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SYLLABUS  
OF  
ELEMENTARY DYNAMICS

PART I.  
LINEAR DYNAMICS

WITH AN APPENDIX

ON  
THE MEANINGS OF THE SYMBOLS IN PHYSICAL EQUATIONS

*Prepared by the  
Association for the Improvement of Geometrical Teaching*



London  
MACMILLAN & CO.  
AND NEW YORK

1890

Price One Shilling

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## NOTICE.

THE following Syllabus of <sup>60707</sup>Linear Dynamics, originally prepared for the Association for the Improvement of Geometrical Teaching by its late President, R. B. Hayward, M.A., F.R.S., and subsequently submitted for criticism to the Members of the Association and revised by a Committee of the same, is now published in accordance with a resolution passed at the fifteenth General Meeting of the Association held in January, 1889. The Appendix originally prepared by Prof. A. Lodge, M.A., having been similarly discussed and revised, is published under the same authority.

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## SYLLABUS OF ELEMENTARY DYNAMICS.

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### I.

#### INTRODUCTORY.

[This introduction is intended, not for the beginner, but to explain the scope of the following syllabus and the principles on which it has been drawn up.]

THE Science of Dynamics treats of *Force*, *Matter* and *Motion*, their measures and mutual relations.

*Motion* when considered in itself, apart from the physical nature of the moving object and the forces which produce the motion, forms the subject of the preliminary Science of *Kinematics*.

Dynamics naturally divides itself into different branches, according to the greater or less simplicity of the masses or systems of matter, on and between which the forces are supposed to act.

1. The simplest case is that of a single *particle*, or mass of matter of insensible dimensions, and therefore such that it may be treated as a geometrical point.

2. Next in simplicity is the case of a (so-called) *Rigid* body, or a mass whose position (like that of a definite geometrical figure) is determined when the positions of three of its points (not in the same straight line) are determined.

3. Then follow cases of greater complexity, according to the supposed nature of the forces connecting the parts of the system. — Flexible Strings, Elastic Strings, Fluid Masses, Elastic Solids, &c.

Each branch of Dynamics is conveniently subdivided into two parts, *Statics* and *Kinetics*.

In *Statics* are considered all those relations of the forces acting on the system, which are independent of the element of *time*. It includes, therefore, the case of the system remaining at rest or moving with steady (i. e. unchanging) motion, in which cases the forces are said to be in *equilibrium*.

In Kinetics are considered the relations between the forces and the changes of motion produced by them in the system on which they act.

Hence Dynamics includes—

- (a) The Statics and Kinetics of a particle.
- (β) The Statics and Kinetics of a rigid body.
- (γ) The Statics and Kinetics of strings, of fluids (Hydrostatics and Hydrokinetics), of elastic solids, of viscous masses, &c., &c.

The fundamental principles of Dynamics are clear and simple. Their application to the various forms of aggregation of matter, solid, fluid, viscous, &c., requires all the resources of Mathematical Science.

*Elementary* Dynamics aims at giving a knowledge of the principles themselves, and their applications to such simple cases, as can be treated with the aid of elementary mathematical processes.

Elementary Dynamics is conveniently divided into the following branches:—

#### 1. *Linear* Dynamics.

In which the forces and motions considered are limited to *given straight lines*. The moving object is a particle, or some point of a body or system which defines its motion of translation. Hence Arithmetic and a little Elementary Algebra is all the *necessary* mathematical knowledge demanded, with the use of the signs + and — to distinguish *sense*.

#### 2. Uniplanar (or Plane) Dynamics.

In which the forces and motions considered are limited to directions parallel to a given plane, and so involving translations parallel to that plane and rotations about axes perpendicular to the plane. Hence Arithmetic, Elementary Algebra, Elementary Plane Geometry, and for the fuller developments, Plane Trigonometry and Conic Sections, are the requisite mathematical equipment for this branch.

#### 3. Solid (or three-dimensional) Dynamics (Rigid Dynamics).

In which the forces and motions considered may have any directions in space, but the masses on which the forces act are *rigid*, or such that their positions are defined when the positions of three points (not in the same straight line) are determined. In addition to the before-mentioned mathematical subjects, a knowledge of Geometry of three dimensions is required for this branch.

#### 4. Non-rigid Dynamics.

The questions in this division which can be treated without the aid

of higher mathematical methods are very few. They are chiefly some simple cases of the equilibrium of *Fluids*, and of Flexible strings and Elastic springs.

## II.

### LINEAR DYNAMICS.

[Clauses within square brackets may be omitted for the beginner.]

Special Introduction.

Subject of this Division. A single particle capable of moving along a straight (or other defined) line acted on by a force or forces in that line.

The nature and measures of motion of a point along a line must first be treated, apart from the nature of the particle and the forces which act on it. Hence the first chapter on *Linear Kinematics*.

### CHAPTER I.

#### LINEAR KINEMATICS.

1. Motion involves a combination of the conceptions of *time* and *space* or *distance*.

A point  $P$  is in motion with respect to a point  $O$ , if the distance  $OP$ , or its direction or both, change with the lapse of time. All motion relative. Absolute rest, even if conceivable, not realisable.

Motion of a point along a given line is its change of position with respect to a point supposed fixed in that line.

2. *Position*.

If  $O$  be a fixed point in a given line and  $P$  any other point in the same, the position of  $P$  is defined by the *distance*  $OP$  along the line and the *sense* (forwards or backwards) in which it is measured. The distance measured in terms of a unit of length, *foot*, *mile*, *metre*, &c., the sense by the sign  $+$  or  $-$  prefixed.

The distance measured in the sense from  $O$  to  $P$  will be denoted by  $\overline{OP}$ ; then that measured from  $P$  to  $O$  must be denoted by  $\overline{PO}$  and  $\overline{PO} = -\overline{OP}$  or  $\overline{OP} + \overline{PO} = 0$ .

If, when  $\overline{PQ}$  is moved along the line till  $P$  comes to  $R$ ,  $Q$  comes to  $S$ , then  $\overline{PQ} = \overline{RS}$ .

For all positions of  $P, Q, R$ ,  $\overline{PQ} + \overline{QR} = \overline{PR}$ .

(*Ex.* If  $\overline{PQ} = + 6$  feet,  $\overline{QR} = - 5$  feet,  $\overline{PR} = (+ 6 - 5)$  feet =  $+ 1$  foot, &c., &c.).

The signs  $+$  and  $-$ , as indicating *sense* are thus seen to obey the same laws as when they indicate addition and subtraction.

[If  $M$  be the middle point between  $P$  and  $Q$ ,  $\overline{PM} + \overline{QM} = 0$ , and  $\overline{OM} = \frac{\overline{OP} + \overline{OQ}}{2}$ .

So  $G$  is the *mean point* of  $n$  points  $P_1, P_2, \dots P_n$ , if

$$\overline{GP}_1 + \overline{GP}_2 + \dots + \overline{GP}_n = 0,$$

whence 
$$\overline{OG} = \frac{\overline{OP}_1 + \overline{OP}_2 + \dots + \overline{OP}_n}{n}.$$

If of these points  $p$  coincide in  $P$ ,  $q$  in  $Q$ ,  $r$  in  $R$ , &c.,

$$\overline{OG} = \frac{p \cdot \overline{OP} + q \cdot \overline{OQ} + r \cdot \overline{OR} + \dots}{p + q + r + \dots} . ]$$

### 3. *Units of Length.*

The unit-length is arbitrary.

The actual standards are the *yard* and the *metre*.

The units, used in this syllabus, generally the *foot* or third part of the yard denoted by **F**, or the *centimetre* ( $\frac{1}{100}$  of the metre) denoted by **C**.

When the unit is left undefined, it is denoted by **L**.

### 4. *Time.*

An *à priori* test of the equality of two different intervals of time perhaps impossible.

It is *assumed* that the time of a complete rotation of the earth about its axis is constant. A unit derived from this is the *mean solar day*, which is the mean of the intervals between two consecutive transits of the sun's centre across the meridian. The *second*, which is  $\frac{1}{86400}$  of the *mean solar day*, is the unit generally adopted in Dynamics. It is here denoted by **S**.

When the unit-time is left undefined, it is denoted by **T**.

### 5. *Velocity—Speed.*

The motion of a point at any instant is defined by its *velocity*, a magnitude involving the two elements *direction* and *rate of motion or speed*. In Linear Kinematics the *direction* is given, and the *speed* alone is the subject of investigation. The motion may be either forwards or backwards along the line, so the *speed* will have the sign  $+$  or  $-$  to express the *sense* of the motion. To explain how speed is estimated numerically, we must consider

6. *Uniform Motion.*

(1) Definition.

Speed proportional to the length traversed in a given time.

(2) The unit-speed not assumed arbitrarily, but with a definite relation to the unit-length and unit-time.

DEF.—The unit-speed is the speed of a point moving at the rate of the unit-length per unit-time.

If  $V$  denote the unit-speed, this may be expressed thus :

$V = L/T$ , the 'solidus' or mark / standing for the word 'per.'

We shall also denote  $L/T$  by  $\dot{L}$ .

The British unit-speed is  $F/S$  or  $\dot{F}$ : read, 'foot per second.'

The C. G. S. unit-speed is  $C/S$  or  $\dot{C}$ : read, 'centimetre per sec.'\*

(Examples. Change of 'miles per hour' to  $\dot{F}$  or  $\dot{C}$ , and the reverse, &c., &c.)

(3) [Change of units generally,

If  $L'$ ,  $T'$  be new units of length and time, such that  $L' = mL$  and  $T' = nT$ , then

$$L'/T' = \frac{m}{n} L/T, \text{ or } V' = \frac{m}{n} V.$$

Hence speed is of the *dimensions* 1 with respect to length and  $-1$  with respect to time.

Examples.]

(4) In uniform motion if, with the speed  $v$  feet per sec. (or  $\dot{L}$ ),  $l$  feet (or  $L$ ) are traversed in  $t$  seconds (or  $T$ ),

$$l = vt \text{ or } v = \frac{l}{t}.$$

(Numerical Examples.)

(5) *Relative motion of two points along the same line.*

If the points  $P$ ,  $Q$  have speeds  $u \dot{F}$ ,  $v \dot{F}$  respectively, the speed of  $Q$  relative to  $P$  is  $(v-u) \dot{F}$ .

Discussion of different cases, according to the signs of  $u$  and  $v$  and their relative magnitude.

(Numerical Examples.)

(6) [The speed of the mean point of any number of points is the mean of the speeds of those points.]

7. *Variable Motion.*

(1) *Mean speed.* If a point move along a given line from the point  $P$  to the point  $Q$  in  $t$  seconds, the uniform speed which would

\* The British Association Committee on Units suggest that a speed of 1 cm. per sec. should be called a *kine*.

carry it from  $P$  to  $Q$  in the same time is  $\frac{\overline{PQ}}{t}$  feet per sec. (or  $C/S$  or  $L/T$ ), and this is called the *mean speed* of the point during the time  $t$  seconds.

(2) If the motion is uniform, the ratio  $\frac{\overline{PQ}}{t}$  is constant, whatever be the length of the interval of time. If it is variable, this ratio, or the mean speed, changes with the length of the interval of time, but as  $t$  is made less and less, and consequently  $\overline{PQ}$  less and less, without limit, the mean speed approaches without limit to a finite value. This *limiting value of the mean speed* is regarded as the *actual speed* of the point, when it is at the point  $P$ .

(Various Illustrations.)

(3) (*Acceleration.*) *Quickening.*

When the speed of a point along a line is increasing or diminishing, it is said to be *quickened* (accelerated), and the rate at which the speed is changing, reckoned positive if increasing and negative if diminishing, is termed the *quickening* (acceleration) of the point.

If the speed  $u$   $F/S$  changes to  $v$   $F/S$  in the interval  $t$   $S$ , the *mean quickening* during the interval is  $\frac{v-u}{t}$   $F/S$  per second.

If the mean quickening is the same, whatever be the length of the interval  $t$  seconds; that is, if the mean rate of increase or decrease of speed is the same over whatever interval of time it is estimated, the point is said to be *uniformly quickened* (accelerated), or its quickening is constant or uniform. In other cases the *quickening* is *variable*.

Very few cases of variable quickening can be dealt with by elementary mathematical methods. The case of uniformly quickened (accelerated) motion alone is here treated.

[Note.—*Velocity* may change either by change of speed or of direction or of both. *Acceleration* is the rate at which velocity is added (or destroyed). If the velocity added is in a direction different from that of the existing velocity, there is a change of direction of velocity as well as (in general) of *speed*. For that part of the acceleration which produces change of speed only, the term *quickening* has been here introduced.]

(4) *Units of Quickening.* (Acceleration.)

*Quickening* (uniform) proportional to the speed added (or destroyed) in a given time.

*Unit-Quickening* assumed with reference to unit-speed and unit-time.

DEF.—The *Unit-Quickening* is that quickening which adds the unit-speed in the unit-time.

If  $A$  denote the *unit-quickening*, this may be expressed thus :

$A = V/T$  or  $A = (L/T)/T$ , which may be written  $L/TT$  ;

Or  $A = \dot{L}/T$ , which may be expressed as  $\ddot{L}$ .

The British unit-quickening is then  $F/SS$  or  $\ddot{F}$ , which may be read 'foot per second per second.'

The C. G. S. unit-quickening is  $C/SS$  or  $\ddot{C}$ , which may be read 'centimetre per second per second.'

(Numerical examples of changes as of 'yards per min. per min.' to  $F/SS$ , &c., &c.)

(5) [Change of units generally.

If  $L'$ ,  $T'$  be new units of length and time, such that  $L' = mL$  and  $T' = nT$ , then

$$L' T' T' = \frac{m}{n^2} \cdot L/TT \text{ or } A' = \frac{m}{n^2} A.$$

Hence *quickening* is of the dimension 1 with respect to length and  $-2$  with respect to time.

Examples.]

8. *Uniformly quickened (accelerated) motion.*

(1) If with the constant quickening  $a\ddot{F}$  (or  $a\ddot{C}$  or  $a\ddot{L}$ ) the speed changes in  $tS$  (or  $T$ ) from  $u\dot{F}$  (or  $\dot{C}$  or  $\dot{L}$ ) to  $v\dot{F}$  (or  $\dot{C}$  or  $\dot{L}$ ),

$$a = \frac{v - u}{t} \text{ or } v = u + at. \tag{A}$$

(2) The mean speed during any interval of time is the mean of the speeds at the beginning and end of that interval.

Or, if during the  $tS$  (as above)  $lF$  (or  $lC$  or  $lL$ ) are traversed, the

$$\text{mean speed} = \frac{l}{t} = \frac{u + v}{2}.$$

*Outline of Proof.* The speeds  $t'S$  after the beginning and before the end of the interval are respectively  $u + at'$  and  $v - at'$ , and the lengths described during an interval  $\tau S$  with these speeds being  $(u + at')\tau F$  and  $(v - at')\tau F$ , the sum of a pair of such lengths  $= (u + v)\tau F$ : hence, if there be  $n$  pairs of such intervals of  $\tau S$  in the whole interval of  $tS$ , so that  $2n\tau = t$ ,

$$\text{the total length in } tS = \frac{u + v}{2} \cdot tF.$$

This is true however large  $n$ , and however small (consequently)  $\tau$ , is taken, and hence up to and therefore at the limit

$$l = \frac{u + v}{2} \cdot t. \tag{B}$$

(3) Combining the formulae of (1) and (2),

$$l = ut + \frac{1}{2} at^2. \quad (\text{C})$$

Also  $l = \frac{u+v}{2} \cdot \frac{v-u}{a} = \frac{v^2 - u^2}{2a}$  or  $v^2 = u^2 + 2al$ . (D)

(4) If the point start from a state of rest,  $u = 0$ , and the formulae become

$$v = at, \quad l = \frac{1}{2} vt = \frac{1}{2} at^2, \quad v^2 = 2al.$$

(5) The lengths traversed in successive equal intervals of time are in Arithmetical Progression.

Starting from rest, the lengths are as the successive odd numbers 1, 3, 5, 7.

(Examples.)

### 9. *Falling Bodies.*

(1) A body (particle) left to itself near the earth's surface falls in a definite direction, known as the *vertical*, with uniformly quickening speed (except so far as it is affected by the resistance of the air, which acts differently on different bodies), and the quickening is the same for all bodies at the same place. This is proved by observation and experiment, and it is found that the quickening, which is called the quickening due to gravity and usually denoted by  $g$ , varies to a small extent from place to place according to the latitude, and to a lesser extent (for any accessible heights) according to the height above the sea-level. The value of  $g$  at sea-level varies between  $32.09 \ddot{\text{F}}$  at the equator and  $32.35 \ddot{\text{F}}$  at the poles, its value at Greenwich being  $32.19 \ddot{\text{F}}$ : or, expressed in C. G. S. units, between  $978.10 \ddot{\text{C}}$  and  $983.11 \ddot{\text{C}}$ , the value at Greenwich being  $981.17 \ddot{\text{C}}$ .

In the examples  $g$  may be taken as  $32 \ddot{\text{F}}$  or  $981 \ddot{\text{C}}$ .

(2) If length, speed, and quickening be all reckoned positive in the *upward* vertical direction, the formulae (A), (B), (C), (D) become adapted to falling (or rising) bodies by putting  $a = -g$ , so that

$$v = u - gt, \quad l = ut - \frac{1}{2} gt^2, \quad v^2 = u^2 - 2gl.$$

It follows that a body thrown vertically upwards with the speed  $u \ddot{\text{F}}$  rises for  $\frac{u}{g} \text{ S}$ , and attains the greatest height  $\frac{u^2}{2g} \text{ F}$ . Also that in the descent it has the same position and is moving with the same speed downwards at a given time after its greatest height as it had at the same time before reaching it.

(Examples and Exercises.)

[10. The mean point of any number of points moving in the same line with uniform quickening moves with uniform quickening equal to the mean of the quickening of the several points.]

## CHAPTER II.

## FORCE AND MASS.

1. Linear Kinematics has treated of the motion of a point without reference to the object moved or the forces producing motion.

Dynamics proper treats of the relations between Forces, the bodies on and between which they are exerted, and the motions they produce.

In *Linear* Dynamics the body is a particle, the motion along a straight (or other defined) line, the forces those which change the motion along the line.

2. The fundamental principles of Dynamics are best summed up in Newton's three Axioms, known as the 'Laws of Motion.' A statement of these Laws, followed by comments introducing the definitions and explanations requisite for their full comprehension, will be a convenient mode of expounding those principles.

In Newton's words:—

'Lex Prima. Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus illud a viribus impressis cogitur statum suum mutare.

'Lex Secunda. Mutationem motus proportionalem esse vi motrici impressae et fieri secundum lineam rectam qua vis illa imprimitur.

'Lex Tertia. Actioni contrariam semper et aequalem esse reactionem: sive corporum duorum actiones in se mutuo semper esse aequales et in partes contrarias dirigi.'

Translated:—

Law i. Every body persists in its state of rest or of uniform motion along a straight line, except in so far as it is compelled by impressed (i.e. external) forces to change that state.

Law ii. Change of motion is proportional to the impressed moving force and takes place along the straight line in which that force acts.

Law iii. To every action there is an equal and contrary reaction: or the mutual actions of two bodies on one another are always equal and directed in contrary senses.

3. These laws are too abstract to be directly deduced from or verified by experiment, though many familiar facts may be adduced giving a presumption of their truth. Their *proof* depends on consequences deduced from them, which can be compared with experiment.

4. In Law i. the *body* must be primarily a *particle* or mass of insensible dimensions, its motion being along a *line*.

(The results obtained for a particle may be extended to a finite body or system of particles, either if the motions of all the particles are the same or, when their motions are different, if the motion of a certain point, called the *Centre of Mass*, defining the motion of the system as a whole or its motion of *translation* is alone considered. That such a point can always be found, whether the body be a solid figure or a system of disconnected masses, as a flock, an army, a flight of birds, &c., is afterwards proved.)

5. Law i. implies that *motion*, like *rest*, is a *state* or *condition* of a body. Not *motion*, but *change* of motion has to be accounted for by force.

It asserts the *Inertia* of *Matter*, or that property by reason of which a material body can be set in motion or reduced to rest, or have its motion altered only by the action of Force—a fact with which we are familiar from the consciousness of *effort*, when we try to move a body or reduce it to rest. (Illustrations.)—This property is fundamental, and hence the following definitions:

*Matter* is that which possesses *Inertia*.

*Force* is a cause which changes a body's state of rest or motion.

6. Law i. also asserts that the forces which change a body's state of rest or motion are *impressed* forces, that is, forces exerted on the body *from without* or *external* forces: not forces between the parts of the body. (A man cannot jump without something to press against, &c., &c.)

The important distinction of *external* and *internal* forces made clear by Law iii.

Law iii. asserts that if *A* exerts a force on *B*, *B* exerts an equal force in the opposite sense on *A*. (Illustrations.) Such a pair of forces is termed a *Stress*.

The stresses between the different parts of a body or system are the *internal* forces of the system.

A force which acts on the body and which is one of the elements of a stress between it and another body is an *external* or *impressed* force on the first. (Illustrations.)

7. *Measure of Force.*

By Law ii. equal changes of motion in a given mass are due to equal forces.

Equal changes of motion in a given mass are equal changes of speed in the same time.

Hence,

DEF. *Equal Forces* are such as applied to the same mass produce equal quickenings (accelerations): i.e. generate equal speeds in the same time.

It follows that equal forces applied in exactly contrary senses to the same mass leave its state of rest or motion unchanged.

Hence the equivalent definition :

*Equal Forces* are such as applied to the same mass in opposite senses balance one another : i.e., leave its state of rest or motion unchanged.

Hence the measure of any force in terms of some Standard Unit of Force, by finding how many of such units, or definite parts of such unit, it will balance.

The most familiar force is *Weight*, or the force which urges a body downwards in the vertical direction at any place on the earth's surface, and the most obvious unit is *the weight of some definite body*. The objection to this is that the weight of the same body varies from place to place. This variation however is so small as to be of minor importance in many applications, and so for ordinary terrestrial purposes such a unit is commonly used. It is termed a *Gravitation Unit*.

The Standard Gravitation Units are the weight of a pound in the British System, and that of a gramme (or a cubic centimetre of pure water at max. density) in the Metrical System.

An absolute Standard Unit of Force, independent of locality and time, defined later.

#### 8. *Single Proposition of Linear Statics.*

A number of forces acting on the same particle in the same line are equivalent to a single force equal to their algebraical sum.

If that sum be zero, or if the sum of the forces in one sense is equal to that of the forces in the opposite sense, the forces balance one another or are in equilibrium.

#### 9. *Mass.*

Equal forces applied to different bodies produce different quickenings (accelerations). (Illustrations.)

Such bodies are said to *differ in Mass*. Hence,

DEF. *Equal Masses* are such as are equally quickened (accelerated) by equal forces.

From this, and taking the mass of a body as the sum of the masses of its parts, mass can be measured in terms of a standard unit of mass.

When the same quickening (acceleration) is produced in different masses by different forces, the forces are proportional to the masses.

(For if any number of equal particles close to one another are acted on by *equal* forces, they are equally quickened, and if they have no

relative motion at first, they will not alter their relative positions. If then  $m$  of them are supposed to coalesce into a single mass and  $n$  of them into another single mass, these two masses are in the ratio of  $m$  to  $n$ , and the forces acting on them also in the same ratio.)

Hence the mass of a body may be measured by comparing the force which produces in it a given quickening with the force that produces the same quickening in the standard mass, assumed as the unit.

10. *Weight proportional to Mass.*

Gravity, or the weight of a body, produces at a given place on the earth's surface the same quickening for all bodies: hence at the same place

The weights of bodies are proportional to their masses.

The *weight* of a body changes from place to place, and if the body were moved far enough from the earth, would become insensible, its *mass* however is at all places and under all circumstances the same.

(Illustrations.)

11. *Mass the measure of Inertia and of Quantity of Matter.*

The Inertia of a body is proportional to the force required to generate or destroy in it a given change of motion: it is therefore proportional to the mass. Also Inertia being the characteristic property of matter, the Quantity of Matter in a body is proportional to its Inertia and therefore to its mass.

12. *Relation between Force, Mass, and Quickening (Acceleration).*

Since for a given quickening,

The force is proportional to the mass moved:

and by Law ii. for a given mass,

The change of motion, that is, the quickening, is proportional to the force;

Therefore generally,

The force is proportional to the mass  $\times$  the quickening.

Or if  $f$  units of force produce in  $m$  units of mass  $a$  units of quickening, and  $f'$  units of force in  $m'$  units of mass  $a'$  units of quickening,

$$f : f' :: m a : m' a'.$$

By assuming the units, so that  $f' = 1$ ,  $m' = 1$ , and  $a' = 1$ , or that the unit of force produces in the unit of mass the unit of quickening, this becomes

$$f = m a.$$

13. *Units of Mass and Force.*

The relation between the units of Force, Mass, and Quickening implied in the equation

$$f = m a$$

is universally adopted.

The unit of Quickening has been taken as,

The foot per sec. per sec. ( $F/SS$  or  $\ddot{F}$ ) in the British system, or the centimetre per sec. per sec. ( $C/SS$  or  $\ddot{C}$ ) in the C. G. S. system, or generally  $L/TT$  or  $\ddot{L}$ , when  $L$  is the unit-length and  $T$  the unit-time.

Thus the unit-mass being assumed, the unit-force is defined, and the unit-force so determined is an *absolute* unit.

The unit-mass is,

In the British system the Imperial Pound, herein denoted by  $P$ , and in the C. G. S. the gramme, herein denoted by  $G$ .

Hence in the British system,

The unit-force is that force which, applied to a mass of one pound, produces the acceleration of one foot per sec. per sec. Its fitting signature, therefore, is  $P\ddot{F}$  or  $PF/SS$ .

In the C. G. S. system,

The unit-force is that force which, applied to a mass of one gramme produces the acceleration of one centimetre per sec. per sec. Its fitting signature, therefore, is  $G\ddot{C}$  or  $GC/SS$ .

If  $M$  denote any unit-mass, the signature of the unit-force generally is  $M\ddot{L}$  or  $ML/TT$ . Or, if  $\Delta$  ( $\Delta\nu\nu\nu\nu\nu\nu$ ) denote the unit-force in a system in which  $M$ ,  $L$ ,  $T$  denote respectively the units of mass, length, and time,

$$\Delta = M\ddot{L} \text{ or } ML/TT.$$

The unit of force in the C. G. S. system is called a *dynes*: that in the British system has been called a *poundal*.

If we denote the dyne by  $\Delta_G$  and the poundal by  $\Delta_P$ ,

$$\Delta_P = P\ddot{F} \text{ or } PF/SS, \quad \Delta_G = G\ddot{C} \text{ or } GC/SS.$$

#### 14. Comparison of absolute units with gravitation units.

If the force acting on a body is its weight equal to  $w$  absolute units of force, the acceleration produced is  $g$  units, and the equation  $f = ma$  becomes

$$w = mg.$$

In the British system  $g = 32.2 \ddot{F}$ , approximately, and for the mass of a lb.  $m = 1$ , therefore

the weight of a pound = 32.2 poundals,

$$\text{and 1 poundal} = \frac{1}{32.2} \times \text{weight of a pound};$$

$$= \text{weight of } \frac{1}{2} \text{ oz. nearly,}$$

so that to convert *poundals* to *pound-weights* we must divide by 32.2.

In the C. G. S. system  $g = 981 \text{ C}$ , approximately, and hence

the weight of a gramme = 981 dynes,

and 1 dyne =  $\frac{1}{981} \times$  weight of a gramme,

= 1.02  $\times$  weight of a milligramme,

so that to convert dynes to gramme-weights we must divide by 981.

(Numerous examples, chiefly numerical, of the forces required to produce given motions, motions produced by given forces, &c., &c., in a single mass.)

15. [Change of units generally.]

If  $M'$ ,  $L'$ ,  $T'$  are respectively units of mass, length, and time, such that  $M' = pM$ ,  $L' = qL$ ,  $T' = rT$ ,

$$\Delta' = M'L'/T'T' = \frac{pq}{r^2} ML/TT = \frac{pq}{r^2} \Delta.$$

Hence force is of the dimensions 1 with respect to mass and length, but of the dimensions  $-2$  with respect to time.

Examples.]

16. *Application of the foregoing principles to masses connected by a string.*

The string is supposed inextensible and of insensible mass, and to pass round smooth pegs, or pulleys of insensible mass which turn on smooth axles.

(1) A mass  $m' \mathbf{P}$  (or  $m' \mathbf{G}$ ) is dragged along a smooth table by a string which, passing over its edge, is attached to a mass  $m \mathbf{P}$  (or  $m \mathbf{G}$ ), hanging freely.

The mass  $m'$  has a quickening,  $a \ddot{\mathbf{F}}$  (or  $a \ddot{\mathbf{C}}$ ) produced by the tension of the string, and the mass  $m$  the same quickening produced by the excess of the weight  $mg$  poundals over the tension. Whence

$$a = \frac{m}{m + m'} g, \text{ and the tension} = \frac{mm'}{m + m'} g \text{ poundals.}$$

(Numerical examples.)

(2) Two masses,  $m \mathbf{P}$ ,  $m' \mathbf{P}$  hang freely by a string passing over a pulley.

The mass  $m \mathbf{P}$  falls and the mass  $m' \mathbf{P}$  rises with the same quickening  $a \ddot{\mathbf{F}}$ , the former owing to the excess of the weight  $mg$  poundals ( $\mathbf{P}\ddot{\mathbf{F}}$ ) over the tension, the latter owing to the excess of the tension over the weight  $m'g$  ( $\mathbf{P}\ddot{\mathbf{F}}$ ) whence

$$a = \frac{m - m'}{m + m'} g, \text{ and the tension} = \frac{2mm'}{m + m'} g \mathbf{P}\ddot{\mathbf{F}}.$$

(Numerical examples.)

(3) Both the foregoing cases may be treated as a single system of two masses, in which case the tension becomes an internal stress, and

may be neglected in determining the quickening. Thus, in the first case, the only external force (excluding those forces which are *forces of constraint* defining the path along which the particle moves) is the weight

$mg$ , and the mass moved is  $m + m'$ , therefore  $a = \frac{mg}{m + m'}$ .

### 17. Atwood's Machine.

This is a contrivance for realising experimentally the motion investigated in the second of the foregoing examples, by reducing the friction of the axle of the pulley and the mass of the pulley so that they may be neglected without much error.

Every student should make numerous experiments, comparing the motion observed by means of the vertical scale and the seconds' pendulum with that calculated from the foregoing formulae for the known masses.

Professor Willis's modification of Atwood's machine is to be recommended as cheaper and sufficiently accurate for purposes of illustration.

By taking masses such that  $\frac{m - m'}{m + m'}$  is a moderately small fraction, the acceleration due to gravity is so reduced as to become easily observed.

The machine is adapted to compare forces, masses, and the motions produced in many ways, the most important of which, as illustrating the foregoing principles, are

- (1) To show that the quickening is *uniform*.
- (2) To show that, the mass moved being the same, the quickening is proportional to the moving force.
- (3) To show that, the force being the same, the quickening is inversely proportional to the mass.
- (4) To show that when the quickening is the same for different loads, the force is proportional to the mass moved.
- (5) To obtain an approximation to the value of  $g$ .

### 18. Examples of stresses between the parts of a moving system.

(1) The pressure of a heavy mass on an ascending or descending platform.

a. Speed uniform. The pressure = the weight.

β. Acceleration uniform and =  $a$  downwards,

$$\text{pressure} = \left(1 - \frac{a}{g}\right) \times \text{weight.}$$

(2) Pressure of one weight on another attached to one end of the string in Atwood's machine, &c.



19. Miscellaneous examples involving the direct application of the foregoing principles.

## CHAPTER III.

### FORCE AND TIME.

#### *Momentum—Impulse—Direct Collision.*

1. Suppose a force,  $f$  poundals (or dynes), acting on a mass,  $m$  pounds (or  $\mathbf{G}$ ), to produce the quickening  $a\ddot{\mathbf{F}}$  (or  $a\ddot{\mathbf{C}}$ ), which in  $t\mathbf{S}$  changes its speed from  $u\dot{\mathbf{F}}$  to  $v\dot{\mathbf{F}}$ , then since

$$f = ma \quad (\text{ii. } \S 12)$$

$$\text{and} \quad v - u = at, \quad (\text{i. } \S 8)$$

$$mv - mu = ft, \text{ or } f = \frac{mv - mu}{t}.$$

2. DEF. The product of the mass of a particle and its speed at any instant is termed its *momentum*.

The above result may then be stated thus :

The change of momentum of a body acted on by a constant force is proportional to the force and to the time during which it acts.

Or thus :

When a body is acted on by a constant force, that force is equal to the rate of change of its momentum.

3. The unit of momentum is the momentum of the unit-mass moving with the unit-speed: that is, one pound moving with the speed  $1\dot{\mathbf{F}}$ , or 1 gramme with the speed  $1\dot{\mathbf{C}}$ .

Its signature then is generally  $\mathbf{ML}$ ,  $\mathbf{PF}$  for British units, and  $\mathbf{GC}$  for C.G.S. units.

No name has generally been assigned to this unit, but it will conduce to clearness to have such a name. It is proposed therefore to express it by adding the syllable -em to the name of the mass-unit.

Then  $\mathbf{PF}$  may be called a *poundem*,  $\mathbf{GC}$  a *grammem* \*.

[4. If the force be variable, suppose the whole interval  $t\mathbf{S}$  to be divided into intervals  $t_1, t_2, \dots, t_n \mathbf{S}$ , during which the forces acting uniformly are respectively  $f_1, f_2, \dots, f_n$  poundals, then the total change of momentum is  $f_1 t_1 + f_2 t_2 + \dots + f_n t_n$  poundems, which may be denoted by  $\Sigma(ft)$ , and is called the *time-integral* of the force. Hence the change of momentum during any time is equal to the *time-integral* of the force through that time.

\* The B.A. Committee on Units suggest the term *bole* for this C.G.S. unit.

5. If  $\bar{f}$  denote a force such that  $\bar{f}t = \Sigma(ft)$ ,  $\bar{f}$  is the *mean-force* acting during the time  $t$ , and the change of momentum during the time  $t$  being equal to  $\bar{f}t$ ,  $\bar{f}$  is also the mean rate of change of momentum during that time.]

6. *Impulse.*

If the force acting on a mass be very large, it will produce a finite change of momentum in a very short time. If the time is so short that there is no sensible change of position of the body during that time, the total change of momentum, which is equal to the time-integral of the force for that interval, is all that is required to be known to determine the motion.

DEF. The aggregate effect (or time-integral) of a force acting for any time, measured by the change of momentum it produces, is termed an *Impulse*. When the force is very great and the time of its action extremely small its impulse is called a Blow.

The unit-impulse is that impulse which generates or destroys a unit of momentum, and its signature is  $M\dot{L}$  or  $P\dot{F}$  or  $G\dot{C}$  in the different systems. An impulse may thus be expressed in poundems or grammems\*.

(Examples of impulses, and of the *mean force* of impulses on different suppositions as to the duration of the impulse, &c.)

7. *Impulsive actions between two masses which have relative motion.*

In all such cases when a stress arises between the masses, the action and reaction generate equal and opposite momenta, so that the total momentum of the system is unchanged. The effect of the stress is to transfer momentum from one part of the system to another.

(1) Masses connected by an inextensible string, which suddenly becomes tight.

If the masses  $mP$ ,  $m'P$  were moving with speeds  $u\dot{F}$ ,  $u'\dot{F}$ , they move on with a common speed  $v\dot{F}$ , such that

$$v = \frac{mu + m'u'}{m + m'},$$

and the impulse =  $\frac{mm'}{m + m'}(u - u')$  poundems ( $P\dot{F}$ ).

- (2) Impact between inelastic balls.
- (3) Impact between imperfectly elastic balls.
- (4) Limiting case of perfect elasticity.
- (5) Limiting case, where one of the masses is infinite. Reflexion from a fixed surface. (Examples.)

\* It has been suggested that the impulse of a poundal acting for a second should be called a poundal-second.

## CHAPTER IV.

## FORCE AND LENGTH.

*Work—Power—Energy.*

1. When the particle (or point of a body) to which a force is applied moves in the line in which the force acts, the force is said to do *Work*, or to have *Work done against it*, according as the motion is in the sense of the force or in the opposite sense. In the former case the work done is reckoned as *positive*, and in the other as *negative*, and the quantity of work done is greater as the distance through which the point moves is greater, and also as the force is greater.

A force which acts in the direction in which its point of application moves is called an *Effort*, and a force which acts in a direction opposite to that in which its point of application moves is called a *Resistance*.

(Illustrations.)

We may take therefore the equation,

Work done = Force  $\times$  the distance in the line of action of the force through which the point where it is applied moves, this distance being reckoned positive or negative as the motion is in the same or opposite sense to the force.

2. *Units of Work.*

A unit-work is the work done by the unit-force acting through the unit-length—or that of a poundal through a foot, called a *pounderg*, or of a dyne through a centimetre, called an *erg*—the respective signatures are therefore

$$W = \Delta L \equiv MLL/TT, \quad W_P = \Delta_P F \equiv PFF/SS, \\ W_G = \Delta_G C \equiv GCC/SS.$$

It is more usual, however, to measure work by the gravitation units, the *foot-pound* or the *kilogrammetre*, which are respectively the work done against gravity in the ascent (or that done by gravity in the descent) of one pound through one foot, or one kilogramme through one metre. Hence, the foot-pound = 32.2 poundergs, and the kilogrammetre =  $981 \times 10^5$  ergs.

(Examples.)

3. If the force be variable, the work done through a given length is the *line-integral* of the force through that length; and the mean force over that length is the line-integral divided by the length, or the mean rate of doing work per unit-length.

4. If the motion of the point at which a force acts is perpen-

dicular to the direction of the force, there is no motion in that direction, and therefore the force does no work.

(Illustrations.)

When the motion is in a direction inclined at an acute or obtuse angle to that of the force, the force does *some* work (positive or negative), but the consideration of this case is beyond the province of Linear Dynamics.

5. *Principle of Work in a Simple Machine.*

A simple machine may be defined as a contrivance by which a force applied at one part is made to overcome a resistance at another part, the other external forces acting on the system being merely forces of constraint which do no work, positive or negative, and the internal stresses being also such as (under the conditions of the working of the machine) do no work.

For such a machine *working steadily*, that is at a uniform speed, it is a *general Principle* that the work done by the effort applied is equal and opposite to that done by the resistance.

(NOTE.—This principle is stated by Newton in a more comprehensive form in the Scholium with which he closes his comments on the Laws of Motion. He regards it as an exemplification of the Third Law, the work done by the force being the *Action* and that by the resistance as the *Reaction*. As it is important, however, to prevent the confusion which would arise from applying the same term to a Force and to Work done by a force, it is perhaps best to regard the Third Law as relating to Forces only, as is done in this syllabus, and to enunciate the Principle of Work as a distinct principle.

In its most general form this principle contains in itself the whole of Dynamics, and Lagrange in the 'Mecanique Analytique' has deduced all his results therefrom.)

6. Since a small force working through a considerable distance may do the same amount of work as a much larger force through a less distance, a machine may, in accordance with the principle of work, be made by means of a small force to overcome a great resistance. The ratio of the resistance to the effort, when the machine is working steadily (or remains at rest), is termed the *Force-ratio* of the Machine, and the ratio of the speed of the point where the resistance acts, in the direction of the resistance, to that of the point where the effort acts, in the direction of the effort, is termed the *Velocity-ratio*.

7. *Force-ratio of some simple machines.*

(1) Wheel and Axle.

By the construction of the machine, while the force applied by a string round the wheel, works through a length equal to the circumference of the wheel, the weight suspended by the string round the axle is raised through a height equal to the circumference of the axle, therefore by the principle of work

the force  $\times$  circum. of wheel = resistance  $\times$  circum. of axle,

$$\begin{aligned} \text{whence the force-ratio} &= \frac{\text{circum. of wheel}}{\text{circum. of axle}} \\ &= \frac{\text{rad. of wheel}}{\text{rad. of axle}}. \end{aligned}$$

If the radii of wheel and axle are equal, this ratio = 1, or the effort = the resistance, and the system is reduced to the case of a single fixed pulley.

(Examples.)

(2) Systems of Pulleys.

a. Single Moveable Pulley.

If the weight attached to the string is descending, that attached to the pulley ascends through half the distance: therefore the effort =  $\frac{1}{2} \times$  the resistance, or the force-ratio is 2.

$\beta$ . Other systems.

Examples.

(3) Inclined Plane.

If a force acting along the plane pulls up a weight resting on the plane at a constant speed, the distance along the plane : height through which the weight rises :: the length of the plane : its height ; whence the force : the weight :: the height of plane : its length.

Examples.

(4) Lever with suspended weights.

(5) Roberval's Balance.

8. It should be observed that all the above machines are ideal, and not capable of being realized in practice, as it is impossible to get rid entirely of friction and other resistances which absorb work, and so reduce the amount which is applicable to effect the work which the machine is designed to do. The ratio of the useful work done to the work applied is termed the *efficiency* of the machine. This is always a fraction less than 1, and as the fraction is nearer to 1, so is the machine nearer to the perfect machine, as we have assumed it above.

9. *Power*

Is the rate \* of doing work, measured by the work done per unit of time.

\* The *Power* of a machine may be regarded as its *maximum* rate of doing work ; when it is working at a less rate, the term *Activity* might be used instead of Power.

The absolute unit of power is therefore a unit of work (erg or pounderg) per unit of time (second). The gravitation unit is 1 foot-pound per sec.

Power, however, is more frequently measured by a conventional unit, called a *Horse-power*. A Horse-power is that power which does 33000 foot-pounds per min. or 550 foot-lbs. per sec. It is, therefore,  $550g \times$  the absolute unit, or

$$550g \Delta_P F/S \text{ or } 550g PFF/SSS.$$

(Examples.)

#### 10. *Energy*.

*Energy* is a general term for the *capability of doing work* which from any cause a mass, or different masses in their relation to one another, may possess.

Energy may exist in various forms. Like matter it may be transformed, but wherever energy in one form disappears, the same quantity of energy in another form invariably arises. This is the great principle of 'Persistence (or conservation) of Energy,' which asserts that 'Energy may be transformed, but cannot be originated or annihilated.'

To discuss all the forms of Energy, would be to treat of every branch of Physical Science. Here it must suffice to enumerate some of its principal forms, and then develop the notion in its purely Dynamical relations.

#### 11. Different forms of Energy.

##### (1) *Kinetic Energy*.

The power which a body in motion has of doing work in virtue of that motion, that is, the power which it has in virtue of its motion of overcoming a definite resistance through a definite space, is termed its *Kinetic Energy*. The kinetic energy of a particle depends on its mass and its speed in a manner which we shall presently demonstrate. The kinetic energy of a body is the sum of the kinetic energies of its particles.

Kinetic Energy is an essentially positive magnitude, of which there may be more or less in a given body or system, and of which we may conceive it to be totally deprived, but beyond this zero there is no negative.

(Illustrations.)

##### (2) *Potential Energy* or *Energy of Position* or *Static Energy*.

When there is a stress exerted between two masses, work is done when they approach or recede from one another, and their capability of doing work in virtue of that stress is increased or diminished accord-

ing as the work so done is negative or positive. Hence they have a power of doing work dependent on their relative position, and this is termed *Potential Energy* or *Energy of Position*.

When one of the masses is regarded as fixed, the potential energy is referred to the other mass.

(Illustrations—weights at different levels, springs, &c.)

*Kinetic and Potential Energy* are the purely dynamical forms of energy, and all purely dynamical problems resolve themselves into investigating their relations and the changes in a system from one form to the other. Probably all forms of energy may prove ultimately reducible to these two, or even to kinetic energy alone; but in the present state of science it is necessary to recognise other forms. Foremost among these is

(3) *Heat*.

Heat is, according to modern views, the energy of the insensible motions of the molecules of which matter is composed.

It has been proved experimentally by Joule that when dynamical energy is employed solely in generating heat, a definite quantity ( $H$ ) of heat is produced by a definite quantity ( $W$ ) of dynamical energy, and conversely, when heat is employed to produce dynamical energy, the quantity of such energy developed will be  $W$  when the quantity of heat which disappears is  $H$ .

(General illustrations.)

The numerical relation between heat and dynamical energy has been found by Joule to be this:

The quantity of heat, which raises the temperature of 1 lb. of water by  $1^{\circ}$  Fahrenheit, is equivalent, when transformed into mechanical energy, to about 772 foot-pounds or 24850 poundergs.

(Examples.)

(4) Other forms of energy are those due to chemical affinity, and to electric and magnetic attractions and repulsions.

12. *Kinetic Energy*.

Suppose a force of  $p$  poundals acting on a particle, whose mass is  $m$  pounds, through the distance  $l$  feet to change its speed from  $u$  feet per sec. to  $v$  feet per sec., then  $a$  feet per sec. per sec. being the acceleration due to the force,

$$p = ma,$$

and the work done by the force  $= pl = mal$  poundergs,

also from kinematics  $v^2 - u^2 = 2al$ ,

therefore the work done by the force  $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ . Hence  $\frac{1}{2}mv^2$  is the total work done by the mass  $m$  against a resistance which

reduces it to rest, and is therefore the measure of its kinetic energy when it is moving with the speed  $u \dot{\mathbf{F}}$ .

The above equation may then be expressed thus :

The change in kinetic energy between two positions of a particle is equal to the work done by the force acting along its path between these positions.

Hence the rate of change of kinetic energy per unit of length, or the *line-rate* of change of kinetic energy is equal to the force.

[13. If the force is variable, and the whole interval  $l \mathbf{F}$  is divided into parts,  $l_1, l_2, \dots, l_n \mathbf{F}$ , through which the forces  $p_1, p_2, \dots, p_n$  poundals respectively act, the total change in kinetic energy is equal to  $p_1 l_1 + p_2 l_2 + \dots + p_n l_n$ , which is denoted by  $\Sigma (p l)$ , and is called the *line-integral* of the force. Hence the change of kinetic energy over any distance is equal to the *line-integral* of the force over that distance.

14. If  $\bar{p}$  denote a force such that  $\bar{p} l = \Sigma (p l)$ ,  $\bar{p}$  is the mean force acting through the distance  $l$ , and  $\bar{p}$  is equal to the mean *line-rate* of change of kinetic energy of the particle. It should be observed that this mean force is not in general the same as the *mean force during the time* of describing the interval, which (as we have seen) is equal to the mean time-rate of change of momentum. These, however, have the same limit, when the time and therefore the distance are diminished without limit, being then the force acting at the instant. They are also equal, when the force is constant.]

#### 15. Units.

Since kinetic energy = work done, the unit of kinetic energy is that kinetic energy which is equal to the unit of work, and is therefore that of 2 units of mass moving with the unit-speed: that of 2 lbs. with the speed of 1 foot per sec., or that of 2 grammes with the speed of 1 centimetre per sec.

16. *Illustrations of the principle of energy by the solution of simple problems.*

(1) Rising or falling body.

If  $u \dot{\mathbf{F}}$ ,  $u' \dot{\mathbf{F}}$  be the speed of the mass  $m \mathbf{P}$  at two different heights  $h \mathbf{F}$ ,  $h' \mathbf{F}$  measured vertically from the same point and positive upwards, the change of kinetic energy in passing from the height  $h$  to  $h'$  is  $\frac{1}{2} m u'^2 - \frac{1}{2} m u^2$ , and the work done is  $mg(h-h')$ , so that

$$\frac{1}{2} m u'^2 - \frac{1}{2} m u^2 = mg(h-h'),$$

or 
$$\frac{1}{2} m u'^2 + mgh' = \frac{1}{2} m u^2 + mgh.$$

The quantity  $mgh$  is the potential energy of the mass  $m$  due to its weight, relatively to the level whence the heights are measured, and the above equation expresses that in the motion the sum of the

kinetic and potential energies of the mass remains constant, which is the form in this case of the principle of the Persistence of Energy.

As the body rises, its kinetic energy diminishes and potential energy increases, while as it falls, the kinetic energy increases and the potential energy diminishes.

(2) Bodies rising and falling under constraint.

If besides the force of gravity no other force is acting but one of constraint, guiding the particle along a certain line from one level to another, this force doing no work, the same relations hold as in the last case.

(3) Atwood's Machine.

Suppose  $m$  to descend and  $m'$  to ascend through the distance  $h$ , then the decrease of potential energy =  $mgh - m'gh$ , and the kinetic energy acquired from the state of rest =  $\frac{1}{2}mv^2 + \frac{1}{2}m'v^2$ , therefore

$$\frac{1}{2}(m+m')v^2 = (m-m')gh,$$

or

$$v^2 = 2 \frac{m-m'}{m+m'} gh,$$

so that the acceleration is  $\frac{m-m'}{m+m'}g$ , as before shewn.

(In this, the kinetic energy communicated to the wheel, and the energy lost (or rather converted into heat) by friction are neglected.)

(4) Wheel and Axle loaded with any weights.

(Mass of Wheel and Axle neglected, and also Friction.)

If  $m$  descend through  $h$  and  $m'$  ascend through  $h'$ ,

$$\begin{aligned} \text{since } \frac{h}{h'} = \frac{r}{r'}, \text{ the decrease of potential E.} &= mgh - m'gh', \\ &= \frac{1}{2}(m - m' \frac{r'}{r}) gh, \end{aligned}$$

$$\begin{aligned} \text{and since } \frac{v}{v'} = \frac{r}{r'}, \text{ the increase of K. E.} &= \frac{1}{2}mv^2 + \frac{1}{2}m'v'^2, \\ &= \frac{1}{2}(m + m' \frac{r'^2}{r^2}) v^2, \end{aligned}$$

$$\text{therefore } \frac{1}{2}(m + m' \frac{r'^2}{r^2}) v^2 = (m - m' \frac{r'}{r}) gh,$$

or

$$v^2 = 2 \frac{(mr - m'r')r}{mr^2 + m'r'^2} gh.$$

If  $mr = m'r'$ , the masses  $m$  and  $m'$  balance as before.

(Examples.)

17. Relations between Momentum and Kinetic Energy.

If a mass  $m$   $\mathbf{P}$  moving with the speed  $v$   $\dot{\mathbf{F}}$  has the momentum  $I$  poundems and the kinetic energy  $E$  poundergs,

$$I = mv \text{ and } E = \frac{1}{2}mv^2,$$

whence  $E = \frac{1}{2}Iv = \frac{1}{2}\frac{I^2}{m}$  and  $I = \frac{2E}{v} = \sqrt{2mE}$ .

Hence, of two masses having the same momentum, that has the greater kinetic energy which has the greater speed and therefore the less mass.

Thus in firing a gun, by Law iii. the momentum of the ball\* forward is equal to that of the gun backward, but the kinetic energy of the ball is greater than that of the recoiling gun in the ratio in which the mass of the gun is greater than that of the ball.

Again, by communicating a given amount of momentum to a larger mass the kinetic energy is diminished. Instances.

Examples on all the foregoing results.

## CHAPTER V.

### SYSTEM OF TWO OR MORE MASSES.

#### *Centre of Mass.*

1. The *mean point* of a series of points in a line has been defined in Kinematics.

If at the points we conceive particles of equal mass placed, and then suppose  $m_1$  of the particles to coalesce at  $P_1$ ,  $m_2$  at  $P_2$  ..., then  $m_1$ ,  $m_2$ , ... will measure the masses at  $P_1$ ,  $P_2$  ..., and if  $G$  be the mean point, now to be called the *centre of mass*, and  $O$  a fixed point in the line,

$$\overline{OG} = \frac{m_1\overline{OP}_1 + m_2\overline{OP}_2 + \dots}{m_1 + m_2 + \dots}.$$

If  $O$  coincides with  $G$ ,  $\overline{OG}$  becomes 0, and we have

$$m_1\overline{GP}_1 + m_2\overline{GP}_2 + \dots = 0.$$

In the case of two masses  $m_1$ ,  $m_2$ , since  $m_1\overline{GP}_1 + m_2\overline{GP}_2 = 0$ ,  $\overline{GP}_1/\overline{GP}_2 = -m_2/m_1$  or  $G$  divides  $P_1P_2$  in the inverse ratio of the masses at  $P_1$ ,  $P_2$ .

\* The mass of the powder being neglected.

2. If at any instant  $m_1, m_2, \dots$  have speeds  $v_1, v_2, \dots$   $\vec{F}$  respectively, the centre of mass will have the speed  $\bar{v}$   $\vec{F}$ , such that

$$\bar{v} = \frac{m_1 v_1 + m_2 v_2 + \dots}{m_1 + m_2 + \dots}.$$

Thus  $\bar{v}$   $\vec{F}$  is the mean speed of the particles, and the momentum of the total mass of the system moving with the mean speed or that of the centre of mass is equal to the sum of the momenta of the separate particles or the total momentum of the system.

If  $v_1', v_2', \dots$   $\vec{F}$  be the speeds of  $m_1, m_2, \dots$  relatively to the centre of mass, so that  $v_1 = \bar{v} + v_1', v_2 = \bar{v} + v_2', \dots$ , it follows that

$$m_1 v_1' + m_2 v_2' + \dots = 0,$$

or the total momentum of the system relatively to the centre of mass is 0.

3. Since the internal stresses of a system can only transfer momentum from one part to another of the system without altering its total amount, the motion of the centre of mass of the system is unaffected by any collisions or mutual actions of the particles. Its motion may therefore be fitly taken as defining the motion of the system as a whole, or its motion of translation relatively to external objects.

4. The statements as to speed and momentum, made in § 2 above, are equally true as to quickening and external forces, and the centre of mass will move as if all the forces were applied to the total mass of the system collected at that point.

5. Kinetic Energy of the system.

The total kinetic energy =  $\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \dots$  (poundergs)

If, as in (2), we put  $\bar{v} + v_1'$  for  $v_1$ , &c.,

the total kinetic energy =  $\frac{1}{2} (m_1 + m_2 + \dots) \bar{v}^2 + \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 + \dots$ ,  
since  $m_1 v_1' + m_2 v_2' + \dots = 0$ .

Hence the total kinetic energy is equal to that of the whole mass moving with the speed of the centre of mass (or the kinetic energy of translation) *plus* that of the particles relatively to the centre of mass (or the kinetic energy of their motion relative to their centre of mass).

The kinetic energy of translation is unaffected by internal stresses, such as collisions, explosions, or mutual attractions or repulsions, and can be changed by external forces only. The kinetic energy of relative motion may be increased by the change of potential energy into kinetic, or decreased by the opposite change. It will also be diminished by its conversion into heat, when this escapes and is lost to the system. Such ultimate loss of kinetic energy to the system occurs in all cases

of collisions unless the particles are perfectly elastic, as we proceed to shew for two particles.

6. *Loss of kinetic energy in collisions.*

The kinetic energy of two masses  $m_1, m_2$  moving with speeds  $u_1, u_2$  respectively may be put into the form

$$\frac{1}{2} (m_1 + m_2) \bar{u}^2 + \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (u_1 - u_2)^2.$$

The first term is unchanged by collision, the second becomes, if  $v_1, v_2$  be the speeds after collision,

$$\frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (v_1 - v_2)^2,$$

but  $v_1 - v_2 = e(u_2 - u_1)$ , therefore the second part, or the relative kinetic energy, is diminished by

$$(1 - e^2) \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (u_1 - u_2)^2.$$

Hence the kinetic energy lost (or converted into heat) is the fraction  $1 - e^2$  of the original relative kinetic energy, which fraction varies from 0 for perfectly elastic to 1 for perfectly inelastic bodies.

(Examples.)

7. In a machine working steadily by which one weight raises another, the centre of mass of the two weights neither rises nor falls.

(Examples.)

## CHAPTER VI.

### MISCELLANEOUS EXAMPLES AND PROBLEMS.

# STANDARD ABSOLUTE UNITS—THEIR NAMES AND SIGNATURES.

BRITISH.		C. G. S. (Centimetre-gramme-second.)	
<i>Magnitude.</i>	<i>Name.</i>	<i>Signature.</i>	<i>Signature.</i>
Fundamental Units.	1. Length (Material Standard).	Foot . . . . . F	Centimetre . . . . . C
	2. Mass (Material Standard).	Pound . . . . . P	Gramme . . . . . G
	3. Time (Astronomical Observation).	Second . . . . . S	Second . . . . . S

F = 30.48 C  
P = 453.6 G  
C = .03281 F  
G = .002205 P.

Derived Units.	4. Area . . . . .	Square-foot . . . . . FF or F <sup>2</sup>	Square-centimetre . . . . . CC or C <sup>2</sup>	LL or L <sup>2</sup>
	5. Volume . . . . .	Cubic-foot . . . . . FFF or F <sup>3</sup>	Cubic-centimetre . . . . . CCC or C <sup>3</sup>	LLL or L <sup>3</sup>
	6. Speed . . . . .	Foot per second . . . . . F/S or $\dot{F}$	Centimetre per second . . . . . C/S or $\dot{C}$	L/T or $\dot{L}$
	7. Quickening (acceleration)	Foot per sec. per sec. . . . . F/SS or $\ddot{F}$	Centimetre per sec. per sec. . . . . C/SS or $\ddot{C}$	L/TT or $\ddot{L}$
	8. Momentum Impulse . . . . .	Poundem . . . . . U <sub>P</sub> ≡PF/S or P $\dot{F}$	Grammen . . . . . U <sub>G</sub> ≡GC/S or G $\dot{C}$	U≡ML/T or M $\dot{L}$
	9. Force . . . . .	Pound-dyne (Poundal) Δ <sub>P</sub> ≡PF/SS or P $\ddot{F}$	Dyne . . . . . Δ <sub>G</sub> ≡GC/SS or G $\dot{C}$	Δ≡ML/TT or M $\dot{L}$
	10. Work or Energy . . . . .	Pounderg W <sub>P</sub> ≡PFF/SS or P $\dot{F}$ F or Δ <sub>P</sub> F	Erg . . . . . W <sub>G</sub> ≡Δ <sub>G</sub> C or GCC/SS or G $\dot{C}$ C	W = ΔL or MLL/TT or M $\dot{L}$ L
	11. Power . . . . .	Pounderg per sec. . . . . W <sub>P</sub> /S or W <sub>P</sub> $\dot{F}$	Erg per sec. . . . . W <sub>G</sub> /S or W <sub>G</sub> $\dot{C}$	W/T or $\dot{W}$

*General Signature.*

L  
M  
T

## GRAVITATION AND CONVENTIONAL UNITS.

Quickening due to Gravity in Vacuo.	At Equator.	At Greenwich.	At Pole.	At Equator.	At Greenwich.	At Pole.
	32.09 $\dot{F}$	32.19 $\dot{F}$	32.26 $\dot{F}$	978.10 $\dot{C}$	981.17 $\dot{C}$	983.11 $\dot{C}$
Weight	Pound-weight = 32.19 pound-dynes	Gramme-weight = 981 dynes	Weight of unit-mass = $g\Delta$			
	1 Pound-dyne = 217 grains-weight = $\frac{1}{16}$ oz. weight (roughly)	1 Dyne = .00102 gramme-weight	. . . . . Δ = $\frac{1}{g}$ × weight of unit-mass.			
Work	Foot-pound (the work of raising one pound through one foot against gravity).	Kilogramme (the work of raising one kilogramme through one metre against gravity).				
	1 Foot-pound = 32.19 pounder-gs	1 Kilogramme = 981 × 10 <sup>5</sup> ergs.				
Power	Horse Power = 550 foot-pounds per sec. = 17704 pounder-gs per sec.	Force de Cheval = 75 kilogrammetres per sec. = 73.6 × 10 <sup>8</sup> ergs per sec.				

*At Greenwich.*

## APPENDIX.

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### ALTERNATIVE MODE OF REGARDING SYMBOLS IN PHYSICAL EQUATIONS.

IN the foregoing syllabus the small italic letters used in the formulæ have been regarded as expressing the numerical multipliers of the several standards or units in terms of which the magnitudes involved are measured. This necessitates, of course, an explicit statement of some particular relation between the standards of measurement employed, and therefore, in order to avoid this necessity and to make the formulæ as general as possible and applicable to all methods of measurements and sets of standards, many physicists prefer to regard these symbols, when used in fundamental equations, as representing the *magnitudes themselves*, that is, numerical multipliers *and* standards together, instead of merely the numerical multipliers. They consider that by this method of interpretation the equations are rendered more truly complete and fundamental statements of mechanical and physical equivalences, such equations being unaffected by any variations in systems of standards.

It should be clearly understood that the adoption of this interpretation involves the extension of the notion of *ratio* to that of two magnitudes of *different kinds*, and of a *product* to that of two magnitudes, both concrete, whether of the same or of different kinds.\* Thus, a speed involves the elements *length* and *time*, being measured by the distance travelled per certain time, and, as it is directly proportional to the distance travelled, and inversely proportional to

\* See paper on 'the Multiplication and Division of Concrete Quantities,' read by Mr. A. Ledge before the Association in January 1888, reprinted in 'Nature' July 19th, 1888, and in the 14th Report of the Association, January, 1888.

the time occupied, would be represented by the ratio or quotient  $\frac{\text{distance}}{\text{time}}$ . Again, the *work* done by an effort is directly proportional to the effort and to the distance through which its point of application moves, and is represented by the product of this distance into the effort,  $\text{distance} \times \text{effort}$ .

In applying the formulæ, thus understood, to particular cases, we shall sometimes be led to expressions such as (minute)<sup>2</sup>, (feet)<sup>5</sup>,..., to which by themselves we *may* not be able to apply any specific meaning, but we know from the mode in which we have framed the primary definitions that we can apply the ordinary rules of proportion to them without stopping to enquire into their physical meaning. Further, as we cannot *equate* two quantities of different kinds, it will follow that in any valid equation the concrete factors can be arranged so as to involve only pure ratios, and the impossibility of doing this in any case would be therefore a certain indication of error. The application of this test to a result is often extremely useful.

For practical application of physical principles to numerical examples it is often convenient to deduce, from the fundamental relations, such numerical equations referred to specified units as may be found practically convenient, though even in practical applications it would often be useful, particularly in cases of mixed standards, or change of standards, to use the *full* values of the various quantities involved, standards and all. The student, or practical man, ought to be able to use either method at will. For example :

(i) The distance ( $d$ ) of the marine horizon from a man standing with his eye at the height  $h$  above the sea is given by the approximate formula  $d^2 = 2Rh$ , where  $R$  is the earth's radius. This is a fundamental relation, independent of any method of measurement we like to adopt. To deduce from this a convenient numerical equation, we may say : Let  $h = a$  feet,  $d = x$  miles,

then  $(x \text{ miles})^2 = (8000 \text{ miles}) \times (a \text{ feet})$  ;

$$\begin{aligned} \therefore x^2 &= 8000 a \times \frac{1 \text{ foot}}{1 \text{ mile}}, \\ &= \frac{8000}{5280} a, \\ &= 1\frac{1}{2} a. \end{aligned}$$

$\therefore$  the square of the distance of the horizon, measured in miles, is half as much again as the height of the observer's eye above the

surface, measured in feet. This is a convenient *numerical* relation, deduced from the original fundamental one.

This illustration clearly exemplifies the difference between *numerical* equations in certain specified units and the *fundamental* relations which are independent of units, and shows the essential *limitation* of the numerical equation as compared with the generality of the other, but shows also the greater convenience of the numerical equation for practical application.

The following example will show, on the contrary, the advantage which sometimes accrues from taking the standards into the formulæ.

[The solution of this and some of the following examples is given in duplicate in parallel columns illustrating two different ways of expressing the concrete quantities, the one employing ratios and products as above explained, the other rising the relations between the units of the different magnitudes involved as explained in the Syllabus.]

(ii) A steamer moves at the rate of 20 knots against a constant resistance equal to the weight of 55 tons. Find the amount of coal per day required, given that the energy of combustion of a pound of coal is 5000 foot tons-weight, and that the engines can utilize 5 per cent. of this energy. (A knot = 6080 feet per hour.)

Let  $x$  = required daily consumption of coal,

then  $\frac{x}{1\text{ lb.}} \times 250 \text{ feet} \times \text{tons-weight}$  is the utilised energy each day,

and  $\frac{x}{1\text{ lb.}} \times \frac{250 \text{ feet} \times \text{tons-weight}}{1 \text{ day}}$  is the power exerted.

This power must be equal to the product of the resistance and velocity, namely  $55 \text{ tons-weight} \times 20 \text{ knots}$ ,

$$\begin{aligned} \therefore \frac{x}{1\text{ lb.}} \times \frac{250 \text{ feet} \times \text{tons-weight}}{24 \text{ hrs.}} \\ = 55 \text{ tons-weight} \times \frac{20 \times 6080 \text{ feet}}{1 \text{ hr.}}; \end{aligned}$$

$$\begin{aligned} \therefore x &= \frac{55 \times 20 \times 6080 \times 24}{250} \text{ lbs.,} \\ &= \frac{55 \times 20 \times 6080 \times 24}{250 \times 2240} \text{ tons,} \\ &= 286.6 \text{ tons.} \end{aligned}$$

Suppose the daily consumption of coal to be  $x$  tons.

Then since 1 lb. of coal yields  $\frac{1}{25}$  of 5000 or 250 *foot-tons* of useful work,  $x$  tons yields  $2240 x \times 250 \text{ foot-tons}$ : and the work of the resistance per hour =  $55 \times 20 \times 6080 \text{ foot-tons}$ .

Hence, equating the energy utilized to the work done in one day,

$$2240 \times 250 x = 24 \times 55 \times 20 \times 6080;$$

$$\therefore x = \frac{24 \times 55 \times 20 \times 6080}{2240 \times 250} = 286.6.$$

It should be noted that, in the left-hand solution of the above

example,  $x$  is the quantity of coal irrespective of methods of measurement, and that when the number of lbs. in this quantity is required, the ratio of  $x$  to 1 lb.,  $\left(\frac{x}{1\text{lb.}}\right)$ , has to be taken. This is an important principle, and may be very usefully employed in attacking problems such as the following, where a strictly numerical equation referred to one set of standards is to be adapted to another set. (It is adopted in the left-hand solution below.)

(iii) The number of grammes in a volume of air occupying a volume of  $v$  litres at a temperature of  $t^\circ$  centigrade under a pressure equal to  $h$  millimetres of mercury

$$= .4645 \frac{vh}{273 + t}.$$

Find the mass in grains, when the volume is given in cubic inches, the pressure in inches of mercury, and the temperature in degrees Fahr.

*First method.* The full expression for the above formula is

$$\text{The mass} = .4645 \frac{\frac{\text{volume}}{\text{one litre}} \times \frac{\text{height of barometer}}{\text{one mm.}}}{\frac{\text{absolute temperature}}{\text{one } C^\circ}} \text{ grammes,}$$

$$\begin{aligned} \text{and a gramme} &= 15.43 \text{ grains,} \\ \text{a litre} &= 61.02 \text{ cubic inches,} \\ \text{a mm.} &= .03937 \text{ inches,} \\ 1 C^\circ &= 1.8 F^\circ; \end{aligned}$$

$$\begin{aligned} \therefore \text{The mass} &= .4645 \frac{\frac{\text{volume}}{61.02 (\text{in.})^3} \times \frac{\text{height}}{.03937 (\text{in.})}}{\frac{\text{absolute temperature}}{1.8 F^\circ}} \times 15.43 \text{ grains,} \\ &= \frac{.4645 \times 1.8 \times 15.43}{61.02 \times .03937} \times \frac{\frac{\text{volume}}{(\text{in.})^3} \times \frac{\text{height}}{(\text{in.})}}{1 F^\circ} \text{ grains,} \end{aligned}$$

of which expression the numerical coefficient reduces to 5.37;

$\therefore$  the number of grains is given by the formula

$$5.37 \frac{vh}{459 + t},$$

if the volume is  $v$  cubic inches, the height of the barometer  $h$  inches, and the temperature  $t^\circ$  Fahr.

*Second method.* Let  $m'$ ,  $v'$ ,  $h'$  be the measures in grains, cubic inches, and inches respectively of the mass, volume, and barometric height, and  $t'^{\circ}$  the temperature Fahrenheit.

$$\text{Then } m' \text{ grains} = \frac{m'}{15.43} \text{ grammes,}$$

$$v' \text{ c. in.} = \frac{v'}{61.02} \text{ litres,}$$

$$h' \text{ inches} = \frac{h'}{.03937} \text{ mm.,}$$

$$\text{and } t'^{\circ}F = \frac{5}{9}(t' - 32)^{\circ}C;$$

hence by the given equation

$$\frac{m'}{15.43} = \frac{.4645 v' h'}{61.02 \times .03937 \times \{273 + \frac{5}{9}(t' - 32)\}},$$

$$\text{or } m' = \frac{15.43 \times .4645 \times 9}{61.02 \times .03937 \times 5} \times \frac{v' h'}{459 + t'} = 5.37 \times \frac{v' h'}{459 + t'}.$$

(iv) In what distance would a constant force bring a train of 100 tons, moving with the speed of 30 miles per hour, to rest in 40 seconds? and what is the force?

The distance

$$= \text{average velocity} \times \text{time,}$$

$$= \frac{15 \text{ miles}}{1 \text{ hour}} \times 40 \text{ sec.,}$$

$$= \frac{15 \times 40}{60 \times 60} \text{ miles,}$$

$$= \frac{1}{3} \text{ of a mile,}$$

$$= 880 \text{ feet.}$$

Also, Force  $\times$  time = Mass  $\times$  change of velocity,

$$\therefore \text{Force} = \frac{100 \text{ tons} \times 30 \frac{\text{miles}}{\text{hour}}}{40 \text{ sec.}},$$

$$= \frac{100 \times 30 \times 5280 \text{ tons} \times \text{feet}}{40 \times 60 \times 60 (\text{sec.})^2},$$

$$= \frac{100 \times 30 \times 5280}{40 \times 60 \times 60 \times 32} \text{ tons-weight,}$$

$$= 3 \frac{1}{16} \text{ tons-weight,}$$

$$= 7700 \text{ pounds-weight.}$$

The speed = 30 miles/hr.,

$$= \frac{30 \times 5280}{60 \times 60} \text{ F/S,}$$

$$= 44 \text{ F/S;}$$

and by the formula  $l = \frac{1}{2} vt$ ,

$$\text{the dist.} = \frac{1}{2} \times 44 \times 40 \text{ feet,}$$

$$= 880 \text{ feet or } \frac{1}{3} \text{ mile.}$$

Also the momentum

$$= 100 \times 2240 \times 44 \text{ PF/S,}$$

which is destroyed in 40 S by the force

$$= \frac{100 \times 2240 \times 44}{40} \text{ PF/SS,}$$

$$= 110 \times 2240 \text{ poundals,}$$

$$= \frac{110 \times 2240}{32} \text{ pounds-weight,}$$

$$= 7700 \text{ pounds-weight.}$$

(v) What uniform power would stop the train in the same time ?

$$\begin{aligned} \text{Power required} &= \frac{\text{Force} \times \text{distance}}{\text{time}}, \\ &= \frac{7700 \text{ pounds-weight} \times 880 \text{ feet}}{40 \text{ sec.}}, \\ &= 7700 \times 22 \text{ foot lbs.-weight per sec.}, \\ &= \frac{7700 \times 22}{550} \text{ Horse-power}, \\ &= 308 \text{ Horse-power.} \end{aligned}$$

Or, Power = Rate of change of Kinetic energy;

$$\begin{aligned} \therefore \frac{\text{Power}}{1 \text{ HP}} &= \frac{50 \text{ tons} \times \left(\frac{30 \text{ miles}}{\text{hr.}}\right)^2 \div 40 \text{ sec.}}{550 \text{ ft.} \times \text{lbs. weight} \div 1 \text{ sec.}}, \\ &= \frac{2240 \times 30^2 \times 5280^2}{11 \times 60^4 \times 32 \times 40}, \\ &= 308, \end{aligned}$$

$\therefore$  Power = 308 HP.

$$\begin{aligned} \text{The kinetic energy} &= \frac{1}{2} \times 100 \times 2240 \times 44^2 \frac{\text{PFF}}{\text{SS}} \text{ or poundergs,} \\ \text{and the power} &= \text{rate of decrease of k. e.} \\ &= \frac{50 \times 2240 \times 44^2}{40} \text{ poundergs per sec.} \end{aligned}$$

Also 1 HP.

$$= 550 \text{ foot-lbs. per sec.}$$

$$= 550 \times 32 \text{ poundergs per sec.,}$$

$\therefore$  the power

$$= \frac{50 \times 2240 \times 44^2}{40 \times 550 \times 32} \text{ HP.}$$

$$= 308 \text{ HP.}$$

Two other examples are appended, similar respectively to the first and second of the examples already given.

(vi) Required the numerical equation giving the number of gallons of water discharged per minute through a pipe of length  $l$  feet and diameter  $d$  inches, the frictional loss of head being supposed known and equal to  $h'$  feet.

The general formula (Cotterill's *Applied Mechanics*, p. 462), gives the rate of discharge as equal to

$$\frac{\pi}{4} \sqrt{\frac{2g}{4f}} \sqrt{\frac{\text{frictional loss of head}}{\text{length of pipe}}} (\text{diameter})^{\frac{5}{2}},$$

where  $f$  is a coefficient of fluid friction, an average value of which may be taken as .0075, so that  $4f = .03$ .

$\therefore$  Rate of discharge

$$\begin{aligned} &= \frac{\pi}{4} \sqrt{\frac{2 \times 32 \times 12 \text{ inches}}{.03 (\text{sec.})^2}} \sqrt{\frac{h'}{l}} (d \text{ inches})^{\frac{5}{2}}, \\ &= 40 \pi \sqrt{\frac{h'}{l}} d^{\frac{5}{2}} \frac{(\text{inches})^3}{\text{sec.}}. \end{aligned}$$

Now 1 cubic inch = .0036 gallons,

$$\begin{aligned} \therefore \text{Rate of discharge} &= 40 \pi \times .0036 \times 60 \sqrt{\frac{\bar{k}'}{l}} d^{\frac{5}{2}} \frac{\text{gallons}}{\text{min.}} \\ &= 27.2 \sqrt{\frac{\bar{k}'}{l}} d^{\frac{5}{2}} \text{gallons per minute;} \end{aligned}$$

that is, the required formula giving the number of gallons discharged per minute is

$$27.2 \sqrt{\frac{\bar{k}'}{l}} d^{\frac{5}{2}}.$$

(vii) Find the maximum deflection of a timber beam of rectangular section, 28 feet long, 18 inches deep, and 6 inches wide, supported at its ends, and carrying a uniform load of  $\frac{1}{4}$  ton-weight per foot run, Young's modulus of elasticity ( $E$ ) being 750 tons-weight per square inch.

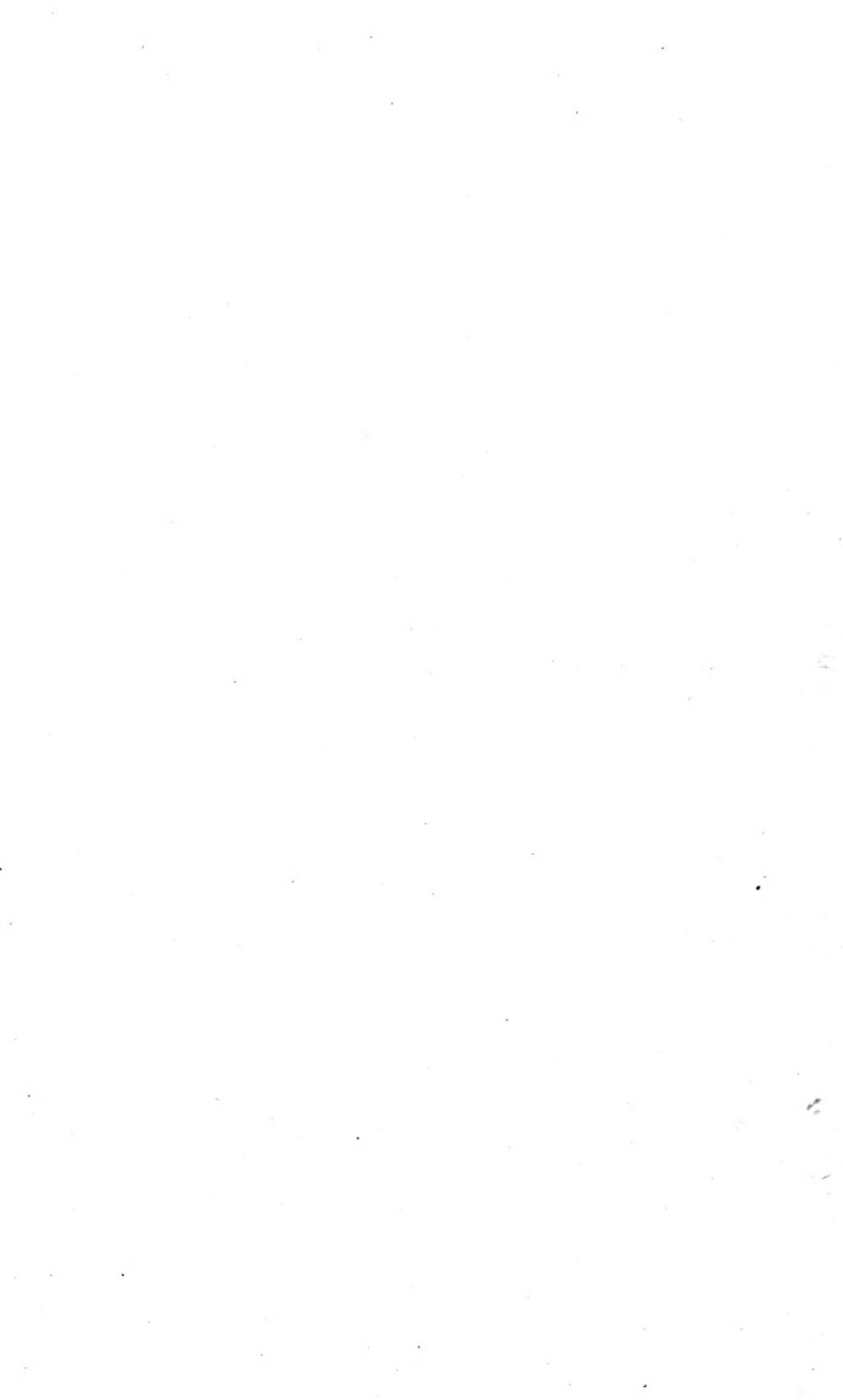
The formula is

$$\text{Maximum deflection} = \frac{5}{384} \text{ of } \frac{\text{load} \times (\text{length})^3}{E \times \frac{1}{12} \text{ of } (\text{width})(\text{depth})^3}.$$

Hence, in this example,

$$\begin{aligned} \text{Maximum deflection} &= \frac{5}{384} \times \frac{7 \text{ tons-weight} \times (28 \text{ feet})^3}{750 \frac{\text{tons-weight}}{(\text{inch})^2} \times \frac{1}{12} (6 \text{ in.}) (18 \text{ in.})^3} \\ &= \frac{5 \times 7 \times (28 \times 12) \times 12}{384 \times 750 \times 6 \times 18^3} \text{ inches} \\ &= 1.58 \text{ inches.} \end{aligned}$$







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