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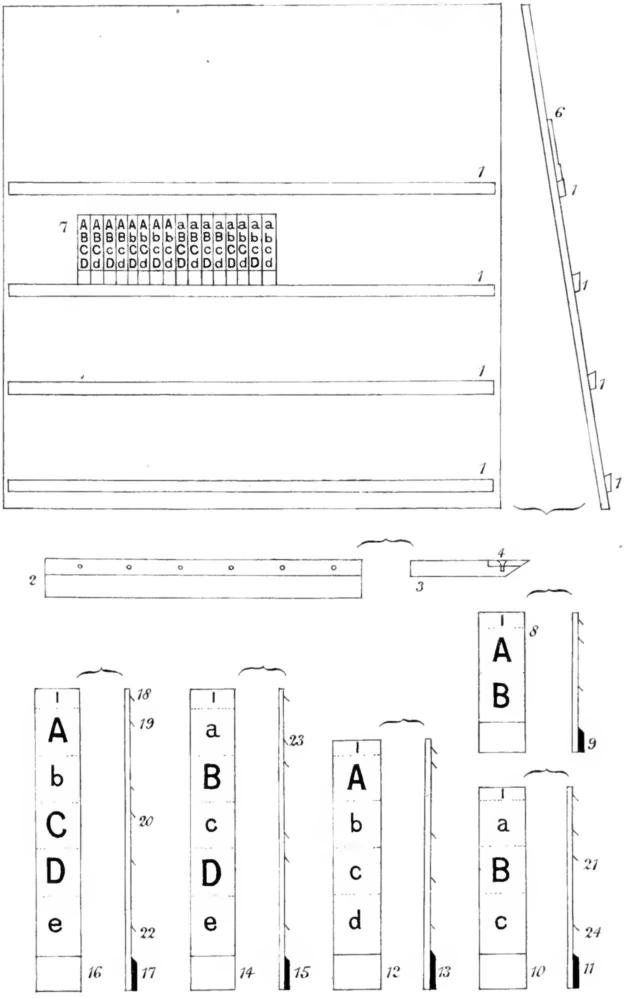
THE LOGICAL ABACUS.



SEE DESCRIPTION P 55 AND APPENDIX

THE

SUBSTITUTION OF SIMILARS,

The True Principle of Reasoning,

DERIVED FROM A MODIFICATION OF ARISTOTLE'S DICTUM.

BY

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PREFACE.

In this small treatise I wish to submit to the judgment of those interested in the progress of logical science a notion which has often forced itself upon my mind during the last few years. All acts of reasoning seem to me to be different cases of one uniform process, which may perhaps be best described as the *substitution of similars.* This phrase clearly expresses that familiar mode in which we continually argue by analogy *from like to like*, and take one thing as a representative of another. The chief difficulty consists in showing that all the forms of the old logic, as well as the fundamental rules of mathematical reasoning, may be explained upon the same principle; and it is to this difficult task I have devoted the most attention. The new and wonderful results of the late Dr.

Boole's mathematical system of Logic appear to develop themselves as most plain and evident consequences of the self-same process of substitution, when applied to the Primary Laws of Thought. Should my notion be true, a vast mass of technicalities may be swept from our logical text-books, and yet the small remaining part of logical doctrine will prove far more useful than all the learning of the Schoolmen.

CONTENTS.

THE SUBSTITUTION OF SIMILARS,

THE

TRUE PRINCIPLE OF REASONING.

ARISTOTLE is, perhaps, the greatest of human authors, but we may apply to him the words of Bacon, "Let great authors have their due, as Time, the author of authors, be not deprived of his due, which is farther and farther to discover truth." Aristotle has had his due in the obedience of more than twenty centuries, and Time must not be deprived of his due. Men, whose birthright is the increasing result of reason, are not to be bound for ever by the *dictum* of a thinker who lived but a little after the dawn of scientific thought. We are not to be persuaded any longer to look upon the highest of the sciences as a dead science. Logic is the science of the laws of thought itself, and there is no sphere of observation and reflection which is more peculiarly open to any inquirer, than the inquirer's own mind as engaged in the process of reasoning. It is from reflection on the operations of his own mind that Aristotle must have drawn the materials of his memorable Analytics. But Bentham's mind, as he himself remarked, was equally open to Bentham,[1] and it would be slavery indeed if any *dictum* of the first of logicians were to deprive all his successors of the liberty of inquiry.

2. It may be said, perhaps, that the weaker cannot possibly push beyond the stronger, and it is willingly allowed that among us moderns can few or none be found to equal in individual strength of intellect the great men of old. But Time is on our side. Though we reverence them as the ancients, they really lived in the childhood of the human race, and *these* times are, as Bacon would have said, the ancient times.[2] We enjoy not only the best

[1] "Essay on Logic," Bentham's works, vol. viii. p. 218.

[2] "De antiquitate autem, opinio quam homines de ipsa fovent, negligens omnino est, et vix verbo ipsi congrua. Mundi enim senium et grandævitas pro antiquitate vere habenda sunt; quæ temporibus nostris tribui debent, non juniori ætati mundi, qualis apud antiquos fuit. Illa enim ætas respectu nostri, antiqua et major; respectu mundi ipsius, nova et minor fuit. Atque revera quemadmodum majorem rerum humanarum notitiam, et maturius judicium, ab homine sene expectamus, quam a juvene, propter experientiam, et rerum quas vidit et audivit, et cogitavit, varietatem et copiam; eodem modo et à nostra ætate (si vires suas nosset, et

intellectual riches of the Greeks and Romans, but also the wonderful additions to the physical and mathematical sciences made since the revival of letters. In our time we possess an almost complete comprehension of many parts of physical science which seemed to Socrates, the wisest of men, beyond the powers of the human mind. We have before us an abundance of examples of the modes in which solid and undoubted truths may be attained, and it is absurd to suppose that among such successful exertions of the human intellect we can find no materials for a newer analytic of the mental operations.

3. The mathematics especially present the example of a great branch of abstract science, evolved almost wholly from the mind itself, in which the Greeks indeed excelled, but in which modern knowledge passes almost infinitely beyond their highest efforts. Intellects so lofty and acute as those of Euclid or Diophantus or Archimedes reached but the few first steps on the way to the widening generalizations of modern mathematicians; and what reason is there to suppose

experiri et intendere vellet) majora multo quam a priscis temporibus expectari par est; utpote ætate mundi grandiore, et infinitis experimentis et observationibus aucta et cumulata."—*Novum Organum*, Lib. i. Aphor. 84.

that Aristotle, however great, should at a single bound have reached the highest generalizations of a closely kindred science of human thought?

4. Kant indeed was no intellectual slave, and it might well seem discouraging to logical speculators that he considered logic unimproved in his day since the time of Aristotle, and indeed declared that it could not be improved except in perspicuity. But his opinions have not prevented the improvement of logical doctrine, and are now effectually disproved. A succession of eminent men,—Jeremy Bentham, George Bentham, Sir William Hamilton, Professor De Morgan, Archbishop Thomson, and the late Dr. Boole,—have shown that in the operations and the laws of thought there is a wide and fertile area of investigation. Bentham did more than assert our freedom of inquiry; in his uncouth logical writings are to be found most original hints, and in editing his papers his nephew George Bentham pointed out the all-important key to a thorough logical reform, the *quantification of the predicate*.[1] Sir William Hamilton, Archbishop Thomson, and Professor De Morgan rediscovered and developed the same new idea. Dr. Boole, lastly, employing this fundamental idea as his starting

[1] See "Outline of a New System of Logic," by George Bentham, Esq., London, 1827, p. 133 *et seq.*

point, worked out a mathematical system of logical inference of extraordinary originality.

5. Of the logical system of Mr. Boole Professor De Morgan has said in his "Budget of Paradoxes:"[1] "I might legitimately have entered it among my *paradoxes*, or things counter to general opinion: but it is a paradox which, like that of Copernicus, excited admiration from its first appearance. That the symbolic processes of algebra, invented as tools of numerical calculation, should be competent to express every act of thought, and to furnish the grammar and dictionary of an all-containing system of logic, would not have been believed until it was proved. When Hobbes, in the time of the Commonwealth, published his 'Computation or Logique,' he had a remote glimpse of some of the points which are placed in the light of day by Mr. Boole. The unity of the forms of thought in all the applications of reason, however remotely separated, will one day be matter of notoriety and common wonder; and Boole's name will be remembered in connexion with one of the most important steps towards the attainment of this knowledge."

6. I need hardly name Mr. Mill, because he has expressly disputed the utility and even the truthfulness of the reforms which I am considering, and

[1] No. xxiii. *Athenæum.*

has evolved most divergent opinions of his own in a wholly different direction from the eminent men just mentioned.

7. In the lifetime of a generation still living the dull and ancient rule of authority has thus been shaken, and the immediate result is a perfect chaos of diverse and original speculations. Each logician has invented a logic of his own, so marked by peculiarities of his individual mind, and his customary studies, that no reader would at first suppose the same subject to be treated by all. Yet they treat of the same science, and, with the exception of Mr. Mill, they start from almost the same discovery in that science. Modern logic has thus become mystified by the diversity of views, and by the complication and profuseness of the formulæ invented by the different authors named. The quasi-mathematical methods of Dr. Boole especially are so mystical and abstruse, that they appear to pass beyond the comprehension and criticism of most other writers, and are calmly ignored. No inconsiderable part of a lifetime is indeed needed to master thoroughly the genius and tendency of all the recent English writings on Logic, and we can scarcely wonder that the plain and scanty outline of Aldrich, or the sensible but unoriginal elements of Whately,

continue to be the guides of a logical student, while the works of De Morgan or of Boole are sealed books.

8. The nature of the great discovery alluded to, *the quantification of the predicate*, cannot be explained without introducing the technical terms of the science. A proposition, or judgment expressed in words, consists of a *predicate* or *attribute* united by a *copula* to a *subject*. In this proposition,

All metals are elements,

the predicate *element* is asserted of the subject *metal*, and the force of the assertion consists, as usually considered, in making the class of metals a part of the class of elements. The verb, or copula, *are*, denotes *inclusion* of the metals among the elements. But the subject only is quantified; for it is stated that *all metals* are elements, but it is not stated what proportion of the elements may be metals. Now the quantification of the predicate consists in giving some indication of the quantity or portion of the predicate really involved in the judgment.

All metals are some *elements*

is the same proposition thus quantified, and, though the change seems trifling, the consequences are momentous. The proposition no longer asserts the inclusion of one class in the other, but the identity of group with group. The proposition becomes *an equation* of subject and predicate, and the significance of this change will be fully apparent only to those who see that logical science thus acquires a point of contact with mathematical science. Nor is it only in a single point that the two great abstract sciences meet. Dr. Boole's remarkable investigations prove that, when once we view the proposition as an equation, all the deductions of the ancient doctrine of logic, and many more, may be arrived at by the processes of algebra. Logic is found to resemble a calculus in which there are only two numbers, 0 and 1, and the analogy of the calculus of quality or fact and the calculus of quantity proves to be perfect. Here, in all probability, we shall meet a new instance of the truth observed by Baden Powell, that all the greatest advances in science have arisen from combining branches of science hitherto distinct, and in showing the unity of principles pervading them.[1]

9. And yet any one acquainted with the systems of the modern logicians must feel that something is still wanting. So much diversity and obscurity are no usual marks of truth, and it is almost

[1] Baden Powell, "Unity of the Sciences," p. 41.

incredible that the true general system of inference should be beyond the comprehension of nearly every one, and therefore incapable of affecting ordinary thinkers. I am thus led to believe that the true clue to the analogy of mathematics and logic has not hitherto been seized, and I write this tract to submit to the reader's judgment whether or not I have been able to detect this clue.

10. During the last two or three years the thought has constantly forced itself upon my mind, that the modern logicians have altered the form of Aristotle's proposition without making any corresponding alteration in the *dictum* or self-evident principle which formed the fundamental postulate of his system. They have thus got the right form of the proposition, but not the right way of using it. Aristotle regarded the proposition as stating the inclusion of one term or class within another; and his axiom was perfectly adapted to this view.

The so-called *Dictum de omni* is, in Latin phrase, as follows—

Quicquid de omni valet, valet etiam de quibusdam et singulis.

And the corresponding *Dictum de nullo* is similarly—

Quicquid de nullo valet, nec de quibusdam nec de singulis valet.

In English these *dicta* are usually stated somewhat as follows :—

Whatever is predicated affirmatively or negatively of a whole class may be predicated of anything contained in that class. Or, as Sir W. Hamilton more briefly expresses them, *What pertains to the higher class pertains also to the lower.*[1]

These *dicta*, then, enable us to pass from the predicate to the subject, and to affirm of the subject whatever we know or can affirm of the predicate. But we are not authorized to pass in the other direction, from the subject to the predicate, because the proposition states the inclusion of the subject in the predicate, and not of the predicate in the subject.

The proposition,

All metals are elements,

taken in connexion with the *dictum de omni* authorizes us to apply to *all metals* whatever knowledge we may have of the nature of *elements*, because metals are but a subordinate class included among the elements; and, therefore, possessing all the properties of elements. But we commit an obvious fallacy if we argue in the opposite direction, and infer of elements what we know only of

[1] "Lectures on Logic," vol. i. p. 303.

metals. This is neither authorized by Aristotle's *dictum*, nor would it be in accordance with fact. Aristotle's postulate is thus perfectly adapted to his view of the nature of a proposition, and his system of the syllogism was admirably worked out in accordance with the same idea.

11. But recent reformers of logic have profoundly altered our view of the proposition. They teach us to regard it as an equation of two terms, formerly called the subject and predicate, but which, in becoming equal to each other, cease to be distinguishable as such, and become convertible. Should not logicians have altered, at the same time and in a corresponding manner, the postulate according to which the proposition is to be employed? Ought we not now to say that whatever is known of either term of the proposition is known and may be asserted of the other? Does not the *dictum*, in short, apply in both directions, now that the two terms are indifferently subject and predicate?

12. To illustrate this we may first quantify the predicate of our own former example, getting the proposition,

All metals are some *elements*,

where the copula *are* means no longer *are contained among*, but *are identical with;* or availing ourselves of the sign = in a meaning closely analogous to that which it bears in mathematics, we may express the proposition more clearly as,

All metals = some elements.

It is now evident that whatever we know of a certain indefinite part of the elements we know of all metals, and whatever we know of all metals we know of a certain indefinite part of the elements. We seem to have gained no advantage by the change; and if we are asked to define more exactly what part of the elements we are speaking of, we can only answer, *Those which are metals.* The formula

All metals = all metallic elements

is a more clear statement of the same proposition with the predicate quantified; for while it asserts an identity it implies the inclusion of metals among elements. But it is an accidental peculiarity of this form that the *dictum* only applies usefully in one direction, since if we already know what metals are we must know them to be *metallic* elements, the adjective *metallic* including in its meaning all that can be known of metals; and from knowing that metals are metallic elements we gain no clue as to what part of the properties of metals belong to elements. But it is hardly too much to say that Aristotle committed the greatest and most lamentable of all mistakes in the history of science when he took this kind of proposition as the true type of all propositions, and founded thereon his system. It was by a mere fallacy of accident that he was misled; but the fallacy once committed by a master-mind became so rooted in the minds of all succeeding logicians, by the influence of authority, that twenty centuries have thereby been rendered a blank in the history of logic.

13. Aristotle ignored the existence of an infinite number of definitions and other propositions which do not share the peculiarity of the example we have taken. If we define elements as *substances which cannot be decomposed*,[1] this definition is of the form—

Elements = undecomposable substances;

and since the term *element* does not occur in the second member, we may apply the *dictum* usefully in both directions. Whatever we know of the term *element* we may assert of the distinct term *undecomposable substance;* and, *vice versâ*, whatever we know of the term *undecomposable substance* we may assert of *element.*

[1] In strictness we should add, *by our present means.*

The example,

Iron is the most useful of the metals;

hardly needs quantification of the predicate, for it is evidently of the form—

Iron = the most useful of the metals,

the terms being both singular terms, and convertible with each other. We may evidently infer of both terms what we know of either. If we join to the above the similar proposition,

Iron = the cheapest of the metals,

we are easily enabled to infer that the *cheapest of the metals = the most useful of the metals*, since by the *dictum* we know of iron that it is *the cheapest of the metals;* and this we are enabled to assert of *the most useful*, and *vice versâ*. These are almost self-evident forms of reasoning, and yet they were neither the foundation of Aristotle's system, nor were they included in the superstructure of that system. His syllogism was therefore an edifice in which the corner-stone itself was omitted, and the true system is to be created by supplying this omission, and re-erecting the edifice from the very foundation.

14. I am thus led to take the equation as the fundamental form of reasoning, and to modify

Aristotle's *dictum* in accordance therewith. It may then be formulated somewhat as follows :—

Whatever is known of a term may be stated of its equal or equivalent.

Or in other words,

Whatever is true of a thing is true of its like.

I must beg of the reader not to prejudge the value of this very evident axiom. It is derived from Aristotle's *dictum* by omitting the distinction of the subject and predicate; and it may seem to have become thereby even a more transparent truism than the original, which has been condemned as such by Mr. J. S. Mill and some others. But the value of the formula must be judged by its results; and I do not hesitate to assert that it not only brings into harmony all the branches of logical doctrine, but that it unites them in close analogy to the corresponding parts of mathematical method. All acts of mathematical reasoning may, I believe, be considered but as applications of a corresponding axiom of quantity; and the force of the axiom may best be illustrated in the first place by looking at it in its mathematical aspect.

15. The axiom indeed with which Euclid begins to build presents at first sight little or no resem-

blance to the modified *dictum*. The axiom asserts that

Things equal to the same thing are equal to each other.

In symbols,

$$a = b = c$$

gives $a = c$.

Here two equations are apparently necessary in order that an inference may be evolved; and there is something peculiar about the threefold symmetrical character of the formula which attracts the attention, and prevents the true nature of the process of mind from being discovered. We get hold of the true secret by considering that an inference is equally possible by the use of a single equation, but that when there is no equation no inference at all can be drawn. Thus if we use the sign $\backsim$ to denote the existence of an inequality or difference, then one equality and one inequality, as in

$$a = b \backsim c,$$

enable us to infer an *inequality*

$$a \backsim c.$$

Two inequalities, on the other hand, as in

$$a \backsim b \backsim c,$$

do not enable us to make any inference concerning the relation of a and c; for if these quantities are equal, they may both differ from b, and so they may if they are unequal. The axiom of Euclid thus requires to be supplemented by two other axioms, which can only be expressed in somewhat awkward language, as follows:—

If the first of three things be equal to the second, but the second be unequal to the third, the first is unequal to the third.

And again:—

If two things be both unequal to a third common thing, they may or may not be equal to each other.

16. Reflection upon the force of these axioms and their relations to each other will show, I think, that the deductive power always resides in an equality, and that difference as such is incapable of affording any inference. My meaning will be more plainly exhibited by placing the symbols in the following form:—

$$\begin{array}{ccc} a = b & & a \\ \quad \Vert & \text{hence} & \Vert \\ \quad c & & c. \end{array}$$

Here the inference is seen to be obtained by substituting a for b by virtue of their equality as expressed in the first equation $a = b$, the second equation $b = c$ being that in which substitution is effected. One equation is *active* and the other is *passive*, and it is a pure accident of this form of inference that either equation may be indifferently chosen as the active one. Precisely the same result happens in this case to be obtained by a similar act of reasoning in which $b = c$ is the active equation, as shown below:—

$$\begin{array}{ccc} b = c & & c \\ \| & \text{hence} & \| \\ a & & a. \end{array}$$

My warrant for this view of the matter is to be found in the fact that the negative form of the axiom is now easily brought into complete harmony with the affirmative form, except that, since it has only one equation to work by, there can be only one active equation and one form in which the inference can be exhibited as below:—

$$\begin{array}{rcc} a = b & & a \\ \wr & \text{hence} & \wr \\ c & & c. \end{array}$$

Inference is seen to take place in exactly the same manner as before by the substitution of a for b, and the negative equation or difference $b \backsim c$ is the part in which substitution takes place, but which has itself no substitutive power. Accordingly we shall in vain throw two differences into the same form, as in

$$\begin{array}{rcccl} a \backsim & b & & b & \backsim c \\ & \wr & \text{or} & \wr & \\ & c & & a, & \end{array}$$

because we have no copula allowing us to make any substitution.

17. I am confirmed in this view by observing that, while the instrument of substitution is always an equation, the forms of relation in which a substitution may be made are by no means restricted to relations of equality or difference. If $a = b$, then in whatever way a third quantity c is related to one of them, in the same way it must be related to the other. If we take the sign $\backsim$ to denote any conceivable kind of relation between one quantity and another, then the widest possible expression of a process of mathematical inference is shown in the form—

$$\begin{array}{rccc} a = & b & & a \\ & \S & \text{hence} & \S \\ & c & & c. \end{array}$$

If in one case we take the sign $\backsim$ as denoting that c is a multiple of b, it follows that it is a multiple of a; if it is the nth multiple of one, it is the nth multiple of the other; if it is the nth submultiple, or the nth power, or the nth root of one, it similarly follows that it stands in the same relation to the other; or if, lastly, c be greater than b by n or less than c by n, it will also be greater or less than a by n. In this all-powerful form we actually seem to have brought together the whole of the processes by which equations are solved, viz. equal addition or subtraction, multiplication or division, involution or evolution, performed upon both sides of the equation at the same time. That most familiar process in mathematical reasoning, of substituting one member of an equation for the other, appears to be the type of all reasoning, and we may fitly name this all-important process the *substitution of equals.*

18. An apparent exception to the statement that all mathematical reasoning proceeds by equations may perhaps occur to the reader, in the fact that reasoning can be conducted by *inequalities.* A chapter on the subject of inequalities may even be found in most elementary works on algebra, and it is self-evident that *a greater of a greater is a greater,* and *what is less than a less is less.* Thus we certainly seem to have in the two formulæ,

$$a > b > c \quad \text{hence} \quad a > c,$$

and

$$a < b < c \quad \text{hence} \quad a < c,$$

two valid modes of reasoning otherwise than by equations. But it is apparent, in the first place, that the use of these signs $<$ and $>$ demands some precautions which do not attach to the copula $=$; the formulæ,

$$a > b < c,$$
$$a < b > c,$$

do not establish any relation between a and c; and I think the reader will not find it easy to explain why these do not and the former do, without implying the use of an equation or identity. The truth is, that the formulæ,

$$a > b > c,$$
$$a < b < c,$$

involve not only two differences, but also one identity in the direction of those differences, whereas the formulæ,

$$a > b < c,$$
$$a < b > c,$$

appear to fail in giving any inference because they involve only differences both of direction and quantity.

Strength is added to this view of the matter by observing that all reasoning by inequalities can be represented with equal or superior clearness and precision in the form of *equalities*, while the contrary is by no means always true. Thus the inequality

$$a > b$$

is represented by the equality

$$(1) \quad a = b + p,$$

in which p is any positive quantity greater than zero; and the inequality

$$b > c$$

is similarly represented by the equality

$$(2) \quad b = c + q,$$

in which q is again a positive quantity greater than zero. By substituting for b in (1) its value as given in (2), we obtain the equation

$$a = c + p + q,$$

which, owing to the like signs of p and q, is a representation in a more exact and clear manner of the conclusion

$$a > c.$$

On the other hand, the formula

$$a > b < e$$

would evidently lead to the equation

$$a = e + p - r,$$

in which p is the excess of a over b, and r the excess of e over b. Now this equation, taken in connexion with the former one, seems to give much clearer information as to the conditions under which inference is possible than do the formulæ of inequalities, and I entertain no doubt at all that, even when an inference seems to be obtained without the use of an equation, a disguised substitution is really performed by the mind, exactly such as represented in the equations. But I can only assert my belief of this from the examination of the process in my own mind, and I must submit to the reader's judgment whether there are exceptions or not to the rule, that we always reason by means of identities or equalities.

19. Turning now to apply these considerations to the forms of logical inference, my proposed simplification of the rules of logic is founded upon an obvious extension of the one great process of substitution to all kinds of identity. The Latin word *æqualis*, which is the original of our *equal*, was not restricted in signification to similarity of quantities, but was often applied to anything which was unvaried or similar when compared with another. We have but to interpret the word *equal* in the older and wider sense of like or equivalent, in order to effect the long-desired union of logical and mathematical reasoning. For it is not difficult to show that all forms of reasoning consist in

repeated employment of the universal process of the *substitution of equals*, or, if the phrase be preferred, *substitution of similars.*

20. To prevent a confusion of mathematical and logical applications of the formula, it will be desirable to use large capital letters to denote the things compared in a logical sense, but the copula or sign of identity may remain as before. Thus the symbols

$$A = B$$

denote the identity of the things represented by the indefinite terms or names A and B. Thus A may be taken in one case to mean *Iron*, when B might mean *the cheapest of the metals*, or *the most useful of the metals.* In another example which we have used A would mean *element*, and B *that which cannot be decomposed*, and so on. The fundamental principle of reasoning authorizes us to substitute the term on one side of an identity for the other term, *wherever this may be encountered*, so that in whatever relation B stands to a third thing C, in the same relation A must stand to C. Or, using the sign $\backsim$ to denote any possible or conceivable kind of relation, the formula

$$\begin{array}{ccccc} A & = & B & & A \\ & & \S & \text{hence} & \S \\ & & C & & C \end{array}$$

represents a self-evident inference. Thus,

If C *be the father of* B, C *is father of* A ;
If C *be a compound of* B, C *is a compound of* A ;
If C *be the absence of* B, C *is the absence of* A ;
If C *be identical with* B, C *is identical with* A ;

and so on.

21. We may at once proceed to develop from this process of substitution all the forms of inference recognised by Aristotle, and many more. In the first place, there cannot be a simpler act of reasoning than the substitution of a definition for a term defined; and though this operation found no place in the old system of the syllogism, it ought to hold the first place in a true system. If we take the definition of element as

Element = undecomposable substance,

we are authorized to employ the terms *element* and *undecomposable substance* in lieu of each other in whatever relation either of them may be found. If we describe iron as a *kind of element*, it may also be described as a *kind of undecomposable substance.*

22. Sometimes we may have two definitions of the same term, and we may then equate these to each other. Thus, according to Mr. Senior,

(1) *Wealth = whatever has exchangeable value.*

(2) *Wealth = whatever is useful, transferable, and limited in supply.*

We can employ either of these to make a substitution in the other, obtaining the equation,

Whatever has exchangeable value = whatever is useful, transferable, and limited in supply.

Where we have one affirmative proposition or equation, and one negative proposition, we still find the former sufficient for the process of inference. Thus

(1) *Iron = the most useful metal.*

(2) *Iron* $\backsim$ *the metal most early used by primitive nations.*

By substituting in (2) by means of (1) we have

The most useful metal $\backsim$ *the metal most early used by primitive nations.*

23. But two negative propositions will of course give no result. Thus the two propositions,

Snowdon $\backsim$ *the highest mountain in Great Britain,*
Snowdon $\backsim$ *the highest mountain in the world,*

do not allow of any substitution, and therefore do not give any means of inferring whether or not the highest mountain in Great Britain is the highest mountain in the world.

24. Postponing to a later part of this tract (§ 36) the consideration of negative forms of inference, I will now notice some inferences which involve combinations of terms. However many nouns, substantive or adjective, may be joined together, we may substitute for each its equivalent. Thus, if we have the propositions,

Square = equilateral rectangle,
Equilateral = equal-sided,
Rectangle = right-angled quadrilateral,
Quadrilateral = four-sided figure,

we may by evident substitutions obtain

Square = equal-sided, right-angled, four-sided figure.

25. It is desirable at this point to draw attention to the fact that the order in which nouns adjective are stated is a matter of indifference. A *four-sided, equal-sided figure* is identically the same as *an equal-sided, four-sided figure ;* and even when it sometimes seems inelegant or difficult to alter the order of names describing a thing, it is grammatical usage, not logical necessity, which stands in the way. Hence, if A and B represent any two names or terms, their junction as in A B will be taken to

indicate anything which unites the qualities of both A and B, and then it follows that

$$AB = BA.$$

This principle of logical symbols has been fully explained by Dr. Boole in his "Laws of Thought" (pp. 29, 30), and also in my "Pure Logic" (p. 15); and its truth will be assumed here without further proof. It must be observed, however, that this property of logical symbols is true only of adjectives, or their equivalents, united to nouns, and not of words connected together by prepositions, or in other ways. Thus *table of wood* is not equivalent to *wood of table;* but if we treat the words *of wood* as equivalent to the adjective *wooden,* it is true that a *table of wood* is the same as a *wooden table.*

26. We may now proceed to consider the ordinary proposition of the form

$$A = AB,$$

which asserts the identity of the class A with a particular part of the class B, namely the part which has the properties of A. It may seem when stated in this way to be a truism, but it is not, because it really states in the form of an identity the inclusion of A in a wider class B. Aristotle happened to treat it in the latter aspect only, and the extreme incompleteness of his syllogistic system is due to this circumstance. It is only by treating the proposition as an identity that its relation to the other forms of reasoning becomes apparent.

27. One of the simplest and by far the most common form of argument in which the proposition of the above form occurs is the mood of the syllogism known by the name *Barbara.*

As an example, we may take the following :—

(1) *Iron is a metal,*
(2) *A metal is an element,* therefore
(3) *Iron is an element.*

The propositions thus expressed in the ordinary manner become, in a strictly logical form :—

(1) *Iron = metallic iron,*
(2) *Metal = elementary metal.*

Now for *metal* or *metallic* in (1) we may substitute its equivalent in (2) and we obtain

(3) *Iron = elementary, metal, iron ;*

which in the elliptical expression of ordinary conversation becomes *Iron is an element,* or Iron is *some kind* of element, the words *an* or *some kind* being indefinite substitutes for a more exact description.

The form of this mode of inference must be stated in symbols on account of its great importance. If we take

$$A = \textit{iron},$$
$$B = \textit{metal},$$
$$C = \textit{element},$$

the premises are obviously,

$$(1)\quad A = AB,$$
$$(2)\quad B = BC,$$

and substituting for B in (1) its description in (2) we have the conclusion

$$A = ABC,$$

which is the symbolic expression of (3).

28. The mood *Darii*, which is distinguished from *Barbara* in the doctrine of the syllogism by its particular minor premise and conclusion, cannot be considered an essentially different form. For if, instead of taking A in the previous example = *iron*, we had taken it

$$A = \textit{some native minerals};$$

B and C remaining as before, we should then have the conclusion

$$A = ABC,$$

denoting

some native minerals are elements;

which affords an instance of the syllogism Darii exhibited in exactly the same form as Barbara.

29. The *sorites* or chain of syllogisms consists but in a series of premises of the same kind, allowing of repeated substitution. Let the premises be—

(1) *The honest man is truly wise,*
(2) *The truly wise man is happy,*
(3) *The happy man is contented,*
(4) *The contented man is to be envied,*

the conclusion being—

(5) *The honest man is to be envied.*

Taking the letters A, B, C, D, and E to indicate respectively *honest* man, *truly wise*, *happy*, *contented*, and *to be envied*, the premises are represented thus:—

(1) $A = AB$,
(2) $B = BC$,
(3) $C = CD$,
(4) $D = DE$,

and successive substitutions by (4) in (3), by (3) in (2), and by (2) in (1), give us

$C = CDE$,
$B = BCDE$,
$A = ABCDE$.

Or we may get exactly the same conclusion by substitution in a different order, thus:—

$$A = AB = ABC = ABCD = ABCDE.$$

The ordinary statement of the conclusion in (5) is only an indefinite expression of the full description of A given in A = ABCDE.

30. All the affirmative moods of the syllogism may be represented with almost equal clearness and facility. As an example of *Darapti* in the third figure we may take

(1) *Oxygen is an element,*
(2) *Oxygen is a gas,*
(3) *Some gas, therefore, is an element.*

Making A = *gas,*
B = *oxygen,*
C = *element,*

the premises become

(1) B = BC,
(2) B = BA.

Hence, by obvious substitution, either by (1) in (2) or by (2) in (1), we get

(3) BA = BC.

Precisely interpreted this means that *gas which is oxygen* is *element which is oxygen;* but when this full interpretation is unnecessary, we may substitute the indefinite adjective *some* for the more particular description, getting,

Some gas is some element,

or, in the still more vague form of common language,

Some gas is an element.

31. The mood *Datisi* may thus be illustrated :—

(1) *Some metals are inflammable,*
(2) *All metals are elements,*
(3) *Some elements are inflammable.*

Taking

A = *elements*, C = *inflammable*,
B = *metals*, D = *some*,

we may represent the premises in the forms

$$(1)\quad DB = DBC$$
$$(2)\quad B = BA.$$

Substitution, in the second side of (1), of the description of B given in (2) produces the conclusion

$$(3)\quad DB = DBCA,$$

or, in words,

Some metal = some metal element inflammable.

In this and many other instances my method of representation is found to give a far more full and strict conclusion than the old syllogism; but ellipsis or a substitution of indefinite particles or adjectives easily enables us to pass from the strict form to the vague results of the syllogism: it would be in vain that we should attempt to reach the more strict conclusion by the syllogism alone. But I must beg of the reader not to judge the validity of my forms by any single instance only, but rather by the wide embracing powers of the principle involved. Even common thought must be condemned as loose and imperfect if it should be found in certain cases to be inconsistent with a generalization which holds true throughout the exact sciences as well as the greater part of the ordinary acts of reasoning.

32. Certain forms of so-called *immediate inference*, chiefly brought into notice in recent times by Dr. Thomson, are readily derived from our principle.

Immediate inference by added determinant[1] consists in joining a determining or qualifying adjective, or some equivalent phrase, to each member of a proposition, a new proposition being thus inferred. Dr. Thomson's own example is as follows—

A negro is a fellow-creature;

whence we infer immediately,

[1] "Outline of the Laws of Thought," § 87.

A negro in suffering is a fellow-creature in suffering.

To explain accurately the mode in which this inference seems to be made according to our principle, let us take

A = *negro*,
B = *fellow-creature*,
C = *suffering*.

The premise may be represented as

$$A = AB.$$

Now it is self-evident that AC is identical with AC, this being a fact which some may think to be somewhat unnecessarily laid down in the first of the primary laws of thought (see § 41).

In the symbolic expression of this fact,

$$AC = AC,$$

we can substitute for A in the second member its equivalent AB, getting

$$AC = ABC.$$

This may be interpreted in ordinary words as,

A suffering negro is a suffering negro fellow-creature,

which differs only from the conclusion as stated by Dr. Thomson by containing the qualification *negro* in the second member.

33. *Immediate inference by complex conception* closely resembles the preceding, and is of exceedingly frequent occurrence in common thought and language, although it has never had a properly recognised place in logical doctrine until lately.[1]

Its nature is best learnt from such an example as the following :—

Oxygen is an element,
Therefore a pound weight of oxygen is a pound weight of an element.

This is a very plain case of substitution; for if we make

$$O = \textit{oxygen},$$
$$P = \textit{pound weight},$$
$$Q = \textit{element},$$

we may represent the premise as

$$O = OQ.$$

Now it is self-evident that

$$P \textit{ of } O = P \textit{ of } O,$$

and substituting in the second member the description of O we have

$$P \textit{ of } O = P \textit{ of } OQ.$$

[1] Thomson's "Outline," § 88.

34. In an exactly similar manner we may solve a common form of reasoning which the authors of the "Port Royal Logic" described as the *Complex Syllogism*, remarking how little attention logicians had in their day given to many common forms of reasoning.[1] I will employ their example, which is as follows :—

(1) *The sun is an insensible thing,*
(2) *The Persians worship the sun,*
(3) *The Persians, therefore, worship an insensible thing.*

Making

A = *sun*,
B = *insensible thing*,
C = *Persians*,
D = *worshippers*,

we may represent the above by the symbols

(1) A = AB
(2) C = CD *of* A.

Hence, by substitution for A in (2) by means of (1),

(3) C = CD *of* AB.

35. I regard hypothetical propositions as only differing from categorical propositions in the acci-

[1] "Port Royal Logic," translated by Mr. Spencer Baynes, p. 207.

dental form of expression. It is well known to readers of the ordinary handbooks of logic, that hypothetical propositions can always be represented in the categorical form by altering the phraseology; and the fact that the alteration required is often of the slightest possible character seems to show that there is no essential difference. Thus the proposition,

If iron contain phosphorus, it is brittle,

is hypothetical, but exactly equivalent to the categorical proposition,

Iron, containing phosphorus, is brittle;

which is of the symbolic form,

$$AB = ABC.$$

But propositions such as,

If the barometer falls, a storm is coming,

cannot be reduced but by some such mode of expression as the following:—

The circumstances of a falling barometer are the circumstances of a storm coming.

Nevertheless, sufficient freedom in the alteration of expression being granted, they readily come under our formulæ.

36. I have as yet introduced few examples of negative propositions, because, though they may be treated in their purely negative form, it is usually more convenient to convert them into affirmative propositions. This conversion is effected by the use of *negative terms*, a practice not unknown to the old logic, but not nearly so much employed as it should have been. Thus the negative proposition,

A *is not* B

or $A \backsim B$,

is much more conveniently represented by the affirmative proposition or equation,

$$A = Ab,$$

in which we denote by b the quality or fact of differing from B. The term b is in fact the name of the whole class of things, or any of them, which differ from B, so that it is a matter of indifference whether we say that A differs from B and is excluded from the class B, or that it agrees with b and is included in the class b. There are advantages, however, in employing the affirmative form.[1]

[1] It may seem to the reader contradictory to condemn the negative proposition as sterile and incapable of affording inferences, and shortly afterwards to convert it into an affirmative proposition of fertile or inferential power. But on trial it will be found that the

37. The syllogism *Celarent* is now very readily brought under our single mode of inference. Take the example

(1) *All metals are elements,*
(2) *No element can be transmuted,*
(3) *No metal, therefore, can be transmuted.*

To represent this symbolically, let

A = *metal*,
B = *element*,
C = *transmutable*,
c = *untransmutable.*

Then the premises are

$$(1)\quad A = AB$$
$$(2)\quad B = Bc.$$

propositions thus obtained yield no conclusions inconsistent with my theory. Thus the negative premises,

A *is not* B,
B *is not* C,

yield the affirmative propositions or equations,

$$A = Ab$$
$$B = Bc.$$

And when these premises are tested, whether on the logical slate, abacus, or logical machine referred to in a later page, they are found to give no conclusion concerning the relation of A and C. The description of A is given in the equation,

$$A = AbC \cdot|\cdot Abc,$$

from which it appears that A may indifferently occur with or without C.

Substituting in (1) by means of (2) we get

$$(3) \quad A = ABc;$$

or, *metals = metals, elementary, untransmutable.*

38. Before proceeding to other examples of the syllogism, it will be well to point out that every affirmative proposition or equation gives rise to a corresponding equation between the negatives of the terms of the original. The general proposition of the form

$$A = B,$$

treated by the fundamental principle of reasoning, informs us that in whatever relation anything stands to A, in the same relation it stands to B, and similarly *vice versâ*. Hence, whatever differs from A differs from B, and whatever differs from B differs from A. Now the term b denotes what differs from B, and a denotes what differs from A; so that from the single original proposition we may draw the two propositions—

$$a = ab$$
$$b = ab.$$

But as these propositions have an identical second member, we can make a substitution, getting

$$a = b.$$

This form of inference, though little if at all noticed in the traditional logic, is of frequent occurrence and of great importance. It may be illustrated by such examples as

happy = *contented*,
hence *unhappy* = *not-contented*;

or again,

triangle = *three-sided rectilinear figure*,

hence *what is not a triangle* = *what is not a three-sided rectilinear figure.*

The new proposition thus obtained may be called the *contrapositive* of the one from which it was derived, this being a name long applied to a similar inference from the old form of proposition.

39. Though the details of this new view of logic may not yet have been perfectly worked out, much evidence of the truth of the system is to be found in the simplicity, variety, and universality of the forms of reasoning which can be evolved out of a single law of thought,—the *similar treatment of similars.*

The old system of the syllogism, indeed, was nominally founded on a single, or rather double, axiom or law, the *dicta* of Aristotle, but the mode in which these *dicta* led to conclusions was so far from being evident, that the logical student could not be trusted with their use. A cumbrous system of six, eight, or more rules of the syllogism was therefore made out, in order that the validity of an argument might thereby be tested ; but, as even then the task was no easy or self-evident one, logicians formed a complete list of the limited number of forms obeying these artificial rules, and composed a curious set of mnemonic lines by which they might be committed to memory. These lines, the venerable *Barbara celarent*, &c., were no doubt creditable to the ingenuity of men who lived in the darkest ages of science, but they are altogether an anachronism in the present age. What should we think now of a writer of mathematical textbooks, who should select about a score of the commonest forms of mathematical equations, and invent a mnemonic by which both the forms of the equations and the steps of their solution might be carried in the memory? Instead of such an absurdity, we now find, even in purely elementary books, that the general principles and processes are impressed upon the pupil's mind, and he is taught by practice to apply these principles to indefinitely numerous and varied examples. So it should be in logic ; the logical student need only acquire a thorough comprehension of the principle of substitution and the very primary laws of thought, in order to be able to analyse any argument and develop any form of reasoning which is possible. No subsidiary rules are needful, and no mnemonics would be otherwise than a hindrance.

40. I have yet a striking proof to offer of the truth of the views I am putting forward ; for when once we lay down the primary laws of thought, and employ them by means of the principle of substitution, we find that an unlimited system of forms of indirect reasoning develops itself spontaneously. Of this indirect system there is hardly a vestige in the old logic, nor does any writer previous to Dr. Boole appear to have conceived its existence, though it must no doubt have been often unconsciously employed in particular cases. This indirect or negative method is closely analogous to the *indirect proof*, or *reductio ad absurdum*, so frequently used by Euclid and other mathematicians, and a similar method is employed by the old logicians in the treatment of the syllogisms called *Baroko* and *Bokardo*, by the *reductio ad impossibile.* But the incidental examples of the indirect logical method which can be found in any book previous to the "Mathematical Analysis of Logic" of Dr. Boole give no idea whatever of its all-commanding

power; for it is not only capable of proving all the results obtained already by a direct method of inference, but it gives an unlimited number of other inferences which could not be arrived at in any other than a negative or indirect manner. In a previous little work[1] I have given a complete, but somewhat tedious, demonstration of the nature and results of this method, freed from the difficulties and occasional errors in which Dr. Boole left it involved. I will now give a brief outline of its principles.

41. The indirect method is founded upon the law of the substitution of similars as applied with the aid of the fundamental laws of thought. These laws are not to be found in most textbooks of logic, but yet they are necessarily the basis of all reasoning, since they enounce the very nature of similarity or identity. Their existence is assumed or implied, therefore, in the complicated rules of the syllogism, whereas my system is founded upon an immediate application of the laws themselves. The first of these laws, which I have already referred to in an earlier part of this tract (p. 35), is

[1] "Pure Logic, or the Logic of Quality apart from Quantity: with Remarks on Boole's System, and on the Relation of Logic and Mathematics." By W. Stanley Jevons, M.A. London: Edward Stanford, 1864.

the LAW OF IDENTITY, that *whatever is, is,* or *a thing is identical with itself;* or, in symbols,

$$A = A.$$

The second law, THE LAW OF NON-CONTRADICTION, is that *a thing cannot both be and not be,* or that *nothing can combine contradictory attributes;* or, in symbols,

$$Aa = 0,$$

—that is to say, what is both A and not A does not exist, and cannot be conceived.

The third law, that of *excluded middle,* or, as I prefer to call it, the LAW OF DUALITY, asserts the self-evident truth that *a thing either exists or does not exist,* or that *everything either possesses a given attribute or does not possess it.*

Symbolically the law of duality is shown by

$$A = AB \cdot\!|\!\cdot Ab,$$

in which the sign ·|· indicates alternation, and is equivalent to the true meaning of the disjunctive conjunction *or.* Hence the symbols may be interpreted as, A *is either* B *or not* B.

These laws may seem truisms, and they were ridiculed as such by Locke; but, since they describe the very nature of identity in its three aspects, they must be assumed as true, consciously or unconsciously, and if we can build a system of inference upon them, their self-evidence is surely in our favour.

42. The nature of the system will be best learnt from examples, and I will first apply it to several moods of the old syllogism. *Camestres* may thus be proved and illustrated :—

(1) *A sun is self-luminous,*
(2) *A planet is not self-luminous,*
(3) *A planet, therefore, is not a sun.*

Now it is apparent that a planet is either a sun or it is not a sun, by the law of duality. But if it be a sun, it is self-luminous by (1), whereas by (2) it is not self-luminous; it would, if a sun, combine contradictory attributes. By the law of non-contradiction it could not exist, therefore, as a sun, and it consequently is not a sun.

To represent this reasoning in symbols take

A = *sun.*
B = *planet.*
C = *self-luminous.*

Then the premises are

(1) $A = AC$
(2) $B = Bc$

By the law of duality we have

$$B = BA \cdot\!|\!\cdot Ba,$$

and substituting this value in the second side of (2) we have

$$B = BcA \cdot\!|\!\cdot Bca.$$

But for A in the above we may substitute its expression in (1), getting

$$B = BcAC \cdot\!|\!\cdot Bca;$$

and striking out one of these alternatives which is contradictory we finally obtain

$$B = Bca.$$

The meaning of this formula is that *a planet is a planet not self-luminous, and not a sun*, which only differs from the Aristotelian conclusion in being more full and precise.

43. The syllogism *Camenes* may be illustrated by the following example :—

(1) *All monarchs are human beings,*
(2) *No human beings are infallible,*
(3) *No infallible beings, therefore, are monarchs.*

This is proved by considering that every infallible being is either a monarch or not a monarch; but if a monarch, then by (1) he is a human being, and by (2) is not infallible, which is impossible; therefore, no infallible being is a monarch.

Or in symbols, taking

A = *monarch*,
B = *human being*,
C = *infallible being*,

the premises are

$$(1) \quad A = AB$$
$$(2) \quad B = Bc.$$

Now by the law of duality

$$C = aC \cdot\!|\!\cdot AC.$$

Substituting for A its value as derived from both the premises, we have

$$C = aC \cdot\!|\!\cdot ABCc;$$

and, striking out the contradictory term,

$$C = aC.$$

44. By the indirect method we can obtain and prove the truth of the contra-positive of the ordinary proposition A is B, or

$$(1) \quad A = AB.$$

What we require is the description of the term not-B or *b;* and by the law of duality this is, in the first place, either A or not-A:

$$(2) \quad b = Ab \cdot\!|\!\cdot ab.$$

Substituting for A in (2) its value as given in (1) we obtain

$$b = ABb + ab.$$

But the term A B*b* breaks the law of non-contradiction (p. 46), so that we have left only

$$b = ab,$$

or whatever is not-B is also not-A.

Thus, if A = *metal*,
B = *element*,
from the premise

All metals are elements

we conclude *that all substances which are not-elements are not metals;* which is proved at once by the consideration, symbolically expressed above, that if they were metals they would be elements, or *at once elements and not-elements,* which is impossible.

45. It is the peculiar character of this method of indirect inference that it is capable of solving and explaining, in the most complete manner, arguments of any degree of complexity. It furnishes, in fact, a complete solution of the problem first propounded and obscurely solved by Dr. Boole :—

Given any number of propositions involving any number of distinct terms, required the description *of any of those terms or any combination of those terms as expressed in the other terms, under condition of the premises remaining true.*

This method always commences by developing all the possible combinations of the terms involved according to the law of duality. Thus, if there are three terms, represented by A, B, C, then the possible combinations in which A can present itself will not exceed four, as follows:—

$$(1) \quad A = ABC ·|· ABc ·|· AbC ·|· Abc.$$

If we have any premises or statements concerning the nature of A, B, and C, that is, the combinations in which they can present themselves, we proceed to inquire how many of the above combinations are consistent with the premises. Thus, if A is never found with B, but B is always found with C, the two first of the combinations become contradictory, and we have

$$A = AbC ·|· Abc,$$

or, A is never found with B, but may or may not be found with C.

This conclusion may be proved symbolically by expressing the premises thus:—

$$A = Ab$$
$$B = BC,$$

and then substituting the values of A and B wherever they occur on the second side of (1).

46. As a simple example of the process, let us take the following premises, and investigate the consequences which flow from them.[1]

"From A follows B, and from C follows D; but B and D are inconsistent with each other."

The possible combinations in which A, B, C, and D may present themselves are sixteen in number, as follows:—

A B C D	*a* B C D
A B C *d*	*a* B C *d*
A B *c* D	*a* B *c* D
A B *c d*	*a* B *c d*
A *b* C D	*a b* C D
A *b* C *d*	*a b* C *d*
A *b c* D	*a b c* D
A *b c d*	*a b c d*

Each of these combinations is to be compared with the premises in order to ascertain whether it is possible under the condition of those premises. This comparison will really consist in substituting for each letter its description as given in the

[1] See De Morgan's "Formal Logic," p. 123.

premises, which may thus be symbolically expressed :—

$$(1)\quad A = A\,B,$$
$$(2)\quad C = C\,D,$$
$$(3)\quad B = B\,d.$$

The combination $A\,b\,C\,D$ is contradicted by (1) in substituting for A its value; $A\,B\,C\,d$ by (2), $a\,B\,c\,D$ by (3), and so on. There will be found to remain only four possible combinations :—

$$ABcd,$$
$$abCD,$$
$$abcD,$$
$$abcd.$$

Now, if we wish to ascertain the nature of the term A, we learn at once that it can only exist in the presence of B and the absence of both C and D.

We ascertain also that D can only appear in the absence of both A and B, but that C may or may not be present with D. Where D is absent, C must also be absent, and so on.

47. Objections might be raised against this process of indirect inference, that it is a long and tedious one; and so it is, when thus performed. Tedium indeed is no argument against truth; and if, as I confidently assert, this method gives

us the means of solving an infinite number of problems, and arriving at an infinite number of conclusions, which are often demonstrable in no simpler way, and in fact in no other way whatever, no such objections would be of any weight. The fact however is, that almost all the tediousness and liability to mistake may be removed from the process by the use of mechanical aids, which are of several kinds and degrees. While practising myself in the use of the process, I was at once led to the use of the *logical slate*, which consists of a common writing slate, with several series of the combinations of letters engraved upon it, thus :—

A B	A B C	A B C D	A B C D E	A B C D E F
A *b*	A B *c*	A B C *d*	A B C D *e*	A B C D E *f*
a B	A *b* C	A B *c* D	A B C *d* E	A B C D *e* F
a b	A *b c*	A B *c d*	A B C *d e*	A B C D *e f*
	a B C	A *b* C D	A B *c* D E	A B C *d* E F
	a B *c*	...	...	...
	a b C	...	...	...
	a b c	*a b c d*	*a b c d e*	*a b c d e f*

When fully written out, these series consist respectively of 4, 8, 16, 32, and 64 combinations, and that series is chosen for any problem which just affords enough distinct terms. Each combination is then examined in connexion with each of the premises, and the contradictory ones are struck through with the pencil.

48. It soon became apparent, however, that if these combinations, instead of being written in fixed order on a slate, were printed upon light moveable slips of wood, it would become easy by suitable mechanical arrangements to pick out the combinations in convenient classes, so as immensely to abbreviate the labour of comparison with the premises. This idea was carried out in the *logical abacus*, which I constructed several years ago, and have found useful and successful in the lecture-room for exhibiting the complete solution of logical arguments.

49. This *logical abacus* has been exhibited before the members of the Manchester Literary and Philosophical Society, and the following description of it is extracted from the Proceedings of the Society for 3d April, 1866, p. 161.

"The abacus consists of—

"1. An inclined black board, furnished with four ledges, 3 ft. long, placed 9 in. apart.

"2. Series of flat slips of wood, the smallest set four in number, and other sets, 8, 16, and 32 in number, marked with combinations of letters, as follows:—

FIRST SET.

A	A	*a*	*a*
B	*b*	B	*b*

SECOND SET.

A	A	A	A	*a*	*a*	*a*	*a*
B	B	*b*	*b*	B	B	*b*	*b*
C	*c*	C	*c*	C	*c*	C	*c*

"The third and fourth sets exhibit the corresponding combinations of the letters A, B, C, D, *a*, *b*, *c*, *d*, and A, B, C, D, E, *a*, *b*, *c*, *d*, *e*.

"The slips are furnished with little pins, so that, when placed upon the ledges of the board, those marked by any given letter may be readily picked out by means of a straight-edged ruler, and removed to another ledge.

50. "The use of the abacus will be best shown by an example. Take the syllogism in *Barbara*:—

Man is mortal,
Socrates is man,
Therefore Socrates is mortal.

"Let

A = *Socrates,*
B = *man,*
C = *mortal.*

"The corresponding small italic letters then indicate the negatives,

a = *not-Socrates,*
b = *not-man,*
c = *not-mortal,*

and the premises may be stated as

A is B,
B is C.

"Now take the second set of slips containing all the possible combinations of A, B, C, *a*, *b*, *c*, and ascertain which of the combinations are possible under the conditions of the premises.

"Select all the slips marked A; and as all these ought to be B's, select again those which are not-B or *b*, and reject them. Unite the remainder, and, selecting the B's, reject those which are not-C or *c*. There will now remain only four slips or combinations:

A	*a*	*a*	*a*
B	B	*b*	*b*
C	C	C	*c*

"If we require the description of *Socrates*, or A, we take the only combination containing A, and observe that it is joined with C: hence the Aristotelian conclusion, *Socrates is mortal.* We may also get any other possible conclusion. For instance, the class of things *not-man* or *b* is seen from the two last combinations to be always *a* or *not-Socrates*, but either *mortal* or *not-mortal* as the case may be.

51. "Precisely the same obvious system of analysis is applicable to arguments however complicated. As an example, take the premises treated in Boole's 'Laws of Thought,' p. 125.

"'(1.) *Similar figures consist of all whose corresponding angles are equal, and whose corresponding sides are proportional.*

"'(2.) *Triangles whose corresponding angles are equal have their corresponding sides proportional, and* vice versâ.'

"Let

A = *similar*,

B = *triangle*,

C = *having corresponding angles equal*,

D = *having corresponding sides proportional.*

"The premises may then be expressed in Qualitative Logic, as follows:—

$$A = CD,$$
$$BC = BD.$$

"Take the set of 16 slips: out of the A's reject those which are not CD; out of the CD's reject those which are not A; out of the BC's reject those which are not BD; and out of the BD's reject those which are not BC. There will remain only six slips, as follows :—

A	A	*a*	*a*	*a*	*a*
B	*b*	B	*b*	*b*	*b*
C	C	*c*	C	*c*	*c*
D	D	*d*	*d*	D	*d*

"From these we may at once read off all the conclusions laboriously deduced by Boole in his obscure processes. We at once see, for instance, that the class *a*, or '*dissimilar figures*, consist of *all triangles* (B) *which have not their corresponding angles equal* (*c*) *and sides proportional* (*d*), *and of all figures not being triangles* (*b*) *which have either their angles equal* (C) *and sides not proportional* (*d*), *or their corresponding sides proportional* (D) *and angles not equal, or neither their corresponding angles equal nor corresponding sides proportional.*' (Boole, p. 126.)

52. "The selections as made upon the abacus are of course subject to mistake, but only one easy step is required to a logical machine, in which the selections shall be made mechanically and faultlessly by the mere reading down of the premises upon a set of keys, or handles, representing the several positive and negative terms, the copula, conjunctions, and stops of a proposition."

53. In the last paragraph I alluded to a further mechanical contrivance, in which the combination-

slips of the abacus should not require to be moved by hand, but could be placed in proper order by the successive pressure of a series of keys or handles. I have since made a successful working model of this contrivance, which may be considered *a machine capable of reasoning*, or of replacing almost entirely the action of the mind in drawing inferences. When I have an opportunity of describing the details of its construction, I think it will be found to afford a physical proof, apparent to the eyes, of the extreme incompleteness of the Aristotelian logic. Not only are the syllogisms and other old forms of argument capable of being worked upon the machine, but an indefinite number of other forms of reasoning can be represented by the simple regular action of levers and spindles.

54. The most unfortunate feature of the long history of our present traditional logic has been the divorce existing between the logic of the schools and the logic of common life. There has been no apparent connexion whatever between the formal strictness of the syllogistic art and the more loose but useful suggestions of analogy from particulars to particulars. It is owing to this separation, as I apprehend, that a succession of English writers from Locke down to Mr. J. S. Mill have been led to underestimate the value of the syllogism. In

Mr. Mill's system of logic the syllogism occupies a very anomalous position—that of an extraneous form of proof which may be employed when we wish to ensure correctness of inference, but which is useless for the discovery of truth. I believe that the new view of the syllogism which I am now proposing will remedy this lamentable disconnexion of the parts of what should be one most harmonious and consistent whole. There is no subject in which we might expect more perfect unity and system to exist, and more wide-ruling generalizations to be discoverable, than in the science of the laws of thought; and I conceive that a prime object of any logical reform should be to reconcile the strict doctrine with the looser forms of ordinary thought. This reconciliation will really be effected, I believe, by adopting as the fundamental principle the modified axiom of Aristotle which I have called the *substitution of similars.* I hope at some future time to explain fully the results which seem to follow from the principle and the harmony which it creates between the several branches of logical method, and I will only attempt in this tract a few slight illustrations.

55. The most frequent mode of inference in common life is that known as reasoning from analogy or resemblance, by which we argue from any thing

or event we have known to a like thing or event encountered on another occasion. This seems to be Mr. Mill's view of the ordinary process of reasoning, for in discussing the functions and value of the syllogism he says:[1]—"From instances which we have observed, we feel warranted in concluding that what we found true in those instances holds in all similar ones, past, present, and future, however numerous they may be." And again he explains more fully:[2]—"I believe that, in point of fact, when drawing inferences from our personal experience, and not from maxims handed down to us by books or tradition, we much oftener conclude from particulars to particulars directly, than through the intermediate agency of any general proposition. We are constantly reasoning from ourselves to other people, or from one person to another, without giving ourselves the trouble to erect our observations into general maxims of human or external nature. When we conclude that some person will, on some given occasion, feel or act so and so, we sometimes judge from an enlarged consideration of the manner in which human beings in general, or persons of some particular character, are accustomed to feel and act; but much oftener from

[1] "System of Logic," vol. i. p. 210, fifth edition.
[2] Ibid. p. 212.

merely recollecting the feelings and conduct of the same person in some previous instance, or from considering how we should feel or act ourselves. It is not only the village matron who, when called to a consultation upon the case of a neighbour's child, pronounces on the evil and its remedy simply on the recollection and authority of what she accounts the similar case of her Lucy."

56. Mr. Mill expresses as clearly as it is well possible that we argue in common life, as he thinks, not by the syllogism, but directly from instance to instance by the similarity observed between the instances. But this argument from similars to similars is the identical process which I have called the substitution of similars, and which I have shown to be capable of explaining the syllogism itself, and much more. In fact we find Mr. Mill enunciating this principle himself in another chapter, where he is treating of argument from analogy or resemblance. After noticing the stricter meaning of analogy as a *resemblance of relations*, he continues:[1]—

"It is on the whole more usual, however, to extend the name of analogical evidence to arguments from any sort of resemblance, provided they do not amount to a complete induction: without

[1] "System of Logic," vol. ii. p. 86.

peculiarly distinguishing resemblance of relations. Analogical reasoning, in this sense, may be reduced to the following formula;—Two things resemble each other in one or more respects; a certain proposition is true of the one; therefore it is true of the other. But we have nothing here by which to discriminate analogy from induction, since this type will serve for all reasoning from experience. In the most rigid induction, equally with the faintest analogy, we conclude, because A resembles B in one or more properties, that it does so in a certain other property."

57. If this be, as Mr. Mill so clearly states, the type of all reasoning from experience, it follows that the principle of inductive reasoning is actually identical with that which I have shown to be sufficient to explain the forms of deductive reasoning. The only difference I apprehend is, that in deductive reasoning we know or assume a similarity or identity to be certainly known, and the conclusion from it is therefore equally certain; but in inductive arguments from one instance to another we never can be sure that the similarity of the instance is so deep and perfect as to warrant our substitution of one for the other. Hence the conclusion is never certain, and possesses only a degree of probability, greater or less according to the circumstances of

the case; and the theory of probabilities is our only resource for ascertaining this degree of probability, if ascertainable at all.

58. It is instructive to contrast mathematical induction with the induction as employed in the experimental sciences. The process by which we arrive at a general proof of a problem in Euclid's "Elements of Geometry" is really a process of generalization presenting a striking illustration of our principle. To prove that the square on the hypothenuse of a right-angled triangle is equal to the sum of the squares on the sides containing the right angle, Euclid takes only a single example of such a triangle, and proves this to be true. He then trusts to the reader perceiving of his own accord that all other right-angled triangles resemble the one accidentally adopted in the points material to the proof, so that any one right-angled triangle may be indifferently substituted for any other. Here the process from one case to another is certain, because we *know* that one case exactly resembles another. In physical science it is not so, and the distinction has been expressed, as it seems to me, with admirable insight by Professor Bowen in his well-known "Treatise on Logic, or the Laws of Pure Thought."[1] He says of mathe-

[1] Cambridge, United States, 1866, p. 354.

matical figures:—"The same measure of certainty which the student of nature obtains by intuition respecting a single real object, the mathematician acquires respecting a whole class of imaginary objects, because the latter has the assurance, which the former can never attain, that the single object which he is contemplating in thought is a *perfect representative* of its whole class: he has this assurance, *because the whole class exists only in thought*, and are therefore all actually before him, or present to consciousness. For example: this bit of iron, I find by direct observation, melts at a certain temperature; but it may well happen that another piece of iron, quite similar to it in external appearance, may be fusible only at a much higher temperature, owing to the unsuspected presence with it of a little more or a little less carbon in composition. But if the angles at the base of this triangle are equal to each other, I know that a corresponding equality must exist in the case of every other figure which conforms to the definition of an isosceles triangle; for that definition excludes every disturbing element. The conclusion in this latter case, then, is universal, while in the former it can be only singular or particular."

This passage perfectly supports my view that all reasoning consists in taking one thing as a *representative*, that is to say, as a *substitute*, for another, and the only difficulty is to estimate rightly the degree of certainty, or of mere probability, with which we make the substitution. The forms and methods of induction and the calculus of probabilities are necessary to guide us rightly in this; but to show that the principle of substitution is really present and active throughout inductive logic is more than I can undertake to show in this tract, although I believe it to be so.

59. Though I have pointed out how consistent are many of Mr. Mill's expressions with the view of logic here put forward, and how clearly in one place he describes the principle of substitution itself, I cannot but feel that his system is full of anomalies and breaches of consistency. These arise, I believe, from the profound error into which he has fallen, of undervaluing the logical discovery of the quantification of the predicate. Of Sir W. Hamilton's views he says:[1]—"If I do not consider the doctrine of the quantification of the predicate a valuable accession to the art of logic, it is only because I consider the ordinary rules of the syllogism to be an adequate test, and perfectly sufficient to exclude all inferences which do not follow from the premises. Considered, however,

[1] "System of Logic," fifth ed. vol. i. p. 196 note.

as a contribution to the *science* of logic, that is, to the analysis of the mental processes concerned in reasoning, the new doctrine appears to me, I confess, not merely superfluous, but erroneous; since the form in which it clothes the propositions does not, like the ordinary form, express what is in the mind of the speaker when he enunciates the proposition. I cannot think Sir William Hamilton right in maintaining that the quantity of the predicate is 'always understood in thought.' It is implied, but is not present in the mind of the person who asserts the proposition." Again, he says of Mr. De Morgan's ingenious logical discoveries, to which every logical writer is so deeply indebted:—" Since it is undeniable that inferences, in the cases examined by Mr. De Morgan, can legitimately be drawn, and that the ordinary theory takes no account of them, I will not say that it was not worth while to show in detail how these also could be reduced to formulæ as rigorous as those of Aristotle. What Mr. De Morgan has done was worth doing once (perhaps more than once), as a school exercise; but I question if its results are worth studying and mastering for any practical purpose." In these and many other places Mr. Mill shows a lamentable want of power of appreciating the principles involved in the quantification of the predicate. As regards the most original discoveries of Dr. Boole, there is not, so far as I have been able to discover, a single word in Mr. Mill's edition of his "Logic" published in 1862, to indicate that he was conscious of the publication of Mr. Boole's "Mathematical Analysis" in 1849, and of his great work, "The Laws of Thought," in 1854. Although accounted a disciple and potent supporter of the doctrines of Jeremy Bentham, he appears unaware that the doctrine of the quantification of the predicate is traceable to his great master, or at all events to the work of a nephew founded upon the manuscripts of Bentham.

60. I ought not to omit to notice that Dr. Thomson substantially adopts the principle of substitution in treating of what he calls the *syllogism of analogy*. He states the canon in the following manner :[1]—"The same attributes may be assigned to distinct but similar things, provided they can be shown to accompany the points of resemblance in the things and not the points of difference." This means that one thing may be substituted for another like to it, provided that their likeness really extends to the point in question, which can often only be ascertained with more or less probability

[1] "Laws of Thought," fifth ed. p. 251.

by inductive inquiry. He adds, that the expression of the agreement must consist of a qualified judgment of identity, or a proposition of the form U, by which symbol he indicates a proposition denoted in this tract by the expression A = B. This exactly agrees with my view of the matter.

61. The principle of substitution of similars seems to throw a clear light upon the infinite importance of classification. For classification consists in arranging things, either in the mind or in cabinets of specimens, according to their resemblances, and the best classification is that which exhibits the most numerous and extensive resemblances. The purpose and effect of such arrangement evidently is, that we may apply to all members of a class whatever we know of any member, *so far as it is a member.* All the members of a class are mutual substitutes for each other as regards their common characteristics, and a natural classification is that which gives the greatest probability that characters as yet unexamined will exhibit agreements corresponding to those which are examined. Classification is thus the infinitely useful mode of multiplying knowledge, by rendering knowledge of particulars as general as possible, or of indicating the greatest possible number of substitutions which may give rise to acts of inference.

62. I need hardly point out that not only in our reasonings, but in our acts in common life, we observe the principle of similarity. Any new kind of action or work is performed with doubt and difficulty, because we have no knowledge derived from a similar case to guide us. But no sooner has the work been performed once or twice with success than much of the difficulty vanishes, because we have acquired all the knowledge which will guide us in similar cases. Mankind, too, have an instinctive respect for precedents, feeling that, however we act in one particular case, we ought to act similarly in all similar cases, until strong reason or necessity obliges us to make a new precedent. The whole practice of law in English courts, if not in all others, consists in deciding all new causes according to the rule established in the most nearly similar former causes, provided any can be found sufficiently similar. No ruler, too, but an absolute tyrant can perform any public act but under the responsibility of being called upon to perform a similar act, or make a similar concession, in similar circumstances.

63. At the present day, for instance, the Government is called upon to take charge of the telegraphs and railways, because great benefit has resulted from their management of the post-office. It is

implied in this demand that the telegraphs and railways resemble or are even identical with the post-office, in those points which render Government control beneficial, and the public mind inevitably leaps from one thing to anything which appears similar. The whole question turns, of course, upon the degree and particular nature of the similarity. Granting that there is sufficient analogy between the telegraph and the post-office to render the Government purchase of the former desirable, we must not favour so gigantic an enterprise as the purchase of the railways until it is clearly made out that their successful management depends upon principles of economy exactly similar to the case of the post-office.

64. The great immediate question of the day is the Disestablishment of the Irish Church. The opponents of the measure argue against it by the indirect argument, that if the Irish Church ought to be disestablished, so ought the English Church; but as this ought *not*, neither ought the Irish Church. They are answered by pointing out that the Irish and English Churches are not *similarly situated;* the one possesses the sympathy of the great body of the people, and the other does not. This is an all-important point, which prevents our applying to one what we apply to the other. But

on either side it is unconsciously, if not expressly, allowed that similars must be similarly treated. Almost the whole of our difficulties in the government of Ireland arise from the different national characters of the Irish and English, which renders laws and institutions suited to the one inapplicable to the other. Yet such is the tendency of indiscriminating public opinion to run in the groove of similarity, that it requires a bold legislator to repeal laws for Ireland which it is not intended or desired to repeal for England.

65. Before closing, I should notice that at some period in the obscurity of the Middle Ages an attempt seems to have been made to assimilate in some degree the logical and mathematical sciences, by inventing a logical canon analogous to the first axiom of Euclid. Between the *dictum de omni et nullo* of Aristotle, which had so long been esteemed the primary and perfect rule of reason, and the axiom concerning equal quantities, there was no apparent similarity. Logicians accordingly adopted a syllogistic canon which seems closely analogous to the axiom in question, and which was thus stated in the textbook of Aldrich :—

Quæ conveniunt in uno aliquo eodemque tertio, ea conveniunt inter se.

This was supplemented by a corresponding canon concerning terms which disagree :—

Quorum unum convenit, alterum differt uni et eidem tertio, ea differunt inter se.

The excessive subtlety of logical writers of past centuries even led them to invent six separate canons to express the principle which seems to be sufficiently embodied in our one rule. Whately considers two of these canons to be a sufficient rule of reason, which he thus translates :—

If two terms agree with one and the same third, they agree with each other; and

If one term agrees, and another disagrees, with one and the same third, these two disagree with each other.

"No categorical syllogism can be faulty which does not violate these canons: none correct which does."[1]

66. Though Wallis spoke of these canons as an innovation in his day, Mr. Mansel has traced them back to the time of Rodolphus Agricola.[2] They were well known to Lord Bacon, for he appears to

[1] Whately, "Elements of Logic," Book ii. chap. iii. sec. 2.

[2] Born 1442; his logical work, "De Inventione Dialecticæ," was printed at Louvain in 1516.

have been greatly struck with the apparent analogy between these canons and the axioms of mathematicians, and he introduces it as an instance of conformity or analogy in his "Novum Organum"[1] in the following passage:—

Postulatum mathematicum, ut quæ eidem tertio æqualia sunt, etiam inter se sint æqualia, conforme est cum fabrica syllogismi in logica: qui unit ea quæ conveniunt in medio.

67. It is a truly curious fact in the history of Logic, that these canons should so long have been adopted, and yet that the only form of proposition to which they correctly apply should have been almost wholly ignored until the present century. It is only when applied to propositions of the form A = B that these canons prevent us from falling into error, but when used with the propositions of the old Aristotelian system they allow the free commission of fallacies of undistributed middle. It has been well pointed out by Mr. Mansel,[2] that "these canons are an attempt to reduce all the three figures of syllogism directly to a single principle; the *dictum de omni et nullo* of Aristotle, which was universally adopted by the scholastic logicians,

[1] Book ii. Aphorism 27.
[2] Artis Logicæ Rudimenta, p. 65.

being directly applicable to the first figure only. This reduction, so long as the predicate of propositions has no expressed quantity, is illegitimate; the terms not being equal, but contained one within another, as is denoted by the names *major* and *minor*. Hence, as applied to the first figure, the word *conveniunt* has to express, at one and the same time, the relation of a greater to a less, and of a less to a greater,—of a predicate to a subject, and of a subject to a predicate."

Thus in the syllogism of the mood *Barbara*,

Metals are elements,
Iron is a metal,
Iron, therefore, is an element,

the terms *elements* and *iron* are both said to agree with *metals*, the third common term, although *elements* is a wider term, and *iron* a much narrower one, than *metals*. Nothing can be more unscientific and fallacious than such an application of the same word in two distinct meanings. And if we avoid this fallacy by taking the meaning of the word *agreement* in the same manner in each premise, we fall into the fallacy of undistributed middle. Thus

Metals are elements,
Oxygen is an element,
Oxygen, therefore, is a metal,

would conform precisely to the canon, because *oxygen* agrees with *element* exactly in the same sense in which *metals* agree with *elements*, and yet the result is an untrue and fallacious conclusion. Doubtless this absurdity may be explained away by pointing out that *metals* and *oxygen* do not really agree with the same part of the class *elements*, so that there is no really common third term; but the so-called supreme canon of syllogism is unable to indicate when this is the case and when it is not. Other rules have to be assumed in order to overrule the supreme rule, and these involve the principle of quantification, because they depend upon the inquiry as to what parts of the middle term are *identical* respectively with the major and minor terms. Yet for centuries logicians failed to acknowledge that identity is at the bottom of the question.

68. To sum up, we may say that the logicians attempted to reconcile logical with mathematical forms of reasoning, by assuming a canon which is true when applied to quantified propositions; but, as they applied the canon to unquantified propositions, they failed in producing anything but a fallacious appearance of conformity. In the present century logicians have abundantly recognised the importance of quantifying the proposition; but they have either adhered to the old

form of canon, or they have omitted altogether to inquire into the axioms which must be adopted as the groundwork of the reasoning process. I have long felt persuaded of the truth enounced by that most clear thinker, Condillac—that "équations, propositions, jugemens, sont au fond la même chose, et que, par conséquent, on raisonne de la même manière dans toutes les sciences;"[1] and it has been my endeavour at once to transform the proposition into the equation, and to employ it with an axiom of adequate simplicity and generality, not spoiling good new material with old tools.

69. I write this tract under the discouraging feeling that the public is little inclined to favour or to inquire into the value of anything of an abstract nature. There are numberless scientific journals and many learned societies, and they readily welcome the minutest details concerning a rare mineral, or an undescribed species, the newest scientific toy, or the latest observations concerning a change in the weather. All these things are in public favour because they come under the head of physical science. Mathematicians, again, are in favour because they help the physical philosophers: accordingly the most incomprehensible specula-

[1] La Logique; Œuvres de Condillac, vol. xxii. p. 173.

tions concerning a quintic, or a resolvent, or a new theory of groups, are readily (and deservedly) printed, although not a score of men in England can understand them. But Logic is under the *ban* of metaphysics. It is falsely supposed to lead to no *useful works*—to be mere speculation; and, accordingly, there is no journal, and no society whatever, devoted to its study. Hardly can a paper on a logical subject be edged into the Proceedings of any learned society except under false pretences. This state of things is doubtless due to an excessive reaction against the former pre-eminence of logical studies. Bacon, in protesting against the absurdities of the scholastic logicians, and the deference paid to an ancient author, placed himself at the head of this reaction. Were he living now, he would probably see that the slow pendulum of public opinion has swung to the opposite extreme, and would employ his great intellect in showing how absurd it is to cultivate the branches of the tree of knowledge, and neglect the root—which root is undoubtedly to be found in a true comprehension of logical method.

APPENDIX.

DESCRIPTION OF THE LOGICAL ABACUS.

ALTHOUGH a brief account of the abacus is given in the text (p. 55), it seems desirable to add a more minute description, which, in connexion with the drawings placed in front of the title-page, will enable copies of the abacus to be made with ease. The contrivance is of so simple a character, that an instrument-maker, or even an ordinary cabinet-maker, would probably be able to construct it from the figures and description.

The abacus consists, in the first place, of an ordinary black board of deal wood, such as is used in schools or lecture-rooms. This board should be about $3\frac{1}{2}$ feet square, and must have four ledges (1, 1) of wood, about 1 inch deep and $\frac{1}{3}$ inch thick, fixed across it at equal and parallel distances, a space of about 15 inches being left at the upper part of the board. The ledges may be made to extend quite across the width of the board, and they should be painted, like the board, of a dull black colour. When in use, the board is supported on a suitable stand in a slightly inclined position, as shown by the side view (6) in the figure, so that the slips of wood placed upon the ledges, as at (6, 7) will stand securely.

It is convenient to have altogether four sets of lettered slips, namely :—

4 of the size shown at	(8)	...	3¼ inches long.
8 „ „	(10)	...	4½ „
16 „ „	(12)	...	5½ „
32 „ „	(14) and (16)	—	6½ „

At (9, 11, 13, 15, and 17) are shown side views of the same slips. They are made of the best baywood, ⅛ inch thick, 1 inch broad, and of the lengths stated above, so as to give a surface of one square inch for each letter. Each wooden slip is marked with a different combination of letters, printed upon white paper and pasted on the face of the slip.[1] The nature of the combinations will be readily gathered from pp. 52, 54, and 56, and a set of sixteen of the slips is shown in the figure at (7), resting upon one of the ledges in the usual manner.

In the face of each wooden slip are fixed pins of thin brass or steel wire, projecting from the wood about ⅙ inch in an inclined direction. Every slip has a pin near to its upper end, as at (18), but the positions of the other pins are varied according to the combination of letters represented on the slip. Each large capital or positive letter is furnished with a pin in the upper part of the space allotted to it, as at (19, 20, 21, &c.), while each small or negative letter has a pin in the lower part of its space, as at (22, 23, 24, &c.). At the lower end of each

[1] I shall be happy to send a set of the printed letters to any person who may desire to have an abacus constructed.

slip and at the front is fixed, as at (9, 11, 13, 15, 17), a thick square piece of sheet lead, weighing from ½ oz. to 1 oz., so adjusted that each slip will hang in stable equilibrium and in an upright position when lifted by any of the pins. The lead may be covered at the front side with white paper.

The only other requisite is a flat straight-edge or ruler of hard wood about 16 inches long, 1½ inch wide, and ⅙ inch thick. It is shown in the figure at (2), and an enlarged section at (3), where the sharp edge will be seen to be strengthened with a slip of brass plate (4). It will be desirable to have a box made to hold all the slips in proper order, arranged in trays, so that any set may readily be taken out by the aid of the straight-edge, inserted under the row of top pins.

In using the abacus, one or other of the series of slips is taken out, according as the logical problem to be solved contains more or less terms. If there be only two terms, the set of four is used; if three, the set of eight; if four, the set of sixteen; and if five, the set of thirty-two slips. Thus the syllogism *Barbara* would require the set of eight slips (see p. 56, &c.). These must be set side by side upon the topmost ledge, as at (6): the order in which they are placed is not of any essential importance, but it is generally convenient for the sake of clearness that every positive combination should be placed on the left of the corresponding negative, and that the order shown at (7), and at pp. 52, 54, and 56, should be as much as possible maintained. When a series of the slips is resting on one of the ledges, it is evident that we may separate out those marked with A or any capital letter, by inserting the straight edge horizontally beneath the proper row of pins, and then raising the slips and removing them to another ledge. The corresponding negative slips will be left where they were, owing to the absence of pins at the point where the straight-edge is placed. We have always the option, too, of removing either the A's or the *a*'s, the B's or the *b*'s, and so on. Successive movements will enable us to select any class or group out of the series: thus, if we took the series of sixteen, and removed first the *a*'s and then the *b*'s, we should have left the class of A B's, four in number. Dr. Boole based his logical notation upon the successive selection of classes, and it is this operation of thought which is represented in a concrete manner upon the abacus.

The examples given in the text (pp. 56—59) will partly serve to illustrate the use of the abacus, but I will minutely describe one more instance. Let the premises of a problem be—

(α) A is either B or C; but
(β) B is D; and
(γ) C is D.

Let it be required to answer any question concerning the character of the things A, B, C, D, under the above conditions.

(1) Take the set of sixteen slips and place them on the topmost or first ledge of the board.
(2) Remove the A's to the second ledge.
(3) Out of the A's, remove the B's back to the first ledge.

(4) Out of what remain, remove the C's back to the first ledge.

(5) What still remain are combinations contradicted by the first premise (α), and they are to be removed to the lowest ledge, and left there.

(6) The others having been joined together again on the first ledge, remove the B's to the second ledge.

(7) Out of the B's, remove the D's to the first ledge again, and

(8) Reject to the lowest ledge the B's which are not-D's, as contradictory of (β).

(9) Similarly, in treating (γ), remove the C's to the second ledge, return those which are D's, and reject the C's which are not-D's to the lowest ledge.

The combinations which have escaped rejection are all which are possible under the conditions (α) (β) and (γ), and they will be found to be the following :—

A	A	A	*a*	*a*	*a*	*a*	*a*
B	B	*b*	B	B	*b*	*b*	*b*
C	*c*	C	C	*c*	C	*c*	*c*
D	D	D	D	D	D	D	*d*

To obtain the description of any class, we have now only to pick out that class by the straight-edge, and observe their nature. Thus, when the A's are picked out, we find that they always bring D with them; that is to say, all the A's are D's; this being the principal result of the problem. But we may also select any other class for examination. Thus the *d*'s are represented by only one combination, which shows that what is not-D is neither C, B, nor A.

Even when the common conclusion of an argument is self-evident, it will be found instructive to work it upon the abacus, because the whole character of the argument and the conditions of the subject are then exhibited to the eye in the clearest manner; and while the abacus gives all conclusions which can be obtained in any other way, it often gives negative conclusions which cannot be detected or proved but by the indirect method (see p. 44). It also solves with certainty problems of such a degree of complexity that the mind could not comprehend them without some mechanical aid. In my previous little work on Pure Logic (see above, § 40, p. 45) I have given a number of examples, the working of which may be tested on the abacus, and other examples are to be found in Dr. Boole's "Laws of Thought."

THE END.

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