THE
STEAM ENGINE

THE STEAM ENGINE

CONSIDERED AS

A THERMODYNAMIC MACHINE

THE STEAM ENGINE

CONSIDERED AS

A THERMODYNAMIC MACHINE

A TREATISE ON THE THERMODYNAMIC EFFICIENCY OF STEAM ENGINES

ILLUSTRATED BY

DIAGRAMS, TABLES, AND EXAMPLES FROM PRACTICE

BY

JAMES H. COTTERILL, M.A., F.R.S.

PROFESSOR OF APPLIED MECHANICS IN THE ROYAL NAVAL COLLEGE

THIRD EDITION, REVISED

London:

E. & F. N. SPON, 125 STRAND

New York:

SPON & CHAMBERLAIN, 12 CORTLANDT STREET

1896

PREFACE.

The present volume originated in some rough notes on the thermodynamics of the steam engine, drawn up as long ago as 1871, for the use of students of engineering and others interested in steam engines. These notes, having proved useful, were expanded into a complete treatise published in December 1877, which was intended to serve as an introduction to applied thermodynamics, while at the same time an attempt was made to study, more completely than had hitherto been done, the process of the conversion of heat into work in steam engines.

This treatise having been for several years out of print, the Author has undertaken the preparation of a new edition with great reluctance, feeling that, considering the progress made since 1877, a new book rather than a new edition of an old one would be required to deal with the subject in a manner at all commensurate with its difficulty and importance. It seemed, however, on consideration, that the ground occupied and the method of treatment being in many respects different from those adopted by others, the book might with some changes and additions still prove useful, and this opinion would not have been altered if the Author had had the advantage of being able to consult the two able American treatises by Professors Peabody and De Volson Wood, which appeared some time after the greater part of this volume was in type.

It is hoped that in many ways this edition will be found an improvement on the original work. In reprinting the first nine chapters considerable additions have been made in order to illustrate more fully the principles of thermodynamics. These additions are described in the Appendix and need not here be further noticed. In other respects little change has been found necessary in these chapters. Had the Author been writing a new book, it is probable that a larger value would have been adopted for the mechanical equivalent of heat, since it is now generally admitted that the long established value, 772, is somewhat too small. This has been explained in the notes at the end of the book, but pending an exact determination of this important constant, there is little inducement to transfer the correction from the notes to the text.

The tenth and eleventh chapters have been completely re-written and greatly enlarged. In 1877 it was already obvious that cylinder condensation was the rule in steam engines, and its absence the rare exception, but so far as experimental research had then proceeded, no opinion could be formed as to the law which it followed. Since that time a large amount of work has been done, and much attention has been drawn to the question by the labours of Messrs. Mair & Willans, Colonel English, and others. In 1887 Major (now Colonel) English read a paper before the Institution of Mechanical Engineers, describing a series of experiments on an engine consuming a large amount of steam, and found that the theoretical law of the inverse square root of the speed might be applied in comparing this engine with others. The Author, on examining the various experiments on single-expansion non-jacketed engines, has been led to concur in this view; and he has, therefore, described carefully the theory of the conduction of heat, and explained the difficulties which are encountered in applying it to the case of a steam cylinder. While engaged on this task he found that a simple formula, the form of which was suggested by theoretical considerations, would, with certain limitations, give a fair approximation to the amount of steam wasted in single expansion non-jacketed engines by initial condensation and other causes. As might be expected, the formula involves a constant to be determined by experience for each type of engine, but the values of the constant in a large number of examples of engines of widely different types appear fairly consistent. The causes of cylinder condensation are complicated, and the facts relating to it may be interpreted in more than one way, but this uncertainty does not affect the value of the formula, if it should be found on further examination to represent the facts with sufficient approximation. How far this may be the case, the Author must leave to the judgment of those interested in this question.

In the eleventh chapter, other questions relating to the efficiency of steam engines as thermodynamic machines have been considered much more fully than in the original book, and the whole put into a shape which it is hoped will be found practically useful. The action of feed-water heaters, a part of the subject which has been much neglected, is also dealt with. The few observations made on compound engines and on jacketing, are intended chiefly as suggestions for further investigation, since it seems clear that no real progress can be made until the question of cylinder condensation in a single-expansion non-jacketed engine has been more thoroughly studied.

The obligations of the Author to the various treatises and original memoirs from which information has been obtained, are fully stated in the text or in the notes at the end of the book, and it need only be added that if the present edition should be received as favourably as the original work, the Author will be fully rewarded for the not inconsiderable amount of time and labour which has been spent in its preparation. His best thanks are due to his assistant, Mr. J. H. Slade, R.N., for the correction of various errors in the earlier chapters.

Royal Naval College, Greenwich:
July, 1890.

NOTE TO THE THIRD EDITION.

In this edition a few passages in the text have been re-arranged and modified, chiefly where the Author's meaning appeared liable to misapprehension, and some slight changes and additions have been made in the Appendix. For the most part, however, the book has been reprinted without substantial alteration. In the Author's judgment, the time has not yet come when a thoroughgoing revision of the two concluding chapters could be attempted with advantage, even if circumstances had rendered it possible for him to undertake the task at the present time.

Royal Naval College, Greenwich:
November 6, 1895.

CONTENTS.

CHAPTER I.

PHYSICAL PROPERTIES OF STEAM.

CHAPTER II.

CONVERTIBILITY OF HEAT AND WORK. INTERNAL WORK.

CHAPTER III.

THEORY OF THE STEAM ENGINE (PRELIMINARY).

CHAPTER IV.

AIR AND GAS ENGINES.

CHAPTER V.

PERFECT THERMODYNAMIC MACHINES.

CHAPTER VI.

GENERATION AND EXPANSION OF STEAM.

CHAPTER VII.

PERFECT ENGINES WITH ANY GIVEN CYCLE.

CHAPTER VIII.

LOSSES OF EFFICIENCY IN HEAT ENGINES.

CHAPTER IX.

CLEARANCE AND WIRE-DRAWING.

CHAPTER X.

ACTION OF THE SIDES OF THE CYLINDER AND OF WATER REMAINING AFTER EXHAUST.

CHAPTER XI.

EXPERIMENTS ON STEAM ENGINES. CONDITIONS OF ECONOMICAL WORKING.

TABLES.

APPENDIX.

NOTES AND ADDENDA.

LIST OF ADDENDA INCLUDED IN THE NOTES TO THE TEXT.

ERRATUM.

Page 346, line 10 from bottom, *for* proportional *read* inversely proportional.

THE STEAM ENGINE

CONSIDERED AS A THERMODYNAMIC MACHINE.

CHAPTER I.

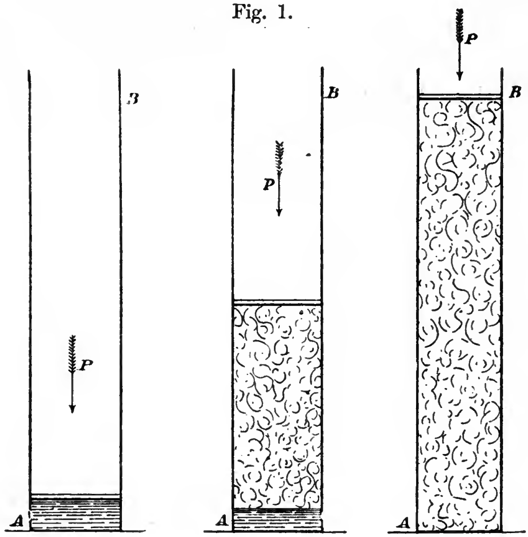
PHYSICAL PROPERTIES OF STEAM.

1. Our knowledge of the properties of steam is chiefly derived from experiments made by Regnault at the Paris Observatory, under the authority of the French Government, for the express purpose of ascertaining the numerical data necessary in calculations respecting steam and other heat engines. The experiments relating to steam are published in the twenty-first volume of the *Memoirs of the Institute of France* (Paris, 1847), to which the reader is referred for all details which it is not absolutely necessary for our purpose to mention: and such a reference is very desirable to obtain an idea of the immense labour and ingenuity employed in rendering the experiments as perfect as possible.

To understand precisely what Regnault ascertained, some preliminary explanations and definitions are requisite, as follows:—

In the figure, $A\ B$ is a cylinder open at the top, and containing a piston: the piston is loaded with weights, which with the atmospheric pressure are equivalent to P lbs. per square foot of the area of the piston, and rests on the surface of a mass of water placed below it; the quantity of water is immaterial, but, for convenience, will be supposed 1 lb.: the temperature of the water is supposed that of melting ice, or 32° on Fahrenheit's scale. If now heat be applied to the water, the temperature rises, becoming greater and greater the more heat is added, the piston remaining stationary (save a very small rise due to the expansion of the water) until a limiting temperature has been attained, the value of which depends on the pressure: the temperature then remains stationary at that limit value, and the formation of steam commences, the piston rising as

Fig. 1.



more and more of the water is evaporated; finally, when sufficient heat has been added to convert the whole of the water into steam, the temperature commences once more to rise, and may be raised to any amount if sufficient heat is added. These successive stages of the process are represented in the figure, which shows the piston in three positions, and for a complete theory of the steam engine a thorough knowledge of all three is indispensable. Such a thorough knowledge has not yet been attained as regards the third stage, in which the temperature of the steam is raised above that at which it was originally formed, but by the aid of Regnault's experiments almost all the needful information can be obtained respecting the first two stages, to which we shall confine ourselves in the present chapter.

Connection between Pressure and Temperature.

2. It was stated above that evaporation takes place when the temperature reaches a certain value depending on the pressure;

now if the constitution of fluid bodies was completely understood, it might be possible to determine the relation between pressure and temperature by theoretical considerations; at present, however, this cannot be done, and direct experiment is our only resource. Such experiments had been made by various experimentalists; but the results showed considerable discrepancy, and hence Regnault's first object was to set the question at rest by a thorough investigation. His apparatus consisted of a boiler containing, when half full, about thirty-three gallons of water, a condenser of suitable dimensions to condense the steam as fast as it was formed, and an air chamber three times the size of the boiler, provided with force pumps by means of which any desired pressure could be produced at pleasure. Pressures were measured by means of a column of mercury open to the atmosphere, an arrangement admitting of greater accuracy than the manometers of compressed air employed by others, but involving the manipulation of a column of mercury nearly 50 feet high at the greatest pressures experimented on. The air chamber and condenser enabled any desired pressure to be maintained for any length of time.

The principal difficulty to be overcome is, however, in the measurement of temperatures, which requires to be effected, especially at high pressures, with extreme accuracy. Now, a mercurial thermometer is not an exact measure of temperature unless it has been graduated by comparison with some standard instrument: differences in the quality of the glass and the mode of construction producing sensible differences in the indications, especially at high temperatures, differences which were probably the most important cause of the discrepancy in the results of the earlier experiments on the elastic force of steam. Hence, in the measurement of temperature, Regnault employed as a standard, not a mercurial, but an air thermometer, an instrument which will be referred to further in a subsequent chapter.

Regnault's experiments at pressures above the atmospheric extended to pressures of twenty-eight atmospheres, or more than 400 lbs. per square inch, while those at pressures below the atmosphere made with a different apparatus extended not only to what is commonly called steam, but likewise to the vapour given off by water at all temperatures, even the lowest. His results are given in degrees centigrade, and millimetres of mercury, and are not merely stated in tables, but expressed graphically by means of a curve drawn on copper with extreme care and accuracy. In reducing them to English measures, it has to be remembered that 100° centigrade corresponds to the pressure 760 millimetres of mercury, or 29·922 inches, at the temperature 32°, and at a height of 60 metres in the latitude of Paris above the level of the sea: at any other level and in any other latitude, 100° centigrade will, on account of the variation of the force of gravity, correspond to a column of mercury of somewhat different height. Now, 212° on a British standard thermometer corresponds to 30 inches of mercury at the equator, 29·922 inches in the south of France, or 29·905 inches at London, and hence lies a little below 100° centigrade; so that 1° Fahrenheit is not exactly $\frac{5}{9}$ths of 1° centigrade, but is a little less; and thus the reduction from French to English measures requires considerable calculation. The reduction has been made with great care and accuracy by Professor Dixon in his valuable *Treatise on Heat* (Dublin, 1849), and the table at the end of this book (Table I*a*) has been deduced from that given in his work, omitting the lower part of the table as not required for purposes connected with the theory of the steam engine, and converting inches of mercury into lbs. per square inch. In the reduction, it has been supposed that the British standard thermometer shows 212° at the pressure 14·7 lbs. per square inch, which is exact in the south of France, and near enough at any point of the earth's surface. The ratio which the pressure at any temperature bears to the pressure at 212° is the same everywhere, and it is this which for theoretical purposes it is important to know with accuracy. The table shows the pressure corresponding to each degree Fahrenheit, from 93° to 432° in lbs. per square inch. A supplementary table (Table I*b*), extracted directly from Dixon's work, shows the same pressure from 70° to 150° in inches of mercury. The third column in the principal table shows the rise of pressure consequent on an increment of temperature of 1°.

The general result of the experiments is to show that the pressure increases with the temperature, and that the more rapidly, the greater the pressure. For example, at 212° the pressure is 14·7 lbs. per square inch, and the increase of pressure for a rise of temperature of 1° is about ·29 lb.; at 247° the pressure has increased to 28·34, and the difference for 1° to about $\frac{1}{2}$ lb.; at 300° the pressure reaches

67·22 and the difference 1 lb.; while at 432° the pressure is no less than 350·73, and the difference is 3·64. Thus, at 350 lbs. per square inch, the pressure increases about thirteen times as rapidly as it does at the atmospheric pressure.

Many formulæ have been devised for the purpose of representing algebraically the results of experiments on the elastic force of steam at a given temperature, a brief account of which will be found in the Appendix.

The simplest of these formulæ, which represents the results with tolerable accuracy, is—

$$\log p = 5 \cdot \frac{t - 212}{t + 367} + \log 14 \cdot 7,$$

where p is the pressure in lbs. per square inch, and t is the temperature in degrees Fahrenheit. In the absence of the table this formula may safely be used, unless for some special purpose minute accuracy is required.

For such purposes and for ready numerical calculation the table is more useful; examples will be found attached to the table. It was mentioned above that Regnault constructed a curve graphically representing his results; this may be done by setting off the temperatures as abscissæ, and the corresponding pressures as ordinates; and the reader will find it a useful exercise to construct such a curve for himself, using the numerical values given in the table, so as to familiarise himself with the general character of the relation between pressure and temperature.

Before leaving this part of the subject, some circumstances must be noticed which modify the results now given in certain cases.

If perfectly quiescent water, perfectly free from air or other foreign substance, be heated in a clean glass vessel, the temperature may be raised far above 212° without occasioning ebullition; and when ebullition does take place it is effected, not regularly and quietly, but by fits and starts, producing what is called "bumping." This effect, which is much more manifest when sulphuric acid is used instead of water, is due to molecular cohesion; for particulars, the reader is referred to Professor Clerk Maxwell's treatise on the *Theory of Heat*, page 269. If such an effect could be produced in the circumstances of an ordinary steam boiler it would be a great source of danger, for suppose a boiler constructed to work at a pressure of 67 lbs. per square inch absolute, say 52 lbs. above the atmosphere, this corresponds to a temperature of about 300° Fahr.; if now the temperature could be raised to 320° Fahr. without the corresponding increase of pressure to 75 lbs. above the atmosphere taking place in the usual way, the slightest change of circumstances might produce explosive ebullition, accompanied by a great and sudden increase of pressure. The subject requires further investigation, but although it is certainly possible that some of the numerous cases of explosion which have occurred immediately after starting an engine may be accounted for in this way, yet the circumstances under which the effect is produced are rather those which occur in a laboratory than in actual practice.

Secondly, if a salt be dissolved in water the temperature of ebullition is varied; thus ordinary sea water contains one thirty-second part by weight of common salt, the temperature of the steam produced under the atmospheric pressure is not 212°, but 213°·2, and it is said that if more salt be added the boiling point of the brine is raised by 1°·2 for each thirty-second part of salt which is added. The steam in such cases is quite free from any admixture of salt, but probably has the temperature of the boiling brine, and is therefore to some degree "superheated," a term the meaning of which will be explained presently.

Subject to these observations, the elastic force of steam is always connected with its temperature, as shown by the table, so long as it remains in contact with water, no matter how the steam has been produced; thus if, instead of supposing the water confined in a cylinder provided with a piston which rises as the steam is formed, we suppose the steam to be produced in a closed steam boiler, then the temperature and pressure will keep rising as more and more heat is added, instead of remaining stationary; but the relation between pressure and temperature remains precisely the same so long as any water is left.

Specific Heat of Water.

3. Returning to our cylinder and piston, and considering the first stage of the process before the production of steam commences, we have now determined the limit temperature ($t°$) in terms of the pressure on the loaded piston, and we next consider, in order to

complete our knowledge of the first stage, the quantity of heat which must be added to the water in order to produce the change in question, or, in other words, to raise its temperature from 32° to t°. Now quantities of heat are measured in thermal units—that is, by the quantity of heat which is necessary to raise a lb. of water through 1° at its temperature of maximum density, or from 39° to 40° Fahr.; if then the same quantity of heat were required to raise a lb. of water through 1° at any other part of the scale, say from 212° to 213°, then the amount of heat required would be $t - 32$ simply, and this is what is usually assumed by practical writers on the subject of the steam engine.

Regnault, however, has shown, by a series of direct experiments, that the quantity of heat in question is always greater than $t - 32$, the difference becoming greater and greater as the temperature t° is higher. His results changed into English measures are given in Table II*a* at the end of the book, for every 27° from 77° to 401°, in which the first column gives the temperature t°, and the second $t - 32$, while the third gives the quantity of heat in question as determined by Regnault's experiments. A knowledge of this quantity of heat is continually required in the course of our work, and hence a special symbol (h) is used for it, and it must be understood that h in this work always means the quantity of heat necessary to raise a lb. of water from 32° to t°, and is never used for any other purpose.

As just stated, the value of h is given by the table at the temperatures indicated in the first column; at any intermediate temperature interpolation is necessary, for which purpose the mean difference for 1° is given in the fourth column of the table. These numbers represent the mean quantities of heat required to produce a rise of temperature of 1°, or, in other words, the mean specific heat of water between the temperatures indicated, whence it will be seen that the specific heat of water increases very considerably at high temperatures, becoming as much as 1·04 at the temperature 375°, corresponding to a pressure of 185 lbs. per square inch. Examples of the process of interpolation will be found at the end of the tables.

Regnault's results require certain corrections in order to make them precisely applicable to our purpose, that is, to make them represent with absolute accuracy the heat expended in the first stage of the process we are considering. These corrections are, however, undoubtedly much less than the deviation of the specific heat of water from unity, and will not be considered here ; for further information the reader is referred to the Appendix. Unless we have to do with steam of very high pressure the difference between h and $t - 32$ may frequently be safely disregarded, but much depends upon the particular question considered.

The quantity of heat requisite to raise a lb. of water through 1° has of late not unfrequently been called a "pound degree" by writers on the steam engine. If this expression be adopted, it must be remembered that it expresses a different quantity of heat for each particular temperature, so that for steam of 185 lbs. pressure it is about 4 per cent. greater than at low temperatures. Hence to make the "pound degree" a definite unit of measurement, the temperature employed as a standard must be indicated. There seems no advantage, however, in abandoning the well-understood term "thermal unit," used in measuring quantities of heat.

Total and Latent Heat of Evaporation.

4. We next go on to consider the second stage of the process, that is, the evaporation of the water, which takes place gradually as heat is added, the piston steadily rising, and the cylinder remaining at the constant temperature already investigated.

Let us first suppose that so much heat has been added that every drop of the water is evaporated, and the cylinder contains nothing but steam of the same constant pressure under which it was originally formed, then the first question to be considered is the quantity of heat required to evaporate the water as described. This quantity of heat is called the *latent heat of evaporation* of water, a term the origin of which will be explained hereafter. If further we consider the quantity of heat expended in the first and second stages together, that quantity of heat is called the *total heat of evaporation* of water. Thus the *total heat of evaporation of water is the quantity of heat requisite to raise a pound of water from 32° to a particular temperature, and evaporate it at that temperature*, while the *latent heat of evaporation of water is the quantity of heat requisite to evaporate a pound of water at a given temperature.* The first of these quantities will in this work invariably be denoted by H, and the second by L, symbols which will be used for no other purpose.

The values of H and L in the present state of our knowledge can only be determined by experiment, and as the results obtained by the earlier investigations of Watt and Southern were discrepant, a second principal object of Regnault's experiments was to set this question also at rest by a thorough investigation.

Regnault's apparatus consisted, as before, of a boiler, condenser, and air chamber, arranged so that a perfectly steady evaporation could be maintained for any length of time required under any desired pressure, the steam being conducted to the condenser and condensed as fast as it was formed in the boiler, quite independently of the calorimeters, mentioned farther on, used to measure the heat given out in condensation. The steam pipe conducting the steam from the boiler to the condenser and calorimeters passed into the boiler below the water line, and after several convolutions terminated in the centre of the steam space, which was large; while outside the boiler the pipe was thoroughly steam-jacketed and clothed; and hence thoroughly dry steam was secured without any possibility of superheating.

Suitable steam of a given temperature being thus obtained, is conducted into a calorimeter consisting of a pair of copper globes surrounded by cold water. Condensation of the steam in the globes immediately takes place, the heat given out being abstracted by the cold water, the rise of temperature of which furnishes a measure of the quantity of heat, while the condensed water issuing from the globes gives the weight of steam condensed; hence the heat given out by each pound of condensing steam is fully determined.

Great care is necessary in conducting experiments of this kind to secure accuracy, a special difficulty being to find the quantity of heat lost by radiation from the calorimeter while the experiment is proceeding. For details I must refer to the original Memoirs. It is sufficient to say that all difficulties were overcome by Regnault, whose results are universally accepted as being as perfect as the nature of the case permits.

If now we carefully consider the way in which Regnault's experiments were made, it will be seen that the result he obtained is no other than the total heat of evaporation as defined above; for if, after the water has been completely evaporated by the application of heat to the cylinder, we imagine some cold body to be applied to take away the heat again, the steam will begin to condense and the piston to descend under its constant load, a process which will go on till all the steam is condensed and there remains nothing but water: a further abstraction of heat causes the temperature to fall till finally we have the pound of water at 32°, with which the process began. And the whole heat taken away when the steam is condensed is the same as the whole heat added when the water is evaporated. Now while the steam was being condensed in the globes of Regnault's calorimeter, a constant pressure was maintained throughout the apparatus during the whole period of the experiment, and thus we are sure that the circumstances of the experiment were just the same as in the case of our hypothetical cylinder and piston. The necessity for insisting on this point will be understood when we come to the next chapter; for the present it is sufficient to say that the values of H are certainly given to a great degree of accuracy by these experiments, which, together with the two other series already mentioned, form the experimental basis of the theory of the steam engine.

Regnault's experiments on the total heat of evaporation extended from a pressure of one-fifth of an atmosphere to a pressure of fourteen atmospheres, say 3 lbs. to 200 lbs. per square inch, and the general result is that H increases slowly with the temperature by ·305 thermal unit for each degree Fahr., so that it may be expressed by either of the formulæ

$$\begin{aligned} H &= 1091{\cdot}7 + {\cdot}305\,(t - 32), \\ &= 1082 \quad + {\cdot}305\,t, \\ &= 1146{\cdot}6 + {\cdot}305\,(t - 212). \end{aligned}$$

Below one-fifth of an atmosphere the difficulty of securing a regular steady ebullition prevented Regnault from obtaining thoroughly reliable results. It is, however, usual to suppose that the same formula applies to all cases.

Table II*a* shows the results of the formula for every 27°, from 77° to 401°, and also at 32°, the fifth column giving the differences, which in this instance are constant. Intermediate values may be obtained either directly from the formula or by interpolation.

In our definition of the total heat of evaporation, it has been supposed that the temperature of the water was 32° when the heating commenced; in practice, however, it generally happens that the water originally has some other temperature t_0, we then speak of the total heat of evaporation *from* t_0 *at* t. The tabular values of H and h enable the result in this case to be easily obtained, for if Q be the required quantity of heat, we shall have

$$Q = H - h_0,$$

where h_0 signifies the heat necessary to raise a lb. of water from 32° to the temperature t_0. For example, to find the total heat of evaporation of water *from* 104° *at* 293° : on referring to the table we find for the value of h at 104°, 72·09, and for the value of H, at 293°, 1171·3, hence

$$Q = 1171\cdot3 - 72\cdot09 = 1099\cdot21.$$

In this case, if the temperatures are not those given in the tables, H and h_0 must be found separately by interpolation ; but we may almost always simplify by using $t - 32$ for h. Thus in the present example the error of so doing is less than one-tenth of a thermal unit, a quantity which is inappreciable compared with the value of h.

From the total heat of evaporation H we can at once deduce the latent heat of evaporation L, for it is clear that

$$L = H - h;$$

so that we have only to subtract the tabular value of h from the tabular value of H in order to find the value of L. The seventh column of table II*a* has been formed in this way, and shows the latent heat of evaporation of water for every 27° from 77° to 401°, while the eighth column shows the differences for 1°, from which the latent heat at any other temperature can be found by interpolation. The table shows that the latent heat L diminishes as the temperature increases, the rate of diminution not being exactly constant, but increasing with the temperature. Unless, however, special accuracy is necessary, the formula

$$L = 966 - \cdot71\ (t - 212°)$$

gives results which are amply sufficiently approximate, according to which the latent heat diminishes by rather more than seven-tenths of a thermal unit for each degree Fahr., and is 966 thermal units at the temperature 212°.

Density of Steam.

5. To complete our knowledge of the first two stages of the process we are considering, it is now only necessary to know what is the volume of the resulting steam, or how high the piston will have risen at the instant when the last drop of water is evaporated. Unfortunately, in the present state of our knowledge, this question cannot be answered with the same degree of accuracy with which we know the elastic force of steam or the total heat of evaporation. Two distinct methods have been adopted: first, by a series of direct experiments; secondly, by a calculation based on the principles of thermodynamics from the data already given.

No experiments on the density of saturated steam have been published by Regnault, and the only investigation possessing any claim to be considered reliable was made by Messrs. Tate and Unwin, under the auspices of the late Sir W. Fairbairn, and published in a paper read by the latter before the Royal Society in 1860. An abridged account of these experiments is given in Fairbairn's *Mills and Mill Work*, Part I. p. 207, to which the reader is referred for details; we shall here only mention the principle of the investigation and its results.

A glass globe was provided with a long stem, say 32 inches long, which was filled with clean mercury, and inverted in a dish of mercury, the mercury being previously boiled to secure the absence of air. A bubble of glass containing the water to be experimented on was then introduced into the globe floating on the top of the mercury. If now heat be applied to the globe the water vapourises, the mercurial column descends, and the capacity of the globe having been previously measured, furnishes the means of measuring the volume of steam produced from the known weight of water in the bubble at a known pressure and temperature. Two special difficulties occur when attempting to measure the density of steam by such a method.

First, it is impossible to tell directly the exact instant at which all the water is evaporated, and it is clear that if the volume be observed before all the water is evaporated there will be no means of determining the weight of water turned into steam; while if the volume is observed after the water is all evaporated, the volume measured will not be that of steam in contact with water, but of steam which is more or less superheated. Now this first difficulty was overcome by replacing the dish of mercury by a long glass tube projecting below a copper boiler containing water, in which the glass globe was immersed. On heating the boiler, steam was produced outside the globe of the same temperature as the steam inside the globe, and hence so long as any water remained in the bubble the pressure outside and inside the globe was the same, as shown by the mercury standing at the same height in the stem of the glass globe and the outer tube connected with the boiler. The moment the steam inside the globe becomes superheated the pressure inside the globe becomes less than the pressure outside, and this is at once indicated by a rise of the mercurial column in the stem of the glass globe, and the precise instant at which the volume should be measured was thus determined; hence this first difficulty was successfully overcome.

The second special difficulty which besets experiments of this kind is the cohesive attraction between water and the glass vessels in which it is contained, in consequence of which a glass vessel may be heated considerably without being at once dried, and in all probability steam is condensed on a glass surface as hot or hotter than itself. This source of error remains in the results of these experiments, and it is probable that the densities determined by them are somewhat too large.

The experiments extended from 2½ lbs. on the square inch to 70 lbs. on the square inch, and their results are given in Table III. for a whole series of pressures. The fourth column of this table gives the weight in lbs. of a cubic foot of steam, at the pressure indicated in the first column, as determined by these experiments.

The second method of obtaining the density is by calculation on the principles of thermodynamics from the values of the latent heat of evaporation given above. This method will be fully considered hereafter; it is sufficient to say at present that certain needful numerical data are not as yet known with absolute exactness, and that consequently the results of the calculation are not free from possible error. The fifth column in Table III. gives the weight of a cubic foot of steam as determined by this method, and the approximate agreement with the results obtained by direct experiment shows that neither method can be very far wrong. The calculation values, which are the smaller, are to be preferred, and it is highly improbable that they can be so much as 1 per cent. in error.

The general results of calculation and experiment on the density of steam are that the weight (w) of a cubic foot increases nearly in proportion to the pressure (p), but at a somewhat slower rate, as shown by column 6, which gives the differences per lb. from which the values of w for pressures not given in the table can easily be found by interpolation.

The reciprocal of w, the weight of a cubic foot, is the volume in cubic feet occupied by 1 lb. of steam, a quantity which in this work will always be denoted by v when steam is under consideration. The value of v is given in the second column of the table for the pressures indicated in the first column; and we shall call v the specific volume of the steam.

It is common to compare the volume of the steam with the volume of the water from which it is produced, which may be done by multiplying v by 62·4, which is nearly the weight of a cubic foot of water at ordinary temperatures; this may be called the relative volume of the steam, though by some writers the term "specific volume" is used in this sense.

Various formulæ have been devised for the purpose of connecting the pressure and density of steam, of which we shall here give two.

First, we place the formula given in Fairbairn's paper to represent his experiments, viz.

$$v = \cdot 41 + \frac{389}{p + \cdot 35},$$

altering the constants to suit the case when the pressure is given in lbs. per square inch, and the volume in cubic feet. This formula* gives the results obtained by direct experiment, and the values as stated above are probably somewhat too small for perfectly dry steam below 110 lbs. per square inch. At pressures much exceeding this limit the formula gives too large a result.

Secondly, the results of calculation are represented with great accuracy by a formula of the form

$$p\, v^n = \text{constant}$$

* In Fairbairn's *Mill Work*, Part I. p. 214, this formula is quoted with a wrong sign in the denominator of the fraction. The error has been copied by Rankine in his work on *Shipbuilding*, where the formula has been applied to a numerical example. In Rankine's *Useful Rules and Tables* the formula is quoted correctly.

where n is an index. To test this and to determine the value of n let the formula be written

$$\log p + n \,.\, \log v = \text{constant},$$

and let the values of $\log v$ and $\log p$ be plotted on a large scale on paper as horizontal and vertical ordinates. It will be found that the extremities of these ordinates lie very exactly in a straight line, showing that the formula is correct in form. The index n will be the tangent of the angle which this straight line makes with the horizontal axis, and will be found to be

$$n = 1 \cdot 0646.$$

The constant is found to be 479 for pressures in lbs. per square inch and volumes in cubic feet, so that the formula becomes

$$p\, v^{1 \cdot 0646} = 479.$$

When employed for finding the volume corresponding to a given pressure, it is most conveniently written

$$\log v = 2 \cdot 5174 - \cdot 9393 \log p,$$

and when, as is sometimes the case, we require the pressure corre sponding to a given volume, it becomes

$$\log p = 2 \cdot 6800 - 1 \cdot 0646 \log v.$$

Neither of these formulæ rest on any theoretical basis, but both are empirical formulæ employed to represent in a simple form the results of calculation and experiment. The second formula is that which will be chiefly employed in this work, when a formula is necessary; for numerical calculations the table may also be employed; examples are given in the Appendix.

Partial Evaporation—Superheating.

6. In all that has been said in the two preceding divisions of this chapter, it has been supposed that the process of evaporation has been carried on until every drop of the water has been evaporated, and there remains nothing but steam, while care has been taken to stop the application of heat at the instant the water has all disappeared, so that the temperature is still stationary.

In such a condition the steam has the greatest density possible in perfectly dry steam at that pressure, and is hence said to be steam of "maximum density," or otherwise it is said to be "saturated." Likewise the temperature of such steam is the lowest possible at that particular pressure. Steam, however, seldom exists in a perfectly dry and saturated condition; either it is more or less mixed with water, or else its temperature is greater than that corresponding to the saturated condition.

In the first place, the steam supplied by an ordinary steam boiler is probably rarely perfectly dry; much obscurity rests on this point from the absence of any easy means of testing steam so as to ascertain the proportion of suspended water, but there can be little doubt that if from any cause the ebullition is irregular, such as, for instance, is the case when the irregularity in the consumption of steam always existing is aggravated by small steam space and rapid evaporation—that water is carried over from the boiler along with the rushing steam. This effect is called "priming," and is often produced on a large scale by impurity of water and other causes which we need not here consider. And even if the steam from the boiler be originally dry, it almost always condenses to a greater or less extent on entering the cylinder.

Thus, in a theory of the steam engine it is not sufficient to confine ourselves to the consideration of dry steam, we must likewise consider steam containing a certain amount of moisture. The amount of moisture in steam is estimated by the amount of pure steam (x) contained in a pound of the actual steam, then x is a fraction which is smaller the wetter the steam, and which may be called the dryness-fraction of the steam. Let s be the volume in cubic feet of 1 lb. of water, then since 1 lb. of the steam contains x lbs. of dry steam and $(1 - x)$ lbs. of water, it is clear that the specific volume V must be

$$V = v\,x + (1 - x)\,s,$$

where v as before is the specific volume of dry steam considered in the preceding section.

We may write this

$$V = x\,(v - s) + s.$$

Now s is a small fraction, being $\cdot 016$ at ordinary temperatures and less than 20 per cent. greater (see Appendix, Note B) at the highest temperatures possible in practice, we may therefore safely neglect it; further, unless x be small, the remaining term s may be neglected, and we obtain simply

$$V = v\,x.$$

These simplifications cannot be made at very high pressures, nor when (as is sometimes the case) we have to do with mixtures of steam and water, consisting chiefly of water. To show the amount of error involved, the value of $v - s$ is given in the third column of Table III.

So much for the density of moist steam. Next, for its total heat of evaporation, we have only to consider that for each lb. of such steam a lb. of water has been raised from 32° to t, but only x lbs. have been evaporated; hence if Q be the heat expended,

$$Q = h + x\,L;$$

or if the water originally be at t_0 instead of 32°

$$Q = h - h_0 + x\,L;$$

a formula which, by the aid of tabulated results given previously, enables us to find the total heat of evaporation very readily when x is known.

The heat necessary to produce dry steam from and at 212° is 966 thermal units, and the total heat of evaporation under any circumstances may conveniently be expressed by stating the equivalent evaporation from and at 212°. Suppose we call this E, then

$$E = \frac{Q}{966} = \frac{h - h_0 + x\,L}{966} \text{ per lb. of steam.}$$

For example, suppose a boiler to supply steam with 10 per cent. of suspended moisture, the evaporation taking place from 100 at 320°, then

$$E = \frac{220 + \cdot 9 \times 888}{966} = \frac{1019}{966} = 1 \cdot 055,$$

which is the factor by which the actual evaporation must be multiplied to obtain the equivalent evaporation from and at 212°.

The possibility of the steam generated by a boiler containing suspended moisture, and thus requiring less heat to produce it, should not be lost sight of when the efficiency of the boiler or the evaporative power of the fuel is being considered, for important errors may easily be produced in this way.

Not only may steam be wet, but it frequently happens, either by direct application of heat or by other causes to be considered hereafter, that its temperature is raised above the limit value, which is the lowest possible at the pressure considered, and in that case it is said to be superheated. Such is the case in the third stage of the process we have been considering, in which after the water has been all evaporated the application of heat is continued. The temperature then rises continually, instead of remaining stationary as before. Our experimental knowledge of this third stage is very imperfect, and the little that is known with certainty cannot advantageously be introduced here; we shall therefore proceed in the succeeding chapter to explain the fundamental principle upon which, together with the results of experiment now given, all successful reasoning on the subject of the steam engine must necessarily be based.

CHAPTER II.

CONVERTIBILITY OF HEAT AND WORK. INTERNAL WORK.

7. WHEN a resistance is overcome by the action of force, the effect of the force, considered as acting through space, is measured by the magnitude of the resistance multiplied by the distance traversed, estimated in the direction of that resistance. The force is then said to do work, and the work done is numerically equal to the product of the resistance and the space.

The power of doing work is called energy, and a body or system of bodies possessing this power is said to "possess energy," an expression which implies that energy is treated as if it were something independent of the bodies through which it is manifested, capable, like matter, of measurement in quantity, and, as we shall see presently, like matter, indestructible. In simple mechanics energy is of two kinds, energy of position and energy of motion, otherwise called potential energy and kinetic energy, exemplified by the simple cases of a raised weight and a rotating wheel, each of which possesses the power of doing work: the one, in virtue cf its position at a certain height above the earth's surface; and the other, in virtue of its motion.

When the force applied is just sufficient, and no more, to overcome the resistance, the energy exerted is exactly equal to the work done; and this is true not merely of a single force applied directly so as to overcome the resistance, but to any number of forces applied by means of a machine of any degree of complexity, so that we may say in any case in which the forces just balance the resistance,

$$\text{Energy exerted} = \text{Work done.}$$

This is the principle of work as applied to balanced forces, and is identical with the older principle of virtual velocities explained and applied in all text books of elementary mechanics.

It will, however, rarely happen that the forces applied exactly balance the resistance overcome; let us suppose that the applied forces are the greater, then the unbalanced part of these forces takes effect by causing the parts of the machine to move quicker and quicker, and thus to increase their energy of motion; hence the energy exerted by the applied forces is not all employed in doing work, but partly takes effect in increasing the kinetic energy of the parts of the machine. In this case the principle of work is equivalent to the statement,

Energy exerted = Work done + Kinetic Energy accumulated in the moving parts of the machine;

a statement which, by supposing the accumulation of energy negative, will include also the case in which the applied forces are in themselves insufficient to overcome the resistance, so that a part of the work is done at the expense of the kinetic energy of the moving parts. Now, as the expression accumulated energy implies, the energy employed in altering the velocity of a particle or machine is not lost, but merely transferred to the particle or machine, existing there in the form of kinetic energy or energy of motion. And further, when the work done consists in raising weights or other similar operations, it is clear that the energy exerted is not lost, but exists in the weights raised, which possess, by falling, a capacity of doing work, or potential energy, exactly equal to the work done in raising them; so that if we confine ourselves to such operations we may assert that energy, when exerted is not destroyed, but simply transferred from one body to another.

In all cases, however, *some*, and in many cases *all*, the work done consists of mechanical operations in which energy is to all appearance lost, a principal instance of this being friction. When one surface rubs against another, work is done in overcoming friction; and although no doubt there is a certain amount of wear, so that some energy might be imagined to be regained by replacing the abraded particles in their original positions, yet it is certain that but a small amount of the whole energy exerted is thus accounted for. Thus, confining ourselves to simple mechanics, energy is not indestructible, and we cannot go farther than to say that energy cannot be created out of nothing, but must be obtained from some store of previously existing energy. To this extent the principle of work is equivalent to the statement that a "perpetual motion is impossible," an axiom which is actually forced on all those engaged in mechanical operations, and which was practically known to our great engineers of the last century, long before the principle of work was formally stated by Poncelet in the *Mécanique Industrielle.*

We now, however, can go much farther than this, since we know that mechanical energy is only one of several forms in which energy may exist, and that the processes by which energy is to all appearance lost are really processes by means of which mechanical energy is transformed into one or other of those forms, and that when account is taken of all the results of the processes in question, we shall find that the energy which has disappeared is in nowise lost, but is merely transferred from one body to another, and altered in form, not substance.

The principal other form into which mechanical energy is capable of being transformed, and the only one which concerns us, is that powerful agent in producing physical changes which we call heat. It is unnecessary here to trace the steps by which the idea arose that heat and mechanical energy are quantities of the same kind, capable of conversion the one into the other; I shall content myself with enunciating the FIRST LAW of Thermodynamics, that is to say, of the science of the relations between work and heat, as follows:—

Heat and mechanical energy are mutually convertible, a unit of heat corresponding to a certain fixed amount of work, called the mechanical equivalent of heat.

Thus, when mechanical energy is expended in overcoming friction, it is a matter of common experience that heat is produced, and the law just enunciated tells us that this heat is merely the energy expended in a different form. Assuming, which is very approximately the case, that the surfaces remain in the same state as before, the energy expended bears a fixed proportion to the heat generated, that fixed proportion being the mechanical equivalent of heat, and by comparing the heat generated with the energy expended the mechanical equivalent of heat may be determined.

The first attempt to establish a connection between work and heat was made by Count Rumford in his celebrated experiments on the heat generated during the boring of a cannon, but the first accurate determination of the mechanical equivalent of heat was made by Joule, and hence we commonly speak of Joule's equivalent. Joule experimented on the heat produced when a given amount of energy was employed in agitating water, and obtained the number 772, signifying that the heat necessary to raise a lb. of water from 39° to 40° would, if wholly converted into work, raise a lb. weight through 772 feet. This value, originally obtained by Joule in 1849, has been since verified by a repetition of the experiments. Some further particulars will be found in the Appendix.

Since heat and mechanical energy are merely different forms of the same thing, it follows that quantities of heat may be expressed in foot pounds, and conversely quantities of work may be expressed in thermal units; thus the total and latent heat of evaporation of water may be expressed in foot pounds, as is shown by Table II*b* derived from Table II*a*, by multiplication by 772. And again a horse-power of 33,000 foot pounds per minute is equivalent to 33,000/772 = 42·75 thermal units per minute, or 2565 thermal units per hour, numbers convenient to remember in working examples.

Internal and External Work done during Evaporation under Constant Pressure.

8. Having now obtained a principle by means of which heat expended can be compared with work done, let us return to the case of the evaporation of water beneath a piston, which we considered at some length in the preceding chapter. It is clear that, when the piston rises during the evaporation, work is done in overcoming the pressure P with which the piston is loaded, and it is easy to find from the data of the last chapter the amount of that work. For let A be the area of the piston in square feet, y_0 the depth of water in the cylinder before evaporation begins, y the height of the piston above the bottom of the cylinder when the water is all turned into steam, then clearly $A\,y_0$ is the volume of the water in the cylinder, and $A\,y$ the volume of the resulting steam, or, using the symbols of the preceding chapter,

$$A\,y_0 = s; \qquad A\,y = v;$$

but the piston rises through the space $y - y_0$ overcoming a pressure of P lbs. per square foot, or a total pressure $P A$, hence the work done is given by

$$\text{Work done} = P A (y - y_0) = P (v - s).$$

Since both P and v are known, and s is the constant fraction ·016, the work done can be calculated for any pressure or temperature from the experimental data of the preceding chapter. The results are exhibited in Table IV*a*, the fourth column of which gives in foot pounds the value of $P (v - s)$, or the work done, for every 27° of temperature from 104° to 401°, while the fifth column gives the differences per 1°, which enable the result for any other temperature to be readily found by interpolation. It appears from this table that the work done during evaporation of a lb. of water increases with the temperature, the rate being slow and slowly diminishing as the temperature rises. At the atmospheric pressure the result is 55,730 foot pounds.

Our next object is to compare this work with the amount of heat expended, in order to do which we have only to find the heat-equivalent of the work done, or else the work equivalent of the heat expended. Let us choose the first course, and obtain the heat-equivalent of the work done by dividing the values just found by 772, the results are given in the sixth column of the same table (Table IV*a*), while the seventh column shows the differences needed for interpolation. Now let us compare the result with the heat expended during evaporation, that is, with what we have previously called the latent heat of evaporation, and given in the seventh column of Table II*a*, and we are at once struck with the great difference which exists, the heat-equivalent of the work done forming but a small fraction of the heat expended. For example, take the temperature 293°: here the latent heat of evaporation is about 908 thermal units, and the heat-equivalent of the work done is only 78·6 thermal units, or about one-twelfth. The principle of work tells us that the work done cannot have been produced out of nothing, but must have been done at the expense of an equivalent amount of heat which has disappeared, while the numerical values, just obtained, show that the heat thus disappearing is comparatively small, and that the greater part of the heat must have been employed in producing changes within the water itself.

We are thus introduced to a conception of great importance, namely the conception that work may be done, not only in raising weights or performing other visible operations in which the resistances overcome are manifest to our senses, but also in overcoming resistances caused by molecular forces invisible to us, if I may use the expression, and only manifest by their results. Thus, in the present example the difference between 908 and 78·6 is 829·4, which is the heat-equivalent of the work done in overcoming the molecular cohesion of the particles of water resisting its conversion into steam. Work done in this way is called "internal work," because the changes considered take place within the body itself, whereas in contradistinction the work done in raising the piston is called "external work," because the change considered takes place, not in the body itself, but in external bodies.

The internal work done during evaporation at constant temperature obtained by subtraction, as in the example just given, will be found in foot pounds and thermal units in the second and fourth columns of Table IV*b*; it is always denoted in this work by the symbol ρ. The differences needful for interpolation are given in the third and fifth columns, and examination of their values shows a nearly constant result, hence the value of ρ is given with very considerable accuracy in foot pounds and thermal units by the formulæ exact at 212°:

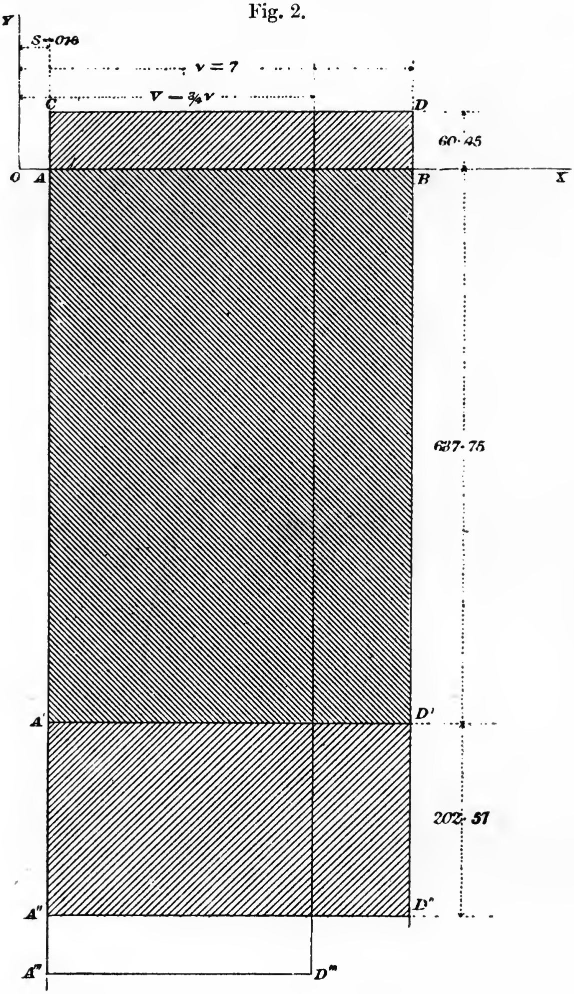
$$\rho = 819{,}330 - 611\, t \text{ foot pounds.}$$
$$\rho = 1061{\cdot}4 - {\cdot}792\, t \text{ thermal units.}$$

The sixth column shows the proportion (k) which the internal work bears to the external work, and exhibits in a striking manner the magnitude of the internal forces with which we have to deal, for we see that the internal work is from nine to sixteen times as great as the external.

9. We may with great advantage exhibit this graphically. In the figure (Fig. 2) OX is a line on which are measured the volumes occupied by the steam and water during the process of evaporation, or, what is the same thing, the volumes swept out by the piston as it rises, on which line we set off $OA = s = {\cdot}016$ cubic foot, and $OB = v$ cubic feet. Let the ordinates parallel to OY represent pressures, and draw a horizontal line CD, the ordinate AC of which represents the pressure P, with which the piston is loaded; then if

we complete the rectangle $A\,D$, its area will represent the external work done in raising the piston. Now to represent the internal

Fig. 2.



work we have only to draw a corresponding rectangle below $O\,X$ on the same base $A\,B$: the height of this rectangle will be $k\,P$, in order that its area may be k times the area of the upper rectangle. Thus the internal work is represented as the work which would be done in overcoming a pressure $k P$ on the piston, an ideal pressure which may be called the pressure equivalent to the internal work, or for brevity, "the internal-work-pressure." In practice pressures are always stated in lbs. per square inch, not lbs. per square foot, and the internal-work-pressure is consequently to be stated in like manner, as so many lbs. on the square inch. For example, take the temperature 293°, at which evaporation takes place under a pressure of 60·45 lbs. per square inch: the molecular resistance to evaporation is equivalent to a pressure 10·55 times as great, say to a pressure of 637·75 lbs. per square inch of the piston area. The figure (Fig. 2) is drawn to scale for this case, except that $O A$, being only ·016 cubic foot, is for clearness set off on a much larger scale than $O B$, which represents 7 cubic feet, the volume of dry steam at the pressure 60·45 lbs. on the square inch.

For any other pressure of evaporation, a corresponding internal-work-pressure exists, usually denoted by $\overline{P}$, which may be found from the tables already given by the method just indicated. But this pressure being frequently required, a special table has been calculated (by a different method) which gives it for any desired pressure. In Table V. the internal-work-pressure is given in lbs. per square foot, and in lbs. per square inch for pressures ranging from 4 lbs. per square inch to 250 lbs. per square inch, together with the differences necessary for interpolation.

Since the heat expended is the equivalent of the internal work and the external work taken together, it appears that the heat expended may be represented as overcoming a pressure on the piston equal to $(k + 1) P$: in the numerical example this pressure is 698·2 lbs. on the square inch; it may be called the pressure equivalent to the expenditure of heat, or, more briefly, the heat-pressure. The last column of the table (Table V.) shows the heat-pressure. In the figure the heat expended is represented by the whole area $C D^1$ of the two rectangles. When quantities of heat are represented by rectangles, the heights of which are pressures in lbs. per square inch and the bases volumes in cubic feet, the numerical value of the area is to be multiplied by 144, to express the quantity of heat in foot pounds, or divided by 772/144, that is to say, 5·36 to express it in thermal units.

Internal Work done during Rise of Temperature.

10. Hitherto we have considered exclusively the second stage of the process, namely, that in which water at a given temperature is converted into steam of the same temperature under the corresponding constant pressure; but internal work is also done during the first stage, in which the temperature of the water is raised from some lower temperature to the temperature at which evaporation commences. This is shown by the heat expended, which must be expended in producing molecular change of some kind, the nature of which it is needless for us to enquire into. Strictly speaking, indeed, external work is also done, for the water expands as its temperature rises and so raises the piston, but the amount of this is so small as to be quite insensible as compared with the heat expended, the value of which can be found in foot pounds or thermal units from Table II., as previously explained. Hence practically the whole heat expended is employed in producing molecular changes, or, as we express it, in doing internal work. (Comp. Art. 15.) Thus, in the numerical example given above, in which the steam is formed at 60·45 lbs. per square inch, the heat required to raise the water from 32° to 293° is 263·4 thermal units, or 203,300 foot pounds, all of which is spent in internal changes.

The proportion which the internal work so done bears to the external work done during evaporation is obviously got by dividing one by the other; thus, in the example, the ratio is

$$k' = \frac{263\cdot 4}{78\cdot 6} = 3\cdot 351\,;$$

and hence we may represent this internal work also as equivalent to raising the piston through its whole height against a pressure equal to k' times the actual piston load. In the example this pressure is $3\cdot 351 \times 60\cdot 45$, or 202·57 lbs. on the square inch.

To treat the questions graphically, we have only to prolong $B\,D'$ (Fig. 2) to D'', making $D'\,D''$ equal to 202·57, and complete the rectangle $A'\,D''$, then the area of that rectangle represents the internal work done during the rise of temperature of the water from 32°o.

The height of the rectangle may, however, conveniently be found thus without the calculation of k', for any initial temperature of the water. The heat requisite to raise a lb. of water from t_0 to t_1 is $h_1 - h_0$, calculated as previously explained, but may usually be taken as $t_1 - t_0$ thermal units. Let P' be the corresponding pressure in lbs. per square foot, which would do an equivalent amount of work upon the piston, then

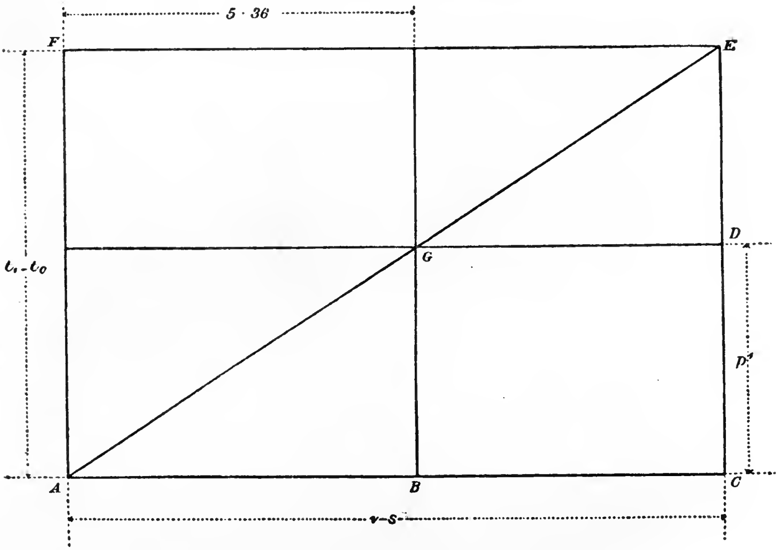
$$P'.(v - s) = (t_1 - t_0)\,772\,;$$

or if p' be the same pressure in lbs. per square inch,

$$p'.(v - s) = \frac{772}{144}.(t_1 - t_0) = 5 \cdot 36\,(t_1 - t_0).$$

In Fig. 3 set off $A\,B = 5 \cdot 36$ cubic feet and $A\,F$ equal to $t_1 - t_0$ reckoned as lbs. per square inch, and complete the rectangle $B\,F$.

Fig. 3.



Now set off $A\,C = v - s$ cubic feet, complete the rectangle $A\,E$, and join $A\,E$; further complete the rectangle $A\,D$ by drawing a parallel through G, then the rectangles $B\,F$ and $A\,D$ are equal; therefore $C\,D$ represents p', the required pressure. This construction will be found of great use in the graphic treatment of questions relating to the theory of the steam engine, as will be seen hereafter when we consider the expansion of steam.

Total Internal Work.

11. The total amount of internal work done is of course obtained by adding the two results together : hence

$$\text{Internal Work} = h + \rho = h + L - P(v - s)$$
$$= H - P(v - s);$$

thus in the numerical example for steam at 293° the internal work done during the rise of temperature is 263·4 thermal units, and during evaporation 829·3 thermal units ; hence the whole amount of internal work done in turning water at 32° into steam at 293° is 1092·7 thermal units, a result which may also be obtained by subtracting the heat-equivalent of the external work (78·6) from the total heat of evaporation (1171·3). Table IV*c* shows the results of this simple calculation for the same range of temperature as in the previous tables: these results are given in foot pounds and thermal units, together with the differences, which facilitate interpolation and serve other purposes.

These results show that the whole heat expended in internal changes increases with the temperature, though at a rate which is still slower than that of the total heat of evaporation.

Partial Evaporation.

12. In the three preceding sections it has been supposed that the evaporation is complete, so that the result of the operation is dry saturated steam. Let us now imagine the evaporation stopped when x lbs. of water have been evaporated, as in Art. 6, Chapter I.

In this case the specific volume is

$$V = x(v - s) + s;$$

and by reasoning similar to that in Art. 8, it is clear that

$$\text{External Work} = P(V - s)$$
$$= xP(v - s);$$

thus the external work done is simply x times what it would have been had the evaporation been complete. The heat expended during evaporation is clearly $x L$, and therefore bears the same proportion to the external work as if the evaporation had been complete: in the same way the internal work will be ρx during evaporation, but will be h, as before, during the rise of temperature Thus the whole internal work from water at 32° is given by

$$I = h + \rho x.$$

Or we may express our result in terms of the internal-work-pressure $\overline{P} = k P$, for just as the external work is $P (V - s)$, so we shall have during evaporation

$$\text{Internal Work} = \overline{P} \ (V - s);$$

$$\therefore \ \text{Total Internal Work} = h + \overline{P} \ (V - s).$$

In graphically representing the process of evaporation, the rectangle representing the internal work during evaporation will be of the same height as before, but described on the base V instead of the base v; while the rectangle which refers to the rise of temperature is not of the same height, but of the same area as before: it is constructed as in the last article, except that the base is V instead of v.

In the figure (Fig. 2, Art. 9) the construction is shown supposing the volume V three-fourths that of dry steam at the same pressure.

Formation of Steam in a Closed Vessel.

13. We have now thoroughly considered the whole process of evaporation, when conducted in a cylinder, beneath a loaded piston which rises as the evaporation proceeds, and we proceed to a different case, by supposing that the evaporation takes place not beneath a rising piston, but in a closed vessel of given capacity.

Let a lb. of water at the temperature 32° be placed in a closed vessel of known capacity, and let heat be gradually applied, then the temperature of the water will gradually rise as before, but instead of the formation of steam commencing at some definite temperature, as in the previous case, steam will be formed at once, and the pressure will keep rising as more and more steam is formed. The pressure is still connected with the temperature by the same invariable law as before, but the evaporation now takes place at a gradually rising temperature, instead of a certain fixed temperature. If sufficient heat be applied, every drop of the water will at length be evaporated, and we shall have nothing but steam: the pressure of that steam will depend on the volume of the vessel, which must be supposed large to avoid excessive pressure. The magnitude of the pressure is found by reference to the table of density (Table III.): thus, for instance, suppose that the volume of the vessel is 4 cubic feet, then the lb. of dry saturated steam occupies 4 cubic feet, therefore its pressure must be that corresponding to a volume of 4 cubic feet, which a reference to the table shows to be almost exactly 110 lbs. on the square inch. On reference to the temperature table (Table I*a*) the corresponding temperature is found to be about $334\frac{1}{2}°$, which is the temperature to which the vessel has risen when every drop of the water is evaporated. If the application of heat be still continued, the steam will become superheated; we, however, suppose it stopped before this takes place, and the question proposed for consideration is to find how much heat is spent in evaporating the water under these circumstances.

Now the essential difference between the two cases is, that in the first case, work is done by raising the loaded piston, while, in the second case, no work is done, and hence, in the first case, we have done two things instead of one; not only has the water been evaporated, but a certain amount of work has been done on external bodies: this amount of work cannot have been produced out of nothing, but must have been obtained at the expense of the energy applied to the water in the shape of heat. Hence, unless some difference be imagined in the amount of heat requisite to produce internal changes, we shall be obliged to conclude that the heat expended in the second case is less than the heat expended in the first case by the exact amount of the heat-equivalent of the external work, which *is* done in the first case, and *is not* done in the second. There is, however, no reason to believe that any such difference can exist; the steam produced is the same in both cases, and the water from which the steam is formed is likewise the same, and consequently the amount of internal change must be precisely the same, and we are entitled to conclude that when water is evaporated in a closed vessel, the heat expended is the same as that expended in internal work, when the water is evaporated beneath a loaded piston.

It is therefore given by the table of internal work just calculated (Table IV*c*).

The reader will now understand why, in defining the total heat of evaporation, it is necessary to specify that the evaporation is supposed conducted at a fixed temperature, and why it was necessary to examine the method in which Regnault carried out his experiments to see if the prescribed condition was satisfied. If that condition be not satisfied, it will be necessary, in order to find the heat expended in the production of steam, to consider how much external work has been done during its formation, and to add its heat-equivalent to the heat just found to be necessary to produce the steam itself. In fact, whenever external work is done in the formation of steam, heat flows out of the steam under the form of mechanical energy, just as really as when it escapes in the form of heat by the process called radiation.

Similarly when steam is condensed, the heat which it gives out is not always the same, but depends upon the circumstances under which the steam is condensed: if it be condensed under constant pressure, the heat given out will be what we have previously defined as the total heat of evaporation, and will include not only the energy given out by the steam during its contraction into water, but likewise the mechanical energy exerted by the pressure of the piston, which will appear in the form of heat in the condenser. But if it be condensed under any other circumstances, the heat given out will be different, because a different amount of energy will be supplied from external sources. In the next chapter we shall have ample illustrations of this in the working of a steam engine.

In Regnault's experiments we have a process of evaporation and a process of condensation carried on simultaneously under given pressure in vessels of constant volume. One of these processes is exactly the reverse of the other, and if we consider the two together, the final result is zero whether internal work or external work or heat expended. But the formation of steam takes place exactly as if that steam had been employed to drive a piston, the external work representing energy applied to condense the steam under constant pressure. It must be remembered that external work simply means work done in overcoming resistances external to the mass of matter which happens for the moment to be under consideration.

Similarly, in the process of formation of steam under rising pressure in a closed vessel; on the whole, no external work is done, but if the evaporation of a small part of the liquid is considered, external work is done in compressing the steam already formed. We shall return to this question hereafter; but it may be stated here, in anticipation, that the mixture of steam and water must be supposed so treated that the temperature throughout is sensibly uniform. If the experiment were tried without proper precautions, the steam would probably be found to be of higher temperature than the water, that is, it would be superheated.

Internal Work in General.

14. As in the case of steam, so in the most general case of the action of heat on any body whatever: we must always separate the external and visible work, done on external bodies, from the internal and invisible work, done in changing the state of the body. The second part, which we call the internal work, depends upon the change of state alone, and not upon the way in which the change of state is produced; while the first part represents energy, which has passed out of the heated body into external bodies, and may have any value according to the way in which the change is accomplished. The heat expended is the sum of these two amounts of work as expressed by the equation:

$$\text{Heat Expended} = \text{Internal Work} + \text{External Work};$$

which is the general statement of the principle of work as applied to such cases.

In the case of a heat engine, it frequently happens that the change considered is of such a kind that heat is added to the steam or other fluid during one part of the process, and taken away during another part: the heat added is then usually called the "heat expended," and the heat taken away, the "heat rejected," and the statement of the principle takes the form:

$$\text{Heat Expended} = \text{Internal Work} + \text{External Work} \\ + \text{Heat Rejected}.$$

If, moreover, the change considered be such that the steam or other fluid, after going through any number of intermediate changes, finally returns to its original state, then the internal work done is evidently zero, and we write simply:

Heat Expended = External Work + Heat Rejected.

Thus, for example, in the steam engine, if the feed water be taken from the condenser, forced by the feed pump into the boiler, there evaporated, and finally, after passing through the cylinder, be returned to the condenser in the shape of water of the same temperature as before, the internal work done in the whole operation is zero, and we shall have:

Heat Expended = Useful work done + Heat rejected into the condensation water.

A change of this latter kind is called a "cycle of operations," because the fluid goes through a cycle of changes, returning to its original state. The conception of a cycle of operations is due to Carnot, and, we shall find, was a most important step towards the true theory of heat engines. Besides, whenever, from deficiency of experimental information, we have no means of telling directly or indirectly the work spent in internal changes, we are obliged to resort to a cycle of operations as being the only case in which we can find the relation between heat expended and work done. For instance, when superheated steam is in question, we cannot reason except in this way, because our experimental knowledge of superheated steam is still very scanty, so that we have no means of knowing with certainty the work done in internal changes.

In the case of saturated steam and permanent gases, the internal work is known with tolerable certainty, and we are not obliged to confine ourselves to the consideration of cycles of operation. Thus, in a steam engine, it will sometimes happen that the condensed steam is not at the same temperature as the feed water from which it was originally produced; then the original state of the steam was water of temperature t_1 say, and its final state is water of temperature t_2, so that the internal work done in changing from the original state to the final state is not zero, but $t_2 - t_1$, thermal units, whence we see that the principle of work takes the form:

Heat Expended = $t_2 - t_1$ + External Work + Heat Rejected;

the external work being expressed by its heat-equivalent in thermal units. For an example the reader is referred to the discussion of various experiments in a subsequent chapter.

15. When the temperature of a body remains constant during the application of heat, that heat is said to be "latent." So long as heat was supposed to be a material substance, such an expression as the "latent heat of steam" was strictly appropriate; but if used now, it must be distinctly understood that a part of the heat is latent, not in the steam, but in external bodies in the form of mechanical energy. In this work the term will only be used in the phrase "latent heat of evaporation," which has the well-understood conventional meaning defined in Art. 4.

In contradistinction to "latent," the word "sensible" was formerly applied to heat which is effective in raising the temperature of the heated body: while the sum of the "sensible" and the "latent" heat was the *total* heat. It is now advisable to employ the expression "total heat" to signify the sum of the quantities of heat expended in internal and external work respectively; and when steam is formed in any way we shall call the whole heat expended its TOTAL HEAT OF FORMATION, while the phrase "total heat of evaporation" will always be used, in accordance with the definition already given, for the particular case in which the steam is formed under constant pressure.

The expression "internal work" has been employed throughout to signify energy expended in internal changes, without any distinction between different kinds of internal change. Writers on thermodynamics, however, often distinguish between internal changes consequent on change of temperature and internal changes consequent on change of molecular position, and confine the term "internal work" to the latter kind of changes only. It seems better, however, to use the term in a sense capable of exact explanation, without any hypothesis as to the nature of the changes considered. The whole of the reasoning in this chapter depends solely on the principle that heat and mechanical energy are merely different forms of the same thing, and are, therefore, completely interchangeable. No hypothesis is involved as to the ultimate nature of heat or the constitution of matter. Another very expressive term was introduced by Rankine, which is directly applicable to the case in which a heated body is a source of energy. Every such body possesses, in virtue of the heat which has been applied to it, a store of energy which is precisely equal to the internal work done during the heating, and which consequently depends upon the state of the heated body alone, and not upon the circumstances under which the body was heated, or upon its relation to external bodies. This store of energy is therefore called the *intrinsic* energy of the body, and when the body returns to its original state it is always given out, either wholly as heat, or partly as heat and partly as external work done upon a piston during expansion. Thus, intrinsic energy and internal work are the same, differing only in sign. For example, the total internal work done in producing dry saturated steam from water at 32° (given in Table IV*c*) may likewise be considered as the intrinsic energy of that steam. We shall occasionally use this term hereafter, but "internal" energy may also conveniently be used in the same sense.

CHAPTER III.

THEORY OF THE STEAM ENGINE (PRELIMINARY).

16. THE principle of work is not, by itself, sufficient to answer many of the most important questions which arise respecting the operation of heat engines; but we shall nevertheless go on at once to consider such parts of the theory of the steam engine as can be conveniently treated here, reserving other parts which are complex, or which require the application of a second equally important principle, till a later period.

In studying a difficult problem of any kind it is necessary to commence with the most simple cases, and pass gradually on to the more complex, in order that we may be enabled to deal with the difficulties of the subject one at a time. These simple cases are ideal, being formed by abstracting, from cases actually occurring, a number of disturbing causes which complicate the problem in practice. When once such cases are thoroughly understood, it is comparatively easy to estimate the effect of each disturbing cause separately in modifying the result of the preliminary investigation.

In dealing with the steam engine, then, we, in the first instance, make certain suppositions, never exactly, and sometimes not nearly, realised in practice, as follows :—

(1) In the first place, the supply of steam is supposed uniform, which cannot be the case in practice on account of the varying speed with which the piston moves. At the beginning and end of the stroke no steam passes from the boiler to the cylinder, and in expansive engines this stoppage lasts during a considerable part of the stroke: hence the evaporation is necessarily irregular, and that the more so the greater the expansion and the smaller the steam space in the boiler as compared with the capacity of the cylinder. No attempt has yet been made to estimate quantitatively the effect

of irregular ebullition, but there is no reason to think it important, except as a cause of the production of moist steam.

(2) The effect of clearance is neglected, and also that of wire drawing during the passage from the boiler to the cylinder. Both these disturbing causes always exist and exert considerable influence on the working of the engine: they will consequently be considered hereafter.

(3) The exhaust is supposed to open suddenly, exactly at the end of the stroke, and the mean value of the "back pressure" always existing behind the piston is supposed given. In practice some lead is usually given to the exhaust, and the back pressure depends on various complicated circumstances not yet reduced to a complete theory.

(4) The action of the sides of the cylinder is either neglected altogether, or some simple supposition is made respecting it. In practice this action has a most important prejudicial influence, and hence will form hereafter the subject of a special chapter.

Subject to these observations, we proceed to discuss various cases, commencing with the simplest.

Non-Expansive Engines.

17. When the steam port is open throughout the stroke the engine works without expansion, the pressure remaining constantly that of the boiler (Art. 16 [2]). In this case the process of evaporation is the same as in the simple case of cylinder and piston considered in the two preceding chapters; the only difference being that the evaporation of the water takes place in a boiler connected with the cylinder by a pipe instead of in the cylinder itself, and that the piston, instead of moving continuously in one direction, moves backwards and forwards. Neither of these circumstances has any influence on the heat expended on, or the energy exerted by, each pound of steam, which are accordingly given by the preceding rules.

The energy exerted in driving the piston is, however, somewhat greater, as is seen thus. Let x be the length in feet described by the piston in a given time, say $1'$, A the area of the piston in square feet, then $A x$ is the volume swept through by the piston per $1'$ in cubic feet. If P be the pressure in lbs. per square feet, $P A x$ will be the work done per 1′, and supposing the engine uses N lbs. of steam per 1′, the volume of each of which is v,

$$Nv = Ax,$$

since for each cubic foot swept through by the piston a cubic foot of steam must pass from the boiler to the cylinder. Thus the work done per lb. of steam in driving the piston is Pv instead of $P(v-s)$, which is the true value of the energy exerted by 1 lb. of steam during evaporation. The reason of this is that part of the energy exerted in driving the piston is obtained by the action of the feed pump, which for each lb. of steam used forces 1 lb. of water into the boiler, and, in doing so, does an amount of work represented by Ps. Thus the true energy exerted by 1 lb. of steam is the difference between the energy exerted in driving the steam piston and the piston of the feed pump. This distinction, though theoretically interesting, is unimportant in practice, so far as the steam engine is concerned, on account of the smallness of s, as compared with v, in consequence of which, except at very high pressures, Pv is sensibly equal to $P(v-s)$ as is shown in Table IV*a* by the values of these quanties there given.

The energy exerted in driving the steam piston is, however, by no means the same as the useful work done by the engine; this is always less, and often much less, on account of "back pressure." Back pressure consists of three parts: (1), the pressure corresponding to the temperature of the condenser, or the pressure of the atmosphere if there be no condenser; (2), the pressure of the air always contained in the water of the condenser or present through leakage; (3), the difference of pressure between the cylinder and condenser. The first may be taken on the average as 1 lb. per square inch where there is a condenser, and 14·7 where there is none; the other two depend on the speed of the piston, the state of the steam, the size of the ports, and other circumstances, but are probably seldom less than 1 or (under normal circumstances) more than say 3 lbs. on the square inch. Thus for non-condensing engines the back pressure may range from 16 to 18 lbs. on the square inch, and for condensing engines from 2 to 4 lbs. on the square inch, but these values may be much increased by improper construction and management.

Let now P_b be the back pressure, then the effective pressure is

$P - P_b$, and the useful work done per lb. of steam is $(P - P_b)\ v$, which is less than if there were no back pressure in the proportion $P - P_b : P$, a fraction which is at most ·9 in non-condensing engines and ·95 in condensing engines. The annexed table gives the effective work of 1 lb. of steam in an engine working without expansion at various boiler pressures. The results are given in thermal units and foot pounds in columns 2 and 3, while the fourth column shows the number of pounds of steam required per indicated horse-power per hour, which is readily obtained by dividing 1,980,000 by the work done by 1 lb. of steam.

The expenditure of heat per lb. of steam is simply the total heat of evaporation *from* the temperature of the feed water *at* the temperature of the boiler; it is given in column 5 in thermal units, while column 6 shows the heat expended, in thermal units per indicated horse-power per 1′, a mode of stating the expenditure of heat which is often convenient; the circumstances chosen are described in the table.

PERFORMANCE OF A NON-EXPANSIVE ENGINE.

Remarks.	Pressure (absolute) lbs. per square inch.	Effective work per lb.		Lbs. Steam per I.H.P. per hour.	Heat expended in thermal units.		Efficiency.
		Thermal units.	Foot pounds.		Per lb. of Steam.	Per I.H.P. per 1′.	
Non-condensing back pressure, 16 lbs. Feed heated to 212°	160	75·3	58,100	34·6	1012	582	·074
	80	64·	49,400	40·1	996	668	·064
	55	55·6	42,900	46·2	989	763	·056
Condensing back pressure, 2 lbs. Feed taken from condenser at 104°	60	76·2	58,800	33·7	1099	620	·069
	30	70·1	54,100	36·6	1086	663	·064
	20	66·2	51,100	38·8	1079	701	·061

The ratio which the useful work done bears to the heat expended is called the Efficiency of the Steam, and is given in the last column of the table, from which it appears that only from $5\frac{1}{2}$ to $7\frac{1}{2}$

per cent. of the heat expended is converted into useful work, the remainder being dissipated in the atmosphere or condenser. Yet the performance indicated is better than will actually be realised in practice, on account of the disturbing causes mentioned above, which always have a more or less prejudical influence, though to a less extent in the present case than in cases where the engine works at a high rate of expansion.

18. Since the useful work amounts to from $5\frac{1}{2}$ to $7\frac{1}{2}$ per cent., the remainder of the heat expended is from $94\frac{1}{2}$ to $92\frac{1}{2}$ per cent. of the whole. This remainder is conveniently called the "heat rejected," and appears in the condenser, where there is one, in the form of heat. When an injection condenser is used, this heat is employed in raising the temperature of the injection water up to the temperature of the condenser. Let θ be the rise of temperature, n the number of pounds of injection water per lb. of steam, then each n pounds of that water abstracts from 1 lb. of steam $n\theta$ thermal units nearly, whence

$$n = \frac{\text{Heat rejected}}{\theta},$$

which determines the amount of condensation water per lb. of steam, from which it is easy to deduce the amount per indicated horse-power per hour. The annexed table shows the result of such a calculation for the same pressures as before, assuming a rise of temperature of 40°

CONDENSATION WATER.

Pressure (absolute) p.	Heat rejected.		Lbs. of condensation water.	
	Thermal units per lb.	Thermal units per I.H.P. per 1′.	Per lb. of Steam.	Per I.H.P. per 1′.
60	1023	577	25·6	14·4
30	1016	625	25·4	15·6
20	1013	658	25·3	16·4

It is here supposed that the feed water is (as usual) taken from the condenser, which, in the numerical calculations, was supposed, as before, to be at the temperature 104°; if this be not the case, let Δt be the difference of temperature, then Δt must be subtracted from the heat rejected, when the temperature of the feed is lower than that of the condenser, and added if it be higher, as will be seen on referring to Art. 14. Again, when a surface condenser is used, the temperature of the condensing steam is higher than that of the condensation water, and unless the feed be taken from the condensed steam a correction will, strictly speaking, be necessary. Minute accuracy here, however, is only useful for the sake of practice in reasoning correctly on these questions, since in practice ample margin must be allowed for leakage and contingencies.

Measurement of the quantity of heat given out in the condensation water of an engine furnishes an excellent test, which has of late been practically introduced, of its efficiency. We shall return to this in a later chapter; for the present it is sufficient to say that the difference between the heat expended and the work done as shown by the indicator, a quantity which we have here called the heat rejected, and tabulated in the table last given, is nearly the same as the heat discharged from the condenser, differing from it only on account of piston friction and radiation, and the difference of temperature (if any) of the feed water from that of the water *entering* the condenser.

The size of cylinder required for a given power is expressed by the number of cubic feet which the piston must sweep through per 1′ for each indicated horse-power: this is easily obtained from the consumption of steam per hour, or directly by the formula

$$C = \frac{33000}{P - P_b} = \frac{229}{p - p_b}$$

varying inversely as the effective pressure of the steam. The actual dimensions of the cylinder will then be fixed by the piston speed and the stroke. The capacity of the feed pump (when single-acting) will be to the capacity of the cylinder in the proportion which unity bears to half the relative volume of steam at the boiler pressure, but allowance, of course, has to be made for leakage and contingencies.

19. The calculations made in Arts. 17, 18, presuppose that the boiler supplies dry steam; if this be not the case, then the total heat

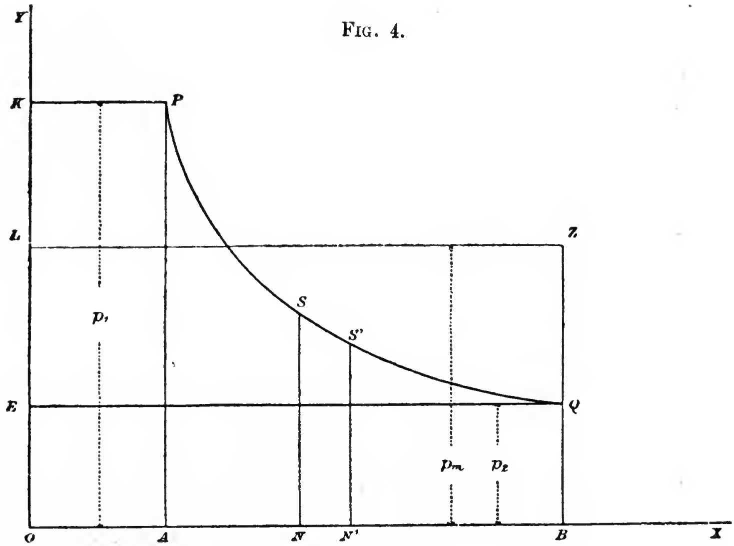
of evaporation, and the useful work done, must be calculated according to the method explained in Arts. 6 and 12. The general effect is that both the work done, and the heat expended, per lb. of steam, will be diminished, but the former in a greater proportion than the latter ; hence the number of pounds of steam per indicated horse-power per hour is increased, and the efficiency diminished, but the diminution of efficiency is but small, being occasioned solely by the fact that *all* the feed water must be raised up to the temperature of the boiler, though only part of it is afterwards evaporated. If however, liquefaction takes place after admission to the cylinder, then the work is diminished, while the heat expended remains unaltered, and the efficiency is diminished in precise proportion to the liquefaction.

Work done during Expansion.

20. The calculations just made respecting the performance of an engine working without expansion show that some 40 lbs. of steam will be required per indicated horse-power per hour, and that in the most favourable circumstances hardly 7 per cent. of the heat expended will be utilised. An additional amount of work may, however, be obtained without increasing the heat expended in the same proportion, by cutting off the steam when a certain part of the stroke has been performed, and allowing it to expand, thus obtaining from each pound of the steam energy which would otherwise be uselessly dissipated in the condenser.

In the figure (Fig. 4) OX is a line on which are set off the volumes of the whole amount of steam shut up in the cylinder, the corresponding ordinates representing pressures which in the first instance are supposed expressed in pounds per square foot. OA represents the volume, and PA the pressure at the beginning of the expansion, that is to say, at the instant when the steam is cut off; while OB, BQ represent the same quantities at the end of the stroke. The cut-off is supposed instantaneous, and the pressure then falls regularly from P to Q; the ratio $OB : OA$ is the ratio of expansion. At any intermediate point of the stroke the volume and pressure will have intermediate values ON, NS: the point S lying on the expansion curve PSQ, a curve which is the same as that drawn by the pencil of a good ordinary indicator, when properly set, because the volumes of the steam are proportional to the spaces traversed by the piston. It has already been shown that, if the pressure be constant, the work done in driving a piston is $P\,V$, where V is the volume swept out by the piston (Art 17): take now two points $N\,N'$ very near together, then $N\,N'$ is the increase of volume of the steam, that is, the volume swept through by the piston as it advances through a small space, and therefore $P.\,N\,N'$ is the work done if P be supposed constant. But P may be made as nearly

FIG. 4.



constant as we please by sufficiently diminishing $N\,N'$, and hence the area of the strip $S\,N'$ represents the work done during the small advance of the piston considered. Whence, dividing the whole area into similar strips, it appears that that area represents the whole work done, during the expansion, by the whole mass of steam shut up in the cylinder. By similar reasoning the rectangle $A\,K$ represents the work done during admission, and the whole area $K\,P\,Q\,B\,O$ consequently represents the total work done by the steam during admission and expansion.

The work done in driving a piston is usually and very conveniently expressed by finding the *mean* pressure of the steam, that is, the pressure which, if it remained constant throughout the stroke, would do the same amount of work as actually is done. In the figure set up $B Z$ so that

$$\text{Rectangle } O Z = \text{Area } O K P Q B,$$

then $B Z$ is the mean pressure required; it is usually denoted by P_m or p_m, according as it is given in pounds per square foot, or pounds per square inch, and called the "mean *forward* pressure," to distinguish it from the "mean *effective* pressure," $P_m - P_b$, found by subtracting the mean back pressure P_b. If the horse-power of the engine be calculated by use of the mean forward pressure instead of the mean effective pressure, the result is what is sometimes called the *total* horse-power of the engine, as distinguished from the *indicated* horse-power. The term is a misleading one, and will not be used in this treatise.

21. The reasoning of the last article is supposed to be already familiar to every reader of this work, and is here repeated for the sake of pointing out that the question is one of pure mechanics, and not at all respecting the nature of the fluid employed, or the circumstances of the expansion. The result of the calculation depends solely on the form of the curve, and the total volume of the cylinder in which the expansion takes place.

When, however, we wish to discuss questions relating to efficiency by comparing work done with heat expended, it is indispensably necessary to know directly or indirectly not merely the work done by a given *volume* of the fluid, but the work done by a given *weight*, for simplicity 1 lb. For this purpose let N be the number of lbs. weight of steam contained in the cylinder: then the work done by 1 lb. will be $1/N^{\text{th}}$ the total amount, and the volume occupied by 1 lb. will be $1/N^{\text{th}}$ the total volume; hence if N be known, a simple alteration of the scale on which volumes are measured will cause the diagram to represent the relation between volume and pressure, and the work done by 1 lb. of steam. It only remains to determine N, and that is to be done either by direct measurement of the consumption of steam by the engine, or by an independent knowledge of the state of the steam at some one point of the stroke. Suppose, for instance, that we know that, at the end of the stroke, the steam contains a given percentage of suspended moisture, then by Art. 6

the volume of 1 lb. of that steam can be calculated, and if the result be compared with the total volume of the cylinder, the value of N will manifestly be determined.

For the purposes of this preliminary investigation it will be supposed, in the first instance, that the steam is dry at the end of the stroke, a condition which ought always to be aimed at, for reasons which will afterwards appear, but which cannot generally be realised in practice, especially at high rates of expansion. The value of N is then found by dividing the whole volume of the cylinder by the specific volume (v) of dry steam at the known terminal pressure; and we may now take the diagram as representing the changes of volume and pressure of, and the work done by, 1 lb. of steam. In the second place, it will be supposed that the steam contains a given percentage of moisture at the end of the stroke, determined by considerations to be explained presently. The sole effect of this is to alter the scale on which volumes are measured, so that when the diagram represents 1 lb. of steam, it is smaller in the proportion $x : 1$ where x is the weight of pure steam in 1 lb. of the actual steam. The area of the diagram, and consequently the work done per lb. of steam, is then diminished in the same proportion. Experience tells us that the expansion curve does not differ widely from a common rectangular hyperbola, the product of the pressure and the volume remaining nearly constant. This implies (Art. 71) that the steam is initially moist and becomes drier and drier as it expands, an effect due mainly to the action of the sides of the cylinder, which condenses on its relatively cold surface a part of the steam admitted, which then forms a film of moisture on the sides of the cylinder, afterwards re-evaporated during the expansion. Without here entering on this question, we shall suppose that the curve is a common hyperbola, and leave other cases to be discussed in a later chapter.

22. In the common hyperbola PQ (Fig. 4) the rectangles OP, OQ are equal, while the area $PQBA$ is found by multiplying either rectangle by $\log_\epsilon r$, where r is the ratio of expansion $OB \div OA$, and the logarithm is of the kind called hyperbolic, from this very property of the hyperbola, found either by multiplying the common logarithm by 2·3026, or by a special table such as is given at the end of this book. Hence, taking first the case of dry steam, since OB is now v_2, the volume of dry steam at the known

terminal pressure p_2, the area of the whole figure, or the energy exerted by each pound of steam in driving the piston is

$$\text{Energy Exerted} = P_2\, v_2\, \{1 + \log_\epsilon r\}. \qquad (1)$$

The effective work is, of course, somewhat less, being found by subtracting from the foregoing result the quantity $P_b\, v_2$, where P_b is, as before, the back pressure, and consequently,

$$\text{Effective Work} = P_2\, v_2 \left\{1 + \log_\epsilon r - \frac{p_b}{p_2}\right\}. \qquad (2)$$

Secondly, if the steam be moist at the end of the stroke, let $1 - x_2$ be the weight of moisture in 1 lb. of the steam, as in Art. 6, then neglecting s (·016) the specific volume is $x_2\, v_2$, and hence the energy exerted on the piston is

$$\text{Energy Exerted} = P_2\, x_2\, v_2\, (1 + \log_\epsilon r), \qquad (3)$$

while the effective work becomes

$$\text{Effective Work} = P_2 x_2\, v_2 . \left\{1 + \log_\epsilon r - \frac{p_b}{p_2}\right\} \qquad (4)$$

The greater the expansion, other things being equal, the greater the amount of work done by the steam, until it is carried so far that the terminal pressure has fallen to the back pressure; in that case the expansion may be said to be *complete.* The effective work per lb. in complete expansion is evidently $P_2\, x_2\, v_2$, $\log_\epsilon r$ where x_2 is unity when the steam is dry. In practice, the prejudicial action of the sides of the cylinder, and other causes, render a moderate amount of expansion preferable, as will be seen hereafter.

The mean forward pressure in hyperbolic expansion is given (whatever be the state of the steam) by the formula

$$p_m = p_1 . \frac{1 + \log_\epsilon r}{r},$$

where $p_1 = r\, p_2$ is the initial pressure.

The amount of steam passing through the cylinder per I.H.P. per hour, and the condensation water, can now be found, as in Arts. 17, 18. (See also Note in the Appendix.)

Expenditure of Heat in an Expansive Engine.

23. The heat expended in an engine is, of course, all primarily employed in the evaporation of water in the boiler, but it is nevertheless not all used in the same way. When a steam jacket is used a part of the steam is condensed in the jacket, and the steam so used represents heat expended, although none of it passes through the engine and does work. Again, without anticipating what will be said in a subsequent chapter respecting the action of the sides of the cylinder, it is clear that heat will be transmitted to the exhaust steam as it leaves the cylinder, and that heat will be radiated continually to external bodies by the hot cylinder. Of these two causes of loss of heat the last is comparatively unimportant, when the cylinder is well clothed; but the first, being chiefly due to evaporation of a film of moisture deposited on the internal surface of the cylinder, is frequently great. We shall therefore call the loss of heat occasioned in these ways the "EXHAUST WASTE."

The cylinder, then, is continually being cooled by the exhaust waste, and heated by the steam jacket, if there is one; and the difference between the two must be subtracted from or added to the steam, during its passage from the boiler to the end of the stroke, according as the exhaust waste, or the supply of heat by the jacket, is the greater. Hence it is not sufficient to consider the heat expended on the working steam during evaporation in the boiler, but we must also consider the heat added or subtracted from that steam during its passage from the boiler to the end of the stroke. Now if the state of the steam at the end of the stroke be known, it will be possible to find the heat so added or subtracted, and thus the heat supplied by the jacket over and above the exhaust waste will be known; and, conversely, if the heat added or subtracted be known, it will be possible to find the state of the steam at the end of the stroke.

24. In the first place, suppose that the steam is dry at the end of the stroke; then if that steam were formed in a boiler by evaporation at constant pressure, the heat expended would be simply the total heat of evaporation (as defined in Chapter I.) *from* the temperature of the feed water *at* the temperature corresponding to the given pressure. The external work then done would be $P_2 v_2$, and the

heat-equivalent of that work is included in the total heat of evaporation.

But now the steam is formed in a much more complicated way, It is generated in the boiler at a much higher pressure than that at which we find it at the end of the stroke; it passes from the boiler to the cylinder and expands, while, as it does so, heat is added or subtracted. During this process external work is done by it, in driving the piston, represented by $P_m v_2$, the heat equivalent of which must form part of its total heat of formation. Hence (comp. Art. 13) its actual total heat of formation must be greater than the total heat of evaporation at P_2 by the heat-equivalent of the difference between $P_m v_2$ and $P_2 v_2$. We must therefore have

$$\text{Total Heat of Formation} = H_2 - h_0 + (P_m - P_2) v_2,$$

where $H_2 - h_0$ is, as in Chapter I., the total heat of evaporation of water *from* t_0 *at* t_2 and $(P_m - P_2) v_2$ is expressed in thermal units. Or if Q be the total heat of formation,

$$Q = H_2 - h_0 + \left(\frac{p_m}{p_2} - 1\right) P_2 v_2,$$

where $P_2 v_2$ is given in thermal units by Table IV*a*.

The same results may be reached by comparing this case with the case of a non-expansive engine, working with the same *terminal* pressure, when it will be seen that, although the circumstances under which the steam is *formed* are different, the circumstances under which it is *condensed* are identical. To fix our ideas, imagine an engine working at 60 lbs. pressure (absolute), and cutting off at one-tenth, assuming the common law of expansion, the terminal pressure is 6 lbs. per square inch, and the steam is dry by hypothesis; therefore, the cylinder at the end of the stroke is full of dry steam, of pressure 6 lbs. per square inch, which, when the exhaust is opened, rushes out into the condenser and is there condensed.

Now compare this with the case of an engine, working without expansion, at the pressure 6 lbs. per square inch with the same back pressure, a case which, though not occurring in practice, we are entitled to assume for our purpose; then at the end of the stroke in this case also, we have a cylinder full of dry steam at pressure 6 lbs. per square inch, which rushes into the condenser when the exhaust is opened, and is there condensed. Clearly it is impossible to distinguish the two cases. They are identical, and the reader who has carefully considered Chapter II. will perceive that we are entitled to conclude that the heat *rejected* is the same in the two cases; there is, in fact, no possible reason for any difference.

The heat expended, however, is not the same, because the steam is formed by a different process in the two cases; in the first case it is generated in the boiler at 60 lbs., passes into the cylinder, and there expands till its pressure has fallen to 6 lbs., while in the second case it is generated in the boiler at 6 lbs. But since we know the heat rejected by the rule previously given for a non-expansive engine, we can find the heat expended in the expansive engine by simply adding to that rejected heat the useful work done, for the two together make up the whole heat expended.

Using the same notation as before (comp. Art. 18), the heat rejected is given by

$$R = H_2 - h_0 - (P_2 - P_b)\, v_2,$$

while the useful work done is $(P_m - P_b)\, v_2$: adding which two together we find as before,

$$Q = H_2 - h_0 + (P_m - P_2)\, v_2.$$

This general formula is applicable to any case in which the steam is dry at the end of the stroke, and from it we can find how much heat is to be supplied to the jacket, independently of the exhaust waste, which for dry steam is probably small. For $H_1 - h_0$ is the heat supplied in the boiler to each pound of the working steam, assuming the boiler to supply dry steam; therefore if J be the jacket heat per lb. of working steam,

$$J = H_2 - H_1 + (P_m - P_2)\, v_2$$

$$= P_2 v_2 \left(\frac{p_m}{p_2} - 1\right) - \cdot 305\, (t_1 - t_2).$$

By division by $H_1 - h_0$ we find how much steam must be supplied to the steam jacket besides that required to provide for the exhaust waste, in which is included the waste heat of the liquefied steam discharged from the jacket.

Whatever the form of the expansion curve, this must be true, but if we suppose, as before, that curve to be an hyperbola, we get

$$J = P_2 v_2 . \log_\epsilon r - \cdot 305\ (t_1 - t_2),$$

which will give the required results, in conjunction with formula (2) of Art. 22, for any ratio of expansion r: taking $P_2 v_2$ in thermal units from Table IV*a*, and remembering that $t_1 - t_2$ is the difference of temperature at the beginning and end of the expansion found by Table I*a* from the corresponding pressures.

PERFORMANCE OF AN EXPANSIVE ENGINE. CASE I.

Expansion.	Effective work per lb. in thermal units.	Lbs. of Steam per I.H.P. per hour.			Heat expended per I.H.P. per 1′.	Efficiency.
		Working.	Jacket.	Total.		
None	79·0	32·5	0	32·5	594	·072
$r = 2$	126·3	20·3	·73	21·0	382	·112
$r = 5$	180·1	14·2	1·15	15·3	280	·153
$r = 9$	206·7	12·4	1·33	13·7	250	·171
$r = 13$	218	11·8	1·44	13·2	242	·177
Complete.	228	11·2	1·77	13·0	236	·181

Remarks.—Condensing engine, initial pressure 95 lbs., back pressure 3 lbs. absolute. Steam dry at the end of the stroke. Exhaust waste neglected. Temperature of feed 116°.

The annexed table has been calculated to show the performance of a condensing engine of initial pressure 95 lbs. per square inch, working at various rates of expansion, with back pressure 3 lbs. The results show that the efficiency may be more than doubled by expanding the steam five times, and still further increased, though only to a small extent, by a further increase in the expansion. Not all the gain here shown can be realised in practice, chiefly on account of the action of the sides of the cylinder, which occasions greater and greater loss as the ratio of expansion is increased; till, at a certain limit, there is no longer any advantage, but a positive loss in expansion. The reason of this cannot be discussed here. (See Chapter X.)

25. The amount of heat indicated by the table as necessary to be supplied to the steam, together with that required for the exhaust waste, is so considerable, that the steam jacket will, in most cases, be unable to supply it, and, in that case, the steam cannot be dry at the end of the stroke, but must contain a certain portion of moisture, either distributed over the internal surface of the cylinder or suspended throughout the whole mass of steam. In the latter case it will rush out into the condenser with the exhausting steam, but in the former it will remain on the internal surface of the cylinder and piston during the exhaust, and will be evaporated by heat abstracted from the cylinder. Thus the exhaust waste will in general be much increased when the steam is wet at the end of the stroke, and will increase the difficulty of supplying enough heat by the jacket. We shall now consider two cases in which the steam is wet. First, we shall imagine the cylinder jacketed, but that the jacket supplies only just enough heat to provide for the exhaust waste, so that the steam obtains none, and expands as in a non-conducting cylinder, except that the expansion curve is still supposed approximately an hyperbola. This case sometimes occurs in practice.

The steam is then moist at the end of the stroke; let the dryness-fraction be x_2, then the total heat of evaporation of the steam in its terminal state is, by Chapter I.,

$$\text{Total heat of Evaporation} = h_2 - h_0 + x_2 L_2,$$

of which $P_2 x_2 v_2$ is due to external work: while during formation in the present case, by Art. 21, the work done in driving the piston is $P_m x_2 v_2$, hence the total heat of formation must be

$$Q = h_2 - h_0 + x_2 L_2 + (P_m - P_2) x_2 v_2.$$

Now in the present case the steam obtains no heat during its passage from the boiler to the end of the stroke, and consequently its total heat of formation must be equal to the heat supplied in the boiler, therefore

$$H_1 - h_0 = h_2 + x_2 L_2 - h_0 + (P_m - P_2) x_2 v_2,$$

or remembering that the expansion curve is supposed an hyperbola,

$$H_1 = h_2 + x_2 L_2 + x_2 P_2 v_2 \, . \log_\epsilon r,$$

an equation which enables us to find out how much steam is liquefied at the end of the stroke : for

$$x_2 = \frac{H_1 - h_2}{L_2 + P_2 v_2 \, . \log_\epsilon r} .$$

Having found x_2, the useful work done can now be found by formula (4) Art. 22, and the efficiency is deduced by dividing by $H_1 - h_0$.

The annexed table shows the results of this calculation under the same circumstances as in the preceding example.

PERFORMANCE OF AN EXPANSIVE ENGINE. CASE II.

Expansion.	Wetness.	Lbs. of Steam per I.H.P. per hour.	Heat expended per I.H.P. per 1′.	Efficiency.
None.	0	32·5	594	·072
$r = 2$	·04	21·2	385	·112
$r = 5$	·08	15·5	281	·152
$r = 9$	·1	13·8	250	·171
$r = 13$	·12	13·4	243	·176
Complete.	·16	13·2	240	·178

Remarks.—Condensing engine, initial pressure 95 lbs., back pressure 3 lbs. absolute. Jacket just supplies the exhaust waste, which is not included. Temperature of feed 116°.

It will be seen that the results are nearly the same as in the preceding case : the loss by liquefaction being almost exactly compensated by the circumstance that the steam considered all passes through the cylinder instead of being partly used in the jacket, or we may express it roughly by saying that the liquefaction takes place in the cylinder instead of in the steam jacket. But no doubt in the second case the exhaust waste is greater than in the first case, and this has to be provided for by jacket steam.

26. Although the two preceding cases may possibly occur in practice, yet it can hardly be doubted that it is much more common that the exhaust waste is greater than the jacket supply. In this case the difference is abstracted from the steam during the passage from the boiler to the end of the stroke, and it is obvious that, in non-jacketed cylinders, this must always be so.

We shall now consider a particular case of this kind by supposing that the exhaust waste is greater than the jacket supply by 20 per cent. of the whole heat expended in the boiler. This proportion is far from unusual in practice, in non-jacketed cylinders. Then, all that heat is abstracted from the steam during its passage from the boiler to the end of the stroke: a circumstance which is expressed algebraically by the equation,

$$\tfrac{4}{5}(H_1 - h_0) = Q = h_2 - h_0 + x_2 L_2 + (P_m - P_2)\, x_2 v_2,$$

the notation being as in the preceding article. Whence we get an equation for x_2, replacing as before $(P_m - P_2)\, v_2$ by $P_2 v_2 \log_\epsilon r$; namely,

$$x_2 = \frac{\tfrac{4}{5}(H_1 - h_0) - (h_2 - h_0)}{L_2 + P_2 v_2 \,.\, \log_\epsilon r},$$

from which may be found the amount of water mixed with the steam at the end of stroke, and the useful work done may be calculated as before. The results are given in the annexed table for the same data as in the two preceding cases. The steam is now very wet at the end of the stroke, as is shown in column 2, and the performance is much inferior on account of the exhaust waste being included.

The assumption of a waste of 20 per cent. is, of course, an arbitrary one introduced to fix our ideas. As a matter of fact, the waste is in general greater at high rates of expansion than at low ones, for reasons which will appear hereafter. So also the expansion curve is not always an hyperbola; but at low expansion, the pressure falls quicker, and at high expansion slower, than is indicated by a hyperbolic curve.

Performance of an Expansive Engine. Case III.

Expansion.	Wetness.	Lbs. of Steam per I.H.P. per hour.	Heat expended per I.H.P. per 1′.	Efficiency.
None.	·243	42·9	782	·055
$r = 2$	·264	27·6	503	·085
$r = 5$	·285	19·9	362	·118
$r = 9$	·291	17·4	319	·134
$r = 13$	·312	16·9	308	·139
Complete.	·323	16·7	303	·141

Remarks.—Condensing engine, initial pressure 95 lbs., back pressure 3 lbs. absolute. Not jacketed: exhaust waste assumed 20 per cent. Temperature of feed 116°.

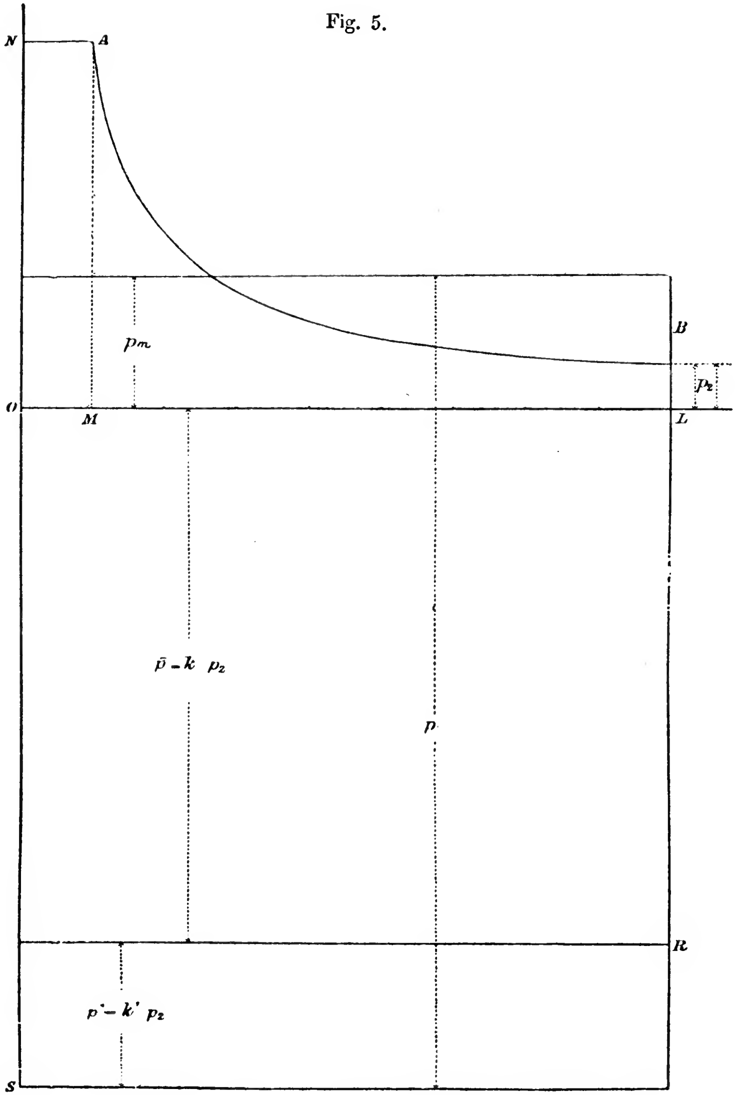
In the present case, the consequence of a uniform curve and a uniform percentage of waste being supposed at all rates of expansion, has been that the consumption of steam in Case III. has been increased, in the fixed proportion of 25 per cent., approximately, as compared with Case II., the advantage of expansion remaining the same, but this will of course not hold good in practice.

Graphical Representation of the Heat Expended. Equivalent Pressure.

27. The graphical method, explained in the last chapter, of representing the internal work done during the evaporation of water under constant pressure, may with great advantage be extended to the present case.

In the figure (Fig. 5), $NABLO$ represents the expansion diagram, showing, as in Art. 21, the energy exerted by the steam in driving the piston, during admission, and expansion according to any law from volume OM to volume OL, the pressure at the same time falling from AM to BL. Then it was shown that the internal work done during the formation of dry steam, at the terminal pressure p_2, could be represented by the area of a pair of rectangles,

Fig. 5.



the base of which is $O\,L$, the volume of the steam, and the heights $k\,p_2$, $k'\,p_2$ respectively, k, k' being numbers calculated for any pressure and temperature of feed by easy rules. (See Arts. 9 and 10.) Or the heights of the rectangles themselves may be found without a previous determination of k, k', as explained in detail in the same articles. In the figure these rectangles are OR and RS respectively.

Now, as has been sufficiently explained, the energy expended in internal changes in forming steam of given quality is always the same, and to find the total heat of formation we have only to add the energy exerted on external bodies during the process of formation. But this energy is represented by the area of the expansion diagram, whatever the form of the curve may be, and thus it is clear that the heat expended must be represented by the area of the whole figure $NABLS$, made up by the rectangles and the curve.

Moreover, it was shown that the internal work might be represented to our minds as equivalent to overcoming an ideal pressure on the piston, which we called the "internal-work-pressure," the said pressure being represented by $(k + k')\, p_2$. Now the energy exerted is equivalent to overcoming a mean pressure p_m on the piston where p_m as usual is the mean forward pressure, and consequently the heat expended must be represented by a pressure p_h on the piston given by

$$p_h = p_m + (k + k')p_2.$$

This pressure is what Rankine called the "pressure equivalent to the expenditure of heat," but for brevity it may be also called the "heat-pressure"; it may always be calculated by the preceding formula whatever be the treatment of the steam on its way from the boiler to the cylinder, provided the steam be dry at the end of the stroke. If it be not dry, then the same formula serves with the same value of k, but a modified value of k', as explained in Art. 12 of the last chapter. Two examples will now be given of the calculation of p.

First let us suppose

$$p_2 = 5 \; : \; t_0 = 90°,$$

being data which might frequently occur in condensing engines.

Here the temperature corresponding to p is found by Table I*a* to be 162°, and we must calculate

$$k = \frac{\rho}{Pu},$$

taking the values of ρ and Pu from Table IVa, whence we have

$$\rho = 933 \qquad Pu = 67 \cdot 7$$
$$\therefore k = 13 \cdot 78.$$

Next, to find k' we take the formula

$$k' = \frac{t - t_0}{Pu} = \frac{162 - 90}{67 \cdot 7} = \frac{72}{67 \cdot 7} = 1 \cdot 06$$
$$\therefore k + k' = 14 \cdot 84.$$

Again, let

$$p_2 = 25 : t_0 = 62°,$$

being data which might occur in a non-condensing engine in which no special provision was made for heating the feed. Then

$$t_2 = 240° : \rho = 871 : Pu = 74 \cdot 6 : t_2 - t_0 = 178,$$

Hence

$$k = 11 \cdot 68 : k' = 2 \cdot 37,$$
$$\therefore k + k' = 14 \cdot 05.$$

These examples show that $k + k'$ does not vary very much under a great variety of circumstances; hence Rankine recommended the formulæ

$$p_h = p_m + 15 p_2 \text{ (condensing)},$$
$$p_h = p_m + 14 p_2 \text{ (non-condensing)},$$

and no formulæ equally simple and general have, in fact, hitherto been given. It must always, however, be remembered that the exhaust waste is not included, and that hence the results obtained are too small unless in the (probably) exceptional cases in which the steam is dry at the end of the stroke: also it is better to calculate $k + k'$, or use the direct method of construction, than to trust to the average values given.

If the steam be known to be wet at the end of the stroke, then k' is replaced by k'/x_2 where x_2 is the corresponding dryness-fraction, but the modification so introduced is not important. The principal difficulty in the application lies in the determination of p_2, which ought to be known with accuracy; whereas in general it can only be determined to a rough degree of approximation. An example will now be given in which the data are taken from an experiment on the *Dexter*, a non-jacketed engine indicating about 200 horse-power with speed of piston of 366 feet per minute as follows:

Terminal Pressure	= 16·87,
Temperature of Feed	= 114°,
Mean Effective Pressure	= 37·54,
Mean Back Pressure	= 3·65.

The value of $k + k'/x_2$ here is 14·2, the value of x_2 being about ·72, whence

$$p_h = 41 \cdot 2 + 14 \cdot 2 \times 16 \cdot 87 = 281 \text{ nearly.}$$

Dividing the mean effective pressure by 281, we get for the efficiency,

$$\text{Efficiency} = \cdot 134.$$

The thermal equivalent of 1 horse-power is 42·75, whence dividing by ·134 we get 319 thermal units per I.H.P. per 1′, approximately, as the expenditure of heat. This result requires a small correction for clearance, but is enough to show a very considerable exhaust waste, the actual expenditure of heat being probably more than 400 thermal units per indicated horse-power per 1′.

The method followed in the present article is capable of great extension, but we must postpone further consideration of the numerous and important questions relating to expansion till the chapter specially devoted to that purpose.

The other calculations relating to an expansive engine are just the same as in a non-expansive engine working at the same terminal pressure. (See Art. 18.)

Expansion in General. Elementary Compound Engine.

27. The work done by steam during expansion has been already considered in Art. 20, so far as concerns the simple case of a single cylinder within which the mass of steam is wholly contained. It is, however, essential to consider the question from a more general point of view, in order to be able to deal with cases of a more complicated character.

Instead of supposing the steam to be shut up in a cylinder behind a piston, the motion of which causes changes of volume, let us imagine the steam or other elastic fluid to be contained in a bag of any size or shape which expands, retaining or not its original form as the case may be. Then considering, first of all, a very small increase of size and change of form, it is clear that the surface of the bag may be supposed divided into an indefinite number of small portions, each of which may be conceived to be the base of a small cylinder, in which the corresponding portion of the surface moves as a piston when the bag becomes a little larger. Hence, whatever the size or shape of the bag, if P be the pressure, V the volume through which the surface sweeps, or in other words the increase of volume of the bag, $P V$ will be the work done in overcoming a constant pressure P. If P be variable, then if we set off on a horizontal line abscissæ to represent volumes, and ordinates to represent the corresponding pressures, it appears by reasoning similar to that of Art. 20, that the area of the expansion curve represents the work done by the expanding fluid, just as in the particular example of a cylinder and piston. Accordingly that work depends solely on the changes of volume and pressure, and not at all on the changes of shape which the bag may have undergone. Again, when a fluid expands, the internal changes it undergoes are sometimes a source of energy, by means of which the whole or a part of the external work is done, and sometimes require a supply of energy in order to produce them. In the first case, the difference between the energy and the work is abstracted from, or supplied to, the expanding fluid in the shape of heat, according as the energy or the work is the greater; while, in the second case, heat enough must be supplied from without to do both the internal and the external work. In neither case is there any reason to believe that change of shape (unless made with very extreme rapidity) can have any sensible influence on the internal work done or the intrinsic energy exerted during a given change of pressure and volume, and consequently it follows that the form of the expansion curve depends solely on the nature of the fluid and the amount of heat (if any) which it receives during expansion. This will be illustrated fully in the case of air and steam in a later chapter; for the present I content myself with the very important conclusion that—

The work done by a given quantity of expanding fluid does not depend in any way on the particular machinery by means of which the expansive energy is utilised.

Thus when 1 lb. of water is forced into the boiler and evaporated, the resulting steam, expanded, exhausted, and finally condensed, the work done by it does not depend on the number of cylinders through which it passes during the series of changes it undergoes, but simply upon the pressure of admission, the ratio of expansion, and the amount of heat it receives during the process.

In a compound engine it frequently happens that the series of changes undergone is very complex; the steam is admitted to the high-pressure cylinder during a part of the stroke, is then cut off, and expands till the stroke is completed; on release, instead of passing at once to the condenser, it is exhausted into a second cylinder, either at once or through an intermediate reservoir, and only reaches the condenser after a complicated series of changes of pressure. Nevertheless, if our object is merely to find the power of the engine, we have no occasion to consider these changes at all: we have merely to discover how much steam is used and how many times it expands.

Let the large cylinder or cylinders, if there be more than one, be n times the small cylinder, and let the steam be cut off in the small cylinder at $1/r^{\text{th}}$ part of the stroke, then it is clear that the volume finally occupied by the steam, immediately before exhaust, is to the original volume in the ratio $n\,r : 1$; therefore if R be the number of times the steam expands,

$$R = n\,r,$$

and all the calculations for work done, and heat expended per lb. of steam, are to be conducted just as if it were used in a simple engine with a ratio of expansion R.

The weight of steam used depends upon the size of the large cylinder or cylinders alone, because at every stroke the volume of steam discharged is that of the large cylinder, and hence *the power and efficiency of a compound engine are (other things being equal) the same as if the steam were used in the large cylinder alone with the same total expansion.*

The addition of a high-pressure cylinder, then, has in itself no influence on the power or the efficiency of the engine; it is merely a device for partially overcoming some of the difficulties which attend the use of high grades of expansion.

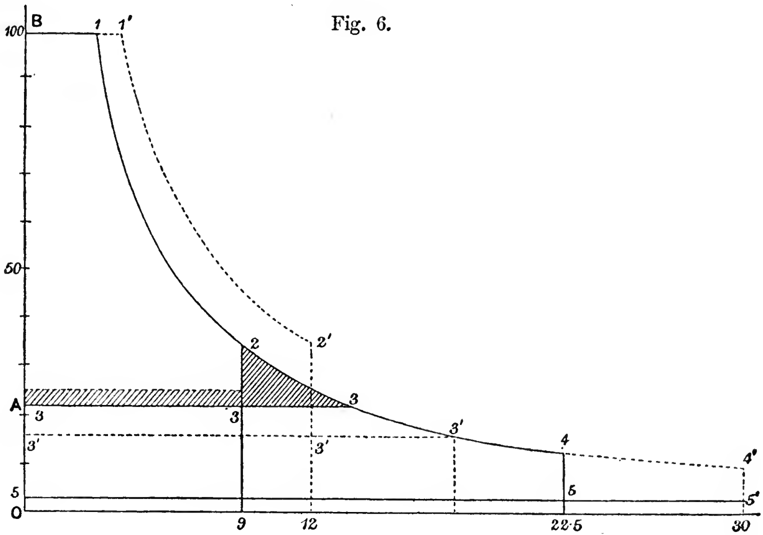
The foregoing statements, however, must be understood with certain qualifications as expressed by the saving clause, "other things being equal." When steam is "wire-drawn," or when communication is opened between vessels containing steam of different pressures, a part of the expansive energy of the steam is employed in generating violent motions in the particles of the steam itself; which motions are ultimately destroyed by fluid friction, and the corresponding kinetic energy transformed into heat. This energy is nearly all lost, so far as any useful purpose is concerned, and consequently the steam does not do all the work which it might have been made to do by a different arrangement. Since unbalanced expansion occurs to a greater extent in a compound engine than in a simple engine, the compound engine is at a disadvantage in this respect; the actual increase of efficiency generally produced by compounding being chiefly due to a diminution in the prejudicial influence of the sides of the cylinder, so that the exhaust waste is greatly diminished.

28. The type of compound engine which it is most useful to consider in a preliminary investigation, is that in which an intermediate reservoir exists, through which the steam passes on its way from the high-pressure to the low-pressure cylinder. In practice the reservoir is of moderate dimensions as compared with the cylinders, in consequence of which the pressure within it varies in a complicated way; but for the purpose in question, it is supposed so large that the addition or subtraction of a cylinder full of steam makes no sensible difference in the pressure, which accordingly remains constant, so long as the weight of steam supplied by the high-pressure cylinder in one revolution of the engine is equal to that drawn out of it by the low-pressure cylinder. Variation of pressure in the reservoir makes all the calculations relating to the engine much more complicated, without making any important change in the conditions on which efficiency depends. We describe the engine in this case as an Elementary Compound Engine.

Assuming then that the pressure in the reservoir has a constant value (p_3) to be determined presently, and neglecting the resistance of the passages connecting it with the cylinders, the back pressure in the high-pressure cylinder will have the constant value p_3 while the low-pressure cylinder will be supplied with steam at pressure p_3, just as it might be from a boiler. In each cylinder the steam is admitted, cut off, and expended, just as if there were two independent engines; the only connection between them is, that they each use the same weight of steam per minute.

Fig. 6 shows the indicator diagram of such an engine, in which *O B* represents the pressure at which steam is admitted to the high-pressure cylinder from the boiler, while *O A* represents the pressure

Fig. 6.



in the reservoir. The line 3 3 3 passing through it, is at once the back pressure line in the high-pressure cylinder and the admission line in the low-pressure cylinder, so that *B* 1 2 3 *A* is the diagram which would be drawn by an indicator applied to the high-power cylinder. The line 5 5 near the base of the diagram is the back pressure line of the low-pressure cylinder due to the pressure (p_5) in the condenser into which it exhausts its steam, and *A* 3 3 4 5 5 is the diagram which would be drawn by an indicator applied to it. The two diagrams are here combined into one, which shows the pressure and volume of a given weight (1 lb.) of steam as it passes through the cylinders in succession, by enlarging the low-pressure diagram, so that its base may be greater than that of the high-pressure diagram in the ratio of the cylinders. In Fig. 6 the ratio of cylinders is $2\frac{1}{2}$, so that 9 cubic feet of steam filling the high-pressure cylinder becomes 22·5 cubic feet at release from the low-pressure cylinder.

The expansion curve in each cylinder may, as in previous examples, be taken as a common hyperbola, and in the first instance we suppose that the two hyperbolæ form part of one and the same curve, a case represented in Fig. 6 by the full curve 1 2 3 4, of which 1 2 is the high-pressure part, and 3 4 the low-pressure. The pressure in the reservoir can on this hypothesis be at once determined, for supposing p the pressure at a point in the diagram indicated by the suffix attached, m the ratio of expansion in the low-pressure cylinder, and the other notation as in the last article,

$$p_3 = m \, . \, p_4 = \frac{m}{n\,r}\, p_1.$$

The work done in each cylinder is now determined by the same rules as before, from the area of its indicator diagram, and it will be observed that the sum is less than the area of the complete diagram B 1 2 3 4 5 5, by the small triangular portion 2 3 3 shaded in the figure. This triangle represents a loss due to the "sudden drop" of pressure on exhaust of the high-pressure cylinder into the reservoir given by

$$\text{Loss by sudden drop} = \frac{(P_2 - P_3)(V_2 - V_3)}{2} \text{ nearly}$$

where the symbol V is used as usual for volumes at points on the diagram indicated by the suffix.

Remembering that the product $P\,V$ is the same at all points, this equation is easily seen to be equivalent to

$$\text{Loss by sudden drop} = \frac{1}{2} P\,V\left(1 - \frac{m}{n}\right)\left(\frac{n}{m} - 1\right).$$

In addition to this, the resistance of the passages will cause a loss indicated by the shaded rectangle on the base 3 3, for the actual pressure in the reservoir will be greater than the admission pressure in the low-pressure cylinder, and less than the back pressure in the high-pressure cylinder. But for these losses, as already pointed out, the total work done in the compound engine would be the same as in a simple engine with the same total expansion and the same liquefaction.

It should carefully be noticed that the total expansion, and therefore the total work done (neglecting losses) is not dependent on the cut-off in the low-pressure cylinder. The only effect of altering this, is to change the reservoir pressure and the distribution of power between the cylinders. The earlier the cut-off in the low-pressure cylinder, the higher the reservoir pressure, and the less the work done in the high-pressure cylinder. But the relative liquefaction in the two cylinders has also a great influence on the reservoir pressure and distribution of power, as will be seen immediately.

29. When diagrams taken from the cylinder of an actual compound engine of this type are combined in the manner described, due account being taken of the effect of clearance, it will seldom be found that the expansion curves of the high-pressure and low-pressure cylinders agree; the high-pressure curve either intersects the low-pressure diagram or passes completely over. The reason of this is that agreement is only possible when the liquefactions in the two cylinders bear a certain proportion to each other. Hence the value of PV, though approximately constant in each cylinder, changes on passing from one cylinder to the other. When a compound engine is working to best advantage the high-pressure curve generally passes over the low-pressure, so that the value of PV is greatest in the high-pressure cylinder.

In Fig. 6 (p. 63) this case is represented by dotted lines; 1′ 2′ 3′ 3′ being the high-pressure diagram now much enlarged because the steam is much drier. In constructing such diagrams the horizontal ordinates represent the volume of a lb. of steam at pressures shown by the corresponding vertical ordinate. Thus, for example, in Fig. 6 at the end of the stroke in the high-pressure cylinder, the pressure in both cases is one-third the pressure of admission because the cut-off is supposed one-third, but the volume, which before was 9 cubic feet, is now 12 cubic feet, in consequence of the smaller amount of water contained in the steam.

The low-pressure diagram is represented by the dotted lines 3′ 3′ 4′ 5′ 5 in which 3′ 4′ the expansion curve is supposed part of the same hyperbola as before, a supposition which implies that the steam in the low-pressure cylinder is a little, though a very little, drier than before. The terminal volume in the low-pressure cylinder must be $2\frac{1}{2}$ times that in the high-pressure, because the ratio of cylinders is $2\frac{1}{2}$, and is therefore 30 cubic feet instead of $22\frac{1}{2}$. From the hyperbola 1 2 3 4 4′ the terminal pressure is now immediately found, and thence the pressure in the reservoir. If the low-pressure curve had not been part of the same hyperbola, it would have been necessary to know the dryness fraction from which the volume of dry steam can be found corresponding to 30 cubic feet of wet steam, and the pressure deduced by the usual formula.

PERFORMANCE OF AN ELEMENTARY COMPOUND ENGINE.

	Pressure in Reservoir.	Percentage of Water at End of Stroke.		Percentage of Power developed in each Cylinder.		Pounds of Steam passing through Cylinders per I.H.P. per Hour.	Loss per cent. by sudden drop.
		H.P.	L.P.	H.P.	L.P.		
Case I. ..	$22\frac{1}{4}$	27	22	52·6	47·4	16·9	3
Case II. ..	$16\frac{2}{3}$	2	21	63·7	36·3	13·7	5

Remarks.—Compound engine working with admission pressure of 100 lbs. (absolute). Ratio of cylinders $2\frac{1}{2}$. Steam cut off in h.p. cylinder at one-third, in l.p. cylinder at three-fifths the stroke. In Case I. the percentage of water at the end of the stroke in h.p. cylinder is supposed 27, while in Case II. the steam is nearly dry. In the l.p. cylinder the percentage is about 20 per cent. in both cases. (Compare Fig. 6, p. 63.) Back pressure in l.p. cylinder 3 lbs. The steam liquefied in the jackets (if any) is not included in the calculation.

The areas of the diagrams can now be found by the usual rules, so that the work done in each cylinder is known, and it will be seen that the percentage of work done in each cylinder is very different, and the total amount much greater, points which are illustrated by the annexed table of numerical results.

29A. The foregoing example is ideal, but it will serve the purpose of conveying a general idea of the working of a compound engine, of the conditions on which its efficiency depends, and the reason why it is in many practical cases more economical than a simple engine. In a simple engine working with ratio of expansion 7·5, as experience shows, the consumption of steam, even after allowing for losses, would not be less than in Case I. of the table above. The expansion curve would probably not be hyperbolic, but an hyperbola coinciding with the curve at its lower part, would fall outside it at the upper, so that less work per lb. of steam would be done notwithstanding the "loss by sudden drop" in the compound engine. Now, without anticipating what will be said in a subsequent chapter, it may here be stated that a principal factor in the production of liquefaction in a steam cylinder, is the range of temperature in that cylinder. In the high-pressure cylinder of a compound engine working as represented by Fig. 6, the range of temperature is less than one-half that of the simple engine, and unless there be some special cause, the liquefaction would be much less considerable, though it probably would be greater than in the ideal Case II. Hence the working of a compound engine as compared with a simple engine, is in many practical cases represented by Case II. as compared with Case I., the simple engine consuming 25 per cent. more steam, the cause of the superior economy of the compound being the greatly diminished liquefaction in the first stage of the expansion. This advantage, however, may be, and often is, thrown away by cutting off the steam too early in the high-pressure cylinder, or in some other way occasioning an unnecessary amount of liquefaction there. Whether a simple or a compound engine is the more economical when each is working under the most favourable conditions, with the same boiler pressure and vacuum, at the same speed, is a doubtful and, it may be added, usually an unpractical question. The great economy realised by the introduction of the compound engine for marine purposes must, in the first instance, be ascribed to the employment of steam of high-pressure with an amount of expansion which could hardly be attempted in a single cylinder.

The principle of compounding may be carried still further by the use of triple instead of double expansion; and if the full advantage of steam of 150 lbs. pressure or upwards is to be realised in a condensing engine, triple expansion is a necessity.

In making the calculations the expenditure of heat has not been found. This, however, can always be done by the methods described in preceding articles, which may be applied to each cylinder separately, or to the two combined.

Cycle of Operations in a Steam Engine, Diagram of Energy.

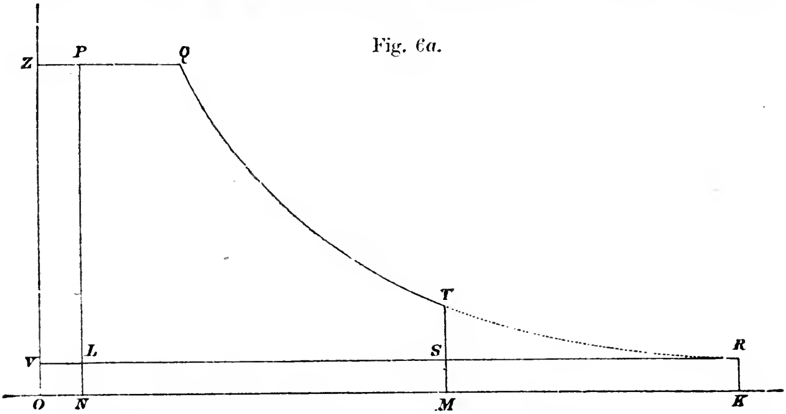
30. The construction of an indicator, and the general nature of the diagram obtained by its use, are supposed to be already familiar to every reader of this work. Suffice it to say that, when properly set, the motion of the drum precisely corresponds with the motion of the piston, so that the diagram drawn on the card shows the pressure of the steam at any point of the stroke: during the forward stroke in front of the piston, and during the backward stroke at the back of the piston. In practice, the diagram is generally considered as showing the total effective pressure urging the piston forwards at each point of the stroke; it being usual to take a diagram from each end of the cylinder and combine the upper half of each with the lower half of the other; thus obtaining figures often called "true" indicator diagrams, which show the true driving force on the piston.

No doubt, if the object is to draw a true curve of crank effort when studying the variation of the twisting moment on the crank shaft, or the fluctuation of energy of the engine, this course must be adopted; but for the purposes of a theory of the steam engine, the indicator diagram may be looked at from an entirely different point of view; it may, or rather must, be regarded as the graphic representation of the series of changes undergone by the feed water in the course of its transformation into steam, passage through the engine, and subsequent condensation.

In the figure (Fig. 6*a*), suppose ON to represent the volume of a small quantity of water contained in a bag according to the conception of the preceding article, and let PN represent the boiler pressure: then the vertical line PN represents the rise of pressure which takes place as the water is forced into the boiler by the action of the feed pump. Next, neglecting wire-drawing, the horizontal line PQ represents the gradual increase of volume of the bag as the water within it gradually evaporates and finally becomes all steam. Imagine the steam suddenly cut off when the bag is wholly within the cylinder, then the bag gradually expands, as represented by the curve QT, till precisely at the end of the stroke (suppose) the exhaust opens.

When the exhaust is opened, the bag undergoes a sudden expansion as the steam rushes into the condenser, followed by an almost coincident contraction as the steam within it is condensed. The process is not exactly the same for all particles of steam, but may be sufficiently nearly represented by the dotted expansion curve *T R*, and the horizontal line of condenser pressure *L S R*. During this expansion the only work the bag does is in overcoming the condenser pressure, the excess of expansive energy being employed in generating kinetic energy, ultimately reappearing as heat in the condenser. During condensation, energy is exerted by the steam piston in compressing the bag, which energy, after allowing for the work done by it in the sudden expansion, is represented by the rectangle *S N*. Hence, referring to Art. 21, the effective energy exerted by the bag, which has now returned to its original state, is

Fig. 6*a*.



represented by the area *P Q T S L*, being the difference between *P Q T M N*, the energy exerted *on* the piston, and *S N*, the energy exerted *by* the piston.

The figure thus drawn, representing the changes of volume and pressure of the bag, may be called the Diagram of Energy of a lb. of steam. If clearance and wire-drawing be neglected, the figure is identical with that drawn by an actual indicator, except that actual indicator diagram includes the very small rectangle *O P*, representing, as previously shown, the work done by the feed pump (Art. 17). The changes themselves are described as a Cycle of Operations or briefly a Cycle. In all heat engines the working fluid goes through a cycle of changes, and the nature of the engine depends chiefly on the nature of the cycle, which is represented graphically by its indicator diagram.

The effect of clearance and wire-drawing is that all particles of steam do not go through the same series of changes; some are never condensed at all; some are more and some less wire-drawn; there are, therefore, many distinct diagrams, each representing its own particle of steam. The figure drawn by the indicator may be regarded as representing the average changes undergone by the steam, as will be seen more clearly from the chapter (Chapter IX.) devoted to this part of the subject. In a compound engine the figures drawn by the indicator, when combined by a well-known process, employed in Fig. 6, p. 63, show the changes of state of the steam, care being taken that the figures combined are taken from corresponding ends of the two cylinders, so as to show the changes of the same mass of steam; but while the steam is passing into the low-pressure cylinder, before it is cut off in that cylinder, it must be remembered that the weight of steam in the low-pressure cylinder does not remain constant, nor does it usually vary in proportion to the volume swept out, as during admission to a simple cylinder.

Returning to the supposition that clearance is neglected, and, further, assuming the expansion complete, observe that the diagram may be separated into parts in two distinct ways.

(1) The useful work (U) done by the steam, as represented by the area $P\ Q\ R\ L$ of the diagram of energy, is the difference of the *effective* energy (W) exerted in the working cylinder, as shown by the indicator card $Z\ Q\ R\ V$, and the work (F) done in driving the feed-pump, represented by the rectangle $Z\ L$, which may be regarded as the card of the pump. This is expressed by the equation

$$U = W - F,$$

where the quantities W, F, have the same meaning as in the case of the Joule air engine described in the next chapter.

(2) Again, the useful work is the difference of the *total* energy (E) exerted in the working cylinder, represented by the area $Z\ Q\ R\ V$, and the work (C) done in overcoming back-pressure, as represented by the rectangle $V\ K$. That is,

$$U = E - C.$$

In this case, E is the whole energy exerted by the fluid during enlargement of volume, and C the whole work done during contraction of volume, as on p. 128. These symbols are also used in the next chapter with the same meaning.

CHAPTER IV.

AIR AND GAS ENGINES.

It has been shown in the last chapter that, in a steam engine, even when working under very favourable conditions, less than one-fifth of the whole heat expended is transformed into mechanical energy, and it is natural to inquire into the causes of this unfavourable result to discover, if possible, whether those causes are removable, either in the case of a steam engine or by the employment of some other kind of heat engine. A full answer to this question will be given in the next chapter, but a preliminary study is necessary of engines of a much simpler character, which employ air or some other permanent gas as a working agent. The present chapter will, therefore, be devoted to an outline of the theory of air and gas engines. The chapter is divided into four parts, commencing with—

Part I.—Physical Properties of the Permanent Gases.

31. The simplicity of air engines, from a theoretical point of view, lies in the fact that the constitution of air and other permanent gases is incomparably simpler than that of any other body in nature; all permanent gases (that is, gases which cannot easily be liquefied by cold and pressure) being subject approximately to certain very simple laws of which it will be necessary, before proceeding farther, to give a brief summary, referring for fuller information to Clerk Maxwell's '*Theory of Heat.*'

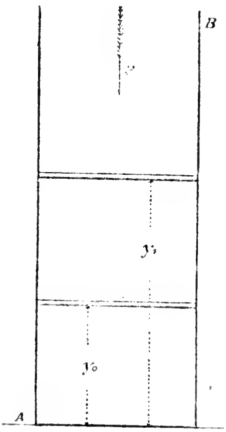
1. Boyle's Law.—*The product $P V$ of the pressure P and volume V is a constant quantity when the temperature remains constant.*

The value of this constant is known with great accuracy for different gases by the experiments of Regnault, for certain standard temperatures, and more especially at 32° Fahr., being the temperature of melting ice. For air, if P_0 be the standard pressure of the atmosphere, here taken as 14·7 lbs. per square inch, or 2116·8 lbs. on the square foot, V_0 the volume of 1 lb., found by direct experiment,

$$P_0 V_0 = 26{,}214 \text{ foot-lbs.}$$

For any other value of P the same value of $P V$ should be preserved, according to the first law, so long as the temperature remains that of melting ice, and in fact, though the deviations are perceptible, they are very small except at extreme pressures.

Fig. 7a.



If the temperature be altered, however, the value of this product will alter; two cases of which, amongst others, may be specially noticed.

(1) Let the air be enclosed in a cylinder $A B$ (Fig. 7a), beneath a loaded piston which originally, when the air is at 32°, stands at a height y_0 above the bottom. Now let heat be applied, the load on the piston remaining equivalent to a given constant pressure P, then the temperature of the air rises, and at the same time the air expands, causing the piston to rise. Suppose heat to be continually applied till the temperature is that at which water boils under the standard atmospheric pressure, and let the piston then have risen to the height y_1. Then clearly, since the pressure is the same, that is $P_1 = P_0$,

$$\frac{P_1 V_1}{P_0 V_0} = \frac{y_1}{y_0}.$$

(2) Let the air be enclosed in a vessel of invariable volume and let heat be applied to it, then the pressure increases, as heat is applied, and the temperature rises from 32° to 212°. Since the volume is constant we shall have in this case

$$\frac{P_1 V_1}{P_0 V_0} = \frac{P_1}{P_0},$$

so that if P_1 be determined by experiment the ratio of products will be known.

Hence the ratio of products may be determined by observing either the ratio P_1 / P_0 or the ratio y_1 / y_0, and if the first law were strictly true the result of the two experiments ought to be identical.

This is, in fact, very nearly the case, and, moreover, the result is found to be very approximately the same if any other permanent gas be used in place of atmospheric air—a fact included in

2. CHARLES'S LAW.—*Under constant pressure all gases expand alike.*

That is to say, in the first operation supposed above, if the temperatures, initially and finally, be any whatever, instead of 32° and 212°, the ratio y_1 / y_0, or, by the first law, the ratio of the products $P\,V$, will be the same for all gases.

It must be repeated that these two laws are only approximations for actual gases, but the deviations are so small that we are justified in regarding them as essential characteristics of a *perfectly* gaseous body, and the deviations may be considered as caused by actual gases being more or less imperfect. In an absolutely perfect gas, between the temperatures 32° and 212°, the ratio is about 1·3654, a value which is exceeded in actual gases by quantities, which in permanent gases are very minute, but are more considerable in liquefiable gases.

Since at different temperatures the product $P\,V$ has different values, it evidently may be taken as a measure of temperature. Temperature is ordinarily measured by a mercurial thermometer, that is, by the expansion of mercury in a glass tube, but there is no reason, except that of mere convenience, why this should be so; a bar of iron might also be used by observing the changes of length which take place when heat is applied. Now the choice of the product $P\,V$ has this advantage, which is at once obvious—namely, that the measurement is (by the second law) independent of the kind of gas employed; while in other thermometers—unless specially graduated by reference to a standard instrument—the indications are different for each different thermometer; hence, when measured by the expansion of a perfect gas, temperature is said to be "absolute," an expression which hereafter will be justified by much more cogent reasons (see Art. 60). Assuming temperature to be measured in this way, let t be a temperature reckoned on Fahrenheit's scale, then manifestly

$$P\,V = P_0\,V_0 + \frac{\cdot 3654}{180}(t - 32)\,.\,P_0\,V_0$$

$$= P_0\,V_0\,.\,\frac{180 + \cdot 3654\,(t - 32)}{180}$$

$$= \frac{P_0\,V_0}{492\cdot 6}(460\cdot 6 + t).$$

We can now make this simpler by measuring temperature, not from the purely arbitrary zero used by Fahrenheit, but from a zero 460·6° lower, so that if T be the absolute temperature,

$$T = 460{\cdot}6 + t,$$

$$\therefore\ P V = \frac{P_0 V_0}{492{\cdot}6} \cdot T.$$

The zero in question is called the "absolute zero," and according to certain experiments by Joule and Thomson, some account of which will be found in the Appendix, is 460·66 below the zero of Fahrenheit's scale; in all our subsequent work we shall adopt the nearest whole number—viz. 461, and write

$$P V = \frac{P_0 V_0}{493} \cdot T,$$

where T is measured from a zero 461° below the zero of Fahrenheit's scale. Rankine adopted 461·2, corresponding to 274° Centigrade, while writers on thermodynamics now usually adopt 273° Centigrade.

A thermometer of this kind gives results not differing materially at ordinary temperatures from those of a mercurial thermometer, a fact usually included, by implication, in statements of the second law. When great accuracy is required in the measurement of temperature, an air thermometer is actually used under one of the two forms mentioned above.

A perfect gas, then, is represented by the formula

$$P V = c\, T,$$

where c is a number depending on the density of the gas at 32°; for atmospheric air the value of c is about 53·2,

$$\therefore\ P V = 53{\cdot}2 \,.\, T$$

represents the properties of a perfect gas, the density of which at 32° is the same as that of air, and when in the course of the present and succeeding chapters "air" is mentioned, it is to be understood that such a perfect gas is intended, unless otherwise expressly mentioned. Actual air when perfectly dry differs from such a gas by very small quantities, but when containing moisture, as it always does in practice, the differences are more considerable.

3. REGNAULT'S LAW.—*The specific heat at constant pressure of a gas is the same at all temperatures.*

By "specific heat" is to be understood the amount of heat necessary to raise 1 lb. of the substance through 1°—temperature being measured by the expansion of the gas itself as explained above—a quantity which has different values according to the way the change of temperature is accomplished, depending on the amount of external work done. When the gas expands at constant pressure, as in the first process mentioned above, the value has been determined by Regnault with great accuracy for various gases, and the result agrees so closely with this third law, that it also may be regarded as an essential characteristic of a perfect gas; the numerical value for air is ·2375 thermal units, or 183·4 foot-lbs.

The heat expended in raising the temperature of air at constant pressure is partly expended in doing external work; for, let the original volume of the air be V_1, and the final volume V_2, then the work done in raising the piston is given by

$$\text{Work done} = P\,(V_2 - V_1) = c\,(T_2 - T_1).$$

Hence, since the heat expended is $K_p\ (T_2 - T_1)$ where K_p is the specific heat at constant pressure,

$$\text{Internal Work} = (K_p - c)\,(T_2 - T_1).$$

Thus the internal work is proportional to the change of temperature whenever the change takes place at constant pressure, and for air is 183·4 – 53·2 = 130·2 for each degree Fahrenheit. This result is, of course, in foot-lbs.; when changed into thermal units it becomes ·1686.

The process may very conveniently be represented graphically. Let $O\,N$ (Fig. 7*b*) represent the volume before, and $O\,M$ the volume after, heat has been applied, while $P\,N$ represents the constant pressure: then the rectangle $P\,M$ represents the external work done. Now set downwards $N\,Q$, such that

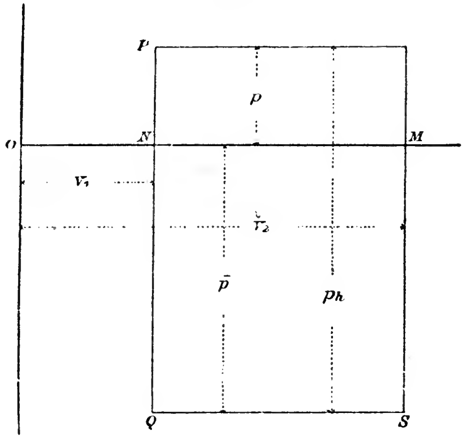
$$\frac{N\,Q}{P\,N} = \frac{\text{Internal Work}}{\text{External Work}}$$

$$= \frac{K_p - c}{c} = 2\cdot451 \text{ (for air)},$$

then the rectangle $N\,S$ represents the internal work, and the rectangle $P\,S$ the whole heat expended. As in previous cases, we may regard the heat expended in internal work as overcoming a pressure on the piston represented by NQ, while NQ and PN together represent the heat-pressure, hence

$$p_h = 3 \cdot 451\ p.$$

Fig. 7b.



4. JOULE'S LAW.—*If a gas expand without doing external work its temperature is unaltered.*

When air expands, overcoming the resistance of a piston, its temperature falls, as appears by experience; the kind of expansion indicated here is, however, different. Let two vessels be provided, one of which is empty, and the other contains air of any pressure: let the vessels be connected by a pipe provided with a stop-cock which, when opened, allows the air to flow from one vessel to the other; then, at first, the expansive energy of the air is employed in generating violent motions in the air, which, however, quickly subside from fluid friction when equilibrium of pressure has been attained, after which, if the temperature be observed before any heat is lost by radiation, it will be found sensibly the same as before the cock was opened.

Now the difference between this case and the first case supposed is, that no external work of any kind is done, so that the air has lost no energy, and the result therefore shows that the intrinsic energy (Art. 15) possessed by the air is always the same at the same temperature, whatever be its pressure, a fact which enables us at once to show that however air changes its state, the internal work done must be proportional to the change of temperature.

For let 1 lb. of air expand in the way supposed from pressure P_1 volume V_1, till its volume is V_2; its temperature by hypothesis remains the same (T_1, say) and its pressure is therefore $\frac{V_1}{V_2} \cdot P_1 = P_2$. Let it now expand at this constant pressure until it reaches any other volume, and its temperature has risen to T_2; heat must be added to render this possible, and the internal work done has been shown to be $(K_p - c)(T_2 - T_1)$, but during the free expansion no internal work is done, therefore the air has been changed from pressure P_1, temperature T_1, to pressure P_2, temperature T_2, with an expenditure of heat in internal work of $(K_p - c)(T_2 - T_1)$ which depends solely on the difference of temperature. Hence, by Art. 14, the internal work done in changing the temperature of air is always given by

$$\text{Internal work} = (K_p - c)(T_2 - T_1).$$

This reasoning applies directly to the case where P_2 is less than P_1, and by a slight modification also when P_1 is less than P_2.

Now, in the second method of changing the product $P\,V$ mentioned above (p. 72), the volume of the air remains constant, so that no external work is done; and we learn that when air is heated at constant volume the expenditure of heat for 1° rise of temperature is $K_p - c$, a constant quantity. Hence the specific heat at constant volume is, like the specific heat at constant pressure, a constant quantity, and moreover, if it be K_v, we have

$$K_p - K_v = c,$$

a relation which is a necessary consequence of the fourth experimental result. In the case of air, from the values given above, we find

$$K_v = 183{\cdot}4 - 53{\cdot}2 = 130{\cdot}2 \text{ foot-lbs.} = {\cdot}1686 \text{ thermal units.}$$

Now, if we could make experiments on the specific heat of air at constant volume, we should be able to verify this result. This has not yet been done directly, but we can obtain a value indirectly by means of the knowledge we possess of the ratio K_p / K_v, a number on which the velocity of sound depends, irrespectively of any special theory of heat, and which for air and some other simple gases has been shown to be about 1·408, whence

$$K_v = \frac{183 \cdot 4}{1 \cdot 408} = 130 \cdot 25,$$

agreeing closely with the previous calculation. Thus we might have assumed as our fourth experimental result the equation

$$K_p - K_v = c,$$

from whence can be shown without difficulty that the temperature is unaltered by free expansion. The ratio just mentioned is commonly denoted by γ, and this equation shows that, for gases which have the same value of γ, the specific heat at constant pressure is inversely proportional to the density; a result which has been experimentally verified.

The fourth experimental result is not to be regarded as exact for actual gases any more than the three others, but merely as an approximation so close that we are justified in regarding it as another essential characteristic of a perfectly gaseous body.

The value of γ for air is believed to be slightly greater than 1·4, and is frequently assumed, as above, to be 1·408, but it is not known with minute accuracy, and in calculations may be taken as 1·4. As stated before, air in practice always contains moisture, and this circumstance alone renders absolute accuracy impossible.

Completely Superheated Steam.

32. Saturated steam is not a perfect gas, as is sufficiently shown by the fifth column of Table IV*a*, which shows the differences $\Delta P u$ which are roughly approximately the same as $\Delta P v$. These differences would be constant if the steam followed the perfectly gaseous laws, whereas they actually diminish very considerably as the temperature rises. There is, however, little doubt that, when sufficiently superheated, steam becomes sensibly a perfect gas, and it may then conveniently be said to be "completely" superheated. It then follows the law expressed by

$$P V = c T,$$

like atmospheric air, but with a different value of the constant c. As to the amount of superheating necessary, much uncertainty exists; but there can be no doubt that the higher the temperature of saturated steam the greater is the rise of temperature needful to produce from it completely superheated steam. In the intermediate state in which steam is neither saturated nor a perfect gas, its properties are not fully known, but such experiments as exist show that its rate of expansion is much greater than that of a perfect gas near the saturation point, but quickly diminishes as the superheating progresses. In order to deal with the steam in this state, hypotheses not fully warranted by experiment are necessary.

The density of steam as calculated from its chemical composition is ·622, nearly, that is, a cubic foot of steam should weigh this fraction of the weight of a cubic foot of air at the same pressure and temperature (see *Dixon's Treatise on Heat*, p. 187), whence

$$c = \frac{53 \cdot 2}{\cdot 622} = 85 \cdot 5,$$

so that the theoretical equation for completely superheated steam is

$$P V = 85 \cdot 5\, T.$$

The density of dry saturated steam at low temperatures has not hitherto been determined in a satisfactory way, but the results usually given indicate that, below 104°, saturated steam is sensibly a perfect gas of density (·63) rather greater than the theoretical value. The theoretical values of $P V$ are given for every 27° from 104° to 431° in Table IVa, for the sake of comparison with the same values for saturated steam, which, it will be seen, are smaller, and at high temperatures considerably smaller.

The value of K_p is generally taken at ·48 in thermal units, or in foot-lbs. 370·56, whence $K_v = 285 \cdot 03$ and $\gamma = 1 \cdot 3$.

Thermodynamics of a Perfect Gas.

33. From the four experimental results expressing the physical properties of a perfect gas, it is possible to give a complete theory of the action of heat in such a body. For in the most general case of the action of heat in a body we found in Chapter II. that

Heat Expended = Internal Work + External Work;

and further it was shown above that the internal work done is always $K_v(T_2 - T_1)$ where T_1 T_2 are the temperature at the beginning and end of the operation.

$$\therefore \text{ Heat Expended} = K_v(T_2 - T_1) + \text{External Work.}$$
$$= \frac{1}{\gamma - 1}(P_2 V_2 - P_1 V_1) + \text{External Work,}$$

an equation from which all questions can be answered about the action of heat in a perfect gas.

First. Suppose the temperature constant, then the air expands, when heat is added, according to the hyperbolic law, and the internal work done is zero, so that the whole of the heat expended is employed in doing external work, and flows out of the air in this shape as fast as it enters. If the ratio of expansion be r, it was shown in Chapter III. that the work done during hyperbolic expansion is $P V . \log_\epsilon r$; hence, if Q be the heat expended

$$Q = P V . \log_\epsilon r = c T \log_\epsilon r,$$

where T is the constant absolute temperature of the air, and the same equation serves for any gas by substitution of the proper value of c. For air, $c = 53 \cdot 2$ also $\log_\epsilon r = 2 \cdot 302 \log_{10} r$.

$$\therefore Q = 122 \cdot 5 . T \log r \text{ foot-lbs.}$$
$$= \cdot 1584 . T \log r \text{ thermal units,}$$

where the logarithm is now common.

Expansion at constant temperature is called "isothermal" expansion, and the curve is the isothermal curve which for perfect gases is therefore an hyperbola.

Secondly. Let the expansion of the air take place according to the law expressed by the equation

$$P V^n = \text{const.}$$

Curves of this kind occur constantly in the theory of the steam engine; thus the relation between the pressure and volume of saturated steam is expressed very approximately by such an equation in which $n = 1 \cdot 0646$, and we shall have numerous instances hereafter of such curves, with various values of n. Their general appearance is that of an hyperbola, which is indeed a particular case in which

$n = 1$. In the Appendix it is shown that the area included between two ordinates (Fig. 8, page 84), $A\,N$, $B\,M$, the curve $A\,P\,B$ and the base $N\,M$, is given by the equation,

$$\text{Area} = \frac{P_1 V_1 - P_2 V_2}{n - 1},$$

a rule which fails when $n = 1$, when we must resort to that previously given for the case of the hyperbola. In any other case then we have

$$\text{External Work} = \frac{P_1 V_1 - P_2 V_2}{n - 1} = \frac{c}{n - 1}(T_1 - T_2),$$

so that in this law of expansion the external work done is proportional to the change of temperature, which is a rise when n is less than unity, and a fall when n is greater than unity. Placing this value in the equation for Q the heat expended,

$$Q = K_v (T_2 - T_1) + \frac{c}{n - 1}(T_1 - T_2)$$

$$= \left(K_v - \frac{K_p - K_v}{n - 1}\right)(T_2 - T_1)$$

$$= \frac{n\,K_v - K_p}{n - 1} \cdot (T_2 - T_1),$$

which shows that the heat expended is also proportional to the change of temperature, or, in other words, that the specific heat (K) is a constant, given by

$$K = \frac{n\,K_v - K_p}{n - 1} = K_v \cdot \frac{n - \gamma}{n - 1}.$$

When $n = \gamma$ we have a very important case, for then the heat expended is zero, showing that when a gas expands without either gaining heat or losing heat the expansion curve is given by

$$P\,V^{\gamma} = \text{const.},$$

a curve which is called the *adiabatic* curve, and the expansion is said to be *adiabatic* expansion. In this case the external work is done at the expense of the internal energy stored up in the gas, therefore the temperature falls from T_1 to T_2, say, where the ratio $T_2 : T_1$ is readily found from the equations

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \quad : \quad P_1 V_1 = c\,T_1 \quad : \quad P_2 V_2 = c\,T_2,$$

whence if $r = \frac{V_2}{V_1}$ be the ratio of expansion,

$$T_2 = T_1 . \left(\frac{1}{r}\right)^{\gamma - 1.}$$

The accompanying table shows how the pressure and temperature fall during an expansion to double its volume of air, the initial pressure of which is 100 lbs. per square inch, and temperature 539° Fahr.; the corresponding pressures during hyperbolic expansion being given for comparison :

ADIABATIC EXPANSION OF DRY AIR.

Temperature.		Pressure.		Ratio of Expansion.
Ordinary.	Absolute.	Adiabatic.	Isothermal.	
539	1000	100·0	100·0	1·
501	962	87·5	90·9	1·1
467	928	77·3	83·3	1·2
437	898	69·1	76·9	1·3
410	871	62·2	71·4	1·4
386	847	56·5	66·7	1·5
363	824	51·5	62·5	1·6
343	804	47·3	58·8	1·7
326	787	43·7	55·6	1·8
307	768	40·4	52·6	1·9
293	754	37·7	50·	2·0

Remark.—In calculating this table the value of γ has been supposed 1·408.

The table applies to any other initial temperature and pressure by multiplication by the initial absolute temperature, and dividing by 1000 for the absolute temperature and by multiplying by the initial pressure and dividing by 100 for the pressure. To become familiar with the curve the reader should construct it to scale for some convenient initial pressure and temperature, obtaining the initial volume from the formula

$$P_1 V_1 = c\, T_1.$$

The table shows in a striking manner the rapid rate at which the temperature falls; the fall being as much as 246° in the moderate expansion of 2 : 1.

It must, however, be understood that the air is supposed absolutely dry, a case which only occurs when it is artificially dried by heat or by the use of chloride of calcium or other substance with a strong affinity for moisture. When air in its ordinary condition expands, first the dew-point and then the snow-point is speedily reached, and the heat given out prevents the temperature from falling to anything like the same extent.

The work done during the expansion is found by multiplying the fall of temperature by K_v. By use of a table of squares the results given can be extended to higher rates of expansion. The curve can also be constructed graphically, by a modification of the process employed further on (Art. 35).

34. Returning to the general case where n has any value, we have

$$\text{External Work} = \frac{c}{n-1} \cdot (T_1 - T_2) = \frac{K_p - K_v}{1-n} \cdot (T_2 - T_1),$$

$$\text{Internal Work} = K_v (T_2 - T_1),$$

therefore we see that the internal work always bears a fixed proportion to the external work, namely, $1 - n : \gamma - 1$, a result which is represented graphically in Figs. 8*a* and 8*b* (see next page).

In Figs. 8*a*, 8*b*, $A\,B$ represents the expansion curve, which when $n > 1$ (Fig. 8*b*) falls below, and when $n < 1$ (Fig. 8*a*) falls above, the hyperbola through A, represented in both figures by the dotted line $A\,B_0$; $P\,P^1$ are two points close together, the ordinates of which are $P\,S$, $P^1\,S^1$; then, as has been previously shown, the external work done during expansion from S to S^1 is represented by the area of the strip $P\,S^1$.

Taking first the case (Fig. 8*a*) in which the curve falls above the hyperbola, it is clear that in this case the temperature rises as the expansion proceeds, and internal work must be done in order to produce this rise of temperature. Set, therefore, downwards $S\,Q$ such that

$$\frac{S\,Q}{S\,P} = \frac{\text{Internal Work}}{\text{External Work}} = \frac{1 - n}{\gamma - 1},$$

and carry out the same construction at every step of the expansion; a curve $C\,Q\,D$ is thus obtained, the area of a strip $S\,Q^1$ of which

Fig. 8*a* ($n < 1$).

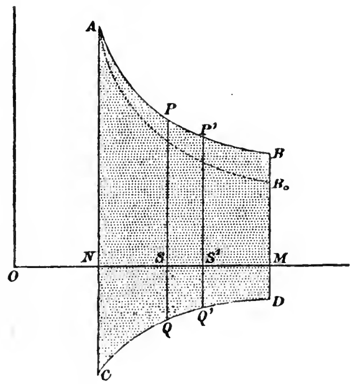
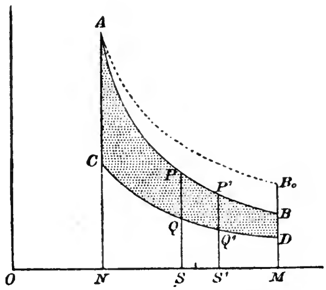


Fig. 8*b* ($n > 1$).



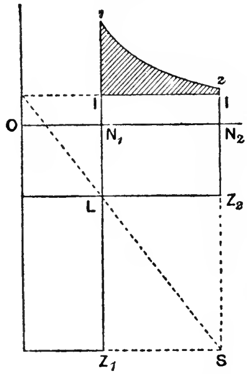
represents the internal work done during expansion from S to S^1 on the same scale that the strip $S\,P^1$ of the expansion curve represents the external work, and the internal work done may be represented as overcoming at every instant a pressure on the piston, represented by $S\,Q$, which may be called the Internal-Work-Pressure. The heat expended is the sum of the internal work and the external work, and is represented by the area of both curves shaded in the figures; the pressure (p_h) equivalent to it is clearly represented by $S\,Q + S\,P$, and we have, if the external pressure be p,

$$p_h = p + p \cdot \frac{1 - n}{\gamma - 1} = p \cdot \frac{\gamma - n}{\gamma - 1}.$$

In the second case when $n > 1$ (Fig. 8*b*) the temperature falls and part of the external work is done at the expense of the intrinsic energy of the expanding air; we must then set off $S\,Q$ upwards instead of downwards, and the heat expended is shown by the difference of area of the curves shaded in the figure. If $n = \gamma$ the two curves coincide, no heat being added or taken away; if $n > \gamma$, the expansion curve falls below the adiabatic curve, in which case heat must be taken away from the air as it expands, and this would be shown on the figure by the curve of internal work $C\,Q\,D$ lying above the expansion curve $A\,B$.

When gas is compressed, its pressure increases, and the relation between volume and pressure, represented graphically by a compression curve, depends, as in the case of expansion, solely on the mode in which heat is added or subtracted. If the circumstances of the compression are the same as those of the expansion, the compression curve will be identical with the expansion curve, but otherwise not. Thus, when a gas is compressed without the addition or subtraction of heat, the pressure and temperature rise according to the law expressed by the table given above for adiabatic expansion. The work done in compressing the gas is then proportional to the rise of temperature. But if the rise of temperature be prevented by the continual abstraction of heat as the compression progresses, the compression curve will be an hyperbola, and the energy exerted in compressing the gas will be equivalent to the heat abstracted, and will be given by the same formulæ as in isothermal expansion.

Fig. 9.



35. Another method of exhibiting graphically the relation between internal work, external work, and heat supply, is shown in Fig. 9, which applies in any case whatever.

The points 1, 2 in this diagram show the pressure and volume of a lb. of air at the beginning and end of any change of state. On the base $O\,N_1$ construct a rectangle $O\,Z_1$ below the volume axis, the height of which is $2 \cdot 45$ times the pressure at 1; then the area of that rectangle is equal to $2 \cdot 45\ P_1\,V_1$, and therefore to $K_v\,T_1$, that is to I_1, the internal energy of the air at 1 reckoned from the absolute zero. A similar rectangle $O\,Z_2$ represents I_2 the internal energy at Z. Now complete the rectangle $Z_1\,Z_2$ and draw the other diagonal $S\,L$, producing it to meet the pressure axis, and through the intersection draw the horizontal $I\,I$. Then it is easily proved that the rectangle $N\,I$ represents the difference $I_1 - I_2$ of the rectangles $O Z_1$, $O Z_2$, and whatever the form of the expansion curve, the area (shaded in the diagram) between it and the horizontal line $I I$ must represent the heat supply. We shall have occasion to use this construction hereafter in the case of steam.

In the construction of diagrams exhibiting graphically the changes of volume and pressure of an elastic fluid, the method of logarithmic plotting may also sometimes be adopted with advantage. On this system, instead of setting off the volumes and pressures themselves as horizontal and vertical ordinates, the logarithms of these quantities are employed, and consequently all curves of the form $P V^n$ = constant become straight lines the tangent of the inclination of which to the axis is the index n. Moreover, in permanent gases subject to the law $P V = c T$ the sum of the logarithms of the pressure and volume, after subtracting a constant, is equal to the logarithm of the absolute temperature, so that the temperature at any point is at once found by adding the ordinates of the point. In particular a line inclined at 45° to the axis is an isothermal, and at the angle $\tan^{-1} 1 \cdot 4$, is an adiabatic, line.

Fig. 10.

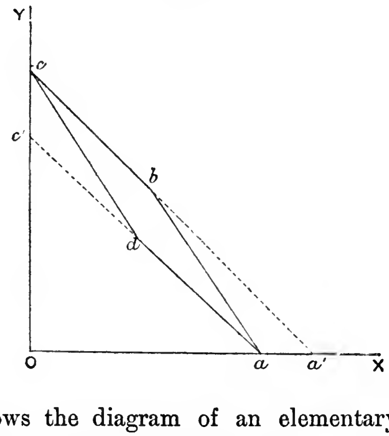


Fig. 10 shows the diagram of an elementary perfect engine (Art. 36) plotted on this system. The lines $c\,b\,a'$, $a\,d\,c'$ represent the isothermals corresponding to the temperatures T_1 T_3 of the hot and cold bodies respectively while the lines $a\,b$, $c\,d$ represent the adiabatic expansion and compression of the air. The indicator diagram is simply the parallelogram *a b c d*, in which the difference of the horizontal ordinates of *b* and *c*, or of *a* and *d*, gives the logarithm of the ratio of isothermal expansion, and similarly the logarithm of the ratio of adiabatic expansion may be found. The intercepts *a a′* or *c c′* give the logarithm of the temperature-ratio of the corresponding isothermals.

In logarithmic diagrams the area of the diagram has no physical meaning, and the work done by the fluid is not therefore directly represented. This work done can, however, always be derived from quantities given by the diagram, and this is especially easy in the permanent gases, because in all cases where the expansion follows the law $P V^n$ = constant it depends directly on the temperatures of the fluid at the beginning and end of the expansion.

It is further of importance to remark that in all cases of the expansion of a gas, the internal work, external work, and heat-supply depend on the volume, not on the weight of fluid considered. This is evident from the graphical construction given above, and so far as regards external work, has already been pointed out on page 45. Thus in the formula

$$\text{External Work} = \frac{P_1 V_1 - P_2 V_2}{n - 1}$$

employed in the last article the volumes V_1 V_2 are supposed in the first instance the volumes of 1 lb. of gas; but this restriction is unnecessary, they may be the volumes of any other quantity. Let, for example, a cubic foot of gas expand adiabatically to an indefinite extent, the work done is $2 \cdot 5\ P_1$. This is also the absolute intrinsic energy per cubic foot of a quantity of air. At the atmospheric pressure the amount is about 2½ foot-tons.

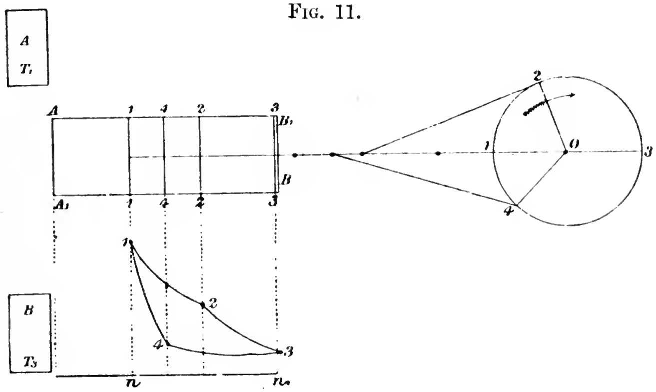
PART II.—AIR ENGINES.

We now proceed to consider some simple thermo-dynamic machines which employ air or some other approximately perfect gas as their working agent. Such machines may be heat engines the object of which is to convert heat-energy into mechanical energy, but are also used for the converse purpose, as will be seen in the fourth division of this chapter. The term "thermo-dynamic" machine is a general one, employed when it is unnecessary or inconvenient to specify the purpose for which it is employed.

36. The simplest form of heat engine is constructed as follows :—

A B (Fig. 11) is a working cylinder containing a given quantity, say 1 lb., of air or some other perfect gas, always included between the cylinder cover *A A'* and a piston, successive positions of which are represented in the figure by 1 1, 2 2, 3 3, 4 4, and which may be supposed connected in the usual way with a crank, corresponding positions of which are $O_1\ O_2\ O_3\ O_4$. The right-hand portion of the cylinder between the cover *B B'* and the piston is empty.

FIG. 11.



A and *B* are two bodies of temperatures (absolute) T_1 and T_3, capable of communicating to, or abstracting from, any body placed in contact with them, indefinite amounts of heat.

Suppose the crank moving in the direction of the arrow, and, initially, let it be in the position O_1, and let the pressure volume and temperature of the air be P_1, V_1, T_1, respectively. Then as the crank rotates the volume of the air increases, and if no heat were applied to it the temperature would fall; but this is prevented by placing the body *A* in contact with the cylinder, which is to be supposed a perfect conductor, so that the slightest depression of the temperature of the air below T_1 causes heat to flow from *A* into the air so as to keep the temperature of the air constantly at T_1 so long as *A* is in contact. During this first operation, then, the air expands at constant temperature, and the expansion curve 1 2 in the indicator diagram below is a common hyperbola.

The expansion having reached some convenient point, 2, the body *A* is to be removed from the cylinder, so that no more heat is received by the air; the temperature then falls instead of remaining constant, and the expansion curve 2 3 of the indicator diagram is now an adiabatic curve given by $P V^{\gamma}$ = constant. This goes on until the piston has reached the end of its stroke and begins to return, so as to compress the air again and raise its temperature at the same rate at which it fell during expansion.

In order to prevent this rise the body B is applied, the temperature of which is T_3, the temperature of the air at the end of the stroke, and which abstracts heat from the air the instant its temperature rises above T_3. The effect of this is that the air is compressed at constant temperature T_3, and the compression curve 3 4 on the diagram is consequently a common hyperbola.

This compression goes on till the piston reaches the position 4, depending on 2 in a manner to be explained presently, when the body B is removed and the temperature allowed to rise. The air is then compressed without gain or loss of heat, and the compression curve 4 1 on the diagram is consequently an adiabatic curve. If the point 4 has been properly chosen, the air at the end of the return stroke of the piston will have the pressure (P_1) the volume (V_1) and the temperature T_1 with which the operations commenced. The same cycle may now be repeated as often as we please.

The position of the point 4 is readily determined thus, let $P_2 V_2$, $P_3 V_3$, $P_4 V_4$, be the pressure and volume of the air at the points 2, 3, 4 then since 1 2, 3 4 are common hyperbolas,

$$\frac{P_1}{P_2} = \frac{V_2}{V_1} \text{ and } \frac{P_3}{P_4} = \frac{V_4}{V_3},$$

and since 2 3, 1 4 are adiabatics,

$$\frac{P_2}{P_3} = \left(\frac{V_3}{V_2}\right)^{\gamma} \text{ and } \frac{P_4}{P_1} = \left(\frac{V_1}{V_4}\right)^{\gamma}.$$

Hence multiplying all four equations together

$$V_1 V_3 = V_2 V_4,$$

from which we find

$$\frac{V_3}{V_2} = \frac{V_4}{V_1} \text{ and } \frac{V_2}{V_1} = \frac{V_3}{V_4}.$$

The first of these equations shows that the ratio of adiabatic compression is equal to the ratio of adiabatic expansion. This common ratio of expansion and compression will be denoted by r', it depends solely on the temperatures of A and B being given, as is easily seen by

$$r' = \left(\frac{T_1}{T_3}\right)^{\frac{1}{\gamma - 1}}.$$

The second equation shows that the ratios of expansion and compression at constant temperature are also equal. This common ratio of isothermal expansion and compression will be denoted by r, it is given by

$$r = \frac{V_2}{V_1} = \frac{V_2}{V_3} \cdot \frac{V_3}{V_1} = \left(\frac{T_3}{T_1}\right)^{\frac{1}{\gamma - 1}} \cdot \frac{T_3}{T_1} \cdot \frac{P_1}{P_3}$$

$$= \frac{P_1}{P_3} \cdot \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{\gamma - 1}} = \frac{V_3}{V_1} \cdot \left(\frac{T_3}{T_1}\right)^{\frac{1}{\gamma - 1}}.$$

This ratio therefore depends on the ratio of greatest and least pressures in the cylinder, or on the total expansion employed. Unless one of these quantities be given it is arbitrary.

Let us now examine the manner in which this engine does work and the amount of heat it expends.

(1) During the operation 1 2 the air is expanding at constant temperature and is exerting energy on the working piston. The amount of energy exerted according to the last article is $c\,T_1 \log_\epsilon r$. During the operation 2 3 the air is expanding adiabatically and the energy exerted is (page 83) equal to $K_v(T_1 - T_3)$ the loss of intrinsic energy consequent on the fall of temperature. Thus the whole energy exerted during the forward stroke of the piston is

$$E = c\,T_1 \log_\epsilon r + K_v (T_1 - T_3).$$

During the backward stroke the piston compresses the air by means of energy supplied by a fly-wheel or other external agency. Reasoning as before, the whole work done is

$$C = c\,T_3 \log_\epsilon r + K_v (T_1 - T_3)$$

The difference between E and C represents an excess of energy which may be employed to do useful work given by

$$U = E - C = c\,(T_1 - T_3) \log_\epsilon r.$$

Graphically the area 1 2 3 n n in the diagram represents E and the area 1 4 3 n n represents C, while the difference, namely the area of the diagram 1 2 3 4, represents U, the useful work done.

(2) Again, during the operation 1 2, the air receives heat from A at constant temperature T_1 of amount equal to the energy exerted (page 80), but during the operation 2 3 receives no heat. Hence the whole heat received in the forward stroke is

$$Q = c\,T_1 \log_\epsilon r.$$

During the operation 3 4 heat is abstracted by the body B at constant temperature T_3, but during the operation 4 1 no heat is abstracted. Hence the whole heat rejected by the air during the backward stroke is

$$R = c\,T_3 \log_\epsilon r.$$

The difference between these two quantities is the heat which disappears during the cycle of operations represented by the double stroke of the working piston. This is given by

$$U = Q - R = c(T_1 - T_3) \log_\epsilon r,$$

being equal to the useful work done as found above. Evidently this ought to be the case according to the principles explained in a preceding chapter. (See page 34).

The efficiency of the engine is now, as before (page 40), found by comparing the useful work and heat expended, that is

$$\text{Efficiency} = \frac{U}{Q} = \frac{T_1 - T_3}{T_1},$$

This simple rule, moreover, must be true not only for the ideal engine which we have chosen for consideration precisely on account of its simplicity, but also for any engine, however complex, working with air or any other perfect gas, which receives and rejects heat in the same way. For it was shown in Art. 28 that the work done by a given quantity of expanding fluid is not dependent on the particular machinery by which its expansive energy is utilised, but solely on the law of expansion and the degree of expansion, which again depend only on the way in which the fluid receives heat. In the present case, the fluid expands partly at constant temperature, and partly adiabatically, the ratios of expansion being fixed by the total expansion admissible, and by the ratio of absolute temperatures. If these be given, the power and efficiency of the engine will be the same whatever its construction, if no part of the expansion takes place according to any other law. But, as in the article cited, this statement is subject to some qualification when a part of the expansion is partly or wholly unbalanced.

We are here introduced for the first time to a conception which it is the principal object of the present and next following chapters to explain and illustrate, namely, the conception that the power and efficiency of heat engines depend on the temperature of a hot body or source of heat from which heat is derived, and a cold body or refrigerator into which it passes; the efficiency being greater the greater the difference of these temperatures.

As an example, let us suppose that the temperature of the source of heat is 660° Fahr. and that of the refrigerator 32° Fahr., then

$$\text{Efficiency} = \frac{660 - 32}{660 + 461} = \cdot 56.$$

It is then ideally possible to transform 56 per cent. of the heat expended into mechanical energy, and if a greater range of temperature were available the efficiency might be still greater. We shall show that no engine whatever can have a greater efficiency if the temperatures remain the same, and in anticipation of this conclusion we describe an air engine receiving and rejecting heat in this way as a PERFECT Air Engine.

37. Returning now to the first method of calculating the useful work by taking the difference between the energy exerted in the forward stroke and the work done in the backward, we observe that the first of these quantities given by

$$E = cT_1 \log_\epsilon r + K_v (T_1 - T_3)$$

is the sum of two parts, the first of which is the heat expended. The whole, therefore, is greater, and generally much greater, than the heat expended; the excess being derived from a store of energy $K_v(T_1 - T_3)$ accumulated in the air by compression at the end of the return stroke. Hence the useful work is the difference of two large quantities and forms, but a small fraction of the energy originally exerted on the working piston. An air engine of this type is consequently a very bulky machine unless the pressures employed are exceedingly great, much greater than are at all convenient in practice. It is easy to verify this assertion by calculating the maximum value which the mean effective pressure could bear to the maximum pressure when the temperatures are given, but it will be sufficient to refer to the annexed diagram (Fig. 12),

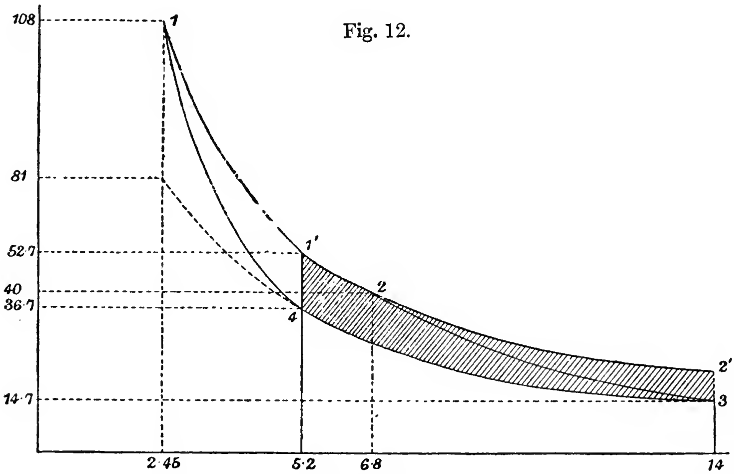


Fig. 12.

and to a table of numerical results to be given presently. In Fig. 12 the indicator diagram is drawn to scale for the very moderate temperature-ratio 1·333 and isothermal ratio of expansion 2·7. The numbers on the diagram give the volumes and pressures at different positions of the piston. The mean effective pressure will be found to be 6 lbs. per square inch, and the maximum pressure 108 lbs. per square inch absolute, when the minimum pressure is that of the atmosphere. The efficiency in this case is only ·25, and if a wider range of temperature be used to obtain a greater value, the maximum increases much faster than the mean effective, so that their ratio is much smaller

Let us now alter the working of the engine by supposing that when the piston reaches the point 4 in the return stroke and is just about to commence adiabatic compression that it is there stopped, and the hot body *A* at once applied. The temperature then rises immediately from T_3 to T_1 while the volume of the air remains constant. This operation is represented by the vertical line 4 1′ in Fig. 12. The crank radius having been previously altered so as to shorten the stroke, let the piston now make its working stroke, starting from 1′ instead of 1, and let the hot body A be in contact throughout the whole stroke instead of being taken away at 2. The expansion curve is now the hyperbola 1′ 2 2′ and the energy exerted on the piston is

$$E = c\,T_1 \log_\epsilon r.$$

At the end of the stroke the cold body B is applied and the temperature falls suddenly from T_1 to T_3, while the volume of the air remains constant. This operation is represented by the vertical line 2′ 3 in the diagram. The piston next makes its return stroke, the cold body B remaining in contact throughout. The hyperbolic compression curve 3 4 is then described and the work done is

$$C = c\,.\,T_3 \log_\epsilon r.$$

The piston has now reached the point 4 at which the cycle commenced.*

The indicator diagram is 1′ 2′ 3 4, and its area, shaded in the figure, is

$$U = E - C = c\,(T_1 - T_3) \log_\epsilon r.$$

This quantity is the useful work done by the engine, which remains the same notwithstanding a great reduction in the maximum pressure and an increase in the mean effective. The ratio

$$\frac{U}{E} = \frac{T_1 - T_3}{T_1}$$

is the same as the efficiency in the previous arrangement. For given limits of pressure the engine is much less bulky.

38. Next, considering the heat expended, it is clear that the heat drawn from A in changing the temperature of the air at 4 from T_3 to T_1 is $K_v\,(T_1 - T_3)$ while the heat expended during expansion remains the same as before, we therefore have

$$Q = c\,T_1 \log_\epsilon r + K_v\,(T_1 - T_3).$$

* In Fig. 12 the pressure at 4 is inadvertently stated as 36·7 ; it should be 39·7, being very slightly less than that at 2.

And the heat passing to B at $2'$ when the temperature falls from T_1 to T_3 is $K_v(T_1 - T_3)$ while the heat rejected during compression remains unaltered. We have then

$$R = cT_3 \log_\epsilon r + K_v(T_1 - T_3).$$

Thus the heat changed into work is

$$U = Q - R = c(T_1 - T_3)\log_\epsilon r,$$

agreeing with the result found above, but the heat expended is increased by the quantity $K_v(T_1 - T_3)$ which passes into B when the temperature is lowered and has to be replaced from A when the temperature is raised. The efficiency is in this way greatly reduced, becoming

$$\frac{U}{Q} = \frac{c(T_1 - T_3)\log_\epsilon r}{cT_1 \log_\epsilon r + K_v(T_1 - T_3)}$$

which is the same as the value of U/E in the original arrangement.

These points are illustrated by the table (p. 96) of numerical results, showing the performance of the two engines, with two different temperature-ratios. The ratio of isothermal expansion chosen is that which gives the greatest mean effective pressure (referred to the total volume of cylinder) for a given temperature-ratio and maximum pressure. The two engines may be described as "elementary," and for brevity are referred to in what follows as *Type P* and *Type S*.

It will be seen that in *Type S* the maximum pressure is reduced from 450 to 80 in the first case, and from 108 to 53 in the second case, while the mean effective pressure is considerably increased. But this advantage is only obtained at the sacrifice of efficiency, which is reduced from ·5 to ·222 or from ·25 to ·154. The next article will show how this loss is in a great measure avoided.

39. In the primitive form of heat engine we have been studying, the changes of temperature of the air are supposed produced by external application of bodies of different temperatures to the whole cylinder containing the air. This operation could hardly be carried out in practice, and if it were, the process would be very wasteful. The necessary changes can, however, be carried out as follows :—

In Fig. 13, *A A B B* is a closed cylindrical chamber within which a piston *D* moves, fitting easily in a cylinder of smaller diameter.

The annular space between the two cylinders is fitted with a large number of layers of wire gauze, near each other but not in contact, while the piston is of considerable depth, and filled internally with non-conducting material. The effect of this is that the end *A* of the

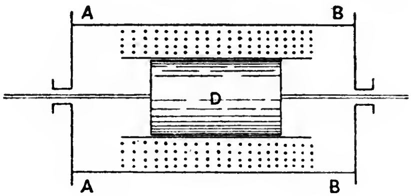
PERFORMANCE OF AN ELEMENTARY AIR ENGINE.

Temperatures.	Type.	Pressures.		Efficiency.	Expenditure of Heat, Therm. Units per I.H.P. per 1'.	Percentage of Energy exerted in		Ratios of Expansion.	
		Highest	Mean Effective.			Useful Work.	Compression.	Adiabatic.	Isothermal.
Highest = 525° F. Lowest = 32° F.	P	450	15·6	·5	85·5	22·2	77·8	5·66	2·7
T_1 = 986 T_3 = 493 Ratio = 2	S	80	23·3	·222	193	50	50		2·7
Highest = 287° F. Lowest = 100° F.	P	108	6	·25	171	15·3	84·7	2·05	2·7
T_1 = 748 T_3 = 561 Ratio = 1·333	S	53	8	·154	277	25	75		2·7

Remarks.—Type *P* receives and rejects heat at constant temperature only. Type *S* changes temperature of air at constant volume, as in a Stirling engine, but without regenerator. Minimum pressure in cylinder 14·7 lbs. per square inch.

cylinder may be kept hot by a furnace or other source of heat, while the end *B* is kept cold by a coil of piping through which a stream of cold water flows, or otherwise. Heat will flow by conduction from *A* to *B*, but the rate of flow will be very small, and may be neglected for the present. In the figure, the piston *D* is shown at the middle of its stroke; if it now be moved to the end *A*, air will pass through the gratings to the end *B*. The air in *A* has the temperature T_1, of *A*, but as it passes each grating in succession it parts with some of its heat, till it reaches the end *B*, when its temperature has fallen to T_3 the temperature of B. The successive gratings will have temperatures gradually diminishing by equal decrements from T_1 to T_3. Next move D back from A to B; nearly all the air in B will pass into A, and in passing each grating will take up heat,

Fig. 13.



so that its temperature will rise from T_3 to T_1. The gratings of gauze are called a Regenerator, and they play the part (1) of a heat valve or trap, obstructing the passage of heat from A to B, while permitting air to pass freely; and (2) of a store of heat which supplies heat to the air when the piston D is moved one way, and is replenished when it returns. Since the volume of air remains the same, its pressure increases and diminishes in proportion to the temperature, so that when the air is in B the pressure is low, and when in A the pressure is high. The piston D requires little or no force to move it, because the pressure in the chamber is everywhere nearly the same if the motion is not too rapid, and the only effect of moving it is to transfer a certain amount of energy from the regenerator to the air and back again. This process cannot of course be carried out entirely without loss by friction and leakage of heat, but the loss is very small when the apparatus is properly constructed.

We have now only to connect the end B of this chamber by a pipe with the working cylinder, and we are enabled to produce the necessary changes of temperature and pressure at the ends of the stroke in the engine, *Type S*, without drawing any heat from the source, and the efficiency of the engine at once becomes nearly equal to that of *Type P*. Not only so, but the working cylinder always remains cool, being constantly at the temperature of the refrigerator. Thus we obtain the advantages of *Type S*, in reduction of bulk and maximum pressure, without sacrifice of efficiency.

The whole of the air contained in the regeneretor chamber and cylinder does not in this case undergo the necessary changes of temperature; a small part at the hot end, and a larger part at the cold end of the chamber and in the cylinder, never enters the regenerator, and therefore expands and contracts without doing useful work. There is a certain diminution of efficiency in this, for the changes of pressure and volume cannot be produced without some loss, but the principal effect is to reduce the ratio of expansion of the working air. The *efficiency* is not reduced by this, for it is not at all dependent on the ratio of expansion, but the work done is reduced, an evil which can be counteracted by using the air at a sufficiently high pressure.

The arrangement here described is in all essential respects that of the *Stirling* engine, first patented in 1827, and subsequently improved in 1840, by R. & J. Stirling. The details of construction of this celebrated machine need not be considered here; it is sufficient to say that an engine of this kind, of about 20 horse-power (average), and occasionally worked at a much higher power, was in use for some years at Dundee about 1843. It proved economical in fuel when the working pressure was sufficiently high, and was only abandoned from the difficulty of constructing a heating chamber which would withstand for any length of time the heat of the furnace. For an account of various small engines now in use which are modifications of the original design, the reader is referred to a lecture by the late Prof. Jenkin.* The original patent of the Stirling engine is reprinted in an appendix to this lecture.

The regenerator is an apparatus of great theoretical interest, and we shall frequently recur to it as we proceed.

40. In order to heat or cool, with sufficient rapidity for the purposes of a heat engine, air contained in a receiver of any kind, by the external application of a body of different temperature, an extensive surface is required over which the air may circulate in thin sheets so as to come into direct contact with the surface; and it is far from easy to avoid overheating. To meet this difficulty, which is the most serious obstacle to the employment of air engines in practice, *internal* combustion may be resorted to; the furnace being placed within the receiver containing the hot air supplied to the engine. The fuel must be smokeless, so that the products of com-

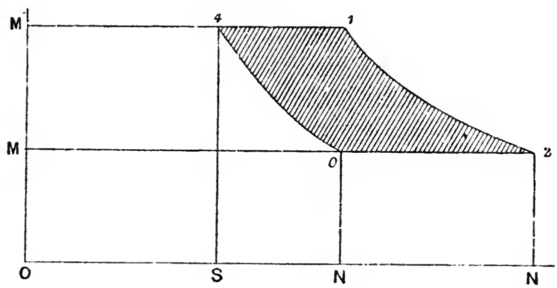
* *Lectures on Heat in its Mechanical Applications*, Inst. C.E., 1885.

bustion, largely diluted with air, may pass through the working cylinder without inconvenience. As the air is necessarily supplied under pressure, the furnace is fed through a small chamber on the top of the receiver provided with two doors, one above opening outwards, the other below opening inwards. In feeding the furnace the upper door is opened and the charge placed in the chamber, after which it is closed. The inner door is then opened by levers from without, and the charge drops into the furnace placed vertically below.

The principal parts of the engine are a feed pump drawing air from the atmosphere and supplying it to the receiver at a pressure which in actual engines is generally from 1 to 2 atmospheres, and a working cylinder which simultaneously draws air from the receiver and discharges it into the atmosphere. The working of such an engine is highly instructive, both in itself and in comparison with a steam engine, which in some respects it closely resembles, while in others it is entirely different.

In Fig. 14 $O\,N$ represents the volume V_0 of 1 lb. of air at the atmospheric pressure P_0, which is drawn into the feed pump and compressed as shown by the adiabatic curve 0 4, till it reaches the

Fig. 14.



pressure P_4, in the receiver containing the heated air. A valve then opens, and (neglecting clearance) as the pump piston moves up to its cylinder cover, the whole of the air, the volume of which is $O\,S = V_4$ is driven into the receiver. At the same time the working cylinder is drawing air from the receiver, so that the pressure in it remains approximately constant. The air supplied by the feed pump is, therefore, heated at constant pressure to a temperature T_1 by mixture with the products of combustion of the fuel. By this mixture the volume at a given pressure and temperature is very slightly altered. When the working cylinder has drawn at constant pressure P_1 a volume of air represented by 1 M on the diagram, admission is stopped, and the air expands adiabatically, as represented by the curve 1 2, when the end of the stroke is reached and exhaust takes place into the atmosphere. If arranged to work to best advantage, the expansion will be *complete*, that is the air will be discharged at the atmospheric pressure $P_2 = P_0$.

Let us now calculate the useful work and heat expended:

(1) The work done per lb. of air in driving the feed pump is represented by the area M 4 0 M in the diagram, which is the figure which would be drawn by an indicator. If F be this area

$$\begin{aligned} F &= N\,0\,4\,S + P_4 V_4 - P_0 V_0 \\ &= K_v (T_4 - T_0) + c\,(T_4 - T_0) \\ &= K_p (T_4 - T_0) \end{aligned}$$

where T_4 is the temperature to which the air is raised during compression in the pump from pressure P_0 to pressure $P_4 = P_1$.

Similarly when the air is admitted to the working cylinder, cut off, and expanded as just described, the energy exerted in driving a crank shaft, with which the working piston may be supposed connected, is graphically represented by the area M 1 2 M of the indicator card of this cylinder drawn on the same scale of *volumes* as the card of the pump. If this area be W, then reasoning as before,

$$W = K_p (T_1 - T_2)$$

where T_2 is the temperature of the air on exhaust into the atmosphere.

The temperatures T_2 T_4 are found as follows, observing that $P_2 = P_0$ and $P_4 = P_1$, and reasoning as on page 90.

From the adiabatic curves 1 2 and 0 4

$$\frac{T_2}{T_1} = \frac{P_0 V_2}{P_1 V_1} = \left(\frac{P_0}{P_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

and

$$\frac{T_4}{T_0} = \frac{P_1 V_4}{P_0 V_0} = \left(\frac{P_1}{P_0}\right)^{\frac{\gamma - 1}{\gamma}}$$

The temperatures T_0 T_1 are the absolute temperatures of the atmosphere and the receiver, and may be supposed known. Comparing the equations and referring to the diagram, it is easily seen that if m be the ratio of the volumes of the working cylinder and feed pump,

$$m = \frac{T_2}{T_0} = \frac{T_1}{T_4} = \frac{T_1 - T_2}{T_4 - T_0}.$$

The working cylinder and the feed pump must be supposed connected with the same crank shaft, and the useful work done by the engine must therefore be represented by the shaded area 4 1 2 0, which is the difference of the two cards. In symbols, if U be the useful work,

$$U = W - F = K_p (T_1 - T_2 - T_4 + T_0),$$

which may also be written

$$U = K_p\left(T_1 + T_0 - m\,T_0 - \frac{T_1}{m}\right).$$

(2) The heat expended in the receiver is applied in raising the temperature of the air at constant pressure from T_4 to T_1, that is

$$Q = K_p (T_1 - T_4),$$

and the heat rejected is given out as the air cools after exhaust into the atmosphere from T_2 to T_0, that is

$$R = K_p (T_2 - T_0).$$

The difference gives the same value as before for U the useful work done by the engine, and the efficiency from the result previously given is

$$\text{Efficiency} = \frac{U}{Q} = 1 - \frac{T_2 - T_0}{T_1 - T_4} = \frac{T_1 - T_2}{T_1}.$$

Since the air is always discharged at a temperature T_2 greater than T_0, this result shows that the efficiency is less than of the engines previously considered when supposed working between the temperatures T_0 and T_1. It only approaches equality when the size of the pump is nearly equal to that of the working cylinder; but this reduces the work done by a lb. of air and increases the bulk of the engine, an evil so great that it would probably be preferable to take m the ratio of cylinders so as to make U greatest, which from the formula just given appears to be when

$$m = \sqrt{\frac{T_1}{T_0}}$$

which gives

$$T_2 = T_4 = \sqrt{T_1 T_0}$$

and reduces the efficiency to

$$\text{Efficiency} = \frac{\sqrt{T_1} - \sqrt{T_0}}{\sqrt{T_1}}.$$

It should be observed that W, the effective energy exerted on the working piston, is less than the total energy exerted by the air upon it by the amount necessary to overcome the pressure of the atmosphere at the back. And similarly F, the work done by the crank shaft in driving the feed pump, is less than the total work done in forcing the air into the receiver by the energy exerted by the atmosphere pressing against the feed pump piston. The difference between work done upon, and energy exerted by, the atmosphere is graphically represented by the rectangle N 0 2 N. Hence, besides the useful work, waste work is done upon the atmosphere given by

$$\text{Waste Work} = c\,(T_2 - T_0),$$

which reappears as heat when the exhaust air cools under constant pressure from T_2 to T_0.

In any case, when working with complete expansion, the efficiency is found to be

$$\text{Efficiency} = \frac{T_1 - T_2}{T_1} = \frac{T_4 - T_0}{T_4},$$

the second value showing that it depends only on the temperature to which the air is raised by compression in the feed pump, that is on the pressure to which the air is compressed, being greater the higher the pressure in the receiver.

The various points considered are illustrated by the annexed table of numerical results, in which it will be observed that unless the air is compressed to a greater extent than is at all convenient in practice, the best efficiency is 30 per cent. and the mean effective pressure, representing the useful work when referred to the working cylinder, hardly exceeds 6 lbs. per square inch.

PERFORMANCE OF A JOULE ENGINE.

Maximum Pressure above Atmosphere.	Temperatures Fahr.				Mean effective pressures referred to working cylinder.			Efficiency.
	Working Cylinder		Feed Pump.					
	Admission.	Exhaust.	Atmosphere.	Feed.	Working cylinder.	Feed pump.	Useful Work.	
14·7	539°	359°	39°	149°	11·3	6·9	4·4	·18
35	539°	246°	39°	246°	21	14·8	6·2	·293

Remarks.—Air drawn from atmosphere at temperature 39° F., and admitted to working cylinder at temperature 539° F. The table also refers to an internal combustion (Brown) engine.

It should also be understood that the ideal efficiency in this case includes the efficiency of the process of heating the air as well as the efficiency of the working of the engine, whereas in external combustion engines the calculated efficiency refers to the working alone. Unfortunately this is an advantage very difficult to realise in practice : the temperature is difficult to regulate, and unless the air is cooled beforehand, a process involving loss, it will generally be exhausted from the working cylinder at a much higher temperature than that given in the table.

The idea of constructing an internal combustion air engine is an old one, and several inventors have endeavoured to carry it out. The best known example is an air engine employed in several cases by the Trinity House for working the machinery of lighthouses, where an absence of water and other reasons render the use of steam inconvenient. For the details of construction of one of these engines described as a "Brown engine," the reader is referred to a paper * by Sir J. Douglass. It appears to be the same as the engine

* *Proceedings of the Institution of Civil Engineers*, vol. 57.

described by the late Prof. Jenkin as a "Buckett engine" in his lecture cited elsewhere in this chapter.

The mechanical difficulties attending internal combustion engines appear to have been very successfully overcome in the Brown engine, and if the consumption of fuel be considered without reference to friction, they are fairly economical. The friction, however, is necessarily excessive, because the useful work is the difference of the energy exerted in the working cylinder and the work done in the feed pump. This point is well illustrated by reference to the experiments on a Brown engine made by Dr. Hopkinson.*

Diagrams from the working cylinder and feed pump showed a horse-power of 33·1 and 17·7 respectively. The difference of these, namely 15·4, is the true indicated power of the engine, but of this 6·3 horse-power, being more than 40 per cent., was wasted by friction, the brake power being found by experiment to be only 9·1. The consumption of coke per horse-power per hour is stated as 4 lbs., the brake horse-power apparently being meant, showing that the efficiency of the engine as a thermodynamic machine must have been considerable.

An engine of this class was proposed by Dr. Joule, as an external combustion engine, and is often consequently described as a "Joule Engine." It will be referred to under this title in what follows.

41. When the working fluid in a heat engine is permanently enclosed within the cylinder, so that the same mass of fluid goes the same cycle of changes indefinitely, the heat engine is described as Closed. The Stirling engine is closed, for the working cylinder communicates with a closed chamber to which the atmosphere has no access. In such an engine any pressure consistent with safety may be used, and in the example of a Stirling engine, referred to in the last article, the engine was worked at different powers by pumping in air or allowing it to escape. In most heat engines, however, the atmosphere is admitted at some point of its action, and the engine is then described as Open. Thus, in the engine we have just considered, the air is exhausted from the working cylinder into the atmosphere, and a fresh supply drawn in through the feed-pump.

* *Proceedings of the Institution of Civil Engineers*, vol. 87, p. 253.

On examination of the working of an open engine, it is, however, found that the fluid goes through a cycle of changes just as much as if it were closed, the difference being that part of the cycle takes place outside the engine, and that the particular mass of fluid operated on in one cycle is replaced by an exactly similar mass in the next. In the case we are now considering the air is drawn from the atmosphere into the feed pump, passes into the receiver, is heated, admitted to the working cylinder, expanded, and finally exhausted into the atmosphere, where it is cooled. Now all these operations might, ideally, have been performed within the working cylinder. For instead of exhausting the air, suppose it gradually cooled in the cylinder, while at the same time the piston returns so as to maintain the pressure constant. As soon as the temperature has fallen to T_0, which will happen when the piston has reached a point in the return stroke given by the ratio of compression,

$$r = \frac{T_2}{T_0} = \frac{V_2}{V_0},$$

let the cooling be stopped and the temperature allowed to rise by compression of the air behind the piston till the pressure has risen to P_1 just as it does in the feed pump. These two operations are represented by 2 0 and 0 4 on the diagram. Next, let the piston make its working stroke, which now will be shorter than before, the temperature of the air rising by application of heat so as to keep the pressure constant till the temperature T_1 is reached, when the source of heat is removed and the air expands adiabatically to the temperature T_2 and pressure P_2 with which we commenced. We have now a closed engine the indicator diagram of which is 4 1 2 0 and the cycle of which is the same as in the actual engine except that the part represented by 2 0 is performed within the cylinder instead of in the atmosphere. The area of the diagram representing the cycle, or as we may describe it, the Diagram of Energy of the cycle, always represents the useful work. This diagram is the indicator diagram of the Equivalent Closed Engine, and in an engine which is open, is not the figure drawn by an indicator applied to the working cylinder, but is a combination of two or more such figures taken from all the working cylinders, if there be more than one, and all the pumps playing the part of a feed pump.

In a non-condensing steam engine the mechanical action of the fluid closely resembles that of the air in the type of engine we have just dealt with, and to obtain the true diagram of energy of the steam we have to combine not only diagrams from the several working cylinders if it be a double or triple expansion engine, but also strictly speaking a diagram from the feed pump, as will be seen on reference to Art 30, page 69. This article should here be carefully considered, and it should especially be noticed that the compression which occurs in steam cylinders after the exhaust is closed forms no part of the cycle of the working steam, but only of the steam shut up in the clearance space.

In a condensing steam engine with surface condensation, if leakage of air into the condenser, and of steam from the cylinder and passages, could be avoided, the "air pump" draining the condenser would be simply a feed pump, which by proper arrangements might be used to force the condensed steam back into the boilers. The engine would then be closed. In the actual engine the air pump must be enlarged, being required for other purposes, but it also plays the part of a feed pump in combination with the actual feed pump, which performs the final part of the process. Any part of the feed pump diagram $Z L$ in Fig. 6*a*, page 68, below the atmospheric line must be supposed obtained from the air pump.

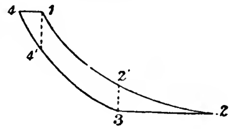
To avoid misapprehension it may here be added that though the mechanical action is the same the thermal action is different, so that the air engine in question is not thermodynamically similar to the steam engine. This will be fully considered hereafter.

42. The largest air engine hitherto constructed is probably that designed by the famous engineer Ericsson, and employed by him for the propulsion of a vessel over 2000 tons burden which made the voyage from New York to Alexandria.

This engine, like the Stirling, receives and rejects heat at constant temperature, but the changes of temperature of the air are produced at constant *pressure* instead of constant *volume*. Its indicator diagram, therefore, consists of two hyberbolic curves 1 2, 3 4 (Fig. 15) connected by *horizontal* lines 4 1, 2 3, whereas in the Stirling engine the connecting lines are the verticals 1 4′, 3 2′. A regenerator is added which, as in the Stirling engine, raises the ideal efficiency to that of the "perfect" engine.

The form of the diagram is such that more work is done by each lb. of air than is possible in any other engine working between the same limits of pressure and temperature, but it does not follow that the engine is less bulky than the Stirling, because the maximum volume of a lb. of air is so much greater as shown in the figure. Practically it is more bulky, especially when it is considered that high pressure cannot be used. For the purposes of this work it is not necessary to describe this type of air engine, as no fresh point in the theory of heat engines is illustrated by it. It has never found favour on this side of the Atlantic, but small engines of the class are still in use in America for the special purposes to which air engines can properly be applied.

Fig. 15.



43. On comparing the performance of the air engines considered in the present chapter with that of the steam engine as found in Chapter III., we find that, ideally, the air engine has the advantage. If we make some allowance for the higher pressure at which steam can be, and often is, used than is supposed in the cases discussed, we may estimate the best ideal efficiency of steam engines at 20 per cent., losses connected with the boiler being excluded. In air engines we have just found it to be 30 per cent., though, for various reasons already alluded to, it is difficult to approach the ideal efficiency in an air engine, and the losses in the process of heating the fluid are much greater in air than steam.

Taking, however, the ideal value of the efficiency, we see that at least 70 per cent. of the heat used is wasted by an air engine, and 80 per cent. in a steam engine. The cause of the waste is indeed very different in the two cases, being in the steam engine due to an enormous amount of heat-energy being required to effect the internal changes involved in turning water into steam, while in the air engine it is mainly due to the power required to work a large feed pump forming an indispensable part of the apparatus, or to overcome a back pressure on the working piston. To discover whether the waste of more than two-thirds the heat employed is necessary, or if it can be avoided by the adoption of some other form of heat engine, will be the principal object of the next chapter; but it is first advisable to consider some other examples of thermodynamic machines which may assist us in forming a correct judgment on this very important question.

A description of the Ericsson and Stirling engines, with calculations and comparison with experiment, will be found in Rankine's *Steam Engine and other Prime Movers.* With this should be read four articles on *Air Engines* contained in *Engineering*, vol. xix., which form a valuable commentary.

PART III.—EXPLOSIVE GAS ENGINES.

44. Gas engines form a class of thermodynamic machines of constantly increasing importance. The best known type of gas engine will now be briefly noticed for the sake of the illustration of thermodynamical principles which it affords. The fluid employed is coal gas, such as is used for lighting purposes, mixed with air.

The composition of coal gas varies considerably according to the quality of coal from which it is manufactured. For the present purpose its density may be taken as ·44, so that 1 lb. of it at temperature 59° F. occupies about 30 cubic feet instead of 13, as it would do if its density were the same as that of air. The value of c therefore in the formula $P V = c T$ is about 121. For complete combustion a cubic foot of gas requires about 6 cubic feet of air, and heat is developed, which in gas of this quality amounts to about 650 thermal units.

If a cubic foot of gas is mixed with not more than 14 cubic feet of air a mixture is obtained which on ignition explodes with considerable violence. The explosive force is not due to increase of volume at a given temperature—on the contrary a small contraction occurs of about 2 or 3 per cent.—but to a rise of temperature consequent on the development of heat. If we assume that the mixture is subject to the ordinary gaseous laws, we can calculate the pressure and temperature produced when fired in a close chamber. For let N cubic feet of air be mixed with 1 cubic foot of gas, then, remembering that the weight of a cubic foot of air at the atmospheric pressure is $40 / T_0$ where T_0 is the absolute temperature of the atmosphere, we have

$$\frac{40}{T_0}(N + \cdot 44) \,.\, K_v (T - T_0) = 650.$$

Hence, since the volume of the mixture is constant, if P be the pressure obtained, P_0 that of the atmosphere,

$$\frac{P}{P_0} = \frac{T}{T_0} = 1 + \frac{65}{4\,(N + \cdot 44)\,K_v}.$$

The value of the specific heat, at constant volume at moderate temperatures, can be very little greater than that of air, and may be assumed as ·18 in thermal units. Supposing it the same at high temperatures, we are enabled to calculate P and T. On making the calculation for $N = 7$ for example, we find that a pressure of about 14 atmospheres (absolute) should be developed.

The question has been thoroughly studied experimentally by Mr. Clerk, by means of an indicator applied to the closed chamber, the drum of this indicator rotating uniformly by clockwork, so that the curve drawn on the card shows the pressure at any time after the explosion. The general character of this curve is similar to the curve shown in Fig. 17 further on, which applies to a different purpose. A certain maximum pressure is developed in a small but measurable fraction of a second, which afterwards drops rapidly by cooling. The maximum pressure by experiment, however, when there is no initial compression, never exceeds 100 lbs. per square inch above the atmosphere, or say 8 atmospheres (absolute), and is generally less than one-half that found by the calculation just made. This difference may be due in part to the failure of the gaseous laws at such excessive temperatures, and is certainly partially due to cooling, but neither of these causes is sufficient; it can hardly be doubted that the principal reason is that the combustion is not complete. It may plausibly be conjectured that beyond a certain limit of temperature, combustion cannot take place.

45. The first gas engine in all respects efficient was the Otto engine, and this is still the type which is most widely used. It consists of a working cylinder of length $O\,N_2 = (1 + k)\,s$, Fig. 16, in which moves a piston, the stroke of which is $N_1\,N_2 = s$, so that when the piston is at one end of its stroke a space of length $O\,N_1 = k\,s$ is left between the piston and cylinder cover.

The engine goes through its cycle in the course of two revolutions or four strokes. During the first outward stroke, commencing from the position just named, a mixture of gas and air at atmospheric pressure is drawn into the cylinder and completely fills it, always at the same pressure, when the end of the stroke is reached. The piston now returns, making the first inward stroke, compressing the charge behind it as represented by the compression curve 0 4 on the

Fig. 16.

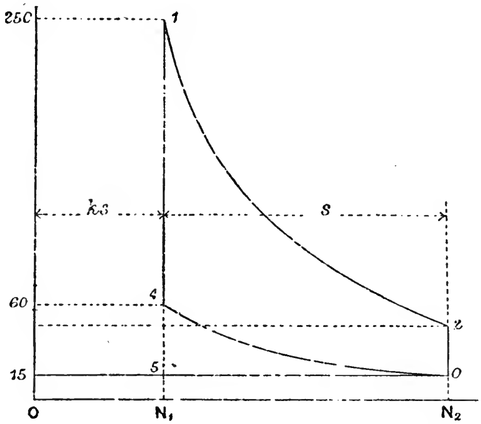


diagram. At the end of this stroke the clearance space $k s$ is filled with the compressed charge, which is now fired, so that the pressure suddenly rises from P_4 to P_1 as represented by the vertical 4 1. The pressure ratio is found to be about the same as if the gaseous mixture had been initially at the atmospheric pressure, being from 4 to 5. The piston now makes its second outward stroke, during which the gaseous mixture expands as shown by the curve 1 2 on the diagram. At the end of this stroke the pressure P_2 is still much greater than the atmospheric pressure P_0. Exhaust now takes place, and the burnt gases are expelled during the second inward stroke, at the end of which the cycle recommences.

Though communicating with the atmosphere at the end of every cycle, this engine is almost completely closed, the operations all taking place within the same cylinder, and the figure drawn by the indicator is the diagram of energy of the cycle as in a closed engine with the addition of the horizontal line 0 5 representing the operations of exhaust and renewal of the charge. The area 1 2 0 4, therefore represents the useful work done in the cycle. The object of compressing the charge before ignition is to obtain a higher initial pressure, which enables a greater ratio of expansion to be used: if this is not done the pressure falls below that of the atmosphere with a very low grade of expansion, and the engine is much less economical. To effect this compression, however, an additional double stroke is required by the Otto type of engine, so that there is only one working stroke in two revolutions. By use of a heavy fly-wheel sufficient uniformity is obtained for most purposes, and the engine is regulated by a centrifugal governor acting on the supply of gas so that an explosion is omitted when more power is developed than is required.

This brief sketch may serve to give some idea of the working of this engine, but for details the reader is referred to the works cited at the end of this chapter.

The expansion curve 1 2 and compression curve 0 4 are found by experience to be approximately adiabatic curves, a matter to be further considered presently. Assuming this, the useful work is calculated thus:—

(1) From the diagram the energy exerted upon the piston during expansion is graphically represented by the area of the adiabatic curve 1 2 N_2N_1 and is, therefore, as in previous cases,

$$E = K_v (T_1 - T_2),$$

and the work done by the piston during compression is graphically represented by the area 4 0 N_1N_2 given by

$$C = K_v (T_4 - T_0).$$

The useful work is therefore

$$U = K_v (T_1 - T_2 - T_4 + T_0).$$

(2) The expenditure of heat, so far as accounted for by the diagram, is

$$Q_1 = K_v (T_1 - T_4),$$

being the heat necessary to produce the rise of temperature from T_4 to T_1 on ignition of the charge, as represented by the vertical line 4 1. As the gas is discharged at a pressure greater than that of the atmosphere, as in the case of the steam engine (Chap. III.), the heat rejected cannot be directly calculated, but only found from the value just obtained of the useful work. The efficiency is, considering only the heat accounted for by the diagram,

$$\text{Efficiency} = \frac{U}{Q_1} = 1 - \frac{T_2 - T_0}{T_1 - T_4}.$$

The relation between the several temperatures is found by the equations

$$r^{\gamma - 1} = \frac{T_4}{T_0} = \frac{T_1}{T_2} = \frac{T_1 - T_4}{T_2 - T_0},$$

where r is the ratio of expansion, which is clearly the same as the ratio of compression. Hence

$$\text{Efficiency} = \frac{T_4 - T_0}{T_4} = 1 - \left(\frac{1}{r}\right)^{\gamma - 1} = \frac{T_1 - T_2}{T_1},$$

results quite analogous to those obtained for the type of air engine last considered (p. 101), showing that the efficiency depends on the temperature of compression T_4 irrespective of the temperature developed by explosion; and that it must be less than in a "perfect" air engine working between the limits T_1 and T_0. In the Otto engine as actually used, the value of k is about ·6, giving a ratio of expansion 1·6 / ·6 or about 2·7, hence by numerical work we find that the apparent efficiency is ·33.

The expenditure of heat here considered is only about one-half that actually required, on account of the large escape of heat by conduction through the metal of the cylinder. Careful experiments by Professor Thurston on a 6 horse-power Otto engine showed that the indicated work of the engine accounted for 17 per cent. of the total heat of combustion of the gas used, and that 52 per cent. passed through the metal into the water-jacket which in all gas engines is necessary to keep the cylinder sufficiently cool. The results obtained by Dr. Slaby were nearly the same. In addition to this, the gas exhausted from the cylinder had a temperature between 700° and 800° Fahr., and must have carried away a large amount of heat. These facts appear to show conclusively that combustion, which, as above stated, is not nearly complete in the first instance, goes on during expansion, an explanation which appears to account for the fact that the expansion curve is nearly adiabatic; the outflow of heat during expansion, to the metal of the cylinder, being balanced approximately by additional heat generated by combustion. It has not as yet been determined by experiment whether combustion be complete before discharge of the burnt gases, but if this be assumed to be the case the actual efficiency may be taken as about one-sixth, being about one-half the ideal value found above.

To obtain a horse-power of 42·75 thermal units per minute, about 4 cubic feet of gas per hour of the quality considered in this article would be required, if the heat of combustion could be fully utilised : this gives an actual consumption of 24 cubic feet per indicated horse-power per hour. In the most recent types of engine the consumption has been reduced to 20 cubic feet, or even lower.

The necessity of a water-jacket to keep the cylinder cool is easily understood on calculating the temperature equivalent to the actual pressure produced by explosion of the gaseous mixture, which will be found to be over 3000° Fahr. Any such temperature as this would be immediately destructive of the internal surface of the cylinder, and as the surface remains uninjured we conclude that it is protected by a comparatively cool layer of gas which exchanges heat with the central mass by intense convection currents. This fact we may have occasion to refer to hereafter.

The history of the gas engine and the endeavours made to improve on the Otto type are very interesting, but the special purpose of this treatise renders it impossible to dwell on the subject.*

Action of Gunpowder in a Gun.

46. When gunpowder is exploded the products of combustion may be separated into two distinct parts—a gaseous part consisting of a mixture of certain approximately permanent gases, and a non-gaseous part which at moderate temperatures is solid. This latter part is in a very finely divided state intimately mixed with the gases, so that the whole has the appearance of smoke.

The physical properties of these products have been exhaustively studied by Captain Noble and Sir F. Abel, from whose results,†

* Some notice will be found in the Appendix of the Atkinson type of gas engine.

† *Phil. Trans.*, Part I., 1880.

changing thermal units into foot-lbs., we have for the specific heats of gases and solid :

$$K_p = \cdot 2324 \times 772 = 179 \cdot 4$$
$$K_v = \cdot 1762 \times 772 = 136$$
$$K = \cdot 45 \times 772 = 347 \cdot 4.$$

The ration (β) of the weight of the solid matter to the weight of the gases, they found to be 1·2957, showing that about 43 per cent. of the whole weight of powder becomes gaseous, while 57 per cent. remains solid.

When such an intimate mixture of gas and solid matter expands while doing work, the temperature of the solid matter must fall at the same rate as that of the gas, and assuming the specific heat (K) of the solid matter constant, we have a case of expansion such as was considered on page 81, in which the gas receives heat at a uniform rate as the temperature falls. Considering a fall of temperature of 1°, the heat given out by the non-gaseous products per lb. of gas will be βK, and referring to the page cited we shall have

$$\beta K + K_v = \frac{K_p - K_v}{n - 1},$$

where n is the index of the expansion curve, from which by numerical calculation with the values just given for the constants we find

$$n = 1 \cdot 074.$$

The expansion, then, of the products of explosion, while exerting energy upon the shot which it propels, is represented by Fig. 8*b*, page 84, with this value of n, from which we conclude that 77 per cent. of the energy exerted is due to heat stored up in the non-gaseous products.

In such a mixture as we are considering, the internal energy depends solely on the temperature, just as it does in a permanent gas taken by itself, and hence, when expansion takes place without doing work, the temperature remains constant. So also when a given weight of powder is exploded in a closed vessel, the same temperature is reached whatever the size of the vessel. Let V be the volume of 1 lb. of powder-gas at the temperature of explosion, v_0 the volume of the charge of powder which produces the gas, $a v_0$ the volume of the non-gaseous products at the temperature of explosion. The value of a is believed to be about ·57, being approximately the same as would be obtained if the density of the non-gaseous products were the same at this temperature as that of the powder. The temperature being, as just remarked, always the same, $p\,V$ must be the same where p is the pressure, so that

$$p\,V = p_0\,V_0,$$

in which formula the suffix refers to the case where the powder completely fills the closed vessel in which it is exploded. But evidently

$$V = v - a\,v_0 \qquad V_0 = (1 - a)\,v_0,$$

so that the formula

$$p = p_0 \cdot \frac{(1 - a)\,v_0}{v - a\,v_0}$$

gives the pressure on firing in a closed vessel of volume v a charge of powder of volume v_0, in terms of p_0, the pressure found by experiment when the vessel is completely full. It is unnecessary to say that no instrument of the nature of an indicator can be employed in the present case; in its place a "crusher gauge" is used, consisting of a small cylinder of copper in which a known permanent set is produced by a known crushing stress. In this way the value of p_0 is found to be 43 tons per square inch, from which the value of p can be calculated in any given case. For example, let the charge half fill the vessel, then $v = 2\,v_0$, and taking $a = {\cdot}57$ we find the pressure produced to be 13 tons per square inch. The actual value of p is shown by experiment to be slightly less, say about 11·7, a difference probably due to the cooling influence of the sides of the vessel.

When the explosive products expand while doing work, the energy exerted is

$$U = \frac{P_0\,V_0 - P_2\,V_2}{{\cdot}074},$$

where V_0 is the specific volume of the gaseous portion of the charge at the initial pressure P_0 expressed in tons per square foot, while $V_2 = r\,V_0$ is the volume, and P_2 the pressure after expanding r times. The value of r, the ratio of expansion of the gases, is connected with r', the ratio of expansion of the total charge, by an equation derived from the equations just now given

$$r = \frac{r' - a}{1 - a},$$

and the final pressure P_2 is connected with P_0 by the equation

$$P_2 = P_0 r^{-n}$$

in which, as before, $n = 1 \cdot 074$.

The density of dry powder closely packed is about the same as that of water, and using the above equations the value of U can now be calculated. The result is the work done by a lb. of powder-gas, and since the gases are 43 per cent. of the whole, the work done by 1 lb. of powder is immediately determined.

It is in this way that tables of the theoretic effect of gunpowder are calculated. If the expansion be supposed to be carried on till the atmospheric pressure is reached, and the work done in overcoming the atmospheric pressure is deducted, the total ideal effect of a lb. of powder is found to be about 250 foot-tons. In actual guns expansion is not carried on far enough to realise more than a fraction of this, the calculated effect being from 50 to 90 foot-tons per lb. of powder. Of this about 80 or 90 per cent. is usefully employed upon the shot, the rest being dissipated by friction and other causes. The effect of the inertia of the products of combustion will be noticed in a later chapter.

47. A gun, when fired, becomes a thermodynamic machine, deriving its energy from the heat generated during the explosion of the powder, and its peculiarities will now be briefly noticed.

Fig. 17.

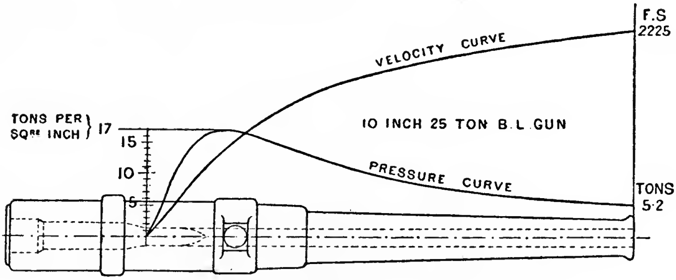


Fig. 17, taken from a lecture by Captain Noble, relates to a gun of modern type in which slow burning powder is used. The pressure curve shown is an indicator diagram similar to that of other heat engines, though the pressures are far too great to be determined by an indicator. They are actually obtained by "crusher gauges" placed at different parts of the bore, and this tentative process is checked by comparison with the velocities of the shot at the same points, as found by an instrument called a Chronoscope, and graphically exhibited by the velocity curve shown on the diagram. The maximum pressure is shown by the diagram to be reached after the shot has moved a short distance, its value being about 17 tons on the square inch, being little more than half that obtained with the quick burning powder formerly used. Expansion then takes place nearly according to the theoretical curve, and as the shot leaves the gun the pressure falls to 5·2 tons per square inch.

The total heat of combustion of powder is found by experiment to be about 1300 thermal units per lb., the mechanical equivalent of which is about 450 foot-tons. Comparing this with the results previously given, we see that the best efficiency is somewhat less than 20 per cent., the loss being partly due to the discharge of the products from the gun at a pressure which is still very high, but mainly to the high temperature which they retain even when discharged at atmospheric pressure.

As a thermodynamic machine a gun is peculiar in that its cycle is not completely closed. In the internal combustion air engine and in the gas engines, the gases employed return very nearly to their original volume, pressure, and temperature; the only change which has occurred is in the re-arrangement of the ultimate atoms by the chemical combustion which generates the heat. But in the gun, the products of combustion on returning to their original pressure and temperature do not return to their original volume, and in consequence a permanent displacement of the atmosphere occurs, involving the expenditure of energy in producing it. The amount is probably about 7 foot-tons per lb. of powder.

For further information on gas engines and on the action of gunpowder, the reader is referred to a treatise on the *Gas Engine*, by Mr. Dugald Clerk, and a lecture by Captain Noble, forming one of the series of lectures on *Heat in its Mechanical Application*, already cited on p. 98.

Part IV.—Reversed Air Engines.

48. Returning to the elementary air engine with which we commenced, and referring to Fig. 11, p. 88, suppose the piston to be in the position marked 3, and to be commencing its return stroke; in the original arrangement the cold body B was here applied to maintain the temperature at T_3: but now let the air be compressed behind the returning piston without abstraction of heat, so that the compression curve is the original expansion curve 2 3. As soon as the position 2 is reached let the body A be applied to prevent the temperature rising above T_1; the isothermal curve 3 1 will now be traced while a quantity of heat Q passes into the body A. When the piston has reached the position 1, and is about to commence the forward stroke, remove A so that the expansion of the air takes place without addition of heat, then the same curve 1 4 will be traced, which originally was a compression curve. Finally, when 4 is reached, let a fall of temperature below T_3 be prevented by application of the body B, from which a quantity of heat R is drawn as the piston completes its stroke. The cycle then recommences.

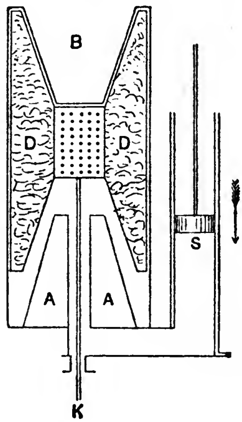
The whole action of the engine is now exactly reversed: by application of a certain amount U of mechanical energy to the crank shaft a quantity of heat R is drawn from the cold body B, and the quantity $R + U = Q$ passes into the hot body A, so that here mechanical energy is converted into heat, and, consequent on the conversion, a certain quantity of heat is made to flow into a body hotter than itself. The relation between U, R, and Q remains the same, whether the engine works forwards or backwards, the same formulæ being applicable to either case.

49. Every heat engine is capable of being thus reversed, though the reversal is not exact as in the preceding example, unless certain conditions are satisfied, to be considered hereafter. Such reversed heat engines may be employed either to supply heat by a mechanical process, replacing the ordinary chemical process of combustion, or for the purpose of cooling bodies below the temperature of surrounding objects. In the second case they form a class of machines known as Refrigerating Machines, used for making ice or maintaining a low temperature in a chamber for the preservation of articles of food.

Two examples of refrigerating machines which employ dry air as the working agent will now be described. The first is especially interesting to the student of applied thermodynamics, as having been expressly designed by Mr. Kirk as a reversed Stirling engine, the correspondence being so close that almost the same description would apply to both.*

In Fig. 18, *S* is the piston of an air compressing pump, corresponding to the working cylinder of a Stirling engine, which forces air into the regenerator chamber till its pressure is about double the atmospheric, and allows it to expand to its original pressure. The chamber is made with conical ends projecting inwards. The upper cone *B* contains the fluid to be cooled, which corresponds to the cold body of the heat engine, while the lower one *A* is closed at the bottom and filled with water, circulating through it from without by pipes not shown in the figure. The circulating water has the atmospheric temperature, and plays the part of the hot body in the heat engine. The displacing piston *D D* is made with hollow conical ends, which fit the corresponding cones of the chamber. Like the chamber, it is constructed of very thin tin plate, and its interior is filled with sawdust or other non-conducting material. The regenerator in this instance occupies the central part of the displacing piston and moves with it. The displacing piston is moved by a rod *K*, its stroke is short, and in the figure it is shown in its highest position; the air pump piston is then about the middle of its stroke, and is compressing air through a central pipe into the space below *D* surrounding *A A* in a thin sheet. The large surface exposed by *A A* now enables the circulating water within to abstract heat from the air and keep its temperature down. The displacer *D D* is connected with a shaft, not shown in the figure, by an eccentric and rod, and when the air pump piston *S* is near the

Fig. 18.



* *Mechanical Production of Cold*, by A. C. Kirk, *Minutes of Proceedings of the Institution of Civil Engineers*, vol. xxxvii., 1874. This valuable paper will be frequently referred to further on.

bottom of its stroke it moves rapidly downwards, causing the air to flow through the regenerator at its centre to the upper end B of the chamber, being cooled on its way through the successive gratings as previously described. At the same time S moves upwards and allows expansion to take place, while the cold fluid in B supplies the air with heat. Thus by the action of the air pump the brine or other fluid in B is cooled, it may be as much as 100° below the temperature of the atmosphere. The only limit to the fall of temperature which can be produced in this way is the degree in which conduction can be prevented, which in practice causes a leakage of heat from A to B. In a model apparatus constructed as here described mercury has been frozen into a solid mass which could be worked by a wooden mallet.

In this machine, if the regenerator worked perfectly, every step of the process in the heat engine would be exactly reversed, just as in the elementary engine previously considered, and the same relations would exist between U, Q, and R. In Mr. Kirk's paper a great number of experiments on regenerators are described, from which it appears that when properly designed the loss by imperfect action is very small. From these experiments the important conclusion is drawn that the efficiency of surfaces employed for the heating or cooling of air increases in direct proportion to the density of the air. This conclusion appears to be borne out by common experience with air engines, and will be referred to hereafter.

There are two serious difficulties in working this machine on a large scale; first, the air must be artificially dried by the use of chloride of calcium or some other means, otherwise much snow is produced, which chokes the regenerator. Secondly, it is not easy to obtain surface enough exposed to the air to enable the exchange of heat to be carried out with sufficient rapidity. The cooling action is then greatly diminished. This will be again referred to presently.

50. Let us next return to the second type of elementary heat engine, described as *Type S* (p. 95), being a Stirling engine with the regenerator omitted, and attempt to reverse its action.

Commencing at the point 4 and supposing the piston to return while the cold body B remains in contact, the isothermal 4 3 will be described, and the quantity of heat $c\,T_3 \log_\epsilon r$ will be abstracted from B. So far the cycle of the heat engine is exactly reversed, but on attempting to reverse the operation represented by the vertical line 3 2 we find that to raise the temperature of the air it will be necessary to apply the hot body A, and draw from it the quantity of heat $K_v (T_2 - T_3)$, whereas in the heat engine the body B was applied, and the quantity of heat in question abstracted by it. The next operation represented by 2 1 is exactly reversed, but the last represented by the vertical 1 4 is performed with B in contact instead of A, so that the quantity of heat $K_v (T_1 - T_4)$ is supplied to B instead of being abstracted from A. If then the whole heat abstracted from B be R, and the whole heat supplied to A be Q, we shall have

$$Q = c\,T_1 \cdot \log_\epsilon r - K_v (T^2 - T_3)$$
$$R = c\,T_3 \cdot \log_\epsilon r - K_v (T_1 - T_4),$$

whereas in the heat engine the sign of the second term is in both cases positive. On comparing these results with those already found for the first type it will be seen that with a given amount of mechanical energy U, when worked forwards the flow of heat from A to B is greater in *Type S*, but when worked backwards the flow of heat from B to A is less. In both directions the machine is less efficient, and we thus learn to connect the efficiency of an engine with its reversibility. An engine which is perfectly reversible is also an engine of maximum efficiency. The performance of such an engine will be considered in detail in the next chapter.

If, moreover, we consider the difference between those parts of the cycle which are reversible and those which are not, we shall find that in the first there is no sensible difference of temperature between the air and bodies with which it exchanges heat, whereas in the second the air receives heat from or rejects heat into a body of different temperature from itself, so that one condition of reversibility and, presumably, of maximum efficiency is that the air must receive all its heat at the temperature of the source of heat, and reject all its heat at the temperature of the body into which it passes.

51. When a Joule engine (page 103) is reversed, the working cylinder becomes a compressing pump which draws air from the atmosphere and compresses it to a certain pressure, while the original feed pump becomes an expansion cylinder in which the air exerts energy. The excess of work done in the working cylinder

over energy exerted in the expansion cylinder is supplied from without by a motor of some kind applied to the crank shaft which is common to the two cylinders.

Referring to Fig. 14, and commencing at the point 2, at which the air has its greatest volume and lowest pressure, at the end of the stroke of the working piston, let this piston return without application of any cold body (B) to draw heat from the air, the compression curve 2 1 is described indentical with the original expansion curve. On application of the hot body A, which in the heat engine has the temperature T_1, we, however, find that it cannot draw heat from the air as it should if the process were reversible, unless its temperature be less than before; and if the flow of heat into A is to last to the end of the stroke at 4 the temperature of A must not be greater than T_4. If this condition be satisfied the same quantity of heat Q flows into A as originally flowed out. Having reached 4, A is now removed, and adiabatic expansion takes place in the expansion cylinder according to the curve 4 0, which is identical with the original compression curve. When the piston completes its stroke, starting from 0 we find that the temperature of B cannot be T_0 as it was in the heat engine, but must not be less than T_2 in order that it may be possible for heat to flow out of B into the air as the horizontal line 0 2 is described. If, however, B has the temperature T_2 the same amount of heat R will flow into the air from B as originally flowed out of the air into B. When therefore this engine is reversed the indicator diagram and the quantities U, Q, R may be the same, but in order that this may be so it is necessary to suppose that the hot body is cooler and the cold body hotter than in the original heat engine. If they retain their original temperatures the indicator diagram will be different, the engine then is not exactly reversible.

It has hitherto been supposed that the bodies A and B have uniform temperatures throughout, but this need not be the case; let us, instead, suppose that each is split up into a number of gratings placed as in a regenerator, so that the air may pass through them in succession. The gratings of A have temperatures commencing at T_1 and gradually descending till T_4 is reached by infinitely small steps. In the same way the gratings of B commence at the lowest temperature T_0, and gradually rise till the last, which has the temperature T_2. Each grating must be supposed capable of abstracting or giving out indefinite quantities of heat. The process of receiving and rejecting heat at constant pressure represented by the horizontal lines 1 4 and 0 2 may now be carried out by passing the air through the gratings, the same quantities of heat Q and R will pass as before, but the process will be exactly reversible. Comparing this reversible engine with the original non-reversible one we see that the flow of heat is the same for the same amount of mechanical energy U, but that in the non-reversible engine the flow of heat is through a wider range of temperature when worked forwards, and through a narrower range when worked backwards. In other words, the non-reversible engine is in both cases less efficient.

The two examples which have been considered in the present and the next preceding articles may be considered as typical, and the conclusions drawn are applicable in all cases.

52. A reversed Joule engine is much used as a refrigerating machine. Though not nearly so economical in power as some other types, as will be seen clearly as we proceed, it possesses certain advantages for use on board ship and other similar cases. The working of this machine is highly instructive and will now be further considered. It occurs in two forms, one employing dry air and the other wet air, but otherwise substantially alike. The calculations which follow apply directly to a dry air machine, the formulæ for which are the same as for the heat engine (p. 100) if we suppose

T_1 = Temperature of air after compression,
T_2 = " of atmosphere,
T_4 = " of air on entering expansion cylinder,
T_0 = " of air on discharge from expansion cylinder,

the temperatures are absolute, and refer to the four corners 0, 1, 2, 4 of the indicator diagram, as shown in Fig. 14, p. 99.

The relations between U, Q, R, are given by the formulæ of the heat engine, from which we readily find

$$\frac{R}{U} = \frac{T_0}{T_4 - T_0} = \frac{T_2}{T_1 - T_2}.$$

The quantity R here is the amount of heat which the cold air discharged from the expansion cylinder abstracts from the body which it is employed to cool, or from the atmosphere into which it escapes. Its value is

$$R = K_p (T_2 - T_0).$$

The machine is employed to maintain a low temperature (T_5) in a cold chamber or "chill room," in which meat and other perishable objects are stored, notwithstanding a constant leakage of heat from the warm air outside. In bringing carcases from Australia the temperature of the chill room must be about 25° F., in order that the meat may be frozen hard. On shorter voyages a higher temperature is admissible. The expansion cylinder discharges its air into the chamber at a far lower temperature T_0 and the quantity of heat

$$R_1 = K_p (T_5 - T_0)$$

is consequently abstracted. The ratio

$$\frac{R_1}{U} = \frac{T_0}{T_4 - T_0} \cdot \frac{T_5 - T_0}{T_2 - T_0},$$

though generally greater than unity, may properly be described as the efficiency of the refrigerating machine. We have now two cases to consider.

First, let the air be allowed to escape from the cold chamber into the atmosphere, and let the cooling of the air from T_1 to T_4 before entering the expansion cylinder be produced by water of the atmospheric temperature surrounding the pipes through which the compressed air passes, then $T_4 = T_2$ and $T_1 \cdot T_0 = T_2^2$ (page 101), using which values, the efficiency can be found by substitution.

Secondly, instead of allowing the air to escape from the cold chamber into the atmosphere, it may be caused to flow along the outside of tubes through which the compressed air passes in the opposite direction, after having undergone a preliminary cooling as before to T_2. This apparatus, invented by the late Dr. Siemens, is called an "interchanger," its action is identical with that of the regenerator of a heat engine, and it might be described as such. Its action is no doubt much less perfect than that of a regenerator, but assuming it perfect the effect is to lower the temperature of entering the expansion cylinder from T_2 to T_5, the temperature of the cold chamber, and the temperature T_0 is consequently greatly lowered. Putting $T_4 = T_5$ the efficiency now becomes

$$\frac{R_1}{U} = \frac{T_0}{T_2 - T_0},$$

depending solely on the lowest temperature of the air. The lower this temperature the less the efficiency.

The temperature T_0 actually produced by these machines is very low, it is said that it occasionally reaches 100° below zero (Fahrenheit). With a perfect interchanger and no losses, such a temperature by calculation would be produced in absolutely dry air by initial compression to about 30 lbs. above the atmosphere, with atmospheric temperature of 90°. In the actual machine a compression to 45 lbs. or more is required.

Assuming the temperature of the atmosphere 90° and the value of T_0 to be 367, being 94° below zero, we find that the efficiency is 2. Taking the latent heat of fusion of ice as 140 thermal units and remembering that 1 horse-power is equivalent to 2565 thermal units per hour, we find that about 36½ lbs. of ice *from* and *at* 32° should be produced per horse-power per hour. This result, though small compared with the performance of an ideally perfect machine, is more than double that of the actual machine, owing to the various losses omitted in the calculation.

The low temperature is due (1) to the employment of a great compresion, (2) to the action of the interchanger. As regards the first cause, the thermal efficiency by the above formula would be much greater if a smaller compression were used so as to make T_0 nearly equal to T_5 the temperature of the cold chamber, but the quantity of air required would then become so great that the bulk and friction of the machine become excessive. In a machine of this class by Mr. Coleman the friction was found to be 20 per cent. of the indicated power of the steam engine employed to drive it. With a low compression it would be much greater as in the corresponding heat engine (page 104).

The action of the interchanger lowers the temperature and diminishes the bulk of the machine without diminishing its efficiency, but it has a still greater advantage, the moisture contained in the compressed air being nearly all deposited before entering the expansion cylinder, so that the diminution of the cooling of the air due to the production of snow is much diminished. So great is this advantage that in the absence of an interchanger the air before expansion may be cooled by passing it through pipes contained in the cold chamber, a process which with perfectly dry air would be wasteful.

The difficulty of obtaining sufficient surface for heating and cooling air has been already alluded to. With large quantities of air it is so great as to render it often advisable to employ a damp air machine in which the air is heated or cooled by injection. In the Kirk machine * water at the atmospheric temperature is injected into the hot part of the machine, and brine, which is the liquid to be cooled, into the cold part. The regenerator in this case has to be made of much larger size, and its action appears to be much less perfect. Much power is spent in forcing the damp air through the gratings, so that the internal friction of the machine is great.

In the machine described in this article, water is injected into the compressing cylinder and receiver of compressed air, to absorb the heat produced by compression. Less power is required for compression without any loss, for no use is made of the high temperature generated in the dry air machine. The machine in this form is due to Mr. Coleman.†

* See Mr. Kirk's paper already cited. A recent example of this machine is described and illustrated in *Engineering* for Dec. 7th, 1888.

† For details of design, arrangement, and working of this machine the reader is referred to a paper by Mr. Coleman in the *Minutes of the Proceedings of the Institution of Civil Engineers*, vol. lxviii., 1882.

CHAPTER V.

PERFECT THERMODYNAMIC MACHINES.

53. Mechanical power is produced from heat through the agency of an elastic fluid, such as steam or air, capable of assuming different volumes under the action of heat and cold, and exerting mechanical energy on external bodies during such changes of volume.

Observation of the action of heat engines, combined with reasoning of the same character as that already given in the case of steam and air, leads us to certain general conclusions as follows:

(1) The changes consist in a continual repetition of operations of the same kind, whether on the same mass of fluid or on a continual succession of exactly similar masses.

(2) Each repetition includes, *first*, a period during which the fluid on the whole increases in volume, and receives heat while exerting energy on a working piston; *secondly*, a period during which the fluid, on the whole, contracts in volume and rejects heat, while work is done upon it either by the working piston overcoming a back pressure, or by some special compressing apparatus, or by both these causes combined. The final result is that the fluid returns to the same state as before; that is to say, the changes in question constitute a Cycle of Operations.

(3) The energy exerted on the working piston, and the work done during contraction, do not at all depend on the particular mechanism by means of which the changes of volume of the fluid are carried out, but solely on the quantity of fluid, and the nature of the changes it undergoes.

Hence a heat engine implies, (1) a source or sources from which the fluid is supplied with heat; (2) a working piston or other means of utilising the expansive energy of the fluid; (3) a refrigerator capable of abstracting heat from the fluid; and (4) a compressing apparatus, in consequence of which the fluid returns to its original state; and it is to be especially remarked that the contraction of volume and rejection of heat is just as indispensable as the enlargement of volume and reception of heat.

The useful work done by the engine is the difference between the energy exerted by the fluid during enlargement of volume and the work done upon it during contraction; in some engines, as for instance the ideal air engines of Chapter IV., the work done during contraction is large, so that the useful work is a small fraction of the energy exerted; in others, as for instance the steam engine, the work done during contraction is comparatively small; in all cases, if U be the useful work done, E the energy exerted, C the work done during contraction,

$$U = E - C,$$

where the ratio $E : C$ may have any value according to the nature of the fluid.

But, further, the useful work done is also the work-equivalent of heat which disappears during the process; that is to say, the heat supplied by the source or sources is greater than the heat abstracted by the refrigerator exactly by the equivalent of the useful work done, so that, if Q be the heat expended, R the heat rejected, we have necessarily,

$$U = Q - R.$$

The ratio $Q : R$, however, is not capable of variation in the way that the ratio $E : C$ is, according to the nature of the fluid; on the contrary, it will be shown that, whatever the fluid be, that ratio will always have the same value provided the changes of state of the fluid follow a certain prescribed law. Hence the efficiency $U : Q$ of the engine is to a great extent independent of the nature of the fluid, just as much as of the mechanism of the engine (see Art. 27), being chiefly dependent on the way in which the fluid is supplied with heat; and whatever the engine be, a large amount of heat always passes into the refrigerator, being merely conveyed there from the source by the agency of the fluid.

In the present chapter we shall suppose that the supply of heat proceeds from a single external source at a given fixed temperature.

54. Again, bodies may be cooled by mechanical agency, and when a machine employed for this purpose is examined it is found to consist of the same essential parts as a heat engine. In both cases there is a working fluid which goes through a cycle of changes as it alternately receives and rejects heat: the difference is that in the refrigerating machine the fluid, instead of supplying energy, is constrained, by energy received from without, to go through its cycle in a reverse direction, drawing heat from a cold body, and supplying it to a comparatively hot body. In other words, a refrigerating machine is simply a heat engine which works backwards.

The cycle in the heat engine may be *completely* reversible, and when it is so, the engine will be of maximum efficiency, as has been already shown in air engines. To secure this important characteristic, two conditions are necessary, and, if the fluid offers no sensible resistance to change of shape (page 60), will also be sufficient.

In the first place, the reception of heat from the source, and the rejection of heat into the refrigerator, must take place at temperatures not sensibly different from those of the bodies themselves. This point has already been fully considered in the case of air engines, and we shall return to it again presently.

In the second place, in order to secure reversibility, it is necessary that the expansive force of the fluid should be exactly balanced by the resistance which is being overcome. If it should be greater than the resistance, then the excess takes effect by generating kinetic energy in the particles of the fluid, which are thrown into violent motion. In order exactly to reverse such a process, it would be needful to direct the motion of the particles so as to compress the fluid again without the application of any other external force than that originally overcome when the fluid expanded. Such direction is obviously impossible, nor is it less impossible to set the particles of fluid in motion by the direct action of heat reversing the process by which kinetic energy is transformed into heat by fluid friction; hence unbalanced expansion is necessarily irreversible. When, for instance, the exhaust is opened at the end of the stroke in a s eam cylinder, the steam rushes violently into the condenser, and the greater part of its expansive energy is employed in generating kinetic energy, which is afterwards changed into heat by fluid friction; to take hold of the particles of steam, and direct their motions so as to cause them to enter the cylinder again without the application of pressure, is impracticable, even were it possible to set them in motion by the direct action of heat, and the ordinary steam engine in which the expansion is incomplete is consequently irreversible.

55. Whether completely reversible or not, however, a heat engine may be conceived to work backwards, and when it does so, the original source of heat plays the part of a receiver of heat, and the original refrigerator becomes a source from which heat is derived. Hence, by a proper application of mechanical energy, heat can be taken away from a body of low temperature, and supplied to a body of high temperature. The refrigerating machines, of which examples have already been given, actually perform the operation.

Now this is essentially an artificial or non-natural effect; when we speak of one body as hot and another as cold, no other meaning can be ascribed to these words than that heat tends to flow from the hot body to the cold one, and will certainly do so if no external cause prevent; much more then are we justified in saying that in the natural order of things heat will not pass from a cold body to a hot one, but only under the influence of some external agency. This is expressed in formal terms by the annexed statement of the Second Law of Thermodynamics.

Heat cannot pass from a cold body to a hot one by a purely self-acting process. (Clausius' Statement.) (See Appendix.)

It is easy to see what enormous consequences the denial of the principle would involve in the theory of the steam engine, for all the heat expended in the boiler which is not transformed into mechanical energy—that is to say, at least five-sixths of the whole amount—appears in the condenser, being employed in heating the condensation water, and if it were possible by some self-acting contrivance to cause that heat to flow from the condenser into the boiler, it is manifest that the said five-sixths of the consumption of heat might be saved. It is certain, however, that this is impossible, but that to cause the heat to flow from the condenser into the boiler we must have recourse to some artificial process which, like working a heat engine backwards, involves in some way or other, directly or indirectly, the expenditure of energy to as great or greater amount than we can recover by utilising that heat in the boiler; and the second law of thermodynamics merely amounts to a statement of this impossibility.

By aid of this law we are able to prove an extremely important theorem, due to Carnot, which may be thus enunciated:

CARNOT'S PRINCIPLE.—*The efficiency of all reversible engines, working between given limits of temperature, is the same.*

For, let us imagine two engines, A and B, of which in the first instance we suppose B reversible in the sense explained above, and let the power of these two engines be the same, then the engine A may be employed to drive the engine B backwards, and the combination of the two engines will be self-acting, requiring no energy derived from without to drive them, but continuing in motion (neglecting friction) for ever when once set going. Let Q_A, R_A be the heat expended and rejected respectively by the engine A, and let Q_B R_B be corresponding quantities for the engine B, so that by A the heat Q_A is taken from the hot body, and the heat R_A added to the cold body, while by B the heat Q_B is added to the hot body, and the heat R_B taken away from the cold body. Then the final result of the working of the combination is that $Q_A - Q_B$ has been taken away from the hot body, and $R_A - R_B$ has been added to the cold body. But since the power of the engines is the same,

$$Q_A - R_A = Q_B - R_B,$$
$$\therefore\ Q_A - Q_B = R_A - R_B;$$

so that the result of the working of the combination is that an amount of heat $Q_A - Q_B$ has passed from the hot body to the cold one. Now, since the combination is self-acting, the second law of thermodynamics tells us that Q_A cannot in any case be less than Q_B, but must be either equal or greater, for if Q_B were the greater, the heat $Q_B - Q_A$ would pass from a cold body to a hot one through the agency of a self-acting machine. But, the efficiencies of the engines are $\frac{Q_A - R_A}{Q_A}$ and $\frac{Q_B - R_B}{Q_B}$, of which fractions the numerators are equal, and hence we learn that the efficiency of the engine B cannot be less, though it may be greater, than that of the engine A.

Next imagine not only the engine B, but also the engine A to be reversible, and suppose the direction of the combination reversed, so that B works forward and A works backward, then manifestly the same reasoning shows that the efficiency of A cannot be less but may be equal to that of B, and consequently when both A and B are reversible we must conclude that their efficiencies must be equal.

Moreover, we conclude that the efficiency of an irreversible engine cannot be greater, so that an engine which is reversible is also an engine of maximum efficiency. In fact an engine which is not reversible has probably always a lower efficiency, as is illustrated by the example of the last chapter, and shown more fully presently.

56. The foregoing reasoning may be put in a somewhat different form. Instead of supposing the two engines to be of the same power, let the refrigerating machine (say B) supply, at the higher limit of temperature, an amount of heat which is just enough to supply the heat engine A, or in other words let $Q_A = Q_B$. Then if the efficiencies of the two engines are not the same, suppose that of A the greater, so that $Q_A - R_A$ is greater than $Q_B - R_B$. Evidently R_A must be less than R_B, so that the amount of heat $R_B - R_A$ is drawn from the cold body by the combined action of the two engines and converted into work, without any heat being taken from the source of heat. If the efficiency of A is less instead of greater than B, the same conclusion is arrived at by reversing the whole arrangement. Unless, therefore, Carnot's principle be true, we must admit that a machine can be constructed which not only will draw heat from a cold body, however low its temperature, but will actually convert the whole of that heat into mechanical energy. A refrigerating machine on this hypothesis would be capable, not only of working without motive power, but might even serve as a source of motive power. The impossibility of this is expressed by an alternative statement of the second law.

Mechanical Energy cannot be obtained from heat by cooling a body below the temperature of surrounding objects. (Thomson's Statement.)

Whichever statement we adopt, the law merely expresses the fact that the value of a quantity of heat for mechanical purposes entirely depends upon its temperature. If its temperature is low, the heat-energy, though existing in its full amount, will be unavailable for mechanical purposes. That energy may exist in a form which is useless to us, is a matter of common experience. For example, each cubic foot of the air in a room, if it were allowed to expand while driving a piston till its pressure falls to zero, would exert energy amounting to about 2½ foot tons. The whole room therefore contains an enormous amount of energy, none of which is available for any useful purpose.

57. We can now state certain general conclusions respecting the action of heat engines of maximum efficiency, or, as we may express it, of PERFECT heat engines, when supplied with heat from a single external source.

Each repetition of the series of changes which the fluid undergoes includes four periods.

(1) Expansion at constant temperature accompanied by reception of heat from the source.

(2) Expansion at falling temperature without gain or loss of heat.

(3) Compression at constant temperature accompanied by rejection of heat.

(4) Compression at rising temperature without gain or loss of heat.

But we have already considered a reversible air engine in which the cycle is of this kind, and we found that the efficiency was (page 91)—

$$\frac{U}{Q} = \frac{Q - R}{Q} = \frac{T_1 - T_3}{T_1},$$

where T_1, T_3 are the absolute temperatures of the hot and cold body. We therefore conclude that every engine in which the working fluid goes through this cycle must have an efficiency expressed by this formula, whatever the fluid be; and, moreover, that no non-reversible engine can have a greater efficiency.

In the steam engine, as will be explained farther on more fully, the first period is that of the evaporation of the water in the boiler, the second is that of expansion in the cylinder, the third that of condensation in the condenser, while the fourth is that of forcing the water into the boiler; but inasmuch as the ordinary steam engine is not a perfect engine, but can only be made so by certain ideal arrangements to be explained farther on, none but the first satisfy the needful conditions for maximum efficiency, and even the first is usually imperfect in practical cases.

Let now I_1 be the internal work done during the first period, and E_1 the energy exerted on the working piston, then

$$Q = I_1 + E_1;$$

so, if I_3 be the internal energy given out during compression in the third period, and C_3 the corresponding work done upon the fluid,

$$R = I_3 + C_3.$$

In the second period, a portion of the internal energy stored up in the fluid is utilised in performing external work by expansion; while, in the fourth period, energy derived from external sources is employed in compressing the fluid and increasing its store of internal energy.

The equation just given connecting U, Q, and R may be written thus:

$$\frac{I_1 + E_1}{T_1} = \frac{I_3 + C_3}{T_3};$$

that is to say, although the internal and external work may vary greatly according to the nature of the fluid, yet they are not altogether independent of each other, but must satisfy the above equation, which furnishes a restiction on the possible variation of constitution of fluids.

In the case of air, the quantites I_1 I_3 are zero, since no change of internal energy takes place when perfect gases expand or contract at constant temperature, all the heat received or rejected being represented by the energy exerted or work done; hence,

$$\frac{E_1}{T_1} = \frac{C_3}{T_3};$$

that is to say, in fluids so constituted, although all the heat expended is transformed into mechanical energy in the first instance, yet the power required to work the indispensable compressing apparatus is so great as to absorb the greater part of it, leaving only a fraction available for useful purposes. On the other hand, if instead of air we employ steam, then the power required to work the compressing apparatus, that is to say, to overcome back pressure, is comparatively small; but then the quantities I_1 I_3, representing heat expended in internal changes, so far from being zero, are very large, so that in this case also but a small fraction of the heat expended is available for useful purposes.

Our general reasoning shows that this difficulty can never be overcome, and that if the heat be used in any other way than that just described, the result will be still more unfavourable.

Analogy between a Heat Engine and a Water-power Engine. Auxiliary Heat Engines.

58. A reader who for the first time is introduced to the reasoning just now given, will complain, and not altogether without reason, of its abstract and difficult character. This is in part due to its great generality, for all reasoning is more difficult to grasp the greater number of cases it covers; but it is also due to the novelty of the ideas involved, which are in many respects different from anything to be found in any other branch of science. It may be then of some service, if we draw a parallel between the case of a heat engine and that of an engine driven by falling water, which although far from perfect, is nevertheless so far true as to be some help in the comprehension of the ideas in question.

In order to produce mechanical energy by means of heat, we must, in the first place, have bodies of different temperatures, since all changes produced by heat are due to the passage of heat from one body to another, for which passage difference of temperature is an indispensable condition. In the second place, in addition to the hot body or source of heat, and the cold body or receiver of heat, we must have a third intermediate body, by the agency of which heat is transferred from the hot body to the cold body, and through the changes of volume of which mechanical energy is exerted. If no such third body exist, the heat simply passes from the hot to the cold body without doing any work, and when once it has done so, the second law just enunciated tells us that the opportunity of doing work, which might have been utilised, has been irretrievably lost.

Now, when work is done by means of falling water, it is in the first place necessary to have a fall, that is, a passage of water from a high to a low level, and in the second, a suitable machine to receive the falling water, and transfer it mechanically from the high level to the low level; if no such intermediate agency exists, the water descends indeed, just as the heat flows from the hot body to the cold one, but none of the energy of the falling water is changed into useful work.

Hence heat may be described as descending from a high temperature to a low one and doing work by the agency of steam or air in the course of its descent, while the power of doing work by such descent, if not properly utilised, is lost in the water-power engine, just as in the heat engine. Hence also the conditions of perfect efficiency may be described in terms to a great extent identical. For, in the water-power engine, in order that every particle of the energy of the falling water may be employed in useful work, the transfer of the water from a high level to a low level must be wholly produced by the artificial agency of the water wheel; if the water pours from a certain height on to the wheel, or pours off the wheel on to the lower level, the difference of level thus represented is wholly or partially wasted. So in the heat engine, if the heat descends from the source to the steam or air, or from the steam or air to the cold body, through a sensible interval of temperature, then the difference of temperature in question, which might have been utilised, is wasted, or, in other words, for maximum efficiency the steam or air must receive heat at the constant temperature of the source of heat, and reject it at the constant temperature of the cold body or receiver of heat. And, still further, the condition for maximum efficiency of a water machine may be described as consisting in the machine being *reversible*, for that condition consists in the water entering the wheel without shock, and leaving it without velocity, a condition which, if exactly satisfied, would enable us, by reversing the motion of the water wheel, exactly to reverse the process to which the water is subjected, raising it without loss of energy from the low level to the high level by means of energy drawn from external sources. An imperfect water wheel may in like manner be described as non-reversible.

The parallel here drawn is due to the celebrated Carnot, to whom we also owe the conception of a reversible heat engine: but it applies more completely to Carnot's conception of the action of a heat engine than to its true action as now understood. Carnot was a believer (like most persons of his time) in the material nature of heat, and hence, when he speaks of heat descending from one level to another, none of it is regarded as disappearing in the process, but the work done is conceived as done at the expense of the difference of temperature: heat of high temperature possessing more energy (to use the language of modern science), than heat of low temperature. We now know that this is not so; the mechanical equivalent of the heat in the condenser of an engine is just the same as that of the heat in the boiler, and hence difference of temperature does not in itself constitute energy, but is merely an essential condition that heat may be changed into work. But although temperature is not in itself energy, yet it is temperature which gives to heat its value for all useful purposes, so that the parallel in question still to a great extent holds good. The parallel is by some writers made closer by the introduction of certain ideal quantities called "heat-weights" found by dividing quantities of heat by the corresponding absolute temperatures, but the "heat weight" appears to be such a purely artificial conception that we have not thought it desirable to introduce it here.

59. An additional help towards the comprehension of the principles we have been trying to explain occurs in considering the action of the steam and ether engine actually used for marine propulsion, and only abandoned on account of the practical difficulties involved.

Ether is a fluid which evaporates and produces vapour of considerable pressure at temperatures not exceeding the temperature of an ordinary condenser. Accordingly, the surface condenser of a steam engine may be used as a boiler of an ether engine, which will do work without the expenditure of any heat except that furnished by the exhaust steam of the steam engine; the work so done will be so much clear gain, and the efficiency of the combined steam and ether engine will be greater than that of the steam engine alone. Now the reason of this is clearly that we make use of the difference of temperature, otherwise wasted, between the condenser of the steam engine and surrounding bodies.

So, again, the hot gases of a furnace have a very high temperature, far higher than the temperature of the steam boiler by which the heat is utilised. If then we imagine a fluid to exist which, evaporated at or near that temperature, produces vapour of considerable pressure, then that fluid might be used in an auxiliary engine, which received heat from the hot gases, and rejected heat into the steam boiler, which would serve as its condenser. In such a case the work done by the combination and the heat expended would be increased by *equal* quantities, and the efficiency would consequently be greater than that of the simple steam engine.

Such auxiliary engines may involve practical difficulties, rendering them incapable of being brought into practical use, but their conception alone is sufficient to enable us to see how completely the power of a heat engine is dependent on difference of temperature, and how certain it is that the greater the difference of temperature the more efficient the engine must be, other things being equal. Moreover, we can see that every time difference of temperature is wasted, efficiency is wasted, or, in other words, that among the conditions of maximum efficiency must be those already stated, namely, that the steam, air, or other fluid *must receive heat at the constant temperature of the source of heat, and reject heat at the constant temperature of the receiver of heat.* To put the same thing in other words: we must utilise, as far as possible, every available difference of temperature, just as, in the case of an hydraulic machine, we carefully utilise every part of the available fall.

The other condition, previously stated to be included in reversibility, has not perhaps been absolutely proved to be in all cases indispensable to maximum efficiency, but no doubt in all practical cases it is necessary that no part of the expansive energy of the fluid should be converted into kinetic energy by wholly or partially unbalanced expansion.

Every possible condition, of whatever kind, however, is always included in the one word *reversibility*; every heat engine which is completely reversible will be of maximum efficiency, though it may be difficult to prove, conversely, that the efficiency of every conceivable non-reversible engine is necessarily less.

60. Let us now imagine the temperature T_1 of the hot body, or source of heat, to be divided into n equal parts, and let us imagine a quantity of heat Q to flow from that body to a second body, the temperature of which is $T_1\left(1-\frac{1}{n}\right)$, then our results show that a quantity $\frac{Q}{n}$ of mechanical work is capable of being produced, and that, consequently, if such conversion be effected, the quantity of heat $\frac{n-1}{n}\cdot Q$ will pass into the second body. Now imagine a third body, the temperature of which is $T_1\left(1-\frac{2}{n}\right)$, and let this heat

pass from the second body to the third body; then the heat capable of being turned into work is

$$\frac{n-1}{n} \cdot Q \cdot \frac{T_1}{n} \cdot \frac{1}{T_1\left(1-\frac{1}{n}\right)};$$

that is $\frac{Q}{n}$ as before. This process may be continued indefinitely, and we thus see that—

If the temperature of a source of heat be divided into any number of equal parts, then the effect of each of these parts in causing work to be performed is the same.

It is in this form that Rankine enunciates the second law of thermodynamics, and his view may be illustrated by the analogy just considered between the difference of level, which causes work to be performed by an hydraulic machine, and the difference of temperature which causes work to be performed by a heat engine. Each foot of fall in a water wheel is equally effective in doing work by means of the water wheel, and just so each degree of temperature passed through during the passage of heat from a hot body to a cold one is equally effective in causing work to be done by a heat engine working by means of this heat.

The temperatures in question are, as our investigation shows, to be measured on the perfect gas thermometer, which is therefore entitled to be considered as a definite measure of temperature. Temperature, being by its nature incapable of direct measurement, can only be measured by considering some physical effect which difference of temperature produces; thus the ordinary mode of measurement is by observing the expansion which bodies undergo when their temperature is raised. This, however, is inconvenient for scientific purposes, since no two thermometers give exactly the same results; for instance, the mercurial and air thermometers, if graduated to indicate correctly the temperatures of melting ice and of water boiling under a given pressure, will be found to differ at intermediate temperatures. We must, therefore, consider some other physical effect due to difference of temperature, and the only one known to be independent of the particular body operated on is the power, of which we have just been speaking, which difference of temperature possesses of converting heat into work. If equal intervals of temperature be understood to mean equal capability of converting heat into work, we get a scale of temperature which may, with propriety, be called *absolute*, and which, as our investigation shows, coincides with that of the perfect gas thermometer, that is (sensibly) with the air thermometer; and the term "absolute," already applied to temperature measured in this way, is hereby justified.

Performance of a Perfect Heat Engine. Application to the Steam Engine.

61. Thus we see that if a heat engine be perfect, its efficiency is $\frac{T_1 - T_3}{T_1}$, where T_1 T_3 are the absolute temperatures between which the engine works; or if U be the work done in a given time, Q the heat expended in the same time,

$$Q = \frac{T_1}{T_1 - T_3} \times U,$$

hence the expenditure of heat per H.P. per minute is given by

$$Q = 42 \cdot 75 \, . \, \frac{T_1}{T_1 - T_3} \, . \text{ (thermal units)},$$

a formula which gives the least amount of heat necessary to produce the given power, so long as we are restricted to work within the given limits of temperature. Let us now consider what those limits of temperature are in the case of the steam engine.

The inferior limit T_3 can in no case be less than the temperature of the atmosphere, and in the case of the condensing steam engine may be taken as 100° F., which is about the temperature of the condenser. Du Tremblay, indeed, virtually lowered this temperature to 60° F., or thereabouts, by the addition of an ether engine working between the temperature of the exhaust steam and the temperature of condensation of ether; but on account of the practical difficulties attending the use of ether, this plan is not likely to come into ordinary use. In the non-condensing steam engine, T_3 is the temperature corresponding to the atmospheric pressure, that is to say, 212° F.

The superior limit T_1 in a simple steam engine, is the temperature of the boiler; for although the hot gases of the furnace have a vastly higher temperature (say t), yet the power of turning heat into work, due to the difference of temperature $t - T_1$, cannot be realised by the arrangements of an ordinary steam engine. In order to realise it, it would be necessary to have a fluid which evaporates at a temperature t of say 1000° F., and condenses at a temperature a little above that of an ordinary steam boiler. Such temperatures are impracticable in practice, and hence the superior limit must be, as stated, the temperature of the boiler.

The table on page 142 shows the performance of various descriptions of perfect heat engines working under various circumstances, expressed in thermal units per I.H.P. per minute, and also in terms of the consumption of coal.

The consumption of coal will, of course, depend on the quality of the coal and the efficiency of the boiler. In seeking a theoretical limit to the amount of power which can be produced from a pound of coal, it is perhaps proper to consider the efficiency of the boiler unity, in which case, if we adopt pure carbon as the standard quality of coal, the consumption will be given by

$$C = \frac{60\,Q}{14{,}500}.$$

The total heat of combustion of actual coal is sometimes nearly 10 per cent. greater than that of carbon, but is more often less.

The table, then, shows the consumption of a perfect engine and boiler under the circumstances indicated: to provide for losses connected with the boiler, from 30 to 50 per cent. must be added to the numbers given.

The sixth column shows the efficiency, from which it appears that, in the best possible steam engine, at least two-thirds of the whole heat expended is wasted, the waste arising from no fault in the construction or nature of the engine, but solely from the narrow limits of temperature within which we are restricted to work. To obtain a better result it will be indispensable to overcome in some way or other the practical difficulties which exist in employing unusual temperatures.

Again, the expenditure of heat in a perfect steam engine, as shown by the fourth column, is much less than that of an actual engine under the same circumstances, showing that faults in the construction of the engine or the treatment of the steam must exist, which, at least theoretically, are remediable. This is shown in a striking manner by comparing the performance of a perfect engine working at 95 lbs. per square inch absolute, with that of the same engine working as in Chapter III. (page 51). Here the boiler pressure is the same and also the condenser temperature, yet the expenditure of heat is, even at the greatest expansion theoretically available, 236 thermal units per I.H.P. per minute, instead of 150 as in the perfect engine. The exhaust waste is here not included, and the loss arises from improper application of heat and excess back pressure, as will be explained fully in a later chapter.

PERFORMANCE OF A PERFECT HEAT ENGINE.

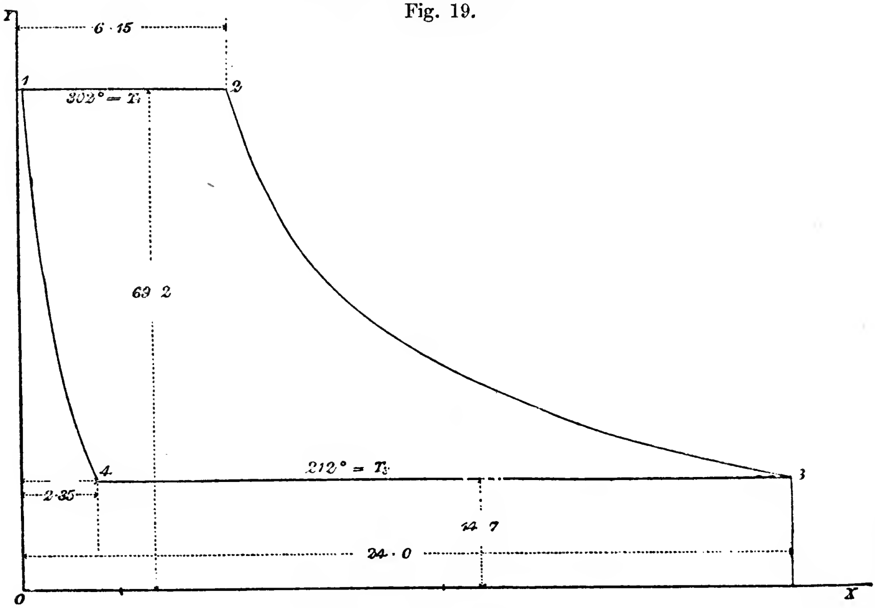
Description of Engine.	Superior Temperature.	Boiler Pressure absolute.	Thermal Units per I.H.P. per Minute.	Pounds of Carbon per I.H.P. per Hour.	Efficiency.	Heat rejected per I.H.P. per Minute.
Non-condensing steam engine. Inferior temperature, 212°.	401°	250	195	·806	·219	152
	363	160	233	·964	·183	190
	341	120	266	1·10	·161	223
	312	80	329	1·36	·130	286
	287	55	427	1·77	·100	384
Condensing steam engine. Inferior temperature, 100°.	341	120	143	·592	·299	100
	324	95	150	·621	·285	107
	293	60	167	·691	·256	124
	250	30	203	·840	·211	160
	228	20	230	·952	·186	187
Steam and ether engine. Inferior temperature, 60°.	341	120	122	·505	·351	79·3
	293	60	138	·571	·309	95·3
Air Engine. Inferior temperature, 60°.	660	..	79·8	·33	·536	37·1

The table further shows that the gain by the use of steam of high pressure is not very great, because the temperature of steam at high pressure increases but slowly with the pressure. The theoretical gain by the addition of a condenser, on the other hand, is very large;

but it will be seen hereafter that the condensing engine is a much more imperfect machine than the non-condensing, so that much of that gain cannot be realised in practice. When the temperature is low the pressure is low, and this renders it more difficult to approach in practice the theoretical efficiency.

In all cases considered in this chapter the heat is supposed to be supplied at a given temperature; if, as is generally the case, a part or the whole is supplied at varying temperature the efficiency will be less, the *maximum* temperature being supposed the same in the two

Fig. 19.



cases. This question is fully considered in a later chapter, but to avoid misapprehension it may here be stated that a closer estimate of the maximum efficiency may be made by replacing the *maximum* temperature T_1 by the *average* temperature of supply T_m, this average temperature being calculated by a rule hereafter to be explained.

62. We will now consider the case of a steam engine working under conditions of maximum efficiency.

The figure (Fig. 19) shows the indicator diagram of an engine of maximum efficiency, $O X$ being the line from which pressures are measured, and $O Y$ the line from which volumes are measured. At the point 1 the pressure and volume of 1 lb. of water in the boiler are represented, which water, during admission, is evaporated at constant pressure, as represented by the straight line 1 2, the other extremity 2 of which represents the pressure and volume of the steam produced. The steam is then cut off, and expansion takes place without gain or loss of heat till the pressure has fallen to 3, which must be supposed the pressure in a surface condenser. At the end of the stroke the piston returns, and condensation takes place under that same constant pressure.

So far the diagram is exactly the same as an ordinary indicator diagram in which expansion has been carried to its extreme limit, namely, till the pressure has fallen to the pressure of the condenser. But now, instead of the condensation being complete, we must imagine it stopped at a suitable point 4, and the mixture of steam and water compressed without gain or loss of heat until it becomes once more water of the pressure and temperature of the water in the boiler. Then, if we suppose the condensed steam returned into the boiler, the process may be repeated indefinitely. The diagram is drawn to suit the particular case in which the engine is working between the temperatures 302° and 212°: the possibility of the supposed operations and the manner in which the numerical data are obtained will be shown hereafter (page 217).

Evidently this engine satisfies the conditions of maximum efficiency, for it receives all its heat at the constant temperature (T_1) of the boiler, and rejects all its heat at the constant temperature (T_3) of the condenser, and is moreover in all other respects *reversible*. Hence its efficiency must be $\frac{T_1 - T_3}{T_1}$, but the heat expended per 1 lb. of steam is L_1, the latent heat of evaporation of water at the absolute temperature T_1, therefore we have

$$\text{Work done per lb of steam} = \frac{T_1 - T_3}{T_1} \cdot L_1,$$

which work is represented geometrically, as usual, by the area 1, 2, 3, 4 of the indicator diagram,

$$\therefore \text{ Area of Diagram} = \frac{T_1 - T_3}{T_1} \cdot L_1 .$$

This equation leads to an extremely important result, for let us suppose that T_1 is very nearly equal to T_3, then the area of the diagram is very nearly

$$(P_1 - P_3)(v_1 - s),$$

where P_1 P_3 are the pressures corresponding to the temperatures T_1 T_3 of saturated steam, v_1 the specific volume of saturated steam at T_1, and s the specific volume of liquid water. Then we have

$$(P_1 - P_3)(v_1 - s) = L \, . \, \frac{T_1 - T_3}{T_1};$$

or if ΔP be the rise of pressure corresponding to a very small rise of temperature ΔT,

$$\Delta P (v - s) = L \, . \, \frac{\Delta T}{T},$$

in which equation the suffixes are omitted as no longer required.

The consumption of steam in a perfect steam engine, working as just described, is given by

$$N = \frac{60 \, Q}{L_1}$$

where L_1 is the latent heat of evaporation at T_1. The consumption of steam is a measure of performance which is always inaccurate, because a pound of steam requires a different amount of heat to produce it, according to the temperature of evaporation and the temperature of the feed water. In the present case the feed is raised in temperature by compression instead of by direct application of heat, and consequently each pound of steam requires much less heat to produce it. Hence an engine might be conceived which consumed less steam while at the same time, not being perfect, its expenditure of heat was greater. The total heat of evaporation $H_1 - h_0$ is sometimes used, for this reason, as a divisor instead of L_1, but the best measure of the performance of an engine is the quantity of heat expended per I.H.P. per minute; the consumption of steam should only be used as a measure of *comparative* performance when the temperature of the feed is nearly the same in the cases compared.

Vapour Engines.

63. The indicator diagram just given (page 143) for the case of a steam engine working under conditions of maximum efficiency

applies to all engines employing a volatile fluid which, during the cycle of the engine, is alternately evaporated and condensed. The maximum efficiency of all such engines is given by the same formula, and the latent heat of evaporation is connected with the pressure, density, and temperature in the same way, so that with the same notation as in the preceding article we find, from the result just given,

$$L = (v - s) T \frac{dP}{dT}$$

a very important formula, true for all fluids, and which may be even employed in the case of the fusion of a solid.

Engines which employ a volatile fluid as the working agent may be described generally as Vapour Engines; they form a distinct class of thermodynamic machines, of which the steam engine is the principal example. They possess the great advantage that the compression part of their cycle absorbs only a small part of the energy given out in the expansion part. So far as thermodynamic efficiency is concerned this is unimportant, but the bulk and friction of the machine are much diminished, as may be seen from what was said in the case of air engines.

A vapour engine, therefore, is generally the most economical heat engine. The kind of fluid makes no difference so far as thermodynamic efficiency is concerned, but in practice many things have to be considered of which the pressure due to a given temperature is one of the most important. If that pressure be too low for the temperature it is intended to use, the bulk of the machine and various losses of the nature of friction will be too great. Hence the kind of fluid chosen will depend chiefly on the range of temperature it is desired to utilise. For such temperatures as are used in non-condensing steam engines no fluid is so suitable as water, the efficiency of such engines in practice being very considerable as compared with that of a perfect engine working between the same temperatures. In condensing steam engines the pressure at the lower part of the range of temperature is too low and a different fluid would be advantageous, while to utilise differences of temperature below 100° F. a more volatile fluid is indispensably necessary. No such fluid has hitherto been found suitable for the purpose, and such differences of temperature are consequently at present altogether wasted. To utilise high temperatures, say above 400°, some form of gas engine must be employed, for in steam engines such temperatures are injurious to the working parts. And it must further be remembered that the action of the sides of the cylinder might possibly be much diminished by the use of a working fluid other than steam. Apart from considerations such as these, the kind of fluid is immaterial.

Performance of a Reversed Heat Engine. Reversed Vapour Engines.

64. A thermodynamic machine is capable of being worked either forwards or backwards, and most actual machines occur in both forms. In the first case a current of heat flows from a hot body to a cold body, and a fraction of it is converted into mechanical energy : the machine is then a heat engine, by means of which we derive mechanical energy from heat. In the second case the direction of the current is reversed, so that the heat flows from a cold body to a hot body in consequence of a supply of mechanical energy from external bodies. The machine is then commonly employed for refrigerating purposes, and for this reason is called a "refrigerating machine"; but it might also be called a "warming machine," being capable of being used for the production of warmth by heat derived from cold surrounding objects.

When the thermodynamic machine is perfect, the relation between the quantity of heat flowing in the current and the mechanical energy generated or absorbed, is exactly the same in whichever direction the machine is worked, being dependent solely on the interval of temperature bridged over by the machine and the temperature of the current. All the results given in the table, page 142, are, therefore, capable of being used for a reversed heat-engine when properly interpreted. As, however, the limits of temperature are generally quite different in the two cases, being much lower in the reversed engine, it is advisable to make a fresh calculation.

The annexed table gives numerical results in two cases. In the first a perfect refrigerating machine is employed to abstract heat from some liquid not easily frozen, at a temperature of 19° F., and discharge it into the atmosphere at 67° F. The table shows the heat abstracted per H.P. per minute, corresponding to the "heat rejected" in the heat engine of the preceding table. The cold liquid is supposed to be employed to produce ice from water at the atmospheric temperature, and the table shows the weight of ice per H.P. per hour.

PERFORMANCE OF A REVERSED HEAT ENGINE.

Temperatures.		Thermal Units per H.P. per Minute.		Lbs of Ice per H.P. per Hour.	Cubic Feet of Warm Air per Minute.	Efficiency.
T_1	T_3	Abstracted.	Supplied.			
528 = 67° F.	480 = 19° F.	427	470	146·6	..	10
553 = 92° F.	493 = 32° F.	351	394	..	382	9·22

In the second case a reversed heat engine is employed to warm air derived from the atmosphere at 32°, the temperature desired being 92° F., so that the air is warmed 60°. The table shows the heat supplied per H.P. per minute corresponding to the "heat expended" in the heat engine, and also the number of cubic feet of air which, theoretically, might be warmed at *constant pressure* by this amount of heat. A comparison of these results with the corresponding results for the heat engine is not without interest, for it shows that, ideally, a horse-power may be produced by an expenditure of heat of about 80 thermal units per minute, and employed to generate about 400 thermal units per minute, that is, about five times as much, the difference being drawn from the atmosphere. When, therefore, we warm our houses by the direct action of heat derived from combustible bodies, we waste by far the greater part of it by making no use of the high temperature at which the heat is generated, a small quantity of heat at high temperature being ideally capable of raising a large quantity to a moderate temperature.

The last column of the table shows the "efficiency"—using this term in the same sense in which it is used in the case of a heat engine. In the warming machine we see that the supply of heat is 9·22 times the heat equivalent of the mechanical energy supplied by water-power or some other motor, a number which is the reciprocal of the "efficiency" of the corresponding heat engine. In the refrigerating machine the heat abstracted, which in this case measures the value of the machine, is 10 times the heat equivalent of the power of the motor. Further remarks will be made on this point at the close of the present chapter.

65. As already explained at some length, the difficulty of economically heating and cooling large masses of air, and internal friction, render a reversed air engine an inefficient refrigerating machine so far as regards economy of power. They are, therefore, frequently replaced by reversed vapour engines.

The diagram of a reversible vapour engine has already been given (Fig. 19, page 143), and we have only to choose a suitable vapour to employ.

(1) A reversed steam engine may be used as an ice-making machine, working between the limits of 32° and say 95°, but the corresponding pressure limits are very low, being only about $12\frac{1}{4}$ and 116 lbs. on the *square foot*. Hence the machine is only suitable for use on a small scale.*

(2) In the case of ether the pressures at the limits of temperature just mentioned are much more convenient, being about $3\frac{1}{2}$ and 15 lbs. on the square inch respectively, and an ether machine has consequently been used to a considerable extent. The principal objection appears to be the inflammable character of the fluid, which has occasioned several accidents.

(3) More recently anhydrous ammonia has been used to a great extent, though in warm climates the pressure is somewhat excessive, being about $4\frac{1}{4}$ and $13\frac{1}{2}$ *atmospheres* at the limits stated above. This machine will now be briefly noticed.

The essential parts of the machine are (1) a double-acting, compressing pump, driven by a steam engine, which forces ammonia gas into (2) a condenser consisting of a coil of iron piping surrounded by circulating water which abstracts the necessary amount of heat. These operations are represented in the diagram, Fig. 19, by the compression curve 32 and the horizontal line 21. A small orifice at the further end of the condensing coil now allows the liquefied ammonia to escape at high pressure into a continuation pipe which leads to (3) the refrigerator, consisting of a second coil of piping placed in a refrigerating tank containing brine. During this operation the liquid ammonia evaporates, in consequence of the fall of pressure, and

* A description, with illustrations, of one of these machines by Messrs. Southby & Blythe, will be found in the *Enjineer* for December 28th, 1888.

absorbs heat from the brine, which is thereby cooled to any desired extent. The first part of this operation, which would otherwise be represented by the curve 14 in the diagram, is irreversible, but the second part is represented by the horizontal line 43. The gas now passes back into the compressing pump and the cycle recommences.

The cycle of this machine is by no means perfect, and besides there are losses from the friction of the long coils of piping and leakage of heat into the refrigerating tank. A machine of this class on the Lindé system is said to require 30 H.P. (effective) to produce 1 ton of ice per hour, which is equivalent to about 75 lbs. per H.P. per hour. If this estimate be correct, we find, on comparison with the result given above for an ideally perfect machine, that the "true" efficiency is about 50 per cent.

The irreversibility of a part of the cycle is due to the absence of a pump similar to the feed pump of a steam engine: if this could be provided the machine might evidently be used as a heat engine in cold climates. Little has been as yet done to utilise differences of temperature such as that between the condenser of a steam engine and the atmosphere. An ammonia engine must be very perfectly closed from the nature of the gas, and the mechanical difficulties of making it so appear to have been completely overcome in the Lindé machine, the loss of ammonia in which is very small.

Efficiency of Thermodynamic Machines in general.

66. We are now enabled to give a complete answer to the question which forces itself on our attention on first commencing the study of heat engines, as to the cause of the low apparent efficiency of these machines. We find that no direct method of converting heat energy into mechanical energy exists or is even conceivable: such direct conversion would involve a manipulation of the ultimate atoms of which matter is composed, which may be compared with an attempt to reverse the unbalanced expansion of an elastic fluid (page 129) but which is still less conceivable. The only way in which the conversion can be accomplished is by a machine which utilises some difference of temperature which is available to us, by means of a current of heat, the magnitude of which is greater the smaller the difference of temperature, and which is always greater than the mechanical energy generated. The ratio of

the energy generated to the flowing current which, in the first instance, we call the "efficiency" of the engine, though it may be regarded as a measure of *absolute performance*, is in no sense a measure of *perfection*, that is, it is not the *true* efficiency. It has no other signification than the values greater than unity which we obtain for the apparent efficiency in a refrigerating machine, and would more properly be described as a Co-efficient of Performance. The true efficiency of a thermodynamic machine can only be obtained by comparison with that of a reversible machine working between the same limits of temperature.

Thus we find that a unit of heat, though it always possesses a certain definite mechanical *equivalent*, the same in all circumstances, has a mechanical *value* which is only definite when the temperatures are known between which the heat can be used. This value is the amount of energy exerted by a perfect engine working between those temperatures, and with given temperatures is entirely independent of the kind of machine employed.

And, conversely, a given quantity of mechanical energy, though always *equivalent* to a certain definite amount of thermal energy, has a thermal *value* which is much greater, being the sum of the equivalent and a quantity of heat, generally much more considerable, which by a perfect thermodynamic machine can, ideally, be drawn out of cold bodies and applied to useful purposes at any temperature which we desire.

Finally, it is of much importance to remark that, as far as air and gas engines are concerned, the conclusions we have arrived at are independent of the indirect reasoning by which Carnot's principle is established, and might have been derived at once from the investigations relating to these machines in the preceding chapter. All that the reasoning in question shows is, that all bodies alike must be so constituted as to render these conclusions true in all cases.

CHAPTER VI.

GENERATION AND EXPANSION OF STEAM.

A DETAILED comparison between the operation of a perfect steam engine and the steam engine as it actually exists will be instituted in a later chapter: for the present, however, we proceed to consider, more fully, various questions relating to the formation of steam and the thermodynamic changes occurring in saturated steam and mixtures of steam and water during expansion in a steam cylinder.

Calculation of the Density of Steam.

67. First of all it will be shown that the formula already obtained (page 145)—

$$\Delta P (v - s) = L \cdot \frac{\Delta T}{T}$$

may be used to find the density of dry saturated steam of given pressure.

In Table I*a* the pressure in lbs. per square inch is given for any temperature, and, besides, the rise of pressure for 1° rise of temperature. By multiplication by 144 we get ΔP, when $\Delta T = 1°$, and by the addition of two consecutive results the value of ΔP is obtained for 2°, corresponding approximately to the intermediate temperature: but L is known from Regnault's experiments, hence $v - s$ is determined.

For example, to calculate the volume of 1 lb. of steam at a pressure of 25 lbs. on the square inch. Here the corresponding temperature is 240° Fahr., and the value of ΔP for 2° is

$$\Delta P = (\cdot 458 + \cdot 452) \, 144 = 131 \text{ lbs. on the square foot;}$$

also by Table II*b* the latent heat of evaporation per lb. is

$$L = 730{,}700 - 550 = 730{,}150;$$

therefore, using the formula found above,

$$131\,(v - s) = \frac{730{,}150 \times 2}{461 + 240}\,;$$

$$\therefore\ v - s = \frac{1{,}460{,}300}{701 \times 131} = 15{\cdot}902,$$

whence putting $s = {\cdot}016$ we get to two places of decimals,

$$v = 15{\cdot}92,$$

closely agreeing with the result given in the density table. To obtain very accurate results by this method it is necessary, in order to obtain a more approximate value of $\Delta\,P$, to use a formula derived from one of the formulæ representing the relation between pressure and temperature, instead of resorting to the table, because the differences are not given by the table with sufficient accuracy and (especially at high temperature) the interval of 2° is not sufficiently small. This is further explained in the Appendix.

It is by this method that the density of steam is found by calculation, as mentioned in Chapter I. The possible errors in the calculation are as follows :—

(1) The mechanical equivalent of heat is taken as 772. As stated in Chapter II. this value is certainly very approximate, though, as will be explained in the Appendix, there is good reason to believe that it may be somewhat too small. From the formula it is clear that the calculated values of the density of steam are all in the same proportion, subject to any error occasioned in this way.

(2) The temperatures were measured by Regnault in his experiments on a thermometer of real air, whereas in the formula temperatures are measured by a thermometer in which a perfectly gaseous body is used. The error here cannot be estimated precisely, and will be different at different parts of the scale; but there is reason to believe that such error is very small. Also the position assumed for the absolute zero is open to possible error not exceeding a degree centigrade. Some particulars on these points will be found in the Appendix.

Subject to these observations the density of steam is calculated with the same degree of accuracy that Regnault's experiments were made; and although no calorimetrical experiments can be expected to be free from minute errors, yet it is certain that the accuracy actually attained was very great: so that the density of steam determined in this way must be within (probably we may say) one per cent. of the truth for steam of the same degree of dryness as that experimented on by Regnault. The precautions taken by Regnault to secure dry steam were explained in a former chapter, and it is probable that the steam from an ordinary steam boiler is usually not free from suspended moisture, in which case its latent heat will be less and its density greater, as before stated.

The densities of steam at different pressures are tabulated by Rankine in his work on the Steam Engine; they differ by minute quantities from the results given by continental writers who mostly base their work on Clausius' investigations made independently at about the same time. The densities given in Table III., and certain quantities in Table IV. dependent on them, are derived from Rankine's results, as more exact values are not attainable in the present state of our knowledge. It is much to be wished that a further direct experimental investigation should be made of the density of steam.

In Chapter II. the internal work done during evaporation at constant temperature was expressed by means of an equivalent pressure on the piston, called, for brevity, the "internal work pressure." The same equation which furnishes the density of steam likewise furnishes the value of this pressure. For let $\overline{P}$ be this pressure, then

$$\begin{aligned} \text{Internal Work} &= \overline{P}\,(v - s)\,; \\ \text{External Work} &= P\,(v - s)\,; \\ \text{Heat Expended} &= L\,; \end{aligned}$$

$$\therefore L = (\overline{P} + P)\,(v - s) = (\overline{P} + P) \cdot \frac{L}{T} \cdot \frac{\Delta T}{\Delta P}$$

whence

$$\overline{P} = \frac{T \Delta P}{\Delta T} - P,$$

an equation which furnishes an easy means of calculating $\overline{P}$, which, it will be observed, is just the same whether the evaporation be partial or whether it be complete: that is to say, the values of $\overline{P}$ are not subject to any such uncertainty as may be considered to exist respecting the density of steam, except that the scale of Regnault's air thermometer is supposed identical with that of a perfect gas thermometer.

Table V. gives the internal-work-pressure during evaporation at various constant pressures in lbs. per square foot and lbs. per square inch, together with the differences needful for interpolation. It has been calculated from a formula derived from Rankine's formula for the pressure of steam (see Appendix), but a result in close agreement may be obtained by the use of Table I*a* together with the formula just now given.

The ratio $\overline{P} : P$ is given in another column of the same table, it is the number denoted by k in Chapters II. and III., and given in Table III., for various temperatures. This ratio may likewise be obtained direct from Table I*a*, for from the above equation

$$k = \frac{\overline{P}}{P} = \frac{T}{P} \cdot \frac{\Delta P}{\Delta T} - 1,$$

which also may be written

$$k = \frac{\Delta (\log P)}{\Delta (\log T)} - 1.$$

Numerical examples will be found at the end of the table. A simple formula for k is given on page 175.

Generation of Steam in a Closed Boiler.

68. The question of the evaporation of water in a closed vessel has been already considered in the simple case in which the quantity of water is so small in proportion to the size of the vessel that all the water can be converted into steam without producing excessive pressure. It was then shown that the heat required completely to evaporate the water could be found without difficulty; but it was not shown how to find the heat required partially to evaporate the water, a case of some interest, involving as it does the expenditure of heat in getting up steam to a given pressure in a steam boiler, and the rate at which the pressure will rise when the safety valves are fastened down and the engine is standing.

Let the water-room in a boiler be m times the steam room, then if s be as usual the volume of a lb. of water, it is clear that for each cubic foot of water in the boiler before sensible evaporation commences there will be $1 + 1/m$ cubic feet of total room for water and steam together, and the volume of 1 lb. of water and steam together must therefore be

$$V = \frac{m + 1}{m} \cdot s,$$

which volume remains constantly the same during the whole operation. Now it has been already shown, in Art. 12, that when water is partially evaporated at *constant* pressure, the internal work done, reckoned from water at 32°, is

$$I = h + \overline{P}(V - s),$$

where $\overline{P}$ is the internal-work-pressure in lbs. per square foot, V is the volume of 1 lb. of the mixture of steam and water, and h has the usual meaning. Substituting for V

$$I = h + \frac{s}{m} \cdot \overline{P},$$

a formula which gives the internal work done in producing 1 lb. of the mixture from water at 32°, when the process takes place at constant pressure. But it has been repeatedly explained that the same amount of internal work is done, however the process of evaporation is conducted, and therefore, since in the present case no external work is done, this formula gives for each lb. of weight of the contents of the boiler the heat expended in getting up steam from water at 32°. If the water originally have any other temperature t_0, then the corresponding value of I must be subtracted, so that the heat expended (Q) is

$$Q = h - h_0 + \frac{s}{m} \cdot (\overline{P} - \overline{P}_0) \text{ foot lbs.}$$

It will be convenient to have the result in thermal units, which is easily obtained by division by 722, whence we have, when the temperature t_0 is so low that $\overline{P}_0$ is small enough to be disregarded

$$Q = h - h_0 + \frac{s}{772\,m} \cdot \overline{P} \text{ thermal units,}$$

in which h_1 h_0 must now of course be taken from the thermal unit table. Finally, by writing

$$\overline{P} = 144\,\overline{p} \text{ and } s = \cdot 016$$

the convenient formula

$$Q = h - h_0 + \frac{\overline{p}}{335\,m}$$

is obtained for the expenditure of heat in getting up steam to temperature t from water at temperature t_0. The first two terms in this formula represent the heat which would have been expended if the formation of steam had been completely prevented by the application of sufficient pressure to the surface of the water; the second term (always relatively small) is the correction necessary to provide for the formation of steam.

In steam boilers it appears that m is seldom, if ever, less than unity: that is to say, that the boiler is rarely less than half full, and is usually much more. Putting then $m = 1$ as the case in which the correction is greatest, and taking the values of $\bar{p}$ from Table V. at the end of the book, we obtain the results shown in the annexed table, assuming the boiler when cold to be at temperature 60°.

p (Absolute).	Q (Thermal Units).	$t - t_0$	Δ Q From 50 lbs. pressure.	θ'.
250	353·7	341	129	98
140	301·2	293	76·5	58
90	265·8	260	41·1	31·
50	224·7	221		
25	182·2	180		

If the temperature of the boiler when cold is not 60°, but a different temperature, then the results of the table are to be modified by the addition of the difference when below, or the subtraction when above, 60°. The third column of the table shows the value of $t - t_0$, which is less, partly on account of the specific heat of water being greater than unity, so that $h - h_0$ is greater than $t - t_0$, and partly on account of the correction spoken of above. It will be seen that the increase of Q due to these two causes is not of great importance, and hence for practical purposes may often be neglected.

The heat necessary to raise steam at one given pressure to steam at another pressure is found by taking the difference of the corresponding values of Q. Thus, for example, let us suppose a boiler working at 50 lbs. pressure absolute, and let it be asked what amount of heat is required to raise the pressure by a given amount; then we have only to subtract the value of Q for 50 lbs. from its value for the new pressure. The fourth column of the table shows the result for the pressures indicated, from which we see that a comparatively small amount of heat is required to produce a great increase of pressure.

The time occupied in raising the pressure can be found when we know the amount of heat furnished by the furnace, and the cubic contents of the boiler. The proportion which the water contained in a boiler bears to the evaporation in a given time appears to vary a good deal, according to the size and type of boiler, and even in boilers of the same size and type. According to Armstrong's rule for flue boilers, the water-room of a boiler should be 13½ times the volume of water evaporated per hour, or 810 times the volume evaporated per minute, in which case the weight of water in the boiler would also be 810 times the weight evaporated per minute. Now, the total heat of evaporation of water from 132° at the temperature corresponding to 50 lbs. pressure is 1068 thermal units; and therefore, assuming the evaporation equally active and efficient when the stop valves and safety valves are closed, the time in minutes necessary to raise the pressure to p lbs. on the square inch will be found by multiplying by 810 and dividing by 1068, the results of which operation appear in the fifth column of the table, headed θ'; from which it appears that in half an hour the pressure will have risen to almost 90 lbs. per square inch; in an hour, to more than 140 lbs. per square inch; and in an hour and forty minutes, to over 250 lbs. per square inch.

The last result shows the rapidity with which the pressure rises when we have to do with high-pressure steam, the increase of *temperature* being approximately proportional to the time, but the increase of *pressure* far more rapid.* The time occupied in getting up steam to 50 lbs. pressure from water of 60° is, on the same principle, about 2¾ hours. These results are confirmed by experience for stationary boilers of the type indicated.

In tubular boilers, the water-room is generally much less than that given by Armstrong's rule, and the times required are con-

* An experiment made by the late Sir W. Fairbairn in 1853, on a locomotive, agrees closely with this theoretical conclusion. See *Useful Information for Engineers*, 2nd edition, p. 321.

sequently less than those given. The principles explained are, however, sufficient to enable any particular case to be calculated at pleasure.

69. If in the formula for Q we consider a rise of temperature of 1°, we then get the "specific heat at constant volume" of a mixture of steam and water. The formula may then be written.

$$\Delta Q = \Delta h + \frac{\Delta \bar{p}}{335\, m},$$

in which the value of Δh is given in the table for h, being the specific heat of water at the temperature considered, and $\Delta \bar{p}$, which for an increase of 1 lb. is given in Table V., is easily found by multiplying the tabular result by the value of Δp given in Table I*a* for the temperature indicated. Thus, for example, to find the specific heat at constant volume of the contents of a boiler at pressure 100 lbs. per square inch. Here, from the tables,

$$\Delta h = 1 \cdot 03, \qquad \Delta p = 1 \cdot 39,$$

and the tabular value of $\Delta \bar{p}$ is $8 \cdot 66$,

$$\therefore \Delta Q = 1 \cdot 03 + \frac{1 \cdot 39 \times 8 \cdot 66}{336\, m} = 1 \cdot 03 + \frac{\cdot 036}{m};$$

or if the boiler be half full, as before,

$$\Delta Q = 1 \cdot 066 \text{ nearly.}$$

70. Another question closely connected with the subject of the present chapter is to find the amount of heat necessary to dry a given volume of moist steam. Let $1 - x$ be the weight of suspended moisture in a lb. of steam of pressure P^1, then, assuming the value of $P\, v$ approximately constant,

$$P^1\, v^1 = P\, v : v = V^1,$$

where P is the pressure when the steam has become dry, v the corresponding volume, $V^1 = v^1\, x$ the original volume, hence

$$P = \frac{P^1}{x}$$

$$\text{and } \Delta P = P^1 \cdot \frac{1 - x}{x}$$

gives the increase of pressure, very approximately, necessary to dry the steam at constant volume.

To obtain the heat needful we have

$$\Delta Q = \Delta h + V_1 \Delta \overline{P}$$
$$= \Delta h + x v^1 \Delta \overline{P}$$

from which numerical results can readily be computed by aid of the tables.

For example, suppose steam of 60 lbs. pressure to contain 10 per cent. of suspended moisture, how much heat is required to dry it at constant volume? Here $x = \cdot 9$ and hence

$$\Delta p = \frac{60}{9} = \frac{20}{3} = 6\tfrac{2}{3},$$

so that the pressure rises to $66\frac{2}{3}$ approximately. The corresponding rise of temperature is 7° nearly, whence, referring to Table II*b*, the value of Δh is

$$\Delta h = 7 \times 793 = 5551 \text{ foot lbs.}$$

Also referring to Table V., we find for $\Delta \overline{P}$

$$\Delta \overline{P} = \frac{20}{3} \times 1327 = 8847 \text{ lbs. on the square foot.}$$

$$\therefore \Delta Q = 5551 + \cdot 9\, v^1 \,.\, 8847.$$

Also v^1 at the pressure of 60 lbs. is 7 cubic feet nearly.

$$\therefore \Delta Q = 5551 + 6 \cdot 3 \times 8847$$
$$= 5551 + 55{,}736 = 61{,}287 \text{ foot lbs.}$$
$$= 79 \cdot 4 \text{ thermal units nearly,}$$

which determines the required amount of heat. It will be seen that the term representing the evaporation of the water is much greater than that representing the elevation of temperature.

Expansion of Dry Saturated Steam.

71. As explained in Chapter III. and confirmed by universal experience, the expansion curve of steam is approximately a common hyperbola; but I now propose to consider this question more fully by investigating the law of expansion under given circumstances,

and conversely the circumstances under which steam will expand according to a given law.

In the case of air, it has been already shown that the law of expansion depends on the amount of heat (if any) received or lost by the air at each step of the expansion. Precisely the same thing takes place in the case of steam, and the question reduces itself to this: to find the law of expansion when the heat received or lost is given, and conversely, to find the heat which must be supplied or abstracted in order that the expansion curve may be of given form. Of this question we shall now consider certain ideal cases, commencing as usual with the simplest; the cases most often occurring in practice being too complex to be discussed at the commencement of the subject.

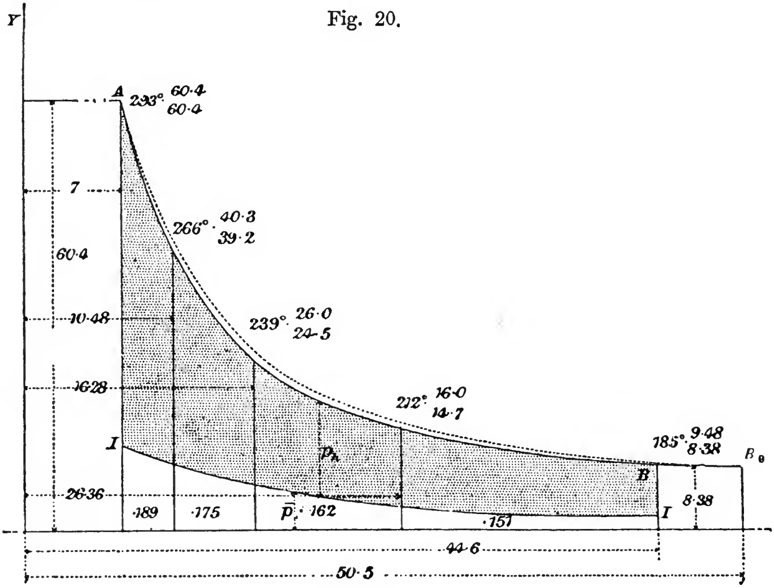
First in order of simplicity is the case in which the steam is supposed always dry and saturated during the whole progress of the expansion; and the question to be answered will be as to the amount of heat, to be supplied from without, to keep the steam in the state supposed.

Here the law of expansion is expressed approximately by the equation

$$p\, v^n = \text{constant},$$

being the approximate relation which, according to Art. 5, always connects the pressure and volume of saturated steam so long as it remains dry. Probably the easiest way of constructing the curve is to take the volumes corresponding to given pressures from Table III., and setting them off along the volume axis $O\,X$ (Fig. 20), to set up the corresponding pressures as ordinates; then a curve drawn through the extremities of the ordinates will be the expansion curve, which, as the form of the equation shows, does not differ greatly from an hyperbola: though other methods may also be adopted, one of which will be mentioned presently. In drawing the curve for a very great range of pressure, it is advisable to represent the upper part and lower part on different scales for the sake of distinctness. Fig. 20 represents the part of the curve which lies between the pressures 60·4 and 8·38 lbs. on the square inch absolute; in this case the volume increases from 7 cubic feet to 44·6 cubic feet, showing a ratio of expansion of about 6·37. The dotted curve $A\,B_0$ lying above the expansion curve $A\,B$ represents an hyperbola drawn through A, and shows that the pressure of the steam is always less than if it followed the hyperbolic law, the diminution being represented at each point by the vertical distance between the two curves. As the pressure falls, the temperature falls too, according to the law expressed by Table I., and this is shown on the figure at various

Fig. 20.



points of the expansion by giving the values of the temperature attached; the other numbers giving the pressures for the hyperbola and the actual curve respectively. Thus when the volume has increased from 7 cubic feet to 26·36 cubic feet, that is when the steam has expanded 3·77 times, the temperature has fallen to 212° and the pressure to 14·7, while the pressure according to the hyperbolic law would have been 16, showing a diminution of pressure of 1·3 lb. Hence if the steam really expand according to the hyperbolic law from the dry and saturated state at *A*, it must become superheated, and that the more the greater the expansion; also the ratio of expansion in the hyperbola, in order that the pressure may fall by a given amount, must be greater: thus in the figure the steam must expand till its volume is 50·5 in place of 44·6 before its pressure can fall to 8·38.

Hyperbolic expansion will be discussed in the next section; at present we have to do with dry steam, and the question is to find what heat, if any, is to be supplied in order that it may always be dry and yet not superheated. For this purpose it is only necessary to apply the equation,

$$\text{Heat Expended} = \text{Internal Work} + \text{External Work},$$

by considering how much work of either kind is done during each step of the expansion.

First, as to the external work: it is shown in the Appendix that the area of any curve of the form

$$P V^n = \text{constant},$$

included between two ordinates and the base, is

$$\text{Area} = \frac{P_1 V_1 - P_2 V_2}{n - 1};$$

but in the present case the value of n is approximately 1·0646, and for V we write v, because the volumes are those of dry saturated steam: hence

$$\text{Area} = 15{\cdot}5\,(P_1 v_1 - P_2 v_2),$$

and thus, to find the work done on the piston as the steam expands from volume v_1 to volume v_2, we have only to take the difference of the values of $P v$ at the beginning and end of the portion of the expansion considered and multiply by 15·5. The values of $P v$ may either be found from Table IV*a*, or be calculated by one of the formulæ

$$\log P v = 4{\cdot}6758 + {\cdot}0607 \log p$$
$$= 4{\cdot}8384 - {\cdot}0646 \log v$$

which may be derived from the formula (page 15) for the specific volume of dry saturated steam. These formulæ give the result in foot-lbs., but by subtraction of log 772 it may be found in thermal units if desired. It will be convenient to divide the expansion into stages during each of which the fall of temperature is 27°; the figure represents four of these stages, namely, from 293° to 185°: the annexed table, in the column headed $15{\cdot}5\ \Delta\ P v$, shows the external work done during each stage, not merely for these four, but for the whole range of pressure from 250 lbs. to $4\frac{1}{2}$ lbs. on the square inch; the absolute values are divided by 27 to obtain the mean value per 1°.

Next, for the internal work it is only necessary to consider the values explained in Chapter II., Art. 11, and tabulated in Table IV*c*, for the total internal work (I) done in changing water at 32° into dry steam of any pressure; for, since the amount of internal work done during any change does not depend on the way in which the change is produced, it is clear that to change dry steam at one pressure into dry steam at another pressure, we have only to take the difference of the corresponding values of I. Inspection of the table shows that I increases slowly with the temperature: thus, to form dry steam at 212° from water at 32° requires 1074·4 thermal units, while if the steam be formed at 401°, 1119·2 themal units are necessary, whence it follows that to change dry steam at 212° into dry steam at 401° we must expend in internal work 1119·2 − 1074·4, say 44·8 thermal units. Conversely, when steam expands always remaining dry a part of the external work done is done at the expense of the internal energy of the steam, that is to say so much as is equivalent to the corresponding diminution of I, so that we have only to take the work-equivalent of I to find out how much that is.

EXPANSION OF DRY STEAM.

$t°$.	$15{\cdot}5\,\Delta\,Pv$ per 1°.	$\Delta\,I$ per 1° foot-lbs.	p_i/p.	Δ Q. foot-lbs. per 1°.	Δ Q. Thermal units per 27°.
401°					
374°	700	197	·281	503	17·7
347°	751	192	·256	559	19·6
320°	802	187	·233	615	21·6
293°	860	182	·211	678	23·6
266°	911	178	·195	733	25·7
239°	962	174	·181	788	27·5
212°	1015	170	·167	845	29·6
185°	1072	166	·155	906	31·7
158°	1124	162	·144	962	33·6

The column headed ΔI shows the result of this calculation for each stage of the expansion, from which it appears, taking the third stage for example, in which the temperature falls from 347° to 320°, corresponding to a fall of pressure from 130 lbs. to 90 lbs., that the external work per 1° is on the average 802 foot-lbs. and the corresponding diminution of the work-equivalent of I is 187 foot-lbs., the difference of 615 foot-lbs. is the external work done by the agency of heat supplied from without, and if that heat be not supplied the steam will not remain dry, but some of it will be condensed. The columns headed ΔQ are thus calculated, and show, the first in foot-lbs. per 1°, and the second in thermal units for the whole stage of 27° the heat which must be supplied from without during each stage of the expansion to keep the steam dry.

The column headed p_i / p shows the proportion which the internal work bears to the external work, and enables us to exhibit the whole process graphically by constructing a curve of internal work in the same manner as was explained in detail in Chapter IV. for the case of air (see Art. 34). Let p_i be the internal-work-pressure, p_i will be to the external pressure p in the same proportion that the internal work during a very small part of the expansion bears to the external work, and the numbers given in that colnmn will therefore be the average values of p_i / p during the stage of expansion indicated. Hence the curve of internal work showing the internal-work-pressure at each point is readily drawn : in the figure that curve is represented by $I\,I$, its area gives the internal work just as the area of the expansion curve gives the external work, and the difference of areas (shaded in the figure) shows the heat supplied during expansion.

Conversely, if steam be compressed, in order that it may remain in a saturated condition, heat must be taken away from it at each step of the compression as its temperature and pressure rise, otherwise it will become superheated. The heat so taken away per 1° of rise of temperature is called the "specific heat" of steam when that term is used without qualification : it is said to be negative because heat is subtracted, not added, as the temperature rises. The column headed ΔQ shows the work-equivalent of the mean specific heat of steam during the interval of temperature in which it occurs. The second column headed ΔQ shows in thermal units per 27° the same amount of heat; thus in the case illustrated by the figure where the steam expands from 60·4 to 8·38, by adding the numbers given for each stage we find, for the four stages, 114·5 thermal units as the heat required to keep the steam dry. The numerical results here found are only approximations, for any minute error in the determination of the density of steam by the empirical formula $p v^n =$ const. and of the products $P v$ by the tables is multiplied many times by the process of calculation; but they are without doubt a tolerable approximation, and they show that the heat required to keep steam dry varies from three-fourths to five-sixths of the external work done during expansion.

Another formula (Fairbairn and Tate's) was given in Art. 5, namely,

$$v = \cdot 41 + \frac{389}{p + \cdot 35},$$

which suggests a method of drawing the curve in a simple way without reference to the tables. Draw new axes of reference for measurement of abscissæ and ordinates distant to the right of and below the old ones by ·41 cubic foot and ·35 lb. on the square inch respectively, and with these new axes describe an hyperbola, then the form of the formula shows that this hyperbola considered relatively to the old axes will be the required curve, for the formula may be written

$$(v - \cdot 41)(p + \cdot 35) = 389 = \text{constant}.$$

The curve may likewise be drawn, and the mean pressure calculated or graphically constructed by the general method applicable to all curves of the form $P V^n =$ const., explained in the Appendix.

The curve being constantly used in our subsequent work, will for the sake of a name be called the "saturation curve." When plotted logarithmically, a method which is often very useful, it becomes a straight line (page 15) which may be called the "saturation line."

71A. An exact formula for the specific heat of steam may be found as follows:—

If Q be the heat supplied during expansion,

$$\Delta Q = \Delta I + P \Delta v,$$

where ΔI is the increment of intrinsic energy consequent on expansion through the volume Δv. But we know that for dry saturated steam

$$I = H - Pu,$$
$$\therefore \Delta I = \Delta H - P\Delta u - u\Delta P,$$

where as usual u is written for $v - s$, and consequently Δu is the same thing as Δv, hence

$$\Delta Q = \Delta H - u\Delta P;$$

but it was shown in Art. 62, page 145, that

$$u\Delta P = \frac{L}{T} \cdot \Delta T;$$

therefore dividing by ΔT, and proceeding to the limit,

$$\frac{dQ}{dT} = \frac{dH}{dT} - \frac{L}{T} = \cdot 305 - \frac{L}{T}.$$

The specific heat is the heat necessary to *raise* the temperature 1°, and is therefore $+ dQ/dT$

$$\therefore \text{Specific Heat} = \cdot 305 - \frac{L}{T} \text{ thermal units.}$$

Inspection of Table VI. shows that the result is always negative. For example, at temperature 278°,

$$T = 278 + 461 = 739 : L = 918\cdot 7,$$
$$\text{Specific Heat} = \cdot 305 - 1\cdot 243$$
$$= - \cdot 938.$$

This calculation shows that the specific heat of steam is negative, as already found. By multiplication by 772 we obtain the value in foot-lbs., that is − 724 foot-lbs. The difference between this result and − 733 given in the table above, as the mean value between 266° and 293°, is due to the method of calculation of the table, which is not suited to secure precise accuracy, but exhibits the nature of the process more clearly.

To find the heat supplied during a given fall of temperature, we may replace L by its very approximate value.

$$L = 966 - \cdot 71\,(t - 212) = 1444 - \cdot 71\,.\,T;$$

then by substitution,

$$-\frac{dQ}{dT} = \frac{1444}{T} - 1\cdot 015;$$

therefore by integration between the limits T_1 T_2,

$$-Q = 1444 \log_\epsilon \frac{T_1}{T_2} - 1 \cdot 015 \{T_1 - T_2\}.$$

For example, for the expansion between 293° and 185°, considered in the preceding article, on making the calculation, we find 113·6 thermal units instead of the approximate value 114·5 thermal units there given.

The method here adopted of showing that the specific heat of steam is negative, does not essentially differ from that which precedes. For we have no accurate knowledge of the density of steam apart from the formula employed above, which connects it with the latent heat of evaporation. The result, however, was first reached by Rankine without this accurate knowledge, on the assumption that steam was subject to the laws of a perfect gas. Almost at the same time the exact formula was found by Clausius. Whichever method we adopt, the result is certain, and we must conclude that when steam expands while driving a piston, some of it is condensed unless heat is supplied from without to keep it dry. The conclusion has been verified by Hirn, Cazin, and others, who find that when steam, originally perfectly transparent, expands in a glass cylinder, a cloud at once appears, indicating the presence of moisture.

It must, however, be distinctly understood, that condensation is not in the nature of things a *necessary* accompaniment of the doing of work by an expanding vapour. On the contrary, fluids exist in which, not condensation, but superheating occurs. Such a fluid is ether, in which, on making the calculation in the same way as in steam, a *positive* value of the specific heat is obtained. And this result also has been experimentally verified, no cloud appearing when ether expands, as it does in steam, and most other vapours.*

Conversely, as already remarked, when dry saturated steam is compressed, it becomes superheated unless heat is abstracted during compression. This circumstance renders it necessary in considering the formation of steam in a closed space, as in Art. 13, p. 33, and Art. 68, p. 155, to suppose that the operation is carried out in such a

* On these points see *Mechanical Theory of Heat*, by R. Clausius (Browne's translation, Macmillan, 1879), pp. 135-141. On p. 136 negative signs, by a printer's error, have been prefixed to the numerical values given for the specific heat of ether, thus rendering Clausius' language unintelligible.

way, that no sensible difference of temperature can occur in different parts of the whole mass of steam and water. There will be a constant tendency of the temperature of the steam already formed, to rise faster than that of the water from which it proceeds.

Hyperbolic Expansion.

72. The next case to be considered is that in which the expansion curve is an hyperbola, that is to say where

$$p\,V = \text{constant} = p_1\,V_1,$$

in which, as usual, p is the pressure, and V the volume of a lb. of steam. In this case the steam becomes drier and drier as it expands if it be originally moist, and becomes superheated if it be originally dry, for by Art. 6

$$V = u\,x + s = v\,x \text{ (approximately)},$$

hence since

$$p\,v^n = p_1\,v_1{}^n$$

$$x = x_1\left(\frac{p_1}{p}\right)^{\frac{n-1}{n}}$$

but if r be the ratio of expansion,

$$r = \frac{V}{V_1} = \frac{p_1}{p},$$

$$\therefore\; x = x_1 \,.\, r^{\frac{n-1}{n}};$$

in which formulæ, as before, $n = 1{\cdot}0646$.

This shows that, if the expansion starts from a point indicated by the suffix 1, x is greater than x_1; so that if the expansion be carried far enough, the steam becomes dry, and if still farther, superheated—a result which agrees with the last article.

In the figure (Fig. 21), if $A\,Z\,B$ be the hyperbolic expansion curve, $A_0\,Z\,B_0$ the saturation curve considered in the last article: these curves cross one another at a point Z, easily determined if the amount of moisture in the steam at any given pressure is known. Above this point the steam is wet, below, it is superheated.

Taking first the expansion above Z, the amount of moisture at any point 1 is given by putting $x = 1$, then

$$x_1 = r^{-\frac{n-1}{n}}.$$

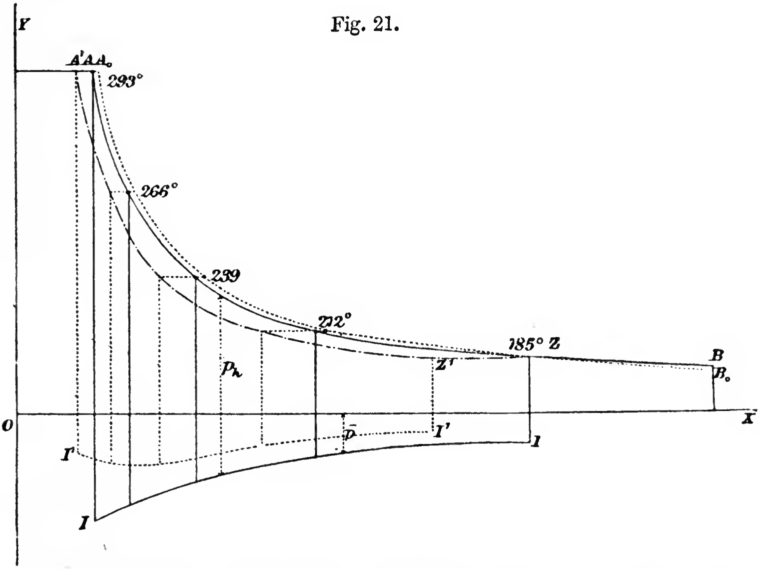
Thus, for example, let steam of pressure 60·4 lbs. expands till its pressure has fallen to 8·38 lbs., that is to say, with a ratio of expansion 7·22, let it be dry at that pressure, then

$$x_1 = (7 \cdot 22)^{-\frac{n-1}{n}} = \cdot 886,$$

so that the steam must contain originally 11·4 per cent. of moisture.

The same result may be obtained graphically if the figure be accurately drawn on a large scale, for the horizontal distance between the curves always represents the volume of steam existing in a state of moisture.

Fig. 21.



The temperature always falls with the pressure, according to the same invariable law, until superheating commences, and it is convenient, as before, to split up the expansion into stages during each of which the fall of temperature is 27°. The figure shows four of these stages from 293° to 185°, just as in the previous case, and we have to find the external and internal work done during each stage. First, as to the external work: let p be the pressure at the commencement, and p_1 at the conclusion of any stage, then by Art. 22

$$\text{External Work} = P\,V \,.\, \log_\epsilon \frac{p}{p_1},$$

where PV is the constant product (in foot-lbs.) of the pressure and volume. At Z the volume V is that of dry steam at the corresponding pressure; so that PV is the same as Pv, which by Table IV*a* we find to be at 185°, 53,900 foot-lbs.

$$\therefore \text{ External Work } = 53{,}900 \log_\epsilon \frac{p}{p_1},$$

from which formula, by division by 27 and substitution of the pressures, the external work per 1° is found for each of the four stages, and tabulated in the second column of the annexed table. To find the internal work, the formula

$$I = h + \overline{P}(V - s) \quad \text{(Art. 12)}$$

is to be used, which shows the internal work done in forming V cubic feet of steam at any pressure by any process, or neglecting s as usual

$$I = h + \overline{P}V = h + kPV,$$

in which last form kP is written for $\overline{P}$.

Then PV is constant, and k is known from the tables, or by a formula given on page 175, so that the change in I is

$$\Delta I = PV \,.\, \Delta k - \Delta h,$$

where Δk, Δh are the differences of k and h for each stage. It is to be observed that Δh is subtracted, because h diminishes as the temperature falls. Writing for PV its value 53,900 foot-lbs. we get from Tables II*b* and IV*b* in the example considered the values of ΔI shown in the third column of the Table in foot-lbs. per 1°; from which it appears that when steam expands according to the hyperbolic law, not only is heat required to do the external work but also to produce internal changes in the steam itself.

This heat, which must be supplied from without, in order that the steam may expand exactly in accordance with the hyperbolic law, is shown in the columns headed ΔQ, in the first column in foot-lbs. per 1°, in the second in thermal units for the whole stage of 27°. As before, the numbers obtained are only approximations, but are undoubtedly close approximations, to the actual facts.

The value of p_i/p is given in another column, and the whole process exhibited graphically in Fig. 21, as in the previous case. And as before the method of logarithmic plotting may also be used with advantage.

Hyperbolic Expansion.

Remarks.	$t°$.	External work per 1°.	Internal work per 1°.	p_i/p	Value of ΔQ. foot-lbs. per 1°.	Value of ΔQ. Thermal units per 27°.
Steam dry at 185°.	293°					
	266°	860	310	·360	1170	41
	239°	940	434	·462	1370	48
	212°	1020	577	·565	1600	56
	185°	1120	660	·589	1780	62
Steam contains 20 per cent. moisture at 185°.	293°					
	266°	688	89	·130	777	27
	239°	752	188	·250	940	33
	212°	816	300	·368	1116	39
	185°	896	365	·407	1261	44

If the expansion be carried beyond Z the steam becomes superheated, and from the existing deficiency in experimental data we cannot say with exactness what takes place; it is, however, clear that the temperature will go on falling till it reaches a limit given by the equation

$$85{\cdot}5\,(t + 461) = PV = 53{,}900 \quad . \quad \text{(Art. 32)},$$

whence

$$t = 170°,$$

that is to say, till the temperature has fallen 15° more; the steam will then be completely superheated, and the curve of internal work will reach the axis of volumes (see Art. 34). But how far expansion must proceed to realise this, cannot at present be determined with any certainty. If B be the point where the steam becomes completely superheated, the internal-work-curve will reach the axis at B if not before; according to the supposition of Hirn that the isodynamic curve (Art. 68) is an hyperbola not only when the steam is completely superheated but even when the superheating is only partial, it would follow that the internal-work-curve reaches the axis immediately superheating commences, and in that case the heat supplied from without during that part of the expansion in which the steam is superheated is simply the heat-equivalent of the external work done during that part.

73. If, instead of supposing the steam dry at the end of the expansion, as in the preceding example, it be supposed wet, then the results obtained will be somewhat modified. For example, imagine the steam to contain 20 per cent. moisture at 185°, then to find the amount of moisture initially,

$$x_1 = \cdot 8 \,.\, r^{-\frac{n-1}{n}},$$

or, since the ratio of expansion will be unaltered,

$$x_1 = \cdot 8 \times \cdot 886 - \cdot 709,$$

that is, the initial amount of moisture is 29·1 per cent., showing that 9·1 per cent. is evaporated during expansion instead of 11·4 per cent.

The value of $P\,V$ is now ·8 × 53,900, or 43,120, and the external work during each stage is diminished in like proportion. For the internal work

$$\Delta I = 43{,}120 \,.\, \Delta k - \Delta h,$$

the results of which formula are shown in the table.

In Fig. 21 the process is graphically represented : $A^1 Z^1$ is the expansion curve, which is an hyperbola four-fifths the size of the original. The horizontal distance between the expansion curve and the saturation curve still represents the volume of steam existing as moisture, which of course now is everywhere much greater than before. The dotted curve $I^1 I^1$ below is the internal-work-curve, which now is no longer always convex towards the axis, but attains a maximum distance and then approaches the axis again. This always happens when the steam contains much water.

74. If, now, we compare the results obtained, when the steam expands according to the hyperbolic law, and when it remains always dry and saturated, it appears that an apparently trifling difference in the law of expansion makes a great difference in the heat needful to produce it. Thus it was found above that in expansion

from 293° to 185°, while remaining dry, the heat supplied is 114·5 thermal units; but in hyperbolic expansion, on adding the numbers in the last column of the last table, it will be found to be in the first case 207 thermal units, and in the second case 143 thermal units. Hence, conversely, a great change in the heat supply produces a small change in the law of expansion; and if it is further considered that the indicator tells us nothing about the absolute size of the expansion curve, so that the two cases of hyperbolic expansion just considered would appear identical, it will not be surprising that the expansion curve of steam does not appear to vary much in practice in the most various circumstances.

It is sometimes supposed that hyperbolic expansion in steam is the same as hyperbolic expansion in a perfect gas; in fact, however, the two cases are very different. When a perfect gas expands according to the hyperbolic law, its temperature remains constant, and the heat supply is just that needful to perform the external work; whereas when steam so expands, its temperature falls rapidly, and the heat supply must be from one-fourth to one-half greater than that equivalent to the external work done.

75. In the last articles it has been found convenient, as in the case of air (Art. 34), to represent the internal work which is being done during expansion, by means of an ideal pressure on the piston, just as the external work is represented by the real pressure; yet it must always be remembered that this ideal pressure depends not merely on the actual state of things at the instant considered, but also on the law of expansion, that is, on the way in which that state varies from instant to instant.*

Thus in the two kinds of expansion just considered the same actual pressure on the piston corresponds to two different internal pressures; in the first, a pressure in the same direction as, and forming part of, the actual steam pressure; while in the second it forms a resistance to be overcome, or, so to speak, a back pressure. It usually varies from point to point of the expansion, as shown by the curve of internal work, and its mean value may be found just

* For this reason in seeking an abbreviation for the phrase "pressure equivalent to internal work," we have preferred the rather cumbrous expression "internal-work-pressure" to the briefer term "internal pressure," which might prove misleading.

in the same way as the mean actual pressure on a piston, and may be expressed either with reference to the whole stroke, including both admission and expansion, as is usually done, or with reference to the expansion alone. Examples will occur hereafter of both ways of expressing it.

When steam is formed by evaporation under constant pressure, the internal-work-pressure is constant, and is given for each external pressure by Table V. We may also sometimes find it convenient to use the formula

$$k = \frac{11{,}530}{T} - 4{\cdot}7,$$

which gives the ratio k with great accuracy in terms of the absolute temperature. From this the difference Δk, used on page 171, may readily be found when required.

When steam is formed in any way from water of the same temperature the internal-work-pressure is not generally constant, but its mean value is the same as if the steam were formed under constant pressure.

APPENDIX—ADDITIONAL EXAMPLES.

We now proceed to consider a number of additional examples of the expansion of steam, in which either the form of the expansion curve or the supply of heat to the steam is given. The method employed is the same as that by which the two preceding cases have been treated. Though simple in principle it is frequently too laborious in practice for ordinary use. Readers to whom the subject is new are recommended to pass at once to the next chapter, referring to the results when necessary.

Any given Expansion Curve.

76. We shall first show that, if a perfectly accurate indicator diagram be given together with the weight of steam used per stroke, it will be possible to determine the curve of internal work, and to deduce the heat supplied to the steam at each step of the expansion.

In this case it clearly may be supposed that the pressures at the beginning and end of the expansion are exactly known. Then, by Table I., the temperatures are likewise known, and the corresponding values of h and $\overline{P}$ in the formula

$$I = h + \overline{P}(V - s)$$

can be found by Tables II. and V. Moreover, dividing the volume of the cylinder by the weight of steam used per stroke, the terminal value of V is known, and the initial value can then be found by division by the ratio of expansion. Thus, the internal work reckoned from water at 32° is determined both for the beginning and end of the stroke. Subtraction now furnishes the internal work done during expansion, and by adding the heat-equivalent of the area of the expansion curve which represents the external work, the supply of heat is obtained.

The effects (often very important) of clearance and wire drawing are throughout this chapter wholly neglected, being reserved for discussion at a later period.

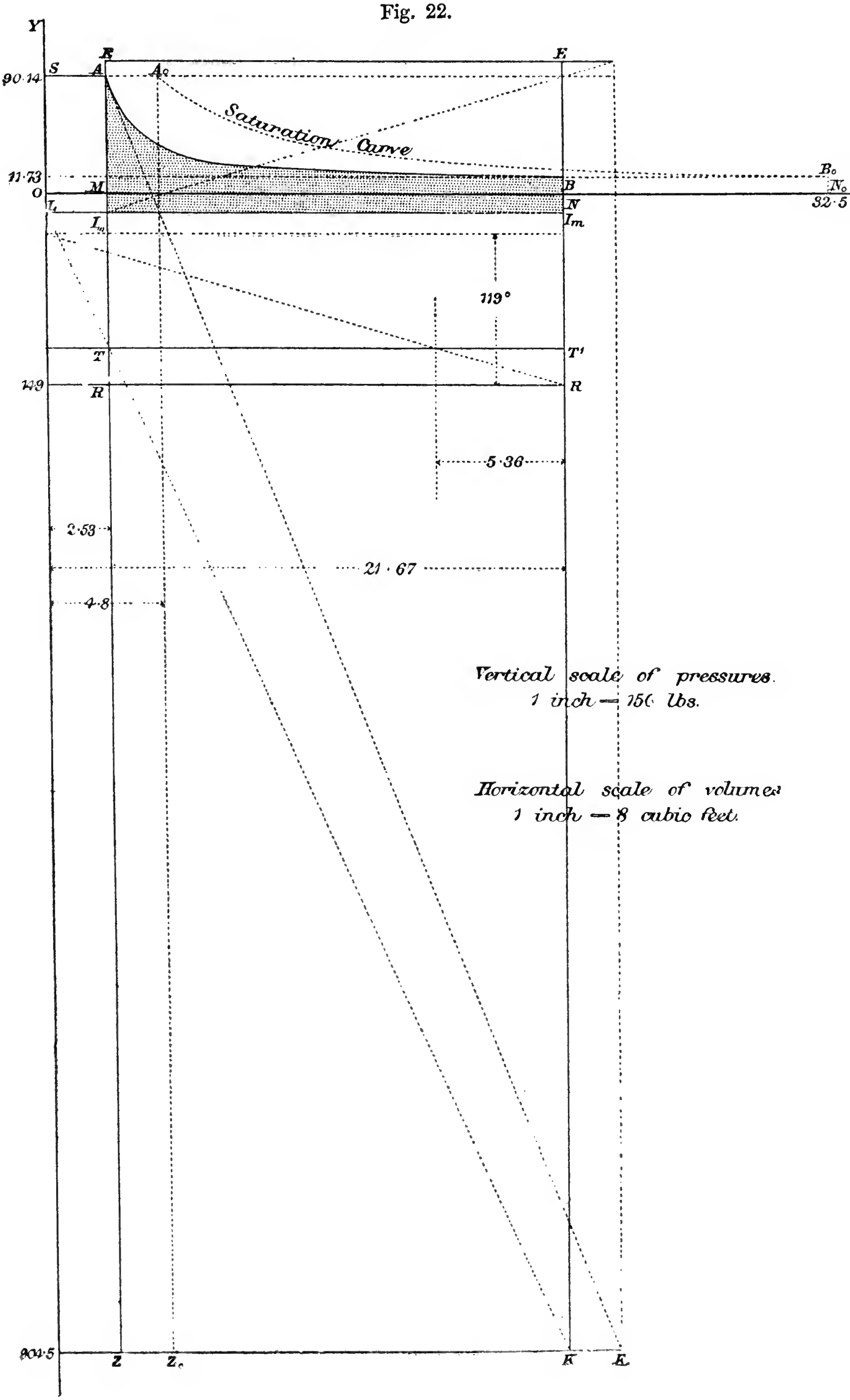
77. The graphical method of Chapter II. not only enables us to represent the process on the diagram, but also to obtain readily definite results.

Fig. 22 represents the admission line $S\,A$, and the expansion curve $A\,B$, neglecting all effects of wire-drawing. To fix our ideas, take as data the results of one of the experiments on the *Bache* (Chapter XI.), and suppose the initial pressure 90·14 and the terminal pressure 11·73 lbs. on the square inch; further, we suppose the ratio of expansion 8·57. Also, we shall suppose the terminal volume of 1 lb. of the steam two-thirds the volume of dry steam at the terminal pressure, as was probably approximately the case in the experiment in question. Now, the volume of dry steam at 11·73 lbs. per square inch is 32·5 cubic feet nearly, taking two-thirds of which, 21·67 is obtained for the actual volume at the end of the stroke. In the figure then, $O\,N$, the base of the diagram, represents 21·67 cubic feet, and $O\,M$ represents 21·67 / 8·57, or 2·53 cubic feet, which is the initial volume of the steam. The accuracy of these data is not here the question, we have merely to show the results deducible when accurate data are attainable.

First, to find the state of the steam at any point of the expansion, lay off $O\,N_0$ = 32·5 cubic feet, and trace the saturation curve $A_0\,B_0$ (see last articles), thus showing the volume of dry steam at any pressure, then the horizontal distance between this curve and the expansion curve shows the amount of water in the steam at any point of the expansion. In particular, the volume $S\,A_0$ of dry steam at the initial pressure is 4·8 cubic feet, therefore $A\,A_0$ = 2·27 cubic feet, showing that 47·4 per cent. of steam was in the state of water at the commencement of the expansion. The form of the expansion curve shown in the figure is ideal, the diagram not being given, but only the mean pressure, so that the wetness of the steam cannot be determined at any other point of the stroke.

Next, to find the internal work according to the graphical process of Arts. 9, 12, it is only necessary to refer to Table V., from which it appears that the internal-work-pressure corresponding to 90·14 is 904·5 nearly. Then, constructing the rectangle $O\,Z$ on the base $O\,M$, which represents the actual volume of the steam initially, the internal work done in evaporation at 90·14 is represented by the area of this rectangle; this gives the initial value of the internal work reckoned from water at the initial temperature of 320° corresponding to 90·14. In the same way the terminal value of the internal work is represented by the area of the rectangle $O\,R$ on the base $O\,N$, and of height $N\,R$ = 149, corresponding by Table V. to

Fig. 22.


Saturation Curve
90·14
11·73
149
32·5
119°
5·36
2·53
21·67
4·8
Vertical scale of pressures.
1 inch = 150 lbs.
Horizontal scale of volumes
1 inch = 8 cubic feet.

11·73, the terminal pressure, this work being reckoned from 201°, the temperature corresponding to 11·73. We have now to take the difference, but before doing so, it is necessary that the initial and terminal values of the internal work should be reckoned from water at the *same* temperature. The most convenient temperature to choose is the initial temperature of 320°; hence we take the difference 320° − 201°, or 119°, and perform the construction of Fig. 3, Arts. 10, 12, as indicated in Fig. 22, remembering that the difference of 119° must be reckoned negative, because we are now reckoning from the higher temperature. Thus the rectangle $O\,T'$ is obtained, which represents the terminal value of the internal work reckoned from water at 320°.

We have now only to find the difference of these values, and for this a simple construction suffices. Complete the rectangle $O\,K$; join $K\,T$, and produce it to meet the pressure axis in L; then draw the horizontal line $L\,I_m\,I_m$, as shown in the figure; the required difference is simply the rectangle $M\,I_m$. For, by a well-known proposition of Euclid, the rectangles $L\,T'$, $L\,Z$, are equal, and consequently the difference of $O\,T'$, $O\,Z$, must be the same as the difference of $L\,N$, $L\,M$. Hence, it follows that the horizontal line $I_m\,I_m$ is the line of mean internal-work-pressure during the expansion. In the present case this line falls below the volume axis, showing that the internal work is greater at the end of the stroke than at the beginning, and $O\,L$, the internal-work-pressure in question, is found to be 15·77 lbs. per square inch.

The external work on the same scale is represented by the area $A\,B\,N\,M$ of the expansion curve, and consequently the heat supplied is represented by the area, shaded in the figure, comprised between the expansion curve and the line $I_m\,I_m$. This will be true, whatever be the form of the curve, provided only that the data remain the same at the beginning and end of the expansion.

The numerical value can be found in the present example, because the mean forward pressure is given as 30·31, whence

$$\begin{aligned} \text{Total area of diagram} &= 30{\cdot}31 \times 21{\cdot}67 \\ &= 657 \text{ nearly;} \\ \text{Admission area} &= 90{\cdot}14 \times 2{\cdot}53 \\ &= 228 \text{ nearly;} \end{aligned}$$

$$\therefore \text{ Expansion area} = 429.$$

Also for the internal work,

$$\text{Area of rectangle } M_1\, I_m = 15 \cdot 77 \times (21 \cdot 67 - 2 \cdot 53)$$
$$= 303 \text{ nearly};$$
$$\therefore \text{ Area } A\, B\, I_m\, I_m = 732 \text{ nearly}.$$

The areas are converted into foot-lbs. by multiplication by 144, or into thermal units by division by 5·36 (see Art. 9). Adopting the latter course, we obtain

$$\text{Heat supplied} = 136 \text{ thermal units}.$$

The steam then must have received from the cylinder during the expansion about 136 thermal units per lb.; a result which requires certain corrections for the effect of clearance to be explained hereafter. Also the influence of wire-drawing—in some cases considerable—is neglected, but in the present case the result is probably substantially correct.

If now we ask from whence the cylinder derived so much heat, our first idea would, probably, be, that it was derived from the steam jacket which was in operation during this experiment; if, however, the observed quantity of steam liquefied in the jacket be considered, it will be found that only 16 thermal units were so derived, and the remainder must, therefore, have been obtained from some other source, and that source can only be the steam during admission. Thus at least 120 thermal units were so derived, but in fact much more steam must have been liquefied during admission than is represented by this quantity of heat; for although there are no data from which the state of the steam can be determined as it left the boiler, yet there is no reason to suppose it contained any considerable amount of water; let us assume it dry, then complete the rectangle $A\, Z_0$ in Fig. 22, then that rectangle represents the amount of heat abstracted from the steam during admission. To make comparison more easy, reduce this rectangle by the constructions indicated in the figure to the base $I_m\, I_m$, then we obtain the rectangle $E\, I_m$, which is greater than the heat supplied during expansion by the area between $E\, E$ and the expansion curve $A\, B$. This last area represents the whole heat abstracted by the cylinder from the steam during its passage from the boiler to the end of the stroke, which heat, in addition to that furnished by the jacket, is chiefly transmitted to the exhaust during the return stroke (compare Art. 24). This is true, whatever be the form of the curve; in the present case

Heat abstracted = 421 − 136 = 285
Jacket supply .. .. = 16
∴ Exhaust waste = 301 thermal units.

If the result be compared with the whole heat expended, as shown by the quantity of water evaporated, it is found that the exhaust waste in the present case was 27 per cent. of the whole; and though the effect of clearance modifies this conclusion to some extent, as will be seen hereafter (Chapter IX.), yet there is little doubt of its substantial accuracy, if the data are correct. We shall return to this subject in a later chapter.

78. So far we have only considered the initial and terminal state of the steam, on which alone the position of the line $I_m I_m$ of mean internal-work-pressure depends; if, however, it be desired to know, not merely how much heat on the whole is supplied to the steam, but according to what law it is supplied, it will then be necessary to have an exact expansion curve, which will give the exact value of the pressure at every point; then, dividing the expansion into small portions, and going through the construction of the last article for each portion, a series of lines $I_m I_m$ will be obtained, each of which refers to its own part of the expansion. Trace now a curve through the middle points of all these short lines, and the result will be an internal-work-curve, such as was found in the two kinds of expansion first considered in this chapter. The reader will find it a useful exercise to construct in this way the diagrams for the two cases in question.

To each particular expansion curve corresponds its own curve of internal work, showing the law of supply of heat; but, as has been shown in the preceding cases, a very small change in the expansion curve causes a very great change in the position of this curve, so that it will rarely happen that the curve drawn by an indicator can be relied on sufficiently for the purpose.

79. As another example of the method just explained of finding the heat supplied during expansion, when the expansion curve is

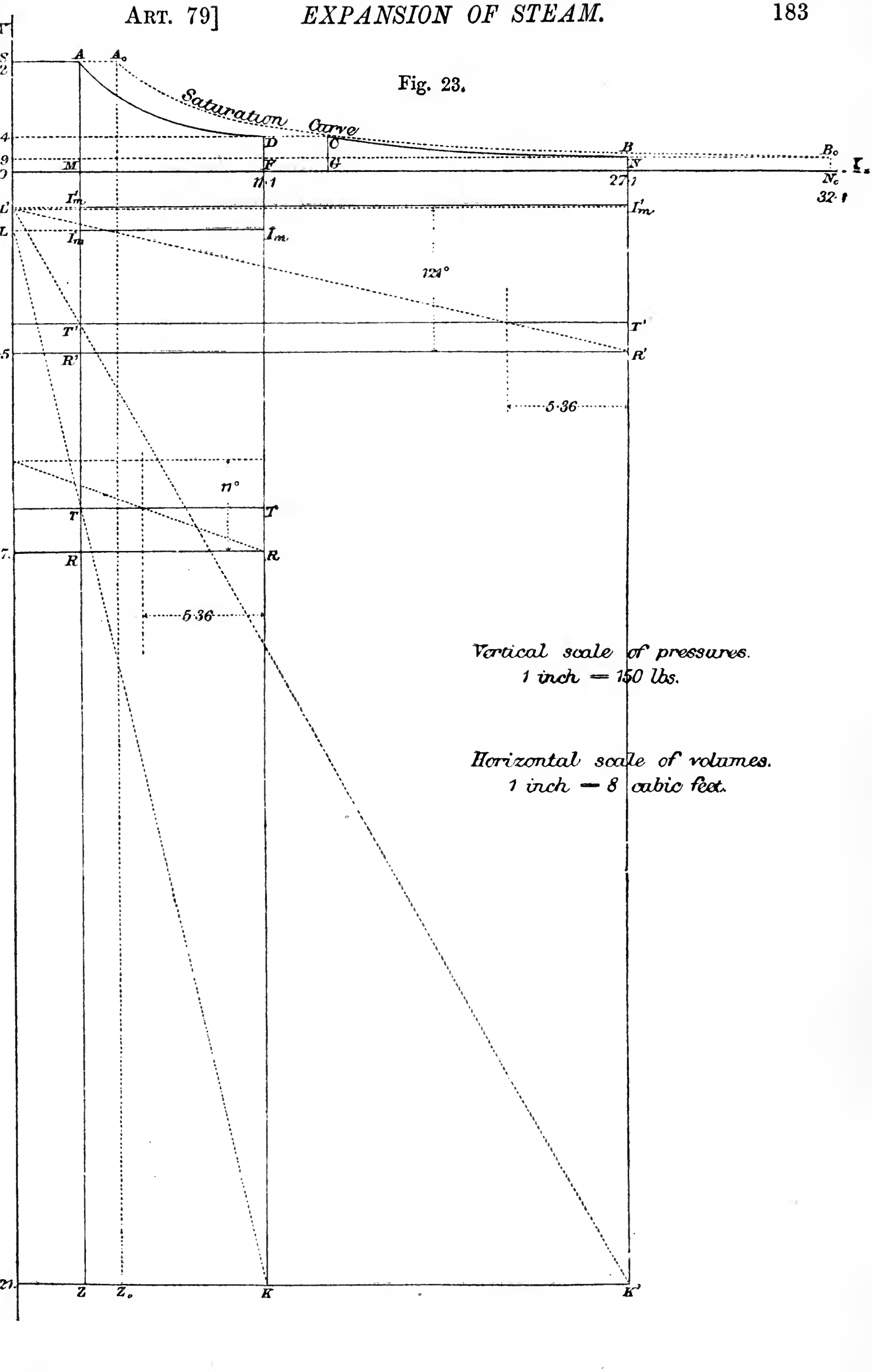
given, let us take as data one of the *Bache* experiments (Chapter XI.), when the engine was tried as a compound engine, as follows:

Initial pressure (small cylinder)	..	..	=	92·0
Terminal " " "	..	..	=	27·4
" " (large cylinder)	..	..	=	11·9
Total expansion	..	..	=	9·19
Ratio of cylinders	..	..	=	2·44
Water in steam at end of stroke (large cylinder)			=	·156

In Fig. 23 lay off $O N_0$ to represent the volume (32·1) of dry steam at the end of the stroke in the large cylinder, and trace the saturation curve $A_0 B_0$ as before, which shows the volume of dry steam at any pressure; then, multiplying by ·844 the proportion of dry steam, we get for the volume of the actual steam 27·1 cubic feet; this is represented on the diagram by $O N$, and the true expansion curve therefore starts from B. Divide now 27·1 by 2·44 the ratio of the cylinders, and 11·1 is obtained for the volume of 1 lb. of the steam at the end of the stroke in the high-pressure cylinder; this is represented by $O F$ in the figure, while $D F$ represents the terminal pressure. Again, dividing 27·1 by 9·19, we obtain 2·95, which is represented in the figure by $O M$, while $A M$ represents the corresponding pressure of 92.

These points being determined by the data of the question, expansion curves are now to be drawn, which, as before, are ideal, except that the mean pressures are given, as will be explained presently. The full curve $A D$ represents the expansion in the high-pressure cylinder, while $D C B$ represents the expansion in passing from the high-pressure cylinder to the end of the stroke of the low-pressure cylinder. For simplicity it is supposed that the reservoir is very large, and wire-drawing is neglected in drawing the ideal curve $D C B$; although in the subsequent calculation of areas the actual mean pressures are used as found by experiment: thus $C D$ is a straight line representing an increase of volume at the constant pressure of the reservoir consequent on the steam being partially dried by the exhaust heat of the high-pressure cylinder, while $C B$ is the expansion in the low-pressure cylinder. Then, as before, the horizontal distance between the whole expansion curve $A D C B$, and the saturation curve represents the amount of water in

Fig. 23.


Saturation Curve
A
A₀
D
C
B
B₀
M
F
G
N
N₀
11·1
27·1
32·1
124°
5·36
11°
5·36
T'
R'
T
R
Z
Z₀
K
K'
Vertical scale of pressures.
1 inch = 150 lbs.
Horizontal scale of volumes.
1 inch = 8 cubic feet.

the steam at each point of its passage through the engine from the beginning of the high-pressure stroke to the end of the low-pressure stroke.

Next, the heat supplied to the steam during the expansion, is found by performing the construction of Art. 77 with reference, first, to the high-pressure expansion; secondly, to the total expansion: this is shown in Fig. 23, in detail, with the same letters attached as in Fig. 22, so that it is unnecessary to describe the process further. The calculations are now conducted as follows, commencing with the high-pressure cylinder:

$$\begin{aligned}\text{Internal work area } F\,I_m &= 47{\cdot}7 \times (11{\cdot}1 - 2{\cdot}95) \\ &= 389.\end{aligned}$$

For the external work it is necessary to estimate the area $A\,D\,F\,M$, which cannot be done exactly, because the mean forward pressure is not given, but only the mean effective pressure for the high-pressure cylinder: it can, however, be approximately estimated without fear of serious error, by taking the mean of the values of $P\,V$ at the beginning and end of stroke, and multiplying by the value of $\log_\epsilon r$,

$$\begin{aligned}\text{Initial value of } P\,V &= 92 \times 2{\cdot}95 = 271{\cdot}4 \\ \text{Terminal ,, } P\,V &= 27{\cdot}4 \times 11{\cdot}1 = 304{\cdot}1 \\ \therefore \text{ Mean value of } P\,V &= 287{\cdot}7.\end{aligned}$$

The ratio of expansion in the high-pressure cylinder is $9{\cdot}19 \div 2{\cdot}44 = 3{\cdot}76$, the hyperbolic logarithm of which is $1{\cdot}32$, hence we obtain

$$\text{Expansion area} = 380 \text{ nearly.}$$

Adding which to the internal work area found above, we obtain

$$\text{Total area} = 769,$$

whence, by division by $5{\cdot}36$,

$$\text{Heat supplied} = 143 \text{ thermal units approximately.}$$

Assuming now that the boiler supplied dry steam, then, as before, the rectangle $A\,Z_0$ is the heat abstracted from the steam during admission:

$$\begin{aligned}\text{Area } A\,Z_0 &= (4{\cdot}70 - 2{\cdot}95) \times 1013 \\ &= 1773\end{aligned}$$

whence, by division by 5·36,

Heat abstracted = 333 thermal units.

The cylinder was not jacketed, hence, neglecting radiation and conduction, 333 − 143 or 190 thermal units must have been transmitted to the exhaust steam on its passage out of the high-pressure cylinder.

Passing on to consider the total expansion, the internal work done, while the steam passes from its initial pressure in the high-pressure cylinder to its terminal pressure in the low, is represented by the rectangle $M\,I'_m$, and this is true, whatever amount of wire-drawing exists between the cylinders, and whatever be the treatment of the steam in the reservoir. The external work is equal to the whole area of the diagram, known from the mean pressures given by the experiment, diminished by the admission area $S\,M$.

Total mean effective pressure = 27·52 (reduced to L.P.)
Back pressure .. .. .. = 3·05 approximately
Mean forward pressure .. = 30·57

∴ Area of diagram = 30·6 × 27·1
= 829·3;

but

Admission area = 271·4;
∴ Total expansion area = 558 nearly
Internal work area = 30 (27·1 − 2·95)
= 724
Total area = 724 + 558 = 1282

∴ Heat supplied during total expansion = 239 thermal units.

Let us now trace the process from the instant when the steam is cut off in the high-pressure cylinder to the instant when it exhausts from the low-pressure cylinder into the condenser.

Heat received during high-pressure expansion = 143
" " " " exhaust = 190

∴ Total heat received before finally leaving the high-pressure cylinder = 333 thermal units;

but

Total heat received during total expansion = 239;

∴ Heat abstracted in passage through reservoir and low-pressure cylinder = 94 thermal units

The heat is partly abstracted in the reservoir, but probably the greater part is abstracted in the low-pressure cylinder by the action of its sides. If we add the heat supplied by the steam jacket of the low-pressure cylinder, we shall obtain the exhaust waste.

The steam jacket in the present case supplied far more heat than in the case previously described in Art. 77. It appears from the experiment that about 7·8 per cent. of the working steam was the additional supply required to replace the liquefaction in the jacket: a result which shows that about 67 thermal units per lb. of working steam was supplied to the low-pressure cylinder; hence

$$\text{Exhaust waste} = 161 \text{ thermal units,}$$

which is about $13\frac{1}{2}$ per cent. of the whole heat expended, as shown by the total amount of water evaporated. As before, the numbers given require certain corrections on account of clearance, and are not intended as precisely accurate results, but they are probably substantially correct unless the data are rejected as wholly unreliable, a point which is not in question in the present chapter.

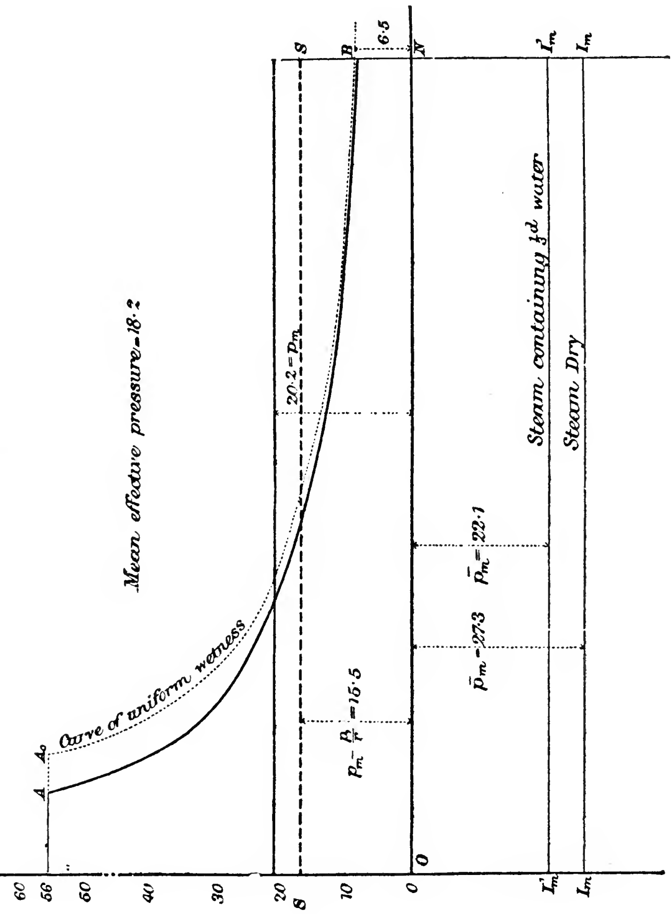
80. It has been already stated that complete and exact results can only be obtained when the weight of steam passing through the engine per stroke is precisely known, in addition to the data furnished by an indicator diagram. The determination of this quantity is by no means easy, and therefore in most cases this essential datum is wanting, so that it will be desirable to examine what results can be obtained in its absence.

As an example, take a diagram from a Corliss engine working at *Saltaire*, given in *Naval Science*, vol. iii. p. 160.

Fig. 24 shows this diagram, so far as the admission line and expansion line are concerned, from which it appears that the engine was working with initial pressure 56, and ratio of expansion 12; the terminal pressure is not easy to estimate exactly, but is taken at 6·5 lbs. per square inch. The consequence of an error of $\frac{1}{2}$ lb. will be mentioned presently. Clearance and wire-drawing (Chapter IX.) are throughout neglected except in measuring the volumes.

The expansion curve AB now gives the pressure at each point of the stroke, but since the weight of steam is not known, the volume per lb. cannot be found. The volume of dry steam at 6·5 lbs. per square inch is 56·6 cubic feet; let us take the base ON of the diagram to represent it, then if the steam were dry, the diagram would represent the changes gone through by 1 lb. of steam, and

Fig. 24.



hence, if we imagine the steam to contain at the end of the stroke $1 - x_2$ lbs. of water, it is clear that the diagram will actually represent the changes undergone by $1 / x_2$ lbs. of steam.

From Table III. set off the volumes of one lb. of dry steam at various pressures, then if the steam were everywhere equally wet, the resulting curve would be the actual expansion curve $A\,B$. Instead of this, however, the dotted curve $A_0\,B$, is obtained, which may be called a curve of uniform wetness; were the steam dry at the end of the stroke, it would be the saturation curve drawn in preceding examples. This curve falls above the actual expansion curve $A\,B$, and the horizontal distance between the two curves represents the excess water in $1 / x_2$ lbs. of steam at the point considered; hence these distances multiplied by x_2 represent the excess water in 1 lb. of steam. For example, at the beginning of the stroke, the volume of the steam considered is $56 \cdot 6 / 12$, or $4 \cdot 72$, while the volume of steam in the terminal state of wetness is $7 \cdot 51$ cubic feet, therefore in the $1 / x_2$ lbs. of steam $2 \cdot 79$ cubic feet exist as water, in addition to the water at the end of the stroke, if any. Let then $1 = x_1$ be the amount of water initially in 1 lb. of the steam, then

$$x_2 - x_1 = \frac{2 \cdot 79}{7 \cdot 51} \, . \, x_2 = \cdot 371 \, . \, x_2.$$

Hence, if the steam be dry at the end of the stroke, 37 per cent. of moisture must have been evaporated during expansion, or if it then contain one-third water, 25 per cent. This calculation, of course, presumes that there is no valve leakage, which might be suspected if this were a solitary instance of the kind.

Next, to find the heat supplied during expansion we might resort to the graphical construction of the preceding articles, but, for the sake of variety, let us proceed differently.

Taking the difference of the two values of I found by the formula of Art. 13, the change of internal work will be

$$I_2 - I_1 = h_2 - h_1 + \overline{P}_2 V_2 - \overline{P}_1 V_1,$$

where the suffix 2 refers to the end, and the suffix 1 to the beginning of the stroke, while $s = \cdot 016$ is omitted, and $V_2 \, V_1$ are the actual volumes of 1 lb. of steam:

$$\therefore \frac{I_2 - I_1}{V_2} = \frac{h_2 - h_1}{V_2} + \overline{P}_2 - \frac{\overline{P}_1}{r},$$

since $V_2 \div V_1$ is the ratio of expansion r.

Now let $\overline{p}_m$ be the mean pressure which working throughout the stroke would do the same work, then :

$$\overline{p}_m \,.\, 144 \,.\, V_2 = I_2 - I_1,$$

and

$$h_2 - h_1 = -772\,(t_1 - t_2) \text{ nearly ;}$$

thus

$$\overline{p}_m = \overline{p}_2 - \frac{\overline{p}_1}{r} - \frac{5 \cdot 36\,(t_1 - t_2)}{V_2}.$$

In the present example, if we seek t_1 t_2 from Table I., the difference will be found to be 119°, also by Table V.

$$\overline{p}_2 = 87 \cdot 4 : \overline{p}_1 = 596 \,;$$

moreover,

$$V_2 = x_2 v_2 = 56 \cdot 6\, x_2 : r = 12,$$

hence we find

$$\begin{aligned} \overline{p}_m &= 37 \cdot 7 - \frac{10 \cdot 4}{x_2} \\ &= 27 \cdot 3 \ (x_2 = 1) \\ &= 22 \cdot 1 \ (x_2 = \tfrac{2}{3}). \end{aligned}$$

These results give the line of mean internal-work-pressure I_m I_m for two cases which may be regarded as extremes. These lines differ from the corresponding lines in the preceding diagrams only in the circumstance that the pressures are supposed reduced to the whole stroke instead of, as before, referring to the expansion alone. Should an error of $\frac{1}{2}$ lb. have been made in estimating the terminal pressure, the effect would be to shift the lines up or down by about 6 lbs. pressure.

Now the external work is represented on the same scale by the mean forward pressure p_m which may be taken at about 20·2 lbs per square inch, diminished by p_1 / r, which represents the admission. work. This difference is 15·5, and is shown by the dotted line S S.

The heat expended is the sum of the internal work and the external work, and is represented by a pressure on the piston of 42·8 if the steam be dry at the end of the stroke, or 37·6 if it contain one-third water. Now, the mean effective pressure was 18·2, and hence we learn that the heat supplied during expansion must have been 2·35 or 2·07 times the heat-equivalent of useful work done, and hence must have amounted to 100, or 88 thermal units per I.H.P. per 1′.

Again, as in previous examples, the heat abstracted by the cylinder during liquefaction at admission is represented by a rectangle the height of which is the internal-work-pressure + the actual pressure, and the base is the volume of steam condensed. This must be reduced to a rectangle, the base of which is $O N$, then the height of that rectangle will be the equivalent pressure on the piston.

First, let the steam be dry at the end of the stroke, then the volume of steam condensed is represented by $A A_0$ on the same scale that $O N$ represents the terminal volume; thus, since it was shown above that $A A_0$ is 2·79 cubic feet when $O N$ is 56·6 cubic feet, it appears that in the reduction we must multiply by ·0493. Referring to Table V. the internal-work-pressure is found to be 596, and hence

$$\text{Required pressure} = (596 + 56) \times \cdot 0493 = 32 \cdot 14.$$

This represents an abstraction of heat equivalent to 1·76 times the useful work, or about 75 thermal units per I.H.P. per 1′. The difference between this and the heat supplied during expansion is 25 thermal units per I.H.P. per 1′, which must have been supplied by the steam jacket in addition to the exhaust waste, which, if the steam were really dry at the end of the stroke, may be supposed small.

Secondly, if the steam be supposed to contain one-third water at the end of the stroke, no doubt an extreme supposition under the circumstances, the volume of steam condensed in admission, assuming as in previous cases that the boiler supplied dry steam, is found from

$$x_2 - x_1 = \cdot 371\, x_2 \text{ (see above, p. 158)};$$

$$\therefore\ x_1 = \cdot 63\, x_2 = \cdot 43, \text{ or } 1 - x_1 = \cdot 57.$$

$$\text{Volume of steam condensed} = \cdot 57 \times 7 \cdot 51$$
$$= 4 \cdot 28 \text{ cubic feet per lb.}$$

$$\text{Terminal volume} \quad .. \quad .. \quad = 56 \cdot 6 \times x_2 = 56 \cdot 6 \times \cdot 666$$
$$= 37 \cdot 7 \text{ cubic feet per lb.}$$

$$\therefore \text{Required pressure} \quad .. \quad .. \quad = 652 \times 4 \cdot 28 \div 37 \cdot 7$$
$$= 73 \cdot 8.$$

Thus the heat abstracted during admission is 4·06 times the useful work done, or 174 thermal units per I.H.P. per 1′. Subtracting the heat supply, we obtain 86 thermal units per I.H.P. per 1′, which, together with the jacket heat, would form the exhaust waste.

These results will show how far it is possible to go without further information than is furnished by an accurate indicator diagram. The accuracy of the diagram is not discussed here, and the complicated effect of clearance and wire-drawing here neglected may have considerable influence on the results.

81. Before leaving this part of the subject, another formula may be mentioned which may often conveniently be employed.

In Chapter III., Art. 25, it was shown that the total heat of formation of steam at the end of the stroke of an engine is given by

$$Q = H_2 - h_0 + (P_m - P_2)\, v_2$$

if the steam be then dry, while if it then be wet,

$$Q = h_2 - h_0 + x_2 L_2 + (P_m - P_2)\, x_2 v_2 \quad \text{(Art. 26)},$$

the notation being as explained in the articles cited.

Now there is no reason to restrict this formula to the end of the stroke, the reasoning used being applicable in any case, only P_m must now mean the mean pressure exerted on the piston during the part of the stroke considered, while P_2 becomes the pressure at the end of that part. Let the part considered then be the admission, then $P_m - P_2$ vanishes and $Q_1 = h_1 - h_0 + x_1 L_1$ is the total heat of formation initially, where the suffix 1 corresponds to the commencement of the stroke. This may also be seen by considering that the total heat of formation is in this case identical with the total heat of evaporation.

If now S be the heat supplied during expansion, S must be identical with $Q - Q_1$,

$$\therefore\ S = h_2 - h_1 + L_2 x_2 - L_1 x_1 + (P_m - P_2)\, x_2 v_2,$$

which determines the heat supplied to 1 lb. of steam during expansion, when the initial and terminal state of the steam is known, together with the mean forward pressure exerted on the piston during admission and expansion. The results of the last articles can also be obtained by use of this formula.

The examples given in the present section show sufficiently how to deal with cases in which the law of expansion is known, by experiment or otherwise, and it is required to find the law of supply, and at the same time furnish materials for subsequent consideration. We next go on to the converse question, where the supply of heat is supposed known, and it is required to find the law of expansion.

Expansion of Steam under a Given Supply of Heat.

82. When the supply of heat is given for each step of the expansion, it is then possible, at least theoretically, to construct the expansion curve step by step, so that the given supply may be equal to the internal work + the external work. The data of the question would be, either the heat supplied during each degree of the fall of temperature which always takes place whatever the law of expansion be, or else the supply of heat as the volume increases by a given amount: it is the first case which is the most simple, and at the same time generally the most important, and to that we confine ourselves in the present chapter.

Case I.—First, suppose that no heat is supplied to the steam during its expansion, then the expansion curve is, as in the case of air (Chapter IV.), called the "adiabatic" curve, the form of which it is our object to investigate. Adiabatic expansion does not occur in practice, for it presupposes a perfectly non-conducting cylinder, but its consideration is nevertheless indispensable in any theory of the steam engine, both as an interesting ideal case, and because, although the whole mass of steam cannot expand adiabatically, yet the central portion, not in immediate contact with the sides, probably does so. The general nature of the curve can be foreseen from what was said at the commencement of this chapter, when we considered the expansion of dry and saturated steam; for it was there shown that to keep steam dry, heat must be supplied from without, and the necessary inference is that, if the heat be not supplied, the steam will condense, and hence the adiabatic curve must fall below the saturation curve. On the other hand, it cannot fall much below, for it has been already seen what a small difference in the expansion curve corresponds to a great difference in the heat supply.

To construct the curve it is only necessary to remember that the whole external work done in expansion must now be derived from the internal energy stored up in the steam itself, that is to say, it must be equal to a diminution of internal energy, which must take place during the expansion. Thus, if the steam expand from a point 1 to a point 2,

$$I_1 - I_2 = \text{Area of curve};$$

or with the previous notation (Art. 80),

$$5 \cdot 36\ (t_1 - t_2) + \bar{p}_1 V_1 - \bar{p}_2 V_2 = \text{Area of curve};$$

where areas are supposed expressed by the product of a pressure in lbs. per sq. inch and a volume in cubic feet. The construction of the curve is now to be carried out so as to satisfy this equation.

In Fig. 25, OX is as usual the volume axis and OY the pressure axis, from which lines are drawn with ordinates representing the pressures 15, 20, 25, 30, 40, 50, 70 lbs. per square inch, and any other pressures which may be required; for convenience, such pressures are chosen as occur in Tables III. and V. Corresponding lines in the lower part of the figure show the internal-work-pressures taken from Table V.; these lines are distinguished by the numerical values of the pressures in question being written against them.

Imagine, for example, that we have dry steam at 70 lbs. pressure, and take A on the corresponding pressure-line so as to represent its volume, that is to say, 6·09 cubic feet. Let that steam expand according to the curve AB, till its pressure has fallen to 50 lbs. per square inch without gain or loss of heat, it is required to find the corresponding volume, which we already know to be less than 8·35 cubic feet, the volume of dry steam at that pressure.

Complete the internal-work-rectangle corresponding to A precisely as in previous questions, but let the internal work be reckoned from water at the lower temperature of B instead of from the upper temperature of A; the construction is shown in the figure, resulting in the line SS which forms the base of A's rectangle: while the base of B's rectangle is simply the line of internal-work-pressure corresponding to B, which in the figure meets the vertical through A in G. Now draw the line $ZI_m I_m$ midway between A and B, the ordinate of this line must represent the internal-work pressure, corresponding to the expansion from A to B wherever B is, exactly if the expansion curve were a straight line between A and B, and very approximately if, as is actually the case, the line be curved. Hence, by the same reasoning as in previous questions, it is quite clear that the other side of *B*'s rectangle will be determined by joining *Z G* and producing it to meet *S S* produced in *K*, then a vertical through *K* must determine *B*; the difference of internal work at *A* and *B* being then equal to a rectangle, the area of which is very approximately the same as the aréa of the expansion curve *A B*.

The construction can now be repeated as often as desired, and the adiabatic curve is thus determined. The figure shows expansion by successive stages, from 70 lbs. pressure to 15 lbs. pressure; the numbers written below the axis give the volumes of the expanding steam at the pressures indicated by the ordinates, while the numbers written above the expansion curve on the level of the line of 70 lbs. pressure represent the corresponding volumes of dry steam, as shown by the saturation curve $A\ D^1$ dotted in the diagram; thus, the volume of steam condensed at each step of the expansion is the difference of these numbers.

The process here described requires considerable care to obtain accurate results; hence, when great exactness is desired in the determination of the condensation, numerical calculation is preferable, according to a formula to be explained in the next chapter, or by the method followed in Case II. immediately following; the numbers given in the figure were obtained in this way. The results show that in the whole expansion the ratio of expansion is 3·88 and the steam condensed 2·3 cubic feet; that is to say, when steam expands adiabatically 3·88 times from a pressure of 70 lbs. per square inch, the terminal pressure, instead of being 18·1, as would be the case if the expansion were hyperbolic, is about 3 lbs. less and about 9 per cent. of the steam is liquefied. Inspection of the figure gives a clear idea of the gradually increasing liquefaction as the pressure falls. The mean pressure in adiabatic expansion can always be found when the terminal volume and pressure are known, by inverting the construction of the present article and applying it to the total expansion.

83. *Case II.*—Secondly, as before, let the expansion be adiabatic, but let the steam be initially wet, then the construction is in general identical with that just given, and the general results can be foreseen. The position of the lines of internal-work-pressure is unaltered, but the rectangle representing the effect of difference of

temperature is of smaller breadth, and consequently greater height; thus, the lines $S\,S$ are all shifted downwards, and consequently the points K (for the same initial volume) shifted to the right; that is to say, the volume of the steam is greater (relatively to the initial volume) than if the steam had been initially dry. Thus wet steam does not condense so fast as dry steam when expanding adiabatically.

The extreme case of wet steam is when there is no steam, but only water initially, and then it is obvious that water must be evaporated, not steam condensed, on diminution of pressure. In Fig. 25 let $O\,N$ represent ·016, the volume of 1 lb. of water enclosed in a cylinder behind a piston, and let the piston be loaded with 70 lbs. per square inch (say); take $A\,N$ to represent that pressure, and draw horizontal lines to represent various pressures as in the previous case. Now imagine the temperature of the water to be 303°, corresponding to 70 lbs. per square inch, and then suppose that pressure gradually to diminish, the water will gradually evaporate and its temperature fall; it is required to find the expansion curve. We might employ the purely graphical method of Case I., introducing a suitable modification, but instead of this, we may proceed differently by a method likewise applicable in Case I. and the following Case III. Let the point of departure A be denoted by 1, and let the next point, corresponding to a pressure of 60 lbs., be denoted by 2, as shown on the diagram, then if, as usual,

$$I = h + \overline{P}(V - s),$$

we can now no longer neglect s, because V is a small quantity; hence, since $V_1 = s$, the formula employed in the last article becomes

$$5 \cdot 36\,(h_1 - h_2) - \overline{p}_2 \,.\, (V_2 - s) = \text{Area of curve},$$

in which $h_1 - h_2$ is not replaced by its approximate value $t_1 - t_2$, because the error thus introduced is much greater than in previous cases. Now, for the area of the curve between 1 and 2, it is sufficient to consider the curve as a straight line between these points, then

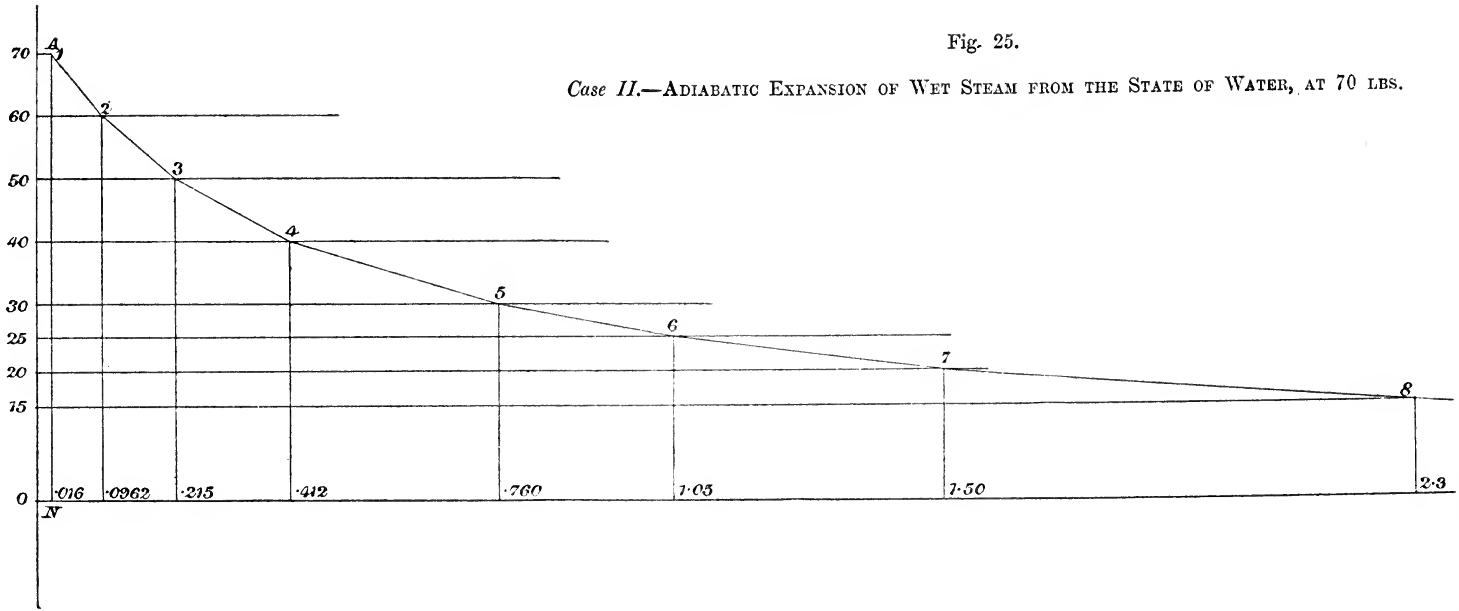
$$\text{Area} = \frac{p_1 + p_2}{2} \,.\, (V_2 - s) = 65\,(V_2 - s);$$

introducing which into the foregoing equation, and replacing $\overline{p}_2$ by its value, namely, 633·4 lbs. on the square inch,

$$(V_2 - s)\,(633 \cdot 4 + 65) = 5 \cdot 36\,(h_1 - h_2).$$

Fig. 25.

Case II.—Adiabatic Expansion of Wet Steam from the State of Water, at 70 lbs.



On referring to Table II*a* it will be found that the specific heat of water between 303° and 293° is about 1·025, hence

$$h_1 - h_2 = 1{\cdot}025\,(t_1 - t_2) = 10{\cdot}25 \text{ nearly};$$

substituting and performing the numerical calculations,

$$V_2 - s = {\cdot}0802, \qquad V_2 = {\cdot}0962,$$

so that the volume of the mixture of steam and water is about ·0962 cubic foot. The point 2 can now be laid off on the diagram, and shows the volume at a pressure of 60 lbs. per square inch. Now pass on to the point 3 on the line of 50 lbs. pressure, then considering that area as a pair of trapezoids,

$$\text{Area of curve up to } 3 = 65 \times {\cdot}0802 + (V_3 - V_2)\,55;$$

$$\therefore \text{ Area} = 5{\cdot}2 + 55\,(V_3 - V_2),$$

and the equation becomes

$$5{\cdot}2 + 55\,(V_3 - V_2) = 5{\cdot}36\,(h_1 - h_3) - \bar{p}_3\,(V_3 - s),$$

or

$$5{\cdot}2 + (V_3 - V_2)\,(\bar{p}_3 + 55) = 5{\cdot}36\,(h_1 - h_2) - \bar{p}_3\,(V_2 - s).$$

Replacing $\bar{p}_3$ by its value, namely, 540 lbs. per square inch, and $V_2 - s$ by its value just found,

$$V_3 - V_2 = \frac{5{\cdot}36\,(h_1 - h_3) - 540 \times {\cdot}0802 - 5{\cdot}2}{595}.$$

Assume the specific heat of water by Table II*a* as 1·02, then

$$h_1 - h_2 = 1{\cdot}02\,(t_1 - t_3) = 1{\cdot}02\,(303° - 281°);$$

substituting which and performing the arithmetical operations,

$$V_3 - V_2 = {\cdot}119,$$

whence

$$V_3 = {\cdot}119 + {\cdot}0962.$$

This process can be repeated indefinitely, and hence the volume of the steam is determined after a fall of pressure of any extent; the results are shown so far as 15 lbs. in the diagram by the numbers placed against the ordinate above the volume axis; whence it appears that the volume of the mixture, after the pressure has fallen to 15 lbs., is about 2·3 cubic feet.

Thus it appears that when steam is very wet, instead of condensation taking place during expansion, just the reverse is true, some of the water being evaporated. Of course it follows that some proportion of steam to water must exist for which neither evaporation nor condensation takes place. This proportion can be found either by graphic construction or by an approximate calculation of the kind just made, or by a formula which will be given in the next chapter; it varies for each particular case, but never differs greatly from half.

84. If, instead of supposing steam, or a mixture of steam and water, to expand, we imagine conversely that it is compressed by a gradual increase of the pressure on the piston, then the effect produced is exactly reversed. In the case of moderately moist steam, the moisture is gradually evaporated as the compression proceeds, and when sufficiently compressed the steam becomes superheated. In the case of very wet steam containing more than half its weight of water, condensation takes place gradually as the compression goes on, and that the more rapidly the wetter the steam; thus, if the compression be sufficient, the steam is wholly condensed, and this is what was supposed in Art. 62, Chap. V., when considering the action of a perfect steam engine.

85. *Case III.*—Next, instead of supposing, as in the two preceding cases, that no heat is supplied or abstracted during expansion, let us imagine that heat is added as the temperature falls by equal quantities for each degree. This will be realised if the steam be supposed to expand in a non-conducting cylinder, but with a thin metallic plate attached to the piston; then the temperature of the plate will closely follow that of the steam, and if its specific heat be supposed constant, it will supply the steam with equal quantities of heat as the temperature of the steam falls through each degree. Let the weight of the plate be m times that of the steam and its specific heat c, then the heat supplied per lb. of steam will be $m c$.

The supposition of such a metallic plate is very interesting theoretically, because it imitates more closely than any other case, sufficiently simple to be thoroughly investigated, the real action of the sides of the cylinder.

Let t' be the temperature of the exhaust steam, and t be the temperature initially in the cylinder; then on admission the plate has to be heated from t' to t by the fresh steam from the boiler, whereby $mc(t - t')$ thermal units are subtracted from that steam, and $1 - x$ lbs. are liquefied, given by

$$1 - x = \frac{mc(t - t')}{L}.$$

This liquefied steam is deposited on the surface of the plate as a film of moisture, which is afterwards re-evaporated, partly during expansion and partly during exhaust. This process is highly instructive, and will be carefully examined hereafter; for the present we are only concerned with its effect on the expansion curve.

Let Q be the amount of heat furnished by the plate as the expansion proceeds from a point 1 to a point 2, then by the general principle

$$\text{Heat Expended} = \text{Internal Work} + \text{External Work},$$

$$Q = I_2 - I_1 + \text{Area of expansion curve};$$

or using the same formula as in Art. 61,

$$Q = h_2 - h_1 + \overline{P}_2 \cdot (V_2 - s) - \overline{P}_1 (V_1 - s) + \text{Area of curve}.$$

Omitting s, and writing as usual

$$h_1 - h_2 = 5\cdot 36\,(t_1 - t_2),$$

to correspond with pressures in lbs. per square inch,

$$Q = \overline{p}_2 V_2 - \overline{p}_1 V_1 - 5\cdot 36\,(t_1 - t_2) + \text{Area of curve};$$

but on the same scale

$$Q = mc(t_1 - t_2) \times 5\cdot 36,$$

and therefore

$$5\cdot 36\,(1 + mc)(t_1 - t_2) + \overline{p}_1 V_1 - \overline{p}_2 V_2 = \text{Area of curve}.$$

The construction of the curve is now to be carried out so as to satisfy this equation. On comparing the equation from which the adiabatic curve was constructed in Art. 32, it will be seen to differ solely in $t_1 - t_2$ being replaced by $(1 + mc)\ (t_1 - t_2)$, and the construction must therefore be the same save a slight modification. To fix our ideas, let us imagine the plate to be of iron, the specific heat of which is ·12, and let the weight of the plate be 8·33 times that of the expanding steam; then $mc = 1$, and in the construction we have only to use $2\ (t_1 - t_2)$ in place of $t_1 - t_2$.

This has been done in Fig. 25, which shows the construction on the further supposition that the steam contains initially one-third water; the inner strongly-dotted curve $a\,b\,d$ is the expansion curve. As before, to obtain very exact numerical results, a calculation method is preferable, such as the process adopted in Case II., or the formula given in a subsequent chapter (Chapter VII.). The numbers given in the diagram at the various points of the curve $a\,b\,d$ were obtained in this way; those immediately above the volume axis representing the actual volumes of the expanding steam, and those in the upper part of the figure giving the volumes of steam containing one-third water, as shown by the faintly-dotted curve of uniform wetness $a\,d^1$. The first set of numbers are the greater, showing that the action of the plate is sufficient, not merely to prevent the condensation which would otherwise take place, but make the steam considerably drier. Thus, at the end of the stroke the steam occupies 18·8 cubic feet, instead of 17·27 cubic feet, as it would do did it still contain as much as one-third water. The ratio of expansion is in this case 4·63, and the curve rises very nearly indeed up to the hyperbola.

The steam contains, on the whole, at the end of the stroke, 27·4 per cent. of water; but it is highly probable that not all of this is deposited on the plate, but that the central mass of the steam condenses as it would do if the plate were not there. If this be the case, then by Case I., 8·9 per cent. of two-thirds the whole mass is distributed uniformly throughout the mass, while the rest represents the film still existing on the plate at the end of the expansion. This will be discussed further hereafter.

The curve of internal work may be easily derived by graphic construction for

$$Q = mc\,(t_1 - t_2),$$

and hence, since the area included between the curves of internal work and external work represents the heat expended, all that is necessary is to reduce a rectangle representing Q to a base representing the corresponding increase of volume. Or, by calculation, let V_1 V_2 be two volumes corresponding to the temperatures t_1 t_2, then if p_h be the height of such a rectangle in lbs. per square inch,

$$p_h = \frac{m\,c\,.\,5\cdot 36\,.\,(t_1 - t_2)}{V_2 - V_1}.$$

In the example $m\,c = 1$, then, commencing with the first stage of the expansion from 70 to 50 lbs., it has been already shown that

$$V_2 - V_1 = 5\cdot 71 - 4\cdot 06 = 1\cdot 65;$$

$$\therefore p_h = 5\cdot 36\,.\,\frac{303^\circ - 281^\circ}{1\cdot 65} = \frac{5\cdot 36 \times 22}{1\cdot 65},$$

or

$$p_h = 71\cdot 5.$$

A similar calculation is made for each of the five other stages into which the expansion is divided, whence is obtained

$$71\cdot 5 : 51\cdot 4 : 38\cdot 8 : 28\cdot 5 : 23\cdot 8 : 17\cdot 1,$$

numbers which show the mean pressures equivalent to the heat expended during each of the six stages of the expansion, from which the curve is readily constructed. It evidently falls a little below the axis at the higher pressures, and almost coincides with it at the lower; to avoid confusion it is not represented in the diagram. In adiabatic expansion the curves of internal work and external work obviously coincide.

88. The expansion of steam in contact with a thin plate does not differ materially from the expansion of wet steam; indeed the cases would be identical if the specific heat of the metal varied according to precisely the same law as the specific heat of water. For it is clear that the material of the plate does not influence the result in any other way, and consequently we may just as well suppose a mass of water as a mass of metal. The water, however, will follow the temperature of the steam more readily than the metal when the expansion is rapid; an important consideration, as will be seen hereafter.

Isodynamic Expansion. General Remarks.

89. One other case of expansion remains to be mentioned; namely, that in which the supply of heat is just equivalent to the external work done, so that the internal energy of the expanding steam suffers no change. This is called isodynamic expansion, and in perfect gases is the same as isothermal expansion (Art. 33); so that the expansion curve, or, as it is called, the isodynamic curve, is a common hyperbola. The curve of internal work then coincides with the volume axis, and, consequently, on comparing Articles 70, 72, of the present chapter, it appears that the isodynamic curve must lie between the saturation curve and the common hyperbola, and hence differs very little from either; moreover it follows that in this kind of expansion the steam becomes drier as it expands, though not so rapidly as in hyperbolic expansion. This curve likewise represents the relation between the volume and the pressure of saturated steam when expanding without doing any external work.

The isodynamic curve is graphically constructed by an easy modification of the process adopted for the adiabatic curve (Art. 82); we have only to draw the radiating lines (Fig. 25) through the fixed point O instead of through the middle points of the corresponding pressure intervals; then a curve will be determined for which the internal-work-rectangles are constant, and this curve, by the definition, is the isodynamic curve.

90. The principal object of the present chapter has been to point out the connection between the expansion curve of steam and the supply of heat during the expansion, as had been already done in the case of air in Chapter IV.; and we see clearly that in both steam and air the law of expansion depends solely on the treatment of the fluid as regards the reception or rejection of heat, as has been already shown in general terms in Art. 29.

The graphical methods employed for steam, when used for air, are much more simple, because the internal energy of air, reckoned from the absolute zero, is always represented by a rectangle constructed on the base V, with a height of $2 \cdot 45\ P$. (See page 80.) With this modification the construction representing the heat supplied during expansion, or for the adiabatic curve, may be carried out exactly as explained in detail in the case of steam.

CHAPTER VII.

PERFECT ENGINES WITH ANY GIVEN CYCLE.

In all that was said in Chapter V. it was supposed that the engine received all its heat at a fixed temperature from a single source of very slightly higher temperature, while it rejected heat at another temperature very slightly above that of a refrigerator. The cycle of the engine is, then, of that simple kind described in Art. 57, page 133. It may be called an elementary cycle, and the engine an elementary heat engine. The terms "Carnot Cycle," "Carnot Engine," are also not unfrequently used.

In actual heat engines, however, there are sometimes several sources of heat, like the gratings of a regenerator, or the source of heat may be within the working fluid itself, as in the explosive gas engines, and vary in temperature. Even when the heat is all ultimately derived from a single source of fixed temperature, it will generally happen that the fluid does not receive the whole of it at that temperature, and it is upon the temperature of *reception* and *rejection* of heat that the cycle depends, not upon the temperature of the ultimate source, which may be, and often is, much higher.

In actual engines, then, the cycle is never elementary, but is much more complex, and we now resume the subject with the object of studying such cases.

Generalisation of Carnot's Principle.

91. Take a simple engine, such as that of Art. 36, and imagine, instead of a single source of heat and a single refrigerator, various sources of temperature, T_1, T_2, &c., the temperature of the refrigerator being T_0. Suppose the engine working with any fluid, as, for instance, a mixture of steam and water, and, at the beginning of the operation, let the temperature of the fluid be T_1, the temperature of the corresponding source of heat: then, if that source be applied,

the fluid receives heat and increases in volume. In Fig 26 it is supposed that the fluid is initially water, and evaporates partially, till its volume has increased from $Z A$ to $Z B$, during reception of a quantity of heat Q_1 from the first source; the pressure, of course, in this case remains constantly at its initial value. Now remove the

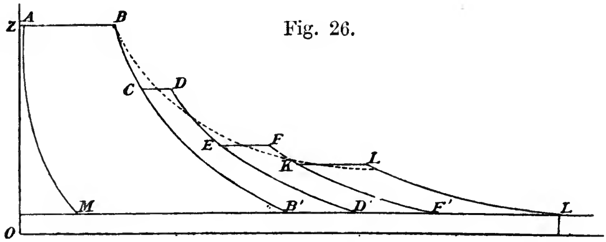


Fig. 26.

first source and allow the fluid to expand without gain or loss of heat: the adiabatic curve $B C$ is described, while the temperature falls from that of the first source to that of the second (say T_2); next, instead of allowing the expansion to proceed, and the temperature to fall, apply the second source of heat, which causes a further evaporation at the constant pressure, corresponding to T_2, during the reception of a quantity of heat from that source, which may be called Q_2: the corresponding increase of volume is represented in the figure by $C D$. Repeat this process for all the sources of heat, the last adiabatic curve being $L L^1$, while the temperature finally falls to T_0, the temperature of the condenser; then let heat be abstracted by the refrigerator, till the volume $M L^1$ of steam has been condensed, the point M being so taken, as in the simpler case of Art. 62, that the mixture of steam and water returns, after adiabatic compression, represented by the curve $M A$, to the state of water at temperature T_1 As in all other heat engines, the area of the diagram represents the energy exerted by the fluid during the cycle of charges through which it passes; and the only question is to find the relation between that area and the quantities of heat Q_1, Q_2, Q_3, &c., received at each temperature. This can easily be done by reference to Chapter V.; for imagine the adiabatic curves $B C$, $D E$, &c., prolonged to meet the horizontal line $M L^1$ in B^1, D^1, F^1, &c., then each of the curved quadrilaterals $A B^1$, $C D^1$, &c., may be considered as the indicator diagram of a simple engine, such as was considered in Chapter V., which receives heat from its own source and rejects heat into the refrigerator, satisfying the conditions of maximum efficiency for engines working between given limits of temperature. But from Art. 62 it appears that the area of such a diagram must be $Q \cdot \frac{T - T_0}{T}$ where Q is the heat received, and T the temperature of the source; hence, if U be the whole area of the complete diagram, it follows that

$$U = Q_1 \cdot \frac{T_1 - T_0}{T_1} + Q_2 \cdot \frac{T_2 - T_0}{T_2} + Q_3 \cdot \frac{T_3 - T_0}{T_3} + \dots \quad \text{(A)}$$

an equation which shows that the area of the diagram, that is to say, the work done by the engine, depends solely on the quantity of heat supplied from each source, and the temperatures at which it is supplied.

Moreover, let Q be the whole heat supplied from the various sources, and R the heat which passes into the refrigerator, then

$$U + R = Q = Q_1 + Q_2 + Q_3 + \dots$$

whence by subtraction of the preceding equation and division by T_0, we obtain the general relation

$$\frac{R}{T_0} = \frac{Q_1}{T_1} + \frac{Q_2}{T_2} + \frac{Q_3}{T_3} + \dots \quad \text{(B)}$$

In drawing Fig. 26, and in the description, it was supposed that the engine was a steam engine, but this is not at all necessary; precisely the same reasoning applies to any engine, but the lines *A B*, *C D*, *E F*, &c., of the figure will now no longer be straight, but will be the isothermal curves proper to the particular fluid considered: thus for a perfect gas these lines would be rectangular hyperbolæ. Hence the equations (A) and (B) are true for any engine receiving heat in the way supposed, provided only that none of the expansive energy of the fluid is dissipated by unbalanced expansion. (Art. 54) It should be understood that any of the quantities of heat Q may be negative, representing heat abstracted instead of added.

92. A fluid may be made to receive heat from a succession of sources of different temperatures by passing it through gratings similar to those of a regenerator, as described in Art. 39 and elsewhere in Chapter IV. In the Stirling air engine the volume of the

fluid remains constant during the process, but this is not necessary, it may change in any way as the pressure and temperature change; thus, for example, in the Ericsson air engine, or in the ideal Joule engine of Art. 51, the pressure remains constant during the passage through the gratings. In this way a fluid may be made to expand according to any law we please, receiving at each step the corresponding amount of heat necessary from a grating, the temperature of which differs insensibly from its own, so that the process is reversible. Similarly, when a fluid is compressed, the compression curve may be made of any form we please, the needful amount of heat being abstracted at each step by the gratings. In a number of cases the amount of heat so added or subtracted has been found for air and steam in Chapters IV. and VI.

Let us suppose that we have a single principal source of heat of temperature T_1, a refrigerator of temperature T_0, and let a grating of temperature (absolute) t supply the fluid with amount of heat q, while the same or a different grating subsequently abstracts the heat r. Equation (A) becomes

$$U = Q \cdot \frac{T_1 - T_0}{T_1} + \Sigma \frac{t - T_0}{t} \cdot (q - r),$$

the summation extending to all the gratings.

In the case of a regenerator $q = r$, and the formula reduces to

$$U = Q \cdot \frac{T_1 - T_0}{T_1},$$

just as for a perfect engine without regenerator working between the same limits of temperature. The reason of this is that the fluid is compressed during change of temperature according to the same law as it originally expanded, so that, apart from losses (page 97), each individual grating is simply a store-house of heat of that particular temperature, alternately drawn upon during compression and replenished during expansion. The only effect of the regenerator is to alter the expansion and compression curves of the indicator diagram without altering its area. The bulk of the engine is thus diminished in a manner already fully explained in the case of the Stirling air engine.

If the law of expansion during change of temperature is not the same as the law of compression—for example, if the fall of temperature is at constant volume while the rise is at constant pressure—then q is not equal to r, and the formula just given serves to determine the efficiency. In the ideal Joule engine of Art. 51, which should here be carefully considered, one set of gratings form the only source of heat, while another set form the refrigerator, and, proceeding as in Art. 94, further on, the equation may be verified by comparing its results with those previously obtained in Chapter IV. In this case both expansion and compression are at constant pressure, but the first is the greater.

93. Engines rarely receive heat from various sources in the way supposed, and the great importance of the result arises from the fact that it is immaterial from whence the heat is derived, provided that the fluid *receives* heat at the temperatures supposed. For example, in the perfect steam engine of Art. 62, the heat is derived from the hot gases of the furnace at a temperature much higher than that of the boiler: yet, in considering the efficiency of the engine, it is the temperature of the boiler which is considered, not that of the furnace, because it is at the temperature of the boiler that the fluid receives heat.

Thus the results are applicable to any engine whatever, receiving heat at any temperatures, provided only that the whole expansive energy of the fluid is completely utilised. Subject to this proviso, the following general statements may be made, which are equivalent to the foregoing equations.

FIRST.—*The efficiency of every possible heat engine depends solely on the mode in which it is supplied with heat, and not at all on the nature of the fluid or arrangement of the engine.*

This principle is the last step of a gradual generalisation, of which the principle laid down in Art. 27, Chapter III., and Carnot's principle explained in Art. 55, Chapter V., are special cases. In Chapter III. it was shown that it is a necessary consequence of the principle of work, that the energy exerted by a given quantity of a *given kind* of fluid, is independent of the particular kind of machinery by means of which that energy is utilised. In Chapter V. we found that although the magnitude of the energy exerted varies according to the nature of the fluid, yet, when it receives and rejects heat at *given fixed temperatures*, the proportion which that energy bears to the heat expended, that is to say, the efficiency of the engine, is the same for all fluids, and consequently for all possible engines. Now we go a step farther, and assert that this will be the case, not only when the engine receives heat at one fixed temperature and rejects heat at another fixed temperature, but also when it receives heat at *any* number of temperatures, provided only that the quantities of heat received at the various temperatures are in the same proportion.

Thus, if the law according to which heat is supplied be supposed given, an engine will be of maximum efficiency, and hence may be said, in a certain sense, to be "perfect," even though it receives heat at varying temperature from a source the temperature of which is much higher, the only condition of maximum efficiency being that there shall be no unbalanced expansion. With a single source of fixed temperature, that temperature is the true datum of the problem, and then it is only engines which receive the whole of their heat at that temperature which can be considered as "perfect" in the full sense of the word: all others are of less efficiency, because they receive some of their heat at a lower temperature.

We have supposed for simplicity, as being sufficiently general for the purpose, that the heat is all rejected at a single temperature, but this restriction is not necessary for the truth of the reasoning employed.

SECONDLY.—*If the heat received at any temperature by a heat engine be divided by that temperature (absolute), the sum of the quotients is unaltered by the passage of the heat through the engine.*

This principle is merely the expression in words of equation (B), and amounts to saying, that although heat disappears (being changed into work), during its passage through a heat engine, yet that a certain quantity, found by dividing the heat by the temperature at which it is used, is unchanged, if the opportunity of turning heat into work, presented by the available difference of temperature, has been duly utilised.

Zeuner has called this quantity a "*heat-weight*," thus developing further Carnot's analogy between a heat engine and a water-power engine.* Let W be the weight of water used in a perfect water-wheel, $h - h_0$ the fall, then

$$U = W(h - h_0)$$

* *Grundzüge der Mechanischen Wärmetheorie*, p. 68.

is an equation which gives the work done by the machine: or, if there be various quantities of water, falling through various heights,

$$U = W_1 (h_1 - h_0) + W_2 (h_2 - h_0) + \dots .$$

Now compare this with the general formula (A),

$$U = \frac{Q_1}{T_1}(T_1 - T_0) + \frac{Q_2}{T_2}(T_2 - T_0) + \dots .$$

and it will be seen that just as the difference of temperature corresponds to the fall, so the quotient Q / T corresponds to the weight.

In the same way a refrigerating machine may be considered as a "heat-pump" employed to raise a "heat-weight" to a higher level of temperature.

We shall use this term "heat-weight" occasionally as a name for the quotient in question; but analogies of this kind, though useful at the outset of the subject, must not be pushed too far, and should be considered merely as aids to the imagination: the things compared are essentially different. In particular, the analogy fails for imperfect engines; in such engines the "heat-weight" increases during the passage through the engine. A different interpretation will be found further on.

94. In Art. 91 it was supposed that the supply of heat took place by successive steps at successive fixed temperatures: it will, however, seldom happen that this is the case, the engine receiving some or all of its heat, in general, continuously during a gradual change of temperature. By taking the successive temperatures near enough, however, this case becomes equivalent to that originally supposed: let us suppose that the amount of heat ΔQ is supplied at temperature T, then equation (B) becomes

$$\frac{R}{T_0} = \Sigma \frac{\Delta Q}{T} + \frac{Q_1}{T_1} + \frac{Q_2}{T_2} + \dots$$

Q_1, Q_2, &c., being supplied as before at fixed temperatures T_1, T_2... Thus in Fig. 26, just now given to show the operation of a steam engine receiving heat at varying temperature: let us suppose the sources of heat indefinitely multiplied, then the broken line *B C D E F K L* becomes a continuous curve, dotted in the figure, representing the expansion of steam under a continuous supply of heat, the form of the curve depending, as shown in the last chapter, on the quantity of heat supplied at each step of the expansion.

An important case of this is, when the heat supplied during a continuous change of temperature is, at every step, proportional to the change, as in Case III. of the expansion of steam in Art. 87 of the last chapter. Here we shall have

$$\Delta Q = K \, . \, \Delta T,$$

where K is some constant, whence

$$\frac{R}{T_0} = K \, . \, \Sigma \, . \, \frac{\Delta T}{T} + \&c.$$

Proceeding to the limit and integrating,

$$\frac{R}{T_0} = K \, . \, \log_\epsilon \frac{T}{T'} + \&c. \, ;$$

and thus it appears that when a quantity of heat is applied continuously, in exact proportion to the change of temperature, its "heat-weight" is equal to $K \log_\epsilon \frac{T}{T'}$ where T, T' are the limits of temperature within which the heat is supplied: or, to put the same thing in other words, it is the same as if the heat were supplied at the mean temperature (T_m) given by

$$T_m = \frac{T - T'}{\log_\epsilon \frac{T}{T'}} \, .$$

When the difference $T - T'$ is not very great,

$$T_m = \frac{T + T'}{2} \text{(nearly)}.$$

95. An important example of the supply of heat at rising temperature occurs in the explosive gas engines briefly discussed in Chapter IV. Here the principal part of the heat is supplied by combustion before expansion begins, that is at constant volume, while the temperature rises from T_4 to T_1, the amount being (page 111)—

$$Q_1 = K_v (T_1 - T_4).$$

The theoretical maximum efficiency is therefore

$$\text{Efficiency} = \frac{T_m - T_0}{T_m}$$

where T_0, as in the article cited, is the temperature of the atmosphere and T_m is given by the equation found in the last article.

For example, let $T_1 = 3000°$ and $T_4 = 750°$, then

$$T_m = \frac{2250}{\log_\epsilon 4} = 1623°,$$

and assuming the temperature of the atmosphere 62° F. or 523° absolute, the theoretic maximum efficiency is found to be about 68 per cent. In the article cited it is shown that the calculated efficiency so far as accounted for by the diagram is ·33, and the "true" efficiency apart from the large loss by leakage of heat through the cylinder walls is therefore 48 per cent. The losses here are first, incomplete expansion, and secondly, rejection of heat at a temperature far above that of the atmosphere. The first of these is much diminished in the Atkinson type of gas engine, some notice of which will be found in the Appendix. The second occurs in all actual gas engines and is difficult to avoid, being, unless a regenerator is used, a necessary consequence of the cooling of the burnt gases at the atmospheric pressure instead of during compression. If rejection of heat at constant pressure, without a regenerator, be regarded as a necessity from the nature of the engine, an average temperature of rejection of heat might be found on the same principle that we have just determined the average temperature of supply. An ideal efficiency is thus obtained which represents more closely what is practically possible. It would be of little use to make such a calculation by the method we are now considering, for, as elsewhere pointed out, no results are deducible from Carnot's principle, either in its simple form or as generalised in the present chapter, which cannot also be obtained so far as air and gas engines are concerned by direct calculation. The question is here considered for the sake of pointing out (1) that the average temperature of the heat supplied must generally be much less than the maximum temperature of the source of heat; and (2) that in each class of heat engine the cycle of the ideally perfect engine will be to some extent arbitrary, depending on what losses are considered as inherent in the nature of the engine and what losses are ideally avoidable. All that can be said is that when this point is decided the maximum quantity of work which can be done by a given quantity of heat will be definitely determined, and the ratio which the work done by the actual engine bears to it will be the "true" efficiency of the engine as distinguished from the "apparent" or "absolute" efficiency, which, as pointed out in Art. 66, is nothing more than a coefficient of performance.

In any case, the margin for improvement in gas engines, though no doubt much greater than in other heat engines, is not nearly so great as might be supposed judging merely from the maximum temperature in the cylinder.

Similarly, in reversed heat engines the effective range of temperature is often much less and, consequently, the theoretical maximum efficiency much greater than might be supposed from the extreme temperatures reached. Take, for example, the warming machine considered in Art. 64, page 148. Here heat drawn from the atmosphere at 32° is delivered at 92° by an elementary reversed heat engine, and employed to raise the temperature of air by 60°. Evidently, during the process of heating, the air gradually rises in temperature, and if heat at 92° is employed for the purpose there is a difference of temperature which we fail to utilise. In fact the average range of temperature through which the heat has to be elevated is 30° not 60°, and an ideally perfect machine would have an efficiency of 18·44 instead of 9·22. It would be very difficult to construct such a machine which would be practically efficient, but that is not here the question, the point is that in a reversed engine the margin for possible improvement is much greater than it might appear at first sight.

Nature of the Cycle in an ordinary Steam Engine.

Adiabatic Equation.

96. We are now in a position to discuss the process undergone by the steam in an ordinary steam engine, and at the same time to complete the study of the expansion of steam, left partly unfinished in the last chapter.

We shall begin by supposing a non-conducting cylinder, complete expansion down to the pressure of the condenser, clearance and wire-drawing neglected: also the back pressure is imagined that which corresponds to the temperature of the condenser. Then the indicator diagram is shown in Fig. 20*a*, where $O\ N$ represents s ($=\ \cdot 016$) the volume of 1 lb. of water, $A\ N$ the rise of pressure as the water is forced into the boiler, $A\ B$ the evaporation of the water till the volume has increased from $A\ S$ to $B\ S$, $B\ C$ the expansion of the steam from volume $B\ S$ to volume $C\ R$, $C\ M$ the condensation of the steam which is carried on till *all* the steam is condensed and it becomes once more water.

Fig. 27*a*.

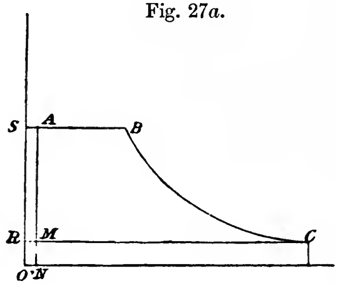
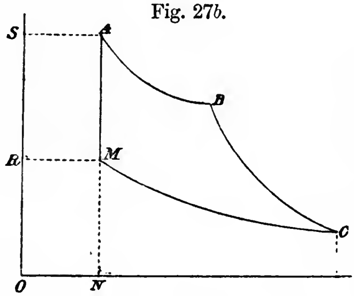


Fig. 27*b*.



The efficiency of this engine is now to be found by comparing it with an air engine, which receives and rejects heat in the same way. In Fig. 27*b* the diagram of such an engine is shown: $M\ A$ represents a rise of pressure at constant volume during reception of heat, $A\ B$ an increase of volume at constant temperature during a further application of heat, $B\ C$ expansion without gain or loss of heat till temperature has fallen to that of the refrigerator, $C\ M$ compression at constant temperature till the air has returned to its original state. Different as the two diagrams may appear on paper, yet from a thermodynamical point of view they are precisely similar, provided the temperatures be the same, and the quantities of heat supplied at each temperature be in the same proportion: hence the efficiency of the two engines must be the same according to the generalised form of Carnot's principle given in Art. 93.

Now in the air engine the heat expended or rejected at each step of the process is, employing the notation of Chapter IV., per lb. of air,

During elevation of temperature	.. ..	$K_v(T_1 - T_0)$
,, expansion	.. ..	$c\,T_1 \log_\epsilon r$
,, compression	.. ..	$c\,T_0 \log_\epsilon R$

where T_1, T_0 are the temperatures at which the air receives and rejects heat during its expansion $A\ B$ and compression $C\ M$ at constant temperatures, r, R, the ratios of expansion and compression (isothermal).

In the steam engine, on the other hand, the heat expended per lb. of steam is

During elevation of temperature	.. .. ..	$T_1 - T_0$
,, evaporation	.. ..	$x_1 L_1$
,, condensation	.. ..	$x_0 L_0$

where $1 - x_1$, $1 - x_0$ are the weights of water mixed with the steam at the beginning and end of the expansion respectively, and L_1, L_0 are as usual the latent heats of evaporation at $T_1\, T_0$.

We must now suppose the air engine so arranged that the quantity of heat received at constant temperature is in the same proportion to that received at constant volume as in the case of the steam engine, that is to say, we must suppose

$$\frac{c\,T_1 \log_\epsilon r}{K_v (T_1 - T_0)} = \frac{x_1 L_1}{T_1 - T_0}$$

but by Art. 31, page 82,

$$\frac{c}{K_v} = \gamma - 1\,;$$

$$\therefore\ (\gamma - 1)\, T_1 \,.\, \log_\epsilon r\,. = x_1 L_1\,.$$

This condition being satisfied, the air engine and the steam engine will receive heat, at the same temperatures, in the same proportions, which is the needful condition that the efficiency may be the same: thus it will necessarily follow that the heat rejected in the two cases is in that same proportion, so that by similar reasoning

$$(\gamma - 1)\, T_0 \,.\, \log_\epsilon R = x_0 L_0\,;$$

but, if r' be the ratio of adiabatic expansion in the air engine, it is clear that

$$R = r\,r',$$

And by Art. 36 we know that

$$r' = \left(\frac{T_1}{T_0}\right)^{\frac{1}{\gamma - 1}}\,;$$

$$\therefore\ (\gamma - 1) \log_\epsilon r' = \log_\epsilon \frac{T_1}{T_0},$$

and

$$(\gamma - 1) \log_\epsilon R = (\gamma - 1) \log_\epsilon r + \log_\epsilon \frac{T_1}{T_0};$$

whence replacing $(\gamma - 1) \log_\epsilon r$, $(\gamma - 1) \log_\epsilon R$ by their values just given, it is clear that

$$\frac{x_0 L_0}{T_0} = \frac{x_1 L_1}{T_1} + \log_\epsilon \frac{T_1}{T_0} \qquad \text{(I)}$$

97. The method employed in the last article is based on the first statement of Art. 93, page 207. It is very instructive as an illustration of thermodynamical principles, but the very important equation deduced, which is called the Adiabatic Equation, is merely a particular case of equation (B), page 205, being obtained by writing the proper values of Q and R in that equation, and performing the summation for the supply of heat at rising temperature as on page 210.

It should be remarked that the variation in the specific heat of water (Art. 3) is neglected, and it is interesting to remark where the reasoning of Art. 96 would fail if it were taken into account. If the specific heat of water vary, the same amount of heat will not be received at each degree of rise of temperature, and hence the processes in the air engine and the steam engine will not precisely correspond; hence the resulting equation is not quite exact. For some purposes it is desirable to take this into account, which may be done, with sufficient approximation, by employing a mean value, greater than unity, for the value of the specific heat, chosen, by Table II., according to the range of temperature under consideration.

Again, it is possible to make equation (I) more general, so as to include the case in which a plate of metal is supposed in contact with the steam, as in Case III., Art. 85, of the last chapter. If $m c$ be the heat supplied to the steam, as its temperature falls, as explained in detail in the article cited, then by Art. 94, its "heat-weight" is $m c \log_\epsilon \frac{T_1}{T_0}$, and thus, if q be the mean value of the specific heat of water, we obtain the more general equation

$$\frac{x_0 L_0}{T_0} = \frac{x_1 L_1}{T_1} + (q + m c) \log_\epsilon \frac{T_1}{T_0}, \qquad \text{(II)}$$

which applies to all the three cases treated in Articles 82-85. It was there shown how to determine the expansion curve of a mixture of steam and water by applying a graphical or arithmetical process in successive stages; the formula now given enables us to obtain a final result with accuracy without going through the intermediate stages.

The adiabatic curve, then, is determined by the equation, derived from equation (I) by omitting the suffix 0,

$$x\frac{L}{T} = x_1\frac{L_1}{T_1} + \log_\epsilon\frac{T_1}{T} \qquad \text{(approximately)},$$

from which the amount of water in the steam can be determined, after expansion till the pressure (and therefore the temperature) has fallen by a given amount. Let V be the volume of a lb. of steam at the end of the expansion, then

$$V = (v - s)\,x + s = v\,x + s \qquad \text{(approximately)}$$
$$= v\,x \text{ (nearly, unless the steam is nearly all water)};$$

and substituting for x we determine the final volume of the steam after expanding till the pressure has fallen by a given amount. In using the equation it is necessary to suppose the final pressure given, and then to find T from Table I*a*, of course adding 461 as the temperatures are absolute. The calculation is simplified by finding x first and then V. The equation is applicable, however much water be mixed with the steam initially, or even when there is no steam but only water, as in Case II., Art. 83; it differs from Rankine's equation given in his work on the *Steam Engine* (p. 385) only in notation and in being more general, as Rankine has considered only the case in which the steam is originally dry. The formula was discovered by Rankine and Clausius working independently.* Two numerical examples will show how it is applied.

(1) Dry Steam at 302° expands without gain or loss of heat till its temperature has fallen to 212°, or, what is the same thing, its pressure to 14·7 lbs. on the square inch; it is required to find how much moisture it contains, and what is its value. Here we have

$$x_1 = 1 : T_1 = 302° + 461° = 763°;$$
$$T = 212° + 461° = 673°.$$

* The extension of the formula to the case where heat is supplied to the steam in exact proportion to the fall of temperature appears to be due to Zeuner. See *Grundzüge der Mechanischen Wärmetheorie*, p. 357.

From Table II. the value of L_1 is 902 thermal units, and of L 966 thermal units;

$$\therefore x \cdot \frac{966}{673} = \frac{902}{763} + \log_\epsilon 763 - \log_\epsilon 673.$$

Performing the numerical calculations, for which Table VI. is useful,

$$x = \cdot 911,$$

showing that rather less than 9 per cent. of the steam is condensed.

(2) Let the steam, instead of being initially dry, contain initially 30 per cent. of water, then $x_1 = \cdot 7$, and the other data are unaltered;

$$\therefore x \cdot \frac{966}{673} = \cdot 7 \cdot \frac{902}{763} + \log_\epsilon 763 - \log_\epsilon 673,$$

whence

$$x = \cdot 662,$$

showing that about 4 per cent. by weight of steam is condensed, being less than when the steam is initially dry, as already explained in the Appendix to Chapter VI.

(3) Let 1 lb. of water at 302° expand, as in Art. 83, till the pressure has fallen to 14·7, then $x_1 = 0$ and the other data are unaltered, whence

$$x = \cdot 0872,$$

a result which signifies that about $8\frac{3}{4}$ per cent. of the water has evaporated.

The volumes of the steam in these three cases are now found by the equation just mentioned,

$$V = vx + s,$$

in which s may be disregarded in the first two cases, but *not* in the third, whence

$$V = 24 : V = 17 \cdot 5 : V = 2 \cdot 35.$$

The first and last results serve to fix the four corners of an indicator diagram of a perfect steam engine working with initially dry steam between the temperatures 302°, 212°, that is to say, the pressures 69·21 and 14·7 lbs. per square inch. It is this diagram which is shown in Fig. 19, Art. 62.

The calculation here described is for some purposes inconvenient, because the form of the curve is not ascertained directly, there being no direct relation between p and V, but only an indirect one by means of the temperature. Hence, if it be required to find the pressure after expanding r times, this can only be done by trial and error. To avoid this inconvenience it was suggested by Rankine that the expansion curve might be represented approximately by the equation

$$p V^n = \text{constant} = p_1 V_1^n,$$

if the index n be found by trial so as to give results agreeing with the foregoing calculation.

To test this suggestion, we have the equation

$$\log p + n \cdot \log V = \log p_1 + n \cdot \log V_1,$$

from which is obtained

$$n = \frac{\log p_1 - \log p}{\log V - \log V_1}.$$

If the suggestion is correct, we ought to find the same value of n, whatever be the values of p V, provided only the expansion start from the same point represented by the suffix 1.

For example, let us calculate n in the first of the two preceding cases; then we have

$$n = \frac{\log 69 \cdot 21 - \log 14 \cdot 7}{\log 24 - \log 6 \cdot 153} = 1 \cdot 138.$$

Now if the calculation be repeated with a different value of the terminal pressure, the result ought to be the same.

On trial it is found to be so approximately, and the result is also nearly the same if the initial pressure be, not 69·21, but some other pressure, provided the steam be initially dry. Zeuner has examined the question with great care and accuracy, and finds the best average value of n to be 1·135. But if the steam be initially wet, a smaller result is obtained; thus, if as in the second example given above, the steam contain 30 per cent. moisture, then

$$n = 1 \cdot 1 \text{ (nearly)}.$$

By making numerous calculations of this kind, Zeuner found the best average value of the index to be given by the empirical formula.*

$$n = 1 \cdot 035 + \frac{x_1}{10},$$

where x is the initial dryness-fraction of the steam supposed not less than $\cdot 7$; and Zeuner's calculations have since been verified by Grashof.† When Rankine suggested the equation $p\,V^n$ = constant he gave the value 10/9 or $1 \cdot 11$ for the index, on what grounds cannot now be determined; it is certain that numerical calculations from his own equations give a larger result, except when the steam is wet initially.

When the method of logarithmic plotting is used (page 86), it is clear that the adiabatic curve, like the saturation curve, becomes a straight line.

The equation $p\,V^n$ = constant is of less value than might be supposed, for it will be seen hereafter that when adiabatic expansion has to be considered in practical questions, the data of the question are, nearly always, not the initial pressure and ratio of expansion, but the initial and final pressures, in which case the equation may just as well be employed in its original form.

98. Although the adiabatic curve, for a mixture of steam and water, is of a very complicated character, incapable of being expressed exactly by any algebraical equation, yet all adiabatic curves are connected together in such a way that, when any two are given between given limits of pressure, all the others are at once determined by a calculation of the simplest character.

Let $V_1\,\overline{V}$ be the volumes corresponding to a given pressure p of a mixture of steam and water for the two extreme cases in which the mixture is wholly steam, or wholly water, at some other greater pressure; and let us suppose these volumes already known. Then, to find the corresponding volume (V'), when the mixture consists at the same initial pressure of x_1 lbs. of steam, and $1 - x_1$ lbs. of water, it is easily showh that

$$V' = x_1 V_1 + \overline{V}(1 - x_1),$$

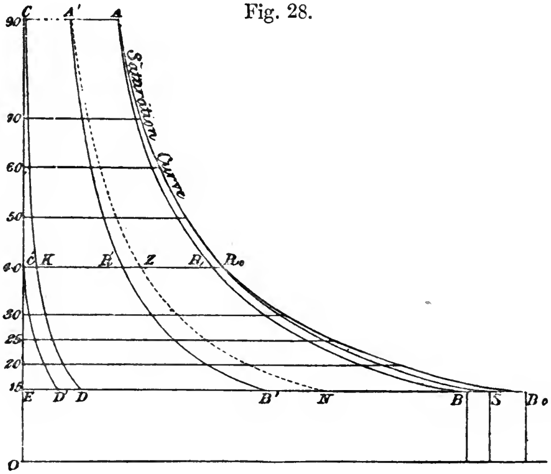
* *Grundzüge der Mechanischen Wärmetheorie*, p. 342.

† *Theoretische Maschinenlehre.* Band 1, p. 175.

an equation which determines all the other adiabatic curves: thus, when, by methods already given, V_1 $\overline{V}$ have been determined, the volume of steam in any given state initially, after adiabatic expansion down to pressure p, is readily calculated; and the formula may be extended to cases in which the steam expands in contact with a metallic plate.

The nature of the connection between the curves may, however, be most easily seen by reference to a figure. Fig. 28 shows various

Fig. 28.



adiabatic curves lying between 90 lbs. and 15 lbs. pressure, of which we will suppose known in the first instance, the curve $A\ B$, $C\ D$, corresponding to dry steam at 90, and water at 90, respectively. Then, to find the adiabatic curve for a mixture containing, at 90 lbs. pressure, one-half steam and one-half water, it is only necessary to draw a curve, $A'\ B'$, dividing the intercepts between the original curves into equal parts. Or, again, to determine the adiabatic curve corresponding to dry steam at 40 lbs. pressure, draw the saturation curve $A\ R_0\ B_0$, then the required curve starts from R_0, the point in $A\ B_0$ which corresponds to 40 lbs. pressure, and divides externally the horizontal intercepts between the two original curves in a fixed ratio; for example, the point S, where that curve cuts the line of 15 lbs. pressure, is found by the proportion

$$D\,S : D\,B :: K\,R_0 : K\,R;$$

and, on the same principle, $C'' D'$, the curve corresponding to water at 40 lbs. pressure, or any other required curve, is at once found.

Still further, the expansion curve of steam in contact with a metallic plate may be found: take, for instance, a mixture of steam and water, containing originally one-half water, and let it expand in contact with $8\frac{1}{3}$ times its weight of iron, as in the example of Chapter VI., then the expansion curve is found by setting off at every pressure, such as, for example, 40 lbs. in the figure, $R^1 Z$ equal to $C^1 K$, then the required curve is $A\ Z\ N$ in the figure.

99. The equation given in the last article is equivalent to saying that, when a mixture of steam and water expands, the water evaporates, and the steam condenses, just as each would do when taken alone: so that the actual total condensation or evaporation is the difference between the condensation of the steam and the evaporation of the water. When a certain proportion exists between steam and water, the evaporation of the water exactly compensates for the condensation of the steam: a proportion which is given by the equation

$$x = \frac{q \log_\epsilon \dfrac{T_1}{T_2}}{\dfrac{L_2}{T_2} - \dfrac{L_1}{T_1}},$$

where T_1 T_2 are the initial and final temperatures between which the change is supposed to take place. For a small change at temperature T, the above formula is very approximately equivalent to

$$x = \frac{T}{1440}.$$

At 38 lbs. on the square inch, this gives ·5 as the value of x, which at high pressures will be at most ·6, and at low pressures at least ·45: if a greater change of temperature be considered, the variation in x will be less, and we may say generally that when a mixture of steam and water in equal proportions by weight expands without gain or loss of heat, the evaporation and condensation approximately balance one another, so that the total change is very small: and further, if water predominates, evaporation takes place, but if steam predominates, condensation.

Similar conclusions may be drawn in the case where a mixture of steam and water expands in contact with a metallic plate: the state of things is the same as if the water received the whole amount of heat given out by the cooling plate, and expanded alone without connection with the steam, while the steam condenses as if the plate were not there. When the water forms a film on the surface of the plate, it is probable that this is really what takes place—a question which we shall resume in a later chapter.

Again, when a mixture of steam and water expands, either adiabatically, or in contact with a plate, we may, if we please, separate mentally any part of the water from the rest, and consider it as a solid plate giving out heat as its temperature falls; so long as any of the remaining water remains unevaporated, the process of expansion will be quite unaltered by this supposition. This is an observation of great importance in the theory of the steam engine, for, generally, it is impossible to determine the *total* amount of water contained in a steam cylinder by direct observation: all that can be done is, to find the quantity of water discharged from the cylinder per stroke, either as suspended moisture distributed throughout the whole mass of steam discharged, or by re-evaporation during exhaust. We now see, however, that any water remaining in the cylinder after exhaust plays the part of a metallic plate, but with much greater effect, weight for weight, on account of the great specific heat of water.

100. To complete the study of the expansion of steam, in the way supposed in preceding articles, it is now only necessary to find the useful work done per lb. of steam in the cycle of the steam engine considered in Art. 96, represented graphically by Fig. 27*a*. From the results given on page 214 we have at once, assuming the specific heat of water unity,

$$\text{Heat expended} = T_1 - T_0 + x_1 L_1 = Q,$$
$$\text{,, rejected} = x_0 L_0 \qquad = R.$$

Of these two quantities Q is known from the data of the question, and R is calculated from the adiabatic equation. Their difference must be the useful work (U), which is therefore readily found.

For example, let the initial pressure be 60·4 corresponding to a temperature 293° F., and let the condenser temperature be 102° F. Also let the steam be initially dry, so that x_1 is unity, then

$$T_1 = 754 : T_0 = 563 : L_1 = 908,$$
$$\therefore Q = 191 + 908 = 1099.$$

Also from the adiabatic equation we have

$$x_0 L_0 = T_0 \left\{ \frac{L_1 x_1}{T_1} + \log_\epsilon \frac{T_1}{T_0} \right\}$$

which on substitution of the numerical values gives 843 as the value of $x_0 L_0$. Hence, in thermal units

$$U = Q - R = 1099 - 843 = 256.$$

The result of this calculation is important as being the greatest amount of work which can ideally be obtained from a lb. of steam in an engine which has a feed pump of the ordinary kind. The efficiency of the engine is U / Q, or, in the present case, ·233, and is the same as it would have been if the heat had all been supplied at the mean temperature (absolute)

$$T_m = \frac{Q}{\frac{L_1 x_1}{T_1} + \log_\epsilon \frac{T_1}{T_0}}.$$

When the steam from the boiler is dry, the value of Q is $H_1 - h_0$, otherwise the formula given above for Q must be used. In the present example the mean temperature T_m is 734° absolute, or 273° F., being 20° less than the maximum temperature.

The calculation of these and all other results connected with the adiabatic equation is facilitated by the use of Table VI. at the end of the book, which gives the values of L / T.

Temperature-Entropy Diagrams.

101. Returning to equation (B), page 205, observe that it may be written in the abbreviated form

$$\frac{R}{T_0} = \Sigma \frac{\Delta Q}{T},$$

where ΔQ is the heat added, or (if negative) abstracted at temperature T and the sign of summation extends through the whole cycle of the engine except the rejection of heat into a refrigerator at constant temperature T_0.

Referring now to Fig. 27, page 213, let $A\ B\ C\ M$ be an indicator diagram representing the cycle. Any point in that diagram represents the working fluid in one particular state as defined by its pressure and volume. Any part of that diagram represents a change of state: for example, $M\ A\ B$ represents a change of state from M to B, while $B\ C\ M$ represents a change back again from B to M, completing the cycle. Let $B\ C$ be an adiabatic curve, so that no heat is added or abstracted between these points, and $C\ M$ an isothermal curve during the operation represented by which, the temperature remains T_0, and the quantity of heat R is abstracted. For a given fluid this quantity R is necessarily the same, whatever the form of the part of the diagram $M\ A\ B$ representing the mode in which the fluid changes its state in passing from M to B. If then we write

$$\phi = \frac{R}{T_0} = \Sigma \frac{\Delta Q}{T}$$

the quantity ϕ depends on the initial state M, and the final state B of the fluid, but not at all on the way in which the change of state has been produced. This may vary to any extent, according to the amount of heat supplied at each step of the process.

Thus, for a given point B, representing a certain definite state of a given fluid, the quantity ϕ has a definite value which can be calculated if we know the heat supplied at each step when the fluid changes its state from M to B in any one way. If the position of B be changed, the value of ϕ will change unless the change be adiabatic. Along an adiabatic curve the value of ϕ must be constant.

We have already used the term "heat-weight" for the sum of the quotients in question (page 208), but we are here considering this quantity in relation to some given fluid, of which it may be regarded as an attribute like its temperature or its internal energy. By Rankine this quantity ϕ was called the Thermodynamic Function, and by Clausius, the Entropy, of the liquid. This last is the term which we shall adopt. A fluid in any state represented by B possesses a certain "entropy" compared with some initial state M from which the entropy is reckoned.

102. The state of a fluid is definitely known when we know the numerical values of two independent quantities upon which all its

other properties are made to depend. The two quantities generally selected for the purpose are its pressure and its volume, upon which its temperature, its internal energy, and, as we now find, another property, namely its *entropy*, depend. To represent the state graphically, the volume and pressure are plotted as horizontal and vertical ordinates. But as its other properties have definite numerical values for a given state, one or more of these values may be selected instead, just as in logarithmic plotting (page 84), the quantities plotted are the logarithms of the pressure and volume. Thus the ordinary indicator diagram is a pressure-volume diagram, but the same cycle might be represented by a temperature-volume diagram, in which the vertical ordinate represented the temperature, not the pressure, at the point considered.

An indefinite number of such diagrams may be constructed, but by far the most important is the temperature-entropy diagram in which the vertical ordinate represents temperature, and the horizontal that quantity (ϕ) which we call "entropy."

From the definition it at once follows that if $\Delta\,\phi$ be the change in ϕ consequent on the supply of a small quantity of heat $\Delta\,Q$ at temperature T we shall have

$$\Delta\,Q = T\,.\,\Delta\,\phi,$$

or performing the summation through any change of state produced by application of the heat Q,

$$Q = \Sigma\,T\,.\,\Delta\,\phi\,;$$

from which it appears that the area of any part of a temperature-entropy curve represents the heat supplied during the change it represents, just as the area of an ordinary expansion curve represents the energy exerted on the piston by the expanding fluid. The entropy of a fluid therefore corresponds to its volume, and may even be described as its Thermal Volume.

The cycle of an engine is represented on such a diagram by a closed figure, the area of which represents the heat utilised during the cycle, while the area between any part of it and the base line from which temperatures are measured represents heat supplied or abstracted during that part of the cycle.

(1) Take first a Carnot Cycle, such as is described on page 133. In Fig. 29 (page 228), $Z\,Z$ represents a zero line from which

absolute temperatures are set up as ordinates: to save room, it is placed much nearer the rest of the figure than it would usually be if plotted to scale. The horizontal lines 1 2, 3 4 are drawn at heights representing T_1 T_0, the temperatures of receiving and rejecting heat. The horizontal ordinates are measured from any convenient zero line $Z\ Y$, and represent the value of ϕ. For an adiabatic change ϕ is constant (page 224), and in the present case, therefore, the changes of temperature are represented by vertical lines 2 3, 1 4. Thus the temperature-entropy diagram for a Carnot Cycle is the rectangle 1 2 3 4. This is true, whatever the fluid be, though the pressure-volume diagram, as represented by Fig. 11, page 88, for air, and Fig. 19, page 143, for steam, varies greatly, according to the nature of the fluid.

The heat supplied or rejected during any part of the process is now represented by the area of the corresponding part of the diagram. Thus if Q be the heat supplied during the operation 1 2 and R the heat rejected during the operation 3 4,

$$Q = \text{Area } 1\ 2\ N\ N = T_1 \{\phi_2 - \phi_1\},$$
$$R = \text{Area } 3\ 4\ N\ N = T_0 \{\phi_3 - \phi_4\}.$$

The difference is the heat utilised by being transformed into useful work; this is represented by the area 1 2 3 4 of the closed figure representing the cycle just as in an ordinary indicator diagram. The values of ϕ, which in the figure are measured from an arbitrary zero line $Z\ Y$, are obtained in terms of the pressure, volume, or temperature, by employing the known values of Q, R for an isothermal change of the particular fluid considered. For example, if the fluid be air, referring to page 91,

$$\phi_2 - \phi_1 = \phi_3 - \phi_4 = c \cdot \log_\epsilon r,$$

where r is the ratio of expansion or compression.

(2) Take next the cycle of the ordinary steam engine as represented in Fig. 27, and already considered at some length.

We must now, in addition to the isothermal changes, consider a change of temperature produced by supplying heat uniformly as the temperature rises. In Fig. 29 draw a horizontal line at a height above $Z\ Z$, representing T, the absolute temperature at any stage of the rise, and upon it take a point B lying on a curve $O\ B\ 1$ defined by the equation

$$\text{Area } O\ B\ M\ Z = a \{T - T_0\}$$

where a is the heat supplied for each degree of the rise. The horizontal ordinate $O\ L$ being the quantity ϕ, we have

$$\Delta\phi = \frac{a \, . \, \Delta T}{T},$$

which by integration gives immediately

$$\phi = a \log_\epsilon \frac{T}{T_0},$$

so that the curve $O\ B\ 1$ is a logarithmic curve, and the horizontal ordinate $O\ 4$ of the point 1 is

$$\phi_1 = a \, . \, \log_\epsilon \frac{T_1}{T_0}.$$

We have here, as is convenient, adopted the line $O\ Y$ drawn at this distance from 1 4 as a zero line to reckon the value of ϕ from. Using as before the symbols Q, R for heat expended and rejected.

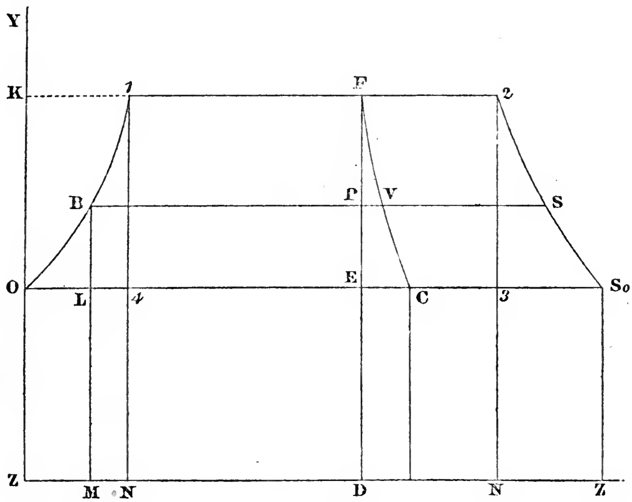
$$Q = \text{Area } Z\ O\ 1\ 2\ N = a\,\{T_1 - T_0\} + T_1\,\{\phi_2 - \phi_1\}$$
$$R = \text{Area } Z\ O\ 3\ N = T_0 \, . \, \phi_3.$$

The difference is the heat transformed into useful work, graphically represented by O 1 2 3, the area of the diagram representing the cycle. And as before, the values of ϕ are found by substitution.

103. Referring now to Chapter V., pages 127, 128, it will be seen that in every heat engine the working fluid goes through a series of changes in which (1) heat is alternately received and rejected, and (2) mechanical energy is alternately exerted and absorbed. In other words, there is a cycle of *thermal* operations and a cycle of *mechanical* operations. The mechanical cycle is graphically represented by the ordinary indicator diagram or by some combination of indicator diagrams (Art. 41), and we now find that the thermal cycle may also be graphically represented by a thermal indicator diagram in a strictly analogous way. In both diagrams the area of the closed figure represents the same quantity of energy, namely, the heat utilised or the useful work done; but in the thermal diagram the area of any part represents heat added or subtracted, whereas in the mechanical diagram it represents mechanical energy exerted on or supplied by external bodies. In other words, the

thermal diagram represents the outflow and inflow of heat in the same way as the mechanical diagram represents the outflow and inflow of mechanical energy. By the addition of a set of adiabatic curves to an ordinary indicator diagram, it may be made to show the thermal cycle as well as the mechanical, and this was the way in which Rankine treated the question.* This method, however, is not of practical use ; it is far better to construct a separate temperature-entropy diagram to represent the thermal cycle, as proposed by Professor J. Willard Gibbs, who was the first to point out its advantages.† It is the graphical expression of the Second Law of Thermodynamics in a simple but perfectly general form.

Fig. 29.



104. It has already been pointed out that the quantity ϕ for each state of the fluid has a value which, relatively to some given initial state, is perfectly definite and may be calculated. We now proceed to perform the calculation for air and steam.

Returning to Fig. 29, take any point P representing the fluid in a state defined by its temperature T and entropy ϕ. These quantities

* See Rankine's *Steam Engine and other Prime Movers*, 6th edition, page 343.

† *Graphical Methods in the Thermodynamics of Fluids.* Transactions of the Connecticut Academy of Arts and Sciences, vol, ii. part 2, April 1873.

are represented by the vertical ordinate $P D$ and horizontal ordinate $Z D$. Reasoning as before

$$\phi = Z D = O L + B P$$

$$= a \log_\epsilon \frac{T}{T_0} + \frac{Q}{T},$$

where Q is now the heat necessary to produce the isothermal change $B P$.

(1) Taking the case of air

$$a = K_v ; \; c = K_v (\gamma - 1) ; \; \frac{Q}{T} = c \, . \log_\epsilon r$$

where $r = V/V_0$ is the rate of expansion. Hence

$$\phi = K_v \, . \log_\epsilon \frac{T}{T_0} + K_v (\gamma - 1) \log_\epsilon \frac{V}{V_0} .$$

This gives the "entropy" reckoned from a state represented by the origin O on the diagram, expressing it in terms of the volume and temperature in the state considered. For an adiabatic change the value of ϕ must be constant, and this will be found to give the equation of the adiabatic curve for air already found in Chapter IV. As applied to air, however, the present method of treating thermo-dynamical questions presents no special advantage.

(2) Next taking the case of steam, we write for a the mean specific heat of water during the change considered, or, approximately, unity, and for Q, we write $L x$ where x is the dryness-fraction of the steam considered, then

$$\phi = \log_\epsilon \frac{T}{T_0} + \frac{L x}{T},$$

which gives the entropy reckoned from water at T_0 of any mixture of steam and water at temperature T.

To complete the diagram, take now from Table VI.

$$B S = \frac{L}{T};$$

the point S will be a point in a curve which, as in a pressure-volume diagram, may be described as the "saturation curve." In the figure this curve is $2 \, S \, S_0$, starting from the point 2, which is supposed to represent dry steam. The dryness-fraction x is evidently

$$x = \frac{BP}{BS}.$$

Now the value of x is represented by a similar ratio in an ordinary indicator diagram, on which a saturation curve has been plotted as shown by many examples in Chapter VI., and corresponding points in the mechanical and thermal diagrams are in this way immediately found. See also Fig. 28, page 220.

(3) It thus appears that any expansion curve on an ordinary indicator diagram can at once be plotted on the thermal diagram, and conversely. For example, in the thermal diagram all adiabatic curves are represented by vertical straight lines, hence any such curves may be plotted at once on the mechanical diagram much more simply than by the direct method employed in Chapter VI.

Again, take the case of steam receiving heat from a metal plate or otherwise at the rate of $m\,c$ thermal units for each degree that the temperature falls. In Fig. 29 take

$$EC = mc \,.\, O4$$

and trace a logarithmic curve FVC, this will be the expansion curve starting from the point F and,

$$\text{Heat supplied during Expansion} = \text{Area } FDNC.$$

The diagram in this form is due to Mr. Macfarlane Gray, and is a valuable simplification of the thermodynamics of the steam engine, especially in questions relating to the exchange of heat between the cylinder and the steam it contains (See Appendix). Most of the results already obtained may also be derived from this diagram; as an example, the reader may for exercise find the value of the specific heat of steam as given by the formula on page 167.

In such cases as occur in steam engines the logarithmic curves may generally be replaced by straight lines, and the diagram becomes one of great simplicity, considering the nature of the subject.

CHAPTER VIII.

LOSSES OF EFFICIENCY IN HEAT ENGINES.

105. When a heat engine works between given limits of temperature, it has already been sufficiently explained that the conditions of maximum efficiency are, that the engine shall utilise the whole difference of temperature between the source and the refrigerator, and that none of the expansive energy of the fluid shall be dissipated by unbalanced expansion. Hence, any diminution of efficiency below that maximum value—common to all engines satisfying those conditions, and hence called "perfect"—may be considered as due to losses of efficiency, which may be ranged in two great classes:—

(1) Losses by non-utilisation of the whole available difference of temperature.

(2) Losses by non-utilisation of the whole available expansive energy of the fluid.

We propose to consider briefly these losses as regards heat engines in general, before passing on to study in detail the losses of efficiency in the steam engine in particular.

Class I.—The first class of losses arises from *misapplication* of heat, the engine not receiving the whole of its heat at the superior limit of temperature, as it should for maximum efficiency, but at some lower temperature; and not rejecting the whole of its heat at the lower limit, but at some higher temperature. Losses of this kind are independent of the particular engine considered, provided that the mode of application of the heat be the same, as appears from Art. 93.

Let us imagine T_1 T_0 the limits of temperature, and let us suppose that a certain quantity of heat, Q, instead of being received by the engine at the temperature T_1, as it should, is received at the lower temperature t, then the *value* of that heat (page 151) in the first case is

$$U = Q \cdot \frac{T_1 - T_0}{T_1},$$

while the actual value in an engine, otherwise perfect, is

$$U' = Q \cdot \frac{t - T_0}{t};$$

$$\therefore \quad \text{Loss} = U - U' = Q \cdot \left\{\frac{T_0}{t} - \frac{T_0}{T_1}\right\} = T_0 \left\{\frac{Q}{t} - \frac{Q}{T^1}\right\},$$

a loss quite independent of the particular nature of the engine, and depending only on the temperatures.

Thus, for example, let the limits of temperature be 320° and 120° Fahrenheit, or 781 and 581 absolute, and let 1000 thermal units be supplied to the engine at temperature 250°, instead of 320°; then

$$U = 1000 \cdot \frac{200}{781} = 256 \text{ thermal units.}$$

$$U' = 1000 \cdot \frac{130}{711} = 183 \quad \text{"} \quad \text{"}$$

$$\therefore U - U' = 73 \quad \text{"} \quad \text{"}$$

that is to say, if the whole available difference of temperature—namely, 200°—had been utilised, we might have transformed 256 thermal units out of the 1000; but, actually, by wasting the 70° difference of temperature between 320° and 250°, we have obtained only 183 thermal units, or 71·4 per cent., the remaining 28·6 per cent. being wasted by non-utilisation of the whole available difference of temperature during the reception of heat.

It may be that the heat is not all rejected at the lowest available temperature, and, in that case, there will be a corresponding loss during the rejection of heat, which may be calculated in terms of the amount of heat rejected, and the difference of temperatures on the same principles. For example, suppose that in a steam engine the heat discharged from the condenser is 300 thermal units per I.H.P. per 1′, at the temperature of 100°, while the temperature of the atmosphere is 60°, then the loss by wasting the interval of 40° between the condenser and the atmosphere is 300 × 40/561, or 21·4 thermal units per I.H.P. per 1′, being about one-half a horse-power.

When the heat is received at various temperatures, the value of that heat is the sum of the values of the various parts, that is to say,

$$U' = \Sigma \Delta Q \cdot \frac{t - T_0}{t} = Q - T_0 \Sigma \frac{\Delta Q}{t},$$

where t is the *absolute* temperature at which the part ΔQ is supplied, and the summation extends to the whole supply of heat. Hence the loss is

$$U - U' = T_0 \left\{ \Sigma \frac{\Delta Q}{t} - \frac{Q}{T_1} \right\}.$$

When, as in Art. 94, the heat Q is supplied in exact proportion to the change of temperature

$$U - U' = K T_0 \left\{ \log_\epsilon \frac{t_1}{t_2} - \frac{t_1 - t_2}{T_1} \right\},$$

where t_1, t_2 are the extreme values of t.

In terms of ϕ (Art. 102) the general formula may be written

$$U - U' = \frac{T_0}{T_1} \{ T_1 \phi - Q \},$$

a result which shows that all losses of this kind are shown graphically on a thermal indicator diagram, such as Fig. 29, in a very simple way. This point will be illustrated by examples as we proceed.

Class II.—The second class of losses arises from the fluid not being allowed to expand steadily throughout the whole available difference of pressure overcoming a resistance, and thus doing useful work. Instead of this, the resistance to expansion is wholly or partially removed before the pressure has fallen to the lowest available limit; the expansive energy of the fluid is then employed, wholly or partially, in generating violent motions, the kinetic energy of which is ultimately transformed into heat by fluid friction. Energy is not destroyed by this process, but it is invariably rendered partially unavailable to us; that is to say, it is no longer possible to do as much work at the expense of the internal energy of the fluid as might have been done had the unbalanced expansion been avoided. Again, even though the whole expansive energy of the fluid be duly exerted on a piston, it may be partially employed in overcoming useless resistances.

This second class of losses differs from the first in being different for each kind of fluid, and hence cannot usefully be considered apart from the particular kind of engine which is being examined. Its magnitude is determined graphically by comparing the area of the actual indicator diagram with that of the indicator diagram of an engine, receiving heat according to the same law, in which no such unbalanced expansion takes place, and no such useless resistances occur.

Losses of Efficiency in Steam Engines.

106. As in other heat engines, so in the steam engine the principal loss of efficiency proceeds from the narrow limits of temperature within which we are practically compelled to work. In a steam engine at least two-thirds the heat is always wasted in this way, but inasmuch as it is a loss which necessarily occurs whatever be the nature of the engine or its arrangement, an engine in which no other loss exists is conventionally said to be "perfect," and the efficiency of an engine working between given limits of temperature may consequently be properly estimated with reference to that of a perfect engine. When we have to compare engines which work between different limits of temperature, a somewhat different course is advisable, but such cases will be reserved to a later section of this chapter. The present section will be devoted to the comparison between an actual steam engine and an ideally perfect engine working between the same limits of temperature.

No actual steam engine is perfect in the sense just described, and its losses of efficiency may be classified thus:

(1) By radiation to external bodies.
(2) By transmission of heat to the exhaust steam.
(3) By clearance and wire-drawing.
(4) By misapplication of heat to the feed water.
(5) By misapplication of heat during expansion.
(6) By incomplete expansion.
(7) By excess back pressure.

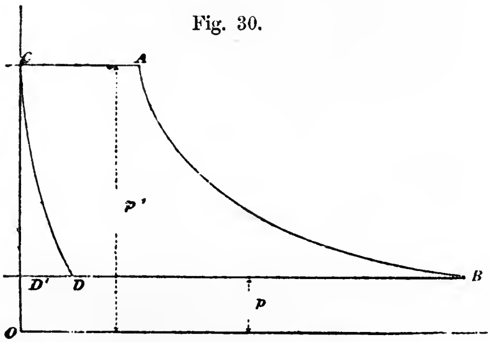
The first of these causes of loss of efficiency can only be estimated experimentally, and what is known about it will be considered in a later chapter; the second will only be treated incidentally here, as it properly belongs to Chapter X. on the action of the sides of the cylinder; the third will be discussed in a special chapter (Chapter IX.), and there remains to be considered in detail in the present chapter the last four, which will be treated in order.

107. If the action of a perfect steam engine (Art. 62) be examined, it will be found that an essential part of the process is, that

the temperature of the feed water should be raised by compression and not by application of heat; for which purpose it is necessary that the condensing steam should be taken from a surface condenser before the condensation is complete, and by means of a pump playing the part of an air pump as well as a feed pump, compressed until the condensation is complete and the pressure has risen to that of the boiler. That such a process is theoretically possible is manifest from what has already been said, but it is very difficult to carry out, without introducing evils greater than that against which it would be intended to provide. In actual steam engines the condensation is completed at the condenser temperature, and the feed water is raised in temperature by direct application of heat; no doubt a smaller feed pump is then required, and consequently less power is consumed in driving it, but this advantage is more than counterbalanced by the heat necessary to raise the temperature of the water.

In Fig. 30 the indicator diagram of a perfect engine is represented by $C\,A\,B\,D$, while $C\,A\,B\,D\,D'$ represents the diagram of an engine in which the compression part of the process is not carried

Fig. 30.



out. Then, the cases differ in this, that more work is done and more heat expended in the second case than in the first. It is clear that we shall have

Excess work done .. .. .. $= \text{Area } C\,D\,D'$;
Excess heat expended .. .. $= T_1 - T_0$;

in which last result the temperatures of boiler and condenser are supposed T_1, T_0 respectively, and the deviation from unity of the specific heat of water is neglected.

Now, proceeding as in Art. 100,

$$\text{Area } C\,D\,D' = T_1 - T_0 - T_0 \log_\epsilon \frac{T_1}{T_0},$$

and the excess heat, if properly utilised in a perfect engine, would have produced an amount of work expressed by

$$\text{Available heat} = (h_1 - h_0)\frac{T_1 - T_0}{T_1} = (T_1 - T_0)\frac{T_1 - T_0}{T_1} \text{ (nearly)};$$

hence the loss by misapplication of heat must be

$$U_1 = T_0 \log_\epsilon \frac{T_1}{T_0} - \frac{T_0}{T_1}(T_1 - T_0).$$

This kind of loss belongs to Class I. of the general causes of loss considered in Art. 105, and the result might have been written down at once from the general formula there given; its absolute value is the same whether or not the engine is in other respects perfect.

For example, suppose the boiler pressure 95 lbs. per square inch, and the condenser temperature 120°, then we have

$$T_1 = 325° + 461° = 786°;\quad T_0 = 581°;$$
$$p_1 = 95;\quad p_0 = 1{\cdot}68.$$

Hence we obtain

$$\text{Log}_\epsilon \frac{T_1}{T_0} - \frac{T_1 - T_0}{T_1} = {\cdot}3022 - {\cdot}2608 = {\cdot}0414,$$

and, multiplying by T_0,

$$U_1 = {\cdot}0414 \times 581 = 24{\cdot}05 \text{ thermal units.}$$

which is the loss of heat required, being the thermal equivalent of the additional work which might have been done had the heat been properly used.

In the thermal indicator diagram, Fig. 29, page 228, the curve $O\,B\,1$ represents the supply of heat to the feed water at rising temperature, and, therefore, in applying the general formula of Art. 105,

$$\text{Rectangle } N\,K = T_1\,\phi.$$
$$\text{Area } O\,1\,N\,Z = T_1 - T_0;$$

so that the area OKI, when multiplied by the factor T_0/T_1, represents the loss.

To find the percentage of loss, when the boiler supplies dry steam, it is sufficient to remember that

$$\text{Total heat expended} = H_1 - h_0 = 1093,$$

being simply the total heat of evaporation from 120° at 325°, of which in a perfect engine would be utilised

$$\text{Available heat} = \frac{T_1 - T_0}{T_1} \times 1093 = \cdot 261 \times 1093 = 285.$$

The loss, then, is 24 thermal units out of 285, or about 8·5 per cent. The maximum value of this loss in practical cases is 10 per cent., but, unless the pressure be high, it is much less than the value now found, as is also generally the case in non-condensing engines. If the boiler pressure be 60·4 instead of 95, and the temperature of the condenser 102° instead of 120°, the loss will be found to be 21·85, a result which will be employed in a subsequent article.

The loss here considered must not be confounded with the loss by non-utilisation of the waste heat of a furnace in heating the feed water. The heat utilised in a feed-water heater is no doubt better employed than if it were altogether wasted, but it would be still better employed if it could be used in the boiler to generate steam from water at the boiler temperature. A feed-water heater of this type, then, is to be considered as increasing the efficiency of the boiler, not that of the engine. A feed-water heater, however, may be so constructed as to play the part of a regenerator, as will be seen hereafter.

The compression shown in this article to be theoretically advantageous is entirely different from the compression taking place in actual steam cylinders; this kind of compression is in general also advantageous, but from quite different causes (Chap. IX., X.).

108. The next cause of loss of efficiency, namely, misapplication of heat during expansion, likewise belongs to Class I. of Art. 105, being due to supplying the steam with heat after its temperature has fallen. The heat in question is supplied by the cylinder, which itself obtains that heat partly from the steam jacket, if there is one, but chiefly from the steam condensed during admission. Its value

is found by the general formula of the article cited, that is to say, if U_2 be the loss,

$$U_2 = T_0\left\{\Sigma \, . \, \frac{\Delta Q}{t} - \frac{Q}{T_1}\right\},$$

in which the whole heat Q supplied during expansion is supposed divided into parts ΔQ, each supplied at a corresponding temperature t; thus, t is the mean absolute temperature of the expanding steam during a part of the expansion, in which the heat supplied is ΔQ.

For example, in Art. 53, in considering hyperbolic expansion from 60·4 to 8·5, that is to say, from 293° to 185° the whole expansion was divided into four stages, thus

293°		266°		239°		212°		185°
	41		48		56		62·3	

in which the heat supplied during expansion in thermal units per stage of 27° was found to be as shown by the numbers placed below. Then taking for t the mean absolute temperature for each stage, we find

$$\Sigma \frac{\Delta Q}{t} = \frac{41}{741} + \frac{48}{714} + \frac{56}{687} + \frac{62 \cdot 3}{659} = \cdot 2986\,;$$

and further,

$$\frac{Q}{T_1} = \frac{41 + 48 + 56 + 62 \cdot 3}{754} = \frac{207 \cdot 3}{754} = \cdot 2750.$$

Hence, if the condenser temperature be 102°,

$$U_2 = 563 \times \cdot 0236 = 13 \cdot 3 \text{ thermal units,}$$

which represents an amount of heat which might have been wholly transformed into work had the heat Q been properly used. Had the heat Q been used in a perfect engine, its value would have been

$$\text{Available part of } Q = 207 \cdot 3 \times \frac{293 - 102}{754} = 52 \cdot 51\,;$$

thus 25·3 per cent. of the heat supplied during expansion is wasted.

So it is in every case where heat is supplied during expansion; that heat is not wholly wasted, for it increases the work done by the expanding steam, but this increase is by no means as great as if the same heat had been applied in the boiler to generate more steam.

The actual amount of the loss will vary considerably, according to the nature of the expansion curve, and will be greater the greater the fraction of the whole which is supplied near the end of the stroke. If the expansion curve be accurately given, then, by the methods of Chapter VI., the heat supplied can be found for each step of the expansion, and hence the process just now used in the particular case of hyperbolic expansion can be applied; but the final result can also be obtained by a different method in any case in which the area of the expansion curve and the terminal state of the steam are known. For, by formula (B) Art. 91 applied to the operation represented in the diagram, Fig. 30, by $A\,B\,D'\,C$,

$$\text{Log}_\epsilon \frac{T_1}{T_2} + \frac{L_1 x_1}{T_1} + \Sigma \,.\, \frac{\Delta Q}{t} = \frac{L_2 x_2}{T},$$

the suffix 1, as usual, referring to the initial, and the suffix 2 to the final, state of the steam. Also, comparing the whole heat expended with the area of the diagram,

$$T_1 - T_2 + L_1 x_1 + Q = L_2 x_2 + (P_m - P_2)\,V_2,$$

where P_m is as usual the mean forward pressure, and the term containing it is reckoned in thermal units. Divide this last equation by T_1, and subtract it from the first, then

$$\Sigma \frac{\Delta Q}{t} - \frac{Q}{T_1} = L_2 x_2 \left\{\frac{1}{T_2} - \frac{1}{T_1}\right\} + \frac{T_1 - T_2}{T_1} - \log_\epsilon \frac{T_1}{T_2}$$
$$- (P_m - P_2)\,V_2 \,.\, \frac{1}{T_1};$$

and thus U_2 is found by the equation

$$\frac{U_2}{T_0} = \frac{T_1 - T_2}{T_1}\left\{1 + \frac{x_2 L_2}{T_2}\right\} - \log_\epsilon \frac{T_1}{T_2} - \frac{(P_m - P_2)\,V_2}{T_1}.$$

Thus, in the example of hyperbolic expansion just considered,

$$T_1 = 754^\circ : T_2 = 646^\circ : x_2 = 1 : r = 7{\cdot}22;$$

hence we obtain

$$\frac{T_1 - T_2}{T_1} = {\cdot}1432 : \frac{x_2 L_2}{T_2} = 1{\cdot}525 : \log_\epsilon \frac{T_1}{T_2} = {\cdot}1546.$$

Also, since the expansion is hyperbolic,

$$(P_m - P_2)\,V_2 = P_2 V_2 \log_\epsilon r = 138{\cdot}4 \text{ thermal units},$$

dividing which by T_1, we obtain ·1835; hence, performing the calculation,

$$U_2 = \cdot 0235\ T_0,$$

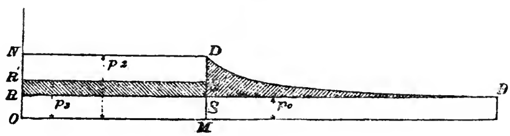
a result nearly identical with that found above.

It is to be remarked that when, as is sometimes the case, heat is taken away during expansion, the loss, as calculated by this formula, may prove negative, and then expresses the saving occasioned by taking away heat from the steam during expansion, instead of at the higher temperature of admission. The waste occasioned by the action of the sides of the cylinder is the sum of this loss and the loss by re-evaporation during exhaust, and hence is greater or less than that indicated by the re-evaporation during exhaust according as this loss is positive or negative This point will be considered hereafter, but the calculation is only given here for the sake of illustration of principles.

109. The next cause of loss is incomplete expansion: in a perfect engine expansion is carried on till the pressure has fallen to that corresponding to the temperature of the condenser, but in actual engines this is never possible; in the first place, the back pressure behind the piston is always greater than that corresponding to the temperature of the condenser, for reasons to be explained presently, and it never can be profitable to expand the steam below that limit, while practically the greatest expansion is still further limited by other causes. Hence the work obtained from 1 lb. of steam in actual engines is always less than in a perfect engine from this cause: and the magnitude of the loss is thus investigated.

In Fig. 31 let DM represent the pressure and DN the volume of 1 lb. of steam at the end of the stroke, while RB shows the line

Fig. 31.



of condenser pressure corresponding to the temperature of the condenser. Through D trace an adiabatic curve by the preceding rules till it reaches that line in B: then the area DBS represents an amount of work which would have been done in the perfect engine and which is not done in the actual engine, and is consequently the loss by incomplete expansion. This loss belongs to Class II. of Art. 105, and takes effect by generating kinetic energy in the exhausting steam, which is afterwards changed into heat by fluid friction and becomes part of the waste heat given out in the condenser. Proceeding as in Art. 100,

$$\text{Area } NDBR = T_2 - T_0 + \frac{x_2 L_2}{T_2} . (T_2 - T_0) - T_0 . \log_\epsilon \frac{T_2}{T_0},$$

in which the suffix 2 refers to the terminal state of the steam and the suffix 0 to the condenser. But the area of the rectangle DR in thermal units is

$$\text{Rectangle } DR = \frac{(p_2 - p_0)}{5 \cdot 36} . V_2 = \left(1 - \frac{p_0}{p_2}\right) P_2 V_2,$$

where V_2 is the actual volume of 1 lb. of steam; hence for the loss (U_3) we have

$$U_3 = (T_2 - T_0)\left\{1 + \frac{x_2 L_2}{T_2}\right\} - T_0 \log_\epsilon \frac{T_2}{T_0} - \left(1 - \frac{p_0}{p_2}\right) P_2 V_2,$$

in which formula $P_2 V_2$ is supposed expressed in thermal units.

For example, let the expansion terminate when $p_2 = 8 \cdot 4$ or $T_2 = 461° + 185° = 646°$, and let $T_0 = 102° + 461° = 563°$ be the temperature of the condenser corresponding to $p_0 = 1$, then assuming the steam dry at the end of the stroke or $x_2 = 1$, we have, as in a previous example,

$$P_2 V_2 = 70; \qquad \frac{x_2 L_2}{T_2} = 1 \cdot 525;$$

$$\therefore U_3 = 83 (1 + 1 \cdot 525) - 563 . \log_\epsilon \frac{646}{563} - 70\left(1 - \frac{1}{8 \cdot 4}\right)$$

$$= 209 \cdot 57 - 77 \cdot 42 - 61 \cdot 17$$

$$= 70 \cdot 48 \text{ thermal units,}$$

which is the loss by incomplete expansion.

The loss here calculated depends solely on the state of the steam at the end of the stroke and the temperature of the condenser: it is rarely less than 15 per cent., and more often from 20 to 30 per cent., in condensing engines. In non-condensing engines it may be, and often is, comparatively small, because in these the pressure p_0 is the pressure of the atmosphere.

110. Lastly, the process in an actual engine differs from that of a perfect engine in the back pressure (p_3) being greater than that (p_0) corresponding to the temperature of the condenser. This difference is due partly to frictional resistance in the exhaust ports, coupled with the difference of pressure necessary to exhaust the cylinder in the short space of time in which that operation takes place, and partly to the presence of air in the condenser. It may conveniently be called the *excess* back pressure: its value depends, as formerly stated, on the speed of piston, the dimensions of the exhaust ports, and on the terminal pressure.

In Fig. 31 let the line $R^1 S^1$ be the line of mean back pressure, then the loss by excess back pressure is clearly represented by the rectangle $S^1 R$, or if U_4 be the loss in thermal units,

$$U_4 = \frac{(p_3 - p_0)}{5 \cdot 36} V_2 = \frac{p_3 - p_0}{p_2} . P_2 V_2 .$$

For example, let the terminal pressure be as before $8 \cdot 4$, and the temperature of the condenser 102°, so that p_0 is unity, while p_3 the real mean back pressure is 3, then we have

$$U_4 = 70 \times \frac{2}{8 \cdot 4} = 16 \cdot 67 \text{ thermal units.}$$

111. Let us now add together the four losses just found for the particular case considered of an engine working with hyperbolic expansion of about $7\frac{1}{4}$ times from initial pressure $60 \cdot 4$, the steam being dry at the end of the stroke and the temperature of the condenser 102°: we have

		Thermal units.
Loss by heating feed	=	21·85
„ during expansion	=	13·30
„ by incomplete expansion	=	70·48
„ by excess back pressure	=	16·67
∴ Total loss	=	122·30

Now the useful work per lb. of steam is given by the formula

$$\text{Useful work} = (P_m - P_3) V_2 \text{ foot lbs.}$$

$$= P_2 V_2 . \frac{p_m - P_3}{p_2} . \text{ thermal units,}$$

if $P_2 V_2$ be supposed in thermal units; but

$$p_m = \frac{1 + \log_\epsilon 7 \cdot 22}{7 \cdot 22} \cdot p_1 = 24 \cdot 9,$$

the expansion being hyperbolic, and thus

$$\text{Useful work} = 182 \cdot 5 \text{ thermal units.}$$

This result added to the total loss gives about 305 for the number of thermal units per lb. of steam which could have been turned into work had the engine been perfect. We have a means of testing the truth of this result, for by Art. 25 the total heat of formation is given by the equation

$$Q = H_2 - h_0 + (P_m - P_2) V_2;$$

but since the temperature of the condenser is 102° and the temperature at the end of the stroke 185°,

$$H_2 - h_0 = 1138 - 70 = 1068,$$

while $(P_m - P_2) V_2$ has already been shown to be 138 thermal units, nearly;

$$\therefore \quad Q = 1068 + 138 = 1206,$$

in which calculation the jacket heat is included Now of this there would have been changed into work in a perfect engine the amount

$$U = 1206 \times \frac{T_1 - T_0}{T_1} = 1206 \times \cdot 2433$$

$$= 305 \cdot 5 \text{ thermal units,}$$

or practically the same.

The coincidence is not accidental, for if to the algebraical values of the four losses given in the preceding articles we add the algebraical value of the useful work done, the result will be found to be the general value of Q, the total heat of formation, multiplied by the fraction $\frac{T_1 - T_0}{T_1}$, showing that the whole heat expended in producing the steam, in the state in which we find it at the end of the stroke, is fully accounted for by the useful work done and by the four losses which we have been considering.

We can now express the expenditure of heat in useful work and the losses of heat in terms of the work of an ideally perfect engine by dividing by 305, whence we obtain

		Per cent.
Useful work done ..	.. .. =	59·9
Loss in heating feed	.. .. .. =	7·2
" during expansion	.. .. .. =	4·3
" in complete expansion	.. .. =	23·1
" in excess back pressure	.. .. =	5·5
Available heat	.. .. =	100·0

The difference between the expenditure of heat in an engine such as that considered in Case I., Art. 24, Chapter III., and that in a perfect engine given in Art. 62, Chapter V., is thus accounted for. The calculations of Chapter III., given in the table p. 51, show that little is gained by increasing the ratio of expansion beyond a moderate limit. The reason of this is that the loss by misapplication of heat during expansion, and excess back pressure, rapidly increases with the ratio of expansion, and thus the direct saving by increased expansion is counterbalanced.

The exhaust waste, together with clearance and wire-drawing, is here neglected, as was also the case in the articles cited: the effect of these causes is so considerable that condensing engines rarely utilise more than 40 or 50 per cent. of the available heat, and often much less. Non-condensing engines, however, have in general a greater *relative* efficiency, many of the causes of loss of efficiency being less influential in their case: hence they utilise sometimes as much as 75 per cent. of the whole amount of heat, which could be turned into work in a perfect engine working between the same limits of pressure.

Unavoidable Losses. Conditions of Practical Economy.

112. In the preceding articles the useful work and various losses in a heat engine have been compared with the useful work of an ideally perfect engine working between the same limits of temperature. This method is highly instructive as an illustration of thermodynamical principles, but it is not sufficient when taken by itself. For these limits are in reality the temperatures at which the engine

receives and rejects heat. The ultimate source of heat has a temperature much higher than the upper limit, and the atmosphere into which the heat ultimately passes has a temperature much below the lower limit, so that the limits in question depend on the class of engine considered. The practical economy of the engine depends on its absolute performance, not solely on the success with which it utilises the heat which is available within the working limits of temperature. As a measure of absolute performance we adopt the expenditure of heat in thermal units per I.H.P. per minute; the ratio of work done to heat expended, which we commonly call the "absolute" or "apparent" efficiency, may also be used, but it has already been pointed out that this ratio (page 151) has no special significance, and the term "efficiency" is better avoided. Of this total expenditure 42·75 is converted into useful work, and the rest constitutes the loss which may be distinguished into the *avoidable* and the *unavoidable* loss.

Let T_0 be the temperature at which heat is rejected, which generally has in each class of engine some definite value, and, in the first instance, suppose the engine capable of receiving all its heat at a temperature T_1, then there is an unavoidable flow of heat into the refrigerator given by

$$\text{Unavoidable loss} = \frac{T_0}{T_1 - T_0} \times 42{\cdot}75.$$

This transfer of heat is a necessary consequence of the nature of the process by which a heat engine converts heat into work; in a refrigerating machine it is a gain, not a loss, and, as pointed out on page 148, waste occurs in almost every case in which heat generated by combustion is used for practical purposes, by not making use of the available difference of temperature to produce such a transfer.

A loss to this extent is absolutely unavoidable, but in addition it will frequently happen that the nature of the engine renders some further loss practically necessary. In explosive gas engines the heat is necessarily supplied at rising temperature, and generally must be rejected at falling temperature. The loss thus occasioned may properly be reckoned as part of the unavoidable loss, and the total amount is most conveniently estimated by replacing $T_1 - T_0$, the total range of temperature, by the *effective* range found by calculating average temperatures of supply and rejection of heat as explained in the last chapter.

Adding 42·75 to the total unavoidable loss, we obtain the unavoidable expenditure of heat, and the difference between this and the actual expenditure, as determined by experiment or estimated by calculation, gives the *avoidable* loss. This it is which shows the margin for possible improvement in an engine of the class considered. As already remarked, however, Art. 98, the question of what constitutes an "avoidable" loss must often be to some extent a matter of convention.

113. In non-condensing steam engines the temperature of rejecting heat is taken at 212°, for the difference between this and the temperature of the atmosphere is made no use of. Writing then $T_0 = 673$, we obtain in thermal units per I.H.P. per 1′,

$$\text{Unavoidable Loss} = \frac{28{,}800}{T_1 - T_0},$$

where $T_1 - T_0$ is the range of temperature. If any conceivable engine is considered possible, T_1 is the temperature of the boiler, but, if the loss by "misapplication of heat to the feed water" (Art. 105) is considered as unavoidable in consequence of the absence in all actual steam engines of the necessary pump, we must replace T_1 by T_m, the average temperature at which heat is supplied as found by the equation on page 223.

In condensing steam engines the temperature of the condenser varies considerably, but when below a certain limit, its variations have no real influence on the performance of the engine. Hence a fictitious temperature may properly be chosen as in non-condensing engines, and the value we shall adopt is 102° F., giving

$$\text{Unavoidable Loss} = \frac{24{,}000}{T_m - T_0}.$$

Taking for example the case we have been considering in this chapter, and referring again to page 223, it will be seen that $T_m = 734$, $T_0 = 563$, and therefore,

$$\text{Unavoidable Loss} = 140 \text{ thermal units per I.H.P. per } 1',$$

which includes the loss by "misapplication of heat to the feed." Had this not been included the range of temperature would have been taken as 191° instead of 171°, and the loss would have been 126 instead of 140.

CHAPTER IX.

CLEARANCE AND WIRE-DRAWING.

114. By "clearance" was originally meant the distance between the piston and the cylinder cover when the piston stands at the end of its stroke, some small interval being necessary to provide against a possible variation in the stroke due to wear of the connecting-rod brasses. The term is, however, now employed in the theory of the steam engine to signify a volume, being the whole volume included between the piston and the slide valve at the instant when the stroke commences. Clearance is expressed as a fraction of the whole piston displacement; thus, if X be the piston displacement, $c X$ is the clearance, where c is a fraction which ranges from ·02 to ·3, according to the size and type of engine. Clearance modifies to a greater or less extent almost every calculation relating to the steam engine, and its neglect may give rise to serious errors, but we have thought it advisable to reserve its consideration to a special chapter, on account of the complexity thereby introduced if the question be considered at all thoroughly.

Together with clearance it is convenient to consider two other intimately-connected subjects—first, that of compression; secondly, wire-drawing. In practice, the exhaust steam is rarely permitted to escape from the cylinder throughout the return stroke; on the contrary, the exhaust port closes before the end of the stroke, and the steam still remaining in the cylinder is compressed by the advancing piston till at the end of the stroke the clearance contains steam of higher pressure than before, though (usually) of lower pressure than that of the boiler. Again, if the pressure shown by the indicator during the admission be compared with the boiler pressure, a difference, always perceptible and sometimes very great, will be observed: the steam is then said to be "wire-drawn." The amount of wire-drawing depends on the speed of piston, the state of the steam, and the magnitude of the admission ports: its influence is very complicated and cannot be treated exactly, but nevertheless it is necessary to have some idea of its general character.

As usual, we commence with the simplest case, which is that where compression and wire-drawing are left out of account: the problem is thereby rendered far easier, and its solution offers no considerable difficulty.

Effects of Clearance considered alone.

115. In condensing engines with no sensible compression, the clearance at the end of the return stroke is full of steam of low pressure, not more than 2 or 3 lbs. per square inch, and in this section we suppose that steam wholly neglected, that is to say, the clearance is regarded as absolutely empty. On this supposition, when the admission valve opens, the steam rushes in and fills the empty space with steam of the boiler pressure, which then presses against the piston and exerts energy upon it as if there were no clearance.

Now imagine a cylinder without clearance of the same volume as the original cylinder including clearance, then, if the state of the steam were exactly the same in the two cases, the only difference would be that in the cylinder with clearance the steam would not do as much work during admission as in the cylinder without clearance, because the piston does not move through the clearance space.

Actually, if the boiler steam be supposed precisely the same in the two cases, the cylinder steam will often be rather drier with clearance than without clearance; but, reserving this point for future consideration, we shall suppose the steam in the same state, for which it is only necessary to suppose that the boiler steam is slightly wetter for the cylinder with clearance than for the cylinder without clearance.

In Fig. 32 (page 253), $NS = X$ represents the piston displacement for a cylinder the clearance of which is represented by $ON = cX$: then $OS = (1 + c) X$ represents the total volume of the actual cylinder, and also the volume of the cylinder without clearance with which the comparison is to be made. NM represents the part of the stroke traversed before the steam is cut off:

then if $NM = m.NS$, m is the cut-off and $1/m$ is the apparent ratio of expansion. But the real ratio of expansion (r) is $OS : OM$,

$$\therefore (1 + c) X = r (c + m) X,$$

or,

$$r = \frac{1 + c}{c + m};$$

so that r is no longer the reciprocal of the cut-off, as when clearance is neglected, but has a value which is smaller the greater the clearance.

For example, suppose the steam cut off at $\frac{1}{10}$ and the clearance also $\frac{1}{10}$, then

$$r = \frac{1 \cdot 1}{\frac{1}{10} + \frac{1}{10}} = \frac{11}{2} = 5 \cdot 5.$$

The example has been purposely chosen to show how great the influence of clearance may be: but, though not often so great as this, it is always so great as to render a determination of the clearance indispensable before the real ratio of expansion used in an engine can be determined with any approach to accuracy.

Next, to compare the mean pressures on the piston in the two cases considered, it is only necessary to compare the areas $EABSN$ and $DABSO$, the first of which represents the energy exerted on the piston when there is clearance, and the second when there is none. The question arises in two forms.

(1) Being given the ratio of expansion and the law of expansion, it is required to find the actual mean pressure on the piston when the clearance is known. Here, from the data of the question the mean pressure (P_m) is known on the piston of an engine working without clearance: that is to say,

$$P_m \,.\, OS = \text{Area } DABSO.$$

But if P'_m be the actual mean pressure,

$$P'_m \,.\, NS = \text{Area } EABSN;$$

$$\therefore P_m (1 + c) X - P'_m \,.\, X = \text{Area } DENO$$
$$= P_1 c X.$$

where P_1 is the initial pressure: thus,

$$P'_m. = (1 + c) P_m - c P_1.$$

For example, let the steam be cut off at $\frac{1}{10}$, and the clearance be $\frac{1}{10}$, then the real ratio of expansion is, as shown above, 5·5, and assuming hyperbolic expansion,

$$P_m = P_1 . \frac{1 + \log_\epsilon 5 \cdot 5}{5 \cdot 5} = \cdot 49 . P_1,$$

which gives the mean pressure when there is no clearance with the same ratio of expansion, and

$$P'_m = 1 \cdot 1 . P'_m - \tfrac{1}{10} . P_1 = \cdot 439\ P_1,$$

which gives the true mean forward pressure to be used in conjunction with the area of the piston and the stroke in calculating the horse-power.

(2) But instead of this the datum of the question may be the actual mean pressure on the piston as determined by an indicator diagram in the usual way, and it may be required to find the equivalent mean pressure in an engine of the *same power* without clearance. In that case let P'_m be the mean pressure in question, and P_m the equivalent pressure, then

$$P_m . OS = P'_m . NS,$$

or

$$P_m = \frac{P'_m}{1 + c}.$$

The object of this process is to reduce the mean pressure from the piston displacement to the volume of the steam. In practice the base of the indicator diagram always represents the stroke, and the mean pressure is found by dividing the area by the stroke. Hence, when the abscissæ of the diagram are taken to represent volumes, the mean pressure refers to the piston displacement instead of to the volume of the steam as it ought to do when questions relating to the expenditure of heat are under consideration.

116. From what has been said it is clear that the volumes represented by the abscissæ of the indicator diagram are to be reckoned not from EN, the commencement of the stroke, but from DO, so as to include the clearance, and when the diagram is taken to represent the changes of state of 1 lb. of steam this must be borne in mind.

Thus, in considering the expenditure of heat, OS is to be taken as the base of the diagram, and not the stroke NS.

In finding graphically or by calculation the expenditure of heat, the only difference is that the area of the diagram representing the external work done during the formation of the steam is diminished by the area of the rectangle DN: the internal work done in producing steam in a given condition being quite independent of clearance. Hence the graphic construction of Art. 28 performed on the base OS will be unaltered, and the result in combination with the actual area of the indicator diagram will represent the total heat of formation. The heat pressure representing it, however, will be different, according as the base OS or the base NS is chosen: in the first case it will compare with the reduced mean pressure of case (2) of the last article: in the second case it will compare with the actual mean pressure on the piston.

The formula for the total heat of formation,

$$Q = h_2 - h_0 + x_2 L_2 + (P_m - P_2) V_2,$$

will be unaltered if by P_m we understand the reduced mean pressure of case (2) of the preceding article.

The loss of work per lb. of steam by clearance when, as in the present section, there is no compression, is

$$\text{Loss} = \text{Area } DN = \frac{c P_1 V_2}{1 + c} = \frac{c r}{1 + c} \cdot P_1 V_1,$$

understanding by V_1, V_2 as usual the actual volumes of 1 lb. of steam, and by r the real ratio of expansion, and remembering that the volume V_2 corresponds to the total volume of the cylinder.

Compression.

117. The compression of the steam behind the piston during the return stroke has no important influence on the total energy exerted on the steam piston, but only on the back pressure and the consumption of steam. The ratio of expansion is just the same whatever the compression, and the only difference in the forward action on the piston consists in the state of the steam being somewhat different according to the amount of the compression. When the boiler steam rushes into the partly empty clearance it becomes drier, as

previously stated, but the amount of drying is less, and that the fuller the clearance, and hence the state of the steam depends on the compression. This source of complication is for the present to be avoided by supposing the boiler steam to be slightly altered according to the amount of compression, so that the state of the steam in the various cases considered may be the same after reaching the cylinder.

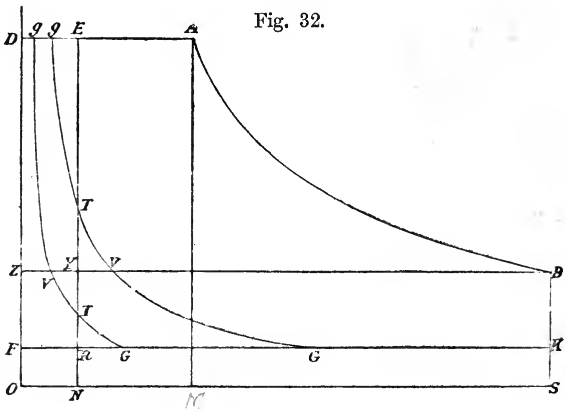
Let the exhaust port be imagined to close at a known point of the return stroke, then a certain quantity of steam will be enclosed behind the piston: this steam is conveniently called the "cushion" steam, to distinguish it from the steam discharged from the cylinder, and its volume, at release, as compared with the total volume of the cylinder, will be denoted by n. The value of n may be found approximately from the indicator diagram, as will be seen presently. If the whole contents of the clearance be reckoned part of the cushion steam, the ratio of weights will often be much greater than the ratio of volumes, on account of the water remaining after exhaust (Chapter X.): but it will be more convenient to include in the cushion steam only so much of the whole contents of the clearance as is necessary to make the two ratios equal, and to treat the excess water separately, as virtually forming part of the sides of the cylinder (Art. 108).

If we trace the changes undergone by the cushion steam from the instant when the exhaust port closes, it appears that, in the first place, it is compressed by the piston till the end of the stroke, or a little before, when it fills the clearance space with steam at a pressure, usually less than that of the boiler, which may be called the cushion pressure; in the second place, it is compressed further by the entrance of fresh steam from the boiler; in the third place it enters the cylinder, and, after cut-off, expands, doing work upon the piston; in the fourth place, it suddenly expands, doing no work, except in overcoming back pressure; while, finally, either it or a precisely similar mass resumes its original place, and recommences its cycle of changes.

Thus the whole mass of steam in the cylinder may be separated into two parts, the cushion steam and the working steam, of which the latter goes through nearly the same change as if there were no clearance, and the former goes through a series of changes without being condensed at all. The consumption of steam per stroke is, of course, only the working steam, and not the whole amount of steam contained in the cylinder after cut-off.

Let us now compare the action of the steam in two cylinders of the same *total* volume, one without clearance, and the other with clearance and compression. The ratio of expansion is the same in the two cases, and the mean forward pressure, being independent of the compression, is found by the preceding rules. The consumption of steam is, however, diminished in the proportion $1 - n : 1$, while the back pressure is increased; thus, the energy exerted on the steam piston per lb. of steam is increased in the proportion $1 : 1 - n$, while the useful work done is altered in a more complicated way, according to the amount of compression.

Fig. 32.



To exhibit the process graphically, let OS, Fig. 32, represent, as before, the total volume of the cylinder, and ON the clearance, while KRF is the line of mean back pressure, assumed uniform till the exhaust closes at G; then FG represents the volume of the cushion steam at its lowest pressure. The curve $GVTg$ represents the compression of the cushion steam from the exhaust volume GF to the initial volume Dg, of which compression, part, namely GT, is produced by the steam piston, and the remainder gT is produced by the influx of boiler steam into the clearance. The fraction n is given by the equation

$$n = \frac{ZV}{BZ},$$

where V is the point of intersection of the compression curve and the line $B\,V\,Z$ corresponding to the terminal pressure. The figure represents two compression curves, one for the case in which n is small, and the other where n is large; in the first case the point V lies within the clearance, and the consumption of steam is greater than in a cylinder without clearance with the *same piston displacement;* in the second case V lies without the clearance, and the consumption of steam is less instead of greater. The first case is common in condensing engines, and the second in non-condensing. The area $E\,A\,B\,K\,G\,T$ is the area of the diagram, and represents the useful work done per stroke—that is to say, by a weight of steam less than if there were no clearance, in the proportion $1 - n : 1$. The space $Y\,V$ may be called the *effective* clearance, the consumption of steam being the same as if there were no compression with clearance $Y\,V$. If we express the effective clearance as a fraction (c^1) of the piston displacement,

$$c^1 = c - n\,(1 + c),$$

a quantity which may be positive, zero, or negative, according to circumstances.

118. If the base $O\,S$ of the diagram be now understood to represent the volume of 1 lb. of steam in its terminal state, then

$$\text{Useful work per lb. of working steam} = \frac{\text{Area } E\,A\,B\,K\,G\,T}{1 - n}.$$

The value of $1 - n$, that is to say, the comsumption of steam, diminishes when the compression is increased, while the area expressing the whole useful work done also diminishes; thus the useful work done per lb. is altered, but the alteration may be an increase or a diminution, according to circumstances. In fact, a value can be found for the compression which shall make that useful work a maximum, as may be seen by the following general reasoning.

The n lbs. of cushion steam are alternately compressed and expanded; now the compression, as far as the cushion pressure, is performed by the steam piston, and takes effect as a diminution of the useful work done, while the expansion exerts energy on the piston only till the terminal pressure is reached, when the expansive energy of the cushion is dissipated by sudden expansion. Thus the cushion steam absorbs energy from the steam piston at every stroke, and this loss is greater the greater n. On the other hand, the consumption of steam is less the greater n, and consequently there must be some value of the compression for which the loss is least. A loss there must always be, however, except in one particular case—namely, the purely ideal case, in which the expansion is carried on till the terminal pressure is the same as the back pressure, and the compression is so great that the cushion pressure coincides with the initial pressure, so that, at the end of the stroke, the clearance is full of steam at the boiler pressure. Under these circumstances the useful work done per lb. of steam is unaltered, the diminution in total work being exactly balanced by the diminution in the consumption of steam.

To obtain definite results it is now necessary to make some assumption regarding the form of the compression curve of the cushion steam and the value of n.

In the first place, it is to be observed that the expansion (or compression) curve of steam is always approximately an hyperbola, unless the quantity of water mixed with the steam be excessive; and we may therefore fairly suppose the curve VTG (Fig. 32) to be an hyperbola, starting from G, the point where the exhaust closes.

Let now the ratio of compression—that is to say, $FG : FR$—be denoted by R, then, returning to the supposition that the base of the diagram represents the total volume of the actual cylinder,

$$FG = RcX,$$
$$GK = (1 + c)X - RcX;$$

therefore the area $TGKSN$ representing the whole back pressure work done

$$= P_3(\overline{1 + c} \,.\, X - RcX) + P_3 \,.\, RcX \,.\, \log_\epsilon R$$
$$= P_3 X(1 + c - cR + cR\log_\epsilon R)$$

where P_3 is the back pressure before the exhaust closes. Thus the mean back pressure on the piston for the whole stroke is

$$P_B = P_3(1 + c - cR + cR\log_\epsilon R).$$

The value of R is easily found when the point where compression begins is known. For example, let compression begin at ·8 of the return stroke, then

$$1 + c - cR = \cdot 8;$$

or assuming $c = \frac{1}{10}$, as in a previous example,

$$R = (\cdot 2 + \cdot 1)\, 10 = 3\,;$$

hence

$$P_B = P_3 (1 + \cdot 1 - \cdot 3 + \cdot 3 \,.\, \log_\epsilon 3)$$
$$= 1 \cdot 129 \,.\, P_3.$$

Having thus found the mean back pressure, the mean effective pressure is to be obtained by subtracting the result from the value of the mean forward pressure found in Art. 115, then

$$P'_m - P_B = (1 + c)(P_m - P_3) - c P_1 + c R P_3 (1 - \log_\epsilon R),$$

and the influence of clearance and compression is thus determined. The total useful work done, expressed graphically by the area $E\,A\,B\,K\,G\,T$, is found by multiplication by X.

119. So far no important error can arise from the assumption that the compression curve is an hyperbola through G: in calculating n, however, the error of the assumption may be greater, and it will be desirable to diminish it by finding the point T directly from the diagram, making an allowance for the effect of lead; for assuming T known, we have only to suppose $T\,V$ an hyperbola, a supposition which cannot be far from the truth.

Then, since $V\,T$ is an hyperbola,

$$\frac{Z\,V}{F\,R} = \frac{P_c}{P_2},$$

where P_c is the cushion pressure represented by $N\,T$ in the figure, and P_2 is, as usual, the terminal pressure;

$$\therefore Z\,V = c \,.\, \frac{P_c}{P_2} \,.\, X, \quad n = \frac{c}{1 + c} \,.\, \frac{P_c}{P_2}.$$

The "effective" clearance (see last article) is consequently

$$c^1 = c - c \,.\, \frac{P_c}{P_2} = c \,.\, \frac{P_2 - P_c}{P_2}.$$

In general we may write $P_c = R\,P_3$, and thus

$$n = \frac{c\,R}{1 + c} \,.\, \frac{P_3}{P_2},$$

a value of n which we shall have occasion to use presently.

120. The useful work now becomes (page 254),

$$\text{Useful work} = X \cdot \frac{P'_m - P_B}{1 - n};$$

or if V_2 be the terminal volume of the steam per lb.,

$$\text{Useful work} = \frac{V_2}{1 + c} \cdot \frac{P'_m - P_B}{1 - n} \text{ (per lb.)}$$

$$= V_2 \cdot \frac{P_m - P_3 - \frac{c}{1 + c} \cdot P_1 + \frac{c}{1 + c} R P_3 (1 - \log_\epsilon R)}{1 - n}.$$

If there had been no clearance the useful work would have been

$$\text{Useful work} = V_2 (P_m - P_3),$$

and therefore the loss is given by the equation

$$U_5 = V_2 \cdot \frac{- n (P_m - P_3) + \frac{c}{1 + c} (P_1 - R P_3 + R P_3 \log_\epsilon R)}{1 \quad n}.$$

It has been already pointed out that for some value of the compression the loss will be least. This value is found by applying the usual rules for a maximum or minimum to the value of the useful work per lb. of steam just obtained, the variable being R and the value of n expressed in terms of R by the equation given above. The result of this operation is given by the equation

$$(1 + c) P_2 \log_\epsilon R - c R P_3 = (1 + c)(P_m - P_3) - c P_1,$$

which may be put in a simple form if it be supposed that the expansion is hyperbolic, so that

$$P_m = P_1 \cdot \frac{1 + \log_\epsilon r}{r},$$

whence, substituting and simplifying, the equation reduces to

$$\log_\epsilon \frac{R}{r} = 1 - \frac{r P_3}{P_1} - \frac{c r}{1 + c}\left(1 - \frac{R P_3}{P_1}\right), \qquad \text{(A)}$$

and the value of the greatest work becomes simply

$$\text{Greatest work} = P_1 V_1 \log_\epsilon R, \qquad \text{(B)}$$

where in (B) R has a value previously found, by trial, from (A). Corresponding values may be found if the expansion be not hyperbolic, but the result on this supposition is sufficiently approximate for the purpose.

For example, let us take the data

$$r = 4 : c = \frac{1}{15} : P_3 = \frac{1}{8} P_1,$$

equation (A) becomes

$$\log_\epsilon \frac{R}{4} = 1 - \frac{4}{8} - \frac{4}{16}\left\{1 - \frac{R}{8}\right\},$$

or

$$\log_\epsilon \frac{R}{4} = \frac{1}{4} + \frac{R}{32},$$

from which, trying a few values of R by aid of the table of hyperbolic logarithms, we easily find

$$R = 6\tfrac{1}{4} \text{ (nearly)}$$

or, again, if we take a higher rate of expansion, say

$$r = 8 : c = \frac{1}{15} : P_3 = \frac{1}{16} \cdot P_1,$$

we obtain

$$\log_\epsilon \frac{R}{8} = \frac{R}{32}, \text{ or } R = 11 \cdot 5 \text{ (nearly)}.$$

Placing the value in equation (B),

Useful work per pound of steam $= 1 \cdot 832\, P_1 V_1 : (R = 6\tfrac{1}{4})$;

" " " " $= 2 \cdot 442 \,.\, P_1 V_1 : (R = 11 \cdot 5)$.

Suppose now we calculate the work when $c = 0$, that is to say, when there is no clearance, then

$$\text{Useful work} = V_2(P_m - P_3) = P_1 V_1 \left\{1 + \log_\epsilon r - \frac{r P_3}{P_1}\right\},$$

which, in the two preceding cases, becomes

$$\text{Work} = 1 \cdot 886 \,.\, P_1 V_1 \left\{r = 4 : P_3 = \frac{1}{8} P_1\right\}$$

$$= 2 \cdot 579\, P_1 V_1 \left\{r = 8 : P_3 = \frac{1}{16} P_1\right\};$$

the difference between these results and the former ones gives the least possible loss by clearance.

$$\therefore \text{Loss} = \cdot 054 \,.\, P_1 V_1, \text{ or } \cdot 137 \, P_1 V_1.$$

Hence the percentage of total work lost by clearance is at least 3 per cent. in the first case, and 5·3 per cent. in the second. The loss will of course be greater if the clearance be greater, as is not uncommon in practice.

The point in the stroke at which compression should begin for least loss is obtained from the equation

$$z = 1 + c - cR,$$

where z is the fraction of the stroke required, whence, taking the cases considered above,

$$z = \cdot 65, \text{ or } z = \cdot 33,$$

results which show that in condensing engines it is usually inconvenient or impossible to commence compression sufficiently early to reduce the loss to a minimum. In non-condensing engines P_3 is at least 15 lbs. per square inch, and a moderate ratio of compression produces a great cushion pressure.

Let us next suppose no compression, and inquire how great the loss will be by clearance in the special cases considered. Putting $R = 1$ in the general value for the useful work:

$$\text{Useful work} = \frac{P_m - P_3 - \dfrac{c}{1 + c}(P_1 - P_3)}{1 - n} V_2,$$

where

$$n = \frac{c}{1 + c} \cdot \frac{P_3}{P_2},$$

which, still supposing the expansion hyperbolic, and taking the numerical data above given, becomes

$$1 \cdot 721 \, P_1 V_1, \quad \text{or} \quad 2 \cdot 178 \, P_1 V_1.$$

Comparing these values with those just given, when there is no clearance, we find for the loss

$$\cdot 165 \, P_1 V_1, \quad \text{or} \quad \cdot 401 \, P_1 V_1,$$

showing a loss of $8\frac{3}{4}$, or $15\frac{1}{2}$ per cent.

The numerical value assumed for the clearance in the preceding examples is often exceeded, especially in recent marine engines; the results obtained therefore show that the loss by clearance is very considerable when there is no compression, and may be diminished by compression, conclusions which are doubtless in the main correct, though the value of the loss cannot be exactly determined, and may be somewhat exaggerated in the preceding calculations. In the next chapter it will be shown that compression has the important additional advantage of diminishing the action of the sides of the cylinder. The partial compensation in consequence of the drying effect of clearance will be considered presently.

The fact that there always is a loss by clearance, except in the ideal case where the expansion is complete, was first pointed out by Mr. Macfarlane Gray, in a paper read before the Institution of Naval Architects in 1874; and in the same paper it was shown that for a particular value of the compression the loss was reduced to a minimum. On the Thom slide valve, see Appendix.

121. In finding, graphically or otherwise, the heat expended on the steam when clearance and compression are taken into account, $B\ V$ (Fig. 32) must be regarded as the base of the diagram, because it is $B\ V$ which determines the consumption of steam and represents its terminal volume. If this be done, all internal-work-pressures will be unaltered, but external-work-pressures—such as, for instance, the mean forward pressure—must be reduced, when necessary, from the base $N\ S$, representing the piston displacement, to the base $B\ V$, representing the terminal volume of the steam.

The energy exerted by the working steam is employed, partly in compressing the cushion steam and partly in doing work on the steam piston; while the energy exerted by the cushion steam during its expansion is also employed in doing work on the steam piston; hence the whole work done on the steam piston is equal to the energy exerted by the working steam, subject to a correction corresponding to the expansion or compression $T\ V$, occasioned by the compression $T\ g$ of the cushion steam by the working steam being not exactly equal to its expansion $g\ V$. This correction is positive or negative, according as V lies within or without the clearance—that is to say, according as n is small or large; it is, however, of small amount, and may usually be disregarded. In applying the formula for the total heat of formation, the mean forward pressure is therefore to be estimated as if there was no compression, and then reduced as just described.

Drying Effect of Clearance.

122. It has been already explained that, in the preceding articles, the steam has been supposed in the same state, when comparing together the case with clearance and the case without clearance, and that to realise this it is necessary that the boiler steam should be imagined of various degrees of dryness, according to the circumstances of each special case. The reason of this is that when the steam rushes into an empty or partly empty clearance the expansive energy is employed in generating kinetic energy which, *if time enough elapses*, is absorbed by fluid friction, and appears as heat. Any complete investigation of the effect of clearance and compression must then include the consideration of the drying effect of clearance, at any rate so far as to ascertain the magnitude of the error produced by its neglect.

The effect in question is greatest when there is no compression, as in Art 115: in that case the admission work is diminished by the quantity $c P_1 X$; that is to say,

$$\text{Diminution of admission work} = \frac{c P_1 V_2}{1 + c} = \frac{c r}{1 + c} \cdot P_1 V_1$$

for each 1 lb of steam admitted.

Now the total heat of formation of the steam in its initial state is diminished by an exactly equal amount, and hence this result, expressed in thermal units, is the difference between the heat necessary to produce the boiler steam and the heat necessary to produce the actual steam in the state in which it actually enters the cylinder after being dried by absorption of kinetic energy by fluid friction.

Let, then, x_1 be the actual dryness-fraction after admission, and x'_1 the boiler dryness-fraction: $x_1 - x'_1$ represents the number of lbs. of moisture evaporated per lb. of steam, and clearly

$$x_1 - x'_1 = \frac{c r}{1 + c} \cdot \frac{P_1 V_1}{L_1},$$

where L_1 is as, usual, the latent heat of evaporation expressed in the same units as $P_1 V_1$: thus,

$$x_1 - x'_1 = \frac{c r}{1 + c} \cdot \frac{x_1}{1 + k},$$

where k is the number given by the formula on page 175, or by Tables III. and V.—a simple result for the drying effect of clearance where there is no compression. Evidently it increases greatly when the expansion increases. The extreme case possible, and that only ideally, is when $c\,r = 1$, so that no admission work at all is done : in that case,

$$x_1 - x'_1 = \frac{x_1}{1 + k} \text{ (nearly).}$$

Referring to the Tables for the value of k, it will be seen that the drying effect of clearance may be as much as 8 per cent. in this extreme case. In all ordinary cases it is far less, but nevertheless is sufficient perceptibly to diminish the loss by clearance.

For example, let us take

$$c = \frac{1}{15} : r = 4 : p_1 = 60 \cdot 4 \, ;$$

$$\therefore x_1 - x'_1 = \frac{1}{4} \cdot \frac{x_1}{11\frac{1}{4}} = \cdot 022 \, . \, x_1,$$

or,

$$x'_1 = \cdot 978 \, x_1,$$

and the heat expended per lb. of steam will be approximately diminished by clearance in the proportion $\cdot 978 : 1$. Now the useful work done per lb. of steam was shown to be $1 \cdot 886 \, P_1 V_1$ without clearance, and $1 \cdot 721 \, P_1 V_1$ with clearance, which last result must now be divided by $\cdot 978$ to enable us to make a true estimate of the effect of clearance. Thus the effective work is $1 \cdot 76 \, P_1 V_1$, instead of $1 \cdot 72 \, P_1 V_1$; and the true loss is $\cdot 126 \, P_1 V_1$, or $6 \cdot 7$ per cent., instead of $8\frac{3}{4}$ per cent., as was found to be the case when the drying effect of clearance was omitted.

The partial compensation due to the cause here considered is proportionally less when there is compression than when there is none : it can be calculated in each special case by a proper modification of the foregoing process, but enough has been said to render the method of procedure intelligible. No great reliance can be placed on the results of the calculations, for reasons to be stated presently (Art. 127).

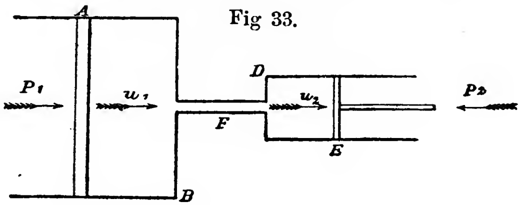
Steady Wire-drawing.

123. As already stated, steam is said to be "wire-drawn" when a sensible difference of pressure exists between the boiler and the

cylinder during the admission of the steam. Such a difference proceeds partly from frictional resistances consequent on the formation of eddies, and partly from the fact that the motion of the piston is not uniform, but gradually increases from the commencement to the middle of the stroke, and of these two causes we consider the frictional resistances first.

In Fig. 33, $A B$ is a large cylinder, and $D E$ a small one, the two being connected by a pipe, F, provided with a stop-cock which

Fig 33.



can be opened or closed at pleasure. The large cylinder contains steam of pressure P_1, confined by a suitably loaded piston, which flows through the pipe into the small cylinder $D E$, the piston of which is loaded with pressure P_2, the difference between P_1 P_2 depending on the frictional resistance overcome in the pipe F, which may be made to vary at pleasure by opening the stop-cock. The pistons are imagined to move forward with velocities u_1 u_2, which in the present case are uniform.

Under these circumstances, let a time be considered during which 1 lb. of steam flows from the large cylinder to the small one; then, clearly, the only change produced in the whole mass of steam included between the pistons, is due to the transfer of 1 lb. of steam from the large cylinder to the small one, the remainder of the steam remaining in the same condition as before.

Applying the principle of work, we have

$$\text{Energy exerted} = \text{Work done} + \frac{u_2^2 - u_1^2}{2g},$$

the last term being the change of kinetic energy consequent on the increased velocity with which the lb. of steam moves. But the energy exerted is $P_1 V_1$, where V_1 is the specific volume of steam in the large cylinder; and the work done consists partly of external work, $P_2 V_2$, done in overcoming the resistance to motion of the small piston, and partly of internal work, which is given by rules previously stated. Hence we obtain

$$P_1 V_1 = P_2 V_2 + I_2 - I_1 + \frac{u_2^2 - u_1^2}{2g},$$

which may be written

$$\frac{u_2^2 - u_1^2}{2g} = P_2 V_2 + I_2 - P_1 V_1 - I_1;$$

but with the notation employed in previous chapters (Art. 12),

$$I = h + \rho x = h + (L - Pv) x = h + Lx - PV;$$

thus,

$$\frac{u_2^2 - u_1^2}{2g} = h_2 - h_1 + L_2 x_2 - L_1 x_1;$$

which is a general formula applicable to any case in which the motion is *steady*—that is to say, when the pistons move uniformly under uniform pressure. The only suppositions made are, that sufficient time elapses to permit the kinetic energy, corresponding to the rushing motion through the pipe F, to be converted into heat by fluid friction when the eddying motions subside into the simple motion of translation of the small piston, and that any heat generated by friction against the sides of the pipe passes into the steam, not into the metal of the pipe.

The very important general formula here given is applicable to many special cases, of which that to be considered in the present article is when $u_1 u_2$ are small, while the difference of pressures is not very small. Let the velocity of the small piston be not more than 16 feet per second, or 960 feet per minute, then the height due to that velocity is under 4 feet, and the kinetic energy is consequently less than 4 foot-lbs. and may be neglected; then, as u_1 is in any case less than u_2 we have simply

$$0 = h_2 - h_1 + L_2 x_2 - L_1 x_1,$$

a formula which enables us to find x_2, the final dryness-fraction of the steam when its initial dryness is known, and also the pressures in the two cylinders.

Hence

$$x_2 = \frac{h_1 + L_1 x_1 - h_2}{L_2} = \frac{t_1 - t_2 + L_1 x_1}{L_2},$$

where L_1 L_2 are expressed in thermal units, a formula which is applicable to every case of wire-drawing which is steady and not too rapid.

By subtracting x_1 from each side,

$$x_2 - x_1 = \frac{t_1 - t_2 - x_1 (L_2 - L_1)}{L_2};$$

or using the formula for L (Art. 4),

$$x_2 - x_1 = \frac{t_1 - t_2}{L_2}(1 - \cdot 71\, x_1),$$

a formula which shows that steam is made drier by wire-drawing, and enables us to calculate the amount of the drying.

For example, let steam at 60·4 lbs. per square inch be wire-drawn to 14·7 lbs. per square inch, and let it contain initially 5 per cent. of moisture, so that $x_1 = \cdot 95$, then

$$t_1 = 293^\circ : t_2 = 212^\circ : L_2 = 966;$$

$$\therefore\ x_2 - x_1 = \frac{81}{966}(1 - \cdot 95 \times \cdot 71)$$

$$= \cdot 0274,$$

showing that more than half the moisture is evaporated. It will be seen, however, that the amount of drying is nearly proportional to the difference of temperature, and hence is generally much less than in the extreme case here supposed.

In the investigation just given the large cylinder represents the boiler or other reservoir from which the steam is supplied; for, on entering the cylinder, the steam is pressed from behind just as it would be by the ideal piston introduced for the sake of clearness. The small cylinder is the working cylinder in which the steam is used, and the difference of pressure between it and the boiler and the steam pipe connecting the two can be roughly estimated.

Let the ratio of areas of the cylinder and steam pipe be m, then $m\, u_2$ is the velocity through the pipe, and neglecting u_1 in comparison $(1 + F)\dfrac{m^2 u^2_2}{2g}$ = head due to difference of pressure. Let ΔP be the loss of pressure in the steam pipe, and V the mean specific volume of the steam entering from the boiler, then $V . \Delta P$ is the head in question, and dividing by $P V$

$$\frac{\Delta P}{P} = \frac{m^2 u^2_2 (1 + F)}{2 g . P V}.$$

For example, if the velocity through the pipe be 100 f. s., taking $P V$ as 60,000 ft. lbs.,

$$\frac{\Delta P}{P} = \frac{1 + F}{386}.$$

The value of F, the coefficient of resistance, can be estimated for a pipe of given length and diameter.

On passing into the cylinder there would be a gain of pressure, instead of a loss, if the pipe simply led into the cylinder; but the contraction at the steam port probably occasions a further loss, difficult to estimate with any approach to exactness. In any case, observe that the drop of pressure due to this cause is proportional to the square of the velocity of the piston, and also to the density of the steam.

124. When clearance is combined with wire-drawing, the effect in drying the steam is of course much greater. It will be sufficient to consider one case, namely, that in which there is no compression and the clearance is regarded as sensibly empty when admission commences.

The difference between this case and the last is, that now the piston does not move through the clearance space, and that, consequently, the work done in driving the piston is diminished in the proportion

$$\frac{1}{r} : \frac{1}{r} - \frac{c}{1 + c}, \text{ or } 1 : 1 - \frac{c r}{1 + c},$$

the volume of the cylinder being regarded as unity for the purposes of the calculation. Hence the equation of the last article becomes, omitting the velocities for the reasons previously stated,

$$P_1 V_1 = P_2 V_2 \left(1 - \frac{c r}{1 + c}\right) + I_2 - I_1;$$

or, making the same changes as before,

$$L_1 x_1 = L_2 x_2 + t_2 - t_1 - \frac{c r P_2 V_2}{1 + c};$$

that is,

$$x_2 = \frac{t_1 - t_2 + L_1 x_1}{L_2} + \frac{c\,r}{1+c} \cdot \frac{P_2 V_2}{L_2}$$

$$= \frac{t_1 - t_1 + L_1 x_1}{L_2} + \frac{c\,r\,x_2}{1+c} \cdot \frac{1}{k+1},$$

from which x_1 may be calculated.

For example, let

$$c = \frac{1}{15} : r = 4 : x_1 = \cdot 95;$$

and let the wire-drawing be from 66 to 60 lbs. per square inch, then

$$t_1 - t_2 = 6 \cdot 2^\circ : L_2 = 909 : L_1 u_1 = 859 \cdot 4;$$

$$\therefore\ x_2 = \frac{866}{909} + \frac{1}{46} \cdot x_2;$$

$$x_2 = \cdot 973.$$

From this example it appears that the effect of clearance is much greater than that of moderate wire-drawing in drying the steam; but, as before, it is much diminished by compression.

125. The loss by wire-drawing may be differently stated, according to the particular cases which are compared together. The most natural case to take is, perhaps, to compare the performance of an engine working as in Art. 100, page 223, with one which is in all respects similar except that the steam is wire-drawn down to a given pressure.

Let $T_1\ T_2$ be the absolute temperatures, and let the other notation be as before, then, writing down the values of the useful work (U) done in the cycle as given by the formulæ on page 223 for initial temperatures T_1, T_2, and condenser temperature T_0, the difference will be found to be, remembering that $L_2 x_2 + T_2 = L_1 x_1 + T_1$ (p. 264),

$$\text{Loss} = U_1 - U_2 = T_0 \left\{ \frac{x_2 L_2}{T_2} - \frac{x_1 L_1}{T_1} - \log_\epsilon \frac{T_1}{T_2} \right\}.$$

This may either be written (see page 229)

$$\text{Loss} = T_0 \{\phi_2 - \phi_1\}$$

or may be reduced to a simple form by substitution for $L_2 x_2$, and replacing the logarithm by its approximate algebraic value. We thus obtain

$$\text{Loss} = T_0\left(\frac{1}{T_2} - \frac{1}{T_1}\right)L_1 x_1.$$

For example, let

$$T_1 = 764^\circ : T_2 = 754^\circ : T_0 = 563^\circ : x_1 = \cdot 95,$$

corresponding to wire-drawing from 70 to 60 lbs. per square inch, then $L_1 x_1 = 856$;

$$\therefore \text{ Loss} = 8 \cdot 39 \text{ thermal units.}$$

The loss here, as in Chapter VIII., means the loss of available heat; that is to say, the heat-equivalent of work which might have been done in a perfect engine working at 70 lbs., but is not, in consequence of wire-drawing. It must consequently be compared, as in the case of the other losses considered in the chapter cited, not with the whole heat expended, but with the fraction available for mechanical purposes. Other things being equal, it is proportional approximately to the fall of temperature, *not* to the fall of pressure.

The loss here calculated is less than the direct loss during admission due to the loss of pressure by wire-drawing. This may be taken as $V_1 \,.\, \Delta P$, where V_1 is the mean specific volume of the steam admitted, and ΔP is the drop of pressure. In the example its value is 12·1 thermal units, and the difference between this and the result (8·4) just found is the saving occasioned by the drying effect of wire-drawing. In practical cases, however, the saving would not be so great.

Unsteady Wire-drawing. Influence of the shortness of the Period of Admission.

126. Wire-drawing in practice is much more complicated than in the comparatively simple cases which have just been considered, on account of the varying motion of the piston. This has a two-fold effect on the difference of pressure between boiler and cylinder, the result being that, at the beginning of the stroke, the

difference is not large, but that it goes on increasing as the stroke progresses, the admission line drawn by the indicator exhibiting a gradually falling pressure, with a rounded cut-off corner instead of a constant pressure with sharp cut-off. Moreover, the period of admission is so short that it frequently happens that there is not sufficient time for the absorption of kinetic energy by fluid friction, as supposed in the preceding section.

In the first place, the acceleration of the piston has a direct influence apart from frictional resistances, an idea of which may be obtained by considering the simpler case of an incompressible fluid flowing through a pipe into a cylinder provided with a piston which moves with velocity u_2. Let the ratio of areas be m, then, $m\,u_2$ is the velocity of the fluid flowing through the pipe. Further, let the piston be connected with a uniformly-revolving crank, then if the obliquity of the connecting rod be neglected, the acceleration of the piston is well known to be $4\,\pi^2\,n^2\,y$, where y is the distance of the piston from the middle of its stroke, and n is the revolutions per 1″. Then since the acceleration (f) of the water in the pipe is obviously m times the acceleration of the piston, we must have

$$f = m\,\omega^2\,y = m\,4\,\pi^2\,n^2\,y.$$

Now take two points in the pipe, A and B, distant l from each other, then, if the water moved uniformly, there could be no difference of pressure between A and B, except that due to frictional resistances; but if there be an acceleration in the direction $A\,B$, P_A must be greater than P_B in order to produce that acceleration. Let A be the area of the pipe, w the weight of a cubic foot of the fluid, then

$$(P_A - P_B)\,A = \frac{w}{g}\,.\,A\,l\,.\,f\,;$$

or putting $l = 1$,

$$P_A - P_B = m\,\frac{w}{g}\,4\,\pi^2\,n^2\,y,$$

a formula which gives the difference of pressure (so far as due to acceleration) for each foot length of the pipe in pounds per square foot.

In the theory of reciprocating pumps, attention has to be paid to this effect of unsteady motion, which restricts considerably the speed at which it is possible to drive them, and renders an air vessel necessary to avoid shocks. Some idea of its magnitude, in the case of a steam cylinder, may be obtained by considering that at moderate speeds a maximum value of $4\pi^2 n^2 y$ would be about 100, when the units are feet and seconds, while w, the weight of a cubic foot of steam, will not often exceed ·33, hence a maximum value of the fall of pressure per foot length of pipe would be m lbs. per square foot. If now there were no initial condensation, m would be a little, but not much, greater than the ratio of areas of piston and port, and hence would not exceed 20, so that the fall of pressure per foot length of pipe should not in any case exceed ·14 lb. per square inch at the commencement of the stroke, gradually diminishing as the piston approaches half stroke. In high speed engines the value of $4\pi^2 n^2 y$ might exceed 2000; but since the loss of pressure in question is largely reduced by the elasticity of the fluid, it is probable that it seldom becomes large.

Frictional resistances are doubtless, usually, far more influential in the production of a difference of pressure between the boiler and the cylinder; they vary as the square of the velocity with which the steam rushes into the cylinder, and hence, if there were no initial condensation, would be zero at the beginning of the stroke, and gradually increase up to half stroke if cut-off has not previously occurred. The line drawn by the indicator pencil during admission, often called the "steam line" by writers on the indicator, is accordingly not horizontal, but falls as the stroke proceeds. When the ports and passages are too small for the speed of piston, the fall is very marked. When there is any great amount of initial condensation, the velocity of the steam through the ports, at a particular instant, will depend in great measure on the rate of condensation at that instant, and will be increased; thus the fall of pressure is increased, and is not perhaps *necessarily* greater at the end of the admission than at the beginning, though no doubt that generally occurs in practice.

127. The other reason, mentioned above, why wire-drawing, in an actual steam cylinder, is more complicated than in the simple case considered in the preceding section, is the shortness of the time allotted for admission, in consequence of which the eddying motions, due to frictional resistances and especially to the sudden expansion

from the area of the port to the area of the cylinder, have not time to subside, but continue for some instants after cut-off, or even perhaps, in some cases, till the end of the stroke.

At any point in the stroke, either before or after cut-off, imagine the piston suddenly held fast, and, if open, the port suddenly closed, the whole mass of steam in the passages and cylinder will be in a state of violent motion, and its pressure will not be uniform throughout, but will generally be greater in the passages than in the cylinder. In a very short space of time, however, the pressure will be equalised, and the kinetic energy of the motion will be absorbed in the form of heat: thus the pressure in the cylinder will rise slightly, and there remain stationary. Suppose this operation carried out at every point of the stroke, and the corresponding pressures laid off on a diagram, which may be called the "equilibrium" diagram, that diagram will in general lie above the actual diagram, and will not coincide with it until after the steam is cut off, and where the piston speed is high may even, in extreme cases, be conceived to deviate from it at the end of the stroke.

In the investigations relating to the drying action of clearance and wire-drawing in the present chapter, and to the heat supplied during expansion in Chapter VII., it has been presupposed that there was no sensible difference between the two diagrams, and each particular case must be carefully examined and allowance made for possible error before relying on the results of such calculations. It may be in some cases that instead of clearance making the steam drier, it makes it actually wetter than it was in the boiler. And the heat supplied during expansion may be in some cases considerably less than would appear from an actual diagram, however carefully taken.

The question here considered may also be dealt with without any reference to the "equilibrium" diagram, by treating the kinetic energy not yet absorbed by fluid friction as part of the internal energy of the steam, in addition to the energy stored up in steam of the same quality at rest. The heat supplied during expansion, found by the processes of Chapter VII., is therefore to be corrected by the subtraction of the difference between the kinetic energy of the steam in its initial and final states respectively; or if, as will usually be the case, the kinetic energy in the terminal state may be neglected, the correction will simply be the initial kinetic energy of the steam. Hence it is possible to find a maximum value of the correction so far as due to wire-drawing: for, if the difference of pressure between boiler and cylinder be $P - P^1$, and the mean specific volume of the steam admitted be V_0, the kinetic energy in question must be less, and will, probably, be much less, than is given by the formula

$$\text{Kinetic energy} = (P - P^1)\, V_0 = \frac{p - p^1}{p_0} \cdot P_0 V_0,$$

which readily may be calculated in thermal units by use of Table IV*a*. Now in the American experiments (Chapter XI.), from which the data were taken employed in Chapter VII., the difference of pressure between boiler and cylinder does not appear to have exceeded one-tenth the boiler pressure (absolute), and therefore seven or eight thermal units is an excessive estimate of the correction in question. Where the pressure is reduced by throttling to one-half the boiler pressure or less, the case of course is very different. It is more difficult to find a maximum limit in the case of clearance, but, as the correction cannot exceed a small fraction of the work done, it seems that if the data are given correctly there is no reason to doubt the substantial accuracy of the results given in the examples in question.

The influence of the shortness of the time allotted for admission on the effects of wire-drawing was first pointed out by Zeuner, in a paper in the *Civil Ingénieur* for 1875, to which we shall have occasion to refer hereafter. In the case of a gun the kinetic energy of the products of combustion has a very similar influence on the work done during their expansion. This question will be considered in the Appendix.

The formula and graphical process of Chapter III. for determining the total heat of formation of the steam at the end of the stroke are quite unaffected by the circumstances considered in the present article, but the exhaust waste in some extreme cases may include kinetic energy stored up in the exhausting steam, instead of being exclusively due to re-evaporation during exhaust and external radiation.

128. In Art. 30, Chapter III., it has already been explained that the figure drawn by the pencil of an indicator represents the average changes of state of the whole mass of steam shut up in the cylinder,

but that, when clearance and wire-drawing are taken into account, these changes are not exactly the same as the actual changes of state of any particular portion of the steam. The diagram of energy of a particle of steam can never have any compression curve differing greatly from the expansion curve, nor can there be a rounded cut-off corner, such as appears in actual indicator diagrams.

In compound or triple expansion engines the diagrams taken from each cylinder are effected in different degrees by the clearance and compression in that cylinder; and the proper method of combining into a single diagram, such as was given in Art. 28, page 63, will be considered in Chapter XI. The clearance of the cylinders of recent marine engines, especially of the high pressure cylinders of triple expansion engines, is generally very large. A number of examples of the value of the clearance-ratio, and also of the surface-factor μ employed in the next chapter, will be found in the Appendix.

CHAPTER X.

ACTION OF THE SIDES OF THE CYLINDER AND OF WATER REMAINING AFTER EXHAUST.

129. WHEN the volume of steam actually delivered from a steam cylinder at release is compared with the volume of dry steam at the terminal pressure, corresponding to the amount of feed water used, after deduction of the jacket supply, it is always, or nearly always, found that the first is far less than the second, showing that at the end of the stroke the steam discharged from the cylinder must contain more or less water, which is either re-evaporated during exhaust, or is carried out with the exhaust steam in the shape of suspended moisture. Some of this effect is no doubt due to the fact that the steam supplied by the boiler is rarely dry; but in general the difference in question is far too great to be thus accounted for, and it is therefore necessary to suppose that liquefaction takes place after the steam enters the cylinder. Moreover, when the expansion curve drawn by an indicator is examined, it is almost always found, even when the greatest care has been taken to eliminate disturbing causes, to show that evaporation takes place during expansion.

Now, these unquestionable facts can only be explained by supposing that liquefaction takes place during the admission of the steam to the cylinder, and evaporation during expansion and exhaust. This alternate liquefaction and evaporation may be supposed to be due either to the action of the metal of the cylinder, or to the effect of water remaining in the cylinder after the exhaust is completed. Let us first consider the action of the metal.

Initial Condensation.

130. It is in the first place clear that the amount of steam liquefied during admission to a steam cylinder, by contact with a comparatively cold surface, must depend on the area of the surface.

It is true that other circumstances must have great influence, and especially the time during which the contact lasts, together with the condition and temperature of different parts of the surface, but the first consideration is the actual area of surface.

Now, on entering the cylinder, the steam first comes in contact with the piston, the cylinder cover, and the passages, which make up the clearance space. The total area of these parts may be described as the "clearance surface," and it is evident that if we write

$$\text{Clearance surface} = \mu \cdot \frac{\pi}{2} d^2,$$

where d is the diameter of the cylinder and μ a coefficient, the value of this coefficient must always be greater than unity. Its actual value in three engines experimented on by Major English, ranges from 1·85 to 2·1, and in many types of engine is still greater; in the cylinders of three large marine engines employed in vessels of war, it was found to range from 1·5 to 3, and it is only in engines provided with Corliss or other similar valve gear, that it is as small as 1·1. Let m be the cut-off, as on page 249, and λ the stroke, then, as the piston advances to the point of cut-off, the surface $\pi m \lambda d$ is exposed, and at the end of the admission the total surface in contact with the steam is consequently

$$\text{Admission surface} = \mu \cdot \frac{\pi}{2} d^2 + m \pi \lambda d.$$

The volume of steam then in the cylinder is—

$$\text{Volume of steam} = \frac{\pi}{4} \lambda d^2 (c + m),$$

and its weight, inclusive of the fresh steam which enters the cylinder to supply the place of that liquefied, is

$$\text{Weight of steam} = \frac{\pi}{4} \lambda d^2 \frac{c + m}{V_1},$$

where V_1 is the specific volume of the wet steam filling the admission space. Hence, by division,

$$\text{Admission surface per lb. of steam} = \frac{4 V_1}{c + m} \left\{ \frac{\mu}{2 \lambda} + \frac{m}{d} \right\}.$$

The formula may conveniently be written in the abbreviated form

$$\text{Admission surface per lb.} = \frac{4\,V_1}{d}(1 + \sigma\,r),$$

where σ is a numerical coefficient given by

$$\sigma = \frac{\dfrac{\mu\,d}{2\,\lambda} - c}{1 + c},$$

and r, as usual, is the real ratio of expansion.

The surface thus found is the only part of the whole internal surface of the cylinder and piston which comes into contact with the fresh steam entering from the boiler, and therefore the only part which can directly affect the total consumption of steam. The surface exposed after cut-off may be described as the "expansion surface;" the loss occasioned by it takes effect by lowering the mean effective pressure, not by increasing the quantity of steam used per stroke. In compound engines the admission surface of the H.P. cylinder alone affects the consumption of steam; the condensation in the other cylinders diminishes the work done in them without altering the quantity of steam used.

The formula here obtained shows that the admission surface per lb. of steam is greatly increased by shortening the stroke, and that the clearance surface is, unless the stroke be long and the cut-off late, by far the most important part of the whole. It will, of course, be understood that it is only the actual area of surface which is here spoken of; the different parts of the whole may, and generally will, be very different as regards condition of surface, the effect of compression, and other circumstances.

In the Appendix to Chapter VI. methods have been explained by means of which the quantity of heat abstracted from each lb. of steam during admission may be determined, and hence we are in a position to find the heat abstracted per square foot of admission surface. It is true that the results of these methods are not free from possible errors of considerable magnitude, occasioned by difficulties of observation, ignorance of the quality of the steam supplied by the boiler, and certain effects of wire-drawing detailed in the last chapter. Moreover, the value of the coefficient μ is only known in exceptional cases. A rough approximation is, however, all that is needed for our present purpose.

The annexed table gives some examples of the results of such calculations in several of the American experiments, the value of μ being assumed as 2.* The engine of the *Bache* was operated as a simple engine in the particular experiments here considered; the cylinder was jacketed, but the jacket does not appear to have supplied much heat, as the liquefaction in it was small. The engine of the *Dallas* was simple, and the cylinder was not jacketed.

INITIAL CONDENSATION IN STEAM CYLINDERS.

Dimensions of Cylinder.	Ratio of Expansion.	Initial Pressure.	Revolutions.	Surface per Lb.	Initial Condensation, per cent.	Heat Abstracted.		
						Per Lb.	Per Sq. Ft.	Per Sq. Ft. per min.
Bache	12·6	89	40	49·2	62·8	558	11·4	2736
Stroke = 2′	8·6	90	46	48·2	47·4	421	8·8	2112
Diam. = 2′ 1″	5·1	91	54	47·4	30·3	269	6·7	1608
Dallas	5·1	47	49	60·0	34·5	317	5·3	1272
Stroke = 2′ 6″	3·9	47	57	53·6	26·4	243	4·5	1080
Diam. = 3′	2·9	46½	64	46·6	21·5	198	4·2	1008

These results are only rough estimates, for reasons already stated, but they are enough to show that the heat abstracted during the condensation which takes place as the steam enters the cylinder is exceedingly great when compared with the area of condensing surface and the time of contact. The table shows the rate of abstraction on the supposition that the time of contact was one-fourth of a second; the actual time varied, but was generally considerably less. That more that 25 per cent. of the steam supplied by the boiler may be condensed on entering the cylinder is a fact established by many conclusive experiments, and, indeed, at high rates of expansion in a single cylinder, the proportion may exceed 50 per cent. We have only to compare the surface of the cylinder with the surface of an

* In the first edition of this work the value of μ was taken as unity, and the estimates of the amount of heat abstracted per square foot of surface were much greater.

ordinary surface condenser to see that the former is far more efficient as a condenser than the latter. And if, instead of the condensation, we consider the re-evaporation which immediately follows, we shall be led to the conclusion that a steam cylinder is by far the most efficient evaporating apparatus which is known. Hence attempts have often been made to explain away the facts, or, if that is impossible, to attribute them exclusively to the action of water contained in the cylinder. For the present we are concerned with the action of the metal alone, and we shall now show that in the alternate condensation and re-evaporation which occurs in a steam cylinder, the action of the metal is far more powerful than it would be in a continuous process of either kind.

Flow of Heat by Conduction.

131. When heat is transmitted from a medium A to a medium B through a metallic plate which separates them, the quantity of heat transmitted in a given time depends, first, on the area of the surface in contact; secondly, on the difference of temperature. Let us suppose the temperatures T_A and T_B, then all the heat passing from A to B must pass, first, *into* the plate; second, *through* the plate; and third, *out of* the plate; and each of these three passages requires a certain difference of temperature. Thus the temperature t_A, of the outer surface of the plate is less than T_A, while that of the inner surface is less than t_A, but greater than T_B. The present and next following articles will be devoted to the consideration of the passage of heat *through* the plate, which takes place by conduction, for which purpose we suppose the temperatures t_A and t_B known.

If these temperatures have remained the same for some little time, the same quantity of heat per minute flows out of A into the metal, and out of the metal into the medium B. The flow of heat is then said to be *steady*, and is governed by very simple laws, for if F be the flow per square foot per minute, y the thickness of the plate,

$$F = f \cdot \frac{t_A - t_B}{y}$$

where f is a coefficient of conductivity, which is known roughly for iron by the experiments of Forbes and others. Forbes found that a difference of temperature of 1° would cause the transmission per square foot per minute through a plate 1 foot thick of as much heat as would raise about ·01 cubic foot of water through 1°. Hence, if y be reckoned in inches,

$$f = 7 \cdot 5.$$

Forbes' results show considerable discrepancies in different bars experimented on, owing probably to differences in the quality of the iron, and also show that the conductivity of iron diminishes as the temperature rises. Lorenz and also Angstrom independently obtained results which at the same temperature are not very different from those of Forbes. Smaller values of f are given by some experimentalists (page 283), and it should be noticed that in curved surfaces the results are modified. We may, however, probably take the formula

$$F = 7 \cdot 5 \,.\, \frac{t_A - t_B}{y}$$

as giving approximately an average value of the rate of transmission of heat by conduction through iron and steel at temperatures such as occur in steam cylinders. Some comparative result for other metals will be given further on.

Fig. 34.

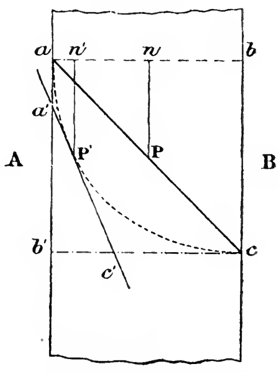


Fig. 34 shows a section of a plate through which heat is flowing: a is a point on the hotter surface, b on the cooler, while P is a point within the plate taken for convenience on the straight line $a\,P\,c$ at a perpendicular distance y from the hotter surface. When the flow is steady the formula applies to all points, and shows that the ordinate $P\,n$ graphically represents the difference of temperature $t_A - t$ between P and a on the same scale that $b\,c$ represents $t_A - t_B$. The tangent of the angle $c\,a\,b$—that is to say, $P\,n\,/\,a\,n$—measures what we may conveniently call the "thermal gradient" in degrees to the inch. If i be the thermal gradient, we have

$$F = f \cdot i,$$

a definite value of F corresponding to any given value of i.

As an example, suppose the flow of heat to be 5244 thermal units per square foot per hour, or 87·4 per minute. Writing this last value for F, and taking y as unity, we find

$$t_A - t_B = \frac{87 \cdot 4}{7 \cdot 5} = 11 \cdot 65,$$

showing a thermal gradient of less than 12° per inch. This particular example is given as being the mean flow through the plates of a marine boiler experimented on recently by Professor Kennedy. The smallness of the result shows that a very small part of the thermal resistance to the passage of the heat from the hot gases of a furnace to the water in a boiler can be due to the metal, illustrating the well-known fact that thickness and material of boiler plates have little influence on the efficiency of heating surface of the boiler.

Turning now to the results given in the table on page 277, we find that the smallest value given for the rate of abstraction of heat by a square foot of cylinder surface is about 1000 thermal units per minute, corresponding to a thermal gradient of about 140° per inch. If the thermal gradient were uniform, as in the case of steady flow, such a result would hardly be possible in a cylinder of ordinary thickness, but in a steam cylinder it is evident that, so far from being steady, the flow must fluctuate periodically from the changes of temperature in the cylinder in the course of a revolution of the engine.

132. Returning to Fig. 34, suppose, in the first instance, the temperatures of the two surfaces the same, the whole plate having the temperature t_B. Next imagine a hot body applied, raising the surface A immediately to the temperature t_A; then, after the lapse of some little time, the flow becomes steady as before, as indicated by the thermal gradient $c\,a\,b$. But before this can be the case the plate must be heated, and this requires time. In the first place, then, more heat will flow into the plate at the surface A than flows out of it at the surface B, the difference producing a gradual rise of temperature at each point of the metal, and thus at each instant and at each point of the plate the temperature varies. We may express

this by a temperature curve for each instant considered. One of these curves $a\,P'\,c$ is shown in the figure. At first they are nearly horizontal in the neighbourhood of B, and nearly vertical in the neighbourhood of A, but they rapidly become more and more nearly straight, in a short time sensibly coinciding with the straight line $a\,P\,c$ of steady flow.

The slope of each curve at any point graphically represents the thermal gradient, at first extremely great at points near it, but rapidly diminishing as time passes and as the interior of the plate is penetrated. Thus the flow of heat from A into the metal is for the first moment of the change very much greater than the steady flow under the same difference of temperature.

The thermal gradient i at any point P' is the tangent of the angle $a'\,c'\,b'$ made with the horizontal by the tangent $a'\,P'\,c'$ to the temperature curve at P', and we shall have in all cases

$$F = f\,.\,i = -f\,.\,\frac{d\,t}{d\,y}$$

where the flow F now varies from instant to instant and from point to point of the metal.

We now proceed to show that these two distinct kinds of variation are necessarily connected together.

In Fig. 34 imagine the plate divided into vertical layers of thickness $\Delta\,y$, and consider the layer in which P' lies. At this point F has a definite value, but it changes as the thermal gradient changes on passing through the layer, the curvature at P' showing that the thermal gradient is greater on entrance than on exit. Hence more heat enters the layer than leaves it, and the excess must be employed in raising the temperature of the layer. For our purpose time is most conveniently measured by the angle turned through by a uniformly rotating crank. If N be the number of revolutions per minute, θ the angle turned through from some given position in circular measure, $\theta\,/\,2\,\pi\,N$ is the corresponding time in minutes. Any small interval of time may now be measured by the corresponding angle $\Delta\,\theta$ turned through by the crank, and the quantity of heat flowing through the layer in that interval will be $F\,.\,\Delta\,\theta\,/\,2\,\pi\,N$.

Now suppose

w = weight of unit volume of the metal,

s = specific heat of the metal,

then $w\,s\,.\,\Delta\,y$ is the quantity of heat necessary to raise the temperature of unit area of the layer through 1°, and the rise of temperature Δt of the layer in the interval must be given by

$$-\frac{\Delta F \,.\, \Delta \theta}{2\pi N} = ws \,.\, \Delta y \,.\, \Delta t,$$

an equation which may be written

$$-\frac{dF}{dy} = 2\pi N \,.\, ws \,.\, \frac{dt}{d\theta}.$$

Finally, by differentiating the value of F previously found, and substituting, we obtain the general equation of the conduction of heat in a steam cylinder, namely,

$$2\pi N \,.\, \frac{dt}{d\theta} = \frac{f}{ws} \,.\, \frac{d^2 t}{dy^2} = k \,.\, \frac{d^2 t}{dy^2},$$

where f / ws is, for brevity, replaced by k.

This general equation, which is known as "Fourier's equation," connects the rate of change of temperature with the temperature at different points of the mass of metal. The coefficient k which it contains is that form of the coefficient of conductivity which must be used when we are considering changes of temperature of the metal, which changes evidently depend not merely on the rate of transmission of quantities of heat, but also on the specific heat per unit volume (ws) of the material. In the present case the unit of volume to be considered is a plate 1 foot square and 1 inch thick, which in wrought iron weighs 40 lbs., whence, taking the specific heat of iron as ·12, we have

$$k = \frac{f}{ws} = \frac{7 \cdot 5}{4 \cdot 8} = \frac{25}{16},$$

which is the absolute value of k for wrought iron, to be adopted with the units we employ. The comparative values, assuming iron as unity for different materials, are given in the annexed table.

It may be remarked, however, that the results given by different experimentalists differ considerably, partly from the difficulty of making accurate experiments on conductivity, but no doubt in great measure from differences in quality of material. In the analogous case of electric conductivity it is well known that great differences are produced by impurities in the metal. The value given by Rankine*, on the authority of Peclet, for the conductivity of iron, which, expressed in the units we employ, is about 3·8, is known

* *Steam Engine*, 6th edition, p. 259.

to be much too small, probably because the thermal resistances at entrance and exit were not completely eliminated by the method adopted. The ratio of the conductivities of lead and iron given by Rankine agrees with more recent experiments by Lorenz and others.

A value for cast iron derived from Mr. Bryan Donkin's experiments will be found in the Appendix.

COMPARATIVE CONDUCTIVITY OF MATERIALS.

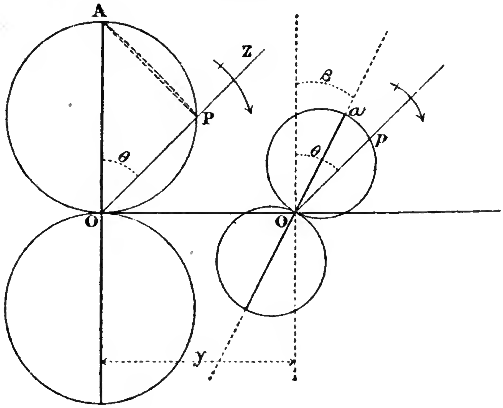
Material.	f.	$k = \frac{f}{ws}$.	$\sqrt{k}$.	$\frac{f}{\sqrt{k}}$.
Copper	5	5·4	2·33	2·14
Brass	2	2·5	1·58	1·27
Iron	1	1	1	1
Lead	·5	1·5	1·2	·4
Glass	·01	·02	·14	·07
Water	·01	·0087	·093	·107

133. The working fluid in a heat engine in all cases goes through a cycle of changes of temperature, and when the engine has been in steady work for some time, each particle of the metal of the cylinder must do the same, returning to its original temperature at the end of every complete period. The cycle for the internal surface in contact with the working fluid is not the same as the cycle of the fluid, the changes being smaller and occurring later. In the gas engine the difference must be very great; in the steam cylinder it must exist, and probably may be very considerable, but in every case there must be a definite cycle at the internal surface, which, at the present stage of our work, must be supposed known. Within the metal the changes must be smaller and must lag behind more and more the greater the distance from the surface, because time is required to transmit the changes from the surface to the interior of the metallic mass.

In Fig. 35, OZ is a line rotating with the crank, and inclined at an angle θ to a given position OA, for which a particle of metal at the internal surface has its highest temperature t_1, and the length OA is taken to represent $t_1 - t_0$ where t_0 is the mean temperature of the particle. On $O\,A$ as diameter describe a circle cutting $O\,Z$ in P, and suppose the cycle such that

$$t - t_0 = O\,P = (t_1 - t_0) \cos \theta.$$

Fig. 35.



Such a cycle is called an Harmonic Cycle, and in all periodic changes is of fundamental importance. The diagram graphically represents such a cycle, in the same way as Zeuner's Valve Diagram shows the position of a slide valve for any position of the crank; the motion of a slide valve indeed is properly described as an "harmonic motion." The second circle below represents temperatures below the mean, occurring when θ is greater than 90° and less than 180°.

As a necessary consequence of the surface cycle, as will be seen presently, particles at any distance y from the surface go through a similar cycle, but the changes are smaller and occur later the greater y. In the figure the cycle at o, a point such that $O\,o = y$, is represented by two circles constructed on a line $a\,o\,a$ inclined at an angle β, the diameters (d) of these circles representing the half-range of temperature at o on the same scale that $O\,A$ ($= D$) represents the half-range at O.

The value of β is proportional to y, and we shall write

$$\beta = m\,y$$

where m is a quantity to be found presently, and the diameters d, D are connected by the equation

$$d = D\,.\,\epsilon^{-m\,y} = D\,.\,\epsilon^{-\beta}.$$

The temperature at any point distant y from the surface is therefore given by

$$t - t_0 = D \,.\, \epsilon^{-my} \,.\, \cos(\theta - my).$$

The truth of this solution is now verified and the value of m found by substitution in the general equation of the last article. Performing this operation we obtain

$$m = \sqrt{\frac{\pi N}{k}}.$$

The angle β gives the fraction of a period by which the changes at depth y lag behind the changes at the surface. If $\beta = 2\pi$ they will lag behind by a complete period, and, therefore, will agree with those at the surface. The corresponding depth (y_0) is given by

$$y_0 = \frac{2\pi}{m} = \sqrt{\frac{4\pi k}{N}},$$

which, for iron, becomes, by substitution for k

$$y_0 = \sqrt{\frac{20}{N}} \text{ (approximately)},$$

a formula which gives the depth in inches at which the changes at N revolutions per minute agree with those at the surface: at half this depth they would be exactly reversed, the metal being hottest when the surface is coldest, and conversely.

The extent of the fluctuation of temperature diminishes with great rapidity as the depth increases, according to the same law as the orbit radii of the particles of water in an oscillating wave. On making the calculation by substitution of 2π for β, we find that the range of temperature at the depth y_0 mentioned above is less than 1/500th of the range at the surface, and even at half that depth it is only 1/23rd. A table of depths will be given presently.

134. We next proceed to find the quantity of heat which enters and leaves the metal in the course of a revolution. First, by differentiating the equation for t, and afterwards putting $y = o$, we obtain

$$F = mf\{-\sin\theta + \cos\theta\}(t_1 - t_0),$$

a formula which gives the rate at which heat is entering or leaving the metal at any point of the revolution. Its maximum value occurs when the value of θ is $-\pi/4$; that is, heat enters the metal most rapidly when the crank is 45° behind the position at which the surface has its highest temperature. In the same way, heat is leaving the metal most rapidly when the crank is 45° behind the position at which the metal has its lowest temperature. This maximum rate in both cases is given by

$$F_{max.} = \sqrt{2} \,.\, m f (t_1 - t_0) = \sqrt{\frac{\pi N}{2}} \,.\, \frac{f}{\sqrt{k}} \,.\, 2 (t_1 - t_0).$$

When θ lies between $-3\pi/4$ and $+\pi/4$, heat is flowing into the metal, and the total amount so entering is found by integration to be

$$Q = \frac{\sqrt{2} \,.\, m f}{\pi N} (t_1 - t_0) = \frac{f}{\sqrt{k}} \sqrt{\frac{2}{\pi N}} \,.\, (t_1 - t_0).$$

On substitution for f and k of their values for iron, we find in round numbers

$$Q = \sqrt{\frac{6}{N}} \times 2 (t_1 - t_0).$$

The same quantity of heat leaves the metal in the second half revolution, and the cycle then recommences. This formula gives the heat alternately abstracted from and supplied to the liquid filling the cylinder by a square foot of a surface of iron at N revolutions per minute, the quantity $2(t_1 - t_0)$ being the range of temperature, not of the fluid, but of the *metal*, at the internal surface of the cylinder.

ACTION OF METAL OF CYLINDER.

Revolutions per minute = N ..	25	50	100	200	300	400	600
Penetration in inches .. = y_0 ..	·894	·632	·447	·316	·258	·224	·182
Heat abstracted per sq. ft. per 100° = Q	49	34·6	24·5	17·3	14·1	12·3	10
Jacket ratio = $\frac{J}{Q}$	·156	·110	·078	·055	·045	·039	·032

In the annexed table the second line gives the penetration in inches at the revolutions given in the first line, as calculated from the formula for y_0, the result being for wrought iron. It will be seen that at 100 revolutions the result is about half an inch; at a depth of one-quarter of an inch, then, the range of temperature is only 1/23rd that at the surface. For other materials the result may be obtained by multiplying by the number given in the column headed $\sqrt{k}$ in the table of conductivity (page 283). The second line gives the heat alternately abstracted and given out by the metal for a range of temperature of 100°. As the result is proportional to the range, it can readily be found for any other range. Thus, at 100 revolutions and a range 150°, the heat abstracted by the metal in one-half the revolution and given out in the other half is about 37 thermal units per square foot. The multiplier for other materials than iron is given in the column headed $f / \sqrt{k}$ in the table of conductivity.

135. Hitherto we have tacitly supposed that the mean temperature of the metal is the same at all points, a supposition which implies that the cylinder is thoroughly clothed so as to prevent the escape of heat, but is not supplied with heat at the external surface by a steam jacket or otherwise. In the equation employed in the last article,

$$t - t_0 = D \cdot \epsilon^{-my} \cdot \cos(\theta - my),$$

t_0 is then an absolute constant. But the Fourier equation is satisfied if t'_0, the mean temperature at any depth y, is given by the formula

$$t'_0 = t_0 + i_0 \cdot y,$$

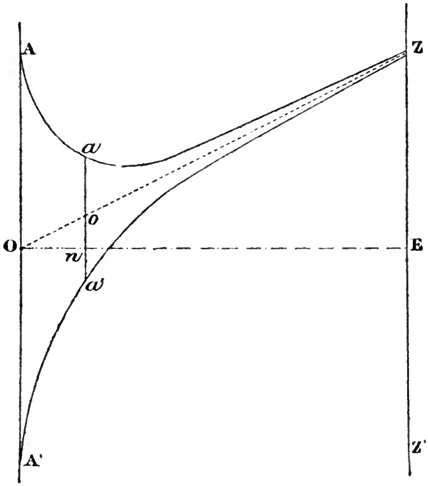
where i_0 is a constant. In Fig. 36, OE represents the thickness of the cylinder; AA' is the internal surface, ZZ' the external. ZZ' is kept at a given temperature t_5 by a steam jacket, or in a gas engine by a stream of cold water. The case of a steam cylinder is the one shown in the figure. On ZZ' take

$$EZ = t_5 - t_0$$

where t_0 is the mean temperature of the internal surface AA'. Joining OZ the ordinate on represents the mean temperature of a particle of metal at a distance $On = y$ from the internal surface. The range of temperature at this point is just the same as before, and is represented in the figure by the two logarithmic curves, AaZ

which is a curve of *maximum* temperature, and $A'\,a'\,Z$, which is a curve of *minimum* temperature. The changes of temperature are exactly the same as before; it is only the mean temperature which is altered.

Fig. 36.



The flow of heat through a cylinder of thickness e is now obtained by superposing a steady flow from without,

$$F_0 = f \cdot i_0 = f \cdot \frac{t_5 - t_0}{e},$$

upon the same variable flow as before. The effect of this is that the steam jacket supplies in the course of half a revolution the amount of heat

$$J = \frac{F_0}{2N} = \frac{f}{2Ne} \cdot (t_5 - t_0),$$

and thus the fluid within the cylinder during the hot part of the cycle gives out the heat $Q - J$, and during the cold part receives the heat $Q + J$, where Q is the quantity of heat given by the equation previously found, that is (p. 286),

$$Q = \frac{f}{\sqrt{k}}\sqrt{\frac{2}{\pi N}} \cdot (t_1 - t_0).$$

Two cases will now be considered, which may be regarded as typical.

(1) Let $t_5 = t_1$; that is, let the temperature of the external surface be kept by a steam jacket at the maximum temperature of the internal surface, a condition of things which is aimed at though not often realised in an ordinary jacketed steam cylinder, then

$$\frac{J}{Q} = \frac{\sqrt{k}}{2e} \cdot \sqrt{\frac{\pi}{2N}}.$$

In an iron cylinder 1 inch thick this becomes $\cdot 783 / \sqrt{N}$, and the numerical results for various cases are given in the preceding table. The multiplier for difference of material is given in the column headed $\sqrt{k}$ in the table of conductivity, and if the cylinder be not 1 inch thick we have only to divide by the thickness in inches. Thus in a cylinder half an inch thick the results are doubled. They are the same whatever the range of temperature.

(2) In the cylinder of a gas engine the external surface is kept by the water jacket at a temperature much below the lowest temperature of the internal surface. If we assume then that $t_0 - t_5$ is a certain multiple of $t_1 - t_0$, then the ratio J / Q will be that same multiple of the fractions given in the table. In this case the fluid in the cylinder gives out the heat $Q + J$ in the hot part of the cycle, and receives the heat $Q - J$ in the cold part.

In either of these two cases we see that a certain definite connection exists between the heat continuously supplied or abstracted at the external surface, and the alternate out-flow and in-flow of heat at the internal surface of the cylinder.

136. The results given in the table (p. 286) for the action of a metallic surface are obtained by supposing :—

(1) That the variation of temperature of the external surface of the mass of metal is small.

(2) That the variation of temperature of the internal surface is of the simple harmonic kind.

The first of these hypotheses is undoubtedly very approximately correct, but not so the second. In a simple harmonic variation, the time of receiving heat is the same as the time of giving out heat. Now, whatever the connection between the temperature of the steam and the temperature of the metal, it is quite clear that the time during which the metal receives heat must be much less than the time during which it gives out heat. In fact, most of the condensation occurs in a small fraction of the peroid when the steam is first admitted, and the temperature-cycle of the metal at the internal surface must be very complex. It is, however, well known that any cycle whatever may be considered as made up of elementary cycles, each of which is of the simple harmonic kind, the period of each being some aliquot part of the period of the compound cycle. This principle was demonstrated by Fourier, and applied by him to many questions relating to the conduction of heat. It has also been extensively employed in other branches of physical science, and especially in *Acoustics.** It immediately follows that in any cycle whatever the heat (Q) which alternately enters and leaves the metal must be given by a formula of the form

$$Q = \sqrt{\frac{B}{N}} \times \text{Range of Temperature},$$

where B is a number which can be found for any known cycle, being for a single harmonic cycle in wrought iron equal to 6, as shown on page 286. This question has been considered at some length by Dr. Kirsch. † In one example worked out by him for a cycle resembling the cycle of the steam in an ordinary steam cylinder—the metal receiving heat for about one-third of the period—his results may be shown to give a value of B, about the same as if the cycle were of the simple harmonic kind. The principal difference is that the mean temperature of the cylinder is *lowered*, being now *less* than the arithmetic mean of the highest and lowest temperatures. The rapidity with which the temperature rises as the steam is admitted favours the abstraction of heat by the metal, and this about compensates for the shorter time in which the abstraction takes place. In the present state of our knowledge, however, the practical utility of such calculations appears to be limited, because the cycle of the metal will generally differ by an unknown amount from the cycle of the steam. Moreover, the approximations used are not entirely free from difficulty when the rise of temperature takes place almost instantaneously (p. 292), as in a steam cylinder during admission.

The action of the metal is greatest when the temperature is changed most rapidly and the period of exposure to the extreme temperatures

* A very clear account of Fourier's Theorem will be found in the late Professor Donkin's *Treatise on Acoustics*. Macmillan, 1870.

† *Die Bewegung der Wärme in den Cylinder Wandungen.* Leipzig, 1886.

is most prolonged, that is when the admission of steam is prolonged to the end of the stroke. The values of Q are then probably greater than those given in the table, but in ordinary cases, with a cycle like that of the steam, these numbers may be considered as rough estimates of the amount of heat which actually enters and leaves the metal during a revolution, and comparing them with the results of experience as presented in the table on page 271, we are at once led to the conclusion that *the temperature-cycle of the metal must generally be very different from that of the steam.* This conclusion will be further developed as we proceed.

Difference between the Temperature Cycles of the Steam and the Metal.

137. We have hitherto been considering exclusively the transmission of heat from the inner surface of the cylinder to the exterior, and conversely: let us now turn our attention to the exchange of heat between the steam and the metal. Evidently there must be a difference of temperature to render this exchange possible, and the difference may be large, as in the case already referred to of the hot gases of a furnace and the exposed surface of the furnace chamber and flues. Yet, when such an expression is used, it must not be supposed that there is a finite difference of temperature between particles of fluid and particles of metal in actual contact. It is probable that no such discontinuity occurs in nature, and what is really meant is that a steep thermal gradient exists in the fluid in the immediate neighbourhood of the surface. This remark is sufficient to show that any sensible difference between the temperature of saturated steam and that of a perfectly clean surface on which it is condensing, must be due to the obstruction occasioned by the film of water deposited by the condensation; the temperature of a mass of saturated steam being necessarily very nearly the same throughout. The obstruction, however, of a layer of water is very great, even when the thickness is very small, on account of the very low conductivity of water. When the flow is steady, a film $\frac{1}{100}$th of an inch thick will have the same effect as a plate of iron one inch thick; and hence, in any process of condensation by passing steam through tubes, the efficiency of the condensing surface will depend chiefly on the rapidity with which the surface can be cleared of the water

deposited, and on the activity of the circulation of the cold water on the outside of the tubes. Thickness and material of tubes, as already stated in the corresponding case (p. 280), of the heating surface of a boiler are of minor importance. The presence of air, either in the steam or in the cooling water, checks condensation, by preventing the contact between steam and metal.

In the case of a steam cylinder, the obstructive effect of the film is far less, because the coefficient to be considered is now $\sqrt{k}$ for penetration, or $f/\sqrt{k}$ for flow (p. 287). Moreover, from the table of results already given, it will be seen that the amount of water deposited, if spread uniformly over the surface, will not usually exceed $\frac{1}{100}$th of a pound per square foot, that is, the film will not be more than $\frac{1}{500}$th of an inch thick, corresponding to a thickness of about $\frac{1}{50}$th of an inch of iron. And this thickness is of course only gradually deposited as the condensation proceeds. Thus, even if we made a large allowance for unequal distribution, it is probable that the temperature of the metal (when clean) before the end of the admission, rises nearly to the temperature of the steam, but that the rise is not quite instantaneous.

Similarly, when the temperature of the steam falls in consequence of the fall of pressure due to expansion, the temperature of the metal can hardly be supposed much greater than that of the film of water deposited by condensation, and therefore it is only when the surface becomes dry that a considerable difference of temperature between steam and metal appears to be possible. And as the range of temperature of the metal has been shown to be much less than that of the steam, we must conclude that the surface actually is dry during the greater part of the exhaust. This conclusion, however, may be considerably modified by the effect of grease or other impurities.

138. When a steam pipe or other hot body is placed within a chamber containing air or some other gaseous fluid of temperature the same as that of the walls of the chamber, an escape of heat takes place which is proportional to the exposed surface of the body. When the surface is dry, the escape is due to two causes: (1) radiation, (2) convection. The conductivity (f) of gases is exceedingly small, and its influence may be neglected.

Radiation would occur even though the chamber were empty, and the escape of heat due to it is then the difference between the heat radiated from the body to the sides of the chamber, and that radiated from the sides of the chamber to the body. It is dependent on the condition of the surface, being greater for a blackened surface than when the surface is polished. Convection is due to the presence of the fluid. The particles of the fluid actually in contact with the surface acquire the temperature of the surface, and in consequence their density is diminished; the action of gravity then sets up a current which removes them from the surface, their place being supplied by others, which in their turn acquire the temperature of the surface. Thus heat is carried off by a mechanical process. The escape of heat due to each of these causes is not proportional to the difference of temperature; but is, proportionally, greater for large differences of temperature than small ones. It is also dependent on the actual temperature, being greater for high temperatures than low ones, and on other circumstances. It is not necessary for our purposes to state what is known on the subject, it is sufficient to say that at ordinary temperatures, when air and (probably) dry steam is the fluid in question, the total escape of heat from a dry surface does not exceed ·05 thermal units per square foot per minute for each degree of difference of temperature. This, however, is only so long as the air is at rest; if a current of air circulates over the surface, the escape is greatly increased by the mechanical carrying off of heat, which may properly be described as "convection," even though not due to gravity only.

If a gaseous fluid be contained in a chamber the sides of which are at a uniform temperature above that of the fluid, the quantity of heat received by the fluid will depend partly on the power of absorbing radient heat possessed by the fluid, and partly on the process of convection. Hence the escape of heat from the sides cannot be supposed to exceed the limit just mentioned, and may be considerably less so long as the convection currents are due to gravity only; but, as before, it may be much increased by causing the fluid to circulate. If the temperature of the fluid change periodically, as in a steam cylinder, the transmission of these changes, measured by the coefficient k (p. 282), is rapid, on account of the very small density of gases, but the *quantity* of heat transmitted by conduction is still very small.

In a steam cylinder at release the internal surface is probably in most cases still covered wholly or in part with a portion of the film of water deposited by initial condensation. The sudden fall of pressure on exhaust is necessarily accompanied by a corresponding fall of temperature, which, on account of its suddenness, penetrates the metal to a very small distance. Heat therefore flows out of the metal with great rapidity, as represented by a steep thermal gradient; and the re-evaporation of the remaining water may therefore be supposed to be nearly instantaneous, when the amount is not too great. The flow of heat will be obstructed as soon as the surface becomes dry, and the fall of temperature of the surface will therefore be momentary only, being followed by a rapid rise, the dry surface then having a temperature considerably above the exhaust temperature. During nearly the whole of the return stroke, till compression begins, the dry surface will now lose heat at a rate which will be much increased by the agitation of the steam, the suddenness with which the effect takes place, and, doubtless, also by the moisture produced by expansion, which renders the absorption of heat greater and more rapid. The quantity of heat thus escaping cannot be estimated, even approximately, but must be many times greater than the estimate given above for a continuous process with dry air or steam.

Adopting the highest possible estimate, however, it is difficult to suppose that more than a very small fraction of the action of the sides of the cylinder in steam engines can be due directly to this cause. Its true influence is indirect, by widening the range of temperature of the metal in a manner to be explained presently.

Conditions on which the Intensity of the Action of the Sides depends.

139. From what has been said on the subject of conduction, it appears that the action of the metal is quite adequate to produce the condensation and re-evaporation actually observed in steam cylinders; and that, in fact, the effects would be much greater than they are if the range of temperature of the metal were as great as that of the steam. Further, we find that the action of the metal must be almost completely *superficial*, heat being drawn from a film deposited during condensation and given out again almost exclusively to the same film as it re-evaporates during expansion and exhaust. The great mass of steam not in immediate contact with

the metal must therefore expand adiabatically, and partially condense, as it would in a non-conducting cylinder. The water in the cylinder at release therefore consists of two parts: (1) the suspended moisture produced by adiabatic expansion, (2) the portion (if any) of the film which has not been re-evaporated during expansion. The first of these will be carried out of the cylinder with the exhaust steam; and in some cases a part of the second may also be swept off the surface by the rush of steam, but the greater part will either be re-evaporated at release, as described above, or remain in the cylinder. When the re-evaporation is complete, so that the surface becomes dry, a small amount of heat (S) will be given out by the metal in addition to that required for re-evaporation. Hence the action of each square foot of the metal must be subject to two general conditions.

(1) The weight of steam (W) condensed during the formation of the film must be equal to the weight of water re-evaporated during expansion and exhaust, together with the weight of the part of the film (if any) swept off the surface by the exhaust steam, or remaining behind in the cylinder.

(2) The heat supplied to the metal during condensation must be greater than the heat drawn from the metal during re-evaporation, by the amount $S - J$ where S is the amount of heat which escapes after the surface becomes dry, and J is the amount of heat received by the part of the surface considered, either by lateral conduction from other parts or by conduction from the outside of heat supplied by a steam jacket or otherwise.

If, now, the re-evaporation were conducted under the same conditions as the condensation, the corresponding quantities of heat would be necessarily equal; but the conditions are frequently such that less heat is required for re-evaporation, the difference being a small quantity, Z. The second condition is then expressed by the equation

$$Z = S - J,$$

which will only be satisfied for some definite range of temperature of the metal which determines the intensity of its action.

When the engine is working without expansion a calculation can be made which will illustrate this point. Assuming

t_1 = Temperature of Admission,
t_3 = Temperature of Exhaust,

then the temperature of the metal will rise to t_1 (nearly) during admission, but at exhaust will not sink to t_3 but will remain at some higher temperature which we will call t. The range of temperature of the metal is now $t_1 - t$, and the amount of heat supplied to the metal during the formation of the film is

$$Q = W \cdot L_1 = \sqrt{\frac{B}{N}}\,(t_1 - t),$$

where L_1, as usual, is the latent heat of evaporation at temperature t_1. At the end of the stroke the pressure and temperature are suddenly lowered, and the film adhering to the surface may be supposed to re-evaporate under the exhaust pressure (p. 294), absorbing the same amount of heat as would be required if the water were placed in a cylinder below a piston which is held fast till the temperature has fallen, and then allowed to rise under the exhaust pressure. On this hypothesis, the heat abstracted during re-evaporation is therefore less than Q, being given by

$$Q' = W(L_3 - (t_1 - t_3)).$$

Hence we find

$$S - J = Q - Q^1 = W(t_1 - t_3 - (L_3 - L_1)),$$

and substituting for L_3, L_1

$$S - J = \cdot 3\, W(t_1 - t_3) = Q \cdot \frac{\cdot 3\,(t_1 - t_3)}{L_1}.$$

Since $\cdot 3\,(t_1 - t_3)$ will always be a small fraction of L_1, we see that a very small amount of heat, $S - J$, by disturbing the balance between condensation and re-evaporation, is capable of producing a considerable action of the metal, as represented by the quantity of heat Q received during condensation. And it seems clear that this general conclusion must be true, even if (as is possible) the small quantity of heat $Q - Q^1$ be not given with precise accuracy by the above formula.

If the quantities of heat S and J can be estimated, the initial condensation can be found. As regards S we may provisionally assume

$$S = \frac{K(t - t_3)}{2\,N};$$

where K is a co-efficient representing the escape of heat from a dry surface of metal, after the re-evaporation is complete, per degree of difference of temperature per minute, and then, assuming J zero,

$$t - t_3 = W \cdot \frac{\cdot 6\,(t_1 - t_3)\,N}{K}$$

$$t_1 - t = W \cdot L_1 \cdot \sqrt{\frac{N}{B}}.$$

By addition t disappears, and we thus obtain the formula

$$W = \frac{t_1 - t_3}{L_1 \sqrt{\frac{N}{B}} + \frac{\cdot 6\,(t_1 - t_3)}{K} N},$$

which determines the weight of steam condensed initially per square foot of surface per revolution, in terms of two constants, B and K, of which B depends mainly on conductivity, and K on the escape of heat from a dry surface. The value of these constants could only be found by comparison with experiment. As regards K, its value probably depends on the density of the steam at exhaust and release, for we know that in air engines the efficiency of the heating and cooling surfaces is proportional to the density of the air (p. 120): and it is probably also greater the greater the difference of temperature and the higher the speed.

Neglecting the effect (sometimes considerable) of lateral conduction, the value of J is connected with that of Q. If the difference between the temperature of the external surface and the mean temperature of the internal surface of the cylinder be $\frac{1}{2}\,(t_1 - t_3)$ as on page 288, the value of J will be

$$J = Q \cdot \frac{\cdot 783}{e\sqrt{N}},$$

and on substitution it will be found that the effect of the steam jacket is the same as if the conductivity of the metal, as measured by the constant B, had been reduced. By raising the temperature of the external surface of the cylinder, it is evidently possible to make $S - J$ zero, and initial condensation will then be avoided altogether. This has actually been done in some recent experiments by Mr. Donkin, the cylinder being heated by gas flames.

If the cylinder be without covering, J will be negative, representing heat which escapes from the external surface. The action of the metal is greatly increased.

140. When the engine is working expansively the calculation becomes much more complicated, and, for the purposes of illustration, we shall suppose the cylinder non-conducting, and the action of the sides represented by a thin metallic plate attached to the piston. The influence of such a plate upon the expansion curve has been thoroughly considered in Chapters VI. and VII., and in a former article it has been shown that the true action of the metal bears a general resemblance to that of such a plate. Let C be the heat supplied by a square foot of plate per degree, then, with the same notation as was employed in the last article,

$$Q = W \cdot L_1 = C(t_1 - t).$$

The weight of water W deposited as a film covering the plate now falls in temperature as the expansion proceeds, and while doing so receives the total amount of heat C (or the quantity C/W per lb.) for each degree of the fall. Applying the equation given on page 215, the fraction of W evaporated will be

$$x = \frac{\left(q + \frac{C}{W}\right) \log_\epsilon \frac{T_1}{T_2}}{\frac{L_2}{T_2}} = \frac{\left(q + \frac{L_1}{t_1 - t}\right) \log_\epsilon \frac{T_1}{T_2}}{\frac{L_2}{T_2}},$$

where the capital letters, as usual, refer to absolute temperatures, and the suffix 2 to the end of the expansion.

The values of x obtained from this equation are, of course, greatest when t is greatest, that is, when $t = t_2$, and even then are always fractional, showing that the whole of the film produced by initial condensation will not be evaporated during expansion, but that a certain portion $1 - x$ will remain, which can be calculated by the above formula. When the exhaust opens the portion in question wholly or partially evaporates, and the temperature of the metallic plate falls until enough heat has been supplied to produce that evaporation. If the expansion has been considerable, the lowest temperature of the plate will not be so much above the exhaust temperature as to render it necessary to take into account the quantity of heat S which escapes after the surface has become dry; and, therefore, when a permanent *regime* has been attained, the plate returns exactly to its original temperature t, and the whole process will then be repeated indefinitely. When this is the case, the exhaust waste will be $C(t_2 - t)$ thermal units per square foot of plate per revolution.

Now, if the fraction $1 - x$ be evaporated by a fall of temperature $t_2 - t$, we must have, since the absolute weight of that fraction will be $W(1 - x)$, an abstraction of heat from the plate given, as before, by

$$Q_1 = C(t_2 - t) = W(1 - x)(L_3 - (t_2 - t_3))$$

whence, substituting for C/W, as before,

$$1 - x = \frac{t_2 - t}{t_1 - t} \cdot \frac{L_1}{L_3 - t_2 + t_3},$$

an equation which, in combination with the preceding, will give the value of t. That value will always be less than t_2, and should it be greater than t_3, represents the actual temperature of the plate during exhaust, which will be lower and lower the greater the expansion. But if the expansion proceed far enough, the calculated value of t will be equal to or less than t_3; in the first case the range of temperature of the plate will be the same as that of the steam, and in the second, as the plate cannot sink lower, the conclusion is that the initial condensation is always greater than the re-evaporation during expansion and exhaust. Water then accumulates in the cylinder, forming a permanent film on the plate, which increases in thickness at every revolution, until the conditions are altered in such a way as to enable a permanent *regime* to exist.

As a numerical example, suppose that the initial pressure in the cylinder is 90 lbs. per square inch, and the back pressure $3\frac{3}{4}$ lbs. per square inch, corresponding to an initial temperature (t_1) of 320°, and an exhaust temperature (t_0) of 150°; then, so long as the terminal pressure is not less than about 20 lbs. per square inch, the lowest temperature (t) of the plate will be above the exhaust temperature, and the effect of the plate will be to produce an initial condensation and exhaust waste, which gradually increase as the expansion increases. Any water which may happen to be in the cylinder at first will gradually evaporate till the permanent *regime* has been reached, and the minimum exhaust waste will be 78 C thermal units per square foot of plate. But if it be attempted to carry expansion beyond the critical pressure of 20 lbs. per square inch, accumulation of water will begin, and the action of the plate will be indefinitely increased.

141. The effect of the actual surface is much more complex than that of the thin plate considered in the preceding article, because the supply of heat to the film is not the same for each degree that the temperature falls, but varies greatly. Moreover, the outflow and inflow of heat always lag behind (p. 286) the corresponding changes of temperature, so that re-evaporation may continue through a part or the whole of the return stroke. It is probable, however, that we may safely conclude—(1) that the re-evaporation is not complete at the end of the expansion, and (2) that if the expansion is not too great the surface at release becomes rapidly dry. At low ratios of expansion the action therefore may be supposed to resemble that considered in Article 139. Initial condensation increases as the expansion increases, and at high ratios of expansion becomes large, on account of the great range of temperature of the metal, which is probably seldom, if ever, less than the range of temperature during expansion. The effect of speed is to diminish initial condensation, but no precise law can be stated. The action of the steam jacket will be to diminish condensation in a manner analogous to that already described.

When the pressures at release and exhaust are such that the corresponding difference of temperature is small, re-evaporation must continue through a considerable fraction of the return stroke, and that fraction will vary with the speed. This is often the case in high-pressure cylinders, and the laws of cylinder condensation may therefore be different in simple and compound engines.

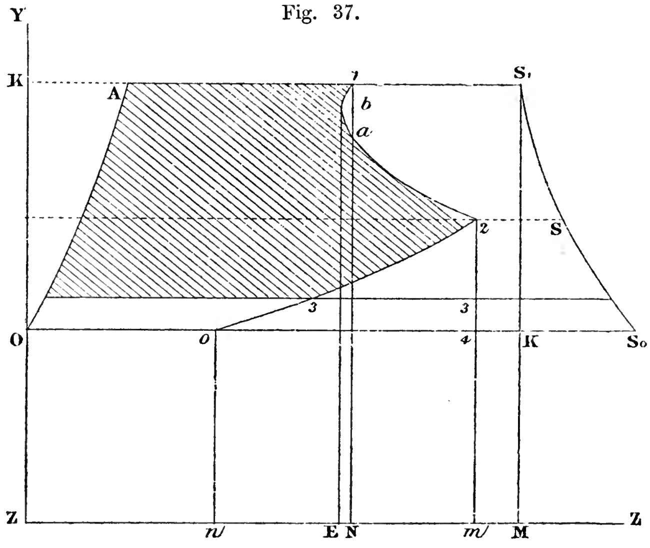
It is further probable that cases may occur in which the surface never becomes dry. Condensation then predominates over re-evaporation, and the excess tends to accumulate in the cylinder. As already stated, a film of water of very small thickness would be sufficient to prevent any sensible communication of heat to the metal; and if we imagine such a film permanently to cohere to the surface, its action will be the same as that of a metal of very low conductivity, and will consequently be much less than that of a surface not thus protected, notwithstanding the greater range of temperature to which it will be subject. The same remark applies to any water which may collect in angles and corners of the surface, if it be supposed to form a coherent mass. Thus, accumulation of water, when cohering in this way, does not imply increased action, but rather the contrary. Nor does there seem any reason why the accumulation in this case should cease. Coherent water, however, rarely, if ever, exists in steam cylinders, and it is probable that a permanent *regime* is reached in these cases, by the breaking up of the water into spray by the rush of the exhausting steam, and in the case of the barrel surface by the returning piston sweeping off the film into the clearance space. The excess of condensation over evaporation will ultimately be carried off with the exhaust steam, but part of the broken water will remain behind in the clearance space, and increase the initial condensation. A small quantity of water when broken up, will have the same effect as a thin plate of metal. This view of the question has been taken by some writers, notably by Mr. Illeck in a paper published in the *Civil Ingenieur* for 1876, where it is also pointed out that priming water entering the cylinder at intervals may be a potent cause of condensation. A similar view is advocated by Professor Zeuner, who regards the clearance contents as being always the principle cause of condensation.

Under what circumstances, and to what extent, water accumulates in steam cylinders is still uncertain, and it must be distinctly understood the calculations given in Arts. 139, 140, are useful for the purposes of illustration only; they cannot become the basis of a formula of practical value until the subject has been more thoroughly studied experimentally. It must also be remembered that, as already remarked (page 276), the different parts of the whole exposed surface will be very different as regards time of exposure and condition of surface; in particular, the central part of the barrel surface is exposed twice to steam and twice to exhaust in each revolution, that is, the period of its temperature-cycle is only one-half that of the clearance surface, and its range of temperature is, of course, much less. We shall therefore return to the question in the next chapter when we come to consider in detail the results of experiment.

Losses of Efficiency.

142. Let us now return to the question of the losses of efficiency in a steam engine, considering especially the loss directly due to the action of the sides and the manner in which the other losses are effected by it. A number of examples of this action have already been given in the appendix to Chapter VI., but we shall now proceed by a different method, employing the thermal diagram introduced by Mr. Macfarlane Gray, which is well adapted for the purpose, because (p. 228) the outflow and inflow of heat are directly represented by the area of the several parts of the diagram. Clearance and wire-drawing are for the present neglected.

Fig. 37.



In Fig. 37, $O\ A\ S_1\ S_0$ represents such a diagram, constructed as in Fig. 29, p. 228, for the evaporation of water in a boiler at temperature T_1 and condensation at T_0, $S_1\ S_0$ being the saturation curve and $O\ A$ the logarithmic curve for the rise of temperature from T_0 to T_1 on entering the boiler. The point S_1 therefore represents dry saturated steam of temperature T_1 which (neglecting priming water) we may suppose supplied by the boiler.

The vertical line $S_1 K$ would represent adiabatic expansion, but instead of this we now suppose that condensation takes place during admission in the manner already explained, so that the state of the steam at the end of the admission is actually represented by the point 1, such that

$$x_1 = \frac{A\,1}{A\,S_1}$$

where $1 - x_1$ is the initial condensation. The heat abstracted from the steam, as shown on page 226, is represented by the rectangle $1\,S_1\,M\,N$, the base of which is the zero line $Z\,Z$.

The initial condensation is supposed large in consequence of the steam being cut off early, and in such cases the condensation probably not unfrequently continues after cut-off, the expansion curve, as shown on an ordinary indicator diagram, actually for a short distance falling below the adiabatic curve. When plotted on a thermal diagram, as described on page 230, the expansion curve then falls within the vertical $1\,N$ representing adiabatic expansion. As soon as abstraction of heat ceases the curve becomes vertical, as shown at the point b in the figure. The total amount of heat abstracted from the steam by the metal of the cylinder or the water it contained before condensation commenced is

$$Q_1 = \text{Area } E\,b\,1\,S_1\,M,$$

the small amount $E\,b\,1\,N$ being abstracted after cut-off. Re-evaporation now commences, becoming large as the expansion proceeds, so that the steam soon becomes drier than it was initially. Thus, the expansion curve, when plotted on a thermal diagram, is of the form $1\,b\,a\,2$, cutting the vertical at the point a and terminating at the point 2, which represents the terminal state of the steam. The heat supplied to the steam during the process of re-evaporation during expansion is given by

$$Q_2 = \text{Area } b\,a\,2\,m\,E.$$

The steam is now released, and rushes into the condenser. This part of the process is not mechanically reversible, and our method is not directly applicable, we therefore replace it by an ideal process which is reversible and gives the same final result. Let us imagine that the steam is retained in the cylinder and gradually condensed by abstraction of heat until the temperature has fallen to T_0. The specific volume of the mixture of steam and water during this operation remaining always constant, we shall have at each point,

$$v\,x = v_2\,x_2,$$

where, as usual, x is the dryness-fraction, v the specific volume of dry steam of the pressure considered, and the suffix 2, as in the diagram, refers to the *terminal* state of the steam. Hence it follows that at every step in the condensation,

$$x = \frac{v_2\,x_2}{v};$$

and knowing x, we can, as on page 230, plot on the diagram (Fig. 37) the corresponding curve 2 3 0, which is a curve of *constant volume.* Or we can at once plot this curve, by the process given on the page just cited, directly from the vertical line which represents it on the mechanical diagram. The area 0 3 2 $m\,n$ between this curve and the zero line $Z\,Z$ now represents the heat abstracted during this ideal process of condensation, which, because the volume is constant, is also the amount of internal energy lost by the steam in passing from the state 2 to the state 0. The condensation is now completed at the temperature T_0, the additional heat abstracted being represented by the rectangle $O\,Z\,n\,o$. If, now, we examine the difference between the ideal process and the actual one in which the steam is released at the point 2, and suddenly expands until its pressure has fallen to that due to the temperature of the condenser, the condensation being then wholly carried under that pressure, we at once see that exactly the same amount of energy is supplied by external bodies in the two cases, and therefore, the heat rejected must be the same. Hence, when there is no excess back pressure (p. 241), the area, $O\,A$ 1 2 0, represents the useful work. To represent the effect of excess back pressure we have only to draw the horizontal line 3 3, at a height 3 m, representing the corresponding temperature T_3, then, reasoning as before, the area A 1 2 3, which is shaded in the figure, shows the useful work, and the area O 0 3, the loss by excess back pressure. The loss by incomplete expansion is given by the area 2 0 4, for if the steam had expanded adiabatically from the state 2, to the condenser temperature, the expansion curve would have been the vertical 2 4, and the additional work 2 0 4 would have been done. On calculating the areas $0\,0\,3$ and $2\,0\,4$, the same formulæ will be obtained as have already been given in Articles 109, 110, page 241.

It is, however, sometimes practically more convenient to include in the loss by excess back pressure that part of the loss by incomplete expansion which corresponds to expansion below the pressure p_3. This loss is then represented in Fig. 31, page 240, by that part of the whole shaded area which lies below the back pressure line $R'\,S'$, when prolonged to meet the curve, and in Fig. 37 by the very approximately trapezoidal strip on the base $O\,K$. If T_3 be the absolute temperature corresponding to the back pressure, the area of this strip gives

$$\text{Loss} = (T_3 - T_0)\left(\log_\epsilon \frac{T_1}{T_0} + \frac{L_1}{T_1} - \tfrac{1}{2}\cdot\log_\epsilon \frac{T_3}{T_0}\right),$$

in which formula the last term may generally be neglected, representing as it does the small triangle at O by which the trapezoid differs from a rectangle on $C\,K$. On this supposition the result differs from the area $O\,A\,S_1\,K$, which gives the work of a perfect engine, only in the factor $(T_m - T_0)$ being replaced by $T_3 - T_0$, and therefore

$$\text{Fractional loss} = \frac{T_3 - T_0}{T_m - T_0},$$

being simply equivalent to wasting the part $T_3 - T_0$ of the range of temperature theoretically available. This result can easily be corrected if necessary for the omitted term. The same result may be obtained, though less easily, from the mechanical diagram, by the formulæ for U_3, U_4, on pages 241, 242.

143. It thus appears that if by the process of Art. 104, page 230, a complete indicator diagram be plotted as a thermal diagram, the area of the resulting closed figure will represent the useful work, and the areas of its several parts will represent the outflow and inflow of heat, notwithstanding the fact that parts of the process are not mechanically reversible. Hence, it is clear that such a diagram shows graphically the efficiency, whether absolutely or relatively, to an ideally perfect engine.

(1) Taking first the absolute efficiency, we have only to observe that (page 227)

$$\text{Heat expended} = H_1 - h_0 = \text{Area } O\,A\,S_1\,M\,Z,$$

and therefore

$$\text{Absolute efficiency} = \frac{\text{Area } A\,1\,2\,3\,3}{\text{Area } O\,A\,S_1\,M\,Z}.$$

(2) Next, admitting that the cycle of the engine is the best attainable (page 246), the useful work done by an ideally perfect engine is represented by the area $O\,A\,S_1\,K$ (page 227), and therefore

$$\text{Relative efficiency} = \frac{\text{Area } A\,1\,2\,3\,3}{\text{Area } O\,A\,S_1\,K}.$$

It is this last fraction which is the "true" efficiency, the difference between the two areas in question being the margin for possible improvement. The effect of a steam jacket will be considered presently.

Again, it was found above that the amounts of heat stored up in the cylinder during condensation by action of the sides, and restored during re-evaporation, were (page 303),

$$Q_1 = \text{Area } E\,b\,1\,S_1\,M,$$
$$Q_2 = \text{Area } E\,b\,a\,2\,m.$$

The difference of these is

$$Q_1 - Q_2 = \text{Area } 1\,b\,a\,2\,m\,M\,S_1,$$

which represents an amount of heat lost by the cylinder, either in consequence of re-evaporation during exhaust, or by the escape of heat in some other way, that is, the area in question graphically represents the Exhaust Waste. Of this, the rectangle $m\,K$ below the condenser line $O\,K$ represents an "unavoidable loss," and the remainder gives the avoidable

$$\text{Loss by Action of Sides} = \text{Area } 1\,b\,a\,2\,4\,K\,S_1.$$

As thus estimated, the loss includes (1) a loss by "misapplication of heat during expansion" (Art. 108), and (2) a loss by leakage of heat into the condenser without doing work, of which, as pointed out on page 240, the first might (at least in ideal cases) be negative. It is, however, not practically useful to make this distinction.

Another way of stating the question is—

Loss by initial condensation = Area $1\,b\,E\,M\,S_1$.
Saving by re-evaporation during expansion = Area $b\,E\,m\,2\,a$.

Taking the difference of these,

$$\text{Loss by Action of Sides} = \text{Area } 1\,b\,a\,2\,m\,M\,S_1,$$

of which, as before, the rectangle $K\,m$ forms part of the unavoidable loss.

When the cylinder is jacketed and the jacket is supplied with steam from the boiler, a separate diagram, placed to the right of the vertical $S\,M$, should be constructed for the steam supplied to the jacket, the horizontal scale of the subsidiary diagram being less than that of the original in the ratio of the weight of the jacket steam to the weight of the boiler steam. The exhaust waste will now include the heat supplied by the jacket.

CHAPTER XI.

EXPERIMENTS ON STEAM ENGINES. CONDITIONS OF ECONOMICAL WORKING.

144. THE more important parts of the theory of the steam engine having been discussed in the preceding chapters, let us now go on to apply the theory in further detail to the working of steam engines, making use of data furnished by some of the numerous experiments which have been made.

Let the data be the indicated power, and *either* the amount of feed water used in a given time, *or* the condensation water, with its rise of temperature, *or*, preferably, both these last data combined.

The indicated power must be determined from a number of diagrams, accurately taken every few minutes, for a considerable time; a single diagram is of little value where accuracy is desired. The feed water is to be determined by direct measurement during the same period, and care must be taken that the water level in the boiler is precisely the same at the beginning and end of the experiment; the loss of water, frequently occasioned by imperceptible leakage, is, so far as possible, to be estimated and allowed for; and the boiler pressure, height of barometer, and temperature of the feed, are to be frequently noted, so that mean values can be obtained. The case where the condensation water is the datum will be mentioned presently.

From these data it is possible to find approximately the performance of the engine, both absolutely and relatively to a perfect engine working between the same limits of temperature. The result will be exact, instead of approximate, if the amount of water carried over with the boiler steam be known—a matter to be considered presently.

As an example, we select an experiment made with the compound engine of the U.S. steamer *Rush*. (See Art. 148.) The data are :—

Water per I.H.P. per hour	= 18·38 lbs.
Average boiler pressure above atmosphere	= 69·06 lbs. per sq. in.
Average barometer	= 14·81 " "
Average temperature of feed	= 114°.

First.—Suppose the boiler to supply dry steam, then

$$\text{Heat expended} = H_1 - h_0,$$

where the suffix 1 refers to the boiler temperature (315°), corresponding to the absolute boiler pressure (83·87). Referring to Tables I. and II.,

$$H_1 = 1178\,;\ h_0 = 114 - 32 = 82.$$

$$\therefore \text{ Heat Expended} = 1096 \text{ thermal units per lb.}$$

$$\text{Ditto per I.H.P. per } 1' = \frac{1096 \times 18\cdot38}{60} = 336.$$

The number thus obtained is the best measure of the absolute performance of the engine. We may, if we please, compare it with the thermal equivalent of a horse-power, and thus obtain

$$\text{Absolute efficiency} = \frac{42\cdot75}{336} = \cdot127\,;$$

but the fraction thus obtained has no special significance (pp. 151, 245), and is better described as the Coefficient of Performance. We may also conveniently estimate the consumption of coal, either by taking pure carbon as a standard (see page 141) or by assuming a probable value for the available heat of a pound of coal of good quality. Adopting the second method, we take the value given by Mr. Mair, in a paper cited further on, which is 11,000 thermal units when the boilers are not forced, and of good design; and hence we find

$$\text{Consumption of Coal} = \frac{336 \times 60}{11000} = \frac{336}{183} = 1\cdot84.$$

The three results thus obtained are merely three ways of stating the same thing, of which the first may be considered the best.

The temperature of the feed water to be assumed in the calculation is to be taken before being raised by heat supplied by a feed water heater, in case the heat is derived from the hot gases of the furnace, and not directly or indirectly from the steam supplied by the boiler.

The corresponding performance of a perfect heat engine working between the limits of temperature 114° and 315° is (page 140),

$$Q = \frac{776 \times 42 \cdot 75}{201} = 165;$$

and therefore, by division

$$\text{Relative Efficiency} = \frac{165}{336} = \cdot 491.$$

The fraction thus obtained is in a much truer sense the *efficiency* of the engine, being the ratio of the heat actually converted into work to the heat which could be converted into work by any engine whatever, however perfect, supplied with heat at 315° and rejecting it at 114°. A Carnot cycle, however, when employed in a steam engine, requires the use of a specially designed feed pump (p. 235), never applied to any actual steam engine. Moreover, if a steam engine were constructed with such a cycle, there would be an unavoidable loss connected with the boiler which theoretically need not exist in a steam engine of ordinary construction. The boiler of a steam engine is not supplied with heat from an indefinite source of fixed temperature, but from hot gases which are cooled while supplying heat. An essential part, therefore, of a theoretically perfect boiler is a feed water heater analogous to a Siemens interchanger (page 124), which shall raise the temperature of the feed water while cooling the hot gases below the temperature of the boiler.

If the cycle be not that of Carnot, the engine, as will be seen hereafter, can be made equally efficient by the addition of a properly constructed feed water heater. This, however, is best considered separately, and in our perfect engine we therefore suppose heat supplied as in the simple cycle of Art. 96. The mean temperature of supply is consequently given by the formula on page 223, which for dry steam becomes

$$T_m = \frac{H_1 - h_0}{\frac{L_1}{T_1} + \log_\epsilon \frac{T_1}{T_0}}.$$

In the present case, if we suppose the limits of temperature as before to be 114° and 315°, we find by Table VI.,

$$\frac{L_1}{T_1} = 1 \cdot 1495 \; : \; \log_\epsilon \frac{T_1}{T_0} = \cdot 2998.$$

$$\therefore \quad T_m = \frac{1096}{1 \cdot 4493} = 756.$$

Thus, the mean temperature at which heat is supplied is 756 − 461, or 295° F. instead of 315°, and the performance of a perfect steam engine is now

$$Q = \frac{756 \times 42 \cdot 75}{295 - 114} = 178 \cdot 5.$$

By division we now find the efficiency to be ·531 instead of ·491. This result supposes that the lower limit of temperature is 114°. The right value to adopt will be considered further on.

Secondly.—If the boiler supply moist steam, the total heat of evaporation must be calculated by the formula on page 17.

Hence it appears that, to obtain exact results, when the amount of feed water used is the datum, the amount of priming water in the boiler steam must be known. A number of methods have been employed to determine this quantity. The most common is as follows:—

A known weight of steam from the boiler is allowed to escape into a tank of cold water, the weight of which is known, and the rise of temperature noted. Otherwise it is condensed in an actual surface condenser, or a calorimeter playing the part of a small surface condenser, and the heat given out to the condensation water determined. In either case the experiment is one on the total heat of evaporation of water, and requires for minute accuracy all the precautions described by Regnault in his memoirs referred to in Chapter I. Some of the errors may be rendered insensible by making the experiments on a large scale, as has been done by Mr. Willans, but the difficulty of measuring the mean temperature of the water at the beginning and end of the experiment with sufficient exactness is rather increased than diminished. A very minute error will cause a relatively considerable error in the determination of the amount of priming water. If the experiment is carried out with care however, the dryness fraction may be determined with tolerable exactness—say within 1 per cent.—and enough such experiments have been made to render it certain that, unless some special cause of priming exists, the steam from an ordinary boiler contains less than 5 per cent. of moisture. The actual amount no doubt varies from time to time, according to the circumstances of the evaporation, and probably does not usually exceed 2 or 3 per cent.*

145. Let us next suppose that, instead of the feed water being measured, the quantity of water discharged from the condenser per minute is noted, and the difference of temperature between the water entering and the water leaving. The observation has hitherto been made for injection condensers only, and in this case the water leaving the condenser consists partly of condensed steam, and partly of the water entering the condenser, which, while rising in temperature, absorbs the heat given out by the condensing steam.

Let W be the weight of water leaving the condenser in pounds per I.H.P. per minute, and W' the weight of the condensed steam, while t_0 is the temperature on exit, and θ_0 on entrance, then

$$(W - W')(t_0 - \theta_0) = W' R,$$

where R is the heat given out by each pound of condensing steam.

But if Q be the heat expended, reckoned from θ_0,

$$Q = U + R + t_0 - \theta_0 \qquad \text{(Art. 14)},$$

where U is the useful work done per pound of steam, so that

$$W' U = 42 \cdot 75.$$

Substituting these values,

$$W' Q = W(t_0 - \theta_0) + 42 \cdot 75.$$

Thus the heat discharged from the condenser in thermal units per I.H.P. per minute, when added to the constant 42·75, gives the heat expended *reckoned* from the temperature of the water on *entering* the condenser.

This result is quite independent of the quality of the steam

* For an account of various other methods which have been employed to determine the amount of priming water in boiler steam, the reader is referred to the *Report of the British Association for* 1894.

supplied by the boiler, but does not include any loss by external radiation, and, on the other hand, the work done per pound of steam is not exactly the indicated work, but is a smaller quantity, because the friction of the piston generates heat, which forms part of that which appears in the condenser.

For example, in an engine of 46·21 I.H.P., the quantity of water discharged from the condenser was observed to be 408·3 lbs. per minute. The water, on entrance, had a temperature of 53°, and on exit of 89°·54, so that the rise of temperature was 36°·54. Dividing 408·3 by 46·21, and multiplying by 36·54, we obtain 322·86 as the thermal units per I.H.P. per minute, discharged from the condenser; and adding 42·75, we obtain 365·6 as the expenditure of heat when the feed water is drawn from the water *entering* the condenser. When the feed water has a different temperature, the expenditure of heat cannot be calculated exactly without knowing the quantity of steam used. If the boiler pressure is known, the quantity of steam used can be found approximately by neglecting priming water. For, assuming that the boiler supplies dry steam, the expenditure of heat per lb. is the total heat of evaporation *from* the temperature from which the expenditure is reckoned *at* the temperature of the boiler. Thus, in the present example the expenditure of heat per lb. is 1158 thermal units, and the weight of steam used is therefore 365·6/1158, or ·316 lb. per minute, or 19 per hour per I.H.P. This process may be called "estimating the consumption of steam from the condensation water." Though not exact, the weight of steam thus found is sufficiently accurate to enable us to obtain the correction necessary to find the expenditure of heat when the feed has any given temperature. In the present case, if the feed is taken from the water leaving the condenser, the correction is ·316 × 36·54, or 11·6, and the corrected expenditure of heat is 354 thermal units per I.H.P. per minute. Theoretically, the weight of steam used might be found by measuring the water entering the condenser, a method which was actually adopted in some of the French experiments; but this measurement is not practically easy to carry out, and any error will be multiplied many times, because the weight of condensation water is fully twenty times the weight of the feed water.

The measurement of the heat discharged from the condenser has long been used as a practical method of testing the performance of steam engines by Messrs. B. Donkin and Co., and the test is, no doubt, a valuable one, being to a great extent independent of the performance of the boiler. For details of the mode of practically carrying out the method, we must refer to an article in *Engineering* for February 5th, 1875.

The heat discharge from a condenser reckoned in thermal units per I.H.P. per minute is often called the "Farey-Donkin Constant." When corrected as just described, it gives correctly the heat rejected by an engine. In the example the Farey-Donkin Constant is 322·8, or when corrected 311·2, which number measures the actual flow of heat into the condenser. Of this, a part, calculated as described in a former chapter (page 246), is the unavoidable flow which forms a necessary part of the working of every steam engine, however perfect, and the remainder is a loss which is theoretically avoidable.

146. The two observations of feed water and condensation water may be made at once, and in that case we have a means of verifying the first law of thermodynamics, by showing experimentally that the heat appearing in the condenser is actually less than the heat supplied in the boiler; and at the same time by use of the known value of the mechanical equivalent of heat, we can obtain some idea of the possible magnitude of the external radiation and of the amount of priming water in the steam supplied by the boiler.

As an example, take an experiment made by Messrs. B. Donkin and Co. on a 60-H.P. compound engine at the Hele Works, described in *Engineering* for November 3rd, 1871. The necessary data are as follows:—

Boiler pressure (absolute)	=	67·7
Water evaporated per I.H.P. per 1′	=	$\frac{20\cdot55}{60} = \cdot3425$
Water discharged from condenser	=	606·5 lbs. per 1′
Rise of temperature in condenser	=	31·66
Indicated horse-power	=	56·88
Heat discharged from condenser..	=	337·6 {thermal units per I.H.P. per 1′.

Reckoning the temperature of the feed as 51°·66, being the mean initial temperature of the injection water, for reasons explained in the last article, we find for the total heat of evaporation 1153 thermal units, and hence

$$\text{Heat expended per I.H.P. per } 1' = 394\cdot 9 \text{ thermal units.}$$

But, on the second mode of reckoning,

$$\text{Heat expended} = 337\cdot 6 + 42\cdot 75 = 380\cdot 35,$$

a result which shows that 14·55 units of heat per I.H.P. per minute, together with the heat generated by friction, were wasted in radiation, inclusive, probably, of the heat discharged in the water from the steam jacket. When this water is not returned into the condenser, and so reckoned as part of the discharged water, the difference of temperature between it and the water entering the condenser has to be allowed for. This difference in the present case appears to have been about 122°, and the quantity discharged was 102 lbs. per hour, or ·0298 per I.H.P. per minute; thus 122 × ·0298, or 3·63 of the above difference must be subtracted, leaving 10·92 units per I.H.P. per minute, which is the difference between the external radiation and the heat supplied by friction of pistons and valves. If we estimate the friction at one-twentieth of the engine power, it would amount to 2·1 thermal units per I.H.P. per minute, leaving 13 units for radiation, or about one-thirtieth of the whole heat supplied. It is to be observed, however, that this is a maximum result, proceeding from the supposition that the steam supplied by the boiler was perfectly dry. It would be reduced one-half by supposing that steam to contain 2 per cent. priming water. The experiment here made lasted ten hours, and careful observations were frequently made throughout that time, so as to obtain average results.

In France many such experiments have been made by M. Hirn, with quite analogous results, and it may therefore be considered as experimentally demonstrated that the amount of heat appearing in the cendenser of an engine is really less than that supplied in the boiler, and if Joule's value of the mechanical equivalent of heat be assumed, we may further conclude that, when an engine is thoroughly clothed, the loss by external radiation is not important, and that the steam from an ordinary boiler need not contain any important amount of priming water—conclusions which are confirmed by direct experiments on the radiation from surfaces, and on the amount of water contained in boiler steam.

More recently some valuable experiments of the kind now under consideration have been made by Mr. J. G. Mair on a number of engines of different types.* The loss by radiation was found to be from 1 to 3 per cent., being generally less than 2 per cent. Mr. Mair, like M. Hirn, found that when such experiments are carried out with accuracy, a small fraction of the whole heat expended remains unaccounted for. One reason of this may be that given by Hirn—namely, that the air contained in the steam carries with it a certain proportion of vapour; but it seems also possible that a minute amount of leakage may be unavoidable.

147. The expenditure of heat, reckoned from the temperature of the condenser in the example just considered, is—

$$\text{Heat expended} = 395 - \cdot 3424 \times 31 \cdot 66$$
$$= 384 \text{ thermal units per I.H.P. per } 1'.$$

Taking the temperature of the condenser as 84°, and assuming a Carnot cycle, the relative efficiency is found to be ·391. Apart from the question of the cycle, however, this fraction, when compared with that obtained for the *Rush* (·491), does not measure correctly the comparative economy of the two engines. The condensed steam always contains air, and a certain amount of leakage of air into the condenser in unavoidable. Hence, as already remarked in Chapter III., p. 79, the absolute pressure in a condenser is always greater than that which corresponds to its temperature. There is, in consequence, an excess back pressure, giving rise to a loss depending on the imperfection of the vacuum, not on the action of the steam. The pressure in a condenser is seldom, if ever, less than 1 lb. per sq. in., however low its temperature, and temperatures below 102° are therefore always useless. In comparing different engines we consequently adopt 102° in the first instance as the lower limit of temperature, instead of the actual temperature of the condenser.

The subjoined table shows the performance of a number of condensing engines, calculated on the supposition that the value of T_0 is 102 + 461, that is, 563. The six engines selected are as follows:—

I. A compound beam engine, by Mr. Leavitt, employed for

* *Proceedings of the Institution of Civil Engineers*, vol. lxx., p. 313; also vol. lxxix.

PERFORMANCE OF CONDENSING ENGINES.

	I.	J.	K.	R.	N.	G.
EXPERIMENTAL RESULTS.						
Indicated horse-power	250	127	2000	266	120	95
Revolutions per minute	13	24	72	70	20	41
Speed of piston	105	264	576	315	240	206
Boiler pressure (absolute)	114	76	160	84	60	26·3
Condenser pressure (inches)	2·1	—	5·56	3·68	—	4·88
Lbs. of steam per I.H.P. per hour	13·9	14·8	15	18·4	21·3	33·3
Temperature of feed	96°	59°	163°	114°	90°	119°
RANGE OF TEMPERATURE.						
Mean available range ($T_m - 563$)	207°	186°	228°	191°	171°	132°
Loss of range by imperfect vacuum ($t_3 - 102°$)	1°	0	36°	21°	24°	32°
Fractional loss	0	0	·154	·11	·14	·24
EXPENDITURE OF HEAT.						
Thermal units per I.H.P. per minute from 102°. Actual	258	274	281	340	391	603
Thermal units per I.H.P. per minute from 102°. Necessary	159	172	148	168	183	225
Ratio of Expenditures	·62	·63	·53	·50	·47	·37
PERFORMANCE OF ENGINE.						
Actual expenditure of heat	259	285	266	336	395	594
Coefficient of performance	·166	·15	·16	·127	·108	·072
Estimated coal	1·41	1·55	1·45	1·84	2·16	3·25
Efficiency	·62	·63	·63	·56	·55	·49

pumping sewage at Boston, U.S.A. A detailed description is given by Professor Peabody,* from which the experimental results are taken.

J. A compound beam rotative engine of the receiver type. The trial was made by Mr. Mair, and described by him in the first of the two papers cited above. The reference letter (J) is that used by Mr. Mair.

K. Triple expansion engine of the steamer *Meteor*. The trial was made under the direction of Professor Kennedy, and is described by him in a paper read before the *Institution of Mechanical Engineers* in May, 1889.

R. Compound engine (receiver type) of the U.S. revenue steamer *Rush*. This is one of the American experiments frequently referred to in this work. They were made in the year 1874–5 on the machinery of the five steamers *Bache*, *Rush*, *Dexter*, *Dallas*, and *Gallatin*. Their results, and the conclusions to be drawn from them, are the subject of various reports by Mr. Emery, which have been reprinted in *Engineering*, vols. xix., xxi. (See also *Peabody*, p. 268).

N. A single cylinder beam rotative engine. Trial described by Mr. Mair in the second of the two papers cited above under the same reference letter.

G. Single cylinder direct acting engine of the *Gallatin*. One of the American experiments referred to above.

The mean temperature of supply (T_m) is calculated as on p. 311, taking $T_0 = 563$, and hence the mean available range of temperature $T_m - T_0$ is found. The necessary expenditure of heat is now obtained from the formula (p. 246),

$$Q = \frac{24{,}000}{T_m - T_0} + 42 \cdot 75.$$

the ratio of which to the actual expenditure, reckoned from 102°, is the efficiency when the standard of comparison is an engine working with the best vacuum (about 28 inches) practically attainable, corresponding to a pressure of 1 lb. per sq. inch. The actual pressure was very different in the different engines. In Mr. Mair's engines,

* *Thermodynamics of the Steam Engine*, page 298. This excellent work will be quoted occasionally in the following pages by the word *Peabody*.

J and N, it is not given, but can be estimated from the total back pressure; in the others it forms part of the observed results. The difference between the corresponding temperature and 102° is the loss of range by imperfect vacuum, and the corresponding fractional loss in a perfect engine working with the actual vacuum is obtained (very approximately, p. 305) by dividing by the total range. In I the loss is insensible, and in J it must have been very small; in the rest it is very considerable, amounting in G to as much as 24 per cent. Subtract now these fractions from unity and divide the ratio previously obtained by the results; we thus obtain the efficiency of the engines when the standard of comparison is a perfect engine working with the same vacuum and boiler pressure. The temperature of the feed water is supposed 102°, but the result does not in any way depend on the temperature of the feed, because the heat supplied to a lb. of steam is the same in the perfect and the imperfect engines: it is only the work done by the steam that is different. If the standard of comparison therefore be an engine working with the same boiler pressure, vacuum, and temperature of the feed, the results will be the same whatever that temperature. They are the same (very approximately)* as would be obtained by calculating the mean temperature (T_m) and the expenditure of heat from the temperature corresponding to the actual vacuum, but the method here described appears preferable for simplicity of calculation, and as showing more distinctly the great influence of variations in the vacuum. When calculating the absolute performance, the temperature of the feed must be taken account of as before. The effect of a feed water heater will be considered hereafter.

Taking first the three engines I, J, K, we observe that the efficiency is the same in all, being about 63 per cent., while the consumption of steam is greater in K, notwithstanding the much higher boiler pressure employed. The consumption in I, J is the lowest yet recorded by thoroughly trustworthy experiments, and we see that it is in great part due to the excellence of the vacuum. In K the vacuum was good for a marine engine, yet the difference amounts to about $1\frac{3}{4}$ lb. additional back pressure, corresponding to a waste of 15·4 per cent. of the mean available range of temperature. Thus the mean effective range was only 192° in K, with a pressure of

* If the vacuum is very bad, this direct method would be sensibly more accurate.

160 lbs., while in I it was 207°, with a pressure of 114 lbs. That lowering the back pressure has much greater effect than raising the boiler pressure is sufficiently obvious, but it should be carefully noticed that this is in strict conformity with the thermodynamical principle that the power of a heat engine depends on the available difference of temperature. The work of the air pump, here left out of account, will be considered subsequently.

Turning next to R, N, G, we find that the efficiency is much lower, ranging from 49 to 56 per cent., while their absolute performance is much inferior. The loss by imperfect vacuum is relatively much greater in a low-pressure engine, and the mean effective range of temperature is small. In N the vacuum is assumed as about 26 inches, but may have been somewhat better.

148. An efficiency of from 49 to 63 per cent. appears low, yet the engines considered are mostly examples of the lowest consumption of steam in engines of their type with the supposed boiler pressure and vacuum. Let us, therefore, now consider the various ways in which efficiency is lost. In previous chapters it has been seen that these losses are partly thermal and partly mechanical. The mechanical losses are those which are occasioned by waste of the expansive energy of the steam—at admission by clearance and wire-drawing — during expansion by "sudden drop" and wire-drawing, between the cylinders of a compound engine—and at exhaust by incomplete expansion and excess back pressure. These losses may be called collectively the Waste Work, and are best expressed in terms of the Useful Work as found by experiment. By far the most important part of the waste work in condensing engines is generally the Waste Work at Exhaust, and it is this which we shall first consider. It is represented graphically by the shaded area in Fig. 31, p. 240, in which the line RSB corresponds to the actual condenser pressure now to be denoted by p_0, $R'S'$ to the total mean back pressure (p_3) and ND to the terminal pressure (p_2) of which the last two must be taken from an average indicator diagram (p. 308). Strictly speaking, the curve DB is an adiabatic curve, and the area should be determined by the exact formulæ given in the articles cited. We may, however, with sufficient approximation, suppose DB a curve of the form $pV^n = \text{Const.}$ with a value of n which, for simplicity of calculation, we take as

10/9, and it is then easily seen that the area in question will be represented by a pressure on the piston given by

$$\text{Waste Pressure} = p_2 \left\{ 9 - 10 \left(\frac{p_0}{p_2} \right)^{\frac{1}{10}} \right\} + p_3,$$

just as the useful work is represented by the mean effective pressure $p_m - p_3$, which is also known from the diagram. Thus the ratio of the waste work at exhaust to the useful work is obtained by division by $p_m - p_3$, and subsequent multiplication by the efficiency already found gives the fraction of the work of a perfect engine wasted at exhaust. Taking the *Rush* as an example, the data for which are given in the table farther on,

$$\text{Waste Pressure} = 9 \cdot 22 \left\{ 9 - 10 \left(\frac{1 \cdot 8}{9 \cdot 22} \right)^{\frac{1}{10}} \right\} + 3 \cdot 46,$$

which will be found to be about 8·1 lbs. per sq. inch. The efficiency as already given is ·56, and the mean effective pressure for the two cylinders when referred to the low pressure cylinder is 22·6.

$$\therefore \quad \text{Fractional loss} = \frac{8 \cdot 1 \times \cdot 56}{22 \cdot 6} = \cdot 20.$$

We thus see that no less than 20 per cent. of the work in a perfect engine is wasted at exhaust, and adding ·56, 76 per cent. is accounted for. The remaining 24 per cent. will be described as the Missing Work. A part of this indeed is in reality waste work, being wasted by clearance and wire-drawing. If there be no compression, the loss by clearance at admission can be represented by the addition of the term $c\,(p_1 - p_3)$ to the waste pressure as calculated above, p_1 being as usual the admission pressure, and the loss at exhaust already found can be corrected by multiplication by $1 + c$. These corrections may be considerable when, as in so many marine engines, the clearance is large. They are, however, much diminished by compression, which should be *included* in estimating the mean back pressure p_3. In high speed engines it must also be remembered that the loss by wire-drawing may be very considerable. As explained in Chapter IX., clearance and wire-drawing have a complex influence on the state of the steam, and it is of little use attempting to separate the loss they occasion from the other elements of the missing work. Radiation, priming, and leakage account for another fraction of the missing work, but the greater part of this quantity is generally due to liquefaction in the cylinder, as described in Chapter X., of which it is to some extent a measure.

MISSING WORK IN CONDENSING ENGINES.

	J.	K.	R.	N.	G.	F.	E.
Absolute Pressure—							
Boiler	76	160	84	60	26·3	86	71
Condenser (p_0)	1	2·73	1·8	2	2·4	2·18	2
Total back (p_3)	1·67	3·3	3·46	2·9	3·6	3·6	2·9
Terminal (p_2)	4·57	7·7	9·22	13·8	12	12·1	22·8
Mean effective	15·83	30	22·6	29	16·8	28	40·2
Waste	3·5	3·2	8·1	13·3	9·4	10·6	29·3
Fraction of Available Heat—							
Useful Work..	·63	·63	·56	·55	·49	·51	·315
Waste Work ..	·14	·07	·20	·25	·27	·19	·23
Missing Work	·23	·30	·24	·20	·24	·30	·455

The annexed table gives the results of calculations made as just described for various engines, of which the first five are the same as given in the preceding table under the same letters of reference. I is omitted, the indicator diagrams not being furnished. No corrections of any kind have been made for clearance, so that the tabulated values of the missing work include all losses of this kind. In the two compound engines J and R the missing work is about 24 per cent.; in the single cylinder engines N it is less, being only 20 per cent. In the triple expansion engine K it is greater, reaching 30 per cent.; if it had been only 20 per cent., as in N, the consumption of steam would have been only 13 lbs. per I.H.P. per hour. The difference between G and N is chiefly due to a greater loss by clearance. On the whole, making some allowance for clearance and wire-drawing, we may say that in the best condensing steam engines about 80 per cent. of the steam consumed is accounted for by the useful work and waste work which they do. The waste work is inordinately large at low pressure, and this cause of loss is unavoidable so long as steam is employed as the working fluid, to utilise differences of temperature much below 212°. The great economy of J is in great measure due to an excellent vacuum and a reduction of the waste work by lowering the terminal pressure; but even then the waste work reaches 14 per cent. In K the waste work at exhaust is only 7 per cent., but then the boiler pressure is so high that a much smaller part of the effective range is below 212°. Engines F and E are added as examples of single cylinder engines, in which the missing work is large, for reasons to be explained hereafter. The large missing work in K is partly due to the feed water heater employed, a point fully considered further on.

149. In non-condensing engines the efficiency may be calculated as in condensing engines, the lower limit now being taken as 212°, instead of 102, and the number 24,000 in the formula for Q being replaced by 28,800 (p. 246). The result obtained is much higher than in condensing engines, being generally about ·75, unless some special cause of waste exists. The reason of this is that the waste work at exhaust is now of small amount, being only 2 or 3 per cent., unless the terminal pressure is exceptionally high, or the boiler pressure exceptionally low.

In a valuable paper read before the *Institution of Civil Engineers*,* Mr. Willans has described a number of experiments on a non-condensing engine, which could be arranged to work as a single, a double, or a triple expansion engine. The standard of comparison adopted is that of a perfect engine, without feed water heater, working with the same back pressure and mean admission pressure. The effect of adopting the mean admission pressure, instead of the boiler pressure, as the upper limit, is approximately to eliminate the direct effect of wire-drawing at admission, which, in consequence of the great number of revolutions per minute (400) was in most of the experiments very large. The adiabatic curve is supposed of the form $p\,V^n$ = Const. (p. 219), with a value of n, which appears somewhat large; in other respects the calculation is based on the same principle as that adopted in the present chapter for condensing

* *Proceedings of the Institution of Civil Engineers*, March 1889.

engines. The waste work at exhaust being small, however, is not separately calculated. This paper will be again referred to further on.

150. It has already been explained, in Chapter III., that when steam is formed in a particular way, a certain definite amount of heat is required to form it, depending on the amount of external work done by it during formation: and hence that when steam is found to be in a particular state at the end of the stroke in an engine of known power, it is possible to find the total amount of heat which has been supplied to it in the boiler and during the passage from the boiler to the end of the stroke. This heat, which we called the total heat of formation, is given by the formula

$$Q = h_2 - h_0 + x_2 L_2 + (P_m - P_2) V_2,$$

or by the graphical method employed in Chapter III., and the result of the calculation is not in any way hypothetical, provided the data are correctly given. This formula can also be derived from the thermal diagram given on page 302.

Now the heat (Q^1) supplied in the boiler, supposing in the first instance that the steam supplied is dry, is known when the circumstances of the evaporation are known, and evidently it follows that $Q^1 - Q$ must be subtracted, or $Q - Q^1$ added, as the steam passes from the boiler to the end of the stroke. This difference is due partly to external radiation, but chiefly to waste of heat by re-evaporation during exhaust, as has already been repeatedly explained, and hence $Q^1 - Q$ was called the exhaust waste: its effects have been considered in hypothetical cases in Chapter III. We propose in the present section to show how the necessary data may be ascertained by experiment, and to give some results of the calculation.

The first step is to find the weight of steam discharged from the cylinder per stroke, and this is done by measuring the feed water, as in the preceding section, and also the water discharged from the steam jacket, if any; then by subtraction and division by the number of strokes observed in the given time, the result must be the required weight. These observations require the greatest care, but may be carried out without any important error.

Next, let the volume at the terminal pressure of the cushion steam remaining in the cylinder after exhaust be found, as in Art. 84, Chapter IX., and let that volume be subtracted from the total volume of the cylinder, including clearance, then the result must be the volume occupied at release by the steam discharged from the cylinder per stroke, and division by the weight found as above gives the volume of 1 lb. of it.. Now, if the terminal pressure be known, it will be possible to find the corresponding volume of dry steam, and thus by division the proportion (x_2) of dry steam in the steam contained in the cylinder at the end of the stroke is found.

The determination of the volume of cushion steam cannot be made with accuracy, and the error thus occasioned, when the compression is large, may be of considerable importance, but the principal source of error is in the determination of the terminal pressure. The indicator used must be carefully tested before and after the experiment; every precaution must be taken to avoid oscillations as far as possible, and a mean value should be obtained from a large number of diagrams, rejecting those which show exceptional variation from the mean. It appears advisable to select a period from the whole duration of the experiment during which the diagrams vary little, showing an approach to uniformity in the conditions, but of course the feed water used during the period must be separately measured. At low terminal pressures an error of one-tenth of a lb. per square inch will cause a considerable error in determining x_2. Any determination of the terminal pressure must be less certain than that of the mean effective pressure, and allowance must be made for possible error. When, as is generally the case, release occurs before the forward stroke is completed, an estimate is to be made of what the terminal pressure would have been had the release been postponed till the completion of the stroke.

We now proceed to give two examples, the data of which are :—

	Dexter.	*Gallatin.*
Boiler pressure (absolute)	= 81·92	86·4
Temperature of feed	= 114°	115
Terminal pressure	= 16·87	12·14
Cushion pressure	= 8·27	9·93
Diameter of cylinder	= 26 ins.	34·41
Stroke	= 36 „	30
Clearance	= ·0537	·066
Total weight of feed water..	= 178,867	8961
Revolutions, using same	= 125,197	6809

First consider the *Dexter*.

Here the weight of feed water per stroke = ·714 lb., and this would be the weight of steam discharged at each stroke, if there was no loss of any kind: the loss in the case of the *Dexter* was accurately ascertained to be 4·96 per cent., but although the piston appears to have been perfectly tight, it does not follow that all this should be deducted; we, however, assume this, and hence the actual weight of steam per stroke is reduced to ·679 lb.

From the data it follows that the volume of the cylinder, including clearance, was 11·652 cubic feet, of which the fraction

$$n = \frac{c}{1+c} \cdot \frac{P_c}{P_2} = \cdot 0249 \quad \text{(Art. 85)}$$

was occupied by cushion steam, so that the volume of the ·679 lb. of working steam was 11·38 cubic feet nearly, and hence for the volume of 1 lb.

$$V_2 = 16 \cdot 76 \text{ cubic feet.}$$

If now we seek the volume of dry steam corresponding to the terminal pressure of 16·78 lbs., we find from Table III. that the value is 23·25 cubic feet, whence, by division,

$$x_2 = \cdot 72 \text{ (nearly),}$$

a value which has been already used in Chapter III. in finding the total heat of formation, neglecting clearance, by the graphical method. We shall now employ a formula just quoted to obtain the same result.

The mean forward pressure P_m is given by the experiment as 41·19 lbs. on the square inch, and this, referring as it does to the piston displacement (11·06), and not to the terminal volume of the working steam, must be diminished in the proportion 11·06 : 11·38; thus the corrected value is

$$p_m = 40 \cdot 03.$$

The correction in question does not apply to P_2, and therefore in thermal units

$$(P_m - P_2) V_2 = \frac{(40 \cdot 03 - 16 \cdot 87) \times 16 \cdot 76}{5 \cdot 36}$$

$$= 72 \cdot 4 \text{ thermal units.}$$

Now the temperature corresponding to the terminal pressure is found from Table I*a* to be 219° nearly, and the corresponding value of L is 961 : hence

$$h_2 - h_0 = 219 - 114 = 105 \text{ thermal units.}$$

and

$$x_2 L_2 = \cdot 72 \times 961 = 691 \cdot 9 \text{ thermal units.}$$

therefore

$$\begin{aligned} Q &= 105 + 691 \cdot 9 + 72 \cdot 4 \\ &= 869 \cdot 3 \end{aligned}$$

is the total heat of formation.

But the heat expended in the boiler is the total heat of evaporation of water from 114° at the boiler temperature of 313°, that is to say, 1088 thermal units ;

$$\therefore \text{ Exhaust waste } = 218 \cdot 4 \text{ thermal units,}$$

or rather more than 20 per cent. This calculation does not include the loss of water mentioned above, and signifies that the cylinder abstracts from each pound of the steam passing through it 218·4 thermal units, which is afterwards given out by external radiation and re-evaporation during exhaust.

Terminal pressures near to the atmospheric pressure, as in the present instance, are perhaps difficult to measure with accuracy, but as a difference of half a pound does not make a difference of more than 20 thermal units, the calculation is probably very near the truth ; supposing only that the boiler supplied dry steam.

The quality of the steam was not tested, but there is no reason to think there was any important amount of priming water. To see the effect of priming water on the result, let us imagine the amount to be 5 per cent. ; then the whole calculation of the total heat of formation is unaltered, but the total heat of evaporation becomes

$$\begin{aligned} Q^1 &= h_1 - h_0 + x_1 L_1 \\ &= 313 - 114 + \cdot 95 \times 893 \\ &= 190 + 848 = 1047 \text{ thermal units ;} \\ &\therefore \text{ Exhaust waste } = 178, \end{aligned}$$

or about 17 per cent.

The conclusion then seems inevitable that there must have been a very considerable amount of heat wasted in this way.

The cylinder in this instance was clothed, but not steam jacketed. Let us next consider a trial made with the machinery of the U.S.

steamer *Gallatin*, with steam jacket in use, the data of which are given above, in addition to which we require the quantity of water received from the steam jacket, steam chest, &c.

This amounted to 460 lbs., so that the total weight of steam passing through the cylinder is 8501 lbs., and the weight per stroke consequently ·624 lb.; the volume of the cylinder, without clearance, is 15·86 cubic feet, and, including clearance, 16·91 cubic feet. The fraction of this volume occupied by cushion steam was

$$n = \frac{c}{1+c} \cdot \frac{P_c}{P_2} = \cdot 0506,$$

and therefore the volume of the ·624 lb. of working steam was 16·05 cubic feet, so that dividing by ·624,

$$V_2 = 25\cdot 72 \text{ cubic feet.}$$

From Table III. we find the volume of dry steam at the terminal pressure of 12·14 lbs. to be 31·45 cubic feet;

$$\therefore x_2 = \cdot 818.$$

The tabulated mean forward pressure is 31·684, which being diminished as before to reduce it to the volume of the steam, gives

$$p_m = 31\cdot 3;$$

from which is obtained, as before,

$$(P_m - P_2)\,V_2 = 92\cdot 1 \text{ thermal units.}$$

But the temperature corresponding to 12·14 is 202°; hence

$$h_2 - h_0 = 202 - 115 = 87; \qquad x_2 L_2 = 973 \times \cdot 818 = 796;$$

therefore

$$Q = 87 + 796 + 92\cdot 1 = 975.$$

We must now estimate the heat supplied in the boiler; and in doing so we must include the steam supplied to the jacket, that is to say, for each lb. of working steam we must reckon 1·054 lb. of water evaporated in the boiler. The total heat of evaporation, calculated as usual, is 1096;

$$\therefore Q' = 1\cdot 054 \times 1096 = 1155,$$

and hence

$$\text{Exhaust waste} = 180 \text{ thermal units,}$$

being about $15\frac{1}{2}$ per cent. of the whole heat supplied. No allowance is made for loss of water of the kind mentioned in the previous case.

151. As has already been repeatedly explained, the prejudicial action of the sides of the cylinder takes effect partly by re-evaporation during exhaust, and partly by waste during admission and expansion. One method of graphical representation has already been given by means of a thermal diagram, we now use a mechanical diagram for the same purpose.

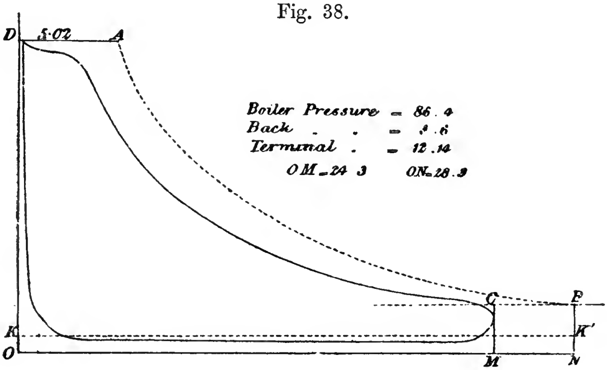
The necessary data are—(1) the total weight of feed water used in a given time, including the liquefaction in the jacket, which need not be separately measured; (2) an average indicator diagram showing the average pressure at each point of the stroke; (3) the average boiler pressure and height of barometer; (4) the amount of priming water in the steam supplied by the boiler.

From (3) and (4) may be found the specific volume of the steam supplied by the boiler, which is to be set off on a diagram on which are laid down as ordinates the boiler pressure and the terminal pressure shown by the indicator. In Fig. 38, DA, BC, are lines of boiler pressure and terminal pressure, and DA represents the specific volume of the boiler steam. Now, by the construction of Chapter VII., or otherwise, draw the adiabatic curve through A, terminating in B on the line of terminal pressure, then if KK^1 be the line of mean back pressure shown by the diagram, the figure $DABK^1K$ will be the indicator diagram of an engine with non-conducting cylinder, in which the back pressure and the terminal pressure are the same as in the actual engine. The losses of work in such an engine are approximately the same as in the actual engine, except so far as due to clearance, wire-drawing, and the action of the sides of the cylinder, and hence this diagram may for many purposes be adopted as a standard of comparison.

The data furnished by (1) and (2) are sufficient to enable us to find, by a process exactly similar to that of Art. 110, the proportion between the volume of steam discharged per stroke, and the volume of the whole weight of steam (including jacket steam) used per stroke, supposed dry at the terminal pressure. This proportion is often called the fraction of the whole consumption "accounted for by the indicator," and in many experiments on steam engines forms part of the tabulated results. Now multiply the specific volume of dry steam at the terminal pressure by the fraction just found, and set off on the diagram $O\,M$ to represent the result: then on $O\,M$ as base construct the actual indicator diagram, which will now compare with the indicator diagram of the ideal engine with non-conducting cylinder, the *total* consumption of steam being the same in the two engines.

Fig. 38 shows the construction for the trial of the *Gallatin*, cited in Art. 150, the jacket being in operation, and the ratio of expansion about $7\frac{1}{4}$. The data already given show that the *total* weight of feed water per stroke was ·66 lb., and the effective volume of the cylinder, as before, 16·05 cubic feet: hence the volume of the cylinder per

Fig. 38.



pound of steam is 24·3 cubic feet. Take then $O\,M$ 24·3 cubic feet, and allowing for clearance in a manner to be described in a subsequent article (Art. 163, p. 357), construct the actual diagram to scale. Next set off $O\,D = 86 \cdot 4$, which was the boiler pressure, and assuming the boiler to supply dry steam, lay off $A\,D = 5 \cdot 02$ cubic feet; trace the adiabatic curve $A\,B$, which terminates in B, such that $O\,N$, the terminal volume, is 28·3 cubic feet, and draw $K\,K^1$ corresponding to the actual back pressure of 3·6 lbs. per square inch. The difference between the real and the ideal diagram represents approximately the loss through the action of the sides combined with clearance and wire-drawing. By continuing the adiabatic curve $A\,B$ to the line of back pressure, and drawing a new adiabatic curve through C, the comparison may be made exact.

When the fraction (f) of steam consumed "accounted for by the indicator" forms part of the tabulated results of an experiment, all that is necessary in applying this method is to construct the adiabatic curve, starting from the boiler pressure on a scale enlarged in the proportion $1 : f$, the diagram having been previously corrected for clearance.

The diagram here reproduced from the first edition of this work should be carefully compared with the thermal diagram given in Fig. 37, page 302, of the last chapter. It will be seen that they exactly correspond line by line, the curve $A B$ of Fig. 38 corresponding to the vertical $S M$ in Fig. 37, $D C$ to $1\ b\ a\ 2$, and $C M$ to $2\ 3$. In the thermal diagram, however, no wire-drawing is included. The missing work of Art. 148 is represented in both diagrams by the space between the real and the ideal expansion curves. This "missing work" is not the mechanical equivalent of the *whole* exhaust waste or "missing heat," but only of that part of it which could ideally have been converted into work.

In the case of the *Gallatin* which we have been considering, the whole exhaust waste is 180 thermal units (page 328), of which a part is unavailable (page 304). The unavailable part may be calculated by a formula which may readily be derived from the thermal diagram, when completed (page 307) by the addition of a subsidiary diagram for the jacket steam. We have not space to give the process in detail, but the amount will be found to be about 108·6 thermal units. Subtracting this from 180, the missing work per pound is 71·4 thermal units. Finally, comparing this with the available heat of a pound of steam (282 thermal units), the percentage of missing work is found to be 25·3. In the table on page 322 this case is given in the column headed F, and the missing work will be seen to be 30 per cent., the difference (4·7 per cent.) being due to the effect of clearance and wire-drawing, which, as before stated, is included in the tabulated value of the missing work.

Cylinder Condensation.

152. From the results given in preceding articles, it appears that, in the most economical steam engines, 80 per cent. of the whole heat expended is accounted for, not more than about 20 per cent. of

the work theoretically possible being "missing." This missing work is partly, no doubt, due to radiation and certain small losses connected with the clearance, which are not completely allowed for, and is also in part only apparent in consequence of priming and imperceptible leakage being neglected altogether or insufficiently allowed for; but the greater part is due to liquefaction in the cylinder, as before explained. Moreover, if most other engines are examined working with the same boiler pressure and vacuum, it is found that the consumption of steam is much greater. Sometimes this is due to the waste work being a greater fraction of the useful work, but on making the calculation as before, it will generally be found that the missing work is largely increased. Engines F and E, the results for which are given in the tables, are examples, the missing work being 30 per cent. in the first, and as much as 45 per cent. in the second.

Cylinder condensation is due to two causes—(1) the action of the metal; (2) to the mixture of steam and water which fills the clearance space before admission, and which must have its temperature raised by liquefaction of some of the fresh steam arriving from the boiler. Both the metal and the clearance contents are real causes which must operate to some extent in all cases, but an attentive consideration of the facts leads us to believe that the metal is the principal agent, and the clearance contents a subsidiary cause, in by far the greater number.

The action of the metal, as sufficiently explained in Chapter X., is exceedingly powerful, so much so as to render it clear that the range of temperature of a large part or the whole of its surface must be much less than that of the steam. Hence the actual magnitude of the effect of the metal depends chiefly on the obstruction offered to the passage of heat from steam to metal, and, conversely, from metal to steam; an obstruction of the same kind (pp. 291, 292) as in a boiler or surface condenser, differing only in degree. This renders the question one of such intricacy that it can only be studied to any purpose by checking our theoretical conclusions step by step, by reference to experience. This we now proceed to do, making use of a number of the various valuable experiments which have been made. It should be understood, however, that precise and certain conclusions cannot be reached solely by comparing together experiments made under differing circumstances on engines of different types; a method which we are obliged to employ, because very few experiments have been made systematically for the sole purpose of discovering the laws of cylinder condensation.

153. The first question to be considered is the influence of size and speed. That small engines are less economical than large, and low speed than high, will be readily admitted; and to ascertain the law according to which cylinder condensation varies with size and speed, is the first step needful in order to interpret the results of experiment. If the temperature-cycle of the metal were the same as that of the steam, or even if it had any fixed relation to it, the question would be easy, for the heat abstracted by a square foot of metal is (p. 276),

$$Q = \sqrt{\frac{B}{N}} \times \text{Range of temperature},$$

where B is a number depending on the nature of the cycle of the metal. Hence, for a given cycle of the steam, the initial condensation would vary as the exposed surface (S) per lb. of steam at admission, and inversely as the square root of the revolutions. Conceive now two engines in every respect similar, but of different dimensions, working at the same initial pressure and ratio of expansion, and running at such speeds that the initial condensation is the same, it may evidently be provisionally supposed that the obstructive influences are the same in the two cases, and, in consequence, that the temperature-cycles of the metal are the same. If the supposition is correct, the ratio $S / \sqrt{N}$ must be the same in the two engines. Since everything else is the same, the surface S varies inversely as the linear dimensions, that is, inversely as the stroke (λ), or the diameter (d). For reasons which will be seen presently, it is best to take the diameter of cylinder as the measure of size, and consequently the product $d \sqrt{N}$ as the measure of size and speed. This product will be described as the Speed-Factor. If our law of corresponding speeds is correct, we ought to find that in two exactly similar engines for which the speed-factor is the same, the initial condensation should be the same. High speed-factors should give a small condensation, and *vice versâ*.

The annexed table gives some examples of single cylinder engines without jackets.

M. One of Mr. Mair's experiments, similar to N, given in a preceding table, but with jacket out of use.

GK. One of the experiments made by Messrs. Gateley and Kletzsch, on an engine of 250 H.P. Being carried out on a systematic plan, they are of considerable importance, and will be referred to frequently as we proceed. They are described in a series of articles in the *Journal of the Franklin Institute*, commencing October 1885.

D. One of the American experiments made on the steamer *Dallas*, referred to on page 318.

EA. One of Colonel English's experiments, already given in the table on page 322 in the column headed E.

HH. An experiment by MM. Hirn and Hallauer, described in M. Hirn's treatise *Thermodynamique*, tome 2, p. 24.

Influence of Size and Speed.

	M	GK	D	EA	HH
Cycle—					
Initial pressure	56	62	47	71	55
Ratio of expansion ..	3·8	3·8	3·9	3·4	3·9
Dimensions—					
Stroke	5·5	3·5	2·5	1·5	5·6
Diameter	2·67	1·5	3	1·33	2
Speed—					
Speed of piston	223	476	285	120	336
Revolutions	20·3	68	57	40	30
Speed-factor	12	12·4	22·6	8·5	11
Initial Condensation—					
Per lb. of steam admitted	·37	·38	·29	·513	·30
,, ,, uncondensed	·59	·61	·42	1·13	·43
Value of $y_1 . d \sqrt{N}$ ..	7·1	7·6	9·5	9·1	4·6

The first two experiments, M and GK, are fairly comparable, the engines being similar. In GK, no trial was made at the ratio of expansion 3·84, given in the table, but from the results given for various expansions at the same initial pressure, the initial condensation can be safely estimated as ·38, being nearly the same as in M. On referring to the table, it will be seen that the value of the speed-factor is about the same in the two engines, while the speed of piston in GK is more than double that of M. Thus the speed-factor in this case is a much better measure of the influence of size and speed than the simple speed of piston, and there is every reason to believe that this is generally the case in single cylinder non-jacketed engines.

154. Next let us compare together M and D. These engines differ widely in type, the stroke of D being less than half that of M, while the diameter is somewhat greater. Hence, for the same value of the surface factor μ, the admission surface per pound of steam, as given by the formula on the top of page 276, in D was about double that in M. The speed of D, it is true, was much greater, the ratio of the values of $\sqrt{N}$, being about 1·66, but even after allowing for this we should find the condensation in D the greater. Instead of this, the table shows that it was considerably less, showing that shortening the stroke has the effect of diminishing μ. This may probably actually be the case, the length of the passages being in most cases diminished; but it is also conceivable that the obstructive influences are much greater in the case of the clearance surface than in that of the barrel surface. In many cases we know that it must be so, much of the clearance surface being coated with a layer of grease impervious to heat, and the passages scoured by a rush of steam, which sweeps off the water deposited by condensation before it can abstract heat by re-evaporation. When this is so the value of the coefficient σ will be small, and can only be determined by experiment.

Whatever the true cause, comparison of experimental results appears to show clearly that the diameter of the cylinder is, for the present purpose, a better measure of the size of an engine than the stroke. We provisionally assume this, leaving the errors of the supposition to be determined by further reference to experience, and it is for this reason that $d\sqrt{N}$ was chosen for the speed-factor in the preceding article. It may here be remarked, that if we wish to take into account the exposed surface of the piston-rod, we have only to take for d, the difference between the diameter of the cylinder, and the radius (or where there is a tail-rod, the diameter) of the rod, instead of the simple diameter of the cylinder.

The smaller condensation in D is now explained as being a consequence of its much higher speed-factor, which is 22·6, instead of 12. On the other hand, the greater condensation in EA is seen to be due to its low speed-factor, which is only 8·5. In HH, we have an example in which a somewhat lower speed-factor (11) is accompanied by a smaller condensation. The causes of this will be considered further on. The example is given here for the sake of illustrating the necessity for caution in drawing conclusions from isolated examples.

155. In order to connect the initial condensation with the action of a square foot of metal, we have only to observe that

$$Q = (1 - x_1)\,\frac{L_1}{S},$$

where $1 - x_1$ is the initial condensation, reckoned as usual as a fraction of the *total* amount of steam admitted. Now S is proportional to V_1, that is to $x_1\,v_1$, where as usual v_1 is the specific volume of dry steam at the admission pressure. Hence, by substitution for S (pp. 276, 333),

$$y_1 = \frac{1 - x_1}{x_1} = \frac{4\,v_1}{d\,.\,L_1}\,.\,(1 + \sigma\,r)\,\sqrt{\frac{B}{N}} \times \text{Range}.$$

This fraction y_1 is the initial condensation expressed as a fraction of the steam remaining uncondensed at the end of the admission, or, as we may express it, of water to steam: it will be described as the Condensation Ratio. The action of the metal is proportional to y_1, not to $1 - x_1$.

In the preceding article we compared two engines, M and G K, in which the condensation is the same, and supposed the obstructive influences the same. This amounts to supposing that the condensation is some function of the speed-factor. We now go a step further, and suppose the obstructive influences such that for a given initial pressure and expansion there is at all speeds the same fixed relation between the cycles of the steam and the metal. If this be so, we ought to find that $y_1 \, d \sqrt{N}$ has a constant value in the examples just considered. The values of this quantity are given in the table, and it will be seen that (except H H) there is a general agreement. The differences might be accounted for by small changes in the co-efficient σ, or in another way to be mentioned subsequently.

Again, if the same engine be tried at different speeds with the same initial pressure and the same expansion, we ought to find that $y_1 \sqrt{N}$ remains unaltered. Very few speed trials have been made, but in such as exist, the results for single cylinder non-jacketed engines are believed to be in accordance with this law. Especially may be mentioned the single cylinder speed trials made by Mr. Willans, and described in a paper already referred to. The results of these trials show a somewhat greater condensation at low speeds than would be given by the law in question, which may be due to the cause pointed out in Chapter X. (p. 297), or which may also be partially due to the effect of an imperceptible leakage, a source of error extremely difficult wholly to eliminate.

156. Hitherto we have supposed the cycle of the steam to be the same in the cases compared, let us now suppose it to vary. This may occur in three ways, by changing—(1) the back pressure; (2) the initial pressure; (3) the ratio of expansion.

(1) Taking first the back pressure, consider Colonel English's experiments already referred to. These experiments were made in pairs, the initial pressure and expansion being the same in each pair, but one condensing, the other non-condensing. Out of a total condensation exceeding 50 per cent., the diminution in the non-condensing experiments was only from 3 to 5 per cent., or less than one-tenth; yet the total range of temperature of the steam in the condensing experiments was about 170°, whereas in the non-condensing it was only about 100°. Thus the temperature of the steam at exhaust has in itself little effect on the condensation in a cylinder, and this conclusion is borne out by experiments on non-condensing engines, and the high-pressure cylinders of compound engines. If the initial pressure and expansion be the same, the condensation is little different. These facts show that one principal difference

between the cycle of the steam and that of the metal must be that the temperature of the metallic surface does not sink very far below the temperature at the end of the expansion. Moreover, it appears that the escape of heat (p. 294) from a dry surface does not greatly influence the amount of condensation. It must be remembered, however, that the ratio of expansion in Colonel English's experiments was not less than $3\frac{1}{2}$. When the admission is prolonged to half stroke, or beyond, the result may possibly be different.

(2) In Colonel English's experiments, the initial pressure was about the same in the cases compared; but now, suppose it to increase, the ratio of expansion remaining the same. The exposed surface per lb. of steam now diminishes, because the weight of steam in the cylinder is increased in nearly the same proportion as the pressure, and we might suppose that the percentage of steam condensed would be correspondingly reduced. On reference to experience, however, we find that there is little change. In some of the experiments by Messrs. Gateley and Kletzsch, already cited, the initial pressure varied from 27 to 80 lbs. per square inch, while the ratio of expansion remained about the same; the condensation was but little diminished, although the pressure increased threefold. At first sight we might suppose that increased range of temperature was the cause of this; but, for reasons already stated, we must reject this explanation. The true explanation must be that the re-evaporation during exhaust increases in intensity nearly in proportion to the density, and alters the temperature-cycle of the metal. The range of temperature of the metal is increased, and it remains longer at the lowest temperature. Thus the value of B increases with the pressure, and only at some limiting pressure, to be considered hereafter, reaches the value corresponding to the cycle of the steam. Perhaps the coherence of the film obstructs re-evaporation more at low temperatures than at high.

(3) Next, let us suppose that the initial pressure and exhaust temperature remain the same, while the ratio of expansion is increased. A large number of experiments of this kind have been made, and they all show that the initial condensation increases rapidly according to some regular law. The circumstances here are exceedingly complicated, for we have to consider—(1) the increase of the clearance surface per lb. of steam nearly in proportion to the ratio of expansion; (2) the diminution of the intensity of re-evapo-

ration due to the lowering of the terminal pressure; (3) the drainage of heat from the admission part of the barrel surface of the cylinder to the central part of that surface, consequent on its lower temperature; (4) the increased range of temperature of the metal which must follow an increase in the range of temperature during expansion. On examination, however, of the experimental results, we find that the action of the metal in this case is, for a limited but sufficient range of expansion, proportional to the fall of temperature of the steam during expansion, showing that the other causes mentioned approximately neutralise each other. Since the fall of temperature during expansion is approximately proportional to the logarithm of the ratio of expansion, we are thus led to the semi-empirical formula

$$y_1 = C \cdot \frac{\log_\epsilon r}{d\sqrt{N}}$$

for the condensation-ratio, r being, as usual, the ratio of expansion, and C a constant, which will be called the Condensation Constant, to be determined by experiment for each type of engine. The formula is limited to values of r, not generally less than 2, and not generally greater (sometimes less) than about 8. Outside these limits the condensation is greater, as will be seen presently. Within these limits the formula gives fairly good results when C has been determined for a given type of engine working at a given initial pressure.

The annexed table gives values of C for a number of engines of various types. In similar engines they are believed to be somewhat smaller at high pressures than at low, and in non-condensing than in condensing. In different types one of the most important features appears to be the ratio of stroke to diameter, the value of C being usually smaller the longer the stroke, as might be expected from what has already been said on p. 335 (see also page 352). The true action of the clearance surface is still uncertain. Where it is to a great extent coated with grease, it is easy to understand that its effect will be small; but this explanation will not apply to all cases, and no difference has as yet been noticed dependent on its condition. Perhaps, when clean, the surface is scoured by the incoming steam, so that the film is swept towards the periphery of the cylinder as fast as it is formed. Wherever this is the case, the metallic surface will have little action, for, as before shown (p. 294), re-evaporation upon the surface is the only way in which large quantities of heat can be abstracted from the metal. Water thus driven to the periphery will, in general, soon evaporate, unless the re-evaporation be hindered by molecular cohesion.

Some experiments were made by Colonel English * on a portable engine with cylinder 10 inches diameter, in which the piston was

CYLINDER CONDENSATION CONSTANTS.

Authority for Experimental Data.	Reference Letter.	Absolute Initial Pressure.	Ratio of Stroke to Diameter.	Value of C for Non-jacketed Cylinders.	
				Con-densing.	Non-condensing.
Hirn & Hallauer	H H	55	2·8	3·4	
J. W. Hill ..	H	90–100	2·67	3·7	3·4
H. W. Spangler	S	95	2	..	4
Gateley & Kletzsch	G K	80	..	4·4	
		40–70	2·33	5·8	
		27	..	6	
Mair	M	56	2·06	5·3	
Emery & Loring	D	47	·833	7	
	G	55	·88	7·5	5
Isherwood ..	M I	35	2·67	7·5	
English	E A	70	1·12	7·5	6
	E B	50–75	1·4	4·7	
Willans 400 rev.	W	50–100	·43	..	6·5
Willans 200 ,,					7
Willans 100 ,,					8

Remarks.—These values of C are for diameters in feet and revolutions per minute, the logarithm being hyperbolic.

* *Proceedings of the Institution of Mechanical Engineers*, September 1887, p. 503.

blocked at the end of the stroke, and the steam admitted to and exhausted from the clearance space by the slide valve. The valve was driven from the crank shaft in the usual way, but by an independent engine. A certain weight of steam was thus admitted to the clearance space at every revolution of the shaft, which could be measured by connecting the exhaust port with a surface condenser and collecting the resulting water. The difference between this and the weight of dry steam necessary to fill the clearance space furnishes a measure of the steam condensed. Nearly the same results were obtained as in an engine working in the usual way, from which the conclusion was drawn that the clearance surface is the leading factor in the production of cylinder condensation. The conditions of the exhaust, however, were so different in these experiments from those in an actual cylinder, and the probability of water adhering to the metal in the angles of the clearance space so great, that the conclusion does not seem necessary.

The data relating to the experiments by Messrs. J. W. Hill and H. W. Spangler have been taken from a table by Professor Marks.* Mr. Hill's experiments were made with great care on three engines of the Corliss type at various expansions. Nearly the same value of C is obtained from all. Of the two engines tried by Colonel English, E A has been already referred to (p. 334); E B is a double cylinder engine, with cylinders horizontal and attached to a frame beneath the boiler. A large number of experiments were made at ratios of expansion ranging from 5 to 8·5. Excluding those made with jacket in use, and some in which superheating was evident, they nearly all give very consistent results, the value of C being 4·7; this somewhat low value being probably due to the steam supply being enveloped in the smoke-box, so that the steam was thoroughly dried.† In two sets, however, the value of C is found to be 6. Such isolated discrepancies in a series of experiments otherwise consistent frequently occur, and are possibly due to a small amount of water contained in the angles of the clearance space. The large value of C (7·5) obtained in E A may be due to the same cause, or possibly to priming from the boiler. A small amount of priming or leakage makes a great difference in the value of the constant.

* *Limitation of the Expansion of Steam.* Philadelphia, 1887.

† *Proceedings of the Institution of Mechanical Engineers,* October 1889.

The very small value (3·4) of the constant for H H requires special notice. Notwithstanding the length of stroke, it might have been expected to be 20 per cent. greater, when account is taken of the low pressure of the steam. This engine, one of the trials of which has already been considered (p. 334), has four independent flat valves moved by cams with an extremely small clearance,* apparently only ·01, an arrangement which diminishes condensation, since the value of the surface factor μ little exceeds unity. But part of the difference is very possibly due to the more complete elimination of losses by leakage. These trials are among those especially relied on by Hirn in his controversy with Zeuner, no pains being spared to discover losses of this kind. In all the other results, similar small losses probably existed in different degrees, and it must be understood that the effect of the clearance contents is included in all cases, so that they cannot be considered as exactly measuring the action of the metal.

As showing the effect of difference of type in cases where the ratio of stroke to diameter is the same, M I is instructive as compared with H. These trials were made in 1861 by Mr. Isherwood, on the engines of the paddle steamer *Michigan.* The results are fairly consistent, giving at ratios of expansion less than 7 a value of C of about 7·5. After making full allowance for the lower pressure, this is much greater than that (3·7) found for H. The difference may be attributed to a deposit of water, the steam pipes being so inclined as to drain into the cylinder.† Yet it should be remarked that the clearance surface must have been much greater in M I, and, as already stated, this is a point of great importance. Whatever be the real cause of the larger constant required, it appears not to affect the form of the formula, though, as might be expected, the results are less regular when the condensation is large.

In the table, values of C are added for the *Dallas* (D) and the *Gallatin* (G), two of the engines tried by Messrs. Emery and Loring (page 318), but the pressures at cut-off not being given, these values are only rough estimates. Mr. Willan's engine has already been referred to (page 323).

* *Réfutations d'une Critique de M. G. Zeuner,* par G. A. Hirn et O. Hallauer. Paris, 1881, p. 17. See also *Peabody,* p. 302.

† *Peabody,* p. 244.

157. Writing for brevity $z = C / d \sqrt{N}$, and expressing y_1 in terms of x_1, the dryness fraction at the point of cut-off, we find

$$x_1 = \frac{1}{1 + y_1} = \frac{1}{1 + z . \log_\epsilon r} .$$

Supposing the expansion hyperbolic, an assumption sufficiently approximate for our present purpose, and referring to page 47, Chapter III., we find that the effective work of a pound of steam is given by a formula which by substitution of P_1, v_1, x_1, for P_2, x_2, v_2, and using the value of x_1 just found, becomes

$$\text{Effective Work} = P_1 v_1 . \frac{1 + \log_\epsilon r - \frac{r p_b}{p_1}}{1 + z . \log_\epsilon r} .$$

If z approaches unity, this formula gives nearly the same result whatever the ratio of expansion. Cases are very common in which the consumption of steam is about the same whatever the expansion, and we now see that this arises from the speed being too low for an engine of that size and type. To gain anything by expansion the revolutions must be considerably greater than is given by the formula

$$N = \frac{C^2}{d^2}$$

where C is the constant proper to an engine of that type. Again, if z be given, we can find the best ratio of expansion. For, applying the usual rule for a maximum, when r varies, we find the greatest value of the effective work to be when

$$\log_\epsilon r = \frac{1 - z}{z} \left(\frac{p_1}{r p_b} - 1 \right)$$

an equation which can be solved by trial when the ratio p_1 / p_b of the initial and back pressures is known. Let us suppose that the engine makes four times the number of revolutious given above, so that $z = \cdot 5$, then putting

$$r = 2, \quad 3, \quad 4, \quad 5, \quad 6,$$

we find

$$\frac{p_1}{p_b} = 3 \cdot 6, \quad 6 \cdot 3, \quad 9 \cdot 5, \quad 13, \quad 17.$$

The importance of cylinder condensation was pointed out long ago by Mr. D. K. Clark, and subsequently, in 1861, Mr. Isherwood determined the best ratio of expansion in the low-pressure marine engines of that day to be about 2. Later, in the engines tried by Messrs. Emery and Loring (p. 318), the best ratio of expansion was found to be given by the empirical formula,

$$r = 1 + \frac{p_1}{22},$$

which gives about the same result as the theoretical formula, when the back pressure in the latter is taken at from 6 to 7 lbs., while z, as above, is ·5. In applying the theoretical formula, a fictitious value must always be assumed for the back pressure, in order to allow for clearance, the loss by which increases with the expansion. On referring to page 251, it will be seen that when there is no compression, the estimated back pressure should be increased by the amount,

$$\text{Additional Pressure} = \frac{c}{1 + c} \cdot p_1,$$

and there will be a similar increase of smaller amount in other cases. Thus the best ratio of expansion depends on clearance and compression, as well as on the speed as measured by the fraction z. With a small clearance, high speed, and low back pressure, the best ratio of expansion may be considerably greater than is given by Emery's formula. In small engines running at moderate speed it may be much less.

158. The simplicity of the condensation formula we have been using is due to the neglect of a number of disturbing causes which to a great extent compensate each other. Hence it is only approximate within certain limits, which we now proceed to consider.

(1) If r be less than about 2, the range of temperature of the metal has no longer a definite relation to the fall of temperature during expansion. It becomes relatively greater as the expansion diminishes, and has a definite value when the admission is prolonged throughout the stroke. The cause of this is to be looked for in the effect of the escape of heat from a dry surface, described in the last chapter, which indirectly, by cooling the metal, produces an action of

the metallic surface where otherwise there would be none. Three speed trials were made by Messrs. Gateley and Kletzsch, at revolutions 34, 50, 63, the initial pressure being 28, and the admission prolonged through very nearly the whole stroke. Their results are expressed pretty closely by the formula

$$y_1 = \frac{4 \cdot 2}{d \sqrt{N}},$$

being about the same as would have been obtained for $r = 2$. So far as they go they indicate, what is not in itself improbable, that the coefficient K, in the formula on page 296, varies in such a way that the law of the inverse square root of the speed is satisfied.

(2) If r is greater than a certain limit, not sharply defined, but probably about 7, or in some cases 8 or more, the condensation usually becomes much greater than is given by the formula, a result which may be attributed to the effect of the clearance contents, and probably also to a cause considered further on.

(3) In non-condensing engines and the high-pressure cylinders of compound and triple expansion engines, the total range of temperature may be too limited to permit of so great a condensation as is indicated by the formula, and we must then resort to the general value of y given on page 336, in which we now suppose that the range is the *total* range of temperature within the cylinder, and assume a maximum value of B (say 6, see p. 290). Replacing L_1 / v_1 by $(k + 1) p_1 / 5 \cdot 36$, where k is a number given by the tables or by the formula on page 175; we obtain

$$y_1 = \frac{52 \cdot 5}{(k + 1) p_1} \cdot \frac{t_1 - t_3}{d \sqrt{N}} (1 + \sigma r ,$$

where $t_1 - t_3$ is the *total* range. If this formula gives a smaller result than the original, it must be employed instead. In order that it may do so, the pressure must exceed the value given by the equation,

$$(k + 1) p_1 = \frac{52 \cdot 5}{C} \cdot \frac{t_1 - t_3}{\log_e r} (1 + \sigma r),$$

in using which it is convenient to notice that $(k + 1) p_1$ is given in the last column of Table V., being the same as p_h. Let us, for example, suppose $r = 2 \cdot 72$, corresponding to a cut-off of about one-third the stroke, and take $C = 5 \cdot 25$, then supposing σ zero we have

$$p_1 = 60, \quad 70, \quad 90, \quad 110, \quad 140,$$
$$t_1 - t_3 = 70°, \quad 80°, \quad 99°, \quad 119°, \quad 147°.$$

If the initial pressure be greater than the value given for the range of temperature in the cylinder stated below, the condensation will be limited to the amount given by the formula of the present article, as far as the condensation is due to the metallic surface. It is certain that some limitation of this kind must exist, though the limiting pressure must generally be much greater than that given by this calculation.

(4) In all cases where the steam is exhausted from a cylinder at high pressure we have, however, a limitation of a somewhat different character, arising from the fact that the drop of temperature at release is small unless the drop of pressure be considerable. The condensation formula under discussion rests mainly on the supposition that the surface at release becomes suddenly dry : the temperature of the metal rising after its sudden fall, at first rapidly and then gradually, above the temperature of release. But, as already pointed out on page 300, re-evaporation must continue through a considerable fraction of the return stroke when there is no drop of temperature at release, and that fraction will depend on the quantity of water to be re-evaporated. Hence the cycle of the metal depends on the speed, and the law connecting condensation with speed becomes different.

In the remarkable series of experiments referred to on page 323 the late Mr. Willans found that in his compound trials the condensation was proportional to the speed simply, and not to the square root of the speed. The truth of this was questioned at the time, but without sufficient reason, and a second series of experiments subsequently showed that the result was correct, and that under certain conditions there might be a continuous passage from one law to the other. The connection of the condensation with the pressure must also be different.

The abnormal increase of the condensation when the expansion exceeds a certain limit may also possibly be partly explained by the diminution of the range of temperature between release and exhaust.

159. Let us now turn our attention to the reservoir type of compound engine, already considered briefly in Chapter III., p. 63. It was there pointed out that the working of the engine depends greatly on the relative liquefaction in the two cylinders, and that one reason for its economy may be that the liquefaction in the high-pressure cylinder is less than in a simple engine working with the same total expansion (see p. 67). To enable us to form some idea of the conditions of economical working, let us suppose that the coefficient in the formula for condensation we have been using is the same in the two cases, and let us first ascertain under what conditions the liquefaction in the high-pressure cylinder of the compound will be a given fraction (f) of that in the simple engine.

With the same notation as in the articles cited, and further supposing the diameter of the large cylinder to be D, and that of the small cylinder d, we have, by application of our condensation formula,

$$f \cdot \frac{\log_\epsilon (n r)}{D} = \frac{\log_\epsilon r}{d},$$

or since $D = \sqrt{n} \cdot d$

$$\log n = \left(\frac{\sqrt{n}}{f} - 1\right) \log r,$$

where the logarithms may now, if we please, be common instead of hyperbolic. This equation determines the ratio of expansion in the cylinder for a given ratio of cylinders, in order that the compound may have a given advantage over the simple, or if both ratios are given, enables us to find the advantage obtained.

(1) Let $n = r$, then

$$\sqrt{n} = 2f \text{ and } n = 4f^2;$$

thus no advantage will be gained by compounding if n is equal to 4, and if f is not to exceed ·75, n must be less than $2\frac{1}{4}$, and the total expansion must not therefore be greater than about 5.

(2) Again, let $n = 2$, then assuming

$$r = 2, \quad 2{\cdot}5, \quad 3, \quad 4,$$

we find

$$f = {\cdot}70, \quad {\cdot}80, \quad {\cdot}86, \quad {\cdot}94.$$

These examples show that the total expansion which can be employed in a compound engine, without a large amount of condensation, is limited, as in a simple engine, and in particular that the expansion in the high-pressure cylinder must not exceed a certain limiting value, which is smaller the greater the ratio of cylinders, results on the whole borne out by experience. The formula here employed is, however, only applicable to single-cylinder engines for reasons discussed at the close of the preceding article. The actual condensation in high-pressure cylinders varies greatly. In some, perhaps all, cases where the action of the metal alone is concerned, the condensation is much less than would be given by the formula with the same value of the coefficient as would be used for a single cylinder. On the other hand, cases are very common in which the condensation is large in consequence, probably, of there being a strong tendency to the accumulation of water (page 301).

As regards the low-pressure cylinder, it must be remembered that, unless there be a reheater, the steam with which it is supplied will contain suspended moisture. For, as explained in earlier chapters, the central mass of steam not in immediate contact with the sides of a cylinder probably expands adiabatically, or nearly so (compare p. 352). This forms part of the water in the low-pressure cylinder, and like priming water supplied continuously (in moderate amount) from the boiler, is not an indication of waste, nor to any sensible extent a cause of waste. In the low-pressure cylinders of triple expansion marine engines of large size, the speed factor is so large that almost the whole of the water they contain must be due to this cause, except so far as may be due to the clearance contents. Thus, in the low-pressure cylinder of the *Meteor*, the steam not accounted for by the indicator was 24·7 per cent., but of this a part was condensed in the jackets, the drain from which was not measured, and 14 per cent. of the steam contained in the high-pressure cylinder at cut-off would have been condensed by adiabatic expansion from the initial pressure of 140 lbs. to the terminal pressure of about 11 lbs., at which the measurement was taken.

160. According to the view of the causes of cylinder-condensation, which has been presented in preceding articles, it is only necessary to increase the speed sufficiently to reduce the corresponding loss to any extent. Thus, the initial condensation will not exceed 20 per cent., if y_1 is not greater than ·25, that is, if the speed factor $d\sqrt{N}$ is not less than $4\,C \log_\epsilon r$. Taking, for example,

$r = 4 \cdot 5$, $C = 5$, the condition will be satisfied if the revolutions be greater than

$$N = \frac{900}{d^2},$$

the diameter d being, as before, in feet.

But the speed is often limited by external conditions, and excessive speed is not conducive to economy, so that it is very desirable, if possible, to reduce the loss by condensation in some other way. The effect of compounding has just been noticed, but a considerable reduction may also be effected by the addition of a steam-jacket, or by the employment of superheated steam.

It has been shown in Chapter X. that the intensity of the action of the metal is regulated by the equality which is necessary between (1) the weight of steam condensed and re-evaporated upon the surface, and (2) the heat abstracted and given out by the surface. The effect of a steam-jacket, as explained in Art. 135, and graphically exhibited in Fig. 36, page 288, is to supply a continuous current of heat flowing through the metal from without, which may be regarded as superposed on the alternate in-flow and out-flow (Q), already existing. During the in-flow the quantity of heat J_1 passes through the cylinder in the opposite direction, reducing the actual in-flow to $Q - J_1$, and during the out-flow a quantity J_2 passes, increasing the actual out-flow to $Q + J_2$, the jacket supplying the total amount $J_1 + J_2$ during the revolution. The quantities J_1 J_2 are not equal, as in the article cited, but are in the ratio of the times of in-flow and out-flow, so that $J_1 < J_2$. If the condition of the cylinder remained unaltered, the weight of steam condensed at admission would be less than the weight of water re-evaporated in the preceding revolution, and the balance can only be restored by a change in the cycle, which reduces the action of the metal, the change consisting in a rise of mean temperature, and a diminution of mean range. A small quantity of heat supplied by a jacket thus reduces the action of the metal in a much greater proportion. For the purposes of illustration, a calculation might be made for the case of a simple plate, proceeding as in Art. 140, p. 298. But an analogous action has already been described in Art. 139, p. 297, and as the circumstances are much more complex in an actual cylinder, we shall at once pass on to consider experimental results, referring especially to experiments made by Mr. Mair. The trials now to be noticed were made on jacketed single-cylinder engines of about the same size, type and speed, at various rates of expansion, ranging from 1·95 to 6·85. In each case the initial condensation $(1 - x_1)$ was determined and plotted on a diagram, the horizontal ordinate in which represented the ratio of expansion.* The result is very approximately a straight line, the algebraical equation of which will be found to be

$$1 - x_1 = \cdot 059\, r + \cdot 03.$$

One of the engines (M) was tried with jacket out of use, and the condensation-constant for it is given in the table (p. 340) as 5·3. We have thus the means of comparing the action of the metal as measured by the fraction y at various expansions with and without jacket. The results of this calculation are for

$r =$	3,	4,	5,	6,	
$y =$	·26,	·36,	·46,	·62	(jacketed),
$=$	·48,	·61,	·71,	·79	(not jacketed),
Ratio $=$	·54,	·59,	·67,	·79.	

It will be seen that when $r = 3$, the action of the metal is reduced by the jacket to 54 per cent. of its original amount, to effect which, without a jacket, an increase of speed from 20 to 70 revolutions, would have been necessary. At higher ratios of expansion, the effect of the jacket is less, corresponding for $r = 6$, to an increase of the speed to 32 revolutions only. The condensation in the jacket was only 4 to 5 per cent. of the whole consumption, and there was, therefore, at the best expansion, a considerable saving by its use. The increase in the expenditure of heat with jacket out of use, is estimated by Mr. Mair as 20 per cent. at the ratio of expansion employed in trial M (about 4).

Taking trial N, already discussed (p. 322), and referring to page 340 of Mr. Mair's paper already cited, we find the condensation per stroke in the jacket to be ·0528, corresponding to a supply of heat of 48 thermal units. At twenty revolutions, this gives 1920 thermal units per minute. Of course the whole of this does not flow inwards, but if we make this assumption and further suppose

* *Proceedings of the Institution of Civil Engineers,* vol. lxxix. p. 331.

the surface through which it flows to be the internal surface of the cylinder (42·4 square feet), we obtain a flow of 45 thermal units per square foot per minute, corresponding to a thermal gradient of only (p. 279) 6° to the inch. If the reduction of the action of the surface was the same throughout the mean temperature of the cylinder must have been below the arithmetic mean (274°), of the initial and terminal temperatures, while the temperature of the jacket was 292°, showing that the temperature of the external surface of the cylinder was more—possibly much more—than 12° below the temperature of the jacket: if these temperatures had been equal, the condensation in the jacket would have been three times as great.

According to this view of the facts the difference is simply due to the fact that the rate of condensation on a surface depends on the rapidity with which it can be cleared of the water deposited by condensation. Jackets are sometimes supplied with steam passing through on its way to the cylinder. This method is objected to by engineers, not without reason, because some of the condensed steam will be carried into the cylinder, but it has a countervailing advantage in increasing the condensation in the jacket. Jackets are sometimes supplied with steam from a separate boiler at a higher pressure, but little is generally gained by this, the condensation being limited as just described; if a strong current of steam were made to pass through the jacket, the result would probably be different. For the same reason, perhaps, the jacket condensation per stroke, in Mr. Mair's experiments, was nearly the same at all expansions, and therefore, using the same notation, and remembering that Q varies as y,

$$\frac{J_1 + J_2}{Q} = \frac{q \cdot d}{e \cdot \log_\epsilon r \sqrt{N}},$$

e being the thickness of the cylinder, and q a coefficient depending chiefly on the activity of the circulation in the jacket. It may be conjectured that this fraction is a measure of the percentage reduction in the action of the metal occasioned by the jacket, but with active circulation q should increase as the expansion increases, because the mean temperature of the cylinder is lowered. It may also depend on the ratio J_1 / J_2, which again depends on the expansion.

The numbers however just found for the reduction of the action of the metal can of course only represent the average reduction through the whole extent of the admission surface ; and the actual reduction must be different at different points. Some authorities, notably Rankine, have supposed that when thoroughly efficient the effect of jacketing is to raise the temperature of the whole mass of metal nearly to the temperature of admission, thus reducing its action almost to zero. The numbers in question would then mean the fraction of the whole surface which was sensibly unaffected by the jacket. In the same way the difference of temperature calculated above can only be an average difference, and if so, the facts may also, and perhaps more probably, be explained by supposing the jacketing of part of the surface inoperative.

All that has here been said applies to the admission surface alone, but the action of the expansion surface is also reduced, so that the saving by evaporation is increased (p. 353). This effect is greater the greater the expansion.

It must not be forgotten that a jacket will only be economical when the gain by a reduction in the action of the metal is greater than the loss by steam condensed in the jacket. According to many authorities, among whom may be especially mentioned M. Le Dieu,* a jacket may supply too much heat. As already remarked (p. 298), it is possible to avoid condensation altogether by maintaining the external surface of the cylinder at a sufficiently high temperature. This was pointed out by Mr. D. K. Clark, as a result of the very earliest experiments on cylinder condensation, and the question has also been studied by Mr. Bryan Donkin (p. 298,) and others. When the internal surface is dry the escaping heat may penetrate the whole mass of steam and even superheat it.

Another method of reducing the action of the metal is by the employment of superheated steam. Steam when superheated gives out more heat as it condenses than saturated steam, so that less water has to be re-evaporated for the same amount of heat supplied by the metal. The balance between condensation and re-evaporation is thus altered in the same way as by the use of a jacket. Cylinder condensation may be greatly reduced in this way, but the amount of superheating must be considerable; and, from the difficulty of

* *Étude de Thermodynamique Expérimentale sur les Machines à Vapeur.* Paris, 1881. This able memoir is recommended to the notice of those interested in the subject.

regulating the temperature, the method has until recently been little used.

The drainage of heat from the ends of the cylinder liner to the centre, in consequence of its lower temperature, has already been referred to; the action here is analogous to that of a steam jacket, but in the opposite direction, increasing the initial condensation. There can be no doubt that this is one reason why condensation-constants are smaller the longer the stroke, since the thermal gradient is diminished in proportion to the distance of the ends from the centre.

Consumption of Steam. Re-evaporation.

161. Before cut-off, steam condensed is replaced by fresh steam admitted, and the total weight consumed per stroke is obtained by multiplying the weight of dry steam by (p. 343) a factor $1 + y_1$, which in single-cylinder non-jacketed engines is given by the equation

$$1 + y_1 = 1 + \frac{C \log_\epsilon r}{d \sqrt{N}}$$

and which may be described as the Condensation Factor.

After cut-off, more steam is condensed on the fresh surface exposed, but this is replaced by re-evaporation from the admission surface or by steam already in the cylinder. In such a case as that represented in Fig. 37, p. 302, the condensation is at first greater than the re-evaporation, which only predominates at a later point in the stroke. Thus a loss is caused by the direct action of the expansion surface, but it takes effect by diminishing the saving due to re-evaporation described on page 307, and thus diminishing the work done per stroke, not by increasing the consumption of steam. In compound engines the consumption of steam per stroke depends on the condensation which occurs in the H.P. cylinder before cut-off; after cut-off in this cylinder the consumption is fixed and cannot be affected by the condensation in any cylinders through which the steam passes on its way to the condenser. The pressure in these cylinders is, however, lowered throughout, and the mean effective pressure correspondingly reduced, since the condensation increases the density of the steam without altering the quantity used.

The relation between the actual mean effective pressure and that which would exist on a non-condensing cylinder is an important question which we shall not enter on here, but it may be remarked that the principal cause of the economy due to compounding at high rates of expansion is generally not so much reduced initial condensation, though that occurs in some cases, as increased saving by re-evaporation at an earlier stage of the total expansion. In a triple expansion engine, for instance, the work done in the intermediate cylinder is greater than that done in a single cylinder during the corresponding stage of the whole expansion. The best ratio of expansion is that which gives the lowest consumption per horse-power, and, for this reason, is always much greater in a compound than in a single engine. When a comparison is made between the two at best ratio of expansion, the result is by no means always in favour of the compound, and the advantage of the compound is in most cases small.

Clearance and Compression.

162. Hitherto the term initial condensation has been used without any attempt to explain precisely the way in which the quantity signified is connected with the results obtained by experiments on cylinder condensation. It will now be necessary to consider this point, together with some others connected with clearance and compression. It is convenient to adopt a method differing in some respects from that employed in Chapter IX.

Supposing the piston at any point of its stroke, the volume of steam in front of it can be calculated exactly by adding to the volume swept through by the piston the volume of the clearance space. At this point the pressure will be known from the indicator diagram, and the corresponding specific volume of dry steam at that pressure can be found. Let us now write

$$A = \frac{\text{Total Volume of Steam}}{\text{Specific Volume of Dry Steam}},$$

then A is what the weight of steam would be if it were dry. This may be described as the Indicated Weight of the contents of the cylinder. The total weight, however, will be the sum of this and the weight (W) of water, which is either suspended through the whole mass, or spread over the surface as a film, or deposited in small portions in the angles of the clearance surface. Strictly speaking, the volume ·016 W of this water ought to be subtracted in calculating A, but this correction may generally be neglected. Thus we have

$$\text{Cylinder Contents} = A + W.$$

If two positions of the piston be now considered after cut-off, the weight of the cylinder contents must be the same in both, unless the piston or valves leak, and, denoting them by the suffixes 1, 2,

$$W_2 - W_1 = A_1 - A_2;$$

that is, the weight of steam condensed, or water re-evaporated during any stage of the expansion, can always be found, being the difference of the indicated weights at the beginning and end of the stage.

Next, consider any position of the piston during the return stroke after the exhaust has closed: the steam is now being compressed, and we can find in the same way the indicated weight (a) of the clearance contents, and write

$$\text{Clearance Contents} = a + w,$$

where w is the weight of water contained in the clearance in one or more of the three forms mentioned above. Considering any stage of the compression, we have, as before, until the steam port commences to open,

$$w_2 - w_1 = a_1 - a_2.$$

Take now the difference between the cylinder contents at any point of the expansion and the clearance contents at some given point of the compression, which may be described as the Point of Reference, and chosen at pleasure. This difference must be the weight of steam which has entered the cylinder before cut-off, and is evidently the weight of feed-water used per stroke after subtracting the jacket supply. Calling this F,

$$F = A - a + W - w,$$

in which equation F is known by experiment, $A - a$ is the part of F "accounted for by the indicator," or, as we may otherwise describe it, the Indicated Weight of feed-water per stroke, while

$$1 - x = \frac{W - w}{F}$$

is the fraction of the feed-water, which is not accounted for by the indicator at that point of the expansion. This fraction then gives, not the whole quantity of water in the cylinder, but the difference between it and the weight of water in the cylinder when the piston passes the point of reference. It is, in fact, the quantity of water which has entered the cylinder as priming water, or has been condensed in the cylinder since the piston passed the point in question.

If the weight of water (w) remained the same during compression, the position of the point of reference would be immaterial. In some experiments * by Prof. M. F. Fitzgerald on compression curves, drawn by a special apparatus suitable for exhibiting them on a large scale, the weight w was found to be constant. Generally, however, we cannot assume this, compression curves often showing re-evaporation to go on as the compression proceeds, a result which would be expected, unless the clearance contained water, in which case the reverse might be true. Any reasoning which depends on the compression curve being exactly given must be uncertain, the chances of error by leakage being so great. In the method adopted in Chapter IX., and illustrated by examples on p. 326-328, the point of reference, unless the compression is large, is fictitious, being taken on an ideal prolongation of the compression curve into the clearance, and the error in this must be very small, but any point of reference may be chosen according to convenience. The chances of an error by leakage are evidently increased when the ports are nearly open, but, on the other hand, it is sometimes advisable to take the point low down and sometimes high up, as will be seen presently.

Let us now consider the point of cut off, which we may indicate by the suffix 1. If $1 - x_1$ be the fraction of the feed not accounted for,

$$1 - x_1 = \frac{W_1 - w}{F}.$$

Let $\overline{w}$ be the weight of water in the clearance at the instant the steam port commences to open, then $W_1 - \overline{w}$ is the sum of three parts—(1) the priming water W_0 entering with the boiler steam, (2) the steam ($\overline{W}$) condensed in raising the temperature of the clearance contents, (3) the weight of steam (W) condensed at admission by the metal of the cylinder. Thus,

$$1 - x_1 = \frac{W_0 + W + \overline{W} + \overline{w} - w}{F}.$$

* *Industries*, Oct. 5, 1888.

Of these, W_0 may be determined by calorimetric experiments, and if we suppose the boiler to supply dry steam, will be zero. W depends on $\bar{w}$ and could be definitely calculated if we knew this quantity; if $\bar{w}$ were very small it might be negative, as pointed out in Chapter IX. (p. 261). We have no certain means of knowing $\bar{w}$ directly, and if it were large, W might be reduced to zero. That is, initial condensation may be accounted for by supposing the water left behind in the cylinder a large quantity. An active controversy has arisen on this question,* and might be carried on indefinitely if it were not that the broad results of experience appear to show that the metal is the principal agent in cylinder condensation, that is, W must be the principal part of the whole, and $\overline{W}$ of small importance, except in cases where, as previously described, small quantities of water cohere in the angles of the clearance surface. It has already been pointed out that water in this form can hardly be distinguished from the metal itself. If,

$$W = w - \bar{w},$$

$$1 - x_1 = \frac{W}{F} \text{ (omitting priming),}$$

and it is only in this case that the initial condendsation calculated from experimental data is exactly the same thing as the condensation produced by the action of the metal. In general, no doubt, $\overline{W}$ is greater then $w - \bar{w}$, but it is conceivable that strong re-evaporation may, in some cases, exist in the clearance during compression (p. 300), and then $\overline{W}$ may be less than $w - \bar{w}$. The indicated initial condensation may in such cases be less than the actual action of the metal. This is a reason for taking the point of reference high up. On the other hand, if condensation takes place during compression, and it is desired to find the action of the metal, not during admission alone, but through the whole period during which it abstracts heat from the contents of the cylinder, it will be best to take the point of reference low down, so as to include the condensation during compression. When the clearance and compression are large, as is usually the case in high-pressure cylinders, the position of the point of reference is a

* *Réfutations d'une Critique de M. G. Zeuner*, par G. A. Hirn et O. Hallauer. Paris, 1881, 1883.

matter of importance, and the interpretation of experimental data is frequently far from easy.*

163. An indicator diagram gives directly only the pressure existing in the cylinder at a given point of the stroke. The extreme length of the diagram represents the stroke, and when the motion is correct, the ordinate at any point of the base gives the pressure at the corresponding point of the stroke. By the addition of certain lines, and by interpreting the horizontal ordinates of the diagram as volumes of steam, instead of as distances traversed by the piston, the diagram may, however, be made to exhibit graphically the behaviour of the steam. In the absence of clearance and compression, this has already been fully considered, but it remains to explain briefly the way in which these disturbing causes affect the interpretation of the diagram.

Let v as usual be the specific volume of dry saturated steam at the pressure considered, then the equation for F given above may be written,

$$v(F + a) = vA + v(W - w).$$

Fig. 32, p. 253, here reproduced (Fig. 39), shows an indicator diagram which though ideal is sufficient for our purpose. In this diagram NS represents the piston displacement, and ON the clearance volume set off on the same scale. Two compression curves are shown as described on page 254.

The point V may conveniently be adopted for the point of reference, unless there be some special reason for a change.

If the point of reference be not the real or ideal intersection of the compression curve with BZ, then V must be found by drawing a saturation curve through the point of reference chosen, for we shall now for our present purpose suppose the curve GVg in the diagram a saturation curve so drawn, not the actual compression curve. This being understood, va will be the horizontal ordinate of the curve in question, which may be described as the Curve of Reference, while vA is, of course, the horizontal ordinate of the expansion curve AB. Measure now from the curve of reference

* On this point the reader is referred to the discussion on a paper by Professor Osborne Reynolds, in the *Proceedings of the Institution of Civil Engineers*, ol. xcix.

$G V g$ horizontal distances representing $v F$ the volume of the actual weight of steam per stroke, and plot the corresponding curve; this curve will be the true saturation curve corresponding to the actual expansion curve, and it will be seen that its exact position depends

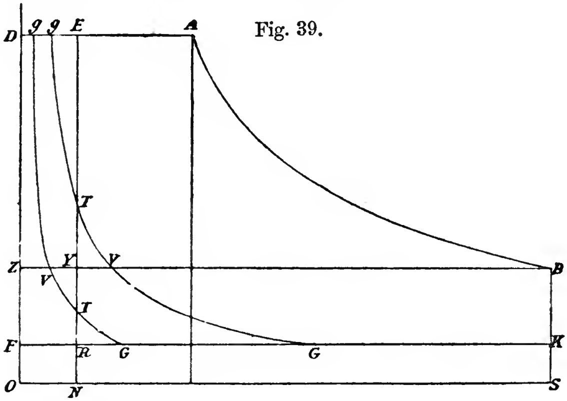


Fig. 39.

on the point of reference chosen. When correctly laid down the horizontal distance between it and the actual expansion curve represents $v(W - w)$, that is, the difference between the amount of water in the cylinder at any point during expansion, and that at the point of reference.

For purposes of comparison, however, it is best to proceed in a manner suggested by writing the equation in the form

$$v F = v(A - a) + v(W - w).$$

The term $v(A - a)$ is now the difference between the horizontal ordinates of the actual expansion curve $A B$, and the curve of reference $G V g$, both measured from the volume axis $O D$. Plotting this difference we get an expansion curve corrected for clearance, while the saturation curve is obtained by plotting $v F$ from the volume axis $O D$ instead of from the curve $G V g$. The convenience of this is that the saturation curve is the same whatever the clearance; it is only the corrected expansion curve which differs according to the clearance and the point of reference.

In applying the graphical method of Art. 151, p. 330, as shown in Fig. 38, the complete diagram should be corrected for clearance in the manner here described.* The area of this corrected diagram

* The correction is not shown in the diagram.

will obviously be the same as that of the original, whatever the clearance or point of reference. The curve of reference will be the volume axis, and that part of the original diagram which lies to the left of it will be to the left of that axis. An adiabatic curve can now be plotted, starting from a point A in the saturation curve corresponding to the boiler pressure as described in the article cited. To construct an adiabatic curve we may plot a succession of horizontal ordinates obtained by calculation, or derive it from the vertical straight line which represents it on a thermal diagram, as described in Chapter VII., p. 230. But by far the simplest method, and one which is sufficiently accurate for all practical purposes, is to employ the principle (see Appendix), that the horizontal deviation of an adiabatic curve from the saturation curve is approximately the same as that of a hyperbola from the saturation curve, the three curves starting from the point where the expansion commences. If then, we plot a saturation curve by use of Table II. or otherwise, and also a common hyperbola, we have only to set off from the saturation curve in the opposite direction, the horizontal distance between the two, and the resulting curve will be the required adiabatic curve. It must not be forgotten that the weight of steam per stroke used in plotting the saturation curve now *includes* the jacket supply.

In combining the diagrams for the various cylinders of a double or triple expansion engine of the receiver type, each diagram should be corrected for clearance as just described, and reduced to the same scale of volumes by expanding the horizontal ordinates in the ratio of the cylinders. The saturation curve will then be the same for all the cylinders, and the amount of water in each cylinder, in excess of that at the point of reference chosen for that cylinder, will be shown as before by the horizontal distance between the saturation curve and the expansion curve. It only remains to explain how the exact ratio of cylinders is to be found. For this purpose consider the end of the expansion in any cylinder a point which we shall denote by 2, and take for the point of reference the point in the return stroke at which the exhaust port closes, which we may denote by 0; then

$$1 - x_2 = \frac{W_2 - w_0}{F}.$$

Now W_2 is the weight of water in the cylinder at release, and

w_0 at the instant exhaust ceases; therefore $W_2 - w_0$ must be the quantity of water discharged with the exhaust steam, either as suspended moisture or by re-evaporation during exhaust. Using the same equation as before,

$$v_2 F = v_2 (A_2 - a_0) + v (W - w_0).$$

Returning to Fig. 32, suppose the point V determined by a saturation curve through o, the point of reference, then

$$B V = v_2 (A_2 - a_0)$$

is the volume at release of the steam and water discharged from the cylinder. Hence the effective volume of the cylinder is found, and in expanding the diagrams, the *effective*, not the actual volume of the cylinder is to be taken. It is, however, only for this purpose that the point of reference need be chosen low down; for other purposes it may be chosen as may be necessary or convenient.

It should be carefully observed that the processes described in this article furnish no additional information respecting the admission and exhaust parts of the cycle of the steam, but only for the expansion and compression parts. In a compound engine of the simple Woolf type, where the steam is not cut off in either cylinder, the expansion part of each diagram taken separately does not occur. Each diagram may be corrected for clearance, and the two combined as before, with a saturation curve added; but such a diagram will only show the water in the cylinder at the end of the stroke in each cylinder, not at intermediate points. Remembering that the weight of steam and water shut up between the high-pressure and low-pressure pistons must remain the same; a curve may be plotted which shows the division between steam and water as before, but the distribution of that water between the cylinders, and the clearance space between them, cannot be found without making questionable assumptions.

If the steam condensed in the jackets has not been separately measured, a saturation curve may still be drawn, using the *total* feed per stroke; but the liquefaction shown will then include the steam condensed in all the jackets. If the feed has not been measured, an ideal saturation curve may be drawn touching, but not intersecting, the combined diagram. Such a curve would be a curve of "uniform wetness," as shown in Fig. 24, p. 187. The information furnished in such cases, is, of course, imperfect, but it is nevertheless valuable, the relative liquefaction in the cylinders being roughly indicated.

In addition to the saturation curve, adiabatic curves may with advantage be added to the diagram. First, we may employ the method of Art. 151 just mentioned, to show graphically the "missing work." But when the jacket supply has been measured, it is also useful to draw an adiabatic curve, as well as a saturation curve, which corresponds to the weight of steam passing through the cylinders. Such an adiabatic curve shows what may be described as the "natural" liquefaction, that is, the liquefaction which would take place in a non-conducting cylinder, and which, as previously explained, probably actually does take place in the central mass of steam not in immediate contact with the metal unless the surface be dry and at a high temperature.

164. When steam is compressed behind a piston the temperature rises according to the usual law connecting it with the pressure if the steam be saturated. If the steam be superheated its temperature will be higher, and it may be considerably higher. As already explained in Chapter X., the temperature of a surface of metal in contact with saturated steam cannot be sensibly less, though it may be greater than that of the steam, and hence it follows that the range of temperature during admission, and, in consequence, the initial condensation, must be reduced by compression. On the other hand, the mean effective pressure will also be diminished, because the re-evaporation after cut-off will be reduced. It is probable, however, that not only the absolute consumption but also the consumption per horse-power will be reduced by compression, and in particular cases this has been proved experimentally.

Feed-water Heaters.

165. As already fully explained, the efficiency of a theoretically perfect heat engine working between given limits of temperature is

$$\text{Maximum efficiency} = \frac{T_1 - T_0}{T_1},$$

while that of an engine also in a certain sense perfect, but receiving heat in the way a steam engine usually does receive it, is

$$\text{Maximum efficiency} = \frac{T_m - T_0}{T_m},$$

the difference between the two results consisting in T_1, the maximum temperature, being replaced by T_m, the average temperature at which heat is supplied during the entrance of the feed-water into the boiler, and its subsequent evaporation.

It remains to consider to what extent the supply of heat in this way is really unavoidable. If we could avoid it, the heat necessary for a given quantity of work would be reduced, in the case of the *Rush* (p. 311), in the proportion ·491/·531, which is equivalent to a saving of about 7½ per cent. In the *Meteor* (K, p. 317) the saving would be as much as 10·7 per cent.

Now, as already remarked (p. 310), if the boiler be theoretically perfect there will be a feed-water heater supplied with heat from the hot gases of the furnace, by which the temperature of the feed is raised above that of the condenser. The saving due to such a feed-heater belongs to the boiler, not to the engine; and so far as such a saving is possible there can of course be no saving in the case of the engine. That is, either the engine or the boiler must be imperfect when the comparison is made between what is actually possible and what would be possible if an indefinite source of heat existed at the boiler temperature. If, however, as is generally the case, the hot gases of the furnace are not made use of, then the saving just indicated becomes theoretically possible, and the question whether it is really attainable becomes of practical importance.

In its simplest form a theoretically perfect heat engine, working between given limits of temperature, is one which employs a Carnot cycle, the objections to which have been already stated. But a Carnot cycle is not essential, as is readily seen on considering the case of a Stirling air engine. The efficiency here is the same as if a Carnot cycle had been used, although the actual cycle is very different. The reason is that the changes of temperature, which in the Carnot cycle are produced by compression and expansion, are in the Stirling engine produced by heat alternately supplied by and abstracted from the regenerator (pp. 97, 206) without recourse to the source from which heat is derived. The question, then, is whether in the steam engine also the supply of heat to the feed as its temperature rises cannot be carried out without recourse to heat derived direct from the furnace. A little consideration shows that it not only may be done, but that it actually is done to some extent in practice.

For example, take the case of a large pumping engine of the Worthington type, some trials of which were made by Professor

Unwin, and described by him in a valuable report.* As is usual in large engines, the feed-pump was a small independent engine taking steam from the main boiler; and in this case the exhaust of this small engine was condensed in a coil placed in the feed-tank and returned to the boiler. The effect of this arrangement is that the work done and the heat expended are each increased by the same amount. Thus, if U be the work done, and Q the heat expended in the main engine, and e the work done by the auxiliary engine, the efficiency of the combined engines will be

$$\text{Efficiency} = \frac{U + e}{Q + e},$$

which is necessarily greater than the efficiency U / Q of the main engine alone. In fact, the work done by the pump is simply the mechanical equivalent of the additional heat expended. In the present example the saving was trifling, and the arrangement was obviously not adopted for economical reasons, but it is evident that the principle may be carried much further; for air-pumps, circulating pumps, and other machinery can also be driven by auxiliary engines exhausting into the feed-tank. Now, the saving thus obtained would be equally a saving, even though the main engine were perfect according to the standard previously used, and can only be accounted for by observing that the heat supplied to the feed is not derived from the furnace but from the steam, just as in the air-engine the heat required for a rise of temperature is derived through the regenerator from the air. Hence we see the limit to the possible saving, and the way in which the auxiliary engines must be used to make it as great as possible. Our theoretically perfect steam engine will now consist of a main engine, combined with an indefinite number of small auxiliary engines, the exhaust of each of which is employed to raise the temperature of the feed by a small amount till at last the temperature of the boiler is reached. The process may be carried out ideally by passing the feed through gratings similar to the gratings of the regenerator of an air engine, but tubular, each grating serving as the condenser of one of the auxiliary engines. No heat is now derived from the furnace except at the temperature of the boiler, and no heat is received by the feed as its temperature rises, except from steam of its own temperature. The main engine

* *Engineering*, Dec. 1888.

and the set of auxiliary engines taken together are completely reversible, and consequently form a theoretically perfect heat engine, the efficiency of which is necessarily the same as if a Carnot cycle had been employed, as may also be shown by direct calculation of the work done and heat expended by the auxiliary engines. (See Appendix.)

The theoretical maximum saving by the use of auxiliary engines is that just given as due to the employment of a Carnot cycle; but, of course, the whole of this cannot be realised. It should be remarked, however, that for given values of e and Q (p. 363) the saving is greater the smaller U; that is, the possible saving is relatively greater in the actual imperfect engine than in the ideal perfect engine.

The method here explained of heating the feed by the exhaust of auxiliary engines has been extensively employed by Mr. J. G. Weir, who has also patented a well-known feed-heater in which the steam used is drawn from the receivers in compound or triple expansion engines. The steam used by the feed-heater may in this case be considered as belonging to an auxiliary engine which is combined with the main engine, and in estimating the efficiency of the main engine by the method previously employed, the steam consumed and the work done by this ideal engine should be subtracted from the corresponding total amount. The saving due to the feed-heater may then be estimated independently. For example, in the *Meteor* (K. p. 317), the feed was heated from 120° to 163° by steam supplied from the valve chest of the intermediate cylinder. The weight of steam thus employed, not having been measured, is unknown, but may be estimated as about 5 per cent. of the total consumption. The horse-power of the high-pressure cylinder, through which alone this steam passed, was 662, and therefore that of the ideal auxiliary engine was $662 \times \cdot 05$, or about 33; being $1 \cdot 65$ per cent. of the whole. Thus the work done per lb. of steam in the main engine taken alone, is greater in the proportion $\cdot 9835 / \cdot 95$, being an increase of $3\frac{1}{2}$ per cent. The real efficiency of K therefore was $\cdot 63 \times 1 \cdot 035$, or $\cdot 652$ instead of $\cdot 63$ as previously found. The same correction applies to the waste work, and consequently the missing work (p. 323) is reduced from 30 per cent. to $27\frac{1}{2}$ per cent.

The saving due to this feed-heater must have been very trifling, a little more than 1 per cent. The actual saving by Weir's feed-heaters is said to amount to 4 per cent., and there is nothing improbable in this, the feed being raised to a higher temperature by steam drawn from the low-pressure receiver. It may be conjectured that the performance of many compound engines might be improved by taking steam from the reservoir to heat the feed, the quantity necessary being provided by cutting off later in the high-pressure cylinder, thereby diminishing cylinder condensation.

The method adopted in the present chapter of estimating the efficiency of a steam engine as if there was no feed-heater, and subsequently correcting the result if necessary, is that which appears to the author to be the most convenient; but we may also, as in Chapter VIII., start from a theoretically perfect heat engine, working between given limits of temperature, and estimate the loss by "misapplication of heat to the feed-water," which of course will be less than is stated in the chapter cited when there is a properly constructed feed-heater.

Utilisation of Low Temperatures.

166. The correction for the effect of a feed-heater just made in the case of K is not required for the other examples given in the table, p. 322; and we find, therefore, that in engines which run at a sufficient speed for their size, type, and grade of expansion, the "missing work" ranges from 20 to 27 per cent. of the total theoretical amount. Since this includes (1) priming, leakage, and radiation, (2) cylinder condensation, (3) clearance and wire-drawing, the last of which increases with the speed, there is little probability of any material reduction, and it follows that the performance and efficiency of engines working with a given boiler pressure and vacuum at sufficient speed, will depend on the waste work at exhaust, as is well shown by the values given in the table. This waste work in non-condensing engines is small, the actual efficiency of these machines reaching, and in the best examples probably exceeding, 75 per cent. That is, for temperatures such as occur in non-condensing engines: a steam engine is an efficient machine, and not much is likely to be gained by recourse to any other fluid than steam, except in cases where the engine cannot conveniently be run at a sufficient speed. In condensing engines the case is different, and the reason of this is that for temperatures below 212° the pressure of steam is too low. On reference to page 321 it will be seen that the waste work at exhaust depends on the *ratio* of the terminal pressure p_2 to the condenser pressure p_0. Now, the terminal pressure cannot be reduced below a certain value, depending on the

frictional resistances of the engine itself and the machinery it drives. A large part of these resistances may be represented by an increased back pressure on the piston, which is independent of the power transmitted, and the terminal pressure must not be less than the total. In non-condensing engines the ratio p_2 / p_0 is even then not much greater than unity, because p_0 is nearly 15 lbs. per square inch, and the waste work at exhaust is therefore small, a remark which likewise applies to the loss by "sudden drop" in a compound engine. But in condensing engines the case is different; the ratio p_2 / p_0 is seldom less than 3, and a loss is therefore necessary, which is relatively greater the lower the mean effective pressure. This is well shown by the values given in the table for J and K, the waste work at exhaust being 14 per cent. in the first, and 7 per cent. in the second, although the terminal pressure in the first was much the smaller. In the low-pressure engine G, the waste is as much as 24 per cent. Again, the use of steam as the working fluid for temperatures below 212° involves pressures less than that of the atmosphere; in other words, a condenser is necessary, with its accompanying air-pumps and circulating pumps. The power required to drive these pumps is not inconsiderable, and is, of course, so much deducted from the useful work of the engine. Hence it appears that a condensing steam engine is necessarily a much less efficient machine than the non-condensing, unless high presssures are used; or, in other words, steam is an unsuitable fluid for the purpose of utilising temperatures below 212°. The great economy obtained by the use of high-pressure steam is quite as much due to the better utilisation of the upper part of the range of temperature, as to the actual increase of range. For temperatures below 100° steam is absolutely useless; yet, as pointed out on page 232, the discharge from the condenser of a steam engine is theoretically capable of increasing the power of the engine by 50 per cent., if the temperature of the atmosphere be 60°.

We conclude, therefore, that for temperatures below 212° some other fluid than steam (see also p. 146) should be employed. The idea of a binary vapour engine is an old one, a steam and ether machine having been actually tried by M. du Tremblay (p. 140), with a certain degree of success. So far as pressure is concerned, ether is a suitable fluid, for at temperatures below 212° its pressures are about the same as those used in non-condensing steam engines; but its inflammable character is a serious objection, which, together with its comparatively low pressure, has caused its disuse in refrigerating machines (p. 149). Anhydrous ammonia, which has replaced it, has a pressure too great for our present purpose; but the extent to which it is used shows conclusively that the difficulties attending the use of highly volatile fluids are not insuperable. Some years ago petroleum spirit was tried by Messrs. Yarrow & Co.,* with considerable success, in the engine of a small launch. The composition of the vapour of petroleum varies; but in this case the boiling-point under a pressure of one atmosphere appears to have been 130°, and at 220° its pressure is as much as 50 lbs. per square inch. It should, therefore, be possible to utilise temperatures much below those which can be utilised in a steam engine. As compared with a small steam engine running at a moderate speed, in which case cylinder condensation is always large, it may be further conjectured that it presents considerable advantages, since the loss by cylinder condensation when fluids of this class are used, may very possibly be small.

If a suitable fluid could be discovered, it is certain that a greatly increased economy would be possible in our heat engines. Mr. Willans has shown that in a non-condensing engine with a boiler pressure of about 180 lbs., the consumption of steam need not exceed 18 lbs. per horse-power per hour. Now, the range of temperature here little exceeds 150°, and is, therefore, no more than that between 212° and the average temperature of the atmosphere in our climate. Thermo-dynamic science tells us that if properly utilised by a suitable working fluid, this is worth as much as the same range above 212°; and it must therefore be practically possible to double the horse-power with the same consumption of steam, thus reducing the consumption to 9 lbs. per horse-power per hour. Much attention has been paid to the utilisation of high temperatures by the employment of very high-pressure steam, and further by gas engines; but it appears to the author that the question of utilising low temperatures is hardly less important. All known heat engines are very imperfect in consequence of its neglect.

* *Transactions of the Institution of Naval Architects for* 1888. See also *Gas and Petroleum Engines*, by W. Robinson, M.E. Spon, 1890.

TABLES OF THE PROPERTIES OF SATURATED STEAM.

TABLE I*a*.—RELATION BETWEEN PRESSURE AND TEMPERATURE.

Temperature Fahrenheit, *t*.	Pressure in Lbs. per Sq. In. at the Level of the Sea in Lat. 44°, *p*.	Difference, Δ *p*.	Temperature Fahrenheit, *t*.	Pressure in Lbs. per Sq. In. at the Level of the Sea in Lat. 44°, *p*.	Difference, Δ *p*.
° 432	350·73		° 414	289·48	
		3·64			3·15
431	347·09		413	286·33	
		3·61			3·12
430	343·48		412	283·21	
		3·58			3·09
429	339·90		411	280·12	
		3·55			3·07
428	336·35		410	277·05	
		3·53			3·04
427	332·82		409	274·01	
		3·50			3·02
426	329·32		408	270·99	
		3·47			2·99
425	325·85		407	268·00	
		3·44			2·97
424	322·41		406	265·03	
		3·42			2·94
423	318·99		405	262·09	
		3·39			2·92
422	315·60		404	259·17	
		3·36			2·89
421	312·25		403	256·28	
		3·33			2·87
420	308·92		402	253·41	
		3·31			2·84
419	305·61		401	250·57	
		3·28			2·82
418	302·33		400	247·75	
		3·25			2·79
417	299·08		399	244·96	
		3·23			2·77
416	295·85		398	242·19	
		3·20			2·75
415	292·65		397	239·44	
		3·17			2·72

Table Ia—*continued.*

t	p	Δp	t	p	Δp
396°	236·72	2·70	374°	182·63	2·21
395	234·02	2·67	373	180·42	2·19
394	231·35	2·65	372	178·23	2·17
393	228·70	2·63	371	176·07	2·15
392	226·07	2·61	370	173·92	2·13
391	223·46	2·58	369	171·79	2·11
390	220·88	2·56	368	169·69	2·09
389	218·32	2·53	367	167·60	2·07
388	215·79	2·51	366	165·53	2·05
387	213·28	2·49	365	163·49	2·03
386	210·79	2·47	364	161·47	2·01
385	208·33	2·45	363	159·46	1·99
384	205·88	2·42	362	157·48	1·97
383	203·46	2·40	361	155·51	1·95
382	201·06	2·38	360	153·56	1·93
381	198·68	2·36	359	151·63	1·91
380	196·32	2·34	358	149·72	1·89
379	193·98	2·32	357	147·82	1·87
378	191·67	2·29	356	145·95	1·85
377	189·38	2·27	355	144·10	1·83
[illegible]	187·11	2·25	354	142·27	1·82
[illegible]	…4·85	2·23	353	140·45	1·80

Table Ia—*continued.*

t	p	Δ p	t	p	Δ p
°			°		
352	138·65	1·78	330	103·43	1·41
351	136·87	1·76	329	102·02	1·40
350	135·11	1·74	328	100·62	1·39
349	133·37	1·73	327	99·23	1·37
348	131·64	1·71	326	97·86	1·35
347	129·93	1·70	325	96·51	1·34
346	128·23	1·68	324	95·17	1·32
345	126·55	1·66	323	93·85	1·31
344	124·89	1·64	322	92·54	1·29
343	123·26	1·63	321	91·25	1·28
342	121·63	1·61	320	89·97	1·27
341	120·02	1·59	319	88·70	1·25
340	118·43	1·57	318	87·45	1·24
339	116·86	1·56	317	86·21	1·22
338	115·30	1·54	316	84·99	1·21
337	113·76	1·52	315	83·78	1·19
336	112·24	1·50	314	82·59	1·18
335	110·74	1·49	313	81·40	1·17
334	109·25	1·48	312	80·23	1·15
333	107·77	1·46	311	79·08	1·14
332	106·31	1·45	310	77·94	1·13
331	104·86	1·43	309	76·81	1·12

Table I*a—continued.*

t	*p*	Δ *p*	*t*	*p*	Δ *p*
308°	75·69	1·10	286°	54·24	·843
307	74·59	1·09	285	53·39	·832
306	73·50	1·08	284	52·56	·821
305	72·42	1·06	283	51·74	·810
304	71·36	1·05	282	50·93	·799
303	70·31	1·04	281	50·13	·790
302	69·27	1·03	280	49·33	·780
301	68·24	1·018	279	48·55	·771
300	67·22	1·006	278	47·78	·761
299	66·22	·994	277	47·02	·752
298	65·23	·982	276	46·27	·743
297	64·25	·970	275	45·53	·733
296	63·29	·957	274	44·79	·724
295	62·33	·945	273	44·07	·714
294	61·38	·933	272	43·35	·705
293	60·45	·921	271	42·65	·696
292	59·53	·909	270	41·96	·687
291	58·62	·898	269	41·27	·678
290	57·72	·887	268	40·60	·669
289	56·83	·876	267	39·93	·660
288	55·96	·865	266	39·27	·651
287	55·09	·854	265	38·62	·642

Table I*a—continued.*

t	p	Δ p	t	p	Δ p
264°	37·98		242°	25·92	
		·633			·465
263	37·35		241	25·46	
		·624			·458
262	36·72		240	25·00	
		·615			·452
261	36·11		239	24·55	
		·607			·446
260	35·50		238	24·11	
		·599			·439
259	34·90		237	23·67	
		·591			·433
258	34·31		236	23·25	
		·583			·426
257	33·73		235	22·82	
		·575			·419
256	33·15		234	22·40	
		·567			·413
255	32·59		233	21·99	
		·559			·407
254	32·03		232	21·59	
		·551			·400
253	31·48		231	21·19	
		·543			·394
252	30·94		230	20·80	
		·535			·388
251	30·41		229	20·41	
		·527			·382
250	29·88		228	20·03	
		·520			·376
249	29·36		227	19·66	
		·513			·370
248	28·85		226	19·29	
		·506			·364
247	28·34		225	18·93	
		·499			·358
246	27·84		224	18·57	
		·492			·352
245	27·35		223	18·22	
		·485			·346
244	26·87		222	17·87	
		·478			·340
243	26·39		221	17·53	
		·471			·335

Table Ia—*continued.*

t	*p*	Δ *p*	*t*	*p*	Δ *p*
220°	17·20	·330	198°	11·05	·229
219	16·87	·325	197	10·82	·225
218	16·54	·320	196	10·60	·221
217	16·22	·315	195	10·38	·217
216	15·91	·310	194	10·16	·213
215	15·60	·304	193	9·95	·209
214	15·29	·299	192	9·74	·205
213	14·99	·294	191	9·53	·202
212	14·70	·290	190	9·33	·199
211	14·41	·285	189	9·13	·195
210	14·12	·280	188	8·94	·191
209	13·84	·276	187	8·75	·188
208	13·57	·271	186	8·56	·185
207	13·30	·266	185	8·37	·181
206	13·03	·262	184	8·19	·178
205	12·77	·258	183	8·01	·175
204	12·51	·254	182	7·84	·171
203	12·26	·249	181	7·67	·168
202	12·01	·245	180	7·50	·165
201	11·76	·241	179	7·34	·162
200	11·52	·236	178	7·17	·159
199	11·29	·232	177	7·01	·156

Table Ia—*continued.*

t	p	Δp	t	p	Δp
176°	6·85	·153	154°	4·091	·0991
175	6·70	·150	153	3·992	·0969
174	6·55	·147	152	3·895	·0948
173	6·40	·144	151	3·800	·0928
172	6·26	·141	150	3·707	·0909
171	6·12	·139	149	3·616	·0890
170	5·98	·136	148	3·527	·0872
169	5·85	·134	147	3·440	·0854
168	5·71	·131	146	3·354	·0836
167	5·58	·129	145	3·270	·0818
166	5·45	·126	144	3·188	·0801
165	5·32	·123	143	3·108	·0784
164	5·20	·121	142	3·030	·0767
163	5·08	·119	141	2·953	·0751
162	4·961	·1166	140	2·878	·0735
161	4·844	·1144	139	2·805	·0720
160	4·730	·1123	138	2·733	·0705
159	4·618	·1101	137	2·663	·0690
158	4·508	·1079	136	2·594	·0675
157	4·401	·1057	135	2·526	·0660
156	4·295	·1035	134	2·461	·0645
155	4·192	·1013	133	2·397	·0630

Table Ia—*continued.*

t	p	Δ p	t	p	Δ p
° 132	2·334		° 112	1·342	
		·0615			·0382
131	2·273		111	1·304	
		·0601			·0372
130	2·212		110	1·267	
		·0588			·0363
129	2·154		109	1·230	
		·0574			·0355
128	2·096		108	1·195	
		·0561			·0346
127	2·040		107	1·160	
		·0548			·0337
126	1·985		106	1·127	
		·0535			·0328
125	1·932		105	1·094	
		·0522			·0319
124	1·880		104	1·062	
		·0509			·0311
123	1·829		103	1·031	
		·0497			·0302
122	1·779		102	1·001	
		·0485			·0294
121	1·731		101	·971	
		·0475			·0287
120	1·683		100	·942	
		·0464			·0280
119	1·637		99	·914	
		·0454			·0274
118	1·591		98	·887	
		·0444			·0267
117	1·547		97	·860	
		·0433			·0260
116	1·504		96	·834	
		·0423			·0252
115	1·462		95	·809	
		·0412			·0246
114	1·421		94	·785	
		·0402			·0239
113	1·381		93	·761	
		·0392			

TABLE I*b*.—RELATION BETWEEN PRESSURE AND TEMPERATURE.

Temperature.	Force in Inches of Mercury at 32° at Sea Level. (Lat. 53° 21′.)	Temperature.	Force in Inches of Mercury at 32° at Sea Level. (Lat. 53° 21′.)	Temperature.	Force in Inches of Mercury at 32° at Sea Level. (Lat. 53° 21′.)
°	ins.	°	ins.	°	ins.
150	7·540	123	3·720	96	1·6971
149	7·354	122	3·619	95	1·6457
148	7·173	121	3·520	94	1·5958
147	6·996	120	3·423	93	1·5471
146	6·822	119	3·329	92	1·4998
145	6·651	118	3·237	91	1·4537
144	6·485	117	3·147	90	1·4088
143	6·322	116	3·059	89	1·3652
142	6·162	115	2·974	88	1·3228
141	6·006	114	2·890	87	1·2815
140	5·854	113	2·809	86	1·2413
139	5·704	112	2·729	85	1·2023
138	5·558	111	2·652	84	1·1643
137	5·415	110	2·576	83	1·1274
136	5·275	109	2·502	82	1·0915
135	5·139	108	2·430	81	1·0566
134	5·005	107	2·360	80	1·0227
133	4·874	106	2·292	79	0·9898
132	4·747	105	2·225	78	0·9577
131	4·622	104	2·160	77	0·9266
130	4·500	103	2·097	76	0·8964
129	4·381	102	2·035	75	0·8671
128	4·264	101	1·975	74	0·8386
127	4·150	100	1·917	73	0·8109
126	4·039	99	1·8595	72	0·7841
125	3·930	98	1·8039	71	0·7580
124	3·824	97	1·7498	70	0·7327

TABLE II*a*.—TOTAL AND LATENT HEAT OF EVAPORATION IN THERMAL UNITS.

t	$t-32$	h	Δh	H	ΔH	L	ΔL
401°	369	375·16		1204·2		829·1	
			1·043		·305		·738
374	342	347·0		1196·0		849·0	
			1·037		·305		·732
347	315	319·0		1187·8		868·8	
			1·032		·305		·727
320	288	291·14		1179·5		888·4	
			1·027		·305		·722
293	261	263·41		1171·3		907·9	
			1·023		·305		·718
266	234	235·8		1163·1		927·3	
			1·0185		·305		·713
239	207	208·3		1154·8		946·5	
			1·0144		·305		·709
212	180	180·9		1146·6		965·7	
			1·011		·305		·706
185	153	153·6		1138·4		984·8	
			1·008		·305		·703
158	126	126·4		1130·1		1003·8	
			1·006		·305		·701
131	99	99·2		1121·9		1022·7	
			1·004		·305		·699
104	72	72·09		1113·7		1041·6	
			1·002		·305		·697
77	45	45·03		1105·4		1060·4	
			1·000		·305		·695
32	0	0		1091·7		1091·7	

Explanation of Symbols.

H.—Total Heat of Evaporation (Art. 4) in thermal units.

h.—Heat necessary to raise a lb. of water from 32° to t° (Art. 3) in thermal units.

$L = H - h$.—Latent Heat of Evaporation (Art. 4) in thermal units.

The results given in this table are not altered by a change in the mechanical equivalent of heat.

TABLE II*b*.—TOTAL AND LATENT HEAT OF EVAPORATION IN **FOOT-POUNDS.**

t	$\frac{h}{100}$	Δh	$\frac{H}{100}$	ΔH	$\frac{L}{100}$	ΔL
401	2896		9297		6401	
		805		235		570
374	2679		9233		6554	
		801		235		565
347	2463		9170		6707	
		797		235		561
320	2247		9106		6858	
		793		235		557
293	2033		9042		7009	
		790		235		554
266	1820		8979		7159	
		786		235		550
239	1608		8915		7307	
		783		235		547
212	1396		8852		7455	
		780		235		545
185	1186		8788		7603	
		778		235		543
158	975·8		8725		7749	
		777		235		541
131	765·8		8661		7895	
		775		235		540
104	556·5		8598		8041	
		773		235		538
77	347·5		8534		8186	
		772		235		536
32	0		8428		8428	

EXPLANATION OF SYMBOLS.

The symbols in this table have the same meanings as in Table II*a*, but quantities of heat are expressed in foot-pounds instead of thermal units. The differences in both tables are the mean differences per degree between the temperatures indicated.

The mechanical equivalent of heat is taken as 772; if it be increased, the numbers given in this table must be increased in the same proportion.

TABLE III.—DENSITY AND SPECIFIC VOLUME OF DRY STEAM.

Pressure in Lbs. per Sq. In., p.	Volume of 1 lb. in Cub. Feet, v.	$v - s = u$	Weight of a Cubic Foot in Lbs. by direct Experiment.	Weight of a Cubic Foot in Lbs. by Calculation, w.	Difference per Lb. Pressure, Δw.
250	1·841	1·825	..	·5432	
					·002056
200	2·270	2·254	..	·4404	
					·002080
170	2·645	2·629	..	·3780	
					·002107
140	3·177	3·161	..	·3148	
					·002130
110	3·986	3·970	..	·2509	
					·002150
90	4·810	4·794	..	·2079	
					·002185
70	6·090	6·074	·1682	·1642	
					·002210
60	7·037	7·021	·1457	·1421	
					·002230
50	8·347	8·331	·1228	·1198	
					·002270
40	10·30	10·28	·09937	·0971	
					·002297
30	13·49	13·47	·07550	·07413	
					·002332
25	16·01	15·99	·06339	·06247	
					·002364
20	19·74	19·72	·05117	·05065	
					·002398
15	25·87	25·85	·03878	·03866	
					·002437
12	31·9	..	·03131	·03135	
					·002470
10	37·8	..	·02630	·02641	
					·002490
8	46·6	..	·02126	·02143	
					·00253
6	61·1	..	·01620	·01637	
					·00258
5	72·5	..	·01258	·01379	
					·00262
4	89·5	..	·01112	·01117	

TABLE IV*a*.—EXTERNAL WORK.

t	PV Steam Gas.	Pv Foot-Pounds.	Pu Foot-Pounds.	ΔPu	Pn Thermal Units.	ΔPu
401°	73,670	66,250	65,600		85	
				38		·0492
374	71,360	65,030	64,570		83·6	
				43		·0557
347	69,050	63,720	63,400		82·1	
				48		·0622
320	66,750	62,320	62,100		80·4	
				53		·0687
293	64,440	60,820	60,670		78·6	
				57		·0739
266	62,130	59,230	59,130		76·6	
				61		·0791
239	59,830	57,550	57,480		74·5	
				65		·0842
212	57,520	55,780	55,730		72·2	
				69		·0894
185	55,210	53,910	53,880		69·8	
				73		·0946
158	52,910	51,950	51,930		67·3	
				77		·0998
131	50,600	49,890	49,880		64·6	
				80		·104
104	48,290	47,740	47,740		61·8	

EXPLANATION OF SYMBOLS.

The third column of Table IV*a* gives the product in foot-pounds of the pressure (P), and the volume (v) of dry saturated steam at the temperature indicated in the first column. The fourth, fifth, sixth, and seventh columns give Pu, that is to say, $P(v-s)$ in foot-pounds and thermal units, together with the differences needed for interpolation. If the mechanical equivalent of heat is supposed greater than 772, the values of Pu in foot-pounds must be increased in the same proportion. The same remark applies to the values of u in Table III., but the values of w are diminished in the same proportion. Results in thermal units are unchanged.

TABLE IV*b*.—INTERNAL WORK DURING EVAPORATION.

t	ρ Foot-Pounds.	$\Delta\rho$	ρ Thermal Units.	$\Delta\rho$	$k = \frac{\rho}{Pu}$
401°	574,500		744·1		8·75
		608		·787	
374	590,900		765·4		9·16
		608		·788	
347	607,300		786·7		9·59
		609		·789	
320	623,800		808·0		10·05
		610		·791	
293	640,300		829·3		10·55
		611		·792	
266	656,800		850·7		11·10
		611		·792	
239	673,300		872·1		11·71
		612		·793	
212	689,800		893·5		12·39
		614		·795	
185	706,300		915·0		13·11
		616		·797	
158	722,900		936·4		13·92
		618		·800	
131	739,600		958·1		14·82
		620		·803	
104	756,400		979·8		15·78

Explanation of Symbols.

Table IV*b* gives π the internal work done in producing dry saturated steam of a given temperature, and hence, by Table I., of a given pressure from water of the same temperature, or, what is the same thing, the intrinsic energy of dry saturated steam reckoned from water at the same temperature. The results are given in foot-pounds and thermal units with the differences necessary for interpolation. The last column shows the proportion (k) which the internal work bears to the external work, a number also given in Table V. for a series of pressures.

TABLE IV*c*.—TOTAL INTERNAL WORK.

t	I Thermal Units.	ΔI	$I \div 100$ Foot-Pounds.	ΔI
401°	1119·2		8641	
		·256		197
374	1112·4		8587	
		·249		192
347	1105·7		8536	
		·242		187
320	1099·2		8485	
		·236		182
293	1092·7		8435	
		·231		178
266	1086·5		8378	
		·226		174
239	1080·3		8340	
		·221		170
212	1074·4		8295	
		·216		166
185	1068·6		8249	
		·210		162
158	1062·8		8205	
		·205		158
131	1057·3		8162	
		·201		155
104	1051·9		8121	

Explanation of Symbols.

Table IV*c* gives I the total internal work done in producing dry steam at any given temperature from water at 32°, or, what is the same thing, the intrinsic energy of dry saturated steam reckoned from water at 32°. As in the preceding tables, the differences are given for a difference of temperature of 1°.

The mechanical equivalent of heat is taken as 772; if it is supposed greater, results expressed in foot-pounds in Tables IV*b*, IV*c* must be increased in the same proportion. Results expressed in thermal units are unaltered.

TABLE V.—TABLE showing the INTERNAL-WORK-PRESSURE and the HEAT-PRESSURE during Evaporation under a constant External Pressure.

External Pressure, p.	Internal-Work-Pressure.				$k = \frac{\bar{p}}{p}$	Heat-Pressure, p_h.
	Lbs. per Sq. Foot, $\bar{P}$.	Difference, $\Delta \bar{P}$.	Lbs. per Sq. Inch, $\bar{p}$.	Difference, $\Delta \bar{p}$.		
250	315,100		2188		8·75	2438
		1100		7·64		
200	260,100		1806		9·03	2006
		1130		7·83		
170	226,200		1571		9·24	1740
		1162		8·07		
140	191,300		1329		9·49	1469
		1208		8·39		
110	155,100		1077		9·79	1187
		1247		8·66		
90	130,200		903·6		10·04	993·6
		1282		8·90		
70	104,500		725·5		10·36	795·5
		1327		9·21		
60	91,200		633·4		10·56	693·4
		1347		9·36		
50	77,750		539·8		10·80	589·8
		1391		9·65		
40	63,850		443·3		11·08	483·3
		1435		9·97		
30	49,500		343·6		11·45	373·6
		1480		10·28		
25	42,100		292·2		11·69	317·2
		1518		10·54		
20	34,500		239·5		11·98	259·5
		1563		10·85		
15	26,670		185·2		12·35	200·2
		1612		11·20		
12	21,840		151·7		12·64	163·7
		1645		11·42		
10	18,550		128·8		12·88	138·8
		1688		11·72		
8	15,170		105·4		13·17	113·4
		1723		11·97		
7	13,450		93·43		13·35	100·43
		1740		12·13		
6	11,710		81·30		13·55	87·30
		1780		12·34		
5	9,930		68·96		13·79	73·96
		1822		12·63		
4	8,110		56·33		14·08	60·33

TABLE IV.—VALUES OF $\frac{L}{T}$.

t	$\frac{L}{T}$	Diff. ·00.	t	$\frac{L}{T}$	Diff. ·00.
401°	·9618		257°	1·3005	
		199			284
392	·9797		248	1·3261	
		203			292
383	·9980		239	1·3524	
		208			299
374	1·0167		230	1·3793	
		212			306
365	1·0358		221	1·4068	
		217			313
356	1·0553		212	1·4350	
		221			321
347	1·0752		203	1·4639	
		226			330
338	1·0955		194	1·4936	
		231			340
329	1·1163		185	1·5242	
		236			350
320	1·1375		176	1·5557	
		241			361
311	1·1592		167	1·5882	
		247			371
302	1·1814		158	1·6216	
		252			381
293	1·2041		149	1·6559	
		258			392
284	1·2273		140	1·6912	
		264			403
275	1·2511		131	1·7275	
		271			
266	1·2755				
		278			

EXPLANATION OF THE TABLES OF THE PROPERTIES OF SATURATED STEAM.

In order to apply theoretical principles to questions relating to the steam engine, tables are indispensable; and it will be convenient here to make some observations respecting the tables given in this book, and to give examples of the method of using them.

Tables Ia, Ib.—Tables connecting the pressure and temperature of steam may be arranged either by equal intervals of temperature, or by equal intervals of pressure. The second method is not more practically useful than the first—for, in the results of experiment, pressures of an even number of pounds per square inch do not often occur—and it has certain disadvantages which render the first method preferable. To avoid interpolation as much as possible, and to render the use of logarithms unnecessary when it cannot be avoided, the table extends to every degree. For many purposes a determination to the nearest degree is quite sufficient, but when greater accuracy is desired, the tabulated differences are used, as follows, to obtain more exact results:—

Example 1. Find the pressure corresponding to a temperature of 318°·4.

Here
$$\Delta p = 1 \cdot 25 \text{ for } 1^\circ$$
$$= \cdot 5 \text{ for } \cdot 4^\circ$$
$$\therefore p = 87 \cdot 45 + \cdot 5 = 87 \cdot 95.$$

Example 2. Find the temperature corresponding to a pressure of 124 lbs. per square inch.

Here the temperature lies between 343° and 344°, and at 344° the pressure is 124·89. Difference = ·89. $\Delta p = 1 \cdot 64$ for 1°.

$$\therefore \text{Difference of temperature} = \frac{\cdot 89}{1 \cdot 64} = \cdot 54$$

$$\therefore \text{Temperature} = 344^\circ - \cdot 54^\circ = 343^\circ \cdot 46.$$

The accuracy with which the pressure is determined for a given temperature depends mainly on the accuracy with which temperature can be measured. Regnault states that some of his thermometers were capable of measuring $\frac{1}{200}$th of a degree Centigrade; if with Dixon we regard $\frac{1}{50}$th of a degree Fahrenheit as the limit of accuracy, the maximum probable error will be found by dividing the corresponding tabular difference by 50: this amounts to ·07 lb. per square inch at the highest pressure given in the table, but to less than ·01 below 28 lbs.; in no case does the probable error reach $\frac{1}{10}$ per cent.

The values of the differences given are not nearly so accurate, proportionally, the last place of decimals not being always reliable; one of the formulæ given in the Appendix is therefore to be used when great accuracy is necessary. For the purpose of interpolation they are always sufficiently accurate.

As stated in the text, the standard atmosphere adopted in Dixon's table, from which the present table is reduced, is 30 inches of mercury at 32° at the sea-level at the Equator. This corresponds to 14·73 lbs. per square inch at the Equator, 14·66 at the Pole, and 14·7 in latitude 44°. It is the last value which has been adopted in the present work, and the table is therefore exact in latitude 44°, and amply sufficiently approximate at all other points on the earth's surface.

Tables IIa, IIb.—To avoid the necessity of interpolation altogether in determining the total and latent heat of evaporation, it would be necessary that the tables should extend to every degree at least. On the other hand, the differences of the quantities in question vary so slowly that simple interpolation is amply sufficient for wide intervals of temperature. The values are therefore given by the table for every 27°, and for intermediate values interpolation is used. A single example will suffice.

Example 3. Find in foot pounds and thermal units the latent heat for the pressure 70 lbs. per square inch, also the heat necessary to raise the feed water from 100°.

By temperature table the temperature to the nearest degree is 303°. (The nearest degree is always sufficient.)

293°	$h = 263{\cdot}4$		$\Delta h = 1{\cdot}027$
Diff. for 10°	$= 10{\cdot}27$		10
$\therefore$ for 303°	$h = 273{\cdot}7$		$10{\cdot}27$
but for 100°	$h = 68{\cdot}1$		
	$\therefore h_1 - h_2 = 205{\cdot}6$		

Again, in foot pounds—

$293° \, h$ =	203,300	$104° \, h$ =	55,650
Diff. for 10° =	7,930	Diff. for 4° =	4×775
		=	3,100
	211,230		
	52,550		52,550
$\therefore h_1 - h_2$ =	158,680		

A similar process obtains the latent heat of evaporation in foot pounds or thermal units.

Table III.—The volume of steam varies so rapidly with the pressure that it could only be determined directly by a very extensive table: it is therefore preferable to tabulate the weight of a cubic foot of steam, instead of the volume of 1 lb. The differences in this case vary so slowly that wide intervals are sufficient, especially at high pressures. In the table the interval varies so as not to exceed one-fifth of the pressure itself, and the differences given are the mean differences per lb. within the interval.

Example 4. Find the weight (w) of a cubic foot of dry steam at the pressure 33 lbs. per square inch. Here

$$p = 30 \qquad w = 07413 \qquad \Delta w = \cdot 002297$$

$$\therefore \text{difference for 3 lbs.} = 3 \times \cdot 002297 = \cdot 006891$$

$$w = \cdot 07413 + \cdot 00689$$

$$= \cdot 08012$$

The reciprocal of w is the volume in cubic feet of 1 lb. of steam, and is readily found by a table of reciprocals or division to be 12·34 in this example.

At pressures less than 5 lbs., or when accuracy is required, it is better to use Table IV., as explained below.

Tables IVa, IVb, IVc.—In these Tables the results are given for every 27°, for the same reasons as in Table II., and the process of interpolation by means of which results are obtained for intermediate temperatures is precisely similar, so that it need not be further illustrated. Table IV*a*, however, may be used for other purposes, as follows :—

First. To find the specific volume of steam at low pressures, or when special accuracy is desirable.

Example 5. Find the specific volume of steam at 70 lbs. pressure.

Here the corresponding temperature is 303°, and the value of $P u$ by the table is

$$P u = 60{,}670 + 530 = 61{,}200$$

$$\therefore u = \frac{61{,}200}{70 \times 144} = 6 \cdot 071$$

$$v = 6 \cdot 071 + \cdot 016 = 6 \cdot 087$$

Secondly. To calculate the value of $(P_1 - P_2) V_3$ in thermal units, where the pressures are given in lbs. per square inch, and V_3 is the volume of dry steam corresponding to some pressure P_3, which may or may not be the same as either of the others.

Here we might proceed by finding the pressures in lbs. per square foot and the volume V_3 separately, then by multiplication and division by 772 the result would be obtained. It is better, however, to proceed thus :—

$$(P_1 - P_2) V_3 = \frac{P_1 - P_2}{P_3} \cdot P_3 V_3 = \frac{p_1 - p_2}{p_3} \cdot P_3 V_3,$$

which can now be calculated by use of Table IV*a* with facility. This method is constantly used in the text.

Table V.—The meaning of the term "internal-work-pressure," and the use of this quantity in the graphical treatment of questions in the theory of the steam engine, have been sufficiently explained in the text. In order that a small table may suffice, the tabulated intervals are unequal, being arranged so as not to exceed one-fifth of the pressure under consideration.

Example 6. Find the internal-work-pressure during the formation of steam at 65 lbs. pressure.

By Table V. we find

$$p = 60 : \bar{p} = 633 \cdot 4 \ : \ \Delta p = 9 \cdot 21$$

$$\text{Diff. for 5 lb.} = 46 \cdot 05 \qquad\qquad 5$$

$$\therefore \text{for } p = 65 : \bar{p} = 679 \cdot 45 \qquad\qquad 46 \cdot 05$$

Nearly the same results may be obtained direct from the temperature table, by use of the tabulated values of the differences. For by Art. 67, p. 154,

$$\bar{p} = T \Delta p - p.$$

Example 7. Find the equivalent pressure at 12 lbs. on the square inch.

By Table $t = 202° \therefore T = 663°$,

$$\Delta p = \frac{\cdot 249 + \cdot 245}{2} = \frac{\cdot 494}{2} = \cdot 247,$$

$$\therefore \text{Pressure equivalent} = 663 \times \cdot 247 - 12$$
$$= 163 \cdot 76 - 12 = 151 \cdot 76.$$

By table the result is 151·7, showing practical identity.

The ratio which this pressure bears to the external pressure is

$$\frac{\delta (\log p)}{\delta (\log T)} - 1.$$

Example 8. Find the ratio for pressure 5 lbs. on the square inch.

$t = 163°$	$\log p = \cdot 70586$	$\log T = \cdot 79518$
$t = 161°$	$\log p = \cdot 68529$	$\log T = \cdot 79379$
	$\cdot 02057$	$\cdot 00139$

$$\therefore \text{Ratio} = \frac{2057}{139} - 1 = 13 \cdot 8.$$

Table VI.—This table of the value of L / T has been added to the present edition in order to facilitate the calculation of T_m, the average temperature of supply (p. 311) and ϕ the entropy (p. 229). It will also be found useful in all calculations where the adiabatic equation (p. 215) is used.

TABLE OF HYPERBOLIC LOGARITHMS.

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
1·01	·00995	1·36	·3075	1·71	·5365
1·02	·01980	1·37	·3148	1·72	·5423
1·03	·02956	1·38	·3221	1·73	·5481
1·04	·03922	1·39	·3293	1·74	·5539
1·05	·04879	1·40	·3365	1·75	·5596
1·06	·05827	1·41	·3436	1·76	·5653
1·07	·06766	1·42	·3507	1·77	·5710
1·08	·07696	1·43	·3577	1·78	·5766
1·09	·08618	1·44	·3646	1·79	·5822
1·10	·09531	1·45	·3716	1·80	·5878
1·11	·1044	1·46	·3784	1·81	·5933
1·12	·1133	1·47	·3853	1·82	·5988
1·13	·1222	1·48	·3920	1·83	·6043
1·14	·1310	1·49	·3988	1·84	·6098
1·15	·1398	1·50	·4055	1·85	·6152
1·16	·1484	1·51	·4121	1·86	·6206
1·17	·1570	1·52	·4187	1·87	·6259
1·18	·1655	1·53	·4253	1·88	·6313
1·19	·1739	1·54	·4318	1·89	·6366
1·20	·1823	1·55	·4382	1·90	·6418
1·21	·1906	1·56	·4447	1·91	·6471
1·22	·1988	1·57	·4511	1·92	·6523
1·23	·2070	1·58	·4574	1·93	·6575
1·24	·2151	1·59	·4637	1·94	·6627
1·25	·2231	1·60	·4700	1·95	·6678
1·26	·2311	1·61	·4762	1·96	·6729
1·27	·2390	1·62	·4824	1·97	·6780
1·28	·2469	1·63	·4886	1·98	·6831
1·29	·2546	1·64	·4947	1·99	·6881
1·30	·2624	1·65	·5008	2·00	·6931
1·31	·2700	1·66	·5068	2·01	·6981
1·32	·2776	1·67	·5128	2·02	·7031
1·33	·2852	1·68	·5188	2·03	·7080
1·34	·2927	1·69	·5247	2·04	·7129
1·35	·3001	1·70	·5306	2·05	·7178

N.	Logarithm.	*N.*	Logarithm.	*N.*	Logarithm.
2·06	·7227	2·46	·9002	2·86	1·0508
2·07	·7275	2·47	·9042	2·87	1·0543
2·08	·7324	2·48	·9083	2·88	1·0578
2·09	·7372	2·49	·9123	2·89	1·0613
2·10	·7419	2·50	·9163	2·90	1·0647
2·11	·7467	2·51	·9203	2·91	1·0682
2·12	·7514	2·52	·9243	2·92	1·0716
2·13	·7561	2·53	·9282	2·93	1·0750
2·14	·7608	2·54	·9322	2·94	1·0784
2·15	·7655	2·55	·9361	2·95	1·0818
2·16	·7701	2·56	·9400	2·96	1·0852
2·17	·7747	2·57	·9439	2·97	1·0886
2·18	·7793	2·58	·9478	2·98	1·0919
2·19	·7839	2·59	·9517	2·99	1·0953
2·20	·7884	2·60	·9555	3·00	1·0986
2·21	·7930	2·61	·9594	3·01	1·1019
2·22	·7975	2·62	·9632	3·02	1·1053
2·23	·8020	2·63	·9670	3·03	1·1086
2·24	·8065	2·64	·9708	5·04	1·1119
2·25	·8109	2·65	·9746	3·05	1·1151
2·26	·8154	2·66	·9783	3·06	1·1184
2·27	·8198	2·67	·9821	3·07	1·1217
2·28	·8242	2·68	·9858	3·08	1·1249
2·29	·8286	2·69	·9895	3·09	1·1282
2·30	·8329	2·70	·9933	3·10	1·1314
2·31	·8372	2·71	·9969	3·11	1·1346
2·32	·8416	2·72	1·0006	3·12	1·1378
2·33	·8459	2·73	1·0043	3·13	1·1410
2·34	·8502	2·74	1·0080	3·14	1·1442
2·35	·8544	2·75	1·0116	3·15	1·1474
2·36	·8587	2·76	1·0152	3·16	1·1506
2·37	·8629	2·77	1·0188	3·17	1·1537
2·38	·8671	2·78	1·0225	3·18	1·1569
2·39	·8713	2·79	1·0260	3·19	1·1600
2·40	·8755	2·80	1·0296	3·20	1·1632
2·41	·8796	2·81	1·0332	3·21	1·1663
2·42	·8838	2·82	1·0367	3·22	1·1694
2·43	·8879	2·83	1·0403	3·23	1·1725
2·44	·8920	2·84	1·0438	3·24	1·1756
2·45	·8961	2·85	1·0473	3·25	1·1787

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
3·26	1·1817	3·66	1·2975	4·06	1·4012
3·27	1·1848	3·67	1·3002	4·07	1·4036
3·28	1·1878	3·68	1·3029	4·08	1·4061
3·29	1·1909	3·69	1·3056	4·09	1·4085
3·30	1·1939	3·70	1·3083	4·10	1·4110
3·31	1·1969	3·71	1·3110	4·11	1·4134
3·32	1·2000	3·72	1·3137	4·12	1·4159
3·33	1·2030	3·73	1·3164	4·13	1·4183
3·34	1·2060	3·74	1·3191	4·14	1·4207
3·35	1·2090	3·75	1·3218	4·15	1·4231
3·36	1·2119	3·76	1·3244	4·16	1·4255
3·37	1·2149	3·77	1·3271	4·17	1·4279
3·38	1·2179	3·78	1·3297	4·18	1·4303
3·39	1·2208	3·79	1·3324	4·19	1·4327
3·40	1·2238	3·80	1·3350	4·20	1·4351
3·41	1·2267	3·81	1·3376	4·21	1·4375
3·42	1·2296	3·82	1·3403	4·22	1·4398
3·43	1·2326	3·83	1·3429	4·23	1·4422
3·44	1·2355	3·84	1·3455	4·24	1·4446
3·45	1·2384	3·85	1·3481	4·25	1·4469
3·46	1·2413	3·86	1·3507	4·26	1·4493
3·47	1·2442	3·87	1·3533	4·27	1·4516
3·48	1·2470	3·88	1·3558	4·28	1·4540
3·49	1·2499	3·89	1·3584	4·29	1·4563
3·50	1·2528	3·90	1·3610	4·30	1·4586
3·51	1·2556	3·91	1·3635	4·31	1·4609
3·52	1·2585	3·92	1·3661	4·32	1·4633
3·53	1·2613	3·93	1·3686	4·33	1·4656
3·54	1·2641	3·94	1·3712	4·34	1·4679
3·55	1·2669	3·95	1·3737	4·35	1·4702
3·56	1·2698	3·96	1·3762	4·36	1·4725
3·57	1·2726	3·97	1·3788	4·37	1·4748
3·58	1·2754	3·98	1·3813	4·38	1·4770
3·59	1·2782	3·99	1·3838	4·39	1·4793
3·60	1·2809	4·00	1·3863	4·40	1·4816
3·61	1·2837	4·01	1·3888	4·41	1·4839
3·62	1·2865	4·02	1·3913	4·42	1·4861
3·63	1·2892	4·03	1·3938	4·43	1·4884
3·64	1·2920	4·04	1·3962	4·44	1·4907
3·65	1·2947	4·05	1·3987	4·45	1·4929

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
4·46	1·4951	4·86	1·5810	5·26	1·6601
4·47	1·4974	4·87	1·5831	5·27	1·6620
4·48	1·4996	4·88	1·5851	5·28	1·6639
4·49	1·5019	4·89	1·5872	5·29	1·6658
4·50	1·5041	4·90	1·5892	5·30	1·6677
4·51	1·5063	4·91	1·5913	5·31	1·6696
4·52	1·5085	4·92	1·5933	5·32	1·6715
4·53	1·5107	4·93	1·5953	5·33	1·6734
4·54	1·5129	4·94	1·5974	5·34	1·6752
4·55	1·5151	4·95	1·5994	5·35	1·6771
4·56	1·5173	4·96	1·6014	5·36	1·6790
4·57	1·5195	4·97	1·6034	5·37	1·6808
4·58	1·5217	4·98	1·6054	5·38	1·6827
4·59	1·5239	4·99	1·6074	5·39	1·6845
4·60	1·5261	5·00	1·6094	5·40	1·6864
4·61	1·5282	5·01	1·6114	5·41	1·6882
4·62	1·5304	5·02	1·6134	5·42	1·6901
4·63	1·5326	5·03	1·6154	5·43	1·6919
4·64	1·5347	5·04	1·6174	5·44	1·6938
4·65	1·5369	5·05	1·6194	5·45	1·6956
4·66	1·5390	5·06	1·6214	5·46	1·6974
4·67	1·5412	5·07	1·6233	5·47	1·6993
4·68	1·5433	5·08	1·6253	5·48	1·7011
4·69	1·5454	5·09	1·6273	5·49	1·7029
4·70	1·5476	5·10	1·6292	5·50	1·7047
4·71	1·5497	5·11	1·6312	5·51	1·7066
4·72	1·5518	5·12	1·6332	5·52	1·7084
4·73	1·5539	5·13	1·6351	5·53	1·7102
4·74	1·5560	5·14	1·6371	5·54	1·7120
4·75	1·5581	5·15	1·6390	5·55	1·7138
4·76	1·5602	5·16	1·6409	5·56	1·7156
4·77	1·5623	5·17	1·6429	5·57	1·7174
4·78	1·5644	5·18	1·6448	5·58	1·7192
4·79	1·5665	5·19	1·6467	5·59	1·7210
4·80	1·5686	5·20	1·6487	5·60	1·7228
4·81	1·5707	5·21	1·6506	5·61	1·7246
4·82	1·5728	5·22	1·6525	5·62	1·7263
4·83	1·5748	5·23	1·6544	5·63	1·7281
4·84	1·5769	5·24	1·6563	5·64	1·7299
4·85	1·5790	5·25	1·6582	5·65	1·7317

N.	Logarithm.	*N.*	Logarithm.	*N.*	Logarithm.
5·66	1·7334	6·06	1·8017	6·46	1·8656
5·67	1·7352	6·07	1·8034	6·47	1·8672
5·68	1·7370	6·08	1·8050	6·48	1·8687
5·69	1·7387	6·09	1·8066	6·49	1·8703
5·70	1·7405	6·10	1·8083	6·50	1·8718
5·71	1·7422	6·11	1·8099	6·51	1·8733
5·72	1·7440	6·12	1·8116	6·52	1·8749
5·73	1·7457	6·13	1·8132	6·53	1·8764
5·74	1·7475	6·14	1·8148	6·54	1·8779
5·75	1·7492	6·15	1·8165	6·55	1·8795
5·76	1·7509	6·16	1·8181	6·56	1·8810
5·77	1·7527	6·17	1·8197	6·57	1·8825
5·78	1·7544	6·18	1·8213	6·58	1·8840
5·79	1·7561	6·19	1·8229	6·59	1·8856
5·80	1·7579	6·20	1·8245	6·60	1·8871
5·81	1·7596	6·21	1·8262	6·61	1·8886
5·82	1·7613	6·22	1·8278	6·62	1·8901
5·83	1·7630	6·23	1·8294	6·63	1·8916
5·84	1·7647	6·24	1·8310	6·64	1·8931
5·85	1·7664	6·25	1·8326	6·65	1·8946
5·86	1·7681	6·26	1·8342	6·66	1·8961
5·87	1·7699	6·27	1·8358	6·67	1·8976
5·88	1·7716	6·28	1·8374	6·68	1·8991
5·89	1·7733	6·29	1·8390	6·69	1·9006
5·90	1·7750	6·30	1·8405	6·70	1·9021
5·91	1·7766	6·31	1·8421	6·71	1·9036
5·92	1·7783	6·32	1·8437	6·72	1·9051
5·93	1·7800	6·33	1·8453	6·73	1·9066
5·94	1·7817	6·34	1·8469	6·74	1·9081
5·95	1·7834	6·35	1·8485	6·75	1·9095
5·96	1·7851	6·36	1·8500	6·76	1·9110
5·97	1·7867	6·37	1·8516	6·77	1·9125
5·98	1·7884	6·38	1·8532	6·78	1·9140
5·99	1·7901	6·39	1·8547	6·79	1·9155
6·00	1·7918	6·40	1·8563	6·80	1·9169
6·01	1·7934	6·41	1·8579	6·81	1·9184
6·02	1·7951	6·42	1·8594	6·82	1·9199
6·03	1·7967	6·43	1·8610	6·83	1·9213
6·04	1·7984	6·44	1·8625	6·84	1·9228
6·05	1·8001	6·45	1·8641	6·85	1·9242

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
6·86	1·9257	7·26	1·9824	7·66	2·0360
6·87	1·9272	7·27	1·9838	7·67	2·0373
6·88	1·9286	7·28	1·9851	7·68	2·0386
6·89	1·9301	7·29	1·9865	7·69	2·0399
6·90	1·9315	7·30	1·9879	7·70	2·0412
6·91	1·9330	7·31	1·9892	7·71	2·0425
6·92	1·9344	7·32	1·9906	7·72	2·0438
6·93	1·9359	7·33	1·9920	7·73	2·0451
6·94	1·9373	7·34	1·9933	7·74	2·0464
6·95	1·9387	7·35	1·9947	7·75	2·0477
6·96	1·9402	7·36	1·9961	7·76	2·0490
6·97	1·9416	7·37	1·9974	7·77	2·0503
6·98	1·9430	7·38	1·9988	7·78	2·0516
6·99	1·9445	7·39	2·0001	7·79	2·0528
7·00	1·9459	7·40	2·0015	7·80	2·0541
7·01	1·9473	7·41	2·0028	7·81	2·0554
7·02	1·9488	7·42	2·0042	7·82	2·0567
7·03	1·9502	7·43	2·0055	7·83	2·0580
7·04	1·9516	7·44	2·0069	7·84	2·0592
7·05	1·9530	7·45	2·0082	7·85	2·0605
7·06	1·9544	7·46	2·0096	7·86	2·0618
7·07	1·9559	7·47	2·0109	7·87	2·0631
7·08	1·9573	7·48	2·0122	7·88	2·0643
7·09	1·9587	7·49	2·0136	7·89	2·0656
7·10	1·9601	7·50	2·0149	7·90	2·0668
7·11	1·9615	7·51	2·0162	7·91	2·0681
7·12	1·9629	7·52	2·0176	7·92	2·0694
7·13	1·9643	7·53	2·0189	7·93	2·0707
7·14	1·9657	7·54	2·0202	7·94	2·0719
7·15	1·9671	7·55	2·0215	7·95	2·0732
7·16	1·9685	7·56	2·0229	7·96	2·0744
7·17	1·9699	7·57	2·0242	7·97	2·0757
7·18	1·9713	7·58	2·0255	7·98	2·0769
7·19	1·9727	7·59	2·0268	7·99	2·0782
7·20	1·9741	7·60	2·0281	8·00	2·0794
7·21	1·9755	7·61	2·0295	8·01	2·0807
7·22	1·9769	7·62	2·0308	8·02	2·0819
7·23	1·9782	7·63	2·0321	8·03	2·0832
7·24	1·9796	7·64	2·0334	8·04	2·0844
7·25	1·9810	7·65	2·0347	8·05	2·0857

N.	Logarithm.	*N.*	Logarithm.	*N.*	Logarithm.
8·06	2·0869	8·46	2·1353	8·86	2·1815
8·07	2·0881	8·47	2·1365	8·87	2·1827
8·08	2·0894	8·48	2·1377	8·88	2·1838
8·09	2·0906	8·49	2·1389	8·89	2·1849
8·10	2·0919	8·50	2·1401	8·90	2·1861
8·11	2·0931	8·51	2·1412	8·91	2·1872
8·12	2·0943	8·52	2·1424	8·92	2·1883
8·13	2·0956	8·53	2·1436	8·93	2·1894
8·14	2·0968	8·54	2·1448	8·94	2·1905
8·15	2·0980	8·55	2·1459	8·95	2·1917
8·16	2·0992	8·56	2·1471	8·96	2·1928
8·17	2·1005	8·57	2·1483	8·97	2·1939
8·18	2·1017	8·58	2·1494	8·98	2·1950
8·19	2·1029	8·59	2·1506	8·99	2·1961
8·20	2·1041	8·60	2·1518	9·00	2·1972
8·21	2·1054	8·61	2·1529	9·01	2·1983
8·22	2·1066	8·62	2·1541	9·02	2·1994
8·23	2·1078	8·63	2·1552	9·03	2·2006
8·24	2·1090	8·64	2·1564	9·04	2·2017
8·25	2·1102	8·65	2·1576	9·05	2·2028
8·26	2·1114	8·66	2·1587	9·06	2·2039
8·27	2·1126	8·67	2·1599	9·07	2·2050
8·28	2·1138	8·68	2·1610	9·08	2·2061
8·29	2·1150	8·69	2·1622	9·09	2·2072
8·30	2·1163	8·70	2·1633	9·10	2·2083
8·31	2·1175	8·71	2·1645	9·11	2·2094
8·32	2·1187	8·72	2·1656	9·12	2·2105
8·33	2·1199	8·73	2·1668	9·13	2·2116
8·34	2·1211	8·74	2·1679	9·14	2·2127
8·35	2·1223	8·75	2·1691	9·15	2·2138
8·36	2·1235	8·76	2·1702	9·16	2·2148
8·37	2·1247	8·77	2·1713	9·17	2·2159
8·38	2·1258	8·78	2·1725	9·18	2·2170
8·39	2·1270	8·79	2·1736	9·19	2·2181
8·40	2·1282	8·80	2·1748	9·20	2·2192
8·41	2·1294	8·81	2·1759	9·21	2·2203
8·42	2·1306	8·82	2·1770	9·22	2·2214
8·43	2·1318	8·83	2·1782	9·23	2·2225
8·44	2·1330	8·84	2·1793	9·24	2·2235
8·45	2·1342	8·85	2·1804	9·25	2·2246

N.	Logarithm.	*N.*	Logarithm.	*N.*	Logarithm.
9·26	2·2257	9·51	2·2523	9·76	2·2783
9·27	2·2268	9·52	2·2534	9·77	2·2793
9·28	2·2279	9·53	2·2544	9·78	2·2803
9·29	2·2289	9·54	2·2555	9·79	2·2814
9·30	2·2300	9·55	2·2565	9·80	2·2824
9·31	2·2311	9·56	2·2576	9·81	2·2834
9·32	2·2322	9·57	2·2586	9·82	2·2844
9·33	2·2332	9·58	2·2597	9·83	2·2854
9·34	2·2343	9·59	2·2607	9·84	2·2865
9·35	2·2354	9·60	2·2618	9·85	2·2875
9·36	2·2364	9·61	2·2628	9·86	2·2885
9·37	2·2375	9·62	2·2638	9·87	2·2895
9·38	2·2386	9·63	2·2649	9·88	2·2905
9·39	2·2396	9·64	2·2659	9·89	2·2915
9·40	2·2407	9·65	2·2670	9·90	2·2925
9·41	2·2418	9·66	2·2680	9·91	2·2935
9·42	2·2428	9·67	2·2690	9·92	2·2946
9·43	2·2439	9·68	2·2701	9·93	2·2956
9·44	2·2450	9·69	2·2711	9·94	2·2966
9·45	2·2460	9·70	2·2721	9·95	2·2976
9·46	2·2471	9·71	2·2732	9·96	2·2986
9·47	2·2481	9·72	2·2742	9·97	2·2996
9·48	2·2492	9·73	2·2752	9·98	2·3006
9·49	2·2502	9·74	2·2762	9·99	2·3016
9·50	2·2513	9·75	2·2773	10·00	2·3026

Explanation of Table.

The table gives the hyperbolic logarithm of all numbers between 1 and 10, increasing by ·01, from which can be derived all multiples by 10 by adding 2·3026 once, and all multiples by 100 by adding the same number twice.

Example 1. Find $\log_\epsilon 53 \cdot 1$:—

$$\begin{aligned} \log_\epsilon 5\cdot 31 &= 1\cdot 6696 \text{ (by table)} \\ & \ 2\cdot 3026 \\ \hline \log_\epsilon 53\cdot 1 &= 3\cdot 9722 \end{aligned}$$

Example 2. Find $\log_\epsilon 531$:—

$$\begin{aligned} \log_\epsilon 5\cdot 31 &= 1\cdot 6696 \text{ (by table)} \\ & \ 2\cdot 3026 \\ & \ 2\cdot 3026 \\ \hline \log_\epsilon 531 &= 6\cdot 2748 \end{aligned}$$

NOTES AND ADDENDA.

APPENDIX.

NOTES AND ADDENDA.

CHAPTER I. (ARTS. 1–6).

PAGE 4.—It is now not uncommon to find temperature measured on the Centigrade scale, while British units are in other respects adhered to. There is no doubt that the artificial zero of Fahrenheit's scale is inconvenient, especially to beginners, but this disadvantage disappears when absolute temperatures are being used; and the smaller magnitude of the Fahrenheit degree is a very considerable advantage, for it enables us to dispense almost altogether with fractions of a degree. The Centigrade degree is too large to allow this to be done.

PAGE 5.—Among the earliest and simplest of the numerous formulæ which have been employed to represent the relation between the temperature and pressure of steam we find

$$p = \left(\frac{a + t}{b}\right)^n$$

where a, b, n are constants determined by comparison with the table. Thus, for example, Tredgold, who was one of the first to use this formula, expressed the elastic force of steam by writing, $a = 100$, $b = 177$, $n = 6$, for Fahrenheit degrees and inches of mercury. All formulæ of this class are very defective in form, and consequently apply only to a limited portion of the table for which their constants have been specially determined. In particular, the index n is smaller the higher the pressure, the value 6 being applicable only for pressures below the atmosphere. At four atmospheres it is reduced to 5, and at high pressures to 4·5. At pressures near that of the atmosphere the best value is 5·5. The simplicity of this formula renders it convenient for some purposes, notwithstanding its deficiencies.

On examining the table we find the pressure increases much faster than the temperature, a first rough approximation being given by Dalton's Law, that the pressure increases in *geometrical* when the temperature increases in *arithmetical* progression. If, for example, we take a series of temperatures increasing by equal increments of 20° from 212° and find the ratio of each pressure to that which immediately precedes it, we find

Pressure =	14·7	21·6	30·9	43·3	59·5	80·2
Ratio =		1·47	1·43	1·40	1·37	1·35

If Dalton's Law were exact, the ratios would be equal, and the formula connecting pressure and temperature would be either

$$p = A\,\epsilon^{bt} \quad \text{or} \quad \log p = B\,(t - a)$$

where, A, b and B, a are constants. Dalton's Law is not exact, as is shown by the numbers just given, which diminish slowly as the pressure rises; but it is sufficiently nearly true to explain why all formulæ representing with tolerable accuracy the elastic force of steam, are either exponential or logarithmic in form. Regnault adopted a formula of the first kind, but the most useful are of the logarithmic type. That given in the text is an example of Roche's formula, the values of the constants being those employed by Tate. It is the best formula to use in the absence of the tables, being simple and quite sufficiently accurate for all pressures occurring in practice. A full account of all the older formulæ will be found in Dixon's *Treatise on Heat*, cited in the text. We shall give other formulæ employed for theoretical purposes further on.

Page 7.—Strickly speaking, the thermal unit is the quantity of heat necessary to raise the temperature of 1 lb. of water through 1°, at some specified temperature. In the text, following Rankine, the temperature chosen is 39° F., the critical temperature at which water has its maximum density; by physicists generally 32° F. has been hitherto adopted, it being, in fact, assumed on Regnault's authority, that the specific heat of water is practically constant below about 100° F. It is now, however, known from experiments by Rowland, subsequently confirmed by Griffith, that the specific heat of water in reality varies very perceptibly at low temperatures, and the temperature specified should be about 60° F.

The volume of 1 lb. of water at its temperature of maximum density is

$$s = \cdot 01602 \text{ cubic foot};$$

at any other temperature,

$$s = (1 + m) \times \cdot 01602,$$

where m is a coefficient, which at temperatures below 212° is given by a table found in all works on physics, and which we need not repeat here. Above 212° experiments made by M. Hirn give the following results:*—

Temp.	1 + m.
392° Fahr.	1·159
356° „	1·127
284° „	1·0795
212° „	1·0431

* *Mémoire sur la Thermodynamique*, p. 18, par G. A. Hirn. Paris, 1867.

the value of s therefore increases from ·01602 to ·01671 at 212°, and to ·01857 at 392°: strictly speaking, this dilatation ought to be taken into account in some of the calculations of this book; but the error so introduced is very minute, far less than the least probable error of the most accurate calorimetrical experiments.

The quantity of heat necessary to raise the temperature of a pound of water through 1° increases with the temperature, as has already been explained: a very minute part of this increase is due to the external work corresponding to the dilatation of the water, but the greater part represents a true increase in the energy requisite to produce the internal changes corresponding to the rise of temperature. The quantity of heat (h) necessary to raise 1 lb. of water from 32° to t° was determined by Regnault in the following way:—The boiler used for the experiments on the elastic force of steam was filled three-quarters full, and maintained at a steady pressure by use of the air chamber as before (Art. 2, p. 3). A small pipe, curved downwards into the water, and provided with a stopcock, connected the boiler with a vessel filled with water and open to the atmosphere, which served as a calorimeter: the pipe projected inwards into this vessel, and was pierced with small holes, the end of the pipe being closed. On opening the stop-cock, the difference of pressure between the boiler and the atmosphere causes hot water to flow into the calorimeter, the rise of temperature of the water in which furnishes a measure of the heat given out by the cooling water. Rankine has pointed out * that the value of h thus found by Regnault is too large, because the energy exerted by the difference between the boiler pressure and the atmospheric pressure will be employed in generating kinetic energy and overcoming frictional resistances, and will appear as heat in the calorimeter. The energy in question will—neglecting the dilatation of water—be given by

$$\text{Energy} = \frac{\cdot 016\,(P - P_0)}{772} \text{ thermal units per pound,}$$

where P is the boiler pressure, P_0 the atmospheric pressure, both in pounds per square foot; which becomes, reducing the pressures to pounds per square inch,

$$\text{Energy} = \frac{p - 14\cdot 7}{336} \text{ thermal units per pound.}$$

Rankine accordingly diminishes the values of h given by Regnault by small quantities calculated in this way, and gives a formula representing the corrected results. It is not, however, clear that the whole amount of this correction ought to be subtracted, for the pipe being long and small, the greater part of the energy exerted will be spent in overcoming frictional resistances, and it does not seem evident that all the heat thus generated will reach the calorimeter. Moreover, Rankine's formula at the lower temperatures appears to deviate more from the experiments than is justified by the correction in question.

* *Transactions of the Royal Society of Edinburgh*, vol. xxi.

In any case the correction is small, as the above formula shows, and is not the only correction to which the values of h should, strictly speaking, be subjected, though no doubt it is the most important; we have therefore employed Regnault's values of h, as has been done by other writers on the subject, notwithstanding that they are slightly too large at high pressures.

Since the first edition of this work was published, Regnault's results have been questioned, on account of discrepancies between them and the data from which they are stated to be calculated. This discrepancy may probably only be apparent, not real.*

PAGE 15.—The curves given by the equation $P\,V^n$ = const. where P is the ordinate, V the abscissa, and n an index, occur so frequently in the theory of the steam engine, that it is convenient to investigate their geometrical properties without reference to the particular application to be made of them.

Area.—Let $A\,B$ (Fig. 40, p. 406) be any plane curve, $L\,N$, $K\,M$ two ordinates $P_1\,P_2$, the corresponding abscissæ $O\,N$, $O\,M$ being $V_1\,V_2$, then the area $L\,K\,M\,N$ comprised between the curve, the ordinates, and the axis, is given by the formula

$$\text{Area} = \int_{V_1}^{V_2} P\,d\,V;$$

but in the present case

$$P\,V^n = P_1\,V_1^n,$$

whence by substitution

$$\text{Area} = P_1\,V_1^n \int_{V_1}^{V_2} \frac{d\,V}{V^n}.$$

Performing the integration, we find

$$\text{Area} = P_1\,V_1^n \cdot \frac{V_2^{1-n} - V_1^{1-n}}{1-n},$$

which, remembering that

$$P_1\,V_1^n = P_2\,V_2^n,$$

may be written

$$\text{Area} = \frac{P_1\,V_1 - P_2\,V_2}{n-1},$$

which is the result so frequently used in the text. When $n = 1$ the formula fails: the curve in that case is a common hyperbola, and its area is given by

$$\text{Area} = P_1\,V_1 \,.\, \log_\epsilon \frac{V_2}{V_1}.$$

* See a paper by Mr. W. Sutherland, *Phil. Mag.*, vol. xxvi. 1888.

If to the area thus calculated we add the rectangle $D N$, we obtain

$$\text{Area } D L K M O = P_1 V_1 + \frac{P_1 V_1 - P_2 V_2}{n - 1}$$

$$= \frac{n}{n - 1} \cdot P_1 V_1 - \frac{1}{n - 1} \cdot P_2 V_2.$$

When the curve is regarded as an expansion curve, the ratio $V_2 : V_1$ is the ratio of expansion, usually denoted by r : P_1 is the initial, and P_2 the terminal pressure, so that we have

$$P_2 = \frac{P_1 V_1^n}{V_2^n} = P_1 \cdot r^{-n},$$

and if P_m be the mean forward pressure,

$$P_m = \frac{\text{Area } D L K M O}{V_2} = \frac{n}{n - 1} \cdot \frac{P_1}{r} - \frac{1}{n} \cdot P_2.$$

For example, suppose the expansion curve to be the saturation curve, then $n = 1 \cdot 0646$,

$$P_m = 16 \cdot 47 \frac{P_1}{r} - 15 \cdot 48 \, P_2.$$

When $n = 1$, the rule fails, and we have

$$P_m = P_1 \cdot \frac{1 + \log_\epsilon r}{r}.$$

Geometrical Construction.—The curve may be constructed approximately on the following general method (see Fig. 40). Starting from L, draw any horizontal line $Q C$, at a small distance below L, then the question is to find S, the point on the curve which lies on $Q C$. For this purpose, set downwards

$$O T = \frac{D O}{n - 1} : O R = \frac{O Q}{n - 1},$$

and complete the rectangle $N T$ as shown in the figure, also draw the horizontal line $R F$ to meet the ordinate $L N H$ in F, as shown in the figure. Then bisect $D Q$ in Z, join $Z F$, and prolong it to meet the horizontal through T in E: a vertical through E will be the new ordinate very approximately, and by its intersection with $Q C$ will determine S.

For, completing the rectangle $T S^1$, as shown by the dotted lines in the figure, the rectangles $Z H$, $Z G$ are equal, and $Z N$ is common,

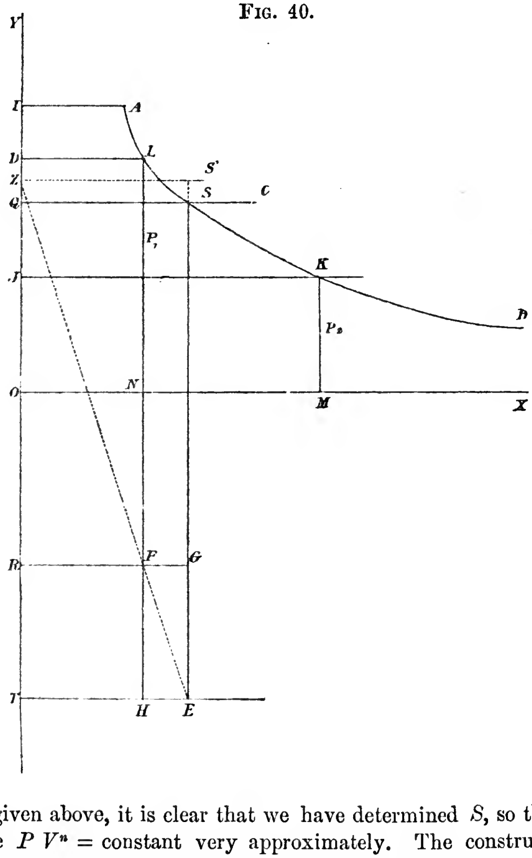
$$\therefore \; O H = O G + N S^1;$$

that is to say,

$$\text{Rectangle } N S^1 = \text{Rectangle } O H - \text{Rectangle } O G.$$

Now if the points $L S$ be taken near enough together, the area of the rectangle $N S^1$ may be made to differ as little as we please from the area of the strip of the curve $L S N$, and the rectangles $O H$, $O G$ are equal to $P_1 V_1$, $P_2 V_2$, respectively divided by $n - 1$; hence, referring to the formula

FIG. 40.



for the area given above, it is clear that we have determined S, so that it lies on the curve $P V^n$ = constant very approximately. The construction can now be repeated as many times as we please, so as to obtain more points on the curve. The physical interpretation of this construction, when applied to the adiabatic curve for air, will be understood on referring to Art. 35 in Chapter IV.

The mean pressure is found graphically by inverting the construction. For let us suppose $L S$ to be two given points not necessarily near together, but anywhere on the curve: then, inverting the construction, the line $Z S^1$ will be determined, the ordinate of which must be the mean pressure during expansion.

Deviation from an Hyperbola and from the Saturation Curve.—All curves of this class resemble an hyperbola the more closely the nearer n is to unity. When n is nearly equal to 1 the deviation is very small, and it is often important to know it, the more so as the construction just given then becomes cumbrous.

Taking the equation

$$P V^n = P_1 V_1^n,$$

to find the vertical distance between two curves, differing slightly in the value of n, we have only to differentiate, assuming V constant and n variable, then

$$\delta P = - P \cdot \log_\epsilon V \,.\, \delta n + P \,.\, \log_\epsilon V_1 \,.\, \delta n,$$

the curves being supposed to start from the same initial point indicated by the suffix 1. Or if r be the ratio of expansion $V : V_1$,

$$\delta P = - P \,.\, \log_\epsilon r \,.\, \delta n,$$

which becomes, if one of the curves be an hyperbola,

$$\delta P = (1 - n) P \,.\, \log_\epsilon r.$$

For example, the vertical distance of the saturation curve of steam from an hyperbola is approximately

$$- \delta P = \cdot 0646 \,.\, P \log_\epsilon r,$$

where r is the ratio of expansion, reckoned from an initial point, from whence the two curves start.

The horizontal distance of two curves near each other may in like manner be found by differentiation, considering P constant and V variable; whence is obtained, by a similar process,

$$\delta V = - \frac{V}{n} \,.\, \log_\epsilon r \,.\, \delta n.$$

CHAPTER II. (Arts. 7–15).

Page 22.—Joule's second series of experiments for the purpose of determining the mechanical equivalent of heat are described in a paper published in the *Philosophical Transactions* for 1878 (vol. 169, p. 365). The method employed was the same as before, and the value obtained was practically the same. Shortly afterwards, in 1879, Rowland made some important experiments by Joule's method, in a manner calculated to show the variation in the specific heat of water (if any) at low temperatures (p. 402). A clear and brief account of these experiments will be found in Prof. Peabody's work, cited on page 318. Their results at the same temperature are about one-quarter per cent. greater than those of Joule, accepting which and adopting a minute correction for latitude and another of more importance for thermometric measurement, the value 772 is raised to 778, an increase of rather less than 1 per cent. At the lowest temperatures the value obtained was greater than at higher, from which the conclusion drawn was that the specific heat of water was greater at low temperatures, the minimum being at about 80° F.

[1895.] The precise value of the mechanical equivalent of heat depends on the thermometer employed as a standard, the number 772 obtained by Joule, as will be seen from the paper cited referring to a special mercurial thermometer and a temperature of 60° F. The standard now used by physicists is the nitrogen thermometer of the Bureau International at Paris, referred to which Joule's number becomes 775 and Rowland's (probably) 777·3. Experiments made by Miculesca in 1892, employing a method similar in principle to that of Rowland and Joule, give 776·6. On the other hand, researches have been carried out by Mr. Griffith in 1893, and by Professor Schuster and Mr. Gannon in 1894, in which a totally different method was employed, a given mass of water being heated by an electric current of known strength. Both experimentalists obtained practically the same result, the first being 780·2 and the second 779·7, referred to the same thermometer and the same temperature as before. The particulars now given are taken from Schuster and Gannon's paper, read before the Royal Society in November 1894, and published in the 'Proceedings.' Till the discrepancy between the results of the two methods is cleared up, the precise value of the equivalent must be considered still uncertain. As a convenient provisional value 780 may be taken, a number just 1 per cent. greater than the old value, and convenient for corrections.

When the value 780 is adopted, the thermal equivalent of a horse-power becomes 42·3 instead of 42·75, and the constant in the formula for the specific volume of steam is changed from 479 to 484 (p. 15). The important divisor by which areas represented in the usual way are converted into thermal units may be taken as 5·4 instead of 5·36.

PAGES 31–35.—In the original notes of 1871, on which this work was based, the Author laid great stress on the conception of internal work, and remarked with some surprise that, though universally admitted, the principle was not placed in the rank of a law of thermodynamics by authorities on the subject. In his recent valuable treatise (*Thermodynamique*, Paris, 1887), Bertrand states it as a fundamental law, ascribing it to Mayer. Subsequently the Author inclined to the generally received view that the principle may be considered as axiomatic. Is it not implied in the statement that the state of a body is completely defined by any two of the three quantities, pressure, volume, and temperature?

CHAPTER III. (Arts. 16–30).

Page 40.—The term "efficiency" here, and constantly throughout the book, used, as Rankine used it, for the ratio of work done to heat expended, is undoubtedly objectionable. Some name is necessary for this ratio, but it appears to the Author that the term "co-efficient of performance," suggested in Chapter V., page 151, and employed in Chapter XI., is far preferable.

Pages 48–58.—These articles (23–26) are better omitted on first reading the subject; they may with advantage be postponed till Chapter VI. has been commenced.

Page 63.—The elementary articles here introduced into the present edition, on the compound engine of the receiver type, are intended chiefly to call the attention of the reader at the earliest possible moment to the enormous influence on the working of a compound engine of the relative liquefaction in the cylinders. The Author has for many years past laid great stress on this point in his teaching, being convinced that the practice, common in the best treatises on the steam engine, of making calculations without any reference to this matter, is most misleading.

Page 67.—Thanks to the introduction of gas engines, the word "cycle" has become part of the vernacular of engineering science. In the present edition (1890) the term has, therefore, from the commencement, been used more freely.

CHAPTER IV. (Arts. 31–52).

Page 74.—The experiments referred to are described in a paper on the "Thermal Effects of Fluids in Motion," originally published in the *Philosophical Transactions* for 1854, and reprinted in vol. i. of his *Mathematical and Physical Papers*, by Sir W. Thomson. The conclusion arrived at was that the absolute zero was, within two or three-tenths of a degree," 273°·7 C. below the melting point of ice. In his article on "Heat" in the *Encyclopædia Britannica*, Sir W. Thomson places it half a degree lower, at 273°·1. This value, which corresponds to 459°·5 F., is now universally received; but for the purposes of a theory of the steam engine the alteration is not of much importance. The same remark applies to certain small changes consequent on the adoption of a larger value of the mechanical equivalent of heat.

Page 77.—If we assume that a body exists, for which the relation between pressure volume and temperature is—

$$P\,(v+b)=c\,T,$$

where b is a constant which is zero for a perfect gas, and if we further suppose the specific heats constant and given by the same equation as in a perfect gas, namely—

$$K_p - K_v = c,$$

it will be found by reasoning, as in the text, that

$$\text{Internal Work} = K_v (T_2 - T_1)$$

depending solely on the temperature, and the thermodynamics of such a body can be treated as on page 81. A mixture of a gas and a finely-divided solid, furnishes an example of a body so constituted, as will be seen further on in this chapter (Art. 46, p. 113).

Page 78.—The assumption that the vapour of a liquid is a perfect gas, though untrue for any actual vapour, and, perhaps, physically impossible, leads to results of importance.

Let Q be the total heat of evaporation at temperature T, and let the vapour be superheated to the temperature t at constant pressure P, its volume then being V. Finally, suppose it further superheated at constant volume to a given temperature θ. If s be the volume of the liquid, the internal work done must be

$$\text{Internal Work} = Q + K_p (t - T) + K_v (\theta - t) - P (V - s).$$

Omitting s as very small, and remembering that $PV = cT$, where $c = K_p - K_v$ this becomes

$$\text{Internal Work} = Q + K_v \theta - K_p T.$$

Now the final temperature θ and volume V of the vapour being given, the internal work must also be given, that is, it cannot depend on the temperature of evaporation T, and we learn that

$$Q = \text{Constant} + K_p . T.$$

It appears, therefore, that if the vapour of a liquid is a perfect gas, its total heat of evaporation must increase uniformly with the temperature, the rate of increase being K_p, the specific heat of the vapour at constant pressure. In a few liquids, an important example being water, the rate of increase is uniform, but the specific heat of superheated steam is not ·305. The demonstration here given is taken, with slight modifications, from Bertrand's *Thermodynamique*, p. 77. We shall return to the question further on.

Page 92.—The object of the articles on air and gas engines, most of which have been added to the present (1890) edition, is not of course to convey to the reader practical knowledge of these machines, which would be impossible in so small a space, but to illustrate thermodynamical principles. For this purpose, the thermal cycle and the mechanical cycle have been separately traced in full for each case. In no other way can a true idea of the way in which a heat engine works be so easily obtained. Many questions relating to a steam engine cannot be understood without reference to the simpler case of an air engine.

PAGE 108.—The articles on gas engines here introduced are intended chiefly for reference in Chapter VII. On first reading the subject they may be passed over.

PAGE 118.—Refrigerating machines are seldom mentioned in treatises on thermodynanics ; yet it seems clear that this aspect of a thermodynamic machine is, in principle, quite as important as that in which the conversion of heat into work is considered. Moreover, the manufacture of these machines has of late years become a great industry. The articles here introduced in this (1890) edition on reversed air engines should be carefully studied before going further, so that the idea of reversibility may be fully grasped before the difficulties connected with Carnot's principle are encountered. It is too often supposed that Carnot's reversed heat engine is a purely ideal machine.

CHAPTER V. (ARTS. 53–56).

PAGE 130.—The statement that heat cannot pass from a cold body to a hot one was worded by Clausius in two different ways; in the earlier the qualifying words were "of itself," in the latter, "without a compensation." If a heat motor with a wide range of temperature drives a heat pump with a narrow range of temperature, the arrangement as a whole is self-acting, and yet in a certain sense causes heat to pass from a cold body to a hot one, and for this reason, Clausius preferred the later wording. The Author has adhered to the earlier, as being, in the first instance, more easily intelligible. By many writers Carnot's principle is regarded as the second law, but it is difficult to see how a statement capable of proof by reference to some still more fundamental axiom can properly be regarded as a law. The question is of course one of words only; the Author here follows Maxwell.

That there is a difference in degree between the first and second laws is generally admitted. In the preface to the first edition of this book, the Author, again following Maxwell, stated that the impossibility of converting heat of low temperature into heat of high temperature might perhaps be evaded if we could "operate on the ultimate molecules of which bodies are composed." On this point the reader is referred to *Application of Dynamics to Physics and Chemistry*, chapter xviii., by Professor J. J. Thomson.

PAGE 132.—The alternative statement here made was given by Sir W. Thomson (now Lord Kelvin), in 1851, in a paper which is reprinted in vol. i. of his *Mathematical and Physical Papers.* Sir W. Thomson remarks in this paper (p. 181), that his own and Clausius' statement differ only in form, either being a necessary consequence of the other.

Page 136.—A translation of Carnot's work, published in 1824, *Reflexions sur la Puissance Motrice du Feu*, by Professor Thurston, has recently (1890) been announced. Some rough notes, published in 1871, show that Carnot had, before his death in 1827, arrived at the conclusion that heat and mechanical energy were convertible, and had actually obtained a correct value of the mechanical equivalent of heat.

Page 139.—The conception of an absolute scale of temperature is due to Sir W. Thomson, and the fundamental experiments by which the close coincidence of this scale with that of the air thermometer was established were made by him in conjunction with Dr. Joule. They have already been referred to on p. 409.

Page 145.—It is shown in Chapter XI. that a steam engine may be so constructed as to be a theoretically perfect heat engine, not only by use of a Carnot cycle, but also by the addition of a properly constructed feed-water heater. The consumption of heat and steam are then given by the formulæ here obtained.

Page 146.—The statements here made are examined and verified in Chapter XI.

Page 148.—Like so much else in thermodynamic science, the idea of a warming machine is due to Sir W. Thomson. His original paper, published in 1852, will be found at page 515 of the work already more than once cited. The numerical results obtained by him differ from those given in the table, for reasons explained in Chapter VII.

Page 149.—Refrigerating machines form a very large subject. The best comprehensive account of the various processes is perhaps that given by Mr. Lightfoot in a paper read before the Institution of Mechanical Engineers in 1886.

CHAPTER VI. (Arts. 67–89).

Page 153.—For the purpose of finding the density of steam, Roche's Formula (p. 402) is not sufficient. Though giving approximate results for a wide range of pressure, it is undoubtedly incorrect in form, discrepancies appearing on minute examination, which become large errors if the formula is applied to very low pressures. For theoretical investigations a formula more exact in form is required, expressed in terms of the *absolute* temperature. Of these, the first to be mentioned is Rankine's formula

$$\log p = A - \frac{B}{T} - \frac{C}{T^2}$$

where T is the *absolute* temperature, and A, B, C are constants. This formula represents the experimental results very closely, not only for the vapour of water, but also, with suitable modifications in the constants, in the case of other liquids. The values of the constants employed by Rankine in the case of steam were

$$A = 6{\cdot}1007 : \log B = 3{\cdot}43642 : \log C = 5{\cdot}59873,$$

in which it is to be noted that the pressure is supposed in pounds per square inch, and that the temperature must be reckoned from a zero 461°·2 below the zero of Fahrenheit's scale.

Hence by differentiation we find

$$k + 1 = \frac{T}{P} \cdot \frac{dP}{dT} = \frac{B'}{T} + \left(\frac{C'}{T}\right)^2,$$

where B' C' are other constants easily derived from B and C, their values being

$$\log B' = 3{\cdot}79864 : \log C' = 3{\cdot}13099.$$

The meaning of this number k, and its connection with the internal-work-pressure, are explained in Chapter VI., page 155. It was by this formula that Table V. was calculated.

Again, it was shown in Chapter V., page 146, that

$$L = u\,T\frac{d\,P}{d\,T},$$

where u is, as usual, written for $v - s$;

$$\therefore\ u = \frac{L}{T} \cdot \frac{d\,T}{d\,P}.$$

Proceeding as above, a formula is obtained by which u is found in terms of the latent heat of evaporation L. It was in this way that Rankine calculated the densities of steam registered in his tables, from which Table III. in this book has been calculated by interpolation.

Rankine's formula is of the semi-empirical kind, having been obtained by reasoning from a certain supposition as to the constitution of matter called the "hypothesis of molecular vortices." This hypothesis has not obtained acceptance; moreover, on close examination discrepancies may be detected, which may, perhaps, be diminished, but probably cannot be removed, by a fresh determination of the constants. Of much more interest is Dupré's formula, obtained by supposing steam a perfect gas (p. 410), and writing

$$L = a - b\,T, \qquad P\,u = c\,T,$$

where a, b, c are constants. The theoretical values of a and b are those given further on (p. 167), multiplied by 772, and that of c (neglecting $s = \cdot 016$) is 85·5 (p. 79). Substituting and dividing by T^2, the formula employed in this article becomes

$$\frac{1}{p} \cdot \frac{d\,p}{d\,T} = \frac{a}{c\,T^2} - \frac{b}{c\,T}.$$

Integrating we find

$$\log_\epsilon p = A - \frac{a}{c\,T} - \frac{b}{c} \log T,$$

where A is the constant of integration. Now, if we employ in this formula the theoretical values of a, b, c, we obtain results which are not exact indeed, but, as Dupré pointed out in his *Théorie Mécanique de la Chaleur* (Paris, 1869), p. 97, they are not very far from the truth for steam and other vapours. Recently, Bertrand, in his work already cited (p. 408), has examined Dupré's formula, and shown that by a proper determination of the constants it may be made to represent the (complete) relation between pressure and temperature with great accuracy in the case of sixteen different vapours, as determined experimentally by Regnault. Writing the formula in the form

$$\log p = A - \frac{B}{T} - C \,.\, \log T,$$

the values of A, B, C are determined for each vapour by comparison with the experimental results. These values are for the vapour of

	A.	*B.*	*C.*
Water	17·44324	2795	3·8682
Ether	13·42311	1729·97	1·9787
Ammonia	13·37156	1449·83	1·8726

pressures being supposed in millimetres of mercury, temperatures in degrees Centigrade from a zero 273° below the zero of the Centigrade scale, and the logarithms being common, not hyperbolic. In cases where a formula is required for theoretical purposes, Dupré's may be considered the best, being very approximately accurate in form, and the derived formula very simple.

When a liquid is not homogeneous, but is a mixture of liquids of different degrees of volatility, the relation between pressure and temperature is of a different character. A formula of the exponential type (p. 402) would probably be found more suitable for representing it. Petroleum is a case of this kind.

PAGE 153A.—The possible errors referred to here have already been noticed (pp. 408, 409). It will be seen that the probable error in the value of the mechanical equivalent of heat is greater than is stated in the text, and that consequently the density of steam may be as much as 1 per cent. less, and its specific volume 1 per cent. greater than as calculated by Rankine.

It is of importance to remark that the external work $P\,u$, together with the other properties of saturated steam, when expressed in thermal units, are not affected by a change in the mechanical equivalent.

PAGE 172.—An extensive series of experiments on superheated steam were made by Hirn, and are described by him in his *Théorie Mécanique de la Chaleur*. Attempts have been made to connect the results of these experiments with the properties of saturated steam, and express them by a modification of the formula for the permanent gases, which may be written

$$P\,(v + \beta) = c\,T.$$

The modification consists in the introduction of a quantity β, which depends in some way on the temperature volume or pressure, and is determined by comparison with experiment. Partly by reasoning of a not very satisfactory character, and partly by experiments on sudden expansion, Hirn was led to the conclusion stated in the text, that the isodynamic curve for superheated steam is a common hyperbola. The simplest case of this is when the internal energy is given by the formula

$$I = I_0 + a\,.\,P\,V,$$

where I_0 and a are constants. This equation should be true for saturated steam, and on trial by aid of the tables it will be found that it is true, within probable limits of experimental error, an average value of a being 3. Reasoning as on page 80, it immediately follows that the adiabatic curve for superheated steam is of the form $P\,V^n$ = const. where

$$n = 1 + \frac{1}{a} = \frac{4}{3}.$$

Further, when steam is superheated from state 1 to state 2, we shall have

$$\begin{aligned}\text{Heat expended} &= a\,V(P_2 - P_1) \quad (\text{when } V \text{ is constant}),\\ &= (a+1)\,P\,(V_2 - V_1) \quad (\text{when } P \text{ is const.})\end{aligned}$$

The specific heats at constant volume (K_v) and at constant pressure (K_p) are now one or both variable, and are given by

$$K^v = a\,V\,\frac{d\,P}{d\,T};\; K_p = (a+1)\,P\,\frac{d\,V}{d\,T}.$$

We must now make one further assumption on experimental grounds, and according to Zeuner we ought to take

$$K_p = \cdot 48 \times 772,$$

the value ·48 being that obtained by Regnault for the specific heat at constant pressure of superheated steam, by experiments which, it may be remarked, were only made at atmospheric pressure. Substitute in the equation for K_p just found, and integrate, then

$$P\,V = 92\cdot 6\;T - f(P),$$

where $f(P)$ is a function of P determined by reference to the adiabatic equation found above, and assuming that the temperature in adiabatic expansion falls as it would do in a permanent gas. We thus finally obtain the equation

$$P\,V = 92\cdot 6\;T - 950\,\sqrt[4]{P},$$

the constant 950 being found by observing that the equation must be true for saturated steam. This formula represents what is known about the properties of superheated steam with sufficient accuracy for practical purposes, but it, nevertheless, can only be regarded as empirical.

According to Zeuner, then, β is a function of the pressure only, while the specific heat at constant volume varies, diminishing as the temperature rises. Hirn, on the other hand, supposed the specific heat at constant volume constant, from which hypothesis, reasoning as before, we conclude that β is a function of the volume only, and that the specific heat at constant pressure increases as the temperature rises. The facts may, probably, so far as they are known at present, be represented equally well by either hypothesis, but Zeuner's equation is that most used.

[1895.] In a paper on the *Rationalisation of Regnault's Experiments on Steam*, read at Paris in the summer of 1889 before the Institution of Mechanical Engineers, Mr. Macfarlane Gray makes the same assumptions as on the preceding page with a value of a which is taken as $2\frac{1}{2}$: but the further assumption made is that β is a function not of the pressure or volume, but of the *temperature*. It is supposed that

$$\beta = \frac{b}{T^a}$$

where b is constant and a is the constant already employed.

The characteristic equation is now

$$P\,V = c\,(1 - z)\,T$$

where z is a function given by the equation

$$z = \frac{b}{c} \cdot \frac{P}{T^{a+1}}.$$

If ΔQ be the supply of heat during a small change of state we have, using the preceding notation,

$$\Delta Q = \Delta I + P \,.\, \Delta V = a \,\Delta (P V) + P \,.\, \Delta V$$
$$= (a + 1) \,\Delta (P V) - V \,.\, \Delta P.$$

Now ΔP can be derived in terms of z from the equation just given for z, and $\Delta (P V)$ from the equation for $P V$. Hence by substitution

$$\Delta Q = - a c T \,.\, \Delta z - c T \frac{\Delta z}{z} \,.$$

Divide by T and integrate, the resulting value of the entropy is

$$\phi = \phi_0 - c (a z + \log z).$$

This equation, obtained by Mr. Gray in a manner not very easy to follow, shows that the supposed value of β is consistent with the primary assumption, and, in fact, it is the only value which is also consistent with the supposition that β is a function of the temperature only. The specific heats at constant pressure and constant volume are found from the equations at the bottom of page 415; they are neither of them constant, but diminish as the temperature rises.

The properties of the gas are thus completely defined and are consistent with each other. It has not, however, been shown that these properties are possessed by superheated steam or any actual gas, and, in fact, there is no reason to think that such is the case; the interest of the investigation lies in the formula for steam pressure which is deducible therefrom. Let us suppose that a vapour does possess these properties, and let us proceed as on page 410. Starting from water at 0, let Q be the total heat of evaporation at pressure P and let any point 1 be reached by superheating first at constant pressure, then at constant volume, then for the internal energy

$$I = Q - (a + 1) P V + P s + a P_1 V_1.$$

As this must be independent of P

$$Q = (a + 1) P V + I_0 - P s$$

where I_0 is constant. Regarding s as negligible,

$$Q = S + (a + 1) P V = S + (a + 1) c (1 - z) T$$

where S is very approximately constant. This is Mr. Gray's formula for the total heat of evaporation, which differs from Regnault's by quantities within the limits of experimental error.

A formula for the relation between the pressure and temperature of steam is now obtained by equating the value of ϕ for the vapour to that previously obtained in terms of z for the gas. To do this it is necessary to know the specific heat of the liquid at different temperatures, and it is in fact implicitly assumed that the specific heat is constant.

Assuming $a = 2\frac{1}{2}$ the resulting formula represents the relation between the pressure and temperature of saturated steam with great accuracy, for the whole range of temperature from the temperature of melting ice up to the neighbourhood of the critical temperature. But this is far from sufficient to show that the assumptions on which it is based are correct. The formula is a modified form of Dupré's (p. 414), and should be compared with that obtained by M. Kraevitch in a paper which will be found in the *Philosophical Magazine* for January 1894.

Page 161.—While reading the articles on the expansion of steam, that part (pp. 48–58) of Chapter III. which refers to the total expenditure of heat in an engine should be considered, and the method applied to the case of an engine working with steam which always remains dry and saturated. This case, though hardly ever occurring in practice, is nevertheless instructive, and has the advantage of being more completely definite than any other. It is sometimes adopted as a standard of efficiency. The case where the index of the expansion curve has a value (say ·9) less than unity is also worth investigation, as it represents many actual expansion curves.

Though not practically used, the formulæ (p. 57) given by Rankine for the pressure equivalent to the expenditure of heat are of some interest.

Page 175.—This formula for k may be derived from Dupré's formula (p. 414). The constants employed were obtained direct from the table; they agree very closely, but are not identical with those derived from Bertrand's coefficients given previously.

Page 176.—The additional examples of the expansion of steam placed here as an appendix to Chapter VI. formed the second half of Chapter VII. in the first edition, the first half being now incorporated with Chapter VI. The Author has reprinted these examples with some hesitation, the method adopted, though of some interest, not being now often used. The examples are, however, required for reference, and moreover the method has been recently employed in a memoir by M. Dwelvshauser-Déry, on the exchange of heat between steam and metal, in the *Bulletins de la Société Industrielle de Mulhouse* for 1888. In the Diagram of Exchanges given in this memoir, the ordinate of the curve drawn, showing the inflow and outflow of heat, is the difference of the ordinates of the curves of internal and external work according to the method of this chapter, and, as will be seen on reference to page 181 (page 150 of the first edition), the idea of drawing a curve of internal work for an actual expansion curve occurred to the Author. In the memoir referred to diagrams are drawn for eight examples; two are with saturated steam, and six with superheated steam. The results are highly instructive, and are, doubtless, trustworthy, the diagrams relied on being taken by Hallauer, whose accuracy in such work is so well known.

The Author is inclined to think, however, that a method based on the thermal diagram described in a later chapter would be more easy in application.

CHAPTER VII. (Arts. 91–104).

Page 203.—This chapter formed the first half of Chapter VIII. in the first edition. The principles which the Author has here attempted to explain are well illustrated by the action of the feed-water heater of a steam engine discussed further on (pp. 362, 364).

Page 204.—This formula for mean temperature was given in the first edition, but the Author at that time by no means realised the importance of the idea involved. The conception of mean temperature in applied thermodynamics is as useful as that of mean pressure.

Page 211.—In the absence of a regenerator, the cycle of the Atkinson engine exactly corresponds to the mode of supply and rejection of heat in gas engines. A description of this machine, which presents many points of interest, with full illustrations, will be found in Prof. Robinson's *Gas and Petroleum Engines*, recently published. Trials were first made by Prof. Unwin, which have been described by him in a report published in *Engineering* for 1887. More recently (*Journal of the Society of Arts* for February 1889), three gas engines, together with a portable steam engine, were reported on by Dr. Hopkinson, Prof. Kennedy, and Mr. Beauchamp Tower, acting as judges for the Society of Arts. To these valuable reports the reader is referred for further information on gas engines.

Page 217.—The formula given in this appendix, page 407, for the deviation of two curves of the type $P V^n$ = constant, differing slightly in the value of n, leads to a very useful formula for the condensation in adiabatic expansion, for it is clear that if the ratio of expansion be r we shall have

$$1 - x = a \,.\, \log r,$$

where a is a constant which may be found by application to a particular case. In the case considered in the text, $r = 24/6{\cdot}15$, so that $\log r = {\cdot}5913$. The value of x is found to be ·911, that is, the condensation is ·089. Hence we find :—

$$1 - x = {\cdot}15 \log r.$$

If the pressures be given, then r must be understood to mean the pressure-ratio, and

$$1 - x = \cdot 13 \log r.$$

For example, if steam at 140 lbs. expands till its pressure falls to 11 lbs., $r = 140/11$, and

$$1 - x = \cdot 13 \times 1 \cdot 105 = \cdot 144.$$

These formulæ were given in the first edition of this work, but the Author has since found that when the constant is properly determined the approximation is much better than he then supposed. Exact results cannot be expected, because the index of the adiabatic curve varies, as explained in the text. The logarithms in the formulæ are common, not hyperbolic.

Again, the same formulæ (p. 407) show that the deviations of two curves from a third are in a constant ratio. If then we determine the ratio for one pressure, we shall know it for all. Thus, in the example of the text, the final volume of the steam is,

For hyperbolic expansion	..	28·96 cubic feet.
When dry and saturated	..	26·36 „ „
For adiabatic expansion	..	24·00 „ „

The deviations from the saturation curve of the adiabatic curve and the common hyperbola are therefore approximately in the ratio 12 to 13. As stated on page 359, this result furnishes a useful method of constructing an adiabatic curve.

In the 4th edition (1895) of the author's treatise on *Applied Mechanics* a very simple formula for the available energy in a steam engine cycle is derived by use of this principle.

PAGE 222.—Professor Thurston suggests that the steam engine cycle of Arts. 96 and 100 might properly be described as a RANKINE Cycle, and the author is inclined to agree with the suggestion.

PAGE 228.—The function (ϕ) which we describe as "entropy," besides determining the inflow and outflow of heat, may be employed for an entirely different purpose. Referring to page 224, if T_0 be the lowest temperature of surrounding objects, R the heat rejected at that temperature,

$$R = T_0 \cdot \phi.$$

Now the heat R thus rejected at the lowest possible temperature cannot be converted into mechanical energy, and may consequently be described as "unavailable energy." Hence, since R is proportional to ϕ, the entropy of a body is a measure of its unavailable energy. In fact the term is better adapted to this interpretation than to that given to the function by Rankine. Many authorities formerly employed the term in this way, and it would seem that Clausius, who introduced it, regarded this interpretation as the primary one, while fully accepting the other. Whichever we adopt it must always be remembered that entropy is not in itself energy, but only becomes so when multiplied by a temperature.

PAGE 230.—It should be mentioned that temperature-entropy diagrams had been previously used extensively for steam engine purposes by Professor Lindé at the Polytechnicum at Munich, but their importance in this connection appears to have been not generally understood, for in the edition of Zeuner's work on Thermodynamics published in 1890 they are used only in the case of the permanent gases.

CHAPTER VIII. (ARTS. 105–113).

PAGE 235.—This chapter has been reprinted with slight modifications from the first edition, and it was only in 1890 that the Author found that the loss by misapplication of heat to the feed-water can be avoided, not only by use of a Carnot cycle, but also by the employment of a properly constructed feed-heater. This question is considered in a later chapter (p. 362), but it may be useful to give here an independent investigation.

Let us suppose that the supply pipe through which the feed-water passes on its way from the hot well to the boiler contains a series of gratings similar to those of a regenerator. Each grating, however, is to be imagined tubular, so as to act as a condensing coil for the exhaust steam of an auxiliary engine, small orifices at the lower end permitting the drainage of the coil to mingle with the feed. The condensing pressure in the coils is so regulated that the temperatures of the gratings form an ascending series, commencing with the temperature of the hot well and ending with that of the boiler. The effect of these arrangements is that the feed-water is gradually heated during its passage to the boiler, and finally, together with the drainage of the gratings, enters the boiler at the boiler temperature.

Considering any grating, let y be the condensation in all the gratings which precede it for each lb. of feed of the main engine, and Δy the condensation in the grating itself, whereby $1 + y$ lbs. of water are raised through the small interval of temperature (ΔT) by which the grating in question is separated from the next preceding, then

$$(1 + y) \Delta T = x L \, . \, \Delta y,$$

where L, as usual, is the latent heat of evaporation at temperature T, at which the condensation in the grating takes place. Referring to page 223, and remembering that the auxiliary engines are all supplied (with dry steam suppose) from the main boiler at temperature T_1

$$x L = T \left\{ \frac{L_1}{T_1} + \log_\epsilon \frac{T_1}{T} \right\}.$$

Substituting this value of $x L$,

$$\frac{\Delta y}{1+y} = \frac{\Delta (\log_\epsilon T)}{\frac{L_1}{T_1} + \log_\epsilon T_1 - \log_\epsilon T}.$$

Integrating

$$(1+y)\left\{\frac{L_1}{T_1} + \log_\epsilon \frac{T_1}{T}\right\} = \text{Const.}$$

The constant is found by observing that when $T = T_0$ the temperature of the hot well, $y = 0$, and then putting $T = T_1$, we find,

$$(1+y_1)\frac{L_1}{T_1} = \frac{L_1}{T_1} + \log_\epsilon \frac{T_1}{T_0},$$

or

$$y_1 = \frac{T_1}{L_1} \cdot \log_\epsilon \frac{T_1}{T_0},$$

a formula which gives the weight of steam used by the auxiliary engines.

Since only the latent heat of evaporation is supplied by the boiler, the heat expended for all the auxiliary engines and the main engine taken together must be

$$\text{Heat expended} = T_1 \cdot \log_\epsilon \frac{T_1}{T_0} + L_1.$$

To find the work done, observe that the main engine alone rejects heat into the condenser, therefore, again referring to page 223,

$$\text{Heat rejected} = T_0 \left\{\frac{L_1}{T_1} + \log_\epsilon \frac{T_1}{T_0}\right\}.$$

The work done is found by subtraction, and hence we obtain

$$\text{Efficiency} = \frac{T_1 - T_0}{T_1},$$

being the same as if a Carnot cycle had been used.

The demonstration here given is added only for the sake of completeness, for it adds nothing to the force of what is said on page 362.

It is sometimes asked whether the cycle of an ordinary steam engine is or is not reversible, and the answer depends on the nature of the feed-heater, if there be one. If it be such as is here described, the process is clearly reversible, each of the auxiliary engines being reversible. And if it be an "interchanger," through which the hot gases of the furnace pass in the opposite direction to the feed-water, so as to be cooled down to the temperature of the hot well, the process is still reversible. But if there be no feed-heater the process is not reversible, and there is a loss by misapplication of heat to the feed as described in this chapter. We have here a good illustration of the thermodynamical principle that reversibility is the true test of maximum efficiency.

PAGE 237.—With the high pressures now in common use in condensing engines, the maximum loss by misapplication of heat to the feed is somewhat more than 10 per cent., say 11 or 12 per cent. (p. 362).

PAGE 238.—It will be found a useful exercise to work out the loss by misapplication of heat during expansion when the steam remains always dry and saturated. We have not space to give the investigation.

CHAPTERS IX., X. (ARTS. 114–143).

PAGE 260.—In the ingenious valve invented by Mr. Thom a portion of the steam at release is trapped and employed to fill the clearance at the other end of the cylinder. The losses by clearance can in this way be avoided, it being only necessary to compress the cushion steam from the pressure of release to the initial pressure.

PAGE 273.—The subjoined table shows the clearance ratios for the engines of some vessels of war. The surface-factors were calculated for the Author by Mr. Slade, from drawings furnished by the Admiralty. The volume coefficients are selected from a number of examples, in some of which the value is even greater. A coefficient less than ·1 is very exceptional.

Vessel.		Surface Factor (μ).	Volume Coefficient (c).
ANSON	H.P.	2·1	
	L.P.	2	
BENBOW	H.P.	2·5	·22
	L.P.	1·5	·10
AURORA	H.P.	3	
	I.P.	2·3	
	L.P.	2·1	
WASP	H.P.	..	·23
	I.P.	..	·14
	L.P.	..	·13

PAGE 281.—To avoid misapprehension it may be added that the actual form of the temperature curve in the neighbourhood of the surface A (Fig. 39),

where the heat enters, will depend on the rate at which heat can be supplied by external bodies. This is usually limited, and consequently the slope at a has a certain maximum steepness. In the first instance, then, a point of contrary flexure exists which in a few moments moves up to a and disappears.

PAGE 283.—To obtain an exact value of the coefficient of conductivity for the metal of a steam cylinder, it would be necessary to make experiments on similar material at a similar temperature. The comparative conductivity of slightly different materials no doubt varies greatly, and the values given in the table can only be considered as rough estimates. The conductivity of water is known with some accuracy by the experiments of Mr. J. T. Bottomley, (*Phil. Trans.*, 1881). The comparison with iron is made at low temperatures. Sir W. Thomson (now Lord Kelvin) takes the ratio as $\frac{1}{80}$th.

[1895.] Experiments on the penetration of heat in steam cylinders have been made by Mr. Bryan Donkin, and are described by him in a paper on "The Condensation of Steam," which will be found in vol. cxv. *Proc. Inst. C.E.* In these experiments steam was alternately admitted and exhausted from a vertical cylinder, without piston, by two valves—one for steam, the other for exhaust—worked by cams driven from an independent engine. Vertical temperature holes were drilled in the cylinder walls at different distances from the internal surface. These were filled with mercury and small mercurial thermometers inserted. The oscillation of the mercury in each thermometer during the period of admission and exhaust showed, on a reduced scale, the variation of temperature of the metal, which was greatest near the surface and insensible at a short distance from it. These experiments are of interest as showing that the temperature of the metal actually does change in the manner described in the text. Mr. Donkin kindly determined for the Author the coefficient of conductivity from some experiments on steady flow, in which the external and internal surfaces of the cylinder were maintained at known temperatures. The value for cast iron proved to be 6, being 20 per cent. less than that (7·5) adopted in the text as a standard.

PAGE 285.—The calculation here given will probably appear to many readers arbitrary and inconclusive, for it seems at first sight as if the result was assumed rather than proved. Yet the process is one which is necessarily adopted in all similar questions. The reason is that the differential equation furnishes but one general condition which is common to the particular problem considered and to an indefinite number of widely different problems. A tentative method is therefore employed, which consists in selecting from a number of solutions of the equation that particular solution which corresponds to the particular case considered. If it is possible to find a solution which satisfies the equation while at the same time it fully represents the physical problem considered, that, and that alone, will be the true solution; no other is possible.

PAGE 286.—The application of the simple harmonic formula to obtain the values of y and Q for a steam cylinder was given by the Author in his course at the Royal Naval College, in 1881. The value of the coefficient in the formula for Q is only a round number, the exact value, corresponding to the value (7·5) of f, adopted, being 5·7. For reasons just stated, the coefficient is not known exactly, and may be as low as 5, in which case the tabular values would be reduced by about 10 per cent. The formulæ were published by Grashof in 1884. The values of the coefficients given by Kirsch in his work cited on page 290, are nearly the same as those adopted in the text.

PAGE 291.—The great effect on the efficiency of a condensing surface produced by a rush of steam which clears the surface of the water deposited by condensation, is well shown by some important experiments by Mr. Charles Lang, which are described by him in a paper read before the Institution of Engineers and Shipbuilders in Scotland, in March 1889. The condenser employed was one designed by Messrs. Weir, in which the tubes are contracted at the outlet ends, through which the condensed steam, together with a small quantity remaining uncondensed, pass into a chamber, from which they return through the condenser by a single "return tube." The effect of this arrangement is that the pressure in the chamber is much lower than at the entrance of the tubes, and a strong current of steam is produced through all the tubes. The condensation per square foot of surface was so great as fully to bear out the statement in the text, that the deposit of water produced by condensation is the principal cause which obstructs condensation.

Thus, in a continuous process of condensation, the scour of the surface produced by a rush of steam increases condensation. As will be seen from what is said further on, the effect is exactly the reverse in a process of alternate condensation and re-evaporation, where the heat enters and leaves the metal at the same surface, instead of passing through and out at the opposite side.

PAGE 294.—The extreme slowness of the escape of heat from a dry surface, as compared with the rate of conduction through metal, is best understood by observing that, according to the formula for conduction given on page 278, a flow of ·05 thermal units per square foot per minute corresponds to a thermal gradient of $\frac{1}{150}$th, that is of 1° F. in 12·5 *feet*. If we suppose that in consequence of the moisture suspended in the steam and its violent agitation, the escape is twenty times as great; it is still small when compared with the intensity of the exchange of heat between steam and metal which we know goes on in steam cylinders.

Although for this reason it is believed that the action of the metal in ordinary cases is mainly superficial, it is not intended to assert that by sufficiently heating the external surface of the cylinder the steam may not be prevented from condensing, not merely superficially, but throughout its whole mass. When once the internal surface has become dry its temperature may, and generally does, rise above that of the steam, and the action becomes entirely different. (Compare p. 352.)

PAGE 297.—Mr. Donkin's experiments here referred to are described in a paper which will be found in the *Proc. Inst. C.E.*, vol. xcviii.

PAGE 298.—The investigation here given (Art. 140) is taken, with some slight modifications in notation introduced for the sake of clearness, from the first edition of this work. That which precedes (Art. 139), which is its necessary complement, is now (1890) added. As stated further on (p. 301), they are only intended for the purpose of illustration.

PAGE 301 [1895].—The interesting experiments by Mr. Donkin already referred to (p. 424), show clearly the extreme mobility of the water occurring in steam cylinders, at any rate when existing in any considerable quantity.

PAGE 302.—Curves of constant volume are nearly straight, but deviate from the straight line in the opposite direction to that shown in the curve 230 of Fig. 37, which was one of the first figures of the kind constructed. The position of the point b in the expansion curve shown in this figure has been determined experimentally in some cases. See a paper by Professor Thurston, published in the *Transactions of the Institution of Naval Architects* for 1895.

The horizontal distance of a curve of constant volume from the curve A 0 in Fig. 37 is proportional to Δp the rise of pressure for 1° of temperature, a quantity given in Table I. This rule furnishes the simplest method of constructing such curves.

PAGE 305.—The Author has not as yet attempted to construct a thermal diagram which shall include the effect of clearance and wire-drawing; but it is probable that this may be done without much difficulty.

CHAPTER XI. (ARTS. 144-165).

PAGE 312.—See, also, a paper by Professor Unwin, on the *Determination of the Dryness of Steam*, read before the Institution of Mechanical Engineers in January 1895.

PAGE 317.—In this chapter, as throughout the book, the calculations are made on the supposition that the mechanical equivalent of heat is 772. The effect of adding a small percentage is to reduce the *necessary* (not the actual) expenditure of heat and the calculated efficiency in the same proportion. The missing work given in the next table (p. 322), is increased in consequence of the reduction in the useful and waste work. The comparison of the heat expended with the sum of the heat rejected and the useful work (p. 315), is also affected, though to a very small extent. The difficulties attending the exact estimation of priming water render such corrections of little importance, provided the same value of the equivalent is constantly adhered to. The same remark applies to the corresponding change in the density of steam.

PAGE 321.—If A be the available energy per lb. of steam for a given boiler pressure and vacuum, $1 - k$ the fraction which is "missing" according to the definition here given, then the actual work per lb. of steam is

$$W = k\,A - W_0,$$

where W_0 is the waste work at exhaust, also reckoned per lb. of steam. If x_2 be the dryness fraction at release, and the effects of clearance be disregarded,

$$W_0 = x_2\,P_2\,v_2\left\{9 - 10\left(\frac{p_0}{p_2}\right)^{\frac{1}{10}} + \frac{p_3}{p_2}\right\},$$

which can be calculated if x_2 and the pressures are known. The consumption of steam (taking the mechanical equivalent of heat at 780) will now be

$$\text{Lbs. per I.H.P. per hour} = \frac{2538}{W}.$$

Taking, for example, a vacuum of 1 lb. per sq. inch, a terminal pressure of 10 lbs., and a back pressure of $1\frac{1}{2}$ lbs. absolute, W_0 will be found to be 85 x_2. Assuming, further, $x_2 = \cdot 8$,

$$W = k\,A - 68.$$

In the best single cylinder engines it will be seen that k may be taken as ·8, and with so high a terminal pressure as 10 lbs., it is probable that the same is true for bi-compound and tri-compound engines, notwithstanding the extra losses by clearance and wire-drawing to which they are subject. Adopting this value of k, the annexed table of numerical results is obtained for various boiler pressures.

Boiler Pressure.	A	W	$\frac{2538}{W}$	$\frac{W_0}{A}$	$\frac{W}{A}$
350	373	230	11	·182	·618
180	328	194	13·1	·207	·593
84	279	155	16·4	·244	·556
60	258	138	18·4	·264	·536
20	187	82	31	·363	·437

The last two columns of the table give the efficiency and the fraction of the available energy wasted at exhaust by incomplete expansion and excess back pressure.

Somewhat lower consumptions have been obtained in recent engine tests, as will be seen on reference to a table given by Professor Unwin in his Forrest lecture (*Proc. Inst. C.E.*, vol. cxxii., p. 184) for 1895, for when the lowest possible consumption per I.H.P. per hour is alone aimed at, expansion may no doubt with advantage be carried below the limit of 10 lbs. per sq. inch.

Of late the vacuum here and in the text adopted as a standard (28 inches) has been frequently exceeded, even 29 inches being occasionally recorded. If p_3 be always taken as the true back pressure there is little reason for changing the standard, since a change alters A and W_0 by nearly the same amount.

PAGE 334.—The report of their experiments by Messrs. Gateley & Kletzsch, cited in the text, bears obvious marks of hasty preparation, and in one or two points may reasonably excite distrust. The Author, however, sees no reason for rejecting these experiments altogether. It is not intended to attach any importance to a close coincidence in the results obtained in different cases (p. 332). A very small error in the experimental data will frequently give rise to a large one in the calculated result. It should be remarked that the clearance is the same in the two engines, M and GK, a very important point in making a comparison.

PAGE 337.—The experiments by Major (now Colonel) English here referred to are described in the first of the two papers cited further on. They were made on a pumping engine with a pair of horizontal cylinders. One of the trials is denoted by EA in the table on page 334, and by E on page 322. In this paper a formula is given for cylinder condensation, based on the supposition that it follows the law of the inverse square root of the speed, but in other respects differs entirely from that adopted in the text.

PAGE 342.—It has already been remarked that the law of the inverse square root of the speed is not exact, even for single cylinder non-jacketed engines, somewhat larger constants being obtained at low speeds than high. Two reasons for this have already been given, and a third may be added in the effect of the drainage of heat from the ends to the centre of the cylinder described on page 353. This effect is evidently greater at low speeds than high. In the slow moving engine of the *Michigan* the revolutions were sixteen per minute, but in H four or five times as great. In selecting a constant, therefore, some regard must be had to the speed as well as the type and the initial pressure.

PAGE 343.—The equation here given for the best ratio of expansion may also be written

$$\frac{p_2}{p_b} = 1 + \frac{z}{1 - z} \log_\epsilon r.$$

Assuming $z = \cdot 5$ as in the text, this gives for

$$r = 2, \quad 3, \quad 6, \quad 9,$$

$$\frac{p_2}{p_b} = 1\cdot 8, \quad 2\cdot 1, \quad 2\cdot 8, \quad 3\cdot 2,$$

showing, that for a given back pressure the best terminal pressure (p_2) increases as the expansion increases. In this respect the formula agrees with Emery's, given on the following page. It appears to be not unfrequently assumed now by steam engine authorities that the best terminal pressure may be considered as independent of the initial pressure.

An important point, which must not be lost sight of, to which attention has been called by Professor Thurston and others, is that the best ratio of expansion is less when account is taken of the frictional resistances. It is well known that the friction of an engine may in great part be represented by a pressure on the piston which is equivalent to an increase in the back pressure. If, then, a suitable addition be made to p_3 in the formula for W_0 given above, the result of the calculation will be the consumption of steam per brake horse power per hour, and the reasoning of the present article shows that the best terminal pressure will be raised. In an interesting paper on "Steam Engine Economy" (*Proc. Inst. C.E.*, vol. cxxii., p. 11) Mr. Davey states his opinion that under average conditions a terminal pressure of 10 lbs. per sq. inch gives the best practical result, and it is for this reason that this value was chosen in estimating the consumption of steam above.

PAGE 352.—If steam be superheated θ° above the saturation temperature T_1 (absolute), the quantity of heat supplied during superheating will be $c\,\theta$, where c is the mean specific heat at constant pressure, and the mean temperature of supply will be approximately $T_1 + \frac{1}{2}\theta$. Hence the addition to the available energy of a lb. of steam will be

$$\Delta A = c \,.\, \theta \,.\, \frac{T_1 + \frac{1}{2}\theta - T_0}{T_1 + \frac{1}{2}\theta},$$

where T_0 is the lower limit of temperature. Since c is less than unity, the direct effect of superheating is small; but its indirect effect in diminishing cylinder condensation is without doubt very considerable. It appears probable that cylinder condensation may be practically reduced to zero in this way.

The annexed table gives the result of supposing $k = \cdot 95$ and $x_2 = 1$ in the formula for W already given (p. 427), the value of c being supposed $\cdot 5$, the superheat 100° F., and A replaced by $A + \Delta A$.

Boiler Pressure.	Δ A.	A + Δ A.	W	$\frac{2538}{W}$
350	22	395	290	8·8
180	18	346	244	10·4
84	16	295	195	13·8
60	15	273	174	14·6
20	12	199	104	24·4

From the table given by Professor Unwin, already referred to, it will be seen that a consumption of 10·17 has actually been reached in a compound engine of 76 H.P., with steam of 180 lbs. pressure.

The saving of fuel to be obtained by superheating is not so great as appears from the diminished consumption of steam, for each lb. of steam requires more heat to produce it. Besides which a dry cylinder requires a considerable amount of lubricant, which is not only a source of expense in itself, but, by finding its way into the feed-water, diminishes considerably the efficiency of the heating surface of the boiler.

PAGE 353.—Two completely distinct questions may be asked in reference to a proposed engine of given dimensions and speed:—(1) What will be the consumption of steam, and (2) What will be the mean effective pressure?

Of these two questions an attempt has been made to answer the first, for the case of a single cylinder non-jacketed engine, the difficulty being reduced to that of determining the proper constant to employ. The second question is obviously much more difficult, and no theoretical solution of the problem has yet been attempted. In practice it is answered by assuming that the expansion is hyperbolic, and allowing for clearance and other disturbing causes by multipliers determined by experience for each type of engine.

PAGE 360.—The question of the right method of combining diagrams has given rise to a considerable amount of discussion. The method adopted in the text is believed to be the same in all essential respects as that employed by various other writers. The Author has, however, endeavoured to state the question as clearly as possible, and for this purpose has ventured to introduce the terms "point of reference," "curve of reference," so as to make the statement more definite.

PAGE 364.—The nature of the feed-heater employed in the *Meteor* is not stated in the original paper, but Professor Kennedy kindly informed the Author that the steam supplied to it was taken from the valve-chest of the intermediate cylinder.

LONDON: PRINTED BY WILLIAM CLOWES AND SONS, LIMITED,
STAMFORD STREET AND CHARING CROSS.