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A

SYLLABUS OF LOGIC,

IN WHICH

THE VIEWS OF KANT ARE GENERALLY ADOPTED, AND THE LAWS OF SYLLOGISM SYMBOLICALLY EXPRESSED.

BY

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PREFACE.

The object which I have proposed to myself in writing this Treatise on Logic, is the combination of a brief but complete account of the Aristotelian system, with some of Kant's philosophical views of the nature and divisions of the science. With respect to the first-mentioned part of my task, I have endeavoured to give a strictly *à priori* character to the derivation of the fundamental laws of syllogism, and the results of their combination in the various forms of reasoning. This is attempted, partly by employing a method in the derivation of these laws of a rather more exhaustive character than that which has usually been adopted by logicians, and partly by introducing mathematical analysis, for the exhibition of the symmetry in their forms. Symbolical representation is also employed in the second section of the Introduction, for the purpose of explaining the nature of abstract conceptions, and the method of thinking some of them purely, or independently of sense..

I have derived from the *deductio ad absurdum* the grounds for a division of the twenty-four categorical syllogisms into eight systems of three each. By means of this arrangement, which to the best of my knowledge is entirely new, the equality of the number of sound moods in the first three figures, and several other properties of categorical syllogism, may be demonstrated *à priori* without the assistance of symbolical reasoning. And although these results admit of no immediate application to practice, yet are they useful in giving the student such firm hold upon the fundamental principles of the science, that they will never afterwards desert him. Besides, it should be considered a sufficient merit that they add a theoretical completeness to the science, which it can never obtain when the symmetry of its results is arrived at by empirical methods alone.

There is another subject which has been very much neglected in all the works on Logic with which I am acquainted. Aristotle's Analytics is the only book in which I have discovered any attempt at a theory of the modality of syllogism, and on this particular point he appears to me to have failed. I have accordingly devoted an entire section to the consideration of this subject, at the end of the second book, in which I have endeavoured to expose the fallacious nature of his reasoning. It must, moreover, be remembered, that this subject is one of the most important connected with Logic, for any misconception respecting it may give birth to fallacies of a very complicated nature, and extremely difficult to detect. Perhaps not a few of the errors on the nature of the will might be ultimately traced to this source.

I will now mention as briefly as may be the extent to which I am indebted to Kant, or rather his translators, for it is by their means alone that I have had any access to his works.

The division of the science into Transcendental and Universal, (for the latter of which terms I have substituted 'Formal' as being more generally intelligible) is adopted from the Criticism, and cannot perhaps be entirely comprehended without a reference to that work. The first part of the second section in the Introduction, and a great part of the third, are little more than an abridged paraphrase of some portion of his Logic. There are also several other places throughout the Introduction which have at least originated in some idea derived from his works, though it is impossible individually to specify them here.

In the first book, which contains the merely formal analysis, I have adopted the opinion of Kant respecting the distinct nature of Categorical, Hypothetical, and Disjunctive propositions. But as the latter part of the first book is entirely opposed to his opinion on the philosophical correctness of the distinction of figures, it will be necessary to account for my having thought proper to retain the scholastic theory on that subject. Many other reasons might probably be urged in favour of this view, but one alone is amply convincing to myself. The *deductio ad absurdum* can only be applied to some syllogisms in the first figure, by the introduction of those moods of other figures which cannot be reduced to the first by conversion; and the indirect proof of these syllogisms would only restore the original syllogisms in the first figure. There are one or two other points in which I have not followed Kant, and which are immediately consequent upon the adoption of the system of figures. These it is not necessary to specify here; for if the reader is acquainted with the works of that philosopher, he will readily detect them for himself; and if he is not, he would not understand my explanation.

Material fallacies, ambiguous terms, and many other similar subjects which are usually considered in works on this science, have found no place in the following pages. All these subjects have been already discussed by Dr. Whately in such a very able and lucid manner, that nothing more remains to be said about them. And even had this not been the case, my silence respecting them could not have been considered an omission, as they never entered into the plan of my work.

I have attempted little more than an analysis of the formal laws of reasoning, and how far this attempt has been attended with success, the reader can now determine for himself.

Cambridge, May 13, 1839.

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ERRATA.

Page 4, *lines* 5 and 7, from the bottom, *for* "represented," *read* "imaged."

Page 40, *line* 1, *for* "cognitions," *read* "representation."

Page 89, *lines* 5, 6, in the table, dele "Subject Premiss."

Page 108, *line* 10 from bottom, insert "not" between "B" and "being."

A

SYLLABUS OF LOGIC.

INTRODUCTION.

SECTION I.

ON THE NATURE AND DIVISIONS OF THE SCIENCE.

(1.) THE subjection of the Understanding to certain invariable laws is the first indispensable condition to all knowledge. Kant has shewn, in his Criticism on the pure Reason, that the mind legislates for matter, or in other words, that the laws we discover in the external world derive their very possibility from the laws of Mind. But let us for an instant imagine the possibility of nature following fixed laws as an object of our senses, quite independently of any laws in our understanding, which for argument's sake we will suppose to be without them. It is evident that upon this hypothesis the Understanding could never take cognizance of these laws of nature, nor even of the existence of the objects they concerned. For these objects can only become known to us by means of certain laws, according to which we can severally distinguish them from each other: and how could we even distinguish between ourselves and nature, or between the 'me' and the 'not me,' without some law in the faculty by which objects are known?—Every change in our representations might either arise from a change in nature, or from a change in our own indeterminate state. But even could we separate ourselves from nature, there would still remain the question, 'How can we conjoin any phenomena in a synthesis for the purposes of knowledge, unless we are conscious of something fixed and determinate to which we can refer all the phenomena to be conjoined, and also of some law for the mode of their conjunction?' Without these requisites the cessation of the phenomena and of their conjunction in the mind must be simultaneous. And, moreover, even while the phenomena lasted, no act of conjoining them in any one instant of time could ever be considered the same as, or be united with a similar act of conjoining them in any succcessive instant, unless we were to grant something fixed in our consciousness, and quite independent of time, as a common ground for the unity of these successive syntheses. Hence, upon every hypothesis, the legality of the knowing faculty is a necessary condition to its use; and it now only remains to extend this remark to the other logical faculty —the Constructive Reason.

(2.) The Understanding has been defined by Kant, as 'the faculty of rules;' by Coleridge, as 'the faculty of judging according to sense.' Its operation may be considered as two-fold, accordingly as it dissects a representation by analysis, or conjoins several representations in a synthesis. But there can be no act of analysis without the consciousness of a prior synthesis; and hence it follows, that the latter is an indispensable condition to every act of the faculty. Now the Reason, considered as to its logical use, (its transcendental use has nothing to do with our present purpose, and need not be considered here,) differs from the Understanding in this: Whereas the Understanding merely conjoins the diversity of representations in synthesis, or dissects them by analysis, in either case referring this diversity to the unity of consciousness; the Constructive Reason on the other hand conjoins the very unities of these syntheses, which unities are acts in the consciousness or logical functions of the Understanding, and entirely distinct from the diverse representations contained in them.* But as this kind of reasoning is rather difficult to comprehend in the abstract, we shall endeavour to explain our meaning by an example:—If any person were to state the two propositions, 'all tyrants are unhappy,' and 'Nero was a tyrant,' my Understanding alone could never have enabled me to discover that 'Nero was unhappy.' For this faculty unaided by the Reason could only conjoin certain conceptions according to a rule. For instance, I

* If we might be permitted to offer an illustration of so abstract a subject, we would represent the synthesis of representations in the Understanding by the arc of a circle connecting two points, whose centre or unity of the syntheses may be of course at a finite distance. But if another point be taken in the same right line with the first two, we can only join all these points, and make the centres of the two arcs correspond, by placing the common centre at an infinite distance, where it will represent the Reason conjoining the unities of the syntheses of the Understanding. It must also be observed, that as on the one hand the centre of this circle is infinitely distant, but its arc, which is a straight line joining the three points, is finite and determinate; so on the other hand the rational act of conjoining the unities of the syntheses of the Understanding transcends all possible experience, and cannot be represented, but can only be thought, whereas the synthesis, which results from the act of reason, is quite as easily represented as those from which it was originally derived. We are perfectly aware of the manifold objections which might be urged against this illustration, but we are inclined to think it may, in some measure, assist the reader in comprehending the idea we wish to convey.

may severally conjoin the conceptions of unhappiness and Nero with the conception of tyrant, as in each case belonging to the same subject: but in order to conjoin the two extreme parts of these judgments, namely, Nero and unhappiness, I must simultaneously reflect on my previous conjunction of each of them with the conception of tyrants, and therefore on the two acts of the Understanding by which these judgments take place. Before then I can arrive at any conclusion from the two given propositions, I must possess some faculty by which I can conjoin the very acts of the Understanding that are contained in them, and this is what is meant by the Constructive Reason.

The ease with which the Reason arrives at a conclusion from judgments that have been reduced to their most simple logical form, and the perfect similarity of such a conclusion when obtained to any immediate act of the Understanding, are circumstances which have rendered the peculiar function of the Reason, in its logical use, extremely liable to be overlooked. But a little attention to the subject will make it evident, that the conjunction of the acts of the Understanding requires quite another faculty than that of the Understanding itself, which merely conjoins the parts of possible experience according to rules.*

Having given this short account of the logical use of the Reason, we shall now proceed to shew that its functions are entirely determined by laws. For as the conclusions of the Reason are constructed with materials derived from the use of the Understanding, they can only concern the objects of the latter faculty, and must in their intrinsic nature be possible as its immediate acts. But this harmony of the two faculties can only be secured by the subjection of the Reason to laws. Here then at length we are justified in assuming, that the whole use of our intellectual and rational faculties is based upon certain universal laws, and that the Science which treats of them is not a mere chimera of the imagination, but founded on a reality which is the primary condition to all other knowledge whatever.

* After the author had written his account of the logical use of the Reason, he met with the following passage from Kant. It refers, however, to the whole use of the faculty :—

" The Understanding may be a faculty of the unity of phenomena by means of rules ; Reason is thus the faculty of the unity of the rules of the Understanding under principles. Reason, therefore, never refers directly to experience, or to an object, but to the Understanding, in order to give to the diverse cognitions of this, unity *à priori* by means of conceptions, which may be termed unity of Reason, and which is of quite another kind to that which can be derived from the Understanding."—*Anonymous Translation of Kant's Criticism.*

(3.) The laws of the use of the Understanding must be *à priori,* and cannot be derived by the method of induction from any number of mental phenomena. For it has been shewn that they are necessary to all use of the Understanding, (as it cannot even determine an object without them,) and consequently to experience itself; and we should therefore be guilty of a glaring circle in our reasoning, if we endeavoured to derive from experience those laws which have previously been made the very grounds of its possibility.*

But if the laws of the Understanding are

* The commonest example of this most fallacious attempt to deduce the pure conceptions of the Understanding from experience, is to be found in the grounds that are sometimes given for the causal relation of phenomena. For instance: *Q.* Why do I expect a spark from the concussion of flint and steel? *A.* Because in all past time such a concussion has been immediately succeeded by a spark.—*Q.* But why does this succession having taken place in all past time, give me any right to expect that it will take place again? *A.* Because any succession that has taken place very often, has been afterwards observed to take place again. And if we were to ask for a reason, why the uniformity of nature up to the present moment should be considered as any guide for the future, we should get precisely the same answer, or in other words, the fact would be stated as the ground for itself. And we might thus continue the question and answer to infinity, without ever getting any nearer the point. The difficulties of this subject were first brought to light by the subtilty of Hume. And although he never succeeded in offering any satisfactory solution of them, yet his "sceptical doubts" aroused the attention of Kant, and became the occasion of the Criticism on Pure Reason, in which the difficulty is satisfactorily explained.

independent of all empirical matter, *à fortiori* must this be the case with the formal laws of the Reason, which simply regard the conjunction of the acts of the former faculty, and are therefore removed one step farther from their empirical contents. Hence it follows that both Reason and Understanding are entirely self-regulated, or subject to *à priori* laws. And the determination of these methodically, and their arrangement in a system, is the business of Logic in the widest acceptation of the word.

Division of Logic.

(4.) Logic is divided by Kant into Transcendental, and Universal or Formal.*

Transcendental Logic agrees with Formal in excluding all consideration of particular objects, but differs from it in admitting that of the pure conception of an object in general.

Formal Logic entirely excludes all consideration of the objects thought, and merely re-

* The reader who has no acquaintance with the Kantian system, must not be surprised if he does not clearly understand the distinction between these branches of the science. As, however, Formal Logic is exclusively the subject of the first book, and a few considerations from Transcendental are introduced in the second—the two books together may throw some light upon the nature of the division in question.

gards the form of our judgments, and their relations to each other.

It will be seen from the above definitions that Transcendental Logic embraces a very wide field, and includes within its limits much that is metaphysical, and entirely foreign to the other branch of the science.*

As Formal Logic alone is the proposed subject of this treatise, we should have been justified in taking leave of Transcendental Logic here: but certain limitations are imposed upon the very forms of the judgments of the Understanding by conceptions which are peculiar to the last-mentioned science. And although they may be justly assumed in every treatise on Formal Logic, yet, as they are the origin of several very peculiar results,† we thought the notice of them in our second book might be deemed not wholly superfluous.

(5.) This apology has been offered for an intended reference to what properly lies out of the sphere of the subject, on account of the great injury which the sciences must in-

* For instance, the conceptions of substance, causality, &c.

† Some of the limitations are alluded to in the commencement of the first book. The absence of a formal proposition of identity is one of them. For if we say A is B, we do not know but that other things may be B also. One of the peculiar results is the "possibility of a true conclusion from false premises in every form of reasoning."

variably incur whenever their boundaries are not strictly recognised. Formal Logic has, perhaps, suffered more on this score than any other science: for, to say nothing of many encroachments on the side of Transcendental Logic, it has been a common custom with logicians to introduce into their treatises a great deal of matter derived entirely from empirical psychology, which is a distinct branch of knowledge. Thus, the choice and arrangement of arguments, the best application of particular syllogistic forms, and other similar considerations, have frequently been made the subject of rules which can only be derived from practice, and should never be mixed up with Logic, which is an *à priori* science. Observations of this kind can never form part of a system, but are merely an aggregate of information, which is very useful no doubt, but belongs more particularly to the Art of Rhetoric. Knowledge of this kind may properly be termed an art, and so far the old logicians were at least consistent. But this term can never be applied to Logic without an absurdity as manifest as if we were to speak of the Art of the Differential Calculus, or Conic Sections.*

* We have thought it better to put a few observations on Dr. Whately's Definitions of Logic in a notice by themselves at the end of this section.

(6.) Logic is not an organum of the sciences; for it does not contain a single reference to any one branch of knowledge, and therefore has no resting-place or fulcrum from which to commence its investigations: it is however of the greatest use in testing the work of another organum, by exposing all its results. When a new law has been arrived at by Induction, Logic will determine all its consequences; and should one of these prove at variance with truth, it is certain that the induction is erroneous, and that new observations are necessary.

Notice on Whately's Logic.

(7.) As the distinction between the theory of Logic and its application to practice has been well explained in the very valuable treatise by the Archbishop of Dublin, we cannot help feeling in some measure surprised that he has united these very different subjects in the same definition. In the introduction to his work, Dr. Whately defines Logic as the "Science, and also the Art of Reasoning;" and afterwards explains in a note, "that as a science is conversant about knowledge only, an art is the application of knowledge to practice." Now it follows from the latter unobjectionable definition, that the rules for the art must be merely practical, and derived from the experience of "how we reason best," or in other words from empirical psychology; for if they were *à priori,* they would *ipso facto* become laws of the science. Hence we see that the rules of the art, which are all empirical and depend on observation, differ from the laws of the science, which are *à priori* and derived from the reason, not only in their use and nature, but also in their origin. The question however is not one of fact, or even of theory, as Dr. Whately has himself introduced the very distinction for which we are contending: but the question *is,* whether that arrangement can be considered conducive to the interests of science, which combines under the same name two branches of knowledge whose nature and origin are equally distinct from each other.

(8.) There are two other passages in Dr. Whately's work, which appear to the author to contradict each other, and to convey equally false notions of the nature of the science. Soon after the definition upon which we have just commented, Dr. Whately adds, "Its most appropriate office however is, that of instituting an analysis of the process of the mind in reasoning." Now it is not the process of the mind in reasoning, but the principles with which that process must accord, that is the proper object of

Logic.* We hardly ever reason to ourselves in syllogism, but only in a manner which agrees with its laws. For instance, if I think of the future death of a hale and hearty man, in whom there do not appear the slightest symptoms of decay, I do not consider it a merely probable circumstance, but morally certain. Now this conclusion could only be logically deduced by my virtually assuming the proposition that 'All men must die,' and therefore that John being a man must die also. But although the actual law which connects humanity and mortality must have been thought in my mind, when I first considered it as morally certain that John must die, yet the proposition that 'all men must die' (which is, as it were, the exponent of the law) had, in all probability, never occurred to me: perhaps, indeed, so far from thinking of all mankind, I had not thought of any other person than John. We see then that the object of Logic is not the actual process of our reasoning, but rather the principles to which that process can always be referred.† And it must be remembered that

* Dr. Whately admits that Logic "investigates the principles on which augmentation is conducted;" and had his definition terminated here, it would not have been liable to any objections.

† The simple state of the case is this: In the actual process of our minds we generally connect the mere conceptions in a law, without thinking of all that is contained in their sphere. But

this is no unimportant point in the definition of Logic, as an erroneous conception on this head is extremely calculated to bring the science into disrepute. For, if any one discovers that the process of reasoning in his own mind is rarely, if ever, in the form of a complete syllogism, he immediately leaves the science with the conviction that it certainly does not contain the principles of all reasoning, and probably not of any.

(8.) The other passage to which we would allude, is to be found in a note near the commencement of the second book, in which Dr. Whately states, that "Logic is entirely conversant about language."

This appears to us to be at variance with the passage we have just quoted, in which the most appropriate office of Logic is said to be the analysis of the process of the mind in reasoning. Now, that language follows certain laws, is unquestionably true. But whence do these laws arise, unless from the necessity

in the logical development we consider the aggregate of individuals that come under the conception contained in the subject of an universal proposition. As, for instance, in the above example, if I look at John and think of his future death, I merely connect the conception of death with that of humanity, to the conditions of which I see that he corresponds. But if I am asked to express logically my reasons for expecting his death, I immediately commence with 'All men die,' &c.

of language conforming itself to the mental laws of which it is the exponent? If language did not receive its stamp from mind (the laws of which we assume to be universally the same in all countries and ages), we could never be certain that we might not at some future time meet with a people whose language required quite a new logic, and in that case the science would lose its *à priori* character, and rest on probability alone.

SECTION II.

ON COGNITION.

(10.) Cognition is the generic name for all representations that are sufficiently completed for the logical use, and must therefore contain a reference to our consciousness on the one hand, and to an object on the other.

If a cognition contain sensation, it is empirical; if it does not, it is pure.

Cognition may be divided into the two species—Intuition and Conception, each of which may be either pure or empirical.

Intuition is a merely sensual* cognition of an object.

Conception is an intellectual cognition, and refers to many objects.

Perhaps we can exhibit these co-divisions of cognition in a more intelligible manner by means of the following scheme:—

COGNITION TABLE.

	INTUITION.	CONCEPTION.
Pure........	A straight line.	Substance.
Empirical..	A horse that is seen.	Abstract conception of a horse.

* Pure Intuitions concern the mere forms of the Sensitivity, Space, and Time, and are therefore sensual without containing sensation.

To investigate the ultimate original of cognition, and shew by what process a merely sensual representation obtains objective validity,* and thus becomes an intuition, is entirely foreign to our present purpose, as it belongs to the science of Metaphysics. We shall therefore only explain that formal property of our cognition upon which its logical perfection mainly depends, and conclude the section with investigating those logical acts of the understanding by which it arrives at conceptions.

(11.) Cognition may be divided into distinct and indistinct.

Indistinct cognition is that in which a diversity is thought, that is not exhibited objectively to the consciousness.

Distinct cognition, on the contrary, is that in which no diversity is thought, which is not exhibited objectively to the consciousness.

Let us take an example of an indistinct intuition.

While walking on the sea-shore, I perceive a large object in the offing, which, from its general appearance, I immediately know to be a ship. Now, from my previous knowledge of

* We give objective validity to a representation, when we conceive it to arise from something independent of the peculiar state or nature of our own mind, and therefore believe that other people would view it as we do.

the construction of a ship, I know that it must have a rudder, shrouds, and a great variety of tackle, which are not to be found in the image as it appears to me, although I must really see them: I therefore say that my intuition of the ship is indistinct. If, on the other hand, I had never seen a ship before, and had no conception of the nature of the object presented to me, I should say that my intuition of the object was distinct; and this would arise from my ignorance of what there was in it to distinguish. But if any person then asked me 'if I could see what the object was?' the question 'what?' would superadd in my mind a conception of this object, having some nature peculiar to itself, or having been made for some particular purpose, which was not exhibited to me in my intuition, and I should therefore answer that I could not see distinctly what it was.

Hence we see that an intuition is indistinct, when it suggests a conception of the diverse, which it does not exhibit objectively to the consciousness. And it is the relation of our conception to the representation, and not the latter alone, upon which distinctness must depend. For if distinctness were defined as the consciousness of the diversity of the representation, without introducing any limitation to this diversity, distinctness would be impossible, as the diversity in any object is infinite, and can never be exhausted.*

In the same manner we say that a conception is indistinct, when we attach to it the conception that an analysis of it is possible, or in other words that it contains a diversity, and yet cannot dissect, and severally represent to our consciousness† the parts of this diversity. For instance, we may be said to have an indistinct conception of the nature of the human mind, when we attach to it the conception that some analysis of its different powers is possible, and yet cannot exhibit these powers separately to our consciousness.

Conceptions.

(12.) The logical acts by which the understanding arrives at conceptions are three,—Comparison, Reflection, and Abstraction.

Comparison is that act of the understanding by which several representations are referred to the consciousness simultaneously.

* It is evident from this that distinctness is relative, not positive. And we have dwelt the more upon this, as it does not seem to be very clearly laid down in Kant's Logic.

† If the remarks in the text are founded on truth, it will follow that an increase of knowledge may sometimes render a previously distinct conception indistinct, by destroying the equality between that which we know to be in it, and that of which we are conscious.

Reflection is that act of the understanding by which we determine what is common to several representations, and consequently how we may embrace them in one conception.

Abstraction is that act of the understanding by which we separate all that is not common to several representations.

For example, if I compare several men, I reflect on their resemblance as bipeds that walk erect, and abstract their differences as to height, complexion, &c., and thus arrive at a conception that answers to all of them.

Matter and Sphere of Conceptions.

Conceptions may be considered either as to their matter, or as to their sphere.

(13.) The matter of a conception consists of the various representations contained in it.

The sphere of a conception consists of the things that come under it, or answer its conditions.

As any addition to the matter of a conception is a new condition to be answered, and as a part of the original sphere of the conception probably does not answer this new condition; it follows that the conception thus altered by an addition to its matter will have a less sphere than that of the original conception. Hence it is evident that the sphere and matter of a conception vary inversely, and that the more matter there is in a conception, the less is its sphere. Let us take an example. The conception of a horse has more matter than the conception of an animal: for the former conception must contain all that is contained in the latter conception, and something more besides. But the sphere of the conception of a horse is less than the sphere of the conception of an animal; as there are many more things that answer the conditions of the latter than the conditions of the former conception.

Rank of Conceptions.

(14.) Superior and inferior conceptions are merely relative terms. The inferior conception contains all the matter of the superior conception, and something more besides, but is itself contained under the superior conception. For example, the conception 'horse' is inferior to that of 'animal.'

A superior conception is also termed a genus, and an inferior conception a species. There can be a highest genus, but there cannot be a lowest species; for we may continue to abstract matter from a conception till we have left so little in it, that the next step would take the conception away entirely. This conception must then be the highest, for it is impossible to think of another superior to it, under which it might rank. But we can never arrive at a lowest species; for though we continue to increase the matter of the conception, and thereby lessen its sphere, we can never be certain that we have exhausted all the possible partial representations, which may make distinct species under this conception, and thus render it a genus: for instance, I may gradually abstract from my conception of 'horse' through the steps of 'quadruped,' 'animal,' and 'organic being,' till at last I arrive at mere being, from which I cannot abstract any thing more. This conception must therefore be considered as a highest genus. But if I take the conception of 'horse,' and continue gradually to increase its contents, I can never be certain that there may not be other unknown distinctions in horses, which might constitute the grounds of a division into still lower species.

But although we can never arrive at a lowest species (which must of course be a conception), we can very easily complete the determination of a cognition by fixing its individuality in Time and Space.

Symbolical expression of the nature of Abstract Conceptions.

(15.) A conception may be symbolically represented as the common measure of the representations from which it is derived. For if we express the various parts of a representation by the letters a, b, c, &c., and the whole representation as their product,* we may consider the following quantities as the expressions for three different representations,

$$a\,b\,c\,d\,e\,f, \quad b\,c\,d\,f, \quad a\,b\,c\,d\,e,$$

where it is evident that the conception obtained in the manner already explained, will be symbolically represented by the common measure $b\,c\,d$.

It frequently happens that we cannot image to our minds the conceptions we obtain by reflection and abstraction; for sometimes the representations from which we wish to obtain a conception, have not a single sensible partial representation in common, but only a law connecting their parts. And although this law may always be thought, yet it cannot be imaged to the mind, as the image could only

* In the following symbolical exposition of the nature of abstract conceptions, we have thought it better to retain the names for the operations that are suggested by arithmetical algebra. The terms, 'product,' 'multiplication,' &c. must therefore be understood as merely referring to the corresponding symbolical operations.

represent a particular case. If, for instance, I witness a great many cases of the rebound of elastic bodies from a smooth surface, perhaps the only thing these cases have in common is the equality of the angles of incidence and reflection, which cannot be imaged to the mind, but can only be thought. For an image could only give us the representation of the equality of two particular angles, but the conception of the equality generally could only be thought in the understanding. This may be symbolically expressed as follows.

Let the representations be

$$b \,.\, \phi(b), \quad e \,.\, \phi(e), \quad h \,.\, \phi(h).$$

Now, in arithmetical algebra these quantities would have no common measure, but in symbolical algebra we may separate the symbol of affection ϕ, and consider that as the symbolical representation of the conception of the law. And it must be observed, that as a general law may be thought and employed in reasoning, but cannot be imaged (which is only possible for a particular case), in like manner the function ϕ, which is a symbol of affection, may be employed in analysis as a medium of reasoning, but can never be itself interpreted in all its generality in any subordinate science, as its interpretation is only possible by its union with some particular symbol of quantity.

(16.) It is impossible to obtain a correct abstract conception of the organic productions of nature. For let us take as an example the conception of man. If we were to proceed in accordance with the simple method already proposed, and abstract every thing that is not common to all men, we should obtain a conception in which no one could ever recognise the least semblance of humanity: it must possess neither arms, legs, eyes, ears, nose, teeth, hair, or the power of speech,—for men exist who are separately deprived of each of these things. Our abstract conception of the human form divine would therefore contain a trunk without a single limb, and a scull without a single feature.

Again, if we determine in our own minds the greater part of a horse (all but the tail for instance), in conformity with our general conception of that animal, we may allow variations in that one part between very wide limits, and yet consider the whole result as coming under our conception of a horse. In the same manner we might allow any other part of the horse to vary between very wide limits, provided the rest resembled the corresponding parts of horses generally, or of any well made horse in particular: but were we to introduce all these variations simultaneously, the result would be a monster, to which our conception of 'horse' would be no longer applicable. Hence we see that the several parts of our conception must be considered as functions of other parts, and all mutually dependent on one another. And retaining the symbolical language we have already employed, where u, v, w, &c. express the partial variable representations, we might write

$$\text{abstract conception} = F(u, v, w, x, y, z)$$
$$\text{where } \phi\,(u, v, w, x, y, z) = 0\,;$$

and every set of values that answers the conditions of the equation will correspond to an individual of the species.

(17.) A friend once suggested to the author that there was a great analogy between an abstract conception and an enveloping curve; and after a little consideration he discovered that the symbolical expression for the envelope is not a mere illustration, but a strictly correct representation of the nature of an abstract conception.

Instead of representing each individual of a species by a function of a particular set of values for u, v, w, &c., let us consider the whole locus*

* As the original (*i.e.* geometrical) signification of the word 'locus' is necessarily confined to functions containing not more than three variables, it may be as well to remark, that we have adopted it here in the strictly analogous sense of 'interpretation of an equation.'

of an equation as but one individual. If then this equation contains a constant (a)* to which we give continuously successive values, we shall obtain a family of these equations to individuals whose loci will intersect each other (in a locus of one dimension less than themselves), and the envelope or locus of these intersections will be a correct symbolical representation of the conception abstracted from them all. For we may consider an abstract conception as one which contains all the points of resemblance between any individual, and those immediately preceding and succeeding it.

We therefore discover a perfect analogy between abstract conceptions and enveloping surfaces or curves: and the only difference in their respective symbolical representations consists in the number of variables which we are justified in introducing in either case. For the equations whose interpretations relate to space cannot of course have more variables

* It may be as well to remind the reader, that though the constant (a) is a symbol of quantity in the function considered simply symbolically, yet it becomes a symbol of affection (in fact a part of the function) in the interpretation: the symbol (a) is therefore to be considered as a part of the law between the members of the individual; just in the same manner as a constant in the equation to a surface or curve is only interpreted through its affecting the variables.

than space has dimensions, and are therefore confined to three: whereas the variables introduced in our abstract conceptions concern many other things besides space, and are therefore unlimited in number. We subjoin the symbolical expression for the enveloping or abstract conception.

Let $F(u, v, w, x, y, z, a)$ represent the locus of an individual: then the locus of the enveloping conception will be expressed by the equations

$$F(u, v, w, x, y, z, a) = 0 \quad . \quad . \quad . \quad (1)$$

$$\frac{dF(u, v, w, x, y, z, a)}{da} = 0 \quad . \quad . \quad . \quad (2).$$

And if a is eliminated from (1) and (2) we shall have an equation of the form

$$\phi(u, v, w, x, y, z) = 0,$$

which represents the locus of the abstract conception.

(18.) Before we conclude this section, it may be as well to add a few remarks on the method of thinking certain conceptions purely or independently of sense.

The idea of a limit, which is the basis of the differential calculus, supplies the only possible means of thinking the conceptions Substance and Causality purely, *i. e.* independently of all empirical matter. For in order to think the conception of Substance at all, I must imagine certain phenomena, and then conceive the objective existence of a substratum for their support. But this alone is insufficient, for although the conception of Substance is obtained, yet it is not pure as long as it contains any sensation or phenomenon. I must, therefore, diminish my sensation till it approximate to zero. This, however, must not be effected by lessening its extension in space, for in that case the object itself would vanish, as its identity and objective existence can only be determined in that form of sensation. Neither, on the other hand, may the sensation be diminished as to its intensity: for although this operation would only affect the conception of its being an object for me, and not of its being an object at all, yet this mode of diminution would introduce the conception of successive states, and therefore other matter besides the bare conception of Substance.

The only way that remains is that of diminishing the time in which the phenomenon is viewed, till we arrive at the limit. If then we consider time as the independent variable, and the phenomenon as a function of it, the pure conception of Substance will be properly represented as the limiting ratio between the phenomenon and the time in which it is viewed.

Let S = the pure conception of Substance, P = phenomenon, and T = time in which the phenomenon is viewed, we shall then have

$$S = \frac{dP}{dT}.$$

And as the conception of Substance contains permanence, it follows that S is constant. We may therefore consider

$$P = S\,T.$$

as the equation to a phenomenon to which the pure conception of Substance alone is applicable.

(19.) Very nearly the same reasoning will apply to the conception of Causality. This can only be imaged to the mind in a succession of phenomena occupying finite portions of time. But while our conception contains sensible phenomena, it is not pure: we therefore diminish the conceived duration of the times in which they exist, till they approximate to zero. When they arrive at this limit the conception becomes pure, for the phenomena will only occupy successive instants of time; and must therefore vanish. Hence we see that the two cases resemble each other, inasmuch as they are each represented by the limiting ratio of the phenomenon to the variable time. They differ however in this: whereas the phenomenon from which the conception of Substance alone was derived did not vary in time, but bore a constant relation to it; in the latter case the phenomenon will vary, and consequently be some unknown function of time. If then the pure conception of Causality = C, and the other symbols retain their former signification, we shall have

$$P = \phi(T, S),$$

$$\text{and } C = \frac{d\phi(T, S)}{dT}.$$

(20.) If the reader will examine his conception of $\frac{dy}{dx}$ and also of one of these categories, the analogy between them will become immediately evident. The ratio $\frac{dy}{dx}$ cannot be imaged to our minds (because we cannot image the ratio between two nothings), but it can be thought. For we can very well understand that there may be such a relation between two quantities, as that the nearer they approximate to zero, the nearer does their ratio approximate to equality with some given ratio, and that the two approximations are completed simultaneously. Exactly in the same manner the pure conception of Substance cannot be imaged;* for we cannot image that,

* "The invisible was assumed as the supporter of the apparent, τῶν φαινομένων—as their substance, a term which in any other interpretation expresses only the striving of the imaginative power under conditions that involve the necessity of its frustration."—*Coleridge's Friend.*

which, considered as to space and time, and the sensation contained in them, is equal to zero. But it may be thought: for we can very well understand that there should be in our minds a law or *à priori* relation between phenomena and the time in which they are viewed, and that this law should retain its signification even when the phenomena and their time both $= 0$. It is, however, quite beyond the powers of symbolical representation to give the nature of these conceptions themselves. Nothing more is here intended, than to express, by an analogy to the mathematical idea of a limit, the only method by which these conceptions can be thought pure, and independent of the possible experience to which they are necessary.

SECTION III.

JUDGMENTS.

(21.) A judgment is that act of the understanding by which it determines how certain representations may be conjoined in the consciousness according to some rule of its own.

Kant has divided the consideration of judgments as to their form into the four points of Quantity, Quality, Relation, and Modality; and each of these again into three subdivisions or moments,* from which he derives the twelve categories, or pure conceptions of the understanding.

Quantity.

(22.) The three moments of Quantity are the Universal, the Particular, and the Singular. These determine the quantity of the subject of a judgment, and refer respectively to the whole sphere of a conception, to a part of the sphere

* Our choice of the word 'moment,' was entirely determined by its having been employed in this signification by the anonymous Translator of the Criticism.

of a conception, or to an individual; *e. g.* All men are mortal; some men are mortal; John is mortal.

In Transcendental Logic we distinguish between the Universal and Singular, because the subject of the former is in respect of its formal quantity unlimited, whereas the subject of the latter is completely determined. If, for instance, I speak of 'all men,' there is nothing in my bare conception of man which can put any limit to the number of individuals that may come under it, or answer its conditions; and the quantum of the sphere of the conception is therefore formally indeterminate: whereas, if I speak of an individual, John, my cognition contains all that is found in the conception of man, and also the possibility of his complete determination in space and time, for on that his identity depends.* But For-

* That our conception of identity requires us to think the possibility of determining the object in a particular space at a particular time, is evident from the consideration that there is no limit to the number of individuals who may exactly resemble each other in all other respects. Now the particularity of space and time is determined by a reference to ourselves; for we can only think of a particular time as being at such a period before or after the present time, or that in which we are thinking, and of a particular space as being at a certain distance from the present space, or that in which we are now sensible: hence, the identity of all external objects ultimately depends on a reference to the identity of the thinking subject. But if I endeavour to represent my own identity, I must have recourse to the identity of external objects;

mal Logic takes no cognizance of the object thought, and consequently does not recognise the distinction: for in speaking of an individual man, I speak of all that answers to my cognition, and that is all that Formal Logic can require for an Universal. This science therefore acknowledges only the two moments, Universal and Particular.

Quality.

(23.) The three moments of Quality are—the Affirmative, the Negative, and the Indefinite.

The first, as its name implies, affirms the predicate of the subject; the second denies it; but the third affects the matter of a conception which it limits by entirely excluding it from some particular conception. Thus, 'not A' is indefinite, as it applies to any thing that lies out of the sphere of A. As every thing

e. g. I may represent myself to myself as the person who was in a particular room at a certain time: hence it follows, that I think of the identity of other objects by means of a reference to myself, and of myself by means of a reference to other objects. There is, then, between the 'me' and the 'not me' a kind of polarity, which existing in space and time according to some undiscoverable law of the consciousness, gives rise to the conception of identity. This may be illustrated by representing space and time as the axes of co-ordinates, and our own mind as the origin from whence any values of x and y may be measured for the determination of an object. Or if we think of our own identity by means of that of another object, we must place the origin at the object, and consider our own mind as the point to be determined.

must be either A or 'not A,' it is the same thing whether 'not A' be affirmed or A be denied of any subject; and for this reason the Indefinite is not considered in Formal Logic, as the Negative moment can always take its place.

Relation.

(24.) The three moments of Relation are—the Categorical, the Hypothetical, and the Disjunctive.

The first of these considers the relation of cognitions to the same substratum or substance. The second considers the dependence of one cognition as consequent upon another as antecedent. The third considers a reciprocity of relation between cognitions in such a manner that any one of them can be known by the determination of all the rest. The most general form of a Categorical judgment would be 'A is B,' or 'No A is B,' in which the conception B is asserted to belong, or not to belong, to the subject A. An Hypothetical is of the form 'If A is B, C is D,' in which the judgment that 'C is D' is made to depend as consequence upon the judgment that 'A is B' as antecedent. Either 'A is B, or 'C is D,' is a Disjunctive judgment, and the determination of either part would determine the other.

On these three points of a judgment we touch but lightly here, because they affect the laws of Formal Logic, and are therefore more fully considered in another place.

Modality.

(25). Modality concerns the manner in which we think a judgment with regard to its truth, and is divided into the three moments of Problematical, Assertive, and Necessary. The first degree of holding true is that of problematical, which merely signifies the possibility of the judgment, inasmuch as it does not violate any of the universal laws of thinking.

The second degree accords to a judgment the agreement with the matter of the senses, as well as with their necessary forms. The third degree makes some *à priori* law of thinking the matter of the judgment.

A problematical judgment merely implies a formal, but not a material possibility. For instance, the conception of an elderly lady riding through the air on a broomstick, is formally possible, as it involves no contradiction, but is not generally considered materially possible.

A judgment may be assertive without its contents having come immediately under the cognizance of the person who forms it. Nothing more is meant by the above definition, than that assertive judgments must have sufficient and empirical grounds for their truth.

The third moment, Necessity, is applicable to any *à priori* law, whether rational, intellectual, or intuitive. For instance, that A is C, if A is B, and B is C, is a necessary truth of reason. That no change in phenomena can be self-originated, or that every effect must have its cause, is a necessary truth of the understanding. That the straight line is the shortest between two given points, is a necessary truth of intuition.

(26.) Propositions are judgments whose modality is either assertive or necessary: for a problematical judgment, so far as its modality alone is concerned,* merely implies that a certain judgment is not necessarily false, but bears no positive testimony to its truth. And we could not therefore say with propriety that any thing is *proposed* in a problematical judg-

* It was necessary to introduce this restrictive clause; for problematical judgments, considered independently of their modality, are generally intended to imply some anticipation of a truth, and their main use is as a step to the assertive. For instance, there may be such a person as Mr. Stiggins in New York, but no man would ever make such a judgment unless he had some grounds for believing it true.

ment, as that term has evidently a positive signification.

Propositions alone can enter into the consideration of Formal Logic; for the laws of Reason require something determinate for the matter of the judgments whose combinations they regulate, and are not conversant about mere possibilities which have only a negative value in relation to truth. This science moreover regards all propositions as assertive: for it cannot admit the distinction between assertive and necessary modality, as this is determined by the matter of judgments, and is quite independent of their form.

Synthetical and Analytical Judgments.

(27.) Judgments may be divided, as to their matter, into Synthetical and Analytical.

Synthetical judgments are those whose predicates are not contained in the conception of their subjects.

Analytical judgments, on the other hand, predicate of their subjects something that is already contained in them, or, in other words, their predicate is a superior conception to their subject.

Hence we see that synthesis, or the principle of synthetical judgments is the conjunction of two different cognitions, neither of which is contained in the other.

Analysis, or the principle of analytical judgments, is the dissection of a conception into its partial representations.

As the matter contained in the conceptions of the subject and predicate determine the nature of the judgment in respect of the above division, it follows that the same proposition may express an analytical judgment to one person, and a synthetical judgment to another. The proposition 'all members of the University are members of some particular college,' is an analytical judgment to a person who is well acquainted with the constitution of the University, and whose conception of a member of the University already contains the conception of his belonging to some particular college. But the same proposition is synthetical to a person who is entirely ignorant of that constitution, and is therefore unacquainted with the necessary conditions to being a member of the University.

(28.) But the most remarkable instance of confusion between synthesis and analysis arises from the circumstance, that what is analysis considered objectively, is very frequently a synthesis if considered subjectively. Thus the process by which a chemist examines any substance for the purpose of discovering its qualities, is rightly named analysis when referred to the object examined, but is synthesis when considered in relation to himself. For instance, if a person, who is perfectly ignorant of the constituent elements of water, separates the two gases by means of some chemical process, the investigation is correctly styled analysis, if considered in reference to the object 'water,' as he has disjoined its component parts: but the result in the operator's own mind is a synthesis; for his previous conception of water contained nothing more than fluidity, the absence of all colour and taste, and perhaps the property of dissolving a great many salts. But by the analysis of the object he has increased this conception, by adding to it, that water is composed of oxygen and hydrogen; and consequently his mental act is a synthesis.

Definition.

(29.) Definition is a judgment which determines all the partial representations contained in any conception, and is therefore equivalent to a completed analysis.

There is a specious resemblance of definition which is grounded on a synthesis, and has really no right to the name.

Definition, in the strict acceptation of the word, regards only the matter of a conception, and not its sphere. But sometimes the term is improperly applied to the exposition of any conception whose sphere is the same with that of the conception to be defined. Let us suppose that a conception *a b*, or one which contains the partial representations *a* and *b*, is connected by a certain law with the conception *c d*, and in such a manner that the sphere of *a b* is identical with the sphere of *c d*. In this case it would not be a correct definition of the conception *a b*, to state that it contained the partial representations *c* and *d*; though such a judgment would lead us to the same individuals as if the correct definition had been given. Such a judgment would be a true synthetical judgment, but not a definition.

(30.) It follows from this, that two persons may give different definitions of the same thing, and yet both of them be equally correct: for though the things be the same in each case, yet the conceptions by which each individual may recognise these things may be very different, and perhaps have hardly a single point in common. Let us take as an example the conception of water. The common conception contains little more than that water is a perfectly tasteless and colourless liquid. Now these properties are connected by a law of nature with a combination of oxygen and hydrogen, in certain proportions and under certain circumstances; and a definition which contained either account of water would equally refer to the same thing, but to very different conceptions. The rustic would probably not have much faith in the chemist's proof that a certain liquid was water by his exhibiting the gases in a separate state; neither would the chemist have much faith in the rustic's definition: and yet the definition would be correct in each case, as it would express that conception by which each was in the habit of recognising the thing itself. The example in Mr. Newman's Logic is a very good one. He observes that it would be extremely incorrect to define man as a cooking animal, although it is highly probable that man, and man only, answers to that description. But as we do not recognise men by this peculiarity in their nature—in short, as it does not form part of our conception of humanity, such a definition would only mark out the sphere, and by no means determine the matter of our conception of man. When two conceptions are united by some law which is immediately and universally recognised, it becomes a matter of indifference from which of these conceptions we derive our definition. For instance, the common definition of a triangle is 'a figure of three sides,' whereas in strictness it should be 'a figure of three angles.' The usual definition is in reality a pure synthetical judgment of intuition.

FORMAL LOGIC.

BOOK I.

SECTION I.

PROPOSITIONS.

(30.) Propositions are of three kinds—Categorical, Hypothetical, and Disjunctive.

Form of Categorical Propositions.

Categorical propositions consist of two terms and a copula, of which the terms designate the matter, and the copula their relation to each other. Their most general form may be represented by the proposition 'A is B,' in which the terms A and B are connected by the copula 'is.' Particular examples are, 'horses are animals,' 'men are not monkies.'

Many categorical propositions do not come immediately under this simple form, but are easily reduced to it by a periphrasis; *e. g.* the proposition 'I took a ride this morning' may be put under the form, 'I am a person who took a ride this morning.'

The terms are susceptible of no other* formal variations than the moments of quantity, which determine to how much of the sphere of a conception the proposition refers.

There can be only two such formal variations—the whole, and less than the whole, or part. For we evidently cannot speak of more than the whole, as the idea involves an absurdity: neither can we make any distinction between different quantities that are less than the whole, without introducing the consideration of the matter of a conception as well as of its form. It follows, therefore, that 'the whole' and 'less than the whole' are the only variations of quantity which Formal Logic can recognise: *e.g.* we may speak of 'all albinos,' *i. e.* the whole sphere of the conception 'albino,'

* It is sometimes convenient to consider the matter of a proposition with respect to its quality as well as quantity, and to speak of 'not A,' 'not B,' &c.: in this case, the 'not A' is called the external sphere of A. The attribute of quality is, however, but rarely accorded to the terms: for if the predicate is an external sphere, the form of the proposition is precisely the same as if the negation had been applied to the copula. Thus, A is a 'not B' is exactly equivalent to A 'is not' B. And an external sphere is but rarely introduced in the subject for other reasons, as well as on account of its extreme awkwardness. For instance, we do not say, 'not men' are 'not Englishmen,' but 'all Englishmen are men,' though the meaning of the two propositions is precisely the same.

or of 'some albinos,' *i. e.* a part of the sphere of that conception.

When the whole of either term is compared with the other, it is said to be distributed; when a part only is so compared, it is said to be undistributed. For instance, in the proposition 'All A is B,' the term A is distributed; but in the proposition 'Some A is B,' it is undistributed.

The only formal variations of which the copula is susceptible, are two moments of quality, affirmation, and negation. For if we compare any two terms A and B in a categorical proposition, we can only affirm or deny the one of the other: *e. g.* A is B, A is not B. In the first example the quality is said to be affirmative, in the latter negative.

Hence, the two terms and the copula, which constitute the three elements of a categorical proposition, severally admit of two gradations or variations in form: and if there were no law for their limitation, the number of possible combinations of these elements would $= 2^3 = 8$. These eight combinations would be,

* All A is Some B.	All A is All B.
* Some A is Some B.	Some A is All B.
* All A is not All B.	All A is not Some B.
* Some A is not All B.	Some A is not Some B.

(32.) But we find by experience, that of these eight forms of categorical propositions, only four are ever introduced in practice: for the quantity of the term that is placed last in the general categorical form (*i.e.* A is B) is entirely determined by the quality of the copula.* The variations of the copula and of this latter term must therefore be taken together, and the whole number of combinations for categorical propositions will $= 2^2 = 4$.

As the first and last† terms of a categorical proposition do not bear precisely the same relation to the copula, independently of their mere position, they are distinguished respectively by the names of *subject* and *predicate:*

predicate

* The law for the dependence of the quantity of the ~~subject~~ upon the quality of the copula, can only be assumed in Formal Logic: for the *à priori* grounds upon which it rests are to be found in the consideration of one of the categories or conceptions of the understanding, and therefore belong to the Transcendental branch of the science. This subject will be considered in a future section, but a cursory view of it may not be wholly out of place here. The category of substance can never be introduced in the predicate of any proposition, and this term must therefore be a conception. But the quantum of the sphere of a conception is entirely indeterminate, as there is no formal limit to the number of individuals that may answer its conditions. It is therefore impossible to compare the limits of the sphere of the predicate with those of the subject, as those of the former are unknown. The subject then can only be placed wholly out of, or wholly in, the sphere of the predicate, and the latter will be distributed or undistributed accordingly.

† *i.e.* the first and last in the simplest form of a categorical proposition, *e.g.* 'All A is B.'

and the propriety of introducing such a distinction into Formal Logic depends entirely on this formal difference of relation, and not at all on the real nature of the distinction, which is a subject of Transcendental Logic, and cannot therefore be considered here. Neither does it at all depend on the order in which the terms are arranged;* for this is frequently reversed in some languages, and occasionally even in English.

Of the eight combinations given in the last page, those marked by an asterisk are the four legitimate Categorical propositions. As, however, the quantity of the predicate is a known function of the quality of the copula, it is never expressed, but always understood; the four propositions will therefore assume the following form:

All A is B.	No A is B.
Some A is B.	Some A is not B;

where it must be observed that the proposition 'No A is B' is equivalent to the proposition

* This order is seldom introduced in English, except in poetry, emphatic diction, or where the subject is a sentence. The following are examples of these three cases:

"Oh, many are the Poets that are sown
By Nature!—*Wordsworth.*

i. e. the poets are many that, &c. "Brave indeed is that man who," &c. *i. e.* that man who, &c. is brave indeed. "It is very easy to say a great deal in a letter that cannot be hinted at in a personal interview," *i. e.* to say a great deal in a letter, &c. is very easy.

'All A is not B,' or the entire exclusion of A from B.

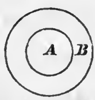
As the difference in form of these four propositions arises from variations in the subject and copula, they will admit of two corresponding co-divisions. One of these is into Universal and Particular, and respects the quantity of the subject; the other is into Affirmative and Negative, and respects the quantity of the copula. They are thus divided into Universal Affirmative represented by A; Universal Negative by E; Particular Affirmative by I; and Particular Negative by O:

	AFFIRMATIVE.	NEGATIVE.
Universal.	A. All A is B.	E. No A is B.
Particular.	I. Some A is B.	O. Some A is not B.

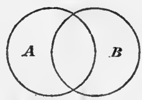
(33.) The law by which the number of legitimate combinations is reduced to four, may be stated as follows:

Affirmative copulas have undistributed predicates; Negative copulas have distributed predicates. For if B is predicated of A, A is subsumed under the conception B as being a part of its sphere; for instance, in the proposition 'all horses are animals:' the subject 'horses' is subsumed under the conception of the predicate 'animals,' as being a part of the sphere of that conception. But it is evident that the proposition does not refer to the whole sphere of the conception 'animals,' for in that case there could be no other animals than horses.

The annexed diagram is a general illustration of the law:



All A is B.

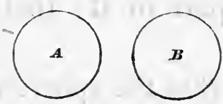


Some A is B.

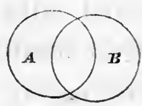
In each of these diagrams we see that A is compared with only a part of B; that is to say, B is undistributed.

It frequently happens that the sphere of the predicate is not any larger than the sphere of the subject: this however is incidental, and can only arise from the peculiar nature of the matter of the proposition, and never from its logical form: for in the example 'all carnivorous animals have teeth of a certain form,' it may be equally true that 'all animals with teeth of this form are carnivorous.' But the latter proposition requires additional knowledge of the subject matter, and cannot be deduced from the mere form of the original statement.

If, on the other hand, the predicate B is denied of the subject A, the sphere of A is entirely excluded from the whole sphere of B: for instance, in the proposition 'no durable friendship can be based on a participation in crime,' friendship is entirely excluded from all those things which can be based on a participation in crime: or in the particular proposition 'some of the most talented men do not possess the best private characters; these 'some men' are entirely excluded from all who possess the best private characters. In each case then the predicate is distributed, as the proposition refers to the whole of its sphere. The diagrams will take the following forms:



No A is B



Some A is not B

in which it is evident that the whole of B lies out of as much of A as is introduced in the subject.

The accompanying table contains the results of these remarks:

		SUBJECTS.	PREDICATES.
Universal Affirmative	A ...	Distributed —	Undistributed.
Particular Affirmative	I ...	Undistributed—	Undistributed.
Universal Negative	E ...	Distributed —	Distributed.
Particular Negative	O ...	Undistributed—	Distributed.

SECTION II.

MUTUAL RELATIONS OF CATEGORICAL PROPOSITIONS.

(34.) The mutual relations of the four propositions A, E, I, O, have been usually classed by logicians under the generic term of Opposition. But as there is no opposition whatever in some of these relations, it may be more correct to restrict the acceptation of the word to those cases which contain its meaning, and apply that of Subalternation to the rest.

Opposition.

Opposition is of three kinds—Contradictory, Contrary, and Sub-contrary.

Contradictory opposition is the relation which exists between two propositions that simply contradict each other. One of them therefore must be false, and the other true. For each of them must be either true or false, and if one of them is true, the other which asserts that it is false, must be itself false; and if one of them is false, the other which asserts that it is false, must be itself true.

Contradictories differ from each other in the quality of their copulas; for as one denies what the other asserts, the former must be affirmative, and the latter negative.

They differ also in the quantity of their subjects. For a contradictory contains nothing more than the falsity of that proposition to which it is opposed, and must therefore express the change that takes place in the relation of the terms, when its opposite first ceases to be true. But the first formal change affecting the truth of an universal proposition, affects only a part of its subject; as that cannot be true of the whole, which is false of a part. And the first formal change affecting the truth of a particular proposition, must necessarily affect the whole of the subject; as that cannot be false of every part, which is not at the same time false of the whole.

These remarks will be better understood by the assistance of the accompanying diagrams:

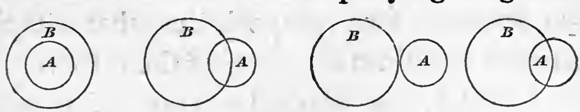


Fig. 1. *a*. Fig. 1. *b*. Fig. 2. *a*. Fig. 2. *b*.

Fig. 1. *a*, represents the proposition, All A is B; and Fig. 2. *a*, represents the proposition, No A is B. Now it is very evident that the first changes which will render these propositions no longer true, must take place when the sphere of A first begins to emerge from

that of B, as in Fig. 1. *b*, or when it first begins to impinge on it, as in Fig. 2. *b*. And the propositions which express these changes must necessarily be the respective contradictories of the original propositions. 'Some A is not B' will therefore contradict the proposition 'All A is B,' and 'Some A is B,' the proposition 'No A is B.' Contradictories are therefore opposed to each other both in quality and quantity: the universal affirmative A is opposed to the particular negative O; and the universal negative E, to the particular affirmative I. Example: 'Some Englishmen are as light-hearted as Frenchmen;' 'No Englishmen are as light-hearted as Frenchmen.'

(35.) Contrary opposition exists between two propositions which contain each other's contradictory, and something more besides. Hence it follows that they cannot be both true, but may be both false: for if one is false, that part of its contrary which merely contradicts it, must be itself true; but the other part, or surplus statement, may be either true or false.

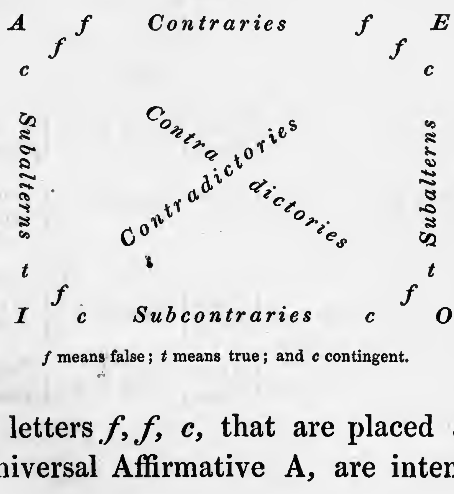
As contraries state more than each other's contradictories, the quantity of their subjects must be greater, and therefore distributed; whence it follows that A and E are the only propositions between which this species of opposition exists. Examples of this opposition are—'All smuggling is dishonest'; 'No smuggling is dishonest.' 'All Englishmen are haughty'; 'No Englishmen are haughty,' &c.

(36.) Subcontrary opposition, as its name in some measure indicates, is the opposition of propositions contained under contraries. The subjects of the subcontraries must of course be less than the subjects of the contraries which contain them, and are therefore undistributed. As I and O the sub-contraries are contradictories of the two contraries E and A, and as the contraries cannot be both true, but may be both false, it follows that the subcontraries may be both true, but cannot be both false. Examples are, 'Some men are liars'; 'Some men are not liars': both of which are true. 'Some men are perfect'; 'Some men are not perfect': of which propositions one is false.

Subalternation.

(37.) Subalternation is the relation which exists between an universal proposition, and the particular that is contained in it. The former is usually called the Subalternant, the latter the Subalternate.

The truth of the subalternant necessarily involves the truth of its subalternate, as what is true of the whole, must be also true of a part; and the falsity of the subalternate involves the falsity of the subalternant, as what is false of a part cannot be true of the whole. But the converse of these propositions does not hold; for what is true of a part, is not necessarily true of the whole, and what is false of the whole, is not necessarily false of a part. Hence, we cannot infer the truth of the universal from that of the particular subalternate to it, nor the falsity of the particular from that of its subalternant universal. Examples are—'All dogs are animals,' whence it follows, 'Some dogs are animals.' 'Some politicians are no better than they should be,' whence it cannot be logically inferred that 'All politicians are no better than they should be.' The annexed scheme is usually employed to elucidate the relations of categorical propositions:—



f means false; *t* means true; and *c* contingent.

The letters *f*, *f*, *c*, that are placed against the Universal Affirmative A, are intended to represent that A is false when E or O are true, and contingent when I is true, and similarly of the rest of the table.

Perhaps the following table is more immediately intelligible —

			If one is	The other is
Opposition.	Contradictions.	Between A and O.	True	False.
			False	True.
		Between E and I.	True	False.
			False	True.
	Contrary.	Between A and E.	True	False.
			False	Contingent.
	Subcontrary.	Between I and O.	False	True.
			True	Contingent.

Subalternation.	Between A & I.	If A is	True	Then I is	True.
			False		Contingent.
		If I is	True	Then A is	Contingent.
			False		False.
	Between E & O.	If E is	True	Then O is	True.
			False		Contingent.
		If O is	True	Then E is	Contingent.
			False		False.

Conversion.

(38.) Conversion is the transposition of the terms of a proposition, in such a manner that the subject becomes the predicate, and the predicate the subject. Conversion is of two kinds, simple and limited. The former is the simple transposition of the terms, retaining the previous quantities; the latter requires the limitation of the subject.

In all conversion, no term must be distributed in the converted proposition, that is not distributed in the original proposition: hence E and I are simply convertible, as in E both terms are distributed, and in I, neither. No term then is distributed in the converted proposition, that was not previously distributed in the original proposition. But A is not simply convertible, as its predicate is undistributed, and would become distributed by simple conversion. It admits however of limited conversion, and then becomes I; for in I, no terms are distributed, and therefore none can be distributed in it, which are undistributed in the original proposition. E may also be converted by limitation, as well as simply, and then becomes O. O can never be converted, as its subject is undistributed, and therefore can never become the predicate of a negative proposition, which is always distributed.

CONVERSION.

Simple.	Limited.
E...... to .. ...E	A...... to I
I... ... to I	E...... to ......O

Examples :—

No cats are cows. All dogs are animals.
No cows are cats. Some animals are dogs.

SECTION III.

HYPOTHETICAL AND DISJUNCTIVE PROPOSITIONS.

(39.) An hypothetical proposition consists of two judgments and a copula.

The judgments constitute the matter of the proposition, and the copula expresses their relation, or that function of the understanding by which they are conjoined in one consciousness.

The whole proposition represents that one of these judgments which is called the antecedent, contains all the necessary grounds for the truth of the other, which is accordingly called the consequent.

The most general form of these propositions is the following. 'If A is B, C is D,' in which the judgment 'A is B' is the antecedent, 'C is D' the consequent, and the word 'if' the copula.

(40.) As an hypothetical proposition merely asserts such a connexion between two judgments, that the truth of the one may always be inferred from the truth of the other,* it follows that both the form and the matter of these judgments constitute the matter alone of the hypothetical, and cannot be the means of introducing any variations into its form. Neither can such variations arise from the form of the copula; for the conception of a law connecting a consequent with its grounds, may be thought, or not thought, with regard to any judgments, but will neither admit of a negative quality, or of a quantum or degree. Hence there is but one form for hypothetical propositions, and all the variations that take place in them must be considered as affecting their matter alone. For instance, the proposition 'if some A is B, some C is D,' does not differ in form from the proposition 'if all A is B, all C is D.' It is true that the categorical judgments in these examples have formal differences, as in the former they are both particular, and in the latter both universal. But in either case, such a relation is asserted to exist between them, that the truth of the one must be the invariable consequence

* The understanding merely asserts a relation of such a nature between the judgments that the truth of the one *may* be inferred from that of the other. But in order that this result should be *actually* inferred, another judgment is necessary which shall assert the truth of the antecedent, and then the *reason* will deduce the truth of the consequent.

of the truth of the other; and it is this universality alone, and not that of the judgments themselves, with which the form of an hypothetical proposition is concerned.

(41.) From these observations it will readily be seen that an hypothetical proposition cannot have an hypothetical contradictory: for a contradictory states nothing more than the falsity of that proposition to which it is opposed, and, if this latter is an hypothetical, merely denies that there is any law by which the truth of the consequent can be rightly inferred from that of the antecedent. But this does not establish any similar relation between any other two judgments, and is not therefore hypothetical in its nature. The contradictory must accordingly be a categorical proposition, in which I predicate of my consciousness, that it does not give objective validity to such a relation between two judgments, that one contains the grounds for the other.

(42.) Many logicians have considered the hypothetical proposition as merely another form of a categorical; and this error has been rather favoured by the seeming ease with which an hypothetical may be put under such a categorical form as shall answer all the logical ends of the original proposition. But the two propositions have been shewn by

Kant to be based on fundamentally different acts of the understanding, which ought never to be confounded with each other. For instance, the hypothetical proposition 'if A is, B is,' appears to be fully expressed in the categorical form, 'all the cases of A being, are cases of B being;' and so far as any logical deductions from either one or the other are concerned, the propositions are equivalent. But there is this fundamental difference between them: the former represents the dependence of one judgment upon grounds contained in the other, and therefore enunciates a law to which the understanding has accorded its assent as being universally valid; whereas the latter form either merely asserts the fact 'that all the cases of A being are cases of B being,' without superadding the conception that they are so universally by a law; or else, if it means that 'all possible cases of A being are necessarily cases of B being,' it has only placed what constituted the form in the hypothetical proposition in the matter of the categorical, but has by no means transfused the virtue of the form of the one into the form of the other. In fact, this method is no more a formal reduction of an hypothetical to a categorical proposition, than if we merely said, "the hypothetical proposition 'if A is, B is,' is true."

Disjunctive Propositions.

(43.) Disjunctive propositions consist of any number of judgments, which they disjoin in the relation of reciprocal dependence. Hence the whole conception of their possibility is divided between them, and the truth of either may be inferred from the falsity of the others, or inversely, the falsity of all the others from the truth of one. Their general form is 'either A is B, or C is D.' All the observations that have been made on hypothetical propositions, in order to prove the impossibility of any variations in their form, apply with equal force to the disjunctive: and as the disjunctive expresses a distinct function of thinking, any attempt to bring it into a categorical form will meet with no better success than in the case of hypotheticals. It is true that we can express the meaning of a disjunctive in a categorical form; but then, what constitutes the form in the one proposition must be introduced in the matter of the other. For instance, the disjunctive proposition 'A is either B or C' may be expressed thus: 'the cases of A being B are identical with the cases of A being not C.' But here we see that the completeness of the division, which is the real principle of disjunctive propositions, is expressed in the conception of the identity of one judgment with the negation of another, which constitutes the matter of the categorical proposition. And hence, as in the case of the hypothetical, we have no more succeeded in the transformation of the disjunctive to the categorical form than, if we had said, "the proposition 'A is either B or C' is true."

(44.) Disjunctives resemble hypothetical propositions in not admitting a contradictory of the same form with themselves: for the contradictory of a disjunctive must merely contain the falsity of that proposition, and therefore denies that a certain cognition is exactly divided out among the members of the disjunction, but by no means gives a new and correct division. Hence the contradictories to both these forms of propositions are simply categorical in their nature.

SECTION IV.

SYLLOGISM.

(45.) Kant has defined syllogism as that function of thinking by which we derive one judgment from another, and has in this manner included all those judgments which may be formally derived from another single judgment by the methods of opposition, subalternation, &c.

These syllogisms, styled *immediate* from their wanting a middle term, he arranges under the title of syllogisms of the understanding, and the propriety of such a classification, as far as the meaning alone is concerned, is evident from the nature of the faculty employed. But as it is rather questionable whether the derivation of the word will bear out this general acceptation, we shall follow the usual practice of logicians, and restrict its signification to those syllogisms which Kant defines as the syllogisms of reason.

Syllogism, then, is that function of the reason by which a third judgment is derived from the union of two others, neither of which contain it when taken separately.

(46.) The first division of which syllogism is susceptible, is determined by the nature of the judgments in respect of their moments of relation. These have already been shewn to be three—the categorical, hypothetical, and disjunctive; and as there are a great many combinations of judgments of these several moments, from all of which other judgments may be deduced, it follows that there must also be a great many different syllogistic forms of ratiocination. But as the principles involved in all of them are the same as those in the three simplest forms (which are usually named after the three moments), it will be sufficient to investigate the laws which regulate these alone, and, with one or two exceptions, leave the consideration of the rest to the ingenuity of the reader. In these three species of syllogism, the categorical, the hypothetical, and the disjunctive, a categorical conclusion is deduced from one judgment of categorical form, and another of that form from which the syllogism takes its name. We shall now proceed to the separate consideration of each of these three forms of reasoning.

Categorical Syllogism.

(47.) In this syllogism the two given judgments, which are also called premises, and the derived judgment or conclusion, are all categorical propositions. The nature of this species of argument may be popularly stated as follows: 'If two cognitions are severally compared with a third cognition, (that is, objectively the same in each case,*) they may afterwards be compared † with one another.‡

* Perhaps an objection may be brought against this exposition, on the grounds of its admitting negative premises. But it must be observed that when both premises are of this quality, the real middle is an external sphere, and consequently undistributed, or virtually two middles. And this fallacy is quite excluded by the clause in parentheses.

† The affirming or denying one cognition of another is what is here intended by comparison. For if two cognitions are referred to the consciousness simultaneously, they must either be thought as belonging to the same subject or substratum, in which case one may be affirmed of the other; or as not belonging to the same, in which case one may be denied of the other.

‡ As it is our present object to discover the universal laws of all categorical syllogism, we have preferred this simple though not very elegant definition to the celebrated '*dictum de omni aut nullo*' of Aristotle, which is merely a particular case of it, and may be stated thus: 'What is affirmed or denied of all, is affirmed or denied of each;' but this would only have given a particular class of categorical syllogisms, and is therefore insufficient for our present purpose. Were we however to admit either the empirical considerations of the use of the reason, or the metaphysical considerations of the conceptions of the understanding, we should then be justified in confining our attention to the dictum alone. For in the actual use of the reason we discover the fact, that this form of syllogistic ratiocination is at once more natural

This is the principle of all categorical syllogism; and whenever the conditions are really answered, and the comparison made according to certain laws which we are about to determine, the conclusion is necessary, and the reasoning incontrovertible. Let us take as an example, 'Animals of the same species are supposed to have originated from the same pair; all dogs are animals of the same species, therefore all dogs are supposed to have originated from the same pair.' Here, dogs and animals supposed to have originated from the same pair, are severally compared with animals of the same species, and are afterwards compared with one another.

(48.) We must now determine those laws, in conformity with which the various forms of categorical propositions may be so combined as to answer the required conditions of syllogism.

For this purpose we must not give a mere aggregate of rules, even though those rules should be in themselves sufficient. This method, which has been adopted in most (if not

and intelligible than the rest, and in the nature of one of the conceptions of the understanding, we can discover the reason of the fact. But we must not regard what lies out of the field of Formal Logic, and have therefore given a definition which will include all the forms of categorical syllogism that are possible in theory, however awkward some of them may be in practice.

all) works on Logic, is exceedingly unscientific, as it only shews that certain rules are requisite and sufficient to provide against certain forms of paralogism, but by no means proves that these are the only forms to which a syllogism is exposed. Now the method of exhaustion will secure our reasoning against all fallacy, so far at least as mere Formal Logic is concerned.* We shall therefore dissect categorical syllogism into its ultimate elements, and discover what rules are necessary and sufficient for the prevention of fallacy in each of them.

(49.) Every categorical syllogism contains three terms—the major term, the minor term, and the middle term; and three propositions—the major premiss, the minor premiss, and the conclusion. In the major premiss, the major term is compared with the middle term. In

* It may be as well to notice here, the very prevalent custom of introducing into treatises on Logic, the consideration of material as well as formal fallacies. In all the works on this science that have come under our observation, (with the exception at least of Kant's,) nearly the first rule for syllogism is to the effect that the middle term must not have different meanings in the two premises. Now Logic merely considers the formal laws of reasoning, but has nothing whatever to do with its matter; and the introduction of such a rule as this in a work on that science, is something like beginning a treatise on Geometry with an injunction to the student to draw his circles correctly: as if mathematical reasoning could be at all affected by the perfection of the diagram.

the minor premiss, the minor term is compared with the middle term. And in the conclusion, the major and minor are compared together. The major term is always the predicate, and the minor the subject of the conclusion. Hence may be deduced the following rules for distinguishing the different terms and premises in any given syllogism :—

1st. The term that is common to the two premises is the middle term.

2nd. The term that is the predicate of the conclusion is the major term ; and the premiss that contains it, the major premiss.

3rd. The term that is the subject of the conclusion is the minor term ; and the premiss that contains it, the minor premiss.

The three propositions are usually placed in the following order : the major premiss, the minor premiss, and the conclusion. And the particular form of a syllogism, as far as it depends on the particular categorical forms of its three propositions, is termed its mood : though this name is also given to any ternary combination of the symbols A, E, I, O, without reference to its conformity to the syllogistic laws.

(50.) For the simplification of the subject we shall divide it into the three following questions:—

1st. What forms of premiss are sufficient for a conclusion of the reason,* without considering its capability of being expressed in any of the four categorical forms, (*i.e.* A, E, I, O)?

2nd. When can the conclusion be expressed in any of the legitimate forms, and when can it not?

3rd. What laws are sufficient and necessary to secure the legitimacy of the conclusion?

As the first of these questions excludes all reference to the categorical form of the conclusion, and only seeks for premises that give a conclusion valid for the reason, it does away with the distinction of subject and predicate, so far as the conclusion is concerned; and therefore with the distinction of major and minor premiss. Hence, in this part of our investigation, both premises are on exactly the same footing.

* As it is impossible that the reader should understand the following pages, unless he has a clear conception of what is meant by a conclusion 'possible for the reason, but not expressible in any of the four legitimate categorical forms,' we subjoin the following example. If the premises are 'some B is A, some C is not B,' the reason may logically deduce that some C is not some A. But this conclusion is not in one of the four legitimate forms, and is therefore styled a conclusion only for the reason.

We may also discard all considerations of the extremes. For any peculiarities in the form of these terms, can merely require corresponding peculiarities when they recur in the conclusion, as they constitute the matter only of that judgment. They may therefore affect the nature of the conclusion, but cannot affect the possibility of its existence.

All that remains then for our consideration, is the middle term, and the copula in both the premises.

(51.) Now the middle term admits of no variations but those of quantity. Every law therefore regarding it must respect this, and this alone. And we find accordingly that the middle term must be distributed in at least one of the premises.

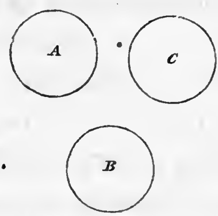
For if the middle term is undistributed, a part of it only is compared with each premiss. And as it cannot be formally known that the major and minor are compared with the same parts, the middle term becomes virtually two terms, and the major and minor terms cannot be considered as formally compared with the same. But it is not necessary that the middle term should be distributed in more than one premiss. For if the whole middle term is compared with one of the extremes in one premiss, and only a part of it with the other extreme in the other premiss, that part must be compared with both extremes, and in this case therefore the middle term cannot be considered as two terms, or ambiguous.

(52.) The first law for the quality of the premises may be derived from the law for the distribution of the middle term. This could not be the case if every possible comparison of the spheres of two terms, both internal and external, positive and negative, found a corresponding expression among the categorical forms: for in that case, a law regarding quantity could not affect quality. This however has been shewn not to be the case, as external spheres are never made subjects; and hence arises the possibility of deducing from the law for the distribution of the middle term the following law for the quality of the premises.

No conclusion is possible from two negative premises. For in premises of this form, the extremes are each placed in the external sphere of as much of the middle term as is compared with them, and consequently the external sphere becomes the real term with which both extremes are compared. But the external sphere is not distributed in any categorical form, and consequently the virtual middle term would be undistributed, and no conclusion possible.

No. B is A. No. C is B.

Fig. 1.



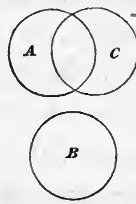


Fig. 2.

In the accompanying diagram we see that A and C are each placed in the external sphere of B; but as the external sphere is undistributed in both propositions, we do not know that A and C are compared with the same parts of it, and cannot therefore compare them with one another. In Fig. 1. we find that 'no C is A,' and in Fig. 2. the contradictory 'some C is A.'

As the external spheres of conceptions do not admit the variations of quantity, or bear the same mutual relations in any categorical propositions as the internal spheres, they are never understood unless they are expressly mentioned. It is for this reason that the law for the distribution of the middle term is generally understood to relate to the internal spheres only, and that the law against two negative premises cannot be subsumed under it as a particular case. It is therefore given as a separate law, that at least one premiss must be affirmative.

The extreme term contained in the affirmative premiss will of course agree and coincide with the middle term, and will therefore agree or disagree with all with which the middle term agrees or disagrees in the other premiss. Hence the latter premise may be either affirmative or negative, and there is no other law for the quality of the premises, than that given above.

(53.) We have now exhausted the elements of the premises, (for we have shewn that our present inquiry does not involve the distinction of the premises, or the consideration of the major and minor terms,) and have arrived at the following laws for the middle term, and the quality of the two copulas.

In order that a conclusion for the reason, though not necessarily categorically expressible, be possible from any particular forms of premises, it is only necessary—

1st. That these categorical forms contain at least one distributed term.

2nd. That one of these forms be affirmative.

As at present there is no distinction between the premises, we are not considering permutations but combinations; and as each proposition may be combined with itself, the number will be $\frac{5 \cdot 4}{1 \cdot 2} = 10$, of which we shall find by the following table, that 6 are unobjectionable, that 3 are excluded by their negative premises, and 1 by the want of a distributed term.

	Conclusion.	Forms of premises from which no conclusion is possible. No distributed term.	Forms of premises from which no conclusion is possible. No affirmative term.	Forms of premises from which a conclusion at least for the reason, is possible.
A combined with	A	——	——	A A
	E	——	——	A E
	I	——	——	A I
	O	——	——	A O
E combined with	E	——	E E	——
	I	——	——	E I
	O	——	E O	——
I combined with	I	I I	——	——
	O	——	——	I O
O combined with...	O	——	O O	——

The above table is only intended to state, that some conclusion, though perhaps not categorically expressible, is sometimes possible in each of the six forms in the last column, but always impossible in the four forms in the other two columns. And it must be observed that we can only say sometimes possible; for though all these six forms contain one or more distributed terms, yet if the middle is not one of those terms, it will be undistributed, and no conclusion possible. For instance, let B be the middle term, and A I the form of the premises, if we say 'All B is A,' 'Some C is B,' the conclusion that 'Some C is A,' is strictly deducible. But if we say 'All A is B,' 'Some C is B,' therefore 'Some C is A,' our conclusion would not be logically correct, as the middle term B would be undistributed in each premiss. We repeat, therefore, that the last table merely indicates the possibility of deducing a conclusion for the reason from some premises of certain forms, and the impossibility of deducing any conclusion whatever from premises of certain other forms.

(54.) We may now dismiss the first part of our present investigation, and consider the second question. When can the conclusion be expressed in any of the legitimate forms, and when can it not?

As the variations of quality have not been allowed to affect the terms * in the categorical forms of the admissible premises, the conclusion from them that is possible for the reason, but not categorically expressible, cannot be so restricted, on account of its being valid for

* Perhaps the following explanation will be more easily understood than the text. As external spheres have not been admitted as terms in the forms of premises given in the above table, it is impossible that they should appear in the conclusion; the impracticability therefore of these conclusions cannot arise from the appearance of an external sphere in either of their terms.

only the external sphere of a conception. It must therefore belong to one of the four rejected forms (mentioned in Art. 32,) whose predicates do not as functions of the copula follow the same law with the accepted forms. In other words, the conclusion must belong to one of these four forms, in which the predicate of an affirmative is distributed, and of a negative undistributed. But it cannot belong to a proposition of the former class, as in that case one of the legitimate categorical propositions, in which the predicate is undistributed, would state less than this sound conclusion, and would therefore be contained in it, and be itself true; but this is contrary to the hypothesis. Hence the only case that remains in which a conclusion can be valid for the reason, but not categorically expressible, is when a conclusion whose quality is negative has a predicate whose quantity ought to be undistributed. And we shall accordingly find that the rule for the quantity of the predicate of the conclusion will exclude all those premises which give conclusions only possible for the reason.

(55.) Our next object is to determine what laws are sufficient and necessary to secure the legitimacy of the conclusion.

For this, let us examine the three elements of the conclusion—the subject, the predicate, and the copula.

As the only variations of which the terms admit are those of quantity, the laws concerning them must respect that, and that only.

As, moreover, logic can only consider the formal quantity of a term, either extreme may contain in the conclusion the same quantity as in its premiss, but not more. Hence the only rule for the terms is this:

If the major or minor terms are undistributed in the premises, the predicate and subject must be respectively undistributed in the conclusion.

When this rule is violated in a syllogism whose major term is undistributed in the major premiss, but whose predicate is distributed in the conclusion, the resulting fallacy is called an illicit process of the major.

When the minor term is undistributed in the minor premiss, and the subject is distributed in the conclusion, the resulting fallacy is called an illicit process of the minor.

(56.) With regard to the quality of the copula, the only law for the conclusion is this:

If either premiss be negative, the conclusion must also be negative; but if both premises are affirmative, the conclusion must also be affirmative.

For as one premiss must be affirmative, the extreme term which it contains must agree with the middle term in that premiss, and therefore agrees or disagrees in the conclusion, with whatever the middle term agrees or disagrees with in the other premiss. Whatever therefore is the quality of this latter premiss must also be the quality of the conclusion.

(57.) The elements of a categorical syllogism have now been completely exhausted, and it is absolutely certain that if all the given rules are preserved inviolate, a formally incorrect conclusion can never be obtained.

All then that is necessary to ensure the legitimacy of a syllogism, is comprised in the five following rules :—

1. There cannot be more than one negative premiss.
2. If there is one negative premiss, the conclusion is negative; if there is no negative premiss, the conclusion is affirmative.
3. The middle term must be distributed in at least one of the premises.
4. If the predicate of the conclusion is distributed, the major term must be distributed in the major premise.
5. If the subject of the conclusion is distributed, the minor term must be distributed in the minor premiss.

The rules that have been given for the premises will only ensure a conclusion possible for the reason, and hence it will follow that the rules for the conclusion will in certain cases affect the premises also. For if the conclusion is negative, its predicate must be distributed; and therefore, by Rule 4, the major term must also be distributed in the major premiss.

(58.) Although the five given rules are quite sufficient in themselves, yet two others are derived from them, with which the student should be well acquainted, as they are of very easy application. They are—

1. No conclusion can be drawn from particular premises.
2. Only a particular conclusion can be drawn where one of the premises is particular.

As the middle term must always be distributed in one premiss at least, and as no term can be distributed in the conclusion that is not distributed in the premises, it follows that there must be at least one more distribution in the premises than in the conclusion. But there can be only one distribution in the predicates of the premises, as there can be only one negative premiss, and in that case there must be a distribution in the predicate of the conclusion, as that must also be negative. Hence it follows that the number of distributions in the subjects of the premises must exceed by at least unity, the number of distributions in the subject of the conclusion. If then both premises are particular, there will be no distributions in the subjects of the premises; and as the above condition will not be answered, there can be no conclusion. And if one premiss is particular, there will be only one distribution in the subjects of the premises, and therefore there can be none in the subject of the conclusion, *i. e.* the conclusion must be particular.

(59.) We shall now be able to extend our table for the premises only, and determine the moods in which a syllogism, perfect in all its parts, is possible. It must however be remembered, that as the table of sound premises paid no respect to the distinction of major and minor premiss, the order of the premises was there indifferent. But as the conclusion is here considered in its legitimate categorical forms, the distinction holds, and we must therefore examine the two permutations of each of those forms of premises.

Premises.	Conclusion.	Moods in which Paralogism is unavoidable.			Moods in which Syllogism is possible.
		Illicit Process of Minor.	Undistributed Middle.	Illicit Process of Major.	
A A with	A	—	—	—	A A A
	I	—	—	—	A A I
A I with	A	A I A	—	—	—
	I	—	—	—	A I I
I A with	A	I A A	or I A A	—	—
	I	—	—	—	I A I
A E with	E	—	—	—	A E E
	O	—	—	—	A E O
E A with	E	—	—	—	E A E
	O	—	—	—	E A O
A O with	E	A O E	or A O E	or A O E	—
	O	—	—	—	A O O
O A with	E	O A E	or O A E	or O A E	—
	O	—	—	—	O A O
E I with	E	E I E	—	—	—
	O	—	—	—	E I O
I E with	E	—	—	I E E	—
	O	—	—	I E O	—
I O with	E	I O E	or I O E	I O E	—
	O	—	—	I O O	—
O I with	E	O I E	O I E	or O I E	—
	O	—	O I O	or O I O	—

	SUBJECT.	PREDICATE.
A	Distributed.	Undistributed.
E	Distributed.	Distributed.
I	Undistributed.	Undistributed.
O	Undistributed.	Distributed.

The following example will shew how the above table should be studied.

Let us take the mood O A E. In this mood the predicate is distributed in the major premiss, and the subject in the minor premiss, and both subject and predicate in the conclusion. Hence it follows that either some term must be distributed in the conclusion which was not distributed in the premises, or else the middle term cannot be distributed in either premiss. We cannot therefore determine at once which form the fallacy will take, but may be quite certain that there must be either an illicit process of major or minor, or else an undistributed middle. Again, in the mood O I E, both subject and predicate are distributed in the conclusion, whereas no term is distributed in the minor premiss, and it therefore follows that there must be an illicit process of the minor. It is also evident that the middle term cannot be distributed in the minor premiss, and that if it is distributed in the major premiss, the major term must be undistributed, and consequently there must be a fallacy either of undistributed middle or illicit major.

(60.) It will be seen from the last column in the table, that there are eleven moods in which syllogism is possible, or in which the syllogistic rules may be observed. But it does not follow that the rules of syllogism are necessarily observed in these eleven moods, as paralogism is possible in all but two of them;* for in the premises of some of the moods we have but one distributed term, and in others two or more; and unless the position of the terms is such as to obey the laws for the distribution of the middle term, and prevention of illicit process, fallacy will still be the inevitable result.

The position of the terms will accordingly give rise to a further classification of categorical syllogisms. And as each premise admits of two permutations, in one of which the middle term is predicate, and in the other subject, there will be four combinations determined by its position, which are usually called the four syllogistic figures.

In the first figure, the middle term is the subject of the major premiss, and predicate of the minor.

In the second figure, the middle term is the predicate of both premises.

In the third figure, the middle term is the subject of both premises.

In the fourth figure, the middle term is the

* These two moods are E A O, and E I O, which are true in all the four figures.

predicate of the major, and subject of the minor premiss.

The following scheme will enable the reader to understand the distinction at a glance.

Let S represent the minor term (or subject of conclusion), M the middle term, and P the major term, (or predicate of conclusion).

	1st Fig.	2nd Fig.	3rd Fig.	4th Fig.
Major Premiss	M — P	P — M	M — P	P — M
Minor Premiss	S — M	S — M	M — S	M — S
Conclusion	S — P	S — P	S — P	S — P

In each of these figures several moods are sound, and others unsound. For instance, the mood A A A, which is sound in the first figure, in the second figure would have its middle undistributed in each premiss, while the mood A E E, which is sound in the second figure, would contain an illicit process of the major in the first figure. The accompanying table will shew what moods are sound, and what unsound, in each figure, and in the latter case, the peculiar form of the fallacy.

TABLE OF CATEGORICAL SYLLOGISM.

Name of Mood.	Form of Mood.	1st Figure.	2nd Figure.	3rd Figure.	4th Figure.
	Subject Premiss	M — P S — M S — P	P — M S — M S — P	M — P M — S S — P	P — M M — S S — P
A A A	D — U D — U D — U	Sound	Undistributed Middle	Illicit Minor	Illicit Minor
A A I	D — U D — U U — U	Sound	Undistributed Middle	Sound	Sound
A I I	D — U U — U U — U	Sound	Undistributed Middle	Sound	Undistributed Middle.
I A I	U — U D — U U — U	Undistributed Middle	Undistributed Middle	Sound	Sound
A E E	D — U D — D D — D	Illicit Major	Sound	Illicit Major	Sound
A E O	D — U D — D U — D	Illicit Major	Sound	Illicit Major	Sound
A O O	D — U U — D U — D	Illicit Major	Sound	Illicit Major	Undistributed Middle
E A E	D — D D — U D — D	Sound	Sound	Illicit Minor	Illicit Minor
E A O	D — D D — U U — D	Sound	Sound	Sound	Sound
E I O	U — D D — U U — D	Sound	Sound	Sound	Sound
O A O	U — D D — U U — D	Undistributed Middle.	Illicit Major	Sound	Illicit Major

In the first column of this table, the name of the mood is given. In the second, its form with regard to the distribution of its terms, where D signifies distributed, and U signifies undistributed. In the other four columns, which are headed by the forms of the four figures, the nature of each mood in each figure is expressed by the word 'sound,' if the reasoning is unobjectionable, and if not so, by the name of its particular fallacy.

By comparing the form of one of the moods with the general form for any one of the figures given in the upper line of the table, we shall see if any D in the one corresponds in position with an M in the other, in which case the middle will be distributed. We must also observe if either S and P in the conclusion correspond with a D in the form of the mood, and if so, they must correspond respectively with a D in the premises, or else there will be an illicit process. With this explanation there will be no difficulty in understanding the manner in which the table is formed. For instance, let us take the mood A A A. We find in the form of the mood that the subject in each premiss is distributed, and upon turning to the form of the first figure, we find that the subject of the major premiss is the middle term, and are therefore justified in concluding that the mood A A A in the first figure has its middle distributed. Again, we find that the only term in the conclusion corresponding to a D, is S, which also corresponds to a D in the premises. We are therefore certain that no term is distributed in the conclusion that is not also distributed in the premises, or, in other words, that there is no illicit process; and as the middle is distributed, it follows that the syllogism A A A in the first figure is sound.

If however we compare the form of the mood A A A, with the form of the third figure, we find that S the subject of the conclusion corresponds to D, and is therefore distributed; but that S the predicate of the minor premiss corresponds to U, and is therefore undistributed: whence it follows that A A A in the third figure has an illicit process of the minor.

(61.) Several laws may be obtained for each figure, by an examination of their peculiar forms.

In the first figure, the minor premiss must be affirmative: for if it were negative, the major premiss must be affirmative, and therefore have its predicate, which is the major term, undistributed. But the conclusion must be negative, and therefore its predicate would be distributed, which would accordingly give rise to an illicit process of the major.

As the minor premiss must be affirmative, the middle term, which is its predicate, must be undistributed in that premiss, and therefore distributed in the other premiss, in which it holds the place of subject. Hence it follows that the major premiss must always be universal in the first figure.

In the second figure, the middle term is the predicate in each premiss; and as it must be distributed in one of them, one premiss must be negative, and therefore the conclusion also. In the third figure the minor premiss must always be affirmative for the same reasons as in the first figure; and therefore its predicate (which is the minor term), being undistributed, the conclusion must be particular, for were it universal, there would be an illicit process of the minor. There are several other rules which may be derived from the primary laws, but as all of them may be more simply evolved by means of algebraical symbols, we shall postpone their consideration for the present.

Transformation of the figures of Syllogism.

(62.) It has been already observed, that Aristotle's dictum (or the first figure) is the form in which our reasoning appears the most natural, and is most easily comprehended. This is the reason why the other three figures have been considered as unnatural deviations from it, and that laws have been laid down for the reduction of all syllogisms to the original form of the first. But as this peculiarity of the dictum is based on the laws of an understanding-conception, and as Formal Logic can pay no attention to a distinction which originates in grounds that lie out of its field, it will be more correct to consider the laws for transforming a syllogism from any one figure into another, especially as they will include the laws for reduction (or transformation to the first) as a particular case. As the distinction of figure depends entirely on the position of the terms, conversion, by which alone this can be altered, is the only method of transformation. But this operation is not always possible, as in many of the moods the propositions are not of a convertible form: *e. g.* it is impossible to transform the mood A E O from the second figure into the first; for the major premiss is not simply convertible, and limited conversion would give the mood I E O, which contains an illicit process of the major. The following table can therefore only give those conditions which must be answered by any syllogism in each figure for its transformation into any other; but in order to know if a syllogism is capable of answering these conditions, it will be necessary to examine its particular mood.

TRANSFORMATION TABLE.

Figure to be transformed.	Propositions to be converted for transformation into the			
	First figure	Second figure	Third figure	Fourth figure
First		Major	Minor	Conclusion * or Major & Minor
Second	Major		Major and Minor	Minor or Major & Conclusion
Third	Minor	Major and Minor		Major
Fourth	Conclusion or Major & Minor	Minor	Major	

(63.) It appears from the table at Art. 60, that there are six moods in each figure which answer all the required conditions for a sound categorical syllogism. Five of these have been very generally rejected by logicians, on the grounds of their having particular conclusions, although universal are warranted by the premises.† But this regard to a more prac-

* It is hardly necessary to observe, that whenever the conclusion is converted, the major and minor terms, which are respectively its predicate and subject, must be interchanged, and therefore the premises that contain them.

† These five are the moods A A I in the first figure, E A O in the first and second, and A E O in the second and fourth.

tical utility is always very unphilosophical in an *à priori* science, and especially if it be at all destructive of the symmetry of our results. The mathematician will readily acknowledge his obligations to symmetry for the light it throws upon truths already known, as well as for its efficiency as an organum for the discovery of new ones: and the same regard should be paid to it in every pure science, but more particularly in one which is intended to increase the accuracy and rigid strictness of our thought. In the present case there are six syllogisms in each of the four figures, and this reduction would have only nineteen, four in the first and second, six in the third, and five in the fourth. Some of the old schoolmen carried their veneration for Aristotle so high, as to reject the fourth or Galenic * figure entirely, and thus reduced the whole number to fourteen. But this reduction is nearly as unphilosophical as the other, for the only rational ground upon which it can rest will apply to the second and third figures also, though not perhaps to the same extent. We shall therefore submit to the reader the whole

* The fourth figure was first recognised by Galen, the great medical philosopher, who flourished in the second century.

twenty-four syllogisms, collected under their respective figures.*

Fig. 1. A A A, A A I, A I I, E A E, E A O, E I O.
Fig. 2. A E E, A E O, A O O, E A E, E A O, E I O.
Fig. 3. I A I, A A I, A I I, O A O, E A O, E I O.
Fig. 4. A E E, A A I, I A I, A E O, E A O, E I O.

Hypothetical Syllogism.

(64.) One premiss of a hypothetical syllogism is a hypothetical proposition; the other premiss is a categorical proposition, and either asserts

* There is a barbarous practice of naming the various forms of a categorical syllogism by certain words which constitute mnemonic hexameters. A subject which should always be treated in a rational point of view (*i. e.* as appertaining to the reason), is in this manner degraded to a mere historical record of the deductions of others, and draws upon the memory alone. Having entered my protest against these lines, I still think it proper to subjoin them, in order that the student may understand the allusion when he hears such phrases as 'a syllogism in barbara,' &c. The vowels contained in the words of these lines give the names of the moods, and the consonants refer to their other peculiarities,—such, for instance, as the methods of reducing them to the first figure. The two first lines give the nature of the four categorical propositions.

Asserit A, negat E, verum generaliter ambæ.
Asserit I, negat O, sed particulariter ambæ.

Barbara, Celarent, Darii, Ferioque, prioris
Cesare, Camestres, Festino, Baroko, secundæ.
Tertia, Darapti, Disamis, Datisi, Felapton,
Bokardo, Ferison habet: Quarta insuper addit
Bramantip, Camenes, Dimaris, Fesapo, Fresison:
Quinque Subalterni, totidem Generalibus orti,
Nomen habent nullum, nec, si bene collegis, usum.

the antecedent, or denies the consequent. In the former case, which is called the modus ponens, the conclusion infers the truth of the consequent; in the latter case, which is called the modus tollens, the conclusion infers the falsity of the antecedent. The general forms of these two cases are, 'If A is, B is; but A is; therefore B is;' and 'If A is, B is; but B is not; therefore A is not.' Example, 'If what we learn from the bible is true, we ought not to do evil that good may come; but what we learn from the bible is true; therefore we ought not to do evil that good may come.'

These are the only two forms which a hypothetical syllogism can assume. For no variation can enter on the side of the hypothetical (which is usually styled the major) premiss, as there is but one form of such propositions; neither can there be any other form for the categorical (which is usually styled the minor) premiss than those already mentioned, for nothing can be inferred by denying the antecedent or asserting the consequent. As, moreover, the latter premiss only concerns the truth or falsity of the members of the hypothetical, all variations in its form, when considered merely as a categorical proposition, must affect its matter and not its form when considered as the minor premiss.

Disjunctive Syllogism.

(65.) In this syllogism, we commence with a disjunctive judgment, and proceed either by asserting the truth of one member of the division, and thence inferring the falsity of all the rest, which is called the 'modus ponens,' or else by asserting the falsity of all the members but one, and hence inferring the truth of that one, which latter method is called the 'modus tollens.' The general form of these two cases will be, 'Either A is, or B is, or C is; but A is; therefore neither B is, nor C is.' And 'Either A is, or B is, or C is; but neither B is, nor C is; therefore A is.' We may take as an example 'Either the Pope is infallible, or there is at least one great error in the Romish church; but the Pope is not infallible; therefore there is at least one great error in the Romish church.'

These may be shewn to be the only forms of a disjunctive syllogism by reasoning very similar to that employed in the case of the hypothetical.

Dilemma, &c.

(66.) Besides the three simple forms of syllogism already mentioned, there are several others, of which perhaps the Dilemma is the most important. This is an hypothetical syllogism, whose consequent is divided into members by a disjunctive judgment. Thus, 'if A is B, either C is D, or E is F' is the general form of the major premiss of a dilemma; and as it is hardly ever used except in the modus tollens, the minor premiss and conclusion will be 'But neither C is D, nor E is F, therefore A is not B.'

(67.) The following are examples of combinations of premises which differ from those already given in respect of the moments of relation. Thus, two hypotheticals will give—

'If A is B, C is D; but if C is D, E is F; therefore if A is B, E is F.'

Or, 'If A is B, C is D; but if E is F, C is not D; therefore if E is F, A is not B.'

'If A is B, C is D; and if A is not B, E is F; therefore either C is D, or E is F.'

In the same manner two disjunctives will give—

'A is either B or C; but B is either D or E; therefore A is either C or D or E.'

Or a categorical and disjunctive—

'A is either B or C; D is A; therefore D is either B or C.'

Enthymeme.

(68.) The Enthymeme is a syllogism abridged by the suppression of one of its premises, which is nevertheless understood, as the argument would not be valid without it. For, as the conclusion and either premiss are sufficient to indicate what the other premiss must be, we rarely express both premises in practice, but generally leave one of them to the hearer or reader to supply. The following is an example of a categorical enthymeme:

'The science of Logic is very useful, as it enables us to detect the formal fallacies in the arguments of our adversaries'. Here the major premiss is suppressed. The completed syllogism will stand thus—

'Whatever enables us to detect the formal fallacies in the arguments of our adversaries is very useful; the science of Logic enables us, &c.; therefore the science of Logic is very useful.' Had the minor premiss been suppressed, the enthymeme would have been—

'The science of Logic is very useful, for any thing is useful that enables us to detect the formal fallacies in the arguments of our adversaries.'

The following is an example of an hypothetical enthymeme:—

'It will certainly rain, for the sky looks very black.'

In this case the major premiss is suppressed. The syllogism when completed would stand thus—

'If the sky looks black, it will certainly rain.
'The sky does look black.
'Therefore it will certainly rain.'

Had the minor premiss been suppressed, the enthymeme would have been of the following form :—

'It will certainly rain, for it always rains if the sky looks black.'

The following examples are two disjunctive enthymemes in which the major and minor premises of the same syllogism are respectively suppressed :—

'He must be in York, for he is not in London.'

The suppressed premiss is, 'he must be either in London or York.'

If the minor premiss is suppressed the enthymeme will become—

'He must be in York, for he must be either in London or York.'

Sorites.

(69.) In a Sorites the conclusion of a syllogism is not expressed, but made the suppressed premiss of an enthymeme whose conclusion may be made the suppressed premiss of another, and similarly for any number of enthymemes. Thus, 'B is A, C is B, D is C, E is D;

therefore E is A' is a categorical Sorites. Again, 'If A is, B is; if B is, C is; if C is, D is; but D is not; therefore A is not,' is a specimen of an hypothetical Sorites.

SECTION V.

THE DEDUCTIO AD ABSURDUM, OR INDIRECT PROOF.

(70.) THIS name is given to a circuitous method of proving one proposition from two or more others, by means of at least three syllogisms. As, however, the principle of the proof is quite independent of the number of given propositions, our present object will be fully answered by an investigation of that case, in which they are limited to two. And, moreover, as the forms of syllogism contain all the principles or functions of the reason, by which one proposition can be thought as necessarily connected with several others, it follows, that any conclusion at which we can arrive by the *deductio ad absurdum,* might also have been obtained by the direct application of one of the regular syllogistic forms of ratiocination. We may therefore consider the given propositions, and the one to be deduced from them, as the premises and conclusion of a syllogism, and proceed to shew how this conclusion may be obtained by a different chain of reasoning.

The method, then, consists first, in assuming the falsity of the conclusion and truth of one premiss, and deducing from these propositions as premises, the falsity of the other premiss, as a conclusion; secondly, in taking this conclusion as the consequent, and the premises of the last syllogism as the antecedent in a hypothetical syllogism, and inferring the falsity of the antecedent by the modus tollens; thirdly, in dividing this falsity in a disjunctive syllogism into its three members, viz. the falsity of each proposition separately, or of both together, and inferring the falsity of that member of the division which is the contradictory of the original conclusion by the modus ponens. But the reader will understand this method more easily by examining the following general form:

Let A and B represent the premises, and C the conclusion of any syllogism. In order to prove C by the indirect method, we commence with assuming that C is not true. The three syllogisms may be then stated as follows:

First syllogism: 'A is; C is not; therefore B is not.

Second syllogism: 'If A is, and C is not, it follows that B is not; but B is; therefore it is false that "A is and C is not.'"

Third syllogism: 'Either both propositions "A is" and "C is not" are false, or else one of them is false; but that "A is" is not false; therefore that "C is not" is false, (*i. e.* C is').

The hypothetical syllogism is rarely if ever expressed in practice; the disjunctive, perhaps, never. But, in an analysis of the indirect proof, it would be just as unreasonable to neglect the consideration of these syllogisms on the ground of their being rarely expressed, as it would be in an analysis of syllogism generally to neglect the consideration of one of the premises on the ground of our usually reasoning in enthymemes.

Although the *deductio ad absurdum* requires premises from which the conclusion might have been deduced by the direct method, yet is it frequently very useful when these premises are of such a nature as not to admit of a very convenient syllogistic form. It is on this account not unfrequently used in the propositions of geometry, though Euclid has occasionally introduced it when the direct proof would have been equally simple. An instance of this will be found in the fourth proposition of the third book, in which it is required to prove that 'If in a circle two straight lines cut one another which do not both pass through the centre, they do not bisect each the other.' The indirect proof assumes that they bisect each other, and then shews that they must both be perpendicular to a line joining the centre and the point of their intersection, which is absurd; therefore, &c. Whereas the direct proof states that all chords which are bisected by the line drawn from the centre to the point of intersection must be perpendicular to that line; but both of these chords are not perpendicular to that line; therefore both of these chords are not bisected by the line drawn from the centre to the point of their intersection, which cannot therefore be the point of their bisection.* But every proposition which admits the *deductio ad absurdum,* will also admit of a direct proof from the same data, though in a great many cases the direct proof would be exceedingly clumsy, and not nearly as simple as the indirect. There is a very striking instance of this in the usual proof of the falsity of a hypothetical or disjunctive proposition from two categorical premises. The reasoning is rather abstruse, but it bears too completely upon the present subject to be entirely

* This syllogism is the mood A O O in the second figure.

overlooked, and is therefore subjoined in a note.*

* Hypothetical and disjunctive differ from categorical propositions in not possessing the quality of negation objectively. For if the cognition upon which any one of them is grounded (*i. e.* the dependence of one particular cognition upon another, or its division into members) does not hold in nature, there is no other hypothetical or disjunctive proposition which can simply assert this want of objectivity. This remark has been made already (Art. 42—44) where it was observed that a proposition which simply denies the dependence of a particular cognition on another, or the completeness of the division of a cognition, in neither case gives any new dependence of cognitions, or any new exhausting division, and consequently cannot be either hypothetical or disjunctive, but must be simply categorical. If then two categorical propositions are granted as premises, from which we are to deduce as conclusion the falsity of an hypothetical or disjunctive proposition, these two premises must be referred to the thinking subject as the only middle term by which they can be united in an act of reason, for all common objective grounds are denied them by the very nature of the case. This necessity for a subjective reference renders the reasoning so much more abstruse, that the mind naturally chooses the other method of *deductio ad absurdum*, which, from its objective nature, is much easier of comprehension. In this method we assume the falsity of our desired conclusion (which conclusion is, in this case, the contradictory of an hypothetical or disjunctive proposition) and therefore assume the truth of the hypothetical or disjunctive, and conjoining it with one of our given premises, deduce the falsity of our other premiss; in this manner we entirely avoid the necessity of any reference to the thinking subject. We will exemplify this reasoning as follows:—

Let the given premises be, 'A is; B is not,' from which we are to deduce the falsity of the hypothetical proposition,

'If A is, B is.'

If I would deduce this falsity (which being the contradictory of a hypothetical must be contained in a categorical proposition) by the direct method, I must refer the two premises to the thinking subject 'I' in some such manner as the following:—

(71.) The *deductio ad absurdum* supplies the principle upon which may be founded a rather pretty and symmetrical arrangement of the twenty-four categorical syllogisms. It also suggests a method of proving the necessary equality of the moods true in the first three figures without the aid of mathematical analysis, and of shewing the reason why the number of negative syllogisms is exactly double the number of the affirmative. And although we do not propose to derive any particular practical advantage from its consideration, yet anything that tends to give additional order and theoretical completeness must always have a sufficient value in a pure science to warrant its insertion.

As in every categorical syllogism we may

'Whatever is in my present consciousness is conjoined in it with my assent to the conception of B not being.

'My assent to the conception of A being is in my present consciousness.

'Therefore my assent to the conception of A being is conjoined in my present consciousness with my assent to the conception of B not being.'

This conclusion is the categorical statement of the falsity of the hypothetical which can only be shewn directly by this or some other equally clumsy method. The indirect proof however, is simple enough, and may be stated thus:

Let us assume that the hypothetical is true, or that 'if A is, B is; but A is; therefore B is; but B is not, therefore,' &c.

As we have already stated, the simplicity of the proof in this case arises from every premiss contained in it having an objective reference, which the contradictory of a hypothetical has not.

either employ the major premiss and contradictory of the conclusion to disprove the minor premiss, or the minor premiss and contradictory of the conclusion to disprove the major premiss, it follows that there must be two distinct syllogisms, with either of which we may commence an indirect proof. The forms which they respectively assume will readily appear from the following table :—

	Major Premiss and Contradictory of Conclusion being Premises.	Minor Premiss and Contradictory of Conclusion being Premises.
First Figure. M — P S — M becomes S — P	Second Figure. M — P S — P S — M	Third Figure. S — P S — M M — P
Second Figure. P — M S — M becomes S — P	First Figure. P — M S — P S — M	Third Figure. S — M S — P P — M
Third Figure. M — P M — S becomes S — P	Second Figure. S — P M — P M — S	First Figure. S — P M — S M — P
Fourth Figure. P — M M — S becomes S — P	Fourth Figure. S — P P — M M — S	Fourth Figure. M — S S — P P — M

From this table it is immediately evident that for every sound mood in the first figure, there must also be a sound mood in the second and third; that for every sound mood in the second figure, there must also be one in the first and third; and for every sound mood in the third figure, one in the first and second. Hence it follows, that the number of moods that are sound in each of the first three figures must be the same. But the form of the fourth figure is such, that it only admits of an indirect proof by syllogisms in the same figure; and the equality, therefore, of the number of its moods to that of the moods in the other three figures, is not susceptible of this method of proof.

This table will supply the grounds of a division of categorical syllogisms into eight systems, containing three each. Six of these systems have one mood in each of the first three figures. The other two are contained entirely in the fourth. The systems are as follows:—

Figure	1	2	3	4	5	6
First	A A A	A A I	A I I	E A E	E A O	E I O
Second	A O O	A E O	A E E	E I O	E A O	E A E
Third	O A O	E A O	E I O	I A I	A A I	A I I

Fourth Figure.

7	8
A A I	I A I
A E O	A E E
E A O	E I O

Each system must contain one affirmative and two negative syllogisms. For the contradictory of the conclusion of every affirmative syllogism must constitute one of the premises in its two complementary syllogisms, or those contained in the same system with itself; and as this contradictory must be a negative proposition, the two syllogisms that contain it must be negative also. Hence, every affirmative syllogism must have two complementary negative syllogisms. And as the affirmative contradictory of the conclusion of a negative syllogism may be combined with either its negative or its affirmative premiss, its two complementary syllogisms must themselves also be the one negative, and the other affirmative. As, then, every system contains one affirmative, and two negative syllogisms, it is evident that the whole number of negative must double the number of affirmative syllogisms.

SECTION VI.

SYMBOLICAL EXPRESSION FOR THE LAWS OF CATEGORICAL SYLLOGISM.

(72.) The conditioning laws of categorical syllogism admit of a very simple analytical expression from which all its properties may be readily obtained. But the more especial object in treating this subject mathematically, is the exhibition of that symmetry, from which, the equality of the number of moods that are true in the first three figures, may be derived *à priori* to all consideration of the moods themselves. Before, however, we proceed to make any assumptions, it is necessary to remind the reader, that the usual arithmetical interpretation of the symbols employed has no natural connection with the subject under consideration, but can merely be useful as an index to the laws of their combination. If, for instance, any sets of symbols be juxtaposed, and equated to zero, the equation will indicate that at least one of these factors may itself be singly equated to zero; and, in that case, all its constituent symbols must receive whatever interpretations may arise from the transposition of terms, or any other operations, that are formally analogous to those of arithmetical algebra, and have been admitted in our particular application of the science. In the present case the conception of homogeneity is not introduced, and it will therefore follow, that, if several symbols are equated to several others, each of those on one side of the equation must be considered as equated to one on the other, but no division of a symbol into parts will be considered admissible.

The laws of categorical syllogism have been already once stated, but are nevertheless repeated here, as a trifling alteration in the manner of expressing those respecting quality will be necessary for their reduction to a mathematical form. As the latter will be represented in a single equation, it will be more convenient to throw them into one rule, which may be stated as follows:—

(1.) At least one of the premises must have an affirmative copula, and the quality of the conclusion will be the same as that of the remaining premiss.

As this law respects the quality of the copulas, which will always determine the quantity of the predicates, it may be written thus:

1. The predicate of at least one of the premises is undistributed, and the predicate of the conclusion will be of the same quantity as the predicate of the other premiss.

The other three laws are—

2. The middle term must be distributed in at least one of the premises.

3. The major term must not be distributed in the conclusion, unless it has been distributed in the major premiss.

4. The minor term must not be distributed in the conclusion, unless it has been distributed in the minor premiss.

Let u represent an undistributed term.

Let d represent a distributed term.

Also, let p_1, p_2, p_3, and s_1, s_2, s_3, represent the predicates and subjects respectively of the three propositions as expressed below.

	SUBJECTS.	PREDICATES.
Major premiss	s_1	p_1
Minor premiss	s_2	p_2
Conclusion	s_3	p_3

where each of these symbols must $= d$ or u.

Now, observing from rule (1), that at least one of the predicates of the premiss (i. e. p_1, p_2) must be undistributed or $= u$, we may express the other predicate as $= p_1 + p_2 - u$.

But by the latter clause of law (1) we also find that whatever is the quantity of that predicate, must also be that of the predicate of the conclusion, and we shall therefore have

$$p_3 = p_1 + p_2 - u, \; . \; . \; . \; . \text{ law } (1);$$

and this equation will be a correct expression for law (1).

Now from law (2) we find that the middle term must be distributed (i.e. $=d$), in at least one premiss, and as s_1 and p_2 are the middle term in the first figure, one or both of the following equations must be true in that figure,

$$\begin{aligned}(s_1 - d) &= 0\\ (p_2 - d) &= 0;\end{aligned}$$

and we shall therefore have

$$(s_1 - d)\,(p_2 - d) = 0 \; . \; . \; . \; . \; . \text{ law } (2)$$

as an equation expressing the second law for the first figure; and as the same reasoning will apply to all the four, we shall have

$$\left.\begin{aligned}\text{Fig. } (1) \; . \; . \; . \; . \; (s_1 - d)\,(p_2 - d) &= 0\\ \text{Fig. } (2) \; . \; . \; . \; . \; (p_1 - d)\,(p_2 - d) &= 0\\ \text{Fig. } (3) \; . \; . \; . \; . \; (s_1 - d)\,(s_2 - d) &= 0\\ \text{Fig. } (4) \; . \; . \; . \; . \; (p_1 - d)\,(s_2 - d) &= 0\end{aligned}\right\} . \; . \text{ law } (2).$$

Again, from law (3) we know that either the predicate of the conclusion, i. e. p_3, must be undistributed or $= u$, or else the major term which is either s_1 or p_1 according to the figure must be distributed or $= d$; and by reasoning similar to the preceding we shall have

$$\left.\begin{array}{l} \text{Fig. (1)} \ldots\ldots (p_1 - d)(p_3 - u) = 0 \\ \text{Fig. (2)} \ldots\ldots (s_1 - d)(p_3 - u) = 0 \\ \text{Fig. (3)} \ldots\ldots (p_1 - d)(p_3 - u) = 0 \\ \text{Fig. (4)} \ldots\ldots (s_1 - d)(p_3 - u) = 0 \end{array}\right\} \ldots \text{law (3).}$$

In precisely the same manner we may obtain the following expressions for law (4).

$$\left.\begin{array}{l} \text{Fig. (1)} \ldots\ldots (s_2 - d)(s_3 - u) = 0 \\ \text{Fig. (2)} \ldots\ldots (s_2 - d)(s_3 - u) = 0 \\ \text{Fig. (3)} \ldots\ldots (p_2 - d)(s_3 - u) = 0 \\ \text{Fig. (4)} \ldots\ldots (p_2 - d)(s_3 - u) = 0 \end{array}\right\} \ldots \text{law (4).}$$

We shall now assume the symbols p'_3 and s'_3 of such a nature that

$$p'_3 + p_3 = d + u \ldots\ldots\ldots (5).$$
$$\text{and} \quad s'_3 + s_3 = d + u \ldots\ldots\ldots (6).$$

And as it has been already stated that homogeneity is not introduced in the conception of our symbols, it will follow from these equations that p'_3 and s'_3 are each $= d$ or u. They will also be respectively complementary to p_3 and s_3 in such a manner that when one $= u$, the other $= d$, and *vice versa*; and they will therefore be of precisely the same nature as the other six symbols, p_1, p_2, &c.

From (5) and (6) we obtain for the values of p_3, s_3 respectively

$$p_3 = d + u - p'_3$$
$$s_3 = d + u - s'_3.$$

And substituting these values of p_3 and s_3 in the equations (1), (3), and (4), we shall get the following sets of equations for the four figures of categorical syllogism.

$$\left.\begin{aligned} {}^{*}\ p_1 + p_2 + p'_3 &= d + 2u \ldots (1)\\ (s_1 - d)(p_2 - d) &= 0 \ldots\ldots\ldots (2)\\ (p_1 - d)(p'_3 - d) &= 0 \ldots\ldots\ldots (3)\\ (s_2 - d)(s'_3 - d) &= 0 \ldots\ldots\ldots (4) \end{aligned}\right\}\text{First figure.}$$

$$\left.\begin{aligned} p_1 + p_2 + p'_3 &= d + 2u \ldots (1)\\ (p_1 - d)(p_2 - d) &= 0 \ldots\ldots\ldots (2)\\ (s_1 - d)(p'_3 - d) &= 0 \ldots\ldots\ldots (3)\\ (s_2 - d)(s'_3 - d) &= 0 \ldots\ldots\ldots (4) \end{aligned}\right\}\text{Second Figure}$$

$$\left.\begin{aligned} p_1 + p_2 + p'_3 &= d + 2u \ldots (1)\\ (s_1 - d)(s_2 - d) &= 0 \ldots\ldots\ldots (2)\\ (p_1 - d)(p'_3 - d) &= 0 \ldots\ldots\ldots (3)\\ (p_2 - d)(s'_3 - d) &= 0 \ldots\ldots\ldots (4) \end{aligned}\right\}\text{Third Figure.}$$

$$\left.\begin{aligned} p_1 + p_2 + p'_3 &= d + 2u \ldots (1)\\ (p_1 - d)(s_2 - d) &= 0 \ldots\ldots\ldots (2)\\ (s_1 - d)(p'_3 - d) &= 0 \ldots\ldots\ldots (3)\\ (p_2 - d)(s'_3 - d) &= 0 \ldots\ldots\ldots (4) \end{aligned}\right\}\text{FourthFigure}$$

(73). If we examine the equations to the first three figures, we shall find each set per-

* The following is the most general symbolical expression for the syllogistic laws.

Let x and y represent the middle term in the major and minor premises respectively, and m and n the major and minor terms. The equations will then be equally applicable to all the four figures, and will assume the following forms:

$$\begin{aligned} p_1 + p_2 + p'_3 &= d + 2u \ldots (1).\\ (x - d)(y - d) &= 0 \ldots\ldots\ldots (2).\\ (m - d)(p'_3 - d) &= 0 \ldots\ldots\ldots (3).\\ (n - d)(s'_3 - d) &= 0 \ldots\ldots\ldots (4).\\ \text{where}\quad x + m &= p_1 + s_1 \ldots\ldots\ldots (5).\\ \text{and}\quad y + n &= p_2 + s_2 \ldots\ldots\ldots (6). \end{aligned}$$

fectly symmetrical with respect to the other two. For in equation (1), which is common to all the three figures, the symbols p_1, p_2, p_3 are perfectly symmetrically involved, and the only assumptions that have been made respecting s_1, s_2, s_3 are that each of them must either = d or u. Hence any interchange among the three symbols s_1, s_2, s'_3 or the three p_1, p_2, p'_3 will not in the least affect the form of the equations. But if we interchange p_2 and p_3' in the equations to the first figure, we shall obtain the equations to the second; and if we interchange s_1 and s'_3 in the first figure, we shall obtain the equation to the third; or lastly, if we interchange p_2 and p'_3, also s_1 and s_3', in the equations to the third figure, we shall obtain those to the second; and of course the same interchanges will reproduce the first from the second, the first from the third, &c.: hence it follows that the sets of equations to the first three figures are perfectly symmetrical with regard to each other.*

If however it were required to obtain the

* Although the sets of equations to the first three figures are perfectly symmetrical with respect to each other, yet does it by no means follow that the corresponding equations represent the same laws. The equation that contains the law against illicit major in the first figure, corresponds to the similar equation in the third; but in the second figure it corresponds to the equation containing the law for the distribution of the middle term; and similarly of the others.

equations to the fourth figure from those to either of the other three, it would be necessary to interchange one or more of the three symbols s_1, s_2, s_3' with one or more of the symbols p_1, p_2, p_3'; and this cannot be admitted, as the three latter symbols are all involved in equation (1), whereas the three former are neither involved in that or any other corresponding equation; and hence it follows that the equations to the fourth figure are not symmetrical with those to the other three.

(74.) As the symbols are symmetrically involved in the equations to the first three figures, we can know *à priori* to all other considerations that there must be the same number of solutions, and therefore the same number of true moods for all of them. But as the equations to the fourth figure do not involve the symbols symmetrically with the equations to the other three, we cannot say at once that the number of their solutions must be the same as in those to the other figures, but can only shew that it is so by determining the number of solutions in each case. For this purpose it will be sufficient to investigate separately the number of solutions for the first and fourth figures.

We will commence by examining the first figure. It is evident from equation (1), that one of the symbols p_1, p_2, p_3' must $= d$, and the other two each $= u$; and from equation (3) that either p_1 or p_3' must $= d$. Hence there will be two solutions for equation (3) (accordingly as p_1 or $p_3 = d$), which, will correspond to two solutions for equation (1).

But as equation (3) requires that either p_1 or p'_3 should $= d$, it follows from equation (1) that p_2 must always $= u$, and therefore from equation (2) that $s_1 = d_1$, or otherwise neither factor in equation (2) would vanish. Hence there is but one solution for equation (2), and only two solutions for the equations (1) and (3), as the latter are mutually dependent on each other. Equation (4) will have three solutions, accordingly as both together or either separately of the symbols s_2 and $s_3' = d$. And as the solutions of equation (4) are quite independent of the symbols involved in the equations (1), (2), (3), it follows that any one of the three solutions of the former may be combined with either of the two solutions of the other three equations, and thus produce six sets of solutions which will correspond to the six moods that are sound in the first figure.*

* Great care must be taken, in the interpretation of these equations into their corresponding moods, not to confound p'_3 with p_3, or s'_3 with s_3. The values of p'_3, and s'_3 are first determined

In the fourth figure we shall have three solutions for equation (1), accordingly as either of the three symbols p_1, p_2, or p'_3 may $= d$. Of the other three equations (2), (3), and (4), that which contains the particular symbol of the three p_1, p_2, p'_3 which $= d$, will admit of two solutions; but the two equations which respectively contain those two of the three symbols p_1, p_2, p'_3 that are $= u$, will admit but of one solution. Hence, each of the three solutions of equation (1) may be combined with the two solutions of one of the other three equations, and thus produce six combinations which will respectively answer to the six moods of the fourth figure.

Although we have only been able to prove that the number of moods in the fourth figure is equal to that in the other three, by the numerical tentative method already given, yet can we at least shew that no other method is possible. For it is evident that the only method by which such an equality can be shewn independently of numbers, is that of form, which has been already employed in be-

by the four equations to the figure under consideration, and from them the values of p_3 and s_3 are deduced by means of the equations

$$p_3 + p'_3 = d + u,$$
$$s_3 + s'_3 = d + u,$$

and will indicate the peculiar categorical form of the conclusion.

hoof of the first three figures. But the equations to those three figures completely determine two of their six symbols (s_1 and p_2 in the first; s_1 and p_3' in the second; and p_2 and s'_3 in the third), whereas the equations to the fourth do not determine a single one, but admit variations in all. Hence it follows that no artifice can bring the equations to the fourth figure under the same form as the equations to the other three, and consequently that all formal proof of the equality of the number of the solutions is absolutely impossible.

(75.) Among other advantages in the symbolical expression of the laws of categorical syllogism, we may mention the facility with which the derived secondary laws may be obtained, and the peculiar fallacies exposed which their violation entails. For instance, we know from equation (1), that one alone of the symbols p_1, p_2, p'_3 can $= d$; and moreover from equation (3) in the first figure, that either p_1 or p_3' must $= d$. We may therefore conclude that $p_2 = u$, that is to say, the predicate of the minor premiss in the first figure is undistributed, and the quality of its copula affirmative. But if $p_2 = u$, it will follow from equation (2) in the first figure that $s_1 = d$, or in other words that the subject of the major premiss is distributed, and that premiss an universal proposition. Should either of these secondary laws be violated, the conditions of one of the equations (2) or (3) cannot be fulfilled, and there must either be a fallacy of undistributed middle, or illicit process of the major. In the same manner it may be proved, that in the second figure $s_1 = d$, and $p'_3 = u$, (or $p_3 = d$,) in other words, that the major premiss must be universal, and the conclusion negative. But these examples are sufficient to enable the reader to derive the other laws for himself.

Truth of Premises.

(76.) To assert that ‘if the premises are true, the conclusion that is deduced from them must be true likewise,’ is a mere tautology; for the very definition of a conclusion is ‘that proposition, the truth of which follows necessarily from the truth of the premises.’ But the converse of this proposition is by no means true, for it does not follow that if the conclusion is true, the premises from which it is deduced must be true also. A more satisfactory explanation of this subject will be given in a future section: the following, however, is an unobjectionable proof by the method of the *deductio ad absurdum.**

Let us assume that if the conclusion is true, the premises must be true also; it will follow that if the premises are not true, the conclusion will not be true either. Let any sound mood in any figure be represented by the general symbols $x\,y\,z$, and let x', y', z', represent the formal contradictories of the propositions x, y, z respectively. Let one or both of the premises x and y be false; it follows from our present hypothesis, that z must be false also. If then, for example, x is false, and its falsity is sufficient to ensure the falsity of z, and if we substitute for x a proposition which merely states that falsity, we may also substitute for z a proposition which merely states its falsity, and thus change the form without affecting the soundness of the reasoning. But the contradictories of x and z respectively state the falsity of those propositions, and consequently a syllogism of the form $x'\,y\,z'$ must represent

* This proof will only strictly apply to categorical syllogisms. It may, however, be extended to the hypothetical and disjunctive by converting their form into the matter of a categorical. Thus the hypothetical, 'if A is, B is; A is, therefore B is,' may have its form put into the matter of a categorical in the usual way, commencing with 'all the cases of A being,' &c., and as this categorical syllogism may have false premises and true conclusion, it is evident the hypothetical may also.

an unobjectionable mood in the same figure as that of $x\ y\ z$. And as we might have assumed that y was false instead of x, or that both were false together, we shall have the three moods $x'\ y\ z'$, $x\ y'\ z'$, and $x'\ y'\ z'$ all sound in the same figure as that of $x\ y\ z$.

But contradictories differ from each other in the quantity both of their predicates and subjects. If, then, we recur to the symbolical expression of the syllogistic laws (Art. 73), we must find that for every solution of the equations there given, three corresponding solutions may be obtained by exactly reversing the values of s'_3 and p'_3 together with the values of one or both pairs of symbols s_1, p_1 and s_2, p_2 in such a manner that those which $= u$ should $= d$, and *vice versâ*. But it was shewn in Art. 74, that in the first figure the values of s_1 and p_2 are determined by the equations (1), (2), and (3), and consequently neither of the pairs of symbols s_1, p_1 or s_2, p_2 can have their values reversed in that figure. In the second figure we find the symbol p'_3 determined by the equations (1) and (2), and in the third figure the symbol s'_3 determined by the equations (1), (3), and (4), and consequently the changes of values cannot take place in these two figures any more than in the first. Again, it appears from the equations to the fourth figure, that when s_1 can change its value, p_1 is determined, and when s_2 can change its value, p_2 is determined, and consequently in neither case can one of the pairs s_1, p_1 or s_2, p_2 both change their values simultaneously. Hence it follows that the conclusions at which we have arrived are all false, and consequently that the hypothesis which we assumed must be false also; and that true conclusions may be deduced from false premises in every figure of categorical syllogism.

FORMAL LOGIC.

BOOK II.

SECTION I.

LIMITATIONS UPON THE FORM OF JUDGMENTS, &c.

(77.) In the present section, it is proposed to trace the limitations upon the form of judgments, the superiority of the first figure of categorical syllogism, and the possibility of true conclusions from false premises, to a common *à priori* ground in the very constitution of the understanding itself. The ground in question is a simple property of the understanding-conception, Substance, and may be stated as follows. Substance, or the substratum of phenomena, (*i.e.* the thing that is,—but is not phenomenon,) can never become a predicate.

For the conception of Substance may be defined as that which is thought as remaining when all possible predicates have been abstracted from it. It is therefore impossible to make it a predicate, for nothing is left which can become its subject; and were we to attempt to predicate it of phenomena, we must previously think a substratum for these phenomena, and should therefore only be predicating the simple conception-substance of itself, which is absurd.*

Law between the predicate and copula of categorical propositions.

(78.) This property of the understanding-conception, Substance, will immediately explain the reason of the law between the predicate and copula of categorical propositions. This law,

* It is necessary to put the reader on his guard against a certain species of categorical judgments which appear to militate against the observations in the text, and to contain the conception of substance in the predicate as well as subject. The predicate in these judgments contains matter of such a nature as to determine it to a particular object, *e. g.* 'that man travelled with me yesterday.' In this judgment, taken alone, I really only think of the existing man before me, and predicate of him all that is contained in my conception of his having travelled with me yesterday. But the instant after making such a judgment as this, I might very probably convert it in my own mind, and think first of the existing man with whom I travelled yesterday, and afterwards predicate of him that he then stood before me. And the case with which the understanding can thus at pleasure make either term a subject containing the conception of substance, very naturally produces a false semblance of that conception being in the predicate.

which has already been stated in Art. 33, is repeated here :—

Affirmative copulas have undistributed predicates ; negative copulas have distributed predicates.

For, inasmuch as the predicate cannot contain the conception of substance, it is not thought in respect of its sphere, or the things that are contained under it, but in respect of its matter, or the representations contained in it. This term is accordingly a mere conception, which is never formally determined as to its quantity, and the precise limits of its sphere must always remain unknown. The limits of the sphere of the subject cannot, therefore, be exactly compared with those of the sphere of the predicate, but the former term must either be placed wholly in or wholly out of the latter. If it is placed wholly in, it is compared with only a part of the sphere of the predicate ; if it is placed wholly out, it is compared with the whole. From these considerations it is immediately evident, that the quantity of the predicate is always undistributed in affirmative, and distributed in negative judgments.

There is no formal proposition of identity.

(79.) For if the subject is single, it evidently cannot be identical with the predicate which is a conception. But if the subject is a conception, it must be considered either as to its partial representations, *i. e.* what is thought in it, or else as to its sphere, *i. e.* what is thought under it. In the first case the conception in the subject must be literally the same as the conception in the predicate, and the result would be a tautology, but not a judgment: in the second case the subject, which refers to the sphere or aggregate of individuals, cannot of course be identical with the predicated conception, the quantum of whose sphere is formally indeterminate. In neither case therefore could there be a formal proposition of identity. It need hardly be observed, that in the proposition 'A is identical with B,' the identity is expressed in the matter, and not in the form.

Figures of Categorical Syllogism.

(80.) The great advantages in elegance and perspicuity that the first possesses over the other three figures, have been already mentioned in the first book; and the ground of this superiority may now be derived from the considerations introduced at the commencement of this section.

It has been shewn in Art. 77, that the conception substance is never placed in the predicate of a categorical proposition. Hence a conception, which in the subject is considered as to its sphere, (or the aggregate of individuals that are contained under it,) in the predicate is considered as to its contents, (or the representation that is contained in it as a mere conception). Any change then in the situation of a term introduces the necessity of a change in the manner in which it is thought. Now in the first figure, both subject and predicate of the conclusion retain the same position which they held in the premises, and consequently no such change in the manner of thinking them is necessary. But in the second figure, the major term changes its place from subject in the major premiss to predicate in the conclusion; and in the third figure the minor term changes its place from predicate in the minor premiss to subject in the conclusion, and consequently a change takes place in the manner of thinking one extreme in each of these figures. Again, in the fourth figure neither of the extremes hold the same position in the conclusion which they hold in the premises, and consequently two changes in the nature of the terms take place. In this manner, then, it is sufficiently easy to account for the superiority of the first, and inferiority of the fourth, to all the other figures. For all but the first involve the necessity of some change in the manner of our thinking one at least of the extremes; and the fourth involves two such changes. If the reader will make the experiment of different moods in different figures, he will become immediately conscious that their comparative merits entirely depend on the cause alleged. Perhaps the mood E I O is the fittest for the experiment, as it is sound in all figures.

True Conclusions from False Premises.

(81.) True conclusions may be logically deduced from false premises in every correct form of categorical syllogism.

This proposition has been formally demonstrated in the first book, by means of the symbolical expression of the syllogistic laws.

The following is another rather preferable proof, which could not with propriety have been introduced there, as it rests upon certain considerations from Transcendental Logic.

As no categorical proposition is either formally identical, or formally exhaustive, (which latter form would really be negatively identical, since it would assert the identity of $\overline{\text{not B}}$ with A,) it follows that no categorical proposition completely determines the whole of one term with respect to the whole of the other. For instance, in the proposition 'No A is B,' each term is placed in the external sphere of the other, but whereabouts in it, is quite undetermined. Again, in the proposition 'All A is B,' B is a conception, the excess of whose sphere above that of A is also undetermined, as it may vary from nothing to infinity. Now the indeterminate parts of the premises can never be introduced in the conclusion, which must follow from them necessarily, and therefore depend on the determinate alone. Hence it is evident that the materially necessary parts of the premises are invariably less than those which are formally necessary. If then we suppose an error in the indeterminate, and therefore unavailable parts of the premises, they will of course be false, although the conclusion, which is materially dependent on the remaining parts of those premises, is itself true.

It ought to be, if it is not, an axiom in an *à priori* science, that the general proof should invariably precede all reference to particular examples. But the reasoning in the last paragraph may now be elucidated, by taking as an example the mood A A A. The general form of a syllogism in this mood is the following :

All B is A,
All C is B,
Therefore All C is A.

In this syllogism, the indeterminate parts of the terms are the excess of A above B, and of B above C, and these parts are accordingly unavailable in the conclusion. For it is only so far as C agrees with B, that it agrees with what B agrees ; and in the same manner it is only so far as B agrees with A, that C, which agrees with B, can be concluded to agree with A. Let us then assume an error in the comparison of that part of B, which exceeds C, with A. Still A will be predicated of so much B as agrees with C, and consequently the conclusion will be true, though the major premiss is false.

The following is an example of a sound syllogism, of the form A A A in the first figure, in which the major premiss is false, though the conclusion is true :—

'All animals are quadrupeds,'
'All horses are animals,'
Therefore 'All horses are quadrupeds.'

We subjoin three diagrams of this form of syllogism, in which the conclusion is true, though one or both premises are false. In

Fig. 1, both premises are false; in Fig. 2, the major premiss is false; and in Fig. 3, the minor premiss is false.

Fig. 1.

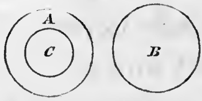


Fig. 2.

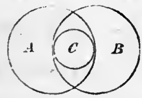
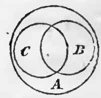


Fig. 3.



False, 'All B is A.'	False, 'All B is A.'	True, 'All B is A.'
False, 'All C is B.'	True, 'All C is B.'	False, 'All C is B.'
True, 'All C is A.'	True, 'All C is A.'	True, 'All C is A.'

These diagrams will also represent many other forms of syllogism, in which the conclusion is true, though one or both premises are false. For instance, if A be taken as the middle term, B as the major, and C the minor, Fig. 1 will represent the mood A E E in the second figure with a true conclusion, though both its premises are false. Again, if C be taken as middle term, A as major, and B as minor, Fig. 2 will represent the mood E I O in the first figure having its conclusion and minor premiss true, but its major premiss false. And similarly a great many other cases might be adduced, to which the above diagrams would be equally applicable.

This manner of proof may easily be extended to the other forms of syllogism. For instance, in the hypothetical syllogism the antecedent contains the grounds of the truth of the consequent; but as it may also contain much more, their exact limits cannot be determined. The truth therefore of the consequent will be only formally, and not materially dependent on some part of the antecedent; and if an error is introduced in the minor premiss of a syllogism in the modus ponens, and in that part of it which does not constitute one of the grounds of the consequent, the result will be a true conclusion, though one of the premises is false: *e. g.* 'If the whole of the —— regiment were going to Canada, Captain A. would go; but the whole of the —— regiment are going to Canada; therefore Captain A. will go.' Now the antecedent contains all the necessary grounds for the truth of the consequent, and a great deal more besides; and consequently our minor premiss, which states that the whole regiment is going, might be false, and yet the conclusion that 'Captain A. is going' true.

As the conclusion must always be true if the premises are true, and will sometimes be true when they are false, it follows that the probability of the truth of the conclusion must always be rather greater than the probability of the truth of the premises.

Conclusions of the Reason.

(82.) The limitation which the understanding imposes upon the actual use of the reason, is very well calculated to place in a clear and strong light the distinct functions of the two faculties in every categorical syllogism; it is therefore briefly re-considered here.

The reason can deduce a conclusion from premises of the form I O in the first figure, by precisely the same mental operation as from the premises A A, or premises of any other legitimate form. These propositions are of the form

'Some B is A;
'Some C is not B:'

from which the conclusion may be logically deduced, that 'Some C is not some A.'

But this conclusion, though derived by the same function of the reason as any other legitimate conclusion (*i. e.* by a conjunction in the consciousness of two acts of the understanding), is nevertheless absolutely worthless, as it may be predicated *à priori* of any objects of which the understanding can think. For whatever be the nature of the hypothesis respecting the relation of C and A, (*e. g.* let 'All C be identical with all A,') still will it be true that 'Some C is not some

A.' And here therefore we find the logical reason performing its regular office in complete blindness, and quite independently of the nature of the result when considered in reference to the understanding.

SECTION II.

MODALITY OF SYLLOGISM.

(83.) Modality has been already defined as the determination of a judgment in respect of its relation to truth. If the matter of the judgment is merely in accordance with the *à priori* laws of thinking, the judgment is, in respect of its modality, problematical, or of the lowest degree: if it is in accordance not only with those laws, but also with the matter of the senses, it is assertive, or of the second degree; but if one of these *à priori* laws becomes the matter of the judgment, the latter is, in respect of its modality, necessary, or of the highest degree.*

As the formal use of the reason is quite independent of the matter of judgments, which at the same time determines their modality, it follows that this point of judging cannot have any influence on the nature of the conclusion in respect of its categorical form. But this remark does not extend to the modality of

* Vide Arts. 25, 26, on this subject.

the conclusion, which may in some measure be determined by that of the premises, and the extent to which one is a criterion of the other will supply the topic of the present section.

(84.) In order to place this subject in as clear a light as possible, it will be necessary to say a few words on the different sides from which the modality of a conclusion may be viewed. And in the first place, great care must be taken never to confound the modality of this proposition, 'that such a conclusion follows from such premises,' with the modality of the conclusion considered merely as to its own matter, and quite independently of its grounds.

The former modality is always of the highest degree, as it constitutes the very essence of a conclusion that it follows necessarily from its premises; whereas the real modality of the conclusion depends on the peculiar nature of its matter, and is determinable in the same manner as that of any other judgment.

But this distinction will become more apparent, when the above proposition assumes its appropriate form of a hypothetical, in which the conclusion is the consequent, and the premises the antecedent. It may then be stated thus: If such and such premises are both true, it will follow that such a conclusion is true. Now the relation between these judgments is unquestionably necessary, as it consists in those laws of the reason by which the syllogism is known to be a correct one; but this is no criterion whatever for the modality of the judgments themselves, which constitute the matter of the hypothetical, and are therefore only propounded problematically. Before proceeding any further, an example will throw some light upon the foregoing remarks.

'All men are liars;'
'Obadiah is a man;'
Therefore 'Obadiah is a liar.'

Now the modality of this conclusion, considered in reference to the premises, is of the highest degree, or necessary; for the proposition that 'Obadiah is a liar' follows necessarily from the two propositions that 'all men are liars,' and that 'Obadiah is a man.' But the modality of this conclusion considered by itself is by no means necessary, since no necessity is contained in the proposition that 'Obadiah is a liar,' for no contradiction is involved in the supposition that he always tells the truth.

As we shall have occasion to recur to these two views of the modality of a conclusion, we shall designate them by the terms 'proper,' or that of any judgment considered merely as to its own nature, and 'consequential,' or that which depends on its reference to the judgments from which it is deduced.

(85.) The proper modality of a conclusion cannot be fully determined by that of the premises, for the modality of the premises depends on their matter, and therefore on the middle term which occurs in each of them. But the conclusion will be unaffected as to matter, and therefore as to modality, by any change in the matter of the middle term, provided its formal quantity is preserved. Hence it follows that a change is possible in the modality of the premises, while that of the conclusion remains the same, and therefore that the modality of the one can never be fully determined by that of the other. This will appear from the following example. Let the two premises be

'All animals are organic beings;'

'All horses are animals.'

These propositions are analytical, and therefore necessary; and the conclusion derived from them, that 'All horses are organic beings,' is also analytical and necessary. But provided the formal quantity of the middle term 'animals, be retained in each premiss, any other matter may be substituted for it without affecting the legitimacy of the reasoning. Thus let the term 'cows' be substituted for the middle term 'animals,' 'All cows are organic beings; all horses are cows; therefore all horses are organic beings.' Now the same conclusion has been logically deduced from these as from the original premises, and consequently its modality is the same in each case. But in the former syllogism the modality of both the premises was necessary; whereas in the latter, one of them is so far from being necessary that it is not even true. Hence it is evident that the nature of the premises can never be a complete criterion for the proper modality of the conclusion. There is, however, one law which will enable us to arrive at some sure knowledge on this subject, and may be stated thus: The modality of the conclusion is never of a lower degree than the lowest in either premiss.

(86.) Previous, however, to the consideration of this law, it will be necessary to shew why the case of two problematical judgments must be entirely put out of the question: for two judgments of this modality always leave the possibility of such a disjunctive relation existing between them, that they are never assertively true simultaneously.* No conclusion,

* It is necessary to distinguish between events that are 'simultaneously possible,' and 'possible simultaneously.' The first signifies that 'each is at the same time possible to happen;' the

therefore, can be deduced from them: but the truth of this remark will become more evident by an example. Let us suppose that a box contains only black and white balls, of which I abstract several, and put them in a bag without observing their colour: I may then state as problematical judgments—

'All the balls in the bag may be white; several black balls may be in the bag.' But I cannot deduce from these judgments even the problematical conclusion that 'several black balls may be white,' although such a syllogism in respect of its mere rational from (*i. e.* A I I in the first figure) would be perfectly legitimate. It is evident in this example that a disjunctive relation exists between the two premises, and consequently that they can never be true simultaneously, though either may be true when taken alone. As however the assertive modality of either premiss would destroy the possibility of such a relation, the objection will only apply to those cases in which both are problematical.

other, that 'both may possibly happen at the same time.' If I put my hand into a bag and draw out only one ball, I may draw out either a black one or a white one, and these events are therefore simultaneously possible. But as one ball cannot be both a black ball and a white ball too, these events are not possible simultaneously. The judgments in the example in the text, and also in every disjunctive proposition, are simultaneously possible, but not possible simultaneously.

(87.) After the exclusion of this particular combination of premises, five others will remain to which no such objection can be offered. These will accordingly come under the law, that the modality of the conclusion is never of a lower degree than that in the weakest* premiss.

To attempt a strict *demonstration* of any law respecting the operations of the Reason, involves the absurdity of making that faculty both judge and defendant in its own case: for in every proof the reason must tacitly assume the validity of its own laws, and any pretended demonstration of them from themselves is open to the objection of reasoning in a circle. It is useless therefore to sue the reason in its own court. All that can be done in a question respecting its functions, is to place it in as axiomatic a light as possible, and admit as final the decision of the faculty itself. In the present instance it is perhaps sufficient to observe, that inasmuch as the connection between the major and minor terms in the conclusion is entirely dependent on their previous comparison with the middle term in their respective premises, this connection must be of at least as high a degree as

* By the term 'weakest' that premiss is intended, whose modality is of at least as low a degree as that of the other premiss.

the lowest between the middle term and either extreme. And the same reasoning will also make evident the impossibility of logically deducing from the premises a higher degree of modality for the conclusion,* though such may perhaps exist. It will therefore be necessary to introduce the distinction of 'proper' and 'derived' modality, for the purpose of avoiding those long periphrases which would otherwise be perpetually recurring.

(88.) Perhaps the only great logical error of which Aristotle is guilty in his Analytics, refers to this very point of the modality of syllogism; and as an exposure of the fallacy will supply the best explanation of the last few paragraphs, we shall briefly consider it here.

The following passage will be found in the ninth chapter of the first book of the Former Analytics :—

Συμβαίνει δέ ποτε καὶ τῆς ἑτέρας προτάσεως ἀναγκαίας οὔσης ἀναγκαῖον γίνεσθαι τὸν συλλόγισμόν, πλὴν οὐχ ὁποτέρας ἔτυχεν, ἀλλὰ τῆς πρὸς τὸ μεῖζον ἄκρον, οἷον εἰ τὸ μὲν Α τῷ Β ἐξ ἀνάγκης εἴληπται ὑπάρχον ἢ μὴ ὑπάρχον, τὸ δὲ Β τῷ Γ ὑπάρχον μόνον' οὕτως γὰρ εἰλημμένων τῶν προτάσεων ἐξ ἀνάγκης τὸ Α τῷ Γ ὑπάρξει ἢ οὐχ ὑπάρξει. Επεὶ γὰρ παντὶ τῷ Β ἐξ ἀνάγκης ὑπάρχει ἢ οὐχ ὑπάρχει τὸ Α, τὸ δὲ Γ τι τῶν Β ἐστί, φανερὸν ὅτι καὶ τὸ Γ ἐξ ἀνάγκης ἔσται θάτερον τούτων.

* This law is precisely analogous to the mechanical axiom that every chain is of the same strength as its weakest link.

(*Translation.*)—It happens in some cases that if one premiss is necessary the syllogism becomes necessary (*i. e.* the conclusion of the syllogism), not however either premiss at random, but that which involves the major extreme; just as if, for instance, B is assumed to be A or not A, of necessity, and C is assumed merely to be B (*i. e.* assertively, not necessarily); for the premises being thus assumed, C will be A, or will not be A, of necessity. For since 'All B is A, or is not A,' of necessity, and 'C is a part of B,' it is evident that C will be one of these things (*i. e.* 'A or not A') of necessity.

Now as merely assertive judgments can only have empirical grounds for their truth, it follows that their predicates are stated of the sphere or individuals contained under the conception of the subject, and not of the conception itself. For a conception can only be predicated of another conception, (prior at least to the empirical consideration of the sphere of the latter,) either by being a superior conception, in which case the judgment is analytical, or else by some *à priori* law; but in either case the judgment is necessary, and therefore more than merely assertive.

Now, by the hypothesis, the minor premiss in the above syllogism is assertive, and consequently the middle term B is only predicated of the minor term C in respect of its sphere, and not of its matter. But the truth of the conclusion (considered as such) depends entirely on this minor premiss, and in that judgment, therefore, A must be predicated of C in respect of its sphere and not its matter. But when a conception is predicated only of the sphere of another, the judgment cannot be necessary, as it involves no *à priori* law of thinking, and conveys no certainty that an exception may not be found on some future occasion. Hence the derived modality of the conclusion that 'all C is A, or is not A,' can never exceed the assertive or second degree, which is that of the weakest premiss. Let us take the following example:

'All members of Caius College are members of the University. Several Norfolk men are members of Caius College. Therefore several Norfolk men are members of the University.'

Now the major premiss of this syllogism is analytical, and therefore necessary, for it constitutes a part of the conception of a member of a college that he should be a member of the University. But the minor premiss on the contrary is merely assertive, as it is by no means necessary to our conception of Norfolk men that several of them should be members of Caius College. And the conclusion is also merely assertive, as it does not necessarily enter into our conception of Norfolk men that they must be members of the University.

It is evident from this example, that only an assertive conclusion can be deduced from a necessary major and an assertive minor premiss. But this law regards the derived modality alone, and leaves the proper modality quite undetermined. For a trifling alteration in the example just given will render the proper modality of the conclusion necessary, and leave that of the premises unaltered. Let the minor term be changed into 'Several members of the University who reside in Norfolk;' the minor premiss will still be only assertive, as it is not necessary that any one of these men should be a member of Caius College; but the conclusion will be analytical, and therefore its proper modality will be necessary.

But, that Aristotle never intended to say that the conclusion from premises so assumed might sometimes* be necessary in its own nature, and sometimes only assertive, appears from his giving this proof in the general symbols A, B, C; and also from his predi-

* The restriction contained in the word ποτε, evidently alludes to the limitation of the law to those cases in which the major premiss is necessary, and the minor assertive.

cating necessity of the syllogism, and not merely of the conclusion. Neither is he merely alluding to the consequential modality of the conclusion, for he distinguishes between the proper and consequential modalities more than once, and particularly in the middle of the following chapter. So that his evident meaning is this: if a necessary major premiss and an assertive minor premiss be assumed in the first figure, that modality which can be derived from that of the premises for the conclusion as a judgment considered by itself, will be necessary and not merely assertive.

(89.) A fallacy involving a breach of the same law is to be found at the commencement of the nineteenth chapter of the first book of the Former Analytics.* Aristotle here lays down as a general rule, that if one premiss is necessary and negative, and the other problematical and affirmative, an assertive conclusion may be deduced from them.

"For let 'All B be necessarily not A,' and 'All C be possibly A:' if the negative premiss is converted, 'All A will be necessarily not B;' but 'All C may possibly be A;' there will therefore be a syllogism in the first figure to the effect that 'All C is possibly not B.' But

* This passage is at page 97 of Bekker's 8vo. Edition, Oxford reprint: 'Ει δ' ἡ μὲν ἐξ ἀνάγκης ——— τεθείη τὸ στερητικόν.

it is evident, moreover, that 'All C will *be* not B.' For let us assume that 'Some C *is* B:' if then 'All B is necessarily not A,' and 'Some C is B,' it follows that 'Some C cannot be A,' which is absurd, as by the original hypothesis, 'All C may be A.' Therefore, &c."

This reasoning is extremely ingenious, but not less sophistical. It will be found to hinge upon the very fallacy which has just been exposed in the last paragraph. For the conclusion which Aristotle wishes to deduce, is assertive, viz. 'All A *is* B:' he accordingly assumes the assertive contradictory of this conclusion as the minor premiss of the first syllogism in an indirect proof, and from this assertive minor, and the original necessary major premiss, fallaciously deduces a necessary conclusion, to the effect that, 'Some C cannot be A.' And as this conclusion is at variance with the other premiss that 'All C may be A,' he infers that his assumption of the contradictory of the proposition 'All A is B' was unwarranted, and therefore concludes that 'All A is B.' Now the necessity of the conclusion that 'Some C cannot be A' was merely consequential and not proper, and the legitimate conclusion that 'Some C is not A' is not at all opposed to the original minor premiss that 'All C is possibly A.' For this, as well as every other problematical conclusion, contains the conception of the possibility of the contrary. The proposition 'All C is possibly A' is therefore perfectly compatible with the proposition that 'Some C is possibly not A.' And it must be observed that the assertive modality of the conclusion that 'Some C is not A,' arises from the assumption of the assertive premiss that 'Some C is B,' which latter proposition states more than the premises would warrant in the conclusion of the original syllogism, though it is by no means contradictory to it. We will bring the question to an *experimentum crucis,* which will render the fallacy immediately evident. This will be most readily effected by taking a particular example, and carrying out the whole argument in precisely the same form as that in which it is given by Aristotle.

Let a bag contain several balls whose colour is unknown, and may therefore possibly be white or black, or any other colour. The necessary and negative premiss may then be, 'No white balls are black balls,' and the problematical affirmative, 'All the balls in the bag may be white balls;' from which Aristotle's reasoning would deduce as conclusion, that 'none of the balls in the bag *are* black balls.' His proof would run as follows: 'Let some of the balls in the bag be black balls; as no black balls can be white balls, it follows that some of the balls in the bag cannot be white balls. But this is absurd, for by the hypothesis all the balls in the bag may be white. Therefore, &c.'

This species of fallacy will become yet more apparent by stripping it of the syllogism with which it is connected, and proving by its means that every judgment which is true problematically, must also be true assertively. Let the judgment be, 'All A may possibly be B.' It will follow from this that 'All A is B.' For if it is false that 'All A is B,' it must necessarily be true that 'Some A is not B.' But this is absurd, as by the hypothesis 'All A may be B.' Hence it follows, that the assumed falsity of the proposition 'All A is B,' must itself be false, and therefore that 'All A is B.'

Both of these fallacies of Aristotle are of considerable importance, as they violate this fundamental principle of the modality of syllogism, namely, that the derived modality of the conclusion is the same as that of the weakest premiss.

(90.) There is however another fallacy, very near the end of the twenty-eighth chapter of the same book, which is introduced here rather as a curiosity than for any other reason, as in all probability it is the only instance of a false mood and figure in the whole work. It is stated in this passage that the denying B of H is exactly equivalent to identifying B with some T, where T has been previously assumed to represent the whole external sphere of a certain term E, which is itself a predicate of H. This paralogism is of the form A E E in the first figure, which has an illicit major, or E A E in the fourth figure, which has an illicit minor.*

It is rather remarkable, that, notwithstanding Aristotle's entire rejection of the fourth figure, he has introduced it once in this very chapter, in the mood A A I.†

* Page 115, Τὸ γὰρ μὴ ἐνδέχσθαι τὸ B, κ. τ. λ.

† Page 112, Εἰ δὲ τῷ H τὸ B ταὐτόν, κ. τ. λ.

APPENDIX.

The following are a few examples in which the reader can try his skill in detecting fallacies, determining the peculiar form of syllogisms, and supplying the suppressed premises of enthymemes. The arguments alone have been adopted from the different authors whose names are attached, as alterations in the mode of expressing them have invariably been found necessary to bring them a little nearer the simple syllogistic forms. Several of the examples contain more than one syllogism.

(1.) None but those who are contented with their lot in life can justly be considered happy. But the truly wise man will always make himself contented with his lot in life, and therefore he may justly be considered happy.

(2.) All nations, whose commerce has been very extensive, have reached a great height in refinement and luxury. The Romans reached a great height in refinement and luxury. Therefore the Romans must have had a very extensive commerce.

(3.) A really terrible enemy would never excite so small a degree of fear that any other passion could master the feeling. But there is no passion so weak that it cannot master the fear of death. Therefore death is no very terrible enemy.—*Bacon.*

(4.) None but the contented are happy; the good are happy; therefore they are contented.

(5.) If there is a possibility of the existence of God, nothing can be more evident than that men ought to live virtuously and piously, and that vice is the most absurd thing in nature. But that God exists is perfectly possible, as no demonstration can be given of the contrary. Therefore men ought to live virtuously and piously, and vice is the most absurd thing in nature.—*Clarke.*

(6.) If all cats are animals, and all animals are organic beings, it follows that all cats are organic beings. But all cats are organic beings. Therefore all cats are animals, and all animals are organic beings.

(7.) A statesman should particularly avoid anything that tends to bring him into contempt. Idle and frivolous duels will probably have this effect, and he should therefore particularly avoid them. — *Taylor's Statesman.*

(8.) All intelligible propositions must be either true or false. The two propositions 'Cæsar is living still,' and 'Cæsar is dead,' are both intelligible propositions; therefore they are both true, or both false.

(9.) God acts according to laws because he knows them, he knows them because he has made them, he has made them because they bear a certain relation to his wisdom and power; therefore God acts according to laws because they bear a certain relation to his wisdom and power.—*Montesquieu.*

(10.) Blessed are the pure in heart, for they shall see God.

(11.) None but the good are really great, and all the good are happy. The slaves of passion are never really great, and therefore they are never happy.

(12.) No man who is in London can be in York. As no person has told me where Mr. A is, for all I know to the contrary he may be in York. Therefore I conclude that he is not in London. For if this conclusion is false, it must be true that he is in London. But no man that is in London can be in York. Therefore Mr. A is not in York, which is contrary to the original hypothesis that he may be in York; and therefore the second hypothesis, that he is in London, must be false, and Mr. A is not in London.

(13.) All God's gifts are intended for use. Our foresight and power over the future are God's gifts. Therefore it is intended that we should use our foresight and power over the future. But Slavery precludes all possibility of the exercise of these powers; it is therefore opposed to the intentions of Providence —*Channing.*

(14.) Many things are more difficult than to do nothing. Nothing is more difficult to do than to walk on one's head. Therefore many things are more difficult than to walk on one's head.

(15.) If God does not grant his grace upon the same conditions to all mankind, he is a respecter of persons. But God is no respecter of persons. Therefore God grants his grace upon the same conditions to all mankind.

(16.) He that is of God heareth God's words: ye therefore hear them not, because ye are not of God. —*John*, c. viii, v. 47. *Quoted from Whately's Logic.*

(17.) It is highly probable that all persons who have established a new religion entirely subversive of the old, have suffered persecution. The first preachers of Christianity were such persons. It is therefore highly probable that they suffered persecution. —*Paley's Evidences.*

(18.) The men who barter their eternal welfare for temporary gratifications are very deficient in real wisdom. But the men who are thus deficient are by no means few. Therefore the men who barter their eternal welfare for temporary gratifications are by no means few.

(19.) None but Whigs vote for Mr. B. All who vote for Mr B. are ten-pound householders. Therefore none but Whigs are ten-pound householders.

(20.) The waking state succeeds the sleeping, and the sleeping succeeds the waking; things become cold from having been hot, and hot from having been cold; men can only become taller from having been shorter, and shorter from having been taller. Thus all contraries mutually produce, and are produced from, each other. But the states of life and death are contrary to each other. Therefore the state of life succeeds that of death, as that of death does that of life.—*Plato's Phædo.*

(21.) If the Mosaic account of the cosmogony is strictly correct, the sun was not created till the fourth day. And if the sun was not created till the fourth day, it could not have been the cause of the alternation of day and night for the first three days. But either the word 'day' is used in Scripture in a different sense to that in which it is commonly accepted now, or else the sun must have been the cause of the alternation of day and night for the first three days. Hence it follows that either the Mosaic account of the cosmogony is not strictly correct, or else the word 'day' is used in Scripture in a different sense to that in which it is commonly accepted now.

(22.) Men who would peril their own lives and those of their fellow-creatures on no better pretext than that of maintaining a reputation for courage, would unquestionably engage in duels upon very slight provocation. But none except very weak men would ever peril any human life with such an object alone, and we therefore conclude that none but very weak men would engage in duels upon very slight provocation.

(23.) Laws, in the widest acceptation of the word, are the necessary relations which derive their origin from the nature of things. The Deity, therefore, has his laws, for he bears a certain relation to the universe as its creator and preserver.—*Montesquieu.*

(24.) Suffering is a title to an excellent inheritance; for God chastens every son whom he receives. (Quoted literally from Jeremy Taylor's Holy Living and Dying.) This sentence may be put under the following form: All whom God receives, he chastens; all who suffer, God chastens; therefore, all who suffer, God receives.

(25.) To do a certain evil for a problematical good is contrary to the spirit of Christianity. But to hurry a great criminal into the presence of his Creator, and this for the sake of the very questionable advantage which capital may have over other severe punishments in deterring others from crime, is to do a certain evil for a problematical good. Therefore capital punishment is contrary to the spirit of Christianity.

ADDITION TO NOTE p. 117.

The complete elimination of the indeterminate symbols x, y, m, n from the equations on note to p. 117, will furnish additional equations representing the general laws of Categorical Syllogism in their simplest form: each of whose solutions will correspond to a possible mood.

It is unnecessary to insert the process of elimination; but it will be seen without difficulty that the equations thus obtained are the following:—

Quality............ $p_1 + p_2 + p'_3 = d + 2u$.........(1).

$$\begin{array}{l} \text{Laws of particular} \\ \text{premises and par-} \\ \text{ticular conclusion.} \end{array} \left\{ \begin{array}{l} (s_1 - d)(s_2 - d) = 0 \\ (s_1 - d)(s'_3 - d) = 0 \\ (s_2 - d)(s'_3 - d) = 0 \end{array} \right\} \ldots\ldots\ldots\ldots (2).$$

$$\begin{array}{l} \text{I E O excluded as} \\ \text{leading to illicit} \\ \text{major.} \end{array} \left\{ (s_1 - d)(p_1 - d)(p'_3 - d) = 0 \ldots (3). \right.$$

(1.) Implies that two of the quantities $p_1\ p_2\ p'_3$ must be $= u$, and a third $= d$, and admits therefore of 3 solutions.

(2.) Imply that any two or all three of the quantities $s_1\ s_2\ s'_3$ must $= d$ . . and admit therefore of 4 solutions.

The whole number of solutions therefore admissible from (1) and (2) is $3 \times 4 = 12$.

Of these one is rendered inadmissible by (3). The whole number of possible moods is therefore 11.

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