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The Science Series

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THE STARS

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A STUDY OF THE UNIVERSE

BY

SIMON NEWCOMB

RETIRED PROFESSOR U. S. NAVY

"*Hæc sunt fastigia mundi*"

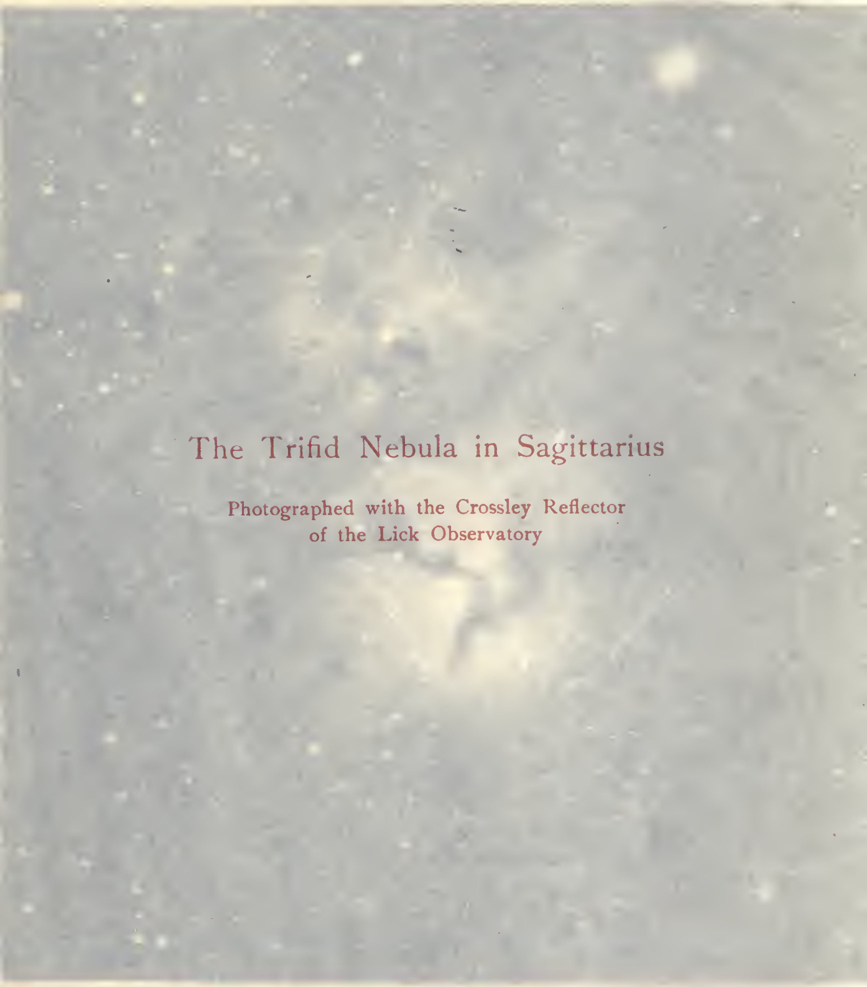
NEW YORK

G. P. PUTNAM'S SONS

LONDON

JOHN MURRAY

1902



The Trifid Nebula in Sagittarius

Photographed with the Crossley Reflector
of the Lick Observatory

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PREFACE

WHEN the author accepted the flattering invitation of the editor to prepare a volume for the present "Science Series," he supposed that it would be an easy task to sketch in simple language for the lay as well as the scientific reader the wonderful advances of our generation in the knowledge of the fixed stars. But, as the work went on, it became more evident at every step that such was not the case. The problem was, now to study whole chapters of observations and researches on some minute branch of the subject, and condense their gist into a few sentences; now to search volumes of periodicals, perhaps in vain, to find who was first in some field, or what result some investigator had reached; now to do justice to the respective works of students of the same subject; now to recast or rewrite passages in the light of some newly published research. The author must say in all candour that he has failed to surmount the difficulties thus arising in a way satisfactory to himself, and that in consequence the professional reader, if any such shall take up the book, will find defects that may seem to him serious in nearly every chapter. In palliation can be only pleaded the extent and complexity of the subject, and the impossibility of entering far into technical details in a work designed mainly for the general use.

In treating such a subject it is impossible always to avoid the use of language more or less technical, except at the expense of precision and completeness of statement. An effort has however been made to limit the use of such language to the necessities of the case.

The most gratifying experience associated with the work has been the cordial assistance and support rendered by a number of the author's friends and colleagues, who have supplied him with the material necessary to the presentation of their latest researches. Professor Campbell has supplied nearly all the material relating to spectroscopic binary systems, completed and revised the list of those objects, and freely placed at the author's disposal photographs taken at the Lick Observatory, including the frontispiece to the volume. Professor Kapteyn has supplied a large mass of material, published and unpublished, relating to his researches in stellar statistics, of which, however, only inadequate use could be made. Professor Pickering has permitted the free use of the treasures contained in the circulars and other publications of the Harvard Observatory, and Sir William Huggins has communicated the results of his latest studies in the life-history of the stars. Sirs A. A. Common and Isaac Roberts have each supplied a specimen of their photographs of nebulæ, and Father Sidgreaves,

S. J., of his photographs of spectra taken at the Stonyhurst College Observatory. Professor Barnard has allowed the use of his photographs of the Milky Way.

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THE STARS

CHAPTER I

REVIEW OF RECENT PROGRESS

These are thy glorious works, Parent of good,
Almighty, thine this universal frame,
Thus wondrous fair.—MILTON.

WE begin our study of the stars by a glance at the structure of the universe. What are familiarly known as the heavenly bodies belong to two classes which are very different as regards their relation to our earth. Those nearest to us form a sort of colony far removed from all the others, called the solar system. The principal bodies of this system are the sun and eight great planets, with their moons, revolving round it. On one of these planets, small when compared with the great bodies of the universe, but large to our every-day conceptions, we dwell. The other planets appear to us as stars. Four of them, Venus, Mars, Jupiter, and Saturn, are distinguished from the fixed stars by their superior brightness and characteristic motions. Of the remaining three, Mercury will rarely excite notice, while Uranus is nearly invisible to the naked eye, and Neptune quite so.

The dimensions of the solar system are vast when compared with any terrestrial standard. A cannon-shot going incessantly at its usual speed would be five hundred years in crossing the orbit of Neptune from side to side. But vast as these dimensions are, they sink into insignificance when compared with the distances of the stars. Outside the solar system are spaces which, so far as we know, are absolutely void, save here and there a comet or a meteor, until we look far outside the region which a cannon-shot would cross in a million of years. The nearest star is thousands of times farther away than the most distant planet. Scattered at these inconceivable distances are the bodies to which our attention is directed in the present work. If we are asked what they are, we may reply that the stars are suns. But we might equally well say that the sun is one of the stars; a small star, indeed, surrounded by countless others, many of which are much larger and brighter than itself. We shall treat our theme as far as possible by what we may call the natural method, beginning with what, being most obvious to the eye, was first noticed by man, or will be first noticed by an observer, and tracing knowledge up step by step to its present state.

Several features of the universe of stars will be evident at a glance. One of these is the diversity of the apparent brightness, or, in technical language, of the magnitudes of the stars. A few far outshine the great mass of their companions. A greater number are of what we may call medium brightness; there is a yet larger number of fainter ones, and about one-half of all those seen by a keen eye under favourable conditions are so near the limit of visibility as to escape ordinary notice. Moreover, those which we see are but an insignificant fraction of the number revealed by the telescope. The more we increase our optical power, the greater the number that come into view. How many millions may exist in the heavens it is scarcely possible even to guess. The photographic maps of the heavens now being made probably show more than fifty millions, perhaps one hundred millions, possibly twice this number.

Another evident feature is the tendency of the brighter stars to cluster into groups, known as constellations. The latter are extremely irregular, so that we cannot always decide where one constellation should end and another begin, or to which constellation a certain star may belong. Hence, the definition and mapping out of the constellations and the division of the stars among them are somewhat arbitrary proceedings.

A third feature is the Milky Way or Galaxy, which to ordinary vision appears as an irregular succession of cloud-like forms spanning the heavens. We now know that these seeming clouds are really congeries of stars too faint to be individually visible to the naked eye. We shall hereafter see that the stars of the Galaxy form, so to speak, the base on which the universe appears to be constructed.

Each of these three features will be considered in its proper place. In the present chapter we shall make a rapid survey of what has been done in our time to advance our knowledge of the stars.

A natural result of the northern hemisphere being the home of civilised peoples was that, until recent times, the study of the southern heavens had been comparatively neglected. It is true that the curiosity of the enquiring astronomers of the past would not be satisfied without their knowing something of what was to be seen south of the equator. Various enterprises and establishments had therefore contributed to our knowledge of the region in question. As far back as 1677, during a voyage to St. Helena, Halley catalogued the brighter stars in the region near the South Pole. About 1750 Lacaille, of France, established an observing station at the Cape of Good Hope, and made a catalogue of several thousand stars, which has remained a handy-book for the astronomer up to the present time. In 1834–38 Sir John Herschel made a special voyage to the Cape of Good Hope, armed with the best telescopes which his father had shown him how to construct, for the purpose of doing for the southern heavens as much as possible of what his father had done for the northern. The work of this expedition forms one of the most important and interesting chapters in the history of astronomic science. Not only is Herschel's magnificent volume a classic of astronomy, but the observations which it contains are still as carefully and profitably studied as any that have since been made. They may be said to form the basis of our present knowledge of the region which they included in their scope.

Herschel's work may be described as principally in the nature of an exploration. He had no instruments for accurately determining the positions of stars. In the latter field the first important contributions after Lacaille were made by Sir Thomas Brisbane, Governor of New South Wales, and Rumker, his assistant, at Paramatta. Johnson, of England, about 1830, introduced modern accuracy into the construction of a rather limited catalogue of stars which he observed at St. Helena. About the same time the British Government established the observatory at the Cape of Good Hope, which has maintained its activity to the present time. About the middle of the century the Government of New South Wales established, first at Williamstown and then at Melbourne, an observatory which has worked in the same field with marked success.

An American enterprise in the same direction was that of Captain James M. Gilliss, who, in 1849, organised an astronomical expedition to Chili. The principal motive of this enterprise was the determination of the solar parallax by observations upon Venus and Mars about the time of their nearest approach to the earth. As these observations would take but a small part of his time, Gilliss determined to take with him instruments for determining the positions of the stars. He established his observatory at a point near Santiago, where he continued his observations for nearly three years. He was an excellent practical observer, but an untoward circumstance detracted from the value of his work. His observatory was built upon a rocky eminence, a foundation which seemed to afford the best possible guaranty of the stability of his instruments. He made no attempt to reduce his observations till after his return home. Then it was found that the foundation, through the expansion and contraction due to the heat of the sun, was subject to a diurnal change which made it extremely difficult to derive good results from his careful work. It was not until 1896, more than thirty years after his death, that the catalogue of the stars observed by him was at last completed and published.

We do not derogate in any way from the merit of these efforts in saying that they could not lead to results comparable with those of the score of richly equipped northern observatories which the leading nations and universities of Europe had endowed and supported for more than a hundred years. Only within the last thirty years has it been possible to bring our knowledge of the southern heavens up to a satisfactory stage. Now, however, the progress of southern astronomy, if we may use the term, is such that in several points our knowledge of the southern heavens surpasses that of the northern ones. If we measure institutions by the importance of the work they are doing, there are several in the southern hemisphere which must to-day be placed in the first rank.

The history and work of the Cordoba Observatory are of special interest. In 1870 Dr. B. A. Gould, who might fairly be considered as the father of modern American astronomy, conceived the idea of establishing an observatory of the first class in South America. He found the President and Government of the Argentine Republic ready to support his scheme with a liberality well fitted to impress us with a high sense of their standard of civilisation. In a year or two the observatory at Cordoba was in active operation. The discussions to which its work gives rise belong to a subsequent chapter. But there is one branch that is worthy of special mention in the present connection. The *Uranometria Argentina*, published in 1879, in a quarto volume, with a large atlas, must be regarded as one of the most remarkable contributions yet made to our knowledge of the southern sky. It is concerned exclusively with the objects visible to the naked eye, or at most with an opera-glass. These were studied, described, catalogued, and mapped with a minuteness of detail exceeding anything yet done in that line for the northern sky. The notes to the catalogue alone comprise fifty pages, but being duplicated in the English and Spanish languages, really fill more than a hundred. A particular watch was kept up for variable stars; and the evidence, conclusive or doubtful, for variability, takes up an important part of the notes. These are followed by a discussion of the distribution of the stars, primarily of the southern hemisphere, but incidentally including the northern, which must still be regarded as a standard study of the subject. Dr. Gould continued in active charge of the Cordoba Observatory until 1885, when he returned home, and was succeeded by Thome, the present director.

A few years after Gould went to Cordoba, Gill was made director of the Royal Observatory at the Cape of Good Hope. The rapid growth of this institution to one of the first rank is due no less to the scientific ability of the new director than to the unflagging energy which he has devoted to the enlargement of the resources of the institution. The great fact which he sought to impress upon his supporters was that the southern celestial hemisphere was as large as the northern, and therefore equally worthy of study.

In any general review of the progress of stellar astronomy during the past twenty years, we should find the Harvard Observatory before us at every turn. What it has done will be seen, though in an imperfect way, in subsequent chapters. Not satisfied with the northern hemisphere, it has established a branch at Arequipa, Peru, in which its methods of observation and research are extended to the south celestial pole. Its principal specialty has been the continuous exploration of the heavens. Celestial photography, photometry, and spectroscopy sum up its fields of activity. For more than ten years it might be almost said that a sleepless watch of the heavens has been kept up by an all-seeing photographic eye, with an accuracy of which the world has hardly had a conception. The completeness with which its work has been done has recently been shown in a striking way.

Our readers are doubtless acquainted with the singular character of the minor planet Eros, whose orbit passes through that of Mars, as one link of a chain passes through another, and which comes nearer the earth at certain times than any other celestial body, the moon excepted. When the character of the orbit became established, it was of interest to know whether the planet had ever been observed as a fixed star at former oppositions. Chandler, having computed the path of the planet at the most important of the oppositions, beginning with 1892–94, communicated his results to Director Pickering, and suggested a search of the Harvard photographs to see if the planet could be found on them. The result was the discovery of the planet upon more than a score of plates taken at various times during the preceding ten years.

New stars were formerly supposed to be of very rare occurrence, but since the Harvard system of photographing the heavens has been introduced, five or six have been known to burst forth.

The Spectroscope and Photographic Plate.

Although the first application of the spectroscope to the study of the heavenly bodies was made within the memory of the present generation, its results have been only less epoch-making than those of the telescope. The two instruments differ in that the one, bringing all the light from a star which falls on its object-glass to one focus, sends it all into the eye, thus multiplying it hundreds or thousands of times, and bringing into view a universe of stars formerly invisible. The newer instrument operates by analysing the light collected by the telescope into its separate colors or kinds, which it arranges, as it were, on a sheet. The sheet is simply the retina of the eye on which the spectrum is formed. Thus the eye is enabled to see the quantity of light on every part of the sheet, and by the immense variety of arrangement which the method admits of, remarkable conclusions respecting the constitution and motion of the body that emits the light can be drawn. The most distinctive feature of the spectroscopic method arises from the fact that the composition of the light is independent of the distance of the body. The spectroscopist can therefore draw conclusions as to the constitution and motion of the most distant star, as readily as he can about those of the flame within his laboratory.

Spectroscopy has, in recent times, been re-enforced by photography. In the early '40's, Dr. Draper took a daguerreotype of the moon. As the photographic art was developed, astronomers naturally occupied themselves with photographing celestial bodies by the light which they emitted. For this purpose the telescope could be used as a camera. The first important step in this direction was taken by Bond at Harvard. The next great advance was made by Rutherfurd of New York, who photographed clusters of stars and used the plates in determining the positions of the individual bodies of the cluster.

When more sensitive chemicals were introduced into photography, another step in advance was made by combining the spectroscopic and the sensitive plate into a spectrograph. In all the more serious spectroscopic work of the present day the spectrum is photographed, and the astronomer, or astrophysicist as he now calls himself, can study and measure the plates at his leisure.

The great revelations of our times have come through the application of this method to the measurement of motions in the line of sight from us to a star. No achievement of the intellect of man would have seemed farther without the range of possibility to the thinker of half a century ago than the discoveries of invisible bodies which are now being made by such measurements. The revelations of the telescope take us by surprise. But if we consider what the thinker alluded to might regard as attainable, they are far surpassed by those of the spectroscope. The dark bodies, planets we may call them, which are revolving round the stars, must be for ever invisible in any telescope that it would be possible to construct. They would remain invisible if the power of the instrument were increased ten thousand times. And yet if there are inhabitants on these planets, our astronomers could tell them more of the motions of the world on which they live than the human race knew of the motions of the earth before the time of Copernicus.

The men and institutions which have contributed to this result are so few in number that it will not be tedious to mention at least the principal actors. The possibility of measuring the motions of the stars in the line of sight by means of the spectroscope was first pointed out by Mr. now Sir William Huggins. He actually put the method into operation. As soon as its feasibility was demonstrated it was taken up at

Greenwich. In these earlier attempts, eye methods alone were used, and the results were not always reliable. Then spectrum photography was applied at the German astrophysical observatory of Potsdam by Vogel, who introduced into the method a degree of precision which had never before been reached. His measures of the motions of the stars in the line of sight opened up the last era in science. Applying the method to the variable star Algol, he proved that the loss of light which it undergoes at intervals of nearly three days is merely a partial eclipse by a dark planet, almost as large as itself, revolving round it. Thus was discovered a new order of bodies in the universe, telescopic binary systems, pairs of stars, or stars and planets, revolving round each other by their mutual gravitation; although no telescope that it is possible to make would ever show that more than a single body was present. Thence the photographic method soon spread to Meudon and Pulkova. But, as often happens when new fields of research are opened, we find them cultivated in quarters where we should least expect. The successful application of the method requires not only the best spectroscope, but the most powerful telescope at command. Ten years ago the most powerful telescope in the world was at the Lick Observatory. A few years later Mr. D. O. Mills put at its eye end the best spectrograph that human art could make at that time, the work of Brashear. It is Campbell who, with this instrument, has inaugurated a series of discoveries in this line which are without a parallel. He finds that about one star in thirteen has a planet revolving round it, so massive as to change the motion of the star by an amount visible in the spectroscope. The more or less eccentric orbits of these bodies are being determined. The final conclusion from all his work is that isolated stars may be the exception rather than the rule; that possibly a great majority at least of the stars are composed of two or more bodies revolving round each other, though they appear in our telescopes as single.

The study of variable stars from being little more than a scientific amusement, has grown into an important branch of astronomical science. It has now joined hands with spectroscopy to make it probable that in most cases the variations of light in a star are due to changes in its constitution produced by invisible planets revolving round it.

All these results naturally involve a great increase in the number of men who are devoting themselves to astronomical research. When we study the work of this small army of investigators, and compare the possibilities of the field they are exploring with what has been done in the past, we feel that astronomy, although the oldest of the sciences in years, is reaching a stage of vigorous youth, and that the twentieth century will open up views of the universe of which quite possibly we, at its beginning, have no conception.

A mere survey of what has been done in the various lines we have mentioned would be far from giving an idea of the real significance of the advance we are considering. Cataloguing the stars, estimating their magnitudes, recording and comparing their spectra, and determining their motions might be considered as, after all, barren of results of the highest human interest. When we know the exact position of every star in the heavens, the direction in which it is moving, and the character of its spectral lines, how much wiser are we?

What could hardly have been foreseen fifty years ago, is that these various classes of results are now made to combine and converge upon the greatest problem which the mind of man has ever attempted to grasp—that of the structure of the universe. The study of variable stars has suddenly fallen into line, so to speak, so that now it is uniting itself to the study of all the other celestial objects, to give us at least a faint conception of what the solution of this problem may be.

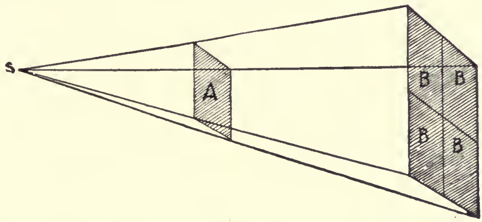
One of the principal objects of the present work is to make a comparison of these various researches, and discuss the views respecting the constitution of the stars individually, as well as of the universe as a whole, to which they lead us. But there are a number of details to be considered singly before we can combine results in this way. Our early chapters will, therefore, be devoted to the special features and individual problems of stellar astronomy which have occupied the minds of astronomers from the beginning of their work to the present time. Keeping these details in mind, we can profitably proceed to the consideration of the general conclusions to be drawn from them.

CHAPTER II

MAGNITUDES OF THE STARS

And one star differeth from another star in glory.—PAUL.

THE apparent brightness of a star, as we see it from the earth, depends upon two causes—its intrinsic brilliancy, or the quantity of light which it actually emits, and its distance from us. It follows that if all the stars were of equal intrinsic brightness we could determine their relative distances by measuring the respective amounts of light which we receive from them. The quantity of light in such a case varies inversely as the square of the distance. This will be seen by the figure, where S represents the position of a star,



regarded as a luminous point, while A and B B B B are screens placed at such distances that each will receive the same amount of light from the star. If the larger screen is twice as far as the screen A, its sides must be twice as long in order that it shall receive all the light that would fall on A. In this case, its surface will be four times the surface of A. It is then evident that each quarter of the surface marked B will receive one-fourth as much light as the surface A. Thus, an eye or a telescope in the position B will receive from the star one-fourth as much light as in the position A, and the star will seem one-fourth as bright.

The fact is, however, that the stars are very unequal in their actual brightness, and in consequence the apparent magnitude of a star gives us no clue to its distance. Among the nearer of the stars are some scarcely, if at all, visible to the naked eye, while among the brighter ones are several whose distances are immeasurably great. A remarkable example is that of Canopus, the second brightest star in the heavens.

For these reasons astronomers are obliged to content themselves, in the first place, with determinations of the actual amount of light that the various stars send to us, or their apparent brilliancy, without regard to their distance or actual brilliancy. The ancient astronomers divided all the stars they could see into six classes, the number expressing the apparent brightness being called the magnitude of the star. The brightest ones, numbering in all about fourteen, were said to be of the first magnitude. The fifty next in brightness were said to be of the second magnitude. Three times as many, an order fainter, were of the third magnitude. The progression was continued up to the sixth magnitude, which included those which were barely visible.

As the stars are actually of every degree of apparent brilliancy, no sharp line of demarkation could be drawn between those of one magnitude and those of the magnitude next higher. Hence, different observers made different estimates, some calling a star of the second magnitude which others would call of the first, and designating as of the third magnitude one which others would call of the second. It is therefore impossible to state, with absolute numerical precision, what number of stars should be regarded as of one magnitude and what of another.

An idea of the magnitude of a star can be readily gained by the casual observer. Looking at the heavens on almost any cloudless evening, we may assume that the two, three, or more brightest stars which we see are of the first magnitude. As examples of those of the second magnitude, may be taken the five brightest stars of the Dipper, the Pole Star, and the brighter stars of Cassiopeia. Some or all of these objects can be seen on any clear night of the year in our latitude. Stars of the third magnitude are so numerous that it is difficult to select any one for comparison. The brightest star of the Pleiades is really of this magnitude, but it does not appear so in consequence of the five other stars by which it is surrounded. At a distance of 15° from the Pole Star, Beta Ursa Minoris is always visible, and may be distinguished by being slightly redder than the Pole Star; it lies between two fainter stars, the brighter of which is of the third and the other of the fourth magnitude. The five readily visible but fainter stars of the Pleiades are about of the fourth magnitude. Of the fifth magnitude are the faintest stars which are easily visible to the naked eye, while the sixth comprises those which are barely visible with good eyes.

Modern Conception of Magnitude.

Modern astronomers, while adhering to the general system which has come down to them from ancient times, have sought to give it greater definiteness. Careful study showed that the actual amount of light corresponding to the different magnitudes varied nearly in geometrical progression from one magnitude to another, a conclusion which accords with the well-known psychological law that the intensity of sensation varies by equal amounts when the exciting cause varies in geometrical progression. It was found that an average star of the fifth magnitude gave between two and three times as much light as an average one of the sixth; one of the fourth gave between two and three times as much light as one of the fifth; and so on to the second. In the case of the first magnitude, the diversity is so great that it is scarcely possible to fix an average ratio. Sirius, for example, is really six times as bright as Altair, which is commonly taken as a standard for a first-magnitude star. To give precision to their estimates, modern astronomers are gradually seeking to lay the subject of magnitudes on an exact basis by defining a change of one unit in the magnitude as corresponding to an increase of about two and one-half times in the amount of light.

If the practice of separating the visible stars into only six orders of magnitude were continued without change, we should still have the anomaly of including in one class stars of markedly different degrees of brightness. Some more than twice as bright as others would be designated as of the same magnitude. Hence, to give quantitative exactness to the results, a magnitude is regarded as a quantity which may have any value whatever, and may be expressed by decimals—tenths or even hundredths. Thus, we may have stars of magnitude 5.0, 5.1, 5.2, etc., or we may even subdivide yet further and speak of stars having magnitudes 5.11, 5.12, etc. Unfortunately, however, there is as yet no way known of determining the amount of light received from a star except by an estimate of its effect upon the eye. Two stars are regarded as equal when they appear to the eye of equal brilliancy. In such a case the judgment is very uncertain. Hence, observers have endeavoured to give greater precision to it by the use of photometers,—instruments for measuring quantities of light. But even with this instrument the observer must depend upon an estimated equality of light as judged by the eye. The light from one star is increased or diminished in a known proportion until it appears equal to that of another star, which may be an artificial one produced by the flame of a candle. The proportion of increase or diminution shows the difference of magnitude between the two stars.

As we proceed to place the subject of photometric measures of star-light on this precise basis we find the problem to be a complex one. In the first place, not all the rays which come from a star are visible to our eyes as light. But all the radiance, visible or invisible, may be absorbed by a dark surface, and will then show its effect by heating that surface. The most perfect measure of the radiance of a star would therefore be the amount of heat which it conveys, because this expresses what is going on in the body better than the amount of visible light can do. But unfortunately the heating effect of the rays from a star is below what can be measured by an instrument. We are therefore obliged to abandon any thought of determining the total amount of radiation and confine ourselves to that portion which we call light.

Here, when we aim at precision, we find that light, as we understand it, is properly measured only by its effect on the optic nerve, and there is no way of measuring this effect except by estimation. Thus, all the photometer can do is to give us the means of increasing or diminishing the light from one star, so that we can make it equal by estimation to that from some other star or source of light.

The difficulty of reaching strict results in this way is increased by the fact that the stars differ in color.

Effect of Colour on Magnitude.

Two lights can be estimated as equal with greater precision when they are of the same colour than when their colours are different. An additional source of uncertainty is brought in by what is known as the Purkinje phenomenon, after the physicist who first observed it. He found that if we took two lights of equal apparent brightness, the one red and the other green, and then increased or diminished them in the same proportion, they would no longer appear equal. In other words, the geometrical axiom that halves or quarters of equal quantities are themselves equal, does not apply to the effect of light on the eye. When the lights are diminished the green will look brighter than the red. If we increase them in the same proportion, the red will look brighter than the green. In other words, the red light will, to our vision, increase or fade away more rapidly with a given amount of change than the green light will.

It is found in recent times that this law of change does not extend progressively through all spectral colours. It is true that as we pass from the red to the violet end of the spectrum the yellow fades away less rapidly with a given diminution than does the red, and the green still less rapidly than the yellow. But when we pass from the green to the blue, it is said that the latter does not fade out quite so fast as the green.

One obvious conclusion from all this is that two stars of different colours which look equal to the naked eye will not look equal in the telescope. The red or yellow star will look relatively brighter in a telescope; the green or bluish one relatively brighter to the naked eye.

Photographic Magnitudes

In recent times stars have been photographed on a large scale. Their magnitudes can then be determined by the effect of the light on the photographic plate, the impression of the star, as seen in a microscope, being larger and more intense as the star is brighter. But the magnitude thus determined is not proportional to the apparent brightness as seen by the eye, because the photographic effect of blue light is much greater than that of red light having the same apparent brightness. In fact, the difference is so great that, with the chemicals formerly used, red light was almost without photographic effect. Even now, what we measure in taking the photograph of a star is almost entirely the light in the more refrangible portions of the spectrum. It appears therefore that when a blue and a yellow star, equally bright to the naked eye, are photographed, the impression made on the negative by the blue star will be greater than that made by the yellow one. A distinction is therefore recognised between photographic and visual magnitudes. The bluer the star, the brighter will be its photographic as compared with its visual magnitude.

The photographic magnitudes of the stars are now being investigated and catalogued on a scale even larger than that on which we have studied the visual magnitudes. Yet we have to admit the non-correspondence of the two systems. The most that can be done is to bring about the best attainable agreement between them in the general average of all the stars.

Fortunately the differences between the colours of the stars are by no means so great as those between the colours of natural objects around us. All the stars radiate light of all colours ; and although the colouring is quite appreciable by the eye, it is not so great as it would have been were the variations in colour as wide as in the case of terrestrial objects.

Two comprehensive surveys of the heavens, intended to determine as accurately as possible the magnitudes of all the brighter stars, have recently been undertaken. One of these is the Harvard photometry, commenced by Professor Pickering at the Harvard Observatory, and now extended to the southern hemisphere by the aid of the branch establishment at Arequipa, Peru.

Surveys of the Heavens

The instrument designed by Professor Pickering for his purpose is termed a meridian photometer, and is so arranged that the observer can see in the field of his telescope a reflected image of the Pole Star, and, at the same time, the image of some other star while it is passing the meridian. By a polarising apparatus the image of the star to be measured is made to appear of equal brightness with that of the Pole Star, and the position of a Nicol prism, which brings out this equality, shows the ratio between the magnitudes of the two stars.

The other survey, with the same object, is now being made at the Potsdam Astrophysical Observatory, near Berlin. In the photometer used by the German astronomers the image of one star is compared with an artificial star formed by the flame of a candle. The work is performed in a more elaborate way than at the Harvard Observatory, and in consequence only that part of the heavens extending from the equator to 40° north declination has been completed and published. A comparison of the results of the German astronomers with those of Professor Pickering shows a curious difference depending on the colour of the star. In the case of the reddest stars, the estimates are found to be in fairly close agreement, Pickering's being a little the fainter. But in the case of the white or bluish stars, the estimates of the German astronomers are more than one-fourth of a magnitude greater than those of Pickering. This corresponds to a change of nearly one-fourth in the brightness. Whether this difference is to be regarded as purely psychological, or as due to the instruments used, is an interesting question which has not yet been settled. It is difficult to conceive how different instruments should give results so different. On the one hand, the comparisons made by the Germans make it difficult to accept the view that the difference is due purely to the personality of the observers. There are two German observers, Drs. Müller and Kempf, whose results agree with each other exactly. On the other hand, Pritchard, at Oxford, made quite an extensive photometric survey, using an instrument by which the light of one star was cut down by a wedge-shaped dark glass, whereby any gradation of light could be produced. A comparison shows that the results of Pritchard agree substantially with those of Pickering. It is quite possible that the Purkinje phenomenon may be the cause of the difference, the source of which is eminently worthy of investigation.

It must not be supposed from this that such estimates are of no value for scientific purposes. Very important conclusions, based on great numbers of stars, may be drawn even from these uncertain quantities. Yet, it can hardly be doubted that if the light of a star could be measured from time to time to its thousandth part, conclusions of yet greater value and interest might be drawn from the measures.

We have said that in our modern system the aim has been to so designate the magnitudes of the stars that a series of magnitudes in arithmetical progression shall correspond to quantities of light ranging in geometrical progression. We have also said that a change of one unit of magnitude corresponds to a multiplication or division of the light by about 2.5. On any scale of magnitude this factor of multiplication is called the light-ratio of the scale. In recent times, after much discussion of the subject and many comparisons of photometric measures with estimates made in the old-fashioned way, there is a general agreement among observers to fix the light-ratio at the number whose logarithm is 0.4. This is such that an increase of five units in the number expressing the magnitude corresponds to a division of the light by one hundred. If, for example, we take a standard star of magnitude 1 and another of magnitude 6, the first would be one hundred times as bright as the second. This corresponds to a light-ratio slightly greater than 2.5.

When this scale is adopted, the series of magnitudes may extend indefinitely in both directions so that to every apparent brightness there will be a certain magnitude. For example, if we assign the magnitude 1.0 to a certain star, taken as a standard, which would formerly have been called a star of the first magnitude, then a star a little more than 2.5 times as bright would be of a magnitude one less in number, that is, of magnitude 0. The one next brighter in the series would be of magnitude – 1. So great is the diversity in the brightness of the stars formerly called of the first magnitude that Sirius is yet brighter than the star just supposed, the number expressing its magnitude being – 1.4.

Stellar Magnitude of the Sun.

This suggests what we may regard as one of the capital questions in celestial photometry. There being no limit to the extent of the photometric scale, what would be the stellar magnitude of the sun as we see it when expressed in this way on the scale? Such a number is readily derivable when we know the ratio between the light of the sun and that of a star of known magnitude. Many attempts have been made by observers to obtain this ratio; but the problem is one of great difficulty, and the results have been extremely discordant. Amongst them there are three which seem less liable to error than others: those of Wollaston, Bond, and Zöllner. Their results for the stellar magnitude of the sun are as follows:

Wollaston	–26.6
Bond	–25.8
Zöllner	–26.6

Of these, Zollner's seems to be the best, and may, therefore, in taking the mean, be entitled to double weight. The result will then be:

Stellar magnitude of sun..... –26.4

From this number may be readily computed the ratio of sunlight to that of a star of any given magnitude. We thus find:

The sun gives us:

10,000,000,000 times the light of Sirius.
91,000,000,000 times the light of a star of magnitude 1.
9,100,000,000,000 times the light of a star of magnitude 6.

The square roots of these numbers show the number of times we should increase the actual distance of the sun in order that it might shine as a star of the corresponding magnitude. These numbers and the corresponding parallax are as follows:

Sirius;	Distance=	100,000 :	Parallax=	2″.06
Mag. 1	"	302,000 :	"	0″.68
" 2	"	479,000 :	"	0″.43
" 3	"	759,000 :	"	0″.27
" 4	"	1,202,000 :	"	0″.17
" 5	"	1,906,000 :	"	0″.11
" 6	"	3,020,000 :	"	0″.07

These parallaxes are those that the sun would have if placed at such a distance as to shine with the brightness indicated in the first column. They are generally larger than those of stars of the corresponding magnitudes, from which we conclude that the sun is smaller than the brighter of the stars.

or less brilliant

CHAPTER III

CONSTELLATION AND STAR NAMES

Now came still evening on, and twilight grey
Had in her sober livery all things clad.
. . . now glowed the firmament
With living sapphires; Hesperus that led
The starry host rode brightest.—MILTON.

IT is strongly recommended to the reader to study the constellations for himself. If he desires to feel all the sublimity associated with them, he must not be satisfied with the hurried glance or occasional survey to which one commonly confines himself in his evening walk. What he should do is, on a clear and moonless summer evening, to escape from his usual surroundings, and go to a place, whether field or housetop, where there is nothing to obstruct his vision, or disturb the current of his thoughts. There he must recline on his back, so as to take in as much as possible of the starry vault at one view. One doing this for the first time will be surprised at the magnificence of the spectacle. As he looks upon the "universal frame" and reflects that it has stood as he now sees it through ages compared with which the whole period of human history is but a fleeting moment, the mind will be filled with a consciousness of infinity and eternity which never before entered it. Other sights become stale from custom, but this can never lose its relish. It can be enjoyed without knowing the name of a constellation, but is more impressive when one reflects that the eyes of man have gazed upon and studied it ever since our race appeared on earth.

In ancient times the practice was adopted of imagining the figures of heroes and animals to be so outlined in the heavens as to include in each figure a large group of the brighter stars. In a few cases some vague resemblance may be traced between the configurations of the stars and the features of the object they are supposed to represent; in general, however, the object chosen seems quite arbitrary. One animal or man could be fitted in as well as another. There is no historic record as to the time when the constellations were mapped out, or of the process by which the outlines were traced. The names of heroes, such as *Perseus*, *Cepheus*, *Hercules*, etc., intermingled with the names of goddesses, show that the time was probably during the heroic age. No maps are extant showing exactly how each figure was placed in the constellation; but in the catalogue of stars given by Ptolemy in his *Almagest*, the positions of particular stars on the supposed body of the hero, goddess, or animal are designated. For example, Aldebaran is said to have formed the eye of the Bull. Two stars marked the right and left shoulders of Orion, and a small cluster marked the position of his head. A row of three stars in a horizontal line showed his belt, three stars in a vertical line below them his sword. From these statements the position of the figure can be reproduced with a fair degree of certainty.

In the well-known constellation *Ursa Major*, the Great Bear, familiarly known as "the Dipper," three stars form the tail of the animal, and four others a part of his body. This formation is not unnatural, yet the figure of a dipper fits the stars much better than that of a bear. In *Cassiopeia*, which is on the opposite side of the pole from the Dipper, the brighter stars may easily be imagined to form a chair in which a lady may be seated. As a general rule, however, the resemblances of the stars to the figure are so vague that the latter might be interchanged to any extent without detracting from their appropriateness.

In any case, it was impossible so to arrange the figures that they should cover the entire heavens; blank spaces were inevitably left in which stars might be found. In order to include every star in some constellation, the figures have been nearly ignored by modern astronomers, and the heavens have been divided up, by somewhat irregular lines, into patches, each of which contains the entire figure as recognised by ancient astronomers. But all are not agreed as to the exact outlines of these extended constellations, and, accordingly, a star is sometimes placed in one constellation by one astronomer and in another constellation by another.

The confusion thus arising is especially great in the southern hemisphere, where it has been intensified by the subdivision of one of the old constellations. The ancient constellation *Argo* covered so large a region of the heavens, and included so many conspicuous stars, that it was divided into four, representing various parts of a ship—the sail, the poop, the prow, and the hull.

The Southern Constellations.

Dr. Gould, while director of the Cordoba Observatory, during the years 1870 to 1880, constructed the *Uranometria Argentina*, in which all the stars visible to the naked eye from the south pole to a parallel of declination ten degrees north of the celestial equator were catalogued and mapped. He made a revision of the boundaries of each constellation in such a way as to introduce greater regularity. The rule generally followed was that the boundaries should, so far as possible, run in either an east-and-west or a north-and-south direction on the celestial sphere. They were so drawn that the smallest possible change should be made in the notation of the conspicuous stars; that is, the rule was that, if possible, each bright star should be in the same constellation as before. The question whether this new division shall replace the ancient one is one on which no consensus of view has yet been reached by astronomers. Simplicity is undoubtedly introduced by Gould's arrangement; yet, in the course of time, owing to precession, the lines on the sphere which now run north and south or east and west will no longer do so, but will deviate almost to any extent. The only advantage then remaining will be that the bounding lines will generally be arcs of great circles.

When the heavens began to be carefully studied, two or three centuries ago, new constellations were introduced by Hevelius and other astronomers to fill the vacant spaces left by the ancient ones of Ptolemy. To some of these rather fantastic names were given; the *Bull of Poniatowski*, for example. Some of these new additions have been retained to the present time, but in other cases the space occupied by the proposed new constellation was filled up by extending the boundaries of the older ones.

At the present time the astronomical world, by common consent, recognises eighty-nine constellations in the entire heavens. In this enumeration *Argo* is not counted, but its four subdivisions are taken as separate constellations.

A glance at the heavens will make it evident that the problem of designating a star in such a way as to distinguish it from all its neighbours must be a difficult one. If such be the case with the comparatively small number of stars visible to the naked eye, how must it be with the vast number that can be seen only with the telescope? In the case of the great mass of telescopic stars we have no method of designation except by the position of the star and its magnitude; but with the brighter stars, and, indeed, with all that have been catalogued, other means of identification are available.

Naming the Stars.

It is but natural to give a special name to a conspicuous star. That this was done in very early antiquity we know by the allusion to Arcturus in the Book of Job. At least two such names, Castor and Pollux, have come down to us from classical antiquity, but most of the special names given to the stars in modern times are corruptions of certain Arabic designations. As an example we may mention Aldebaran, a corruption of *Al Dabaran* — The Follower. There is, however, a tendency to replace these special names by a designation of the stars on a system devised by Bayer early in the seventeenth century.

This system of naming stars is quite analogous to our system of designating persons by a family name and a Christian name. The family name of a star is that of the constellation to which it belongs. The Christian name is a letter of the Greek or Roman alphabet or a number. As any number of men in different families may have the same Christian name, so the same letter or number may be assigned to stars in any number of constellations without confusion.

The work of Bayer was published under the title of *Uranometria*, of which the first edition appeared in 1601. This work consists mainly of maps of the stars. In marking the stars with letters on the map, the rule followed seems to have been to give the brighter stars the earlier letters in the alphabet. Were this system followed absolutely, the brightest star should always be called Alpha; the next in order Beta, etc. But this is not always the case. Thus in the constellation *Gemini*, the brightest star is Pollux, which is marked Beta, while Alpha is the second brightest. What system, if any, Bayer adopted in detail has been a subject of discussion, but does not appear to have been satisfactorily made out. Quite likely Bayer himself did not attempt accurate observations on the brightness of the stars, but followed the indications given by Ptolemy or the Arabian astronomers. As the number of stars to be named in several constellations exceeds the number of letters in the Greek alphabet, Bayer had recourse, after the Greek alphabet was exhausted, to letters of the Roman alphabet. In this case the letter *A* was used as a capital, in order, doubtless, that it should not be confounded with the Greek α. In other cases small italics are used. In several catalogues since Bayer, new italic letters have been added by various astronomers. Sometimes these have met with general acceptance, and sometimes not.

Flamsteed was the first Astronomer Royal of England, and observed at Greenwich from 1666 to 1715. Among his principal works is a catalogue of stars in which the positions are given with greater accuracy than had been attained by his predecessors. He slightly altered the Bayer system by introducing numbers instead of Greek letters. This had the advantage that there was no limit to the number of stars which could be designated in each constellation. He assigned numbers to all the brighter stars in the order of their right ascension, irrespective of the letters used by Bayer. These numbers are extensively used to the present day, and will doubtless continue to be the principal designations of the stars to which they refer. It is very common in our modern catalogues to give both the Bayer letter and the Flamsteed number in the case of Bayer stars.

The catalogues by Flamsteed do not include quite all the stars visible to the naked eye; but various uranometries have been published which were intended to include all such stars. In such cases the designations now used frequently correspond to the numbers given in the uranometries of Bode, Argelander, and Heis.

In recent times these uranometries have been supplemented by censuses of the stars, which are intended to include all the stars to the ninth or tenth magnitude. I shall speak of these in the next section; at present it will suffice to say that stars are very generally designated by their place in such a census.

There is still here and there some confusion both as to the boundaries of the constellations and as to the names of a few of the stars in them. I have already remarked that, in drawing the imaginary boundaries on a star map, as representing the celestial sphere, different astronomers have placed the lines differently. One of the regions in which this is especially true is in the neighbourhood of the north pole, where some astronomers place stars in the constellation *Cepheus* which others place in *Ursa Minor*. Hence in the Bayer system the same star may have different names in different catalogues. Again, in extending the names or numbers, some astronomers use names which others do not regard as authoritative. The remapping of the southern hemisphere by Dr. Gould changed the boundaries of most of the southern constellations in a way already mentioned.

I have spoken of the subdivision of the great constellation *Argo* into four separate ones. Bayer having assigned to the principal stars in this constellation the Greek letters alpha, beta, gamma, etc., the general practice among astronomers since the subdivision has been to continue the designation of the stars thus marked as belonging to the constellation *Argo*. Thus, for example, we have *Alpha Argus*, which after the subdivision belonged to the constellation *Carina*. The variable star *Eta Argus* also belongs to the constellation *Carina*. But in the case of stars not marked by Bayer, the names were assigned according to the subdivided constellations, *Vela*, *Carina*, etc. Confusing though this proceeding may appear to be, it is not productive of serious trouble. The main point is that the same star should always have the same name in successive catalogues. Still, however, it has recently become quite common to ignore the constellation *Argo* altogether and use only the names of its subdivisions. The reader must therefore be on his guard against any mistake arising in this way in the study of astronomical literature.

In star catalogues the position of a star in the heavens is sometimes given in connection with its name. In this case the confusion arising from the same star having different names may be avoided, since a star can always be identified by its right ascension and declination. The fact is that, so far as mere identification is concerned, nothing but the statement of a star's position is really necessary. Unfortunately, the position constantly changes through the precession of the equinoxes, so that this designation of a star is a variable quantity. Hence the special names which we have described are the most convenient to use in the case of well-known stars. In other cases a star is designated by its number in some well-known catalogue. But even here different astronomers choose different catalogues, so that there are still different designations for the same star. The case is one in which uniformity of practice is unattainable.

CHAPTER IV

CATALOGUING AND NUMBERING THE STARS

Canst thou bind the sweet influences of Pleiades, or loose the bands of Orion? Canst thou bring forth Mazzaroth in his season? Or canst thou guide Arcturus with his sons?—JOB.

A CATALOGUE or list of stars is a work giving for each star listed its magnitude and its position on the celestial sphere, with such other particulars as may be necessary to attain the object of the catalogue. If the latter includes only the more conspicuous stars, it is common to add the name of each star that has one; if none is recognised, the constellation to which the star belongs is frequently given.

The position of a star on the celestial sphere is defined by its right ascension and declination. These correspond to the longitude and latitude of places on the earth in the following way: Imagine a plane passing through the centre of the earth and coinciding with its equator, to extend out so as to intersect the celestial sphere. The line of intersection will be a great circle of the celestial sphere, called the celestial equator. The axis of the earth, being also indefinitely extended in both the

Right Ascension and Declination.

north and the south directions, will meet the celestial sphere in two opposite points, known as the north and south celestial poles. The equator will then be a great circle 90° from each pole. Then as meridians are drawn from pole to pole on the earth, cutting the equator at different points, so imaginary meridians are conceived as drawn from pole to pole on the celestial sphere. Corresponding to parallels of latitude on the earth we have parallels of declination on the celestial sphere. These are parallel to the equator, and become smaller and smaller as we approach either pole. The correspondence of the terrestrial and celestial circles is this:

To *latitude* on the earth's surface corresponds *declination* in the heavens.

To *longitude* on the earth corresponds *right ascension* in the heavens.

A little study of this system will show that the zenith of any point on the earth's surface is always in a declination equal to the latitude of the place. For example, for an observer in Philadelphia, in 40° latitude, the parallel of 40° north declination will always pass through his zenith, and a star of that declination will, in the course of its diurnal motion, also pass through his zenith.

So also to an observer on the equator the celestial equator always passes through the zenith and through the east and west points of the horizon.

In the case of the right ascension, the relation between the terrestrial and celestial spheres is not constant, because of the diurnal motion, which keeps the terrestrial meridians in constant revolution relative to the celestial meridians. Allowing for this motion, however, the system is the same. As we have on the earth's surface a prime meridian passing from pole to pole through the Greenwich Observatory, so in the heavens a prime meridian passes from one celestial pole to the other through the vernal equinox. Then to define the right ascension of any star we imagine a great circle passing from pole to pole through the star, as we imagine one to pass from pole to pole through a city on the earth of which we wish to designate the longitude. The actual angle which this meridian makes with the prime meridian is the right ascension of the star, as the corresponding angle is the longitude of the city on the earth's surface.

There is, however, a difference in the unit of angular measurement commonly used for right ascensions in the heavens and longitude on the earth. In astronomical practice, right ascension is very generally expressed by hours, twenty-four of which make a complete circle, corresponding to the apparent revolution of the celestial sphere in twenty-four hours. The reason of this is that astronomers determine right ascension by the time shown by a clock so regulated as to read 0 h. 0 m. 0 s. when the vernal equinox crosses the meridian. The hour-hand of this clock makes a revolution through twenty-four hours during the time that the earth makes one revolution on its axis, and thus returns to 0 h. 0 m. 0 s. when the vernal equinox again crosses the meridian. A clock thus regulated is said to show sidereal time. Then the right ascension of any star is equal to the sidereal time at which it crosses the meridian of any point on the earth's surface. Right ascension thus designated in time may be changed to degrees and minutes by multiplying by 15. Thus, one hour is equal to 15°; one minute of time is equal to 15′ of arc, and one second of time to 15″ of arc.

It may be remarked that in astronomical practice terrestrial longitudes are also expressed in time, the longitude of a place being designated by the number of hours it may be east or west of Greenwich. Thus, Washington is said to be 5h. 8m. 15s. west of Greenwich. This, however, is not important for our present purpose.

Ancient and Mediæval Catalogues of Stars.

The first astronomer who attempted to make a catalogue of all the known stars is supposed to be Hipparchus, who flourished about 150 B.C. There is an unverified tradition to the effect that he undertook this work in consequence of the appearance of a new star in the heavens, and a desire to leave on record, for the use of posterity, such information respecting the heavens in his time that any changes which might take place in them could be detected. This catalogue has not come down to us—at least not in its original form.

Ptolemy, the celebrated author of the *Almagest*, flourished A.D. 150. His great work contains the earliest catalogue of stars which we have. There seems to be a certain probability that this catalogue may either be that of Hipparchus adopted by Ptolemy unchanged, or may be largely derived from Hipparchus. This, however, is little more than a surmise, due to the fact that Ptolemy does not seem to have been a great observer, but based his theories very largely on the observations of his predecessors. The actual number of stars which it contains is 1030. The positions of these are given in longitude and latitude, and are also described by their places in the figure of the constellation to which each may belong. Not unfrequently the longitude or latitude is a degree or more in error, showing that the instruments with which the position was determined were of rather rough construction.

So far as the writer is aware, no attempt to make a new catalogue of the stars is found until the tenth century. Then arose the Persian astronomer, Abd-Al-Rahman Al-Sufi, commonly known as Al-Sufi, who was born A.D. 903 and lived until 986. Nothing is known of his life except that he was a man celebrated for his learning, especially in astronomy. His only work on the latter subject which has come down to us is a description of the fixed stars, which was translated from the Arabic by Schjellerup and published in 1874 by the St. Petersburg Academy of Science. This work is based mainly on the catalogue of Ptolemy, all the stars of which he claimed to have carefully examined. But he did not add any new stars to Ptolemy's list, nor, it would seem, did he attempt to redetermine their positions. He simply used the longitudes and latitudes of Ptolemy, the former being increased by 12° 42′ on account of the precession during the interval between his time and that to which Ptolemy's catalogue was reduced. The translator says of his work that it gives a description of the starry heavens at the time of the author and is worthy of the highest confidence. The main body of the work consists of a detailed description of each constellation, mentioning the positions and appearances of the stars which it contains. Here we find the Arabic names of the stars, which were not, however, used as proper names, but seem rather to have been Arabic words representing some real or supposed peculiarity of the separate stars, or arbitarily applied to them.

Four centuries later arose the celebrated Ulugh Beigh, grandson of Tamerlane, who reigned at Samarcand in the middle of the fifteenth century. Baily says of him:

> "Ulugh Beigh was not only a warlike and powerful monarch, but also an eminent promoter of the sciences and of learned men. During his father's lifetime he had attracted to his capital all the most celebrated astronomers from different parts of the world; he erected there an immense college and observatory, in which above a hundred persons were constantly occupied in the pursuits of science, and caused instruments to be constructed of a better form and greater dimensions than any that had hitherto been used for making astronomical observations."

His fate was one which so enlightened a promoter of learning little deserved: he was assassinated by the order of his own son, who desired to succeed him on his throne, and, in order to make his position the more secure, also put his only brother to death. A catalogue of the stars bears the name of this monarch; he is supposed to have made many or most of the observations on which it is founded. Posterity will be likely to suppose that a sovereign used the eyes of others more than his own in making the observations. However this may be, his catalogue seems to have been the first in which the positions of the stars given by Ptolemy were carefully revised. He found that there were twenty-seven of Ptolemy's stars too far south to be visible at Samarcand, and that eight others, although diligently looked after, could not be discovered. It is curious that, like Al-Sufi, he does not seem to have added any new stars to Ptolemy's list.

Next in the order of time comes the work of Bayer, whose method of naming the stars has already been described. The main feature of his work consists in maps of all the constellations. Previous to his time, celestial globes, made especially for the use of the navigator, took the place of maps of the stars. The first edition of this book was published in 1601, and is distinguished by the fact that a list of stars in each constellation is printed on the backs of the maps. Bayer did not confine himself to the northern hemisphere, but extended his list over the whole celestial sphere, from the north to the south pole.

The catalogue of the celebrated Tycho Brahe, prepared toward the end of the sixteenth century, though of great historic value, is of no special interest to the general reader at the present time. A supplement to it, continuing its list of stars to the south pole, was published by Halley, who made the necessary observations during a journey to St. Helena in 1677.

The catalogue of Hevelius, published in 1690, offers no feature of special interest, except the addition of several new constellations, which he placed between those already known. Having the aid of the telescope, he was able to include in his catalogue stars which had been invisible to his predecessors.

Modern Catalogues of Stars.

Modern catalogues of the stars may be divided into two classes: Those which include only stars of a special class, or stars of which the observer sought to determine the position or magnitude with all attainable precision; and catalogues intended to include all the stars in any given region of the heavens, down to some fixed order of magnitude. It may appear remarkable that no attempt of the latter sort was seriously made until more than two centuries after the telescope had been pointed at the heavens by Galileo. A reason for the absence of such an attempt will be seen in the vast number of stars shown by the telescope, the difficulty of stopping at any given point, and the seeming impossibility of assigning positions to hundreds of thousands of stars. The latter difficulty was overcome by the improved methods of observation devised in modern times.

Catalogues intended to be complete down to some given magnitude are of two classes: Those which include only the stars visible to the naked eye, or with a small opera-glass, and those which take in all the stars to the 9th or 10th magnitude.

Those of the first class are mostly published in connection with star maps, and are sometimes called "uranometries." For that portion of the sky visible in our latitudes the best work of this kind is Heis's *Atlas Cœlestis*, which extends to magnitude 6.3.

About the middle of the nineteenth century the celebrated Argelander commenced the work of actually cataloguing all the stars of the northern celestial hemisphere to magnitude 9½. This work was termed a *Durchmusterung* of the northern heavens, a term which has been introduced into astronomy generally to designate a catalogue in which all the stars down to the 9th or 10th magnitude are supposed to be mustered, as if a census of them were taken. The work fills three quarto volumes and contains more than 324,000 stars, between the north pole and 2° of south declination, of each of which the magnitude and the right ascension and declination are given. This work was extended by Schönfeld, Argelander's assistant and successor, to 22° of south declination.

In the latitudes in which the great observatories of the northern hemisphere are situated, that part of the celestial sphere within 40° or 50° of the south pole always remains below the horizon. Above this invisible region a belt of somewhat indefinite breadth, 10° or more, can be only imperfectly observed, owing to the nearness of the stars to the horizon, and the brevity of the period between their rising and setting. Up to the middle of the nineteenth century, the few observatories situated in the southern hemisphere were too ill-endowed to permit of their undertaking a complete census of their part of the sky.

The first considerable work emanating from the Cordoba Observatory, under Gould, was the *Uranometria Argentina*, already mentioned, which comprised a catalogue of all the stars down to the 7th magnitude from the south pole to 10° of north declination. Another work, which was not issued until after Dr. Gould's death, was devoted to photographs of southern clusters of stars.

The work of Argelander is being continued at the Cordoba Observatory as a *Durchmusterung* of the southern heavens. It commences at 22° of south declination, where Schönfeld's work ended, and is to be continued to the south pole. This work is still incomplete, but three volumes have been published by Thome, extending to 51° of south declination. It is expected that the fourth is approaching completion. This catalogue is, in one point at least, more complete than that of Argelander and Schönfeld, as it contains all the stars down to the tenth magnitude. The three volumes give the positions and magnitudes of no less than 489,827 stars, nearly 175,000 more than the catalogue of Argelander gives for the entire northern hemisphere. If the remaining part of the heavens, from 42° to the south pole, is equally rich, it will contain about 350,000 stars, and the entire work will comprise more than 800,000 stars.

The Royal Observatory of the Cape of Good Hope, under the able and energetic direction of Dr. David Gill, has carried out a work of the same kind, which is remarkable for being based on photography. The history of this work is of great interest. In 1882

Gill secured the aid of a photographer at the Cape of Good Hope to take pictures of the brilliant comet of that year, with a large camera. On developing the pictures the remarkable discovery was made that not only all the stars visible to the naked eye, but telescopic stars down to the ninth or tenth magnitude were also found on the negatives. This remarkable result suggested to Gill that here was a new and simple method of cataloguing the stars. It was only necessary to photograph the heavens and then measure the positions of the stars on the glass negatives, which could be done with much greater ease and certainty than measures could be made on the positions of the actual stars, which were in constant apparent motion.

The Cape Durchmusterung.

As soon as the necessary arrangements could be made and the necessary instruments devised and put into successful operation, Gill proceeded to the work of photographing the entire southern heavens from 18° of south declination to the celestial pole. The results of this work are found in the *Cape Photographic Durchmusterung*, a work in three quarto volumes, in which the astronomers of all future time will find a permanent record of the southern heavens towards the end of the nineteenth century. The actual work of taking the photographs extended from 1887 to 1891. This, however, was far from being the most difficult part of the enterprise. The more arduous task of measuring the positions of a half-million of stars on the negatives, and determining the magnitude of each, was undertaken by Professor J.

C. Kapteyn, of the University of Groningen, Holland, and brought to a successful completion in the year 1899.[1]

What the work gives is, in the first place, the magnitude and approximate position of every star photographed. The determining of the magnitude of a star from its photograph is an important and delicate question. There is no difficulty in determining, from the diameter of the image of the star as seen in the microscope, what its photographic magnitude was at the time of the exposure, as compared with other stars on the same plate. But can we rely upon similar photographic magnitudes on different plates corresponding to similar brightnesses of the stars? In the opinion of Gill and Kapteyn we cannot. The transparency of the air varies from night to night, and on a very clear night the same star will give a stronger image than it will when the air is thick. Besides, slightly different instruments were used in the course of the work. For these reasons a scale of magnitude was determined on each plate by comparing the photographic intensity of the images of a number of stars with the magnitudes as observed with the eye by various observers. Thus on each plate the magnitude was reduced to a visual scale.

It does not follow from this that the magnitudes

[1] This work of Kapteyn offers a remarkable example of the spirit which animates the born investigator of the heavens. Although the work was officially that of the British Government, the years of toil devoted to it were, as the writer understands, expended without other compensation than the consciousness of making a noble contribution to knowledge, and the appreciation of his fellow astronomers of this and future generations.

are visual, and not photographic. It is still true that a blue star will give a much stronger photographic image than a red star of equal visual brightness. In a general way, it may be said that the category includes all the stars to very nearly the tenth magnitude, and on most of the plates stars of 10.5 were included. In fact, now and then is found a star of the eleventh magnitude.

A feature of the work which adds greatly to its value is a careful and exhaustive comparison of its results with previous catalogues of the stars. When a star is found in any other catalogue the latter is indicated. Most interesting is a complete list of catalogued stars which ought to be on the photographic negatives, but were not found there. Every such case was exhaustively investigated. Sometimes the star was variable, sometimes it was so red in colour that it failed to impress itself on the plate, sometimes there were errors in the catalogue.

The great enterprise of making a photographic map of the heavens, now being carried on as an international enterprise, having its headquarters at Paris, is yet wider in its scope than the works we have just described. One point of difference is that it is intended to include all the stars, however faint, that admit of being photographed with the instruments in use. The latter are constructed on a uniform plan, the aperture of each being 34 centimetres, or 13.4 inches, and the focal length 343 centimetres. Two sets of plates are taken, one to include all the stars that the instrument will photograph, and the other only to take in those to the eleventh magnitude. Of the latter it is intended to prepare a catalogue. Some portions of the German and English catalogues have already been published, and their results will be made use of in the course of the present work.

Numbering the Stars.

Closely connected with the work of cataloguing the stars is that of enumerating them. In view of what may possibly be associated with any one star — planets with intellectual beings inhabiting them — the question how many stars there are in the heavens is one of perennial interest. But beyond the general statement we have already made, this question does not admit of even an approximate answer. The question which we should be able to answer is this: How many stars are there of each distinguishable magnitude? How many of the first magnitude, of the second, of the third, and so on to the smallest that have been estimated? Even in this form we cannot answer the question in a way which is at the same time precise and satisfactory. One magnitude merges into another by insensible gradations, so that no two observers will agree as to where the line should be drawn between them. The difficulty is enhanced by the modern system — very necessary, it is true — of regarding magnitude as a continuously varying quantity and estimating it with all possible precision. In adjusting the new system to the old one, it may be assumed that an average star of any given magnitude on the old system would be designated by the corresponding number on the new system. For example, an average star of the fourth magnitude would be called 4.0; one of the fifth, 5.0, etc. Then the brightest stars which formerly were called of the fourth magnitude, would now be, if the estimate were carried to hundredths, 3.50, while the faintest would be 4.50. What were formerly called stars of the fifth magnitude would range from 4.50 to 5.50, and so on. But we meet with a difficulty when we come to the sixth magnitude. On the modern system, magnitude 6.0 represents the faintest star visible to the naked eye; but the stars formerly included in this class would, on the average, be somewhat brighter than this, because none could be catalogued except those so visible.

The most complete enumeration of the lucid stars by magnitudes has been made by Pickering (*Annals of the Harvard Observatory*, vol. xiv.). The stars were classified by half-magnitudes, calling

			M.		M.
Mag. 2.0	all	from	1.75	to	2.25
" 2.5	"	"	2.25	to	2.75, etc.

For the northern stars, Pickering used the Harvard Photometry; for the southern, Gould's *Uranometria Argentina*. A zone from the equator to 30° south declination is common to both; for this zone I use Gould. The number of each class in the entire sky, north and south of the celestial equator, is as follows:

Number of Stars.

Mag.	Northern Hemisphere. Pickering.	Southern Hemisphere. Gould.	Total.
1±	9	14	23
2.0	17	15	32
2.5	17	24	41
3.0	37	41	78
3.5	61	74	135
4.0	114	126	240
4.5	228	234	462
5.0	450	426	876
5.5	787	681	1468
6.0	789	1189	1978
Sum	2509	2824	5333

It would seem from this that the number of lucid stars in the southern celestial hemisphere is 315 greater than in the northern. But this arises wholly from a seemingly greater number of stars of magnitude 6. In the zone 0° to 30° S., Pickering has 214 stars of this class fewer than Gould. Hence it is not likely that there is really any greater richness of the southern sky.

The total number of lucid stars is thus found to be 5333. But it is not likely that stars of magnitudes 6.1 and 6.2 should be included in this class, though this is done in the above table. From a careful study and comparison of the same data from Pickering and Gould, Schiaparelli numerated the stars to magnitude 6.0. He found:

North pole to 30° S.	3113	stars
30° S. to south pole	1190	"
Total lucid stars	4303	

For most purposes a classification by entire magnitudes is more instructive than one by half-magnitudes. From the third magnitude downward we may assume that forty per cent. of the stars of each half-magnitude belong to the magnitude next above, and sixty per cent. to that next below. We thus find that of

		Total.
Mag. 0 and 1	there are 21 stars	21
" 2	" 52 "	73
" 3	" 157 "	230
" 4	" 506 "	736
" 5	" 1740 "	2476
" 6	" 5171 "	7647

Here it is to be remarked that under magnitude 6 are included many other than the lucid stars, namely, all down to magnitude 6.4. The last column gives the entire number of stars down to each order of magnitude.

It will be remarked that the number of stars of each order is between three and four times that of the order next brighter. How far does this law extend? Argelander's *Durchmusterung*, which is supposed to include all stars to magnitude 9.5, gives 315,039 stars for the northern hemisphere, from which it would be inferred that the whole sky contains 630,000 stars to the ninth magnitude. Comparing this with the number, 7647, of stars to the magnitude 6.5, we see that it is fortyfold, so that it would require a ratio of about 3.5 from each magnitude to the next lower. But it is now found that Argelander's list contains, in the greater part of the heavens, all the stars to the tenth magnitude.

On the other hand, Thome's Cordoba *Durchmusterung* gives 340,380 stars between the parallels $-22°$ and $-42°$. This is 0.14725 of the whole sky, so that, on Thome's scale of magnitude, there are about 2,311,000 stars to the tenth magnitude in the sky. This is more than three times the Argelander number to the ninth magnitude. There is, therefore, no evidence of any falling off in the ratio of increase up to the tenth magnitude.

CHAPTER V

THE SPECTRA OF THE STARS

No unregarded star
  Contracts its light
Into so small a character,
  Removed far from our humane sight,

But if we steadfast looke
  We shall discerne
In it, as in some holy booke,
  How man may heavenly knowledge learne.

HABINGTON.

THE principles on which spectrum analysis rests can be stated so concisely that I shall set them forth for the special use of such readers as may not be entirely familiar with the subject. Everyone knows that when the rays of the sun pass through a triangular prism of glass or other transparent substance they are unequally refracted, and thus separated into rays of different colours. These colours are not distinct, but each runs into the other by insensible gradations, from deep crimson through red, scarlet, orange, yellow, green, and blue to a faint violet.

Principles of Spectrum Analysis.

This result is due to the fact that the light of the sun is made up of an indiscriminate mixture of rays of an infinite number of wave-lengths, or, in simpler language, of an infinite number of tints of colour, since to every wave-length corresponds a definite tint. Such a spreading out of elementary colours in the form of a visible sheet is called a *spectrum.* By the spectrum of an incandescent object is meant the spectrum formed by the light emitted by the object when passed through a refracting prism or otherwise separated into its elementary colours. The interest and value which attach to the study of spectra arise from the fact that different bodies give different kinds of spectra, according to their constitution, their temperature, and the substances of which they are composed. In this manner it is possible, by a study of the spectrum of a body, to reach certain inferences respecting its constitution.

In order that such a study should lead to a definite conclusion, we must recall that to each special shade of colour corresponds a definite position in the spectrum. That is to say, there is a special kind of light having a certain wave-length and therefore a certain shade which will be refracted through a certain fixed angle, and will therefore fall into a definite position in the spectrum. This position is, for every possible kind of light, expressed by a number indicating its wave-length.

If we form a spectrum with the light emitted by an ordinary incandescent body, a gaslight for example, we shall find the series of colours to be unbroken from one end of the spectrum to the other. That is to say, there will be light in every part of the spectrum.

Such a spectrum is said to be continuous. But if we form the spectrum by means of sunlight, we shall find the spectrum to be crossed by a great number of more or less dark lines. This shows that in the spectrum of the sun light of certain definite wave-lengths is wholly or partly wanting. This fact has been observed for more than a century, but its true significance was not seen until a comparatively recent time.

Spectrum Analysis.

If, instead of using the light of the sun, we form a spectrum with the light emitted by an incandescent gas, say hydrogen made luminous by the electric spark, we shall find that the spectrum consists only of a limited number of separate bright lines, of various colours. This shows that such a gas, instead of emitting light of all wave-lengths, as an incandescent solid body does, principally emits light of certain definite wave-lengths.

It is also found that if we pass the light of an incandescent body through a sufficiently large mass of gas cooler than the body, the spectrum, instead of being entirely continuous, will be crossed with dark lines like that of the sun. This shows that light of certain wave-lengths is absorbed by the gas. A comparison of these dark lines with the bright lines emitted by the same gas when incandescent led Kirchhoff to the discovery of the following fundamental principle:

Every gas, when cold, absorbs the same rays of light which it emits when incandescent.

An immediate inference from this law is that the dark lines seen in the spectrum of the sun are caused by the passage of the light through gases either around the sun or forming the atmosphere of the earth. A second inference is that we can determine what these gases are by comparing the position of the dark lines with that of the bright lines produced by different gases when they are made incandescent. Hence arose the possibility of spectrum analysis, a method which has been applied with such success to the study of the heavenly bodies.

So far as the general constitution of bodies is concerned, the canons of spectrum analysis are these:

Firstly, when a spectrum is formed of distinct bright lines, the light which forms it is emitted by a transparent mass of glowing gas.

Secondly, when a spectrum is entirely continuous the light emanates either from an incandescent solid, from a body composed of solid particles, which may be ever so small, or from a mass of incandescent gas so large and dense as not to be transparent through and through.

Thirdly, when the spectrum is continuous, except that it is crossed by fine dark lines, the body emitting the light is surrounded by an atmosphere formed of gases cooler than itself. The chemical constitution of these gases can be determined by the position of the lines.

Fourthly, if, as is frequently the case, a spectrum is composed of an irregular succession of bright and shaded portions, the body is probably a gaseous mass under great pressure.

It will be seen from the preceding statement that a mass of gas so large as not to be transparent may not be distinguishable from a solid body. It is therefore not strictly correct to say, as is sometimes done, that an incandescent gas always gives a spectrum of bright lines. It will give such a spectrum only when it is transparent through and through.[1]

A gaseous mass, so large as to be opaque, would, if it were of the same temperature inside and out, give a continuous spectrum, without any dark lines. But the laws of temperature in such a mass show that it will be cooler at the surface than in the interior. This cooler envelope will absorb the rays emanating from the interior, as in the case when the latter is solid. We conclude, therefore, that the fact that the great majority of stars show a spectrum like that of the sun, namely, a continuous one crossed by dark lines, does not throw any light on the question whether the matter composing the body of the star is in a solid, liquid, or gaseous state. The fact is that the most plausible theories of the constitution of the sun lead to the conclusion that its interior mass is really gaseous. Only the photosphere may be to a greater or less extent solid or liquid. The dark lines that we see in the solar spectrum are produced

[1] As this principle is not universally understood, it may be well to remark that it results immediately from Kirchhoff's law of the proportionality between the radiating and absorbing powers of all bodies for light of each separate wave-length. When a body, even if gaseous in form, is of such great size and density that light of no colour can pass entirely through it, then the consequent absorption by the body of light of all colours shows that throughout the region where the absorption occurs there must be an emission of light of these same colours. Thus light from all parts of the spectrum will be emitted by the entire mass.

by the absorption of a comparatively thin and cool layer of gas resting upon the photosphere. Analogy as well as the general similarity of the spectra lead us to believe that the constitution of most of the stars is similar to that of the sun.

Description of the Spectrum.

The visible spectrum, as commonly described, terminates with the red at one end and the violet at the other. But the termination is by no means sharp at either end. Especially is this the case with the violet, where, if extraneous light be shut off, a faint extension known as the ultra-violet, to which no definite limit can be assigned, will become visible. Moreover, it is found that the heating effect does not terminate with the red end of the spectrum, but that if a sensitive thermometer be held in the seeming darkness beyond the red end a heating effect is produced. It is also found that a photographic effect is produced by rays scarcely, if at all, visible in the ultra-violet.

These three different effects were formerly attributed to three different kinds of rays, those of heat, those of light, and those which, affecting the photographic plate, were called chemical or actinic rays. But it is now known that heat, light, and photographic effects are all due to one and the same agency, which we may call radiance. The radiance from an incandescent body like the sun may be of all wave-lengths ; at least we can set no definite limit to the wave-length. These lengths may be expressed in millionths of a millimetre, or, as is now more commonly done, in ten millionths. This measure is sometimes called the tenth-metre, meaning the metre divided by the tenth power of ten. To give a general idea of wave-length we remark that near the brightest part of the spectrum the wave-length is 5000 tenth-metres or 500 millionths of a millimetre, the latter being nearly $\frac{1}{25}$ of our inch. The wave-length in question is therefore about $\frac{1}{50,000}$ of an inch. As we pass toward the violet end of the spectrum, commonly called the upper end, this wave-length diminishes ; as we pass toward the lower or red end it increases. As we approach wave-length 7500, the effect on the eye as light gradually dies away with a sensation of very deep red ; below that point only the heat effect is produced, except that with certain chemicals a faint photographic effect may still be obtained.

The more refrangible parts of the spectrum are now studied almost entirely by photography. The astro-physicist can photograph not only the visible spectrum at pleasure, but the higher parts of the spectra of bodies even when so faint as to be invisible to the eye. The photograph has the additional advantage that it forms a permanent record which can be measured and studied at pleasure.

The farthest exploration into the ultra-violet region has been made by Dr. V. Schumann, who has examined it up to W. L. 1620. The higher region is very rich in lines, of which he found more than six hundred, separated into fifteen groups. As we approach its limit the air becomes opaque to radiation. A layer of one millimetre in thickness was found to absorb all the radiance shorter than 1700.

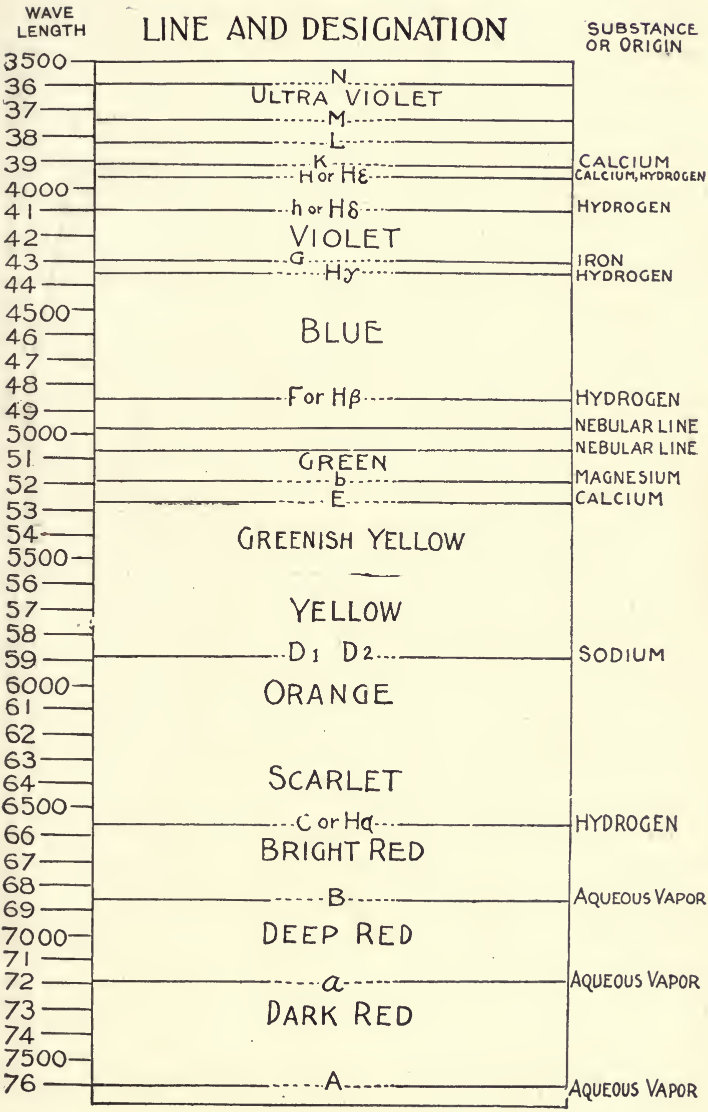
The strongest dark lines of the spectra were studied and laid down by Wollaston about 1800. He designated the strongest by the capital letters A, B, C, D, E, and F, to which some small letters were subsequently added. As the exploration of the spectrum extended into the violet additional letters were added. It has been found convenient in recent times to replace some of those letters by symbols expressing the substance which produces the line. Thus, the line which Wollaston called C, being produced by hydrogen, is now frequently called $H\alpha$. The other lines produced by this substance are designated as $H\beta$, $H\gamma$, etc.

Extensive maps of the solar spectrum have been published, of which that of Rowland surpasses all others in the completeness of its details. The number of spectral lines as found on this map mount high into the thousands, so that the great mass of them can be designated only by their wave-length. Thus the line C or $H\alpha$ may be designated as 6561.7. Maps or tables of the spectra of the various chemical elements are found in special treaties on the subject.

While some substances, notably the lightest and most permanent gases, have few lines in their respective spectra, in other substances the lines are very numerous. The metal which gives the richest spectrum is iron. Thalén has recorded not less than 1200 lines in its spectrum between wave-lengths 4000 and 7600.

It is now found that the spectra of most substances vary with the physical condition of the substance in such a way that detection may become doubtful or difficult. The general rule is that, when a gas is subjected to pressure, the lines, if dark, become blacker and broader, being sometimes changed into bands with more or less ill-defined borders. Commonly the line broadens only on one side, thus leading to a displacement of its apparent position with the pressure. Not less than three distinct spectra have been found as due to argon. These changes have not, up to the present time, been expressed by any uniform and general law. It is not alone the thickness of the lines which changes; it frequently happens that a line visible under one condition will disappear under another, while a second will be better seen. These seeming anomalies may sometimes make our conclusions from the spectral analysis of the heavenly bodies uncertain; but it may be hoped that when they are fully understood, they will give us more precise knowledge than we yet possess of the exact physical constitution of these bodies.

A complete map of the spectrum is too full of detail to well serve the purpose of the general reader or student. We therefore give on the opposite page a plan of the visible spectrum, giving the wavelengths, the arrangement of colours, and a few of the stronger lines with the substances to which they are due. It must not, however, be supposed that the solar spectrum with its lines is so simple as might appear from this plan. For the most part, what are drawn and lettered as lines really consist of groups of lines of different degrees of intensity. Whether they shall appear as a simple ill-defined line or a group depends


WAVE LENGTH
LINE AND DESIGNATION
SUBSTANCE OR ORIGIN
3500
36
37
38
39
4000
41
42
43
44
4500
46
47
48
49
5000
51
52
53
54
5500
56
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6000
61
62
63
64
6500
66
67
68
69
7000
71
72
73
74
7500
76
N
ULTRA VIOLET
M
L
K
H or Hε
h or Hδ
VIOLET
G
Hγ
BLUE
F or Hβ
GREEN
b
E
GREENISH YELLOW
YELLOW
D1 D2
ORANGE
SCARLET
C or Hα
BRIGHT RED
B
DEEP RED
a
DARK RED
A
CALCIUM
CALCIUM, HYDROGEN
HYDROGEN
IRON
HYDROGEN
HYDROGEN
NEBULAR LINE
NEBULAR LINE
MAGNESIUM
CALCIUM
SODIUM
HYDROGEN
AQUEOUS VAPOR
AQUEOUS VAPOR
AQUEOUS VAPOR

largely on the resolving power of the spectroscope. With every increase of power new lines are brought out. It will also be seen from the photographs which we reproduce that the spectra of the heavenly bodies, whether stars or sun, do not consist of uniform sheets of light crossed by dark lines, but that one part runs into another with slight and nearly imperceptible gradations of shade. These are due partly to innumerable lines not visible singly, and partly to the varying and irregular absorption to which the light has been subject.

Particularly irregular is the absorption produced by the aqueous vapour of the atmosphere. The strongest lines and groups of lines in the red, notably those between A and B, as well as irregular shadings in the bright parts of the spectrum, are due to the absorption of this agent.

It thus happens that the individual Wollaston lines cannot as a general rule be considered as each due to some one substance, most of them being composed of a number of lines produced by different substances whose lines chance to fall very close together.

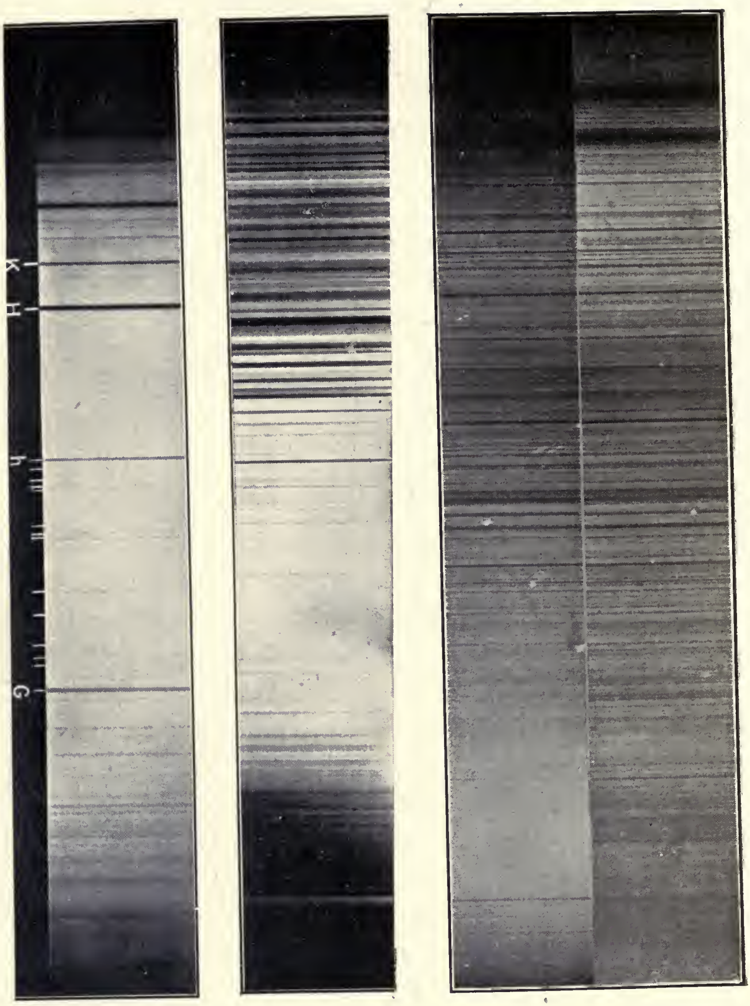
In connection with the lines and the wave-lengths we have also named the spectral colours. One of these shades into the other so gradually that no precise line of demarkation can be drawn. In fact, the change of colour is continuous from one end of the spectrum to the other. The red, green, blue, and violet are the only colours which, to the eye, seem unchanged through any perceptible space in their central portions.

Different authorities, and perhaps different eyes, may therefore assign different boundaries to the colours. For these reasons we have not attempted to draw any demarkations of the colours, but have simply shown the central parts of those colours which are best marked.

Quite possibly different eyes may have slightly different impressions of the spectral colours. To those of the writer, the yellow of the spectrum is in no way comparable in depth and intensity with the yellow of such flowers as the buttercup. The shading from a tinge of red on the one side to a tinge of green on the other takes place without what seems like a pure bright yellow.

Classification of Stellar Spectra.

When the spectra of thousands of stars were recorded for study, such a variety was found that some system of classification was necessary. The commencement of such a system was made by Secchi in 1863. It was based on the observed relation between the colour of a star and the general character of its spectrum. Arranging the stars in a regular series, from blue in tint through white to red, it was found that the number and character of the spectral lines varied in a corresponding way. The blue stars, like Sirius, Vega, and Alpha Aquilæ, had the F lines strong, as well as the two violet lines H, but had otherwise only extremely fine lines. On the other hand, the red stars, like Alpha Orionis and Alpha Scorpii, show spectra with several broad bands. Secchi was thus led to recognise three types of spectra, as follows :



α Cygni
Type I

α Tauri
Type II

α Aurigæ
Type II

Sun
Type II

EXAMPLES OF STELLAR SPECTRA

The first type is that of the white or slightly blue stars, like Sirius, Vega, Altair, Rigel, etc. The typical spectrum of these stars shows all seven spectral colours, interrupted by four strong, dark lines, one in the red, one in the bluish green, and the two others in the violet. All four of these lines belong to hydrogen. Their marked peculiarity is their breadth, which shows that the absorbing layer is of considerable thickness, or is subjected to a great pressure. Besides these broad rays, fine metallic rays are found in the brighter stars of this type. Secchi considers that this is the most numerous type of all, half the stars which he studied belonging to it.

The second type is that of the somewhat yellow stars, like Capella, Pollux, Arcturus, Procyon, etc. The most striking feature of the spectrum of these stars is its resemblance to that of our sun. Like the latter, it is crossed by very fine and close black rays. It would seem that the more the star inclines toward red, the broader these rays become and the easier it is to distinguish them. We give a figure showing the remarkable agreement between the spectrum of Capella, which may be taken as an example of the type, and that of the sun.

The spectra of the third type, belonging mostly to the red stars, are composed of a double system of nebulous bands and dark lines. The latter are fundamentally the same as in the second type, the broad, nebulous bands being an addition to the spectrum. Alpha Herculis may be taken as an example of this type.

It is to be remarked that, in these progressive types, the brilliancy of the more refrangible end of the spectrum continually diminishes relatively to that of the red end. To this is due the gradations of colour in the stars.

To these three types Secchi subsequently added a fourth, given by a comparatively few stars of a deep red colour. The spectra of this class consist principally of three bright bands, which are separated by dark intervals. The brightest is in the green; a very faint one is in the blue; the third is in the yellow and red, and is divided up into a number of others.

To these types a fifth was subsequently added by Wolf and Rayet, of the Paris Observatory. The spectra of this class show a singular mixture of bright lines and dark bands, as if three different spectra were combined, one continuous, one an absorption spectrum, and one an emission spectrum from glowing gas. Less than a hundred stars of this type have been discovered. A very remarkable peculiarity, which we shall discuss hereafter, is that they are nearly all situated very near the central line of the Milky Way.

Vogel proposed a modification of Secchi's classification, by subdividing each of his three types into two or three others, and including the Wolf-Rayet stars under the second type. His definitions are as follows:

Type I is distinguished by the intensity of the light in the more refrangible end of the spectrum, the blue and violet. The type may be divided into three subdivisions, designated *a*, *b*, and *c*:

In I*a* the metallic lines are very faint, while the hydrogen lines are distinguished by their breadth and strength.

In I*b* the hydrogen lines are wanting.

In I*c* the lines of hydrogen and helium both show as bright lines. Stars showing this spectrum are now known as helium stars.

According to Vogel, the spectra of type II are distinguished by having the metallic lines well marked and the more refrangible end of the spectrum much fainter than in the case of type I. He recognises two subdivisions:

In II*a* the metallic lines are very numerous, especially in the yellow and green. The hydrogen lines are strong, but not so striking as in I*a*.

In II*b* are found dark lines, bright lines, and faint bands. In this subdivision he includes the Wolf-Rayet stars, more generally classified as of the fifth type.

The distinguishing mark of the third type is that, besides dark lines, there are numerous dark bands in all parts of the spectrum, and the more refrangible end of the latter is almost wanting. There are two subdivisions of this type:

In III*a* the broad bands nearest the violet end are sharp, dark, and well defined, while those near the red end are ill defined and faint. In III*b* the bands near the red end are sharp and well defined; those toward the violet, faint and ill defined. The character of the bands is therefore the reverse of that in subdivision *a*.

This classification of Vogel is still generally followed in Germany and elsewhere. It is found, however, that there are star spectra of types intermediate to all these defined. Moreover, in each type the individual differences are so considerable that there is no well-defined limit to the number of classes that may be recognised. Other designations frequently occur in literature. The stars of type II are sometimes termed Capellan stars, or solar stars. The stars which show the lines of helium are known as helium stars.

A classification far more minute than either of the preceding was made by Miss Antonio C. Maury, of the Harvard Observatory, and has been adopted in the Draper Memorial work of that institution.[1] The classification is too extended for us to give more than its principal features. In the main it recognises a regular progression in the character of the spectra. The principal feature is the addition of an extended type called the Orion type, because the stars showing it abound in the constellation Orion, though not confined to it. It is marked principally by what are called Orion lines, which include most of the lines of hydrogen, and nearly one hundred others. Few or none of the latter can be recognised as solar lines, nor can they certainly be ascribed to any known substances. The peculiar feature of the type is that the Orion lines are strong and numerous, declining in the

[1] *Annals Harvard Observatory*, vol. xxviii., No. 1.

later groups. The hydrogen lines are of moderate intensity, inclining toward those of the first type. Of the two main calcium lines, K is often, and H generally, absent.

This Orion type is divided into five groups: type I into five, types II and III each into four. Besides these there are several intermediate groups, and a group each for the fourth and fifth types, the whole number of such groups being twenty-two. Each group is still further subdivided into classes.

There are many star spectra which cannot be included in any of the classes we have described. Up to the present time these are generally described as stars of peculiar spectra.

As the present chapter is confined to the more general side of the subject, we shall not attempt any description of special spectra. These, especially the peculiar spectra of the nebulæ, of new stars, of variable stars, etc., will be referred to, so far as necessary, in the chapters relating to those objects.

Results of Spectrum Analysis.

The most interesting conclusion drawn from observations with the spectroscope is that the stars are composed, in the main, of elements similar to those found in our sun. As the latter contains most of the elements found on the earth and few or no others, we may say that earth and stars seem to be all made out of like matter. It is, however, not yet easy to decide to what extent elements unknown on the earth exist in the heavens. It would scarcely be safe to assume that, because the line of some terrestial substance is found in the spectrum of a star, it is produced by that substance. It is quite possible that an unknown substance might show a line in appreciably the same position as that of some substance known to us. The evidence becomes conclusive only in the case of those elements of which the spectral lines are so numerous that when they all coincide with lines given by a star there can be no doubt of the identity.

CHAPTER VI

PROPER MOTIONS OF THE STARS

I 'm constant as the Northern Star,
Of whose true-fixed and vesting quality
There is no fellow in the firmament.—SHAKESPEARE.

WE may assume that the stars are all in motion. It is true that only a comparatively small number of stars have been actually seen to be in motion; but as some motion exists in nearly every case where observations would permit of its being determined, we may assume the rule to be universal. Moreover, if a star were at rest at any time it would be set in motion by the attraction of other stars.

In dealing with the subject, the astronomer commonly expresses the motion in angular measurement, as so many seconds per year or per century. The keenest eye would not, without telescopic aid, be able to distinguish between two stars whose apparent distance is less than 2′ or 120″ of arc. The pair of stars known as Epsilon Lyræ are over 3′ apart; yet to ordinary vision they appear as a single star. To appreciate what 1″ of arc means we must conceive that the distance between these two stars is divided by 200. Yet this minute space is easily distinguished and accurately measured by the aid of a telescope of ordinary power.

Statements of the motion from different points of view illustrate in a striking way the vast distance of the stars and the power of modern telescopic research. If Hipparchus or Ptolemy should rise from his sleep of two thousand years — nay, if the earliest priests of Babylon should come to life again and view the heavens, they would not perceive any change to have taken place in the relative positions of the stars. The general configurations of the constellations would be exactly that to which they were accustomed. Had they been exact observers they might notice a slight change in the position of Arcturus; but not in that of any other star.

Apparent and Real Motions.

Slow as the angular motion is, our telescopic power in the course of a few years makes its detection frequently possible—in the case of Arcturus even in a few weeks. As accurate determinations of positions of the stars have been made only during a century and a half, no motions can be positively determined except those which would become evident to telescopic vision in that period. Only about three thousand stars have been accurately observed so long as this. In the large majority of cases the interval of observation is so short or the motion so slow that nothing can be asserted respecting the law of the motion.

Contrast these apparently slow motions with the actual motions. Swift indeed are these when measured by terrestrial standards. Arcturus has been moving ever since the time of Job at the rate of probably more than two hundred miles per second—

possibly three hundred miles. Generally, however, the motion is much smaller, ranging from an imperceptible quantity up to forty miles a second.

The great mass of stars seem to move only a few seconds per century, but there are some whose motions are exceptionally rapid. The general rule is that the brighter stars have the largest proper motions. This is what we should expect, because in the general average they are nearer to us, and therefore their motion will subtend the greatest angle to the eye. But this rule is only one of majorities. As a matter of fact, the stars of largest proper motion happen to be low in the scale of magnitude. It happens thus because the number of stars of smaller magnitudes is so much greater than that of the brighter ones that their very small proportion of large proper motions exceeds in actual number those among the brighter stars.

The discovery of the star of greatest known proper motion was made by Kapteyn, of Groningen, in 1897, co-operating with Gill and Innes, of the Cape Observatory. While examining the photographs of the stars made at this institution, Kapteyn was surprised to notice the impression of a star of the eighth magnitude which at first could not be found in any catalogue. But on comparing different star lists and different photographs it soon became evident that the star had been previously seen or photographed, but always in different positions. An examination of the observed positions at various times showed that the star had a more rapid proper motion than any other yet known. Yet, great though this motion is, it would require nearly 150,000 years for the star to make a complete circuit of the heavens if it moved round the sun uniformly at its present rate.

The following is a list of the annual proper motions of nine stars exceeding 4″. We add the positions and magnitudes of the stars.

STAR	POSITION			MAG.	PROP. MOT.
	R. A.		DEC.		
	h	m	°		″
Z. C. 5^h, 243	5	7	−45.0	8.5	8.70
Groomb. 1830	11	47	+38.4	6.4	7.04
Lacaille 9352	22	59	−36.4	7.1	7.00
Cord. 32,416	0	0	−37.8	8.5	6.07
61 Cygni	21	2	+38.2	4.8	5.20
Ll. 21,185	10	58	+44.3	7.3	4.76
ε Indi	21	56	−57.2	4.8	4.68
Ll. 21,258	11	0	+44.0	8.7	4.41
o^2 Eridani	4	11	− 7.8	4.5	4.06

The fact that the stars move suggests a very natural analogy to the solar system. In the latter a number of planets revolve round the sun as their centre, each planet continually describing the same orbit, while the various planets have different velocities. Around several of the planets revolve one or more satellites. Were civilised men ephemeral, observing the planets and satellites only for a few minutes, these bodies would be described as having proper motions of their own, as we find the stars to have. May it not then be that the stars also form a system; that each star is moving in a fixed orbit, performing a revolution around some far-distant centre in a period which may

Moving Systems of Stars.

be hundreds of thousands or hundreds of millions of years? May it not be that there are systems of stars in which each star revolves around a centre of its own while all these systems are in revolution around a single centre?

This thought has been entertained by more than one contemplative astronomer. Lambert's magnificent conception of system upon system will be described hereafter. Mädler thought that he had obtained evidence of the revolution of the stars around Alcyone, the brightest of the Pleiades, as a centre. But, as the proper motions of the stars are more carefully studied and their motion and direction more exactly ascertained, it becomes very clear that when considered on a large scale these conceptions are never realised in the actual universe as a whole. But there are isolated cases of systems of stars which are shown to be in some way connected by their having a common proper motion. We shall mention some of the more notable cases.

The Pleiades are found to move together with such exactness that up to the present time no difference in their proper motions has been detected. This is true not only of the six stars which we readily see with the naked eye, but of a much larger number of fainter ones made known by the telescope. It is an interesting fact, however, that a few stars apparently within the group do not partake of this motion, from which it may be inferred that they do not belong to the system. But there must be some motion among themselves, else the stars would ultimately fall together by their mutual attraction. The amount and nature of this motion cannot, however, be ascertained except by centuries of observation.

Another example of the same sort is seen in five out of the seven stars of Ursa Major, or The Dipper. The stars are those lettered β, γ, δ, ε, and θ. All five have a proper motion in R. A. of nearly 8″ per century, while in declination the movements are sometimes positive and sometimes negative; that is to say, some of the stars are lessening their distance from the pole, while others are increasing it. But when we project the motions on a map we find that the actual direction is very nearly the same for all five stars, and the reason why some move slightly to the north and others slightly to the south is due to the divergence of the circles of right ascension. It is worthy of remark that the community of motion is also shown by spectroscopic observations of the radial motions described below.

The five stars in question are all of the second magnitude except Delta, which is of the third. It is a curious fact that no fainter stars than these five have been found to belong to the system.

From a study of these motions Höffler has concluded that the five stars lie nearly in the same plane and have an equal motion in one and the same direction. From this hypothesis he has made a determination of their relative and actual distances. The result reached in this way cannot yet, however, be regarded as conclusive.

There are three stars in Cassiopeia, Beta, Eta, and

Mu, each having a large proper motion in so nearly the same direction that it is difficult to avoid at least a suspicion of some relation between them. The angular motions are, however, so far from equal that we cannot regard the relation as established.

In the constellation Taurus, between Aldebaran and the Pleiades, most of the stars which have been accurately determined seem to have a motion which is positive in R. A. and negative in declination. But these motions are not equal, as they should be if the stars belonged to one system, and we cannot draw any definite conclusion from them. They show a phenomenon which Proctor very aptly designated as star-drift.

Another curious case is that of A Ophiuchi and a smaller star of the seventh magnitude, about 14′ from it, having an equal proper motion, showing the two to form a connected system.

The systems we have just described comprise stars situated so far apart that, but for their common motion, we should not have suspected any relation between them. The community of origin which their connection indicates is of great interest and importance, but this subject belongs to a later chapter.

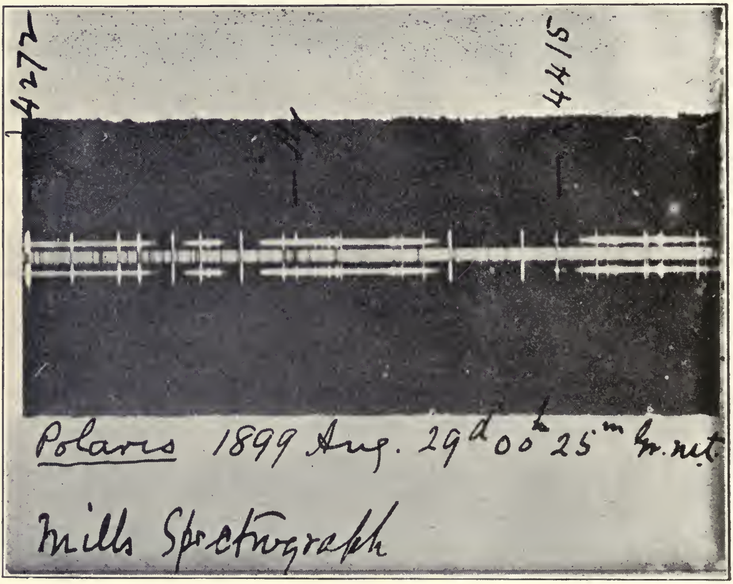
Radial Motions of the Stars.

No achievement of modern science is more remarkable than the measurement of the velocity with which stars are moving to or from us. This is effected by means of the spectroscope through a comparison of the position of the spectral lines produced by the absorption of any substance in the atmosphere of the star with the corresponding lines produced by the same substance on the earth. The principle on which the method depends may be illustrated by the analogous case of sound. It is a familiar fact that if we stand alongside a railway while a locomotive is passing us at full speed and at the same time blowing a whistle, the pitch of the note which we hear from the whistle is higher as the engine is approaching than after it passes. The reason is that the pitch of a sound depends upon the number of sound-beats per second.

Now, we may consider the waves which form light, when they strike our apparatus, as beats in the ethereal medium which follow each other with extraordinary rapidity, millions of millions in a second, moving forward with a definite velocity of more than 186,000 miles a second. Each spectral line produced by a chemical element shows that that element, when incandescent, beats the ether a certain number of times in a second. These beats are transmitted as waves. Since the velocity is the same whether the number of beats per second is less or greater, it follows that, if the body is in motion in the direction in which it emits the light, the beats will be closer together than if it is at rest; if moving away they will be farther apart. The fundamental fact on which this result depends is that the velocity of the light-beat through the ether is independent of the motion of the body causing the

beat. To show the result, let A be a luminous body at rest; let the seven dots to the right of A be the crests of seven waves or beats, the first of which, at the end of a certain time, has reached X. The wave-length will then be one-seventh the distance A X. Now, suppose A in motion toward X with such speed that when the first beat has reached X, A has reached the point B. Then the seven beats made by A while the first beat is travelling from A to X, and A travelling from A to B, will be crowded into the space B X, so that each wave will be one-seventh shorter than before. In other words, the wave-lengths of the light emitted by any moving body will be less or greater according as the motion is in the direction in which its light is transmitted, or in the opposite direction.

The position of a ray in the spectrum depends solely on the wave-length of the light. It follows that the rays produced by any substance will be displaced toward the blue or red end of the spectrum, according as the body emitting or absorbing the rays is moving towards or from us. This method of determining the motions of bodies to or from us has been perfected by photographing the spectrum of a star, or other heavenly body, side by side with that of a terrestrial substance rendered incandescent in the tube of a telescope. The rays of this substance pass through the same spectroscope as those from the star, so that, if the wave-lengths of the lines produced by the substance were the same as those found in the star spectrum, the two lines would correspond in position. The minute difference found on the photographic plate is the measure of the velocity of the star in the line of sight called its radial motion.



SPECTROGRAM OF POLARIS TAKEN BY CAMPBELL AT THE LICK OBSERVATORY
The bright cross-lines are those of the comparison-spectrum of iron

These measures require apparatus and manipulation of extraordinary delicacy, in order to avoid every possible source of error. The displacement of the lines produced by the motion is in fact so minute that great skill is required to make it evident, unless in exceptional cases.

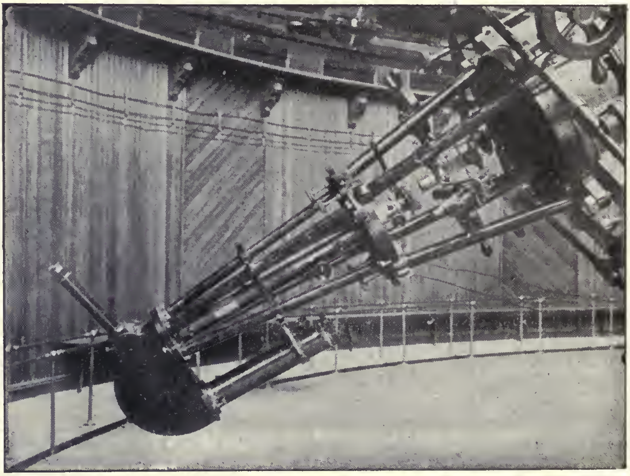
It will be seen that the conclusion as to radial motion depends on the hypothesis that the position of any ray produced by a substance is affected by no cause but the motion of the substance. How and when this hypothesis may fail is a very important question. It is found, for example, that the position of a spectral ray may be altered by compressing the gas emitting or absorbing the ray; and it may be inquired whether the results for motion in the line of sight may not be vitiated by the absorbing atmosphere of the star being under heavy pressure, thus displacing the absorption line.

To this it may be replied that, in any case, the outer layers of the atmosphere, through which the light must last pass, are not under pressure. How far the inner portions may produce an absorption spectrum we cannot discuss at present, but it does not seem likely that serious errors are thus introduced in many cases.

In the measures made by Vogel at Potsdam the substance used for comparison was generally hydrogen, the lines of this substance being frequently very sharp in the spectrum of the stars. The spectrum of iron can also be used for comparison. The stars measured by Vogel are forty-seven in number, all brighter than the third magnitude, this being about the limit which his instrument could reach. Out of his forty-seven stars he found four to be affected with a periodic inequality and therefore to belong to the class of binary systems to be described in a subsequent chapter.

About 1892 Belopolsky of Pulkova continued Vogel's work with a much larger instrument, detecting several other periodic motions. One of his most interesting discoveries was a periodic motion in the star

Eta Aquilæ corresponding in period to the variations of its light. He also detected in Castor a variation with a period of about three days. Another of his discoveries was the very rapid motion of seventy kilometres per second in the motion of Zeta Herculis. This, however, is exceeded by the motion of eighty-seven kilometres which Campbell discovered in a star of Cepheus. Large though these motions are, they fall much below those that belong to Arcturus and 1830 Groombridge.



THE MILLS SPECTROGRAPH OF THE LICK OBSERVATORY

During the last few years another step forward has been made by Campbell of the Lick Observatory with the Mills spectrograph.[1] In order to reach

[1] It may be remarked in this connection that Mr. D. O. Mills, the donor of this instrument, was one of the original trustees charged by Mr. Lick in 1874 with the construction of his Observatory.

fainter stars than ever before, a longer exposure of the photographic plate was necessary. A difficulty is met with in the prolonged exposure, owing to the change of temperature of the apparatus, which alters the refracting power of the prisms. This difficulty was obviated by protecting the apparatus from such changes. With this great increase in photographic power and time of exposure it is now possible to photograph the spectra of stars down to the 6th or 7th magnitude. But it is not all stars that can thus be measured, because, in many cases, the spectral lines of the star are not sufficiently sharp and well defined.

The Motion of the Sun.

When a star is found to be seemingly in motion, as described in the last section, we may ascribe the phenomenon to a motion either of the star itself or of the observer. In fact no motion can be determined or defined except by reference to some body supposed to be at rest. In the case of any one star, we may equally well suppose the star to be at rest and the observer in motion, or the contrary. Or we may suppose both to have such motions that the difference of the two shall represent the apparent movement of the star. Hence our actual result in the case of each separate star is a relation between the motion of the star and the motion of the sun.

I say the motion of the sun and not of the earth, because, although the observer is actually on the earth, yet the latter never leaves the neighbourhood of the sun, and, as a matter of fact, the ultimate result in the long run must be a motion relative to the sun itself, as if we made our observations from that body. The question then arises whether there is any criterion for determining how much of the apparent motion of any given star should be attributed to the star itself and how much to a motion of the sun in the opposite direction.

If we should find that the stars, in consequence of their proper motions, all appeared to move in the same direction, we would naturally assume that they were at rest and the sun in motion. A conclusion of this sort was first reached by Herschel, who observed that among the stars having notable proper motions there was a general tendency to move from the direction of the constellation Hercules, which is in the northern hemisphere, towards the opposite constellation Argo, in the southern hemisphere.

Acting on this suggestion, succeeding astronomers have adopted the practice of considering the general average of all the stars, or a position which we may regard as their common centre of gravity, to be at rest, and then determining the motion of the sun with respect to this centre. Here we encounter the difficulty that we cannot make any absolute determination of the position of such a centre. The latter will vary according to what particular stars we are able to include in our estimate. What we can do is to take all the stars which appear to have a proper motion, and determine the general direction of that motion. This gives us a certain point in the heavens toward which the solar system is travelling, and which is now called the *solar apex*, or "the apex of the solar way."

The apparent motion of the stars away from the apex, and due to this motion of the solar system, is now called their *parallactic motion*, to distinguish it from the actual motion of the star itself.

The interest which attaches to the position of the solar apex has led a great number of investigators to determine it. Owing to the rather indefinite character of the material of investigation, the uncertainty of the proper motions, and the additions constantly made to the number of stars which are available for the purpose in view, different investigators have reached different results. Until quite recently, the general conclusion was that the solar apex was situated somewhere in the constellation Hercules. But the general trend of recent research has been to place it in or near the adjoining constellation Lyra. This change has arisen mainly from including a larger number of stars, whose motions were determined with greater accuracy.

Former investigators based their conclusions entirely on stars having considerable proper motions, these being, in general, the nearer to us. The fact is, however, that it is better to include stars having a small proper motion, because the advantage of their great number more than counterbalances the disadvantage of their distance.

The conclusions reached by some recent investigators of the position of the solar apex are as follows: We call A the right ascension of the apex; D its declination.

Prof. Lewis Boss, from 273 stars of large proper motion, found:

$$A = 283^\circ.3 \,;\; D = 44^\circ.1.$$

If he excluded the motions of 26 stars which exceeded 40″ per century the result was

$$A = 288^\circ.7 \,;\; D = 51^\circ.5.$$

A comparison of these numbers shows how much the result depends on the special stars selected. By leaving out 26 stars the apex is changed by 5° in R. A. and 7° in declination.

It is to be remarked that the stars used by Boss are all contained in a belt four degrees wide, extending from 1° to 5° north of the equator.

Dr. Oscar Stumpe, of Berlin, made a list of 996 stars having proper motions between 16″ and 128″ per century. He divided them into three groups, the first including those between 16″ and 32″; the second between 32″ and 64″; the third between 64″ and 128″. The number of stars in each group and the position of the apex derived from them are as follows:

Gr. I, 551 stars ;	A = 287°.4 ;	D = +45°.0
II, 339	282°.2	43°.5
III, 106	280°.2	33°.5

Porter, of Cincinnati, made a determination from a yet larger list of stars with results of the same general character.

These determinations have the advantage that the stars are scattered over the entire heavens, the southern as well as the northern ones. The difference of more than 10° between the position derived from stars with the largest proper motions, and from the other stars, is remarkable.

The present writer, in a determination of the precessional motion, incidentally determined the solar motion from 2527 stars contained in Bradley's Catalogue which had small proper motions, and from about 600 more having larger proper motions. Of the latter the declinations only were used. The results were:

From small motions:	A = 274°.2;	D = +31°.2
From large motions:	276°.9	31°.4

Quite recently Campbell has made a determination of the position of apex from the radial motions of 280 stars, mostly measured by himself. The result is:

$$A = 277^\circ.5$$
$$D = +20^\circ.0$$

From all these results it would seem that the most likely apex of the solar motion is toward a point in

Right Ascension, 280°
Declination, 35° north

This point is situated in the constellation Lyra, about 4° from the first-magnitude star Vega. The uncertainty of the result is as much as this difference, 4° or 5° at least. We may therefore state the conclusion in this form:

The apex of the solar motion is in the general direction of the constellation Lyra, and perhaps near the star Vega, the brightest of that constellation.

It must be admitted that the wide difference between the positions of the apex from large and from small proper motions, as found by Porter, Boss, and Stumpe, requires explanation. Since the apparent motions of the stars are less the greater their distance, these results, if accepted as real, would lead to the conclusion that the position of the solar apex derived from stars near to us was much farther south than when derived from more distant stars. This, again, would indicate that our sun is one of a cluster or group of stars having, in the general average, a different proper motion from the more distant stars. But this conclusion is not to be accepted as real until the subject has been more fully investigated. The result may depend on the selection of the stars; and there is, as yet, no general agreement among investigators as to the best way of making the determination.

The next question which arises is that of the velocity of the solar motion. The data for this determination are more meagre and doubtful than those for the direction of the motion. The most obvious and direct method is to determine the parallactic motion of the stars of known parallax. Regarding any star 90° from the apex of the solar motion as in a state of absolute rest, we have the obvious rule that the quotient of its parallactic motion during any period, say a century, divided by its parallax, gives the solar motion during that period, in units of the earth's distance from the sun. In fact, by a motion of the sun through one such unit, the star would have an apparent motion in the opposite direction equal to its annual parallax. If the star is not 90° from the apex we can easily reduce its observed parallactic motion by dividing it by the sine of its actual distance from the apex.

Since every star has, presumably, a proper motion of its own, we can draw no conclusion from the apparent motion of any one star, owing to the impossibility of distinguishing its actual from its parallactic motion. We should, therefore, base our conclusion on the mean result from a great number of stars, whose average position or centre of mass we might assume to be at rest. Here we meet the difficulty that the stars measured for parallax are generally those having a proper motion away from the apex. This will make the result derived in this way too large.

A second method is based on measures of the motion of stars in the line of sight. A star at rest in the direction of the solar apex would be apparently moving towards us with a velocity equal to that of the solar motion. Assuming the centre of mass of all the stars observed to be at rest, we should get the solar motion from the mean of all. In the investigation just referred to, Campbell has derived the velocity, 19.89 kilometres per second, with a probable error of 1.52 kilometres. A speed of 19 kilometres per second would carry our system over almost exactly four radii of the earth's orbit in a year, and we may regard this as the most likely value of the speed in question.

CHAPTER VII

VARIABLE STARS

—And the moist star . . .
Was sick almost to doomsday with eclipse.—SHAKESPEARE.

IT is a curious fact that the ancient astronomers, notwithstanding the care with which they observed the heavens, never noticed that any of the stars changed in brightness. The earliest record of such an observation dates from 1596, when the periodical disappearance of Omicron Ceti was noticed. After this, nearly two centuries elapsed before another case of variability in a star was recorded. During the first half of the nineteenth century Argelander so systematised the study of variable stars as to make it a new branch of astronomy. In recent years it has become of capital interest and importance through the development of spectroscopic research.

Students who are interested in the subject will find the most complete information attainable in the catalogues of variable stars published from time to time by Chandler in the *Astronomical Journal*. His third catalogue, which appeared in 1896, comprises more than three hundred stars whose variability has been well established, while there is always a long list of "suspected variables"—whose cases are still to be tried. The number to be included in the established list is continually increasing at such a rate that it is impossible to state it with any approximation to exactness. The possibility of such a statement has been yet further curtailed by the recent discovery at the Harvard Observatory that certain clusters of stars contain an extraordinary proportion of variables. Altogether at the time of the latest publication, 509 such stars were found in twenty-three clusters. The total number of these objects in clusters, therefore, exceeds the number known in the rest of the sky. They will be described more fully in a subsequent chapter. For the present we are obliged to leave this rich field out of consideration and confine our study to the isolated variable stars which are found in every region of the heavens.

Variable stars are of several classes, which, however, run into each other by gradations so slight that a sharp separation cannot always be made between them. Yet there are distinguishing features, each of which marks so considerable a number of these stars as to show some radical difference in the causes on which the variations depend.

We have first to distinguish the two great classes of irregular and periodic stars. The irregular ones increase and diminish in so fitful a way that no law of their change can be laid down. To this class belong the so-called "new stars," which at various periods in history have blazed out in the heavens, and then in a few weeks or months have again faded away. It is a remarkable fact that no star of the latter class has ever been known to blaze out more than once. This fact distinguishes new stars from other irregularly variable ones.

Periods of Variable Stars.

Periodic stars are those which go through a regular cycle of changes in a definite interval of time, so that, after a certain number of days, sometimes of hours, the star returns to the same brightness. But even in the case of periodic stars, it is found that the period is more or less variable, and in special cases the amount of the variation is such that it cannot always be said whether we should call a star periodic or irregular.

The periodic stars show wide differences, both in the length of the period and in the character of the changes they undergo. In most cases they increase rapidly in brightness during a few days or weeks, and then slowly fade away, to go through the same changes again at the end of the period. Some stars are distinguished more especially by their maxima, or periods of greatest brightness, while others are more sharply marked by minima, or periods of least brightness. In some cases there are two unequal maxima or minima in the course of a period.

Chandler's third catalogue of variable stars gives the periods of 280 of these objects, which seem to have been fairly well made out. Mr. A. W. Roberts has added an important number of southern stars in a list found in the *Astronomical Journal*, xxi., p. 84. A classification of these periods, as to their length, will be interesting. The first set of numbers in the following table, headed C., are the periods of Chandler's catalogue, the next, headed R., are the additional periods given by Roberts. There are of periods

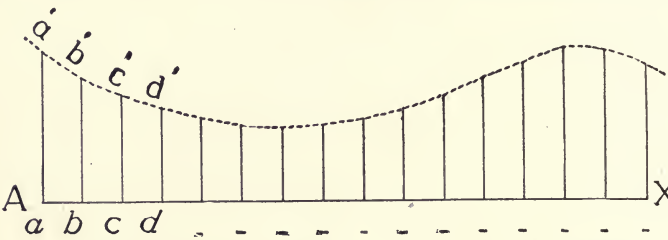
	C.	R.	Sum.
Less than 50 days	63	10	73 Stars.
Between 50 and 100 days	6	2	8 "
" 100 " 150 "	9	3	12 "
" 150 " 200 "	18	4	22 "
" 200 " 250 "	29	12	41 "
" 250 " 300 "	40	5	45 "
" 300 " 350 "	44	5	49 "
" 350 " 400 "	44	6	50 "
" 400 " 450 "	18	2	20 "
" 450 " 500 "	6	0	6 "
" 500 " 550 "	1	0	1 "
" 550 " 600 "	1	1	2 "
" 600 " 650 "	1	0	1 "

It will be seen from this that, leaving out the cases of very short period, the greater number of the periods fall between 300 and 400 days. From this value the number falls off in both directions. Only four periods exceed 500 days, and of these the longest is 610 days. We infer from this that there is something in the constitution of these stars, or in the causes on which their variation depends, which limits the period. This limitation establishes a well-marked distinction between the periodic stars and the irregular variables to be hereafter described.

Returning to the upper end of the scale, the contrast between the great number of stars less than 50 days, and the small number between 50 and 100 seems to show that we have here a sharp line of distinction between stars of long and those of short period. But when we examine the matter in detail we find that the statistics of the periods do not enable us to draw any such line. Among isolated stars about ten periods are less than one day, and the number of this class known to us is continually increasing. Forty or fifty are between one and ten days, and from this point upwards they are scattered with a fair approach to equality up to a period of 100 days. There is, however, a possible distinction, which we shall develop presently.

The law of change in a variable star is represented to the eye by a curve in the following way: We draw a straight horizontal line *A X* to represent the time. A series of equidistant points, *a*, *b*, *c*, *d*, etc., on this line will represent moments of time. One of the spaces, *a*, *b*, etc., may represent an hour, a day, or a month, according to the rapidity of change.

Light-curve of a Star.



We take *a* to represent the initial moment, and erect an ordinate, *a a'*, of such length as to represent the brightness of the star on some convenient scale at this moment. At the second moment, *b*, which may be an hour or a day later, we erect another ordinate, *b b'*, representing the brightness at this moment. We continue this process as long as may be required. Then we draw a curve, represented by the dotted line, through the ends of all the ordinates. In the case of a periodic star it is only necessary to draw the curve through a single period, since its continuation will be a repetition of its form for any one period.

We readily see that if a star does not vary, all the ordinates will be of equal length, and the curve will be a horizontal straight line. Moreover, the curve will take this form through any portion of time during which the light of the star is constant.

Types of Variable Stars.

There are three of the periodic stars plainly visible to the naked eye at maximum, of which the variations are so wide that they may easily be noticed by anyone who looks for them at the right times, and knows how to find the stars. These stars are:

Omicron Ceti, called also *Mira Ceti.*
Beta Persei, or Algol.
Beta Lyræ.

It happens that each of these stars exemplifies a certain type or law of variation.

The Ceti Type.

On August 13, 1596, David Fabricius noticed a star in the constellation Cetus which was not found in any catalogue. Bayer, in his *Uranometria*, of which the first edition was published in 1601, marked the star Omicron, but said nothing about the fact that it was visible only at certain times. Fabricius observed the star from time to time until 1609, but he does not appear to have fully and accurately recognised its periodicity. But so extraordinary an object could not fail to command the attention of astronomers, and the fact was soon established that the star appeared at intervals of about eleven months, gradually fading out of sight after a few weeks of visibility. Observations of more or less accuracy having been made for more than two centuries, the following facts respecting it have been brought to light:

Its variations are somewhat irregular. Sometimes, when at its brightest, it rises nearly or quite to the second magnitude. This was the case in October, 1898, when it was about as bright as Alpha Ceti. At other times its maximum brightness scarcely exceeds the fifth magnitude. No law has yet been discovered by which it can be predicted whether it will attain one degree of brightness or another at maximum.

Its minima are also different. Sometimes it sinks only to the eighth magnitude; at other times to the ninth or lower. In either case it is invisible to the naked eye.

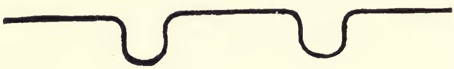
As with other stars of this kind, it brightens up more rapidly than it fades away. It takes a few weeks from the time it becomes visible to reach its greatest brightness, whatever that may be. It generally retains this brightness for two or three weeks, then fades away, gradually at first, afterwards more rapidly. The whole time of visibility will, therefore, be two or three months. Of course, it can be seen with a telescope at any time.

The period also is different in a somewhat irregular way. If we calculate when the star ought to be at its greatest brightness on the supposition that the intervals between the maxima ought to be equal, we shall find that sometimes the maximum will be thirty or forty days early and at other times thirty or forty days late. These early or late maxima follow each other year after year, with a certain amount of regularity as regards the progression, though no definable law can be laid down to govern them. Thus, during the period from 1782 to 1800 it was from thirteen to twenty-four days late. In 1812 it was thirty-nine days late. From 1845 to 1856 it was on the average about a month too early. Several recent maxima, notably those from 1895 to 1898, again occurred late. Formulæ have been constructed to show these changes, but there is no certainty that they express the actual law of the case. Indeed, the probability seems to be that there is no invariable law that we can discover to govern it.

Argelander fixed the length of the period at 331.9 days. More recently, Chandler fixed it at 331.6 days. It would seem, therefore, to have been somewhat shorter in recent times. It was at its maximum toward the end of October, 1898. We may therefore expect that future maxima will occur in June, 1902; May, 1903; April, 1904; March, 1905, and so on, about a month earlier each year. During the few years following 1903 the maxima will probably not be visible, owing to the star being near conjunction with the sun at the times of their occurrence.

The Algol Type.

The star Algol, or Beta Persei, as it is commonly called in astronomical language, may, in northern latitudes, be seen on almost any night of the year. In the early summer we should probably see it only after midnight, in the north-east. In late winter it would be seen in the north-west. From August until January one can find it at some time in the evening by becoming acquainted with the constellations. It is nearly of the second magnitude. One might look at it a score of times without seeing that it varied in brilliancy. But at certain stated intervals, somewhat less than three days, it fades away to nearly the fourth magnitude for a few hours, and then slowly recovers its light. This fact was first discovered by Goodrick in 1783, since which time the variations have been carefully followed. The law of variation thus defined is expressed by a curve of the following form:



The idea that what we see in the star is a partial eclipse caused by a dark body revolving round it, was naturally suggested even to the earliest observers. But it was impossible to test this theory until recent times. Careful observation showed changes in the period between the eclipses, which, although not conclusive against the theory, might have seemed to make it somewhat unlikely. The application of the spectroscope to the determination of radial motions enabled Vogel, of Potsdam, in 1889, to set the question at rest. His method of reasoning and proceeding was this:

If the fading out which we see is really due to an eclipse by a dark body, that body must be nearly or quite as large as the star itself, else it could not cut off so much of its light. In this case, it is probably nearly as massive as the star itself, and therefore would affect the motion of the star. Both bodies would, in fact, revolve around their common centre of gravity. Therefore when, after the dark body has passed in front of the star, it has made one-fourth of a revolution, which would require about seventeen hours, the star would be moving towards us. Again, seventeen hours before the eclipse, it ought to be moving away from us.

The measurement of six photographs of the spectrum, of which four were taken before the eclipses and two afterward, gives the following results:

Before eclipses: Velocity *from* the sun equals 39 km. per second.

After eclipses: Velocity *toward* the sun equals 47 km. per second.

These results show that the hypothesis in question is a true one, and afforded the first conclusive evidence of a dark body revolving around a distant star. A study of the law of diminution and recovery of the light during the eclipse, combined with the preceding motions, enabled Vogel to make an approximate estimate of the size of the orbit and of the two bodies. The star itself is somewhat more than a million of miles in diameter; the dark companion a little less. The latter is about the size of our sun. Their distance apart is somewhat more than three millions of miles; the respective masses are about one-half and one-fourth that of the sun. These results, though numerically rather uncertain, are probably near enough to the truth to show us what an interesting system we here have to deal with. We can say with entire certainty that the size and mass of the dark body exceed those of any planet of our system, even Jupiter, several hundredfold.

The period of the star is also subject to variations of a somewhat singular character. These have been attributed by Chandler to a motion of the whole system around a third body, itself invisible. This theory is, however, still to be proved. Quite likely the planet which causes the eclipse is not the only one which revolves around this star. The latter may be the centre of a system like our solar system, and the other planets may, by their action, cause changes in the motion of the body that produces the eclipses. The most singular feature of the change is that it seems to have taken place quite rapidly about 1840. The motion was nearly uniform up to near this date; then it changed, and again remained nearly uniform until 1890. Since then not enough of observations have been published to test the laws of change conclusively.

It is found that several other stars vary in the same way as Algol; that is to say, they are invariable in brightness during the greater part of the time, but fade away for a few hours at regular intervals. This is a kind of variation which it is most difficult to discover, because it will be overlooked unless the observer happens to notice the star during the time when an eclipse is in progress, and is thoroughly aware of its previous brightness. One might observe a star of this kind very accurately a score of times, without hitting upon the right moment. On the principle that like effects are due to like causes, we are justified in concluding that in the cases of all stars of this type, the eclipses are caused by the revolution of a dark body round the principal star.

A feature of all the Algol variables is the shortness of the periods. The longest period is less than five days, while three are less than one day. This is a result that we might expect from the nature of the case. The nearer a dark planet is to the star, the more likely it will be to hide its light from an observer at a great distance. If, for example, the planet Jupiter were nearly as large as the sun, the chances would be hundreds to one against the plane of the orbit being so nearly in the line of a distant observer that the latter would ever see an eclipse of the sun by the planet. But if the planet were close to the sun, the chances might increase to one in ten, and yet further to almost any extent, according to the nearness of the two bodies.

Still, we cannot set any definite limit to the period of stars of this type; all we can say is that, as the period we seek for increases, the number of stars varying in that period must diminish. This follows not only from the reason just given, but from the fact that the longer the interval that separates the partial eclipses of a star of the Algol type, the less likely they are to be detected.

The Beta Lyræ Type.

The star Beta Lyræ shows variations quite different in their nature from those of Algol, yet having a certain analogy to them. Anyone who looks at the constellation Lyra a few nights in succession, and compares Beta with Gamma, a star of nearly the same brightness in its neighbourhood, will see that while on some evenings the stars are of equal brightness, on others Beta will be fainter by perhaps an entire magnitude.

A careful examination of these variations shows us a very remarkable feature. On a preliminary study, the period will seem to be six and one-half days. But, comparing the alternate minima, we shall find them unequal. Hence the actual period is thirteen days. In this period there are two unequal minima, separated by equal maxima. That is to say, the partial eclipses at intervals of six and one-half days are not equal. At the alternate minima the star is half as bright again as at the intermediate minima.

It is impossible to explain such a change as this merely by the interposition of a dark body, and this for two reasons. Instead of remaining invariable between the minima, the variation is continuous during the whole period, like the rising and falling of a tide. Moreover, the inequality of the alternating minima is against the theory.

Pickering, however, found from the doubling of the spectral lines that there were two stars revolving round each other. Then Prof. G. W. Myers, of Indiana, worked out a very elaborate mathematical theory to explain the variations, which is not less remarkable for its ingenuity than for the curious nature of the system it brings to light. His conclusions are these:

Beta Lyræ consists of two bodies, gaseous in their nature, which revolve round each other, so near together as to be almost in contact. They are of unequal size. Both are self-luminous. By their mutual attraction they are drawn out into ellipsoids. The smaller body is much brighter than the other. When we see the two bodies laterally, they are at their brightest. As they revolve, however, we see them more and more end on, and thus the light diminishes. At a certain point one begins to cover the other and hide its light. Thus the combined light continues to diminish until the two bodies move across our line of sight. Then we have a minimum. At one minimum, however, the smaller and brighter of the two bodies is projected upon the larger one, and thus increases its apparent brilliancy. At the other minimum, it is hiding behind the other, and therefore we see the light of the larger one alone.

This theory receives additional confirmation from the fact, shown by the spectroscope, that these stars are either wholly gaseous, or at least have self-luminous atmospheres. Some of Professor Myers's conclusions respecting the magnitudes are summarised as follows:

The larger body is about 0.4 as bright as the smaller.

The flattening of the ellipsoidal masses is about 0.17.

The distance of centres is about $1\frac{7}{8}$ the semi-major axis of the larger star, or about 50,000,000 kilometres (say 30,000,000 miles).

The mass of the larger body is about twice that of the smaller, and $9\frac{1}{2}$ times the mass of the sun.

The mean density of the system is a little less than that of air.[1]

It should be remarked that these numbers rest on spectroscopic results which need further confirmation. They are therefore liable to be changed by subsequent investigation. What is most remarkable is that we have here to deal with a case to which we have no analogy in our solar system, and which we should never have suspected, had it not been for observations of this star.

The gap between the variable stars of the Algol type and those of the Beta Lyræ type is at the present time being filled by new discoveries in such a way as to make a sharp distinction of the two classes difficult. It is characteristic of the Algol type proper that the partial eclipses are due to the interposition of a dark planet revolving round the bright star. But suppose that we have two nearly equal stars, *A* and *B*, both bright, revolving round their common centre of gravity in a plane passing near our system. Then *A* will eclipse *B*, and, half a revolution later, *B* will eclipse *A*, and so on in alternation. But when the stars are equal we may have no way of deciding which is being eclipsed, and thus we shall have a star of the Algol type so far as the law of variation is

[1] *Astrophysical Journal*, vol. vii., January, 1898.

concerned, yet, as a matter of fact, belonging rather to the Beta Lyræ type. If the velocity in the line of sight could be measured, the question would be settled at once. But only the brightest stars can, so far, be thus measured, so that the spectroscope cannot help us in the majority of cases.

The most interesting case of this kind yet brought to light is that of Y Cygni. The variability of this star, ordinarily of the fourth magnitude, was discovered by Chandler in December, 1886. The minima occurred at intervals of three days. But in the following summer he found an apparent period of 1 d. 12 h., the alternate minima being invisible because they occurred during daylight, or when the star was below the horizon. With this period the times of minima during the summer of 1888 were predicted.

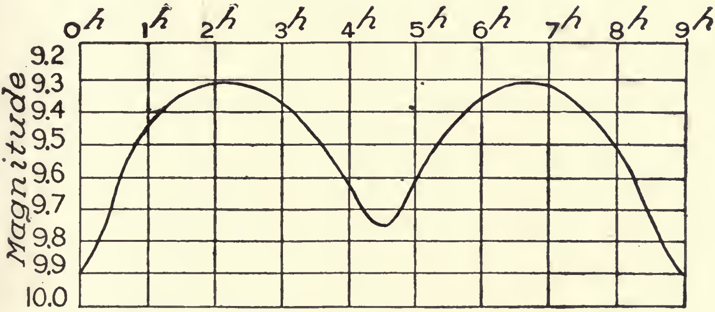
It was then found that the times of the alternate minima, which, as we have just said, were the only ones visible during any one season, did not correspond to the prediction. The period seemed to have greatly changed. Afterward, it seemed to return to its old value. After puzzling changes of this sort, the tangle was at length unravelled by Dunér, of Lund, who showed that the alternate periods were unequal. The intervals between minima were 1 d. 9 h., 1 d. 15 h., 1 d. 9 h., 1 d. 15 h., and so on, indefinitely.

This law once established, the cause of the anomaly became evident. Two bright stars revolve round their common centre of gravity in a period of nearly three days. Each eclipses the other in alternation.

The orbit is eccentric, and, in consequence, one half of it is described in a less time than the other half. If we could distinguish the two stars by telescopic vision, and note their relative positions at the four cardinal points of their orbit, we should see the pair alternately single and double, as shown in the following diagram :

		A		B
Position (1), stars at pericentre..................		*		*
Interval, 16 hours.				
Position (2), A eclipses B......................			*	
Interval 19 hours.				
	B			A
Position (3), stars at apocentre..................	*			*
Interval, 20 hours.				
Position (4), B eclipses A......................			*	
Interval 17 hours.				
		A		B
Position (1) is repeated........................		*		*

U Pegasi is a star which proved as perplexing as Y Cygni. It was first supposed to be of the Algol type, with a period of about two days. Then it was found that a number of minima occurred during this period, and that the actual interval between them was only a few hours. The great difficulty in the case arises from the minuteness of the variation, which is but little more than half a magnitude between the extremes. The observations of Wendell, at the Harvard Observatory, with the polarising photometer, enabled Pickering to reach a conclusion which, though it may still be open to some doubt, seems to be the most likely yet attainable. The star is of the Beta Lyræ type; its complete period is 8 hours 59 minutes 41 seconds, or 19 seconds less than 9 hours; during this period it passes through two equal maxima, each of magnitude 9.3, and two unequal minima, 9.76 and 9.9, alternately.



LIGHT-CURVE OF U PEGASI, OF THE BETA LYRÆ TYPE.

The difference of brightness of these minima, 0.14 mag., is less than the errors which ordinarily affect measures of a star's magnitude with the best photometers. Some scepticism has, therefore, been felt as to the reality of the difference; which, if it does not exist, would reduce the periodic time below 4½ hours, the shortest yet known. But Pickering holds that, in observations of this kind upon a single star, the precision is such that the reality of the difference, small though it be, is beyond serious doubt.

Taking Pickering's law of change as a basis, Myers has represented the light-curve of U Pegasi on a theory similar to that which he constructed for Beta Lyræ. His conclusion is that, in the present case, the two bodies which form the visible star are in actual contact. A remarkable historic feature of the case is that Poincaré has recently investigated, by purely mathematical methods, the possible forms of revolving fluid masses in a condition of equilibrium, bringing out a number of such forms previously unknown. One of these, which he calls the apioidal form, consists of two bodies joined into one, and it is this which Myers finds for U Pegasi.

Quite similar to these two cases is that of Z Herculis. This star, ordinarily of the seventh magnitude, was found, at Potsdam, in 1894, to diminish by about one magnitude. Repeated observations elsewhere indicate a period of very nearly four days. Actually it is now found to be only ten minutes less than four days. The result was that during any one season of observation the minima occur at nearly the same hour every night or day. To an observer situated in such longitude that they occur during the day, they would, of course, be invisible.

Continued observations then showed a secondary minimum, occurring about half-way between the principal minima hitherto observed. It was then found that these secondary minima really occur some two hours earlier than the mid-moment, so that the one interval would be between forty-six and forty-seven hours and the other between forty-nine and fifty. The time which it takes the star to lose its light and regain it again is about ten hours. More recent observations, however, do not show this inequality, so that there is probably a rapid motion of the pericentre of the orbit.

It will be seen that this star combines the Algol and Beta Lyræ types. It is an Algol star in that its light remains constant between the eclipses. It is of the Beta Lyræ type in the alternate minima being unequal. Dunér subjected the observations of this star to a very careful discussion. His conclusion is as follows:

Z Herculis consists of two stars of equal size, one of which is twice as bright as the other. These stars revolve around their common centre of gravity in an elliptic orbit whose semiaxis major is six times the diameter of the stars. The plane of the orbit passes through the sun; the eccentricity is 0.2475, and the line of apsides is inclined at an angle of 4° to the line of sight (*Astrophysical Journal*, vol. i.).

From a careful study, Seliger and Hartwig derived the following particulars respecting this system:

Diameter of principal star, 15,000,000 kilometres.
" smaller " 12,000,000 "
Mass of the larger star, 172 times sun's mass.
Mass of the smaller star, 84 times sun's mass.
Distance of centres, 45,000,000 kilometres.
Time of revolution, 3 d. 23 h. 49 m. 32.7 s.

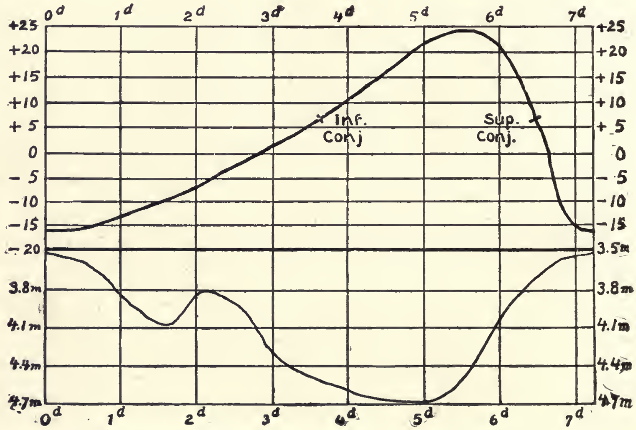
It must be added that the data for these extraordinary numbers are rather slender and partly hypothetical.

Beta Lyræ is always of the same brightness at the same hour of its period, and Algol has always the same magnitude at minimum. It is true that the length of the period varies slowly in the case of these stars. But this may arise from the action of other invisible bodies revolving around the visible stars. This general uniformity is in accord with the theory which attributes the apparent variations to the various aspects in which we see one and the same pair of revolving stars.

Another variable star showing some unique features is Eta Aquilæ. What gives it special interest is that spectroscopic observations of its radial motion show it to have a dark body revolving round it in a very eccentric orbit, and in the same time as the period of variation. It might therefore be supposed that we have here a star of the Algol or Beta Lyræ type. But such is not the case. There is nothing in the law of variation to suggest an eclipsing of the bright star, nor does it seem that the variations can readily be represented by the varying aspects of any revolving system.

Variation of Eta Aquilæ.

The orbit of this star has been exhaustively investigated by Wright from Campbell's observations of the radial motion. The laws of change in the system are shown by the curves below, which are reproduced, in great part, from Wright's paper in the *Astrophysical Journal.*



LIGHT- AND VELOCITY-CURVES OF η AQUILÆ COMPARED.

The lower curve is the light-curve of the star during a period of 7.167 days. Starting from a maximum of 3.5 mag., it sinks, in the course of 5 days, to a minimum of 4.7 m. It was found by Schwab that the diminution is not progressive, but that a secondary maximum of 3.8 m. is reached at the end of the second day. After reaching the principal minimum it rises rapidly to the principal maximum in 2¼ days.

The upper curve shows the radial velocity of the star during the period of variation. It will be seen that the epoch of greatest negative velocity, which, referred to the centre of mass of the system, is 16.2 km. per second, occurs at the time of maximum brightness. The greatest positive velocity, 23.9 km., occurs during the sixth day of the period, just after the time of minimum brightness.

Finally, the moments of inferior and superior conjunction of the dark body with the bright one are neither of them an epoch of minimum brightness, which takes place half-way between the two.

The case of Delta Cephei is not dissimilar to that of Eta Aquilæ. This star is regularly variable in a period of 5.366 days. Its magnitude at maximum is 3.7; at minimum 4.9. It was found by Belopolsky to be a spectroscopic binary with a period the same as that of its variation of the light. He finds that, as in the case of the other star, there appears to be nothing in the nature of an eclipse. The orbit is, however, very eccentric. The epoch of minimum is one day earlier than that of perihelion passage.

Its slight variation, as in the case of Eta Aquilæ, is much more rapid during the increase than during the decrease. From Schur's table it seems that the whole time of rise, from minimum to maximum, is 1.6 d., which is less than one-third the entire period. Moreover, the larger part of this change takes place in less than a day.

Classification of Variable Stars.

A classification of variable stars, based on the period of variation and the law of change, was proposed by Pickering. It does not, however, seem that a hard-and-fast line can yet be drawn between different types and classes of these bodies, one type running into another, as we have found in the case of the Algol and Beta Lyræ types. Yet the discovery of the cause of the variation in these types makes it likely that a division into four great classes, dependent on the cause of variation, is possible. These classes are:

(1) Stars, or systems appearing to us as a single star, of which the apparent variability arises solely or mainly from the rotation of the system as a whole, or from the revolution of its components around each other. In this case the variations of light are purely the effect of perspective, arising from the various aspects which the system presents to us during the revolution of its components. There is no real variation either in the constitution of the star or in the actual amount of light which it emits. If we could change our point of view so that the plane of the orbit of an Algol star no longer passed near our system, the star would cease to appear variable. Under the same

circumstances the apparent variations of a star of the Beta Lyræ type would be smaller than they are, and would disappear entirely if the axis of rotation were directed toward our system. The stars of this class are also distinguished by the uniformity and regularity with which they go through their cycle of change.

(2) The second class comprises stars in which the changes of light are real and arise from some cycle of change going on in the star, but which may be due to the action of an external body. This class may be divided into two or three subclasses, as has been done by Pickering, depending on the length of the period and the character of the variation. But it does not appear that we can yet sharply define the subdivision, because, as already stated, one class runs into the other by insensible gradations. Perhaps the best defined class is that of the Omicron Ceti type. There are certain general laws of variation and irregularities of brightness which stars of this class go through. Starting from the time of the minimum, the increase of light is at first very slow. It grows more and more rapid as the maximum is approached, near which there may be as great an increase in two or three days as there formerly was in a month. The diminution of light is generally slower than the increase. The magnitude at corresponding times in different periods may be very different. Thus, as we have already remarked, Omicron Ceti is ten times as bright at some maxima as it is at others. The periods also, so far as they have been made out, vary more widely than those of stars of the other types. The most remarkable

feature of this type is found in its spectrum. Nearly all these stars have spectra of the third type in which the hydrogen lines are bright at the time of maximum. So well defined is this peculiarity that stars are recognised as variable at the Harvard Observatory merely by this feature of the spectrum.

From what has been said, it will be seen that, although a sharp line cannot be drawn, there seems to be some distinction between the stars of short and long periods. The number of stars which have been known to belong to the first class is quite small, only about fifteen all told. On the other hand, there are still left some stars having a period less than ten days, which are otherwise not distinguishable from the Omicron Ceti type.

The discovery that Delta Cephei and Eta Aquilæ have dark bodies revolving around them in a period equal to that of the variation of light, suggests the idea that in perhaps all this class of stars the variations of light are due to the varying action of a revolving planet as it moves around in a very eccentric orbit.

The periodic stars of short period which have not been recognised as of the Algol or Beta Lyræ type form an interesting subject of study. Although the separation between them and the stars of long period is not sharp, it seems likely to have some element of reality in it. But no conclusions on the subject can be reached until the light-curves of a large number of them are carefully drawn; and this requires an amount of patient and accurate observation which cannot be carried out for years to come.

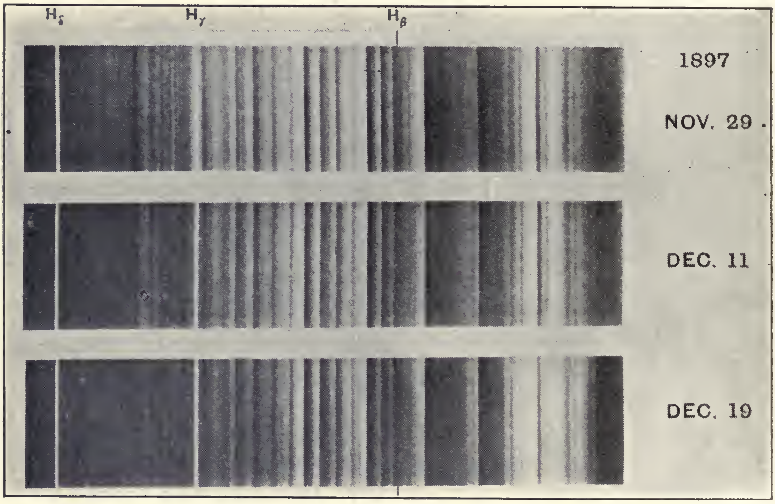
(3) The third class comprises stars subject to small and irregular but frequently recurring fluctuations of light. The range of variation is commonly only a fraction of a magnitude. The following are the most noteworthy examples of this class :

α Cassiopeæ,	range	in	mag.	2.2	to	2.8
ρ Persei,	"	"	"	3.4	"	4.2
α Orionis,	"	"	"	1.0	"	1.4
α Herculis,	"	"	"	4.6	"	5.4
μ Cephei,	"	"	"	4.0	"	5.0
β Pegasi,	"	"	"	2.2	"	2.7

(4) The fourth class are the "novæ," or new stars, which, so far as is known, blaze out but once in history. They will be described in the next chapter.

Spectra of Variable Stars.

It might be supposed that the changes in the light of the variable stars, at least in those cases where they are not caused by a mere partial eclipsing of the star, would be accompanied by wide changes in their spectra, following some definable law. Many studies have been made on this subject, but it is difficult to formulate any general conclusion from them. The investigation is a difficult one, because the most interesting cases are those in which the diminution of light at minimum is very great, and the spectrum cannot be well studied. The star Omicron Ceti has perhaps been more carefully studied from this point of view than any other. Campbell found that near the time of maximum, the bright hydrogen line $H\gamma$ was very strong and overexposed on all the plates. He found that two minutes sufficed to obtain an impression of this line, at a stage of brightness when an hour is wanted for the rest of the spectrum. Under the same circumstances, the line Hδ is triple. The central component of this triple system is much stronger than the two others, which are about equal. As the spectrum grows fainter, the components occupy nearly the position of certain iron lines, but nothing definite can be ascertained about them.



SPECTRUM OF O CETI NEAR THE MAXIMUM OF 1897, PHOTOGRAPHED BY FATHER SIDGREAVES AT THE STONYHURST COLLEGE OBSERVATORY.

Suspected Variations in the Colour of Stars.

The question whether certain stars vary in colour without materially changing their brightness has sometimes been raised. This was at one time supposed to be the case with one of the stars of Ursa Major. This suspected variation has not, however, been confirmed, and it does not seem likely that any such changes take place in the colour of stars not otherwise variable.

Possible Secular Variations in the Brightness of Stars.

All the variations we have hitherto considered take place with such rapidity that they can be observed by comparisons embracing but a short interval of time—a few days or months at the outside. A somewhat different question of great importance is still left open. May not individual stars be subject to a slow variation either in their colour or their brightness, which are sensible in the course of only one generation of men, but admit of being brought out by a comparison of the brightness of the stars at widely distant epochs? Is it certain that, in the case of stars which we do not recognise as variable, no change has taken place since the time of Hipparchus and Ptolemy? This question has been investigated by C. S. Pierce and others. The conclusion reached is that no real evidence of any change can be gathered. The discrepancies are no greater than might arise from errors of estimates.

There is, however, an aspect of the question which is of great interest and has been much discussed in recent times. In several ancient writings the colour of Sirius is described as red. This fact would, at first sight, appear to afford very strong evidence that, within historic times, the colour of the brightest star in the heavens has actually changed from red to bluish white.

Two recent writers have examined the evidence on this subject most exhaustively and reached opposite conclusions. The first of these was Prof. T. J. J. See, who collated a great number of cases in which Sirius was mentioned by ancient writers as red or fiery, and thus concluded that the evidence was in favour of a red colour in former times. Shortly afterwards, Schiaparelli examined the evidence with equal care and thoroughness and reached an opposite conclusion, showing that the terms used by the ancient authors which might have indicated redness of colour were susceptible of other interpretations ; they might mean fiery, blazing, etc., as well as red in colour, and were therefore probably suggested by the extraordinary brightness of Sirius and the strangeness with which it twinkled when near the horizon. In this position a star not only twinkles, but changes its colour rapidly. This change is not sensible in the case of a faint star, but if one watches Sirius when on the horizon, it will be seen that it not only changes in appearance, but seems to blaze forth in different colours.

It seems to the writer that this conclusion of Schiaparelli is the more likely of the two. From what we know of the constitution of the stars, a change in the colour of one of these bodies in so short a period of time as that embraced by history is so improbable as to require much stronger proofs than any that can be adduced from ancient writers. In addition to the possible vagueness or errors of the original writers, we have to bear in mind the possible mistakes or misinterpretations of the copyists who reproduced the manuscripts.

CHAPTER VIII

NEW STARS

It may be glorious to write
Thoughts that shall glad the two or three
High souls, like those far stars that come in sight
Once in a century.—LOWELL

THE stars considered in the preceding chapter go through their changes of light in a limited and generally more or less regular period, so that a prediction of their brightness at future epochs is in most cases possible. They are distinguished by the remarkable fact, pointed out at the beginning of the chapter, that the period seems to be limited, none so long as two years being yet known.

New stars, or "Novæ" as they are frequently called, are distinguished from the irregularly variable stars already described by their blazing forth, so far as is yet known, only once in the period of their history.

The limitation of the period seems to form a well-marked distinction between periodic stars and the irregularly variable ones now to be considered, and to indicate some radical difference in the cause of variability.

The most remarkable among these stars is undoubtedly Eta Argus, which, though now invisible to the naked eye, was, at various times between 1830 and 1850, of the first magnitude. It falls so closely on a line between the new or temporary stars and those which are irregularly variable that it may form a distinct class. Being in 58° of south declination it is not visible except in latitudes south of 32°. For this reason it could not be made a subject of observation in northern European countries. Of the greatest interest is the question whether it was visible in early historic times. On this question no decisive evidence can be gathered. The catalogues of Ptolemy and Ulugh Beigh are among the earlier authorities which we consult on the subject. Much confusion, however, is found in the data to be consulted. In Halma's edition of Ptolemy's catalogue, two stars in the constellation Argo are marked as having the Bayer letter Eta. But neither of these is near the position of the star under consideration. In fact, Ptolemy's constellation Argo seems scarcely to extend as far east as the point in question. The same remark applies to the mediæval catalogue of Ulugh Beigh. The only conclusion we can draw on the subject is that the star was probably not so conspicuous in early historic times as to excite the attention of observers.

On Bayer's charts, published about 1600, there is a star marked Eta, but this is nowhere near the place of the modern Eta, nor is there any star shown in the position of the latter. The fact appears to be that

Bayer's maps of this constellation are so erroneous that little correspondence can be found between his figures and the heavens, and the certain identification of any particular star scarcely seems possible, except in the case of Canopus and possibly a few other bright ones. Near the position of the modern Eta are several small stars marked *d*, but from what has been said we have no reason to identify these with the star in question.

The first authentic observation of the star is found in Halley's catalogue, made at St. Helena in 1677, where it appears as of the fourth magnitude. The next observation is by Lacaille, who observed it at the Cape of Good Hope about 1750. In the catalogue at the end of his *Cælum Australe Stelliferum*, the star is given as of the second magnitude; but in the original observations it is marked of magnitude 2.3. It may be added that Lacaille was the first one to assign the symbol Eta. From a remark at the end of the catalogue, it seems that he assigned these symbols in accordance with Bayer only when the Bayer stars could be identified, but it would seem that there could have been few such identifications in Argo. In catalogues made between the years 1822 and 1832 it still appears as of the second magnitude; whether this magnitude was an independent one or merely taken from Lacaille may be an open question, but we cannot suppose that the variation from Lacaille's estimate was at all striking. A traveller named Birchell noted it as of the first magnitude in 1827, but this seems doubtful in view of the records of other observers.

Our next authority on the subject is Sir John Herschel, who, during his residence at the Cape of Good Hope, in 1834, noted Eta Argus as of magnitude between first and second. It remained without exciting any suspicion of change to near the end of 1837. In December of this year Herschel's astonishment was excited by the appearance of "a new candidate for distinction among the very bright stars of the first magnitude, in a part of the heavens with which being perfectly familiar, I was certain that no such brilliant object had before been seen." This was soon found to be identical with Eta Argus, of which the light had nearly trebled. It decidedly surpassed Procyon, Alpha Orionis, and even Rigel, which was nearest to it. It continued to increase until the beginning of January, 1838, when it was equal to Alpha Centauri. Then it began slowly to fade, but on April 14th, which seems to have been the date of Herschel's last observation, it was still about equal to Aldebaran, and therefore of the first magnitude. It seems to have blazed up again, according to the testimony of observers, in 1843, when it was fully as bright as Canopus, and could not therefore have been far below Sirius. It fluctuated during the following ten years, and then began to fade away slowly. In 1868 it was estimated by Mr. Tebbut as only of the sixth magnitude, and gradually disappeared from vision by the naked eye in the year following. During the last fifteen or twenty years it has generally been of the seventh magnitude, or fainter, and there is no evidence of any approaching renewal of its bright stage of half a century ago. I quote the following list of determinations from Mr. R. T. A. Innes (M. N. R. A. S., lix., 570.)

Year 1886. 2	Mag.	7.60	(Finlay)
1896. 4	"	7.58	(Innes)
1897. 2	"	7.60	(See)
1899. 5	"	7.71	(Innes)

We now pass to the class of new or temporary stars properly so called. A distinguishing feature of a star of this class is that it blazes up, so far as is known, only once in the period of its history, then gradually fades away to its former magnitude, which it commonly retains with, so far as is yet known, little or no subsequent variation.

It was formerly supposed that stars of this class were new creations which went out of existence after a span of life which would have been brief even for a human being, much less for a star. It is hardly necessary to say that such a view as this can find no place in modern science.

Miss Clerke, in her *System of the Stars*, gives a list of ten such stars which appeared between B.C. 134 and the end of the fifteenth century. Accepting all these as real there would be an average of one such star in about 160 years. In the few cases where the duration of the appearance is given it varies from three weeks to eight months. The following list of such stars which have appeared since 1500 is compiled from the circulars of the Harvard Observatory:

Year.	Constellation.	Position, 1900.		Mag.	Discoverer.
		R. A. H. M.	Dec.		
1572....	Cassiopeia.	0.19.2	+63° 36′	Br.	Tycho.
1600....	Cygnus.	20.14.1	+37 43	3	Janson.
1604....	Ophiuchus.	17.24.6	—21 24	Br.	Kepler.
1670....	Vulpecula.	19.43.5	+27 4	3	Anthelm.
1848....	Ophiuchus.	16.53.9	—12 44	5	Hind.
1860....	Scorpius.	16.11.1	—22 44	7	Auwers.
1866....	Corona Bor.	15.55.3	+26 12	2	Birmingham.
1876....	Cygnus	21.37.8	+42 23	3	Schmidt.
1885 ...	Andromeda.	0.37.2	+40 43	7	Hartwig.
1887....	Perseus.	1.55.1	+56 15	9	Fleming.
1891....	Auriga.	5.25.6	+30 22	4	Anderson.
1893....	Norma.	15.22.2	—50 14	7	Fleming.
1895....	Carina.	11. 3.9	—61 24	8	Fleming.
1895....	Centaurus.	13.34.3	—31 8	7	Fleming.
1898....	Sagittarius.	18.56.2	—13 8	5	Fleming.
1901....	Perseus.	3.22	+44 0	0	Anderson.

Among all these the first, sometimes called Tycho's Star, was the most brilliant. It was first noticed on November 7, 1572,[1] by Lindaeur at Winterthur. It was first seen by Tycho Brahe four days later, when it had attained the first magnitude. It continued to increase in brilliancy, at length becoming equal to Venus and visible in full daylight. In December it began to diminish, faded gradually away, and finally disappeared from view in May. As the telescope was then unknown, it was impossible to follow it further.

During the period of its visibility Tycho not only made all the observations he was able to on its appearance, but measured its position relative to other stars. It is now found that a star of magni-

[1] *System of Stars*, page 97.

tude 10.5 is situated within a minute of the position derived from Tycho's observations. In view of this fact there is a strong presumption that this is the star. It has therefore been watched occasionally to detect evidences of variability, but, although some change was strongly suspected by Hind, it does not appear that observations upon it have been made systematically enough to establish any actual change at the present time.

Of Janson's Star of 1600 little is known; a star called P Cygni is supposed to be identical with it, but on what authority I do not know.

The star of 1604 in Ophiuchus has a history not unlike that of Tycho. It was first seen in October, when it had attained the first magnitude. In a few days it became as bright as Jupiter, but began to fall off during the winter. It seems to have been remarkable for its duration, having been visible to the naked eye during the whole year 1605. Early in 1606 it disappeared from view. A very full history of this star has been left by Kepler.

Nearly two centuries now elapse before we have any record of another appearance of the kind. On April 28, 1848, Mr. Hind, then in charge of a private observatory in London, noticed a star of between the fourth and fifth magnitude where none had been seen April 5th. For some days it seems to have fluctuated between the fifth and sixth magnitudes. Soon it began to diminish and fade away year after year until it sank to magnitude 12.5, at which it seems to have remained for more than thirty years.

The Auwers Star of 1860 was discovered in the cluster Messier 80. It only reached the seventh magnitude, soon faded away, and has not since been recognised. It is mainly of interest in connection with the recent discovery at the Harvard Observatory of great numbers of variable stars in clusters.

The star T Coronæ, which appeared in May, 1866, attained the greatest brilliancy of any new star since that of Kepler, having been nearly or quite of the second magnitude. One of the most interesting questions connected with it is the rapidity with which such a star may blaze up, a question which is not yet fully settled. The facts on record are that on the 12th and 13th of May it was remarked independently by at least five observers in Europe and America. On May 12th Schmidt of Athens, who was scanning the heavens, asserts in the most positive manner that the star could not have been visible without his having noticed it. If we accept this negative testimony as conclusive the star must have risen from some low magnitude, probably fainter than the fifth, to the second, within a few hours.

The star is of special interest as the first of which the light could be analysed with the spectroscope. This was done by Mr. Huggins on the first evening after he received notice of the strange object. He found the spectrum to be a singularly composite one, leading to the conclusion that two distinct spectra were superimposed, and that the light had emanated from two different sources, each forming its own spectrum. The principal spectrum was analogous to that of the sun. It indicated light emitted by an incandescent photosphere which suffers partial absorption by passing through a vaporous atmosphere. Beginning at the red end of the spectrum the first dark line was a little more refrangible than the hydrogen line C. Next came a shaded group of lines, then a faint line coincident with D. In the higher regions of the spectrum, the lines were stronger and extended as far as the spectrum could be traced.

The second spectrum was composed of five bright lines. One of these seemed to coincide with line C; still brighter was one coinciding with F, then two fainter lines. The fifth bright line was near G. All the bright lines were much more apparent than the continuous spectrum. It would follow that the gas which emitted them must have had a temperature higher than that of the stellar photosphere from which the light forming the other spectrum emanated.

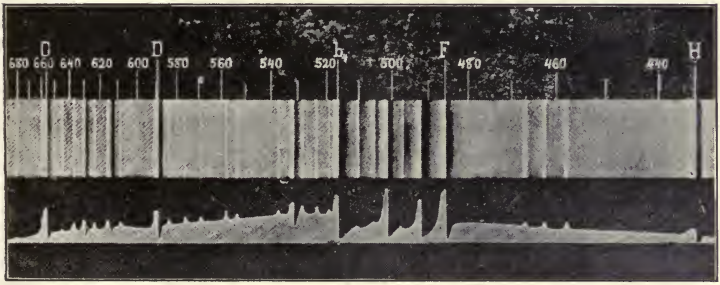
Mr. Huggins compared the spectrum of the star with that of hydrogen. It seemed quite apparent that two of the brighter lines were entirely coincident with the lines C and F of hydrogen. The conclusion, therefore, was that the great brilliancy of the star was due to an outburst of incandescent hydrogen, giving rise to a volume of flame of such magnitude as to be visible at the vast distance of our system.

The star faded away with great rapidity. In twelve days it fell from the second to the eighth magnitude, so that no opportunity was afforded for a continuous study of its spectrum.

The stars which have subsequently appeared have naturally been studied by a greater number of observers and with much detail. Among them Nova Aurigæ, which appeared in February, 1892, long held the first place, on account of the length of time during which it remained bright enough for favourable examination. A citation of the observations and researches would fill a small volume. Within our limited space we can only summarise the principal conclusions of Campbell, Sidgreaves, and Vogel.

The star was first noticed by Dr. Anderson, a diligent watcher of the heavens, at Edinburgh about the end of January, 1892. As it has been in some noteworthy cases since, the region occupied by the star was found to have been photographed at the Harvard Observatory before the star was noticed by Dr. Anderson. On November 2, 1891, the star was not shown on a plate where those of the eleventh magnitude were impressed. On December 1st it would have been shown had it been brighter than the sixth magnitude. The first plate on which it was found bore the date December 16th, when the magnitude was the sixth. Two days previously it was invisible on a plate taken at a European observatory. It must therefore have blazed up within a period of two or three days. It seemed to vary from night to night — at least the magnitudes assigned by the observers were very different. Early in March it began to fade rapidly. By the middle of the month it had sunk to the eighth magnitude, and, by the end, to the twelfth. For several months it was supposed to have sunk almost out of sight, as the minutest object visible in the most powerful telescopes. But in August new interest was excited by its again blazing up to the ninth magnitude. From this time it seems to have fluctuated in a very irregular way for nearly a year before it finally sunk into its former insignificance.

Its spectrum was of course photographed by every astronomer who had the means of doing so. Lock-



SPECTRUM OF NOVA AURIGÆ PHOTOGRAPHED BY CAMPBELL

yer and Huggins in England, Vogel in Germany, and Campbell at Mount Hamilton are the investigators on whom we shall mainly depend. Lockyer found that all the lines in the spectrum were broad, although they showed perfectly sharp in the spectrum of Arcturus. There was no falling off of intensity at the edges of the bright lines. The hydrogen lines and the K line of calcium were very bright and accompanied by dark lines on their more refrangible sides.

As Campbell had the best optical means for photographing the spectrum, we reproduce one of his photographs, taken on February 28th. It is accompanied by an intensity curve, showing the intensity of the light in the various parts of the spectrum by the length of the ordinate with greater accuracy than it can be inferred from the figure of the spectrum. The numbers on the spectrum are the wave-lengths in millionths of a millimetre.

The apparent superposition of at least two spectra, one continuous with dark lines, the other consisting of bright lines, was noticed both by Campbell and Vogel. The latter found the spectrum to extend far into the violet, showing many bright and broad lines, among which the whole range of hydrogen lines were especially noticeable; but on the more refrangible side most of these were broad, dark lines, whose distances from the bright lines increased in going toward the violet in proportion to the increasing dispersion of the prism, and whose identity with the bright lines is thereby established. On February 20th Vogel compared the spectrum with that of hydrogen, showing with seeming certainty that this element was principally concerned in forming the spectrum. The main difference was that the lines were bright in the spectrum of the star and were perceptibly brighter and more sharply defined on the side toward the violet than on that toward the red. Besides being three or four times brighter than the lines of hydrogen, they were displaced strongly toward the red, showing a rapid motion away from the earth. On the other end the dark lines which accompanied the bright ones were so much displaced toward the violet that they could be readily distinguished. A remarkable fact noticed by Vogel was that a number of the lines coincided with those of the sun's chromosphere as catalogued by Young.

On March 19th the continuous spectrum was very faint and fell off rapidly beyond F. The latter was now the brightest line in the spectrum, but several were occasionally glimpsed in the green.

After the star again brightened up in September there was a change in the spectrum, which now consisted principally of a bright line in the green and a faint continuous spectrum. This continued without change until March, 1893. The lines coincided with the brighter ones in the spectrum of the nebulæ, but there was also a very faint continuous spectrum. It was noted by other observers that the spectrum now became identical with that of a planetary nebula.

The remarkable opposite displacement of the lines during the early period of the star's visibility is shown in the following condensed summary of the results of Vogel's measures: Taking the first four bright lines of hydrogen and calcium the following velocities away from the earth were derived from the four lines:

K	243	kilometres	per	second
H	265	"	"	"
H_δ	402	"	"	"
H_γ	457	"	"	"

It will be seen that the calcium lines agree fairly well in giving a motion of 250 kilometres, while the hydrogen lines, especially H , give a considerably larger motion, the mean of the two being 430 kilometres per second.

Very different was it with the accompanying dark-line spectrum. The two hydrogen lines agreed in giving a motion toward the earth of about 780 kilometres per second. The difference of these two results is enormous, more than 600 English miles per second.

The problem of reconciling these rapid motions with any easily conceivable constitution of the body or bodies was no easy one, and the proposed solutions can hardly be considered as better than speculations. The view most generally received was that two bodies had suddenly approached very closely together, perhaps come into collision, and then separated. While this view is by no means impossible, it is far from being established. The great change in the character of the spectrum, while not conclusive against it, certainly seems to throw difficulties in the way of its reception. The history of the star leaves us in great doubt on the question whether, even if the displacement of the lines was due to a rapid motion, the latter was the integral motion of a body. It might have been only that of an incandescent gas escaping from under pressure, in a direction from our system, in fact, an eruption of hydrogen and calcium vapours. If these vapours, after cooling, fell back again in such a way as to cut off the light of the brighter region beyond, they would absorb the dark lines and give the spectrum of a dark body moving toward us.

The most recent investigations showing to what changes the form, position, and brightness of spectral lines are subject through changes in the physical condition of the bodies which emit the light lead to great caution in attributing the displacement of broadened lines in any spectrum to motion.

The fact that these objects blaze up only once in their history shows that the phenomenon is due to some cataclysm of a rather extraordinary kind. The first and most interesting question raised by this fact is whether one star is more likely to be subject to such a cataclysm than another. If new stars were known to vary, or to have any special kind of spectrum before their sudden outburst, we should know that the latter was a catastrophe to which only a particular kind of star is subject. If we could find no peculiarity in the spectrum of the star we should conclude that the catastrophe was due to some external cause. But unfortunately we have thus far no record of any new star before its appearance except, in a very few cases, its position in the heavens. It is true that the star may be studied after it has settled down again, but if the catastrophe was due to an external cause, we have no reason to suppose that it had relapsed to its former condition. Quite likely the cataclysm might have made a permanent change in its constitution.

Perhaps the most natural theory at first sight is that the outburst is due to a collision. It seems probable that stars like our sun, which are in a state of considerable condensation, have somewhat the character of masses of gas confined under enormous pressure, as if they were hollow globes of highly heated and compressed gas. We do not mean by this that the shell is solid; what is possible is that it is composed of divided matter probably denser than the gases below, and compressing the latter by its weight rather than by its tension. If, by the fall of a foreign body, an opening is suddenly made in the shell, the interior gases will burst forth. What magnitude the outburst might assume it is impossible to say, and cautious thinkers will decline to accept this or any other solution until we have had more experience on the subject.

A general fact that seems supported by the most recent observations is that after their outbursts of light these bodies settle down to a nebular condition. This was the case with Nova Aurigæ, and the recent Nova Aquilæ of 1900. Campbell found the spectrum of the latter to consist of extremely faint continuous light in the green, and three bright bands in the positions of the three nebular lines.

The New Star of 1901 in Perseus.

On the night of February 21–22, 1901, Dr. Anderson of Edinborough noticed a previously unknown star of magnitude 2.7, in the constellation Perseus. In the course of the next two days it increased so rapidly as to become about the third brightest star in the sky, being a little brighter than Capella. Then it began slowly to fade away. Early in March it was again of the third magnitude, and before the middle of April had dropped to the fifth.

It seems to have blazed out with extraordinary rapidity. It happened most fortunately that the region had been photographed at the Harvard Observatory several times during the month of February, the last photograph having been taken on the 19th. The plate showed stars as faint as the eleventh magnitude. It must therefore have risen from some magnitude below the eleventh to the first within about three days. This difference corresponds to an increase of the light ten thousandfold.

Its spectrum shows the mixture of dark and bright bands characteristic of new stars. But, in the beginning, Campbell found that the sodium lines were faint and dark. He was thus enabled to determine the radial velocity of the star, which was six kilometres per second away from the sun.

Nova Persei, as the star will hereafter be called, is the brightest new star that has been recorded since the time of Kepler. But it is not impossible that, before the heavens were so carefully watched by observers, such an object might have reached an equal degree of brightness without exciting notice. The complete history of this star cannot yet be written, and there is no reason to suppose that it will differ very widely from that of Nova Aurigæ. Indeed on June 25, 1901 Professor Pickering reported that its spectrum had been gradually changing into that of a gaseous nebula.

CHAPTER IX

THE PARALLAXES OF THE STARS

> These mathematic men have thoughts that march
> From sphere to sphere and measure out the blue
> Of infinite space like roods of garden ground.
>
> BLACKIE.

IT needs only the most elementary conceptions of space, direction, and motion to see that, as the earth makes its vast swing from one extremity of its orbit to the other, the stars, being fixed, must have an apparent swing in the opposite direction. The seeming absence of such a swing was in all ages before our own one of the great stumbling-blocks of astronomy. It was the base on which Ptolemy erected his proof that the earth was immovable in the centre of the celestial sphere. It was felt by Copernicus to be a great difficulty in the reception of his system. It led Tycho Brahe to suggest a grotesque combination of the Ptolemaic and Copernican systems, in which the earth was the centre of motion, round which the sun revolved, carrying the planets with it.

With every improvement in their instruments, astronomers sought to detect the annual swing of the stars. Each time that increased accuracy in observations failed to show it, the difficulty in the way of the Copernican system was heightened. How deep the feeling on the subject is shown by the enthusiastic title, *Copernicus Triumphans*, given by Horrebow to the paper in which, from observations by Roemer, he claimed to have detected the swing. But, alas, critical examination showed that the supposed inequality was produced by the varying effect of the warmth of the day and the cold of the night upon the rate of the clock used by the observer, and not by the motion of the earth.

Hooke, a contemporary of Newton, published an attempt to determine the parallax of the stars, under the title *An Attempt to Prove the Motion of the Earth*, but his work was as great a failure as that of his predecessors. Had it not been that the proofs of the Copernican system had accumulated until they became irresistible, these repeated attempts might have led men to think that perhaps, after all, Ptolemy and the ancients were somehow in the right.

The difficulty was magnified by the philosophic views of the period. It was supposed that Nature must economise in the use of space as farmer would in the use of valuable land. The ancient astronomers correctly placed the sphere of the stars outside that of the planets, but did not suppose it far outside. That Nature would squander her resources by leaving a vacant space hundreds of thousands of times the extent of the solar system was supposed contrary to all probability. The actual infinity of space; the consideration, that one had only to enlarge his conceptions a little to see spaces a thousand times the size of the solar system look as insignificant as the region of a few yards round a grain of sand, does not seem to have occurred to anyone.

Considerations drawn from photometry were also lost sight of, because that art was still undeveloped. Kepler saw that the sun might well be of the nature of a star; in fact, that the stars were probably suns. Had he and his contemporaries known that the light of the sun was more than ten thousand million times that of a bright star, they would have seen that if placed at one hundred thousand times its present distance the sun would still shine as a bright star. If, then, the stars are as bright as the sun, they must be one hundred thousand times as far away, and their annual parallax would then have been too small for detection with the instruments of the time. Such considerations as this would have removed the real difficulty.

The efforts to discover stellar parallax were, of course, still continued. Bradley, about 1740, made observations on Gamma Draconis, which passed the meridian near his zenith, with an instrument of an accuracy before unequalled. He thus detected an annual swing of 20″ on each side of the mean. But this swing did not have the right phase to be due to the motion of the earth; the star appeared at one or the other extremity of its swing when it should have been at the middle point, and *vice versa*. What he saw was really the effect of aberration, depending on the ratio of the velocity of the earth in its orbit to the velocity of light. It proved the motion of the earth, but in a different way from what was expected. All that Bradley could prove was that the distances of the stars must be hundreds of thousands of times that of the sun.

An introductory remark on the use of the word parallax may preface a statement of the results of researches now to be considered.

In a general way, the change of apparent direction of an object arising from a change in the position of an observer is termed *parallax*. More especially, the parallax of a star is the difference of its direction as seen from the sun and from that point of the earth's orbit from which the apparent direction will be changed by the greatest amount. It is equal to the angle subtended by the radius of the earth's orbit, as seen from the star. The simplest conception of an arc of one second is reached by thinking of it as the angle subtended by a short line at a distance of 206,265 times its length. To say that a star has a parallax of 1″ would therefore be the same thing as saying that it was at a distance of 206,265 times that of the earth from the sun. A parallax of one-half a second implies a distance twice as great; one of one-third, three times as great. A parallax of 0.″20 implies a distance of more than a million times that of our unit of measure.

First Measures of Parallax.

The first conclusive result as to the extreme minuteness of the parallax of the brighter stars was reached by Struve, at Dorpat, about 1830. In the high latitude of Dorpat the right ascension of a star within 45° of the pole can be determined with great precision, not only at the moment of its transit over the meridian, but also at transit over the meridian below the pole, which occurs twelve hours later. He, therefore, selected a large group of stars which could be observed twice daily in this way at certain times of the year, and made continuous observations on them through the year. It was not possible, by this method, to certainly detect the parallax of any one star. What was aimed at was to determine the limit of the average parallax of all the stars thus observed. The conclusion reached was that this limit could not exceed one-tenth of a second and that the average distance of the group could not, therefore, be much less than two million times the distance of the sun. If, perchance, some stars were nearer than this, others were more distant.

By a singular coincidence, success in detecting stellar parallax was reached by three independent investigators almost at the same time, observing three different stars.

To Bessel is commonly assigned the credit of having first actually determined the parallax of a star with such certainty as to place the result beyond question. The star having the most rapid proper motion on the celestial sphere, so far as known to Bessel, was 61 Cygni, which is, however, only of the fifth magnitude. This rapid motion indicated that it was probably among the stars nearest to us, much nearer, in fact, than the faint stars by which it is surrounded.

After several futile attempts, he undertook a series of measurements, the best in his power to make, with a heliometer, in August, 1837, and continued them until October, 1838. The object was to determine, night after night, the position of 61 Cygni relative to certain small stars in its neighbourhood. Then he and his assistant, Sluter, made a second series, which was continued until 1840. All these observations showed conclusively that the star had a parallax of about 0″.35.

While Bessel was making these observations, Struve, at Dorpat, made a similar attempt upon Alpha Lyræ. This star, in the high northern latitude of Dorpat, could be accurately observed throughout almost the entire year. It is one of the brightest stars of the northern heavens and has a proper motion. There was, therefore, reason to believe it among the nearest of the stars. The observations of Struve extended from 1835 to August, 1838, and were, therefore, almost simultaneous with the observations made by Bessel on 61 Cygni. He concluded that the parallax of Alpha Lyræ was about one-fourth of a second. Subsequent investigations have, however, made it probable that this result was about double the true value of the parallax.

The third successful attempt was made by Henderson, of England, astronomer at the Cape of Good Hope. He found from meridian observations that the star Alpha Centauri had a parallax of about 1″. This is a double star of the first magnitude which, being only 30° from the south celestial pole, never rises in our latitudes. Its nearness to us was indicated not only by its magnitude, but also by its considerable proper motion. Although subsequent investigation has shown the parallax of this body to be less than that found by Henderson, it is, up to the time of writing, the nearest star whose distance has been ascertained.

The great difficulty of detecting an annual change in the direction of a star amounting to only a fraction of a second will be obvious to the reader. He will be still more impressed with it if, looking through a powerful telescope at any star, he sees how it flickers in consequence of the continual motions going on in the air through which it is seen, and considers how difficult it must be to fix any point of reference from which to measure the change of direction.

The latter is the capital difficulty in measuring the parallax. How shall we know that a star has changed its direction by a fraction of a second in the course of six months? There must be for this purpose some standard direction from which we can measure.

The most certain of these standard directions is that of the earth's axis of rotation. It is true that this direction varies in the course of the year, but the amount of the variation is known with great precision, so that it can be properly allowed for in the reduction of the observations. The angle between the direction of a star and that of the earth's axis, the latter direction being represented by the celestial pole, can be measured with our meridian instruments. It is, in fact, the north polar distance of the star, or the complement of its declination. If, therefore, the astronomer could measure the declination of a star with great precision throughout the entire year, he would be able to determine its parallax by a comparison of the measures. But it is found impossible in practice to make measures of so long an arc with the necessary precision. The uncertain and changing effect of the varying seasons and different temperatures of day and night upon the air and the instrument quite masks the parallax in all ordinary cases. After several attempts with the finest instruments, handled with the utmost skill, to determine stellar parallax from the declinations of the stars, the method has been practically abandoned.

Modern Methods.

The method now practised is that of relative parallax. By this method the standard direction is that of a small star apparently alongside one whose parallax is to be measured, but, presumably, so much farther away that it may be regarded as having no parallax. In this assumption lies the weak point of the method. Can we be sure that the smaller stars are really without appreciable parallax? The latest researches make it probable that we can. It is now considered quite safe to assume that the small stars without proper motion are so far away that their parallax is insensible.

Until recent times it was generally supposed that the magnitude of the stars afforded the best index to their relative distances. If the stars were of the same intrinsic brilliancy, the amount of light received from them would, as already pointed out, have been inversely as the square of the distance. Although there was no reason to suppose that any such equality really existed, it would still remain true that, in the general average, the brighter stars must be nearer to us than the fainter ones. But when the proper motions of stars came to be investigated, it was found that the amount of this motion afforded a better index to the distance than the magnitude did. The diversity of actual or linear motion is not so wide as that of absolute brilliancy. Stars have, therefore, in recent times, been selected for parallax very largely on account of their proper motion, without respect to their brightness.

Ever since the time of Bessel the experience of practical astronomers has tended toward the conclusion that the best instrument for delicate measurements like these is the heliometer. This is an equatorial telescope of which the object-glass is divided along a diameter into two semicircles, which can slide along each other. Each half of the object-glass forms a separate image of any star at which the telescope may be pointed. By sliding the two halves along each other, the images can be brought together or separated to any extent. If there are two stars in proximity, the image of one star made by one-half of the glass can be brought into coincidence with that of the other star made by the other half. The sliding of the two halves to bring about this coincidence affords a scale of measurement for the angular distance of the two stars.

The most noteworthy forward steps in improving the heliometer are due to the celebrated instrumentmakers of Hamburg, the Messrs. Repsold, aided by the suggestions of Dr. David Gill, astronomer at the Cape of Good Hope. The latter, in connection with his coadjutor, Elkin, made an equally important step in the art of managing the instrument and hence in determing the parallax of stars. The best results yet attained are those of these two observers, and of Peter, of Germany.

Yet more recently, Kapteyn, of Holland, has applied what has seemed to be the unpromising method of differences of right ascension observed with a meridian circle. This method has also been applied by Flint, at Madison, Wis. Through the skill of these observers, as well as that of Brünnow and Ball, in applying the equatorial telescope to the same purposes, the parallaxes of nearly one hundred stars have been measured with greater or less precision.

A rival method to that of the heliometer has been discovered in the photographic telescope. The plan of this instrument, and its application to such purposes as this, are extremely simple. We point a telescope at a star and set the clock-work going, so that the telescope shall remain pointed as exactly as possible in the direction of the star. We place a sensitised plate in the focus and leave it long enough to form an image both of the particular star in view and of all the stars around it. The plate being developed, we have a permanent record of the relative positions of the stars which can be measured with a suitable instrument at the observer's leisure. The advantage of the method consists in the great number of stars which may be examined for parallax, and in the rapidity with which the work can be done.

The earliest photographs which have been utilised in this way are those made by Rutherfurd in New York during the years 1860 to 1875. The plates taken by him have been measured and discussed principally by Rees and Jacoby, of Columbia University. Before their work was done, however, Pritchard, of Oxford, applied the method and published results in the case of a number of stars.

One of the pressing wants of astronomy at the present time is a parallactic survey of the heavens for the purpose of discovering all the stars whose parallax exceeds some definable limit, say $0''.1$. Such a survey is possible by photography, and by that only. A commencement, which may serve as an example of one way of conducting the survey, has been made by Kapteyn on photographic negatives taken by Donner at Helsingfors.

These plates cover a square in the Milky Way about two degrees on the side, extending from 34° 50′ in declination to 36° 50′, and from 20h. 1m. in R. A. to 20h. 10m. 24s. Three plates were used, on each of which the image of each star is formed twelve times. Three of the twelve impressions were made at the epoch of maximum parallactic displacement, six at the minimum six months later, and three at the following maximum. The parallaxes found on the plates can only be relative to the general mean of all the other stars, and must therefore be negative as often as positive. The following positive parallaxes, amounting to 0″.1, came out with some consistency from the measures:

Star, B. D., 3972	Mag. 8.6	R. A. 20h. 2m. 0s.	Dec. +35°.5	Par. +0″.11
Star, B. D., 3883	Mag. 7.1	R. A. 20h. 2m. 3s.	Dec. +36°.1	Par. +0″.18
Star, B. D., 4003	Mag. 9.2	R. A. 20h. 4m. 58s.	Dec. +35°.4	Par. +0″.10
Star, B. D., 3959	Mag. 7.0	R. A. 20h. 9m. 14s.	Dec. +36°.3	Par. +0″.10

Against these are to be set negative parallaxes of −0″.09, −0″.08, and several a little smaller, which are certainly unreal,

The presumption in favour of the actuality of one or more of the above positive values, which is created by their excess over the negative values, is offset by the following considerations: The area of the entire sky is more than 40,000 square degrees, or 10,000 times the area covered by the Helsingfors plates. We cannot well suppose that there are 1000 stars in the sky with a parallax of 0″.10 or more without violating all the probabilities of the case. The probabilities are therefore against even one star with such a parallax being found on those plates. Yet the cases of these four stars are worthy of further examination, if any of them are found to have a sensible proper motion.

On an entirely different plan is a survey recently concluded by Chase with the Yale heliometer. It includes such stars having an annual proper motion of 0″.50 or more as had not already been measured for parallax. The results, in statistical form, are these:

2 stars have parallaxes between + 0″.20 and + 0″.25.
6 stars have parallaxes between + 0″.15 and + 0″.20.
11 stars have parallaxes between + 0″.10 and + 0″.15.
24 stars have parallaxes between + 0″.05 and + 0″.10.
34 stars have parallaxes between 0″.00 and + 0″.05.

8 stars have parallaxes between $-0''.05$ and $0''.00$.
5 stars have parallaxes between $-0''.10$ and $-0''.05$.
2 stars have parallaxes between $-0''.15$ and $-0''.10$.

92, total number of stars.

It will be understood that the negative parallaxes found for fifteen of these stars are the result of errors of observation. Assuming that an equal number of the smaller positive values are due to the same cause, and substracting these thirty stars from the total number, we shall have sixty-two stars left of which the parallax is real and generally amounts to $0''.05$ more or less. The two values approximating to $0''.25$ seem open to little doubt. We might say the same of the six next in the list. The first two belong to the stars 54 Piscium and Weisse, 17h., 322.

A table of all the well-determined parallaxes of stars which the author has been able to find in astronomical literature will be found in the Appendix to the present work.

CHAPTER X

SYSTEMS OF STARS

> ——— and other suns perhaps,
> With their attendant moons thou wilt descry,
> Communicating male and female light,
> Which two great sexes animate the world.—MILTON.

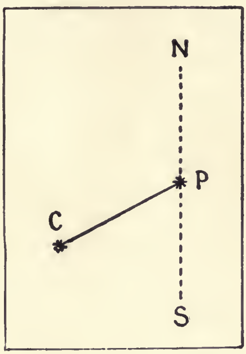
SIR WILLIAM HERSCHEL was the first to notice that many stars which, to the unaided vision, seemed single, were really composed of two stars in close proximity to each other. The first question to arise in such a case would be whether the proximity is real or whether it is only apparent, arising from the two stars being in the same line from our system. This question was speedily settled by more than one consideration. If there were no real connection between any two stars, the chances would be very much against their lying so nearly in the same line from us as they are seen to do in the case of double stars. Out of five thousand stars scattered at random over the celestial vault the chances would be against more than three or four being so close together that the naked eye could not separate them, and would be hundreds to one against any two being as close as the components of the closer double stars revealed by the telescope. The conclusion that the proximity is in nearly all cases real is also proved by the two stars of a pair moving together or revolving round each other.

Altogether there is no doubt that in the case of the brighter stars all that seem double in the telescope are really companions. But when we come to the thousands or millions of telescopic stars, there may be some cases in which the two stars of a pair have no real connection and are really at very different distances from us. The stars of such a pair are called "optically double." They have no especial interest for us and need not be further considered in the present work.

After Herschel, the first astronomer to search for double stars on a large scale was Wilhelm Struve, the celebrated astronomer of Dorpat. So thorough was his work in this field that he may fairly be regarded as the founder of a new branch of astronomy. Armed with what was, at that time (1815–35), a remarkable refracting telescope, he made a careful search of that part of the sky visible at Dorpat, with a view of discovering all the double stars within reach of his instrument. The angular distance apart of the components and the direction of the fainter from the brighter star were repeatedly measured with all attainable precision. The fine folio volume, *Mensuræ Micrometricæ*, in which his results were published and discussed, must long hold its place as a standard work of reference on the subject.

Struve had a host of worthy successors, of whom we can name only a few. Sir John Herschel was rather a contemporary than a successor. His most notable work on double stars was done during his expedition to the Cape of Good Hope, where he discovered a great number of these objects in the southern heavens with the great telescope at his command. Herschel, South, and Dawes, of England, were among the greatest English observers about the middle of the century. Otto Struve, son of Wilhelm, continued his father's work with zeal and success at Pulkowa. Later one of the most industrious observers was Dembowski, of Italy. During the last thirty years one of the most successful cultivators of double-star astronomy has been Burnham, of Chicago. He is to-day the leading authority on the subject. Enthusiasm, untiring industry, and wonderful keenness of vision have combined to secure him this position.

Let P be the principal star and C the companion. Let N S be a north and south line through P, or an arc of the celestial meridian, the direction N being north and S south from the star P.



Then, the angle N P C is called the *position-angle* of the pair. It is counted round the circle from 0° to 360°. The angle drawn in the figure is nearly 120°. Were the companion C in the direction S the position-angle would be 180°; to the right of P it would be 270°; to the right of N it would be between 270° and 360°.

The *distance* is the angle P C between the components which is expressed in seconds of arc.

The following definitions and explanations will be useful to the general reader. The two stars of a pair are called its *components*. The lesser is called the *companion* of the brighter. To *separate* a pair means to distinguish the two stars of the pair. The particulars which the careful observer of a double star should record are the position-angle and distance of the components and their respective magnitudes. To these Struve added their colours; but this has not generally been done.

We cannot set any well-defined limit to the range of distance. The general rule is that the greater the distance beyond a few seconds the less the interest that attaches to a double star, partly because the observation of distant pairs offers no difficulty, partly because of the increasing possibility that the components have no physical connection, and so form only an optically double star. With every increase of telescopic power so many closer and closer pairs are found that we cannot set any limit to the number of stars that may have companions. It is therefore to the closer pairs that the attention of astronomers is more especially directed.

The difficulty of seeing a star as double, or, in the familiar language of observers, of "separating" the components, arises from two sources, the proximity of the companion to the principal star, and the difference in magnitude between the two. It was only in rare cases that Struve could separate a pair as close as half a second. Now Burnham finds pairs whose distance is less than one-quarter of a second; indeed the limit of a tenth of a second is being approached. It goes without saying that a very minute companion to a bright star may, when the distance is small, be lost in the rays of its brighter neighbour. For all these reasons no estimate can be made of the actual number of double stars in the heavens. With every increase of telescopic power and observing skill more difficult pairs are being found, without any indication of a limit.

The great interest which attaches to double stars arises from the proof which they afford that the law of gravitation extends to the stars. Struve, by comparing his own observations with each other, or with those of Herschel, found that many of the pairs which he measured were in relative motion; the position-angle progressively changing from year to year, and sometimes the distance also. The lesser star was therefore revolving round the greater, or, to speak with more precision, both were revolving round their common centre of gravity. To such a pair the name *binary system* is now applied.

There can be no reasonable doubt that the two components of all physically connected double stars revolve round each other. If they did not their mutual gravitation would bring them together and fuse them into a single mass. We are therefore justified in considering all double stars as binary systems, except those which are merely optically double. For reasons already set forth, the pairs of the latter class which are near together must be very few in number; indeed, there are probably none among the close double stars whose brightest component can be seen optically by the naked eye.

The time of revolution of the binary systems is so long that there are only about fifty cases in which it has yet been determined with any certainty. Leaving out the "spectroscopic binaries," to be hereafter described, the shortest period yet fully established is eleven years. In only a small minority of cases is the period less than a century. In the large majority either no motion at all has yet been detected, or it is so slow as to indicate that the period must be several centuries, perhaps several thousand years.

There is great difficulty in determining the period with precision until the stars have been observed through nearly a revolution, owing to the number of elements, seven in all, that fix the orbit, and the difficulty of making the measures of position-angle and distance with precision. It thus happens that many of the orbits of binary systems which have been computed and published have no sound basis. Two cases in point may be mentioned.

The first-magnitude star Castor or Alpha Geminorum is seen to be double with quite a small telescope. The components are in relative motion. Owing to the interesting character of the pair it has been well observed, and a number of orbits have been computed. The periodic times found by the computers have a wide range. The fact is, nothing is known of the period except that it is to be measured by centuries, perhaps by thousands of years.

The history of 61 Cygni, a star ever memorable from being the first of which the parallax was determined, is quite similar. Although, since accurate observations have been made on it, the components have moved through an apparent angle of 30°, the observations barely suffice to show a very slight curvature in the path which the two bodies are describing round each other. Whether the period is to be measured by centuries or by thousands of years cannot be determined for many years to come.

In his work on the *Evolution of the Stellar Systems*, Prof. T. J. J. See has investigated the orbits of forty double stars having the shortest periods. There are twenty-eight periods of less than one hundred years.

In considering the orbits of binary systems we must distinguish between the actual and the apparent orbit. The former is the orbit as it would appear to an observer looking at it from a direction perpendicular to its plane. This orbit, like that of a planet or comet moving round the sun, is an ellipse, having the principal star in its focus. The point nearest the latter is called the periastron, or pericentre, and corresponds to the perihelion of a planetary orbit. The point most distant from the principal star is the apocentre. It is opposite the pericentre and corresponds to the aphelion of a planetary orbit. The law of motion is here the same as in the case of a body of the solar system; the radius vector joining the two bodies sweeps over equal areas in equal times.

The apparent orbit is the orbit as it appears to us. It differs from the actual orbit because we see it from a more or less oblique direction. In some cases the plane of the orbit passes near our system. Then to us the orbit will appear as a straight line and the small star will seem to swing from one side of the large one to the other like a pendulum, though the actual orbit may differ little from a circle. In some cases there may be two pericentres and two apocentres to the apparent orbit. This will be the case when a nearly circular orbit is seen at a considerable obliquity.

It is a remarkable and interesting fact that the law of areas holds good in the apparent as in the actual orbit. This is because all parts of the plane of the orbit are seen at the same angle, so that the obliquity of vision diminishes all the equal areas in the same proportion and thus leaves them equal.

Binary Systems of Sirius and Procyon.

The two most interesting binary systems are those of Sirius and Procyon. In the case of each the existence and orbit of the companion were inferred from the motions of the principal star before the companion had been seen. Before the middle of the century it was found that Sirius did not move with the uniform proper motion which characterises the stars in general; and the inequality of its motion was attributed to the attraction of an unseen satellite. Later Auwers, from an exhaustive investigation of all the observations of the star, placed the inequality beyond doubt and determined the elements of the orbit of the otherwise unknown satellite. Before his final work was published the satellite was discovered by Alvan G. Clark, of Cambridgeport, Mass., son and successor of the first and greatest American maker of telescopes. Additional interest was imparted to the discovery by the fact that it was made in testing a newly constructed telescope, the largest refractor that had been made up to that time. The discoverer was, at the time, unaware of the work of Peters and Auwers demonstrating the existence of the satellite. The latter was, however, in the direction predicted by Auwers, and a few years of observation showed that it was moving in fairly close accordance with the prediction.

The orbit as seen from the earth is very eccentric, the greatest distance of the satellite from the star being about ten seconds, the least less than three seconds. Owing to the brilliant light of Sirius the satellite is quite invisible, even in the most powerful telescopes, when nearest its primary. This was the case in the years 1890–92 and will again be the case about 1940, when another revolution will be completed.

The history of Procyon is remarkably similar. An inequality of its motion was suspected by Peters, but not proved. Auwers showed from observations that it described an orbit seemingly circular, having a radius of about 1″. There could be no doubt that this motion must be due to the revolution of a satellite, but the latter long evaded discovery, though carefully searched for with the new telescopes which were from time to time brought into use. At length in 1895 Schaeberle found the long-looked-for object with the

36-inch telescope of the Lick Observatory. It was nearly in the direction predicted by Auwers, and a year's observation by Schaeberle, Barnard, and others showed that it was revolving in accordance with the theory.

If the conclusion of Auwers that the apparent orbit of the principal star is circular were correct, the distance of the satellite should always be the same. It would then be equally easy to see at all times. The fact that neither Burnham nor Barnard ever succeeded in seeing the object with the Lick telescope would then be difficult to account for. The fact is, however, that the periodic motion of Procyon is so small that a considerable eccentricity might exist without being detected by observations. The probability is, therefore, that the apparent orbit is markedly eccentric and that the satellite was nearer the primary during the years 1878–92 than it was when discovered.

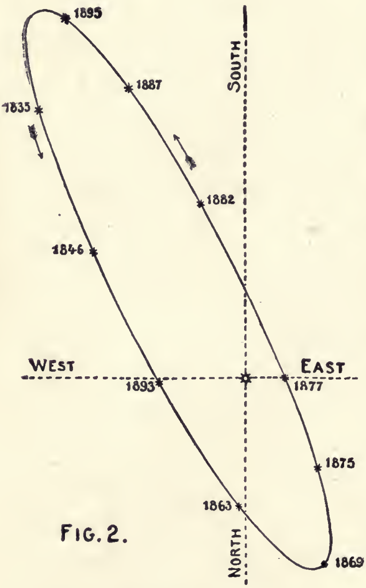


FIG. 2.

APPARENT ORBIT OF α CENTAURI, BY PROFESSOR SEE

One very curious feature, common to both of these systems, is that the mass of each satellite, as compared with that of its primary, is out of all proportion to its brightness. The remarkable conclusions to be drawn from this fact will be discussed in a subsequent chapter.

The system of Alpha Centauri is interesting from the shortness of the period, the brightness of the stars, and the fact that it is the nearest star to us, so far as known. We reproduce a diagram of the apparent orbit from Dr. See's work. The period of revolution found by Dr. See is eighty-one years. The major axis of the apparent orbit is 32″; the minor axis 6″.

Special interest attaches to binary systems of short period. Omitting Capella, which will be described later, it does not seem that a well-established period of less than eleven years is known, though several are suspected. Among the pairs of which the period of revolution is the shortest are these:

					YEARS
κ Pegasi:	R. A. = 21h. 40m.;	Dec. =	+ 25° 11′;	Period	= 11.42
δ Equulei:	" = 21h. 10m.;	"	+ 9° 37′;	"	= 11.45
β 883:	" = 4h. 45m.;	"	+ 10° 52′;	"	= 15.80
ξ Sagittarii:	" = 18h. 56m.;	"	− 30° 1′;	"	= 18.85
ρ Argus:	" = 7h. 47m.;	"	− 13° 38′;	"	= 22.00
85 Pegasi:	" = 23h. 57m.;	"	+ 26° 34′;	"	= 24.00

Shorter periods than these have been suspected in the cases of κ Pegasi and β 883. Dr. See considers that the period of β 883 is only five and one-half years, but the extreme difficulty of the observations still leaves room for question.

Triple and Multiple Systems.

Systems of three or more stars so close together that there must be a physical connection between them are quite numerous. There is every variety of such systems. Sometimes a small companion of a brighter star is found to be itself double. A curious case of this sort is that of Gamma Andromedæ. This object was observed and measured by Struve as an ordinary double star, of which the companion was much smaller than the principal star. Some years later Alvan Clark found that this companion was itself a close double star, of which the components, separated by about 1″, were nearly equal. Moreover, it was soon found that these components revolved round each other in a period not yet accurately determined, but probably less than a century. Thus we have a binary system revolving round a central star as the earth and moon revolve round the sun.

In most triple systems there is no such regularity as this. The magnitudes and relative positions of the components are so varied that no general description is possible. Stars of every degree of brightness are combined in every way. Observations on these systems extend over so short an interval that we have no data for determining the laws of motion that may prevail in any but one or two of the simplest cases. They are, in all probability, too complicated to admit of profitable mathematical investigation. There is, therefore, little more of interest to be said about them.

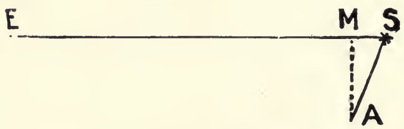
There is a very notable multiple system known as the Trapezium of Orion from the fact that it is composed of four stars. They are so close together as to appear like a single star to the naked eye, but may be well separated in the smallest telescope. There are also two other very faint stars, each of which seems to be a companion of one of the bright ones. This system is situated in the great nebula of Orion, to be described in the next chapter, a circumstance which has made it one of the most interesting objects to observers. No motion has yet been certainly detected among the components.

Spectroscopic Binary Systems.

Among the many striking results of recent astronomical research it would be difficult to name any more epoch-making than the discovery that great numbers of the stars have invisible dark bodies revolving round them of a mass comparable with their own. The existence of these revolving bodies is made known not only by their eclipsing the star, as explained in the chapter on Variable Stars, but by their producing a periodic change in the radial motion of the star. How this motion is determined by means of the spectroscope has been briefly set forth in a former chapter. As a general rule the motion is uniform in the case of each star. We have described in a former chapter the periodic character of the radial motion of Algol, discovered by Vogel. This was followed by the discovery that Alpha Virginis, though not variable, was affected by a similar inequality of the radial motion, having a period of four days and nineteen minutes. The velocity of the star in its apparent orbit is very great, —about ninety-one kilometres, or fifty-six English miles, per second. It follows that the radius of the orbit is some three million miles. The mass of the invisible companion must, therefore, be very great.

A new form of binary system was thus brought out, which, from the method of discovery, was called the spectroscopic binary system. But there is really no line to be drawn between these and other binary systems. We have seen that as telescopic power is increased, closer and closer binary systems are constantly being found. We naturally infer that there is no limit to the proximity of the pairs of stars of such systems, and that innumerable stars may have satellites, planets, or companion stars so close or so faint as to elude our powers of observation.

The actual orbit of such a system cannot be determined with the spectroscope, because only one component of the motion, that in the direction of the earth, can be observed. In the case of an orbit of which the plane was perpendicular to the line of sight from the earth to the star the spectroscope could give us no information as to the motion. The motion to or from the earth would be invariable. To show the result of the orbit's being seen obliquely, let E be the earth and A S be the plane of the orbit seen edgewise. Drop the perpendicular A M upon the line of sight. Then, while the star is moving from S to A the spectroscope will measure the motion as if it took place from S to M. Since S M is less than A S, the measured velocity will always be less than the actual velocity, except in the rare case when the motion of the star is directed toward the earth. Since the spectroscope can give us no information as to the inclination under which we see the orbit, it follows that the actual orbital velocities of the spectroscopic binaries must



RADIAL MOTION OF A BINARY SYSTEM

remain unknown. We can only say that they cannot be less, but may be greater to any extent, than that shown by our measures.

If the components of a spectroscopic binary system do not differ greatly in brightness, its character may be detected without actually measuring the radial velocities. Since the motion is shown by a displacement of the spectral lines, and since in any binary system the two components must always move in opposite directions, it follows that the displacements of the spectral lines of the two stars will be in opposite directions. Hence, when one of the stars, say A, is moving towards us, and the other, say D, from us, all the spectral lines common to the two will appear double, the lines made by A being displaced toward the blue end of the spectrum and those by B toward the red end. After half a revolution the motion will be reversed and the lines will again be double; only the lines of star A will now be on the red side of the others. Between these two phases will be one in which the radial velocities of the two stars are the same; the lines will then appear single.

The first star of which the binary character was detected in this way is Xi Ursæ Majoris. The discovery was made at the Harvard Observatory.

The perfection of the spectroscopic method is of so recent date that only binary systems of comparatively short period have so far been certainly detected. It is quite likely that nearly all double stars so bright that their spectrum can be accurately measured for the purpose of radial motion will eventually be investigated with the spectroscope. But, so far, there has been no time to determine an orbit of long period from the radial motion. There has therefore been a wide gap between the shortest period of a visual binary system and the longest of a spectroscopic binary.

Quite recently, however, this gap has been filled in a remarkable way. Early in 1900 it was found by Campbell, and independently by Newall, at Cambridge, that Capella was a spectroscopic binary in whose spectrum two types were superimposed. There was first the regular spectrum of the second type, offering a remarkable resemblance to that of our sun; superimposed on this was a second spectrum similar to that of Procyon. Between the lines of these two spectra a relative motion was found with a period of 104 days.

With the new 28-inch telescope of the Greenwich Observatory the observers have been able to see the duplicity of Capella and, measuring the position-angle from time to time, found a period substantially the same as that derived from the radial motions. The components were too close together to admit of their distance being accurately measured. The best estimates that could be made placed it at less than one tenth of a second, probably about 0″.08. This is about equal to the parallax of the star, as measured by Elkin. The two stars did not seem to differ much in brightness. The conclusion to be drawn is that the actual distance of the components is not very different from the distance between the earth and sun. The fact that they revolve in less than one-third the time that our earth does shows that the combined mass of the two bodies must be about ten times that of the sun.

It is very remarkable in this connection that the observations at Greenwich have not, so far, been confirmed at Mount Hamilton, where the telescope is more powerful and the conditions of seeing supposed to be of the best, nor at the Yerkes Observatory.

Star-clusters

A star-cluster is a bunch or collection of stars separated from the great mass of stars which stud the heavens. The Pleiades, or "Seven Stars" as they are familiarly called, form a cluster of which six of the components are easily seen by the naked eye while five others may be distinguished by a good eye without a telescope.

About 1780 Michell, of England, raised the question whether, supposing the stars visible to the naked eye to be scattered over the sky at random, there would be a reasonable possibility that those of the Pleiades would all fall within so small a space as that filled by the constellation. His correct conclusion was in the negative. It follows that this cluster does not consist of disconnected stars at various distances, which happen to be nearly in a line from our system, but is really a collection of stars by itself. Besides the stars visible to the naked eye, the Pleiades comprise a great number of telescopic stars, of which about sixty have been catalogued and their relative positions determined. The principal star of the cluster is Alcyone or Eta Tauri, which is of the third magnitude. The five which come next in the order of brightness are not very unequal, being all between the fourth and fifth magnitudes. Six are near the sixth magnitude. The remainder, so far as catalogued, range from the seventh to the ninth.

In this case there is a fairly good method of distinguishing between a star which belongs to the cluster and one which probably lies beyond it. This test is afforded by the proper motion. We have stated in Chapter VI that all the stars of the group have a common proper motion in the same direction. The amount of this motion is about 7″ per century. The first accurate measures made on the relative positions of the stars of the cluster were those of Bessel, about 1830. In recent years several observers have made yet more accurate determinations. The most thorough recent discussion is by Elkin. One result of his work is that there is as yet no certain evidence of any relative motion among the stars of the group. They all move on together with their common motion of 7″ per century, as if they were a single mass.

A closer cluster, which is plainly visible to the naked eye and looks like a cloudy patch of light, is Præsepe in the constellation Cancer. It is very well seen in the early evenings of winter and spring. Although there is nothing in the naked-eye view to suggest a star, it is found on telescopic examination that the individual stars do not fall far below the limit of visibility, several being of about the seventh magnitude.

Another notable cluster of the same general nature is that in Perseus. This constellation is situated in the Milky Way, not far from its region of nearest ap-

THE GREAT CLUSTER IN HERCULES, AS PHOTOGRAPHED WITH THE CROSSLEY REFLECTOR OF THE LICK OBSERVATORY

proach to the pole. In the figure of the constellation the cluster forms the handle of the hero's sword. It may be seen in the evening during almost any season except summer. To the naked eye it seems more diffused and star-like than Præsepe ; in fact, it has two distinct centres of condensation, so that it may be considered as a double cluster.

The two clusters last described may be resolved into stars with the smallest telescopes. But in the case of most of these objects the individual stars are so faint that the most powerful instruments scarcely suffice to bring them out. One of the most remarkable clusters in the northern heavens is that of Hercules. To the naked eye it is but a faint and insignificant patch, which would be noticed only by a careful observer, but in a large telescope it is seen to be one of the most interesting objects in the heavens. Near the border the individual stars can be readily distinguished, but they grow continually thicker toward the centre, where, even in a telescope of two feet aperture, the observer can see only a patch of light, which is, however, as he scans it, suggestive of the countless stars that must there be collected. By the aid of photography, Professor Pickering nearly succeeded in the complete resolution of this cluster, and Keeler was even more successful with the Crossley reflector of the Lick Observatory.

In many cases the central portions of these objects are so condensed that they cannot be visually resolved into their separate stars, even with the most powerful telescopes. A closer approach to complete resolution has been made by photography. We reproduce photographs of two noted clusters which show their appearance in a powerful telescope.

The cluster which, according to Pickering, may be called the finest in the sky, is Omega Centauri. It lies just within the border of the Milky Way, in right ascension 13h. 20.8m., and declination −46° 47′. There are no bright stars near. To the naked eye it appears as a hazy star of the fourth magnitude. Its actual extreme diameter is about 40′. The brightest individual stars within this region are between the eighth and ninth magnitudes. Over six thousand have been counted on one of the photographs, and the whole number is much greater. (See Figure on page 175.)

The most remarkable and suggestive feature of the principal clusters is the number of variable stars which they contain. This feature has been brought out by the photographs taken at the Harvard Observatory and at its branch station in Arequipa. The count of stars and the detection of the variables was very largely made by Professor Bailey, who for several years past has been in charge of the Arequipa station.

The results of his examination of the photographs are given in the table below.[1] In this table, the first number is that of the new general catalogue of Dreyer. The second column gives the usual designation of the cluster, generally its number in Messier's list. The next two columns give the position referred to the equinox of 1900. Next follows the approximate number of stars examined. The other columns are sufficiently explained by their headings.

[1] *Harvard College Observatory Circular No. 33.*

VARIABLE STARS IN CLUSTERS

DESIGNATION.	POSITION 1900 R. A.	DEC.	NO STARS EX-AMINED	AREA EX-AMINED.	NO. OF VAR.	PROPORTION. FRACT.	1 IN.
	h. m.	° ′		sq. min.			
104 47 Tucanæ	0 19.6	−72 38	2000	1257	6	.003	333
362	0 58.9	−71 23	675	314	14	.021	48
869	2 12.0	+56 41	1050	10800	1	.001	1050
884	2 15.4	+56 39					
1904 Messier 79	5 20.1	−24 37	200	79	5	.025	40
3293	10 29.6	−57 40	724	314	0	.000	——
4755 κ Crucis	12 47.7	−59 48	555	314	0	.000	——
5139 ω Centauri	13 20.8	−46 47	3000	1257	125	.042	24
5272 Messier 3	13 37.6	+28 53	900	1257	132	.147	7
5904 Messier 5	15 13.5	+ 2 27	900	1257	85	.094	11
5986	15 39.5	−37 26	289	314	1	.003	289
6093 Messier 80	16 11.1	−22 44	145	79	2	.014	72
6205 Messier 13	16 38.1	+36 39	1000	177	2	.002	500
6266 Messier 62	16 54.9	−29 58	960	218	26	.027	37
6397	17 32.5	−53 37	487	218	2	.004	244
6626 Messier 28	18 18.4	−24 55	900	314	9	.010	100
6656 Messier 22	18 30.3	−23 59	1550	218	16	.010	97
6723	18 52.8	−36 46	900	314	16	.018	56
6752	19 2.0	−60 8	600	218	1	.002	600
6809 Messier 55	19 33.7	−31 10	440	218	2	.005	220
7078 Messier 15	21 25.2	+11 44	900	1257	51	.057	18
7089 Messier 2	21 28.3	− 1 16	600	218	10	.017	60
7099 Messier 30	21 34.7	−23 38	275	218	3	.011	92
			19050	20380	509		

It will be seen from this table that the proportion of variables is very different in different clusters. In the double cluster 869–884, only one has been found among a thousand stars. The richest in variables is Messier 3, in which one variable has been detected among every seven stars. It might be suspected that the closer and more condensed the cluster the greater the proportion of variables. This, however, does not hold universally true. In the great cluster of Hercules only two variables are found among a thousand stars.

Very remarkable, at least in the case of Omega Centauri, is the shortness of the period of the variables. Out of 125 found, 98 have periods less than twenty-

THE CLUSTER ω CENTAURI, PHOTOGRAPHED BY GILL AT THE CAPE OBSERVATORY.

four hours. On the subject of the law of variation in these cases, Pickering says:

"The light curves of the ninety-eight stars whose periods are less than twenty-four hours may be divided into four classes. The first is well represented by No. 74. The period of this star is 12h. 4m. 3. and the range in brightness two magnitudes. Probably the change in brightness is continuous. The increase of light is very rapid, occupying not more than one-fifth of the whole period. In some cases, possibly in this star, the light remains constant for a short time at minimum. In most cases, however, the change in brightness seems to be continuous. The simple type shown by No. 74 is more prevalent in this cluster than any other. There are, nevertheless, several stars, as No. 7, where there is a more or less well marked secondary maximum. The period of this star is 2d. 11h. 51m. and the range in brightness one and a half magnitudes. The light curve is similar to that of well-known short-period variables, as Delta Cephei and Eta Aquilæ. Another class may be represented by No. 126, in which the range is less than a magnitude and the times of increase and decrease are about equal. The period is 8h. 12m. 3. No. 24 may perhaps be referred to as a fourth type. The range is about seven-tenths of a magnitude and the period is 11h. 5m. 7. Apparently about 65 per cent. of the whole period is occupied by the increase of the light. This very slow rate of increase is especially striking from the fact that in many cases in this cluster the increase is extremely rapid, probably not more than 10 per cent. of the whole period. In one case, No. 45, having a period of 14h. 8m., the rise from minimum to maximum, a change of two magnitudes, takes place in about one hour, and in certain cases, chiefly owing to the necessary duration of a photographic exposure, there is no proof at present that the rise is not much more rapid."

The periods of 63 of the 85 variables in Messier 5 have been determined by Professor Bailey. Their most remarkable feature is the approach of a majority of them to half a day. Of the number, 39, or more than three-fifths, are contained between the limits 10h. 48m. and 15h.

The regularity in the period of these stars is remarkable. Several have been studied during more than a thousand, and one during more than five thousand, periods without irregularities manifesting themselves.

It may be added that this regularity of the period, taken in connection with the case of Eta Aquilæ, already mentioned, affords a strong presumption that the variations in the light of these stars are in some way connected with the revolution of bodies round them, or of one star round another. Yet it is certain that the types are not of the Algol class and that the changes are not due merely to one star eclipsing another. That such condensed clusters should have a great number of close binary systems is natural, almost unavoidable, we might suppose. It is probable that among the stars in general, single stars are the exception rather than the rule. If such be the case, the rule should hold yet more strongly among the stars of a condensed cluster.

Perhaps the most important problem connected with clusters is the mutual gravitation of their component stars. Where thousands of stars are condensed into a space so small, what prevents them from all falling together into one confused mass ? Are they really doing so, and will they ultimately form a single body ? These are questions which can be satisfactorily answered only by centuries of observation ; they must, therefore, be left to the astronomers of the future.

CHAPTER XI

NEBULÆ

Some tumultuous cloud
Instinct with fire and nitre.
MILTON.

THE first nebula, properly so called, to be detected by an astronomical observer was that of Orion. Huyghens, in his *Systema Saturnium*, gives a rude drawing of this object, with the following description:

"There is one phenomenon among the fixed stars worthy of mention which, so far as I know, has hitherto been noticed by no one, and, indeed, cannot be well observed except with large telescopes. In the sword of Orion are three stars quite close together. In 1656, as I chanced to be viewing the middle one of these with the telescope, instead of a single star, twelve showed themselves (a not uncommon circumstance). Three of these almost touched each other, and, with four others, shone through a nebula, so that the space around them seemed far brighter than the rest of the heavens, which was entirely clear, and appeared quite black, the effect being that of an opening in the sky, through which a brighter region was visible."

For a century after Huyghens made this observation it does not appear that these objects received special attention from astronomers. The first to observe them systematically on a large scale was Sir

Wm. Herschel, whose vast researches naturally embraced them in their scope. His telescopes, large though they were, were not of good defining power and, in consequence, Herschel found it impossible to draw a certain line in all cases between nebulæ and clusters. At his time it was indeed a question whether all these bodies might not be clusters. This question Herschel, with his usual sagacity, correctly answered in the negative. Up to the time of the spectroscope, all that astronomers could do with nebulæ was to discover, catalogue, and describe them.

Several catalogues of these objects have been published. The one long established as a standard is the *General Catalogue of Nebulæ and Clusters*, by Sir John Herschel. With each object Herschel gave a condensed description. Recently Herschel's catalogue has been superseded by the general catalogue of Dreyer, based upon it and published in the Memoirs of the Royal Astronomical Society.

Some of the more conspicuous of these objects are worthy of being individually mentioned. At the head of all must be placed the great nebula of Orion. This is plainly visible to the naked eye and can be seen without difficulty whenever the constellation is visible. Note the three bright stars lying nearly in an east and west line and forming the belt of the warrior. South of these will be seen three fainter ones, hanging below the belt, as it were, and forming the sword. To a keen eye, which sharply defines the stars, the middle star will appear hazy. It is the nebula in question. Its character will be strongly brought out by the smallest telescope, even by an opera-glass. Drawings of it have been made by numerous astronomers, the comparison of which has given rise to the question whether the object is variable. It cannot be said that this question is yet decided; but the best opinion would probably be in the negative. In recent times the improvements of the photographic process

THE GREAT NEBULA OF ORION, AS PHOTOGRAPHED BY A. A. COMMON, F.R.S., WITH HIS FOUR-FOOT REFLECTOR

have led to the representation of the object by photography. A photograph made by Mr. A. A. Common, F.R.S., with a reflecting telescope, gives so excellent an impression of the object that by his consent we reproduce it.

The most remarkable feature connected with the nebula of Orion is the so-called Trapezium, already described. That these four stars form a system by themselves cannot be doubted. The darkness of the nebula immediately around them suggests that they were formed at the expense of the nebulous mass.

Great interest has recently been excited in the spiral form of certain nebulæ. The great spiral nebula M. 51 in Canes Venatici has long been known. We reproduce a photograph of this object and another. It is found by recent studies at the Lick Observatory that a spiral form can be detected in a great number of these objects by careful examination.

THE GREAT SPIRAL NEBULA M. 51, AS PHOTOGRAPHED WITH THE CROSSLEY REFLECTOR AT THE LICK OBSERVATORY

Another striking feature of numerous nebulæ is their varied and fantastic forms, of which we give a number of examples. The "Triphid nebula," figured in our frontispiece, is a noted one in this respect.

The great nebula of Andromeda is second only to that of Orion. It also is plainly visible to the naked eye and can be more readily recognised as a nebula

THE GREAT NEBULA OF ANDROMEDA, PHOTOGRAPHED BY DR. ISAAC ROBERTS, F.R.S.

than can the other. It has frequently been mistaken for a comet. Seen through a telescope of high power, its aspect is singular, as if a concealed light were seen shining through horn or semi-transparent glass.

Another nebula which, though not conspicuous to the naked eye, has attracted much attention from astronomers, is known, from the figure of one of its branches, as the Omega nebula. Sir John Herschel, who first described this object in detail, says of it: "The figure is nearly that of the Greek capital Omega, somewhat distorted and very unequally bright." From one base of the letter extends out to the east a long branch with a hook at the end, which in most of the drawings is more conspicuous than the portion included in the Omega. The drawings, however, vary so much that the question has been raised whether changes have not taken place in the object. As in other cases, this question is one which it is not yet possible to decide. The appearance of such objects varies so much with the aperture of the telescope and the conditions of vision that it is not easy to decide whether the apparent change may not be due to these causes. It is curious that in a recent photograph, the Omega element of it, if I may use the term, is far less conspicuous than in the older drawings, and is, in fact, scarcely recognisable.

Among the most curious of the nebulæ are the annular ones, which, as the term implies, have the form of a ring. It should be remarked that in such cases the interior of the ring is not generally entirely black, but is filled with nebulous light. We may, therefore, define these objects as nebulæ which are brighter round their circumference than in the centre. The most striking of the annular nebulæ is that of Lyra. It may easily be found from being situated about half-way between the stars Beta and Gamma. Although it is visible in a medium telescope, it requires a powerful one to bring out its peculiar features in a striking way. Recently it has been photographed by Keeler with the Crossley reflector of the Lick Observatory, who found that the best general impression was made with an exposure of only ten minutes.

The ring, as shown by Keeler's photograph, has a quite complicated structure. It seems to be made up of several narrower bright rings, interlacing somewhat irregularly, the spaces between them being filled with fainter nebulosity. One of these rings forms the outer boundary of the preceding end of the main ring. Sweeping around to the north end of the minor axis, it becomes very bright, perhaps by superposition on the broader main ring of the nebula at this place. It crosses this ring obliquely, forming the brightest part of the whole nebula, and then forms the inner boundary of the main ellipse toward its following end. The remaining part of the ring is not so easily traced, as several other rings interlace on the south end of the ellipse.

The central star of this nebula has excited some interest. Its light seems to have a special actinic power, as the star is more conspicuous on the photographs than to the eye.

There are several other annular nebulæ which are fainter than that of Lyra. The one best visible in our latitudes is known as H IV. 13, or 4565 of Dreyer's catalogue. It is situated in the constellation Cygnus, which adjoins Lyra. Both Herschel and Lord Rosse have made drawings of it. It was photographed by Keeler with the Crossley reflector on the nights of August 9 and 10, 1899, with exposures of one and two hours, respectively. Keeler states that the nebula, as shown by these photographs, "is an elliptical, nearly circular ring, not quite regular in outline, pretty sharply defined at the outer edge." The outside dimensions are:

Major axis	42″.5
Minor axis	40.″5
Position angle of major axis	32°.

The nebula has a nucleus with a star exactly in the center. This is very conspicuous on a photograph, but barely if at all visible with a 36-inch reflector.

Another curious class of nebulæ are designated as *planetary*, on account of their form. These consist of minute, round disks of light, having somewhat the appearance of a planet. The appellation was suggested by this appearance. These objects are for the most part faint and difficult.

It is impossible to estimate the number of nebulæ in the heavens. New ones have from time to time been discovered, located, and described by many observers during the last thirty years. Among these

Lewis Swift is worthy of special mention as one of the most successful discoverers of these objects.

NEBULOUS MASS IN CYGNUS, INCLUDING H. V. 14 AND H. 2093, PHOTOGRAPHED AT THE LICK OBSERVATORY

But in recent times photography has gone far toward replacing the eye in this field. On photographing the sky near the galactic pole with the Crossley reflector, Keeler found no less than seven of these objects in a space of about one-half a square degree. He therefore estimates the whole number in the heavens capable of being photographed at several hundred thousand. It may be assumed that only a moderate fraction of these are visible to the eye, even aided by the largest telescopes.

Among the most singular of these objects are large diffused nebulæ, sometimes extending through a region of several degrees. A number of these were discovered by Herschel. Barnard, W. H. Pickering, and others have photographed these for us. One of the most remarkable of them winds around in the constellation Orion in such a way that at first sight one might be disposed to inquire whether the impression on the photographic plate might not have been the result of some defect in the apparatus or some reflection of the light of the neighbouring stars, which is so apt to occur in these delicate photographic operations. But its existence happens to be completely confirmed by independent testimony. It was first detected by W. H. Pickering and afterwards independently by Barnard.

A curious fact connected with the distribution of nebulæ over the sky is that it is in a certain sense the reverse of that of the stars. The latter are vastly more numerous in the regions near the Milky Way and fewer in number near the poles of that belt. But the reverse is the case with the nebulæ proper. They are least numerous in the Milky Way and increase in number as we go from it in either direction. Precisely what this signifies one would not at the present time be able to say. Perhaps the most obvious suggestion would be that in these two opposite nebulous regions the nebulæ have not yet condensed into stars. This, however, would be a purely speculative explanation.

On the other hand, star-clusters are more numerous in the galactic region. This, however, is little more than saying that in the regions where the stars are so much more numerous than elsewhere many of them naturally tend to collect in clusters. It is, however, a curious fact that, so far as has yet been noticed, the large diffused nebulæ which we have mentioned are more numerous in or near the Milky Way. If this tendency is established it will mark a curious distinction between them and the smaller nebulæ.

The most interesting question connected with these objects is that of their physical constitution. When, about 1866, the spectroscope was first applied to astronomical investigation by Huggins he found that the light of the great nebula of Orion formed a spectrum of bright lines, thus showing the object to be gaseous. This was soon found to be true of the nebulæ generally. There is, however, a very curious exception in the case of the great nebula of Andromeda. This object gives a more or less continuous spectrum. The bright lines in the spectrum of a nebula are seldom or never more than four in number. The wave-lengths are 4341, 4861, 4957, 5004. The first of these is the violet, is very faint, and visible only in the brightest nebulæ. The last is the brightest, and in faint nebulæ is the only one that can be distinguished. None of these lines can be certainly identified with those of any terrestrial substance.

The supposed matter which produces them has, therefore, been called *nebulum*.

Beyond the general fact that the light of a nebula does not come from solid matter, but from matter of a gaseous or other attenuated form, we have no certain knowledge of the physical constitution of these bodies. Certain features of their constitution can, however, be established with a fair approach to accuracy. Not only the spectroscopic evidence of bright lines but the aspect of the objects themselves show that they are transparent through and through. This is remarkable when taken in connection with their inconceivable size. Leaving out the large diffused nebulæ which we have mentioned, these objects are frequently several minutes in diameter. Of their distance we know nothing, except that they are probably situated in the distant stellar regions. Their parallax can be but a small fraction of a second. We shall probably err greatly in excess if we assume that it varies between one-hundredth and one-tenth of a second. To assign this parallax is the same thing as saying that at the distance of the nebulæ the dimensions of the earth's orbit would show a diameter which might range between one-fiftieth and one-fifth of a second, while that of Neptune would be more or less than one second. Great numbers of these objects are, therefore, thousands of times the dimensions of the earth's orbit, and probably most of them are thousands of times the dimensions of the whole solar system. That they should be completely transparent through such enormous dimensions shows their extreme tenuity. Were our solar system placed in the midst of one of them, it is probable that we should not be able to find any evidence of its existence.

A form of matter so different from any that can be found or produced on the surface of the earth can hardly be explained by our ordinary views of matter. A theory has, however, been propounded by Sir Norman Lockyer, so ingenious as to be at least worthy of mention. It is that these objects are vast collections of meteorites in rapid motion relatively to each other, which come into constant collision. Their velocity is such that at each collision heat and light are produced. In the language of our progenitors, who in the absence of matches used flint and steel, they "strike fire" against each other. The idea of such a process originated with Prof. P. G. Tait, in an attempt to explain the tail of a comet, but it was elaborated and developed by Mr. Lockyer in his work on the *Meteoritic Theory*.

The objections to this theory seem insuperable. A velocity so great, at such a distance from the centre of the nebulæ, would be incompatible with the extreme tenuity of these objects. Every time that two meteors came into collision they would lose velocity, and, therefore, if the mass was sufficient to hold them from flying through space, would rapidly fall toward a common centre. The amount of light produced by the collision of two such objects is only a minute fraction of the energy lost. The meteors which fall on the earth are mostly of iron, and, were the theory true, numerous lines of iron should be most conspicuous in the spectrum.

CHAPTER XII

CONSTITUTION OF THE STARS

Doubt thou the stars are fire.—SHAKESPEARE.

THE spectroscope shows that, although the constitution of the stars offers an infinite variety of detail, we may say, in a general way, that these bodies are suns. It would, perhaps, be more correct to say that the sun is one of the stars and does not differ essentially from them in its constitution. The problems of the physical constitution of the sun and stars may, therefore, be regarded as one, all these being bodies of the same general nature, consisting of vast masses of incandescent matter at so exalted a temperature as to shine by their own light.

Diversities among the Stars.

This similarity in general constitution does not, however, preclude very great differences in detail. The spectra of the stars show that hardly any two are exactly alike in the substances of which they are composed, and in the temperature and density of these substances. Most remarkable is the diversity of their actual luminosities or the amount of light and heat which they individually emit. The whole tendency of recent research has been to accentuate this diversity. It was once supposed that the brighter stars must all be among the nearer ones to us. But as parallaxes were measured with greater and greater accuracy, it became more and more certain that this is not always the case.

The last step in this direction has been taken by Gill in his measures of the parallaxes of the southern stars of the first magnitude. Of two at least, Canopus and Rigel, the parallaxes are so small as to elude certain detection. Most extraordinary is the case of Canopus, the second brightest star in the heavens. A long-continued series of measures, sufficient to make evident a parallax of one hundredth of a second, converged to a value of $0''.000$! Canopus is doubtless situated among the small stars of the eighth magnitude around it, of which we have every reason to believe the parallax to be only a few thousandths of a second. In all likelihood, it is more than ten thousand times as bright as the sun. A planet as near to it as we are to the sun would become red hot under its radiation.

At the other extreme we have the minute stars of large proper motion whose parallaxes have been measured. These seem to be of only about one-fiftieth the brightness of the sun. It therefore seems certain that some stars emit hundreds of thousands—nay, millions of times as much light as others.

Masses and Densities of the Stars.

It has long been known that the mean density of the sun is only one-fourth that of the earth, and, therefore, less than half as much again as that of water. In a few cases an approximate estimate of the density of stars may be made. The method by which this is done can be rigorously set forth only by the use of algebraic formulæ, but a general idea of it can be obtained without the use of that mode of expression.

Let us set forth in advance an extension of Kepler's third law, which applies to every case of two bodies revolving around each other by their mutual gravitation The law in question, as stated by Kepler, is that the cubes of the mean distances of the planets are proportional to the squares of their times of revolution. If we suppose the mean distances to be expressed in terms of the earth's mean distance from the sun as a unit of length, and if we take the year as the unit of time, then the law may be expressed by saying that the cubes of the mean distances will be equal to the squares of the periods. For example, the mean distance of Jupiter is thus expressed as 5.2. If we take the cube of this, which is about 140, and then extract the square root of it, we shall have 11.8, which is the period of revolution of Jupiter around the sun expressed in years. If we cube 9.5, the mean distance of Saturn, we shall have the square of a little more than 29, which is Saturn's time of revolution.

We may also express the law by saying that if we divide the cube of the mean distance of any planet by the square of its periodic time we shall always get 1 as a quotient.

The theory of gravitation and the elementary principles of force and motion show that a similar rule is true in the case of any two bodies revolving around each other in virtue of their mutual gravitation. If we divide the cube of their mean distance apart by the square of their time of revolution, we shall get a quotient which will not indeed be 1, but which will be a number expressing the combined mass of the two bodies. If one body is so small that we leave its mass out of consideration, then the quotient will express the mass of the larger body. If the latter has several minute satellites moving around it, the quotients will be equal, as in the case of the sun, and will express the mass of this central body. If, as in the case we have supposed, we take the year as a unit of time and the distance of the earth from the sun as a unit of length, the quotient will express the mass of the central body in terms of the mass of the sun. It is thus that the masses of the planets are determined from the periodic times and distances of their satellites, and the masses of binary systems from their mean distance apart and their periods. To express the general law by a formula we put

a, the mean distance apart of the two bodies, or the semi-major axis of their relative orbit in terms of the earth's mean distance from the sun;

P, their periodic time;

M, their combined mass in terms of the sun's mass as unity.

Then we shall have:

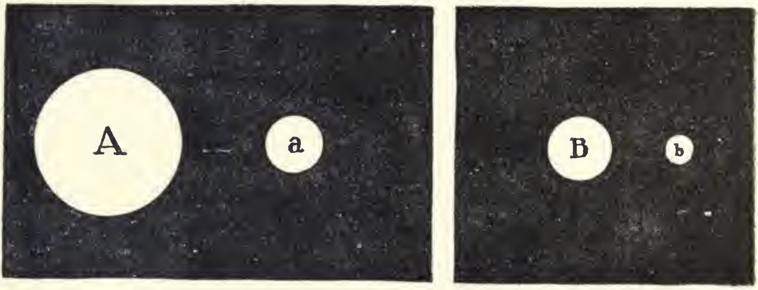
$$M = \frac{a^3}{P^2}$$

Another conclusion we draw is that if we know the time of revolution and the radius of the orbit of any binary system, we can determine what the time of revolution would be if the radius of the orbit had some standard length, say unity. To do this we have only to divide the actual period by the cube of the square root of the actual radius of the orbit.

We cannot determine the dimensions of a binary system unless we know its distance from us. But there is a remarkable law which, so far as I know, was first announced by Pickering, by virtue of which we can determine a certain relation between the surface brilliancy and the density of a binary system without knowing its distance.

Let us suppose a number of bodies of the same constitution and temperature as the sun — models of the latter we may say—differing from it only in size. To fix the ideas, we shall suppose two such bodies, one having twice the diameter of the other. Being of the same brilliancy, we suppose them to emit the same amount of light per unit of surface. The larger body, having four times the surface of the smaller, will then emit four times as much light. The volumes being proportional to the cubes of their diameters, it will have eight times its volume. The densities being supposed equal, it will have eight times the mass.

Suppose that each has a satellite revolving around it, of which the size is proportional to that of its primary, as shown in the figure, and that the orbit of the satellite of the larger body is twice the radius of that of the smaller one. Calling the radius of the nearer satellite 1, that of the more distant one will then be 2 The cube of this number is 8. It follows from the extension of Kepler's third law, which we have cited that the times of revolution of the two satellities will be the same. Thus the two bodies, A and B, with their satellites, a and b, form two binary systems whose proportions and whose periods are the same, only the linear dimensions of B are all double those of A. In other words, we shall have a pair of binary systems which will look alike in every respect, only one will have double the dimensions and eight times the mass of the other.



TWO BINARY SYSTEMS ON THE SAME MODEL, ONE HAVING TWICE THE LINEAR DIMENSIONS OF THE OTHER

Now, let us suppose the larger system to be placed twice as far away from us as the smaller. The two will then appear of the same size, and, if stars, will appear of the same brightness, while the two orbits will have the same apparent dimensions. In a word, the two systems will appear alike when examined with the telescope, and the periodic times will be equal.

Near the end of the second chapter we have given a little table showing the magnitude that the sun would appear to us to have were it placed at different distances among the stars. The parallaxes we have there given are simply the apparent angles which would be subtended by the radius of the earth's orbit at different distances. It follows that, were the stars all of similar constitution to the sun, the numbers given in the last column of the table referred to would, in all cases, express the apparent distance from the star of a companion, having a time of revolution of one year. From this we may easily show what would be the time of revolution of any binary system of which the companions were separated by 1″, if the stars were of the same constitution as the sun.

Periods of binary systems whose components are separated by 1″ and whose constitution is the same as that of the sun.

Mag.	Period, Years.	Annual Motion.
1	1.8	200°
2	3.5	102
3	7.0	51
4	14.1	25
5	28.1	13
6	56.0	6
7	112.	3.2
8	223.	1.6

It will be seen that the periods are very nearly doubled for each diminution of the brilliancy of the star by one magnitude. Moreover, the value of the photometric ratio for two consecutive magnitudes is a little uncertain, so that we may, without adding to the error of our results, suppose the period to be exactly doubled for each addition of unity to the magnitude. A computation of the period for any magnitude, m, may be made with all necessary precision by the formula :

$$P = 0^{y}.88 \times 2m;$$

$$\text{or,} \qquad \log. P = 9.944 + 0.3m.$$

It will now be of interest to compare the results of this theory with the observed periods of binary systems with a view to comparing their constitution with that of our sun. There are, however, two difficulties in the way of doing this with precision.

The first difficulty is that there are very few binary systems of which the apparent dimensions of the orbits and the periods are known with any approach to exactness. This would not be a serious matter were it not that the systems of short, and, therefore, known, periods belong to a special class, that having the greatest density. Hence, when we derive our results from such systems we shall be making a biassed selection from this particular class of stars.

The next difficulty is that the theory which we have set forth assumes the mass of the satellite either to be very small compared with that of the star, or the two bodies to be of the same constitution. If we apply the theory to systems in which this is not the case, the results which we shall get will be, in a certain way, those corresponding to the mean of the two components. Were it a question of masses, we should get with entire precision the sum of the masses of the two bodies. The best we can do, therefore, is to suppose the two companions fused into one having the combined brilliancy of the two. Then, if the result is too small for one, it will be too large for the other.

To show the method of proceeding, I have taken the six systems of shortest period found in Dr. See's *Researches on Stellar Evolution.* The principal numbers are shown in the table below.

The first column, a'', after the name of the star, gives the apparent semi-major axis of the orbit in seconds of arc. The next column gives the period in years. Column Mag. gives the apparent magnitude which the system would have were the two bodies fused into one. Column P′ gives the period in years as it would be were the radius of the orbit equal to one second. It is formed by dividing the actual period by $a''^{\frac{3}{2}}$. The next column gives the period as it would be were the stars of similar constitution to the sun. The last column gives the square of the ratio of the two periods, which, if the stars had the same surface brilliancy as the sun, would express the ratio of density of the stars to that of the sun. Actually, it gives the product: Density × brilliancy $^{\frac{3}{2}}$.

	A″	PER.	MAG.	P′	SUN'S PER.	STAR'S DENSITY.
	″	Years.		Years.	Years.	
κ Pegasi	0 .42	11.4	4.2	41.9	16.2	0.15
ξ Equulei	0 .45	11.4	4.6	37.8	21.0	0.31
ξ Sagittarii	0 .69	18.8	2.9	32.7	6.7	0.04
F9 Argus	0 .65	22.0	5.5	42.0	39.7	0.90
42 Comæ	0 .64	25.6	4.4	50.0	18.5	0.14
β Delphini	0 .67	27.7	3.7	50.4	11.4	0.51

The numbers in the last column being all less than unity, it follows that either these stars are much less dense than the sun or they are of much greater surface brilliancy. Moreover, these stars belong to a selected list in which the numbers of the last column are larger than the average.

To form some idea of the result of a selection from the stars in general, we may assume that the average of all the measured distances between the components of a number of binary systems is equal to the average radius of their orbits, and that the observed annual motion is equal to the mean motion of the companion in its orbit. Taking a number of cases of this sort, I find that the number corresponding to the last number of the preceding table would be little more than one-thousandth.

A very remarkable case is that of Zeta Orionis. This star, in the belt of Orion, is of the second magnitude. It has a minute companion at a distance of 2″.5. Were it a model of the sun, a companion at this apparent distance should perform its revolution in fourteen years. But, as a matter of fact, the motion is so slow that even now, after fifty years of observation, it cannot be determined with any precision. It is probably less than 0°.1 in a year. The number expressing the comparison of the density and surface brilliancy of this star with those of the sun is probably less than .0001.

The general conclusion to be drawn is obvious. The stars in general are not models of our sun, but have a much smaller mass in proportion to the light they give than our sun has. They must, therefore, have either a less density or a greater surface brilliancy.

We may now inquire whether such extreme differences of surface brilliancy or of density are more likely. The brilliancy of a star depends primarily, not on its temperature throughout, but on that of some region near or upon its surface. The temperature of this surface cannot be kept up except by continual convection currents from the interior to the surface. We are, therefore, to regard the amount of light emitted by a star not merely as indicating temperature, but as limited by the quantity of matter which, impeded by friction, can come up to the surface, and there cool off and afterwards sink down again. This again depends very largely on internal friction, and is limited by that. Owing to this limitation, we cannot attribute the difference in question wholly to surface brilliancy. We must conclude that at least the brighter stars are, in general, composed of matter much less dense than that of the sun. Many of them are probably even less dense than air and in nearly all cases the density is far less than that of any known liquid.

An ingenious application of the mechanical principle we have laid down has been made independently by Mr. A. W. Roberts, of South Africa, and Mr. H. N. Russell, of Princeton, in another way. If we only knew the relation between the diameters of the two companions of a binary system, and its dimensions, we could decide how much of the difference in question is due to density and how much to surface brilliancy. Now this may be approximately done in the case of variable stars of the Algol and Beta Lyræ types. If, as is probably the most common case, the passage of the stars over each other is nearly central, the ratio of their diameter to the radius of the orbit may be determined by comparing the duration of the eclipse with the time of revolution. This was one of the fundamental data used by Myers in his work on Beta Lyræ, of which we have quoted the results. Without going into reasoning or technical details at length, we may give the results reached by Roberts and Russell in the case of the Algol variables.

For the variable star X Carinæ, Roberts finds, as a superior limit for the density of the star and its companion, one-fourth the density of the sun. It may be less than this is, to any extent.

In the case of S Velorum the superior limits of density are:

Bright star..................................	0.61
Companion..................................	0.03

In the case of RS Sagittarii the upper limits of density are 0.16 and 0.21.

It is possible, in the mean of a number of cases like these, to estimate the general average amount by which the densities fall below the limits here given. Roberts's final conclusion is that the average density of the Algol variables and their eclipsing companions is about one-eighth that of the sun.

The work of Russell was carried through at the same time as that of Roberts, and quite independently of his. It appeared at the same time.[1] His formulæ and methods were different, though they

[1] *Astrophysical Journal*, vol. x, no. 5.

rested on similar fundamental principles. Taking the density of the sun as unity, he computes the superior limit of density for 12 variables, based on their periods and the duration of their partial eclipses. The greatest limit is in the case of Z Herculis and is 0.728. The least is in the case of S Cancri and is 0.035. The average is about 0.2. As the actual density may be less than the limit by an indefinite amount, the general conclusion from his work may be regarded as the same with that from the work of Roberts.

The results of the preceding theory are independent of the parallax of the stars. They, therefore, give us no knowledge as to the mass of a binary system. To determine this we must know its parallax, from which we can determine the actual dimensions of the orbit when its apparent dimensions are known. Then the formula already given will give the actual mass of the system in terms of the sun's mass.

There are only six binary systems of which both the orbit and the parallax are known. These are shown in the table below. Here the first two columns after the stars named give the semi-major axis of the orbit and the measured parallax. The quotient of the first number by the second is the actual mean radius of the orbits in terms of the earth's distance from the sun as unity. This is given in the third column, after which follow the period and the resulting combined mass of the system. The last column shows the actual amount of light emitted by the system, compared with that emitted by the sun.

	A.″	PAR.	*A.*	PERIOD.	MASS.	LIGHT.
	″	″		y.		
η Cassiopiæ......	8.21	0.20	41.0	195.8	1.8	1.0
Sirius...........	8.03	0.37	21.7	52.2	3.7	32.0
Procyon.........	3.00	0.30	10.0	40.0	0.6	8.5
α Centauri.......	17.70	0.75	23.6	81.1	2.0	1.7
70 Ophiuchi.....	4.55	0.19	24.0	88.4	1.8	0.7
85 Pegasi........	0.89	0.05	17.8	24.0	9.0	2.2

Even in these few cases some of the numbers on which the result depends are extremely uncertain. In the case of Procyon, the radius of the orbit can be only a rough estimate. In the case of 85 Pegasi the parallax is uncertain. In the case of Eta Cassiopiæ the elements are still doubtful.

So far as we have set forth the principles involved in the question, we do not get separate results for the mass of each body. The latter can be determined only by meridian observations, showing the motion of the brighter star around the common centre of gravity of the two. This result has thus far been worked out with an approximation to exactness only in the cases of Sirius and Procyon. For these systems we have the following masses of the companions of these bodies in terms of the sun's mass:

Companion of Sirius.........................	1.2
Companion of Procyon.......................	0.2

It will now be interesting to compare the brightness of these bodies with that which the sun would have if seen at their distance. In a former chapter we showed how this could be done. The results are:

At the distance of Procyon the apparent magnitude of the sun would be $2^m.8$. At the distance of Sirius, it would be $2^m.3$. Supposing the sun to be changed in size, its density remaining unchanged, until it had the same mass as the respective companions of Procyon and Sirius, its magnitudes would be:

For companion of Procyon....................	3.9
For companion of Sirius......................	2.9

These numbers are the magnitudes the companions would show were they models of our sun. Their actual magnitudes cannot be estimated with great precision, owing to the effect of the brilliancy of the star. From the estimate of the companion of Sirius, by Professor Pickering, its magnitude was about the eighth. It is probable that the magnitude of the companion of Procyon is not very different. It will be seen that these magnitudes are very different from those which they would have were they models of the sun. What is very curious is that they differ in the opposite direction from the stars in general, and especially from their primaries. Either they have a far less surface brilliancy than the sun or their density is much greater. There can be no doubt that the former rather than the latter is the case.

This great mass of the two companions as compared with their brilliancy suggests the question whether they may not shine, in part at least, by the light of their primaries. A very little consideration will show that this cannot be the case. To shine as brightly as it does by reflected light, the diameter of the companion of Sirius would have to be enormous, at least one-thirtieth its distance from Sirius. Moreover, its apparent brightness would vary so widely in different parts of its orbit that we should see it almost as well when near Sirius as when distant from it. The most likely cause of the great dimness is the low temperature of the bodies.

Gaseous Constitution of the Stars.

All these results point to the conclusion that the stars, or at least the brighter among them, are masses of gas, enormously compressed in their interior by the gravitation of their outer parts. We have now to show how this result was arrived at, at least in the case of the sun, from different considerations, before the spectroscope had taught us anything of the constitution of these bodies.

We must accept, as one of the obvious conclusions of modern science, the fact that the sun and stars have, for untold millions of years, been radiating heat into space. We refrain from considering the basis on which this conclusion rests, not so much because it must be considered unquestionable, as because the discussion would be too long and complex for the present work.

One of the great problems of modern science has been to ascertain the source of this heat. Before the theory of energy was developed this problem suggested no difficulty. In the time of Newton, Kant and even of La Place and Herschel, no reason was known why the stars should not shine forever without change. Now we know that when a body radiates heat, that heat is really an entity termed *energy*, of which the supply is necessarily limited. Kelvin compared the case of a star radiating heat to that of a ship of war belching forth shells from her batteries. We know that if the firing is kept up, the supply of ammunition must at some time be exhausted. Have we any means of determining how long the store of energy in sun or star will suffice for its radiation?

We know that the substances which mainly compose the sun and stars are similar to those which compose our earth. We know the capacity for heat of these substances, and we also have determined how much heat the sun radiates annually. From these data, it is found by a simple calculation that the temperature of the sun would be lowered annually by more than two degrees Fahrenheit, if its capacity for heat were the same as that of water. If this capacity were only that of the substances which compose the great body of the earth, the lowering of temperature would be from 5° to 10° annually. Evidently, therefore, the actual heat of the sun would only suffice for a few thousand years' radiation, if not in some way replenished.

When the difficulty was first attacked, it was supposed that the supply might be kept up by meteors falling into the sun. We know that in the region round the sun, and, in fact, in the whole solar system, are countless minute meteors, some of which may from time to time strike the sun. The amount of heat that would be produced by the loss of energy suffered by a meteor moving many hundred miles a second would be enormously greater than that which would be produced by combustion. But critical examination shows that this theory cannot have any possible basis. Apart from the fact that it could at best be only a temporary device, there seems to be no possibility that meteors sufficient in mass can move round the sun or fall into it. Shooting stars show that our earth encounters millions of little meteors every day; but the heat produced by the collisions is absolutely insignificant.

It was then shown by Kelvin and Helmholtz that the sun might radiate the present amount of heat for several millions of years simply from the fund of energy collected by the contraction of its volume through the mutual gravitation of its parts. As the sun cools it contracts; the fall of its substance toward the centre, produced by this contraction, generates energy, which energy is constantly turned into heat. The amount of contraction necessary to keep up the present supply may be roughly computed; it amounts in round numbers to 220 feet a year, or four miles in a century.

Accepting this view, it will almost necessarily follow that the great body of the sun must be of gaseous constitution. Were it solid, its surface would rapidly cool off, since the heat radiated would have to be conducted from the interior. Then, the loss of heat no longer going on at the same rate, the contraction also would stop and the generation of heat to supply the radiation would cease. Even were the sun a liquid, currents of liquid matter could scarcely convey to the surface a sufficient amount of heated matter to supply the enormous radiation. Thus the reason of the case combines with observation of the density of the sun to show that its interior must be regarded as gaseous rather than solid or liquid.

A difficult matter, however, presents itself. The density of the sun is greater than we ordinarily see in gases, being, as we have remarked, even greater than the density of water. The explanation of this difficulty is very simple: the gaseous interior is subject to compression by its superficial portions. The gravitation on the surface being twenty-seven times what it is on the earth, the pressure increases twenty-seven times as fast when we go towards the centre as it does on the earth. We should not have to go very far within its body to find a pressure of millions of tons on the square inch. Under such pressure and at such an enormous temperature as must there prevail, the distinction between a gas and a liquid is lost; the substance retains the elasticity of a gas, while assuming the density of a liquid.

It does not follow, however, that the visible surface of the sun is a gas, pure and simple. The sudden cooling which a mass of gaseous matter undergoes on reaching the surface may liquefy it or even change it into a solid. But, in either case, the sudden contraction which it thus undergoes makes it heavier and it sinks down again to be remelted in the great furnace below. It may well be, therefore, that the description of the sun as a vast bubble is nearly true. It may be added that all we have said about the sun may very well be supposed true of the stars. We have now to consider the law of change as a sun or star contracts through the loss of heat suffered by its radiation into space.

This subject was very exhaustively developed by Ritter some years since.[1] It is not practicable to give even an abstract of Ritter's results in the present work, especially as every mathematical investigation of the subject must either rest on hypotheses more or less uncertain, or must, for its application, require data impossible to obtain. We shall, therefore, confine ourselves to a brief outline of the main points of the subject. A fundamental proposition of the whole theory is Lane's law of gaseous attraction, which is as follows:

When a spherical mass of incandescent gas contracts through the loss of its heat by radiation into space, its temperature continually becomes higher as long as the gaseous condition is retained.

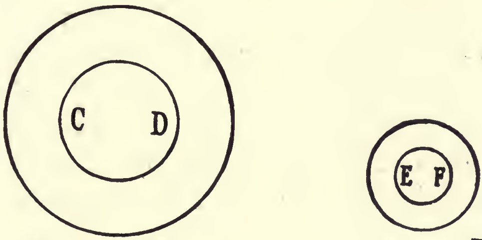
The demonstration of this law is simple enough to be understood by anyone well acquainted with elementary mechanics and physics, and it will also furnish the basis for our consideration of the subject.

We begin by some considerations on the condition of a mass of gas held together by the mutual attraction of its parts. This attraction results in a certain hydrostatic pressure, capable of being expressed as so many pounds or tons per unit of surface, say a square inch. This pressure at any point

[1] Wiedemann's *Annalen der Physik u. Chemie*, 1878 to 1883, etc.

is equal to the weight of a column of the gas having a section of one square inch and extending from the point in question to the surface. It is a law of attraction, in a sphere of which the density is the same at equal distances from its centre, that if we suppose an interior sphere concentric with the body, the attraction, of all the matter outside that interior sphere, on any point within it, is equal in every direction, and, therefore, is completely neutralised. A point is, therefore, drawn towards the centre only by the attraction of matter inside the sphere on the surface of which it lies.

At every point in the interior the hydrostatic pressure must be balanced by the elastic force of the gas. In the case of any one gas this force is proportional to the product of the density into the absolute temperature. This condition of equilibrium must be satisfied at every point throughout the mass.



A B

Let the two circles in the figure represent gaseous globes of the kind supposed. The larger, A, represents the globe in a certain condition of its evolution; the smaller, B, its condition after its volume has contracted to one-half. The temperature in each case will necessarily increase from the surface to the centre. The law of this increase is incapable of accurate expression, but is not necessary for our present purpose.

Let the inner circle, C D, represent a spherical shell of the matter forming the body, situated anywhere in the interior of the mass, but concentric with it. Let E F be the corresponding shell after the contraction has taken place. The case will then be as follows:

The two shells will by hypothesis have the same quantity of matter, both in their own substance and throughout their interior.

In case B, the central attraction, being as the inverse square of the distance from the centre, will be four times as great for each unit of matter in the shell.

This force of attraction, tending to compress the shell, is, in case B, exerted on a surface one quarter as great, because the surface of a shell is proportional to the square of its diameter.

Hence the hydrostatic pressure per unit of surface is sixteen times as great in case B as in case A.

The elastic force of the gas, if the two bodies were at the same temperature, would be eight times as great in case B as in case A, being inversely as the volume.

The hydrostatic pressure being sixteen times as great, while the elastic force to counterbalance it is only eight times as great, no equilibrium would be possible. To make it possible, the absolute temperature of the gas must be doubled, in order that the elastic force shall balance the pressure. The temperature of the spherical surface E F will therefore be double that of the surface C D.

That a mass can become hotter through cooling, may, at first sight, seem paradoxical. We shall, therefore, cite a result which is strictly analogous. If the motion of a comet is hindered by a resisting medium, the comet will continually move faster. The reason of this is that the first effect of the medium is to diminish the velocity of the object. Through this diminution of velocity, the comet falls towards the sun. The increase of velocity caused by the fall more than counterbalances the diminution produced by the resistance. The result is that the comet takes up a more and more rapid motion, as it gradually approaches the sun, in consequence of the resistance it suffers. In the same way, when a gaseous celestial body cools, the fall of its mass towards the centre changes an amount of energy greater than that radiated away from a potential to an actual form.

The critical reader will see a weak point in this reasoning, which it is necessary to consider. What we have really shown is that if the mass, being in equilibrium when it has the volume A, has to remain in equilibrium when it is reduced to the volume B, then its temperature must be doubled. But we have not proved that its temperature actually will be doubled by the fall. In fact, it cannot be doubled unless the energy generated by the fall of the superficial portions towards the centre is sufficient to double the absolute amount of heat. Whether this will be the case depends on a variety of circumstances, including the mass of the body, and the capacity of its substance for heat. If we are to proceed with mathematical rigour, it is, therefore, necessary to determine in any given case whether this condition is fulfilled. Let us suppose that in any particular case the mass is so small or the capacity for heat so considerable that the temperature is not doubled by the contraction. Then the contraction will go on further and further, until the mass becomes a solid. But in this case let us reverse the process. The body being supposed nearly in a state of equilibrium in position A, let the elastic force be slightly in excess. Then the gas will expand. In order that it shall be reduced to a state of equilibrium by expansion, its temperature must diminish according to the same law that it would increase if it contracted. When its diameter doubles, its temperature should be reduced to one-half or less by the expansion, in order that the equilibrium shall subsist. But, in the case supposed, the temperature is not reduced so much as this. Hence, it is too high for equilibrium by a still greater amount and the expansion must go on indefinitely. Thus, in the case supposed, the hypothetical equilibrium of the body is unstable. In other words, no such body is possible.

This conclusion is of fundamental importance. It shows that the possible mass of a star must have an inferior limit, depending on the quantity of matter it contains, its elasticity under given circumstances, and its capacity for heat. It is certain that any small mass of gas, taken into celestial space and left to itself, would not be kept together by the mutual attraction of its parts, but would merely expand into indefinite space. Possibly this might be true of the earth, if it were gaseous. The computation would not be a difficult one to make, but I have not made it.

In what precedes, we have supposed a single mass to contract. But our study of the relations of temperature and pressure in the two masses assumes no relationship between them, except that of equality. Let us now consider any two gaseous bodies, A and B, and suppose that the body B, instead of having the same mass as A, is another body with a different mass.

Since the mass B may be of various sizes, according to the amount of contraction it has undergone, let us begin by supposing it to have the same volume as A, but twice the mass of A. We have then to inquire what must be its temperature in order that it may be in equilibrium. We have first to inquire into the hydrostatic pressure at any point of the interior. Referring to either of the bodies in the figure of p. 211, a spherical shell like CD will now, in the case of the more massive body, have double the mass of the corresponding shell of A. The attraction will also be doubled, because the diameter of the spherical shell is the same, while the amount of matter within it is twice as great. Hence the hydrostatic pressure per unit of surface will be four times as great, or will vary as the square of the density. The elasticity at equal temperatures being proportional to the density, it follows that, were the temperature the same in the two masses, the elasticity would be double in the case of mass B; whereas, to balance the hydrostatic pressure, it should be quadrupled. The temperature of B must, therefore, be twice as great as that of A. It follows that in the case of stars of equal volume, but of different masses, the temperature must be proportional to the mass or density.

But how will it be if we suppose the density of the two bodies to be the same, and, therefore, the mass to be proportional to the volume? In this case the attraction at a given point will be proportional to the diameter of the body. If, then, we suppose one body to have twice the diameter of the other, but to be of the same density, it follows that at corresponding points of the interior, the hydrostatic pressure will be twice as great in the larger body. The density being the same, it follows that the temperature must be twice as high in order that equilibrium may be maintained. It follows that the stars of the greatest mass will be at the highest temperature, unless their volume is so great that their density is less than that of the smaller stars.

CHAPTER XIII

STELLAR EVOLUTION

As yet this world was not, and Chaos wild
Reigned where these heavens now roll, where earth now rests.

MILTON.

Und Stürme brausen um die Wette
Vom Meer aufs Land vom Land aufs Meer
Und bilden wüthend eine Kette
Der tiefsten Wirkung ringsumher.

GOETHE.

IT follows from the theory set forth in the last chapter that the stars are not of fixed constitution, but are all going through a progressive change —cooling off and contracting into a smaller volume. If we accept this result, we find ourselves face to face with an unsolvable enigma,—How did the evolution of the stars begin? To show the principle involved in the question, I shall make use of an illustration drawn from another work. An inquiring person, wandering around in what he supposes to be a deserted building, finds a clock running. If he knows nothing about the construction of the clock, or the force necessary to keep it in motion, he may fancy that it has been running for an indefinite time just as he sees it, and that it will continue to run until the material of which it is made shall wear out. But if he is acquainted with the laws of mechanics, he will know that this is impossible, because the continued movement of the pendulum involves a constant expenditure of energy. If he studies the construction of the clock, he will find the source of this energy in the slow falling of a weight suspended by a cord which acts upon a train of wheels. Watching the motions, he will see that the scape-wheel acting on the pendulum moves very perceptibly every second, while he must watch the next wheel for several seconds to see any motion. If the time at his disposal is limited, he will not be able to see any motion at all in the weight. But an examination of the machinery will show him that the weight must be falling at a certain rate, and he can compute that at the end of a certain time the weight will reach the bottom, and the clock will stop. He can also see that there must have been a point from above which the weight could never have fallen. Knowing the rate of fall, he can compute how long the weight occupied in falling from this point. His final conclusion will be that the clock must in some way have been wound up and set in motion by an external force a certain number of hours or days before his inspection, and must be again wound up by such a force unless it is to stop.

If we accept the theory that the heat of the stars is kept up by their slow contraction we must think of the universe of stars as of a clock which is running down. As we can see by the eye of reason that the weight of the clock was higher yesterday than it is to-day, so we can compute that the stars must have been larger in former times, and that there must have been some finite and computable period when they were all nebulæ. Not even a nebula can give light without a progressive change of some sort. Hence, within a certain finite period the nebulæ themselves must have begun to shine. How did they begin? This is the unsolvable question.

The process of stellar evolution may be discussed without considering this question. Accepting as a fact, or at least as a working hypothesis, that the stars are contracting, we find a remarkable consistency in the results. Year by year laws are established and more definite conclusions reached. It is now possible to speak of the respective ages of stars as they go through their progressive course of changes. This subject has been so profoundly studied and so fully developed by Sir William and Lady Huggins that I shall depend largely on their work in briefly setting it forth.[1] At the same time, in an attempt to condense the substance of many folio pages into so short a space, one can hardly hope to be entirely successful in giving merely the views of the original author. The following may, therefore, be regarded as partly the views of Sir William Huggins, condensed and arranged in the order in which they present themselves to the writer's mind, and partly those of the writer himself.

There is an infinite diversity among the spectra of the stars; scarcely two are exactly alike in all their details. But the larger number of these spectra, when

[1] Publications of Sir William Huggins's Observatory, vol. i., London, 1899.

carefully compared, may be made to fall in line, thus forming a series in which the passage of one spectrum into the next in order is so gradual as to indicate that the actual differences represent, in the main, successive epochs of star life rather than so many fundamental differences of chemical constitution. Each star may be considered to go through a series of changes analogous to those of a human being from birth to old age. In its infancy a star is simply a nebulous mass; it gradually condenses into a smaller volume, growing hotter, as set forth in the last chapter, until a stage of maximum temperature is reached, when it begins to cool off. Of the duration of its life we cannot form an accurate estimate. We can only say that it is certainly to be reckoned by millions and probably by tens of millions or even hundreds of millions of years. We thus have in the heavens stars ranging through the whole series from the earliest infancy to old age. How shall we distinguish the order of development? Mainly by their colours and their spectra. In its first stage the star is of a bluish white. It gradually passes through white into yellow and red. Sir William gives the following series of stars as an example of the successive stages of development:

Sirius; *α* Lyræ.
α Ursæ Majoris.
α Virginis.
α Aquilæ.
Rigel.
α Cygni.
. . .
Capella; the sun.

Arcturus.
Aldebaran.
α Orionis.

The length of the life of a star has no fixed limit; it depends entirely on the mass. The larger the mass, the longer the life; hence a small star may pass from infancy to old age many times more rapidly than a large one.

At the same time, up to at least the yellow stage, the star continually grows hotter as it condenses. A difficulty may here suggest itself in reconciling this order with a well-known physical fact. As a radiating body increases in temperature, its color changes from red through yellow to white, and the average wave-length of its light continually diminishes. We see a familiar example of this in the case of iron, which when heated is first red in color and then goes through the changes we have mentioned. The ordinary incandescent electric light is yellow, the arc light, the most intense that we can produce by artificial means, is white. When the spectrum of a body thus increasing in temperature is watched, the limit is found to pass gradually from the red toward the violet end. It would seem, therefore, that the hotter stars should be the white ones and the cooler the yellow or red ones.

There are, however, two circumstances to be considered in connection with the contracting star. In the first place, the light which we receive from a star does not emanate from its hottest interior, but from a region either upon or, in most cases, near its surface. It is, therefore, the temperature of this region which determines the colour of the light. In the next place, part of the light is absorbed by passing through the cooler atmosphere surrounding the star. It is only the light which escapes through this atmosphere that we actually see.

In the case of the sun all the light which it sends forth comes from a comparatively shallow bounding layer, the photosphere. The most careful telescopic examination shows no depth to this layer, which would rapidly cool off were it not for convection currents bringing up heated matter from the interior. It might be supposed that such a current would result in the surface being kept at nearly as high a temperature as the interior; but, as a matter of fact, the opposite is the case. As the volume of gas rises, it expands from the diminished pressure and it is thus cooled in the very act of coming to the surface, as well as by the rapid radiation when it reaches the surface.

In the case of younger stars, there is probably no photosphere, properly so called. The light which they emit comes from a considerable distance in the interior. Here the effect of gravity comes into play. The more the star condenses, the greater is gravity at its surface; hence the more rapidly does the density of the gas increase from the surface toward the interior. In the case of the sun, the density of any gas which may immediately surround the photosphere must be doubled every mile or two of its depth until we reach the photosphere. But if the sun were many times its present diameter, this increase would be very much slower. Hence, when the volume is very great the increase of density is comparatively slow; there being no well-defined photosphere, the light reaches us from a much greater depth from the interior than it does at a later stage.

The gradual passing of a white star into one of the solar type is marked by alterations in its spectrum. These alterations are especially seen in the behaviour of the lines of hydrogen, calcium, magnesium, and iron. The lines of hydrogen change from broad to thin; those of calcium constantly become stronger.

Of the greatest interest is the question, At what stage does the temperature of the star reach its maximum and the body begin to cool? Has our sun reached this stage? This is a question to which, owing to the complexity of the conditions, it is impossible to give a precise answer. It seems probable, however, that the highest temperature is reached in about the stage of our sun. Accepting Sir William Huggins's view, the reason the light is not then bluest is that it suffers a strong selective absorption by the gases surrounding the photosphere. We know this to be the case with the sun. According to Vogel, the removal of the sun's atmosphere would make its light two-and-a-half times as bright at the blue-violet end of the spectrum.

The general fact that every star has a life history —that this history will ultimately come to an end— that it must have had a beginning in time—is indicated by so great a number of concurring facts that no one who has most profoundly studied the subject can have serious doubts upon it. Yet there are some unsolved mysteries connected with the case, which might justify a waiting for further evidence, coupled with a certain degree of scepticism. Of the questions connected with the case the most serious one is raised by the geologists.

On the theory set forth in the last chapter, that the radiant energy sent out is balanced by the continual loss of potential energy due to the contraction, the age of the sun can be at least approximately estimated. About twenty millions of years is the limit of time during which it could possibly have radiated anything like its present amount of energy. But this conclusion is directly at variance with that of geology. The age of the earth has been approximately estimated from a great variety of geological phenomena, the concurring result being that stratification and other geological processes must have been going on for hundreds—nay, thousands of millions of years. This result is in direct conflict with the only physical theory which can account for the solar heat.

The nebulæ offer a similar difficulty. Their extreme tenuity and their seemingly almost unmaterial structure appear inadequate to account for any such mutual gravitation of their parts as would result in the generation of the flood of energy which they are constantly radiating. What we see must, therefore, suggest at least the possibility that all shining heavenly bodies have connected with them some source of energy of which science can, as yet, render no account. Facts are accumulating which converge to the view that forms of substance exist which are neither matter nor ether, but something between the two—perhaps primeval substance from which matter itself was evolved. In this ethereal substance is stored an almost exhaustless supply of energy, the withdrawal of which results in the condensation of the substance into matter. More than this it seems hard to say until we have either seen the nebulæ contracting in volume, or have made such estimates of their probable masses that we can compute the amount of contraction they must undergo to maintain the supply of energy.

CHAPTER XIV

THE STRUCTURE OF THE HEAVENS

He who through vast immensity can pierce,
See worlds on worlds compose one universe,
Observe how system into system runs,
What other planets circle other suns,
What varied being peoples every star,
May tell why Heaven has made us as we are.—POPE.

THE problem of the structure and duration of the universe is the most far-reaching with which the mind has to deal. Its solution may be regarded as the ultimate object of stellar astronomy, the possibility of reaching which has occupied the minds of thinkers since the beginning of civilisation. Before our time the problem could be considered only from the imaginative or the speculative point of view. Although we can to-day attack it to a limited extent by scientific methods, it must be admitted that we have scarcely taken more than the first step toward the actual solution. We can do little more than state the questions involved, and show what light, if any, science is able to throw upon the possible answers.

First, we may inquire as to the extent of the universe of stars. Are the latter scattered through infinite space, so that those we see are merely that portion of an infinite collection which happens to be within reach of our telescopes, or are all the stars contained within a certain limited space? In the latter case, have our telescopes yet penetrated to the boundary in any direction? In other words, as, by the aid of increasing telescopic power, we see fainter and fainter stars, are these fainter stars at greater distances than those before known, or are they smaller stars contained within the same limits as those we already know? Otherwise stated, do we see stars on the boundary of the universe?

Secondly, granting the universe to be finite, what is the arrangement of the stars in space? Especially, what is the relation of the galaxy to the other stars? In what sense, if any, can the stars be said to form a permanent system? Do the stars which form the Milky Way belong to a different system from the other stars, or are the latter a part of one universal system?

Thirdly, what is the duration of the universe in time? Is it fitted to last for ever in its present form, or does it contain within itself the seeds of dissolution? Must it, in the course of time, in we know not how many millions of ages, be transformed into something very different from what it now is? This question is intimately associated with the question whether the stars form a system. If they do, we may suppose that system to be permanent in its general features; if not, we must look further for our conclusion.

The first and third of these questions will be recognised by students of Kant as substantially those raised by the great philosopher in the form of antinomies. Kant attempted to show that both the propositions and their opposites could be proved or disproved by reasoning equally valid in either case. The doctrine that the universe is infinite in duration and that it is finite in duration are both, according to him, equally susceptible of disproof. To his reasoning on both points the scientific philosopher of to-day will object that it seeks to prove or disprove, *a priori*, propositions which are matters of fact, of which the truth can be therefore settled only by an appeal to observation. The more correct view is that afterward set forth by Sir William Hamilton, that it is equally impossible for us to conceive of infinite space (or time), or of space (or time) coming to an end. But this inability merely grows out of the limitations of our mental power, and gives us no clue to the actual universe. So far as the questions are concerned with the latter, no answer is valid unless based on careful observation. Our reasoning must have facts to start from before a valid conclusion can be reached.

The first question we have to attack is that of the extent of the universe. In its immediate and practical form, it is whether the smallest stars that we see are at the boundary of a system, or whether more and more lie beyond to an infinite extent. This question we are not yet ready to answer with any approach to certainty. Indeed, from the very nature of the case, the answer must remain somewhat indefinite. If the collection of stars which forms the Milky Way be really finite, we may not yet be able to see its limit. If we do see its limit, there may yet be, for aught we know, other systems and other galaxies, scattered through infinite space, which must for ever elude our powers of vision. Quite likely the boundary of the system may be somewhat indefinite, the stars gradually thinning out as we go farther and farther, so that no definite limit can be assigned. If all stars are of the same average brightness as those we see, all that lie beyond a certain distance must evade observation, at least as individual stars, for the simple reason that they are too far off to be visible in our telescopes.

There is a law of optics which throws some light on the question. Suppose the stars to be scattered through infinite space in such a way that every great portion of space is, in the general average, about equally rich in stars. Then imagine that, at some great distance, say that of the average stars of the sixth magnitude, we describe a sphere having its centre in our system. Outside this sphere, describe another one, having a radius greater by a certain quantity, which we may call S. Outside that let there be another of a radius yet greater by S, and so on indefinitely. Thus we shall have an endless succession of concentric spherical shells, each of the same thickness, S. The volume of each of these regions will be nearly proportional to the square of the diameters of the spheres which bound it. Hence, supposing an equal distribution of the stars, each of the regions will contain a number of stars increasing as the square of the radius of the region. Since the amount of light which we receive from each individual star is as the inverse square of its distance, it follows that the sum-total of the light received from each of these spherical shells will be equal. Thus, as we include sphere after sphere, we add equal amounts of light without limit. The result of the successive addition of these equal quantities, increasing without limit, would be that if the system of stars extended out indefinitely the whole heavens would be filled with a blaze of light as bright as the sun.

Now, as a matter of fact, such is very far from being the case. It follows that infinite space is not occupied by the stars. At best there can only be collections of stars at great distances apart.

The nearest approximation to such an appearance as that described is the faint, diffused light of the Milky Way. But so large a fraction of this illumination comes from the stars which we actually see in the telescope that it is impossible to say whether any visible illumination results from masses of stars too faint to be individually seen. Whether the cloud-like impressions which Barnard has found on long-exposed photographs of the Milky Way are produced by countless distant stars, too faint to impress themselves individually even upon the most sensitive photographic plate, is a question which cannot yet be answered. But even if we should answer it in the affirmative, the extreme faintness of the light shows that the stars which produce it are not scattered through infinite space; but that, although they may extend much beyond the limits of the visible stars, they thin out very rapidly. The evidence, therefore, seems to be against the hypothesis that the stars we see form part of an infinitely extended universe.

But there are two limitations to this conclusion. It rests upon the hypothesis that light is never lost in its passage to any distance, however great. This hypothesis is in accordance with our modern theories of physics, yet it cannot be regarded as an established fact for all space, even if true for the distances of the visible stars. About half a century ago Struve propounded the contrary hypothesis that the light of the more distant stars suffers an extinction in its passage to us. But this had no other basis than the hypothesis that the stars were equally thick out to the farthest limits at which we could see them. It might be said that he assumed an infinite universe, and, from the fact that he did not see the evidence of infinity, concluded that light was lost. The hypothesis of a limited universe and no extinction of light, while not absolutely proved, must be regarded as the one to be accepted until further investigation shall prove its unsoundness.

The second limitation arises from the possible structure of an infinite universe. The mathematical reader will easily see that the conclusion that an infinite universe of stars would fill the heavens with a blaze of light, rests upon the hypothesis that every region of space of some great but finite extent is, on the average, occupied by at least one star. In other words, the hypothesis is that, if we divide the total number of the stars by the number of cubic miles of space, we shall have a finite quotient. But an infinite universe can be imagined which does not fill this condition. Such will be the case with one constructed on the celebrated hypothesis of Lambert, propounded in the latter part of the eighteenth century. This author was an eminent mathematician who seems to have been nearly unique in combining the mathematical and the speculative sides of astronomy. He assumed a universe constructed on an extension of the plan of the solar system. The smallest system of bodies is composed of a planet with its satellites. We see a number of such systems, designated as the Terrestrial, the Martian (Mars and its satellites), the Jovian (Jupiter and its satellites), etc., all revolving round the sun, and thus forming one greater system, the solar system. Lambert extended the idea by supposing that a number of solar systems, each formed of a star with its revolving planets and satellites, were grouped into a yet greater system. A number of such groups form the great system which we call the galaxy, and which comprises all the stars we can see with the telescope. The more distant clusters may be other galaxies. All these systems again may revolve around some distant centre, and so on to an indefinite extent. Such a universe, how far so ever it might extend, would not fill the heavens with a blaze of light, and the more distant galaxies might remain for ever invisible to us.

But modern developments show that there is no scientific basis for this conception, attractive though it be by its grandeur.

So far as our present light goes, we must conclude that, although we are unable to set absolute bounds to the universe, yet the great mass of stars is included within a limited space the extent of which we have as yet no evidence. Outside of this space there may be scattered stars or invisible systems. But if these systems exist, they are distinct from our own.

The second question, that of the arrangement of the stars in space, is one on which it is equally difficult to propound a definite general conclusion. So far, we have only a large mass of faint indications, based on researches which cannot be satisfactorily completed until great additions are made to our fund of knowledge.

A century ago Sir William Herschel reached the conclusion that our universe was composed of a comparatively thin but widely extended stratum of stars. To introduce a familiar object, its figure was that of a large, thin grindstone, our solar system being near the centre. Considering only the general aspect of the heavens, this conclusion was plausible. Suppose a mass of a million of stars scattered through a space of this form. It is evident that an observer in the centre, when he looked through the side of the stratum, would see few stars. The latter would become more and more numerous as he directed his vision toward the circumference of the stratum. In other words, assuming the universe to have this form, we should see a uniform, cloud-like arch spanning the heavens—a galaxy in fact.

This view of the figure of the universe was also adopted by Struve, who was, the writer believes, the first astronomer after Herschel to make investigations which can be regarded as constituting an important addition to thought on the subject. To a certain extent we may regard the hypothesis as incontestable. The great mass of the visible stars is undoubtedly contained within such a figure as is here supposed.

To show this let Fig. 1 represent a cross section of the heavens at right angles to the Milky Way, the

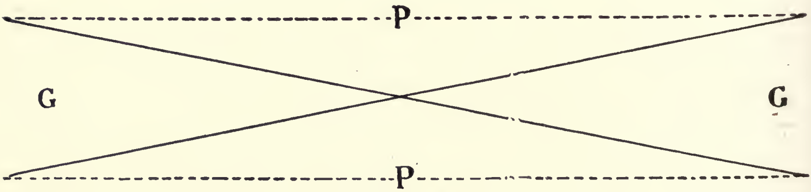


FIG. 1.

solar system being in the centre. It is an observed fact that the stars are vastly more numerous in the galactic regions G G than in the regions P P. Hence, if we suppose the stars equally scattered, they must extend much farther out in G G than in P P. If they extend as far in the one direction as in the other, then they must be more crowded in the galactic belt. It will still remain true that the greater number of the stars are included in the flat region G G P P, those outside this stratum being comparatively few in number.

But we cannot assume that this hypothesis of the form of the universe affords the basis for a satisfactory conception of its arrangement. Were it the whole truth, the stars would be uniformly dense along the whole course of the Milky Way. Now, it is a familiar fact that this is not the case. The Milky Way is not a uniformly illuminated belt, but a chain of irregular cloud-like aggregations of stars. Starting from this fact as a basis, our best course is to examine the most plausible hypotheses we can make as to the distribution of the stars which do not belong to the galaxy, and see which agrees best with observation.

Let figure 2 represent a section of the galactic ring or belt in its own plane, with the sun near the

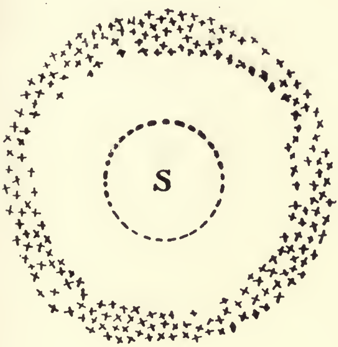


FIG. 2.

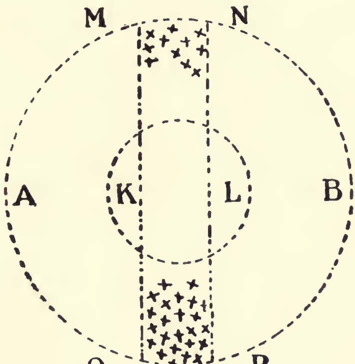


FIG. 3.

centre, S. To an observer at a vast distance in the direction of either pole of the galaxy,[1] the latter would appear of this form. Let Fig. 3 represent a cross

[1] Regarding the galaxy as a belt spanning the heavens, the central line of which is a great circle, the poles of the galaxy are the two opposite points in the heavens everywhere 90° from this great circle. Their direction is that of the two ends of the axle of the grindstone, as seen by an observer in the centre, while the galaxy would be the circumference of the stone

section as viewed by an observer in the plane of the galaxy at a great distance outside of it. How would the stars that do not belong to the galaxy be situated? We may make three hypotheses:

1. That they are situated in a sphere (A B) as large as the galaxy itself. Then the whole universe of stars would be spherical in outline, and the galaxy would be a dense belt of stars girdling the sphere.

2. The remaining stars may still be contained in a spherical space (K L), of which the diameter is much less than that of the galactic girdle. In this case our sun would be one of a central agglomeration of stars, lying in or near the plane of the galaxy.

3. The non-galactic stars may be equally scattered throughout a flat region (M N P Q), of the grindstone form. This would correspond to the hypothesis of Herschel and Struve.

There is no likelihood that either of these hypotheses is true in all the geometric simplicity with which I have expressed it. Stars are doubtless scattered to some extent through the whole region M N P Q, and it is not likely that they are confined within limits defined by any geometrical figure. The most that can be done is to determine to which of the three figures the mutual arrangement most nearly corresponds.

The simplest test is that of the third hypothesis as compared with the other two. If the third hypothesis be true, then we should see the fewest stars in the direction of the poles of the galaxy; and the number in any given portion of the celestial sphere, say one square degree, should continually increase, slowly at first, more rapidly afterwards, as we went from the poles toward the circumference of the galaxy. At a distance of 60° from the poles and 30° from the central line or circumference we should see perhaps twice as many stars per square degree as near the poles.

Were it possible to determine the distance of a star as readily as we do its direction, the problem of the distribution of the stars in space would be at once solved. This not being the case, we must first study the apparent arrangement of the stars with respect to the galaxy, with a view to afterward drawing such conclusions as we can in regard to their distance.

CHAPTER XV

APPARENT DISTRIBUTION OF THE STARS IN THE SKY

Zwei Dingen erfüllen das Gemuth mit immer neuer und zunehmender Bewunderung und Erfurcht, je öfter und anhaltender sich das Nachdenküng damit beschäftigt. der bestirnte Himmel über mir und das moralische Gesetz in mir.—KANT.

OUR question now is, How are the stars, as we see them, distributed over the sky? We know in a general way that there are vastly more stars round the belt of the Milky Way than in the remainder of the heavens. But we wish to know in detail what the law of increase is from the poles of the galaxy to the belt itself.

In considering any question of the number of stars in a particular region of the heavens, we are met by a fundamental difficulty. We can set no limit to the minuteness of stars, and the number will depend upon the magnitude of those which we include in our count. As already remarked, there are, at least up to a certain limit, three or four times as many stars of each magnitude as of the magnitude next brighter. Now, the smallest stars that can be seen, or that may be included in any count, vary greatly with the power of the instrument used in making the count. If we had any one catalogue, extending over the whole celestial sphere, and made on an absolutely uniform plan, so that we knew it included all the stars down to some given magnitude, and no others, it would answer our immediate purpose. If, however, one catalogue including the stars in a certain part of the sky should extend only to the ninth magnitude, while another, covering another part, should extend to the tenth, we should be led quite astray in assuming that the number of stars in the two catalogues expressed the star density in the regions which they covered. The one would show three or four times as many stars as the other, even though the actual density in the two cases were the same.

If we could be certain, in any one case, just what the limit of magnitude was for any catalogue, or if the magnitudes in different catalogues always corresponded to absolutely the same brightness of the star, this difficulty would be obviated. But this is the case only with that limited number of stars whose brightness has been photometrically measured. In all other cases our count must be more or less uncertain. One illustration of this will suffice:

I have already remarked that in making the photographic census of the southern heavens, Gill and Kapteyn did not assume that stars of which the images were equally intense on different plates were actually of the same magnitude. Each plate was assumed to have a scale of its own, which was fixed by comparing the intensity of the photographic impressions of those stars whose magnitudes had been previously determined with these determinations, and thus forming as it were a separate scale for each plate. But, in forming the catalogue from the international photographic chart of the heavens, it is assumed that the photographs taken with telescopes of the same aperture, in which the plates are exposed for five minutes, will all correspond, and that the smallest stars found on the plates will be of the eleventh magnitude.

Distribution of the Lucid Stars.

In the case of the lucid stars this difficulty does not arise, because the photometric estimates are on a sufficiently exact and uniform scale to enable us to make a count, which shall be nearly correct, of all the stars down to, say, magnitude 6.0 or some limit not differing greatly from this. Several studies of the distribution of these stars have been made; one by Gould in the *Uranometria Argentina*, one by Schiaparelli, and another by Pickering. The counts of Gould and Schiaparelli, the former having special reference to the Milky Way, are best adapted to our purpose. The most striking result of these studies is that the condensation in the Milky Way seems to commence with the brightest stars. A little consideration will show that we cannot, with any probability, look for such a condensation in the case of stars near to us. Whatever form we assign to the stellar universe, we shall expect the stars immediately around us to be equally distributed in every direction. Not until we approach the boundary of the universe in one direction, or some great masses like those of the galaxy in another direction, should we expect marked condensation round the galactic belt. Of course we might imagine even the nearest stars to be most numerous in the direction round the galactic circle. But this would imply an extremely unlikely arrangement, our system being as it were at the point of a conical region richer in stars than the region around it. It is clear that if such were the case for one point, it could not be true if our sun were placed anywhere except at this particular point. Such an arrangement of the stars round us is outside of all reasonable probability. Independent evidence of the equal distribution of the nearer stars will hereafter be found in the proper motions. If, then, the nearer stars are equally distributed round us, and only distant ones can show a condensation toward the Milky Way, it follows that among the distant stars are some of the brightest in the heavens, a fact which we have already shown to follow from other considerations.

As we have to study the distribution of the stars with respect to the galaxy, the precise position of the latter enters into our problem. There is no difficulty in mapping out its general course by unaided eye observations of the heavens or a study of maps of the stars. Looking at the heavens, we shall readily see that it crosses the equator at two opposite points; the one east of the constellation Orion, between 6h. and 7h. of right ascension; the other at the opposite point, in Aquila, between 18h. and 19h. It makes a considerable angle with the equator, somewhat more than

60°. Consequently it passes within 30° of either celestial pole. The point nearest of approach to the north pole is in the constellation Cassiopeia.

Its position can readily be determined by noting the general course of its brighter portions on a map of the stars, and then determining, by inspection or otherwise, the circle which will run most nearly through those portions. It is thus found that the position is nearly always near a great circle of the sphere. From the very nature of the case the position of this circle will be a little indefinite, and probably the estimates made of it have been based more on inspection than on computation. The following positions have been assigned to the pole of the galaxy:

Gould...........	R. A. =	12h. 41m.	Dec. =	$+27°\ 21'$
Herschel, W	" "	12h. 29m.	" "	$+31°\ 30'$
Seeliger	" "	12h. 49m.	" "	$+27°\ 30'$
Argelander......	" "	12h. 40m.	" "	$+28°\ 5'$

The author, with the assistance of Mr. Wm. T. Carrigan, has made an independent determination by finding the great circle which will pass nearest to some 40 of the brightest regions of the galaxy. The result is different according as we include or omit the divergent branch toward the west between Cygnus and Aquila. Including the branch, the position of the galactic pole is,

$$\text{R. A.} = 12\text{h. } 44\text{m.} \quad \text{Dec.} = 26°\ 48'$$

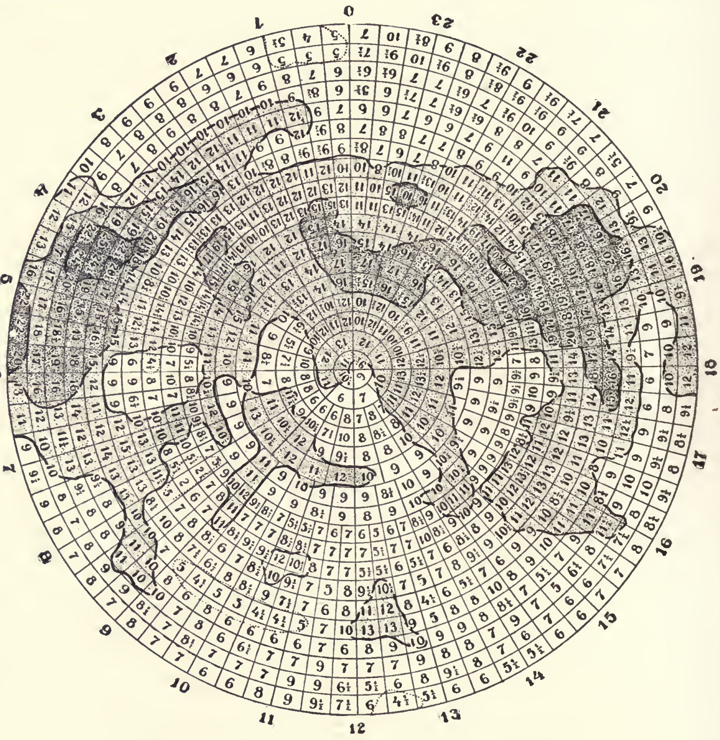
Excluding the branch it is,

$$\text{R. A.} = 12\text{h. } 51\text{m.} \quad \text{Dec.} = 27°\ 12'$$

Very remarkable is the fact, first pointed out by Sir J. Herschel, and more fully developed by Gould, that a belt of bright stars encircles the heavens but does not exactly coincide with the Milky Way. It intersects the galaxy at the points nearest the celestial poles, one node being near the Southern Cross and the other in Cassiopeia. This belt includes the brightest stars in a number of constellations, from Canis Major through the southern region of the heavens and back to Scorpius. In the northern heavens the brightest stars in Orion, Taurus, Cassiopeia, Cygnus, and Lyra belong to it. It would not be safe, however, to assume that the existence of this belt results from anything but the chance distribution of the few bright stars which form it. In order to reach a definite conclusion bearing on the structure of the heavens, it is advisable to consider the distribution of the lucid stars as a whole.

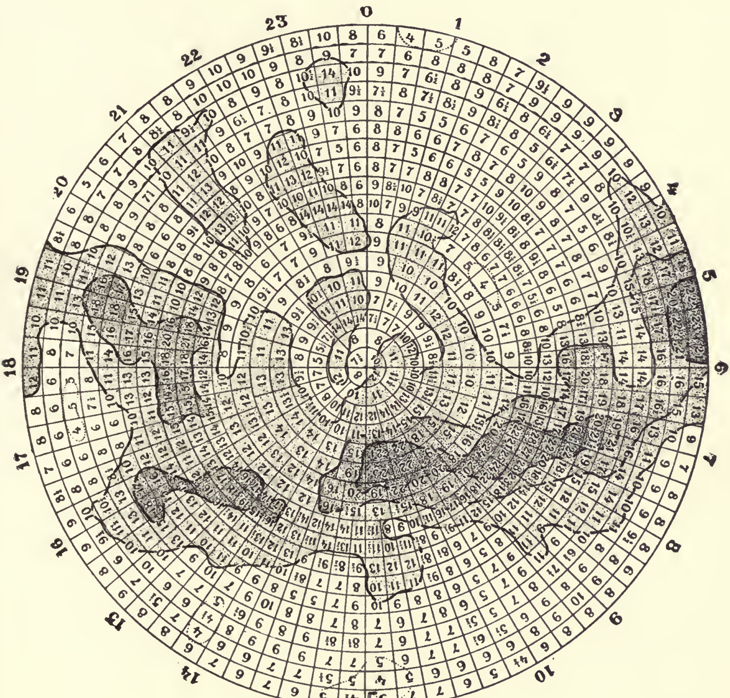
Dr. Gould found that the stars brighter than the fourth magnitude are arranged more symmetrically relatively to the belt of bright stars we have just described than to the galactic circle. This and other facts suggested to him the existence of a small cluster within which our sun is eccentrically situated, and which is itself not far from the middle plane of the galaxy. This cluster appears to be of a flattened shape and to consist of somewhat more than 400 stars of magnitudes ranging from the first to the seventh. Since Gould wrote, the extreme inequality in the intrinsic brightness of the stars has been brought to light and seems to weaken his explanation of the fact.

A very thorough study of the subject, but without considering the galaxy, has also been made by Schiaparelli. The work is based on the photometric measures of Pickering and the *Uranometria Argen-*



STAR-DENSITY OF THE NORTHERN HEMISPHERE

tina of Gould. One of its valuable features is a series of planispheres, showing in a visible form the star density in every region of the heavens for stars of various magnitudes. We reproduce on a reduced scale two of these planispheres. They were constructed by Schiaparelli in the following way: The entire sky was divided into 36 zones by parallels of declination 5° apart. Each zone was divided into spherical trapezia by hour-circles taken at intervals of 5° from the equator up to 50° of north or south declination;



STAR-DENSITY OF THE SOUTHERN HEMISPHERE

of 10° from 50 to 60; of 15° from 60 to 80; of 45° from 80 to 85, while the circle within 5° of the pole was divided into four regions. In this way 1800 areas, not excessively different from each other, were formed.

The star-density, as it actually is, might be indicated by the number of stars of these regions. As a matter of fact, however, the density obtained in this way would vary too rapidly from one area to the adjoining one, owing to the accidental irregularities of distribution of the stars. An adjustment was, therefore, made by finding in the case of each area the number of stars contained in 1 / 200 of the entire sphere, including the region itself and those immediately around it. The number thus obtained was considered as giving the density for the central region. The total number of stars being 4303, the mean number in 1 / 200 of the whole sphere is 21.5, and the mean in each area is 10.4.

The numbers on the planisphere given in each area express the star density of the region, or the number of stars per 100 square degrees, expressed generally to the nearest unit, the half-unit being sometimes added.

A study of the reproduction which we give will show how fairly well the Milky Way may be traced out round the sky by the tendency of those stars visible to the naked eye to agglomerate near its course. In other words, were the cloud-forms which make up the Milky Way invisible to us, we should still be able to mark out its course by the crowding of the lucid stars toward it. As a matter of interest, I have traced out the central line of the darker shaded portions of the planispheres as if they were the galaxy itself. The nearest great circle to the course of this line was then found to have its pole in the following position:

R. A.; 12h. 18m.
Dec. $+ 27°$.

This estimate was made without having at the time any recollection of the position of the galaxy given by other authorities. Compared with the positions given in the last chapter by Gould and Seeliger, it will be seen that the deviation is only 5° in right ascension, while the declinations are almost exactly similar. We infer that the circle of condensation found in this way makes an angle with the galaxy of less than 5°.

Distribution of the Fainter Stars.

The most thorough study of the distribution of the great mass of stars relative to the galactic plane has been made by Seeliger in a series of papers presented to the Munich Academy from 1884 to 1898. The data on which they are based are the following:

1. The *Bonner Durchmusterung* of Argelander and Schönfeld, described in our third chapter. The two works under this title are supposed to include all the stars to the ninth magnitude, from the north pole to 24° of south declination. But there are some inconsistencies in the limit of magnitude which we shall hereafter mention.

2. The "star gauges" of the two Herschels. These consisted simply in counts of the number of stars visible in the field of view of the telescope when the latter was directed toward various regions of the sky. Sir William Herschel's gauges were partly published in the *Philosophical Transactions*. A number of unpublished ones were found among his papers by Holden and printed in the publications of the Washburn Observatory, vol. ii. The younger Herschel, during his expedition to the Cape of Good Hope,

continued the work in those southern regions of the sky which could not be seen in England.

3. A count of the stars by Celoria, of Milan, in a zone from the equator to 6° N. Dec., extending round the heavens.

From what has been said, the first question to occupy our attention is that of the distribution of the stars with reference to the galactic plane, or, rather, the great circle forming the central line of the Milky Way.

The whole sky is divided by Seeliger into nine zones or regions, each 20° in breadth, by small circles parallel to the galactic circle. Region I. is a circle of 20° radius, whose centre is the north galactic pole. Round this central circle is a zone 20° in breadth, called zone II. Continuing the division, it will be seen that zone V. is the central one of the Milky Way, extending 10° on each side of the galactic circle. VI. is the zone next south of the galaxy, and so on to IX., which is the circle 40° in diameter round the south galactic pole.

The condensed result of the work is shown in the following table.

Column "Area" shows the number of square degrees in each region, so far as included in the survey. It will be remarked that the catalogues in question do not include the whole sky, as they stop at 24° S.Dec.

Column "Stars" shows the number of stars to magnitude 9.0 found in each area.

Column "Density" is the quotient of the number of stars by the area, and is, therefore, the mean number of stars per square degree in each region. In the last column these numbers are corrected, for certain anomalies in the magnitudes given by the catalogues, so as to reduce them to a common standard.

Region.	Area. Degrees.	Stars.	Density.	Corrected Density.
I	1,398.7	4,277	3.06	2.78
II	3,146.9	10,185	3.24	3.03
III	5,126.6	19,488	3.80	3.54
IV	4,589.8	24,492	5.34	5.32
V	4,519.5	33,267	7.36	8.17
VI	3,971.5	23,580	5.94	6.07
VII	2,954.4	11,790	3.99	3.71
VIII	1,790.6	6,375	3.56	3.21
IX	468.2	1,644	3.51	3.14

A study of the last two columns is decisive of one of the fundamental questions already raised. The star-density in the several regions increases continuously from each pole (regions I. and IX.) to the galaxy itself. If the latter were a simple ring of stars surrounding a spherical system of stars, the star-density would be about the same in regions I., II., and III., and also in VII., VIII., and IX., but would suddenly increase in IV. and VI. as the boundary of the ring was approached. Instead of such being the case, the numbers 2.78, 3.03, and 3.54 in the north, and 3.14, 3.21, and 3.71 in the south, show a progressive increase from the galactic pole toward the galaxy itself.

The conclusion to be drawn is a fundamental one. The universe, or, at least, the denser portions of it, is really flattened between the galactic poles, as supposed by Herschel and Struve. In the language of Seeliger: "The Milky Way is no merely local phenomenon, but is closely connected with the entire constitution of our stellar system."

This conclusion is strengthened by a study of the data given by Celoria. It will be remarked that the zone counted by this astronomer cuts the Milky Way diagonally at an angle of about 62°, and, therefore, does not take in either of its poles. Consequently, regions I. and IX. are both left out. For the remaining seven regions the results are shown as follows: We show first the area, in square degrees, of each of the regions, II. to VIII., included in Celoria's zone. Then follows in the next column the number of stars counted by Celoria, and, in the third, the number enumerated in the *Durchmusterung*, in these portions of each region. The quotients show the star-density, or the mean number of stars per square degree, recorded by each authority:

	Area.	Number of Stars.		Star-Density.	
Region.	Degrees.	Cel.	D. M.	Cel.	D. M.
II	404.4	27,352	1,230	67.6	3.04
III	284.6	22,551	932	79.3	3.28
IV	254.6	29,469	1,488	115.7	5.83
V	284.6	41,820	1,833	146.9	6.44
VI	284.6	31,706	1,472	111.4	5.22
VII	329.5	25,618	1,342	77.7	4.07
VIII	314.5	22,264	1,184	70 8	3.77

It will be seen that the law of increasing star-density from near the galactic pole to the galaxy itself is of the same general character in the two cases. The number of stars counted by Celoria is generally between 18 and 25 times the number in the *Durchmusterung*.

An important point to be attended to hereafter is that the star-density of the Milky Way itself, as found by Celoria and the authors of the *Durchmusterung*, is between two or three times that near the galactic poles. Very different is the result derived from the Herschelian gauges, which is this:

Region....I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
Density...107	154	281	560	2019	672	261	154	111

From the gauges of the Herschels it follows that the galactic star-density is nearly 20 times that near the galactic poles. At these poles the Herschels counted only about 50 per cent. more stars than Celoria. In the galaxy itself they counted 14 for every one by Celoria. There is little doubt as to the principal cause of this discrepancy. The observations by the first two authorities were made with smaller telescopes than that of Herschel, and they failed to count all the visible stars of the Milky Way. The recent comparisons of the *Durchmusterung* with the heavens, mostly made since Seeliger worked out the results we have given, show that the limit of magnitude to which this list extends is far from uniform, and varies with the star-density. In regions poor in stars, all of the latter to the tenth magnitude are listed; in the richer regions of the galaxy the list stops, we may suppose, with the ninth magnitude, or even brighter. Yet, in all cases, the faintest stars listed are classed as of magnitude 9.5. Thus a ninth-magnitude star in the galaxy, according to the *Durchmusterung*, is markedly brighter than one of this magnitude elsewhere.

Distribution of the Stars having Sensible Proper Motion.

Having found that the stars of every magnitude show a tendency to crowd toward the region of the Milky Way, the question arises whether this is true of those stars which have a sensible proper motion. Kapteyn has examined this question in the case of the Bradley stars. His conclusion is that those having a considerable proper motion, say more than 5″ per century, are nearly equally distributed over the sky, but that when we include those having a small proper motion, we see a continually increasing tendency to crowd toward the galactic plane.

It seems to the writer that the uncertainty as to the smaller proper motions of the Bradley stars renders this result quite unreliable. To reach a more definite conclusion, we must base our work on lists of proper motions which are as nearly complete within their limits as it is possible to make them. Such lists have been made by Auwers and Boss, their work being based on their observations of zones of stars for the catalogue of the *Astronomische Gesellschaft*. The zone observed by Auwers was that between 15° and 20° of N. Dec.; while Boss's was between 1° and 5°. To speak more exactly, the limits were from 14° 50′ to 20° 10′ and 0° 50′ to 5° 10′, each zone of observation overlapping 10′ on the adjoining one. Thus the actual breadths were 5° 20′ and 4° 20′. Within these respective limits, Auwers, by a comparison with previous observations, found 1300 stars having an appreciable proper motion, and Boss 295. But Boss's list is confined to stars having a motion of at least 10″; of such the list of Auwers contains 431. The number of square degrees in the two zones is 1556 and 1830, respectively. The corresponding number of stars with proper motions extending 10″ is for each 100 square degrees:

In Boss's zone, 18.9.
In Auwers's zone, 23.9.

The question whether the greater richness of nearly 25 per cent. in Auwers's zone is real is one to which it is not easy to give a conclusive answer. The probability, however, seems to be that it is mainly due to the greater richness of the material on which Auwers's proper motions are based. Happily, the question is not essential in the present discussion.

We now examine the question of the respective richness of proper-motion stars in this way:

Each of these zones cuts the galaxy at a considerable angle in two opposite regions. Each zone, as a matter of course, has a far greater richness of stars per unit of surface in the two galactic regions than in the intermediate regions. We, therefore, divide each zone in four strips, two including the galactic regions and two the intermediate regions. The boundaries are somewhat indefinite; we have fixed them by the richness of the total number of stars. For the galactic strips we take in Boss's zone the strip between 5h. and 8h. of R. A. and that between 17h. and 20h. Each of these strips being 3h. in length, the two together comprise one quarter the total surface of the zone. If the proper-motion stars crowd towards the galaxy like others do, then the numbers in the galactic region should be proportional to the total number observed in the region. But if they are equally distributed, then there should be only one quarter as many in the galactic region as in the other regions.

In the case of Boss's zone, the total number of stars observed, and of those having a proper motion, found in the four regions described, are as follows :

	Total Number Observed.	Proper Motions.	Star-Density per hour.
Galactic strip, 5h. to 8h.	1,614	24	8
Galactic strip, 17h. to 20h.	1,340	36	12
Intermediate strip, 8h. to 17h.	2,458	124	12
Intermediate strip, 20h. to 5h.	2,831	111	12

The last column contains the average number of proper-motion stars per hour in each of the four strips. There is evidently no excess of richness in the galactic strips, but rather a deficiency in the strip near 6h., which we may regard as accidental.

In the case of Auwers's zone, the galactic strips are those between 5h. and 8h., and again between 18h. and 21h. Here, as in the other case, the galactic strips include one quarter of the whole area. But, owing to the greater richness of the sky, they include nearly forty per cent. of the whole number of stars. Then, if the-proper motion stars are equally distributed, one-quarter should be found in the galactic regions, and if they are proportional to the number of stars observed, forty per cent. should be within these regions. Grouping the regions outside the galaxy together, as we need not distinguish between them, the result is as follows:

	Stars Observed.	Proper Motions.	Star Density per hour.
Galactic strip, 5h. to 8h.	1,797	155	52
Galactic strip, 18h. to 21h.	1,984	202	67
Outside the galaxy	6,008	901	50

We see that in the galactic strip from 5h. to 8h. there is contained almost exactly one-eighth the whole number of proper-motion stars. That is, in this region the stars are no thicker than elsewhere. In the region from 18h. to 21h. there is an excess of 45 stars having proper motions, or 15 per hour. Whether this excess is real may well be doubted. It is scarcely, if at all, greater than might be the result of accidental inequalities of distribution. Were the proper-motion stars proportional to the whole number, there ought to be 240 within the strip. The actual number is 38 less than this.

It is to be remembered that Auwers's proper motions are not limited to a definite magnitude, as were Boss's, but that he looked for all stars having a sensible proper motion. The question, what proper motion would be sensible, is a somewhat indefinite one, depending very largely on the data. It may, therefore, well be that the small excess of 45 found within this strip is due to the fact that more stars were observed and investigated, and, therefore, more proper motions found. Besides this, some uncertainty may exist as to the reality of the minuter proper motions.

The conclusion is interesting and important. If we should blot out from the sky all the stars having no proper motion large enough to be detected, we should find remaining stars of all magnitudes; but they would be scattered almost uniformly over the sky, and show little or no tendency to crowd toward the galaxy, unless, perhaps, in the region near 19h. of R. A.

From this again it follows that the stars belonging to the galaxy lie farther away than those whose proper motions can be detected.

Distribution of Fifth-type Stars.

Pickering found that the stars of the fifth spectral type, or of Vogel's class II *b*, are mostly distributed along the central line of the Milky Way. An exception occurs in the case of a group situate in the "Magellanic clouds," a cloud-like mass of small stars too far south to be visible in our latitudes, and detached from the main course of the Milky Way itself. The total number of the stars in question is 91, of which 70 are in the Milky Way and 21 in the Magellanic clouds.

An interesting question now is whether the 70 stars along the Milky Way are arranged independently of the latter, or belong to its agglomerations. In the latter case we should expect to find most of the stars in the densest portions of the galaxy; in the former case they would be arranged independently of the galactic masses.

The actual distribution is not decidedly in favour of either view. Groups of the stars are found here and there in the densest spots of the galaxy; but there are also a number in the very darkest regions of the central line. The mean distance of the 70 stars from the central galactic circle is 2°. 6; the mean distance of 42 of the brightest regions of the galaxy from the same circle is 2°.3. The central circle which passes most nearly through the 71 stars has its pole in the position

R. A. = 18h. 44m., Dec. = + 26°. 6

The coincidence of this with the galactic circle is very close, the deviation being only a quarter of a degree.

Most curious is the unequal distribution of these stars around the galactic circle. Starting from the point where this circle crosses the equator near 18h. 40m. of R. A., and going toward the north there are

In	the	first quadrant		15	stars
"	"	second	"	3	"
"	"	third	"	21	"
"	"	fourth	"	31	"

Thus there are 18 stars in the first semicircle against 52 in the second. They are sometimes bunched together; thus in R. A. 10h. and Dec. — 60° there are 13h. of the stars in a region 5° square.

CHAPTER XVI

THE CLUSTERING OF THE STARS

The stars in deep amaze
Stand fixed in steadfast gaze,
Bending one way their precious influence
And will not take their flight
For all the morning light
Or Lucifer that often warned them thence.

A STUDY of Schiaparelli's planispheres, found in the last chapter, shows that some regions of the heavens are especially rich in lucid stars and others especially poor.

Neither telescope nor planisphere is necessary to show that many of those stars are collected in clusters. That the Pleiades form a group of stars by itself is clear from the consideration that six stars so bright would not fall so close together by accident. This conclusion is confirmed by their common proper motion, different from that of the stars around them. The singular collection of bright stars which form Orion, the most brilliant constellation in the heavens, and the little group called Coma Berenices—the Hair of Berenice—also suggest the problem of the possible connection of the stars which form them.

The question we now propose to consider is whether these clusters include within their limits an important number of the small stars seen in the same direction. If they and all the small stars which they contain within their actual limits were removed from the sky, would important gaps be left? The significance of this question will be readily seen. If important gaps would be left, it would follow that a large proportion of the stars which we see in the direction of the clusters really belong to the latter, and that, therefore, most of the stars would be contained within a limited region. The clusters which we shall especially study from this point of view are the Pleiades, Coma Berenices, Præsepe, and Orion.

The Pleiades.—In the case of this cluster the question was investigated by Professor Bailey, by means of a Harvard photograph 2° square, having Alcyone near its centre. It was divided into 144 squares, each 10′ on a side. The brighter stars of the cluster were included within 42 of these squares. The count of stars gave the results:

Within cluster: 1012 stars, or 24 per square.
Without cluster: 2960 stars, or 29 per square.

It therefore seems that the portion of the heavens covered by the cluster is actually poorer in stars than the region around it.

Two opposite conclusions might be suggested by this fact. Assuming that the difference is due to the presence of the cluster, we might suppose that the latter was formed of material that otherwise would have gone into numerous smaller stars. Accepting this view, it would follow that the material in question was a sheet so thin that the thickness of the space filled by the cluster was an important fraction of that occupied by the stars. In other words, one fifth of the stars of the region would be contained in a thin sheet. This result seems too unlikely to be accepted. The other and more likely conclusion is that the number of very minute stars included in the cluster is no greater than that in the surrounding regions, and that the lesser number in the region is to be regarded as accidental.

Coma Berenices.— This cluster, which may be seen east, south, or west of the zenith on a spring or summer evening, contains seven stars visible to the naked eye, each of the fifth magnitude. It may be considered as comprised within the limits 12h. 13m. and 12h. 25m. of R. A., and 25° to 29° of declination, an area of 10°.5. The existence of seven lucid stars within so small an area suggests that they belong together, and may have smaller stars belonging to the group, making the star-density of this area greater than that of the sky in general.

The question whether there is any corresponding excess of richness in the fainter stars will be decided by a count of those contained in Graham's section of the A. G. Catalogue, which extends to the ninth magnitude. Within the area above defined this catalogue gives 71 stars. Subtracting the 7 lucid stars, we have 64 small stars left within the area. To the same belt of declination 336 stars are listed in the twelfth hour of R. A., giving an average of 67 stars to an area equal to that of the cluster. The small stars are, therefore, no thicker within the area of the cluster than around it. It may be added that the seven lucid stars do not seem to have any common proper motion, so that their proximity is probably an accident.

Præsepe.—This object, situate in the constellation Cancer, appears to the naked eye as a patch of nebulous light. It is actually a condensed group of stars, of which the brightest are of the seventh magnitude. The stars of the ninth magnitude included within the area of the group probably belong, for the most part, to it, but they are too few to serve as the base for any positive conclusion.

Orion.—I find by measurement and count that a circle 20° in diameter, comprising the brightest stars of this constellation, contains 80 stars to magnitude 6.3. Of these, 6 are of the first or second, leaving 74 from the third to the sixth. The resulting richness is 24 to 100 square degrees, about the average richness along the borders of the galaxy. It follows that this remarkable collection of bright stars has no unusual collection of faint stars associated with it.

A very natural inquiry is whether the bright stars in Orion have any common proper motion, indicating that they form a system by themselves. The answer is shown in the following statement of the proper motions in a century :

Star.	Mag.	Proper Motions. R. A.	Proper Motions. Dec.
		″	″
Rigel	1	+0.1	0.0
η Orionis	3	+0.1	—0.3
γ Orionis	2	—0 6	—1.7
δ Orionis	2	0.0	—0.2

Star.	Mag.	Proper Motions. R. A. ″	Dec. ″
ε Orionis	2	0.0	+0.1
ζ Orionis	2	0.0	—1.4
κ Orionis	2	+0.1	—0.3
α Orionis	1	+3.0	+0.9

For the most part these motions are too small to be placed beyond doubt, even by all the observations hitherto made. In the case of Alpha Orionis the motion is established ; in those of Gamma and Zeta it is more or less probable, but not at all certain ; in all the other cases it is too small to be measured.

This minuteness of the motion makes it probable that these stars are very distant from us, an inference which is confirmed by the smallness of their parallaxes. The careful and long-continued measures of Gill show no parallax to Rigel, while Elkin finds one of only 0″.02 to Alpha Orionis.

The general conclusion from our examination is this : *The agglomeration of the brighter stars into clusters does not, in the cases where it is noticeable to the eye, extend to the fainter stars.*

Let us now study the question on the opposite side. Schiaparelli's planispheres show regions of great paucity in lucid stars ; is there in these regions any paucity of telescopic stars ?

The two regions of greatest paucity are near the equator ; one extends through the hour 0 of R. A. ; the other from 12h. 20m. to 12h. 40m. The richness of these and of the adjoining regions may be inferred from Boss's zone of the A. G. Catalogue, including a belt from 1° to 5° of declination. The number of stars observed by Boss in each hour from 23h. to 3h. is as follows:

In 23h.: 271 stars.
In 0h.: 293 stars.
In 1h.: 299 stars.
In 2h.: 295 stars.

These numbers show no paucity in the hour 0, and no excess in the hour 2, which is much richer in lucid stars than the hour 0.

In the strip from 12h. 20m. to 12h. 40m. the catalogue contains 78 stars, a richness of 234 to the hour. In the hour preceding there are 211 stars; in that following, 225. There is, therefore, no paucity in the strip in question.

We conclude from all this that the separate stars of a cluster do not range through a scale of brightness so wide as the stars in general, and that they are limited in number. The numerous small stars seen in the same direction have no connection with them. But we shall see that this rule does not apply to the clusters of the galaxy.

CHAPTER XVII

THE STRUCTURE OF THE MILKY WAY

> A broad and ample road whose dust is gold,
> And pavement stars, as stars to thee appear
> Seen in the galaxy, that milky way
> Which nightly as a circling zone thou seest
> Powdered with stars.—MILTON.

THE most salient problems suggested by the appearance of the Milky Way are to be approached on lines quite similar to those followed in the last chapter. We begin with a description of this wonderful object as it appears to the observer. It can be seen through some part of its course at some hour on any clear night of the year, and in the evening of any season except that of early summer. In consequence of its obliquity to the equator, its apparent position on the celestial sphere, as seen in our latitude, goes through a daily change with the diurnal rotation of the earth. In the language of technical astronomy, every day at 12h. of sidereal time, it makes so small an angle with the horizon as to be scarcely visible. If the air is very clear, we might see a portion of it skirting the northern horizon. This position occurs during the evenings of early summer. At oh. of sidereal time, which during autumn and early winter fall in the evening, it passes nearly through our zenith, from east to west, and can, therefore, then best be seen. We begin with the portion which will be visible in the late summer or early autumn. We can then trace its course southward from Cassiopeia in the northwest. It passes a little east of the zenith down to Sagittarius, near the south horizon. This portion of the belt is remarkable for its diversity of structure and the intensity of the brighter regions.

In Cassiopeia it shows nothing remarkable; but above this constellation, in Cepheus, we notice in the midst of the brighter region a nearly circular and comparatively dark patch several degrees in diameter. A little farther along we notice a similar elongated patch in Cygnus lying across the course of the belt. In this region the brighter portions are of great breadth, more than 20°.

In Cygnus begins the most remarkable feature of the Milky Way, the great bifurcation. Faintly visible near the zenith, as we trace it towards the south, we see it grow more and more distinct, until we reach the constellation Aquila, near the equator. Between Cygnus and Aquila the western branch seems to be the brighter and better marked of the two, and might, therefore, be taken for the main branch. About Aquila the two appear equal, but south of this constellation we see the western branch diverge yet farther toward the west, leaving the gap between it and the eastern yet broader and more distinct than before. This branch finally terminates in the constellation Ophiuchus, while the eastern branch, growing narrower, can still be followed toward the south.

Between the equator and the southern horizon we have the brightest and most irregular regions of all. Several round, bright patches of greater or less intensity are projected on a background sometimes moderately bright and sometimes quite dark. If the night is quite clear and moonless we shall see that, after a vacant stretch, the western branch seems to recommence just about the constellation Scorpius. In this constellation we have again a bifurcation, a dark region between two bright ones.

This is about as far as the object can be well traced in our middle latitudes. From a point of view nearer to the equator it can be traced through its whole extent. South of Scorpius and Sagittarius it becomes broad, faint, and diffused through the constellations of Norma and Circinus. It reaches its farthest southern limit in the Southern Cross, where it becomes narrower and better defined. The most remarkable feature here is the "coal sack," a dark opening of elliptical shape in the central line of the stream. West and north of this, in the constellation Argo, is the broadest and most diffused part of the whole stream, the breadth reaching fully 30°. Here we again reach the portion which rises above our horizon.

Returning now to our starting-point, we shall notice that, as we make our observations later and later in the autumn, the southern part, which we have been mostly studying, is seen night by night lower down in the west, while new regions are coming into view in the north-east and east. These regions rise earlier every evening, and, if we continue our observations to a later hour, we shall see more and more of them above the eastern or south-eastern horizon. By midwinter Cassiopeia will be seen in the north-west, and we can readily trace the course of the galaxy from that constellation in the opposite direction from that which we have been following. South of Cassiopeia we see, near the central line, the well-known cluster forming the sword-handle of Perseus. Farther south the belt grows narrower and fainter; although the irregularities of structure continue, they are far less striking than on the other side. On a moonlight evening it will scarcely be visible at all. If the moon is absent and the air clear we shall see that it grows slightly brighter toward the southern horizon, near which will be the narrowest part of its entire course. Below is the broad and diffused region in Argo already mentioned.

One conclusion from the inequalities of structure which we have described will be quite obvious. The Milky Way is something more than the result of the general tendency of the stars to increase in number as we approach its central line. There must be large local aggregations of stars, because, as we have already pointed out, there cannot be such diversity of structure shown in a view of a very widely stretched stratum of stars.

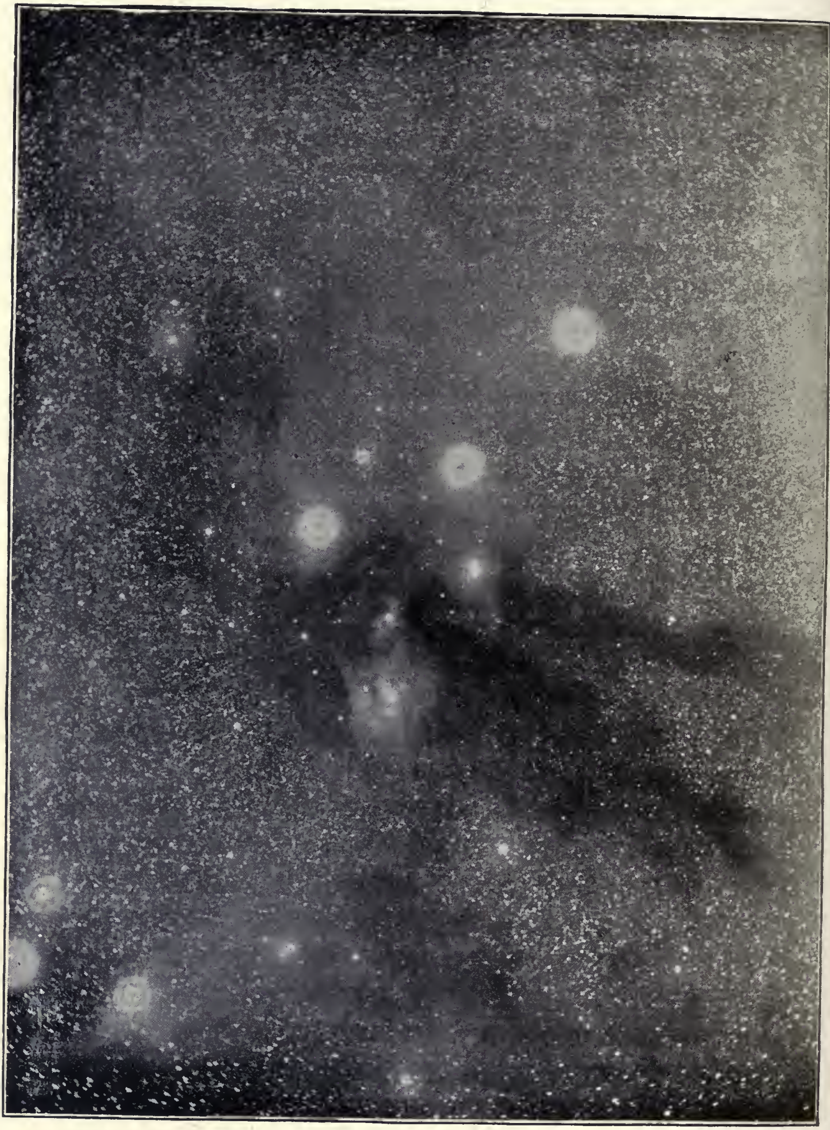
When, instead of a naked-eye view of the belt, we study the photographs of the Milky Way, we find

PHOTOGRAPH SHOWING STRUCTURE OF THE MILKY WAY, BY BARNARD.

this evidence of clustering to grow still stronger. It is seen very strikingly in the photograph by Barnard showing the singular rifts in the Milky Way in the constellation Ophiuchus. Yet more singular are three small openings very close together in the constellation Aquila, the positions of which are:

(1) R. A. = 19h. 35.0m.; Dec. = + 10° 17′.
(2) " = 19h. 36.5m.; " = + 10° 37′.
(3) " = 19h. 37.2m.; " = + 11° 2′.

The fundamental question which we meet in our further study of this subject is: At what magnitude do these agglomerations of stars begin? Admitting, as we must, that they are local, are they composed altogether of faint stars, or do they also include the brighter stars within their limits? We consider this question in a way quite similar to that in which we discussed the clustering of the stars in the last chapter. We mark out on a map of the Milky Way the brightest regions—that is, those which include the densest agglomeration of very faint stars. We also mark out the darkest regions, including the coal sack. For this purpose I have taken the maps found in Heis's *Atlas Cœlestis* for the northern portion of the Milky Way and the Atlas of Gould's *Uranometria Argentina* for the southern portion. In order to enable anyone to repeat and verify the work I give the position of the central part of each patch or region studied. This serves simply for the purpose of indentification. The outlines can be drawn by anyone when the patch is identified. In the



RIFTS IN THE MILKY WAY, PHOTOGRAPHED BY BARNARD.

third column of the table is given, approximately, the number of square degrees in the patch as outlined. Then follows the number of stars found on the map. Here are included stars somewhat fainter than those regarded as lucid. Heis maps all stars down to about magnitude 6.2 or 6.3. Gould gives the places of all stars to magnitude 7.

A.—*Number of lucid stars in certain bright regions or patches of the Milky Way.*

I.—Northern portion, from Heis.

Position of patch.		Area.	Number
R. A.	Dec.	sq. deg.	of stars.
19h. 10m.	+ 35°	60	21
20h. 0m.	+ 37°	25	11
20h. 20m.	+ 47°	20	11
21h. 5m.	+ 45°	12	4
0h. 20m.	+ 60°	25	9
2h. 20m.	+ 55°	60	16
3h. 30m.	+ 36°	32	7
3h. 40m.	+ 44°	43	12
Sums.............		277	91

II.—Southern portion, from Gould :

Position.		Area.	
R. A.	Dec.	sq. deg.	Stars.
8h. 4m.	— 47°	10	14
2h. 24m.	— 44°	9	7
10h. 35m.	— 58°	12	19
11h. 40m.	— 62°	10	11
16h. 10m.	— 53°	7	7
18h. 0m.	— 28°	25	9
18h. 10m.	— 18°	8	5
18h. 42m.	— 8°	16	5
Sums.............		97	77

B.—*Number of lucid stars in the darker regions or patches of the Milky Way.*

I.—Northern part, from Heis.

Position. R. A.	Position. Dec.	Area. sq. deg.	Stars.
21h. 0m.	+ 50°	26	10
22h. 0m.	+ 67°	33	7
22h. 25m.	+ 60°	30	12
0h. 0m.	+ 69°	56	10
4h. 0m.	+ 55°	98	19
4h. 20m.	+ 35°	98	13
6h. 15m.	+ 18°	86	17
6h. 12m.	+ 4°	48	9
Sums		475	97

II.—Southern part, from Gould.

Position. R. A.	Position. Dec.	Area. sq. deg.	Stars.
7h. 22m.	— 38°	18	8
7h. 28m.	— 38°	12	5
8h. 0m.	— 22°	11	4
8h. 40m.	— 50°	30	16
9h. 0m.	— 45°	12	6
10h. 0m.	— 52°	11	5
12h. 40m.	— 63°	18	2
15h. 10m.	— 56°	31	3
17h. 30m.	— 27°	18	3
18h. 10m.	— 35°	18	7
18h. 0m.	— 22° }	24[1]	10
18h. 30m.	— 8° }		
18h. 50m.	— 5°	16	5
Sums		219	74

[1] A long narrow region between the limits defined in the first two columns.

To derive the best conclusions from these numbers we must compare them with the mean star-density for the sky in general, and for the regions near the galactic plane. Heis has 3903 stars north of the equator; Gould, 6755 south of it. The area of each hemisphere is 20,626 square degrees. It will be convenient to express the various star-densities in terms of 100 square degrees as the unit of area. Thus we have the following star-densities according to the two authorities:

	His.	Gould.
Star-density of the entire hemisphere	19.0	32.7
Star-density of the darker galactic regions	20.4	33.8
Star-density of the bright galactic regions	32.9	79.4

The first two pairs of numbers lead to the remarkable and unexpected conclusion that the darker regions of the Milky Way are but slightly richer in lucid stars than the average of the whole sky; certainly no richer than is due to the general tendency of all the stars to crowd toward the galactic plane. On the other hand, the bright areas are 60 per cent. richer according to Heis, and more than 100 per cent. richer according to Gould, than the darker areas seen among and around them. The conclusion is that an important fraction of the lucid stars which we see in the same areas with the agglomerations of the Milky Way is really in those agglomerations and form part of them.

A study quite similar to this has been made by Easton for the portions of the Milky Way between Cygnus and Aquila, where the diversities of brightness are greatest. His count of the stars in the bright and dark regions differs from that made above principally by including all the stars of the *Durchmusterung*, which we may suppose to extend to about the ninth magnitude.[1]

He divides the regions studied into six degrees of brightness. For our present purpose it is only necessary to consider three regions, the brightest, the faintest, and those intermediate between the two. Besides the count from the *Durchmusterung* he made a count of the same sort from Dr. Wolf's photographs and from Herschel's gauges of the heavens. In the following table I have reduced all his results so as to express the number of stars in a square degree in the three separate regions. At the top of each column is given the authority, whether Argelander, Wolf, or Herschel. Wolf had two sets of photographs, one supposed to include all the stars to the eleventh, the other to the twelfth magnitude. The magnitudes included are given in the second line. That Herschel's count extends to the fifteenth magnitude is by no means certain; but we can judge from the great number of his stars that it goes considerably beyond Wolf's in the faintness of the stars included. Below this we give, in the regions A, B, and C, which are respectively those of least, of medium, and of greatest brightness, the number of stars per square degree according to each of the authorities:

[1] Easton's work is given in detail in the *Astronomische Nachrichten*, vol. 137, and the *Astrophysical Journal*, vol. i, no. 3.

Authority	Arg.	Wolf(A)	Wolf(B)	Herschel.
Magnitude	1—9	1—11	1—12	1—15(?)
Region A	23	72	224	405
Region B	33	134	764	4114
Region C	48	217	1266	6920
C—A	25	145	1042	6425
Ratio C : A	2.1	3.0	5.7	14.0

The vastly greater number of individual stars per square degree in the brighter regions is what we should expect from the studies we have made of the lucid stars. But what is of most interest in the table is the continual increase in the proportion of faint stars in the separate regions. We notice that, when we consider only the stars of the ninth magnitude, there are twice as many in the brightest as in the darkest portions. When we go to the eleventh magnitude, as shown by Wolf's photograph A, we find the number of stars in the brighter regions to be threefold. When the twelfth magnitude is included we find that there are between five and six times as many stars in the bright regions as the dark ones. Finally, when we come to stars from Herschel's gauges there are fourteen times as many stars per square degree in the brighter regions as in the dark.

At first sight this result seems to show a great difference between the clusters of stars described in the last chapter, and the collections of the Milky Way, in that the former include few or no faint stars, while the latter include a greater and greater number as we ascend in the scale of magnitude. This difference is important as showing a vastly greater range of actual brightness among the galactic stars than among those which form the scattered clusters. Allowing for this difference, the results from the two classes of objects can be brought to converge harmoniously toward the same conclusion.

We have collected abundant evidence that, separate from the accumulations of stars in the Milky Way, perhaps extending beyond them, there is a vast collection of scattered stars, spread out in the direction of the galactic plane, as already described, which fill the celestial spaces in every direction. We have shown that when, from any one area of the sky, we abstract the stars contained in clusters, this great mass is not seriously diminished. We have also collected abundant evidence that the distances of this great mass are very unequal; in other words, there is no great accumulation, in a superficial layer, at some one distance. The question which now arises is whether the darker areas which we see in the Milky Way are vacancies in this mass. Although some of the counts seem to show that they are, yet a general comparison leads to the contrary conclusion. In the darkest areas of the Milky Way, when of great extent, the stars are as numerous as on each side of the galactic zone. Our general conclusion is this:

If we should remove from the sky all the local aggregations of stars, and also the entire collection which forms the cloud-forms of the Milky Way, we should have left a scattered collection, constantly increasing in density toward the galactic belt.

CHAPTER XVIII

THE PROGRESSION IN THE NUMBER OF STARS AS THE BRIGHTNESS DIMINISHES

Hither, as to their fountain, other stars
Repairing, in their golden urns draw light.—MILTON.

WE mentioned in an earlier chapter that, when we compare the number of stars of each successive order of magnitude with the number of the order next lower, we find it to be, in a general way, between three and four times as great. The ratio in question is so important that a special name must be devised for it. For want of a better term, we shall call it the star-ratio. It may easily be shown that there must be some limit of magnitude at which the ratio falls off. For a remarkable conclusion from the observed ratio for the stars of the lower order of magnitude is that the totality of light received from each successive order goes on increasing. Photometric measures show, as we have seen, that a star of magnitude *m* gives very nearly 2.5 times as much light as one of magnitude $m+1$. The number of stars of magnitude $m+1$ being, approximately from 3 to 3.75 times as great as those of magnitude *m*, it follows that the total amount of light which they give us is some 40 or 50 per cent. greater than that received from magnitude *m*. Using only rough approximations, the amount of light will be about doubled by a change of two units of magnitude; thus the totality of stars of the sixth magnitude gives twice as much light as that of the fourth; that of the eighth twice as much light as that of the sixth; that of the tenth twice as much again as of the eighth, and so on as far as accurate observations and counts have been made.

To give numerical precision to this result, let us take as unity the total amount of light received from the stars of the first magnitude. The sum-total for this and the other magnitudes, up to the tenth, will then be:

Mag. 1	Light	=	1.0
" 2	"	=	1.4
" 3	"	=	2.0
" 4	"	=	2.8
" 5	"	=	4.0
" 6	"	=	5.7
" 7	"	=	8.0
" 8	"	=	11.3
" 9	"	=	16.0
" 10	"	=	22.6
Total			74.8

That is, from all the stars to the tenth magnitude combined, we have more than seventy times as much light as from those of the first magnitude.

There must, evidently, be an end to this series, for, were this not the case, the result would be that which we have shown to follow if the universe were infinite; the whole heaven would shine with a blaze of light like the sun. At what point does the rate of increase begin to fall off?

We are as yet unable to answer this question, because we have nothing like an accurate count of stars above the ninth, or at most, the tenth magnitude. All we can do is to examine the data which we have and see what evidence can be found from them of a diminution of the ratio.

It must be pointed out, at the outset, that the ratio must be greater in the galactic region than it is in other regions. This follows from the fact that the proportion of small stars increases at a more rapid rate in the galaxy than elsewhere. This is shown by the comparisons we have already made of the Herschelian gauges with the counts of the brighter stars. While the galactic region is less than twice as dense as the remaining regions for the brighter stars, it seems to be ten times as dense for the Herschelian stars. If we knew the limiting magnitude of the latter, we could at once draw some numerical conclusion. But unfortunately this is quite unknown. All we know is that they were the smallest stars that Herschel could see with his telescope.

The ratio in various regions of the heavens has been very exhaustively investigated by Seeliger, in the work already quoted. The bases of his investigations are the counts of stars in the *Durchmusterung*. Instead of taking the ratio for stars differing by units of magnitude, as we have done, Seeliger divides them according to half-magnitudes. The reproduction of his numbers in detail would take more space than we can here devote to the subject and would not be of special interest to our readers. I have, therefore, derived their general mean results for different parts of the sky with reference to the Milky Way and for stars of the various orders of magnitude. The following table shows the conclusions:

Zone.	Ratio of increase. D. M.	S. D.	Diff.	Concluded result.
I.	2.99	—	—	3.24
II.	3.00	3.49	0.49	3.25
III.	3.07	3.72	0.65	3.37
IV.	3.32	3.85	0.53	3.58
V.	3.55	4.15	0.60	3.85
VI.	3.28	3.68	0.40	3.48
VII.	3.23	3.55	0.32	3.37
VIII.	3.44	3.56	0.12	3.40
IX.	—	3.49	—	3.24

In the first column we have the designation of the zone or region of the sky, as already given.

In the second and third columns we have the mean ratio of increase for whole magnitudes as derived from the *Durchmusterung* and the *Southern Durchmusterung*, respectively. It will be recalled that region I., around the north galactic pole, is entirely wanting in the S. D., while the adjoining regions, II. and III., are only partially found, and that, in like manner, the D. M. includes none of region IX. around the south galactic pole, and but little of the adjoining region.

It will be seen that there is a very remarkable systematic difference between the two lists, the ratio of the number of faint to that of bright stars being much greater in the S. D. This difference is shown in the fourth column. I have assumed that the two systems are equally good, and so diminished all the ratios of the S. D. by 0.25, and increased those of the D. M. by the same amount. The mean of the two corrected results was then taken, giving the principal weight to the one or the other, according to the number of stars on which they depend.

It will be seen that the increase of the ratio from either galactic pole to the Milky Way itself is as well marked as the increase of the richness of the respective regions in stars in general. We may condense the results in this way:

In the galactic zone,	ratio = 3.85
In zones IV. and VI.,	" = 3.53
In polar zones I., II., VIII., and IX.,	" = 3.28

It will be recalled that zone V. is a central belt 20° broad, including the Milky Way in its limits. But the latter, as seen by the eye, especially its brightest portions, does not fill this zone. These portions, as we know, comprise the irregular collection of cloud-like masses described in the last chapter. Seeliger has investigated the ratio within these masses, and compared it with the stellar density, or the number of stars per square degree. The mean results are:

In that portion of the galaxy extending from Cassiopeia to the equator near 6h. of R. A., ratio = 4.02.

In that portion from Cassiopeia in the opposite direction to near 19h. of R. A., in Aquila, ratio = 3.70.

These remarkable results are derived from the D. M., and will be yet more striking if corrected by half the difference between it and the S. D., as we have done for the sky generally. They will then be 4.27 and 3.95, respectively.

As might be expected, the regions of greater star density have generally, though not always, the higher ratio. The highest of all is in a patch south of Gemini, between 6h. and 7h. of R.A., and near + 5° of declination. Here it amounts to 5.94, showing that there are eighty-six stars of magnitude 9.0 to every one of magnitude 6.5.

The D. M. does not stop at magnitude 9, as the above numbers do, but extends to 9.4, while the S. D. extends to magnitude 10. For these magnitudes Seeliger finds a yet higher ratio. This is, however, to be attributed to the personal equation of the observers, and need not be further considered.

The only available material for estimating the ratio of increase above the ninth magnitude is found in the Potsdam photographs for the international chart of the heavens, which extend to magnitude 11. These are published only for a few special regions. Five of the published plates fall in regions not far from the galactic pole. I have made a count by magnitudes of the 312 stars contained in these plates. An adjustment is, however, necessary from the fact that the minuter fractions of a magnitude could not be precisely determined from the photographed images. The results are practically given to fourths of a magnitude, although expressed in tenths. But it is found that the numbers corresponding to round magnitudes and their halves are disproportionately more frequent than those corresponding to the intermediate fourths. For example, there are only 19 stars of magnitude 10.7 and 10.8 taken together; while there are 49 of 10.5. Under these circumstances I have made an adjustment to half-magnitudes by taking the stars of quarter-magnitudes and dividing them between half-magnitudes next higher and next lower. The number of stars of the several magnitudes is then as follows :

Mag.	Stars.
6.5	2
7.0	2
7.5	4
8.0	11
8.5	15
9.0	29
9.5	33
10 0	39
10.5	64
11.0	115

It is difficult to derive a precise value of the star-ratio from this table, owing to the small number of stars of the brighter magnitudes, which are insufficient to form the first term of the ratio. Assuming, however, that the ratio is otherwise satisfactorily determined up to the ninth magnitude, we find that there is but a slight increase from the ninth up to the tenth. The number of the eleventh magnitude is, however, nearly three times that of the tenth and nearly double that of 10.5.

Another way to consider the subject is to compare the total number of stars of the fainter magnitudes with the number of lucid stars corresponding, which, in the general average, will be found in the same space. We may assume that near the poles of the galaxy there is about one lucid star to every ten square degrees. The five belts included in the above statement cover about thirteen square degrees. The region is, therefore, that which would contain about one star of the sixth magnitude. An increase of this number by somewhat more than 100 times in the five steps from the sixth magnitude to the eleventh would indicate a ratio somewhat less than 3; about 2.5. But the comparison of the photographic and visual magnitudes renders this estimate somewhat doubtful. Besides this, it is questionable whether we should not reckon among stars of the eleventh magnitude those up to 11.5, which would greatly increase the number. It is a little uncertain whether we should regard the limit of magnitude on the Potsdam plates as 11.0 or 11 plus some fraction near to one half.

Altogether, our general conclusion must be that up to the eleventh magnitude there is no marked falling off in the ratio of increase, even near the poles of the galaxy.

I have not made a corresponding count for the galactic region, but the great number of stars given on the plates show, as we might expect, that there is no diminution in the ratio of increase.

The question where the series begins to fall away is, therefore, still an undecided one, and must remain so until a very exact count is made of the photographs taken for the international photographic chart of the heavens, or of the Harvard photographs.

There is also a possibility of applying a photometric study of the sky to the question. The background of the sky itself is by no means black. The question to be investigated is whether a considerable fraction of the apparently smooth and uniform light of the nightly sky comes from countless telescopic stars, perhaps from stars too faint to be found on the most delicate photographs, or whether it is mostly reflected by our atmosphere from the stars. It may seem questionable whether the latter is the case, because the fraction reflected in a clear atmosphere is not supposed to exceed one tenth the total amount of light of the stars themselves. On the other hand, the seemingly blue colour of the sky might seem to indicate reflected light, since the average colour of all the stars is white rather than blue. The subject is an extremely interesting one and requires investigation before a definitive conclusion can be reached.

CHAPTER XIX

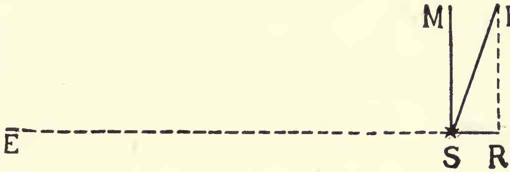
STATISTICAL STUDIES OF PROPER MOTIONS

How charming is divine philosophy,
Not harsh and crabbed as dull fools suppose,
But musical as is Apollo's lute,
And a perpetual feast of nectared sweets
Where no crude surfeit reigns.—MILTON.

THE number of stars now found to have a proper motion is sufficiently great to apply a statistical method to their study. The principal steps in this study have been taken by Kapteyn, who, in several papers published during the past ten years, has shown how important conclusions may be drawn in this way.

We must begin our subject by showing the geometrical relations of the proper motion of a star, considered as an actuality in space, to the proper motion as we see it. The motion in question is supposed to take place in a straight line with uniform velocity. Leaving out the rare cases of variations in the motion due to the attraction of a revolving body, there is nothing either in observation or theory to justify us in assuming any deviation from this law of uniformity. The direction of a motion has no relation to the direction from the earth to the star. That is to say, it may make any angle whatever with that direction.

Let E be the position of our solar system, and S that of a star moving in the direction of a straight line, S D. It must not be understood that the length of this line is taken to represent the actual motion; the latter would be infinitesimal as compared with its length; we use it only to show direction. We may, however, use the line to represent on a magnified scale the actual amount of the motion during any unit of time, say one year. It may be divided into two components: one, S R, in the direction of the line of sight from us to the star, which for brevity we shall call the radial line, and the other, S M, at right angles to that line.



COMPONENTS OF PROPER MOTION.

It must be understood that, as the term "proper motion" is commonly used, only the component S M can be referred to, because the radial component, S R, does not admit of being determined by telescopic vision. As we know from the preceding chapters, it can in the case of the brighter stars be determined by spectroscopic measurement of the radial motion.

The visible component, S M, can also be resolved into two perpendicular components, the one east and west on the celestial sphere, the other north and south. The former is the proper motion in right ascension (the measured motion in this co-ordinate being multiplied by the cosine of the declination to reduce it to a great circle), and the other is the proper motion in declination. In star catalogues these two motions are given, so far as practicable. Thus, altogether, the actual motion of a star in space may be resolved into three components: that of right ascension, that of declination, and the radial component.

An additional consideration is now to be added. The proper motion of a star, as observed and given in catalogues, is a motion relative to our system. It has been shown in a former chapter that the latter has a proper motion of its own. When account is taken of this, and the motions are all reduced as well as we can to a common centre of gravity of the whole stellar system, we conceive the observed proper motion of the star to be made up of two parts, of which one is the actual motion of the star relative to the common centre, and the other due to the motion of the sun, carrying the earth with it. The direction of the latter appears to us opposite that of the motion of the sun. The sun's motion being directed to the constellation Lyra, it follows that the component in question in the case of the stars is directed toward the opposite constellation, Argo. This component, as we know, is termed the parallactic motion, being dependent on the distance or parallax of the star.

As in the case of other proper motions, we may measure the parallactic motion either in angular measure, as so many seconds per century, or in linear measure, as so many kilometres per second. The relation of the two measures depends on the distance of a star. The simplest conception of the relation may be gained by reflecting that the linear speed of the parallactic motion must be equal to that of the sun.

We have cited Campbell's result for the speed of the solar motion, which is between 19 and 20 km. per second, or 4 radii of the earth's orbit per year. Accepting this speed we shall have the following rule:

The parallax of a star lying in a direction nearly at right angles to that of the solar motion is equal to one fourth of its annual parallactic motion.

In the case of stars in other directions, the parallactic motion for a given parallax would be less in proportion to the sine of the angle between the direction of the star and the solar apex.

If the stars were at rest this rule would enable us immediately to determine the distance of any star by its proper motion, which would then be simply the parallactic motion itself. Unfortunately, in the case of any one star considered individually, there is no way of deciding how much of its motion is proper to itself and how much is the parallactic motion. But when we consider the great mass of stars, it is possible in a rough way to make a distinction between the two motions in a general average.

The direction or motion of any particular star, having no reference to that of the sun, is as likely to be in the direction of one of the three components we have described as of any other. Hence, in the average of a great number of stars we may conclude that these components are equal.

One of the simplest applications of this law will enable us to compute the mean parallax of the stars whose radial motions have been determined. As this application is, in the present connection, made only for the purpose of illustration, I shall confine myself to the 47 stars of which the radial motions have been measured by Vogel. The mean annual proper motions of these stars, taken without any regard to their signs, are:

	Including Arcturus.	Omitting Arcturus.
	″	″
In right ascension...	0.163	0.144
In declination.......	0.155	0.118

The difference of the mean motions in right ascension and declination is to be regarded as accidental. The velocity of Arcturus is so exceptionally great that we ought, perhaps, to leave it out in taking the mean.

Now, the mean of the radial motions as found by Vogel is 16 kilometres per second. By hypothesis the actual motion in the radial line is in the general average the same as in the other two directions. We have, therefore, to determine what must be the parallax of a star in order that, moving with a velocity of 16 kilometres per second, its angular proper motion may have one of the above values. This result is by a simple computation found to be:

	″		″
From the mean motion in R. A..........	0.049	or	0.043
From the mean motion in Dec..........	0.046	or	0.035

The difference of these results, which depends on the omission or exclusion of Arcturus, shows the amount of uncertainty of the method. Our general conclusion, therefore, is that the mean parallax of the Vogel stars, which may be regarded as corresponding approximately to the mean parallax of all the stars of the second magnitude, is about 0″.04.

We have spoken of the two components of the apparent motion as those in right ascension and declination, respectively. But there is no particular significance in the direction of these co-ordinates, which have no relation to the heavens at large. For some purposes it will be better to take as the two directions in which the motions are to be resolved that of the parallactic motion and that at right angles to it. That is to say, taking the solar apex as a pole, we conceive an arc of a great circle drawn upon the celestial sphere from it to the star, and resolve the apparent motion into two components, the one in the direction of this arc, the other at right angles to it. The former, which we may call the *apical motion*, is affected by the parallactic motion; the latter, which we call the *cross motion*, is not, and therefore shows the true component of the motion of the star itself in the direction indicated.

Kapteyn has gone through the labour of resolving all the proper motions of the Bradley stars given by Auwers, in this way. His assumed position of the solar apex was:

Right ascension..............	276° = 18h. 24m.
Declination [1].................	+34°

[1] This work of Kapteyn is unpublished. The author is indebted to his

The radically new treatment in his discussion of the distribution of the stars in space embraces three points. The first consists in the distinction between the spectral types of the different stars and the separate study of the proper motions peculiar to each type. The next point is the reference of the motions to the solar apex. The third is the study of the relations of the stars to the galactic plane.

A remarkable relation existing between the spectral type of stars and their proper motions[1] was brought out by these investigations. The stars of Type I. have, in the general mean, smaller proper motions than those of Type II. The following table is made up from Kapteyn's work. First we give the limits of proper motion; then on the same line the number of stars of the respective Types I. and II. having proper motions within these limits:

Centennial prop. motions. ″ ″	Number of stars. Type I.	Type II.
0 to 5	786	474
6 to 9	203	194
10 to 19	159	223
20 to 29	25	86
30 to 49	13	71
50 and more	3	58
Total	1189	1106

courtesy for a manuscript copy, with permission to use it. Kapteyn's researches based on this material are contained in a series of papers communicated to the Amsterdam Academy of Science. An abstract in English of one of the earlier papers is found in *Knowledge* for June 1, 1893.

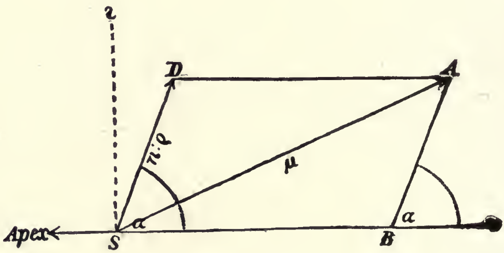
[1] The author believes that Monck, of England, independently pointed out this relation, perhaps in advance of Kapteyn.

It will be seen that in the case of stars having proper motions of less than 5″ per century a large majority are of Type I. In the case of proper motions between 6″ and 9″ the number is nearly equal. Between 10″ and 20″ there is a large majority of Type II. Between 30″ and 49″ the number of Type II. is nearly five times that of Type I. Finally, only three stars of Type I. have proper motions exceeding 50″, while fifty-eight stars of Type II. have a proper motion exceeding this limit.

We may make two hypotheses on this subject: one, that the stars of Type II. really move more rapidly than those of Type I.; the other, that their actual motion is the same, but that the stars of Type I. are more distant stars. The last conclusion seems much more probable, and is strengthened by the much greater condensation of stars of Type I. toward the Milky Way.

Let us now consider the principles by which we may study a great collection of proper motions statistically. There are scattered around us in the stellar spaces, in every direction from us, a large number of stars, each moving onward in a straight line and in a direction which, with rare exceptions, has nothing in common with the motion of any other star. The velocities of the motion vary from one star to another in a way that cannot be determined, some moving slowly and some rapidly. Is it possible from such a maze of motions to determine anything? Certainly we cannot learn all that we wish, yet we may learn something that will help us to form some idea of the respective distances of the stars and the actual velocity of their motions. An obvious remark is that the more distant a star the slower it will seem to move. We must, therefore, distinguish between the linear or actual motion of a star, expressed as so many kilometres per second, and its apparent or angular motion of so many seconds per year, derived by measuring its change of direction as we see it with our instruments.

We shall now endeavour to explain Kapteyn's method in such a way that the reasoning shall be clear without repeating the algebraic operations which it involves. Let us conceive that the following Fig. is drawn on the celestial sphere as we look up at the heavens. S is the direction of a star in the sky as we see it. Let us also suppose



that the solar apex, situated in the constellation Lyra, lies anywhere horizontally to the left of the star, in the direction of the arrow-head marked *Apex*. Suppose also that, were the solar system at rest, we should see the star moving along the line S. D. Let the length of the line S D represent the motion in some unit of time, say, one year. Next, suppose the star at rest. Then in consequence of the motion of the solar system, by which we are carried toward the apex, the star would seem to be moving with its parallactic motion in the direction S B, away from the apex. Let the length of this line represent the parallactic motion in one year. Then by the theory of composition of motions, the star, moving by its real motion from S to D, and by the motion of the earth having an apparent motion from S to B, will appear to us to move along the diagonal S A of the parallelogram. Thus, the line S A will represent the annual proper motion of the star as we observe it with our instruments, and which can be resolved into the apical motion, in the direction S B, and its cross-motion in the direction S.

The apical motion consists of two parts, one the parallactic motion, equal to S B; the other real, and due to the motion of the star itself along the line S D, and equal to the distance of D from the line S τ.

We have now to inquire how, in the case of a great number of stars, we may distinguish between these two parts of the apical motion.

We must make the general hypothesis that, in the average of a great number of stars, actual motions have no relation to the direction of our sun from the star. Then the components of the actual motion, S D, will in the general average have equal values, positive and negative motions cancelling each other. Hence, if we take the mean of a great number of motions along the apical line it will give us the value of S B due to the motion of the earth, and, hence, the mean parallactic motion of all the stars considered.

The problem now becomes one of averages. We wish to form at least a rude estimate of the average speed of a star in miles or kilometres per second. To show how this may be done let us suppose that we observe the proper motions of a great number of stars at some distance from the solar apex, so that their parallactic motion shall be observable. Stumpe and Ristenpart, the German astronomers, as well as Kapteyn, have considered the relation between the two motions in the following way: We divide the stars observed into classes, taking, say, one class having small but easily measured proper motion; another having a proper motion near the average, and a third, of large proper motion. Sometimes a fourth class is added, consisting of stars having exceptionally large proper motions. From each of these classes we can determine, as already shown, the average motion from the direction of the solar apex; that is to say, the average parallactic motion. This will be inversely as the average distance of the stars.

Stumpe's three classes were: I., proper motions ranging from 16″ to 32″ per century; II., between 32″ and 64″ per century; III., between 64″ and 128″ per century; IV., greater than 128″. The average of the proper motions in each class, the average of the apparent apical motions, and the ratio of the two are these:

Class.	Prop. Mot. ″	Par. Mot. ″	Quotient.
I.	0.23	0.142	1.6
II.	0.43	0.286	1.5
III.	0.85	0.583	1.4
IV.	2.39	2.057	1.1

It will be seen that the ratio of the proper motion of the star to the parallactic motion diminishes as the former increases.

The same thing was found by Ristenpart from the proper motions of the Berlin zone, as shown below:

Class.	Prop. Mot.	Par. Mot.	Quotient.
	″	″	
Small	0.128	0.061	2.1
Medium	0.197	0.109	1.8
Large	0.374	0.279	1.3

The smaller value of the quotient from stars near to us than from the more distant stars was supposed to lead to the conclusion that the latter had a more rapid real motion than the former. A little thought will show that, while this is quite true of the stars included in the list, this does not prove it to be true for the stars in general. We cannot, as already pointed out, determine the motion of any star unless it exceeds a certain limit. Hence, in the case of the more distant stars we can observe the proper motions only of those which move most rapidly, while in the case of the nearer ones we may have measured them all. We should, therefore, naturally expect that the more distant stars in our list will show too large a value of the proper motion, for the simple reason that those having small proper motion are not included in the average. There is, therefore, no evidence that the more distant stars move faster than the nearer ones.

An error in the opposite direction occurs through the method of selecting stars of given proper motion. We have already pointed out that in the case of any individual star we cannot determine how much of its apparent apical motion may be that of the star itself, and how much the parallactic motion arising from the motion of the earth. What we have done is to assume that in the case of a great number of stars the actual apical motions will be equal, and in opposite directions, so as to cancel each other in the average of a great number, leaving this average as the parallactic motion. Now, to fix the ideas, suppose that two stars have an equal apical motion, say three radii of the earth's orbit in a year, but in opposite directions. The apical motion of the earth being four radii per year, it follows that the star which is moving in the same direction as the earth will have a relative apical motion of only 1, and will, therefore, not appear in our list as a star of large proper motion. On the other hand, the star moving with equal speed in the opposite direction will have a motion of seven radii per year, and will, therefore, be included among stars of considerable proper motion. Thus, a bias occurs, in consequence of which we include many stars having a motion away from the solar apex, while the corresponding ones, necessary to cancel that motion, will be left out of the count. Thus, the parallactic motion will, in the average, be too large in the case of the stars of large apparent proper motion. Now, this is exactly what we see in the above tables. As we take the classes with larger and larger proper motions, the supposed parallactic motion, which is really the mean of the apical motions, seems to increase in a yet larger degree. It is, therefore, impossible to determine from comparisons like these what the exact ratio is.

This error is avoided when we do not arrange and select the stars according to the magnitude of their proper motions, but take a large list of stars, determine their proper motions as best we can, and draw our conclusions from the whole mass. This has been done by Kapteyn in the paper already quoted. By a process too intricate to be detailed in the present work he has reached certain conclusions as to the ratio of the actual motion of the sun in space to the average motion of the stars. His definitive result is:

Average speed of a star in space
= Speed of solar motion × 1.86.

This I shall call the straight-ahead motion of the star, without regard to its direction. But the actual motion as we see it is the straight-ahead motion, projected on the celestial sphere. The two will be equal only in cases where there is no radial motion to or from the earth. In all other cases the motion which we observe will be less than the straight-ahead motion. By the process of averaging, Kapteyn finds:

Linear projected speed of a star
= Speed of solar motion × 1.46.

This projected motion, again, may be resolved into two components at right angles to each other. It follows that the average value of either component will be less than that of the projected motion. The components may be the motions in right ascension or declination, or the apical motion and the motion at right angles to it. In any case, the mean value of a component will be:

Speed of solar motion $\times$ 0.93.

I have used Kapteyn's numbers to obtain the same relation by a somewhat different and purely statistical method.

Imagine the proper motion of a star situated nearly at right angles to the direction of the solar motion. Although we cannot determine how much of its apical motion is actual and how much is parallactic, we can determine whether its motion, if toward the solar apex, exceeds that of the sun. In fact, all stars the apical component of whose motion is in the same direction and greater than that of the sun, whatever the distance of the star, appear to us as moving toward the apex, a direction to which we assign a negative algebraic sign. All stars moving more slowly than this, or in the opposite direction from the sun, will have apparent motions away from the apex, which we regard as algebraic ally positive. We can, therefore, by a simple count separate the stars moving in the same direction as the sun, and with greater speed, from all the others.

I have classified the stars in this way, not only as a whole, but also with reference to their cross motion—motion at right angles to that of the sun. That is to say, I have taken the stars whose cross motion, τ, is 2″ per century or less and counted their apical-motions as positive, negative, and zero. Then I have done the same thing with cross motions of 3″ or 4″, then with cross motions ranging from 5″ to 7″, and so on. All cross motions above 13″ we put together.[1] The results of this work are shown, so far as described, in the first four columns of the table below. We have here, for the various values of τ, the number of positive, negative, and zero apical-motions.

Table, showing the number of positive and negative apical motions for different values of the crossmotion.

Values of τ	Apical Motions, σ					Percentage.	
	Pos.	Zero.	Neg.	P′.	N′.	P.	N.
0, ± 1, 2	1,013	261	425	1,143	555	0.67	0.33
± 3, 4	360	56	160	388	188	0.67	0.33
± 5 to 7	285	37	107	303	125	0.71	0.29
± 8 to 12	215	7	52	218	55	0.80	0.20
± 13 + ...	216	2	61	217	62	0.78	0.22
Totals	2,089	363	805	2,269	985	0.70	0.30

The first question that arises in connection with this table is, how to count the motions that come out zero; that is to say, those which are too small to be certainly observed. The most probable distribution we can make of them is to suppose that they are equally divided between positive and negative motions. I have, therefore, added one-half of the zero motions to the positive and one-half to the negative column, thus getting the results given in columns P′ and N′. The

[1] The author should say that the greater part of the work on these countings was done with great care and accuracy by Mrs. Arthur Brown Davis.

percentages of positive and negative motions thus resulting are given in the last column.

We see that there is a fairly regular progression in the percentage, depending on the value of the cross motion. In the case of the small cross motions, which presumably belong to the more distant stars, the percentage of negative apical motions is markedly greater than it is in the case of the nearer stars which have larger values of τ; the diminution in the number of zero motions is still more remarkable. This arises from the fact that in the case of the nearer stars the apical motions are necessarily larger, whether positive or negative.

In the preceding table all the stars were counted, without reference to their distance from the solar apex. The result of this will be that the mean of the apical motions is taken as we see it projected on the sphere, which does not correspond to the actual motion in space except when the direction of the star is at right angles to that of the apex. I have, therefore, made a second partial count, including only stars between 60° and 120° from the apex. These stars were selected in opposite regions of the heavens, so as to eliminate any constant error depending on the right ascension. The result of a count of 733 stars is:

Number of positive motions	530
" " zero "	50
" " negative "	153

If we proceed as before, dividing the zero motions equally between the positive and negative ones, we shall find the respective numbers to be 555 and 178.

The percentage of negative motions is, therefore, 24. This will still be slightly too large, owing to the obliquity under which many of the stars were seen. We may estimate the most likely percentage as 23.

We conclude that when the motions of all the stars are so resolved that one component shall be that in the direction of the apex, 23 per cent. of the stars will be found moving towards the apex with a greater speed than that of the sun. It may, therefore, be assumed that in the general average an equal number are moving in the opposite direction with a greater speed than that of the sun. We conclude that the resolved motion of 46 per cent. of the stars is greater than that of the sun, leaving 54 per cent. less.

In the absence of an exact knowledge of the relation between the magnitude and the number of motions, we shall not be far wrong in assuming that one-half the stars move to or from the apex with more than the average speed, and one-half with less. Comparing this with the percentage found, we may conclude that the average motion of a star is less than that of the sun, in the ratio 46 : 50; or that it is found by multiplying the motion of the sun by the factor 0.92. This is almost exactly the number which we have quoted from Kapteyn.

We have already stated that the actual speed of the solar motion, still somewhat uncertain, may be estimated at 20 kilometres per second, or 4 radii of the earth's orbit in a year. For our present purposes the latter method of expressing the velocity is the more convenient. Multiplying this speed by the factors already found, we have the following results for the average proper motions of a star in space expressed in kilometres per second, and radii of the earth's orbit, called R, in a year:

Straight-ahead motion........	35km.	= 7.4R.
Projected motion	28km.	= 5.8R.
Motion in one component....	18km.	= 3.7R.

The motion of 19km. or 4R. assigned to the sun is its straight-ahead motion. This is little more than half the average. It follows that our sun is a star of quite small proper motion.

CHAPTER XX

THE DISTRIBUTION OF THE STARS IN SPACE

Hoc opus immensi constructum corpore mundi
Membraque naturæ diversa condita forma.
Æris atque ignis terræ pelagique jacentis,
Vis anima divina regit—

MANILIUS.

WE shall now bring the lines of thought which we have set forth in the preceding chapters to converge on our main and concluding problem, that of the distribution of the stars in space. While we cannot reach a conclusion that can claim numerical exactness, we may reach one that will give us a general idea of the subject. The first question at which we aim is that of the number of stars within some limit of distance. It is as if, looking around upon an extensive landscape in an inhabited country, we wished to estimate the average number of houses in a square mile. On the general average, what is the radius of the sphere occupied by a single star? If we divide the number of cubic miles in some immense region of the heavens by the number of stars within that region, what quotient should we get? Of course, cubic miles are not our unit of measure in such a case. It will

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be more convenient to take as our unit of volume a sphere of such radius that, from its centre, supposed to be at the sun, the annual parallax of a star on the surface would be 1″. The radius of this sphere would be 206,265 times that of the earth's orbit. We may use round numbers, consider it 200,000 of these radii, and designate it by the letter R.

Now let us conceive drawn around the sun as a centre concentric spheres of which the radii are R, 2R, 3R, and so on. At the surfaces of these respective spheres the parallax of a star would be 1″, half of a second, one-third of a second, and so on. The volumes of spheres being as the cubes of their radii, those of the successive spheres would be proportional to the numbers 1, 8, 27, 64, etc.

If the stars are uniformly scattered through space, the numbers having parallaxes between the corresponding limits will be in the same proportion.

The most obvious method of determining the number of stars within the celestial spaces around us is by measurement of their parallaxes. It is possible to reach a definite conclusion in this way only in the case of parallaxes sufficiently large to be measured with an approach to accuracy. In the case of a small parallax the uncertainty of the latter may be equal to its whole amount. In this case the star may be at any distance outside the sphere given by its measured parallax, or far within that sphere, so that no conclusion can be drawn. It is, on the whole, useless to consider parallaxes less than 0″.10; even those having this value are quite uncertain in most of the cases.

The data at command for our purpose are the known individual parallaxes and the statistical summary given by Dr. Chase as the result of his survey and quoted in our chapter on the parallaxes of the stars. This survey was confined to stars whose parallax was not already measured, and it brought out no parallax exceeding 0″.30.[1]

The most careful search has failed to reveal any star with a parallax as great as 1″, and it is not likely that any such exists. It is, therefore, highly probable that the first sphere will not contain a single star except the sun in its centre.

Within the third sphere, the parallax at the surface of which is 0″.33, we may place the following four stars:

α Centauri	Par.=	0″.75
Ll. 21,185	"	0.46
61 Cygni	"	0.39
Sirius	"	0.37

There are two other cases in which the parallax is doubtful, though the measures as made bring the stars within the sphere 3R. They are:

η Herculis	Par.=	0″.40
O. A. 18,609		0.35

In the case of Eta Herculis the proper motion is so small that the presumption is strongly against so large a parallax, and the doubtful parallax of the last

[1] The results of this survey were communicated to the Astronomical and Astrophysical Society of America toward the end of June, 1900, and published in *Science* with the Proceedings of the Society.

star is so near the limit that it may be left out of the count. The doubt in its case may be set off against a doubt whether the parallax assigned to Ll. 21,185 is not too large. We assume, therefore, that four stars are contained within the sphere 3R, the volume of which is $3^3=27$. This would give, in whole numbers, one star to 7 unit spheres of space.

When we come to smaller parallaxes we find a great deficiency in the number measured in the Southern Hemisphere. The policy of Gill, under whose direction or with whose support all the good measures in that hemisphere were made, was to make a few very thorough determinations rather than a general survey. Between the limits 0″.20 and 0″.33 are found :

In the Southern Hemisphere	4	meas.	(Gill)
" Northern "	2	"	(Chase)
" " "	12	"	(others)
Total	18		

Of the northern results three are exactly on the limit, 0″.20, and several others are doubtful, and probably too large. The most likely number for the Northern Hemisphere seems to be 12, and if we estimate an equal number for the Southern Hemisphere we shall have 24 in all. Adding the four stars within the sphere 3R, we shall then have a total of 28 within the sphere 5R, of which the volume is 125. This gives between 4 and 5 space units to a star.

Let us now consider the space between the spheres 5R and 10R, including all stars whose parallax lies between the limits 0″.10 and 0″.20. Of these the numbers are:

Southern Hemisphere	....	6	(Gill)
Northern "	....	15	(Chase)
" "	....	15	(others)

Reasoning as before, we may assume that the number of stars between the assigned limits is 60, making a total of 88 within the sphere 10R. The volume of space enclosed being 1000 units, this will give one star to 12 units of space.

How far can we rely on this number as an approximation to the actual number of stars within the tenth sphere? The errors in the estimate are of two classes, those affecting the parallax itself and those arising from a failure to include all the stars within the sphere. The very best determinations are liable to errors of two or three hundredths of a second, the inferior ones to still larger errors. Thus, it may happen that there are stars with a real parallax larger than the limit, of which the measures fall below it and are not included, and others smaller than the limit, which, through the errors of measurement, are made to come within the sphere. As we have seen in the chapter on the parallaxes, it is quite possible that there may be a number of stars with a measurable parallax whose proximity we have never suspected on account of the smallness of the proper motion. We can only say that the nearer a star is to us the more likely its proximity is to be detected, so that we are much surer of the completeness of our list of large parallaxes than of small ones. Hence, there may well be a number of undetermined parallaxes upon or just above the limit 0″.10.

The most likely conclusion we can draw from this examination seems to be that in the region around us there is one star to every 8 units of space; or that a sphere of radius 2R, equal to 412,500 radii of the earth's orbit, corresponding to a parallax of 0″.50, contains one star. This is a distance over which light would pass in 6½ years.

We next see how far a similar result can be derived from statistics of the proper motions. It seems quite likely that nearly all proper motions exceeding 1″ annually have been detected. The number known is between 90 and 100, but it cannot be more exactly stated because there is some doubt in the case of a number which seem to be just about on the limit. In this value, 1″, is included the effect of the parallactic motion, which, on the general average, increases the apparent proper motion of a star. To study this effect let us call the list of 90 or more stars actually found List A. Were it possible to observe the proper motions of the stars themselves separate from the parallactic motion, we should find that, when we enumerate all having a proper motion of more than 1″, we should add some to our List A and take away others. The stars we should add would be those moving in the same direction as the sun, whose motions appear to us to be smaller than they really are, while we should take away those moving in the opposite direction, whose motions appear to us larger than they really are. On the average, we should take away more than we added, thus diminishing slightly the number of stars whose motion exceeds 1″. Making every allowance, we may estimate that probably 80 stars have an actual proper motion on the celestial sphere of 1″ or more. We have found that the average linear proper motion of a star, as projected on the sphere, is about 6 radii of the earth's orbit annually. A star having this motion would have to be placed at the distance 6R to have, as seen by us, an angular motion of 1″. The parallax corresponding to the surface of this sphere is 0″.167. The volume of the sphere is 216, and according to our estimate from the parallaxes it would contain only 27 stars. Thus the proper motions seem to give a greater density of the stars than do the measured parallaxes; that is to say, they indicate that there are still a large number of measurable parallaxes undetermined. But the fact is that the number of stars estimated as within a given sphere by the proper motions will be in excess, owing to the actual diversity of these proper motions, which may range from 0 to a value several times greater than the average. In consequence of this, our list of stars with a proper motion exceeding 1″ will contain a number lying outside the sphere 6R, but having a proper motion larger than the average. We are also to consider that within the sphere may actually lie stars having a proper motion less than the average, which will, therefore, be omitted from the list. Of the number of omitted and added stars the latter will be the greater, because the volumes of spheres increase as the cubes of their radii. For example, the space between the spheres 6R and 9R is more than double that within 6R, and our list will include many stars in this space. Thus arises a discrepancy between the parallaxes and the proper motions.[1]

Let us see what the result is when we take stars of smaller proper motion. The most definite limit which we can set is 10″ per century. We have seen that Dr. Auwers, in his zone, found 23.9 stars per 100 square degrees having a proper motion of 10″ or more. This ratio would give about 10,000 for the whole heavens. The sphere corresponding to this limit of proper motion is 60R. On our hypothesis as to star-density this sphere would contain 27,000 stars, nearly three times the number derived from Auwers's work. But it is not at all unlikely that this sphere contains three times as many proper-motion stars as have been detected. Great numbers of the more distant stars will not have been catalogued, owing to their faintness, because a star at the distance 60R will shine to us with only one per cent. the light of one at distance 6R. This corresponds to a diminution of five magnitudes; that is to say, a star of the sixth magnitude

[1] The principle involved in the case may be more fully stated thus: If we take all the stars that lie within a given sphere, and determine their proper motions and parallaxes, we shall get the correct relation between the proper motions and parallaxes. But if we take all stars whose proper motion exceeds a certain limit, and determine their parallaxes, the mean of these parallaxes will be disproportionately small, owing to the omission of stars with proper motions below the limit, but lying within the sphere of measurement. It thus happens that the proper motions found in our Appendix II. are, in the general average, much more than six times the parallax.

at distance 6R would only be of the eleventh magnitude at distance 60R, and would, therefore, not be catalogued at all. There is, therefore, no reason for changing our estimate of star-density, which assigns to each star around us 8 units of volume in space.

This fact suggests another important one. Owing to the great diversity in the absolute magnitude of the stars, those we can observe with our telescopes will naturally be more crowded in the neighbourhood of our system than they will at greater distances.

Some further results as to the mean parallax of the stars may be derived from a continuation of the statistical study of the proper motions. Kapteyn's investigation in this direction includes a determination of the mean parallactic motion of the stars of each magnitude for the first and second spectral types separately. From this he obtains the following mean parallaxes for stars of the different magnitudes :

Mean parallaxes of stars of different magnitudes, and of the two principal types, as found from their parallactic motions :

Mag.	Type I.	Type II.
	″	″
2.0	.0315	.0715
3.0	.0223	.0515
4.0	.0157	.0357
5.0	.0111	.0253
6.0	.0079	.0179
7.0	.0056	.0126
8.0	.0039	.0089
9.0	.0028	.0063
10.0	.0020	.0045
11.0	.0014	.0032

Using the value 4 for the solar motion, instead of 3.5, found by Kapteyn, all these parallaxes should be diminished by one eighth of their amount.

Unfortunately, owing to the great diversity in the absolute brightness of the stars, and the resulting great difference in the distances of stars having the same magnitude, these numbers can give us no idea of the actual parallaxes. Let us take, for example, the stars of the sixth magnitude. A few of these are doubtless quite near to us and have a parallax several times greater than that of the table. Excluding these from the mean, an important fraction of the remainder, perhaps a great majority, may have a parallax smaller than that of the table to any extent — may, in fact, be on the very confines of the universe.[1]

We get a slightly more definite result by studying another feature of the proper motions. We may consider the Bradley stars, whose motions have been investigated, as typical in the general average of stars of the sixth magnitude. By a process of reasoning from the statistics, of which I need not go into the details at present, it is shown that the parallactic motion of a large number of these stars, probably one-sixth of the whole, is less than 1″ per century. To

[1] Since the present work was prepared for the press, Kapteyn has published a number of careful and intricate researches on stellar statistics, bearing on the subject discussed in this and the next chapter. One of these papers, forming No. 8 of the *Publications of the Astronomical Laboratory at Groningen*, is "on the mean parallax of stars of determined proper motion and magnitude"; another, published in the Proceedings of the Amsterdam Academy for April 20, 1901, is "on the luminosity of the fixed stars." So far as the results worked out in these papers bear on the problem of the extent of the universe, the reasoning is too abstruse and the results too mathematical to be easily presented in the present work.

this motion corresponds a parallax of 0″.0025, corresponding to the sphere of radius 400R.

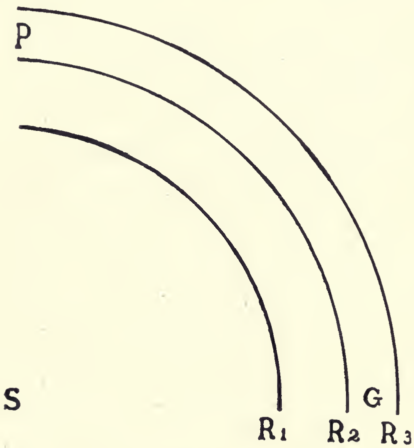
The statistics of cross motions lead to a similar conclusion. One-half the Bradley stars have a cross motion of less than 2″.5 per century. To this motion would correspond a sphere of radius 200R and a parallax of 0″.005. Stars at this distance must be hundreds of times the absolute brightness of the sun to be seen as of the sixth magnitude. Yet the conclusion seems unavoidable that the sphere of lucid stars extends much beyond 400R.

We shall next make an estimate based on the number of the stars. All the facts we have reviewed lead to the belief that, out to a great distance, the stars are scattered without any great and well marked deviation from uniformity. This belief rests upon the remarkable equality in the number of stars in opposite directions from us. We do not detect any marked difference between the numbers lying round the two opposite poles of the galaxy, nor, so far as known, between the star density in different regions at equal distances from the Milky Way. Accepting this view, the question how far we must place the boundary of a sphere in order that it may contain a given number of stars admits of a definite answer. We have only to extract the cube root of the number, and multiply it by 2. Consequently the sphere of radius $2n$R will contain n^3 stars. Thus a sphere of

Radius	400R	will contain	8,000,000	stars
"	600R	"	27,000,000	"

Radius 800R will contain 64,000,000 stars
" 1000R " " 125,000,000 "

The minutest counts of stars that have been made, and the photometric law shown in the beginning of Chapter XVIII. lead us to suppose that the actual number of non-galactic stars, visible and invisible, probably falls within the limits of the above numbers. We have therefore no reason to believe that, away from the Milky Way, the stars extend far beyond the sphere 1000R, at whose boundary the parallax is 0″.001, and the average proper motion of a star about 0″.6 per century. But the phenomena of the Milky Way show that around the region of the galactic belt, there is a distance at which the law of uniform density ceases to be true. Let S be the sun, R1 a portion of the surface of the outer sphere of uniform distribution, and R2 and R3 two contiguous spheres passing through the galactic region G, of which the pole is in the direction P. It is quite certain that the star-density is greater around G than around P. This may arise either from the density at G being greater, or from that at P being less than the density within the sphere R1. From the enormous number of stars



collected in the galactic regions, we can scarcely doubt that the former alternative is the correct one. But there must be a sphere at which the second alternative is also correct, because we find the number of stars, even of the lucid ones, to continuously increase from P toward G.

Can we form any idea where this difference begins, or what is the nearest sphere which will contain an important number of galactic stars? A precise idea, no; a vague one, yes. We have seen that the galactic agglomerations contain quite a number of lucid stars, and that, perhaps, an eighth of these stars are outside the sphere 400R. We may, therefore, infer that the Milky Way stars lie outside this sphere. Considerations based on the proper motions lead us to place these stars even outside the sphere 1000R. It seems certain that the blue stars of the constellation Orion have a proper motion of only a small fraction of a second per century—a few tenths or less. Although these do not belong to the Milky Way itself, there is reason to believe that they do not lie beyond it, and that the proper motions of the stars of the Milky Way are equally small. This would place the stars of the Milky Way at a greater distance than the probable confines of the universe in the direction of the galactic poles.

So far as we can judge from the enumeration of the stars in all directions, and from the aspect of the Milky Way, our system is near the centre of the stellar universe. That we are in the galactic plane itself seems to be shown in two ways: (1) the equality in the counts of stars on the two sides of this plane all the way to its poles, and (2) the fact that the central line of the galaxy is a great circle, which it would not be if we viewed it from one side of its central plane.

Our situation in the centre of the galactic circle, if circle it be, is less easily established, because of the irregularities of the Milky Way. The openings we have described in its structure, and the smaller density of the stars in the region of the constellation Aquila, may well lead us to suppose that we are perhaps markedly nearer to this region of its centre than to the opposite region; but this needs to be established by further evidence. Not until the charts of the International Photographic Survey of the heavens are carefully studied does it seem possible to reach a more definite conclusion than this.

One reflection may occur to the thinking reader, as he sees these reasons for deeming our position in the universe to be a central one. Ptolemy showed by evidence which, from his standpoint, looked as sound as that which we have cited that the earth was fixed in the centre of the universe. May we not be the victims of some fallacy, as he was?

The following is a summary of more or less probable conclusions, drawn from facts developed in the present work:

1. The stars differ enormously in their actual luminosity. Some are thousands or tens of thousands of times more luminous than the sun; others only one-hundredth or one-thousandth as luminous.

2. The more luminous stars are generally the hotter, the bluer, and the rarer in their constitution. They are, as it were, inflated masses of rare and intensely incandescent gas. Hence the stars do not differ in mass so widely as in luminosity.

3. The bluest and most luminous stars are situate mainly in the region of the Milky Way. There is some reason to suspect that in this region the more densely the stars are agglomerated the larger and more luminous they are.

4. That collection of stars which we call the universe is limited in extent. The smallest stars that we see with the most powerful telescopes are not, for the most part, more distant than those a grade brighter, but are mostly stars of less luminosity, situate in the same regions. This does not preclude the possibility that far outside of our universe there may be other collections of stars of which we know nothing.

5. The boundary of our universe is probably some what indefinite and irregular. As we approach it, the stars may thin out gradually. The parallax at the boundary is probably nowhere greater than 0″.001, and may be much less. The time required for light to pass over the corresponding interval is more than three thousand years.

6. The universe extends farther around the girdle of the Milky Way than toward the poles of that girdle. But, in every direction, it extends beyond the limit within which the proper motions of the stars have yet been determined.

7. It does not yet seem possible to decide whether the agglomerations of the Milky Way lie on the boundary of the universe or not. The number of lucid stars which they contain might seem to militate against the view, though not strongly because of the possible great luminosity of the galactic stars.

8. The total number of the stars is to be counted by hundreds of millions.

9. Outside the galactic region the stars in general show no tendency to collect into systems or clusters, but are mostly scattered through space with some approach to uniformity.

APPENDIX

In this appendix are found lists of the individual names of certain stars, of parallaxes and large proper motions, and of spectroscopic binary systems.

The list of names seems to require no explanation.

List of parallaxes and proper motions.

The parallaxes in this list are derived, for the most part, from a combination of all the investigations or authorities on the subject.

A colon after a parallax indicates that it is subject to more doubt than usual; two colons, that it is entirely unreliable.

The numbers and letters in the column "light" are intended to show the luminosity of the star, or the ratio of the actual amount of light emitted from its entire surface to that emitted from the entire surface of the sun. The numbers cannot lay any claim to exactness, owing to the uncertainty as to the star's exact distance from us, and are intended only to give a general idea of the actual magnitude or luminosity of the star.

Where the letters XM are used in this column they mean that no numerical statement is possible except that the star is thousands and perhaps tens or even hundreds of thousands of times brighter than the sun.

List of spectroscopic binary systems established to July, 1901.

This is a list of stars for which a variability of the radial motion supposed to be due to the action of a companion or the duplicity of the star has been established.

The period is given in days.

The orbital velocity is the extreme deviation of the observed orbital velocity from the mean, smoothed off where the observations are sufficiently numerous. In those cases where an orbit has been computed from the observed velocities, the velocity given is that derived from the elements.

It will be noted that in many cases the period and velocity are not yet determined.

The author is indebted to Professor Campbell for most of the particulars given in the list, and for its final revision.

I. Names of individual stars found in astronomical literature, with their approximate positions for 1900.

Individual Name.	Name on Bayer System.	Position for 1900. R. A.		Dec.	
		h	m	°	′
Achernar	α Eridani	1	34.0	−57	55
Alcor	80 Ursæ Majoris	13	21.2	+55	30
Alcyone	η Tauri	3	41.5	+23	48
Aldebaran	α Tauri	4	30.2	+16	18
Algenib	γ Pegasi	0	8.1	+14	38
Algol	β Persei	3	1.7	+40	34
Alioth	ε Ursæ Majoris	12	49 8	+56	29
Altair	α Aquilæ	19	45.9	+ 8	36
Antares	α Scorpii	16	23 3	−26	13
Arcturus	α Bootis	14	11.1	+19	42
Bellatrix	γ Orionis	5	19 8	+ 6	16
Betelguese	α Orionis	5	49.8	+ 7	23
Canopus	α Argûs (Carinæ)	6	21.7	−52	38
Capella	α Aurigæ	5	9 3	+45	54
Caph	β Cassiopeiæ	0	3.8	+58	36
Castor	α Geminorum	7	28.2	+32	6
Cor Caroli	α Canum Venaticorum	14	51.3	+38	52
Deneb	α Cygni	20	38.0	+44	55
Denebola	β Leonis	11	44.0	+15	8
Dubhe	α Ursæ Majoris	10	57.6	+62	17
Fomalhaut	α Piscis Australis	22	52.1	−30	9
Markab	α Pegasi	22	59.8	+14	40
Mira Ceti	o Ceti	2	14.3	− 3	26
Mizar	ζ Ursæ Majoris	13	19.9	+55	27
Polaris	α Ursæ Minoris	1	22.5	+88	46
Pollux	β Geminorum	7	39.2	+28	16
Procyon	α Canis Minoris	7	34.1	+ 5	29
Regulus	α Leonis	10	3.0	+12	27
Rigel	β Orionis	5	9.7	− 8	19
Sirius	α Canis Majoris	6	40.7	−16	35
Spica	α Virginis	13	19.9	−10	38
Vega	α Lyræ	18	33.6	+38	41

II. List of parallaxes of stars and of proper motions exceeding 100″ per century.

Star.	Position, 1900.		Parallax.	Magnitude.	Luminosity. ⊙=1.	Annual Proper Motion.	
	R. A.	Dec.				R. A.	Dec.
	h m	° ′	″	M		s.	″
β Cassiopeiæ	0 3 8	+58 36	0.15	2.4	5	+0.068	−0.18
Gr. 34	0 12.7	+43 27	0.30:	8.1	0.01	+0.262	+0.39
ζ Tucani	0 14.9	−65 28	0.06	4.3	6	+0.273	+1.16
β Hydri	0 20.5	−77 49	0.13	2.9	5	+0.703	+0.32
82 B Ceti	0 32.2	−25 19		5.6		+0.100	0.00
α Cassiopeiæ	0 34.8	+55 59	0.04:	2.4	50	+0.006	−0.03
η Cassiopeiæ	0 43.0	+57 17	0.20	3.6	1	+0.143	−0 48
147 B Piscium	0 43.1	+ 4 46		5.7		+0.048	−1.13
γ Cassiopeiæ	0 50.7	+60 10	0.01.	2.3	1000	+0.004	0.00
μ Cassiopeiæ	1 1.6	+54 26	0.14	5.2	0.5	+0.391	−1.55
Polaris	1 22.6	+88 46	0.06	2.1	50	+0.136	0.00
α Eridani	1 34.0	−57 45	0.04	0.5		+0.010	−0.04
τ Ceti	1 39.4	−16 28	0.31	3.7	0.5	−0.120	+0.86
Lac. 661	2 6.4	−51 19		6.4		+0.225	+0.72
δ Trianguli	2 11.0	+33 46		5.0		+0.091	−0.23
128 H.[1] Ceti	2 30.6	+ 6 25		5.9		+0.120	+1.46
Lac. 5490	2 56.0	+61 20		6.7		+0.094	−0.68
ι Persei	3 1.8	+49 14		4.2		+0.133	−0.10
ζ[1] Reticuli	3 15.6	−62 58		6.2		+0.195	+0.65
e Eridani	3 15.9	−43 27	0.14	4.3	1	+0.281	+0.76
ζ[2] Reticuli	3 16.0	−62 53		5.8		+0.192	+0.66
ε Eridani	3 28.2	− 9 48		3.8		−0.066	+0.02
Ll. 6888	3 40.2	+41 9		8.2		+0.053	−0.12
Ll. 7443	3 56.5	+35 2		8.5		+0.143	−1.34
50 Persei	4 1.9	+37 47	0.04::	5.5	5	+0.014	−0.18
o[2] Eridani	4 10.7	+17 48	0.18	4.5	57	−0.148	−3.44
Aldebaran	4 30.2	− 6 18	0.11	1.1	45	+0.005	−0.19
Weisse, 1189	4 55.8	− 5 52		6.4		+0.040	−1.13
C. Z.Vh, 243	5 7.7	−45 3		8.5		+0.621	−5.70
Capella	5 7.7	+45 54	0.09	0.2	120	+0.009	−0.43
Rigel	5 9.7	− 8 19	0.00	0.3	XM	0.000	0.00
Weisse, 592	5 26.4	− 3 42		8.7		+0.046	−2.12
π Mensæ	5 45.1	−80 33		6.5		+0.087	+1.09
α Orionis	5 49 8	+ 7 23	0.02	0.9	500	+0.002	+0.01
β Aurigæ	5 52 2	+44 56	0.06:	2.1	48	−0.004	−0.01
Canopus	6 21.7	−52 38	0.00	−1.0	XM	+0.002	+0.01
ψ[5] Aurigæ	6 39.5	+43 41	0.11	5.3	0.7	+0.002	+0 16
Sirius	6 40.7	−16 35	0.37	−1.4	30	−0.037	−1.21
51 H. Cephei	6 53.7	+87 12	0.03:	5.2	11	+0.057	−0.04
Castor	7 28.2	+32 6	0.20:	1.6	7	−0 014	−0.08
Procyon	7 34.0	+ 5 29	0.30	0.5	8	−0.047	−1.04

Star.	Position, 1900. R. A.	Position, 1900. Dec.	Parallax.	Magnitude.	Luminosity ⊙ = 1.	Annual Proper Motion. R. A.	Annual Proper Motion. Dec.
	h m	° ′	″	M		s.	″
Pollux	7 39.2	+28 16	0.06	1.2	100	−0.047	−0.06
Lac. 2957	7 41.8	−33 59		5.4		−0.021	+1.67
Ll. 15290	7 47.2	+30 55	0.02	8.2	1.6	+0.058	−1.82
Ll. 15565	7 54.3	+29 31		7.0		−0.012	−1.17
Ll. 16304	8 13.6	−12 18		6.0		+0.017	−0.99
Lac. 3386	8 29.0	−31 11		6.4		−0.088	+0.69
Fed. 1384	8 46.0	+71 11		8.5		−0.280	−0.35
ε Ursæ Maj	8 52.4	+48 26	0.13:	3.1	3	−0.044	−0.25
10 Ursæ Maj	8 54.2	+42 11	0.20	4.2	0.6	−0.039	−0.26
Fed. 1457–8	9 7.6	+53 7	0.15	8.0	0.03	−0.175	−0.62
θ Ursæ Maj	9 26.2	+52 8	0.07	3.3	110	−0.103	−0.54
Ll. 19022	9 37.1	+43 10	0.06	8.0	0.2	+0.002	−0.80
Weisse, 954	9 46.2	−11 49		9.3		+0.085	−1.50
20 Leo Min	9 55.2	+32 25	0.06	5.5	2	−0.042	−0.44
Regulus	10 3.0	+12 27	0.02	1.3	1000	−0.017	0.00
Gr. 1618	10 5.2	+49 58	0.18	6.8	0.07	−0.140	−0.52
Gr. 1646	10 21.9	+49 19	0.10	6.5	0.3	+0.011	−0.89
Gr. 1657	10 27.7	+49 42	0.04	7.6	0.7	+0.024	+0.11
Ll. 21185	10 57.9	+36 38	0.46	7.6	0.005	−0.044	−4.74
Ll. 21258	11 0.5	+44 2	0.22	8.5	0.01	−0.402	+0.95
Fed. 1831	11 8.6	+74 1	0.15	7.2	0.07	−0.105	+0.13
O. A. 11677	11 14.8	+66 23	0.27:	9.0	0.004	−0.503	+0.24
Brad. 1584	11 29.6	−32 18	0.03	6.0		−0.053	+0.84
Σ 1561	11 33.5	+45 40	0.02	6.3		−0.060	+0.03
Gr. 1822	11 40.3	+48 14		7.8	4	−0.061	−0.28
Lac. 4887	11 41.8	−39 57		5.0	2	−0.133	+0.39
Gr. 1830	11 47.2	+38 26		6.4	0.2	+0.341	−5.78
Lac. 4955	11 53.0	−27 8		7.2		−0.074	−0.70
Gr. 1855	12 4.6	+40 49	0.06	7.4	0.4	−0.029	−0.06
Ll. 22954	12 10.0	− 9 44	0.14	6.0		+0.005	−1.01
β Comæ	13 7.2	+28 23	0.11	4.3	1.9	−0.060	+0.88
61 Virginis	13 13.2	−17 45		4.8		−0.075	−1.07
Auwers A. G. 4999	13 40.2	+18 20		9.2		+0.027	−1.85
Aron	13 40.7	+15 26		8.5		+0.125	−1.47
β Centauri	13 56.8	−59 53	0.05	0.8	220	−0.003	−0.03
Arcturus	14 11.1	+19 42	0.03	0.3	1000	−0.078	−2.00
α Centauri	14 32.8	−60 25	0.75	0.2	1.7	−0.485	+0.73
Ll. 27026	14 46.0	−23 53		7.8		−0.066	−0.48
43 B Libræ	14 51.6	−20 58		5.8		+0.074	−1.79
Fed. 2544	14 52.4	+54 4	0.08	7.7	0.1	−0.110	+0.48
O. A. 14318	15 4.7	−15 59		9.3		−0.067	−3.64
O. A. 14320	15 4.7	−15 54		9.2		−0.066	−3.63
Ll. 27744	15 8.8	− 0 58		6.7		−0.085	−0.93
Ll. 28607	15 37.7	−10 36		7.3		−0.076	−0.34
γ Serpentis	15 51.8	+15 59		4.0		+0.021	−1.29

Star.	Position, 1900.		Par-allax.	Magni-tude.	Lumi-nosity ⊙=1.	Annual Proper Motion.	
	R. A.	Dec.				R. A.	Dec.
	h *m*	° ′	″	M		s.	″
Antares	16 23.3	−26 13	0.02	1.3	900	−0.001	−0.03
Ll. 30044	16 25.6	+ 4 26		7.6		−0.030	−0.39
η Herculis	16 39.5	+39 7	0.40:	3.7	0.3	+0.003	−0.09
Ll. 30694	16 47.9	+ 0 11		6.8		−0.049	−1.49
Weisse, 906	16 50.1	− 8 9		8.8		−0.063	−0.87
Ll. 31055	16 59.8	− 4 54		7.9		−0.062	−1.15
A. Ophiuchi	17 9.2	−26 27		4.6		−0.037	−1.17
Brad. 2179	17 10.1	−26 24		6.7		−0.038	−1.14
π Herculis	17 10.9	+36 55	0.11:	3.3	4.7	−0.001	0.00
Lac. 7215	17 11.6	+34 53		5.9		+0.096	−0.21
8 Herculis	17 12.2	+24 57	0.05:	3.2	25	+0.002	−0.16
ω Herculis	17 16.9	+32 36		5.4		+0.009	−1.05
Weisse, 322	17 20.8	+ 2 14		8.0		−0.040	−1.22
γ[1] Draconis	17 30.2	+55 15	0.32	4.9	0.1	+0.018	+0.05
O. A. 17415	17 37.0	+68 26	0.22	7.9	0.02	+0.069	−1.25
70 Ophiuchi	18 0.4	+ 2 31	0.19	4.1	0.7	−0.018	−1.12
δ Ursæ Min.	18 4.5	+86 37	0.03:	4.4	22	+0.019	+0.05
α Lyræ	18 33.6	+38 41	0.11	0.1	90	+0.018	+0.28
Anon	18 41.7	+59 29	0.35:	8.9	0.007	−0.171	+1.87
Anon	18 53.1	+ 5 48		9.3		−0.016	−1.22
31 Aquilæ	19 20.2	+11 44	0.06	5.3	2.5	+0.050	+0.63
σ Draconis	19 32.6	+69 29	0.26:	4.8	0.2	+0.100	−1.76
α Aquilæ	19 45.9	+ 8 36	0.23	0.9	10	+0.036	+0.38
Lac. 8267	19 55.6	−67 34		6.6		+0.186	−0.67
δ Pavonis	19 58.9	−66 26		3.6		+0.192	−1.13
Ll. 38383	19 59.7	+23 5		7.2		−0.074	−0.94
Lac. 8362	20 4.6	−36 21		5.4		+0.037	−1.64
Lac. 8381	20 9.0	−27 20		5.8		+0.094	−0.24
O. A. 20452	20 17.7	−21 40		8.2		+0.037	−1.10
α Cygni	20 38.0	+44 55	0.00	1.3	XM	0.000	0.00
Lac. 8620	20 51.0	−44 29		7.5		−0.050	−0.99
61 Cygni	21 2.4	+38 15	0.39	4.8	0.1	+0.350	+3.24
Lac. 8760	21 11.4	−39 15		6.8		−0.280	−1.22
α Cephei	21 16.2	+62 10	0.06:	2.6	30	+0.022	+0.05
Weisse, 562	21 24.5	−12 56		9.1		+0.070	−0.28
ε Indi	21 55.7	−57 12	0.20	4.8	0.4	+0.479	−2.58
α Gruis	22 1.9	−47 27	0.02	1.9	500	+0.011	−0.18
γ Indi	22 16.0	−72 44		5.8		+0.280	−0.74
Fomalhaut	22 52.1	−30 9	0.13	1.3	21	+0.025	−0.17
Lac. 9352	22 59.4	−36 26	0.28	7.4	0.02	+0.573	+1.15
Brad. 3077	23 8.5	+56 37	0.15	5.6	0.3	+0.253	+0.30
Weisse, 175	23 11.9	−14 22		8.2		−0.035	−1.21
Ll. 46650	23 44.0	+ 1 52		8.7		+0.065	−1.00
85 Pegasi	23 57.0	+26 33	0.05	5.8	2.2	+0.062	−0.99
Cord. 32416	23 59.5	−37 51		8.5		+0.485	−2.58

III. List of spectroscopic binary systems.

Name of Star.	Position, 1900.		Mag.	Period Days.	Orbital Velocity km. sec.	Authority or Discoverer
	R. A.	Dec.				
	h m	° '				
η Andromedæ..	0 52	+22 52	4.6		14	Campbell
Polaris	1 22	+88 46	2.1	3.97	3	Campbell
ξ Piscium.....	1 48	+ 2 42	4.7			Campbell
ξ_1 Ceti	2 8	+ 8 23	4.4			Campbell
12 Persei	2 36	+39 46	4.9			Campbell
τ Persei	2 47	+52 22	4.0			Campbell and Miss Maury
β Persei.......	3 2	+40 34	2.5	2.87	41	Vogel
λ Tauri.......	3 55	+12 12	Var.			Belopolsky
α Aurigæ.....	5 9	+45 54	0.2	104.0	26	Campbell and Newall
δ Orionis......	5 27	− 0 22	2.4	1.9	70	Deslandres
β Aurigæ.....	5 52	+44 56	2.1	3.98	120	Miss Maury
ζ Geminor	6 58	+20 43	Var.	10.15	13	Belopolsky and Campbell
α^1 Geminor...	7 28	+32 7	2.0	2.91	11	Belopolsky
A. G. C. 10534	7 55	−48 50	5.0	3.12	305	Pickering
ε Hydræ	8 42	+ 6 48	3.6			Campbell
ξ Leonis	9 36	+10 21	3.8	14.5	56	Campbell and Miss Maury
o Ursæ Maj...	11 13	+32 6	3.8			Wright
93 Leonis.....	11 43	+20 46	4.6		18	Campbell
ζ Ursæ Maj...	13 20	+55 27	2.4	52	80	Pickering
α Virginis	13 20	−10 38	1.2	4.01	91	Vogel
ζ Centauri	13 49	−46 48	2.7	8.02		Mrs. Fleming
d Bootis.......	14 6	+25 34	4.8		40	Wright
β Lupi.......	14 52	−42 44	2.8			Mrs. Fleming
ε Libræ	15 19	− 9 57	5.2	240±	13	Campbell
π Scorpii.....	15 53	−25 49	3.1	1.57		Miss Cannon
θ Draconis	16 0	+58 50	4.2	9	25	Campbell
β Herculis....	16 26	+21 42	2.8	412±	12	Campbell
μ Scorpii.....	16 45	−37 52	3.6	1.45	230	Bailey
h Draconis....	16 55	+65 17	4.7		16	Campbell
ε Ursæ Min...	16 56	+82 12	4.5		25	Campbell
ω Draconis...	17 38	+68 48	4.9			Campbell
χ Draconis....	18 23	+72 42	3.7	282	18	Campbell
2 Scuti........	18 37	− 9 9			6	Wright
6 H. Scuti....	18 42	− 4 51	4.4		7	Wright
β Lyræ.......	18 46	+33 15	Var.	12.91	181	Belopolsky
113 Herculis..	18 50	+22 32	4.6		10	Wright
υ Sagittarii...	19 16	−16 8	4.7			Campbell and Miss Maury
η Aquilæ. ...	19 47	+ 0 45	Var.	7.18	20.6	Belopolsky
o^1 Cygni	20 10	+46 26	3.8			Campbell and Miss Maury
β Capricorni..	20 15	−15 5	3.4	1000±		Campbell and Miss Maury
α Equulei.....	21 11	+ 4 50	4.0			Campbell and Miss Maury
κ Pegasi......	21 40	+25 11	4.2		40	Campbell
ι Pegasi.......	22 2	+24 51	4.0	10.2	45	Campbell
δ Cephei......	22 25	+57 54	Var.	5.37	20	Belopolsky
η Pegasi	22 38	+29 42	3.1	818.0	14	Campbell
π Cephei.....	23 5	+74 51	4.5			Campbell
λ Andromedæ.	23 33	+45 56	4 0	20	8	Campbell

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