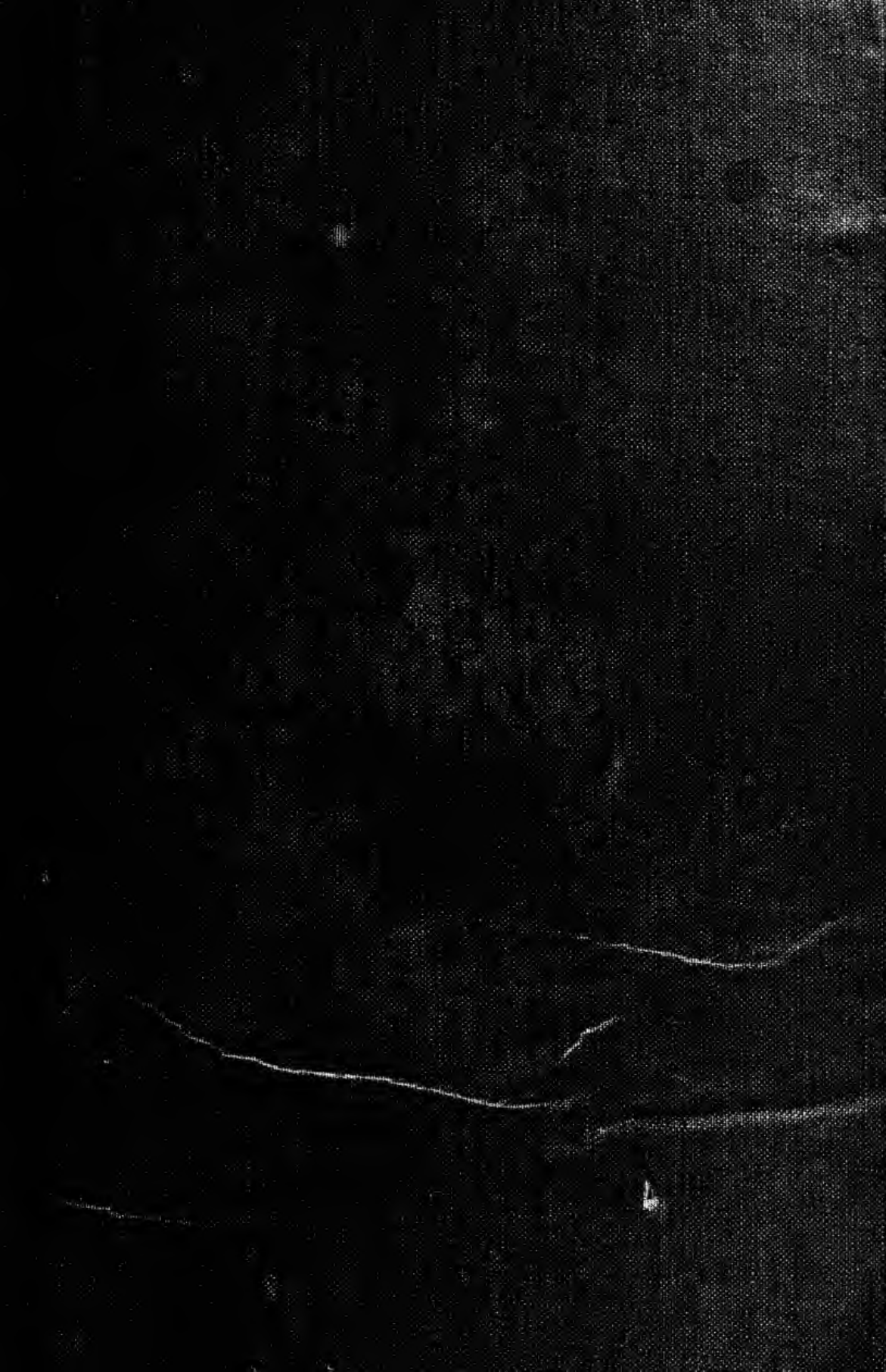

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Bulletin No. 9 (Part A)*

A System for Teaching Column Additions

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A System for Teaching Column Additions

BY FREDERIC BURK, SAN FRANCISCO STATE NORMAL SCHOOL.

The following is a system in detail of teaching column addition to beginners. It is the system now employed in the Elementary Department of the State Normal School at San Francisco. The essential principles have been taken from that invented and put into operation in the Oakland schools by Superintendent J. W. McClymonds. The writer has made departures from Mr. McClymonds' system, in matters of detail,—in the order of introducing the various combinations and in certain devices of method—but no claim is made for any originality. The fundamental principle is contributed by Superintendent McClymonds, to-wit: that no combination should be learned until it is used in column addition, and that drill should be focussed upon one set of combinations until its use in column addition is thoroughly mastered.

Before taking up the combinations and column addition certain preliminary work is necessary in the matters of learning to count, to write and read figures, to visualize the figures of any number when its name is thought, and to learn two or three language phrases.

Preliminary Work

I. The pupil must be able to count orally.

II. The pupil must be able to write and read figures at least to 120. To the end of getting a visual picture of the number series the following scheme is suggested:—

9	19	29	39	49	59	69	79	89	99	109
8	18	28	38	48	58	68	78	88	98	108
7	17	27	37	47	57	67	77	87	97	107
6	16	26	36	46	56	66	76	86	96	106
5	15	25	35	45	55	65	75	85	95	105
4	14	24	34	44	54	64	74	84	94	104
3	13	23	33	43	53	63	73	83	93	103
2	12	22	32	42	52	62	72	82	92	102
1	11	21	31	41	51	61	71	81	91	101
0	10	20	30	40	50	60	70	80	90	100

Such a scheme should be written permanently upon the board and pupils may be given seat work in copying it, and finally in

writing it from memory; or the figures may be written on small cardboard squares and these can then be given to pupils to sort out and arrange in their proper order.

III. The pupil must be trained to see, *in mental imagery,* the figures of any number so that when he thinks this number he will see the figures. Before pupils have learned to read or write figures they, of course, have no such mental pictures, and a great deal of drill is necessary before this end can be accomplished. Writing from oral dictation is probably the chief device. Send a section of the class to the board with an eraser in the left hand and a piece of chalk in the right. Begin by telling them to write, for example, 26; then direct them to change it to 36 by erasing only one figure and replacing it by a figure which shall make the number 36; then change to 32 by erasing only one figure, and so on. This device, which the writer learned from Deputy State Superintendent Job Wood, Jr., is an excellent one for our present purpose, for it continually focuses the attention upon the place value of each number. Follow this by oral drills in requiring pupils to tell you what a given number "ends in"; e, g., What does 34 end in? what does 20 end in? what does 12 end in? At first the teacher should stand at the board, writing the numbers whenever a pupil hesitates in his answer, and repeatedly explaining what is meant by the question.

As a second device, send the pupils to the board and tell them to write numbers which end in any given number, as for example, 4, or 6; also use the phrases, "Write the 4's"; "Write the naughts," etc., meaning they should write the numbers ending in these figures. When the pupils readily understand the meaning of these phrases, drill them in the sequence. For example, require them to write, in an ascending series, the 0's, the 4's, the 6's, etc., thus:

etc.,	etc.,	etc.,	etc.,
50	54	56	58
40	44	46	48
30	34	36	38
20	24	26	28
10	14	16	18
0	4	6	8

The third step is to train pupils to recognize the sequence place of any number when its name is called, and to be familiar with a corresponding language form, that is, they must see, for example, that "24 is the first 4 above 20, that the next naught above 24 is 30, etc. By referring to the beginning lessons of column addition,

it will be noted that the first facts of this kind to be used will be the following:

That 10 is the first naught above 6 and 4.
That 14 is the next 4 above 10.
That 16 is the next 6 above 10.
That 20 is the next 0 above 14.
That 20 is the next 0 above 16.
That 24 is the next 4 above 20.
That 26 is the next 6 above 20.
That 30 is the next naught above 24.
That 30 is the next naught above 26.
That 34 is the next 4 above 30.
That 36 is the next 6 above 30, etc., etc.

As a device, after considerable drill has been given to the oral questions, the teacher may dictate or write several numbers upon the boards as follows:

6	6	4	4	0	0	0	6
24	14	10	20	16	36	24	10

The pupil may be required to ask himself the question, "What 6?" "What 4?" or "What 0?" and then write the left hand figure. If pupils have difficulty in grasping this form, the following device may be used: Write the numbers between 10 and 45, for example, in ascending column, and have pupils discover what 0 comes next after (or above) 16; what 4 after (or above) 20, etc.

The pupils should be trained to use these language forms and to ask the questions themselves, to-wit: What does 24 (or any number) end with? What naught (or any figure) next above 26?

Column Addition and Combinations

6
4
6
4
6
4
6
4
6
4
—

If we go through the usual process of adding this column, 10, 14, 20, 24, 30, 34, 40, 44, 50, we note there are two series of difficulties — the difficulty of finding, in each successive sum, the left hand figure, and the difficulty of finding that the right hand, or "end" figure. If we find the right hand figure, or 4, we find the left hand figure by asking the question "what 0?" or "what 4?" and locating it in the series, just as the devices in the preliminary work given require. Let us therefore begin with the difficulty of finding the right hand figure.

In the given column the right hand figure is always obtained by a knowledge of just two combinations, 4 and 6, 0 (i. e. ends in 0); and 0 and 4, 4. These two combinations are therefore all that are necessary in order to add the end figure, and the left hand figure is obtained by knowledge the children have already acquired by a preliminary device.

Method

6	6	4
4	4	0
6	—	—
4	0	4
6		
4		
6		
4		
6		
4		
—		

Step I.— (a) Write on the board the column as given. At the top and right, write the two combinations to be learned.

(b) Give some drill, of a minute or so, to say, pointing to the figures as named, 4 and 6, 0; and 0 and 4, 4.

(c) Then erase the answers and require the form as before. If the slightest hesitation appears rewrite the answers. Drill until pupil readily can repeat this form.

Step II.— (a) Take the chalk and beginning at the foot of the column say, "4 and 6, 0," pointing to the numbers as named, and write the 0 to the right of the 6, "what 0? 10"; write 1 to the left of the 0 just written. "0 and 4, 4," pointing to the 0 and the 4 and then writing 4 to the right of the 4 in the column.

		6	4
6	50	4	0
4	44	—	—
6	40	0	4
4	34		
6	30		
4	24		
6	20		
4	14		
6	10		
4			
—			
50			

"What 4? 14," writing 1 to the left of the 4.

Have the pupils in concert or individually repeat this form with you. Continue the process until the top is reached, and finally write the answer, 50, at the foot of the column. Drill the pupils in concert, and individually until they can go through this form without error. If there is any hesitancy about the combinations point to the combination above so that they may learn where to find the correct form if they should forget.

(b) After this process and language form seem established, write similar columns for each pupil upon the board with instructions for him to do the example for himself. The teacher may now pass from one to another, hearing each give the form. As a pupil finishes let him exchange examples with another pupil, first erasing the side columns. To avoid confusion, it is well to write two or three more examples in excess of the number in the class, so that no pupil need wait. As a further convenience, it

may be found helpful for the pupil who finishes a column to write his name underneath it. The teachers passing around later erases the answer and the side columns, and writes "c" (correct) or "o" (wrong) after his name. The place is then ready for another pupil.

4
6
4
6
4
6 40
4 24
6 20
4 14
6 10
4

Caution.— With a certain number of pupils there will be a continual tendency to make mistakes in the left hand figure, to write 34 instead of 24, etc. In such cases the pupil has not learned the preliminary form for recognizing the place order. The following device will generally be found helpful: Suppose, in an example given he has written 40 instead of 30. Have him write, in ascending column the numbers beginning with 24 until he reaches a naught. Or, draw a line under 24 and ask, "What naught next above 24?" Drill him upon each partial sum in this way and if necessary enlarge his general form to the following: "4 and 6, 0; what 0? *The 0 above 4,-10*. 0 and 4, 4; what 4? The 4 above 10-14," etc.

(c) When this general form seems thoroughly learned, introduce the reverse combination $\begin{array}{r} 4 \\ 6. \\ \hline 0 \end{array}$ This will require, for the column work, also $\begin{array}{r} 6 \\ 0. \\ \hline 6 \end{array}$ Teach, in precisely the same manner as given in *a* and *b*.

(d)
4 4 6
4 6 0
6 — —
6 0 6
4
4
6
4
6
6
4
—

(d) Now construct a column which shall contain both $\begin{array}{r} 6 \\ 4 \\ \hline 0 \end{array}$ and $\begin{array}{r} 4 \\ 6 \\ \hline 0 \end{array}$ as for example the accompanying column (*d*):

Caution.—Spend a few minutes each day drilling upon the combinations. Write all the combinations thus far used upon the board in promiscuous order, without answers. Then point to each and require pupils to give the forms, "4 and 6, 0"; "0 and 4, 4," etc. If there is any hesitancy write the answer, for the pupil can not legitimately get the answer by thinking; either he knows it or he does not.

(e) After there is no hesitation about these combinations, write in the same promiscuous manner the form of the partial sums, as—

6	4	6	6	6	
24	30	14	10	34	
—	—	—	—	—	
4	4	4	6	4	
40	26	10	54	36	etc.
—	—	—	—	—	

(1) Ask the question, pertaining to any given form, "What is the combination?" and train the pupil to answer (e. g., $\begin{array}{r} 6 \\ 24 \\ \hline \end{array}$) "4 and 6, 0," etc. Then ask "What 0?" Pupil's reply: "30."

(2) Next, orally, in quick, snappy work, say "24 and 6—what combination?" Pupil: "4 and 6, 0." Teacher: "What 0?" Pupil: "30." A few minutes, not more than three or four, should be spent at the beginning of each recitation in this type of drill. Confine the work to forms which are to be used in the day's column work.

Caution.—In asking questions, as $\begin{array}{r} 6 \\ 4 \\ \hline \end{array}$, do not permit pupils to give the answer "0." Insist the complete form, 4 and 6, 0, be given. We need to establish a rhythmic form.

(f) The next combination to be taught is $\begin{array}{r} 8 \\ 6, \\ \hline 4 \end{array}$ but do not introduce it until the form for adding, developed by the column $\begin{array}{r} 6 \\ 4, \\ \hline 0 \end{array}$ has been thoroughly learned, so that pupils go through the entire process without hesitation. The first thing to be done is to

8	8	6
6	6	0
6	—	—
8	4	6
6		
6		
8		
6		
6		
8		
6		
—		

learn this form and it will lead to loss of time to introduce any new combination while they are learning the mere form.

Write the accompanying column upon the board (For the construction of this and other columns see later paragraphs upon the construction of columns),

and the two combinations, $\begin{array}{r} 8 \\ 6 \\ \hline 4 \end{array}$ and $\begin{array}{r} 6 \\ 0. \\ \hline 6 \end{array}$ Proceed

precisely as in previous cases. It is not necessary to drill much upon the combination at first, but teach pupils to look up to the combination when they forget.

(g) Next, introduce the reverse four $\begin{array}{r} 6 \\ 8 \\ \hline 4 \end{array}$ with $\begin{array}{r} 8 \\ 0, \\ \hline 8 \end{array}$ see colmun (g). Then construct a column containing both $\begin{array}{r} 8 \\ 6 \\ \hline \end{array}$ and $\begin{array}{r} 6 \\ 8, \\ \hline \end{array}$ see column (h); finally construct review columns containing $\begin{array}{r} 8 \\ 6, \\ \hline \end{array}$ $\begin{array}{r} 6 \\ 8 \\ \hline \end{array}$ and also $\begin{array}{r} 6 \\ 4 \\ \hline \end{array}$ and $\begin{array}{r} 4 \\ 6, \\ \hline \end{array}$ see columns (i) and (j).

(g)			(h)	(i)	(j)
6	6	8	8	8	6
6	8	0	6	6	6
8	—	—	6	6	6
6	4	8	8	4	8
6			6	6	6
8			8	6	8
6			6	8	6
6			6	4	6
8			6	6	4
—			8	—	—
			—		

Continue drills with each lesson upon all combinations, and also this type:

6	6	8	6	8	6
14	20	36	18	26	24 etc.
—	—	—	—	—	—

Step III. The form of finding the partial sums placed at the side of the column should now be firmly fixed as a mental habit. Its purpose has been to drill the pupils in learning new combinations and in visualizing the end figure of the successive partial sums. The form should be continued in teaching new combinations (at least for the first twelve combinations). But with the introduction of the fourth combination we are ready to commence upon the step of adding without writing the partial sums at the side and to train the pupil to add by imagining this scaffolding. Begin with the original column for

6
4
—

and require pupil to add directly, thus: 10, 14, 20, 24, 30, etc. If he makes mistakes, have the pupil, in imagination, go through the form of the partial sums in side columns, without actually writing them. First attempts will be slow, but in a few exercises he will gradually learn to depend upon his own visual imaging. Proceed in this way to add all other columns in review.

The Construction of Columns

The columns for exercising the pupils are made up of "cycles" — cycles of 10 and cycles of 20.

Cycles of 10.	9	8	7	6	5
These are	1	2	3	4	5
	—	—	—	—	—
	10	10	10	10	10
	1	2	3	4	
and their reverses	9	8	7	6	
	—	—	—	—	
	10	10	10	10	

It is clear we may make up a column of indefinite height by merely stacking one of these cycles repeatedly upon itself, and only two combinations will be employed. In the first column of

6 4
4 and 0 are repeated alternately.
— —
0 0

Cycles of 20.

All of the other combinations the sum of whose figures exceed 10, require cycles of 20. The combination $\begin{matrix}8\\6\\-\end{matrix}$ requires 6 more to make 20, thus $\begin{matrix}6\\8\\-\end{matrix}$. We may now stack this form upon itself repeatedly requiring only 3 combinations to be known, to wit, $\begin{matrix}8\\6\\-\end{matrix}$ and $\begin{matrix}6\\4\\-\end{matrix}$ and $\begin{matrix}6\\0\\-\end{matrix}$. Of these $\begin{matrix}6\\4\\-\end{matrix}$ and $\begin{matrix}6\\0\\-\end{matrix}$ are in review.

$\begin{matrix}0\\-\end{matrix}$ $\begin{matrix}6\\-\end{matrix}$

The Order of Introducing the Combinations

The order of introducing the various combinations is somewhat an arbitrary matter, but experience in teaching by system, and repeated changes, seem to justify, as the most economical in time and the easiest stages of learning for pupils, the following series:

$$\begin{matrix}6\\4\\-\\0\end{matrix}\ \begin{matrix}8\\6\\-\\4\end{matrix};\quad \begin{matrix}7\\3\\-\\0\end{matrix}\ \begin{matrix}7\\6\\-\\3\end{matrix};\quad \begin{matrix}8\\2\\-\\0\end{matrix}\ \begin{matrix}6\\6\\-\\2\end{matrix};\quad \begin{matrix}9\\1\\-\\0\end{matrix}\ \begin{matrix}5\\6\\-\\1\end{matrix};\quad \begin{matrix}4\\3\\-\\6\end{matrix}\ \begin{matrix}3\\2\\-\\6\end{matrix};$$

$$\begin{matrix}6\\2\\-\\8\end{matrix}\ \begin{matrix}6\\3\\-\\9\end{matrix};\quad \begin{matrix}7\\4\\-\\1\end{matrix}\ \begin{matrix}7\\5\\-\\2\end{matrix};\quad \begin{matrix}7\\7\\-\\4\end{matrix}\ \begin{matrix}5\\5\\-\\0\end{matrix};\quad \begin{matrix}8\\7\\-\\5\end{matrix}\ \begin{matrix}5\\2\\-\\7\end{matrix};\quad \begin{matrix}4\\3\\-\\7\end{matrix}\ \begin{matrix}7\\2\\-\\9\end{matrix};$$

$$\begin{matrix}8\\3\\-\\1\end{matrix}\ \begin{matrix}8\\4\\-\\2\end{matrix};\quad \begin{matrix}8\\5\\-\\3\end{matrix}\ \begin{matrix}8\\8\\-\\6\end{matrix};\quad \begin{matrix}5\\3\\-\\8\end{matrix}\ \begin{matrix}5\\4\\-\\8\end{matrix};\quad \begin{matrix}5\\4\\-\\9\end{matrix}\ \begin{matrix}3\\2\\-\\5\end{matrix};$$

These 28 combinations exhaust the series of possible combinations excluding the reverse forms and the combinations involving 0, 1 and 9. The 0's have really been included, for in the earlier

columns we treat them as combinations, but pupils soon realize that, 0 and any number, makes that number. The 1's need not be considered for the same reason, although some drill upon the fact that, any number and 1, is one more than the number. In the case of the combinations involving 9, except 1 and 9, it will be found more economical, after all other combinations in column practice have been learned, to teach the mental habit that 9, in any combination will give a sum whose end figure is 1 less than the other number. Columns involving 9's should then be given for much practice; in this way we really save the teaching of eight combinations.

The corresponding series of cycles by which these 28 combinations may be used in columns, and constructed as previously explained, are as follows:

	6		7		8		9	4	4
6	8	7	7	8	6	9	5	4	3
4	6	3	6	2	6	1	6	2	3
—,	—;	—,	—;	—,	—;	—,	—;	—,	—;
10	20	10	10	10	20	10	20	10	10

2	1	9	8	6		5	3	3
6	6	7	7	7	5	8	5	4
2	3	4	5	7	5	7	2	3
—;	—;	—;	—;	—;	—,	—;	—,	—,
10	10	20	20	20	10	20	10	10

1	9	8	7	4	2	2	1	5
7	8	8	8	8	5	4	5	3
2	3	4	5	8	3	4	4	2
—,	—,	—,	—,	—,	—,	—,	—,	—,
10	20	20	20	20	10	10	10	10

The construction of these cycles explains some reasons for the order of the introduction of the various combinations. It will be noted that the first eight combinations are in pairs; $\frac{6}{4}$ having been learned, the next combination, $\frac{8}{6}$, ends in 4, and thus we can immediately construct a column by using $\frac{8}{6}$, and $\frac{6}{4}$ in review. The same will be found true of each of the pairs.

Another point to be noted that the combination is always begun with the smaller number first, thus 2 and 8, not 8 and 2. This is done to offer difficulty to the pupil's tendency to count serially upon the fingers or by means of dots. We therefore begin the series,

6 8

not with 3 and 2, but with combinations 4 and 6, which offer this difficulty at the outset as a prevention against serial counting. Another determining feature of the order is that the combinations

6 7 8 9

which make 10, to-wit, 4, 3, 2, 1, are introduced in the first eight

— — — —

7

combinations and become review parts of later combinations; 4,

8

and 3 both give sums ending in 1 and by means of the combination,

—

9

1, already learned, we can form column cycles without further difficulty.

Column Groups and Reviews

Upon the thoroughness of the reviews, rather than upon the mere *knowing of the* combination, depends the success of ability to add accurately. To teach a child *to know* that "6 and 8 ends in 4" is a matter of merely a few moments, but so to drill him that whenever 6 and 8 come together in a column he will instantly think "4," is a matter of months and endless drill. For this reason the review and its method are really the kernel of our problem. To this end, the system includes a method of systematic reviews which require a pupil daily to use every combination he has previously learned. Teachers should construct a series of review groups upon a plan to be explained. The first four groups, however, will be given as the simplest means of explaining the plan. The height of a column however is immaterial. In the earlier lessons columns should not exceed 50, as a sum, but after the form is learned it is best to run them up to about 110, which of course is easily done by stacking additional cycles. To save space in printing the columns will be made short.

GROUP I

(a)	(b)	(c)
		6
4	6	4
6	4	6
4	6	4
6	4	4
4	6	6
6	4	4
4	6	6
6	4	6
4	6	4
—	—	—

GROUP II

(a)	(b)	(c)	(d)	(e)
				6
	6		6	4
8	6	6	8	6
6	8	8	4	8
6	6	6	6	6
8	6	6	6	6
6	8	8	8	6
6	6	6	6	8
8	6	8	6	4
6	8	6	4	6
—	—	—	—	—

GROUP III

(a)	(b)	(c)	(d)	(e)
3	7	7	6	7
7	3	7	8	3
3	7	3	3	6
7	3	7	7	6
3	7	3	6	8
7	3	3	8	6
3	7	7	6	4
7	3	7	7	3
3	7	3	3	7
—	—	—	—	—

GROUP IV

(a)	(b)	(c)	(d)	(e)	(f)
					6
7	7	7	8	7	7
7	6	7	6	3	7
6	7	6	7	6	6
7	7	7	7	6	7
7	6	7	6	8	6
6	7	7	6	7	6
7	7	6	4	3	8
7	6	7	3	6	7
6	7	3	7	4	3
—	—	—	—	—	—

It is intended that a class shall have at least one group for its daily lesson; this will mean that he shall have in review every combination he has learned. It does not mean, however, that he should have a new group each day. It will require about six months to do well his first twelve combinations. The (a) column in each group is thus constructed by stacking the cycle to which the new combination belongs upon itself repeatedly, thus 0 in Group I, the

$$\begin{matrix} 6 \\ 4 \\ - \end{matrix}$$

(a) column stacks; in Group II the (a) column stacks

$$\begin{matrix} 6 \\ 8 \\ - \end{matrix}$$

; in Group III the (a) column stacks

$$\begin{matrix} 7 \\ 3 \\ - \end{matrix}$$

; in Group IV, the (a) column

7
stacks 6. The (b) column introduces the reverse of the new com-
—

bination and stacks itself exclusively. The (c) column always combines the new combination and its reverse, but it is made up of Cycles of these exclusively. The other columns commencing with (d), in all groups, is composed of cycles in review; thus, in Group IV observation will show that in columns (d), (e) or (f),

6 4 8 6 7 3
every old combination, 4 6 6 8, 3 7, as well as the new combina-
7 6 — — — — — —
tion, 6 and 7, have been introduced. The number of review columns depends upon the advancement in the series. The greater number of combinations taken, the greater will be the number of review columns necessary. The teacher may easily construct the groups of columns for each new combination upon the plan for here given.

Review Without Cycles

The cycle columns have their advantages in beginning new combinations, but it is necessary later to have columns without cycles. In a short time brighter pupils will discover the cycles and will add by tens and twenties if given opportunity of seat work. In the beginning stages of teaching new combinations, the cycle system is advantageous because it permits us to focus attention upon new combinations and the filling between them is composed of combinations already thoroughly learned, and consequently requiring slight attention. Thus, 6 and 6, 4 and 8, 7 and 5, and their reverses all end in 2, and it is a convenience and assistance to pass by means of 8 to 0. But later we must have a review column free from cycles which may be used as seat or board work, without the teacher's surveillance. This type cannot be easily introduced until after about the first twelve combinations. After this point is reached we may readily construct columns without cycles for we

4 3 2 2 6 6
have the combinations 2, 3, 4, 6, 2, 3. Space will not permit their writing here but teachers may easily construct them and, having done so, they may be preserved in note books.

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