage of mixed clubs means that resource requirements could be greater or smaller in a mixed club. The upshot is that the RHS of (14) is ambiguous in sign while the RHS of (15) is negative.

To check for satisfaction of (14) or (15), consider first the case where $\alpha > 0$ and $f'(\theta) > 0$. Since the LHS's of the inequalities are respectively positive and negative in this case while the RHS's are ambiguous and negative, neither (14) nor (15) is guaranteed to hold. It is important to realize that if neither inequality is satisfied, then CM is possibly though not necessarily preferred to PM. Although CM is sure to be preferred if the maximized objective function for the problem is concave in σ on either side of θ, nonconcavity means that PM could be preferred even though small movements away from CM are undesirable. If (14) or (15) happens to hold, on the other hand, then there exist PM configurations that dominate CM. Note that (14) and (15) may hold simultaneously.

A stronger statement can be made when $f'(\theta) \le 0$, as follows:

Proposition 6. Suppose that $\alpha > 0$ and that $f'(\sigma) \le 0$ holds for $\sigma \ge \theta$. Then, if the optimal configuration contains mixed clubs, it must be a PM configuration with $\sigma < \theta$.

To prove this result, note first that (15) is guaranteed to hold when $f'(\theta) \le 0$, so that there exist PM configurations with $\sigma < \theta$ that are superior to CM. To show further that no PM configuration with $\sigma > \theta$ can be optimal under the given assumptions, suppose to the contrary that such a configuration is in fact optimal. Then note that for mixed clubs to exist at the optimum, the derivative of the Lagrangean with respect to Q must be positive at the given σ (otherwise the positive corner solution cannot be optimal). This derivative has the same sign as

$$(1-\sigma)[(x^{ah} + C^{a}/n^{a}) - (x^{a} + C/n)]$$
$$+ \sigma[(x^{bh} + C^{b}/n^{b}) - (x^{b} + C/n)]. \tag{16}$$

Intuitively, (16) must be positive for mixed clubs to be optimal because moving people into homogeneous clubs must then increase per capita consumption.[15] Since $x^{bh} + C^{b}/n^{b} < x^{b} + C/n$ holds from above, resources can always be saved by relocating b-types in this manner. Therefore, if mixed clubs are to be optimal, relocating the a's must consume extra resources, making the first term in (16) positive. The final step is use this fact to show that the σ-derivative of the Lagrangean at the supposedly optimal $\sigma > \theta$ is negative, which is a contradiction. This derivative is negative whenever the LHS of (14) (with σ replacing θ throughout) is less than the RHS. Since the LHS and RHS are respectively nonpositive and positive given the previous result and $f' \leq 0$, the contradiction follows, implying that an optimal configuration with $\sigma > \theta$ is impossible under the given assumptions.

The ultimate goal of the analysis is to identify the club configuration that is optimal under given circumstances. While none of the propositions by itself yields this information, the results can be merged to help identify the optimum. Generally, whenever circumstances are such that CM is preferred to H at a given v, then the optimal configuration contains mixed clubs. These clubs coexist with homogeneous clubs if there are PM configurations preferred to CM at the given v. Otherwise, the optimal configuration contains only mixed clubs. Suppose, for example, that utility is transferable and that preferences are "similar" in the sense of Proposition 2, and suppose further that $f'(\sigma) \leq 0$ for $\sigma \geq \theta$. Then it follows from Propositions 2 and 6 that a PM configuration with $\sigma < \theta$ is optimal at the given v. If, however, $f'(\theta) > 0$ holds under these circumstances, then CM could replace PM as the mixed-club optimum. Similarly, when the mixing constraints hold at a given v or when α exceeds the critical value $\alpha^*(v)$, then by Propositions 1 and 3, the optimal configuration contains mixed clubs, with either CM or PM optimal.

When H is preferred to CM, on the other hand, the identity of the optimal configuration is not clear. While H is optimal in this case if CM dominates PM, H could be better or worse than PM otherwise. Since Proposition 5 does not offer a sufficient condition for CM to be preferred to PM, the Propositions as a whole do not yield a sufficient conditions for the optimality of H. By focusing on the general choice problem, however, it is easy to establish the obvious result that H must be optimal in the absence of local altruism ($\alpha = 0$). This is done by referring to the L_Q expression (16). If (16) is zero or positive for some σ between zero and one when $Q = 0$,[16] then per capita consumption of resources can be reduced by moving people out of homogeneous clubs into a mixed club with the given σ. Therefore, a necessary and sufficient condition for the optimality of H is that (16) be negative for all σ between zero and one. Recalling the above discussion, it is clear that negativity will hold when $\alpha = 0$. Since there is then no gain to the a-types to offset the public consumption inefficiency of mixed clubs, more resources must be consumed than in a homogeneous club to reach a given utility level. With the same conclusion holding for the b-types, the negativity of (16) (and the optimality of H) follows. It may also be shown that H is optimal in the transferable utility case whenever the type-a preference parameter Ω from Proposition 2 is sufficiently large (indicating a substantial difference in preferences).[17]

The preceding analysis of the general choice problem can be incorporated into Figure 1 by replacing the H and CM frontiers with a general utility frontier showing for each v the value of u at the problem's solution. If H or CM is optimal at a given v, then the general frontier coincides with either the H or CM frontier at that point. Otherwise, the general frontier is higher than either of the other frontiers.

While it was seen above that local governments can handle income redistribution under the CM configuration, a national redistribution program must exist to carry out the interclub transfers that will typically be required in a PM configuration. Since local governments, however, can handle part of the redistributive duties under PM, such a configuration can be supported by a dual system involving both local and national transfers.

3. Positive Analysis

The key feature of standard models of altruism is that through the voluntary action of individuals in the economy, the utility of the poor group is raised above the level corresponding to the original distribution of income. The purpose of the positive analysis in this section of the paper is to see whether this outcome obtains in the present model. The goal is to determine the level of v that actually emerges as a result of decentralized behavior (v, of course, was parametric in the planning problem). The analysis is carried out under the assumption that clubs are organized by competitive developers, as in Berglas and Pines (1980, 1981). A key feature of the competitive model is that developers, being small operators, are not able to make interclub transfers. This means that PM configurations and all H configurations except one are not attainable in the model. The only feasible configurations are the CM configuration and the H configuration based on the original distribution of income, neither of which involves interclub transfers. The unattainability of some club configurations means that equilibrium in the model may not be efficient. To reduce the likelihood of this outcome, one possible source of inefficiency is removed by the assumption that generalized altruism is absent ($\delta = 0$), which means that the H frontier in Figure 1 is always downward sloping. This rules out a situation in which the one attainable H configuration, denoted NR for "no redistribution", is automatically Pareto-inefficient as a result of being dominated by one of the unattainable configurations on the H frontier.

A critical additional assumption is that in forming mixed clubs, developers are required (by law, perhaps) to mix the a's and the b's in accordance with the overall composition of the population, forming clubs with type-b proportions σ equal to θ. This requirement will be referred to as the "club composition constraint." As will be seen below, equilibrium may not exist when developers are allowed to choose σ.

The analysis first derives the features of the homogenous and mixed clubs organized by developers. The discussion then identifies the club structure (homogeneous or mixed) that actually emerges in equilibrium. Consider the problem faced by a developer organizing a homogeneous type-a club.[18] His choice variables are the public good level in the club and the size of its population. Being competitive, the developer is a "utility-taker," which means that the club entry fee P^a that he charges must allow the a's to achieve at least the prevailing utility u for their type. Recalling that $\delta = 0$ and that homogeneous clubs must reflect the original distribution of income, P^a therefore satisfies $U(I^a-P^a,z^a,0) \geq u$. In equilibrium, this relationship will hold as an equality, implicitly determining the entry fee as a function of z^a and u. Differentiation shows that P^a is an increasing function of the the public good level (P_z^a = U_z/U_x) and a decreasing function of u. The developer's problem is then to maximize the following profit expression by choice of z^a and n^a:

$$\pi^a = n^a P^a(z^a,u) - C(z^a,n^a). \qquad (17)$$

The first-order conditions are the Samuelson condition (3) and $P^a - C_n^a = 0$. In a zero-profit equilibrium, the latter condition reduces to the per capita cost-minimization condition (5) and (substituting the budget constraint) $\pi^a = 0$ reduces to the resource constraint of the club (note that u becomes endogenous when the zero-profit condition is added). The equilibrium and planning conditions are therefore identical, implying that developer-organized clubs are efficient. Since exactly the same argument applies to the creation of type-b clubs, it follows that the utilities in competitively-organized homogeneous clubs lie on the H frontier of Figure 1 at point NR.

Now consider the problem of the mixed-club developer, who provides a common level of the public good to both the a's and the b's. While the a's may voluntarily enter a mixed club to benefit from the presence of the b's, the b's themselves have no such incentive and will require compensation in the form of a transfer payment to join a mixed club. The transfer T is provided by the developer, who collects the necessary funds from the a's. While the two groups pay a common club entry fee P, the presence of the transfer makes their net costs of joining the club different.

For a mixed club to attract residents, the formal requirement is that P and T assume values that allow each individual to achieve at least the prevailing utility level for his type. P and T are then jointly determined by the following conditions:

$$U(I^{a}-P-\theta T/(1-\theta),z,\alpha f(\theta)v) = u \tag{18}$$

$$V(I^{b}-P+T,z) = v. \tag{19}$$

(note in (18) that the per capita tax on the a's depends on the type-b proportion, which is set at θ to satisfy the club composition constraint).[19] Total differentiation of (18) and (19) shows that $P_z = (1-\theta)U_z/U_x + \theta V_z/V_x$ and that P is decreasing in u.[20] As before, the developer chooses z and n to maximize

$$\pi = nP(z,u,v) - C(z,n), \tag{20}$$

with first-order conditions being the Samuelson condition (8) and $P - C_n = 0$. As before, the latter condition reduces to the per capita cost-minimization condition (9) in a zero-profit equilibrium. Also, after multiplying the type-a and type-b budget constraints by $(1-\theta)$ and θ respectively, adding, and substituting the implied P into (20), the zero-profit condition gives back the mixed-club resource constraint. As a result, the equilibrium and planning conditions are once again equivalent, implying that competitively-organized mixed clubs are efficient.

While utility levels for both the a's and the b's were determined endogenously in the creation of homogeneous clubs, there are only three conditions to determine four unknowns (z,n,u,v) in the mixed-club case. As a result, one of the parametric utilities (say v) remains undetermined in this case. The efficiency result therefore means that clubs are efficient conditional on v, implying that the economy reaches some point on the CM frontier as a result of the activities of developers.

Having looked separately at homogeneous and mixed clubs, it is now possible to analyse the club structure that emerges in equilibrium. For a club configuration to be an equilbrium, it must accomodate the economy's population and there must exist no alternative clubs that are both profitable (yielding positive profit for the developer) and viable (offering potential residents higher utility than they enjoy in the given configuration). With this in mind, the following result can be established:

> **Proposition 7**. Assume that generalized altruism is absent and that clubs are organized by competitive developers who are subject to the club composition constraint, as described above. If there are no points on the CM frontier satisfying $u \geq u_{NR}$ and $v \geq v_{NR}$, where u_{NR} and v_{NR} are the utility levels at point NR on the H frontier, then the equilibrium club configuration is the homogeneous configuration corresponding to NR. If the CM frontier contains points satisfying $u > u_{NR}$ and $v > v_{NR}$, then multiple equilibria are possible. Each equilibrium configuration is mixed, with any point on the CM frontier satisfying $u \geq u_{NR}$ and $v \geq v_{NR}$ and not Pareto-dominated by some other point corresponding to a possible equilibrium.

To prove the first statement in the proposition, the first step is to recall that in the homogeneous case, club entry fees (and hence the profit levels of developers) are decreasing functions of the group utility levels. Next, note that under the assumption that no points on the CM frontier satisfy $u \geq u_{NR}$ and $v \geq v_{NR}$, a developer-organized mixed club must yield a utility level lower than the NR level for at least one group. Suppose without loss of generality that such a club has $u = u' < u_{NR}$. This club cannot be part of an equilibrium configuration because a homogeneous type-a club offering a utility u" between u' and u_{NR} would attract away the a's and earn its developer a positive profit (this follows because $\pi_u^a < 0$ and $\pi^a = 0$ when $u = u_{NR}$). With a mixed-club equilibrium ruled out by this argument, consider the homogeneous configuration corresponding to NR.

First, since any viable alternative homogeneous club must offer a utility higher than the NR level to one group, it will lose money. Similarly, since a viable mixed club must also improve on the NR utilities, it must lie above the CM frontier. But since mixed-club profit is zero on the frontier and $\pi_u < 0$, such a club also loses money. This establishes that the homogeneous NR configuration is the equilibrium under the given assumptions.[21]

Now suppose that the CM frontier passes to the northeast of NR, so that some of its points satisfy $u > u_{NR}$ and $v > v_{NR}$. In this case, the argument used above to rule out mixed-clubs shows that all CM points with at least one utility less than the NR level are not equilibria (viable and profitable homogeneous clubs exist). Furthermore, the homogeneous NR configuration cannot be an equilibrium in this case since a mixed club offering a utility pair below the CM frontier but still to the northeast of NR will offer higher utilities than NR and be profitable. This leaves points on the CM frontier to satisfying $u \geq u_{NR}$ and $v \geq v_{NR}$ as candidates for equilibria. Suppose one of these points with coordinates (u',v') is Pareto-dominated by another such point with coordinates (u'',v''), a possiblity given that the CM frontier may contain upward-sloping segments. In this case, (u',v') cannot be an equilibrium because a mixed club offering a utility pair slightly below (u'',v'') would be viable and profitable. The remaining undominated points, however, satisfy the requirements of equilibrium. First, since any profitable homogeneous club must offer its group a utility below the NR level, such a club will not be viable relative to any point to the northeast of NR. Second, any mixed club (profitable or otherwise) offering a utility pair not to the northeast of NR will not be viable relative to a point northeast of NR since one of its utilities will be lower. Finally, consider profitable mixed alternative clubs to the northeast of NR. Relative to an undominated point, at least one of the utilities for any such club must be lower since profitable clubs lie below the CM frontier and the undominated point is by definition not dominated by any point on the frontier. This means that such clubs are not viable relative to an undominated point, establishing that any such point is an equilibrium.

Note that, for simplicity, Proposition 7 does not cover the case where the CM frontier intersects the H frontier at NR. In this case, it is easy to see that the CM and H configurations corresponding to NR are both equilibria. The second part of Proposition 7 is illustrated in Figure 1, where the CM frontier is shown passing to the northeast of NR. The Pareto-undominated CM points in this range, which comprise the set of equilibria, are contained in the segment JM of the frontier.

This analysis shows that when CM configurations exist that are Pareto-superior to the status-quo point NR, decentralized behavior drives the economy to one of these configurations. Since it is easily seen that the transfer T received by the b's is positive in any such equilibrium, the outcome is identical to that in a standard altruism model, where one group voluntarily relinquishes income to help the other.[22]

If CM is always preferred to PM, then equilibrium in the model is efficient in that no alternative club configuration is Pareto-superior to any equilibrium configuration (this is clear from Figure 1). However, if PM is sometimes superior to CM, then the general utility frontier will sometimes lie above the CM frontier. In this case, both the mixed- and homogeneous-club equilibria of the model may be inefficient (the general frontier could pass above either type of equilibrium point). This potential inefficiency is a consequence of the inability of competitive developers to make interclub transfers.

A difficulty with the conclusions of this analysis is that they depend critically on the presence of the club composition constraint, which might be viewed as an unrealistic requirement in a decentralized economy. Without this constraint, σ becomes a choice variable of the developer and equilibrium may fail to exist. To see this, note that the entry fee P now explicitly includes σ as a argument, with profit maximization requiring $nP_\sigma = 0$. This condition reduces to

$$(1-\sigma)^2 U_k \alpha f'(\sigma) v/U_x - T = 0. \tag{21}$$

Unless (21) holds at $\sigma = \theta$, the previous mixed-club equilibria lose their equilibrium status since developers can find profitable and viable clubs with σ's different from θ. Since such clubs do not accommodate the population, no equilibrium exists under the circumstances that previously led to mixed-club equilibria. In the alternate situation where the CM frontier passes below NR, equilibrium may or may not exist depending on whether the u value in a mixed club satisfying (21) and offering $v = v_{NR}$ exceeds u_{NR}. If this is not the case, then the H configuration corresponding to NR is the equilibrium. Otherwise, no equilibrium exists.

It is interesting to note that in the absence of the club composition constraint, equilibrium (when it exists) involves no income redistribution. This brings to mind the frequently-enunciated concern that local income redistribution (which is the only kind possible in the model) cannot be implemented because of the phenomenon of poor chasing rich. While implementation fails in the present model, the reason is somewhat different. The problem is not a result of consumer mobility compromising the redistribution process but is rather a consequence of developers' attempts to mix the groups in a manner that is not feasible in order to best exploit consumer altruism.[23]

As a final point, it is interesting to ask how the economy would be organized if the a-types could specify the club configuration. Under such an arrangement, the goal of the a's would be to maximize their own utility subject to the constraint that the b's are willing to participate in the chosen configuration. Formally, this problem amounts to finding the point on the general utility frontier that maximizes u subject to the requirement that v exceeds or equals v_{NR}. Unless this latter constraint is satisfied, the b's will decline to participate in the chosen configuration, retreating instead to homogeneous clubs based on the original distribution of income. A difficulty with this choice process, however, is that cannot be viewed as decentralized.

4. Conclusion

Local redistribution is practiced in the U.S. on a variety of different levels. For example, unequal sharing of the costs of running school districts and providing other public services leads to extensive implicit redistribution among households at the community level. Moreover, the fact that state contributions to federal welfare programs are substantial and not at all uniform shows that the welfare system involves an important element of local redistribution. What can be said about such policies on the basis of the discussion in this paper? The main practical lesson of the paper is that while local redistribution may be consistent with efficiency in the presence of spatially-limited altruism, the pursuit of such policies could involve a substantial welfare cost. Since there is no reason to think that real world economy mimics the developer model in avoiding undesirable equilibria, the economy could conceivably benefit from homogenization of communities and reassignment of the redistributive function to higher levels of government. While such a conclusion follows from the model, it cannot be taken too seriously as a policy prescription. The main reason is that complementarities in production (as in Berglas (1976a)) are probably important enough in the real world to invalidate any call for community homogenization based on public-sector considerations. In spite of this, awareness of the potential welfare loss from pursuit of local redistribution can only be beneficial in the analysis of policy questions related to fiscal federalism.

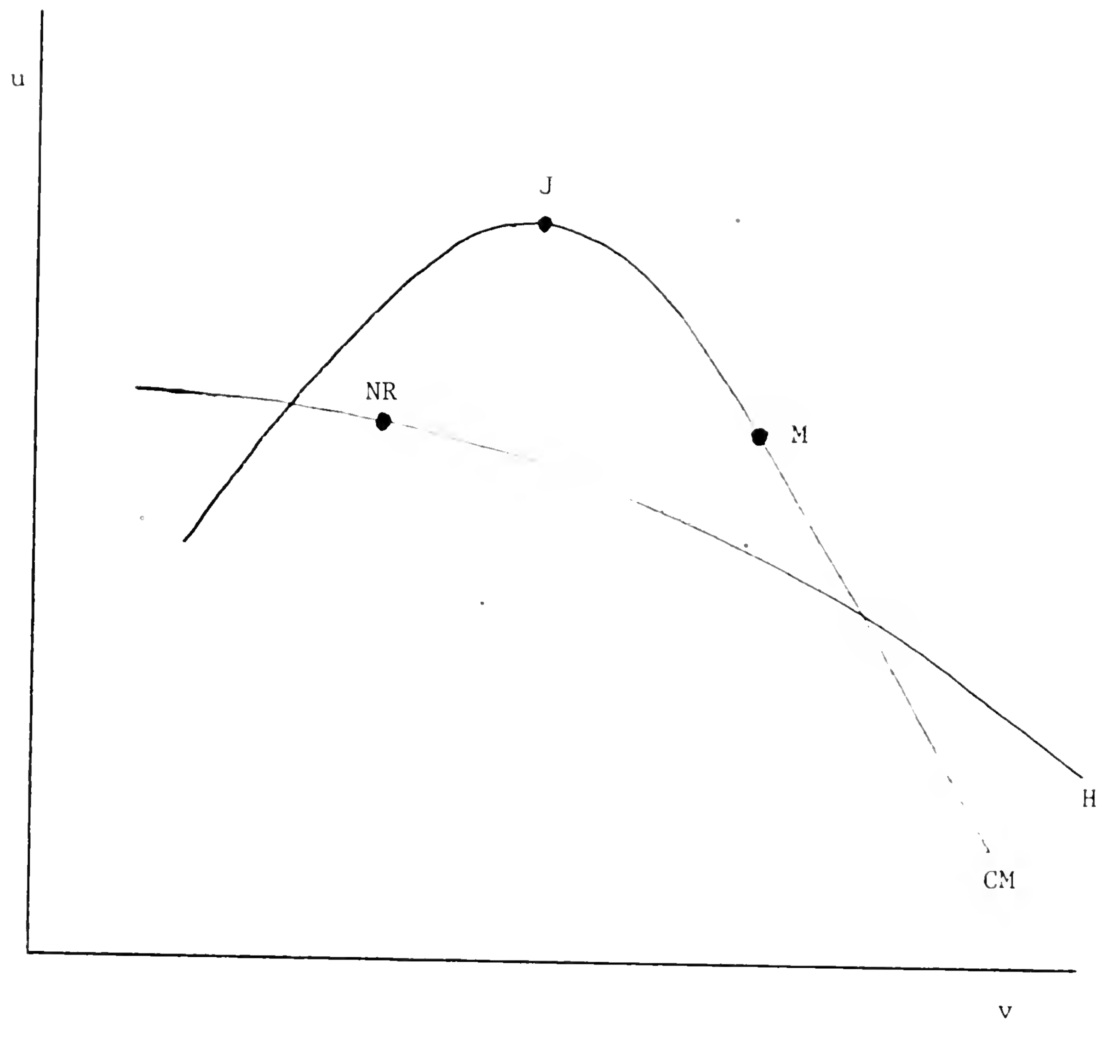

FIGURE 1.

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Footnotes

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[1]Brown and Oates (1987) provide a different type of equilibrium analysis in a model essentially the same as Pauly's.

[2]Brueckner (1988) used a similar approach to analyse local redistribution in the absence of altruism.

[3]See Berglas and Pines (1981) and Berglas (1984).

[4]Recall that a previous assumption on C guarantees an interior solution to (5).

[5]In a mixed-club problem without altruism, Brueckner (1988) shows that $x^a = 0$ must hold in the upper range of possible v values, with $x^b = 0$ holding in the lower range of v values. The present problem exhibits a similar outcome, with nonnegativity constraints on x^a binding for large values of v. It is also possible that x^b may be zero at the CM solution, although the presence of altruism precludes any definite statement. While the Samuelson condition (8) is altered when nonnegativity constraints are binding, this has no effect on any of the results derived below.

[6]It should be noted that in contrast to Pauly's characterization of Pareto-efficiency, the welfare of both the rich and the poor is taken into account in the above discussion (Pauly's Pareto-optimum was defined relative to the nonpoor members of the club).

[7]This can be seen by comparing the resource constraint resulting from complete expropriation of the a's ((2) with $x^a = z^a = 0$) to the mixed-club constraint with $x^a = 0$ (the maximum value of v is found by maximizing type-b utility subject to this constraint). The latter constraint is more restrictive, implying that the maximal v is higher under H than under CM.

[8]It should be noted that when (10) holds at the H solution, the nonnegativity constraints on the x^i cannot be binding at the same v in the CM problem (this follows because the solutions are identical and the H solution is interior).

[9]The derivative is equal to $nS'(1-\Omega)C_{nn}/D$, where $D = -C_{nn}[(\Omega(1-\theta) + \theta)nS'' - C_{zz}] - (C_{zn} - C_z/n)^2$. C_{nn}/D must be positive for the second-order condition to be satisfied.

[10]Note that the utility differential could be positive for all $0 < \Omega < 1$, in which case $\mu_1(v) = 1$.

[11]Note that the curves in Figure 1 reflect the difference discussed above in the set of feasible v's under H and CM.

[12]The integer problem is again ignored.

[13]It should be noted that since homogeneous clubs disappear from the choice problem when $\sigma = \theta$, the n^i, z^i, and x^{ih} variables in Proposition 5 are in fact undetermined. This problem is handled by defining shadow values of these variables equal to their limits as σ approaches θ. It is easy to see that the limiting values are those that minimize the per capita cost of providing the CM utilities in homogeneous clubs (were they to exist). It is interesting to note that the disappearance of the homogeneous-club variables from the resource constraint (13) results in zero values for the multipliers of (11) and (12) when $\sigma = \theta$.

[14]To see this, consider the case where $\sigma \geq \theta$. Letting D denote the cost of displacing one a-type, total displacement cost equals D times the number of a's outside mixed clubs, which is $(1-\theta)N - \theta N(1-\sigma)/\sigma$. To find the displacement cost per a-type remaining in mixed clubs, this must be divided by $\theta N(1-\sigma)/\sigma$, which yields $D(\sigma-\theta)/\theta(1-\sigma)$. The derivative of this expression with respect to σ (with D held fixed) evaluated at $\sigma = \theta$ is the LHS of (14). A similar argument applies to (15).

[15]If the residents of a mixed club were relocated to homogeneous clubs, they would consume resources equal to $(1-\sigma)n(x^{ah} + C^a/n^a) + \sigma n(x^{bh} + C^b/n^b)$. For this to be greater than mixed club consumption $(1-\sigma)nx^a + \sigma nx^b + C$ on a per capita basis, (16) must be positive.

[16]Note that the problem encountered earlier appears here as well in that the x^i, z, and n variables are undetermined when $Q = 0$. Once again, these variables at $Q = 0$ are set equal to their limiting values as Q approaches zero. These limiting values minimize the cost of providing the H utilities in a mixed club (were one to exist).

[17]This follows because, for any σ, the derivative expression (16) is a decreasing function of Ω when $\Omega > 1$, which means that the expression can be made negative for any σ by choosing Ω to be sufficiently large.

[18]Note that it is assumed that developers can identify people by type.

[19]Note that T is in fact unrestricted in sign.

[20]T_z is proportional to $U_z/U_x - V_z/V_x$.

[21]It should be noted that this discussion (as well as that below) relies on the absence of generalized altruism in that the type-a utility level in an alternative club does not depend on the prevailing type-b utility in the original club configuration. With generalized altruism, by contrast, the k value in a alternative mixed club would depend on the club's own v value as well as the v level in the original configuration. With type-b utilities nonuniform, the profitability of such a club could not be evaluated by referring to the CM frontier, which presumes uniformity. It is easy to see, however, that since the v^{away} part of

the k argument disappears, the CM and H frontiers are appropriate for evaluating alternative clubs in the absence of generalized altruism.

[22] T is positive because the b's reach a higher utility than in a homogenous club in spite of the efficiency loss of joint public consumption. As a result, $x^b + C/n$ must exceed consumption in a homogeneous club, which equals I^b. But since $x^b = I^b - P + T$, it follows that $T > 0$.