BEBR

FACULTY WORKING
PAPER NO. 986

Stratified Sampling Using a Stochastic Model

Donald M. Roberts

College of Commerce and Business Administration
Bureau of Economic and Business Research
University of Illinois, Urbana-Champaign

BEBR

FACULTY WORKING PAPER NO. 986

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

November 1983

Stratified Sampling Using a Stochastic Model

Donald M. Roberts, Professor
Department of Business Administration

Abstract

A superpopulation modelling approach is used to represent the audited amounts within a population of balances or transactions. When monetary errors represent overstatements, an upper bound on the expected variance of the stratified difference estimator is derived. This result is used to stratify the population, determine an appropriate sample size, and determine a decision rule for evaluating sample results.

Stratified Sampling Using a Stochastic Model

Introduction

According to SAS No. 39, planning a statistical substantive test of details requires specifying a tolerable monetary error and an allowable risk of incorrect acceptance. The tolerable error represents the maximum monetary error that can exist in the population without causing the financial statements to be materially misstated. The risk of incorrect acceptance is the risk of the sample's supporting the conclusion that the total monetary error in the population does not exceed the tolerable error when, in fact, the total monetary error does not exceed the tolerable error. The auditor may also elect to control the risk of incorrect rejection. This is the risk of the sample's supporting the conclusion that the total monetary error in the population exceeds the tolerable error when, in fact, the total monetary error is less than the tolerable error.

Statistically, the auditor may formulate this audit problem as a statistical test of hypothesis (Elliott and Rogers [1972], Roberts [1978]). This involves specifying an hypothesis and an alternative. One hypothesis would state that the total monetary error in the population exceeds the tolerable error, while the alternative would state that the total monetary error in the population is less than the tolerable error.

To test these hypotheses, the auditor must specify the sample size, how the sample is to be selected, and how the sample results are to be evaluated. One commonly used selection method is stratified random sampling. This entails dividing the recorded amounts into several strata, and choosing a random sample from each stratum. To evaluate the results, a decision rule based on an estimate of the total monetary error can be developed.

An exact statistical solution to this testing problem requires knowing the sampling distribution of the estimated total monetary error under the hypothesis and under the alternative. If the sampling distribution were known as a function of the total monetary error in the population, a sample size and decision rule could be determined that would have the allowable risk of incorrect acceptance and, if desired, the allowable risk of incorrect rejection at some specified small amount of monetary error.

Because the sampling distribution as a function of the total monetary error is unknown, only approximate solutions are possible. The currently used testing procedures that are based on classical statistical estimators regard the sampling distribution as being approximately a normal distribution.

Even in situations where the normal approximation is appropriate, a difficulty arises because the variability of the sampling distribution (the standard error of the estimate) is related to the amount of error in the population. The fact that the variability of the population of audited or error amounts changes as the total monetary error is changed, was observed by Duke [1980] and Duke, Neter, and Leitch [1982] in their study of power characteristics. Their study demonstrates some of the difficulties encountered when the auditor uses a procedure that does not recognize this changing variability.

Is it possible to say anything about the variability of audited or difference amounts relative to the variability of recorded amounts as a function of the error population distribution? In this paper we suggest this question may be answered "yes" provided we are willing to employ a plausible model for the population of audited or difference amounts.

The modelling technique entails regarding the audited amounts associated with any particular population of recorded amounts as being a realization of a particular type of chance mechanism. This technique, known in the statistical literature as a superpopulation model, permits us to derive relationships on an expected value basis.

The particular case where all monetary errors represent overstatements allows us to determine upper and lower limits for the expected variability of audited and difference amounts. The upper limits are then used to provide an approximate solution to the testing problem.

The suggested procedure provides a method for stratifying the population, determining the sample size, and evaluating the sample results. Because this procedure uses an upper bound based on a worst case situation, it is robust.

Superpopulation Model. Cassel, Särndal, and Writman [1977] observe that many recent important contributions to the problem of inference in finite populations have used the superpopulation approach. In the auditing context, taking a superpopulation approach means that the observed audited amounts are regarded as realized outcomes of a prescribed random process. Superpopulation models have long been used in sampling research. Early users include Cochran [1939] and [1946], Deming and Stephan [1941], and Madow and Madow [1944].

The model for the audited amounts in the population used here is

$$(1) \qquad \underset{\sim}{X}_j = (1 - \underset{\sim}{\Theta}_j)Y_j, \qquad j = 1, \ldots, N.$$

In this model, the recorded amount (Y_j) is not regarded as a random variable, but the associated audited amount (X_j) is the outcome of a random process. The random variable $\underset{\sim}{X}_j$ is generated from the recorded amount Y_j by multiplying by the factor $(1 - \underset{\sim}{\Theta}_j)$. $\underset{\sim}{\Theta}_j$ is a random variable which takes on the value zero (0) with probability $(1-\pi)$, and with probability π, takes on a value governed by a distribution function F.

Conceptually, each recorded amount is accorded an equal chance, π, of being in error. If a monetary error exists, the magnitude of the error is determined by the value of Θ, which measures the relative error ($\underset{\sim}{\Theta}_j = \frac{Y_j - \underset{\sim}{X}_j}{Y_j}$) in the recorded amount. The relative errors are considered to be generated from the same distribution function. When this distribution is confined to the interval from zero to one, all the monetary errors are overstatements. Negative values for $\underset{\sim}{\Theta}$ would correspond to understatement errors.

This conceptual model reasonably reflects the situation the auditor faces in a substantive test of details. The auditor knows the recorded amounts but the associated audited amounts are unknown. An unknown fraction of the recorded amounts contain monetary errors. Because the auditor generally has no knowledge of which items are in error, it seems reasonable to suppose that each item is equally likely to contain an error. The size of the monetary error may be expressed relative to the magnitude of the recorded amount.

Kaplan [1973A] used a similar model. The chief difference, other than notation, is that he considered a second stage of randomization in which the audited amount was associated with a recorded amount selected at random from the population. His expectations were taken relative to both the random selection process and the error producing process. Because we want to examine the structure of the audit population generated from the error producing process, the superpopulation model defined here does not include random selection as part of the model.

Expectation and Variance of Audited Amounts. The model may be used to derive the expectation and variance of audited amounts. Using the symbol E_Θ to represent the expectation operator with respect to the random variable $\underset{\sim}{\Theta}$, the following relationship holds:

$$(2) \qquad E_\Theta \underset{\sim}{X}_j = Y_j(1-\pi\mu_\Theta)$$

where μ_Θ denotes the mean of the distribution of relative errors. By adding over all population items, it follows that

$$(3) \qquad E_\Theta(\sum_1^N \underset{\sim}{X}_j) = (\sum_1^N Y_j)(1-\pi\mu_\Theta).$$

This says that the expected total audited amount equals the total recorded amount multiplied by the factor $(1-\pi\mu_\Theta)$, or equivalently, the difference between the total recorded amount and the expected total audited amount equals the total recorded amount multiplied by $\pi\mu_\Theta$ (the faction of accounts in error times the mean relative error). Because of the large size of N, a realization of the random process would yield a value of the actual sum of audited amounts very close to its expectation, or

$$\sum_1^N X_j \doteq \sum_1^N Y_j(1-\pi\mu_\Theta),$$

where $\doteq$ denotes approximate equality.

The expected variance of audited amounts is

$$\text{(4)} \qquad E_\Theta \text{Var}\ \underset{\sim}{X} \doteq [\pi\sigma_\Theta^2+\pi(1-\pi)\mu_\Theta^2+(1-\pi\mu_\Theta)^2]\text{Var}\ Y + [\pi\sigma_\Theta^2+\pi(1-\pi)\mu_\Theta^2]\overline{Y}^2$$

where $\text{Var}\ \underset{\sim}{X} = \dfrac{\sum_1^N(\underset{\sim}{X}_j-\overline{\underset{\sim}{X}})^2}{N}$, σ_Θ^2 is the variance of the relative error,

and the symbol $\doteq$ denotes approximate equality. The approximation arises from substituting one (1) for the quantity $(\frac{N-1}{N})$, and consequently the expression on the right slightly overstates the expected variance.

Of special interest is the magnitude of the expected variance when the total monetary error equals the tolerable error. The following inequality holds when all monetary errors represent overstatement errors.

$$\text{(5)} \qquad E_\Theta \text{Var}\ \underset{\sim}{X} < (1-\pi\mu_\Theta)\text{Var}\ Y + \pi\mu_\Theta(1-\pi\mu_\Theta)\overline{Y}^2$$

Using the symbol TE to represent the tolerable error, and Y to represent the total recorded amount, the following inequality is the result of imposing the condition that the expected total monetary error equals TE($\pi\mu_\Theta Y$ = TE),

$$\text{(6)} \qquad E_\Theta \text{Var}\ \underset{\sim}{X} < (1-TE/Y)\text{Var}\ Y + \frac{TE}{Y}(1-TE/Y)\overline{Y}^2$$

This upper bound is realized when all monetary errors represent 100 percent overstatements.

A lower bound on the expected variance of audited amounts when the total monetary error equals the tolerable error corresponds to the situation where the relative error is concentrated at a single value, making $\sigma_\Theta^2 = 0$. This can be seen by examining (4) and setting $\pi\mu_\Theta = \frac{TE}{Y}$. The following inequality then holds:

$$(7) \qquad E_\Theta \text{Var } \underset{\sim}{X} > (1-\frac{TE}{Y})\text{Var } Y$$

This lower bound is realized when each account is overstated by a constant percentage.

While these results have been derived without considering the effect of stratifying the recorded amounts, similar relationships hold when the recorded amounts are stratified provided the probability of an item's being in error is not affected by the stratification. Examining each of the relationships (2)-(7), the only change is that all hold for each stratum. To illustrate this (3) and (4) become, for the k^{th} stratum,

$$(3') \qquad E_\Theta(\sum_1^{N_k} \underset{\sim}{X}_{jk}) = (\sum_1^{N_k} Y_{jk})(1-\pi\mu_\Theta)$$

and

$$(4') \qquad E_\Theta \text{Var } \tilde{X}_k = \pi\sigma_\Theta^2+\pi(1-\pi)\mu_\Theta^2+(1-\pi\mu_\Theta)^2\text{Var } Y_k + (\pi\sigma_\Theta^2+\pi(1-\pi)\mu_\Theta^2)\overline{Y}_k^2$$

where the subscript k indicates the restriction to the k^{th} stratum.

<u>Expectation and Variance of Differences</u>. Defining the difference as the recorded amount minus the audited amount, analogous results may be derived concerning the expected difference and the expected variance of the difference. From the basic definition of the model, the difference, $\underset{\sim}{D}$, may be represented as

(8) $$\underset{\sim}{D}_j = Y_j - \underset{\sim}{X}_j$$

$$= \underset{\sim}{\Theta}_j Y_j.$$

Taking the expectation with respect to the random variable $\underset{\sim}{\Theta}$, the following relationship holds:

(9) $$E\underset{\sim}{D}_j = \pi\mu_\Theta Y_j.$$

By adding overall population items, it follows that

(10) $$E_\Theta(\sum_1^N \underset{\sim}{D}_j) = \pi\mu_\Theta(\sum_1^N Y_j)$$

The expected variance of difference amounts is

(11) $$E_\Theta \text{Var } \underset{\sim}{D} \doteq \pi(\mu_\Theta^2+\sigma_\Theta^2)\text{Var } Y + (\mu_\Theta^2\pi(1-\pi)+\pi\sigma_\Theta^2)\overline{Y}^2$$

As anticipated, this expected variance is always smaller than the expected variance of audited amounts whenever the expected audited amount is at least fifty percent of the recorded amount.

When all monetary errors represent overstatements and the total monetary error equals the tolerable error (TE), an upper bound for the expected variance of differences is

(12) $$E_\Theta \text{Var } \underset{\sim}{D} \leq \frac{TE}{Y} \text{ Var } Y + \frac{TE}{Y}(1-\frac{TE}{Y})\overline{Y}^2$$

If the distribution of monetary error with the largest variance is called the least favorable, then it follows from (12) that when all monetary errors represent overstatements, the least favorable distribution of monetary error selects a proportion of the items in the population to contain the error, and each item selected is 100% overstated.

This notion of a least favorable distribution was introduced by Teitlebaum [1973] in connection with dollar unit sampling.

The inequality (12) is obtained from (11) by maximizing the variance of the relative error (σ_Θ^2) subject to the conditions that $\pi\mu_\Theta = \frac{TE}{Y}$ and Θ takes its values between zero and one. More generally, for any value of $\pi\mu_\Theta$, the inequality may be written

$$(13) \qquad E_\Theta \text{Var } D \leq \pi\mu_\Theta \text{Var } Y + \pi\mu_\Theta(1-\pi\mu_\Theta)\overline{Y}^2$$

It is also possible to obtain a lower bound for the expected variance of differences under these same conditions. This relationship is expressed as

$$(14) \qquad E_\Theta \text{Var } \underset{\sim}{D} \geq (\frac{TE}{Y})^2 \text{Var } Y$$

This inequality corresponds to the situation where every population item is overstated by the same relative monetary error, $\frac{TE}{Y}$.

From (12) when all monetary errors represent overstatements and the total monetary error equals the tolerable error it follows that the variance of recorded amounts exceeds the expected variance of differences whenever the square of the coefficient of variation of recorded amounts is greater than $\frac{TE}{Y}$. That is,

$$\frac{\text{Var } Y}{\overline{Y}^2} > \frac{TE}{Y}$$

Because these results apply to each stratum in a stratified design, it follows that as long as the square of the coefficient of variation of recorded amounts within any stratum is larger than $\frac{TE}{Y}$, the variance of the stratum recorded amounts exceeds the expected variance of differences within the stratum.

APPLICATION TO TESTING. In this section we shall apply the approximate expected variances to the problem of testing whether the monetary error exceeds the tolerable error (TE). We suppose that the auditor expects some monetary error (EE), and that all monetary errors represent overstatements.

Figure 1 illustrates the situation. On the left is the sampling distribution of the estimated monetary error, $\hat{D}$, under the hypothesis that the total monetary error equals EE, the expected error, and on the right is the sampling distribution of $\hat{D}$ under the hypothesis that the total monetary error equals TE, the tolerable error. Note that the variability of the sampling distribution on the right as measured by its standard deviation, S(TE), is larger than the standard derivation on the left, S(EE). S(TE) is the standard error of the estimated difference when the population monetary error equals TE; S(EE) is the standard error of the estimated difference when the population monetary error equals EE.

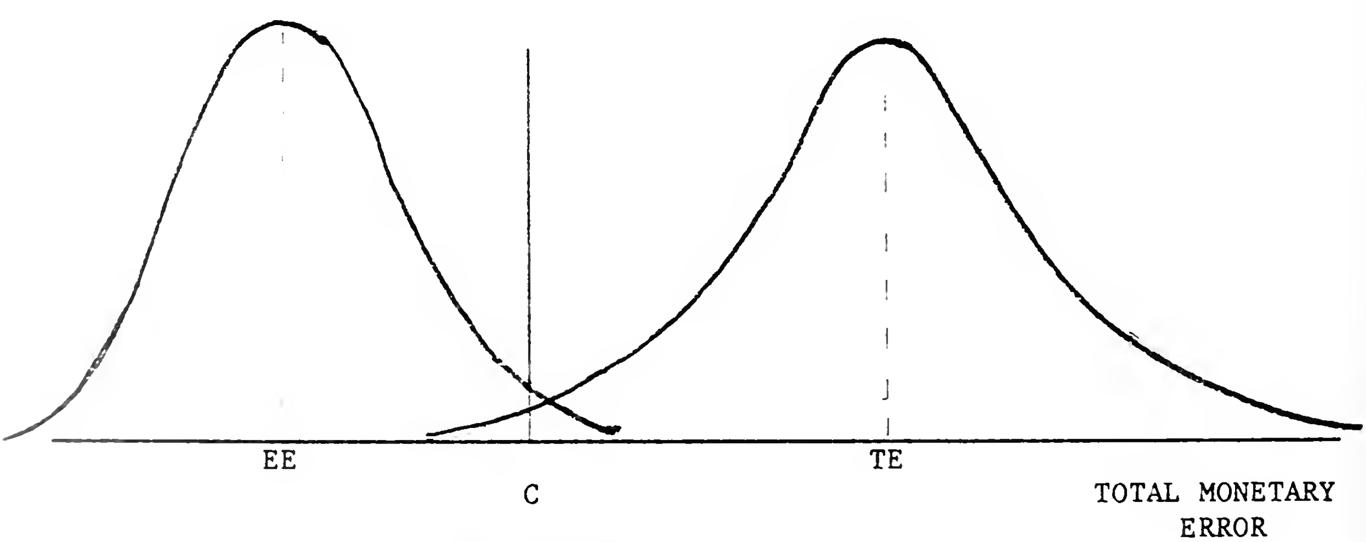

FIGURE 1

For a specified risk of incorrect acceptance (β) and a specified risk of incorrect rejection (α), the auditor must determine a sample size n and a critical amount C. If the estimated difference, $\hat{D}$, exceeds C, the auditor decides that the monetary error may be larger than the tolerable error (TE), and if the estimated difference, $\hat{D}$, is less than or equal to C, the auditor decides that the monetary error does not exceed the tolerable error.

The risk of incorrect acceptance is determined as the probability that $\hat{D}$ is less than or equal to C when the total monetary error equals TE; the risk of incorrect rejection is determined as the probability that $\hat{D}$ exceeds C when the total monetary error equals EE. Using the symbols z_β to represent the normal table value corresponding to a risk of incorrect acceptance equal to β, and z_α to represent the normal table value corresponding to a risk of incorrect rejection equal to α, the equations for the sample size, n, and the critical amount, C, are

$$(15) \qquad TE - EE = z_\alpha S(EE) + z_\beta S(TE)$$

and

$$(16) \qquad C = TE - z_\beta S(TE)$$

Solving these equations for n and C requires knowing the standard derivations S(EE) and S(TE). To obtain an approximate solution, we can use the inequality (13) developed in the previous section for the case where all monetary errors represent overstatements. Neglecting stratification and the finite population correction factor for the moment, the following inequalities hold,

$$S(EE) \leq \sqrt{\frac{1}{n} \cdot \frac{EE}{Y}} \sqrt{\text{Var } Y + (1-\frac{EE}{Y})\bar{Y}^2}$$

$$\text{and} \quad S(TE) \leq \sqrt{\frac{1}{n} \cdot \frac{TE}{Y}} \sqrt{\text{Var } Y + (1-\frac{TE}{Y})\bar{Y}^2}$$

Replacing S(EE) and S(TE) by these upper limits, we can solve the following equation for the sample size:

$$(17) \qquad n = \frac{\left[z_\alpha \sqrt{\frac{EE}{Y}} \sqrt{\text{Var } Y + (1-\frac{EE}{Y})\bar{Y}^2} + z_\beta \sqrt{\frac{TE}{Y}} \sqrt{\text{Var } Y + (1-\frac{TE}{Y})\bar{Y}^2} \right]^2}{(TE - EE)^2}$$

This expression can be simplified and made somewhat larger by substituting (1-EE/Y) for (1-TE/Y) and rewriting as

$$(18) \qquad n = \frac{\left[z_\alpha \sqrt{\frac{EE}{Y}} + z_\beta \sqrt{\frac{TE}{Y}} \right]^2 \left[\text{Var } Y + (1-EE/Y)\bar{Y}^2 \right]}{(TE - EE)^2}$$

Now we are ready to consider the situation where stratification is used. We suppose that the stratification is based on the recorded amounts, by using some acceptable technique such as the square-root of the cumulative frequencies. As in the unstratified case, the two formulas (15) and (16) are to be solved to determine the required sample size and critical number.

An additional decision to be made when a stratified plan is used is how to allocate the sample is to the strata. Neyman allocation, in which the sample is divided among the strata in proportion to the product of the stratum population size times the stratum standard deviation, is a commonly used procedure. Using this allocation method, the question is what standard derivation to use. One choice would be to use the standard deviation of recorded amounts. A better choice would be to use the standard deviation associated with either the expected error amount (EE) or the tolerable error amount (TE).

While neither of the latter two is known, the inequality (13) provides a useful upper limit for the situation when the monetary errors all represent overstatements. Using the inequality, the two possible allocations are

$$(19) \qquad n_h = n \frac{N_h \sqrt{\text{Var } Y(h) + (1-\frac{EE}{Y})\overline{Y}^2(h)}}{\sum_1^L N_h \sqrt{\text{Var } Y(h) + (1-\frac{EE}{Y})\overline{Y}^2(h)}}$$

and

$$(20) \qquad n_h = n \frac{N_h \sqrt{\text{Var } Y(h) + (1-\frac{TE}{Y})\overline{Y}^2(h)}}{\sum_1^L N_h \sqrt{\text{Var } Y(h) + (1-TE/Y)\overline{Y}^2(h)}} .$$

In these equations, h represents stratum h and there a total of L strata.

As a numerical example, we adapt an example described in Roberts [1978], p. 98. A population of 10,000 items with a total recorded amount of $4,000,000 is divided into four strata. Table 1 gives the facts for this example. Additionally, we assume that TE = $200,000 (TE/Y = .05), and EE = $20,000 (EE/Y = .005).

STRATUM	N_h	Var Y(h)	$\overline{Y}_h$
1	5500	6,400	$203.64
2	3000	22,500	316.67
3	1000	40,000	1050.00
4	500	168,100	1760.00

The following table shows the allocations using each formula as well as the standard deviation of recorded amounts.

PERCENTAGE OF SAMPLE

STRATUM	EE = \$20,000	TE = \$200,000	RECORDED AMOUNTS
1	28.48	28.48	34.00
2	24.88	24.92	34.67
3	25.27	25.24	15.33
4	21.37	21.36	16.00

These results illustrate that the allocation differs little between using EE and TE, but both of these give a different allocation from that based on the recorded amounts. Consequently, we shall use Neyman allocation calculated using the upper limit at EE. This choice simplifies the formulas for calculating the sample size and critical number without affecting the resulting allocation very much.

The following expression represents the upper limit for the standard error of stratified estimates $\hat{D}_S$, based on L strata when the monetary error equals TE:

$$(21) \qquad S(TE) < \sqrt{\frac{TE}{Y}} \sqrt{\sum_{1}^{L}\left(\frac{N_h^2}{n_h} - N_h\right)\left(\text{Var } Y(h) + (1-TE/Y)\overline{Y}^2(h)\right)}$$

The corresponding inequality for S(EE) is of the same form, but with EE replacing TE.

The formula (15) can now be used to determine the sample size. Using that formula, we replace the stratum sample sizes n_h by the expression (19) and use the inequalities for S(TE) and S(EE) represented by (21). After some simplification and replacing (1-TE/Y) by (1-EE/Y) in the bound for S(TE), the formula for n can be expressed as

$$(22) \qquad n = \frac{\left(z_\alpha\sqrt{\frac{EE}{Y}} + z_\beta\sqrt{\frac{TE}{Y}}\right)^2 \left(\sum_1^L N_h\sqrt{\text{Var } Y(h) + (1-\frac{EE}{Y})\bar{Y}^2(h)}\right)^2}{(TE-EE)^2 + \left(z_\alpha\sqrt{\frac{EE}{Y}} + z_\beta\sqrt{\frac{TE}{Y}}\right)^2 \sum_1^L N_h(\text{Var } Y(h)+(1-\frac{EE}{Y})\bar{Y}^2(h))}$$

When this formula is applied to the previous numerical example with $\alpha = \beta = .05$, the resulting sample size is 125, allocated among the four strata as 35, 31, 31, and 28. For comparison, suppose the sample size is determined by using the recorded amounts. In that case, the required sample size is 518. This large difference is caused by two factors: (1) in this case the standard deviation of the stratum recorded amounts is larger than the standard deviation of differences under EE, and, except for stratum 3, under TE, and (2) using the recorded amounts does not permit using the fact that the standard deviation under EE is smaller than under TE.

Having determined an appropriate sample size for a stratified design, the critical amount C may be obtained by using inequality (21) as a proxy for S(TE) in formula (16). The stratified difference estimator, $\hat{D}_S$, is compared to the critical amount as described earlier in the paper. The critical amount C is determined by the following equation:

$$C = TE - z_\beta\sqrt{\frac{TE}{Y}}\sqrt{\sum_1^L\left(\frac{N_h^2}{n_h} - N_h\right)(\text{Var } Y(h) + (1 - \frac{TE}{Y})\bar{Y}^2(h))}$$

Continuing the numerical example, the critical amoung C = \$65848. The decision rule is to decide that the total monetary error exceeds \$200,000 when the stratified estimator of the monetary difference exceed \$65,848.

SUMMARY AND CONCLUSIONS. Adopting a superpopulation approach to modelling the distribution characteristics of audited amounts provides a useful basis for planning and evaluating stratified random samples. When the monetary errors represent overstatments we have derived upper bounds to the expected variance of the stratified difference estimator. Using this upper bound, it is possible to stratify the population, determine an appropriate sample size, and determine a decision rule for evaluating the sample results.

Using this modelling approach when faced with overstatemetns we are able to avoid some of the difficulties associated with the more commonly used stratified sampling designs. The foremost of these is the problem of observing very few monetary errors in the sample. Neter and Loebbecke [1975] observed this in their simulation study. The model approach does not depend upon the sample to provide an estimate of the standard deviation of population differences, and hence will perform well regardless of the number of errors observed in the sample.

Another difficulty noted in the literature is the failure of the standardized estimator (defined as the estimator minus the mean divided by the standard error) to be approximately normally distributed. Kaplan [1973b] observed that the correlation between the estimator and the estimated standard error was responsible for this failure. The model approach developed here depends only on the approximate normality of the estimator. Examining the results of Neter and Loebbecke [1975] for the populations 3 and 4, we see evidence that the stratified difference estimator has a distribution that is reasonably close to being normal when the population error percentage is at least five. Consequently, the use of normal table factors should produce reasonably good results.

Finally, the modelling approach presented here overcomes some of the theoretical difficulties caused by the fact that the standard deviation of audited amounts (or difference amounts) increases as the amount of monetary increases.

REFERENCES

Cassel, C., C. Särndal, and J. Wretman (1977), Foundations of Inference in Survey Sampling. New York: Wiley.

Cochran, W. G. (1939), "The Use of Analysis of Variance in Enumeration by Sampling," Journal of the American Statistical Association, 34, pp. 492-510.

Cochran, W. G. (1946), "Relative Accuracy of Systematic and Stratified Random Sampling for a Certain Class of Population," Annals of Mathematical Statistics, 17, pp. 164-177.

Deming, W. E. and F. Stephen (1941), "On the Interpretation of Censuses as Samples," Journal of the American Statistical Association, 60, pp. 750-771.

Duke, G. (1980), An Empirical Investigation of the Power of Statistical Sampling Procedures Used in Auditing under Different Models of Change of Error Patterns. Unpublished Ph.D. Dissertation, University of Georgia.

Duke, G., J. Neter, and R. Leitch (1982), "Power Characteristics of Test Statistics in the Auditing Environment: An Empirical Study," Journal of Accounting Research (Spring), pp. 42-67.

Elliott, R. K. and J. R. Rogers (1972), "Relating Statistical Sampling to Auditing Objectives," The Journal of Accountancy, July, pp. 46-55.

Kaplan, R. S. (1973A), "A Stochastic Model for Auditing," Journal of Accounting Research, Autumn, pp. 38-46.

Kaplan, R. S. (1973B), "Statistical Sampling in Auditing with Auxiliary Information Estimates," Journal of Accounting Research, Autumn, pp. 238-258.

Madow, W. G. and L. H. Madow (1944), "On the Theory of Systematic Sampling," Annals of Mathematical Statistics, 15, pp. 1-24.

Neter, J. and J. K. Loebbecke (1975), Behavior of Major Statistical Estimates in Sampling Accounting Populations. New York: American Institute of Certified Public Accountants.

Roberts, D. M. (1978), Statistical Auditing. New York: American Institute of Certified Public Accountants.

Teitlebaum, A. D. (1973), "Dollar-unit Sampling in Auditing," unpublished paper presented at 1973 American Statistical Association annual meeting.