Structural Iron and Steel

W. N. TWELVETREES

STRUCTURAL IRON AND STEEL.

"The Builder" Student's Series.

STRUCTURAL IRON AND STEEL.

A TEXT BOOK FOR ARCHITECTS, ENGINEERS, BUILDERS, AND SCIENCE STUDENTS.

With numerous Examples, Illustrations, Diagrams, and Tables.

BY

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LONDON:

WHITTAKER & CO., 2, WHITE HART STREET, PATERNOSTER SQUARE, E.C.

AND

66, FIFTH AVENUE, NEW YORK.

1900.

LONDON:
PRINTED BY LOVE AND MALCOMSON, LIMITED,
GREAT QUEEN STREET, W.C.

PREFACE.

In this elementary treatise the Author has endeavoured to present in regular sequence some of the more important details relative to iron and steel as applied to structural work.

Examination has been made of processes attending the production and manufacture of iron and steel, because such matters properly constitute the starting-point of knowledge, and also for the reason that familiarity with details of this kind affords a useful clue to the probable behaviour of materials when they appear in manufactured forms. Further, the Author has attempted to furnish in a manner convenient for ready reference the most reliable information as to the physical properties of iron and steel, to explain the nature of stresses and strains, and to indicate the methods adopted for their measurement.

Joints and connexions have naturally occupied a considerable share of attention, for such details are of the utmost importance in every department of constructional work. The principal forms of structural iron and steel have been described and illustrated, as well as the manner in which they may be practically applied as ties, struts, columns, and beams. Roofs and other framed structures have not come under notice, for no incidental reference to so comprehensive a subject could possibly be useful or satisfactory.

So far as building construction is concerned, there can be no doubt whatever that the age of steel is still in its infancy, and if the architect of the future is to be, as he ought to be, the master of his profession in all its branches, he must inevitably make it his business to become thoroughly familiar with all the details of iron and steel construction.

The greater portion of this work originally appeared in the columns of *The Builder*, and although primarily written for architectural students, it is hoped that it may also prove useful to others who may be professionally concerned with structural iron and steel construction.

W. N. TWELVETREES.

47, VICTORIA STREET, WESTMINSTER,
October, 1900.

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INTRODUCTION.

Each succeeding year brings fresh evidence of the changes taking place in connexion with the practice of various arts and sciences. Amongst these architecture exhibits comparatively little indication of advancement, either in theory or in practice. We say comparatively little, because the real progress which has been made is dwarfed by the enormous concurrent development of other arts and sciences. Architecture as a fine art was known, practised, and brought to a state of what appears to have been perfection many centuries ago; as a practical science it has always provided, more or less satisfactorily, for the application of materials to the requirements of climate, to the customs of a period, and to the necessities of an occasion. Thus it is that, surrounded by the wonders of latter-day science and by the opening up of new departments of engineering, the art of building seems to be almost at a standstill. Ministering, as he does, to the wants of human nature, which changes not, the architect is not called upon to exploit new wonders, and his mission is fulfilled if he present comfort, convenience, sanitation, and scientific construction in an artistic form. At the present time there is sufficient evidence to show that this mission is being accomplished, and that adaptation is being made of resources rendered available by the progress of engineering science. It is beyond the scope of this work to discuss the general question of building construction, and inquiry will be limited to a consideration of the applicability of iron and steel to such work. Iron construction, as practised in the United States, has not hitherto been introduced into this country, and, as a matter of fact, it would not be economically possible without modification of existing building regulations. Nevertheless, very considerable use is made of both iron and steel in all classes of buildings, and it is therefore desirable that the architect should be familiar not only with the properties of the materials themselves, but also with the practical details of their manufactured forms which are suitable for adaptation to particular requirements. In dealing with some details of structural iron and steel we do not propose to treat, unless incidentally, of either applied mechanics or constructional design, because these comprehensive subjects are included in the ordinary educational curriculum of the architect and the engineer. Our intention is rather to present, in a natural sequence, such information with regard to iron and steel work as may be readily applied in practice by those who have studied, or are studying, the art of building, but who possibly are not intimately acquainted with the details of constructional ironwork. There is every reason why the architect should be perfectly familiar with this kind of work. Strictly speaking, it comes within his province, for it is his function to conceive, in its entirety, the harmonious combination constituting the final solution of diverse constructive problems. At the present time, the unpaid advice of contracting engineers is too frequently relied upon by the architect, who in such cases is paid for the work he does not perform, whilst the contractor performs work for which he is not supposed to be paid. If his price does not cover remuneration for his trouble, he suffers injustice; and if it does, then the architect's client has to pay twice, which, as Euclid tersely remarks, "is absurd." The assumed position considered from an ethical standpoint cannot be regarded as satisfactory to the architect, to the client, or to the contractor. As the adoption of iron framework in buildings grows more general in this country, there will concurrently arise the necessity for increased technical knowledge on the part of the designer. In the United States, the usual practice is for a civil engineer to act in conjunction with the architect in the design and superintendence of all important metal-framed buildings. Whether a similar condition will ultimately prevail in England probably depends very much upon the future training of the architect.

One of the chief problems of modern constructional work is that of employing materials to the greatest possible advantage. The designer must invariably ascertain to what straining actions the structure will be exposed, what is the best disposition of the material, and the least quantity necessary to resist the ascertained strains. These considerations naturally involve knowledge of the properties of materials, of the nature of stresses and strains, of data ascertained as the result of experimental research with regard to strength, of the joining together of individual members forming structures, and of the arrangement calculated to contribute most adequately to the stability of the structure as a whole. Our object is now to traverse the ground thus indicated in the hope that the inference of practical conclusions may thereby be facilitated.

PART I.

PHYSICAL PROPERTIES OF IRON AND STEEL

STRUCTURAL IRON AND STEEL.

CHAPTER I.

IRON PRODUCTION.

1. Varieties of Iron Ore. — An important matter affecting the suitability of iron for any given purpose is the source from which it is derived. It forms no part of our present duty to enter upon a detailed discussion of this subject, but a brief summary of the chief kinds of iron ore may conveniently be made. Iron ores are divisible into three main groups—where the metal occurs chiefly as sulphide, as carbonate, or as oxide. The chief forms of ore coming within these groups are described in the following summary :—

§ (*a*) *Sulphides.*—Pyrites is never used for the manufacture of iron, owing to the deleterious effects of sulphur; but the residue remaining after the mineral has been burnt in the course of other industrial processes is sometimes usefully employed in puddling furnaces.

§ (*b*) *Carbonates.*—Spathose ore is a crystalline and comparatively pure form of ferrous carbonate, though magnesium, manganese, and calcium carbonates are usually present. In the United Kingdom the chief sources of supply are in the Weardale and the Exmoor districts; on the Continent, extensive deposits occur in Germany and in Spain. Owing to its freedom from phosphorus, this ore is largely used in the production of steel.

Clay ironstone, or argillaceous ore, consists of ferrous carbonate, mixed with clay. A variety of clay ironstone, containing carbonaceous or bituminous matter, is known as Blackband ore. The former type is found in the Cleveland beds of Yorkshire, in Shropshire, Derbyshire, and in South

Wales; also in France, Germany, and various districts of the United States. The latter type occurs chiefly in the coal measures of Staffordshire, Scotland, and Wales, as also in Germany and the United States. Clay ironstone is the most abundant variety of British iron ore, but it contains a somewhat large proportion of phosphorus—a metal which is disadvantageous except in minute quantities.

§ (*c*) *Oxides.*—Red hematite ores include several varieties, such as "kidney" ore, so called from the shape of the nodules in which it is deposited; specular ore, a crystalline form with a bright mirror-like surface; titaniferous ore, or ilmenite, a variety somewhat resembling specular ore; and micaceous ore, a crystalline mineral. Kidney and specular ores are the forms chiefly used for iron manufacture, and both of them are freely distributed in Great Britain, notably in Cumberland and Lancashire. There are also considerable deposits in Sweden, Norway, Belgium, Germany, Austria, Brazil, Elba, and in the United States.

Brown hematite varies much in appearance and in purity, and includes the following types: limonite and bog-iron ore, both of earthy and concretionary nature; göthite, a crystalline substance; and others, which need not be specifically mentioned. The principal sources of brown hematite for iron production in the United Kingdom are Northamptonshire and the adjoining counties; considerable quantities are also imported from Spain. Important deposits occur in other countries of Northern Europe and in Canada.

Magnetic iron ores include forms described as franklinite, loadstone, and iron-sand. Comparatively small quantities are found in this country, but extensive deposits exist in Sweden, Norway, France, Germany, Spain, Portugal, in the Ural Mountains, in India, in Africa, and in America. Being remarkably free from phosphorus and sulphur, magnetic ore is well adapted for the making of the finest qualities of iron and steel.

2. Chemical Constitution of Iron Ores.—In the foregoing classification only such characteristics as are germane to the purposes of our inquiry are mentioned;

more precise details being obtainable from various treatises dealing with the metallurgy of iron. Analyses of iron ores show that each kind is of somewhat variable constitution, not only in different localities, but also in any individual place. A very comprehensive set of tables would therefore be necessary for the purpose of showing even the average composition of different classes of ore.

Table I. indicates the comparative values of the principal ores found in different parts of the world, and Table II. shows the approximate composition of ores used commercially.

Table I.—Average Percentages of Metallic Iron in Ores of various kinds.

I. SULPHIDE CLASS :—		
Iron Pyrites	Spain	41·92
	Portugal	44·28
II. CARBONATE CLASS :—		
Spathose Ores	Weardale	38·95
	Somerset	34·67
	Germany	42·59
	France	37·45
	Nova Scotia	42·76
Clay Ironstone	Scotch Blackband ...	32·00
	Staffordshire	36·14
	Cleveland (Yorkshire) ...	31·42
	Dowlais (South Wales) ...	34·72
	Abercorn Blackband ...	36·40
	Ohio (U.S.A.)	41·89
III. OXIDE CLASS :—		
Red Hematite Ores	Ulverston	63·66
	Elba	61·81
	Lake Superior	62·92
	Nova Scotia	43·40
	Missouri	59·15
Brown Hematite Ores... ...	Northamptonshire ...	37·00
	Sweden	47·32
	Spain	45·87
	France	49·00
	Kentucky	48·95
	Nova Scotia	58·22
Magnetic Ores	Yorkshire...	49·17
	Sweden	62·60
	Canada	66·73
	Wyoming...	45·49
	New Jersey	64·86
	Lake Champlain... ...	69·51

Table II.—Approximate Composition of Iron Ores used in Manufacture.

Constituents.	Carbonate Class.		Oxide Class.		
	Spathose.	Ironstone.	Red Hematite.	Brown Hematite.	Magnetic.
Ferric oxide	0– 5	0–10	60–95	50–90	30–70
Ferrous ,, ...	20-60	30-45	0– 5	—	15–55
Manganous oxide	1–25	0– 2	0– 2	0– 2	0– 1
Magnesia	0–10	1–10	0– 1	0– 2	0– 2
Alumina	0– 5	1–10	0– 5	1–10	0–10
Lime	0–25	1–10	0– 3	0– 5	0– 5
Silica	0– 5	2–25	1–25	1–30	0–25
Carbon dioxide	35–40	20–35	0– 2	0– 5	0– 5
Phosphoric anhydride ...	—	0– 3	0– 3	0– 3	0– 2
Sulphur	—	0– 2	0– 1	0– 1	0 - 2
Water	0– 5	0– 4	0– 5	5–20	0– 5

§ (*a*) *Foreign Ingredients.*—Without entering further upon the question of chemical constitution, it may be remarked that foreign substances are invariably found in iron ores, in addition to the compounds of the metal, which furnish a convenient means of grouping the different varieties of mineral. Clay, sand, and the carbonates of calcium and magnesium are substances very generally occurring in iron ores. Others, existing in small quantities, are compounds of carbon, manganese, arsenic, titanium, sulphur and phosphorus. Most of these are injurious, only carbon and manganese being of advantage, the former aiding reduction of the ore and the latter imparting strength to manufactured steel. Further notes on this subject will be found in Art. 11, § *a*.

3. Extraction of Iron from Ores.—Modern methods for the extraction of iron from its ores may be said to come under one of two heads: (1) where cast iron is produced by smelting; and (2) where malleable iron or steel is obtained directly from the ore. Preliminary preparation of the ore for smelting is always desirable, in order that the maximum surface of the mineral may be exposed to the action of the furnace gases, and that volatile constituents may be eliminated. The ore is first crushed in a stone-breaker or mill into fragments about 2 in. in diameter; it is then mixed with suitable fuel and roasted in kilns, or in heaps exposed to the open air. In this way fragments of impure ferric oxide are obtained. Blackband contains its own fuel, and if properly stacked merely requires lighting with a small quantity of coal. The process of roasting is essential in the case of most ores, and even when the raw material consists of oxide, roasting has the effect of simplifying the subsequent operation of smelting.

4. **Roasting Kilns.**—Roasting kilns vary considerably both as to design and dimensions. In a general way they may be likened to limekilns, and heat is furnished by solid fuel, by producer-gas, or by waste gas from a blast furnace. In the construction of a roasting kiln it is always desirable that sharp angles in the interior should be avoided, because combustion is always less rapid in the angles than in the centre, therefore if heat were regulated so as to produce its proper effect in the middle of the kiln, calcination would not take place in the corners.

§ (*a*) *Ordinary Roasting Kiln.*—Figs. 1 and 2 are given as illustrations of two satisfactory forms of roasting kilns, and incidentally these exemplify the application of cast iron in structural work. In each figure the kiln is built of masonry with a fire-brick lining, and the floor is made of cast-iron plates 2 in. in thickness. The average dimensions of such furnaces may be taken as 20 ft. long by 9 ft. wide at the top by 18 ft. high; a cast-iron frame is provided on the upper edge, so that the brickwork may be protected from injury during the operation of charging. This frame is about 12 in. wide, with a flange 6 in. high, and it may project either above or below the upper course of brickwork. The front of the kiln is built with two arches having openings communicating at the floor level with the interior. Through these, calcined ore can be drawn prior to being loaded into barrows for removal to

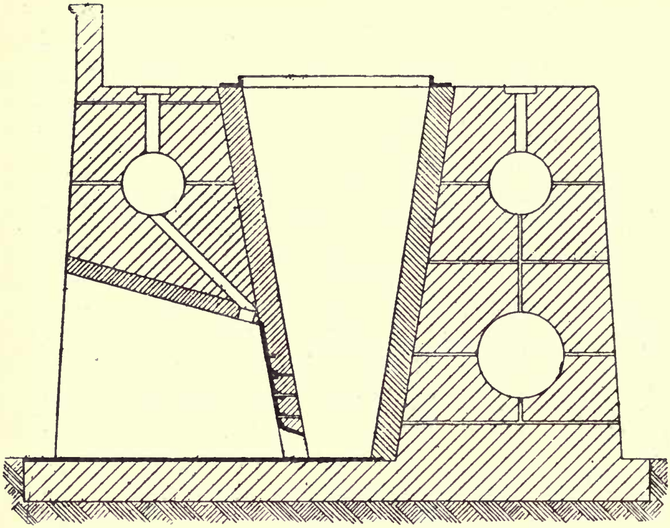


Fig. 1.

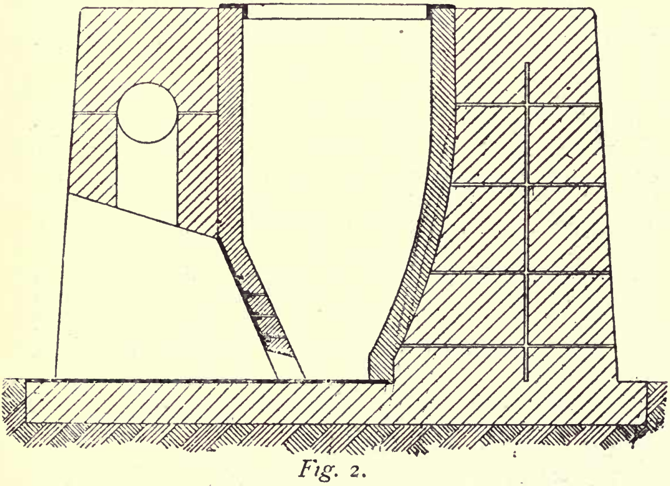


Fig. 2.

the blast furnace, and above the openings four or more holes 6 to 8 in. square are provided, with suitable arrangements for the regulation of draught.

§ (*b*) *The Gjers Roasting Kiln.*—Another type of roasting kiln is that first used at Middlesbrough, and erected from the designs of Mr. Gjers. One of these kilns is shown in fig. 3, in elevation and section. The Gjers kiln is circular in form, being built of fire-brick, with a shell of wrought iron resting on an annular cast-iron entablature, supported

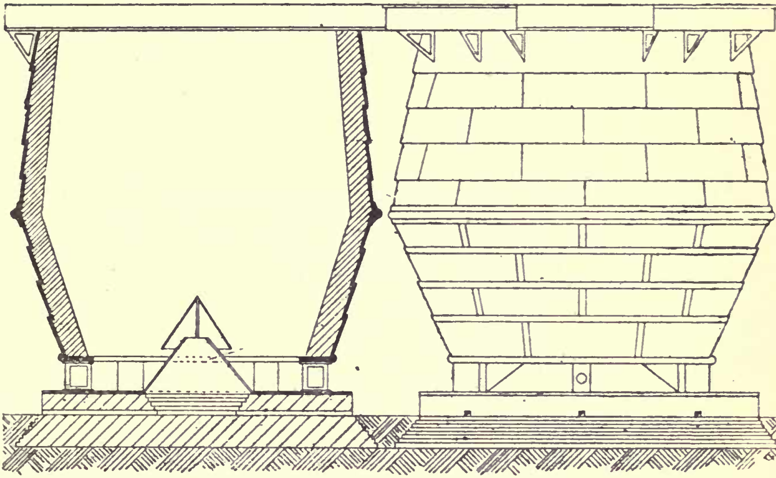
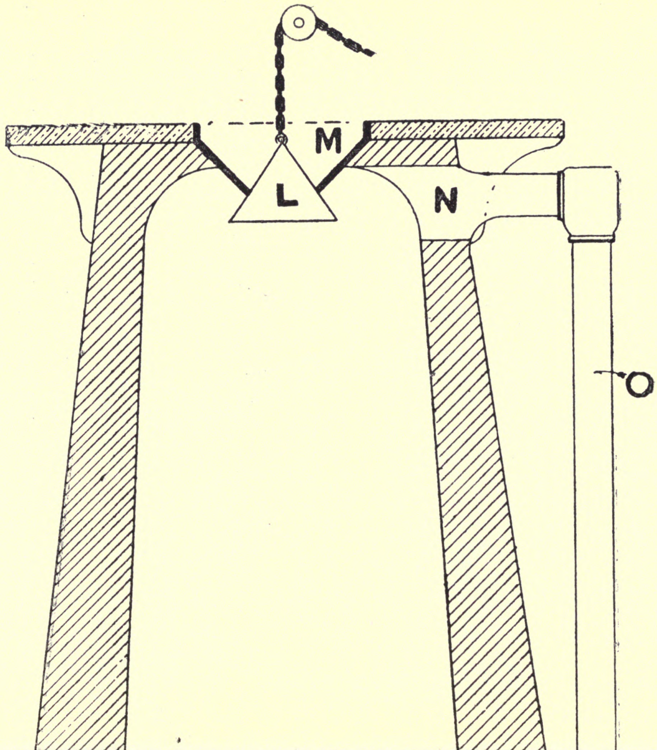


Fig. 3.

by cast-iron pillars, between which space is afforded for the raking out of calcined ore and for the admission of air. Beneath the pillars is a foundation of brickwork having additional air passages communicating with the double cast-iron cone in the centre. Air is also admitted through orifices shown in the lower portion of the kiln. The central cone has the effect of spreading the calcined ore in the direction of the spaces between the pillars, and it also helps to break up any partly-fused lumps which descend from the upper part of the kiln. Some-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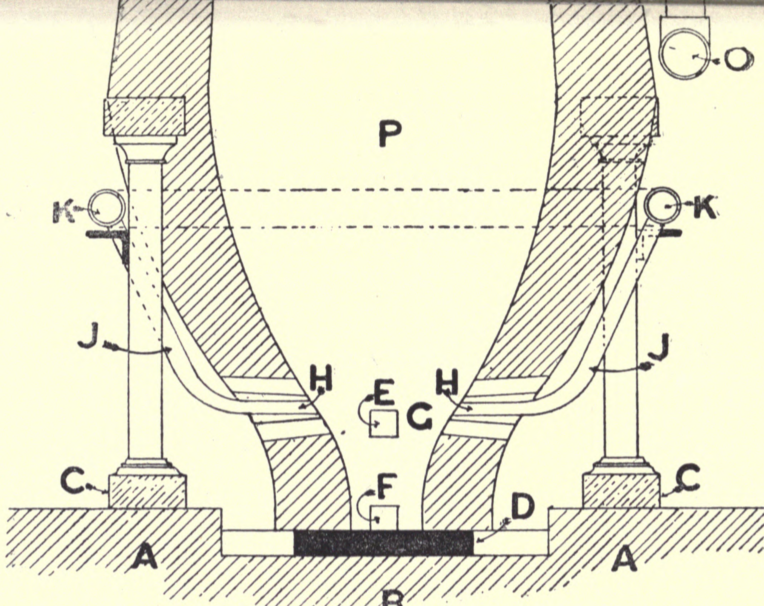


Fig. 4.

times the annular space between the two sections of the cone serves the purpose of admitting waste gas from the blast furnace, and in any case it is available for the supply of air in aid of combustion. As a general rule the maximum internal diameter of the furnace is 20 ft., the height 24 ft., the capacity more than 5,000 cubic feet, and the cone is about 8 ft. diameter by 8 ft. high. In some works the kilns are connected together in groups of four by galleries of cast iron with wrought-iron brackets, and in others they are arranged in a continuous line, above them being a tramway, by the aid of which waggons may be caused to discharge their contents directly into the kilns. Besides the forms of kiln mentioned above there are many others, but these need not be described in detail, as no variation of general principle is involved.

5. Smelting.—Next in order comes the process of smelting, performed in a blast furnace, where the following changes take place. The fuel used for the purpose of reduction combines with oxygen of the air supplied, forming first carbon dioxide (CO_2), and then by the absorption of more carbon, carbon monoxide (CO). This, coming in contact with the heated iron oxide, is again converted into carbon dioxide, leaving the iron in a metallic state. With the object of aiding the separation of earthy impurities contained in the ore, and of promoting their fusion, various classes of ore are frequently mixed together, so that suitable relations may exist between compounds of silicium, calcium, and aluminium contained therein. More generally, however, limestone, ferruginous clay, or poor ores containing aluminous compounds are added to serve the purpose of a flux. Limestone ($CaCO_3$), when subjected to heat, parts with CO_2, leaving as the residue calcium oxide or quicklime (CaO). This, combining with silicious compounds and other impurities in the molten iron, forms a double silicate of calcium and aluminium—really a crude type of glass—which, owing to its smaller specific gravity, rises to the surface, and when cold constitutes what is known as iron-slag. The melted metal collects on the furnace hearth, whilst the fused slag floats on its surface. Slag is drawn off from openings in the upper part of the hearth, and the metal is run from below into channels formed in sand, each channel of iron being termed "a pig," and hence originates the expression "pig-iron."

6. Blast Furnaces.—Considerable differences are exhibited with regard to the constructional details of blast furnaces, but so far as British practice is concerned, it seems to be generally agreed that the most suitable height is from 60 ft. to 80 ft., and that the configuration of the inner surface should exhibit easy flowing curves from top to bottom. Furnaces have been built as high as 103 ft., but it does not appear that any advantage is gained by the adoption of a greater height than 80 ft., which is that generally used in the Cleveland district. In Lancashire and Cumberland, blast furnaces are usually from 60 ft. to 70 ft. high; in the United States, they vary from 40 ft. to 70 ft. high, according to the description of ore to be operated upon; and the latter dimensions also apply to most of the furnaces on the Continent. Without considering in detail the mechanical and other auxiliaries of a blast furnace, we may with advantage examine its essential points.

§ (*a*) *Ordinary Blast Furnace.*—Taking, first, the furnace shown in section by fig. 4, and which may be regarded as fairly typical of the ordinary blast furnace, we find the foundation up to the ground line to be of brickwork (B) resting on clay. A circular case of fire-brick (A) (A) is formed round the hearth-bottom (D), and on this casing is a stone curb (C) supporting the cast-iron columns, which bear the greater part of the structure. The columns are twelve in number, about 18 ft. high by 2 ft. 4 in. diameter, and of 2-in. metal. Fire-brick is the material used in the construction of the furnace shell, which is cased externally with wrought-iron plates varying from $\frac{3}{8}$ in. to $\frac{1}{2}$ in. thick, and internally with fire-lumps 5 in. thick. These are carefully dressed both on the beds and joints for a short distance above the hearth or crucible (G), which occupies the neck of the funnel-shaped portion of the furnace (P), known as the "boshes." The hearth-bottom is made of fire-brick or other infusible material, laid in such a manner as to counteract the tendency of the molten metal and slag, first to undermine, and then to exert upward pressure on the bottom. All fire-brick used

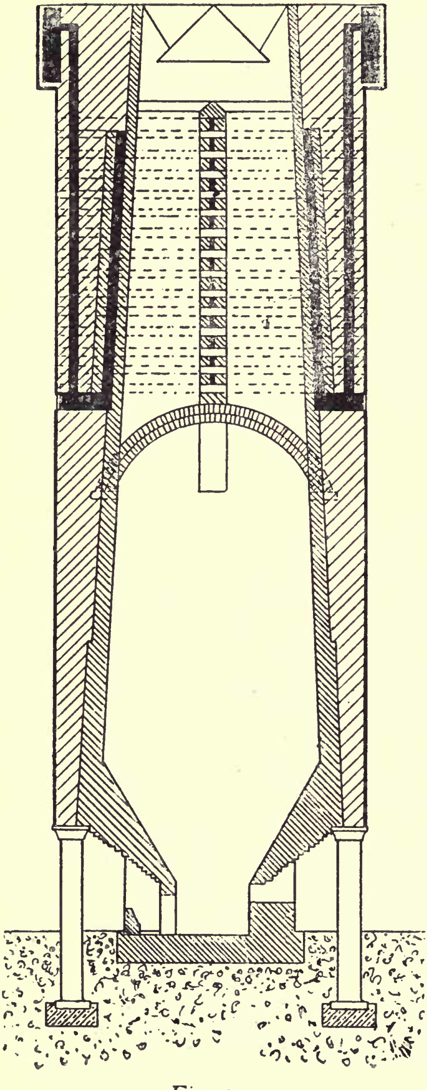


Fig. 5.

is, as far as possible, free from iron, so as to avoid the fluxing action of slag. At the bottom of the hearth a tapping-hole (F) is left in the brickwork, through which metal may be drawn off as required, by the removal of a stopping of clay and sand. Near the top of the hearth is a channel, known as the cinder-notch (E), through which slag may flow out from the surface of the liquid iron. In the upper part of the hearth, four tuyère-holes are built in the wall of the furnace, and the jets or tuyères (H) (H) for the supply of air pass into the holes, their nozzles being generally protected from the heat by a water-jacket continuously fed with cold water. Each tuyère is joined by a branch connexion or "goose-neck" (J) with the blast pipe (K), which is carried partly or entirely round the furnace, and which receives its supply of air, either cold or hot as the case may be, from a blowing engine. The top, or throat, of the furnace is closed by a bell (L) and hopper (M) of cast iron, by means of which materials are introduced from time to time, without interfering with the egress of gases from the opening (N) at the side of the furnace. Into this opening is fitted the end of a riveted iron pipe (O) which serves to convey the heated gases to boiler fires, to superheaters, or to calcining furnaces. Thus the products of combustion can be utilised for steam raising, for the provision of a hot-air supply to the blast furnace, or for roasting ore, as may be preferred. The top of the furnace is finished with a platform or gallery, supported by brackets; and the main blast-pipe is upheld by braces attached to the cast-iron columns. In constructing a blast furnace, the upper portion is built on the columns before the hearth and its enveloping brickwork are added. This mode of procedure permits any settlement of the main structure to take place before the hearth-work is built. It may, therefore, be seen that the bulk of the weight is borne, as before stated, by the cast-iron columns, and that no unnecessary strain is caused to the brickwork on which depends the successful consummation of smelting operations and the safety of workmen engaged therein. Before leaving the furnace illustrated in fig. 4 it may be interesting to give the chief dimensions, which are as follows:—Height from hearth-bottom to platform, 85 ft.; internal diameter of hearth, 8 ft.; internal diameter at widest part, 28 ft.; diameter of bell opening, 13 ft. The cubical capacity is about 30,000 cubic ft., and the pig-beds will hold 1,200 moulds.

§ (*b*) *The Ferrie Blast Furnace.*—A modified form of apparatus is known as the Ferrie self-coking blast furnace, in which raw coal is used in place of coke as ordinarily employed. Fig. 5 is a vertical section of this furnace which, so far as the lower portion is concerned, exhibits no special features, but in the upper part four chambers, or retorts, are formed by radial and circumferential walls resting on arches. Part of the products of combustion are suitably collected at the top of the furnace and led down the outermost internal flues shown in the figure to the level of the coke retort bottom. Here they mingle, in predetermined proportions, with atmospheric air, and, being ignited, the mixture passes upwards through and around the inner series of flues. The heat thus communicated, together with that ascending from the lower part of the furnace, effectually cokes the coal during its passage down the retorts. By the use of the Ferrie furnace the temperature of the ore and flux is raised with a minimum expenditure of fuel, and the iron produced is of superior quality. All the auxiliary arrangements in connexion with the furnace resemble those ordinarily used.

7. Cold- and Hot-blast Processes.—With regard to the relative values of "cold-blast" and "hot-blast" iron, it may be well to make a few remarks. At one time there was a widespread belief that iron produced by the cold-blast process possessed a marked superiority over hot-blast iron, but no satisfactory evidence has ever been forthcoming for the substantiation of such a creed. Many ironmasters formerly considered that the colder the blast the better was the quality of the iron, and the greater was the quantity produced per furnace. The only foundation for this opinion was the observation that furnaces gave better results in winter than in summer, and consequently a fictitious importance was given to the lower air temperature prevalent in the former season. As a matter of fact, subsequent investigation demonstrated satisfactorily that hygrometric, and not thermometric, differences were the governing factors. The economic advantages of the hot-blast process are now thoroughly appreciated by English and Scotch manufacturers, and in a less degree by Welsh smelters, because ores produced in Wales are so readily reducible, that the saving possible by the aid of the newer method is comparatively small.

§ (*a*) *Relative Efficiency of Hot and Cold Blast.*—As regards the question of efficiency, Sir William Fairbairn stated some years ago that the quality of iron had been greatly improved since the introduction of the hot blast. This he attributed to the higher temperature attained, which in his opinion tended to volatilise phosphorus, sulphur, and other injurious ingredients which at a lower temperature would combine with the iron. In the course of his investigations, Sir William discovered very little difference between hot and cold-blast iron. It is, of course, quite possible that cold-blast iron of a particular make may be better than samples made elsewhere, but the difference will be due to natural superiority of the ore, and not to the method of smelting.

8. Products resulting from Smelting. — The chemical changes taking place in a blast furnace were briefly described in Article 5, and it will now only be necessary for us to consider the nature of the main and by-products resulting from the operation of smelting. Blast-furnace slag collecting above the iron is run off through the cinder-notch into waggons, and is practically a waste product, though comparatively small quantities are utilised for paving, for cement manufacture, and for the production of slag-wool. This substance, which is largely used for lagging steam boilers and pipes, is similar in appearance and constitution to glass-wool, and is prepared by subjecting a stream of molten slag to the action of a jet of air under pressure.

§ (*a*) *Ordinary Pig-iron.*—Crude iron drawn from the furnace is of variable quality. In the pig-moulds a part of the carbon taken up by the metal separates and rises to the surface in the form of graphite, known to furnacemen as "kish." Grey or graphitic iron is that from which the bulk of the carbon has separated during cooling from the state of solution in which it was held when hot. Mottled iron results when partial separation has taken place; and white or carburetted iron is produced when iron and carbon are in a state of chemical combination. Grey iron is produced by furnaces where silicious ores are treated with a maximum of fuel at a high temperature, whilst white iron results from ores containing little silicon and smelted with a minimum of fuel.

§ (*b*) *Special Pig-iron.*—By intentional variations in working the furnace the character of iron yielded may be considerably modified; by the smelting of silicious ores pig-iron may be made containing a large proportion of silicon; by the reduction of manganiferous ores pig-iron can be produced containing a high percentage of carbon, and from ten to eighty-five per cent. of manganese ; chrome iron ore yields a compound containing from sixty to seventy per cent. of chromium ; and phosphoric pig-iron, with from five to seven per cent. of phosphorus, is made by the smelting of ores rich in phosphorus, or by adding slag containing basic compounds to an ordinary charge.

§ (*c*) *Uses of Different Pig-irons in Manufacture.*—The above-mentioned variations of pig-iron are classified for reference in Table III., where some indication is given of the purposes for which they are employed in iron and steel works.

Table III.—Ordinary and Special Varieties of Pig-iron.

Style.	Proportion of Characterising Element.	Chief Use in Manufacture.
Grey iron ...	Graphitic carbon 3·30 per cent.	Foundry purposes and steel-making.
White iron ...	Combined carbon 3·20 per cent.	Malleable or wrought iron production.
Ferro-silicon ..	Silicon 10 per cent. ...	Conversion of white into grey iron and preventing combination of carbon in cast iron.
Spiegeleisen ...	Manganese 10 per cent.	Special castings and steel making.
Ferro-manganese	Manganese up to 85 per cent.	Special castings and steel making.
Ferro-chromium...	Chromium 60 to 70 per cent.	Steel making.
Phosphoric iron...	Phosphorus 5 or 7 per cent.	,, ,,

So far we have traced the methods adopted for the extraction of iron from the ore, and the resultant pig-iron is found to be a mixture or an alloy of varying composition from which other compounds, known as cast iron, malleable or wrought iron, and steel, are subsequently made.

CHAPTER II.

THE MANUFACTURE OF CAST IRON.

9. **Foundry Practice.**—It is frequently said that the manufacture of cast iron is completed when the metal leaves the smelting works in the form of pig-iron. In one sense this statement is perfectly correct, for the primary object of subsequent operations carried on in the foundry is to melt and mould the metal into such shapes as may render it suitable for practical application. Nevertheless, it is the fact that chemical changes actually take place in the metal as the direct result of foundry work. We, therefore, propose to dwell briefly upon the operations coming within the province of foundry practice. Then we shall see in what respects manufactured cast iron differs from the raw material as delivered by the smelter, and a convenient opportunity will be afforded for the consideration of matters with which the student ought to be familiar, because they have an important bearing upon constructive work. Foundry practice includes metal mixing, furnace work, pattern-making, moulding, casting, and other minor matters.

10. **Metal Mixing.**—Unless under exceptional circumstances, ironfounders never make castings from simply one kind of iron, not merely because judicious blending tends to facilitate their work, but for the additional and more important reason that a mixture of several brands of iron is invariably stronger than the average collective strength, each taken separately. The exact proportions of a mixture vary according to the form of the castings required, to the work for which they are intended, and to the stresses they may have to withstand. The art of the founder is one involving an intimate knowledge of cast iron in its different varieties, and of the ascertained results yielded by their admixture. It is very unlikely that any architect will be

found to possess the qualifications necessary to enable him to prescribe what should be the exact constitution of the metal in the castings which he may require, and he will, therefore, do well to limit his demands to the expression of tests and of general conditions which are to be observed. At the same time it is desirable that the principles of foundry work should be duly appreciated. We have seen (Art. 8, §*a*) that pig-iron is roughly classified by the descriptive terms grey, mottled, and white, but, as a matter of fact, there is no boundary line separating the three varieties. It is true that between typical grey and white irons an important difference exists, for the former contains carbon in a state of mechanical mixture, and the latter exhibits the same substance chemically combined. Grey iron, however, may be said to merge gradually into white iron, and the metal at the intermediate stage is that known as mottled iron. Thus we see that the distinctions in question are purely empirical, and of no real value to the iron-founder. Consequently, eight grades have been established for commercial purposes, each being recognised by a numeral. Numbers 1 to 4 are applied to grey iron, No. 5 to mottled iron, and Nos. 6 to 8 to white iron. Pig-iron is further designated by, and its value estimated according to, the district from which the ore is derived, and, as mentioned in Art. 8, §*b*, there are special varieties of pig-iron, each named in accordance with its characterising element.

§ (*a*) *Metal for Various Purposes.*—No. 1 iron is readily fused, and being very fluid in this state it is suitable for fine ornamental castings, but is deficient in hardness and strength; No. 2, being stronger and harder than No. 1, is well adapted for heavier ornamental castings; No. 3 is tough and strong, producing castings capable of resisting heavy weights and sudden strains; No. 4, known as "foundry pig," is very hard and tenacious, and is most useful for making heavy castings which require little workshop preparation before use. The higher numbers are used almost entirely for the manufacture of malleable iron and steel. Combinations of Nos. 1, 3, and 4 are considered to be very suitable for structural purposes, giving castings which are both clean and strong. So far as geographical distinctions are concerned, Scotch iron is generally and deservedly popular for the production of castings for architectural work; Cleveland and Lincolnshire iron are suitable for large castings, but they are much used for mixing purposes, as also is Cumberland hematite. Welsh iron is of exceedingly good quality, and is frequently employed to increase the strength of other brands. Wrought iron and steel scrap are added with advantage to iron mixtures in proportions depending upon the intended application of the castings.

11. Furnace Work.—Having settled the ingredients to be used, the next process for the founder is to melt the metal. For this operation a furnace is used somewhat resembling a blast furnace (Fig. 4), and known as a cupola. This is built in cylindrical form, having an external shell of cast or wrought iron, lined with fire-clay or fire-brick. As the cupola is not worked continuously, after the manner of a blast furnace, the lining need not be particularly thick. In the case of a furnace working from three to four hours daily, a thickness of 9 in. is ample; if the time of work extends throughout the day, the thickness may be increased up to 12 in. Air is introduced into the furnace by means of tuyères, and the blast is well distributed, for the double purpose of reducing pressure and of securing a large surface contact between the air and the fuel.

§ (*a*) *Effect of Foreign Ingredients.*—To the iron and fuel used in the cupola, limestone is usually added with the object of separating any earthy matters which are present. These combine with the limestone to form slag, which floats on the surface of the fused metal. Ferro-silicon is sometimes introduced with ordinary pig-iron into the furnace, in order to keep the iron grey by hindering the chemical combination of iron and carbon, but the proportion of silicon in the resulting metal should not exceed 2·5 per cent. The effects produced by various proportions of carbon in combination with cast iron are as follows:—

Maximum softness	.. .. ..	0·15 per cent.
,, tensile strength	.. ..	0·47 ,, ,,
,, general ,,	.. ..	0·50 ,, ,,
,, transverse ,,	.. ..	0·7 ,, ,,
,, crushing ,, over	.. ..	1·0 ,, ,,

Manganese should not exceed 1 per cent., or the iron will become white, and weak except as regards crushing strength. Phosphorus in small quantities induces fluidity, and although this condition is desirable for thin castings, the metal has the disadvantage of being brittle.

§ (*b*) *Effect of Repeated Meltings.*—Re-melting cast iron is beneficial, but repetitions of the process more than about ten times cause diminution of tensile and transverse strength, though crushing strength and hardness are augmented. These changes are indicative of altered constitution, carbon being combined, and silicon eliminated. Thus, as the repetition of re-melting takes place, the iron so treated gradually assumes the condition in which it is known as white iron. Bearing these things in mind, we find that furnace practice in the foundry has an important influence upon the character of iron, although some writers say that none is intended.

12. Pattern-making.—For the successful practice of pattern-making, a fair knowledge of practical plane and solid geometry is required. It is also necessary that the pattern-maker should be acquainted with the nature of woods or other materials used, with the characteristics of the metal to be cast, and with the details of operations performed in the foundry. Those who are wise in their generation will, as far as possible, use castings from stock patterns, such as are available in great variety for many classes of work.

§ (*a*) *Importance of Correct Design.*—Patterns cannot be satisfactory unless the draughtsman has a sufficient practical knowledge of the operations conducted in the foundry. Important castings are occasionally known to fail entirely, because of weakness due to imperfect design, and others, again, are often observed whose excessive bulk does not necessarily present any advantages except to the ironfounder. Defects of this nature indicate that the designer has attempted work beyond his powers. If the architect has not studied the details of pattern-making, he will do well to employ some master of the craft, and confine his own efforts to giving general directions. Of course, occasions constantly arise where it is absolutely necessary that the architect should furnish designs for castings, especially in connexion with ornamental work; but in such cases it is advisable that the pattern-maker should be allowed as much latitude as may be possible, so that sentimental and practical art may be equally exemplified. A competent pattern-maker may always be trusted to make due allowance when required for shrinkage of the iron during cooling; he will also see that patterns are "drafted," so as to permit of their ready withdrawal from the mould; but it is not always within his power to avoid the expense which a comparatively insignificant alteration of the design might render possible, nor is he invariably at liberty to modify features whose only effect may be the production of weakness, as the result of unequal molecular strain. It would be difficult to overstate the importance of designing castings so that their lines may be in general harmony with the natural laws of crystallisation brought into operation during the cooling of the metal.

§ (*b*) *Allowance for Shrinkage.*—The contraction rule used by pattern-makers is about one-eighth of an inch per foot longer than the standard measurement, and although this rule may be relied upon in ordinary cases, it is frequently necessary that the pattern-maker should think for himself. If a wood pattern has to be made for a casting intended for an iron pattern, there must of course be an additional allowance for shrinkage. Under ordinary conditions, and where the metal is approximately 1 inch thick, iron castings will shrink about $\frac{1}{10}$ inch, and steel castings about $\frac{1}{4}$ inch per foot. Under similar conditions the shrinkage of castings thicker or thinner than 1 inch will be inversely proportional to thickness. It should always be remembered that the quality of the metal, and the nature of the methods adopted in moulding and in cooling the castings may exercise a considerable influence upon the actual amount of shrinkage (Art. 34).

§ (*c*) *Laws of Crystallisation.*—Without intending to enter fully upon this question we think it very desirable that attention should be directed to the following interesting points. Cast iron is a body which, in the act of cooling, follows the laws of crystallisation, more or less exactly according to attendant circumstances, and it is found that the planes of crystallisation assume a direction identical with that along which heat has passed outward from the cooling metal. In a cylindrical bar the crystals radiate from the centre; in a square bar they are perpendicular to the sides; consequently a kind of mitre-joint exists at each corner forming a line of weakness, and a similar effect occurs in a flat plate having square corners. From these facts the reader will at once draw the inferences, (1) that the forms best calculated to promote strength are those most nearly related to the circle, or, better still, to the sphere; (2) that all sharp angles should be avoided, in order that the metal may be in curves of easy gradation; and (3) that uniformity of thickness should be preserved as far as possible. As we shall afterwards show, the ironfounder sometimes has it within his power to assist the designer with regard to the direction in which crystallisation takes place (Art. 14, § *b*).

13. Moulding.—As practised in the iron foundry moulding includes the formation of matrices into which the molten metal is poured. The moulds can be used only once, as they are broken up for the removal of the casting after cooling. Iron is generally moulded either in sand or in loam. When the former material is employed, a pattern is required; when the latter is used, a pattern is usually dispensed with, the mould being built up and shaped in accordance with templets by hand-tools and mechanical appliances. Green-sand moulding is the method adopted in the case of all ordinary castings, and the term implies the fact that the sand is used in its natural condition, and that the mould is not dried or baked before use. Dry-sand moulding is practised when heavy castings of exceptional solidity are to be produced. The sand is then specially prepared, and the finished mould is dried or baked in a stove. Loam-moulding is specially suitable for the production of such castings as columns, cylinders, pipes, &c., for which patterns may be either costly or unnecessary, or both. Finished moulds can generally be removed to the stove for drying; but when this course proves to be impracticable, they are dried by the aid of fires made beneath them in the moulding shop.

The work of the moulder may be much facilitated by the careful pattern-maker, who makes all parts of the model smooth, so that the moulds may be perfect. His patterns are made so that certain parts requiring exceptionally good metal shall be cast in a position where the greatest pressure exists during pouring, and that impurities shall therefore rise to less important parts. He marks on the pattern the special treatment required for different parts; thus the moulder may know where extra care is necessary

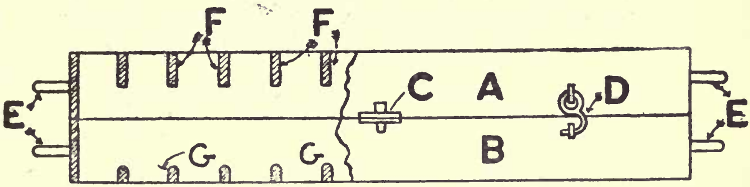


Fig. 6.—Elevation of Moulding Boxes.

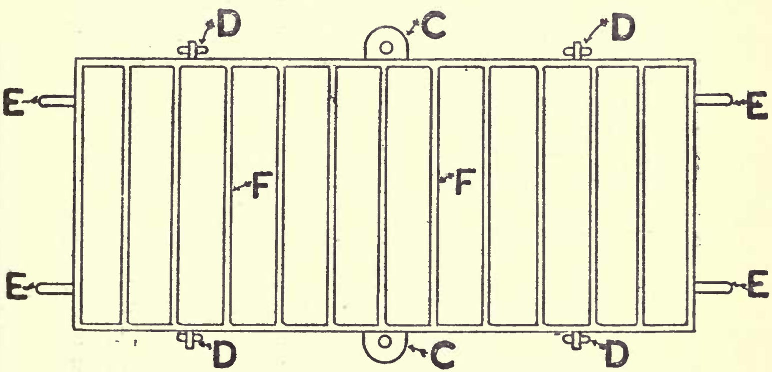


Fig. 6a.—Plan of Moulding Boxes.

in the preparation of the mould, and where his gate or gates may be most suitably placed.

§ (*a*) *Moulding Boxes, or "Flasks."*—In the ordinary way, moulding is accomplished by the aid of rectangular frames, fitting together in pairs or in stacks of three or more, and provided with dowel pins so that they may be separated and re-united in their previously relative positions.

These frames are termed moulding-boxes, or "flasks," being made of cast iron with lugs by means of which they may be readily lifted. Small boxes are entirely open both at top and bottom; but in the case of large boxes, cross bars are formed in such a manner as to aid the retention of sand. Figs. 6 and *6a* illustrate a pair of moulding-boxes where (A) is the top flask or "cope," (B) is the bottom flask or "drag," (CC) are dowel pins, (DD) hooks and eyes for holding the flasks together, (EE) are lugs, (FF) are cross-bars on the cope, and (GG) are cross-bars on the drag.

§ (*b*) *Method of Moulding in Flasks.*—With properly made patterns of good design, moulding is of course easy and reasonably inexpensive. When an object of fairly symmetrical shape has to be moulded, a cope is laid bottom part uppermost and filled with sand, which is rammed tightly and struck off level. A portion of the sand is then scooped out so that the pattern may be embedded, approximately, as far as the centre line; some sand, free from clay and described in the shop as "parting-sand," is sprinkled over the surface for preventing adhesion between the two halves of the mould, and the drag is laid in place. "Facing-sand" is then sifted over the projecting portion of the pattern until it is completely covered, and the box is filled from the top with old sand, which is well rammed in and struck off level. Provision is made for the escape of gas through vents pierced in the sand by a piece of steel wire. The two flasks are inverted upon a level surface, and the cope, in which the pattern was first embedded, is taken off and emptied. A parting-line is then established by the addition or removal of damp facing-sand. This line may be either straight or otherwise, according to circumstances, but it must always be formed so that the embedded and exposed portions of the pattern slope away from it towards its vertical axis. From the parting-line the sand extends as a plane or sloping surface to the edge of the flask; over this surface parting-sand is sprinkled, the cope is replaced, and after a "runner-stick" has been placed in a suitable position, facing-sand is introduced and the box is filled and rammed as before described. Vents are then made, and the removal of the runner-stick leaves an opening known as the "gate," through which metal is poured into the mould after the pattern has been removed.

§ (*c*) *Cores.*—When hollow castings are required it becomes necessary to use cores, and if these are properly supported in position, the castings should be of the anticipated thickness at all parts. Unfortunately, like other things animate and inanimate, cores do not always behave exactly as may be expected of them, and are liable to become misplaced. This eventuality is one for which the architect or engineer must be prepared, and as the chief weight of a building often rests upon cast-iron columns, it is very important that such members should be subjected to careful examination before use (Arts. 66 and 79, § *a*).

14. Casting.—The work of the founder depends largely upon both the pattern-maker and the moulder. Considered separately, the intrinsic strength of the metal depends upon the suitable combination of various qualities of iron, and upon the work done in the cupola. The strength of the casting is governed chiefly by intrinsic strength, partly by the suitability of the pouring, feeding, and other gates, and partly also by the judgment and skill evinced by the founder with regard to pouring and cooling. From a purely constructive point of view, the strength of the casting depends, of course, entirely upon the designer, although it is quite possible for the intention of a good design to be frustrated by indifferent metal and workmanship; or, on the other hand, for a bad design to triumph over the best material and the most careful manipulation.

§ (*a*) *Effect of Temperature when Pouring.*—It is the duty of the founder to provide metal possessing the requisite strength and the quality of flowing readily into all parts of the mould; he must also exercise judgment as to the proper temperature at which the metal should be poured. Pouring at too high a temperature tends to reduce the closeness of the grain and the specific gravity of the metal, as well as to exaggerate unavoidable lines of weakness. On the other hand, if poured at too low a temperature the metal is apt to be "cold-short," which means that continuity of the casting is interrupted by seams or imperfect joints, resulting from rapid congelation in the mould.

§ (*b*) *Modification of Weakness due to Crystallisation.*— The founder should do all that is in his power to modify the formation of lines of weakness due to crystallisation (see Art. 12, § *c*) by accelerating the cooling of some portions of a casting, and by retarding that of others. This procedure is not always possible, but by the exercise of discrimination injurious action may often be minimised.

For the purpose of making this matter clear, sketches are given in fig. 7 of four typical castings. In the case of A,

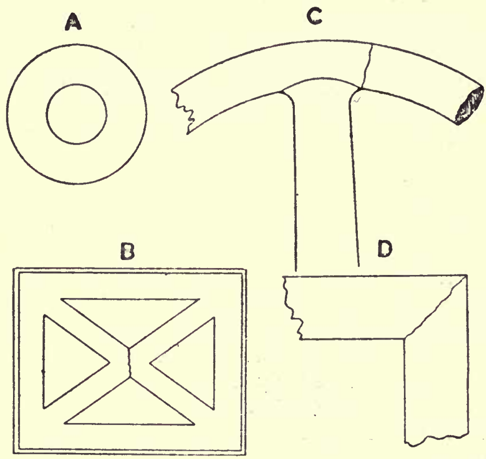


Fig. 7.

representing any thick cylinder, the outer portions of the metal are naturally the first to cool; and, owing to the subsequent cooling of the inner parts, there will be unequal strain, the outer parts being under compression and the inner under tension. If, however, arrangements can be made for retarding the cooling of the outer layers of metal, and simultaneously for accelerating that of the inner layers, the cylinder will be considerably strengthened.

Taking a casting such as B, we find the diagonals tend to shrink less rapidly than the outer rigid frame, and consequently fracture is likely to occur at the centre.

A different effect occurs during the cooling of a wheel casting, C. If the arms cool first they will act as rigid struts resisting contraction of the rim, which may be ruptured on the line indicated. On the other hand, if the wheel has a thin rim cooling more quickly than the arms, the latter may be broken by tension.

The sketch D shows the line of weakness in a flat casting with square corners, resulting from crystallisation in directions perpendicular to the surfaces. In such a case the founder can do very little in aid of a badly-designed casting.

§ (*c*) *Chilled Castings.*—By the use of special moulds designed to convey heat very rapidly from the molten metal, castings are produced in which the surface is of extreme hardness. The outer portions of the castings assume a state similar to that existing in white iron, whilst their interior parts remain in the form of grey iron. Chilled castings made in this way have great resistance to crushing stress. Although such a process is not intentionally applied to castings used in building construction, yet a similar effect, though in a very modified degree, is caused by the ordinary methods of foundry practice. All castings have a skin, which is really a chilled surface, and this adds considerably to their strength, besides offering a better resistance to oxidising influences.

§ (*d*) *Malleable Cast Iron.*—Another useful and convenient modification of cast iron is obtained by heating castings of specially mixed metal to a red heat, in contact with powdered red hematite ore, for a period of four or five days, at the expiration of which they are allowed to cool very gradually. During the process the quantity of carbon in the cast iron is diminished, owing to combination with oxygen contained in the hematite, and the result is the formation of a kind of steel, or semi-steel, which is to some extent malleable, and may be bent or hammered without injury.

§ (*e*) *Approximate Composition of Cast Iron.*—Cast iron is of so variable a nature, that its composition can only be stated in a very casual manner. For the purpose of comparing cast iron with other composite substances, such as malleable iron, steel, and so-called steel, the following table may, however, be found of service :—

Table IV.—Approximate Composition of Cast Iron.

Carbon, combined . .	0·15 to 1·25 per cent.
„ uncombined . .	1·85 „ 3·25 „
Silicon	·15 „ 5·0 „
Sulphur	·0 „ ·5 „
Phosphorus	·0 „ 1·3 „
Manganese	·0 „ 1·5 „
Iron	90·0 „ 95·0 „

CHAPTER III.

THE MANUFACTURE OF WROUGHT IRON AND STEEL.

15. Manufacture of Wrought Iron. — Malleable iron, or wrought iron, as it is more generally called, exemplifies a nearer approach of the metal to a chemically pure condition than is evidenced by any of the substances spoken of as iron or steel. In its more perfect commercial forms wrought iron contains not less than 99 per cent. of pure iron, and its ductility depends directly upon the degree of purity attained.

Wrought iron is made either by the elimination of carbon and various foreign ingredients from pig iron, or by the direct reduction of iron ore. All processes of the first class involve the selective oxidation of impurities, although variations of their details are particularly marked. Up to 1784 the conversion of cast into wrought iron was performed by charcoal finery, of which the best known form was the Walloon process. Although superseded long ago by the introduction of "puddling," a few isolated examples of charcoal hearths survived until a comparatively recent date. The characteristics of iron produced by charcoal finery were fineness of grain, combined with remarkable strength and ductility, but the small quantity of metal treated at one time, and the relatively large area of the hearth, made the cost of production unnecessarily high.

16. Puddling.—"Dry-puddling," as formerly practised, involved two distinct operations—(*a*) treatment in the refinery hearth, where the percentage of carbon and silicon was reduced; and (*b*) puddling, by means of which the metal, being gradually freed from impurities, "came to nature," and was raked together into a pasty mass, termed a "ball." Dry-puddling has now been almost entirely superseded by the more modern process, which, in the picturesque language of the furnace-man, is described as "pig-boiling," the term being suggested by the effervescence of carbon monoxide at a certain stage of the operation.

§(*a*) *Pig-boiling.*—In puddling proper, or pig-boiling, no use is made of the refinery hearth. The crude metal is placed directly in puddling-forges, which are constructed in varied forms, but all of them represent modifications of the original appliance invented by Cort in 1784. Fig. 8 is a sectional dia-

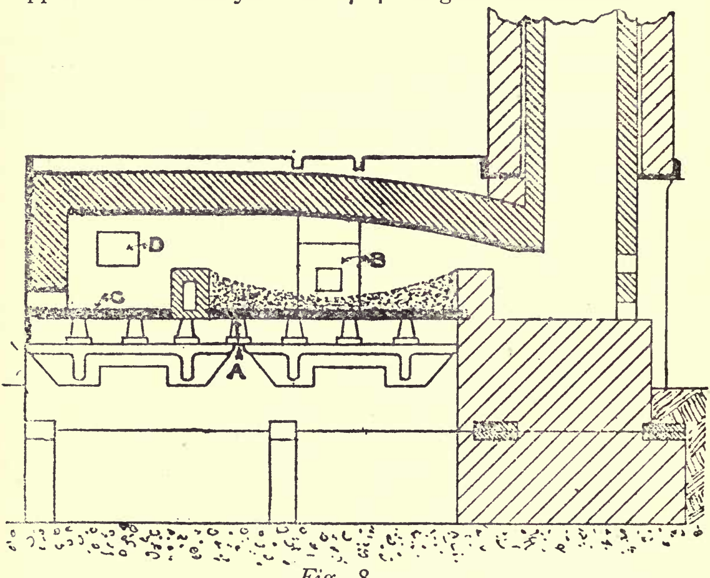


Fig. 8.

gram, by the aid of which we may follow the general nature of the operations conducted in pig-boiling. The bed (A) is of cast iron, kept cool by circulation of air at the under surface; on the top a "fettling" of slag, covered with a thin layer of hematite ore, forms a hollowed surface, upon which the pig iron is placed by means of the working door (B). A fire in the compartment (C) is charged through the stoking door (D), and provides gaseous fuel, which passes through the converting chamber to the chimney shaft. When the metal is fluid it is stirred or "rabbled" by implements put in at the working door (B), so that the hematite may be incorporated. Slag containing iron oxide is added as a flux for the silica resulting from oxidation of silicon in the pig iron. Lime is sometimes introduced with a similar object, and chloride of sodium is also used for removing sulphur and phosphorus. As the iron becomes purer, the temperature is raised proportionately with increase of the melting-point, and the charge is "rabbled" towards the outer parts of the bed. After further raking, the metal is rolled into puddle-balls, which are taken from the furnace at a welding heat. Mechanical rabbles and revolving cylindrical furnaces have been devised to dispense with hand labour, but such appliances are not entirely satisfactory.

17. Production of Mercantile Forms of Iron.—When removed from the furnace a puddle-ball is permeated with slag, the bulk of which is extracted by "shingling" under a hammer, or in a squeezing-machine. In this way a compact "bloom" of wrought iron is produced, which is passed through the roughing-rolls of a rolling-mill, and then through the finishing-rolls, whence it emerges as a "puddle-bar," and such bars are cut into lengths by powerful shears. Their transformation into various forms of iron used in constructional work is effected by placing a number of the bars one on top of another to make a "pile," which is reheated in a reverberatory furnace, and afterwards rolled into bars, plates, and rails of any desired section by rolls with suitably-shaped grooves.

Much depends upon the manner in which piling is performed in the rolling-mill. As far as possible the pieces should be of uniform thickness, so that the heat may be equally diffused, and that pressure may be transmitted through the whole mass of metal. Many different modes of piling are adopted, according to the form of section required and to the preference of the manufacturer. Sketches of some of these are given in fig. 9. Want of care in rolling seriously affects the quality of joists; the webs and flanges may be buckled or otherwise damaged by unequal working, and the flanges are sometimes defective owing to the imperfect filling of spaces between the rolls.

We have now traced the evolution of puddled wrought iron from pig iron to finished forms of the material, and it will be observed that four distinct stages are successively reached :—

1. Production of the "ball," being wrought iron permeated with fused slag.

2. Squeezing or hammering the "ball" into a "bloom."

3. Rolling the "bloom" into "puddle-bars."

4. Rolling "puddle-bars" into mercantile forms of iron.

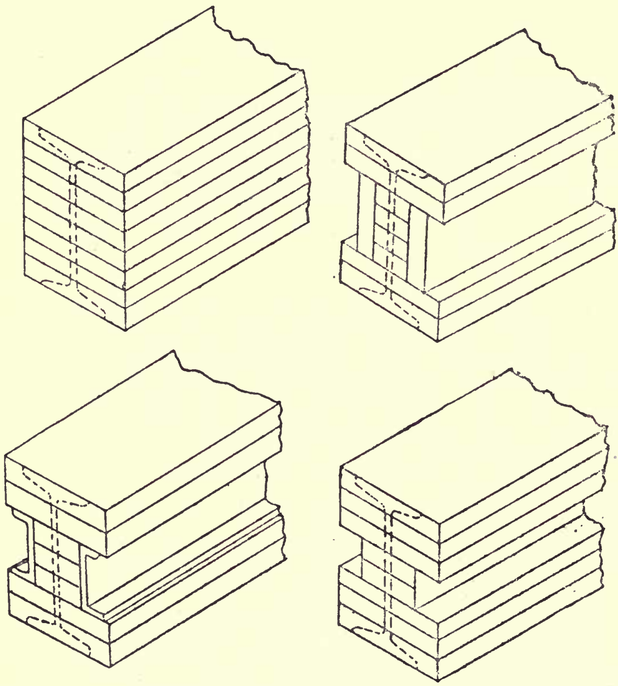


Fig. 9.—Modes of Piling.

18. Other Modes of Manufacture.—Other methods are available for the manufacture of wrought iron, which can be sufficiently explained by the selection of typical examples.

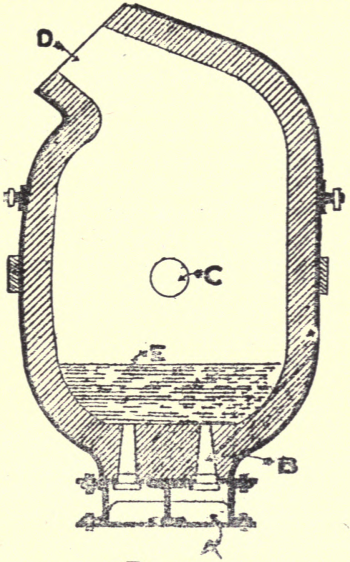


Fig 10.

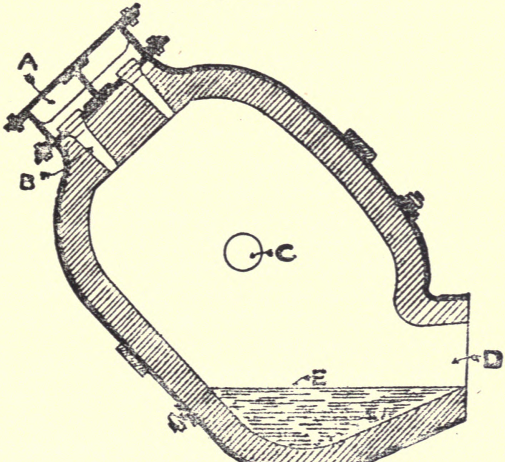


Fig. 10*a*.

§ (*a*) *Bessemer Process.*—In the Bessemer process, liquid metal from one or more blast furnaces is transferred to a suitable vessel or "converter," shown in two positions in figs. 10 and 10*a*. The vessel is formed of steel plates lined with refractory material and mounted on trunnions (C). It is made red-hot, and the fuel is turned out before the charge is run in. The converter is turned down, as in fig. 10*a*, so that the fireclay tuyères (B) will not be filled with iron, and the charge (E) is then inserted at the mouth (D). When the vessel has been turned up, as in fig. 10, air under pressure is blown through the melted iron from the tuyères, which are fed from the tuyère-boxes (A). Oxygen in the air combines with the carbon and silicon of the pig iron, and incidentally produces heat enough to maintain the metal in a state of fusion until it is reduced to the malleable condition, when it is run into "ingots" of homogeneous constitution, which are rolled into mercantile forms. As originally worked, the Bessemer process was only applicable with success to pig irons free from sulphur and phosphorus, hence the term "Bessemer iron" has been used to indicate crude metal possessing this qualification.

§ (*b*) *Bloomery Process.*—Wrought iron made directly from the ore by one form or another of the "Catalan forge" is obtained in blooms, hence establishments where the process is practised are called "bloomeries." The method is still used in America and elsewhere, and is only suitable for ores naturally rich and pure.

§ (*c*) *Siemens Process.*—The Siemens "precipitation" process is one by which wrought iron can be made directly from the ore at one operation, and its essential features are (1) the reduction of ore in a regenerative furnace; and (2) the separation of wrought iron from the crude metal whilst still in a state of fusion. Purified iron appears in the form of a pasty ball, surrounded by fluid slag, and the ball is subjected to treatment similar to that followed in the case of balls produced by puddling.

19. Types of Wrought Iron.—Two distinct classes of wrought iron are produced by the processes which have been mentioned.

§ (*a*) *Fibrous Wrought Iron.*—Methods of purification not causing complete fusion yield metal which, after squeezing and rolling, consists of parallel fibres and laminæ, more or less permeated with slag. Such iron is fibrous and non-homogeneous in structure, and the strength of its manufactured forms depends very much upon the extent to which the peculiarities of the metal are recognised by the ironworker.

§ (*b*) *Homogeneous Wrought Iron.*—Methods of purification involving absolute fusion produce homogeneous ingots of uniform quality, into the composition of which slag enters to a very limited degree. Metal of this kind is sometimes described as ingot iron.

20. Manufacture of Steel.—Steel is the third and last compound of iron for consideration, and the term is applied to metals differing so widely one from another in strength and other qualities, that some confusion exists as to what may really be meant by the word "steel." Until recent times steel, made by cementation from wrought iron, was a substance having well-defined characteristics, but since the development of modern processes a new material has made its appearance, which, although called steel, does not possess the distinctive property of tempering, and the proportion of contained carbon ranges from a percentage equal to that in cementation steel down to the percentage existing in ordinary wrought iron. It is impossible, in fact, to determine where wrought iron ends or where steel begins.

§ (*a*) *Parallel Series of Irons and Steels.*—In Germany it is becoming usual to describe iron and steel produced by puddling as weld metal, and to distinguish the product of processes where fusion takes place as ingot metal. Thus two parallel series exist which were first clearly defined by M. Greiner, of Seraing.

These two series are shown in the subjoined table :—

Table V.—Parallel Series of Irons and Steels.

Carbon (percentage).	Iron Series.	Steel Series.
0·0 to 0·15	Ordinary iron.	Extra soft steel.
0·15 ,, 0·45	Granular iron.	Soft steel.
0·45 ,, 0·55	Puddled steel.	Medium steel.
0·55 ,, 1·50	Cemented steel.	Hard steel.

The above classification is not altogether satisfactory, because it seems to mix up irons and steels, and also because it does not recognise the property of tempering as a point of difference between the two metals. For architectural purposes, tempered steel is only incidentally used, and we have chiefly to do with "mild steel," a material which has practically superseded wrought iron so far as structural work is concerned.

§ (*b*) *Varieties of Steel.—Blister, or cemented, steel* is made by imbedding bars of wrought iron in charcoal, and heating them at high temperature for some days. Carbon is absorbed and the surface of each bar is converted into steel, but the interior part is not fully transformed. The term "blister" as applied to steel is descriptive of the appearance of the surface of the metal.

Shear, or tool, steel is made by heating piles of blister steel bars and rolling them at a welding heat, so that uniformity of structure and constitution may be secured. This quality is chiefly used in the manufacture of cutting tools.

Cast, or crucible, steel is made by melting blister steel with a relatively small quantity of carbon. Other grades of cast steel, sometimes called "homogeneous metal," are made by melting wrought iron with carbon.

Puddled steel is made in the puddling forge by arresting the process when sufficient carbon has been removed to secure the conversion of pig iron into steel, and before the metal reaches the condition of wrought iron. The ball of steel taken from the furnace is finished as described in Art. 17.

Bessemer steel is made by restoring carbon to Bessemer wrought iron (Art., 18 § *a*) by the addition of either spiegeleisen or ferro-manganese. If the Bessemer converter be lined with "ganister," a material composed largely of free silica, and therefore capable of acting as an acid at high temperatures, only the purest qualities of pig iron may be used. By the employment of a basic lining of dolomite, the Bessemer process, as modified by Thomas and Gilchrist, is rendered suitable for the treatment of phosphoric pig iron. Steel produced is run into ingots, which are hammered and rolled like blooms of puddled iron (Art. 17). The success of the Bessemer process was largely due to Mushet, whose patent rights lapsed through neglect in the third year, and whose name is now either unknown or nearly forgotten.

Siemens and Siemens-Martin steels are produced by the open-hearth processes, and the hearth may be *acid* or *basic*, as preferred. In the Siemens process, pig iron and ore are used, whilst in the Siemens-Martin process a mixture of pig and scrap iron is employed. In either case the raw material is first reduced to the malleable state, and the proportion of carbon is regulated by the addition of spiegeleisen.

Varieties of special steel are made for engineering requirements, and such kinds include in their composition manganese, nickel, aluminium, tungsten, chromium, or any metal which may be chosen.

Compressed steel is made by the application of pressure to the fluid metal, and the resulting steel is of superior density and tenacity, besides being free from blow-holes.

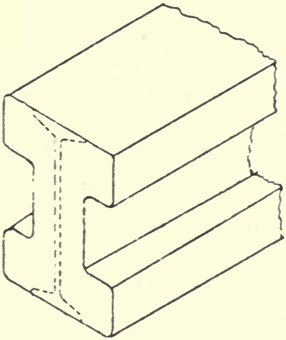


Fig. 11.

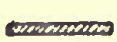
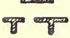
§ (*c*) *Rolling.*—Steel is formed into bars, rails, and beams in the rolling mill, but at a lower temperature than that necessary for puddled iron. Girders are rolled from rectangular "billets," but for sections of more than 12 in. in depth the billets are shaped in the roughing rolls to the form shown in fig. 11.

The operation of rolling necessitates from twelve to eighteen passes through the rolls according to the size of the girder. Owing to this prolonged treatment and to the comparatively low temperature observed, the stiffness and strength of the material are considerably increased (Art. 35, § *a* and *b*).

Cold-rolled steel is shaped roughly when hot, and finished after cooling. In this way the metal is rendered harder and more elastic, but plasticity is diminished (Art. 35, § *a*).

21. Unit Forms of Wrought Iron and Steel.—Wrought iron and steel, which can only be moulded to definite shapes on a large scale by powerful machinery, is necessarily produced in a variety of unit forms or sections, from which the structural engineer can select such as may be most appropriately used either in their existing state or after suitable manipulation. Ordinary iron and steel plates, sheets and bars are obtainable of any dimensions which are likely to be demanded by the architect, and there are numerous other sections more particularly intended for structural work. Some of the latter are illustrated and described in the following table :—

Table VI.—Standard Sections of Rolled Iron and Steel.

Section and Style.		Transverse Dimensions.		Length.	
		Minimum.	Maximum.	Ordy.	Max.
	Round bars	¾″ dia.	8″ dia.	24 ft.	60 ft.
	Square ,,	¾″ sq.	4″ sq.	24 ft.	60 ft.
	Flat ,,	1½″×¼″	18″×1″	40 ft.	60 ft.
	Half-rd. bars	1½″×⅜″ thick.	3½″×½″ thick	—	—
	,, hollow	1½″×¾″			
	,, solid	2″×½″	3½″×1¾″	—	—
	Pillar section	3″ radius ½″ thick	6″ radius ¾″ thick	—	60 ft.
	Equal angles	1″×3/32″	8″×½″	50 ft.	60 ft.
	Unequal ,,	2″×1½″×3/16″	7″×4″×⅝″	50 ft.	—
	Tees (Iron)	1″×1″	—	50 ft.	60 ft.
	,, (Steel)	1½″×1½″×¼″	6″×3″×½″	50 ft.	—
	Bulb-tees	4½″×6″×9/20″	6½″×12″×14/20″	—	—
	Bulb-bars	6″×6/20″	12″×14/20″ 7″×1⅛-1½″	—	—
	Bulb-angles	3¾″×2½″×8/20″	9″×3½″×10/20″	—	—
	Hatchway sections	2½″×1⅞″ (over all).	6″×2 11/16″ (over all).	—	—
	Zed-angles	4″×3″×3″×⅜″ web.	8″×3½″×3½″×⅝″ web.	—	—
	I-beams	3″×1¼″×⅛″ web.	24″×8″×⅝″ web.	36 ft.	50 ft.
	Channels	3½″×1½″×7/16″ web.	20″×7½″×⅝″ web.	36 ft.	50 ft.
	Trough decking	12″×4″×¼″	36″×17¼″×1″	—	—

CHAPTER IV.

EXTERNAL FORCES, STRESSES AND STRAINS.

22. External Forces.—The nature of the forces to which iron and steel are commonly subjected, the relations existing between external forces and internal molecular forces, and the effects produced by the application of external forces are matters forming part of that very comprehensive subject, the Strength of Materials, as to which much is written, but of which comparatively little is known. It is not our intention to enter upon the region of molecular physics, nor to discuss the strength of materials, except to the limited degree which may be requisite for a clear and comparative statement of strength of the various kinds of iron and steel used in structural work. A load is said to be the force, or combination of forces, acting in any direction or directions upon any part or the whole of a body or structure.

§ (*a*) *Definitions of Various Loads.—External and Internal Loads.*—Loads may be described as internal when the force exerted arises from the weight of the parts themselves; and as external when they are due to the influences of other bodies. Strictly speaking, all loads are due to external forces, although they may in some cases be first evidenced within the boundaries of a body. Although the force of gravity itself is actually an external one, its direct manifestation may be inside a given body, and we may therefore conveniently say that it occasions an internal load. Looking now at another aspect of the case, we find the weight of one body acting as a whole and through its centre of gravity upon another body below, exerts pressure which is entirely external in its nature, and is additional to any action produced by gravity upon internal particles of the body itself. Therefore it is appropriate to speak of the pressure as constituting an external load. Whilst internal load is a quantity directly governed by the specific gravity of the material, external load is subject to variation from case to case, or to fluctuation in any given case.

Permanent or Dead Load.—A structure of iron or steel frequently has to support its own weight as well as the weight of walls, roof, floors, and heavy fixtures. In any individual structure, however, such weights are permanent, and their sum forms the permanent or dead load.

Moving or Live Load.—Pressure—resulting from human beings, animals, merchandise, vehicles, and other weighty bodies—which is either intermittent or changeable as to position, constitutes moving or live load.

Total load includes weight of every kind which may be imposed upon a body or structure.

Breaking load is coincident with the ultimate or breaking strength of a body, and is the load causing fracture in some specified manner.

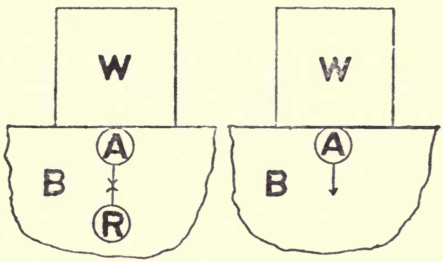
Proof load is the greatest load applicable without risk of injury to the material, and is equivalent to the proof strength of the material. In actual tests a lower standard is always adopted, otherwise damage might unintentionally be caused.

Working load is the load occurring in actual practice, and coincides with working strength.

23. Definitions of Stresses and Strains.—It is next desirable to look at the nature of the effects produced by loads upon solid bodies. At the outset we are confronted by some difference of opinion amongst writers of standard text-books as to the proper signification of the terms *stress* and *strain*. Not so many years ago the word strain was understood to be expressive of force tending to effect some change in the form of a body or to break it asunder. A more elaborate system of terminology has since been evolved in which strain bears a different signification.

§ (*a*) *Rankine's and Thomson's Theories Compared.*—Rankine first introduced the term stress into mechanics about the year 1855, and he gave the following explanation of its meaning and of the relations existing between strain and stress:—"The word *strain* will be used to denote the change of volume and figure constituting the deviation of a molecule of a solid from that condition which it preserves when free from the action of external forces; and the word *stress* will be used to denote the force, or combination of forces, which such a molecule exerts in tending to recover its free condition, and which, for a state of equilibrium, is equal and opposite to the combination of the external forces applied to it."

Unfortunately, it happened in the following year that Sir William Thomson, now Lord Kelvin, applied the word *stress* as being synonymous with *pressure*, and his definition is thus enunciated:—"It will be seen that I have applied



Rankine's Theory. *Fig.* 12. *Thomson's Theory.*

the word *stress* to the direct action experienced by a body from the matter round it, and not to the elastic reaction of the body equal and opposite to that action." With the object of making matters perfectly plain, the two rival theories are diagrammatically represented in fig. 12, where, in the first sketch, W=load or combination of external forces, B=the body upon which force is exercised, A=direct action produced by W, and R=reaction equal and opposite to A; and in the second sketch, W=load, B=the body on which force is exercised, and A=direct action due to W.

Rankine's theory is to be preferred as presenting a truer picture of the interactive forces involved, but the other is more convenient and equally as serviceable for all practical purposes. Some writers adopt one theory, and some the other; but it is generally the case that each class speaks of a body as being "subject to stress" when under the influence of a load, and of strain as a change of form coincident with stress. Therefore the difference is seen to be purely theoretical.

§ (*b*) *Simple and Compound Stresses.*—There are three forms of simple stress :—

1. Tensile stress, tending to pull asunder.

2. Compressive stress, tending to push together or crush.

3. Shearing stress, tending to cut through.

Besides these, compound stresses are frequently found, the most usual of which are :—

1. Transverse or bending stress, composed of tensile and compressive stresses.

2. Torsional or twisting stress, which, in a short bar, is substantially the same as shearing stress; and, in a long bar, is composed of shearing and tensile stresses.

§ (*c*) *Elastic Strain and Permanent Set.*—In the modern acceptation of the term, strain signifies change in the form of a body; according to Rankine, strain precedes stress, but according to Thomson, it is the direct product of stress. In either case the practical result is the same, namely, that force acting upon a body induces change of form, however minute the change may be. When stress is applied to a solid body gradually and up to a certain limit, strain is approximately proportional to stress intensity; and this relationship, enunciated in 1676 by Robert Hooke, is known as Hooke's Law. Strain which disappears on removal of stress is called *elastic strain*. On the other hand, strain which remains after stress has been removed is known as *permanent set*.

24. Properties of Iron and Steel under Stress.—The mechanical properties exhibited by bodies under stress constitute a subject for study which is as interesting as it is important. Our present object is merely to define and to mention briefly the chief physical attributes of iron and steel, with the view of preparing the way for the consideration of details given hereafter.

§ (*a*) *Elastic Limit.*—There is some point up to which a body is elastic, and beyond which it is more or less plastic. This point varies according to the nature of the body, and marks the *elastic limit*, or, as it is alternatively called, the *limit of elasticity*. Corresponding with the three forms of simple stress, there are three elastic limits. So far as iron and steel are concerned, the limits for tension and compression are generally taken as being of equal value, although they are not precisely identical under any circumstances; and the third, for shear, is about two-fifths of either of the others. Some materials when tested by any form of stress may give one value in one direction, and a different value in another direction.

§ (*b*) *Isotropy.*—A substance which is characterised as *isotropic* is one having the same properties in all directions. The processes of iron and steel manufacture are conducted at temperatures at which the material is almost perfectly plastic, and there are no very great differences in their elastic properties in different directions. These materials are therefore treated as if they were *isotropic*, although the condition of isotropy is only approximate, being less nearly attained in rolled iron than in rolled steel.

§ (*c*) *Yield Point.*—Substances such as glass are almost perfectly elastic up to the breaking point, exhibiting no appreciable *permanent set;* others, like cast iron, have little or no elastic limit, taking a small set with comparatively light loads, and the set increases proportionately with the loads up to the breaking point. Others again, as for instance wrought iron and steel, show deformation in a constant ratio per ton per square inch up to the elastic limit, but shortly after this is passed, deformation is suddenly augmented without increase of the load, and the deformation constitutes permanent set. The point just above the limit of elasticity at which this change takes place is termed the *yield point*. It will be at once evident that this point indicates a limit beyond which stress should never be imposed on any part of a structure, whether in test or in actual use, notwithstanding the fact that actual fracture could only be caused by much greater stress. Iron or steel strained beyond the yield point is practically ruined, although not necessarily broken. If, therefore, the working stresses be kept well within the elastic limit, safety will be assured.

§ (*d*) *Resilience.*—Work done in stretching or compressing a body up to the elastic limit is denoted by the term resilience. Energy thus absorbed may be restored when the body resumes its normal form on the removal of stress.

§ (*e*) *Fatigue.*—Metals are subject to the change of condition, or disorder, known as *fatigue*, and, as usually applied, this term indicates the effect produced either by repeated changes in stress, or by the cumulative effect of blows. Experiments extending over many years have been made by Fairburn, Hodgkinson, Wöhler, Bauschinger, Baker, and others as to the effect of repetitions of stress, and the results so obtained are remarkably consistent. From them it appears that the number of repetitions possible before fracture bears a definite relation to the range of stress variation, and the number is reduced if stress alternates from tension to compression, or *vice versâ*. When such changes of stress are caused beyond the elastic limit, a bar of wrought iron or steel may be broken by a comparatively small number of repetitions, but if the stresses be kept well within the elastic limit, there may be no appreciable deterioration. So far as structures are concerned carrying only dead loads, there is no reason to believe that injury of any kind will occur with the lapse of time. No doubt there is some limiting point below which fatigue will not be experienced, and above which it will be produced; but as no such point has hitherto been differentiated, experience and judgment have largely to be relied upon for ensuring absolute safety. Turning now to the cumulative effect of blows or sudden shocks, we find that the energy exerted by a blow is frequently greater than the resiliency of the metal; local hardening may consequently occur in some particular part of the material, which will gradually lose its power of resisting the repetition of such shocks. Impact tests have not received the amount of attention which their importance deserves, but it may be remarked that in the United States a new form of impact testing machine, breaking specimens of metal at one blow, has recently been perfected, and some of the results obtained thereby seem to indicate that the measurement of shock resistance is likely to furnish a new factor in judging the reliability of iron and steel.

§ (*f*) *Modification of Elastic Limit and Yield Point.*—By adding a load to iron or steel, gradually and between intervals of rest, a much greater weight can be borne without rupture than if the load were continuously applied. After overstraining, the effect of rest is to restore elasticity, concurrently with the establishment of a higher *elastic limit* and a higher *yield point;* but the new conditions only apply to the kind of stress produced, and both the elastic limit and yield point may be lower for other forms of stress. Conversely, the elastic limit may be lowered by loading and straining in other directions. Again, hardening caused by overstrain of any kind may be removed, and the original properties of the metal be restored by annealing. These characteristics are not sufficiently considered by some professional men, but it is very desirable that they should be studied by every one who desires to be master of his profession. In ordinary practice the use of a factor of safety is supposed to render impossible the existence of stress intensity sufficient to produce any material alteration of normal condition. Whether this supposition is always justified must depend upon the knowledge and judgment exhibited by the person selecting any particular factor.

25. Measurement of Stresses and Strains.—Stress, whether resulting from the application of external load, or from an internal molecular condition, is expressed in units of weight per unit of sectional area of the body exposed to stress. Co-efficients of strength are quantities indicating the stress intensity causing the rupture of a body, and there are as many co-efficients as there are kinds of stress. Moreover, co-efficients of the same order exhibit variations in the case of non-isotropic bodies, according to the direction in which stress is exerted.

§ (*a*) *Units of Stress Measurement.*—Stress intensity and co-efficients of strength are expressed in units of pounds or tons avoirdupois per square inch of sectional area. Thus, if a tensile stress intensity of 30 tons per square inch breaks asunder a bar of steel which, at the commencement of the test, had a sectional area of 1 square inch, we say 30 tons per square inch is the co-efficient of strength. It should be noticed, however, that as the metal elongates under the influence of tensile stress, its sectional area is much less when the bar finally gives way, and the stress intensity per square inch must consequently be more than 30 tons. Therefore the co-efficient of strength, being based upon the original sectional area, is more or less fictitious as to value. There are, however, practical reasons which make it desirable that calculations should be based upon the original instead of upon the ultimate dimensions.

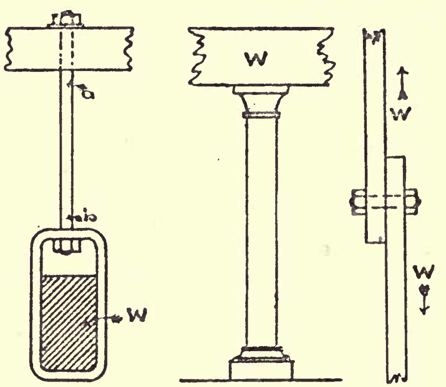


Fig. 13. *Fig.* 14. *Fig.* 15.

§ (b) *Examples of Stress Calculations.*

Example 1. Tensile Stress.

If, as in fig. 13, an iron bolt (ab) of (x) square inches sectional area be suspended from the end (a), and a load of (W) tons be hung from the end (b), then the bar will be under a tensile stress of (W) tons throughout its entire length.

The unit stress will be $(W \div x)$ tons, and if $x = 0{\cdot}8$ square inch and $W = 3$ tons, the stress intensity will be $(3 \div 0{\cdot}8) = 3{\cdot}75$ tons per square inch.

Example 2. Compressive Stress.

If, as in fig. 14, a column of (x) square inches sectional area rest upon a solid foundation, and a weight of (W) tons be placed on the top, a compressive stress of (W) tons will be established.

The unit stress at any given transverse section will be $(W \div x)$ tons, and if $x = 30$ square inches and $W = 3$ tons, the stress intensity will be $(3 \div 30) = 0{\cdot}1$ ton per square inch.

Example 3.—Shearing Stress.

If, as in fig. 15, two bars of iron be connected by an iron bolt of (x) square inch sectional area, and if equal forces of (W) tons act upon the bars in opposite directions, the bolt will be subjected to a shearing stress of (W) tons.

The unit stress will be $(W \div x)$ tons, and if $x = 0{\cdot}44$ sq. inch and $W = 3$ tons, the stress intensity will be $(3 \div 0{\cdot}44)$ $= 6{\cdot}8$ tons per square inch.

From the above it may readily be deduced that if the co-efficient of strength of any metal for any kind of stress be known, it will be easy to calculate the total load necessary to cause rupture by direct stress of any given bar or piece of material.

Thus, if $F =$ co-efficient of strength, $a =$ sectional area, and $W =$ total load,

Then

$$W = F \times a \quad \ldots\ldots\ldots\ldots\ldots (1).$$

Consequently, we see the necessity for data as to the strength of materials, such as those contained in Chapter V.

§ (*c*) *Units of Strain Measurement.*—Strains of the kind experienced in structural work are expressed in terms of lineal measure, or of percentage. For instance, a unit of tensile strain may be indicated in parts of an inch per lineal inch of the body under strain, or by the percentage which actual elongation bears to the original length of the body measured along the axis of the strain.

Strain within the elastic limit, being approximately proportional to stress, can be calculated. Beyond the elastic limit, deformation increases to a marked degree, and measurement of the change taking place affords a valuable index to the suitability of the material for any particular purpose. For instance, in a structure subject to shocks caused by moving loads, ultimate strength is not the only desideratum. There is often advantage to be gained by using materials which, when exposed to heavy loads, are capable of considerable strain before being broken, and such materials are preferred to those which suddenly break when their ultimate strength is reached. Therefore the condition frequently occurs in specifications that materials shall exhibit certain percentages of elongation and contraction, in addition to possessing a given co-efficient of ultimate strength (Art. 33, § *c*; Appendix I.).

Here, again, the published results of numerous experiments furnish most valuable data for the guidance of the constructional engineer.

26. Co-efficients of Elasticity and Resilience.—A co-efficient or modulus of elasticity is a quantity expressive of the ratio existing between stress and strain within the elastic limit. It has been argued that there are no less than twenty-one different co-efficients, but only four of these need be mentioned; only two are usually necessary for denoting the elastic properties of iron and steel, and very frequently only one is used. The latter co-efficient is variously described, as the modulus of direct elasticity; the modulus of longitudinal extensibility; the stretch modulus; Young's modulus; sometimes inexactly as the modulus of elasticity; and symbolically as "E." The four co-efficients to which we refer are:—

(E), Young's modulus of direct elasticity;

(σ) Poisson's ratio, the ratio of sectional contraction to elongation;

(C), Modulus of rigidity, or shear modulus;

(K), Modulus of cubic compressibility.

These four constants bear such relations one to another, that if any two of them be ascertained, the others may be found by calculation.

§ (*a*) *Young's Modulus.*—The modulus (E), which is universally employed in connexion with structural work, may be arrived at by the rule:—

$$E = (P \times L) \div l \qquad \ldots\ldots\ldots\ldots \qquad (2).$$

Where P = stress in tons per square inch, L = length of a bar of material in inches, and l = difference of length in inches due to P.

For a bar of the unit length of 1 inch, the modulus may be more simply expressed by the equation:—

$$E = P \div l \qquad \ldots\ldots\ldots\ldots \qquad (2a).$$

No two materials, or samples of the same material, necessarily possess exactly the same modulus, and the constant (E) is often employed as a standard by which materials may be judged.

Suppose, for instance, that we require the value of "E" for some particular kind of steel shall be not less than 13,000 tons per square inch. A bar of the material, say, 10 in. long by 1 sq. in. sectional area, might be taken as a test specimen, and if subjected to tensile stress within the range of the elastic limit, it would exhibit uniform extension per ton per square inch. Assuming the measured extension for 1 ton to be 0·0008 inch, the co-efficient of elasticity will be found by the aid of rule (2).

Thus:

$$E = (1 \times 10) \div 0{\cdot}0008 = 12{,}500 \text{ tons per sq. in.}$$

Consequently, the material does not comply with the assumed requirement.

The elasticity of structural iron and steel is an important index of quality, and as we shall afterwards see, Young's modulus E is an essential factor in various problems relative to the strength of girders and other members.

§ (*b*) *Modulus of Resilience.*—For the purpose of calculating the amount of work performed in extending or compressing a bar of iron or steel up to the elastic limit, the modulus of resilience R is a useful and convenient co-efficient.

In the following consideration we have used inch units of measurement and pound units of weight. The stress producing a given elastic strain is denoted by P; the strain due to P is denoted by l; consequently, as the modulus R represents the product of the stress and the strain, its value is obtained in inch-pound units, as shown by the equation:

$$R = P \times l \text{ inch-pounds} \ldots\ldots\ldots \quad (3).$$

By rule (2*a*), $E = P \div l$; therefore $l = P \div E$. Hence $P \times l = P \times P \div E$, and the modulus of resilience becomes

$$R = P^2 \div E \text{ inch-pounds} \ldots\ldots\ldots \quad (3a).$$

Example 4.—Find the modulus of resilience for a steel bar, 2 in. square, subjected to a tensile stress of 60 tons. Let E = 13,000 tons per sq. in.

With a stress of 60 tons on the bar, the stress per sq. in.

$$P = 60 \div 4 = 15 \text{ tons.}$$

By rule (3*a*):

$$R = \frac{(15^2)}{13{,}000} \times 2{,}240 = 38{\cdot}769 \text{ inch-pound units.}$$

§ (*c*) *Measurement of Resilience.*—In expressing the actual quantity of work done, inch-pound or foot-pound units can be used, but the latter notation is adopted in practice. As the stress applied commences at zero and ends at the value P, it is evident that the effective stress is of the mean intensity represented by P ÷ 2, and resilience, denoted by the symbol *R*, is equal to the mean stress intensity multiplied by the strain in inches or feet.

Using inch and pound units as before,

$$R = (P \div 2) \times l \text{ inch-pounds} \quad \ldots\ldots \quad (4).$$

We have already shown that $l = P \div E$; therefore $(P \div 2) \times l = \frac{P}{2} \times \frac{P}{E}$, and it consequently follows that

$$R = P^2 \div 2E \text{ inch-pounds} \ldots\ldots\ldots \quad (4a).$$

These rules can be applied to the determination of work done, or of resilience in bars of any length and area by including the necessary dimensions; and the results will be given in foot-pound units, if the values first obtained be divided by 12, or if the factor *l* be made to express length in feet (Table X, Appendix).

Example 5.—The measured extension of a bar 1 inch square is 0·1 inch with a stress of 10 tons. Find the work done, (*a*) in inch-pounds, (*b*) in foot-pounds.

By rule (4):

(*a*) Work done $= (10 \times 2{,}240 \div 2) \times 0{\cdot}1 = 1{,}120$ inch-lbs.

(*b*) „ „ $= \frac{(10 \times 2{,}240 \div 2) \times 0{\cdot}1}{12} = 93{\cdot}33$ foot-lbs.

(*b*) „ „ $= (10 \times 2{,}240 \div 2) \times {\cdot}0083 = 93{\cdot}33$ „

Example 6.—Find the resilience in foot-pounds for a steel bar 10 ft. long by 2 in. square subjected to a tensile stress of 60 tons, equal to 15 tons per sq. inch. Let E = 13,000 tons per sq. inch; *a* = sectional area of the bar in inches, and L′= length of the bar in feet. Adding these factors to rule (4*a*), we get $R = P^2 \div 2E \times aL'$.

Therefore,

$$R = \frac{15^2}{26{,}000} \times 2{,}240 \times (4 \times 10) = 775{\cdot}38 \text{ foot-pounds.}$$

Resilience can also be calculated in a still more simple manner if the modulus of resilience be known, because it is equal to R multiplied by half the product of the area in square inches and the length in feet. Thus :

$$R = \mathrm{R} \times aL' \div 2 \quad \ldots\ldots\ldots\ldots \quad (5).$$

Example 7.—Find the resilience in foot-pounds for the bar in Example 6 and under similar conditions. By Example 4, the co-efficient of resilience is 38·769 inch-pounds. Then

$$R = 38{\cdot}769 \times \frac{4 \times 10}{2} = 775{\cdot}38 \text{ foot-pounds.}$$

The determination of resilience is generally limited in practice to such structures as may be exposed to concentrated loads applied at or near one point, and co-efficients of resilience are, therefore, seldom required in connexion with ordinary buildings.

27. Co-efficient of Transverse Rupture.—Another co-efficient, termed the modulus of transverse rupture, is applied to denote the ultimate strength of bars or beams in respect of resistance to bending, and is commonly indicated by the symbol (*f*). The modulus of rupture does not coincide either with the tensile strength or with the compressive strength, and its value varies with the form of the cross-section of the material. Several theories have been propounded in explanation of this peculiarity, though none of them are entirely satisfactory. Rankine assumes one cause to be the fact that the resistance of a material to direct stress is increased by preventing or diminishing the alteration of its transverse dimensions. He also suggests that when a bar is directly torn asunder, the strength indicated is that of the centre part, which is the weakest, whilst when it is broken transversely the strength indicated is that of the outer part, which is the strongest. In any event, the difference really exists, and the modulus of rupture can only be determined experimentally. The unit value of the co-efficient (f) is expressed in pounds or tons avoirdupois per square inch (Art. 99, § *c*).

§ (*a*) *Internal Stress in the Absence of External Load.*—A state of internal stress may exist in a material in the absence of external load, and upon the presence or absence of this condition the value of any particular piece very much depends (Arts. 75, § *a*; 79, § *a*; and 82). As pointed out when speaking of crystallisation in connexion with castings (Arts. 12 and 14), weakness may result from unequal cooling of the metal, and this weakness is entirely due to internal stress, which could only be measured by a direct test of each individual casting. Internal stress may also result from deformation produced by previously applied forces, and can be removed by annealing (Arts. 24, § *f*, 34, 35, § *d*; 43, § *a*). Material free from internal stress is said to be in a state of ease.

28. Factors of Safety.—We have now summarised the principal units adopted for measuring or denoting the mechanical properties of bodies under stress, and before proceeding to examine existing records which furnish multiples showing the actual values for different kinds of iron and steel it will be convenient that factors of safety for these metals should be numerically expressed. As commonly understood, a factor of safety is the ratio of ultimate strength to working stress, and the standard so arrived at is not only meaningless, but is based on entirely fallacious premises. Its existence, like that of "horse-power," as applied to a steam boiler, is hallowed by custom, and its use will, no doubt, be continued for years to come, both by those who know its absurdity and by those who do not. Bearing in mind the fact that under no conditions should direct stress be sufficient to produce strain beyond the elastic limit or to occasion fatigue, it is evident that the factor of safety should be some ratio between the elastic limit and the working stress, more especially as we know that elastic limit bears no definite relation to ultimate strength in the case of different metals or of varieties of the same metal. Those who object to the elastic limit as a basis for the calculation of working strength rely chiefly upon the fact that an artificial elastic limit might be established, as mentioned in Art. 24, § *f*, for the purpose of deceiving the inspector sent to test the material. We do not think it probable any respectable iron or steel master would practise dishonest manipulation of this kind, nor would it always be practicable. Moreover, it is obvious that precautions could easily be taken to guard against the success of such a manœuvre.

In whatever way the factor of safety may be calculated, the following contingencies have always to be considered: (*a*) the possibility of errors in estimating loads; (*b*) the differences between the strength of materials in bulk and in tested specimens; (*c*) possible errors in the calculation of stresses and strains; (*d*) the occurrence of unexpected strains through indifferent workmanship; (*e*) the risk of deterioration from various causes; (*f*) the establishment of oscillative stresses; and (*g*) the production of fatigue due to vibration or shock. If we use the basis mentioned above, the factor of safety = (ultimate strength ÷ working stress), which may be expressed thus: $S = (F \div x)$, where S = factor of safety, F = ultimate strength, and x = working stress. As the quantity (x) is always more or less variable, the choice of a factor depends almost entirely upon individual judgment and experience. Taking ultimate strength at the value indicated by tested specimens, and assuming the working stress to be carefully calculated, the factors given in the subjoined table should be found to cover the several sources of error, and, in addition, to provide a sufficient margin of safety.

Table VII.—Factors of Safety.

Material.	Dead Load.	Live Load
Wrought Iron and Steel, Girders..	3—4	5—6
" " " Columns and Struts	4—5	6—7
Cast Iron, Girders..	5—6	8—9
" " Columns	5—7	8–10

CHAPTER V.

STRENGTH OF IRON AND STEEL.

29. Strength of Cast Iron.—§ (*a*) *Tensile Strength.*—Undoubtedly the most useful tests of cast iron for structural work are those showing its tensile strength. When very careful examination is desired, the test specimens ought to be cast upon the castings themselves, so that they may be broken off and shaped for the testing machine. The average tensile strength of cast iron is about eight tons per square inch, and it is in no way decreased by removal of the skin.

Table VIII.—Tensile Strength of Cast Iron.

Authority.	No. of Tests.	Ultimate Strength in Tons per sq. in.		
		Highest.	Lowest.	Mean.
Hodgkinson	—	9·76	6·0	7·37
,, ...	51	10·5	4·9	6·83
Woolwich	53	15·3	4·2	10·4
Wade	6	—	—	13·7
,, ...	4	—	—	9·1
Turner	—	15·7	4·75	—
Rosebank Foundry ...	23	18·2	6·5	15·3

§ (*b*) *Compressive Strength.*—The resistance of cast iron to compressive stress diminishes as the excess of height over diameter is increased, but strength may be taken as being fairly uniform for any given quality when the height is between one and three times the diameter. The average compressive strength of cast iron is between thirty-eight and fifty tons per square inch.

Some of Mr. Hodgkinson's experiments are summarised in the following table :—

Table IX.—Compressive Strength of Cast Iron (Hodgkinson).

Kind of Metal.	Number of Tests.	Number of Brands.	Ultimate Strength in Tons per sq. in.		
			Highest.	Lowest.	Mean.
Ordinary	81	16	52·502	25·198	38·525
Stirling's (C.I. and W.I. mixed) ...	—	2	70·827	53·329	59·522
Cold Blast	—	6	51·50	36·50	44·30
Hot ,, ...	—	6	64·90	36·90	45·80

§ (*c*) *Transverse Strength.*—A very convenient and easily applied test for cast iron consists in placing a bar of the material on two supports a certain distance apart, and loading it at the centre with gradually increasing weights. Tensile and compressive stresses then result partly from the external load and partly from the weight of the material itself, and mechanical principles are also involved which need not be considered now, as at present our attention is confined to comparative records of strength. In the annexed table the breaking weights for bars of various dimensions are given, together with the corresponding co-efficients of transverse rupture (Arts. 27 ; 99, § *c*).

Table X.—Transverse Strength of Cast Iron.

Authority.	Dimensions in Inches. *l.* *b.* *d.*	Centre Breaking Weight in Tons.	Co-efficient of Transverse Rupture (f). Tons per sq. in.
Hodgkinson ...	54 × 1 × 1	·219	17·73
	54 × 1 × 3	1·736	15·62
	54 × 1 × 4	4·600	23·28
Barlow	60 × 1 × 1	·226	20·34
	60 × 2 × 2	1·541	17·33
Clark	54 × 1 × 1	·252	20·41
	27 × 3 × 1	1·376	18.58
	54 × 3 × 2	2·410	16·27
Millar	36 × 1 × 2	1·670	22·6
	36 × 1 × 1	·358	19·6

§ (*d*) *Shearing Strength.*—Very little information is available with regard to the resistance of cast iron to shearing, and it is not easy to ensure uniform distribution of stress on the section during tests. Shearing strength varies from 6 to 13 tons per square inch.

Table XI.—Shearing Strength of Cast Iron.

Authority.	Shearing Strength in Tons per sq. in.	Condition of Bar.
Rankine	12·37	—
Stoney	8 to 9	—
University College ...	5·18	Turned.
,, ,, ...	3·92	Rough.

30. Elastic Properties of Cast Iron.—The following tables are calculated from Hodgkinson's experimental results obtained by testing bars of cast iron 10 ft. long by 1 in. square, and from them it will be seen that cast iron has no definite elastic limit, as a perceptible permanent deformation is caused with comparatively small loads, and the set increases regularly with every addition to the load up to the breaking-point. The extent to which *elastic strain* takes place may be found by deducting set from extension or compression.

Table XII.—Elastic Properties of Cast Iron in Tension, Bars 10 feet long (Hodgkinson).

Stress in Tons per sq. in.	Total Extension in inches.	Total Set in inches.	Co-efficient of Elasticity (E).
·94	·0186	·0006	6067
1·88	·0391	·0018	5775
2·82	·0613	·0037	5525
3·76	·0859	·0066	5257
4·70	·1136	·0106	4968
5·64	·1448	·0161	4677
6·6	·1859	·0241	4262

Table XIII.—Elastic Properties of Cast Iron in Compression, Bars 10 *feet long* (*Hodgkinson*).

Stress in Tons per sq. in.	Total Compression in inches.	Total Set in inches.	Co-efficient of Elasticity (E).
·92	·0188	·0005	5879
1·84	·0388	·0023	5701
2·76	·0598	·0040	5549
3·68	·0788	·0060	5611
4·60	·0994	·0085	5560
5·32	·1203	·0109	5516
6·45	·1416	·0141	5467

31. **Strength of Malleable Cast Iron.**—Published tests relative to malleable cast iron are somewhat divergent in their nature, probably owing to differences of form and quality. The undermentioned data are therefore only useful for general guidance :—

Table XIV.—Strength of Malleable Cast Iron.

	Tons per sq. in.
Tensile strength	14 to 17
Compressive strength	48 „ 70
Transverse strength	20 „ 30
Elastic limit	1 „ 4

32. **Strength of Wrought Iron.**—For structural work wrought iron is now comparatively little used, although for some special purposes it is preferred to steel. From experimental records bearing on the question of strength, we have selected only such as may serve to illustrate characteristics which ought not to escape attention, and which, for the sake of clearness, will be dealt with under separate sections.

§ (*a*) *Tensile Strength.*

Table XV.—Tensile Strength of various kinds of Wrought Iron (Styffe).

Material.	Percentage of Carbon.	Ultimate Strength. Tons per sq. in.	Contraction of Area per cent.	Elongation in 5 ft. per cent.
Lowmoor iron	·21	24·73	53	19·6
Cleveland	·07	26·81	42	17·3
Dudley	·09	22·3	20	8·5
Rolled charcoal iron ...	·12	26·74	16	7·2
,, ,, ,, ...	·07	22·67	65	18·4
Swedish iron	·07	20·92	70	21·1

Table XVI.—Average Tensile Strength of Structural Wrought Iron.

Material.	Ultimate Strength in Tons per sq. in.	Contraction of Area per cent.
Plates	18 to 22	7 to 10
Angles, &c.	20 ,, 23	12 ,, 16
Bars, flat	21 ,, 24	18 ,, 22
,, round	19 ,, 22	13 ,, 18
Rivet iron	23 ,, 25	30 ,, 40

§ (*b*) *Compressive Strength.*—Wrought iron, used under compressive stress in structures, seldom fails from insufficiency of strength, but rather from want of proper stiffening. The strength of wrought iron to resist compression is approximately the same as its tensile strength, and the average is usually taken at from 16 to 20 tons per square inch.

§ (*c*) *Shearing Strength.*—According to data furnished by tests made on behalf of the United States Navy, the shearing strength of wrought iron should not be taken at more than 80 per cent. of the ultimate tensile strength, and for ordinary brands 75 per cent. is a safer estimate.

§ (*d*) *Elastic Strength.*—The co-efficient of elasticity for wrought iron is naturally a movable quantity, some of the most reliable values of which are summarised below :—

Table XVII.—Co-efficients of Direct Elasticity of Wrought Iron.

Material.	Co-efficient E in Tons per square inch.	Authority.
Iron plate (longitudinal)... ...	11,200	Kupffer
,, (transverse)	12,160	,,
Rolled bar iron (English)... ...	12,850	,,
Lowmoor iron	14,280	Styffe
Dudley ,, ...	12,260	,,
,, ,, ...	12,680	,,
Bar iron (medium quality) ...	10,000	Barlow

As before explained (Art. 24, § *f*), the elastic limit of wrought iron varies with successive loading and with the manner in which the loads are imposed. The averaged results in the following table will afford some indication as to the co-efficient which may safely be used for purposes of calculation :—

Table XVIII.—Elastic Limits of Wrought Iron.

Material.	Elastic Limit, Tons per square inch.	Authority.
Yorkshire bar iron	13·0	Steel Committee
S. C. Crown	11·8	—
Swedish	11·05	Kirkaldy
Yorkshire plate	12·2	—
Bar iron (medium)	10·0	Barlow

33. Strength of Steel.—Mild steel used in constructional work is about 50 per cent. stronger than wrought iron. It is proportionately superior in elastic strength and ductility, and is much more uniform in quality. The following tables, compiled from the various data available, will sufficiently demonstrate the chief characteristics of structural steel :—

§ (*a*) *Mechanical Properties.*

Table XIX.—Mechanical Properties of Steel (Bauschinger).

Carbon per cent.	Elastic Limit, Tons per sq. in.	Tensile Strength, Tons per sq. in.	Elongation per cent. in 2-in. length.	Contraction per cent. of area.
·19	21·02	30·4	20·1	41·7
·46	21·90	33·8	18·1	30·5
·57	21·02	35·6	18·4	30·7
·78	23·80	41·1	11·4	19·1
·87	27·24	46·7	8·1	16·5
·96	30·90	52·7	6·6	10·0

Table XX.—Co-efficients of Elasticity of Steel (Bauschinger).

Co-efficient E.				Co-efficient C.	Poisson's Ratio.
Carbon per cent.	Tension Tests.	Pressure Tests.	Bending Tests.	Torsion Tests.	$\sigma = \frac{E}{2C} - 1.$
0·19	13,780	16,540	13,020	5,575	·32
0·46	14,300	14,640	13,090	5,420	·30
0·57	13,720	14,290	13,080	5,320	·30
0·78	14,980	14,480	13,460	5,405	·34
0·87	13,880	14,100	13,590	5,400	·29
0·96	13,820	14,640	13,080	5,560	·26

§ (*b*) *Shearing Strength.*—The ultimate resistance of steel to shearing stress averages from 70 to 75 per cent. of the ultimate tensile strength.

§ (*c*) *Standards for Mild Steel.*—For the purpose of indicating more precisely the qualities of steel which are most suitable for the architect, some of the requirements of Government and other specifications are quoted below in tabular form. (See also Appendix I.)

Table XXI.—Standards for Mild Steel.

Name.	Tensile Strength in tons per sq. in.	Elongation in 8-in. length * (in 10-in.)
British Admiralty	26 to 30 tons	20 per cent.
French Admiralty	28 to 28·5 ,,	20 ,,
Board of Trade	26 to 32 ,,	20* ,,
Lloyd's	27 to 31 ,,	20 ,,
Lancashire and Yorkshire Railway ...	26 to — ,,	20 ,,

As a general rule, rolled steel joists, angles, and bars ought to exhibit characteristics such as the following: tensile strength, 30 tons; elastic limit, 18 to 20 tons; co-efficient of elasticity (E), 13,000 tons; elongation about 20 per cent., and contraction of area from 30 to 40 per cent.

34. Strength of Steel Castings.—As steel castings are now frequently employed in place of cast iron, malleable cast iron, and sometimes of wrought iron, it may be useful to state the average measurements of strength deducible from various experiments. Ultimate tensile strength may be anything between 15 and 45 tons per square inch, and, other things being equal, 20 tons is probably a safe average. The mean elastic limit is about 13 tons, but it will be safer to assume this quantity a little lower. The mean co-efficient of elasticity (E) is about 10,000 tons. Owing to the fact that the melting-point of cast steel is considerably higher than that of cast iron, greater contraction takes place in the foundry; consequently steel castings should always be annealed for a period of about twenty-four hours at a temperature of about 1,700 deg. Fahr. Annealing somewhat reduces tensile strength, but largely increases ductility, and, for the reasons previously stated, removes internal stress.

35. Notes upon Iron and Steel.—§ (*a*) *Effect of Rolling.*—The ratio existing between the area of a finished bar and the area of the pile from which it was rolled exercises influence upon tenacity and elastic limit; and the temperature at which rolling is conducted has a similar effect. These phenomena are evidenced by the tables given below;—

Table XXII.—Effect of Reducing Iron Bars by Rolling (United States Testing Board).

Diam. in.	Area of Bar. (*Area of Pile=100.*)	Tensile Strength, Tons per sq. in.		Elastic Limit, Tons per sq in.	
		Rough Bar.	Turned Bar.	Rough Bar.	Turned Bar.
$2\frac{1}{2}$	6·1	21·1	21·1	13·25	13·3
2	4·4	21·4	21·6	16·0	14·2
$1\frac{1}{2}$	4·9	22·7	23·1	15·6	16·3
1	3·1	23·1	22·8	17·4	17·2

Table XXIII.—Effect of Cold Rolling.

Material.	Elastic Limit.	Tensile Strength,	Elongation.
Iron plate	14·48	23·75	15·0
,, ,, cold rolled	26·42	29·78	7·0
Dudley bar iron	—	26·0	20·3
,, ,, cold rolled	—	38·4	8·0
Mild steel	18·80	33·34	18·0
,, ,, cold rolled	26·92	34·61	11·5

§ (*b*) *Effect of Repeated Rolling.*—If experimental results are to be accepted as establishing a general rule, repeated rolling improves the quality of wrought iron up to a certain point, beyond which, however, injury ensues, as shown below :—

Table XXIV.—Effect of Repeated Rolling.

Number of Times Worked.	Tensile Strength, Tons per sq. in.	
	(Clark)	(Clay)
One	—	19·6
Two	22·38	23·5
Three	25·60	26·6
Four	25·97	—
Five	26·65	—
Six	—	27·5
Nine	—	26·0
Twelve	—	19·6

§ (*c*) *Effect of Hammering.*—As a general rule it may be taken that the effect of hammering or forging ductile materials such as wrought iron and steel, at temperature below that of plasticity, is to increase ultimate strength and to reduce ductility proportionately.

§ (*d*) *Effect of Re-heating.*—The effect of re-heating, or annealing, wrought iron and steel is to reduce tensile strength whilst increasing ductility. Annealing removes the effects of over-strain, and causes the metal to revert to its natural condition. Consequently, ironwork in a building is not necessarily injured by exposure to fire; on the contrary, it may be improved in quality, if originally rolled at too low a temperature, or if subsequently over-strained.

§ (*e*) *Effect of Temperature.*—Very little diminution of strength is experienced by wrought iron and steel when subjected to low temperatures; in fact, some experiments indicate improvement at −2 deg. Fahr. With regard to abnormally high temperatures, some investigators consider that wrought iron exhibits increase of strength up to about 400 deg. or 500 deg. Fahr., but this conclusion is not entirely in accordance with the following data:—

Table XXV.—Effect of Temperature on Strength.
(Normal Strength = 100.)

Temperature.	Wrought Iron (Roelker).	Bessemer Steel (Roelker).	Wrought Iron (Franklin Institute).
0 deg. F.	100	100	96
100 ,,	100	100	102
300 ,,	97	100	106
500 ,,	92·5	98·5	104
700 ,,	81·5	68	92·5
1,000 ,,	26	31	36
1,500 ,,	10	12	—
2,000 ,,	3·5	5	—

§ (*f*) *Effect of Sudden Stress.*—Comparatively few records are available as to this point, but the average results obtained by Mr. Kirkaldy with regard to wrought iron indicate a difference of about 18 per cent. diminution of tensile strength under a suddenly applied load, and a similar decrease of elongation.

§ (*g*) *Effect of Galvanising.*—Experiments made upon wrought iron plates appear to show no perceptible difference in any way between galvanised and ungalvanised plates, but there is a popular impression that the process of galvanising is not calculated to improve the quality of either iron cr steel, and no possible harm can result from the use of a somewhat more liberal factor of safety when calculating the strength of material so treated.

PART II.

JOINTS AND CONNEXIONS

INTRODUCTORY NOTE.

A STRUCTURE in iron or steel consists of two or more members, or pieces, which touch each other and are connected together at portions of their surfaces called joints. Each individual piece may be a simple member, consisting of metal in some unit form, such as a rolled joist; or it may be a compound member, composed of two or more unit forms joined together, as may be most suitable and convenient. For example, columns and stanchions are frequently constituted by the joining together of castings or of rolled sections by means of bolts, rivets, or other fastenings. Completed columns and stanchions are connected one to another, or to other parts of a structure, and they are provided with suitable arrangements for the attachment of various details. Girders, again, are often built up of several sections of iron and steel, and the completed members are joined together, or connected with other members. Similar treatment is practised with regard to roof construction and other matters of detail.

Moreover, in many of these operations it is compulsory that something should be done in addition to the joining together of two or more pieces of metal, inasmuch as the parts have often to be cut, forged, welded, or otherwise manipulated in order that they may be suitable for the specific purpose in view. As previously expressed, our wish is to preserve a natural and logical sequence with regard to the matters under consideration, and we therefore propose to relegate the discussion of different typical connexions to future chapters, where they will be examined in conjunction with the integral parts of buildings for which they are more especially adapted. By the adoption of this course, we shall now be free to review in a general manner the theoretical aspects of the subject of joints, to summarise experimental data, and to refer briefly to some points connected with workshop practice.

CHAPTER VI.

METHODS OF MAKING JOINTS.

36. Welding.—When the question arises as to how two pieces of metal are to be joined along two edges in the most satisfactory and efficient manner, it appears that the only perfect method is to be found in welding. There are, however, many practical difficulties, especially where long welded joints are concerned, which prevent the attainment of theoretical efficiency.

Welding is effected by raising the temperature of two pieces of iron or steel until a white heat is almost attained, and then by pressing or hammering them firmly together. In the case of steel, the temperature necessary varies with the percentage of carbon present in the metal. When the proportion of carbon is above 1 per cent., welding is difficult, if not impossible. Welded joints made by the aid of a steam hammer are found to be about 5 per cent. stronger for wrought iron and 10 per cent. stronger for mild steel than similar joints made by hand. The strength of a weld depends very much upon the suitability of the metal, upon the form of joint, upon the care taken to bring the surfaces joined into close contact, and upon their freedom from oxide, cinder, &c.

The welding together of steel and cast iron is not difficult if the steel be of suitable quality; but such a practice is not adopted for the connexion of structural iron and steel.

§ (*a*) *Electrical Welding.*—Some tests by Professor Unwin of electrically welded steel bars of small dimensions yielded the following results:—

Table XXVI.—Strength of Electrical Welded Steel (Unwin).

Carbon per cent.	Ratio of Strength (original strength = 100).	
	Series A.	Series B.
·75	59	68
1·00	76	51
1·00	50	84
1·15	89	113
1·15	75	117
1·25	75	86

These figures do not appear to indicate that varying proportions of carbon exercise any marked influence upon the susceptibility of steel to welding, but in ordinary welding it is found, as before stated, that as the percentage of carbon increases, so does the difficulty of making a weld, and the operation should only be attempted in the case of mild steels.

Electrical welding is a process which is extremely useful in special cases, though it cannot be practised on the site of ordinary building operations, and there are not many firms who possess the necessary plant for carrying it on in their works.

§ (*b*) *Efficiency of Welding.*—Experiments were made in 1860 by Mr. Bertram upon two forms of welded joint, the *scarf-weld* and *lap-weld* (figs. 16 and 17), where the lap was $1\frac{1}{4}$ in., the material being Staffordshire iron plate 24 in. long by 4 in. wide, and of three thicknesses.

Fig 16.—*Scarf-weld.*

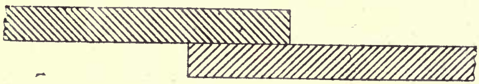


Fig 17.—*Lap-weld.*

The results observed in these tests are tabulated below :—

Table XXVII.—Strength of Welded Joints (Bertram).

Description of Joint.	Comparative Strength (original plate = 100).			
	$\frac{1}{2}$-in. Plate.	$\frac{7}{16}$-in. Plate.	$\frac{3}{8}$-in. Plate.	Average.
Scarf-weld	Faulty	106	102	104
Lap-weld	50	69	66	62

According to Mr. Kirkaldy, the loss of strength in welded joints is from 15 to 30 per cent., and experiments made more recently by Professor Bauschinger upon mild steel and ingot iron bars varying in area from 0·15 to 2·0 sq. in., indicated the strength of steel at the weld to be from 57 to 105 per cent. of the original bar, whilst the average was 89 per cent. In the case of wrought iron, the strength at the weld was from 83 to 102 per cent., with an average of 95 per cent.

It must be remembered that all the above were purely scientific experiments, in which every precaution would naturally be taken to ensure perfect connexions. In the course of ordinary workshop practice it is highly probable that the strength of a welded joint seldom exceeds 60 or 70 per cent. of the strength of the original material.

§ (*c*) *Welded Joints in Structural Work.*—Not only because of undesirable loss of strength, but also on account of cost, welded joints should be avoided as far as possible in structural work. When such a joint seems to be the only practicable method of connecting two parts, let us say, of a round, square, or narrow flat bar, the ends should be upset, or swelled out, in order that by the increase of sectional area some compensation may be provided for the diminution of unit-strength.

Welding cannot always be dispensed with, especially when connexions have to be made involving the use of "stiffeners," similar to those shown in fig. 18. Cranks of this kind can only be made in one of two ways—either the surplus metal, protruded by the act of bending, must be taken up in thickening the web, or V-shaped pieces

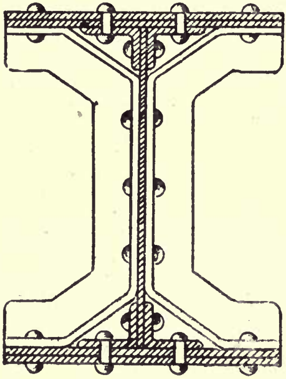


Fig. 18.

equal to the estimated surplus must be cut out, as in fig. 19, and the two edges be welded together after bending.

Fig. 19.

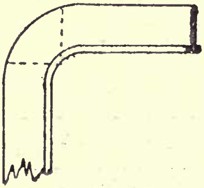


Fig. 20.

The former mode of treatment is much to be preferred, as the greater thickness of the web will add rigidity to the stiffener; and if the web be cut, it is morally certain that the joint will be considerably weaker than the original metal. Sometimes it may happen that a tee or angle bar is required to be cranked with the web outwards, as in fig. 20, and then the only course for adoption is to cut out a portion of the web, and to make good the deficiency by the insertion of a piece of plate, as indicated by dotted lines in the figure.

The Board of Trade rules for steam boilers contain the stipulation that steel plates which have been welded shall not be passed if subject to a tensile stress, and that they shall be efficiently annealed, subject to a compressive stress.

37. Burnt-on Joints.—A process sometimes adopted in foundry practice consists in the joining together of two cast-iron surfaces, and, being somewhat akin to welding, it may conveniently be noticed in the present connexion. Occasionally it happens that some part of a casting, such as the bracket of a column, has cracked off. The ironfounder may then possibly suggest that a new bracket should be "burnt on." This operation is conducted by remoulding the bracket, and by placing the column, previously heated to a high temperature at the desired point, in such a position as to close the mould. The molten metal added to form the new bracket is supposed to raise the temperature of the surface of the column to the melting point, so that the two parts may unite to form a homogeneous casting. It is by no means certain that this result will follow, and a far safer practice is to have a new bracket cast separately, and to bolt it to the column.

38. Joints for Sheet Metal.—Besides welding there are some ingenious methods of connecting together iron plates by folded, hammered, and soldered joints, but these are not generally applicable to work of the kind we are now considering, although they are used in some minor details of construction involving the employment of very thin plates or sheets of metal.

39. Riveted and Bolted Joints.—Seeing that the joining together of structural iron and steel by means of welding is only permissible or practicable under exceptional circumstances, and that other methods mentioned are equally inapplicable, we have to look elsewhere for a mode of connecting such sections as are commonly used in constructional work. Some forms of material may be placed so that the edge of one piece overlaps the edge of another, and the two can then be joined by the insertion of rivets, bolts, pins, or similar fastenings. There are, however, some varieties of structural iron and steel—I-joists, for example—which cannot conveniently be caused to overlap in any approved manner, and any two pieces which it may be necessary to connect must be laid end to end and held fast by the aid of flat plates, riveted or otherwise fixed to the flanges or webs of the two separate pieces.

It should be remembered that joints, where perforation of plates or bars is necessary, are not all of the same class; some are intended to make rigid connections, whilst others are designed to permit a certain amount of movement about fixed points. (Art. 60.) It is not needful for the moment that we should enter upon detailed consideration of these two types of joints, but it will be advantageous that some features should be pointed out which are common to both. The strength of any joint between two pieces of metal, which have been perforated and are held together by transverse fastenings, largely depends upon two essential factors, (*a*) the tensile and compressive strength of the material, and (*b*) the shearing and tensile strengths of the fastenings, whether these be rivets, bolts, or pins. We already know from tables in Chapter V. what is the average unit-strength of iron and steel, but have now to ascertain in what manner and degree this is affected by manipulations incidental to the process of joint-making.

CHAPTER VII.

THE PERFORATION OF PLATES.

40. Methods of Perforation.—Plates of metal may be perforated either by punching or by drilling, but although these are the only two methods adopted in practice, subsequent auxiliary treatment is often applied. In the case of punched holes the plates are sometimes annealed to restore the original quality of the material, or the holes are either drilled out or countersunk to remove the ring of hardened metal wholly or in part. The sharp edges of drilled holes are generally removed, or the holes are slightly countersunk, so that the rivets shall not be cut. The relative advantages and disadvantages of these manipulations are considered in the present chapter, only so far as they are evidenced by plates perforated, but not riveted, and in Chapter IX. as they are evidenced by plates in riveted joints. It has been found desirable to divide the discussion of the subject in this way, because the data presented in the latter chapter would be inappropriate for presentation at the present stage.

41. Punching.—Punches and dies for forming rivet-

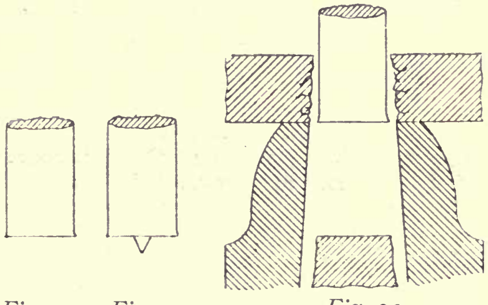


Fig. 21. *Fig.* 22. *Fig.* 23.

holes do not vary much in shape. A centre-point, as in fig. 22, assists the operator in finding the centres of holes marked on the plate, and tends to ensure greater accuracy than is obtainable with a plain-faced punch, as fig. 21. In every case the punch is slightly larger in diameter at the face than in the shank, and the face is usually concave, so that a clean cut may be obtained.

It is not advisable that the punch should fit the die without clearance, nor that the clearance should be excessive, as illustrated in fig. 23, for in either case the result will be prejudicial. A close die, producing a cylindrical hole, causes greater loss of strength than an open die, by which a conical hole is made. It is usual in practice to use a punch about $\frac{1}{16}$ in. larger in diameter than the rivet, and to have the die hole larger than the punch in the proportion of $\frac{1}{8}$ in. to 1 in. thickness of the plate. The following table shows these relations for some of the most ordinary rivets and plates.

Table XXVIII.—Proportions for Diameter of Rivets, Punches, Die Holes, and Rivet Holes.

Thickness of plate.	Diameter of rivet.	Diameter of punch.	Diameter of die hole.	Diameter of rivet hole.	
				Top.	Bottom.
$\frac{3}{8}$ in.	$\frac{3}{4}$ in. $=\frac{48}{64}$	$\frac{52}{64}$	$\frac{55}{64}$	$\frac{52}{64}$	$\frac{55}{64}$
$\frac{1}{2}$,,	$\frac{3}{4}$,, $=\frac{48}{64}$	$\frac{52}{64}$	$\frac{56}{64}$	$\frac{52}{64}$	$\frac{56}{64}$
$\frac{3}{4}$,,	$\frac{3}{4}$,, $=\frac{48}{64}$	$\frac{52}{64}$	$\frac{58}{64}$	$\frac{52}{64}$	$\frac{58}{64}$
$\frac{7}{8}$,,	1 ,,	$1\frac{1}{16}$	$1\frac{1}{8}$	$1\frac{1}{16}$	$1\frac{1}{8}$
1 ,,	1 ,,	$1\frac{1}{16}$	$1\frac{3}{16}$	$1\frac{1}{16}$	$1\frac{3}{16}$

§ (*a*) *Punching Machines.*—As a general rule, punching machines for structural iron and steel are provided with a single punch, and they are operated by hand, steam, or hydraulic power. Some steam machines are driven by belt, and others by engines attached to the main frames of the machines. In marking off the rivet holes a template is generally used in which holes are drilled wherever holes are required in the metal plate. The template is then clamped to the plate, and a wooden plug, the end of which has been dipped in white lead, is pressed into each hole, making a mark to indicate the position of the rivet hole.

Instead of marking off the holes by means of the wooden plug, it is better to use a centre punch, so that the centre of each required rivet hole may be shown by a slight conical depression. When the latter plan is adopted, a centre-point punch (as fig. 22) is fitted to the machine, and the "puncher" has no difficulty in ascertaining the exact centre of the hole to be punched.

When an automatic-feed punching machine is available, the process of marking off is unnecessary, as after each stroke of the punch the plate is carried forward the exact distance for which the machine has been set. Multiple punching machines are made in which as many as 70 or 80 holes can be punched at once, but such tools are chiefly intended for tank and similar work, in which the plates are not more than $\frac{1}{4}$ in. thick. Other multiple machines have been made capable of punching holes of variable pitch and arrangement, but their cost is so great that very few tools of this type are to be found in operation.

§ (*b*) *Force Required for Punching.*—Experiments have shown that resistance to punching is practically of the same value as resistance to tensile strain, and the force required in any given case may be found by the following rule :—

$$P = a \times t \times F_t.$$

Here P = force required, a = area of the hole, t = thickness of the plate in inches, F_t = tensile strength of the material in tons per square inch.

§ (*c*) *The "Flow" of Metal during Punching.*—Perhaps it may at first sight appear that there is nothing particularly interesting in connexion with the punching of a hole in a piece of iron or steel, but some remarkable phenomena are nevertheless involved. As formulated by M. Tresca, in a paper on "The Flow of Solids," three periods exist with regard to bodies under the influence of stress :—(1) the period of perfect elasticity; (2) the period of imperfect elasticity; and (3) the period of fluidity. In the first of these, strain is proportional to stress; and in the second, strain increases more rapidly than the stress which produces it (Art. 24 §*a* and §*c*). The third condition, as defined by M. Tresca, is one in which a given force would continue to produce strain, or deformation, without limit—as may be observed in the drawing of lead. This period is said to be "more extended for plastic substances; it is more restricted, and will even disappear altogether for some vitreous or brittle substances. But it is perfectly developed and extremely extended in the case of clays and of the most malleable metals." An exemplification of M. Tresca's theory will be furnished if a piece of plastic material be placed on a die and perforated by a punch. It will then be found that the thickness of the "burr," or piece punched out, will be less than the thickness of the original block, as indicated in figs. 24 and 25. As it has been demonstrated that no increase of density takes place in the burr under such circumstances, it is evident that a large proportion of the metal is absorbed laterally into the block.

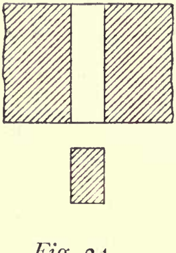


Fig. 24.

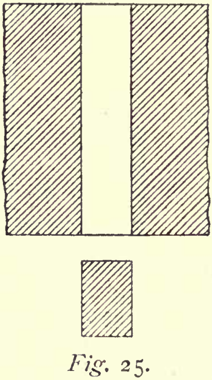


Fig. 25.

In a case like this a very distinct side movement of the particles of the metal commences before direct severance occurs. This fact was established some years ago by Messrs. Hooper and Townsend, of Philadelphia, when punching material $1\frac{3}{4}$ in. thick with a $\frac{1}{2}$-in. punch (fig. 24.) The thickness of the resulting burr was only $\frac{13}{16}$ in., and the greater part of the difference represented side flow. Another remarkable case of "flow" (represented by fig. 25) is mentioned by M. Tresca, in which a hole 2 cm. diameter was punched through a block of metal 10 cm. thick, and the resulting burr was only 3 cm. thick. Such results as these are of course altogether exceptional, and must not be looked for in the course of ordinary work.

For thinner plates the flow is proportionately less, but it always exists. In another experiment by M. Tresca, several piled-up pieces of lead were punched through, and when the burr was taken apart it was found that the discs near the bottom were nearly of the same thickness as the original metal, whilst the upper layers were considerably thinner. Although more recent experience indicates that the effects of punching a piled specimen are not precisely identical with those experienced in the case of a solid block, the experiment cited will serve to confirm the statement that the greater part of the metal which flows laterally into a block during punching comes from the higher layers of the material, and, consequently, compression is most considerable near the cut edge of a perforation. This part of the metal is in a state of internal compressive stress, and the thicker the plate which has been punched the greater must be the injury caused.

42. Effect of Punching on Strength.—As mentioned in the preceding paragraph, one effect of punching is to produce compression of the material, especially near the upper edge of the hole. Speaking generally, it is found that plates which have been perforated by punching are more or less damaged. Sometimes there appears to be an increase of strength, but this is only attained at the expense of ductility. At other times a distinct loss of strength is observable. The general concensus of opinion is that the loss of tenacity in iron and steel plates due to punching varies from 5 to 25 per cent. of the original strength of the solid plate.

The percentage in any individual case depends upon various attendant conditions, which for the sake of convenience we have discussed in the following separate sections of this article.

§ (*a*) *Thick and Thin Plates.*—As we have already seen (Art. 41, §*c*), the thickness of a plate is directly responsible for a difference of strength; but when plates are under ½ in. in thickness, the loss due to punching is not very serious, and many authorities are inclined to permit such material to be punched if it is to be used in ordinary structural work. In the case of very mild Landore steel not exceeding $\frac{3}{8}$ in. thick, it was demonstrated by Professor Kennedy that punching actually caused an increase of unit-strength ; and the late Sir William Siemens always maintained that punching did not injure mild steel, although experimenters with other brands found the reverse to be the case.

After an exhaustive series of experiments, Martell arrived at the conclusion that the loss of strength due to punching plates above $\frac{1}{2}$ in. thick ranged from 20 per cent. to 23 per cent. for iron, and from 22 per cent. to 23 per cent. for steel. Table XXXI. shows that there is a general loss of tenacity, increasing directly with the thickness of the plates ; and the concurrent diminution of contraction indicates a serious loss of ductility.

From the facts stated above it will be safe to infer that the loss of tenacity resulting from punching plates of different thicknesses to be as follows :—

Thickness of plate ..	$\frac{1}{4}$ in.	$\frac{1}{2}$ in.	$\frac{3}{4}$ in.	1 in.
Loss per cent.	6·0	8·5	17·6	25·0

§ (*b*) *Narrow and Wide Plates.*—Under some conditions the width of a plate has an important bearing upon the question of punching, and this will be best understood if we state the effect observed in the course of some tests by M. Barba.

Table XXIX.—Tests upon Wide and Narrow Steel Bars Perforated by Punching (Barba).

(Thickness = 0·28 in., Diameter of hole 0·68 in., Normal Tensile Strength = 32·7 tons.)

Width of bars, inches ...	1·28	2·0	2·72	3·44	4·16	4·88
Tenacity—cylindrical holes	27·1	25·9	25·3	22·7	24·3	23·1
„ conical „	31·7	28·3	26·3	22·4	22·9	23·7

It has already been stated that the metal immediately around the punched hole is compressed, especially in the cut edge, but the injury, as shown by the investigations of

M. Barba, M. Considère, and others, is confined to a narrow zone of about $\frac{1}{8}$ in. wide. If this ring of metal be cut out, it will be found to be extremely brittle and incapable of bending, and, assuming the holes to be spaced close together, the annuli of hardened metal will nearly touch each other, and the tenacity of the material between the holes will be considerably raised. In this way the strength of metal in narrow bars full of holes becomes much increased, but as the effect is to create a brittle material out of a ductile one, the element of danger is distinctly perceptible, and such bars should never be used unless perforation be performed without altering the natural characteristics of the material.

This point is illustrated by the results obtained by M. Considère by testing narrow bars punched less than half-way through from each side, so that a strip of metal was allowed to remain between the semi-perforations.

Table XXX.—Tests upon Narrow Steel Bars partly Perforated by Punching (Considère).

(Longitudinal Pitch of Semi-holes stated on the Upper Line of Table.)

Make of Steel.	Normal Tensile Strength.	Tensile Strength after punching.					
		·2	·24	·32	·56	1·2	2·0
Martin	32·7 tons	42·6	41·5	40·6	33·3	28·6	27·2
Bessemer	38·1 ,,	—	—	46·8	39·6	33·6	30·6

When the bars are wider and the pitch of the holes is greater, a totally different state of things exists, for the annuli of hardened metal do not meet, and on the application of stress, they do not stretch so much as the intermediate parts. Consequently, the annuli are ruptured before the intermediate metal has come fully under the influence of tension, and the unit-strength of the perforated plate is, therefore, less than that of an entire plate.

§ (*c*) *Hard and Mild Metal.*—The extent to which iron and steel are affected by punching depends very much upon the quality of the metal. Puddled iron plates exhibit varying diminution of strength according as the textile pull is applied longitudinally or transversely, the difference averaging about five per cent. Hard metal is apt to develop cracks when punched, and to suffer reduction of tenacity in proportion to hardness, but mild steel, on the other hand, is sometimes improved in strength by punching. Thus, the experiments of Professor Kennedy, mentioned in § *a* above, showed that the increased tenacity of very mild steel punched plates was very much greater than in the case of harder steel.

§ (*d*) *Form and Condition of Punch and Die.*—Strength is considerably affected by the form and condition of the punch and die. Bad tools may cause great injury to the metal, and develop weakness along a line of rivet holes. Experiments made for Lloyd's upon steel plates as to the effect of increasing the taper of punched rivet holes gave the following results:—

Taper of hole, $\frac{1}{16}$ in. Loss of tenacity, 17·8 per cent.
" " " $\frac{1}{4}$ " " " " 12·3 " "
" " " $\frac{1}{3}$ " " " " 24·5 " "

43. Removing Injury due to Punching.—Table XXXI. affords a general indication as to the manner in which the injurious effects of punching may be obviated, and some notes upon the same subject are contained in the section following.

Table XXXI.—Strength of Perforated Steel Plates (Board of Trade).

Condition of plate.	$\frac{1}{4}$-in. plate.		$\frac{1}{2}$ in. plate.		$\frac{3}{4}$-in. plate.		1-in. plate.	
	B	C	B	C	B	C	B	C
Unperforated ...	30·17	53·3	27·69	50·0	27·84	39·4	28·17	38·8
Punched ...	28·32	24·1	25·33	19·1	22·94	13·0	21·26	5·4
Punched and annealed ...	31·6	41·3	29·39	37·4	29·31	30·7	29·12	24·7
Punched and drilled ...	31·23	24·0	30·86	28·9	28·81	19·4	29·15	15·9
Drilled	31·46	36·6	31·37	32·7	30·23	32·1	28·43	33·2

B—Ultimate tensile strength in tons per square inch of initial net section.
C—Contraction per cent.

§ (*a*) *Annealing.*—Strength lost during the operation of punching can be restored by annealing. There is ample testimony to show that annealing will entirely remove the prejudicial effects caused by punching, and that in some cases greater relative strength may be obtained than originally existed. Annealing is performed by raising the temperature of the plate to a dull red and covering it with ashes or sand until cold; but a better plan is to leave the plate in the annealing oven until it is cold. In the case of mild steel bars used for structural work, annealing is not considered necessary if the thickness does not exceed $\frac{1}{2}$ in. When the punched material is between $\frac{1}{2}$ in. and $\frac{3}{4}$ in. thick, either annealing or some equally restorative treatment is very desirable.

The benefit derived from annealing is clearly evident in Table XXXI.

§ (*b*) *Drilling-out and Countersinking.*—The practice of punching holes about $\frac{1}{4}$ in. less in diameter than the required size and then of boring out the remaining metal is distinctly advantageous, as also is the practice of countersinking after punching. In either case the unit tensile strength of the plate may be fully restored, but as neither operation removes the whole of the hardened metal, the plate is not left with ductility quite equal to that in "punched and annealed" or in drilled plates.

44. **Drilling.**—Drilling is always to be recommended as the best mode of effecting the perforation of plates, and, considering the great advances which have been made in connexion with tackle available for the purpose, it is probable that the operation can be performed almost, if not quite, as economically as punching. As will be seen by reference to Table XXXI., the effect of drilling is clearly to increase the unit-strength of metal between the holes, and according to Professor Kennedy, the explanation of this apparent paradox is that contraction of area at the hole may alter the usual flow of the metal when under severe stress, rendering it more uniform than it would be in an unperforated plate, so that the stress can be more evenly distributed throughout its area. Mr. Kirkaldy attributed the phenomenon to suppression of elongation, and consequently of contraction of area. This theory is shared by other

experimentalists, to whose work we need make no detailed reference, for the fact itself is really sufficient for our purpose. It must be borne in mind, however, that the total strength of a plate which has had a portion, or portions, removed cannot be so great as that of an entire or untouched plate, although its unit-strength may possibly be greater. Table XXXI. shows that the increase of strength due to drilling is inversely proportionate to the thickness of the plate, as we found the diminution of strength to be in the case of punched plates.

CHAPTER VIII.

RIVETS AND RIVETING.

45. **Forms of Rivets.**—Rivets are manufactured from round bars of iron or steel, of tough and ductile quality, and of suitable diameter. The bars are heated in a furnace, cut up into pieces of the required length, and each piece is pressed in a die, which upsets one end of the metal, so as to form a head. This part, for the purpose of distinguishing it from the "head" made in the process of riveting, is technically described as the "tail," and the other portion of the rivet is known as the "shank." Rivet shanks are generally parallel, but sometimes they are tapered towards the point, so that they may be easily placed into the rivet holes of a perforated plate. There are many different shapes in which rivets are made, and some of the most common of these are illustrated in figs. 26 to 30.

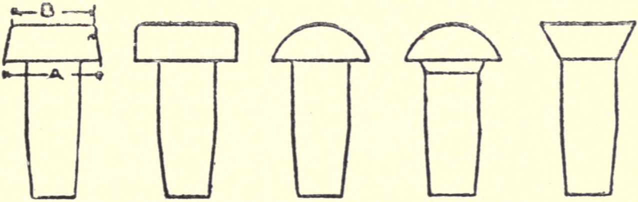


Fig. 26. *Fig.* 27. *Fig.* 28. *Fig.* 29. *Fig.* 30.

§ (*a*) *Proportions of Rivets.*—Unfortunately, no standard dimensions have yet been agreed upon with regard to the proportions of rivets, but those contained in Table XXXII. may be taken as fairly representative of good practice.

Table XXXII.—Average Proportions of Rivets.
(D=diameter of rivet.)

Fig. No.	Description.	Dia. of Tail.	Depth of Tail.	Dia. of Neck.	Depth of Neck.	Dia. of Point.
26	Pan-tail ...	(a) $D \times 1 \cdot 5$ (b) $D \times 1 \cdot 3$	$D \times \cdot 67$	—	—	$D \times \cdot 93$
27	Flat or "Cheese"-tail	$D \times 1 \cdot 5$	$D \times \cdot 67$	—	—	$D \times \cdot 93$
28	Cup or "Snap"-tail	$D \times 1 \cdot 6$	$D \times \cdot 65$	—	—	$D \times \cdot 93$
29	Do. do. with conical neck...	$D \times 1 \cdot 7$	$D \times \cdot 66$	$D \times 1 \cdot 125$	$D \times \cdot 5$	$D \times \cdot 93$
30	Countersunk-tail..	$D \times 1 \cdot 5$	$D \times \cdot 66$	—	—	$D \times \cdot 93$

In the process of riveting, the rivet is heated to a suitable temperature and is then placed in the hole. The head is formed by hammering, aided in some cases by the use of a riveting-set; or a riveting machine actuated by steam, hydraulic, or pneumatic pressure may take the place of hand labour. The necessary length of shank projecting through the plates for the formation of a satisfactory head is governed by the thickness of the plates, by the diameter of the rivet, and by the method of riveting adopted.

§ (*b*) *Length of Shank.*—As the shank of the rivet is swelled out during the process of riveting so as to completely fill the hole, it is obvious that the length of the projecting portion must be in some measure governed by the collective thickness of the plates. The diameter of the rivet is always proportioned to the thickness of the plates and to the kind of joint made; and as a general rule the diameter of the rivet is the basis from which the length of projection is calculated. Thus in the case of countersunk heads the projection is equal to the diameter of the rivet, to $1\frac{1}{4}$ or $1\frac{1}{2}$ times the diameter for raised heads in hand riveting, and to $1\frac{1}{2}$ or $1\frac{3}{4}$ times the diameter for similar heads in machine riveting. Fig. 31 represents a rivet projecting through two plates and the dotted arc shows the ultimate form of the head.

46. Proportions and Forms of Rivet Heads.—In Table XXXIII. the average proportions of various rivet heads are given, together with references to the figures in which the completed rivets are illustrated. The rivet heads are at the top in every instance.

Table XXXIII.—Average Proportions of Rivet Heads.
(D=diameter of rivet.)

Fig. No.	Description.	Diameter of Head.	Depth of Head.
32	Conical head	D×2	D×·75
33	Snap head	D×1·6	D×·63
34	Conoidal head	D×2	D×·75
35	Countersunk head	D×1·5	D×·5 to ·7
37	Countersunk elliptical head	D×1·5	D×·5 within the plate. D×·25 above the plate.

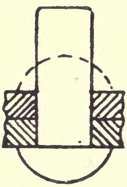


Fig. 31.

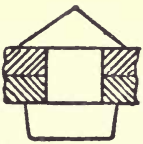


Fig. 32.

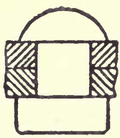


Fig. 33.

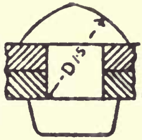


Fig. 34.

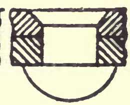


Fig. 35.

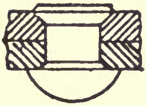


Fig. 36.

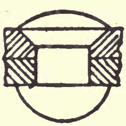


Fig. 37.

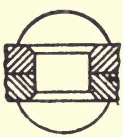


Fig. 38.

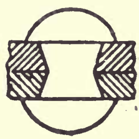


Fig. 39.

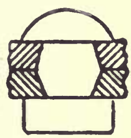


Fig. 40.

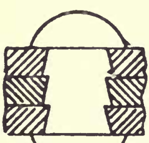


Fig. 41.

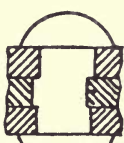


Fig. 42.

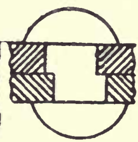


Fig. 43.

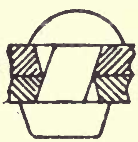


Fig. 44.

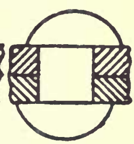


Fig. 45.

§ (*a*) *Modes of Making Various Heads.*—The conical head, illustrated in fig. 32, is made by hammering down the rivet point by the use of light hammers, and finishing the head with a riveting-set or die.

The cup, or snap, head, shown in fig. 33, is formed by hammering down the point of the rivet, and finishing the work by the aid of a cup-shaped die, commonly called a "snap."

The conoidal head, shown in fig. 34, has a radius equal to diameter of the rivet × 1·8; but this form is now very little used.

The countersunk head (fig. 35,) is formed by knocking down the rivet point into a conical hole, and sometimes, as shown in fig. 36, the head is finished so that it projects above the surface of the plate, in order to give an increase of strength. Another type of countersunk head is illustrated in fig. 37, where the height of the head above the surface of the plate equals the rivet diameter × ·25. Sometimes the plate is slightly countersunk in such a manner as to form a neck both at the head and tail, as in fig. 38, thereby adding to the strength of the rivet, and diminishing the cutting effect of the perforated edge of the metal.

47. Examples of Defective Riveting.—Riveting is an operation requiring a sufficient amount of attention to various points of detail, and if these be neglected, serious diminution of strength must ensue. Therefore, it is desirable that we should briefly examine the most frequent causes of defective work.

§ (*a*) *Wrong Arrangement of Plates.*—When two punched plates are to be joined, they should always be placed so that the holes come together, as in fig. 39; the holes are then completely filled by the rivet, and as the tension due to contraction is equally distributed throughout the whole length of the hole instead of being concentrated at the ends, the stress on the rivet head and tail is not excessive, and the configuration of the rivet is such that there is always a grip on the plates, even if the head or tail should be accidentally detached.

The converse arrangement of the plates is shown in fig. 40, where the rivet takes a barrel-like shape, which is in every way undesirable.

§ (*b*) *Non-cylindrical Hole through Two or more Plates.*—Supposing three or more plates have to be riveted, they should be clamped together in their proper relative positions, and drilled through at one operation so that the hole may be truly cylindrical throughout. If perforation be performed by punching, it is impossible for the hole through a series of plates to be cylindrical, because each individual hole is tapered, and if riveted up in that state the result would be a joint similar to that depicted in fig. 41. Supposing the individual plates are punched, and afterwards drilled or rimered to size, the separate holes will be cylindrical, but it is quite possible that the collective hole may more or less resemble the pattern of that indicated in fig. 42. In either event, the riveting would be bad, for, supposing the rivet were made so hot that the hole became entirely filled during the operation of closing-up, there would be a series of sharp angles in the shank tending to assist shearing stress. On the other hand, if the rivet were not heated sufficiently to permit of its substance being forced into all the eccentricities of the hole, a full bearing would not be obtained, and the rivet would become loosened in time, especially under the influence of live loads. Another example of defective riveting is shown in fig. 43, where a rivet is seen filling a partly obscured or "blind" hole, formed by the imposition of two inaccurately punched plates. If a hole of the last-named kind has to be encountered in the course of hand-riveting, it is generally the case that the space will not be completely filled, and the finished rivet will probably acquire a shape somewhat like that seen in fig. 44.

§ (*c*) *Badly Worked Rivets.*—A badly-headed rivet, as in fig. 45, can only result from carelessness. Rivets should not be used when above a dull red heat, as they are liable to injury if worked at too high a temperature. They ought to be quickly heated in a clean fire, in which they should not be allowed to remain for too long a time. The presence of dirt, or of scale, hinders the metal from filling the hole in a proper manner. If the length projecting through the plate before closing up be insufficient, the head will be too thin, and will be liable to curl up round the edge. On the other hand, if an excess of length be left projecting, a collar will be formed round the head. This should be allowed to remain, as any attempt to cut it away may damage the plate. Care should always be taken to avoid any injury to the plate when using a riveting-set.

48. Essentials of Good Riveting.—In a perfect riveted joint the various mechanical points to which we have referred must have received due attention, and the metal must entirely fill the hole, not only when the rivet is hot, but also after cooling. The latter condition is manifestly impossible, owing to the lateral contraction which must inevitably take place as the metal cools. Probably the most perfect joint possible would be one with drilled holes in which the rivets were turned and closed quite cold. In this case, however, the benefit resulting from longitudinal contraction would be lost. This effect is considered in Art. 57, in connexion with the strength of riveted joints.

§ (*a*) *Relative Suitability of Various Rivets.*—No hard-and-fast rule can be made as to the most suitable form of rivet for adoption in any particular case, because convenience, cost, and appearance have generally to be considered in addition to the question of strength. Conical heads are chiefly used on the fronts of steam boilers because of their neat appearance, but they are not so strong as snap-heads, which are very largely used in boiler, girder, and general engineering work. For locomotive boiler work snap-heads are generally used, but in ship-building practice countersunk rivets are largely adopted, their employment being essential in the outer plates of a vessel.

§ (*b*) *Comparative Strength.*—So far as comparative strength is concerned, there appears to be divergence of opinion, but the subjoined table will convey some useful, though not altogether consistent, information on the subject. The figures have been calculated from experiments conducted by Mr. Wildish at Pembroke Dock.

Table XXXIV.—Relative Values of Rivets.

1 in. dia. in $\frac{3}{4}$-in. Plates; $\frac{3}{4}$ in. dia. in $\frac{1}{2}$-in. Plates.

Description of Rivet.	1 in. Dia.		$\frac{3}{4}$ in. Dia.	
	Frictional Resistance in Tons.	Ratio.	Frictional Resistance in Tons.	Ratio.
Snap-head and tail	6·4	1·00	4·72	·738
Conical head and pan tail ...	7·36	1·15	4·52	·704
Countersunk head and pan tail	8·55	1·33	6·25	·976
Countersunk head and tail ...	9·04	1·41	4·95	·773

49. Methods of Riveting.—§ (*a*) *Hand Riveting.*—Riveting was formerly done entirely by hand, and the operation requires the aid of three men. The rivets are hung over a fire by means of a perforated plate, which prevents the tails from being overheated. One rivet is taken at a time and placed in the hole, the tail being kept close up to the plate by one man, with a hammer or "holder up," whilst the two other men form the head by hammering down the point, and finishing the work by the use of a die, or "snap." A lad is generally employed for the purpose of attending to the furnace, and of keeping the riveters supplied with hot rivets. Machine-riveting is now the rule, and hand-riveting the exception.

§ (*b*) *Steam Riveters.*—The older types of machine-riveters are driven by steam, acting upon a piston, at the end of which is a plunger, arranged so as to strike a sudden blow upon the rivet. Two dies are provided, one fitting over the tail and the other over the point of the rivet, so that upon the application of the necessary force the head is formed. Pressure exerted by a steam riveter is directly governed by the area of the piston and the pressure of the steam admitted.

§ (*c*) *Mechanical Riveters.*—A few steam riveting machines still survive, although as a class they were practically superseded some time ago by mechanical riveters, in which the hammer is moved backwards and forwards by means of a cam or other suitable device, and the machine is driven from shafting by means of a belt. This appliance is not approved for high-class work, because pressure cannot be varied, and no ready means exists for ascertaining what force is really being exerted.

§ (*d*) *Hydraulic Riveters.*—The two forms of apparatus above mentioned have been to a large extent displaced by the hydraulic riveter, which embodies several important advantages. For instance, pressure may be gradually applied to the rivet; variations of power may be made as required; and the exact force used can always be ascertained. Portable forms of the apparatus are made, which can easily be handled and taken to almost any part of a structure where work has to be done. This adaptability is highly advantageous in constructional ironwork, and it is also deservedly popular in the erecting shop. Water pressure is supplied by a flexible pipe, and the pressure varies from 1,000 lbs. to 1,750 lbs. per square inch. Owing to the high pressure employed there is always some risk of leakage, and in outdoor work during exceptionally cold weather there is a possibility that the water may be frozen. Another drawback is to be found in the cost of the machine itself and of auxiliaries, such as pumps and accumulators.

§ (*e*) *Pneumatic Riveters.*—Riveting apparatus worked by compressed air constitutes the most recent application of power to the needs of the structural engineer. The pneumatic riveter, like many other ingenious inventions, has been introduced into this country from the United States, and it appears to be regarded with a considerable amount of favour, especially so far as the more portable forms are concerned. Compressed air is taken from a receiver fed by a pump, and is led by means of a flexible pipe to the riveting machine, where it acts directly upon the piston. The piston-rod is connected with the plunger by a series of levers arranged in such a manner that the force exerted upon the ram continually increases during the progress of each stroke. As the pressure required is only from 100 lbs. to 125 lbs. per square inch, very little inconvenience is experienced in connexion with the conveyance of air from the receiver to the machine, and there can be no risk of freezing. An advantage is to be found in the fact that the first cost of the apparatus is considerably less than that of a hydraulic riveter.

§ (*f*) *Advantages of Machine Riveting.*—Riveting by the aid of mechanical power is in every way preferable to hand riveting. One reason, which appeals very forcibly to contractors, is to be found in the fact that rivets costing 3*l.* 15s. per thousand when put in by hand, will not average more than about 17s. 6d. when the work is done by a machine. Another advantage of much more importance is to be found in the greater strength of machine riveting, resulting from the heavier and more equable pressure obtained. The plates are therefore brought into closer contact, and a larger quantity of the metal composing the rivet shank is pressed into the hole. Further, if it should be thought desirable, the pressure upon the newly-formed rivet head may be sustained, so that the plates may be pressed together during the cooling of the metal; consequently, the grip of a rivet upon the plates is maintained without the risk of drawing apart whilst the rivet is still hot. Closely riveted plates offer considerable resistance to the penetration of moisture which would cause corrosion, and to the buckling of plates between the rivets under longitudinal compressive stress. Moreover, the friction produced tends to relieve the rivets from shearing stress. (Art. 57.)

§(*g*) *Relative Efficiency of Various Machines.*—Experiments made by Messrs. Greig and Eyth are indicative of differences in the strength and in other characteristics of riveting done by various machines. For instance, they show that "the plate, especially if soft, is much less injured by hydraulic riveting, and that this method has therefore a decided advantage where the plate is the weaker part; but that the rivet itself is stronger when put in by the steam-riveter, owing probably to the greater compactness of the rivet material obtained by the sudden shock." The following table embodies the results of some experiments as to the relations between pressure applied and the shearing resistance of the rivets put in by various machines.

Table XXXV.—Riveting by Various Machines.

(Steel rivets $\frac{5}{8}$ in. diameter, $\frac{11}{16}$ in. drilled holes.)

Kind of Machine.	Pressure on Rivet.	Shearing Resistance per sq. in.
Steam riveter	37 tons.	25·74 tons.
Power riveter (light)	31 ,,	22·5 ,,
,, ,, (heavy)	52 ,,	23·76 ,,
Hydraulic riveter (portable) ..	20 ,,	22·78 ,,
,, ,, (stationary) ...	39 ,,	23·80 ,,

CHAPTER IX.

RIVETED JOINTS.

50. Forms of Joints.—There are two primary methods of connecting together bars or plates by means of rivets or similar fastenings. The arrangement by which the edge of one piece overlaps the edge of another occurs in a *lap-joint*, and the disposition of the pieces end to end is exemplified in a *butt-joint*. When a lap-joint is made, continuity of the members connected is secured by the insertion of rivets; if a butt-joint is adopted there is no direct continuity, and a distinct break occurs, which is bridged over, so to speak, by super-imposed strips of material termed *cover-plates* or *butt-straps*. So far as actual strength is concerned, there is not much difference between the two kinds of joint. Lap and butt-joints are often made by the aid of bolts, but in describing and illustrating the chief varieties of such connexions, we shall now only make detailed reference to those in which rivets are employed.

§ (*a*) *Lap-joints.*

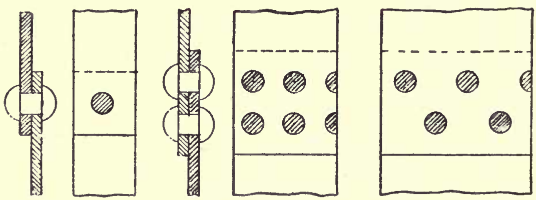


Fig. 46. *Fig.* 47. *Fig.* 48. *Fig.* 49. *Fig.* 50.

A *single-riveted* lap-joint, the simplest form, is illustrated in section by fig. 46, and in plan by fig. 47. A *double-riveted* lap-joint is seen in section in fig. 48, and two forms of rivet arrangement are indicated by figs. 49 and 50, the former joint being *chain-riveted*, and the latter *zig-zag-riveted.*

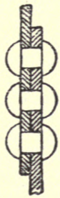
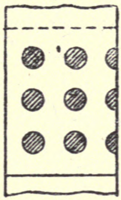
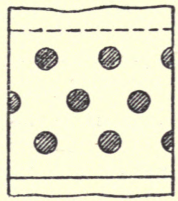
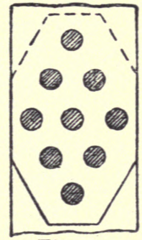





Fig. 51. *Fig.* 52. *Fig.* 53. *Fig.* 54.

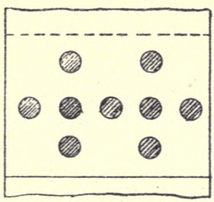
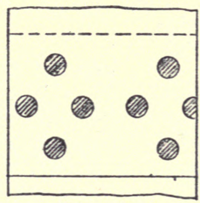
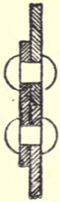
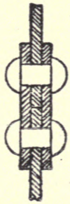
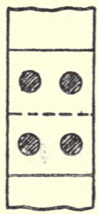
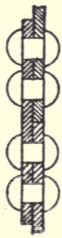
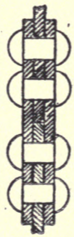
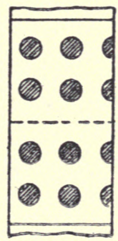
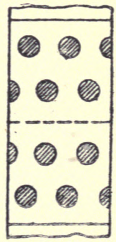



Fig. 55. *Fig.* 56.

Fig. 51 shows a *triple-riveted* lap-joint, and figs. 52 and 53 illustrate chain and zig-zag riveting respectively. It is unnecessary to illustrate further multiplication of the rows of rivets, except for the purpose of directing attention to fig. 54, which represents a very good type of lap-joint. Lap-joints with rivets of uneven pitch are shown in figs. 55 and 56.

§ (*b*) *Butt-joints.*

57. 58. 59. 60, 61, 62, 63.

Turning now to butt-joints, we see in fig. 57 the section of a joint with one cover-plate, and in fig. 58 one with two cover-plates. Fig. 59 is the plan of a single-riveted butt-joint. Sections of double-riveted butt-joints with one and with two cover-plates are given in figs. 60 and 61. A double-riveted butt-joint, with chain riveting, is shown in plan by fig. 62, and a similar joint, zig-zag-riveted, in fig. 63.

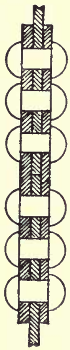
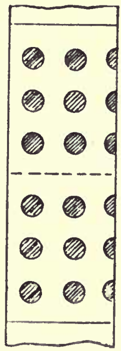
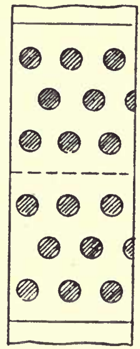
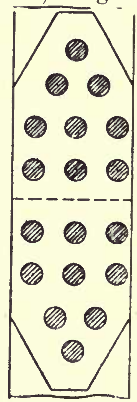
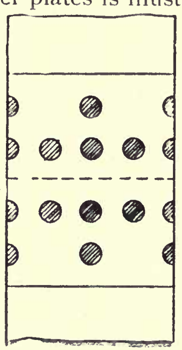






Fig. 64. *Fig* 65. *Fig.* 66. *Fig.* 67. *Fig.* 68.

Triple-riveted butt-joints with one and with two cover-plates are shown sectionally in figs. 64 and 65, and plans of chain and zig-zag riveting are to be seen in figs. 66 and 67. A useful and efficient type of butt-joint with two cover-plates is illustrated in fig. 68.

Fig. 69.



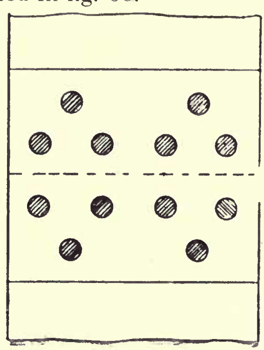


Fig. 70.

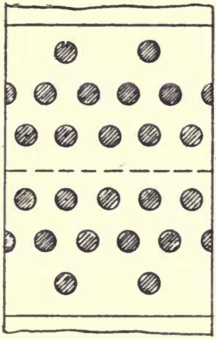


Fig. 71.

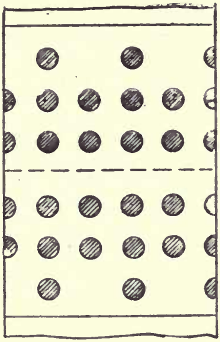


Fig. 72.

Butt-joints with rivets of uneven pitch are shown in plan by the figures following:—Fig. 69 is a double-riveted butt-joint, which would be chain-riveted if alternate rivets were not omitted from the outer rows; fig. 70 is a double-riveted butt-joint on the zig-zag system, but with the omission of alternate rivets in the outer rows; figs. 71 and 72 are examples of similar treatment as applied to triple-riveted butt-joints. It should be remarked that in all ordinary joints, whether butt or lap, the rivets are usually arranged in rows of even pitch.

51. Modes of Failure.—Riveted joints may fail in one of several ways, of which typical examples will now be brought to notice.

§ (*a*) *Shearing of the Rivet.*—The rivet in fig. 73 is in *single-shear*, a condition attaching to all lap-joints and to butt-joints with one cover-plate; the rivet in fig. 74 is in *double-shear*, a state associated with butt-joints having two cover-plates; fig. 75 is the plan of a sheared joint. A very natural and logical inference is that the strength of a rivet in double-shear ought to be exactly twice the strength of a similar rivet in single-shear. This actually happens in the case of steel rivets, but the resistance of iron rivets in double-shear is only ·75 more than that of rivets in single-shear. Further, the strength of rivets, whether in single or in double-shear, is never the same as that posessed by the material before being fixed in the joint. Thus in all calculations relative to riveted joints, the shearing strength of rivet material should be qualified by co-efficients indicating the ratios of loss due to specific conditions existing in joints. Numerous experiments have been made for the purpose of ascertaining the chief differences of resistance, and the classification of results so obtained affords valuable guidance to the structural engineer. Complicated calculations may, therefore, be avoided in ordinary practice, as the average shearing strength of iron or steel in joints can be taken directly from tables, some of which are given later. For

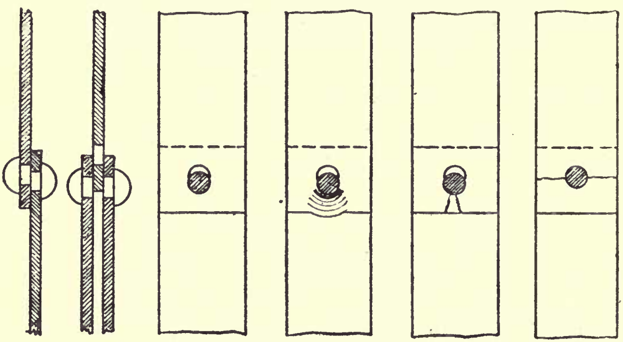


Fig. 73. *Fig.* 74. *Fig.* 75. *Fig.* 76. *Fig.* 77. *Fig.* 78.

our present purpose, the shearing strength per square inch of rivet section may be represented by the symbol f_s, and if the rivet area in inches be expressed as (dia.^{2} × ·7854) or ·7854d^2, then the force P required to fracture the joint by shearing of the rivet can be indicated thus:

$$P = \cdot 7854 d^2 f_s \quad \ldots\ldots\ldots\ldots \quad (1)$$

§ (*b*) *Crushing of the Plate.*—Supposing the rivet to be stronger than the plate, the latter may be crushed, as shown by fig. 76. Resistance to strain under such a condition is higher than the ordinary crushing strength of the material, because the part most severely strained is supported by the surrounding metal, and by the head and tail of the rivet. The value of the crushing strength in this

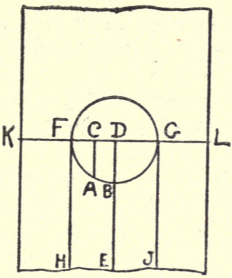


Fig. 79.

position is known as the result of experiments, and as the resistance of any portion AB of the circumference (fig. 79) equals the resolved portion CD perpendicular to the line of strain, the force required to cause failure is :—

$$P = f_c dt \quad \dots\dots\dots\dots \quad (2)$$

where P = the force, f_c = crushing strength, d = diameter of the rivet, and t = thickness of the plate.

§ (*c*) *Bursting of the Plate.*—A joint may fail by rupture of the plate, as in fig. 77, which is equivalent to breakage along the line BE (fig. 79). In this case the part FHJG opposing the rivet can be likened to a continuous girder under a uniform load, and the ultimate strength equals *thickness of plate* × *depth* BE^2 ÷ *length* FG × (*a*).

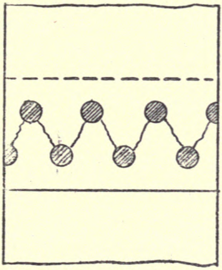


Fig. 80.

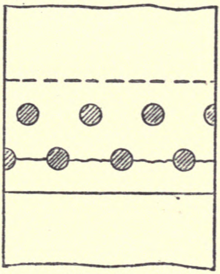


Fig. 81.

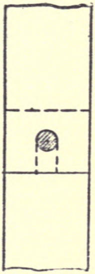


Fig. 82.

The value of the constant (*a*) has not been the subject of much inquiry, but as it is stated by Fairbairn to be 38 tons for wrought iron, the value for steel should be at least 48 tons.

Denote FG by *d*, BE by *h*, thickness of plate by *t*, and the force P, required to rupture the plate at its end will be found by the equation :

$$P = \frac{th^2}{d} \times a. \quad \dots\dots\dots\dots \quad (3)$$

§ (*d*) *Tearing of the Plate.*—One or both of the plates may be torn along the line of rivets, as indicated in figs. 78, 80, and 81.

Referring to fig. 79, if the line of fracture be KL, and if the thickness of the plate be t, the effective sectional area for the resistance of strain will be $(KF+GL) \times t$.

Let $b = (KF+GL)$, and f_t = tensile strength of the plate, then the force necessary to cause fracture is thus expressed:—

$$P = f_t t b \quad \ldots\ldots\ldots\ldots \quad (4)$$

§ (*e*) *Shearing of the Plate.*—It is possible, though not probable, that a rivet might force its way out of a joint in the manner shown by fig. 82. If such an effect were produced, the required force would be measured thus:

$$P = f_s t\,(FH+GJ) \quad \ldots\ldots\ldots\ldots \quad (5)$$

§ (*f*) *Examples*:—With the object of demonstrating the practical value of the above-mentioned rules, an example of each will now be calculated upon the following basis:—

Lap-joint, as fig. 46, with one rivet 1 in. dia.; steel bars 3 in. wide by ·5 in. thick; distance of hole from end of bar $(= h)$, 1 in.; P = force required to cause failure of joint; f_s = 22 tons; f_c = 40 tons; f_t = 30 tons; a = 48 tons.

			Tons.
Ex.—1.	$P = \cdot 7854 d^2 f_s$	$= \cdot 7854 \times 1 \times 22$	$= 17{\cdot}27$
,, 2.	$P = f_c d t$	$= 40 \times 1 \times \cdot 5$	$= 20{\cdot}00$
,, 3.	$P = \dfrac{th^2}{d} \times a$	$= \dfrac{\cdot 5 \times 1}{1} \times 48$	$= 24{\cdot}00$
,, 4.	$P = f_t t b$	$= 30 \times \cdot 5\ (1+1)$	$= 30{\cdot}00$
,, 5.	$P = f_s t\,(FH+GJ)$	$= 22 \times \cdot 5\ (1{\cdot}5 + 1{\cdot}5)$	$= 33{\cdot}00$

Similar calculations may be applied to every form of lap and butt-joint.

In practice only three modes of failure need be taken into account—viz., (1), (2), and (4). The others (3) and (5) are interesting, but seldom, if ever, occur.

It should be said that, if conditions occur such as those in Example 4, the metal is liable to unequal strain, and for this reason it might be thought safer to take f_t at a somewhat lower value than 30 tons, if it were not for the fact that the factor of safety is always applied to the rivet strength, which is by far the weaker element of the joint.

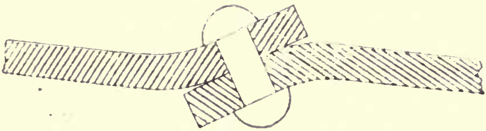


Fig. 83.

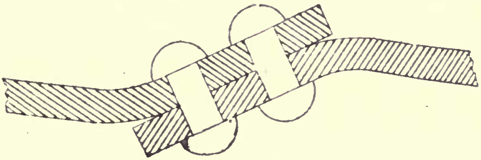


Fig. 84

§ (*g*) *Injury to Lap-Joints by bending.*—Single-riveted lap-joints are apt to bend considerably when under severe stress, as in fig. 83, and tearing of the plates is liable to occur, partly because of bending along the line of holes, and partly owing to the establishment of unequal compressive stress. In addition to the endurance of shear, the rivets have to withstand an excessive amount of tensile strain in consequence of bending, and it is quite possible their heads may thereby be forced off. Less harm is done to double-riveted lap-joints in this way, as they are very slightly bent. (See fig. 84.)

52. Theoretical Proportions of Joints.—Summing up what has been said above with regard to riveted joints, we see that failure of the rivet by shearing is affected by its diameter; that failure by crushing of the plate is affected by the diameter of the rivet and the thickness of the plate; and that failure by tearing of the plate is affected by its thickness and its width on each side of the rivet. The various form of resistance should be as nearly as possible equal, for excess of strength in one direction can only be obtained by diminution of strength in another direction. Thus, if the number of rivets be increased, the resistance to shearing will be higher, but the effective plate area along the line of rivets must be diminished; if the number of rivets be diminished, the strength of the plate will be increased, but resistance to shearing must be less; if again the number of rivets be reduced and their diameter be increased, resistance both to shearing and to tearing will be augmented, but resistance to crushing will be at the same time reduced. Therefore, the necessity for striking a happy medium becomes evident.

Correct theoretical proportions between the diameter of the rivet and the thickness of the plate may be ascertained by comparing modes of failure (1) and (2); and the pitch or distance of the rivets one from another can be found by comparing modes (2) and (4) or modes (1) and (4) (Art. 51). Thereby equations can be evolved by the aid of which the theoretical proportions of riveted joints are easily calculated.

§ (*a*) *Rules for Theoretical Proportions.*—In all rules of the kind now given the following symbols are used as required:—

d = diameter of the rivet in inches,
t = thickness of the plate in inches,
p = transverse pitch in inches,
f_t = tensile strength per sq. in. of net plate area between the holes,
f_s = shearing strength per sq. in. of rivet section,
f_c = crushing strength per sq. in. of the plate,
c = co-efficient for double shear = 1·75 for iron,
= 2·00 for steel.

Assuming failure to result from simultaneous shearing of the rivet and crushing of the plate, the equation of modes (1) and (2) shows that $\cdot 7854 d^2 f_s = f_c dt$. Consequently, the subjoined rules are deducible:—

To determine the diameter of a rivet in single shear in a single or double riveted joint:—

$$d = \frac{f_c t}{\cdot 7854 f_s} \quad \ldots\ldots\ldots\ldots \quad (6).$$

To find the ratio between rivet diameter and thickness of the plate.

$$\frac{d}{t} = \frac{f_c}{\cdot 7854 f_s} \quad \ldots\ldots\ldots\ldots \quad (7).$$

To determine the diameter of a rivet in double shear in a single or double riveted joint :—

$$d = \frac{f_c t}{c \times \cdot 7854 f_s} \quad \ldots\ldots\ldots\ldots \quad (8).$$

To find the ratio between diameter and thickness :—

$$\frac{d}{t} = \frac{f_c}{c \times \cdot 7854 f_s} \quad \ldots\ldots\ldots\ldots \quad (9).$$

Supposing failure to result from simultaneous crushing and tearing of the plate, we find, by comparing modes (2) and (4) (Art. 51), that $f_c dt = f_t tb$; and as b is equivalent to $(p - d)$, the following rules are obtained :—

To determine the pitch in single riveted joints in single or in double shear :—

$$p = \frac{f_c + f_t}{f_t} d \quad \ldots\ldots\ldots\ldots \quad (10).$$

To find the ratio between pitch and rivet diameter :—

$$\frac{p}{d} = \frac{f_c + f_t}{f_t} \quad \ldots\ldots\ldots\ldots \quad (11).$$

To determine the pitch in double riveted joints in single or in double shear :—

$$p = \frac{2f_c + f_t}{f_t} d \quad \ldots\ldots\ldots\ldots \quad (12).$$

To find the ratio between pitch and rivet diameter :—

$$\frac{p}{d} = \frac{2f_c + f_t}{f_t} \quad \ldots\ldots\ldots\ldots \quad (13).$$

The pitch of rivets may be calculated in another way by assuming failure to result from simultaneous shearing of one rivet and tearing of the plate area between two holes. Then, by comparison of modes (1) and (4), $\cdot 7854 d^2 f_s = f_t tb$.

As $b = (p - d)$, we have the rules :—

To determine the pitch in single riveted joints in single shear :—

$$p = \frac{\cdot 7854 d^2 f_s + f_t td}{f_t t} \quad \ldots\ldots \quad (14).$$

To determine the pitch in single riveted joints in double shear :—

$$p = \frac{c \times \cdot 7854 d^2 f_s + f_t td}{f_t t} \quad \ldots\ldots \quad (15).$$

To determine the pitch in double riveted joints in single shear :—

$$p = \frac{2 \times \cdot 7854 d^2 f_s + f_t td}{f_t t.} \quad \ldots\ldots \quad (16).$$

To determine the pitch in double riveted joints in double shear :—

$$p = \frac{c \times 2 \times \cdot 7854 d^2 f_s + f_t t d}{f_t t} \quad \ldots . \quad (17).$$

None of the foregoing equations can be applied without the aid of data obtained by experiment, and even then the calculated proportions are not always strictly suitable for adoption in actual practice. These are matters which will be discussed in the next chapter. In the meantime, the following approximate values are given of the strength of iron and steel under the various conditions involved.

*Table XXXVI.—Strength of Materials in Riveted Joints in tons per square inch.**

Description of Joint.	Tensile Strength. f_t		Shearing Strength. f_s		Crushing Strength. f_c	
	Iron.	Steel.	Iron.	Steel.	Iron.	Steel.
Single riveted lap	18	30	19	22	30	40
Double riveted lap	19	29	19	23	30	40
Single riveted butt (1 cover)	18	30	18	21	30	40
Double riveted butt (2 covers)	19	29	19	23	40	50

* (Iron plates punched, steel plates drilled.)

§ (*b*) *Examples :—*

Ex. 6.—Find the diameter of a rivet in single shear in a single riveted lap-joint, of ½-in. iron plate :—

By Rule 6 :—

$$d = \frac{f_c t}{\cdot 7854 f_s} = \frac{30 \times \cdot 5}{\cdot 7854 \times 19} = 1 \cdot 005 \text{ in.}$$

Ex. 7.—Find the rivet diameter in a similar joint, of ½-in. steel plate :—

By Rule 6 :—

$$d = \frac{f_c t}{\cdot 7854 f_s} = \frac{40 \times \cdot 5}{\cdot 7854 \times 22} = 1 \cdot 151 \text{ in.}$$

Ex. 8.—Find the pitch for a single riveted lap-joint, in ½-in. iron plate; rivet diameter = say, 1 in. (example 6) :—

By Rule 10 :—

$$p = \frac{f_c + f_t}{f_t} d = \frac{30 + 18}{18} \times 1 = 2 \cdot 66 \text{ in.}$$

By Rule 14:—

$$p = \frac{\cdot7854d^2f_s + f_t td}{f_t t} = \frac{\cdot7854 \times 19 + 18 \times \cdot5}{18 \times \cdot5} = 2\cdot658 \text{ in.}$$

Ex. 9.—Find the pitch for a similar joint, in steel plate rivet diameter $= 1\cdot15$ (example 7):—

By Rule 10:—

$$p = \frac{f_c + f_t}{f_t t} d = \frac{40 + 30}{30} \times 1\cdot15 = 2\cdot68 \text{ in.}$$

By Rule 14:—

$$p = \frac{\cdot7854d^2f_s + f_t td}{f_t t}$$

$$= \frac{(\cdot7854 \times 1\cdot15^2 \times 22) + (30 \times \cdot5 \times 1\cdot15)}{30 \times \cdot5} = 2\cdot673 \text{ in.}$$

53. Practical Strength of Joints.—The practical, as distinguished from the theoretical, proportions of riveted joints vary according to the class of work for which they are intended. We have already seen that the theoretical efficiency of a joint is calculated upon the shearing strength of the rivet, the tensile strength of the plate, and the crushing strength of the plate. The strength of an iron or steel rivet when in the joint is not necessarily the same as that exhibited by a bar of the material, or by a rivet before it has been fixed in place. Similarly, the unit strength of iron and steel plate differs according as the plate is (1) unperforated, (2) perforated but not riveted, (3) perforated and riveted, and (4) according to differences of mechanical treatment in the acts of perforation and riveting. Our business is now to find the strength of rivets and plates when actually in the joint, by comparison of existing records; and to deduce some simple standard which may be applied to the necessities of everyday work. So many variations are possible in the quality of material, and in the mode of mechanical treatment, that no one can be surprised if somewhat conflicting testimony appears to be furnished by the labours of experimentalists. This state of things is accentuated by the fact that no standard basis of procedure has been established, and the significance of ascertained results is thereby considerably discounted. If, however, information from various sources be collected together, data will be found for the calculation of fairly reliable averages.

Speaking in general terms it may be said that the shearing strength of iron and steel rivets is about 75 per cent. of their tensile strength. The mode of perforation adopted affects the practical strength of rivets in the joint. The tensile strength of rivets may suffer reduction in consequence of heating and working; thus a diminution of strength to the extent of 12 per cent. frequently occurs in the case of iron rivets, and of 24 per cent. in that of steel. On the other hand, if the rivets be carefully heated, and worked at a comparatively low temperature, their tensile strength should not be diminished, and may even be increased by the effect of mechanical treatment.

54. Strength of Rivets in the Joint.—§(*a*) *Shearing Strength.*—Commencing with the shearing strength of rivets, considerable divergencies are noticeable in recorded results, some of which are summarised by a joint process of analysis and condensation in the tables following.

Table XXXVII.—Shearing Strength of Iron Rivets in the Joint, in Tons per Square Inch of Rivet Area.

Authority.	Mode of Riveting.			Holes.		Single Shear Joints.			Double Shear Joints.
	Hand.	Steam.	Hydraulic.	Drilled.	Punched.	Single Riveted Lap.	Double Riveted Lap.	Single Riveted Butt.	Single & Double Riveted Butt.
Denny ..	—	—	19·30	19·30	—	—	—	—	19·30
,, ...	18·70	—	—	18·70	—	—	—	—	18·70
Greig & Eyth	—	17·87	19·35	18·43	—	18·43	—	—	—
Kirkaldy ...	—	17·92	—	—	17·92	—	—	17·92	—
Knight ...	—	—	—	—	18·60	—	18·60	—	—
,, ...	—	—	—	—	19·36	—	19·36	—	—
,, ...	—	—	—	—	19·54	—	19·54	—	—
Martell ...	—	—	—	—	19·20	—	19·20	—	—
Stoney ...	18·28	—	—	18·28	—	18·28	—	—	—
,, ...	18·84	—	—	—	18·84	18·84	—	—	—
,, ...	19·35	—	—	—	19·35	—	19·35	—	—
Average...	18·79	17·89	19·31	18·67	18·97	18·51	19·21	17·92	19·00

According to Table XXXVII., no great difference exists in the shearing strength of iron rivets closed by hand, steam, or hydraulic power. Steam riveting appears in an unfavourable light, and curiously enough, partly owing to the inclusion in the table of results given by Greig & Eyth, whom we have previously quoted as having expressed the general opinion that "the rivet itself is stronger when put in by the steam riveter" (Art. 49 § *g*). Of course, it must be borne in mind that riveting for experimental work is more carefully performed than it would be in the ordinary way. Therefore, unless tests are made for the specific purpose of ascertaining the relative values of various modes of riveting, it is not clear that any very marked differences will be evidenced, so far as shearing strength is concerned. With

Table XXXVIII.—Shearing Strength of Steel Rivets in the Joint, in Tons per Square Inch of Rivet Area.

Authority.	Mode of Riveting.			Holes.		Single Shear Joints.			Double Shear Joints.
	Hand.	Steam.	Hydraulic.	Drilled.	Punched.	Single Riveted Lap.	Double Riveted Lap.	Single Riveted Butt.	Single & Double Riveted Butt.
Denny ...	—	—	—	23·95	—	—	—	—	23·95
,, ...	22·45	—	—	22·45	—	—	—	—	22·45
Greig & Eyth	—	25·74	23·29	23·71	—	—	—	—	23·71
,,	—	—	—	—	25·62	25·62	—	—	—
,,	—	—	—	23·67	—	23·67	—	—	—
,,	—	—	—	22·72	—	—	—	22·72	24·93*
Kennedy ...	22·02	—	—	22·02	—	22·02	—	—	—
,, ...	24·80	—	—	24·80	—	—	24·80	—	—
,, ...	—	—	21·33	21·33	—	—	21·33	—	—
Kirkaldy ...	—	—	—	—	23·20	23·20	—	—	—
,, ...	—	—	—	—	26·00	—	26·00	—	—
,, ...	—	—	—	—	—	—	—	—	24·60
Martell ...	—	—	—	—	24·10	—	24·10	—	—
,, ...	—	—	—	24·25	—	—	24·25	—	—
Average...	23·09	25·74	22·31	23·21	24·73	23·62	24·09	22·72	23·92

* Double riveted butt.

regard to methods of perforation and their influence upon shear, a slight advantage is presented by punched over drilled holes, unless the "arrises," or sharp edges, of the latter be scraped or slightly countersunk.

As for the form of the joint, the results are fairly proportionate, except in the case of single-riveted butt joints.

Turning to steel rivets, we find in Table XXXVIII. that the relative effects of the three modes of riveting are not very clearly demonstrated, but in other respects the averages are more correctly proportionate than those observed in the previous table. It is probable that the approximate values in Table XXXIX. should not be exceeded in calculations of the shearing strength of iron and steel rivets in joints.

Table XXXIX.—Approximate Shearing Strength of Iron and Steel Rivets in Joints, per Square Inch of Rivet Area.

Form of Joint.	Iron.	Steel.
SINGLE SHEAR.		
Single-riveted lap joint, punched holes	19 tons	23 tons
,, ,, ,, ,, drilled ,, ...	18 ,,	22 ,,
Double ,, ,, ,, punched ,, ...	19 ,,	24 ,,
,, ,, ,, ,, drilled ,, ...	18 ,,	23 ,,
Single-riveted butt ,, punched ,, ...	18 ,,	22 ,,
,, ,, ,, ,, drilled ,, ...	17 ,,	21 ,,
Double ,, ,, ,, punched ,, ...	19 ,,	24 ,,
DOUBLE SHEAR.		
Double-riveted butt-joint, punched ,, ...	19 ,,	24 ,,
,, ,, ,, ,, drilled ,, ...	18 ,,	23 ,,

§ (*b*) *Tensile Strength.*—Rivets are always subjected to tensile stress caused by contraction of the metal in the act of cooling, and when the rivets are of considerable length the heads are sometimes drawn off. As contraction ought to be proportionate to length, it is not perfectly clear why failure should result in this way; but the most likely explanation is that tension may be concentrated beneath the head, where heat is less rapidly dissipated than in other parts. Again, rivets are subjected to tensile stress when a lap-joint becomes bent, and when employed in details of structural work so that they have to support weights. It would never be safe to take the tensile strength of the material as representing the actual resistance in any of these cases, for the true measure must be the strength of the weakest part. Data for the establishment of this measure are very scarce, but Stoney says that in some experiments made by him, with $\frac{3}{4}$-in. iron rivets in punched holes, the heads flew off with an average pull of 7 tons per rivet, or 12·32 tons per square inch of rivet area, and he adopts a factor of safety of 5 for this class of work in girders. If rivets be used for supporting weights, care should always be taken to make the heads of adequate proportions. Countersunk heads are particularly unsuitable, as they are liable to be drawn through the holes by a pressure less than that required to fracture the rivets themselves.

55. Strength of Plates in the Joint.—Some experiments made by Stoney for the Royal Irish Academy upon iron plates illustrate in a striking manner the differences of tensile strength exhibited by unperforated, perforated, and riveted perforated plates. Samples taken from the same plate were perforated, and in some of them hot dummy rivets were inserted and closed up. Each sample was 8 in. wide, $\frac{3}{8}$ in. thick, and perforated with five holes ·82 in. diameter. Compared with a test specimen of the solid plate the variations of tensile strength were as follows :—

Table XL.—Tensile Strength of variously treated Plates per Square Inch of Net Section (Stoney).

Condition of Plate.	Tons per sq. in.
Unperforated plate	24·0
Punched plate, without rivets	22·1
Drilled plate ,, ,, ...	23·8
Punched plate, with rivets..........	25·16
Drilled plate ,, ,, ...	28·84

The increased strength of the riveted plates may be attributed to one or more of three causes: (1) to the contraction of the rivets, causing friction on the metal around the holes; (2) to the annealing effect of the hot rivets on the plate; and (3) to the suppression of elongation under longitudinal stress. Whatever be the correct explanation, the effect itself is important to the structural engineer.

§ (*a*) *Iron Plates.*—The tensile strength of iron plates in the joint is more fully shown by a collection of data in Table XLI. :—

Table XLI.—Tensile Strength of Perforated Iron Plates in Various Joints, in Tons per Square Inch of Net Sections.

Form of Joint.	Holes.		Original Plate (calculated.)	Authority.
	Punched.	Drilled.		
Single-riveted lap ...	17·55	—	25·7	Fairburn.
	17·96	—	25·0	Hendry.
	17·16	—	21·4	Master Mechanics' Asso.
	22·30	—	26·7	Stoney.
	—	19·39	21·9	,,
	16·80	—	22·2	Greig & Eyth.
	—	19·75	22·2	,, ,,
Double-riveted lap ...	23·50	—	25·7	Fairbairn.
	25·57	—	22·0	Kirkaldy.
	16·35	—	18 7	Easton & Anderson.
	12·08	—	21·4	Knight.
	—	21·17	23·3	Greig & Eyth.
Single-covered butt ...	24·07	—	25·7	Fairbairn.
	19·95	—	22·0	Martell.
	—	18·07	22·2	Greig & Eyth.
Double-covered butt...	21·44	—	25·7	Fairbairn.
	19·37	—	22·4	Kirkaldy.
	—	20·65	22·2	Greig & Eyth.
	17·52	—	19·4	Knight.
Average	19·4	19·8	22·93	

From this table we find that strength is influenced by the form of joint as well as by the mode of perforation. Unfortunately, experimental evidence on these points is by no means of uniform character, and it is absolutely essential that allowance must be made for some preponderating results if reliable averages are to be deduced. We see that tenacity is more reduced by punching than by drilling, but as some high entries in the first column are not balanced by corresponding figures in the second, the real difference of tenacity is not fairly represented by the general averages of the two columns. The same trouble arises when the respective values of various kinds of joint are sought, and allowance must again be made for the fragmentary nature of existing records, but the figures stated in Table XLII. may safely be adopted when calculating the strength of iron plates in the joint.

Table XLII.—Approximate Tensile Strength of Iron Plates in Joints, per Square Inch of Net Section.

Form of Joint.	Tons per sq. in.	
	Punched.	Drilled.
Single-riveted lap	18	19
Double ,, ,, ...	19	20
Single-covered butt	18	19
Double ,, ,, ...	19	20

§ (*b*) *Steel Plates.*—Turning to records bearing upon the tensile strength of steel plates in the joint, we find it more easy to arrive at satisfactory conclusions, because investigation has been conducted in a more complete and therefore more intelligible manner.

Table XLIII. contains some general data furnished by well-known experimentalists, and it will be seen that a very marked difference occurs between the tensile strength of punched and drilled plates in the joint. Even omitting from consideration 1-in. plates, which exhibit exceptionally low results, punched plates have an average tensile strength of only about 29 tons as against about 32 tons in the case of drilled plates, the average unit-strength of the former being approximately equal to that of the unperforated plate, whilst the average unit-strength of the latter exceeds that of the original plate by about 3 tons. Examination of the Board of Trade tests in the same table will show that this excess of strength diminishes as the thickness of the plate is increased. We have already noticed a similar effect in perforated but unriveted plates (Arts. 42, § *a* and 44).

Table XLIII.—Tensile Strength of Perforated Steel Plates in Various Joints in Tons per Square Inch of Net Section.

Form of Joint.	Thickness.	Holes. Punched.	Holes. Punched and Annealed.	Holes. Drilled.	Original Plate.	Authority.
Single-riveted lap	in. $\frac{3}{8}$	—	—	32·73	29·71	Kennedy.
	$\frac{3}{4}$	—	—	30·73	27·18	,,
Double-riveted lap (zig-zag) ...	$\frac{3}{8}$	—	—	33·16	29·97	,,
,, ,, ...	$\frac{3}{4}$	—	—	30·68	28·46	,,
,, ,, ...	$\frac{5}{16}$	29·17	—	—	29·1	Kirkaldy.
	$\frac{7}{16}$	25·99	—	—	28·4	,,
Triple ,, (chain)	$\frac{1}{4}$	—	—	35·96	31·6	Board of Trade.
	$\frac{1}{2}$	—	—	34·18	29·1	,,
	$\frac{3}{4}$	—	—	30·46	28·6	,,
	1	—	—	29·44	27·7	,,
	$\frac{1}{4}$	31·7	32·9	—	29·9	,,
	$\frac{1}{2}$	30·3	32·2	—	28·0	,,
	$\frac{3}{4}$	27·8	30·6	—	28·7	,,
	1	13·6	29·5	—	28·4	,,
Double-riveted butt with two covers (zig-zag)	$\frac{3}{8}$	—	—	30·1	28·24	Kennedy.
Double-riveted butt with two covers (chain).	$\frac{11}{16}$	—	—	28·7	27·0	Martell.
Average ...	—	26·42	31·3	31·61	28·75	

The standards given in Table XLIV. below may be adopted as applicable to the tensile strength, in the joint, of steel plates up to $\frac{3}{4}$ in. thick :—

Table XLIV.—Approximate Tensile Strength of Steel Plates in Joints per Square Inch of Net Section.

Form of Joint.	Tons per square inch. Drilled.	Tons per square inch. Punched.
Single-riveted lap	30	27
Double ,, ,, ...	29	27
Single ,, butt-one cover ..	30	27
Double ,, butt-two covers ..	29	27

The above figures are based upon a strength of 28 tons in the original plate, and allowance is made in the case of double-riveted joints with drilled holes, for smaller increments of strength, because less benefit is derived from perforation, owing to the greater pitch of the rivets. The excess in the tenacity of the plate probably disappears entirely when the pitch of the rivets is as wide as that generally adopted in building work.

56. Bearing Pressure.—Another matter for consideration is the effect produced upon plates of iron and steel by the bearing pressure of rivets. When joints are tested to the breaking point, the bearing pressure per square inch does not as a rule exceed :—

$$\left.\begin{array}{l}\text{30 tons for iron} \\ \text{40 ,, ,, steel}\end{array}\right\} \text{in single shear.}$$

$$\left.\begin{array}{l}\text{40 ,, ,, iron} \\ \text{50 ,, ,, steel}\end{array}\right\} \text{in double shear.}$$

If these pressures be exceeded, it is believed that tenacity of the plate becomes reduced, and although rupture may not actually ensue the plate will probably be crippled. Heavy bearing pressure has also a detrimental effect on the resistance of rivets to shearing stress, and there is every reason for fixing a limit beyond which strain should not be permitted. It is quite possible that excessive bearing pressure may unintentionally be produced as the result of insufficient care in calculating the rivet area necessary for resisting the stress resulting from a given load. That this contingency may very easily arise will be evident if we remember that the plate resists bearing pressure in proportion to the diameter of the rivet × thickness of the plate ($d \times t$), whilst the rivet resists shearing stress proportionately to its area (a). Therefore, if the thickness of the plate be constant, the shearing area of the rivet may increase much more rapidly than does the bearing area.

§ (a) *Examples.*—To illustrate this point, let us take three examples of iron rivets and plates of lap-joints, in which the following rules are used :—

Shearing area of rivet $= a$ .... (18)

,, strength ,, $= a \times f_s$ .... (19)

Bearing area ,, $= d \times t$ .... (20)

Crushing strength of plate $= (d \times t) \times f_c$ .... (21)

Ex. 10.—Rivet $\frac{1}{2}$ inch diameter in joint of $\frac{1}{4}$-inch plate.

Shearing area $=(\cdot 5)^2 \times \cdot 7854 = \cdot 1963$ sq. in.
Shearing strength $= \cdot 1963 \times 19 = 3\cdot 72$ tons.
Bearing area $= \cdot 5 \times \cdot 25 = \cdot 125$ sq. in.
Crushing strength $= \cdot 125 \times 30 = 3\cdot 75$ tons.

Here the proportion is quite safe and fairly in accordance with theory.

Ex. 11.—Rivet $\frac{3}{4}$ inch diameter in $\frac{1}{4}$-inch plate.

Shearing area $=(\cdot 75)^2 \times \cdot 7854 = \cdot 44$ sq. in.
Shearing strength $= \cdot 44 \times 19 = 8\cdot 36$ tons.
Bearing area $= \cdot 75 \times \cdot 25 = \cdot 1875$ sq. in.
Crushing strength $= \cdot 1875 \times 30 = 5\cdot 62$ tons.

Here a considerable deficiency of strength is observable in the plate as compared with the rivet.

Ex. 12.—Rivet 1 inch diameter in $\frac{1}{4}$-inch plate.

Shearing area $=(1)^2 \times \cdot 7854 = \cdot 7854$ sq. in.
Shearing strength $= \cdot 7854 \times 19 = 14\cdot 92$ tons.
Bearing area $= 1 \times \cdot 25 = \cdot 25$.
Crushing strength $= \cdot 25 \times 30 = 7\cdot 5$ tons.

Again a deficiency is to be seen, but much more marked in degree.

In these examples the ultimate shearing and crushing strengths are used, but in practice a factor of safety of from 4 to 5 would, of course, be adopted.

57. Friction of Plates in the Joint.—Friction of the plates in a joint, due to contraction of the rivets in cooling, is often sufficient to prevent any slip so long as the stress is no greater than that prevailing under ordinary working conditions. Thus the rivets may be entirely exempt from shearing stress, and the plates may be similarly protected from crushing, or crippling, pressure. Even if this state of things exists, the inference must not be drawn that the ultimate strength of a joint is in any way increased thereby. The extent to which friction is caused depends upon the nature of the material, the shape of the rivet, and the mode of riveting.

§ (*a*) *Co-efficient of Friction.* — Widely varying statements are made by different writers as to the value of the co-efficient of friction (f) for iron and steel plates.

Experiments by Morin upon plane surfaces for some time in contact, showed that $(f) = 0\cdot 19$.

Clark, in his work on the Britannia and Conway Tubular Bridges takes the value of (f) as 0·18. Others suggest that $(f) = \cdot 25$, and Stoney states that in some experiments made by him upon ordinary steel plates pressing on each other, the co-efficient proved to be as high as 0·6. This estimate is not representative of the value of (f) for clean and absolutely plane surfaces of the metal, and ought only to be accepted on the basis:—$(f) = 0 \cdot 6$ for (steel + inequalities + dirt). The contractile strength of hand-made iron rivets was stated in Art. 54 §*b*, as 12·32 tons per square in., and the frictional resistance of steel plates held together by iron rivets ought not to be greater than $0 \cdot 6 \times 12 \cdot 32 = 7 \cdot 39$ tons per square in. of rivet section.

§ (*b*) *Friction as Determined by Visible Slip.*—In order that the above computation may be compared with ascertained results, we subjoin Table XLV., in which all records are expressed in uniform terms of tons per square inch of rivet area.

From this table the following approximate conclusions may apparently be drawn:—

(1.) That the average frictional resistance of riveted joints is about 8·8 tons per square inch, both for iron and for steel;

(2.) That the ratio of hand to machine riveting is as 1 : 2·4;

(3.) That the ratio of single to double riveting is as 1 : 1·9;

(4.) That if the value of $\frac{3}{4}$-in. rivets per square inch of area = 1; then the values of $\frac{7}{8}$ in. and 1 in. rivets are 0·9 and 0·75 respectively.

(5.) That differently-shaped rivet heads do not affect frictional resistance to a very serious degree.

§ (*c*) *Slipping Loads for Various Forms of Riveting.*—Further experiments are needed for the complete establishment of a basis from which the friction of joints may be correctly calculated. In the meantime, it is probable that fairly accurate calculations may be made by the use of co-efficients obtained by slightly modifying the ratios mentioned above. Table XLVI. has been calculated by taking the value of a $\frac{3}{4}$-in. rivet at 5·6 tons per square in., as derived from the experiments of Professor Kennedy, and the following modified ratios have been adopted for different modes of treatment, viz.:—

Table XLV.—Friction of Riveted Joints, as judged by Visible Slip, in Tons per Square Inch of Rivet Area.

Authority and Metal.	Diameter of Rivet.	Mode of Riveting.		Form of Riveting.		Shape of Rivet Head.		
		Hand.	Machine.	Single.	Double.	Snap.	Conical.	Countersunk.
Iron.								
Clark ...	$\frac{7}{8}$ in.	—	—	—	9·3	—	—	—
Reed ...	$\frac{3}{4}$ in.	—	—	—	10·6	10·6	—	—
	,,	—	—	—	11·3	—	11·3	—
	,,	—	—	—	9·4	—	—	9·4
	1 in.	—	—	—	7·2	7·2	—	—
	,,	—	—	—	8·9	—	8·9	—
	,,	—	—	—	5·4	—	—	5·4
Average...	—	—	—	—	8·8	8·9	10·1	7·4
Steel.								
Kennedy ...	$\frac{3}{4}$ in.	5·6	—	5·6	—	—	—	—
	,,	7·2	—	—	7·2	—	—	—
	,,	—	15·9	—	15·9	—	—	—
Reed ...	,,	—	—	—	9·0	9·0	—	—
	,,	—	—	—	10·8	—	10·8	—
	,,	—	—	—	8·0	—	—	8·0
Wildish ...	,,	—	—	—	10·7	10·7	—	—
	,,	—	—	—	10·2	—	10·2	—
	,,	—	—	—	12·7	—	—	12.7
Kennedy ...	1 in.	4·0	—	4·0	—	—	—	—
	,,	5·4	—	—	5·4	—	—	—
	,,	—	11·4	—	11·4	—	—	—
Reed ...	,,	—	—	—	7·5	7·5	—	—
	,,	—	—	—	6·9	—	6·9	—
	,,	—	—	—	6·3	—	—	6·3
Wildish ...	,,	—	—	—	8·1	8·1	—	—
	,,	—	—	—	9·3	—	9·3	—
	,,	—	—	—	11·1	—	—	11·1
Average...	—	5·5	13·6	4·8	9·4	8·8	9·3	9·5

Table XLVI.—Calculated Slipping Loads of Riveted Joints, in Tons per Square Inch of Rivet Area.

Diameter of Rivet.	Single Riveting.		Double Riveting.	
	Hand.	Machine.	Hand.	Machine.
$\frac{3}{4}$ in.	5·6	11·2	7·28	14·56
$\frac{7}{8}$ in.	4·48	8·96	5·82	11·64
1 in.	3·92	7·84	5·09	10·18

single-riveting : double-riveting as 1 : 1·3 ; hand-riveting : machine-riveting as 1 : 2 ; rivet diameter $\frac{3}{4}$ in. = 1·0, $\frac{7}{8}$ in. = 0·8, 1 in. = 0·7.

For the purpose of comparison we append Table XLVII., which is based on one by Professor Kennedy ; the only alteration being that the values are stated in tons per square in., instead of in tons per rivet.

Table XLVII.—Approximate Loads at which a Riveted Joint will commence to slip visibly, in Tons per Square Inch of Rivet Area (Kennedy).

Diameter of Rivet.	Single Riveting.		Double Riveting.	
	Hand.	Machine.	Hand.	Machine.
$\frac{3}{4}$ in.	5·6	—	6·8 to 7·9	15·9
1 in.	4·0	—	5·4	10·1 to 12·7

With regard to all experimental data, it must be remarked that slipping of the plates is assumed to be visible, and Professor Kennedy suggests it is possible that the observed slip may consist almost wholly of shear. Friction is never taken into account when the proportions of a joint are being calculated, as it may disappear entirely in process of time, owing to vibration and other causes.

58. Practical Proportions of Joints.—Engineers are quite agreed upon the desirability of conforming as far as possible to the theoretical proportions of riveted joints, but it is neither practicable nor convenient for the correct standard to be approached when the plates exceed $\frac{5}{8}$ in. in thickness. Theory provides for equality of resistance both in the plate and in the rivet. Every one is ready to admit that if, whilst possessing sufficient strength for their work, the rivets fail more readily than the plates, plate material must be wasted ; and, conversely, that if the plates fail more readily than the rivets, a waste of rivet material must be involved. But no one has succeeded, or is likely to succeed, in making practice invariably coincide with theory. Taking rivet diameter, for instance, we find by Rule 6, that with 1 in. plate the theoretical diameter of the rivet might be 2 in. for iron, and 2·3 in. for steel, in a single riveted lap-joint. Rivets of either diameter would be much too large for manipulation, unless by the aid of exceptionally powerful machinery. In boiler-making shops rivets are seldom used of more than $1\frac{1}{8}$ in. or $1\frac{1}{4}$ in. diameter, and in structural work there is a very obvious objection to the employment of unnecessarily large rivets. Looking at the theoretical pitch of the same joint, we find by Rule 10, that the rivets might be spaced 5·3 in. and 5·36 in., centre to centre, for iron and steel respectively. Neither of these pitches would be inappropriate for girder work, but the boiler-maker would object to them most strongly because of the impossibility of making steam-tight joints with the rivets so far apart. Therefore, in using thick plates it is undesirable to conform with theoretical rules. Hence we find that various standards have grown up, each being suited for its particular purpose, and only conforming to theory so far as may be convenient.

§ (*a*) *Proportions for Boilermaking.*—So far as boiler-making is concerned, the practice is to adopt the widest pitch consistent with tightness, to use rivets of the greatest practicable diameter, and to proportion the strength of the shell plates to the weaker element of the joint, whether it be represented by the rivet or the plate. Again, when single and double riveting both occur in the same boiler, one diameter is adopted for all the rivets for reasons of practical expediency.

Table XLVIII.—Boilermakers' Proportions for Riveted Iron Joints (Wilson).

All measurements are in inches.

Thickness of Plate.	Single Riveted Lap-joints.		Double-Riveted Lap-joints and Butt-joints, with Single Cover-plate.		Double Riveted Butt-joints with Double Cover-plate.		
	Rivet Dia.	Pitch.	Rivet Dia.	Pitch.	Rivet Dia.	Pitch.	Thickness of Cover.
$\frac{1}{4}$	$\frac{1}{2}$	$1\frac{1}{4}$	$\frac{1}{2}$	$1\frac{3}{4}$	—	—	—
$\frac{3}{8}$	$\frac{11}{16}$	$1\frac{3}{4}$	$\frac{11}{16}$	$2\frac{1}{4}$	$\frac{5}{8}$	$2\frac{1}{2}$	$\frac{1}{4}$
$\frac{1}{2}$	$\frac{3}{4}$	$1\frac{7}{8}$	$\frac{3}{4}$	$2\frac{1}{2}$	$\frac{5}{8}$	$2\frac{3}{4}$	$\frac{5}{16}$
$\frac{5}{8}$	$\frac{7}{8}$	$2\frac{1}{8}$	$\frac{7}{8}$	$2\frac{3}{4}$	$\frac{3}{4}$	3	$\frac{3}{8}$
$\frac{3}{4}$	1	$2\frac{1}{4}$	1	$3\frac{1}{4}$	$\frac{7}{8}$	$3\frac{1}{4}$	$\frac{7}{16}$
$\frac{7}{8}$	1	$2\frac{1}{4}$	1	$3\frac{1}{4}$	1	$3\frac{5}{8}$	$\frac{1}{2}$
1	$1\frac{1}{8}$	$2\frac{1}{2}$	$1\frac{1}{8}$	$3\frac{1}{2}$	$1\frac{1}{8}$	4	$\frac{9}{16}$

Table XLIX.—Boilermakers' Proportions for Riveted Steel Joints (Board of Trade).

All measurements are in inches.

Thickness of Plate.	Single Riveted Lap-joints.			Double Riveted Lap-joint.			Double Riveted Butt-joint. Two Cover-plates.		
	Rivet Dia.	Pitch.	Lap.	Rivet Dia.	Pitch.	Lap.	Rivet Dia.	Pitch.	Cover Plate.
$\frac{3}{8}$	$\frac{13}{16}$	$1\frac{15}{16}$	$2\frac{7}{16}$	$\frac{3}{4}$	$2\frac{11}{16}$	$3\frac{3}{4}$	—	—	—
$\frac{1}{2}$	$1\frac{1}{16}$	$2\frac{17}{32}$	$3\frac{3}{16}$	$\frac{7}{8}$	$2\frac{7}{8}$	$4\frac{3}{8}$	$\frac{11}{16}$	$2\frac{13}{16}$	$6\frac{7}{8}$
$\frac{5}{8}$	$1\frac{3}{16}$	$2\frac{21}{32}$	$3\frac{9}{16}$	$1\frac{1}{16}$	$3\frac{13}{32}$	$5\frac{5}{16}$	$\frac{13}{16}$	$3\frac{1}{4}$	$8\frac{1}{8}$
$\frac{3}{4}$	—	—	—	$1\frac{1}{4}$	$3\frac{15}{16}$	$6\frac{1}{4}$	$\frac{15}{16}$	$3\frac{19}{32}$	$9\frac{3}{8}$
$\frac{7}{8}$	—	—	—	—	—	—	$1\frac{1}{8}$	$4\frac{3}{8}$	$11\frac{1}{4}$
1	—	—	—	—	—	—	$1\frac{1}{4}$	$4\frac{13}{16}$	$12\frac{1}{2}$

§ (*b*) *Proportions for Ship-building.*—Special requirements are evidenced in connection with ship-building, and in this industry the practical proportions of riveted joints differ from those adopted in boiler-making and in structural work.

Table L.—Shipbuilders' Proportions for Riveted Iron Joints.

All measurements are in inches. Rivet pitch = 4 diameters.

Thickness of Plates.	Rivet Diameter.	Breadth of Lap.			Breadth and Thickness of Cover Plates.	
		Single Riveting.	Double Riveting.		Double Riveting.	Triple Riveting.
			Zig-zag.	Chain.		
$\frac{1}{4}$	$\frac{1}{2}$	$1\frac{3}{4}$	$2\frac{3}{4}$	3	$6\frac{1}{4} \times \frac{1}{4}$	$9\frac{1}{4} \times \frac{5}{16}$
$\frac{3}{8}$	$\frac{5}{8}$	$2\frac{1}{8}$	$3\frac{3}{8}$	$3\frac{3}{4}$	$7\frac{1}{2} \times \frac{3}{8}$	$11\frac{1}{4} \times \frac{7}{16}$
$\frac{1}{2}$	$\frac{3}{4}$	$2\frac{3}{4}$	4	$4\frac{1}{2}$	$9 \times \frac{1}{2}$	$13\frac{1}{2} \times \frac{9}{16}$
$\frac{5}{8}$	$\frac{13}{16}$	—	$4\frac{7}{16}$	$4\frac{7}{8}$	$9\frac{3}{4} \times \frac{5}{8}$	$14\frac{5}{8} \times \frac{3}{4}$
$\frac{3}{4}$	$\frac{7}{8}$	—	$4\frac{3}{4}$	$5\frac{1}{4}$	$10\frac{7}{8} \times \frac{3}{4}$	$16 \times \frac{7}{8}$
$\frac{7}{8}$	1	—	$5\frac{7}{16}$	6	$12\frac{1}{4} \times \frac{7}{8}$	$18\frac{1}{4} \times 1$
1	$1\frac{1}{8}$	—	$6\frac{1}{8}$	$6\frac{3}{4}$	—	$20\frac{1}{4} \times 1\frac{1}{8}$

§ (*c*) *Proportions for Structural Work.*—In joints for structural work theory is similarly disregarded. The pitch of the rivets may be decided with the especial object of avoiding undue reduction in the strength of the plates by numerous transverse perforations, and reasons of a practical nature may fix $\frac{3}{4}$ in. as the most suitable rivet diameter. Assuming these conditions, a sufficient number of rivets must be put in to withstand the stress coming upon the joint, and their arrangement will necessarily be governed by the available width of the plates. Several rows of rivets may therefore be required, and the number of rows (n) may be determined by the following rule :—

$$n = \frac{(p - d)\,t}{\cdot 7854 d^2}. \quad \ldots\ldots\ldots\ldots \quad (22)$$

A joint of this kind may very easily constitute a violation of theory, but it should be judged more by the criterion of suitability for the duty required than by abstract principles.

Rivets used in girder work may be of the sizes adopted by boiler-makers, but $\frac{3}{4}$-in. rivets are more largely employed than any others. As joints in structural ironwork have not to be caulked, the distance of rivets from centre to centre may be from 3 to 6 inches, but it should not exceed six diameters of the rivet, or about ten times the thickness of the plate, otherwise moisture will be apt to penetrate between the plates, causing rust, which may ultimately damage the joint. Thus, the maximum spacing for rivets of ordinary sizes should be:—$\frac{1}{2}$ in. (dia.), (p) = 3 in.; $\frac{5}{8}$ in. (dia.), (p) = $3\frac{3}{4}$ in.; $\frac{3}{4}$ in. (dia.), (p) = $4\frac{1}{2}$ in.; $\frac{7}{8}$ in. (dia.), (p) = $5\frac{1}{4}$ in.; 1 in. (dia.), (p) = 6 in. In builders' ironwork these pitches are frequently exceeded, possibly with a view to economy, but the practice should always be discouraged. Very often it will be necessary that the pitch should be very much less than the maximum spacing mentioned above.

§ (*d*) *Notes on Different Forms of Joints.*—Single-riveted joints are weakened by unequal tension, which produces a loss of efficiency estimated at about 20 per cent. of the net section. Double-riveted joints are therefore in every way to be recommended, and they may be either chain-riveted, as fig. 62, or zig-zag-riveted, as fig. 63 (Art. 50). In the former style of riveting the distance between the two rows of rivets is generally two diameters of the rivet; but if the holes are punched, it is safer to make the distance equal to $2\frac{1}{2}$ diameters, to allow for the zones of weakness caused by this method of perforation. Fracture in zig-zag-riveted joints often takes place, as shown in fig. 80, and the stress along this zig-zag line is composed of tensile and shearing stress. Professor Kennedy states that in order to ensure a straight fracture the net metal measured zig-zag should be from 30 to 35 per cent. in excess of that measured straight across. Consequently the diagonal pitch becomes

$$\frac{2}{3}p + \frac{d}{3} \quad \ldots\ldots\ldots\ldots\ldots \quad (23)$$

where $p =$ the straight pitch and $d =$ diameter of the rivet hole. When setting out a double-riveted zig-zag joint, the best course is to settle the ordinary pitch, and then to determine from it the distance between the rows of rivets.

§ (*e*) *Proportions of Cover-plates.*—We have seen that the bending of a joint causes injury owing to the establishment of unequal stress, and as experience shows that a similar result follows the bending of a single-riveted butt-joint with one cover-plate, an advantage is gained by making the cover somewhat thicker than the plates connected thereby. Professor Unwin recommends that single cover-plates should be $1\frac{1}{8}$ times the thickness of the plate, and the Board of Trade rules state that single butt-straps with punched holes must be one-eighth thicker than the plates they cover. When two cover-plates are fitted, no necessity exists for additional strength, and the covers need not be more than five-eighths or two-thirds the thickness of the plates covered.

§ (*f*) *Margin and Lap of Plates.*—Care always has to be taken to leave a sufficient margin outside the rivets of any joint, otherwise the plate might burst, as shown in fig. 77. Ample safety is ensured by making the margin equal to the rivet diameter, except in butt-joints with double covers, and in joints where the plate edges are roughly trimmed and the holes are punched. In such cases as these the margin should be equal to at least $1\frac{1}{4}$ times the rivet diameter. The extent to which one plate should overlap another, or to which two plates should be covered by butt-straps, is governed partly by the margin necessary, partly by the number of rows of rivets, and partly by the distances necessary between the rows. As narrow bars offer less resistance to splitting action than plates, the lap may advantageously be increased to $1\frac{1}{2}$ diameters of the rivet.

59. Practical and Theoretical Efficiency of Riveted Joints.—Some fifty or sixty years ago there was a popular superstition to the effect that riveted joints were actually stronger than the solid plate. No person of average intelligence would venture to express such an opinion in the present day, but very likely there are many who do not fully recognise the serious diminution of strength which must take place if a plate is cut, and the two pieces are then joined by riveting. The theoretical percentage of the strength of a perforated plate at any joint as compared with the original plate is :

$$\text{Theoretical percentage} = \frac{p-d}{p} \times 100 \quad \ldots\ldots\ldots\ldots\ldots (24)$$

where p = pitch of rivets, d = diameter of rivets, both in inches.

When determined by the shearing resistance of the rivet, the theoretical percentage must depend upon the effective shearing area and strength of the rivet, considered in connexion with the section and strength of the original plate.

If exact theoretical proportions be taken, so that the stresses tending to rupture or cripple the plate and to shear the rivet are exactly balanced, the theoretical efficiency of the joint will be about 61 per cent. But proportions such as these are not always observed, and the steel joint taken in Art. 51, § *f*, is probably as near an approach to theory as one is likely to meet with in ordinary architectural practice. Calculating the theoretical percentage by Rule 24 above, and taking the value of p as the width of the plate, we get the following result :—

$$\frac{p-d}{p} \times 100 = \frac{3-1}{3} \times 100 = 66{\cdot}6 \text{ per cent.}$$

In this particular joint, however, the rivet is by far the weaker element, and the greater strength of the plate does not make the joint any stronger. Turning back to the examples in Art. 51, § *f*, we find the resistance of this joint to different modes of failure to be by no means uniform. We will assume the strength of the original plate to have been 28 tons per square inch, 2 tons having been added in Table XXXVI. to represent increase of strength due to drilling.

Therefore the strength of the original plate was $(3 \times \cdot 5) \times 28 = 42$ tons. Judging the strength of the joint by this standard, we obtain the following percentages of calculated practical strength :—

Shearing	strength of	rivet,	17·27 tons	= 41 per cent.
Crushing	,,	plate,	20·00 ,,	= 47 ,,
Bursting	,,	,,	24·00 ,,	= 57 ,,
Tearing	,,	,,	30·00 ,,	= 71 ,,
Shearing	,,	,,	33·00 ,,	= 78 ,,

If the same kind of joint were made in iron, original strength, say, $(3 \times \cdot 5) \times 22 = 33$ tons, the figures would be as follows :—

Shearing	strength of	rivet,	14·9 tons	= 45 per cent.
Crushing	,,	plate,	15·0 ,,	= 45 ,,
Bursting	,,	,,	19·0 ,,	= 57 ,,
Tearing	,,	,,	18·0 ,,	= 54 ,,
Shearing	,,	,,	28·5 ,,	= 86 ,,

By these figures we see that the iron joint would have a higher percentage of efficiency, though it would actually be weaker than, and cost quite as much as, a steel joint. In either case the lowest value must be taken as representing calculated efficiency.

§ (*a*) *Estimates of Practical Efficiency.*—Table LI. contains the approximated values of iron and steel joints as compared with the solid plate. The figures are derived from the experiments which have been quoted and from other sources, but it should be remarked that in order to make a safe estimate of the efficiency of joints such as are used in structural work, fully 15 per cent. ought to be deducted from the percentages stated in the table.

Table LI.—Approximate Values of Iron and Steel Riveted Joints. (Original Solid Plate=100.)

Form of Joint.	Iron.	Steel. Thickness of Plate. ¼ in. to ⅜ in.	Steel. Thickness of Plate. ½ in. to ⅝ in.	Steel. Thickness of Plate. ¾ in. to ⅞ in.
Lap-joint, single riveted, punched	45	50	45	40
,, ,, ,, drilled	50	55	50	45
,, double ,, punched	60	75	70	65
,, ,, ,, drilled	60	80	75	70
Butt-joint, one cover, single riveted ...	45	—	—	—
,, ,, ,, double ,, ...	60	—	—	—
,, two covers, single riveted ...	55	—	—	—
,, ,, ,, double ,, punched	66	75	70	65
,, ,, ,, ,, ,, drilled..	66	80	75	70

CHAPTER X.

BOLTS AND OTHER FASTENINGS.

60. Bolts and Screws.—Any of the joints which have already been considered might be made by means of bolts or screws, and some of them could be made by pins. Bolts and screws are apt to become loosened under the influence of vibration, however tightly they may have been fixed at the outset. Therefore, friction of the plates or bars cannot be relied upon as an element of resistance in any joint formed in this manner. Simple pins are still more likely to become loose, whether they have been driven into tapered or parallel holes; and although they may be formed so that they cannot actually fall out, effective contractile force is never exercised upon the pieces of metal joined together. Further, the shank of a bolt, screw, or pin cannot fill the hole in a plate so completely as a rivet which has been closed up by hammering, or by heavy pressure in a riveting machine. Finally, the cost of a bolt or screw is always more than that of a rivet, and some pins are more costly than bolts or screws. Nevertheless, there are often occasions when it would be inconvenient or impracticable to use rivets in the making of joints, sometimes because access for riveting would be difficult, and at others because the length and diameter necessary far exceed the dimensions up to which rivets can be worked. Very frequently a bolt or a pin is intended to act as a pivot upon which one or more of the parts connected may move freely in a plane perpendicular to the axis of the bolt or pin. Here, of course, no other kind of fastening would be so suitable. (Art. 39.)

§ (*a*) *Forms of Bolts and Nuts*:—

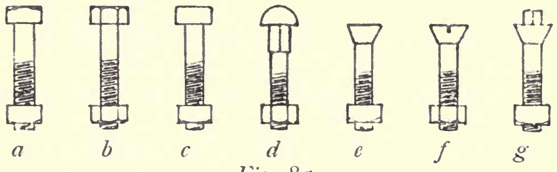
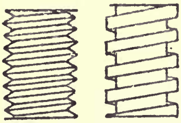


Fig. 85.

Bolts are made in various shapes, some of which are shown in fig. 85. The heads may be square, hexagonal, cylindrical, or spherical, the shanks may be cylindrical throughout, or with square necks, and the nuts may be square or hexagonal. The threads are usually triangular, as *a*, fig. 86, but when great strength is necessary, square threads, as *b*, are used.

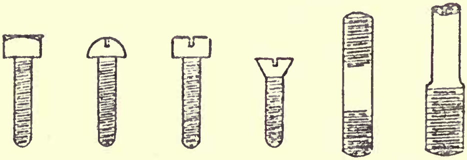


a *b*

Fig. 86.

For girder and similar work choice is practically limited to square and hexagonal-shaped heads and nuts (*a* and *b*, fig. 85); countersunk bolts (*e* and *f*) are occasionally necessary when projecting heads would not be permissible; and bolts with cylindrical or with spherical heads (*c* and *d*) are chiefly used in timber, being usually made with square necks so that the bolt shall be prevented from turning whilst the nut is being tightened up. The countersunk head with projecting square (*g*) is useful when a slotted head would be too light; the square end is screwed down by a spanner and afterwards cut off.

§ (*b*) *Set-screws* :—



h *j* *k* *l* *m* *n*

Fig. 87.

Set-screws, or tap-bolts, as they are indifferently termed, (*h*, *j*, *k*, *l*, fig. 87), are similar in construction to ordinary bolts, except that no nuts are provided. Stud-bolts are often required (as *m*), so that nuts may be fitted at each end; and sometimes, for the sake of strength, the end of a bolt is upset, or enlarged before screwing, as *n*. As a general rule, bolts are passed through plain or "clearing-holes," but in the case of set-screws it is necessary that at least one of the holes should be threaded by means of a screw-tap. This forms what is known as a "tapping-hole."

§ (*c*) *Application of Bolts and Screws*:—

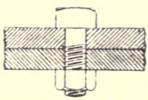


Fig. 88.

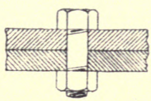


Fig. 89.

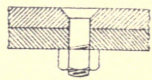


Fig. 90.

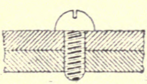


Fig. 91.

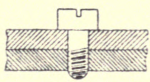


Fig. 92.

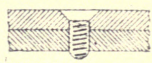


Fig. 93.

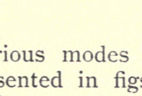


Fig. 94.

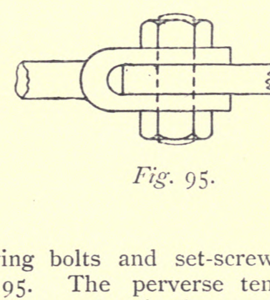


Fig. 95.

Various modes of applying bolts and set-screws are represented in figs. 88 to 95. The perverse tendency exhibited by nuts to unscrew themselves is the cause of a good deal of unnecessary worry to engineers. Many efforts have been made to overcome the difficulty in a more refined manner than is represented by simply driving a pin through the nut, as shown in plan and elevation by fig. 96, but no entirely satisfactory solution has yet been achieved.

§ (*d*) *Methods of Securing Nuts* :—

The familiar device of placing two nuts at the end of a bolt, and of screwing one down upon the other, as in fig. 97, until jamming friction results, is not sufficiently reliable ;

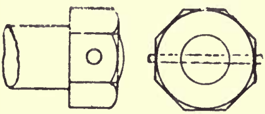


Fig. 96.

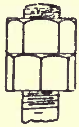


Fig. 97.

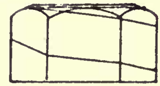


Fig. 98.

many so-called lock-nuts are sold which in reality are only jam-nuts, as fig. 98, and no one has hitherto succeeded in placing a genuine lock-nut upon the market.

§ (*e*) *Turnbuckles and Swivels* :—

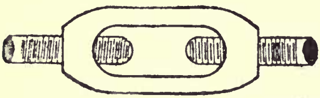


Fig. 99.

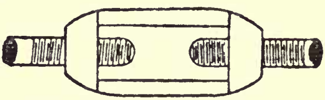


Fig. 100.

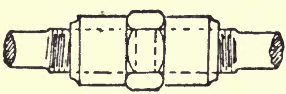


Fig. 101.

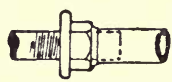


Fig. 102.

Fig. 99 shows two tie-rods, with right and left-hand threads, screwed into an open drop-forged turnbuckle; fig. 100 shows two tie-rods connected by means of a turnbuckle; fig. 101 illustrates an equivalent arrangement, where the connector is termed a pipe-swivel ; and fig. 102 shows how a tie-rod may be joined to a tubular bar.

§ (*f*) *Whitworth's Standard for Bolts and Nuts.*—The application of Whitworth's standard to the dimensions of bolts and nuts is practically universal in the present day. Table LII. gives the standard dimensions for some of the most useful sizes of bolts and nuts. Bolts with square threads have one-half the number of threads used in the triangular form.

Table LII.—Whitworth's Standard for Bolts and Nuts: Hexagonal Heads and Nuts, Triangular Threads.

Dia. of Bolt.	Dia. at Bottom of Thread.	Number of Threads per inch.	Thickness of Head.	Thickness of Nut.	Breadth across the flats.
Inches.	Inches.	Threads.	Inches.	Inches.	Inches.
$\frac{1}{4}$	·186	20	·219	$\frac{1}{4}$	·525
$\frac{3}{8}$	·295	16	·328	$\frac{3}{8}$	·709
$\frac{1}{2}$	·393	12	·437	$\frac{1}{2}$	·919
$\frac{5}{8}$	·508	11	·547	$\frac{5}{8}$	1·101
$\frac{3}{4}$	·622	10	·656	$\frac{3}{4}$	1·301
$\frac{7}{8}$	·733	9	·766	$\frac{7}{8}$	1·479
1	·840	8	·875	1	1·670
$1\frac{1}{4}$	1·067	7	1·094	$1\frac{1}{4}$	2·048
$1\frac{1}{2}$	1·286	6	1·312	$1\frac{1}{2}$	2·413
2	1·715	$4\frac{1}{2}$	1·75	2	3·149
$2\frac{1}{4}$	1·930	4	1·969	$2\frac{1}{4}$	3·546
$2\frac{1}{2}$	2·180	4	2·187	$2\frac{1}{2}$	3·894
3	2·634	$3\frac{1}{2}$	2·625	3	4·531

§ (*g*) *Tensile Stress.*—Bolts and screws in joints are liable to experience both tensile and shearing stress, but pins are only exposed to shearing stress. Taking tensile stress first, we note that the resistance of a bolt is governed (*a*) by the tenacity of the material, and (*b*) by the effective diameter. The effective diameter of a screwed bolt is measured at the bottom of the thread, and this dimension can always be ascertained from a table of the Whitworth standard dimensions.

§ (*h*) *Excessive Strain sometimes Caused.*—Whilst speaking of tensile stress it may be just as well to remark that excessive strain is very often put upon bolts by the conscientious but mistaken efforts of workmen, who are apt to think their labours will be more satisfactorily performed if the nuts are screwed down as tightly as possible. This practice is very likely to lead to accidents, as, owing to the enormous mechanical advantage derivable from the screw, the strain put upon the bolt may easily be higher than the elastic limit of the material. We have already pointed out the injurious effects of straining material beyond the elastic limit, and there is no reason for the imposition of such a strain, for when the parts which have to be connected by bolts are brought into close contact, and when the nuts have a uniform solid bearing against the surface, no further screwing down is necessary. (Art. 24, § *c.*)

§ (*j*) *Safe Loads for Bolts.*—We can now proceed to find the safe load which may be imposed on a bolt, let us say, of 1 in. diameter. By Table LII. the effective diameter $=\cdot840$ in., the area being:

$$(\cdot840^2 \times \cdot7854) = \cdot5542 \text{ square inch.}$$

Taking the tensile strength of iron at 22 tons per square inch, and of steel at 28 tons per square inch, the ultimate strength of a 1-inch bolt would be 12·2 tons in iron and 15·5 tons in steel. Using 6 as the factor of safety, the working loads would be about 2 tons for iron and 2·5 tons for steel, or about 3·6 and 4·5 tons per square inch respectively.

The safe load for iron bolts not subject to much strain is frequently taken at 4 tons per square inch of the effective section; for bolts moderately tightened, at 2 tons per square inch; and for bolts liable to severe strain after being very tightly screwed up, at $1\frac{1}{2}$ tons per square inch.

Long bolts, or tie-rods, should always have the screwed ends enlarged as *n*, fig. 87, so that the effective area shall be equal to the area of the unscrewed portion.

In well-proportioned bolts the resistance of the threads is fully equal to that of the bolt itself. During some experiments made by Brunel it was observed that most of the bolts broke at the base of the screwed part, and others, with enlarged ends, broke in the shank.

§ (*k*) *Shearing Stress.*—Shearing stress experienced by a bolt, when tightly fixed in position, is to be calculated exactly as in the case of a rivet, but if the bolt be not held tightly in its place, stress will not be uniformly distributed throughout the sectional area, and its incidence will be governed by the laws which relate to the distribution of shearing stress in beams (Chapter XVI.). The maximum intensity of shearing stress will then be greatest at the neutral axis, diminishing to zero at the top and bottom of the section.

For example, if $F =$ shearing action of the load at a given cross-section of the bolt, and $a =$ area of the cross-section, at which it acts, then

$$\text{mean intensity} = F \div a$$

The ratio in which maximum intensity exceeds mean intensity in the case of a circular bolt or pin is as 4 : 3, and if the bolt is loosely fitted at first, or is likely to become so afterwards, its sectional area must be increased in this proportion. If shearing stress should be imposed upon the screwed part of a bolt, the effective, and not the nominal, diameter must be used in calculations.

61. Pins and Cotters.—A pin may be either a cylindrical or tapered peg, as fig. 103; or it may be a short

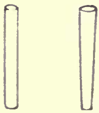


Fig. 103.

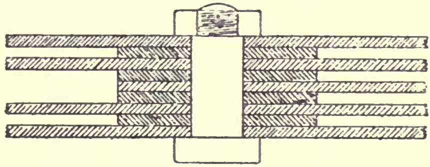


Fig. 104.

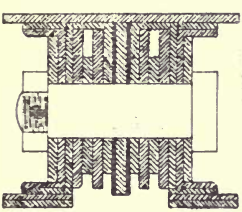


Fig. 105.

Fig. 106.

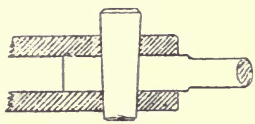


Fig. 107.

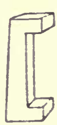


Fig. 108.

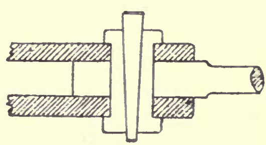


Fig. 109.

shaft, a portion of which serves as a journal, permitting movement of the parts joined in one direction.

Pins of this kind are shown in figs. 104 and 105, each having a nut which cannot be screwed beyond the shoulder formed on the shank; consequently no lateral pressure can be exerted upon the parts connected. Each nut is secured by a small pin.

§ (*a*) *Types of Pins.*—A cotter, key, or wedge is merely a tapered rectangular piece of iron or steel, as illustrated in fig. 106. Sometimes the cotter is simply driven into slots cut in the two parts to be joined, as in fig. 107; at other times one or two gibs (fig. 108) are used in conjunction with a cotter, as in fig. 109.

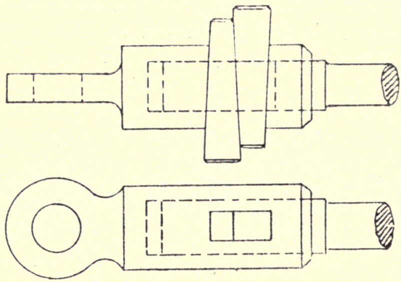


Fig. 110.

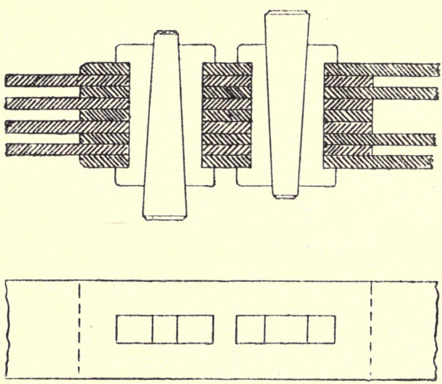


Fig. 111.

Fig. 110 shows the plan and elevation of two cotters employed for connecting the ends of tie-bars in the roof of St. David's Station, Exeter, and fig. 111 illustrates the use of cotters and gibs as applied to the ends of tie-bars in the roof of Lime-street Station, Liverpool.

One of the chief advantages presented by cotters is that they furnish ready means for the regulation of the tension of tie-bolts, tie-bars, and other members. The angle of obliquity in a cotter should not exceed 4 deg., which is the angle of repose of iron upon iron, allowing for the accidental presence of oil or grease upon the surfaces.

§ (*b*) *Distribution of Shearing Stress.*—So long as they are perfectly tight in their seats, pins and cotters are exposed to equally distributed shearing stress, but if they become loosened, stress will be distributed in accordance with the laws relating to shearing stress in beams (Chap. XVI.). The ratio in which maximum intensity exceeds mean intensity in the case of a rectangular key or wedge is as 3 : 2, and if the contingency of slackness has to be encountered, due provision must be made by proportionately increasing the sectional area of the cotter.

62. Eye-bars.—When rods or bars are connected by bolts or pins, their ends are usually swelled out into the form of an eye, and it is important that the eyes should be of sufficient strength to resist the stress to which they will necessarily be exposed.

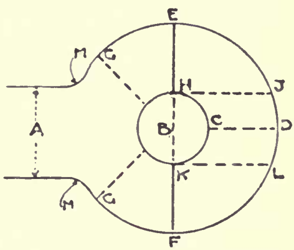


Fig. 112.

§ (*a*) *Modes of Failure.*—A joint into which such a bar enters may fail in one of the six different directions stated below, the various parts of the eye being indicated in fig. 112 :—

1. By shearing or bending of the bolt (B).
2. By crushing of the link at (C).
3. By bursting of the link along the line (C D).
4. By tearing of the link along the line (E F).
5. By tearing of the link through the shoulders (G G).
6. By shearing of the link along the lines (H J, K L).

With the exception of No. 5, all these alternative methods of failure are very similar to those described in Art. 51, as applying to ordinary riveted joints; but the conditions here are so different as regards the distribution of stress, that the most suitable proportions for the eye cannot be determined theoretically.

§ (*b*) *General Proportions.*—Theoretical proportions are not observed in practice with regard to bars which are slotted for the reception of cotters. The simple practical rule is to make the sectional area of the bar, taken through the slot in the head, from 25 to 50 per cent. greater than the sectional area of the bar itself. The following minimum dimensions are recommended by Mr. Berkley for wrought-iron flat eye-bars :—

Width of bar	(A)	=	1·00
Diameter of bolt ..	(B)	=	·75
Depth beyond bolt hole..	(CD)	=	1·00
Net width across eye	(EH + KF)	=	1·25
Radius of shoulders ..	(G, G)	=	1·00
Radius of neck curves ..	(M, M)	=	1·50

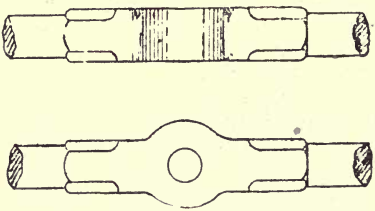


Fig. 113.

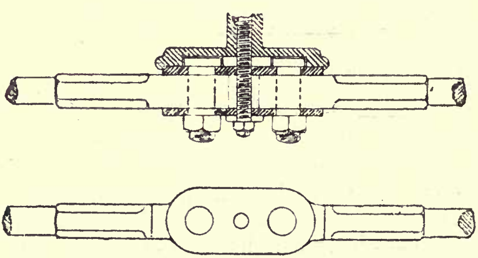


Fig. 114.

Fig. 113 illustrates the eye in the middle of a tie-rod, and fig. 114 shows one method of connecting an eye-bar of this kind.

63. Various Bolts and Fastenings.—Although not used for making joints between metal and metal, there are some other forms of bolts and fastenings which may now be conveniently mentioned.

Fig. 115.

Fig. 116.

Fig. 117.

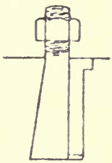


Fig. 118.

Lag, or coach screws (fig. 115), are very useful for attaching metal plates to timber. Expansion bolts, single and double (figs. 116 and 117), are for fastening ironwork to wall or other smooth surfaces. It is only necessary to have a hole of sufficient diameter and depth for the insertion of the bolt end. Expansion takes place when the head of the bolt is turned, and the bolt can be removed at any time without injury to the surface of the work. Fig. 118 shows a Lewis bolt which also is secured by expansion.

Fig. 119.

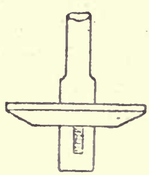


Fig. 120.

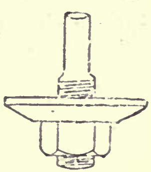


Fig. 121.

Fig. 119 is a representation of the ordinary rag bolt much used for the attachment of ironwork to stone, concrete, &c. Foundation bolts are sometimes made with a cotter through the end supporting the foundation plate or washer (fig. 120), and at other times the end is screwed and fitted with a nut (fig. 121). In either case the end should be swelled, so that the diameter is $1\frac{1}{4}$ times that of the remainder of the bolt.

64. Anchors.—The illustrations in fig. 122 show some of the principal varieties of fastenings used for securing iron and steel girders to walls, and for fulfilling other structural requirements.

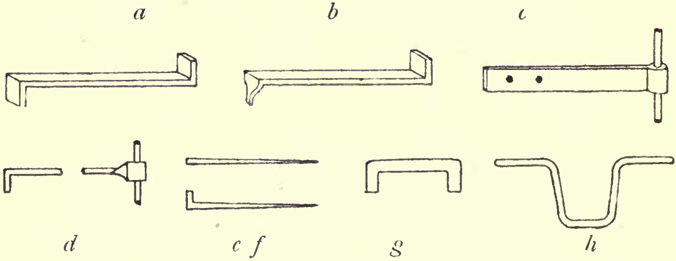


Fig. 122.

Ashlar anchors (*a*, *b*) are useful ties for ashlar facing or for tying lining walls to main walls when air spaces are required. Beam anchors (*c*) are for securing girders, which rest upon walls, but angle brackets are frequently used for the same purpose. Hook angles (*d*) are intended for tying a wall to timber beams parallel to the wall, and are also used to tie front, side, and interior walls. Drive anchors (*e*) are useful for connecting new brick linings to old walls. Wedge anchors (*f*) are used for making tight the junction between new and old work, when new foundation walls have been formed in an old building. A coping anchor is shown in *g*, and the V-shaped anchor (*h*) is used for holding girders, the loop being passed through a hole drilled near the end of the web.

CHAPTER XI.

THE STRENGTH OF COLUMNS.

65. Definition.—From the academic point of view there are clearly-marked distinctions between columns, pilasters, stanchions, and struts. These differences are more important in decorative than in constructive design. Art, on the one hand, seeks variety of form for its due expression, whilst Science, on the other, strives to disregard merely external considerations, and to look at the essential nature of things. Thus, from the strictly scientific standpoint, all members of a structure, which are exposed to direct compressive stresses in the direction of their lengths, belong to one class, and are subject to one set of rules. Such members may either be isolated or built into walls, or they may be parts of a framed structure. They may be of different shapes and be called by different names, but their inherent properties are not thereby affected except in degree.

For these reasons we shall include in this elementary consideration of columns all members designed for the resistance of pressure applied in the direction of their length, and the rules which will be given for the calculation of strength must be understood to apply generally to all such parts employed in structural work.

66. Cast-Iron Columns.—Columns of cast iron are very extensively used in all kinds of buildings, and will doubtless continue to be used. Circumstances which conduce very much to their popularity are the readiness with which they may be procured, and their general adaptability to the exigencies of architectural design. The last-named recommendation is to some extent a demerit, for the reason that it tends to encourage the employment of unprotected ironwork in a manner entirely contrary to the most approved methods of fireproof construction. In cast-iron columns there is always present the risk of flaws, blow-holes, irregularities of section, and variations of quality due to foundry operations. Such contingencies are inseparable from the use of cast-iron columns, and a further drawback is to be found in the fact that brackets and lugs, which are cast on for the connexion of other members, may break off without warning of any kind. Failures of cast-iron columns are usually attended by serious damage, and the utmost vigilance should be exercised with regard to all details connected with their design, manufacture, and erection. (Arts. 13, § *c*, and 79, § *a*.)

§ (*a*) *Resistance to High Temperatures.*—The statement is frequently made that cast iron is more readily affected than wrought iron or steel by the action of fire. It appears, however, to be the fact that unprotected cast iron is practically unharmed by temperatures as high as 1,300 or 1,500 deg. Fahr., whilst, as shown in Art. 35, § *c*, wrought iron and steel suffer serious injury at temperatures 1,000 deg. Fahr. In support of these statements we may cite the experiments of Professor Bauschinger, of Munich; of the New York Committee representing the Architectural League of New York, the American Society of Mechanical Engineers, and the Tariff Association of New York; and, further, the official reports upon the fires in the Ames Building in Boston, U.S.A., and in the Horne Building, Pittsburg. It should never be forgotten that protection of cast iron is necessary, because in most fires the temperature would be much greater than any which could be safely endured by cast-iron columns. The same remark applies more forcibly to wrought iron and steel because of their comparatively smaller resistance to heat.

§ (*b*) *Corrosion.*—So far as corrosion is concerned, it is generally admitted that cast iron is not affected in the same degree as wrought iron and steel, and as the metal is always thicker, the proportion of rust to sectional area is necessarily smaller. Cast iron is protected by the silicious skin received in the sand mould (Art. 14, § *c*), and the surface is comparatively free from rivet and bolt holes, projections and joints, which aid the collection of moisture. Moreover, the superficial area is always less in a cast-iron column than in a wrought-iron column of equal strength, and consequently there is a smaller surface exposed to corrosive influences. These reasons point to the superiority of cast iron; but as a matter of fact the advantage is more apparent than real, because if wrought iron and steel are reasonably protected, they are very little more susceptible than cast iron to corrosion.

67. Euler's Investigation.—Exhaustive investigation has been made by able mathematicians, and notably by Euler, with the object of demonstrating the laws which govern the strength of columns, but the results so achieved apply to the ideal, and not to the practical column. Consequently, the rules used in practice for determining the strength of columns are considerably qualified by empirical co-efficients intended to represent the results afforded by experimental inquiry.

Euler assumed the existence of a certain ratio of length to diameter below which the material constituting a column would be fractured by direct compression, and above which it would fail by reason of transverse strain. Euler's theory may be thus illustrated:—Assume that a straight column of uniform section, and symmetrical as to elasticity, is fixed in such a way that when loaded axially it shall be free to bend along its entire length. Supposing curvature to be produced by the momentary application of a transverse force, there will be a bending moment at every section. If the load be sufficiently great, the deflection will continue or will increase; whilst if the load be not sufficiently great, the column will resume its normal shape. In this assumed case the influence of the load is resisted by the elasticity and inertia of the column. The modulus of elasticity depends upon the nature of the material; the moment of inertia is governed by the form of its cross-section; and both these measurements are qualified by the ratio between length and diameter. Euler arrived at the conclusion that the strength of a long column would be directly proportional to the 4th power of its diameter, and inversely proportional to the square of its length. His theory, therefore, demonstrates the desirability of adopting sections in which the moment of inertia is proportionately large.

68. Hodgkinson's Experiments.—Hodgkinson's experiments, conducted in the year 1840, still constitute one of our chief sources of knowledge as to the strength of columns. Briefly summarised, the following are the conclusions to be derived from his inquiry. Short columns, in which the height is not greater than four times the diameter, are fractured by actual crushing of the material; sometimes, as shown in fig. 123, the end slides off in the form of a wedge, the height of which is rather less than 1·5 diameters, and when the height is less the sides are split, as in fig. 124.

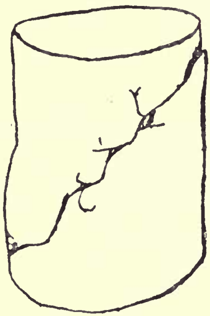


Fig. 123.

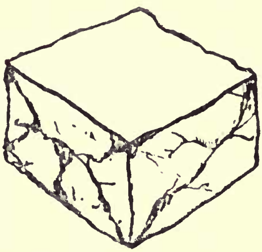


Fig. 124.

Medium columns, whose length is less than 30 and more than 5 diameters, are distinctly affected by bending stress, but the weight required to cause fracture in this manner is so great, that crushing force becomes manifest, and the column yields to the joint action of the two forces. Long columns, whose length is more than 30 diameters, fail by flexure, and although pressure tends to cause failure by vertical compression, the direct breaking weight is far below the crushing strength of the material.

§ (*a*) *Classification of Columns.*—Hodgkinson's experiments also showed the strength of a column to be affected by the form of the ends and the manner in which they were fixed. He therefore divided columns into three main classes:—

1. With both ends spherical.
2. With one end spherical and the other end flat.
3. With both ends flat.

The relative strengths of these three classes are 1, 2, and 3 respectively, and an example of each type is shown diagrammatically in figs. 125, 126, and 127, where the line of flexure is approximately indicated by dotted lines, and the point or points of fracture are shown by arrows.

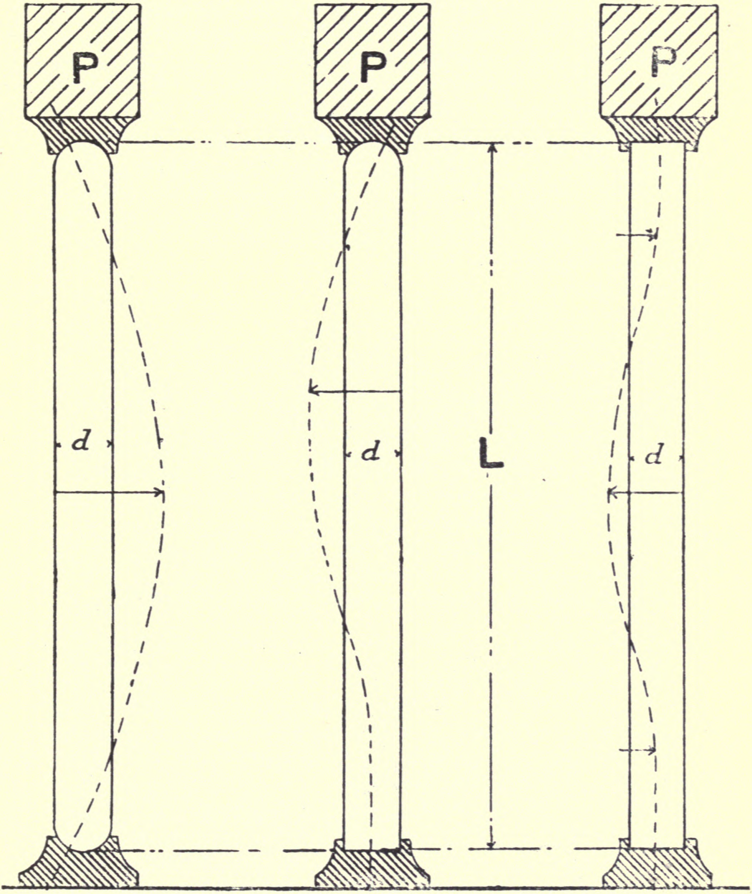


Fig. 125. *Fig.* 126. *Fig.* 127.

When flat-ended columns have discs cast upon the ends, strength depends less upon the presence of the discs than upon the manner in which they are fixed. If the tops and bases be faced in the lathe, and the foundations be dressed off level, the maximum strength will be attained. If the column be irregularly fixed, so that the pressure is taken diagonally, only one-third of the maximum strength will be developed. If the ends be roughly faced by chipping off irregularities with a cold chisel, inequalities of the bearing surfaces will cause the weight to come upon a few points at the ends, which cannot then be truly perpendicular to the axis. In such a case strength should not be taken as more than two-thirds of the maximum. It is now usual for engineers to draw distinctions between columns with round, pivoted, hinged, pin, flat, and fixed ends. Within certain limits, the value of each of the first four of these types is taken as 1, and of the last two as 2.

The properties observed by Hodgkinson apply to pillars of wrought iron, steel, and wood, as well as to those of cast iron, although strength is governed in each case by the nature of the material and other factors.

§ (*b*) *Hodgkinson's Formula.*—As the general result of his investigation, Hodgkinson found that the strength of a column depended upon the nature of the material, and also that it varied directly as the 3·6th power of its diameter, and inversely as the 1·7th power of its length. Hodgkinson's formula bears a sort of family resemblance to that of Euler, and in its simplest form is :

$$P = m \times \frac{d^{3\cdot6}}{l^{1\cdot7}} 12 \ldots\ldots\ldots\ldots\ldots (1)$$

Variations of the rule, and of the value expressed by the co-efficient m, are necessary for computing the strength of columns of different forms, and as tables of the 3·6th and 1·7th powers, or alternatively of logarithms, are also required, Hodgkinson's formula has never been popular, and is very seldom used by engineers.

69. Gordon's Formula.—Various attempts have been made to embody the valuable data obtained by Hodgkinson in the form of a simple rule which might be convenient for general use. A formula of this kind was first suggested by Professor Lewis Gordon, and it assumes failure to result from a combination of crushing and bending stress. Gordon's rules are thus written :—

(a) For columns with both ends flat and fixed:

$$P = \frac{f\,S}{1 + a\left(\frac{l}{d}\right)^2} \quad \ldots\ldots\ldots\ldots(2)$$

(b) For columns with rounded or jointed ends:

$$P = \frac{f\,S}{1 + 4a\left(\frac{l}{d}\right)^2} \quad \ldots\ldots\ldots\ldots(3)$$

The following are the significations of the symbols used:—

P = breaking load of the column in tons.
S = sectional area of the column in square inches.
l = length of the column in inches.
d = least diameter of the column in inches.
f = strength of the material in tons per square inch.
a = a constant depending upon the sectional form of the column.

§ (a) *Co-efficients for Various Materials and Sections.*— By altering the value of the co-efficients f and a, Gordon's formula may be employed for calculating the strength of cast iron, wrought iron, and steel columns of various sections. Table LIV. gives the most generally accepted values of the co-efficients f and a used in connexion with Gordon's formula:—

Table LIV.—Values of Co-efficients for Gordon's Rule.

Section of Column.	Metal.	Value of f.	Value of a.
Round, solid	Cast iron	36 tons.	·0025
,, hollow	,,	36 ,,	·0025
Rectangular, solid	,,	36 ,,	·002
,, hollow	,,	36 ,,	·002
Round	Wrought iron	16 ,,	·00033
Rectangular	,,	16 ,,	·00033
Rolled Joists and Bars. (Unwin)	,,	19 ,,	·0011
Round, solid (Baker)	Steel	30 ,,	·0007
Rectangular solid ,, ...	,,	30 ,,	·0004

In applying Professor Unwin's modification of the rule for miscellaneous sections of wrought iron, it should be noticed that the diameter to be used is found by taking the shortest diameter of a rectangle or triangle circumscribing the section. As the quality of steel supplied on ordinary building contracts is not usually equal to that demanded in important engineering works, it is preferable to calculate the strengths of steel columns by the aid of the same co-efficients as are employed for wrought iron. In this connexion it may be noticed that the values of f adopted by the New York Building Department are 80,000 lbs. $= 35 \cdot 7$ tons for cast iron; and 40,000 lbs. $= 17 \cdot 85$ tons for both wrought iron and steel.

70. Rankine's Formula.—Modifications of the Gordon rule, applicable to columns of any section, were recommended in the following forms by the late Professor Rankine:—

(*a*) For a column fixed at both ends:

$$P = \frac{f S}{1 + \frac{l^2}{c r^2}} \qquad \ldots\ldots\ldots\ldots\ldots \quad (4)$$

(*b*) For a column with both ends rounded or jointed:

$$P = \frac{f S}{1 + \frac{4 l^2}{c r^2}} \qquad \ldots\ldots\ldots\ldots\ldots \quad (5)$$

(*c*) For a column with one end fixed and the other rounded or jointed:

$$P = \frac{f S}{1 + \frac{16 l^2}{9 c r^2}} \qquad \ldots\ldots\ldots\ldots\ldots \quad (6)$$

Here, the symbols and their significations are:—

$P =$ breaking load of the column in tons.

$S =$ sectional area of the column in square inches.

$l =$ length of the column in inches.

$r =$ least radius of gyration of the section in inches.

$f =$ strength of the material in lbs. or tons per sq. inch.

$c =$ a co-efficient depending on the nature of the material.

Table LV.—Values of Co-efficients for Rankine's Rule.

Metal.	Value of f.	Value of c.
	lbs. tons.	
Cast Iron	80,000 = 35·7	3,200
Wrought Iron	36,000 = 16·07	36,000
Mild Steel	48,000 = 21·42	30,000

71. Angular Motion.—In order that the meaning of the term *radius of gyration* may be duly appreciated, some acquaintance with fundamental mechanical principles is necessary. Gyration, or angular motion, is readily perceptible when exemplified in a machine or structure where a part is, and is intended to be, in visible motion about a given axis. It is not quite so easy to realise gyration as existing in the part of a structure which is intended to be motionless, and is apparently so. But the same principles apply to either case. For instance, if part of the axis A B (fig. 128) of a column be bent so that it assumes the position A C, the plane a a will rotate about the axis A until it assumes the direction a_1 a_1, and incidentally there will be induced a state of compression in the fibres at one side, and of tension in those at the other. In a vertical column, bending under the influence of a load, motion of the plane surface constituting a transverse section must be considered in connexion with its axis. The *axis of rotation* is the imaginary line about which gyration takes place, and the *centre of gyration* is that point where the energy of gyration may be conceived to be concentrated.

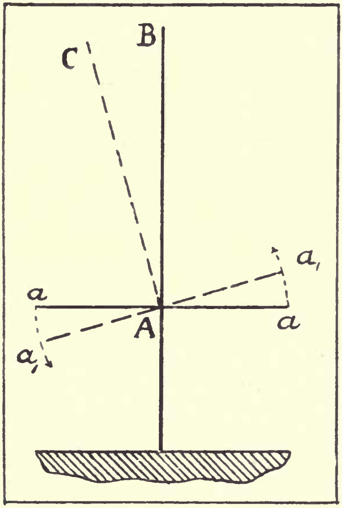


Fig. 128.

§ (*a*) *Radius of Gyration.*—The radius of gyration is the distance of the centre of gyration from the axis of rotation, and this radius, denoted by the symbol r, is used as the index to an effect producible in a given form of column section. Consideration must be given to inertia, that power by virtue of which matter tends to resist any change of state; and more particularly to the *moment of inertia*, because it is an index to the resistance afforded by a given form of column section to a change of state. As action and reaction are equal and opposite, so there is a definite relation between the radius of gyration and the moment of inertia of a given surface. Denoting the moment of inertia by I, and the area of the surface by a, the relationship is thus expressed :—

$$r = \sqrt{\frac{I}{a}}, \text{ and } r^2 = \frac{I}{a} \quad \ldots\ldots\ldots\ldots \quad (7)$$

From this it appears that if the value of I is known, the values of r and r^2 can readily be ascertained.

Table LVI. contains rules by which some values of r^2 may be approximately obtained in an expeditious manner.

Table LVI.—Values of r^2 for various Sections (*Rankine*).

Form of Section.	$r^2 = \frac{I}{a}$
Solid rectangle, least side $= h$	$h^2 \div 12$
Thin hollow rectangle, breadth $= b$, height $= h$	$\frac{h^2}{12} \times \frac{h + 3b}{h + b}$
Thin hollow square, side $= h$	$h^2 \div 6$
Solid cylinder, dia. $= h$	$h^2 \div 16$
Thin hollow cylinder, dia. $= h$	$h^2 \div 8$
I-iron, breadth of flanges $= b$, joint flange area $=$ A, web area $=$ B	$\frac{b^2}{12} \times \frac{A}{A + B}$
Channel-iron, depth of flanges $+ \frac{1}{2}$ thickness of web $= h$; area of web $=$ B; of flanges $=$ A	$h^2 \left\{ \frac{A}{12 (A + B)} + \frac{A B}{4 (A + B)^2} \right\}$

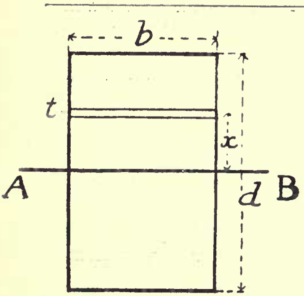


Fig. 129.

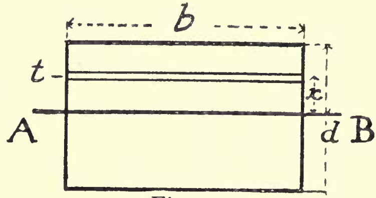


Fig. 130.

§ (*b*) *Moment of Inertia.*—To find the moment of inertia of a rectangular surface, such as that represented in fig. 129, it is necessary to consider the surface to be made up of an infinite series of narrow areas parallel to the axis A B. Let t be the thickness of one such area, and x its distance from A B. Then its area is $b\,t$, and its moment of inertia with regard to A B $= b\,t\,x^2$. The moment of inertia of the whole rectangle equals the sum of the moments of the infinite series, hence

$$I = \Sigma\, b\,t\,x^2 = \frac{b\,d^3}{12} \quad \ldots\ldots\ldots\ldots \quad (8)$$

If the breadth b and the depth d be expressed in inches, the value for I will be found in inch units.

The moment of inertia in respect of an axis in the position and direction A B is said to be the vertical, or greatest, moment of inertia. In the rectangle shown in fig. 130 the moment of inertia in respect of the axis A B is much smaller, and is known as the horizontal, or least, moment of inertia. This is the moment which must always be taken when calculating the strength of a column, because the axis is that about which bending is most likely to occur.

Table LVII.—Least Moments of Inertia of various Sections.

Reference.	Form.	Value of I.
Fig.		
131	Solid rectangle	$bd^3 \div 12$
132	Hollow ,,	$(bd^3 - b_1d_1^3) \div 12$
133	Solid square	$b^4 \div 12$
134	Hollow square	$(b^4 - b_1^4) \div 12$
135	T-section	$(bd^3 + b_1d_1^3) \div 12$
136	Simple I-section	$\left\{ bd^3 - b_1(d^3 - d_1^3) \right\} \div 12$
137	Compound I-section	$\left\{ b2d^3 - (b_1 2d_1^3 + b_2 2d_2^3 + b_3 2d_3^3) \right\} \div$
138	Cruciform section	$bd^3 + 2b_1d_1^3) \div 12$
139	Channel section	$\left\{ b(d^3 - d_2^3) + 2b_1(d_2^3 + d_1^3) \right\} \div 3$
140	Solid cylinder	$\cdot 7854 r^4$
141	,, ,,	$\cdot 7854 (r^4 - r_1^4)$

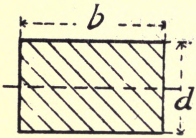


Fig. 131.

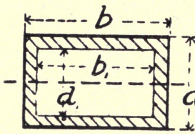


Fig. 132.

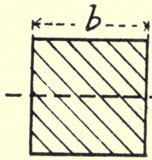


Fig. 133.

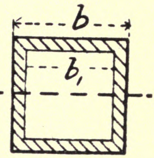


Fig. 134.

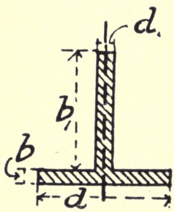


Fig. 135.

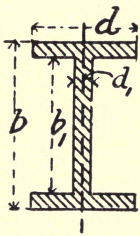


Fig. 136.

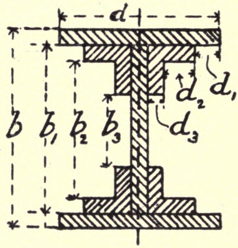


Fig. 137.

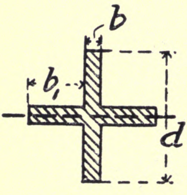


Fig. 138.

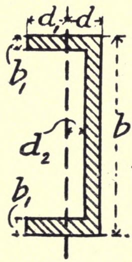


Fig. 139.

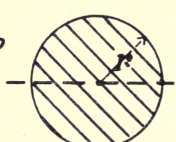


Fig. 140.

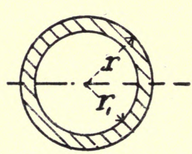


Fig. 141.

When the actual area of a surface is less than the product of its extreme breadth and depth, deduction must be made for the area included in these dimensions, which forms no part of the metal section. Modifications of the simple equation $I = \frac{b\,d^3}{12}$ are also necessary for calculating the value of I in the cases of circular and of unequally-formed sections. Table LVII. gives equations for the least moments of inertia of various sections, illustrated in figs. 131 to 141, relative to the axes there shown.

72. Application of Gordon's and Rankine's Rules.—Reverting to rules for calculating the strength of columns, we find that neither the Gordon nor the Rankine formula expresses experimental data with any marked precision, and results given by the two formulas are apt to exhibit considerable differences. It is not necessary to occupy space by comparative calculations, as the student can perform these at his leisure. The following typical examples are simply intended to illustrate the application of the rules.

Ex. 1.—Find the breaking weight of a solid cylindrical column of cast iron with fixed ends, length 20 ft., diameter 6 in. :—

In Gordon's rule (2): $f = 36$ tons, $a = \cdot 0025$, $S = 28\cdot 27$, sq. in., $l = 240$ in., $\left(\frac{l}{d}\right)^2 = 1{,}600$.

Hence,

$$P = \frac{36 \times 28\cdot 27}{1 + (\cdot 0025 \times 1{,}600)} = 203\cdot 5 \text{ tons.}$$

Ex. 2.—Find the breaking weight of a hollow cylindrical column of wrought iron with fixed ends, length 10 ft., diameter 5 in., thickness of metal, ·25 in. :—

In Gordon's rule (2): $f = 16$ tons, $a = \cdot 00033$,

$S = (19\cdot 635 - 15\cdot 904) = 3\cdot 731$, $l = 120$ in. $\left(\frac{l}{d}\right)^2 = 576$.

Hence,

$$P = \frac{16 \times 3\cdot 731}{1 + (\cdot 00033 \times 576)} = 50\cdot 1 \text{ tons.}$$

In Rankine's rule (4): $f = 16{\cdot}07$, $c = 36{,}000$, $l = 14{,}400$, $r^2 = 3{\cdot}125$ (Table LVI.).
Hence,

$$P = \frac{16{\cdot}07 \times 3{\cdot}731}{1 + \dfrac{14{,}400}{36{,}000 \times 3{\cdot}125}} = 53{\cdot}1 \text{ tons.}$$

Factors of safety for cast and wrought-iron columns were stated in Art. 28, but these may have to be varied to suit individual cases. Moreover, it is desirable that the factor should be increased as the ratio of length to diameter increases.

73. Computation of Strength from Tables.—If existing formulas are not perfect, those who use them have at least the satisfaction of knowing what they are doing, so far as may be, but when the strength of columns is found by the aid of tables, the inquirer frequently exhibits a degree of blind confidence which may or may not be justified. Tabular records published in this country as to experiments on columns are frequently more remarkable for antiquity than for applicability to modern requirements, and for some unknown reason British iron and steel manufacturers appear to think it desirable that practical details as to their goods should be withheld as much as possible. Ironfounders seldom, if ever, state dimensions from which the strength of their standard columns may be calculated. Similarly, steel manufacturers rarely publish more than the weights and over-all measurements of rolled sections, and even these dimensions are not often given for built-up sections. Everything, in fact, is left undone which would tend to encourage the use of standard productions, and everything is done which tends to confirm engineers in the habit of specifying special castings and special sections. In the United States a very different practice is followed, and the handbooks of engineering firms contain such precise details, and are of so comprehensive a nature, that they are actually used as text-books in more than one of the leading technical colleges. For the purpose of illustrating the mode of procedure adopted when such aids are available for the computation of strength, we append some tables taken from American sources.

§ (*a*) *Hollow Cast-iron Columns.*—Table LVIII. gives the ultimate strength of cylindrical and square cast-iron columns, as judged by the ratio of length to least diameter. A suitable factor of safety must, of course, be applied in every calculation, and attention must be paid to the form and condition of the ends.

Ex. 1.—Find the safe load for a cast-iron column with fixed ends, length 15 ft., diameter 10 in., thickness of metal 1 in., factor of safety = 8.

Here, $\frac{l}{d} = \frac{180}{10} = 18$, the equivalent strength for which in the table is 44,200 lbs. per square inch. The area of the column is $\cdot 7854\ (10^2 - 8^2) = 28\cdot 27$. Therefore the total safe load is:

$$\frac{44{,}200 \times 28\cdot 27}{8} = 156{,}191 \text{ lbs., or } 69\cdot 7 \text{ tons.}$$

Table LVIII.—Ultimate Strength of Hollow Cast-iron Columns (Birkmere) in pounds per square inch of sectional area.

Length ÷ diam.	Round.	Square.	Length ÷ diam.	Round.	Square.
5	75,300	76,200	17	46,444	50,700
6	73,400	74,630	18	44,200	48,540
7	71,270	72,860	19	42,100	46,460
8	68,970	70,920	20	40,000	44 450
9	66,530	68,850	21	38,100	42,510
10	64,000	66,670	22	36,200	40,650
11	61,420	64,410	23	34,460	38,870
12	58,820	62,110	24	32,790	37,175
13	56,240	59,890	25	31,220	35,560
14	53,860	57,470	26	29,740	34,010
15	51,200	55,170	27	28,340	32,550
16	48,780	52,910	28	27,030	31,150

§ (*b*) *Wrought-iron Struts and Columns.*—Table LIX. gives safe loads for wrought-iron struts and columns as calculated by Mr. Birkmere, of New York. Before this table can be used for any given column, the radius of gyration must be ascertained, either by calculation or from one of the numerous tables always available in the United States, but rarely so in this country. Suppose we wish to find the safe load for an I-joist used as a column, with flat ends; length 10 ft., size 10 in. by 5 in., area 8·53 in. The value of r is found by calculation to be 1·14; therefore $\frac{l}{r} = 120 \div 1{\cdot}14 = 105$, and, taking the strength opposite to the ratio next higher in value than 105 in the first column of the table, the safe load per square inch is found to be 6,840 lbs. Multiplying this by the area, we have $6{,}840 \times 8{\cdot}53 = 58{,}345$ lbs., or 26 tons total safe load. Similar calculations can readily be made for any other form of section.

Table LIX.—Calculated Safe Loads for Wrought-iron Struts and Columns (Birkmere) in pounds per square inch of sectional area.

(For round columns add 10 per cent. to the figures given below; for square columns add 5 per cent.)

Length ÷ least Radius of Gyration.	Flat Ends.	Fixed Ends.	Hinged Ends.	Round Ends.
20	14,380	14,380	13,940	13 330
30	13,030	13,030	12,460	11,670
40	11,760	11,760	11,110	10,140
50	10,860	10,860	10,130	8,930
60	10,000	10,000	9,230	7,820
70	9,190	9,190	8,330	6,850
80	8,420	8,420	7,500	5,950
90	7,920	7,950	6,840	5,230
100	7,450	7 500	6,220	4,560
110	6,840	7,070	5,620	3,980
120	6,260	6,670	5,060	3,440
130	5,790	6,220	4,580	2,960
140	5,340	5,800	4,120	2,510
150	4,830	5,390	3,570	2,120
160	4,350	5,000	3,060	1,760
170	3,920	4,570	2,640	1,530
180	3,500	4.170	2,250	1,310
190	3,190	3,830	2 020	1,150
200	2,900	3,500	1,800	1,000

§ (*c*) *Phœnix Columns.*—Table LX. gives particulars of the well-known Phœnix columns, which are built up of pillar sections (Table VI., and figs. 169, 170). It will be observed that the fullest details are given on all essential points, so that strength can easily be estimated by the usual rules, or by any reliable table showing either the ultimate strength, or the safe load corresponding to an ascertained value of the ratio $\frac{l}{r}$.

Table LX.—Dimensions of Phœnix Columns.
Columns A, B, and C have four Segments.
Least Radius of Gyration = D × ·3636.

One Segment.		Diameters in inches.			One Column.			
Thickness in inches.	Weight in lbs. per yard	d Inside.	D Outside	D¹ Over Flanges	Area of Cross Section square in.	Weight per foot in lbs.	Least Radius of gyration in inches.	Size of Rivets.
$\frac{3}{16}$	$9\frac{1}{2}$	A	4	$6\frac{1}{16}$	3·8	12·6	1·45	$\frac{3}{8} \times 1\frac{1}{8}$
$\frac{1}{4}$	12	$3\frac{3}{8}$	$4\frac{1}{8}$	$6\frac{3}{16}$	4·8	16·0	1·50	$1\frac{1}{4}$
$\frac{5}{16}$	$14\frac{1}{2}$		$4\frac{1}{4}$	$6\frac{5}{16}$	5·8	19·3	1·55	$1\frac{3}{8}$
$\frac{3}{8}$	17		$4\frac{3}{8}$	$6\frac{7}{16}$	6·8	22·6	1·59	$1\frac{1}{2}$
$\frac{1}{4}$	16	B	$5\frac{5}{16}$	$8\frac{1}{16}$	6·4	21·3	1·92	$\frac{1}{2} \times 1\frac{5}{8}$
$\frac{5}{16}$	$19\frac{1}{2}$	$4\frac{13}{16}$	$5\frac{7}{16}$	$8\frac{1}{8}$	7·8	26·0	1·96	$1\frac{3}{4}$
$\frac{3}{8}$	23		$5\frac{9}{16}$	$8\frac{1}{4}$	9·2	30·6	2·02	$1\frac{3}{4}$
$\frac{7}{16}$	$26\frac{1}{2}$		$5\frac{11}{16}$	$8\frac{3}{8}$	10·6	35·3	2·07	$1\frac{7}{8}$
$\frac{1}{2}$	30		$5\frac{13}{16}$	$8\frac{7}{16}$	12·0	40·0	2·11	$1\frac{7}{8}$
$\frac{9}{16}$	$33\frac{1}{2}$		$5\frac{15}{16}$	$8\frac{1}{2}$	13·4	44·6	2·16	2
$\frac{5}{8}$	37		$6\frac{1}{16}$	$8\frac{5}{8}$	14·8	49·3	2·20	$2\frac{1}{8}$
$\frac{1}{4}$	$18\frac{1}{2}$	C	$6\frac{7}{16}$	$9\frac{1}{8}$	7·4	24·6	2·34	$\frac{1}{2} \times 1\frac{5}{8}$
$\frac{5}{16}$	$22\frac{1}{2}$	$5\frac{15}{16}$	$6\frac{9}{16}$	$9\frac{1}{4}$	9·0	30·0	2·39	$1\frac{3}{4}$
$\frac{3}{8}$	$26\frac{1}{2}$		$6\frac{11}{16}$	$9\frac{5}{16}$	10·6	35·3	2·43	$1\frac{3}{4}$
$\frac{7}{16}$	$30\frac{1}{2}$		$6\frac{13}{16}$	$9\frac{3}{8}$	12·2	40·6	2·48	$1\frac{7}{8}$
$\frac{1}{2}$	$34\frac{1}{2}$		$6\frac{15}{16}$	$9\frac{1}{2}$	13·8	46·0	2·52	$1\frac{7}{8}$
$\frac{9}{16}$	$38\frac{1}{2}$		$7\frac{1}{16}$	$9\frac{5}{8}$	15·4	51·3	2·57	2
$\frac{5}{8}$	$42\frac{1}{2}$		$7\frac{3}{16}$	$9\frac{11}{16}$	17·0	56·6	2·61	$2\frac{1}{8}$

§ (*d*) *Z-bar Columns.*—Table LXI. is one of a series contained in the handbook of Messrs. Carnegie, Phipps & Co., and is a fair example of the kind of information furnished by American manufacturers. (For Z-bars see Table VI.; and for Z-bar column see fig. 167).

Table LXI.—Safe Loads in tons of 2,000 lbs. of Steel Z-bar Columns, square ends (Carnegie, Phipps, & Co.).

Allowed strains per square inch for steel, safety factor 4 :—

12,000 lbs. for lengths of 90 radii or under. $17{,}100 - 57\frac{l}{r}$ for lengths over 90 radii.

20-in. Steel Z-bar Columns.

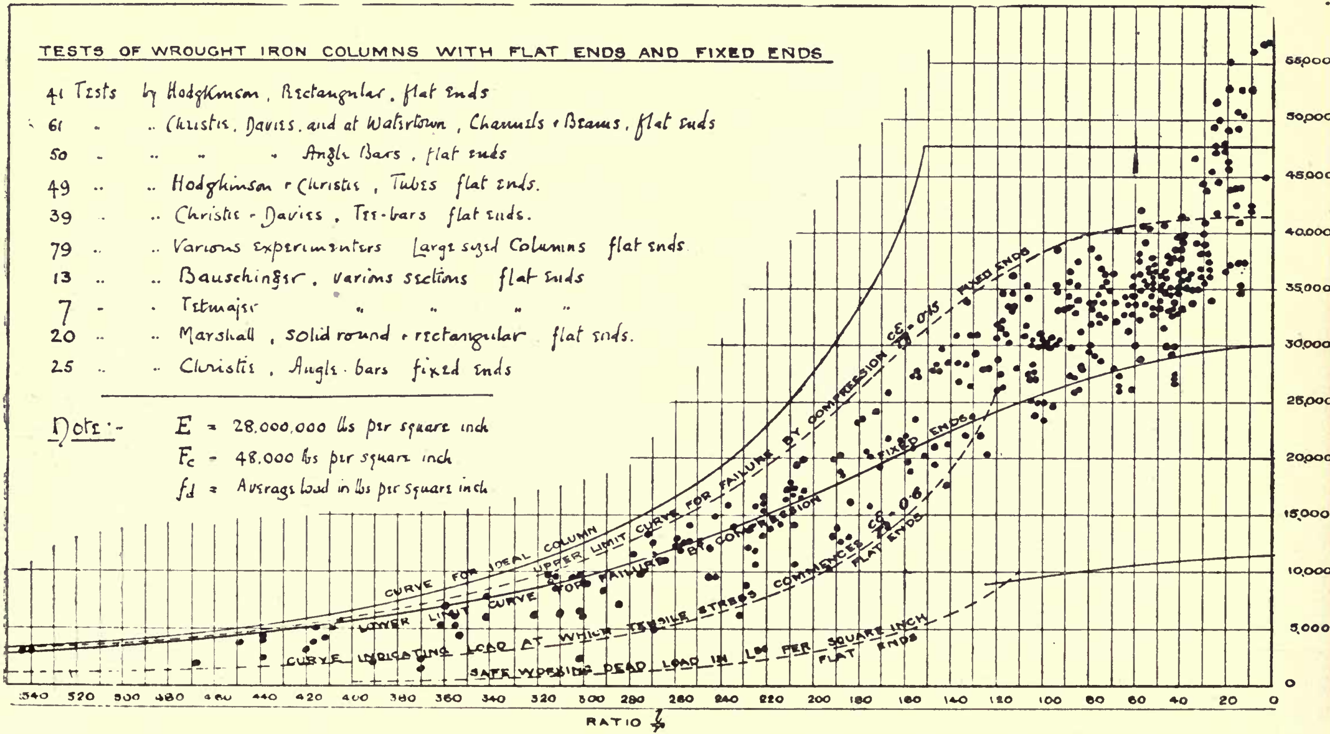
Section: 4 Z-bars $6\frac{1}{8}$ in. × $\frac{7}{8}$ in. 1 Web Plate 14 in. × 1 in. 6 Side Plates 20 in. wide.

Length of Column in feet.	20 × 2 plates = 458·0 lbs. = 134·7 square in. r (min.) = 5·92	20 × $2\frac{1}{16}$ plates = 466·5 lbs. = 137·2 square in. r (min.) = 5·92	20 × $2\frac{1}{8}$ plates = 475·0 lbs. = 139·7 square in. r (min.) = 5·92	20 × $2\frac{3}{16}$ plates = 483·5 lbs. = 142·2 square in. r (min.) = 5·92	20 × $2\frac{1}{4}$ plates = 492·0 lbs. = 144·7 square in. r (min.) = 5·91	20 × $2\frac{5}{16}$ plates = 500·5 lbs. = 147·2 square in. r (min.) = 5·91	20 × $2\frac{3}{8}$ plates = 509·0 lbs. = 149·7 square in. r (min.) = 5·91	20 × $2\frac{7}{16}$ plates = 517·5 lbs. = 152·2 square in. r (min.) = 5·91	20 × $2\frac{1}{2}$ plates = 526·0 lbs. = 154·7 square in. r (min.) = 5·90
44 and under ...	808·1	823·1	838·1	853·1	868·1	883·1	898·1	913·1	928·1
46	793·7	808·3	823·0	837·5	852·1	866·7	881·2	895·8	910·4
48	778·2	792·5	806·9	821·2	835·5	849·7	864·0	878·3	892·6
50	762·6	776·7	790·8	804·7	818·7	832·8	846·7	860·7	874·7

§ (*e*) *Remarks on the Use of Tables.*—So far as the use of tables is concerned, it may be said that absolute reliance should never be placed upon such aids to calculation, and the architect should always know exactly what is the true purport of all data that he is employing. Tables of dimensions, whether concrete or abstract, are usually reliable when issued by responsible firms, but information as to the ultimate strength, and especially as to the safe load, of columns ought to be accepted with some reserve, unless their origin and the conditions to which they apply are fully known. It is preferable to adopt tables showing ultimate strength, as the operator can then add his own factor of safety according with the nature of the case under consideration. Even then great care must be taken in the selection of tables, for many of the tests by different experimenters do not in themselves afford any reliable basis for practical work. It is only when records furnished by experiments covering a sufficiently wide range are gathered together in large numbers and submitted to close scrutiny and classification, that their real teaching becomes evident. More harm than good is likely to result from the indiscriminate repetition of data in various text-books and collections of tables; and no really comprehensive and intelligible summary of all authoritative records has hitherto existed. This deficiency has to some extent been made good by recent investigations, to which attention will be directed in the next article.

74. Moncrieff's Investigation. — An exhaustive inquiry as to the strength of columns has been made by Mr. J. M. Moncrieff, M.Inst.C.E., whose practical conclusions are fully stated in a paper recently communicated to the American Society of Civil Engineers. All the most important experimental records are dealt with, including those of Bauschinger, Christie, Considère, Hodgkinson, Marshall, Tetmajer, and others furnished by various official tests conducted in the United States. Mr. Moncrieff has subjected the data at his disposal to analytical examination, and all results, except those involving defects or special conditions, are plotted in a series of diagrams in such a manner that the relationship between the direct load and the ratio $\frac{l}{r}$ may be seen at a glance.

Upwards of thirty diagrams of this kind are given in the paper, containing the results of no fewer than 1,789 experiments indicative of the behaviour of cast iron, wrought iron, steel, and timber columns. Curves are drawn in each diagram, showing the strength of the ideal column, the upper and lower limits of strength of the practical column, and the safe working dead load of the latter, *Fig.* 143.—*The Moncrieff Theory, and various Experimental Results.*



These curves have been calculated by a formula, now made public by Mr. Moncrieff, which exhibits features worthy of careful consideration.

For the purpose of showing the manner in which Mr. Moncrieff has presented his facts, we reproduce one of his diagrams (fig. 143), where full details of a large number of tests are plotted, and curves are given showing how far the formulas are consistent with experimental results.

75. The Moncrieff Theory.—The theory forming the basis of the formula relies chiefly upon two assumptions:—(1) that a perfectly centred column, of perfect material and straightness, is an ideal conception seldom or never realised in practice, and (2) that the various disturbing influences preventing its realisation are practically all capable of being represented, as regards their ultimate effect, by an equivalent eccentricity of loading. Any theory founded on these principles should be expressed by a formula from which the strength of the ideal column may be calculated, if the factor representing eccentricity be reduced to zero. To the first proposition no one will be likely to object, and as regards the second, we may remark that the investigations of Bauschinger, Christie, Considère, and Marshall show that the physical and geometrical axes of a column do not often coincide, and that a column may possess greater strength when loaded in an apparently eccentric manner than when loaded in an apparently central manner. So far, then, the theory appears to be reasonable and consistent with ascertained facts.

§ (*a*) *Accidental Eccentricity.*—An important factor in the formula is ε, representing *equivalent eccentricity*. The value of ε cannot be determined beforehand, for it depends upon defects of material or of workmanship, and upon the effects of manipulation during manufacture (Arts. 79, § *a*; 82). As employed by Mr. Moncrieff, ε finds a place in the expression $\frac{c\,\varepsilon}{r^2}$, denoting *accidental eccentricity*. The value of $\frac{c}{r}$ could easily be calculated, because c is the distance of the extreme fibres in a column section from the neutral axis—the distance, for instance, of *a* from A in fig. 128, p. 145—and r is the radius of gyration of the section. In the present state of our knowledge it appears to be desirable that one constant value should be assigned to the quantity $\frac{c\,\varepsilon}{r^2}$, as determined by experimental evidence. Any column of medium length having the least eccentricity of loading, or the least degree of initial curvature, will immediately commence to bend under load, and the bending will increase more rapidly than the increase of load. Therefore we see that any expression for the strength of a column should not only be in harmony with the laws governing deflection, but ought also to include the corrective factor $\frac{c\,\varepsilon}{r^2}$, for accidental eccentricity.

§ (*b*) *Incipient Tension.*—We already know that when flexure takes place both compressive and tensile stresses are induced, but it should be remembered that the maximum compressive stress in a column of symmetrical section has a greater value in pounds per square inch than the maximum tensile stress; consequently tension does not become the controlling factor unless a considerable difference exists between the compressive and tensile strengths of the material. It is necessary, however, to take tension into account to a certain extent when determining the strength of a column with flat ends which are not fixed, because a distinct line is to be drawn between the column with flat ends and the column with fixed ends. In the former type no tensile stress can be developed at the ends, and if tension were attempted to be set up at these points, the ends would begin to rotate on their bearing surfaces, and the column would be in a highly unstable condition. In a column with fixed ends, the bending moment at each end is theoretically equal to that at the centre. So long as no tensile stress is established in a column with flat ends, it will behave exactly the same as a column with fixed ends, that is to say, up to the point of loading where

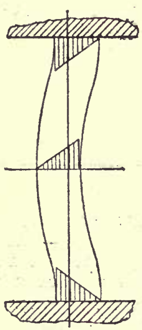


Fig. 142.

stress in the column at its ends and centre is as shown in fig. 142, when a small increase of load will probably result in failure. The accuracy of this view is demonstrable by considering the case of a column whose ends are fixed, in which tensile stresses have been developed at the ends and centre. If the fibres in tension at the ends could be cut, whilst under load, so as to transform the column into a flat-ended column, we should naturally expect to find an immediate alteration of curvature, and it is highly probable that the column would fail with the same or a slightly augmented load. The substantial truth of these views with regard to flat-ended columns appears to be clearly evidenced in Mr. Christie's experiments, and Mr. Moncrieff provides for the effect of incipient tension by a rule derived from his formula for failure by tensile stress.

76. Moncrieff's Formulas.—This modified rule is intended to determine the critical condition of incipient tension, and if flat-ended columns are proportioned in accordance therewith, no tension can be set up, and they will behave similarly to columns with fixed ends.

In their unabridged forms the two equations for estimating the strength of columns are thus written :—

For failure by compression :—

$$\frac{l}{r} = \sqrt{\frac{48\,E}{5\,F_c + f_d\left(\frac{c\,\varepsilon}{r^2} - 5\right)}\left[\frac{F_c}{f_d} - 1 - \frac{c\,\varepsilon}{r^2}\right]} \quad \ldots (9)$$

For incipient tension in flat-ended columns :—

$$\frac{l}{r} = 2\sqrt{\frac{48\,E\left(1 - \frac{c\,\varepsilon}{r^2}\right)}{f_d\left(\frac{c\,\varepsilon}{r^2} + 5\right)}} \quad \ldots\ldots (10)$$

§ (*a*) *Precise Uses of the Formulas.*—Formula (9) applies to columns with round, hinged, or pin ends without alteration; when used for columns with flat or fixed ends the value ascertained must be multiplied by two; but formula (10) must be applied to columns with flat ends when the ratio of $\frac{l}{r}$ is reached, at which incipient tension becomes the controlling factor, The following are the significations of the symbols used in the formulas stated above :—

$\frac{l}{r}$ = length in ins. ÷ least radius of gyration, in. ins.

E = modulus of elasticity, in pounds per square inch.

F_c = maximum compressive stress, in pounds per sq. in.

f_d = direct load, in pounds per square inch.

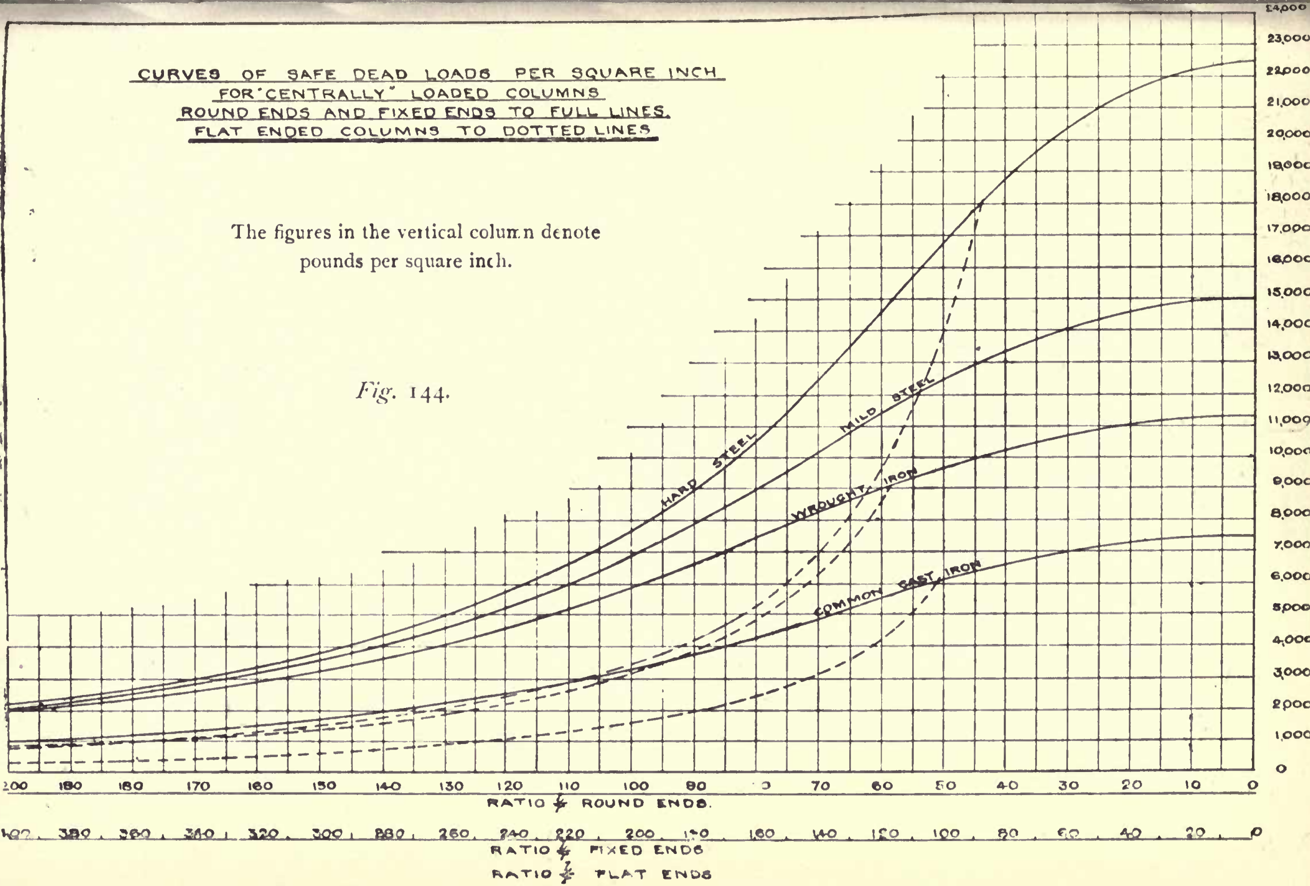
$\frac{c\,\varepsilon}{r^2}$ = a constant, representing accidental eccentricity.

The precise use of equation (9) is as follows :—Let it be assumed that in a given section a certain value of maximum compressive stress F_c is not to be exceeded; if the numerical equivalent of this be inserted, together with the values of f_d, E, and $\frac{c\,\varepsilon}{r^2}$, we have at once the proper ratio of $\frac{l}{r}$, corresponding to the direct load f_d per square inch.

The formulas can be used, as they have been in connexion with fig. 143, for ascertaining ultimate strength and for calculating the strength of the ideal column.

For actual practice they are intended to indicate the correct ratio of $\frac{l}{r}$ for any given direct load, at the same time fulfilling the conditions that the maximum fibre stress shall be within safe limits as regards the strength of the material, and that the working load shall not be more than one-third of that under which instability would result by reason of insufficient stiffness of the material. The first condition is secured by assigning to F_c a sufficiently low value, and the second by the insertion of the factor of safety K. These qualifications, the manner in which they are effected, are explained in the next article.

§ (*b*) *Working Formulas.*—Two forms of the Moncrieff formula (9) and (10) have already been given, but it is desirable that the rules should be simplified and modified for use in every-day work. Each equation is qualified by the insertion of suitable values for the various factors, and the co-efficient of safety K is added. The following are the values assigned by Mr. Moncrieff for different materials, but others may be used if desired ;—



The figures in the vertical column denote pounds per square inch.

Fig. 144.

Table LXII.—Co-efficients for Moncrieff Formulas.

Factors.	Cast Iron.	Wrought Iron.	Mild Steel.	Hard Steel.
Modulus of elasticity (E), lbs. per sq. in. ...	14,000,000	28,000,000	30,000,000	30,000,000
Maximum compressive fibre stress, F_c, lbs. per sq. in.	12,000	18,000	24,000	36,000
Accidental eccentricity, $\frac{c\,\varepsilon}{r^2}$	0.6	0·6	0·6	0·6
Factor of safety against instability K	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Units of 10,000 lbs. are used for E and F_c, and when the values have all been substituted, working formulas are obtained for the calculation of safe dead loads. The only modification necessary in dealing with live loads instead of dead loads obviously consists of a reduction in the values of F_c, and an increase in the value of K. Mr. Moncrieff suggests as an alternative that the moving load should be increased by a percentage dependent upon the character of the load, and that the result should then be treated as a dead load.

Table LXIII.—Moncrieff Working Formulas for Columns under "Central Loading."

Material.	Formula (a) for round-ended columns. (Results to be doubled for fixed-ended columns.)	Formula (b) for incipient tension in flat-ended columns.
Cast iron ...	$\frac{l}{r}=10\sqrt{\frac{224}{6-4\cdot4f_d}\left(\frac{1\cdot2}{f_d}\right)-1\cdot6}$	$\frac{l}{r}=2\cdot138\sqrt{\frac{1{,}400}{f_d}}$
Wrought iron	$\frac{l}{r}=10\sqrt{\frac{448}{9-4\cdot4f_d}\left(\frac{1\cdot8}{f_d}\right)-1\cdot6}$	$\frac{l}{r}=2\cdot138\sqrt{\frac{2{,}800}{f_d}}$
Mild steel ...	$\frac{l}{r}=10\sqrt{\frac{480}{12-4\cdot4f_d}\left(\frac{2\cdot4}{f_d}\right)-1\cdot6}$	$\frac{l}{r}=2\cdot138\sqrt{\frac{3{,}000}{f_d}}$
Hard steel ...	$\frac{l}{r}=10\sqrt{\frac{480}{18-4\cdot4f_d}\left(\frac{3\cdot6}{f_d}\right)-1\cdot6}$	$\frac{l}{r}=2\cdot138\sqrt{\frac{3{,}000}{f_d}}$

§ (*c*) *Examples.*—The diagram in Fig. 144 represents various curves obtained by the use of the working formulas stated in Table LXIII. The following examples will serve to explain how the curves are calculated. Units of 10,000 lbs. are used for f_d the direct load.

Ex. 1.—If the dead load per square inch be limited to 5,000 lbs. in a cast-iron column, find the correct ratios of $\frac{l}{r}$ for round and fixed ends respectively.

(1) For round ends by formula (*a*):—

$$\frac{l}{r} = 10\sqrt{\frac{224}{6 - 4{\cdot}4\,({\cdot}5)}\left(\frac{1{\cdot}2}{{\cdot}5}\right) - 1{\cdot}6} = 68{\cdot}6.$$

(2) For fixed ends the ratio is

$$68{\cdot}6 \times 2 = 137{\cdot}2.$$

Ex. 2.—With the same limit of load as in Ex. 1, find the correct ratio of $\frac{l}{r}$ for flat ends. By formula (*b*):—

$$\frac{l}{r} = 2{\cdot}138\sqrt{\frac{1{,}400}{{\cdot}5}} = 113{\cdot}1$$

It may not seem to be clear when formula (*a*) and when formula (*b*) should be applied to columns with flat ends, because the point at which the curves would intersect is not necessarily known. As, however, the lower ratio is always that which must be accepted, we have an easy clue to the selection of the proper rule.

These equations may be used for the purpose of obtaining a series of curves for general guidance, or they can be applied to individual cases.

Perhaps the following example will serve to illustrate the practical application and convenience of the formula, or of the curves obtained therefrom.

A wrought-iron column, 15 ft. long, with flat or with fixed ends, has to carry 30 tons dead load = 67,200 lbs., and the maximum weight is not to exceed 10,000 lbs. per square inch of section. We find either by direct calculation or from the diagram of safe dead loads that the ratio $\frac{l}{r} = 90$; therefore $r = \frac{180}{90} = 2$. As the sectional area should be $\frac{67,200}{10,000} = 6{\cdot}72$ sq. in., any column may be selected whose radius of gyration is not less than 2 in., and whose sectional area is not less than 6·72 sq. in.

77. Misconception as to Fixity of Columns.—Before leaving this part of our subject it will be well to direct attention to the fact that a serious misconception frequently exists with regard to the requirements necessary for realising the conditions of fixity in the ends of a column. Columns are often put up and fixed in a fashion which makes them nothing more than flat-ended columns so far as their behaviour and strength are concerned. To make this matter perfectly plain, let us take two examples. In the first, let it be assumed that a row of columns, of which two are drawn in fig. 145, are properly fixed to rigid foundations, and the upper ends of the columns are secured to a strong and rigid girder. Of course, the imposition of a load on any span of the girder will cause deflection, but the deviation will be small, and the stiffness of the girder, as compared with that of the columns, will be relatively so high, that fixity will be practically, though not theoretically, attained. In the second example, let us assume the columns to be spaced some distance apart, and that their upper ends are fixed to a girder which is only moderately rigid. Then the girder, when loaded, will deflect considerably, as shown to an exaggerated degree by dotted lines in fig. 146, and the columns will experience severe bending stresses in addition to their direct loads. Thus, it should be clear that an injudicious attempt to fix the ends of a column is not likely to afford the expected augmentation of trength.

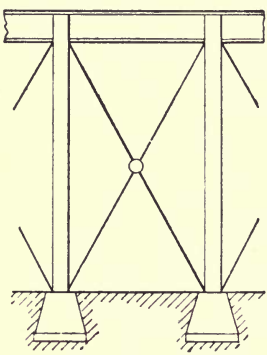


Fig. 145.

If there be any uncertainty as to the fixity of the ends, strength should be calculated as for a flat-ended column,

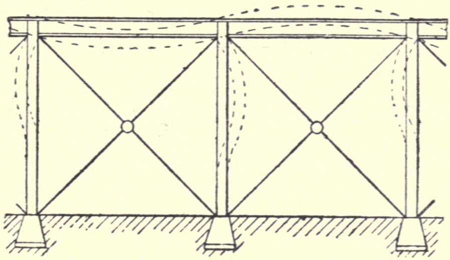


Fig 146.

with due allowance for bending stresses, as mentioned above, if any such may reasonably be anticipated.

CHAPTER XII.

CAST-IRON COLUMNS AND CONNEXIONS.

78. Typical Forms.—Cast-iron columns and stanchions are made with sections of so varied a nature, that no difficulty occurs in choosing a form suitable for any required purpose. A channel section is much used for carrying the ends of main girders entering party walls, and is then generally embedded in the brickwork. Rectangular stanchions or pilasters are employed for similar purposes, but are generally fixed against the wall surfaces. One of the most common forms of stanchion is of H-section, and it may be made either with solid or hollow web. This section is largely used in mills and factories, chiefly because of the convenience it offers for the attachment of brackets for shafting and of other mechanical features. When exceptional heavy loads have to be carried the addition of a third flange will be found convenient. The cruciform section is considerably used, although it is one of the weakest types which could be designed, and is therefore the most expensive so far as the weight of metal is concerned. One advantage it possesses, in common with other types having open flanges and webs, is that any flaws or defects in the metal may readily be detected. Judged by the criterion of strength per unit weight of metal, the hollow cylindrical column is undoubtedly the best and most economical of all. It is, of course, the least desirable form for building into the walls, and is more difficult to fireproof than the other shapes to which reference has been made.

§ (*a*) *Values of Different Sections.*—The relative strengths of three simple forms of column are stated below, and the student may find it instructive to calculate the resistance of various sections by the rules which have previously been explained :—

Hollow cylindrical section	100
H-section	75
Cruciform section	44

All columns or stanchions with open webs should be strengthened by transverse webs called "stiffeners." These are usually cast at intervals of 3 ft. apart, and when the hollows of the column are to be filled with cement or fine concrete, holes may advantageously be cast in the stiffeners so that continuity of the filling may be preserved.

§ (*b*) *Columns Enclosed in Brickwork.*—When a column or stanchion is adjacent to, or enclosed by, a wall, care must be taken to see that the load comes wholly upon the iron member, and that it does not rest partly upon the wall itself. If this point be overlooked, cracking of the brickwork will most likely result. Therefore, it is desirable (1) that an enclosed column should be independent of the brickwork, so far as relates to vertical movement, and (2) that the top course of the wall should be slightly below the cap or other part of the column upon which the load is intended to be imposed.

§ (*c*) *Ornamental Columns.*—Ornamental cast-iron columns and pilasters are still used, though not to the same extent as formerly. A well-known firm of ironfounders, in some remarks prefatory to illustrations of ornamental columns, suggests cast iron as a material not liable to injury from fire. This expression ought only to be accepted in a comparative sense, and whilst fully recognising the merits of cast iron, we feel it necessary to advise the invariable application of proper fireproof protection to all supports formed of this material.

79. Practical Proportions.—The thickness of metal in hollow cast-iron columns should not be less than one-twelfth of the diameter, and in practice it generally varies between this fraction and one-sixth. Morin suggested the following proportions for the length and thickness of columns :—

Length 7 to 10 ft.,	maximum	thickness	0·5 in.
,, 10 ,, 13 ft.	,,	,,	0·6 ,,
,, 13 ,, 20 ft.	,,	,,	0·8 ,,
,, 20 ,, 27 ft.	,,	,,	1·0 ,,

§ (*a*) *Defects.*—As pointed out in Art. 13, § *c*, careful examination of hollow castings is always desirable. Very little useful information is to be derived from external contemplation, and if the architect or engineer wishes to be quite sure that the metal is uniformly of the specified thickness, he must cause holes to be drilled at different points, so that actual measurements may be taken. The practice of casting columns horizontally is not entirely obsolete, and when it is adopted, considerable eccentricity may be expected from shifting of the core. Unequal internal strains, productive of initial curvature, are also likely to be developed. By the more modern system of casting columns in a vertical position these defects are less undesirably evidenced, and the metal is of better quality because gases are forced out by the head of metal.

§ (*b*) *Bases and Base Plates.*—Whatever may be the profile of a column, the base should be spread somewhat for the purpose of securing lateral stiffness. The column shown in fig. 147 is one of those supporting the roof of Lime-street Station, Liverpool. It has a length of 20 ft., a mean diameter of 3 ft., and the thickness of metal in the shaft is 2 in. In this example the base is only thickened at the extreme end, and the column is secured by means of an annular ring held in its place by foundation bolts. None of the load is supported by the apparent base and capital, which are merely added for architectural effect. Flat base-plates, as fig. 149, are suitable for light loads, but should not be used for heavy loads, or the weight will not be distributed equally over the foundation stone. When the load is considerable, a ribbed base-plate, such as that shown in fig. 150, ought to be employed. Plates of this description should be of sufficient area for distributing the weight as may be necessary for safety and stability. The bottom plate and the body of the base should be the same thickness as the column to be supported; the ribs must be of similar thickness in order that the casting may not be strained in the act of cooling. A hole is left in the centre through which grout may be run, and smaller holes are provided between the ribs for the purpose of indicating whether the operation of grouting has been properly performed. Very often the base of a

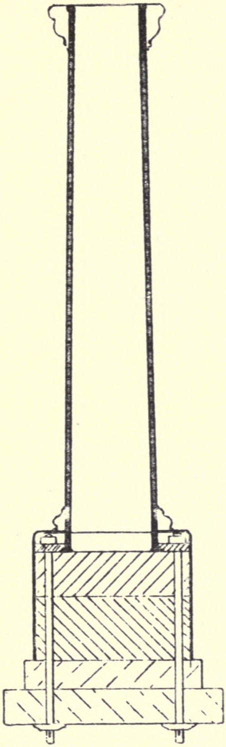


Fig. 147.

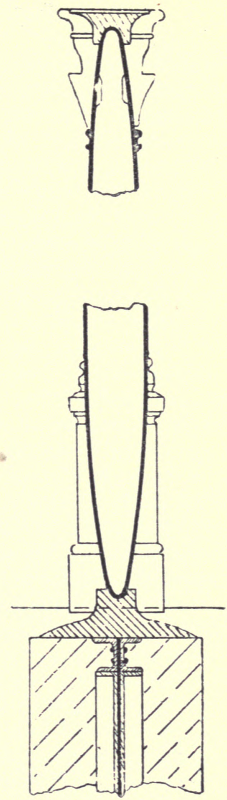


Fig. 148.

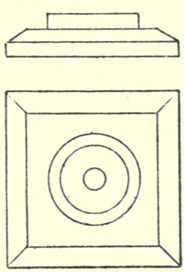


Fig. 149.

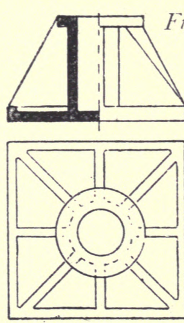


Fig. 150.

column is merely bolted to the foundation plate at the ground level, where it is more or less subject to corrosive influences. A better plan is to make the joint a little below the surface, as in fig. 147, so that it may be adequately protected. In all important work the joints should be faced in the lathe, but frequently the faces are merely chipped off level, and a sheet of lead or felt is inserted for the purpose of compensating for deficiencies of workmanship. Sheet lead may cause serious galvanic action, and felt is almost certain to crumble into powder in course of time. We have previously pointed out the fact that corrosion is most apt to take place at joints (Art. 58, §*c*); therefore it is obvious that the bolts by which a column is secured should be protected in some adequate manner. If the base should become unstable through corrosion of the bolts, a considerable loss of strength will ensue, as the condition of fixity will cease to obtain, and the column will virtually become one with flat ends.

§ (*c*) *Round-ended Columns.*—An interesting example of a column with rounded ends is given in fig. 148. This particular form of support was designed for the inner portion of the gallery at Olympia, Kensington, by the joint engineers, Mr. A. T. Walmisley and Mr. Max am Ende. The whole gallery acts as an abutment to the roof arch, and the ball-and-socket joint was adopted so that bending moment on the column might be obviated. Columns with rounded ends are sometimes used in structures where considerable variations of length are expected in consequence of expansion and contraction. Columns or struts with round, pivoted, and pin ends are, of course, most generally used in connexion with bridge and roof work.

80. Column Connexions.—In large buildings the columns usually extend from the basement to the roof, and arrangements must be made at each floor for the connexion of girders. The columns may, therefore, be supplied in lengths governed by the distance from floor to floor, and the different sections can conveniently be joined together near the points at which the girders are connected.

§ (*a*) *Forms of Joints.*—Sometimes a simple spigot and faucet joint is used, similar to that adopted for water pipes, but it is far better that bolts should also be applied, as

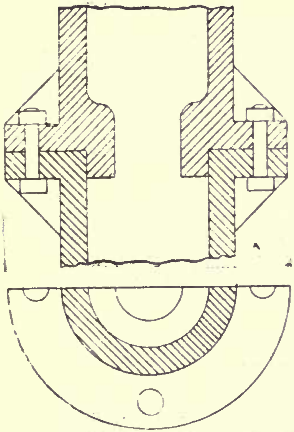


Fig. 151.

shown in fig. 151. For the best class of work the surface should invariably be turned and bored so that a true joint of metal to metal may be ensured. If the spigot and faucet be omitted, the bolt-holes ought to be drilled, and the bolts themselves should be turned to fit the holes; cast holes with loosely-fitting bolts would be entirely inadmissible, as prejudicial to the stability of the joint. Various artifices are adopted for hiding the actual joint in a column, and some of these are described below. As we have seen in figs. 147 and 148, false bases and capitals may be attached, and a similar mode of treatment can be followed in buildings where the connexion is between the ceiling of one story and the floor of another.

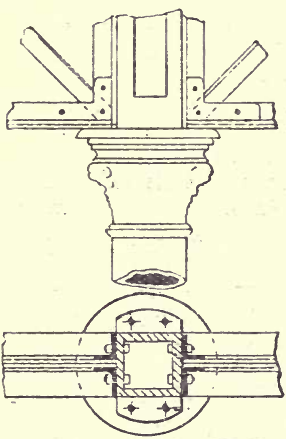


Fig. 152.

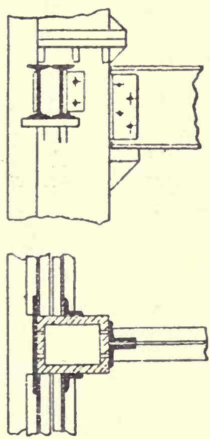


Fig. 153.

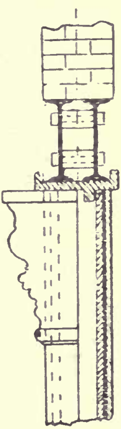


Fig. 154.

At other times no artifice is necessary. For instance, in fig. 152 the capital is useful as well as ornamental, for it serves to support two girders, of which the ends are shown in the illustration, and it also provides a joint for the extended portion of the column.

§ (*b*) *Beam Supports.*—Fig. 153 shows the elevation and plan of a column detail in the Jackson Building, New York. The columns measure 16 in. by 12 in., and are placed in the side walls of the building, about 15 ft. apart; the main girders are 20 in. deep, and rest upon brackets cast with the columns, whilst they are connected by angle knees and bolts. Smaller girders are similarly fixed at each side of the columns for the purpose of supporting curtain walls. A so-called fireproof column is shown in fig. 154, where a cast-iron shell-covering surrounds the shaft, and the intervening space may be filled with incombustible material. The capital is loose, and may be in two halves, so that it can be fixed after the column has been erected. This column is supporting an upper wall, which rests directly upon a pair of girders connected by distance pieces and bolts.

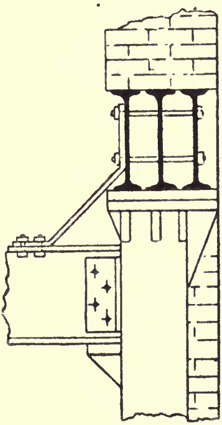


Fig. 155.

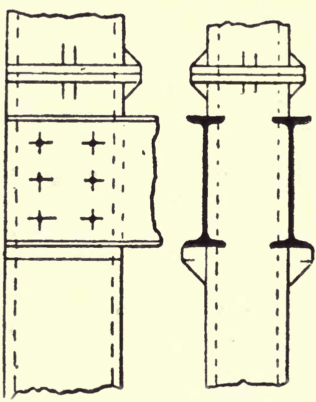


Fig. 156.

A similar arrangement is illustrated in fig. 155, where an upper wall is built upon a three-beam girder; the main girder at the left hand rests on a cast bracket, being secured to the column by angle brackets and to the upper girder by a 6-in. by $\frac{1}{2}$-in. strap. Sometimes it may be inconvenient to use a deep main girder, and in such a case the connexion shown in fig. 156 will be useful. Here two I-beams are fixed, one on each side of the column. This arrangement provides the necessary rigidity, and as the girders need only be of moderate depth, the ceiling below can be perfectly level.

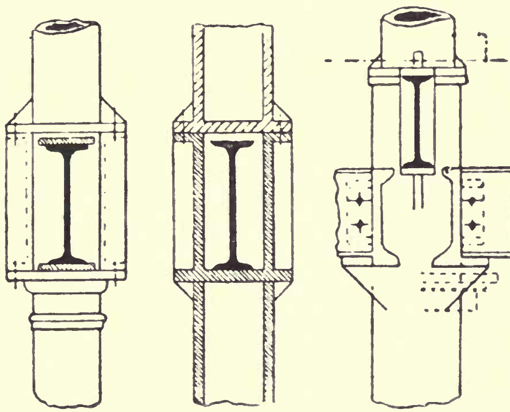


Fig. 157. *Fig.* 158. *Fig.* 159.

When a main girder has to extend through a column or stanchion, provision should be made as indicated in figs. 157 and 158. In the first of these diagrams the two separate lengths of the column are connected by independently cast jaws of sufficient height to permit the passage of the girder. In the second illustration the jaws form part of the lower section of the column. These methods of construction can be applied either to round or to square columns, but the connecting part between the cap of the lower length and the base of the upper length must be of rectangular section. A mode of connexion suitable for internal columns is shown in fig. 159, where the floor joist rests on a bracket and is secured by bolts to a lug cast with the column. This mode of fixing is by no means so desirable as that in which angle knees are used, for the reasons that the lug may break off, and that the number of bolts is necessarily more limited. The main girders at each side of the column are connected to similar lugs, and if the girder consist of two joists placed side by side, the lug can be cast in the form of a hollow rectangle, and of suitable size, so that it may act as a separator or distance piece. Dotted lines in the figure indicate the positions of the capital and base respectively of the lower and upper lengths of the column.

CHAPTER XIII.

WROUGHT-IRON AND STEEL COLUMNS. CONNEXIONS.

81. Typical Forms.—In the present chapter no distinction will be drawn between wrought-iron and steel columns, because construction in either material is precisely the same. As a matter of fact, the employment of mild steel is now practically universal, and wrought-iron columns are out of date so far as new work is concerned.

§ (*a*) *Simple Sections.*—Stanchions may consist of a single I-joist or channel, providing the load is not too heavy.

The simplest possible form of steel column can be made by using a weldless tube, to which an angle ring is riveted at the bottom to serve as a connexion to the base, and another ring at the top for connecting the shaft with the cap. Tubes of this kind are readily obtainable up to 20 in. in diameter by $\frac{3}{4}$ in. thick, and they are made of round, square, octagonal, hexagonal, and other sections.

Some new sections of seamless tubes, manufactured at the Duisburg Iron and Steel Works, are illustrated in fig. 160, and it will be observed that they comply generally with the conditions necessary for strength and economy, although the second example does not appear to be of particularly desirable shape.

Fig. 160.

These sections are produced in lengths up to 33 ft. and up to 20 in. in diameter, so there are no longitudinal joints to favour corrosion and to cause weakness and expense. It is said that columns formed of these sections have already been somewhat extensively used on the Continent, and the attention of our own manufacturers might advantageously be turned in a similar direction.

§ (*b*) *Built-up Sections.*—Columns are most generally built up of various commercial forms of steel, and a great variety of sections can thus be produced, of which a few examples are illustrated in figs. 161 to 166.

Fig. 161.

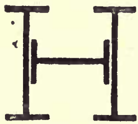


Fig. 162.

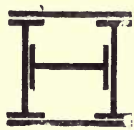


Fig. 163.

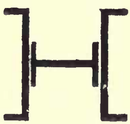


Fig. 164.

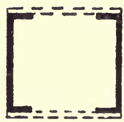


Fig. 165.

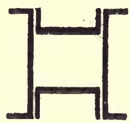


Fig. 166.

§ (*c*) *Strength and Suitability.*—Two conditions which should always govern the form given to the completed column are strength and suitability for the attachment of other members of a structure. Strength can easily be calculated by the rules given in Chapter XI. and suitability for the connexion of other members depends entirely upon the arrangement adopted, which should always permit the load to fall as equally as possible on all parts of the column.

§ (*d*) *Economy.*—It is always policy to select sections which can be riveted together with a minimum expenditure of time and material. Economy of manufacture is ensured by using sections providing the necessary strength, but requiring a small number of rivet lines.

§ (*e*) *Cruciform Section.*—A cruciform column may be made, as shown in fig. 161, by the use of three I-joists; the two smaller members are riveted through their flanges to the web of the larger one, and only two lines of rivets are required. Although the cruciform shape is not in itself economical because of its relative weakness, this drawback is considerably neutralised by savings in manufacture. The same kind of section can be obtained by building up the column of angles and plates, but eight lines of rivets are then necessary.

§ (*f*) *H-section.*—The H-section (fig. 162) is formed of three I-joists, joined by four lines of rivets. If cover plates are added, as in fig. 163, twelve rows of rivets are necessitated, and the metal inside the square is of small theoretical value, for it is always the case that metal near the neutral axis of a column contributes very little strength. Another type of H-section is shown in fig. 164. The only difference between this and the column illustrated in fig. 162 is that the outer members are of channel instead of I-section; by turning the channel so that the flanges face inwards, a more pleasing appearance is secured. Sometimes a built-up I-beam, as fig. 137 (Art. 71), is used for a column, and when the lines of rivets are not more than six in number this form is in every way satisfactory. In either of the last two examples, four lines of rivets are required.

§ (*g*) *Rectangular Section.*—Square columns may be constructed in several ways by the use of angles, channels, and plates. The channels may be riveted to solid side plates, or, if preferred, the sides can be latticed (fig. 165). Angles are frequently employed in making square columns, the side plates being fixed on the inside or outside of the angles according to fancy. The former arrangement is certainly more sightly than the other, though not quite so convenient for riveting. Examples of columns so made can be seen on some stations on the Metropolitan Railway and elsewhere. Square columns formed with channels only need four lines of rivets, whilst those with angles take eight lines. A good rectangular section is seen in fig. 166, made entirely of channels, and only necessitating four lines of rivets. Several modifications of this shape are obvious. For instance, the upper and lower channels might be displaced by two flat plates, or channels could be discarded altogether in favour of plates and angles.

§ (*h*) *Z-bar Section.*—A column made up of Z-bars is considered by some engineers to be superior to all other types. It certainly requires a comparatively small expenditure of labour, and provides facilities for the suitable attachment of girders. As will be seen in fig. 167 (Art. 83), only two lines of rivets are used, and all the surfaces of the column are readily accessible.

§ (*j*) *Phœnix Section.*—Columns are also made up of rolled segments, as shown in figs. 169 and 170 (Art. 83). Most of the large rolling mills in England manufacture this kind of section, which is well known in the United States as the "Phœnix" section, and, being patented, can only be obtained from one firm in that country. Any desired sectional area within reasonable limits can be secured by placing plates between the segments. In practice it is always desirable to use such plates if the column is formed of two or more lengths, as connexions cannot satisfactorily be made by any other means.

82. Weakness due to Internal Stresses.—One of the influences which tend to produce weakness in built-up columns by reason of internal stresses is the effect of machine riveting. We do not mean to imply, of course, that the individual rivet joints are weak as compared with hand-riveted joints, but to direct attention to the undoubted fact that when a compound member is being riveted up, the different parts tend to stretch and creep past each other, in varying degrees. Consequently the member may exhibit slight curvature on completion, or it may be apparently straight though subject to considerable internal stress. An instance of this effect is cited by Mr. Moncrieff, in connexion with a bridge over the river Tyne. In order to guard against the creeping tendency, the engineer's specification demanded that the large columns of the river piers should have their butt-joint ends machined over the full section, after having been riveted up in lengths at the contractor's works, the object being that the butts should have full bearing over the entire sectional area. The columns were of cruciform section, built of plates and angles, and the total length of each was about 81 ft., consisting of three separate lengths. Owing to some oversight in the case of some of the columns, the terms of the specification were not complied with, and instead, the ends of each bar and plate were machined to a good fit before riveting was commenced. Tacking rivets were put in for the purpose of keeping the different parts in their correct relative positions, and riveting was then proceeded with throughout the length of 27 ft. On completion, it was discovered that the angle-bars and plates had crept past each other to such an extent that the previous fitting was entirely nullified, and the joint had to be redressed by hand. The web-plate ends were also found to be hollow to the extent of $\frac{1}{16}$ in. in the half-width of column between the angle bars, thus evidencing internal stresses in the column as made. The columns appeared to be quite straight and true as a whole, and the material was all made and tested under the same specified requirements of tensile strength, viz., twenty-eight to thirty-two tons per square inch, with ultimate elongation of at least 20 per cent. This creeping tendency is one which cannot be avoided in riveted members, though it can be allowed for when estimates of strength are being made.

83. Column Connexions.—Columns of wrought iron and steel may be joined together in various ways, the simplest being that in fig. 172, where the top of one length and the bottom of another are fitted with angle-bar flanges which can be joined by bolts passing through a separating plate.

In fig. 167 a somewhat similar mode of connexion is illustrated, and the figure also shows how girders may suitably be supported. The two portions of the column are separated by a plate, the thickness of which must depend upon the weight transmitted by the girders. The two small joists are intended to carry a curtain wall, and they stand upon castings secured to the separating plate by bolts which pass through the flanges of the joists, as well as through the castings and the plate. The large girder rests directly upon the separating plate, and the bolts which hold it in position pass through two angle brackets serving to stiffen the plate and to connect the two portions of the column. The plate is also strengthened by brackets at each side, and a cover-plate is provided at the back of the column, where in this case there would be no room for an angle bracket, because the column is intended to be situated in one of the outer walls of a building.

The column, of which front and side elevations and sectional plan are shown in fig. 168, is of a type used in the New Netherland, New York. This building stands close to the entrance of Central Park, and has seventeen stories above the street level. It will be noticed that the girders, composed of two I-joists, are supported on angle brackets riveted to the column, and that they are also riveted to angle knees placed at each side of the webs of the joists. The inside knees were fixed to the column in

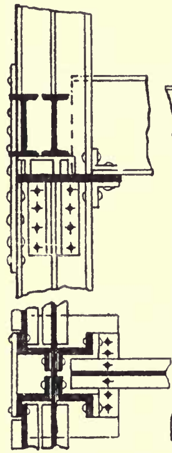


Fig. 167.

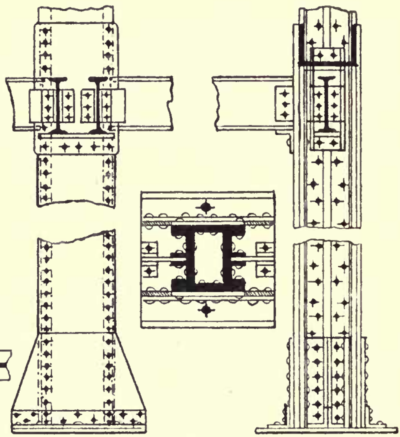
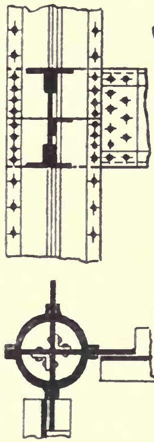


Fig. 168.

the contractor's yard, and the outer knees were left loose so that they could be riveted on when the girder had been fixed in position. A similar arrangement was made for the other girders shown. In connecting one portion of the column to another, a separating plate was employed as before described, only in this case it is above instead of below the girders, and it does not extend so far beyond the sides of the column. Two sides of the joint are made by the aid of angle brackets, and at the other sides cover-plates are provided extending about 2 ft. above and below the joint. As the upper length of column is of smaller area than that to which it is connected, filling plates are necessary between the cover-plates and the sides of the upper column.

One method of attaching girders to a column made of segmental section is shown by fig. 169. Here bars are placed between the flanges throughout the entire length of the column, but at the part where the girders are attached, and the different lengths of the column are connected one to another, the bars are displaced by plates, one extending through the column, and two others reaching to the centre, where they are attached to the first plate by angle brackets. The segments are riveted to these plates, and the girders are riveted to their projecting ends as indicated. In this manner it is possible to form a column continuous from the bottom to the top of a building, so that the joints are quite as strong as, if not stronger than, the remainder of the shaft. The angle bar drawn at the side of one girder is for supporting the floor arch, the other end of which would be carried by a joist riveted to the other girder. In fig. 170 a column of eight segments is represented. The girders are supported on knees, and the upper flange of each girder is not connected by the usual angle bracket, but a wedge is driven between the end of the girder and the column, so that the space shall be entirely closed. A cap plate is fitted to each length of column and connexion is made by rivets passing through the plate and angle plates riveted to the upper length.



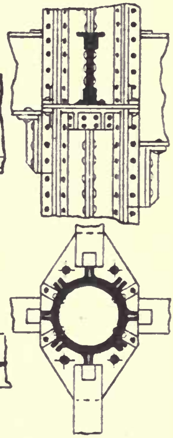


Fig. 169. *Fig.* 170.

When a very heavy load has to be carried by a girder, the latter member may advantageously be supported by a triangular bracket riveted to the column, as illustrated in fig. 171. This bracket can generally be situated in a partition wall, and thus any disfigurement of the interior apartments may be avoided. The column in this figure rests upon the girder below, which transmits the weight to the cast iron base. By this arrangement the centre line of pressure is removed farther away from the outer wall than it would be if the column were supported in the usual manner. A cast-iron base plate suitable for a heavy steel column is shown in fig. 172.

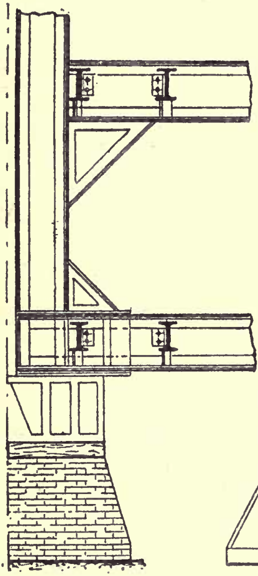


Fig. 171.

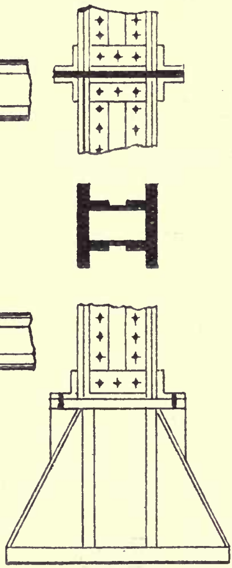


Fig. 172.

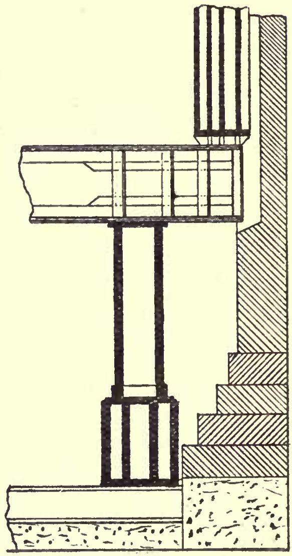


Fig. 173.

§ (*a*) *Cantilever Construction.* — Fig. 173 illustrates an interesting example of cantilever construction occurring in the Old Colony Building, Chicago.

This mode of support became necessary for some of the columns because of a difficulty with the owner of the adjoining property. The lower column is 3 ft. $6\frac{3}{4}$ in. from the party line, and rests on a box girder, 2 ft. 6 in. by 2 ft. 6 in., which is placed on a series of 15-in. I-beams bedded in concrete. The end of the box-girder cantilever supporting the segmental upper column is also shown in the figure.

CHAPTER XIV.

EXTERNAL LOADS ON BEAMS.

84. **Definitions.**—As employed in connexion with structural work, the term *beam* is the name given to any member exposed to transverse stresses, whatever may be the material of which it is composed. The term *girder* is generally restricted in application to flanged beams of iron or steel, but as a matter of principle it is not necessary to draw any distinction between a beam and a girder.

§ (*a*) *Effects of Load.*—A *simple beam* is supported at its ends and loaded at one or more points between the supports. A *semi-beam* or *cantilever* is fixed at one end and free at the other. A continuous beam is supported at three or more points. The ends of a beam may be simply *supported*, or they may be *fixed*. In the latter case the beam is described as an *encastrè* beam. When a beam is supported at one end or at both ends, and a load is imposed upon the beam, several results follow :—

1. The weight of the beam is divided between the supports if there be two. If only one support be used, it must of course carry the entire weight.

2. The load placed upon the beam is transmitted to the supports, though not necessarily in equal proportions.

3. The beam is bent in some measure by its own weight, and by the weight placed upon it. At the same time shearing force is developed, tending to cut the beam asunder in a vertical direction.

4. Internal stresses are caused, acting upon the fibres of the material of the beam, and these stresses vary from point to point along the beam.

From this brief statement it will be clear that a distinction must be drawn between forces acting externally on the beam and those which are set up in the fibres of the material.

§ (*b*) *Positive and Negative Forces.*—When a beam is loaded at one or more points, the loads cause downward vertical action, and also develop upward vertical reaction at the point or points of support. It is usual to designate these forces as positive and negative respectively. As the forces to which we now refer are all parallel and act in one plane, the following conditions are necessary if the beam is to be kept in a state of equilibrium:—(1) The sum of the negative forces must be equal to the sum of the positive forces; (2) the sum of the moments of the forces tending to turn the beam in one direction round any fixed point must be equal to the sum of the moments tending to turn it in the opposite direction round the same point.

85. Classification of Loads.—Loads are generally divided into three classes:—

1. *Concentrated* loads, generally assumed to be concentrated at one or more points, although they must actually occupy some finite length of the beam.

2. *Uniformly distributed* loads, spread over the whole or part of the length of the beam, and generally measured in pounds or tons per lineal foot.

3. Loads which are partly concentrated and partly distributed.

These three classes of load must come under one or other of the following divisions:—

a. Dead loads, comprising the weight of the beam itself and weight due to some of the other parts of the structure, such as small beams, joists, floor arches, floor boards, timber upon which flooring is nailed, ceilings, partitions, tanks with the water contained therein, and, in general, anything forming a part of the building and its permanent contents. *b. Live loads*, due to the weight of inmates, of furniture, of goods, and of any objects whose positions may be changed. *c. Intermittent loads*, which are repeatedly or suddenly applied and removed, and which are exemplified in the case of hoists and cranes.

86. Distribution of Load on Supports.—The position occupied by any load on a beam affects the proportion in which weight is divided between the supports, and the leverage due to bending of the beam affects the amount of strain suffered by the walls or by the columns supporting the beam. The position of the load upon a *semi-beam* or *cantilever* cannot, of course, make any difference in the weight carried by the supporting wall or column, but the leverage exerted must by no means be overlooked. At the present stage we are chiefly concerned with the manner in which the load is divided between the supports of a beam, and flexure, due to leverage, will not now occupy our attention. In the case of semi-beams, however, some notice will be taken of leverage, but the members themselves will be assumed to be perfectly rigid. In the following discussion of external loads the weight of the beams themselves is omitted, as this part of the load is nearly always uniformly distributed, and it would produce unnecessary complication to introduce this element into calculations of a purely explanatory nature.

§ (*a*) *Examples of Differently Loaded Beams :—*

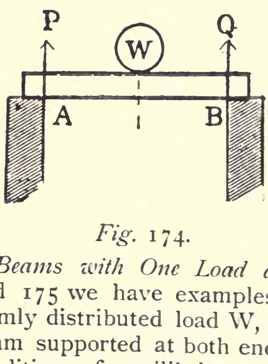


Fig. 174.

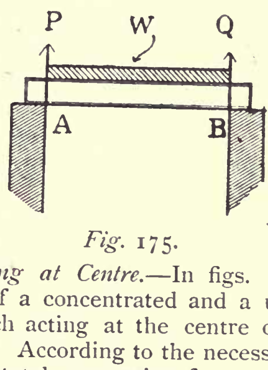


Fig. 175.

Beams with One Load acting at Centre.—In figs. 174 and 175 we have examples of a concentrated and a uniformly distributed load W, each acting at the centre of a beam supported at both ends. According to the necessary conditions of equilibrium, the total supporting force $= W$. Using the symbols P and Q to represent the reactions at the abutments A and B, we find that $P + Q = W$. Therefore

$$P = Q = \tfrac{1}{2} W. \quad \ldots\ldots\ldots\ldots (1).$$

Beams with One Unsymmetrical Load.—In fig. 176 a concentrated load is at the distance *a* from one support, and at the distance *b* from the other. Here, again, $P + Q = W$, but the values of the supporting forces at A and B are not the same. Thus,

$$P = Wb \div l; \; Q = Wa \div l \quad \ldots\ldots\ldots\ldots (2).$$

Similarly, if the load be uniformly distributed over part of the beam, as in fig. 177, it is only necessary to find the centre of gravity of the load and to treat it as a concentrated load acting at that point.

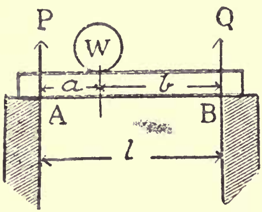


Fig. 176.

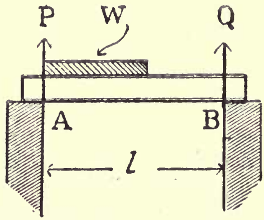


Fig. 177.

Ex. 1.—A beam of 20 ft. span supports a load of 24 tons, whose centre of gravity is over a point 8 ft. from the abutment A. Find the supporting forces at the two abutments.

Here $\quad P = 24 \times 12 \div 20 = 14{\cdot}4$ tons.

$\qquad\quad Q = 24 \times 8 \div 20 = 9{\cdot}6$ tons.

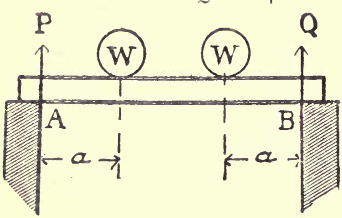


Fig. 178.

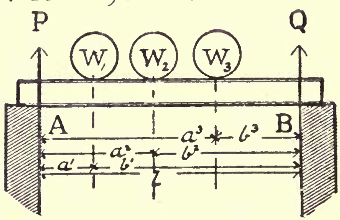


Fig. 179.

Beams with Two or more Loads.—Fig. 178 shows a beam supported at both ends, and carrying two symmetrical loads, each at the distance a from the nearest abutment. In this case, $P \times Q = 2W$; hence,

$$P = Q = W \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3).$$

Fig. 179 illustrates a beam whose length $= l$, carrying three weights, W_1, W_2, W_3, placed at the distances a_1, a_2, and a_3 from the abutment A, and at the distances b_1, b_2, b_3 from the abutment B. Here:

$$\left.\begin{array}{l} P = (W_1\, b_1 + W_2\, b_2 + W_3\, b_3) \div l, \text{ or } P = \Sigma\, W\, b \div l \\ Q = (W_1\, a_1 + W_2\, a_2 + W_3\, a_3) \div l, \text{ or } Q = \Sigma\, W\, a \div l \end{array}\right\} (4)$$

Ex. 2.—A beam of 20 ft. span supports loads of 2, 4, and 6 tons, concentrated at points dividing the span into four equal parts. Find the pressure on each support.

$$P = (2 \times 15) + (4 \times 10) + (6 \times 5) \div 20 = 5 \text{ tons},$$
$$Q = (2 \times 5) + (4 \times 10) + (6 \times 15) \div 20 = 7 \text{ tons}.$$

Beams Projecting over One Support.—When a beam projects over one of its supports, as in fig. 180, and the projecting end C is loaded, the load W and the weight of the projecting end tend to cause a turning moment about B, and to lift the end A from its bearing. Let P represent the upward force of the beam at the abutment A; Q the reaction of the support B; W the load at C; Y_1 the weight of the part B C, Y_2 the weight of the part A B of the beam; l the total length of the beam; l_1 the length of A B, and l_2 the length of B C. If the parts A B, B C, be of equal weight, then

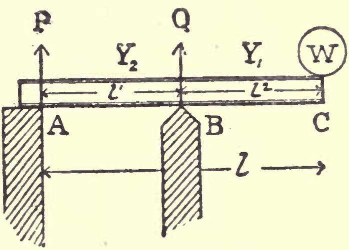


Fig. 180.

$$\left.\begin{aligned} P &= Wl_2 \div l_1 \\ Q &= Wl \div l_1 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (5.)$$

It may sometimes be the case that the end of the beam will be of such length, that its weight may be sufficient to exert a downward pressure on A; or a similar result may be attained by the imposition of a weight on the part A B. In either event, the end A may not require to be held down, and the reaction P is then said to be positive.

Taking into account the weight of the beam as represented by Y_1 and Y_2, we have

$$\left.\begin{aligned} P &= \left\{ (Wl_2 + Y_1 \tfrac{1}{2}l_2) - Y_2 \tfrac{1}{2}l_1 \right\} \div l_1 \\ Q &= \left\{ Wl + Y_1(l - \tfrac{1}{2}l_2) + Y_2 \tfrac{1}{2}l_1 \right\} \div l_1 \end{aligned}\right\} \quad \ldots\ldots (6.)$$

Ex. 3.—A beam 20 ft. long, weighing 56 lb. (·025 ton) per foot, rests upon two supports A and B (fig. 180), the distance A B being 14 ft., and B C = 6 ft. At the point C is a concentrated load of 5 tons. Find the reaction of the support B and the effect produced at A.

Taking moments about A, we have :—

$$Q = \left\{ (5 \times 20) + (\cdot 15 \times 17) + (\cdot 35 \times 7) \right\} \div 14 = 7\cdot 5 \text{ tons.}$$

Taking moments about B, we find :—

$$P = \left\{ (5 \times 6) + (\cdot 15 \times 3) - (\cdot 35 \times 7) \right\} \div 14 = 2 \text{ tons.}$$

The reaction at A is therefore upward *or negative*, and the end of the beam must be properly secured. If it be decided to hold down the end by means of anchor bolts, we can readily ascertain the proper diameter by reference to Chapter X. Assuming the load at C to be intermittent, the working strain on the bolts should not exceed $1\frac{1}{2}$ tons per square inch. Consequently their effective area should be, say, 1·6 sq. in., and if two bolts be used, we find by Table LII. that the nominal diameter must be $1\frac{1}{4}$ in.

§ (*b*) *Continuous Beams.*—A *continuous* beam is one resting upon three or more supports, as illustrated in fig. 181. The reactions at the various points of support are not the same as they would be if the beam consisted of a number of separate parts. If the beam were cut so as to form two separate beams, on which the total load = 2 W, the load on the central support would be ($\frac{1}{2}$W + $\frac{1}{2}$W), and on each abutment $\frac{1}{2}$W. As the beam is continuous, the distribution of the load, 2 W, is quite different. Suppose the central pier to be slightly higher than the abutments, then the total load will rest upon the pier, and none of it will be carried by the side supports. Again, suppose the pier to be a little lower than the abutments, there will be no pressure on the pier. In practice a mean between these extreme cases is adopted, and the pressures are modified accordingly. The methods by which they are determined involve calculations of a somewhat abstruse nature, and which would be out of place in this treatise. The subjoined table states the result of

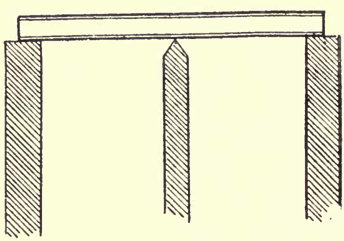


Fig. 181.

such calculations up to four spans, all of equal length, and carrying equal and uniformly distributed loads.

Table LXIX.—Loads on the Supports of Continuous Beams.
Length of Span $= l$, Load per Unit Length $= w$.

Number of Spans.	Abutment.	Pier 1.	Pier 2.	Pier 3.	Abutment.
1	$\frac{1}{2}\ wl$	—	—	—	$\frac{1}{2}\ wl$
2	$\frac{3}{8}\ wl$	$\frac{5}{4}\ wl$	—	—	$\frac{3}{8}\ wl$
3	$\frac{4}{10}\ wl$	$\frac{11}{10}\ wl$	$\frac{11}{10}\ wl$	—	$\frac{4}{10}\ wl$
4	$\frac{11}{28}\ wl$	$\frac{32}{28}\ wl$	$\frac{26}{28}\ wl$	$\frac{32}{28}\ wl$	$\frac{11}{28}\ wl$

By this table it will be seen that pressure on the abutments is considerably reduced, whilst it is greatly increased on the central support or supports. This condition consequently entails special precaution with regard to the foundations of continuous beams, partly because pressure upon the piers is increased, and partly for the reason that if subsidence of the foundations should take place, the nature and amount of the stresses in the beams would be undesirably modified.

87. Cantilevers.—As shown in figs. 182 and 183, semi-beams, or cantilevers, necessarily transmit the entire weight to their supports, but the leverage exerted depends upon the distance of the centre of gravity of the load from the support.

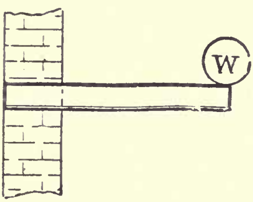


Fig. 182.

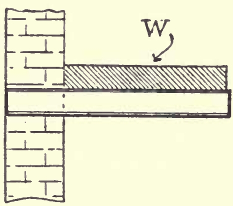


Fig. 183.

When a beam, assumed to be perfectly rigid, rests upon two supports so that the load of each is central, compression is the factor chiefly affecting the supports. In the case of a cantilever, however, the lateral load will inevitably produce a bending moment, as in fig. 184, causing internal stresses in the material of which the support is constructed.

§ (*a*) *Effect upon Supporting Columns and Brickwork.*—In an iron or steel column or stanchion the safe load may very easily be reduced by 30 or 40 per cent., owing to a side load, and it will therefore be clear that some amount of consideration is desirable before cantilevers or brackets are connected to existing columns, or applied to new ones. Still greater care is necessary when cantilevers are to be fixed to brick walls, because brickwork possesses practically no tensile strength.

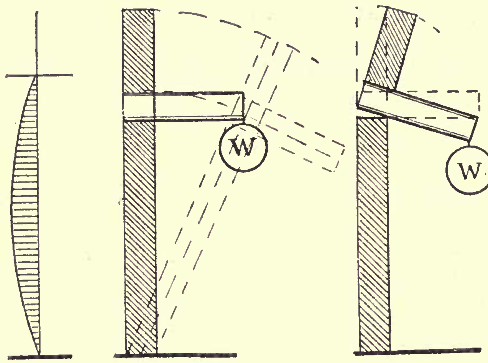


Fig. 184. *Fig.* 185. *Fig.* 186.

The conditions necessary for stability will cease to exist if a heavy lateral load be applied to a wall of comparatively little width and height, and the wall might fall bodily, as in fig. 185, if the brickwork were sufficiently adhesive. It is more probable, however, that a portion of the wall would be torn out or lifted off, as in fig. 186, allowing the cantilever and its load to fall to the ground. If the upper part of the wall possessed sufficient weight to counteract the leverage of the cantilever and its load, it is still possible that a portion of the brickwork would be crushed by reason of excessive compressive stress. Such contingencies as these do not always appear to be considered by architects, and perhaps more particularly in connexion with walls which have to support shafting and other engineering details.

CHAPTER XV.

BENDING MOMENT. SHEARING FORCE.

88. Definition of Bending Moment.—So far our consideration of external forces on beams has been limited to the effects produced upon the supports. We have now to devote attention to the influences exerted by external forces upon the beams themselves.

Bending moment is the product of the intensity of the resultant of all the forces tending to bend a beam, multiplied by the distance from the line of action of that resultant, of any transverse plane with reference to which the moment is taken.

§ (*a*) *Modes of Expression.*—The forces resulting from weights may be expressed in pounds, tons, or in any convenient unit of weight, and the distance can be expressed in inches, feet, or other units of measure. When inches and pounds are used the moment is stated in inch-pounds; when feet and tons are used the moment is stated in foot-tons; and similarly there may be inch-tons, foot-pounds, &c. The basis involved in nearly every handbook published in this country of safe loads for joists and girders is the maximum bending moment under a distributed load. Books of this kind are largely used by people who do not stop to consider the effect of different systems of loading upon the strength of a beam; and as uniformly distributed loads are by no means the rule, it is probable that girders are frequently employed whose proportions are entirely inadequate for the duties required. In most cases of this kind the factor of safety no doubt prevents any mishap, but it is clearly undesirable that the user should believe he is working on a factor of, say, 5, when, owing to the manner of loading, it may actually be only 3 or perhaps less.

89. Graphical Diagrams.—We have not space for a complete exposition of all possible systems of loading beams, but some of the chief of these will now be noticed. As the amount of bending moment is governed by the distance between the force and the point about which it acts, it is evident that the moments vary at different sections in the length of a beam. The maximum bending moment is more frequently considered than any other, but occasions often arise when it becomes necessary to know what effect is produced at different points along a beam. In ordinary cases the bending moment can readily be found by calculation, but the solution of a problem may sometimes be effected more expeditiously by a graphical diagram. The latter method certainly has the merit of demonstrating most clearly the exact effect produced at every section of the beam, and it would not be possible to obtain by any other means so satisfactory a conception of what may be expected in a loaded beam. A diagram of bending moments can be produced by purely graphical methods, or it may be to some extent based upon preliminary calculations.

Any convenient scale of feet may be used in representing the length and depth of the beams and the width of the distributed loads; whilst the same or a different scale can be adopted for measuring the bending moment at any point in the length of the beams. Typical examples of differently loaded beams are illustrated in Art. 90, and for the purpose of aiding comparison, the beams with one exception are under loads of equal weight, and are of equal dimensions. A diagram of bending moments is drawn immediately below each beam, and the second diagram in each case represents shearing force, drawn to the same scale of foot-tons. Shearing force is considered in Art. 91.

90. Bending Moments due to different Systems of Loading.—Symbols used in connexion with figs. 187 to 197 :—

M = Bending moment, in foot-tons.
W = Total load on span in tons = $w\,l$.
w = Unit weight per foot, in tons.
l = Length of span, in feet.
P = Reaction at left abutment (Art. 86).
Q = „ „ right „ „

Ex. 1.—*Cantilever fixed at one end and loaded at the other.*—The cantilever (fig. 187) is 6 ft. long, and has a load of two tons resting on its free end. According to the definition previously stated, the maximum bending moment,

$$M = Wl \quad \ldots\ldots\ldots\ldots \quad (1)$$

and the bending moment at any distance x from the free end,

$$M_x = Wl_x \quad \ldots\ldots\ldots\ldots \quad (2)$$

Similarly, the bending moment might be calculated by using the reaction P instead of W. Thus,

$$M = Pl \ldots\ldots\ldots\ldots\ldots \quad (3)$$
$$M_x = Pl_x \quad \ldots\ldots\ldots\ldots \quad (4)$$

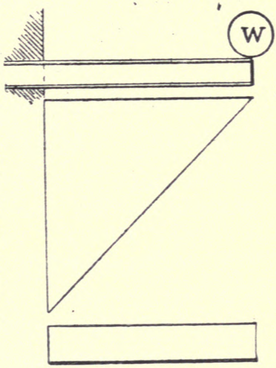


Fig. 187.

At the fixed end of the beam,

$$M = 2 \times 6 = 12 \text{ foot-tons};$$

at the centre of the beam,

$$M = 2 \times 3 = 6 \text{ foot-tons},$$

and at the point of application of the load,

$$M = 0 \times 6 = 0 \text{ foot-tons};$$

The horizontal line of the diagram is equal to the length of beam; the vertical line $= M$, and is proportioned by the scale of foot-tons; the diagonal completes the triangle, and any ordinate drawn from the horizontal to the diagonal line will denote the moment at the corresponding section of the cantilever.

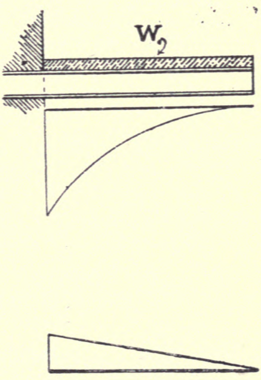


Fig. 188.

Ex. 2.—*Cantilever fixed at one end and loaded uniformly.*—In fig. 188 the cantilever, 6 ft. long, is loaded with a weight of two tons equally distributed over the whole length. This load may be assumed to be concentrated at its centre, and the maximum bending moment is therefore

$$M = Wl \div 2, \text{ or } Pl \div 2 \quad \ldots\ldots\ldots\ldots(5)$$

Hence in this example

$$M = 2 \times 3 = 6 \text{ foot-tons.}$$

At any point of the distance x from the free end,

$$M_x = w\, l_x^2 \div 2 \quad \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

This equation indicates that the diagonal completing the diagram of bending moment is a parabolic curve, and an ordinate drawn through the triangle as before will give the measure of the moment in the corresponding section of the beam.

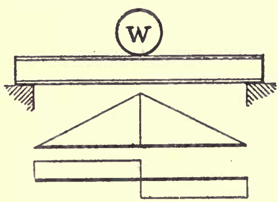


Fig. 189.

Ex. 3.—*Beam supported at both ends, with a concentrated load in the middle.*—In fig. 189 the span of the beam is 6 ft., the central load is 2 tons, and for the maximum bending moment we have

$$M = Wl \div 4 \quad \ldots\ldots \quad (7)$$

or

$$M = P \times \tfrac{1}{2}l = Q \times \tfrac{1}{2}l \quad \ldots\ldots\ldots\ldots(8)$$

Consequently we have by Rule 7,

$$M = 2 \times 6 \div 4 = 3 \text{ foot-tons,}$$

and by Rule 8,

$$M = 1 \times 3 = 3 \text{ foot-tons.}$$

At any section x between the load and the left abutment,

$$M_x = Pl_x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

At any section y between the load and the right abutment,

$$M_y = Ql_y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

In the diagram of bending moments the central vertical line = 3 foot-tons, and ordinates may be drawn showing the moments at any corresponding sections of the beam.

Ex. 4.—*Beam supported at both ends and loaded uniformly.*—In this case the load of 2 tons is equally distributed over the span of 6 ft., as shown in fig. 190, and the maximum bending moment,

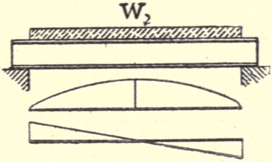


Fig. 190.

$$M = Wl \div 8 \quad \ldots\ldots\ldots \quad (11)$$

or

$$M = P \times \tfrac{1}{4}l = Q \times \tfrac{1}{4}l \quad \ldots (12)$$

Hence, by Rule 11,

$$M = 2 \times 6 \div 8 = 1{\cdot}5 \text{ foot-tons};$$

and by Rule 12,

$$M = 1 \times 1{\cdot}5 = 1{\cdot}5 \text{ foot-tons}.$$

To determine the moment at any distance x from the left-hand abutment, let w = unit-weight per foot. Then,

$$M = \frac{w\,x}{2}(l - x) \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

The curve of bending moments given by this equation is a parabola; the maximum moment is reached when $x = \frac{1}{2}l$, and the value of the expression is then $(wl^2 \div 8) = (Wl \div 8)$. In the diagram the ordinate 1·5 foot-tons in length is the axis of the parabola, and the upper end its vertex.

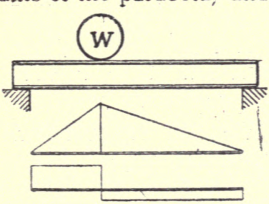


Fig. 191.

Ex. 5.—*Beam supported at both ends with an unsymmetrical concentrated load.*—The beam in fig. 191 has a load of 2 tons at the distance $a = 2$ ft. from the left-hand abutment; and b the distance from the right-hand abutment is 4 ft. The maximum moment

$$M = Wab \div l \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Therefore we find

$$M = (2 \times 2 \times 4) \div 6 = 2{\cdot}66 \text{ foot-tons},$$

or

$$M = Pa = (1{\cdot}33 \times 2) = 2{\cdot}66 \text{ foot-tons}.$$

Or again,

$$M = Qb = ({\cdot}66 \times 4) = 2{\cdot}66 \text{ foot-tons}.$$

If we now desire to determine the moment at some point x, it is necessary to consider whether x is greater or less than a. In the former event the moment at x will be

$$M_x = Qy,$$

where y is the distance of x from Q, and in the latter case

$$M_x = Px.$$

For the given position of W, the maximum is reached when $x = a$, and then

$$Pa = Px = Qb = Qy.$$

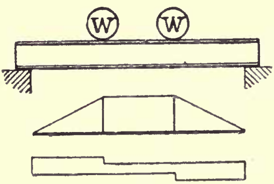


Fig. 192.

Ex. 6.—*Beam supported at both ends with two symmetrical loads.*—A case of simple bending is shown in fig. 192, where the load consists of two weights, W, each equal to 1 ton, situated at the distance a, = 2 feet, from P and Q respectively.

Here

$$M = Wa = Pa = Qa \quad \ldots\ldots\ldots\ldots (15)$$

In this example,

$$M = 1 \times 2 = 2 \text{ foot-tons.}$$

The diagram of bending moments is constructed by first drawing a triangular figure, as in fig. 188, in which the height of the central ordinate is 3 foot-tons. Vertical lines are then drawn corresponding to the points a, a, in the beam; the points at which these verticals intersect the sides of the triangle are joined by a line parallel to the base of the triangle, and the resulting polygon represents the bending moments of the beam.

Ex. 6a.—*Beams supported at both ends with two or more unsymmetrical loads.*—A case of this kind is illustrated in fig. 179, Art. 86, and it may be useful to inquire how the bending moments may be ascertained at each point of application. For the sake of uniformity, we will take the length of the beam at 6 ft., and divide the load W into three weights, $W_1 = \cdot 4$ ton, $W_2 = 1 \cdot 0$ ton, $W_3 = \cdot 6$ ton, placed at a_1, a_2, a_3, 15, 33, and 50 in. respectively from the abutment A. Proceeding as in Ex. 2, Art. 86, we find

$$P = 1 \cdot 04 \text{ tons, and } Q = \cdot 96 \text{ ton.}$$

Therefore

$$M_1 = 1{\cdot}04 \times 15 = 15{\cdot}6 \text{ inch-tons, or } 1{\cdot}3 \text{ foot-tons;}$$
$$M_2 = 1{\cdot}04 \times 33 = 34{\cdot}32 \text{ inch-tons, or } 2{\cdot}86 \text{ foot-tons;}$$
and $$M_3 = {\cdot}96 \times 22 = 21{\cdot}12 \text{ inch-tons, or } 1{\cdot}76 \text{ foot-tons.}$$

It will be seen that the maximum bending moment is slightly less than if the load had been concentrated in the centre of the beam, and the figures also indicate that the diagram of moments must assume a polygonal form. To construct the diagram, the triangle for each weight should be drawn as in fig. 191, exactly as if the load under consideration were the only one upon the beam. An ordinate should then be drawn at a_1, and the sum of the moments occurring there will represent the total bending moment at a_1 for the three loads. A length should therefore be set off on the ordinate equal to the three separate moments. When a similar course has been followed at a_2 and a_3, a polygon can be formed showing the curve of bending moment produced by simultaneous action of the three weights.

Ex. 7.—*Beam supported at both ends, with a uniformly distributed load over a part of its length.*—This case is illustrated in fig. 193, where the beam, 6 ft. long, has a load of 2 tons, distributed over a length of 2 ft. Here the portions which are clear of the load are seen to be affected just as if the load were concentrated in the centre of the beam, but immediately below the load a second moment is produced, equal to that which would occur if the same load were supported by a beam of 2 ft. span supported at both ends. Let a represent either portion clear of the load, and b the portion under the load. Then,

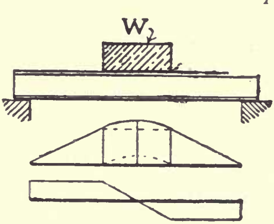


Fig. 193.

$$M = \tfrac{1}{2}Wa + (Wb \div 8) \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

Therefore in this example

$$M = (1 \times 2) + (2 \times 2 \div 8) = 2\tfrac{1}{2} \text{ foot-tons.}$$

Ex. 8.—*Beam supported at one end and in the middle, with a concentrated load at the free end.*—The beam in fig. 194 is 6 ft. long measured from the support at the left hand, where the end is assumed to be held down by a weight to counterbalance the load at the other extremity. Practically this case is the same as that of a beam resting on two supports (Ex. 3), only that the moments are inverted or *negative.* Therefore by Rule 7 we have the maximum bending moment

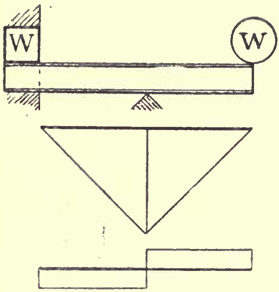


Fig. 194.

$$M = 4 \times 6 \div 4 = 6 \text{ foot-tons.}$$

In arriving at this result the load has been doubled because there are really two weights, one at the free end and the other at the left-hand support.

This beam might be regarded as consisting of two cantilevers, each 3 ft. long, fixed at the middle and loaded at the two free ends. Taking this view of the case, the maximum bending moment is found by Rule 1.

Thus:

$$M = Wl = 2 \times 3 = 6 \text{ foot-tons, as before.}$$

Ex. 9.—*Beam with both ends fixed, and loaded at the centre.*—When a beam is fixed, so that the ends are prevented from curving under the influence of a load, its strength is considerably increased. This condition is rarely, if ever, completely attained in practice. There are two points of *contrary flexure* in a beam of this class, situated one quarter of the length from each support, and at each ot these points there is theoretically no bending moment. We may therefore regard the beam as consisting of three separate parts—a central beam and two cantilevers. Referring to fig. 195, the central part $= \frac{1}{2}l$, and as the total length is 6 ft., and the load is 2 tons, we have for the central part $M_{cen} = (W \times \frac{1}{2}l) \div 4$ ....(17)

$= (2 \times 3) \div 4 = 1{\cdot}5$ foot-tons.

Each of the two end parts supports half the total load and is of the length $\frac{1}{4}l$; therefore

$M_{end} = \frac{1}{2}W \times \frac{1}{4}l$ ....(18)

$= 1 \times 1{\cdot}5 = 1{\cdot}5$ foot-tons.

The diagram of bending moments (fig. 195) is constructed in a manner similar to those in figs. 187 and 189, the three triangles being drawn upon a line representing the length of the beam as shown in fig. 195. If the bending moments were calculated in terms of total weight and total length, the rule for all three parts would become $Wl \div 8$ (as Rule 11), and it will consequently be evident that the strength of a beam fixed in the way described is exactly double the strength of one supported in the ordinary manner, whose bending moment $= Wl \div 4$ (as Rule 7).

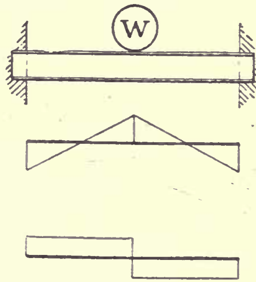


Fig. 195.

Ex. 10.—*Beam with both ends fixed and loaded uniformly.*— In fig. 196 we have a beam similar to that in the last example, but owing to the difference of loading, the points of contrary flexure do not come in quite the same positions. The length of each end portion from the support to the point of contrary flexure is $\cdot 211l$, and the central part is $\cdot 578l$. In terms of total load and total length, the bending moments are thus expressed:

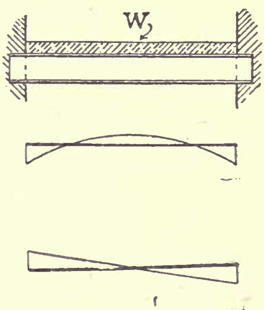


Fig 196.

$M_{cen} = Wl \div 24, \quad M_{end} = Wl \div 12$ .... (19)

Thus in our case we have for the maximum bending moments $M_{cen} = 2 \times 6 \div 24 = \cdot 5$ foot-ton, and

$M_{end} = 2 \times 6 \div 12 = 1$ foot-ton.

As in the case of other beams with distributed loads, the diagram of bending moments is formed by a parabolic curve. The strength of a beam fixed and loaded in the manner described is 1·5 times that of a similar beam supported in the ordinary manner.

Ex. 11.—*Continuous beam of two equal spans uniformly loaded.*

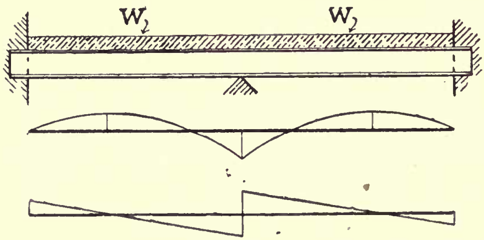


Fig. 197.

The beam illustrated in fig. 197 has two spans of 6 ft., each carrying a uniformly distributed load of 2 tons. Assuming the ends are not rigidly fixed, we may regard each span as consisting of two portions—(1) a beam supported at both ends, and extending from the abutment to the point of contrary flexure, occurring at $\frac{3}{4}l$ from the abutment; and (2) a cantilever extending from the central pier to the point of contrary flexure. The moments for these parts are readily found by considering the proportion of weight which each section of the beam has to carry. In terms of total load and length for each span the maximum bending moments are:—

$$(1) \qquad M = 9Wl \div 128 \quad \ldots\ldots\ldots\ldots\ldots (20)$$

$$(2) \qquad M = Wl \div 8 \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

In the beam now under consideration, the moment of the portion adjoining each abutment is

$$M = (9 \times 2 \times 6) \div 128 = \cdot 8437 \text{ foot-tons,}$$

and of the cantilever part at the pier the moment is

$$M = (2 \times 6) \div 8 = 1 \cdot 5 \text{ foot-tons.}$$

§ (*a*) *General Conclusions.*—From the above examples it will be clearly seen that the strength of a beam depends very greatly upon the system of loading adopted. In some cases the strength is very much less than that of a beam with a uniformly distributed load, whilst in others a considerable accession of strength is evidenced. The obvious lessons to be learnt are that the conditions to which a beam is intended to be subject should always be fully considered, and that tables of calculated safe loads are only to be relied upon if the working conditions are to be the same as those assumed for the purpose of calculation.

91. Definition of Shearing Force.—Shearing force in a beam tends to cut the beam asunder, and is developed by the downward action of the load and the upward reaction of the support or supports. Consequently, the maximum shearing force F must be at the abutments of beams supported in the ordinary manner, and at one or more of the piers of a continuous beam.

§ (*a*) *Positive and Negative Shearing Force.*—It is convenient to differentiate between positive and negative shearing force. The diagrams contained in figs. 187 to 197 conform with this distinction, and it will be understood that in every case the shearing force in tons is equal to the supporting force P or Q at the corresponding abutment.

Referring to the diagrams in figs. 187 to 197, it will be noticed that whenever the load is *concentrated*, shearing force is constant throughout each portion into which the diagram is divided by the transverse section, and that a sudden change of force appears to take place in each beam at the point where the load is applied. This does not mean that two different values exist at a single section of the beam, for there can be but one value at each place under any given system of loading. The abnormal condition apparently indicated is entirely owing to the assumption that the load is applied at a point, whereas in practice any load, however concentrated it may be, must occupy some finite length over which the transition from positive to negative shearing force is gradually manifested.

Referring again to the diagrams, we see that when the load is uniformly distributed, the positive and negative shearing forces gradually diminish from the abutments to a point immediately below the centre of gravity of the load in the case of a beam supported at each end, and to the free end in the case of a cantilever.

§ (*b*) *Shearing Force in Encastré and Continuous Beams.* —When beams with fixed ends or *encastré* beams (figs. 195, 196) and continuous beams (fig. 197) are examined, it will be observed that the portion which behaves as a beam on two supports is also subject to shearing forces diminishing from the points of contrary flexure to the centre. The lines denoting this decrease of force towards the centre, or increase of force away from the centre, serve, when continued, to delineate the shearing force of the end portions of the beam. These end portions, as we remarked in the last chapter, are practically cantilevers, although, as they have to carry the central load as well as their own, the minimum shearing forces at the points of contrary flexure do not fall to zero, but to the intensity corresponding with that at the ends of the central portion. Similar analytical consideration will enable the student to ascertain the extent of the shearing forces in beams such as those drawn in figs. 192 and 193.

§ (*c*) *Connexion between Shearing Force and Bending Moment.*—A clearly defined connexion exists, and is readily traceable, between bending moment and shearing force. Thus, taking the supporting force P, and the maximum shearing force F, we have $P = F$. In the case of a cantilever, $W = P = F$, and the maximum bending moment $M = Wl$. Therefore

$$M = Fl, \quad \text{and } F = M \div l \qquad (22)$$

92. Modes of Calculation. — The rules given in Art. 86, §*a*, for finding the values of the supporting forces P and Q at the abutments also serve to indicate the maximum shearing forces.

In the majority of cases the maximum shearing force is all that need be studied, but there are some structures in connexion with which it is desirable that the shearing force throughout the length of a beam should be known and duly considered. The mode of calculation then adopted is indicated by the following examples :—

Cantilever uniformly loaded.—Let w = unit weight per foot; then at any section at the distance x from the support of a uniformly loaded cantilever, the shearing force,

$$F_x = w\,(l-x) \quad \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

Ex. 12.—Assuming the cantilever, as in fig. 188, to be of 6 ft. length, the total load W to be 2 tons, and the distance x to be 3 ft.; then the unit weight per foot, $w = (2 \div 6) = {\cdot}33$. Hence the shearing force,

$$F_x = {\cdot}33\,(6-3) = 1 \text{ ton.}$$

Ex. 13.—If the point x, in the same beam, under the same load as that stated above, be 6 ft., then

$$F_x = {\cdot}33\,(6-6) = 0.$$

Beam supported at both ends and uniformly loaded.—Here the shearing force at any section at the distance x from either abutment is

$$F_x = w\,(\tfrac{1}{2}\,l-x) \quad \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

Ex. 14.—If the point x, in the beam drawn in fig. 190, be 2 ft. distant from the left-hand abutment, the span being 6 ft. and the unit weight ·33 ton, then the shearing force

$$F_x = {\cdot}33\,(3-2) = {\cdot}33 \text{ ton.}$$

Ex. 15.—In the same beam, if $x = \frac{1}{2}l$, the value of

$$F_x = {\cdot}33\,(3-3) = 0.$$

93. **Beams of Uniform Strength.**—Many books of reference contain plans and elevations representing beams of uniform strength in which the cross sections are varied according to the bending moments due to the loads.

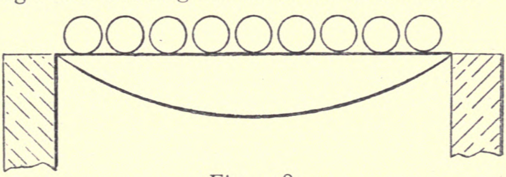


Fig. 198.

Fig. 198 is reproduced from one of these drawings indicating diagrammatically the proportions of a beam of uniform strength and breadth. It should be remarked, however, that no account is here taken of shearing force which may be calculated by rule 24.

Fig. 199 represents the correct theoretical and practical proportions of a similar beam, and in this diagram due provision has been made for the effect of shearing force in

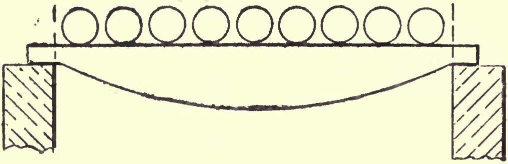


Fig. 199.

addition to that of bending moment. Beams such as this are frequently to be observed in structural work, and in the fire-bar of a steam boiler the student will find a familiar and homely example.

94. Summary of Points relative to External Forces.—With regard to the manner of beam-loading, we find

(1) That it affects the design of the supporting walls, columns, and foundations;

(2) That it exercises a marked influence upon the development of bending moment, which is really a multiplication of the direct force of the load; and

(3) That it is responsible for different manifestations of shearing force from case to case.

These three questions of external load, bending moment, and shearing force are, of course, applicable to all beams, whatever be the material of which they are composed.

Finally, if the student will consider the diagrams of bending moments previously given, he will see how suggestive is each figure of the shape most suitable for the proper support of the load, although, in practice, the exact dimensions must depend upon the strength and form of the material and the weight to be carried. In these diagrams may be traced the outlines of many familiar engineering and architectural details, the accuracy of whose general proportions can be gauged by their degree of correspondence with graphical diagrams representing combinations of bending moments and shearing forces.

CHAPTER XVI.

INTERNAL STRESSES. STRENGTH OF BEAMS.

95. Bending Stress.—Transverse or bending stress is directly due to bending moment, and, as pointed out in Art. 23, it is composed of tensile and compressive stresses. Take the case of a beam supported at both ends and loaded uniformly. Concurrently with the bending moment produced there must be deflection, causing the upper part of the beam to be compressed and the lower part to be stretched. Thus we have compressive stress and tensile stress, the amounts of which at any section are proportionate to the amount of the bending moment. As compressive stress consists of push, and tensile stress of pull, there must be a neutral axis in the beam where the opposing forces are reduced to zero.

§ (*a*) *Areas of Tension and Compression.*—Fig. 200 shows part of a beam, and the neutral axis is indicated by a full

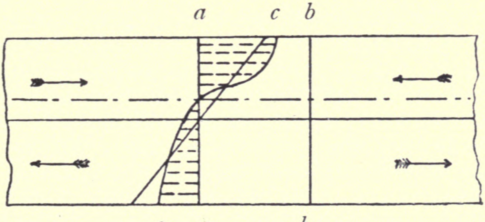


Fig. 200.

line passing through the centre of gravity of the section. Let the lines *a a*, *b b* enclose a portion of the beam before bending, and let the lines *a a*, *c c* enclose a portion after bending; then the triangles represent areas of compression and tension respectively, each of equal value and together forming a *couple*. Within the elastic limit the stresses and their correlative strains are distributed in the uniformly varying manner implied by the triangles (fig. 200), but when the beam is over-strained, and if the elastic limit be higher for compression than for tension, the neutral axis may assume a new position, as shown by a dotted line in the figure, and the stress diagrams will probably take irregular forms such as those enclosed by the dotted triangles. Turning now to fig. 201, we have part of a beam in which deflection is much exaggerated, so that the figures representing stresses may

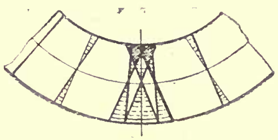


Fig. 201.

be of intelligible dimensions. At the middle of the beam, where bending moment reaches a maximum, the stresses are greater than those indicated at two adjacent points, and very much greater than those suggested by the pair of triangles to the extreme left, which look almost like a single line of uniform thickness. At the ends of the beam, not shown in the figure, where bending moment becomes a minimum, the stresses, of course, cease to exist. Several *couples* are drawn at the middle of the beam with the object of showing the overlapping areas of compressive stress due to shortening of the upper part of the beam, and the equivalent areas of tensile stress caused by lengthening of the lower part of the beam. The neutral axis, it should be observed, retains its original length, and is therefore exposed to no stress of any kind.

§ (*b*) *Distribution of Stress.*—Inasmuch as compressive stress results from shortening of the parts above the unaltered neutral axis, and as tensile stress is due to lengthening of the parts below the neutral axis, it is evident that the stress on any longitudinal layer of the beam must be proportionate to the distance of the layer from the neutral axis. The incidence of stress is consequently demonstrable by the application of geometrical principles. For the purposes of our present inquiry, the following method is more suitable. In fig. 202 let p denote the intensity of

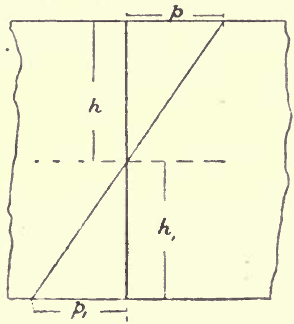


Fig. 202.

compressive fibre stress, and p_1 the intensity of tensile fibre stress. Then at any transverse section x, of a beam, and at any distance h from the neutral axis—

$$p = Mh \div I \quad \text{and } p_1 = Mh_1 \div I \; \ldots\ldots (1)$$

Here M = bending moment at the section x, and I = moment of inertia of the beam, assuming it to be of uniform section, as is usually the case.

96. Greatest Moment of Inertia.—Rules were given in Table LVII. for finding the horizontal or least moments of inertia for various sections used as columns, but the moments then considered were those in respect of the axis about which bending was most likely to occur. A beam is almost invariably disposed so that its greatest strength is opposed to bending moment. Therefore the vertical or greatest moment of inertia is appropriately used in calculations.

§ (*a*) *Rules for Calculation.*—Table LXX. contains rules for ascertaining the greatest moments of inertia for various sections of beams, and figs. 203 to 209 are explanatory of the measurements involved.

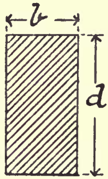


Fig. 203.

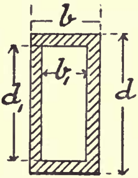


Fig. 204.

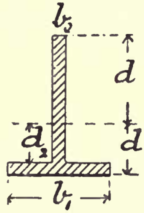


Fig. 205.

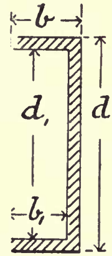


Fig. 206.

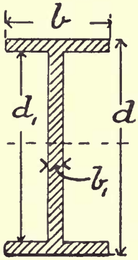


Fig. 207.

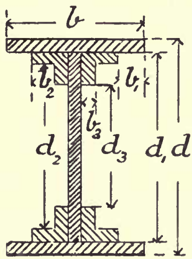


Fig. 208.

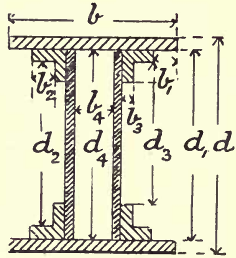


Fig. 209.

Table LXX.—Vertical or Greatest Moments of Inertia of Beams.

Fig. No.	Form of Section.	Value of I.
203	Solid rectangle ...	$bd^3 \div 12$
204	Hollow ,, ...	$(bd^3 - b_1d_1^3) \div 12$
205	Tee	$[bd^3 + b_1d_1^3 - (b_1 - b)d_2^3] \div 3$
206	Channel ,, ...	$(bd^3 - b_1d_1^3) \div 12$
207	I, Simple ...	$[bd^3 - (b - b_1)d_1^3] \div 12$
208	I, Compound ...	$[bd^3 - (2b_1d_1^3 + 2b_2d_2^3 + 2b_3d_3^3)] \div 12$
209	Compound Girder	$[bd^3 - (2b_1d_1^3 + 2b_2d_2^3 + 2b_3d_3^3 + 2b_4d_4^3)] \div 12$

The values of I for square, cruciform, and circular sections are the same as those in Table LVII.

§ (*b*) *Suitable forms of Beam section.*—A consideration of the foregoing statement, perhaps aided by a few experimental calculations, will serve to indicate, though not completely, what forms of beam section are good and what are bad. For instance, as the fibres at the top and bottom surfaces suffer the most severe strain, whilst those near the neutral axis suffer very little, it is evident that any solid beam of rectangular section must contain a proportionately large quantity of absolutely useless material. A solid beam of circular section, having a still greater proportion of its material concentrated near the neutral axis, would be much more wasteful than a rectangular beam. The economic advantage of a deep and narrow rectangular beam over one of square section becomes obvious when we consider that when the area is constant the stress for a given bending moment varies inversely as I, the governing factor in which is depth. In order that the form may be suitable for the resistance of compressive and tensile stresses developed by bending moment, the material should be concentrated at the places where stress intensity is greatest. Therefore a flanged beam of I-section, having the bulk of the material at a distance from the neutral axis, is to be recommended. Part of the bending moment falls upon the web, but the bulk is taken by the flanges, one of which is in compression and the other in tension, and in well-designed beams the stress intensity at any given section is approximately uniform over the whole of each flange at that section. In every case, the sectional areas of the compression and tension flanges should be inversely proportional to the compressive and tensile strength of the material. Beams of wrought iron and steel have flanges of equal sectional area, because F_c is approximately equal to F_t; but cast-iron beams, including members called by other names though really constituting beams, must be differently proportioned. For cast iron F_c has an average value of 50 tons, and F_t an average value of 8 tons. Therefore the ratio 6·25 : 1 governs the relative areas of the compression and tension flanges.

97. Distribution of Shearing Stress.—Before we can fully realise the correct theoretical proportions of a beam it is necessary to know in what manner the distribution of shearing stress takes place over the section. In a longitudinal section shearing stress varies from point to point with the shearing force (Art. 91); and in any cross-section the intensity of shearing stress attains its maximum value at the neutral axis, and falls to zero at the top and bottom of the section. In a beam of rectangular section the maximum value is one and a half times the mean intensity. In a properly designed I-beam shearing stress is approximately uniform throughout the web, whilst very little is experienced in the flanges owing to their much greater width and greater distance from the neutral axis.

98. Considerations Affecting the Design of Beams.—We have already shown that part of the bending stress in a beam of I-section must necessarily be resisted by the web, therefore the strength of this part must be sufficient to withstand the combined effect of bending and shearing stress, the former of which increases as the flanges are approached. Hence the web should be additionally strengthened in such regions, and, moreover, it must in no part be too thin to resist buckling due to compressive stress.

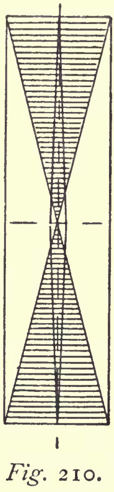


Fig. 210.

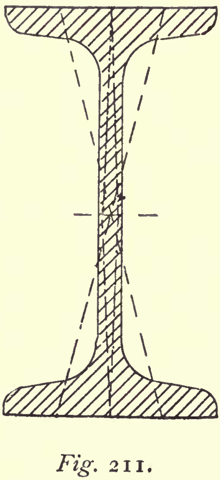


Fig. 211.

Fig. 210 is a sketch of a rectangular beam where bending stress is indicated by shaded triangles, shearing stress by narrow triangles in the centre, and waste material by the unshaded parts outside the stress diagrams. Fig. 211 shows an I-beam to which the stress areas from fig. 210 are applied. Both of these diagrams are purely hypothetical, but they show that the I-section fairly complies with the assumed conditions, the only parts where strength is lacking being the ends of the web adjoining the flanges.

If a beam were designed in strict accordance with the principles we have now very briefly reviewed, it would present the following characteristics :—

1. Breadth and depth throughout the length would be proportionate to variations of bending and shearing stress from point to point.

2. The form of cross-section would diminish towards the neutral axis until the thickness at the centre became little more than that sufficient for the resistance of shearing stress.

Features such as these are to some extent evidenced in large girders employed on important engineering works, but in ordinary structures beams are usually of uniform section, more or less preserving the essential qualities of the I-beam, which we have seen conforms to a very considerable degree with the conditions necessary for strength and economy.

99. The Principal Stress Occurring in Beams.—Stress resulting from bending moment constitutes the chief factor in problems relative to the practical strength of beams. The essential nature of a beam is such that some amount of bending stress must always be present, for even in the absence of any other weight, that of the material itself is really a load. Theoretically, there is no maximum limit to the amount of bending moment, but practically the maximum is attained when the breaking point of the beam is reached. The value of M, the bending moment for any given beam, may represent the effect under any of the following conditions :

(1) Under an ascertained working load ;

(2) Under an estimated load ;

(3) Under a load sufficient to break the beam.

§ (*a*) *Moment of Resistance.*—Bending moment, regarded as the resultant of downward forces, is opposed by the resultant of upward forces in the beam, and known as the *moment of resistance.* This we will denote by the symbol R. Then in every case $M = R$ ....(2)

As the moment of resistance always balances the bending moment, R equals the degree of resistance called into play in any specific case.

In Art. 90 we found the equations representing bending moment under certain specific conditions; but, speaking in general terms, it may be said that M is equal to weight multiplied by length, or Wl_x.

Similarly, we may now say generally that R is equal to strength multiplied by breadth and depth. In order that a more convenient expression may be found for the value of R, (1) let the element of strength be represented by (F), the ultimate tensile or compressive fibre-stress at the distance (h) from the neutral axis; and (2) let the element of breadth and depth be represented by (I), the moment of inertia (Art. 96).

Then,

$$R = FI \div h \text{ ....(3)}$$

But M is equal to R, consequently $M = FI \div h$.

Hence

$$F = Mh \div I \text{ ....(4)}$$

§ (*b*) *Intensity of Fibre-stress.*—The load W is not always sufficient to occasion the bending moment which would break a beam, and in such a case R would not represent the ultimate resistance of the beam. Therefore the fibre-stress could not be equal to F, which is the ultimate strength, and we will use the symbol p to represent the prevailing intensity of fibre-stress. Of course it may be that the actual stress will be such that $p = F_t$ or F_c, and although this condition is never intentionally caused in structural work, it is frequently assumed for the purpose of calculation. In the case of cast iron it would be necessary to distinguish between tensile and compressive stress. (Art. 96, §*b*.) As we shall be chiefly concerned with wrought iron and steel, which are practically isotropic (Art. 24§*b*), needless complication will be avoided by using the symbol p to indicate fibre-stress under all conditions. The above-mentioned expressions can therefore be written in the following terms:—

Bending moment, in inch-tons,

$$M = Wl_x \text{ ....(5)}$$

Moment of resistance, in inch-tons,

$$R = p\,I \div h \quad \ldots\ldots\ldots\ldots\ldots (6)$$

Unit-stress on the extreme fibres, in tons,

$$p = Mh \div I \quad \ldots\ldots\ldots\ldots\ldots (7)$$

We are now furnished with equations by the aid of which the strength of a beam under any system of loading, and the intensity of fibre-stress at any distance h from the neutral axis should be calculable, providing the values of M and I be known. The precise expression for M under any given conditions must be taken as in Art. 90.

§ (c) *Modulus of Transverse Rupture.*—Some beams, chiefly those of solid rectangular section, behave in an unreasonable manner when exposed to transverse stress. Hence the hypothetical stress f was invented by the late Professor Rankine, and was called by him the *modulus of transverse rupture.* This quantity is represented thus:—$f = M_x\,h \div I$, where M_x is the amount of bending moment which is sufficient to rupture the beam. Reference to the modulus f was made in Arts. 27 and 29 § c, and we now append in Table LXXI. some values for beams of different kinds.

Table LXXI.—Approximate Values of the Modulus of Transverse Rupture for Beams of different kinds.

Section of Beam for each Material.	Values of f in tons.
CAST IRON.	
Rectangular bars (not exceeding 1 in. wide)... ...	20·4
,, ,, (3 in. wide)...	13·5
Round bars (1 in. diameter)	23·0
,, ,, (2 in. ,,)	20·0
I-beams (according to section) from	7·5
,, ,, to	15·0
WROUGHT IRON.	
Rectangular bars	23·0
I-beams	27·0
T-bars, with flange at top	24·0
,, ,, ,, bottom	23·0
STEEL.	
Rolled rectangular bars	51·0

The modulus f cannot be regarded as a desirable factor, because its value is only to be ascertained by experiments, and there are different values for beams of the same material according to the form of section. In the absence of a better aid to calculation, the modulus of transverse rupture is of value, and if it were not used, the excess of strength given by ordinary rules would inevitably lead to considerable waste of material.

Although the use of the solid rectangular form is chiefly restricted to timber beams, yet there are often occasions when bars of cast and wrought iron are employed in such a manner that they are essentially beams.

100. The Strength of Solid Rectangular Beams.—In calculating the strength of solid rectangular beams the expression $R = p\ I \div h$ may be simplified, and instead of p we must use f, the modulus of rupture.

As $I = bd^3 \div 12$, and $h = d \div 2$, we find that

$$R = f\ bd^2 \div 6 \quad \ldots\ldots\ldots\ldots \quad (8)$$

And if the section of the beam be square, we have

$$R = f\ b^3 \div 6 \quad \ldots\ldots\ldots\ldots\ldots \quad (9)$$

§ (*a*) *Examples.*—Ex. 1 : A cast-iron bar of solid rectangular section, 2 in. breadth by 3 in. depth, rests upon two supports placed 60 in. apart. Find the weight which will break the bar, assuming the load to be concentrated at the centre, and the value of f to be 13 tons. Here

$$R = f\ bd^2 \div 6 = (13 \times 2 \times 9) \div 6 = 39 \text{ inch-tons.}$$

Again, $M = Wl_x$, the precise expression for which by Rule 7, Art. 90, is $Wl \div 4$; hence

$$M = W \times 60 \div 4 = 15\ W \text{ inch-tons.}$$

As $R = M$; 39 inch-tons $= 15$ W inch-tons, and $W = 39 \div 15 = 2{\cdot}6$ tons, the required breaking weight.

Calculating the above example by the same rule, but using, instead of f, the ultimate tensile strength F_t of the material, say 8 tons, we arrive at a very different result :—

$$R = (8 \times 2 \times 9) \div 6 = 24 \text{ inch-tons};$$

$$M = W \times 60 \div 4 = 15\ W \text{ inch-tons};$$

and for the breaking weight we have

$$W = 24 \div 15 = 1{\cdot}6 \text{ tons.}$$

This load would not, as a matter of fact, cause failure of the bar unless the distance between the supports were increased to 97·5 in., when the bending moment would be responsible for a stress equal to 13 tons, which in this case is the value of f, the modulus of rupture.

Ex. 2.—A wrought-iron bar of solid section, 2 in. square, rests upon two supports, placed 60 in. apart. Find the weight which will break the bar, assuming the load to be concentrated at the centre, and the value of f to be 23 tons.

Here

$$R = fb^3 \div 6 = (23 \times 8) \div 6 = 30{\cdot}66 \text{ inch-tons};$$
$$M = W \times 60 \div 4 = 15 \text{ W inch-tons};$$

and for the breaking weight we have

$$W = 30{\cdot}66 \div 15 = 2{\cdot}044 \text{ tons}.$$

If calculated on the basis F_t = say 20 tons, the following incorrect result would be obtained:—

$$R = (20 \times 8) \div 6 = 26{\cdot}66 \text{ inch-tons};$$
$$M = 15 \text{ W inch-tons};$$

and

$$W = 26{\cdot}66 \div 15 = 1{\cdot}77 \text{ tons}.$$

Again, this weight would be insufficient to cause rupture of the bar.

101. The Strength of I-Beams.—When a material such as wrought iron or mild steel, in which the values of F_t and F_c are approximately equal, is manufactured in the form of an I-beam, where the web is thin and the flanges constitute the chief part of the sectional area, the modulus of rupture is found to differ very little from F_t or F_c. Consequently, all calculations relative to strength may be based on rules which accord both with theory and with practice.

§ (*a*) *Characteristics of some Section Books.*—Reference has been made in a previous chapter to the probability that erroneous conclusions may be drawn from the accurate information contained in some of the commercial section books published in this country. In these books the estimated safe permanent distributed loads are given for various simple and compound joists and girders, but the factor of safety is only occasionally stated. Sometimes the loads are calculated on the assumption that the ends of the beams will be firmly fixed. This stipulation is usually stated in a foot-note of the most unobtrusive character, which may be very easily overlooked. As the condition of fixity is rarely attained in practice, it is very undesirable that it should be indiscriminately adopted in calculated data, because it may lead to serious misunderstanding. Again, there is often a foot-note stating that in no case should the span exceed 20 or $22\frac{1}{2}$ times the depth of the joist. This is a very proper stipulation, having for its object the avoidance of excessive deflection. But if the compilers of these books expect the rule to be observed, it seems unnecessary that they should print long lists of safe loads for spans which it is stated ought not to be employed. If these various pitfalls are to be avoided considerable care is necessary, and in many cases it will be less trouble for the architect to make the calculations entirely on his own account. For this purpose very little information is obtainable from the ordinary mercantile hand-book, for the moments of inertia and of resistance are seldom stated, and even the ultimate strength of the material is not infrequently a matter for conjecture. There are one or two section books to which these criticisms do not apply, and from one of which the beams mentioned in the following examples have been selected so that all the dimensions quoted shall be practical instead of supposititious.

§ (*b*) *Mode of Calculation.*—In the following examples it is required to find the safe load per lineal foot, under different systems of loading, for a rolled steel joist of the following dimensions:—Breadth 5 in., depth 10 in., web thickness ·48 in., mean flange thickness 6 in.; ultimate strength of steel, $F_t = 32$ tons per square inch. By the rule in Table LXX., the moment of inertia for this section $I = 159{\cdot}97$; or say 160, to simplify our explanatory calculations. By Rule 6, $R = p\,I \div h$; p being in this case equivalent to F_t or F_c, which is 32 tons; $h = d \div 2 = 5$ in.

Consequently, for the moment of resistance of the section we have

$$R = 32 \times 160 \div 5 = 1{,}024 \text{ inch-tons.}$$

Sometimes the moment of resistance is expressed in square inches, being represented by $I \div h$, so that it may be used for any variable value of F_t. We shall use the symbol R to denote the moment of resistance in this form. In our case $I \div h = 32$, and as $F_t = 32$ tons,

$$R = R \times 32 = 1{,}024 \text{ inch-tons, as before.}$$

Having settled these preliminaries, we can readily find the safe load under different systems of loading.

Ex. 3.—*Uniformly distributed load, beam supported both ends :—*

By Rule 11 (Art. 90),

$$M = Wl \div 8 = W \times 12 \div 8 = 1{\cdot}5\ W \text{ inch-tons;}$$

and as $M = R$, $1{\cdot}5$ W inch-tons $= 1{,}024$ inch-tons;

hence, $W = 1{,}024 \div 1{\cdot}5 = 682{\cdot}6$ tons.

Using a factor of 5, the safe load would be, say, 136·5 tons. If the ends of the beam could be rigidly fixed, the load might be increased by 50 per cent. (Art. 90, Ex. 10).

Ex. 4.—*Concentrated load in centre, beam supported at both ends :—*

By Rule 7 (Art. 90), as $M = Wl \div 4$, the safe load must be half that found in Ex. 3 above, or say 68·25 tons. If the ends of the beam could be rigidly fixed, the load might be increased by 100 per cent. (Art. 90, Ex. 9.)

Ex. 5.—*Cantilever uniformly loaded :—*

By Rule 5 (Art. 90), $M = Wl \div 2$, and the safe load must be one-fourth that in Ex. 3, or 34·125 tons.

Ex. 6.—*Cantilever loaded at free end :—*

By Rule 1 (Art. 90), $M = Wl$, and the safe load must be one-eighth that in Ex. 3, or 17·06 tons.

The simple cases taken above show clearly how the safe load for a beam may be calculated for a unit length of 1 ft. To find the safe load for any desired span, it is only necessary to divide the load per unit length by the length of the span in feet. Summarising the results obtained above, we have the following tabular statement :—

Safe Loads in Tons for Steel I-*beam, Size* 10 *in. by* 5 *in.*

$R = 1{,}024$ inch-tons. Factor of safety $= 5$.

Type of Beam.	Distributed Load.		Concentrated Load.	
	Span 1 ft	Span [illegible]	Span [illegible]	Span [illegible]
Supported at both ends ...	136·5	13·65	68·25	6·825
Ends rigidly fixed	204·75	20·475	136·50	13·650
Cantilever	34·125	3·412	17·06	1·706

§ (*c*) *Comparison of "Safe" Loads.*—These figures obviously point to the necessity for the most careful consideration with regard to the manner in which any beam is to be used, and with the object of further emphasising the point, we append the "safe" distributed loads for 10-in. by 5-in. beams of 10-ft. span, as given in the hand-books of four different firms :—

Weight per ft.	F_t.	Ends.	Factor.	Distributed "Safe Load." fixed ends.	Load for Supported Ends, Factor 5.
lbs.	tons.				
(A) 29	28-32	Fixed	{ 3	20·9	—
			(*5*)	(*12·54*)	*8·3*
(A^1) 30	query	,,	query	18·66	*12·4, perhaps 7·46*
(B) 30	,,	,,	,,	18·66	*12·4, ,, 7·46*
(C) 30	,,	query	,,	18·31	*12·2, ,, 7·32*
(D) 35	28-32	supported	5	—	13·65

The statistics in the foregoing table constitute a very interesting example of commercial mathematics, by the aid of which it will be observed the least desirable sections are made to appear the most desirable. Only in one case are the figures entirely satisfactory. In calculating the loads for supported ends, printed in italics, we have assumed that a factor of 5 was originally employed, except where the factor is stated. If the factor of 3 is involved in the uncertain cases, the smaller loads in the last column will be those which should be used for the purpose of comparison. The steel in examples A^1, B, and C is presumably of foreign manufacture, and such sections are largely bought by those who imagine them to be much cheaper than British sections. No particular harm can be done if care be taken to use foreign sections which are equal to British sections in strength, but beams of insufficient strength may easily be selected in the absence of critical comparison.

102. Beams suitable for Given Loads.—Hitherto we have assumed the dimensions of the beam to be known, and the load to be unknown. As these conditions are converse to those usually obtaining in practice, it will be useful to select examples in which the load and the system of loading are known, and from these data to ascertain the dimensions of the required beams. The operation is readily performed by the aid of a section book stating for each section the ultimate strength of the material and the values of the moments of resistance.

$$R = (F_t \times I \div h), \quad \text{or } R = (I \div h).$$

The latter values may be specially calculated if desired, but it will save trouble to obtain a hand-book in which one or the other is given. For the purpose of preserving continuity in our inquiry, and of avoiding unnecessary repetition, the following examples are based as far as possible upon those in Art. 90. A factor of safety of 5 is applied throughout; the ultimate strength of the material is taken at 32 tons per square inch, and suitable standard sections are selected from the hand-book of Messrs. Dorman, Long & Co., Ltd., of Middlesbrough. The weights of the beams are not taken into account, and M = maximum bending moment in every case.

§ (*a*) *Calculations.*

Ex. 7.—Cantilever 6 ft. long, having a load of 2 tons at the free end. $M = 12$ foot-tons $= 144$ inch-tons. As $M = R$, the moment of resistance of the section selected should be $144 \times 5 = 720$ inch-tons, or $720 \div 32 = 22{\cdot}5$ square inches. Referring to the table of joists in the section book, we find the nearest approach to our requirement is represented by a section measuring 8 in. × 5 in., whose moment of resistance R is 23·46 square inches. This will give a slight surplus of strength.

Ex. 8.—Cantilever 6 ft. long, having a load of 2 tons equally distributed over the whole length. $M = 6$ foot-tons $= 72$ inch-tons. R should be $72 \times 5 = 360$ inch-tons, or $R = 11{\cdot}25$ square inches. In this case the most suitable section measures 7 in. × $3\frac{3}{4}$ in., and its moment $R = 11{\cdot}99$ square inches, again giving a small surplus of strength.

Ex. 9.—Beam supported at both ends, span 6 ft. and a concentrated load of 2 tons in the centre. $M = 3$ foot-tons $= 36$ inch-tons; hence R should be $36 \times 5 = 180$ inch-tons and $R = 5{\cdot}625$ square inches. The section whose moment of resistance is nearest to this measures $4\frac{5}{8}$ in. × 3 in., and the value of R is 5·68 square inches.

Ex. 10.—Beam supported at both ends, span 6 ft., uniform load 2 tons. $M=1 \cdot 5$ foot-tons $= 18$ inch-tons. R should be 18×5 inch-tons, and $R=2 \cdot 812$ square inches. The smallest section fully complying with the required conditions measures $3\frac{1}{2}$ in. $\times$ 3 in., and its value of $R=3 \cdot 35$ square inches.

Ex. 11.—Beam supported at both ends, span 6 ft., concentrated load of 2 tons placed 2 ft. from left-hand abutment. $M=2 \cdot 66$ foot-tons $=31 \cdot 92$ inch-tons. R should be $31 \cdot 92 \times 5=159 \cdot 6$, and $R=4 \cdot 98$ square inches. The nearest section for use is a 5-in. $\times$ 3-in. size, and its value of $R=5 \cdot 46$ square inches.

Ex. 12. — Beam supported at both ends, span 6 ft., two loads of 1 ton placed 2 ft. from right and left-hand abutments. $M = 2$ foot-tons $= 24$ inch-tons. R is to be $24 \times 5 = 120$ inch-tons, and R to be $3 \cdot 75$ square inches. The selected joist measures 4 in. $\times$ 3 in., and its value of $R=4 \cdot 51$ square inches.

Ex. 13.—Beam supported at both ends, span 6 ft., total load 2 tons, made up of three separate loads at various points. $M = 2 \cdot 86$ foot-tons $=34 \cdot 32$ inch-tons, $R=34 \cdot 32 \times 5=171 \cdot 6$ inch-tons, and $R=5 \cdot 36$ square inches. A 5-in. $\times$ 3-in. joist whose moment of resistance in square inches$=5 \cdot 46$ is most suitable in this case.

Ex. 14.—Beam supported at both ends, span 6 ft., load 2 tons distributed over a length of 2 ft. in the middle portion of the span. $M=2 \cdot 5$ foot-tons$=30$ inch-tons. R must be 150 inch-tons, and $R=4 \cdot 68$ square inches. The best section for this case measures $5\frac{1}{2}$ in. $\times$ 2 in., and its value of $R=4 \cdot 69$ square inches.

Ex. 15.—Beam supported at one end and at centre, span 6 ft., with a concentrated load of 2 tons at free end. Here $M = 6$ foot-tons ; consequently the same section as used for Ex. 8 is suitable.

Ex. 16.—Beam with both ends fixed, span 6 ft., concentrated load of 2 tons at centre. $M = 1 \cdot 5$ foot-tons, therefore the section used in Ex. 10 is suitable. It should be observed that, owing to the fixity of the ends, the force exerted by the load is half that produced by the similar load in Ex. 9.

Ex. 17.—Beam with both ends fixed, span 6 ft., uniform load of 2 tons. M = 12 inch-tons; R must be $12 \times 5 = 60$ inch-tons, or $R = 1 \cdot 875$ square inches. The nearest corresponding section measures 4 in. $\times\ 1\frac{3}{4}$ in., and its value of $R = 2 \cdot 58$ square inches. Owing to the ends being fixed, the force exerted upon this beam is only two-thirds of that produced by the same load in Ex. 10.

Ex. 18.—Continuous beam ends supported, two spans of 6 ft., each carrying a uniform load of 2 tons. The maximum value of M is the same as in Ex. 10, and a similar section is therefore suitable.

Some of the sections selected for the above examples are rather stronger than is really necessary, and in one or two cases it might be permissible to use a joist of slightly less strength, especially as the factor of 5 is taken.

The reader may find it interesting and instructive to calculate the stress intensity for each example by Rule 7, $p = Mh \div I$. Let us take Ex. 12 as an illustration.

Here

$$p = 24 \times 2 \div 9 \cdot 03 = 5 \cdot 315 \text{ tons per square inch,}$$

consequently the factor of safety is actually $32 \div 5 \cdot 315 = 6 \cdot 02$. If the next weaker joist had been selected, viz., $4\frac{3}{4}$ in. $\times\ 1\frac{3}{4}$ in., $R = 3 \cdot 57$ square inches, the following result would have been obtained:—

$$p = 24 \times 2 \cdot 375 \div 8 \cdot 48 = 6 \cdot 7217 \text{ tons per square inch.}$$

This reduces the factor of safety to $32 \div 6 \cdot 7217 = 4 \cdot 76$, which is less than the stipulated standard, but would probably be satisfactory in practice.

103. Deflection.—Deflection must always be considered, for a beam that may be of strength sufficient for the support of a given load may deflect to an undesirable degree. In bridge girders deflection is usually supposed to vary from $\frac{1}{125}$ to $\frac{1}{200}$ of the span; in building construction it ought not to exceed $\frac{1}{40}$ in. per foot of clear span, though the maximum is sometimes stated as $\frac{1}{30}$ in. If the span does not exceed twenty times the depth of the beam, deflection will be sufficiently prevented.

§ (*a*) *Method of Calculation.*—In the following rules for the determination of deflection the following symbols are used:—D = deflection in inches, W = load in tons, l = length of span in inches, E = modulus of elasticity (12,000 tons for steel), I = moment of inertia of beam:—

Beam supported at both ends, single load at centre,

$$D = Wl^3 \div 48\ EI \quad \ldots\ldots\ldots \quad (10)$$

Beam supported at both ends, uniform load,

$$D = Wl^3 \div 76{\cdot}8\ EI \quad \ldots\ldots \quad (11)$$

Cantilever, single load at free end,

$$D = Wl^3 \div 3\ EI \quad \ldots\ldots\ldots \quad (12)$$

Cantilever, uniform load,

$$D = Wl^3 \div 8\ EI \quad \ldots\ldots\ldots \quad (13)$$

The deflection resulting from a combination of loads can be found by considering the deflection due to each separate load and adding the amounts together, as in the case of bending moments.

Example 19.—Take the beam selected in Ex. 9. Here $W = 2$ tons, $l = 72$ in., $E = 12{,}000$ tons, and $I = 13{\cdot}17$. Therefore,

$$D = 2 \times (72)^3 \div (48 \times 12{,}000 \times 13{\cdot}17) \doteq {\cdot}098 \text{ in.}$$

This represents less than $\frac{1}{60}$ in. deflection per foot, and the amount is small because the span is little more than fifteen times the depth of the beam.

CHAPTER XVII.

BEAM DETAILS AND CONNEXIONS.

104. Simple and Compound Beams. — Many of the simple forms of section illustrated in Table VI. can be used as beams, but for the reasons already stated (Arts. 96 and 97), the I-section is the most suitable, and at the same time it is generally the most convenient. There are two main considerations which justify the employment of compound beams—(1) the circumstance that sections are rarely, if ever, rolled in this country of a depth greater than 20 in., and (2) the fact that beams of the maximum depth cannot always be employed owing to the exigencies of architectural design. In either case it becomes necessary to build up girders from simple sections, so that the necessary strength may be secured without exceeding the permissible limits of height. Built-up girders can be treated as coming under one of the three following classes:—

(*a*) I-beams joined together, or used singly, but strengthened by plates riveted to the flanges;

(*b*) Beams formed by the combination of channels, or angles with plates; and

(*c*) Beams with latticed webs.

The last-named type of beam is used in building construction when great strength is necessary, but is more generally found in bridges and in similar works. A lattice girder is essentially a framed structure in itself, and a complete consideration of the different forms of such members is beyond the scope of this treatise, partly because the subject is a very large one, and partly for the reason that we are concerned with details rather than with questions of design. Our discussion of beam details will, therefore, be limited to classes (*a*) and (*b*).

105. Details of Compound Beams. — Two elementary types of strengthened I-beam are illustrated by figs. 212 and 213. One or more plates may be riveted to one or to each flange, but it would serve no useful purpose to strengthen the flanges beyond the limit at which the web could be safely used for the resistance of shearing stress.

Fig. 212.

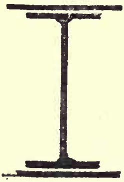


Fig. 213.

Fig. 214.

A beam treated as shown in fig. 212 is well adapted for the support of an upper wall, because the plate riveted to the top flange affords a bearing for the brickwork, but, considered on its merits as a beam, this compound section derives very little advantage from the enlarged area of the top flange, for the additional strength secured will not much exceed 15 per cent., whilst the cost will probably be increased by nearly 50 per cent. If a similar plate be also riveted to the bottom flange, as in fig. 213, the increase of strength will be approximately proportionate to the increase of cost. The reason why so small an advantage results from an increase in the sectional area of one flange was fully explained in Arts. 96 and 97. The section illustrated in fig. 214 is by no means perfect, as the lower flanges are not held in any way.

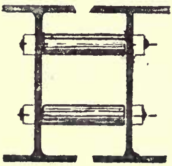


Fig. 215.

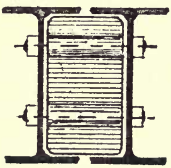


Fig. 216.

Fig. 217.

A simple mode of joining two beams by means of tie-bolts and pipe separators is shown in fig. 215. This arrangement is distinctly undesirable, and cannot be said to produce a compound girder in the true sense of the term. A much better mode of joining I-beams without plates is indicated in fig. 216, where a cast-iron separator is shown accurately fitting the space between the two joists so that a moderate degree of rigidity may be ensured; the use of large outside washers is recommended. Castings to suit all rolled I-sections can be procured, and should always be employed rather than pieces of pipe. The girder in fig. 217 has the appearance of being in every way a satisfactory arrangement, but it is not really so. It is manifestly impossible for rivets to be inserted continuously along both of the inside flanges in the case of long girders; consequently there is a considerable loss of strength by reason of irregular distribution of longitudinal stress, and the beam is more or less of a shaky nature.

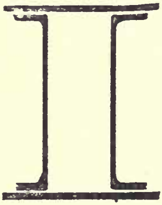


Fig. 218.

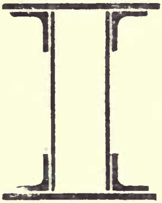


Fig. 219.

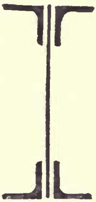


Fig. 220.

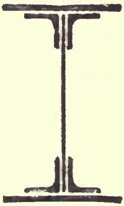


Fig. 221.

The sections shown in figs. 218 and 219 are much to be preferred, as riveting can be performed in a more uniform and satisfactory manner. Figs. 220 and 221 are examples of girders built up of angles and plates.

Fig. 222 illustrates a compound section which can be readily built up, though the mode of construction is not otherwise advantageous. There is too much metal at the middle, where it is of little use, and considerable strain must fall upon the rivets passing through the outside flanges. Rigidity would, of course, be added by using a tie-bolt (as in fig. 215) at the inner ends of each pair of I-beams, but the central weight of metal would thereby be further increased.

Figs. 223 and 224 represent types of plate girders with different forms of stiffeners for the webs.

Many more varieties of compound beams are made, but with the exception of lattice girders they do not essentially differ from those which we have mentioned. In every case where the flanges consist entirely or partly of plates, it should be remembered that each of the plates need not necessarily extend the whole length of the beam,

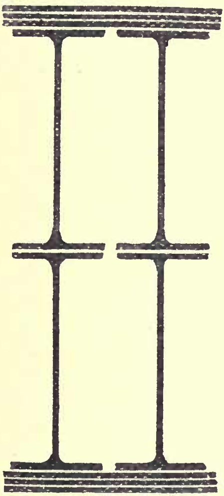


Fig. 222.

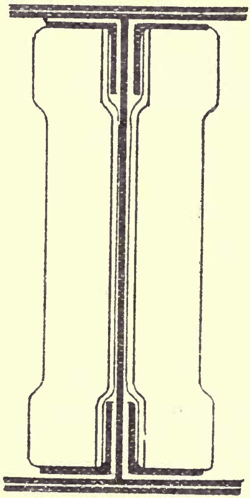


Fig. 223.

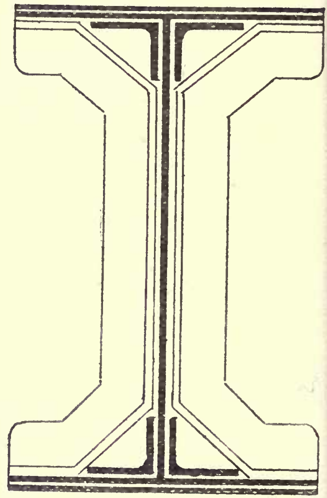


Fig. 224.

because, as we have already seen, the stresses due to bending moment diminish towards the ends. In order that the plates may be accurately proportioned it is necessary that stress intensities should be known at several points, or throughout the length of the beam under consideration. Besides due attention to this matter consideration must be given to the proportions of rivets and to the loss of strength due to rivet holes and riveting, subjects which have been fully discussed in Chapter IX.

106. Joint Plates.—For connecting two separate lengths of rolled I-section, joint or fish-plates are usually employed. Illustrations of four typical plates are given in fig. 225. The area of the plate and its thickness will, of course, vary with the dimensions and duty of the beam. Standard joint-plates are made by some manufacturers to suit all regular sections, and the proportions of such plates are calculated so that the maximum possible strength may be obtained. Whenever practicable, joints of this nature should be riveted, but if, as sometimes happens, bolts must be used, the bolt holes should be drilled, and the greatest care ought to be taken to see that the bolts are

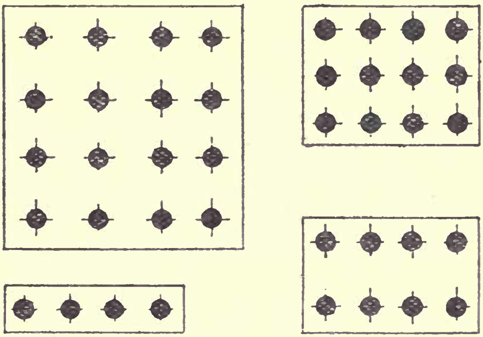


Fig. 225.

accurately fitted. Failing these precautions, the joints cannot possibly be of a satisfactory character.

Beams constructed largely of plates are generally built in the desired length at the contractor's yard, and the outer flange-plates are arranged so as to cover any joints which are necessary. Similarly, the joints of plates forming the webs are covered by flat bars or by stiffeners.

107. Angle Brackets.—A kind of connexion more frequently required than any other is the joining of main girders with floor beams and joists. As a rule, it would be highly inconvenient to place one beam on top of the other, and the weight of one beam ought not to rest upon the lower flange of another, because the load would then be non-axial, and, moreover, the sloping and curved flange does not lend itself to the support of a horizontal plane surface. For these reasons the rule is to connect girders by the aid of angle brackets riveted or bolted to the web of each member. Standard angle brackets are obtainable for all sections of rolled joists, and the use of such is to be recommended, as uniform spacing of the rivet holes will be secured for all sections of equal dimensions.

108. "Joggled" and Notched Joints.—Several different modes of preparing girders for connexion are in general use.

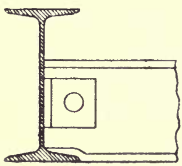


Fig. 226.

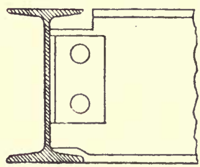


Fig. 227.

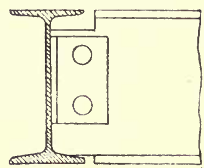


Fig. 228.

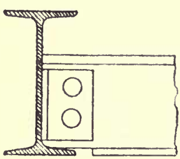


Fig. 229.

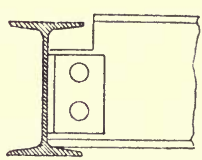


Fig. 230.

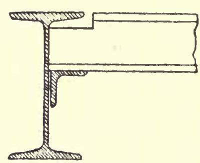


Fig. 231.

The transverse member may be "joggled" (Art. 109) at the bottom flange, as in fig. 226; the top and bottom flanges may be notched and "joggled" respectively, as shown in fig. 227; both flanges may be notched, as in fig. 228; the bottom flange may be notched, as in fig. 229; or the top flange notched, as in figs. 230 and 231.

In all these examples the chief support is afforded by the angle brackets and the webs, although the under part of one beam is usually shaped so that it is partly supported upon the lower flange of the other.

§ (*a*) *Avoidance of Torsional Stress.*—A better plan is to cut away the lower flange and part of the web of the supported girder so as to prevent any weight from being thrown upon the flange of the other beam. The adoption of this course obviates the creation of torsional stress at that part of the supporting beam where the web meets the flange. This is the very worst place for any such strain. Of course, if two equally loaded small girders rest upon the right and left hand flanges of a larger girder, torsional stress of the kind mentioned will not be caused, but if the loads be unequal and variable, the alternations of stress will be of a most destructive character.

Notched joints may not look so strong as "joggled" joints, but when properly proportioned angle brackets are used, the former type of connexion may be really stronger and in every way more desirable than the latter. As far as possible, rivets should be relied upon in preference to bolts, and as portable riveters, of the kind described in Art. 49, are now readily obtainable, there is very little excuse for the continued employment of bolts, except, of course, in special situations and for special purposes. Under the most favourable circumstances bolting can never be so secure as riveting, and as in building construction bolts rarely fit their holes properly, bolted joints usually permit a degree of freedom which does not add to the rigidity of a structure.

109. "Joggling."—Strictly speaking, a joggled joint is one where a projection on one member fits any notch on another member, but as the term is applied to beams and other iron and steel sections, it is understood to imply that the member has been bent or forged to the desired shape. When a large number of bars have to be shaped, a cast-iron die attached to a hydraulic press affords a ready means of performing the work rapidly and accurately. This mode of procedure is generally adopted in iron and steel works, but in small establishments joggling is generally done by a smith, and the work is then expensively and indifferently executed.

§ (*a*) *Workshop Practice.*—T-bars are comparatively easy to handle in joggling, as the web is equally supported on each side during the process. Channels and angles are not so easy of manipulation, as the vertical limbs tend to yield laterally when hammered, owing to inequality of support. Moreover, it is always impossible to ensure really clean, sharp work by the hammer, which is necessarily applied from point to point of the bend, instead of acting simultaneously upon the whole as occurs when a press is employed. Rolled I-sections cannot be readily joggled in the ordinary way because the web would buckle on the application of pressure. It is, therefore, usual to slot out a piece from this part, and to hammer up the flange, so that it shall be in contact with the edge of the web. As no actual connexion is generally made between the two parts, the so-called joint has no practical value, and the hammered flange simply acts as a packing plate, preventing the web of the supported girder from cutting into the flange of the supporting girder. The flange might advantageously be welded to the web, but it is doubtful whether this is ever done in ordinary work. Speaking generally, it is well to avoid all working of material after it has left the rolling mill, because, as explained in Art. 35, the effect of such manipulation is inevitably to modify the quality of the metal, and sometimes to induce a state of internal stress which causes a serious diminution of strength.

110. Connexion to Columns and Walls.—When describing and illustrating column connexions, in Art. 83, several diagrams were given which incidentally included beam connexions. It is unnecessary to make further reference to these, as the general mode of procedure should be sufficiently plain.

For the support of fireproof floors, channel sections are sometimes attached to columns, as shown in fig. 232; and at other times a compound girder formed of two I-beams is used, as in fig. 233. Various modifications of these arrangements can, of course, be devised to suit the requirements of any building. Proper care should always be taken to protect the metal from air and moisture, and due regard should be given to the essential principles of fireproof construction when beams are applied in this manner.

Girders are frequently expected to serve as ties for the external walls of buildings, in addition to performing the duty of carrying floor joists and floor loads. In such cases, anchors must be used, of the kind illustrated in Art. 64, or if preferred a pair of angle brackets can be riveted to the ends of the girder. One mode of anchoring a beam is indicated in fig. 234.

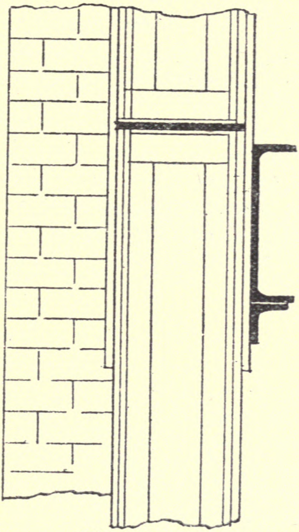


Fig. 232.

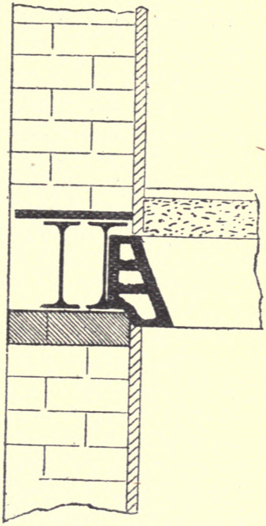


Fig. 233.

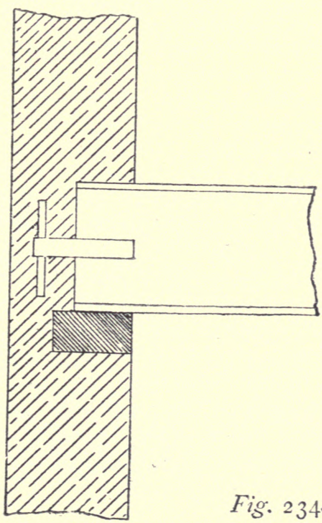


Fig. 234.

III. Expansion and Contraction.—The architect ought not to lose sight of expansion and contraction due to variations of temperature. Although structural iron or steel-work inside a building may not be exposed to such a range of temperature as occurs in the open air, it is nevertheless the fact that a certain amount of irresistible movement must take place. If the ends of girders were absolutely fixed in the walls of a building, the structure would be alternately exposed to push and pull as the girders expanded and contracted with variations of temperature. Very possibly distortion of the girders might be caused during expansion, thus lessening the outward thrust, but the full effect of contraction would inevitably be felt by the walls. By Table VIII.—(Appendix) the longitudinal expansion of steel for 1 deg. Fahr. is ·0000068 inch ; for 50 deg. we have $\cdot 0000068 \times 50 = \cdot 00034$ inch. As the extension of mild steel averages ·00007 inch per ton of tensile stress, the force exerted by expansion due to a difference of 50 deg. Fahr. is $\cdot 00034 \div \cdot 00007 = 4 \cdot 85$ tons per square inch. In the case of a beam having a sectional area equal to 50 square inches, the force would be such as no wall could possibly resist. At the same time, the expansion measured in lineal inches would be small. Thus, a beam 30 ft. long, and exposed to 50 deg. Fahr. increase of temperature, would only expand $\cdot 0000068 \times 360 \times 50 = \cdot 122$ in. Even supposing the beam to be 60 ft. long and the range of temperature to be 100 deg., the expansion would be less than half an inch, or about one quarter of an inch at each wall. If built into the brickwork in the ordinary manner, the necessary amount of clearance would most probably be available so that no damage could result, although movement of the walls in an outward direction would still be prevented. In case of fire, the expansion of metal work would be much greater, and if the beams once failed the anchors might pull in the walls, and thus cause serious injury to the building.

CHAPTER XVIII.

FLOOR-FRAMING.

112. Different Action of Flooring Systems.—So many systems of flooring are now adopted, that the work of designing a building is somewhat more complicated than it was a few years ago. Before the structural iron or steel work can be planned the architect must decide exactly what type of floor is to be employed. This course is necessary because the weights and forms of construction of floors differ very much one from another; consequently there must be considerable variation in the amount and nature of the stresses caused. Some floors merely act as distributed loads exerting force in a downward direction, and others exert force or forces laterally against the girders or the walls of a building. In the latter case, additional strength must be provided in some suitable way, of which the architect should naturally be the most competent judge. Cases are not altogether unknown where buildings have first been designed, and plans have then been issued to various manufacturers of special types of flooring, so that each might prepare an estimate and specification of the floors recommended. It is to be hoped that practice such as this is rare, for all the parts of a building cannot be in complete harmony with one another if important features are designed by two persons, neither of whom necessarily understands the work of the other.

113. Computation of Dead and Live Loads.—For the calculation of dead loads on floors it is necessary that the architect should be familiar with the weight per square foot of the various forms of ordinary and special floors, including floor beams and girders. Many of these can be found by calculation, and others will be readily furnished by the makers. Live loads must be estimated by the aid of assumed data, as no one can prophesy the exact load which will be imposed on any floor. In this country a considerable difference of opinion seems to prevail as to the proper allowance to be made for live loads.

§ (*a*) *Floor Loads for Different Buildings.*—According to one well-known authority, the following provisional weights are usual :—

(*a*) Dwelling-houses ....140 lbs. per square foot.
(*b*) Public buildings ....168 ,, ,, ,, ,,
(*c*) Warehouses ......280 to 336 ,, ,, ,, ,,

Judging from the average suburban residence, we may reasonably infer that the enterprising builder has far more confidence in the strength and rigidity of materials than is implied by the above-mentioned figures. To enable the student to arrive at any trustworthy idea of the probable live load, it is necessary that he should make a few simple calculations for himself. He will then see that vague rules are not particularly useful, and that they are likely to conduce to the waste of material on the one hand, or possibly to deficiency of strength on the other. Take the case, for instance, of an ordinary dwelling-room. The number of occupants per square yard cannot well average more than three, and, taking the weight of each person at 12 stone, or 168 lbs., the load per square foot would not be higher than $(168 \times 3) \div 9 = 56$ lbs. In parts of the room where bending moments are greatest the furniture will not weigh more than the persons who could occupy the same space, and heavy furniture is always situated near the walls where bending moments are smallest. Therefore, it would be quite safe to take the maximum live load at 70 lbs. per square foot. In bedrooms the load cannot well be more than half this amount, unless exceptionally heavy furniture be used. Rooms occupied as offices have to carry heavier loads, and provision ought to be made for at least 100 lbs. per superficial foot.

In public buildings allowance must generally be made for vibration and oscillative stresses caused by dancing, drilling, or gymnastic exercises, in addition to the weight of the persons themselves. Experimental evidence shows that a square yard of floor surface will not conveniently afford space for more than six men weighing 12 stone, or 168 lbs. each. If crowded together, as they would then necessarily be, there could be practically no movement, and consequently no vibration. The maximum live load is, therefore, $(168 \times 6) \div 9 = 112$ lbs. per square foot. Vibration caused by movement may be said to produce an effect about equivalent to that due to the weight of the persons, and as the weight of soldiers in close marching order does not exceed 80 lbs. per square foot, we can safely assume 160 lbs. to be an adequate allowance for live load. In the case of churches and assembly rooms, where the occupants are always supposed to be seated, an allowance of 120 lbs. per square foot ought to be amply sufficient.

Warehouses are sometimes specially designed for particular purposes, and if so, it is an easy matter to calculate what load will come upon the various floors. Frequently, however, warehouses are built with the intention of being leased to any tenants who may find the premises suitable, and the architect must then be sure to fix his hypothetical live load at an ample value. In ordinary warehouses an allowance of from 200 lbs. to 250 lbs. per square foot ought to suffice; but if heavy machinery has to be provided for, the estimate should be increased to 350 lbs. or even 400 lbs. per square foot.

114. Beams Suitable for Given Floor Loads.—The determination of suitable sections for floor girders and joists is by no means a difficult matter, as the two following examples will show.

Example 1.—Suppose that a floor has to be constructed for a building and is to be supported by rolled steel girders. The greatest live load is assumed to be 150 lbs. per square foot, uniformly distributed; the distance from wall to wall is 20 ft.; the main girders rest upon the walls, and are to be 10 ft. apart. Small joists spaced 5 ft. apart are to be supported by the main girders, and these joists are to carry a fireproof floor, the weight of which, complete with concrete, flooring boards, and ceiling, is taken at 59 lbs. per square foot. In the first place, the load on one joist should be ascertained. The dead weight of the flooring is

$$59 \text{ lbs.} \times (10 \times 5) = \text{say, } 1{\cdot}32 \text{ tons};$$

the live load is

$$150 \text{ lbs.} \times (10 \times 5) = 3{\cdot}35 \text{ tons}.$$

Total, excluding the weight of the joist, 4·67 tons.

Referring to a table of breaking loads for beams and using a factor of 4, or to a reliable table of safe loads in which the same factor is employed, we find a suitable section for our 10-ft. span to be one measuring 7 in. × $3\frac{3}{4}$ in., the permissible load for which is 5·48 tons, and the weight is 16 lbs. per foot run. This section possesses a sufficient margin of strength to cover its own comparatively insignificant weight, which does not amount to ·08 ton for a length of 10 ft.

In the next place, the load on one main girder should be calculated. The dead weight of the flooring is

$$59 \text{ lbs.} \times (20 \times 10) = 5{\cdot}27 \text{ tons};$$

the dead weight of five joists supporting the four floor spans is

$$16 \text{ lbs.} \times (5 \times 10) = {\cdot}36 \text{ ton};$$

the live load is

$$150 \text{ lbs.} \times (20 \times 10) = 13{\cdot}4 \text{ tons}.$$

Total, excluding the weight of the main girder, 19·03 tons.

Referring again to the table of beams, we find a suitable section for the span of 20 ft. to be 14 in. × 6 in., the safe load for which, with a factor of 4, is 19·8 tons. Again we have a sufficient margin of strength to cover the weight of the girder itself, which is less than ·51 ton for a length of 20 ft.

When the weight of a beam is small as compared with that of the load to be carried, the surplus strength ensured by choosing a suitable stock section is very frequently sufficient to provide for the dead load caused by the beam. This point should not, however, be left to chance, as it may happen that a slightly stronger section will be found necessary when the dead weight of the beam is considered.

Ex. 2.—In this case we will assume that the floor to be constructed is to be carried, as before, by steel girders, but that the flooring is to consist of timber 3 in. thick and weighing 42 lbs. per cubic foot. Let the live load be 150 lbs. per square foot, the distance from wall to wall 20 ft., and the main girders 10 ft. apart. Proceeding as in the previous example, and assuming the joists to be 30 in. apart, we find the dead load of timber on one joist to be

$$42 \text{ lbs.} \times (3 \div 12) \times (10 \times 2{\cdot}5) = {\cdot}117 \text{ ton};$$

the live load is

$$150 \text{ lbs.} \times (10 \times 2{\cdot}5) = 1{\cdot}674 \text{ tons}.$$

Total, excluding weight of the joist, 1·79 tons.

If this were the total load, and if deflection had not to be considered, the most economical section for use would be one measuring $4\frac{3}{4}$ in. × $1\frac{3}{4}$ in., and weighing 10 lbs. per foot run. But as a safeguard against excessive deflection, the depth of the joist ought not to be less than 10 ft. ÷ 20 = 6 in. Consequently the smallest permissible section is 6 in. × 2 in., weighing 12 lbs. per foot run, and as with a factor of 4 this is capable of carrying 2·9 tons, we have an ample margin of strength for the span of 10 ft. If economy be an object, as we believe it very often is, the spacing of the joists may be increased to 3 ft. 4 in., and even then the total load on one joist will be less than 2·5 tons, which is well below the carrying capacity of the selected joist.

Taking now the load on one main girder, the dead weight of the timber is

$$42 \text{ lbs.} \times (3 \div 12) \times (20 \times 10) = \cdot 94 \text{ ton};$$

the weight of seven joists, 3 ft. 4 in. apart, is

$$12 \text{ lbs.} \times (7 \times 10) = \cdot 38 \text{ ton};$$

the live load is

$$150 \times (20 \times 10) = 13 \cdot 4 \text{ tons}.$$

Total, excluding weight of main girder, 14·72 tons.

The most suitable section is one measuring 14 in. × 6 in., weight 46 lbs. per foot run, the safe load for which, using a factor of 4, is 16·09 tons. Including the weight of the girder, which is ·41 ton for the span of 20 ft., the total load is 14·72 × ·41 = 15·13 tons, and the selected girder provides a sufficient margin of strength.

§ (*a*) *Comparison of Examples.*—Comparing the total weights of the two floors suggested in the above examples, we find in the first case that one span of 20 ft. by 10 ft. weighs 19·54 tons. Consequently, the load on the pier supporting one end of each main girder is approximately 10 tons, and if there be five such floors the load per pier will be 50 tons, in addition to the other loads which must be expected. In the second case, one span of 20 ft. by 10 ft. weighs 15·13 tons. Therefore the load on each pier is only about 8 tons, or 40 tons if there be five floors. The difference of 10 tons on each pier of the building is an important item, and one which emphasises the necessity for determining beforehand exactly what system of flooring is to be adopted.

115. Extract from New York Building Law.—In concluding our consideration of floor framing, the following extract from the New York Building Law of 1892 may be usefully quoted :—"All iron or steel trimmer beams, headers, and tail beams shall be suitably framed and connected together, and the iron girders, columns, beams, trusses, and all other ironwork of all floors and roofs shall be strapped, bolted, anchored, and connected together and to the walls in a strong and substantial manner. Where beams are framed into headers, the angles which are bolted to the tail beams shall have at least two bolts for all beams over 7 in. in depth, and three bolts for all beams 12 in. and over in depth, and these bolts shall not be less than $\frac{3}{4}$ in. in diameter. Each of such angles or knees when bolted to girders shall have the same number of bolts as stated for the other leg. The angle iron in no case shall be less in thickness than the header or trimmer to which it is bolted, and the width of angle in no cases shall be less than one-third the depth of beam, excepting that no angle knee shall be less than $2\frac{1}{2}$ in. wide, nor required to be more than 6 in. wide. All wrought-iron or rolled-steel beams 8 in. deep and under shall have bearings equal to their depth if resting on a wall; 9 to 12-in. beams shall have a bearing of 10 in., and all beams more than 12 in. in depth shall have bearings not less than 12 in. if resting on a wall. Where beams rest on iron supports and are properly tied to the same, no greater bearings shall be required than one-third the depth of the beams. Iron or steel floor beams shall be so arranged as to spacing and length of beams that the load to be supported by them, together with the weight of the materials used in the construction of the said floors, shall not cause a deflection of more than one-thirtieth of an inch per lineal foot of span; and they shall be tied together at intervals of not more than eight times the depth of the beam."

116. Lateral Stays for Floor Beams.—With regard to the last stipulation, it may be well to remark that as the lateral stiffness of a beam depends upon width rather than on depth, the spacing of stays should be governed by the ratio of length to width. Thus, when the span exceeds about twenty times the width of the beam, lateral stays may be required. It should, however, be remembered that in actual practice sufficient bracing is usually afforded by other constructional details.

APPENDIX.

I.

Full text of Specification for Structural Steel presented by the American Section at the Congress of the International Association for Testing Materials, Paris, 1900, *with a view to their future adoption as a basis for International Standard Specifications.*

STRUCTURAL STEEL FOR BUILDINGS.

Process of Manufacture.

1. Steel may be made by either the open-hearth or Bessemer process.

Chemical Properties.

2. Each of the two classes of structural steel for buildings shall not contain more than 0·10 per cent. of phosphorus.

Physical Properties.

3. *Classes.*—There shall be two classes of structural steel for buildings, namely: RIVET STEEL and MEDIUM STEEL, which shall conform to the following physical qualities;

I.—Specification for Structural Steel (continued).

4. *Tensile Tests.*—

	Rivet steel.	Medium steel.
Tensile strength, pounds per square inch.	50,000 to 60,000	60,000 to 70,000
Yield Point, in pounds per square inch shall not be less than.....................	30,000	35,000
Elongation, per cent. in eight inches shall not be less than...........................	26	22

5. *Modifications in elongation for thin and thick material.*—For material less than five-sixteenths inch ($\frac{5}{16}''$), and more than three-fourths inch ($\frac{3}{4}''$) in thickness, the following modifications shall be made in the requirements for elongation :

(*a*). For each increase of one-eighth inch ($\frac{1}{8}''$) in thickness above three-fourths inch ($\frac{3}{4}''$), a deduction of one per cent. (1%) shall be made from the specified elongation.

(*b*). For each decrease of one-sixteenth inch ($\frac{1}{16}''$) in thickness below five-sixteenths inch ($\frac{5}{16}''$) a deduction of two and one-half per cent. ($2\frac{1}{2}$%) shall be made from the specified elongation.

(*c*). For pins the required elongation shall be five per cent. (5%) less than that specified in paragraph No. 4, as determined on a test specimen the centre of which shall be one inch (1″) from the surface.

6. *Bending Tests.*—The two classes of structural steel for buildings shall conform to the following bending tests ; and for this purpose the test specimen shall be one and one-half inches ($1\frac{1}{2}''$) wide, if possible, and for all material three-fourths inch ($\frac{3}{4}''$) or less in thickness the test specimen shall be of the same thickness as that of the finished material from which it is cut ; but for material more than three-fourths inch ($\frac{3}{4}''$) thick the bending test specimen may be one-half inch ($\frac{1}{2}''$) thick :

Rivet rounds shall be tested of full size as rolled.

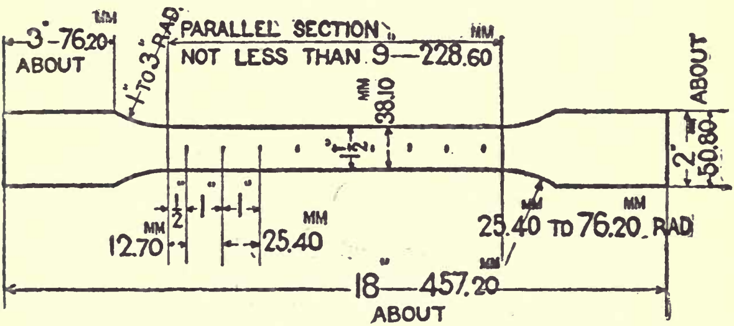
(*d*). Rivet steel shall bend cold 180° flat on itself without fracture on the outside of the bent portion.

(*e*). Medium steel shall bend cold 180° around a diameter equal to the thickness of the specimen tested, without fracture on the outside of the bent portion,

I.—Specification for Structural Steel (continued).

Test Pieces and Methods of Testing.

7. *Test Specimen for Tensile Test.*—The standard test specimen of eight inch (8″) gauged length, shall be used to determine the physical properties specified in paragraphs Nos. 4 and 5. The standard shape of the test specimen for sheared plates shall be as shown by the following sketch:



PIECE TO BE OF SAME THICKNESS AS THE PLATE.

For other material the test specimen may be the same as for sheared plates, or it may be planed or turned parallel throughout its entire length, and in all cases where possible two opposite sides of the test specimen shall be the rolled surfaces. Rivet rounds and small rolled bars shall be tested of full size as rolled.

8. *Number of Tensile Tests.*—One tensile test specimen shall be taken from the finished material of each melt or blow; but in case this develops flaws, or breaks outside of the middle third of its gauged length, it may be discarded and another test specimen substituted therefor.

9. *Test Specimen for Bending.*—One test specimen for bending shall be taken from the finished material of each melt or blow as it comes from the rolls, and for material three-fourths ($\frac{3}{4}$″) and less in thickness this specimen shall have the natural rolled surface on two opposite sides. The bending test specimen shall be one and one-half inches

I.—Specification for Structural Steel (continued).

($1\frac{1}{2}''$) wide, if possible, and for material more than three-fourths inch ($\frac{3}{4}''$) thick the bending specimen may be one-half inch ($\frac{1}{2}''$) thick.

Rivet rounds shall be tested of full size as rolled.

(*f*). The bending test may be made by pressure or by blows.

10. *Annealed Test Specimens.*—Material which is to be used without annealing or further treatment shall be tested for tensile strength in the condition in which it comes from the rolls. For material which is to be annealed or otherwise treated before use, a full-sized section of tensile test specimen length shall be similarly treated before cutting the tensile test specimen therefrom.

11. *Yield Point.*—For the purposes of this specification, the yield point shall be determined by the careful observation of the drop of the beam or halt in the gauge of the testing machine.

12. *Sample for Chemical Analysis.*—In order to determine if the material conforms to the chemical limitations prescribed in paragraph No. 2 herein, analysis shall be made of drillings taken from a small test ingot.

Variation in Weight.

13. The variation in cross section or weight of more than $2\frac{1}{2}$ per cent. from that specified will be sufficient cause for rejection, except in the case of sheared plates, which will be covered by the following permissible variations:—

(*g*). Plates $12\frac{1}{2}$ pounds per square foot or heavier, when ordered to weight, shall not average more than $2\frac{1}{2}$ per cent. variation above or $2\frac{1}{2}$ per cent. below the theoretical weight.

(*h*). Plates under $12\frac{1}{2}$ pounds per square foot, when ordered to weight, shall not average a greater variation than the following:—

Up to 75 inches wide, $2\frac{1}{2}$ per cent. above or $2\frac{1}{2}$ per cent. below the theoretical weight.

75 inches and over, 5 per cent. above or 5 per cent. below the theoretical weight.

(*i*). For all plates ordered to gauge, there will be per mitted an average excess of weight over that corresponding

I.—Specification for Structural Steel (continued).

to the dimensions on the order equal in amount to that specified in the following table :—

Tables of Allowances for Overweight for Rectangular Plates when Ordered to Gauge.

The weight of one cubic inch of rolled steel is assumed to be 0·2833 pound.

Plates 1/4 inch and over in thickness.

	Width of Plate.		
Thickness of plate. Inch.	Up to 75 ins. Per cent.	75 to 100 ins. Per cent.	Over 100 ins. Per cent.
1/4	10	14	18
5/16	8	12	16
3/8	7	10	13
7/16	6	8	10
1/2	5	7	9
9/16	4½	6½	8½
5/8	4	6	
over 5/8	3½	5	6½

Plates under 1/4 inch in thickness.

	Width of Plate.	
Thickness of plate. Inch.	Up to 50 inches. Per cent.	50 inches and above. Per cent.
1/8 up to 5/32	10	15
5/32 „ 3/16	8½	12½
3/16 „ 1/4	7	10

Finish.

14. Finished metal must be free from injurious seams, flaws or cracks, and have a workmanlike finish.

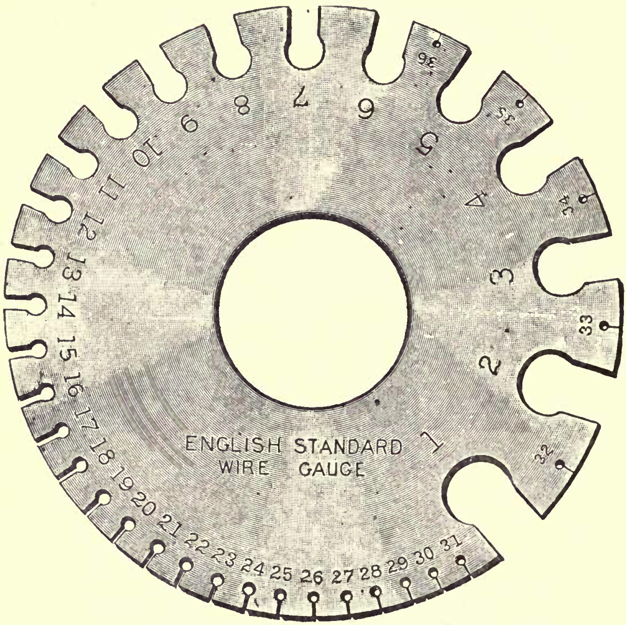
Branding.

15. Every finished piece of steel shall be stamped with the melt or blow number, except that small pieces may be shipped in bundles securely wired together with the melt or blow number on a metal tag attached

Inspection.

16. The inspector representing the purchaser shall have all reasonable facilities afforded to him by the manufacturer to satisfy him that the finished material is furnished in accordance with these specifications. All tests and inspections shall be made at the place of manufacture prior to shipment.

II.—Sheet Iron and Wire Gauges.



English Standard Wire Gauge (*Full Size*).

The Birmingham Wire Gauge (B.W.G.) represented the first attempt to secure the adoption of a uniform standard of measurement for metal sheets and wire. It is still largely used.

Since March, 1884, the only legalised gauges for these materials have been, the Imperial Standard Wire Gauge (S.W.G.), for wire, and the gauge known as B.G., for metal sheets and hoops. The latter two gauges are sanctioned by the Board of Trade, under the provisions of the Weights and Measures Act, 1878; and the values have been officially determined to the ten-thousandth part of an inch. Table III. shows the comparative values of the different numbers for each of these three gauges.

III.—Thickness and Weight of Iron Sheets.

Gauge No.	Thickness in inches.			Weight of Sheet Iron per square ft. in lbs.*	
	Sheet Iron Gauge "B.G."	Standard Wire Gauge "S.W.G."	Birmingham Wire Gauge "B.W.G."	"B.G."	"B.W.G."
1	·3532	·300	·300	14·294	12·150
2	·3147	·276	·284	12 758	11·543
3	·2804	·252	·259	11·340	10·490
4	·2500	·232	·238	10·125	9·639
5	·2225	·212	·220	9·032	8·910
6	·1981	·192	·203	8·739	8·222
7	·1764	·176	·180	7·128	7·290
8	·1570	·160	·165	6·359	6·683
9	·1398	·144	·148	5·670	5·994
10	·1250	·128	·134	5·063	5·427
11	·1113	·116	·120	4·496	4·860
12	·0991	·104	·109	4·014	4·415
13	·0882	·092	·095	3·572	3·848
14	·0785	·080	·083	3·179	3·362
15	·0699	·072	·072	2 831	2·916
16	·0625	·064	·065	2·531	2.633
17	·0556	·056	·058	2 252	2·349
18	·0495	·048	·049	2·005	1·985
19	·0440	·040	·042	1·782	1·701
20	·0392	·036	·035	1·588	1·418
21	·0349	·032	·032	1 413	1 296
22	·03125	·028	·028	1·268	1 134
23	·02782	·024	·025	1·126	1·013
24	·02476	·022	·022	1·004	·891
25	·02204	·020	·020	·891	·810
26	·01961	·018	·018	·794	·729
27	·01745	·0164	·016	·709	·648
28	·015625	·0148	·014	·632	·567
29	·0139	·0136	·013	·563	·527
30	·0123	·0124	·012	·498	·486
31	·0110	·0116	·010	·445	·405

* For steel add 2 per cent.

IV.—Weight of Metal Plates, in Pounds per Square Foot.

Thickness in Inches.	Iron.	Steel.	Brass.	Copper.	Lead.	Zinc.
1/16	2·5	2·6	2·7	2·9	3·7	2·3
1/8	5·	5·2	5·5	5·8	7·4	4·7
3/16	7·5	7·8	8·2	8·7	11·1	7·
1/4	10·	10·4	11·	11·6	14·8	9·4
5/16	12·5	13·	13·7	14·5	18·5	11·7
3/8	15·	15·6	16·4	17·2	22·2	14·
7/16	17·5	18·2	19·2	20·	25·9	16·4
1/2	20·	20·8	21·9	22·9	29·5	18·7
9/16	22·5	23·4	24·6	25·7	33·2	21·1
5/8	25·	26·	27·4	28·6	36·9	23·4
11/16	27·5	28·6	30·1	31·4	40·6	25·7
3/4	30·	31·2	32·9	34·3	44·3	28·1
13/16	32·5	33·8	35·6	37·2	48·	30·4
7/8	35·	36·4	38·3	40·	51·7	32·8
15/16	37·5	39·	41·2	42·9	55·4	35·1
1	40·	41·6	43·9	45·8	59·1	37·1

V.—Weight of Flat Bar Iron, in Pounds per Lineal Foot.

(*For Steel add 2 per cent.*)

Width.	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3½	4
Thickness ⅛ in.	·42	·53	·63	·74	·84	·95	1·05	1·16	1·26	1·47	1·68
¼ ,,	·84	1·05	1·26	1·47	1·68	1·90	2·11	2·32	2·53	2·25	3·37
⅜ ,,	1·26	1·58	1·90	2·21	2·53	2·84	3·16	3·47	3·79	4·42	5·05
½ ,,	1·68	2·11	2·53	2·95	3·37	3·79	4·21	4·63	5·05	5·89	6·74
⅝ ,,	2·11	2·63	3·16	3·68	4·21	4·74	5·26	5·79	6·32	7·37	8·42
¾ ,,	2·53	3·16	3·79	4·42	5·05	5·68	6·32	6·95	7·58	8·84	10·10
⅞ ,,	2·95	3·68	4·42	5·16	5·89	6·83	7·37	8·10	8·84	10·32	11·79
1 ,,	3·37	4·21	5·05	5·89	6·74	7·58	8·42	9·26	10·10	11·79	13·48

VI.—Weights of Square and Round Bars of Wrought Iron, in Pounds per Lineal Foot.

(*For Steel add* 2 *per cent.*)

Thickness or Diameter in Inches.	Square Bars.	Round Bars.	Thickness or Diameter in Inches.	Square Bars.	Round Bars.
0			2	13·33	10·47
1-16	·013	·010	1-16	14·18	11·14
1-8	·052	·041	1-8	15·05	11·82
3-16	·117	·092	3-16	15·95	12·53
1-4	·208	·164	1-4	16·88	13·25
5-16	·326	·256	5-16	17·83	14·00
3-8	·469	·368	3-8	18·80	14·77
7-16	·638	·501	7-16	19·80	15·55
1-2	·833	·654	1-2	20·83	16 36
9-16	1·055	·828	9 16	21·89	17·19
5-8	1·302	1·023	5 8	22·97	18·04
11-16	1·576	1·237	11-16	24·08	18·91
3-4	1·875	1·473	3-4	25·21	19 80
13-16	2·201	1·728	13-16	26·37	20·71
7-8	2·552	2·004	7-8	27·55	21·64
15-16	2·930	2·301	15-16	28·76	22·59
1	3·333	2·618	3	30 00	23·56
1-16	3·763	2·955	1-16	31·26	24·55
1-8	4·219	3·313	1-8	32·55	25·57
3 16	4·701	3·692	3-16	33·87	26·60
1-4	5·208	4·091	1-4	35 21	27·65
5-16	5·742	4·510	5 16	36 58	28·73
3-8	6·302	4·950	3-8	37·97	29·82
7-16	6·888	5·410	7-16	39·39	30·94
1-2	7·500	5·890	1-2	40·83	32·07
9-16	8·138	6·392	9-16	42·30	33·23
5-8	8·802	6·913	5-8	43·80	34·40
11-16	9·492	7·455	11-16	45·33	35·60
3-4	10·21	8·018	3 4	46·88	36·82
13-16	10·95	8·601	13-16	48·45	38·05
7-8	11·72	9·204	7-8	50·05	39·31
15-16	12·51	9·828	15-16	51·68	40·59

VII.—Weights of various substances.

Names of substances.	Pounds per cubic foot.
Asphalte	150·
Bath stone	123·
Brick, common, hard	125·
Brickwork, pressed hard	140·
,, London Stock	115·
Chalk, in lumps	80·
Cement, Portland	90·
Clay	119·
Concrete, ordinary	119·
,, in cement	136·
Earth, common loam, dry, loose	76·
,, ,, ,, ,, moderately rammed	95·
,, as a soft flowing mud	108·
Gneiss, common	168·
Granite	170·
Glass, Crown	157·
,, flint	192·
Ice at 0° C.	57·2
Lime, quick	53·
Limestone Blue Lias	154·
Masonry, of granite or limestone, well dressed.	165·
Mortar, hardened	103·
Mud, dry, close	80 to 100.
Quartz	165·4
Pitch	70·
Portland stone	151·
Sand, dry pit	100·
,, damp	118·
,, quartz	170·
,, river	117·
,, Thames	102·
Shingle	88·
Slate	175·
Tile, common	115·
Wood, birch	43·7
,, oak	46·8
,, pine	31·2
,, teak	50·
Water at 32° F	62·418
,, ,, 39·1° F	62·425
,, ,, 50° F	62·409
,, ,, 60° F	62·367
,, ,, 70° F	62·302
,, ,, 80° F	62·218
,, ,, 90° F	62·119

VIII.—Weights, Melting Points, and Expansion of Metals.

Name.	Weight in lbs. per cubic foot.	Melting Point in deg. Fahr.	Co-efficient of Expansion per deg. Fahr.
Brass	525	1700	·00001047
Copper	540	1930	·00000887
Gun Metal	530	—	—
Iron (cast)	450	2000	·00000616
,, (wrought)	480	2900	·00000657
Lead	710	612	·00001555
Steel	490	2500	·00000680
Tin	455	446	·0000121
Zinc	437	736	·00001636

IX.—Decimal Equivalents of Pounds, in Parts of a Ton.

lbs.		tons.	lbs.		tons.	lbs.		tons.	lbs.		tons.
1	=	·000446	29	=	·012946	57	=	·025446	85	=	·037946
2	=	·000893	30	=	·013393	58	=	·025893	86	=	·038393
3	=	·001339	31	=	·013839	59	=	·026339	87	=	·038839
4	=	·001786	32	=	·014286	60	=	·026786	88	=	·039286
5	=	·002232	33	=	·014732	61	=	·027232	89	=	·039732
6	=	·002679	34	=	·015179	62	=	·027679	90	=	·040179
7	=	·003125	35	=	·015625	63	=	·028125	91	=	·040625
8	=	·003571	36	=	·016071	64	=	·028571	92	=	·041071
9	=	·004018	37	=	·016518	65	=	·029018	93	=	·041518
10	=	·004464	38	=	·016964	66	=	·029464	94	=	·041964
11	=	·004911	39	=	·017411	67	=	·029911	95	=	·042411
12	=	·005357	40	=	·017857	68	=	·030357	96	=	·042857
13	=	·005804	41	=	·018304	69	=	·030804	97	=	·043304
14	=	·006250	42	=	·018750	70	=	·031250	98	=	·043750
15	=	·006696	43	=	·019196	71	=	·031696	99	=	·044196
16	=	·007143	44	=	·019643	72	=	·032143	100	=	·044643
17	=	·007589	45	=	·020089	73	=	·032589	101	=	·045089
18	=	·008036	46	=	·020536	74	=	·033036	102	=	·045536
19	=	·008482	47	=	·020982	75	=	·033482	103	=	·045982
20	=	·008929	48	=	·021429	76	=	·033929	104	=	·046429
21	=	·009375	49	=	·021875	77	=	·034375	105	=	·046875
22	=	·009821	50	=	·022321	78	=	·034821	106	=	·047322
23	=	·010268	51	=	·022768	79	=	·035268	107	=	047768
24	=	·010714	52	=	·023214	80	=	·035714	108	=	·048214
25	=	·011161	53	=	·023661	81	=	036161	109	=	·048661
26	=	·011607	54	=	·024107	82	=	·036607	110	=	·049107
27	=	·012054	55	=	·024554	83	=	·037054	111	=	·049554
28	=	·012500	56	=	·025000	84	=	·037500	112	=	·05

X.—Decimal Equivalents of Inches, in parts of a Foot.

Inches.	Feet.	16ths of an inch.	Feet.	32nds of an inch.	Feet.
1	·08333	1	·00521	1	·00260
2	·16666	2	·01041	3	·00781
3	·25	3	·01562	5	·01302
4	·33333	4	·02083	7	·01823
5	·41666	5	·02604	9	·02344
6	·5	6	·03125	11	·02864
7	·58333	7	·03646	13	·03385
8	·66665	8	·04166	15	·03906
9	·75	9	·04687	17	·04427
10	·83333	10	·05208	19	·04948
11	·91666	11	·05729	21	·05469
		12	·06250	23	·05989
		13	·06771	25	·06510
		14	·07292	27	·07031
		15	·07813	29	·07552
				31	·08073

XI.—Decimal Equivalents and Squares of Fractions.

32nds.	Dec. equiv.	Square.	16ths.	Dec. equiv.	Square.
1	·03125	·000977	1	·0625	·003901
3	·09375	·008789	2	·125	·015625
5	·15625	·024414	3	·1875	·035156
7	·21875	·047852	4	·25	·0625
9	·28125	·0791	5	·3125	·09766
11	·34375	·11817	6	·375	·14063
13	·40625	·16504	7	·4375	·19141
15	·46875	·21973	8	·5	·25
17	·53125	·28223	9	·5625	·31641
19	·59375	·35254	10	·625	·39063
21	·65625	·43067	11	·6875	·47266
23	·71875	·51661	12	·75	·5625
25	·78125	·61035	13	·8125	·66016
27	·84375	·71192	14	·875	·76563
29	·90625	·82129	15	·9375	·87891
31	·96875	·93848			

XII.—Metric Conversion Table.

Measures of Length.

Millimetres × ·03937 = inches.
Millimetres ÷ 25·4 = inches.
Centimetres × ·3937 = inches.
Centimetres ÷ 2·54 = inches.
Metres = 39·37 inches.
Metres × 3·281 = feet.
Metres × 1·094 = yards.
Kilometres × ·621 = miles.
Kilometres × 3280·7 = feet.

Measures of Surface.

Square Millimetres × ·0155 = square inches.
Square Millimetres ÷ 645·1 = square inches.
Square Centimetres × ·155 = square inches.
Square Centimetres ÷ 6·451 = square inches.
Square Metres × 10·764 = square feet.
Square Kilometres × 247·1 = acres.
Hectares × 2·471 = acres.

Measures of Capacity.

Cubic Centimetres ÷ 16·383 = cubic inches.
Cubic Metres × 35·315 = cubic feet.
Cubic Metres × 1·308 = cubic yards.
Cubic Metres × 220·1 = gallons.
Litres × 61·022 = cubic inches.
Litres × ·2201 = gallons.
Litres ÷ 4·5436 = gallons.
Litres ÷ 28·316 = cubic feet.

Measures of Weight.

Grammes × 15·432 = grains.
Grammes (water) ÷ 29·57 = fluid ounces.
Grammes ÷ 28·35 = ounces avoirdupois.
Grammes per cubic c.m. ÷ 27·7 = pounds per cubic inch.
Kilograms × 2·2046 = pounds.
Kilograms × 3.53 = ounces avoirdupois.
Kilograms ÷ 1016 = tons (2240lbs.)
Kilograms per square cent. × 14·223 = pounds per square inch.
Tonnes × ·9842 = tons (2240 lbs.)

Measures of Power.

Kilo-watts ÷ 1·34 = horse power.
Watts × 746 = horse power.
Calorie × 3·968 = B. T. U.
Cheval vapeur × ·9863 = horse power

XIII.—Comparison of Thermometers.

Fahrenheit to Centigrade.

($(F^\circ.-32) \times \frac{5}{9} =$ Degrees C.)

Fahrenheit.	Centigrade.	Fahrenheit.	Centigrade.	Fahrenheit.	Centigrade.	Fahrenheit.	Centigrade.	Fahrenheit.	Centigrade.
50	10·	61	16·1	72	22·2	83	28·3	94	34·4
51	10·6	62	16·7	73	22·8	84	28·9	95	35·
52	11·1	63	17·2	74	23·3	85	29·4	96	35·6
53	11·7	64	17·8	75	23·9	86	30·	97	36·1
54	12·2	65	18·3	76	24·4	87	30·6	98	36·7
55	12·8	66	18·9	77	25·	88	31·1	99	37·2
56	13·3	67	19·4	78	25·6	89	31·7	100	37·8
57	13·9	68	20·	79	26·1	90	32·2		
58	14·4	69	20·6	80	26·7	91	32·8		
59	15·	70	21·1	81	27·2	92	33·3		
60	15·6	71	21·7	82	27·8	93	33·9		

Centigrade to Fahrenheit.

($\frac{9}{5}\,C^\circ + 32 =$ Degrees F.)

Centigrade.	Fahrenheit.	Centigrade.	Fahrenheit.	Centigrade.	Fahrenheit.	Centigrade.	Fahrenheit.
10	50·	18	64·4	26	78·8	34	93·2
11	51·8	19	66·2	27	80·6	35	95·
12	53·6	20	68·	28	82·4	36	96·8
13	55·4	21	69·8	29	84·2	37	98·6
14	57·2	22	71·6	30	86·	38	100·4
15	59·	23	73·4	31	87·8	39	102·2
16	60·8	24	75·2	32	89·6	40	104·
17	62·6	25	77·	33	91·4		

XIV.—Squares, Cubes, Square Roots, and Cube Roots.

No.	Sq're.	Square Root.	Cube.	Cube Root.	No.	Square.	Square Root.	Cube.	Cube Root.
1	1	1·0	1	1·0	51	2,601	7·14143	132,651	3·7084
2	4	1·41421	8	1·2599	52	2,704	7·21110	140,608	3 7325
3	9	1·73205	27	1·4422	53	2,809	7·28011	148,877	3·7563
4	16	2·0	64	1·5874	54	2,916	7·34847	157,464	3·7798
5	25	2·23607	125	1·7100	55	3,025	7·4162	166,375	3·8030
6	36	2·44949	216	1·8171	56	3,136	7·48331	175,616	3·8259
7	49	2·64575	343	1·9129	57	3,249	7·54983	185,193	3·8485
8	64	2·82843	512	2·0	58	3,364	7·61577	195,112	3 8709
9	81	3·0	729	2·0801	59	3,481	7·68115	205,379	3·8930
10	100	3·16228	1,000	2·1544	60	3,600	7·74597	216,000	3·9149
11	121	3·31662	1,331	2·2240	61	3,721	7·81025	226,981	3·9365
12	144	3·46410	1,728	2·2894	62	3,844	7·87401	238,328	3·9579
13	169	3·60555	2,197	2·3513	63	3,969	7·93725	250,047	3·9791
14	196	3·74166	2,744	2·4101	64	4,096	8·0	262,144	4·0
15	225	3·87298	3,375	2·4662	65	4,225	8·06226	274,626	4·0207
16	256	4·0	4,096	2·5198	66	4,356	8·12404	287,496	4·0412
17	289	4·12311	4,913	2·5713	67	4,489	8 18535	300,763	4·0615
18	324	4·24264	5,832	2·6207	68	4,624	8·24621	314,432	4 0817
19	361	4·35890	6,859	2·6684	69	4,761	8·30662	328,509	4·1016
20	400	4·47214	8,000	2·7144	70	4,900	8·36660	343,000	4·1213
21	441	4·58258	9,261	2·7589	71	5,041	8·42615	357.911	4·1408
22	484	4·68012	10,648	2·8020	72	5,184	8·48528	373,248	4·1602
23	529	4·79583	12,167	2·8439	73	5,329	8·54400	389,017	4·1793
24	576	4 89898	13,824	2·8845	74	5,476	8·60233	405,224	4·1983
25	625	5·0	15,625	2·9240	75	5,625	8·66025	421,875	4·2172
26	676	5·09902	17,576	2·9625	76	5,776	8·71780	438,976	4·2358
27	729	5·19615	19,683	3·0	77	5,929	8·37496	456,533	4·2543
28	784	5·29150	21,952	3·0366	78	6,084	8·83176	474,552	4·2727
29	841	5·38516	24,389	3·0723	79	6,241	8 88819	493,039	4·2908
30	900	5·47723	27,000	3·1072	80	6,400	8·94427	512,000	4·3089
31	961	5·56776	29,791	3·1414	81	6,581	9·0	531,441	4·3267
32	1,024	5·65685	32,768	3·1748	82	6,724	9·05539	551,368	4·3445
33	1,089	5·74456	35,937	3·2075	83	6,889	9·11043	571,787	4·3621
34	1,156	5·83095	39,304	3·2396	84	7,056	9·16515	592,704	4·3795
35	1,225	5·91608	42,875	3·2711	85	7,225	9 21954	614,125	4·3968
36	1,296	6-0	46,656	3·3019	86	7,396	9·27362	636,056	4·4140
37	1,369	6·08276	50,653	3·3322	87	7,569	9·32738	658,503	4·4310
38	1,444	6·16441	54,872	3·3620	88	7,744	9·38083	681,472	4·4480
39	1,521	6·245	59,319	3·3912	89	7,921	9·43398	704,969	4·4647
40	1,600	6·32456	64,000	3·4200	90	8,100	9·48683	729,000	4·4814
41	1,681	6·40312	68,921	3·4482	91	8,281	9·53939	753 571	4·4979
42	1,764	6·48074	74,088	3·4760	92	8,464	9·59166	778,688	4·5144
43	1,849	6·55744	79,507	3·5034	93	8,649	9·64365	804,357	4·5307
44	1,936	6 63325	85,184	3·5303	94	8,836	9·69536	830,584	4·5468
45	2 025	6·70820	91,125	3 5569	95	9,025	9 74679	857,375	4 5629
46	2,116	6 78233	97,336	3 5830	96	9,216	9 79796	884,736	4·5789
47	2,209	6 85565	103,823	3 6088	97	9.409	9 84886	912,673	4 5947
48	2,304	6·92820	110,592	3·6342	98	9,604	9·89949	941,192	4·6104
49	2,401	7·0	117,649	3·6593	99	9,801	9·94987	970,299	4·6261
50	2,500	7·07107	125,000	3·6840	100	10,000	10·0	1,000,000	4·6417

XV.—Circumferences and Areas of Circles.

(*From* 1 *to* 25.)

Diam.	Circumf.	Area.	Diam.	Circumf.	Area.	Diam.	Circumf.	Area.
1-32	·098175	·00077	**1.** 13-16	5·69414	2·5802	**4.** 9-16	14·3335	16·349
1-16	·196350	·00307	7-8	5·89049	2·7612	5-8	14·5299	16·800
3-32	·294524	·00690	15-16	6·08684	2·9483	11-16	14·7262	17·257
1-8	·392699	·01227	**2.**	6·28319	3·1416	3-4	14·9226	17·721
5-32	·490874	·01917	1-16	6·47953	3·3410	13-16	15·1189	18·190
3-16	·589049	·02761	1-8	6 67588	3·5466	7-8	15·3153	18·665
7-32	·687223	·03758	3-16	6·87223	3·7583	15-16	15·5116	19·147
1-4	·785398	·04909	1-4	7·06858	3·9761	**5.**	15·7080	19·635
9-32	·883573	·06213	5-16	7·26493	4·2000	1-16	15·9043	20·129
5-16	·981748	·07670	3-8	7·46128	4·4301	1 8	16·1007	20·629
11-32	1·07992	·09281	7-16	7·65763	4·6664	3-16	16·2970	21·135
3-8	1·17810	·11045	1-2	7·85398	4·9087	1-4	16·4934	21·648
13-32	1·27627	·12962	9-16	8·05033	5·1572	5-16	16·6897	22·166
7-16	1·37445	·15033	5-8	8·24668	5·4119	3-8	16·8861	22·691
15-32	1·47262	·17257	11-16	8·44303	5·6727	7-16	17·0824	23·221
1-2	1·57080	·19635	3-4	8·63938	5·9396	1-2	17·2788	23·758
17-32	1·66897	·22166	13-16	8·83573	6·2126	9-16	17·4751	24·301
9-16	1·76715	·24850	7-8	9·03208	6·4918	5-8	17·6715	24·850
19-32	1·86532	·27688	15-16	9·22843	6·7771	11-16	17·8678	25·406
5-8	1·96350	·30680	**3.**	9·42478	7.0686	3-4	18·0642	25·967
21-32	2 06167	·33824	1.16	9·62113	7·3662	13-16	18·2605	26·535
11-16	2·15984	·37122	1-8	9·81748	7·6699	7-8	18·4569	27·109
23-32	2·25802	·40574	3-16	10·0138	7·9798	15-16	18·6532	27·688
3-4	2·35619	·44179	1-4	10·2102	8·2958	**6.**	18·8496	28·274
25-32	2·45437	·47937	5.16	10·4065	8 6179	1-8	19·2423	29·465
13-16	2·55254	·51849	3-8	10·6029	8·9462	1-4	19·6350	30·680
27-32	2·65072	·55914	7-16	10·7992	9·2806	3-8	20·0277	31·919
7-8	2·74889	·60132	1-2	10 9956	9·6211	1-2	20·4204	33·183
29-32	2·84707	·64504	9-16	11·1919	9·9678	5-8	20·8131	34·472
15-16	2·94524	·69029	5-8	11·3883	10·321	3-4	21·2058	35·785
31-32	3·04342	·73708	11-16	11·5846	10·680	7-8	21·5984	37·122
1.	3·14159	·78540	3-4	11·7810	11·045	**7.**	21·9911	38·485
1-16	3·33794	·88664	13-16	11·9773	11·416	1-8	22·3838	39·871
1-8	3·53429	·99402	7-8	12·1737	11·793	1-4	22·7765	41·282
3-16	3·73064	1·1075	16-16	12·3700	12·177	3-8	23·1692	42·718
1-4	3·92699	1·2272	**4.**	12·5664	12·596	1-2	23·5619	44·179
5-16	4·12344	1·3530	1-16	12·7627	12·962	5-8	23·9546	45·664
3-8	4·31969	1·4849	1-8	12·9591	13·364	3-4	24·3473	47·173
7-16	4·51604	1·6230	3-16	13·1554	13·772	7-8	24·7400	48·707
1-2	4·71239	1·7671	1-4	13·3518	14·186	**8.**	25·1327	50·265
9-16	4·90874	1·9175	5-16	13·5481	14·607	1-8	25·5254	51·849
5-8	5·10509	2·0739	3-8	13·7445	15·033	1-4	25·9181	53·456
11-16	5·30144	2·2365	7-16	13·9408	15·466	3-8	26·3108	55·088
3-4	5·49779	2·4053	1-2	14·1372	15·904	1-2	26·7035	56·745

XV.—Circumferences and Areas of Circles—continued.

Diam.	Circumf.	Area.	Diam.	Circumf.	Area.	Diam.	Circumf.	Area.
8. 5-8	27 0962	58·426	**14.** 1-8	44·3750	156 70	**19.** 5-8	61·6538	302·49
3-4	27·4889	60·132	1-4	44·7677	159·48	3-4	62·0465	306·35
7-8	78·8816	61·862	3-8	45·1604	162·30	7-8	62·4392	310 24
9.	28·2743	63·617	1-2	45·5531	165·13	**20.**	62·8319	314·16
1-8	28·6670	65·397	5-8	45·9458	167·99	1-8	63·2246	318·10
1-4	29·0597	67·201	3-4	46·3385	170·87	1-4	63 6173	322·06
3-8	29·4524	69·029	7-8	46·7312	173·78	3-8	64·0100	326·05
1-2	29 8451	70 882	**15.**	47·1239	176·71	1-2	64·4026	330·06
5-8	30·2378	72·760	1-8	47·5166	179·67	5-8	64·7953	334·10
3-4	30 6305	74·662	1-4	47 9093	182·65	3-4	65·1880	338·16
7-8	31·0232	76·589	3-8	48 3020	185·66	7-8	65·5807	342·25
10.	31·4159	78·540	1-2	48·6947	188·69	**21.**	65·9734	346·36
1-8	31·8086	80·516	5-8	49·0874	191·75	1-8	66·3661	350·50
1-4	32·2013	82·516	3-4	49·4801	194·83	1-4	66·7588	354·66
3-8	32·5940	84·541	7-8	49·8728	197·93	3-8	67·1515	358·84
1-2	32·9867	86·590	**16.**	50·2655	201·06	1-2	67·5442	363·05
5-8	33·3794	88·664	1-8	50·6582	204·22	5-8	67·9369	367·28
3-4	33·7721	90·763	1-4	51·0509	207·39	3-4	68·3296	371·54
7-8	34 1648	92 886	3-8	51·4436	210·60	7-8	68·7223	375·83
11.	34·5575	95·033	1-2	51·8363	213·82	**22.**	69·1150	380·13
1-8	34·9502	97·205	5-8	52·2290	217 08	1-8	69·5077	384·46
1-4	35·3429	99·402	3-4	52·6217	220·35	1-4	69·9004	388·82
3-8	35·7356	101·62	7-8	53·0144	223 65	3-8	70·2931	393·20
1-2	36·1283	103·87	**17.**	53·4071	226·98	1-2	70·6858	397·61
5-8	36·5210	106 14	1-8	53·7998	230·33	5-8	71·0785	402·04
3-4	36 9137	108·46	1-4	54·1925	233 71	3-4	71·4712	406·49
7-8	37·3064	110·75	3-8	54·5852	237·10	7-8	71·8639	410·97
12.	37·6991	113·10	1-2	54·9779	240·53	**23.**	72·2566	415·48
1-8	38·0918	115·47	5-8	55·3706	243·98	1-8	72·6493	420·00
1-4	38·4845	117·86	3-4	55·7633	247·45	1-4	73 0420	424·56
3-8	38·8772	120·28	7-8	56·1560	250·95	3-8	73·4347	429·13
1-2	39·2699	122·72	**18.**	56·5487	254·47	1-2	73·8274	433·74
5-8	39·6626	125·19	1-8	56·9414	258·02	5-8	74·2201	438·36
3-4	40·0553	127 68	1-4	57·3341	261·59	3-4	74·6128	443·01
7-8	40·4480	130 19	3-8	57·7268	265·18	7-8	75·0055	447·69
13.	40·8407	132·73	1-2	58 1195	268·80	**24.**	75·3982	452 39
1-8	41·2334	135·30	5-8	58·5122	272·45	1-8	75·7909	457·11
1-4	41·6261	137·89	3-4	58 9049	276·12	1-4	76·1836	461·86
3-8	42·0188	140·50	7-8	59 2976	279·81	3-8	76·5763	466·64
1-2	42·4115	143·14	**19.**	59·6903	383·53	1-2	76·9690	471·44
5-8	42·8042	145·80	1-8	60·0830	287·27	5-8	77 3617	476·26
3-4	43·1969	148·49	1-4	60·4757	291·04	3-4	77·7544	481·11
7-8	43 5896	151·20	3-8	60·8684	294·83	7-8	78·1471	485·98
14.	43·9823	153·94	1-2	61·2611	298·65	**25.**	78·5398	490·87

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