

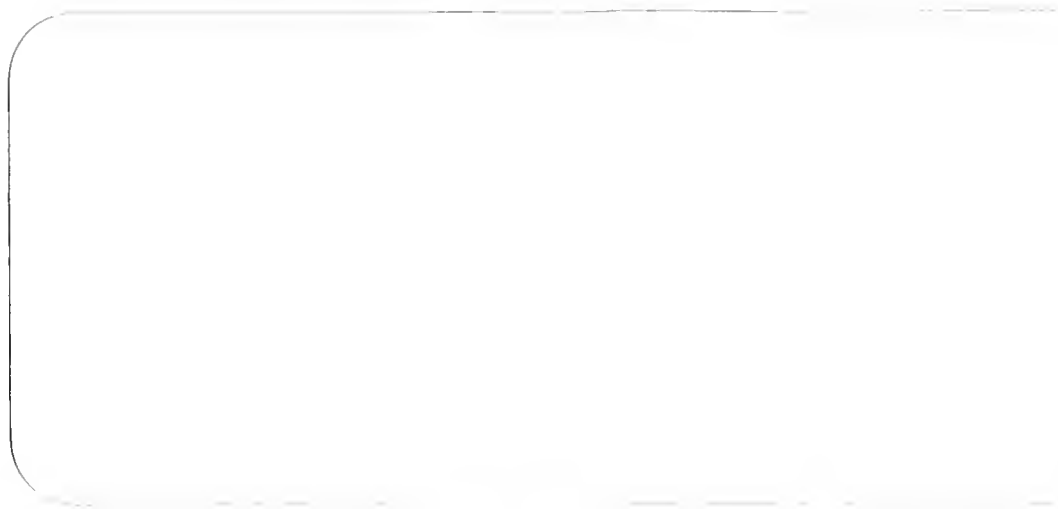
Faculty Working Papers

A STOCHASTIC MODEL OF THE
INTERNAL CONTROL SYSTEM

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College of Commerce and Business Administration
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ROLE OF INTERNAL CONTROL IN AUDITING

The primary purpose of incorporating a set of internal controls in the accounting system is to maintain a high probability of preventing and/or eliminating errors, irregularities and fraud in the financial information process. The reliability of the system of internal control not only provides evidence as to the bonafides of the output of the system, but also influences the nature and extent of the auditor's examination.

The first step the auditor undertakes, therefore, is to establish to what extent the financial information system is supported by internal controls and to what degree such a system is reliable. It has become axiomatic that the effectiveness of internal controls must be taken into account in determining the extent and nature of the audit procedures appropriate in a given examination.¹ The more reliable the system is, the less extensive the tests the auditor need conduct. Recognizing this inverse relation between effectiveness of internal controls and audit scope, the American Institute of CPA's requires all auditors to first evaluate the reliability of internal controls as a matter of audit standards.² Recently, the Committee on Auditing Procedures of the AICPA released several statements on the subject of internal controls which re-emphasize the importance of the study of the reliability of internal controls.³ Despite the emphasis on evaluating the reliability of internal

¹R. K. Mautz and H. A. Sharaf, The Philosophy of Auditing (American Accounting Association, 1961), p. 140.

²The American Institute of CPA's, Statements on Auditing Procedure No. 33 (New York: The AICPA, 1963), p. 16.

³For example, see Statement on Auditing Procedure Nos. 49, 52, 54 (New York: The AICPA, 1971-2).



controls, the auditor has been devoid of means that enable him to quantitatively evaluate the reliability of the internal control system. Conventionally, the auditor uses questionnaires, flow charts, and tests of transactions for evaluation purposes. While these methods have merit, they do not result in an objective, quantitative evaluation.

Trueblood and Cyert stated one and one-half decades ago,

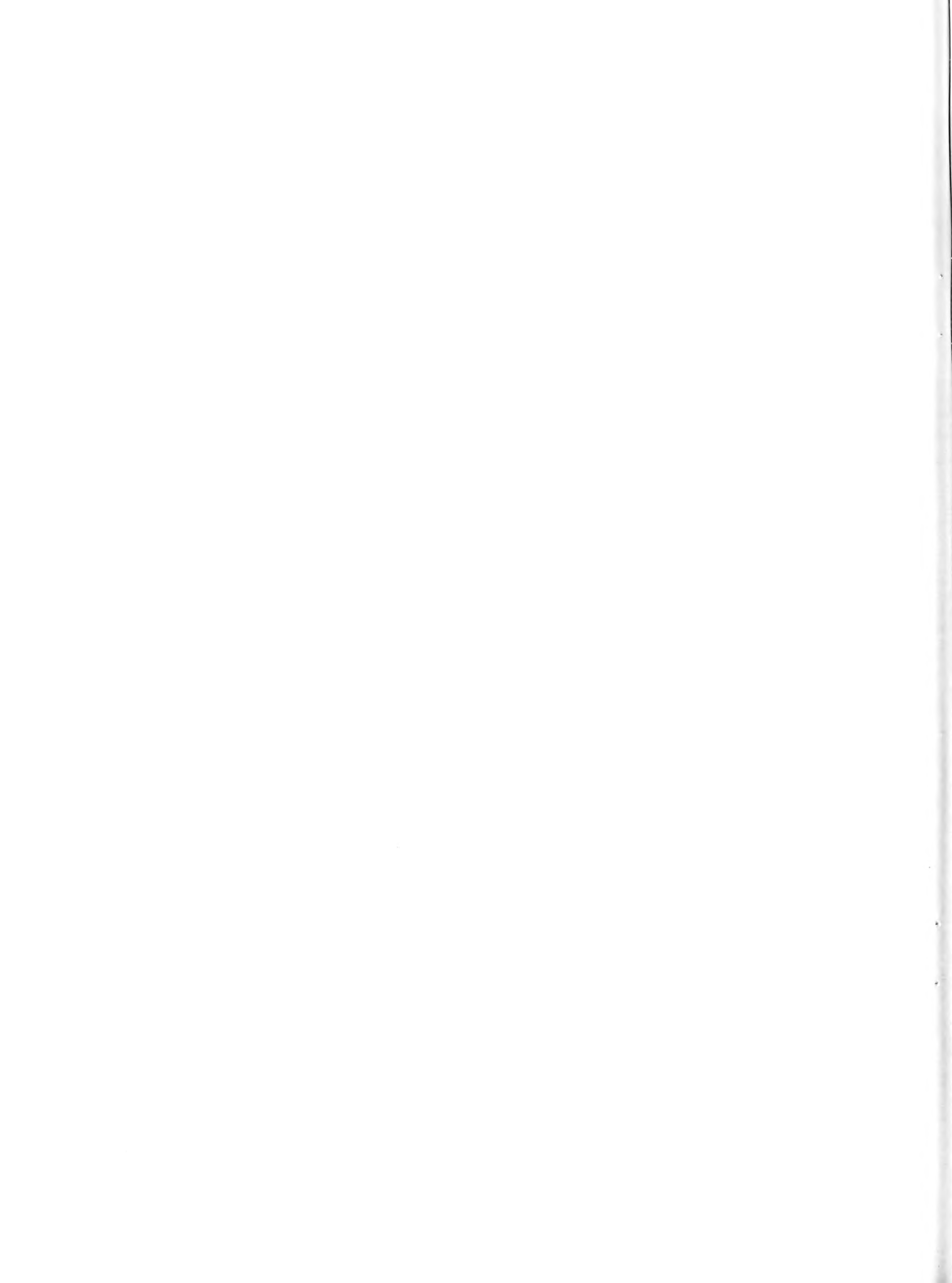
The auditor typically deals with many subjective evaluations such as the appraisal of the overall functioning of the system of internal control. . . . perhaps only if the fabric of internal control has been formulated into mathematical models will the complete chain of reasoning be demonstrated by objective means. . . .⁴

The purpose of this paper is to propose a model that might be able to serve as the basis for an objective, quantitative evaluation of the reliability of the internal control system. Tying in this evaluation with the evaluation of the bonafides of the accountá is not considered in this paper.

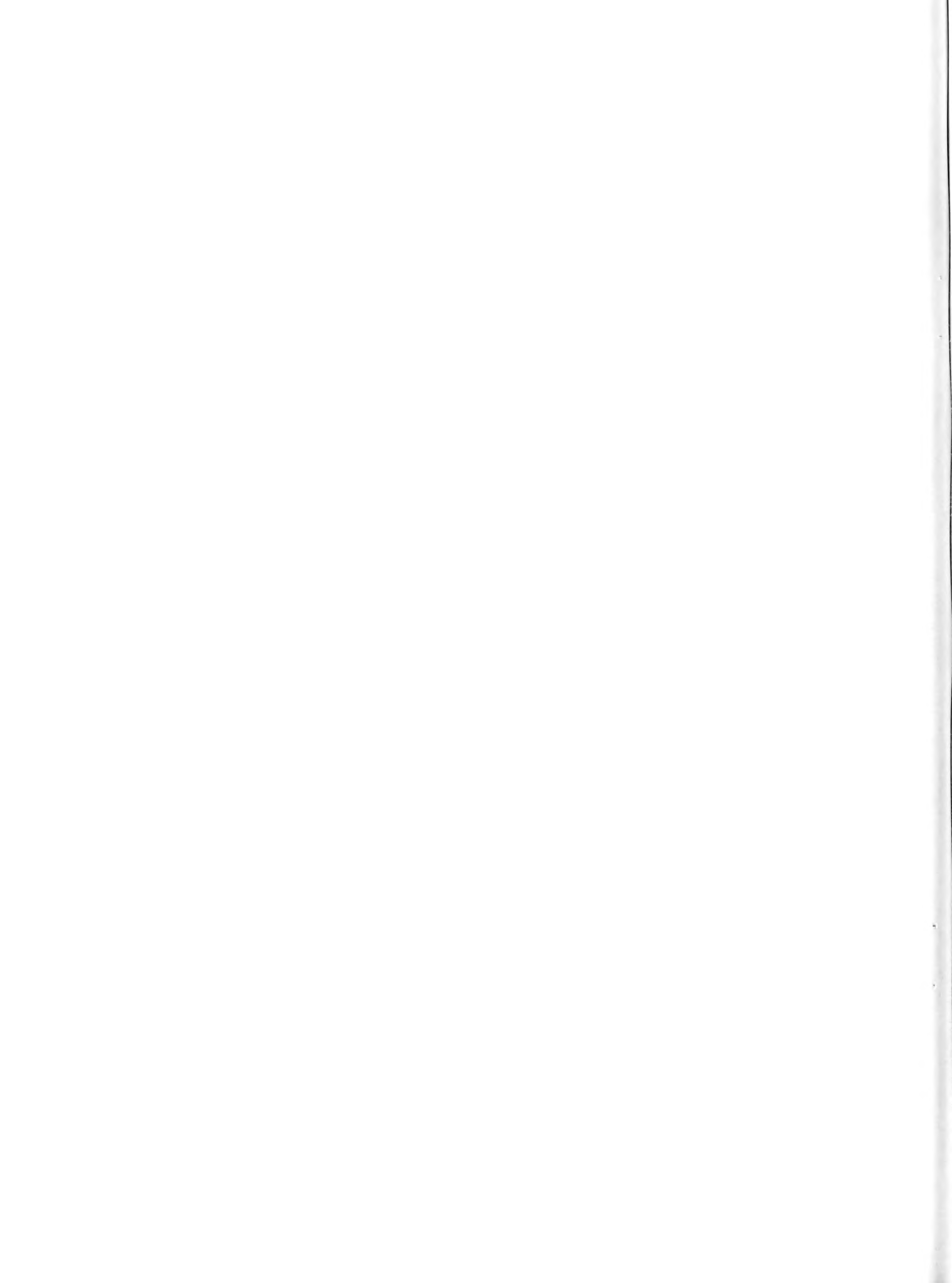
NEED FOR STOCHASTIC SYSTEMS EVALUATION

The financial information system consists of many operating elements, including internal controls, which are methodically connected so that the system can produce, whenever inputs are given, some type of outputs. The quality of the output depends on the quality of performance of the individual elements, but, generally, the probability that such performance is perfect is less than one. That is, each element in the system has some propensity for introducing errors and/or for not eliminating them. Four aspects of the financial information system are of key importance in developing a model describing the system of internal control:

⁴R. M. Trueblood and R. M. Cyert, Sampling Techniques in Accounting (Englewood Cliffs: Prentice-Hall, 1957), p. 53.



1. Internal control measures and devices are integrated and meshed into the financial information system. This implies that the evaluation of the system of internal controls cannot be done by itself in a vacuum but should be done in relation to the overall system's reliability.
2. A system's reliability depends on the quality of performance of the operating elements. No matter how many internal controls are employed, if the persons who operate the controls do not perform their duties properly, the internal controls do not assure high system reliability.
3. The quality of the accounting documents changes as they are processed through the system. Each operating element can alter the quality by either adding or correcting errors. In a payroll system, for example, a time clock attendant makes an error in recording work hours (i.e., hour error only at this point); next, the error-ridden time card is used to compute gross wage (i.e., hour error and gross wage error at this point); on the basis of the computed gross wage, taxes are deducted (i.e., hour error, gross wage error, and deduction error at this point). Next, the internal auditor reviews the original time card and happens to detect the hour error and rectifies all the related errors (i.e., no error at this point). This situation not only shows how the quality of the documents may change from one step to the next, but also indicates that the quality of the accounting output may be strongly affected by the quality of the input.
4. Since the operating elements add or correct errors typically in a fashion that may be viewed as probabilistic, the quality of the accounting data may be regarded as a stochastic variable and the movement of error states of the accounting data may be described as a stochastic



process.

In this paper, the term reliability is defined as the probability of the internal control system eliminating errors, irregularities, or fraud in the financial information process under the operating conditions present during a given period.

MODELING OF INTERNAL CONTROL SYSTEM

In order to facilitate the presentation, we will discuss the proposed modeling process and the nature of the problems encountered for a small segment of the accounting system, namely in terms of the payroll system. The reason the payroll system is chosen is: (1) it is a system found in every business organization; (2) it is a self-contained system that has limited interactions with other segments of the accounting system; and (3) the payroll is usually a significant portion of total business expenses.

A hypothetical manual payroll system is presented in the form of a flow chart in Figure 1. This payroll system has the basic payroll control measures: time card punching, foreman's review, payments by checks, review by the controller, the use of the imprest account, and monthly bank reconciliation.

Definition of Operating Elements

Since we shall view the quality of accounting data processing as a stochastic process, we first need to define the operating elements of the payroll system. For our purposes, an operating element will be a performance unit that can affect the state of error or quality of documents. Operating elements may be classified according to various criteria that the auditor considers most appropriate for his purposes. The criteria



that appear to be useful for our purpose are "by people" and "by function." Using these criteria, sixteen separate operating elements are obtained for the payroll system, and they are shown in the flow chart of Figure 1. The rationale for the two criteria are: (1) the performance of the elements can be easily observed and evaluated when they are classified by people and function because many documentary evidences in accounting are easily identifiable by these classifications; (2) in a financial information system where possibilities of intentional as well as unintentional errors and irregularities are of major concern, the arrangement of people and functions can have a significant impact on the system reliability.

A constraint, however, must be taken into consideration in defining the payroll system operating elements by the two criteria: independent relations between defined elements must be maintained. The reason is that the mathematical model to be suggested requires independent relations. This restriction is not a serious one, however, because independent relations are typically found in financial systems to help reduce and control errors, intentional and others.

Definitions of Errors and Error States

Next, we need to define the kinds of errors the operating elements may commit during the operating processes. Errors in payroll accounting are made typically in work hours, employee names, deductions, over-or-under-payments, pay rates, and payments to non-existent employee. Errors may arise singly or in combination. Furthermore, an error of one kind may induce other kinds of errors. For example, an error in employee name may lead to an error in pay rate and deductions for another employee.

To make the demonstration of the model simple, we will define just

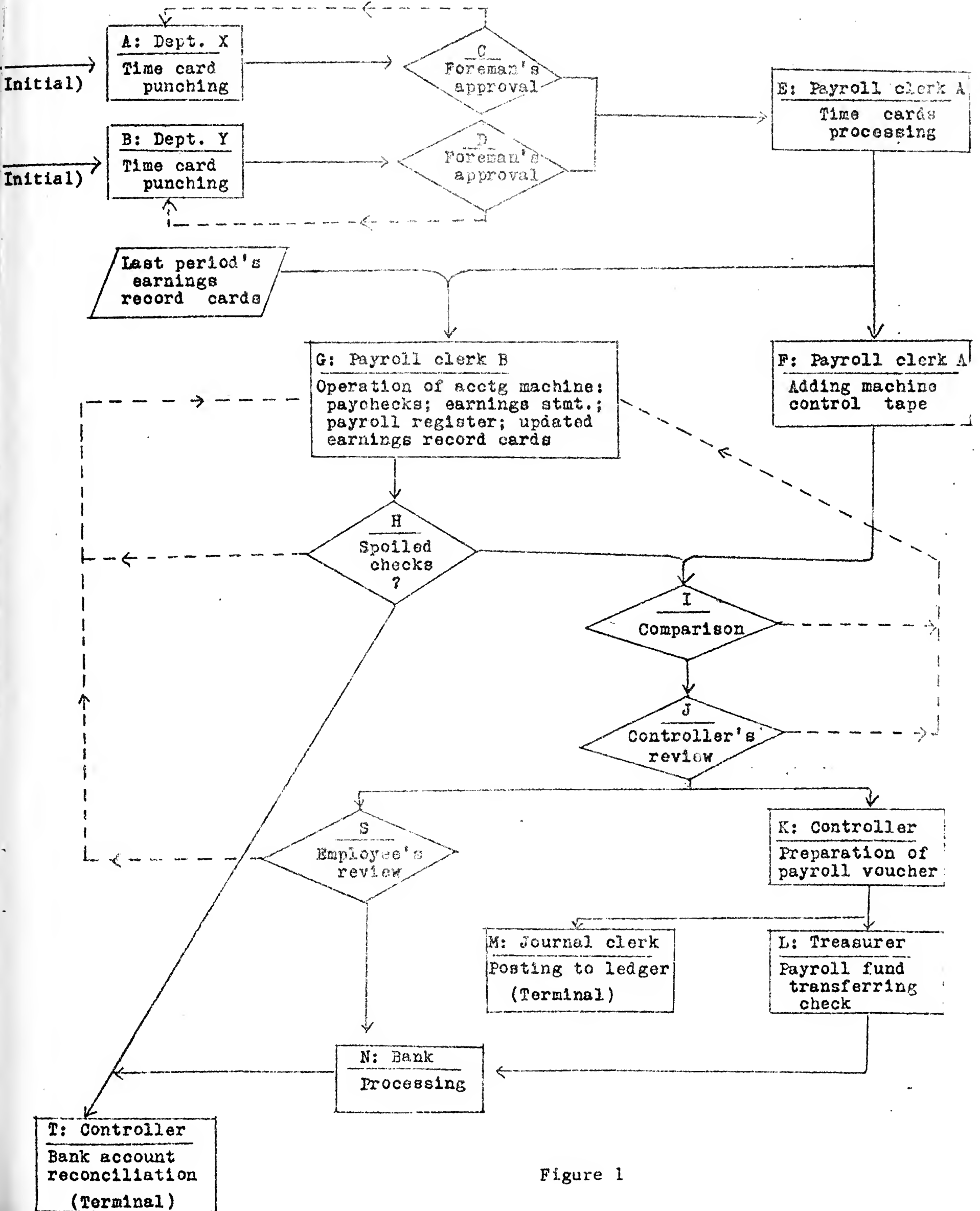


Figure 1

two kinds of errors: (1) monetary errors, and (2) non-monetary errors. Monetary errors include any error that is represented by a \$ sign. For example, errors in pay rate, tax deductions, net pay, and gross pay fall in this category. Non-monetary errors include all other errors such as errors in name, social security number, work hours, etc. We can therefore enumerate four distinct categories that describe the quality state of an accounting document after the processing by any operating element:

Category 1: No error of either kind

Category 2: Monetary error only

Category 3: Non-monetary error only

Category 4: Monetary as well as non-monetary errors

As the payroll operation proceeds from one element to the next, the payroll documents will move between these four error states.

Consider a vector of the form (r_1, r_2) where r_1 is an indicator variable taking on the values 1 and 0 to represent the presence or absence, respectively, of monetary errors, and r_2 is defined likewise for non-monetary errors. We can therefore describe the four categories as follows:

$s_1 = (0, 0)$: Absence of any errors

$s_2 = (1, 0)$: Presence of monetary error only

$s_3 = (0, 1)$: Presence of non-monetary error only

$s_4 = (1, 1)$: Presence of monetary and non-monetary errors

In general, we may define as many relevant kinds of errors as the audit situation warrants. We simply need to enumerate, according to the above rules, all possible combinations of error states, one of which each accounting document must occupy at any point in the processing.

Operation Probability Matrix

Operating elements perform basically two types of functions:⁵

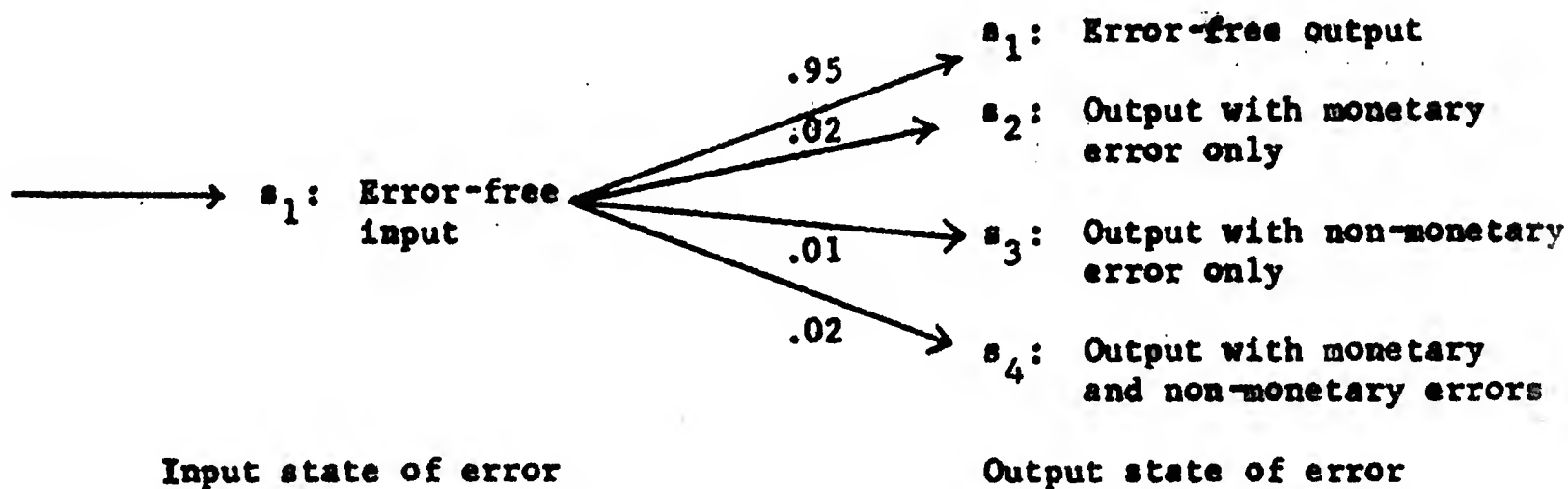
(1) transformation operations, and (2) decision operations. A transformation operation converts input data into output in accordance with certain rules. For example, time card punching, preparation of paychecks from punched time cards, and posting to payroll ledger are transformation operations. These are shown in rectangular boxes in the flow chart of Figure 1. A decision operation, on the other hand, is basically a sorting operation. It distinguishes one input from another and takes a separate action for each, like a sieve or gravel grader. For example, a foreman reviews time cards and rejects any unusual ones as incorrect; or the controller reviews employees' earnings statements and rejects for correction those he judges as incorrect statements. Decision operations are shown in diamonds in the flow chart of Figure 1.

While each operating element is supposed to follow a set of deterministic rules, its performance is not always perfect, resulting in some deviations that are commonly called errors. Each operator has some propensity to introduce errors, as well as a propensity to change and eliminate errors.⁶ These propensities are a function of the operator's skill, the quality of the input, and the characteristics of the transformation or decision operation. One way of describing the propensities is by use of the probability concept.

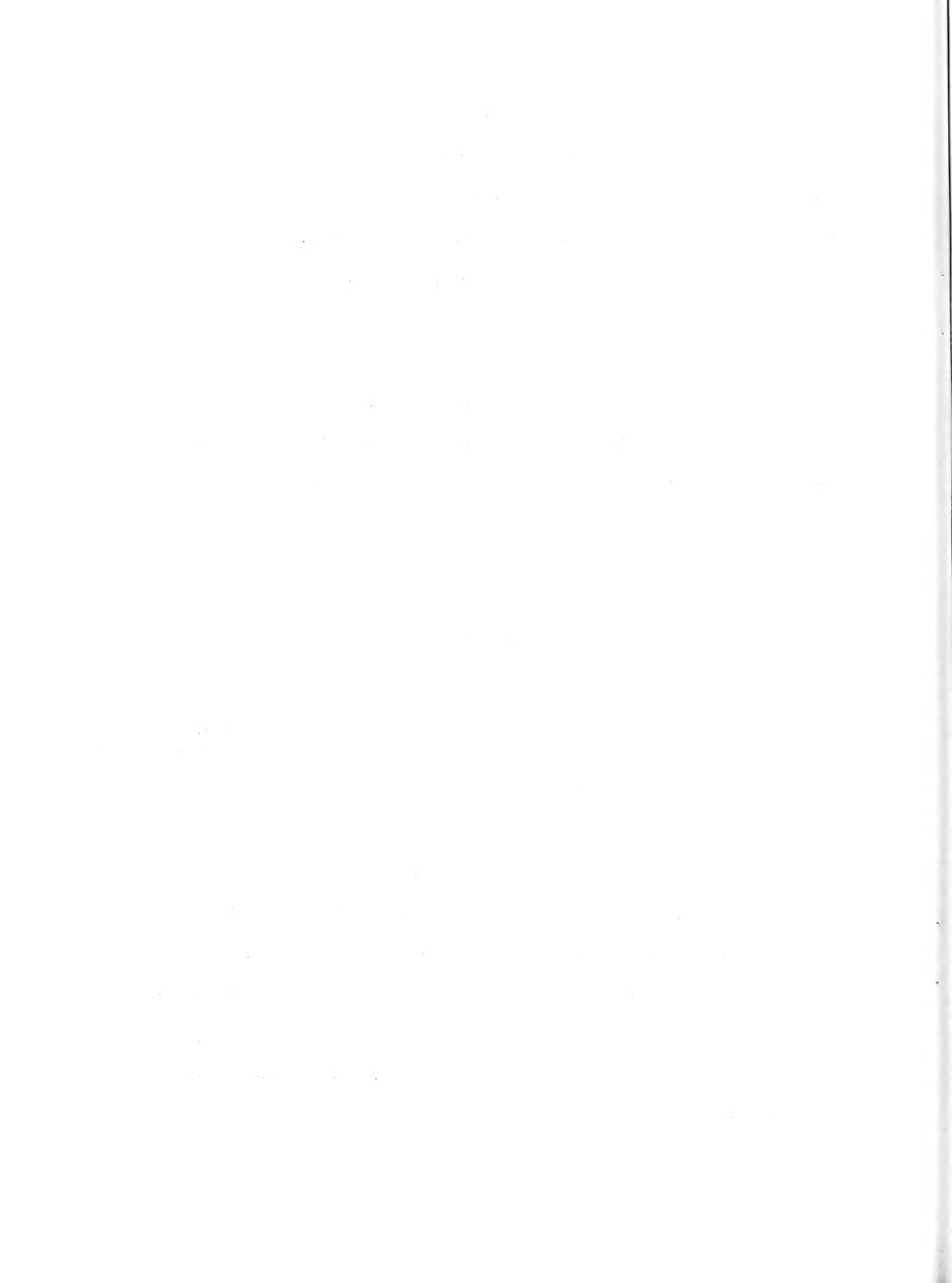
⁵V. C. Hare, Jr., Systems Analysis: A Diagnostic Approach (New York: Harcourt, Brace & World, 1967), pp. 27, 41.

⁶R. Leasing, "A Performer-Oriented Approach to Systems Quality," Management Science, 16, 1969, p. B-264.

To illustrate how probabilities can be used to model these propensities, consider a payroll clerk who prepares employees' earnings statements and paychecks from time cards and earnings record cards which are assumed to have no errors. In other words, the payroll clerk performs a transformation operation, and the documents reach him in state s_1 . The payroll clerk may produce error-free paychecks and earnings statements (state s_1) most of the time, say with a probability of .95. Occasionally, he produces outputs containing monetary errors only (state s_2), non-monetary errors only (state s_3) or both (state s_4), say with probabilities of .02, .01, .02, respectively. The situation can be shown by a tree diagram:



We indicated earlier that the operator's propensities typically depend on the quality of the input. Thus the probability that the output document occupies any specific error state may be different from the one shown in the tree diagram if the input state is not s_1 . Thus, we need four different tree diagrams, as there are four different input error states in our example. The presentation of four different trees can best be summarized by a matrix:

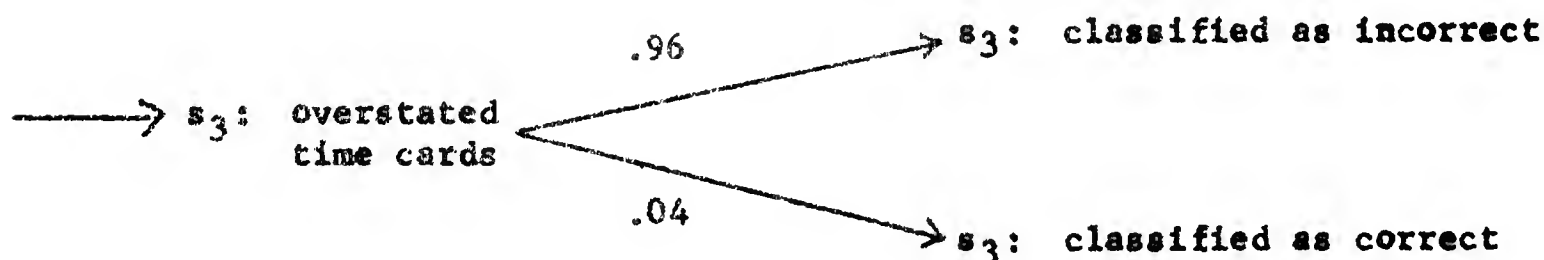


$$\begin{array}{cccc}
 s_1 = (0, 0) & s_2 = (1, 0) & s_3 = (0, 1) & s_4 = (1, 1) \\
 s_1 = (0, 0) & \left\{ \begin{array}{cccc} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{array} \right. & = & P \\
 s_2 = (1, 0) & & & \\
 s_3 = (0, 1) & & & \\
 s_4 = (1, 1) & & &
 \end{array}$$

The matrix P is called a Transformation Probability Matrix. The subscript i of p_{ij} represents the input state of error, corresponding to s_1, s_2, s_3, s_4 , respectively. The second subscript, j, represents the output state of error. The symbol p_{ij} is the probability, given an input state s_i , that the transformation operator will produce an output state s_j . The symbol p_{ij} is called a transition probability, and must satisfy the conditions:

$$0 \leq p_{ij} \leq 1, \quad \sum_j p_{ij} = 1.$$

A decision operation may be described in a similar way. Consider a foreman who reviews the time cards at the end of each period and classifies them as either approved or rejected before they are forwarded to the payroll department. His decision will not always be perfect, but subject to some probabilistic pattern. If he receives time cards reflecting over-stated work hours (state s_3), he may classify them as incorrect with a probability of, say, .96, and as correct with probability .04. This may be shown as follows:





or, in vector notation:

classified as incorrect				classified as correct					
s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4		
s_3 :	(0,	0,	.96,	0)	s_3 :	(0,	0,	.04,	0)

A similar expression can be obtained for the other incoming quality states. Combining these, we may obtain a Decision Probability Matrix Q as follows:

	Incorrect category				Correct category				
	s_1	s_2	s_3	s_4	s_1	s_2	s_3	s_4	
s_1	q_1^i	0	0	0	q_1^c	0	0	0	= Q
s_2	0	q_2^i	0	0	0	q_2^c	0	0	
s_3	0	0	q_3^i	0	0	0	q_3^c	0	
s_4	0	0	0	q_4^i	0	0	0	q_4^c	

where $q_j^i + q_j^c = 1$, and $q_j^i, q_j^c \leq 1$ for every j , ($j= 1, 2, 3, 4$). We shall denote the first and second components of Q as Q^i and Q^c , respectively.

Basic Operation on Input Vector

To illustrate how the elements discussed so far are combined, suppose the punched time cards forwarded to Payroll Clerk A (Figure 1) are distributed among the error states according to the following probability distribution:

$$W_I = \begin{matrix} & s_1 & s_2 & s_3 & s_4 \\ \begin{matrix} W_I \\ \text{---} \end{matrix} & (.966 & 0 & .034 & 0) \end{matrix}$$

This vector signifies that the time cards are free from errors with probabilities of .966; have non-monetary error only with probability .034; and have no errors otherwise. This vector is called an input vector.

Payroll Clerk A performs his payrolls processing on this input. Suppose

Payroll Clerk A has the following transformation probability matrix:

$$P = \begin{matrix} & \begin{matrix} s_1 & s_2 & s_3 & s_4 \end{matrix} \\ \begin{matrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{matrix} & \begin{pmatrix} .94 & 0 & .06 & 0 \\ 0 & 1 & 0 & 0 \\ .44 & 0 & .56 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{matrix}$$

Then, the output quality resulting from Payroll Clerk A's operation is obtained by matrix multiplication $W_I \cdot P$ or:

$$W_O = W_I \cdot P = \begin{pmatrix} .925 & 0 & .075 & 0 \end{pmatrix}$$

W_O is called the output vector. It shows that Payroll Clerk A has a tendency to increase the non-monetary error rate from .034 to .075. This output vector is then used as an input vector for the next operation (Payroll Clerk B's transformation) to obtain another output vector.

This process is continued until the last operation. As a result, we obtain a sequence of output vectors showing how the quality of the documents changes between the four error states as they are processed through the system. However, the basic operation linking the input vector and the transformation probability matrix:

$$W_I \cdot P = W_O$$

is subject to some modification because not all operations are connected in one series. Sometimes, the documents are branched for different processing routes and then merged, sometimes merging of various types of documents takes place, and sometimes documents are returned for correction. We consider now how these more complicated cases are handled.

Branching operations. Branching generally arises due to internal controls such as reviews, approvals, or comparisons that result in sorting between acceptable and unacceptable documents. In our payroll example, the Controller reviews employees' paychecks, payroll register and updated earnings record cards and rejects any erroneous items before he signs the paychecks. Suppose his decision probability matrix Q has the following form:

$$\begin{array}{c}
 Q': \text{ Approved as correct } \qquad \qquad \qquad Q'': \text{ Rejected as incorrect } \\
 \begin{array}{cccc|cccc}
 & s_1 & s_2 & s_3 & s_4 & s_1 & s_2 & s_3 & s_4 \\
 s_1 & \left(\begin{array}{c} .99 \\ 0 \\ 0 \\ 0 \end{array} \right. & \begin{array}{c} 0 \\ .87 \\ 0 \\ 0 \end{array} & \begin{array}{c} 0 \\ 0 \\ .92 \\ 0 \end{array} & \begin{array}{c} 0 \\ 0 \\ 0 \\ .65 \end{array} & \left. \begin{array}{c} \begin{array}{c} .01 \\ 0 \\ 0 \\ 0 \end{array} \\ \begin{array}{c} 0 \\ .13 \\ 0 \\ 0 \end{array} \\ \begin{array}{c} 0 \\ 0 \\ .08 \\ 0 \end{array} \\ \begin{array}{c} 0 \\ 0 \\ 0 \\ .35 \end{array} \right) \\
 s_2 \\
 s_3 \\
 s_4
 \end{array}
 \end{array} = Q$$

Also assume the documents the Controller receives from the payroll clerks have the error distributions:

$$W_I = (.894 \quad .011 \quad .046 \quad .049).$$

Then, by multiplying W_I and Q , we obtain,

$$\begin{aligned}
 W_I \cdot Q &= W_I \cdot (Q', Q'') \\
 &= (W_I \cdot Q', W_I \cdot Q'') \\
 &= (W_0', W_0'') \\
 &= \left((.885 \quad .01 \quad .042 \quad .032), (.009 \quad .001 \quad .004 \quad .017) \right),
 \end{aligned}$$

where the first and second component vectors are denoted as W_0' and W_0'' , respectively. Note that the Controller, among the 89.4% of perfect documents, classifies 99% (i.e., $.885/.894$) as correct, and 1% (i.e., $.009/.894$) as incorrect, and similarly for documents in the other error categories. Thus, a set of payroll documents is branched, with certain probabilities,

for different routes and actions.

There is another type of branching that arises when there are multi-copies of the same document for a given transaction. An example may be found outside the payroll system: the sales department prepares three copies of a sales transaction; one copy is routed to the customer, the second to the accounts receivable clerk, and the third to the inventory clerk. This type of branching is not real branching, as each party receives the identical documents of the identical quality. In this case, the basic operation of the form $W_I \cdot P$ is applicable for each route.

Merging operations. Documents that are branched or processed separately may merge together again. There are two types of merging operations. With the first type of merging, documents representing separate transactions of different departments are merged together. In the payroll example, the time cards of Department X employees are merged with those of Department Y employees in order for Payroll Clerk A to process them together. This kind of merging is handled by adding the two output vectors from each department, with suitable normalization. Suppose 50 time cards from Department X and 100 time cards from Department Y are merged, and that the output vectors are:

$$W_x = (.938 \quad 0 \quad .062 \quad 0)$$

$$W_y = (.980 \quad 0 \quad .020 \quad 0),$$

respectively. The merging of these two sets is obtained by adding the vectors with appropriate normalization, which is necessary to maintain the characteristic of the probability vector after merging:

$$\begin{aligned} W_x / 3 + 2W_y / 3 &= W_{x,y} \\ &= (.966 \quad 0 \quad .034 \quad 0). \end{aligned}$$

This direct addition is justified because each vector represents different transactions.

The second type of merging, which corresponds to the second case of branching discussed earlier, arises when the input vectors coming from different routes represent the identical set of transactions or when different documents pertaining to the same transaction are combined to form one complete document. For example, Payroll Clerk B receives two sets of documents for a given employee's payroll processing: the earnings record card showing the accumulated payroll information up to the last pay period and the current period's time card. These are combined to constitute one record.

One important problem with this kind of merging is that the error state of a given transaction document coming from one route may be different from the error state of the transaction document from another route. When merged, the outcome state may be still another type of error state. For example, consider an employee's earnings record card that contains some monetary error: $s_2 = (1, 0)$. Also suppose that the same employee's time card has only non-monetary error: $s_3 = (0, 1)$. When Payroll Clerk B puts these two documents together for the employee's paycheck preparation, he processes with an input document having both monetary and non-monetary errors: $s_4 = (1, 1)$. Let W_1 and W_r denote the input vectors representing the qualities of the earnings record cards and the time cards, respectively; and let their components be

$$\begin{aligned} W_1 &= (w_1 \quad w_2 \quad w_3 \quad w_4) \\ W_r &= (w'_1 \quad w'_2 \quad w'_3 \quad w'_4). \end{aligned}$$



Now, these two vectors are going to merge and produce one input vector W_b for Payroll Clerk B's operation. Let its components be

$$W_b = (w_1'' \quad w_2'' \quad w_3'' \quad w_4'')$$

Assuming the merging act is but a collating act and therefore does not alter the quality of the documents, an error before merging remains also an error after merging. Therefore, merging of, say, $s_1 = (0, 0)$ and $s_2 = (1, 0)$ results in $s_2 = (1, 0)$. Applying this rule, we can obtain a states combination table that shows the merged error state for any combination of separate states:

	$s_1 = (0,0)$	$s_2 = (1,0)$	$s_3 = (0,1)$	$s_4 = (1,1)$	States from time cards
	w_1	w_2	w_3	w_4	
$s_1 = (0,0)$	(0,0) ----- $w_1 \cdot w_1^i$	(1,0) ----- $w_2 \cdot w_1^i$	(0,1) ----- $w_3 \cdot w_1^i$	(1,1) ----- $w_4 \cdot w_1^i$	
$s_2 = (1,0)$	(1,0) ----- $w_1 \cdot w_2^i$	(1,0) ----- $w_2 \cdot w_2^i$	(1,1) ----- $w_3 \cdot w_2^i$	(1,1) ----- $w_4 \cdot w_2^i$	
$s_3 = (0,1)$	(0,1) ----- $w_1 \cdot w_3^i$	(1,1) ----- $w_2 \cdot w_3^i$	(0,1) ----- $w_3 \cdot w_3^i$	(1,1) ----- $w_4 \cdot w_3^i$	
$s_4 = (1,1)$	(1,1) ----- $w_1 \cdot w_4^i$	(1,1) ----- $w_2 \cdot w_4^i$	(1,1) ----- $w_3 \cdot w_4^i$	(1,1) ----- $w_4 \cdot w_4^i$	

States from the last
period's earnings
record cards

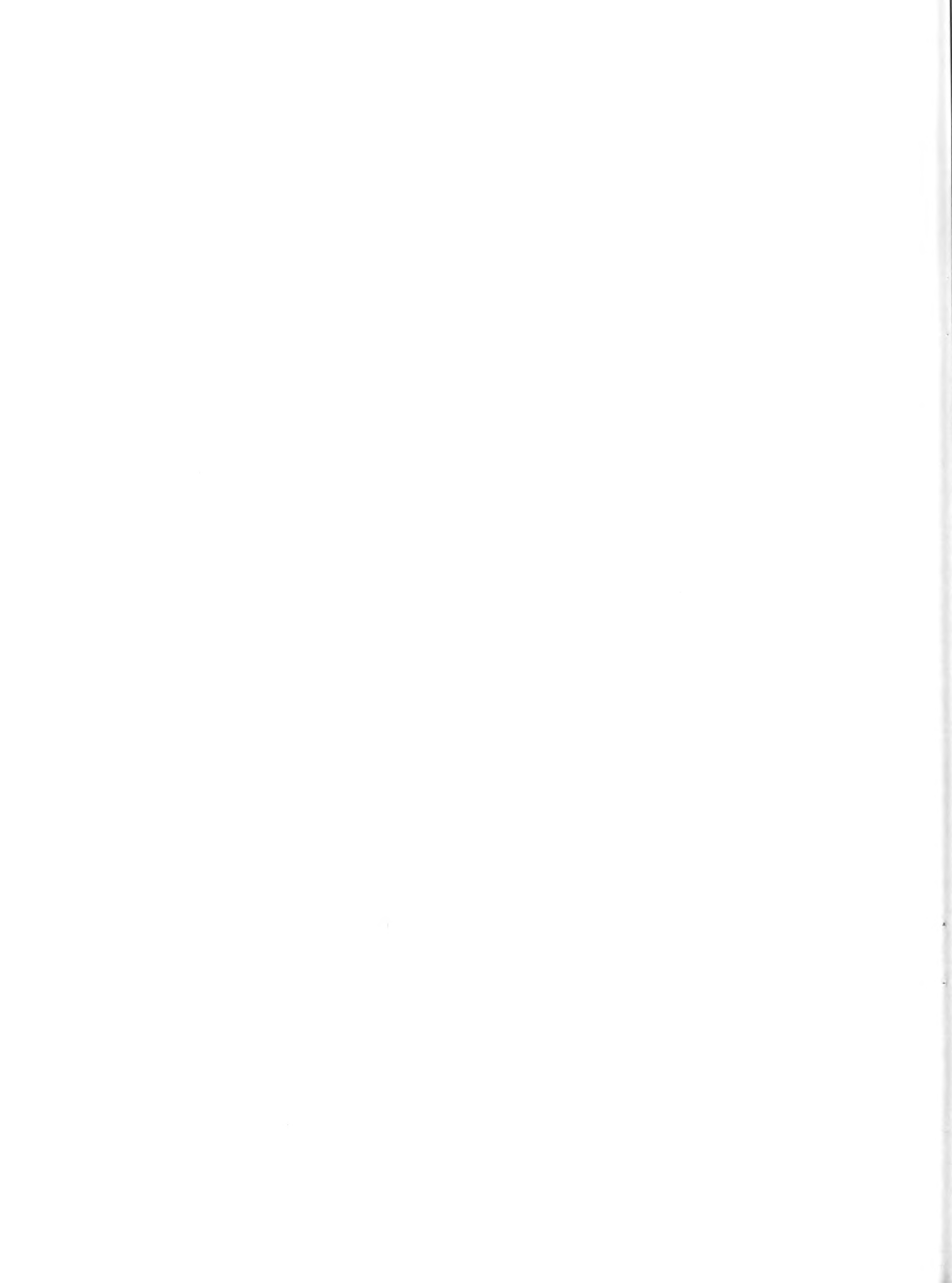
Figure 2

A mechanical method of preparing the table is to follow the rule:

$$(i,j) + (h,k) = (m,n),$$

where $i, j, h, k, m, n = 0, \text{ or } 1$; and $m, n = 1$ if the vector addition results in a number greater than or equal to one; otherwise $m, n = 0$.

Since the operating elements are defined to have independent



relations, the table of Figure 2 can be used as a joint probability table, each cell occurring with probability $w_i \cdot w_r$, ($i, r = 1, 2, 3, 4$). Now grouping the joint probabilities $w_i \cdot w_r$ according to the newly formed error states in the table, the following merged vector $W_b = (w_1'' \quad w_2'' \quad w_3'' \quad w_4'')$ is obtained:

$$\begin{aligned}
 s_1 = (0,0): \quad w_1'' &= (w_1 \cdot w_1) \\
 s_2 = (1,0): \quad w_2'' &= (w_1 \cdot w_2) + (w_2 \cdot w_1) + (w_2 \cdot w_2) \\
 s_3 = (0,1): \quad w_3'' &= (w_1 \cdot w_3) + (w_3 \cdot w_1) + (w_3 \cdot w_3) \\
 s_4 = (1,1): \quad w_4'' &= (w_2 \cdot w_3) + (w_3 \cdot w_2) + (w_1 \cdot w_4) \\
 &\quad + (w_2 \cdot w_4) + (w_3 \cdot w_4) + (w_4 \cdot w_4) \\
 &\quad + (w_4 \cdot w_1) + (w_4 \cdot w_2) + (w_4 \cdot w_3).
 \end{aligned}$$

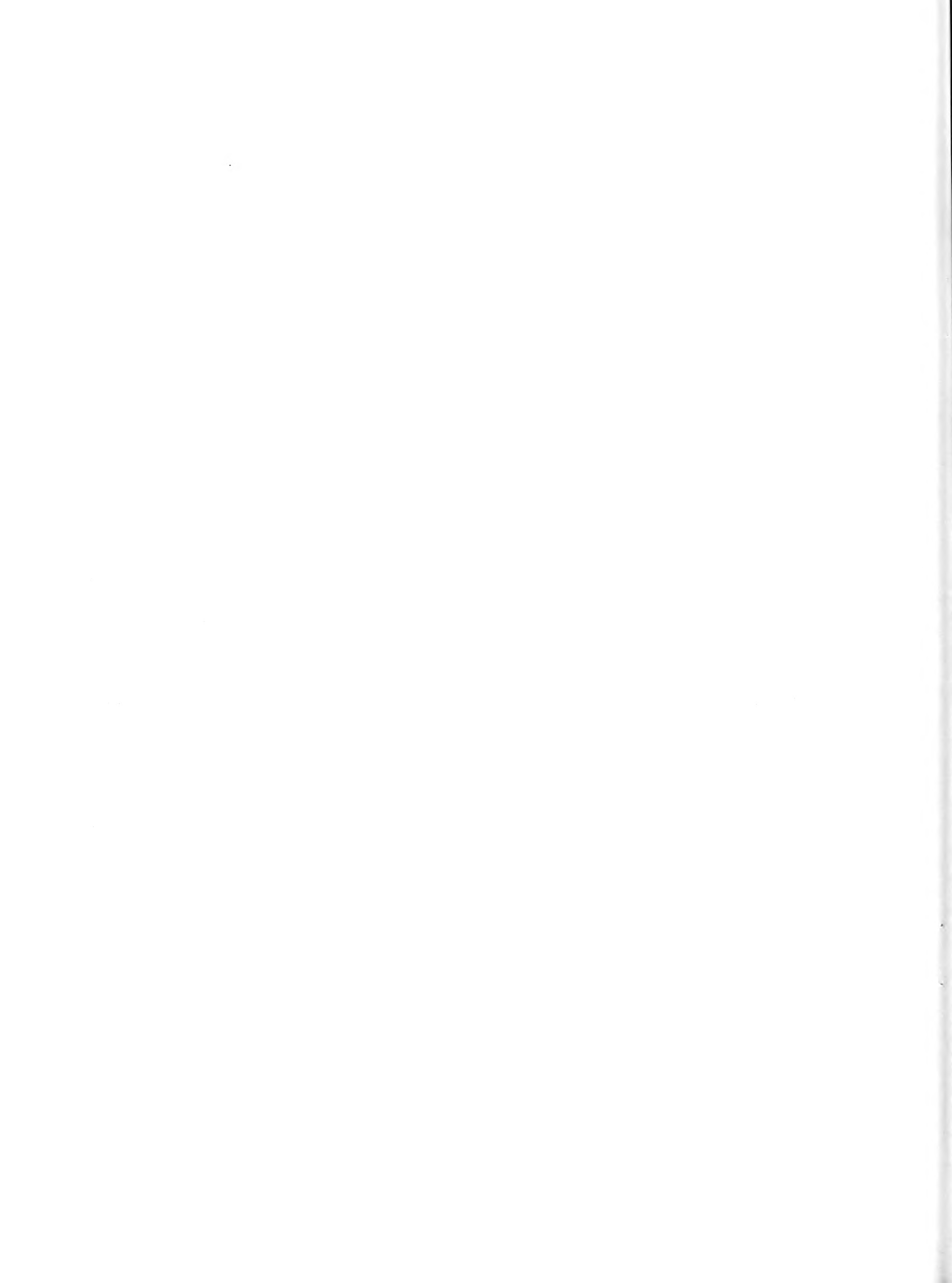
Feedback operation for correction. A feedback operation typically arises when an internal control element detects errors and returns the error-ridden document back to an appropriate operator for correction. This feedback operation always requires branching (i.e., separation of error documents) and merging (i.e., reunion with the corrected documents).

Returning to the example used for explaining a branching operation, suppose the Controller reviews paychecks and payroll register and separates between correct and incorrect documents as follows:

$$\text{Correct documents: } W_0' = (.885 \quad .01 \quad .042 \quad .032)$$

$$\text{Incorrect documents: } W_0'' = (.009 \quad .001 \quad .004 \quad .017).$$

Since in accounting data processing, most of the errors detected by internal control measures receive special attention, we assume that all detected errors emerge eventually as correct documents. To reflect this, we use a



special transformation matrix R:

$$\begin{array}{c}
 s_1 \\
 s_2 \\
 s_3 \\
 s_4
 \end{array}
 \begin{array}{c}
 s_1 \quad s_2 \quad s_3 \quad s_4 \\
 \left[\begin{array}{cccc}
 1 & 0 & 0 & 0 \\
 1 & 0 & 0 & 0 \\
 1 & 0 & 0 & 0 \\
 1 & 0 & 0 & 0
 \end{array} \right]
 \end{array}
 = R$$

to which the rejected output vector W''_0 is multiplied. For our example, we obtain:

$$\begin{aligned}
 W''_0 \cdot R &= (.009 \quad .001 \quad .004 \quad .017) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \\
 &= (.031 \quad 0 \quad 0 \quad 0).
 \end{aligned}$$

Now, this vector is rejoined to W'_0 :

$$W'_0 + W''_0 \cdot R = (.916 \quad .01 \quad .042 \quad .032).$$

This merged output vector reflects the quality of the documents passed through the controller's inspection.

When the documents returned for correction receive less than perfect attention and consequently still contain errors, the problem becomes more complicated, involving repeated feedback loops.

An overall view of the system. We have explained how accounting data processing, including systems of internal control, can be described by the basic operations of branching, merging, and feedback, and have discussed how to model these processes. These processes have been combined to model the payroll system used as an example in this paper, with



hypothetical transformation and decision matrices provided in the appendix. The results are shown in Figure 3. We discuss the implications of this model next.

USES OF THE MODEL

The auditor can use the proposed model for internal control for at least two purposes: (1) for the probabilistic evaluation of the system reliability; and (2) for assisting in the design of the internal control system.

Probabilistic Evaluation of the System Reliability

The immediate purpose of reviewing the internal control system is to obtain information that will show how much confidence can be placed in the system. This information is summarized in the terminal output vector produced from the model. In the payroll example, we have two terminal output vectors:

$$\begin{array}{rcccc} & s_1 & s_2 & s_3 & s_4 \\ W_t = & (.992 & .002 & .004 & .002) \\ W_m = & (.871 & .056 & .042 & .031), \end{array}$$

where W_t and W_m denote the output vectors from the bank reconciliation and the journal entries to the payroll expense accounts, respectively.

The closer the first component of each vector is to 1, the higher the probability that the system produces reliable results. If the first component is not of high probability, we may wish to examine the third component of the vector, which is associated with non-monetary errors and thus not related to the monetary bonafides of the payroll expense, to see if most of the final errors are of the non-monetary type.

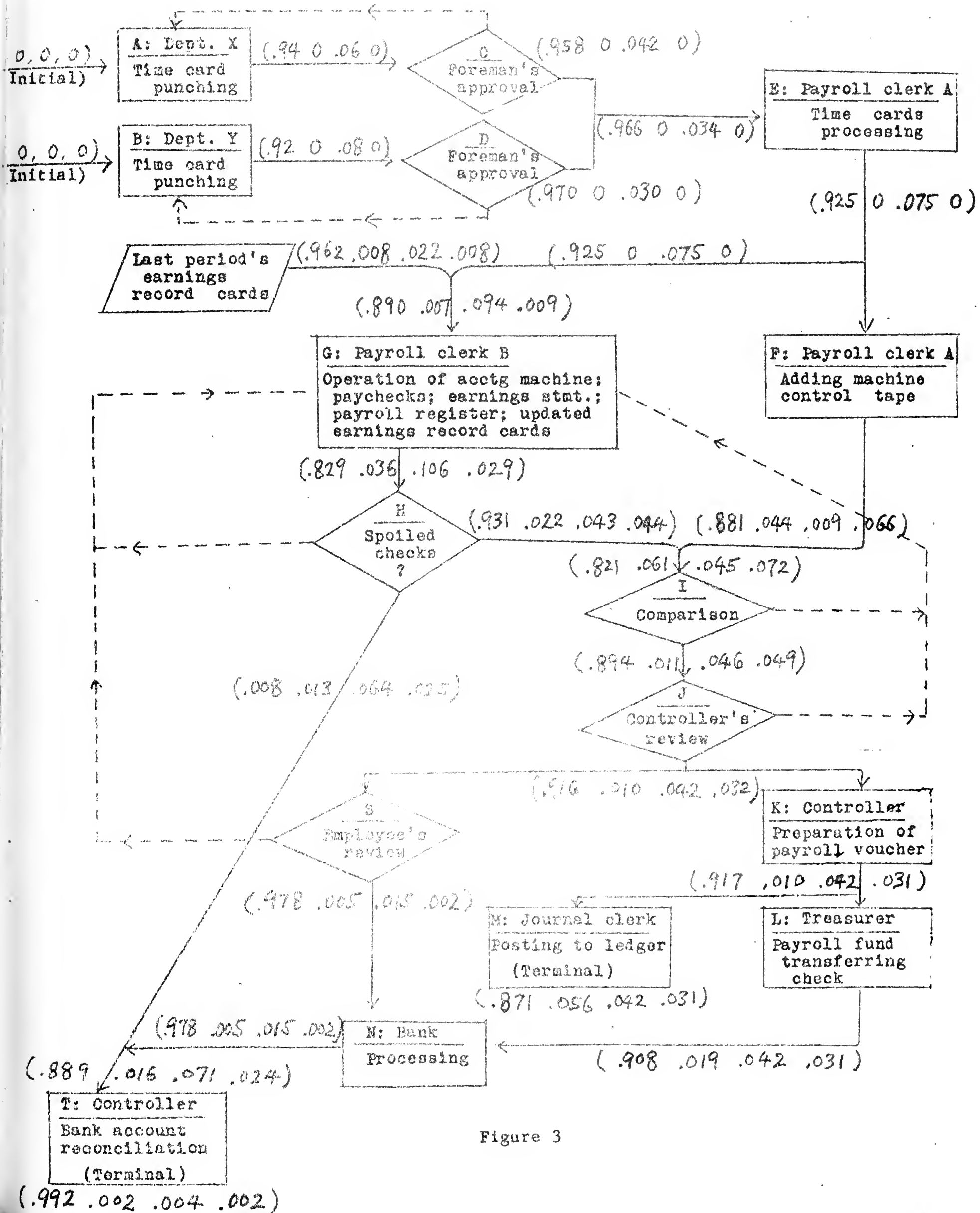


Figure 3

If we reach the conclusion that the internal control system is unreliable, we may want to analyze the system further to find the area or elements that need closer investigation. The system may be analyzed by one or more of the following approaches: (a) by observing the sequence of output vectors; (b) by observing a set of terminal vectors, each vector representing the case where an error is introduced at a given point in the system's processing line; and (c) by observing the output vectors when the probability matrices P and/or Q are changed.

Sequence view of error states. We can study the sequence of output vectors most easily by a graph. Let the vertical axis of the graph represent probability, and let operating elements be represented on the horizontal axis. We then plot the component elements of each output vector as shown in Figure 4. The curves, $\text{Prob}(s_1)$, $\text{Prob}(s_2)$, etc. show the sequence of error state probabilities leading toward the terminal output of the bank reconciliation. A similar graph may be obtained for the sequence leading to the other terminal output of journal entries. We see from Figure 4 that the quality of the documents (in terms of s_1) reaches the lowest level at operation G (operation of accounting machine by Payroll Clerk B) and improves as the subsequent internal controls designated as I, J, S, and T find and rectify errors.

Observation of a set of terminal vectors. One of the most important questions that a person who analyzes an internal control system asks is how reliably the system can work under unusual conditions. One way of analyzing this is to observe the effect on the terminal output vectors when a specified error type is placed into the system.

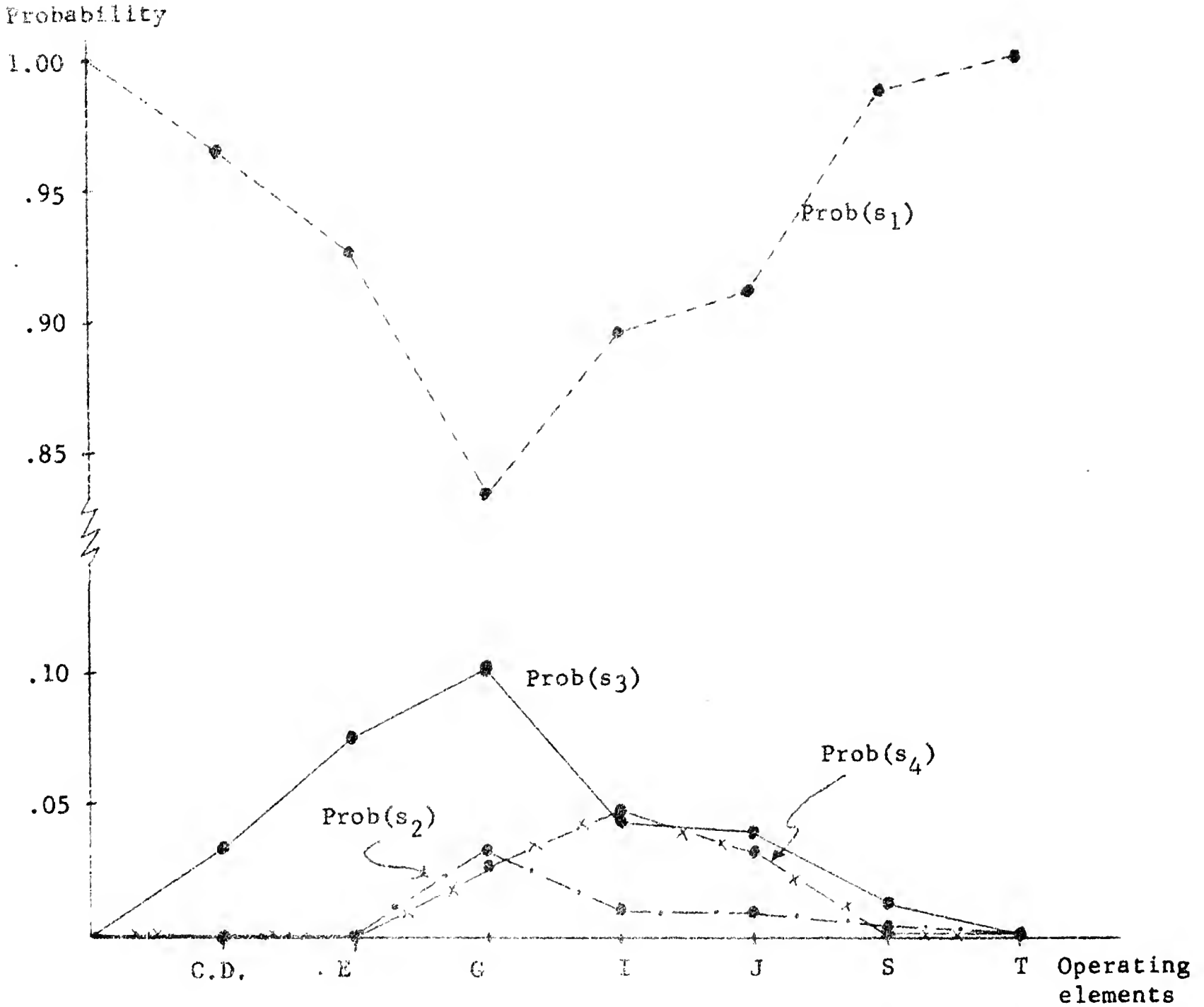


Figure 4

Suppose we want to see the impact of a non-monetary error introduced at a given point in the process on the system's reliability. The introduction of such an error into the system may be simulated by using a probability vector of the form:

$$W(s_3) = \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

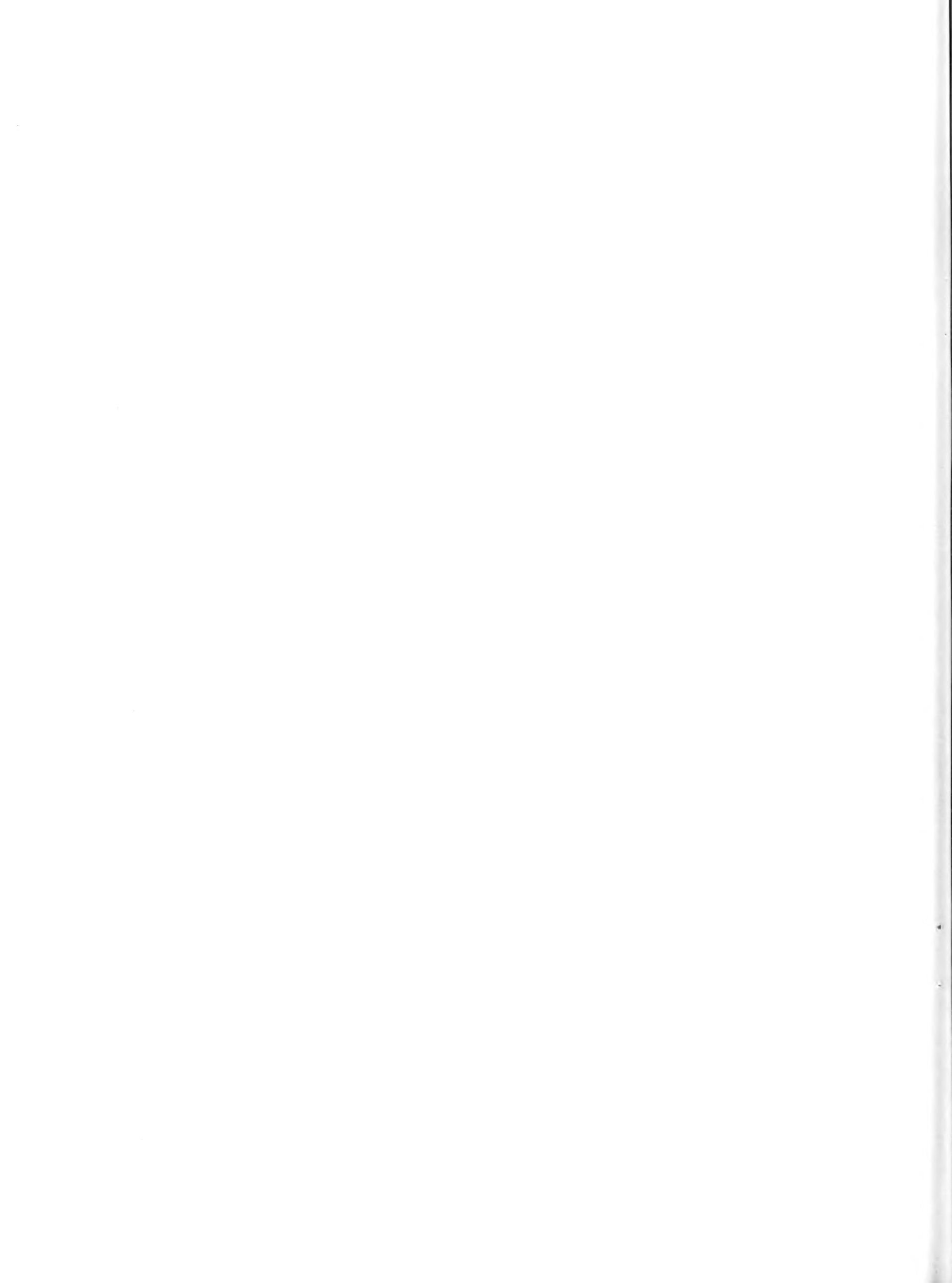
which signifies the existence of an error of the non-monetary type (s_3)

with probability one. To study the impact of such an error at the beginning of the system, we use this vector as the input vector for the first operating element of the system and go through the usual process to come out with a terminal output vector. We can study the impact of a non-monetary error at any other point in the system in similar fashion. Such analyses can be most helpful. For instance, suppose that introducing a non-monetary error in the preparation of the payroll register leads to low terminal quality. Hence, a close examination of such errors at the stage of the payroll register preparation is most important.

Effect of changes in operating probability matrices. The model can also be used for investigating the effect of unusual performance of operating elements. For example, hiring of new employees, breakdown of accounting machines, intentional disturbance of operation by dishonest employees, intensive employee education, etc., would result in different performances. To see the impact of such changes on the quality of the terminal outputs, we would need to obtain different operation probability matrices reflecting the new situation and replace the old matrices with the new ones in computing the sequence of output vectors. This new set of vectors may be evaluated on its own or compared to the set obtained under the old situation.

Design of System

Another area where we can apply the model is to help design the internal control system. The design problem may involve some changes in the existing system's elements and/or configuration. For example, a company may want to use accounting machines instead of manual preparation,



eliminate internal auditor's review at a specific point in the system, drop a control measure that has been used for a long period, or use a specific processing method ~~form~~ hitherto not used. To evaluate the changes in design, we simply apply the model both before and after the change and observe how the two systems differ in terms of the sequence of the output vectors. Suppose we want to know the effectiveness of the bank reconciliation as an internal control in the payroll example of Figure 3. What we need to do is to compare the terminal output vectors with and without the bank reconciliation:

With: $W_0 = (.992 \quad .002 \quad .004 \quad .002)$

Without: $W_0 = (.889 \quad .016 \quad .071 \quad .024).$

It is clear that the bank reconciliation performs a positive role in the system.

When using the model for designing a system, one must be careful to note whether the change of any internal control measure under study affects the "independent relations" between the defined elements. Independent relations are required for the proposed model to be valid.

DISCUSSION AND CONCLUSIONS

The proposed model demonstrates the possibility of analyzing the system of internal control in objective, quantitative terms. It enables the fabric of operating and control elements to be mathematically structured and quantitatively evaluated. As a result, the auditor may be better able to: (1) analyze the internal control system and explicitly find out the weak and strong areas, (2) assess the impact of the weaknesses and strengths on the quality of accounting data in probabilistic terms,

(3) advise his client about system problems, and (4) properly adjust his audit programs to meet the situation.

A consequence of the use of the proposed model in auditing is that it provides a new purpose to statistical sampling methods in auditing. Conventionally, statistical sampling methods are used in a direct attempt to evaluate the system reliability. However, the applications are on piecemeal bases, and the objective integration of the statistical tests applied to various segments of the internal control system has been difficult. With the stochastic process model, statistical sampling methods largely will be used for estimating the probability matrices.

The proposed model is also applicable to a computerized accounting system. Such a system typically involves processes such as preparation of source documents, transmission of documents to the EDP department to be merged with other documents, input conversion to machine-readable form, disposition of error messages from the computer, and distribution of outputs to appropriate users.⁷ During each of these operating processes, the quality of documents may change, so that such a process may be described by the kind of model discussed in this paper.

The use of the proposed model may open a new road toward the quantification of the auditor's judgment as to the bonafides of account balances, for account balances are directly influenced by the system's reliability.

Like many new models, the proposed one is not free of implementation difficulties. Problems like definitions of operating elements and errors need careful consideration, because too detailed definitions might greatly

⁷Gordon B. Davis, Auditing and EDP (New York: The AICPA, 1968), pp. 103-116.

increase the complexity of the flow chart and error states, while too coarse definitions might fail to reveal significant information. For example, if we define just three kinds of errors instead of two, the number of error states according to our error states definition would be 8 (2^3 instead of 2^2). Since the increase of the number of error states is exponential, a detailed definition of error kinds would easily result in a prohibitively large number of error states and consequently too large a probability matrix size.

A large probability matrix size not only causes computational problems, but also creates various estimation problems. Even though the statistical estimation of transition probabilities is conceptually clear,⁸ the cost and feasibility of estimation may be a practical constraint.

We do not believe these problems are insurmountable, but extensive work will be required to implement our proposed approach.

⁸ For example, see T. W. Anderson and L. A. Goodman, "Statistical Inference About Markov Chains," The Annals of Mathematical Statistics, 28, 1957, pp. 89-110.



APPENDIX

The following hypothetical data were used in generating the sequence of output vectors shown in Figure 4 in accordance with the model discussed in this paper. The order of the elements in the vectors and matrices corresponds to the definition of the error states, i.e., s_1 , s_2 , s_3 , s_4 .

1. The initial input to time punching operation

$$= (1 \ 0 \ 0 \ 0)$$

2. Transformation and Decision Probability Matrices:

A = Time card punching in
Dept. X (50 employees)

$$P(A) = \begin{pmatrix} .94 & 0 & .06 & 0 \\ 0 & 1 & 0 & 0 \\ .97 & 0 & .03 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

B = Time card punching in Dept.
Y (100 employees)

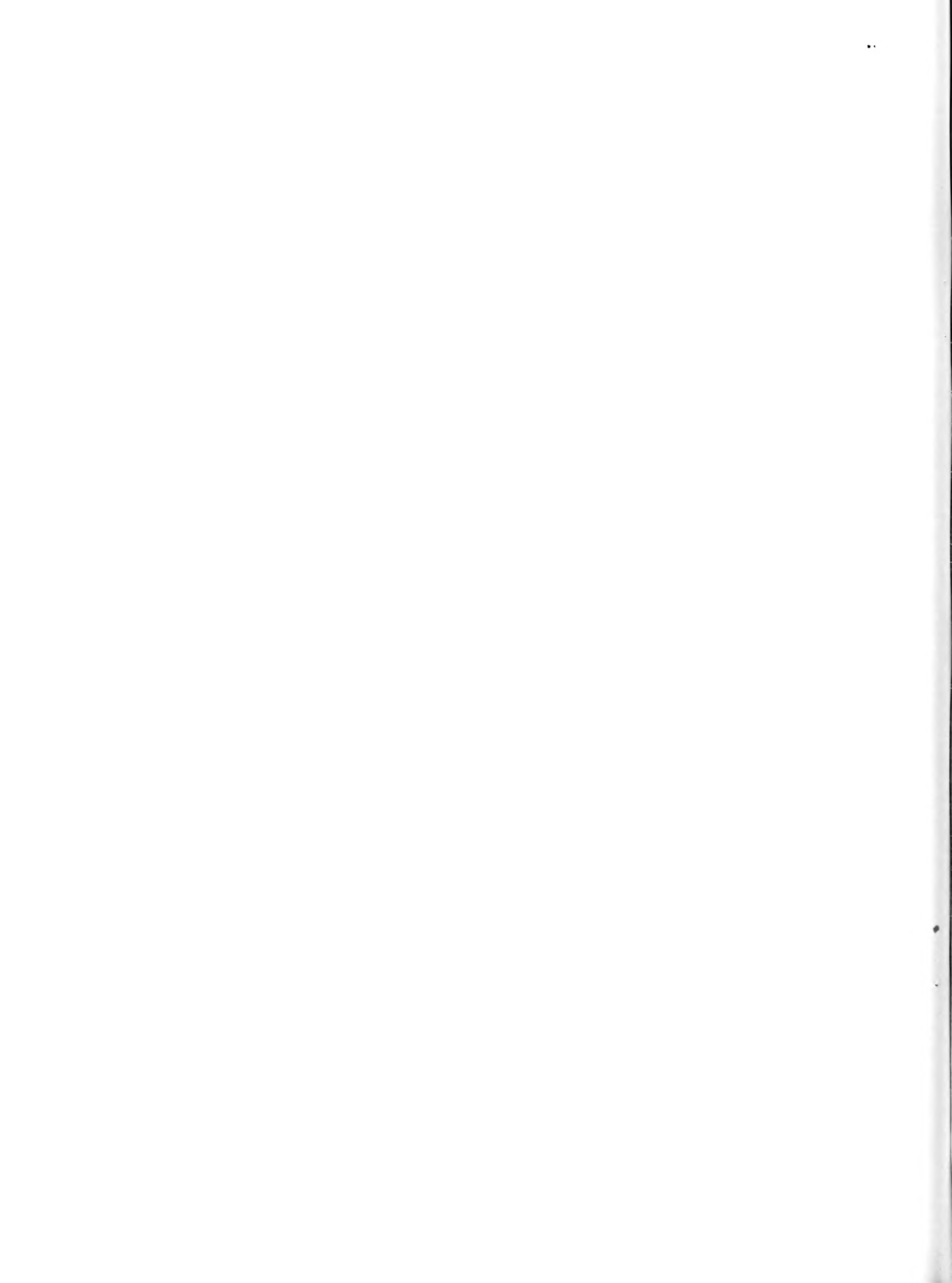
$$P(B) = \begin{pmatrix} .92 & 0 & .08 & 0 \\ 0 & 1 & 0 & 0 \\ .98 & 0 & .02 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

C = Dept X foreman's approval of time cards

(Approve as correct)

(Reject as incorrect)

$$Q(C) = \begin{pmatrix} .92 & 0 & 0 & 0 \\ 0 & .45 & 0 & 0 \\ 0 & 0 & .37 & 0 \\ 0 & 0 & 0 & .36 \end{pmatrix} \begin{pmatrix} .08 & 0 & 0 & 0 \\ 0 & .55 & 0 & 0 \\ 0 & 0 & .63 & 0 \\ 0 & 0 & 0 & .64 \end{pmatrix}$$



D = Dept. Y foreman's approval of time cards

$$Q(D) = \begin{matrix} & \begin{matrix} \text{(Approve as correct)} & & & \end{matrix} & \begin{matrix} \text{(Reject as incorrect)} & & & \end{matrix} \\ \begin{matrix} .96 & 0 & 0 & 0 \\ 0 & .41 & 0 & 0 \\ 0 & 0 & .70 & 0 \\ 0 & 0 & 0 & .19 \end{matrix} & \begin{matrix} | & & & | \\ | & & & | \\ | & & & | \\ | & & & | \end{matrix} & \begin{matrix} .04 & 0 & 0 & 0 \\ 0 & .59 & 0 & 0 \\ 0 & 0 & .30 & 0 \\ 0 & 0 & 0 & .81 \end{matrix} & \begin{matrix} | & & & | \\ | & & & | \\ | & & & | \\ | & & & | \end{matrix} \end{matrix}$$

E = Entering hours, rates, deductions to time cards by Payroll Clerk A

F = Preparing an adding machine control tape by Payroll Clerk A

$$P(E) = \begin{matrix} .94 & 0 & .06 & 0 \\ 0 & 1 & 0 & 0 \\ .44 & 0 & .56 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

$$P(F) = \begin{matrix} .95 & .04 & .01 & 0 \\ 0 & 1 & 0 & 0 \\ .03 & .09 & 0 & .88 \\ 0 & 0 & 0 & 1 \end{matrix}$$

G = Operation of accounting machine by Payroll Clerk B, producing paychecks and earnings statements, updated earnings record cards, and payroll register.

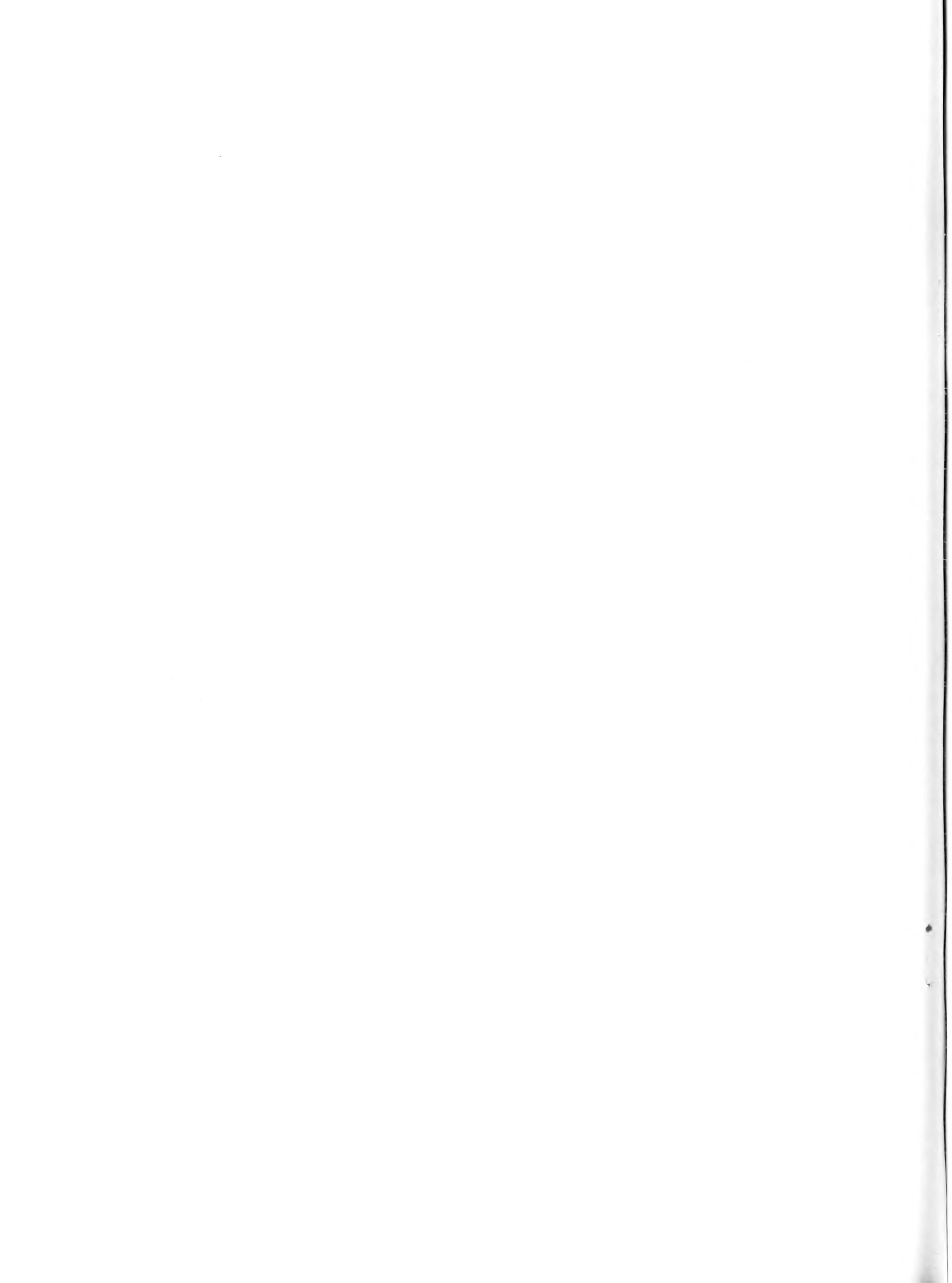
K = Preparation of payroll voucher by controller

$$P(G) = \begin{matrix} .93 & .03 & .02 & .02 \\ .02 & .89 & .01 & .08 \\ .01 & .02 & .94 & .03 \\ .01 & 0 & .02 & .97 \end{matrix}$$

$$P(K) = \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & .95 & 0 & .05 \\ 0 & 0 & 1 & 0 \\ .01 & .04 & 0 & .95 \end{matrix}$$

H = Sorting spoiled checks by Payroll Clerk B

$$Q(H) = \begin{matrix} & \begin{matrix} \text{(Accept as correct)} & & & \end{matrix} & \begin{matrix} \text{(Reject as spoiled)} & & & \end{matrix} \\ \begin{matrix} .99 & 0 & 0 & 0 \\ 0 & .62 & 0 & 0 \\ 0 & 0 & .40 & 0 \\ 0 & 0 & 0 & .15 \end{matrix} & \begin{matrix} | & & & | \\ | & & & | \\ | & & & | \\ | & & & | \end{matrix} & \begin{matrix} .01 & 0 & 0 & 0 \\ 0 & .38 & 0 & 0 \\ 0 & 0 & .60 & 0 \\ 0 & 0 & 0 & .85 \end{matrix} & \begin{matrix} | & & & | \\ | & & & | \\ | & & & | \\ | & & & | \end{matrix} \end{matrix}$$



I = Comparison of payroll register with the controlling tape by Payroll Clerk A

	(Accept as agreed)	(Reject as discrepant)
Q(I) =	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & .18 & 0 & 0 \\ 0 & 0 & .98 & 0 \\ 0 & 0 & 0 & .69 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & .82 & 0 & 0 \\ 0 & 0 & .02 & 0 \\ 0 & 0 & 0 & .31 \end{pmatrix}$

J = Review of the paychecks and earnings statements, updated earnings record cards, and payroll register by controller

	(Approve as correct)	(Reject as incorrect)
Q(J) =	$\begin{pmatrix} .99 & 0 & 0 & 0 \\ 0 & .87 & 0 & 0 \\ 0 & 0 & .92 & 0 \\ 0 & 0 & 0 & .65 \end{pmatrix}$	$\begin{pmatrix} .01 & 0 & 0 & 0 \\ 0 & .13 & 0 & 0 \\ 0 & 0 & .08 & 0 \\ 0 & 0 & 0 & .35 \end{pmatrix}$

L = Preparation of the fund transfer check by treasurer to transfer fund to the payroll bank account

P(L) =

$$\begin{pmatrix} .99 & .01 & 0 & 0 \\ .01 & .99 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ .01 & 0 & 0 & .99 \end{pmatrix}$$

M = Journal entries of the payroll expenses by the general ledger bookkeeper

P(M) =

$$\begin{pmatrix} .95 & .05 & 0 & 0 \\ .01 & .95 & .02 & .02 \\ 0 & 0 & .99 & .01 \\ .01 & .01 & .01 & .97 \end{pmatrix}$$

N = Processing of the paychecks by the bank

P(N) =

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

T = Monthly reconciliation of the payroll bank account by controller

P(T) =

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ .85 & .15 & 0 & 0 \\ .95 & 0 & .05 & 0 \\ .90 & 0 & 0 & .1 \end{pmatrix}$$

S = Employee receives the paycheck and earnings statement. He rejects the check if it is understated according to his own computation.

$$Q(S) = \begin{array}{cc} \text{(Accept as correct)} & \text{(Reject as incorrect)} \\ \left[\begin{array}{cccc} .99 & 0 & 0 & 0 \\ 0 & .45 & 0 & 0 \\ 0 & 0 & .36 & 0 \\ 0 & 0 & 0 & .08 \end{array} \right] & \left[\begin{array}{cccc} .01 & 0 & 0 & 0 \\ 0 & .55 & 0 & 0 \\ 0 & 0 & .64 & 0 \\ 0 & 0 & 0 & .92 \end{array} \right] \end{array}$$

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