SULLIVAN'S

NEW HYDRAULICS

Consisting of

New Hydraulic Formulas

and

The Rational Law of Variation of Coefficients

Flow and Resistance to Flow in all Classes of Rivers, Canals, Flumes, Aqueducts, Sewers, Pipes, Fire Hose. Hydraulic Giants, Power Mains, Nozzles, Reducers, etc. with Extensive Tables and Data of Cost of Pipes and Trenching and Pipe Line Construction.

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SULLIVAN'S NEW HYDRAULICS.

ERRATA.

Page 36—Under table of channels, add the following:

Divisor : quotient :: Dividend : quotient squared.

$P : r :: a : r^2$

or $r : r^2 :: p : a$, in any possible case,

Page 40—9th line from bottom. "If the value of the coefficient etc." This should be corrected so as to read: As the loss of effective head or slope is inversely as $\sqrt{d^3}$ or $\sqrt{r^3}$ for any constant head,it is evident that a change in the value of d or r cannot affect the value of the coefficient n, for as the loss of head per foot length, S'', decreases directly as $\sqrt{d^3}$ or $\sqrt{r^3}$ increases, the effective head or slope will increase as $\sqrt{d^3}$ or $\sqrt{r^3}$ and this will result in a like increase of v^2. As n is the ratio of $\frac{S''\sqrt{d^3}}{v^2}$, and as S'' varies *inversely* with $\sqrt{d^3}$, it is evident that v^2 varies *directly* with $\sqrt{d^3}$ or $\sqrt{r^3}$. Hence the ratio of $\frac{S''\sqrt{d^3}}{v^2}=n$, will be constant for all diameters and all slopes and velocities, and will not be affected by anything except a change in roughness of perimeter. A similar correction should be applied to similar errors occurring from page 40 to page 51. See for a general correction of such errors, pages 237 to 241.

Page 42— $\sqrt{\frac{1}{m}}=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$ *should be* $C=\sqrt{\frac{1}{m}}$

or $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$

Page 42—$v=C\sqrt[4]{r^3\sqrt{S}}$ *should be* $v=C\sqrt[4]{r^3}\times\sqrt{S}$

Page 42—9th line from bottom, "Varies with $\sqrt[4]{r^3}$" should be

"Varies *inversely* with $\sqrt[4]{r^3}$. But see pages 237 241.

Page 45—18th line from top. "His n might be made & c" *should be* "His C might be made to vary &c.

Page 46—Bottom line. v^3 *should be* v^2.

Page 48—9th line from top. $S\sqrt{r^2}$ *should be* $S\sqrt{r^3}$.

Page 52—Equation (18) should be $m = \frac{S' + Sv}{v^2} \times \sqrt{r^3}$

Page 85—River Elbe. C=49.80 *should be* C=32.51, and C=52 *should be* C=38.40.

Page 86—Top line. 1885 *should be* 1855.

Page 109, 110—Remark. Add the following:—As large cast iron pipes are full of swellings and contractions, a 48 inch pipe is not really 48 inches diameter. As the effective value of any constant head or slope varies directly with the *actual* value of $\sqrt{d^3}$, if we credit the pipe as being 48 inches, when in fact it is not, the result is to make the value of $S\sqrt{d^3}$, apparently too large for the corresponding value of v^2. Hence m will be too large or C will be too small for a pipe which is really 48 inches diameter throughout. The irregularity of diameter in *large* cast iron pipes reduces the value of C by from 3 to 5 per cent for diameters greater than 36 inches. It is not the fault of the formula, nor is it a peculiarity in the law of flow. It is simply the fault in casting pipes of large diameters.

Page 114—4th line from bottom. $AC \times \sqrt[4]{d^3} \frac{q}{\sqrt{s}}$ *should be* $A\ C \times \sqrt[4]{d^3} = \frac{q}{\sqrt{s}}$

Page 149—$d = \sqrt[11]{\frac{0000004205 \times q^4}{S^3}}\ 2632\sqrt[11]{\frac{q^4}{S^2}}$ *should be*

$$d = \sqrt[11]{\frac{.0000004205\ q^4}{S^3}} = .2632\sqrt[11]{\frac{q^4}{S^3}}$$

PREFACE

HYDRAULICIANS and engineers have long been aware that there is some element or law governing the flow and resistance to flow of water which is not provided for in any of the formulas presented up to this time. This is made evident by the fact that the results as computed by formula do not agree with actual results, and by the further fact that no two formulas will give the same result for like conditions. Nearly all writers give one theory of flow and resistance to flow in pipes and closed channels, and an entirely different theory and formula for flow in open channels. The usual formulas for flow in pipes give results too high for all diameters smaller than about fourteen inches, and too low for all greater diameters.

The loss of head or pressure by resistance as computed by the ordinary formulas is much too small in small pipes and greatly in excess of the truth in large pipes. The reason of these erroneous results is that the coefficients were determined for pipes of medium diameter and do not vary correctly so as to meet the requirements of varying diameters. Hence the greater the diameter varies from the medium, either below or above, the greater will be the error in the computed result. The usual formulas for flow in open channels are equally faulty, but not in the same way.

The acknowledged fault in all the formulas so far presented is due to the failure of hydraulicians to discover the rational law of variation of the coefficient. This law has been sought in vain since the beginning of the sixteenth century, at which time Galileo discovered the law of gravity and undertook to formulate rules for flow in rivers. The failure to discover the true law of variation of the coefficient has been generally conceded by all, and the possibility of its discovery has been denied by many. Ganguillet and Kutter observe that "more than a century ago, Michelotti and Bossut established the true principle that the formulæ for the movement of water must be ascertained from the results of observation, and not by abstract reasoning." In the Translator's preface (Hering & Trautwine) to the work of Ganguillet and Kutter the following observation occurs:

"As $V=C\sqrt{RS}$ will most likely remain the fundamental expression for such formulæ, the attention of hydraulicians will be turned chiefly to the more accurate determination of the variable coefficient C. A number of authors have endeavored to establish laws for its variation, and among them Ganguillet and Kutter appear so far to have been the most successful." Ganguillet and Kutter, however, do not claim to have discovered the true law of variation, as many unthinking persons have supposed, but on the contrary they announce the belief that it cannot be discovered by abstract reasoning, and in default of its discovery they propose a formula which is entirely empirical. They observe (page 105): "The formula (Kutter's) rests only upon actual guagings. * * * Be-an empirical formula, it is confined to the limits occurring in nature and makes no claim whatever to absolute perfection. In spite of the large number of available guagings, it cannot be denied that our knowledge of the elements and laws of the motion of water still need extension and correction."

Webster's definition of the word empirical is, "used and applied without science." The Ganguillet and Kutter formula does not claim to be scientific or rational, and yet it is a considerable improvement on some of the older formulas, but is quite complicated and not simple and easy of application. In the search for the true law of variation of the coefficient the greatest mistake has been made in assuming that $V= C\sqrt{RS}$ is the fundamental expression for the formula. The $\sqrt{S}$ is the factor which expresses the unimpeded and constant effect of gravity, while R is an expression for a factor which modifies the effect of gravity. The constant effect of gravity should be expressed separately and should not be confused in the same expression with other factors which are variable and which impede or modify the effect of gravity. All the variable factors should be included in the coefficient formula for C, and then all elements which affect or modify the law of gravity will be included in the value of C, and the fundamental form of the formula then becomes simply $V = C\sqrt{S}$.

This is evidently true, because the value of R or D has no connection with the law of gravity, which is constant. The value of R or D has to do directly with the law of resistance, and as R or D varies the resistance to flow varies. C is supposed to include this resistance, and hence the value of R or D should be included in the formula for C. If there were no resistance to flow whatever, then the velocity would be directly as $\sqrt{S}$ regardless of the value of R or D. But as the resistance to flow does vary with the value of R or D, it is evident that C must vary with R or D, and if we write $V = C\sqrt{S}$ simply, we thereby have all modifying factors included in the value of C, while the constant law of gravity is expressed by $\sqrt{S}$. Thus we prevent confusing the opposing laws of gravity and of resistance in one combined factor, and clear the way for ascertaining the true law which governs the variation of the value of C.

By the law of gravity we know that if there were no resistance whatever the velocity would be equal in all diameters, regardless of dimensions, where the values of $\sqrt{S}$ were equal. But by experiment we find that the smaller the diameter becomes, the smaller the velocity becomes for equal values of $\sqrt{S}$. It is therefore evident that the resistance to flow must vary with some function of the diameter or of the hydraulic mean radius. As the resistance to flow is the variable factor, and is separate and distinct from and directly opposed to, the acceleration of gravity which is a constant, we know that $\sqrt{S}$ has nothing to do with the value or variation of C. We therefore narrow the field of investigation by writing $V = C\sqrt{S}$, and then searching for the law of variation in C, which we know is some function of the diameter or hydraulic radius. After years of diligent experiment and observation the writer discovered that for any constant degree of roughness of wetted perimeter, either in pipes or open channels, the value of C varied as $\sqrt[4]{R^3}$ or as $\sqrt[4]{D^3}$ In other words, if K is a constant which represents the given degree of roughness of perimeter, then $C = K \times \sqrt[4]{R^3}$, and $V = C\sqrt{S}$. But if we confuse the element of resistance, D or R with the element of acceleration $\sqrt{S}$, by writing $V = C\sqrt{RS}$ then we find C varies as $\sqrt[4]{R}$ simply, and if K represents the degree of roughness, and we write $V = C\sqrt{RS}$, then we have $C = K \times \sqrt[4]{R}$ and $V = C\sqrt{RS}$.

As all hydraulic formulas are necessarily based on the laws of gravity and of friction, the correctness of such formulas must depend upon their accordance with these laws. If our present understanding and acceptance of these laws is correct, then any formula which violates either of these laws must necessarily be incorrect. It is the object of this volume to present the rational law of variation of the coefficient in accordance with our present understanding of the laws of gravity and friction. If those laws are yet unknown it must remain for some future investigator to supply a theoretically correct hydraulic formula. The discussion in the following pages is based on the assumption that those laws are correctly known, or nearly so.

In the case of open channels and rivers of irregular cross section, and where the banks alternately diverge and converge, and the perimeter varies in roughness at different depths of flow, the correct application of any formula will be difficult. In sections 13 and 83 methods are pointed out for ascertaining the value of C in such cases.

The law of resistance in nozzles and convergent pipes, as herein stated has been very thoroughly tested and its correctness established by hundreds of experiments. This law will be found of great service to hydraulic miners and firemen and also in determining the coefficient for flow in converging reaches of rivers and other channels.

In the course of experiments of the writer which has extended over a period of six years, it was discovered that the discharges over weirs and through orifices, as computed by the usual formulas, and with the tabulated direct coefficient were frequently erroneous, especially if the weir used did not correspond exactly in breadth and depth with that from which the coefficient was determined. Interpolation for intermediate conditions was certain to result in error. On investigation it was found that the difficulty lay in the fact that the law of variation of the coefficient of contraction has never been discovered. In order to avoid this difficulty until the law of contraction shall be understood, an appendix has been added to this volume in which the difficulties are pointed out and the suggestion made that the position of the weir be reversed in order to prevent contraction from taking place at all. New weir and orifice formulas are proposed and the writer hopes that other experimentalists will perfect the theories there suggested. It is proper that attention should here be called to the fact that our coefficient m or C, as used for determining the flow in pipes includes all resistances to flow, including the resistance to entry into the pipe. No separate provision was made in the formula for the resistance to entry because it is a matter of no practical consequence under ordinary circumstances, or in any case except for high velocities in very short pipes. (See remarks under Group No. 1, § 14.) The writer hopes that the theory of coefficients and the law of their variation as herein presented may contribute something new and valuable to hydraulics as a science.

For each degree of roughness of perimeter there is a unit value of the coefficient from which unit value the coefficient varies as $\sqrt[4]{R^3}$, or as $\sqrt[4]{D^3}$, when $V=C\sqrt{S}$; or C varies as $\sqrt[4]{R}$ or $\sqrt[4]{D}$ if we write $V=C\sqrt{RS}$. At section 20 the various methods of writing the formula are given. As the old theories and formulas are generally admitted to be erroneous they have been given no space in this volume except in a few instances, where the defects in the best of them have been pointed out in the course of demonstrating the new principles herein presented.

Perhaps the great variety of theories of variation of the coefficient, and of formulas for flow will best exhibit the present confused and uncertain knowledge of hydraulics. A few of the leading formulas for flow in pipes, and a few of the leading formulas for flow in open channels are here given in order to illustrate the general confusion with which every student of hydraulics has met. It will be noted that all, or nearly all, these formulas may be reduced to the form $V=C\sqrt{RS}$. Hence the main difference in them lies in the different theories of the variation of C. We find in most cases the same author gives a different law of variation in C for pipes from that given for open channels, as though the law depended upon the form of the channel, and changed with the change in form of channel. Others adopt a constant unit value of C for all classes and degrees of roughness of both open channels and pipes, and make C vary with $\sqrt{R}$ only. Hence the same value of $\sqrt{RS}$ will give the same result by such formulas for open channels as for pipes, and for rough as for smooth perimeter. If, in the following formulas, the cofficient formula for C be separated from the formula for V, the various theories for variation in C at once appear, and we at once see why it is that scarcely any two formulas will give like results for the same conditions. It is evident, therefore, that if one is right all the others are wrong.

$$V=\sqrt{\frac{2\,gHd}{4\,m\times l}}$$ for pipes......................(Fanning)

$$V=\sqrt[1.833]{\frac{d^{1.33}\,H}{.00038021\,l}}$$ for pipes..........(W. E, Foss)

$$V=\sqrt{\frac{3600Hd}{l}+.890625}-0.625$$ for pipes..(Wm. Cox)

$$V=\sqrt{\frac{155256d}{12d+1}}\times\sqrt{RS}$$ for pipes............(D'Arcy)

$$V=\sqrt{\frac{RS}{.00007726+\frac{.00000162}{R}}}$$ for pipes.......(D'Arcy)

$V=105.926\,(RS)^{\frac{11}{21}}$ for pipes..........(Saint Vennant)

$V=(9579\,RS+.00813)^{\frac{1}{2}}-0.0902$ for pipes............ ....(D'Aubuisson)

$V=47.913\sqrt{Sd}$, for pipes................. (Blackwell)

$V=100\sqrt{RS}$ for pipes......................(Leslie)

$V=\sqrt{(11703.95\,RS+.01698)}-0.1308$ for pipes....... ....(Eytelwein)

$V=48.045\sqrt{\frac{dH}{l+54d}}$ for pipes......... ..(Hawkesley)

$V=\sqrt{(9978.76\,RS+.02375)}-0.15412$ for pipes...... ....(Prony)

$V=\sqrt{\frac{HR}{.0234\,R+.0001085\,l}}$ for pipes...........(Neville)

$$V=\left(\frac{41.6+\frac{1.811}{n}+\frac{.00281}{S}}{1+\left(41.6+\frac{.00281}{S}\right)\frac{n}{\sqrt{R}}}\right)\times\sqrt{RS},$$ for pipes and channels................................(Kutter)

In the Kutter formula n represents the degree of roughness, and

$$N=\sqrt{\frac{l\sqrt{R}}{BC}+\frac{1}{4}\left(\frac{C-B}{BC}\right)^2 R}-\frac{1}{2}\left(\frac{C-B}{BC}\right)\sqrt{R}$$

In the formula for n

$l=1.811$; $C=\frac{V}{\sqrt{RS}}$; $B=41.6+\frac{.00281}{S}$ for feet measure.

It is to be observed in regard to the variation of Ganguillet and Kutter's C that

First—In all cases of pipes or open channels, where the hydraulic mean radius (R) is less than 3.281 feet, an increase in slope (S) will increase the value of C.

Second—In all cases of pipes or open channels where R is greater than 3.281 feet, an increase in S will cause C to decrease.

Third—Where R=3.281 feet exactly, the value of C will be constant for all slopes and will equal $\frac{1.811}{n}$

> At page 106 of Ganguillet and Kutters work (Hering and Trautwine's translation) an unsatisfactory attempt is made to explain this reverse variation of the coefficient. In the translator's preface it is pertinently stated "that the laws of flowing water must be the same whether the channel is large or small, slightly inclined or precipitous." In this remark the writer fully concurs. The above variations of C would lead to the conclusion that the law of gravity reversed itself at the point where R=3.281 feet. Such variation is clearly erroneous and is unsupported by any sound theory or facts.

In order that the various theories of flow in open channels may be compared with the theories of flow in pipes, and their differences noted, a collection of the most prominent formulas for flow in open channels will here be given:

$\sqrt{V} = A\sqrt[3]{R} \times \sqrt[4]{S}$ ....(Gauckler.)

In Gauckler's formula A is supposed to be constant for any given roughness, and the coefficient varies as $\sqrt[3]{R}$.

$V = 92.20\sqrt{RS}$............(Brahms & Eytelwein).

In Brahm's formula the coefficient varies only as $\sqrt{R}$

$V = 4.9\,R\sqrt[5]{S}$ for small streams.......(Hagen.)

Here the coefficient varies directly with R.

$V = 3.34\sqrt{R} \times \sqrt[5]{S}$ for large streams...(Hagen)

Here the coefficient varies with $\sqrt{R}$.

$V = \sqrt{\frac{1.0}{A + \frac{B}{R}}} \times \sqrt{RS}$.... ....(Bazin)

In Bazin's formula A and B are constant for any given degree of roughness of perimeter.

$V=\left(\frac{1000s}{.08534R+.35}\right)^{\frac{1}{2}}$ ....(D'Arcy and Bazin)

$V=\sqrt{\frac{2gRS}{m}}$ ....(Fanning)

In Fanning's formula $m=\frac{2gRS}{V^2}$. It is a direct coefficient which decreases as V^2 increases or as the roughness decreases and also varies with R. The mean values of m for channels in earth of ordinary roughness vary from 0.05 for R= 0.25 to m =0.002 for R=25.00. For very rough channels the value of m would be greater because V^2 would decrease as the roughness increased. To show the theory of this formula it should be written $V=\sqrt{\frac{2gR}{m}}\times\sqrt{S}$ and $m=\frac{S}{V^2}\times 2gR$ The value of $\frac{S}{V^2}$ depends upon the degree of roughness only and varies with R instead of $\sqrt{R^3}$ as it should.

$V=140\sqrt{RS}-11\sqrt[3]{RS}$ ....(Neville)

$V=\sqrt{1067.02\,RS+0.0556}-0.236$ ....(Prony)

$V=\sqrt{10567.80RS+267}-1.64$ ....(Girard)

$V=\sqrt{8976\,50\,RS+0.012}-0.109$ ....(D'Aubuisson)

$V=\sqrt{8975.43\,RS+0.011589}-0.1089$ ......(Eytelwein)

$V=100\sqrt{RS}$ ....(Pole, Leslie, Beardmore)

$V=\left(\sqrt{0.0081m+\sqrt{225\,R\sqrt{S}}}-0.09\sqrt{m}\right)^2$ .....
.............................(Humphreys & Abbot)

In Humphreys & Abbot's formula R $\frac{\text{Area}}{\text{Wet P+Width}}$ and m $=\frac{1.69}{\sqrt{R+1.5}}$

It will be noted that a majority of these formulas make no provision whatever for different degrees of roughness or different classes of wet perimeter, and hence the computed results will be the same for rough, stony channels of irregular cross section as for smooth channels in firm earth with uniform cross sections. The experimental coefficients developed from actual guagings of different classes of streams, and tabulated in the following pages of this work show that the value of C varies from 27 to 75 in channels of like dimensions and slope, but of different degrees of roughness of wet perimeter. The different classes of perimeter must therefore be classified and the unit value of C for each class must be established experimentally by actual guagings. When the proper unit value of the coefficient for each degree of roughness is thus established, it must thereafter vary correctly with all changes of conditions as to slope, hydraulic mean depth,etc.,so that the one unit coefficient will accurately apply to all channels in the same class of roughness, regardless of dimensions and slope or velocity of flow.

Ganguillet & Kutter recognized the necessity of classifying the degrees of roughness of wet perimeter and of.establishing the unit value of the coefficient for each class. They adopted the unit value of their coefficient of roughness, n, for each degree of roughness, and this value of n was to apply to all perimeters of like roughness, regardless of the dimensions of the channel, but they failed to discover the law which governs the true variation of n and hence the value of their C will not vary correctly with changing conditions.

Mr. J. T. Fanning also recognized the necessity of classifying coefficients in accordance with the degree of roughness of the perimeter, but he adopted a system of direct coefficients for each velocity and for each hydraulic mean depth, instead of determining the unit value for each class of perimeter. The difficulty with such direct coefficients is that they will apply only to cases exactly similar to the conditions under which they were determined. The result is that we must have a separate coefficient for each velocity in the same channel, and for each change in hydraulic mean depth, and for each degree of roughness. It is a fixed, inflexible quantity whose value must be ascertained by experiment for every change in any of the conditions. When the student of hydraulics investigates and compares the conflicting theories of flow and of the variation of the coefficient as set forth in the old formulas, he is simply bewildered and discouraged, for he can discover no satisfactory reason for adopting any one of them in preference to another. The writer therefore hopes to be pardoned for offering what he conceives to be the rational solution of these difficulties.

MARVIN E. SULLIVAN.

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"There is in this world but one work worthy of a man, the production of a truth, to which we devote ourselves, and in which we believe."—Taine.

INTRODUCTORY.

The evolution of the formula and discussion of the present available data of flow, with an explanation of the reverse variation of coefficients,

The general formula for flow as herein finally presented may justly be called the result of the combined labors of Galileo and all subsequent writers and experimentalists, including the present writer. The present writer has accepted and adopted from all former writers on hydraulics such principles and theories as have been thoroughly proven true and general, and has rejected all theories of doubtful or uncertain value and supplied the deficiencies thus arising by original investigations and experiments. The foundation of the formula was the discovery, early in the seventeenth century, of the law of falling bodies by Galileo. In his investigations of flow in rivers Galileo failed to recognize the nature of the resistance of the solid wet perimeter and the difference between the resistance of a solid in contact with a liquid, and that of two solid bodies in contact. His investigations therefore resulted in failure. Later it was discovered by Torricelli, a pupil of Galileo, that resistances aside, the square of the velocity is directly proportional to the head or inclination, or in other words, that the velocity would be as the square root of the head or slope.

Brahms discovered that the acceleration which would occur according to the law of gravity did not actually occur, but that the velocity of flow became constant. His investigations established the fact that the solid wet perimeter offered a resistance to the flow which opposed the acceleration that would otherwise occur, and he assumed that this resistance was directly proportional to the hydraulic mean radius, or to the area of cross-section of the column of water divided by the wet girth. In the latter part of the eighteenth century, Du Buat instituted a series of experiments from the results of which he discovered that the velocity of flow depended upon the slope of the water surface or head, and that in channels of uniform area and grade, when equilibrium was attained, the flow became uniform and the resistance equalled the acceleration of gravity. Thus each investigator has contributed some valuable discovery or fact which has been able to stand, while many of their assumptions have been proven wholly erroneous.

Du Buat also discovered that the resistance of a solid in contact with a liquid, was in no manner increased or decreased by a change of pressure between the liquid surface and the solid surface. In other words he discovered that the pressure with which a liquid is pressed upon a solid does not affect the friction between them. Du Buat and Prony each discovered, as a consequence of the law of gravity, that the head or slope had no influence whatever upon the value or variation of the coefficient, but they erroneously assumed also that the character of the wet perimeter had no influence upon the coefficient. It was the opinion of Du Buat (and adopted by Prony) that the nature of the walls and bottom of a channel could not affect the coefficient because, as Du Buat observed, "A layer of water adheres to the walls, and is therefore to be considered as the wall proper which surrounds the flowing mass." With this view, he supposed all perimeters to be practically "water perimeters", and consequently equally smooth. It remained for D'Arcy and Bazin to demonstrate by many practical experiments that this latter assumption of Du Buat and Prony was entirely without foundation.

As a result of the experiments of D'Arcy and Bazin the fact was established that the coefficient of flow would vary directly as the degree of roughness or smoothness of wet perimeter. Bazin stated also that the coefficient varied with the value of the hydraulic mean radius, thus confirming the observations of Brahms. D'Arcy and Bazin recognized that the slope or head had no influence upon the value or variation of the coefficient, and hence omitted that feature in their formula, and assumed with Brahms that the total resistance for any given degree of roughness would be directly proportional to R. the hydraulic mean radius. They were correct in their assumptions thus far, but they failed to go one step farther and provide for the acceleration as well as the resistance, or in other words to ascertain the mean resistance of all the particles of the entire cross-section by taking the product of total retardation by total acceleration. They adopted and embodied in their formula simply the feature of total retardation without modification by the acceleration. Hence they failed to ascertain the mean resistance of the entire cross-section, and as a necessary result of adopting total resistance instead, their formula gives results too low in large pipes or channels and the larger the pipe or channel, the greater the error will become.

Ganguillet and Kutter proposed a formula based partly on Bazin's formula and partly upon the results of some ill assorted guagings. While this latter formula has become popular and is considered as standard by many engineers, it is really based upon theories which are directly contradictory of both the laws of friction and of gravity,and a short investigation will expose the fact that it can be applied with accuracy only to open channels of very slight inclination, and whose mean radii approach closely to unity. The discussion of coefficients will point out the reasons why this is true.

The writer would also remark that the published tables of data in relation to flow in pipes and open channels are, in a large majority of cases,wholly unreliable, as many contracting engineers have recently discovered to their great cost.

Coefficients should never be based upon data of uncertain value, as the results must depend upon the correctness of the coefficient used.

The data of guagings of rivers at different stages and for various depths of flow are nearly all worthless for scientific purposes for one or more of the following reasons:

1. The data fail to show whether the stream was rising or falling or stationary when the mean velocity and corresponding slope of water surface were ascertained.

   Where a stream is either rising or falling with considerable rapidity, there is little or no relation between the slope of the water surface and the actual mean velocity then prevailing. The same depth of flow or guage height at the same point does not necessarily always produce the same slope of water surface. The slopes are usually thus recorded as corresponding to a certain guage height without actual measurement. The same slope and guage height on a rising river will cause a much greater velocity and give a higher coefficient of flow than upon a falling river. The difference in value of the velocity of flow and of the coefficient will depend upon the magnitude of the freshet—the distance it extends up stream—the suddenness and rapidity of the rise or fall. In a rising or falling stream equilibrium is lost and the actual effective slope which is generating the velocity at such times is very different from the apparent slope, and can be ascertained only from the mean velocity actually existing at the time, and from a previous knowledge of the degree of roughness of the stream in that locality. The effective slope may then be found by formula for S.

2. Guaging stations are always located at narrow, deep sections of the stream, and the hydraulic radius thus roughly measured is given as the mean hydraulic radius of the stream. This is never the true nor even scarcely approximate, mean hydraulic radius, except at the particular point where measured.

3. In many cases the mean velocities tabled are deduced from the surface velocities by some absurd formula which is based upon the theory that there is a constant ratio between the maximum surface velocity and the mean velocity, and that this ratio is the same in all classes and dimensions and degrees of roughness of channels.

4. The general slope is usually taken and is assigned as the local slope. They are usually very different except at extreme high water, when the general and local slope are nearly equal.

5. In turbulent streams, or in very large streams it is impossible, for many reasons to ascertain the slope of water surface, especially the high water slope. The slopes assigned in such cases are simply the record of a guess, and have no value for scientific purposes.

6. The method of ascertaining the mean velocity as finally tabled, is frequently by mid-depth floats. These floats vary in the time of passing over the same course by as much as 25 per cent, depending upon the number of whirls, boils and cross-currents encountered. The mean is taken as the actual mean velocity, or as bearing a given fixed ratio thereto, regardless of formation of the channel bed. The inequalities of the bottom of the stream make it impossible to adjust a float to mid depth. If the mid depth velocity were absolutely known, it is not known what relation it bears to the actual mean velocity. The mean velocity may be either above or below middepth. That will depend upon the magnitude and roughness of the channel at the given place.

7. Many rivers are affected by gulf tides, as the Mississippi river as far up as Donaldsonville, and the river Seine at Poissy, Triel and Meulon, where the water surface fluctuated as much as two feet during the time of the guagings there. This is so marked on

the Mississippi river at Carrollton, La., as to actually produce reverse slopes, as noted by the river engineers.

8. Data are frequently published of the gaugings of a channel at different stages where the value of the hydraulic mean radius increases four or five hundred per cent., and the discharge increases by a thousand per cent. or more, and yet the same slope is assigned for all stages! As an example of this kind, see the ten gaugings of the Saone under the direction of M. Leveille, 1858-9. Here the hydraulic radius varies from R=3.88 to R=15.83, and yet the slope of water surface is given as S=.00004 for each of the ten gaugings. The gaugings of the Weser by Funk are of no value. Bazin remarks; "It is to be noted that Funk has almost always adopted the same slope for an entire group of experiments." Bazin also states that in the experiments of Brunning on the Rhine, "the slopes were not measured at all, but subsequently computed so as to make the results accord with the formula."

It is well known that the gaugings of the Mississippi River under the direction of Humphreys and Abbot during the year 1858 are of very doubtful value. The areas had been measured at the gauging stations seven years before, and were assumed to have remained constant ever afterward, when in fact the area at a given point in that river is frequently altered by as much as 14,000 square feet within twenty-four hours by scour. In 1858 the velocities were taken at a depth of only five feet below surface, and the mean velocities were calculated by an empirical formula of no value. Du Buat's mean velocities as tabled for the Canal du Jard, were deduced from surface float velocity by Du Buat's formula. Du Buat's formula for ascertaining the mean from the surface velocity was based on his experiments on a very small wooden trough of smooth perimeter, and has long since been discarded as being of no value.

If the present available tables of data were assorted carefully and the worthless were rejected, there would little remain. These remarks are made here in order to call attention to the need of obtaining new and accurate data, and to prevent too great reliance upon the value of such data as are now available. The data relating to flow in pipes and conduits are equally bad and untrustworthy. The data relating to asphaltum coated pipes except those of Hamilton Smith Jr., and to wrought iron pipes are especially of uncertain value, and no expensive enterprise should be based upon them without additional experiment. Some of the more recent data relating to flow at different depths in large masonry conduits of comparatively short length, show by the value of the slope of water surface as compared with the slope of the bottom of the conduit, that there would have been no water in the upper end of the conduit at all. It is probable that equilibrium had not been attained at the time of gauging. Any other explanation renders the data absurd, and this explanation renders them worthless. In a channel of uniform grade, cross-section and roughness of perimeter, the slope of the water surface will be the same as the slope of the channel bed as soon as uniform flow is established. If this were not the case, uniform flow could never occur, because the water would be of greater depth at one point than at another, and the velocities would be inversely as the depths or wetted areas. An investigation of the data of flow which is now available is discouraging to a degree. It is a misfortune common to us all. In large rivers where the roughness of perimeter and the area of cross-section vary at almost every foot in length, and where scour or fill is constantly in progress, it is impossible that equilibrium in its true sense, should ever be established. The flow is alternately checked by rough perimeter and increased area due to scour, and accelerated by reaches of straight, smooth perimeter where the area is con tracted. The flow is similar to that in a compound pipe made up of lengths alternately large and small, and alternately smooth and rough. The velocity is necessarily inversely as the areas, in case the supply of water is constant.

For these reasons the local slope over a very short length of channel, at normal stages of the stream, is the slope that must be used in the application of a slope formula. Otherwise the result by formula will be of no value.

The value of the coefficient will usually decrease with increase in depth of flow at any given point along a natural channel, after the depth exceeds the usual depth of flow. This is due to the simple fact that the bed is silted and worn smoother up to the depth of ordinary flow than the sides above the usual flow. Hence as depth of flow increases the proportion of the rougher side wall perimeter increases also, and thereby decreases the value of the coefficient as depth of flow and ratio of rough perimeter increase. It frequently occurs, however, that the reverse of this is true, as for example in channels having very rough, stony bottoms and comparatively regular side walls, In this latter case the coefficient will increase as depth of flow increases because with each gain in depth there is a gain of the smooth over the rough perimeter, and the mean of the roughness of the perimeter considered as a whole becomes less and less at each successive increase of depth. In either class of channels the value of the coefficient must vary directly with the mean of the roughness of the entire wet perimeter taken as a whole. It therefore follows that if the whole perimeter be of equal and uniform roughness or smoothness throughout, the coefficient will remain absolutely constant for all depths of flow.

Large masonry conduits, not being adapted to withstand pressure from within, are built on small grades or inclinations and given free discharge. It is nearly always found that the coefficient in such conduits is greater for very small depths of flow than for greater depths. This is explained by the fact that much cement or mortar is dropped upon the invert or bottom during construction and is ground into the pores and joints of the brick by the tramping of the masons. The floor is worn smooth by reason of this, and the slight inclination of the conduit and low velocity permit of the deposit of a very fine, dense silt upon the bottom which settles in and fills up all the irregularities and depressions along the bottom—thus presenting a smooth, uniform, continuous bottom perimeter to the flow. The side walls, although of the same material, are much rougher than the invert or bottom, because the rough projections of sand along the sides of the brick are not rubbed off, and the pores and small cavities are not filled and plugged by mortar tramped in under pressure, and by the deposit and settlement of fine silt, as occurs on the bottom. This is, however, not true of open canals with paved bottoms and masonry side walls where the slope is sufficiently great to generate a velocity at the bottom sufficient to prevent the deposit of silt or to scour out the joints of the masonry floor. In this latter case the bottom has no advantage of the sides so far as relates to roughness, unless it is better constructed, or is composed of smoother material, or is in better repair than the side walls. In any given case the value of the coefficient will be directly as the mean roughness of the entire wetted portion of the perimeter, or as the ratio of smooth to rough perimeter as the depth of the flow varies.

With these general introductory remarks upon the evolution of the formula for flow, the uncertain value of the present available data of flow, and the causes of contrary variation in the value of the coefficient, the reader will be better prepared to understand what follows.

CHAPTER I.

Of the Laws of Gravity and the Laws of Friction Between a Liquid and a Solid.

1. The Law of Falling Bodies—As the law of falling bodies, or of gravity, and the law of friction between a liquid and a solid include all the elements of the flow of water, it is of prime importance that these laws should never be lost sight of in any investigation or application of hydraulic formulas. The moment that a theory or a formula deviates from the requirements of any one of these natural laws, that moment it must fail. These laws must be observed in their entirety. No provision must be either excluded or violated. The penalty is certain failure to the extent of the evasion or violation.

Let g=feet per second by which gravity will accelerate the descent of a falling body. g=32.2 at sea level.

2g=64.4.
v =velocity in feet per second.
H=height in feet, total fall in feet, or total head in feet.
t=time in seconds.

A body falling freely from rest will descend 16.1 feet in the first second of time,(t) and will have acquired a velocity, at the end of the first second, of 32.2 feet per second, and will be accelerated in each succeeding second 32.2 feet, so that for each additional second of time consumed in falling, there will be a gain in the rate of descent equal to 32.2 feet. At the end of the first second the rate of velocity will be 32.2 feet per second, at the end of the second second of time, the rate of velocity will be 64.4 feet per second, and so on for any number of seconds, adding 32.2 feet to the rate of velocity for each second of time.

Velocity is the rate of motion. Acceleration is the gain in this rate. The acceleration or gain in rate of motion in the case of a body falling freely, is 32.2 feet per second, and consequently the velocity, (v) at any time (t) is equal to $g \times t$, or to the acceleration per second (g) multiplied by the number of seconds of time (t). The distance in feet (H) fallen through by a body in the first second is 16.1 feet, or one half g, and the distance fallen in any given time(t) is as the square of that time (t^2). Consequently the velocities are as the square roots of the distances or vertical heights fallen through, or as $\sqrt{H}$. As gravity produces the velocity of 2 in falling through the height 1, the height in feet fallen multiplied by 2g will equal the square of the velocity in feet per second or

$$v^2 = 2gH \text{ ....} (1)$$

From this fundamental law of gravity we have

$$v = \sqrt{2gH} = \sqrt{64.4H} = 8.025\sqrt{H} \text{ ....} (2)$$

$$H = \frac{v^2}{2g} = \frac{v^2}{64.4} = v^2 \times .015536 \text{ ....} (3)$$

$$v = gt \text{ ....} (4)$$

$$g = \frac{v}{t} = \frac{v^2}{2H} = 32.2 \text{ at sea level................} (5)$$

$$t = \frac{v}{g} = \frac{\sqrt{2H}}{g} = v \times .031056 = .2492\sqrt{H} \text{ ....} (6)$$

The velocity head, or that portion of the head which is producing the velocity of flow in any case is therefore

$$hv = \frac{v^2}{2g} \text{ ....} (7)$$

And the velocity generated by any velocity head is equal

$$v = \sqrt{2g\ hv} = 8.025\sqrt{hv} \text{ ....} (8)$$

2. The Laws of Friction as Applied to a Liquid in contact with a Solid Surface—The results of many very careful experiments establish the correctness of the following rules:

I. The friction on any given unit of surface will be directly as the roughness or smoothness of that surface.

II. The total resistance will be as the total number of units of friction surface.

III. The friction on any given unit of surface will be augmented as the square of the velocity with which the liquid is impelled along that surface.

IV. The friction between the molecules or particles of the liquid themselves, is infinitely small, and may be entirely neglected.

V. The friction between a liquid and a solid is not affected by the pressure with which the liquid is pressed perpendicularly upon the solid. The friction is entirely independent of the amount of radial pressure.

VI. The mean resistance of all the particles of the entire cross section of the liquid vein considered as a whole, will be as the total retardation or loss of velocity by resistance, as modified by the total acceleration or free and unretarded flow, or as the product of total retardation by total acceleration. Total acceleration will be as the square root of the net free head. Total retardaation will be as the square root of the head consumed or lost by resistance.

The mean resistance, or mean loss of head, of all the particles of the entire cross section taken as a whole will be as the product of total retardation by total acceleration.

The mean velocity of all the particles in a cross-section will be as the square root of the mean head of all the particles.

CHAPTER II.

Of Coefficients and their Variation.

3. Properties of the Circle—In order to exhibit the properties of the circle, and the relations of area to friction surface in both open and closed channels, and the relations common to both open channels of any form and to circular closed channels or pipes, and to also show the relation of these common properties to the value and variation of the coefficients, the following tables of circles and of open channels of various forms will be referred to. The notation here given will be followed throughout:

H=total head in feet.

h″=friction head, or head required to balance the total resistence.

h_v =velocity head in feet in the total length *l*.

l=length in feet of pipe or channel.

v=mean velocity in feet per second.

d=diameter in feet.

a=area in square feet.

p=wet perimeter in lineal feet, or friction surface.

$r=\frac{a}{p}$=hydraulic mean radius in feet.

n=coefficient of friction or of resistance.

m=coefficient of flow or of velocity.

$S=\frac{\text{total head in feet}}{\text{total length in feet}}=\frac{H}{l}$=sine of slope=total head per foot.

$S''=\frac{h''}{l}$=friction head per foot length of channel or pipe

S_v =Velocity head per foot length of pipe or channel.

In full pipes or circular closed channels $r=\frac{d}{4}=d\times .25$, and $d=4r$. $a=d^2\times .7854$, or $a=(4r)^2\times .7854=r^2\times 12.5664$.

TABLE OF CIRCLES.

d FEET	$\sqrt{d}$ FEET	r FEET	a SQ FEET	p FEET	RELATI'N OF p to a
0.50	0.707	0.125	0.19635	1.5708	p=8a
1.00	1.000	0.25	0.7854	3.1416	p=4a
2.00	1.4142	0.50	3.1416	6.2832	p=2a
4.00	2.000	1.00	12.5664	12.5664	p= a
8.00	2.828	2.00	50.2656	25.1328	p=½a
16.00	4.000	4 00	201.0624	50.2656	p=¼a
32.00	5.657	8.00	804.2496	100.5312	p=⅛a

It will be observed that the diameter d, is doubled here each time, and that the result of doubling d is to also double r and p. It follows therefore that in circular closed channels and pipes d, r and p vary in the same ratio. As d, r and p all vary exactly in the rame ratio, and as the area varies as d^2 or r^2, and as d or r must therefore vary as the square root of the area, it follows that the friction surface p, must also vary as the square root of the area. It will be observed that when d, r or p is doubled, the area is increased four times. Hence if the friction surface p is doubled the area is increased four times. The right hand column of the table shows how rapidly the friction surface p gains on the area as the diameter is reduced from four feet, and also the reverse gain of area over friction surface as the diameter is increased above four feet. In pipes or circular closed channels flowing full, the the same value of d or r always represents the same length of wet perimeter, because the perimeter or circumference of a circular closed channel or pipe is always equal to $d\times 3.1416$, or to $r\times 12.5664$. As the area is always as d^2 or r^2 and as the friction surface varies as d, r, p or $\sqrt{a}$, it follows that the same value of d, r or p in circular closed channels and pipes flowing full, will always represent the same value of the area and of the wet perimeter or circumference. Such circular full channels may therefore be compared one with another, by simple proportion, because in such channels,

$a : a :: d^2 : d^2$, or $a : a :: r^2 : r^2$

$r : r :: d : d$, or $r : r :: p : p$

$p : p :: d : d$, or $\sqrt{a} : \sqrt{a} :: p : p$

In open channels, however, r is not necessarily an index of either the extent of the area or of the perimeter, and therefore open channels of different forms cannot be thus compared one with another, but in all cases r expresses the ratio of a to p in the given case.

TABLE OF OPEN CHANNELS.

Channel	Area, a	Perimeter, p	$r=\frac{a}{p}$	r^2
Flume 10'x20'.......	200.00	40.00	5.00	25.00
Mississippi River...	15911.00	1612.00	9.87	97.427
Lauter Canal........	56.40	31.00	1.81935	3.31
River Seine.........	9522.00	518.00	18.382	337.898
Chazilly Canal......	11.30	10.80	1.0462	1.09453

While we cannot compare these open channels one with another as in the case of pipes, yet if we take the data for any one pipe or for any one open channel it will be found in any case that

$$p : r :: a : r^2$$

In other words the friction surface p varies as r, and the area varies as r^2 in any possible shape or form of open or closed channel. As $p : r :: a : r^2$ in any form of channel it follows that r bears the same relation to a and also to p in any given channel or pipe, as it does in any other channel or pipe, for $r = \frac{a}{p}$ in any given case. The properties which are common to all shapes and classes of channels and pipes are that, in any given case, the area varies as r^2 and the friction surface varies as r or as $\sqrt{a}$, regardless of the shape of the pipe or channel. These properties which are common to all classes and forms of channels and pipes are the only two which affect the coefficients. Hence the form or shape or size of the pipe or channel does not affect the application of the coefficients which vary with these properties which are common to all possible forms of waterways. The tables were given not only to illustrate the above facts, but for other reasons which will be referred to when the application of certain formulas to open channels is discussed.

Brahms discovered and announced these common relations as early as the middle of the eighteenth century, but like his successors, he mistook the total resistance for the mean resistance or in other words he did not modify the effects of total resistance by that of total acceleration. Since that time, coefficients have been made to vary either as r or as $\sqrt{r}$, and also as some other factor such as slope or velocity.

4. Coefficients of Friction or of Resistance—In general terms a coefficient may be defined as the constant amount or per cent by which the head per foot length of pipe or channel must be reduced on account of loss by frictional resistances.

In any constant diameter, or in any open channel of constant hydraulic radius, the friction will be directly as the total friction surface and directly as the roughness of that surface, and will increase directly as the square of the velocity. As the amount of resistance per foot length of pipe or channel is always directly proportional to v^2 in any given case, it follows that the amount of friction head per foot length (S') required to balance it must also always be directly proportional to v^2—otherwise one could not balance the other, and uniform flow could never occur. In any given pipe or channel, if the head or slope increases, the square of the velocity, and consequently the friction, will increase in the same ratio, for v^2 is always directly proportional to the head or slope. Hence the ratio of friction head per foot length S', to the square of the velocity, v^2, is necessarily a constant for all heads, slopes and velocities. The coefficient of resistance n, in any given diameter or hydraulic radius is simply the expression of this ratio of S' to v^2. It follows therefore, as this ratio is necessarily a constant, that a change of slope or velocity can have no possible effect upon the value of the coefficient. As long as the diameter or mean hydraulic radius remains constant, the coefficient of resistance is $n=\frac{S''}{v^2}$ for all slopes and velocities, and will be constant for all slopes and velocities because S'' and v^2 must always vary exactly at the same rate. It is evident that any formula which causes the coefficient to vary in any manner, or to any extent whatever, with a change of head, slope or velocity, violates the law of gravity which shows that H or S must be directly proportional to v^2 in all cases. It also violates the law of friction which declares the friction to be always proportional to v^2. The results computed by such a formula for any given diameter with different slopes, or for any given open channel with different slopes must necessarily be erroneous at least to the extent that the value of the coefficient was made to vary with changing slopes. The value of the coefficient for any given diameter or for any given hydraulic radius will depend upon the degree of roughness of the wet perimeter. A rough perimeter will offer great resistance to the flow and will require a considerable head or inclination to generate a small velocity. In such case the ratio (n) of $\frac{S''}{v^2}$ will be large. because S'' will be large and v^2 will be small. This ratio will, however, be constant for any given degree of roughness in a pipe or channel where r is constant.

5. Coefficient of Velocity—Where the discharge is free in a pipe or channel, and the value of r remains constant, the totalhead will be consumed in balancing the resistance and in generating the velocity of flow. The resistance must be balanced before flow can ensue. The resistance being as v^2 in all cases, the coefficients of velocity represents the ratio of total head H, to v^2, or rather S to v^2 if the discharge is free If the discharge is throttled so that a portion of the head is converted into radial pressure, then this pressure head is neither converted into velocity nor lost by resistance. In this latter case the coefficient of velocity is the ratio of $\frac{S_v+S''}{V^2}$, in which S_v is the velocity head per foot length, and S'' is the friction head per foot length. Where the discharge is free, S= total head per foot length, and the coefficient of velocity, $m=\frac{S}{V^2}$. As H or S is always directly proportional to v^2 ($v^2=2g\,H$) it follows that if S be increased in any given diameter or hydraulic radius, v^2 will also increase at the same rate, and hence $m=\frac{S}{V^2}$ will necessarily be a constant in any given hydraulic radius regardless of the value of the slope or velocity. If it be admitted that the fundamental laws of gravity, $V^2=2\,g\,H$, and $V=\sqrt{2g\,H}$ are correct, and that the frictional resistances on any given surface will increase as v^2, it must also be admitted that the ratio (m) of S to v^2 is always constant after equilibrium is attained, and that as a necessary result, the head, slope or velocity can have no possible effect upon the value of the coefficient of velocity.

6. The Law of Variation of Coefficients—The coefficient of resistance n, and the coefficient of velocity m, have so far been considered only as applied to a constant hydraulic radius or constant diameter, and it has been shown that in no possible case can the slope or velocity affect the value of the coefficients in any diameter or hydraulic radius. The effect of variation of hydraulic radius or of diameter, upon the value of the coefficient will next be investigated.

As the area in any given open channel varies with r^2 and in any given circular closed channel or pipe as r^2 or d^2, and as the wet perimeter or friction surface in any shape of pipe or open channel varies as r, d or p, it follows that the friction surface p (which is as r or d) must vary in any given case as $\sqrt{a}$. Area is as d^2 or r^2. Consequently $\sqrt{a}$ is as d or r. The total resistance will be directly as the total friction surface which is as d, r or $\sqrt{a}$. The total area or free flow in any given case varies with d^2 or r^2. As area gains over wet perimeter a greater number of particles of water are set free from the resistance of the perimeter, or acquire a head equal to the slope of the channel or water surface. This does not increase the head of the particles already free of resistance, but simply adds to their number. The number of these unresisted particles will increase directly as the area, or as d^2 or r^2. Total acceleration will increase as the square root of the net free head or free flow, which is therefore as d or r. Total retardation will be as the square root of the total head lost by resistance. The head lost by resistance will be directly as the amount of friction surface, which is as d or r, but the total retardation or loss of acceleration due to this loss of head, will be as the square root of the head lost, or as $\sqrt{d}$ or $\sqrt{r}$.

Then total acceleration is directly proportional to the square root of the area or net free head, or to d or r, or $\sqrt{a}$. Total retardation or loss of velocity is directly proportional to the square root of the loss of head, or to $\sqrt{d}$ or $\sqrt{r} = \sqrt[4]{a}$. It follows therefore that the mean loss of head, or mean resistance, of all the particles of the cross-section taken as a whole, will be as the total acceleration modified by the total retardation, or the mean loss of head will be inversely as $d\sqrt{d}$ or $r\sqrt{r} = \sqrt{r^3}$, or $\sqrt{d^3}$.

The mean loss of head will be inversely as $\sqrt{r^3}$ or $\sqrt{d^3}$ because the acceleration increases as $\sqrt{a}$ or d or r while retardation is only as $\sqrt{d}$ or $\sqrt{r}$, or $\sqrt[4]{a}$. If the value of the coefficient of resistance n be found for any degree of roughness when d or r=1, then it will vary from this unit point inversely as $\sqrt{r^3}$ or $\sqrt{d^3}$, and the general formula for the value of the coefficient becomes, $n = \frac{S''}{V^2} \times \sqrt{d^3}$, or $n = \frac{S''}{V^2} \times \sqrt{r^3}$.

These expressions are equivalent to $n = \frac{h'' r\sqrt{r}}{l\, v^2}$, or $n = \frac{h'' d\sqrt{d}}{l\, v^2}$.

It is apparent that the coefficient of resistance or friction n, will vary for any given degree of roughness, only with the variation in the value of $\sqrt{d^3}$ or $\sqrt{r^3}$. For the same degree of roughness, no other factor will affect its value in any manner.

Now as the mean loss of head is inversely as $\sqrt{d^3}$ or $\sqrt{r^3}$ the mean gain in head of the entire cross section taken as a whole, will be directly as $\sqrt{d^3}$ or $\sqrt{r^3}$. The mean velocity of the entire cross section, or the gain in the mean velocity, will be directly proportional to the square root of the net mean head, or to the square root of the gain in the net mean head, or $\sqrt{\sqrt{d^3}} = \sqrt[4]{d^3}$ or $\sqrt{\sqrt{r^3}} = \sqrt[4]{r^3}$.

The friction head h'' is therefore inversely as $\sqrt{r^3}$ or $\sqrt{d^3}$

The velocity head is directly as $\sqrt{r^3}$ or $\sqrt{d^3}$.

The velocity of flow is directly as $\sqrt[4]{d^3}$ or $\sqrt[4]{r^3}$. It is understood that in these cases the slope remains constant.

The coefficient of velocity, m, if determined for any given degree of roughness when d or r = 1, will therefore vary from this unit point directly as $\sqrt{r^3}$ or $\sqrt{d^3}$.

$$n = \frac{h'' \; r\sqrt{r}}{l \, v^2} = \frac{S''}{v^2} \times \sqrt{r^3} \quad \dots\dots\dots\dots\dots\dots (9)$$

$$n = \frac{h'' d \sqrt{d}}{l \, v^2} = \frac{S''}{v^2} \times \sqrt{d^3} \quad \dots\dots\dots\dots\dots\dots (10)$$

$$m = \frac{H \; r\sqrt{r}}{l \, v^2} = \frac{S}{v^2} \times \sqrt{r^3} \quad \dots\dots\dots\dots\dots\dots (11)$$

$$m = \frac{H \; d\sqrt{d}}{l \, v^2} = \frac{S}{v^2} \times \sqrt{d^3} \quad \dots\dots\dots\dots\dots\dots (12)$$

As these coefficients for any given degree of roughness vary only with $\sqrt{d^3}$ or $\sqrt{r^3}$, the value of d or r may be in inches or feet, while the constant $\frac{S}{v^2}$ may remain in feet in either case, or all the values may be expressed in metres.

The value of any given constant slope is made more effective directly as $\sqrt{r^3}$ increases, because the mean gain in head increases directly as $\sqrt{r^3}$ or the number of unresisted particles increases as $\sqrt{r^3}$. The mean head and mean velocity both gain without a change in slope. This does not affect the constant ratio $\frac{S}{v^2}$ because as S is made more effective by an increase in $\sqrt{r^3}$, v^2 increases in the same ratio.

7. Formula for Mean Velocity—By transposition in equation (11) we have

$$v=\left(\frac{H}{l}\times\frac{\sqrt{r^3}}{m}\right)^{\frac{1}{2}}=\sqrt{\frac{S\sqrt{r^3}}{m}}\cdots\cdots\cdots\cdots\cdots\cdots(13)$$

But for each given degree of roughness of friction surface m is a constant equal to the ratio $\frac{S}{v^2}$, and varies only as $\sqrt{r^3}$. Hence we may take the square root of the reciprocal and write $\sqrt{\frac{1}{m}}=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$, whence

$$v=C\sqrt[4]{r^3}\sqrt{S}\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(14)$$

In this case C is a constant which applies to all pipes or open channels of the given degrees of roughness represented by C. In other words C represents $\sqrt{\frac{v^2}{S}}$ which is constant for any given roughness and only varies with $\sqrt[4]{r^3}$. If we replace C by K in equation (14) and reduce the formula to the Chezy form, that is, if we write

$$v=C\sqrt{rs}\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(15)$$

then, $C=K\sqrt[4]{r}$, which, when multiplied by $\sqrt{rs}$ equals $C\sqrt[4]{r^3}\sqrt{S}$. In the Chezy form it is seen that $C=K\sqrt[4]{r}$, or in other words, that for any given degree of roughness of friction surface represented by K, the coefficient C will vary only as $\sqrt[4]{r}$.

If we write simply, $v=C\sqrt{S}$, then $C=K\sqrt[4]{r^3}$.

The result is the same in either case.

The value of C in any formula in the Chezy form must vary only as the roughness and as $\sqrt[4]{r}$ or $\sqrt[4]{d}$.

If a series of pipes or open channels of equal roughness be selected, it will be found that $C : C :: \sqrt[4]{r} : \sqrt[4]{r}$ regardless of the slopes or dimensions of the channels. If C fails to vary only as $\sqrt[4]{r}$ in the Chezy form of formula for a series of channels of equal roughness, then it will be found that $C\sqrt{rs}$ will not equal v. This will be illustrated by the following pairs of open channels—each pair being nearly equal in roughness, but varying in the values of the slope and hydraulic radius.

8. Variation of C Illustrated—In the following tables we shall give the values of our C as found by the formula $C=\sqrt{\frac{v^3}{S\sqrt{r^3}}}$, for each channel. We will also give the value of the Chezy C for each channel which is required to make $C\sqrt{rs}=v$. It will be found in each case that barring the slight difference in the degree of roughness in each pair of channels the Chezy C which will cause $C\sqrt{rs}$ to equal v, will vary only as $\sqrt[4]{r}$, and their values in such class of channels may be found or compared by the simple proportion $C : C :: \sqrt[4]{r} : \sqrt[4]{r}$. The values of the Chezy C were determined by the formula, $C=\frac{v}{\sqrt{rs}}$ in all the following tables. The values as given in the translation of Ganguillet and Kutter are only approximate, being sometimes in error by as much as 12 or 15. The values of Kutter's n, or constant coefficient of roughness, are also given for each channel. These values of n are transcribed from Hering & Trautwines Translation, second edition. These values of n show that it is not a constant, but is an auxiliary quantity which must be used as r and s vary in order to balance the erroneous variation of C with the slope.

Pair No. 1. Slopes equal. Radii vary slightly. R> 1 meter.

NAME OF CHANNEL	R FEET	$\sqrt[4]{R}$ FEET	S SLOPE	V FEET SEC.	SULLIVAN'S C	KUTTER'S n	CHEZY C
Seine (Paris)	9.50	1.7556	.00014	3.37	52.85	.0240	92.40
Seine (Paris)	10.90	1.8170	.00014	3.741	52.71	.0238	95.77

The channel where R=10.90 is very slightly rougher, as shown by Sullivan's C, than the first channel. As the slopes are equal, Kutter's n has to vary only with the slight difference in values of r, in order that $c\sqrt{rs}$ will equal v, and in order that c: c:: $\sqrt[4]{r}$: $\sqrt[4]{r}$.

Pair No. 2. Slopes nearly equal. Small difference in R. R > 1 meter.

NAME OF CHANNEL	R FEET	$\sqrt[4]{R}$ FEET	S SLOPE	V FEET SEC.	SULLIVAN'S C	KUTTER'S n	CHEZY C
Seine (Triel)	12.40	1.876	.00060	2.359	46.12	.0295	86.43
Seine (Poissy)	15.90	2.000	.00062	2.911	46.43	.0285	92.70

The Chezy C varies only as $\sqrt[4]{r}$ in Pair 2. It is not affected by the slight difference in slope. Where R=15.90 the channel is very slightly smoother than where R=12.40. Yet Kutter's n must be reduced because of the slight increase of slope and hydraulic radius. As Kutter's C will increase with decrease in slope where R is greater than 1 meter, or 3.281 feet, n must be increased where R=12.40 because this slope is least, and if n were taken as a constant for both channels, it would make C too great for the first channel.

Pair No 3. R > 1 meter. R and S vary. Roughness equal nearly.

NAME OF CHANNEL	R FEET	$\sqrt[4]{R}$ FEET	S SLOPE	V FEET SEC.	SULLIVAN'S C	KUTTER'S n	CHEZY C
La Fourche	15.70	1.988	.0000438	2.798	53.40	.0205	106.30
Mississippi River	72.00	2.913	.0000205	5.929	53.00	.0277	154.30

Note the difference in value of Kutter's n for these two channels of equal roughness.

In Pair No. 3 R is greater than one meter, and in this case Kutter's C will increase as slope decreases. As the slope of the Mississippi river is much less than that of Bayou La Fourche, if n were used as a constant for both, the value of Kutter's C would be greatly too large for the Mississippi. Therefore, in order to balance the error of increase in Kutter's C with decrease in slope, the value of n must be increased in proportion as slope decreases. Otherwise his $C\sqrt{rs}$ will not equal v. It is seen from Pair No. 3 that when the required value of the Chezy or Kutter C is obtained which will make $C\sqrt{rs}=v$, then $C:C::\sqrt[4]{r}:\sqrt[4]{r}$, regardless of the difference in slope. Kutter admits, in his work on Hydraulics (pages 99 and 132) that n is not a constant for the same degree of roughness if there is much variation in the dimensions of the channels to which it is applied. His n might be made a constant like our C for each degree of roughness, and regardless of the dimensions of the channels, if it were made to vary only as $\sqrt[4]{R}$, for all slopes and all dimensions of channels, whether R were greater or less than one meter. It is absurd that C, and consequently the velocity, should be proportionately less for a steep slope in a large channel than for a small slope. Of course the value of $\sqrt{S}$ remains in any case, but decrease in C as S increases in large channels amounts to reducing the actual value of S by the amount that C is there made to decrease. It cannot be justified upon any sound theory, and the above tables show that it is not sustained by fact. It is equally erroneous that C will increase with an increase in slope in small channels where R is less than one meter, and in which the ratio of friction surface to the quantity of water passed is much greater than in large channels. The laws of gravity and of friction do not reverse themselves at the point where R=1 meter, nor at any other value of R. As Kutter's n is not a constant for the same degree of roughness where the slopes vary or where R varies, it is very misleading when viewed as an index of roughness, which is supposed to be its special function.

Pair No. 4. R< 1 Meter. Roughness Equal. R and S Vary

Name of Channel	R Feet	$\sqrt[4]{R}$ Feet	S Slope	v FtSec	Sullivan's C	Kutter's n	Chezy C
Rhine Forest	0.42	.8051	.0142	2.332	37.50	.0337	30.13
Simme Canal	1.32	1.072	.0170	5.993	37.37	.0361	40.06

In Pair No. 4, R is less than one meter in either channel. For this reason Kutter's C will increase with increase of slope. Hence the steeper the slope becomes where R is less than one meter, the greater we must increase the value of his n in order to cut down this unnatural increase in C. We find by simple proportion in Pair No. 4, as in all other cases where the roughness is equal, that C:C: : $\sqrt[4]{r}:\sqrt[4]{r}$, simply, and regardless of difference in slope. Kutter's n must be trimmed or increased in such manner as to cause C to vary only as $\sqrt[4]{r}$, otherwise his $C\sqrt{rs}$ will not equal v. It is therefore neither a constant nor an index of roughness, but is an uncertain and misleading quantity. See Kutter's discussion of the variation of his n at pages 99, 110 and 132 of Hering and Trautwine's edition of Kutter's work. Also see Translators preface.

Pair No. 5. R and S vary. Roughness Equal.

Name of Channel	R Feet	$\sqrt[4]{r}$ Feet	S Slope	v FtSec	Sullivan's C	Kutter's n	Chezy C
Grosbois Canal	1.71	1.143	.000441	1.51	48.08	.0284	55.50
Seine (Paris)	14.50	1.951	.00014	4.232	48.12	.0255	93.92

In Pair No. 5, the value of R is less than 1 meter in one case, and greater in the other, and there is a difference in slope also. Notwithstanding both these facts, C must vary only as $\sqrt[4]{r}$ as shown in the table, or $C\sqrt{rs}$ will not equal v.

Sullivan's C in all the above tables is $C=\sqrt{\frac{v^3}{S\sqrt{r^3}}}$; and applies in the formula, $v = C\sqrt[4]{r^3}\ \sqrt{S}$. Its unit value is constant for all slopes and all dimensions of pipes or open channels of the same degree of roughness. It is simply the square root of the reciprocal of m. It has been shown that slope or velocity cannot affect the value of m, as it is the expression of the ratio $\frac{S}{v^2}$. Its numerical value depends only upon the degree of roughness of perimeter. The formula for m or n or C as heretofore given, will give the unit value of the coefficient directly, that is, its value for r or d=1. It therefore does not matter whether the formula for ascertaining the coefficient is applied to the data of a very small or very large channel, the result will be the value of the coefficient for r=1, or d=1, as the case may be. From this unit point the coefficient varies with the inverse value of $\sqrt{r^3}$ or $\sqrt{d^3}$ if it is n that is sought. The coefficient m of velocity, varies from the unit value as found by formula for m, directly as $\sqrt{d^3}$ or $\sqrt{r^3}$. The variation of C will be as the $\sqrt[4]{r}$ if the formula is written $v = C\sqrt{rs}$, or if it written $v = C \times \sqrt[4]{r}\ \sqrt{rs}$. If we write $v = C\sqrt{S}$ then C must vary as $\sqrt[4]{r^3}$. This latter form is equivalent to the form $v = C \times \sqrt[4]{r^3}\ \sqrt{S}$, in which C is the constant for any given degree of roughness of perimeter. This last form has been adopted in all the foregoing and following tables. For the reason that m or C, as found by formula from the data of guagings will be the unit value, and will differ in value only as the degree of roughness differs, the mere development of the unit values of the coefficient for a series of pipes or open channels will at once classify such pipes or channels, and exhibit their relative degrees of roughness. Those which give like values of the coefficient are of similar degrees of roughness, because the unit value of the coefficient is not affected by any element or factor except the degree of roughness.

The coefficient C or m does not, and should not, vary, except as the roughness of perimeter varies. For this reason our m or C is an absolute index of the roughness for it cannot vary with any other factor. We have shown that the effective value of the slope S is increased as $\sqrt{r^3}$ increases, because the net mean head, or net gain in area over friction surface is as $\sqrt{r^3}$. But whatever increases or makes the mean head, or S, more effective, must also increase the value of v^2 in the same ratio.

The effective slope S, is as $S\sqrt{r^3}$, and the mean velocity is as $\sqrt{S\sqrt{r^3}}$. Now in the formula for m or C. $m=\frac{S\sqrt{r^3}}{v^2}$ and $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$. In either formula an increase in the value of $S\sqrt{r^3}$ will cause the value of v^2 to increase in the same ratio. It is then apparent that where the values of $S\sqrt{r^3}$ are equal, the velocities must be equal unless the resistances caused by roughness of perimeter are greater in the one case than in the other. It is also apparent from an inspection of the formula for m or C that as v^2 will increase in the same ratio as $S\sqrt{r^3}$ increases, m or C will be constant for all values of r or d if the roughness of perimeters is the same.

In the velocity formula, $v=C\sqrt[4]{r^3}\times\sqrt{S}$, we see that the mean velocity increases not only as $\sqrt{S}$ but also as the square root of $\sqrt{r^3}$, which is $\sqrt[4]{r^3}$, not because m or C varies, but because the value of S is made more effective as $\sqrt{r^3}$ increases.

9. Practical Determination of Coefficients of Resistance. The resistance to flow, or loss of head by friction, is exactly equal to the amount of head, pressure, or force required to balance it. In a pipe of uniform diameter and roughness the friction will be the same in one foot length of pipe as in any other foot length, hence the total friction will be directly as the length and roughness of the pipe. Friction in any given diameter and roughness of pipe will increase with the square of the velocity. Hence the head lost by friction, or the head which is consumed in balancing friction, must also increase as the square of the velocity. The friction or loss of head for any given velocity in different diameters will be inversely as $\sqrt{d^3}$ or $\sqrt{r^3}$, because total acceleration is proportional to the square root of the area, or to d or r, while total retardation is proportional only to $\sqrt{d}$ or $\sqrt{r}$. Hence the mean loss of head of all the particles of water will be inversely proportional to the resultant of total acceleration and total retardation, or to $d\sqrt{d} = \sqrt{d^3}$, or $r\sqrt{r} = \sqrt{r^3}$. (See columns headed d, $\sqrt{d}$, and "Relation of P to A," in table of circles, ante, §3).

The mean of many experiments shows that a cast iron pipe of ordinary density or specific gravity, one foot in diameter and clean, will require a total head of one foot in a length of 2,500 feet, in order to cause it to generate a velocity of one foot per second. The discharge being free, it is evident that the total head of one foot has been lost by resistance except that part of the one foot head which remained to generate the mean velocity of one foot per second. As the velocity head is not lost by resistance, and as we wish to determine the numerical value of the coefficient of resistance n, the velocity head must be deducted from the total head of one foot in order to find the total head lost by friction. By the law of gravity we find that the head which generates any given velocity is $hv = \frac{v^2}{2g} = \frac{v^2}{64.4}$

In the case we are now considering $v^2=1$, and consequently the velocity head $hv = \frac{1.00}{64.4} = .01552795$ feet. Deducting this velocity head, which was not lost, from the total head of one foot, and we find that the total head lost by friction in the 2,500 feet of 12-inch pipe while $v^2=1$ was equal to $1.00 - .01552795 = .98447205$ feet. Therefore the head lost per foot length of pipe while $v^2=1$, and $d=1$, was $\frac{.98447205}{2500}$

=.00039379 feet=n. As the friction will be as the number of feet length of the constant diameter, and will increase as v^2, then, as long as d remains constant, the total head in feet lost by friction, $h''=n\times l\times v^2$. But if the value of d changes, or the formula is to be applied to a pipe of like roughness, but of a different diameter, we have seen that the friction will be inversely as $\sqrt{d^3}$. Hence the general formula which will apply equally to all diameters of this given degree of roughness will be

$$h''=\frac{n\,l\,v^2}{\sqrt{d^3}}=\frac{.00039379\,l\,v^2}{\sqrt{d^3}}\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

We might have found the value of n directly by applying formula (10) (§ 6).

$$n=\frac{h''\,d\,\sqrt{d}}{l\,v^2}=\frac{S''}{v^2}\times\sqrt{d^3}=m\times.9845\ldots\ldots(10)$$

As the ratio of $\frac{S''}{v^2}$ is always constant for any given degree of roughness, regardless of slope or velocity, and as it varies from the unit point, or d=1, and $v^2=1$, only as $\sqrt{d^3}$ varies, we may find the unit value of the coefficient from any diameter and velocity whatever.

It is simply necessary to find the ratio $\frac{S''}{v^2}$ in any case, and when the value of $\frac{S''}{v^2}$ is multiplied by $\sqrt{d^3}$, the result will be the unit value of n. When this unit value of n is inserted in formula (16) it is made to vary inversely as $\sqrt{d^3}$ as exhibited in formula (16). To make it appear more clearly we write

$$h''=\frac{n}{\sqrt{d^3}}\times l\times v^2\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

It consequently does not matter what head, diameter or velocity we may select for the purpose of finding the unit value of n. The formula for n will always give the unit value, regardless of the size of the pipe to which the formula is applied. As the unit value of n is not affected by any factor except the degree of roughness, it is a faithful index of roughness, and when the value of n for a series of different classes of perimeter has been found, it exhibits the direct difference in roughness per unit of perimeter, between the different classes.

10. Conversion of the Coefficient.—The coefficient may be determined in terms of diameter in feet, or diameter in inches, or in terms of r instead of d, or in terms of cubic feet or gallons. If the value of n has been found for any given degree of roughness, it may be converted to any desired terms. Thus, if the value of n has been found in terms of d in feet, as above, it may be converted to terms of r in feet by simply multiplying it by 0.125 or dividing by eight. If n was originally found in terms of r, and it is desired to convert it to terms of d in feet, multiply by eight. If n is in terms of d in feet, it may be converted to terms of d in inches by multiplying by $\sqrt{(12)^3}=41.5692$. As n, for any given degree of roughness, varies only with $\sqrt{d^3}$, the value of d may be in meters, inches or feet, as may be most convenient. h″, l and v^2 may remain in feet or meters.

$m=\frac{n}{.9845}$; and $n=m\times.9845$, for any given degree of roughness.

11. Determination of Coefficients of Velocity.—We have just seen that a coefficient of resistance (n) represents only the head per foot length of pipe which is lost or consumed in balancing the resistance to flow. A coefficient of velocity, however, must represent not only the head per foot length required to balance the resistance, but also the head per foot length required to generate the velocity of flow, or it must represent $S''+Sv$ in any case. If the diameter or hydraulic radius is constant, and the discharge is free and full bore, the total head per foot length S, will be converted into velocity of flow except that part of S which is consumed in balancing friction. In this case, $S''+Sv=S$, and S must be used in the formula for determining the value of m—the coefficient of velocity. Where the discharge is partially throttled, as by a reducer at discharge, or by a valve partly closed, only a part of the total head per foot length will be consumed by resistances and in generating velocity, and the remainder of the head will remain as radial pressure within the pipe. As the head due to this pressure is neither lost by resistance nor engaged in generating velocity of flow, it has no connection with the value of the coefficient of velocity m. If the discharge is free, then

$$m=\frac{H\sqrt{r^3}}{l\,v^2}=\frac{S}{v^2}\times\sqrt{r^3}=\frac{S\sqrt{r^3}}{v^2}=\frac{n}{.9845}\cdots(17)$$

If the discharge is throttled, then

$$m=\frac{h''+hv\sqrt{r^3}}{l\,v^2}=\frac{S''+Sv}{v^2}+\sqrt{r^3}\ldots\ldots\ldots\ldots\ldots(18)$$

For the ordinary cast iron pipe described in section 9, the coefficient of velocity would be

$$m=\frac{H\sqrt{d^3}}{l\,v^2}=\frac{1}{2500}=.0004,\text{ in terms of d in feet.}$$

The coefficient m may be converted to terms of d in inches, or r in feet or to any other terms in the same manner, and by the use of the same multipliers, as n may be converted. (See § 10)

The velocity coefficient m applies to open or closed channels alike and its unit value depends only on the degree of roughness of perimeter. The value of m as found by the formula is always the unit value, and is equally as accurate an index of roughness as is the coefficient n. The remarks in regard to n in this respect (§ 9) apply to m with equal force.

The coefficient m is to be used in the formula,

$$v=\sqrt{\frac{H\sqrt{r^3}}{l\,m}}=\sqrt{\frac{S\sqrt{r^3}}{m}}=\sqrt{\frac{\sqrt{r^3}}{m}}\times\sqrt{S}\ \ldots(19)$$

$$\text{Or } v=\sqrt{\frac{H\sqrt{d^3}}{l\,m}}=\sqrt{\frac{S\sqrt{d^3}}{m}}\quad\sqrt{\frac{\sqrt{d^3}}{m}}\times\sqrt{S}\ .(20)$$

If m was determined in terms of r, it must not be used in formula (20) which is in terms of d, until it has been converted to like terms with those in the formula. If m is in terms of d in inches, then d in the formula must also be in inches. In other words m must be in the same terms as the formula in which it is used is expressed.

The value of m in terms of d in feet for average cast iron pipe is m=.0004. If it is desired to use C instead of m then

$$C=\sqrt{\frac{1}{m}}=\sqrt{\frac{1}{.0004}}=\sqrt{2500}=50.00 \text{ and}$$

$$v=C\sqrt[4]{d^3}\,\sqrt{S}.$$

The value of C may be found directly and without reference to m by the formula

$$C=\sqrt{\frac{v^2}{S\sqrt{r^3}}} \text{ or } C=\sqrt{\frac{v^2}{S\sqrt{d^3}}}\ldots\ldots\ldots\ldots\ldots\ldots(21)$$

This will give the unit value of C directly, and C is a constant like m or n, which depends on the roughness of perimeter.

If we have m=.0004 for ordinary cast iron pipe, in terms of diameter in feet, we may convert it to terms of r in feet by simply dividing by 8. We then have $\frac{.0004}{8.}=.00005=m$ in terms of r in feet. We may convert m to C in terms of r in feet by taking the square root of its reciprocal in terms of r, and we have

$$C=\sqrt{\frac{1}{m}}=\sqrt{\frac{1}{.00005}}=\sqrt{20000}=141.42=C \text{ in terms of r.}$$

Then, $v=C\sqrt[4]{r^3}\,\sqrt{S}.$

The unit values of n, m and C may be found in all classes of pipes and channels, and may be converted at pleasure as shown. The law governing the flow of water and the value and variation of the coefficients, is exactly the same in open channels as in pipes. The same formulas apply to all equally well so far as the coefficients and the formulas for flow are concerned. Of course the unit value of the coefficient must be found experimentally for each class or degree of roughness of friction surface. When the unit value of the coefficient is determined for any given degree of roughness, it then applies to all forms and dimensions of pipes and channels which fall within that degree of roughness. These remarks apply to n, m and C alike. The roughness or smoothness of perimeter affects the flow in a large river in the same manner as in a small canal. In a large, deep river the area of the cross-section of the column of water is greater in proportion to the wet perimeter than in a small stream, and hence the ratio of free particles of water is greater than in small channels, but the effect of roughness of perimeter is the same in both cases. The unit value of m and C distinctly establish these facts. It is the influence of the great values of r in large rivers that has led some hydraulicians to conclude that the character of the perimeter does not materially affect the flow in such streams.

12.—Coefficients Affected by Specific Gravity, or Density of Material.—In a series of experiments with new, clean cast iron pipes the writer was perplexed by the fact that one 12 inch new, clean pipe would not generate the same mean velocity as another new, clean 12 inch pipe, when the conditions were exactly the same in each case. The difference was so great in the case of one pair of 12 inch new pipes, that the experiment was repeated a number of times, but always with the same result. As no other explanation could be given the writer concluded to ascertain if it was caused by the difference in density or specific gravity of the two pipes, which were from different foundries. The shells were of equal thickness, but on weighing a few lengths of the pipe from each lot, it was found that the pipe which generated the least velocity was much lighter than the other. The investigation thus begun led to experiments with pipes of different metals and different specific gravities. The results then obtained seem to confirm the correctness of the view that the density of the friction surface has a marked influence upon the flow and upon the value of the coefficient. There may be some difference also between the values of the coefficient for a surface of granular metal and a surface of fibrous metal, although the specific gravities of the two metals may be equal. It appears that the flow over earthen perimeters of equal regularity of cross-section will be affected by the nature and specific gravity of the particular kind of earth. The flow in a cement lined pipe or channel which is clean and free of fine silt, will be affected by the fineness of the cement and also of the sand used, as well as by the proportion of sand to cement in the mortar lining. Even in pure cement linings, it is noticed that the flow will be affected by the quality and fineness of the cement used. Classification of perimeters is therefore difficult.

It is stated by Professor Merriman that "it is proved by actual gaugings that a pipe 10,000 feet long and one foot in diameter discharges about 4.25 cubic feet per second under a head of 100 feet. The mean velocity then is

$v=\frac{4.25}{0.7854}=5.41$ feet per second." ("Treatise on hydraulics." page 165, fifth edition.) It will be noted that the character of the pipe, whether cast iron, wrought iron, riveted or welded, coated or uncoated, is not mentioned. It was certainly a remarkably smooth pipe. If the value of the coefficient m is developed for this pipe we shall have

$$m=\frac{S}{v^2}\times\sqrt{d^3}=.00034165, \text{ in terms of d in feet.}$$

$$m=\frac{S}{v^2}\times\sqrt{r^3}=.00004270625, \text{ in terms of r in feet.}$$

The average value of m for clean cast iron pipe is

m=.01662768, in term of d in inches.

m=.0004 in terms of d in feet.

m=.00005 in terms of r in feet.

The writer made a number of experiments with 6", 12" and 24″ cast iron pipes which were new and absolutely clean and of the greatest density that the writer has ever discovered before or since in cast iron pipes. The water was pure mountain water from the melting snow on the granite hills. The pipes were laid straight and perfectly jointed, and the discharge was perfectly free, into a large measuring tank. Under these perfect experimental conditions, the value of m as developed by the three pipes was

m=.000368 in terms of d in feet.

m=.000046 in terms of r in feet.

Such favorable conditions as these scarcely ever occur in actual water works building, and do not continue if they originally exist.

In later experiments with new clean cast iron pipes of inferior quality and very low specific gravity, the values of the coefficient of flow developed were

m=.01721 in terms of d in inches:—C=7.622.

m=.000414 in terms of d in feet:—C=49.14.

New, clean cast iron pipe of average weight per cubic unit as long as it remains clean gives,

m=.01663 in terms of d in inches:—C=7.755.

m=.0004 in terms of d in feet:—C=50.00.

m=.00005 in terms of r in feet:—C=141.42.

It is therefore evident that where the pipes are made of the same class of metal and are new and clean, the value of the coefficient will bear a close relation to the specific gravity, or density, of the pipe metal. The fact that clean leaden or brass pipe will generate a much greater velocity of flow under the same conditions than will a clean iron pipe of equal diameter can be accounted for in no other manner than the difference in specific gravity of the different metals.

These facts demonstrate the important influence of even very small degrees of roughness of perimeter upon the flow and consequently upon the value of the coefficients. Low specific gravity in metal indicates that it is porous and its surface is affected by innumerable small cavities, rendering it irregular. The specific gravity of cast iron varies from 6.90 to 7.50; of steel, from 7.70 to 7.90; of wrought iron from 7.60 to 7.90.

While the specific gravity of a metal, or of stone or brick, or earth where the cross section is equally uniform, undoubtedly affects the flow, yet other substances of much less specific gravity, when applied as a lining or coating, will greatly increase the flow. Thus the specific gravity of asphaltum varies from 1 to 1.80 according to its purity, and an asphaltum coated pipe will generate a much higher velocity of flow than a clean iron pipe. The coefficients developed by asphaltum coated pipes, however, vary like cement lined pipes, with the quality of the material, or the proportion of pure asphaltum to the other ingredients used in the manufacture of the coating compound. It would appear therefore that while the specific gravity of one metal may be compared with that of another metal, or the specific gravity of one class of asphaltum coating compound may be compared with another, as to its probable resistance to flow, we cannot compare materials of wholly different natures with each other, and judge of the relative resistance by the respective densities. The values of m for asphaltum coated double riveted wrought iron pipe when new varies with quality of the coating as follows:

m=.000036 in terms of r in feet, to m=.000044.

m=.000288 in terms of d in feet, to m=.000352.

The average value of m for such coating while in prime condition may be taken as m=.00033, in terms of d in feet. The average value of the coefficient of resistance in pipe thus coated is about n=.000325 in terms of d in feet. The average value of n for common cast iron pipe while clean is n=.0003938 in terms of d in feet.

Ordinary lead pipe gives m=.000135 in terms of d in feet, or C=86.07. In terms of r in feet, ordinary lead pipe gives m=.000016875, or C=243.20. Lead pipe varies in specific gravity, and the coefficient varies with the specific gravity. Very dense, smooth lead pipe gives values of C in terms of r as high as C=297.00 before the pipe becomes incrusted or scaled.

13—Value of C Where the Flow is in Contact with Different Classes of Perimeter at the Same Time.—

The sides of a channel may be rough and covered with vegetation while the bottom is smooth and clean. In such case the value of C will decrease as depth of flow increases, because of the gain in ratio of rough to smooth perimeter as depth increases. On the contrary the bottom may be rough, stony and irregular, while the sides are smooth, clean and regular. In the latter case the value of C will increase as depth of flow increases, because of the gain in ratio of smooth to rough perimeter as depth of flow increases. In all such cases it is necessary to arrive at the mean or the average roughness of the combined classes of perimeter. If the flow is two feet deep in a canal six feet wide on the bottom and the sides are smooth and vertical, while the bottom is rough and stony, let us suppose that the sides correspond with C =60, and the bottom with C=30. Then we have the two smooth sides equal 4 feet and the rough bottom equal 6 feet and the whole perimeter equal 10 feet.

Then, $\frac{4}{10}$=Smooth perimeter where C=60.

$\frac{6}{10}$= rough perimeter where C=30.

And $\frac{4\times 60}{10}=\frac{240}{10}=24$; $\frac{6\times 30}{10}=\frac{180}{10}=18$. And 24 +18= 42.

The value of C for this combination of perimeters would be 42.

14.—***Tables of Coefficients.***—In the following tables of coefficients as developed from the published data of experiments, the groups are arranged with reference to smoothness or roughness of wet perimeter. The remarks in regard to the available data for this purpose, which were made in the introductory to this volume, should not be forgotten. Only a part of the available data have been used, and that was simply a choice between evils in many cases. The writer is indebted to Mr. Charles D. Smith, C. E., of Visalia, California, for the data of the guagings by him of sixteen canals in the vicinity of Visalia, California. It is believed that these data, all of which are given the common name of "Visalia Canal," are good and reliable. The writer is also indebted to Mr. J. T. Fanning for a diagram of the results of experiments by him on cast iron pipes of diameters ranging from 4 inches to 96 inches, and exhibiting the average value of the coefficient in such pipes; and for guagings of the New Croton aqueduct recently, made by Mr. Fteley, and for numerous valuable suggestions. The writer is indebted to Mr. Otto Von Geldern, C. E., of San Francisco, for the guagings of the Sacramento river by C. E. Grunsky, C. E.

GROUP NO. 1, STRAIGHT LEAD PIPE. (Rennie.)

L'GTH FEET	DIAM. FEET	S SLOPE	V FEET SEC.	COEFFICIENT $m=\frac{S}{v^2}\times\sqrt{d^3}$	COEFFICIENT $C=\sqrt{\frac{v^2}{S\sqrt{d^3}}}$	$\sqrt{d^3}$ FEET
15.00	0.0417	.26666	5.00	.0000908	105.00	.008515

Straight lead pipe. (W. A. Provis.)

100.00	0.125	.02917	3.09	.0001350	86.07	.04419
80.00	0.125	.03646	3.396	.0001397	84.60	.04419
60.00	0.125	.04861	3.903	.0001410	84.21	.04419

The coefficients for pipes are in terms of diameter in feet.

Straight Lead Pipe. (W. A. Provis.)

L'GTH FEET	DIAM. FEET	S SLOPE	V FEET SEC.	COEFFICIENT $m=\frac{S}{v^2}\times\sqrt{d^3}$	COEFFICIENT $C=\sqrt{\frac{v^2}{S\sqrt{d^3}}}$	$\sqrt{d^3}$ FEET
40.	0.125	.07292	4.759	.0001422	83.86	.04419
20.	0.125	.14583	6.150	.0001703	76.55	.04419

REMARK.—The coefficient m or C, includes all resistances to flow, including the resistance to entry into the pipe. In such very short pipes, where the velocity is considerable, the effect of resistance to entry will materially affect the coefficient. For this reason a general pipe formula for ordinary lengths of pipe will not apply with accuracy to short tubes or very short pipes. A special formula for short pipes or tubes should be applied in such cases. It is not known whether all the above lead pipes of different lengths were of the same quality and in the same condition or not. It is probable that they were, and that the decrease in length of pipe and increase in velocity greatly affected the resistance to entry. The resistance to entry of a pipe cut off square and flush with the inner walls of the reservoir is always equal to .505 of the head generating the velocity of flow through such pipe. Hence in order to obtain the true coefficient of flow due only to the resistance of the inner circumference of the pipe, the entry head should first be deducted. The entry head$=\frac{v^2}{2g}\times .505$.

The data of experiments on very short pipes are not reliable, and should never be relied upon. They have no application to long pipes.

Lead Pipe—(Iben) Example of erroneous data.

L'GTH FEET	TOTAL HEAD FEET	DIAM. FEET	$\sqrt{d^3}$ FEET	ALLEGED VELOCITIES	COEFFICIENT $m=\frac{S}{v^2}\times\sqrt{d^3}$	COEFFICIENT $C=\sqrt{\frac{v^2}{S\sqrt{d^3}}}$
350.30	17.71	0.082	.02384	2.70	.000162[illegible]4	78.36
350.30	122.01	0.082	.02384	9.11	.0001000447	99.97

REMARK—Here are the alleged results of two experiments on the same pipe—the only difference in conditions being a change of head. As the length, diameter and roughness were absolutely the same in both cases, the only possible effect of varying the head would be that the velocity would vary directly as the square root of the head varied, and nothing else.

Where all the other conditions are constant, the velocity will vary directly as the square root of the head, and the resistance, or loss of head by friction, will vary directly as the square of the velocity. If this is not true, then the law of gravity and the law of friction as accepted by the scientists are necessarily erroneous, and all scientific calculations based upon those laws must fail.

In the first experiment with this pipe of constant length, diameter and roughness, the head was 17.71 feet, and velocity was 2.70 feet per second. As all conditions remained constant except an increase in head, then by the law of gravity and of friction we would have

$\sqrt{H} : \sqrt{H} :: v : v$; or 4.148 : 11.08 :: 2.70 : 7 21

In the last experiment, Iben makes v=9.11 instead of 7.21.

If the velocity was correct in the first experiment, or v=2.70, then the head lost by friction for this velocity was equal to the total head minus the head which remained to generate the 2 70 feet velocity. The head required to generate 2.70 feet per second velocity was $hv=\frac{v^2}{64.4}=\frac{(2\ 70)^2}{64.4}=0.1132$ ft. The head lost by friction at this velocity was therefore 17.71 —.1132=17.59 feet, and $v^2=7.29$. Now, if the law of friction is correct, to wit, that friction will increase in a constant diameter and length as the square of the velocity, then the loss of head in feet by friction in this pipe when the velocity increased to 9 11 feet per second, would be

$v^2 : v^2$ :: head lost : head lost, or

7.29 : 83.00 :: 17.59 : 211.00.

In other words in Iben's second experiment where the total head was only 122.00 feet, he was able to lose 211.00 feet by friction, and still have remaining 1.29 feet head to generate the 9.11 feet per second velocity, which is alleged to have occurred. It is conclusive that the laws of friction and of gravity are absurd, or such data are in error.

All correct experimental data of flow for the same length, diameter and roughness of pipe will necessarily develop the same value of either of the coefficients, n, m or C, regardless of all changes in head or velocity, because the ratio $\frac{S}{v^2}$ is necessarily constant in any given pipe. The foregoing illustration is given as a suggestion of a correct method of testing the value of such published data of flow as are now available. Most of such data are furnished by experiments of a century or more ago, and have been translated from one language to another and reduced from one system to another, and printed and reprinted until the accumulated errors, added to the original crude methods in vogue a century ago, render them of very uncertain value. The writer is aware that Fanning and other very eminent hydraulicians have been of opinion that m will decrease or C increase with the velocity in a constant diameter, but this theory is not sustained by the results of Fanning's experiments on a constant diameter (See Group No. 4) nor by the results of experiments by the writer (Group No. 3). That theory cannot be accepted without first rejecting the law of gravity and of resistance as now generally accepted. If C increases with an increased velocity in a constant diameter, it is obvious that resistance does not increase as rapidly as v^2, and hence the ratio $\frac{S}{v^2}$ would not be constant but would vary with the velocity. If that is true, then $v^2 = 2gH$—the fundamental law of gravity—is necessarily untrue, and all our learned discussions of equilibrium and of uniform flow are mere theoretical myths and rubbish. Either that theory or the law of gravity and resistance must be rejected, for both cannot stand. The experimental data now available afford as much evidence to sustain an opposite theory as to sustain the above theory, and hence these opposite results destroy both theories, and prove only the erroneousness of the data. The evidence to sustain one theory destroys that which sustains the opposite theory, and the laws of gravity and of resistance positively refute both theories, and establish the theory that m or C is constant for all velocities in a constant diameter, except as slightly affected by the resistance to entry into the pipe. If the entry to the pipe is in the form of the vena contracta, then the velocity cannot affect the value of C or m at all.

GROUP NO. 2. COATED PIPES.

Asphaltum Coated Pipe. [Hamilton Smith Jr.]

Lgth. Feet	Diam. Feet	$\sqrt{d^3}$ Feet	S Slope	v Feet Sec.	Coefficient. $m=\frac{S\sqrt{d^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{d^3}}}$
1200.00	2.154	3.161	.01641	12.605	.0003265	55.34
700.00	1.056	1.085	.00668	4.595	.0003432	54.00
700.00	1.056	1.085	.01428	6.962	.0003200	55.90
700.00	1.056	1.085	.02219	8.646	.0003220	55.73
700.00	1.056	1.085	.03319	10.706	.0003142	56.40
4440.00	1.416	1.685	.06672	20.143	.0002771	60.07
700.00	0.911	0.8695	.0085	4.712	.0003330	54.80
700.00	0.911	0.8695	.01334	6.094	.0003123	56.58
700.00	0.911	0 8695	.01695	6.927	.0003072	57.05
700.00	0.911	0.8695	.02559	8.659	.0003000	57.73
700.00	1.230	1.364	.01097	6.841	.00032000	55.90
700.00	1.230	1.364	.01227	7.314	.00031264	56.56
700.00	1.230	1.364	.01646	8.462	.00031356	56.48
700.00	1.230	1.364	.02470	10.593	.00030025	57.71
700.00	1.230	1.364	.03231	12.090	.00030150	57.58

REMARK.—The slight variation of C or m in the same diameter and length is due to errors in weir or orifice coefficients used in determining the velocities. The above pipes were double riveted lap seam wrought iron pipes put together like stove-pipe joints. Some of the velocities were determined by weir and others by orifice measurement. The difference in value of C for different diameters is due to difference in quality of the coating. (See § 12). In applying the above coefficients it should be remembered that these pipes were new and laid straight, and had free discharge and high velocities which would prevent any deposit in them. The proportion of asphaltum in the coating is not stated. This is important and should be known.

Cast Iron Asphaltum Coated Pipe.—[Lampe].

Legth Feet.	Diam. Feet	$\sqrt{d^3}$ Feet	S Slope.	v Feet Sec.	Coefficient $m=\frac{S\sqrt{d^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{d^3}}}$
26,000	1.373	1.609	.000594	1.577	.0003840	51.03
26,000	1.373	1.609	.001376	2.479	.0003600	52.69
26 000	1.373	1.609	.00163	2.709	.0003574	52.91
26,000	1.373	1.609	.00195	3.090	.0003300	55.04

REMARK.—This pipe had been in use five years. Velocity was judged of by reservoir contents and pressure guage. The last coefficient is probably the true one. As the velocities tabled in the constant length and diameter do not correspond with the slopes tabled, it is impossible to ascertain whether either of the coefficients are correct or not. Only one of them can be correct. The last one is about the average value of the coefficient for such coated pipes.

Cast Iron Asphaltum Coated Pipe. [D'Arcy].

L'gth. Feet	Diam. Feet	$\sqrt{d^3}$ Feet	S Slope	v Feet Sec	Coefficient $m=\frac{S\sqrt{d^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{d^3}}}$
365.00	0.6168	0.4844	.00027	0.673	.0002890	58.82
365.00	0.6168	0.4844	.00368	2.487	.0002882	58.90
365.00	0.6168	0.4844	.02250	6.342	.0002710	60.74
365.00	0.6168	0.4844	.10980	14.183	.0002644	61.50
365.00	0.6168	0.4844	.14591	16.168	.0002704	60.95

REMARK.—Velocities determined by orifice. Variation in C is due to error in orifice coefficients used. This pipe was quite short, and must have had a remarkably smooth coating. The coefficients developed by this pipe are too high for safe use in ordinary practice. Lap welded wrought iron pipe in long lengths with few joints, when coated with asphaltum and oil, give C=60.00. It will be noted that D'Arcy's data generally give the value of C too high. As would be expected from a series of experiments especially planned with reference to the most favorable conditions.

The weir and orifice coefficients should be standardized in the same manner as m or C, so that a given form of weir or orifice would have a unit coefficient which would vary with $\sqrt{r^3}$ for any dimensions of weir notch or orifice. The results would then be uniform and correct.

Such weir formula might take the form, $q=A\ \frac{2}{3}\sqrt{\frac{2gH\sqrt{r^3}}{m}}$

The value of m would depend upon the form of the weir only, and would apply to all dimensions of weirs of that given form. Before this kind of a weir formula could be successfully adopted, however, it would be necessary to so construct the weir as to suppress all contraction of the discharge, for the contraction seems to follow no law. (See Apendix.)

The Loch Katrine Cast Iron Pipe. Coated with Dr. Smith's Coal Pitch. [Gale].

Lgth 3¾ Miles	Diam. Feet	$\sqrt{d^3}$ Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{d^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{d^3}}}$
3¾ m.	4.00	8.00	.000947	3.458	.0006344	39.70

This pipe probably had large deposits of gravel in it. It was evidently very rough from some cause. We give its coefficient here simply because this particular pipe has been the subject of so much discussion. See Flynn's "Flow of Water," page 34, for remark of Rankine and Humber on this pipe.

GROUP NO. 3.

Clean cast iron pipes—not coated. (See § 12.) (Sullivan.)

L'gth Feet	Diam. Feet.	$\sqrt{d^3}$ Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{d^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{d^3}}}$
2,800	0.50	0.35355	.001	0.98	.00036828	52.08
2,800	1.00	1.00000	.001	1.648	.00036823	52.11
2,800	2.00	2.82842	.001	2.771	.00036837	52.10
2,800	0.50	0.35355	.004	1.960	.00036813	52.11
2,800	1.00	1.00000	.004	3.296	.000368203	52.11
2,800	2.00	2.82842	.004	5.540	.00036862	52.08

REMARK.—These experiments were the foundation of the writer's formula. They were made with the greatest possible care. The writer being aware that a weir or orifice coefficient determined by the use of one degree of convergence of the edges of the plate would not apply to another degree of convergence or divergence, and having discovered discrepancies of several per cent. in velocities thus determined, did not rely on such measurements in the above experiments, but erected a large measuring tank into which the pipe discharged. The velocities were then determined by the formula $v=\frac{\text{cubic feet second}}{\text{area in sq. feet}}$. The pipes were remarkably dense and smooth, and had never before been wet. They were laid straight and perfectly jointed. In doubling the diameters and increasing the head four times, as will be observed in the above table, it was the purpose to test the law of gravity as well as to test the effect upon the flow of doubling the diameter while the head remained constant. A study of the results thus obtained resulted in the formula for flow herein presented.

It may be remarked here that the coefficients developed by the experiments under these exceedingly favorable circumstances with absolutely clean, very dense, straight pipes, are not to be relied on for average weight cast iron pipes laid in the ordinary manner. For average weight new cast iron pipe, as long as it remains clean, m=.0004, and C=50.

The nature of the water which flows in a pipe which is not coated may materially roughen the walls and reduce the value of the coefficient in a very short time. Allowance should always be made for this deterioration by adopting diameters amply large.

GROUP No. 4.

Cement mortar lined wrought iron pipes,—(Fanning.)

L'gth Feet	Diameter Feet	$\sqrt{d^3}$ Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{d^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{d^3}}}$
8171.00	1.667	2.153	.00044	1.488	.0004300	48.22
8171.00	1.667	2.153	.00073	1.925	.0004241	48.56
8171.00	1.667	2.153	.00104	2.329	.0004130	49.20
8171.00	1.667	2.153	.00134	2.598	.0004274	48.38
8171.00	1.667	2.153	.00158	2.867	.0004139	49.15
8171.00	1.667	2.153	.00199	3.271	.0004004	49.97
8171.00	1.667	2.153	.00228	3.439	.0004151	49.08
8171.00	1.667	2.153	.00272	3.741	.0004183	48.92
8171.00	1.667	2.153	.00300	3.920	.0004203	48.78
8171.00	1.667	2.153	.00313	4.000	.0004212	48.72
8171.00	1 667	2.153	.00320	4.040	.0004221	48.67

REMARK.—This was a force main, and velocities were measured at the pump. Considering slight errors in calculations of slip, it is seen how nearly constant the coefficients are. If there were no errors of slip, &c., there would result but one constant value of m and C throughout, The above guagings were remarkably accurate if the conditions under which they were made be considered. They show great care and excellent judgment on the part of the experimentalist. Under more favorable conditions, still closer results would have been had. From the values of the coefficient it is probable that the lining of this pipe was one third sand and two-thirds cement. Neat cement linings develop higher values of C than the above, while the above coefficients agree closely with those for linings of one-third sand and two-thirds cement.

The value of C does not increase with an increased velocity in a constant diameter, as has been claimed by some authors. If so, the last value of C in the above table should be the greatest.

GROUP No. 5.

Wooden conduits, planed poplar, closely jointed. (D'Arcy & Bazin.)

L'gth Feet	R Feet	$\sqrt{R^3}$ Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
230.58	0.505	0.3589	.000475	1.666	.00006143	127.58
230.58	0.505	0.3589	.001076	2.519	.00006080	128.30
230.58	0.505	0.3589	.001899	3.372	.00006000	129.10
230.58	0.505	0.3589	.002911	4.225	.00005853	130.60
230.58	0.505	0.3589	.004272	5.068	.00005970	129.40
230.58	0.505	0.3589	.005063	5.527	.00005948	129.75
230.58	0.505	0.3589	.00576	5.914	.00006000	129.10
230.58	0.505	0.3589	.006614	6.373	.00005845	130.70

REMARK.—This conduit had a bottom width of 2.624 feet and was 1.64 feet in depth. The velocities were determined by weir measurement. The values of C developed illustrate the uncertain application of weir coefficients even in the same small channel and for small differences in head, and when applied by persons of great experience and sound judgment. The value of the true coefficient in this conduit was probably C=129.00 in each case. The value of the coefficient for planed wood surfaces will doubtless vary with the density of the wood. The coefficient will be greater in conduits in which the boards are laid parallel to the flow than where the flow is across the grain of the wood and the joints. Assuming that m=.00006 is the true coefficient in terms of r in feet for planed hard wood surfaces, we may reduce to terms of d in feet (See § 10) by multiplying by 8, and we have m=.00048 or C=45.64 in terms of d in feet. This permits of a direct comparison of the relative degrees of resistance to flow in wooden pipes of planed staves closely jointed, and in iron pipes, Thus

Lead pipes—C=85.00
Asphaltum coated pipes, C=56.00
Clean cast iron pipes, C=50.00
Clean planed hard wood, C=45.64
Cement (one third sand)—C=48.50

} All in terms of diameter in feet.

Wooden conduits, Planed boards. (D'Arcy & Bazin)

Surface Width Feet	R Feet	$\sqrt{R^3}$ Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
3.16	0.390	0.24355	.0015	2.61	.00005363	136.55
3.62	.537	.39350	.0015	3.23	.00005660	132.95
3.89	.632	.50240	.0015	3.71	.00005475	135.35
4.08	.717	.60710	.0015	4.04	.00005580	134.80
4.53	1.015	1.0225	.0015	5.00	.00006131	127.70
4.59	1.148	1.2300	.0015	5.54	.00006011	128.90

REMARK.—The velocities in this table were determined by surface floats and Pitot-D'Arcy tube measurements. The velocities thus determined are undoubtedly too high. The weir measurements given in the preceding table are more nearly correct. A large majority of the guagings by D'Arcy and Bazin were made by surface float and Pitot tube measurements of velocity. They are not reliable when so made. This table is introduced here to show that velocities thus determined are too high, and the fluctuating values of C show that this method of guaging is not at all reliable. Data of flow determined by such methods should be avoided. It is not intended to convey the idea that all of D'Arcy and Bazin's guagings are unreliable, but to show that such guagings as are made by surface floats or by Pitot tube are worthless, whether made by them or any one else. Some of D'Arcy's data are good. Actual tank measurement of the discharge is the only really accurate method of determining the velocity which has so far been adopted. Weir measurement can be made accurate by adopting unit coefficients for weirs similar to m or C as suggested in a remark under Group No, 2, and the Appendix I.

Unplaned boards, well jointed and without battens.

The average value of m=.000070 in terms of r in feet.

C=119.60 in terms of r in feet.

Ordinary Flume 6×6 feet,—Straight. Clarke

Length. Feet.	R Feet	$\sqrt{r^3}$ Feet	S Slope	v Feet Sec	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
2500.	1.45	1.746	.000435	2 94	.000088	106.30

REMARK—This flume is near Boston and is used to convey sewage. The grease and slime may affect the flow considerably, as well as the solid matter mixed with the sewage.

Rough Irrigation Flume. Highline Flume, Colorado. (Wilson)

Length Feet	R Feet	$\sqrt{r^3}$ Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
3000	4.50	9.546	00099432	6.7657	.00020733	69.50

Remark—This is a rough bench flume with many abrupt bends. For a cut and description of this flume see "Irrigation Engineering" by Herbert M. Wilson, C. E., pages 173 and 174. The bends reduce the value of C considerably below its value for a straight flume.

Group No. 6,

Stone and brick lined Channels.—Chazilly Canal. D'Arcy and Bazin.

Surface Width Feet	Depth Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
4.04	0.50	0.41	.0081	5.73	.000064765	124.29
4.10	0.70	0.57	.0081	7.52	.000061634	127.37
4.14	1.00	0.68	.0081	8.19	.000067713	121.52
4.18	1.20	0.77	.0081	8.75	.000071483	118.30

Remark—This canal is lined with smooth ashlar or cut stone. The gaugings were probably by surface floats or Pitot tube which accounts for the fluctuating values of C developed. If this is not the true cause, then the bottom and the sides to a depth of .70 feet must be very much smoother than the walls are above that depth. The last value of C is probably nearest the correct value. See § 13.

Roquefavour Aqueduct. Neat cement bottom. Brick sides. (D'Arcy & Bazin.)

Surface Width Feet	Depth Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
7.40	2.50	1.504	.00372	10.26	.0000652	123.85

Remark.—This aqueduct is nearly rectangular and at this depth of flow the smooth cement bottom forms more than half the wet perimeter. It should therefore develop a greater value of C than the stone lined Chazilly canal of the preceding table. In a smooth bottomed canal similar to this aqueduct where the bottom is much smoother than the sides, the value of C should be greatest for the least depths of flow, because as depth increases the proportion of the rougher side perimeter becomes greater.

Aqueduct de Crau. Hammer dressed stone. (D'Arcy & Bazin.

Surface Width Feet	Depth Feet	R Feet	S Slope	v Feet. Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
8.50	3.00	1.774	.00084	5.55	.0000668	122.57

Sudbury Conduit. Hard brick, well jointed. (Fteley & Stearns, 1880.)

L'gth Feet	Greatest Depth Feet	R Feet	S Slope	v Feet. Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
4,200	1.518	1.078	.0001928	1.827	.00006460	124.33
	2.037	1.385	.0001922	2.139	.00006851	120.82
	2.519	1.628	.0001924	2.372	.00007115	118.60
	3.561	2.049	.0001929	2.720	.00007648	114.35

REMARK.—Velocities measured by weir. Only four of these guagings are given because the slopes of water surface in the others are so different from the slope of the conduit and from each other as to show that equilibrium and uniform flow had not ensued when the guagings were made. The coefficients are remarkably high for a plain brick perimeter. The silt deposit on the bottom also affects the flow.

New Croton Aqueduct. Hard brick, well jointed (Fteley, 1895.)

Depth above center of Invert.	Area Sq. Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
1.10	9.24	0.7434	00013257	1.0969	.0000706267	118.97
1.50	14.12	1.0656	.00013257	1.4338	.0000709351	118.71
2.10	21.57	1.4886	.00013257	1.7731	.0000766000	114.26
3.00	33.04	2.0236	.00013257	2.1281	.0000842174	108.97
4.00	46.14	2.5137	.00013257	2.4102	.0000906000	105.08
5.20	62.20	2.9947	.00013257	2.6560	.0000974027	101.35
6.80	83.89	3.4998	.00013257	2.8894	.000103930	98.09
9.20	115.78	4.0062	00013257	3.0989	.000110689	95.05
11.00	136.93	4.1417	.00013257	3.1519	.000112500	94.28
12.50	150.55	4.0031	.00013257	3.0977	.000110600	95.09
12.842	152.81	3.9161	.00013357	3.0625	.000109530	95.55

REMARK.—Mr. Fteley states in his report that the velocities for depths below 1.90 feet are not as accurate as those for greater depths, as the bottom or invert has slight silt deposits. It is evident that the bottom is very much smoother than the sides, or the guaging apparatus was greatly at fault. With the assistance of a very smooth silted bottom the side walls and arch are apparently so rough as to run the value of C below its value for common brick masonry. This is a conduit of the horse shoe form and the velocities were measured by meter. The result does not commend meter guagings. From the slope of water surface it appears that uniform flow was attained in each case before the guagings were made. See § 13.

First Class Brick Conduits Washed Inside With Cement.* (Fteley.)

Name of Conduit	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
Sudbury	2.4588	.00020	3.029	.000084000	109.11
Cochituate	1.4170	.0000496	1.000	.000083637	109.34

*See "Water Supply Engineering" by J. T. Fanning, p. 445, Ninth Edition.

Washington, D. C., Aqueduct, Brick Conduit. Completed 1859. See Fanning, P 445.)

		R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
		1.8735	.00015	1.893	.000107218	96.60

REMARK.—This is about the correct value of C for ordinary brick perimeters after several year's use. Where specially smooth or scraped brick are used or a cement wash is applied the value of C will be greater. Pure cement linings in channels of uniform cross section and good alignment develop an average value of C=150.00 in terms of r in feet. The value of C will vary somewhat with different qualities and fineness of pure cement linings, and uniformity of the walls.

Sudbury Conduit.—Hard Brick With Surfaces Scraped. (Fteley & Stearns 1880.)

Greatest Depth Feet	R Feet	S Slope	v Ft. Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
0.719	0.493	.0001640	1.079	.000048763	143.21
1.055	0.762	.0001742	1.423	.000057200	132.28
1.076	0.778	.0000983	1.098	.000055950	133.65
1.187	0.858	.0000246	0.550	.000064450	124.85
1.224	0.885	.0001715	1.577	.000057600	131.79
1.328	0.957	.0000746	1.064	.000061700	127.75
1.415	1.016	.0000140	0.443	.000073000	117.10

Remark.—This conduit is of the horse shoe form and 600 feet in length. Velocities were determined by weir. The conduit has a grade S=.00016. Compare the slopes in the above table with that of the conduit. Also compare the depths of flow with the corresponding velocities tabled. It is quite remarkable to note the great changes in S for such very small changes in R in a uniform channel with a grade S=.00016. As the slope of water surface is so different from that of the bottom of the conduit, it necessarily follows that the depth of flow must have been different at each successive point along the conduit, and the value of r was different at each different point. The velocities were inversely as the depths or wetted cross sections and hence were greatest where the depths were least. Uniform flow had not occurred and hence the effective value of S could not be known.

A comparison of the values of C for this conduit with those for carefully dressed poplar conduits (Group No. 5) and for average weight clean cast iron pipes would show this brick surface to be smoother than either of the others. This is, of course, not the fact. Because of the great number of joints and resulting small irregularities of a brick wall, it is scarcely possible that such wall should be more uniform and smooth than a carefully constructed conduit of unplained boards of hard wood, unless the wall were coated. In the latter case the wetted perimeter would consist of the coating and not of brick. Such data, although from eminent authority, cannot be accepted. The last value is nearest correct.

Brick lined channel. (D'Arcy and Bazin)

Area Sq. Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
6.22	0.7554	.0049	6.69		118.67

REMARK.—Velocity determined by surface float and Pitot tube which almost invariably gives the mean velocity much too high. This error results in giving too great a value to C for ordinary plain brick perimeters.

Croton Aqueduct. Brick. Completed 1842. (See Fanning, Page 445).

	2.3415	.00021	2.218	.0001677	77.50

REMARK.—This conduit is of the horse shoe form. It probably contains deposits of gritty material which reduce C to so low a value.

Brooklyn Conduit. Brick. Completed 1859. (See Fanning Page 445)

	2.5241	.00010	1.588	.000159	79.30

REMARK.—As masonry conduits are permanent investments it is best to adopt a coefficient value low enough to allow for deposits and future deterioration of perimeter.

Concrete Conduits.—Old.—Different stages of ruin. (See Fanning, page 445).

Name of Conduit	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
Metz	0.915	.00100	2.783	.00011175	94.87
Pont du Gard	1.250	.00040	2.000	.00014000	84.52
Pont Pyla	0.6109	.00166	2.950	.00009110	104.77
Montpellier	0.25	.00030	0.716	.00007310	117.00

REMARK.—In response to a recent inquiry of the writer Mr. Fanning states that he visited these conduits a few years ago and that some of them appeared to be in excellent repair. They are constructed of hydraulic concrete, and are rectangular in form.

Spillway of Grosbois Reservoir. Ashlar laid in Cement. (D'Arcy and Bazin)

Surface Width Feet	Depth Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
5.98	0.36	0.324	.101	12.29	.00012331	90.05
6.01	.55	.467	.101	16.18	.00012313	90.12
6.05	.71	.580	.101	18.68	.00012786	88.45
6.07	.84	.662	.101	21.09	.00012230	90.42

Covered with a slimy deposit.

Tail race Grosbois reservoir. Ashlar laid in cement. (D'Arcy & Bazin.)

Surface Width Feet	Dep'h Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
6.00	0.49	0.424	.037	9.04	.00012499	89.45
6.10	.77	.620	.037	11.46	.00013750	85.28
6.10	.97	.745	.037	13.55	.00012958	87.85
6.10	1.16	.852	.037	15.08	.00012800	88.30

Covered with a light slimy deposit. Joints partly damaged. Surface float.

Grosbois Conduit. Horseshoe form. Stone masonry set in mortar. (D'Arcy & Bazin.)

6.35	1.66	0.98	.000305	1.32	.00017000	76.70
6.40	2.21	1.29	.000308	1.90	.00012503	89.44
6.46	2.75	1.49	.000331	2.12	.00013368	86.50
6.50	3.12	1.60	.000347	2.47	.000115118	93.27

REMARK.—Bottom is rougher than sides. No deposit. Joints not damaged. As D'Arcy & Bazin nearly always give the slope of the bottom of the conduit, it is probable that these guagings were made at different places along the conduit, as the slopes are different. The values of C may be attributed to the rough bottom and smooth sides and also to errors in guaging with Pitot tube and floats. For ascertaining the correct value of C for any given depth in such channels see § 13.

Groisbois Canal. Roughly hammered stone masonry. (D'Arcy &Bazin.)

Sur-face Wdth Feet	D'pth Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
3.50	0.90	0.62	.0600	13.93	.00015610	80.04
3.50	1.20	.71	.0290	11.23	.00013757	85.26
3.60	1.30	.84	.0141	8.36	.00015420	80.53
3.90	1.60	.88	.0121	7.58	.00017400	75.82

Remark.—From the difference in slope it is probable that these guagings were at different places where the roughness was different. Otherwise the guagings are at fault. C should be constant, unless the roughness of perimeter was different at different depths of flow.

Grosbois Canal. Stone Masonry. Broken Stones on the Bottom. (D'Arcy & Bazin.)

Sur-face Wdth Feet	D'pth Feet	R Feet	S Slope	v Feet Sec.	Coefficient $=m\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
6.80	1.50	0.88	.000648	1.47	.00024755	63.55
6.90	2.00	1.23	.000671	2.02	.00022434	66.76
6.90	2.40	1.40	.000683	2.34	.0002066	69.57
7.00	2.70	1.50	.000683	2.78	.0001624	78.45

Remark.—The effect of the loose, broken stones and mud deposits on the bottom, is to reduce the value of the coefficient C in the ratio that the mean of the different degrees of roughness increases. If the depth of flow were reduced to .50 foot, the value of C would not exceed 45, because the rough bottom perimeter would control. It is probable that if the sides alone were considered apart from the rough bottom the value of C would be 90.00. The value of C for different depths of flow in such channels will be different for each depth, and may be determined by the rule given in § 13. Compare with New Croton Aqueduct.

Grosbois Canal.—Stone Masonry in Bad Order. (D'Arcy and Bazin)

Surface Width Feet	Depth Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
6.80	1.60	1.07	.00030	1.12	.00026373	61.58
6.90	2.40	1.38	.00035	1.69	.00019865	70.95
7.00	2.90	1.57	.00033	1.92	.00017610	75.36
7.00	3.30	1.71	.00030	2.18	.00014115	84.18

REMARK.—Broken stones and mud on the bottom. Sides fairly smooth. Rule given in § 13 applies.

Solani Embankment. Sides of Stone Masonry, Stepped. (Cunningham).

Surface Width Feet	Depth Feet	R Fee	S Slope	v Feet Sec,	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
150.00	1.50	1.69	.000090	0.44	.00102128	31.28
150.00	2.30	2.26	.000148	0.87	.00066400	38.81
160.00	4.10	4.07	.000215	1.79	.00055090	42.61
164.00	9.10	7.84	.000215	3.43	.000401162	49.92
170.10	11.00	9.31	.000227	4.02	.000400944	49.94

REMARK—This channel has a bottom width of 150 feet. The side slopes are of stone masonry, built in steps. The steps are broken and sunken in many places. The bottom is of clay and boulders, very irregular, with bars of brick and boulders built across at frequent intervals to prevent scour. As depth of flow increases the ratio of smoother side perimeter increases, and the mean of the different degrees of roughness becomes of a lesser degree of roughness. Hence the value of C will increase with each increase in depth. See §13.

GROUP NO. 7, RUBBLE AND RIP-RAP.

Chazilly canal. Bottom of earth. One side wall of mortar rubble and the other side wall of dry laid rubble. (D'Arcy & Bazin.)

Surf. Wdth Feet	D'pth Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
8.50	1.30	1.00	.000525	1.01	.00051465	44.08
9.50	2.00	1.36	.000450	1.38	.00037470	51.66
9.80	2.40	1.54	.000462	1.58	.00035347	53.20
10.20	2.70	1.67	.000487	1.74	.00034730	53.66

REMARK—If these guagings were all made at the same point, the values of C show that the earth bottoms and rubble footings were much rougher than the masonry side walls higher up. See § 13.

Turlock canal rock cut. Partly excavated in slate rock, other parts of dry laid rubble and rip rap. ("IrrigationEngineering" by H. M. Wilson, p. 82-83.)

Surf. Wdth Feet	Dpth Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
50.00	10.00	5.90	.0015	7.50	.000382	51.18

This rock cut is 6,200 feet in length and forms part of the Turlock canal, California.

River Waal. Gravel bottom. Sides revetted with dry laid rubble. (Krayenhoff.)

1329	17.25	11.10	.0001044	3.165	.00038607	50.88

Description of this rubble,etc. from Beaumont's Geology, Paris, 1845.

Head Race, Kapnikbanya, Hungary. (Rittinger) Bottom is paved. Sides are of dry laid rubble not smooth as bottom.

	Depth Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
	0.26	0.213	.0038	1.37	.000200	70.71
	0.45	0.344	.0038	1.83	.000229	66.08

REMARK.—The value of C for rubble will depend on the size and shape of the stones. If the stones are small and laid closely the coefficient will be much greater than if large, irregular stones are laid with large spaces and projections.

Tail Race. Staukau. Hungary, Bottom paved. Sides of dry laid rubble. (Rittinger)

	Depth Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
	0.42	0.289	.0025	1.257	.0002457	63.80
	0.56	0.359	.0025	1.491	.0002390	64.73
	0.69	0.419	.0025	1.643	.0002500	63.25

Mill Race. Pricbram, Hungary. Very rough, irregular bottom of earth.
Side walls of dry laid rubble. Bottom 2.07 feet width. (Rittinger).

Bottom Width Feet	Depth Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
2.07	0.41	0.316	.0022	0.3 9	.00258236	19.675
2.07	0.44	0.336	.0022	0.588	.001239473	28.400
2.07	0.70	0.472	.0022	0.953	.0007855758	35.670
2.07	0.80	0.548	.0022	1.135	.000692858	37.987
2.07	0.86	0.560	.0022	1.190	.000651000	39.200
2.07	0.90	0.566	.0022	1 269	.000581962	41.450

REMARK.—This is a good illustration of the effect of a combination of perimeters of different degrees of roughness, which is referred to in § 13, The rough bottom and large rough rubble footings at the bottom of the side walls almost prevented any bottom velocity of flow.

Grosbois Canal.—Rough, trapezoidal canal. The bottom of earth; one side slope rip-rapped, the other of earth with some little vegetation.
The bottom is covered with stones and loose boulders. (D'Arcy and Bazin)

Surface Width Feet	Depth Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
9.10	1.70	1.05	.000936	1.08	.00087200	33.86
11.20	2.30	1.37	.000936	1.37	.00080000	35.36
12.40	2.60	1.52	.000957	1.56	.00073100	36.98
13.40	2.90	1.64	.000964	1.71	.00069231	38.00

GROUP NO. 8. IRRIGATION CANALS.

A series of California Irrigation Canals carefully guaged. By Charles D. Smith, C. E., of Visalia, California. (1895).

No. 1. A new canal in common loam just completed. Weir measurement.

		R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
		0.99	.0006	1.33	.000334	54.72

See Group No. 11.

No. 2. An old canal in common loam. In only fair condition. By meter.

R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
0.85	.001	1.52	.0003392	54.30

No. 3, An old canal in common loam recently cleaned but not punned. By weir.

0.93	.0004	1.11	.000291	58.62

No. 4. A canal in sandy gravel, good repair. Firm gravel. By meter.

1.40	.00302	4.02	.00030951	56.87

No 5. A new canal. In river sand. By meter.

1.16	.00175	2.33	.0004026	49.83

Nos. 6 and 7. Canals in clay with loose gravel on the bottom, otherwise clean. By meter.

1.34	.00177	3.46	.000229254	66.04
1.32	.00194	3.51	.000238620	64.73

Nos. 8, 9 and 10.—Canal in firm earth with clay bottoms. Good condition. By meter.

2.00	.0004	2.255	.00022249	67.08
3.35	.00001	0.531	.00021740	67.22
3.34	.0000375	1.032	.00021486	68.22

Nos. 11, 12 and 13.—Canals in very heavy, smooth earth, recentiy cleaned, trimmed and punned and put in excellent order. Guaged by weir.

0.92	.001165	2.36	.000184553	73.73
1.09	.0006	1.87	.000195000	71.61
1.13	.0006	1.88	.000203960	70.02

Nos. 14 and 15. Old canals grown up with weeds reaching nearly to the surface. Weir and meter,

1.13	.00060	0.868	.00095658	32.33
1.77	.00035	0.845	.001154	30.00

No. 16 —Very crooked old slough in firm earth. By meter.

0.91	.0030	3.086	.00027354	60.47

Group. No. 9.—Smooth Canals in Earth.

Mill race, Kagiswyl, Switzerland. Side slopes of firm earth, smooth. Bottom covered with fine gravel. Guaged by meter. (Epper.)

R Feet	S Slope	v Feet Sec.	Coefficient $m = \frac{S\sqrt{r^3}}{v^2}$	Coefficient $C = \sqrt{\frac{v^2}{S\sqrt{r^3}}}$
1.040	.001754	2.817	.00023447	65.30
1.387	.001255	3.139	.00020774	69.40
1.410	.001200	3.221	.000193711	71.85

Remark.—Gravelly bottom reduces C as depth decreases. See § 13.

Clean canal in firm earth and in best order. Straight. (Watt.)

Surf. Wdth Feet	Dpth Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m = \frac{S\sqrt{r^3}}{v^2}$	Coefficient $C = \sqrt{\frac{v^2}{S\sqrt{r^2}}}$
18.00	4.00	2.40	.0000631	1.134	.00018216	74.10

Mill race. Flachau, Hungary. Clean ditch in firm earth. (Rittinger.)

	0.55	0.467	.0020	1.953	.00016742	77.46
	0.86	0.703	.0020	2.199	.00024350	64.58

Remark.—This is an example of remarkably bad guaging. The writer has never known the smoothest and firmest perimeters of earth with best alignment to develop a higher value of C than 75.00. In a clean canal with earth perimeter there should be very little variation in C, especially for so small a change in depth. Rittinger's experimental data of flow are usually much better than the average of such data, but the above is evidently untrustworthy.

Linth canal. Grynau. Clean canal in common loam. Side slopes a little irregular. Rod floats. (Legler.)

Surf. Wd th Feet	Dpth Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m = \frac{S\sqrt{r^3}}{v^2}$	Coefficient $C = \sqrt{\frac{v^2}{S\sqrt{r^2}}}$
123.00		5.14	.00029	3.414	.0002900	58.74
		7.12	.00032	4.418	.0003115	56.70
		8.87	.00036	5.40	.0003278	55.30
		9.18	.00037	5.53	.00033648	54.52

Remark.—The slight irregularities of the side slopes cause the value of C to decrease as the depth increases and includes a greater proportion of side slope perimeter which is rougher than the bottom. The results of guagings in such channels as this are probably what led Kutter to suppose that the value of C would decrease with an increase in slope where R>1 meter: The perimeter above the usual depth of flow in a channel of any size whatever is exposed to freezing and thawing, the burrowing of insects and the growth of vegetation. The change of slope or of hydraulic radius has no effect upon the roughness. The value of C depends upon the mean of the different degrees of roughness. See Nos. 8, 9, 10, Group No. 8, and Solani Embankment, Group No. 6, where the hydraulic radii are both less and greater than one meter or 3.281 feet, and where the slopes increase with R. It will be seen that it is the roughness of perimeter alone that affects the unit value of C and that C varies with $\sqrt[4]{r^3}$ only, from its unit value as tabled for the same degree of roughness. Kutter's C should vary only as $\sqrt[4]{r}$ for any given degree of roughness, and for different degrees, it should vary as the mean of the roughness and as $\sqrt[4]{r}$, but should not be affected by the slope at all, because $\frac{S}{v^2}$ is necessarily constant for all slopes. The recent gaugings of the Mississippi entirely explode Kutter's theory.

GROUP No. 10. RIVERS.

Mississippi River, Carrolton, La. Bottom is fine sand and the sides of alluvium, fairly stable. (Miss. River Com. Report, 1882.)

Surface Width Feet	Depth Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt[4]{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt[4]{r^3}}}$
2647.00	93.00	63.10	.0000165	5.90	.00024000	64.54
2565.00	90.00	63.40	.0000127	5.08	.00025000	63.25
2582.00	92.00	57.20	.0000139	4.46	.00032220	58.31
2359.00	86.00	57.60	.0000097	2.95	.00048745	44.78
2423.00	89.00	57.70	.0000112	3.73	.00035300	59.72

REMARK.—The values of R were taken as nearly equal as could be selected from the Report so that slope alone would show its effect in connection with the various degrees of roughness at different depths. The writer acknowledges that he has little confidence in the correctness of these guagings, but the various slopes and velocities tabled probably bear some relation to the actual slopes and velocities. It does not appear that the value of either Kutter's C or that of the writer decreases as slope increases. The values of Kutter's C for the above guagings are given below, as transcribed from his work.

Surface Width Feet	Depth Feet	R Feet	S Slope	v Feet Sec.	Kutter's C	Kutter's n
2359.00	86.00	57.60	.0000097	2.95	124.8	.0452
2423.00	89.00	57.70	.0000112	3.73	146.7	.0354
2582.00	92.00	57.20	.0000139	4.46	158.2	.0290
2565.00	90.00	63.40	.0000127	5.08	179.0	.0261
2647.00	93.00	63.10	.0000165	5. 90	182.9	.0218

REMARK.—The value of the writer's C for a depth of 86 feet, in the first table above, corresponds with the average value of C for rough, sandy perimeters in rivers, and is probably about the true value for this place. These are double float, or mid-depth guagings. Kutter's n is not as constant or as good an index of roughness as is chaimed for it.

Sacramento River, Freeport, California. Bottom of shifting sand. Sides of earth. Straight reach. Guaged by meter. (C. E. Grunsky.)

Date of Guagings	Area Sq. Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
March 11th	14,540	23.45	.0000744	3.994	.00053000	43.45
" 14th	14,920	23.99	.0000786	4.157	.00052867	43.49
" 17th	14,880	23.92	.0000675	3.974	.00050000	44.72
" 18th	14.750	23.79	.0000750	3.897	.00057290	41.78
" 19th	14,690	23.69	.0000713	3.741	.00058720	41.27
" 28th	14,570	23.54	.0000778	3.383	.00077637	35.89
May 26th	12,160	19.93	.0000580	2.879	.00062246	40.08

REMARK.—The low water area at this place is 4,590 square feet. From the dates and areas given it will be seen that the guagings were made during high water, and that the river was not stationary, or that continual scour or fill was going on. Mr. Grunsky says in his report: "The river bottom is sand. The river is there (at Freeport) surcharged with sand brought in by its tributaries in quantities greater than the water can assort, according to volume and velocitiy of flow. At the high stages of the river the changes in the contours of the bottom are rapid and sometimes sudden. Boils are of frequent occurrence. The river is full of whirls" (Report, p. 86.) At pages 96, 97 of his report Mr. Grunsky says: To prepare a scale of discharge representing the volume of the river's flow at various elevations of the water surface, for a locality such as Freeport, was, in view of the shifting position of the river bottom, an uncertain undertaking. * * * Neither could any reasonably correct relation between water surface elevation and velocity be established."—Report Com. Public Works to Governor of California, 1894.

River Rhine in Rhine Forest. Bed of coarse gravel. (La Nicca.)

R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
0.42	.0142	2.332	.000710	37.50
0.76	.0142	4.526	.000483	45.50
1.21	.0142	6.032	.000520	44.47

Simme Canal. Canton Berme. Very coarse gravel. (Wampfler.)

1.32	.0170	5.993		37.37
1.36	.0116	4.491		40.38
1.82	.0065	4.92		38.94
1.87	.0070	5.373		40.16

Mississippi River. Columbus, Ky. Rocky bluffs and gravel. (Humphreys & Abbott.)

65.90	0000658	6.957	.0007516	36.48

Mississippi River, Vicksburg, Miss. Rocky bluffs and Gravel. (Humphreys & Abbot.

64.10	.0000638	6.949	.000678	38.40

River Izar. Coarse gravel bed. (Grebenau.)

6.04	.0025	7.212	.00071	37.54

River Rhine. Boulders and gravel. Large stones on the bottom.

21.65	.001	8.858	.001284	27.92

River Seine at Paris. Fairly regular reach. Guaged while rising. (Poiree.)

Area Sq. Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
1978	5.70	.000127	2.094	.0003940	50.38
2570	7.10	.000133	2.264	.0004909	45.13
3176	8.40	.000135	2.418	.0005620	42.18
3692	9.50	.000140	3.370	.0003578	52.85
4421	10 90	.000140	3.741	.0003600	52.71
5108	12.20	.000140	3.816	.0004090	49.44
6372	14.50	.000140	4.232	.0004316	48.12
6929	15.00	.000140	4.512	.0004000	50.00
8034	15.90	.000172	4.682	.0004974	44.84
8668	16.80	.000131	4.800	.0003915	50.54
9522	18.40	.000103	4.689	.0003700	51.98

REMARK.—The guagings were made by floats. Bazin says they are good. It is seen from the areas recorded that the river was rising. Considering the different degrees of roughness of the sides as the water rose above its usual depth of flow, and the great difficulty of ascertaining the true slope on a rapidly rising river the results are quite satisfactory The slope for r=15.90 is probably an error. See the discussion of these and other data by Gen. H. L. Abbot in the Journal of the Franklin Institute for May, June, July, 1873. The guagings of the Seine at Triel, Menlon and Poissy have been condemned because the water surface was affected by tidal oscillations as great as two feet while the guagings and slopes were taken. It was also rising at that time as shown by the areas. The slope of the water surface under such conditions could not be determined with any accuracy.

Ohio River, Point Pleasant, W. Va., Mid-depth floats. (Ellet).

Area Sq. Feet.	R Feet	S Slope	v Ft Sec	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
7218.00	6.72	.0000933	2.515	.000257	62.38

Great Nevka River, Surface floats. 8-10 rule applied. (Destrem).

15554.00	17.40	.0000149	2.049	.00025748	62.32

Mississippi River. Quincy, Ill. Sandy alluvium. (T. C. Clarke).

15911.00	9.87	.00007434	2.941	.00026526	61.40
51610.00	16.27	.00007434	3.898	.00032135	55.78

REMARK.—The first gauging at Quincy was at low water when the flow was entirely in contact with its usual perimeter which is somewhat smoother and less irregular than the banks above the usual low water depth. The second gauging was at high water after permanent high water conditions had obtained. The slope of water surface was the same for both stages of the river, showing that stationary conditions had occurred.

Speyerbach Creek. Firm earth bed. (Grebenau)

Area Sq. Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
30.20	1.54	.0004666	1.814	.00026931	60.93

River Neva. Surface floats. 8 10 rule applied. (Destrem).

43461.00	35.40	.000139	3.23	.00028065	59.70

River Elbe. Steep banks. Coarse gravel and small boulders. (Harlacher.)

Surface Width Feet	Depth Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
343 00	6.20	3.51	.00038	2.49	0004030	49.80
452.00	11.80	7.77	.00041	4.95	.0003708	52.00

REMARK.—In a channel like this with gravel and small boulders on the bottom the value of C for a depth of 1.50 feet would not exceed 40 if the channel were narrow. In a wide bottomed rough channel with steep banks smoother than the bottom, it will require a considerable depth of flow to include sufficient side wall to balance the rougher bottom perimeter. The above guagings were by meter.

River Salzach, Bavaria. At different places and stages. Meter. (Reich.)

R Feet	S Slope	V Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
3.45	.000280	2.686	.0002487	63.40
3.52	.000348	3.618	.0001760	75.40
4.96	.000290	3.510	.0002600	62.00
5.00	.000607	5.543	.0002190	67.60
5.20	.000410	5.094	.0001800	74.53
7.00	.00036	4.118	.000393	50.44

REMARK.—These guagings were made in 1885 with the meters then in use. The nature of the perimeter is not stated, but it is safe to state that no natural channel will develop a value of C as high as 75. The mill race at Pricbram with its masonry side walls and smooth clay bottom, and the smoothest, best aligned canals in firm earth and in perfect order, only give C=75. See next group.

GROUP No. 11—CANALS.

Mill race at Pricbram, Hungary. Side walls of masonry. Smooth clay bottom 1.88 feet width. Trapezoidal. (Rittinger.)

Depth Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
0.54	0.373	.0010	1.127	.00017937	74.67
0.66	0.425	.0010	1.254	.00017621	75.33

See Nos. 11, 12 and 13 in group No. 8, also see group No. 9.

Realtore canal. Common loam bed in only fair condition. (D'Arcy & Bazin.)

Surface Wdth Feet.	D pth Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
19.70	4.50	2 87	.00043	2.54	.000324667	55.51

Marseilles canal. Common loam bed in only fair condition. (D'Arcy & Bazin.)

		2.90	.00043	2.536	.00033	55.04

Henares canal, Spain. Common loam bed in fair condition. (See Fanning.)

	4.92	2.95	.000326	2.296	.000313328	56.40

Lauter canal. Gravelly soil. Bed in fair condition. (Strauss.)

29.50		1.82	.000664	2 106	.00036758	52.16

See Group No. 8, California canals.

Rivers, creeks and canals grouped according to roughness of perimeter at the given depths. Areas are given in all cases where known. Perimeters are described as fully as available information will permit. The guagings are good, bad and worthless. It is difficult to separate them without more precise knowledge of the exact conditions under which they were made. The slopes of some rivers were measured while the stream was rising and the velocities were taken when the stream was falling. A fair average value of C may be arrived at for each class of perimeters from what has already been shown together with the following groups.

Group No. 12.

Shallow canals grown up in weeds reaching nearly to the water surface.

Name of Channel	Area Sq. Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
Visalia Canal		1.77	.00035	0.845	.001154	30.00
Visalia Canal		1.13	.00060	0.868	.00095658	32.33

Group No. 13.

Large canals with quantities of weeds and bushes on the margins and shallow places.

Name of Channel	Area Sq. Feet	R Feet	S Slope	v Feet Sec.	Coefficient m	Coefficient C
Cavour Canal	799.10	5.58	.000357	2.60	.0006964	37.85
C. & O. C. Feeder	119.00	3.70	.0006985	2.723	.0006700	38.64
C. & O. C. Feeder	121.00	3.70	.0006985	3.032	.0005410	43.00

Group No. 14.

Large streams with very rough banks and with large stones and gravel on the bottom.

Name of Channel	Area Sq. Feet	R Feet	S Slope	v Feet Sec.	Coefficient m	Coefficient C
River Rhine	13725.50	21.65	.00100	8.858	.001284	27.92
River Rhine	4650.10	7.67	.00125	4.921	.001096	30.20
River Izar	1063.40	6.04	.00250	7.212	.000710	37.54

GROUP No. 15.

Shallow channels with very coarse gravel and small boulders on the bottom.

Name of Channel	Dpth Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
Schwarza River	0.95	0.80	.0090	2.528	.001007	31.47
Schwarza River	1.20	0.99	.0052	2.544	.0007914	35.55
Solani Embankment	1.50	1.69	.000090	0.440	.00102128	31.28
Simme Canal		1.32	.0170	5.993	.00072	37.37
River Rhine		0.42	.0142	2.332	.00071	37.50

GROUP No. 16.

Channels with one rough, stony bank and with gravel bottoms. One bank of earth.

Name of Channel	Dpth Feet	R Feet	S Slope	v Feet	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
M'ssissippi River*	88.00	65.90	.0000680	6.957	.0007516	36.48
M'ssissippi River†	100.00	64.10	.0000638	6.949	.0006780	38.40
Grosbois Canal‡	1.70	1.05	.0009360	1.080	.0008720	33.86
Grosbois Canal	2.30	1.37	.0009360	1.370	.0007997	35.36
Grosbois Canal	2.60	1.52	.000957	1.560	.0007310	36.98
Grosbois Canal	2.90	1.64	.000964	1.710	.0006923	38.00

*At Columbus, Kentucky. Bluff on left bank composed of strata of coarse sand, coarse brown clay, blue clay, fine sand, coarse gravel, limestone, pudding stone, iron ore.

†At Vicksburg, Miss. Bluff forms left bank and is composed of strata of blue clay, logs, carbonized wood, marine shells, sand full of shells, sandstone. See "Levees of the Mississippi River," by Humphreys & Abbot, pages 28, 29. (1867.)

‡Gravel and pebbles on the bottom. One side slope rip-rapped with rough stone—the other side slope of earth.

GROUP No. 17.

Channels in firm earth with low stumps and roots on the bottom.

Name of Channel	Area Sq. Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
Bayou Plaquemine	3560.00	18.30	.0002064	5.198	.000598	40.88
Bayou Plaquemine	4259.00	15.30	.0001437	3.959	.00054875	42.69

REMARK.—This bayou was guaged by Humphreys & Abbot with mid-depth floats. It is simply an overflowed coule, which was formerly covered by a thick forest of cypress trees. These trees were cut down and the water brought into the Plaquemine in 1770 by means of a small canal connecting with the Mississippi river. As the dirt washed from around the stumps the Navigation company had them recut. See "Levees of the Mississippi River," page 204, note. This bayou varies in width from 200 to 300 feet, and in depth from 20 to 35 feet. There is luxuriant plant growth along the margins.

GROUP No. 18.

Grosbois canal. Earth bed in bad repair, with many patches of vegetation.

Surface Width Feet	Dpth Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
10.10	1.70	1.06	.00042	0.89	.0005785	40 95
12.30	2 40	1.41	.00047	1.18	.00056505	42.06
13.50	2.80	1.60	.00047	1.31	.000554326	42 47
14.70	3 10	1.76	.00045	1 39	.000544	42.86

GROUP No. 19.

Channels with gravelly bottoms and rough, irregular banks

Name of Channel	Area Sq. Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m = \frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
Mississippi River	150355.00	57.40	.0000481	6.310	.00052400	43.69
Feeder Chazilly	11.30	1.04	.0004450	0.962	.00053100	43.40
Feeder Chazilly	18.80	1.41	.0009930	1.789	.00051450	44.10
River Rhine	19135.00	16 50	.0000997	3.575	.00051235	44.17
Seine (Poissy)	10400.00	17.80	.0000750	3.330	.00050800	44.37

GROUP No. 20.

Rivers and Canals with beds of sand and with irregular side Slopes of earth.

Name of Channel	Area Sq. Feet	R Feet	S Slope	v Feet Sec	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
Seine (Paris)	8034.00	15.90	.000172	4.682	.0004974	44.83
Seine (Paris)	2570.00	7.10	.000133	2.264	.0004909	45.13
Feeder Chazilly	22.20	1.54	.000986	1.959	.00049	45.16
Feeder Chazilly	9.50	0.96	.000792	1.234	.0004878	45.28
Feeder Grosbois	23.00	1.63	.000479	1.434	.00048456	45.40
Mississippi River	179502.00	64.50	.0000436	6.825	.0004845	45.42
Seine (Poissy)	9733.00	16.80	.0000670	3.101	.00048	45.65
Seine (Triel)	6375.00	12.40	.0000600	2.359	.0 047	46.12
Seine (Poissy)	8996.00	15.90	.0000620	2.911	.0004638	46.43
Feeder Chazilly	22.90	1.57	.0004350	1.401	.000456	46.83
Upper Miss River	5010.10	10.85	.0000289	1.509	.0004536	46.95
Saalach	96.76	1.34	.0011640	1.970	.00045238	47.02
Seine (Poissy)	7475.00	13.60	.0000500	2.372	.0004457	47.34

See Sacramento River, Group No, 10.

GROUP NO. 21.

Natural Channels with sandy gravel bottoms and fairly regular sides of earth. Canals in common loam in bad repcir

Name of Channel	Area Sq. Feet	R Feet	S Slope	v Feet Sec	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
Feeder Grosbois	19.40	1.41	.000858	1.815	.00043515	47.90
Feeder Grosbois	18.10	1.38	.00045	1.296	.00043280	48.07
Feeder Grosbois	27.20	1.71	.000441	1.510	.00043230	48.08
Seine (Paris)	6372.00	14.50	.00014	4.232	.0 04316	48.12
Feeder Grosbois	25.40	1.69	.00033	1.296	.00043155	48.14
Seine (Poissy,	7952.00	14.20	.000054	2.595	.00042900	48.28
Izar	300.10	1.85	.0025	3.997	.00042430	48.54
Feeder Grosbois	17.20	1.38	.00045	1.326	.000414	49.18
Feeder Grosbois	20.90	1.56	.000525	1.575	.0004138	49.23
Feeder Grosbois	32.00	1.85	.00 33	1.411	.0004168	49.00
Feeder Grosbois	22.90	1.56	.000842	1.998	.00041	49.39
Feeder Chazilly	14.10	1.18	.000929	1.703	.00041068	49.34
Seine (Paris)	5108.00	12.20	.00014	3.816	.0004 9	49.44
Visalia Canal		1.16	.00175	2.33	.0004026	49.83
Rhine	14149.80	9.72	.000112	2.91	.0004008	49.95
Seine (Paris	6929.00	15.00	.00014	4.512	.0004	50.00
Seine (Paris)	1978.00	5.70	.000127	2.094	.000394	50.38
Salzach	40 5.60	7.00	.00036	4.118	. 00393	50.44
Seine (Paris)	8668.00	16.80	.000131	4.80	.0003915	50.54
Feeder Grosbois	26.80	1.71	.000493	1.683	.00039	50.64
Feeder Grosbois	14.90	1.21	.000808	1.667	.000387	50.83
Feeder Grosbois	25.90	1.71	.000515	1.746	.0003775	51.46
Feeder Grosbois	15.40	1.30	.000555	1.480	.00037557	51.60
Seine (Paris)	9522.00	18.40	.000103	4.689	.00037	51.98
Lauter Canal	56.40	1.82	.000664	2.106	.00036756	52.16
Saalach	86.90	1.38	.0010357	2.155	.000362	52.56
Seine (Paris)	3692.00	9.50	.00014	3.37	.0003578	52.85
Seine (Paris)	4421.00	10.90	.00014	3.741	.00036	52.71

Group No. 22.

Channels revetted with rough angular rubble, dry laid; channels in firm earth with rough, uneven bottoms and irregular side slopes.

Name of Channel	Area Square Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
River Waal*	14782.00	11.10	.0001044	3.165	.00038607	50.88
Turlock Rock Cut		5.90	.0015	7.50	.000382	51.18
Mississippi River	134942.00	52.10	.0000303	5.558	.0003693	52.03
Mississippi River	193968.00	72.00	.0000205	5.929	.000356	53.00
Bayou La Fourche	286[illegible].00	15.70	.0000438	2.789	.000352	53.30
Bear River Canal		5.63	.00018939	2.67	.00035487	53.00
River Rhine	6304.00	11.20	.0000999	3.277	.0003486	53.57
Seine [Meulan]	6488.00	7.70	.000087	2.313	.00034734	53.66

*See Group No. 7.

Group No. 23.

New canals in loam, or light soil, just completed; old canals in similar soil with weeds along the margin; canals in fairly good condition but with pebbles on the bottom.

Name of Channel	Area Square Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
Visalia Canal		0.85	.0010	1.52	.0003392	54.30
Visalia Canal		0.99	.0006	1.33	.000334	54.72
Idaho M & I Canal		7.232	.00037878	4.70	.00033347	54.77
Feeder Grosbois	11.80	1.050	.00031	0.817	.000343	54.03

Remark—The flow in a new canal is never as great at first as it will become after the bed is saturated with water and the loose material is dissolved and deposited in the pores of the earth and in the little depressions and irregularities along the perimeter.

Group No. 24.

Clean channels with bottoms of fine gravel and sand well settled, and with banks of sandy loam; channels in sandy alluvium; canals in ordinary loam in fair condition but not recently cleaned. (Average for canals in ordinary condition).

Name of Channel	Area Sq. Feet	R Feet	S Slope	v Feet Sec.	Coefficient. $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
Marseilles Canal	66.00	2.90	.00043	2.536	.00033	55.04
Realtore Canal		2.87	.00043	2.540	.000324667	55.51
River Tiber	2355.00	9.40	.001306	3.413	.00032335	55.62
Seine (Meulan)	5982.00	7.10	.00009	2.31	.000318876	55.75
River Haine	306.40	5.70	.0001559	2.558	.0003213	55.71
Upper* Miss. River	51610.00	16.27	.00007434	3.898	.00032135	55.78
Henares Canal		2.95	.000326	2.296	.000313328	56.40
Mississippi River	78828.00	31.20	.0000223	3.523	.0003132	56.49
Visalia Canal		1.40	.00302	4.02	.00030951	56.87
Feeder Grosbois	18.00	1.42	.00029	1.26	.000309	56.89

* At Quincy, Illinois, gauged by Thomas C. Clarke, C. E.

Group No. 25.

Rivers and canals in good condition, having beds of fine sand and small pebbles, with fairly regular banks of firm loam.

Name of Channel	Area Sq. Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
Hockenbach Creek	10.30	0.88	.0007966	1.463	.000307287	57.00
Hockenbach Creek	10.50	0.87	.0007783	1.440	.00030463	57.28
Missssissippi River	195349.00	72.40	.0000171	5.887	.000304	57.35
Upper* Miss. River	3441.00	4.42	.0002227	2.611	.00030355	57.40
Feeder Grosbois	30.80	1.78	.000275	1.467	.0003034	57.48
Feeder Grosbois	10.90	0.96	.00025	0.886	.0003	57.74

* At Fort Snelling, Minnesota. It is apparent from the areas and hydraulic mean radii in Group 25, that the effect of rough or smooth perimeter is the same in a very large channel as in a very small one. These perimeters are almost exactly alike, and develop like coefficients, regardless of size or slope.

GROUP No. 26.

Rivers and canals in alluvial soil, or firm earth mixed with fine sand, in good condition, and free of stones and weeds.

Name of Channel	Area Square Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
Bayou La Fourche	3738.00	15.70	.0000447	3.076	.0002948	58.24
Visalia Canal		0.93	.(004	1.110	.000291	58.62
River Rhine	5341.00	7.60	.0001174	2.917	.00028905	58.83
Hubengraben	3.80	0.59	.0013	1.424	.00029	58.74
River Neva	43461.00	35.40	.0000139	3.230	.00028065	59.70
Marseilles Canal		3.386	.000333	2.720	.00028065	59.70

GROUP No. 27.

Canals in heavy loam in excellent repair; natural channels with very fine sand on firm and regular bottoms with sandy loam banks in excellent condition, free of weeds and stones.

Name of Channel	Area Square Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
Speyerbach Creek	30.20	1.54	.0004666	1.814	.00026931	60.93
Upper Miss. River	15911.00	9.87	.00007434	2.941	.00026526	61.40
Feeder Grosbois	24.20	1.57	.000246	1.362	.00026080	61.88
Great Nevka	15554.00	17 40	.0000149	2.049	.00025748	62.32
Ohio River	7218.00	6 72	.0000933	2.515	.00025700	62.38

GROUP No. 28.

Canals in smooth clay with loose pebbles on the bottom.

Name of Channel	Area Square Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
Visalia* Canal		1.32	.00194	3.510	.00023862	64.73
Feeder Grosbois	17.10	1.32	.000275	1.336	.00023366	65.37
Visalia Canal		1.34	.00177	3.460	.000229254	66.04

*A few weeds along the margin in patches.

GROUP No. 29.

Canals in very firm, heavy soil, with clay bottoms worn smooth, but not recently trimmed and punned; natural streams of good alignment with clay bottoms, and fine grained, firm and uniform alluvial banks, free of stones and vegetation.

Name of Channel	Area Sq. Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{\sqrt{v^2}}{S\sqrt{r^3}}}$
Visalia Canal		2.00	.000400	2.255	.00022249	67.08
Visalia Canal		3.35	.000010	0.531	.0002174	67.82
Yssel River	1930.00	15.90	.0001166	2.773	.00021728	67.84
Loch Katrine		2.525	.0001578	1.7126	.0002158	68.05
Visalia Canal		3.34	.0000375	1.032	.00021486	68.22
Bayou La Fourche	3025.00	13.00	.0000373	2.843	.0002145	68.27
Bayou La Fourche	2957.00	12.80	.0000366	2.807	.0002126	68.53

REMARK—The bottom of this portion of Bayou La Fourche is clay, and the banks are leveed. The banks are of heavy, alluvial soil. Its bends are few and gentle. There are no boils, whirls, nor eddies. It resembles an artificial channel very much. For a general description see "Levees of the Mississippi River", page 198.

GROUP No. 30.

Canals in very firm, smooth, dense earth, recently cleaned, trimmed and punned, and put in the best condition.

Name of Channel	Area Sq. Feet	R Feet	S Slope	v Feet Sec.	Coefficient $m=\frac{S\sqrt{r^3}}{v^2}$	Coefficient $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$
Visalia Canal		1.13	.00060	1.88	.00020396	70.02
Visalia Canal		1.09	.00060	1.87	.000195	71.61
Visalia Canal		0.92	.001165	2.36	.000184553	73.73
English Canal	50.00	2.40	.0000631	1.134	.00018216	74.10

15—Roughness of Perimeter Defined.—The foregoing tables of coefficients might be greatly enlarged by the addition thereto of the data of many other pipes and channels, but such matter would be simply cumulative. It is believed that the tables given cover all cases as accurately as the published data will permit, and it was not deemed neccessary to give but a few examples of each class in order to assist in selecting the value of the coefficient in any ordinary case. The inaccuracies which abound in the data of flow in all classes of pipes and channels are due in great part to the failure of weir and orifice coefficients. The writer is aware that a general belief in the accuracy of weir measurement has become very great, but the fact remains that such measurements are very frequently erroneous to a very considerable extent. Meter measurements of velocity are still less reliable When better methods are discovered and adopted, we shall have more reliable data than we now have. In the consideration of the degree of roughness of any given channel, the alignment, uniformity of cross-section, freedom from grit gravel, stones and vegetation, are not, by any means, all that are to be considered. The nature of the material in contact with the flow, as to density and compactness, is as important as any or all other features. The coefficients show that for clean canals in earth, the value varies from about 56 to 75 as the nature of the earth forming the perimeter varies. The amount of sand, and whether coarse or fine, which enters into the majority of different classes of earth, has a great effect upon the flow and upon the value of the coefficient. Every table of data of open channels abundantly proves the incorrectness of the idea that the character of the perimeter has no influence upon flow in very large channels. The Mississippi river at Columbus and Vicksburg with depths of 88 and 100 feet respectively, develop the same value of the coefficient as very small ditches having the same kind of perimeter. (See Group No. 16). There is no reason why this should not be the case, and it would be strange if it were not the the case. The flow in large rivers is nearly always overestimated, especially where meters or surface floats are used for determining the velocity. Insufficient attention has been given the character of the perimeter and its effect upon the flow of the water in contact therewith and affected by the reactions therefrom. The velocity of the film of water in contact with and affected by the sides and bottom has never been considered of great importance in determining the mean velocity of the whole cross section in large streams, and yet if this outer layer of water thus affected were deducted from the whole, at least one fourth the total area would be subtracted.

CHAPTER III,

Of the Deduction of the General Formulas.

16—Formulas in Terms of Diameter in Feet.—For large pipes and circular channels flowing full, a set of formulas in terms of diameter in feet will be most convenient. For small pipes the coefficients may be in terms of diameter in inches.

FORMULA FOR LOSS OF HEAD BY FRICTION.

By formula (10) § 6, the coefficient of friction n, is

$$n = \frac{h'' d\sqrt{d}}{l v^2} = \frac{S''}{v^2} \times \sqrt{d^3} \ldots\ldots (10)$$

By transposition in (10) we have the formula for loss of head in feet by friction

$$h'' = \frac{n\, l v^2}{d\sqrt{d}} = \frac{n l v^2}{\sqrt{d^3}} = \frac{n}{\sqrt{d^3}}\, l v^2 \ldots\ldots (16)$$

In which, h″ = total head in feet lost in the length in feet, *l*.

d = diameter of pipe in feet.

n = coefficient in terms of diameter in feet.

l = length of pipe in feet.

v = velocity in feet per second.

FORMULA FOR HEAD IN FEET LOST PER FOOT LENGTH OF PIPE.

As $n = \frac{S''}{v^2} \times \sqrt{d^3}$, we have by transposition,

$$S'' = \frac{n}{\sqrt{d^3}} \times v^2 \ldots\ldots (22)$$

In which,

S″ = head in feet lost by friction per foot length of pipe.

n = unit value of the coefficient of resistance which increases as v^2, and is inversely as $\sqrt{d^3}$.

FORMULA FOR MEAN VELOCITY OF FLOW.

By equation (12) the coefficient of velocity m, is

$$m=\frac{Hd\sqrt{d}}{l\,v^2}=\frac{S}{v^2}\times\sqrt{d^3}=\frac{S\sqrt{d^3}}{v^2}\ldots\ldots\ldots\ldots\ldots(12)$$

And by transposition in (12) we have,

$$v=\sqrt{\frac{H}{l}}\times\sqrt{\frac{d\sqrt{d}}{m}}=\sqrt{\frac{S\sqrt{d^3}}{m}}=\sqrt[4]{d^3}\times\sqrt{\frac{S}{m}}\ldots\ldots\ldots(23)$$

In which,

v = mean velocity in feet per second.

H = total head in feet, where discharge is free.

H = h″ + hv where discharge is throttled (§ 5).

l = length of pipe in feet.

d = diameter in feet.

m = coefficient of velocity determined in terms of d in feet.

Where the altitude is sufficient to affect the value of **g**, the formula may be written,

$$v=\sqrt{\frac{2gH\sqrt{d^3}}{m\,l}},\ \text{when } m=\frac{2gH\sqrt{d^3}}{l\,v^2};\ H=\frac{m l v^2}{2gd\sqrt{d}}$$

In this case the value of m must be found according to the value of 2gH at the given altitude.

Formula for Total Head Required to Generate a given Velocity.

$$H=\frac{m\,l\,v^2}{d\sqrt{d}}=\frac{m\,l\,v^2}{\sqrt{d^3}}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(24)$$

The slope (S) required to generate a given mean velocity is

$$S=\frac{m\,v^2}{d\sqrt{d}}=\frac{m}{\sqrt{d^3}}\times d^2\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(25)$$

To find the length in feet l, in which there must be the given head in feet H, or fall in feet equal H, in order to generate the given mean velocity v:—

$$l=\frac{H\,d\sqrt{d}}{m\,v^2}=\frac{H}{v^2}\times\frac{\sqrt{d^3}}{m}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(26)$$

FORMULAS IN TERMS OF CUBIC FEET PER SECOND AND DIAMETER IN FEET.

Let q = cubic feet per second discharged, = Area $\times$ velocity.
a = area of pipe in square feet.

Then $a = d^2 \times .7584$, and $v = \sqrt{\frac{H}{l} \times \frac{\sqrt{d^3}}{m}}$ whence

$$q = d^2\, .7854 \sqrt{\frac{H \sqrt{d^3}}{l\, m}} = d^2 .7854 \sqrt{\frac{S \sqrt{d^3}}{m}} \quad \ldots\ldots\ldots (27)$$

$$q \sqrt{l\, m} = d^2\, .7854 \sqrt{H \sqrt{d^3}} \text{ or } q \sqrt{m} = d^2 .7854 \sqrt{S \sqrt{d^3}}$$

$$\sqrt[4]{d^{11}} = \frac{q \sqrt{l\, m}}{.7854 \sqrt{H}}\,;\; d^{11} = \frac{q^4 (l\, m)^2}{.3805 H^2}, \text{ whence}$$

$$d = \sqrt[11]{\frac{(l\, m)^2 q^4}{.3805\, H^2}} \text{ or } d = \sqrt[11]{\frac{q^4\, m^2}{.3805\, S^2}} = \sqrt[11]{\frac{m^2}{.3805}} \times \sqrt[11]{\frac{q^4}{S^2}} \quad \ldots\ldots\ldots (28)$$

$$q = \sqrt{\frac{.61685\, S \sqrt{d^{11}}}{m}} = \sqrt{\frac{.6165}{m}} \times \sqrt{S \sqrt{d^{11}}} \ldots (29)$$

Hence the diameter in feet required to cause the discharge of a given number of cubic feet per second is

$$d = \sqrt[11]{\frac{q^4\, m^2}{.3805\, S^2}} \text{; or } d = \sqrt[11]{\frac{q^4 (l\, m)^2}{.3805\, H^2}} \quad \ldots\ldots\ldots (28)$$

If total loss of head is predetermined then

$$d = \sqrt[11]{\frac{q^4\, n^2\, l^2}{.3805 \times h''^2}} = \sqrt[11]{\frac{n^2}{.3805}} \times \sqrt[11]{\frac{q^4}{h''^2}} \times l^2 \ldots\ldots (\S\, 64)$$

And the slope required to cause a given diameter to discharge a given quantity in cubic feet per second will be

$$S = \frac{m\, q^2}{.61685 \sqrt{d^{11}}} = \frac{m}{.61685} \times \frac{q^2}{\sqrt{d^{11}}} \quad \ldots\ldots\ldots (30)$$

And,

$$H = \frac{q^2\, m\, l}{.61685 \sqrt{d^{11}}} = \frac{m}{.61685} \times \frac{q^2}{\sqrt{d^{11}}} \times l \quad \ldots\ldots\ldots (31)$$

$$h' = \frac{q^2 l n}{.616853\sqrt{d^{11}}} = \frac{n}{.616853} \times \frac{q^2}{\sqrt{d^{11}}} \times l \ldots\ldots (32)$$

$$n = \frac{h''\sqrt{d^{11}}\ .61685}{l\ q^2} \ldots\ldots (33)$$

$$m = \frac{H\ .61685\sqrt{d^{11}}}{l\ q^2} = \frac{S\sqrt{d^{11}} \times .61685}{q^2} \ldots\ldots (34)$$

$$l = \frac{H\ .61685\sqrt{d^{11}}}{m\ q^2} \ldots\ldots (35)$$

$$q = \sqrt{\frac{H.\sqrt{d^{11}}.61685}{m\ l}} = \sqrt{\frac{S\sqrt{d^{11}} \times .61685}{m}} \ldots (36)$$

As $v = \frac{q}{a}$ it appears from an inspection of (29) and (36) that the relative discharges will be as $\sqrt[4]{d^{11}}$.

The slopes, or heads and lengths being equal, then

$q : q :: \sqrt[4]{d^{11}} : \sqrt[4]{d^{11}}$, provided the roughnesses are equal.

(See Table No. 18, § 33.)

17 Formulas in Terms of Pressure, Diameter and Quantity.—Head in feet and pressure in pounds per square inch are convertible terms. Pressure increases directly as head increases, and the velocity will be proportional to either $\sqrt{H}$ or $\sqrt{P}$. When H=2.3041 feet, P=1 lb. per square inch. Hence the coefficients determined in terms of H will not apply in a formula in terms of P. The coefficients may, however, be converted from terms of H or S to terms of P as pointed out in § 10 and as follows:-

$P = H \times .434$, and $H = \frac{P}{.434} = P \times 2.3041$. Hence if we have the value of n or m in terms of H and d in feet, and wish to convert to terms of pressure in pounds per square inch P, and diameter in feet d, we divide the value of n or m in terms of H and d, by 2.3041, and the result is the value of n or m in terms of P and d.

$$n = \frac{P'\sqrt{d^3}}{l\ v^2} = \frac{P' d\sqrt{d}}{l\ v^2} \ldots\ldots (37)$$

$$P' = \frac{n\, l\, v^2}{\sqrt{d^3}} = \frac{n\, l\, v^2}{d\sqrt{d}} \text{ ....}(38)$$

In which,

P'=total lbs. per square inch pressure lost by friction.

n=coefficient of resistance in terms of P' and d.

Let P=total pressure in lbs. per square inch.

P'=total pressure lost by friction.

Pv=velocity pressure.

When the discharge is free, $P'=P-Pv$,

To find the pressure in pounds per square inch required to balance the friction and generate a mean velocity v, in feet per second:—

$$P = \frac{m\, l\, v^2}{d\sqrt{d}} = \frac{m}{\sqrt{d^3}} \times l\, v^2 \text{ ....}(39)$$

$$m = \frac{Pd\sqrt{d}}{l\, v^2} = \frac{P\sqrt{d^3}}{l\, v^2} \text{ ....}(40)$$

$$v = \sqrt{\frac{Pd\sqrt{d}}{l\, m}} = \sqrt{\frac{P\sqrt{d^3}}{l\, m}} = \sqrt[4]{d^3} \times \sqrt{\frac{P}{l\, m}} \text{ ....}(41)$$

$$l = \frac{P\sqrt{d^3}}{m v^2} \text{ ....}(42)$$

$$q = av = \sqrt[4]{d^{11}}\ \sqrt{P} \times .7854 \div \sqrt{m\, l}$$

$$q\sqrt{m\, l} = \sqrt[4]{d^{11}}\sqrt{P} \times .7854, \text{whence}$$

To find the diameter in feet to discharge the quantity q, in cubic feet per second, through the length l, when the total pressure is P:—

$$d = \sqrt[11]{\frac{(l\, m)^2 q^4}{.3805\, P^2}} = \sqrt[11]{\frac{m^2}{.3805}} \times \sqrt[11]{\frac{l^2\, q^4}{P^2}} \text{ ....}(43)$$

To find the total pressure required to balance the resistance and force the discharge of q cubic feet per second through a pipe of given length and diameter in feet (Lifting weight of water not included).

$$P = \frac{l\, m\, q^2}{.616853\sqrt{d^{11}}} = \frac{m}{.616853} \times \frac{q^2}{\sqrt{d^{11}}} \times l \text{ ....}(44)$$

To find the pressure lost by friction while discharging a given quantity in cubic feet per second:—

$$P' = \frac{nlq^2}{.61685\sqrt{d^{11}}} \quad \text{.......... (45)}$$

The length of pipe in feet through which a given pressure in pounds per square inch will force a given quantity of discharge is,

$$l = \frac{P\sqrt{d^{11}} \times .61685}{mq^2} \quad \text{.......... (46)}$$

To find the coefficient of resistance n, in terms of q, d and P:—

$$n = \frac{P'\ \sqrt{d^{11}} \times .61685}{lq^2} \quad \text{.......... (47)}$$

To find the coefficient of velocity in terms of q, d and P:—

$$m = \frac{.61685\sqrt{d^{11}} \times P}{lq^2} \quad \text{.......... (48)}$$

18.—Formulas in Terms of Hydraulic Radius (r) and Slope (S).

$$H = \frac{m\ l\ v^2}{r\sqrt{r}} = \frac{m\ l\ v^2}{\sqrt{r^3}} \quad \text{.......... (49)}$$

$$l = \frac{H\sqrt{r^3}}{mv^2} = \frac{Hr\sqrt{r}}{mv^2} \quad \text{.......... (50)}$$

$$m = \frac{Hr\sqrt{r}}{l\ v^2} = \frac{S\sqrt{r^3}}{v^2} = \frac{S}{v^2} \times \sqrt{r^3} \quad \text{.......... (51)}$$

$$n = \frac{h'' r\sqrt{r}}{l\ v^2} = \frac{S''\sqrt{r^3}}{v^2} = \frac{S''}{v^2} \times \sqrt{r^3} \quad \text{.......... (52)}$$

The length in which there must be a fall of one foot in order to generate any given mean velocity v, is

$$l = \frac{r\sqrt{r}}{mv^2} = \frac{\sqrt{r^3}}{mv^2} \quad \text{.......... (53)}$$

$$v = \sqrt{H \div \left(\frac{m\ l}{r\sqrt{r}}\right)} = \sqrt{\frac{H}{l} \times \frac{\sqrt{r^3}}{m}} = \sqrt{\frac{S\sqrt{r^3}}{m}} =$$

$$\sqrt{r^3} \times \sqrt{\frac{S}{m}} \quad \text{.......... (54)}$$

Area in square feet $=12.5664 \times r^2$. $q=av$.

$$q=12.5664\,r^2 \times \sqrt[4]{r^3} \times \sqrt{\frac{S}{m}} \qquad (55)$$

$q\sqrt{m}=\sqrt[4]{r^{11}} \times \sqrt{S} \times 12.5664$, whence,

$$m=\frac{S\sqrt{r^{11}} \times 157.9144}{q^2} \qquad (56)$$

$$n=\frac{S'' \sqrt{r^{11}} \times 157.9144}{q^2} \qquad (57)$$

$$S''=\frac{q^2 n}{157.9144 \sqrt{r^{11}}}=\frac{n}{157.9144} \times \frac{q^2}{\sqrt{r^{11}}} \qquad (58)$$

$$S=\frac{q^2 m}{157.9144\sqrt{r^{11}}} \qquad (59)$$

$$H=\frac{q^2 m\, l}{157.9144\sqrt{r^{11}}} \qquad (60)$$

$$h''=\frac{q^2 n\, l}{157.9144\sqrt{r^{11}}} \qquad (61)$$

$$S=\frac{mv^2}{r\sqrt{r}}=\frac{m}{\sqrt{r^3}} \times v^2 \qquad (62)$$

$$q=\sqrt{\frac{S\sqrt{r^{11}} \times 157.9144}{m}} \qquad (63)$$

$$r=\sqrt[11]{\frac{q^4 m^2}{24936.958 \times S^2}} \qquad (64)$$

$$d=(4r)=4.\sqrt[11]{\frac{q^4 m^2}{24936.958 \times S^2}} \qquad (65)$$

$$l=\frac{157.9144\, H\sqrt{r^{11}}}{mq^2} \qquad (66)$$

19.—Application and Limitations of the Foregoing Formulas.— As heretofore noted under the table of circles and of open channels (§ 3) r is not necessarily an index of capacity in open channels as it is in pipes and circular channels flowing full. Hence in open channels the formulas (63) and (55) for q will not necessarily give accurate results, unless the value of r was originally determined in terms of q when the channel was designed. In channels having side slopes of 2 to 1 the formula for q will usually apply quite accurately. For the same reason the formula for r does not apply to open channels in general, but only to those in which the value of r was determined, or is to be determined in terms of q. All the formulas apply with exactness to pipes and circular closed channels flowing full. All the formulas in terms of r, except those just noted, apply to all forms of open or closed channels. These exceptions in the case of open channels, do not, however, affect the general application of the coefficients, because the coefficient depends upon the relations of a to p which is always expressed in any given case by r which is equal $\frac{a}{p}$ in all cases, and the friction surface p, always bears the same relation to r that r^2 bears to the area in any given case whether r is an index of capacity or of length of perimeter or not.

20—Formulas in Which C is Used Instead of m.

In the tables of coefficients heretofore developed the values of both m and C were given in order that either form of the formula might be applied in any case at pleasure. All the formulas using the coefficient C instead of m may be deduced from the following:—

$v=C\sqrt[4]{r}\sqrt{r}\sqrt{S}$ .... (67)

$v=C\sqrt[4]{r}\sqrt{rS}$ .... (68)

$v=C\sqrt[4]{r^3}\sqrt{S}=C\sqrt{S\sqrt{r^3}}$ .... (69)

$v=C\sqrt[4]{d}\sqrt{d}\sqrt{S}$ .... (70)

$v=C\sqrt[4]{d}\sqrt{dS}$ .... (71)

$v=C\sqrt[4]{d^3}\sqrt{S}=C\sqrt{S\sqrt{d^3}}$ .... (72)

$v=C\sqrt[4]{d^3}\sqrt{P}\div\sqrt{l}$ .... (73)

$v=C\sqrt[4]{r^3}\sqrt{H}\div\sqrt{l}$ .... (74)

Area in square feet, $A = d^2 \times .7854$

Area in square feet, $A = r^2 \times 12.5664$

q=Av. The same limitations mentioned in the preceding section will also be observed in the formulas in this form relating to q and r in open channels.

FORMULAS IN TERMS OF DIAMETER IN FEET USING C.

By transposition in formula (72) we have,

$$\sqrt{S}=\frac{v}{C\sqrt[4]{d^3}} \quad \cdots\cdots (75)$$

$$S=\frac{v^2}{C^2\sqrt{d^3}} \quad \cdots\cdots (76)$$

$$C=\sqrt{\frac{v^2}{S\sqrt{d^3}}}=\frac{v}{\sqrt[4]{d^3}\times\sqrt{S}} \quad \cdots\cdots (77)$$

$$H=\frac{lv^2}{C^2\times\sqrt{d^3}} \quad \cdots\cdots (78)$$

$$l=\frac{C^2 H\sqrt{d^3}}{v^2} \quad \cdots\cdots (79)$$

$$q=d^2\times .7854\times C\times\sqrt[4]{d^3}\times\sqrt{S}=\sqrt[4]{d^{11}}\sqrt{S}\times C\times .7854 \cdots\cdots (80)$$

$$\sqrt[4]{d^{11}}=\frac{q}{C\sqrt{S}\times .7854};\quad d^{11}=\frac{q^4}{S^2C^4 .3805}$$

$$d=\sqrt[11]{\frac{q^4}{.3805\times C^4\times S^2}} \quad \cdots\cdots (81)$$

21.—Formulas in Terms of Hydraulic Radius in Feet Using C.

By transposition in formulas (69) and (74) we have

$$C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}=\frac{v}{\sqrt[4]{r^3}\times\sqrt{S}} \quad \cdots\cdots (82)$$

$$\sqrt{S}=\frac{v}{C\times\sqrt[4]{r^3}} \quad \cdots\cdots (83)$$

$$S=\left(\frac{v}{C\times\sqrt[4]{r^3}}\right)^2=\frac{v^2}{C^2\times\sqrt{r^3}} \quad \cdots\cdots (84)$$

$$H=\frac{l\,v^2}{C^2\times\sqrt{r^3}} \quad \cdots\cdots (85)$$

$$l=\frac{C^2 H\sqrt{r^3}}{v^2} \quad \cdots\cdots (86)$$

$$q=Av=12.5664\times r^2\times C\times\sqrt[4]{r^3}\times\sqrt{S}= \cdots\cdots$$
$$\sqrt[4]{r^{11}}\times C\sqrt{S}\times 12.5663 \cdots\cdots (87)$$

$$r = \sqrt[11]{\frac{q^4}{24936.958 \times C^4 \times S^2}} \quad \cdots\cdots (88)$$

$$d = (4r) = 4\sqrt[11]{\frac{q^4}{C^4 \ S^2 \ 24936.958}} \quad \cdots\cdots (89)$$

$$S = \frac{q^2}{C^2 \ \sqrt{r^{11}} \ 157.9144} \quad \cdots\cdots (90)$$

$$H = \frac{l \, q^2}{C^2 \ \sqrt{r^{11}} \ 157.9144} \quad \cdots\cdots (91)$$

$$l = \frac{157.9144 \ \sqrt{r^{11}} \ C^2 \ H}{q^2} \quad \cdots\cdots (92)$$

$$C = \frac{q}{12.5664 \ \sqrt{S} \times \sqrt[4]{r^{11}}} \quad \cdots\cdots (93)$$

A set of formulas in terms of pressure in pounds per square inch and diameter in feet or inches may be deduced in like manner from equation (73). It is not deemed necessary to deduce the formula in all its possible forms and terms, as that is a simple matter which may be performed at the pleasure of the person using it, and would require unnecessary space here.

22.—Special Formula for Vertical Pipes.—Because of the relation of H to l in all formulas the ordinary formulas for flow will not apply to a pipe in a vertical or nearly vertical position. In such case H and l increase at tne same rate, and hence $\frac{H}{l}=1$, regardless of the head or length. On account of this fact all the formulas of the different writers on hydraulics will give the same velocity for a head of a hundred feet as for a head of 1,000 feet. It is therefore necessary to use a special formula in such case. In a vertical pipe the water is supported at no point whatever by any portion of the pipe walls. The effect of gravity is not impeded except by the roughness of the pipe walls. In such vertical pipe there is a gain of one foot head for each foot of length. The resistance to entry and the pipe wall friction will be the only loss of head. Hence if the sum of their effects be deducted from the total head, the velocity should equal that due to the remainder of the head. On this theory the following formula is proposed:

$$v = \sqrt{\left(H - \left(\frac{m\,l}{r\sqrt{r}}\right)\right) 2\,g} =$$

$$8.02\sqrt{H - \left(\frac{m\,l}{\sqrt{r^3}}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (94)$$

The head, slope, velocity, or quantity may be found by the principles given in § 52, and table No. 24.

CHAPTER IV.

Of Tables for Rapid Calculation of Velocity and Discharge in Open and Closed Channels, Friction Loss, &c.

23. Table for Velocity and Discharge. Clean, Average Weight Cast Iron Pipes, Not Coated.—In tables No. 1 and No. 2 the diameters are given in inches, the areas in square feet, and the discharge in cubic feet per second.

How to use Tables No. 1 and No. 2.

To find the mean velocity in feet per second:—Multiply the quantity in column No. 5 opposite the given diameter in inches by $\sqrt{S}$. For $\sqrt{S}$, see table No. 15, § 30. For S, see Table No. 16, § 31.

To find the discharge in cubic feet per second:—Multiply the quantity in column No. 6 opposite the given diameter by $\sqrt{S}$.

$v = C \times \sqrt[4]{d^3}\ \sqrt{S}$. $q = AV = AC \times \sqrt[4]{d^3}\sqrt{S}$. Take d in inches.

For average weight clean cast iron pipe, $C = 7.756$ when $d =$ inches.

TABLE. No. 1.

Clean cast iron pipe, not coated.

Col. 1 Diam. Inch's	Col. 2 $\sqrt{d^3}$ Inches	Col. 3 $\sqrt[4]{d^3}$ Inches	Col. 4 Area Sq. Feet	Col. 5 For Velocity $C \times \sqrt[4]{d^3}$	Col. 6 For Discharge $AC \times \sqrt[4]{d^3}$
½	0.35355	0.5946	.001366	4.6117	.006300
¾	0.65227	0.8059	.003068	6.2505	.019176
1.00	1.	1.	.005454	7.7560	.042301
1.¼	1.3975	1.1820	.008522	9.1675	.078125
1.½	1.8360	1.3550	.01227	10.5093	.128949
1.¾	2.3152	1.5210	.01670	11.7968	.197006
2	2.8284	1.6810	.02232	13.0378	.291003
3	5.1961	2.2790	.04909	17.6759	.867709
4	8.	2.8284	.08726	21.9370	1.91422
5	11.1803	3.3439	.13630	25.9352	3.53496
6	14.6969	3.8340	.19635	29.7365	5.83876
7	18.5202	4.3040	.26730	33.3818	8.92295
8	22.6274	4.7570	.34910	36.8952	12.88014
9	27.	5.1960	.44180	40.3001	17.80458
10	31.6227	5.6231	.54540	43.6127	23.78636
11	36.4828	6.0400	.66000	46.8462	30.91849
12	41.5692	6.4470	.7854	50.0029	39.27000
13	46.8721	6.8460	.9218	53.0975	48.94527
14	52.3832	7.237	1.069	56.1301	60.00307
15	58.0747	7.622	1.227	59.1162	72.53557
16	64.	8.	1.396	62.0480	86.61900
17	70.0927	8.372	1.576	64.9332	102.3347
18	76.3675	8.738	1.767	67.7719	119.7529
19	82.8190	9.100	1.969	70.5796	138.8300
20	89.4427	9.457	2.182	73.3484	160.0462
21	96.2340	9.810	2.405	76.0863	182.9876
22	103.189	10.158	2.640	78.7854	208.0000
23	110.304	10.504	2.885	81.4690	235.0380
24	117.575	10.844	3.1416	84.1069	264.2274
25	125.	11.180	3.409	86.7120	295.6012
26	132.574	11.514	3.687	89.3025	329.2583
27	140.296	11.844	3.976	91.8620	365.2433
28	148.162	12.172	4.276	94.4060	403.6800
29	156.169	12.496	4.587	96.9189	444.5670
30	164.316	12.820	4.909	99.4319	488.1112
31	172.600	13.139	5.241	101.9061	534.0898
32	181.0193	13.456	5.585	104.3647	582.8768
33	189.5705	13.768	5.940	106.7846	634.3005
36	216.0000	14.698	7.069	113.9977	805.8497
40	252.8822	15.907	8.726	123.3747	1076.5676
44	291.8629	17.086	10.558	132.5190	1399.1356
48	332.5537	18.237	12.567	141.4461	1777.5531
54	396.8173	19.920	15.905	154.4995	2457.3145
60	464.7580	21.560	19.635	167.2193	3283.3509
72	606.9402	24.710	29.607	191.6507	5674.2022
84	769.8727	27.746	38.484	215.1979	8281.6759
96	940.6040	30.670	50.265	237.8765	11956.8622
120	1314.5341	36.250	78.540	281.1550	22081.9137

REMARK.—In large cast iron pipes, or in thick small pipes there is great liability to blow holes and rough places. The thicker the pipe shell is, the more liable it is to be rough. It might be well to take C=7.65 in terms of diameter in inches for cast iron pipes of 48 inches diameter or greater, and for other and smaller diameters that are equally thick as 48 inch pipe. Large pipes are never as perfect or as smooth as medium diameters and thicknesses.

This fact has led some engineers to conclude that the law of friction in pipes was slightly different in large pipes from what it is in medium diameters. It is claimed that this change occurs at about a diameter of 48 inches. It is due simply to the rougher casting of large pipes which require thickness. There is no change in the law of friction at any diameter whatever. Very small cast iron pipes are also cast thick to prevent breakage in handling and are usually as rough as the very large diameters. Pipes less than six inches diameter and over 36 inches diameter, are usually rougher to some extent than the intermediate diameters.

$$v=C\times\sqrt[4]{d^3}\times\sqrt{S}. \quad q=A\,C\times\sqrt[4]{d^3}\times\sqrt{S}$$

TABLE No. 2.

Asphaltum coated pipes. C=8.67, d=inches.

Col. 1 Diam Inch's	Col. 2 $\sqrt{d^3}$ Inches	Col. 3 $\sqrt[4]{d^3}$ Inches	Col. 4 Area Sq. Feet	Col. 5 $C\times\sqrt[4]{d^3}$ For Velocity.	Col. 6 $AC\times\sqrt[4]{d^3}$ For Discharge
6	14.6969	3.834	.19635	33.2407	6.42681
7	18.5202	4.304	.2673	37.3156	9.97445
8	22.6274	4.757	.3491	41.2432	14.39800
9	27.	5.196	.4418	44.9493	19.85860
10	31.6227	5.623	.5454	48.7522	26.58945
11	36.4828	6.040	.6600	52.3668	34.56208
12	41.5692	6.447	.7854	55.8955	43.90032
13	46.8721	6.846	.9218	59.3548	54.71325
14	52.3832	7.237	1.069	62.7448	67.07419
15	58.0747	7.622	1.227	66.0827	81.08347
16	64.	8.	1.396	69.3600	96.82656
17	70.0927	8.372	1.576	72.5852	114.39427
18	76.3675	8.738	1.767	75.7584	133.86509
19	82.8190	9.100	1.969	78.8970	155.34819
20	89.4427	9.457	2.182	81.9922	178.90698
21	96.2340	9.810	2.405	85.[illegible]527	204.55174
22	103.189	10.158	2.640	88.0698	232.50443
23	110.304	10.504	2.885	91.0697	262.73603
24	117.575	10.844	3.1416	94.0174	295.36541
25	125.	11.180	3.409	96.9306	330.43641
26	132.574	11.514	3.687	99.8264	368.05986
27	140.296	11.844	3.976	102.6874	407.88542
28	148.162	12.172	4.276	105.4312	450.82398
29	156.169	12.496	4.587	108.3403	496.95704
30	164.316	12.820	4.909	111.1494	545.63240

TABLE No. 2—Continued.

Col. 1 Diam. Inch's	Col. 2 $\sqrt{d^3}$ Inches	Col. 3 $\sqrt[4]{d^3}$ Inches	Col. 4 Area Sq. Feet.	Col. 5 $C\times\sqrt[4]{d^3}$ Velocity	Col. 6 $AC\times\sqrt[4]{d^3}$ For Disch'g.
31	172.600	13.139	5.241	113.9151	597.02919
32	181.0193	13.456	5.585	116.6635	651.56576
33	189.5705	13.768	5.940	119.4685	709.64324
34	198.2523	14.081	6.305	122.0822	769.72871
35	207.0628	14.390	6.681	124.7613	833.52624
36	216.	14.698	7.069	127.4316	900.80267
37	225.0622	15.002	7.467	130.0673	963.93283
38	234.2477	15.300	7.876	132.6510	1044.75927
40	252.8822	15.907	8.726	137.9137	1203.43495
44	291.8629	17.086	10.558	148.1356	1564.01587
48	332.5537	18.237	12.567	158.1148	1987.02869
54	396.8173	19.920	15.905	172.7064	2746.89529
60	464.7580	21.560	19.635	186.9252	3670.27630
72	606.9402	24.710	29.607	214.2357	6342.87637
84	769.8727	27.746	38.484	240.5478	9257.24164
96	940.6040	30.670	50.265	265.9089	13365.41086

REMARK—This table relates to asphaltum coated pipes—not to pipes coated with coal tar, nor to compound coatings made of only one part asphaltum. What is meant by asphaltum coated pipes is that class of pipes which have been properly coated with a compound composed of 18 to 20 per cent of crude petroleum and the remainder of asphaltum. The coating compound to be heated to 300 degrees, Fahr., and the pipe to remain submerged in the hot bath until the pipe metal attains the same temperature as that of the bath. Coal tar coatings do not form quite as smooth a surface as the above described coating, and hence do not develop as high values of C. If d is taken in feet, then $m=.00032$, and $C=55.90$ as the average value of the coefficients for asphaltum and oil coated pipes. The value of C or m will vary slightly with the quality or purity of the asphaltum used. (See group No. 2.)

24.—Table for Velocity and Discharge of Brick Lined Circular Conduits or Sewers Flowing Full.—In the following Table No. 3 the diameters are in feet, the areas in square feet, and the discharge in cubic feet per second. The coefficient is in terms of diameter in feet and is based upon the discharge of Washington, D. C., aqueduct,. (See Group 6.)

$v = \sqrt{\frac{S\sqrt{d^3}}{m}}$; or $v = C \times \sqrt[4]{d^3}\ \sqrt{S}$. $m = .0008577$; $C =$ 34.00 in terms of diameter in feet. $q = A\ C \times \sqrt[4]{d^3}\ \sqrt{S}$.

TABLE NO. 3

Circular brick conduits and sewers. C=34.00.

Col. 1 Diam. Feet	Col. 2 $\sqrt{d^3}$ Feet	Col. 3 $\sqrt[4]{d^3}$ Feet	Col. 4 Area Sq Feet	Col. 5 For Vel. $C \times \sqrt[4]{d^3}$	Col. 6 For Disch'g $AC \times \sqrt[4]{d^3}$
1.50	1.837	1.355	1.767	46.070	81.4057
2.00	2.828	1.681	3.142	57.154	179.5778
2.50	3.953	1.988	4.909	67.592	331.8091
3.00	5.196	2.279	7.068	77.486	537.6711
4.00	8.	2.828	12.566	96.166	1208.8016
5.00	11.180	3.344	19.635	113 696	2232.4210
6.00	14.697	3.834	28.274	130.356	3685.6855
7.00	18.520	4.304	38.485	146.336	5631.7410
8.00	22.627	4.757	50 266	161.738	8129.9223
9.00	27.	5.196	63.617	176.664	11238.8337
10.00	31.623	5.623	78.540	191.182	15015.4343
11.00	36.483	6.040	95.033	205.360	19515.9769
12.00	41.569	6.447	113.100	219.198	24791.2938
13.00	46.872	6.846	132.730	232 764	30894.7657
14.00	52.383	7.237	153.940	246.058	37878.1685

NOTE.—Compare the values of the coefficients of the new Croton aqueduct for a depth of 9 feet with those of the Washington aqueduct—both in group No. 6. The above value of C in terms of diameter in feet is about correct for plain brick.

25.—Egg Shaped Brick Sewers and Conduits.—In egg shaped sewers the vertical diameter is one and one-half times the horizontal or greatest transverse diameter. Radius of invert, $\frac{1}{6}$ vertical diameter. Radius of sides equal vertical diameter. Let d=greatest transverse diameter in feet.

a=area in square feet.

p=wetted perimeter in lineal feet.

r=hydraulic mean depth $= \frac{a}{p}$

Then, in egg shaped sewers and conduits,

$a = d^2 \times .284$ for $\frac{1}{3}$ full depth

$a = d^2 \times .755825$ for $\frac{2}{3}$ full depth.

$a = d^2 \times 1.148525$ for full depth.

The wet perimeter in lineal feet will be,

$p = d \times 1.3747$ for $\frac{1}{3}$ full depth.

$p = d \times 2\,3941$ for $\frac{2}{3}$ full depth.
$p = d \times 3.965$ for full depth.

The mean hydraulic depths, $\frac{a}{p} = r$, will be,

$r = d \times .2066$ for $\frac{1}{3}$ full depth.
$r = d \times .3157$ for $\frac{2}{3}$ full depth.
$r = d \times .2897$ for full depth.

See "Hydraulic Tables" by P. J. Flynn; Van Nostrand's Science Series No. 67, and also see "Treatise on Hydraulics" by Prof. Merriman, p. 235. (5th. Edition.)

TABLE FOR VELOCITIES AND DISCHARGES OF EGG SHAPED BRICK CONDUITS AND SEWERS FLOWING TWO-THIRDS FULL DEPTH.

As this class of conduits are not circular in form, the coefficient is in terms of hydraulic mean depth (r) in feet, and the value of the coefficient used in the following table is that developed by the Washington, D. C., aqueduct, (See Group No. 6). This table is to be used in the same manner as Tables Nos. 1, 2 and 3.

$v = C \times \sqrt[4]{r^3}\ \sqrt{S}$, and $q = AC \times \sqrt[4]{r^3}\ \sqrt{S}$. $C = 96.00$

Table No. 4.

Areas, hydraulic depths, velocities and discharges for ⅔ Full Depth. C=96.

Col. 0. Trans. Diam.	Col. 1 r Feet	Col. 1 $\sqrt{r^3}$ Feet	Col. 2 $\sqrt[4]{r^3}$ Feet	Col. 4 Area Sq. Ft	Column 5. For velocity $C \times \sqrt[4]{r^3}$	Column 6 For Dischg $AC \times \sqrt[4]{r^3}$
1.50	0.474	0.3263	0.5713	1.701	54.8443	93.2910
2.00	0.631	0.5012	0.7080	3.025	67.9680	205.6032
2.50	0.789	0.7008	0.8372	4.724	80.3712	379.3535
3.00	0.947	0.9216	0.9600	6.802	92.1600	626.8723
3.50	1.105	1.1615	1.0780	9.259	103.4880	958.1954
4.00	1.263	1.419	1.1910	12.093	114.3360	1382.6652
4.50	1.421	1.694	1.302	15.305	124.9920	1913.0025
5.00	1.579	1.984	1.408	18.895	135.1680	2555.3994
6.00	1.894	2.606	1.614	27.210	154.9440	4216.0262
7.00	2.210	3.285	1.812	37.035	173.9520	6442.3123
8.00	2.526	4.015	2.004	48.373	192.3840	9306.1912
9.00	2.841	4.789	2.188	61.222	210.0480	12859.5586
10.00	3.157	5.610	2.368	75.583	227.3280	17182.1322
11.00	3.473	6.472	2.544	91.455	244.2240	22335.5059

Small sewers should be circular in form. See § 55.

26—Formulas for Use in Connection With the Foregoing Tables.

In the tables for pipes and conduits are the tabular values of

$C\times\sqrt[4]{d^3}$, $C\times\sqrt[4]{r^3}$, $A\ C\times\sqrt[4]{d^3}$ and $A\ C\times\sqrt[4]{r^3}$.

Now $v=C\times\sqrt[4]{d^3}\times\sqrt{S}$. and $q=A\ C\times\sqrt[4]{d^3}\times\sqrt{S}$.

If the slope and mean velocity have been decided upon, then the value of $C\times\sqrt[4]{d^3}=\frac{v}{\sqrt{S}}$ and opposite this value of $C\times\sqrt[4]{d^3}$ is the required diameter to generate the given velocity.

If a given diameter is required to discharge a given number of cubic feet per second, then the grade or slope may be found thus:

$$\sqrt{S}=\frac{q}{A\ C\times\sqrt[4]{d^3}}$$

The grade to generate a given velocity in feet per second may be found thus:

$$\sqrt{S}=\frac{v}{C\times\sqrt[4]{d^3}}$$

If the quantity to be discharged and the grade are given, then the required diameter will be found thus:

$A\ C\times\sqrt[4]{d^3}\frac{q}{\sqrt{S}}$. Look for the diameter which corresponds to the value of $A\ C\times\sqrt[4]{d^3}$ in the table.

The general formulas already given are so simple that resort to these formulas is not necessary.

27—General Table of Values of r or d, With Roots·

TABLE NO. 5

r or d	$\sqrt{r}$ or $\sqrt{d}$	$\sqrt{r^3}$ or $\sqrt{d^3}$	$\sqrt[4]{r^3}$ or $\sqrt[4]{d^3}$
0.20	0.4472	0.089440	0.2990
.22	.4690	.103180	.3212
.24	.4899	.117576	.3429
.26	.5099	.132574	.3641
.28	.5291	.148148	.3849
.30	.5477	.164310	.4053
.32	.5656	.180992	.4244
.34	.5831	.198254	.4452
.36	.6000	.216000	.4647
.38	.6164	.234232	.4840
.40	.6324	.252960	.5030
.42	.6481	.272202	.5217
.44	.6633	.291852	.5402
.46	.6782	.311972	.5585
.48	.6928	.332544	.5767
.50	.7071	·353550	.5946
.52	.7211	.374972	.6124
.54	.7348	.396792	.6299
.56	.7483	.419048	.6473
.58	.7616	.441728	.6646
.60	.7746	.464760	.6817
.62	.7874	.488188	.6987
.64	.8	.512	.7155
.66	.8124	.536184	.7322
.68	.8246	.560728	.7488
.70	.8366	.585620	.7653
.72	.8485	.610920	.7816
.74	.8602	.636548	.7978
.76	.8718	.662568	.8139
.78	.8832	.688896	.8300
.80	.8944	.715520	.8459
.82	.9055	.742510	.8617
.84	.9155	.769020	.8774
.86	.9273	.797478	.8930
.88	.9380	.825440	.9086
.90	.9487	.853830	.9240
.92	.9591	.882372	.9394
.94	.9695	.911330	.9546
.96	.9798	.940608	.9698
.98	.9899	.970102	.9849
1.00	1.	1.	1.
1.02	1.010	1.0302	1.015
1.04	1.020	1.0600	1.029
1.06	1.029	1.0907	1.045
1.08	1.039	1.1521	1.059
1.10	1.049	1.1540	1.074
1.12	1.058	1.1850	1.089
1.14	1.068	1.2175	1.104
1.16	1.077	1.2 94	1.118
1.18	1.086	1.2815	1.132
1.20	1.095	1.3140	1.146
1.22	1.104	1.3469	1.160
1.24	1.114	1.3800	1.175
1.26	1.123	1.4150	1.189

TABLE No. 5.—Continued.

r or d	$\sqrt{r}$ or $\sqrt{d}$	$\sqrt{r^3}$ or $\sqrt{d^3}$	$\sqrt[4]{r^3}$ or $\sqrt[4]{d^3}$
1.28	1 131	1.4480	1.203
1.30	1.144	1.4872	1.217
1.32	1.150	1.5180	1.239
1.35	1.161	1.5673	1.252
1.40	1.183	1.6562	1.287
1.45	1.214	1.7603	1.326
1.50	1.225	1.8375	1.355
1.55	1.245	1.9297	1.399
1.60	1.265	2.0240	1.422
1.65	1.284	2.1186	1.455
1.70	1.304	2.2168	1.488
1.75	1.323	2.2852	1.511
1.80	1.341	2.4138	1.554
1.85	1.360	2.5160	1.568
1.90	1.378	2.6182	1.618
1.95	1.396	2.7122	1.647
2.	1.414	2.8284	1.663
2.05	1.431	2.9335	1.713
2.10	1.459	3.0639	1.750
2.15	1.466	3.1519	1.775
2.20	1.483	3.2626	1.806
2.25	1.500	3.3750	1.837
2.30	1.526	3.5098	1.873
2.35	1.533	3.6025	1.904
2.40	1.549	3.7176	1.928
2.45	1.565	3.8342	1.958
2.50	1.581	3.9525	1.988
2.55	1.597	4.0723	2.018
2.60	1.612	4.1912	2.047
2.65	1.638	4.3407	2.083
2.70	1.643	4.4361	2.106
2.75	1.658	4.5595	2.135
2.80	1.673	4.6844	2.164
2.85	1.688	4.8108	2.193
2.90	1.703	4.9387	2.222
2.95	1.717	5.0651	2.250
3.	1.732	5.1960	2.279
3.05	1.746	5.3253	2.308
3.10	1.761	5.4591	2.336
3.15	1.775	5.5912	2.364
3.20	1.789	5.7248	2.392
3.25	1.803	5.8597	2.421
3.30	1.816	5.9928	2.448
3.35	1.830	6.1305	2.476
3.40	1.844	6.2696	2.504
3.45	1.857	6.4066	2.531
3.50	1.871	6.5485	2.559
3.55	1.884	6.6882	2.586
3.60	1.897	6.8292	2.613
3.65	1.910	6.9715	2.640
3.70	1.923	7.1151	2.667
3.75	1.936	7.26 0	2.694
3.80	1.949	7.4062	2.721
3.85	1.962	7 5537	2.746
3.90	1.975	7.7025	2.775
3.95	1.987	7.8486	2.801

TABLE NO. 5.—Continued.

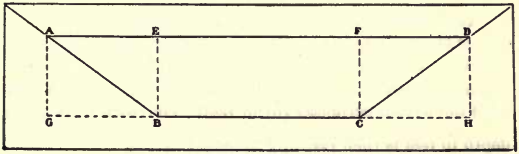
r or d	$\sqrt{r}$ or $\sqrt{d}$.	$\sqrt{r^3}$ or $\sqrt{d^3}$	$\sqrt[4]{r^3}$ or $\sqrt[4]{d^3}$
4.	2.	8.	2.828
4.05	2.012	8.1486	2.854
4.10	2.024	8.2984	2.881
4.15	2.037	8.4535	2.908
4.20	2.049	8.6058	2.933
4.25	2.061	8.7592	2.960
4.30	2.073	8.9139	2.986
4.35	2.085	9.0698	3.012
4.40	2.097	9.2268	3.037
4.45	2.109	9.3850	3.064
4.50	2.121	9.5445	3.089
4.55	2.133	9.7051	3.115
4.60	2.144	9.8624	3.140
4.65	2.156	10.0254	3.165
4.70	2.168	10.1896	3.192
4.75	2.179	10.3502	3.218
4.80	2.191	10.5168	3.243
4.85	2.202	10.6797	3.268
4.90	2.213	10.8437	3.293
4.95	2.225	11.0137	3.319
5.	2.236	11.1800	3.344
5.05	2.247	11.3473	3.369
5.10	2.258	11.5158	3.393
5.15	2.269	11.6853	3.419
5.20	2.280	11.8560	3.444
5.25	2.291	12.0277	3.468
5.30	2.302	12.2006	3.493
5.35	2.313	12.3745	3.518
5.40	2.323	12.5442	3.542
5.45	2.334	12.7203	3.567
5.50	2.345	12.8975	3.592
5.55	2.356	13.0758	3.616
5.60	2.366	13.2496	3.640
5.65	2.377	13.4300	3.665
5.70	2.388	13.6116	3.689
5.75	2.398	13.7885	3.713
5.80	2.408	13.9664	3.737
5.85	2.412	14.1453	3.761
5.90	2.429	14.3311	3.786
5.95	2.439	14.5120	3.810
6.	2.449	14.6940	3.834
6.05	2.460	14.8830	3.858
6.10	2.470	15.0670	3.881
6.15	2.480	15.2520	3.905
6.20	2.490	15.4380	3.929
6.25	2.500	15.6250	3.953
6.30	2.510	15.8130	3.977
6.35	2.520	16.0020	4.000
6.40	2.530	16.1920	4.024
6.45	2.540	16.3830	4.047
6.50	2.550	16.5750	4.071
6.55	2.560	16.7680	4.094
6.60	2.569	16.9554	4.118
6.65	2.579	17.1503	4.140
6.70	2.588	17.3396	4.164
6.75	2.598	17.5365	4.188

TABLE No. 5—Continued.

r or d	$\sqrt{r}$ or $\sqrt{d}$	$\sqrt{r^3}$ or $\sqrt{d^3}$	$\sqrt[4]{r^3}$ or $\sqrt[4]{d^3}$
6.80	2.607	17.7276	4.211
6.85	2.617	17.9264	4.234
6.90	2.627	18.1263	4.257
6.95	2.636	18.3202	4.280
7.	2.645	18.5150	4.304
7.05	2.655	18.7177	4.327
7.10	2.665	18.9215	4.350
7.15	2.674	19.1191	4.373
7.20	2.683	19.3176	4.395
7.25	2.692	19.5170	4.418
7.30	2.702	19.7246	4.441
7.35	2.711	19.9258	4.464
7.40	2.720	20.1280	4.487
7.45	2.729	20.3310	4.510
7.50	2.739	20.5425	4.532
7.55	2.748	20.7474	4.555
7.60	2.756	20.9456	4.578
7.65	2.766	21.1600	4.600
7.70	2.775	21.3675	4.622
7.75	2.784	21.5760	4.642
7.80	2.793	21.7854	4.668
7.85	2.802	22.0000	4.690
7.90	2.811	22.2069	4.712
7.95	2.819	22.4110	4.735
8.	2.828	22.6240	4.759
8.05	2.837	22.8378	4.779
8.10	2.846	23.0526	4.801
8.15	2.855	23.2682	4.823
8.20	2.864	23.4848	4.846
8.25	2.872	23.6940	4.868
8.30	2.881	23.9123	4.890
8.35	2.890	24.1315	4.912
8.40	2.898	24.3432	4.934
8.45	2.907	24.5641	4.956
8.50	2.915	24.7775	4.978
8.55	2.924	25.	5.
8.60	2.932	25.2152	5.013
8.65	2.941	25.4396	5.044
8.70	2.949	25.6563	5.066
8.75	2.958	25.8825	5.088
8.80	2.966	26.1008	5.109
8.85	2.975	26.3300	5.131
8.90	2.983	26.5487	5.153
8.95	2.992	26.7784	5.175
9.	3.	27.	5.196
9.05	3.008	27.2224	5.217
9.10	3.016	27.4456	5.239
9.15	3.025	27.6787	5.261
9.20	3.033	27.9036	5.283
9.25	3.041	28.1293	5.304
9.30	3.049	28.3557	5.325
9.35	3.058	28.5923	5.348
9.40	3.066	28.8204	5.369
9.45	3.074	29.0493	5.390
9.50	3.082	29.2790	5.411
9.55	3.091	29.5190	5.433

TABLE No. 5—Continued.

r or d	$\sqrt{r}$ or $\sqrt{d}$	$\sqrt{r^3}$ or $\sqrt{d^3}$	$\sqrt[4]{r^3}$ or $\sqrt[4]{d^3}$
9.60	3.098	29.7408	5.454
9.65	3 106	29.9729	5.475
9.70	3.114	30.2058	5.496
9.75	3.123	30.4492	5.518
9.80	3.131	30.6838	5.539
9.85	3.138	30.9093	5.560
9.90	3.146	31.1454	5.581
9.95	3.154	31.3823	5.602
10.	3.162	31.6200	5.623



28.—Tables for Velocity and Discharge of Trapezoidal Canals. In Fig. 1, let A, E, F, D, equal the width of the water surface in feet. Let B C equal bottom width of canal in feet, and E B or F C, equal greatest depth of water in feet.

TO FIND THE AREA IN SQUARE FEET.

Multiply E D by E B, or F A by F C. Or secondly: Add together the width of water surface and the bottom width in feet, and divide the sum by 2. Then multiply the quotient by the depth F C or E B in feet. In either case the result will equal the area in square feet.

TO FIND THE LENGTH A E OR F D IN FEET.

If the side slopes A B and D C are 1 to 1, then A E=E B, and F D=F C. If the side slopes are $1\frac{1}{2}$ horizontal to 1 vertical, then A E=E B$\times$1.50. If the side slopes are 2 horizontal to 1 vertical, then A E=E B$\times$2.00.

TO FIND THE WETTED PERIMETER IN LINEAL FEET.

The length of B C, or of the bottom width in feet, is, of course, always known. It is, therefore, only required to find the length in feet of the side slopes A B and D C which when added to B C, will equal the total wetted girth or perimeter. If the side slopes are 1 to 1, then the length A B or D C is equal to the diagonal of a square, or equal to the depth of water E B$\times$1.41421.

The length of either side slope for any rate of slope whatever is the same as the hypotenuse of a right angled triangle, and $A\,B=\sqrt{(A\,E)^2+(E\,B)^2}$ or $D\,C=\sqrt{(F\,D)^2+(F\,C)^2}$.

Adding together the lengths in A B, B C, and C D, we have the wetted perimeter (p) in feet. The hydraulic mean depth in feet is then $r=\frac{\text{area in square feet}}{\text{wet perimeter in feet}}=\frac{a}{p}$

In the following tables of trapezoidal canals the value of the area in square feet, and the hydraulic mean depth r, and of $\sqrt[4]{r^3}$ for each additional half foot depth of water in each canal is given, so that the velocity and discharge for each depth of flow may be readily ascertained. The value of m or C will depend upon the material forming the wetted perimeter, and the condition of the canal as to good or bad repair. The value of m or C may be selected from the tables of values developed in the groups of rivers and canals heretofore given. The following tables show the area for each depth of water. The discharge for any given depth will equal the area for that depth multiplied by the mean velocity. The slope required to generate any desired mean velocity in feet per second for any depth of flow will be

$$S=\frac{mv^2}{\sqrt{r^3}}, \text{ or } s=\left(\frac{v}{C\times\sqrt[4]{r^3}}\right)^2$$

The distance or length in feet (l) of canal in which there must be a fall of 1 foot in order to generate a given mean velocity in feet per second may be found by the formula,

$$l=\frac{\sqrt{r^3}}{mv^2}, \text{ or } l=\frac{C^2\sqrt{r^3}}{v^2}$$

HOW TO USE THE FOLLOWING TABLES.

To find the mean velocity for any given depth, multiply $\sqrt[4]{r^3}$ for that depth by $C\sqrt{S}$, or multiply $\sqrt[4]{r^3}$ by $\sqrt{\frac{S}{m}}$

To find the discharge in cubic feet per second for any given depth of flow, multiply $\sqrt[4]{r^3}$ by area$\times C\sqrt{S}$, or multiply $\sqrt[4]{r^3}$ by area$\times\sqrt{\frac{S}{m}}$. For values of $\sqrt{S}$, see § 30, Table No. 15.

TABLE No. 6.

Trapezoidal canal. Bottom width 2 feet. Side slopes 1 to 1.

Depth of Water in ft.	Area Square Feet	r Feet	$\sqrt{r^3}$ Feet	$\sqrt[4]{r^3}$ Feet
1.00	3.00	0.6213	0.4897	0.6998
1.50	5.25	0.8400	0.7699	0.8774
2.00	8.00	1.0450	1.068	1.034
2.50	11.25	1.240	1.380	1.177
3.00	15.00	1.430	1.710	1.308

REMARK.—In climates where the earth freezes in winter, side slopes of earth will not stand if they are steeper than 1½ to 1 even in very firm earth. In lighter soil in frosty climates the side slopes should vary from 1¾ to 1 to 3 to 1, according to the nature of the soil.

TABLE No. 7.

Trapezoidal canal. Bottom width 4 feet. Side slopes 1 to 1.

Depth of Water in ft.	Area Square Feet	r Feet	$\sqrt{r^3}$ Feet	$\sqrt[4]{r^3}$ Feet
1.00	5.00	0.732	0.6263	0.7914
1.50	8.25	1.000	1.000	1.000
2.00	12.00	1.2426	1.385	1.176
2.50	16.25	1.4700	1.782	1.335
3.00	21.00	1.762	2.339	1.530
3.50	26.25	1.888	2.594	1.611
4.00	32.00	2.089	3.019	1.737
4.50	38.25	2.280	3.443	1.855
5.00	45.00	2.480	3.905	1.976
5.50	52.25	2.670	4.383	2.089
6.00	60.00	2.861	4.836	2.199
6.50	68.25	3.050	5.327	2.308
7.00	77.00	3.235	5.819	2.412
8.00	96.00	3.605	6.844	2.616

TABLE No. 8.

Trapezoidal canal. Bottom width 4 feet. Side slopes 1½ to 1.

Depth of Water in ft	Area Square Feet	r Feet	$\sqrt{r^3}$ Feet.	$\sqrt[4]{r^3}$ Feet.
1.00	5.500	0.7231	0.6149	0.7841
1.50	9.375	1.0000	1.0000	1.000
2.00	14.000	1.2488	1.3960	1.181
2.50	19.375	1.4880	1.815	1.347
3.00	25.500	1.7200	2.256	1.502
3.50	32.375	1.8370	2.490	1.578
4.00	40.000	2.1710	3.099	1.788
4.50	48.375	2.4000	3.718	1.928
5.00	57.500	2.6100	4.216	2.053
6.00	78.000	3.0420	5.306	2.303
7.00	101.500	3.4730	6.472	2.544
8.00	128.000	3.9000	7.702	2.775

TABLE No. 9.

Trapezoidal canal. Bottom width 6 feet. Side slopes 1 to 1.

Depth of Water in ft.	Area Square Feet	r Feet	$\sqrt{r^3}$ Feet.	$\sqrt[4]{r^3}$ Feet.
1.00	7.00	0.7929	0.7060	0.8403
1.50	11.25	1.0980	1.1510	1.0725
2.00	16.00	1.3726	1.6090	1.2680
2.50	21.25	1.6280	2.077	1.4420
3.00	27.00	1.8639	2.545	1.595
3.50	33.25	2.0900	3.022	1.738
4.00	40.00	2.3100	3.511	1.874
4.50	47.25	2.5200	4.000	2.000
5.00	52.00	2.5800	4.144	2.036
5.50	63.25	2.9340	5.025	2.242
6.00	72.00	3.1345	5.548	2.355
7.00	91.00	3.5270	6.624	2.574
8.00	112.00	3.9100	7.731	2.780

TABLE No. 10.

Trapezoidal canal. Bottom width 8 feet. Side slopes 2 to 1.

Depth of Water Feet	Area Square Feet	r Feet	$\sqrt{r^3}$ Feet	$\sqrt[4]{r^3}$ Feet
1.00	10.00	0.8018	0.7179	0.8473
1.50	16.50	1.1210	1.187	1.089
2.00	24.00	1.4164	1.685	1.298
2.50	32.50	1.7000	2.216	1 489
3.00	42.00	1.9600	2.744	1.656
3.50	52.50	2.2200	3.308	1.819
4.00	64.00	2.470	3.882	1.970
4.50	76.50	2.720	4.486	2.118
5.00	90.00	2.964	5.103	2.259
6.00	120.00	3.445	6.394	2.528
7.00	154.00	3.910	7.732	2.781

TABLE No. 11.

Trapezoidal canal. Bottom width 8 feet. Side slopes 1 to 1.

Depth of Water Feet	Area Square Feet	r Feet	$\sqrt{r^3}$ Feet	$\sqrt[4]{r^3}$ Feet
1.00	9.00	0.831	0.7576	0.8704
1.50	14.25	1.164	1.256	1.121
2.00	20.00	1.464	1.771	1.331
2.50	26.25	1.741	2.297	1.516
3.00	33.00	2.000	2.828	1.682
3.50	40.25	2.248	3.370	1.836
4.00	48.00	2.485	3.917	1.979
4.50	56.25	2.710	4.461	2.112
5.00	65.00	2.935	5.029	2.242
5.50	74.25	3.153	5.598	2.366
6.00	84.00	3.364	6.170	2.484
7.00	105.00	3.777	7.341	2.709

TABLE No. 12.

Trapezoidal canal. Bottom Width 10 feet. Side slopes 1 to 1

Depth of Water Feet	Area Square Feet	r Feet	$\sqrt{r^3}$ Feet	$\sqrt[4]{r^3}$ Feet
1.00	11.00	0.8574	0.7939	0.891
2.00	24.00	1.5320	1.896	1.377
2.50	31.25	1.8300	2.475	1.574
3.00	39.00	2.1100	3.065	1.750
3.50	47.25	2.3743	3.658	1.912
4.00	56.00	2.6270	4.258	2.063
4.50	65.25	2.8700	4.862	2.205
5.00	75.00	3.1060	5.474	2.339
5.50	85.25	3.3350	6.090	2.468
6.00	96.00	3.5600	6.717	2.591
7.00	119.00	3.9930	7.979	2.825
8.00	144.00	4.4130	9.271	3.045

TABLE No. 13.

Trapezoidal canal. Bottom width 10 feet. Side slopes 2 to 1.

Depth of Water Feet	Area Square Feet	r Feet	$\sqrt{r^3}$ Feet	$\sqrt[4]{r^3}$ Feet
1.00	12.00	0.8222	0.7455	0.8634
2.00	28.00	1.478	1.797	1.340
2.50	37.50	1.774	2.363	1.537
3.00	48.00	2.049	2.933	1.713
3.50	59.50	2.320	3.534	1.880
4.00	72.00	2.581	4.146	2.036
4.50	85.50	2.840	4.786	2.187
5.00	100.00	3.093	5.440	2.332
5.50	115.50	3.340	6.104	2.471
6.00	132.00	3.584	6.785	2.605
7.00	168.00	4.068	8.493	2.914
8.00	208.00	4.543	9.683	3.111

29.—Table for Velocity and Discharge of Rectangular Channels, Flumes, Masonry Conduits etc.—The value of the coefficient to be used with the following table will depend upon the nature and condition of the lining of the flume or channel. According to the experiments of D'Arcy and Bazin, the average value of C for unplaned board flumes. well jointed, and without strips or battens on the inside is C=119.00, or m=.00007. For nicely dressed lumber flumes, well jointed and without battens on the inside, their experiments give C =128.00 as an average. If we refer to the last two flumes in Group No. 5, one at Boston gives C=106.30, and the Highline in Colorado gives C=70.00. The data of flow in wooden conduits are very unsatisfactory. The density of the wood, the closeness of joints, the alignment of the flume, gritty deposits etc.. all affect the value of C in any case. Where the flume is constructed of rough, very knotty, lumber and has battens on the inside to cover the joints, it is proable that the value of the coefficient will be about C=80.00, if the alignment of the flume is fairly direct. For channels lined with brick, ashlar, rubble etc., see the groups of such channels for value of C. See table No. 15 for value of $\sqrt{S}$.

$$v=C\times\sqrt[4]{r^3}\times\sqrt{S};\quad q=Av=AC\times\sqrt[4]{r^3}\times S$$

TABKE No. 14

Flumes and other rectangular channels.

Width Feet	Depth of Water feet.	Area Sq. Feet.	r Feet.	$\sqrt{r^3}$ Feet.	$\sqrt[4]{r^3}$ Feet.
1.5	0.50	0.75	0.300	0.1643	0.4054
1.5	1.00	1.50	.429	0.2810	0.5301
2.0	0.75	1.50	.429	0.2810	0.5301
2.0	1.50	3.00	.600	0.4647	0.6817
3.0	1.00	3.00	.600	0.4647	0.6817
3.0	1.50	4.50	.750	0.6495	0.8059
3.0	2.00	6.00	.860	0.7975	0.8930
4.0	1.50	6.00	.860	0.7975	0.8930
4.0	2.00	8.00	1.000	1.0000	1.0000
5.0	1.50	7.50	.937	0.9070	0.9524
5.0	2.00	10.00	1.111	1.171	1.082
5.0	3.00	15.00	1.363	1 591	1.261
6.0	2.00	12.00	1.200	1.314	1.147
6.0	2.50	15.00	1.363	1.591	1.261
6.0	3 00	18.00	1.500	1.837	1.355
6.0	4.00	24.00	1.714	2.244	1.498
8.0	3.00	24.00	1.714	2.244	1.498
8.0	4.00	32.00	2.000	2.828	1.682
8.0	5.00	40.00	2.222	3.312	1.820
8.0	6.00	48.00	2.400	3.718	1.928
10.0	4.00	40.00	2.222	3.312	1.820
10.0	5.00	50.00	2.500	3.953	1.988
10.0	6.00	60.00	2.727	4.503	2.122
10.0	7.00	70.00	2.916	4.979	2.231
12.0	4.00	48.00	2.400	3.718	1.928
12.0	5.00	60.00	2.727	4.503	2.122
12.0	6.00	72.00	3.000	5.196	2.279
12.0	7.00	84.00	3.230	5.805	2.410
14.0	5.00	70.00	2.916	4.979	2.231
14.0	6.00	84.00	3.230	5.805	2.410

30. Table of Values of Slopes S and $\sqrt{S}$.

The distance in feet l, in which there is a fall of one foot is

$$l=\frac{1}{S},\text{or } l=\frac{\sqrt{r^3}}{m\,v^2},\text{ or } l=\frac{C^2\times\sqrt{r^3}}{v^2}$$

TABLE No. 15.

Value of S and $\sqrt{S}$.

Fall per Mile in Feet	Fall One In	Slope S	$\sqrt{S}$
0.50	10560.00	.0000947	.009731
0.75	7042.25	.0001420	.011915
1.00	5280.00	.0001894	.013762
1.76	3000.00	.0003333	.018255
2.00	2640.00	.0003788	.019463
2.64	2000.00	.0005000	.022361
3.00	1760.00	.0005682	.023836
3.18	1660.00	.0006024	.024544
3.30	1600.00	.0006250	.025000
3.38	1560.00	.0006410	.025318
3.52	1500.00	.0006667	.025820
3.62	1460.00	.0006849	.026171
3.70	1427.00	.0007007	.026472
3.75	1408.00	.0007102	.026650
3.80	1389.00	.0007199	.026832
3.85	1371.00	.0007294	.027007
3.90	1354.00	.0007385	.027176
4.00	1320.00	.0007576	.027524
4.20	1257.00	.0007955	.028205
4.40	1200.00	.0008333	.028868
4.50	1173.00	.0008525	.029198
4.60	1148.00	.0008710	.029514
4.70	1123.00	.0008905	.029841
4.75	1111.00	.0009000	.030001
4.80	1100.00	.0009090	.030151
4.90	1078.00	.0009276	.030457
5.00	1056.00	.0009469	.030773
5.10	1035.00	.0009662	.031083
5.20	1015.00	.0009852	.031388
5.28	1000.00	.0010000	.031623
6.00	880.00	.0011364	.033710
7.00	754.30	.0013258	.036411
8.00	660.00	.0015151	.038925
9.00	586.60	.0017044	.041286
10.00	528.00	.0018940	.043519
11.00	480.00	.0020833	.045643
12.00	440.00	.0022730	.047673
13.00	406.10	.0024621	.049620
14.00	377.10	.0026515	.051493
15.00	352.00	.0028409	.053300
16.00	330.00	.0030303	.055048
17.00	310.60	.0032197	.056742
18.00	293.30	.0034090	.058388
19.00	277.90	.0035985	.059988
20 00	264.00	.0037878	.061546
21.00	251.40	.0039773	.063066
22.00	240.00	.0041666	.064549
23.00	229.60	.0043560	.066000
24.00	220.00	.0045454	.067419
25.00	211.20	.0047348	.068810
26.	203.10	.0049242	.070173
27.	195.20	.0051136	.071510

TABLE No. 15.—Continued.

Fall Per Mile in Feet	Fall, One In	Slope S	$\sqrt{S}$
28.	188.60	.0053030	.072822
29.	182.10	.0054924	.074111
30.	176.00	.0056818	.075378
31.	170.30	.0058712	.076624
32.	165.00	.0060606	.077850
33.	160.00	.0062500	.079057
34.	155.30	.0064394	.080246
35.	150.90	.0066288	.081417
36.	146.60	.0068182	.082572
37.	142 70	.0070075	.083711
38.	139.00	.0071969	.084835
39.	135.40	.0073864	.085944
40.	132.00	.0075757	.087039
41.	128.80	.0077651	.088120
42.	125.70	.0079545	.089188
43.	122.80	.0081439	.090244
44.	120.00	.0083333	.091287
45.	117.30	.0085227	.092319
46.	114.80	.0087121	.093339
47.	112.30	.0089015	.094348
48.	110.00	.0090909	.095346
49.	107.70	.0092803	.096334
50.	105.60	.0094697	.097312
51.	103.50	.0096591	.098281
52.	101.50	.0098485	.099241
52.80	100.00	.01	.10
55.80	96.00	.0104167	.102060
60.00	88.00	.0113636	.106600
70.00	75.43	.0132576	.115141
80.00	66 00	.0151515	.123091
90.00	58.66	.0170455	.130559
100.00	52.80	.0189394	.137620
120.00	44.00	.022727	.150756
140.00	37.71	.0265151	.162835
160.00	33.00	.0303030	.174077
180.00	29 33	.0340909	.184637
200.00	26.40	.0378787	.194625
240.00	22.00	.0416667	.213200
280.00	18.86	.0530303	.230283
320.00	16.50	.0606060	.246183
360.00	14.66	.0681818	.261116
400.00	13.20	.0757575	.275241
450.00	11.73	.0852273	.291937
500.00	10.56	.0946969	.307729
600.00	8 80	.1136364	.337100
700.00	7.54	.1325757	.364109
800.00	6.66	.1515151	.389249

31.—Table of Slopes for Average Weight Clean Cast Iron Pipes, Showing the Inclination Required in Each Diameter to Generate a Mean Velocity of One Foot per Sec-

ond, from which the Slope Required to Generate any other Mean Velocity may be Found.

$l=\frac{H d \sqrt{d}}{m v^2}$ If in this general formula we assign H=1 foot and v^2=1 foot, we have, $l=\frac{\sqrt{d^3}}{m}$ as the formula for finding the length in which there must be a total head, fall or slope of one foot to generate a velocity of one foot per second. For this class of pipe m is a constant, and in terms of diameter in feet m=.0004, or in terms of diameter in inches m=.0004$\times\sqrt{(12)^3}$=.01662768. Hence the length in feet l, in which there must be a head or fall of one foot in order to generate a mean velocity of one foot per second will be

$l=\frac{\sqrt{d^3}}{.0004}$ if d is taken in feet, and $s=\frac{.0004}{\sqrt{d^3}}$.

$l=\frac{\sqrt{d^3}}{.016628}$ if d is taken in inches, and $S=\frac{.016628}{\sqrt{d^3}}$.

The length in feet in which there must be a head or fall of one foot in order to generate any given or desired mean velocity in feet per second is,

$$l=\frac{d\sqrt{d}}{m\ v^2}=\frac{\sqrt{d^3}}{m\ v^2};\quad \text{whence } S=\frac{m\ v^2}{\sqrt{d^3}};\quad \sqrt{S}=\sqrt{\frac{m\ v^2}{\sqrt{d^3}}}.$$

In which v^2 is the square of the given or desired velocity in feet per second. The coefficient m may be in feet or in inches as above but the mean velocity will be in feet per second in either case.

The required slope S, to generate a mean velocity of one foot is,

$S=\frac{m}{\sqrt{d^3}}$, and to generate any velocity is $S=\frac{m}{\sqrt{d^3}}\times v^2$.

Hence if the slope for any diameter, which causes v=1 be taken from the following table, the required slope to cause any other velocity may be found at once by multiplying this slope from the table by the square of the desired velocity, v^2.

EXAMPLE.

From Table No. 16 it is seen that a slope of S=.0004 for a pipe 12 inches diameter, will generate a mean velocity of one foot per second. Required, the slope of a 12 inch pipe to generate 5 feet per second velocity:

SOLUTION;—From Table 16, take the slope for 1 foot velocity, S=.0004. Multiply this slope by the square of the required velocity, and we have,

$S=.0004\times(5)^2=.01$, and $l=\frac{1}{S}=\frac{1}{.01}=100$ feet. In other words there must be a fall of one foot in a length of 100 feet.

TABLE No. 16.

Table giving the required slope to generate a mean velocity of one foot per second in average weight clean cast iron pipes.

$$\text{For } v=1, \quad S=\frac{.016628}{\sqrt{d^3}}. \quad l=\frac{1}{S}=\frac{\sqrt{d^3}}{m}$$

Diameter Inches	$\sqrt{d^3}$ Inches	S (v=1)	Diameter Inches.	$\sqrt{d^3}$ Inches	S (v=1)
3	5.1961	.003200000	26	132.5740	.0001254242
4	8.	.002078500	27	140.2960	.0001185208
5	11.1803	.001487250	28	148.1620	.0001122285
6	14.6969	.001131156	29	156.1690	.0001064743
7	18.5202	.0008975830	30	164.3160	.0001012000
8	22.6274	.000734861	31	172.6000	.00009633825
9	27.	.000615852	32	181.0193	.0000918570
10	31.6227	.000525825	33	189.5705	.00008771406
11	36.4828	.0004557764	34	198.2523	.00008386790
12	41.5692	.000400000	35	207.0628	.00008030414
13	46.8721	.000354752	36	216.	.00007698100
14	52.3832	.000317239	37	225.0622	.00007388170
15	58.0747	.000286320	38	234.2477	.00007098480
16	64.	.000260000	40	252.8222	.00006575390
17	70.0927	.000237228	44	291.8629	.0000569720
18	76.3675	.0002177366	48	332.5537	.00005000094
19	82.8190	.0002007753	54	396.8173	.00004190341
20	89.4427	.0001859069	60	464.7580	.00003577776
21	96.2340	.0001727870	72	606.9402	.00002740000
22	103.1890	.0001611411	84	769.8727	.00002160000
23	110.3040	.0001516536	96	940.6040	:00001767800
24	117.5750	.0001414254	120	1314.5341	.00001264935
25	125.	.0001330240			

32.—Head in Feet Lost by Friction in Average Weight Clean Cast Iron Pipes for Different Velocities of Flow.

By equation (10) the coefficient of resistance or friction is

$$n=\frac{h'' d\sqrt{d}}{l\ v^2}=\frac{S''}{v^2}\times\sqrt{d^3}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

From which the formula for head lost by friction h″, is

$$h''=\frac{n\ l\ v^2}{d\sqrt{d}}=\frac{n\ l\ v^2}{\sqrt{d^3}}=\frac{n}{\sqrt{d^3}}\times l\ v^2\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

For a constant diameter and velocity the friction loss will be directly as the length in feet (l)of pipe, and will vary as v^2 for different velocities. For constant degrees of roughness of pipe n is a constant

As the friction loss is inversely as $\sqrt{d^3}$ and directly as the length and as v^2, the loss in one foot length of any diameter when $v^2=1$, will be $S''=\frac{n}{\sqrt{d^3}}$ and for any other velocity it will be $S''=\frac{n}{\sqrt{d^3}}\times v^2$and for any length in feet of pipe it will be $\frac{n}{\sqrt{d^3}}\times l\times v^2$. Hence if we form a table which shows the loss of head in feet for one foot length of pipe and for a velocity of one foot per second, the loss for any other length in feet will be found by multiplying the tabular quantity by the given length in feet l, and the loss for any velocity will be found by multiplying by the square of that velocity, v^2. (See § 9 and § 10.)

TABLE No. 17.

Table showing the loss of head in feet by friction in one foot length of clean cast iron pipe when $v^2=1$.

When $v^2=1$, the loss per foot length $=\frac{n}{\sqrt{d^3}}=\frac{.01637}{\sqrt{d^3}}$, when d=inches.

Diam. Inches	$\sqrt{d^3}$ Inches.	Head lost in feet per foot length	Diam. Inches	$\sqrt{d^3}$ Inches.	Hd lost in ft per ft length
3	5.1961	.003150440	25	125.	.00013096000
4	8.	.002046250	26	132.5740	.00012347820
5	11.1803	.0014641820	27	140.2960	.000116681710
6	14.6969	.0011138327	28	148.1620	.000110487170
7	18.5202	.0008839000	29	156.1690	.000104822340
8	22.6274	.0007234590	30	164.3160	.0000996251126
9	27.	.0006062963	31	172.6000	.000094843569
10	31.6227	.0005176661	32	181.0193	.000090435330
11	36.4828	.00044870400	33	189.5705	.000086353094
12	41.5692	.0003937900	34	198.2523	.000082571551
13	46.8721	.00034924806	35	207.0628	.000079058190
14	52.3832	.000312504772	36	216.	.000075787037
15	58.0747	.000281878340	37	225.0622	.000072735444
16	64.	.000255781250	38	234.2477	.000069883290
17	70.0927	.000233547850	40	252.8822	.000064730000
18	76.3675	.000214358200	44	291.8629	.00005608800
19	82.8190	.00019766 000	48	332.5537	.0000492'3100
20	89.4427	.000183022203	54	396.8173	.000041253239
21	96.2340	.000170166200	60	464.7580	.000035222632
22	103.1890	.000158640940	72	606.9402	.0000270000 0
23	110.3040	.000148408036	84	769.8727	.000 212632556
24	117.5750	.000139230278	96	940.6040	.0000174037000

REMARK.—As the loss here tabulated is for one foot length only and for a velocity of only one foot per second, none of the decimals should be cut off especially in case the pipe is of considerable length and the velocity is high, because the loss increases directly as the number of feet in length and also as v^2.

HOW TO USE TABLE No. 17.

The table shows the loss of head in feet by friction for one foot length of pipe of each diameter, and for a velocity of one foot per second. If the pipe is several hundred feet in length, then move the decimal point two places to the right. This will be equivalent to multiplying by 100, and will show the loss of head in feet per 100 feet length for $v^2=1$. Multiply this result by the square of the actual or proposed velocity in feet per second and the result is the actual loss per 100 feet length for that velocity. If the pipe is several thousand feet in length then take out from the table the loss for one foot length and $v^2=1$, and move the decimal point three places to the right. Multiply by the square of the actual or proposed velocity in feet per second. The result will be the actual loss of head in feet per 1,000 feet length of pipe. The loss of head in feet per mile (5280 feet) of pipe equals the loss for 1,000 feet multiplied by 5.28.

EXAMPLE.

What is the loss of head in feet in an 8 inch cast iron pipe 750 feet in length when the velocity is six feet per second ?

SOLUTION.

In table 17, opposite a diameter of 8 inches and in the third column the tabular loss for one foot length of 8 inch pipe when $v^2=1$ is .000723459. Multiplying this by 100 feet length by moving the decimal point two places to the right, and the loss for 100 feet =.0723459 when $v^2=1$. As the actual velocity is to be six feet per second, and as the loss varies as v^2 in any given diameter the last result must be multiplied by $(6)^2=36$, and we have the actual loss per 100 feet length= $.0723459\times36=2.60445$ feet, and for 750 feet the loss will be $2.60445\times7.5=19.5334$ feet.

33.—Formula and Table for Ascertaining the Loss of Head in Feet In any Class of Pipe While Discharging a Given Quantity In Cubic Feet Per Second.

Let h''=total head in feet lost by friction in the length l

d=diameter of pipe in feet.

q= cubic feet per second discharged.

Then the coefficient of resistance is

$$n=\frac{h''\sqrt{d^{11}}\times.616853}{l\,q^2}=\frac{S''\sqrt{d^{11}}\times.616853}{q^2}$$

And the head in feet lost by friction is

$$h''=\frac{l\,n\,q^2}{.61685\sqrt{d^{11}}}=\frac{l\,n}{.616853}\times\frac{q^2}{\sqrt{d^{11}}}.$$ (See equation 32.)

If l be taken =1 foot length of pipe, then n is constant for any given class of pipe, and we may take the quotient of $\frac{n}{.61685}$ as a constant, which, when multiplied by $\frac{q^2}{\sqrt{^{11}}}$ will equal the loss of head in feet per foot of pipe for the given discharge q. As q^2 ond v^2 are convertible terms we use the same coefficient value in terms of either q or v.

The value of n in terms of diameter in feet for ordinary cast iron pipes is n=.00039380.

The loss of head in one foot length is $h'' = \frac{n}{.616853} \times \frac{q^2}{\sqrt{d^{11}}}$. Then $\frac{n}{.616853} = \frac{.00039380}{.616853} = .00063840$. Whence $h'' = .00063840 \times \frac{q^2}{\sqrt{d^{11}}} \times l$. The following table gives value of $\sqrt{d^{11}}$.

The slope required to cause a given diameter to discharge q cubic feet, $S = \frac{m}{.616853} \times \frac{q^2}{\sqrt{d^{11}}}$

From tables Nos. 1 and 2, $q = a\,c \times \sqrt[4]{d^5} \times \sqrt{S}$, and $S = \left(\frac{q}{a\,c \times \sqrt[4]{d^5}}\right)^2$

TABLE No. 18.

Values of $\sqrt{d^{11}}$ when d is taken in feet. (See § 44, 45.)

Diameter Inches	Diameter Feet	$\sqrt{d^{11}}$ Feet	Diameter Inches	Diameter Feet	$\sqrt{d^{11}}$ Feet
3	0.2500	.0004883	24	2.00	45.25
4	0.3333	.002375	25	2.083	56.60
5	0.4167	.00811	26	2.166	70.17
6	0.5	.0221	27	2.25	86.50
7	0.5833	.05157	28	2.333	105.55
8	0.6667	.1075	29	2.416	128.00
9	0.75	.2055	30	2.50	154.40
10	0.8333	.3668	31	2.584	185.20
11	0.9167	.6198	32	2.666	219.90
12	1.0000	1.000	33	2.75	260.80
13	1.083	1.55	34	2.834	307.80
14	1.167	2.338	35	2.916	360.00
15	1.25	3.412	36	3.00	420.90
16	1.333	4.859	38	3.166	566.00
17	1.417	6.800	40	3.333	750.90
18	1.5	9.301	42	3.50	982.60
19	1.583	12.51	44	3.666	1268.00
20	1.667	16.62	48	4.00	2048.00
21	1.75	21.71	54	4.50	3914.00
22	1.833	28.01	60	5.00	6979.00
23	1.917	35.84	72	6.00	19050.00

For asphaltum coated pipes take n=.000325 in terms of diameter in feet. Then for such coated pipes,

$$h'' = \frac{n}{.616853} \times \frac{q^2}{\sqrt{d^{11}}} \times l = .00051864 \times \frac{q^2}{\sqrt{d^{11}}} \times l =$$

$$\frac{.00051864}{\sqrt{d^{11}}} \times q^2 l$$

q=cubic feet discharged per second.

l=length of pipe in feet.

d=diameter of pipe in feet. See § 44.

34.—Asphaltum Coated Pipes. Table for Ascertaining the Loss of Head in Feet for any Velocity.

By formula (16)

$$h'' = \frac{n\,l\,v^2}{d\sqrt{d}} = \frac{n}{\sqrt{d^3}} = \times\, l\,v^2 = \frac{l\,n}{\sqrt{d^3}} \times v^2$$

The average value of n for this class of pipe is n=.00032 in terms of diameter in feet, or n=.013302 in terms of diameter in inches. In order to find the loss of head in feet by friction per 100 feet length of pipe for any velocity, make l= 100, and insert the value of n, and we have

$$\text{Head lost per 100 feet length} = \frac{l\,n}{\sqrt{d^3}} \times v^2 = \frac{100 \times .013302}{\sqrt{d^3}}$$

$$\times v^2 = \frac{1.3302}{\sqrt{d^3}} \times v^2, \text{ if d is in inches, or } \frac{.032}{\sqrt{d^3}} \times v^2, \text{ if d is in feet.}$$

TABLE No. 19.

Table showing loss of head in feet per 100 feet length of asphaltum coated pipe when $v^2=1$. To find the loss for any other velocity multiply the tabular loss by the square of that velocity in feet per second.

Diameter In.	$\sqrt{d^3}$ Inches	Head in Ft. Lost per 100 Feet	Diameter In.	$\sqrt{d^3}$ Inches	Head in Feet Lost per 100 Feet
3	5.1961	.2560000	23	110.3040	.0120594
4	8.	.1662800	24	117.5750	.01131363
5	11.1803	.1190000	25	125.	.01064160
6	14.6969	.0905080	26	132.5740	.01003364
7	18.5202	.0718242	27	140.2960	.009481382
8	22.6274	.0588000	28	148.1620	.009000000
9	27.	.0492667	29	156.1690	.008517700
10	31.6227	.0420600	30	164.3160	.008095377
11	36.4828	.0364610	31	172.6000	.0077068366
12	41.5692	.0320000	32	181.0193	.0073483880
13	46.8721	.0283800	33	189.5705	.0070700000
14	52.3832	.0254000	34	198.2523	.0067096300
15	58.0747	.0229050	35	207.0628	.0064241400
16	64.0000	.02078436	36	216.	.006160000
17	70.0927	.01897770	38	234.2477	.005680000
18	76.3675	.01741840	40	252.8822	.005460000
19	82.8190	.01606140	42	272.2500	.004885950
20	89.4427	.01487200	44	291.8629	.004557600
21	96.2340	.01382250	48	332.5537	.004000000
22	103.1890	.01289000	54	396.8173	.003352000

What is the loss of head in feet by friction in a 22 inch coated pipe 2500 feet in length, when the velocity is six feet per second?

SOLUTION.

From table 19 we see that the loss in one hundred feet length of 22 inch pipe is .01289 feet head when $v^2=1$. If $v=6$, then $v^2=36$, and $.01289 \times 36 = .46404$ feet lost per 100 feet length of pipe. As there are 2,500 feet of pipe the total loss in the whole length will equal the loss for 100 feet length multiplied by the number of 100 feet, or 25, and we have $.46404 \times 25 = 11.601$ feet head lost in 2500 feet length when the velocity is 6 feet per second.

If this asphaltum coated pipe were replaced by an average weight clean cast iron pipe 22 inches in diameter, what would be the loss of head in the cast iron pipe for 6 feet velocity, and what slope would be required to cause the latter pipe to generate 6 feet per second velocity?

SOLUTION.

From table No. 17 the loss of head per one foot length of

22 inch cast iron pipe when $v^2=1$ is .00015864094. The loss per 100 feet =.015864094, and when v=6 the loss per 100 feet will be $.015864094\times(6)^2=.571107384$, and for 2500 feet, $.571107384\times25=14.2777$. The slope or fall in the 2500 feet must therefore be 14.2777—11.601=2.6767 feet greater for the cast iron pipe than for the asphaltum coated pipe.

The slope in either pipe which is required to generate the given velocity is

$$S=\frac{m}{\sqrt{d^3}}\times v^2$$

m=.0004 for cast iron

m=.00033 for asphaltum coating

These values of m are in terms of diameter in feet. The value of m may be converted to terms of diameter in inches by multiplying by $\sqrt{(12)^3}=41.5692$. (See §§ 10, 12 and Group No. 2, § 14.)

The slopes to generate any given velocity may be found from Tables No. 1 and No. 2 by the formula

$$S=\left(\frac{v}{C\times\sqrt[4]{d^3}}\right)^2, \text{ or } S=\left(\frac{q}{AC\times\sqrt[4]{d^3}}\right)^2, \text{or } S=\frac{m}{.61685}\times\frac{q^2}{\sqrt{d^{11}}}$$

Table No. 18 gives the different values of $\sqrt{d^{11}}$. Tables No. 1 and 2 give the values of $\sqrt[4]{d^3}$ and also of $AC\times\sqrt[4]{d^3}$ for each diameter and class of pipe. When d=feet, the slope required to cause a cast iron pipe to discharge a given number of cubic feet per second q, is

$$S=.000648456\times\frac{q^2}{\sqrt{d^{11}}}.$$ (See §§ 42, 43).

From which the diameter in feet required to discharge a given quantity when the slope is given, is

$$d=\sqrt[11]{.0000004205}\times\sqrt[11]{\frac{q^4}{S^2}},$$ for clean cast iron, or

$$d=\sqrt[3]{\frac{q^4}{(AC)^4\times S^2}}$$ when d=inches.

See Tables Nos. 1 and 2 for value of AC, and see formulas § 26.

35.—Flow and Friction In Fire Hose.—Fire hose is made of different material, such as woven hose, lined with rubber, or hose made entirely of leather. The resistance to flow will depend upon the nature of the material which forms the lining. The resistance to flow in rubber lined hose is much smaller than in leather hose, or in iron pipes of equal diameter. Fire hose of all classes are made $2\frac{1}{2}$ inches in diameter, and therefore the area and friction surface are constant. Head in feet and pressure in lbs per square inch increase or vary at the same rate. The quantity discharged per second by a hose of constant diameter increases directly as the velocity. In a constant diameter the velocity or quantity increases as the square root of the head in feet, or as the square root of the pressure in lbs per square inch. The friction increases as v^2 or q^2 in a constant diameter. The pressure or the head is as v^2 or q^2. The coefficient may therefore be determined in terms of head in feet or in terms of pressure in lbs per square inch and in terms of v^2 or q^2. The friction loss will then vary as the head or pressure or as v^2 or q^2 in the constant diameter. As fire hose are all $2\frac{1}{2}$ inches diameter, we may use the direct value of the coefficients m and n instead of the unit values. It is more convenient to have the discharge of fire hose in gallons per minute than in cubic feet per second, hence the formulas will be given in terms of pressure in lbs per square inch and discharge in gallons per minute.

Let P=total guage pressure at hydrant or steamer.

P′=total pressure lost by friction in the length l, in feet of hose.

q=gallons per minute discharged by the hose.

n=coefficient of friction.

As the diameter is constant, the direct value of n will be

$$n=\frac{P' d}{l q^2}, \text{ and } P'=\frac{n}{d}\times q^2\times l.$$

If 200 feet of rubber lined woven hose $2\frac{1}{2}$ inches diameter be laid out straight on a level with one end attached to a hydrant or steamer, and with a smooth nozzle one inch diameter and 18 inches in length at the other end, and a pressure guage at the hydrant end registers 50 pounds, pressure per square inch, another guage attached at the butt of the nozzle on the other end will register only 35 lbs per square inch, and the discharge will be 145 gallons per minute. The pressure lost in the 200 feet of hose, (not including the nozzle), was therefore $P'=50-35=15$ lbs. Then,

$$n=\frac{P'\,d}{l\,q^2}=\frac{15\times 2.50}{200\times(145)^2}=,000008918.$$

$$\text{And } P'=\frac{.000008918}{2.5}\times q^2\times l=.000003567\times q^2\times l.$$

q= gallons per minute.

l=length in feet of hose.

The loss of pressure in lbs per square inch in 2½ inch rubber lined woven hose of any length and for any discharge in gallons per minute will therefore be

$P'=.000003567\times q^2\times l.$

In experiments with this class of hose the writer has observed that the friction increases very slightly for low pressures and decreases slightly for high pressures, because as the pressure within the hose becomes intense, the rubber lining is compressed, enlarging the diameter slightly and also causing the hose to straighten. An experiment on 300 feet length of rubber lined hose with a guage pressure of 156 lbs per square inch at the hydrant end, showed a pressure of 95 pounds at the butt of the nozzle, or a loss by friction of 61 lbs in 300 feet of hose while the discharge was 239 gallons per minute. This gives the formula

$$P'=.00000356\times q^2\times l$$

The difference in the value of the coefficient for very low and very high pressures is so slight as to be of no practical importance. It will be understood that the above formula does not apply to leather hose, nor to any other than 2½ inch rubber lined hose. The coefficient is in its direct form, and consequently applies only to the diameter for which it was determined.

36—Pressure Required at Hydrant or Steamer to Force the Discharge of a Given Quantity in Gallons per Minute. As the hose we are considering was partially throttled by the one inch smooth nozzle at discharge, the total pressure was not all neutralized by resistance nor converted into velocity, but a large portion of it remained to balance the friction in the nozzle and to generate the velocity through the nozzle. Therefore, in order to ascertain the value of the coefficient of velocity m, we must take $P = P' + Pv$ only, for the hose, (not the nozzle).

To do this, we must first find the value of Pv, or the amount of pressure which generates the given velocity in the 2½ inch hose. The quantity passing through the hose was 239 gallons per minute.

This is equal .5311 cubic feet per second. The area of the hose is =.0341 square feet. The velocity in feet per second through the hose was therefore

$$v = \frac{\text{cubic feet}}{\text{area}} = \frac{.5311}{.0341} = 15.57 \text{ feet.}$$

The pressure causing this velocity was

$$Pv = \frac{.434\ v^2}{2\ g} = 1.6337 \text{ lbs per square inch.}$$

Hence, $P = P' + Pv = 61 + 1.6337 = 62.634$ lbs.

$$\text{And } m = \frac{P\ d}{l\ q^2} = \frac{62.634 \times 2.50}{300 \times (239)^2} = .00000914.$$

$$P = \frac{m}{d} \times q^2 \times l = \frac{.00000914}{2.5} \times q^2 \times l = 000003656 \times q^2 \times l.$$

Therefore the total pressure at hydrant or steamer that is required to force the discharge of a given number of gallons per minute, (q) through any length in feet of 2½ inch rubber lined hose, will be

$$P = .000003656 \times q^2 \times l.$$

This does not include the pressure required to balance the friction in the nozzle, nor to lift the weight of the water when the nozzle end of the hoze is elevated. This value of P is that which is required to balance the friction in the hose (not the nozzle) and to generate the velocity of flow in the hose. If the discharge end of the hose is elevated, then sufficient additional pressure must be added to the above value of P to raise the weight of the given quantity to the given height.

The pressure lost by friction in 2½ inch leather hose is

$P' = .0000067464 \times q^2 \times l$

q=gallons discharged per minute.

l=length in feet of hose.

From this value of the coefficient as compared with the value of the coefficient for rubber lined hose, it is seen that the friction loss in leather hose is nearly double that in rubber hose. For this reason leather hose has fallen into disuse and will therefore not be discussed further.

37.—Loss By Friction In Brass Fire Nozzles.—

In conical pipes or nozzles which converge from a larger to a smaller diameter, the velocity is inversely as the constantly changing area and the resistance is inversely as $\sqrt{d^3}$. The velocity and resistance are therefore different at each successive point along the length of such convergent pipe or nozzle. The velocity is greatest in the portion having the least diameter and least in the greatest diameter. If we take the mean of all the varying velocities in such convergent nozzle, it will be found that this mean is very much greater than the mean velocity through a pipe of uniform diameter which uniform diameter is equal to the mean or average diameter of the convergent nozzle. It is therefore evident that the friction in the nozzle will greatly exceed that in the uniform diameter.

From the results of many experiments with very small nozzles and large nozzles of cast iron from 8 to 12 feet in length, the writer has discovered that the friction in a nozzle or convergent pipe is nine times as great as in a pipe of uniform diameter which uniform diameter equals the mean diameter of the convergent pipe, both being of the same material and same length, and discharging equal quantities of water in equal times.

The coefficient of resistance n, for smooth brass in terms of head and diameter in feet is n=.0003268, or nearly the same as for asphaltum coated pipes. A smooth brass fire nozzle 18 inches in length and converging from 2½ inches inside diameter at the butt to a diameter of one inch at discharge, discharged .17134566 cubic feet per second when the guage pressure at the butt of the nozzle was 10 pounds per square inch. As the velocity pressure is parallel to the walls of the pipe, it is not shown by a pressure guage. In order to find the friction loss in the nozzle we must find the total pressure at the butt of the nozzle and then find the pressure which causes the velocity of final discharge from the nozzle. The difference between the total pressure at the butt of the nozzle and the pressure due to the velocity of discharge from the nozzle, is evidently equivalent to the pressure lost by friction in the nozzle.

The pressure causing the velocity in the hose at the butt of the nozzle is to be found and added to the guage pressure at the butt of the nozzle. The velocity in the hose while discharging .17135 cubic feet per second was $v=\frac{q}{a}=\frac{.17135}{.0341}$ =5.0248. The pressure causing this velocity is $Pv=\frac{v^2\times.434}{2g}$ =.17 lb. Add this to guage pressure at butt of nozzle and the total pressure at the butt is P=10.17 lbs. The area in square feet of the one inch discharge of the nozzle is .0055. Consequently the final velocity of discharge from this one inch nozzle was $v=\frac{q}{a}=\frac{.17134566}{.0055}=31.20$ feet per second. The pressure causing this final velocity of discharge from the nozzle was $Pv=\frac{v^2\times.434}{2\,g}=6.55$ lbs. The pressure lost in the nozzle by friction was therefore 10.17—6.55=3.62 lbs, or 8.34 feet head while the discharge was .17135 cubic feet per second. The average or mean diameter of this nozzle was .1458 foot, and the area of this mean diameter was .0167 square foot. Hence the velocity through the mean diameter while discharging .17135 cubic feet per second was $v=\frac{q}{a}=\frac{.17135}{.0167}$ =10.26 feet. The coefficient of resistance of a smooth brass pipe of uniform diameter is n=.0003268. Hence the loss of head in feet in a smooth brass pipe of uniform diameter equal to the mean diameter of this nozzle, and of equal length would be $h''=\frac{.0003268}{\sqrt{d^5}}\times l\times v^2=\frac{.0003268\times1.50\times105.2676}{.0557}=$ .9267 feet. This is equal to only one ninth part of the actual loss in the convergent nozzle. Hence in a formula for friction loss in a conical or convergent pipe or nozzle we must take the square of three times the velocity through the mean diameter $(3\times v)^2$ or $(3\times q)^2=9v^2$ or $9q^2$, or we must find the coefficient of friction n in terms of quantity or velocity and multiply by 9 for a convergent nozzle or pipe. or we must consider the nozzle as a pipe of uniform diameter and as being 9 times as long as the nozzle. If we consider it as a uniform diameter then that diameter must be equal to the average diameter of the nozzle or conical pipe and nine times as long.

Hence the general formula for loss of head in feet by friction in conical pipes, reducers and nozzles will be

$$h''=\frac{(n\times9)}{\sqrt{d^5}}\times l\,v^2,\text{or } h''=\frac{n}{\sqrt{d^5}}\times9\,l\times v^2,\quad\text{or } h''=\frac{n}{\sqrt{d^5}}\times l\times 9v^2$$

In which

d=mean or average diameter of the convergent pipe.

v=velocity in feet per second in the mean diameter.

n=coefficient of friction in same terms as d.

The above value of n is in terms of head and diameter in feet.

In a nozzle of given length and form the loss by friction will vary directly as the head or pressure at the butt of the nozzle, or directly as v^2 or q^2. Hence a constant multiplier may be determined for each form and length of nozzle, by which the loss for any discharge, head or pressure may at once be found. For example, if the formula is in terms of diameter in feet, pressure in lbs. per square inch at butt of nozzle (guage pressure +Pv.) and v^2, then

$$P' = \frac{.0001418 \times l \times 9\,v^2}{\sqrt{d^3}}$$, for brass smooth (not ring) nozzles

From which the following table of constants was calculated:

TABLE No. 20.

Table of multipliers for finding pressure lost by friction in brass smooth (not ring) fire nozzles. For any head or total pressure.

Length of Nozzle Inches	Diameter at Butt Inches	Diameter at Discharge Inches	Lbs. pressure lost equal total pressure at butt multiplied by the decimal below:
18.000	2 1-2	1.	.356
12.000	2 1-2	1.	.2373
3.500	2 1-2	1.	.06922
18.000	2 1-2	1.1-8	.49
12.000	2 1-2	1.1-8	.325
3.204	2 1-2	1 1-8	.087
18.000	2 1-2	1.1-4	.474
12.000	2 1-2	1.1-4	.316
2.9125	2 1-2	1.1 4	.0767

These multipliers exhibit the relative efficiency of fire nozzles of different lengths and forms, and show the importance of making nozzles and reducers of short length. For the least loss and greatest efficiency the rate of convergence in a reducer or nozzle should be one inch in a length of 2.33 inches,which will conform to the shape of the contracted vein or vena contracta. (See § 80.)

If we wish to determine the direct coefficient for a given length and form of nozzle in terms of gallons discharged per minute and pressure in lbs. per square inch, take the experimental data already given, for example, and we have

l=1.5 feet=18 inches

d=.1458 feet=mean diameter=$\frac{2.5+1}{2}$=1.75 inches

P′=10.17—6.55=3.62 lbs.

Discharge=.17135 cubic feet per second=77 gallons per minute

$$n=\frac{P'd}{l\,q^2}=\frac{3.62\times1.75}{1.5\times(77)^2}=.0007123$$

$$P'=\frac{n}{d}\times l\times q^2=\frac{.0007123}{1.75}\times q^2\times l=.000407\times q^2\times l$$

l=feet, and d=mean diameter in inches

q=gallons per minute.

CAUTION. This last formula is in the direct form, and will apply only to the given nozzle for which it was determined. If the direct coefficient, .000407, be multiplied by the constant length in feet l of the given nozzle, then .000407×1.5 =0006105, and the loss of pressure by friction in this given form and length of nozzle for any discharge in gallons per minute is

$$P'=.0006105\times q^2.$$

A direct constant may be found in the same manner for each length and form of nozzle.

It is interesting to compare the values of n for different materials when the unit values of n are all in the same terms. Thus n=0001418 for smooth brass. n.=0000754 for rubber. These are the unit values of n in terms of P′ and diameter in feet, showing that rubber offers less resistance to flow than smooth brass or asphaltum coatings.

In a constant diameter of pipe, or in a constant length and form of nozzle, the friction will increase or decrease directly as the pressure or head. Hence if a total pressure at the butt of the nozzle of 10.17 lbs will cause a loss by friction of 3.62 lbs in the given nozzle, then a total pressure of one lb. at the butt would cause a friction loss=$\frac{3.62}{10.17}$=.356 lb., and any other total pressure at the butt would cause a loss of $P'=P\times.356$, for the given nozzle.

If a slope S=.0004 in a cast iron pipe one foot diameter will cause a loss of .0003938 foot head per foot length of pipe, then the loss of head for any other slope of a one foot pipe would be $=\frac{.0003938}{.0004}=.9845\times S$. And so of any other constant diameter or form of pipe or nozzle.

As friction increases as the square of the quantity discharged, if the loss by friction in the nozzle is 3.62 lbs. while it is discharging 77 gallons per minute, the loss for a discharge of one gallon per minute would be $\frac{3.62}{(77)^2}=\frac{3.62}{5929}=$ .0006105 lbs., and for any other discharge in gallons per minute it would be $=.0006105\times(\text{gallons})^2$.

If the loss of head in feet by friction in each foot length of a 12 inch diameter cast iron pipe is .0003938 foot while the pipe is discharging .7854 cubic foot per second, then the loss for a discharge of one cubic foot per second in such diameter will be $=\frac{.0003938}{(.7854)^2}=.0006384$ foot head per foot length and for any greater or less discharge in cubic feet per second the loss of head per foot length will be

$h''=.0006384\times(\text{cubic feet per second})^2$.

Hence it is a simple matter to find the proper constant in terms of head, pressure, velocity, slope or quantity for any given form of nozzle or for any given diameter.

38—Friction In Ring Fire Nozzles.—On account of the abrupt shoulder or offset caused by the sudden contraction of the diameter by the ring in what is termed a ring fire nozzle, very serious reactions and eddy effects occur in such nozzles, and the loss of head or pressure thus caused is very great. In an experiment with a ring nozzle of brass, 18 inches in total length, with a butt diameter of 2½ inches and a ring one inch diameter, and a total pressure at the butt equal to 23.237 feet head, the nozzle discharged .13333 cubic feet per second. The velocity through the one inch ring was therefore $v=\frac{q}{a}=\frac{.13333}{.0055}=24.243$ feet per second. The head due to this final velocity of discharge from the nozzle was $Hv=\frac{v^2}{2g}=\frac{587.72}{64.4}=9.126$ feet head.

Deducting this from the total head at the butt of the nozzle, and the friction loss in the nozzle was 23,237—9.126= 14.111 feet head or more than half the total pressure at the butt of the nozzle.

39—Hydraulic Giants, Cast Iron Nozzles for Power Mains, Reducers, and Conical Pipes In General.—The writer has made many experiments on cast iron giants or convergent pipes of various dimensions and under heads of 20 to 600 feet at the base of the giant. The results of these experiments confirm the correctness of the general formula heretofore given for finding the loss by friction in nozzles, reducers and convergent pipes—that is to say, the friction in a cast iron giant or convergent pipe, will be nine times as great for the same discharge as it would be in a uniform diameter equal to the mean diameter of the giant, reducer or convergent pipe. Hence the general formula for head in feet lost by friction in such giant or convergent pipe is

$$h''=\frac{n}{\sqrt{d^3}}\times 9\,v^2\times l=\frac{(n\times 9)}{\sqrt{d^3}}\times v^2\times l \ldots\ldots\ldots\ldots (95)$$

In this formula

d=the mean or average diameter of the giant.

v=velocity in the mean diameter in feet per second.

n=the usual coefficient of resistance for the class of cast iron or other material.

l =length in feet of giant.

If d is taken in inches then n must also be in the same terms.

Cast iron giants for discharging water upon impulse water wheels are required to be of the best metal and without flaws. They are usually under high pressure and the velocities through them are terrific. Hence they are scoured and kept clean so the coefficiennt will not increase after long use, unless the water contains sand or gritty matter which cuts the pipe walls and roughens them.

For this very dense, smooth cast iron, as usually found in such nozzles, n= .0003623 in terms of diameter in feet. Using the value of the mean diameter in feet of the cast iron nozzle and the velocity v, through the mean diameter, and the general formula for friction in such cost iron giants is

$$h'' = \frac{(.0003623 \times 9)}{\sqrt{d^3}} \times v^2 \times l = \frac{.0032607}{\sqrt{d^3}} \times v^2 \, l.$$

The coefficient n may be determined in terms of quantity discharged per second or per minute so that the discharge will correspond with a given loss of head. In an experiment with a cast iron giant 8 feet in length and converging from a diameter of fifteen inches at the base to a diameter of 3 inches at discharge, the loss of head in feet by friction while the discharge was 8.845 cubic feet per second was 16.10 feet head.

The mean diameter $= \frac{15+3}{2} = 9$ inches $= .75$ foot.

Area of mean diameter $= (.75)^2 \times .7854 = .4418$ square foot.

Velocity through mean diameter $= \frac{q}{a} = \frac{8.845}{.4418} = 20\,022$ ft. per second.

Velocity in 15-inch diameter =7.209 feet per second.

Velocity of discharge in 3-inch diameter =180.16 feet per second.

The mean of the velocities in all diameters =93.68 feet per second.

Using the value of n applicable to this class of dense cast iron and n=.0003623 in terms of head and diameter in feet. Then,

$$h'' = \frac{.0003623 \times 9}{\sqrt{d^3}} \times v^2 \, l = \frac{.0032607}{.6495} \times (20.022)^2 \times 8 = 16{,}10$$

feet head lost by friction. This corresponds exactly with the actual result.

In this given nozzle therefore, as the loss of head in feet by friction was 16.10 feet while the discharge was 8.845 cubic feet per second, the loss by friction for a discharge of one cubic foot per second would be $h'' = \frac{16.10}{(8.845)^2} = .2058$ foot, and for any other discharge in cubic feet per second it would be $h'' = .2058 \times q^2$. Here q^2=cubic feet per second discharged. A constant for any other length and mean diameter of nozzle or conical pipe may be readily found in the same manner in any terms desired. (See Table No. 25, § 56,)

40.—Table of Multipliers for Determining the Loss of Head in Feet by Friction in Clean Cast Iron Nozzles of Given Dimensions.

TABLE No. 21. (See Table No. 26.)

Length in Feet of Nozzle	Greatest Diameter Inches	Least Diameter Inches	Head in feet lost in nozzle equals effective head at base of nozzle multiplied by the decimal below
8	20	5	.0415
8	20	4	.02113
8	20	3	.00823
8	18	4	.0324
8	18	3	.012033
8	18	2	.003815
8	15	3	.031
8	15	2½	.01768
8	15	2	.0086
8	14	2	.01186
8	12	1½	.0098
8	12	2	.0244
8	12	1	.002394
8	10	1	.0058654

See § 80.

42:—The Total Head in Feet H, or the Slope Required to Cause the Discharge of a Given Quantity in Cubic Feet Per Second in Ordinary Cast Iron Pipes.

By formula (30) the slope required to cause the discharge of a given number of cubic feet per second, is

$$S=\frac{m\,q^2}{.616853\sqrt{d^{11}}}=\frac{m}{.616853}\times\frac{q^2}{\sqrt{o^{11}}}=.000648452\times\frac{q^2}{\sqrt{d^{11}}}$$

The total head in feet in the length in feet l, required to cause a given diameter of common cast iron pipe to discharge a given quantity q, in cubic feet per second, is

$$H=\frac{m}{.616853}\times\frac{q^2}{\sqrt{d^{11}}}\times l=.000648452\times\frac{q^2}{\sqrt{d^{11}}}\times l$$

In these formulas d is expressed in feet. In table No. 18 the values of $\sqrt{d^{11}}$ are given.

43—The Slope or Total Head in Feet Being Given, to Find the Diameter in Feet of Common Cast Iron Pipe Required to Discharge a Given Quantity in Cubic Feet per Second.

From the above formula, $S=.000648452\times\frac{q^2}{\sqrt{d^{11}}}$

Whence,

$$S\sqrt{d^{11}}=.000648452\times q^2$$

$$\sqrt{d^{11}}=\frac{.000648452\times q^2}{S}$$

$$d^{11}=\frac{(.000648452)^2\times q^4}{S^2}$$

$$d=\sqrt[11]{\frac{.0000004205\times q^4}{S^2}}\;.2632=\sqrt[11]{\frac{q^4}{S^2}}$$

Or in terms of total head in feet, for the given value of m.

$$d=.2632\sqrt[11]{\frac{q^4 l^2}{H^2}}$$

44.—Head in Feet Lost by Friction In Different Diameters of Clean, Ordinary Cast Iron Pipe While Discharging Given Quantities in Cubic Feet Per Second.

By formula (32) the head in feet lost by friction for a given discharge is

$$h''=\frac{n\,l\,q^2}{.616863\sqrt{d^{11}}}\;\cdots\cdots\cdots\cdots\;(32)$$

The value of n in terms of diameter in feet for ordinary clean cast iron pipe is n=.0003938. Hence....

$$h''=\frac{n}{.616853}\times\frac{q^2}{\sqrt{d^{11}}}\times l=\frac{.0003938}{.616853}\times\frac{q^2}{\sqrt{d^{11}}}\times l=.0006384\times l\times\frac{q^2}{\sqrt{d^{11}}}.$$

$h''=\frac{.0006384}{\sqrt{d^{11}}}\times q^2\times l.$ It is convenient to have the loss of head per 100 feet length of pipe, and therefore we may make l=100 feet as a constant. The loss of head in feet by friction in each 100 feet length of pipe for any given discharge in cubic feet per second will then be

$h''=\frac{.06384}{\sqrt{d^{11}}}\times q^2$. Now if we take the quotent of $\frac{.06384}{\sqrt{d^{11}}}$ for each diameter of pipe in feet, the result will be a constant for that diameter in feet, and when such constant is multiplied by the square of the discharge in cubic feet per second, the product will equal the loss of head in feet per 100 feet length of that diameter for the given discharge.

To facilitate such calculations the following table of such constants is given.

45—Table of Multipliers for Determining the Loss of Head in Feet by Friction Per 100 Feet Length of Ordinary Clean Cast Iron Pipe for a Given Discharge in Cubic Feet Per Second.—

TABLE No. 22.

Multiply the constant which corresponds with the given diameter on same line in the table by the square of the discharge in cubic feet per second (q^2). The result will be the loss of head in feet per 100 feet length of pipe for that discharge.

Diam. Inch's	Diam. Feet	$\frac{.06384}{\sqrt{d^{11}}}$ Constant	Diam. Inch's	Diam. Feet.	$\frac{.06384}{d\sqrt{11}}$ Constant
2	.1667	1214.611000	23	1.917	0.00178125
3	.25	130.737000	24	2.000	0.00141083
4	.3333	26.880000	25	2.083	0.00112791
5	.4167	7.871700	26	2.166	0.00090979
6	.5	2.900000	27	2.250	0.00073800
7	.5833	1.236000	28	2.333	0.00060483
8	.6667	0.594000	29	2.416	0.00050000
9	.75	0.310657	30	2.500	0.00041347
10	.8333	0.174045	31	2.584	0.000344708
11	.9167	0.103000	32	2.666	0.000290000
12	1.000	0.06384	33	2.750	0.000244785
13	1.083	0.041187	34	2.834	0.000207407
14	1.167	0.027305	35	2.916	0.000177333
15	1.250	0.0187104	36	3.000	0.000151675
16	1.333	0.0131385	38	3.166	0.000112800
17	1.417	0.00938823	40	3.333	0.000085018
18	1.500	0.00686380	42	3.500	0.000064970
19	1.583	0.00510310	44	3.666	0.000050347
20	1.667	0.003841155	48	4.000	0.0000311714
21	1.750	0.00294060	54	4.500	0.0000163100
22	1.833	0.00227919	60	5.000	0.0000900415

For convenience in referring to the table No. 22, the diameters are given first in inches and then in feet. The total head per 100 feet length is equal the loss of head per 100 feet divided by .9845, provided the diameter is constant and

the discharge is free, or $H = h'' \times 1.01573 = \frac{h''}{.9845}$.

46.—Head in Feet Lost by Friction in Asphaltum Coated Pipes While Discharging a Given Quantity in Cubic Feet Per Second.—If we refer to the table of values of m as developed in Group No. 2 from the experiments of Hamilton Smith Jr., and of D'Arcy and Bazin, on this class of pipe, it will be seen that the value of m, the coefficient of velocity, varies from m=.00028 to m=.0003432 in the experiments of Smith, and from m=.000271 to m=.000289 in the experiments of D'Arcy. Smith's experiments were on lap seamed riveted pipes with slip joints like stove pipe joints. D'Arcy's experiments were on cast iron coated pipes which were free of rivet heads and seams, but which were in shorter lengths and required more joints. The coefficient of friction n, for any given class of perimeter, is always equal .9845 per cent of the value of m for the given class of perimeter. Hence where the value of m is known for any class of perimeter, the value of n for that class is $n = m \times .9845$, and $m = \frac{n}{.9845}$. As it is prudent to allow for errors in the experimental data from which the above values of m were deduced and also for inferior quality of the coating, and future deterioration of the coating and slight deposits, we will adopt the value of $n = .00032$ in terms of diameter and head in feet. This should be a safe and reliable value of n for either riveted pipe, welded pipe, or cast iron pipe, which has been coated with asphaltum. The coating material usually covers the rivet heads and fills the longitudinal offset made by the lap of the plate. Hence there should not be a great difference in the value of n or m for either class of pipe after it has been coated. D'Arcy's coefficients are usually too small (that is, m or n, which makes C too high) and the length of pipe used in his experiments was rather short. The experiments of Smith are considered more reliable. They are safer to use in practice at any rate.

TABLE NO. 23.

Table No. 23 is based on the same principle as table No. 22, and its use is fully explained in § 44, 45.

TO USE TABLE NO. 23.

To find the loss of head in feet per 100 feet length of asphaltum coated pipe for any given discharge in cubic feet per second, multiply the constant in 3d column opposite the given diameter by the square of the discharge in cubic feet per second, q^2.

Head in feet lost per 100 feet length, $h'' = \frac{.051864}{\sqrt{d^{11}}} \times q^2$. d is in feet.

Diameter In.	Diameter Feet.	$\frac{.051864}{\sqrt{d^{11}}}$ Constant	Diameter In.	Diameter Feet.	$\frac{.051864}{\sqrt{d^{11}}}$ Constant
2	.1667	986.758	23	1.917	0.00144710
3	.25	106.213	24	2.000	0.001146165
4	.3333	21.417	25	2.083	0.000916325
5	.4167	6.400	26	2.166	0.000739120
6	.5	2.347	27	2.25	0.000599600
7	.5833	1.006	28	2.333	0.000491370
8	.6667	0.482460	29	2.416	0.000405200
9	.75	0.252380	30	2.5	0.000336000
10	.8333	0.141400	31	2.584	0.000280000
11	.9167	0.083680	32	2.666	0.000236852
12	1.000	0.051864	33	2.75	0.000199000
13	1.083	0.033460	34	2.834	0.000170000
14	1.167	0.022183	35	2.916	0.000144070
15	1.25	0.015200	36	3.000	0.000123222
16	1.333	0.0106738	38	3.166	0.0000916325
17	1.417	0.0076270	40	3.333	0.0000690690
18	1.5	0.00557617	42	3.500	0.0000527820
19	1.583	0.00414580	44	3.666	0.0000409000
20	1.667	0.00312060	48	4.000	0.0000253241
21	1.75	0.00240000	54	4.500	0.0000132509
22	1.833	0.00185160	60	5.000	0.0000074314

REMARK—The value of n used in above taole will allow for the reduction of area and diameter by the thickness of the coating, so that the actual diameter before it is coated may be used without any allowance for thickness of the coat.

47.—To Find the Quantity Discharged when the Loss of Head and Diameter are Given.—The quantity in cubic feet per second which is being discharged by any diameter may be found from the loss of head as indicated by pressure guages. We have just seen that the loss of head per 100 feet length of coated pipe for a given discharge in cubic feet per second is $h'' = \frac{.051864}{\sqrt{d^{11}}} \times q^2$. By transposing in this equation we have the formula for finding the quantity discharged in cubic feet per second from the amount of head in feet lost by friction h'', thus,

$$h'' = \frac{.051864}{\sqrt{d^{11}}} \times q^2\text{, whence, } q = \sqrt{\frac{h''\sqrt{d^{11}}}{.051864}} = 4.40\sqrt{h''\sqrt{d^{11}}}$$

d=feet, and h″=head in feet lost per 100 feet length of the pipe.

A similar formula for cast iron pipe may be deduced from the coefficient values given in § 44. The values of $\sqrt{d^{11}}$ will be found in table No. 18. For ordinary cast iron pipe, not coated, we have $h''=\frac{.06384}{\sqrt{d^{11}}}\times q^2$=loss per 100 feet length of pipe. Hence the discharge in cubic feet per second corresponding with this loss of head in feet per 100 feet length of pipe is

$$q=\sqrt{\frac{h''\sqrt{d^{11}}}{.06384}}=3.96\sqrt{h''\sqrt{d^{11}}}$$

48—To Find the Quantity that a Given Slope will Cause a Given Diameter to Discharge.

The slope required to cause the discharge of a given quantity in cubic feet per second is $S=\frac{m}{.616853}\times\frac{q^2}{\sqrt{d^{11}}}=\frac{.00064845}{\sqrt{d^{11}}}\times q^2$, for cast iron when m=.0004.

By transposition we have

$$q=\sqrt{\frac{S\sqrt{d^{11}}}{.00064845}}=39.27\sqrt{S\sqrt{d^{11}}}=39.27\sqrt[4]{d^{11}}\times\sqrt{S},$$

$$S=\frac{H}{l}=\frac{\text{total head in feet}}{\text{total length of pipe in feet}}$$

d=diameter in feet.

q=cubic feet per second discharged.

Table No 18 gives the values of $\sqrt{d^{11}}$, and Table No. 15 gives $\sqrt{S}$.

49—To Find the Total Pressure in Pounds Per Square Inch that must be Exerted by a Pump Piston, or by Other Means, in Order to Cause a Given Diameter of Asphaltum Coated Pipe to Discharge a Given Quantity in Cubic Feet Per Second.

By formula (44)

$$P=\frac{m}{.616853}\times\frac{q^2}{\sqrt{d^{11}}}\times l \cdots\cdots (44)$$

As we have adopted n=.00032 as the safe coefficient of friction in terms of head and diameter in feet for asphaltum coated pipes, the corresponding value of the coefficient of flow would be $m=\frac{n}{.9845}=.00032503$, in terms of head and diameter in feet.

To reduce this value of m to terms of pressure in pounds per square inch and diameter in feet, it is simply necessary to divide by the number of feet head required to cause a pressure of one pound per square inch. $H=P\times2.304$, and $P=\frac{H}{2.304}$.

Therefore if P=1 pound per square inch, then $H=1\times2.304=2.304$ feet.

Hence, $m=\frac{.00032503}{2.304}=.00014107$, in terms of P and d in feet.

Then, from formula (44), $P=\frac{.00014107}{.616853}\times\frac{q^2}{\sqrt{d^{11}}}\times l=\frac{.0002287}{\sqrt{d^{11}}}\times q^2\times l.$

The total pressure to be exerted by the pump is therefore,

$$P=\frac{.0002287}{\sqrt{d^{11}}}\times q^2\times l.$$

d=diameter of coated pipe in feet.

l=length of pipe in feet.

q=cubic feet per second discharged.

See Table No. 18 for values of $\sqrt{d^{11}}$.

CAUTION:—It is assumed in the above formula that the pipe is laid level, or that there is no difference in level between its two ends. If the pipe is laid on a declivity, then this declivity would supply a portion of the head or pressure.

If the discharge end of the pipe is above the pump, then additional pressure will be required at the pump sufficient to raise the weight of the given number of cubic feet per second to a height in feet equal to the difference in level between the pump and the discharge end of the pipe.

50.—To Find the Quantity Discharged From the Pressure.

By transposition in the above formula for P, we have

$$q = \sqrt{\frac{P\sqrt{d^{11}}}{.0002287 \times l}}, \text{—for asphaltum coated pipe.}$$

P=total pressure in pounds per square inch.

d=diameter in feet of pipe.

l=length of pipe in feet.

See "Caution" above. For value of $\sqrt{d^{11}}$ see Table No 18.

For cast iron pipe, not coated, m=.0004 in terms of head and diameter in feet. Hence in terms of P and d, it will be

$$m = \frac{.0004}{2.304} = .000173611. \qquad \text{Therefore } P = \frac{.000173611}{.616853} \times \frac{q^2}{\sqrt{d^{11}}} \times l = \frac{.0002816}{\sqrt{d^{11}}} \times q^2 \times l, \text{ and } q = \sqrt{\frac{P\sqrt{d^{11}}}{.0002816 \times l}} = 59.6\sqrt{\frac{P\sqrt{d^{11}}}{l}}$$

51.—Pounds Pressure Per Square Inch Lost by Friction for a Given Discharge in Cubic Feet Per Second.

By formula (45) the pressure lost by friction for a given discharge is

$$P' = \frac{n}{.616853} \times \frac{q^2}{\sqrt{d^{11}}} \times l \ldots\ldots\ldots\ldots\ldots\ldots (45)$$

As we have just found the values of m in terms of diameter in feet and pressure in lbs per square inch, for coated pipes and for uncoated cast iron pipes, the corresponding values of n will be $n = m \times .9845$.

Hence for asphaltum coated pipe $n = .00014107 \times .9845 = .000138883415$.

For cast iron pipe not coated, $n = .000173611 \times .9845 = .00017092$.

The pressure in lbs per square inch lost by friction for a given discharge in cubic feet per second will be, for coated pipe,

$$P' = \frac{.00017092}{.616853} \times \frac{q^2}{\sqrt{d^{11}}} \times l = \frac{.000225}{\sqrt{d^{11}}} \times q^2 \times l.$$

And for cast iron pipe not coated,

$$P' = \frac{.000138883415}{.616853} \times \frac{q^2}{\sqrt{d^{11}}} \times l = \frac{.000277}{\sqrt{d^{11}}} \times q^2 \times l$$

The quantity discharged may be found from the loss of pressure thus

$$q = \sqrt{\frac{P' \sqrt{d^{11}}}{.000225 \times l}}, \text{for coated pipe.}$$

$$q = \sqrt{\frac{P' \sqrt{d^{11}}}{.000277 \times l}}, \text{ for cast iron pipe not coated.}$$

See table No. 18 for value of $\sqrt{d^{11}}$.

52.—Table for finding the Slope of a Cast Iron Pipe or the Total Head, in Feet Required to Cause a Given Discharge in Cubic Feet per Second.

The quantity discharged by a constant diameter will be directly as the velocity of flow. The velocity of flow will be as $\sqrt{H}$ or $\sqrt{S}$. Hence S or H must vary as v^2 or q^2. If the slope or total head required in any given diameter of pipe, one foot in length, to cause a discharge of one cubic foot per second, be found, then, as S or H must vary as q^2 for that given diameter, it follows that the slope or total head required for any other discharge will be equal to the slope or head which causes a discharge of one cubic foot per second multiplied by the square of the desired discharge in cubic feet per second, q^2.

If the required slope S is found, then the total head in feet for any given length in feet will be $H = S \times l$, for $S = \frac{H}{l}$ = total head required per foot length. $l = \frac{H}{S}$; $H = S \times l$.

By formula (30),

$$S = \frac{m}{.616853} \times \frac{q^2}{\sqrt{d^{11}}} = .00064845 \times \frac{q^2}{\sqrt{d^{11}}}. \quad m = .0004.$$

Hence the slope, or the total head in feet per foot length of any given diameter of ordinary cast iron pipe, not coated, required to cause the discharge of one cubic foot per second will be

$S = \frac{.00064845}{\sqrt{d^{11}}}$. And the slope required to cause the discharge of any greater or less quantity in cubic feet will be

$$S = \frac{.00064845}{\sqrt{d^{11}}} \times q^2$$

In which,

d = diameter of pipe in feet.

q = cubic feet per second.

And the total head in feet required in any given length in feet of pipe will be $H = S \times l$.

TABLE No. 24.

To find the slope required to cause any given diameter in feet of uncoated cast iron pipe to discharge a given quantity in cubic feet per second:—Rule.—Multiply the slope in the following table (No. 24) which is opposite the given diameter, by the square of the desired discharge in cubic feet per second,

$$S = \frac{.00064845}{\sqrt{d^{11}}} \times q^2. \quad H = S \times l.$$

Diameter Feet.	$\sqrt{d^{11}}$ Feet	Slope .00064845 / $\sqrt{d^{11}}$	Diameter Feet	$\sqrt{d^{11}}$ Feet	Slope .00064845 / $\sqrt{d^{11}}$
.1667	.00005256	12.33734	1.917	35.84	.0000180929
.25	.0004883	1.32800	2.000	45.25	.0000143300
.3333	.002375	.27303	2.083	56.60	.0000114567
.4167	.00811	.07996	2.166	70.17	.0000092410
.5	.0221	.0293416	2.25	86.50	.00000743653
.5833	.05157	.0125740	2.333	105.55	.00000614353
.6667	.1075	.0060800	2.416	128.00	.00000506600
.75	.2055	.0031555	2.5	154.40	.0000042000
.8333	.3668	.0017680	2.584	185.20	.0000035000
.9167	.6198	.00104622	2.666	219.90	.0000029500
1.000	1.000	.00064845	2.75	260.80	.00000248638
1.083	1.55	.0004184	2.834	307.80	.00000210672
1.167	2.338	.00027735	2.916	360.00	.00000180120
1.25	3.412	.00019000	3.000	420.90	.00000154060
1.343	4.859	.0001334533	3.166	566.00	.00000114570
1.417	6.800	.0000953600	3.333	750.90	.000000863563
1.5	9.301	.0000697180	3.5	982.60	.000000660000
1.583	12.51	.0000518345	3.666	1268.00	.000000511400
1.667	16.62	.0000390000	4.000	2048.00	.000000316621
1.75	21.71	.0000298687	4.5	3914.00	.00000 165674
1.833	28.01	.0000231500	5.000	6979.00	.000000092916

See Table No. 16. These tables apply to pipes flowing full bore and with free discharge. $q:q::\sqrt{S}:\sqrt{S}$, for a given diameter.

53.—Wooden Stave Pipes.—In the western states, where irrigation is practiced on an extensive scale, and in localities without railway facilities, wooden stave pipe, invented by Mr. J. T. Fanning, and described in his "Treatise on Water Supply and Hydraulic Engineering" page 439. has been adopted in many instances in recent years.

In the very dry atmosphere of the arid west these pipes have not proven satisfactory in many cases where they were laid on the surface or without sufficient covering. In such cases it shrinks and warps and leaks badly. Where properly covered and kept constantly full of water it has been quite satisfactory. It has not been in general use for a sufficient length of time to test its durability. That would, of course, depend upon the kind of wood used in manufacturing the staves, and upon whether it was perfectly seasoned and sound. If perfectly seasoned and treated with tar oil or sulphate of copper, it should be very durable. The quantity of this class of pipe which is being used of late years in the

West for irrigation purposes in cases where there is only small pressure to be sustained, and the general belief that this wooden pipe is smoother and will give a higher discharge under like conditions than uncoated iron pipes, demands that it be given some notice here.

54.—Coefficients of Flow in Wooden Stave Pipes Compared with the Coefficients of Pipes of other Material.

By referring to the coefficient values developed from the data of D'Arcy and Bazin, (See group No 5), it will be seen that the average value of m for wooden conduits made of closely jointed,planed poplar lumber is m=.000060 in terms of hydraulic mean radius in feet and head in feet. The rectangular wooden conduits used in these experiments did not contain the great number of joints which are necessary in forming a circular conduit of wooden staves. It is fair to assume then that the circular wooden conduit built up of narrow staves with its many joints would not present a more uniform surface to the flow than the rectangular conduit or flume of planed, well jointed hard wood.

The nature of the wood of which the staves are made as to density and freedom from knots, will undoubtedly affect the value of the coefficient. It appears from the great number of experiments by D'Arcy and Bazin on such conduits (only a few of which were quoted in Group No. 5) that m=.00006 is about the average value of the coefficient in terms of r in feet.

If this value of m be reduced to terms of diameter in feet, we have for well jointed, planed hard wood conduits, m=.00006×8=.00048, or $C=\sqrt{\frac{1}{m}}=45.64$, in terms of head and diameter in feet. For average cast iron pipe, not coated, m=.00040 and C=50.00. For asphaltum coated riveted pipes, C=56.00. For pipes lined with mortar composed of two-thirds cement to one-third sand, C=48.50. It is therefore apparent that the wooden pipe offers much greater resistance to flow than either of the others, and will therefore require a greater diameter for an equal discharge.

The slope required in a wooden pipe in order to cause it to discharge a given quantity in cubic feet per second would be

$$S=\frac{m}{.616853}\times\frac{q^2}{\sqrt{d^{11}}}=\frac{.00048}{.616853}\times\frac{q^2}{\sqrt{d^{11}}}=\frac{.0007781}{\sqrt{d^{11}}}\times q^2;$$

and $q:q::\sqrt{S}:\sqrt{S}$.

And the diameter in feet required to discharge a given quantity for a given slope will be

$$d=.2721\sqrt[11]{\frac{q^4}{S^2}};\text{ or } d=.2721\sqrt[11]{\frac{q^4\,l^2}{H^2}}$$

55.—Earthenware Or Vitrified Pipe.—This class of pipe is made in very short lengths and consequently requires many joints. It is subject to unequal settlement and leaks unless very great care is taken to secure a firm bearing or foundation upon which to lay the pipe. It also requires care and experience to make and properly cement the joints. If the pipe is made of clay containing a high percentage of aluminum and is thoroughly glazed and properly laid and very carefully jointed, it develops a coefficient m=.00036 or C =52.70, in terms of diameter in feet and slope or head in feet. It therefore offers less resistance to flow than very smooth, dense, clean cast iron pipe, provided all the above conditions as to laying and jointing are complied with. As these conditions are scarcely ever fulfilled, it is not prudent to depend upon a greater discharge from such pipe than from ordinary clean cast iron pipe. Hence all the tables heretofore given for cast iron pipe may be adopted as applying also to earthenware glazed pipe. This class of pipe is very extensively used for house drains, small sewers, land drains and irrigation pur-purposes, and in other rough work where great care and thorough workmanship are not usually exercised. Hence it is not safe to take the value of C greater than C=50, or m= .0004 in terms of head or slope and diameter in feet. For small sewers, not exceeding about 18 inches diameter, this class of pipe serves well. The flow of sewage is probably not so great as that of clear water because of the suspended, solid matter that it carries. The value of C for a sewer would therefore not be quite so great as for pure water flowing in the same class of pipe or conduit. C=50, should be a safe value for fairly well laid and jointed earthenware glazed pipe. In order to prevent deposits the mean velocity of flow in a sewer should never be less than two and half feet per second for small depths of flow. In order to ascertain the mean velocity of flow in such sewer pipe when flowing only part full, the coefficient may be reduced to terms of hydraulic mean depth r, in feet by multiplying m in terms of d in feet by 0.125, or by dividing by 8. Then in terms of r in feet

$m = \frac{.0004}{8} = .00005$, and C=141.42. The mean velocity of flow in a circular conduit, or in a pipe, will be the same for just half full as for full, because $\frac{a}{p}$ is the same for half full as for full.

56—Table of Elementary Dimensions of Pipes.

TABLE No. 25.

Diam. In.	Diam. Feet	Area Sq. Feet	U,S. Gal. In one Ft. Lgth	Diam. In.	Diam. Feet	Area Sq. Feet	U. S, Gal In one Ft. Lgth
¼	.0208	.0003	.0025	4.½	.3750	.1104	.8263
⅜	.0313	.0008	.0057	4.¾	.3958	.1231	.9206
½	.0417	.0014	.0102	5	.4167	.1364	1.020
⅝	.0521	.0021	.0159	6	.5	.1963	1.469
¾	.0625	.0031	.0230	8	.6667	.3491	2.611
⅞	.0729	.0042	.0312	10	.8333	.5454	4.080
1.	.0833	.0055	.0408	12	1.	.7854	5.875
1.¼	.1042	.0085	.0638	14	1.167	1.069	7.997
1.½	.125	.0123	.0918	16	1.333	1.396	10.440
1.¾	.1458	.0167	.1249	18	1.5	1.767	13.220
2.	.1667	.0218	.1632	20	1.667	2.182	16.320
2.¼	1875	.0276	.2066	22	1.833	2.640	19.75
2.½	.2083	.0341	.2550	24	2.	3.142	23.50
2.¾	.2292	.0412	.3085	26	2.167	3.687	27.58
3.	.25	.0491	.3672	27	2.25	3.976	29.74
3.¼	.2708	.0576	.4309	28	2.333	4.276	31.99
3 ½	.2917	.0668	.4998	30	2.5	4.909	36.72
3.¾	.3125	.0767	.5738	32	2.667	5.585	41.78
4.	.3333	.0873	.6528	34	2.833	6.305	47.15
4.¼	.3542	.0985	.7369	36	3.	7.069	52.88

231 cubic inches=1 U. S. gallon. 7.48052 U.S. gallons= 1 cubic foot.

The area in square feet of a pipe is the same as the contents of one foot in length of the pipe in cubic feet. Hence by an inspection of table No. 25, the diameter and also the velocity required to carry a given number of cubic feet or of U. S. gallons may be determined at once. If the velocity is one foot per second in any diameter, the discharge in cubic feet per second will equal the area in square feet of that diameter, or the discharge in gallons per second will equal the number of gallons in one foot length of pipe. For a discharge of 2, 3, 4, etc times that quantity, the velocity must be 2, 3, 4, etc feet per second. Tables No. 16 and 17 and 19 will show the slope or head required to generate the required velocity, and also the amount of head that will be neutralized by friction for that velocity. The dimensions of the very small pipes given in table No. 25 will be found convenient in designing hydraulic giants and nozzles, and in selecting small service pipes, and discharge pipes for small pumps. See also Tables 22 and 23, and 26 and 27. Square inches multiplied by .00695= square feet.

57.—Length in Feet of Small Pipes Required to Hold one U.S. Gallon of 231 Cubic Inches, and Areas Given in Square Inches.

TABLE No. 26.

(1 square inch=.0069444 square feet).

Diam. In.	Area Sq. Inches.	Length In Feet to hold 1 Gallon.	Diam. In.	Area Sq. Inches.	Length In Feet to hold 1 Gallon.
¼	.0490875	407.4356700	2.⅝	5.412	3.5570
½	.1963500	98.0392083	2.¾	5.940	3.2540
¾	.4417875	43.5729833	2.⅞	6.492	2.9651
1.	.7854	24.5098000	3.	7.069	2.7225
1 ¼	1.2271875	15.6862000	3.⅓	7.670	2.5097
1 ½	1.7671500	10.8932000	3.¼	8.296	2.3200
1 ¾	2.4050	8.0041650	3.⅜	8.946	2.1517
1.⅞	2.7610	6.9721000	3.½	9.621	2.
2.	3.1416	6.1274100	3.⅝	10.320	1.8652
2.⅛	3.5470	5.4270000	3.¾	11.040	1.7360
2.¼	3.9760	4.8415000	3.⅞	11.790	1.6166
2 ½	4.9090	3.9214000	4.	12.570	1.5310

Length in feet to hold one gallon equals velocity in feet per second required to discharge one gallon per second. The velocity must be 7.5 times as great to discharge one cubic foot per second.

A 4 inch cast iron pipe cannot supply one fire hydrant with the ordinary supply of 255 gallons per minute without a loss of head in such pipe of nearly one foot in each 10 feet length of 4 inch pipe. Add to this the friction loss in the hydrant, the hose and the nozzle, and the resistance of the atmosphere and wind, and it is apparent that a hydrant will be of little service when attached to a four inch pipe of any considerable length.

58.—Decimal Equivalents to Fractional Parts of one Lineal Inch.

TABLE No. 27.

1-32=.03125	1 8+3-32=.21875	3 8+1-32=.40625	5-8 =.625
1-16=.06250	1-4= .25	3 8+1-16=.4375	5-8+1-16=.6875
3-32=.09375	1-4+1-32=.28125	3-8+3-32=.46875	3-4=.75
1-8 =.125	1-4+1-16=.3125	1-2= .5	3-4+1-16=.8125
1-8+1-32=.15625	1-4+3-32=.34375	1 2+1-32=.53125	7-8=.875
1-8+1-16=.1875	3 8= .375	1-2+1-16=.5625	7-8+3-32=.96875

Fractional inches in equivalent decimals of a foot.

Frac. Inch	Deci. Inch	Equiv dec ft.	Frac. Inch	Deci. Inch	Equiv dec ft.	Frac. Inch	Deci, Inch	Equiv dec ft.
1-32	.03125	.002604	3-8	.375	.03125	23-32	.71875	.059895
1-16	.0625	.005208	13 32	.40675	.033854	3-4	.75	0625
3-32	.09375	.007812	7-16	.4375	.036458	25-32	.78125	.065104
1-8	.125	.010416	15-32	.46875	.039062	13 16	.8125	.067708
5-32	.15625	.010420	1-2	.5	.041666	27-32	.84375	.070312
3-16	.1875	.015625	17-32	.53125	.04427	7-8	.875	.072916
7-32	.21875	.018229	9 16	.5625	.046875	29 32	.90625	.07552
1 4	.25	.020833	19-32	.59375	.049479	15-16	.9375	.078125
9 32	.28125	.023437	5-8	.625	.052083	31-32	.96875	.080729
5-16	.3125	.026041	21-32	.65625	.054607	1.00	1.00	.083333
11-32	.34375	.028645	11-16	.6875	.057291			

Tenths of one foot in equivalent inches.

Foot	Inches	Foot	Inches	Foot	Inches
0.10	1 3-16	0.50	6.00	0.90	10.25-32
0.20	2.3-8	0.60	7.3-16	1.00	12.00
0.30	3.19 32	0.70	8.3-8		
0.40	4.25-32	0.80	9.19-32		

59—Tables for Converting Measures,

TABLE No. 28. Lineal Measure.

Inch's	Feet	Yards	Fath.	Rods	Miles	Metres
1	.083333	.02778	.013889	.005051	.000016	.0254
12	1	.33333	.16666	.060606	.000189	.304797
36	3	1.	.5	.181818	.000568	.914392
72	6	2.	1.	.363636	.001136	1.82878
198	16½	5½	2.¾	1.	.003125	5.02915
7920	660.	220.	110.	40.	.125	201.166
63360	5280.	1760.	880.	320.	1.0	1609.33

TABLE No. 29. Land Measure (Lineal).

Inch's	Links	Feet	Yards	Chains	Miles	Metres
1	.1261261	.083333	.0277778	.0012626	.0000158	.0254
7 23-25	1.	.066666	.222222	.01	.000125	.201166
12	1 17-33	1.	.333333	.0151515	.0001894	.304797
36	4 6-11	3.	1.	.0454545	.0005682	.914392
792	100.	66.	22.	1.	.0125	20.1166
63360	8000.	5280.	1760.	80.	1.	1609.33

TABLE No. 30.* Metrical Equivalents. Lineal Measure.

	Inches	Feet	Yards	Rods	Chains	Miles
1 Millimeter=	.03937	.003281	.001094			
1 Centimeter=	.393704	.032809	.010936	.001988		
1 Meter =	39.370432	3.280869	1.093623	.198841	.04971	.000621
1 Kilometer=		3280.8693	1093.6231	198.84057	49.710141	.621377

	Milli-meters	Centi-meters	Meters	Kilo-meters
1 Inch =	25.399772	2.539977	.253998	
1 Foot =	304.79727	30.47973	.304797	.0003048
1 Yard =	914.391795	91.43918	.914392	.0009144
1 Rod =	5029.15487	502.91549	5.029155	.00502915
1 Chain =		2011.66195	20.11662	.02011662
1 Mile =			1609.32956	1.60933

*See "Rules and Tables" page 92, by Prof. W. J. M. Rankine.

TABLE No. 33. Cubic Measure.

Cubic Inches	Cubic Feet	Cubic Yards	Cubic Meters
1.0	.0005788	.00000214	.000016387
1728.0	1.0	.037037	.0283161
46656.0	27.	1.0	.764534

231 cubic inches=1 U. S. gallon. 7.48052 U. S. gallons=1 cubic foot.

The actual weight of 1 U. S. gallon of water at its maximum density is 8.345008 pounds. The weight is, however, adopted by law as 8.33888 pounds avoirdupois.

1 U. S. gallon=.13368 cubic foot. 1 cubic foot per second =448.8312 gallons per minute, or 26929.872 gallons per hour, or 646316.928 gallons per 24 hours. 1 cubic foot per second=60 cubic feet per minute, or 3600 cubic feet per hour, or 86400 cubic feet per 24 hours. This will cover one acre of ground to a depth of 1.98347 feet, or 1.98347 acres to a depth of one foot in 24 hours, or supply 200 gallons per person per 24 hours for 3,231.58 persons. An 8 inch pipe will carry it at a velocity of 2.864 feet per second.

TABLE No. 34. Metrical Equivalents.—Cubic Measure.

	Cubic In.	U. S. Gal.	Cubic Ft	Cubic yd	Perches*
1 cu. centimtr.	.061025386	.000264179	.000035316		
1 cu. decimetr.	61.025386	.264179	.035316	.001307986	.001426893
1 cubic meter	6'025.386	264.179	35.316	1.307986	1.426893

	Cubic Cent.	Cubic Decm.	Cubic Meters
1 cubic inch	16.386623	.016386623	
1 U. S. gallon	3785.31	3.78531	.00378531
1 cubic foot	28316.0844	28.3160844	.0283160844
1 cubic yard		764.5343	.7645343
1 Perch		700.82309	.70082309

*1 Perch=24.75 cubic feet.

TABLE No. 35.—Pressure. (Thurston).

Pounds per sq. Inch.	Kilograms per Sq. Centimeter	Kilograms per Sq. Centimeter	Pounds per. sq. Inch.
1.0	.07030827	1.0	14.22308

REMARK.—The foregoing conversion tables are given in order that the formulas may be used and coefficients determined either in English or metrical terms.

TABLE No. 31.—Square Measure.

Sq. Inches	Square Feet	Square Yards	Square Rods	Square Roods	Square Acres	Square Metres
1.	.0069444	.0007716	.0000255	.00000064	.00000016	.0006452
144.	1.	.1111111	.0036731	.0000918	.000023	.0929013
1296.	9.	1.	.0330579	.0008264	.0002056	.836112
39204.	272.¼	30.¼	1.	.025	.00625	25.292
1568160.	10890.	1210.	40.	1.	.25	1011.696
6272640.	43560	4840.	160.	4.	1.	4046.782

Acres×.0015625=square miles. 1 square mile=27,878,400 square feet, or 3097600 square yards, or 640 acres, or one section. One acre=16 square chains. The length of one chain is 66 feet, or four rods. This Gunter's chain has fallen into disuse, and a steel tape 100 feet length is used instead. Areas are taken in square feet, and when divided by 43,560, are reduced to acres.

TABLE No. 32*—Metrical Equivalents. Square Measure.

	Square Inches	Square Feet	Square Yards	Acres	Square Miles
1 sq. Centimeter=	.155003	.00107641	.00012		
1 sq. Decimeter=	15.500309	.107541	.011960115		
1 sq. Meter=	1550.030916	10.7641	1.1960115	.00024711	
1 sq. Dekameter=1 Are=	155003.0916	1076.41	119.60115	.02411	.00003861
1 sq. Hectometer=1 Hectare=		1076.41	11960.115	2.4711	.003861
1 Kilometer=			1196011.5	247.11	.3861

	Sq. Centimeters	Sq. Decimeters	Sq. Meters.	Sq. Dekameters or Ares
1 Sq. Inch=	6.451484	.06451484	.0006451484	
1 Square Foot=	929.013728	9.29013728	.0929013728	.000929013728
1 Square Yard=	8361.123554	83.61123554	.8361123554	.008361123554

*See "Conversion Tables," page 40, by Prof. Thurston, and Trautwine's "Civil Engineer's Pocket Book", page 78, Rankine's "Rules and Tables," p. p. 110-114.

CHAPTER V

Of Water Powers, Power Mains and Pipe Lines.

Work is expressed in units of weight lifted through one unit of height; as in pounds lifted one foot, called foot pounds. Here there is no reference to the units of time consumed Power is expressed in units of work done in one unit of time; as in pounds lifted one foot in one second of time, called foot pounds per second.

One horse power is a conventional quantity equal to 550 foot pounds per second, or to 550 pounds lifted one foot in one second, or to one pound lifted 550 feet per second.

As there are 60 seconds in one minute of time, the expres ion of horse power in terms of foot pounds per minute would be $550\times60=33{,}000$. or in foot pounds per hour it woud be $33{,}000\times60=1{,}980{,}000$.

One pound of water falling one foot does work equal to that of raising one pound one foot high. Hence the number of pounds of water falling in one second multiplied by the distance fallen in feet will equal the number of foot pounds per second, and as 550 foot pounds per second equal one horse power, the total number of foot pounds per second divided by 550 will equal the horse power of the water. Expressed as a formula, we have

$$\text{H.P.}=\frac{\text{cubic feet per sec.}\times\text{weight of one cubic foot}\times\text{Head or fall in feet.}}{550}$$

The weight of one cubic foot of water at its maximum density is 62.5 lbs. This is the weight always assigned, in ordinary cases, to one cubic foot of water. The formula may therefore be written

$$\text{H. P.}=\frac{62.5}{550}\times\text{cubic feet per second}\times\text{head or fall in feet.}$$

If we take the quotient of $\frac{62.5}{550}=.1136363$, we have,

H. P.=.1136363×head in feet×cubic feet per second...97

61.—Formula for Cubic Feet Per Second Required to Generate a Given Horse Power.

When the net head or fall in feet is given, then the cubic feet per second required to develop any required horse power will be

$$\text{Cubic feet per sec.}=\frac{\text{Horse Power Desired}}{.1136363\times\text{Head in Feet}}\;\ldots\ldots(98)$$

62.—Formula for Net Head or Fall in Feet Required to Develop a Given Net Horse Power.

The efficiency of a water wheel or other machine is the ratio of effective power recovered from it to the total power applied to it. To find the efficiency, divide the effective power delivered by the machine, by the total power applied to it. The quotient is the efficiency. If a water fall of 100 horse power is applied to a turbine and the turbine develops 80 horse power, then the efficiency of the turbine is $E=\frac{80}{100}=80$ per cent.

The efficiency of the motor being given, then the net head or fall in feet required to develop a given net horse power, will be

$$\text{Net Head}=\frac{\text{Desired net H. P.}\div\text{per cent efficiency of motor}}{.1136363\times\text{Cubic Feet per Second,}}\;\ldots\ldots(99)$$

63.—Head of Water Defined.—By the term head is meant the difference of level between the surface of the water in the reservoir or head race, and the water surface in the tail race, to which must be added the head due to the mean velocity of flow in the head race or stream above the fall.

The head due to the velocity in the head race is $H=\frac{v^2}{64.4}$. (See formula (7) Chap. I.) In a pipe or power main the total head is equal to the difference of level between the water surface at the intake end of the pipe and the upper surface of the jet at discharge. and the net head is equal to the total head less the amount of head neutralized by friction or resistance in the pipe. Hence in a pipe or power main the loss of head by friction must be first deducted from the total head in order to ascertain the effective head at discharge.

64.—To Find the Diameter in Feet of Pipe Required to Carry a Given Quantity of Water with a Given loss of Head in Feet,

Where the total head is known, and it is desired to lay a pipe of such diameter as will convey a given number of cubic feet per second with a predetermined loss of head by friction, so that a given net head will be secured at discharge, such diameter in feet may be found as follows:

$$h''=\frac{n}{.616853}\times\frac{q^2}{\sqrt{d^{11}}}\times l \qquad (32)$$

For ordinary clean cast iron pipe n=.0003938 in terms of diameter in feet.

Hence the formula reduces to

$$h''=\frac{.0006384}{\sqrt{d^{11}}}\times q^2\times l$$

and,

$$\sqrt{d^{11}}=\frac{.0006384\ q^2\times l}{h''}, \text{ whence,}$$

$$d=\sqrt[11]{\frac{.00000040755\times q^4\times l^2}{h''^2}} \qquad (100)$$

If it is an asphaltum coated pipe, then take n=00032 in terms of diameter in feet, and the formula for finding the diameter required to carry a given quantity in cubic feet per second with a given total loss of head in feet by friction in the entire length of pipe line, will be

$$d=\sqrt[11]{\frac{.000000269\ q^4\times l^2}{h''^2}} \qquad (101)$$

In these formulas

d=diameter of required pipe in feet

q=cubic feet per second it is to discharge

l=total length of pipe in feet

h''=head in feet lost by friction in total length, l.

EXAMPLE OF THE USE OF THESE FORMULAS.

There is a total fall of 100 feet in a distance of 3,000 feet. A diameter of asphaltum coated pipe is desired, which will convey one cubic foot of water per second with a loss of head not exceeding 6 feet, so that there shall remain an effective head of 94 feet, at discharge while one cubic foot per second is being drawn from the pipe at its lower end. By the above formula

$$d = \sqrt[11]{\frac{.000000269 \times 1 \times 9000000}{36}} = \sqrt[11]{.06725} = .7824 = d \text{ in}$$

feet,

This diameter has an area $= d^2 \times .7854 = .48078$ square feet. The mean velocity required to discharge one cubic foot per second in this diameter would be, $v = \frac{q}{a} = \frac{1}{.48078} = 2.08$ feet per second.

The result may therefore be tested by the formula for loss of head,

$$h'' = \frac{n}{\sqrt{d^3}} \times l v^2,$$

And we have,

$$h'' = \frac{.00032}{.692} \times 3000 \times 4.3264 = 6.00 \text{ feet head lost.}$$

It will be understood that the area at discharge is such that it will admit of no greater discharge under the given net head than the quantity q. The manner of discharge may be through other small pipes tapped into the main if it is a water works system, or the discharge may be through a reducer or nozzle if the pipe is used as a power main for driving water wheels, or the discharge may be full and the total head lost except the velocity head, as may be desired.

65.—To Find the Area and Diameter of the Nozzle Tip or Aperture Required to Discharge a Given Quantity.

If there is a simple tip on the end of the pipe made in the form of the contracted vein which reduces the diameter at discharge, there will be a very small loss of head by friction in efflux from the tip. The area in square feet of the required aperture in such tip will be found as follows: Assume the diameter of the pipe to be .7824 feet, and net head at discharge to be 94 feet, as in the preceding section, and the quantity to be disharged as one cubic foot per second.

The velocity that will be generated by this net head at discharge will be

$$v=\sqrt{2\ g\ H}=8.025\sqrt{94}=77.8052 \text{ feet per second.}$$

Now, $q=\text{area}\times\text{velocity}=a\sqrt{2g\ H}$. Whence $a=$

$\frac{q}{\sqrt{2\,gH}}=\frac{1.0}{77.8052}=.0128526$ square feet. The diameter in feet is then

$$d=\sqrt{\frac{\text{area}}{.7854}}=\sqrt{\frac{.0128526}{.7854}}=.128 \text{ foot}=1.536 \text{ inches diameter.}$$

See table No. 27, § 58.

If the discharge is to be through a nozzle or reducer of several feet length, there will be considerable loss of head by friction in such nozzle or reducer, for which allowance must be made. This loss will depend upon the length of the convergent reducer or nozzle and its mean or average diameter as well as its smoothness of internal circumference, and the square of the velocity through it. We have seen heretofore (§37, 39) that the loss by friction in a convergent or conical pipe is nine times as great as the loss in a pipe of uniform diameter equal to the mean diameter of the convergent pipe. It is therefore evident that such convergent pipes, reducers or nozzles should be as short as possible,provided they do not converge more rapidly than one inch in a length 2.33 inches, which would make them conform to the form of the contracted vein. Assuming the diameter of the base of the nozzle to be the same as the diameter of the pipe it is to join, and that the net head at the base of the nozzle is 94 feet, and that the reducer or nozzle is to be 6 feet in length, and is required to discharge one cubic foot per second under this net head at the base, the problem now is to determine the area and diameter of the small, or discharge end of the nozzle so that it shall discharge this given quantity per second under the given head at its base.

This will require one or more approximations, for the reason that as the mean diameter of the proposed nozzle is yet unknown we have no means of knowing the friction loss that will occur in the nozzle, and hence do not know the value of the net effective head at the point of final discharge from the nozzle.

For first approximation assume that there will be three feet head lost in the nozzle, leaving an assumed effective head at discharge of 94—3=91 feet. The velocity of discharge under the net head of 91 feet will be $v=8.025\sqrt{91}=76.5535$ feet per second. Then the area in square feet required to discharge the quantity q, in cubic feet per second, will be $a=\frac{q}{\sqrt{2gH}}=\frac{1.0}{76.5535}=.01306276$ square feet. The diameter in feet answering to this area in square feet is

$$d=\sqrt{\frac{area}{.7854}}=\sqrt{\frac{.01306276}{.7854}}=.129 \text{ feet.}$$

The smallest diameter in inches is therefore $.129\times12=1.548$ inches. (See § 58, Table 27).

For first approximation we have then the following dimensions of the nozzle:—Greatest, or butt diameter=.7824 foot=9.388 inches. Smallest, or discharge diameter=.129 foot=1.548 inches. Total length of nozzle=6 feet. The average or mean diameter of the nozzle is therefore,

$$\text{Mean } d=\frac{.7824+.129}{2}=.4557 \text{ foot, or } 5.4684 \text{ inches.}$$

Now, two tests must be applied to this nozzle in order to ascertain whether or not it will fulfill the required conditions:—

(1) It must be tested by the formula (§39) for friction loss in nozzles in order to ascertain the actual loss of head that will occur while discharging the given quantity.

(2) It must then be tested to ascertain whether or not

it will discharge the given quantity under the conditions actually existing. If it fails to meet the requirements, further approximation must be made.

(1) TEST FOR LOSS BY FRICTION.

The formula for loss of head in feet by friction in convergent pipes and nozzles is

$$h''=\frac{n}{\sqrt{d^3}}\times l\times 9v^2. \quad \text{(See §§37,39).}$$

In which

h''=head in feet lost by friction

l=length in feet of convergent pipe or nozzle.

d=average or mean diameter of nozzle

v=mean velocity in the mean diameter of the nozzle

In the nozzle we are now considering the mean diameter is .4557 foot, and the nozzle is required to discharge one cubic foot per second. Hence the required mean velocity in feet per second through this mean diameter to cause the discharge of one cubic foot per second will be

$$v=\frac{q}{a}=\frac{1.0}{(.4557)^2\times.7854}=\frac{1.0}{.1631}=6.1312 \text{ feet per second.}$$

Assuming the nozzle to be made of very dense, solid smooth cast iron, the friction coefficient will be n=.0003623 in terms of diameter in feet. Applying the above formula for loss by friction in this nozzle while discharging one cubic foot per second, the velocity in the mean diameter being 6.1312 feet per second, and we find the actual loss of head in the nozzle to be

$$h''=\frac{.0003623}{.3076}\times 6\times 9\times 37.5916=2.40 \text{ feet head lost.}$$

Hence at the point of discharge the effective head would be 94—2.40=91.60 feet, whereas we had assumed that it would be probably 91.00 feet. But as the assumed loss of head (3 feet) and the actual loss (2.40 feet) are so nearly equal, we will now apply the test for quantity discharged under the actual conditions. For this purpose we have the following:

Area of smallest diameter at discharge=.01306276 square feet.

Effective head at point of discharge from nozzle=91.60 feet.

Velocity due to this net head, $v=\sqrt{2gH} = 8.025\sqrt{91.60}=$ 76.807275 feet per second. Quantity discharged q, will be

$q=a\ v=.01306276=76.807275=1.003315$ cubic feet per second.

If a closer result is desired, the smallest diameter may be reduced by 1-16 inch and all the foregoing tests be again applied to the new proportions of the nozzle thus changed. Table No. 21, § 40, will be of assistance in such calculations.

66.—Pipe Lines of Irregular Diameter.—Where the head or pressure is due to the slope or inclination of a pipe line, and not to a pump, there will be very little pressure within the pipe in the upper portion of the line. In such cases large diameters with thin shells may be adopted in the upper part of the line where the pressure is small. As the line proceeds down the slope and the pressure increases, the diameter is diminished and the pipe shell increased in thickness in proportion to the increase of pressure.

If a pipe line is of uniform diameter and is laid on a uniform grade and has a full and free discharge, there will be no radial pressure in the pipe at any point except the very small pressure due to the vertical depth of the diameter. In this case there is no object in increasing the thickness of pipe shell at its lower end, because the total head or pressure, under these conditions, will be converted into velocity of flow. with the exception of the amount of head lost by friction, and as the velocity head or velocity pressure is always parallel to the pipe walls, it does not tend to burst the pipe.

Where a given pressure or head is to be maintained at the lower end of the pipe, or at any point along its length, while a given supply of water is being drawn from it for domestic purposes, or for driving water wheels, the capacity of the pipe must be such that the mean velocity of flow in it while delivering the given supply, will not cause a loss of head by friction exceeding a predetermined amount. The discharge permitted from such pipe must therefore be regulated by the area of discharge so that it will not exceed the given quantity. If the lower end of a pipe line be entirely closed so there can be no discharge from it and no velocity within it,the pressure at any point along the line will be that due to the total head up to that point, which will be equal to the difference in level between the given point in the pipe and the water surface in the reservoir or source of supply. The pressure at the lower end will be that due to the total head in the pipe line. If a small orifice be opened in the lower end of the pipe, it will at first discharge with a velocity due to the total head, but this discharge will cause a small velocity to be generated throughout the length of the entire pipe, and this velocity will cause a small friction with the pipe walls which will reduce the head by the amount of the friction thus generated, and thus slightly check the velocity of discharge through the orifice. The smaller the orifice relatively to the area and capacity of the pipe. the smaller will be the velocity in the body of the pipe to supply the quantity being discharged; and as the loss of head by friction is as the square of the velocity, the smaller the velocity becomes, the smaller the loss by friction will become. If the orifice is enlarged so that it may discharge a greater quantity per second, then the velocity in the body of the pipe must increase proportionately and loss of head or pressure will also increase as the square of this greater velocity. If the entire end of the pipe be opened so that the discharge is entirely free, then the total head will be lost in friction due to the consequent high velocity, except the small portion of the total head which remains to generate the velocity, and which is $h\,v = \frac{v^2}{64.4}$. It is evident then that if head or pressure is to be preserved the diameter and area of the pipe must be sufficient to convey the required quantity at a low velocity, and the pipe must not be permitted to discharge at anything like its full capacity. As loss of head or pressure is directly as the roughness of the pipe, and directly as the length, and inversely as $\sqrt{d^3}$, it is necessary to take into account not only the diameter and velocity but also the length of pipe, and the nature of the pipe walls with regard to smoothness or roughness, and probable future deterioration. The chemical qualities of the water which is to flow through a pipe, and the effect they have upon different classes of pipe and pipe coatings should be carefully ascertained before the pipe is selected. Some waters, apparently almost pure, will corrode a pipe in a very short time to such an extent as to reduce its capacity by nearly one half. A pipe line made up of different diameters, gradually decreasing as the slope increases, designed to convey a given quantity and to maintain a given pressure, is sometimes less expensive than a pipe line of uniform diameter. The velocities in the different diameters of such irregular pipe lines will be inversely as the areas of the different diameters and the friction loss in each section will be as the square of the velocity in that section and inversely as $\sqrt{d^3}$. The loss of head in such a line must be calculated separately for each different diameter. In case the line is divided into divisions of equal lengths, and each division is of a constant diameter but of a different diameter from the rest of the line, the mean diameter of the whole line cannot be adopted for such calculations, because the mean of all the velocities in the different diameters will greatly exceed the mean velocity in a pipe of uniform diameter equal to the mean diameter of the line composed of different diameters. As the friction is as the square of the velocity, it is evident that it will be much greater in the line of decreasing diameters than in a pipe of uniform diameter equal to the mean diameter of the former. Where the saving of head or pressure is a principal object there are only a few cases in which it is cheaper or advisable to adopt large diameters for the upper portion of the line and smaller ones for the lower portion. What is saved in the cost of constructing such line is lost in head or pressure, which may be of more value than the difference in cost between the two kinds of pipe line. For example a pipe line 5,000 feet in length, made of lap welded pipe and thoroughly coated with asphaltum, in which the first 1,000 feet length has a diameter of three feet, the second

1,000 feet has a diameter of 2.75 feet, the third 1,000 feet has a diameter of 2.5 feet, the fourth a diameter of 2 feet, and the fifth a diameter of 1.5 feet, while discharging 8 cubic feet per second, will have velocities and losses of head in the different diameters as follows:

In section No. 1, $v=\frac{q}{a}=\frac{8}{7.069}=1.1178$ feet, $h''=$ .072 feet

In section No. 2, $v=\frac{q}{a}=\frac{8}{5.94}=1.3470$ feet, $h''=$ .119 "

In section No. 3, $v=\frac{q}{a}=\frac{8}{4.909}=1.6300$ feet, $h''=$ .202 "

In section No. 4, $v=\frac{q}{a}=\frac{8}{3.1416}=2.5400$ feet, $h''=$ .680 "

In section No. 5, $v=\frac{q}{a}=\frac{8}{1.767}=4.5200$ feet, $h''=$ 3.350 "

4.423

The loss of head for this small discharge will be 4.423 feet in the line of different diameters, and the mean of all the velocities in the different diameters will be 2.2771 feet per second.

Now if the sum of these five different diameters is divided by 5 we have the mean diameter 2.35 feet. The area of this mean diameter=4.3374 square feet. Consequently if the entire pipe line had been of the uniform diameter of 2.35 feet, the necessary velocity through it to cause a discharge of 8 cubic feet per second would be

$v=\frac{q}{a}=\frac{8}{4.3374}=1.84442$ feet, and the total loss of head would have been $h''=1.424$ feet. (For this class of pipe $n=.0003$). As the friction is inversely as $\sqrt{d^3}$, and also directly as v^2, it is apparent that a small increase of the discharge would greatly increase the loss by friction in sections No. 4 and 5 of the irregular diameter.

67—A Power Main with Nozzle, and Water Wheel to Run at a Given Speed and Develop a Given Power.—

In mountainous regions are many small torrents, the sources of which are at such great altitudes as to afford almost any head desired when the stream is confined within a pipe or power main so as to preserve the head or pressure by regulating the velocity of flow. Where the quantity of water is small and the head is great, an impulse and reaction water wheel will be much more efficient and satisfactory than a turbine. The loss of head in a power main depends upon the velocity of flow through it and upon its length, diameter and smoothness and freedom from bends. The velocity is governed by the quantity of water the main is permitted to discharge, and the quantity discharged is governed by the area of discharge at the point of the nozzle and by the effective head at discharge.

The greater the length of the pipe line, the smaller the velocity must be, for the loss of head by friction is directly as the length and as the square of the velocity. Such power mains or pipe lines are usually constructed of riveted pipe made of steel or wrought iron plate. The pipe is made in any convenient lengths for transportation, or is made on the ground where it is to be laid. After it is riveted into lengths it is thoroughly coated by being submerged in a tank of hot coating compound composed of 80 per cent asphaltum and 20 per cent crude petroleum which is maintained at a temperature of about 300 degrees Fahr. The pipe is allowed to remain submerged in the hot bath until the pipe metal attains the same temperature as the bath. It is then withdrawn from the bath and allowed to cool. In some cases coal tar 45 per cent and asphaltum 55 per cent is used as a coating with fair results.

The quality or purity of the asphaltum used will determine the best proportion of asphaltum to crude petroleum to use in the compound. The per cent of petroleum required varies from 15 to 20. After the compound has been heated and thoroughly mixed and incorporated it should be tested by dipping into it a small sheet of the pipe metal and allowing it to remain for ten minutes in the hot bath. It is then withdrawn and placed in a large vessel of cold water and allowed to cool. If the coating is too soft after cooling and has a tendency to run or wrinkle, there is too much oil in it, and the quantity of asphaltum should be increased. If it is a mixture of coal tar and asphaltum the coating will be too brittle and easily knocked off with a hammer if the proportion of tar is too great to that of asphaltum.

In any case the coating should be tough and elastic and should adhere to the metal similar to paint. If the bath is too hot, the coating will wrinkle on the inside of the pipe when it is withdrawn and laid aside to cool. The lengths of pipe are put together like stove pipe, by wrapping a cloth around the end of one length and driving it into the end of the length below, the laying always being started at the lower end of the pipe line. This is called a slip joint. In cases where the pressure is considerable a sleeve joint is used. A sleeve joint consists of slipping an iron sleeve over the ends where two pipe lengths join or are butted, and running in melted lead between the sleeve and the pipe, having first packed the joint sufficiently to prevent the lead from running into the pipe where the ends come together.

In rocky, mountainous localities where trenching would be quite expensive, the pipe is usually laid on the surface without any trenching except where it is necessary to secure a substantial bearing or foundation for the pipe.

In very cold weather the pipe is allowed to discharge constantly, which prevents freezing within the pipe, or the water is prevented from entering the pipe and the line left empty when not in use.

Suppose a stream affords 10 cubic feet per second and has a fall of 400 feet per mile, and it is required to construct a water power plant that will develop 200 net horse power, using a water wheel of 85 per cent efficiency. What head will be required and what diameter and length of pipe, and what will be the proportions of the discharge nozzle required?

By formula (99) §62, the net head required will be,

$$H=\frac{200\div .85}{.1136363\times 10}=207.065 \text{ feet.}$$

As there is a fall of 400 feet per mile, it is seen that the line will be a little longer than one-half mile. The fall per foot length will be $S=\frac{H}{l}=\frac{400}{5280}=.075757575$, and the length in which there is a fall of one foot is $l=\frac{1}{S}=\frac{1}{.0757575}$ $=13.20$ feet.

Hence the length of pipe required, not making allowance for friction loss, will be $207.065\times13.20=2733.258$ feet of pipe.

But as there will be loss of head by friction in the pipe line and also in the nozzle, and it is required to have 207.065 feet net head at discharge from the nozzle, we must lengthen the pipe line until the total head will cover these losses and still leave the net head of 207.065 feet at discharge. If the nozzle is to be 8 feet long we will assume that the loss of head in the nozzle will be 6 feet while discharging 10 cubic feet per second, and we will design the pipe line so that the loss of head in the line by friction will be 6 feet also. Hence the line must be extended further down the hill until we have a total head in the whole length of the line including the nozzle$=207.065+12=210.065$ feet. In order to gain this additional 12 feet head the line will have to be extended in length by 158.40 feet, including the nozzle. The nozzle is to be 8 feet in length, and therefore the pipe line without the nozzle will be $(2733.258+158.40)-8=2883.66$ feet in length. It is to be double riveted, asphaltum coated, slip joint pipe, and the total loss of head in the whole line without the nozzle is to be 6 feet while discharging 10 cubic feet per second. What diameter will be required?

By formula (101) § 64, the diameter required will be

$d=\sqrt[11]{\frac{.000000269\times q^4\times l^2}{h''^2}}=1.795$ feet$=21.54$ inches diameter.

We have a net head now at the junction of the pipe line with the nozzle of 213.065 feet. The next step is to ascertain the required dimensions of the nozzle to discharge 10 cubic feet per second under these conditions with a loss of head not to exceed six feet in the nozzle. The method of doing this is explained in §65. For this calculation we have the length of the nozzle and its butt or greatest diameter, and the effective head at the butt of the nozzle. Now if we assume that there will be probably a loss of 6 feet head in the nozzle itself, the net head at discharge would be equal to the head at the butt, less the amount lost in the nozzle, or 213.065 —6=207.065 feet. Hence by the rules given heretofore (§65) the area in square feet of the least diameter at discharge of the nozzle will be

$$a=\frac{q}{\sqrt{2gH}}=\frac{10.}{8.025\sqrt{207.065}}=.0866 \text{ square feet.}$$

The diameter answering to this area is $d=\sqrt{\frac{.0866}{.7854}}=$ .11026 foot=1.323 inches

The mean diameter of the nozzle is$=\frac{1.795+.11026}{2}=$

.95263 foot.

Now we must test this nozzle to ascertain what the actual loss of head will be in it while it is discharging 10 cubic feet per second. If the loss is not so great as six feet, as we have assumed in the nozzle, then we may shorten the pipe line to some extent, or we may reduce the diameter of the pipe line very slightly, and still obtain the required head and power at discharge.

The velocity through the mean diameter of this nozzle in order to discharge ten cubic feet per second would be

$$v=\frac{q}{a}=\frac{10}{(.95263)^2\times.7854}=14.03 \text{ feet per second.}$$

The actual loss of head by friction in the nozzle under these conditions would be (§ § 37, 39, 65).

$$h''=\frac{n\,l\,9\,v^2}{\sqrt{d^3}}=\frac{.0003623\times8\times9\times196.841}{.9298}=5\,5224 \text{ feet.}$$

As the net head at the base of nozzle is 213.065 feet, and the loss in the nozzle is 5.5224 feet, we have a net head at point of discharge from the nozzle of 207.5426 feet. The required net head was 207.065 feet. Hence we have .4776 foot head in excess of exact requirements, which is near enough the desired result.

TEST FOR QUANTITY DISCHARGED.

The area of smallest diameter of nozzle at discharge is .0866 square foot as above found, and the net head at discharge is 207.5426 feet.

Hence the quantity that will be discharged is q=a v, or

$q = .0866 \times 8.025\sqrt{207.54} = 10.000568$ cubic feet per second.

Now the velocity of discharge from the nozzle is $v = 8.025\sqrt{207.54} = 115.48$ feet per second or $115.48 \times 60 = 6,928.80$ feet per minute.

It has been established by experiment and experience that the velocity of greatest efficiency of the circumference of an impulse and reaction water wheel is about one-half the velocity of discharge upon the wheel. The number of revolutions per minute of the water wheel will depend upon its circumference from center to center of the buckets taken as its diameter. The circumference equals the diameter in feet from center to center of buckets multiplied by 3.1416.

The circumference of the wheel when the load is on should travel at one-half the velocity of the discharging water. Hence the diameter of the wheel may be so proportioned to the velocity of discharge as to run any desired number f revolutions per minute. Where high speed is desired under a low head, two or more water wheels of equal diameter may be placed upon one shaft and have separate nozzles. In this way very small diameters of the wheels may be used to secure high speed, and the water divided so as to avoid placing very large buckets on small wheels and to also prevent flooding the wheel. The power developed does not depend upon the diameter of the water wheel, but depends upon its speed with reference to its diameter.

The point of the nozzle should be firmly set beyond the possibility of slipping against the wheel, and should be as close to the buckets as possible not to strike them or to have the jet re-acted upon from the buckets. The distance between the point of the nozzle and the center of the bucket on the wheel will depend upon the diameter at discharge of the nozzle and the velocity of discharge upon the wheel. It should not be so close that the jet will react upon itself on striking the buckets.

68.—Table of Eleventh Roots to Facilitate Calculations of Diameter Required to Discharge Given Quantities.

The following table covers diameters from one inch to 32 inches both inclusive, and will be convenient in conjunction with formulas for ascertaining the diameter in feet required to generate a given discharge (formulas 28, 43, 65, 81, 100, 101) or to cause a given discharge with a given loss of head.

TABLE No. 36.

Number	11th. Root	Number	11th. Root
.000000000001345	.08333	46.24	1.417
.00000002762	.1667	86.50	1.5
.0000002384	.25	156.40	1.583
.000005638	.3333	276.20	1.667
.00006578	.4167	471.50	1.75
.0004883	.5	784.80	1.833
.002659	.5833	1285.00	1.917
.01157	.6667	2048.00	2.000
.04223	.75	3203.00	2.083
.1345	.8333	4948.00	2.167
.3842	.9167	7482.00	2.25
1.0000	1.000	11150.00	2.333
2.404	1.083	16370.00	2.416
5.467	1.167	23840.00	2.5
11.64	1.250	31300.00	2.584
23.62	1.333	48560.00	2.667

REMARK 1.—Where the pipe is to be of uniform diameter and to have free discharge, as in the case of a pipe conveying water from one reservoir to another, there is no object in preserving the head by throttling the discharge, and in such case the total head is consumed in balancing the resistance to flow except that part of the head which is converted into velocity. The diameter of a pipe which is required to convey a given quantity of water per second under such conditions will be

$$d = \sqrt[11]{\frac{q^4 m^2}{.3805 S^2}}, \text{ or } d = \sqrt[11]{\frac{q^4 (l m)^2}{.3805 H^2}}$$

m=coefficient of velocity in terms of diameter in feet.

q=cubic feet per second that pipe is to discharge.

$S=\frac{\text{total head in feet}}{\text{total length in feet}}$=sine of slope

H=total head in feet.

REMARK 2—Where the pipe must convey a given quantity per second to a given point and must maintain a given head or pressure at that point while the given quantity is being drawn from it, then the diameter required will be found as pointed out in § 64, or by the following general formula.

$$d=\sqrt[11]{\frac{n^2 l^2 q^4}{.3805\times h''^2}}$$

This diameter will convey a given quantity with a given loss of head which is pre-determined according to requirements.

In which,

h''=head in feet to be lost in friction.

n=coefficient of resistance applicable to class of pipe.

q=cubic feet per second pipe is to discharge.

l =length of pipe in feet.

REMARK 3.—The results of experiments by the writer on "Converse Patent Lock Joint Pipe" made of wrought iron in lengths of from 15 to 20 feet and lap welded, and coated with asphaltum gave an average value of n=.000299 in terms of diameter in feet. The small value of the coefficient of resistance n, in this pipe is to be attributed to its uniformity of diameter, and to the fact that it is made in long lengths so there are fewer joints per mile of pipe, and the joints are so arranged as to present a continuous and uniform surface to the flow. For this class of pipe take n=.0003 and m=.00030472 in terms of diameter in feet. These values of the coefficients do not allow for future deposits in the pipe, if such should occur, nor for deterioration in the pipe coating. It is not probable that a first class asphaltum coating will deteriorate to any considerable extent for a great number of years. This remark has no reference to coatings made of coal tar compounds.

The diameter (inside) in feet of Converse pipe, asphaltum coated, required to convey a given quantity with a given loss of head, would be

$$d = \sqrt[11]{\frac{n^2 l^2 q^4}{.3805\,h''^2}} = .2498 \sqrt[11]{\frac{l^2 q^4}{h''^2}}$$

Or if the discharge is to be free and full bore, and no attempt made to preserve the head or pressure, the diameter required to carry a given quantity will be

$$d = \sqrt{\frac{q^4\ m^2}{.3805\,S^2}} = \sqrt[11]{\frac{m^2}{.3805}} \times \sqrt[11]{\frac{q^4}{S^2}} = .25055 \sqrt[11]{\frac{q^4}{S^2}}$$

Or

$$d = \sqrt[11]{\frac{m^2 l^2 q^4}{.3805\ H^2}} = .25055 \sqrt[11]{\frac{l^2 q^4}{H^2}}$$

For riveted asphaltum coated pipe $d = .2541 \sqrt[11]{\frac{q^4}{S^2}}$

69.—Head Lost by Friction at Bends in Water Pipes.

The amount of the loss of head produced by a bend in a pipe will depend upon the velocity of flow and the radius of the central arc of the bend, and also upon the number of degrees included in the arc of the bend. Whether the additional head required to overcome the resistance of a bend will be proportional to the square or to the cube of the velocity is doubtful. Weisbach's formula, which is most generally used for determining the resistance of bends, gives results undoubtedly too low in all cases except for a bend of 90° with a radius of central arc of bend equal to one half the diameter.

The resistance at a bend in a pipe or in an open channel is caused by the change of direction of the flow. The more abrupt the change, and the greater the amount of the change in direction, the greater will the resistance be. It is evident therefore that the resistance will be directly as the number of degrees included in the central arc of the bend and inversely as the radius of that arc.

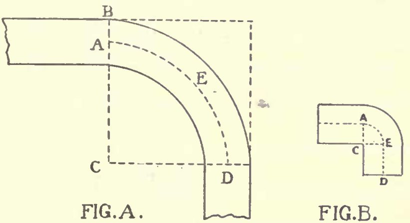


FIG. A. FIG. B.

Fig. A shows a bend of 90°, the radius c a, of the central arc of the bend being equal 6 times the radius a b, of the pipe, or equal three diameters of the pipe. Fig. B shows a bend of 90° with the radius c e of the central arc of the bend equal the radius of the pipe, or equal 1-2 diameter of pipe.

When the radius c a of the central arc of the bend is only equal to the radius of the pipe, or to one half the diameter, then the resistance or amount of head lost at such bend will equal the head in feet which generates the velocity of flow, or $h''=\frac{v^2}{64.4}$. For example suppose the velocity to be 3 feet per second through the pipe, and the bend is as shown in Fig. B, then the head in feet lost by resistance at the bend will be $h''=\frac{v^2}{64.4}=\frac{(3)^2}{64.4}=\frac{9}{64.4}=.14$ foot.

For a bend of 90° or any other constant number of degrees, the amount of change in the direction of the flow will be the same for any length of radius c a, of the arc of the bend, but the distance in which this change is effected will be directly as the len th c a, of the radius of the bend. Hence the shorter the radius of the bend the more abrupt will be the change in direction of flow, and consequently the greater the resistance. The central arc, a e d, of the bend increases in length or becomes more gradual directly as the radius of the bend c a increases in length and hence the longer this radius c a becomes the more gradual will be the change effected in the direction of the flow. The resistance at a bend will therefore be directly as the number of degrees included by the central arc of the bend, and inversely as the length of the radius of the bend and will increase as v^2 (or possibly as v^3). The formula will therefore be

$$h'' = \frac{A}{90} \times \frac{r}{R} \times \frac{v^2}{2g} = \frac{A}{90} \times \frac{.5}{R} \times \frac{v^2}{64.4} = \frac{A}{90} \times \frac{.5}{64.4} \times \frac{v^2}{R}$$

Which reduces to

$$h'' = \frac{A}{90} \times .007764 \frac{v^2}{R} = \frac{A\ v^2\ .007764}{90\ R} \quad \ldots\ldots\ldots\ldots\ldots (102)$$

In this formula (102)

$r = \frac{1}{2}$ diameter of pipe $= .5$

R = radius of central arc of bend in diameters of the pipe and is to be expressed as 1, 2, 3 etc diameters.

A = number of degrees of the arc of bend as 30, 90, 180, etc,

v = mean velocity of flow through the pipe.

EXAMPLE OF THE USE OF THE FORMULA.

It is required to find the resistance at a bend of 180° in an eight inch pipe where the mean velocity is 3 feet per second and the radius of the central arc of the bend is equal 3 diameters.

$$h'' = \frac{A\ v^2\ .007764}{90\ R} = \frac{180 \times 9 \times .007764}{90 \times 3} = .046584 \text{ feet head.}$$

REMARK 1.—The resistance at a bend is in addition to the ordinary frictional resistances of the pipe walls. Hence for a pipe which contains a bend, first calculate the loss of head by friction as for a straight pipe, and then add the loss of head due to the bend.

REMARK 2—It is assumed in all formulas for resistance at bends that the resistance is independent of the diameter of the pipe or width of the open channel, and that the resistance of a bend depends solely upon the velocity, the radius of the bend and the number of degrees included in the central arc of the bend. It is doubtful whether the diameter of a pipe exerts an influence on the resistance at a bend or not. It probably does.

REMARK 3—The force exerted by a column of water impinging upon a fixed surface is as the product of the quantity of water by its head. The quantity is directly as the velocity and the head is as the square of the velocity. Consequently the product is $v \times v^2 = v^3$. It is therefore possible that the force or head or energy absorbed at a bend will vary as v^3 instead of v^2.

70—Formulas of Weisbach and of Rankine for Resistance at Bends in Pipes.

The formula for resistance at bends proposed by Weisbach is

$$h'' = .131 + 1.847 \left(\frac{r}{R}\right)^{\frac{7}{2}} \times \frac{v^2}{2g} \times \frac{A}{180}$$

In which

r=radius of pipe in feet=½ diameter in feet.

R=radius of axis of bend in feet.

A=central angle of bend in degrees.

2g=effect of gravity=64.4.

Professor W. J. M. Rankine's formula is simply a change in form of Weisbach's formula, and is as follows:

$$h'' = \frac{A}{180}\left(.131 + 1.847\left(\frac{d}{2\,r}\right)^{\frac{7}{2}}\right)$$

In which

A=angle of bend in degrees

d=diameter in feet of pipe

r=radius of central arc of bend

To simplify Weisbach's formula, place the coefficient, $.131 + 1.847\left(\frac{r}{R}\right)^{\frac{7}{2}} = Z$. Then

$$h'' = Z \times \frac{A}{180} \times \frac{v^2}{64.4} = \frac{A\,v^2 Z}{11592} = Z.\left(\frac{A\,v^2}{11592}\right)$$

Remembering that in Weisbach's formula, r= half the diameter of the pipe in feet, and R= radius of the central arc of the bend in feet, the following table of values of Z will be readily understood and applied:-

Value of Z in Weisbach's Formula.

$\frac{r}{R}=$	.1	.15	.2	.225	.25	.275	.3	.325	.35	.375	.4	.425
Z=	.131	.133	.138	.145	.15	.155	.16	.17	.18	.195	.206	.225
$\frac{r}{R}=$	.45	.475	.5	.525	.55	.575	.6	.625	.65	.675	.7	.725
Z=	.244	.264	.294	.32	.35	.39	.44	.49	.54	.60	.661	.73
$\frac{r}{R}=$	.75	.775	.80	.825	.85	.875	.9	.925	.95	.975	1.00	
Z=	.806	.880	.98	1.08	1.18	1.29	1.41	1.54	1.68	1.83	2.00	

USE OF ABOVE TABLE.—The velocity in feet per second through an eight inch pipe is 3 feet. There is a bend of 90° with a radius of bend equal 4 inches or half the diameter. What is the loss of head in feet caused by the bend?

We see that as the radius of the central arc of the bend is equal to half the diameter of the pipe; that $\frac{r}{R}=1.00$. Referring to the above table, and it is seen that when $\frac{r}{R}=1.00$, then Z=2.00. Hence by the formula,

$$h''=Z.\ \left(\frac{A\ v^2}{180\times64.4}\right)=\left(\frac{90}{180}\times\frac{9}{64\ 4}\right)\times2=.14\text{ foot.}$$

The radius in feet of an 8 inch pipe=.3333 foot.

The diameter in feet of an 8 inch pipe=.6666 foot.

Suppose the radius of the above bend R=.6666 foot or equal the diameter, and the radius of the pipe is .3333 foot.

$$\text{Then}\ \frac{r}{R}=\frac{.3333}{.6666}=.5$$

From the above table it is seen that when $\frac{r}{R}=.5$, then Z=.294. And in this case Weisbach's formula would give the loss for 3 feet velocity of flow as

$$h''=Z\ \left(\frac{A\ v^2}{11592}\right)=.294\ \left(\frac{90\times9}{11592}\right)=.02\text{ feet.}$$

This latter result is altogether too small.

71.—Comparison of the Results by Weisbach's Formula and by the Formula Herein proposed, for Bends of 90° with Radii Varying form R=½d to R=3d, and Different Velocities.

In the following table the loss of head by friction has been computed by our formula (102) and also by Weisbach's formula for various velocities of flow through a bend of 90° in which the radius of the central arc of the bend varies from R =½ d to R=3d. It is possible that the results by either formula are too small for the reason suggested in remark 3, § 69

TABLE No. 38.

Table of computed results for comparison.

Velocities	2 Feet	3 Feet	4 Feet	5 Feet	6 Feet	7 Feet	8 Feet	
R=½d.....	.062	.14	.248	.388	56	.761	.994	Formula (102)
	.0621	.14	.248	.388	.56	.761	.994	Weisbach
R=d.......	.031	.07	.124	.198	.28	.38	.497	Formula (102)
	.009	.02	.026	.056	081	.11	.144	Weisbach
R=2d......	.0155	.035	.072	.097	.14	.19	.248	Formula (102)
	.005	.011	.019	.029	.042	.057	.074	Weisbach
R=3d......	.0103	.0233	.0413	.0646	.093	.1267	.1656	Formula (102)
	.004	.01	.017	.027	.038	.052	.069	Weisbach

It would appear from an inspection of the results by Weisbach's formula that there is little to be gained by making the radius of the bend greater than twice the diameter of the pipe. This is not true, however, in practice. The radius of a bend should be made as great as the circumstances will permit unless the velocity of flow through the pipe is to be very small. The velocity should be the controlling feature in determining the radius of the bend.

Fanning says "Our bends should have a radius, at axis, equal at least to 4 diameters." Trautwine advises a radius of bend equal to 5 diameters length, or as much longer as it can be made. If the velocity does not exceed 5 feet per second, then a radius of 3 diameters will reduce the loss of head to .0646 foot at a 90° bend.

72.—*Resistance at Bends. Rennie's Fxperiments.*

While the results of experiments by Rennie on leaden pipe one half inch diameter are not of great value as establishing any law of resistance at bends, yet they indicate very clearly that the results by Weisbach's formula are too low.

Rennie experimented with a leaden pipe 15 feet in length and half inch in diameter under a total head of 4 feet. He ob. tained the following results;

The straight pipe before being bent discharged .00699 cubic feet per second.

With one bend at right angles near the end, ... .00556 cubic feet per second.

With 24 right angle bends .... .00253 cubic feet per second.

It will be noted that the bends are described as right angled. This may have crushed the pipe out of form and reduced the area at the bends, This would materially affect the velocity and the resistance through the bend. Whether this occured or not is not stated, From the area in square feet of this half inch pipe and the quantity in cubic feet per second it discharged we find that the velocities of discharge were as follows:

Before the pipe was bent, $v=\frac{q}{a}=\frac{.00699}{.0014}=5$ feet per second.

With one right angle bend, $v=\frac{q}{a}=\frac{.00556}{.0014}=3.971$ feet per second.

With 24 right angled bends, $v=\frac{q}{a}=\frac{.00253}{.0014}=1.80$ feet per second.

In order to prevent confusing the resistance of the pipe walls with that of the bends. we will first find the value of the coefficient of resistance n, of the pipe before it was bent.

The total head was 4 feet, and while the pipe was straight the velocity of discharge was 5 feet per second. The head in feet lost by friction along the walls of the straight pipe under this velocity was equal the total head minus the head due to the velocity of discharge, or was

$$4.-\frac{(5)^2}{64.4}=4.-.3882=3.6118 \text{ feet.}$$

After one bend was made in the pipe, the total head remaining 4 feet, the velocity of discharge was only 3.971 feet per second. Now from the data of flow in the straight pipe before the bend was introduced we find the value of n to be

$$n=\frac{h''\sqrt{d^3}}{l\,v^2}=\frac{3.6118\times.0085}{15\times25}=.000082.$$

After one bend had been introduced the velocity was reduced to 3.971 feet per second, so the friction of the pipe walls exclusive of the resistance of the bend was now

$$h''=\frac{n\,l\,v^2}{\sqrt{d^3}}=\frac{.000082\times15\times15.76884}{.0085}=2.2818 \text{ feet.}$$

But the total loss of head due to pipe walls and one bend combined was equal the total head of 4 feet minus the velocity head, or equal

$$4-\frac{(3.971)^2}{64.4}=4.-.244842=3.755158 \text{ feet.}$$

If we deduct from this total loss the loss due to pipe walls we have 3.755158—2.2818=1.473358 feet head lost by the resistance at the bend; which is equal 6 times the head generating the velocity. This would indicate that the resistance at a bend is more nearly proportional to v^3 than to v^2. as intimated in remark 3, § 69. The resistance at a bend in a very small pipe is probably greater than in large pipes.

The total head remaining 4 feet, after 24 right angled bends were made in this 15 foot length of half inch lead pipe the velocity was 1.80 feet per second as determined from the quantity discharged. The loss of head due to friction of pipe walls, exclusive of the bends, was, for this velocity.

$$h''=\frac{n\,l\,v^2}{\sqrt{d^3}}=\frac{.000082\times15\times3.24}{.0085}=.4688 \text{ feet head.}$$

The total loss of head due both to the 24 bends and the friction of pipe wall was $H-\frac{v^2}{2g}=4.-.05031=3.94969$ feet.

The loss due to the 24 bends alone was therefore equal the total loss minus the loss due to pipe walls=3.94969—.4688 =3.48 feet.

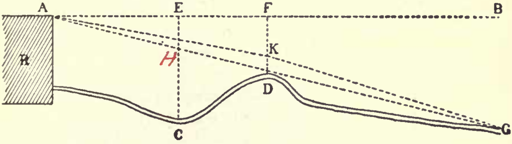
If the loss was equal at each bend, then $h''=\frac{3.48}{24}=$ .145 foot head lost at each bend for a velocity of 1.80 feet per second. In this case the head lost at each bend was only equal 2.88 times the head generating the velocity. It must be remembered that all these bends are described as right angled bends, It is probable that serious contractions of the area of the pipe were produced at each such bend and that the velocity of flow through the contractions was greater than 1.80 feet per second.

Because of the direct action and equal reaction of the water impinging upon the pipe wall at a right angled bend the loss of head at such bend could not be less than twice the head producing velocity, or $h''=2\frac{v^2}{2g}$ According to the above results of Rennie's experiments with 24 right angled bends it appears that the loss at each bend was equal nearly three times the head producing the velocity or $h''=2.88\ \frac{v^2}{2g}$ But it is doubtful what the actual velocity was in the bends as the areas were probably contracted.

Right angled bends or shoulders are, however, never introduced into a water pipe, but the bends are always circular. As a true right angled bend cannot be made without cutting and fitting, or casting, it is probable that Rennie's pipe was bent like Fig. B, §69.

73.—Relation of Thickness of Pipe Shell to Pressure, Diameter and Tensile Strength of Pipe Metal

When a pipe is filled with water and is closed at discharge end so there can be no flow in it, the radial pressure within the pipe tending to burst it will vary as the head of water above any given point along the pipe, and at any given point will be equal H×.434=lbs. pressure on each square inch of the internal circumferance.



In the Figure let R represent a reservoir, the water level in which is A, and a pipe C D G, is laid from it over hills and depressions. When the pipe is closed at G, the pressure within the pipe which tends to burst it will vary as the vertical distance C E, D F, between the given point in the pipe and the level of the water A E F B, in the reservoir. Hence the thickness and strength of the pipe shell must be proportion according to the position it is to occupy in the pipe line. If the vertical distance CE is 130 feet then the pressure at C on each square inch of the internal circumference of the pipe will be 130×.434=56.42 lbs. But the pipe passing over the hill at D is only 80 feet below the level of the water in the reservoir, and consequently the pressure within the pipe at D is equal 80×.434=34.72 lbs. per square inch. A profile of the pipe line showing the distance at all rises and depressions along the line between the pipe and the level A E F B should always be made before the thickness of pipe shell is calculated for any portion of the line. With such profile the thickness and strength of the pipe for each division of the line may be calculated so as to conform to the pressure it must sustain.

The inclined line A, G, is the hydraulic grade line, or line which indicates the hydraulic or running pressure in the pipe when the pipe is open at G and discharging freely.

The hydraulic or running pressure within the pipe at any given point along the pipe line is equal to the distance in feet, measured vertically, from the given point in the pipe to the hydraulic grade line, A G., multiplied by .434. Thus, the running pressure at C in the pipe is equal the vertical distance C H in feet multiplied by .434. The difference in feet between C E and C H shows the loss of head in feet by friction between the reservoir and C. If the pipe were laid on the hydraulic grade line A, G, there would be no pressure in it at all when discharging freely except that due to the depth of the diameter. The pipe must be so laid that no part of it will rise above the hydraulic grade line. If the pipe at D should rise above the line A G, to K, then the line would require to be divided into two divisions, A K, and K G, both as to diameter of pipe and as to the hydraulic grade line. The diameter K G, if the same as A K, would not run full, for the reason that K G would have a greater fall per foot length than A K. Assuming the pipe to be laid as shown by C D G, and that it is closed at G so there is no discharge, then the internal pressure on each square inch at any given point in the pipe will equal the vertical head in feet between the given point in the pipe and the line A E F B, multiplied by .434, and the number of square inches subject to this pressure will be directly as the diameter in inches of the pipe, because the circumference is equal $d \times 3.1416$.

The total pressure on the inner circumference will therefore be $H \times .434 \times d \times 3.1416$.

The pressure of quiet water is equal in all directions. In a circular pipe the pressure radiates from the axis of the pipe to every point in the circumference. The resultant of the pressure on one half the circumference acts through the center of gravity of that half, and equals the products of the pressure into the projection of that half circumference. The projection of half the circumference equals the diameter of the pipe. An equal resultant acts in the opposite direction through the center of gravity of the other half circumference. The resulting strain on the pipe shell at any point in the circumference is equal to the sum of these opposing resultants. If therefore, the thickness and strength of the pipe shell is to be found simply in terms of the pressure resultant of one half the circumference, due to the total head, it is evident that the thickness and strength must equal twice this resultant, or,

$$2\,t\,S = P \times d \ldots\ldots(103)$$

t= thickness of pipe shell in inches.

S= tensile strength in lbs. per square inch of pipe metals.

P= pressure in lbs. per square inch = H.×.434.

d= inside diameter of pipe ininches.

This gives a thickness and strength just sufficient to equal or balance the pressure of the quiet water, as

$$t = \frac{P \times d}{2\,S} \ldots\ldots (104)$$

To be sufficiently strong to withstand the violent shocks and sudden strains caused by water ram, and to provide for defects in casting or in riveting, and to prevent breakage in handling and from unequal settlement of the pipe in the trench, it is necessary to make cast iron pipe very much thicker and heavier than theory would indicate, and wrought iron and steel pipe from three to six times as thick as the quiet pressure alone would actually require. For these reasons the formula (104) must have added to it another factor called the factor of safety, and it then becomes

$$t = \frac{P\,d}{2\,S} \times F \ldots\ldots (105)$$

The factor of safety F, may be equal 2, 3, 4 etc. according to the service the wrought iron or steel pipe is to be put to.

This formula is not used for cast iron pipe for the reason that cast iron pipe is so brittle that it is necessary to give it heavy dimensions regardless of the pressure it is to withstand. Wrought iron and steel pipe being flexible and tough, does not require high factors of safety, but if laid as a permanent line, the shell should be sufficiently thick to prevent pitting through in case the coating is knocked off. The factor of safety of a pipe is found by the formula

$$F = \frac{2tS}{Pd} \ldots\ldots (106).$$

The value of S depends on the net strength of a riveted joint, (See § 74)

Many steel pipes have been in successful use under high pressure for many years with factors of safety as low as 2. These small factors of safety were used, however, where the pipe was not subject to water ram.

For the reason heretofore mentioned, the formulas for the thickness of cast iron pipe are necessarily arbitrary and empirical.

For thickness in inches of cast iron pipe of diameters of less than 60 inches

$$t=(P+100)\times.000142\times d+.33(1.-.01\,d)$$

For thickness in inches of cast iron pipe of 60 inches diameter or greater,

$$t=(P+100)\times.000142\times d.$$

t=thickness of pipe shell in inches.

P=pressure in pounds per square inch.

d=diameter (inside) of pipe in inches.

The tensile strength of cast iron pipe is ordinarily taken as equal to 18,000 pounds per square inch. If made of the best quality of iron and remelted four times, and cast vertically with bell end down, the pipe would have a tensile strength as great as 30,000 pounds per square inch, and would be tough, so that a large part of its superfluous weight might be dispensed with, and the thickness of shell greatly reduced thus reducing the cost of freight, hauling and laying.

74.—Values of S in Water Pipe Formulas.—The value of S to be used in the formula (105) for determining the required thickness and strength of pipe shell depends on the the nature of the pipe, whether steel or iron, and whether welded or riveted, and if riveted, then whether single or double riveted. The net strength of a riveted joint depends on the ratio of shearing strength of rivets to tensile strength of the plate, and also upon whether the riveting is done by hand or by hydraulic power. In hand riveting the work is done with cold rivets and the rivet holes are made from 1-32 to 1-16 inch larger than the diameter of the rivet, and the effect of the hammer in upsetting the rivet is not sufficient to swell the rivet to its full length so as to completely fill the rivet hole. Hand riveting does not leave as substantial a head on the rivet as machine riveting and is inferior to machine riveting in many respects. A formula for fixing the pitch of rivets in a joint is necessarily based on the ratio of the given shearing strength per square inch of the rivet metal to the given tensile strength of the plate metal. The formula must be varied as these factors vary. The tensile strength of wrought iron plates varies from 44,000 to 57,000 lbs per square inch. A good average wrought iron plate should have a tensile strength of 50,000 pounds per square inch before the rivet holes are made in it. The tensile strength of solid steel plate varies from 56,000 to 108,000 lbs per square inch.

The best iron rivets have a shearing strength of only 45,000 lbs. per square inch. The results of a great many experiments made by the Research Committee of the Institution of Mechanical Engineers (London, 1881) showed that the ultimate shearing resistance of steel rivets was 49,280 lbs. per square inch for single riveted joints, and 53,760 lbs. per square inch for double riveted lap joints. It is very probable that iron rivets would not have a greater ultimate shearing resistance than 40,000 lbs. per square inch of rivet area in a single riveted joint riveted by hand. Very high steel of great shearing strength is too brittle for rivets, although riveted hot. Hence there is no advantage in adopting plates of greater tensile strength than rivets of suitable shearing strength can be found for. A steel plate of about 66,000 to 70,000 lbs. per square inch tensile strength is as high as suitable rivets can be obtained for, and plates of this class will require steel rivets of best quality. The value of S to be used in the formula (105) should be the net strength of the joint or pipe shell. We will first give the formula for proportions of riveted joints, and then for testing the strength of such joints. By these means the value of S must be determined in each case. (See § § 75 80.)

75.—Riveted Steel Pipe —For riveting cold, the best grade of steel plate is open hearth mild steel of about 60,000 lbs. per square inch tensile strength to be riveted with best quality swede iron rivets of 45,000 lbs, per square inch shear-resistance. We have then $\frac{45000}{60000}$=75 per cent as the ratio of shearing strength of rivets to tensile strength of plates. In this case $\frac{1}{.75}$=1.33, is the ratio of area of rivets to net plate required to balance the tensile strength of the plate. When the rivet holes are made in the plate it is weakened as a whole by a percentage found thus:

Let S=Original tensile strength of plate, unperforated.

S′=tensile strength of plate after holes are made.

P=pitch, inches, center to center of rivets in one row.

d=diameter in inches of rivet hole (not of rivet).

t=thickness of plate in inches.

Then the per cent strength of the punched plate S′, to the original unpunched plate will be

$$S' = \frac{P-d}{P} = \text{per cent } S$$

The numerical value of S′ will

$$S' = \left(\frac{P-d}{P}\right) \times S \times t$$

We have just seen that in order to make the shearing strength of the rivets equal to the tensile strengh of the plate in this case, the combined area of the rivets must equal 1.33 times the net plate area between holes. The plate area between the rivets holes is

$(P-d)t$

The area of the rivets is $d^2 \times .7854$. Hence the equation

$(P-d) \times t \times 1.33 = d^2 \times .7854$

From which

$$P = \left(\frac{d^2 \times .7854}{t \times 1.33}\right) + d. = .60\frac{d^2}{t} + d,$$ for single riveted joint,

And

$P= \frac{d^2.7854\times2}{t} + d = 1.20 \frac{d^2}{t} + d$, for double riveted joint,

But supose the rivets had been steel rivets of 50,000 plbs. shearing strength, and the plates as above, that is, of 60,000 lbs. per square inch tensile strength. Then the pitch formula would be worked out as follows:

$\frac{50{,}000 \text{ lbs. shearing strength}}{60{,}000 \text{ lbs. tensile strength}} = .833$ per cent.

Hence, $\frac{1}{.833} = 1.20$. That is, the combined area of rivets must be 1.20 times the net plate area between holes.

Then,

$(P-d)t\times1.20 = d^2 .7854$

From which,

$P= \left(\frac{d^2 .7854}{t\times1.20}\right) + d = .6545 \frac{d^2}{t} + d$, for single riveted

And

$P= \left(\frac{d^2\times.7854\times2}{t\times1.20}\right) + d = 1.309 \frac{d^2}{t} + d$, for double riveted

Observe that d=diameter of rivet hole, which is always from 1 32 to 1-16 inch larger than the rivet before the rivet is upset.

We are restricted to the use of the market sizes of rivets, and should select a diameter of rivet equal to from 1.70 to 2.33 thicknesses of the plate. When the diameter of rivet is selected then add 1-32 (.03125 inch) for value of d in the pitch formula

If steel plate of 70,000 lbs. per square inch tensile strength is used, then the best quality of steel rivets of not less than 53,000 lbs. per square inch shearing resistance should be adopted. In this case the combined area of the rivets must exceed the area of the net plate metal between rivet holes by .32075 per cent, as below shown.

$\frac{70000}{53000} = 1.32075$ per cent.

Then,

$(P-d)t\times 1.32075=d^2 .7854$=total area of rivets.

And,

$$P=\left(\frac{d^2 .7854}{t\times 1.32075}\right)+d=.5946\frac{d^2}{t}+d,$$ for single riveted joints

And

$$P=\left(\frac{d^2 .7854\times 2}{t\times 1.32075}\right)+d=1.19\frac{d^2}{t}+d,$$ for double riveted joints.

If the pipe is to sustain an extremely high pressure, or is subject to frequent water ram, it should be triple riveted with a ribbon of lead 1-32 inch thick placed between the lap of the plates. Then for a triple riveted joint with rivets and plates of the above strengths, the pitch formula would be

$$P=\left(\frac{d^2 .7854\times 3}{t\times 1.32075}\right)+d=1.784\frac{d^2}{t}+d,$$ center to center, in one row.

After many tests of riveted joints (steel plates and steel rivets) the Research Committee of the Institution of Mechanical Engineers (London, 1881) reported that: "To attain the maximum strength of joint the breadth of lap must be such as to prevent it from breaking zig-zag. It has been found that the net metal measured zig-zag should be from 30 to 35 per cent in excess of that measured straight across, in order to insure a straight fracture. This corresponds to a diagonal pitch of $\frac{2}{3}P+\frac{d}{3}$, if P be the straight pitch and d=diameter of rivet hole To find the proper breadth of lap for a double riveted joint it is probably best to proceed by first setting this pitch off, and then finding from it the longitudinal pitch, or distance between the centers of the two rivet lines running parallel across the plate."

If the net metal between two rows of rivet holes is equal to twice the diameter of the rivet hole, the joint will be safe.

The distance of the rivet holes from edge of plate should be equal to two diameters of the rivet hole.

In the experiments of the Research Committee they found that a single riveted joint, riveted by hand, (steel rivets and plate) would begin to slip or give when the stress or load per rivet amounted to 6,600 lbs. The plates were 3-8 inch thick and rivets one inch diameter. A similar hand riveted, double riveted joint, began to slip or give when the load per rivet reached 7,840 lbs. whereas a machine riveted joint of similar proportions did not begin to slip until the load per rivet was double that at which the hand riveted joints began to give.

The value of hydraulic riveting is in the fact that it holds the plates more tightly together, and thus doubles the load at which the slip in a joint commences. The size of rivet heads and ends was found of great importance in single riveted joints. An increase of one-third in the weight of the rivets (all the excess weight being in the rivet heads and ends) was found to add 8 1-2 per cent to the resistance of the joint, for the reason that the large heads and ends held the plates firmly together and prevented them from cocking so as to place a tensile strain on the rivets. The committee also found that the effect of punching instead of drilling the rivet holes was to weaken the plates from 5 to 10 per cent in soft wrought iron, and 20 to 25 per cent in hard wrought iron plates. and 20 to 28 per cent in steel plates. This weakening, of course, extended only to the metal immediately around the hole. They also found that the metal between the rivet holes in mild steel plate has a considerably greater tensile strength per square inch than the unperforated metal. The excess tensile strength amounted to from 8 to 20 per cent, being largest where the distance between rivet holes was least.

"A riveted joint may yield in three ways after being properly proportioned, namely, by the shearing of its rivets; or by the pulling apart of the net plate between the rivet holes; or by the crippling (a kind of compression, mashing or crumpling) of the plates by the rivets when the two are too forcibly pulled against each other. It also compresses the rivets themselves transversely at a less strain than a shearing one; and this partial yielding of both plates and rivets allows the joint to stretch considerably before there is any danger of actual fracture. Or in steam or water joints it may cause leaks without further inconvenience or danger."—Trautwine.

In view of the results of the experiments as to the slipping, or giving or "crippling" of joints, as shown by the report of the Research Committee, it is evident that if an absolutely water tight joint is to be made to stand high pressure, the pitch of the rivets must be less than would be indicated by the theory of simply equalizing the shearing strength of rivets and the tensile strength of plates. The crushing or mashing load, within elastic limits, must be observed.

76.—Table of Proportions of Single and Double Riveted Joint, Mild Steel, Water Pipe Joints.—The pitch of the rivets in the following table is for sheet steel of 60,000 lbs. per square inch tensile strength, and for Swede Iron rivets of 45,000 lbs. per square inch shearing strength. The lap for any class or strength of plate in the straight seams should equal 5 diameters of the rivet hole in single riveted joints, and 8 diameters of the rivet hole in double riveted joints. This gives two diameters distance between edge of rivet hole and edge of plate in both single and double riveted joints, and in double riveted joints also gives two diameters (straight distance) between the two rows of rivets, or three diameters straight across from one pitch line to the other. Such lap gives more friction between the plates, is more rigid and less straining on the rivets, and may be scarped down better than a smaller lap. The round seams should have a lap of three times the diameter of the rivet hole, and pitch as for single riveted joint.

TABLE No. 39.

Thickness of Plate	Thickness of Plate	Size of Rivet.	Rivet Hole Diameter.	D² of Hole.	Single Riveted. Pitch C to C	Double Riveted Pitch C to C	Lap. Single R.	Lap. Double R.
B. W. G. No.	In.	In.	In.	In.	In.	In.	In.	In.
5.	0.220	7-16	.46875	.21972656	1.06875	1.66725	2.11-32	3.3-4
6.	.203	3-8	.40625	16503906	0.89405	1.38185	2.1-32	3.1-4
7.	.180	3-8	.40625	.16503906	0.95638	1.50651	2.1-32	3.1-4
8.	.165	3-8	.40625	.16503906	1.00000	1.64285	2.1-32	3.1-4
9.	.148	1-4	.28125	.07910256	0.60195	0.92262	1.13-32	2.1-4
10.	.134	1-4	.28125	.07910256	0.63545	0.98963	1.13-32	2.1-4
11.	.120	1-4	.28125	.07910256	0.67676	1.07227	1.13-32	2.1-4
12.	.109	3-16	.21875	.04785156	0.48215	0.74555	1.3-32	1.3-4
13.	.095	3-16	.21875	.04785156	0.52097	0.82319	1.3 32	1.3-4
14.	.083	3-16	.21875	.04785156	0.56465	0.91058	1.3-32	1.3-4
15.	.072	1-8	.15625	.02441406	0.35970	0.56315	0.25-32	1.1-4
16.	.065	1 8	.15625	.02441406	0.38161	0.60697	0.25-32	1.1-4

77—Table of Decimal Equivalents to Fractional Parts of an Inch.

The following table will greatly facilitate calculations of of riveted joints.

TABLE No. 40.

1-32=.03125	1-4+3-32=.34375	5-8+1-32=.65625
1-16=.0625	3-8=.375	5-8+1-16=.6875
3-32=.09375	3-8+1 32=.40625	5 8+3-32=.71875
1-8=.125	3-8+1-16=.4375	3-4=.75
1-8+1-32=.15625	3 8+3 32=.46875	3-4+1-32=.78125
1-8+1 16=.1875	1-2=.50	3 4+1-16=.8125
1-8+3-32=.21875	1-2+1-32=.53125	3-4+3-32=.84375
1-4=.25	1-2+1-16=.5625	7-8=.875
1-4+1-32=.28125	1-2+3 32=.59375	7-8+1-32=.90625
1-4+1 16=.3125	5-8=.625	7-8+1-16=.9375
		7-8+3 32=.96875

78.—Weight of Each Thickness, Per Square Foot, of Sheet Iron and Steel

TABLE No. 40 A.

B. W. G. No.	Thickness Inches	Wt. 1 Sq. Ft. Iron	Wt. 1 Sq. Ft. Steel.	B. W. G. No.	Thickness Inches	Wt. 1 Sq. Ft. Iron	Wt. 1 Sq. Ft. Steel
1	0.300	12.13 lbs	12.25 lbs	10	0.134	5.416 lbs	5.470 lbs
2	.284	11.48 lbs	11.59 lbs	11	.120	4.850 lbs	4.899 lbs
3	.259	10.47 lbs	10.57 lbs	12	.109	4.405 lbs	4.449 lbs
4	.238	9.619 lbs	9.715 lbs	13	.095	3 840 lbs	3.878 lbs
5	.220	8.892 lbs	8.981 lb	14	.083	3.355 lbs	3.388 lbs
6	.203	8.205 lbs	8.287 lbs	15	.072	2.910 lbs	2.939 lbs
7	.180	7.275 lbs	7.348 lbs	16	.065	2.627 lbs	2.653 lbs
8	.165	6.669 lbs	6.736 lbs	17	.058	2.344 lbs	2.367 lbs
9	.148	5.981 lbs	6.041 lbs	18	.049	1.980 lbs	1.999 lbs

79.—Calculating Weight of Lap-Joint Riveted Pipe.— In measuring the length of a sheet of metal to make a circle of given inside diameter, allowance must be made for the contraction or compression of the metal in bending, This contraction or shortening of the plate in bending equals the thickness of the plate to be bent. Consequently the length of plate required to make a lap riveted pipe of a given inside diameter in inches must be equal to $(d+t)\times 3.1416+$ required lap in inches. $d=$ required inside diameter in inches, and $t=$ thickness of plate to be bent, in inches. The weight of the metal punched or drilled out in making the rivet holes for straight and round seams is about equal to 25 per cent of the weight of the rivets. Consequently take the weight of the solid plate of required dimensions (Table No. 40A) and add 75 per cent of total weight of rivets required. If the pipe is to be coated or flanged, this must also be added to the weight. Allow for lap of each round seam as much loss of length of pipe off each sheet of metal as six times the diameter in inches of the rivet hole, except for the two sheets forming the ends of a length of pipe which will be $d\times 3$. For the straight seams the lap should be

$\text{Lap}=d\times 8$, for double riveted pipe joints.

Lap=d×5, for single riveted pipe joints.

And for round seams d×3=lap at each end of each sheet.

Observe that d= diameter of rivet hole in calculating lap, and in calculating the pitch of the rivets.

80.—Tests for the Strength of a Riveted Lap Joint.—To ascertain the actual net strength of a riveted lap joint proceed as follows:

Let S=tensile strength per square inch of plate before punched.

S′=tensile strength of plate per square inch after punched.

t=thickness of plate in inches or decimals of an inch.

d=diameter in inches of rivet hole.

P=pitch, or distance from center to center of rivets in one row.

Then the net tensile strength of the punched plate will be,

$$S' = \left(\frac{P-d}{P}\right) \times t \times S.$$

EXAMPLE.

The original unpunched plate had a tensile strength of say 60,000 lbs. per square inch, or S=60,000.

The plate was of No. 11 guage steel and .12 inch thick, or t=.12.

The diameter of rivet hole was d=.28125.

It was double riveted and the pitch of the rivets in one row was P=1.07227.

Then,

$$S' = \left\{\frac{1.07227-.28125}{1.07227}\right\} \times .12 \times 60{,}000 = 5{,}311.48 \text{ lbs.}$$

The strength before the rivet holes were made was S=S×t=60,000=.12×7,200 lbs.

Then the actual value of S to be used in the formula for thickness and strength of pipe shell (105) would be

$$\frac{5311.48}{7200} = .7377 \text{ per cent of S, or } 60{,}000 \times .7377.$$

The test for actual strength of plate between rivet holes in one row being satisfactory, we then test the joint for its resistance to shearing of rivets. The area of net plate between two holes in one row was

$$\text{Net plate} = \left(\frac{P-d}{P}\right) \times t.$$

But as the shearing resistance per square inch of rivet metal was only 75 per cent of the tensile strength of the plate metal, we made the rivet area $=\frac{60{,}000}{45{,}000}=1.33$ times the net plate area.

Then, if R= resistance to shear of rivets, we have

$$R = \left(\frac{P-d}{P}\right) \times t \times 1.333 \times 45{,}000 = 5311.48 \text{ lbs.}$$

This shows the tensile and shearing strength to be equal. As to test for "crippling strength of joint, we have Trautwine's rule. N=number of rivets in one inch length of joint.

$N=\frac{1}{P}$ for single riveted joint, and $N=\frac{2}{P}$ for double riveted joint. In this double riveted joint $N=\frac{2}{P}=\frac{2}{1.07227}=1.86522.$

Then,

Crippling strength=N×2 t×d×60,000=7,554 lbs.

The value of S to be used in formula (105) should be the smallest of the three values above found if the pipe is to be absolutely water tight, which in this case was S=60,000 × .7377 per cent.

81.—Testing Plates for Internal Defects.—The quality of iron or steel as to density will of course be determined by the weight per cubic unit of the metal. Light weight indicates weakness and impurities in the metal. Internal laminations may be detected by standing the plate on edge and tapping it all over with a light hammer. If the sound is dull, the plate is laminated internally, but if the ring is clear and sharp the plate is sound. Another test is to place supports under the four corners of the plate and throw a thin layer of dry fine sand upon the plate, and tap it lightly with a hammer. If the plate is defective, the sand will collect over the defective places, but if the plate is sound the vibrations will throw the sand off the plate.

82.—Different Methods of Joining Pipe Lengths.— Cast iron pipe is usually made in lengths of 12 feet, having an enlargement at one end of each length called a bell or hub, to receive the spigot end of the next length. After the spigot is inserted into the bell and adjusted so as to fit up closely at the end and bring the pipe into line, a piece of jute, old rope, or gasket cut long enough to reach around the pipe with a small lap, is forced into the joint to prevent the melted lead from running into the pipe. A fire-clay roll with a rope center is now wrapped around the pipe close to the bell with its two ends turned out along the top of the pipe to guide the melted lead into the joint. The lead is made sufficiently hot to flow freely, and is poured in until the joint is full. The lead is then calked back into the joint all around the pipe with a calking tool.

Lap welded pipe, such as the converse lock joint pipe have hubs similar to cast iron pipe, and the lead is poured by the use of a pouring clamp. Lap welded and riveted pipe are sometimes joined by a butt sleeve joint, In this case the ends of two pipe lengths are butted evenly against each other and an iron or steel sleeve somewhat thicker than the pipe shell, is drawn over the joint, leaving a space of ⅜ inch between the sleeve and pipe. A little packing is then inserted to prevent the lead from running into the pipe, and the space between the sleeve and pipe is then run full of melted lead.

When there is too much water in the trench to permit of pouring hot lead in pipe joints, several pipe lengths may be joined together on the surface and afterward lowered into the trench by the use of several derricks, and these compound lengths may be jointed in the trench by forcing small lead pipe into the joint and setting it up firmly with a calking tool. The method of making a slip joint was described in §67.

83.—Reducers for Joining Pipe Lengths of Different Diameters.

Where a pipe line is made up of different diameters, or where a small pipe is to be connected to a larger pipe, a reducer should be used which is simply a short length of pipe converging from the larger to the smaller diameter. In the investigation of friction in nozzles and converging pipes it was shown that the friction in a converging pipe is much greater than in a uniform pipe whose diameter is equal to the mean or average diameter of the converging pipe. The friction in a converging pipe depends upon its length and mean diameter. Its mean diameter should be as great as possible and its length as short as possible provided it does not converge more rapidly or at a greater angle than the form of the vena contracta or contracted vein. If d is the inside diameter of the larger pipe, then in a length of the reducer equal $\frac{d}{2}$, the diameter should converge to $d'=d\times.7854$. For example a pipe of 20 inches diameter is to be joined to a pipe of 3 inches diameter, and it is required to find the length of the reducer in inches.

Let d=diameter in inches of the large pipe=20.

d'=diameter in inches of the small pipe=3.

Then, in a length$=\frac{d}{2}=\frac{20}{2}=10$ inches, the reducer must converge to a diameter$=d\times.7854=20\times.7854=15.708$ inches. Hence total amount of convergence is $d-d'=20-15.708=4.295$ inches in a length of 10 inches, or the rate of convergence per inch length of the reducer is $\frac{4.292}{10}=.4292$ of an inch per inch length. Or the diameter will converge 1 inch in a length of $\frac{1}{.4292}=2.33$ inches. It shoutd therefore converge from 20 to 3 inches diameter in a length $l=(d-d')\times2.33=(20-3)\times2.33=39.61$ inches. If a diameter of one foot is to be joined to a diameter of .7854 foot, then the length in feet of the reducer should be $l = (d-d') \times 2.33 = (1.-.7854) \times 2.33 = .5$ foot.

In this latter case d and d′ are expressed in feet. All reducers and all nozzles for fire streams or power mains, and conical pipes in general should conform to the foregoing proportions where the most effective delivery and smallest loss by friction and contraction are desired.

The rate of convergence is one inch in 2.33 inches length or one foot in 2.33 feet length of the converging pipe and hence the length of the convergent pipe or reducer will be found by the general formula

$l = (d-d') \times 2.33$ ....(107)

If l is expressed in inches then d and d″ must be in inches

If l=is in feet, then d and d′ must be in feet.

d=largest diameter.

d′=smallest diameter.

CHAPTER VI.

Flow in Open Channels of Uniform Cross Section.

84.—Permanent and Uniform Flow.—Permanent flow may,occur in a channel either of uniform or non-uniform cross section. The flow is said to be permanent when an equal quantity flows through each cross section in equal times. If the cross sections of the channel are of unequal area the velocities will be inversely as the areas, in the case of permanent flow. Uniform flow can only occur in a channel of uniform cross-section and grade. By uniform flow is meant that both the mean velocity and the quantity are equal at all places along the channel. In this case the slope of the water surface and the slope of the bottom of the channel are necessarily the same, otherwise the velocities or quantities passing different points would not be equal. In natural streams with firm beds which are not undergoing scour and fill, the flow will become permanent if the supply of water is constant and uniform. These conditions can scarcely occur in large streams of great length, but may occur in small rivers or creeks. In artificial channels such as irrigation canals and mill races where the area of cross section and grade are uniform, and where the quantity admitted into the canal is constant and uniform, both permanent and uniform flow will occur after sufficient time has elapsed for equilibrium to be established between the acceleration of gravity and the resistances to flow, provided seepage and evaporation are not appreciably great, as sometimes they are.

If different portions of a canal are all of uniform sectional area but the slope is different in the different divisions, the flow may become permanent, but cannot become uniform unless the roughness and resistances in the portions of greatest slope happen to be just enough greater than in the other divisions to equalize the velocity head in all. In such case each division might be considered separately and the flow might be called uniform in and for any given division of the canal in which the area and slope are uniform. With the exception of flumes aqueducts, and canals lined with masonry, there are few open channels in which uniform flow takes place. The variations in grade, area of cross-section and roughness of perimeter may each be slight and yet the effect is marked. In uniform flow the resistances and accelerations of gravity must be constantly equal to each other. If the slope varies the head will be greater in one division than in another. If the sectional area varies the resistances will be inversely as $\sqrt{r^3}$, and will also be increased by cross currents and re actions of the particles of water which impinge upon the irregularities of the perimeter and react therefrom. The resistances due to mere irregularities of perimeter are similar to the resistances of a bend in a pipe or open channel. They deflect the particles of water impinging upon them and thus destroy an amount of head depending upon the angle of deflection and the velocity of the particles affected.

Where the width of a channel is alternately small and then greater, the resistances are similar to those in a convergent or divergent pipe, and will vary with the mean value of $\sqrt{r^3}$ for a given convergent length of channel and with the mean velocity through the section having the mean value of r for the given length considered. It is apparent, therefore, that a coefficient which would apply at one station or to one given short length of a non-uniform channel, will not apply at another station or to another given length unless the same conditions of roughness and convergence of banks obtain at both.

In natural streams containing bends of varying grade, depth and width, there will be what may be termed velocity of approach in many of its divisions which will cause velocities in short straight reaches of the channel which are apparently greater than the velocity due to the apparent slope. A coefficient of velocity C, developed from the data of flow observed at such places will be much too high to be applicable at any other reach or to any other conditions of flow. Such conditions are most common at low water stages, and may not obtain at the same place during medium and high stages of water. In natural and non-uniform channels the areas for different depths of flow and the various angles made by the banks at different heights, and the varying degrees of roughness of the banks above the usual depth of flow, so complicate the conditions for different depths of flow at any given station that it is necessary to find the value of C for the given station under each separate set of conditions. In Section 13 an approximate method of determining C under such conditions has been pointed out. It will require a consideration of the form of the channel above and below the observation station as well as at the station. No one formula without the aid of auxiliary formulas, such as suggested in § 13, supplemented by experience and sound judgment, can be made to apply to the conditions of flow in rivers and irregular channels. With all attainable aids, we can only expect fairly approximate results in such cases. We shall therefore consider the flow in artificial channels of uniform grade and sectional area, or channels in which, by courtesy, these conditions are said to be approximated. It would be closer the truth to say that the flow is permanent to a degree approaching uniform flow in each division of uniform slope.

The closer the actual conditions approach to uniform flow the closer will be the computed results by the formula for flow.

85.—Resistances and Net Mean Head In Open Channels.—In channels of uniform grade and cross section the resistances to flow consist in the friction of the liquid in contact with the perimeter and the internal resistances among the particles of water themselves. The internal resistances are caused by the distortion of the onward course of some of the particles of water causing them to collide with and distort the course of other particles.

These distortions have their origin in the small inequalities or roughnesses along the sides and bottom of the channel against which the moving particles flow, and from which they are hurled off in eddies angling across the path of the parallel flow. Difference in the temperature of different particles of water, which may be caused in part by impact and velocity, also causes upward and downward movements among the particles of water. If each particle of water moved uniformly in a course parallel to the bottom and sides the term resistance to flow would include no element of any importance except what is called the friction of the liquid with the solid perimeter. The results of experiments establish the fact, however, that the sum total of all the resistances whether internal or of friction at the perimeter, are proportional to the extent of wetted perimeter, in channels of uniform cross section and slope. The internal resistances among the particles of water are not caused by friction of one particle with another, but by the collisions and reactions of particles travelling in different directions. There can be no friction as between the particles themselves for they have no roughnesses to interlock or by which they can take hold on each other.

The molecules are independent, free bodies which act upon each other by impact only, and not by friction.

If the flow could occur without any resistance of any nature the effect of gravity would accelerate the flow so that the rate of velocity at any given point down a uniform grade would equal the square root of the total fall in feet between the origin of flow and the given point. The velocity on a uniform grade would therefore constantly increase each second. As this result does not actually occur, but on the contrary the mean velocity becomes uniform throughout the length of such grade, it is evident that the acceleration of gravity has been balanced by and is equal to the combined resistances to flow. It is equally evident that the resistances are as the square of the velocity or are equal to the total head in each foot length. If this were not true there would be a gain in unresisted head in each foot length of channel, and to this extent the acceleration of gravity would cause the velocity to increase in each foot length of channel, and there could be no such thing as uniform flow under any conditions, and all formulas based upon the theory of uniform flow would necessarily fail .Attention was called to this in the discussion of coefficients and the law of variation of coefficients. It is mentioned again here because some hydraulicians of eminent ability contend that the coefficient of friction or rather of resistance will decrease with an increase in the velocity, which means that the acceleration of gravity is greater than the combined resistances to flow. If that contention can be established it must be admitted that uniform flow is an impossibility either in pipes or in channels of uniform grade and cross section. The writer is not yet ready to make that admission. The law governing the flow in pipes of uniform diameter is the same which governs the flow in all uniform channels. The theory of flow and resistance to flow was discussed in general heretofore (§ 3 to 7 inclusive) and need not be repeated here.

It is evident that the velocity of any given film or particle of water will depend upon the net unresisted head of such film or particle after the resistances to its flow have been balanced. It is equally evident that the mean of all the different velocities in a cross section will depend upon the mean net head of all the particles. If the mean net head increases more rapidly than the resistances, it follows that the rate of velocity will increase in every successive foot length of channel; which we know is not the case. In channels of uniform grade and cross section the sum of the resistances per foot length of channel is equal to the head included in each foot length, and thus leave the net unresisted head, or velocity head, a uniform and constant quantity, and the uniform mean velocity is as the square root of this constant net mean head.

There is no friction between the molecules of the atmosphere and the molecules of water at the surface. Before friction can occur between two independent bodies it is necessary that both of the bodies should have projections or roughnesses which would interlock, and require force to separate, When winds occur, the molecules of air are hurled against the molecules of water and thus create resistance by distorting the course of the water from its direct path, if the direction of the wind if not the same as that of the flow,but if the wind follows the direction of the flow of the water with a downward sweep it does not resist, but assists the flow. The small bombardment of the water surface by molecules of air caused by difference in temperature of different air strata does not cause any appreciable resistance to or distortion of the flow. In truth, it may be said that none of the resistances to flow are due to pure friction, but are all due to changes in direction of the courses of different molecules which produces internal collisions and reactions as well as collisions with and reactions from the solid perimeter. The projections and inequalities along the perimeter, however small they may be, distort the course of the molecules of water impinging upon them, and the reaction sends them eddying across the path of the adjacent molecules causing further distortions and reactions among the molecules themselves. Roughnesses along the bottom of a channel cause whirls and boils and vertical currents which spend their energies in reaching the water surface and there spread out inert and without direction or velocity. For this reason the velocity at the surface is less than it is below the surface, which fact has led some persons to believe that there is friction between the atmosphere and the water surface.

Such boils rise above the surface of the water on the same principle that water rises above the surface in a Pitot tube, and when it reaches the height due to its velocity, its energy is spent, and it spreads out in all directions upon the surface. Abrupt bends or changes in the direction of flow produce impact and reaction and cause the formation of whirls and cross currents which are finally overcome by contact with the onward flow at the expense of considerable head, the amount of which will depend upon the angle and the radius of the bend. These remarks in connection with the laws of resistance given at § 2, and the discussion of the relations of area to wetted perimeter and the resulting relations between acceleration and resistance discussed in §§ 3 to 7 both inclusive, it is believed will cover all the important features relating to flow and resistance to flow in channels of uniform grade and sectional area. There are, however certain ratios and relations of surface, to mean and bottom velocities in open channels which demand a separate and more special investigation, as the knowledge of these relations has always been involved in much uncertainty. The writer's theory of these relations is entirely original, and is based upon his theory of coefficients of resistance and upon observation and experiment.

86.—There is no Constant Ratio Between the Surface, the Mean and the Bottom Velocities.

It cannot be denied that the velocity of flow of any given particle of water will depend wholly upon the net unresisted head of such particle.

The conditions under which the motion of any given particle takes place will vary with the relative position of the particle in the cross section with reference to the perimeter, which is the original place of impact and reaction. The distance that a rebounding particle will be projected into and across the flow will depend upon the difference in the velocity along and near the perimeter and the velocity at the center and surface of the cross section, or the difference in the velocity of the rebounding particle and that of the particles with which it comes in collision. The action of a particle of water is similar to that of a billiard ball. When it impinges upon a projection along the perimeter its course is changed so that it travels diagonally toward the opposite bank or surface, but instantly meets the opposition of the particles having a direction of flow parallel to the perimeter.

The force and direction of the reaction is changed and reduced with each successive collision as the rebounding particle travels across the parallel flow, until its direction also becomes parallel and the resistances and collision cease as to that particle.

These impingements and reactions along the sides and bottom are in continual progress and are naturally stronger at the place of their origin along the perimeter than elsewhere and grow weaker and weaker as they approach the center of the volume of flow. The number of these reactions will be directly as the roughness of the perimeter. If the bottom of the channel is corrugated transversely the entire volume of water will rise and fall and reproduce the corrugations on the surface, thus agitating the entire volume of flow.

In such case there will be only a small difference in the surface velocity and that at mid-depth, but the bottom velocity will be almost nothing. If the sides and bottom of the channel are fairly uniform and smooth there will be very little disturbance at the surface and a small number of reactions from the bottom and sides, and the bottom velocity will be proportionately much greater, which will result in increasing the mean velocity. It is well known that the mean velocity will increase very rapidly in uniform channels or diameters, simply by increasing the hydraulic mean radius without increasing the slope. This is accounted for by the fact that as diameter or hydraulic mean radius increases, the area of cross section of the column of water gains very rapidly on solid perimeter and there will be a very large relative quantity passed which, in smooth, uniform channels of large radius, will not come in contact with the perimeter nor any other retarding influence. The result is to increase the rate of mean velocity, not by increasing the bottom velocity but by increasing the area or section of the unretarded portion of the vein, or the number of particles of water having an unresisted head. An increase in hydraulic mean radius or of diameter can not affect the velocity of the water in contact with the perimeter or affected thereby. It does not remove the resistance nor add anything to the net head or freedom of flow of these particles. An increase in hydraulic radius or diameter cannot relieve the roughness of the perimeter nor the reactions therefrom, nor does it add anything to their head. There is no conceivable reason, therefore, why the bottom velocity should increase or decrease with changes in hydraulic mean depth or diameter, because it will be affected by the same retarding influences and resistances regardless of the value of the diameter or hydraulic radius. The velocity along the sides and bottom of a channel will therefore depend solely upon the degree of roughness of the wetted perimeter and the slope of the channel. and will in no manner be affected by an increase in the hydraulic radius or size of the channel. It cannot be maintained that the rapid movement of the upper central core of the liquid vein will assist the flow at the sides and bottom, because the minute globules of water are independent of each other and are without friction among themselves. There are no roughnesses upon these globules of water by which they can take the slightest hold on each other. If there were any roughnesses upon them they would interlock and the flow would become uniform and as great at the perimeter as at the center, or would be brought to rest entirely by friction with the perimeter. There is nothing to affect the velocity of flow of any particular portion of the vein except the constant net head it has remaining after the resistances to its flow have been balanced. As an increase in hydraulic mean radius cannot relieve the roughness and reaction at the perimeter and the consequent loss of head to the portion of the vein thus affected, it cannot therefore increase its velocity which must depend solely upon the inclination of the channel and roughness of perimeter The velocity of the water affected by the perimeter will be the same for the same slope and same degree of roughness regardless of the size of the channel and regardless of the mean and surface velocity. This is directly confirmed by the fact that very high mean and surface velocities may be permitted in large canals without damage by erosion of the bed, while such mean velocity in a small canal would rapidly destroy its bed. The reason is that the small canal would require a steep slope to generate a high mean velocity because the whole volume of water in a small canal is affected by the resistance of and reactions from the perimeter, and consequently the bottom velocity which is controlled by the slope, would be disastrously high.

The smoother the perimeter, the fewer the reactions and disturbances, and the greater the area of cross section unaffected by retarding influences, and as the area of unresisted section increases, the mean velocity will increase. In such case the ratio of surface to mean velocity will be small but the ratio of bottom to mean or surface velocity will be great. The mean velocity is apparently largely controlled by the ratio of area to perimeter as well as by smoothness of perimeter and slope of channel.

The bottom velocity is controlled entirely by the slope and the roughness of perimeter. After the depth of flow is sufficient to remove the water surface from the small reaction from the bottom in a fairly smooth channel, the surface velocity depends only upon the slope and nothing else.

It is evident that there is no fixed ratio between any two of these three velocities. The different velocities are dependent upon separate and distinctly different conditions. The mean velocity gains as area gains over perimeter without any increase of slope, not because the maximum velocity gains, but because a greater number of particles are set free from the retarding influences of the perimeter and thus increase the sectional area of the vein having the higher velocity. This does not affect the bottom velocity because there is no change of slope. If the channel is comparatively deep and has a smooth bottom, a further increase in hydraulic mean depth would not affect the maximum surface velocity which, under these circumstances would be removed from the effects of reactions from the bottom and would therefore only be increased by an increase of slope simply. It is evident that the relation of the maximum surface velocity to the bottom velocity is more constant than the relation of surface to mean or of mean to bottom velocity, and it is also evident that there are so many different influences affecting the one which does not affect the other to an appreciable degree, that it cannot be said that there is any given ratio or relation between any two of them.

The relation between them will be very different in a shallow rough, stony channel from what it will be in a deep smooth channel, and the relation will change in any given channel with changes in depth of flow. It has been demonstrated that the mean velocity will increase as $\sqrt[4]{r^3}$ while all other conditions remain constant. The increase in r does not affect the bottom velocity at all. An increase in r may or may not increase the maximum surface velocity. The various empirical formulas for deducing the mean or the bottom velocity from the surface velocity are therefore totally unreliable, for such a formula can only apply to one set of given conditions. If such formula would apply to a wooden trough two feet wide and one foot deep, it would not apply to a canal five feet wide and three feet deep. If it would apply to a canal with smooth and uniform perimeter it would not apply to a rough canal of like dimensions. Such formulas are therefore not of sufficient importance to demand discussion.

87.—The Eroding Velocity in Unpaved Channels in Earth.

In irrigation engineering there is no one feature of greater importance than the proper adjustment of the eroding velocity, or velocity adjacent to the sides and bottom, to the character of the soil which must form the perimeter of the canal. There is one particular bed velocity best adapted to each different class of earth. From considerations of economy it is desirable to maintain as high a velocity as the nature of the material forming the canal bed will stand without damage by erosion.

The stability of the bed of a canal will depend upon (1) the nature of the material forming the bed, (2) the alignment of the canal. (3) the angle made by the side slopes, (4) the velocity of flow of that portion of the vein adjacent to the sides and bottom, (5) the action of frost, or climatic influences.

The destruction of the side slopes depends as much or more upon the angle made by them as upon the velocity of flow in contact with and adjacent to them. In cold climates where frost penetrates the earth to a depth of several feet the side slopes should be much flatter for the same nature of material than in climates not subject to frost.

The eroding velocity in a majority of cases is only the partial agent of destruction of the bed. Bad alignment and side slopes too steep to withstand the disintergrating action of alternate freezings and thawings are the principal factors in destroying the uniformity and efficiency of the canal.

In a canal of uniform section with direct alignment the only velocity which tends to erode the perimeter is the velocity of the water which is in contact with it, which velocity is governed entirely by the slope and roughness of perimeter and is not affected by the value of the hydraulic mean depth.

On the contrary if the canal has bends and curves, then the surface, mean and bottom, and all intermediate velocities, become eroding velocities at all places where the direction of flow is changed. The outer bank of the curve must form the resistance which forces the change in direction of flow. The amount of this resistance will depend upon the amount of change in direction of flow and the time or distance in which the change is finally effected. It requires work and power. (see §60) The resistance will therefore be distributed along the outer curves over a distance depending upon the abruptness of the curve or upon the distance in which the total curvature is effected. The power expended upon each square unit of area of the outer curve will therefore be directly as the radius of the curve. This is the measure of resistance which each unit of area must be sufficiently stable to offer, otherwise it will be eroded and removed.

A comparison of the coefficients for straight flumes with the coefficient of the crooked Highline flume (Group No. 5) would indicate that the resistance of a bend of 90° with a radius equal one-half the width of the channel would amount to at least twice the head in feet generating the mean velocity of flow. If this ratio of resistance holds good in channels of all widths then the resistance (which is equal to the head required to balance it) would be

$$h''=\frac{A}{90^\circ}\times\frac{2v^2\times.007764}{R}=\frac{A\times2v^2\times.007764}{90\times R}\ldots\ldots(108)$$

In which

A=angle in degrees included in central arc of bend.

R=radius of central arc of bend in widths of the channel, not feet.

For further discussion see § 69 et seq., where the formula is explained in detail.

In channels with converging banks the resistance, which they must be sufficiently stable to offer and withstand is similar to that in a conical or convergent pipe (§§ 37, 39), and therefore will vary as $(3\times v)^2$, when v= the mean velocity through the section of the convergent length at the point where the value of r is the mean or average value of r for the whole length of the convergent channel. If the channel is both curved and convergent at the same place, then the banks must be able to withstand the resistances due to both causes. The necessity of direct alignment and of uniformity of cross-section is therefore apparent, if we would avoid erosion and yet maintain a reasonably high mean velocity. In large rivers which have small slope and frequent bends with cross-sections alternately wide and shallow and then deep and narrow, all the velocities become eroding velocities and their forces vary inversely as $\sqrt{r^3}$. The work done by the impinging water is in the direction of straightening the bends and trimming the sides so the width will be uniform, and in bringing the slope of the bottom to uniform grade. Unfortunately the banks and bends cave in and form new resistances which divert the energies and directions of the water to new quarters, and thus its work is self destructive. In artifical channels this work should be done in advance so that the energies of the water may be employed in a profitable way, and not wasted in building and destroying bars and bends.

88.—Eroding Velocity in Straight Canals of Uniform Section.—Theory and observation both indicate that a depth of flow of one foot upon the perimeter of a straight canal of uniform section will cause as great erosion as a flow of ten feet depth or any greater depth. The power of erosion in a straight, uniform canal varies with the square of the bottom velocity, or as the square of the velocity in contact with the sides and bottom. It has been shown that the velocity along the sides and bottom is controlled by the slope and degree of roughness of perimeter, in straight uniform channels, and that this velocity cannot be affected in such channels by any change in hydraulic mean radius.

As this bed velocity is not affected by the size of the channel, but is the same for the same slope of channel bed and roughness of perimeter without regard to hydraulic mean radius of the channel, we may conceive, for the purpose of determining the eroding velocity in such straight uniform channel, that the central portion of the liquid vein has been removed so that there remains only one foot depth of water upon the sides and bottom of the channel.

Then find the sectional area of this layer of water in square feet, and the length in lineal feet of the wet girth or perimeter.

Then,

$$\frac{\text{area in square feet of the layer of water}}{\text{Wet girth in lineal feet}} = r \text{ or hydraulic}$$

depth, so far as this one foot layer of water is concerned.

Then the velocity of flow of this layer of water one foot depth upon the sides and bottom will be

$$v = \sqrt{\frac{r\sqrt{r}\,S}{m}} = \sqrt[4]{r^3} \times \sqrt{\frac{S}{m}}$$

In channels where the actual depth of flow exceeds one foot, no matter how greatly, the value of r determined as above will be less than unity, but will approach unity. In order to err on the safe side and as a matter of convenience, we assume that r is a constant equal unity in channels where the depth of flow is one foot or greater; and under these conditions the eroding velocity or velocity of this layer of water is

$$v=\sqrt{\frac{S}{m}} \qquad (109)$$

If the channel is so small that the actual value of r for the whole volume of flow is less than r=1.00, then the mean velocity and all other velocities may be considered as equal and may be found by the formula for mean velocity in channels of the given degree of roughness. In either case the actual eroding velocity will not exceed the computed eroding velocity, and the computed result will be a safe guide in determining the grade of the canal.

89.—Slope or Grade of Canal to Generate a Given Bottom or Eroding Velocity.—The stability of the material which forms the perimeter of the canal must be the controlling factor in determining the grade or slope of the canal. Very light soil will not stand a bottom velocity greater than one half foot per second without serious erosion, while other classes of soil will stand much higher bottom velocities without damage. When it has been determined what bottom velocity is best adapted to the material forming the perimeter, then the slope or grade of the canal (without reference to its size) which will be required to generate that given bottom velocity will be

$$S=m\,v^2 \qquad (110)$$

In which,

v^2=the square of the proposed bottom velocity in feet per second.

m=coefficient of velocity applicable to roughness of perimeter.

S=Slope required to generate the given bottom velocity.

If the channel is so small that the value of r for the entire volume of flow is less than r=1.00, then

$S = \frac{m\,v^2}{\sqrt{r^3}}$, and the mean and bottom velocities will be practically the same.

If the bottom velocity$=\sqrt{\frac{S}{m}}$, has been decided, then the mean velocity for any value of r will equal the bottom velocity multiplied by $\sqrt[4]{r^3}$, or $v=\sqrt[4]{r^3}\times\sqrt{\frac{S}{m}}$

The value of m may be selected from the groups of data of flow in open channels heretofore given.

90.—Stability of Channel Bed Materials.—According to the observations of Du Buat a bottom velocity of 3 inches per second will just begin to work upon fine clay fit for pottery; a bottom velocity of 6 inches per second will lift fine sand; 8 inches per second will lift sand coarse as linseed; 12 inches per second will sweep along fine gravel. 24 inches per second will roll along rounded pebbles an inch in diameter; a bottom velocity of 3 feet per second will sweep along shivery, angular stones as large as eggs. Professor Rankine gives the following table of the greatest velocities close to the bed which are consistant with the stability of the materials mentioned:-

Soft clay	0.25	feet	per	second.
Fine sand	0.50	"	"	"
Course sand, and gravel as large as peas	0.70	"	"	"
Gravel as large as French beans	1.00	"	"	"
Gravel one inch diameter	2.25	"	"	"
Pebbles 1½ inches diameter	3.33	"	"	"
Heavy shingle	4.00	"	"	"
Soft rock, brick, earthenware	4,50	"	"	"
Rock, various kinds	6.00	and upwards.		

See also "Civil Engineer's Pocket Book" by Trautwine, pp. 563, 570, and "Irrigation Engineering" by H. M. Wilson page 86, and Fanning, page 622.

The experiments of Du Buat were in a small wooden trough with a smooth bottom so there was little friction between the moving particles of the material and the bottom of the trough. Loose material on a smooth uniform floor would be moved by a smaller bottom velocity than if it were incorporated in the bed of an earthen channel. It is probable that in ordinary earth the bottom velocity should be about .70 foot per second, and the slope should be $S = m\ v^2 = .00031 \times (.70)^2 = .0001519$.

91—Adjustment of Slope Or Grade, Bottom Velocities and Side Slopes of Canals, to the Material Forming the Bed.—In order to preserve the efficiency and delivery of a canal, its cross-section must be uniform, symetrical and free of deposits and plant growth. Caving and sliding banks, due to the action of frost upon side slopes steeper than the natural angle of repose of the material forming the sides of the canal, when such material is reduced to powder by frost in winter when the canal is empty, not only causes the filling up of the canal, but also leaves the banks rough, irregular and ragged, and greatly reduces its area, while it increases and roughens the perimeter. The efficiency or delivery of a canal may be reduced fully one third during one winter from this one cause alone. The extent of damage thus done will not be fully discovered until the water has again been admitted to the canal. All the loose, disintegrated material will then be washed off the sides and deposited in the bottom in irregular heaps. These heaps will be acted upon by the mean velocity in the same manner that a bridge pile or pier is attacked by the flow, and will thus be cut away and redeposited on one side where the velocity is not sufficiently great to keep the material in suspension and in transit. This will change the direction of the current to the deepest part of the cross section next the opposite bank which produces an undercutting and caving at that point and a further deposit on the side opposite the cutting. The thread of the current is caused to cross from one side to the other and thus the energy of the stream is expended in destroying the banks and in transporting material from one point to another. There are few instances in which the bottom of a canal has been scoured and eroded to a serious extent. The silt and deposits nearly always come from the banks which clearly indicates that the side slopes are too steep for the material and for the climate, or that the alignment is bad, for if the alignment is bad and the velocity too high, all the velocities are eroding velocities at the bends, and consequently a very low mean velocity must be adopted or the banks must be protected by paving or otherwise,else the annual expense of cleaning and repairs will be excessive. The proper side slopes of a canal will depend upon the nature of the material forming the perimeter. The side slope should never be steeper, in climates subject to frost, than the natural angle of repose of the material when thrown up in considerable heaps, loose and dry. In climates subject to frost the side slopes will be thoroughly pulverized by alternate freezing and thawing when the canal is empty in winter, or above the water level if the water is not turned out in winter. Under these conditions, if the side slope is steeper than the natural angle of repose of the material when it is perfectly loose and dry, the result is that the material thus pulverized by frost will roll down into the canal at each thawing until the slope finally reaches its natural angle of repose in a rough and irregular way. The method of determining the angle of repose is not by reference to published tables of such angles for different materials, but by throwing up a large heap of the material to be dealt with and allowing it to assume any angle it will. The angle thus assumed by the sides of the heap is as steep as the side slopes of the canal should be in that class of material. The angle of repose will be found to vary widely for different classes of earthy material, and for most kinds the angle will be much steeper if the material is damp or moderately wet than if it is either dry or saturated. Hence the angle should be found when the material is perfectly dry and loose.

The side slopes having been made to conform to the angle of repose thus found, and due attention having been given to the alignment, it is then necessary to so adjust the slope of the bottom of the canal as to cause a bottom velocity of flow most suitable to the material of the perimeter. If the canal is to be of considerable width and to have a depth of flow exceeding one foot, then the grade or slope should be

$S = mv^2$.

Here m is to be selected from the values of m developed for canals in like condition and in like material, given in the groups of data of flow in open channels.

The value of v will depend upon the bottom velocity which the given material of the perimeter will stand without erosion. The suggestions heretofore (§87) given may assist in determining what value should be assigned to v in the above formula.

If the canal is to be comparatively deep and narrow, as it should be where practicable, then the grade should be

$$S = \frac{mv^2}{r\sqrt{r}} = \frac{mv^2}{\sqrt{r^3}}$$

But in this formula the value of r is found not by taking the quotient of the total cross-sectional area of the column of water by the wetted perimeter, but by assuming that there is one foot depth of water adhering to the sides and bottom, the area of which is to be divided by the total wet girth in lineal feet. The resulting value of r is that which is to be used in determining the slope to generate the given bottom velocity.

If the value of r is the true value for total area divided by wet perimeter, and v represents the desired mean velocity, then the last formula will give the required slope to generate the given mean velocity, without reference to bottom velocity.

In very light soil mixed with fine sand the action of waves will reduce the side slopes much flatter than the angle of repose of the material when dry or only damp. If fluming, puddling, or paving cannot be resorted to where the canal passes through such material, then the canal should have a cross-section elliptical in form, and the bottom or scouring velocity should not exceed .45 foot per second, and great care must be taken to avoid bad alignment.

The grade of the canal having been determined with reference to the greatest bottom velocity the material of the bed will safely stand, it then becomes necessary to determine the dimensions of the canal with that given grade which will cause the discharge or carriage of the required quantity of water.

92.—Dimensions of Canals to Carry Given Quantities. —In the case of canals with side slopes of about 2 horizontal to 1 vertical, and of considerable capacity, the value of the hydraulic mean depth $\frac{a}{P}$, may be approximately found by formula (64) which is

$$r=\sqrt[11]{\frac{q^4\ m^2}{24936.958\times S^2}} \ldots\ldots\ldots\ldots\ldots\ldots (64)$$

In this connection see §§ 19 and 3. The required value of r being thus found in terms of cubic feet per second q, then, $a=r^2\times 12.5664$, and wet perimeter, $P=\frac{a}{r}$. For reasons heretofore pointed out these formulas are not generally applicable to all forms of cross-section and capacities of open channels, and when the values of a, p, and r have been calculated in this manner, the general formula for velocity should be applied as a check. When the mean velocity is thus found, then $q=a\times v$.

For example suppose the grade decided upon for a canal is $S=.0002754=1$ in 3631.08, and the value of m applicable to the class of gravelly earth is $m=.00034$. What area in square feet and what wet perimeter and what value of r would be required to cause the canal to discharge 1,000 cubic feet per second, the side slopes being 2 to 1? In the first place find the required value of r by formula (64) which will be $r=5.121$.

Then required area in square feet, $a=r^2\times 12.5664=329.554$.

The required wet perimeter $=\frac{a}{r}=\frac{329.554}{5.121}=64.353$.

Taking 33.1668 feet of the wet perimeter as the bottom width of the canal, there will have to be a depth at center sufficient to take up the remaining 31.1862 feet of wet perimeter which is to be divided equally between the two side slopes. Then the wet perimeter of one side slope will be $=\frac{31.1862}{2}=15.593$ feet.

As the side slopes are 2 horizontal to 1 vertical, a vertical depth of water equal about one half the length of one side slope, or about 7 feet in this case, will be required. So making the depth of water at the center equal 7 feet, and the bottom width as above, equal 33.1668 feet, and the side slopes 2 to 1, we have the length of one side slope $=\sqrt{7^2+14^2}=15.65$ feet. Then total wet perimeter $=15.65+15.65+33.1668=$ 64.466 feet.

The actual area will be 330.1676 feet. The actual value of r will be $=\frac{330.1676}{64.466}=5.121$. Now as a check on this calculation we must apply the general formula for mean velocity to the slope and dimensions above found, and we have $v=\sqrt{\frac{S\sqrt{r^3}}{m}}=\sqrt{\frac{.0002754\times11.59}{.00034}}=\sqrt{9.388}=3.064$. And the quantity in cubic feet per second which will be discharged will be $q=\text{area}\times\text{velocity}=330.1676\times3.064=1011.63$ cubic feet,

Tho bottom velocity in this canal would be $v=\sqrt{\frac{S}{m}}=$ $\sqrt{\frac{.0002754}{.00034}}=.90$ feet.

While it is seen that the dimensions of a canal of this form of cross section and capacity may be closely ascertained by the formulas for r, a and p, as above shown, yet these particular formulas do not apply to small canals nor to rectangular canals, with any degree of accuracy. These particular formulas do apply, however, with exactness to pipes or circular closed channels running full.

93.—Allowance In Cross Section of Canals For Leakage and Evaporation.—The amount of loss by leakage and evaporation from a canal will depend upon the climate, the nature of the soil, the length of the canal, the depth of flow, and above all the position of the canal with reference to the elevations and depressions of the surface of the surrounding country.

If the canal is constructed upon the highest line of the land through which it passes, the leakage from it will be great, and because of its elevated position it can never regain any part of this loss by return seepage. Such location also exposes the water surface to the action of the sun and wind. and thus large losses occur by evaporation, especially if the canal is wide and shallow. In arid regions where irrigation is not general and abundant, the sub-surface water level is at considerable depth below the surface, but after irrigation has been practiced for several years, the earth becomes saturated and the sub-surface water level rises near to the surface. Until this occurs the loss from new canals in such regions will be very great. After irrigation has been practiced for a number of years, and has become general in the given locality, the canals situated along side hills and skirting the valleys will gain vastly more by seepage into the canal than will be lost by leakage and evaporation combined. In some canals in Colorado the gain by seepage into the canal is as great as two thirds the total original quantity admitted into the canal at its head. This occurs only in canals located where irrigation has been practiced for years, and in canals so situated on side hills or along the foot of the hill, as to admit of the seepage flowing into the canal.

The loss by leakage and evaporation from new canals in arid regions varies from 20 to 75 per cent of the quantity admitted into the canal, according to the nature of the soil and the length of the canal. As the canal becomes silted and the sub-surface water level rises, the leakage will decrease, and if the canal is so located as to admit of it, the gain by return seepage will, in the course of a few years, more than balance the loss by leakage and evaporation.

In regions where the rainfall is great it is probable that the seepage into a new canal will offset the leakage from the first opening of the canal, because the sub-surface water level is already very close to the surface of the ground.

In making allowance in cross-sectional area of a canal to cover these losses, it should be by way of extra depth.

94.—Where a Flume Forms Part of a Canal.—Where the course of a canal would pass around on a very steep side hill, or through stretches of very porous earth, or across low depressions, flumes are frequently adopted as portions of the canal for such reaches. In this event the question arises as to the proper ratio of flume cross section to that of the canal, of which the flume forms a part. The determination of this question involves a consideration of the relative degree of roughness of the two classes of channel, and the difference in slope or grade of the flume and the canal, as well as the length of the flume and its alignment. If the flume is short and upon the same grade as that of the canal, and has no vertical fall at its lower end, the water will not acquire a velocity in such short flumes much greater than that in the canal, and therefore the area of the flume under such conditions cannot be reduced much below that of the wetted area of the canal. While the velocity of flow will usually be greater in a flume than in a canal of equal slope, yet at the entry to the flume the water has only the velocity of the canal, and the head due to that velocity. It must flow a sufficient distance in the flume to acquire the greater velocity due to the smoother perimeter before the depth and area of the flume can be materially reduced from that of the connecting canal, otherwise there will be an overflow at the upper junction of the flume with the canal. The flume should converge from the mean width of the canal at the junction, to the standard section adopted for the flume, in a length varying from 50 to 200 feet according to the difference in slope and in roughness of the flume and the canal. The value of C might be 56 for the canal and anywhere from 70 to 130 for the flume, according to the method and materials adopted in its construction and alignment.

A straight canal in firm, dense earth and in best condition develops C= 75.00, while a rough, crooked flume with battens on the inside develops C=70.00. In such cases as this the flume would require an area slightly in excess of that of the canal, or would require an equal area and steeper grade. On the other hand the value of C for a rough canal may be as low as 40, while the value of C for a very smooth well jointed hard wood flume of good aligament might be as high as 130.

The slopes being equal, the velocities will be as $\sqrt[4]{r^3}$ in the one is to $\sqrt[4]{r^3}$ in the other, as modified by the respective values of C, or $v:v::C\sqrt[4]{r^3}:C\sqrt[4]{r^3}$. If the slopes are different then $v:v::C\sqrt[4]{r^3}\sqrt{S}:C\sqrt[4]{r^3}\sqrt{S}$

The value of C may be taken from the data of like flumes and channels given in the groups, Chapter 2.

95—Mean Velocity in Uniform Sections of Canals Found by Floats.

In straight sections of canals of uniform cross-section where the thread of the greatest velocity is midway between banks and just beneath the water surface, the place of mean velocity will be found at .50 of total depth at a point midway between the center of the canal and the bank, unless the depth of flow is less than two feet, in which case the place of mean velocity will be at or just above mid-depth at a point midway between the bank and the middle of the canal, assuming that the sides and bottom of the canal are fairly smooth. In shallow canals with gravel and pebbles along the bottom the place of mean velocity is very near mid-depth, sometimes slightly above, and at one-fourth the width of the canal from the bank. A large tin bucket loaded with gravel and covered, may be suspended by a fine wire at this depth and connected to a flat circular float on the surface no larger than is absolutely necessary to support the submerged bucket at proper depth. This double float is to be placed at some distance above the upper end of a measured length of the canal, and adjusted to proper position with reference to the bank or width of the canal, and with reference to depth, and allowed to travel over the given course a number of times. The average time required for its passage over the given number of feet length of the canal will closely approximate the rate of mean velocity. The difficulty of ascertaining the exact number of seconds which elapse between the time the float crosses the line at the upper station and arrives exactly at the line of the lower station, will probably cause a slight error in the final determination of the mean velocity. For this reason the measured course should be several hundred feet in length. If the channel is rough and winding the float will be cast either too near the bank or into mid-current, and the result is without value. Float measurement of mean velocity is practicable only in channels of uniform width and depth. The surface velocity has no particular relation to the mean velocity, and it is therefore impossible to deduce the mean from the surface velocity. The ratio between surface and mean velocity varies with the form of cross-section, roughness of perimeter, uniformity of cross-section, variation in slope, depth of flow and hydraulic radius and alignment of the channel.

The surface velocity depends mainly on the slope, while the mean velocity depends upon the value of $\sqrt[4]{r^3}$ as well as upon the roughness and slope of the channel. In rough, stony channels of varying cross-section and small depth of flow there is scarcely any difference between surface and mean velocity.

CORRECTION OF TEXT.

It is probable that no one ever turned his manuscript over to the printer without a lively sense of its probable demerits when it shall stare one in the face from the printed page.

The greater part of the book was written several years ago. and portions of it were published in various journals in 1894 and 1895, While the ultimate conclusions reached and formulas deduced, as appear in the text, are correct, yet some of the reasoning is at fault, and not clear. The author would be glad to stop the press and re-write the entire book after having seen half the printed "proof," but it is too late.

He must therefore resort to the alternative of writing a criticism of his own work, and thus forstall the other fellow.

The three important principles which are sought to be established are:-

(I)—That it is the effective value of the head or slope which varies with some function of the diameter or hydraulic mean radius, or mean depth, and not the coefficient that varies.

(II)—That for any given class of wet perimeter, or any given degree of roughness, the coefficient is necessarily a constant for all heads, slopes, velocities, diameters or mean hydraulic radii.

(III)—That the value of the coefficient is governed absolutely by the roughness of wet perimeter, and by nothing else, and is therefore an absolutely reliable index of the roughness of perimeter.

First Proposition.

That the Effective Value of a Constant Head or Slope Varies With Some Function ($\sqrt{d^3}$, or $\sqrt{R^3}$) ***of the Diam-***

eter, or of the Hydraulic Mean Radius, and that the Coefficient does not Vary with the Diameter or Hydraulic Mean Radius at all.

If a series of pipes or open channels of exactly equal roughness of perimeter, but of different diameters, or different hydraulic mean radii, have exactly the same head or slope per foot length, it is well known that the pipe having the greatest diameter, or the open channel having the greatest hydraulic mean depth (R), will generate the greatest velocity of flow, and the pipe having the least diameter, or the open channel having the least mean hydraulic depth, will generate the least velocity of flow. As all these pipes, or all these channels, are of equal roughness, and all have exactly equal heads or slopes, it is evident that the velocity would be the same in each of them if the constant head or slope were not made more effective with an increase in diameter or hydraulic mean depth. This being true, the next inquiry is, what is the ratio of increase in the effectiveness of the given head or slope as diameter or hydraulic mean depth increases?

To solve this problem we must appeal both to the laws of friction or resistance, and of gravity. The resistance, or head lost by resistance, will be directly as the roughness of perimeter, and directly as the extent of perimeter, and also directly as the square of the velocity.

As demonstrated in the text the wet perimeter or extent of friction surface, varies exactly with d or r. (See pp. 34 36, 39, 40.)

But if there were no friction or resistance, then the velocity would be the same for the same actual slope regardless of the value of d or r. While the friction surface and consequently the absolute loss of head by resistance, increases only as d or r, the cross section of the column of water increases as d^2 or r^2, or as the sectional area.

The absolute head or slope therefore increases as the area, or as d^2 or r^2, while the absolute loss of head increases only as d or r. It is evident then, that the absolute head or slope, which varies as d^2 or r^2, must be modified by the absolute loss of head or slope which varies as d or r. Then the mean head, or relative head, of all the particles of water in the cross section will vary with d^2 as modified by d, or with r^2 as modified by r. As r^2 must not be increased by r, but must be modified by r, we must reduce both d and d^2, or r and r^2, in the same ratio, in order to obtain a reducing or modifying multiplier. To accomplish this result, we say that $\sqrt{d}$ bears the same relation to d that d bears to d^2, or that $\sqrt{r}$ bears the same relation to r that r bears to r^2. In other words to maintain the ratio, of r to r^2, or d to d^2, and at the same time obtain a multiplier which will give the combined net effects of d and d^2. or r and r^2, upon the value of H or S, it is necessary to take the square root of both d and d^2, or of both r and r^2. We then say that, relatively, the area or absolute head (d^2 or r^2) varies with $\sqrt{d^2}=d$, or with $\sqrt{r^2}=r$, while the friction surface or absolute loss of head varies with $\sqrt{d}$ or $\sqrt{r}$. and consequently the relative mean head of all the particles in the cross section will vary with the resultant of these two effects, which will be as $d\sqrt{d}$, or as $R\sqrt{R}$. Thus we obtain the modifying multiplier $\sqrt{d}$, or $\sqrt{r}$, while we maintain the correct ratio of friction surface to area, or of loss of head to gain in head as d or r varies for a constant head or slope.

It is evident then that the constant head or slope becomes more effective or less effective as $d\sqrt{d}=\sqrt{d^3}$, or $\sqrt{r^3}$, increases or decreases.

SECOND PROPOSITION.

That for any Given Degree of Roughness of Wet Perimeter, the Coefficient is a Constant for all Heads, Slopes, Velocities, Diameters or Hydraulic Mean Depths.

It was shown in the foregoing discussion that the effective value of the head or slope varies with $\sqrt{d^3}$or$\sqrt{r^3}$. By the law of gravity the square of the velocity must always be pro portional to the head or slope in any given pipe or channel, or $v^2=2gH$. As a necessary consequence of this law, it is obvious that anything which affects the effective value of the head or slope must at the same time equally affect the value of v^2.

When we write $m=\frac{S\sqrt{r^3}}{v^2}$ and remember that the effective value of S increases with $\sqrt{r^3}$, and that any increase in the effective value of S must also increase v^2 in the same ratio, it is evident that as both dividend and divisor increase alike the quotient, m, will continue a constant for all values of r, S and v^2. Their relation is such that we cannot increase the effective value of S without also increasing the value of v^2 in the same ratio. Hence m is necessarily a constant.

THIRD PROPOSITION.

That the Value of the Coefficient is Governed Absolutely by the Roughness of the Wet Perimeter, and by Nothing Else, and is Consequently an Absolutely Corrrect Index of the Roughness.

When we inspect the formula for the coefficient, $m=\frac{S\sqrt{r^3}}{v^2}$ it is apparent that m is simply the expression for the ratio of effective slope to the square of the velocity. If the pipe or channel is rough it will require a large value of the effective slope, $S\sqrt{r^3}$, to generate a small value of v^2. Consequently the ratio, m, of effective slope to v^2, will be large in rough channels. But if the channel is uniform in area, and smooth then a small effective value of slope, $S\sqrt{r^3}$, will generate a relatively large value of v^2, and hence the ratio, m, will be small for smooth perimeters. As m is simply the expression for this ratio, and as this ratio depends exclusively on the roughness of perimeter, it is obvious that m will vary only with the roughness.

The coefficient, $C=\sqrt{\frac{v^2}{S\sqrt{r^3}}}$, is simply the square root of the reciprocal of m, and will consequently be a constant, like m, for any given degree of roughness. But being the square root of the reciprocal of m, C will vary with the roughness in the exact opposite way from m—that is, C will be large for smooth perimeters and small for rough perimeters, while m will be large for rough perimeters and small for smooth perimeters.

As either of the coefficients vary only with the roughness of wet perimeter, but is very sensitive to any change in roughness, it will be found that C will decrease as depth of flow increases in all channels where the sides are rougher than the bottom, and will increase with increase of depth of flow in all channels where the sides are smoother and more uniform than the bottom. In other words C will vary as the mean of the roughness varies. See in this connection §13 page 58, and also p p. 27, 28, 29, 41, 42.

The best form of the formula for general use is,

$$v=C\sqrt{S\sqrt{r^3}}\text{, or } v=\sqrt{\frac{S\sqrt{r^3}}{m}}$$

This form of the formula also shows by mere inspection that the effective value of S varies with $\sqrt{r^3}$.

If the formula is written, $v=C\sqrt[4]{r^3}\sqrt{S}$, the actual result would be the same whether we say that C or $\sqrt{S}$ varies with $\sqrt[4]{r^3}$, but as C insists on being constant, it is evident that it is the effective value of S that varies with $\sqrt{r^3}$, and the writer desires to correct all statements to the contrary, It is somewhat absurd to insist that the coefficient is a constant and at the same time to claim that it varies. The coefficient in our formula can vary only as the average of roughness of the entire wet perimeter. In the Chezy or Kutter form of formula, the coefficient must vary as the roughness and also as $\sqrt[4]{r}$. (See pp. 6, 7, 42, 44.) Hoping that this absurdity is fully corrected in this explanatory note, and asking pardon for having committed such a glaring fault, the author commits the work to the hands of the profession with the further hope that its merits may outweigh its faults.

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Longmont, Colorado,
November, 1st, 1899.

APPENDIX 1

Suggestions Relating to Weir and Orifice Measurements of Flowing Water.

96—Remarks in Relation to Weir Coefficients.—In the third remark under Group No. 2 § 14, a general form of Weir formula was suggested. It is not here intended to discuss the well known theory of flow over measuring weirs with sharp crests and full or partial contraction, any further than to point out what the writer believes would be an improved method of application which is believed would reduce the errors in such determinations. From the nature of a measuring weir it is impossible that the head or depth upon the weir should ever be great, and consequently the velocities are never very high, even in the cases where there is velocity of approach. The amount of resistance to flow (being as v^2) offered by the edges or perimeter of the notch is therefore a small factor in the sum total of the coefficient of discharge. The important factor is the coefficient of contraction. It is usual to combine the coefficient of resistance with the coeffi- of contraction and their product forms the coefficient of discharge, which is usually assigned a mean value of .62. For the reason that these two independent coefficients which combined form the usual weir coefficient of discharge, do not vary in the same manner under similar conditions, it has been found necessary to find their combined value for each given depth upon the weir and for each given length of notch, and for each form of notch. If the length of weir notch remains constant, a small change in depth upon the weir will greatly affect the value of the combined coefficient, or coefficient of discharge. This cannot be attributed, except in very small part, to the resistance at the edges of the notch, for a small change in depth upon the weir does not greatly affect the ratio of area to perimeter of the notch, which may be regarded as a very small fractional length of open channel. The effect upon the combined coefficient of varying the depth upon the weir must therefore be accounted for in the factor representing contraction of the discharge. It is evident from the discussion of coefficients of flow in pipes and open channels (§§ 3 to 7) that the resistance to flow offered by the edges of the notch will vary as H and $\sqrt{r^3}$. But the coefficient of contraction which is the controlling and important factor has no known relation to the value of r. The coefficient of contraction is affected greatly by the position of the weir, the depth upon the weir, the distance from the crest to the bottom of the channel, the distance between the shoulders of the notch and the banks of the channel, and the velocity of flow through the notch.

The experiments of Mr. J. B. Francis upon the same weir of constant length, and where all conditions were constant except the depth upon the weir, show that a change of depth alone upon any given sharp crested weir of the usual form will greatly affect the value of the coefficient of discharge, and further show that the variations of the coefficient of contraction apparently follow no law. The coefficient will decrease as depth increases until a certain depth is reached (depending upon the proportions of the notch) and then increases with a further increase in depth up to a certain point where it will again begin to decrease to a small extent until it becomes nearly constant for great depths (if such were practicable).

To make the usual weir coefficients apply with any degree of accuracy is not a simple matter by any means, for the conditions must be identical with those under which the given coefficient was determined, The ratio of area of notch to area of channel, the depth or height of overfall, the height of crest above the bottom of the channel on the upstream side of the weir, the position of the weir, whether at right angles to the thread of the channel, and vertical, and rigidly straight or allowed to bend under pressure, all affect the coefficient of contraction, in addition to the influence of varying the depth upon the weir. There are so many different influences bearing upon the coefficient of contraction that we can never be certain of its value except under given favorable conditions which do not often occur in actual practice. It is therefore suggested that it would be safer practice where careful determinations are to be made to avoid all these uncertainties by suppressing all contraction. When this is done there remains only the coefficient of resistance of the edges of the notch to be dealt with, and the law of its variation is known.

In order to suppress contraction it is suggested that the notch, whether rectangular, triangular or trapezoidal, should be chamfered on the upstream side of the notch to the form of the vena contracta, instead of placing the chamfered side down stream. As illustrating the desultory manner or variation of the coefficient of discharge of a sharp crested weir the first three columns *l*, H, and q, quoted by Fanning from Francis' experimental data (Table 68, page 288 Water Supply Engineering) are given in the following table, and the column v was computed by the formula $v=\frac{q}{a}$, and from these data the resulting values of m were computed,

The fundamental formula for flow over weirs with sharp crests may be written

$$v=\tfrac{2}{3}\sqrt{\frac{2gH}{m}};\ \text{or}\ v=m\,\tfrac{2}{3}\sqrt{2gH}=5.35\,m\sqrt{H}.$$

Whence

$$m=\frac{28.6225\,H}{v^2},\ \text{if m is used as a divisor,}$$

Or

$$m=\frac{v}{5.35\sqrt{H}}=\sqrt{\frac{v^2}{28.6225H}},\ \text{if m is used as a multiplier.}$$

$$q=\text{Area}\times5.35\sqrt{\frac{H}{m}},\ \text{or}\ q=\text{Area}\times5.35\,m\sqrt{H}=A\,C\sqrt{H}.$$

TABLE. No. 41—Table of Weir Data.

L Feet	H Feet	q Cubic Feet Sec.	A Sq. Feet	v Feet Sec.	v^2 Feet Sec.	R Feet	Coefficient $m=\frac{v}{5.35\sqrt{H}}$
9.997	0.62	16.2148	6.198	2.610	6.8121	0.5515	.6195
9.997	0.80	23.4304	7.997	2.929	8.5790	0.6900	.6121
9.997	1.25	45 5654	12.496	3.645	13.2875	1.0000	.6096
9.997	1.56	62 6019	15 595	4 014	16 1122	1.1880	.6007

L= length in feet of notch.

H= depth in feet upon the weir.

q= cubic feet per second actually discharged.

A=L×H=Area in square feet=depth of water upon the weir×length of notch.

$v=\frac{q}{a}$=mean velocity in feet per second.

R= hydraulic radius in feet of notch=$\frac{a}{P}$

m=Coefficient of discharge=$\frac{v}{5.35\sqrt{H}}$.

In these experiments the conditions all remained constant except the depth H, upon the weir.

In the formula

$v=5.35\ m\sqrt{H}$

if we combine the value of m with the constant $5.35=\frac{2}{3}\sqrt{2g}$, the following values of the coefficient C result:-

H=.62, m=.6195, 5.35×m=C=3.3143.

H=.80, m=.6121, 5.35×m=C=3.2747.

H=1.25, m=.6096, 5.35×m=C=3.2613.

H=1.56, m=.6007, 5.35×m=C=3.2137.

Whence,

$q=\text{Area}\times C\sqrt{H}=C(L\times H)\sqrt{H}$

It is evident, even for different depths upon the same weir, that if the constant value C=3.33 is used, the results must be erroneous. Suppose the velocity of approach is considerable, as on mountain streams, and that the weir notch (rectangular) is nearly as long as the stream is wide, as often becomes necessary, then the sloping banks will approach the submerged corners of the notch and greatly affect the coefficient of contraction, but to what extent, is merely surmise. It is frequently the case that in order to stop the leaks under and around the weir, earth, straw and brush are banked against its upper side, thus training the flow upon the notch and also preventing full contraction. This affects both the real value of H or v^2 and the contraction of the discharge.

The range of experimental coefficients as determined by Francis was very small, being mostly for weirs about 10 feet length with depth upon the weir varying from about six inches to 1.60 feet. The variation of the coefficient of contraction was found so fitful and irregular as the ratio of length to depth was changed and with different depths upon any given length of weir, that Mr. Francis advised caution in the application of his formula and coefficients in cases not falling directly within the experimental conditions. It is assumed that the contraction of the discharge over a sharp crested weir in full contraction is analogous to the contraction of the jet from a sharp edged orifice in thin plate. If the numerous tables of experimental orifice coefficients determined under various heads above the center, and under various proportions of height to width of orifice be investigated, it will be found that each form of orifice, or each ratio of height to width, develops a distinct series of values of the coefficient as the head varies.

The coefficient for an orifice will either decrease or increase with the head upon the center in an irregular and alternating manner which apparently depends upon the ratio of height to length of orifice, as indicated in the following table of experimental coefficients for square edged orifices in thin plate and with full contraction, which were determined by Poncelet and Lesbros.

Coefficients of discharge for square edged orifices in thin plate and with full contraction.

TABLE No. 42—Table from Poncelot and Lesbros.

Head above Center In Inches	Dimensions of Orifice in Inches.						
	8×8	6×8	4×8	3×8	2×8	1×8	0.4×8
0.4							.70
0.8						.65	.69
1.0						.64	.68
1.5					.61	.64	.68
2.0				.60	.62	.64	.68
2.5			.59	.61	.62	.64	.67
3.0			.60	.61	.62	.64	.67
3.5		.57	.60	.61	.62	.64	.66
4.0		.58	.60	.61	.63	.64	.66
4.5	.56	.59	.60	.61	.63	.64	.66
5.0	.57	.59	.61	.62	.63	.64	.66
8.0	.59	.60	.61	.62	.63	.64	.65
12.0	.60	.60	.61	.62	.63	.63	.64
36.0	.60	.60	.61	.62	.62	.63	.63
60.0	.60	.60	.61	.61	.62	.62	.62
120.0	.60	.60	.60	.60	.60	.61	.61

TABLE No. 43—Table From George Rennie.

Head above Center In Feet	Dimensions of Orifice.	Coefficient
1.0	1 inch diameter, circular.	.633
1.0	1×1 inches, square.	.617
1.0	1 square inch area, triangular.	.596
2.0	1 inch diameter, circular.	.619
2.0	1×1 inch, square.	.635
2.0	1 square inch area, triangular.	.577
3.0	1 inch diameter, circular.	.628
3.0	1×1 inch, square.	.606
3.0	1 square inch area, triangular.	.572
4.0	1 square inch area, triangular.	.583
4.0	1×1 inch, square.	.5[illegible]3

TABLE No. 44—Table from Gen. Ellis.

Coefficients of discharge for square edged, circular Orifices in iron plate one half inch thick.

Head above Center In Feet	Diameter of Orifice in Feet.	Coefficient
2.1516	0.50	.60025
9.0600	0.50	.60191
17.2650	0.50	.59626
1.1470	1.00	.57373
10.8819	1.00	.59431
17.7400	1.00	.59994
1.7677	2 00	.58829
5.8269	2.00	.60915
9.6381	2.00	.61530

TABLE No. 45.

Coefficients for square orifice 1×1 foot with curved entrance and discharge slightly submerged. (Gen. Ellis.)

Head above Center In Feet	Dimensions of Orifice.	Coefficient
3.0416	Square, 1×1 Feet	.95118
10.5398	Square, 1×1 Feet	.94246
18.2180	Square, 1×1 Feet	.94364

In these last experiments if the orifice had been in the form of the vena contracta, and the discharge had been entirely free instead of being partially under water, it is probable that the coefficient would have reached .98, and would not have been affected in any manner except by the slight resistance of efflux offered by the perimeter of the orifice. The curving entrance had almost suppressed all contraction of the jet in the above experiments.

The object of these tables and suggestions is to point out the fact that all these uncertainties in the application of weir and orifice coefficients may be easily avoided by so chamfering the inner edges of the weir notch or orifice as to make them conform as nearly as possible to the form of the vena contracta.

In many cases of the practical application of the ordinary weir and orifice coefficients the conditions are such that complete contraction cannot be obtained. In almost any case it is much more convenient to suppress all contraction than to obtain complete contraction, and when contraction is suppressed there is no limit to the range of the remaining coefficient which should be determined in the same manner as the value of m for pipes or open channels. When contraction is suppressed (as pointed out § 83). Then for a submerged orifice.

$$v=\sqrt{\frac{2gH\sqrt{r^3}}{m}}=8.025\sqrt{\frac{H\sqrt{r^3}}{m}};\quad m=\frac{64.4\,H\sqrt{r^3}}{v^2}$$

$$q=\text{Area}\times 8.025\sqrt{\frac{H\sqrt{r^3}}{m}}=A\ C\sqrt{H\sqrt{r^3}}$$

And for weirs,

$$v=5.35\sqrt{\frac{H\sqrt{r^3}}{m}};\quad m=\frac{28.6225\,H\sqrt{r^3}}{v^2}$$

And.

$$q=H\times l\times 5.35\sqrt{\frac{H\sqrt{r^3}}{m}}=A\,C\sqrt{H\sqrt{r^3}}$$

When the numerical value of m is ascertained for any thickness of plate it will apply to any shape or size of orifice or weir notch, and the square root of its reciprocal $\sqrt{\frac{1}{m}}$, may then be taken and combined with the constant 8.025 or 5.35 as the case may be.

APPENDIX II.

Useful Data and Tables Relating to Water Works and the Water Supply of Cities and Towns.

97.—Purposes to Which City Water is Applied.

In planning a water works system for town or city supply, the nature of the chief occupation of the inhabitants must be considered as well as the number of inhabitants at present, and the probable increase in population within the next fifteen or twenty years. The purposes to which city water will be applied will depend upon the humidity of the climate. In the arid portion of the West the city water is demanded for all purposes to which water is applied, such as irrigation of lawns and gardens, and shade trees, street sprinkling, carriage washing, watering horses and cows, water for steam boilers and hydraulic motors, hydraulic lifts or elevators, steam laundries, drinking fountains, ornamental fountains, manufacturing purposes, extinguishment of fires and ordinary household uses. Where manufacturing is the chief business of a town the demand for water will be two or three hundred per cent greater than in towns of equal size and in like climates which are not manufacturing centers. In some manufacturing towns situated on rivers the factories have their own private water supply, and in such cases the city water works is called upon only for water for ordinary purposes. The coast states, and the Eastern and Southern states, have frequent and large rainfalls and except at manufacturing centers, the city water works in these states will not be called upon except for ordinary purposes. In the arid portion of the West the demand on the city water supply is from fifty to one hundred per cent greater than in towns of like population in other parts of the United States.

In non-manufacturing towns in such climates as in Arkansas, Mississipi and Louisana, the demand for all purposes will not exceed 60 gallons per capita per 24 hours, while in Colorado and other arid states the demand in small non-manufacturing towns is from 110 to 150 gallons per capita per 24 hours, and in older and larger cities the demand is from 150 to 200 gallons per capita. Should an essentially manufacturing city spring up in the arid West, it is probable that the demand for water would reach 400 gallons per capita per 24 hours.

98.—Quantity of Water per Capita Required—The quantity of water required per capita per 24 hours for the present given number of inhabitants, and for all purposes, depends upon the section of the country and the chief occupation of the inhabitants, as just pointed out. But in planning a water supply, the very rapid increase in the population of towns and cities in the United States must be amply allowed for. The U. S. census of 1890 shows that our population is fast gathering into the towns and cities. The population of towns and cities, taken collectively, throughout the United States, increased by 61.10 per cent from 1880 to 1890, while the total population of town and country increased only 24.85 per cent. The following table is valuable in this connection.

TABLE No. 46.

Growth of population in cities and in the United States.

Census Year	Total Pop. U.S.	Population in Cities	Increase in total pop. per cent	Per cent of the total pop. living in cities.
1800	5,308,483	210,873		
1810	7,239,881	356,920	36.28	4.93
1820	9,633,822	475,135	33.66	4.93
1830	12,866 020	1,864 509	32.51	6.72
1840	17,069,453	1,453,994	32.52	8.52
1850	23 191,876	2,897,586	35.83	12.49
1860	31,443,321	5,072,256	35.11	16.13
1870	38,558,371	8,071,875	22.65	20.93
1880	50,155,783	11,318,547	30.08	22.57
1890	62,622,250	18,238,672	24.85	29.12

99.—Table Showing the Consumption of Water Per Capita Per 24 Hours in Various Cities and Towns, and the Cost to the Consumer Per 1,000 Gallons, and the Increase In Population in Each City in 20 Years.

The foregoing table shows that the general average increase of population in the towns and cities of the United States was 61.10 per cent from 1880 to 1890. But the rate of increase varies in different sections of the country and also in different classes of cities and towns. The railroad and general manufacturing centers increase most rapidly in all parts of the country, while the general growth of all classes of towns increases most rapidly in the Western states.

TABLE No. 47.

State	City	Census 1870	Census 1880	Census 1890	Gallons Per Day Per Capita.	Cost to Consumer Per 1,000 Gals
Alabama	Birmingham	0	400	26,241	155	8c to 30 c
Cala.	Los Angeles	5,758	11,183	50,394	175	20 c
Colo.	Denver	4,759	35,629	106,670	200	
Conn.	New Britain	9 840	11,800	19,010	87	10 c
Conn.	Norwich	16,653	15.112	16,195	50	15 c to 30 c
Conn.	Hartford	37,180	42,015	53,182	125	7½ c to 30 c
Conn.	New Haven	50 840	62 882	85,981	130	7½ c to 30 c
Georgia	Atlanta	21,789	37,409	65.515	164	13½ c
Georgia	Augusta	15,389	21,891	33,1[illegible]2	106	10 c
Georgia	Macon	10,810	12,479	22,698	70	6 c to 30 c
Illinois	Aurora	11,162	11,873	19,634	60	
Illinois	Chicago	298,977	503,185	1,098,576	131¼	8 c to 10 c
Illinois	Elgin	5 441	8,787	17,429	70	3 c to 8 c
Illinois	Streator	1,486	5,157	6.671	120	10 c to 25 c
Illinois	Freeport	7,889	8,516	10,159	46	10 c to 50 c
Illinois	LaSalle	5,200	7,847	11,610	70	8 c to 15 c
Indiana	Indianopolis	48,224	75,056	107,445	90	6 c to 30 c
Indiana	Richmond	9,445	12,472	16,845	74	5 c to 25 c
Iowa	Cedar Rapids	5,940	10,104	17,997	68	10 c to 30 c
Iowa	Sioux City	3,401	7,366	37,862	43	10 c to 25 c
Iowa	Des Moines	5,241	22 408	50,067	43	20 c to 40 c
Kansas	Atchison	7,054	15,105	14,122	90	10 c to 50 c
Kansas	Minneapolis			2,000*	200	35 c
Kansas	Arkansas City			3,347	46	10 c to 40 c
Ky.	Louisville	100,752	123,758	161,005	[illegible]0	6 c to 15 c
Ky.	Lexington	14,801	16,656	22,355	40	10 c to 25 c
Ky.	Frankfort	5,396	6.958	8,500	100	6 c to 15 c
Ky.	Fulton			4,500*	200	
Md.	Hagerstown	5,779	6,627	11,698	115	8 c to 40 c
Mass.	Adams	12,090	5,591	9,206	84	
Mass.	Fall River	26.766	48,961	74,351	28	
Mass.	Holyoke	10,733	21,915	35,528	78	5 c to 15 c
Mass.	Lowell	40,928	59,475	77,605	75	
Mass.	New Bedford	21,320	26 845	40,705	113	2¼ c to 15 c
Mass.	Newton	12,825	16,995	24,357	53	12 c to 35 c
Mass.	Springfield	26,703	33,340	44,164	87	30 c
Mich.	Battle Creek	5.838	7,063	13,190	31	
Mich.	Bay City	7,064	20,693	27,836	80	5 c to 10 c
Mich.	Detroit	79,577	116,340	205,669	140	6⅜c
Mich.	Sagnaw	7,460	10,525	46,215	100	6 c to 11 c
Miss.	Vicksburg	12,443	11,814	13,298	43	6 c to 35 c
Missouri	Springfield	5,555	6.522	21.842	90	25 c
Missouri	St. Louis	310,864	350,518	460,357	75	10 c to 30 c

TABLE NO. 47 CONTINUED.

State	City	Census 1870	Census 1880	Census 1890	Gallons Per Day Per Capita.	Cost to Consumer Per 1,000 Gals.
Missouri	Butler			4,000*	25	6 c to 60 c
N. Hamp	Nashna	10,543	13 397	19,266	150	10 c to 20 c
N. Hamp	Manchester	23,536	32,630	43,983	50	20 c
N. J.	Bayonne	3.834	9.372	18,996	73	13½ c-23¼ c
N. Y.	Cortland	3,066	4 050	8,561	90	40 c
N. Y.	Elmira	15 863	20 541	28,070	86	7½ c to 45 c
N. Y.	Kingston	6,315	18,344	21,181	80	8 c to 30 c
N. Y.	Olean			7,358	75	10 c to 40 c
N. Y.	Syracuse	43 051	51.792	87.877	300	6 c to 25 c
N. Y.	Brooklyn	396,099	566,663	804,377	100	7½ c-11⅝ c
N. Y.	Yonkers	12.733	18,892	31.942	70	4 c to 25 c
N. Y.	New York City	942,292	1,206,299	1,513,501	92	13⅓ c
Ohio	Dayton	30,473	33 678	58,868	53	8 c
Ohio	Findlay	3,315	4.633	18,672	48	6 c to 12 c
Ohio	Oberlin			4,000	20	30 c
Ohio	Sandusky	13,000	15,838	19,234	154	4 c to 15 c
Ohio	Springfield	12,652	20,730	32,135	90	10 c to 40 c
Ohio	Toledo	31,584	50.137	82,652	70	3 c to 10 c
Ohio	Cincinnati	216.239	255,139	296,309	124	17 c
Oregon	Salem				100	15 c to 25 c
Penn.	Oil City	2.276	7,315	10,943	230	6 c to 25 c
Penn.	McKeesport	2,523	8,212	20,711	110	4½ c to 30 c
Penn.	Williamsport	16,030	18,934	27,107	200	5 c to 10 c
Penn.	Harrisburg	23,104	30,762	40,164	130	2¼ c to 10 c
Penn.	Philadelphia	674 022	847.170	1,046,252	143	4 c
R. I.	Woonsocket	11 527	16,050	20,759	22½	10 c to 30 c
R. I.	Pawtucket	6,619	19,0 0	27,502	79	6 c to 30 c
Texas	Laredo	2,046	3,321	11,313	150	60 c
Texas	Fort Worth	0	6,663	20.725	130	20 c to 65 c
Va.	Richmond	51.038	66,600	80,388	151	7 c to 15 c

*Estimated Population.

The above table will be useful in determining the quantity of water required per 24 hours per person, and in determining what extra capacity of reservoirs and conduits should be provided for the increase in population during the coming 20 years. The capacity of a water supply system should not be based on the present number of inhabitants, but upon the probable number of inhabitants 20 years hence.

What the increase of population will be in any given town or city within any given number of years is a matter which must be considered in the light of the local conditions and surroundings of each given town or city. There are very few cities or towns in the United States which do not increase by 50 per cent within 20 years, and some increase by from 300 to 600 per cent within ten years. The general average increase of population in all cities and towns in the United States for the 10 years, 1880-1890, was 61.10 per cent.

100.—Formulas and Tables for Determining the Diameter of the Conduit or Pipe Required to Convey any Given Number of Gallons Per 24 Hours.—When the total supply of water in gallons per 24 hours has been decided upon, then the required diameter in feet of the circular brick conduit or pipe, or other circular water way, may be at once found by the formula

$$d = \sqrt[11]{\frac{m^2}{.3805}} \times \sqrt[11]{\frac{q^4}{S^2}}$$ (See §§ 43 and 101.)

In this formula the value of m varies with the class or roughness of the internal circumference of the waterway, and the value of m must be in terms of diameter in feet. The value of m for any class of wet perimeter will be found by referring to the different groups of pipes and channels. If the value of m, when found, is in terms of R in feet, it may be converted to terms of d in feet as shown at section 10.

In the above formula q=cubic feet per second, and S =the sine of the inclination of the waterway= $\frac{H}{l}$.

For a constant degree of roughness of perimeter, the value of $\sqrt[11]{\frac{m^2}{.3805}}$ is a constant, and the formula may be simplified accordingly. Thus, if we are going to adopt a double riveted asphaltum coated steel pipe, then m=.00033, and $\sqrt[11]{\frac{m^2}{.3805}}$=0.2541, and the formula for any pipe in this class reduces to $d=.2541\sqrt[11]{\frac{q^4}{S^2}}$. If we adopt an ordinary uncoated, cast iron pipe, then m=.0004, and the formula reduces to

$$d = \sqrt[11]{.0000004205} \times \sqrt[11]{\frac{q^4}{S^2}},=2633\sqrt[11]{\frac{q^4}{S^2}}.$$

If the pipe is to convey water from the distributing reservoir to the street mains, its capacity or diameter should be such as to enable the pipe to maintain a given pressure in lbs. per square inch at the point of juncture with the street system while it is supplying the given quantity of water in cubic feet per second. It is also well to remember that the total supply per 24 hours is usually drawn between 6 o'clock a. m and 9 o'clock p. m., and for this reason the city supply pipe leading from the distributing reservoir to the city must have such diameter as will pass the entire 24 hours' supply within 12 hours' time, and also maintain a given pressure while so discharging. In other words this pipe must carry a given quantity of water within a given time with a given loss of pressure or head at a given point. The formula for finding the required diameter to carry a given quantity with a given or pre-determined loss of head has already been given and discussed. (See § § 64, 68.) The following tables will greatly facilitate all such calculations, and show at once the value of q, or cubic feet per second, corresponding to any supply in gallons per 24 hours. (See also § 102).

TABLE No. 48.*

Gallons per 24 hours=	Cub. feet per second q	Logarithm of q	Value of q^4
1.	.00000154667	$\bar{6}$.189397	.000,000,000,000,000,000,0 0,005,722,563,850,675
10.	.0000154667	$\bar{5}$.189397	.000,000,000,000,000,000,057,225,638,506,75
100.	.000154667	$\bar{4}$.189397	.000,000,000,000,000,572,256,385.067,5
1,000.	.00154667	$\bar{3}$.189397	.000,000,000,005,722,563,850,675
10,000.	.0154667	$\bar{2}$.189397	000,000,057,225,638,506,75
100,000	.154667	$\bar{1}$.189397	.000,572,256,385,067,5
1,000,000.	1.54667	0.189397	5.722,563,850,675
10,000,000.	15.4667	1.189397	57,225.638,506,75
100,000,000.	154.667	2.189397	572,256,385.067,5
1,000,000,000.	1546.67	3.189397	5,722,563,850,675.034,2
10,000,000,000.	15466.70	4.189397	57,225,638,506,750,342.032,1
100,000,000 000.	154667.00	5.189397	572,256,385,067,503,420,321.00

*One cubic foot=7.48 gallons. One gallon=231 cubic inches.

TABLE No. 49.

Gal. per 24 hours =	Cub. feet per second q	Logarithm of q	Value of q^4
10	.0000154667	$\bar{5}$.1893[illegible]8	.000,000,000,000,000,000,057,225,638,506,75
20	.0000309334	$\bar{5}$.490427	.000,000,000,000,000,000,915,720,15
25	.00003866675	$\bar{5}$.587337	.000,000,000,000,000,002,235,673,513
30	.0000464001	$\bar{5}$.666518	.000,000,000,000,000,004,635
40	.0000618668	$\bar{5}$.791458	.000,000,000,000,000,014,65
50	.0000773335	$\bar{5}$.888367	.000,000,000,000,000,035,77
60	.0000928002	$\bar{5}$.967549	.000,000,000,000,000,074,26

TABLE No. 49—Continued.

Gal. per 24 hours	Cub. feet per second q	Logarithm of q	Value of q^4.
70	.00 1082669	$\bar{4}.034495$	.000,000,000,000,000,137,398,72
80	.0001237336	$\bar{4}.092488$	.000,000,000,000,000,234,396,1
90	.0001392003	$\bar{4}.1436399$	.000,000,000,000,000,375,457,278
100	.000154667	$\bar{4}.189398$	.000,000,000,000,000,572,256,385,067.5
200	.000309334	$\bar{4}.490427$	.000,000,000.000,009,157,201,5
250	.0003866675	$\bar{4}.587337$	.000,000,000,000,022,356,735.513
300	.000464001	$\bar{4}.666518$	.000,000,000,000,046,350
400	.000618668	$\bar{4}.791458$	.000,000,000,000,146,500
500	.000773335	$\bar{4}.888367$	.000,000,000,000,357,700
600	.000928002	$\bar{4}.967549$	.000,000,000,000,742,600
700	.001082669	$\bar{3}.034495$	.000,000,000,001,373,987,200
800	.001237336	$\bar{3}.092488$	.000,000,000,002,343,961,00
900	.001392003	$\bar{3}.1436399$	.000,000,000,003,754,572,780
1,000	.00154667	$\bar{3}.189398$	.000,000,000,005,722,563,850,675
2,000	.00309334	$\bar{3}.490427$	.000,000,000,091,572,015
2,500	.003866675	$\bar{3}.587337$	.000,000,000,223,567,351,3
3,000	.00464001	$\bar{3}.666518$	.000,000,000,463,5
4,000	.00618668	$\bar{3}.791458$	.000,000,001,465
5,000	.00773335	$\bar{3}.888367$	.000,000,003,577
6,000	.00928002	$\bar{3}.967549$	.000,000,007,426
7,000	.01082669	$\bar{2}.034495$	.000,000,013,739,872
8,000	.01237336	$\bar{2}.092488$	.000,000,023,439,610
9,000	.01392003	$\bar{2}.5745399$	.000,000,037,545,727,800
10,000	.0154667	$\bar{2}.189398$	.000,000,057,225,638,506,750
20,000	.0309334	$\bar{2}.490427$	.000,000,915,720,015
25,000	.03866675	$\bar{2}.587337$	.000,002,223,567,351,3
30,000	.0464001	$\bar{2}.666518$	.000,004,635

TABLE No. 49—Continued.

Gal. per 24 hours	Cub. feet per second q	Logarithm of q	Value of q^4
40,000	.0618668	$\bar{2}$.791458	.000,014,650
50,000	.0773335	$\bar{2}$.888367	.000,035,770
60.000	.0928002	$\bar{2}$.967549	.000,074,260
70,000	.1082669	$\bar{1}$.034495	.000,137,398,720
80,000	.1237336	$\bar{1}$.092488	.000,234,396,100
90,000	.1392003	$\bar{1}$.5745399	.000,375,457,278
100 000	.154667	$\bar{1}$.189398	.000,572,256,385,067,5
200,000	.309334	$\bar{1}$.490427	.009,157,201,5
250,000	.3866675	$\bar{1}$.587337	.022,356,735,130
300,000	.464001	$\bar{1}$.666518	.046,35
400,000	.618668	$\bar{1}$.791458	.146,5
500,000	.773335	$\bar{1}$.888367	.357,7
600,000	.928002	$\bar{1}$.967549	.742,6
700 000	1.082669	0.034495	1.373,987,2
800,000	1.237336	0.092488	2.343,961
900,000	1.392003	0.1436399	3.754,572,78
1,000,000	1.54667	0.189398	5.722,563,850,675
2,000,000	3.09334	0.490427	91.572,015
2,500,000	3.866675	0.587337	223.567,351,3
3,000,000	4.64001	0.666518	463.500
4,000,000	6.18668	0.791458	1465.00
5,000 000	7.73335	0.888367	3577.00
6,000,000	9.28002	0.967549	7426.00
7,000,000	10.82669	1.034495	13939.872
8,000,000	12.37336	1.092488	23439.61
9,000,000	13.92003	1.1436399	37545.727,8

TABLE 49—Continued.

Gal. per 24 hours	Cub. feet per second q	Logarithm of q	Value of q^4.
10.000,000	15.4667	1.189398	57225.638,506,75
20,000,000	30.9334	1.490427	915720.16
25,000,000	38.66675	1.587337	2235673.513,03
30,000,000	46.4001	1.666518	46350[illegible]0.00
40,000 000	61.8668	1.791458	14650000.00
50,000,000	77.3335	1.888367	35770000.00
60,000,000	92.8002	1.967549	74260000.00
70.000,000	108.2669	2.034495	137398720.00
80,000,000	123.7336	2.092488	234396100.00
90.000,000	139.2003	2.1436399	375457278.00
100 000,000	154.667	2.189398	572256385.067,5
200,000,000	309.334	2.490427	9157201500.000
250,000,000	386.6675	2.587337	22356735130.30

101.—To Find the Diameter in Feet of a Circular Conduit or Pipe With Free Discharge, as From One Reservoir Into Another, which is required to Discharge a given quantity in Cubic Feet Per Second, the Total Head or ths Slope Being Known:—

The general formula for finding the required diameter in feet will be

$$d=\sqrt[11]{\frac{m^2}{.3805}}\times\sqrt[11]{\frac{q^4}{S^2}},\text{ or } d=\sqrt[11]{\frac{m^2}{.3805}}\times\sqrt{\frac{l^2\,q^4}{H^2}}$$

Simplifying the formula as pointed out heretofore (§100) and for the following classes or degrees of roughness of perimeter we have,

(1) For uncoated clean cast iron pipe, $m=.0004$, and

$$d = .2632 \sqrt[11]{\frac{q^4}{S^2}}$$

(2) For uncoated steel or wrought iron, m=.00038, and

$$d = .2608 \sqrt[11]{\frac{q^4}{S^2}}$$

(3) For uncoated wooden stave pipe, made of dressed hard wood, and closely jointed, m=.00048, and

$$d = .2721 \sqrt[11]{\frac{q^4}{S^2}}$$

(4) For cement mortar lined pipe, one third sand and two thirds cement, m=.000424, and

$$d = .2661 \sqrt[11]{\frac{q^4}{S^2}}$$

(5) For riveted pipe, thoroughly dipped and coated with asphaltum and crude petroleum, m=.000325, and

$$d = .2535 \sqrt[11]{\frac{q^4}{S^2}}$$

(6) For cast iron or welded pipes thoroughly asphaltum coated and carefully laid and jointed, m=.000305, and

$$d = .2506 \sqrt[11]{\frac{q^4}{S^2}}$$

For brick perimeters see §24. Always make an extra allowance in the diameter of pipe or conduit for future deterioration and for deposits.

102.—To Find the Diameter in Feet of a Circular Conduit or pipe which is Required to carry a Given Quantity in Cubic Feet Per Second to a Given Point and Maintain a Given Head or Pressure at That Point while Delivering the Required Quantity:—

This formula is very important in designing power mains for water wheels, in which it is required to maintain a given pressure or head at the base of the nozzle which discharges upon the wheel or motor It applies equally well to hydraulic giants used in placer mining, and to fire hose with nozzle attached, and to all other cases where the discharge is partially

throttled, as in the case of a supply pipe leading from the distributing reservoir of a water works system to the street mains. In the latter case it is desirable to so proportion the diameter that it will convey the required quantity of water and at the some time maintain not less than a given head pressure at the point of its juncture with the street mains.
The general formula will be

$$d=\sqrt[11]{\frac{n^2}{.3805}}\times\sqrt[11]{\frac{l^2\,q^4}{h''^2}}$$

In which,

h'' = total head in feet to be lost in friction in the length l.

n=coefficient of resistance, and varies with different classes of wet perimeter.

Simplifying the formula for given classes of perimeters as heretofore pointed out (§§ 64, 68) and

(1) For uncoated clean cast iron, n=.0003938. and

$$d=.2624\sqrt[11]{\frac{l^2\,q^4}{h''^2}}$$

(2) For uncoated clean steel or wrought iron, n=.00037411 and

$$d=.26\sqrt[11]{\frac{l^2\,q^4}{h''^2}}$$

(3) For uncoated wooden stave pipe, made of dressed hard wood and closely jointed, n=.00047256, and

$$d=.2713\sqrt[11]{\frac{l^2\,q^4}{h''^2}}$$

(4) For cement mortar lined pipe, one-third sand and two-thirds cement, n=.0004175, and

$$d=.2653\sqrt[11]{\frac{l^2\,q^4}{h''^2}}$$

(5) For riveted pipe, thoroughly dipped and coated with asphaltum and crude petroleum, n=.00032, and

$$d = .2527\sqrt[11]{\frac{l^2 q^4}{h''^2}}$$

(6) For cast iron and welded pipes, thoroughly coated with asphaltum and oil, and carefully laid and jointed, n=.00030, and

$$d = .2498\sqrt[11]{\frac{l^2 q^4}{h''^2}}$$

REMARK.—For any given class of perimeter $n = m \times .9845$, and $m = \frac{n}{.9845}$, and the difference in value between

$\sqrt[11]{\frac{m^2}{.3805}}$ and $\sqrt[11]{\frac{n^2}{.3805}}$ for any given roughness is equal .0008.

That is to say,

$\sqrt[11]{\frac{n^2}{.3805}}$ is .0008 less than the corresponding ualue of $\sqrt[11]{\frac{m^2}{.3805}}$.

If $\sqrt[11]{\frac{m^2}{.3805}} = .2608$, then $\sqrt[11]{\frac{n^2}{.3805}} = .2600$, and so on.

While the difference in value of m and n is small, yet it must be remembered that m=the head per foot length of pipe to balance the resistance and generate the mean velocity of flow, and n is equal the friction head only, per foot length of pipe. In a pipe of considerable length the difference becomes very considerable. (See, in this connection, §§ 4 and 5)

Formulas (43) and (45) given in § 17 may be adopted instead of the above but in that event the value of m or n must be converted to terms of P as in § 17.

103—Velocities, Discharge and Friction Heads for Slopes and Diameters.

The slope required to generate a velocity of one foot per second in any given diameter with full and free discharge is

$$S=\frac{m}{d\sqrt{d}}.$$

The slope required to generate any other velocity, either greater or less than one foot per second, is

$$S=\frac{m}{d\sqrt{d}}\times v^2$$

In the latter formula v^2 must equal the square of the desired velocity in feet per second. Having found the value of S for $v=1.00$ in any given diameter, then the required value of S to generate any other velocity in the given diameter, will equal the value of S for $v=1.00$ multiplied by the square of the proposed velocity. The distance or length in feet l, of pipe, in which there is a fall of one foot is

$$l=\frac{1}{S}$$

As the value of S shows the total head per foot length of pipe, the fall in feet per 100 feet length is found by moving the decimal point in the value of S two places to the right. The friction head per foot length in any given uniform diameter with full and free discharge is .9845 per cent of the value of S for that pipe. The friction head may therefore be easily found from the value of S. The friction head per 100 feet length of pipe will be

$$h''=S\times98.45, \text{ or } h''=(S\times100)-(S\times100\times.0155).$$

When the friction head per 100 feet is ascertained for a given diameter with $v=1.00$, then the friction head per 100 feet in the given diameter for any other velocity will equal that for $v=1.00$ multiplied by the square of the proposed velocity. The following table (No. 50) is based on $m=.0004$ for all clean iron pipes.

Table No. 50.

Velocities, Discharge and friction heads for given slopes and diameters.

$$\frac{H}{l}=S.\quad S=\frac{m\ v^2}{d\sqrt{d}},\quad l=\frac{d\sqrt{d}}{m}=\frac{1}{S}.$$

$$h''=\frac{n\,l\,v^2}{d\sqrt{d}}.\quad v=\sqrt{\frac{S\,d\sqrt{d}}{m}}$$

Diameter of Pipe.	$\sqrt{d}$ Feet	Slope S	Velocity Feet per Second.	Friction Head per 100 Feet.	Discharge Gallons per minute.	Discharge Cubic feet Second.
3 inches .25 feet	0.500	.0008	0.50	.07876	11.01804	.02455
		.0032	1.00	.31504	22.03608	.04910
		.0072	1.50	.70884	33.05412	.07365
		.0128	2.00	1.26016	44.07216	.09820
		.0200	2.50	1.96900	55.09020	.12275
		.0288	3.00	2.83536	66.10824	.14730
		.0392	3.50	3.85928	77.12628	.17185
		.0512	4.00	5.04070	88.14432	.19640
		.0648	4.50	6.37964	99.16236	.22095
		.0800	5.00	7.87600	110.18040	.24550
		.0968	5.50	9.53008	121.19844	.27005
		.1152	6.00	11.34158	132.21648	.29460
		.1352	6.50	13.31060	143.23452	.31915
		.1568	7.00	15.43715	154.25256	.34370
		.2048	8.00	20.16281	176.28864	.39280
		.3200	10.00	31.50400	220.36080	.49100
4 inches .3333 feet	0.579	.00207850	1.00	.204625	39.18024	.08730
		.00466662	1.50	.459428	58.77036	.13095
		.0083140	2.00	.8185133	78.36048	.17460
		.0129906	2.50	1.278924	97.95060	.21825
		.0187065	3.00	1.841655	117.54080	.26190
		.0254616	3.50	2.506694	136.72096	.30555
		.0332560	4.00	3.254053	155.90120	.34920
		.0420896	4.50	4.143721	175.08144	.39285
		.0519625	5.00	5.115708	194.26168	.43650
		.0628746	5.50	6.190004	213.44192	.48015
		.0748260	6.00	7.368619	232.62216	.52380
		.0878166	6.50	8.645544	251.80240	.56745
		.1018465	7.00	10.026788	270.98264	.61110

TABLE 50.—Continued.

Diameter of Pipe.	√d Feet	Slope S	Velocity Feet per Second.	Friction Head per 100 Feet,	Discharge Gallons per minute.	Discharge Cubic Feet Second.
5 inches 4167 feet	0.645	.00148120	1.00	.14651329	61.17144	.13630
		.00334845	1.50	.32965490	91.757160	.20445
		.00595280	2.00	.58605316	122.342880	.27260
		.00930125	2.50	.91570806	152.92860	.34075
		.01339380	3.00	1.31861961	183.51432	.40890
		.01823045	3.50	1.79276780	214.10004	.47705
		.02381120	4.00	2.34421264	244.68576	.54520
		.03013605	4.50	2.96989412	275.27148	.61335
		.03720500	5.00	3.66283225	305.85720	.68150
		.04501805	5.50	4.43202702	336.44292	.74965
		.05357520	6.00	5.27447844	367.02864	.81780
6 inches .50 foot	0.7071	.001131156	1.00	.1113623	88.14432	.1964
		.002543101	1.50	.2503683	132.21648	.2946
		.004524624	2.00	.4454492	176.28864	.3928
		.007069725	2.50	.6960144	220.36080	.4910
		.010180404	3.00	1.002260	264.43296	.5892
		.013856661	3.50	1.364198	308.50512	.6874
		.018098496	4.00	1.781796	352.57728	.7856
		.022905909	4.50	2.255086	396.64944	8838
		.028278900	5.00	2.784057	440.72160	.9820
		.034217469	5.50	3.368709	484.79376	1.0802
		.040721616	6.00	4.008903	528.86592	1 1784
		.047791341	6.50	4.705057	572.93808	1.2766
7 inches .5833 feet	0.764	.000897583	1.00	.088367	119.96424	.26730
		.002019561	1.50	.198825	179.94636	.40095
		.003590332	2.00	.353468	239.92848	.53460
		.005609893	2.50	.552294	299 91060	.66825
		.008078247	3.00	.795303	359.89272	.80190
		.010995391	3.50	1.081996	419.87484	.93555
		.013961328	4.00	1.374493	479.85696	1.06920
		.018176055	4.50	1.789433	539.83908	1.20285
		.022439575	5.00	2.209176	599.82120	1.33650
		027151885	5.50	2.673103	659.80332	1.47015
8 inches .6667 feet	0.817	.000734357	1.00	.0722975	156.67608	.3491
		.002937428	2.00	.2871888	313.35216	.6982
		.006609213	3.00	.6506771	470.02824	1.0473
		.011749712	4.00	1.1567592	626.70432	1.3964
		.018358925	5.00	1.8074362	783.38040	1.7455
		.026436852	6.00	2.6027091	940.05648	2.0946
		.035983493	7.00	3.5425749	1096.73256	2.4437
		.046998848	8 00	4.6270366	1253.40864	2.7928
		.059482917	9 00	5.8560932	1410.08472	3.1419
		.073435700	10.00	7.2297500	1566.76080	3.4910

TABLE NO. 50 CONTINUED.

Diameter of Pipe	√d Feet	Slope S	Velocity Ft. per. Sec.	Friction head per. 100 feet.	Discharge Gallons per Minute	Discharge Cubic feet Second
9 inches .75 Foot	0.866	.00061585835	1.00	.060631255	198.27984	.4418
		.00246343340	2.00	.242525020	396.55968	.8836
		.00554272415	3.00	.545681193	594.83952	1.3254
		.00985373360	4.00	.970100080	793.11936	1.7672
		.01529645875	5.00	1.505936364	991.39920	2.2090
		.02217090060	6.00	2.182725170	1189.67904	2.6508
		.03017705915	7.00	2.970931474	1387.95888	3.0926
		.03941493440	8.00	3.880400300	1586.23872	3.5344
		.04988452635	9.00	4.911131620	1784.51856	3.9762
		.06158583500	10.00	6.063125500	1982.79840	4.4180
		.07451886035	11.00	7.336381802	2181.07824	4.8598
		.08868360240	12.00	8.730900660	2379.35808	5.3016
10 inches .8333 Ft	0.913	.00052576237	1.00	.051761306	244.77552	.5454
		.00210304948	2.00	.207045222	489.55104	1.0908
		.00473186133	3.00	465851748	734.32656	1.6 62
		.00841219792	4.00	.828180886	979.10208	2.1816
		.01314405925	5.00	1.294 32634	1223.87760	2.7270
		.01892744532	6 00	1.863406992	1468.65312	3.2724
		.02576235613	7.00	2.536303961	1713.42864	3.8178
		.03364879168	8.00	3.312723541	1958.20416	4.3632
		.04258675197	9.00	4.192665732	2202.97968	4.9086
		.05257623700	10.00	5.176130600	2447.75520	5.4540
		.06361724677	11.00	6.263117945	2692.53072	5.9994
12 inches 1.00 Feet.	1.00	.00040000000	1.00	.03938	352.48752	.7854
		.0016	2.00	.15752	704.97504	1.5708
		.0036	3.00	.35442	1057.46256	2.3562
		.0064	4.00	.63008	1409.95008	3.1416
		.0100	5.00	.98450	1762.43760	3.9270
		.0144	6.00	1.41768	2114.92512	4.7124
		.0196	7.00	1.92962	2467.41264	5.4978
		.0256	8.00	2.52032	2819.90016	6.2832
		.0324	9.00	3.18978	3172.38768	7.0686
		.0400	10.00	3.93800	3524.87520	7.8540
		.0484	11.00	4.76498	3877.36272	8.6394
14 inches 1.167 Feet	1.080	.00031736964	1.00	.026817735	479.7672	1.069
		00126947856	2.00	.124980165	959.5344	2.138
		.00285632676	3.00	.281205370	1439.3016	3.207
		.00507791424	4.00	.499920658	1919.0688	4.276
		.00793424100	5.00	.781126000	2398.8360	5.345
		.01142530704	6.00	1.1248208115	2878.6032	6.414
		.01555111236	7.00	1.531007012	3358.3704	7.483
		.02031165696	8.00	1.999682628	3838.1376	8.552
		.02570694084	9.00	2.569280203	4317.9048	9.621
		.03173696400	10.00	3.124504100	4797.6720	10.690
		.03840172644	11.00	3.790649969	5277.4392	11.759

TABLE 50—Continued.

Diameter of Pipe.	√d Feet.	Slope S	Velocity Feet per Second.	Friction Head per 100 Feet.	Discharge Gallons per Minute.	Discharge Cubic Feet Second.
16 Inches 1.333 feet	1.155	.00025980521	1.00	.025577823	626.5248	1.396
		.00103922084	2.00	.103350513	1253.0496	2.792
		.00233824689	3.00	.23020 407	1879.5744	4.188
		.0 415688336	4.00	.409245167	2506.0992	5.584
		.00649513025	5.00	.639445574	3132.6240	6.980
		.00935298756	6.00	.920801626	3759.1488	8.376
		.01273045529	7.00	1.253313324	4385.6736	9.772
		.01662753344	8.00	1.636980668	5012.1984	11.168
		.02104422201	9.00	1.971803657	5638.7232	12.564
		.02598052100	10.00	2.5577823	6265.2480	13.960
		.03143643041	11.00	3.126353005	6891.7728	15.356
18 inches 1.50 feet	1.224	.0002124183	1.00	.02091259	793.0296	1.767
		.0008496732	2.00	.08365033	1586.0592	3.534
		.0019117737	3.00	.18821413	2379.0888	5.301
		.0033986928	4.00	.33460131	3172.1184	7.068
		.0053104575	5.00	.52281455	3965.1480	8.835
		.0076470588	6.00	.75284194	4758.1776	10.602
		.0104084967	7.00	1.02471651	5551.2072	12.369
		.0135947712	8.00	1.33840523	6344.2368	14.136
		.0172 58823	9.00	1.69391912	7137.2664	15.903
		.0212418300	10.00	2.0912590	7930.2960	17.670
		.0257026143	11.00	2.53 42238	8723.3256	19.437
20 inches 1.667 feet	1.291	.000185865228	1.00	.0182984317	979.2816	2.182
		.000743460912	2.00	.0731937268	1958.5632	4.364
		.001672787052	3.00	.1646858853	2937.8448	6.546
		.002973843648	4.00	.2927749072	3917.1264	8.728
		.004646630700	5.00	.4574610000	4896.4080	10.910
		.006691148208	6.00	.6587435411	5875.6896	13.092
		.009107396172	7.0	.8966231532	6854.9712	15.274
		.011895374592	8.00	1.1710996286	7834.2528	17.456
		.015055083468	9.00	1.4821729675	8813.5344	19.638
		.018586522800	10.00	1.82984317	9792.8160	21.820
		.022489692588	11.00	2.2141102353	10772.0976	24.002
24 inches 2.00 feet	1.4142	.0001414227124	1.00	.013923055	1409.35008	3.1416
		.0005656908496	2.00	.055692220	2818.70016	6.2832
		.0012728044116	3.00	.125307495	4228.05024	9.4248
		.0022623633984	4.00	.222768880	5637.40032	12.5664
		.0035355678100	5.00	.348076375	7046.75040	15.7080
		.0050912176464	6.00	.501229930	8456.10048	18.8496
		.0069297129076	7.00	.682229695	9865.45046	21.9912
		.0090510515936	8.00	.891075520	11274.80064	25.1328
		.0114552397044	9.00	1.127767455	12684.15072	28.2744
		.0141422712400	10.00	1.39230550	14093.50080	31.4160
		.0171121482004	11.00	1.684689655	15502.85088	34.5576

TABLE No. 50 CONCLUDED.

Diameter of Pipe	√d Feet	Slope S	Velocity Ft. per. Sec.	Friction head per 100 feet.	Discharge Gallons per Minute.	Discharge Cubic feet Second
27 inches 2.25 Feet	1.5000	.0001185185185	1.00	.011721478	1724.4288	3.976
		.0004740740740	2.00	.046885912	3448.8576	7.952
		.0010666666665	3.00	.105493302	5173.2864	11.928
		.0018962962960	4.00	.187543648	6897.7152	15.904
		.0029629629625	5.00	.293036950	8622.1440	19.880
		.0042671671660	6.00	.421973208	10346.5728	23.856
		.0058074074065	7.00	.574352422	12071.0016	27.832
		.0075851851840	8.00	.750174592	13795.4304	31.808
		.0095999999985	9.00	.949439718	15519.8592	35.784
		.0118518518500	10.00	1.17214780	17244.2880	39.760
		.0143407407385	11.00	1.418298838	18968.7168	43.736
30 inches 2.50 Feet	1.581	.0001012018	1.00	.00996332	2203.1592	4.909
		.0004048072	2.00	.03685328	4406.3184	9.818
		.0009108162	3.00	.08966988	6609.4776	14.727
		.0016192288	4.00	.15941312	8812.6368	19.636
		.0025300450	5.00	.24908300	11015.7960	24.545
		.0036432648	6.00	.35867952	13218.9552	29.454
		.0049588882	7.00	.48820268	15422.1144	34.363
		.0064769152	8.00	.63765248	17625.2736	39.272
		.0081973458	9.00	.80702892	19828.4328	44.181
		.0101218000	10.00	.99633200	22031.5920	49.090
		.0122454178	11.00	1.20556172	24234.7512	54.000
36 inches 3.00 Feet	1.732	.0000769823	1.00	.00757891	3172.5672	7.069
		.0003079292	2.00	.03131564	6345.1344	14.138
		.0006928407	3.00	.06821019	9517.7016	21.207
		.0012317168	4.00	.12126256	12690.2688	28.276
		.0019245575	5.00	.18947275	15862.8360	35.345
		.0027713628	6.00	.27284076	19035.4032	42.414
		.0037721327	7.00	.37136659	22207.9704	49.483
		.0049268672	8.00	.48505024	25380.5376	56.552
		.0062355663	9.00	.61389171	28553.1048	63.621
		.0076982300	10.00	.75789100	31725.6720	70.690
		.0093148583	11.00	.91704811	34898.2492	77.759

104.—Thickness and Weight of Cast Iron Pipe.—There is a great want of uniformity in regard to the thickness of cast iron pipe for any given pressure. Every city seems to have adopted different thicknesses of pipe. The leading formulas for thickness give greatly differing results for the same conditions.

$t=(.000058\ h\ d)+.0152\ d+.312$..........(J. B. Francis)
$t=(.0016\ n\ d)+.013\ d+.32$................(M. Dupuit)
$t=(.00238\ n\ d)+.34$...............(Julius Weisbach)

The following formula gives thickness of cast iron pipe as adopted in recent practice,

$t=(p+100)\ .000142\ d+.33\ (1-.01\ d)$...............(110)

In the above formulas,

t=thickness of pipe shell in inches
d=inside diameter in inches
h=head of water in feet
p=pressure of water=$H\times.434$
n=number atmospheres pressure at 33 feet each.

Fannings formula for the weight per lineal foot of cast iron pipe, including the weight of the bell or hub is, for 12 foot pipes,

$W=12\ (d+t)\times1.08\ t\times3.1416\times.2604$

By a 12 foot pipe is meant a pipe which will actually lay 12 feet, or is 12 feet from bottom of bell to end of spigot. The bell or hub adds about 7½ per cent to the weight of a length of pipe. The above formula allows for the extra weight of bell. For more on weight of pipes, see "Gregory's Practical Mathematics."

105—Dimensions and Weight of Cast Iron Pipe Made by The Colorado Fuel and Iron Company of Denver, for 100 lbs. Pressure.

TABLE No. 51. By W. F. McCue

Diameter Inches	Length over all. Feet—Inches.	Depth of Bell Inches	Will Lay Feet—Inches	Thickness of Shell—Inches*	Inside Diameter of Bell—Inches	Outside Diam. of Bell—Inches	Outside Diam. of Spigot.	Weight per Ft. in Pounds.
3	12—4	3	12—1	13-32	4 1-2	7	4 3-8	15 1-2
4	12—4	3	12—1	7-16	5 1-2	8 1-4	5 3-8	22
6	12—4	3	12—1	1-2	7 5-8	10 7-8	7 1-2	33
8	12—4	3½	12—½	17-32	9 7-8	13 3 8	9 3-4	44
10	12—4	3½	12—½	19 32	11 7-8	15 5-8	11 3-4	63
12	12—4	4	12	5-8	13 3-4	17 3-4	13 5-8	75
16	12—4	4	12	3-4	18	22 1-2	17 7-8	125
20	12—4	4	12	27 32	22 1-8	26 7-8	22	175

*See Table No. 27, §58 for fractional inches in equivalent decimals.

Packing (Jute Hemp) and Lead Required Per Joint For Above Pipe.

Diameters =	3"	4"	6"	8"	10"	12"	16"	20"
Lead, lbs. Per Joint	4 1-2	5 1-2	7	10	12 1-2	17	20 1-2	29
Packing, ozs	3	3 1-2	5	7	9 1-2	12	20	26

106—Weight Per Foot Length of Cast Iron Pipe For 150 and 200 lbs. Pressure, as Made by Colorado Fuel and Iron Co. of Denver, Colorado.

Diameters	3"	4"	6"	8"	10"	12"	16"	20"
Wt. Per. ft. 150 pounds Pressure	17 lbs	23 ½ lbs	36 lbs	48 lbs	70 lbs	85 lbs	140 lbs	210 lbs
Wt. Per. ft. 200 pounds Pressure	19 lbs	26 lbs	42 lbs	55 lbs	78 lbs	94 lbs	155 lbs	222 lbs

Remark—The market prices of pig iron, cast iron and lead and other metals fluctuate so rapidly that tables for estimating the cost of pipe and laying are of no great value except in so far as such tables furnish the data as to the quantity and weight required. The price of pig iron May 12th, 1898, was $6.65, and on July 28th, 1899, the price was $15.25.

The present price of cast iron pipe (August 2nd, 1899) is $33.00 per ton of 2,000 lbs., and of lead, $5.00 per 100 lbs. in Denver.

107—Manufacturers' Standard Cast Iron Water Pipe For 100 lbs. Pressure Per Square Inch.

Table No. 52.

Diameters In.	4	6	8	10	12	14	16	18	20	24	30	36	48
Thickness, In*	1-2	1-2	1-2	9-16	9-16	3-4	3-4	7 8	15-16	1	1 1-8	1 3-8	1 1-2
Wt. per. ft. lb.	22	33	45	60	75	117	125	167	200	250	350	475	775
Wt. per. 12 ft.	264	396	540	720	900	1400	1500	2000	2400	3000	4200	5700	9300

*See Table No. 27, § 58 for fractional inches in equivalent decimals.

108—Cost Per 100 Feet Length, For Labor and Material in Laying Cast Iron Water Pipe in Denver, Colorado, in 1890.

The conditions were:—Top of pipe 5 feet below surface. Depth of trench 5 feet, plus outside diameter of pipe. Easy trenching in sandy loam, Wages, foreman $3.00, calkers $2.50 laborers $1.75 per day of 10 hours, teams $3.00 per day, pipe $33.00 per ton of 2000 lbs.. lead $4.75 per 100 lbs., packing 6 cents per lb. No pavements to tear up. Backfilling done by teams and scrapers. Average water pressure 80 lbs. Thickness of pipe for 120 lbs. hydraulic pressure. Hemp packing. Hauling pipe 60 cents per ton.

TABLE No. 53* Cost per 100 feet.

Diameter In.	Weight Per Ft.	Cost of Pipe 100 Feet.	Unloading 100 Feet	Hauling 100 Feet	Packing 100 Feet	Cost of Lead	Cost of Coal	Hauling Lead, Tools, Specials	Trenching, laying, backfilling.
4	22	$36.30	$0.20	$ 0.66	$0.12	$ 2.15	$0.20	$0.15	$14.00
6	32	52.80	.31	1.05	.18	3.13	.20	.15	14.00
8	45	74.25	.40	1.35	.24	4.03	.20	.15	15.00
10	60	99.00	.54	1.80	.30	4.14	.20	.20	16.00
12	75	123.75	.68	2.25	.35	5.36	.20	.20	20.00
14	117	193.05	.85	3.50	.50	8.07	.25	.25	31.00
16	125	206.25	.94	3.75	.52	9.30	.25	.25	32.00
18	170	280.50	1.28	5.10	.55	11.17	.30	.25	38.00
24	250	412.50	1.88	7.50	.60	14.96	.40	.50	44.00
30	350	577.50	2.63	10.50	.70	16.15	.50	.60	50.00
36	500	825.00	4.50	15.00	.90	23.75	.60	.70	60.00
48	700	1155.00	6.30	21.00	1.10	49.87	.75	1.00	75.00

*Allow 440 joints per mile when estimating cost of laying cast iron pipe. Wrought iron and steel pipe is made in lengths of 15 to 27 feet according to conditions to be met. Cast iron pipe is in lengths of 12 feet. See Remark under Table No. 55.

109—Cost of Pipe Per Foot Laid in Boston.

Axis of pipe is 5 feet below surface. Labor $2.00 per day. Cost of pipe 1½ cents per lb., or $30.00 per ton of 2000 lbs. Special castings 3 cents, lead 5 cents per lb. Cost of excavating rock $3.50 to $5,50 per cubic yard, measured to neat lines.

This table is transcribed from "Details of Water Works etc.", by W, R. Billings.

TABLE No. 54.—Cast Iron Pipe.

Diameter In.	Thickness In.	Weight lbs. Per Foot	Lead per foot Pounds	Cost per foot of pipe and Specials.	Packing, lead and blocking Per foot	Teaming	Trenching, laying, labor	Total cost Per foot
4	0.45	21.7	0.70	$0.38	$0.05	$0.02	$0.25	$0.70
6	0.50	35.0	1.00	0.57	.06	.03	0.27	.93
8	0.55	50.0	1.35	0.83	.08	.05	0.30	1.26
10	0.60	68.0	1.70	1.10	.10	.06	0.34	1.60
12	0.58	78.0	2.00	1.27	.13	.07	0.37	1.84
12	0.65	88.0	2.00	1.42	.13	.07	0.37	1.99
16	0.66	118.0	2.70	1.87	.17	.08	0.45	2.57
16	0.75	135.0	2.70	2.12	.17	.08	0.45	2.82
20	0.73	162.0	3.35	2.55	.21	.09	0.55	3.40
20	0.85	188.0	3.35	2.94	.21	.09	0.55	3.79
24	0.81	216.0	4.00	3.44	.25	.10	0.68	4.47
24	0.94	250.0	4.00	3.95	.25	.10	0.68	4.98
30	0.93	308.0	5.00	4.92	.29	.11	0.80	6.12
36	1.04	410.0	6.00	6.58	.34	.12	1.00	8.04
40	1.12	490.0	6.70	7.80	.40	.15	1.30	9.65
48	1.25	660.0	8.00	10.40	.48	.20	1.75	12.83

REMARK—From the high cost of trenching and the reference to rock excavation, the ground must have been very "hard digging." Compare cost of trenching with that at Omaha for like diameters.

110—Cost of Trenching, Laying, Calking and Backfilling in Omaha, 1889, With wages of Foreman $2.50, Calkers $2.25, Laborers $1.75 Per Day of 10 Hours. (W. F. McCue, C. E., of Colorado Fuel & Iron Co.)

TABLE No. 55.—Cast Iron Pipe.

Diam. of Pipe	Width of trench feet	Depth of trench, feet	Cost of trench, lineal foot	Laying, calking, backfilling, lineal foot.	Cost of labor per lineal foot complete.
4"	1.75	5.666	$0.104	$0.036	$0.140
6	1.75	6.000	0.105	0.036	0.141
8	1.75	6.000	0.107	0.043	0.150
10	2.00	6.083	0.126	0.053	0.179
12	2.00	6.250	0.126	0.056	0.182
16	2.33	7.333	0.175	0.063	0.238

REMARK—Mr. McCue has been in charge of the construction of nearly 700 miles of pipe line in the Eastern and

Western states, In a letter to the writer he says: "We generally employed 60 to 70 men in a gang—enough laborers to excavate the trench ahead of the layers. In laying 4 to 12 inch pipe, we had one yarner aud one calker. In laying pipe 16 inches diameter or larger, we had two yarners and two calkers. In laying pipe larger than 12 inches diameter it is necessary to use a derrick for lowering the pipe into the trench.

One yarner and one calker will make about 60 joints per day of 10 hours in laying 4 or 6 inch cast iron pipe, and about 50 joints of 8 inch, 45 joints of 10 inch and 40 joints of 12 inch pipe. In laying pipe larger than 12 inchs, a derrick is required, and progress is much less. Most of the backfilling is done by team and scraper. The largest days work I ever had done was 80 joints of 8 inch pipe yarned by one man and calked by one man. In 1893, I took one yarner and one calker, the fastest I ever saw, and laid and calked 272 joints of 6 inch pipe in 35 hours. The cost was 1⅛ cents per lineal foot including foreman, kettlemen, and 3 to lay pipe in the trench. We use Jute hemp for packing."

III—Weston's Tables for Estimating Cost of Laying Cast Iron Pipe.

The following tables by E. B. Weston, C. E., of Providence, Rhode Island, were published in Engineering News, June 21, 1890, together with other valuable data of like character. The elements of cost entering these tables are:

Wages, foreman $3.00, calkers $2.25, laborers $1.50, per day. Teams $2.25 per day. Carting $1.00 per ton of 2240 lbs. Depth of trench 4.67 feet plus one-half the outside diameter of pipe. Lead, 5 cents per pound. Tools, blocks and wedges 7 2-10 to 16 1-10 per cent of cost of trenching, laying and backfilling the trench. In the tables the word "trenching" includes excavation and backfilling. "Medium" digging is ground equivalent to gravel and sand. "Hard" digging is ground equivalent to hard or moist clay. Cost of engineering and inspection not included in tables.

TABLE No. 56.
Cost of Laying Cast Iron Pipe Where Digging is "Medium."

Diam. Inches	4"	6"	8"	10"	12"	16"	20"	24"	Cost of labor etc.
1. Trenching	.0597	.0697	.0790	.0883	.0974	.1700	.2100	.3091	$1.50
2. Laying	.0189	.0220	.0249	.0297	.0307	.0440	.0577	.0639	$1.50
3. Foreman	.0180	.0206	.0234	.0265	.0294	.0350	.0373	.0396	$3.00
4. Tools etc.	.0056	.0065	.0075	.0084	.0093	.0154	.0214	.0602	7.2 per cent of 1 and 2*
5. Calking	.0106	.0107	.0108	.0111	.0118	.0159	.0301	.0757	$2.25
6. Lead	.0224	0320	.0431	.0553	.0683	.0950	.1203	.1600	5 cents per lb.
7. Teams	.0070	.0090	.0115	.0136	.0160	.0203	.0216	.0228	$2.25
8. Carting	.0078	.0149	.0208	.0275	.0346	.0518	.0746	.1317	$1.00 per ton [2240 lbs.]
Total per Foot	.1500	.1854	.2210	.2586	.2975	.4474	.6010	.8630	
"Hard" Digging									
1. Trenching	.0860	.0959	.0153	.1147	.1300	.2261	.3264		$1.50
2. Laying	.0271	.0303	.0333	.0362	.0411	.0530	.0669		$1.50
3. Foreman	.0260	.0286	.0314	.0343	.0372	.0428	.0452		$3.00
4. Tools etc.	.0081	.0090	.0099	.0109	.0118	.0201	.0283		7.2 per cent of 1 and 2*
5. Calking	.0106	.0107	.0108	.0111	.0118	.0159	.0301		$2.25
6. Lead	.0224	.0320	.0431	.0553	.0683	.0950	.1203		5 cents per lb.
7. Teams	.0070	.0090	.0115	.0136	.0160	.0203	.0216		$2.25
8. Carting	.0078	.0149	.0208	.0275	.0346	.0513	.0746		$1.00 per ton [2240 lbs.]
Total per Foot	.1950	.2304	.2661	.3036	.3508	.525	.7134		

*The per cent for 24 inch is 16.1 per cent, and includes blocks and wedges.

112—Cubic Yards of Excavation in Trench Per Lineal Foot,—Vertical Sides—Bell Holes Not Included.

TABLE No. 57.

Width in Feet.	Depth of Trench in Feet.											
	3	4	5	6	7	8	9	10	11	12	13	14
1.50	0.166	0.222	0.277	0.333	0.388	0.444	0.500	0.555	0.611	0.666	0.722	0.777
2.	0.222	0.296	0.370	0.444	0.518	0.592	0.666	0.740	0.815	0.888	0.962	1.037
2.50	0.274	0.370	0.463	0.555	0.644	0.740	0.833	0.926	1.018	1.111	1.203	1.288
3.	0.333	0.444	0.555	0.666	0.777	0.888	1.000	1.111	1.222	1.333	1.444	1.555
3.50	0.388	0.518	0.648	0.777	0.876	1.036	1.166	1.296	1.425	1.555	1.654	1.752
4.	0.444	0.592	0.740	0.888	1.037	1.185	1.333	1.481	1.628	1.777	1.926	2.074
4.50	0.500	0.666	0.833	1.000	1.166	1.333	1.500	1.666	1.833	2.000	2.166	2.333
5.	0.555	0.740	0.926	1.111	1.296	1.481	1.666	1.851	2.037	2.222	2.407	2.592
5.50	0.611	0.815	1.015	1.222	1.426	1.630	1.833	2.037	2.237	2.444	2.648	2.852
6.	0.666	0.888	1.111	1.333	1 555	1.777	2.000	2.222	2.444	2.666	2.888	3.111
6.50	0.722	0.963	1.203	1.444	1.685	1.936	2.166	2.407	2.647	2.888	3.130	3.370
7.	0.778	1.037	1.296	1.555	1.814	2.074	2.333	2.592	2.851	3.111	3.370	3.629
7.50	0.844	1.111	1.388	1.666	1.944	2.222	2.500	2.777	3.055	3.333	3.611	3.888
8.	0.888	1.185	1.481	1.777	2.074	2.370	2.666	2.963	3.258	3.555	3.851	4.148
8.50	0.903	1.259	1.574	1.888	2.203	2.518	2.833	3.148	3.462	3.777	4.092	4.406
9.	1.000	1.333	1.666	2.000	2.333	2.666	3.000	3.333	3.666	4.000	4.333	4.666
9.50	1.055	1.407	1.759	2.111	2.462	2.814	3.166	3.518	3.870	4.222	4.573	4.924

REMARK.—The foregoing table (No. 57) will be useful in estimating the cost of sewer work as well as in estimates of cost of pipe laying. It is also the custom of some engineers to excavate irrigation canals with vertical sides and allow for the caving and sliding of the banks until they assume the natural angle of repose. There is nothing to commend this practice, but still it is followed to a considerable extent. In sewer work where the ground is firm, the trench is excavated in alternate sections, and tunnels driven through the short blocks of ground between the excavated sections. This reduces the amount of excavation by about 30 per cent, and saves the cost of sheeting and bracing. In estimating cost of excavation in earth or rock, see Trautwine's "Engineers Pocket Book."

113.—Bell Holes in Trench for Cast Iron Pipes.—In order that the "yarner" and the calker may have room to get at all parts of the joint, the trench should be dug out 8 inches deeper for a distance of four feet in front of the bell or hub, and 8 inches wider on either side for the same distance to give shoulder and striking room. This adds materially to the cost of excavation, especially where the ground has a tendency to cave and slide, or is very wet.

114 —Depth of Trenches for Pipe.—Pipes in which there is a constant flow of water are in little danger from freezing even if laid on the surface of the ground, but in the distribution or street system the flow is almost if not entirely stopped during certain hours of the night when little or no water is being drawn by consumers.

Pipes supplying reservoirs and having a constant discharge may be covered to any convenient depth simply for the protection of the pipe from injury by wagons, falling trees etc., and to prevent too great expansion by heat or contraction by cold, and to get the pipe out of the way.

The general rule in the New England States is to make the trenches for street pipes of such depth as will place the center or axis of the pipe five feet under cover. That is, the trench is five feet plus one half the outside diameter of the pipe. The depth that a street pipe should be covered depends on the climate, the nature of the ground and the diameter of the pipe. Where the temperature gets down to from 25 to 40 degrees below zero (Fahr.) for two or three days at a time, 4 and 6 inch pipes will freeze solid when five feet under cover in sandy and gravelly loam. This occurred in many towns in Colorado in February, 1899. If the earth is dense and free from stones and gravel it is not probable that frost will penetrate to a depth exceeding four and a half feet. Small pipes laid in open, gravelly soil, should have the top of the pipe at least six feet under cover.

115.—Amount of Trenching and Pipe Laying Per Day Per Man.—The number of cubic yards of excavation done per man per day will be less in deep trenches than in comparatively shallow ones because of the extra effort required to throw the dirt out of deep trenches. The nature of the earth or rock to be excavated will, of course, be a controlling element in determining the amount of excavation that can be accomplished per day, by an average laborer. Quicksand, water and caving banks may also be large items of expense and prevent rapid progress. There are so many elements of uncertainty involved in making an estimate of the work that one man will accomplish in a given time that it is best to ascertain what has been actually accomplished under like conditions in the past. By analysis of statements of work actually done in a given time by a given number of men, we can approximate the time required and the cost of doing similar work. Mr. W. R. Billings, superintendent of the Taunton, Mass., Water Works (1887) says*:—

"The following notes of actual work are offered, not in any sense as instances of model performance, but as simple illustrations: Time July 6th 1887; gang 60 men, 16 inch pipe, 2 yarners, 2 calkers, 4 to 10 men digging bell holes, 30 bell holes per day, 400 feet of pipe laid and jointed in 10 hours."

These notes are somewhat incomplete in that they do not disclose the following items:—(1) nature of earth excavated; (2) depth of trench; (3) width of trench; (4) what part of the total 400 feet length of trench and bell holes made on July 6th. (5) Was the trench back-filled for 400 feet on July 6th. (6) How many of the 60 men were in the derrick gang. (7) Did the derrick gang assist in excavating a part of the 400 feet of trench before beginning to lay pipe, or was a part of the trench and bell holes made on the day before. Mr. Billings statement shows that 4 to 10 men working 10 hours made 30 bell holes for 16 inch pipe. 30 bell holes would accommodate only 360 feet of 12 foot pipes. He states that 400 feet were laid. It is therefore evident that some part (at least 40 feet) of trench and bell holes must have been made on some other day.

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*Details of Water Works Construction, p. 55. (Published by "Engineering Record" N. Y.)

In another chapter of Mr. Billings work we find some "Notes on the construction of two miles of 16 inch water main," in 1887. The date shows that it is the same pipe above referred to. From these notes we gather the following facts: The pipe was hauled an average distance of 1½ miles over good roads for 64 cents per ton of 2240 lbs. The first division of the pipe line was 2,927 feet in length. The trenching was in good ground except a short stretch of quicksand and water The total cost of labor for this division of the line was 32.30 cents per lineal foot, including all labor charged on the time book from foreman to water boy, in a gang of 60 men. Another division of 2,100 feet length furnished sandy digging with some tendency to caving. A brook had to be crossed and a blow-off located which required the trench to be 10 or 12 feet deep for 100 feet length. An old 8 inch pipe had to be removed, and 18 services furnished with a temporary supply. The cost of labor per lineal foot for this division was 34.7 cents. In the next division the digging was dry and sandy, and caving of the trench was almost constant. An old 8 inch pipe had to be taken up, and a temporary supply maintained for 53 services. The cost of labor on this division was 41.8 cents per lineal foot.

In the next division the digging was wet and dirty. Old pipe had to be taken up, and temporary supply maintained for 30 services, and four connections made for a manufacturing company. The cost of labor in this division was 47.4 cents per lineal foot. The mill connections being the principal cause of the increased expense.

Mr. Billings states that a detachment of the same gang of men laid 2,000 feet of 8 inch pipe in new ground, good digging, at a cost of 17.3 cents per lineal foot for all labor, and 1060 feet of 4 inch pipe at a cost of 13.10 cents per foot, and 600 feet of 6 inch pipe at 15.38 cents per lineal foot.

While the depth and width of trench and daily wages paid are not stated, it will be near enough to assume that the trenches were 5 feet plus one half the diameter (outside) of the pipe to be laid, and the width of trench 2.333 to 3.00 feet, according to size of pipe (4″, 6″, 8″ and 16″ diameters). For amount of excavation in bell holes, refer to paragraph 110, ante.

Assume wages as follows: Foreman \$3, Calkers and yarners \$2.25, Derrick gang (6 to 10 men) \$1.75, laborers \$1.50 per day of 10 hours. A gang of six men is sufficient to handle the 4, 6, and 8 inch pipe, together with one yarner and one calker. For the 16 inch pipe it will require 2 yarners and 2 calkers and 10 men in the derrick gang. The remainder of the gang of 60 men will be laborers digging trench and bell-holes ahead of the derrick gang. Subtracting the number of calkers and yarners and derrick gang and the foreman from 60, the remainder shows the number of men engaged in trenching and digging bell-holes. The length of trench and bell-holes completed in 10 hours gives a basis of calculating the cubic yards excavated by each man per day, and the wages paid him per day furnishes the data for finding the cost per cubic yard of excavation. In fairly good digging it will be found that one man will make from 5.60 to 6.25 cubic yards of excavation per day of 10 hours. In excavating rock the average will be from, .50 to 1.50 cubic yards of excavation per man per day, depending on the nature of the rock. The excavation of deep, narrow trenches is very much more expensive per cubic yard than railroad and canal work in like earth or rock. See "Remark" under table No. 55, Section 107. With labor at \$2 per day the cost of excavation in rock was \$3.50 to \$5.50 per cubic yard, measured in place, in the City of Boston. This was an average of from .3636 to .57 cubic yard of excavation in rock per man per day. In very wet trenches the digging of sumps, sheet piling and bracing, and pumping out of water is a heavy expense in addition to the ordinary cost, and will amount to from 40 cents to \$1.00 per lineal foot.

By reference to table No. 56, it will be seen that the cost of laying 4″, 6″, 8″ and 16″ pipe as given above by Mr. Billings, is about the same as given in Weston's Table for "medium" digging, and also about the same as shown in table No. 53 for cost of laying pipe in Denver, in sandy loam. Referring to cost of trenching in Omaha (table No. 55) with wages of laborers at $1.75 per day, and we find that a trench, in good digging, 5.666 feet deep and 1.75 feet wide, cost .10$\frac{4}{10}$ cents per lineal foot. In a lineal foot of this trench there were $5.666 \times 1.75 \div 27 = .36724$ cubic yards of excavation, or $\frac{1}{.36724} = 2.723$ lineal feet of trench to the cubic yard of excavation. With the cost at .104 cents per lineal foot of trench, and 2,723 lineal feet to the cubic yard, the cost per cubic yard of excavation was $2.723 \times .104 = .284$ cents.

With wages at $1.75 per day of 10 hours, the average work done by one man in one day was $\frac{\$1.75}{.284} = 6.162$ cubic yards.

One man would therefore average $2.723 \times 6.162 = 16.779$ feet length of trench of those dimensions and in that kind of ground, per day. If wages were reduced to $1.50 per day, the cost of trench would be $\frac{1}{7}$ part less, or .24343 cents per cubic yard of excavation. In stiff clay or cemented gravel, one man will average about 4.50 cubic yards of excavation per day, and the cost at $1.50 per day wages, will be 33⅓ cents per cubic yard, or if wages are $1.75 per day, the cost will be .39 cents per cubic yard of excavation. Hence in stiff clay or cemented gravel the average progress per man per day would be 12.25 lineal feet of trench 5.666 feet deep by 1.75 feet wide. Trenches for larger diameters than 8 inches would be both deeper and wider, and the cubic yards of excavation per lineal foot would be increased in proportion.

One yarner and one calker will joint cast iron pipe about as follows. in an average days work: 720 feet of 4 inch pipe, or 660 feet of 6 inch, or 600 feet of 8 inch, or 540 feet of 10 inch, or 480 feet of 12 inch, or 360 feet of 14 inch, or 200 feet of 16 inch pipe.

The number of joints made will depend on whether the trench is wet or dry or stands up well or caves. The wages of calkers and yarners are usually from 50 to 75 cents per day more than the wages paid to ordinary laborers.

In estimating the cost of completing a cast iron pipe system add five per cent to cost of the pipe in order to allow for breakage. Also add the cost of engineering and inspection.

116—Lead Required Per Joint For Cast Iron Pipe.

The quantity of lead required per joint for cast iron pipe depends on the dimensions of the lead space between the bell and spigot, and also upon the manner in which the joint is yarned or packed. There is no uniform rule observed in the manufacture of cast iron pipe as to the dimensions of the lead space, and consequently no rule can be framed for determining the quantity of lead required per joint. The inner diameter of some bells is uniform while in others it converges. Inside of some bells there is a groove, semi-circular in form, extending around the inner circumference of the bell. Others are plain without grooves. Different foundries adopt different depths and slopes of the lead space, and some yarners will put twice as much yarn into a joint as others. Some engineers adopt the rule of estimating 2 lbs. of lead for each inch diameter, as being approximately the quantity required per joint. The result of this rule is too much for small diameters and not enough for large diameters. The amount of lead per joint used in recent practice is from one-third to one-half less than formerly, and the tendency is to reduce the quantity still more.

In laying 6", 8" and 16" pipe in Taunton, Mass., in 1887, Mr. Billings used 7.68 lbs. per joint for 6 inch pipe, 9.12 lbs., per joint for 8 inch and 21 lbs. per joint for 16 inch pipe.

Trautwine estimates the lead required per joint as follows:

Diam. in inches	2"	3"	4"	6"	8"	10"	12"	14"	16"	18"	20"	24"	30"
lbs. lead per joint	3.67	5.00	6.70	9.30	12.00	15.00	17.70	20.70	23.30	26.00	28.60	34.30	43.00

Mr. J. T. Fanning estimates the lead required per joint as follows:

Diam. in. inches	4"	6"	8"	10"	12"	14"	16"	18"	20"	24"	27"	30"	36"	48"
Lbs. Lead	4.25	6.25	8.25	10.25	13.00	15.00	24.25	27.25	30.75	38.25	51.25	56.25	79.50	111.00

For amount of lead required per joint for cast iron pipes made by the Colorado Fuel & Iron Co., see § 102, under Table No. 51.

The amount of lead per joint used in Denver, Colorado, was:

Diameter in Inches	4"	6"	8"	10"	12"	14"	16"	18"	24"	30"	36"	48"
Lbs. Lead	5.452	7.907	10.181	10.46	13.54	20.388	23.50	28.00	38.00	40.80	60.00	126.00

If bells and lead spaces were designed and specified by engineers with regard to flexibility of joint and economy in the quantity of lead and packing required, one third the lead now used would be saved and the joints would be equally strong and much more flexible and satisfactory. It is generally agreed that the calking tool does not affect or set up the lead to a greater depth than one inch, and the lead beyond this is worse than wasted, for it only stiffens the joint, which is really an element of weakness.

As the circumference of a pipe increases directly as the diameter, and as there is no sufficient reason for increasing the depth of bell as the diameter increases, there is no reason why the quantity of lead required per joint should not vary directly with the diameter if the bell and spigot were properly designed.

Three inches is an ample depth of bell for any diameter, and greater than necessary. Unequal settlement in the trench must be prevented by proper laying anyway.

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