UNCLASSIFIED/LIMITED

DTIC

Technical Report

Statistical filters for smoothing and filtering equally spaced data.

H. M. Linnette.
NAVY, Naval Electronics Laboratory

distributed by

Defense Technical Information Center
DEFENSE LOGISTICS AGENCY
Cameron Station • Alexandria, Virginia 22314

UNCLASSIFIED/LIMITED

NOTICE

We are pleased to supply this document in response to your request.

The acquisition of technical reports, notes, memorandums, etc., is an active, ongoing program at the Defense Technical Information Center (DTIC) that depends, in part, on the efforts and interests of users and contributors.

Therefore, if you know of the existence of any significant reports, etc., that are not in the DTIC collection, we would appreciate receiving copies or information related to their sources and availability.

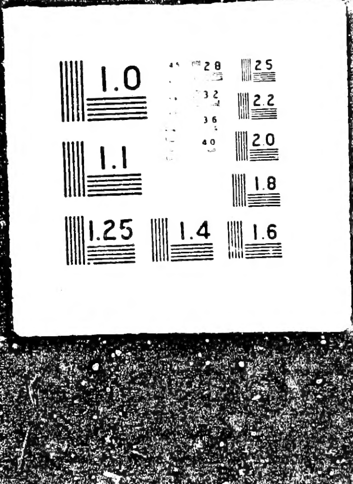
The appropriate regulations are Department of Defense Directive 5100.36, Defense Scientific and Technical Information Program; Department of Defense Directive 5200.20, Distribution Statements on Technical Documents; Military Standard (MIL-STD) 847-A, Format Requirements for Scientific and Technical Reports Prepared by or for the Department of Defense; Department of Defense Regulation 5200.1-R, Information Security Program Regulation.

Our Acquisition Section, DTIC-DDA-1, will assist in resolving any questions you may have. Telephone numbers of that office are: (202) 274-6847, 274-6874 or Autovon 284-6847, 284-6874

June 1982

1 OF 3

AD 471156



RESEARCH REPORT
REPORT 1049
10 July 1961

NEL/Report 1049

FORWARDED BY THE CHIEF, BUREAU OF SHIPS

STATISTICAL FILTERS FOR SMOOTHING AND FILTERING EQUALLY SPACED DATA

H. M. Linnette

Encl. ( ) to BUSHIPS ltr. Ser

U. S. NAVY ELECTRONICS LABORATORY, SAN DIEGO, CALIFORNIA
A BUREAU OF SHIPS LABORATORY

THE PROBLEM

As part of the general study of low frequency ambient sea noise, investigate the characteristics of statistical filters for smoothing of time series data.

RESULTS

1. Smoothing with equally weighted running means is computationally simple and results in a relatively sharp cutoff filter. However, the frequency response decreases continuously within the pass band and high frequency ripples are introduced into the data because of large oscillations in the frequency response above the cutoff.

2. The Gaussian filter does not introduce high frequency ripples into the data since its frequency response approaches zero asymptotically without a finite cutoff. However, its arbitrarily defined 1 per cent cutoff is not very sharp.

3. The defects inherent in the two filters discussed above can be minimized by determining weights for a filter which approximates the square-shaped ideal filter in the least square sense, with corrections for unity gain at zero frequency and a sine termination to reduce oscillations above cutoff. Weights have been determined for 118 such filters which depart from the ideal filter in the pass-band region and below the cutoff by an absolute error less than or equal to 1 per cent. These weights are given in the Appendix.

ADMINISTRATIVE INFORMATION

Work was performed under AS 02101, NE 051600-847.60 (now S-R004 03 01, Task 8119) (NEL L2-4) from July 1960 through April 1961. This report was approved for publication 10 July 1961. The author wishes to thank G. M. Wenz and E. C. Westerfield for advice and helpful suggestions, and R. F. Arenz for assistance relative to computer programming.

CONTENTS

ILLUSTRATIONS

INTRODUCTION

The purpose of this report is to present some of the possibilities associated with statistical filtering of periodic data by smoothing and to provide sets of weights to fit some particular cases, for example, the smoothing of low-frequency, ambient sea-noise data. A brief theoretical discussion is included to explain the basic concepts involved.

A set of data arranged chronologically is called a time series. Time variations in the data may be relatively smooth or of a complex nature devoid of any apparent pattern.

Assuming the Fourier theorem, any variation with time may be considered the result of superposition of a number of simple sinusoidal components, the amplitudes, frequencies, and phases of these components being time-dependent.

In many time series it is assumed that high frequency oscillations in the data are either random noise or are of no significance to the particular purpose for which the data are to be evaluated. Consequently, one important purpose of time smoothing is to attenuate the amplitudes of high frequency components and, at the same time, preserve the low frequency components of immediate interest. Hence, the smoothed value of an experimentally observed time series is an estimate of its true value free from noise and other undesirable high frequency influences originally present.

Smoothing of a time series is a special case of the general process of numerical filtering and is analogous to low-pass filtering of an electrical signal. However, numerical filtering includes band-pass and high-pass filtering as well as low-pass filtering. Thus, if smoothed values are subtracted from the corresponding values in the original unsmoothed time series, only high frequency components will remain. Such an operation is equivalent to high-pass filtering. Band-pass filtering may be achieved by subtracting well smoothed values of a time series from corresponding values smoothed to a lesser extent; only intermediate frequencies will remain, thus giving the equivalent of band-pass filtering.

By use of the above procedures one may separate the oscillations of the time series into particular bands of frequencies, high, intermediate, and low. This report, however, is

primarily concerned with the low-pass case since the high-pass case is easily derived.

LINEAR FILTERS

A system is said to be linear if for all inputs $f(t)$, $g(t)$, and constants a, b

$$S[af(t)+bg(t)]=aS[f(t)]+bS[g(t)]$$

By superposition, these properties extend to any finite number of input functions.

The input and output of a filter can be related by a differential equation, the solution of which gives the output for any input. Notwithstanding, the differential equation description in many cases is not the most convenient for design purposes. More convenient modes of describing a filter, which make use of outputs produced by special types of inputs, employ the following functions:

1. The weighting function.
2. The frequency-response function.
3. The transfer function.

The response of a linear filter to general types of inputs may be described by its weighting function which is defined as the response of the filter to a unit impulse function after a time τ has elapsed. The weighting function $W(\tau)$, frequently called the "impulsive response" of the filter provides a complete characterization of the filter for $W(\tau)$ vanishing when $\tau<0$.

The frequency-response function $R(f)$ relates a sinusoidal input to the output that produces it and, for a stable filter, is the Fourier transform of its weighting function.

The transfer function, a generalization of the frequency-response function, is defined as the Laplace transform of the weighting function.

Theory shows[1,2] that, if $E_i\ (t)$ is the input, the output $E_o\ (t)$ is given by the following relation in which $W_1\ (\tau)$

is a weighting function:

$$E_o(t) = \int_0^\infty W_1(\tau)\, E_i(t-\tau)\, d\tau \tag{1}$$

The output $E_o(t)$ is observed to be a weighted mean of the past inputs. The system is physically realizable if it responds only to inputs which have already occurred, and the system is stable if every bounded input produces a bounded output. The weighting function may be thought of as the unit-impulse-response for the system. The method of description utilizing the concept of the weighting function is based on an analysis in the time domain.

The concept of the frequency-response may be introduced by the following procedure. Let the input $E_i(t)$ be given as follows:

$$E_i(t) = B \exp(j2\pi f t) \tag{2}$$

where $j = \sqrt{-1}$, f is in cycles per unit time, and B is a complex factor involving amplitude and phase. This complex function represents the sinusoidal input. The output is given by substituting this input in equation (1):

$$E_o(t) = \int_0^\infty B\, W(\tau) \exp(j2\pi f(t-\tau))\, d\tau$$

$$E_o(t) = B \exp j2\pi f t \left[\int_0^\infty W(\tau) \exp(-j2\pi f\tau)\, d\tau\right]$$

$$E_o(t) = B\, R_1(f) \exp j2\pi f t \tag{3}$$

where the complex quantity $R_1(f)$ is called the frequency response, and is defined by

$$R_1(f) = \int_0^\infty W_1(\tau) \exp(-j2\pi f\tau)\, d\tau \tag{4}$$

From equations (2) and (3)

$$\frac{E_o(t)}{E_i(t)} = \frac{E_o(t)}{B\exp(j2\pi ft)} = R_1(f)$$

The frequency response function may also be written

$$R_1(f) = |R_1(f)| \exp(j\phi(f))$$

where $|R_1(f)|$, the absolute value of $R_1(f)$, measures the amplitude response of the system to the input frequency f, and $\phi(f)$ represents the corresponding phase response.

Since $W_1(\tau)$ vanishes for $\tau<0$, equation (4) may be written

$$R_1(f) = \int_{-\infty}^{\infty} W_1(\tau)\exp(-j2\pi f\tau)d\tau \qquad (5)$$

The integral (5) exists for stable filters; hence the frequency-response function of a stable filter is the Fourier transform of its weighting function. Let $W_1(\tau)$ be symmetrical about some point τ_a and define a new variable τ such that $\tau=t+\tau_a$. Then $W_1(\tau) = W_1(t+\tau_a) = W(t)$. The frequency $R(f)$, expressed in terms of $W(t)$, is

$$R(f) = \int_{-\infty}^{\infty} W(t)\exp(-j2\pi ft)dt \qquad (6)$$

The weighting function $W(t)$ is given by the inverse Fourier transform:

$$W(t) = \int_{-\infty}^{\infty} R(f)\exp(j2\pi ft)df \qquad (7)$$

The integrals (6) and (7) exist for stable filters, and their application will be basic to the discussions which follow in this report.

An experimental function known only by a set of data sampled

at a constant sampling rate f_s requires the assumption contained in the sampling theorem (for the time domain), if unique representation of the equivalent continuous function is to be realized through equally spaced sampling. If a function $g(t)$ contains no frequencies higher than f_m cycles per unit time, it is completely determined by giving its ordinates at a series of points $1/2f_m$ units of time apart, the series extending throughout the time domain:[3]

$$\Delta t = 1/2f_m = 1/f_s \tag{8}$$

For equally spaced data points, condition (8) limits the frequencies to be considered to the range

$$\left[-0.5f_s \leq f \leq 0.5f_s\right] \tag{9}$$

Frequencies greater than $|0.5f_s|$ cannot be distinguished from those within the range given by (9). This phenomenon is known as aliasing.

It is important to note that once the frequency f_m has been specified and the sampling rate f_s determined by equation (8), frequency components above $0.5f_s$ in absolute value must be filtered out prior to the sampling operation, and if left in the data contribute errors due to spectrum folding (aliasing).

The sufficient condition that filtering functions do not shift the phase of any frequencies is obtained by requiring that the weighting function shall be even, namely $|W(t)| = |W(-t)|$. When this condition is imposed on equation (6) the imaginary part vanishes, and (6) reduces to

$$R(f) = 2\int_0^{\infty} W(t)\cos(2\pi ft)\,dt \tag{10}$$

and the frequency response becomes a pure, real quantity equal to the gain of the filter. For equally spaced data relative to time extending over a finite range, equation (10) is approximated as follows:

$$R(f) = Wo + 2\sum_{k=1}^{N} Wk\cos(2\pi fk) \tag{11}$$

SMOOTHING OF TIME SERIES

Smoothing, which is essentially low-pass filtering, can be accomplished by a numerical operator called the smoothing function.. Such functions consist of a series of fractional values called weights. These weights determine the extent to which each observation of the time series contributes to the smoothed or filtered value. Equation (12) indicates how smoothing is performed by a set of weights. If $\bar{y}_t$ is a smoothed value corresponding to the observation y_t in the time series, the computation is given by the equation below:

$$\bar{y}_t = \sum_{k=-N}^{N} W_k \, y_{t+k} = W_{-n} \, y_{t-N} + W_{-N+1} \, y_{t-N+1}$$

$$+ W_{-1} \, y_{t-1} + W_0 \, y_t + W_1 \, y_{t+1} + \cdot \cdot \cdot \, . W_n \, y_{t+N} \qquad (12)$$

The weight W_0 is called the principal weight and, if the mean of the original series is to be preserved, the sum of the weights composing the weighting function must be unity.

FREQUENCY RESPONSE OF SOME SMOOTHING FUNCTIONS

1."EQUALLY-WEIGHTED RUNNING MEAN"OF 2N+1 CONSECUTIVE WEIGHTS

The weighting function is $W_k = 1/(2N+1)$. Substituting this weighting function in equation (11), the frequency response becomes

$$R(f) = (2N+1)^{-1} \left[1 + 2 \sum_{k=1}^{N} \cos(2\pi fk)\right] \qquad (13)$$

One may use the analytic form of the envelope of the weights $W(t)$ to obtain a convenient approximation to the frequency response of the equally weighted running mean. If T is the

filtering interval, with $W(t) = 1/T$ for $|t| \leq T/2$, and $W(t) = 0$ when $|t| > T/2$, then

$$R(f) = 2\int_0^{T/2} T^{-1} \cos(2\pi f t)\, dt$$

$$R(f) = (\pi f T)^{-1} \sin(\pi f T) \tag{13A}$$

Equation (13A) gives a very good approximation of the frequency response of the equally weighted running mean.

In general, the frequency-response function $R(f)$ expresses the relative amplitude and phase of the input and output as a function of frequency, and is defined only for stable filters.

Since the weighting functions for the equally weighted running mean are even, phase shift is eliminated except for 180° shifts after the first zero crossing of the frequency axis. Because of this 180° shift, undesirable oscillations occur which introduce high frequency ripples into the output. This behavior of the frequency-response function beyond the first zero crossing constitutes the significant disadvantage in smoothing with equal weights. This can be seen by inspection of the curves in figure 1. A brief discussion of the coordinates used in plotting these curves may be instructive.

In requiring the weighting function $W(t)$ to be even, the frequency-response function $R(f)$ becomes a pure, real quantity. A further condition that the mean of the original time function is preserved requires that the sum of the weights of the weighting function $W(t)$ be equal to unity. It follows from this condition and equation (11) that the value of the ordinate of $R(t)$ is unity at zero frequency. The frequency f is plotted as the abscissa in cycles per time interval between data values. This time interval is called the data interval, and frequencies are expressed in cycles per data interval in discussing equally weighted running mean and Gaussian frequency-response functions.

From a study of the curves for the equally weighted running mean type filter in figure 1, a good idea can be obtained of its departure from the ideal low-pass filter with the same

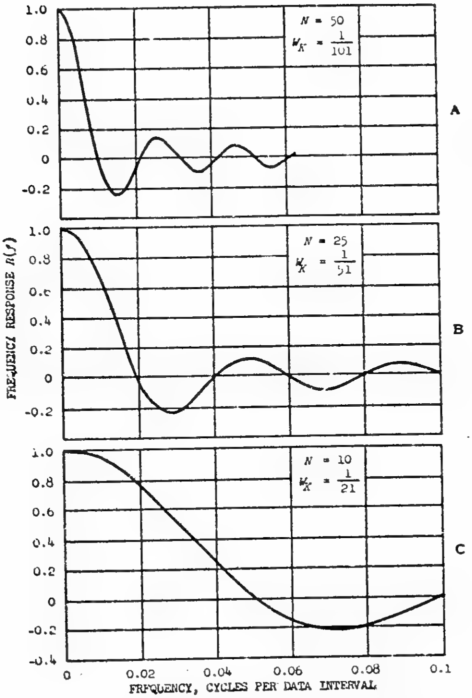


Figure 1. Frequency response of equally weighted running mean weight function computed by equation (13).

cutoff frequency. The cutoff frequency f_c is defined as the frequency at the first zero crossing of the frequency axis by the frequency-response curve.

The ideal low-pass filter leaves all frequency components up to the cutoff frequency f_c unaltered and eliminates completely all frequency components above the frequency f_c.

A brief discussion of figure 1B will reveal the nature of the departure of the equally weighted running mean filter from the ideal low-pass filter. The graph in figure 1B shows the frequency-response curve with cutoff frequency f_c equal to 0.0196 cycle per data interval. At frequency 0.012 cycle per data interval, the frequency response is 0.5 or only 50 per cent. This implies that the filter has removed 50 per cent of the contribution associated with this frequency component in the input data, as compared with the ideal filter (with the same cutoff) which would pass the frequency component at 0.012 cycle per unit data unaltered.

Equation (12) shows the significance of N as related to the total number of weights $(2N+1)$ in the filtering function. The curve in figure 2 gives a graphical description of the cutoff frequency f_c as a function of N. The cutoff frequency is seen to decrease as number of weights in the weighting function increases and, in particular, as N increases.

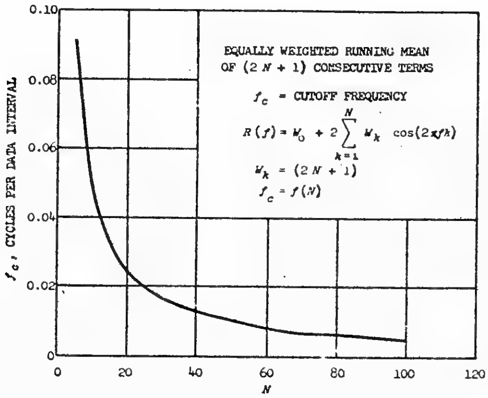


Figure 2. Variation of cutoff frequency f_c with N for the equally weighted running mean filtering function. The curve indicates how the cutoff frequency f_c decreases as N increases. Note that for N greater than 60 decrease in f_c is not very sensitive to increasing N.

2. THE GAUSSIAN FILTER

The undesirable oscillations associated with the frequency-response curve, after the first zero crossing of the frequency axis by the frequency-response curve, in the equally weighted running mean filter can be suppressed if not entirely eliminated by the use of a weighting function in which the weights decrease in magnitude outward from the central or principal weight W_0. One useful set of weights may be made proportional to the ordinates of the normal probability curve. In this case the continuous analytical expression for the weights is available:

$$W(t) = (2\pi\sigma^2)^{-1/2} \exp(-t^2/2\sigma^2) \qquad (14)$$

It is known that the total area under the curve is unity, and the mean of the original time function is preserved. The frequency-response function can be calculated by substituting the expression (14) in the basic equation (10). The expression for the frequency response is given by the integral

$$R(f) = 2 \int_0^\infty (2\pi\sigma^2)^{-1/2} \exp(-t^2/2\sigma^2) \cos(2\pi f t)\, dt \qquad (15)$$

Evaluating the integral (15) gives the frequency-response function for the Gaussian type filter. The result of this integration is given by equation (16) below:

$$R(f) = \exp(-2\pi^2\sigma^2 f^2) \qquad (16)$$

The general features of the function in (16) are shown graphically in figure 3. The frequency response for the normal curve smoothing function decreases smoothly with frequency, approaching zero asymptotically. Theoretically, zero response is never reached. However, the cutoff frequency f_c will be defined arbitrarily as the frequency for which the frequency-response is 1 per cent. It follows from equation (16) that f_c, as defined above, is controlled by the parameter σ. The graph in figure 4 gives some indica-

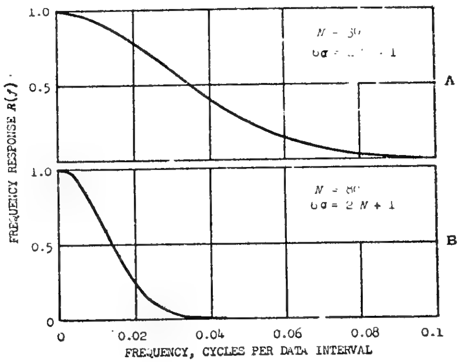


Figure 3. Frequency response of the Gaussian weighting function.

tion of how the cutoff frequency f_c, as defined, varies as a function of σ or N, where $6\sigma = 2N + 1$. Note that since the normal curve ordinates have negligible values beyond 3σ from the origin, the filtering interval for the Gaussian filter is taken as 6σ. With this choice the filtering interval is equal to $(2N+1)$ data intervals, equivalent to 6σ.

For equivalent filtering intervals, a study of figure 5 will indicate the following significant differences between the frequency function for the equally weighted running mean filter of $2N+1$ consecutive weights and the frequency response for the corresponding Gaussian weighting function.

(a) The frequency response for the equally weighted running mean weighting function shows positive and negative values above the frequency of the first zero response point. Such maxima and minima are undesirable because they introduce into the smoothed output misleading high frequency ripples. The frequency response for the normal curve

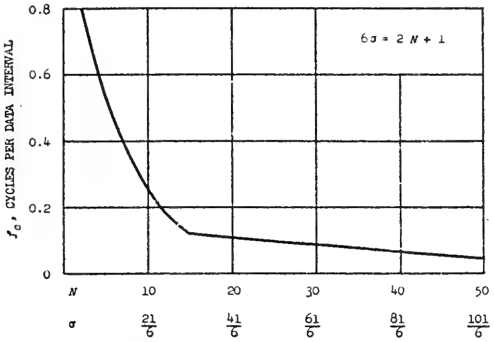


Figure 4. Variation of the cutoff frequency f_c for the Gaussian filtering function. Here the filtering interval T is taken equal to 6σ, since beyond 3σ from the origin the normal curve ordinates have negligible values. The cutoff frequency f_c decreases with increasing σ, and hence is governed by its value.

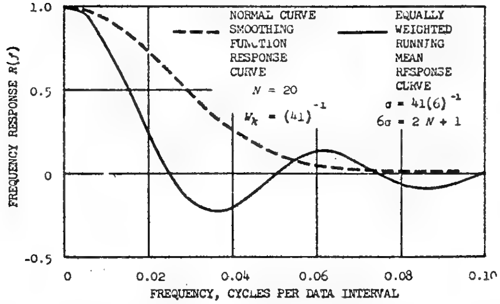


Figure 5. Frequency response of a forty-one term equally weighted running mean (N=20) compared with an equivalent normal curve smoothing function.

smoothing function, on the contrary, decreases smoothly with increasing frequency and asymptotically approaches zero, thus avoiding the generation of undesirable high frequency components in the smoothed output.

(b) If, at pleasure, one defines for practical purposes the cutoff frequency of the Gaussian filter at the point where the frequency response is 1 per cent, it is seen that for equal filtering intervals the equally weighted running mean filter has a lower cutoff frequency than the Gaussian filter. For the smoothing filters shown in figure 5 the equally weighted running mean filter has a cutoff frequency at 0.0248 cycle per data interval, whereas the 1 per cent cutoff point for the corresponding Gaussian filter occurs at 0.074 cycle per data interval.

For some types of smoothing the equally weighted running mean or the Gaussian filter may be satisfactory. Frequently, in other cases of smoothing or filtering the disadvantages of the filters discussed above may render them inadequate. A more satisfactory filter in which the above deficiencies are significantly reduced will be discussed below.

FILTERS WHICH APPROXIMATE IDEAL FILTERS WITH UNITY GAIN AND ZERO PHASE SHIFT

In the preceeding sections we discussed frequency-response functions having specified weighting functions. The procedure may be reversed and filtering or weighting functions having specified frequency-response functions $R(f)$ may be obtained using equation (7):

$$W(t) = \int_{-\infty}^{\infty} R(f) \exp(-j2\pi ft)\, df$$

$$W(t) = \int_{-\infty}^{\infty} R(f)[\cos 2\pi ft - i\sin 2\pi ft]\, df \qquad (17)$$

For an even response $R(f)$ the imaginary part vanishes, and

the expression reduces to

$$W(t) = 2\int_0^{\infty} R(f)\cos 2\pi ft \; df \tag{18}$$

which is observed to be the Fourier cosine transforms of $R(f)$. Here the frequency response can be specified and the weighting or smoothing function computed. If one specifies $R(f)$ with

$$\left.\begin{aligned} R(f) &= 1 \quad 0 \leqq f \leqq f_c \\ &= 0 \qquad f > f_c \end{aligned}\right\} \tag{19}$$

then

$$W(t) = 2\int_0^{f_c} 1\cos\,(2\pi ft)\; df \tag{20}$$

$$W(t) = (\pi t)^{-1} \sin 2\pi f_c \; t$$

The chief disadvantage of this smoothing function is its slow damping which renders it impractical for many important purposes. Furthermore if this function is truncated at some convenient distance on each side of the origin (a necessary procedure in most practical cases), the frequency response of this smoothing function departs significantly from the desired response at most frequencies.

The design of a low-pass numerical filter which eliminates many of the undesired properties of those filters or smoothing functions discussed above has been achieved by determining a set of weights W_k such that the actual frequency response of the filter defined by equation (19) will approximate best in the least square sense the ideal frequency response.

In the design of this filter the weights W_k are determined subject to the following conditions:

1. The phase shift must be identically zero for all frequencies.

2. $(2N+1)$ weights are to be used with $W_k = W_{-k}$.

3. A filter gain of unity from zero frequency to the ideal cutoff is required.

4. The weight calculations are optimized in the least square sense.

5. The following corrections in the weights are made to further approximate the desired properties of the frequency-response function.

(a) The weights must be corrected to minimize the oscillations beyond the first zero crossing of the frequency-response function. This is done by terminating the frequency response by means of a sine function characterized by a parameter h. By selecting h sufficiently large, the oscillations do not exceed the preselected limit. However by increasing the value of h one decreases the sharpness of the desired cutoff, since for $h = 0$ the cutoff is sharp but sharpness of cutoff decreases as h is made larger and larger.

(b) A second correction is made in the weight calculation to insure that the filter gain will be unity at zero frequency.

Formulas for calculating a set of weights, subject to above conditions, and the corresponding frequency response data are given by Marcel Martin.[4] The working formulas for the necessary calculations follow:

$$L_k = \left[\frac{\cos(2\pi kh)}{1-16\ h^2\ k^2}\right]\left[\frac{\sin 2\pi k\ (r_c + h)}{\pi k}\right] \qquad (21)$$

$$\Delta = 1 - \left[L_0 + 2\sum_{k=1}^{N} L_k\right] \qquad (22)$$

$$L_0 = 2\ (r_c + h) \qquad (23)$$

$$W_k = L_k + \frac{\Delta}{2N + 1} \tag{24}$$

$$R(r) = W_0 + 2 \sum_{k=1}^{N} W_k \cos(2\pi k r) \tag{25}$$

The normalized frequency r is defined by the relation $r = f/f_s$ of which the range of r is from $r = 0$ to $r = 0.5$. Here f is in cycles per unit time, and f_s is the sampling rate. Consequently r_c, the normalized cutoff for the ideal filter, is defined as follows:

$$r_c = \frac{f_c}{f_s}$$

where f_c is the ideal cutoff frequency in cycles per unit time and f_s is the sampling rate. The quantity N is defined in terms of the total number of weights in the weighting function, namely $(2N+1)$. The normalized frequency for the first zero crossing of the frequency-response curve is given by r_{ac}.

Equation (21) gives the approximate weights, with sine termination, corrected for high frequency oscillations beyond the first zero crossing of the frequency-response curve. The quantity $\Delta/(2N+1)$ in equation (24) is the correction to insure unity gain at zero frequency. Equation (24) gives the best approximation of the weights for the filter under the conditions specified. Equation (25) gives the best approximation of ordinates for the frequency-response function. The numerical results were obtained with the Datatron 220 digital computer, and the coded programs utilized the basic working formulas (21) through (25) inclusive.

Numerical results were obtained for various combinations of r_c, h, and N. The normalized cutoff frequency for the ideal filter is designated by r_c and h is a parameter which permits variation in the slope of the sine terminations. The use of sine termination avoids sharp discontinuity at r_c, thus greatly reducing the high-frequency oscillations or ripples associated with the Gibbs phenomenon. The number N is associated with the total number of weights in the weighting

function (W_k) of which there are $(2N+1)$. These weights are sequentially arranged as follows for the discrete weighting function.

$$W_{-n},\ W_{-n+1},\ \ldots\ W_{-1},\ W_0,\ W_1\ \ldots\ W_n$$

The smoothed points are obtained by equation (12). For the weighting functions discussed in this section the weights may be positive or negative except the central weight W_0 which is positive. However, any particular W_k is identical with the corresponding W_{-k}, in magnitude and sign.

Table 1 gives in addition to r_c, h, and N, r_{ac} which is the actual cutoff frequency for our approximate filter and, as such, is the frequency for the first zero crossing of the frequency-response-curve with the frequency axis; and E the maximum absolute error which is the departure of the frequency-response ordinate from unity between zero frequency and r_c, or the departure of the ordinate from zero, between the first zero crossing frequency and the frequency limit $r = 0.5$.

Figure 6 is designed to portray the essential characteristics of the frequency-response curve in greater details. The magnified scales at the upper left and the lower right indicate the oscillatory nature of the frequency-response function. The graph in figure 6 shows the desired ideal cutoff frequency r_c and how it is related to the actual cutoff frequency r_{ac} which is the frequency of the first zero crossing of the frequency-response curve. The value E_1 gives the maximum error amplitude in the gain between zero frequency and the ideal cutoff frequency r_c, and E_2 gives the maximum error amplitude between the actual cutoff frequency r_{ac} and the frequency limit $(r = 0.5)$ for this type of filter. In most cases E_1 is approximately equal to E_2. However, in any case the maximum absolute error is taken as E. If $E_1 = E_2$, then $E = E_1 = E_2$.

The two frequency-response curves A and B in figure 7 show the general trend of the function. Both curves have the same ideal cutoff frequency $(r_c = 0.01)$ and both have the same maximum absolute error in the gain $(E = 0.002)$. It is seen in curve A that with $h = 0.08$, which is relatively large, N is only 20 but the penalty is that the actual cutoff frequency, is relatively large namely 0.165. In curve B, with h and r_{ac} relatively small, namely $h = 0.01$, $r_{ac} = 0.0275$, it was necessary to increase N to 100 in order to retain the same maximum absolute error as small as $E = 0.002$. The above

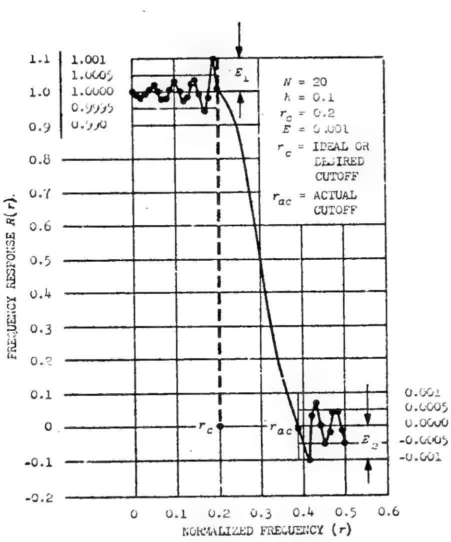


Figure 6. Frequency-response function for the filter. The magnified scale at the upper left gives the magnitude of the oscillations between $r = 0$ and $r_c = 0.2$. The variations show the small departures from the ideal gain of unity. E_1 is the maximum absolute gain error in this portion of the frequency-response function. The portion of the curve between the ideal normalized cutoff frequency (r_c=0.2) and the actual normalized cutoff frequency (r_{ac}=0.395) is plotted with reference to the vertical scale at the extreme left with range $R(r)$ from zero to 1. The magnified scale at the lower right shows the oscillatory nature of the frequency-response function between the first zero crossing and the frequency limit (r=0.5), and E_2 gives the magnitude of the maximum absolute gain error for this part of the response function. The variations in this portion of the frequency-response curve indicate its departure from the ideal value of zero.

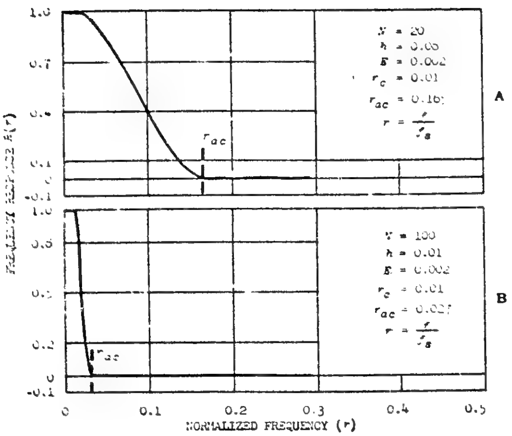


Figure 7. Frequency response $R(r)$ versus the normalized frequency r for two sets of parameters. Curves A and B illustrate that, with r_c fixed, a small h gives a sharper actual cutoff frequency r_{ac}, but only at the expense of increasing N if the maximum absolute gain error in the filter is not permitted to increase.

discussion discloses the significance of increasing N in achieving a small absolute gain error E when the actual cutoff frequency is made sharper by decreasing the parameter h. The gain errors are not observed in the curves A and B because of their small magnitudes relative to the scale used in the plots.

To indicate further the relative influence of the parameters, r_c, h, and N on the gain error E, attention is directed to figure 8. The curves A, B, and C in figure 8 indicate (for a given r_c and N) how the gain error E decreases as h is increased. This decrease in gain error E by increasing h is obtained at a sacrifice of increasing the actual or realizable cutoff frequency r_{ac} which increases with h.

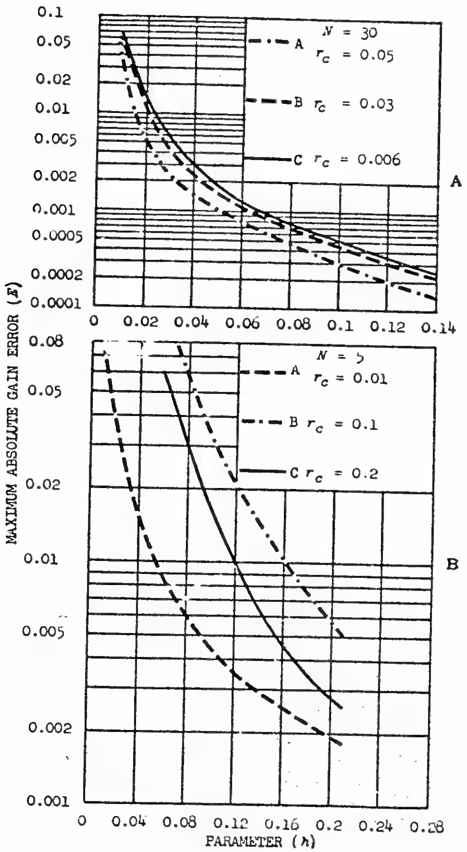


Figure 8. Variation of the maximum absolute gain error E with the parameter h for a given set of parameters r_c and N.

The graphs A, B, and C in figure 9 indicate in a general way the trend of the gain error E when the parameter N increases with h and r_c fixed. Although the gain error E in the frequency-response curve decreases as N increases for a fixed h and r_c (as can be seen from the graph in figure 9), a large N requires more samples of the original time function and necessitates more computer time for the smoothing operation.

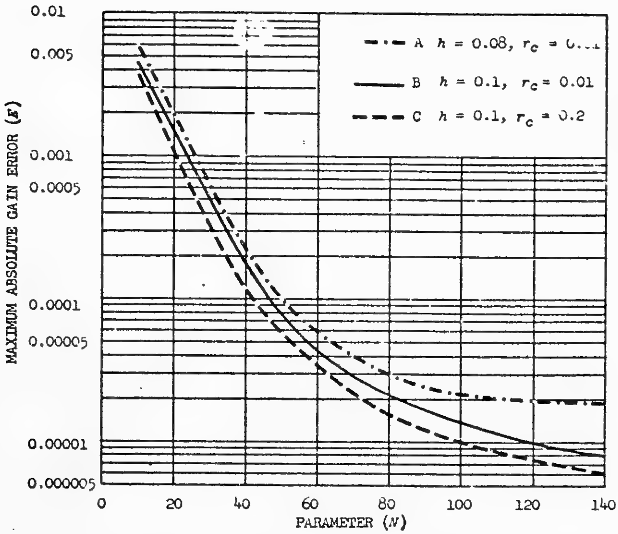


Figure 9. Variation of the maximum absolute gain error E with N for parameters h and r_c held constant. Observe that increasing N beyond 60 or 80 does not significantly decrease the gain error E.

In designing a filter which approximates the ideal filter specified above one should select h and N as small as possible for a chosen r_c such that the resulting gain of the filter does not depart from the ideal gain by more than some predetermined tolerance.

The use of the filter is illustrated by the following example. In figure 10 a spectral density curve is plotted with variance per unit band as the ordinate and frequency in cycles per week as the abscissa. This graph (unfiltered data) shows principal peaks at frequencies of 7 and 14 cycles per week. A numerical filter was designed to eliminate the 14-cycle component.

For the original data f_s the sampling rate was 168 per week. The normalized frequency r is related to the frequency f in cycles per week by the relation $f = rf_s$.

For this problem a filter is needed which will retain the 7-cycle-per-week component and eliminate higher components. Note that filter number 47, table 3, in the Appendix, with $r_c = 0.05$, $h = 0.01$, $r_{ac} = 0.08$, $N = 70$, and $E = 0.002$ can be used.

Then

$$f_c = (0.05)(168) = 8.4 \text{ cycles per week}$$

$$f_{ac} = (0.08)(168) = 13.4 \text{ cycles per week}$$

Frequency components below 8.4 cycles per week are retained but those above 13.4 cycles per week are eliminated. The effectiveness of this filtering operation is seen in figure 11. Observe that the 14-cycle-per-week component has been practically eliminated in the graph (figure 11), but the 7-cycle-per-week component is left unchanged in shape and magnitude.

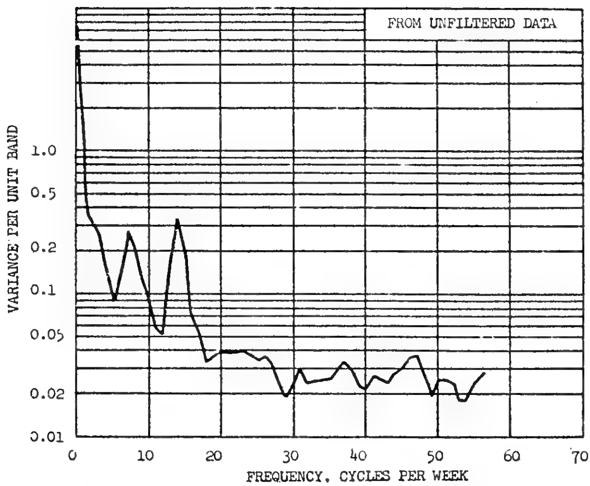


Figure 10. Spectral density function (variance per unit band) versus frequency. Two significant peaks occur at 7 and 14 cycles per week respectively.

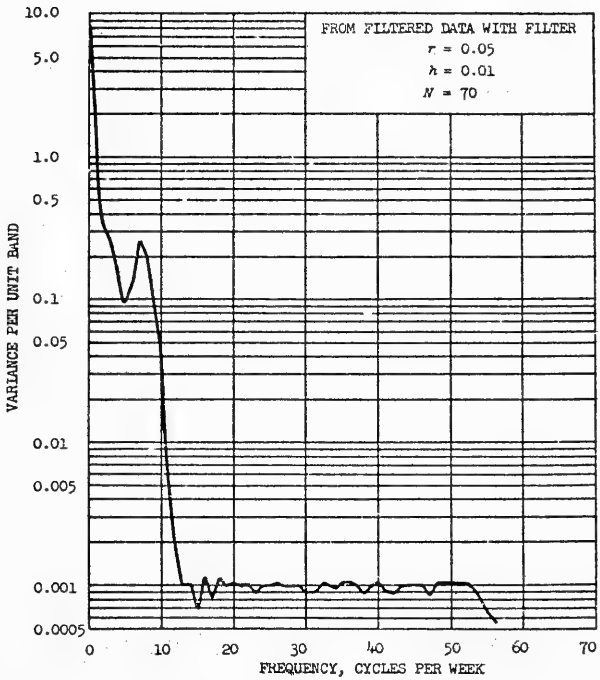


Figure 11. Spectral density function (variance per unit band) versus frequency. The graph was obtained from the data used in plotting figure 14 after filtering. The 14-cycle-per-week component has been practically eliminated leaving the 7-cycle per week component unchanged in shape and magnitude.

SUMMARY AND CONCLUSIONS

Three types of statistical filters have been discussed relative to their effectiveness in performing smoothing or filtering of data in time series.

1. The equally weighted running mean type of $(2N+1)$ consecutive equal weights, has a frequency-response function which decreases smoothly down to its cutoff frequency f_{ac} which is determined by N. This filter is computationally simple and has a relatively sharp cutoff, but its frequency response oscillates above the first zero crossing introducing undesirable ripples in the data. This filter is less critical than desired for many purposes.

2. The Gaussian smoothing function, though devoid of the oscillatory defect in the above type, has a frequency response which drops smoothly approaching zero asymptotically. This function does not have a cutoff but for practical purposes a cutoff may be arbitrarily defined as the frequency for which its frequency response is 1 per cent. With weighting functions of the same N, the equally weighted running mean type has a much sharper cutoff than the Gaussian type. Where the demand for sharp cutoff is not severe, the Gaussian smoothing filter may be used to advantage.

3. In the equally weighted running mean and the Gaussian types of filters the weighting function is specified and from it the frequency response function is evaluated. It is possible to reverse the procedure, by specifying the desired characteristics of the frequency response and from these conditions determining the corresponding weighting function. The weighting function $W(t)$ is the inverse Fourier transform of its corresponding frequency-response function (see equations (6) and (7)). This type of filter discussed earlier specified a gain of unity and zero phase shift for all frequencies. To further improve the fidelity of this filter, the frequency-response was terminated at its ideal cutoff frequency, r_c, by sine termination, and the weights further corrected to insure unity gain at zero frequency.

By a proper selection of the parameters r_c, h, and N (see list of definitions) filters with maximum absolute error less than 1 per cent have been designed. Weights for 118 such filters are given in the Appendix.

Computer programs have been written for the Burroughs Datatron 220 computer and were used in obtaining the numerical results for all filters discussed in this report.

REFERENCES

1. James, H. M. and others, Theory of Servomechanisms, Chapter 22, McGraw-Hill, 1947

2. Bendat, J. S., Principles and Applications of Random Noise Theory, Chapter 1, Wiley, 1958

3. Goldman, S., Information Theory, p.67-71, Prentice-Hall, 1953

4. General Electric Company Document No. 57SD340, Frequency Domain Applications in Data Processing, by M. A. Martin, May 1957

DEFINITIONS

$E_i(t)$ = the filter input.

$E_o(t)$ = the filter output.

f = frequency in cycles per unit time.

f_c = cutoff frequency in cycles per unit time.

f_s = sampling frequency (rate).

N = an integer defined by the total number of weights $(2N+1)$ in the weighting function.

k = an integer whose range is 1, 2, . . . N.

$W(\tau)$ = continuous weighting function

τ = variable time parameter.

W_k = weights of the discrete weighting function.

$R(f)$ = frequency-response function which relates a sinusoidal input to the output that it produces.

$\bar{y}_t$ = variable in the smoothed time series.

T = filtering interval.

$\frac{T}{2N+1}$ = data interval.

r = normalized frequency ratio defined by f/f_s where f is in cycles per unit time and f_s is the sampling rate.

r_c = normalized cutoff frequency for the ideal filter defined by the ratio f_c/f_s where f_c is the cutoff frequency in cycles per unit time.

r_{ac} = the normalized cutoff frequency ratio for the approximate filter.

E = maximum absolute error in the filter gain from zero frequency to the normalized cutoff r_c, and the frequency range between the first zero crossing and the frequency limit, $r = 0.5$. Note the range of r is from $r = 0$ to $r = 0.5$ for the filters discussed.

h = variable parameter which permits variation in the slope of the sine termination and is effective in diminishing the oscillations beyond the first zero crossing of the frequency-response function.

$\mathcal{R}(r)$ = frequency-response function as a function of the normalized frequency ratio r.

APPENDIX: FILTER DATA

Table 1 contains design parameters r_c, h, r_{ac}, E, and N. Various filter parameter combinations are listed relative to increasing values of r_c in the following order 0.01, 0.03, 0.05, 0.08, 0.10, 0.20, and 0.30, and the subgroups are listed relative to increasing values of h. For any particular subgroup it is observed that as N decreases the value of the absolute error E increases. The filter number corresponds to the arrangement of 118 sets of weights given in table 3.

Table 2 gives the filter parameters r_c, h, r_{ac}, E, and N along with the corresponding frequency-response data r and $R(r)$ for filters numbered 1 to 4 inclusive.

Table 3 gives the filter parameters r_c, h, r_{ac}, E, and N and the sets of weights for all the filters which are numbered to 118. Given the filter parameters stated above and the corresponding sets of weights, the frequency-response data are not necessary for designing the filter.

An example of one procedure for using the tables for filter design is as follows. Suppose one has a set of data sampled 12 times per day and it is desired to filter the data retaining all frequency components below 25 cycles per week and rejecting higher frequency components. Also, there is reason to believe that no frequency components exist in the data higher than 0.5 f_s. Then, $f_s = 0.5(12)(7) = 42$ cycles per week.

1. Calculate r_c from the relation

$$r_c = \frac{f_c}{f_s}$$

and

$$r'_c = \frac{25}{7(12)} = 0.298$$

2. Select the value in table 1 nearest r'_c. In the case considered, $r_c = 0.3$. Then $f_c = (0.3)(84) = 25.2$ cycles per week, which is the ideal cutoff frequency f_c for $r_c = 0.3$.

3. Finally select the smallest h (the sharpest cutoff) and the smallest N consistent with the maximum error in the gain

which can be tolerated. In table 1, $r_c = 0.3$, $h = 0.03$, and $N = 20$ will give the cutoff normalized frequency namely $r_{ac} = 0.355$. The absolute error $E = 0.006$ is seen to satisfy the gain requirement. The corresponding f_{ac} (the frequency of the first zero crossing is $(0.355)(84) = 29.8$ cycles per week. However the amplitude of the frequency components between f_c and f_{ac} fall off rapidly in magnitude, the rate of fall depending on the parameter h.

The various filters given in table 1 and corresponding weights in table 3 will satisfy the conditions for many problems. Table 1 is limited and, for some problems, desired r_c values are not sufficiently close to any given in the table. Suppose in the example given above one desired to retain all frequency components less than 3 cycles per week and eliminate all frequency components greater than 3 cycles per week. The calculated $r_c = 0.0357$. The nearest value of r_c in the table is 0.03 which gives the cutoff frequency $f_c = 2.52$ cycles per week. If greater precision is desired, a set of weights may be computed by equations (21) through (24) using the appropriate r_c together with the appropriate values of h and N.

TABLE 1. FILTER DESIGN PARAMETERS

The actual cutoff frequencies, r_{ac}, and maximum errors, E, are listed corresponding to specific choices of cutoff frequency (r_c), parameter h, and the number of weights ($2N+1$). The values of the weights for a particular choice may be found in table 3 in which the filter numbers are the same as in this table.

Filter No.	N	E	r_{ac}	h	r_c
1	100	0.0046	0.0275	0.01	0.01
2	70	.008	.0325		
3	60	.0125	.0275		
4	100	0.0001	0.075	0.03	
5	70	.0003	.065		
6	50	.001	.085		
7	70	0.00007	0.145	0.055	
8	50	.0001	.135		
9	30	.001	.125		
10	20	.006	.115		
11	70	0.00004	0.195	0.08	
12	50	.0001	.185		
13	30	.00064	.165		
14	20	.002	.165		
15	10	.006	.175		
16	30	0.0004	0.205	0.10	
17	20	.001	.205		
18	10	.0045	.205		
19	60	0.000005	0.405	0.20	
20	30	.00004	.415		
21	20	.0003	.405		
22	10	.001	.405		
23	4	.003	.405		
24	3	0.004	0.425		
25	60	0.005	0.0475	0.01	0.03
26	90	0.00006	0.095	0.03	
27	60	.0005	.105		
28	40	.001	.115		
29	30	.0047	.085		

Filter No.	N	$\bar{E}$	r_{ac}	h	r_c
30	70	0.001	0.165	0.055	
31	30	008	.145		
32	20	04	.135		
33	10	09	.135		
34	60	0.00004	0.155	0.08	
35	30	0064	.145		
36	20	01	.185		
37	10	07	.195		
38	60	0.00005	0.225	0.10	
39	30	0045	.235		
40	20	01	.225		
41	10	03	.225		
42	70	0.000006	0.425	0.20	
43	30	0006	.435		
44	20	003	.425		
45	10	006	.435		
46	4	0.008	0.415		
47	70	0.002	0.08	0.01	0.05
48	60	08	.065		
49	90	0.00009	0.115	0.03	
50	50	009	.105		
51	30	026	.115		
52	20	1	.105		
53	70	0.00008	0.145	0.05	
54	40	006	.145		
55	30	011	.145		
56	20	04	.145		
57	60	0.00009	0.205	0.08	
58	30	005	.215		
59	20	01	.205		
60	10	07	.215		
61	50	0.00008	0.245	0.10	
62	30	003	.245		
63	20	01	.245		
64	10	06	.245		

Filter No.	N	E	r_{ac}	h	r_c
65	70	0.000007	0.445	0.20	
66	30	.00007	.445		
67	20	.0002	.445		
68	10	.001	.445		
69	4	0.01	0.455		
70	40	0.0004	0.135	0.03	0.08
71	30	.003	.135		
72	20	.009	.135		
73	60	0.00004	0.255	0.08	
74	30	.0004	.235		
75	20	.001	.235		
76	10	.0006	.245		
77	50	0.00008	0.275	0.10	
78	30	.0002	.285		
79	20	.0008	.275		
80	10	.003	.275		
81	70	0.000007	0.475	0.20	
82	50	.00003	.475		
83	20	.0002	.475		
84	10	.001	.485		
85	4	0.01	0.465		
86	60	0.0006	0.175	0.03	0.10
87	30	.004	.165		
88	20	.009	.155		
89	30	0.00065	0.265	0.08	
90	20	.001	.255		
91	10	.006	.265		
92	60	0.00006	0.295	0.10	
93	30	.00045	.295		
94	10	.005	.295		
95	70	0.000007	0.495	0.20	
96	30	.00006	.495		
97	20	.0002	.495		
98	10	.001	.495		

Filter No.	K	E	r_{ac}	h	r_c
99	60	0.0006	0.275	0.03	0.20
100	20	.008	.255		
101	80	0.00003	0.395	0.08	
102	30	.0003	.365		
103	20	.001	.355		
104	10	.006	.365		
105	50	0.00006	0.395	0.10	
106	30	.0003	.395		
107	20	.00098	.395		
108	10	0.0038	0.395		
109	60	0.0002	0.375	0.03	0.30
110	30	.003	.365		
111	20	.006	.355		
112	90	0.00001	0.465	0.08	
113	70	.0004	.485		
114	10	.006	.465		
115	90	0.000004	0.495	0.10	
116	50	.00007	.495		
117	20	.0008	.495		
118	10	0.004	0.495		

TABLE 2. FREQUENCY RESPONSE DATA FOR FILTERS 1 THROUGH 4.

FILTER NO. 1 - $r_c = 0.01$, $h = 0.01$, $N = 100$, $E = 0.0046$, $r_{ac} = 0.0275$

r	$R(r)$	r	$R(r)$	r	$R(r)$
0.000	1.0000	0.215	0.0000	0.430	-0.0001
.005	0.9960	.220	-0.0001	.435	0.0000
.010	1.0019	.225	0.0000	.440	-0.0001
.015	0.8534	.230	-0.0001	.445	0.0000
.020	0.5008	.235	0.0000	.450	-0.0001
.025	0.1448	.240	-0.0001	.455	0.0000
.030	-0.0001	.245	0.0000	.460	-0.0001
.035	0.0019	.250	-0.0001	.465	0.0000
.040	-0.0014	.255	0.0000	.470	-0.0001
.045	0.0010	.260	-0.0001	.475	0.0000
.050	-0.0008	.265	0.0000	.480	-0.0001
.055	0.0006	.270	-0.0001	.485	0.0000
.060	-0.0005	.275	0.0000	.490	-0.0001
.065	0.0004	.280	-0.0001	.495	0.0000
.070	-0.0004	.285	0.0000	0.500	-0.0001
.075	0.0003	.290	-0.0001		
.080	-0.0003	.295	0.0000		
.085	0.0002	.300	-0.0001		
.090	-0.0002	.305	0.0000		
.095	0.0002	.310	-0.0001		
.100	-0.0002	.315	0.0000		
.105	0.0001	.320	-0.0001		
.110	-0.0002	.325	0.0000		
.115	0.0001	.330	-0.0001		
.120	-0.0001	.335	0.0000		
.125	0.0000	.340	-0.0001		
.130	-0.0001	.345	0.0000		
.135	0.0000	.350	-0.0001		
.140	-0.0001	.355	0.0000		
.145	0.0000	.360	-0.0001		
.150	-0.0001	.365	0.0000		
.155	0.0001	.370	-0.0001		
.160	-0.0001	.375	0.0000		
.165	0.0001	.380	-0.0001		
.170	-0.0001	.385	0.0000		
.175	0.0000	.390	-0.0001		
.180	-0.0001	.395	0.0000		
.185	0.0000	.400	-0.0001		
.190	-0.0001	.405	0.0000		
.195	0.0000	.410	-0.0001		
.200	-0.0001	.415	0.0000		
.205	0.0000	.420	-0.0001		
0.210	-0.0001	0.425	0.0000		

FILTER NO. 2 - $r_c = 0.01$, $h = 0.01$, $N = 70$, $E = 0.$[illegible]6,
$r_{ac} = 0.0325$

r	$R(r)$	r	$R(r)$
0.00	1.0000	0.43	0.0001
.01	0.9967	.44	.0000
.02	.5006	.45	-0.0001
.03	.0022	.46	0.0001
.04	.0010	.47	.0001
.05	.0009	.48	-0.0001
.06	.0001	.49	0.0000
.07	-0.0005	0.50	.0001
.08	0.0000		
.09	.0004		
.10	.0001		
.11	-0.0003		
.12	0.0002		
.13	.0002		
.14	-0.0002		
.15	0.0000		
.16	.0002		
.17	-0.0001		
.18	.0002		
.19	0.0002		
.20	.0001		
.21	-0.0002		
.22	0.0001		
.23	.0002		
.24	-0.0001		
.25	.0001		
.26	0.0001		
.27	.0000		
.28	-0.0001		
.29	0.0001		
.30	.0001		
.31	-0.0001		
.32	.0000		
.33	0.0001		
.34	-0.0000		
.35	.0001		
.36	0.0001		
.37	.0001		
.38	-0.0001		
.39	0.0000		
.40	.0001		
.41	-0.0001		
0.42	0.0001		

FILTER NO. 3 - $r_c = 0.01$, $h = 0.01$, $N = 60$, $E = 0.01$, $r_{ac} = 0.0275$

r	$R(r)$	r	$R(r)$
0.00	1.0000	0.43	0.0002
.01	.0011	.44	-0.0005
.02	0.5037	.45	0.0010
.03	-0.0058	.46	-0.0007
.04	0.0084	.47	0.0005
.05	-0.0011	.48	.0003
.06	.0030	.49	-0.0006
.07	0.0046	0.50	0.0010
.08	-0.0036		
.09	0.0017		
.10	.0007		
.11	-0.0021		
.12	0.0027		
.13	-0.0018		
.14	0.0007		
.15	.0009		
.16	-0.0016		
.17	0.0019		
.18	-0.0011		
.19	0.0002		
.20	.0010		
.21	-0.0013		
.22	0.0015		
.23	-0.0007		
.24	.0000		
.25	0.0010		
.26	-0.0012		
.27	0.0012		
.28	-0.0004		
.29	.0002		
.30	0.0010		
.31	-0.0010		
.32	0.0010		
.33	-0.0002		
.34	.0003		
.35	0.0010		
.36	-0.0010		
.37	0.0008		
.38	.0000		
.39	-0.0004		
.40	0.0010		
.41	-0.0008		
0.42	0.0006		

FILTER NO. 4 - r_c = 0.01, h = 0.03, N = 100, E=0.0001, r_{ac} = 0.075

r	$R(r)$	r	$R(r)$
0.00	1.00000	0.43	-0.00001
.01	1.00010	.44	.00001
.02	0.93300	.45	.00001
.03	.75000	.46	.00001
.04	.50000	.47	.00001
.05	.25010	.48	.00001
.06	.06710	.49	.00001
.07	.00000	0.50	-0.00001
.08	-0.00010		
.09	.00008		
.10	.00005		
.11	.00004		
.12	.00003		
.13	.00003		
.14	.00002		
.15	.00002		
.16	.00002		
.17	.00002		
.18	.00001		
.19	.00001		
.20	.00001		
.21	.00001		
.22	.00001		
.23	.00001		
.24	.00001		
.25	.00001		
.26	.00001		
.27	.00001		
.28	.00001		
.29	.00001		
.30	.00001		
.31	.00001		
.32	.00001		
.33	.00001		
.34	.00001		
.35	.00001		
.36	.00001		
.37	.00001		
.38	.00001		
.39	.00001		
.40	.00001		
.41	.00001		
0.42	-0.00001		

TABLE 5. VALUES OF WEIGHTS RESULTING FROM THE SETS OF PARAMETERS GIVEN IN TABLE 1.

FILTER NO. 1 - $r_c = 0.01$, $h = 0.01$, $N = 100$, $E = 0.0046$, $r_{ac} = 0.0275$

k	Value of W_k	k	Value of W_k	k	Value of W_k
0	0.03998	38	-0.00467	76	-0.00002
1	.03986	39	.00433	77	.00000
2	.03950	40	.00394	78	0.00001
3	.03891	41	.00353	79	.00004
4	.03809	42	.00310	80	.00006
5	.03705	43	.00265	81	.00009
6	.03581	44	.00221	82	.00011
7	.03438	45	.00178	83	.00014
8	.03278	46	.00137	84	.00016
9	.03102	47	.00098	85	.00018
10	.02914	48	.00063	86	.00020
11	.02714	49	.00030	87	.00021
12	.02506	50	.00002	88	.00021
13	.02291	51	0.00023	89	.00021
14	.02072	52	.00044	90	.00021
15	.01852	53	.00060	91	.00020
16	.01632	54	.00073	92	.00018
17	.01415	55	.00082	93	.00017
18	.01203	56	.00088	94	.00014
19	.00998	57	.00091	95	.00012
20	.00011	58	.00091	96	.00009
21	.00615	59	.00088	97	.00007
22	.00440	60	.00084	98	.00004
23	.00279	61	.00078	99	.00000
24	.00131	62	.00071	100	-0.00002
25	-0.00002	63	.00063		
26	.00120	64	.00054		
27	.00223	65	.00046		
28	.00310	66	.00037		
29	.00382	67	.00029		
30	.00440	68	.00022		
31	.00483	69	.00016		
32	.00513	70	.00010		
33	.00530	71	.00006		
34	.00536	72	.00002		
35	.00531	73	-0.00000		
36	.00518	74	.00001		
37	-0.00496	75	-0.00002		

FILTER NO. 2 - $r_c = 0.01$, $h = 0.01$, $N = 70$, $E = 0.006$, $r_{ac} = 0.0325$

k	Value of W_k	k	Value of W_k
0	0.04002	42	-0.00306
1	.03990	43	.00261
2	.03954	44	.00217
3	.03895	45	.00174
4	.03813	46	.00133
5	.03709	47	.00094
6	.03585	48	.00059
7	.03442	49	.00026
8	.03282	50	0.00002
9	.03106	51	.00027
10	.02918	52	.00048
11	.02718	53	.00064
12	.02510	54	.00077
13	.02295	55	.00086
14	.02076	56	.00092
15	.01856	57	.00095
16	.01636	58	.00095
17	.01419	59	.00092
18	.01207	60	.00088
19	.01002	61	.00082
20	.00805	62	.00075
21	.00619	63	.00067
22	.00444	64	.00058
23	.00283	65	.00050
24	.00135	66	.00041
25	.00002	67	.00033
26	-0.00116	68	.00026
27	.00219	69	.00020
28	.00306	70	0.00014
29	.00378		
30	.00436		
31	.00479		
32	.00509		
33	.00526		
34	.00532		
35	.00527		
36	.00514		
37	.00492		
38	.00463		
39	.00429		
40	.00390		
41	-0.00349		

FILTER NO. 3 - $r_c = 0.01$, $h = 0.01$, $N = 60$, $E = 0.01$, $r_{ac} = 0.0275$

k	Value of W_k	k	Value of W_k
0	0.04010	42	-0.00298
1	.03998	43	.00254
2	.03962	44	.00210
3	.03903	45	.00167
4	.03821	46	.00126
5	.03717	47	.00087
6	.03593	48	.00051
7	.03450	49	.00019
8	.03290	50	0.00010
9	.03114	51	.00035
10	.02926	52	.00055
11	.02726	53	.00072
12	.02517	54	.00085
13	.02303	55	.00094
14	.02084	56	.00100
15	.01863	57	.00100
16	.01644	58	.00100
17	.01427	59	.00100
18	.01214	60	.00100
19	.01009		
20	.00129		
21	.00627		
22	.00452		
23	.00291		
24	.00143		
25	.00010		
26	-0.00108		
27	.00211		
28	.00298		
29	.00371		
30	.00428		
31	.00471		
32	.00501		
33	.00519		
34	.00524		
35	.00520		
36	.00506		
37	.00484		
38	.00455		
39	.00421		
40	.00383		
41	-0.00341		

FILTER NO. 4 - $r_c = 0.01$, $h = 0.03$, $N = 100$, $E = 0.0001$,
$r_{ac} = 0.075$

k	Value of W_k	k	Value of W_k	k	Value of W_k
0	0.08000	42	-0.00002	84	0.00003
1	.07889	43	.00007	85	.00001
2	.07564	44	.00012	86	.00001
3	.07046	45	.00014	87	.00000
4	.06364	46	.00015	88	-0.00000
5	.05560	47	.00013	89	.00001
6	.04681	48	.00009	90	.00001
7	.03773	49	.00005	91	.00000
8	.02883	50	.00000	92	.00000
9	.02052	51	0.00004	93	0.00001
10	.01314	52	.00007	94	.00001
11	.00691	53	.00009	95	.00001
12	.00200	54	.00009	96	.00001
13	-0.00165	55	.00007	97	.00001
14	.00403	56	.00005	98	.00001
15	.00530	57	.00003	99	.00000
16	.00566	58	.00001	100	-0.00000
17	.00535	59	-0.00001		
18	.00459	60	.00002		
19	.00361	61	.00002		
20	.00257	62	.00001		
21	.00164	63	0.00001		
22	.00089	64	.00003		
23	.00037	65	.00004		
24	.00009	66	.00006		
25	.00000	67	.00006		
26	.00007	68	.00007		
27	.00022	69	.00006		
28	.00041	70	.00005		
29	.00057	71	.00004		
30	.00068	72	.00002		
31	.00072	73	.00001		
32	.00069	74	.00000		
33	.00059	75	-0.00000		
34	.00046	76	0.00000		
35	.00031	77	.00001		
36	.00016	78	.00002		
37	.00005	79	.00002		
38	0.00003	80	.00003		
39	.00007	81	.00004		
40	.00006	82	.00004		
41	0.00003	83	0.00003		

FILTER NO. 5 - $r_o = 0.01$, $k = 0.03$, $N = 70$, $E = 0.0003$, $r_{ac} = 0.065$

k	Value of W_k	k	Value of Y_k
0	0.08000	42	-0.00001
1	.07890	43	.00007
2	.07565	44	.00011
3	.07046	45	.00014
4	.06364	46	.00014
5	.05561	47	.00012
6	.04681	48	.00009
7	.03774	49	.00004
8	.02884	50	0.00000
9	.02053	51	.00004
10	.01314	52	.00008
11	.00692	53	.00009
12	.00198	54	.00009
13	-0.00165	55	.00008
14	.00402	56	.00006
15	.00529	57	.00003
16	.00566	58	.00001
17	.00534	59	-0.00001
18	.00459	60	.00002
19	.00360	61	.00001
20	.00257	62	.00000
21	.00163	63	0.00001
22	.00089	64	.00003
23	.00037	65	.00005
24	.00008	66	.00006
25	0.00000	67	.00007
26	-0.00007	68	.00007
27	.00022	69	.00006
28	.00040	70	0 .00005
29	.00057		
30	.00068		
31	.00072		
32	.00068		
33	.00059		
34	.00045		
35	.00030		
36	.00016		
37	.00004		
38	0.00004		
39	.00007		
40	.00007		
41	0.00004		

FILTER NO. 6 - $r_c = 0.01$, $h = 0.03$, $N = 50$, $E = 0.001$, $r_{ac} = 0.085$

k	Value of W_k	k	Value of W_k
0	0.08002	42	0.00000
1	.07891	43	-0.00005
2	.07567	44	.00009
3	.07048	45	.00012
4	.06366	46	.00012
5	.05563	47	.00011
6	.04683	48	.00007
7	.03775	49	.00003
8	.02885	50	0.00002
9	.02055		
10	.01316		
11	.00693		
12	.00199		
13	-0.00163		
14	.00400		
15	.00528		
16	.00564		
17	.00533		
18	.00457		
19	.00358		
20	.00255		
21	.00162		
22	.00087		
23	.00035		
24	.00006		
25	0.00002		
26	-0.00004		
27	.00020		
28	.00038		
29	.00055		
30	.00066		
31	.00070		
32	.00067		
33	.00057		
34	.00044		
35	.00028		
36	.00014		
37	.00002		
38	0.00005		
39	.00009		
40	.00009		
41	0.00005		

FILTER NO. 7 - $r_c = 0.01$, $h = 0.055$, $N = 70$, $E = 0.00007$, $r_{ac} = 0.145$

k	Value of W_k
0	0.13000
1	.12499
2	.11066
3	.09004
4	.06597
5	.04226
6	.02194
7	.00694
8	-0.00221
9	.00616
10	.00638
11	.00459
12	.00234
13	.00081
14	0.00018
15	.00015
16	-0.00032
17	.00081
18	.00105
19	.00097
20	.00067
21	.00031
22	.00007
23	0.00000
24	-0.00008
25	.00022
26	.00032
27	.00034
28	.00027
29	.00015
30	.00004
31	0.00001
32	-0.00001
33	.00006
34	.00011
35	.00014
36	.00012
37	.00007
38.	.00002
39	0.00002
40	.00002
41	-0.00000
42	-0.00003
43	.00005
44	.00005
45	.00003
46	.00000
47	0.00002
48	.00003
49	.00002
50	.00000
51	-0.00001
52	.00002
53	.00001
54	0.00000
55	.00002
56	.00003
57	.00002
58	.00001
59	.00000
60	-0.00000
61	.00000
62	0.00001
63	.00002
64	.00002
65	.00002
66	.00002
67	.00001
68	.00000
69	.00000
70	0.00000

FILTER NO. 8 - $r_c = 0.01$, $h = 0.055$, $N = 50$, $E = 0.0001$,
$r_{ac} = 0.135$

k	Value of W_k	k	Value of W_k
0	0.13000	42	-0.00003
1	.12500	43	.00005
2	.11086	44	.00005
3	.09004	45	.00003
4	.06597	46	.00000
5	.04226	47	0.00002
6	.02195	48	.00003
7	.00694	49	.00002
8	-0.00221	50	.00000
9	.00616		
10	.00637		
11	.00459		
12	.00233		
13	-.00061		
14	0.00018		
15	.00016		
16	-0.00031		
17	.00081		
18	.00105		
19	.00097		
20	.00066		
21	.00031		
22	.00006		
23	0.00001		
24	-0.00007		
25	.00021		
26	.00032		
27	.00034		
28	.00027		
29	.00015		
30	.00004		
31	0.00001		
32	-0.00000		
33	.00005		
34	.00001		
35	.00013		
36	.00012		
37	.00007		
38	.00002		
39	0.00002		
40	.00002		
41	0.00000		

FILTER NO. 9 - r_c = 0.01, h = 0.055, N = 30, E = 0.001, r_{ac} = 0.125

k	Value of W_k	k	Value of W_k
0	0.12999	16	-0.00033
1	.12498	17	.00082
2	.11084	18	.00107
3	.09003	19	.00098
4	.06595	20	.00068
5	.04224	21	.00033
6	.02193	22	.00008
7	.00692	23	.00001
8	-0.00222	24	.00009
9	.00618	25	.00023
10	.00639	26	.00034
11	.00461	27	.00036
12	.00235	28	.00028
13	.00063	29	.00016
14	0.00017	30	-0.00006
15	0.00014		

FILTER NO. 10 - r_c = 0.01, h = 0.055, N = 20, E = 0.006, r_{ac} = 0.115

k	Value of W_k	k	Value of W_k
0	0.12989	11	-0.00470
1	.12488	12	.00245
2	.11075	13	.00072
3	.08993	14	0.00007
4	.06586	15	.00004
5	.04215	16	-0.00042
6	.02184	17	.00092
7	.00683	18	.00116
8	-0.00232	19	.00108
9	.00627	20	.00077
10	-0.00648		

FILTER NO. 11 - $r_c = 0.01$, $h = 0.08$, $N = 70$, $E = 0.00004$, $r_{ac} = 0.195$

k	Value of W_k	k	Value of W_k
0	0.18000	42	-0.00003
1	.16651	43	.00003
2	.13070	44	.00001
3	.08431	45	0.00001
4	.04089	46	.00001
5	.01020	47	-0.00000
6	-0.00467	48	.00001
7	.00767	49	.00001
8	.00448	50	0.00000
9	.00084	51	.00001
10	0.00063	52	.00001
11	.00012	53	.00000
12	-0.00090	54	-0.00001
13	.00127	55	.00000
14	.00087	56	0.00000
15	.00024	57	.00001
16	0.00006	58	.00001
17	-0.00008	59	.00000
18	.00035	60	-0.00000
19	.00045	61	.00000
20	.00031	62	0.00001
21	.00009	63	.00001
22	0.00000	64	.00001
23	-0.00006	65	.00000
24	.00017	66	-0.00000
25	.00020	67	0.00000
26	.00013	68	.00001
27	.00004	69	.00001
28	0.00000	70	0.00001
29	-0.00003		
30	.00009		
31	.00010		
32	.00006		
33	.00001		
34	0.00001		
35	-0.00002		
36	.00005		
37	.00005		
38	.00003		
39	0.00000		
40	.00001		
41	-0.00001		

FILTER NO. 12 - $r_c = 0.01$, $h = 0.08$, $N = 50$, $E = 0.0001$, $r_{ac} = 0.185$

k	Value of W_k	k	Value of W_k
0	0.18000	42	-0.00002
1	.16652	43	0.00002
2	.13070	44	.00001
3	.08431	45	.00001
4	.04090	46	.00001
5	.01020	47	.00000
6	-0.00487	48	-0.00001
7	.00767	49	.00001
8	.00448	50	0.00000
9	.00084		
10	0.00063		
11	.00012		
12	-0.00090		
13	.00127		
14	.00086		
15	.00024		
16	0.00006		
17	-0.00008		
18	.00035		
19	.00044		
20	.00030		
21	.00009		
22	.00000		
23	.00006		
24	.00017		
25	.00020		
26	.00013		
27	.00003		
28	.00000		
29	.00003		
30	.00009		
31	.00010		
32	.00006		
33	.00001		
34	0.00001		
35	-0.00002		
36	.00005		
37	.00005		
38	0.00002		
39	-0.00001		
40	0.00001		
41	-0.00000		

FILTER NO. 13 - $r_c = 0.01$, $h = 0.08$, $N = 30$, $Z = 0.0006$, $r_{ac} = 0.165$

k	Value of W_k	k	Value of W_k
0	0.17999	16	0.00005
1	.16650	17	-0.00009
2	.13069	18	.00036
3	.08430	19	.00046
4	.04089	20	.00032
5	.01019	21	.00010
6	-0.00483	22	.00001
7	.00768	23	.00007
8	.00449	24	.00018
9	.00085	25	.00021
10	0.00062	26	.00014
11	.00011	27	.00004
12	-0.00091	28	.00001
13	.00128	29	.00004
14	.00088	30	-0.00010
15	-0.00025		

FILTER NO. 14 - $r_c = 0.01$, $h = 0.08$, $N = 20$, $Z = 0.002$, $r_{ac} = 0.165$

k	Value of W_k	k	Value of W_k
0	0.17995	11	0.00006
1	.16646	12	-0.00095
2	.13064	13	.00133
3	.08426	14	.00092
4	.04084	15	.00029
5	.01015	16	0.00000
6	-0.00492	17	-0.00013
7	.00772	18	.00040
8	.00454	19	.00050
9	.00090	20	.00036
10	0.00057		

FILTER NO. 15 - $r_c = 0.01$, $h = 0.08$, $N = 10$, $E = 0.006$, $r_{ac} = 0.175$

k	Value of W_k	k	Value of W_k
0	0.17949	6	-0.00538
1	.16600	7	.00818
2	.13019	8	.00500
3	.08380	9	.00136
4	.04038	10	0.00011
5	.00969		

FILTER NO. 16 - $r_c = 0.01$, $h = 0.1$, $N = 30$, $E = 0.00009$, $r_{ac} = 0.205$

k	Value of W_k	k	Value of W_k
0	0.22000	16	-0.00041
1	.19541	17	.00010
2	.13419	18	0.00001
3	.06530	19	-0.00013
4	.01519	20	.00024
5	-0.00656	21	.00017
6	.00762	22	.00003
7	.00204	23	.00001
8	0.00091	24	.00010
9	.00015	25	.00013
10	-0.00125	26	.00008
11	.00124	27	.00001
12	.00034	28	.00002
13	0.00012	29	.00007
14	-0.00016	30	.00008
15	-0.00050		

FILTER NO. 17 - $r_c = 0.01$, $h = 0.1$, $N = 20$, $E = 0.001$, $r_{ac} = 0.205$

k	Value of W_k	k	Value of W_k
0	0.21996	11	-0.00127
1	.19538	12	.00037
2	.13416	13	0.00009
3	.06526	14	-0.00019
4	.01515	15	.00053
5	-0.00660	16	.00044
6	.00765	17	.00013
7	.00208	18	.00002
8	0.00027	19	.00016
9	.00011	20	.00028
10	-0.00128		

FILTER NO. 18 - $r_c = 0.01$, $h = 0.1$, $N = 10$, $E = 0.006$, $r_{ac} = 0.205$

k	Value of W_k	k	Value of W_k
0	0.21965	6	-0.00797
1	.19506	7	.00239
2	.13384	8	0.00056
3	.06495	9	-0.00020
4	.01484	10	.00160
5	-0.00691		

FILTER NO. 19 - $r_c = 0.01$, $h = 0.2$, $N = 60$, $E = 0.000005$, $r_{ac} = 0.405$

k	Value of W_k	k	Value of W_k
0	0.42000	42	-0.00000
1	.26465	43	0.00000
2	.03976	44	-0.00000
3	-0.01315	45	.00000
4	0.00225	46	0.00000
5	-0.00131	47	-0.00000
6	.00074	48	0.00000
7	0.00023	49	-0.00000
8	-0.00073	50	0.00000
9	0.00014	51	.00000
10	-0.00030	52	-0.00000
11	.00011	53	0.00000
12	.00003	54	-0.00000
13	.00018	55	0.00000
14	0.00002	56	.00000
15	-0.00012	57	-0.00000
16	.00003	58	0.00000
17	.00003	59	-0.00000
18	.00007	60	0.00000
19	0.00000		
20	-0.00006		
21	.00001		
22	.00003		
23	.00003		
24	.00000		
25	.00003		
26	.00000		
27	.00002		
28	.00001		
29	.00000		
30	.00001		
31	0.00000		
32	-0.00001		
33	.00000		
34	.00000		
35	.00001		
36	0.00000		
37	-0.00001		
38	.00000		
39	.00000		
40	.00000		
41	0.00000		

FILTER NO. 20 - $r_c = 0.01$, $h = 0.02$, $N = 30$, $E = 0.00004$, $r_{ac} = 0.415$

k	Value of W_k	k	Value of W_k
0	0.42000	16	-0.00003
1	.26465	17	.00004
2	.03976	18	.00007
3	-0.01315	19	0.00000
4	0.00225	20	-0.00006
5	-0.00131	21	.00001
6	.00074	22	.00003
7	0.00023	23	.00003
8	-0.00073	24	.00000
9	0.00014	25	.00003
10	-0.00030	26	.00000
11	.00011	27	.00002
12	.00003	28	.00001
13	.00018	29	.00000
14	0.00002	30	-0.00002
15	-0.00012		

FILTER NO. 21 - $r_c = 0.01$, $h = 0.2$, $N = 20$, $E = 0.0003$, $r_{ac} = 0.405$

k	Value of W_k	k	Value of W_k
0	0.41999	11	-0.00012
1	.26464	12	.00004
2	.03975	13	.00019
3	-0.01315	14	0.00001
4	0.00224	15	-0.00013
5	-0.00132	16	.00004
6	.00075	17	.00004
7	0.00022	18	.00008
8	-0.00074	19	.00001
9	0.00013	20	-0.00007
10	-0.00031		

FILTER NO. 22 - r_c = 0.01, h = 0.2, N = 10, E = 0.001, r_{ac} = 0.405

k	Value of W_k	k	Value of W_k
0	0.41992	6	-0.00082
1	.26457	7	0.00015
2	.03969	8	-0.00080
3	-0.01322	9	0.00006
4	0.00217	10	-0.00037
5	-0.00139		

FILTER NO. 23 - r_c = 0.01, h = 0.2, N = 4, E = 0.003, r_{ac} = 0.405

k	Value of W_k	k	Value of W_k
0	0.41922	3	-0.01393
1	.26387	4	0.00147
2	0.03898		

FILTER NO. 24 - r_c = 0.01, h = 0.2, N = 3, E = 0.004, r_{ac} = 0.425

k	Value of W_k	k	Value of W_k
0	0.41964	2	0.03940
1	.26429	3	-0.01351

FILTER NO. 25 - $r_c = 0.03$, $h = 0.01$, $N = 60$, $E = 0.005$,
$r_{ac} = 0.0475$

k	Value of W_k	k	Value of W_k
0	0.07997	42	-0.00333
1	.07910	43	.00339
2	.07652	44	.00323
3	.07235	45	.00289
4	.06675	46	.00241
5	.05995	47	.00183
6	.05220	48	.00121
7	.04382	49	.00060
8	.03511	50	.00003
9	.02640	51	0.00046
10	.01799	52	.00085
11	.01014	53	.00112
12	.00312	54	.00128
13	-0.00291	55	.00133
14	.00781	56	.00128
15	.01149	57	.00115
16	.01395	58	.00096
17	.01522	59	.00073
18	.01539	60	0.00050
19	.01461		
20	.01303		
21	.01084		
22	.00826		
23	.00547		
24	.00268		
25	.00003		
26	0.00231		
27	.00424		
28	.00570		
29	.00663		
30	.00705		
31	.00698		
32	.00648		
33	.00563		
34	.00452		
35	.00324		
36	.00190		
37	.00059		
38	-0.00062		
39	.00165		
40	.00246		
41	-0.00302		

FILTER NO. 26 - $r_c = 0.03$, $h = 0.03$, $N = 90$, $E = 0.00006$,
$r_{ac} = 0.095$

k	Value of W_k	k	Value of W_k	k	Value of W_k
0	0.12000	42	-0.00000	84	0.00001
1	.11678	43	.00003	85	.00002
2	.10108	44	.00009	86	.00003
3	.09313	45	.00014	87	.00003
4	.07523	46	.00017	88	.00002
5	.05561	47	.00017	89	.00001
6	.03614	48	.00013	90	0.00001
7	.01851	49	.00007		
8	.00399	50	0.00000		
9	-0.00662	51	.00006		
10	.01314	52	.00010		
11	.01585	53	.00012		
12	.01547	54	.00010		
13	.01293	55	.00008		
14	.00923	56	.00004		
15	.00530	57	.00002		
16	.00183	58	.00000		
17	0.00074	59	.00000		
18	.00225	60	.00002		
19	.00278	61	.00004		
20	.00257	62	.00006		
21	.00194	63	.00007		
22	.00118	64	.00006		
23	.00053	65	.00005		
24	.00013	66	.00002		
25	.00000	67	-0.00001		
26	.00010	68	.00003		
27	.00031	69	.00005		
28	.00054	70	.00005		
29	.00068	71	.00004		
30	.00068	72	.00003		
31	.00056	73	.00001		
32	.00034	74	.00000		
33	.00008	75	0.00000		
34	-0.00015	76	-0.00000		
35	.00030	77	.00001		
36	.00037	78	.00002		
37	.00035	79	.00003		
38	.00026	80	.00003		
39	.00016	81	.00003		
40	.00006	82	.00002		
41	-0.00001	83	-0.00000		

FILTER NO. 27 - $r_c = 0.03$, $h = 0.03$, $N = 60$, $E = 0.0005$, $r_{ac} = 0.105$

k	Value of W_k	k	Value of W_k
0	0.12000	42	-0.00000
1	.11678	43	.00003
2	.10749	44	.00009
3	.09313	45	.00014
4	.07523	46	.00017
5	.05561	47	.00017
6	.03614	48	.00013
7	.01851	49	.00007
8	.00400	50	0.00000
9	-0.00662	51	.00006
10	.01314	52	.00010
11	.01585	53	.00012
12	.01547	54	.00010
13	.01293	55	.00008
14	.00923	56	.00004
15	.00530	57	.00002
16	.00183	58	.00000
17	0.00074	59	.00000
18	.00225	60	0.00002
19	.00278		
20	.00257		
21	.00194		
22	.00118		
23	.00053		
24	.00013		
25	.00000		
26	.00010		
27	.00031		
28	.00054		
29	.00068		
30	.00068		
31	.00056		
32	.00034		
33	.00008		
34	-0.00015		
35	.00030		
36	.00037		
37	.00035		
38	.00026		
39	.00016		
40	.00006		
41	-0.00001		

FILTER NO. 28 - $r_c = 0.03$, $h = 0.03$, $N = 40$, $E = 0.001$, $r_{ac} = 0.115$

k	Value of W_k	k	Value of W_k
0	0.11999	21	0.00193
1	.11678	22	.00117
2	.10748	23	.00052
3	.09312	24	.00012
4	.07522	25	-0.00001
5	.05560	26	0.00009
6	.03613	27	.00031
7	.01850	28	.00053
8	.00399	29	.00067
9	-0.00663	30	.00068
10	.01315	31	.00055
11	.01586	32	.00033
12	.01548	33	.00008
13	.01293	34	-0.00015
14	.00924	35	.00031
15	.00530	36	.00038
16	.00183	37	.00035
17	0.00074	38	.00027
18	.00224	39	.00016
19	.00278	40	-0.00007
20	0.00257		

FILTER NO. 29 - $r_c = 0.03$, $h = 0.03$, $N = 30$, $E = 0.003$, $r_{ac} = 0.085$

k	Value of W_k	k	Value of W_k
0	0.11997	16	-0.00186
1	.11675	17	0.00071
2	.10746	18	.00222
3	.09310	19	.00275
4	.07520	20	.00254
5	.05558	21	.00190
6	.03611	22	.00114
7	.01848	23	.00050
8	.00396	24	.00010
9	-0.00665	25	-0.00003
10	.01317	26	0.00007
11	.01588	27	.00028
12	.01550	28	.00051
13	.01296	29	.00064
14	.00926	30	.00065
15	-0.00533		

FILTER NO. 30 - r_c = 0.03, h = 0.055, N = 70, E = 0.0001,
r_{ac} = 0.165

k	Value of W_k	k	Value of W_k
0	0.17000	42	-0.00002
1	.16021	43	.00005
2	.13326	44	.00007
3	.09565	45	.00006
4	.05581	46	.00004
5	.02153	47	.00000
6	-0.00216	48	0.00002
7	.01398	49	.00002
8	.01596	50	-0.00000
9	.01205	51	.00002
10	.00638	52	.00002
11	.00187	53	0.00000
12	0.00030	54	.00002
13	.00046	55	.00003
14	-0.00031	56	.00003
15	.00096	57	.00002
16	.00098	58	.00000
17	.00045	59	-0.00000
18	0.00022	60	0.00000
19	.00064	61	.00002
20	.00067	62	.00002
21	.00041	63	.00002
22	.00012	64	.00001
23	-0.00002	65	-0.00000
24	0.00005	66	.00001
25	.00022	67	.00001
26	.00034	68	.00000
27	.00033	69	0.00000
28	.00020	70	0.00000
29	.00005		
30	-0.00005		
31	.00005		
32	0.00001		
33	.00007		
34	.00007		
35	.00002		
36	-0.00005		
37	.00010		
38	.00010		
39	.00006		
40	.00002		
41	-0.00000		

FILTER NO. 31 - r_c = 0.03, h = 0.055, N = 30, E = 0.0008, r_{ac} = 0.145

k	Value of W_k	k	Value of W_k
0	0.16999	16	-0.00099
1	.16020	17	.00046
2	.13325	18	0.00022
3	.09564	19	.00064
4	.05580	20	.00066
5	.02152	21	.00040
6	-0.00217	22	.00011
7	.01399	23	-0.00002
8	.01597	24	0.00004
9	.01206	25	.00021
10	.00639	26	.00033
11	.00188	27	.00032
12	0.00029	28	.00019
13	.00045	29	.00004
14	-0.00032	30	-0.00006
15	-0.00097		

FILTER NO. 32 - r_c = 0.03, h = 0.055, N = 20, E = 0.0004, r_{ac} =0.135

k	Value of W_k	k	Value of W_k
0	0.17007	11	-0.00180
1	.16027	12	0.00036
2	.13333	13	.00052
3	.09572	14	-0.00025
4	.05587	15	.00090
5	.02160	16	.00092
6	-0.00210	17	.00038
7	.01391	18	0.00029
8	.01589	19	.00071
9	.01198	20	0.00073
10	-0.00631		

FILTER NO. 33 - $r_c = 0.03$, $h = 0.055$, $N = 10$, $E = 0.009$, $r_{ac} = 0.135$

k	Value of W_k	k	Value of W_k
0	0.16991	6	-0.00225
1	.16012	7	.01407
2	.13317	8	.01605
3	.09556	9	.01214
4	.05572	10	-0.00647
5	0.02144		

FILTER NO. 34 - $r_c = 0.03$, $h = 0.08$, $N = 60$, $E = 0.00004$, $r_{ac} = 0.205$

k	Value of W_k	k	Value of W_k
0	0.22000	42	-0.00002
1	.19808	43	.00004
2	.14188	44	.00003
3	.07447	45	.00001
4	.01954	46	0.00000
5	-0.01020	47	-0.00000
6	.01654	48	.00002
7	.01044	49	.00002
8	.00313	50	0.00000
9	.00006	51	.00001
10	.00063	52	.00001
11	.00179	53	.00000
12	.00169	54	-0.00000
13	.00062	55	0.00000
14	0.00022	56	.00002
15	.00024	57	.00002
16	-0.00015	58	.00001
17	.00030	59	.00000
18	.00006	60	0.00000
19	0.00025		
20	.00031		
21	.00014		
22	-0.00001		
23	0.00003		
24	.00015		
25	.00020		
26	.00012		
27	.00002		
28	-0.00000		
29	0.00005		
30	.00009		
31	.00006		
32	-0.00001		
33	.00004		
34	.00002		
35	0.00002		
36	.00001		
37	-0.00003		
38	.00005		
39	.00004		
40	.00001		
41	-0.00000		

FILTER NO. 35 - $r_c = 0.03$, $h = 0.08$, $N = 30$, $E = 0.0004$, $r_{ac} = 0.185$

k	Value of W_k	k	Value of W_k
0	0.21999	16	-0.00015
1	.19803	17	.00031
2	.14183	18	.00007
3	.07446	19	0.00024
4	.01953	20	.00030
5	-0.01021	21	.00013
6	.01655	22	-0.00001
7	.01045	23	0.00002
8	.00313	24	.00015
9	.00006	25	.00020
10	.00063	26	.00012
11	.00180	27	.00001
12	.00170	28	-0.00001
13	.00062	29	0.00005
14	0.00021	30	0.00008
15	0.00024		

FILTER NO. 36 - $r_c = 0.03$, $h = 0.08$, $N = 20$, $E = 0.001$, $r_{ac} = 0.185$

k	Value of W_k	k	Value of W_k
0	0.22003	11	-0.00176
1	.19812	12	.00166
2	.14192	13	.00059
3	.07450	14	0.00025
4	.01957	15	.00027
5	-0.01017	16	-0.00012
6	.01651	17	.00027
7	.01041	18	.00003
8	.00310	19	0.00028
9	.00003	20	0.00034
10	-0.00060		

FILTER NO. 37 - $r_c = 0.08$, $h = 0.08$, $N = 10$, $E = 0.007$, $r_{ac} = 0.195$

k	Value of W_k	k	Value of W_k
0	0.21972	6	-0.01683
1	.19780	7	.01072
2	.14160	8	.00341
3	.07418	9	.00034
4	.01925	10	-0.00091
5	-0.01049		

FILTER NO. 38 - $r_c = 0.03$, $h = 0.1$, $N = 60$, $E = 0.00005$, $r_{ac} = 0.225$

k	Value of W_k	k	Value of W_k
0	0.26000	42	-0.00000
1	.22348	43	.00000
2	.13635	44	.00002
3	.04750	45	.00002
4	-0.00517	46	.00000
5	.01717	47	0.00000
6	.00886	48	-0.00001
7	.00110	49	.00001
8	.00033	50	.00000
9	.00210	51	0.00001
10	.00202	52	.00000
11	.00054	53	-0.00000
12	0.00014	54	0.00000
13	-0.00027	55	.00001
14	.00055	56	.00001
15	.00019	57	.00000
16	0.00019	58	.00000
17	.00012	59	.00001
18	-0.00009	60	.00001
19	.00004		
20	0.00015		
21	.00017		
22	.00004		
23	-0.00000		
24	0.00008		
25	0.00013		
26	.00006		
27	-0.00000		
28	0.00002		
29	.00007		
30	.00004		
31	-0.00001		
32	.00002		
33	0.00002		
34	.00002		
35	-0.00002		
36	.00003		
37	.00001		
38	0.00000		
39	-0.00001		
40	.00003		
41	-0.00002		

FILTER NO. 39 - $r_c = 0.03$, $h = 0.1$, $N = 30$, $E = 0.0003$, $r_{ac} = 0.235$

k	Value of W_k	k	Value of W_k
0	0.26000	16	0.00019
1	.22348	17	.00012
2	.13634	18	-0.00010
3	.04750	19	.00005
4	-0.00518	20	0.00014
5	.01717	21	.00017
6	.00886	22	.00004
7	.00110	23	-0.00001
8	.00033	24	0.00008
9	.00210	25	.00012
10	.00202	26	.00006
11	.00055	27	-0.00001
12	0.00013	28	0.00002
13	-0.00027	29	.00006
14	.00055	30	0.00004
15	-0.00019		

FILTER NO. 40 - $r_c = 0.03$, $h = 0.1$, $N = 20$, $E = 0.001$, $r_{ac} = 0.225$

k	Value of W_k	k	Value of W_k
0	0.26002	11	-0.00052
1	.22350	12	0.00016
2	.13637	13	-0.00025
3	.04752	14	.00052
4	-0.00515	15	.00016
5	.01714	16	0.00022
6	.00883	17	.00015
7	.00108	18	-0.00007
8	.00031	19	.00002
9	.00207	20	0.00017
10	-0.00199		

FILTER NO. 41 - $r_c = 0.03$, $h = 0.1$, $N = 10$, $E = 0.003$, $r_{ac} = 0.225$

k	Value of W_k	k	Value of W_k
0	0.25994	6	-0.00891
1	.22342	7	.00116
2	.13629	8	.00039
3	.04744	9	.00215
4	-0.00523	10	.00207
5	.01722		

FILTER NO. 42 - $r_c = 0.03$, $h = 0.2$, $N = 70$, $E = 0.000006$, $r_{ac} = 0.125$

k	Value of W_k	k	Value of W_k
0	0.46000	42	-0.00000
1	.27108	43	.00000
2	.02053	44	.00000
3	-0.01677	45	.00000
4	0.00128	46	0.00000
5	-0.00343	47	-0.00000
6	.00051	48	0.00000
7	.00077	49	-0.00000
8	.00068	50	.00000
9	.00009	51	0.00000
10	.00048	52	-0.00000
11	0.00002	53	0.00000
12	-0.00023	54	-0.00000
13	.00001	55	0.00000
14	.00006	56	.00000
15	.00005	57	.00000
16	0.00003	58	.00000
17	-0.00004	59	.00000
18	0.00005	60	.00000
19	-0.00002	61	-0.00000
20	0.00004	62	0.00000
21	.00002	63	.00000
22	.00001	64	.00000
23	.00003	65	.00000
24	.00000	66	-0.00000
25	.00003	67	0.00000
26	.00000	68	-0.00000
27	.00002	69	0.00000
28	.00001	70	-0.00000
29	.00001		
30	.00001		
31	-0.00000		
32	0.00001		
33	-0.00001		
34	0.00000		
35	-0.00000		
36	.00000		
37	.00000		
38	.00001		
39	0.00000		
40	-0.00001		
41	-0.00000		

FILTER NO. 43 - $r_c = 0.03$, $h = 0.2$, $N = 30$, $E = 0.00006$, $r_{ac} = 0.435$

	Value of W_k		Value of W_k
0	0.46000	16	0.00003
1	.27108	17	-0.00004
2	.02052	18	0.00005
3	-0.01677	19	-0.00002
4	0.00128	20	0.00004
5	-0.00343	21	.00001
6	.00051	22	.00001
7	.00077	23	.00003
8	.00068	24	.00000
9	.00009	25	.00003
10	.00048	26	.00000
11	0.00002	27	.00002
12	-0.00024	28	.00001
13	.00001	29	.00000
14	.00006	30	0.00001
15	-0.00005		

FILTER NO. 44 - $r_c = 0.03$, $h = 0.2$, $N = 20$, $E = 0.0003$, $r_{ac} = 0.425$

	Value of W_k		Value of W_k
0	0.46000	11	0.00003
1	.27108	12	-0.00023
2	.02053	13	.00001
3	-0.01676	14	.00005
4	0.00129	15	.00004
5	-0.00343	16	0.00004
6	.00050	17	-0.00004
7	.00077	18	0.00006
8	.00067	19	-0.00001
9	.00009	20	0.00004
10	-0.00048		

FILTER NO. 45 - $r_c = 0.03$, $h = 0.2$, $N = 10$, $E = 0.0006$, $r_{ac} = 0.435$

k	Value of W_k	k	Value of W_k
0	0.45999	6	-0.00052
1	.27106	7	.00079
2	.02051	8	.00069
3	-0.01678	9	.00011
4	0.00127	10	-0.00049
5	-0.00345		

FILTER NO. 46 - $r_c = 0.03$, $h = 0.2$, $N = 4$, $E = 0.008$, $r_{ac} = 0.315$

k	Value of W_k	k	Value of W_k
0	0.45864	3	-0.01813
1	.26972	4	0.00008
2	.01917		

FILTER NO. 47 - r_c = 0.05, h = 0.01, N = 70, E = 0.002, r_{ac} = 0.08

k	Value of W_k	k	Value of W_k
0	0.12000	42	-0.00045
1	.11714	43	.00164
2	.10879	44	.00246
3	.09569	45	.00285
4	.07895	46	.00280
5	.05999	47	.00237
6	.04034	48	-0.00167
7	.02151	49	.00083
8	.00487	50	0.00000
9	-0.00853	51	.00073
10	.01801	52	.00125
11	.02334	53	.00153
12	.02468	54	.00156
13	.02256	55	.00137
14	.01782	56	.00102
15	.01145	57	.00058
16	.00448	58	.00014
17	0.00211	59	-0.00024
18	.00754	60	.00052
19	.01126	61	.00068
20	.01300	62	.00071
21	.01278	63	.00063
22	.01088	64	.00048
23	.00774	65	.00029
24	.00392	66	.00010
25	.00000	67	0.00005
26	-0.00346	68	.00016
27	.00607	69	.00020
28	.00757	70	0.00020
29	.00788		
30	.00708		
31	.00541		
32	.00319		
33	.00078		
34	0.00147		
35	.00328		
36	.00444		
37	.00486		
38	.00458		
39	.00371		
40	.00243		
41	0.00097		

FILTER NO. 48 - $r_c = 0.05$, $h = 0.01$, $N = 60$, $E = 0.008$, $r_{ac} = 0.065$

k	Value of W_k	k	Value of W_k
0	0.11997	42	-0.00049
1	.11710	43	.00168
2	.10875	44	.00250
3	.09565	45	.00289
4	.07891	46	.00284
5	.05995	47	.00241
6	.04030	48	.00171
7	.02148	49	.00087
8	.00484	50	.00003
9	-0.00856	51	0.00069
10	.01805	52	.00122
11	.02338	53	.00149
12	.02471	54	.00152
13	.02260	55	.00133
14	.01786	56	.00098
15	.01149	57	.00055
16	.00452	58	.00010
17	0.00207	59	-0.00028
18	.00750	60	.00056
19	.01122		
20	.01296		
21	.01275		
22	.01084		
23	.00770		
24	.00388		
25	-0.00003		
26	.00350		
27	.00611		
28	.00761		
29	.00792		
30	.00712		
31	.00545		
32	.00323		
33	.00082		
34	0.00144		
35	.00324		
36	.00440		
37	.00483		
38	.00454		
39	.00367		
40	.00239		
41	.00093		

FILTER NO. 49 - $r_c = 0.05$, $h = 0.03$, $N = 90$, $E = 0.00009$, $r_{ac} = 0.115$

k	Value of W_k	k	Value of W_k	k	Value of W_k
0	0.16000	42	0.00002	84	-0.00004
1	.15283	43	.00003	85	.00003
2	.13258	44	-0.00001	86	.00002
3	.10272	45	.00009	87	.00001
4	.06820	46	.00015	88	0.00000
5	.03437	47	.00018	89	.00001
6	.00588	48	.00016	90	0.00001
7	-0.01414	49	.00009		
8	.02455	50	0.00000		
9	.02617	51	.00008		
10	.02126	52	.00013		
11	.01285	53	.00013		
12	.00392	54	.00010		
13	0.00327	55	.00005		
14	.00748	56	.00001		
15	.00857	57	-0.00001		
16	.00722	58	.00000		
17	.00455	59	0.00001		
18	.00172	60	.00003		
19	-0.00045	61	.00003		
20	.00159	62	.00002		
21	.00175	63	-0.00002		
22	.00130	64	.00005		
23	.00065	65	.00007		
24	.00016	66	.00008		
25	0.00000	67	.00006		
26	-0.00013	68	.00002		
27	.00039	69	0.00001		
28	.00059	70	.00003		
29	.00061	71	.00004		
30	.00042	72	.00003		
31	.00009	73	.00002		
32	0.00026	74	.00000		
33	.00051	75	.00000		
34	.00058	76	.00000		
35	.00050	77	.00002		
36	.00030	78	.00003		
37	.00009	79	.00003		
38	-0.00007	80	.00002		
39	.00013	81	.00000		
40	.00011	82	-0.00001		
41	-0.00004	83	.00003		

FILTER NO. 50 - $r_c = 0.05$, $h = 0.03$, $N = 50$, $E = 0.0009$, $r_{ac} = 0.105$

k	Value of W_k	k	Value of W_k
0	0.16001	42	0.00002
1	.15284	43	.00003
2	.13259	44	-0.00001
3	.10273	45	.00008
4	.06821	46	.00015
5	.03437	47	.00018
6	.00589	48	.00015
7	-0.01413	49	.00008
8	.02455	50	0.00001
9	.02616		
10	.02125		
11	.01285		
12	.00391		
13	0.00328		
14	.00749		
15	.00858		
16	.00722		
17	.00456		
18	.00173		
19	-0.00044		
20	.00158		
21	.00175		
22	.00129		
23	.00064		
24	.00016		
25	0.00001		
26	-0.00012		
27	.00038		
28	.00058		
29	.00060		
30	.00041		
31	.00008		
32	0.00027		
33	.00051		
34	.00059		
35	.00050		
36	.00031		
37	.00010		
38	-0.00006		
39	.00012		
40	.00010		
41	-0.00003		

FILTER NO. 51 - $r_c = 0.05$, $h = 0.03$, $N = 30$, $E = 0.004$, $r_{ac} = 0.115$

k	Value of W_k	k	Value of W_k
0	0.16005	16	0.00727
1	.15288	17	.00460
2	.13263	18	.00177
3	.10277	19	-0.00040
4	.06825	20	.00154
5	.03442	21	.00170
6	.00593	22	.00125
7	-0.01409	23	.00060
8	.02450	24	.00011
9	.02612	25	0.00005
10	.02121	26	-0.00008
11	.01280	27	.00034
12	.00387	28	.00054
13	0.00332	29	.00056
14	.00753	30	-0.00037
15	0.00862		

FILTER NO. 52. $r_c = 0.05$, $h = 0.03$, $N = 20$, $E = 0.01$, $r_{ac} = 0.105$

k	Value of W_k	k	Value of W_k
0	0.15978	11	-0.01307
1	.15261	12	.00413
2	.13236	13	0.00306
3	.10250	14	.00727
4	.06798	15	.00835
5	.03415	16	.00700
6	.00566	17	.00434
7	-0.01436	18	.00150
8	.02477	19	-0.00067
9	.02638	20	.00181
10	-0.02148		

FILTER NO. 53 - r_c = 0.05, h = 0.05, N = 70, E = 0.00008, r_{ac} = 0.145

k	Value of W_k	k	Value of W_k
0	0.20000	42	-0.00008
1	.18536	43	.00006
2	.14578	44	.00002
3	.09268	45	0.00000
4	.04015	46	-0.00001
5	.00000	47	.00004
6	-0.02190	48	.00006
7	.02648	49	.00004
8	.01962	50	0.00000
9	.00893	51	.00003
10	0.00000	52	.00004
11	.00421	53	.00003
12	.00429	54	.00001
13	.00238	55	.00000
14	.00060	56	.00001
15	.00000	57	.00002
16	.00039	58	.00003
17	.00099	59	.00002
18	.00114	60	.00000
19	.00070	61	-0.00001
20	.00000	62	.00002
21	-0.00051	63	.00002
22	.00061	64	.00000
23	.00038	65	0.00000
24	.00011	66	-0.00000
25	0.00000	67	.00001
26	-0.00008	68	.00002
27	.00023	69	.00001
28	.00029	70	0.00000
29	.00019		
30	0.00000		
31	.00015		
32	.00019		
33	.00013		
34	.00004		
35	.00000		
36	.00003		
37	.00009		
38	.00011		
39	.00008		
40	.00000		
41	-0.00006		

FILTER NO. 54 - r_c = 0.05, h = 0.05, N = 40, E = 0.0006, r_{ac} = 0.145

k	Value of W_k	k	Value of W_k
0	0.19999	21	-0.00052
1	.18535	22	.00061
2	.14578	23	.00039
3	.09267	24	.00012
4	.04014	25	.00001
5	-0.00001	26	.00009
6	.02191	27	.00024
7	.02648	28	.00029
8	.01963	29	.00019
9	.00883	30	.00001
10	.00001	31	0.00015
11	0.00421	32	.00018
12	.00426	33	.00012
13	.00237	34	.00003
14	.00060	35	-0.00001
15	-0.00001	36	0.00003
16	0.00038	37	.00008
17	.00098	38	.00011
18	.00113	39	.00007
19	.00069	40	-0.00001
20	-0.00001		

FILTER NO. 55 - $r_c = 0.05$, $h = 0.05$, $N = 30$, $E = 0.001$, $r_{ac} = 0.145$

k	Value of W_k	k	Value of W_k
0	0.20002	16	0.00041
1	.18537	17	.00101
2	.14580	18	.00116
3	.09270	19	.00072
4	.04017	20	.00002
5	.00002	21	-0.00049
6	-0.02188	22	.00059
7	.02646	23	.00036
8	.01961	24	.00009
9	.00881	25	0.00002
10	0.00002	26	-0.00007
11	.00423	27	.00022
12	.00431	28	.00027
13	.00239	29	.00017
14	.00062	30	0.00002
15	0.00002		

FILTER NO. 56 - $r_c = 0.05$, $h = 0.05$, $N = 20$, $E = 0.004$, $r_{ac} = 0.145$

k	Value of W_k	k	Value of W_k
0	0.19991	11	0.00412
1	.18526	12	.00420
2	.14569	13	.00229
3	.09259	14	.00051
4	.04006	15	-0.00009
5	-0.00009	16	0.00030
6	.02199	17	.00090
7	.02657	18	.00105
8	.01971	19	.00061
9	.00892	20	-0.00009
10	-0.00009		

FILTER NO. 57 - $r_c = 0.05$, $h = 0.08$, $N = 60$, $E = 0.00009$, $r_{ac} = 0.205$

k	Value of W_k	k	Value of W_k
0	0.26000	42	0.00001
1	.22653	43	-0.00002
2	.14416	44	.00004
3	.05417	45	.00002
4	-0.00665	46	.00000
5	.02671	47	.00000
6	.01925	48	.00002
7	.00564	49	.00002
8	0.00114	50	.00000
9	.00080	51	0.00002
10	-0.00101	52	.00001
11	.00079	53	.00000
12	0.00069	54	.00000
13	.00135	55	.00001
14	.00079	56	.00002
15	.00009	57	.00001
16	.00007	58	-0.00000
17	.00040	59	.00000
18	.00043	60	0.00000
19	.00009		
20	-0.00019		
21	.00014		
22	0.00001		
23	.00001		
24	-0.00014		
25	.00020		
26	.00011		
27	0.00000		
28	.00001		
29	-0.00006		
30	.00006		
31	0.00002		
32	.00008		
33	.00005		
34	.00001		
35	.00001		
36	.00004		
37	.00006		
38	.00002		
39	-0.00002		
40	.00002		
41	0.00001		

FILTER NO. 58 - $r_c = 0.05$, $h = 0.08$, $N = 30$, $E = 0.0005$, $r_{ac} = 0.215$

k	Value of W_k	k	Value of W_k
0	0.26001	16	0.00008
1	.22654	17	.00041
2	.14416	18	.00044
3	.05417	19	.00009
4	-0.00664	20	-0.00018
5	.02670	21	.00014
6	.01924	22	0.00002
7	.00563	23	.00002
8	0.00114	24	-0.00013
9	.00080	25	.00020
10	-0.00101	26	.00010
11	.00078	27	0.00001
12	0.00070	28	.00001
13	.00136	29	-0.00005
14	.00079	30	.00005
15	.00010		

FILTER NO. 59 - $r_c = 0.05$, $h = 0.08$, $N = 20$, $E = 0.001$, $r_{ac} = 0.205$

k	Value of W_k	k	Value of W_k
0	0.25998	11	-0.00081
1	.22651	12	0.00066
2	.14414	13	.00133
3	.05414	14	.00076
4	-0.00668	15	.00007
5	.02673	16	.00005
6	.01927	17	.00038
7	.00566	18	.00041
8	0.00111	19	.00006
9	.00077	20	-0.00021
10	-0.00104		

FILTER NO. 60 - r_c = 0.05, h = 0.08, N = 10, E = 0.007, r_{ac} = 0.215

k	Value of W_k	k	Value of W_k
0	0.26023	6	-0.01901
1	.22677	7	.00540
2	.14439	8	0.00137
3	.05440	9	.00103
4	-0.00642	10	-0.00078
5	.02648		

FILTER NO. 61 - r_c = 0.05, h = 0.1, N = 50, E = 0.00008, r_{ac} = 0.245

k	Value of W_k	k	Value of W_k
0	0.30000	26	-0.00005
1	.24802	27	0.00001
2	.12993	28	-0.00003
3	.02303	29	.00005
4	-0.02426	30	0.00000
5	.02122	31	.00004
6	.00530	32	.00002
7	0.00064	33	-0.00000
8	-0.00126	34	0.00002
9	.00193	35	.00005
10	0.00000	36	.00002
11	.00103	37	.00000
12	.00035	38	.00001
13	-0.00009	39	.00002
14	0.00036	40	.00000
15	.00061	41	-0.00002
16	.00024	42	.00001
17	-0.00004	43	0.00000
18	0.00010	44	-0.00001
19	.00019	45	.00002
20	.00000	46	.00001
21	-0.00014	47	0.00000
22	.00006	48	-0.00000
23	0.00002	49	.00001
24	-0.00007	50	0.00000
25	-0.00013		

FILTER NO. 62 - $r_c = 0.05$, $h = 0.1$, $N = 30$, $E = 0.0003$, $r_{ac} = 0.245$

k	Value of W_k	k	Value of W_k
0	0.30000	16	0.00024
1	.24803	17	-0.00004
2	.12993	18	0.00011
3	.02303	19	.00020
4	-0.02425	20	.00000
5	.02122	21	-0.00014
6	.00530	22	.00005
7	0.00064	23	0.00002
8	-0.00126	24	-0.00006
9	.00193	25	.00012
10	0.00000	26	.00005
11	.00104	27	0.00001
12	.00036	28	-0.00002
13	-0.00009	29	.00005
14	0.00036	30	0.00000
15	0.00061		

FILTER NO. 63 - $r_c = 0.05$, $h = 0.1$, $N = 20$, $E = 0.001$, $r_{ac} = 0.245$

k	Value of W_k	k	Value of W_k
0	0.29998	11	0.00101
1	.24800	12	.00034
2	.12991	13	-0.00011
3	.02301	14	0.00034
4	-0.02428	15	.00059
5	.02124	16	.00022
6	.00532	17	-0.00006
7	0.00062	18	0.00008
8	-0.00128	19	.00017
9	.00195	20	-0.00002
10	.00002		

FILTER NO. 64 - $r_c = 0.05$, $h = 0.1$, $N = 10$, $E = 0.006$, $r_{ac} = 0.245$

k	Value of W_k	k	Value of W_k
0	0.30023	6	-0.00507
1	.24824	7	0.00086
2	.13015	8	-0.00104
3	.02325	9	.00171
4	-0.02403	10	0.00023
5	-0.02100		

FILTER NO. 65 - $r_c = 0.05$, $h = 0.2$, $N = 70$, $E = 0.000007$,
$r_{ac} = 0.445$

k	Value of W_k	k	Value of W_k
0	0.50000	42	0.00000
1	.27323	43	-0.00000
2	.00000	44	0.00000
3	-0.01803	45	-0.00000
4	0.00000	46	0.00000
5	-0.00424	47	-0.00000
6	0.00000	48	0.00000
7	-0.00121	49	-0.00000
8	0.00000	50	0.00000
9	-0.00021	51	.00000
10	0.00000	52	.00000
11	.00012	53	.00000
12	0.00000	54	.00000
13	.00018	55	.00000
14	.00000	56	.00000
15	.00015	57	.00000
16	.00000	58	.00000
17	.00008	59	.00000
18	.00000	60	.00000
19	.00002	61	-0.00000
20	.00000	62	0.00000
21	-0.00002	63	-0.00000
22	0.00000	64	0.00000
23	-0.00003	65	-0.00000
24	0.00000	66	0.00000
25	-0.00003	67	-0.00000
26	0.00000	68	0.00000
27	-0.00002	69	-0.00000
28	0.00000	70	0.00000
29	-0.00001		
30	0.00000		
31	.00000		
32	.00000		
33	.00001		
34	.00000		
35	.00001		
36	.00000		
37	.00001		
38	.00000		
39	.00000		
40	.00000		
41	-0.00000		

FILTER NO. 66 - $r_c = 0.05, h = 0.2, N = 30, E = 0.00007,$
$r_{ac} = 0.445$

k	Value of W_k	k	Value of W_k
0	0.50000	16	0.00000
1	.27323	17	.00008
2	.00000	18	.00000
3	-0.01803	19	.00002
4	0.00000	20	.00000
5	-0.00424	21	-0.00002
6	0.00000	22	0.00000
7	-0.00121	23	-0.00003
8	0.00000	24	0.00000
9	-0.00021	25	-0.00003
10	0.00000	26	0.00000
11	.00012	27	-0.00002
12	.00000	28	0.00000
13	.00019	29	-0.00000
14	.00000	30	0.00000
15	.00015		

FILTER NO. 67 - $r_c = 0.05, h = 0.2, N = 20, E = 0.0002,$
$r_{ac} = 0.445$

k	Value of W_k	k	Value of W_k
0	0.50000	11	0.00011
1	.27323	12	-0.00000
2	-0.00000	13	0.00018
3	.01804	14	-0.00000
4	.00000	15	0.00014
5	.00425	16	-0.00000
6	.00000	17	0.00008
7	.00122	18	-0.00000
8	.00000	19	0.00002
9	.00022	20	-0.00000
10	-0.00000		

FILTER NO. 68 - $r = 0.05$, $h = 0.2$, $N = 10$, $E = 0.001$, $r_{ac} = 0.445$

k	Value of W_k	k	Value of W_k
0	0.50004	6	0.00004
1	.27328	7	-0.00117
2	.00004	8	0.00004
3	-0.01799	9	-0.00017
4	0.00004	10	0.00004
5	-0.00420		

FILTER NO. 69 - $r_c = 0.05$, $h = 0.2$, $N = 4$, $E = 0.01$, $r_{ac} = 0.455$

k	Value of W_k	k	Value of W_k
0	0.49884	3	-0.01919
1	.27208	4	-0.00116
2	-0.00116		

FILTER NO. 70 - $r_c = 0.08$, $h = 0.03$, $N = 40$, $E = 0.0004$, $r_{ac} = 0.165$

k	Value of W_k	k	Value of W_k
0	0.22000	21	0.00180
1	.20222	22	.00063
2	.15424	23	-0.00014
3	.09020	24	.00026
4	.02775	25	0.00000
5	-0.01807	26	.00020
6	.03960	27	.00009
7	.03811	28	-0.00028
8	.02181	29	.00063
9	.00167	30	.00068
10	0.01314	31	.00039
11	.01819	32	0.00009
12	.01425	33	.00048
13	.00560	34	.00059
14	-0.00272	35	.00042
15	.00729	36	.00011
16	.00733	37	-0.00015
17	.00431	38	.00024
18	.00058	39	.00018
19	0.00194	40	-0.00006
20	0.00257		

FILTER NO. 71 - r_c = 0.08, h = 0.03, N = 30, E = 0.003, r_{ac} = 0.135

k	Value of W_k	k	Value of W_k
0	0.22002	16	-0.00731
1	.20224	17	.00429
2	.15426	18	.00056
3	.09022	19	0.00196
4	.02777	20	.00260
5	-0.01804	21	.00183
6	.03958	22	.00065
7	.03809	23	-0.00012
8	.02179	24	.00024
9	.00165	25	0.00002
10	0.01316	26	.00022
11	.01821	27	.00011
12	.01427	28	-0.00026
13	.00563	29	.00061
14	-0.00270	30	-0.00066
15	.00727		

FILTER NO. 72 - r_c = 0.08, h = 0.03, N = 20, E = 0.009, r_{ac} = 0.135

k	Value of W_k	k	Value of W_k
0	0.22007	11	0.01826
1	.20228	12	.01432
2	.15431	13	.00567
3	.09026	14	-0.00265
4	.02782	15	.00722
5	-0.01800	16	.00726
6	.03953	17	.00424
7	.03804	18	.00052
8	.02174	19	0.00200
9	.00160	20	0.00264
10	0.01321		

FILTER NO. 73 - r_c = 0.08, h = 0.08, N = 60, E = 0.00004,
r_{ac} = 0.255

k	Value of W_k	k	Value of W_k
0	0.32000	42	-0.00003
1	.26238	43	.00002
2	.13070	44	0.00001
3	.01065	45	.00003
4	-0.04089	46	.00001
5	.03140	47	.00000
6	.00487	48	.00001
7	0.00720	49	.00002
8	.00449	50	-0.00000
9	.00033	51	.00002
10	.00063	52	.00001
11	.00185	53	.00000
12	.00090	54	.00001
13	-0.00070	55	.00001
14	.00087	56	.00000
15	.00018	57	0.00001
16	.00005	58	.00001
17	.00041	59	.00000
18	.00035	60	0.00000
19	0.00012		
20	.00031		
21	.00011		
22	.00000		
23	.00013		
24	.00017		
25	-0.00000		
26	.00013		
27	.00008		
28	.00000		
29	.00004		
30	.00009		
31	.00003		
32	0.00006		
33	.00006		
34	.00001		
35	.00001		
36	.00005		
37	.00003		
38	-0.00003		
39	.00004		
40	.00001		
41	-0.00000		

FILTER NO. 74 - $r_c = 0.08$, $h = 0.08$, $N = 30$, $E = 0.0004$, $r_{ac} = 0.235$

k	Value of W_k	k	Value of W_k
0	0.32000	16	-0.00005
1	.26239	17	.00041
2	.13070	18	.00035
3	.01065	19	0.00012
4	-0.04089	20	.00031
5	.03140	21	.00012
6	.00487	22	.00001
7	0.00721	23	.00013
8	.00449	24	.00017
9	.00034	25	.00000
10	.00063	26	-0.00013
11	.00185	27	.00007
12	.00090	28	0.00000
13	-0.00070	29	-0.00004
14	.00086	30	-0.00009
15	-0.00017		

FILTER NO. 75 - $r_c = 0.08$, $h = 0.08$, $N = 20$, $E = 0.001$, $r_{ac} = 0.235$

k	Value of W_k	k	Value of W_k
0	0.32001	11	0.00186
1	.26239	12	.00091
2	.13070	13	-0.00069
3	.01066	14	.00086
4	-0.04089	15	.00017
5	.03139	16	.00005
6	.00486	17	.00040
7	0.00721	18	.00034
8	.00449	19	0.00012
9	.00034	20	0.00031
10	.00063		

FILTER NO. 76 - $r_c = 0.08$, $h = 0.08$, $N = 10$, $E = 0.0006$, $r_{ac} = 0.245$

k	Value of W_k	k	Value of W_k
0	0.32008	6	-0.00480
1	.26246	7	0.00728
2	.13077	8	.00456
3	.01073	9	.00041
4	-0.04082	10	0.00070
5	.03132		

FILTER NO. 77 - $r_c = 0.08$, $h = 0.1$, $N = 50$, $E = 0.00008$, $r_{ac} = 0.275$

k	Value of W_k	k	Value of W_k
0	0.36000	42	0.00000
1	.27739	43	-0.00001
2	.10526	44	.00001
3	-0.01853	45	0.00001
4	.04054	46	.00002
5	.01247	47	.00000
6	0.00434	48	.00000
7	.00205	49	.00001
8	-0.00049	50	-0.00000
9	0.00164		
10	.00202		
11	.00016		
12	-0.00031		
13	0.00024		
14	-0.00008		
15	.00058		
16	.00028		
17	0.00005		
18	-0.00011		
19	.00012		
20	0.00015		
21	.00017		
22	.00001		
23	.00003		
24	.00011		
25	-0.00000		
26	.00008		
27	.00002		
28	.00001		
29	.00007		
30	.00004		
31	0.00003		
32	.00002		
33	-0.00001		
34	0.00003		
35	.00004		
36	.00000		
37	-0.00001		
38	0.00001		
39	-0.00000		
40	.00003		
41	-0.00002		

FILTER NO. 78 - $r_c = 0.08$, $h = 0.1$, $N = 30$, $E = 0.0002$, $r_{ac} = 0.285$

k	Value of W_k	k	Value of W_k
0	0.36000	16	-0.00027
1	.27740	17	0.00005
2	.10527	18	-0.00010
3	-0.01853	19	.00011
4	.04054	20	0.00015
5	.01247	21	.00018
6	0.00435	22	.00002
7	.00205	23	.00004
8	-0.00048	24	.00011
9	0.00164	25	.00000
10	.00202	26	-0.00008
11	.00016	27	.00002
12	-0.00031	28	.00000
13	0.00025	29	.00006
14	-0.00007	30	.00004
15	-0.00057		

FILTER NO. 79 - $r_c = 0.08$, $h = 0.1$, $N = 20$, $E = 0.0008$, $r_{ac} = 0.275$

k	Value of W_k	k	Value of W_k
0	0.36001	11	0.00017
1	.27740	12	-0.00030
2	.10527	13	0.00025
3	-0.01852	14	-0.00007
4	.04053	15	.00057
5	.01246	16	.00027
6	0.00435	17	0.00006
7	.00206	18	-0.00010
8	-0.00048	19	.00011
9	0.00165	20	0.00016
10	0.00203		

FILTER NO. 80 - $r_c = 0.08$, $h = 0.1$, $N = 10$, $E = 0.003$, $r_{ac} = 0.275$

k	Value of W_k	k	Value of W_k
0	0.35994	6	0.00428
1	.27733	7	.00199
2	.10520	8	-0.00055
3	-0.01860	9	0.00157
4	.04060	10	0.00195
5	-0.01254		

FILTER NO. 81 - $r_c = 0.08$, $h = 0.2$, $N = 70$, $E = 0.000007$, $r_{ac} = 0.475$

k	Value of W_k	k	Value of W_k
0	0.56000	42	-0.00000
1	.26839	43	0.00000
2	-0.03038	44	-0.00000
3	.01523	45	0.00000
4	.00182	46	.00000
5	.00250	47	.00000
6	0.00067	48	.00000
7	-0.00030	49	.00000
8	0.00080	50	.00000
9	.00003	51	-0.00000
10	.00048	52	.00000
11	-0.00006	53	.00000
12	0.00018	54	.00000
13	-0.00014	55	.00000
14	0.00003	56	0.00000
15	-0.00014	57	-0.00000
16	.00000	58	0.00000
17	.00008	59	.00000
18	0.00002	60	.00000
19	-0.00002	61	-0.00000
20	0.00004	62	0.00000
21	.00001	63	-0.00000
22	.00003	64	0.00000
23	.00001	65	-0.00000
24	.00001	66	.00000
25	.00000	67	.00000
26	-0.00001	68	0.00000
27	.00001	69	-0.00000
28	.00002	70	0.00000
29	.00000		
30	.00001		
31	0.00000		
32	-0.00000		
33	0.00001		
34	.00000		
35	.00001		
36	-0.00000		
37	0.00001		
38	-0.00001		
39	0.00000		
40	-0.00001		
41	-0.00000		

FILTER NO. 82 - $r_c = 0.08$, $h = 0.2$, $N = 50$, $E = 0.00003$, $r_{ac} = 0.475$

k	Value of W_k
0	0.56000
1	.26839
2	-0.03038
3	.01523
4	.00182
5	.00249
6	0.00067
7	-0.00030
8	0.00080
9	.00003
10	.00048
11	-0.00006
12	0.00018
13	-0.00014
14	0.00003
15	-0.00014
16	.00000
17	.00008
18	0.00002
19	-0.00002
20	0.00004
21	.00001
22	.00003
23	.00001
24	.00001
25	-0.00000
26	.00001
27	.00001
28	.00002
29	.00000
30	.00001
31	0.00000
32	-0.00000
33	0.00001
34	.00000
35	.00001
36	-0.00000
37	0.00001
38	-0.00001
39	0.00000
40	-0.00001
41	-0.00000
42	-0.00001
43	0.00000
44	-0.00000
45	0.00000
46	.00000
47	.00000
48	.00000
49	.00000
50	-0.00000

FILTER NO. 83 - $r_c = 0.08$, $h = 0.2$, $N = 20$, $E = 0.0002$, $r_{ac} = 0.475$

k	Value of W_k	k	Value of W_k
0	0.56000	11	-0.00006
1	.26839	12	0.00018
2	-0.03038	13	-0.00014
3	.01522	14	0.00003
4	.00182	15	-0.00014
5	.00249	16	.00000
6	0.00068	17	.00008
7	-0.00030	18	0.00002
8	0.00081	19	-0.00002
9	.00003	20	0.00004
10	0.00048		

FILTER NO. 84 - $r_c = 0.08$, $h = 0.2$, $N = 10$, $E = 0.001$, $r_{ac} = 0.485$

k	Value of W_k	k	Value of W_k
0	0.55999	6	0.00066
1	.26838	7	-0.00032
2	-0.03040	8	0.00079
3	.01524	9	.00001
4	.00184	10	0.00047
5	-0.00251		

FILTER NO. 85 - $r_c = 0.08$, $h = 0.2$, $N = 4$, $E = 0.01$, $r_{ac} = 0.465$

k	Value of W_k	k	Value of W_k
0	0.56074	3	-0.07101
1	.30600	4	0.03707
2	-0.05243		

FILTER NO. 86 - $r_c = 0.1$, $h = 0.03$, $N = 60$, $E = 0.0006$, $r_{ac} = 0.175$

k	Value of W_k	k	Value of W_k
0	0.26000	42	0.00000
1	.23126	43	-0.00004
2	.15671	44	.00011
3	.06561	45	.00012
4	-0.00945	46	.00002
5	.04730	47	0.00012
6	.04607	48	.00019
7	.02058	49	.00014
8	0.00792	50	-0.00000
9	.02334	51	.00012
10	.02126	52	.00015
11	.00799	53	.00008
12	-0.00580	54	0.00001
13	.01224	55	.00006
14	.00989	56	.00005
15	.00278	57	.00002
16	0.00354	58	-0.00000
17	.00572	59	0.00001
18	.00394	60	0.00003
19	.00068		
20	-0.00159		
21	.00192		
22	.00100		
23	.00005		
24	0.00023		
25	-0.00000		
26	.00018		
27	0.00003		
28	.00046		
29	.00067		
30	.00042		
31	-0.00014		
32	.00059		
33	.00064		
34	.00029		
35	0.00016		
36	.00040		
37	.00033		
38	.00010		
39	-0.00008		
40	.00011		
41	-0.00004		

FILTER NO. 87 - $r_c = 0.1$, $h = 0.03$, $N = 30$, $E = 0.004$, $r_{ac} = 0.165$

k	Value of W_k	k	Value of W_k
0	0.25997	16	0.00351
1	.23123	17	.00569
2	.15668	18	.00391
3	.06558	19	.00065
4	-0.00948	20	-0.00162
5	.04733	21	.00195
6	.04610	22	.00103
7	.02061	23	.00008
8	0.00789	24	0.00020
9	.02331	25	-0.00003
10	.02123	26	.00021
11	.00796	27	.00000
12	-0.00583	28	0.00042
13	.01227	29	.00064
14	.00992	30	0.00039
15	.00282		

FILTER NO. 88 - $r_c = 0.1$, $h = 0.03$, $N = 20$, $E = 0.009$, $r_{ac} = 0.155$

k	Value of W_k	k	Value of W_k
0	0.25989	11	0.00788
1	.23115	12	-0.00591
2	.15660	13	.01235
3	.06549	14	.01000
4	-0.00956	15	.00290
5	.04741	16	0.00343
6	.04618	17	.00561
7	.02069	18	.00383
8	0.00781	19	.00056
9	.02323	20	-0.00170
10	0.02115		

FILTER NO. 89 - $r_c = 0.1$, $h = 0.08$, $N = 30$, $E = 0.0005$, $r_{ac} = 0.265$

k	Value of W_k	k	Value of W_k
0	0.36000	16	-0.00010
1	.28118	17	0.00015
2	.11129	18	.00051
3	-0.02114	19	.00022
4	.05214	20	-0.00019
5	.01941	21	.00015
6	0.00944	22	0.00000
7	.01050	23	-0.00011
8	.00168	24	.00018
9	-0.00063	25	.00000
10	0.00101	26	0.00014
11	.00023	27	.00006
12	-0.00158	28	-0.00001
13	.00123	29	0.00005
14	0.00011	30	0.00005
15	0.00028		

FILTER NO. 90 - $r_c = 0.1$, $h = 0.08$, $N = 20$, $E = 0.001$, $r_{ac} = 0.255$

k	Value of W_k	k	Value of W_k
0	0.35999	11	0.00022
1	.28117	12	-0.00159
2	.11128	13	.00124
3	-0.02114	14	0.00010
4	.05214	15	.00027
5	.01942	16	-0.00011
6	0.00943	17	0.00014
7	.01049	18	.00050
8	.00167	19	.00021
9	-0.00063	20	-0.00020
10	0.00100		

FILTER NO. 91 - $r_c = 0.1$, $h = 0.08$, $N = 10$, $E = 0.006$, $r_{ac} = 0.265$

k	Value of W_k	k	Value of W_k
0	0.35983	6	0.00927
1	.28101	7	.01033
2	.11112	8	.00151
3	-0.02131	9	-0.00080
4	.05231	10	0.00084
5	-0.01958		

FILTER NO. 92 - $r_c = 0.1$, $h = 0.1$, $N = 60$, $E = 0.00006$, $r_{ac} = 0.295$

k	Value of W_k	k	Value of W_k
0	0.40000	42	-0.00000
1	.29156	43	.00000
2	.08030	44	.00002
3	-0.04380	45	.00000
4	.03925	46	0.00002
5	.00000	47	.00000
6	0.00858	48	.00000
7	.00121	49	.00001
8	.00078	50	-0.00000
9	.00228	51	.00001
10	-0.00000	52	.00000
11	.00121	53	.00000
12	.00022	54	.00001
13	.00017	55	.00000
14	.00058	56	0.00001
15	.00000	57	.00000
16	0.00038	58	.00000
17	.00008	59	.00001
18	.00006	60	-0.00000
19	.00023		
20	-0.00000		
21	.00017		
22	.00004		
23	.00003		
24	.00011		
25	.00000		
26	0.00009		
27	.00002		
28	.00002		
29	.00006		
30	-0.00000		
31	.00005		
32	.00001		
33	.00001		
34	.00004		
35	.00000		
36	0.00003		
37	.00001		
38	.00001		
39	.00003		
40	-0.00000		
41	-0.00002		

FILTER NO. 93 - r_c = [illegible], h = [illegible], N = 30, E = 0.0003, r_{ac} = [illegible]

k	Value of W_k	k	Value of W_k
0	[illegible]	16	0.00038
1	.2[illegible]	17	.00007
2	[illegible]	18	.00006
3	-0.04[illegible]	19	.00025
4	[illegible]	20	-0.00000
5	[illegible]	21	.00017
6	0.0[illegible]	22	.00004
7	.00126	23	.00003
8	.00[illegible]	24	.00011
9	.00227	25	.00000
10	-0.00000	26	0.00009
11	.00121	27	.00002
12	.00022	28	.00001
13	.00017	29	.00006
14	.00058	30	-0.00000
15	-0.00000		

FILTER NO. 94 - r_c = 0.1, h = 0.1, N = 10, E = 0.005, r_{ac} = 0.295

k	Value of W_k	k	Value of W_k
0	0.39984	6	0.00842
1	.29141	7	.00105
2	.08014	8	.00062
3	-0.04396	9	.00212
4	.03941	10	-0.00016
5	-0.00016		

FILTER NO. 95 - $r_c = 0.1$, $h = 0.2$, $N = 70$, $E = 0.000007$, $r_{ac} = 0.495$

k	Value of W_k	k	Value of W_k
0	0.60000	42	-0.00000
1	.25986	43	.00000
2	-0.04852	44	.00000
3	.01060	45	.00000
4	.00253	46	0.00000
5	.00000	47	.00000
6	0.00071	48	.00000
7	.00071	49	.00000
8	.00047	50	.00000
9	.00020	51	.00000
10	-0.00000	52	.00000
11	.00011	53	.00000
12	.00014	54	.00000
13	.00011	55	.00000
14	.00005	56	.00000
15	.00000	57	.00000
16	0.00004	58	.00000
17	.00005	59	.00000
18	.00004	60	.00000
19	.00002	61	.00000
20	-0.00000	62	.00000
21	.00002	63	.00000
22	.00002	64	.00000
23	.00002	65	.00000
24	.00001	66	.00000
25	.00000	67	.00000
26	0.00001	68	.00000
27	.00001	69	.00000
28	.00001	70	0.00000
29	.00001		
30	-0.00000		
31	.00000		
32	.00001		
33	.00001		
34	.00000		
35	.00000		
36	0.00000		
37	.00000		
38	.00000		
39	.00000		
40	-0.00000		
41	-0.00000		

FILTER NO. 96 - $r_c = 0.1$, $h = 0.2$, $N = 30$, $E = 0.00006$, $r_{ac} = 0.495$

k	Value of W_k	k	Value of W_k
0	0.60000	16	0.00004
1	.25986	17	.00005
2	-0.04852	18	.00004
3	.01060	19	.00002
4	.00253	20	-0 00000
5	.00000	21	.00002
6	0.00071	22	.00002
7	.00071	23	.00002
8	.00047	24	.00001
9	.00020	25	.00000
10	-0.00000	26	0.00001
11	.00011	27	.00001
12	.00014	28	.00001
13	.00011	29	.00001
14	.00005	30	-0.00000
15	-0.00000		

FILTER NO. 97 - $r_c = 0.1$, $h = 0.2$, $N = 20$, $E = 0.002$, $r_{ac} = 0.495$

k	Value of W_k	k	Value of W_k
0	0.60000	11	-0.00011
1	.25986	12	.00014
2	-0.04852	13	.00011
3	.01060	14	.00006
4	.00253	15	.00000
5	.00000	16	0.00003
6	0.00070	17	.00005
7	.00071	18	.00004
8	.00047	19	.00002
9	.00020	20	-0.00000
10	-0.00000		

FILTER NO. 98 - $r_c = 0.1$, $h = 0.2$, $N = 10$, $E = 0.001$, $r_{ac} = 0.495$

k	Value of W_k	k	Value of W_k
0	0.59997	6	0.00068
1	.25983	7	.00068
2	-0.04854	8	.00044
3	.01063	9	.00018
4	.00561	10	-0.00003
5	-0.00003		

FILTER NO. 99 - $r_c = 0.2$, $h = 0.03$, $N = 60$, $E = 0.0006$, $r_{ac} = 0.275$

k	Value of W_k
0	0.46000
1	.31474
2	.03905
3	-0.09570
4	.03631
5	0.04730
6	.03211
7	-0.02449
8	.02691
9	0.01134
10	.02126
11	-0.00352
12	.01572
13	.00083
14	0.01074
15	.00278
16	-0.00665
17	.00317
18	0.00360
19	.00263
20	-0.00159
21	.00170
22	0.00048
23	.00074
24	-0.00004
25	.00000
26	0.00003
27	-0.00044
28	.00022
29	0.00059
30	.00042
31	-0.00053
32	.00054
33	0.00035
34	.00054
35	-0.00016
36	.00043
37	0.00002
38	.00027
39	.00004
40	-0.00011
41	-0.00002
42	-0.00002
43	.00005
44	0.00008
45	.00012
46	-0.00008
47	.00017
48	0.00005
49	.00019
50	-0.00000
51	.00017
52	.00004
53	0.00012
54	.00005
55	-0.00006
56	.00004
57	0.00002
58	.00001
59	.00000
60	0.00003

FILTER NO. 100 - $r_c = 0.2$, $h = 0.03$, $N = 20$, $E = 0.008$, $r_{ac} = 0.255$

k	Value of W_k	k	Value of W_k
0	0.45997	11	-0.00355
1	.31470	12	.01575
2	.03902	13	.00086
3	-0.09573	14	0.01071
4	.03635	15	.00275
5	0.04727	16	-0.00668
6	.03207	17	.00320
7	-0.02452	18	0.00357
8	.02694	19	.00260
9	0.01131	20	-0.00162
10	0.02123		

FILTER NO. 101 - $r_c = 0.2$, $h = 0.08$, $N = 80$, $E = 0.00003$, $r_{ac} = 0.395$

k	Value of W_k	k	Value of W_k
0	0.56000	42	-0.00003
1	.30525	43	0.00001
2	-0.05317	44	.00003
3	.07175	45	-0.00002
4	0.03633	46	.00001
5	.01941	47	.00000
6	-0.01773	48	.00001
7	.00262	49	0.00002
8	0.00456	50	-0.00000
9	-0.00011	51	.00002
10	0.00101	52	0.00000
11	-0.00089	53	.00000
12	.00144	54	.00001
13	0.00112	55	.00001
14	.00042	56	-0.00002
15	-0.00028	57	.00000
16	0.00002	58	0.00001
17	-0.00042	59	-0.00000
18	0.00013	60	0.00000
19	.00042	61	-0.00000
20	-0.00019	62	.00001
21	.00010	63	0.00001
22	.00002	64	.00000
23	.00005	65	-0.00000
24	0.00020	66	0.00000
25	-0.00000	67	-0.00001
26	.00015	68	0.00000
27	0.00003	69	.00001
28	.00001	70	-0.00000
29	.00004	71	.00000
30	.00006	72	.00000
31	-0.00010	73	.00000
32	.00002	74	0.00001
33	0.00006	75	-0.00000
34	-0.00000	76	.00001
35	0.00002	77	0.00000
36	-0.00002	78	.00000
37	.00005	79	.00000
38	0.00004	80	0.00000
39	.00002		
40	-0.00001		
41	0.00000		

FILTER NO. 102 - $r_c = 0.2$, $h = 0.08$, $N = 30$, $E = 0.0003$, $r_{ac} = 0.365$

k	Value of W_k	k	Value of W_k
0	0.56000	16	0.00002
1	.30525	17	-0.00042
2	-0.05318	18	0.00012
3	.07175	19	.00042
4	0.03633	20	-0.00019
5	.01940	21	.00010
6	-0.01770	22	.00002
7	.00262	23	.00005
8	0.00456	24	0.00019
9	-0.00012	25	-0.00000
10	0.00101	26	.00016
11	0.00089	27	0.00003
12	.00144	28	.00000
13	0.00112	29	.00004
14	.00042	30	0.00005
15	-0.00028		

FILTER NO. 103 - $r_c = 0.2$, $h = 0.08$, $N = 20$, $E = 0.001$, $r_{ac} = 0.355$

k	Value of W_k	k	Value of W_k
0	0.56000	11	-0.00090
1	.30525	12	.00144
2	-0.05318	13	0.00112
3	.07175	14	.00042
4	0.03623	15	-0.00029
5	.01940	16	0.00002
6	-0.01773	17	-0.00042
7	.00262	18	0.00012
8	0.00456	19	.00042
9	-0.00012	20	-0.00019
10	0.00101		

FILTER NO. 104 - $r_c = 0.2$, $h = 0.08$, $N = 10$, $F = 0.006$, $r_{ac} = 0.365$

k	Value of W_k	k	Value of W_k
0	0.55989	6	-0.01784
1	.30514	7	.00270
2	-0.05329	8	0.00444
3	.07186	9	-0.00023
4	0.03622	10	0.00090
5	0.01929		

FILTER NO. 105 - $r_c = 0.2$, $h = 0.1$, $N = 50$, $E = 0.00008$, $r_{ac} = 0.395$

k	Value of $\ddot{w}_k$	k	Value of $\ddot{w}_k$
0	0.00000	42	0.00000
1	.29156	43	-0.00000
2	-0.08030	44	0.00002
3	.04380	45	-0.00000
4	0.03925	46	.00002
5	-0.00000	47	0.00000
6	.00658	48	-0.00000
7	0.00121	49	0.00001
8	-0.00078	50	-0.00000
9	0.00228		
10	-0.00000		
11	.00121		
12	0.00022		
13	-0.00017		
14	0.00058		
15	-0.00000		
16	.00038		
17	0.00008		
18	-0.00006		
19	0.00023		
20	-0.00000		
21	.00017		
22	0.00003		
23	-0.00003		
24	0.00011		
25	-0.00000		
26	.00009		
27	0.00002		
28	-0.00002		
29	0.00006		
30	-0.00000		
31	.00005		
32	0.00001		
33	-0.00001		
34	0.00004		
35	-0.00000		
36	.00003		
37	0.00001		
38	-0.00001		
39	0.00003		
40	-0.00000		
41	-0.00002		

FILTER NO. 106 - $r_c = 0.2$, $h = 0.1$, $N = 30$, $E = 0.0002$, $r_{ac} = 0.395$

k	Value of W_k	k	Value of W_k
0	0.60000	16	-0.00038
1	.29156	17	0.00007
2	-0.08030	18	-0.00006
3	.04330	19	0.00023
4	0.03925	20	-0.00000
5	-0.00000	21	.00017
6	.00858	22	0.00003
7	0.00121	23	-0.00003
8	-0.00078	24	0.00011
9	0.00227	25	-0.00000
10	-0.00000	26	.00009
11	.00121	27	0.00002
12	0.00022	28	-0.00002
13	-0.00017	29	0.00006
14	0.00058	30	-0.00000
15	-0.00000		

FILTER NO. 107 - $r_c = 0.2$, $h = 0.1$, $N = 20$, $E = 0.001$, $r_{ac} = 0.395$

k	Value of W_k	k	Value of W_k
0	0.60000	11	-0.00122
1	.29156	12	0.00021
2	-0.08031	13	-0.00018
3	.04381	14	0.00057
4	0.03924	15	-0.00000
5	-0.00000	16	.00039
6	.00851	17	0.00007
7	0.00120	18	-0.00007
8	-0.00079	19	0.00022
9	0.00227	20	-0.00000
10	-0.00000		

FILTER NO. 108 - $r_c = 0.2$, $h = 0.1$, $N = 10$, $E = 0.005$, $r_{ac} = 0.395$

k	Value of W_k	k	Value of W_k
0	0.59992	6	-0.00870
1	.29148	7	0.00113
2	-0.08038	8	-0.00086
3	.04388	9	0.00220
4	0.03917	10	-0.00008
5	-0.00008		

FILTER NO. 109 - $r_c = 0.3$, $h = 0.03$, $N = 60$, $E = 0.0002$, $r_{ac} = 0.375$

k	Value of W_k	k	Value of W_k
0	0.66000	42	-0.00001
1	.27800	43	0.00007
2	-0.13258	44	-0.00001
3	.00646	45	.00012
4	0.06820	46	0.00016
5	-0.04730	47	-0.00001
6	.00588	48	.00016
7	0.03572	49	0.00017
8	-0.02455	50	.00000
9	.00499	51	-0.00015
10	0.02126	52	0.00013
11	-0.01369	53	.00001
12	.00392	54	-0.00009
13	0.01275	55	0.00006
14	-0.00748	56	.00001
15	.00278	57	-0.00003
16	0.00722	58	0.00001
17	-0.00377	59	-0.00000
18	.00172	60	0.00003
19	0.00358		
20	-0.00159		
21	.00082		
22	0.00130		
23	-0.00041		
24	.00016		
25	0.00000		
26	.00013		
27	.00025		
28	-0.00059		
29	0.00029		
30	.00042		
31	-0.00072		
32	0.00026		
33	.00042		
34	-0.00058		
35	0.00016		
36	.00030		
37	-0.00034		
38	0.00007		
39	.00014		
40	-0.00011		
41	0.00001		

FILTER NO. 110 - $r_c = 0.3$, $h = 0.03$, $N = 30$, $E = 0.003$,
$r_{ac} = 0.365$

k	Value of W_k	k	Value of W_k
0	0.66000	16	0.00722
1	.27800	17	-0.00376
2	-0.13[illegible]	18	.00171
3	.00646	19	0.00359
4	0.06821	20	-0.00816
5	-0.04730	21	.00082
6	.00587	22	0.00130
7	0.03572	23	-0.00041
8	-0.02455	24	.00016
9	.00499	25	0.00001
10	0.02127	26	.00013
11	-0.01368	27	.00025
12	.00391	28	-0.00058
13	0.01275	29	0.00029
14	-0.00748	30	0.00043
15	-0.00278		

FILTER NO. 111 - $r_c = 0.3$, $h = 0.03$, $N = 20$, $E = 0.006$,
$r_{ac} = 0.355$

k	Value of W_k	k	Value of W_k
0	0.66000	11	-0.01369
1	.27800	12	.00392
2	-0.13258	13	0.01275
3	.00646	14	-0.00748
4	0.06820	15	.00278
5	-0.04730	16	0.00722
6	.00858	17	-0.00377
7	0.03572	18	.00172
8	-0.02455	19	0.00358
9	.00499	20	-0.00159
10	0.02126		

FILTER NO. 112 - $r_c = 0.3$, $h = 0.08$, $N = 90$, $E = 0.00001$, $r_{ac} = 0.465$

k	Value of W_k	k	Value of W_k	k	Value of W_k
0	0.76000	42	-0.00001	84	-0.00000
1	.21272	43	0.00003	85	.00000
2	-0.14416	44	-0.00004	86	0.00000
3	0.06548	45	0.00002	87	-0.00000
4	-0.00665	46	.00000	88	.00000
5	.01941	47	.00000	89	0.00000
6	0.01925	48	-0.00002	90	-0.00000
7	-0.00889	49	0.00002		
8	0.00114	50	-0.00000		
9	.00044	51	.00001		
10	.00101	52	0.00001		
11	-0.00168	53	-0.00000		
12	0.00069	54	.00000		
13	.00054	55	.00001		
14	-0.00079	56	0.00002		
15	0.00028	57	-0.00001		
16	.00007	58	0.00000		
17	.00010	59	.00000		
18	-0.00043	60	.00000		
19	0.00045	61	-0.00001		
20	-0.00019	62	0.00000		
21	.00002	63	.00000		
22	.00001	64	-0.00001		
23	0.00014	65	0.00000		
24	-0.00014	66	.00000		
25	.00000	67	.00000		
26	0.00011	68	-0.00001		
27	-0.00009	69	0.00001		
28	0.00001	70	-0.00000		
29	.00001	71	.00000		
30	.00006	72	.00000		
31	-0.00010	73	0.00000		
32	0.00008	74	-0.00000		
33	-0.00001	75	.00000		
34	.00001	76	0.00000		
35	.00002	77	-0.00000		
36	0.00004	78	0.00000		
37	-0.00002	79	.00000		
38	.00002	80	.00000		
39	0.00004	81	-0.00001		
40	-0.00001	82	0.00000		
41	-0.00000	83	-0.00000		

FILTER NO. 113 - $r_c = 0.3$, $h = 0.08$, $N = 70$, $E = 0.0004$,
$r_{ac} = 0.435$

k	Value of W_k	k	Value of W_k
0	0.76000	42	-0.00001
1	.21273	43	0.00003
2	-0.1442	44	-0.00004
3	0.06548	45	0.00002
4	-0.00665	46	.00000
5	.01941	47	.00000
6	0.01925	48	-0.00002
7	-0.00888	49	0.00002
8	0.00114	50	.00000
9	.00044	51	-0.00001
10	.00101	52	0.00001
11	-0.00168	53	-0.00000
12	0.00069	54	.00000
13	.00054	55	.00001
14	-0.00079	56	0.00002
15	0.00028	57	-0.00001
16	.00007	58	0.00000
17	.00010	59	.00000
18	-0.00043	60	.00000
19	0.00045	61	-0.00001
20	-0.00019	62	0.00000
21	.00002	63	.00000
22	.00001	64	-0.00001
23	0.00014	65	0.00000
24	-0.00014	66	.00000
25	0.00000	67	[illegible]
26	.00011	68	-0.00001
27	-0.00009	69	0.00001
28	0.00001	70	-0.00000
29	.00001		
30	.00001		
31	-0.00010		
32	0.00008		
33	-0.00001		
34	.00001		
35	.00002		
36	0.00004		
37	-0.00002		
38	.00002		
39	0.00004		
40	-0.00001		
41	.00000		

FILTER NO. 114 - $r_c = 0.3$, $h = 0.08$, $N = 10$, $E = 0.006$, $r_{ac} = 0.465$

k	Value of W_k	k	Value of W_k
0	0.75991	6	0.01916
1	.21264	7	-0.00897
2	-0.14425	8	0.00105
3	0.06539	9	.00035
4	-0.00674	10	0.00092
5	-0.01949		

FILTER NO. 115 - $r_c = 0.3$, $h = 0.1$, $N = 90$, $E = 0.000004$, $r_{ac} = 0.495$

k	Value of W_k	k	Value of W_k	k	Value of W_k
0	0.80000	42	0.00001	84	0.00000
1	.18020	43	.00001	85	.00000
2	-0.12993	44	-0.00001	86	.00000
3	0.07087	45	.00000	87	.00000
4	-0.02426	46	0.00001	88	.00000
5	.00000	47	-0.00001	89	.00000
6	0.00530	48	.00000	90	0.00000
7	-0.00195	49	0.00001		
8	.00127	50	-0.00000		
9	0.00141	51	.00001		
10	-0.00000	52	0.00000		
11	.00075	53	.00000		
12	0.00035	54	-0.00001		
13	.00028	55	.00000		
14	-0.00036	56	0.00000		
15	.00000	57	-0.00000		
16	0.00024	58	.00000		
17	-0.00012	59	0.00000		
18	.00010	60	-0.00000		
19	0.00014	61	.00000		
20	-0.00000	62	0.00000		
21	.00010	63	.00000		
22	0.00006	64	-0.00000		
23	.00005	65	.00000		
24	-0.00001	66	0.00000		
25	.00000	67	-0.00000		
26	0.00005	68	.00000		
27	-0.00003	69	0.00000		
28	.00003	70	-0.00000		
29	0.00004	71	.00000		
30	-0.00000	72	0.00000		
31	.00003	73	.00000		
32	0.00002	74	.00000		
33	.00002	75	.00000		
34	-0.00002	76	.00000		
35	.00000	77	.00000		
36	0.00002	78	.00000		
37	-0.00001	79	.00000		
38	.00001	80	.00000		
39	0.00001	81	.00000		
40	-0.00000	82	.00000		
41	-0.00001	83	0.00000		

FILTER NO. 116 - $r_c = 0.3$, $h = 0.1$, $N = 50$, $E = 0.00007$, $r_{ac} = 0.495$

k	Value of W_k	k	Value of W_k
0	0.80000	42	0.00001
1	.18020	43	.00001
2	-0.12993	44	-0.00001
3	0.07087	45	.00000
4	-0.02426	46	0.00001
5	.00000	47	-0.00001
6	0.00530	48	.00000
7	-0.00195	49	0.00001
8	.00127	50	-0.00000
9	0.00141		
10	-0.00060		
11	.00075		
12	0.00035		
13	.00023		
14	-0.00036		
15	.00000		
16	0.00024		
17	-0.00012		
18	.00010		
19	0.00014		
20	-0.00000		
21	.00010		
22	0.00006		
23	.00005		
24	-0.00007		
25	.00000		
26	0.00005		
27	-0.00003		
28	.00003		
29	0.00004		
30	-0.00000		
31	.00003		
32	0.00002		
33	.00002		
34	-0.00002		
35	.00000		
36	0.00002		
37	-0.00001		
38	.00001		
39	0.00002		
40	-0.00000		
41	-0.00001		

FILTER NO. 117 - $r_c = 0.3$, $h = 0.1$, $N = 20$, $E = 0.0008$, $r_{ac} = 0.495$

k	Value of W_k	k	Value of W_k
0	0.80000	11	-0.00075
1	.18020	12	0.00035
2	-0.12993	13	.00027
3	0.07087	14	-0.00036
4	-0.02426	15	.00000
5	.00000	16	0.00024
6	0.00530	17	-0.00012
7	-0.00196	18	.00010
8	.00127	19	0.00014
9	0.00140	20	-0.00000
10	-0.00000		

FILTER NO. 118 - $r_c = 0.3$, $h = 0.1$, $N = 10$, $E = 0.004$, $r_{ac} = 0.495$

k	Value of W_k	k	Value of W_k
0	0.79996	6	0.00526
1	.18016	7	-0.00199
2	-0.12996	8	.00130
3	0.07084	9	0.00137
4	-0.02429	10	-0.00004
5	-0.00004		

Navy Electronics Laboratory
Report 1049

STATISTICAL FILTERS FOR SMOOTHING AND FILTERING EQUALLY SPACED DATA, by H. M. Linnette. 126 p., 10 July 1961.
UNCLASSIFIED

An analysis was made of the characteristics of statistical filters for the smoothing of time series data such as are obtained in the study of low frequency ambient sea noise. It was shown that the equally weighted running mean type of filter and the Guassian type of filter have some inherent defects. These defects can be minimized by means of a filter which approximates the square-shaped ideal filter in the least square sense. By use of the Datatron computer, weights were determined for 118 such filters.

I. Linnette, H. M.

1. Electric filters - Test results
2. Filters - Test results

S-R004 03 01, Task 8119
(NEL Problem L2-4)

This card is UNCLASSIFIED.

Navy Electronics Laboratory
Report 1049

STATISTICAL FILTERS FOR SMOOTHING AND FILTERING EQUALLY SPACED DATA, by H. M. Linnette. 126 p., 10 July 1961.
UNCLASSIFIED

An analysis was made of the characteristics of statistical filters for the smoothing of time series data such as are obtained in the study of low frequency ambient sea noise. It was shown that the equally weighted running mean type of filter and the Guassian type of filter have some inherent defects. These defects can be minimized by means of a filter which approximates the square-shaped ideal filter in the least square sense. By use of the Datatron computer, weights were determined for 118 such filters.

I. Linnette, H. M.

1. Electric filters - Test results
2. Filters - Test results

S-R004 03 01, Task 8119
(NEL Problem L2-4)

This card is UNCLASSIFIED.

Navy Electronics Laboratory
Report 1049

STATISTICAL FILTERS FOR SMOOTHING AND FILTERING EQUALLY SPACED DATA, by H. M. Linnette. 126 p., 10 July 1961.
UNCLASSIFIED

An analysis was made of the characteristics of statistical filters for the smoothing of time series data such as are obtained in the study of low frequency ambient sea noise. It was shown that the equally weighted running mean type of filter and the Guassian type of filter have some inherent defects. These defects can be minimized by means of a filter which approximates the square-shaped ideal filter in the least square sense. By use of the Datatron computer, weights were determined for 118 such filters.

I. Linnette, H. M.

1. Electric filters - Test results
2. Filters - Test results

S-R004 03 01, Task 8119
(NEL Problem L2-4)

This card is UNCLASSIFIED.

Navy Electronics Laboratory
Report 1049

STATISTICAL FILTERS FOR SMOOTHING AND FILTERING EQUALLY SPACED DATA, by H. M. Linnette. 126 p., 10 July 1961.
UNCLASSIFIED

An analysis was made of the characteristics of statistical filters for the smoothing of time series data such as are obtained in the study of low frequency ambient sea noise. It was shown that the equally weighted running mean type of filter and the Guassian type of filter have some inherent defects. These defects can be minimized by means of a filter which approximates the square-shaped ideal filter in the least square sense. By use of the Datatron computer, weights were determined for 118 such filters.

I. Linnette, H. M.

1. Electric filters - Test results
2. Filters - Test results

S-R004 03 01, Task 8119
(NEL Problem L2-4)

This card is UNCLASSIFIED.

END

DATE
FILMED

11 65

PLEASE DO NOT RETURN THIS DOCUMENT TO DTIC

EACH ACTIVITY IS RESPONSIBLE FOR DESTRUCTION OF THIS DOCUMENT ACCORDING TO APPLICABLE REGULATIONS.

☆U.S. Government Printing Office: 1982—381-508/1357