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COALITION SITUATIONS

J. Keith Murnighan

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**College of Commerce and Business Administration
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Strength and Weakness in Four Coalition Situations

J. Keith Murnighan

Abstract

Five-person groups formed a total of 12 coalitions in each of four coalition games, with one group playing each of the 24 possible orderings of the games. The frequencies of the different coalitions and the outcomes of the players in the most frequent coalitions were compared to the predictions of four social psychological models of coalition behavior. The results, for both frequencies and for outcomes, supported Bargaining theory (Komorita and Chertkoff, 1973) over the Weighted Probability model, Minimum Resource, and Minimum Power theories. In addition, the overall payoffs of the players (including zero payoffs on trials where the player was excluded from the winning coalition) were compared to the Shapley value, yielding some support for the notion that the Shapley value can be used as a measure of what payoffs a player might expect to receive from playing the game (Roth, 1977). It was also suggested that a player's overall payoffs may be a more appropriate post hoc measure of one's power in the game. Finally, the data clarify the conditions where the "strength is weakness" phenomenon might be expected to occur. The payoffs of the players in positions which have equal Shapley values but different resources, within a particular game, resulted in the "strength is weakness" phenomenon. On the other hand, players in positions with relatively high Shapley values obtained payoffs that were considerably higher than players with low Shapley values, within the same game. In addition, reports of the players concerning their choice of strategies yielded information that may explain the underlying causes of this phenomenon.

The study of coalition behavior has traditionally been approached by discussing power differences among the individuals or parties in coalition situations. Caplow's (1956) original theory, for instance, focused primarily on the use of control. Minimum Power theory, based on notions of pivotal power (Shapley, 1953), also focused directly on the power of the parties involved. Other models of coalition behavior (e.g., Gamson, 1961; Komorita, 1974; Komorita and Chertkoff, 1973) have discussed differences in resources and the strength of the players. Researchers on coalition behavior, however, have most often addressed the accuracy of the models and have only infrequently discussed the relationship between coalition data and the power of the parties in coalition games. The present paper reports data from four coalition games, and discusses the results both in terms of the accuracy of the different models' predictions and the inferences that can be drawn concerning the power of the participants.

Even though the concept of power is central to the study of coalition formation, explicit definitions of the power of the players before and after the play of a game are rare. Early research and theory (e.g., Caplow, 1956; Gamson, 1961; Vinacke and Arkoff, 1957) are not specific about the bases of power, but imply that a player's strength increases with resources. Similarly, the frequency of inclusion in winning coalitions has been implicitly adopted as a post hoc measure of power in coalition games. Thus, Vinacke and Arkoff (1957) reported the frequencies of coalitions in six three-person games, and stated that the triad situation may lead to more frequent inclusion of weaker members (those with less resources) over those who are initially stronger (those with more resources). This result has subsequently been labelled the "strength is weakness" phenomenon, indicating that players with greater resources (a priori strength) are included in few winning coalitions (post hoc weakness).

Unlike the social psychological models, the power of players in coalition situations has been formally defined in the game theoretic literature. The Shapley value (Shapley, 1953), also called pivotal power (Gamson, 1964), has been used as an index of power in many different types of games (Shapley and Shubik, 1954). It is calculated by dividing the number of times a person's resources are "pivotal" (in the sense that a losing coalition is converted to a winning coalition by the addition of this person) by the total number of winning permutations of the players.¹ For instance, in the most often studied three-person game, where the three players are assigned 4, 3 and 2 votes and a majority of 5 votes is needed to form a winning coalition, there are twelve permutations of winning coalitions. Each of the players is pivotal in four of these coalitions; thus, each has a Shapley value of 1/3. Shapley and Shubik (1954) have also shown that the Shapley value can be used to measure the a priori power of the stockholders in a company (where an individual's Shapley value is a function of both the number of shares he holds and the distribution of the remaining shares) or the power of the President in passing legislation, vis a vis the Senate and the House of Representatives.

The Shapley value's initial formulation has been interpreted as resulting in a process of coalition formation that entails adding one player to an existing coalition, and then another, etc., until the coalition of all the players has formed. The average payoffs of the players in this coalition, then, could be compared to the Shapley value to determine its predictive accuracy. Most research, however, has excluded the possibility of the coalition of all the players. Thus, the Shapley value's potential as a predictor of the power of the players has not been realized.

Recently, however, Roth (1977) has shown that the Shapley value equals the proportion of the total payoff that a risk neutral player (i.e., an individual who neither prefers nor avoids economic risk) should expect from the play of a game. Thus, a comparison of a player's overall payoffs in a sequential bargaining game can be compared to the Shapley value to determine if such an expectation is appropriate. In addition, the payoffs the players obtain can be used as a post hoc measure of the players' power. Payoffs can easily be contrasted with the previously used measure of power, frequency of inclusion in the winning coalition through the use of an example. If inclusion is used as the criterion, player X, who is included often but receives relatively low outcomes when he is included, will appear to be as powerful as player Y, who is also included often but receives relatively high outcomes each time. The use of payoffs as a measure of power takes into account both inclusion and the player's outcomes when he is included. Thus, use of the player's payoffs as a measure of their power would result in player Y appearing substantially more powerful than player X.

The social psychological models, on the other hand, have attempted to predict the most frequent coalition and the players' outcomes in such coalitions. Outcomes refer to the points a player receives when he is included in the winning coalition; thus, they differ from payoffs. Agreements where a player is excluded from the winning coalition are not included in the comparison of a player's outcomes (whereas they are included in calculating payoffs). This emphasis on outcomes and inclusion frequency has led to some very inconsistent results, ("strength is weakness") between the generally accepted a priori and the post hoc measures of power.

Ironically, this emphasis on the predicted coalitions and the outcomes of its members has resulted in a series of negative findings (e.g., Komorita and Moore, 1976; Murnighan, Komorita and Sz wajkowski, 1977) for Minimum Power theory, the model that has been derived from the Shapley value (Gamson, 1964). The additional assumptions inherent in Minimum Power theory (see below), however, make tests of Minimum Power theory inconclusive as far as the Shapley value is concerned. Analysis of the players' payoffs would result in a more appropriate test of the model. With the exception of two recent studies that support the Shapley value predictions (Murnighan and Roth, 1977; Note 2), research on coalitions has not reported such tests.

This study, then, assesses the relationship between the expectations of payoffs, that the Shapley value indicates a player might have, with observed payoffs in four coalition games. It also tests four social psychological models of coalition behavior and assesses their ability to predict the frequencies of the different coalitions and the outcomes of the members of these coalitions.

Theories of Coalition Behavior

Although many theories of coalition behavior have been proposed, only those theories that predict both which coalitions are likely to form and the reward division in these coalitions were considered in this study. The restriction excludes some important theories, such as Caplow's (1956) and several normative theories of n-person cooperative games (cf., Luce and Raiffa, 1957; Rapoport, 1970). Four theories meet the above criterion: (1) Gamson's (1961) Minimum Resource theory, (2) Minimum Power theory (Gamson, 1964); (3) Komorita and Chertkoff's (1973) bargaining theory, and (4) Komorita's (1974) Weighted Probability model.

To compare and evaluate the four theories, groups of five persons participated in four coalition situations (games). In each game the resources of the participants were manipulated by assigning each participant a certain number of votes. These votes determined the combinations of players that could form a winning (majority) coalition. The four games were: (1) 20(10-9-8-7-5); (2) 14(10-7-5-3-2); (3) 27(15-14-10-8-5); and (4) 30(24-9-8-7-6), where the first number denotes the number of votes needed to form a winning (majority) coalition, and subsequent numbers denote the number of votes (resources) at the disposal of each player. The players are identified by letter (A, B, C, D, and E) in descending order of resources. The 14(10-7-5-3-2) game will be used to illustrate the different predictions of the four models.

Minimum Resource Theory. Gamson's (1961) theory is based on the assumption of a "parity norm" which specifies that rewards be divided in direct proportion to the resources of the coalition members. Thus, in the 14(10-7-5-3-2) game, if the 10-5 coalition should form, the parity norm specifies that the prize should be divided 10/15 for A and 5/15 for C, hereafter denoted 67-33 (percentage division of the prize). Assuming that individuals are motivated to maximize their share of the reward, the theory predicts the formation of the coalition that minimizes total resources and is just large enough to win. Since the 7-5-2 coalition is just large enough to win (it exactly meets the quota of 14 votes), this coalition should be the most likely to form and should divide the reward 50-36-14.

Minimum Power Theory. Minimum Power theory is based on the Shapley value. For the five parties in the 14(10-7-5-3-2) game, 120 permutations result in winning coalitions. Player A is pivotal in 48 cases (he has a Shapley value of 12/30), players B and C are pivotal in 28 cases (Shapley

values of 7/30 each), and players D and E are pivotal in 8 cases (Shapley values of 2/30 each). If players are motivated to maximize their rewards, and if they believe that rewards should be divided in direct proportion to their Shapley values (Ganson, 1964), the theory predicts that the players will attempt to form a coalition that minimizes the sum of the coalition members' Shapley values. (This approach is very similar to Minimum Resource theory; the focus, however, is on the Shapley value rather than resources.) Thus, the BCD, BCE and ADE coalitions, which result in a total Shapley value of 16/30, are preferred to the two-person coalitions, which total 19/30 Shapley value. In addition, the theory predicts that the reward division should be 43-43-14 for the BCD and BCE coalitions, 74-13-13 for the ADE coalition, and 63-37 for the AB and AC coalitions.

Bargaining Theory. The basic assumption underlying Bargaining theory (Komorita and Chertkoff, 1973) is that, in a given coalition, those members who are "strong" in resources (above average) will expect and demand a share of the rewards based on the parity norm, while those who are "weak" (below average) will demand equality. For an iterated game over trials, the theory makes differential predictions on the initial trial and at the asymptotic level. For the initial trial, the theory predicts that the rewards will be the average of the rewards predicted by the parity and equality norms. At the asymptote, the theory predicts that, for a given coalition, rewards will be divided in direct proportion to each member's maximum expectation in alternative coalitions. For example, if players A and C are negotiating the division of rewards in the 14(10-7-5-3-2) game, A's maximum expectation in alternative coalitions will be 67 (based on parity in the ADE coalition), while C's maximum expectation will be 36 (based on parity in the BCE coalition).

Converting the ratio of these expectations to a base of 100% yields asymptotic predictions of 65-35 for the AC coalition.

The theory postulates that the most likely coalition to form is the one that minimizes coalition members' temptation to defect. This temptation is represented by the quantity $\sum (O_{ij} - E_{ij})$, where O_{ij} denotes the predicted reward of individual i in coalition j ; E_{ij} denotes his maximum expectation in alternative coalitions; and the summation is over the members of coalition j . In the 14(10-7-5-3-2) games, the temptation to defect is minimized in the 10-5 coalition. In the 10-7 coalition (for a 59-41 division), A will be tempted to form 10-5, while in the BCD or BCE coalition (for a 37½-37½-25 split), B and C will be tempted to form a coalition with A. Hence the theory predicts that the 10-5 coalition is most likely to occur.

The Weighted Probability Model. The basic assumption underlying the Weighted Probability model (Komorita, 1974) is that, because of the logistic problem of communicating offers and counteroffers, large coalitions are more difficult to form than small ones. As the number of potential coalition members increases, the severity of the problem of achieving both reciprocity and unanimous agreement on the terms of the offer also increases. The number of potential defectors from the coalition also increases with its size; hence, a large coalition is not only more difficult to form, but may be more difficult to maintain. These hypotheses are supported by the inverse relationship between group size and the cohesiveness of a group reported by Cartwright and Zander (1968, p. 102).

In contrast with Bargaining theory, the Weighted Probability model assumes that an individual's share of the prize should be a function of the number and size of alternative winning coalitions available to him/her,

rather than the quality of these alternatives. In the 14(10-7-5-3-2) game, player A has three alternatives (AB, AC, and ADE), including two two-person coalitions. Players B and C each have three alternatives (AB or AC, BCD and BCE), including one two-person alternative. Players D and E have two alternatives (ADE and BCE or BCE), each of which is a three-person coalition. The present form of the model assigns twice as much of an "advantage" to two-person opportunities as it does to three-person opportunities, and predicts that the player's outcomes should be proportional to this "advantage."² Thus, in the two-person coalitions the outcomes should be 55-44; in the ADE coalition, the outcomes should be 55-22-22; in the BCD and BCE coalitions, the outcomes should be 40-40-20.

Finally, unlike the others, the Weighted Probability model makes an exact probability prediction for each of the minimal-winning coalitions. In a manner similar to the derivation of the reward division predictions, player A should be included more than players B and C, who should be included more than players D and E. For the five minimal-winning coalitions, the predicted probability for AB and AC is .28, for BCD, BCE and ADE it is .14.

Predictions of Four Models. Table 1 shows the predictions of the four models (and the Shapley value) for the four games used in this study. The theories can be differentiated as follows: (1) In the 20(10-9-8-7-5) game,

Insert Table 1 About Here

Minimum Power theory and the Weighted Probability model make predictions for coalition frequencies that differ from those of Minimum Resource and Bargaining theory. Minimum Resource theory's prediction of the outcomes of the included players differ from those of the other models. (2) All of the models can be differentiated from one another on the basis of predictions for both coalition frequencies and outcomes for the 14(10-7-5-3-2)

game. (3) In the 27(15-14-10-8-5) game, Minimum Resource and Minimum Power theories can be differentiated from the other two models on the basis of predictions for both coalition frequencies and outcomes. And (4) all the models make different frequency predictions for the 30(24-9-8-7-6) game, while Bargaining theory and the Weighted Probability model make predictions of outcomes that differ from those of Minimum Resource theory.

The present study also extends the research on coalitions to five-person groups. Most of the previous research has used three-person groups (cf., Chertkoff, 1970; Stryker, 1972). Recently it has been pointed out (Murnighan, Note 1; Murnighan et al., 1977) that the study of power in coalition situations³ requires study of groups with more than three players. While three-person groups result in only one non-veto, non-dictatorship Shapley value distribution, four-person groups result in additional distributions and five-person groups expand the potential distributions even more. The games used in this study, for instance, result in four different non-veto, non-dictatorship Shapley value distributions, thus increasing potential applicability to more complex conflict situations.

Method

Subjects. The participants were 120 advanced undergraduate and graduate student members of three sections of a behavioral science course in a commerce department. The total payoffs students obtained in the bargaining games and completion of a paper analyzing one's own strategies in each of the games accounted for 20% of the course grade.

Individuals participated in five-person groups, yielding a total of 24 groups. Twelve of these groups were composed of five males; six included one female; five included two females; and one included four females. All

of the 20 female participants were graduate students, primarily majoring in commerce.⁴

Design. Each group played each of the four games in the study. Each game was played for 12 trials. The five players in each group were assigned different positions (A, B, C, D, or E) for each game. Thus, using groups as the unit of analysis, the study resulted in three within-groups variables: games (4), trials (12), and player positions (5). Each group played the games in a different order. For the four games, there are 24 different game orders, and each group played the games in one of the 24 orders. Thus, order was completely counterbalanced.

The four games used in the study were those described in Table 1. The second variable, trials, was defined as the formation of a winning coalition. Participants were told that each session would continue for 12 trials. Player positions were designated A, B, C, D and E, with player A having the most resources, player B having the second most resources, etc.

Procedure. The participants were given general instructions about the coalition games in the class prior to the first experimental session. Several examples of the use of the procedure (in games that were not used later) were discussed. The participants were informed that there would be 12 trials in each game, and on each trial, the winning coalition would be allowed to divide a prize of 100 points among its members. They were instructed to do as well as they could (i.e., maximize their points) because their performance would, and did in part, determine 20 percent of their grade in the course. They were told that their payoffs in the games (called exercises in class) would be compared to the payoffs of other students in the same position as themselves. Thus, if they were player A in the 20(10-9-8-7-5) game, their payoffs would be compared to the payoffs of other players

in position A in the same game. It was emphasized, then, that their payoffs would be compared to similar members of other groups rather than to members of their own group.

Students were randomly assigned to groups, with the constraint that they not be acquainted with the other members of their group. The members of each group met for their sessions at the same hour each week for four weeks.⁵ They were told which game they would be playing each week, but were not told which of the five positions they would hold throughout each game. Participants were told that they would be in at least one strong position and at least one weak position in each of the games. They were instructed that they could discuss the games with other members of their class, but not to discuss the games with members of their own group. In addition, the players were encouraged to formulate strategies for each of the positions prior to each game.

During the games, the players were seated around a set of opaque partitions that shielded them from view of each other and the experimenter. At each group's first session, the experimenter read specific instructions about the procedures for the games. The players were told that each of them would make offers on each trial by means of written "offer slips." These slips required each player to "address" his offers (indicating to whom he/she wished to send his/her offers) and also required a proposal regarding the division of rewards for the coalition members. For example, if player X wished to form an XYZ coalition, he/she addressed offers to both persons Y and Z and specified a division of the rewards (e.g., 50-25-25). A player was required to send an offer slip to each of the players included in his/her proposed coalition. If, for instance, a player proposed a three-person coalition, he/she was instructed that the two offers he/she sent must be

identical with regard to the proposed division of rewards; for example, he/she could not send an offer to one person to form one coalition and a second, different offer to another person to form another coalition. This procedure allowed two-, three-, and four-person coalitions to form in a single step. Thus, although large coalitions are more difficult to form, the difficulty is not inherent in the procedure. After the players had completed the offers, the experimenter collected, examined and distributed them to the proper persons.

After receiving an offer, each person could accept or reject it by checking a space marked "Accept" or "Reject" at the bottom of the offer slips. If a person received more than one offer, the person was allowed to accept only one of them, unless the two offers proposed the identical payoff division for the same coalition. Hence, each person could only accept offers to form a single coalition on each trial. Furthermore, in determining a winning coalition, any player's proposal, if accepted, was considered to have priority over any offer he might accept, thus committing him to his own offer. After the players had either accepted or rejected their offer(s), the experimenter collected the offer slips and announced the winning coalition, if one had formed. Subjects were informed that a coalition would be declared the winning one if all the proposed coalition partners accepted the offer. If no coalition formed because at least one person rejected each of the proposed coalitions, the experimenter announced that a coalition had not formed, and the procedure was repeated until a coalition formed.⁶

In order to familiarize the players with the procedure, a practice trial was conducted before the start of the first session. Immediately after the practice trial, the players were reassigned to their position

for that game. A list of the resources (i.e., votes) for each position and a list of the winning coalitions was also provided. No verbal communication was permitted thereafter; hence, the players could not identify each other once the session had begun.

The instructions were summarized for the players at the start of their second, third, and fourth sessions. Practice trials were not run, but information about the player's position, the resources for each of the positions, and the winning coalitions were distributed after the players were seated behind the partitions. Thus, for each game, the players were not informed of the identity of the players in the other positions.

Results

The results are presented in two parts. First, the most frequent coalitions and the mean outcomes of the coalition members are presented and compared to the theoretical predictions. Second, the payoffs of the player positions in the games are analyzed to determine where the power differences lie, both between and within games. The differentiation between outcomes and payoffs is important: Outcomes are defined as a player's rewards when he/she is included in the winning coalition. These figures do not include trials when a player is excluded from the winning coalition. Payoffs, on the other hand, are determined by the total number of points a player accumulates over the 12 trials. A payoff of zero is included for trials where the player is excluded from the winning coalition. Outcomes, then, are relevant for the theoretical tests; payoffs are relevant for comparison with the Shapley value and for discussions of power.

Coalition Frequencies

The four most frequent coalitions (and the near outcomes of the members of these coalitions) are shown in Table 1. In the 20(10-9-8-7-5) game, the

Insert Table 1 About Here

CDE coalition was most frequent, supporting the predictions of Minimum Resource and Bargaining theories. In the 14(10-7-5-3-2) game, the AC coalition was most frequent, supporting Bargaining theory. In the 27(15-14-10-8-5) game, the AB coalition was most frequent, supporting Bargaining theory and the Weighted Probability model. Finally, in the 30(24-9-8-7-6) game, the four two-person coalitions were more frequent than the BCDE coalition, which occurred only 38 times. Because none of the two-person coalitions appears to be more frequent than any other, the Weighted Probability model's prediction is supported.

The data for coalition frequencies, then, supports Bargaining theory in three of four games, the Weighted Probability model in two games, Minimum Resource theory in one game, and Minimum Power theory in none.

The Weighted Probability model also makes predictions for each minimum winning coalition for each game. Previous research (Murnighan, et al., 1977) found strong support for its frequency predictions. For the 30(24-9-8-7-6) game, the model predicts that each two-person coalition should form 66 times. The results are quite close to this prediction, with the exception of the AD coalition. The predictions for the 14(10-7-5-3-2) game are 82 coalitions for AB and AC; although AC is more frequent, the two coalitions approximate the predicted total of 164. The model predicts equal frequencies for each of the three-person coalitions in the 20(10-9-8-7-5) game. The data do not support this prediction. Finally, the model predicts that the AB coalition should form 72 times in the 27(15-14-10-8-5) game, and that each of the

winning three-person coalitions should form at times. Although the AB coalition is most frequent, the entire set of data in this game do not support this prediction.

Coalition Members' Outcomes

The outcomes of the members of the winning coalitions are shown in Table 2. Confidence intervals ($\alpha=.01$) were constructed around the mean outcomes for the players in each of the most frequent coalitions. Theoretical predictions of the players' outcomes that fall within the confidence intervals are counted as support for the theory; predictions falling outside the interval are counted as lack of support. The findings are summarized in Table 3. The results indicate that none of the models made particularly

Insert Table 3 About Here

accurate predictions for the players in positions C, D, and E in the 20(10-9-8-7-5) game. Minimum resource theory made particularly inaccurate predictions. Minimum Power theory and the Weighted Probability model's predictions of equal splits among the players underestimated C's mean outcomes and overestimated position E's mean outcomes. Bargaining theory, on the other hand, was the best predictor for both positions C and E, even though its prediction for position E (i.e., 32,35) fell slightly outside the confidence interval. Similarly, its prediction for position D (also 32,35) also fell just outside the confidence interval.

The comparisons in the other games also favor Bargaining theory over the other models. Its only failing occurred for the AC coalition in the 30(24-9-8-7-6) game, where all the models predicted 75-25 splits and were incorrect. Player A's outcomes in this coalition were higher than expected by any of the models, and were also considerably higher than the other two-person coalitions in this game.

Considering the two criteria, coalition influences and the members' outcomes, indicates that Bargaining theory is decidedly superior on both dimensions to the other models tested in this study. Minimum Power theory appears to be particularly inadequate.

Payoffs

An analysis of variance using games (4), player positions (5), and trial blocks (4 blocks of 3 trials each) as independent variables (all repeated measures) and the players' mean payoffs in each trial block as the dependent variable resulted in significant effects for player position: $F(4,92)=132.17$, $p<.0001$; and for the games by player positions interaction: $F(12,275)=54.87$, $p<.0001$. Other effects were not significant. (It should be noted that it was not possible for the games' main effect to be significant, due to the constant total payoff in each game.) The means for the games by player positions interaction are shown in Table 4. Post hoc tests of this

Insert Table 4 About Here

interaction indicated numerous differences among the player positions in the four games. Players in position A in the 30(24-9-8-7-6) game, for instance, received considerably higher payoffs than any of the other positions. Position A in the 14(10-7-5-3-2) game and positions A and B in the 27(15-14-10-8-5) game also received high payoffs. The significant difference between positions A and B in the 27(15-14-10-8-5) game indicates that, in this situation, the player with fewer resources held the better position, indicative of the "strength is weakness" phenomenon. Similar results were found in the 20(10-9-8-7-5) game, where the "low" resource positions, C, D, and E, obtained higher payoffs than the "high" resource positions, A and B.

The Shapley Value

Comparisons of the player positions' mean payoffs with the Shapley value also utilized confidence intervals with $\alpha=.01$. Comparisons yielded the following results: (1) In the 20(10-9-8-7-5) game, support (where + indicates that the Shapley value fell within the confidence interval and - indicates that it did not) for positions (A, B, C, D, E) was (-, +, -, +, -) respectively; (2) In the 14(10-7-5-3-2) game, support was (-, +, +, +, +), again for (A, B, C, D, E); (3) In the 27(15-14-10-8-5) game, support was (+, -, -, +, +); and (4) In the 30(24-9-8-7-6) game, support was (-, +, -, -, +). Thus, in 11 out of 20 tests, the Shapley value prediction for payoffs was supported.

A comparison of the Shapley value predictions across games was made by correlating the rank ordering of all 20 observed payoffs with the ranked predictions for each position. The results yielded a highly significant correlation: $r(20)=.92$, $p<.0001$. Thus, although the Shapley value's predictions of the absolute magnitudes of the players' payoffs was only marginally supported, its predictions of the relative magnitude of the payoffs in this study were strongly supported.

Discussion

The results of the theoretical comparisons are quite clear. Bargaining theory's predictions surpass those of the other models, for both coalition frequencies and for outcomes. The Weighted Probability model received some support, but did not compare favorably with Bargaining theory, especially in the 14(10-7-5-3-2) game where their predictions differ most. Minimum Resource and, in particular, Minimum Power theories received little support. With the other recent negative findings for both models in games with more

than three players, the applicability of these early models to more complex coalitions situations is very questionable.

Bargaining theory's predictions narrowly missed support by the outcomes of positions D and E in the 20(10-9-8-7-6) game, as discussed earlier, and were not supported by the coalition frequencies in the 30(24-9-8-7-6) game. However, the differences between the predictions of the Weighted Probability model, the only model supported for coalition frequencies in the 30(24-9-8-7-6) game, and Bargaining theory are almost negligible in this case. While the Weighted Probability model predicts that any of the two-person coalitions should form, Bargaining theory predicts that the AE coalition should be more frequent than the other two-person coalitions. Because "more frequent" has not been quantitatively defined by the model, the prediction of the most frequent coalition has been emphasized. In this case, the choice of the AE coalition as being more frequent depends upon relatively minor differences in the players' calculated temptations to defect.⁸ This minor error, then, is all that separates Bargaining theory from being supported in all four games in this study. With the positive results in other recent studies (Komorita and Moore, 1976; Michener, Fleishman and Vaske, 1977; Murnighan, et al., 1977), the model has received strong support. Future research on Bargaining theory, then, might attempt to pinpoint the boundary conditions of the model and increase the likelihood that future theoretical formulations might expand those boundaries.

The comparisons with the Shapley value were not quite so clear. Ironically, where coalition frequencies and players' outcomes did not support the social psychological model fathered by the Shapley value, i.e., Minimum Power theory, the players' payoffs tended to support the predictions of the Shapley value, particularly across the entire set of positions.

Although additional research is necessary before a final judgment of the predictive validity of the model can be rendered, recent research in coalition situations where one player was a monopolist (Murnighan and Roth, 1977; Note 2) also supported its predictions. Thus, although the process attributed to the original formulation (that players are added to coalitions one at a time and receive all marginal payoffs when they join) may not represent the actual bargaining process, the reformulation of the Shapley value as a cardinal utility, indicating what a risk neutral player might expect to receive from playing a game, appears to be correct, empirically as well as mathematically.

Using the players' payoffs as a measure of power indicates that position A in the 30(24-9-8-7-6) game is the most powerful of the 20 positions in this study. The strength of this position was apparent prior to play of the game. However, the use of payoffs as a measure of power allows for comparisons between the payoffs of positions in different games, thus increasing understanding of the relative power of the different position/game combinations. The power (payoffs) of position A in the 39(24-9-8-7-6) game, for instance, appears to be comparable to the combined power (payoffs) of positions A and B in the 27(15-14-10-8-5) game. Within a single game, the power (payoffs) of position A in the 14(10-7-5-3-2) game appears to be comparable to the combined power (payoffs) of positions B and C. Thus, when coalition games are used as models of more complex conflict situations, the payoffs of the different positions can be used as a post hoc measure of the power of those positions, and this measure, because it has a fixed zero point, appears to be a ratio scale.

Strength and Weakness

The "strength is weakness" hypothesis that has been so prevalent in the results of three-person games can be tested by the data in the 20(10-9-8-7-5) game, where any three players can form a winning coalition. Positions C, D, and E received significantly higher payoffs than positions A and B, and position A received the lowest payoff of any of the positions, thus supporting the hypothesis. The "strength is weakness" hypothesis does not, however, apply to all of the games in this study. In the other three games, positions with more resources generally obtained higher payoffs. Thus, to some degree the results also indicate that "strength is strength." However, even in these games, there is evidence of "strength" being "weakness." For instance, position E obtained significantly higher payoffs than position C in the 27(15-14-10-8-5) game, even though E's resources are only half of C's. In the same game, position B obtained significantly higher payoffs than position A. The superiority of positions A and B over positions C, D, and E, however, indicate that greater resources, at times, increase strength. Thus, in this game, "strength is strength" and "strength is weakness." What conditions differentiate these two effects? The data from this study support the conclusion that, in a particular game, when positions have equal Shapley values but different resources, the position with less resources generally obtains equal or higher payoffs than the position with more resources. Thus, in the 20(10-9-8-7-5) game, the "extra" resources held by position A work to his disadvantage. In each of the games, players in positions with identical Shapley values tended to receive equal or lower payoffs than players in positions with less resources. Although all of the comparisons of this type did not yield significant differences, the results are almost always in the right direction. Future research with other games is necessary to substantiate

this conclusion. However, it appears from this study that the "strength is weakness" phenomenon is contingent on identical Shapley values for positions within the same game.

Examination of students' comments in the papers they wrote describing their strategies also yields an answer to the question of why "strength" is sometimes "weakness." Many of the players adopted the parity norm (the basis of Gamson's Minimum Resource theory) in order to structure the situation and formulate strategies. Thus, these players emphasized the resources of the different positions and bargained accordingly. Other players adopted philosophies more in line with the Weighted Probability model and the Shapley value, by focusing on the interchangeability of certain players in different coalitions (e.g., any three players can win in the 20(10-9-8-7-5) game). Neither group of analysts constituted a majority of the players. However, both groups tended to conclude that, at least initially, players with smaller amounts of resources may be more receptive to a given offer than players with more resources. This is true even if the players do not focus on resources: they also recognized this philosophy and the possibility that other players might adopt it. Thus, a player with the least resources necessary to complete a coalition will be chosen over players with more resources,⁹ at least at the start of the game. From a reinforcement point of view, coalitions that form early in the bargaining should become more and more likely as the trials progress. Thus, this early inclination to send offers to those with less resources leads to their more frequent inclusion in the winning coalition, higher payoffs in the game, and support of the "strength is weakness" phenomenon. This effect may erode over trials (e.g., Kelley and Arrowood, 1960) with reference to coalition frequencies; however, the overall effect on payoffs (this study's measure of power) should not change.

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FOOTNOTES

1. This explanation of the Shapley value assumes that winning coalitions receive a constant payoff. If the payoffs vary across coalitions, a more complicated formula must be used to take the value of the different payoffs into account.

2. This "advantage" or weight is determined by the equation

$$P(C_{\underline{j}}) = \frac{1/(n_{\underline{j}}-1)}{\sum 1/(n_{\underline{j}}-1)},$$

where $P(C_{\underline{j}})$ = the probability of coalition \underline{j} , $n_{\underline{j}}$ = the number of members of coalition \underline{j} and the summation is over all minimum winning coalitions. The probability of inclusion for a particular player is equal to the sum of the probabilities that the coalitions of which he/she is a member will form. The players' predicted payoff is proportional to his/her probability of inclusion. Four-person coalitions, by the way, are weighted (give a player the "advantage" of) one-third of a two-person coalition.

3. Gamson (1961) has defined a "full-fledged coalition situation" as consisting of more than two players who are attempting to maximize their share of the rewards, with no player having dictatorial or veto power, and with no condition in which all the players can jointly maximize their payoffs.

4. Vinacke (1959) has hypothesized, and found evidence for, differences in coalition behavior between males and females. A check on the possibility that sex systematically affected the data was conducted. Vinacke's anti-competitive hypothesis predicts that females should do well in relatively weak positions and poorly in relatively strong positions. If a total of 240 points is used as the criterion for determining weak/strong positions in these games (240=total points awarded per session, 1200, divided by five players), females were observed to score above the sample average 16 of 35

times (46%) for strong positions and 23 of 45 times (51%) for weak positions. Thus, it appears that the sex of the players had no appreciable effect on the results.

5. Some scheduling problems did arise. Groups that were unable to meet for four consecutive weeks generally missed one session and completed the games in five weeks. Some groups also participated in two games the same day.

6. In some situations, two or three proposals to form a coalition were accepted on the same trial. For instance, if A sent an offer to C, D sent an offer to A, and both offers were accepted, AC would be declared the winning coalition because A was committed to his/her offer (invalidating his/her acceptance). If three coalitions formed in this manner, with each being invalidated by another, the players were informed of the situation and the trial was rerun.

7. Complete tables of the frequency and outcomes for each of the coalitions are available from the author.

8. The total temptation to defect for players A and E in the AE coalition is .05. For the AD coalition, the total is .10; for AC, .125; for AB, .18. Total temptations to defect for coalitions that are not predicted to form in this study range up to .66. Whether the change from .05 to .18 is sufficiently apparent to the players can be questioned (and studied in future research). Such data might determine whether Bargaining theory can make more specific predictions of the frequencies of the different coalitions.

9. While game theory would describe the tendency to send offers to players with less resources as irrational, such a strategy is subjectively rational (Simon, 1976) for players who emphasize the importance of resources and for players who recognize the possibility that other players might adopt

such a philosophy. For this latter group, directing one's offers to players with less resources increases the probability of their acceptance, and is therefore rational (given equal offers).

TABLE 1

The Predictions of Four Models (and the Shapley Value) for Four Games for the Most Frequent Coalition and Its Payoff Division

Model	20(10-9-8-7-5)	14(10-7-5-3-2)	27(15-14-10-8-5)	30(24-9-8-7-6)
Minimum Resource Theory	CDE 40-35-25	BCE 50-36-14	BDE 52-30-18	AE or BCDE 80-20 25-25-25-25
Minimum Power Theory	any 3-person 33-33-33	BCD, BCE or ADE 43-43-13 74-13-13	AXX or BXX ^a 54-23-23	BCDE 25-25-25-25
Bargaining Theory	CDE 35-32-32 ^b	AC 65-35 ^b	AR ^b 51-49 ^b	AE 76-24 ^b
Weighted Probability Model	any 3-person 33-33-33	AB or AC 55-44	AB 50-50	AB, AC, AD or AE 75-25
Shapley Value (x 100)	20-20-20-20-20	40-23-23-7-7	30-30-13-13-13	60-10-10-10-10

^aAXX refers to ACD, ACE, or ADE; BXX refers to BCD, BCE, or BDE.

^bPredictions listed for Bargaining theory are the asymptotic predictions. In general predictions for the first trial for each game differ from these predictions.

The Four Most Frequent Coalitions, Their Frequencies (in Parentheses)
and the Mean Outcome for the Coalition's Members in Each Game

Results	20(10-9-8-7-5)	14(10-7-5-3-2)	27(15-14-10-8-5)	30(24-9-8-7-6)
Most frequent coalition	CDE(98)	AC (111)	AB(99)	AE(70)
Mean Outcome	36-34-31	64-36	51-49	73-27
2nd Most frequent coalition	ABC(40)	AB(65)	BDE(69)	AB(65)
Mean Outcome	34-33-34	60-40	49-27-24	77-23
3rd Most frequent coalition	ABE(33)	BCF(44)	ADE(37)	AC(61)
Mean Outcome	35-36-30	44-37-18	53-24-23	81-19
4th Most frequent coalition	BDE(31)	ADE(34) and BCD(34)	BCE(36)	AD(53)
Mean Outcome	34-33-32	57-22-22 40-38-23	48-30-23	79-21

Note: The total number of coalitions in each game was 288.⁷

Positive and Negative Support for the Models' Prediction of
the Players' Outcomes in the Most Frequent Coalitions

Game	Coalition	Player Position	Confidence Interval	Support of the Models' Predictions			
				Minimum Resource	Minimum Power	Bargaining Theory	Weighted Probability
20(10-9-8-7-5)	CDE	C	$34.59 \leq \bar{X} \leq 36.91$	-	-	+	-
		D	$32.82 \leq \bar{X} \leq 34.50$	-	+	-	+
		E	$29.32 \leq \bar{X} \leq 31.74$	-	-	-	-
14(10-7-5-3-2)	AC	A ^a	$61.33 \leq \bar{X} \leq 66.95$	+	+	+	-
		B	$49.48 \leq \bar{X} \leq 52.13$	+	+	+	-
30(24-9-8-7-6)	AB	A ^a	$72.43 \leq \bar{X} \leq 80.80$	+	-	+	+
		A ^a	$77.59 \leq \bar{X} \leq 85.03$	-	-	-	-
	AD	$75.17 \leq \bar{X} \leq 82.70$	+	-	+	-	
	AE	$69.24 \leq \bar{X} \leq 76.80$	-	-	+	+	

^aFor two-person coalitions, the endpoints of the confidence intervals for the two players sum to 100. Thus, the second player's confidence interval can be obtained by subtraction, and the support a model obtains is identical to that received for the first player.

TABLE 4
Mean Payoffs for Each Player Position in the Four Games

Player Position	Game			
	20(10-9-8-7-5)	14(10-7-5-3-2)	27(15-14-10-8-5)	30(24-9-8-7-6)
A	14.2 _g	45.0 _b	30.7 _d	67.0 _a
B	17.6 _f	20.4 _f	35.6 _c	8.7 _j
C	24.3 _e	24.0 _e	8.7 _{ij}	7.4 _j
D	21.3 _e	5.3 _j	11.9 _{ghi}	7.5 _j
E	22.7 _e	5.3 _j	13.3 _{gh}	9.8 _{hij}

Note: Means with common subscripts are not significantly different from one another at the 0.5 level using the Newman-Keuls' procedure.



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