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Stochastic Duration and Dynamic Measure of Risk in Financial Futures

Andrew H. Chen
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College of Commerce and Business Administration
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July 1984

Stochastic Duration and Dynamic Measure of Risk in Financial Futures

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Abstract

Combining the contributions of Cox, Ingersoll and Ross (1979, 1981) in stochastic duration of bonds and in equilibrium pricing of futures contracts, this paper develops stochastic duration as a dynamic risk measure for financial futures. Some simulation results are provided and discussed.

Stochastic Duration and Dynamic Measure Of

Risk In Financial Futures

The introduction of futures contracts on several financial instruments into the exchanges in the recent years has generated a great deal of interests in studying the role of financial futures. Most of the studies on financial futures have focused either on the empirical investigation of hedging effectiveness of financial futures or on deriving optimal hedge-ratios in immunization strategies with financial futures using Macaulay's duration as a risk measure.[1] To the best of our knowledge, no study on financial futures to date has explicitly examined the validity of using the traditional duration or proposed any alternative risk measure in the immunization strategies with financial futures.

As Leibowitz (1981) has demonstrated, there are two basic kinds of yield-curve movements--parallel market shifts and yield-curve reshapings--and they lead to fundamentally different types of volatility behavior in the prices of financial futures. In particular, the prices of financial futures have been shown to be extremely sensitive to the yield-curve reshapings even when the cash security's yield remains unchanged. The risk embedded in a financial futures contract is not the same as that of a cash security. Therefore, determining a proper risk measure in financial futures is of significant importance if we attempt to devise effective hedging strategies with financial futures in the management of bond portfolios.

The traditional measures of duration, developed by Macaulay (1938) and Hicks (1939), have been used as measures of basis risk of bonds and as means to devise immunization strategies for bond portfolio management.

The concept of traditional measures of duration has also been extended to assess the risk of other financial assets such as common stocks and financial futures.[2] However, as Cox, Ingersoll and Ross (1979) (CIR hereafter), and Ingersoll, Skelton and Weil (1978) have pointed out, the traditional duration is a valid risk measure only for parallel shifts in the entire yield-curve (i.e., preserving yield-curve shapings). Therefore, applying the traditional measures of duration to financial futures for immunization strategies (e.g., Chance (1982), and Kolb and Chiang (1982)) might lead to improper results. CIR (1979) has developed a "stochastic duration" which has been shown to be a superior alternative for measuring the basis risk of bonds.

The purpose of this paper, combining the contributions of CIR (1979, 1981) in the stochastic duration of bonds and in the equilibrium pricing of futures contracts, is to develop stochastic duration as a dynamic risk measure for financial futures.[3] It is our hope that this paper will increase the understanding about the risk of financial futures and thus give some insights for more efficient immunization strategies in bond portfolio management. Section I reviews the literature on duration of bonds. Section II develops the stochastic duration of financial futures and shows some simulation results. Section III contains a brief summary.

I. Duration of Bonds: Literature Review

Duration of a bond, originally developed by Macaulay and Hicks, is defined as a weighted average of times to maturity. The weight assigned to each period is the present value of the cash flow for that period divided by the current price of the security as follows:

$$D = \Sigma\, tC(t)P(t)/\Sigma\, C(t)P(t) \tag{1}$$

where C(t) is the stream of cash flows (coupons and principal repayment) and P(t) is the present value of \$1.00 to be received at time t. Duration in (1) can also be expressed in the form of an elasticity:

$$-D = [(dB/B)/(dy/y)]/y = [dB/B]\cdot[1/dy] \tag{2}$$

where $B = \Sigma\, C(t)e^{-yt}$ and y is the continuously compounded yield-to-maturity on the bond.

CIR (1979) has demonstrated that measuring the risk of a bond by the elasticity given in (2), which is common in the bond market, is faulty since the result in (2) cannot be used to make cross-sectional comparisons of the riskiness of bonds (p. 52). In addition, Ingersoll, Skelton and Weil (1978) has proved that the duration in (1) can be a valid risk measure only when the entire yield curve is described by proportional shape-preservation under interest rate changes (see also CIR (1979) and Bierwag, Kaufman and Toevs (1982)). Thus it would be misleading if we apply the concept of the traditional duration directly to the financial futures contract since, as Leibowitz (1981) has shown, the futures price is more sensitive to yield curve reshapings than to parallel shifts.[4]

As an alternative, CIR (1979) has proposed stochastic duration as a dynamic measure of risk of bonds with units of time. This concept of duration allows the yield curve changes in shape as well as location. To derive the stochastic duration, CIR assumed that the instantaneous compounding risk-free interest rate, r, follows the first-order autoregressive process as

$$dr = \kappa(\mu - r)dt + \sigma\sqrt{r}\, dz \tag{3}$$

where μ is steady-state mean and κ is the parameter for the speed of adjustment toward μ.

Based upon a general process for interest rate in (3), they derived the stochastic duration as a proxy for basis risk of coupon bonds with the units of time as follows:

$$D \equiv G^{-1}[-B_r/B] = G^{-1}[-\Sigma\, C(t)P_r(t)/\Sigma\, C(t)P(t)] \tag{4}$$

$$= G^{-1}[\Sigma\, C(t)P(t)G(t)/\Sigma\, C(t)P(t)]$$

where $P(\tau)$ = the price of a unit discount bond with time to maturity τ

$$= A(\tau)\exp[-rG(\tau)]$$

$$A(\tau) = \left\{\frac{2\gamma\, \exp[(\gamma + \kappa + \lambda)\tau/2]}{(\gamma + \kappa + \lambda)[\exp(\gamma\tau) - 1] + 2\gamma}\right\}^{2\kappa\mu/\sigma^2}$$

$$G(\tau) = 2/[\kappa + \lambda + \gamma\, \mathrm{Coth}(\gamma\tau/2)]$$

$$\gamma = [(\kappa + \lambda)^2 + 2\sigma^2]^{1/2}$$

$-\lambda$ = the parameter for the market's liquidity preference

$$G^{-1}(x) = \frac{2}{\gamma}\, \mathrm{Coth}^{-1}\left[\frac{2}{\gamma x} - \frac{\kappa + \lambda}{\gamma}\right]$$

CIR (1979) has compared the traditional duration in (1) with the stochastic duration in (4), and concluded that the traditional duration is not theoretically and empirically realistic.

II. Stochastic Duration of Financial Futures[5]

Taking into account the marking-to-market effect in futures contracts explicitly, CIR (1981) has derived the equilibrium pricing formula for the futures contract on a unit discount bond.

Let $F(t,\Delta)$ be the futures price as of time t for a contract with the maturity date s on a discount bond paying one dollar at time T $(t < s < T)$, and let $\Delta = T - s$ and $\tau = T - t$ to be consistent with the notation in section I. Then the equilibrium price of this futures contract is as follows:

$$F(t,\Delta) = A(\Delta)\left[\frac{\eta(s-t)}{G(\Delta) + \eta(s-t)}\right]^{2\kappa\mu/\sigma^2} \cdot \exp\left[-r\left\{\frac{\eta(s-t)G(\Delta)e^{-(\kappa+\lambda)(s-t)}}{G(\Delta) + \eta(s-t)}\right\}\right] \tag{5}$$

where $\eta(s-t) = \dfrac{2(\kappa + \lambda)}{\sigma^2(1 - e^{-(\kappa+\lambda)(s-t)})}$

Using (4) and (5), the stochastic duration of the futures contract on the discount bond (D_F) can be derived as[6]

$$\begin{aligned} D_F &\equiv G^{-1}[-F_r/F] \\ &= G^{-1}\left[\frac{\eta(s-t)\cdot G(\Delta)e^{-(\kappa+\lambda)(s-t)}}{G(\Delta) + \eta(s-t)}\right] \qquad (6) \\ &= G^{-1}(x) \end{aligned}$$

We can also develop the pricing formula for a futures contract on a coupon bond since a coupon bond can be regarded as a portfolio of discount bonds. Consider a coupon bond which pays n constant coupons (C) with the equal time interval (δ) for the period $\Delta = T - s$ and principal of one dollar at time T, i.e., $\Delta/\delta = n$. This coupon bond can be thought of as a portfolio of n discount bonds $(i = 1,2,\ldots,n)$. Let $F(t,i\delta)$ be the futures price on ith discount bond. The futures price on the coupon bond as of time t ($f(t)$) can be written as

$$f(t) = C \sum_{i=1}^{n} F(t,i\delta) + F(t,n\delta) \tag{7}$$

$$= C \sum_{i=1}^{n} A(i\delta)\left[\frac{\eta(s-t)}{G(i\delta) + \eta(s-t)}\right]^{2\kappa\mu/\sigma^2} \cdot$$

$$\exp\left[-r\left\{\frac{\eta(s-t)G(i\delta)e^{-(\kappa+\lambda)(s-t)}}{G(i\delta) + \eta(s-t)}\right\}\right] + F(t,n\delta)$$

Following the same procedure, using (4) and (7), the stochastic duration of the futures contract on a coupon bond can be written as

$$D_f \equiv G^{-1}[-f_r/f]$$

$$= G^{-1}\left[\frac{C\Sigma F(t,i\delta)\left\{\frac{\eta(s-t)G(i\delta)e^{-(\kappa+\lambda)(s-t)}}{G(i\delta) + \eta(s-t)}\right\} + F(t,n\delta)\left\{\frac{\eta(s-t)G(n\delta)e^{-(\kappa+\lambda)(s-t)}}{G(n\delta) + \eta(s-t)}\right\}}{C\Sigma F(t,i\delta) + F(t,n\delta)}\right]$$

$$= G^{-1}(x)$$

Equation (8) is the general form of stochastic duration for financial securities. For instance, if C is zero for discount bonds, then (8) reduces to (6) and we have $D_f = D_F$. In addition, when t is equal to s, a futures contract on a coupon bond becomes a cash coupon bond and thus (8) reduces to (4).

Although the results in (6) and (8) appear to be complicated, their practical application is not as restrictive as it looks once the parameters of the interest rate process in (3) are estimated. For illustration, we have simulated the stochastic durations of financial futures contracts using the parameter values in (3) estimated by CIR (1979). Using a time series of the weekly auction rates on 91-day Treasury bills for 1967-1976, CIR has estimated $\kappa = .692$, $\mu = 5.623\%$, and $\sigma^2 = .00608$.

Table 1 presents the simulation results on stochastic durations of futures contracts on discount bonds and coupon bonds with varying coupon rates and time periods. We have assumed $\mu = r$ and λ (liquidity premium) $= 0$ to see only the effects of uncertainty. We have also used the reversion parameter, $\kappa = .692$, in order to highlight the effect of interest rate process with drift affecting the shape as well as the location of the yield curve, as opposed to the random walk with zero drift affecting the location only.[7]

Table 1 demonstrates that the stochastic duration of futures contracts on bonds decreases as coupon rate increases, which is consistent with the duration of cash bonds. It also shows that as $s-t$ becomes longer for the given period of Δ, the stochastic duration becomes smaller. This result is not surprising, since the futures contract as of time t with the maturity date s on a bond maturing at time T can be viewed conceptually as a portfolio going long in the bond with the maturity date T and at the same time going short in the bond maturing at time s.[8] Thus, the duration of the futures contract can be interpreted as the difference between the duration of the bond maturing at time T and the duration of the bond maturing at time s. In addition, the results in Table 1 are consistent with the notion of CIR (1979) that the stochastic duration need not be an increasing function of maturity.

However, Table 1 is not directly comparable to CIR (1979) because of the different underlying securities. Table 2 presents an indirect comparison between the stochastic duration of cash bonds reported in CIR (1979) and the stochastic duration of futures contracts on the same bonds when the time period until the maturity of the futures contracts is extremely short. As expected, under this circumstance, they are quite similar.

It is, however, important to note that the stochastic duration of financial futures developed in this paper is very sensitive to the reversion parameter. Table 3 demonstrates the sensitivity of the stochastic duration to the reversion parameter κ. This clearly indicates that the effectiveness of the stochastic duration for practical applications critically depends on correct estimates of parameters in the interest rate process specified in (3).

Once the aforementioned stochastic durations for cash bonds and the futures on the bonds are estimated, they can be utilized to calculate the hedge ratios in the immunization strategies with financial futures. Since the stochastic duration for financial futures developed in this paper allows for parallel shifts as well as reshapings in the yield-curve, it must be a better risk measure and it will provide a more effective means in immunization strategies for bond portfolio management. However, the focus of this paper is on developing stochastic duration as dynamic measure of risk of financial futures and thus the effectiveness of the stochastic duration for such practical application is beyond the scope of the current paper.

III. Conclusion

The concept of duration has been commonly used as a measure of basis risk of bonds. However, the usefulness of the traditional duration and its extentions is restrictive both theoretically and empirically because they are valid only for parallel market shifts in the entire yield curve. Since the prices of financial futures contracts are very sensitive to yield-curve reshapings, the traditional duration provides little usefulness in immunization strategies with financial futures. We have developed stochastic duration of a financial futures contract as a proxy for its dynamic measure of risk, based on a more realistic interest rate process allowing changes in shape as well as location of the yield curve suggested by CIR (1979). The simulation results confirm the validity of the aforementioned stochastic duration as a risk measure for financial futures.

Footnotes

[1]See Bacon and Williams (1976), Chance (1982, 1983), Ederington (1979), Hill and Schneeweis (1980), and Kolb and Chiang (1981, 1982).

[2]See Boquist, Racette and Schlarbaum (1975), Bierwag (1977), Bierwag and Kaufman (1979), Chance (1982, 1983), Khang (1979), Kolb and Chiang (1981, 1982), and Williams and Pfeiger (1982).

[3]Futures contracts do not require initial investment. Therefore it seems difficult to interpret the duration of futures contracts. However, as CIR (1981) and Ingersoll (1982) pointed out, although not the price of an asset, a futures price satisfies the same equilibrium as asset prices. The payoffs of a futures contract can be duplicated by a portfolio containing call and put options (see Black (1976)). Also, a futures contract can be interpreted as a portfolio yielding positive and negative cash flows (see Little (1984)): "A long position implies an outflow at the delivery date and subsequent inflows from the delivered instrument" (pp. 285). In any case, the duration of a futures contract can be defined as the duration of an asset, in much the same manner as wealth fractions of futures contracts in investors portfolio are defined in the literature (see Breeden (1979)).

[4]See Kolb and Chiang (1982) for application of the concept of Macaulay's duration to futures contracts.

[5]All arguments about futures contracts (including derivation of stochastic duration) have been done also for forward contracts. The results on forward contracts are not reported here but will be available upon request.

[6]Note that the duration of a futures contract on the discount bond is not equivalent to the duration of the discount bond itself which is equal to the maturity. Also, the correctness of (6) can be easily checked by deriving the duration of cash discount bond with the maturity, s-t

$$
\begin{aligned}
D_s &\equiv G^{-1}\,\frac{C(s)P(s-t)G(s-t)}{C(s)P(s-t)} \\
&= G^{-1}[G(s-t)] \\
&= s-t
\end{aligned}
$$

[7]See CIR (1979) for the effect of κ.

[8]See Little (1984) for the interpretation of futures contracts in much the same way.

References

Bacon, P. and Williams, R. 1976. "Interest Rate Futures: New Tool for the Financial Manager," Financial Management (Spring): 32-38.

Black, F. 1976. "The Pricing of Commodity Contracts," Journal of Financial Economics 3: 169-179.

Boquist, J. A., Racette, G. A., and Schlarbaum, G. G. 1975. "Duration and Risk Assessment for Bonds and Common Stocks," Journal of Finance 30: 1360-1365.

Bierwag, G. O. 1977. "Immunization, Duration and the Term Structure of Interest Rates," Journal of Financial and Quantitative Analysis, Vol. 12 (December): 725-742.

__________ and Kaufman, G. G. 1979. "Coping with the Risk of Interest Rate Fluctuations: A Note," Journal of Business, Vol. 50 (July): 364-370.

__________ and Toevs, A. L. 1982. "Single Factor Duration Models in a Discrete General Equilibrium Framework," Journal of Finance 37: 325-338.

Breeden, D. T. 1979. "Futures Markets and Commodity Options," working paper, University of Chicago.

Chance, D. M. 1982. "An Immunized-Hedge Procedure for Bond Futures," Journal of Futures Markets, Vol. 2: 231-242.

__________. 1983. "Floating Rate Notes and Immunization," Journal of Financial and Quantitative Analysis 18 (September): 365-380.

Cox, J. C., Ingersoll, J. E. Jr., and Ross, S. A. 1978. "A Theory of the Term Structure of Interest Rates," Graduate School of Business, Stanford University.

__________. 1979. "Duration and the Measurement of Basic Risk," Journal of Business 52: 51-61.

__________. 1981. "The Relation Between Forward Prices and Futures Prices," Journal of Financial Economics 9: 321-346.

Ederington, L. H. 1979. "The Hedging Performance of the New Futures Markets," Journal of Finance 34 (March): 157-170.

Hicks, J. R. 1939. "Value and Capital." Oxford, Clarendon Press.

Hill, J. and Schneeweis, T. 1980. "The Use of Interest Rate Futures in Corporate Financing and Corporate Security Investment," proceedings of the International Research Seminar of the Chicago Board of Trade (May).

Ingersoll, J. E. 1982. Discussion on "The Pricing of Commodity-Linked Bonds," by E. S. Schwartz, Journal of Finance 37: 540-541.

__________, Skelton, J., and Weil, R. 1978. "Duration Forty Years Later," Journal of Financial and Quantitative Analysis 13 (November): 627-650.

Khang, C. 1979. "Bond Immunization When Short-term Interest Rates Fluctuate More Than Long-term Rates," Journal of Financial and Quantitative Analysis 14: 1085-1090.

Kolb, R. and Chiang, R. 1981. "Improving Hedging Performance Using Interest Rate Futures," Financial Management 10: 72-79.

__________. 1982. "Duration, Immunization and Hedging with Interest Rate Futures," Journal of Financial Research 5: 161-168.

Leibowitz, M. L. 1981. "The Analysis of Value and Volatility in Financial Futures," Monograph of Solomon Brothers.

Little, P. K. 1984. "Negative Cash Flows, Duration and Immunization: A Note," Journal of Finance 39: 283-288.

Macaulay, F. R. 1938. "Some Theoretical Problems Suggested by the Movements of Interest Rates, Bond Yields and Stock Price Since 1856," National Bureau of Economic Research, New York.

Williams, A. O. and Pfeiger, P. E. 1982. "Estimating Security Price Risk Using Duration and Price Elasticity," Journal of Finance 37: 399-412.

Table 1

Stochastic Duration of Futures Contracts on Discount Bonds and Coupon Bonds*

s−t (Year)	T−s=Δ (Year)	Coupon Rates 0%	4%	6%	8%
.25	.25	.2072	.2072	.2072	.2072
.25	.50	.4079	.4058	.4047	.4037
.25	.75	.6013	.5947	.5916	.5886
.25	1.00	.7866	.7733	.7671	.7611
.25	1.50	1.1304	1.0966	1.0815	1.0673
.25	1.75	1.2876	1.2404	1.2196	1.2004
.25	2.00	1.4345	1.3719	1.3449	1.3204
.25	5.00	2.4144	2.1488	2.0613	1.9916
.25	10.00	2.6186	2.2591	2.1733	2.1128
.25	15.00	2.6252	2.2469	2.1793	2.1354
.25	20.00	2.6254	2.2345	2.1819	2.1499
.50	.25	.1722	.1722	.1722	.1722
.50	.50	.3348	.3331	.3323	.3315
.50	.75	.4872	.4821	.4797	.4773
.50	1.00	.6289	.6189	.6142	.6097
.50	1.25	.7599	.7435	.7360	.7288
.50	1.50	.8799	.8559	.8451	.8350
.50	1.75	.9890	.9566	.9422	.9289
.50	2.00	1.0875	1.0459	1.0278	1.0112
.50	5.00	1.6541	1.5167	1.4688	1.4296
.50	10.00	1.7512	1.5752	1.5299	1.4971
.50	15.00	1.7542	1.5689	1.5331	1.5094
.50	20.00	1.7543	1.5623	1.5345	1.5173
.75	.25	.1434	.1434	.1434	.1434
.75	.50	.2761	.2747	.2740	.2734
.75	.75	.3978	.3938	.3918	.3900
.75	1.00	.5086	.5008	.4972	.4937
.75	1.25	.6088	.5963	.5906	.5852
.75	1.50	.6986	.6808	.6728	.6652
.75	1.75	.7786	.7550	.7445	.7348
.75	2.00	.8495	.8197	.8067	.7947
.75	5.00	1.2285	1.1412	1.1100	1.0843
.75	10.00	1.2884	1.1787	1.1497	1.1285
.75	15.00	1.2902	1.1747	1.1518	1.1365
.75	20.00	1.2903	1.1705	1.1527	1.1416
1.00	.25	.1197	.1197	.1197	.1197
1.00	.50	.2285	.2274	.2268	.2263
1.00	.75	.3267	.3235	.3219	.3205
1.00	1.00	.4147	.4086	.4057	.4029
1.00	1.25	.4930	.4833	.4789	.4747
1.00	1.50	.5622	.5486	.5424	.5366
1.00	1.75	.6229	.6051	.5971	.5897

Table 1 (cont.)

s−t (Year)	T−s=Δ (Year)	Coupon Rates 0%	4%	6%	8%
1.00	2.00	.6760	.6538	.6440	.6350
1.00	5.00	.9484	.8875	.8654	.8471
1.00	10.00	.9895	.9138	.8934	.8785
1.00	15.00	.9907	.9110	.8949	.8841
1.00	20.00	.9908	.9080	.8955	.8877
1.25	.25	.1000	.1000	.1000	.1000
1.25	.50	.1897	.1887	.1883	.1879
1.25	.75	.2695	.2669	.2657	.2645
1.25	1.00	.3402	.3353	.3330	.3308
1.25	1.25	.4024	.3948	.3912	.3879
1.25	1.50	.4567	.4461	.4412	.4367
1.25	1.75	.5040	.4901	.4839	.4782
1.25	2.00	.5449	.5278	.5203	.5133
1.25	5.00	.7490	.7042	.6879	.6743
1.25	10.00	.7790	.7236	.7086	.6976
1.25	15.00	.7799	.7216	.7097	.7017
1.25	20.00	.7799	.7194	.7102	.7044

*The value of parameters used in this table are $r = \mu = 5.623\%$, $\sigma^2 = .00608$ and $\kappa = .692$.

Table 2

Stochastic Duration of Futures Contracts on Coupon Bonds When s-t is Extremely Short Relative to Δ*

s-t (Year)	T-s=Δ (Year)	Coupon Rates 4% Futures	CIR	6% Futures	CIR	8% Futures	CIR
.01	5	3.67	3.81	3.41	3.52	3.23	3.34
.01	10	4.10	4.29	3.79	3.93	3.60	3.73
.01	15	4.05	4.24	3.81	3.95	3.66	3.81
.01	20	4.00	4.18	3.82	3.96	3.71	3.86

*Assumed that $\mu = r = 5.623\%$, $\sigma^2 = .00608$ and $\kappa = 0.692$. The column, CIR presents the stochastic duration of cash coupon bonds with time to maturity, Δ, which was calculated by CIR (1979).

Table 3

Stochastic Duration of Futures Contracts on Discount Bonds for Different Values of κ

s−t (Year)	T−s=Δ (Year)	κ = .001	κ = .100	κ = .692
.25	.25	.2499	.2437	.2072
.25	.50	.4997	.4872	.4079
.25	1.00	.9990	.9733	.7866
.25	4.00	3.9867	3.8640	2.2281
.25	20.00	19.5546	17.3795	2.6254
.50	.25	.2498	.2376	.1722
.50	.50	.4994	.4747	.3348
.50	1.00	.9980	.9473	.6289
.50	4.00	3.9734	3.7341	1.5590
.50	20.00	19.1352	15.5427	1.7543
.75	.25	.2497	.2316	.1434
.75	.50	.4991	.4625	.2761
.75	1.00	.9970	.9221	.5086
.75	4.00	3.9602	3.6099	1.1684
.75	20.00	18.7390	14.1363	1.2903
1.00	.25	.2496	.2258	.1197
1.00	.50	.4987	.4507	.2285
1.00	1.00	.9960	.8977	.4147
1.00	4.00	3.9471	3.4911	.9066
1.00	20.00	18.3639	13.0022	.9908
1.25	.25	.2495	.2201	.1000
1.25	.50	.4984	.4391	.1897
1.25	1.00	.9950	.8739	.3402
1.25	4.00	3.9341	3.3772	.7184
1.25	20.00	18.0078	12.0560	.7799