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SHORT METHODS

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J. S. Sweet

SWEET'S

Hand Book

OF

SHORT METHODS

IN

Arithmetic

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BY

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PREFACE.

The principal object of this little work is to place in the hands of the student, in compact form, many of the briefer methods of rapid calculations. "Time is money," and especially so to many of our young people who are trying to obtain a business education in a brief time and with limited means.

Hoping that many may profit by the suggestions herein contained, I most respectfully dedicate this little volume to the young business people of America.

Santa Rosa, Calif., 1893. J. S. SWEET.

2. *Sums Greater than 9.*

5	4	3	2	1	5	4	3	2
5	6	7	8	9	6	7	8	9

6	5	4	3	6	5	4
6	7	8	9	7	8	9

7	6	5	7	6	8	7
7	8	9	8	9	8	9

8	9
9	9

3. *To Read at Sight.*

When a student sees the figures 1 and 3 written side by side, he instantly recognizes "thirteen" or "thirty-one" according to their positions. The same facility may be acquired in regard to numbers in addition; thus, 4 over or under 8, may be read "twelve" as readily as the figures 1 and 2 side by side. Ten minutes practice daily for one month will accomplish the work.

4. Always add TWO or MORE figures at a time. Never be guilty of adding single figures. Name the results of the following as rapidly as possible:

2	4	6	9	7	5	6	3	4	6	7	4	8	9
3	5	3	2	3	6	7	8	9	8	8	7	2	3

3	8	7	6	5	7	2	5	4	7	5	3	9	9
4	8	7	9	7	9	9	9	8	8	8	7	8	9

5	6	8	5	4	3	2	2	4	6	2	5	3	7
4	9	9	8	7	8	7	3	8	7	8	6	8	9
7	3	7	7	6	2	9	8	8	9	7	7	7	8
8	4	6	9	8	5	4	9	7	3	9	8	9	4

5. *Nine* added to any number is always ONE LESS in its unit's place than the number. Thus,

$8 + 9 = 7$ in its unit's place.
$36 + 9 = 5$ " " "

6. *Eight* added to any number is TWO LESS in its unit's place than the number. Thus,

$7 + 8 = 15, \qquad 15 + 8 = 23.$

7. *To Add by Tens.*

A good method is to add by 10's, carrying the EXCESS in the mind, as in the following:

8^{7}	7^{2}
9	5
6^{3}	9^{5}
7	6
30	27

Here the 3 of the 13 is carried to the 7 of the 17 making three tens in all. Add in this manner the following:

3	9	6	5	9
8	8	8	8	8
7	5	5	7	9
9	9	5	9	6
4	3	4	4	5
6	4	9	8	6

8. *When the Columns are Long.*

When there are two or more columns of considerable length, add each column separately as instructed, and write the sum of each alone, then combine results into one number, as follows:

```
 32476
 58976
 76892
 39428
 73548
 67943
 28745
------
    38
   37
  46
 43
33
------
378008
```

This method is almost indispensable in book-keeping, as an error can be located much more readily than when the separate results are not known.

9. *To Add Two Columns at a Time.*

To add *two* columns at a time practice on the following, by adding the tens' column first, and by reading the units' column, tell at a glance the number to carry:

23	35	66	38	59	88	39
36	44	27	44	71	64	89
72	88	38	86	94	75	85
49	26	79	49	87	89	94

10. Proofs of Addition.

In long columns the best proof is to add them again, up or down, the opposite of your first addition. In short columns and several of them to add, you may prove the work by casting out the 9's as shown below.

25189654 – 4
36972105 – 6
94375517 – 5
15155815 – 4
85310652 – 3
95315175 – 0
352318918 – 4

Casting out the 9's of the first nnmber, we have an *excess* of 4; of the second, 6; of the third, 5; and so on, finally casting out the 9's of these results which gives an *excess* of 4. Also by casting out the 9's of the *sum*, we have 4, we therefore conclude that the work is correct.

NOTE. This is not always a sure test, the answer might be wrong and yet prove by this method.

SUBTRACTION

11. When the forty-five combinations treated of in Addition are thoroughly memorized, the process of subtraction is a very simple one. This consists of being able to discern at a glance the digit which will combine with one of those given to produce the other. Thus,

$$\begin{array}{r} 8 \\ 3 \\ \hline \end{array}$$

are given, and the question is: what number combines with 3 to produce 8? The process is nearly the same as in adding, the only difference is that we must furnish one of the numbers to the combination, the result already being known.

Read the differences as rapidly as possible:

9	8	7	6	7	8	9	6	7	5	7	9	8	8
4	3	2	3	4	4	5	4	3	2	6	3	2	5

15	16	17	14	13	12	18
8	9	8	6	7	8	9

Daily drills in both addition and subtraction should not be neglected. The process of this method is very simple and is readily learned. Practice, only, will perfect it and give value to it.

MULTIPLICATION

12. With Multiplication we begin our *Short Methods*, supposing the student to be sufficiently advanced to know the multiplication table to the 12's. If not, he should learn the following

MULTIPLICATION TABLE:

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	12	84	96	108	120	132	144

13. The following *squares* of numbers should also be memorized :

$13 \times 13 = 169$ $19 \times 19 = 361$
$14 \times 14 = 196$ $20 \times 20 = 400$
$15 \times 15 = 225$ $21 \times 21 = 441$
$16 \times 16 = 256$ $22 \times 22 = 484$
$17 \times 17 = 289$ $23 \times 23 = 529$
$18 \times 18 = 324$ $24 \times 24 = 576$
$25 \times 25 = 625$

14. To multiply any number consisting of two digits by 11.

RULE. Write the sum of the digits between them, the number thus expressed is the product.

EXAMPLES.— 11 times 24 = 264,
11 " 36 = 396,
11 " 57 = 627.

NOTE. When their sum is 10 or more, carry one to the hundred's digit.

EXERCISES.

15. *1.* Multiply 45 by 11. *4.* Multiply 75 by 11.
2. 38 by 11. *5.* 96 by 11.
3. 92 by 11. *6.* 88 by 11.

16. To multiply any number by 11.

RULE. Write the unit's figure; next, write the sum of the units and tens, then the sum of the tens and hundreds, etc., writing the left hand figure last, carrying when necessary.

EXAMPLE.— 11 times 12345 = 135795.

$4 + 5 =$ 9 5
$3 + 4 =$ 7
$2 + 3 =$ 5
$1 + 2 =$ 3
1

EXERCISES.

17. *1.* Multiply 663 by 11. *4.* 6731 by 11.
2. 938 by 11. *5.* 9884 by 11.
3. 734 by 11. *6.* 72596 by 11.

18. *To multiply by 22, 33, etc.*

RULE. Multiply by 11 as above, and then by 2, 3, or 4, etc.

EXAMPLE.— 22 times 234 = 2574 × 2 = 5148.

NOTE. The work should be done mentally, only results being written.

EXERCISES.

19. *1.* Multiply 64 by 22. *4.* 374 by 55.
2. 65 by 33. *5.* 874 by 66.
3. 46 by 44. *6.* 336 by 77.

20. *To multiply by any number between 12 and 20.*

RULE. Multiply by the unit's figure only, writing the result under the number and one place to the right, then add.

EXAMPLES.—13 times 24 = 24
72
312 *Ans.*

14 times 175 = 175
700
2450 *Ans.*

EXERCISES.

21. *1.* Multiply 262 by 13. *5.* 9624 by 17.
2. 382 by 14. *6.* 32694 by 18.
3. 497 by 15. *7.* 27314 by 19.
4. 1824 by 16. *8.* 98794 by 12.

22. To multiply by 21, 31, 41, 51, etc.

RULE. Multiply by the tens only, writing the result under the number and one place to the left, then add.

EXAMPLE.—31 times 24 = 24
72
744

EXERCISES.

23. 1. Multiply 35 by 31. 4. 728 by 51.
2. 46 by 41. 5. 3824 by 61.
3. 245 by 21. 6. 8452 by 71.

24. To multiply by 15.

RULE. Annex one cipher to the number and add its half.

EXAMPLES.— 15 times 28 = 280
½ of 280 = 140
420
15 times 35 = 350
175
525

EXERCISES.

25. 1. Multiply 44 by 15. 4. 248 by 15.
2. 87 by 15. 5. 7634 by 15.
3. 394 by 15. 6. 98768 by 15.

26. To multiply by 51.

RULE. Take one-half the number and write it two places to the left and add.

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EXAMPLES.— 51 times 72 = 72
½ of 72 = 36
4372

51 times 45 = 45
225
2295

EXERCISES.

27. *1.* Multiply 78 by 51. *4.* 1384 by 51.
2. 324 by 51. *5.* 4633 by 51.
3. 723 by 51. *6.* 78254 by 51.

28. To square a number whose unit figure is 5.

RULE. Multiply the tens' digit by one greater and annex 25.

EXAMPLE.— 25 times 25 = 625.
2 times 3 = 6, annex 25 = 625.

EXERCISES.

29. *1.* Multiply 35 by 35. *5.* 75 by 75.
2. 45 by 45. *6.* 85 by 85.
3. 55 by 55. *7.* 95 by 95.
4. 65 by 65. *8.* 105 by 105.

30. To find the product of two numbers whose units' digits are 5's.

RULE. To the product of the tens add one-half their sum and annex 25 if the sum be even; if odd, annex 75.

NOTE. Fractions of one-half are dropped.

EXAMPLES.— 25 times 45 = 1125.
½ of (2 + 4) + 2 × 4 = 11, annex 25 = 1125.
25 times 35 = 875.
½ of (2 + 3) + 2 × 3 = 8, annex 75 = 875.

NOTE. 2 plus 3 is odd.

EXERCISES.

31. *1.* Multiply 25 by 65. *4.* 45 by 35.
2. 25 by 85. *5.* 65 by 35.
3. 105 by 25. *6.* 75 by 65.

32. To find the product of two numbers whose tens' digits are identical and the sum of the units' digits is 10.

RULE. Multiply the tens' digit by one greater and annex the product of the units' digits.

EXAMPLE.— 43 times 47 = 2021.
4 × 5 and annex 7 × 3 = 2021.

EXERCISES.

33. *1.* Multiply 29 by 21. *5.* 38 by 32.
2. 28 by 22. *6.* 37 by 33.
3. 27 by 23. *7.* 49 by 41.
4. 39 by 31. *8.* 48 by 42.

34. To find the product of two numbers whose tens' digits are consecutive, and the sum of the units' digits is 10.

RULE. To the product of the less tens and one more than the greater, annex the complement of the square of the greater number's unit figure.

NOTE. Complement of a number is 100 less the number.

EXAMPLE — 87 times 73 = 6351.
7 × 9 = 63; complement of the square of 7 = 51; annex it to 63 = 6351.

EXERCISES.

35. 1. Multiply 47 by 33. 4. 94 by 86.
2. 56 by 44. 5. 89 by 71.
3. 64 by 56. 6. 84 by 76.

36. *To find the product of two numbers when their tens' digits are the same.*

RULE. Take the product of the units, next the product of the tens times the sum of the units, then the product of the tens, always carrying the tens, if any.

EXAMPLE.— 73 times 75 = 5475

$5 \times 3 = 15$ write 5, carry 1.
$8 \times 7 = 56$ carry 5.
$7 \times 7 = 49$
5475

EXERCISES.

37. 1. Multiply 74 by 72. 4. 97 by 94.
2. 85 by 83. 5. 88 by 89.
3. 67 by 65. 6. 79 by 78.

38. *To find the product of two numbers when the units' digits are identical.*

RULE. Take the product of the units' figures, the sum of the tens times the units, and the product of the tens, carrying when necessary.

EXAMPLE.— 44 times 74 = 3256.

EXERCISES.

39. 1. Multiply 46 by 56. 4. 73 by 63.
2. 54 by 34. 5. 87 by 47.
3. 43 by 53. 6. 98 by 28.

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40. To find the product of any two numbers consisting of two digits.

RULE. Take the product of the units, the sum of the products of each ten times the other unit, and the product of the tens, carrying if necessary.

EXAMPLE.— 47 times 36.

$6 \times 7 = 42$
$6 \times 4 + 3 \times 7 = 45$
$4 \times 3 = 12$
1692

EXERCISES.

41. *1.* Multiply 35 by 27. *4.* 68 by 34.
2. 47 by 34. *5.* 78 by 46.
3. 52 by 46. *6.* 39 by 35.

42. To find the product of numbers when one part of the multiplier is a factor of the other.

RULE. Multiply by the factor, then this product by the quotient of the factor into the other part, and add.

EXAMPLE.—

231
183
Multiply by 3 = 693
" this product by 6 = 4158
42273

423
126
Multiply by 6 = 2538
" this product by 2 = 5076
53298

EXERCISES.

43. *1.* Multiply 1247 by 255.
2. 792 by 279.
3. 3635 by 1089.

44. To multiply by the factors of a number.

RULE. Multiply by one factor and this product by the other.

EXAMPLE.— 21 times 65 = 7 times 65 = 455
and 455 × 3 = 1365.

EXERCISES.

45. *1.* Multiply 73 by 42. *4.* 97 by 14.
2. 83 by 35. *5.* 87 by 36.
3. 123 by 27. *6.* 79 by 49.

46. To multiply by 10, 100, 1000, etc.

RULE. Annex as many ciphers as there are in the multiplier.

EXAMPLES.— 10 times 76 = 760.
100 times 125 = 12500.

47. To multiply by any multiple of 10, 100, 1000, etc.

RULE. Multiply by the digital number and then annex ciphers.

EXAMPLE.— 400 times 123 = 49200.
2000 times 243 = 486000.

48. To multiply by 9, or any number of 9's.

RULE. Annex as many ciphers as there are 9's and subtract the number multiplied.

EXAMPLES.— 9 times 435 = 4350 — 435 = 3915.
99 × 267 = 26700 — 267 = 26433.

EXERCISES.

49. 1. Multiply 47 by 9. 4. 148 by 9.
2. 125 by 9. 5. 725 by 99.
3. 238 by 9. 6. 675 by 999.

50. To multiply by any number ending in 9.

RULE. Multiply by the next greater number and from the product subtract the number multiplied.

EXAMPLE.—382 times 49 = 382 × 50 − 382.

382
50
19100
382
18718

EXERCISES.

51. 1. Multiply 128 by 69. 3. 326 by 599.
2. 245 by 59. 4. 262 by 499.

52. To multiply by any number a little less or a little greater than 100, 1000, etc.

RULE. Annex as many ciphers as there are figures in the multiplier and subtract or add the product of the difference between 100, 1000, etc., and the multiplier.

EXAMPLE.— 423 times 996 = 423000 − 4 × 423.

1692
421308

EXERCISES.

53. 1. Multiply 993 by 624. 5. 9994 by 425.
2. 997 by 529. 6. 9998 by 827.
3. 992 by 895. 7. 99993 by 963.
4. 326 by 104. 8. 1003 by 724.

54. To multiply by any multiple of 9, not exceeding 90.

RULE. Multiply by the multiple of ten next higher than the given multiplier, and subtract its one-tenth.

EXAMPLE.— 454 times 72

454		
80		
36320	product by	80
3632	" "	8
32688	" "	72

EXERCISES.

55. *1.* Multiply 46 by 18. *5.* 288 by 54.
2. 75 by 27. *6.* 384 by 63.
3. 82 by 36. *7.* 772 by 75.
4. 144 by 45. *8.* 1244 by 81.

56. To multiply by complements.

RULE. From either number subtract the complement of the other, and annex the product of the complements.

NOTE. The product should have as many figures as are in both numbers; supply ciphers to make them the same.

EXAMPLES.— 94 comp. 6 999 comp. 1
97 comp. 3 999 comp. 1
9118 998001

EXERCISES.

57. *1.* Multiply 92 by 87. *4.* 996 by 995.
2. 94 by 75. *5.* 993 by 991.
3. 99 by 93. *6.* 998 by 895.

58. *To find the product of two numbers, each of which is a little over 100.*

RULE. From the sum of the numbers subtract 100 and annex the product of the excesses.

EXAMPLE.— 115 times 104 = 11960
115 + 104 — 100 = 119
To 119 annex 15 × 4 = 11960.

EXERCISES.

59. *1.* Multiply 114 by 105. *4.* 144 by 107.
2. 122 by 103. *5.* 160 by 106.
3. 135 by 102. *6.* 138 by 108.

NOTE. Apply the same principle to the following:

1. Multiply 1008 by 1007. *3.* 1250 by 1003.
2. 1125 by 1004. *4.* 1475 by 1002.

60. *To find the product of two numbers one of which is more and the other less than 100.*

RULE. From the sum of the numbers subtract 100, annex two ciphers and subtract the product of the excess and complement.

EXAMPLE.— 108 8 excess.
98 2 complement
10600
16
10584

EXERCISES.

61. *1.* Multiply 102 by 94. *4.* 125 by 92.
2. 103 by 97. *5.* 112 by 99.
3. 115 by 96. *6.* 116 by 95.

NOTE. Apply the same principle to the following:

1. Multiply 1004 by 92. *3.* 1015 by 92.
2. 1008 by 95. *4.* 1025 by 96.

ALIQUOT PARTS.

TABLE.

½ of 100 = 50		⅛ of 100 = 12½	
⅓ " = 33⅓		1/9 " = 11⅑	
¼ " = 25		1/10 " = 10	
⅕ " = 20		1/11 " = 9 1/11	
⅙ " = 16⅔		1/12 " = 8⅓	
1/7 " = 14 2/7		1/16 " = 6¼	
⅜ of 100 = 37½		5/16 of 100 = 31¼	
⅝ " = 62½		7/16 " = 43¾	
⅞ " = 87½		9/16 " = 56¼	
⅔ " = 66⅔		11/16 " = 68¾	
⅚ " = 83⅓		13/16 " = 81¼	
3/16 " = 18¾		15/16 " = 93¾	

62. To multiply by an aliquot part of 100.

RULE. Annex two ciphers, divide by the denominator and multiply by the numerator of the fractional part it is of 100.

EXAMPLES.— 50 times 12 = 7200 ÷ 2 = 3600.

16⅔ times 84 = 8400 ÷ 6 = 1400.

EXERCISES.

63. *1.* Multiply 48 by 25. *5.* 184 by 12½.
2. 33⅓ by 24. *6.* 960 by 8⅓.
3. 35 by 20. *7.* 3603 by 11 1/9.
4. 63 by 14 2/7. *8.* 2560 by 6¼.

1. Multiply 72 by 37½. *4.* 423 by 66⅔.
2. 56 by 12½. *5.* 144 by 83⅓.
3. 96 by 87½. *6.* 216 by 18¾.

64. To multiply by 10 times an aliquot part of 100.

RULE. Annex three ciphers and proceed as before.

Ex.— 166⅔ times 84 = 84000 ÷ 6 = 14000.
83⅓ times 144 = 144000 ÷ 12 = 12000.

EXERCISES.

65. *1.* Multiply 125 by 48. *3.* 112 by 62½.
2. 1236 by 333⅓. *4.* 192 by 83⅓.

66. To multiply by a little more or a little less than an aliquot part.

RULE. Multiply by the nearest aliquot part, as above, and add or subtract the difference times the number.

EXAMPLE.— 13½ times 64 = 864 or

12½ times 64 =	6400 ÷ 8 =	800
1 times 64 =		64
13½		864

EXERCISES.

67. *1.* Multiply 72 by 14½. *4.* 78 by 18⅔.
2. 84 by 15⅔. *5.* 123 by 34⅓.
3. 54 by 17⅔. *6.* 144 by 84⅓.

68. To multiply by 100 and an aliquot part of 100.

RULE. Annex two ciphers and add to the number the portion of it indicated by the aliquot part.

EXAMPLES.— 125 times 128 = 12800 + ¼ of 12800 = 16000.

133⅓ times 36 = 3600 + 1200 = 4800.

EXERCISES.

69. *1.* Multiply 96 by 116⅔. *4.* 72 by 112½.
2. 120 by 137½. *5.* 84 by 114⅔.
3. 345 by 116⅔. *6.* 106¼ by 144.

This same principle may be carried to more than 100 and an aliquot; to 200, 300, and even to thousands. The student will find much in this field for original investigation.

DIVISION

70. *To divide by 5.*

RULE. Multiply by 2 and cut off one figure.

EXAMPLE.— 125 divided by 5 = 125 × 2 = 25.0.

EXERCISES.

71.

1.	Divide 135 by 5.	*4.* 265 by 5.
2.	145 by 5.	*5.* 325 by 5.
3.	175 by 5.	*6.* 875 by 5.

72. *To divide by 25.*

RULE. Multiply by 4 and cut off two figures.

EXAMPLE.— 125 divided by 25 = 125 × 4 = 5.00.

EXERCISES.

73.

1.	Divide 275 by 25.	*4.* 875 by 25.
2.	325 by 25.	*5.* 925 by 25.
3.	475 by 25.	*6.* 975 by 25.

74. *To divide by 125.*

RULE. Multiply by 8 and cut off three figures.

Ex.— 375 divided by 125 = 375 × 8 = 3.000.

EXERCISES.

75. *1.* Divide 500 by 125. *3.* 875 by 125.
2. 625 by 125. *4* 1125 by 125.

76. *To divide by an aliquot part of 100.*

RULE. Multiply by the denominator of the fraction expressing the aliquot part, divide by the numerator and cut off two figures.

EXAMPLES.— $240 \div 5 = 240 \times 20 = 48.00$.
$840 \div 25 = 840 \times 4 = 33.60$.
$1200 \div 12\frac{1}{2} = 1200 \times 8 = 96.00$.
$1350 \div 16\frac{2}{3} = 1350 \times 6 = 81.00$.

EXERCISES.

77. Divide 245 by 25. 820 by $8\frac{1}{3}$.
268 by 20. 725 by $83\frac{1}{3}$.
475 by $33\frac{1}{3}$ 446 by 125.

78. *To divide by 10, 100, 1000, etc.*

RULE. Cut off as many figures as there are ciphers in the divisor.

EXAMPLE.— 1240 divided by 100 = 12.40.

79. *To reduce the divisor to some number of tens, hundreds, thousands, etc.*

RULE. Multiply both divisor and dividend by some number that will make the divisor a multiple of tens, hundreds, thousands, etc., and divide as in short division.

EXAMPLE.— 15)2365
2 2
3.0)473.0
157 and 10 rem.

NOTE. Divide the remainder 20 by 2 to find the true remainder.

EXERCISES.

80. *1.* Divide 3845 by 35. *3.* 8732 by 75.
2. 6492 by 45. *4.* 6288 by 125.

DIVISIBILITY OF NUMBERS.

81. To tell when a number is divisible by 2, 3, 4, 5, 6, 8, 9, 10, etc.

82. All numbers are divisible by 2 when they end in 0, 2, 4, 6, or 8.

83. By 3 when the sum of their digits is divisible by 3.

84. By 4 when the *two* right hand figures express a number divisible by 4.

85. By 5 when they end in 0 or 5.

86. By 6 when divisible by 2 and 3.

87. By 8 when the *three* right hand figures express a number which is divisible by 8.

88. By 9 when the sum of their digits is divisible by 9.

89. By 10 when they end in 0.

90. By 7 or 11 if they consist of four figures, the first and fourth identical and the second and third ciphers.

91. By any composite number if divisible by all of its prime factors.

CANCELLATION.

92. Cancellation is a method of dividing by rejecting equal factors.

RULE. Cancel any or all factors common to both dividend and divisor. Divide tha product of those remaining in the dividend by the product of those remaining in the divisor.

EXAMPLES.— $42 \times 36 \div 24 \times 14 = ?$

Arrange the numbers as follows:

$$\frac{\cancel{6}^{3} \quad \cancel{3}}{\cancel{42} \times \cancel{36}} = \frac{9}{2} = 4\tfrac{1}{2}$$
$$\cancel{24} \times \cancel{14}$$
$$2 \quad \cancel{2}$$

EXERCISES.

93. *1.* Divide 84 times 72 by 36 times 21.

2. 144 times 216 by 56 times 128.

3. 512 times 1728 by 144 times 216.

FRACTIONS

94. To add fractions having a common denominator.

RULE. Add their numerators and write the result over the common denominator.

EXAMPLE.— $\frac{1}{7} + \frac{2}{7} + \frac{3}{7} = \frac{6}{7}$.

EXERCISES.

95. *1.* Add $\frac{2}{9} + \frac{5}{9} + \frac{7}{9}$. *2.* $\frac{5}{11} + \frac{8}{11} + \frac{9}{11}$.
3. $\frac{2}{15} + \frac{4}{15} + \frac{7}{15} + \frac{13}{15} + \frac{14}{15}$.

96. To add two fractions having a common numerator.

RULE. Multiply the sum of the denominators by the common numerator and write the result over the product of the denominators.

EX.— $\frac{1}{2} + \frac{1}{3} = (2 + 3) \times 1$ over $2 \times 3 = \frac{5}{6}$
$\frac{2}{3} + \frac{2}{5} = (3 + 5) \times 2$ over $15 = \frac{16}{15}$.

EXERCISES.

97. *1.* Add $\frac{3}{4} + \frac{3}{5}$. *4.* $\frac{5}{7} + \frac{5}{11}$.
2. $\frac{3}{5} + \frac{3}{7}$. *5.* $\frac{6}{7} + \frac{6}{11}$.
3. $\frac{4}{5} + \frac{4}{9}$. *6.* $\frac{10}{13} + \frac{10}{7}$.

98. *To add fractions not having a common numerator nor common denominator.*

RULE. Multiply each numerator into all the denominators except its own for new numerators, and take the product of all the denominators for a common denominator, then add.

EXAMPLES.— $\frac{2}{3} + \frac{3}{5} = \frac{10 + 9}{15} = 1\frac{9}{15}$.

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} = \frac{12 + 16 + 18}{24} = \frac{46}{24}.$$

EXERCISES.

99. *1.* Add $\frac{3}{5} + \frac{4}{7}$. *3.* $\frac{1}{2} + \frac{3}{4} + \frac{5}{6}$.
2. $\frac{4}{5} + \frac{6}{11}$. *4.* $\frac{2}{5} + \frac{3}{7} + \frac{8}{11}$.

NOTE. When several fractions whose denominators are not prime to each other are to be added, reduce them to their least common denominator and add.

100. *To add mixed numbers.*

RULE. Add whole numbers and fractions separately and then unite results.

EXAMPLE.— $8\frac{2}{3} + 12\frac{2}{5}$.

$8 + 12 = 20$
$\frac{2}{3} + \frac{2}{5} = \frac{16}{15} = 1\frac{1}{15}$
$21\frac{1}{15}$.

EXERCISES.

101. *1.* Add $9\frac{1}{2} + 14\frac{1}{3}$. *4.* $28\frac{3}{5} + 35\frac{4}{5}$.
2. $18\frac{2}{3} + 25\frac{2}{7}$. *5.* $43\frac{1}{5} + 72\frac{4}{7}$.
3. $21\frac{5}{6} + 27\frac{5}{7}$. *6.* $66\frac{2}{3} + 23\frac{1}{4} + 17\frac{4}{5}$.

102. *To subtract fractions having a common denominator.*

RULE. Take the difference of the numerators and write it over the common denominator.

EXAMPLE.— $\frac{5}{6}$ minus $\frac{1}{6} = \frac{4}{6} = \frac{2}{3}$.

EXERCISES.

103. *1.* Solve: $\frac{8}{9} - \frac{4}{9}$. *3.* $\frac{13}{15} - \frac{11}{15}$.
2. $\frac{10}{13} - \frac{5}{13}$. *4.* $\frac{42}{53} - \frac{27}{53}$.

104. To subtract fractions having a common numerator.

RULE. Multiply the difference of the denominators by the common numerator and write the result over the product of the denominators.

EXAMPLE.— $\frac{2}{5} - \frac{2}{7} = \frac{2 \times 2}{35} = \frac{4}{35}$

EXERCISES.

105. *1.* Solve: $\frac{3}{5} - \frac{3}{7}$. *4.* $\frac{8}{11} - \frac{8}{15}$.
2. $\frac{4}{5} - \frac{4}{9}$. *5.* $\frac{20}{21} - \frac{20}{31}$.
3. $\frac{5}{9} - \frac{5}{11}$. *6.* $\frac{45}{52} - \frac{45}{57}$.

106. To subtract fractions having neither common numerators nor common denominators.

RULE. Multiply each numerator into the other denominators, take the difference and write it over the product of the denominators.

EXAMPLE.— $\frac{4}{5} - \frac{3}{7} = \frac{28 - 15}{35} = \frac{13}{35}$.

EXERCISES.

107. *1.* Solve: $\frac{6}{7} - \frac{5}{8}$. *3.* $\frac{5}{6} - \frac{7}{11}$.
2. $\frac{8}{9} - \frac{10}{13}$. *4.* $\frac{9}{11} - \frac{7}{9}$.

108. To subtract mixed numbers.

RULE. Subtract whole numbers and fractions separately, uniting results.

NOTE. If the fraction of the subtrahend is greater than that of the minuend subtract a unit from the minuend and add it to the fraction before taking the difference.

EXAMPLE.— $8\frac{2}{3} - 5\frac{3}{5}$

$8 - 5 = 3$

$\frac{2}{3} - \frac{3}{5} = \frac{1}{15}$

$3\frac{1}{15}$.

$12\frac{1}{3} - 8\frac{1}{2}$

$11 - 8 = 3$

$1\frac{1}{3} - \frac{1}{2} = \frac{5}{6}$

$3\frac{5}{6}$.

EXERCISES.

109. *1.* Solve: $22\frac{4}{5} - 16\frac{2}{3}$. *3.* $89\frac{5}{8} - 35\frac{2}{3}$.
2. $75\frac{6}{7} - 48\frac{3}{4}$. *4.* $95\frac{1}{6} - 74\frac{3}{4}$.

NOTE. A good method is to take the complement of the difference of the fractions when the subtrahend fraction is the greater.

EXAMPLE.— $5\frac{1}{2} - 2\frac{2}{3}$

$4 - 2 = 2$

$\frac{2}{3} - \frac{1}{2} = \frac{1}{6}$ write the complement $\frac{5}{6}$

$2\frac{5}{6}$

EXERCISES.

110. *1.* Solve: $8\frac{1}{4} - 5\frac{1}{3}$. *3.* $25\frac{5}{6} - 17\frac{8}{9}$.
2. $15\frac{2}{3} - 4\frac{3}{4}$. *4.* $44\frac{2}{5} - 31\frac{3}{4}$.

111. To find the square of a mixed number whose fraction is ½.

RULE. Multiply the integer by the next higher number and annex ¼.

EXAMPLES.— $2\frac{1}{2} \times 2\frac{1}{2} = 2 \times 3 + \frac{1}{4} = 6\frac{1}{4}$.

$3\frac{1}{2} \times 3\frac{1}{2} = 3 \times 4 + \frac{1}{4} = 12\frac{1}{4}$.

EXERCISES.

112. *1.* Multiply $4\frac{1}{2}$ by $4\frac{1}{2}$. *3.* $8\frac{1}{2}$ by $8\frac{1}{2}$.
2. $5\frac{1}{2}$ by $5\frac{1}{2}$. *4.* $9\frac{1}{2}$ by $9\frac{1}{2}$.

113. To find the product of two mixed numbers whose fractions are $\frac{1}{2}$.

RULE. *To the product of the integers add $\frac{1}{2}$ their sum and annex $\frac{1}{4}$.*

Ex.— $2\frac{1}{2} \times 4\frac{1}{2} = 2 \times 4 + 3 \times \frac{1}{4} = 11\frac{1}{4}$.

$3\frac{1}{2} \times 4\frac{1}{2} = 3 \times 4 + 3\frac{1}{2} + \frac{1}{4} = 15\frac{3}{4}$.

NOTE. The fraction will be one-fourth if the sum of the two integers is even; if the sum is odd the fraction is three-fourths.

EXERCISES.

114. *1.* Multiply $2\frac{1}{2}$ by $6\frac{1}{2}$. *3.* $3\frac{1}{2}$ by $7\frac{1}{2}$.
2. $3\frac{1}{2}$ by $5\frac{1}{2}$. *4.* $4\frac{1}{2}$ by $5\frac{1}{2}$.

115. To find the product of two mixed numbers whose integers are identical and the sum of whose fractions is a unit.

RULE. *Multiply the integer by the next higher number and annex the product of the fractions.*

Ex.— $2\frac{1}{3} \times 2\frac{2}{3} = 2 \times 3 + \frac{1}{3} \times \frac{2}{3} = 6\frac{2}{9}$.

$3\frac{1}{4} \times 3\frac{3}{4} = 3 \times 4 + \frac{1}{4} \times \frac{3}{4} = 12\frac{3}{16}$.

EXERCISES.

116. *1.* Multiply $4\frac{2}{5}$ by $4\frac{3}{5}$. *4.* $9\frac{4}{7}$ by $9\frac{3}{7}$.
2. $5\frac{4}{5}$ by $5\frac{1}{5}$. *5.* $12\frac{5}{9}$ by $10\frac{4}{9}$.
3. $6\frac{3}{8}$ by $6\frac{5}{8}$. *6.* $15\frac{3}{11}$ by $15\frac{8}{11}$.

117. To find the product of two numbers whose integers are consecutive and the sum of whose fractions is a unit.

RULE. Multiply the greater number increased by 1, by the less; and for the fraction annex the complement of the square of the fraction of the greater number.

EXAMPLE.— $4\frac{1}{3} \times 3\frac{2}{3} = 5 \times 3 + \frac{8}{9} = 15\frac{8}{9}$.

NOTE. The square of one-third equals one-ninth, its complement is eight-ninths.

EXERCISES.

118. *1.* Multiply $5\frac{1}{4}$ by $4\frac{3}{4}$. *4.* $9\frac{4}{7}$ by $8\frac{3}{7}$.
2. $6\frac{3}{5}$ by $5\frac{2}{5}$. *5.* $12\frac{5}{9}$ by $11\frac{4}{9}$.
3. $8\frac{3}{7}$ by $7\frac{4}{7}$. *6.* $20\frac{5}{12}$ by $19\frac{7}{12}$.

119. To find the product of two mixed numbers whose integers are identical.

RULE. To the product of the integers add the product of the sum of the fractions times the common integer and the product of the fractions.

EX.— $6\frac{1}{2} \times 6\frac{1}{3} = 6 \times 6 + 6 \times \frac{5}{6} + \frac{1}{2} \times \frac{1}{3} = 36 + 5 + \frac{1}{6} = 41\frac{1}{6}$.

EXERCISES.

120. *1.* Multiply $8\frac{1}{2}$ by $8\frac{1}{4}$. *4.* $24\frac{3}{8}$ by $24\frac{7}{8}$.
2. $12\frac{1}{3}$ by $12\frac{5}{6}$. *5.* $35\frac{1}{5}$ by $35\frac{3}{5}$.
3. $14\frac{3}{7}$ by $14\frac{6}{7}$. *6.* $45\frac{5}{9}$ by $45\frac{7}{9}$.

121. To find the product of two mixed numbers when the fractions are identical.

RULE. To the product of the integers add the product of the sum of the integers times the common fraction and the product of the fractions.

EX.— $4\frac{1}{3} \times 8\frac{1}{3} = 4 \times 8 \times 12 \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} = 32 + 4 + \frac{1}{9} = 36\frac{1}{9}$.

EXERCISES.

122. *1.* Multiply $6\frac{1}{4}$ by $18\frac{1}{4}$. *3.* $36\frac{1}{8}$ by $44\frac{1}{8}$.
2. $9\frac{1}{3}$ by $15\frac{1}{3}$. *4.* $72\frac{1}{9}$ by $36\frac{1}{9}$.

123. To multiply by an aliquot part of 100.

RULE. Annex two ciphers to the multiplicand and take such a part of it as the multiplier is a part of 100.

EXAMPLE.— $24 \times 16\frac{2}{3} = 2400 \div 6 = 400$.

EXERCISES.

124. *1.* Multiply 39 by $33\frac{1}{3}$. *4.* 54 by $66\frac{2}{3}$.
2. 48 by $12\frac{1}{2}$. *5.* 72 by $37\frac{1}{2}$.
3. 64 by $8\frac{1}{3}$. *6.* 144 by $83\frac{1}{3}$.

125. To multiply a fraction by a fraction.

RULE. Cancel all common factors in numerators and denominators and divide the product of those remaining in the numerator by the product of those in the denominator.

EXAMPLE.— $\frac{3}{4} \times \frac{5}{26} \times \frac{4}{7} \times \frac{3}{210} = \frac{3}{28}$

EXERCISES.

126. *1.* Multiply $\frac{5}{6}$ by $\frac{3}{4}$ by $\frac{6}{35}$.
2. $\frac{8}{9}$ by $21\frac{1}{25}$ by $27\frac{7}{32}$.

127. To divide a fraction by a fraction.

RULE. Invert the divisor and proceed as in multiplication of fractions.

EXAMPLE.— $\frac{3}{4} \times \frac{4}{5} \div \frac{7}{10} \times \frac{9}{16} =$

$$\frac{3}{4} \times \frac{4}{5} \times \frac{\overset{2}{10}}{7} \times \frac{16}{9_3} = \frac{32}{21} = 1\frac{11}{21}.$$

EXERCISES.

128. *1.* Solve: $\frac{5}{6} \times \frac{7}{10} \times \frac{8}{9} \div 21\frac{1}{24} \times \frac{15}{28}$.
2. $\frac{6}{7} \times 11\frac{1}{12} \div \frac{22}{49} \times \frac{3}{4} \times \frac{2}{3}$.

PERCENTAGE

119. To find the percentage when the rate is an aliquot part of 100.

RULE. Take such a part of the number as the rate is a part of 100.

EXAMPLE.— $12\frac{1}{2}$ per cent of 64 = $\frac{1}{8}$ of 64 = 8.

EXERCISES.

130. *1.* Find 50 per cent. of 38. Of 346.
2. $33\frac{1}{3}$ " " 42. Of 543.
3. $16\frac{2}{3}$ " " 96. Of 186.
4. $12\frac{1}{2}$ " " 128. Of 4168.

131. To find the percentage when the rate is an aliquot part of 1000.

Multiply the number by 10, and take such a part of it as the rate is a part of 1000.

Ex.— $83\frac{1}{3}$ per cent of 144 = $\frac{1}{12}$ of 1440 = 120.

EXERCISES.

132. *1.* Find $333\frac{1}{3}$ per cent of 27. Of 279.
2. $166\frac{2}{3}$ " " 66. Of 576.
3. $83\frac{1}{3}$ " " 96. Of 3612.
4. $62\frac{1}{2}$ " " 288. Of 1624.

133. To find the percentage when the rate is any number.

RULE. Multiply the base by the rate and point off two places.

Ex.— 12 per cent of \$400 = 400 × .12 = \$48.00.

EXERCISES.

134. *1.* Find 15 per cent of 500. Of 1879.
2. 22 " " 750. Of 4321.
3. 18 " " 560. Of 8765.
4. 27 " " 1340. Of 9876.

135. To find the base, the rate and percentage being given.

RULE. Divide the percentage by the rate.

EXAMPLE.— Rate = 12 per cent, Percentage = 96.
96 ÷ .12 = 800 Base.

EXERCISES.

136. *1.* Rate 4 per cent, Percentage 52, Base = ?
2. " 9 " " 144 " = ?
3. " 12 " " 176 " = ?

137. To find the rate, the percentage and base being given.

RULE. Divide the percentage by the base.

EXAMPLE.— Base = 400, Percentage = 36.
36 ÷ 400 = .09, or 9 per cent.

EXERCISES.

138. *1.* Base 500, Percentage 35, Rate = ?
2. " 1200, " 72, " = ?
3. " 1800, " 144, " = ?

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139. To find the rate of loss or gain.

RULE. Divide the loss or gain by the cost.

EXAMPLE.— Cost = \$250, Selling price = \$300.
\$300 − \$250 = \$50, Gain,
\$50 ÷ \$250 = 20 per cent., rate of gain.

EXERCISES.

140. Find Rate of Gain or Loss:

1. Cost = \$400, Selling Price, \$500.
2. " = \$279, " \$540.
2. " = \$720, " \$600.

141. The following formulas are a very good illustration of the problems of percentage:

FORMULAS OF PERCENTAGE.

Base × Rate = Percentage.
Percentage ÷ Base = Rate.
Percentage ÷ Rate = Base.

Amount ÷ 1 + Rate = Base.
Difference ÷ 1 − Rate = Base.

By applying the formulas above to these applications, problems of Percentage are very readily solved.

APPLICATIONS OF PERCENTAGE.

Percentage	Base	Rate	Percentage	Amount	Difference
Profit&Loss	Cost	R. of P&L	Profit or L.	Sell'g P.	Selling Price
Trade Disc.	List Price	R. of Dis.	Discount		"
Commiss'n	Sale, Pur. or Coll.	Rate of Com.	Commis-sion	Entire Cost	Net Proceeds
Insurance	Face of P.	Rate of I.	Premium		
Taxes	Ass'd Val.	Rate of T	Tax		
Duties	First Cost	Rate of D	Duty		
Stocks	Par Value	Rate	Prem.or D.	Market Value	

In the above, will be noticed that every quantity considered has its counterpart in nearly all the applications in Percentage.

This table should be studied until the student is able to tell at once the elements of Percentage in any given problem.

INTEREST

CANCELLATION METHOD.

143. EXAMPLE.— Find the interest on $420 for 30 days, at 7 per cent.

$$\begin{array}{r|l} & 35 \\ 12 & \$42\cancel{0} \\ 36\cancel{0} & 3\cancel{0} \text{ days} \\ & .07 \\ \hline & \$2.45 \text{ interest.} \quad \textit{Ans.} \end{array}$$

EXAMPLE.— Find the interest on $540 for 7 months at 9 per cent.

$$\begin{array}{r|l} & \$5\cancel{4}\cancel{0}\ 45 \\ 12 & 7 \text{ months} \\ & .09 \\ \hline & \$28.35 \text{ interest.} \quad \textit{Ans.} \end{array}$$

RULE. Write the principal, rate and time as a dividend, and one year expressed in the same denomination as the time given as a divisor, cancel and reduce.

EXERCISES.

144. Find the interest :

1. Of $1200 for 42 days at 6 per cent.
2. Of $1800 for 33 days at 5 per cent.
3. Of $2250 for 60 days at 7 per cent.
4. Of $8400 for 5 mos. at 8 per cent.
5. Of $9600 for 9 mos. at 6 per cent.
6. Of $9636 for 1 year, 4 mos. at 7 per cent.

145. To find the interest when the time is expressed in months and days.

EXAMPLE.— What is the interest on $240 for 3 months, 12 days at 6 per cent.

3 months, 12 days = 102 days, or,
3 months, 12 days = 3.4 months.

3	$240	2	2	$240	120
360	102 days		12	3.4 months	
	.06	2		.06	
	$4.08 interest.			$4.08 interest.	

RULE. Proceed as in cancellation method, reducing the time to days, or to months and tenths of a month.

NOTE. When the number of days is a multiple of 3 it shortens the work by using months and tenths of a month.

ABBREVIATED METHOD.

146. The cancellation method may be somewhat shortened by omitting the rate and using instead of 360 as a divisor the quotient of the rate into 360. Thus:

When the rate is	2	per cent. use	180.
" "	3	" "	120.
" "	4	" "	90.
" "	5	" "	72.
" "	6	" "	60.
" "	8	" "	45.
" "	9	" "	40.
" "	10	" "	36.
" "	12	" "	30.
" "	18	" "	20.

EXAMPLE.— What is the interest on $720 for 33 days at 5 per cent?

$$\begin{array}{r|l} & 10 \\ 72 & \$720 \\ & 33 \\ \hline \end{array}$$

$3.30 interest.

EXAMPLE.— What is the interest on $1260 for 66 days at 8 per cent?

$$\begin{array}{r|l} 15 & 85 \\ 45 & \$1260 \\ & 66\ 22 \\ \hline \end{array}$$

$18.70 interest.

EXERCISES.

147. Find the interest:

1. Of $840 for 18 days at 6 per cent.
2. Of $960 for 27 days at 8 per cent.
3. Of $1240 for 36 days at 4 per cent.
4. Of $3260 for 63 days at 9 per cent.

BANKERS' METHOD.

148. EXAMPLE.— What is the interest on $1344 for 75 days at 6 per cent?

$13.44 = interest for 60 days.
3.36 = interest for 15 days.
$16.80 = interest for 75 days.

RULE. Point off two places, which will give the interest for the rate and corresponding time as follows:

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2	*per cent for*	180	*days.*
3	"	"	120	"
4	"	"	90	"
5	"	"	72	"
6	"	"	60	"
8	"	"	45	"
9	"	"	40	"
10	"	"	36	"
12	"	"	30	"
18	"	"	20	"

Then take such aliquot parts of this interest as are needed for the given time.

EXERCISES.

149. Find the interest:

1. Of \$810 for 90 days at 4 per cent.
2. Of \$648 for 45 days at 8 per cent.
3. Of \$1232 for 36 days at 10 per cent.
4. Of \$7200 for 37 days at 9 per cent.
5. Of \$963.75 for 80 days at 6 per cent.
6. Of \$2140.50 for 90 days at 8 per cent.
7. Of \$5235.60 for 66 days at 6 per cent.
8. Of \$4840.40 for 72 days at 10 per cent.

PROBLEMS IN INTEREST.

150. The following formulas are illustrative of the four problems of interest.

4. Principal × Rate × Time = Interest.
3. Interest ÷ Principal × Rate = Time.
2. Interest ÷ Principal × Time = Rate.
1. Interest ÷ Time × Rate = Principal.

151. Applications of Percentage involving the element of time are as follows: Interest, Discount, Partial Payments, Insurance, and Stock Investments.

152. To find the time when the principal, rate and interest is given.

EXAMPLE.— Principal = $900; Rate = 8 per cent.; Interest, $6.00; to find the Time.

5	
45	$900
360	(?)
$6.00	.08

30 days, the time.

EXAMPLE.— Principal $720; Rate 6 per cent.; Interest $25.20. Find the time.

12	60
25.20	$720
42	(?)
7	.06

7 months. *Ans.*

RULE. Use the cancellation method as in reckoning interest, using the product of the interest and one year expressed in the proper denomination as a dividend and the product of the principal and rate as a divisor.

153. To find the rate when the principal time and interest are given.

EXAMPLE.— Principal, $960; Time, 45 days; Interest, $8.40. Find the rate.

3	
360	$960 8
7 105	45 days 15
Interest $8.40	(?)

Ans. 7 per cent., Rate

EXAMPLE.— Principal, $1050; Time, 3 months; Interest, $21.00. Find the rate.

	$1050
4 12	3 months
2 21.00	[?]

Ans. 8 per cent., Rate.

RULE. Same as for 152, except that the product of the Principal and Time is used as a divisor.

154. To find the principal, the rate, time and interest being given.

6
360 [?]
75 60 days
Interest $5.25 .07
Ans. $450, Principal.

3
12 [?]
Interest $45.00 9 months
2.50 5.00 .08 2
Ans. $750, Principal.

RULE. Same as for 152, except that the product of the Time and Rate is used as a divisor.

155. To find the Bank Discount of any sum.

EXAMPLE.—Find the bank discount of $840 for 63 days discounted at bank at 10 per cent.

$840 70
63 21
360 10
3

Ans. $14.70 bank discount.

RULE. Find the simple interest for the given time and rate.

156. To find the True Discount of any sum.

EXAMPLE.—What is the True Discount and present worth of a debt of $530, due in one year, discounted at 6 per cent?

$530 ÷ 1.06 = $500 the present worth;
$530 − $500 = $30 the true discount.

RULE. Divide the amount of the debt by 1 plus the rate for the given time, this will give the present worth; subtract the present worth from the debt, the difference is the true discount.

ANALYSIS.

157. The ***first*** step in analysis is to *reduce to the unit* as follows:

If 4 hats cost $20, 1 hat will cost ¼ of $20, or $5.

The ***second*** step is to *reduce to a number:*
If 1 hat cost $5, 7 hats will cost $35.

The ***third*** step combines the *first* and *second:*
If 7 coats cost $84, 1 coat will cost $12; 4 coats will cost $48.

EXERCISES.

158. If 13 hats cost $39, what will 7 hats cost?

2. If 11 pairs of shoes cost $46.50, what will 7 pairs cost?

3. If ⅝ of a ton of hay cost $10, what will ⅞ of a ton cost?

159. Reduce the following first to the fractional unit, then to the integral unit, then to the required number of fractions.

EXAMPLE.—If $\frac{4}{5}$ of a ton of hay cost \$12, what will $\frac{7}{8}$ of a ton cost?

$\frac{4}{5}$ of a ton cost \$12,
$\frac{1}{5}$ of a ton will cost \$3,
$\frac{5}{5}$ or 1 ton will cost \$15,
$\frac{1}{8}$ will cost $\frac{1}{8}$ of 15 or $\frac{15}{8}$,
$\frac{7}{8}$ will cost 7 times $\frac{15}{8} = \frac{105}{8} = 13\frac{1}{8}$.

EXERCISES.

160. *1.* If $\frac{2}{3}$ of a bushel of wheat is worth 72 cents, what are 10 bushels worth?

2. If $\frac{9}{10}$ of an acre of land cost \$108, what will $\frac{5}{8}$ of an acre cost at the same rate?

3. If $\frac{2}{3}$ of $\frac{3}{4}$ of a cord of wood is worth \$3.50, what is $\frac{3}{4}$ of $\frac{4}{5}$ of a cord worth?

161. To find interest on overdrafts.

EXAMPLE.—Overdrafts for the week were as follows:

1.	1200	
2.	1500	
3.	1750	Interest at 10 per cent.
4.	1600	
5.	1600	
6.	1850	
	9500	$\div\ 360\frac{9}{10} = \$2.64.$

RULE. Divide the sum of the daily overdrafts by 360 divided by the rate, and point off two decimal places.

162. How to find errors shown by a trial balance.

1. See that your former *balance* of *balances* is in balance.

2. Be sure that your additions are correct.

3. Find the exact amount out of balance, and look for it and its one-half among the ledger items.

4. If the error is 9 or a multiple of 9, look for reversed figures.

EXAMPLE.—65 written 56 would make a difference of 9; 57 written 75 would make a difference of 2 times 9, or 18; 63 written 36 would make a difference of 27, etc. This may occur in any or all columns.

5. If there is an error of 1 in any column, look for errors in addition.

6. If the error is small, look for it in Interest or Discount.

7. Examine the Bills Receivable and Bills Payable accounts and note that the Debit and Credit entries are *exactly* alike as far as posted.

8. See if your cash account in the Ledger or Cash Book agrees with your Banking Ledger and cash on hand.

9. If the error is in cents column, it is not necessary to add the dollars column.

10. If the above tests will not indicate to you the errors, it will be necessary for you to re-check everything from the previous balance of balances. Do not go over the work without checking, you will waste your time if you do.

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