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STUDIES OF WATERFLOOD PERFORMANCE

II. TRAPPING OIL IN A PORE DOUBLET

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ABSTRACT

This paper discusses the pore doublet, a parallel arrangement of a small- and large-diameter capillary tube, as a model of reservoir rock. The displacement of oil by water is analyzed for the pore-doublet system, and from the results we have developed revised notions about waterflood character and consequences. The conclusions reported are not all in accord with previous assertions of other authors, but we believe them to be consistent with expectations.

Subjects specifically discussed include: 1) displacement efficiency as affected by viscosity ratio, pore texture, and capillary versus pressure-gradient driving forces; 2) explanations for the occurrence of subordinate phase production and the efficiency of imbibition waterflooding; and 3) concepts about fingering phenomena. The main conclusion, which is at variance with what has been previously asserted, is that oil tends to be trapped in the smaller (rather than the larger) tube of the pore doublet.

INTRODUCTION

The Illinois State Geological Survey has undertaken, as part of its research program in petroleum engineering, a series of studies on the flow of fluids through porous media. It is hoped that the results of such studies will provide a better understanding of the waterflooding method of improving oil recovery, which has become of major importance in Illinois.

Because a study of the flow of fluids through porous rock involves complex problems, it may be helpful to choose a simpler system for an analytical treatment. This paper, for example, discusses what will happen in an idealized system of capillary tubes called a "pore doublet." This approach may seem to have nothing to do with oil recovery in the field, but we believe that these indirect methods of analysis will ultimately enable us to understand the basic problems so as to develop better methods for predicting oil recovery, and determine the most efficient rate of water injection to get maximum recovery of oil.

PREVIOUS WORK

Recently Moore and Slobod (1956) have given an interesting and informative, but not altogether accurate, discussion of the VISCAP concept of oil recovery. Their work attempts to summarize the notion that the efficiency of a given process of oil recovery depends to a large extent on the interplay between viscous (shear) forces and capillary forces. The viscous forces act as resistance that must be overcome by the driving force before oil can be displaced and moved towards the production well; but the capillary forces either oppose or add to the driving force in effecting oil recovery. The purpose of this type of approach is to provide a basis for explaining why, for example, waterflooding efficiency should be rate-sensitive. Thus it has been reasoned that, with opposing forces, there might be an optimum intermediate condition that would mark the point at which the maximum amount of oil would be recovered.

We do not object to these proposed principles as the statement of a possibility. Perhaps in time we will know enough about the pore structure, fluid properties, and capillarity of reservoir systems so that optimum operating conditions can be assigned at the beginning of a secondary recovery operation. We would, however, question the application of the pore-doublet model that Moore and Slobod utilized to illustrate the contention that an intermediate rate of flooding will result in maximum oil recovery.

The pore-doublet model has been used widely to evaluate oil recovery. For example, Bartell and co-workers (Benner, Riches, and Bartell, 1943) concluded that the phenomenon of counterflow would be observed in pore doublets where the water-oil interface would be advancing in the smaller pore and receding in the larger. Figure 1 illustrates what they thought would happen in reservoir situations where doublet pore configurations were abundant. As has been discussed elsewhere, their analysis is not relevant to a description of what is likely to occur in the reservoir during waterflooding (Rose, in press). Other authors (for example, Yuster, 1940) also have made use of the pore-doublet model to evaluate the importance of the various factors that determine waterflooding efficiences. We believe, however, that the mechanics of oil displacement in a pore doublet have been incorrectly evaluated in some cases.

ANALYSIS OF THE PORE-DOUBLET MODEL

This paper simply analyzes what will happen in the pore-doublet model, and comes to conclusions different from those presented by other authors. For example, our analysis does not predict the frequent occurrence of Bartell's counter-flow phenomena, nor does it suggest that there is an intermediate rate of waterflooding that gives greater recoveries than either higher or lower rates. On the contrary, our analysis suggests that recovery is greatest when the injection rate (that is, the rate of flood-front advance) is least, rather than at some intermediate value. We consider it premature, however, to imply that any conclusions drawn from an analysis of the pore-doublet model (including ours) has any direct bearing on what will happen in actual reservoir situations. For example, we predict from the pore-doublet model that recovery is increased by employing low flooding velocities in water-wet sands, which may seem quite contradictory to certain field indications (namely at Bradford).

Figure 2 illustrates schematically what we feel to be the case. The pore doublet is indicated by two pores of different size, joined both at the inflow and outflow ends by other pores of arbitrary size. Barrer (1948) correctly gives the rate of interface travel in circular pores as:

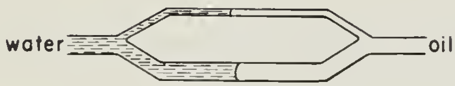

Fig. 1. - The pore-doublet model as originally depicted by Benner et al. (1943).

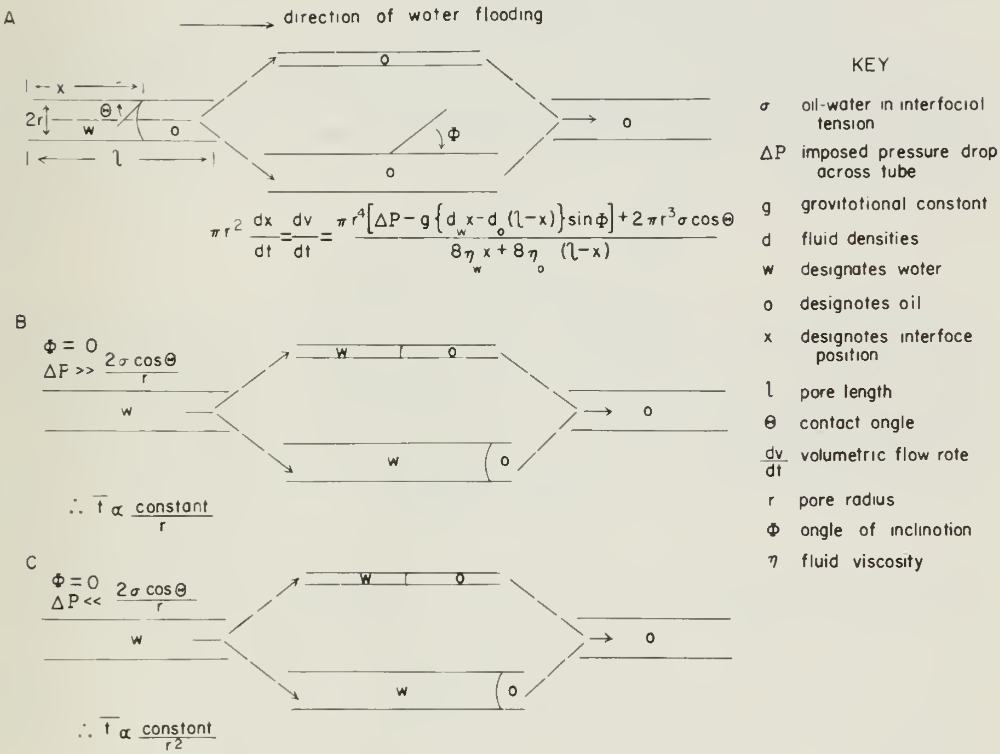

Fig. 2. - The correct representation of the pore-doublet model.

A - The water-oil interface in the inflow tube, advancing as described by Barrer (1948).

B - Oil displacement in the doublet effected by pressure-gradient drive.

C - Oil displacement in the doublet effected by capillary imbibition of water.

$$dx/dt = \frac{r^2[\Delta P - g\{d_w x - d_o(\ell - x)\}\sin\phi] + 2r\sigma\cos\theta}{8\eta_w x + 8\eta_o(\ell - x)} \tag{1}$$

where r is the pore radius
- ΔP is the difference between the pressures as measured at each end of the pore
- g is the gravitational constant
- d_w and d_o are the densities of the water and oil, respectively
- x denotes the interface position, between zero and ℓ where ℓ is the pore length
- ϕ is the angle of inclination with the horizon
- σ is the water-oil interfacial tension
- θ is the contact angle, and
- η_w and η_o are the viscosities of the water and oil respectively.

Hence, considering the simple case of equal oil and water viscosity ($\eta_w = \eta_o$) and zero gravity effect ($\phi = 0$), the time required for the water-oil interface to move the entire length of the tube is:

$$\lim_{x \to \ell} t = \frac{8\eta\ell^2}{r^2(\Delta P + 2\sigma\cos\theta/r)} \tag{2}$$

Assuming a water-wet system with $\theta = 0$, two limiting cases can now be considered. The first is one in which the capillary forces are negligible compared to the viscous forces, ($\Delta P \gg 2\sigma/r$), and in this case the time required for the water-oil interface to move the entire length of the tube is proportional to $1/r^2$ according to:

$$\lim_{\substack{x \to \ell \\ \sigma = 0}} t = \frac{8\eta\ell^2}{\Delta P\, r^2} \tag{3}$$

On the other hand, if capillary forces predominate ($2\sigma/r \gg \Delta P$), then the time for interface travel is proportional to $1/r$ according to:

$$\lim_{\substack{x \to \ell \\ \Delta P = 0}} t = \frac{4\eta\ell^2}{\sigma\, r} \tag{4}$$

Thus, in either case, the larger the tube radius the less time it takes for water to displace oil, no matter whether the driving force is ΔP alone, $2\sigma/r$ alone, or a combination of both the imposed pressure gradient and the inherent capillary forces.

Now, if we are comparing the relative rates of interface travel through two pores in parallel (that is, the pore doublet shown in fig. 2), the only condition to be imposed is that ΔP across each pore is the same. Hence, if the pores are of the same length, the ratio of the times for interface travel through each pore will be equal to the inverse ratio of the pore radii for the case of negligible ΔP; likewise when capillary forces are negligible, the ratio of times for interface travel through each pore will be equal to the inverse ratio of the square of the pore radii. This consequence is contradictory to the discussion given by Moore and Slobod although it follows from their analytic formulations; and it is different from that given by Bartell et al. (as would be expected in this latter case because different boundary conditions have been chosen).

From examination of the pore-doublet model it is thus clear that there is no support for the principle that both low and high injection rates will be less efficient than some intermediate rate.* In fact, it now appears that the pore-doublet model says (if it says anything) that the higher the injection rate, the lower the recovery of oil, as reference to figure 2 and the above equations show. That is, because the water-oil interface (flood-front) moves fastest through the larger pore, no matter what the magnitudes and relative magnitudes of the pressure gradient and the capillary pressure, there will always be residual oil trapped in the smaller pore of the doublet.

We should not lose sight of the fact that although small ΔP favors high ultimate recovery, high ΔP favors an earlier recovery, which therefore may be preferred in practical cases for economic reasons.

Again assuming no gravity effect, zero contact angle, and equal water and oil viscosity, the volume of oil so trapped (expressed as fractional saturation) will be given by:

$$S_o = \left\{ \frac{r_1^2 - \dfrac{r_1^4 [\Delta P + 2\sigma/r_1]}{r_2^2 [\Delta P + 2\sigma/r_2]}}{r_1^2 + r_2^2} \right\} \tag{5}$$

If the two parallel tubes are of different size, analysis shows that the trapped oil volume is minimum when the ratio of ΔP to $2\sigma/r$ is minimum (that is, zero), although (holding other factors constant) it is clear that the saturation of trapped oil approaches an absolute minimum of zero as the ratio of the tube radii approach unity.

In connection with the above equation, it is interesting to note that the residual oil is always trapped in the smallest pore (that is, r_1), and this is in direct contradiction to the Moore and Slobod statement that ". . . for most combinations of properties, the bypassed oil will be left in the larger capillary in the water-wet system." Actually, oil is always trapped in the smaller pore, even if conditions of unequal water and oil viscosity hold, for if only gravity effects are neglected in Equation 1 (that is, ϕ is zero), Equation 2 becomes:

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* Moore and Slobod (loc. cit.) say: ". . . the interplay of capillary and viscous forces determines the efficiency of oil displacement . . ." In the same vein, Benner et al. (1943) assert: ". . . the conditions for optimum recovery of oil depends upon the proper balance between the surface forces and other driving forces . . ." deriving this conclusion from their work with the pore-doublet model. We are prepared to agree that the interplay between capillary and viscous forces may be a determining factor in field recoveries, but we question the methods by which others have attempted to prove this concept by analysis of the pore-doublet model.

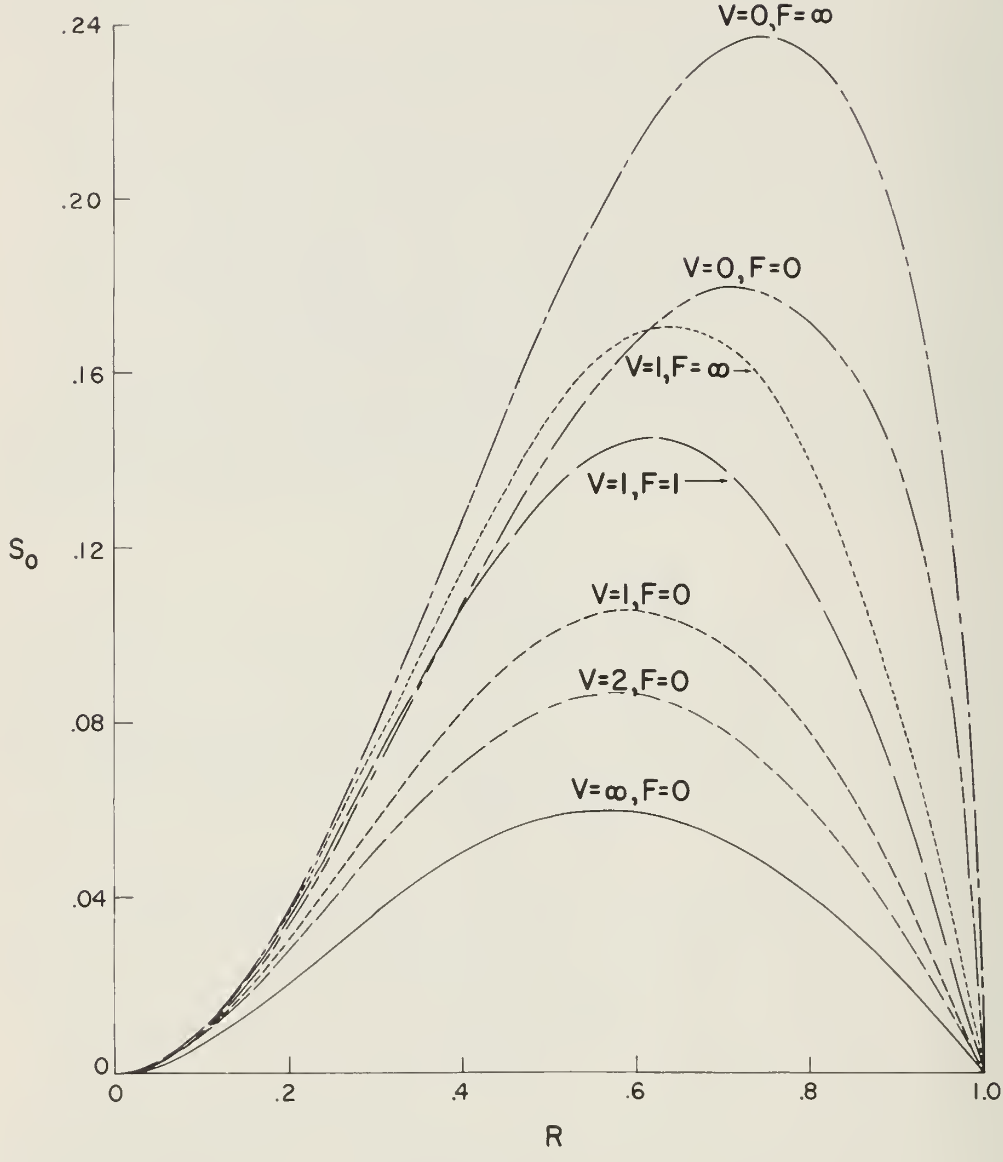


Fig. 3. - Residual oil, S_o, left in the pore doublet at break-through versus R, for various limiting values of V and F.

$$\lim_{x \to \ell} t = \frac{(4\ell^2)(\eta_w + \eta_o)}{r^2 [\Delta P + 2\sigma/r]} \quad (6)$$

Thus it is seen, as before, that the large tube always empties more quickly than the smaller, in a way inversely proportional to the ratio of radii (when capillary driving forces predominate), and inversely proportional to the square of the ratio of radii (when capillary driving forces can be neglected).

Combining Equations 1 and 6 gives the more complete expression for residual oil as:

$$S_o = \frac{R^2 [V - \left\{1 + R^2 \frac{F+1}{F+R} (V^2 - 1)\right\}^{1/2}]}{(R^2 + 1)(V - 1)} \quad (7)$$

where: R is the ratio of r_1 to r_2,
V is the water to oil viscosity ratio, and
F is the ratio of ΔP to $2\sigma/r_1$

Figure 3 gives various families of curves showing the relationship between S_o and R for various conditions of V and F. It is seen that minimum values of residual oil result when R is zero or unity, or when capillary forces control the displacement process (F = 0), or when the water-oil viscosity ratio is extremely high (that is, approaching infinity). This last consequence is especially interesting in that it provides a basis for understanding the relationship between mobility ratio, or "fingering" phenomenon, and oil recovery as discussed by Aronofsky and Ramey (1956).

Clearly, if the water viscosity is considerably greater than the oil viscosity, the movement of the flood-front in the smaller tube more closely approaches the faster rate in the larger tube (than if V were smaller) because there is always more of the lower viscosity oil in the smaller tube to increase its relative conductivity. Likewise, it should not be unexpected that the dominance of capillary forces would favor recovery, or that, when R is zero or unity, the residual oil saturation will be zero. These, in fact, are the intuitively expected results.

More to the point, the question may be appropriately asked: Will the trapped oil stay permanently in the smaller pore (of the pore doublet) in water-wet systems? The answer is clearly no, unless the interfacial curvature (and hence the capillary pressure) at both ends of the trapped oil-leg are strictly identical, and there is no net effective pressure gradient acting across the smaller tube.

This rather improbable situation is depicted schematically in figure 4A, and depends on postulating: 1) that flow of water has ceased in the larger tube (this could happen, for example, if, after the flood-front had moved through the larger pore of the pore doublet, subsequent trapping occurred at the downstream end); and 2) that the advancing and receding contact angles are equal. Or, if a ΔP is allowed across the pore doublet, then the opposing capillary forces must provide an exact balance, either because of difference between advancing and receding contact angle, or slight variation in pore radius of the smaller tube occurring exactly where the ends of the trapped oil-leg happen to be, or variation in the two contact angles resulting from difference in wettability conditions

Fig. 4. - Situations in the pore-doublet model after break-through

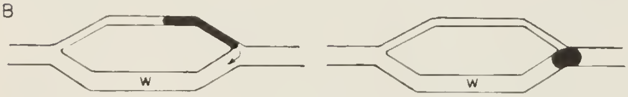
A - There is no net force acting on the trapped oil so it remains immobile.

B - There is counterflow of the oil trapped in the smaller pore so it moves to a more stable configuration in the larger pore.

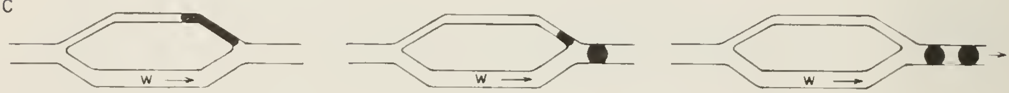
C - During counterflow, globules of the trapped oil are entrained in the water moving through the larger pore.

A

B



C



at different portions of the smaller tube. Such an exact balance of forces might not be encountered, however.

In a more likely case (namely, where advancing and receding contact angles are not equal, and/or a finite pressure difference continues across the pore doublet after breakthrough, and/or there is slight variation in pore size and/or wettability of the smaller tube along its length) it would appear that the oil initially trapped in the smaller pore would ultimately tend to move to some other position. For example, a variation of Bartell's counterflow concept might be responsible for movement of oil from the smaller tube to the larger tube, as depicted schematically in figure 4B. This would occur in response to the tendency of such systems to approach a practical minimum in free energy, for clearly, trapped oil in the smaller tube of the pore doublet has greater interfacial surface area of contact with the water and pore walls than if the same volume of oil were moved (via counterflow) into the larger tube. The latter, of course, would happen only if the water motion in r_2 had ceased, and if there were some net driving force to start this movement.

An interesting observation is that the dimension of the effluent connecting tube that joins the down-stream end of the pore doublet assumes importance, as depicted in figure 4C. For if there is no bottle-neck constriction there (as evidently was assumed in the Benner et al. analysis) to prevent the free entry of oil, it would appear possible that oil globules could be entrained in the moving water stream and be moved (via slug flow) at least until new barriers were met. In such a case it would seem possible that zero residual oil would eventually be left in the subject pore doublet, without reference to the effect of viscosity ratio (V) pore radius ratio (R), or the ΔP versus capillary force ratio (F).

Residual oil, of course, is invariably left in field operations, which probably reflects the fact that the pore doublet itself is too idealized a model of real reservoir situations. That is to say, it seems reasonable that pore doublets occur in nature, but the surrounding environment has more than a little to do with how they function during the recovery process. In this sense, one perhaps can take the values for residual oil, predicted by pore-doublet theory, as maximum values of oil that cannot be produced at the point in the recovery process where oil phase continuity is broken. Observed recoveries may be greater in nature (that is, residual values may be lower) because initial trapping does not mean unalterable trapping; but inasmuch as the practical (economic) end-point of the depletion process always occurs before all the oil that can be moved is produced, concepts regarding pore doublets may prove a useful index of anticipated recoveries.

Another compensating factor results from the fact that pore-doublet theory is speaking of displacement efficiency, whereas actually observed recoveries are always lessened because of sweep efficiency considerations. Thus we consider it not entirely unreasonable as a future possibility that lithologic examination of core samples and knowledge of operating conditions, fluid properties, etc., will permit a reasonable assigning of the R, V, and F terms, so that equally reasonable predictions of recovery can be made by use of Equation 7.

A still more general formulation to use instead of Equation 7, which considers the possibility of more than two parallel paths (but still neglects gravity and non-uniform wettability influences) is:

$$S_o = \frac{\sum^{i} R_i^{\,2} [1 - L_i]}{1 + \sum^{i} R_i^{\,2}} \qquad (8)$$

where:

$$L_i = \frac{\left[1 + R^2 \frac{F + 1}{F + R} (V^2 - 1)\right]^{1/2} - 1}{V - 1}$$

The above analysis admittedly is highly simplified, where, among other things, the flow of water due to a pressure gradient in the larger tube of the doublet has been neglected. Surely, the latter would determine the extent of counterflow that would result or the amount of transfer via slug flow that might be observed; likewise, an exact analysis should consider differences between advancing and receding contact angles, and effects of gravity. But the speculations as presented apparently suggest how to account for part or all of the so-called "subordinate phase of production" discussed in the theory of Buckley and Leverett (1942). The speculations also imply that cases may exist where intermittent water injection might benefit recovery by allowing time for oil trapped in small pores to move (via counterflow) to adjacent larger pores, so that later water injection will carry the oil on further towards the production point (via slug flow).

Perhaps the most interesting observation to be made is that the above considerations present an explanation for the success of the so-called "imbibition" waterflood process in water-wet reservoirs (Brownscombe and Dyes, 1952). This recovery method depends entirely on capillary forces to bring water into the sand for oil displacement, which (in accordance with the theory presented above) gives the oil more time to be displaced from trapped positions in smaller pores so that smaller residuals result.

The foregoing discussion defines (in our view) the usefulness of pore-doublet theory. For example, it provides a qualitative picture that explains the efficiency of imbibition waterfloods, and suggests ideas that help explain subordinate phase oil production and fingering phenomena. We reject, however, many of the conclusions of other authors, even though they may have been based on a model and analytic formulations entirely equivalent to those presented here. For example, Moore and Slobod, in citing an example in which oil is trapped in the larger (instead of the smaller) tube of the pore doublet, chose such a low rate of flow that they were in effect considering a negative (back-pressure) pressure gradient.

These authors cited the example of r_1 and r_2 being one and two microns, ℓ being 5 microns, σ being 30 dyne/cm., oil and water viscosity being one centipoise, and total flow rate through both tubes being 1.6×10^{-5} ccs./ second. From this the pressure gradient, ΔP, can be calculated as having a value of -2×10^5 dynes per square centimeter, which is sufficient to so retard the advance of the water-oil interface in r_2 that displacement only occurs in r_1.

Analysis shows that when ΔP equals $-2\sigma/r_1 + r_2$, the flood front moves at equal velocity through both tubes of the pore doublet so that the residual oil saturation as calculated by Equation 5 [or by Equation 7, letting F equal $-R/R+1$] is zero. However, if the back-pressure is increased to $-2\sigma/r_2$, forward motion of the water-oil interface (namely, displacement in r_2) ceases, and as ΔP is further increased in the negative sense, oil first enters r_2 and then finally enters the smaller tube r_1 so that waterflooding displacement ceases altogether. Discussion of the exact analytics of these situations, however, is beyond the scope of our paper. We simply note in passing that evidently previous contentions that oil recovery is optimum only when there is an ideal balance between capillary and other driving forces appear to have been based on the unusual condition of a back-pressure existing at the downstream end of the pore doublet.

CONCLUSIONS

In conclusion, we state the following as the principal conditions that favor maximum recovery from pore doublets (where again for simplicity gravity effects are neglected, and zero advancing and receding contact angles are assumed):

(1) If both pores of the pore doublet are nearly the same size, or if they are of considerably different sizes, then near-zero residual oil will result (that is, perfect sorting or poor sorting are better than intermediate degrees of sorting).

(2) Recovery is always favored by a high water-oil viscosity ratio.

(3) Examination of Equations (5) and (7) shows that imposing a back-pressure equal to $2\sigma/r_1 + r_2$ makes both pores of the doublet empty simultaneously so that zero oil-residuals result.

(4) If imposing a back-pressure is impractical, then maximum recovery in water-wet systems results when capillary forces alone bring in the displacing water phase (namely, a low ΔP driving force).

(5) The tendency of oil trapped in the smaller pores to seek a lower energy state in the contiguous larger pores would favor additional recovery via slug flow.

Another conclusion which we sense is apropos (although it is not fully demonstrated by the content of this paper) is that one does not expect unlimited success with the pore-doublet model in predicting the performance of actual reservoir systems. This is suggested by the observation that even the more elegant network models of Fatt (1956) leave much to be desired in achieving exact representation of the prototype system.

Thus, although the pore doublet microscopically may be found here and there in nature, its occurrence is not relevant unless and until the surrounding environment is taken into consideration. Our current inquiries, based on the use of complicated network model systems of the Fatt type, demonstrate this as will be shown in a later publication (Rose et al., in preparation). We admit, therefore, the temptation is to disregard what others say about such simple things as pore-doublet models, especially when more powerful methods of analysis are now available; but we must recognize that the projected work with the more complex network models rests to an extent on a correct understanding of what happens in the pore-doublet unit.

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