



UNIVERSITY OF
ILLINOIS LIBRARY
AT URBANA-CHAMPAIGN
BOOKSTACKS

Digitized by the Internet Archive
in 2011 with funding from
University of Illinois Urbana-Champaign

<http://www.archive.org/details/specificationerr676leec>

Faculty Working Papers

SPECIFICATION ERROR, RANDOM COEFFICIENT AND THE
RISK-RETURN RELATIONSHIP TEST IN CAPITAL ASSET
PRICING

Cheng F. Lee, Professor, Department of Finance
Frank J. Fabozzi, Hofstra University
Jack Clark Francis, II, Baruch College, CUNY

#676

College of Commerce and Business Administration
University of Illinois at Urbana-Champaign

FACULTY WORKING PAPERS

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

May 9, 1980

SPECIFICATION ERROR, RANDOM COEFFICIENT AND THE
RISK-RETURN RELATIONSHIP TEST IN CAPITAL ASSET
PRICING

Cheng F. Lee, Professor, Department of Finance
Frank J. Fabozzi, Hofstra University
Jack Clark Francis, II, Baruch College, CUNY

#676

Summary

Both specification error and random coefficient concepts are used to show why the traditional method of testing the risk-return relationship in capital asset pricing may not be appropriate. Empirical studies of capital asset pricing are reviewed in detail to show how the random coefficient behavior can occur in the estimated parameters of the equation used to test the positive theory of capital asset pricing.

SPECIFICATION ERROR, RANDOM COEFFICIENT AND THE RISK-RETURN
RELATIONSHIP TEST IN CAPITAL ASSET PRICING

I. INTRODUCTION

The theory of equilibrium in the capital markets developed independently by Sharpe (1964), Lintner (1965) and Treynor (1961) provides a risk-return relationship for assets and portfolios. This relationship, shown as equation (1), is called the security market line (SML) by Sharpe and the capital asset pricing model (CAPM) by others.

$$(1) \quad E(R_j) = E(R_f) + B_j [E(R_m) - E(R_f)]$$

where $E(R_j)$ denotes the expected rate of return from the j th market asset, $E(R_f)$ represents the expected value of the risk-free interest rate, B_j is called the beta systematic risk coefficient, $E(R_m)$ is the expected return from the market, and

$[E(R_m) - E(R_f)]$ is the theoretical market risk premium.

The risk-return relationship has been used to evaluate portfolio performance, test for market efficiency, determine the required equity return for regulated industries and approximate the hurdle rate for capital investment projects. Because of the pivotal role of the relationship in financial theory, researchers have empirically tested the model to determine if the estimated parameters are equal to the theoretical values. Although the results of researchers have consistently rejected the hypothesis that the estimated and theoretical values are equal, the findings do suggest that systematic risk is a meaningful measure of risk and stocks with high systematic risk yield

correspondingly high rates of return.¹

This paper presents the empirical results of a different test of the validity of the risk return relationship. Rather than estimate the model using a fixed regression coefficient model (as is suggested by theory since the market risk premium is theoretically constant for all securities), a stochastic parameter regression model is tested. With this latter model the market risk premium can be tested to determine if it changes randomly from security to security.

The next section provides theoretical reasons to justify the use of a stochastic parameter regression model to describe the risk-return relationship. Section III formulates the specific stochastic parameter model employed. The data and results are described in the fourth section, followed by the conclusions in section five.

II. JUSTIFICATION FOR EMPLOYING THE STOCHASTIC PARAMETER REGRESSION MODEL

The empirical analogue of equation (1) is shown as equation (2).

$$(2) \quad \bar{R}_j = \lambda_0 + \lambda_1 \hat{B}_j + e_j$$

where

\bar{R}_j = the arithmetic mean return for security j

\hat{B}_j = the estimated systematic risk for security j, and

e_j = the stochastic error for security j

Equation (2) is estimated using cross-sectional data with the beta coefficient estimated from a first-pass regression based on a time

series of historical returns. If equation (1) is a true description of the risk-return relationship, then λ_0 and λ_1 should equal the arithmetic mean return for the risk-free asset (\bar{R}_f) and arithmetic average excess of the market return (\bar{R}_m) over the risk-free rate (that is, the market risk premium). Moreover, the slope should be constant for all securities.

In a survey article about stochastic parameter regressions, Barr Rosenberg (1973, p. 381) states:

The stochastic parameter problem arises when parameter variation includes a component which is a realization of some stochastic process in addition to whatever component is related to observable variables. Thus, stochastic parameter regression is a generalization of ordinary regression. Ideally, a model would be so well defined that no stochastic parameter variation would be present, and no generalization would be needed, but the world is less than ideal.

If the risk-return relationship is well specified, we would not observe that its slope or market risk premium would vary stochastically.

However, there exist both theoretical economic and econometric reasons to suspect the risk-return relationship is, in fact, misspecified.

First, Arditti (1967), Kraus and Litzenberger (1976) and others have published theoretical and empirical work showing that the equilibrium return of an asset is influenced by both the second and third statistical moments of its return distribution. These findings extend the two parameter model to a third parameter, namely, skewness. Since the risk-return relationship ignores the impact of skewness, or skewness related factors, it suffers from omitted variables.²

Second, other studies have suggested the possibility of additional omitted variables. Sharpe (1977), for example, has given the risk-

return relationship a "multi-beta" interpretation. Similarly, Ross (1976, 1977) uses an arbitrage approach to derive a multi-factor risk-return relationship. Brennan (1970) has analyzed the impact of the tax effect due to the different treatment of dividend income and capital gains. He derived a multi-index model including average excess dividend yield as an additional explanatory variable in the risk-return relationship. Bachrach and Calai (1979) have shown that the price of the stock should be included in the risk-return relationship while Lanstein and Sharpe (1978) and Joehnk and Petty (1980) have shown that duration or interest rate risk should also be considered.

Statistically, the multi-index risk-return model can be specified by equation (3).

$$(3) \quad \bar{R}_j = \lambda'_0 + \lambda'_1 \hat{\beta}_j + \lambda_2 \hat{X}_{2j} + \dots + \lambda_n \hat{X}_{nj} + \tau_j$$

where $\hat{X}_2, \dots, \hat{X}_n$ are estimates of omitted factors discussed above, $\lambda'_0, \lambda'_1, \lambda_2, \dots, \lambda_n$ are cross-section regression parameters, and τ_j is the stochastic error for security j . It should be noted that equation (3) is a generalized case of equation (2).

If we use the specification method specified by Theil (1971, pp. 548-549) it can be shown that

$$(4) \quad \hat{\lambda}_1 = \hat{\lambda}'_1 + b_2 \hat{\lambda}_2 + \dots + b_n \hat{\lambda}_n$$

where b_2, b_3, \dots, b_n are so-called auxiliary regression coefficients.³

In addition we also know that

$$5A) \quad \hat{\lambda}_0 = \bar{R}_j - \hat{\lambda}_1 \hat{\beta}_j$$

and

$$5B) \quad \hat{\lambda}'_0 = \bar{R}_j - \hat{\lambda}'_1 \bar{\beta}_j - \hat{\lambda}'_2 \bar{x}_2 - \dots - \hat{\lambda}'_n \bar{x}_n$$

$$\text{where } \bar{R}_j = \frac{n}{\Sigma_{j=1}^n} \bar{R}_j/n, \quad \bar{\beta}_j = \frac{n}{\Sigma_{j=1}^n} \hat{\beta}_j/n$$

If all auxiliary regression coefficients are zero (i.e., all the omitted variables are independent of $\hat{\beta}_j$), then $\hat{\lambda}'_1$ is an unbiased estimate for λ'_1 . However, $\hat{\lambda}'_0$ is no longer an unbiased estimate for λ'_0 .

Therefore, $\hat{\lambda}'_0$ cannot be used to test the null hypothesis that λ_0 is equal to \bar{R}_f if the multi-index model is appropriate. If equation (3) does hold and all auxiliary regression coefficients are approximately equal to zero, $\hat{\lambda}'_1$ may well still be an unbiased estimator of λ'_1 . However, $\hat{\lambda}'_1$ becomes a random instead of a fixed variable.

Third, Roll (1977) and others have suggested beta estimates obtained by regressing returns from common stocks on stock market average returns are a form of partial equilibrium analysis which ignores investment in other capital assets. They suggest a general equilibrium analysis which includes other assets (such as, investments in human capital, commodities, real estate, etc.) should be used to obtain a risk-return tradeoff. If the more general equilibrium analysis suggested by Roll produces a risk-return relationship which departs significantly from the usual partial equilibrium analysis, then all previous empirical estimates of the market risk premium are conceptually flawed and may explain why the model may exhibit the characteristics of a stochastic parameter regression model.

Fourth, Levy (1978) and Hessel (1978) have demonstrated that imperfect capital markets will modify the risk-return relationship

which is predicated on the assumption of perfectly competitive markets. Levy (1978), for example, has developed a generalized risk-return relationship when (i) market participants differ in their investment strategies and do not adhere to the same risky portfolio given by their market portfolio and (ii) do not hold many risky assets in their portfolio. Levy concludes that the true risk index is somewhere between the total variance of the security and the systematic risk implied by capital market theory.

Finally, numerous studies have documented that the explanatory variable in the risk-return relationship, the beta coefficient, is subject to estimation error. The beta coefficient is estimated in the first-pass regression. However, in the first pass regression, the true market model may be a multi-index model rather than a single index model. As indicated in the discussion of the second pass regression above, the estimated beta of the market model will then exhibit the characteristics of a stochastic parameter regression model.⁴

III. TEST FORMULATION

Previous research employed the classical OLS fixed-coefficient approach to estimate equation (2). The purpose here is to determine if a random coefficient relationship between return and systematic risk exists. That is, does the proportionality constant, λ_1 , which represents the market risk premium fluctuate randomly from one security to the next?

There are several stochastic parameter regression models suggested in the literature.⁵ The random coefficient model formulated by Thiel (1971) is used in this investigation. The fixed coefficient model given by equation (2) can be converted to the random coefficient model (RCM) shown by equation (6).

$$(6) \quad \bar{R}_j = \lambda_0 + \lambda_1 \hat{B}_j + w_j$$

where

$$w_j = (\lambda_1 - \lambda_{1j}) \hat{B}_j + e_j$$

and λ_1 is the mean of λ_{1j} . Moreover, it is assumed that the distribution of λ_{1j} is homoscedastic and the e_j values are uncorrelated with the λ_{1j} values.

To test whether the RCM is a description of the risk-return relationship, two statistics must be estimated. First λ_1 of equation (6), and second, the variance of the distribution of λ_{1j} around its mean λ_1 , $\text{var}(\lambda_{1j})$, must be estimated. If no statistically significant variance for λ_{1j} around λ_1 is found, then the RCM can be rejected and the traditional fixed-coefficient model accepted.

Theil (1971, p. 623) has shown that the OLS estimator of λ_1 in equation (6) is unbiased but will result in an inefficient estimator for the variance of the estimate of λ_1 , $\text{var}(\lambda_{1j})$.⁶ The procedure suggested by Theil to estimate λ_1 and $\text{var}(\lambda_{1j})$ is described briefly below.

First, the ordinary least squares residuals, denoted by \hat{e}_j , must be calculated from equation (2). Second, equation (7) must be estimated using OLS.

$$(7) \quad \hat{e}_j^2 = m_0 P_j + m_1 Q_j + f_j$$

where⁷

$$P_j = 1 - \frac{\left(\hat{B}_j^2 \right)}{\left(\sum \hat{B}_j^2 \right)}$$

$$Q_j = \hat{B}_j^2 \left(1 - 2 \frac{\left\{ \hat{B}_j^2 \right\}}{\left\{ \Sigma \hat{B}_j^2 \right\}} + \frac{\left\{ \Sigma \hat{B}_j^4 \right\}}{\left\{ (\Sigma \hat{B}_j^2)^2 \right\}} \right)$$

and

f_j = stochastic error term.

The coefficients m_0 and m_1 are to be estimated. They represent the variance of the error term in equation (6) and $\text{var}(\lambda_{1j})$, respectively. The statistical significance of m_1 (as measured by its t-statistic) then determines whether the RCM is appropriate. However, because of the heteroscedasticity in equation (7), Theil suggests that equation (6) be estimated using generalized least squares (GLS).⁸ The GLS estimate for λ_1 in equation (6) is defined in equation (8).

$$(8) \quad \hat{\lambda}_1 = \left(\frac{\Sigma \hat{B}_j \bar{R}_j}{m_0 + m_1 \hat{B}_j^2} \right) \bigg/ \left(\frac{\Sigma \hat{B}_j^2}{m_0 + m_1 \hat{B}_j^2} \right) .$$

Note that if m_1 is not statistically different from zero, equation (8) reduces to the OLS estimate for $\hat{\lambda}_1$.

IV. EMPIRICAL RESULTS

The securities used to estimate the risk-return relationship are the common stock of 694 New York Stock Exchange companies. For each stock, \hat{B}_j was estimated from the single-index market model, equation (9), using monthly non-compounded price change plus dividend returns for the 72 month period from January, 1966 to December, 1971.

$$(9) \quad R_{jt} = \alpha_j + \beta_j R_{mt} + u_{jt}$$

where

R_{jt} = return on stock j in month t

R_{mt} = market return in month t

u_{jt} = stochastic error term in month t for stock j , and,

α_j and β_j are the parameters to be estimated.

The S&P 500 index with dividends included was used for the market index.

The time period was also partitioned into two non-overlapping 36 month periods--January, 1966 to December, 1968 and January, 1969 to December, 1971. Equation (9) was estimated for both time periods. The market risk premium was positive for the first sample period and negative for the second sample time period.

Estimates of the fixed coefficient OLS model equation (2) for each of the three time periods are presented in Table 1. The theoretical values for λ_0 and λ_1 are also shown in Table 1. For each time period, the signs of the estimated parameters were the same as theory suggests. And, each parameter was significantly different from zero at the 1% level of significance. Other researchers who estimated the risk-return relationship found that the estimated values for the parameters were significantly different from the theoretical values. Table 1 suggests that for each of the three time periods, $\hat{\lambda}_1$ was significantly different from the theoretical value of the market risk premium. For the two 36 month time periods, $\hat{\lambda}_0$ was not statistically different from the theoretical value, R_f .⁹

The results for $\hat{\lambda}_1$ and \hat{m}_1 [= var (λ_{1j})] for the RCM are summarized in Table 1. For the 72 month period, the variance of λ_{1j} was positive and significantly different from zero at the 5% level of significance.¹⁰ This was also found for the 36 month period January, 1966 to December, 1968 in which the market risk premium was positive. Hence, for the two periods in which the market risk premium was positive, the SML was found to exhibit the property of a RCM. However, when the market risk premium was negative, namely, from January, 1968 to December, 1971, the variance of λ_{1j} was not statistically significant.

It is also interesting to note the degree of randomness of the market risk premium for the two cases in which \hat{m}_1 was statistically significant. The coefficient of variation, $\sqrt{\hat{m}_1/\hat{\lambda}_1}$, for the 72 month and 36 month time periods were 1.57 and .97, respectively. This indicates considerable random movement in relation to $\hat{\lambda}_1$. If a 95% confidence interval was constructed for the movements around $\hat{\lambda}_1$ based on $\sqrt{\hat{m}_1}$, the interval would include the theoretical value for the market risk premium.

V. CONCLUSIONS

The risk-return relationship of capital market theory is not simply a model accepted by some academicians. Regulators have used the model to estimate the appropriate return on equity for regulated firms. Corporate management has been encouraged to use the model to evaluate the performance of in-house or independent pension portfolio managers. The performance of an entire industry has been questioned based on empirical results which have used the theoretical model. We must, therefore, continue to evaluate the model both theoretically and empirically.

In this paper, we disclose a disturbing empirical result of the risk-return relationship. Employing a stochastic parameter regression model, we find that the market risk premium varies randomly from one security to the next. Moreover, the observed randomness was substantial. General plausible explanations for such results were suggested. Even if the reader rejects any or all of these arguments, it is difficult to refute the empirical findings. It is worthwhile to note that the empirical results of this paper have indirectly supported Roll and Ross's (1979) empirical results of testing the Arbitrage pricing theory.¹¹ The direct relationship between the results of this study and Roll and Ross's results will be developed in the future research.

FOOTNOTES

¹This is essentially the conclusion reached by Modigliani and Pogue (1974) in their review of the empirical tests of the risk-return relationship. Research subsequent to that reviewed by Modigliani and Pogue has not altered that conclusion.

²Previous research on skewness and the risk-return relationship is summarized in footnote 2 of Kraus and Litzenberger (1976).

³A more precise definition can be found in Theil (1971, p. 549).

⁴This was found true for a substantial number of stocks by Fabozzi and Francis (1978) using the stochastic parameter regression model described in the next section. In such cases, the total risk can be partitioned as follows:

$$\sigma_i^2 = (B_i^2 + \sigma_{B_i}^2) \sigma_m^2 + \sigma_{e_i}^2$$

where

$$\sigma_i^2 = \text{variance for the returns for stock } i$$

$$\sigma_m^2 = \text{variance for the market return}$$

$$\sigma_{e_i}^2 = \text{unsystematic risk for stock } i$$

and

$$\sigma_{B_i}^2 = \text{variance for the systematic risk of stock } i$$

In such a case, equation (1) is then

$$\bar{R}_j = \lambda_0 + \lambda_1 (B_i^2 + \hat{\sigma}_{B_i}^2)^{1/2} .$$

Hence, equation (2) is misspecified in that $\hat{\sigma}_{B_i}^2$ is not considered.

Note also that if the traditional procedure for¹ computing unsystematic risk is employed but the market model is a RCM, then the unsystematic risk would improperly include $\sigma_{B_i}^2 \sigma_m^2$. This might explain why some researchers have found a positive¹ relationship between average returns and unsystematic risk.

⁵See Rosenberg (1973) for a description of various stochastic parameter variation models.

⁶This inefficiency results from the fact that w_j in equation (6) is heteroscedastic [see Theil (1971, pp. 623)]. This may help explain why Miller and Scholes (1972) found heteroscedasticity when they estimated equation (2).

⁷In the equations below \hat{B}_j and \bar{R}_j represent deviations of each variable from their respective means. The summation is over all observations.

⁸Theil (1971) has shown that to estimate the variance-covariance matrix for generalized least squares in this case, the following weights should be used:

$$Z_j = 1/2(\hat{m}_0^2 p_j + \hat{m}_1^2 q_j)^{-2}$$

where Z_j = the weight for the j th observation and \hat{m}_0 and \hat{m}_1 are the ordinary least squares estimates of m_0 and m_1 for equation (7).

⁹This result was somewhat surprising in light of the analytical results derived in Section II. It was shown there that λ_0 will be a biased estimate of \bar{R}_f if the multi-index model is appropriate.

¹⁰A one-tail test is used since the alternative hypothesis is that the variance of λ_{1j} is positive.

¹¹Roll and Ross have found that there exist at least three and probably four "priced" factors in addition to market factor in the generating process of return.

REFERENCES

- Arditti, F. D. (1967), "Risk and the Required Rate of Return," Journal of Finance (March), pp. 19-36.
- Bachrach B. and Galai, D. (1969), "The Risk-Return Relationship and Stock Prices," Journal of Financial and Quantitative Analysis (June), pp. 421-442.
- Brennan, M. J. (1970), "Taxes, Market Valuation and Corporate Financial Policy," National Tax Journal, pp. 417-427.
- Fabozzi, F. J. and Francis, J. G. (1978), "Beta as a Random Coefficient," Journal of Financial and Quantitative Analysis (March), pp. 101-115.
- Hessel, C. A. (1978), "The Effects of Taxes and Imperfect Competition on an Investor's Optimum Portfolio," unpublished doctoral dissertation, New York University.
- Joehnk, M. D. and Petty, J. W. (1980), "The Interest Sensitivity of Common Stock Prices," Journal of Portfolio Management (Winter), pp. 19-25.
- Kraus, A. and Litzenberger, R. H. (1976), "Skewness Preference and the Valuation of Risk Assets," Journal of Finance (September), pp. 1084-1100.
- Lanstein, R. and Sharpe, W. F. (1978), "Duration and Security Risk," Journal of Financial and Quantitative Analysis (November), pp. 653-668.
- Levy, H. (1978), "Equilibrium in an Imperfect Market: A Constraint on the Number of Securities in the Portfolio," American Economic Review (September), pp. 643-658.
- Lintner, J. (1965), "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," The Review of Economics and Statistics 47 (February), pp. 13-37.
- Miller, M. M. and Scholes, M. (1972), "Rates of Returns in Relation to Risk: A Re-examination of Recent Findings," published in Studies in the Theory of Capital Markets, edited by Michael Jensen Praeger, pp. 47-78.
- Modigliani, F. and Rogue, G. A. (1974), "An Introduction to Risk and Return," Financial Analysts Journal (March-April), pp. 68-80; (May-June), pp. 69-86.
- Roll, R. (1977), "A Critique of the Asset Pricing Theory's Tests; Part I: On Past and Potential Testability of the Theory," Journal of Financial Economics, (March), pp. 129-176.

- Roll, R. and S. A. Ross (1979), "An Empirical Investigation of The Arbitrage Pricing Theory," Working paper 15-79, Graduate School of Management, UCLA.
- Rosenberg, B. (1973), "A Survey of Stochastic Regression Parameters," Annals of Economic and Social Measurement, Vol. 2, pp. 381-397.
- Ross, S. A. (1976), "Arbitrage Theory of Capital Asset Pricing," Journal of Economic Theory, 8, pp. 341-360.
- Ross, S. A. (1977), "Return, Risk and Arbitrage," In I. Friend and J. L. Bicksler, Risk and Return in Finance, Vol. 1 (edited), Ballinger Publishing Company.
- Sharpe, W. F. (1964), "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," Journal of Finance, (September), pp. 425-442.
- Sharpe, W. F. (1977), "The Capital Asset Pricing Model: A Multi-Beta Interpretation," in Levy, H. and M. Sarnat, eds., Financial Decision Making Under Uncertainty, (New York: Academic Press, 1977).
- Theil, H. (1971), Principles of Econometrics, John Wiley & Sons.
- Treynor, J. L. (1961), "Toward A Theory of Market Value of Risky Assets," unpublished paper.

TABLE I

SUMMARY OF RESULTS¹

Time Period	Fixed Coefficient Model		Random Coefficient Model		Theoretical Values ²		
	$\hat{\lambda}_0$	$\hat{\lambda}_1$	r^2	$\hat{\lambda}_1$	$\hat{m}_1 = \text{var}(\hat{\lambda}_{1,t})$	$\lambda_0 = \bar{R}_f$	$\lambda_1 = \bar{R}_m - \bar{R}_f$
Jan. 1966 to Dec. 1971	.0025 (2.78)	.0046 (6.27)	.054 (39.50)	.0044 (5.97)	.000048 (2.72)	.0044	.0006
Jan. 1966 to Dec. 1968	.0047 (3.99)	.0105 (11.38)	.158 (129.85)	.0100 (9.67)	.000094 (2.50)	.0040	.0026
Jan. 1969 to Dec. 1971	.0037 (2.82)	-.0036 (-3.58)	.018 (12.68)	-.0045 (-4.31)	.000041 (.93)	.0048	-.0015

¹The T-values are shown below the estimated coefficient. The coefficients of determination is denoted as r^2 . The value below r^2 represents the F-value. All estimated values are significantly different from zero at the 1% level except for the estimate of m_1 for the time period Jan. 1969 to Dec. 1971.

²The average monthly yields from three month Treasury Bills are used to measure the risk-free return, \bar{R}_f .



UNIVERSITY OF ILLINOIS-URBANA



3 0112 060296271