[illegible] fuerit [illegible] [illegible]

me manifest.

A

# SYSTEM

OF

# PRACTICAL ARITHMETIC,

APPLICABLE TO THE

PRESENT STATE OF TRADE,

AND

*MONEY TRANSACTIONS:*

ILLUSTRATED

*BY NUMEROUS EXAMPLES UNDER EACH RULE;*

FOR

THE USE OF SCHOOLS.

BY THE REV. J. JOYCE.

THE FOURTH EDITION, REVISED AND CORRECTED.

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Allowance to Schools.

1812.

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# PREFACE. 1812

In presenting a new System of Arithmetic to the Public, some account of its plan and execution will be expected. It is hoped, that the title of the present Work will briefly explain the views of the Author, who, from his own experience in the business of education, has long been convinced, that, among the excellent introductory books to this science, no one is sufficiently adapted to the occasions of common life. Some are too abstruse for novices, while others are defective in such examples as point out the application of the several rules to transactions of real business.

If the Author of this System of Arithmetic has not deceived himself, he has supplied these deficiencies; and he appeals, without apprehension, to that Public, whose candour and liberality he has already experienced, to decide upon this attempt to render the elementary rules of Arithmetic at once practical and popular.

There are few children who do not experience some disgust in passing through the first four rules; occasioned, without doubt, by the paucity of examples, and by the want of interest in those that are given. The Author has therefore devoted a large portion of this work to the early rules, and has illustrated them by miscellaneous questions, in which will be found much useful information, applicable in the progressive stages of life.

The modes of treating the Rule of Three, of illustrating Vulgar and Decimal Fractions, Practice, &c. &c., will best speak for themselves. But a reason may be demanded for the introduction of Logarithms, and for the particular method adopted in those parts in which the doctrine of

Annuities, Reversions, Leases, &c. is illustrated. The working of Logarithms is extremely simple, and it appears to be high time to introduce that great discovery into the ordinary processes of Arithmetic, and no longer to leave its advantages to those only who have leisure to cultivate the higher branches of the Mathematics. The uses of Logarithms are eminently great in facilitating many calculations —and being, at the same time, of such easy attainment, it is matter of wonder that they have not been taught long ago in the operation of Raising Powers and Extracting Roots, and in their consequent connection with the other parts of Arithmetic. At any rate it appeared to the Author to be highly proper, if not indispensable, to give, in one view, the different modes of calculating Compound Interest, and other similar rules, by COMMON and DECIMAL ARITHMETIC, and also by LOGARITHMS.

A Book of Arithmetic for Schools should contain every thing necessary to be known previously to the study of Algebra; and for the sake of those who wish to proceed to that science, it should be introductory to it. This, it is believed, will be found to be one of the characteristics of the Work now offered to the Public. There are, however, thousands who never trouble themselves to learn beyond the elements of the arithmetic which they acquire at school, who, looking to trade and commerce as the objects of their future lives, seek only for that knowledge which, in some way or other is applicable to those objects. These, in almost every rank and situation, have frequent occasion to calculate the INTEREST of money and the DISCOUNT of bills; and to ascertain the value of ANNUITIES on single and joint lives; of SURVIVORSHIPS; of LEASES; of REVER-

SIONARY INTERESTS; of FUNDED PROPERTY; and of FREEHOLD ESTATES.

On the doctrine and usage of Exchanges, so much of the theory is given as will, it is presumed, render the subject familiar and interesting; and so much of the practice as shall enable a youth, on his entrance into a counting-house, to apply his knowledge to any particular country. In this article, as in that which relates to the buying and selling of Stocks, are given illustrations and explanations of the various items in the Stock and Exchange Tables, which are to be found in the public papers.

For the assistance of Masters, the Author has prepared a Key to the entire Book, containing solutions to many, and answers to all the questions; to which he has subjoined a Practical System of Mental Arithmetic, that may be taught to boys before they leave school: *and in the new edition will be found a new and peculiar method of setting examples in the first rules, now originally published, wholly calculated to relieve the labour of the teacher, as by this means the number of the examples may be increased indefinitely, to which the answers will be instantly known by the preceptor, though to the pupil they will be inscrutable.*

It is the intention of the Author of this work speedily to publish a treatise on Algebra, of the size of the present volume, and on a similar plan of practical utility. In this will be given demonstrations of the rules of common Arithmetic, which necessarily depend on Algebra for solution, and which, on that account could not, with propriety, be introduced into a System of PRACTICAL ARITHMETIC.

*The Attention of the Conductors of Schools, and of Booksellers in general, is invited to the following List*

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| | £. | s. | d. |
|---|---|---|---|
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# ARITHMETIC.

ARITHMETIC is the science which explains the various methods of computing by numbers.

All its operations are performed by *Addition*, *Subtraction*, *Multiplication* and *Division*.

## OF
## NUMERATION OR NOTATION.

*When two or more figures are placed together, the first, or right-hand figure is taken for its simple value; the second to the left signifies so many tens; the third so many hundreds; and the fourth so many thousands; and so on, according to the following Table:* *

| Hundreds of millions. | Tens of millions. | Millions. | Hundreds of thousands. | Tens of thousands. | Thousands. | Hundreds | Tens. | Units. |
|---|---|---|---|---|---|---|---|---|
| 5 | 4 | 3 | 1 | 2 | 6 | 9 | 7 | 8 |

Thus figures, besides their common value, have one which depends upon the place in which they stand when joined to others; 6 and 5 are read six and five; but if they stand together, 65, they are read sixty-five. The figure 5 on the right hand denotes its simple value only, but the 6, from its situation, becomes ten times greater than its simple value, or sixty, therefore the two together are called sixty-five.

If there be three figures, as 978, the first denotes its simple value, as eight; the second a value ten times greater than its simple value, as seventy; and the third is a hundred times greater than its simple value, as nine hundred: the figures together are read nine hundred and seventy-eight.

In this manner, the value of each figure to the left is always ten times greater than it would be if it stood in the next place on the right;

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NOTE.

* The Tutor is recommended to direct the Pupil to commit to memory all the passages which are printed in Italic characters, and likewise all the tables.

thus 6666, the first figure 6 is simply six, the next is sixty, the third six hundred, and the fourth six thousand ; the whole number is read, Six thousand six hundred and sixty-six.

The first six figures in the table above are read, One hundred twenty-six thousand nine hundred and seventy-eight. The whole period of nine figures is thus read, Five hundred and forty-three millions, one hundred and twenty-six thousand, nine hundred and seventy-eight.

The enumeration of figures may be carried much further according to the following Table:

| Hundred thousands of billions. | Ten thousands of billions. | Thousands of billions | Hundreds of billions. | Tens of billions. | Billions. | Hundred thousands of millions. | Ten thousands of millions. | Thousands of millions. | Hundreds of millions. | Tens of millions. | Millions. | Hundreds of thousands. | Tens of thousands. | Thousands. | Hundreds. | Tens. | Units. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 4 | 8 | 7 | 9 | 5 | 1 | 4 | 6 | 2 | 7 | 5 | 3 |

In large numbers it is common to divide them into periods of six figures each, and half periods of three figures. The foregoing three periods are read—One hundred twenty-three thousand, four hundred and fifty-six billions, four hundred eighty-seven thousand, nine hundred and fifty one millions, four hundred sixty-two thousand, seven hundred and fifty-three.*

Hence the following general

**Rule.** *To the simple value of each figure, join the name of its place according to the situation in the series, as hundreds, thousands, millions, billions, trillions, &c.*

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NOTE.

* *The names of the higher periods after Billions, are Trillions, Quadrillions, Quintillions, Sextillions, Septillions, Octillions, and Nonillions, each period consisting of six places of figures. The first three of every period are so many Units of it, and the latter, or left hand part, so many Thousands.*—The following Table contains the whole series:

TABLE.

| Nonillions | Octillions. | Septillions. | Sextillions. | Quintillions. |
|---|---|---|---|---|
| 123,456 | 456,789 | 567,345 | 321,234 | 458,764 |

| Quadrillions. | Trillions. | Billions. | Millions. | Units. |
|---|---|---|---|---|
| 674,321 | 374,532 | 459,876 | 538,764 | 459,579 |

## EXAMPLES IN NUMERATION AND NOTATION.

Read, or write down in words, the value of the following Numbers:

Ex. 1. 19
2. 244
3. 3045
4. 45060
5. 69305
6. 93614
7. 564875
8. 4500342
9. 5687041
10. 6843700

Ex. 11. 40005
12. 324060
13. 400369
14. 765
15. 564001
16. 439764
17. 9300042
18. 70000021
19. 35000
20. 50000000

Ex. 21. 340
22. 436901
23. 36945
24. 9874000
25. 654328
26. 4328764
27. 856540
28. 43760000
29 37004
30. 85600341

Ex. 31. 456074328
32. 5900007643
33. 68670749004
34. 876430786453
35. 1000000843218
36. 34876543218764
37. 594632171834765
38. 87643285176487589
39. 1234567890001259
40. 987654321123456789

Write down the figures answering to the following Examples.

Ex. 1. Thirty-nine.
2. Four hundred and sixty-nine.
3. Two thousand and one.
4. Thirty-five thousand and twenty-eight.
5. Three hundred and seventy-six thousand.
6. One million and fifty-nine.
7. Eighty-seven millions, five hundred and eighty thousand, one hundred and nine.
8. Five hundred seventy-six millions, three hundred twenty-five thousand, three hundred and ninety-one.
9. Eight hundred millions and eighty.
10. Three hundred and three millions and thirty-one.*

---

NOTE.

* Besides these ten examples, it will be desirable that the pupil should, after having written the preceding forty examples into words, write them back again into figures, without the assistance of the book. He should likewise be desired to mention the value of each line in the subsequent examples in Addition, as well as the sum total; by these means Numeration will, in both its parts, become perfectly familiar to him.

### MISCELLANEOUS EXAMPLES.

Ex. 1. By the late enumeration of the people, the number of inhabitants in England is put down at nine millions, three hundred forty-three thousand, five hundred and seventy-eight; and the number found to be in London was eight hundred eighty-five thousand, five hundred and eighty-seven;—How are these numbers expressed in figures?

Ex. 2. The world was created two thousand three hundred and forty-eight years before the Deluge; three thousand two hundred and fifty-one years before the building of Rome; four thousand and four years before the birth of Christ, and five thousand and fourteen years before the present time [1811]:—Let each of these numbers be expressed in figures.

Ex. 3. Express in words the distances of the primary planets from the Sun, which are as follow:

| | | | |
|---|---|---|---|
| Mercury . . . . | 37,000,000 | Venus . . . . | 66,000,000 |
| The Earth . . . | 95,000,000 | Mars . . . . | 145,000,000 |
| Jupiter . . . . | 493,000,000 | Saturn . . . | 903,000,000 |
| The Herschel . . | 1,813,000,000 miles.* | | |

Fractions, or broken numbers, are expressed in the following manner:—A halfpenny is denoted by $\frac{1}{2}$; a farthing, by $\frac{1}{4}$, being the one-fourth of a penny; and three farthings by $\frac{3}{4}$, being three-fourths of a penny. Thus it appears that a fraction is any part or parts of an unit, and is expressed by two numbers separated from each other by a short line. The lower number shows how many parts the unit is divided into, and the upper figure points out what number of these parts are contained in the fraction: thus $\frac{3}{4}$, when standing for

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NOTE.

* The ancient Romans, in their Notation of Numbers, made use of the following five letters: I, V, X, L, and C, which, singly, stood for one, five, ten, fifty, and a hundred. By repeating and combining these, any other numbers were expressed: thus II, signified *two*; III, *three*; XX. *twenty*; CC. *two hundred*, and so on. The rules for Roman Notation are as follow:

1. The *annexing* a letter of a lower value to one of a higher, increases its value, or denotes the sum of both, as VI, signifies six; XII, denotes twelve; LV, fifty-five; LXXVI, seventy-six; CLII, one hundred and fifty-two.

2. The *prefixing* a letter of a *lower* value, to one of a higher, subtracts their values, or shows their difference thus, I prefixed to V, or IV, is four; IX, nine; XL, forty; XC, ninety, &c.

For the sake of abbreviation, the Romans introduced these marks:—IↃ, five hundred; CIↃ, a thousand: these, in process of time, were written D M, so that now the D signifies five hundred, and the M. a thousand; but in the titles of many old books we find the other mode of Notation. The following table will exhibit every thing necessary to be known on this subject:

Table.

three farthings, shows that a penny is divided into four parts, the 3 determines the number of the parts, and we call it three-fourths of a penny.

Inches are usually divided in eighths, or eight parts in each inch; and the fractional parts are thus expressed:

$\frac{3}{8}$ means three-eighths. $\frac{5}{8}$ means five-eighths.
$\frac{7}{8}$ means seven-eighths. $\frac{4}{8}$ means four-eighths, equal to one half.

Sixteenths are likewise in common use, and we say,

$\frac{5}{16}$ five sixteenths. $\frac{11}{16}$ eleven sixteenths.
$\frac{3}{16}$ three sixteenths. $\frac{15}{16}$ fifteen sixteenths.

## TABLE.

| | |
|---|---|
| I | 1 |
| II | 2 |
| III | 3 |
| IV, or IIII | 4 |
| V | 5 |
| VI | 6 |
| VII | 7 |
| VIII | 8 |
| IX | 9 |
| X | 10 |
| XI | 11 |
| XII | 12 |
| XIII | 13 |
| XIV | 14 |
| XV | 15 |
| XVI | 16 |
| XVII | 17 |
| XVIII | 18 |
| XIX | 19 |
| XX | 20 |
| XXI | 21 |
| XXX | 30 |
| XL | 40 |
| XLI | 41 |
| L | 50 |
| LX | 60 |
| LXX | 70 |
| LXXX | 80 |
| XC | 90 |
| C | 100 |
| CI | 101 |
| CCC | 300 |
| IↃ, or D | 500 |
| IↃC, or DC | 600 |
| IↃCCC, or DCCC | 800 |
| IↃCCCC, or DCCCC, or CM | 900 |
| CIↃ, or M | 1000 |
| CIↃC, or MC | 1100 |
| MM, or $\overline{\mathrm{II}}$* | 2000 |
| IↃↃ †, or $\overline{\mathrm{V}}$ | 5000 |
| IↃↃM, or $\overline{\mathrm{VI}}$ | 6000 |
| IↃↃMMM, or $\overline{\mathrm{VIII}}$ | 8000 |
| CCIↃↃ ‡, or $\overline{\mathrm{X}}$ | 10000 |
| CCIↃↃM, or $\overline{\mathrm{XI}}$ | 11000 |
| IↃↃↃ | 50000 |
| IↃↃↃMM | 52000 |
| CCCIↃↃↃM | 101000 |
| CIↃ,IↃCCC,XI, or M,DCCC,XI | 1811 |

* The word thousand is often expressed by a line drawn over the top of a number: thus $\overline{\mathrm{X}}$ signifies ten thousand, and $\overline{\mathrm{M}}$ a thousand thousands.

† The annexing Ↄ to the number IↃ, increases its value ten times: thus IↃↃ is 5000, and IↃↃↃ is fifty thousand.

‡ The prefixing C, and at the same time annexing a Ↄ to the number CIↃ, makes its value ten times greater; CCIↃↃ is 10,000, and CCCIↃↃↃ is 100,000.

# ADDITION.

**Addition** teaches the method of finding the sum or total of several numbers.

**Rule.** (1.) *Place the numbers under one another, so that units may stand under units, tens under tens, &c.*

(2.) *Add up the figures in the row of units: set down what remains above the even tens, or if nothing remains, a cypher, and for the tens carry as many ones to the next column.**

(3.) *Add up the other rows in the same manner, and in the last column put down the whole sum contained in it.* †

**Ex. 1.** What is the sum of 3684, 4863, 365, 29, 56874, and 609?

```
 3684
 4863
  365
   29
56874
  609
─────
```

Answer . . . . 66424 is the sum total.

**Proof.** Add the numbers together in a contrary order, beginning at the top instead of the bottom.

---

NOTES.

* Ten on the right-hand line is equal only to one, or unit, in the next line on the left of it, as we have seen in Numeration: when therefore the sum of any column amounts to, or exceeds ten, or any number of tens, we carry unit for every ten to the next column; for 9 being the highest digit, any number above it requires more thon one place to express it, which is done by removing the tens as so many units to the next place.

† The following Table is thought by some persons to be proper to be committed to memory. The use of it may be easily explained to children of five years old, and when once learnt completely, no difficulty will be found in Addition; for if the pupil knows, at first thought, the sum of any two of the digits, the rest is easy: for instance, if he knows that 6 and 7 are thirteen, he will know that 36 and 7 are 43, because 6

## EXAMPLES.

| | | |
|---|---|---|
| 345 | 8776 | 78329 |
| 489 | 6734 | 87293 |
| 204 | 5709 | 34680 |
| 695 | 9564 | 59417 |
| 731 | 3218 | 21004 |
| 27 | 4507 | 12345 |
| 2491 | 38508 | 293068 |

---

and 7 being 13, he knows there must be a three in the answer to the question of how many are 36 and 7, or 46 and 7, and so on.

### ADDITION TABLE.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 3 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 4 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 5 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 6 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 7 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 8 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 9 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |

To use this table:—Take the greater of the two digits, whose sum is sought, in the upper line, and the lesser on the left-hand column; in the same line with this, and underneath the other, stands the sum sought. If I want to know the sum of 8 and 5, I look for 8 on the head line, and on the same row of figures with 5 on the left hand side stands 13, the sum.

This table may be converted into a SUBTRACTION TABLE, (see p. 12): and the use of it, in this way, is "To find the difference of any two numbers." Look for the largest number in the same line in which the least stands on the left hand column, and the difference will be found in the head line over the largest number. Thus if I want the difference between 7 and 16, I look for 16 in the same line in which 7 stands, in the left hand column, and in the head line above the 16 I find 9, the difference sought.

| Ex. 1. | Ex. 2. | Ex. 3. | Ex. 4. | Ex. 5. | Ex. 6. |
|---|---|---|---|---|---|
| 2 | 4 | 7 | 3 | 4 | 4 |
| 4 | 5 | 8 | 4 | 0 | 6 |
| 6 | 6 | 4 | 5 | 5 | 3 |
| 2 | 3 | 6 | 6 | 9 | 4 |
| 9 | 4 | 2 | 7 | 8 | 1 |
| 2 | 2 | 3 | 8 | 6 | 5 |
| 5 | 8 | 4 | 7 | 7 | 8 |
| 3 | 7 | 5 | 5 | 2 | 7 |
| 2 | 1 | 4 | 9 | 4 | 5 |

| Ex. 7. | Ex. 8. | Ex. 9. | Ex. 10. | Ex. 11. | Ex. 12. |
|---|---|---|---|---|---|
| 8 | 3 | 5 | 1 | 9 | 81 |
| 13 | 7 | 26 | 42 | 8 | 32 |
| 4 | 28 | 7 | 3 | 7 | 3 |
| 56 | 6 | 44 | 4 | 1 | 4 |
| 5 | 5 | 3 | 5 | 2 | 5 |
| 9 | 4 | 9 | 66 | 3 | 96 |
| 5 | 38 | 35 | 2 | 6 | 7 |
| 16 | 21 | 2 | 7 | 5 | 58 |
| 4 | 5 | 1 | 29 | 4 | 39 |

| Ex. 13. | Ex. 14. | Ex. 15. | Ex. 16. | Ex. 17. | Ex. 18. |
|---|---|---|---|---|---|
| 2* | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 |

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NOTE.

* This and the seven following sums may be rendered very useful in shewing the pupil the foundation of the Multiplication Table; thus he may be desired to take two or three rows of each of the eight sums on his slate, and add them up, he will then see how three twos, or 3 times 2 make 6; how three fours, or three times 4 make 12, and so of the rest. When he has done the eight sums consisting of two figures, he may be required to commit the results to memory, which will be rendered a very easy business, when he sees the results before him on the slate. Let him then proceed to the eight sums consisting of three figures each; then with four figures, and so on till the whole nine figures are finished, and the table learnt.

| | | | | | |
|---|---|---|---|---|---|
| Ex. 19. 8 | Ex. 20. 9 | Ex. 21. 24 | Ex. 22. 56 | Ex. 23. 87 | Ex. 24. 25 |
| 8 | 9 | 35 | 65 | 33 | 56 |
| 8 | 9 | 64 | 74 | 29 | 64 |
| 8 | 9 | 28 | 48 | 86 | 88 |
| 8 | 9 | 74 | 82 | 39 | 64 |
| 8 | 9 | 19 | 98 | 45 | 77 |
| 8 | 9 | 45 | 33 | 20 | 25 |
| 8 | 9 | 35 | 66 | 31 | 66 |
| 8 | 9 | 64 | 59 | 99 | 33 |

| | | |
|---|---|---|
| Ex. 25. 5162 | Ex. 26. 7640 | Ex. 27. 49325 |
| 4876 | 39 | 24609 |
| 4008 | 5784 | 37485 |
| 3079 | 4304 | 16004 |
| 1234 | 9865 | 23348 |
| 2341 | 6543 | 32946 |
| 3468 | 2871 | 329 |

| | | |
|---|---|---|
| Ex. 28. 5432 | Ex. 29. 6905 | Ex. 30.*49603 |
| 5789 | 324 | 50792 |
| 1234 | 24 | 4652 |
| 5678 | 9 | 49859 |
| 9123 | 5068 | 654 |
| 4009 | 4081 | 78432 |
| 5746 | 5139 | 20764 |

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NOTE.

* The teacher may, from the three examples in p. 10, form for his pupil an indefinite number, by desiring him to copy on his slate the first three, or four, or five, or any other number of lines: or he may desire him to take only a single column, or half a column, or the half of two or of three columns, according to the progress he has already made.

To make young persons ready and accurate in Addition, which is of vast importance in almost every situation of life, the master may call a class round him, who have the same sum on their slates, and desire them to add each a figure till the sum is done; a place in the class to be lost whenever there is a mistake or a pause. The sum neatest set down to take precedence in the first instance.

| Ex. 31. | Ex. 32. | Ex. 33. |
|---|---|---|
| 67543 | 93217 | 8542 |
| 89678 | 76213 | 39764 |
| 56789 | 34567 | 78912 |
| 22345 | 89002 | 34567 |
| 67890 | 45678 | 91874 |
| 12932 | 345 | 43604 |
| 45764 | 67890 | 51871 |
| 85365 | 45632 | 20302 |
| 12345 | 12349 | 99887 |
| 54321 | 56789 | 44556 |
| 67854 | 48672 | 17280 |
| 58108 | 24 | 59776 |
| 49328 | 51403 | 43509 |
| 98765 | 46795 | 49312 |
| 43200 | 31274 | 56418 |
| 87219 | 45670 | 43004 |

* * *

## MISCELLANEOUS EXAMPLES IN ADDITION.

Ex. 1. Add together the following sums: 98764, 397652, 876, 459321, 21, 80, and 76942.

Ex. 2. Add 39764, 47652, 34291, 225, 48, 764871, and 10000 together.

Ex. 3. What is the sum of thirty-five thousand and four; five hundred and forty thousand, three hundred and nine; four hundred and twenty-seven; fifty thousand nine hundred and eighty; two millions and five; and seven hundred and seventy-seven?

Ex. 4. When will a child, born in 1806, be forty-nine years old?

Ex. 5. How many days are there in the first eight months of the year, when it is not leap-year?

Ex. 6. How old is the world this year, 1808, supposing it was created 4004 years before the birth of Christ?

Ex. 7. A person at his death left 3287*l.* to his widow; to his eldest son he bequeathed 5250*l.*; and to each of five other children he left a thousand pounds less than to the eldest son: he left also to a nephew 105*l.*, and the same sum to be divided among four distant relations; How much money did he leave behind him?

Ex. 8. The lease of my house was granted me in the year 1793, for ninety-nine years; when will it expire?

Ex. 9. How many days will there be between January the first and November the 20th, 1808, being leap year, both days inclusive?

Ex. 10. What do the following sums amount to, $1268+8612+10018+275+919+8+550099$?*

Ex. 11. How many chapters are there in the several books of the New Testament?†

Ex. 12. How many chapters are there in the several books of the Old Testament?

Ex. 13. How many chapters are there in the Bible, which consists of the Old and New Testaments?

Ex. 14. In travelling from London to Bath in a post chaise, for how many miles shall I have to pay? The distance from London to Hounslow is 10 miles, from Hounslow to Maidenhead is 16 miles, from Maidenhead to Reading 13 miles, from Reading to Speenhamland 16 miles, from Speenhamland to Marlborough is 19 miles, from Marlborough to Chippenham is 19 miles, and from Chippenham to Bath is 13 miles.

Ex. 15. How far is it from London to Harwich? To Romford are 11 miles, from thence to Ingatestone 12 miles, from Ingatestone to Chelmsford 6 miles, from Chelmsford to Colchester are 21 miles, and from Colchester to Harwich 20 miles.

Ex. 16. In travelling post to Margate I pay a shilling a mile: How many shillings shall I have paid at the end of the journey? The distance from London to Dartford is 15 miles; from thence to Rochester is 14 miles; from Rochester to Sittingbourne is 11 miles; from Sittingbourne to Canterbury is 15 miles, and from Canterbury to Margate is 17 miles.

---

NOTES.

* The pupil may now be taught that the character +, which is called *plus*, is used to denote Addition, and shews that the numbers between which it stands are to be added together: thus $9+3$ shews that nine is to be added to three. Two lines placed thus =, signify *equal to*, therefore when we write $9+3=12$, it is the same thing as saying in words, *nine* added to *three* are equal to twelve. Again $5+12+4+9=30$; that is, 5 and 12 and 4 and 9 being added together, are equal to 30.

† The learner must refer to the table of contents of his Bible, to enable him to answer this and the two following examples.

# SUBTRACTION.

By **Subtraction** we find the difference between two numbers.*

*Rule* (1). *Place the lesser number under the greater, so that units may stand under units, tens under tens, &c.; begin at the right hand, and take each figure in the lower line from the figure above it, and set down the remainder.*

(2). *If the figure in the lower line be the greater, add ten to the upper one, and then take the lower one from the sum, set down the remainder and carry one to the next lower figure, with which proceed as before.* †

(3). *When the figure in the lower line is equal to that above it, the difference is nothing, for which a cypher must be set down.* ‡

---

NOTES.

* This character —, called *minus*, when placed between any two numbers, denotes that the smaller number is to be subtracted, or taken from the larger: thus $9-5$ shews that 5 is to be taken from the 9, and we say, $9-5=4$; that is, 5 subtracted from 9 is equal to 4. Again, $21504-13695=7809$.

† This operation is commonly called *borrowing*, and as ten in the right-hand line is equal to only one in that which precedes it on the left, one is only *carried* to that line. The pupil may ask why the one is added to the lower line, instead of diminishing the upper line by the one borrowed? The question is very proper: and he will see, if he try it, that either mode of operation produces the same result; but the usual method is thought to be the best in practice: thus, if I have to take 28 from 45, I say 8 from 5 I cannot, but 8 from 15 and there remain 7, I carry 1 to the 2, and say, 3 from 4 and there remains 1; the answer is 17. It will be the same if I say 8 from 15 and there remains 7, and then making the 4 into 3, by taking from it the one I borrowed, I say 2 from 3 and there remains 1; the answer is still 17.

‡ See Table, p. 7, with explanation.

### EXAMPLES.

| | | | |
|---|---|---|---|
| From . . | 874698 | 765087 | 762134 |
| Take . . | 561436 | 425436 | 597082 |
| Remainder | 313262 | 339651 | 165052 |

**Proof.** Add the remainder to the last line, and if the sum be equal to the first, the work is right.

| | | | |
|---|---|---|---|
| From . . | 658742 | 390076 | 431267 |
| Take . . | 346121 | 184193 | 280795 |
| Remainder | 312621 | 205883 | 150472 |
| Proof . . | 658742 | 390076 | 431267 |

### EXAMPLES FOR PRACTICE.

| | | | |
|---|---|---|---|
| Ex. 1. 4867434 | 2. 6789491 | 3. 5876486 | 4. 3390761 |
| 2534213 | 5468354 | 3564214 | 1478490 |
| Ex. 5. 7052673 | 6. 9276807 | 7. 7231607 | 8. 9104008 |
| 3860749 | 4859434 | 5987465 | 9031648 |
| Ex. 9. 6734078 | 10. 5201832 | 11. 6000342 | 12. 1000000 |
| 5943769 | 4876543 | 5999343 | 999999 |
| Ex. 13. 4002103 | 14. 3874205 | 15. 9000123 | 16. 5301864 |
| 3987654 | 1796432 | 8123456 | 99 |
| Ex. 17. 7962038 | 18. 91111118 | 19. 4681035 | 20. 8302697 |
| 6498100 | 80000009 | 93006 | 2912934 |
| Ex. 21. 60001234 | 22. 71216003 | 23. 30061217 | 24. 26013032 |
| 49993490 | 59876543 | 19996642 | 19125346 |

## SUBTRACTION.

the
24

| | | | |
|---|---|---|---|
| Ex. 25. 98743205 | 26. 50237480 | 27. 49764321 | 28. 93816080 |
| 9999999 | 41926321 | 15875492 | 927908 |
| Ex. 29. 94286730 | 30. 92370800 | 31. 42601304 | 32. 27000019 |
| 32199739 | 4812719 | 22500894 | 4102094 |
| Ex 33. 76253922 | 34. 33861400 | 35. 94681039 | 36. 6901090 |
| 344939 | 23713509 | 3041316 | 1860018 |
| Ex. 37. 591040029 | 38. 271216904 | 39. 97348098 | 40. 974689019 |
| 490300019 | 28391767 | 9290412 | 31689247 |
| Ex. 41. 543902742 | 42. 913062138 | 43. 797260839 | 44. 170909009 |
| 312003717 | 44823165 | 62310079 | 24710905 |
| Ex. 45. 99326104 | 46. 19390909 | 47. 30921090 | 48. 111667833 |
| 21281299 | 2109109 | 1937099 | 38103475 |

### MISCELLANEOUS EXAMPLES IN SUBTRACTION.

Ex. 1. The invention of gunpowder was discovered in the year 1302: How long is is it since to the present year, 1811?

2. What is the difference between thirty-five thousand three hundred and nine, and nine thousand and ninety-nine.

3. How much does seven hundred six thousand and four exceed fourteen thousand nine hundred and thirty seven?

4. How much does fifteen thousand and five want of twenty-three thousand?

5. The art of printing was discovered in the year one thousand four hundred forty-nine: How long is it since?

6. Coaches were first used in England in the year 1580: How many years is it since?

7. Needle making was introduced into England from India in the year 1545: How many years was that before the present king came to his throne, which was in 1760?—*See the Book of Trades.*

9. Required the answers of the three following sums: 13045 — 999; 2059 — 928; and 258764 — 49876.

10. How many more chapters are there in the Old Testament than in the New?

# MULTIPLICATION.

MULTIPLICATION is a short method of Addition, and it teaches us to find what a number will amount to, when it is repeated a certain number of times.

RULE. *The number to be multiplied is called the Multiplicand: and the number multiplied is called the Multiplier. The number found is called the Product.*

MULTIPLICATION TABLE.*

| 2 times, or twice | 3 times | 4 times | 5 times | 6 times | 7 times |
|---|---|---|---|---|---|
| 1 are 2 | 1 are 3 | 1 are 4 | 1 are 5 | 1 are 6 | 1 are 7 |
| 2 . . 4 | 2 . . 6 | 2 . . 8 | 2 . . 10 | 2 . . 12 | 2 . . 14 |
| 3 . . 6 | 3 . . 9 | 3 . . 12 | 3 . . 15 | 3 . . 18 | 3 . . 21 |
| 4 . . 8 | 4 . . 12 | 4 . . 16 | 4 . . 20 | 4 . . 24 | 4 . . 28 |
| 5 . . 10 | 5 . . 15 | 5 . . 20 | 5 . . 25 | 5 . . 30 | 5 . . 35 |
| 6 . . 12 | 6 . . 18 | 6 . . 24 | 6 . . 30 | 6 . . 36 | 6 . . 42 |
| 7 . . 14 | 7 . . 21 | 7 . . 28 | 7 . . 35 | 7 . . 42 | 7 . . 49 |
| 8 . . 16 | 8 . . 24 | 8 . . 32 | 8 . . 40 | 8 . . 48 | 8 . . 56 |
| 9 . . 18 | 9 . . 27 | 9 . . 36 | 9 . . 45 | 9 . . 54 | 9 . . 63 |
| 10 . . 20 | 10 . . 30 | 10 . . 40 | 10 . . 50 | 10 . . 60 | 10 . . 70 |
| 11 . . 22 | 11 . . 33 | 11 . . 44 | 11 . . 55 | 11 . . 66 | 11 . . 77 |
| 12 . . 24 | 12 . . 36 | 12 . . 48 | 12 . . 60 | 12 . . 72 | 12 . . 84 |

| 8 times | 9 times | 10 times | 11 times | 12 times |
|---|---|---|---|---|
| 1 are 8 | 1 are 9 | 1 are 10 | 1 are 11 | 1 are 12 |
| 2 . . 16 | 2 . . 18 | 2 . . 20 | 2 . . 22 | 2 . . 24 |
| 3 . . 24 | 3 . . 27 | 3 . . 30 | 3 . . 33 | 3 . . 36 |
| 4 . . 32 | 4 . . 36 | 4 . . 40 | 4 . . 44 | 4 . . 48 |
| 5 . . 40 | 5 . . 45 | 5 . . 50 | 5 . . 55 | 5 . . 60 |
| 6 . . 48 | 6 . . 54 | 6 . . 60 | 6 . . 66 | 6 . . 72 |
| 7 . . 56 | 7 . . 63 | 7 . . 70 | 7 . . 77 | 7 . . 84 |
| 8 . . 64 | 8 . . 72 | 8 . . 80 | 8 . . 88 | 8 . . 96 |
| 9 . . 72 | 9 . . 81 | 9 . . 90 | 9 . . 99 | 9 . . 108 |
| 10 . . 80 | 10 . . 90 | 10 . . 100 | 10 . . 110 | 10 . . 120 |
| 11 . . 88 | 11 . . 99 | 11 . . 110 | 11 . . 121 | 11 . . 132 |
| 12 . . 96 | 12 . . 108 | 12 . . 120 | 12 . . 132 | 12 . . 144 |

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NOTE.

* The above Table must be first learnt in such a manner that the pupil may be able to answer, in an instant, any possible question that can be formed from it; and if he has attended to what was said in Addition, p. 8, the greater part of the difficulty will be already overcome.

## I. When the Multiplier does not exced 12.

**Rule.** *Multiply every figure in the multiplicand from right to left, consider how many tens there are in each product, the remaining units set down under the figure multiplied, and carry the tens as so many ones to the next product. The last product is to be wholly set down.*

### EXAMPLES.

| Ex. 1. | Ex. 2. | Ex. 3. |
|---|---|---|
| 420847 | 94564875 | 3476819 |
| 8 | 5 | 12 |
| 3366776 | 472824375 | 41721828 |

Thus in the first example, I say 8 times 7 are 56, in which there are five tens and six over, I put down the six, and say 8 times 4 are 32, and

---

NOTE.

As soon as the pupil has learnt the table in columns, let him learn it as it stands below.

### MULTIPLICATION TABLE.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 2 | 5 | 4 | 7 | 6 | 9 | 8 | 11 | 10 | 12 |
| 2 | 6 | 4 | 10 | 8 | 14 | 12 | 18 | 16 | 22 | 20 | 24 |
| 3 | 9 | 6 | 15 | 12 | 21 | 18 | 27 | 24 | 33 | 30 | 36 |
| 4 | 12 | 8 | 20 | 16 | 28 | 24 | 36 | 32 | 44 | 40 | 48 |
| 5 | 15 | 10 | 25 | 20 | 35 | 30 | 45 | 40 | 55 | 50 | 60 |
| 6 | 18 | 12 | 30 | 24 | 42 | 36 | 54 | 48 | 66 | 60 | 72 |
| 7 | 21 | 14 | 35 | 28 | 49 | 42 | 63 | 56 | 77 | 70 | 84 |
| 8 | 24 | 16 | 40 | 32 | 56 | 48 | 72 | 64 | 88 | 80 | 96 |
| 9 | 27 | 18 | 45 | 36 | 63 | 54 | 81 | 72 | 99 | 90 | 108 |
| 10 | 30 | 20 | 50 | 40 | 70 | 60 | 90 | 80 | 110 | 100 | 120 |
| 11 | 33 | 22 | 55 | 44 | 77 | 66 | 99 | 88 | 121 | 110 | 132 |
| 12 | 36 | 24 | 60 | 48 | 84 | 72 | 108 | 96 | 132 | 120 | 144 |

To enable the Teacher to exercise his pupil in all the combinations, I shall add the three following series, which will be found very useful in examining boys and girls in classes. Thus he combines all the 12 numbers with the figures in rows, as 8 times 3; 8 times 2, and so on: he may then do the same in columns, as 5 times 3; 5 times 9; 5 times 5, &c.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Series 1. | 3, | 2, | 5, | 7, | 4, | 8, | 6, | 12, | 9, | 11, | 10 |
| Series 2. | 9, | 8, | 10, | 6, | 11, | 4, | 2, | 3, | 10, | 5, | 12 |
| Series 3. | 5, | 11, | 3, | 2, | 7, | 9, | 12, | 6, | 8, | 4, | 10 |

adding the 5 from the last product, I have 37 ; I put down the 7, and carry the 3 for the three tens : I then say 8 times 8 are 64, and 3 are 67, 7 and carry 6 : 8 times 0 is 0, but put down the 6 brought from the last product : 8 times 2 are 16, put down the 6, and then 8 times 4 are 32, and the 1 brought forward are 33, which, as being the last product, must be set down.

## EXAMPLES FOR PRACTICE.

Ex. 1. 4653245
2

Ex. 2. 8756894
3

Ex. 3. 4986587
4

Ex. 4. 3390763
5

Ex. 5. 7052673
6

Ex. 6. 9276807
7

Ex. 7. 7231607
8

Ex. 8. 9134908
9

Ex. 9. 6734078
10

Ex. 10. 5201832
11

Ex. 11. 6893476
12

Ex. 12. 3874095
11

Ex. 13. 83022697
12

Ex. 14. 5391864
11

Ex. 15. 4681953
12

Ex. 16. 98743205
9

Ex. 17. 50947496
8

Ex. 18. 49764329
7

Ex. 19. 5972834
5

Ex. 20. 5097648
6

Ex. 21. 5875496
4

Ex. 22. 5439027
7

Ex. 23. 9999999
8

Ex. 24. 8888888
7

Ex. 25. 9734895
9

Ex. 26. 9237085
8

Ex. 27. 5942867
9

This character ×, which is called St. Andrew's cross; is used to denote Multiplication, and when it stands between two numbers, it signifies that those numbers are to be multiplied into one another: thus 9×6 =54, is read, nine multiplied by six is equal to fifty-four. Again 12× 11=132, that is 12 multiplied by 11 is equal to 132.

### EXAMPLES.*

| | |
|---|---|
| Ex, 1. 528318769 × 5 | Ex. 2. 956728314 × 3 |
| Ex. 3. 825934685 × 7 | Ex. 4. 486875294 × 9 |
| Ex. 5. 496745832 × 9 | Ex. 6. 683637544 × 8 |
| Ex. 7. 578940245 × 2 | Ex. 8. 759654318 × 11 |
| Ex. 9. 987234617 × 6 | Ex. 10. 867122456 × 12 |
| Ex. 11. 716432978 × 9 | Ex. 12. 687649321 × 7 |
| Ex. 13. 795483206 × 11 | Ex. 14. 779368245 × 9 |
| Ex. 15. 91872648 × 12 | Ex. 16. 986049005 × 5 |
| Ex. 17. 85678654 × 4 | Ex. 18. 390057864 × 6 |
| Ex. 19. 894367542 × 8 | Ex. 20. 765438958 × 4 † |

II. To multiply by 10, add an 0 to the multiplicand: thus 567 × 10 is 5670; and 567 × 100 is 56700; and 6489 × 10000 = 64890000. Therefore, to multiply a given number of one denomination, by a number whose significant figures do not exceed 12, having a cypher or cyphers joined to it:

Rule. *Write down the cypher or cyphers for the first part of the product towards the right hand, and then multiply every figure in the multiplicand by the significant figures of the multiplier, as in the preceding case.* Thus

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NOTES.

* If the pupil be not sufficiently ready in multiplying, after he has worked the following twenty sums, the preceptor may, by changing the multipliers only, increase the number of examples to any extent.

† When 13 is the multiplier, the sum may be done in a single line, by multiplying each figure in the multiplicand by 12, and to each product add the number to be carried, if any, and also the figure which is multiplied.

EXAMPLES.

| | | |
|---|---|---|
| Ex. 1. 493642 | Ex. 2. 576287 | Ex. 3. 345682 |
| 13 | 13 | 13 |
| 6417346 | 7491731 | 4493866 |

Here, in the first example, I say 12 times 2 are 24, and 2 are 26; 6 and carry two: 12 times 4 are 48 and 2 are 50, and 4 are 54; 4 and carry 5: 12 times 6 are 72, and 5 are 77, and 6 are 83, &c.

3469456 × 50 = 173472800, and 98765432 × 8000 = 790123456000, for

```
3469456          98765432
     50              8000
---------        ------------
173472800        790123456000
```

EXAMPLES.

Ex. 1. 6754328 × 70
Ex. 2. 987654329 × 800
Ex. 3. 8329674 × 110
Ex. 4. 56780943 × 120
Ex. 5. 6470078 × 9000
Ex. 6. 9237654 × 1100
Ex. 7. 7856423 × 1000
Ex. 8. 7490434 × 600

III. When the multiplier consists of several figures.

RULE. *The multiplicand must be multiplied by each figure of the multiplier separately, beginning with the right-hand figure, and the first figure of every product must stand exactly under the figure multiplied by. Add these products together for the whole product.* *

---

NOTE.

* To multiply by any number between 13 and 19.

RULE. Multiply the units figure of the multiplicand, by the right-hand digit of the multiplier; set down the unit's figure of the product, and remember what is to be carried. Multiply the *second* figure of the multiplicand; to the product, add what was to be carried, and also the first figure of the multiplicand. Then set down the unit's figure, and retain in your mind the number to be carried, as before. Multiply the third figure of the multiplicand; add the number to be carried, and also the second figure of the multiplicand, and so on: thus

```
 74365487596
          17
-------------
1264213289132
```

Here I say 7 times 6 are 42; I put down the 2 and carry 4, and say 7 times 9 are 63, and 4 are 67, then add the 6, which makes 73; put down the 3, and say 7 times 5 are 35, and 7 are 42, to which add the 9, which make 51, put down 1 and carry 5, and so on, till the last figure, when I say 7 times 7 are 49, and 3 to be carried are 52, take in the 4, which make 56, put down 6, and add 7 to the 5, and set down 12.

### EXAMPLES.

| | |
|---|---|
| 37864329 | 35964827 |
| 579 | 846 |
| 520778961 | 215788962 |
| 405050303 | 143859308 |
| 289321645 | 287718616 |
| 33503446491 | 30426243642 |

PROOF. The readiest way of proving the truth of sums in Multiplication is, by *casting out the nines.*

RULE. Make a cross like that which is used to denote Multiplication: add together the figures in the multiplicand, casting out all the nines in the sum as often as they amount to 9, and put the remainder down on one side of the cross; do the same with the multiplier, and put down the remainder on the other side of the cross. Multiply the two remainders together, and casting out the nines of their product, will leave the same remainder as the nines cast out of the answer, when the work is right.*

### EXAMPLES.

| | | | |
|---|---|---|---|
| 4593267 | 0 | 7628954 | 1 |
| 568 | 0 × 1 | 857 | 5 × 2 |
| | 0 | | 1 |
| 36746136 | | 53402678 | |
| 27559602 | | 38144770 | |
| 22963335 | | 61031632 | |
| 2608075656 | | 6538013578 | |

To prove the second example, I say 7 and 6 are 13; 4 above nine, (omit the 9): 4 and 2 are 6 and 8 are 14; 5 above nine, (omit the 9): 5 and 5 are 10, 1 above 9, 1 and 4 are 5: I place the 5 on the left-hand of the cross, and say 8 and 5 are 13, 4 above 9; 4 and 7 are 11, 2 above 9; the 2 I put on the right-hand of the cross. Now 5×2 gives 10, which is 1 above 9, I put the 1 at the top of the cross, and then cast

---

NOTE.

* The method of proof depends on the property of the number 9, which belongs also to the number 3, but to none of the other digits; viz. that any number divided by 9, will leave the same remainder as the sum of digits divided by 9: thus 6769 divided by 9, leaves 1 as a remainder; and so will 6+7+6+7, or 26, divided by 9.

out the 9's of the whole product, and I find the remainder is 1, which answering to the 1 at the top of the cross, leads me to conclude that the operation is right.

IV. When cyphers are intermixed with the figures in the multiplier.

RULE. *Omit the cyphers, and let the first figure of each product be placed under its multiplier.*

EXAMPLES.

Ex. 1. 7650329
600509 1
5 × 2
68852961 1
38251645
45901974
4594091417461

Ex. 2. 4465348
7000608 3
7 × 3
35722784 3
26792088
31257436
31260150931584

Ex. 3. 849275 × 706
Ex. 4. 973648 × 8005
Ex. 5. 597384 × 830004
Ex. 6. 364759 × 2709
Ex. 7. 245918 × 703006
Ex. 8. 609483 × 95007

V. When the multiplier is the product of two or more numbers in the table.

RULE. *Multiply the multiplicand by one of the component parts, and that product by the other, and so on: thus if I have to multiply a given sum by* 64, *I find* 8 × 8 = 64; *instead, therefore, of multiplying by* 6 *and* 4 *in the usual way, I multiply first by* 8, *and then that product by* 8 *again.*

EXAMPLES.

864392 × 64
8 5
6915136 5 × 1
8 5
55321088

39746285 × 168*
7 3
278223995 8 × 6
6 3
1669343970
4
6677375880

---

NOTE.

* Here 168=7×6×4.

### EXAMPLES IN ALL THE CASES.

| | | | |
|---|---|---|---|
| Ex. 1. | 99365497 × 13 | Ex. 2. | 54962874 × 26 |
| 3. | 35729876 × 56 | 4. | 47893062 × 48 |
| 5. | 73167482 × 77 | 6. | 8274386 × 96 |
| 7. | 39745371 × 86 | 8. | 5487962 × 357 |
| 9. | 72983456 × 99* | 10. | 3891307 × 464 |
| 11. | 737394 × 4567 | 12. | 35846 × 4682 |
| 13. | 329357 × 2839 | 14. | 58427 × 3957 |
| 15. | 462875 × 6874 | 16. | 47683 × 3456 |
| 17. | 594326 × 5936 | 18. | 87493 × 7892 |
| 19. | 486752 × 4608 | 20. | 29687 × 3579 |
| 21. | 8739690279 × 397829 | 22. | 7936820056 × 500634 |
| 23. | 2576432874 × 613487 | 24. | 9167403258 × 653000 |
| 25. | 872694325 × 2900008 | 26. | 715976032 × 350736 |
| 27. | 526730469 × 590734 | 28. | 37945687 × 999999 |
| 29. | 74714328 × 345627 | 30. | 46382719 × 50000092 |

### MISCELLANEOUS EXAMPLES.

Ex. 1. Multiply three millions thirty-nine thousand and three, by thirty-five thousand and twenty-eight.

2. Multiply six billions, six hundred thousand and sixty-five, by eight thousand and thirty-nine.

3. There are eleven hundred hackney coaches in London; suppose, on the average, each coach earns thirteen shillings a day, how many shillings will be expended in the hire of these carriages in a year of 365 days, Sundays being excepted?

4. In Jamaica only there were imported, annually, not less than ten thousand eight hundred negroes from the coast of Africa: How many slaves have free-born Englishmen made in that island, since the year 1799 to the year 1807, in which the infamous traffic was abolished.

---

NOTE.

* To multiply by any series of 9's. Add as many cyphers to the multiplicand as there are 9's in the multiplier, and then subtract the original multiplicand.

EXAMPLES.

| | |
|---|---|
| Ex. 1. 843628 × 999 | Ex. 2. 3475962 × 99999 |
| 843628000 | 347596200000 |
| 843628 | 3475962 |
| 842784372 | 347592724038 |

In the first example I add three cyphers, and subtract from the multiplicand, thus increased, the original multiplicand; and the difference is the true answer.

5. A boy can point sixteen thousand pins in an hour: How many will he do in six days, supposing he works eleven clear hours in a day? *See Blair's Universal Preceptor.*

6. What is the continual product of 25, 19, 705, and 999?*

7. How many changes can be rung on twelve bells? †

3. Multiply the difference between 50487 and 30056, by the sums of 850, 9067, and 800?

9. The sum of two numbers is 30355, and the greater number is 25251: What is their product? ‡

10. The sum of two numbers is 4584, and the less is 1876: What is their product?

11. What is the difference between twelve times fifty-seven, and twelve times seven and fifty?

12. How many miles will a person walk in sixty-six years, supposing he travels, one day with another, six miles, and there are 365 days in a year?

13. How many cubic feet does this room contain, which is fifteen feet long, fourteen feet wide, and thirteen feet high? §

---

NOTES.

* The continual product of any given numbers is found by multiplying them into one another: thus $8 \times 5 \times 9 = 360$, is the continual product of 8, 5, and 9.

† The continual product of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, will be the number required: thus the number of changes that can be rung on six bells is $1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$. By the same method we find, that the number of changes that can be made of the twenty-four letters in the alphabet is equal to $\underbrace{621654}_{4}\ \underbrace{561827}_{3}\ \underbrace{891919}_{2}\ \underbrace{360000}_{1}$ The letters in the English alphabet may be reckoned 24, as formerly the *i* and *j* were expressed by the same character, and so were the *u* and *v*.

‡ From the sum subtract the greater number, and the remainder will be the lesser number: the answer will be found by multiplying the greater and lesser numbers together.

§ To find the number of cubic feet in any room, &c., the continual product of the height, length, and width must be found.

# DIVISION.

* * *

By Division, we find how often one number is contained in another of the same denomination; this is a short method of performing Subtraction.

The sum to be divided is called the *dividend;* the figure, or figures, by which we divide, is called the *divisor;* and the result is called the *quotient.*

In this Rule, as in Multiplication, there are several distinct cases.

### I. When the divisor does not exceed 12.

Rule. *Write the divisor on the left-hand side of the dividend, make a curve, and consider how often the divisor is contained in the first figure, or in the first two or three figures, and set the quotient under it; and for every unit remaining after subtraction, carry* TEN *to the next figure of the dividend.*

EXAMPLES.

Ex. 1. 4)78654328
19663582

Ex. 2. 9)85674327
9519369—6

Ex. 3. 11)10876541
988776—5

Ex. 4. 12)11272489
939374—1

In the second example, I say there are 9 nines in 85 and 4 over; I put down the nine and carry the 4, as 40 to the 6, and the 9's in 46, 5 times and 1 over; put down the 5 and carry 1, as 10, and say the 9's in 17, once and 8 over; put down the 1 and carry 8 as 80; 9's in 83, 9 times and two over, and so on: at the last figure there are 6 remaining, this put down beyond a small line.

It is usual, in giving the answer, to make a short line under the remainder, and place under it the divisor: thus the answer to the second sum is 9519369$\frac{6}{9}$: that of the third sum is 988776$\frac{5}{11}$, and that of the fourth 939374$\frac{1}{12}$; and the three remainders are fractions, which we read six-ninths, five-elevenths, and one-twelfth. See p. 4 and 5.

## EXAMPLES.

| | | |
|---|---|---|
| 5)7639487 | 7)440295 | 8)5678943 |
| 1527897$\frac{2}{5}$ | 62899$\frac{2}{7}$ | 709867$\frac{7}{8}$ |

*This character ÷, when placed between two numbers, signifies that the one is divided by the other: thus* 95 ÷ 8 = 11$\frac{7}{8}$; *and we read* 95 *divided by* 8, *gives* 11 *and seven-eighths over; that is, there are* 11-*eights in* 95, *and seven remaining.*

## EXAMPLES.

| | |
|---|---|
| Ex. 1. 5687 ÷ 7 | Ex. 2. 49876 ÷ 3 |
| Here 5687 ÷ 7 = 812$\frac{3}{7}$. | 49876 ÷ 3 = 16625$\frac{1}{3}$. |
| For 7)5687 | 3)49876 |
| 812 — 3 | 16625 — 1 |

| | | | |
|---|---|---|---|
| Ex. 3. | 87240322 ÷ 3[*] | Ex. 4. | 62304678 ÷ 4 |
| 5. | 74009654 ÷ 5 | 6. | 26730217 ÷ 6 |
| 7. | 59234600 ÷ 7 | 8. | 37026541 ÷ 9 |
| 9. | 46872135 ÷ 8 | 10. | 56438752 ÷ 7 |
| 11. | 45900361 ÷ 9 | 12. | 3256487 ÷ 8 |
| 13. | 59764218 ÷ 10[†] | 14. | 3532640 ÷ 11 |
| 15. | 32753742 ÷ 12 | 16. | 3333333 ÷ 12 |
| 17. | 44444444 ÷ 11 | 18. | 5598764 ÷ 12 |
| 19. | 98897603 ÷ 9 | 20. | 9330048 ÷ 8 |

PROOF.—The method of proving the truth of sums in Division, is to multiply the answer by the divisor, and take in the remainder, the result will be equal to the dividend.

---

NOTES.

[*] When any number of the dividend is less than the divisor, put down a cypher, and carry it as a remainder to the next figure, unless it be the last figure in the dividend, then it is put down as a remainder; thus 7226 ÷ 4 = 1806$\frac{2}{4}$; for I say 4 in the 7, once and 3 over; 4 in the 32 goes 8 times; 4 in 2, I cannot, but 4 in 26, 6 times and 2 over.

[†] To divide by 10. Cut off the last figure in the dividend as a remainder, and the thing is done; thus, 76489 ÷ 10 = 7648 — 9. And 694312 ÷ 10 = 69431 — 2.

Ex. 7959467834 ÷ 7.

7)7959467834

Quotient - 1137066833 — 3

7

Proof - - 7959467834

$$\begin{array}{ccc} & 8 & \\ 7 & \times & 2 \\ & 8 & \end{array}$$

Another method is by casting out the nines, as in Multiplication.—Rule. Cast away the nines in the divisor, and put the remainder on one side of the cross; do the same with the quotient, for the other side of the cross: then for the top figure multiply these two numbers together, cast away the nines, and add the excess of nines in the remainder after division, and the excess of nines in this sum will be equal to the excess of nines in the dividend, if the work is right. See the preceding example, where I put down the 7 on one side of the cross; the excess of nines in the quotient is 2, which I put on the other side of the cross, then I say 7 times 2 are 14, and the remainder 3 make 17, which is 8 above nine, this I put at the top of the cross, and I find that 8 is the excess above the nines in the dividend, therefore I conclude the operation is right.

II. To divide a number of one denomination, by another number whose significant figures do not exceed 12, having a cypher or cyphers joined to the right hand.

Rule. *Cut off the cyphers from the divisor, and the same number of figures from the right-hand of the dividend; then divide the remaining figures of the dividend by the remaining part of the divisor, and the result is the answer. To the remainder, if any, join those figures of the dividend, which were first cut off, and the whole will be the true remainder.*[*]

Divide 4685321 by 800; and 326441 by 1200.

8,00)46853,21 12,00)3264,41

5856 — 521 272 — 41

Of course the true answers to these sums are $5856\frac{521}{800}$, and $272\frac{41}{1200}$.

---

NOTE.

[*] When nothing remains after the division, the figures that were cut off will be the true remainder; thus 76543246 ÷ 1200 = $63786\frac{46}{1200}$.

EXAMPLES.

Ex. 1. 3476521 ÷ 60 Ex. 2. 8543009 ÷ 700
3. 2937648 ÷ 800 4. 9003456 ÷ 9000
5. 5620042 ÷ 1100 6. 7641121 ÷ 500
7. 4052079 ÷ 1200 8. 8496531 ÷ 12000
9. 7921164 ÷ 90 10. 9939216 ÷ 8000
11. 46201132 ÷ 700 12. 1234567 ÷ 120

III. To divide a given number of one denomination, by a divisor which is compounded of two or more numbers in the Multiplication Table.

Rule. *Divide the given number by one of those parts, and the quotient by the other component part, and so on till each of the component parts has been used as a divisor;* thus 46875815777 ÷ 105 is performed as follows: the divisor 105 is equal to 7 × 5 × 3; I therefore divide the dividend first by 7, and the quotient by 5, and this second quotient by 3.

```
            7)46875815777
              -----------
            5)6696545111
              -----------
            3)1339309022 — 1 ⎫
              -----------    ⎬*
Answer  - -    446436340 — 2 ⎭
```

EXAMPLES.

Ex. 1. 84596543 ÷ 36 Ex. 2. 345069549 ÷ 42
3. 45897642 ÷ 56 4. 945960542 ÷ 99
5. 39200761 ÷ 66 6. 87932874 ÷ 768
7. 38426587 ÷ 550 8. 44444444 ÷ 121

---

NOTE.

* Here are two remainders, 1 and 2, to be accounted for, and the general rule in finding the value of remainders, in this mode of contracted division, is:

Rule. Multiply the *last* remainder by the *preceding* divisor, and to the product add the preceding remainder, and place the sum thus found over the two divisors multiplied into one another. Here 2, the last remainder, is to be multiplied by 5, which is 10; then add the first remainder 1, and set the 11 over 5 × 3, or 15; the true answer is 446436340$\frac{11}{15}$ = 446436340$\frac{77}{105}$, because I multiply the 11 and the 15 by the first divisor 7.

If there be *many* remainders, instead of two, multiply the sum first found by the next preceding divisor, and to the product add

9. 28476974 ÷ 720 10. 55555555 ÷ 378
11. 56342872 ÷ 132 12. 33992288 ÷ 288
13. 34765982 ÷ 144 14. 98453392 ÷ 432
15. 24853274 ÷ 512 16. 83547552 ÷ 99
17. 43333999 ÷ 343 18. 54954335 ÷ 720
19. 5555556 ÷ 729 20. 25574538 ÷ 343

IV. To divide by a number consisting of two or more digits, which number is not compounded of those in the table.

RULE. (1). *Draw a curved line on the right and left of the dividend, and write the divisor on the left.*

(2). *Find how many times the divisor is contained in as many figures of the dividend as are just necessary, and place the number on the right for a quotient.*

(3). *Multiply the divisor by the quotient figure, and place the product under the above-mentioned figures of the dividend, subtract this product from that part of the dividend under which it stands, and bring down the next figure in the dividend, or more if necessary, to the right-hand of the remainder, and proceed as before, till the whole is finished.* This is called LONG DIVISION.

---

NOTE.

the next preceding remainder, and so on till all the divisors and remainders are gone through: suppose the sum be 8476521 to be divided by 525 = 5 × 5 × 7 × 3.

5)8476521
5)1695304 — 1
7)339060 — 4
3)48437 — 1
16145 — 2

Multiply 2 by 7, and take in 1 = 15. Now 15 × 5 and take in 4 = 79. And 79 × 5 and take in 1 = 396. The true answer, therefore, is $16145\frac{396}{525}$; for the Arithmetician will easily find that the fraction $\frac{396}{525}$ is equal to the sum of the several fractions $\frac{2}{3} + \frac{1}{21} + \frac{4}{105} + \frac{1}{525} = \frac{350}{525} + \frac{25}{525} + \frac{20}{525} + \frac{1}{525}$.

## Ex. 5537049 ÷ 954.

954)5537049(5804 Quotient
4770 . . .
7670
7632
3849*
3816
33 Remainder. Answer $5804\frac{33}{954}$.

Here, the divisor not being contained in the first three figures, I consider how often it is contained in the first four,† and find it to be 5 times, the 5 I put in the quotient, and multiply the divisor by it, setting the product under the dividend. I now subtract this product, and to the remainder 767, I bring down the 0, and find that the divisor is contained 8 times in 7670, the 8 I place in the quotient, and proceed to multiply the divisor by it; the product subtracted leaves only 38; I now bring down the 4, but the divisor not being contained in 384, I put down 0 in the quotient, and bring down the 9, the remaining figure in the dividend, and proceed as before.‡

---

NOTES.

* When more figures than one are brought down to the remainder, to make it larger than the divisor, a cypher must be written in the quotient for every figure so brought down.

† The only difficulty in Long Division, is finding readily how often the divisor is contained in each of the parts of the dividend; in long sums it will be found very convenient to beginners to construct a table of the products of the divisor from 1 to 9, whence the several quotient figures, and their corresponding products will be seen at first sight: thus, if the divisor be 954, the table is as follows:

| | |
|---|---|
| 1 | 954 |
| 2 | 1908 |
| 3 | 2862 |
| 4 | 3816 |
| 5 | 4770 |
| 6 | 5724 |
| 7 | 6678 |
| 8 | 7632 |
| 9 | 8586 |

‡ 1. The pupil will always remember, that as many figures of the dividend must be taken as will contain the divisor; under the last of these let him place a dot, and likewise under every subsequent figure, as he makes use of it, to the end of the dividend: of course

### EXAMPLES.

Ex. 1. 78654321 ÷ 76
Ex. 2. 56943278 ÷ 97
3. 68742164 ÷ 87
4. 84365487 ÷ 69
5. 77755562 ÷ 654
6. 45687403 ÷ 187
7. 53430432 ÷ 7654
8. 56943286 ÷ 429
9. 57678443 ÷ 8439
10. 58456942 ÷ 3279
11. 564320376 ÷ 3976
12. 92876487 ÷ 7392
13. 677744032 ÷ 5186
14. 46859210 ÷ 1437
15. 627432871 ÷ 4967
16. 55555555 ÷ 7777.
17. 44444444 ÷ 5555
18. 888000999 ÷ 999
19. 33333333 ÷ 999
20. 111111111 ÷ 7777

Ex. 21. 487264325876 ÷ 56780909
22. 876842987621 ÷ 90956843
23. 918318296542 ÷ 56400032
24. 567843276549 ÷ 64785321
25. 877896543210 ÷ 92836058
26. 444444444444 ÷ 750000564
27. 2220003330046 ÷ 708385032
28. 540965328762 ÷ 5406057
29. 32899438654 ÷ 10010432
30. 784363254871 ÷ 99834369

After the pupil has gone through all the foregoing examples in Long Division, he should be taught the Italian Method, as it is usually called, of which the following is an example, worked at length; and as the Italian method is so much neater, and with practice full as easy, and taking up only half the space, it is recommended that the learner should repeat the former examples by this mode of operation.

### Ex. Divide 6452800 by 765.

765)6452800(8435
3328 · ·
2680
3850
25 Remainder.

The answer is - $8435\frac{25}{765}$.

---

NOTE.

the number of digits in the quotient will be equal to the number of dots in the dividend.

2. Every remainder must be less than the divisor; for, if it be equal to or greater than the divisor, the quotient figure which has produced the result is too small.

3. When the product of the divisor by the quotient figure is greater than the particular part of the dividend used, the quotient figure is too large.

Illustration.—I find the divisor is contained 8 times in the first four figures, I accordingly put 8 in the quotient, and multiply in this manner, 8 times 5 are 40, and put down as a remainder 2, the difference between 0 the units place of the number gained by multiplication, and the figure in dividend, from which it was to be subtracted, carry 4: 8 times 6 are 48 and 4 are 52, the difference now between the 5 in the dividend and the 2 in 52 is 3, which I put down and carry 5: 8 times 7 are 56 and 5 are 61, between which and 64 is 3, which I put down. Bring down the next figure 8, and proceed: the divisor will now go 4 times, I put 4 in the quotient, and say 4 times 5 are 20, put down 8 and carry 2: 4 times 6 are 24 and 2 are 26, the difference between 26 and 32 are 6, which I put down and carry 3: 4 times 7 are 28 and 3 are 31, which taken from 33 leaves 2. Bring down 0, the divisor now is contained three times in the dividend, the 3 I put in the quotient, and say 3 times 5 are 15, this taken from 20 leaves 5, which put down and carry 2: 3 times 6 are 18 and 2 are 20, which taken from 28 leaves 8, carry 2: 3 times 7 are 21 and 2 are 23, which taken from 26 leaves 3. Bring down the other 0, the quotient is now 5, with which proceed as before.

* * *

## MISCELLANEOUS EXAMPLES.

Ex. 1. Divide fifty millions by four thousand and seventy-nine.

2. The planet Mercury goes round the sun in 88 days, which is the length of her year, how many years of Mercury would make 50 of our years, supposing each year contained exactly 365 days?

3. It is estimated that there are a thousand millions of inhabitants in the known world: if one thirty-third of this number die annually, how many deaths are there in a year?

4. The national debt, at present, cannot be less than five hundred millions sterling: how long would that be in paying off, at the rate of two millions and twenty-five pounds per annum?

5. The taxes annually collected amount to full thirty-three millions of pounds: how many poor families of six persons each would that sum support, supposing the annual expenses of the father and mother to be 20*l.*, and of each child 7*l.*?

6. My friend is to set sail to Jamaica on the first of March, 1812; the distance is reckoned to be 3984 miles: at what rate will he go, supposing he reaches the Island on the 10th day of April, that is, in 41 days?

7. What is the difference between the 12th part of 20,100, and the 5th part of 9110?

8. The prize of 30,000*l.* of the last Lottery became the property of 15 persons: how much was each person's share, after they had allowed 750*l.* to the office-keeper for prompt payment?

9. The sum of two numbers is 1440, the lesser is 48: what is their difference, product, and quotient?

10. The crew of a ship, amounting to 124 men, have to receive, as prize-money, 1890*l.*; but as they are to be paid off, they determine to make their commander and boatswain a present, the one of a piece of plate, value 25*l.*; the other of a whistle, which is to cost 5*l.*: how much will each receive after these deductions are made?

11. In all parts of the world a cubical foot of water weighs 1000 ounces: how many pounds are there, supposing 16 ounces make a pound?

12. A cubical foot of air weighs one ounce and a quarter, how many pounds avoirdupois of air does a room contain, which is 10 feet high, 14 feet wide, and 16 feet long?*

13. Hydrogen gas, or, as it was formerly called, inflammable air, that is, the gas with which balloons are filled, is full nine times lighter than the common air which we breathe: how much less would a balloon, containing 27,000 cubical feet, weigh if filled with hydrogen gas, than if filled with common air?†

14. At what rate per hour and per minute does a place on the equator move, supposing the great circle of the earth to be 25,000 miles, and the earth to turn on its axis exactly in 24 hours?

---

NOTES.

* The number of cubical feet is obtained by multiplying the height, width, and length together, or 10 × 14 × 16, and the product multiplied by $1\frac{1}{4}$ gives the number of ounces, which, divided by 16, is the answer, viz. 175 pounds avoirdupois. This circumstance cannot fail of exciting surprize, to conceive that in a moderate sized room, the air, which is invisible and scarcely observed to exist, should be known to weigh more than three half-hundred weights.

† The answer will be found to be 1875 lb.; of course the balloon would ascend, with several persons in its boat; because it will ascend, when the balloon and persons are together, lighter than an equal bulk of common air.

# REDUCTION.

Reduction is the method of converting numbers from one name, or denomination, to another of the same value; and it is divided into *Reduction descending*, and *Reduction ascending.*

*When numbers of a higher denomination are to be brought to a lower, it is called Reduction* descending, *and it is performed by* Multiplication.

*When numbers of a lower denomination are to be brought to a higher denomination, it is called Reduction* ascending, *and is performed by* Division.

## REDUCTION DESCENDING,

### OR CONVERTING GREAT INTO SMALL.

Rule. *Multiply the given number by as many of the lower denomination as make one of the higher.*

Thus, in reducing 55*l.* into shillings, I multiply the 55 by 20, and the answer is 1100 shillings; in both cases the value is the same, that is, 55*l.* is equal to 1100 shillings.

## REDUCTION ASCENDING,

### OR CONVERTING SMALL INTO GREAT.

Rule. *Divide by as many of the lower denomination as make one of the next higher.*

Thus, in bringing 890 pence into shillings, I divide the number by 12, and the answer is 74 shillings and two pence over.*

---

NOTE.

* The remainders, if any occur, are always of the same denomination as the respective dividends.

### TABLE.

| | | | |
|---|---|---|---|
| 4 farthings (q) | - - make | 1 penny | d. |
| 12 pence | - - - make | 1 shilling | s. |
| 20 shillings | - - - make | 1 pound | £. |
| 21 shillings | - - - make | 1 guinea.* | |

$\frac{1}{4}$ denotes a farthing: $\frac{1}{2}$ two farthings, or a halfpenny: and $\frac{3}{4}$ three farthings.

### EXAMPLES.

£. *s.* *d.*
Ex. 1. Reduce 29 6 $8\frac{3}{4}$ into farthings.
20
586 shillings
12
7040 pence
4
Answer 28163 farthings.

I multiply the 29 by 20, and take in the 6, thus I find 586 shillings are equal to 29*l.* 6*s.* To reduce the shillings to pence, I multiply the 586 by 12, and take in the 8, which give 7040, the number of pence equal to 29*l.* 6*s.* 8*d.* I next multiply 7040 by 4, and take in the $\frac{3}{4}$, and find the answer is 28163 farthings, equal to the given sum 29*l.* 6*s.* $8\frac{3}{4}$*d.*

Ex. 2. In 28163 farthings how many pounds sterling?

4)28163
12)7040 — $\frac{3}{4}$
2,0)58,6 — 8*d.*
Answer £.29 6 $8\frac{3}{4}$

I divide the 28163 farthings by 4, because 4 farthings make a penny; the answer is 7040, and 3 over, which are farthings, because the remainder is always of the same denomination as the dividend. I next divide the 7040 by 12, and the answer is 586 and 8 over, which are pence; and now 586 divided by 20, gives 29 and 6 over, which are shillings; the true answer is therefore 29*l.* 6*s.* $8\frac{3}{4}$*d.*†

Ex. 3. Reduce 28 shillings to pence.
4. Bring 56 pounds into shillings.
5. Reduce 672 pence into farthings.
6. How many pence are there in 105*l.*?
7. In 1000 guineas how many shillings?
8. In 4704*l.* how many pence?

---

NOTES.

* £. *s.* *d.* and *q.* are the initials of the Latin words Libræ, Solidi, Denarii, and Quadrantes; signifying pounds, shillings, pence, and farthings.

† Hence it is evident, that the best method of proving the truth of examples in Reduction is, to reverse the operation. It will also be the best method of teaching the pupil to depend on his own exertions, by making him prove each example, and not permitting him to begin the second till he has made the first come right.

Ex.

9. In 3995*l.* how many farthings?
10. In 7968 guineas, how many farthings?
11. How many farthings are there in 75 guineas?
12.[*] Reduce 576*l.* into farthings.
13. In 99*l.* how many shillings, pence, and farthings?
14. Reduce 567*l.* 9*s.* 9½*d.* into farthings.
15. How many halfpence are there in 157*l.* 7*s.* 7½*d.*
16. In 1084890 pence, how many pounds?
17. In 8410896 pence, how many guineas?
18. In 4808764 farthings, how many pounds?
19. How many seven-shilling pieces are there in a thousand guineas?
20. How many groats are there in a hundred guineas?
21. Bring 3110456 pence into groats, shillings, and pounds.
22. How many crown-pieces are there in 79*l.* 15*s*?
23. How many half-crowns are there in 85*l.* 12*s.* 6*d.*?
24. In 769 guineas, how many sixpences?

# TROY,

## *OR GOLDSMITHS' WEIGHT.*[†]

TABLE.

| | | | |
|---|---|---|---|
| 24 grains (gr.) | make | 1 pennyweight, | dwt. |
| 20 pennyweights | | 1 ounce, | oz. |
| 12 ounces | | 1 pound, | ℔. |

Ex. 1. How many grains of gold are there in a cup weighing 3 lb. 9 oz. 6 dwts. 18 grs.?[‡]

---

NOTES.

[*] These ten examples are all in descending Reduction; but as they furnish the pupil with an equal number in ascending Reduction, it will be advisable that he should make himself perfect in these, before he proceed to others rather less simple.

[†] By this weight, gold, silver, jewels, and precious stones are weighed. It is also used in ascertaining the strength of liquors.

[‡] If the pupil has made himself master of the foregoing examples in money, he will meet with no serious difficulty in this and the following articles in Reduction. *Let him bear in mind, that, in all cases, to reduce a greater name to a lesser he must multiply; and to bring a lesser denomination to a greater he must divide; and the table at the top of each article shows at once what numbers are to be used in the multiplication and division.* If I want to bring pounds troy into grains, I multiply by 12, by 20, and by 24. If I want to bring grains into pounds troy, I reverse the operation, and divide by 24, 20, and 12. For the same reason, as will be seen hereafter, if I want to know how

lb. oz. dwts. gr.
3 9 6 18
12

45
20

906
24

3632
1813

21762

Here I multiply the 3 by 12, and take in 9 for the number of ounces: I then multiply 45 by 20, and take in the 6 for the pennyweights: and afterwards the 906 by 24, and take in 18 for grains. I take the 8 in when I multiply by 4, and the 1 when I multiply by 2.

Ex. 2. How many pounds Troy are there in a million of grains?

4)1,000000
6)250000
2,0)4166,6 — 4* = 16 grains.
12)2083 — 6
173 — 7 Answer 173 lbs. 7 oz. 6 dwts. 16 grs.

Ex.
3. In 36lb. 10 oz. 12 dwts. 16 grs. how many grains?
4. How many pounds troy are there in 5987 pennyweights?
5. In 1434 lb. 0 oz. 0 dwts. 19 grs. how many grains?
6. How many pounds are there in 45065 grains?
7. Reduce 105 lbs. troy into grains.
8. In 495 spoons, weighing 103 lbs. 1 oz. 10 dwts., how many grains? †

---

NOTES.

many minutes there are in 36 days, I multiply that number by 24, because 24 hours make a day; and then by 60, because 60 minutes make an hour.

* Instead of dividing by 24 in long Division, I have divided by the component parts 6 and 4, (see Division, p. 27.) In the second division there is a remainder of 4; to find the value of which I multiply it by the first divisor, of course the true remainder is 16 grains. The result would have been the same, though differently expressed, had I divided by 6 and by 4, instead of 4 and 6: in the former case there would have been two remainders, viz. 4 and 2, to find the value of which I should multiply 2, the last remainder by 6 the first divisor, and add 4, the first remainder, making together 16, as before.

† It will be very desirable, that the pupil should prove the truth of all the examples in Reduction; and if at the end of each article

# AVOIRDUPOIS;

## OR GROCERS' WEIGHT.*

### TABLE.

| | | |
|---|---|---|
| 16 drams (dr.) - | make | 1 ounce, oz. |
| 16 ounces - - - | | 1 pound, ℔. |
| 8 pounds - - - | | 1 stone of meat.† |
| 14 pounds - - - | | 1 stone, horseman's weight.‡ |
| 28 pounds - - - | | 1 quarter, qr. |
| 4 quarters, or 112 lb. - | | 1 hundred weight, cwt. |
| 20 hundred weight - | | 1 ton. |

---

NOTES.

he be not quite expert in working them, he will find an advantage in repeating the operations before he proceed to the next.

* By this weight almost all coarse and heavy goods are weighed; such as butcher's meat, grocery, cheese, butter, &c.; wax, pitch, tallow, and all metals, excepting gold and silver. Avoirdupois weight was first used in Henry VIIIth's reign; and was introduced expressly for weighing butcher's meat, and other coarse and heavy articles.

The *Avoirdupois* ounce is *less* than the *Troy* ounce; but the Avoirdupois pound is greater than the pound Troy. 175 Troy ounces are equal to 192 Avoirdupois ounces; but 144 lb. Avoirdupois are equal to 175 pounds Troy. Therefore, one pound Avoirdupois is equal 1 lb. 2 oz. 11 dwts. 16 gr. Troy.

Hence the following Table:—

144 lb. Avoirdupois = 175 lb. Troy.
192 oz. - - = 175 oz.

| | lb. | oz. | dwts. | gr. | |
|---|---|---|---|---|---|
| 1 lb. - - = | 1 | 2 | 11 | 16 | = 7000 grs. troy. |
| 1 oz. - - = | 0 | 0 | 18 | $5\frac{1}{2}$ | = $437\frac{1}{2}$ |
| 1 dr. - - = | 0 | 0 | 1 | $3\frac{1}{3}$ | = 27.35 |

| | lb. | oz. | dr. |
|---|---|---|---|
| 1 lb. Troy - = | 0 | 13 | $2\frac{3}{4}$ nearly Avoirdupois. |
| 1 oz. - - = | 0 | 1 | $1\frac{1}{2}$ |

Hence the difference between the lb. Avoirdupois, and the lbs. Troy and Apothecaries' weight, is, that the first contains 7000 grs.; both the others only 5760 grs.

† The stone of meat in some counties is 12 lb., in others 14, and even 16 lb.

‡ This stone is the standard at Newmarket; that is, the persons riding races are weighed by this stone, and a jockey that is said to weigh $7\frac{1}{2}$ stone, weighs 105 lb. avoirdupois. By the avoirdupois ounce and lb., hay and bread are weighed, according to the following tables:

Ex. 1. How many drams are there in 225 tons, 17 cwt. 3 qr. 24 lb. 12 oz. 8 dr.?

```
tons, cwt. qr. lb. oz. dr.
 225   17   3  24  12   8
  20
 ----
 4517
    4
 ----
18071
   28
------
144572
36144
------
506012
    16
-------
3036074
506013
-------
8096204
     16
--------
48577232
8096204
---------
129539272
```

I multiply by 20 and take in the 17 cwt., because 20 cwt. make one ton; then by 4 and take in the 3, because 4 quarters make a cwt,; then by 28 and take in the 24, because 28 lb. make a quarter; then by 16 and take in the 12, because 16 ounces make a pound: and again by 16 and take in the 8, because 16 drams make an ounce.

Answer, 129539272 drams.

NOTE.

## HAY WEIGHT.

| | |
|---|---|
| 56 lbs. of old hay - - - - -<br>60 lbs. of new hay - - - - | 1 truss. |
| 36 trusses - - - - - - - | 1 load. |

Of straw, 36 lbs. make the truss, and 36 trusses the load.

| BREAD. | lb. | oz. |
|---|---|---|
| A peck loaf weighs - - - - | 17 | 6 |
| A half ditto - - - - - | 8 | 11 |
| A quartern ditto - - - - - | 4 | 5½ |
| A peck of flour weighs - - - | 14 | 0 |
| A bushel - - - - - - - | 56 | 0 |
| A sack, or 5 bushels - - - - | 280 | 0 |

Ex. 2. How many tons are there in 259078544 drams?

```
4)259078544
 -----------
 4)64769636
 -----------
 4)16192409
  ---------
  4)4048102 — 1 ⎫   oz.
  ---------     ⎬= 9
  7)1012025 — 2 ⎭
  ---------
   4)144575
    -------           lb.
    4)36143 — 3 =  21
    -------
  2,0)903,5 — 3
    -------
     451   15   3   21   9
```

I divide by the same numbers with which I multiplied in the last example, only in the reverse order: and instead of dividing by 16, 16, and 28, by long division, I divide by their component parts, 4 × 4; 4 × 4; 7 × 4.—In bringing the ounces into pounds, I have two remainders, viz. 1 and 2, to find the value of which, I multiply the last remainder 2, by the first divisor 4, and take in the 1, which make 9 ounces. For the same reason the remainder 3 is equal to 21 lb. See note to pages 27 and 28.

| | tons, | cwt. | qrs. | lb. | oz. |
|---|---|---|---|---|---|
| Answer | 451 | 15 | 3 | 21 | 9 |

Ex. 3. In 179 cwt. how many pounds?

Ex. 4. Reduce 8345 tons into quarters.

Ex. 5. How many ounces are there in 4 tons, 15 cwt. 2 qrs. 12 lb.?

Ex. 6. In 233076 ounces of sugar, how many cwt.

Ex. 7. How many drams are there in 53 tons, 14 cwt. 1 qr. 12 lb. 14 oz. 8 dr.?

Ex. 8. In 32384818 drams, how many tons weight?

# APOTHECARIES' WEIGHT.*

## TABLE.

| | | | | | |
|---|---|---|---|---|---|
| 20 grains (gr.) | - | make 1 scruple | ℈ | = | 20 gr. troy. |
| 3 scruples | - - | —— 1 dram | ʒ | = | 60 |
| 8 drams | - - | —— 1 ounce | ℥ | = | 480 |
| 12 ounces | - - | —— 1 pound | ℔ | = | 5760 |

---

NOTE.

* By this weight Apothecaries mix their medicines, but they buy their drugs by Avoirdupois weight. The pound and ounce made use of by Apothecaries, and the pound and ounce Troy weight, are the same, but the smaller divisions are different. See note to Troy weight.

Physicians write their prescriptions according to the following table and characters:

| | | | | |
|---|---|---|---|---|
| 20 grs. troy | - | = 1 scruple | ℈j | |
| 60 —— | - | = 1 dram | ʒj | = ℈iij |
| 480 —— | - | = 1 ounce | ℥j | = ʒviij |
| 5760 —— | - | = 1 pound | ℔j | = ℥xij |

Ex. 1. How many grains are there in 2 lb. 5 oz. 4 dr. 1 scr. 17 gr.?

```
lb. oz. dr. scr. gr.
 2   5   4.  1   17
12
——
29
 8
———
236
  3
———
709
 20
————
14197
```

I multiply the pounds by 12, and take in the 5, because 12 ounces make a pound: afterwards by 8, 3, and 20, taking in the several drams, scruples, and grains, as in the former articles.

Answer - 14197 grains.

Ex. 2. In 42591 grains, how many pounds?

```
2,0)4259,1
    ——————
  3)2129 — 11
    ——————
  8)709 — 2
    ——————
 12)88 — 5
    ——————
     7 . 4   5   2   11
```

The multipliers in the last example are made divisors in this, in the reverse order.

Answer - 7 lb. 4 oz. 5 dr. 2 scr. 11 gr.

Ex. 3. In 51 lb. 2 oz. of rhubarb, how many scruples?

Ex. 4. In 234876 grains, how many pounds?

Ex. 5. How many pounds are there in 1000 ounces of opium?

Ex. 6. In 239 lb. 9 oz. 2 dr. 2 scr. 14 gr., how many grains?

Ex. 7. How many scruples are there in one hundred and three ounces of Peruvian bark?

Ex. 8. In 126794 grains, how many pounds?

---

NOTE.

Apothecaries make use of the following characters also:

℞ recipe, take.
á, áá, or ana, of each the same quantity.
ß, or ss, signifies the half of any thing.
cong, congius, a gallon.
coch. cochleare, a spoonful.
M. manipulus, a handful.
P. pugil, as much as can be taken between the thumb and two fore fingers.
q. s. a sufficient quantity.

## WOOL WEIGHT.

### TABLE.

| | | |
|---|---|---|
| 14 pounds[*] . make . | 1 stone, | st. |
| 2 stone, or 28 pounds, | 1 tod . . | t. |
| $6\frac{1}{2}$ tod, or 13 stone, or | 1 wey, or weigh, | w. |
| 2 weys . . . . . . . . | 1 sack, . . . . . | s. |
| 12 sacks . . . . . . . . | 1 last, . | l. |

Ex. 1. How many stone are there in 6 weys of wool?

$$\begin{array}{r} 6 \\ 6\frac{1}{2} \\ \hline 36 \\ 3 \\ \hline 39 \\ 2 \\ \hline 78 \end{array}$$

In multiplying by $6\frac{1}{2}$, I first take the 6, as usual, and then to multiply by $\frac{1}{2}$, I divide the multiplicand by 2, place the quotient 3 under the former product, and add them together for the answer.

Ex. 2. In 786 stone of wool, how many weys?

$$\begin{array}{l} 13)\overline{786} \\ \quad 60\text{—}6 \end{array}$$

The readiest way of working this example is, to divide the stone by 13; because 13 stone make 1 wey. The answer is, 60 weys and 6 stone, or 60 weys and 3 tods.

Ex. 3. In a pack of wool, weighing 3 cwt. 2 qrs. how many tods are there?

Ex. 4. How many ounces are there in a tod of wool?

* * *

## LONG MEASURE.[†]

### TABLE.

| | | |
|---|---|---|
| 3 barley corns (b.c.) . | 1 inch, | in. |
| 12 inches[‡] . . . . . . . . | 1 foot, | ft. |
| 3 feet, or 36 inches . . | 1 yard, | yd.[‖] |
| 2 yds. or 6 ft. or 72 inch. | 1 fathom, | fth.[§] |
| $5\frac{1}{2}$ yards,[¶] or $16\frac{1}{2}$ ft. | 1 pole, or rod, | p. |
| 40 poles, or 220 yds. | 1 furlong, | fur.[**] |
| 8 furlongs, or 1760 yds. | 1 mile, | m.[††] |
| 3 miles, or 5280 yds. | 1 league, | lea. |

---

NOTES.

[*] The table usually begins with 7 pounds make 1 clove; but the clove differs n different counties, and, besides, is not often used in the wool trade. The stone s also different in different counties: in Gloucestershire it is 15lb., but in Here-ordshire it is only 12lb. By a statute of Hen. VII. it was made 14lb. as above.

[†] The origin of Long Measure is taken from a grain of barley, of which 3, se-ected out of the middle of the ear and well dried, make 1 inch; accordingly, 1 arley-corn is the least measure. But in this, as in other weights and measures, he standard to which all are referred, is preserved at Guildhall, London; and herefore we have no need to look after grains of wheat, or of barley, to get accu-ate weights and measures.

[‡] Four inches make a *hand*, which is used in estimating the height of horses.

[‖] The English yard is said to have been taken from the arm of king Hen. I., the year 1101.

[§] The French *toise* answers to the English fathom, with this difference, that 72 rench inches are equal to 76.736, or nearly $76\frac{3}{4}$ English inches.

[¶] The pole is different in different counties: in Lancashire, 7 yards make a ole, and in Cheshire, an adjoining county, they reckon 8 yards to a pole; and n some counties, 6 yards go to a pole.

[**] A furlong being 220 yards, or 660 feet, or 7920 inches; a chain, used in ind-measuring, is the tenth part of a furlong, or 792 inches; and there being 00 links in a chain, the link is 7.92 inches.

[††] The mile is of different lengths in different countries. The ancient

Ex. 1. How many yards are there between London and Bath, the distance of which is 108 miles?

108
8
864
40
34560
$5\frac{1}{2}$
172800
17280
190080

In multiplying by $5\frac{1}{2}$, I multiply first by the 5 as usual; then to multiply by $\frac{1}{2}$, I divide the multiplicand by 2, & add the quotient so found, to the product already obtained: a shorter way would be to multiply 108 by 1760, the number of yards in a mile: thus . . 1760
108
14080
17600
190080

Answer . . . 190080 yards.

---

NOTES.

Roman, and modern Italian mile, contained 1000 paces, *mille passus*, whence the term *mile* is derived. The following table will shew the length of the mile, or league, in the principal nations of Europe, expressed in yards.

| | yards |
|---|---|
| Mile of Russia . . . . . is . . . | 1100, or 5 furlongs |
| —— Italy . . . . . — . . . | 1467 nearly, or 5-6ths of an Eng. mile |
| —— England . . . — . . . | 1760 |
| —— Scotland and Ireland . . | 2200, or 1 English mile and a quarter |
| Small league of France,* . . . | 2933, or 1 English mile, and 2-3ds of a mile |
| Mean league of ditto . . . . . . | 3666, or 2 English miles, and 1-12th of a mile |
| Great league of ditto .<br>Mile of Poland . . . . | . . . 4400, or 2 English miles and a half |
| —— Spain . . . . . . . . . . | 5028, or 3 English miles nearly |
| —— Germany . . . . . . . | 5866, or 3 English miles and a half |
| —— Sweden . . .<br>—— Denmark . . | . . . 7233, or 4 miles and 1-6th of a mile |
| —— Hungary . . . . . . . . . | 8800, or 5 English miles. |

---

* The French, in all these measures, now make use of the Metre, which is equal to $39\frac{1}{3}$ inches nearly, or to one yard, three inches, and one-third of an inch.

Ex. 2. In 760329 feet, how many leagues?

3)760329  
253443  
2  
11)506886  
4,0)4608,0 — 6 = 3  
8)1152 — 0  
3)144 — 0  
48

I first bring the feet into yards, by dividing by 3; then, as I cannot divide by $5\frac{1}{2}$, I multiply the last quotient by 2, to bring it into *half-yards*, and divide by 11, because there are 11 half-yards in a pole, I find a remainder of 6, which are half-yards, equal to 3 yards.

Ans. 48 lea. 0 m. 0 fur. 0 p. 3 yds.

Ex. 3. How many inches are there in 1009 miles?

Ex. 4. Reduce 57 m. 4 fur. 38 p. 3 yds. 2 ft. 3 in. 1 b. c. into barley-corns.

Ex. 5. In 100004 poles, how many inches?

Ex. 6. In 409683 feet, how many furlongs?

Ex. 7. How often will the wheel of a coach turn round in going from London to Sheffield, or in 160 miles, supposing the circumference of the wheel to be 16 feet?

Ex. 8. Suppose on an average I step two feet and a half: how many steps shall I take in walking from London to Richmond, a distance of 10 miles?

* * *

## CLOTH MEASURE.

### TABLE.

| | | | |
|---|---|---|---|
| $2\frac{1}{4}$ inches - - - - | make | 1 nail, | n. |
| 4 nails, or 9 inches - - | | 1 quarter, | qr. |
| 4 quarters, or 36 inches - - | | 1 yard, | yd. |
| 5 quarters, or 45 inches - - | | 1 English ell, | E.e. |

Ex. 1. How many inches in length are there in 156 ells of cambric?

156  
5  
780  
$2\frac{1}{4}$  
1560  
195  
1755

In multiplying by $2\frac{1}{4}$, I first multiply by the 2, then *divide* the multiplicand by 4, which is the same as multiplying by $\frac{1}{4}$; add the sums thus found for the true answer.

Answer - 1755 inches.

Ex. 2. In 1000 inches of cotton, how many yards are there?

9)1000
——
4)111 — 1
——
27 3 0 1

Here I divide by 9, because 9 inches make a quarter of a yard, and it is easier to divide by 9, than by $2\frac{1}{4}$, and then by 4.

Ans. 27 yds. 3 qr. 0 n. 1 in.

Ex. 3. How many English ells are there in three thousand and fifty-five nails?

Ex. 4. In 15 yds. 2 qr. 3 n. 1 in., how many half inches?

Ex. 5. How many inches are there in 10056 yards?

Ex. 6. Reduce 546 English ells to nails.

* * *

# SQUARE, OR LAND MEASURE.*

TABLE.

| | | |
|---|---|---|
| 144 square inches | - - make | 1 square foot |
| 9 square feet | - - - | 1 square yard |
| 100 square feet | - - - | 1 sq. of flooring, roofing, &c. |
| 100 acres | - - - - | 1 hide |
| $30\frac{1}{4}$ square yards | - - - | 1 perch, p. |
| 40 perches | - - - - | 1 rood, rd. |
| 4 square roods, or 4840 square yds. | | 1 acre† |
| 640 acres | - - - - | 1 square mile. |

---

NOTES.

* Square-measure is used to estimate all kinds of superficies, such as land, paving, plastering, roofing, tiling, and every thing that has length and breadth; thus, if I want to measure a room 25 feet long and 18 broad, I multiply 25 by 18, and find the square measure equal to 450 feet.

Land is measured by a chain, called Gunter's chain, which is 4 poles, or 22 yards, or 66 feet long; and it consists of 100 equal links. Ten chains in length, and one in breadth, make an acre, that is, 100 × 100 × 10 = 100000, equal the number of links in an acre. If therefore I have a field to measure sixty-three chains and fifty-five links long, and twenty-five chains twelve links wide, I put them down thus: 63.55 × 25.12 = 159.63760, the answer is 159 acres, and 63760 links over. The chains and links are separated by a dot, and then they are multiplied as in common Multiplication, taking care to cut off, by a dot, five figures of the product towards the right-hand, which is the same thing as to divide by 100000.

If I wish for greater accuracy, I multiply the remainder 63760 by 4, the number of roods in an acre, and cut off five figures again as a remainder, which must then be multiplied by 40, the number of perches in a rood: thus 63760 × 4 = 2.55040: now 55040 × 40 = 22.01600. The true answer, therefore, is 159 ac. 2 rd. 22 p., and a remainder of 1600.

† A piece of ground, as a field, garden, &c., that measures rather

Ex. 1. How many yards are there in 5604 acres?

5604
4
22416
40
896640
$30\frac{1}{4}$
26899200
224160
27123360

Here, as in a former case, to multiply by $30\frac{1}{4}$, I first multiply by the 30, and then *divide* the multiplicand by 4, which is the same as to multiply by a $\frac{1}{4}$; and adding together the two sums thus found for the answer.

Answer - - 27123360 yards.

Ex. 2. In 6534 square feet, how many perches?

9)6534
726
4
121)2904(24
242
484
484
...

As we cannot divide by $30\frac{1}{4}$, I multiply the 726 yards by 4, to bring them into quarters, and then divide by 121, because there are 121 quarters in $30\frac{1}{4}$ yards.

Answer - 24 perches.

Ex. 3. How many roods are there in 382 perches?
Ex. 4. In 561 acres of ground, how many perches and yards?
Ex. 5. In 2967400 inches, how many acres?
Ex. 6. How many perches are there in 997 acr. 2 rd. 10 p.?

When length, breadth, and thickness, are to be taken into consideration, it is called cubic, or solid measure, which is used to estimate the quantity of stone or marble in blocks, or of timber in trees. Hence the following Measure:

---

NOTE.

more than $69\frac{1}{2}$ yards in length, and as much in breadth, contains just an acre. A garden of half an acre is comprised in a square, whose sides are 49 yards 5 in. long: and the sides of one of a quarter of an acre will be nearly 35 yards long.

A square is a geometrical figure of four equal sides and angles: and a square number is produced by multiplying any number into itself, thus; 49 and 144 are square numbers, being produced by multiplying 7 and 12 into themselves, as $7 \times 7 = 49$, and $12 \times 12 = 144$.

# CUBIC, OR SOLID MEASURE.[*]

## TABLE.

| | | |
|---|---|---|
| 1728 cubic inches | make | 1 cubic foot |
| 27 cubic feet | | 1 yard |
| 40 feet of rough timber | | 1 load, or ton.[†] |
| 50 feet of hewn timber | | |
| 42 feet | | 1 ton of shipping. |

Ex. 1. In 36 solid yards, how many inches?

$$\begin{array}{r} 36 \\ 27 \\ \hline 252 \\ 72\phantom{0} \\ \hline 972 \\ 1728 \\ \hline 7776 \\ 1944\phantom{0} \\ 6804\phantom{00} \\ 972\phantom{000} \\ \hline \end{array}$$

Answer - 1679616 inches.

Ex. 2. How many solid inches are there in 2 tons 12 feet of hewn timber?

Ex. 3. In 1259712 solid inches, how many yards?

---

NOTES.

[*] A cube is a solid body, that has length, breadth, and thickness, of equal dimensions: it contains six equal sides. A cubic number is produced by multiplying any number twice into itself; 27, 125, and 512, are cubic numbers, being produced by multiplying 3, 5, and 8, twice into themselves, as $3 \times 3 \times 3 = 27$; $5 \times 5 \times 5 = 125$; $8 \times 8 \times 8 = 512$.

[†] A cubic yard of earth is called a load: 128 cubic feet, that is, a pile of wood 8 feet long, 4 broad, and 4 deep, make a *cord* of wood; but 108 cubic feet make a *stack*.

# WINE MEASURE.*

### TABLE.

| | | |
|---|---|---|
| 4 gills | make | 1 pint, pt. |
| 2 pints | | 1 quart, qt. |
| 4 quarts | | 1 gallon, gal.† |
| 63 gallons | | 1 hogshead, hhd. |
| 84 gallons | | 1 puncheon |
| 2 hogsheads, or 126 gallons | | 1 pipe, or butt, p. |
| 2 pipes, or 252 gallons | | 1 tun, t. |

Ex. 1. How many gallons are there in 5 pipes of wine?

$$\begin{array}{r} 5 \\ 2 \\ \hline 10 \\ 63 \\ \hline \end{array}$$

Answer - 630 gallons.

---

NOTES.

* Wines, spirits, cider, perry, oil, vinegar, and milk, are bought and sold by this measure, which extends only to the gallons, for in different kinds of wine the measures are very different, as follow:

| | | |
|---|---|---|
| Claret, | 63 gallons | 1 hhd. |
| Madeira, | 110 ditto | 1 pipe |
| Vidonia, | 120 ditto | 1 ditto |
| Sherry, | 130 ditto | 1 ditto |
| Port, | 138 ditto | 1 ditto |
| Lisbon,<br>Bucellas, | 140 ditto | 1 ditto |

The pipe of Port is seldom accurately 138 gallons, and it is customary to charge what the vessel actually contains.

† By an act of parliament passed in the reign of Queen Anne, the wine gallon is fixed at 231 cubic inches.

Hence a pint is - - - 28.875 cubic inches
a quart is - - - 57.75 do.

It is ascertained that 12 wine gallons of distilled water weigh exactly 100 pounds avoirdupois.

The origin of liquid measure, was from Troy-weight: eight pounds Troy of wheat gathered from the middle of the ear, and well dried, were, by a statute made in the reign of Henry III., ordained to be a gallon of wine measure. No other liquor measure but this was used for ages; and it would, perhaps, be difficult to ascertain how the several changes have obtained in the country.

Ex. 2. In 7006 pints, how many gallons?

2)7006

4)3503

875 3 Answer - 875 gal. 3 qts.

Ex. 3. In 31490 pints, how many gallons?
Ex. 4. In 3 tuns, 1 hhd. 49 gallons of claret, how many quarts?
Ex. 5. How many tuns of Port wine are there in 46088 gallons?*
Ex. 6. In ten thousand gills of Sherry, how many hogsheads?

* * *

# ALE AND BEER MEASURE.

## TABLE.

| | | | | |
|---|---|---|---|---|
| 2 | pints | make | 1 | quart, qt. |
| 4 | quarts | | 1 | gallon, gal.† |
| 9 | gallons | | 1 | firkin, fir.‡ |
| 2 | firkins, or 18 gallons | | 1 | kilderkin, kil. |
| 2 | kilderkins, or 36 gallons | | 1 | barrel, bar. |
| 54 | gallons | | 1 | hogshead, hogs. |
| 2 | hogsheads, or 108 gallons | | 1 | butt, bt. |

---

NOTES.

* If the pupil should divide by 9 and 7, instead of 63 in Long Division, he will find two remainders of 8 and 3, the value of which is $3 \times 9 + 8 = 35$. See p. 27.

† One gallon, beer measure, contains 282 solid or cubic inches:
Hence a pint is - - - 35.25 cubic inches.
a quart is - - - 70.5 do.
a barrel, or 36 gallons - 10152.0 do.
a hogshead, or 54 gallons 15228.0 do.

Ten yards of inch pipe, (that is, of pipe whose diameter is one inch,) contain exactly an ale gallon.

A cubic foot of water weighs 1000 ounces; of course 32 cubic feet weigh 2000 lb. which was formerly a ton.

‡ In the year 1689, a statute of excise was passed, which made a firkin of ale or beer, without distinction, to consist of $8\frac{1}{2}$ gallons: this has, however, been long in disuse; and it was customary, till within a few years, to make the firkin of beer to consist of 9 gallons, but that of ale only of 8; custom has now abolished the distinction, and at present for beer and ale the firkin contains 9 gallons, and, of course, we do not, in this work, retain any other measures in the tables than are used in the existing business of life.

Ex. 1. In 506 barrels of ale, how many pints ?

Ex. 2. In 9065 butts of strong beer, how many gallons ?

Ex. 3. How many quarts are there in 79 hogsheads of beer ?

Ex. 4. In 76459 quarts, how many kilderkins ?

Ex. 5. In thirty thousand eight hundred pints of porter, how many hogsheads ?

Ex. 6. How many pints are there in 3 butts of beer ?

# CORN MEASURE.*

## TABLE.

| | | |
|---|---|---|
| 2 pints | make | 1 quart |
| 4 quarts | | 1 gallon |
| 2 gallons | | 1 peck |
| 4 pecks | | 1 bushel |
| 2 bushels | | 1 strike |
| 5 bushels | | 1 load of corn |
| 8 bushels | | 1 quarter† |
| 5 quarters, or 40 bushels | | 1 wey, or load of wheat‡ |
| 2 weys, or 80 bushels | | 1 last |

## EXAMPLES.

Ex. 1. In 57 quarters of corn, how many pecks ?

Ex. 2. In 248456 pecks of oats, how many lasts ?

Ex. 3. How many pints are there in 19 bush. 2 p. of canary seed ?

Ex. 4. In 2 weys and 4 quarters of barley, how many bushels ?

Ex. 5. How many quarters of corn are there in 50,000 gallons ?

---

NOTES.

* One gallon corn measure, contains 268.6 solid inches : Hence

| | | |
|---|---|---|
| A pint is | 33.6 | cubic inches nearly |
| A quart is | 67.2 | do. |
| A peck, or 8 quarts | 537.6 | do. |
| A bushel, or 4 pecks | 2150.4 | do. |
| A heaped bushel is one-third more. | | |
| A quarter, or eight bushels | 17203.2 | do. |

The standard bushel is a cylindric vessel $18\frac{3}{4}$ inches in diameter, and 8 inches deep.

† A quarter of wheat was so called, upon the supposition that it weighed 500 lb. or a quarter of a ton. See note to Ale and Beer measure. The bushel, or the one-eighth of a quarter, is equal to 1000 ounces, or a cubic foot of water.

By this measure, corn, seeds, fruits, sand, salt, Newcastle coals, oysters, &c. are measured and sold. A bushel of wheat on the average weighs 60 pounds ; of barley 50 pounds ; of oats 38 pounds.

‡ It will be observed, that 5 bushels and 40 bushels are both called loads ; the one is reckoned a man's load ; the other to be removed by a cart.

# COAL MEASURE.

## TABLE.

| | | |
|---|---|---|
| 4 pecks | make | 1 bushel |
| 3 bushels | | 1 sack |
| 12 sacks, or 36 bushels | | 1 chaldron |
| 21 chaldrons | | 1 score.* |

### EXAMPLES.

Ex. 1. How many sacks are there in five score of coals?

Ex. 2. How many bushels of coals are there in a vessel containing 15 score?

Ex. 3. In ten thousand and 12 pecks, how many chaldron are there?

Ex. 4. How many chaldron of coals are there in ten thousand and five pecks?

Ex. 5. In three score and ten bushels of coals, how many sacks?

# COMMERCIAL NUMBERS,

## OR ARTICLES SOLD BY TALE.

| | |
|---|---|
| 12 articles of any kind | 1 dozen |
| 13 ditto | 1 long dozen |
| 12 dozen | 1 gross |
| 20 articles of any kind | 1 score |
| 5 score | 1 hundred |
| 6 score | 1 great hundred |
| 12 score | 1 pack of wool |
| 5 dozen skins of parchment | 1 roll |
| 72 words in Common law | 1 sheet |
| 80 —— in the Exchequer | 1 ditto |
| 90 —— in Chancery | 1 ditto |
| 24 sheets of paper | 1 quire |
| 20 quires | 1 ream |
| 21½ quires, or 516 sheets | 1 printer's ream |
| 2 reams | 1 bundle |

---

NOTE.

* In the purchase of coals, to a single chaldron there are 12 sacks only; but if 5 chaldrons be ordered at one time, the seller must send 63 sacks.

| | |
|---|---|
| Folio is the largest size of books, of which, | 2 leaves, or 4 pages, make a sheet. |
| Quarto, 4to. - - | 4 leaves, or 8 pages, make a sheet, |
| Octavo, 8vo. - - | 8 leaves, or 16 pages, ditto |
| Duodecimo, 12mo. - | 12 leaves, or 24 pages, ditto |
| Octodecimo, 18mo.. - | 18 leaves, or 36 pages, ditto |

### EXAMPLES.

Ex. 1. How many long dozen are there in ten thousand oranges?

Ex. 2. How many gross are there in one hundred and fifty thousand corks?

Ex. 3. In seventy thousand quills, how many great hundreds are there?

Ex. 4. I have a deed containing 4 skins of parchment, and each skin contains 850 words; for how many sheets shall I have to pay the person who copies it, reckoning according to the common law charge?

Ex. 5. The writing of an Exchequer cause occupies 315 sheets: for how many words shall I have to pay the clerk who copies it for me?

Ex. 6. A suit has been four years in chancery, and I wish to have a copy of all the proceedings; for how many sheets shall I pay, supposing it occupies 1264 skins of parchment, and each skin 690 words?

Ex. 7. How many sheets are there in 40 reams of paper?

Ex. 8. How many common reams of paper are there in ten thousand printer's reams?

Ex. 9. What number of sheets less are there in 500 common reams of paper, than there are in the same number of printer's reams?

Ex. 10. What number of pages are there in a folio containing 211 sheets?

Ex. 11. What will be the difference in the number of pages whether I print in 12mo. or 18mo., supposing my work will make fourteen sheets?

Ex. 12. What number of words are there in Dr. Gregory's Dictionary of Arts and Sciences, which contains 240 sheets 4to., and each page contains 14784 words?

Ex. 13. How many reams of paper were used in printing that Dictionary, six thousand copies having been taken off?

Ex. 14. How many pens were used in writing the said Dictionary, supposing each pen to write 840 words?

# TIME.

## TABLE.

| | | |
|---|---|---|
| 60 seconds (sec.) . . . . . | make | 1 minute, m. |
| 60 minutes, or 3600 seconds . . . | | 1 hour, h. |
| 24 hours, or 1440 minutes . . . . | | 1 day, d. |
| 7 days, or 168 hours . . . . . . . | | 1 week, w. |
| 4 weeks, or 28 days . . . . . . . | | 1 month, m. |
| 12 calendar months, or 52 weeks, or 365 days, or 8766 hours . . | | 1 year.* |

**Ex. 1.** In 4199 days, how many months and years?

Ex. 2. Reduce 150 days to hours and minutes?

Ex. 3. In 70 years how many days, supposing each year to consist of $365\frac{1}{4}$ days?

Ex. 4. How many minutes, hours, and days, are there in 5960034 seconds?

Ex. 5. How many minutes are there in 1808 years, allowing $365\frac{1}{4}$ days make one year?

Ex. 6. How many seconds has a boy lived, who is 12 years, 9 months, and 13 days old, reckoning 13 lunar months of 28 days each to a year?

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NOTE.

* Thirteen months, each containing 4 weeks, and each week containing 7 days, make only 364 days; but the common year is divided into 12 calendar months, and it consists of $365\frac{1}{4}$ days, some of the months having 30, and some 31 days, and February having only 28 days, excepting on leap year, which is every fourth year, when February has 29 days: this addition of one day in four years, makes the reckoning $365\frac{1}{4}$ days for each year: the following lines will assist the memory in recollecting the length of each particular month:

Thirty days hath September,
April, June, and November;
February has twenty-eight alone,
And all the rest have thirty one.

Though the year is usually reckoned at $365\frac{1}{4}$ days, yet that is not perfectly accurate, it being fully ascertained, that the year consists of 365 days, 5 hours, 48 minutes, 48 seconds.

Leap-year may be found by dividing the year by 4; if there be no remainder it is leap-year; thus 1808 is divisible by 4, without a remainder, and is leap-year. The year 1800 was an exception, and so will 1900, a day dropt, in an hundred years, being necessary to keep the calculations accurate.

# ASTRONOMY.*

## TABLE.

| | | |
|---|---|---|
| 60 seconds (60″) | make | 1 minute, 1′ |
| 60 minutes | . . . . . . . . | 1 degree, 1° |
| 30 degrees | . . . . . . . . | 1 sign |
| 12 signs, or 360 degrees | . | 1 great circle. |

Ex. 1. In 185 degrees how many minutes and seconds?

Ex. 2. How many degrees are there in five thousand and fifty-five seconds?

Ex. 3. How many seconds are there in a great circle?

Ex. 4. In 548056 seconds, how many signs?

Ex. 5. How many seconds are there in 9 s. 4° 55′ 56″?

Ex. 6. In 700809 seconds, how many degrees?

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## MISCELLANEOUS EXAMPLES.

Ex. 1. In 195 pounds, how many shillings, pence, and farthings?

Ex. 2. In 77 guineas, how many shillings, pence, and farthings?

Ex, 3. How many crowns, half-crowns, shillings, and sixpences are there in £354?

Ex. 4. In 4432127 farthings, how many pence, shillings, pounds, and guineas?

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NOTE.

* This table is used in astronomical and geographical calculations. The astronomical day commences at 12 o'clock at noon; but the common or *civil* day begins at 12 o'clock the preceding night: of course the astronomical day begins 12 hours later than the common day.

Ex. 5. In 14 ingots of silver, each weighing 27oz. 5dwts., how many grains?

Ex. 6. In three dozen of table spoons, each weighing 2oz. 9dwts., how many pounds?

Ex. 7. In 78 bags of hops, each weighing 3cwt., how many pounds?

Ex. 8. How many pounds and cwts. of tobacco are there in 75 hogsheads, each containing 3cwt. 1qr. 14lb.?

Ex. 9. In 98465 inches of broad-cloth, how many yards and ells?

Ex. 10. In five thousand yards of cloth, how many nails?

Ex. 11. How many inches are there between London and Bristol, a distance of 120 miles?

Ex. 12. How many barley-corns will reach round the earth, which is a great circle of 360 degrees, and each degree contains 69½ miles? and how many quarters of barley would be necessary to perform this, supposing 9200 barley-corns to fill a pint measure?

Ex. 13. How often will a wheel turn in going from London to York, a distance of 198 miles, if the wheel be 2½ yards in circumference?

Ex. 14. How many perches are there in a field containing 105 acres?

Ex. 15. If a field of 5 acres be taken from one of 56 acres, how many square yards will remain?

Ex. 16. How many pints and gallons are there in 39 hogsheads of cyder?

Ex. 17. How many minutes have elapsed since the creation of the world to the year 1808, supposing the world to have been created 4004 years before the birth of Christ, and each year to consist of 365¼ days?

# COMPOUND ADDITION.

* * *

## ADDITION OF MONEY.

### PENCE AND SHILLING TABLES.

| Pence | | s. | d. | Pence | | s. | d. | Shill. | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | are | 1 | 8 | 12 | are | 1 | 0 | 20 | 1 | 0 | 0 |
| 25 | | 2 | 1 | 18 | | 1 | 6 | 25 | 1 | 5 | 0 |
| 30 | | 2 | 6 | 24 | | 2 | 0 | 30 | 1 | 10 | 0 |
| 35 | | 2 | 11 | 30 | | 2 | 6 | 35 | 1 | 15 | 0 |
| 40 | | 3 | 4 | 36 | | 3 | 0 | 40 | 2 | 0 | 0 |
| 45 | | 3 | 9 | 42 | | 3 | 6 | 50 | 2 | 10 | 0 |
| 50 | | 4 | 2 | 48 | | 4 | 0 | 60 | 3 | 0 | 0 |
| 55 | | 4 | 7 | 54 | | 4 | 6 | 70 | 3 | 10 | 0 |
| 60 | | 5 | 0 | 60 | | 5 | 0 | 80 | 4 | 0 | 0 |
| 65 | | 5 | 5 | 66 | | 5 | 6 | 90 | 4 | 10 | 0 |
| 70 | | 5 | 10 | 72 | | 6 | 0 | 100 | 5 | 0 | 0 |
| 75 | | 6 | 3 | 78 | | 6 | 6 | 110 | 5 | 10 | 0 |
| 80 | | 6 | 8 | 84 | | 7 | 0 | 120 | 6 | 0 | 0 |
| 85 | | 7 | 1 | 90 | | 7 | 6 | 130 | 6 | 10 | 0 |
| 90 | | 7 | 6 | 96 | | 8 | 0 | 140 | 7 | 0 | 0 |
| 95 | | 7 | 11 | 102 | | 8 | 6 | 150 | 7 | 10 | 0 |
| 100 | | 8 | 4 | 108 | | 9 | 0 | 160 | 8 | 0 | 0 |
| 105 | | 8 | 9 | 114 | | 9 | 6 | 170 | 8 | 10 | 0 |
| 110 | | 9 | 2 | 120 | | 10 | 0 | 180 | 9 | 0 | 0 |
| 115 | | 9 | 7 | 132 | | 11 | 0 | 190 | 9 | 10 | 0 |
| 120 | | 10 | 0 | 144 | | 12 | 0 | 200 | 10 | 0 | 0 |

* * *

**Compound Addition** is a method of collecting several numbers of different denominations into one sum.

**Rule.** (1) *Arrange the numbers so that those of the same denomination may stand directly under each other, and draw a line under them.*

(2). *Add the numbers in the lowest denomination together, and find how many units of the next higher denomination are contained in their sum.*

(3). *Write down the remainder, and carry the units to the next higher denomination, and proceed so to the end.*

| | £. | s. | d. |
|---|---|---|---|
| Ex. | 468 | 19 | 4½ |
| | 123 | 16 | 11¾ |
| | 987 | 12 | 9 |
| | 654 | 13 | 7¼ |
| | 123 | 17 | 4½ |
| | 456 | 18 | 10¾ |
| | 439 | 4 | 6½ |
| | 592 | 12 | 4¼ |
| | 3847 | 15 | 10½ |

I first add together the farthings, which I find to be 14, but 14 farthings make 3½*d.*, I put down the ½ and carry the 3 to the column of pence, which I then add together, and find the sum to be 58, but by the table, 55 pence are 4*s.* 7*d.*, therefore 58 pence are 4*s.* 10*d.*, I put down the 10 and carry the 4 to the column of shillings; I now add the shillings together, and find the sum to be 115, but 115 shillings make 5*l.* 15*s.*, I put down the 15, and carry the 5 to the pounds, and proceed as in simple addition.*

## EXAMPLES OF MONEY.

| | £. | s. | d. | | £. | s. | d. | | £. | s. | d. | | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 55 | 3 | 8 | 2. | 67 | 2 | 8 | 3. | 95 | 2 | 9 | 4. | 49 | 9 | 11 |
| | 62 | 6 | 3 | | 24 | 9 | 9 | | 89 | 7 | 8 | | 33 | 8 | 7 |
| | 96 | 2 | 1 | | 38 | 2 | 5 | | 72 | 4 | 3 | | 96 | 12 | 9 |
| | 31 | 8 | 4 | | 42 | 5 | 9 | | 67 | 9 | 2 | | 75 | 3 | 4 |
| | 43 | 7 | 5 | | 78 | 6 | 6 | | 51 | 8 | 9 | | 51 | 8 | 9 |
| | 10 | 9 | 8 | | 64 | 6 | 9 | | 45 | 6 | 4 | | 12 | 19 | 7 |

| | £. | s. | d. | | £. | s. | d. | | £. | s. | d. | | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5. | 58 | 15 | 9 | 6. | 42 | 16 | 9 | 7. | 92 | 13 | 4¾ | 8. | 50 | 19 | 8¼ |
| | 79 | 5 | 5 | | 37 | 15 | 11 | | 84 | 14 | 9 | | 97 | 16 | 7¾ |
| | 61 | 7 | 10 | | 73 | 9 | 9 | | 73 | 18 | 4½ | | 35 | 14 | 2 |
| | 64 | 16 | 3 | | 62 | 10 | 6 | | 69 | 17 | 10 | | 46 | 16 | 8½ |
| | 82 | 15 | 10 | | 29 | 4 | 4 | | 48 | 15 | 7 | | 67 | 19 | 2 |
| | 19 | 12 | 8 | | 19 | 17 | 11 | | 35 | 14 | 11¼ | | 24 | 15 | 9¼ |

| | £. | s. | d. | | £. | s. | d. | | £. | s. | d. | | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 9. | 54 | 17 | 6¾ | 10. | 67 | 16 | 8¼ | 11. | 18 | 14 | 8¼ | 12. | 41 | 15 | 9½ |
| | 93 | 12 | 8 | | 71 | 13 | 9 | | 93 | 15 | 10¾ | | 56 | 10 | 9 |
| | 51 | 6 | 9¼ | | 84 | 11 | 8¾ | | 37 | 6 | 11 | | 62 | 16 | 3¾ |
| | 25 | 10 | 11 | | 32 | 19 | 3 | | 78 | 16 | 5¾ | | 78 | 4 | 11 |
| | 76 | 13 | 10 | | 48 | 10 | 4¼ | | 69 | 12 | 7¼ | | 87 | 13 | 7½ |
| | 44 | 6 | 6¼ | | 55 | 18 | 7¼ | | 43 | 8 | 11 | | 92 | 19 | 0¾ |
| | 33 | 3 | 3 | | 21 | 12 | 4 | | 12 | 17 | 3¼ | | 13 | 16 | 7 |

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NOTE.

* The reason of this rule may be thus illustrated: if I have to receive any sum of money, pence are more convenient than farthings, and shillings than pence; and in large sums bank notes or

| | £. | s. | d. | | £. | s. | d. | | £. | s. | d. | | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 13. | 46 | 2 | 3½ | 14. | 45 | 19 | 9¼ | 15. | 43 | 17 | 10½ | 16. | 52 | 18 | 10 |
| | 65 | 10 | 4½ | | 63 | 17 | 11¼ | | 50 | 14 | 6¼ | | 67 | 12 | 2¼ |
| | 74 | 0 | 10 | | 79 | 13 | 5¼ | | 72 | 6 | 4¼ | | 77 | 14 | 9 |
| | 81 | 17 | 8¾ | | 46 | 10 | 9½ | | 65 | 19 | 7½ | | 82 | 13 | 10¾ |
| | 39 | 15 | 10 | | 35 | 8 | 7 | | 91 | 5 | 1½ | | 98 | 12 | 11½ |
| | 23 | 10 | 8½ | | 47 | 19 | 10½ | | 38 | 19 | 10 | | 21 | 17 | 7¼ |
| | 19 | 14 | 7¼ | | 19 | 14 | 6 | | 29 | 12 | 9½ | | 45 | 12 | 9 |

| | £. | s. | d. | | £. | s. | d. | | £. | s. | d. | | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 17. | 77 | 15 | 4½ | 18. | 57 | 15 | 9¼ | 19. | 446 | 19 | 9¾ | 20. | 48 | 14 | 10½ |
| | 69 | 10 | 9¼ | | 64 | 9 | 2 | | 152 | 15 | 10¾ | | 36 | 13 | 10 |
| | 41 | 0 | 10¼ | | 76 | 17 | 10½ | | 695 | 12 | 0½ | | 74 | 15 | 7¾ |
| | 57 | 13 | 8 | | 97 | 16 | 9 | | 758 | 3 | 5 | | 23 | 18 | 2½ |
| | 87 | 9 | 10¼ | | 39 | 18 | 11¾ | | 338 | 14 | 3½ | | 48 | 9 | 6 |
| | 91 | 16 | 11¾ | | 45 | 10 | 10 | | 166 | 19 | 11 | | 81 | 16 | 4¾ |
| | 75 | 14 | 8 | | 59 | 17 | 9 | | 279 | 12 | 9¾ | | 77 | 11 | 4½ |

| | £. | s. | d. | | £. | s. | d. | | £. | s. | d. | | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 21. | 92 | 19 | 9¾ | 22. | 12 | 14 | 9¼ | 23. | 54 | 11 | 10 | 24. | 414 | 19 | 9 |
| | 56 | 10 | 9 | | 93 | 16 | 10½ | | 22 | 19 | 6¼ | | 627 | 17 | 11¼ |
| | 64 | 18 | 7¾ | | 17 | 12 | 11 | | 61 | 16 | 9½ | | 741 | 6 | 4½ |
| | 38 | 16 | 3 | | 56 | 13 | 7¼ | | 14 | 17 | 0¾ | | 865 | 14 | 8 |
| | 49 | 15 | 11¼ | | 91 | 19 | 11 | | 58 | 12 | 11½ | | 917 | 6 | 10¾ |
| | 64 | 19 | 3¼ | | 76 | 14 | 5¼ | | 72 | 10 | 6 | | 347 | 14 | 10¼ |
| | 92 | 17 | 8½ | | 14 | 11 | 3 | | 76 | 14 | 11 | | 449 | 13 | 4 |

| | £. | s. | d. | | £. | s. | d. | | £. | s. | d. | | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 25. | 427 | 18 | 10¼ | 26. | 548 | 11 | 6 | 27. | 493 | 2 | 8¼ | 28. | 412 | 9 | 11¼ |
| | 941 | 17 | 9 | | 932 | 18 | 4¾ | | 347 | 14 | 3¼ | | 924 | 19 | 6¼ |
| | 712 | 19 | 6 | | 379 | 0 | 6¼ | | 729 | 19 | 5 | | 750 | 11 | 3½ |
| | 625 | 12 | 7¼ | | 414 | 17 | 0½ | | 672 | 5 | 8¾ | | 627 | 19 | 0¾ |
| | 511 | 11 | 10 | | 573 | 4 | 5¼ | | 548 | 10 | 3 | | 438 | 10 | 4¼ |
| | 462 | 10 | 6½ | | 697 | 13 | 9¼ | | 217 | 12 | 8½ | | 363 | 2 | 10½ |
| | 383 | 11 | 9¾ | | 551 | 6 | 11 | | 974 | 16 | 7¼ | | 221 | 15 | 8 |
| | | | | | | | | | 146 | 5 | 0½ | | 147 | 1 | 5 |

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guineas than shillings. If therefore a person sell 7 yards of tape at ¾ farthings per yard, it is more convenient to receive 5 penny-pieces and one farthing, than to have 21 farthings; and so of the higher denominations.

| | £. | s. | d. | | £. | s. | d. | | £. | s. | d. | | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 29. | 152 | 15 | 2¼ | 30. | 504 | 3 | 9½ | 31. | 576 | 14 | 9 | 32. | 827 | 18 | 11¾ |
| | 255 | 18 | 6¼ | | 636 | 19 | 5 | | 613 | 12 | 11½ | | 550 | 11 | 8¾ |
| | 348 | 12 | 9¼ | | 421 | 2 | 7¾ | | 719 | 13 | 4¾ | | 938 | 9 | 4 |
| | 410 | 0 | 10 | | 347 | 12 | 10 | | 914 | 14 | 6¼ | | 344 | 0 | 3 |
| | 566 | 13 | 1¼ | | 383 | 7 | 0½ | | 271 | 10 | 9 | | 615 | 16 | 1¼ |
| | 631 | 6 | 4½ | | 848 | 15 | 2¾ | | 759 | 8 | 5¼ | | 471 | 2 | 7 |
| | 781 | 3 | 10 | | 710 | 0 | 8¼ | | 432 | 15 | 3½ | | 214 | 15 | 10½ |
| | 949 | 16 | 7 | | 483 | 10 | 4½ | | 918 | 11 | 4¾ | | 745 | 19 | 2 |
| | 123 | 15 | 11 | | 426 | 19 | 7 | | 564 | 7 | 2 | | 90 | 9 | 9 |
| 33. | 792 | 19 | 3¼ | 34. | 88 | 16 | 11½ | 35. | 28 | 9 | 4¼ | 36. | 60 | 15 | 5½ |
| | 437 | 14 | 9½ | | 26 | 14 | 5¼ | | 54 | 17 | 9 | | 48 | 13 | 1¾ |
| | 354 | 10 | 10¾ | | 9 | 7 | 2½ | | 6 | 0 | 11 | | 93 | 18 | 6 |
| | 516 | 18 | 4 | | 36 | 12 | 4¾ | | 28 | 13 | 5¾ | | 7 | 7 | 10½ |
| | 209 | 13 | 10½ | | 41 | 18 | 3 | | 65 | 18 | 7¼ | | 35 | 19 | 4¼ |
| | 524 | 17 | 2¾ | | 27 | 3 | 8¼ | | 92 | 6 | 4¾ | | 73 | 6 | 9½ |
| | 739 | 6 | 10 | | 54 | 15 | 11½ | | 7 | 16 | 0½ | | 31 | 17 | 3 |
| | 365 | 2 | 6¼ | | 12 | 19 | 6 | | 14 | 5 | 10 | | 59 | 14 | 10¾ |
| | 147 | 17 | 9 | | 20 | 0 | 10 | | 40 | 0 | 9 | | 60 | 0 | 10 |
| 37. | 94 | 1 | 9¼ | 38. | 53 | 11 | 4½ | 39. | 68 | 19 | 5¾ | 40. | 75 | 12 | 8½ |
| | 88 | 2 | 6½ | | 6 | 2 | 8 | | 84 | 7 | 3½ | | 40 | 0 | 6¼ |
| | 46 | 5 | 11¾ | | 18 | 5 | 3¼ | | 8 | 6 | 5¾ | | 8 | 17 | 4 |
| | 29 | 16 | 3½ | | 26 | 10 | 7½ | | 25 | 11 | 9½ | | 24 | 19 | 5¾ |
| | 48 | 12 | 0 | | 42 | 0 | 4¾ | | 9 | 13 | 7 | | 59 | 15 | 2½ |
| | 5 | 17 | 7 | | 64 | 2 | 2 | | 47 | 15 | 6¾ | | 82 | 6 | 5¾ |
| | 61 | 13 | 3¼ | | 71 | 18 | 10¾ | | 32 | 1 | 3 | | 7 | 18 | 4½ |
| | 7 | 14 | 10¼ | | 3 | 14 | 11½ | | 1 | 18 | 0½ | | 33 | 2 | 9¼ |
| | 12 | 18 | 5½ | | 90 | 0 | 6¾ | | 2 | 16 | 4 | | 8 | 10 | 0¼ |
| 41. | 39 | 14 | 4½ | 42. | 78 | 12 | 5 | 43. | 127 | 10 | 10¾ | 44. | 515 | 14 | 9¾ |
| | 97 | 12 | 2¼ | | 17 | 14 | 8½ | | 356 | 14 | 9¾ | | 943 | 17 | 3¼ |
| | 73 | 15 | 10½ | | 35 | 0 | 6 | | 483 | 9 | 4½ | | 623 | 15 | 11¼ |
| | 6 | 10 | 11¼ | | 28 | 16 | 10¼ | | 849 | 7 | 11 | | 417 | 19 | 3½ |
| | 30 | 2 | 9 | | 11 | 8 | 3½ | | 680 | 18 | 11¼ | | 338 | 14 | 10 |
| | 16 | 12 | 5¾ | | 49 | 15 | 7¼ | | 774 | 19 | 7½ | | 385 | 18 | 11¾ |
| | 58 | 16 | 1½ | | 6 | 11 | 4¼ | | 114 | 6 | 2¾ | | 764 | 13 | 6 |
| | 2 | 13 | 7 | | 62 | 15 | 3 | | 251 | 18 | 9¼ | | 453 | 19 | 9½ |
| | 82 | 0 | 3½ | | 5 | 18 | 4¾ | | 428 | 15 | 6 | | 562 | 18 | 5¾ |
| | 10 | 10 | 10 | | 90 | 0 | 10 | | 567 | 16 | 2 | | 223 | 14 | 2 |

|  | £. | s. | d. |  | £. | s. | d. |  | £. | s. | d. |  | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 45. | 657 | 16 | 10½ | 46. | 491 | 16 | 9 | 47. | 722 | 10 | 9¼ | 48. | 477 | 16 | 4¼ |
|  | 734 | 17 | 4¾ |  | 272 | 15 | 6¼ |  | 966 | 4 | 8¾ |  | 395 | 15 | 2¾ |
|  | 879 | 14 | 3½ |  | 889 | 17 | 10½ |  | 899 | 13 | 6 |  | 736 | 5 | 11 |
|  | 919 | 12 | 10¾ |  | 647 | 19 | 2¾ |  | 248 | 16 | 10¼ |  | 692 | 14 | 9½ |
|  | 131 | 19 | 11 |  | 398 | 16 | 7 |  | 532 | 14 | 9 |  | 565 | 13 | 5¾ |
|  | 235 | 7 | 6¼ |  | 563 | 16 | 10¼ |  | 476 | 19 | 7¼ |  | 937 | 17 | 0 |
|  | 496 | 18 | 3¾ |  | 770 | 0 | 5¾ |  | 744 | 12 | 9½ |  | 441 | 16 | 4¾ |
|  | 587 | 9 | 5 |  | 945 | 17 | 7 |  | 669 | 15 | 7¾ |  | 760 | 18 | 0¾ |
|  | 673 | 11 | 10 |  | 420 | 13 | 9¼ |  | 593 | 15 | 11¼ |  | 672 | 11 | 11 |
|  | 820 | 19 | 4 |  | 150 | 10 | 0 |  | 150 | 10 | 0 |  | 40 | 0 | 10 |

|  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|
| 49. | 494274 | 12 | 9¾ | 50. | 901442 | 16 | 10½ |
|  | 765502 | 6 | 4 |  | 234971 | 5 | 9¼ |
|  | 300089 | 2 | 2¼ |  | 567352 | 14 | 7¾ |
|  | 402193 | 17 | 9 |  | 912261 | 19 | 2½ |
|  | 375451 | 3 | 10 |  | 345512 | 17 | 9½ |
|  | 269440 | 18 | 6¾ |  | 678830 | 12 | 6 |
|  | 123428 | 15 | 10 |  | 912887 | 19 | 10 |
|  | 567865 | 11 | 9½ |  | 456713 | 10 | 3½ |
|  | 910649 | 10 | 6 |  | 891391 | 17 | 8¼ |

|  | £. | s. | d. |  | £. | s. | d. |  | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 51. | 4567 | 14 | 11¾* | 52. | 3256 | 19 | 6½ | 53. | 3567 | 12 | 9¼ |
|  | 4934 | 15 | 9 |  | 4397 | 10 | 11¾ |  | 7960 | 17 | 10 |
|  | 2765 | 16 | 10¼ |  | 1974 | 12 | 9¼ |  | 1234 | 15 | 7¾ |
|  | 9876 | 19 | 11¼ |  | 7246 | 8 | 4 |  | 5678 | 12 | 8½ |
|  | 3497 | 9 | 5 |  | 3942 | 15 | 10¾ |  | 9123 | 14 | 10 |
|  | 1234 | 10 | 8¾ |  | 4567 | 8 | 9¾ |  | 4567 | 13 | 11¼ |
|  | 5678 | 16 | 10 |  | 4567 | 17 | 11¾ |  | 8912 | 17 | 9 |
|  | 4376 | 8 | 9 |  | 9376 | 12 | 8½ |  | 1456 | 9 | 6 |
|  | 2794 | 15 | 4½ |  | 4623 | 2 | 5 |  | 7891 | 10 | 4¾ |
|  | 7921 | 12 | 10¼ |  | 5932 | 5 | 4 |  | 2845 | 6 | 3 |
|  | 1764 | 13 | 9¾ |  | 2487 | 7 | 3 |  | 6789 | 12 | 5½ |
|  | 1805 | 17 | 4 |  | 5764 | 16 | 11½ |  | 2345 | 13 | 11 |
|  | 1764 | 12 | 7 |  | 1234 | 18 | 2 |  | 6789 | 16 | 9¾ |
|  | 3459 | 15 | 11 |  | 5678 | 19 | 9¾ |  | 4972 | 15 | 10 |
|  | 2946 | 16 | 10¾ |  | 9012 | 17 | 10 |  | 3456 | 19 | 5½ |
|  | 1796 | 14 | 10 |  | 3456 | 2 | 2 |  | 7891 | 16 | 7¾ |
|  | 4325 | 16 | 8 |  | 7890 | 14 | 5 |  | 2345 | 14 | 11¼ |
|  | 5678 | 12 | 11¾ |  | 1234 | 13 | 10 |  | 6782 | 12 | 9 |
|  | 4932 | 14 | 6 |  | 5678 | 15 | 7 |  | 4315 | 11 | 7¼ |
|  | 2005 | 9 | 5½ |  | 9123 | 13 | 4 |  | 2105 | 8 | 6 |

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NOTE.

* From these three examples the preceptor may make an almost

## EXAMPLES OF WEIGHTS AND MEASURES.

## TROY WEIGHT.*

| lb. | oz. | dwts. | gr. |
|---|---|---|---|
| 7684 | 9 | 16 | 22 |
| 1234 | 11 | 5 | 19 |
| 9876 | 8 | 11 | 22 |
| 1493 | 9 | 19 | 12 |
| 3587 | 10 | 10 | 3 |
| 2345 | 7 | 6 | 15 |
| 6789 | 9 | 14 | 21 |
| 3257 | 11 | 15 | 8 |
| 36271 | 7 | 1 | 2 |

In adding up the column of grains I find the sum to be 122, which I divide by 24 to bring it into pennyweights; and 122 grains make 5 pennyweights and 2 grains over; the 2 I put down, and carry the 5 to the column of pennyweights; I then add these together, and find the sum to be 101, which I divide by 20 to bring to ounces, I put down the 1 and carry 5 to the column of ounces; then adding the ounces, I find the sum 79, which, by dividing by 12, give 6 lb. 7 oz. the 7 I put down, and carry the 6 to the pounds, and proceed as in simple Addition.†

| | lb. | oz. | dwt. |
|---|---|---|---|
| 1. | 414 | 9 | 14 |
| | 617 | 5 | 13 |
| | 715 | 10 | 9 |
| | 322 | 7 | 15 |
| | 413 | 2 | 10 |
| | 514 | 11 | 15 |
| | 976 | 8 | 7 |

| | lb. | oz. | dwt. | gr. |
|---|---|---|---|---|
| 2. | 410 | 9 | 12 | 19 |
| | 342 | 11 | 16 | 12 |
| | 912 | 3 | 14 | 14 |
| | 751 | 6 | 10 | 22 |
| | 626 | 10 | 17 | 16 |
| | 427 | 4 | 11 | 23 |
| | 123 | 11 | 17 | 12 |

| | lb. | oz. | dwt. |
|---|---|---|---|
| 3. | 526 | 10 | 19 |
| | 712 | 9 | 17 |
| | 944 | 6 | 14 |
| | 633 | 10 | 11 |
| | 319 | 4 | 10 |
| | 247 | 9 | 12 |
| | 123 | 10 | 17 |

| | lb. | oz. | dwt. | gr. |
|---|---|---|---|---|
| 4. | 940 | 10 | 19 | 15 |
| | 738 | 6 | 4 | 23 |
| | 614 | 3 | 17 | 13 |
| | 546 | 7 | 16 | 19 |
| | 321 | 10 | 5 | 22 |
| | 230 | 9 | 15 | 15 |
| | 946 | 11 | 19 | 23 |

| | lb. | oz. | dwt. |
|---|---|---|---|
| 5. | 174 | 11 | 19 |
| | 74 | 10 | 13 |
| | 944 | 9 | 14 |
| | 74 | 11 | 19 |
| | 944 | 10 | 13 |
| | 74 | 11 | 3 |
| | 12 | 4 | 6 |

| | oz. | dwt. | gr. |
|---|---|---|---|
| 6. | 174 | 19 | 23 |
| | 714 | 11 | 14 |
| | 714 | 0 | 18 |
| | 74 | 1 | 22 |
| | 948 | 2 | 21 |
| | 74 | 2 | 12 |
| | 301 | 14 | 4 |

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NOTES.

indefinite number, if the pupil has not already attained to accuracy in adding up the foregoing examples. He may be desired to take on his slate 3 or 4 or more lines of either example, or he may be desired to take 2 or 3 or more lines of each example, and range them under one another for a new example, and so proceed till he has performed the operation as often as necessary.

* The reader is referred to the tables in the preceding pages, which it is hoped he has already committed to his memory.

† This illustration for an example in Troy Weight, will be sufficient for the various examples in the other Weights and Measures, which differ only in the value of the divisors.

| | lb. | oz. | dwt. |
|---|---|---|---|
| 7. | 71 | 11 | 19 |
| | 64 | 8 | 14 |
| | 77 | 0 | 0 |
| | 14 | 3 | 11 |
| | 64 | 2 | 9 |
| | 74 | 6 | 14 |
| | 77 | 2 | 13 |
| | 105 | 9 | 12 |

| | oz. | dwt. | gr. |
|---|---|---|---|
| 8. | 74 | 19 | 23 |
| | 64 | 14 | 17 |
| | 74 | 19 | 11 |
| | 66 | 13 | 9 |
| | 74 | 14 | 11 |
| | 14 | 10 | 3 |
| | 19 | 11 | 14 |
| | 13 | 17 | 5 |

## AVOIRDUPOIS WEIGHT.

| | lb. | oz. | dr. |
|---|---|---|---|
| 1. | 318 | 10 | 10 |
| | 436 | 9 | 8 |
| | 624 | 14 | 6 |
| | 419 | 6 | 15 |
| | 245 | 9 | 7 |
| | 853 | 11 | 10 |
| | 145 | 9 | 8 |

| | tons, | cwt. | qr. | lb. |
|---|---|---|---|---|
| 2. | 416 | 19 | 2 | 26 |
| | 313 | 10 | 0 | 20 |
| | 271 | 11 | 3 | 16 |
| | 725 | 19 | 2 | 18 |
| | 357 | 14 | 2 | 25 |
| | 429 | 17 | 3 | 22 |
| | 235 | 15 | 2 | 19 |

| | lb. | oz. | dr. |
|---|---|---|---|
| 3. | 539 | 13 | 15 |
| | 316 | 14 | 13 |
| | 223 | 12 | 7 |
| | 811 | 9 | 6 |
| | 700 | 6 | 14 |
| | 414 | 12 | 12 |
| | 0 | 0 | 0 |

| | tons, | cwt. | qr. | lb. |
|---|---|---|---|---|
| 4. | 305 | 14 | 2 | 11 |
| | 418 | 18 | 0 | 0 |
| | 336 | 2 | 1 | 14 |
| | 119 | 13 | 3 | 27 |
| | 767 | 16 | 0 | 8 |
| | 782 | 9 | 1 | 16 |
| | 421 | 15 | 3 | 19 |

| | tons, | cwt. | qr. |
|---|---|---|---|
| 5. | 174 | 19 | 3 |
| | 74 | 14 | 2 |
| | 714 | 13 | 1 |
| | 718 | 16 | 2 |
| | 734 | 15 | 2 |
| | 714 | 14 | 1 |
| | 155 | 0 | 3 |

| | cwt. | qr. | lb. |
|---|---|---|---|
| 6. | 174 | 3 | 27 |
| | 724 | 2 | 24 |
| | 149 | 1 | 14 |
| | 719 | 2 | 16 |
| | 407 | 1 | 23 |
| | 149 | 2 | 17 |
| | 76 | 3 | 15 |

| | qr. | lb. | oz. |
|---|---|---|---|
| 7. | 44 | 27 | 15 |
| | 74 | 26 | 14 |
| | 19 | 14 | 13 |
| | 74 | 12 | 14 |
| | 66 | 27 | 13 |
| | 74 | 19 | 10 |
| | 13 | 17 | 5 |

| | lb. | oz. | drs. |
|---|---|---|---|
| 8. | 17 | 15 | 15 |
| | 27 | 14 | 11 |
| | 16 | 13 | 9 |
| | 74 | 14 | 14 |
| | 70 | 0 | 0 |
| | 64 | 13 | 10 |
| | 13 | 4 | 5 |

## APOTHECARIES' WEIGHT.

| | lb. | oz. | dr. |
|---|---|---|---|
| 1. | 314 | 8 | 4 |
| | 210 | 11 | 4 |
| | 766 | 10 | 2 |
| | 555 | 9 | 6 |
| | 417 | 8 | 1 |
| | 324 | 7 | 3 |

| | oz. | dr. | sc. | gr. |
|---|---|---|---|---|
| 2. | 22 | 3 | 2 | 19 |
| | 56 | 0 | 1 | 13 |
| | 43 | 2 | 2 | 11 |
| | 54 | 7 | 0 | 17 |
| | 76 | 5 | 2 | 14 |
| | 45 | 6 | 1 | 0 |

| | lb. | oz. | dr. | sc. | gr. |
|---|---|---|---|---|---|
| 3. | 646 | 11 | 4 | 1 | 19 |
| | 715 | 3 | 7 | 1 | 14 |
| | 934 | 3 | 4 | 0 | 12 |
| | 373 | 10 | 5 | 2 | 9 |
| | 216 | 5 | 1 | 2 | 16 |
| | 159 | 2 | 5 | 0 | 14 |

| | lb. | oz. | dr. |
|---|---|---|---|
| 4. | 47 | 11 | 7 |
| | 94 | 10 | 6 |
| | 74 | 10 | 4 |
| | 75 | 9 | 3 |
| | 69 | 0 | 2 |
| | 57 | 1 | 2 |
| | 18 | 2 | 1 |
| | 19 | 3 | 5 |

| | oz. | dr. | sc. |
|---|---|---|---|
| 5. | 149 | 7 | 2 |
| | 714 | 3 | 0 |
| | 619 | 2 | 1 |
| | 74 | 6 | 2 |
| | 162 | 5 | 2 |
| | 74 | 1 | 2 |
| | 777 | 6 | 1 |
| | 146 | 4 | 0 |

| | dr. | sc. | gr. |
|---|---|---|---|
| 6. | 749 | 2 | 19 |
| | 607 | 1 | 18 |
| | 714 | 2 | 17 |
| | 400 | 0 | 0 |
| | 74 | 1 | 13 |
| | 715 | 2 | 14 |
| | 64 | 1 | 18 |
| | 16 | 0 | 10 |

| | lb. | oz. | dr. |
|---|---|---|---|
| 7. | 84 | 11 | 7 |
| | 74 | 10 | 6 |
| | 37 | 5 | 4 |
| | 19 | 4 | 3 |
| | 74 | 1 | 2 |
| | 79 | 2 | 6 |
| | 19 | 2 | 4 |
| | 13 | 4 | 8 |

## CLOTH MEASURE.

| | yd. | qr. | nl. |
|---|---|---|---|
| 1. | 434 | 3 | 2 |
| | 527 | 1 | 2 |
| | 613 | 2 | 3 |
| | 758 | 3 | 1 |
| | 846 | 1 | 3 |
| | 925 | 2 | 2 |

| | E.e. | qr. | nl. |
|---|---|---|---|
| 2. | 511 | 4 | 2 |
| | 660 | 2 | 0 |
| | 439 | 4 | 2 |
| | 337 | 1 | 2 |
| | 854 | 2 | 3 |
| | 766 | 0 | 2 |

| | E.e. | qr. | nl. |
|---|---|---|---|
| 3. | 565 | 4 | 0 |
| | 626 | 2 | 1 |
| | 724 | 0 | 1 |
| | 882 | 2 | 3 |
| | 933 | 0 | 3 |
| | 227 | 1 | 1 |

| | yd. | qr. | nl. |
|---|---|---|---|
| 4. | 543 | 3 | 2 |
| | 836 | 2 | 2 |
| | 754 | 2 | 3 |
| | 217 | 1 | 3 |
| | 725 | 3 | 2 |
| | 438 | 2 | 2 |

| | E.e. | qr. | nl. |
|---|---|---|---|
| 5. | 120 | 2 | 2 |
| | 394 | 4 | 1 |
| | 110 | 2 | 0 |
| | 481 | 1 | 2 |
| | 556 | 4 | 3 |
| | 664 | 3 | 1 |
| | 779 | 2 | 3 |

| | E.e. | qr. | nl. |
|---|---|---|---|
| 6. | 537 | 0 | 2 |
| | 916 | 3 | 1 |
| | 328 | 3 | 3 |
| | 457 | 1 | 2 |
| | 646 | 3 | 2 |
| | 287 | 4 | 2 |
| | 561 | 2 | 2 |

| | yd. | qr. | nl. |
|---|---|---|---|
| 7. | 74 | 3 | 3 |
| | 64 | 2 | 1 |
| | 74 | 1 | 3 |
| | 49 | 2 | 1 |
| | 74 | 1 | 2 |
| | 44 | 3 | 1 |
| | 16 | 2 | 3 |

| | E.e. | qr. | nl. |
|---|---|---|---|
| 8. | 77 | 4 | 3 |
| | 14 | 3 | 2 |
| | 74 | 2 | 1 |
| | 49 | 1 | 2 |
| | 74 | 2 | 1 |
| | 44 | 1 | 2 |
| | 94 | 0 | 2 |

## LONG MEASURE.

|  | miles | fur. | p. | yds. |  | yds. | ft. | in. | b.c. |  | lea. | mi. | fur. | p. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | 427 | 6 | 23 | 3* | 2. | 214 | 2 | 9 | 0 | 3. | 520 | 1 | 6 | 13 |
|  | 689 | 5 | 26 | 5 |  | 183 | 2 | 11 | 2 |  | 623 | 1 | 7 | 27 |
|  | 322 | 7 | 30 | 2 |  | 597 | 0 | 8 | 1 |  | 721 | 0 | 4 | 16 |
|  | 510 | 2 | 38 | 4 |  | 619 | 2 | 7 | 2 |  | 826 | 1 | 3 | 32 |
|  | 777 | 4 | 0 | 3 |  | 725 | 1 | 6 | 1 |  | 932 | 2 | 6 | 1 |
|  | 888 | 3 | 10 | 4 |  | 930 | 1 | 3 | 0 |  | 315 | 1 | 2 | 28 |
|  | 126 | 0 | 24 | 0 |  | 492 | 1 | 4 | 1 |  | 409 | 1 | 5 | 39 |
|  | 412 | 7 | 39 | 4 |  | 291 | 2 | 10 | 2 |  | 376 | 2 | 7 | 27 |

|  | lea. | m. | fur. |  | fur. | p. | yds. |  | p. | yds. | ft. |  | feet | in. | b.c. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4. | 17 | 2 | 7 | 5. | 147 | 39 | 5 | 6. | 177 | 5 | 2 | 7. | 174 | 11 | 2 |
|  | 14 | 1 | 6 |  | 614 | 37 | 4 |  | 714 | 4 | 1 |  | 49 | 10 | 1 |
|  | 74 | 1 | 7 |  | 714 | 19 | 3 |  | 714 | 1 | 2 |  | 74 | 11 | 2 |
|  | 68 | 2 | 4 |  | 674 | 17 | 1 |  | 615 | 0 | 1 |  | 64 | 9 | 1 |
|  | 74 | 1 | 0 |  | 719 | 27 | 2 |  | 714 | 1 | 2 |  | 74 | 10 | 1 |
|  | 69 | 2 | 1 |  | 197 | 19 | 1 |  | 719 | 1 | 1 |  | 64 | 11 | 2 |
|  | 74 | 1 | 2 |  | 724 | 14 | 3 |  | 437 | 2 | 1 |  | 74 | 10 | 0 |
|  | 96 | 2 | 4 |  | 604 | 29 | 5 |  | 610 | 4 | 0 |  | 94 | 11 | 2 |

## LAND MEASURE.

|  | ac. | r. | p. |  | ac. | r. | p. |  | ac. | r. | p. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | 452 | 2 | 38 | 2. | 982 | 2 | 24 | 3. | 921 | 1 | 29 |
|  | 114 | 1 | 35 |  | 618 | 3 | 14 |  | 604 | 3 | 32 |
|  | 715 | 2 | 16 |  | 100 | 1 | 27 |  | 736 | 2 | 29 |
|  | 430 | 2 | 35 |  | 474 | 2 | 19 |  | 559 | 3 | 28 |
|  | 529 | 3 | 7 |  | 363 | 1 | 31 |  | 265 | 1 | 17 |
|  | 346 | 1 | 23 |  | 755 | 3 | 38 |  | 427 | 0 | 30 |
|  | 661 | 3 | 11 |  | 647 | 0 | 6 |  | 883 | 1 | 39 |
|  | 214 | 2 | 35 |  | 234 | 2 | 29 |  | 291 | 3 | 25 |

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NOTE.

* The pupil will recollect, that to bring yards into poles, he must multiply the yards by 2, and then divide by 11; and if there be a remainder it will be half yards.

|  | ac. | r. | p. |  | ac. | r. | p. |  | ac. | r. | p. |  | ac. | r. | p. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4. | 77 | 3 | 39 | 5. | 714 | 3 | 39 | 6. | 14 | 3 | 39 | 7. | 174 | 3 | 39 |
|  | 64 | 2 | 37 |  | 619 | 1 | 36 |  | 74 | 1 | 19 |  | 714 | 1 | 27 |
|  | 74 | 1 | 24 |  | 714 | 2 | 27 |  | 64 | 2 | 14 |  | 618 | 2 | 12 |
|  | 64 | 2 | 19 |  | 619 | 1 | 34 |  | 74 | 1 | 18 |  | 719 | 1 | 14 |
|  | 74 | 1 | 18 |  | 719 | 2 | 37 |  | 47 | 2 | 24 |  | 734 | 2 | 11 |
|  | 64 | 2 | 17 |  | 719 | 1 | 24 |  | 18 | 1 | 14 |  | 715 | 1 | 24 |
|  | 14 | 1 | 13 |  | 615 | 2 | 14 |  | 74 | 2 | 19 |  | 639 | 2 | 24 |
|  | 94 | 3 | 14 |  | 174 | 3 | 38 |  | 74 | 2 | 24 |  | 714 | 1 | 34 |

## WINE MEASURE.

|  | hhd. | gal. | pt. |  | tuns, | hhd. | g. | qt. |  | tuns, | hhd. | g. | qt. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | 626 | 44 | 7 | 2. | 522 | 1 | 39 | 3 | 3. | 148 | 2 | 25 | 3 |
|  | 753 | 17 | 1 |  | 257 | 3 | 34 | 2 |  | 513 | 0 | 42 | 3 |
|  | 438 | 52 | 6 |  | 763 | 2 | 58 | 3 |  | 614 | 1 | 36 | 1 |
|  | 217 | 13 | 7 |  | 611 | 3 | 43 | 1 |  | 349 | 3 | 43 | 2 |
|  | 135 | 45 | 0 |  | 937 | 1 | 16 | 3 |  | 416 | 2 | 56 | 1 |
|  | 497 | 56 | 2 |  | 238 | 0 | 31 | 2 |  | 952 | 3 | 26 | 0 |
|  | 312 | 11 | 3 |  | 749 | 3 | 7 | 0 |  | 567 | 1 | 19 | 3 |
|  | 256 | 0 | 0 |  | 319 | 2 | 59 | 3 |  | 792 | 3 | 46 | 2 |

|  | tuns, | hhd. | g. |  | pun. | gal. | qt. |  | hhd. | gal. | qt. |  | gal. | qt. | pt. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4. | 714 | 3 | 62 | 5. | 714 | 83 | 3 | 6. | 74 | 41 | 3 | 7. | 14 | 3 | 1 |
|  | 614 | 2 | 61 |  | 615 | 81 | 2 |  | 64 | 40 | 2 |  | 74 | 2 | 1 |
|  | 174 | 1 | 39 |  | 714 | 74 | 1 |  | 74 | 19 | 1 |  | 39 | 2 | 1 |
|  | 164 | 2 | 47 |  | 614 | 18 | 2 |  | 64 | 39 | 2 |  | 17 | 1 | 0 |
|  | 274 | 1 | 49 |  | 713 | 75 | 0 |  | 74 | 40 | 1 |  | 19 | 2 | 0 |
|  | 175 | 2 | 37 |  | 614 | 17 | 1 |  | 69 | 16 | 1 |  | 77 | 1 | 1 |
|  | 375 | 1 | 49 |  | 715 | 14 | 3 |  | 17 | 39 | 2 |  | 39 | 3 | 1 |
|  | 704 | 0 | 64 |  | 919 | 68 | 0 |  | 28 | 44 | 3 |  | 24 | 2 | 0 |

## ALE AND BEER MEASURE.

|  | bar. | fir. | gal. |  | hhd. | gal. | qt. |  | bar. | fir. | gal. | pt. |  | bar. | fir. | g. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | 130 | 3 | 5 | 2. | 666 | 29 | 2 | 3. | 278 | 2 | 0 | 6 | 4. | 381 | 2 | 6 |
|  | 471 | 3 | 7 |  | 883 | 42 | 2 |  | 154 | 3 | 5 | 3 |  | 37 | 3 | 7 |
|  | 348 | 1 | 6 |  | 877 | 53 | 0 |  | 561 | 0 | 8 | 7 |  | 43 | 2 | 5 |
|  | 726 | 2 | 7 |  | 944 | 26 | 3 |  | 923 | 1 | 6 | 4 |  | 16 | 1 | 4 |
|  | 819 | 1 | 4 |  | 278 | 37 | 1 |  | 386 | 2 | 3 | 5 |  | 4 | 3 | 8 |
|  | 455 | 0 | 2 |  | 455 | 18 | 3 |  | 218 | 1 | 5 | 6 |  | 71 | 3 | 0 |
|  | 327 | 2 | 7 |  | 610 | 52 | 1 |  | 117 | 3 | 0 | 2 |  | 5 | 1 | 2 |
|  | 234 | 0 | 1 |  | 757 | 4 | 3 |  | 792 | 2 | 4 | 0 |  | 8 | 3 | 7 |

| | bar. | fir. | gal. | | bar. | fir. | gal. | | hhd. | gal. | qt. | | hhd. | gal. | qt. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5. | 74 | 3 | 8 | 6. | 73 | 3 | 7 | 7. | 714 | 47 | 3 | 8. | 714 | 53 | 3 |
| | 14 | 2 | 7 | | 69 | 2 | 6 | | 614 | 44 | 1 | | 415 | 47 | 2 |
| | 16 | 1 | 4 | | 14 | 1 | 7 | | 374 | 43 | 2 | | 714 | 19 | 1 |
| | 17 | 1 | 3 | | 39 | 2 | 2 | | 157 | 41 | 1 | | 614 | 27 | 1 |
| | 29 | 2 | 2 | | 19 | 1 | 6 | | 719 | 42 | 1 | | 715 | 51 | 2 |
| | 17 | 1 | 7 | | 49 | 2 | 6 | | 874 | 41 | 2 | | 714 | 37 | 2 |
| | 41 | 2 | 6 | | 37 | 1 | 4 | | 174 | 12 | 1 | | 615 | 19 | 1 |
| | 67 | 0 | 6 | | 49 | 2 | 3 | | 419 | 43 | 2 | | 714 | 48 | 3 |

## CORN MEASURE.

| | qr. | b. | pec. | | b. | p. | ga. | | ch. | b. | pec. | | qr. | b. | pec. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | 571 | 6 | 2 | 2. | 506 | 2 | 1 | 3. | 161 | 28 | 2 | 4. | 87 | 5 | 3 |
| | 936 | 4 | 3 | | 524 | 2 | 1 | | 394 | 13 | 1 | | 29 | 4 | 1 |
| | 692 | 7 | 3 | | 914 | 3 | 1 | | 465 | 26 | 3 | | 66 | 5 | 2 |
| | 438 | 6 | 1 | | 393 | 2 | 0 | | 939 | 17 | 1 | | 18 | 4 | 1 |
| | 343 | 4 | 0 | | 746 | 2 | 1 | | 791 | 34 | 2 | | 44 | 6 | 2 |
| | 297 | 2 | 3 | | 673 | 3 | 0 | | 537 | 15 | 2 | | 32 | 3 | 0 |
| | 749 | 1 | 0 | | 252 | 0 | 1 | | 631 | 23 | 1 | | 70 | 7 | 2 |
| | 244 | 5 | 2 | | 438 | 1 | 0 | | 443 | 19 | 2 | | 41 | 5 | 1 |
| | 248 | 3 | 1 | | 927 | 3 | 1 | | 594 | 12 | 3 | | 59 | 7 | 3 |

| | p. | g. | pt. | | ch. | b. | pec. | | ch. | b. | pec. | | score, | ch. | b. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5. | 40 | 1 | 3 | 6. | 91 | 24 | 3 | 7. | 14 | 31 | 3 | 8. | 74 | 20 | 35 |
| | 17 | 1 | 2 | | 74 | 16 | 2 | | 74 | 31 | 2 | | 49 | 19 | 33 |
| | 34 | 1 | 7 | | 42 | 14 | 1 | | 64 | 30 | 1 | | 64 | 17 | 35 |
| | 33 | 0 | 4 | | 56 | 28 | 3 | | 74 | 27 | 2 | | 74 | 14 | 10 |
| | 47 | 1 | 7 | | 23 | 5 | 1 | | 64 | 19 | 2 | | 39 | 13 | 9 |
| | 59 | 0 | 1 | | 78 | 27 | 3 | | 74 | 31 | 1 | | 47 | 16 | 8 |
| | 56 | 1 | 5 | | 99 | 32 | 2 | | 64 | 11 | 1 | | 19 | 17 | 4 |
| | 68 | 1 | 6 | | 10 | 28 | 1 | | 94 | 26 | 2 | | 37 | 12 | 34 |
| | 23 | 1 | 7 | | 9 | 35 | 2 | | 4 | 26 | 2 | | 56 | 13 | 4 |

## TIME.

| | mo. | w. | d. | h. | | w. | d. | h. | mi. | | d. | h. | mi. | sec. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | 19 | 2 | 6 | 19 | 2. | 57 | 4 | 23 | 38 | 3. | 62 | 7 | 47 | 38 |
| | 46 | 1 | 4 | 21 | | 64 | 6 | 13 | 47 | | 18 | 12 | 54 | 56 |
| | 22 | 3 | 5 | 9 | | 15 | 3 | 21 | 19 | | 76 | 21 | 16 | 49 |
| | 57 | 2 | 3 | 21 | | 36 | 2 | 18 | 15 | | 34 | 9 | 20 | 31 |
| | 62 | 1 | 6 | 12 | | 78 | 6 | 9 | 59 | | 99 | 23 | 31 | 46 |
| | 17 | 3 | 2 | 14 | | 49 | 0 | 20 | 6 | | 53 | 22 | 28 | 32 |
| | 11 | 3 | 4 | 16 | | 71 | 5 | 14 | 48 | | 15 | 4 | 58 | 23 |
| | 29 | 1 | 3 | 21 | | 23 | 3 | 7 | 24 | | 64 | 16 | 13 | 16 |

| | yrs. | mo. | w. | | mo. | w. | d. | | days, | hrs. | min. | | hrs. | min. | sec. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4. | 737 | 12 | 3 | 5. | 64 | 3 | 6 | 6. | 714 | 23 | 59 | 7. | 647 | 59 | 59 |
| | 347 | 11 | 2 | | 74 | 1 | 5 | | 74 | 14 | 54 | | 137 | 54 | 54 |
| | 618 | 10 | 1 | | 34 | 2 | 8 | | 94 | 21 | 55 | | 375 | 56 | 56 |
| | 374 | 9 | 2 | | 74 | 1 | 4 | | 74 | 13 | 53 | | 714 | 17 | 19 |
| | 175 | 1 | 1 | | 63 | 2 | 1 | | 69 | 12 | 14 | | 615 | 54 | 54 |
| | 714 | 12 | 3 | | 74 | 1 | 2 | | 74 | 12 | 19 | | 714 | 17 | 13 |
| | 615 | 10 | 1 | | 64 | 2 | 1 | | 37 | 11 | 17 | | 613 | 34 | 56 |
| | 314 | 9 | 3 | | 94 | 2 | 6 | | 46 | 22 | 49 | | 626 | 47 | 49 |

## ASTRONOMY.

| s. | ° | ′ | ″ | s. | ° | ′ | ″ | s. | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 24 | 37 | 41 | 5 | 3 | 26 | 25 | 6 | 9 | 54 | 36 |
| 7 | 12 | 57 | 21 | 9 | 5 | 37 | 56 | 3 | 29 | 59 | 7 |
| 3 | 25 | 13 | 17 | 8 | 24 | 42 | 59 | 11 | 26 | 21 | 19 |
| 4 | 29 | 18 | 29 | 3 | 9 | 12 | 15 | 9 | 24 | 50 | 40 |
| 5 | 16 | 52 | 43 | 4 | 8 | 17 | 41 | 11 | 18 | 29 | 27 |
| 3 | 19 | 47 | 51 | 3 | 26 | 9 | 8 | 5 | 13 | 51 | 46 |
| 11 | 29 | 51 | 36 | 5 | 16 | 8 | 27 | 6 | 7 | 1 | 9 |
| 9 | 18 | 30 | 30 | 11 | 20 | 40 | 50 | 10 | 12 | 24 | 36 |
| 3 | 4 | 44 | 44 | 10 | 9 | 55 | 37 | 7 | 21 | 42 | 56 |
| 7 | 25 | 36 | 51 | 4 | 22 | 44 | 56 | 5 | 23 | 51 | 46 |

## MISCELLANEOUS EXAMPLES IN ADDITION.

1. What is the sum total, in shillings, of 54 guineas, 29 pounds, 36 guineas, and 48 pounds?

2. Add together 16*l.* 12*s.* 2*d.*; 156*l.* 9*s.* 9¼*d.*; 20395*l.* 12*s.*; 24*l.* 19*s.* 11¾*d.*; 37*l.* 6*s.* 7*d.*; 327*l.* 18*s.*; and 100 guineas.

3. In collecting an account of debts owing to me, I find Mr. A. owes me 97*l.* 16*s.*; Mr. B. 125*l.* 13*s.* 9½*d.*; Mr. C. 6*l.* 6*s.*; Mr. D. 50 guineas; and Mr. E. 71*l.* 0*s.* 9*d.*; what is the whole sum due to me?

4. A gentleman ordered a service of plate from his silversmith, and on receiving his bill, he finds that he had dishes and covers weighing 45 lb. 9 oz. 12 dwts.; plates weighing 70 lb. 7 oz. 16 dwts.; spoons of different sizes, and ladles, 24 lb. 9 oz. 12 dwts.; waiters 15 lb. 10 oz.; salts and castors, 4 lb. 4 oz. 3 dwts.; candlesticks, 19 lb. 11 oz. 17 dwts.; and sundry smaller articles 5 lb. 3 oz.; what is the weight of silver he will have to pay for?

5. A carrier brings goods to a shopkeeper, viz. 8 bags of hops weighing 19 cwt. 3 qrs. 14 lb.; cheeses weighing 15 cwt. 1 qr. 21 lb.; butter weighing 12 cwt. 2 qrs.; two chests of tea, weighing 1½ cwt. each; and a sack of salt weighing 8 cwt. 2 qr. 12 lb.; how much weight will the carrier have to charge?

6. The rent of my house is 50*l.* per annum; the house tax is three pounds fifteen shillings; land-tax 5*l.*; windows 15*l.* 12*s.* 0*d.*; poor's rates 10*l.*; lighting, watching, and street rates 3*l.* 9*s.* 3½*d.*; how much therefore do my house and taxes stand me in per annum?

7. The following is an estimate of the repairs wanting to my house; how much is the whole sum? Carpenter's bill 27*l.* 9*s.* 9½*d.*; bricklayer's and plasterer's 17*l.* 7*s.* 6*d.*; mason's 5*l.* 5*s.*; painter's, glazier's, and plumber's, fourteen guineas; smith's, for new rails, 12*l.*; and the slater's 9*l.* 18*s.*

8. A man purchased some goods for the country; the first parcel contained 25 yds. 2 qr. 2 nl. of broad cloth; the second 126 yds. 2 qrs. of serge; the third a thousand yards of green baise; and the fourth 19 yds. 3 qrs. 2 nl. of shalloon; what was the whole quantity?

9. A wine merchant, retiring from business, takes an account of the stock of wines in his cellar, and finds 5 pipes and 50 gallons of port wine, 3 p. 2 hhd. of sherry; 10 pipes of Lisbon; 2 pipes of claret; of Madeira he had 36 gallons; of brandy 50 gallons; of rum two hogsheads; and of Hollands, 1 hhd. and 12 gallons; what quantity of liquor did his cellar contain?

10. A friend in Essex desired me to measure his farm, which he holds on a lease; the three fields at the back of the house measured 59 ac. 2 r. 20 p.; the large piece of ground in the valley measures 74 acres, three others measure each on an average 11 ac. 1 r. 36 p.; the field laid down in clover contains 7 ac. 3 r. 2 p.; one sown with caraways, I find to be 3½ acres; and the ground belonging to the garden, out-houses, &c. makes about 1¼ acres; how many acres ought he to pay for?

11. A merchant sends to his banker on the 2d. day of the month, in money and bills, to the amount of two thousand guineas; on the fifth he sends him 900*l.* 19*s.* 4*d.*; on the eleventh he sends 500*l.*; and in the course of the remaining days of the month he sends 1515*l.* 12*s.* 11½*d.*; how much therefore may he draw as occasion requires?

12. A gentleman's steward received the following sums of money for rents; what was the gentleman's income? Of farmer A he received 394*l.* 12*s.* 6*d.*, of B 97*l.* 14*s.* 9*d.*, of C 175*l.* 10*s.*, of D 99*l.* 4*s*, and of E 139*l.* 12*s.* 4*d.*

13. A person borrows of several friends the following sums of money; of the first 500*l.*; of the second 225*l.* 12*s.*; of the third fifty guineas; of the fourth seventy guineas and 22 crowns; of the fifth he had 150*l.* 7*s.* 6*d.*; how much will he have to pay interest for?

14. A man borrowed a sum of money, and paid at different times 75 guineas, but he still owed 95*l.* 9*s.* 9*d.*; what was the original debt?

# COMPOUND SUBTRACTION

Is the method of finding the difference between two given compound numbers.

RULE I. *Having arranged the numbers so that the smaller may stand under the greater, subtract each number in the lower line from that which stands above it, and write down the remainders.*

2. *When any of the lower denominations are greater than the upper, increase the upper number by as many as make one of the next superior denomination, from which take the figure in the lower line, set down the difference, and carry one to the next number in the lower line, and subtract as before.*

*Ex.* Subtract 595*l.* 17*s.* 9½*d.* from 600*l.* 10*s.* 7¼*d.*

| | £. | s. | d. |
|---|---|---|---|
| | 600 | 10 | 7¼ |
| | 595 | 17 | 9½ |
| | 4 | 12 | 9¾ |
| Proof - | 600 | 10 | 7¼ |

Here I say 2 farthings from 1, I cannot, but I add 4 to the 1, because 4 farthings make a penny, and 2 from 5 and there remains ¾; I carry one to the 9; 10 from 7 I cannot, but I add 12 to 7, because 12 pence make a shilling, and 10 from 19 and there remain 9; I carry 1 to 17; and 18 from 10, I cannot, but I add 20 to the 10, because 20 shillings make a pound, and 18 from 30 and there remain 12; I now carry one to the five, and go on as in simple Subtraction.

The method of proof is the same as in simple Subtraction.

### EXAMPLES.

| | £. | s. | d. | | £. | s. | d. | | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 145 | 19 | 9½ | Ex. 2. | 370 | 17 | 7¾ | Ex. 3. | 450 | 12 | 6½ |
| | 136 | 17 | 6¼ | | 369 | 12 | 4½ | | 371 | 10 | 4 |
| Answer | | | | | | | | | | | |
| Proof - | | | | | | | | | | | |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 4. | 594 | 10 | 9¼ | Ex. 5. | 465 | 12 | 7½ | Ex. 6. | 564 | 12 | 2½ |
| | 374 | 19 | 5½ | | 349 | 17 | 9¾ | | 375 | 18 | 4¼ |

|  | £. | s. | d. |  | £. | s. | d. |  | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 7. | 371 | 19 | 2½ | Ex. 8. | 700 | 0 | 0 | Ex. 9. | 476 | 19 | 4 |
|  | 199 | 17 | 11¾ |  | 376 | 16 | 6½ |  | 374 | 12 | 9 |
| Ex. 10. | 473 | 18 | 7¾ | Ex. 11. | 249 | 9 | 9¾ | Ex. 12. | 376 | 17 | 7 |
|  | 291 | 12 | 7¾ |  | 159 | 19 | 11¼ |  | 299 | 14 | 4¾ |
| Ex. 13. | 594 | 0 | 0 | Ex. 14. | 796 | 12 | 11½ | Ex. 15. | 476 | 17 | 7 |
|  | 593 | 19 | 9¾ |  | 669 | 8 | 3 |  | 399 | 19 | 11¾ |
| Ex. 16. | 399 | 2 | 2 | Ex. 17. | 209 | 18 | 8 | Ex. 18. | 500 | 0 | 0 |
|  | 177 | 12 | 7¾ |  | 159 | 19 | 9½ |  | 499 | 19 | 11¾ |
| Ex. 19. | 422 | 3 | 6¼ | Ex. 20. | 224 | 2 | 6¾ | Ex. 21. | 794 | 15 | 6¾ |
|  | 371 | 15 | 7¾ |  | 156 | 6 | 6¾ |  | 367 | 16 | 4¾ |
| Ex. 22. | 999 | 0 | 0 | Ex. 23. | 764 | 15 | 4¼ | Ex. 24. | 674 | 6 | 0¾ |
|  | 800 | 19 | 11¾ |  | 398 | 12 | 11 |  | 249 | 19 | 9¾ |
| Ex. 25. | 372 | 10 | 6¼ | Ex. 26. | 649 | 12 | 9¾ | Ex. 27. | 341 | 5 | 11½ |
|  | 149 | 6 | 4¾ |  | 597 | 19 | 8¼ |  | 230 | 9 | 4½ |
| Ex. 28. | 846 | 9 | 8¾ | Ex. 29. | 124 | 9 | 10¾ | Ex. 30. | 90441 | 5 | 9¾ |
|  | 375 | 9 | 9½ |  | 109 | 10 | 3¼ |  | 67217 | 13 | 10 |

|  | £. | s. | d. |  | £. | s. | d. |  | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 31. | 438 | 7 | 10 | Ex. 32. | 12427 | 16 | 11¼ | Ex. 33. | 1654 | 12 | 7 |
|  | 399 | 16 | 9½ |  | 7618 | 14 | 9¾ |  | 585 | 9 | 10¼ |
| Ex. 34. | 14476 | 5 | 6½ | Ex. 35. | 222 | 18 | 9¾ | Ex. 36. | 96481 | 16 | 9 |
|  | 7614 | 13 | 8¾ |  | 142 | 7 | 10½ |  | 3768 | 10 | 9¼ |
| Ex. 37. | 164 | 17 | 8½ | Ex. 38. | 18140 | 14 | 0½ | Ex. 39. | 417 | 4 | 10¾ |
|  | 29 | 2 | 9¾ |  | 17216 | 0 | 4¼ |  | 519 | 11 | 7½ |
| Ex. 40. | 20412 | 13 | 9¾ | Ex. 41. | 425 | 18 | 9 | Ex. 42. | 22425 | 14 | 9¼ |
|  | 19911 | 14 | 2½ |  | 139 | 10 | 9¾ |  | 21018 | 8 | 11¾ |
| Ex. 43. | 183 | 9 | 1¼ | Ex. 44. | 24463 | 13 | 11¼ | Ex. 45. | 421 | 16 | 9¾ |
|  | 24 | 14 | 10¾ |  | 17732 | 16 | 9½ |  | 326 | 19 | 0 |
| Ex. 46. | 86473 | 6 | 9¼ | Ex. 47. | 433 | 17 | 2¾ | Ex. 48. | 28446 | 17 | 9 |
|  | 56117 | 13 | 10 |  | 311 | 19 | 4½ |  | 19994 | 14 | 8¼ |
| Ex. 49. | 194 | 12 | 8¼ | Ex. 50. | 80490 | 9 | 9 | Ex. 51. | 474 | 19 | 4¼ |
|  | 117 | 12 | 9 |  | 24689 | 15 | 10½ |  | 362 | 13 | 7¼ |
| Ex. 52. | 26475 | 13 | 9 | Ex. 53. | 4559 | 16 | 9¾ | Ex. 54. | 34487 | 15 | 11½ |
|  | 24716 | 18 | 11¾ |  | 3228 | 9 | 5¼ |  | 31767 | 19 | 10 |

| | £. | s. | d. | | £. | s. | d. | | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 55. | 2139 | 7 | 10 | Ex. 56. | 36492 | 7 | 5¾ | Ex. 57. | 3471 | 19 | 9¼ |
| | 1914 | 13 | 10½ | | 20082 | 0 | 6½ | | 293 | 19 | 9¾ |
| Ex. 58. | 38410 | 14 | 9 | Ex. 59. | 4557 | 18 | 9½ | Ex. 60. | 601273 | 11 | 7 |
| | 28019 | 19 | 10¼ | | 3945 | 17 | 11¾ | | 462104 | 15 | 8¾ |
| Ex. 61. | 5534 | 11 | 3 | Ex. 62. | 424136 | 11 | 6¼ | Ex. 63. | 7860 | 0 | 0 |
| | 559 | 12 | 7 | | 379126 | 10 | 9¾ | | 3271 | 4 | 7 |
| Ex. 64. | 441391 | 6 | 0¼ | Ex. 65. | 6234 | 6 | 6 | Ex. 66. | 1414 | 9 | 9¼ |
| | 389091 | 9 | 8½ | | 309 | 12 | 10¼ | | 729 | 12 | 11¾ |
| Ex. 67. | 1173 | 14 | 9¾ | Ex. 68. | 484760 | 10 | 9 | Ex. 69. | 791 | 5 | 11¼ |
| | 437 | 18 | 11¼ | | 329189 | 19 | 9¼ | | 261 | 19 | 11½ |
| Ex. 70. | 14112 | 0 | 0½ | Ex. 71. | 1345 | 19 | 9¾ | Ex. 72. | 4621 | 15 | 9½ |
| | 4612 | 19 | 1 | | 345 | 17 | 9½ | | 394 | 19 | 0¾ |
| Ex. 73. | 396 | 19 | 9¼ | Ex. 74. | 254 | 14 | 9¼ | Ex. 75. | 1214 | 0 | 5 |
| | 29 | 19 | 9¾ | | 244 | 19 | 10½ | | 880 | 0 | 8¾ |
| Ex. 76. | 564121 | 10 | 10¾ | Ex. 77. | 4465 | 10 | 9½ | Ex. 78. | 4532 | 13 | 9¼ |
| | 379178 | 16 | 10¼ | | 304 | 0 | 11¾ | | 4319 | 15 | 11¾ |

| | | | |
|---|---|---|---|
| Ex. 79. | 408 | 19 | 4¼ |
| | 254 | 1 | 10¾ |

| | | | |
|---|---|---|---|
| Ex. 80. | 60985 | 14 | 4¼ |
| | 1427 | 19 | 9½ |

| | £. | s. | d. |
|---|---|---|---|
| Borrowed | 300 | 0 | 0 |
| Paid at different times | 15 | 15 | 0 |
| | 39 | 7 | 7¾ |
| | 76 | 8 | 1 |
| | 46 | 15 | 10 |
| | 105 | 0 | 0 |
| Paid - | 283 | 6 | 6¾ |
| Remain unpaid | 16 | 13 | 5¼ |

| | £. | s. | d. |
|---|---|---|---|
| Borrowed | 1000 | 0 | 0 |
| Paid at different times | 177 | 16 | 7¾ |
| | 105 | 0 | 3 |
| | 52 | 10 | 11 |
| | 246 | 9 | 9¼ |
| | 300 | 0 | 9 |
| Paid - | 881 | 18 | 4 |
| Unpaid - | 118 | 1 | 8 |

Suppose a person is debtor to sundry persons, in the following sums.

| | £. | s. | d. |
|---|---|---|---|
| | 7678 | 14 | 9¾ |
| | 29 | 17 | 4¼ |
| | 5 | 5 | 0 |
| | 1054 | 12 | 9¼ |
| | 26 | 5 | 0 |
| | 7 | 7 | 0 |
| | 95 | 19 | 9½ |
| | 39 | 11 | 3 |
| Dr. | | | |

And is creditor, by book-debts from different people, in the following sums.

| | £. | s. | d. |
|---|---|---|---|
| | 764 | 14 | 9¾ |
| | 39 | 14 | 4 |
| | 500 | 0 | 0 |
| | 899 | 5 | 9½ |
| | 2500 | 0 | 0 |
| | 5505 | 5 | 11 |
| | 3000 | 0 | 0 |
| Cr. | | | |
| Dr. | | | |

Balance in favour of Cr.

Required the balance of this account?

| Dr. | | | Cr. | | |
|---|---|---|---|---|---|
| £. | s. | d. | £. | s. | d. |
| 764 | 14 | 9 | 397 | 14 | 11¾ |
| 397 | 0 | 10¾ | 267 | 11 | 9 |
| 210 | 19 | 9½ | 726 | 13 | 8¾ |
| 467 | 16 | 7¾ | 464 | 16 | 0 |
| 371 | 14 | 9 | 215 | 12 | 6 |
| 564 | 12 | 6¾ | 345 | 9 | 10¼ |

Required the balance of this account?

| Dr. | | | Cr. | | |
|---|---|---|---|---|---|
| £. | s. | d. | £. | s. | d. |
| 769 | 19 | 10½ | 49 | 12 | 11 |
| 643 | 4 | 4 | 1000 | 17 | 9¾ |
| 248 | 11 | 7 | 1766 | 5 | 5 |
| 591 | 8 | 4 | 4 | 4 | 0 |
| 9 | 19 | 6 | 250 | 12 | 8¾ |
| 300 | 0 | 0 | 1750 | 17 | 0 |

### EXAMPLES OF WEIGHTS AND MEASURES.*

## TROY WEIGHT.

| | lb. | oz. | dwt. | gr. | | lb. | oz. | dwt. | gr. | | lb. | oz. | dwt. | gr. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 187 | 9 | 12 | 20 | 2. | 256 | 6 | 0 | 22 | 3. | 567 | 4 | 0 | 0 |
| | 169 | 6 | 14 | 17 | | 199 | 9 | 3 | 20 | | 379 | 11 | 9 | 9 |

| | lb. | oz. | dwt. | gr. | | lb. | oz. | dwt. | gr. | | lb. | oz. | dwt. | gr. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4. | 254 | 0 | 0 | 0 | 5. | 675 | 3 | 0 | 9 | 6. | 423 | 5 | 15 | 14 |
| | 253 | 11 | 19 | 20 | | 576 | 9 | 17 | 16 | | 246 | 11 | 18 | 23 |

| | lb. | oz. | dwt. | | oz. | dwt. | gr. | | lb. | oz. | dwt. | | oz. | dwt. | gr. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7. | 14 | 11 | 9 | 8. | 74 | 12 | 18 | 9. | 175 | 3 | 10 | 10. | 17 | 10 | 20 |
| | 11 | 10 | 14 | | 64 | 14 | 17 | | 159 | 11 | 14 | | 14 | 11 | 23 |

## AVOIRDUPOIS WEIGHT.

| | tons, | cwt. | qr. | lb. | oz. | dr. | | tons, | cwt. | qr. | lb. | oz. | dr. | | tons, | cwt. | qr. | lb. | oz. | dr. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | 72 | 10 | 3 | 14 | 10 | 12 | 2. | 64 | 15 | 2 | 15 | 10 | 9 | 3. | 25 | 0 | 0 | 0 | 0 | 0 |
| | 9 | 16 | 1 | 25 | 14 | 6 | | 46 | 15 | 3 | 5 | 12 | 14 | | 24 | 0 | 2 | 0 | 0 | 15 |

| | tons, | cwt. | qr. | lb. | oz. | dr. | | tons, | cwt. | qr. | lb. | oz. | dr. | | tons, | cwt. | qr. | lb. | oz. | dr. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4. | 67 | 2 | 1 | 4 | 14 | 2 | 5. | 36 | 7 | 1 | 1 | 1 | 1 | 6. | 76 | 3 | 0 | 0 | 0 | 4 |
| | 29 | 14 | 3 | 2 | 0 | 14 | | 30 | 3 | 2 | 5 | 5 | 5 | | 67 | 12 | 2 | 0 | 14 | 4 |

| | tons, | cwt. | qr. | | cwt. | qr. | lb. | | qr. | lb. | oz. | | lb. | oz. | dr. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7. | 14 | 12 | 2 | 8. | 17 | 1 | 25 | 9. | 143 | 22 | 12 | 10. | 174 | 11 | 10 |
| | 1 | 14 | 3 | | 14 | 2 | 27 | | 74 | 19 | 14 | | 39 | 12 | 13 |

## APOTHECARIES' WEIGHT.

| | lb. | oz. | dr. | scr. | | lb. | oz. | dr. | scr. | | lb. | oz. | dr. | scr. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | 456 | 9 | 4 | 0 | 2. | 269 | 8 | 3 | 2 | 3. | 987 | 4 | 4 | 0 |
| | 309 | 4 | 7 | 2 | | 178 | 11 | 3 | 1 | | 379 | 10 | 5 | 1 |

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NOTE.

* The general student need not work the whole of these examples.

| | lb. | oz. | dr. | scr. | | lb. | oz. | dr. | scr. | | lb. | oz. | dr. | scr. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 4. | 564 | 0 | 0 | 0 | 5. | 375 | 7 | 7 | 1 | 6. | 394 | 2 | 2 | 0 |
| | 469 | 3 | 3 | 2 | | 369 | 4 | 7 | 2 | | 299 | 11 | 7 | 2 |

| | lb. | oz. | dr. | | oz. | dr. | scr. | | dr. | scr. | gr. | | lb. | oz. | dr. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7. | 144 | 10 | 5 | 8. | 27 | 4 | 1 | 9. | 27 | 1 | 14 | 10. | 74 | 10 | 5 |
| | 64 | 11 | 7 | | 14 | 7 | 2 | | 14 | 0 | 19 | | 65 | 11 | 6 |

## CLOTH MEASURE.

| | yds. | qr. | n. | | E.e. | qr. | n. | | yds. | qr. | n. | | yds. | qr. | n. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 218 | 2 | 0 | 2. | 46 | 0 | 0 | 3. | 567 | 1 | 1 | 4. | 459 | 1 | 2 |
| | 176 | 1 | 3 | | 23 | 2 | 2 | | 469 | 0 | 2 | | 399 | 3 | 3 |

| | yds. | qr. | n. | | E.e. | qr. | n. | | E.e. | qr. | n. | | E.e. | qr. | n. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5. | 174 | 2 | 1 | 6. | 174 | 3 | 1 | 7. | 171 | 1 | 3 | 8. | 12 | 1 | 1 |
| | 39 | 3 | 2 | | 49 | 4 | 2 | | 74 | 4 | 2 | | 10 | 4 | 3 |

## LONG MEASURE.

| | yds. | ft. | in. | b.c. | | yds. | ft. | in. | b.c. | | yds. | ft. | in. | b.c. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 456 | 2 | 10 | 1 | 2. | 679 | 0 | 0 | 0 | 3. | 267 | 1 | 1 | 1 |
| | 379 | 1 | 11 | 2 | | 599 | 1 | 1 | 1 | | 199 | 2 | 2 | 2 |

| | lea. | m. | fur. | p. | | lea. | m. | fur. | p. | | lea. | m. | fur. | p. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4. | 470 | 1 | 4 | 19 | 5. | 367 | 0 | 0 | 0 | 6. | 225 | 1 | 1 | 1 |
| | 279 | 2 | 7 | 23 | | 179 | 2 | 5 | 23 | | 167 | 2 | 4 | 4 |

| | lea. | m. | fur. | | fur. | p. | yds. | | p. | yd. | ft. | | ft. | in. | b.c. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7. | 21 | 2 | 4 | 8. | 14 | 34 | 5 | 9. | 14 | 6 | 1 | 10. | 17 | 11 | 2 |
| | 3 | 2 | 6 | | 12 | 39 | 5 | | 9 | 4 | 2 | | 14 | 11 | 1 |

## LAND MEASURE.

| | ac. | r. | p. | | ac. | r. | p. | | ac. | r. | p. | | ac. | r. | p. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 456 | 2 | 25 | 2. | 457 | 1 | 29 | 3. | 356 | 0 | 39 | 4. | 594 | 1 | 1 |
| | 399 | 0 | 29 | | 374 | 3 | 39 | | 279 | 3 | 39 | | 259 | 3 | 17 |

| | ac. | r. | p. | | ac. | r. | p. | | ac. | r. | p. | | ac. | r. | p. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5. | 12 | 0 | 32 | 6. | 112 | 1 | 31 | 7. | 12 | 1 | 25 | 8. | 19 | 1 | 20 |
| | 1 | 3 | 14 | | 74 | 2 | 37 | | 10 | 3 | 39 | | 14 | 2 | 21 |

## WINE MEASURE.

| | tuns, | hhd. | gal. | qt. | pt. | | tuns, | hhd. | gal. | qt. | pt. | | tuns, | hhd. | g. | qt. | pt. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | 456 | 0 | 24 | 1 | 1 | 2. | 257 | 3 | 10 | 1 | 1 | 3. | 467 | 2 | 0 | 0 | 0 |
| | 399 | 3 | 46 | 3 | 1 | | 199 | 0 | 50 | 3 | 1 | | 299 | 3 | 32 | 2 | 1 |

| | tuns, | hhd. | gal. | | punch. | gal. | qt. | | hhd. | gal. | qt. | | gal. | qt. | pt. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4. | 27 | 2 | 54 | 5. | 147 | 14 | 2 | 6. | 14 | 1 | 2 | 7. | 24 | 2 | 2 |
| | 19 | 3 | 62 | | 79 | 83 | 3 | | 12 | 41 | 3 | | 18 | 0 | 1 |

## BEER MEASURE.

| | butts, | hhd. | g. | qt. | pt. | | butts, | hhd. | g. | qt. | pt. | | bar. | fir. | gal. | qt. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | 256 | 0 | 39 | 1 | 1 | 2. | 467 | 0 | 0 | 0 | 0 | 3. | 376 | 2 | 6 | 2 |
| | 198 | 1 | 51 | 3 | 1 | | 299 | 1 | 1 | 1 | 1 | | 371 | 0 | 8 | 3 |

| | bar. | fir. | g. | | bar. | fir. | g. | | bar. | fir. | g. | | bar. | fir. | g. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4. | 14 | 3 | 5 | 5. | 147 | 1 | 3 | 6. | 271 | 1 | 2 | 7. | 143 | 1 | 2 |
| | 12 | 3 | 7 | | 39 | 3 | 8 | | 55 | 2 | 3 | | 97 | 2 | 3 |

## CORN AND COAL MEASURE.

| | qr. | bush. | p. | | ch. | bush. | p. | | ch. | bush. | p. | | qr. | bush. | p. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | 124 | 3 | 2 | 2. | 109 | 18 | 2 | 3. | 529 | 17 | 1 | 4. | 376 | 0 | 0 |
| | 90 | 6 | 3 | | 7 | 29 | 3 | | 297 | 31 | 2 | | 246 | 7 | 3 |

| | ch. | b. | p. | | w. | qr. | b. | | qr. | b. | p. | | score | ch. | b. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5. | 74 | 31 | 3 | 6. | 17 | 3 | 1 | 7. | 147 | 6 | 2 | 8. | 47 | 1 | 12 |
| | 47 | 31 | 2 | | 14 | 3 | 7 | | 94 | 7 | 3 | | 14 | 20 | 35 |

## TIME.

| | d. | hr. | min. | | d. | hr. | min. | sec. | | mo. | w. | d. | hr. | | w. | d. | hr. | m. | s. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | 37 | 2 | 39 | 2. | 74 | 3 | 12 | 14 | 3. | 46 | 1 | 1 | 4 | 4. | 36 | 0 | 0 | 0 | 0 |
| | 29 | 21 | 49 | | 47 | 21 | 54 | 36 | | 29 | 3 | 6 | 21 | | 35 | 6 | 23 | 50 | 59 |

| | yrs. | m. | w. | | m. | w. | d. | | d. | hrs. | m. | | hrs. | min. | sec. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 5. | 17 | 10 | 2 | 6. | 147 | 2 | 3 | 7. | 167 | 21 | 50 | 8. | 174 | 50 | 51 |
| | 14 | 12 | 3 | | 19 | 2 | 4 | | 19 | 23 | 54 | | 94 | 59 | 57 |

## MISCELLANEOUS EXAMPLES IN SUBTRACTION.

1. I borrowed of a friend five hundred guineas, and have paid, at different times, three hundred and ninety pounds six shillings and sevenpence three farthings: what have I still to pay?

2. A horse and his harness are worth 49*l.* 4*s.* 6*d.*; but the harness is worth eleven guineas: I demand the value of the horse?

3. What sum, added to 150 guineas, will make up 199*l.* 9*s.* 9½*d.*?

4. At an eclipse of the sun, the moon is situated between the earth and sun: how far distant is the moon from the sun, supposing the distance between the earth and sun 95 millions of miles, and that between the earth and moon 240 thousands?

5. The great bell at Oxford weighs 7 tons, 11 cwt. 3 qrs. 4 lb.; that at St. Paul's 5 tons, 2 cwt. 1 qr. 22 lb.; and the great *Tom* of Lincoln weighs 4 tons, 16 cwt. 3 qr. 16 lb.: how much heavier than these together is the great bell at Moscow, which is 198 tons?

6. The Royal Exchange cost 80 thousand pounds in building; the Mansion-house 40 thousand; Blackfriars-bridge, 153 thousand; Westminster-bridge, 389 thousand; and the Monument, thirteen thousand pounds; but the Cathedral of St. Paul's cost 800 thousand: how much did this cost more than all the rest?

7. If my income is 367*l.* 8*s.* 4½*d.* and my expenditure be 340 guineas: how much can I lay by?

8. A person, by great losses, was obliged to call his creditors together: he found his whole property amount to 527*l.* 12*s.* 8¾*d.*; but he owed to one man 150*l.*; to another 300 guineas; to a third 20 crowns; to a fourth 55*l.* 8*s.* 9½*d.*; and to a fifth 200 guineas: how much will they be losers?

9. A nobleman leaves, between his two children, 37,000*l.*; to the younger he leaves fifteen thousand guineas: what was the fortune of the elder?

10. An apprentice has served of his term of seven years, three years two months, three weeks, four days, seventeen hours: how much longer has he to serve?

11. From a field of 6½ acres, I take out two gardens, one measuring 4½ roods, and the other 2¼ roods, and a piece of ground for coach-house and stables, that measures 1 rood and 12 perches: what will be the size of the field after these pieces are taken away?

12. A plumber puts lead upon the different parts of my house that weighs 5 cwt. 3 qrs.; and he takes away, in return, old lead weighing 2 cwt. 24lb.: what is the difference in the weight between the new and the old lead?

# COMPOUND MULTIPLICATION

Is the method of finding the amount of any given number of different denominations, by repeating it any number of times:

## I. When the given multiplier does not exceed 12.

Rule. *Write the multiplier under the lowest denomination of the multiplicand, multiply every number of the multiplicand by the multiplier, and bring the several products, as they occur, to the next higher denomination. Write down the remainders, and carry the integers to the next product.*

Ex. Multiply £.768 14*s.* $9\frac{1}{2}$*d.* by 9.

| £. | *s.* | *d.* |
|---|---|---|
| 768 | 14 | $9\frac{1}{2}$ |
| | | 9 |
| 6918 | 13 | $1\frac{1}{2}$ |

I multiply first the $\frac{1}{2}$ by 9, but 18 farthings make $4\frac{1}{2}$*d*, I put down the $\frac{1}{2}$, and carry 4; 9 times 9 are 81, and 4 are 85; 85 pence are 7*s*, 1*d*. I put down the 1, and carry 7; 9 times 14 are 126, and 7 are 133 shillings, or 6*l*. 13*s*., put down the 13, and carry 6; 9 times 8 are 72, and 6 are 78; and so of the rest, as in simple Multiplication.

| | £. | *s.* | *d.* | | | | £. | *s.* | *d.* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 3987 | 4 | $6\frac{1}{2}$ | × | 2 | Ex. 2. | 3564 | 10 | $7\frac{1}{2}$ | × | 3 |
| Ex. 3. | 2987 | 3 | $9\frac{3}{4}$ | × | 5 | Ex. 4. | 2648 | 16 | $8\frac{1}{4}$ | × | 5 |
| Ex. 5. | 3487 | 12 | 8 | × | 6 | Ex. 6. | 3498 | 2 | $6\frac{3}{4}$ | × | 7 |
| Ex. 7. | 5694 | 16 | $11\frac{1}{4}$ | × | 8 | Ex. 8. | 2691 | 18 | $11\frac{1}{4}$ | × | 9 |
| Ex. 9. | 3764 | 12 | $8\frac{1}{2}$ | × | 10 | Ex. 10. | 3465 | 15 | $10\frac{1}{2}$ | × | 11 |
| Ex. 11. | 4610 | 15 | 4 | × | 12 | Ex. 12. | 3591 | 19 | $9\frac{1}{2}$ | × | 4 |
| Ex. 13. | 1456 | 16 | 10 | × | 12 | Ex. 14. | 2761 | 14 | 4 | × | 6 |
| Ex. 15. | 3420 | 13 | $5\frac{1}{2}$ | × | 10 | Ex. 16. | 4694 | 12 | 7 | × | 8 |
| Ex. 17. | 2675 | 19 | $3\frac{3}{4}$ | × | 9 | Ex. 18. | 3476 | 17 | $8\frac{3}{4}$ | × | 5 |
| Ex. 19. | 4675 | 17 | $8\frac{1}{2}$ | × | 11 | Ex 20. | 4900 | 0 | $9\frac{1}{2}$ | × | 7 |

## II. When the multiplier is a composite number, and can be resolved into two or more component parts. See p. 21.

Rule. *Multiply by its component parts successively, and the last product will be the answer.*

Ex. Multiply £.374 10*s.* $11\frac{3}{4}$*d.* by 63.

| | £. | *s.* | *d.* | |
|---|---|---|---|---|
| | 374 | 10 | $11\frac{3}{4}$ | × 63 = 9 × 7 |
| | | | 9 | |
| | 3370 | 18 | $9\frac{3}{4}$ | |
| | | | 7 | |
| Answer | 23596 | 11 | $8\frac{1}{4}$ | |

Here 9 × 7 = 63: I therefore multiply by 9, and that product by seven, which gives the true answer.

### EXAMPLES.

| | £. | s. | d. | | | | £. | s. | d. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 456 | 12 | $9\frac{1}{2}$ | × | 15 | Ex. 2. | 436 | 14 | $3\frac{1}{4}$ | × | 16 |
| 3. | 784 | 15 | 4 | × | 18 | 4. | 397 | 16 | 10 | × | 21 |
| 5. | 674 | 18 | $10\frac{3}{4}$ | × | 22 | 6. | 487 | 16 | $9\frac{1}{2}$ | × | 24 |
| 7. | 245 | 10 | 3 | × | 30 | 8. | 376 | 15 | 11 | × | 30 |
| 9. | 246 | 19 | $9\frac{1}{4}$ | × | 35 | 10. | 489 | 18 | $8\frac{1}{4}$ | × | 42 |
| 11. | 397 | 13 | 3 | × | 48 | 12. | 369 | 10 | 2 | × | 54 |
| 13. | 384 | 15 | $10\frac{1}{2}$ | × | 56 | 14. | 965 | 12 | $9\frac{3}{4}$ | × | 63 |
| 15. | 592 | 12 | 9 | × | 66 | 16. | 800 | 9 | 8 | × | 72 |
| 17. | 911 | 13 | $2\frac{3}{4}$ | × | 84 | 18. | 914 | 16 | 4 | × | 77 |
| 19. | 397 | 4 | $4\frac{1}{4}$ | × | 96 | 20. | 374 | 12 | $5\frac{1}{4}$ | × | 108 |
| 21. | 459 | 9 | $9\frac{3}{4}$ | × | 100 | 22. | 279 | 13 | 3 | × | 120 |
| 23. | 376 | 15 | 4 | × | 121 | 24. | 347 | 3 | 9 | × | 132 |
| 25. | 376 | 4 | $9\frac{3}{4}$ | × | 144 | 26. | 567 | 14 | 7 | × | 45 |
| 27. | 897 | 16 | 6 | × | 108 | 28. | 675 | 13 | $3\frac{3}{4}$ | × | 88 |
| 29. | 487 | 19 | $11\frac{3}{4}$ | × | 121 | 30. | 856 | 12 | 2 | × | 132 |

**III.** When the multiplier is not a composite number.

**Rule.** *Take the composite number which is nearest to it, and multiply by the component parts, as before: then add or subtract as many times the first line, as the composite number is less or greater than the given multiplier.*

(1) Multiply £.324 12*s*. $6\frac{1}{2}$*d*. by 394.

| £. | s. | d. |
|---|---|---|
| 324 | 12 | $6\frac{1}{2}$ × 394 = 8 × 7 × 7 + 2. |
| | | 8 |
| 2597 | 0 | 4 |
| | | 7 |
| 18179 | 2 | 4 |
| | | 7 |
| 127253 | 16 | 4 |
| 649 | 5 | 1 |
| 127903 | 1 | 5 |

The nearest composite number is 392 = 8 × 7 × 7; I accordingly multiply by these three figures, and to the product I *add* twice the original sum, which gives the true answer.

### EXAMPLES.

| | £. | s. | d. | | | | £. | s. | d. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 574 | 12 | $6\frac{1}{2}$ | × | 38 | Ex. 2. | 387 | 18 | $7\frac{1}{4}$ | × | 46 |
| 3. | 325 | 8 | 4 | × | 58 | 4. | 222 | 12 | $8\frac{3}{4}$ | × | 68 |
| 5. | 226 | 18 | $9\frac{3}{4}$ | × | 78 | 6. | 136 | 14 | 5 | × | 94 |
| 7. | 300 | 0 | $3\frac{1}{2}$ | × | 273 | 8. | 249 | 12 | $0\frac{1}{2}$ | × | 356 |

N. B. Compound Multiplication for large numbers, may be performed by the rule of Practice, as will be shewn further on.

| | £. | s. | d. | | | £. | s. | d. | |
|---|---|---|---|---|---|---|---|---|---|
| Ex. 9. | 525 | 16 | $0\frac{3}{4}$ | × 412 | Ex. 10. | 326 | 18 | 3 | × 687 |
| 11. | 239 | 9 | 9 | × 740 | 12. | 560 | 0 | $2\frac{1}{4}$ | × 388 |
| 13. | 660 | 15 | $4\frac{1}{4}$ | × 1004 | 14. | 407 | 13 | 1 | × 1325 |
| 15. | 700 | 0 | $0\frac{3}{4}$ | × 1450 | 16. | 110 | 10 | 11 | × 1208 |

## EXAMPLES OF WEIGHTS AND MEASURES.*

### TROY WEIGHT.

| | lb. | oz. | dwt. | gr. | | | lb. | oz. | dwt. | gr. | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 187 | 9 | 12 | 20 | × 4 | Ex. 2. | 256 | 6 | 0 | 22 | × 5 |
| 3. | 169 | 6 | 14 | 17 | × 6 | 4. | 379 | 11 | 9 | 9 | × 7 |
| 5. | 254 | 3 | 3 | 3 | × 9 | 6. | 253 | 11 | 4 | 20 | × 8 |
| 7. | 675 | 4 | 15 | 10 | × 11 | 8. | 375 | 0 | 0 | 17 | × 12 |

### AVOIRDUPOIS WEIGHT.

| | ton. | cwt. | qr. | lb. | oz. | dr. | | | ton. | cwt. | qr. | lb. | oz. | dr. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 12 | 10 | 3 | 14 | 10 | 12 | × 2 | Ex. 2. | 64 | 13 | 2 | 15 | 6 | 8 | × 4 |
| 3. | 25 | 0 | 2 | 8 | 4 | 4 | × 3 | 4. | 46 | 15 | 3 | 12 | 4 | 4 | × 6 |
| 5. | 75 | 13 | 0 | 18 | 6 | 10 | × 8 | 6. | 39 | 12 | 2 | 16 | 10 | 8 | × 9 |

### APOTHECARIES' WEIGHT.

| | lb. | oz. | dr. | scr. | | | lb. | oz. | dr. | scr. | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 456 | 8 | 4 | 1 | × 5 | Ex. 2. | 748 | 5 | 2 | 2 | × 8 |
| 3. | 534 | 7 | 6 | 2 | × 12 | 4. | 378 | 10 | 0 | 1 | × 11 |
| 5. | 321 | 5 | 4 | 1 | × 10 | 6. | 491 | 5 | 7 | 2 | × 9 |

### CLOTH MEASURE.

| | yds. | qr. | n. | | | E.e. | qr. | n. | | | yds. | qr. | n. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 210 | 2 | 1 | × 4 | 2. | 378 | 4 | 3 | × 7 | 3. | 596 | 3 | 1 | × 12 |
| 4. | 357 | 1 | 3 | × 6 | 5. | 738 | 3 | 2 | × 9 | 6. | 876 | 0 | 3 | × 10 |

### LONG MEASURE.

| | yds. | ft. | in. | b.c. | | | lea. | m. | fur. | p. | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 456 | 2 | 10 | 1 | × 5 | Ex. 2. | 379 | 1 | 6 | 20 | × 7 |
| 3. | 369 | 1 | 9 | 2 | × 8 | 4. | 376 | 2 | 5 | 37 | × 9 |
| 5. | 241 | 2 | 11 | 1 | × 10 | 6. | 674 | 2 | 7 | 18 | × 6 |

### LAND MEASURE.

| | acr. | r. | p. | | | acr. | r. | p. | |
|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 456 | 0 | 25 | × 11 | Ex. 2. | 597 | 3 | 12 | × 12 |
| 3. | 371 | 2 | 18 | × 4 | 4. | 271 | 2 | 25 | × 10 |
| 5. | 189 | 3 | 32 | × 8 | 6. | 430 | 0 | 12 | × 8 |

### WINE MEASURE.

| | tuns, | hhd. | gal. | qts. | p. | | | tuns, | hhd. | gal. | qts. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 456 | 3 | 28 | 2 | 1 | × 4 | Ex. 2. | 456 | 3 | 46 | 2 | × 6 |
| 3. | 374 | 2 | 60 | 3 | 1 | × 8 | 4. | 350 | 2 | 25 | 1 | × 2 |
| 5. | 221 | 1 | 4 | 1 | 0 | × 5 | 6. | 124 | 3 | 50 | 3 | × 10 |

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NOTE.

* The compound rules relative to money are those which are chiefly useful.

## BEER MEASURE.

| | butts. | hhd. | gal. | qts. | pt. | | | bar. | fir. | gal. | qt. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 250 | 1 | 20 | 2 | 0 | × 8 | Ex. 2. | 375 | 8 | 6 | 3 | × 6 |
| 3. | 374 | 2 | 7 | 3 | 0 | × 7 | 4. | 676 | 2 | 8 | 2 | × 9 |
| 5. | 487 | 1 | 50 | 0 | 1 | × 8 | 6. | 169 | 3 | 2 | 1 | × 12 |

## CORN AND COAL MEASURE.

| | qr. | bush. | p. | | | chal. | bush. | p. | | | qr. | bush. | p. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 124 | 3 | 2 | × 4 | 2. | 124 | 17 | 3 | × 6 | 3. | 46 | 7 | 2 | × 5 |
| 4. | 91 | 6 | 3 | × 8 | 5. | 178 | 34 | 2 | × 7 | 6. | 87 | 4 | 0 | × 10 |
| 7. | 594 | 3 | 0 | × 9 | 8. | 476 | 10 | 1 | × 11 | 9. | 31 | 0 | 2 | × 12 |

## TIME.

| | w. | d. | hrs. | m. | sec. | | | yrs. | mo. | w. | d. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 73 | 6 | 10 | 40 | 30 | × 5 | Ex. 2. | 594 | 12 | 3 | 4 | × 7 |
| 3. | 36 | 4 | 12 | 15 | 20 | × 9 | 4. | 364 | 8 | 2 | 6 | × 8 |
| 5. | 98 | 5 | 17 | 13 | 55 | × 12 | 6. | 443 | 10 | 3 | 3 | × 11 |

### MISCELLANEOUS EXAMPLES.

1. What cost 12 lb. of tea, at 7*s.* 6*d.* per lb.?

2. What cost 16½ lb. of sugar, at 1*s.* 1½*d.* per lb.?

3. What is the value of 24 yards of Irish, at 3*s.* 6½*d.* per yard?

4. What will 79 bibles come to, at 4*s.* 7½*d.* each?

5. What is the value of 85 gallons of brandy, at 19*s.* 9½*d.* per gallon?

6. What is the weight of 28 ingots of gold, each weighing 6 lb. 7 oz. 15 dwts. 20 gr.

7. What will 157 oxen cost, at 15*l.* 5*s.* 9*d.* each?

8. What is the value of 576 sheep, at 1*l.* 6*s.* 3*d.* each?

9. How much must I pay for 759 chaldron of coals, at 58*s.* 6*d.* per chaldron?

10. What is the value of 199 firkins of ale, at 12*s.* 6*d.* per firkin?

11. What is the value of 245 yards of broad cloth, at 19*s.* 7*d.* per yard?

12. What is the worth of a stack of hay, containing 75 loads, at 3*l.* 19*s.* 9*d.* per load?

13. What is the worth of 12½ lb. of coffee, at 4*s.* 6*d.* per lb.?[*]

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NOTE.

[*] The pupil will recollect, that to multiply by ½ is to divide by 2, and to multiply by ¼ is to divide by 4; therefore this question is worked thus: 4*s.* 6*d*, and the answer is 2*l.* 16*s.* 3*d.*

```
      12
 ---------
  2 14  0
     2  3
 ---------
  2 16  3
```

14. How many pounds sterling are there in 28 purses, each containing 15 guineas, 15 half-guineas, 15 seven-shilling pieces, and 3 crowns?

15. What is the weight of 1000 guineas, each guinea weighing 5 dwts. 9½ gr.?

16. I bought at a sale 47½ dozen of port wine, at 2*l.* 5*s.* 6*d.* per dozen, how much money must I send to pay for it?

17. What is the value of 85 tons of iron, at 18*l.* 17*s.* 9½*d.* per ton?

18. What do 79 packages of goods weigh, supposing that each package weighs 3 cwt. 3 qrs. 15 lb.?

19. If one ounce of gold cost 3*l.* 16*s.* 8*d.*, what is the value of 436½ ounces?

20. What shall I pay annually for 459½ acres of land, at 2*l.* 12*s.* 6*d.* per acre?

21. What is the price of 185 gallons of rum, at 13*s.* 6½*d.* per gall.?

22. If a man spend 2*s.* 6½*d.* per day, how much does he expend in a year?

23. If a bankrupt pay his creditors 12*s.* 9*d.* in the pound, how much money is required to pay debts to the amount of 1250*l.*?

24. How many pounds sterling are there in a million seven-shilling-pieces?

## BILLS OF PARCELS.

### A MERCER'S BILL.

| | | £. | s. | d. | | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|
| 12 | yards of silk, at - - | 0 | 15 | 2 | per yard | | | |
| 114 | Do. of flowered silk, at - | 0 | 18 | 7½ | - - | | | |
| 16 | Do. of velvet, at - - | 1 | 2 | 4 | - - | | | |
| 12 | Do. of satin, at - - | 0 | 13 | 9 | - - | | | |
| 27 | Do. of brocade, at - - | 0 | 15 | 7 | - - | | | |
| 14 | Do. of lustring, at - - | 0 | 6 | 3 | - - | | | |

### A STATIONER'S BILL.

| | | £. | s. | d. | | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|
| 250 | Reams of paper, at - - | 1 | 3 | 6 | per ream | | | |
| 112 | Do. do. at - - | 2 | 4 | 6 | - - | | | |
| 34 | Do. of imperial brown, at | 1 | 15 | 0 | - - | | | |
| 500 | Dutch quills, at - - | 0 | 3 | 9 | per hdd. | | | |
| 2500 | Do. common, at - - | 0 | 2 | 3 | - - | | | |

---

If I have to multiply by ¾, I first divide the original sum by 2, and then that quotient by 2, and add them together: What is the value of 8¾ yards, at 6*s*, 9*d*. per yard?

| | £. | s. | d. |
|---|---|---|---|
| | | | 8¾ |
| | 2 | 14 | 0 |
| | | 3 | 4½ |
| | | 1 | 8¼ |
| Answer - | 2 | 19 | 0¾ |

### A CARPENTER'S BILL.

| | | s. | d. | | £. | s. | d. |
|---|---|---|---|---|---|---|---|
| 65 | cubic feet of oak, at - - | 4 | 3 | per foot | | | |
| 125 | Do. wrough and framed, at - | 5 | 8 | - - | | | |
| 176 | Do. fir framed and moulded, at | 3 | 6 | - - | | | |
| 15 | square shed roofing, at - - | 5 | 6 | per square | | | |
| 8 | Do. hip and valley roofing, at | 8 | 3 | - - | | | |
| 70 | feet water trunk, at - - | 0 | 10 | per foot | | | |
| 364 | feet ovolo wainscot sashes, at - | 0 | 9 | - - | | | |
| 124 | Do do. mahogany, at - - | 1 | 4 | - - | | | |
| 10 | men's labour, for 25 days, at - | 4 | 8 | per day | | | |

### A BRICKLAYER'S BILL.

| | | £. | s. | d. | | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|
| 39 | rod of grey-stock brick-work, | 13 | 13 | 0 | per rod | | | |
| 7 | Do. in party-wall, at - | 7 | 15 | 0 | - - | | | |
| 105 | feet of 18 inch drain, at - | 0 | 3 | 0 | per foot | | | |
| 1050 | Do. of pointing old work, at | 0 | 0 | 5½ | - - | | | |
| 1500 | grey stocks, at - - | 0 | 4 | 6 | per hdd. | | | |
| 125 | pan-tiles, at - - - | 0 | 0 | 1½ | each | | | |
| 45 | hods of mortar, at - - | 0 | 0 | 7 | - - | | | |
| 13 | Do. of tarras, at - - | 0 | 4 | 2 | - - | | | |
| 15 | bricklayers, 25 days, at - | 0 | 4 | 6 | per day | | | |
| 12 | labourers, ditto, at - | 0 | 3 | 0 | - - | | | |
| 66 | load of rubbish carted away, at | | 2 | 6 | per load | | | |

### A SLATER'S BILL.

| | | £. | s. | d. | | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|
| 9 | square of Westmorland slating, at - - - | 2 | 19 | 6 | per squ. | | | |
| 7 | Do. of Welsh ladies, at - | 1 | 17 | 4 | - - | | | |
| 5 | Do. of Welsh countess, at | 1 | 18 | 3 | - - | | | |
| 35 | Do. ripped and rubbish cleared, at - - - | 0 | 2 | 7 | - - | | | |
| 12 | slaters 7 days, at - - | 0 | 4 | 5 | per day | | | |
| 6 | labourers, do. at - - | 0 | 2 | 9 | - - | | | |
| 5050 | clout nails, at - - - | 0 | 0 | 4 | per hdd. | | | |

### PAINTER AND GLAZIER'S BILL.

| | | s. | d. | | £. | s. | d. |
|---|---|---|---|---|---|---|---|
| 1035 | yards of painting 3 times in oil, at | 0 | 7½ | per yard. | | | |
| 565 | Do. do. and sand, at - | 1 | 3 | - - | | | |
| 36 | sash frames, at - - - | 0 | 11 | each | | | |
| 432 | sash squares, at - - - | 0 | 8½ | per doz. | | | |
| 1265 | feet of best Newcastle glass, at | 1 | 7½ | per foot | | | |
| 356 | Do. large size, at - - - | 2 | 1½ | - - | | | |
| 1000 | Do. in lead work, at - - | 1 | 0½ | - - | | | |

# COMPOUND DIVISION

Is the method of finding how often one given number is contained in another of different denominations; or, to divide a given compound number into any proposed number of equal parts.

### I. When the given divisor does not exceed 12.

RULE. *Place the divisor to the left-hand of the dividend. Divide the highest denomination of the dividend by the divisor, and write down the quotient; reduce the remainder, if any, into the next lower denomination, adding to it the number which stands in that place of the dividend, and divide as before, and so proceed to the end.*

Ex. 1695*l.* 14*s.* $4\frac{1}{2}$*d.* ÷ 8.

| | £. | *s.* | *d.* | |
|---|---|---|---|---|
| 8) | 1695 | 14 | $4\frac{1}{2}$ | |
| | 211 | 19 | $3\frac{1}{2}$ | — 2 |
| | | | 8 | |
| Proof | 1695 | 14 | $4\frac{1}{2}$ | |

I divide the pounds, as in simple Division; the remainder is 7, which I reduce to shillings, that is 140, to which I add the 14, and say, the 8's in 154 will go 19 times and 2 over; I put down the 19, and bring the remainder, 2 shillings, into pence, and add to it the 4; the 8's in 28 will go 3 times, and 4 over; reduce the 4 pence to farthings, and take in the $\frac{1}{2}$, and the 8's in the 18 will go twice, and 2 over: thus the answer is 211*l.* 19*s.* $3\frac{1}{2}$*d.* — 2.

*Proof.* The method of proof is by Compound Multiplication.

EXAMPLES.

| | £. | *s.* | *d.* | | | | £. | *s.* | *d.* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 457 | 8 | $9\frac{1}{2}$ | ÷ | 3 | Ex. 2. | 579 | 18 | $4\frac{1}{2}$ | ÷ | 2 |
| 3. | 396 | 18 | $7\frac{1}{4}$ | ÷ | 4 | 4. | 768 | 2 | $6\frac{1}{2}$ | ÷ | 5 |
| 5. | 474 | 12 | 10 | ÷ | 6 | 6. | 934 | 14 | 5 | ÷ | 7 |
| 7. | 897 | 16 | 4 | ÷ | 8 | 8. | 256 | 17 | $10\frac{3}{4}$ | ÷ | 10 |
| 9. | 759 | 0 | 0 | ÷ | 9 | 10. | 694 | 19 | 6 | ÷ | 12 |
| 11. | 101 | 15 | $9\frac{1}{2}$ | ÷ | 11 | 12. | 496 | 0 | 0 | ÷ | 12 |
| 13. | 900 | 0 | 0 | ÷ | 8 | 14. | 500 | 5 | 5 | ÷ | 4 |
| 15. | 800 | 10 | 2 | ÷ | 7 | 16. | 270 | 17 | $7\frac{1}{2}$ | ÷ | 6 |
| 17. | 464 | 2 | $9\frac{3}{4}$ | ÷ | 6 | 18. | 901 | 1 | 1 | ÷ | 9 |

## II. When the divisor is a composite number.

**Rule.** *Divide by the component parts of the divisor successively, and the last quotient will be the answer.*

### Ex. £148. 8*s*. 8½*d*. ÷ 27 = 3 × 9.

$$\begin{array}{l} \begin{array}{rrr} \text{£.} & s. & d. \\ 3)148 & 8 & 8\frac{1}{2} \\ \hline \end{array} \\ \left.\begin{array}{rrrl} 9)49 & 9 & 6\frac{3}{4} & -\ 1 \\ \hline 5 & 9 & 11\frac{1}{4} & -\ 6 \end{array}\right\} = \frac{19}{27} \end{array}$$

Here the division is first by 3, and then by 9, as in the former example; and I find two remainders; I therefore, as in Simple Division, multiply the last remainder 6, by the first divisor 3, and take in the first remainder 1, and then place under it the common divisor 27. The answer is 5*l*. 9*s*. 11¼*d*.$\frac{19}{27}$. See p. 27.

| | £. | s. | d. | | | | £. | s. | d. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 167 | 12 | 6½ | ÷ | 14 | Ex. 2. | 769 | 9 | 8¼ | ÷ | 20 |
| 3. | 339 | 15 | 8¼ | ÷ | 15 | 4. | 594 | 7 | 6 | ÷ | 25 |
| 5. | 486 | 9 | 9 | ÷ | 16 | 6. | 333 | 10 | 10¼ | ÷ | 28 |
| 7. | 987 | 0 | 0¾ | ÷ | 18 | 8. | 498 | 9 | 9½ | ÷ | 32 |
| 9. | 439 | 5 | 6½ | ÷ | 24 | 10. | 596 | 12 | 7¼ | ÷ | 36 |
| 11. | 379 | 18 | 7 | ÷ | 27 | 12. | 465 | 11 | 11 | ÷ | 44 |
| 13. | 487 | 9 | 9¾ | ÷ | 30 | 14. | 564 | 13 | 5½ | ÷ | 49 |
| 15. | 597 | 4 | 6 | ÷ | 33 | 16. | 678 | 6 | 3 | ÷ | 54 |
| 17. | 854 | 3 | 6¾ | ÷ | 42 | 18. | 999 | 9 | 8 | ÷ | 56 |
| 19. | 327 | 14 | 4 | ÷ | 48 | 20. | 564 | 4 | 6 | ÷ | 63 |
| 21. | 387 | 12 | 11 | ÷ | 72 | 22. | 248 | 3 | 0 | ÷ | 84 |
| 23. | 565 | 13 | 3¾ | ÷ | 88 | 24. | 505 | 5 | 5½ | ÷ | 99 |
| 25. | 674 | 18 | 8½ | ÷ | 108 | 26. | 564 | 2 | 2 | ÷ | 120 |
| 27. | 465 | 3 | 3 | ÷ | 132 | 28. | 888 | 8 | 8 | ÷ | 144 |

### When there are three component parts.

### Ex. £1350. 10*s*. 11*d*. ÷ 240 = 5 × 6 × 8.

$$\begin{array}{l} \begin{array}{rrrl} \text{£.} & s. & d. & \\ 5)1350 & 10 & 11 & \\ \hline 6)270 & 2 & 2 & -\ 4 \\ \hline \end{array} \\ \left.\begin{array}{rrrl} 8)45 & 0 & 4\frac{1}{4} & -\ 2 \\ \hline 5 & 12 & 6\frac{1}{2} & -\ 1 \end{array}\right\}\ 8\ \left\{\ \frac{44}{240}\right. \end{array}$$

The division in this example follows the same rule as before, but there are three remainders, to find the true value of which, I multiply the *third* 1, by the *second divisor* 6, and take in the second remainder, 2, that is, once 6 is 6, and 2 are 8; then this product, 8, I multiply by 5, the first divisor, and take in the first remainder, that is, 8 times 5 are 40, and 4 are 44: so that the true answer is 5*l*. 12*s*. 6½*d*.$\frac{44}{240}$, as will be proved in the next page by Long Division. See also p. 27 and 28.

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NOTE.

* The arithmetician will easily perceive that the fractions $\frac{1}{8} + \frac{2}{48} + \frac{4}{240}$ are equal $\frac{30}{240} + \frac{10}{240} + \frac{4}{240} =$ (because now they have each a common denominator) $\frac{44}{240}$.

**Ex. 1. £5527. 10*s*. $6\frac{1}{2}$*d*. ÷ 243. 2. £18568. 12*s*. $1\frac{1}{2}$*d*. ÷ 1296.**

With these two examples the preceptor may form almost any number, by varying the divisors. The several numbers which were before made use of as multipliers, may now be used as divisors.

### III. When the divisor is greater than 12, and not a composite number?

RULE. *The several quotients must be found by the method of Long Division*, (see pp. 28 and 29), *reducing the remainders to the next lower denomination, and taking in those numbers of the dividend which are of the same denomination.*

Ex. Divide £1350. 10*s*. 11*d*. by 240.

```
      £.  s. d.
240) 1350 10 11(5l.
      1200
      ----
       150
        20
      ----
240) 3010(12s.
      2880
      ----
       130
        12
      ----
240) 1571(6d.
      1440
      ----
       131
         4
      ----
 240) 524(½
      480
      ---
       44
```

Having divided the pounds by 240, I find a remainder of 150, this I reduce to shillings, taking in the 10, and divide again; the next remainder is 130, which I bring into pence, and take in the 11, and then divide again: the remainder now is 131, which I bring into farthings, and divide as before; the last remainder is 44, under which I place the divisor thus, $\frac{44}{240}$. The true answer being 5*l*. 12*s*. $6\frac{1}{2}$*d*.$\frac{44}{240}$.

| | £. | s. | d. | | | | £. | s. | d. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 985 | 18 | 9 | ÷ | 19 | Ex. 2 | 1001 | 12 | $11\frac{1}{2}$ | ÷ | 23 |
| 3. | 465 | 16 | $4\frac{1}{2}$ | ÷ | 29 | 4. | 2468 | 13 | $3\frac{1}{4}$ | ÷ | 39 |
| 5. | 565 | 13 | 3 | ÷ | 37 | 6. | 5746 | 9 | 6 | ÷ | 59 |
| 7. | 800 | 8 | $8\frac{1}{2}$ | ÷ | 41 | 8. | 6321 | 3 | $3\frac{3}{4}$ | ÷ | 61 |
| 9. | 987 | 14 | 4 | ÷ | 46 | 10. | 4268 | 12 | 8 | ÷ | 69 |
| 11. | 598 | 12 | 6 | ÷ | 67 | 12. | 4821 | 9 | $7\frac{1}{2}$ | ÷ | 87 |
| 13. | 483 | 6 | 6 | ÷ | 73 | 14. | 5943 | 16 | 6 | ÷ | 97 |
| 15. | 986 | 5 | $9\frac{3}{4}$ | ÷ | 89 | 16. | 3648 | 4 | 6 | ÷ | 97 |
| 17. | 1485 | 19 | 2 | ÷ | 107 | 18. | 4683 | 15 | $5\frac{1}{2}$ | ÷ | 376 |
| 19. | 2690 | 12 | 3 | ÷ | 166 | 20. | 5649 | 9 | 9 | ÷ | 439 |
| 21. | 6259 | 11 | 6 | ÷ | 215 | 22. | 3604 | 10 | 10 | ÷ | 509 |
| 23. | 9654 | 7 | $7\frac{3}{4}$ | ÷ | 649 | 24. | 6534 | 16 | $3\frac{1}{2}$ | ÷ | 606 |
| 25. | 5942 | 17 | $3\frac{1}{4}$ | ÷ | 757 | 26. | 4593 | 12 | 4 | ÷ | 1585 |
| 27. | 4628 | 5 | 9 | ÷ | 1001 | 28. | 5349 | 0 | 0 | ÷ | 4786 |
| 29. | 1456 | 16 | 7 | ÷ | 3761 | 30. | 9504 | 1 | $1\frac{1}{4}$ | ÷ | 8078 |

### IV. When the divisor consists of a number not exceeding 12, with one or more cyphers.

RULE. *Cut off, by a line, as many places in the pounds as there are cyphers in the divisor, and divide by short division; then reduce the remainder to the next lower denomination, as in the last rule.*

Ex. Divide £5645. 14*s*. 4½*d*. by 1200.

12,00)56,45 14 4½  
£. 4 — 845  
20  
12,00)169,14  
*s*. 14—114  
12  
12,00)13,72  
*d*. 1—172  
4  
688

Having cut off two figures in the pounds to answer to the cyphers in the divisor, I divide by 12; the remainder is 845, which I reduce to shillings, and take in the 14, and divide as before: the second remainder is 114, this I multiply by 12, and take in the 4, and divide: the remainder is now 172, which, reduced to farthings, gives 688; this not being equal to the divisor, I set down the answer 4*l*. 14*s*. 1*d*.—$\frac{688}{1200}$. But as it was obvious, from inspection, that the remainder, 172, would not, when reduced, contain the divisor once, the answer might have stood 4*l*. 14*s*. 1*d*.$\frac{172}{1200}$: for the value of $\frac{172}{1200}$*d*. is equal to $\frac{688}{1200}$*qrs*.*

## EXAMPLES OF WEIGHTS AND MEASURES.†

### TROY WEIGHT.

| | lb. | oz. | dwt. | gr. | | | | lb. | oz. | dwt. | gr. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 287 | 9 | 12 | 20 | ÷ | 4 | Ex. 2. | 356 | 6 | 0 | 22 | ÷ | 5 |
| 3. | 269 | 6 | 14 | 7 | ÷ | 6 | 4. | 379 | 11 | 9 | 0 | ÷ | 7 |
| 5. | 354 | 3 | 3 | 3 | ÷ | 9 | 6. | 356 | 11 | 4 | 20 | ÷ | 8 |
| 7. | 675 | 4 | 15 | 10 | ÷ | 11 | 8. | 775 | 0 | 0 | 17 | ÷ | 12 |

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NOTES.

* When the divisor is 1, with any number of cyphers, there is no division: care is only necessary in cutting off the true number of figures in each separate dividend. *Ex.* 869874*l*. 12*s*. 9*d*. ÷ 1000.

£. *s*. *d*.  
1,000)869,879 12 9  
20  
*s*. 17,592  
12  
*d*. 7,113 Answer - 869*l*. 17*s*. 7*d*.$\frac{113}{1000}$.

In all questions of interest, commission, buying and selling of stock, &c. &c. the divisor is 100; of course care must be taken, in cases of those kinds, to cut off the *two* right-hand figures in each part of the dividend.

† The student need not dwell on these varieties.

## AVOIRDUPOIS WEIGHT.

| | tons. | cwt. | qr. | lb. | oz. | dr. | | | tons. | cwt. | qr. | lb. | oz. | dr. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 412 | 10 | 3 | 14 | 10 | 12 | ÷ 2 | Ex. 2. | 664 | 13 | 1 | 12 | 6 | 8 | ÷ 4 |
| 3. | 529 | 0 | 0 | 18 | 6 | 6 | ÷ 3 | 4. | 464 | 0 | 3 | 27 | 0 | 3 | ÷ 6 |
| 5. | 678 | 2 | 2 | 2 | 8 | 2 | ÷ 8 | 6. | 591 | 5 | 0 | 4 | 3 | 12 | ÷ 9 |

## APOTHECARIES' WEIGHT.

| | lb. | oz. | dr. | sc. | | | lb. | oz. | dr. | sc. | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 591 | 8 | 4 | 1 | ÷ 5 | Ex. 2. | 748 | 5 | 7 | 0 | ÷ 8 |
| 3. | 639 | 1 | 1 | 2 | ÷ 12 | 4. | 392 | 10 | 6 | 0 | ÷ 11 |
| 5. | 487 | 2 | 0 | 0 | ÷ 10 | 6. | 421 | 4 | 5 | 1 | ÷ 9 |

## CLOTH MEASURE.

| | yds. | qrs. | n. | | | E.e. | qrs. | n. | |
|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 5210 | 2 | 1 | ÷ 4 | Ex. 2. | 5964 | 3 | 1 | ÷ 11 |
| 3. | 3976 | 1 | 2 | ÷ 6 | 4. | 7645 | 4 | 2 | ÷ 12 |
| 5. | 4721 | 0 | 0 | ÷ 8 | 6. | 3492 | 0 | 3 | ÷ 9 |

## LONG MEASURE.

| | yds. | ft. | in. | b.c. | | | lea. | m. | fur. | p. | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 5946 | 2 | 10 | 1 | ÷ 5 | Ex. 2. | 3795 | 2 | 7 | 30 | ÷ 7 |
| 3. | 4736 | 1 | 8 | 2 | ÷ 8 | 4. | 4965 | 1 | 3 | 18 | ÷ 9 |
| 5. | 2005 | 0 | 11 | 2 | ÷ 10 | 6. | 6743 | 2 | 6 | 4 | ÷ 6 |

## LAND MEASURE.

| | acr. | r. | p. | | | acr. | r. | p. | |
|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 654 | 2 | 24 | ÷ 11 | Ex. 2. | 958 | 3 | 12 | ÷ 12 |
| 3. | 371 | 0 | 18 | ÷ 4 | 4. | 379 | 0 | 25 | ÷ 10 |
| 5. | 891 | 3 | 32 | ÷ 8 | 6. | 496 | 1 | 1 | ÷ 8 |

## WINE MEASURE.

| | tuns, | hhd. | gal. | qts. | pt. | | | tuns, | hhd. | gal. | qts. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 456 | 3 | 27 | 2 | 1 | ÷ 4 | Ex. 2. | 656 | 3 | 31 | 2 | ÷ 6 |
| 3. | 594 | 0 | 30 | 3 | 0 | ÷ 8 | 4. | 391 | 2 | 25 | 1 | ÷ 3 |
| 5. | 271 | 0 | 0 | 2 | 0 | ÷ 6 | 6. | 421 | 3 | 50 | 3 | ÷ 10 |

## BEER MEASURE.

| | butts, | hhd. | gal. | qts. | pt. | | | bar. | fir. | gal. | qts. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 294 | 1 | 12 | 3 | 1 | ÷ 4 | Ex. 2. | 976 | 3 | 6 | 3 | ÷ 6 |
| 3. | 379 | 1 | 7 | 3 | 0 | ÷ 7 | 4. | 224 | 0 | 0 | 1 | ÷ 9 |
| 5. | 469 | 1 | 50 | 0 | 1 | ÷ 8 | 6. | 796 | 2 | 1 | 0 | ÷ 12 |

## CORN AND COAL MEASURE.

| | quar. | bush. | p. | | | chal. | bush. | p. | | | qr. | bush. | p. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 224 | 3 | 2 | ÷ 4 | 2. | 124 | 17 | 3 | ÷ 6 | 3. | 46 | 7 | 2 | ÷ 5 |
| 4. | 991 | 6 | 3 | ÷ 8 | 5. | 387 | 34 | 2 | ÷ 7 | 6. | 37 | 4 | 0 | ÷ 10 |
| 7. | 954 | 3 | 0 | ÷ 9 | 8. | 476 | 10 | 1 | ÷ 11 | 9. | 31 | 0 | 2 | ÷ 12 |

## TIME.

| | w. | days. | h. | min. | sec. | | | yrs. | mo. | w. | d. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 779 | 6 | 20 | 40 | 25 | ÷ 5 | Ex. 2. | 594 | 12 | 2 | 4 | ÷ 7 |
| 3. | 391 | 4 | 12 | 16 | 12 | ÷ 9 | 4. | 954 | 6 | 3 | 5 | ÷ 6 |
| 5. | 913 | 0 | 4 | 0 | 5 | ÷ 12 | 6. | 348 | 10 | 3 | 3 | ÷ 11 |

* * *

## MISCELLANEOUS EXAMPLES.

Ex. 1. If 17 yards of cloth cost 19*l.* 3*s.* 9*d.*, what is it per yard?

2. What is the price of one pound of sugar, if 8lb. cost nine shillings?

3. The expenses of a journey amounting to 97*l.* 9*s.* 6*d.* are to be defrayed by six persons: how much will each have to pay?

4. I have bought 12 gallons of wine for 7*l.* 16*s.* 6*d.*: how much is that per gallon?

5. Twelve boys are to have a guinea and a half divided among them: what will be each boy's share?

6. A hundred and twenty-five sailors have taken 8465*l.* prize-money: how much will each man be entitled to?

7. I have bought 144 pair of stockings for 27*l.*: at what rate can I sell them so as to gain by each pair one shilling?

8. What did I pay a piece for sheep, having bought 75 for 135*l.*?

9. Cheese at 3*l.* 12*s.* 6*d.* per cwt.: how much is that per lb.?

10. If 81 oxen cost 1781*l.* 12*s.* 6*d.*: what is the value of one?

11. If a pipe of wine cost 95*l.*: how much is that a dozen, which contains three gallons?

12. Bought 50 dozen of wine for a hundred guineas: how much is that per bottle?

13. Divide a thousand guineas between 23 people, and see how much it is for each?

14. If 12 pieces of linen cloth contain 250 yards, what is the length of a single piece?

15. How much can I afford to spend a day, a week, and a month, if my income be 500*l.* per annum?

16. If 12 tea-spoons weigh 9 oz. 17 dwt. 12 gr.: what is the weight of each spoon?

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## *Miscellaneous Questions.*

Ex. 1. It is said that Syrius, or the Dog Star, is the nearest of all the fixed stars, and that its distance is computed at 2,200,000,000,000 miles: how many years, (each containing 365 days, 6 hours exactly,) would a cannon ball be in passing from the earth to Syrius, supposing it travelled at the rate of 480 miles per hour?

Ex. 2. The planet Mercury is about thirty-seven millions of miles from the Sun; Venus sixty-eight millions; the Earth ninety-five millions; Mars a hundred and forty-five millions; Jupiter four hundred and ninety-three millions; Saturn nine hundred and eight, and the Herschel one thousand eight hundred millions of miles from the Sun: put these several distances down in figures, and add them together as a sum in Addition.

Ex. 3. How much nearer the Sun is Mercury than Mars; and how much farther is the Herschel than the Earth? See Ex. 2.

Ex. 4. The beautiful planet Venus travels, in its annual journey round the Sun, at the rate of 75,000 miles in an hour: how many miles does she travel in one of her years, or in 228¼ days?

Ex. 5. The Earth travels, in her annual course, at the rate of 68,400 miles in an hour: how many miles therefore do we move in a second?

Ex. 6. There are in the Old Testament 39 books and 929 chapters, and in the New there are 27 books, and 260 chapters: how many books and chapters are there in the Bible?

Ex. 7. There are 23214 verses in the Old Testament, and 7959 in the New: how much therefore do the verses in the former exceed those in the latter?

Ex. 8. There are 592439 words in the Old Testament, and 181258 in the New: how many words are there in the Bible?

Ex. 9. In the Old Testament there are 2,728,100 letters, and in the New there are 838,380: what are the sum and difference of these two numbers?

Ex. 10. There are in the Bible 3,566,480 letters: how long would a person be in counting them, supposing he could count 200 in a minute?

Ex. 11. A printer charges 5¼*d.* for every 1000 letters that he sets up: how many thousand must he set up to earn 1*l.* 15*s.* per week.

Ex. 12. If a printer set up 8500 letters per day, how long would he be in composing the Old Testament, and how long in composing the whole Bible? See Ex. 9 and 10.

Ex. 13. If a printer be desired to set up the Bible in Latin, how much would he earn in the business, at the rate of 5¾*d.* per 1000 letters, supposing there are as many letters in Latin as there are in English?

Ex. 14. If there be as many letters in the Greek Testament as there are in the English, how much would a printer earn in setting it up at 8¾*d.* per thousand?

Ex. 15. The name of JEHOVAH occurs 6855 times in the Old Testament: what proportion therefore does this word bear to all the other words in that book?

Ex. 16. The word *and* occurs in the Bible 46227 times: what proportion does that bear to the other words? See Answer to Ex. 8.

Ex. 17. There are in the northern side of London 126 houses newly built, and unlet, the average rent of which is 85*l.*; and 75 houses at 50*l.* each, and 68 at 30 guineas each: what is the total annual loss of these empty houses to the proprietors?

Ex. 18. There are 1100 hackney coaches in London, each of which earns on an average 18*s.* per day: how much is expended weekly, monthly, and annually, on these vehicles?

Ex. 19. What are 256 reams of paper worth, at 33*s.* 6*d.* per ream?

Ex. 20. Fifty thousand larks have been sold in a single season in London: what did they fetch, supposing they were bought at 1¼*d.* each?

Ex. 21. The circumference of the Earth, in the latitude of London, is 15,120 miles, which is the space we pass over in 24 hours, by the diurnal motion of the earth: how much space do we pass over in a minute?

Ex. 22. Three thousand ounces of gold are imported into England annually: how many pounds and grains are imported in 50 years, at this rate, and what is the value of it at 3*l.* 18*s.* per ounce?

Ex. 23. To work the silver mines in South America, 40,000 negroes are imported annually: how many of these poor creatures have perished in this work during the last century?

Ex. 24. The duty on hops amounted, at 1½*d.* per lb., in a certain year, to 26,357*l.* 9*s.* 9*d.*: how many hops were grown that season?

Ex. 25. The battering ram employed by Titus to demolish the walls of Jerusalem, weighed 100,000 lbs.: how many tons did it contain?

Ex. 26. The copper mines in the island of Anglesey produce 1500 tons annually, and those in Cornwall 4000 tons: what is the value of the whole at 9½*d.* per lb.?

Ex. 27. Mr. Bolton coined 40,000,000 penny-pieces, each weighing an ounce: how many pounds of copper were used for them: how much was the value of these in pounds sterling; and what was gained by this coinage, supposing the copper and expense of coining to be estimated at 12½*d.* per pound?

Ex. 28. In the year 1794, 43,259,746 yards of Irish linen were exported from Ireland: how many packages did they make, each package containing 20 pieces, and each piece 26½ yards? How many shirts would this linen make, at the rate of 3¾ yards per shirt?

Ex. 29. The circumference of the earth is estimated at 24,912 miles: how many barley-corns, (three of which make an inch), would fill up this space?

Ex. 30. The territory of the United States of America contains a million of square miles, or 640 millions of square acres; of these, about 56 millions are water: what number of acres, roods, and perches of land, do the United States contain, and how many inhabitants will they support, allowing to each 4½ acres?

Ex. 31. There are now in England, Scotland, and Wales, 23 millions of acres of waste land: how many farms might these be divided into, allowing to each 75 acres:—and allowing 5 persons to each farm, how many souls would these waste acres support?*

---

NOTE.

* "England and Wales contain 73,334,400 acres, and 8,873,000 inhabitants, Scotland has 1,600,000, and Ireland about 4,250,000 inhabitants. England and Wales have 152 inhabitants for each square mile: Scotland 55, and Ireland 146.

"England contains 72¼ millions of acres: its rents are estimated

Ex. 32. Between the 5th of July, 1810, and the same day, 1811, there were brewed, by 12 brewers only, 939,900 barrels of porter: how much would this quantity sell for when retailed out at 5*d.* per quart?

Ex. 33. How many hours, minutes, and seconds have elapsed since the birth of Christ, which is 1808 years, supposing 365¼ days in a year?

Ex. 34. It is said the Small-pox carries off in London, by death, 50 persons a week? how many (if the disease is not checked) will it destroy in ten years?

Ex. 35. There are about 10,540 tons of cheese imported into London annually: how much do they sell for at the average price of 7½*d.* per lb.

Ex. 36. It is computed that there are 50,000 tons of butter annually consumed in London: what is the expense, supposing the average price 10¾*d.* per lb.?

Ex. 37. About 120,000 persons are employed in the cotton trade: if of these one-fourth are men, who earn 3*s.* 6*d.* a day, and one-fourth women, who earn 1*s.* 1*d.* a day, and the rest children, who earn, each, 3*s.* per week, how much is earned by manual labour in the cotton manufacture every year?

Ex. 38. There have been 20,000,000 lbs. of tea imported in a single year from China: what was the value of it, supposing the average price 4*s.* 9*d.* per lb.?

Ex. 39. The consumption of tobacco in this country is about 169,000 cwt.: how much is expended on this article at 1¼*d.* per oz.?

Ex. 40. Sir R. Phillips, (the publisher of this Arithmetic), caused to be printed, for various books, between the years 1798 and 1808, as many sheets of paper as would, if joined together, extend round the world. Considering each sheet as 21 inches in length, how many reams of paper did he use in that time? and what was the value of the paper, reckoning it at thirty shillings per ream? See Ex. 29.

Ex. 41. The consumption of milk is not less than 6,980,000 gallons annually, in London: how much is expended on this article at 2*d.* per pint?

Ex. 42. In London alone, 630,000 chaldron of coals are burnt: what is the cost at 4¾*d.* per peck?

Ex. 43. The iron rails round St. Paul's cost 11,202*l.* 0*s.* 6*d.*, and they weighed 200 tons and 81 lbs.: what was the iron charged per lb.?

Ex. 44. Westminster-bridge cost 389,500*l.* in building: how soon would it have been paid for by foot passengers, at a halfpenny each, supposing 2420 went over each day?

---

NOTE.

at about 29 millions, but are in reality 50. The stock on the land is estimated at 145 millions; the money in the country 50; the shipping 190: merchandize and manufactures 60:—of the land 13 millions of acres are inclosed, 11 arable; 6¼ waste in England, 1½ in Wales, 14½ in Scotland. For eight millions of inhabitants, the country produces 11 ounces of wheat, and 7½ of meat per day."—See Middleton's Survey of Middlesex. The above estimate was taken in 1793.

# PROPORTION,

OR

# *THE RULE OF THREE.*

This Rule is called the Rule of Three, because, by three numbers being given we find a fourth; and it is either the Rule of Three *Direct* or *Inverse.*

* * *

## THE RULE OF THREE DIRECT

teaches, from three given numbers, to find a fourth, which shall have the same proportion to the second, as the third has to the first; that is, if the *first* be *greater* than the *third*, the *second* will be *greater* than the *fourth;* and, if the *first* be *less* than the *third*, the *second* will be *less* than the *fourth.*

RULE I. STATE THE QUESTION: that is, *place the given numbers so that the first and third may be of the same kind, and the second the same as the number required.*

2. *Bring the first and third numbers into the same denomination, and the second into the lowest denomination mentioned.*

3. *Multiply the second and third numbers together, and divide the product by the first, and the quotient will be the answer,* * *in the same denomination as that in which the second number was left.*

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NOTE.

* If there be a remainder after division, it is always of the same denomination as that of the middle number, and must be brought into the next lower denomination, and then divide by the first number as before.

Ex. 1. What is the value of a pipe of wine, if 5 gallons cost £4. 17s.?

```
gal. £. s.   pipe.
 5 : 4  17 : : 1
     20        2
     --       --
     97        2
              63
             ---
             126
              97
             ---
             882
           1134
           -----
         5)12222
         -------
   2,0)244,4 — 2
       122,4   12
            5)24
            ----
              4 — 4
              4
             ---
            5)16
             ---
             3/4 — 1
```

In stating the question, I first consider what is known, viz. that 5 galls. cost 4*l.* 17*s.*, and the demand is, what a pipe will cost at the same rate? I therefore say, if 5 galls. cost 4*l.* 17*s.*, what cost 1 pipe? for such is the meaning of the statement: 5 gal. : 4*l.* 17*s.* : : 1 pipe. The first term is gallons; I must accordingly bring the third term, or the pipe into gallons: the second, or middle term, is a mixed number; I bring it therefore to its lowest denomination, or shillings, and then multiply the 126 galls. by 97 shillings, and divide the product by the first term 5, and the answer is, 2444 shillings, because the middle number is shillings, and there is a remainder of 2: this I bring into pence, and divide again by 5; there is now 4 remaining, this I bring into farthings, and divide again by 5, and the answer is 2444*s.* $4\frac{3}{4}$*d.* $\frac{1}{5}$, or by bringing the shillings into pounds, 122*l.* 4*s.* $4\frac{3}{4}$*d.* $\frac{1}{5}$.

Ex. 2. If I can buy 27lb. of sugar for £1. 13s. how much an I purchase for thirty guineas?

```
 s.   lb.  guineas.
 13 : 27 : : 30
             21
            ---
            630
             27
           ----
           4410
           1260    lbs.
       33)17010(515
          165
          ---
           51
           33
          ---
          180
          165
          ---
           15
           16     oz.
       33)240(7
          231
          ---
            9
```

In this example, it is known that 1*l.* 13*s.* will purchase 27 lb., these will therefore be the first and second terms; and as the demand is, how many *pounds* can be purchased for 30 guineas, the second, or middle term, must be *pounds*. Having stated the question, I bring the first and third terms into the same denomination, shillings, and then multiply the second and third terms together, and divide by the first; the quotient, or answer, is 515lb.; but there being a remainder of 15, I multiply this by 16, because 16 ounces make a lb.; dividing again, the quotient is 7 oz., with a remainder of 9, which I might bring into drams; but sugar is never bought or sold with such accuracy. The answer is, therefore, 515lb. 7 oz. $\frac{9}{33}$, or by bringing the lbs. into cwts. and qrs., the answer is, 4 cwt. 2 qrs. 11 lb. 7 oz. $\frac{9}{33}$.

Ex. 3. What is the value of 28 ells of cloth, if 4 ells cost 18*s.*?

ells. shill. ells.
4 : 18 : : 28
18
224
28
4)504
2,0)12,6
6,6

All questions of this kind, in which the first and third terms are of the same denomination, and either of them is a *unit*, may be solved by Multiplication only. Thus if 1lb. cost 9*d.*, what cost 28lb.? I multiply the 28 by 9, and the answer is found in pence.

It often happens that the first or third terms may be *reduced to a unit*, by dividing both by a common number, and then the question is solved by Multiplication only. In the example before us, it is instantly seen, that 4 will divide 4 and 28: then the statement is, 1 : 18 : : 7, and 18 multiplied by 7, gives us 126 shillings, or 6*l.* 6*s.* for the answer as before.

Ex. 4. If six yards of cloth cost 24 shillings, what will 81 yards cost?*

Ex. 5. If 8 bushels of coals cost 9*s.* 6*d.*, what is the value of 35 chaldrons?

Ex. 6. If 5lb. of potatoes cost 4*d.*, what is the worth of 1 cwt. on the same terms?

Ex. 7. If 5lb. of potatoes cost 3*d.*, how many can I buy for 40*s.*?†

Ex. 8. If 10 ells of cloth cost 2*l.* 10*s.*, what is the value of 5 pieces, each containing 26 yards?

Ex. 9. If 16 yards of muslin cost 10 guineas, how many ells can I buy for 45*l.*?

Ex. 10. If I can purchase 25 books for 2*l.* 8*s.*, how many can I have for a 10*l.* note?

Ex. 11. If a servant's wages be 25 guineas a-year, how much has he to receive for 87 day's service?

---

NOTES.

* In this example, the first and second terms may be divided by 6, and then it becomes a question in Multiplication: the original statement is, 6 yds. : 24 shil. : : 81 yds.; but, by Division, it is 1 yd. : 4*s.* : : 81 yds., and the answer is, 324 shillings, or 16*l.* 4*s.* The general rule therefore is, "Divide the first, and *either* the second or third term, but *not both*, by some *common measure*, that is, by some number that will divide the two without leaving a remainder, and use the results instead of the original terms.

† In the statement to this example, viz, 3*d.* : 5lb. : : 40 shillings, neither the second nor third terms are divisible by 3: but when the third, or 40 shillings, is reduced to pence, then it is divisible by 3, and the statement, 3*d* : 5lb. : : 480*d.*, or as 1 : 5 : : 160, and the question is answered by multiplying 160 by 5, which gives 800 lbs., or 7 cwt. 0 qrs. 16lb. for the answer.

From these hints the pupil will frequently see that the labour of the operation may be very much shortened.

Ex. 12. If a servant receive three guineas and a half for 20 weeks service: how long ought he to remain in his place for 12 guineas?

Ex. 13. If I pay half-a-crown for 4 lb. of cheese: how much can I have for three crowns and nine-pence?

Ex. 14. If 2lb. 4oz. of honey cost 3*s.* 9*d.*: what is the value of 28lb.?*

Ex. 15. It is estimated that twelve millions of sheep are fed in this country: now, if 11 sheep produce 28lb. of wool every year, how much wool will there be from the whole number?

Ex. 16 If a dozen of wine glasses cost 10*s.* 6*d.*: what is the value of 500?

Ex. 17. If I can buy 3 pair of shoes for 1*l.* 4*s.* 9*d.*: what must I pay for 17 pair?

Ex. 18. If a cwt. of tobacco cost 8 guineas: what is the value of 7,000,000 of lbs?

Ex. 19. If 6 lb. of different kinds of soap cost 5*s.* 9*d.*: what is the value of a cwt. in the same proportion?

Ex. 20. If I pay 39 shillings per cwt. for lead: how much will it cost to cover the roof of a building with lead that weighs 5505 lb.?

Ex. 21. I want to know how much I have to pay for a cistern 990lbs., at the rate of 2*l.* 2*s.* per cwt., the plumber agreeing to allow me at the rate of 1*l.* 14*s.* per cwt. for the old lead, which weighs 458 lb.?

Ex. 22. If four journeymen dyers can earn 5*l.* 12*s.* in six days: how much will their master have to pay them for 305 days, at the same rate, and how much will each man's income be?

Ex. 23. The brazen statue of Apollo, that was erected by Chares, at Rhodes, weighed 720,000 lbs.: how much did the old brass sell for at four guineas per cwt.?

Ex. 24. If I pay 1*l.* 7*s.* for 18 gallons of porter: how much shall I expend in that article in a year, if my family drink nine gallons of it every week?

Ex. 25 If I buy, at the Custom-house sale, 14 gallons of brandy for 18*l.*: how much must I pay, at the same rate, for four hogsheads, each containing 63 gallons?

Ex. 26. If I buy, at Sheffield, 6 razors for 8*s.* 6*d.*: how much shall I have to pay for twelve dozen, at the same rate? And, how much can I sell them for, so as to gain by the bargain 2½*d.* each razor?

Ex. 27. In building an out house 5050 bricks have been used: how much do they come to at 4*s.* 6*d.* per hundred?

---

NOTE.

* The operation in this example may be shortened thus: the statement is, 2 lb. 4oz. : 3*s.* 9*d.* : : 28 lb. Instead of bringing the first term nto ounces, I bring it into quarters of a lb., by multiplying by 4, and aking in the 4 oz. as *one*, then the statement becomes as 9 qrs : 45*d.* : : 12 qrs.; but the first and second terms are divisible by 9, and the tatement is, 1 : 5 : : 112, and the answer is, 5 × 112, or 560 pence, r 2*l.* 6*s.* 8*d.*

Ex. 28. It requires 32 bricks to pave 9 square feet: how many bricks will be wanted for the pavement of a cellar 24 feet long, and 19 feet wide?*

Ex. 29. It requires 144 Dutch clinkers to pave 9 square feet: how many will be wanted for a court 35 feet long, and 29 feet wide, and how much will they come to at *5s. 6d.* per hundred?

Ex. 30. It requires sixty persons six days to manufacture a pack of wool into cloth: how much wool will they work up in a year, supposing they work 5 days in each week?

Ex. 31. Six children of different ages will earn in five days, at spinning wool, *5s. 9d.*, and the mother will earn *1s. 4d.* per day: how much will they all earn in a year, allowing that they work, one week with another, 5½ days per week?

Ex. 32. At some large iron founderies, they can run off 6000 lbs. of iron in twenty-four hours: how many tons weight will they cast in a year, allowing them to work 298 days, and 16 hours each day?

Ex. 33. By a patent machine for making combs, the teeth of two combs can be cut in three minutes: how many can be manufactured in 28 days, if the machine is worked at the rate of eight hours a day?

Ex. 34. What is the price of a carpet that measures 15 feet each way, at *7s. 6d.* for every 9 feet? †

Ex. 35. If 13 cwt. of fine Lisbon sugar cost me *58l. 10s.*: how much must I pay for 15 casks of the same, each cask weighing 4cwt. 2qr. 12lb.?

Ex. 36. How much hay can I purchase for 155 guineas, at *3l. 10s.* per load?

Ex. 37. If candles sell for *11s. 6d.* per dozen: how much will 250lb. cost?

Ex. 38. If mould candles cost *12s. 6d.* per dozen: how many pounds can I purchase for fifty guineas?

Ex. 39. The best mottled soap is bought at *4l. 6s.* per cwt.: for how much must it be sold per lb., so as to allow a profit of one penny on each pound?

Ex. 40. If I buy 6½ yards of Irish cloth for *1l. 3s. 10d.*: how much must I pay for eight pieces, each containing 26 yards?

Ex. 41. If 40 yards of Irish cloth will make 12 shirts: how many may be made out of 4 pieces, each containing 26 yards?

Ex. 42. If 12 gallons of brandy pay *3l. 18s.* duty at the Custom-house: how much will be paid for 65,873 gallons, which were imported last week?

Ex. 43. The average price of sugar, exclusive of duty was, Aug. 21, 1805, *2l. 11s. 9¼d.* per cwt.: I demand the value of the 9,999,360 lbs. that were imported into London the preceding week?

---

NOTES.

* To find the number of square feet in the cellar, multiply the length by the breadth.

† The size of a carpet, or the number of square feet that it contains, is found by multiplying the 15 by itself, thus $15 \times 15 = 225$.

Ex. 44. The average price of tallow was, on the same day, 4*s.* 2*d.* per stone of 8lb.: what is the worth of 276 tons, imported the preceding week?

Ex. 45. What will 31218 gallons of Port wine, imported last week, sell for, at 2*l.* 18*s.* 6*d.* per dozen, supposing each dozen to contain 3 gallons?

Ex. 46. What is the value of 115 seal-skins, at 3*s.* 6*d.* per lb. supposing the skins to weigh, one with the other, 9 ounces each?

Ex. 47. Ox hides, fit for tanning, were sold on Friday, the 23d of August, at 3*s.* 9*d.* per stone: what did 50 of them fetch, supposing each weighed 96 lb.?

Ex. 48. How much brown Holland can I buy for ten guineas, if I pay 5*s.* 9*d.* for four yards and a quarter?

Ex. 49. Suppose a person save out of his income 5*s.* 6*d.* per week: how long will he be in laying by 100*l.*?

Ex. 50. I want to know the height of a tree, by means of the length, of its shadow; I set up a straight stick that measures, above the ground 3 feet 4 inches; the shadow of this is 5 feet 2 inches, and the shadow of the tree, at the same moment I find to be 79 feet 10 inches? *

Ex. 51. What is the height of a steeple, whose shadow is 148 feet 4 inches, when a shadow 5 feet 3 inches long is projected from a staff 6 feet 4 inches?

Ex. 52. If I pay 4*s.* 9*d.* for a hundred of pens: how many shall I get for 100*l.*?

Ex. 53. If my income is 450*l.* per ann.: how much may I spend in 73 days, supposing I mean to lay by 50 guineas at the year's end?

Ex. 54. What is the value of 57 yards of muslin, at the rate of 12*s.* 6*d.* per ell?

Ex. 55. How many ells of Holland can be bought for 35*l.*, at the rate of 6*s.* 6*d.* per yard?

Ex. 56. I have a tankard that weighs 2lb. 3oz. that cost 10*l.* 2*s.* 6*d.*: how much, at the same rate, will a service of plate cost that weighs 125lb. 9 oz.?

Ex. 57. How much must be paid for 130,544 bushels of wheat, at 4*l.* 8*s.* per quarter? †

Ex. 58. What was paid for 22,275 bushels of flour, at 4*l.* 7*s.* the sack of five bushels?

Ex. 59. What is the value of 6 casks of raisins, each weighing 3 cwt. 2 qrs. 14 lb. at 5*l.* 10*s.* 6*d.* per cwt.?

Ex. 60. How much must I give for a gold snuff-box that weighs 8 oz. 9 dwts., at the rate of 4*l.* 3*s.* 9*d*, per oz.?

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NOTES.

* The question is, if 5 feet 2 inches of shadow is cast from a stick of 3 feet 4 inches, what will be the length of an object whose shadow is 79 feet 10 inches?

† This was the price of wheat, and the quantity sold at Mark-lane, from the 5th to the 10th of August, 1805; the same may be said of the flour in the next question.

Ex. 61. What must I pay to the property-tax for 586*l.* per annum, at the rate of 6¼ per cent.?

Ex. 62. A bankrupt has but 1020*l.* to pay debts to the amount of 3225*l.*: how much can he pay in the pound?

Ex. 63. A merchant failing, his assignees find effects and good debts to the amount of 3335*l.*; but he owes 4225*l.*; the expenses attending his bankruptcy will be 212*l.* 9*s.*: how much, therefore, will he pay in the pound?

Ex. 64. An honest tradesman, through unforeseen misfortunes, is obliged to call his creditors together; he finds his debts to be 4326*l.* and he can pay 14*s.* 6*d.* in the pound: how much has he still left?

Ex. 65. Hops are remarkably cheap, and I have 100*l.* to spare: how many can I purchase at 3*l.* 15*s.* 6*d.* per cwt.?

Ex. 66. If 12lbs. of tea are worth 9*l.* 6*s.*: how much of the same sort can I purchase for 70 guineas?

Ex. 67. What must I pay for the carriage by the canal, from Manchester to Etruria, of 705 tons, 5 cwt. of goods, at 15*s.* per ton; and what is the difference between this and the land-carriage, at 2*l.* 15*s.* per ton?

Ex. 68. What weight of goods can be carried on the canal between Manchester and Birmingham for 85*l.* at the rate of 1*l.* 10*s.* per ton: and how much can be carried the same distance, by land-carriage, at 5*l.* per ton?

Ex. 69. The clothing of a regiment of 760 men comes to 3050*l.*: how much is that per man?

Ex. 70. What may a man spend per week, whose income is 2000*l.* per annum, supposing 52 weeks in a year?

Ex. 71. If, by selling fine Irish cloth at 5*s.* per ell, I gain 8*l* per cent., what will be the rate of my profits if I sell it at 6*s.* 3*d.* per ell?

Ex. 72. If sugar, that cost 9*d.* per lb., be sold at 3lb. for 2*s.* 9*d.*, what is the profit per cent?

Ex. 73. I purchased 5 pieces of Holland, each containing 36 yards, at 4*s.* 9*d.* per yard: how much shall I gain by selling it at 6*s.* 2*d.* per ell? *

Ex. 74. Two persons part at the same time from the same place, the one travels north 24 miles a day, and the other 21 miles a day south: when will they be 1000 miles asunder? †

Ex. 75. If a pack of wool weighs 3 cwt. 2 qrs. 7 lb., what is it worth at 21*s.* 6*d.* per tod of 14 lbs.?

Ex. 76. The rents of a parish amount to 1750*l.*, and a rate for the poor is wanted of 65*l.* 7*s.* 6*d.*: what is that per pound? ‡

---

NOTES.

* The proper method of working this example by the Rule of Three is, by *two* statings; by the first we find how much the Holland cost, and by the second what it sold for; deduct the former from the latter, and the result is the answer.

† Here it is evident, that the distance they both travel in one day must make the first term in the question, and we say, if 45 miles are passed in one day: how many days will it take to pass 1000 miles?

‡ The Author advises the student to pass over the two next rules for the present and pass at once to Vulgar and Decimal Fractions

# THE RULE OF THREE INVERSE.

This rule, like the last, teaches, from three given numbers, to find a fourth, which fourth number shall bear the same proportion to the second, as the first has to the third. Thus, if the question be, If 10 men can mow a certain field in 6 days, how soon can it be done by 20 men? The answer will evidently be in 3 days, because double the number of men will certainly do the same work in half the time: the proportion will therefore stand, 10 men : 6 days : : 20 men : 3 days; and 3 bears the same proportion to 6, that 10 does to 20; that is, the fourth number bears the same proportion to the second, that the first does to the third.

Rule. *State the question, and, when necessary, reduce the terms as before. Multiply the first and second terms together, and divide the product by the third term; the quotient is the answer in the same denomination as the second term;* thus in the foregoing example, $\frac{10 \times 6}{20} = 3$ days.

Ex. 1. If 15 reapers can cut down a field of corn in 4 days, in how long time will the same work be performed by 40 men?

15 : 4 : : 40
4
4,0)6,0
1½ day.

The answer is a day and a half, and the reason of the thing is self-evident, because 40 men must do the same job in much less time than 15 men.*

---

NOTE.

* The best method of proving questions in this rule is, to reverse the operation. Thus, in this example, I now say, if 40 men : 1½ day : : 15 men, and the answer is 4 days, which shews the truth of the former operation.

The precepor will therefore direct his pupil to prove the truth of each example, which, in fact, is the same thing as giving double the number Thus, in the second example, I first consider what is known, viz.

Ex. 2. If the penny loaf weighs 4 ounces when flour is 4*s.* per peck, how much must it weigh when flour is 5*s.* 4*d.* per peck?

Ex. 3. A person lent me 240*l.* for 8 months: in return for his kindness, how much ought I to lend him for eighteen months?

Ex. 4. How many men must be employed to finish a canal in 12 days, which 5 could perform in six weeks, or 36 days?

Ex. 5. If 24 pioneers can make a trench in 12 days, what length of time would the same work employ 9 men?

Ex. 6. The floor of a chapel 96 feet in length and 70 feet in breadth, is to be covered with matting 2 feet six inches broad: how many yards will it require?

Ex. 7. If a person travel 12 hours a day, and finish his journey in three weeks: how long would the same journey take him, if he travelled only 9 hours a day at the same rate?

Ex. 8. If the town and garrison of Bhurtpoor, containing 22,400 persons, have provisions to last three weeks, how many inhabitants must Holkar send away, so as to make the same provisions last 7 weeks, which is as long as General Lake can carry on the siege?[*]

Ex. 9. If a besieged garrison have 4 months provisions, at the rate of 18 ounces per man per day: how long will they be able to hold out, if each man is allowed only 12 ounces per day?

Ex. 10. If there are in a garrison provisions sufficient for 1500 men 10 weeks, which, on account of the rains, is seven weeks longer than the siege can last: how many soldiers may be brought in to defend the place for three weeks, without lessening the quantity of food to any individual?[†]

Ex. 11. If 9 plasterers can finish the inside of a chapel in 10 days: how long will it take 4 men, supposing the other 5 sent away to a new job?

Ex. 12. If $3\frac{1}{2}$ yards of broad cloth, $1\frac{3}{4}$ wide, will make a suit of clothes: how much will be necessary of cloth only $\frac{3}{4}$ wide?

---

NOTES.

that when flour is 4*s.* a peck, the loaf weighs 4 ounces; these are the first and second terms, and the question is, *how much* it will weigh when flour is 5*s.* 4*d.* per peck. The answer, or unknown quantity, is weight. I therefore state it thus: 4*s.* : 4 oz. : : 5*s.* 4*d.* : 3 oz. = the answer. To prove the truth of it I say, if I have 3 oz. of bread when flour is 5*s.* 4*d.* per peck, how much shall I have when it is 4*s.* per peck: thus 5*s.* 4*d.* : 3 oz. : : 4*s.* and I find the answer is 4 ounces.

[*] The answer to this question is the number of people to be sent away; therefore, when I have found how many the provisions will support for 7 weeks, I subtract this number from that given, and the remainder shews what number are to be dismissed, which in this case will be found to be 12,800.

[†] Having found the number of men that may be supported three weeks, subtract from that number the 1500 already in the garrison, and the remainder is the true answer.

Ex. 13. If 52 clerks in the Bank are sufficient to make up the books in a certain office in 15 days, how many clerks would be required to do the same work in 6 days?

Ex. 14. If the carriage of 15½ cwt., for 60 miles, came to 7*s*. 9*d*: how far can I have carried 3¾ cwt. for the same sum?

Ex. 15. The apartment in which the late Duke of Gloucester lay in state previously to his funeral, was 50 feet long, 40 feet broad, and 24 feet in height: how many yards of black cloth, 1½ yards wide, were used in covering the walls, and how much did it cost at 18*s*. per yard.*

Ex. 16. If 12 inches in length, and 12 inches in breadth, make a square foot: what length of board, 8 inches broad, will be equal to the same measure?

Ex. 17. If 220 yards in length, and 22 in breadth, make an acre: what must be the breadth when the length is 121 yards?

Ex. 18. If 5 horses can be maintained when oats are 18*s*. per quarter: how many can be supported at the same cost, when they are 30 shillings per quarter?

Ex. 19. If 250*l*. gain 12*l*. 10*s*. at interest, in 12 months. what principal will gain an equal sum in 5 months?

Ex. 20. There are two rooms, in the floors of which there are an equal number of square feet; the length of the one is 50 feet, and its breadth is 42; but the breadth of the other is 48 feet: what is its length?

Ex. 21. The cock to a large water-tub will empty it in 36 minutes: how many such cocks will empty it in 4½ minutes?

Ex. 22. The sides of a room are found to measure 138 feet in length, and the height is 14 feet 6 inches: how much paper, 2 feet 3 inches wide, will cover it; and what is the value of it at 9*d*. per yard?

Ex. 23. If 50 cows can be kept in a field 17 days: how long will the same pasture feed 70 cows?

Ex. 24. How many Venetian ducats, at 4*s*. 4*d*. each, must I take in payment for 560 English crowns?

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NOTE.

* As there are four sides to the room, add the length to the breadth, and multiply by 2, which gives the length of the sides: then say, as the height of the room is to the length of the sides found, so is the breadth of the cloth to the quantity used. The value of the cloth is found afterwards by the Rule of Three Direct.

# THE DOUBLE RULE OF THREE.

The Double Rule of Three teaches, from five given numbers to find a sixth. Three of the numbers contain the suppositions, and the remaining two are terms of demand.

Rule (1.) Put the terms of supposition one above another in the *first* place, except that which is of the same nature with the term sought, which put in the *second* place.

(2.) Place the terms of demand one above another in the third place, in the same order as the terms of the supposition were put in the first place.

(3.) The first and third term in every row will be of the same nature, and must be reduced to one denomination; and the middle term must be brought to the lowest denomination mentioned.

(4.) Examine each stating separately, using the middle term as common to both, in order to know if the proportion be *direct* or *inverse*. When it is *direct* mark the first term with an asterisk, and when it is *inverse*, mark the third term with an asterisk.

(5.) Multiply the numbers together which are marked for a divisor, and those which are not marked for a dividend, and the quotient will be the answer.

Ex. 1. If 12 persons spend £.160 in 4 months: how much will 32 persons expend in 8 months?

persons. £. persons.
*12 : 160 : : 32
months. months.
* 4 : : : 8
or,
12 × 4 : 160 : : 32 × 8

The terms of *supposition* are, that 12 persons spend, in 4 months, 160*l.*; the 12 and 4 are therefore the first terms; and as the answer will be in money, the 160*l.* is the middle term. The terms of *demand* are, How much 32 persons will expend in 8 months; these are accordingly the third terms It is evident, from inspection, that both the statings in this example. are in direct proportion, because the fourth terms will be greater than the second, that is, 32 persons will expend more than 12, and 8 months expenditure will be greater than 4 months. *

$$\frac{32 \times 8 \times 160}{12 \times 4} = 853l.\ 6s.\ 8d.$$

Ex. 2. If a garrison of 600 men have provisions for 5 weeks, allowing each man 12 ounces per day: how many men can be maintained 10 weeks by the same quantity, if each man is limited to 8 ounces a day?

weeks. men. weeks.
5 : 600 :: 10*
oz. oz.
12 : . : 8*
or,
5 × 12 : 600 :: 10 × 8

In this example the statings are *inverse*, for in the *first*, if the same quantity of provisions is to serve 10 weeks, there must be a smaller number of men: in the *second*, when each man's proportion is reduced from 12 to 8 ounces the same provisions will maintain a greater number.

$$\frac{5 \times 12 \times 600}{10 \times 8} = 450 \text{ men, the Answer.}$$

Ex. 3. If 15 pecks of wheat will last a family of 9 persons 22 days: in how many days will six persons consume 20 pecks?

pecks. days. pecks.
*15 : 22 :: 20
persons. persons.
9 : :: 6*
or,

$$\frac{9 \times 22 \times 20}{15 \times 6} = 44 \text{ days.}$$

In the first stating of this example, the proportion is direct, because a greater quantity of wheat will last a greater number of days. In the second stating the proportion is *inverse*, because a smaller number of people will require more days to eat the same quantity of wheat. The divisors are therefore 15 and 6, and the answer is 44 days.

Ex. 4. If 6 pioneers can dig a ditch 34 yards long in 10 days: how many yards may be dug by 20 men in 15 days?

Ex. 5. If 1050 soldiers consume 250 quarters of corn in 6 months: how many soldiers will 960 quarters serve 4 months?

---

NOTE.

* Examples in the Double Rule of Three may be worked by two statements in the Single method; and this will be a good method of proving the truth of the sums. Thus, in the foregoing example, I say,

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | men. | | £. | s. | d. | | men. | | £. | s. | d. |
| 1st Statement, | 12 | : | 160 | 0 | 0 | :: | 32 | : | 426 | 13 | 4 |
| | months. | | | | | | months. | | | | |
| 2d Statement, | 4 | : | 426 | 13 | 4 | :: | 8 | : | 853 | 6 | 8 |

Thus the fourth term in the proportion, found by the first stating, becomes the middle term of the second stating.

Ex. 6. If a cask of beer last 8 persons 14 days: how many casks will serve 2 persons 365 days?

Ex. 7. If 10 men in six weeks earn 90*l.*: how many weeks must 15 men work to earn 150*l.*?

Ex. 8. Suppose I walk 66 miles in 4 days, of eight hours each day: how many days, of 14 hours each, shall I be in going from London to York, or 196 miles?

Ex. 9. If 3 boats take 6000 herrings in 8 days: how long will 600 boats be in taking 20,000 barrels, each containing 700 herrings? *

Ex. 10. If, against a general mourning, 6 tailors can make 10 suits of clothes in 4 days: how many suits can 600 men make in the 7 days which occur before the mourning is wanted?

Ex. 11. If 12 mantua-makers can make 27 mourning dresses in 4 days: how many persons would be required to make 189 dresses in 8 days?

Ex. 12. If 3000 copies of a History of America, each containing 11 sheets, require 66 reams of paper: how much paper will 5000 take, if the work be extended to $12\frac{1}{2}$ sheets?

Ex. 13. As 12 inches in length, 12 in breadth, and 12 in thickness, make a solid foot: what length of plank, which is 7 inches broad and 3 inches thick, will make the same? †

Ex. 14. If 450 tiles, each 12 inches square, will pave my cellar: how many tiles must I have, if the tiles are 9 inches long and 8 broad?

Ex. 15. If the expense of 3 persons on a tour for five months be 123*l.* 8*s.*: what will 2 persons spend in 9 months?

Ex. 16. If 12 ounces of wool make $2\frac{1}{2}$ yards of very fine cloth, 6 quarters wide: how much wool would be required to 150 yards, 4 quarters broad?

Ex. 17. If 300*l.* gain 9 per cent interest in a year, in what time will 900*l.* gain 150*l.*?

Ex. 18. If an iron bar 4 feet long, 3 inches broad, and $1\frac{1}{2}$ inch thick, weigh 36 lbs.: how much will a bar weigh that is 6 feet long, 4 inches broad, and 2 inches thick? ‡

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NOTES.

* It is asserted, that this number of herrings have been caught in a single season in Loch Fyne, a salt lake communicating with the sea. The fact may be readily credited, when it is added, that at one single haul 12,000 makarel were drawn on shore at Sidmouth, a few years since.

† The statements in this example are,

$$\begin{array}{lllll} \text{broad,} & \text{long.} & & \text{broad.} \\ 12 & : & 12 & :: & 7^* \\ \text{thick.} & & & & \text{thick.} \\ 12 & : & & :: & 3^* \end{array}$$

It is evident these are both inverse proportions, because 7 in breadth will require more length than 12 inches to make an equal surface. The same may be said of the 3 inches in thickness.

‡ The statements are direct in this example.

$$\text{thus, } \left.\begin{array}{lllll} {}^*\;4 & : & 36 & :: & 6 \\ {}^*3 \times 1\frac{1}{2} & & & & 4 \times 2 \end{array}\right\}\text{or, } \frac{36 \times 6 \times 4 \times 2}{4 \times 3 \times 1\frac{1}{2}} = 96 \text{ lb.}$$

## *Miscellaneous Questions on all the foregoing Rules.*

Ex. 1. What three numbers are those, the first of which is 105, the second $\frac{2}{3}$ds of the first, and the third 67 less than the first and second together ?*

Ex. 2. A gentleman left his eldest daughter 1000 guineas more than the youngest, and to three other daughters he left 7000 guineas between them, which was equal to the sum left to the youngest and eldest together: what was each child's fortune ?

Ex. 3. What is the difference in value between five times five and twenty guineas, and five times twenty-five guineas ?

Ex. 4. What was the value of a prize taken by 25 sailors, besides officers, so that each sailor received 19*l.* 9*s.* 9*d.*, and the officers received as much as the sailors ?

Ex. 5. A prize valued at 13,177*l.* 10*s.*, after the officers have had their share, is to be divided among 525 sailors : what would each man have to take ?

Ex. 6. What is a fourth proportional to the numbers 6, 9, and 24 ?†

Ex. 7. What is the value of 4 packs of cloth, each pack containing 4 parcels, each parcel 10 pieces, and each piece 26 yards, at the rate of 2*l.* 8*s.* for 3 yards ?

Ex. 8. How many yards of paper, 3 quarters wide, will be sufficient for a room 48 yards round, and four yards high: and what is the value of the paper, at the rate of 18*s.* per piece of 24 yards ?

Ez. 9. If 100*l.* gain 5*l.* in 12months, what will 75*l.* gain in nine months ?

Ex. 10. If 48 cannon consume, in 3 days, 288 barrels of powder, how much will be spent in 15 days, when 144 cannon are to be supplied ?

Ex. 11. Fifteen people joined to purchase a lottery ticket, for which they gave three shillings less than eighteen guineas: if it came up a prize of 30,000 guineas, what did each man receive, and what was his gain ?

Ex. 12. A tobacconist bought two parcels of tobacco, which weighed 9 cwt. 2 qrs., for a hundred guineas, the difference of the

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NOTES.

* To find $\frac{2}{3}$ds of any number, multiply the number by two, and divide by three: thus, $\frac{2}{3}$ds of a guinea is $\frac{21 \times 2}{3} = 14$ shillings.

† A fourth proportional is found, as is evident, rule, p. 92, by multiplying the second and third numbers together, and dividing by the first, ; thus a fourth proportional to the numbers 8, 12, and 16, is $\frac{12 \times 16}{8} = 24$, or 8 : 12 : : 16 : 24.

parcels in weight was 3 qrs. 12lb., and in value eight guineas: what was their weight and values?

Ex. 13. The clothing of 100 charity children came to 211*l.*, of which 13 5*l.* was expended on 60 boys: what was paid for the 40 girls, and how much did the cloaths of each child cost?

Ex. 14. A great grazier left to his four sons 220 oxen and 1200 sheep: I demand the value of each son's legacy, supposing the oxen worth 18 guineas each, and the sheep 39 shillings each?

Ex. 15. What number is that which, multiplied by 384, will give a product of 3,013,248?

Ex. 16. What is gained by the sale of 436 yards of broad cloth, that was bought at the rate of 5 yards for 4*l.* 1*s.*, and sold at the rate of 6*l.* 4*s.* for 6 yards?

Ex. 17. If 9 printers can set up the New Testament in 22 days, in what time could it be done if 15 were employed?

Ex. 18. If 8 pressmen will, on the average, earn 14*l.* 8*s.* in six days, how much can 15 men earn in 27 days?

Ex. 19. When the quotient is 1083, and the divisor 555, what is the dividend, if there be a remainder of 79?

Ex. 20. There are about 800 *new* books and pamphlets published every year in London, the value for one copy of each work is estimated at 240*l.*: what is the average value of each book?

Ex. 21. If each of the 800 publications contain on an average $15\frac{1}{2}$ sheets, and of each there be 1250 copies printed, I demand how much paper is used in the business, and its value, allowing to each ream of paper 500 sheets, and the price of it at 25*s.* 6*d.* per ream?*

Ex. 22. The silk mill at Derby winds off 73,726 yards of silk every time the great wheel goes round, which is thrice in a minute: how many yards will it wind in a year, allowing that it works every day, except Sundays, 15 hours, and how many skeins will be made, supposing 960 yards go to the skein?

Ex. 23. In the partition of some waste lands in the west of England, A had $59\frac{1}{2}$ acres, B $76\frac{1}{4}$ acres, C 110 acr. 2 r. 12 per., D 15 acres, and E 39 acr. 0 r. 12 per., but these, taken together, were but one-fifth of the whole: how many acres were divided, and what was the value of the whole, supposing each acre worth 15*l.* 9*s.* 6*d.*?

Ex. 24. An island in the West Indies contains 42 parishes, and every parish 76 houses, and each house at the rate of $5\frac{1}{2}$ white persons; besides these, there were 65 negroes to each of 54 plantations: how many people were there on the whole island?

Ex. 25. In the club mentioned in the Spectator (No. 9), there were 15 persons, weighing together 3 tons: how many pounds, ounces, and drams, Avoirdupois, did each man weigh?

Ex. 26. The British possessions in Hindostan contain 212,406 square miles, and the population is estimated at fourteen millions: how many inhabitants are there to a square mile?

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NOTE.

* This, and the preceding question, do not include the new editions of books.

Ex. 27. If 9 lb. of tea cost 3*l.* 7*s.* 7*d.*, what is the worth of four chests, each weighing one hundred and a half?

Ex. 28. What shall I give for a farm containing 256 acres and a half, for which I am to pay at the rate of 95 guineas for three acres?

Ex. 29. What will it cost a young man to come into a farm, for the lease of which he is to pay 1000 guineas; for 22 horses he is to pay at the rate of 18 guineas each; for crops in the ground 354*l.*; for 210 bushels of wheat he is to pay 4*l.* 10*s.* per quarter; the household furniture is appraised to him at 298 guineas, and for farming utensils of all kinds he is to pay 196*l.*?

Ex. 30. The revenue collected in Hindostan, by the British, is reckoned at 3,400,000*l.*, how much is that from each inhabitant, supposing they amount to 14 millions?

Ex. 31. The number of nergoes in Jamaica is estimated at 250,000, and of whites at 20,000, how many slaves are there to a single white man, and what do the planters reckon their property worth in the article of slaves only, supposing each to be worth 93 guineas?

Ex. 32. The population in the United States is estimated at six millions and a half, and the number of slaves still existing in that free country is reckoned to be 697,697, how many free people are there to one slave?

Ex. 33. The extent of China Proper is equal to 1,397,999 square miles, and the population is estimated at 333,000,000, how many inhabitants are there to a square mile?

Ex. 34. In Spain each person pays 10 shillings to government for protection; in France, under the old government, each paid 20*s.* for protection; and in England we pay full three guineas each for the same advantages, how much is the revenue of the three governments, supposing the population of Spain to be 10¼ millions; of France, at the period referred to, 25 millions; and of England and Wales 9,343,173?

Ex. 35. The population of London, Westminster, and Southwark, is 864,865, that of Paris 547,756, how much does the population of London exceed that of Paris?

Ex. 36. How many minutes and seconds have elapsed since the birth of Christ, or 1808 years?

Ex. 37. How long would it require to count five hundred millions sterling, supposing a person were to reckon 150*l.* in a minute, and were to be employed 10 hours each day, and six days a week, till he had finished the job?

Ex. 38. How many barley-corns will reach 'round the earth, supposing the length to be 25,200 miles?

Ex. 39. How many seven-shilling pieces are there in a thousand pounds?

Ex. 40. A French franc is worth 10*d.*, how many francs are there in 100*l.*?

Ex. 41. If 8 men can mow 18 acres in 4 days, how many men will be required to mow 50 acres in six days?

Ex. 42. A balloon has moved at the rate of 6492 feet in a minute, how long would it have been sailing round the earth at the same rate, supposing the circumference of the earth to be 25,200 miles?

Ex. 43. How much oftener will the small wheel of a coach turn than the large one, between London and Bristol, or 120 miles, if the former be 10 feet 8 inches in circumference, and the latter 18 feet 4 inches?

Ex. 44. If my income be 250*l.* per annum, and I have foolishly expended 15*s.* per day, how much shall I be in debt at the year's end, and what may I expend per day the following year, so as to have ten guineas in hand at the conclusion of it?

Ex. 45. It is said the impositions of hackney-coachmen, by overcharges, are equal to one-fourth of what they earn: now, if they earn each on an average 18*s.* per day, and there be 1100 employed 313 days in a year, I demand the amount of their overcharges in a year?

Ex. 46. There were at Vauxhall gardens on the Prince of Wales's birth-day, 1805, 10,059 persons: the admission money was 3*s.* each; now, supposing each person to spend 3*s.* more, the half of which was profit to the proprietor, what would he clear by the night, allowing that the incidental expenses were 250*l.*?

Ex. 47. How much time, in the course of 30 years, does a person gain, who rises at five o'clock in the morning, over another who lies till eight, supposing both go to bed at half past ten at night, and supposing the year to consist of 365¼ days?

Ex. 48. If 3¼ yards of cloth will make a shirt, how much of the same stuff will be wanted to make two shirts for each man of a regiment, consisting of 855 men?

Ex. 49. In November, 1800, 276,334 five-pound bank notes were issued; in December 2,626,700; and in the January following 2,769,160: what was the nominal value of the notes issued in these three months; and what was the cost of white rags, from which they were made, supposing each ounce of rag might be manufactured into twenty five-pound notes, and the rags to be worth 8*d.* per lb.?

Ex. 50. Two persons depart from London for York on the same day; the one walks 19 miles a day, the other only 15½ miles: how far distant will they be from one another after ten days travelling, and when will each get to York, which is 197 miles from London?

Ex. 51. The population of the world is estimated at a thousand millions of human beings: if the face of the earth be re-peopled every 33 years, how many persons are born and die in a year, week, day, and minute?*

Ex. 52. The field opposite my house will serve 50 cows forty days: how long will it afford 220 with equal feed?

Ex. 53. If 10 persons expend 250*l.* in 4 months: how much ought 3 persons to expend in 12 months?

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NOTE.

* The number of persons that are born and die in a year is 1,000,000,000 divided by 33.

# FRACTIONS.

*A Fraction is the part, or parts of a whole, or of any whole quantity expressed by unity, and is expressed by two figures, with a line drawn between them, as $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$.*

*The upper figure of a fraction is called the numerator, and the under one the denominator.*

*The denominator shews how many parts the unit is divided into, and the numerator how many of these parts are to be taken;* thus $\frac{3}{4}$, or three-fourths, shews that the whole is divided into four parts, and that three of those parts are to be taken: and $\frac{5}{8}$, or five-eighths, shew that the whole is divided into eight parts, and that five of these parts are taken.

*There are four sorts of fractions, simple and compound, proper and improper.*

*A simple fraction has only one numerator and denominator, as $\frac{1}{3}$, or $\frac{3}{8}$.*

*A compound fraction consists of two or more parts, and is known by the word* of *placed between them, as* $\frac{3}{4}$ of 6; *or* $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{9}{10}$.

*A proper fraction is, when the numerator is less than the denominator.*

*An improper fraction is, when the numerator is equal to, or greater than, the denominator.*

*A mixed number is formed from an integer and a fraction joined together, as $8\frac{5}{6}$.*

*A complex fraction is one that has a fraction or a mixed number for its numerator, or denominator, or both.*

# REDUCTION OF FRACTIONS.

The method of changing fractions from one form to another, without altering their value, is called Reduction; $\frac{24}{48}=\frac{12}{24}=\frac{6}{12}=\frac{3}{6}=\frac{1}{2}$. Reduction serves to prepare fractions for Addition, Subtraction, Multiplication, and Division.

CASE I. To reduce fractions to their least terms.

RULE. *Divide the terms of the given fraction by any number which will divide them both without a remainder, and the quotients will be the terms of a new fraction, equal in value to the given fraction.* Repeat the operation, till the terms of the reduced fraction are divisible only by 1.

Ex. Reduce $\frac{3136}{3584}$ to its lowest terms.

$$8\Big)\frac{3136}{3584}=\frac{392}{448}\text{ and, }8\Big)\frac{392}{448}=\frac{49}{56},\text{ and }7\Big)\frac{49}{56}=\frac{7}{8}.$$

Reduce the following fractions to their lowest terms.

| Ex. 1. | Ex. 2. | Ex. 3. | Ex. 4. | Ex. 5. | Ex. 6. | Ex. 7. | Ex. 8. |
|---|---|---|---|---|---|---|---|
| $\frac{32}{120}$, | $\frac{208}{684}$, | $\frac{136}{72}$, | $\frac{156}{336}$, | $\frac{360}{708}$, | $\frac{384}{1152}$, | $\frac{5184}{6012}$, | $\frac{2475}{2880}$, |

| Ex. 9. | Ex. 10 |
|---|---|
| $\frac{4032}{4806}$, | $\frac{3105}{15705}$. |

Reduce $\frac{2\times3\times4\times5}{3\times4\times7\times8}$ to its lowest terms.

$$\frac{2\times3\times4\times5}{3\times4\times7\times8}=\frac{10}{56}=\frac{5}{28}.$$

In all compound fractions, if there be the same figure in the denominator as there is in the numerator, they may be omitted in the work; thus we have no further concern with the 3 and 4, because they occur in both terms of the fraction.

Reduce $\frac{3\times8\times9\times2}{4\times3\times14\times36}$ to the lowest terms.

$$\frac{3\times8\times9\times2}{4\times3\times14\times36}=\frac{3\times2\times4\times9\times2}{4\times3\times2\times7\times4\times9}=\frac{2}{28}=\frac{1}{14}.$$

Ex. 11. $\frac{3\times4\times15\times4}{5\times6\times24\times3}$. Ex. 12. $\frac{10\times27\times30\times12.}{15\times9\times55\times30.}$

### CASE II. To find the greatest common measure of a fraction.

RULE. *Divide the greater term by the less, and this divisor by the remainder, then the last divisor will be the greatest common measure of both terms of the fraction.* *

Ex. What is the greatest common measure of the fraction $\frac{918}{1998}$?

```
918)1998(2
    1836
    ----
     162)918(5
         810
         ---
         108)162(1
             108
             ---
              54)108(2
                 108
```

Here 54 being the last divisor, it is the greatest common measure of both terms in the fraction; and to reduce the said fraction to the lowest terms, the numerator and denominator are to be divided by 54, the common measure, thus:

$$54\Big)\frac{918}{1998}=\frac{17}{37}.$$

What is the greatest common measure of the following fractions?

Ex. 1. $\frac{270}{306}$.
Ex. 2. $\frac{1080}{1224}$.
Ex. 3. $\frac{720}{1736}$.
Ex. 4. $\frac{336}{868}$.
Ex. 5. $\frac{3108}{3552}$.
Ex. 6. $\frac{9600}{16800}$.
Ex. 7. $\frac{14960}{18320}$.
Ex. 8. $\frac{4125}{4800}$.

### CASE III. To reduce an improper fraction to an equivalent, whole, or mixed number.

RULE. *Divide the numerator by the denominator, and the quotient will be the integer, or mixed number required:* thus $\frac{35}{8}=4\frac{3}{8}$, and $\frac{45}{9}=5$.

Reduce the following improper fractions to their proper terms.

Ex. 1. $\frac{29}{8}$.
Ex. 2. $\frac{57}{7}$.
Ex. 3. $\frac{69}{8}$.
Ex. 4. $\frac{75}{12}$.
Ex. 5. $\frac{96}{16}$.
Ex. 6. $\frac{101}{13}$.
Ex. 7. $\frac{850}{24}$.
Ex. 8. $\frac{9764}{556}$
Ex. 9. $\frac{5640}{450}$
Ex. 10. $\frac{889}{3}$.

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NOTE.

* A number, ending with an even figure, or a cypher, can be divided by 2, without a remainder.

A number ending with 5 or 0, is divisible by 5.

If a fraction has a cypher, or cyphers, at the right-hand of both its terms, it may be abbreviated by cutting off the cyphers.

### CASE IV. To reduce a mixed number to an equivalent improper fraction.

RULE. *Multiply the whole number by the denominator of the fraction, to the product add the numerator, for a new numerator, under which place the denominator;*

Thus $4\frac{3}{8}=\frac{35}{8}$, and $296\frac{1}{3}=\frac{889}{3}$.

Reduce the following mixed numbers to their equivalent improper fractions.

Ex. 1. $3\frac{5}{8}$. Ex. 2. $8\frac{5}{8}$. Ex. 3. $6\frac{5}{12}$. Ex. 4. $7\frac{10}{13}$.
Ex. 5. $18\frac{7}{9}$. Ex. 6. $435\frac{11}{16}$. Ex. 7. $378\frac{5}{15}$. Ex. 8. $499\frac{3}{15}$.
Ex. 9. $54\frac{2}{33}$. Ex. 10. $67\frac{34}{57}$.*

### CASE V. To reduce a compound fraction to an equivalent simple one.

RULE (1). *If any of the proposed quantities be integers, or mixed numbers, reduce them to their proper terms.*

(2). *Multiply all the numerators together for a new numerator, and all the denominators for a new denominator, and then reduce the fraction to its lowest terms.*

Reduce $\frac{4}{5}$ of 3 of $7\frac{5}{6}$ to a simple fraction.

Operation $\frac{4}{5}\times\frac{3}{1}\times\frac{47}{6}=\frac{2\times2\times3\times47}{5\times1\times2\times3}=\frac{94}{5}$.†

The fraction $\frac{94}{5}$ is already in its lowest terms, because no figure higher than unit will divide both terms of the fraction without a remainder.

Ex. 1. $\frac{7}{9}$ of $\frac{6}{5}$ of 5 of $\frac{3}{4}$. Ex. 2, $\frac{3}{6}$ of 4 of $5\frac{8}{7}$.
Ex. 3. $\frac{5}{11}$ of 8 of $7\frac{6}{9}$ of 12. Ex. 4. $\frac{4}{13}$ of $\frac{3}{18}$ of 12 of $9\frac{8}{9}$.
Ex. 5. $\frac{7}{10}$ of 10 of $\frac{12}{8}$ of $18\frac{2}{3}$. Ex. 6. $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{5}{4}$ of $\frac{6}{10}$.

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NOTES.

* Under this case a whole number may be reduced to an equivalent fraction, having any given denominator: thus, $5=\frac{5}{1}$. For a whole number may be converted into a fraction, by placing under it an unit; and therefore it may be reduced to an improper fraction with any given denominator, by multiplying it by the denominator, and the product will be the numerator required. Thus, to reduce 15 to an equivalent fraction, having the donominator 6, I say, $\frac{15\times6}{6}=\frac{90}{6}$.

† Here, as has already been observed, 2 and 3 being found in the numerator and denominator, are dropped in the work.

CASE VI. To reduce fractions of different denominators to others of equal value, having a common denominator.

RULE. (1). *Multiply each numerator into all the denominators, except its own, for a new numerator, and all the denominators for a common denominator.*

Reduce $\frac{3}{5}$, $\frac{7}{9}$, $3\frac{2}{3}$, and 3, to a common denominator.

Operation, $\frac{3}{5}$, $\frac{7}{9}$, $\frac{11}{3}$, $\frac{3}{1}$.

New numerators.
$3 \times 9 \times 3 \times 1 = 81$
$7 \times 5 \times 3 \times 1 = 105$
$11 \times 5 \times 9 \times 1 = 495$
$3 \times 5 \times 9 \times 3 = 405$
New denom.
$5 \times 9 \times 3 \times 1 = 135$

I multiply the numerator of the first fraction 3, by 9, and 3, and 1: and then 7, the numerator of the second fraction, by 5, and 3, and 1, and so of the others; and for a new denominator. I multiply the 5, and 9, and 3, and 1, together.

Answer, $\frac{81}{135}$, $\frac{105}{135}$, $\frac{495}{135}$, $\frac{405}{135}$.

Ex. 1. Reduce $\frac{2}{3}$, $\frac{5}{4}$, and $\frac{3}{4}$, to a common denominator.*
2. Reduce $\frac{6}{7}$, $\frac{5}{9}$, $\frac{4}{5}$, and $\frac{7}{8}$, to a common denominator.
3. Reduce $\frac{2}{7}$, $\frac{2}{5}$, $\frac{6}{7}$, and 3, to a common denominator.†
4. Reduce $\frac{4}{15}$, $\frac{1}{10}$, 8, and $11\frac{1}{2}$, to a common denominator.
5. Reduce $\frac{3}{11}$, $\frac{1}{2}$, $\frac{2}{7}$, 4, and $2\frac{1}{5}$, to a common denominator.
6. Reduce $\frac{1}{2}$, $\frac{3}{5}$, $\frac{2}{7}$, and $\frac{15}{20}$, to a common denominator.
7. Reduce $\frac{7}{8}$, $\frac{5}{9}$, $\frac{1}{4}$, $\frac{2}{5}$, and 7, to a common denominator.
8. Reduce $\frac{4}{10}$, $\frac{2}{5}$, $\frac{3}{7}$, and $\frac{9}{16}$, to a common denominator.
9. Reduce $\frac{1}{6}$, $\frac{3}{8}$, $\frac{1}{7}$, and $\frac{4}{11}$ of 9, to a common denominator.

(2). To find the *least* common denominator.

*Set down the denominators of the given fractions in a line, and divide as many of them as possible, by any number which will leave no remainder, and set down the quotients,*

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NOTES.

* If the products of the denominators are divided by their greatest common measure, the answer will be in the *least* common denominator, as in this example. See also next rule.

New numerators.
$2 \times 4 \times 4$
$5 \times 3 \times 4$
$3 \times 3 \times 4$
Denominator.
$3 \times 4 \times 4$

Here 4 being common to each new numerator, and to the denominator, may be omitted, and the answer will be

$\frac{8}{12}$, $\frac{15}{12}$, $\frac{9}{12}$ = to the given fractions $\frac{2}{3}$, $\frac{5}{4}$, and $\frac{3}{4}$.

† In the work there is no need to put down the units as multipliers.

*and the undivided numbers below. Repeat the operation till there be no two numbers which can be divided without a remainder. Then the product of all the divisors, and the quotients in the last lines will give the least common denominator. Divide this least common denominator by each of the given denominators separately, and multiply the quotients by their several numerators, their products will be the new numerators.*

Reduce $\frac{3}{5}$, $\frac{7}{9}$, $\frac{11}{3}$, $\frac{3}{1}$, to the *least* common denominator.

$$3\overline{)5,\ 9,\ 3,\ 1}$$
$$5,\ 3,\ 1,\ 1$$

, then $3 \times 5 \times 3 \times 1 \times 1 = 45$, is the common denominator, and 45 divided by the given denominators, 5, 9, 3, 1, give 9, 5, 15, 45; these multiplied by the given numerators, give 27, 35, 165, 135, for new numerators, and the fractions will stand $\frac{27}{45}$, $\frac{35}{45}$, $\frac{165}{45}$, $\frac{135}{45}$.

Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{4}{6}$, and $\frac{3}{8}$, to the least common denominator.

$$3\overline{)3,\ 4,\ 5,\ 6,\ 8}$$
$$4\overline{)1,\ 4,\ 5,\ 2,\ 8}$$
$$2\overline{)1.\ 1,\ 5,\ 2,\ 2}$$
$$1,\ 1,\ 5,\ 1,\ 1$$

The least denominator is, acccordingly,

$$3 \times 4 \times 2 \times 5 = 120;$$
$$120 \div 3, 4, 5, 6, 8 = 40, 30, 24, 20, 15.$$

$40 \times 2$; $30 \times 3$; $24 \times 2$; $20 \times 4$; $15 \times 3$, for new numerators; therefore the fractions required are $\frac{20}{120}$, $\frac{90}{120}$, $\frac{48}{120}$, $\frac{80}{120}$, $\frac{45}{120}$.

The learner may now reduce to the least common denominator, the ten examples given under the first part of the Rule.*

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NOTE.

* To find the least common multiple of two or more given numbers.

RULE. *Find the greatest common measure, by inspection, of* TWO *of the numbers, and divide the product of them by the common measure so found; multiply this quotient by the third number, and divide the product by the common measure of the multiplier and multiplicand, and so proceed to the last number; the last quotient will be the least common multiple.*

Ex. Find the least number that can be divided by 2, 3, 4, 5, 6, and 7, without remainders.

$$\frac{2 \times 3}{1} = 6$$
$$\frac{6 \times 4}{2} = 12$$
$$\frac{12 \times 5}{1} = 60$$
$$\frac{60 \times 6}{6} = 60$$
$$\frac{60 \times 7}{1} = 420$$

The greatest common measure of 2 and 3 is 1, and $2 \times 3$, divided by 1 is 6: the greatest common measure of the 6 just found, and 4, the next given number, is 2, and $6 \times 4$ divided by $2 = 12$: the greatest common measure of 12 so found, and 5, the next figure, is 1; and 60 divided by $1 = 60$, and so on. It is found that 420 is the least number that can be divided by 2, 3, 4, 5, 6, and 7.

Ex. 1. What

### CASE VII. To reduce a fraction of one denomination to the fraction of another denomination of equal value.

RULE. (1). *When it is from the less to a greater denomination, "Multiply the denominator by all the denominations from that given to the one sought."*

Thus, to reduce $\frac{3}{4}$ of a penny to a fraction of a pound, the answer will be $\frac{3}{4 \times 12 \times 20} = \frac{3}{960}$.

(2). *When it is from a greater to a less denomination, "Multiply the numerator by all the denominations, from that given to the one sought."*

Thus, to reduce $\frac{6}{7}$ of a pound to the fraction of a farthing, $\frac{6 \times 20 \times 12 \times 4}{7} = \frac{5760}{7}$.

Ex. 1. Reduce $\frac{288}{9}$ of a farthing to the fraction of a pound.
2. Reduce $\frac{5}{9}$ of a penny to the fraction of a shilling.
3. Reduce $\frac{2}{9}$ of a pound to the fraction of a farthing.
4. Reduce $\frac{4}{15}$ of a pound to the fraction of a penny.
5. Reduce $\frac{11}{14}$ of a pound to the fraction of a farthing.
6. Reduce 3 shillings to the fraction of a pound.
7. Reduce $\frac{4}{9}$ of a dwt. to the fraction of a lb. Troy.
8. Reduce $\frac{5}{7}$ of a cwt. to the fraction of an ounce.
9. Reduce $\frac{7}{8}$ of a week to the fraction of an hour.
10. Reduce $\frac{3}{4}$ of a mile to the fraction of a yard.
11. Reduce $\frac{7}{8}$ of a pipe to the fraction of a gallon.
12. Reduce $\frac{1}{2}$ a pint to the fraction of a hhd. of ale.

### CASE VIII. To find the *value* of a fraction in numbers of inferior denomination.

RULE. *Multiply the integer, or its value in the next lower denomination, by the numerator, and divide by the denominator:*

Thus, the value of $\frac{3}{5}$ of a pound is equal to $\frac{3 \times 20}{5} = 12$ shillings, and $\frac{2}{3}$ of a shilling is equal to $\frac{2 \times 12}{3} = 8$ pence.

---

Ex. 1. What is the least number that can be divided by 4, 6, and 10, without a remainder?

2. What is the least number that can be divided by 3, 5, 8, and 10, without a remainder?

3. What is the least number that can be divided, without a remainder, by 3, 4, 8, 10, and 16?

Ex. 1. What is the value of $\frac{5}{9}$ of a pound ?*

2. What is the value of $\frac{7}{8}$ of a shilling ?

3. What is the value of $\frac{9}{15}$ of half a crown ?

4. What is the value of $\frac{3}{4}$ of a lb. Troy ?

5. What is the value of $\frac{9}{14}$ of a cwt. ?

6. What is the value of $\frac{5}{9}$ of a mile ?

7. What is the value of $\frac{8}{7}$ of a barrel of beer ?

8. What is the value of $\frac{5}{12}$ of a chaldron of coals ?

9. What is the value of $\frac{6}{7}$ of a hogshead of wine ?

### CASE IX. To reduce a complex fraction to an equivalent simple fraction.

RULE. *If the numerator, or denominator, or both, be whole or mixed numbers, reduce them to improper fractions ; and multiply the denominator of the lower fraction into the numerator of the upper, for a new numerator, and the denominator of the upper fraction into the numerator of the lower, for a new denominator.*

Thus, $\frac{4}{\frac{7}{8}} = \frac{\frac{4}{1}}{\frac{7}{8}} = \frac{4 \times 8}{7 \times 1} = \frac{32}{7}$. And $\frac{\frac{4}{10}}{5} = \frac{\frac{4}{10}}{\frac{5}{1}} = \frac{4}{50}$. And $\frac{5\frac{7}{8}}{8} = \frac{\frac{47}{8}}{\frac{8}{1}} = \frac{47}{64}$. And $\frac{9}{3\frac{2}{7}} = \frac{\frac{9}{1}}{\frac{23}{7}} = \frac{63}{23}$. And again $\frac{5\frac{1}{4}}{3\frac{3}{7}} = \frac{\frac{21}{4}}{\frac{24}{7}} = \frac{147}{96}$. No other varieties can happen.

Ex. 1. Reduce $\frac{3\frac{1}{4}}{4}$ to a simple fraction.

2. Reduce $\frac{\frac{3}{4}}{\frac{2}{5}}$ to a simple fraction.

3. Reduce $\frac{3\frac{11}{15}}{19\frac{7}{9}}$ to a simple fraction.

4. Reduce $\frac{15\frac{2}{5}}{53}$ to a simple fraction.

---

NOTE.

* Where there is a remainder we proceed as in Compound Division ; thus $\frac{5}{9}$ of a pound $= \frac{5 \times 20}{9} = \frac{100}{9} =$ 11s. $1\frac{1}{4}$d.—$\frac{3}{9}$. See p. 83.

Ex. 5. Reduce $\dfrac{\frac{3}{8}}{9}$ to a simple fraction.

6. Reduce $\dfrac{7\frac{1}{8}}{9\frac{2}{3}}$ to a simple fraction.

7. Reduce $\dfrac{5}{\frac{4}{7}}$ to a simple fraction.

8. Reduce $\dfrac{4}{19\frac{7}{9}}$ to a simple fraction.

* * *

## ADDITION OF FRACTIONS.

RULE. *Reduce mixed numbers to improper fractions, and compound or complex fractions to simple ones, and bring them all to their least common denominator. Add all the nemerators together, and write the sum over the common denominator.*

Ex. Add $\frac{3}{5}$, $\frac{2}{3}$, $5\frac{1}{2}$, and $\frac{1}{4}$ together; which is thus performed: $\frac{3}{5}$, $\frac{2}{3}$, $\frac{11}{2}$, $\frac{1}{4}$.

$$\left.\begin{array}{r} 3\times3\times2\times4=72\\ 2\times5\times2\times4=80\\ 11\times5\times3\times4=660\\ 1\times5\times3\times2=30\\ \hline 5\times3\times2\times4=120 \end{array}\right\}$$

Therefore $\frac{72}{120}+\frac{80}{120}+\frac{660}{120}+\frac{30}{120}=\frac{842}{120}=7\frac{2}{120}=7\frac{1}{60}$, which is the answer.

This may be performed by bringing the given fractions to the least common denominator: See p. 213.

Thus, $\frac{3}{5}$, $\frac{2}{3}$, $\frac{11}{2}$, $\frac{1}{4}$, then $\dfrac{2)5,\ 3,\ 2,\ 4}{5,\ 3,\ 1,\ 2}$, and the new denominator $= 60$; the fractions will be $\frac{36}{60}+\frac{40}{60}+\frac{330}{60}+\frac{15}{60}=\frac{421}{60}=7\frac{1}{60}$.

Ex. 1. Add $\frac{4}{9}$, $\frac{3}{5}$, and $\frac{6}{7}$ together.

2. Add $\frac{3}{5}$, $\frac{7}{8}$, and $\frac{9}{2}$ together.

3. What is the sum of $\frac{3}{5}$, $\frac{4}{7}$, and $4\frac{1}{2}$?

4. Add together $3\frac{5}{7}$, $4\frac{3}{8}$, and $\frac{2}{5}$.*

---

NOTE.

* When there are two or more mixed numbers, as in the 4th example, the fractions may be first added, and join these to the sum of the whole numbers: thus, I add $\frac{5}{7}$, $\frac{3}{8}$, and $\frac{2}{5}$ together which are $=\frac{417}{280}=1\frac{137}{280}$, and the answer $3+4+1\frac{137}{280}=8\frac{137}{280}$.

Ex. 5. Add $\frac{2}{7}$, $\frac{5}{8}$, $2\frac{3}{4}$, and $5\frac{1}{2}$ together.

6. What is the sum of $7\frac{2}{5}$, $3\frac{1}{2}$, and $\frac{5}{9}$?

7. What is the sum of $\frac{3}{7}$ of a guinea, $\frac{3}{8}$ of a shilling, and $\frac{5}{9}$ of a penny?*

8. What is the sum of $\frac{2}{9}$ of a pound, $\frac{4}{7}$ of a shilling, and $\frac{7}{12}$ of a penny?

9. What is the sum of $\frac{3}{4}$ of a guinea, $\frac{2}{3}$ of a shilling, and $\frac{9}{10}$ of a penny?

10. If I have $\frac{3}{8}$ of a coasting vessel, and purchase another share of $\frac{2}{11}$, what part of her will belong to me?

11. Add $\frac{2}{3}$ of a yard, and $\frac{3}{4}$ of a mile together.

12. What is the sum of $\frac{2}{3}$ of a yard, $\frac{3}{4}$ of a foot, and $\frac{2}{3}$ of an inch?

13. Add $\frac{1}{5}$ of a lb. troy to $\frac{3}{8}$ of an ounce.

14. What is the sum of $\frac{3}{4}$ of a hhd. of beer, and $\frac{3}{5}$ of a barrel?

15. Add $\frac{5}{9}$ of a chaldron to $\frac{5}{7}$ of a bushel?

## SUBTRACTION OF FRACTIONS.

RULE. *Reduce the given fractions to the same denominator, as in Addition, then subtract the lesser numerator from the greater, and under the difference place the common denominator.*

Ex. Take $\frac{3}{9}$ from $\frac{5}{12}$: and $\frac{9}{15}$, from $\frac{11}{16}$.

$$\left.\begin{array}{r} 5 \times 9 \\ 3 \times 12 \\ \overline{\phantom{0}} \\ 9 \times 12 \end{array}\right\} \text{Therefore } \frac{45 - 36}{108} = \frac{9}{108} = \frac{1}{12} \text{ Answer.}$$

---

NOTE.

* To add fractions of different integers, find their respective values by Case VII., and proceed as in Compound Addition: thus,

$\frac{3}{7}$ of a guinea $= \frac{3 \times 21}{7} = 0 \quad 9 \quad 0$

$\frac{3}{8}$ of a shilling $= \frac{3 \times 12}{8} = 0 \quad 0 \quad 4\frac{1}{2}$

$\frac{5}{9}$ of a penny $= \frac{5 \times 4}{9} = 0 \quad 0 \quad 0\frac{1}{2} \; \frac{2}{9}$

Answer - £.0 9 5 $\frac{2}{9}$

$$\left.\begin{array}{r} 11 \times 15 \\ 9 \times 16 \\ \hline 15 \times 16 \end{array}\right\} \text{Therefore } \frac{165-144}{240} = \frac{21}{240}.$$

Ex. 1. From $\frac{7}{8}$ take $\frac{3}{5}$.
2. From $\frac{11}{12}$ take $\frac{4}{7}$.
3. From $\frac{15}{11}$ take $\frac{7}{13}$.
4. From $\frac{12}{5}$ take $\frac{4}{7}$.
5. From $9\frac{3}{4}$ take $4\frac{7}{8}$.*
6. From $12\frac{1}{2}$ take $\frac{2}{3}$ of 17.
7. From $\frac{5}{8}$ of a shilling take $\frac{1}{50}$ of a pound.
8. From $\frac{3}{4}$ of a pound take $\frac{7}{11}$ of a pound.
9. From 1 take $\frac{7}{18}$.†
10. From 1 take $\frac{3}{5}$ of $\frac{4}{9}$.
11. From 12 take $\frac{5}{9}$.‡
12. From 10*l.* take $\frac{5}{9}$ of a pound.§
13. From $\frac{5}{8}$ of a pound take $\frac{3}{10}$ of a pound.
14. From $\frac{2}{3}$ of a pound take $\frac{1}{2}$ of $\frac{5}{8}$ of a shilling.
15. From $\frac{2}{3}$ of 6 lb. avoirdupoise take $\frac{2}{5}$ of 5 lb.
16. Subtract $\frac{8}{24}$ of a ton from $8\frac{4}{9}$ of a ton.

---

NOTES.

* In mixed numbers, the subtraction may frequently be performed without reducing them to improper fractions. After the fractions are brought to a common denominator, subtract the numerator of the lower fraction from the common denominator; to the remainder add the numerator of the upper fraction, and carry one to the lower whole number: thus, $9\frac{3}{4} - 4\frac{7}{8} = 9\frac{6}{8} - 4\frac{7}{8} = 4\frac{7}{8}$.

Here $\frac{3}{4}$ and $\frac{7}{8}$ being brought to a common denominator, as

$$\left.\begin{array}{r} 3 \times 8 \\ 7 \times 4 \\ \hline 4 \times 8 \end{array}\right\} = \frac{24-28}{4 \times 8} = \frac{6-7}{8}; \text{ therefore } \begin{array}{r} 9\frac{6}{8} \\ 4\frac{7}{8} \\ \hline 4\frac{7}{8} \end{array}$$

† To subtract a proper fraction from an unit: *Subtract the numerator from the denominator; the remainder being placed over the denominator, gives the answer required:* thus, take $\frac{3}{5}$ from 1, answer $\frac{2}{5}$.

‡ To subtract a proper fraction from any whole number: *Subtract the numerator from the denominator, and the remainder placed over the denominator, gives the fraction which is to be annexed to the whole number made less by* 1: thus, take $\frac{5}{8}$ from 11, the answer $10\frac{3}{8}$.

§ To subtract fractions of different integers: *Find their respective values, and proceed as in Compound Subtraction*: See p. 68.

From $\frac{3}{9}$ of a pound, take $\frac{5}{8}$ of a shilling.

$\frac{3 \times 20}{9} = \frac{60}{9} = 6s.\ 8d.$; and $\frac{5 \times 12}{8} = 7\frac{1}{2}d.$; therefore the answer will be $6s.\ 0\frac{1}{2}d.$

## MULTIPLICATION OF FRACTIONS.

RULE. *Reduce mixed numbers to improper fractions, and compound fractions to simple ones; multiply all the numerators together for a new numerator; and all the denominators for a common denominator.*

Ex. Multiply $3\frac{5}{8}$, $\frac{3}{4}$, and $\frac{5}{6}$ of 8 together.

$$\frac{29}{8} \times \frac{3}{4} \times \frac{5}{6} \times \frac{8}{1} = \frac{29 \times 3 \times 5 \times 8^*}{8 \times 4 \times 3 \times 2} = \frac{29 \times 5}{4 \times 2} = \frac{145}{8}$$

$= 18\frac{1}{8}$, the answer.

Ex. 1. Multiply $\frac{8}{11}$ by $\frac{4}{9}$; and $\frac{3}{9}$ by $\frac{3}{16}$.
2. What is the product of $\frac{7}{8}$, $\frac{4}{5}$, and $3\frac{1}{2}$?
3. What is the product of 57 by $\frac{9}{11}$?
4. What is the product of $7\frac{2}{5}$ multiplied by 35?
5. What is the product of $\frac{5}{9}$, $\frac{3}{7}$, $12\frac{1}{4}$, and $\frac{3}{5}$ of 10?
6. What is the continued product of $\frac{7}{8}$, $\frac{3}{5}$, $5\frac{1}{9}$, and 6?
7. What is the product of $\frac{2}{7}$ of $\frac{5}{9}$, $\frac{1}{3}$ of $\frac{14}{15}$?
8. What is the product of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{8}{9}$ and $\frac{9}{10}$?
9. How many yards are there in $5\frac{1}{4}$ pieces of Irish, each containing $26\frac{1}{2}$?
10. How many pounds are there in $8\frac{1}{2}$ cheeses, each containing $25\frac{3}{4}$ lb.

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NOTE.

Fractions must be abbreviated, when it can be done, thus we strike out 8 and 3, because they are found in the upper and under line.

*Note.* The learner will, perhaps, be struck with the difference between common multiplication and the multiplication of fractions. By the former, the product of any two numbers above unit is greater than either; but in what is called multiplication of fractions, the product is less than the numbers multiplied together; thus $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, and $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$: and, in general, whatever part of unity the multiplier is, the product will be the same part of the multiplicand: thus, $\frac{1}{3}$ is one third of 1, and $\frac{1}{9}$ is one third part of $\frac{1}{3}$: again, $\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$: here $\frac{3}{4}$ is three fourths of 1, and $\frac{6}{12}$, or $\frac{1}{2}$, is three-fourths of two-thirds: thus, $\frac{3}{4}$ of $\frac{2}{3}$ of a shilling is equal $\frac{1}{2}$ of a shilling, or sixpence, the truth of which is evident, for $\frac{2}{3}$ of a shilling is 8., and $\frac{3}{4}$ of 8*d*. is 6*d*.

# DIVISION OF FRACTIONS.

RULE. *Reduce the fractions, as in Multiplication; then invert the divisor, and proceed as in Multiplication:* thus, $\frac{3}{5}$ to be divided by $\frac{3}{9}$.

$$\tfrac{3}{5} \div \tfrac{3}{9} = \tfrac{3}{5} \times \tfrac{9}{3} = \tfrac{27}{15} = \tfrac{9}{5}.$$

Ex. Divide $\frac{3}{8}$ of $4\frac{3}{5}$ by $\frac{3}{7}$ of $\frac{1}{4}$.

$$\frac{3}{8} \times \frac{23}{5} \div \frac{3}{7} \times \frac{1}{4} \text{ or } \frac{3 \times 23}{8 \times 5} \div \frac{3}{4 \times 7} = \frac{3 \times 23}{4 \times 2 \times 5} \times \frac{4 \times 7}{3}$$

$$= \tfrac{161}{10} = 16\tfrac{1}{10}, \text{ the answer.}$$

### EXAMPLES.

Ex. 1. Divide $\frac{15}{17}$ of 12 by $\frac{4}{3}$.

2. Divide $\frac{7}{11}$ of 8 by $\frac{3}{13}$.

3. Divide $\frac{25}{6}$ by $\frac{11}{15}$.

4. Divide $\frac{2}{9}$ of 54 by $\frac{3}{5}$.

5. Divide $\frac{4}{7}$ of 12 by $3\frac{2}{3}$.

6. Divide $\frac{4}{5}$ of 36 by $3\frac{1}{4}$.

7. Divide $\frac{1}{3}$ of 4 by $\frac{5}{4}$ of 2.

8. Divide $11\frac{3}{7}$ by $\frac{7}{8}$ of $\frac{2}{3}$.

9. Divide $\frac{7}{8}$ of $\frac{2}{5}$ of 5 by $\frac{4}{7}$ of $\frac{1}{5}$.

10. Divide $\frac{7}{11}$ of $\frac{3}{5}$ by $\frac{4}{17}$ of 5.

11. What number multiplied by $\frac{2}{5}$ will give $9\frac{1}{2}$?*

12. What part of 56 is $\frac{5}{12}$ of 3?

13. What number multiplied by $\frac{5}{8}$ of $\frac{2}{3}$ of 15, will produce $\frac{3}{5}$ of 4?

14. From 5 subtract $\frac{3}{4}$ of $\frac{2}{7}$ of 4, and divide the remainder by 4.

15. What is a person's share of a prize of £. 20,000, $\frac{4}{5}$ths of which is to be divided among 13 persons?

---

NOTE.

* The answer to this will be the quotient of $9\frac{1}{2}$ divided by $\frac{2}{5}$.

# PRACTICE.

PRACTICE* is a method of finding the value of any quantity of goods, from the price of an integer being given.

ALIQUOT PARTS of any number or quantity, are such as will exactly divide it without leaving a remainder: thus 7 and 4 are aliquot parts of 28, 4 pence is an aliquot part of a shilling, and 5 shillings is an aliquot part of a pound.

### TABLES OF ALIQUOT PARTS.

Aliquot parts of a £.

| s. | d. | | |
|---|---|---|---|
| 10 | 0 | = | $\frac{1}{2}$ |
| 6 | 8 | = | $\frac{1}{3}$ |
| 5 | 0 | = | $\frac{1}{4}$ |
| 4 | 0 | = | $\frac{1}{5}$ |
| 3 | 4 | = | $\frac{1}{6}$ |
| 2 | 6 | = | $\frac{1}{8}$ |
| 2 | 0 | = | $\frac{1}{10}$ |
| 1 | 8 | = | $\frac{1}{12}$ |
| 1 | 4 | = | $\frac{1}{15}$ |
| 1 | 3 | = | $\frac{1}{16}$ |
| 1 | 0 | = | $\frac{1}{20}$ |

Parts of a shilling.

| d. | | |
|---|---|---|
| 6 | = | $\frac{1}{2}$ |
| 4 | = | $\frac{1}{3}$ |
| 3 | = | $\frac{1}{4}$ |
| 2 | = | $\frac{1}{6}$ |
| $1\frac{1}{2}$ | = | $\frac{1}{8}$ |
| 1 | = | $\frac{1}{12}$ |

Parts of sixpence.

| | | |
|---|---|---|
| $\frac{3}{4}$ | = | $\frac{1}{8}$ |
| $\frac{1}{2}$ | = | $\frac{1}{12}$ |

Parts of 3 pence.

| q. | | |
|---|---|---|
| $\frac{3}{4}$ | = | $\frac{1}{4}$ |
| $\frac{1}{2}$ | = | $\frac{1}{6}$ |
| $\frac{1}{4}$ | = | $\frac{1}{12}$ |

Parts of a penny.

| | | |
|---|---|---|
| $\frac{1}{2}$ | = | $\frac{1}{2}$ |
| $\frac{1}{4}$ | = | $\frac{1}{4}$ |

### I. When the price is less than a penny.

RULE. *Divide the quantity by the aliquot parts in a penny, and the quotient by* 12 *and* 20.

---

NOTE.

* PRACTICE has its name from its general use in business, as it teaches the best and most compendious methods of answering almost all questions that occur in trade and mercantile transactions, and is to be preferred to Compound Multiplication, and to the Rule of Three, whenever the first term is unity.

Ex. What is the value of 7853 yards of tape, at ¾ per yard?

| | | | |
|---|---|---|---|
| ½ | ½ | 7853 | |
| ¼ | ½ | 3926½ | |
| | | 1963¼ | |
| | | 12) 5889¾ | |
| | | 2.0)49.0 | 9¾ |
| Answer, | | £. 24 10 | 9¾ |

In this example I say, ½ is the half of a penny, and a ¼ is the half of a halfpenny. I first divide the number of yards by 2, and the answer is 3926½ pence, or the value of 7853 yards, at ½ per yard; I then divide this sum by 2, which gives 1963¼, or the value of the tape had it been only ¼ per yard. To find the value at ¾ per yard, I add these two sums together, and 5889¾ pence is the value of the tape at ¾ per yard; I then divide this sum by 12, to bring the pence into shillings, afterwards by 20, to bring the shillings into pounds.

### EXAMPLES.

Ex. 1. 4567 at ¼ per yd. 2. 6784 at ½ per lb. 3. 3976 at ¾
4. 7655 at ½ per yd. 5. 7486 at ¾ per lb. 6. 9984 at ½
7. 6327 at ¾ per yd. 8. 5934 at ¼ per lb. 9. 7585 at ½
10. 4767 at ¼ per yd. 11. 6493 at ¾ per lb. 12. 5388 at ¾

## II. When the price is an aliquot part of a shilling.

Rule. *Divide the given number by the aliquot part, and this quotient by* 20*: the answer will be in pounds.*

Ex. What is the value of 2785 lbs. of salt, at 4*d.* per lb?

| | | | |
|---|---|---|---|
| 4*d.* | ⅓ | 2785 | |
| | | 2,0)92,8 | 4 |
| Answer, | | £. 46 8 | 4 |

Four pence being ⅓ of a shilling, I divide the given number by 3, and the answer is 928 shillings and 1 over, that is 928*s.* 4*d.*, because each pound of salt is worth 4*d.*; I then divide by 20 to bring the shillings into pounds.

Ex. 1. 3764 at 2*d.* 2. 5943 at 3*d.* 3. 4953 at 1½*d.*
4. 5943 at 4*d.* 5. 3987 at 3*d.* 6. 5964 at 1*d.*
7. 5684 at 4*d.* 8. 2705 at 2*d.* 9. 3456 at 2*d.*
10. 5924 at 1½*d.* 11. 5964 at 2*d.* 12. 5215 at 4*d.*

## III. When the price is pence and farthings, and no aliquot part of a shilling.

Rule. (1) *Find what aliquot part of a shilling is nearest to the given price, and divide the proposed number by it.* (2) *Consider what part the remainder is of this aliquot part of the given price, and divide the former quotient by it, &c.* (3) *Add the several quotients together, and the answer will be in shillings, which divide by* 20 *to bring into pounds.*

Ex. What is the value of 4277 yards, at $10\frac{3}{4}d.$ per yard?

|  |  |  |  |
|---|---|---|---|
| 6 | $\frac{1}{2}$ | 4277 |  |
| 3 | $\frac{1}{2}$ | 2138 | 6 |
| $1\frac{1}{2}$ | $\frac{1}{2}$ | 1069 | 3 |
| $\frac{1}{4}$ | $\frac{1}{6}$ | 534 | $7\frac{1}{2}$ |
|  |  | 89 | $1\frac{1}{4}$ |
|  |  | 2,0)383,1 | $5\frac{3}{4}$ |
| Ans. £. |  | 191 11 | $5\frac{3}{4}$ |

In this example, I first divide by 2, because $6d.$ is the $\frac{1}{2}$ of a shilling; then I take parts for the $4\frac{3}{4}d.$, and say $3d.$ is the $\frac{1}{2}$ of 6; $1\frac{1}{2}$ is the $\frac{1}{2}$ of $3d.$, and $\frac{1}{4}$ is the $\frac{1}{6}$ of $1\frac{1}{2}$: and of course I divide the first answer by 2, and this quotient by 2, then that last found by 6; and having added the four quotients together, the answer is $3831s.$ $5\frac{3}{4}d.$; which, divided by 20, gives $191l.$ $11s.$ $5\frac{3}{4}d.$

|  | d. |  | d. |  | d. |
|---|---|---|---|---|---|
| Ex. 1. 4784 at | $1\frac{1}{4}$ | 2. 5964 at | $1\frac{3}{4}$ | 3. 4659 at | $2\frac{1}{4}$ |
| 4. 1765 at | $2\frac{1}{4}$ | 5. 4305 at | $2\frac{3}{4}$ | 6. 3694 at | $3\frac{1}{4}$ |
| 7. 7641 at | $2\frac{1}{2}$ | 8. 9875 at | $6\frac{1}{2}$ | 9. 5476 at | $10\frac{3}{4}$ |
| 10. 3592 at | $3\frac{1}{2}$ | 11. 3046 at | $6\frac{3}{4}$ | 12. 3214 at | $11\frac{1}{2}$ |
| 13. 8764 at | $3\frac{3}{4}$ | 14. 5921 at | $7\frac{1}{4}$ | 15. 5178 at | $9\frac{1}{2}$ |
| 16. 9714 at | $4\frac{1}{2}$ | 17. 5643 at | $8\frac{1}{2}$ | 18. 4932 at | $10\frac{1}{4}$ |
| 19. 8934 at | $5\frac{1}{4}$ | 20. 2458 at | $9\frac{3}{4}$ | 21. 8764 at | $11\frac{3}{4}$ |
| 22. 5687 at | $5\frac{3}{4}$ | 23. 1435 at | $10\frac{1}{2}$ | 24. 5842 at | $7\frac{1}{4}$ |
| 25. 5943 at | $9\frac{1}{2}$ | 26. 1876 at | $2\frac{3}{4}$ | 27. 4316 at | $7\frac{3}{4}$ |
| 28. 1956 at | $8\frac{3}{4}$ | 29. 4235 at | $5\frac{1}{2}$ | 30. 1327 at | $9\frac{1}{2}$ |
| 31. 2748 at | 11 | 32. 9374 at | $7\frac{1}{4}$ | 33. 4285 at | $11\frac{1}{4}$ |
| 34. 1594 at | $3\frac{1}{2}$ | 35. 5632 at | 5 | 36. 1114 at | $5\frac{1}{2}$ |

## IV. When the price is more than one shilling, and less than two.

RULE. *Let the given number stand for shillings, and work for the pence and farthings as before.*

Ex. What is the value of 1187 quartern loaves, at $1s.$ $1\frac{3}{4}d.$ each?

|  |  |  |  |
|---|---|---|---|
| $1\frac{1}{2}$ | $\frac{1}{8}$ | 1187 |  |
| $\frac{1}{4}$ | $\frac{1}{6}$ | 148 | $4\frac{1}{2}$ |
|  |  | 24 | $8\frac{3}{4}$ |
|  |  | 2,0)136,0 | $1\frac{1}{4}$ |
| Ans. £. |  | 68 0 | $1\frac{1}{4}$ |

Here I take parts for the $1\frac{3}{4}d.$, that is, $1\frac{1}{2}$ is the $\frac{1}{8}$th of a shilling, and a $\frac{1}{4}$ is $\frac{1}{6}$ of $1\frac{1}{2}$, I first divide the number by 8, and that quotient by 6, and add the two quotients thus found, to the given number which stood for shillings: and the sum thus found, divided by 20, gives the answer in pounds.

|  | s. | d. |  | s. | d. |  | s. | d. |
|---|---|---|---|---|---|---|---|---|
| Ex. 1. 3456 at | 1 | $2\frac{1}{2}$ | 2. 4876 at | 1 | $5\frac{1}{2}$ | 3. 5792 at | 1 | $8\frac{1}{2}$ |
| 4. 2632 at | 1 | $3\frac{3}{4}$ | 5. 4092 at | 1 | $7\frac{1}{4}$ | 6. 2596 at | 1 | 10 |
| 7. 4735 at | 1 | $4\frac{1}{2}$ | 8. 3724 at | 1 | $9\frac{1}{2}$ | 9. 3451 at | 1 | $6\frac{3}{4}$ |
| 10. 7321 at | 1 | $7\frac{3}{4}$ | 11. 5928 at | 1 | 11 | 12. 6542 at | 1 | $8\frac{3}{4}$ |
| 13. 8465 at | 1 | $9\frac{1}{2}$ | 14. 4371 at | 1 | $3\frac{1}{2}$ | 15. 8937 at | 1 | $3\frac{3}{4}$ |
| 16. 1234 at | 1 | 11 | 17. 5629 at | 1 | $1\frac{3}{4}$ | 18. 4516 at | 1 | 2 |
| 19. 5678 at | 1 | $2\frac{3}{4}$ | 20. 9271 at | 1 | $4\frac{3}{4}$ | 21. 5461 at | 1 | 7 |
| 22. 8234 at | 1 | $5\frac{1}{2}$ | 23. 5928 at | 1 | $10\frac{1}{2}$ | 24. 8750 at | 1 | 5 |

### V. When the price is any number of shillings under 20.

RULE. (1) *If the price is an even number, multiply the given quantity by half of it, doubling the first figure to the right-hand for shillings, and the rest are pounds.* (2) *If the price is an odd number, find for the greatest even number, as before, to which add the $\frac{1}{20}$th of the given number for the odd shilling, and the sum is the answer.*

Ex. What is the value of 3456 yards of cloth, at 18*s*. per yard?

3456
9
Ans. £. 3110 8

I multiply the given quantity by 9, and the first product 4, I double for shillings, carrying the 5 to the next figure.

Ex. What is the value of 2592 yards of second cloth, at 11*s*. per yard?

1 = $\frac{1}{20}$ | 2592
5
1296 0
129 12
Ans. £. 1425 12

I multiply by 5, as before, which gives the value of the cloth at 10*s*. per yard: I then divide the quantity by 20, and adding this quotient to the last found gives the answer.

#### EXAMPLES.

Ex. 1. 5975 at 2*s*.
2. 4374 at 3*s*.
3. 5916 at 4*s*.
4. 7591 at 5*s*.
5. 6743 at 6*s*.
6. 9430 at 8*s*.
7. 5734 at 10*s*.
8. 5946 at 11*s*.
9. 3004 at 7*s*.
10. 2935 at 13*s*.
11. 4392 at 14*s*.
12. 5931 at 19*s*.
13. 4917 at 18*s*.
14. 3271 at 9*s*.
15. 9315 at 17*s*.
16. 2514 at 16*s*.
17. 1392 at 10*s*.
18. 5432 at 19*s*.

### VI. When the price is shillings and pence.

RULE. (1) *If they are an aliquot part of a pound, divide the quantity by that part, and the quotient is the answer.* (2) *If they are not an aliquot part, multiply by the shillings, and take parts for the pence.*

Ex. What is the value of 2769 yards of Irish, at 3*s*. 4*d*. per yard?

3*s*. 4*d*. | $\frac{1}{6}$ | 2769
£. 461 10*s*.

3*s*. 4*d*. being $\frac{1}{6}$th of a pound, I divide by 6, and the quotient is the answer.

Ex. What is the value of 3756 yards of muslin, at 12*s*. 9*d*. per yard?

| | | |
|---|---|---|
| 6 | ½ | 3756 |
| | | 12 |
| 3 | ½ | 45072 |
| | | 1878 |
| | | 939 |

2.0)4788.9

Ans. £. 2394 9*s*.

I multiply by 12 for the shillings, and 6*d*. being ½ of a shilling, I divide the given quantity by 2; then 3*d*. being ½ of 6*d*., I divide the last quotient by 2, and add the three sums together, which gives the answer in shillings.

### EXAMPLES.

| | | *s.* | *d.* | | | *s.* | *d.* | | | *s.* | *d.* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 8943 at | 2 | 0 | 2. | 3532 at | 4 | 0 | 3. | 8671 at | 7 | 6 |
| 4. | 2524 at | 3 | 9½ | 5. | 5971 at | 5 | 10 | 6. | 5460 at | 9 | 9¾ |
| 7. | 3764 at | 10 | 0 | 8. | 5638 at | 8 | 11 | 9. | 3745 at | 9 | 11 |
| 10. | 8756 at | 15 | 10 | 11. | 3942 at | 4 | 5 | 12. | 2475 at | 16 | 8 |
| 13. | 5642 at | 18 | 4½ | 14. | 1764 at | 5 | 8 | 15. | 5931 at | 17 | 6 |
| 16. | 9143 at | 6 | 8 | 17. | 7189 at | 3 | 7 | 18. | 4604 at | 19 | 6 |

## VII. When the price is pounds and shillings, or pounds, shillings, pence, and farthings.

RULE. *Multiply the quantity by the pounds, and work the rest by the foregoing rules.*

Ex. What is the value of 5428 hogsheads of ale, at 4*l*. 12*s*. per hogshead?

| | |
|---|---|
| 5428 | |
| 4 | 12 |
| 21712 | |
| 3256 | 16 |

Answer, £. 24968 16*s*.

I multiply first by 4, for the pounds; then 12 being an even number, I multiply by the half, or 6, according to Case V., and add the two sums together for the answer.

Ex. What is the value of 2714 cwt. of sugar, at 3*l*. 12*s*. 9½*d*. per cwt.?

| | | | | |
|---|---|---|---|---|
| 10*s*. | ½ | 2714 | | |
| | | 3 | | |
| | | 8142 | | |
| 2*s*. 6*d*. | ¼ | 1357 | | |
| 3*d*. | ⅒ | 339 | 5 | 0 |
| ½ | ⅙ | 33 | 18 | 6 |
| | | 5 | 13 | 1 |

Answer, £. 9877 16 7

Having multiplied by 3 for the pounds, I take the aliquot parts for 12*s*. 9½*d*., that is, 10*s*. is ½, 2*s*. 6*d*. is the 4th of that, 3*d*. is the one-tenth of that, and ½ is the one-sixth of 3*d*.; then, adding the several sums together, I obtain the answer.

| | | | £. | s. | d. | | | | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 5674 | at | 5 | 17 | 6 | 2. | 6431 | at | 4 | 8 | 4 |
| 3. | 3416 | at | 5 | 11 | 6¾ | 4. | 4931 | at | 9 | 4 | 0 |
| 5. | 3146 | at | 10 | 12 | 9 | 6. | 4316 | at | 10 | 19 | 6½ |
| 7. | 5648 | at | 12 | 18 | 0 | 8. | 1436 | at | 10 | 10 | 6 |
| 9. | 1346 | at | 3 | 13 | 4 | 10. | 2714 | at | 18 | 9 | 0 |
| 11. | 9614 | at | 4 | 14 | 6 | 12. | 5789 | at | 7 | 7 | 7 |
| 13. | 1590 | at | 12 | 12 | 0 | 14. | 6341 | at | 8 | 18 | 6 |
| 15. | 4803 | at | 9 | 9 | 9¾ | 16. | 3465 | at | 8 | 15 | 0 |
| 17. | 7182 | at | 11 | 12 | 10 | 18. | 1604 | at | 4 | 11 | 10 |

## VIII. If there be a fraction in the given quantity.

Rule. *Work for the whole number, according to the preceding rules, to which add ¼, ½, ¾, ⅛, &c. of the price, according to the nature of the question.*

Ex. What is the value of 5354¾ cwt. of soap, at 4*l.* 4*s.* 8*d.* per cwt.?

| | | | | |
|---|---|---|---|---|
| 4*s.* | ⅕ | 5354 | ¾ | |
| | | 4 | | |
| | | 21416 | | |
| 8*d.* | ⅙ | 1070 | 16 | |
| | | 178 | 9 | 4 |
| | | 3 | 3 | 6 |
| Answer, £. | | 22661 | 8 | 10 |

| | | £. | s. | d. |
|---|---|---|---|---|
| ¼ | ½ | 4 | 4 | 8 |
| ¼ | ½ | 2 | 2 | 4 |
| | | 1 | 1 | 2 |
| £. | | 3 | 3 | 6 |

I multiply by the pounds, and take parts for the 4*s.* 8*d.*; viz. 4*s.* is the one-fifth of a pound, and 8*d.* is the one-sixth of 4*s.*; then, to find the value of the ¾, I take parts of the given sum, and adding the 3*l.* 3*s.* 6*d.* thus found, to the other for an answer.

| | | | £. | s. | d. | | | | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 4562¼ | at | 3 | 15 | 9½ | 2. | 6744½ | at | 9 | 9 | 10½ |
| 3. | 2654¾ | at | 7 | 15 | 4 | 4. | 7394¾ | at | 12 | 8 | 8¼ |
| 5. | 4651¼ | at | 5 | 12 | 10 | 6. | 3749½ | at | 16 | 9 | 5 |
| 7. | 3875 | at | 8 | 18 | 6¾ | 8. | 4365¾ | at | 11 | 11 | 11 |
| 9. | 9724¼ | at | 6 | 16 | 4½ | 10. | 3648½ | at | 4 | 4 | 6¾ |

## TABLES OF ALIQUOT PARTS.

Aliquot parts of a ton.

| cwt. | qr. | lb. | | |
|---|---|---|---|---|
| 10 | 0 | 6 | = | ½ |
| 5 | 0 | 0 | = | ¼ |
| 4 | 0 | 0 | = | ⅕ |
| 2 | 3 | 12 | = | 1/7 |
| 2 | 2 | 0 | = | ⅛ |
| 2 | 0 | 0 | = | 1/10 |
| 1 | 0 | 0 | = | 1/20 |

Aliquot parts of a cwt.

| qrs. | lb. | | |
|---|---|---|---|
| 2 | 0 | = | ½ |
| 1 | 0 | = | ¼ |
| | 16 | = | 1/7 |
| | 14 | = | ⅛ |
| | 8 | = | 1/14 |
| | 7 | = | 1/16 |

Aliquot parts of a qr. of cwt.

| lb. | | |
|---|---|---|
| 14 | = | ½ |
| 7 | = | ¼ |
| 4 | = | 1/7 |
| 3½ | = | ⅛ |
| 2 | = | 1/14 |
| 1¾ | = | 1/16 |
| 1 | = | 1/28 |

Aliquot pts. of a lb.

| oz. | | |
|---|---|---|
| 8 | = | ½ |
| 4 | = | ¼ |
| 2 | = | ⅛ |
| 1 | = | 1/16 |

IX. When the given quantity is of several denominations.

Rule. *Multiply the given price by the highest denomination, as in Compound Multiplication, and take parts of the price for the inferior denominations of the given quantity.*

Ex. What is the value of 22 cwt. 3 qr. 21 lb. of hops, at 4*l.* 18*s.* 6*d.* per cwt.?

| | | £. | *s.* | *d.* | | |
|---|---|---|---|---|---|---|
| 2 qr. | ½ | 4 | 18 | 6 | | |
| | | | | 11 | | |
| | | 54 | 3 | 6 | | |
| | | | | 2 | | |
| | | 108 | 7 | 0 | = | value of 22 cwt. |
| 1 qr. | ½ | 2 | 9 | 3 | = | ditto 2 qrs. |
| 14 lb. | ½ | 1 | 4 | 7½ | = | ditto 1 qr. |
| 7 lb. | ½ | | 12 | 3¾ | = | ditto 14 lb. |
| | | | 6 | 1¾ | = | ditto 7 lb. |
| Ans. | £. | 112 | 19 | 4 — 1 | | |

Here, for the 22 cwt., I multiply by 11 and by 2; then I take parts for the 3 qrs. 21 lb., according to the preceding table.

| | cwt. | qr. | lb. | | £. | *s.* | *d.* | |
|---|---|---|---|---|---|---|---|---|
| Ex. 1. | 8 | 2 | 12 | at | 4 | 12 | 7 | per cwt. |
| 2. | 16 | 1 | 21 | at | 3 | 13 | 9 | per cwt. |
| 3. | 37 | 3 | 22 | at | 12 | 11 | 7 | per cwt. |
| 4. | 73 | 2 | 10½ | at | 3 | 16 | 9 | per cwt. |
| 5. | 38 | 1 | 16 | at | 2 | 12 | 6 | per cwt. |
| 6. | 33 | 2 | 8 | at | 39 | 3 | 8 | per cwt. |
| 7. | 84 | 3 | 14 | at | 12 | 11 | 8 | per cwt. |

| | | | | | | £. | *s.* | *d.* | |
|---|---|---|---|---|---|---|---|---|---|
| Ex. 8. | 56 tons, | 4 cwt. | 2 qrs. | 0 lb. | at | 58 | 7 | 6 | per ton. |
| 9. | 39 tons, | 12 cwt. | 1 qr. | 14 lb. | at | 25 | 12 | 8 | per ton. |
| 10. | 124 tons, | 16 cwt. | 2 qr. | 16 lb. | at | 12 | 18 | 7 | per ton. |
| 11. | 16 lb. | 8 oz. | 12 dr. | - - | at | 4 | 3 | 6 | per lb. |
| 12. | 25 lb. | 12 oz. | 4 dr. | - - | at | 8 | 12 | 6 | per lb. |
| 13. | 35 lb. | 4 oz. | 12 dwt. | - - | at | 11 | 9 | 9 | per lb.* |
| 14. | 48 lb. | 8 oz. | 16 dwt. | - - | at | 14 | 4 | 4 | per lb. |
| 15. | 25 lb. | 6 oz. | 5 dwt. | - - | at | 15 | 3 | 9 | per lb. |
| 16. | 18 yds. | 2 qr. | 3 nails | - - | at | 0 | 16 | 8 | per yard. |
| 17. | 55 yds. | 2 qr. | 2 nails | - - | at | 1 | 3 | 9 | per yard. |
| 18. | 15 acr. | 3 rd. | 24 per. | - - | at | 38 | 3 | 6 | per acre. |
| 19. | 25 acr. | 1 rd. | 4 per. | - - | at | 46 | 1 | 0 | per acre. |
| 20. | 39 acr. | 2 rd. | 18 per. | - - | at | 5 | 5 | 0 | per acre. |

---

NOTE.

* The aliquot parts of lbs., yards, ells, acres, &c., are easily found, by dividing the integer, or any part of it, by the quantity, the aliquot part of which is required, and the quotient, if there be no remainder, will be the part sought.

# TARE AND TRET.

**Tare and Tret** are a set of practical rules for deducting certain allowances, made by wholesale dealers in selling their goods by weight.

**Gross Weight** is the whole weight of goods, including package, or whatever contains them.

**Neat Weight** is what remains after all allowances are made.

**Tare** is an allowance to the buyer, for the weight of the package, and is either at so much per barrel, chest, &c., or at so much per cwt., or at so much for the whole.

**Tret** is an allowance of 4 lb. in every 104 lb. for waste, dust, &c. or the $\frac{1}{26}$th part of the whole.

**Cloff** is an allowance, after Tare and Tret are deducted, of 2 lb. upon every 3 cwt. that the weight may hold good when sold by retail.

**Suttle** is when only part of the allowance is deducted from the gross. Thus, after the tare is deducted from the gross, the remainder is called tare suttle.

**Case I.** When the tare is at so much for the whole.

**Rule.** *From the gross weight subtract the tare, and the remainder will be the neat weight required.*

Ex. What is the neat weight of 25 barrels of indigo, weighing 116 cwt. 2 qr. 14 lb., allowing 2 cwt. 3 qr. 12 lb. tare?

| | cwt. | qr. | lb. | |
|---|---|---|---|---|
| | 116 | 2 | 14 | |
| | 2 | 3 | 12 | |
| Answer, - | 113 | 3 | 2 | neat weight. |

Ex. 1. What is the neat weight of 55 barrels of figs, weighing 35 cwt. 2 qr. 15 lb., tare being allowed at 1 cwt. 1 qr. 24 lb.?

Ex. 2. What is the neat weight of 20 casks of Russian tallow, weighing 74 cwt., tare being allowed at 2 cwt. 2 qr. 5 lb?

CASE II. When the tare is at so much per barrel, chest, &c.

RULE. *Multiply the tare by the number of hogsheads, barrels, chests, &c., subtract the product from the gross, and the remainder will be the neat weight required.*

Ex. What is the neat weight of 8 hhds. of tobacco, each weighing 4 cwt. 2 qr. 24 lb. gross, tare being allowed at 2 qrs. 4 lb. per hhd.?

| | | | |
|---|---|---|---|
| | cwt. | qr. | lb. |
| | 4 | 2 | 24 |
| | | | 8 |
| Gross weight | 37 | 2 | 24 |
| | 4 | 1 | 4 |
| Answer, - | 33 | 1 | 20 neat weight. |

| | | |
|---|---|---|
| | qr. | lb. |
| | 2 | 4 |
| | | 8 |
| 4 | 1 | 4 Tare. |

Ex. 1. What is the neat weight of 25 frails of Malaga raisins, each weighing 2 cwt. 3 qrs. 12 lb., when the tare upon each frail is 17 lb.?

Ex. 2. In 79 barrels of figs, each weighing 1 cwt. and 12 lb., and tare 9 lb. per barrel, what is the neat weight?

Ex. 3. What is the neat weight of 24 hhds. of tobacco, the weight of each being $4\frac{1}{2}$ cwt., and tare 67 lb. per hhd.?

Ex. 4. In 18 casks of currants, each weighing 6 cwt. 1 qr. 12 lb., and tare 61 lb. per cask, what is the neat weight?

CASE III. When the tare is at so much per cwt.

RULE. *Take the aliquot part or parts of the whole gross weight that the tare is of a cwt., as in Practice, and subtract the result from the gross weight.*

Ex. What is the neat weight of 24 barrels of figs, each weighing 3 cwt. 2 qrs. 12 lb., and tare 12 lb. per cwt.?

| | | | | |
|---|---|---|---|---|
| | cwt. | qr. | lb. | |
| | 3 | 2 | $12 \times 24 = 6 \times 4$ | |
| | | | 6 | |
| | 21 | 2 | 16 | |
| | | | 4 | |
| Gross weight | 86 | 2 | 8 | oz. |
| Tare, - | 9 | 1 | 2 | $13\frac{1}{2}$ |
| Answer, - | 77 | 1 | 5 | $2\frac{1}{2}$ neat weight. |

| | | | | | |
|---|---|---|---|---|---|
| lb. | | cwt. | qr. | lb. | |
| 8 | $\frac{1}{14}$ | 86 | 2 | 8 | |
| | | | | | oz. |
| 4 | $\frac{1}{2}$ | 6 | 0 | 20 | 9 |
| | | 3 | 0 | 10 | $4\frac{1}{2}$ |
| | | 9 | 1 | 2 | $13\frac{1}{2}$ tare. |

Ex. 1. What is the neat weight of 21 barrels of pot-ash, each barrel weighing 1 cwt. 3 qr. 8 lb., tare being 10 lb. per cwt.?

Ex. 2. What is the neat weight of 35 barrels of anchovies, each weighing 1 qr. 12 lb., tare at 14 lb. per cwt. ?

Ex. 3. Required the neat weight of 15 hhds. of tobacco, each weighing 4 cwt. 2 qrs. 12 lb., tare at 20 lb. per cwt.

Ex. 4. What is the value of 26 hogsheads of tobacco, at 8*l.* 8*s.* per cwt., each hogshead weighing 4½ cwt., and the allowance for tare being 13 lb. per cwt. ?

## CASE IV. When there is an allowance both of tare and tret.

RULE. *Find the tare by the last rule, subtract it from the gross weight, the remainder, or suttle, divided by 26, gives the tret, which being subtracted from the suttle, gives the answer.*

Ex. What is the neat weight of 15 casks of tallow, each weighing 6 cwt. 2 qr. 12 lb., tare being 12 lb. per cwt., and tret as usual ?

| | cwt. | qr. | lb. | |
|---|---|---|---|---|
| | 6 | 2 | 12 | × 15 = 5 × 3. |
| | | | 5 | |
| | 33 | 0 | 4 | |
| | | | 3 | |
| Gross weight | 99 | 0 | 12 | |
| Tare - | 10 | 2 | 12 | |
| 26) | 88 | 2 | 0 | |
| | 3 | 1 | 17 | — 6 |
| Answer, | 85 | 0 | 10 | neat weight.* |

| lb. | | cwt. | qr. | lb. | |
|---|---|---|---|---|---|
| 8 | 1/14 | 99 | 0 | 12 | |
| 4 | ½ | 7 | 0 | 8 | — 12 |
| | | 3 | 2 | 4 | — 6 |
| | | 10 | 2 | 12 | — 18 |

Ex. 1. In 18 cwt. 1 qr. 6 lb. gross, tare 63 lb., and tret as usual, how much neat ?

Ex. 2. In 14 casks of raisins, each 2 cwt. 14 lb. gross, tare 18 lb. per cwt., and tret as usual, what is the neat weight ?

Ex. 3. What is the neat weight of 9 cwt. 2 qr. 17 lb. gross, tare 39 lb., and tret as usual ?

Ex. 4. In 9 chests of sugar, each weighing 8 cwt. 2 qr. 10 lb., tare 14 lb. per cwt, and tret as usual, what is the neat weight ?

## CASE V. When Cloff is allowed.

RULE. *Subtract the tare from the gross, and the tret from the tare suttle; then divide the tret suttle by 168, and*

---

NOTE.

* No account is taken of the remainders; tallow in quantities like this never being weighed to a greater nicety than a pound.

*the result will be the Cloff, which being subtracted from the last suttle, gives the neat weight required.**

Ex. What is the neat weight of 19 cwt. 1 qr. 2 lb. gross, tare 3 cwt. 3 qr. 22 lb., and tret and cloff at the usual rate ?

```
              cwt. qr.  lb.             cwt. qr.  lb.
Gross     -   19    1    2            4)14    2   26 ÷ 168 = 4 × 6 × 7
Tare      -    3    3   22            ----------------
          ---------------             6)3     2   20     8 oz.
           26)15    1    8            ----------------------
Tret      -         2   10               7)2   12   12
          ------------------             ----------------
Tret suttle 14      2   26                      9   13 1/7
Cloff      -             9   13 oz.
           ------------------------
Answer, cwt. 14     2   16    3  neat weight.
```

Ex. 1. What is the neat weight of 224 cwt. 3 qr. 20 lb. of tobacco, tare being 25 cwt. 3 qr., tret and cloff as usual ?

Ex. 2. In 14 hhds. of tobacco, each weighing 5 cwt. 3 qr. 17 lb. gross, tare 11 lb. per cwt., and tret and cloff as usual, what is the neat weight ?

Ex. 3. What is the neat weight of 15 casks of currants, each weighing $5\frac{1}{2}$ cwt. gross, tare 35 lb. per cask, tret and cloff as usual ?

Ex. 4. In 9 chests of sugar, each containing 7 cwt. 2 qr. 12 lb. gross, tare 13 lb. per cwt., tret and cloff as usual, what is the neat weight, and what is the value of it at $9\frac{1}{2}$*d.* per lb. ?

---

NOTE.

* In general, the allowance for cloff is 2 lb. for 3 cwt., according to the foregoing definition (p. 129) ; or, what is the same thing, 1 lb. for every 168 lb. : but other allowances of cloff are made in different places. At the Custom-house, on goods imported, 1 lb. is allowed upon goods weighing less than 1 cwt. ; 2 lb. if they weigh from 1 to 2 cwt. ; 3 lb. from 2 to 3 cwt. ; 4 lb. from 3 to 18 cwt. ; and 9 lb. for all higher weights.

There are cases in which an allowance is made for *damage*, that is, so much in the whole for any part of the merchandize which may have received injury.

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† If an unit of any kind, as one pound, or one hogshead, be divided into 100 equal parts, then 65 represents sixty-five of those parts. If a decimal consists of four figures, one or unity is supposed to be divided into 10,000 parts, of which the decimal represents as many as the number expresses : thus, .0625 is so many parts of an unit divided into ten thousand parts : in this case the 0 is placed before the 6, to shew that the unit is divided into 10,000 ; otherwise, if it stood .625, it would appear that it was divided in 1000 parts only.

# DECIMAL FRACTIONS.

1. **Decimal,** or **Decimated Fractions,** are such as always have 1 with one or more cyphers for their denominators. The denominators are never expressed, being understood to be 10, 100, 1000, &c., according as the numerators consist of 1, 2, or 3 figures: thus, instead of $\frac{2}{10}$, $\frac{24}{100}$, $\frac{211}{1000}$, the numerators only are written, with a dot or inverted comma before them, as .2; .24; .211.

2. If a decimal consists of only one figure, *one* is supposed to be divided into ten equal parts, and the decimal represents as many of those parts as the decimal figure expresses; thus, .7 means seven-tenths of an unit: If it consist of two figures, *one* is supposed to be divided into 100 equal parts, of which the decimal represents as many as the figure expresses; thus, .65 means sixty-five hundredths of an unit. († *See Note to this on the opposite page.*)

TABLE.

| Tens of thousands | Thousands | Hundreds | Tens | Units | Tenth parts of unit | Hundredth, do. | Thousandth, do. | Ten thousandth, do. | Hund. thousandth, do. |
|---|---|---|---|---|---|---|---|---|---|
| 7 | 7 | 7 | 7 | 7, | 7 | 7 | 7 | 7 | 7, &c. |
| **Whole Numbers.** | | | | | **Decimals.** | | | | |

By this table the relative values of whole numbers and decimals are at once seen: thus, take the three first figures of each; in *whole numbers* they express seven hundred and seventy seven; in *decimals* they express seven hundred and seventy-seven parts of an unit.

3. Cyphers to the right-hand of decimals cause no difference in their value, for .5; .50; .500, are decimals of the same value, being each equal to $\frac{1}{2}$; that is, $.5 = \frac{5}{10}$; $.50 = \frac{50}{100}$; $.500 = \frac{500}{1000}$; but if the cyphers are placed on the left-hand of decimals, they diminish their value in a ten-fold proportion, thus .3; .03; .003, are 3-tenths,

3-hundredths; 3-thousandths; and answer to the vulgar fractions $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$, respectively.

4. A whole number and decimal is thus expressed, 85.74 which is equal to $85\frac{74}{100} = \frac{8574}{100}$ and $85.04 = 85\frac{4}{100} = \frac{8504}{100}$, &c.

◆ ◆ ◆

## REDUCTION OF DECIMALS.

**Case I.** To reduce a vulgar fraction to a decimal of an equal value.

**Rule.** *Divide the numerator of the fraction, increased by a cypher, or cyphers, by the denominator, and the quotient will be the decimal sought.*

Reduce $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, to decimals of the same value.

$$\tfrac{1}{2} = \frac{1.0}{2} = .5. \qquad \tfrac{1}{4} = \frac{1.00}{4} = .25. \qquad \tfrac{1}{8} = \frac{1.000}{8} = .125.$$

$$\tfrac{1}{16} = \frac{1.0000}{16} = .625.$$

The cyphers added to the numerators are separated from the original figures by a dot, to shew that they are borrowed for the sake of forming the decimal.

Ex. 1. What decimal expressions answer to the following vulgar fractions, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{2}{9}$, $\frac{11}{15}$?

Ex. 2. Required the equivalent decimals of the fractions $\frac{5}{25}$, $\frac{9}{16}$, $\frac{3}{4}$, $\frac{\text{[illegible]}}{11}$, $\frac{9}{18}$.

Ex. 3. What is the decimal that answers to $\frac{1}{64}$?

$$\tfrac{1}{64} = \frac{1.000000}{64} = .015625.^{*}$$

Ex. 4. What are the decimals answering to the fractions $\frac{5}{128}$, $\frac{15}{256}$, and $\frac{45}{2304}$?

Ex. 5. What decimal expressions answer to $\frac{1}{3}$, $\frac{2}{99}$, and $\frac{41}{333}$? *See note at page* 141.

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NOTE.

* As in this example, the numerator requires two cyphers before it is equal to, or larger than, the denominator: a cypher must be prefixed to the figures in the quotient; for in all cases the number of figures in the quotient must be equal to the number of cyphers made use of in the division.

## CASE II. To reduce numbers of different denominations to their equivalent decimal values.

RULE. (1) *Write the given numbers under each other for dividends, proceeding from the least to the greatest.* (2) *Place on the left side of each dividend, for a divisor, the number that will bring it to the next superior denomination.* (3) *Begin with the uppermost number, and set down the quotient of each division, as decimal parts, on the right-hand of the dividend next below it, and so proceed to the last quotient, which is the decimal required.*

Ex. Reduce 12*s*. $3\frac{3}{4}$*d*. to the decimal of a pound.

| | |
|---|---|
| 4 | 3*qrs*. |
| 12 | 3*d*..75 |
| 20 | 12*s*..3125 |
| | .615625 decimal of a £. |

I divide the $\frac{3}{4}$ by 4, supplying cyphers to the 3 by the imagination; the quotient is .75, which is placed by the side of the 3*d*., and then divide the 3.75 by 12; the quotient, .3125, I set by the side of the 12*s*., and divide by 20, which gives .615625 for the answer: that is, if a pound were divided into 1,000,000 parts, the 12*s*. $3\frac{3}{4}$*d*. would be 615625 such parts, in the same manner as if a penny were divided into 100 parts, $\frac{3}{4}$ would be equal to .75 such parts.

Ex. 1. Reduce 8*s*. $4\frac{1}{2}$*d*. to the decimal of a pound.
Ex. 2. What decimal of a pound are 15*s*. $5\frac{3}{4}$*d*.?*
Ex. 3. What decimal of a pound are 4*s*. $6\frac{1}{4}$*d*.?
Ex. 4. Reduce 18*s*. 6*d*., 8*s*. 2*d*., and 5*s*. to decimals of a pound.
Ex. 5. Reduce 5 oz. 6 dwts. 8 gr. troy, to the decimal of a lb.
Ex. 6. Reduce 3 qrs. 7 lb. 8 oz. avoirdupois, to the decimal of a cwt.
Ex. 7. Reduce 2 qrs. 1 n. to the decimal of a yard.
Ex. 8. Reduce 32 bush. 3 p. to the decimal of a chaldron.

## CASE III. To find the value of any given decimal in terms of the integer. This is the reverse of the last case.

RULE. *Multiply the decimal by the number of parts in*

---

NOTE.

* Here the learner will find, that in dividing by 12 there will be no termination to the quotient; it may be carried to any extent, so as to bring the answer as near the truth as he pleases, but it can never be the exact answer. The same thing will occur in the next example. The two sums together, as they stand in the questions, make a pound; but the two decimals, viz. .77395833 + .22604166, will not, when added together, be a pound, but .99999999 of a pound, which wants the hundred millionth part of a pound only; and by carrying on the division in both examples, the answers may be brought indefinitely near the truth.

*the next less denomination, and cut off as many places to the right-hand, as there are places in the given decimal, and so proceed through each denomination.*

Ex. What is the value of .615625 of a pound?

```
 .615625
      20
---------
12.312500
      12
---------
 3.7500
    4
------
 3.00
```

It may be observed, that as cyphers to the right do not alter the value in decimals, they are omitted in each step of the operation.

Answer, - 12*s.* 3¾*d.*

Ex. 1. What is the value of .625 of a shilling?
Ex. 2. What is the value of .1275 of a pound?
Ex. 3. What is the value of .575 of a cwt.?
Ex. 4. What is the value of .875 of a chaldron of coals?*

* * *

## ADDITION OF DECIMALS.

RULE. (1) *Arrange the numbers under each other, according to their several values.* (2) *Find the sum as in Addition of whole numbers, and cut off, for decimals, as many figures to the right as there are decimals in any one of the given numbers.*

Ex. What is the sum of 23.45, 7.849, 543.2, 8.6234, and 253.004?

```
 23.45
  7.849
543.2
  8.6234
253.004
--------
```

Answer, 836.1264

Ex. 1. What is the sum of 37.035, 4.26, 598.034, 9.3076, 4.321, and 5?

Ex. 2. Find the value of 39.33, 4.2056, .98735, 46.287, 3.7491, and 3.004.

---

NOTE.

* To these examples may be added the answers of the questions in Case II., as in example 3, we say, what is the value of .22604166 of a pound?

## SUBTRACTION OF DECIMALS.

Rule. *Arrange the numbers according to their value; subtract, as in whole numbers, and cut off, for decimals, as in Addition.*

Ex. Subtract 35.87043 from 132.005.

| | |
|---|---|
| | 132.005 |
| | 35.87043 |
| Answer, | 96.13457 |

Ex. 1. What is the difference between 104.326 and 74.05?
Ex. 2. Find the difference between 394.832 and 148.0076.
Ex. 3. From 372.971 take 270.30041.

## MULTIPLICATION OF DECIMALS.

Rule. *Multiply as in whole numbers, and cut off as many figures from the product as there are decimals in the multiplier and multiplicand.*

Ex. Multiply .025 by .045: also 4.82 by 3.53.

| | |
|---|---|
| .025 | 4.82 |
| .045 | 3.53 |
| 125 | .1446 |
| 100 | 2.410 |
| .001125 | 14.46 |
| | 17.0146 |

In the first instance, there being but four figures in the product, and six decimals in the multiplier and multiplicand, two cyphers must be added to the left hand of the product.

Ex. 1. Multiply 76.43 by .875: also .897 by .452.
Ex. 2. Multiply 324.004 by .7872.
Ex. 3. What is the product of 9.57 and .074?
Ex. 4. Multiply .643 by .389.*

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NOTE.

* When the number of decimals in the multiplicand is large, and it is not wished to carry the operation to more than a certain number of decimals in the product, it is done by the following Rule, which I shall illustrate by an example.

Rule. Having arranged the multiplicand, count as many figures from the decimal point, as you intend to keep decimals in the product, and make a * over the last of these, under which, after you have inverted the multiplier, place the units figure of the multiplier thus inverted, and the others in their proper order. Then multiply each figure of the inverted multiplier, beginning, as usual, at the right-hand, and set down the respective products, so that the right-hand figures may fall in a strait line under one another. In multiplying, no attention is to be paid to the figures on the right-hand of that which you multiply by, unless it be with the two preceding figures, to find what number should be carried.

Ex. Required the product of 1.570796, multiplied by 26.3719, with four places of decimals in the product. This, in the usual method, would yield ten places of decimals: by contraction it is thus performed.

|  |  |
|---|---|
| * |  |
| 1.570796 |  |
| 9.17362 |  |
| 3.14159 | = product with 2 regard being had to 2 × 6 |
| 94247 | = —— —— 6 —— —— —— 6 × 9 |
| 4712 | = —— —— 3 —— —— —— 3 × 7 |
| 1099 | = —— —— 7 —— —— —— —— |
| 15 | = —— —— 1 —— —— —— —— |
| 14 | = —— —— 9 —— —— —— 9 × 57 |
| 41.4246 |  |

We will now work the example in the common way.

|  |  |
|---|---|
| 1.570796 |  |
| 26.3719 |  |
| 14 | 137164 |
| 15 | 70796 |
| 1099 | 5572 |
| 4712 | 388 |
| 94247 | 76 |
| 314159 | 2 |
| 41.4248 | 750324 |

From this it will appear plain, why in the contracted form the multiplier is inverted: the last product here being the first there. In the contracted form, the units place is 6; it would however be 8, if the 2 were carried from the 27, obtained in the next line by Addition.

Ex. 2. Multiply 128.678 by 38.24, so as to have but one place of decimals.

| Common method. | Contracted method. |
|---|---|
|  | * |
| 128.678 | 128.678 |
| 38.24 | 42.83 |
| 514712 | 38603 |
| 257356 | 10294 |
| 1029424 | 257 |
| 386034 | 51 |
| 4920.64672 | 4920.5 |

# DIVISION OF DECIMALS.*

**Rule.** (1) *Divide, as in whole numbers, and cut off as many figures in the quotient, as the decimal places in the dividend exceed those of the divisor.* (2) *If there be not figures enough in the quotient, the deficiency must be supplied by prefixing cyphers.* (3) *If there be a remainder, or there be more decimal places in the divisor than in the dividend, cyphers may be affixed to the dividend, and the quotient carried on to any extent.*

### Divide 1.7154 by 1.5; and .37046 by 16.

| | |
|---|---|
| 1.5)1.7154 | 16).37046 |
| 1.1436 | .02315375 |

In the first example, by supplying a single cypher there is no remainder left; but in the second I must supply three cyphers to obtain an even answer; and I find the quotient has one figure less than there are decimals in the dividend so supplied, I must therefore prefix a cypher to the quotient found.

Ex. 1. Divide 25.64 by 3.645.
Ex. 2. Divide 4752 by .9587.
Ex. 3. Divide .865439 by .156.
Ex. 4. Divide 79 by 3965.
Ex. 5. Divide 33.64472 by .882.
Ex. 6. Divide .218 by 7.435.
Ex. 7. Divide 76.42 by 58.
Ex. 8. Divide 88 by .88.

### To find, by inspection, the value of any decimal of a pound sterling.

**Rule.** *Double the first figure for shillings, and if the second figure be 5, or more than 5, add one shilling for that: then reckon the remaining figures in the second and third*

---

NOTE.

* The Contracted method of Division may be thus performed.

Rule. Having determined how many places of whole numbers there will be in the quotient, if any, which is easily known by inspection; if there are none, then consider of what value the first figure in the quotient will be, and proceed as in common Division, only omitting one figure of the divisor at each operation; viz., for every figure of the quotient dot off one in the divisor, remembering *places so many farthings, deducting* one *when the farthings are above* 12, *and* two *when they are more than* 37.

Ex. 1 and 2. What are the values of .125*l.*, and .983*l.* ?

1st Example.

2*s.* 0*d.* = double 1

0 6 = 24 farthings, one being taken as by rule.

0 2 6 £.

2d Example.

18*s.* 0*d.* = double of 9,

1 0 = for the 5 out of 8 in the place of tenths.

0 8 = 32 farthings, one being deducted as by rule.

£. 0 19 8

EXAMPLES.

Ex. 3, 4, 5, and 6. Required the value of .375*l.*; .768*l.*; .494*l.*; and .396*l.*

Ex. 7. Find the value of the following decimals by inspection, and their sum: viz. .567*l.* + .804*l.* + .395*l.* + .061*l.* + .020*l.* + .009*l.*

Ex. 8. What is the value of the following decimals and their difference, .794*l.* and .175*l.* ?

Ex. 9. What is the value of the tenth part of .366. ?

---

NOTE.

to carry for the increase of the figures cut off, as was done in Multiplication.

Ex. Let it be required to divide 23.41 by 7.9863.

| Contracted method. | Common method. | |
|---|---|---|
| 7.9863)23.4100(2.9312 | 7.9863)23.4100 | (2.9312 |
| ..... 15.9726 | 15.9726 | |
| .74374 | .74374 | 0 |
| 71876 | 71876 | 7 |
| .2497 | .2497 | 30 |
| 2395 | 2395 | 89 |
| .101 | .101 | 410 |
| 79 | 79 | 863 |
| .21 | .21 | 5470 |
| 15 | 15 | 9726 |
| .5 | .5 | 5744 |

Here it must be observed, that in each of the subtractions, except the first, unit must be carried to the first figure, as would be the case in the usual course.

### To reduce, by inspection, shillings, pence, and farthings, to an equivalent decimal of a pound.

RULE. *When the number of shillings are even, write half the number for the first decimal; and when the number of shillings are odd, for the remaining shilling write* 5 *in the second place. Reduce the pence and farthings to farthings, and write them in the second and third places, observing to increase the figure in the third place by* 1, *when the farthings are* 12, *or more than* 12, *and by* 2 *when they are* 36 *and upwards.*

Ex. 1. Reduce 14*s*. $6\frac{1}{2}$*d*., and 17*s*. $11\frac{3}{4}$*d*., to the decimals of a pound?

14 0 = .7
0 $6\frac{1}{2}$ = .026
excess of 12 = .001
———
.727

16 0 = .8
1 0 = .05
0 $11\frac{3}{4}$ = .047
excess of 36 = .002
———
.899

Ex. 2. Reduce the following sums to decimals of a pound, by inspection, viz. 16*s*.; 13*s*. 6*d*.; 15*s*. $9\frac{1}{2}$*d*.; 11*s*. $8\frac{1}{4}$*d*.; 6*s*. 8*d*.; 7*s*. 6*d*.; 8*s*. $4\frac{3}{4}$*d*.; and 19*s*. $11\frac{3}{4}$*d*.

---

NOTE.

† The answers to the fractions, Ex. 5, p. 134, will be .333, &c., and .020202, &c., and .123123123, &c. In the first example, one figure, or the .3 is repeated; in the second there are two figures repeated, and in the third there are three figures repeated. These are called *circulating* decimals; and the circulating figures are called *repetends*; if one figure only repeats, it is called a *single* repetend; if more than one repeats, it is a *compound* repetend. There are many other varieties, but they are of too little importance to be introduced into a work which excludes every thing that is not practically useful. Repetends are reduced to fractions by making the repetend the numerator, and for the denominator put as many 9's as there are figures in the repetend: thus $.333 = \frac{333}{999} = \frac{1}{3}$; $.02$ &c. $= \frac{2}{99}$; and $.123$, &c. $= \frac{123}{999} = \frac{41}{333}$.

# INVOLUTION.

INVOLUTION is a method of raising numbers to higher powers.*

A *power* is the product arising from multiplying any given nnmber into itself once, or oftener: thus, $3 \times 3 = 9$ is the second power of 3, and it is denoted in this manner $3^2$.

The number denoting the power is called the *index*, or *exponent* of that power: thus, in $3^2$ the 2 is the index or exponent.

The third power of 4 is $4^3 = 4 \times 4 \times 4 = 64$.

The fourth power of 3 is $3^4 = 3 \times 3 \times 3 \times 3 = 81$.

The sixth power of 5 is $5^6 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 15625$.

The third power of $\frac{1}{4}$ is $\frac{1}{4}|^3 = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$.

The fifth power of .03 is $.03^5 = .03 \times .03 \times .03 \times .03 \times .03 = .0000000243$.

### EXAMPLES.

Ex. 1. What is the sixth power of 6?
Ex. 2. What is the eighth power of 7?
Ex. 3. What is the fourth power of $\frac{3}{5}$?
Ex. 4. What is the fifth power of $\frac{7}{8}$?
Ex. 5. What is the third power of .25?
Ex. 6. What is the fourth power of .05?
Ex. 7. What is the third power of .305?
Ex. 8. What is the ninth power of .9?

*Note.* The 2d power is called the *square*; the 3d is called the *cube*; the 4th is called the *biquadrate*; the rest are generally denominated by the numbers, as the *fifth*, *sixth*, *seventh*, &c.

Ex. 9. What are the squares of 3 and 6; 5 and 10; 6 and 12; 2, 4, 8, and 16?†

Ex. 10. What are the cubes of 3 and 6; 5 and 10; 6 and 12; 2, 4, 8, and 16?‡

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NOTES.

* This rule, when the powers are high, is performed best by means of logarithms, which see further on.

† The solution of these several examples, will lead the pupil to the knowledge of this fact, viz. that *the square of any number is four times as great as the square of half that number:* thus, the square of 6 is 36, which is four times 9, the square of 3: and the square of 10 is 100; but the square of 5 is only 25.

‡ Here we find that *the cube of any number is eight times as great as the cube of half that number.*

# EVOLUTION.

**Evolution** is the method of extracting roots.*

The root of any number, or power, is such a number, as being multiplied into itself once, or oftener, produces that power: thus 3 is the square root of 9, because 3 multiplied into itself gives 9: 4 is the cube root of 64, because 4 multiplied into itself twice, gives 64. The roots are denoted by indices, or exponents, in this manner:

The cube root of 125 is $\sqrt[3]{125} = 5$.

The square root of 81 is $\sqrt[2]{81} = 9$.

The fifth root of 243 is $\sqrt[5]{243} = 3$.†

Ex. 1. What are the square roots of 49 and 64?
Ex. 2. What are the cube roots of 216, 343, 512, and 729?
Ex. 3. What are the fourth roots of 625, 2401, and 4096?
Ex. 4. What are the fifth roots of 3125 and 32768?

## To extract the square root.

Rule. (1) *Divide the given number into periods of two figures each, by placing a dot over units, another over hundreds, and so on.* (2) *Find the greatest square in the first period, and set its root on the right-hand, as a quotient figure in division.* (3) *Subtract the square thus found, and to the remainder annex the succeeding period for a new dividend.* (4) *Double the root for a divisor, and examine how often it is contained in the dividend, exclusive of the place of units, and put the result into the quotient and in the units place of the divisor.* (5) *Multiply the divisor thus increased by the new quotient figure, and subtract the*

---

NOTES.

* This is better done by means of logarithms, which see further on.

† It is evident from these examples, and others that follow, that the root of a *power* of a given number may be found exactly; but there are many numbers, the roots of which can never be accurately determined, as the square-root of 2, 3, 5, &c., because no numbers multiplied into themselves will give 2, 3, 5; but by the help of decimals, the roots of these and of any others may be attained to any degree of exactness.

*product from the dividend.* (6) *Bring down the next period find a divisor as before, by doubling the figures already in the root, and proceed as before.*

The rule will be rendered clear by the following examples:

What are the square roots of 16777216 and 43046721?

```
   . . . .
  16777216(4096
  16
  ----
809)··7772
      7281
     -----
 8186)49116
      49116

   . . . .
  43046721(6561
  36
  ----
 125)704
     625
    ----
 1306)7967
      7836
     -----
 13121)13121
       13121
```

In the first example, having pointed off the periods, I find that the first period of 16 is the square of 4; the 4 I put in the quotient, and its square under the period; as there is no remainder, I bring down the second period 77, and for a new divisor I double the root, 4, found; but the 8 being greater than 7, I bring down the next period, and put a cypher in the quotient; the double of the root is now 80, and this will go 9 times in 777, the 9 I place in the quotient, and also in the divisor, which is now 809; this multiplied by the 9, and the product subtracted from the dividend, leaves 491, to which I bring the other period, and double the root 409 for a new divisor, which is contained 6 times in the four first figures of the dividend, and proceed as before.

The mode of working the second example is evident.

### EXAMPLES.

Ex. 1. What is the square root of 117649?
Ex. 2. What is the square root of 262144?
Ex. 3. What is the square root of 531441?
Ex. 4. What is the square root of 1679616?
Ex. 5. What is the square root of 5764801?
Ex. 6. What is the square root of 1073741824?
Ex. 7. What is the square root of 119550669121?
Ex. 8. What is the square root of 20?*
Ex. 9. What is the square root of 300?
Ex. 10. What is the square root of 1000?
Ex. 11. What are the square roots of $\frac{1}{2}$ ; $\frac{6}{36}$ ; $\frac{25}{144}$?
Ex. 12. What is the square root of .25?

---

NOTE.

* This and the following examples require additional periods of cyphers, in order to be carried to any degree of accuracy: for 4 is less than the root of 20, and 5 is greater than the root, the square

## MISCELLANEOUS EXAMPLES.

Ex. 1. A gentleman desirous of making his kitchen garden, which is to contain 4 acres, a complete square, I demand what will be the length of the side of the garden?

Ex. 2. Six acres of ground are to be allotted to a square garden; but for the sake of more wall for fruit, there is to be a smaller square within the larger, which is to contain 3 acres, I demand the length of the sides of each square?

Ex. 3. What is the mean proportional between 12 and 75?*

Ex. 4. How long must a ladder be to reach a window 30 feet high, when the bottom stands 12 feet from the house?†

## To extract the cube root.

I. **Rule.** (1) *Find, by trials, the nearest cube to the given number, and call it the assumed cube.* (2) *Say, as twice the assumed cube added to the given number, is to twice the number added to the assumed cube, so is the root of the assumed cube to the root required nearly.*

### What is the cube root of 27455?

Here the nearest root that is a whole number is 30, the cube of which is 27000: therefore I say,

As 27000 × 2 + 27455 : 27455 × 2 + 27000 : : 30
or 81455 : 81910 : : 30 : 30.1675.

---

NOTES.

of 4 being 16, and the square of 5 being 25: it must therefore be worked thus:

```
      20(4.4721 &c.
      16
     ----
  84) 400
      336
     -----
 887) 6400
      6209
     ------
8942) 19100
      17884
     -------
8944) 121600
```

By multiplying the root 4.4721 into itself, the answer will be 19.99996, &c., which is very nearly, though not quite, equal to 20; and by carrying the operation still further, greater accuracy would be obtained.

* This is found by multiplying the given numbers together, and taking the square root of the product.

† Square the given numbers, and take the square root of their sum.

It is evident that the true root, omitting the last two figures, is somewhere between 30.16 and 30.17, the former being too little, the latter something too large. By taking the root thus found 30.16, as the assumed cube, and repeating the operation, the root will be had to a still greater degree of exactness.

Ex. 1. What is the cube root of 15625?

Ex. 2. Whrt is the cube root of 140608?

Ex. 3. What is the cube foot of 444194947?

Ex. 4. What is the cube root of the difference between 140608 and 14625?

II. RULE 1. *Separate the given number into periods of three figures each, beginning from units place; then from the first period subtract the greatest cube it contains, put the root as a quotient, and to the remainder bring down the next period for a dividend.* 2. *Find a divisor by multiplying the square of the root by 300, see how often it is contained in the dividend, and the answer gives the next figure in the root.* 3. *Multiply the divisor by the last figure in the root. Multiply all the figures in the root by 30, except the last, and that product by the square of the last. Cube the last figure in the root. Add these three last found numbers together, and subtract this sum from the dividend; to the remainder bring down the next period for a new dividend, and proceed as before.*

Ex. 5. What is the cube root of 444194947?

```
                                  .  .  .
                             444194947(763   Answer.
                             343
                             ------
  7 × 7 × 300 = 14700)101194
                               95976
                             --------
76 × 76 × 300 = 1732800)5218947        1732800 = divisor
    14700 = divisor          5218937              3
        6                    -------   -----------
  -------                    .......   5198400
    88200                                20520 = 76 × 30 × 9
     7560 = 7 × 30 × 36                     27 = 3 × 3 × 3
      216 = 6 × 6 × 6                  -------
  -------                              5218947
    95976                              -------
```

Ex. 6. What is the cube root of 46656?

Ex. 7. What is the cube root of 65939264?

Ex. 8. What is the cube root of 3, carried to 2 places of decimals?

Ex. 9. What is the cube root of $\frac{8}{512}$?

Ex. 10. What is the cube root of $\frac{125}{1728}$?

Ex. 11. What is the cube root of .729?

Ex. 12. What is the cube root of .003375?

### MISCELLANEOUS EXAMPLES.

Ex. 1. A corn-factor requires a cubical bin that shall hold 840 bushels of wheat: I demand the inside length of one of its sides. See Table, p. 49, note.

Ex. 2. In a cubical building that measures 2744 feet, what is the length of a side?

# ARITHMETICAL PROGRESSION.

*When a series of numbers increases or decreases by some common excess, or common difference, it is said to be in arithmetical progression, such as* 1, 3, 5, 7, 9, *&c., and* 12, 10, 8, 6, 4, *&c.*

*The numbers which form the series are called the terms of the progression; of these the first and last are called the extremes.*

| | |
|---|---|
| The first term is called . . | $a$ |
| The last term is called . . | $z$ |
| The number of terms is called . | $n$ |
| The common difference is called | $d$ |
| The sum of all the terms is called | $s$.* |

Any three of these terms being given, the others may be easily found.

### I. When the first term $a$, and the last term $z$, and the number of terms $n$, are given, to find the sum of all the terms, $s$.

RULE. *Multiply the sum of the extremes by the number of terms, and divide by 2, the quotient is the answer:* or

$$\overline{a+z} \times \frac{n}{2} = s.$$

Ex. 1. What is the sum of the terms of an arithmetical series, whose first term is 5, last term 29, and the number of terms 7?

Here $s = \overline{5+29} \times \frac{7}{2} = 34 \times \frac{7}{2} = \frac{238}{2} = 119$, the answer.

---

NOTE.

* These symbols will be easily remembered; $a$ and $z$ being the first and last letters of the alphabet, may properly represent the first and last terms of any series; the reason of the other letters is sufficiently obvious.

Ex. 2. The first and last terms of a series are 3 and 111, and the number of terms 37 : what is the sum?

Ex. 3. How many strokes do the clocks of Venice strike in 24 hours, where they strike from 1 to 24?

Ex. 4. The first and last terms of a series are 1 and 1000, and the number of terms 100: required the sum.

Ex. 5. If 100 stones are placed in a right line, exactly a yard asunder, and the first one yard from a basket, what length of ground will a man go over, who gathers them up, one by one, returning with each to the basket?

Ex. 6. What must a man give for 54 timber trees, for which he pays 5 shillings for the first, and 20*l.* for the last, and the prices of the others being in arithmetical progression?

Ex. 7. A butcher buys a drove of oxen, consisting of 32; for the first he pays 15*s.*, and for the others he is to pay in arithmetical progression, so that for the last he is to pay 38*l.*: what will they all come to?

Ex. 8. A horse-dealer sends to a fair 63 horses, of various kinds and worth, which he is willing to dispose of according to the principles of arithmetical progression, demanding 3*l.* only for the first, provided he had 53*l.* for the last: how much did he receive for the whole, and what was the average value of each horse?

## II. The first and last terms, *a* and *z*, and number of terms being given, to find the common difference, *d*.

RULE. *The difference of the extreme terms divided by the number of terms, less 1, will be the common difference sought:*

or $\dfrac{a \backsim z^{*}}{n - 1} = d.$

Ex. 1. What is the common difference of an arithmetical progression, whose extremes are 8 and 200, and the number of terms 17?

$$d = \frac{8 \backsim 200}{16} = \frac{200 - 8}{16} = \frac{192}{16} = 12.$$

Ex. 2. When the extremes of an arithmetical progression are 6 and 57, and the number of terms 18, what is the common difference?

Ex. 3. A gentleman gives at Christmas, among his 25 poor neighbours, a sum of money in arithmetical progression: to the least needy he gives 5 shillings, and to the poorest, with a very large family, he gives five guineas: what was the common difference?

---

NOTE.

* This mark $\backsim$, is put for the difference of any two numbers, which ever of them is largest; thus, suppose $a$ be 10, and $z$ 4, then $a \backsim z = 6$, or it may be, $a$ is 4, and $z$ 15; then $a \backsim z = 11$.

Ex. 4. A traveller is out on his journey a month, of which he travels 25 days; on the first he rides 7 miles, and on the last, having little to do, he comes 43 miles: how much was the daily increase of his travelling, and how many miles did he ride in the whole?

## III. The extreme terms *a* and *z*, and common difference *d* being given, to find the number of terms *n*.

Rule. *Divide the difference of the extremes by the common difference, and the quotient increased by unity is the number sought:* or $\frac{a \backsim z}{d} + 1 = n.$

Ex. 1. When the extremes are 4 and 106, and the common difference is 3, what is the number of terms?

$$\frac{4 \backsim 106}{3} + 1 = \frac{106 - 4}{3} + 1 = \frac{102}{3} + 1 = 34 + 1 = 35 = n.$$

Ex. 2. If the least term be 6, the greatest 216, and the common difference 5, what is the number of terms?

Ex. 3. What debt can be paid, and in what time, supposing I agree to lay by 3*s.* the first week, 7*s.* the next, 11*s.* the third, and so on in arithmetical progression, till the last saving be four guineas?

Ex. 4. I set out for Hastings, which is 69 miles from this place, and I walk the first day 4 miles, the second 7, increasing every day by 3 miles, and on the last 19 miles: how many days will the journey take?

In addition to the above, the learner may commit to memory the following facts on the subject:

1. *If three numbers are in arithmetical progression, the sum of the extremes is equal to double the mean term;* as 6, 9, 12, where $6 + 12 = 2 \times 9 = 18$.

2. *If four numbers be in arithmetical progression, the sum of the two extremes is equal to the sum of the means;* as 5, 8, 11, 14, where $5 + 14 = 8 + 11 = 19$.

3. *When the number of terms is odd, the double of the middle term will be equal to the sum of the extremes; or of any other two means equally distant from the middle term;* as 3, 8, 13, 18, 23, 28, 33, where $3 + 33 = 2 \times 18 = 13 + 23 = 8 + 28$.

# GEOMETRICAL PROGRESSION.

*A* GEOMETRICAL PROGRESSION *is a series of numbers, the terms of which gradually increase or decrease by the constant multiplication or division of some particular number;* as 1, 3, 9, 27, 81, 243, &c., or 64, 32, 16, 8, 4, 2, 1, $\frac{1}{2}$, &c.

In the *first* case, the series is *increasing* by the constant multiplication of 3; in the *second*, it is a *decreasing* series by the constant division of 2. It is evident that both series may be carried on for ever.

The number by which the series is constantly increased or diminished is called the *ratio*.

| | |
|---|---|
| The first term is called . | $a$ |
| The last term is called . | $z$ |
| The number of terms is called . | $n$ |
| The ratio is called . . . . | $r$ |
| The sum of all the terms is called | $s$. |

Any three of these terms being given or known, the others may be determined.

### I. Given the first term $a$, the last term $z$, and the common ratio $r$, to find the sum $s$.

RULE. *Multiply the last term by the ratio, and from the product subtract the first term, and the remainder divided by the ratio, less one, will give the sum of the series;* or

$$\frac{z \times r - a}{r - 1} = s.$$

Ex. 1. The first term of a series in geometrical progression is 5, the last term is 3645, and the ratio is 3: what is the sum?

$$\text{Here } s = \frac{3645 \times 3 - 5}{3 - 1} = \frac{10935 - 5}{2} = \frac{10930}{2} = 5465.$$

For the terms are 5, 15, 45, 135, 435, 1215, and 3645; which, being added together, make 5465.

Ex. 2. The first and last terms of a geometrical series are 4 and 3294172, and the common ratio is 7: what is the sum?

Ex. 3. The first and last terms of a geometrical progression are 4 and 262144, and the ratio 4: what is the sum?

## II. Given the first term $a$, the number of terms $n$, and the ratio $r$, to find the last term $z$.

The last term may be obtained by continual multiplication; but as that, in a long series, is a tedious process, we shall give the following rule:

### 1. When the first or least term is equal to ratio.

RULE. *Write down some of the leading terms of the geometrical series, over which place the arithmetical series 1, 2, 3, 4, &c., as indices;* find what figures of these indices added together will give the index of the term wanted in the geometrical series; then multiply the numbers, standing under such indices, into each other, and their product will be the term sought.*

Ex. 1. What is the last term of a geometrical series having 13 terms, of which the first is 2, and the ratio 2?

Here the series, with their indices, will stand thus:

$$2^1,\ 4^2,\ 8^3,\ 16^4,\ 32^5,\ 64^6,\ \&c.$$

The number of terms being 13, the index to the last term will be 13 equal to the indices 2 + 5 + 6, which figures standing over 4, 32, and 64, shew that these last are to be multiplied together, and the product is the term sought; thus $4 \times 32 \times 64 = 8192$.

Ex. 2. What is the last term of the series having 9 terms, of which the first is 3, and the ratio 3?

Ex. 3. What did the last of 12 oxen cost, the first of which was sold for 3*s.*, the second for 9*s.*, and so on?

### 2. When the first term $a$, of the series, is *not* equal to the ratio $r$.

RULE. *Write down the leading terms of the series, and place their indices over them, beginning with a cypher, add together the most convenient indices to make an index less one than the number expressing the place of the term sought; then multiply the numbers standing under such*

---

NOTE.

* When the natural numbers 1, 2, 3, 4, 5, &c., are set over a geometrical series, they are called *indices*, or *exponents*, and they shew the distance of any term from unity, or from the first term: thus, in the series $2^1$, $4^2$, $8^3$, $16^4$, $64^5$, $128^6$, &c., 1, 2, 3, &c. are the indices, and shew the distance of any term of the series from the first term; the index 5, for instance, shews that 64 is the fifth term in the series.

*indices, into each other, dividing the product of every two by the first term in the geometrical series; the last quotient is the term required.*

Ex. 1. What is the last term of the series, whose first term is 4, ratio 3, and number of terms 15 ?

$$4^0,\ 12^1,\ 36^2,\ 108^3,\ 324^4,\ 972^5,\ 2916^6,\ \&c.$$

The number of terms being 15, the index sought must be 14 equal to $6 + 5 + 3$, under which stand the terms 2916, 972, and 108, then $\frac{2916 \times 972}{4} = 708588$, and $\frac{708588 \times 108}{4} = 19131876 = z =$ last term.

Ex. 2. The first term of a geometrical series is 2, the number of terms 12, and the ratio 5, required the last term ?

Ex. 3. The first term of a geometrical series is 1, the ratio 2, and the number of terms 25, what is the last term, and also the sum of all the terms ?

Ex. 4. The first term of a series is 5, the ratio 3, and the number of terms 16, what is the last term, and the sum of the terms ?

Ex. 5. A hosier sold 12 pair of stockings, the first pair at 3*d.*, the second 9*d.*, and so on in geometrical progression; for what did he sell the last pair, and how much had he for the whole ?

Ex. 6. What would a horse fetch, supposing it was sold on condition of receiving for it one farthing for the first nail in his shoes, a halfpenny for the second, one penny for the third, and so on, doubling the price of every nail to 32, the number in his four shoes ?

Ex. 7. A husbandman agreed to serve his master during hay-time and harvest, or five-and-forty clear days, provided he would give him a barley-corn only for the first day's work, 3 for the second, 9 for the third, and so on in geometrical proportion; what would he have to receive in money for his labours, supposing there were half a million of grains in a bushel, and each bushel was worth 4*s.* ?

The following facts may be committed to memory :

1. *If three numbers are in geometrical progression, the product of the extremes is equal to the square of the means;* as 3, 9, 27, here $3 \times 27 = 9 \times 9 = 81$.

2. *If four numbers are in geometrical progression, the product of the extremes is equal to the product of the means;* as 2, 4, 8, 16; here $2 \times 76 = 4 \times 8 = 32$.

3. *If the series contain an odd number of terms, the square of the middle term is equal to the product of the adjoining extremes, or of any two terms equally distant from them;* as 3, 9, 27, 81, 243; here $27^2 = 3 \times 243 = 9 \times 81$.

# LOGARITHMS.

Logarithms are artificial numbers, invented for the purpose of facilitating certain tedious arithmetical operations.

If any series of numbers in arithmetical progression beginning with 0, be taken, and a corresponding series of geometrical numbers beginning with 1, the former series will be logarithms to the corresponding numbers in the latter; thus,

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0, | 1, | 2, | 3, | 4, | 5, | 6, | 7, | 8, | 9 | logarithms. |
| 1, | 2, | 4, | 8, | 16, | 32, | 64, | 128, | 256, | 512 | numbers. |

Here 0, 1, 2, &c. are the logarithms of 1, 2, 4, &c., and it will be seen at once, 1. That *Addition in logarithms answers to Multiplication in common numbers:*

Thus, if the logarithms 2 and 6 are added together, the sum is 8 which answers to the logarithm of 256, the number that is obtained by the multiplication of 4 and 64, which are the numbers standing under the logarithms 2 and 6. By adding the logarithms 4 and 5 we have 9, which stands over 512, the number obtained by multiplying together 16 and 32. Hence the addition of logarithms answers to multiplication in common numbers.

2. *Subtraction in logarithms answers to division of common numbers.*

Divide 256 by 8, and you have 32, over which stands $5 = 8 - 3$; the logarithms standing above.

3. *Multiplication in logarithms answers to involution of common numbers.*

Ex. The square of 8 is 64; now 3 is the logarithm answering to 8, and 3 × 2, (because 2 is the index of the square,) is equal to 6, which is the logarithm of 64.

4. *Division in logarithms answers to evolution in common arithmetic.*

Ex. 1. The square root of 256 is 16, over which stands the logarithm 4; which answers to $8 \div 2$, 8 being the logarithm of 256.

Ex. 2. The cube root of 512 is 8; and 9, which is the logarithm of 512, divided by 3, the sign of the cube, gives 3, which is the logarithm of 8.

The same indices will serve for any geometric series; but the logarithms generally made use of are those which increase in a tenfold proportion, as

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0. | 1. | 2. | 3. | 4. | 5. | 6. | &c. |
| 1. | 10. | 100. | 1000. | 10000. | 100000. | 1000000. | |

Here it is evident, that the logarithms of numbers between 1 and 10, are greater than 0, and less than one, as will be seen in the table at the end of the volume, thus the logarithms of 2, 6, 8, &c. are, .3010300, .7781513, .9030900, &c.

The logarithms of the numbers between 10 and 100, are greater than 1, and less than 2 ; thus the logarithm of 15 is 1.1760913, and the logarithm of 95 is 1.9777236.

The logarithms of numbers between 100 and 1000, are greater than 2, and less than 3 ; thus the logarithm of 165 is 2.2174839, and of 984 is 2.9929951.

The logarithms between 1000 and 10000, must be somewhere between 3 and 4 ; between 10.000 and 100.000 they must be between 4 and 5 ; and so on.

The logarithms in the above series are called indices, which are frequently neglected, the decimal part only being put down ; thus, if it be required to find the logarithm of 248, it will be sufficient to put down .3944517, and the number being between 100 and 1000, I know the index is 2. Therefore the rule for finding the index is his :

*The index is always one less than the number of figures in the whole number ; or the figures in the whole number must be always one more than the index.*

| | | | |
|---|---|---|---|
| The logarithm of | 248 | is | 2.3944517 |
| — — — — | 2480 | — | 3.3944517 |
| — — — — | 24800 | — | 4.3944517 |
| — — — — | 24.8 | — | 1.3944517 |
| — — — — | 2.48 | — | 0.3944517 |
| — — — — | .248 | — — | 1.3944517 |
| — — — — | .0248 | — — | 2.3944517 |
| — — — — | .00248 | — — | 3.3944517 |

Here the decimal figures remain the same ; and the only difference is in the indices, which are increased or diminished by unit for every ten-fold increase or decrease of the whole number. It will be observed, that where there is but one whole number, the index will be 0 ; but if the figures be decimals, as .248, the index is minus one, or—1 ; if by the prefixing 0 to the decimal figure, their value is diminished in a ten-fold proportion, then the index is —2, or minus two : if there are two cyphers on the left of the decimal, then the index is—3, minus three, and so on.

We shall now proceed to shew the use of Logarithms and the manner of working by them ; and give some instances of their application.

## First. To find the Logarithms of Numbers.

### I. To find the logarithm of any whole number less than 100.

**Rule.** *Look for the given number in the first page of the table under No., then directly against it you have its logarithm with the proper index:*

Thus, if I want the logarithms of 26, 58, and 87, I find under No. by the side of 26 the logarithm 1.4149733
—— —— 58 ——— 1.7634280
—— —— 87 ——— 1.9395193

Ex. Write down the logarithms of 6, 18, 36, 54, 72, 90, and 99.

### II. To find the logarithm of any whole number between 100 and 1000.

**Rule.** *Seek in the table for the given number in the left-hand column, opposite to which is the logarithm sought, with its proper index* 2.

If I want the logarithms of 176, 375, 684, I find

Opposite to the number 176 the logarithm 2.2455127
—— —— —— 375 ——— 2.5740313
—— —— —— 684 ——— 2.8350561

Ex. Write down the logarithms answering to the numbers 486, 573, 671, 756, 843, 921.

### III. To find the logarithm of any whole number between 1000 and 10000.

**Rule.** *Find, as before, the logarithm belonging to the first three figures; take, from the margin, the difference between this and the next logarithm, which multiply by the fourth figure of the said number; add the product to the logarithm first found, prefix the index* 3, *and it will be the logarithm required very nearly.*

Ex. 1. To find the logarithm of 4528.

In the table, against 452, is the decimal . .6551384
The difference marked in the margin is 959,
which multiplied by 8, the fourth figure, = 7672

Answer . . 3.6559056

If the fourth figure had been 0 instead of 8, then the decimal .6551384, with the index 3, would have been the logarithm sought.

### Ex. 2. To find the logarithm of 8884.

Here the decimal against 888 - - = .9484130
Difference is 488, which multiplied by 4, gives 1952

Answer - - 3.9486082

Ex. 3. Find the logarithms answering to 2465, 4265, 6425, 5387, 3420, 5876, 8464, and 4932.

### IV. To find the logarithm answering to a decimal.

**Rule.** *Work precisely as if it were a whole number, remembering that the index mnst always be one less than the figures counted in the integral part ; that is, one less than the figure before the decimal point :*

Thus, if instead of the foregoing examples, the numbers be 452.8 and 88.84, then the logarithm of 452.8 = 2.6559056, and
— — — 88.84 = 1.9486082

But if they were of still less value, by removing the point forwarder,
then the logarithm of 4.528 = 0.6559056
— — — — .8884 = — 1.9486082

If the given number were a fraction, as $\frac{7}{8}$, or $\frac{15}{16}$, then the fractions must be reduced to decimals, and the logarithms are to be found as before thus (See the Tables)
the logarithm of $\frac{7}{8}$ = .875 = — 1.9420081
— — — — $\frac{15}{16}$ = .9375 = { .9717396
2315*

— 1.9719711

---

NOTE.

* This is the usual method ; but, as is evident, " *The logarithm of a fraction may be found by subtracting the logarithm of the denominator from the logarithm of the numerator.*" Thus,

The logarithm of 7 is .8450980
The logarithm of 8 is . .9030900

— 1.9420080

And, The logarithm of 15 is . . 1.1760913
The logarithm of 16 is . . 1.2041200

— 1.9719713

In these examples, as one is borrowed in the subtraction, we must put down — 1, or minus in the answer.

## Secondly. TO FIND THE NATURAL NUMBER OF ANY GIVEN LOGARITHM.

### I. To find the natural number answering to any given logarithm, to 2 or 3 places of figures.

RULE. *Look in the table for the given logarithm, and if it be exactly found, or nearly so, the natural number stands against it in the left-hand column :*

Thus, the natural numbers answering to 1.3617278, 2.1216834, and 2.5312993, are 23, 132, and 339. The first is found in the table; the others are not to be met with accurately, and therefore we take the natural number belonging to the next nearest logarithm.

Ex. What numbers answer to the following logarithms 2.4650000 2.8148132; 2.5346780; 1.9684829; 2.2454674; and 2.8819765?

### II. When great accuracy is required, and the logarithm is not to bo found very nearly in the table.

RULE. *Seek in the table the difference between the next greater and next less logarithms*, and say,

*As this difference*
*Is to unity or* 1,
*So is the difference between the given logarithm and the next less logarithm*
*To a fourth number.*

This fourth number is to be added to the natural number of the less tabular logarithm, which gives the number sought.

Ex. What is the natural number of 2.4723564;

By the table, I find the number is between 296 and 297,

$$\left.\begin{array}{ll}\text{next less} \;.\;. & 2.4712917\\ \text{next greater} \;. & 2.4727564\end{array}\right\} 14647 = \text{diff.}$$

$$\left.\begin{array}{ll}\text{given logarithm} & 2.4723564\\ & 2.4712917\end{array}\right\} 10647 = \text{2d diff.}$$

As 14647 : 1 : : 10647 : .73 nearly, which added to 296, gives for the true answer 296.73.*

---

NOTE.

* For all common purposes there will be no need of working the numbers: thus it will be seen at once that 10657 is nearly $\frac{3}{4}$ of 14653, and we know that the decimal of $\frac{3}{4}$ is .75, of course .75 might be substituted, which would be sufficiently near. When, however, the index is more than 3, it becomes necessary to be as exact as possible, because the answer is then in whole numbers.

If the index of the given logarithm had been 3 instead of 2, then the answer would have been 2967.3: and if the index had been 4, the answer would have been altogether a whole number, as 29673.

Ex. What are the natural numbers answering to 2.7896453; 3.5648750, and 4.2168435?

---

## MULTIPLICATION BY LOGARITHMS.

### To find the product of two given whole, or mixed numbers.

Rule. *Find the logarithm of each given number, and their sum will be the logarithm of the product, whose corresponding number in the tables is the answer.*

Ex. 1. Multiply 84 = log. 1.9242793
By - 25 = log. 1.3979400

Product 2100 3.3222193, which answers to 210 in the table; but the index being 3, there must be four figures in the answer, or 2100.

Ex. 2. Multiply 41.5 = log. 1.6180481
By - 7.24 = log. 0.8597386

Product 300.4 = 2.4777867*

EXAMPLES.

Multiply 59 by 35 : 14 by 4 : 76.3 by 3.24 : and 2.76 by 345.

---

## DIVISION BY LOGARITHMS.

### To divide a whole or mixed number, by a less whole or mixed number.

Rule. *From the logarithm of the dividend subtract the logarithm of the divisor, and the remainder is the logarithm of the quotient.*

---

NOTE.

* To prove the truth of this, I look into the table, and find the

next less . . 2.4771213 } 14452, 1st diff.
next greater . 2.4785665 }

given log. . 2.4777867 } 6654, 2d diff.
next less . . 2.4771213 }

As 14452 : 1 : : 6654 : .4, which added to 300, gives 300.4.— The same method is applicable in all other cases.

Ex. 1. Divide 624 = log. 2.7951846
By - 26 = log. 1.4149733

Quotient 24 = log. 1.3802113

Ex. 2. Divide 1221 = log. 3.0867143
By - 81.4 = log. 1.9106244*

15 = log. 1.1760899

Ex. 3. Divide 34.86 = log. 1.5423268
By - 8.3 = log. .9190781

4.2 = log. .6232487

Examples. Divide 56 by 4; 8650 by 2.5; and 1870 by 55.

## PROPORTION, OR THE RULE OF THREE BY LOGARITHMS.

Rule. *Add the logarithms of the second and third terms together, and from the sum subtract the logarithm of the first; the remainder is the logarithm of the fourth term.*

Take the 3d, 4th, and 5th examples in the Rule of Three, p. 94.

| | | | | | |
|---|---|---|---|---|---|
| | E. | | shill. | | E. |
| If | 4 | : | 18 | : : | 28 |
| | .6020600 | ; | 1.2552725 | : : | 1.4471580 |

1.2552725

2.7024305
.6020600

2.1003705, this is found by the table to answer to the number 126 shillings, or *6l. 6s.*

---

NOTE.

* I find the log. 122 = 2.0863598; the difference between this and the next is 3545 which is to be added, } = 3545

2.0867143; but the index is now 3. This is not quite accurate, as will be observed by the answer; but it is sufficiently so for common purposes; and in working logarithms to great nicety, where there are more than 3 or 4 figures, larger tables are required than this work admits of. The learner will see the use of them by this specimen, and will, after this, find no difficulty of pursuing the study on a more enlarged scale, by the aid of Hutton's Tables.

| | | | | | | |
|---|---|---|---|---|---|---|
| | | yards. | | shillings. | | yards. |
| Ex. 4. | If | 6 | : | 24 | :: | 81 |
| | | .7781513 | : | 1.3802112 | :: | 1.9084850 |
| | | | | | | 1.3802112 |
| | | | | | | 3.2886962 |
| | | | | | | .7781513 |
| | | | | | | 2.5105449 = 324 shillings, or 16*l.* 4*s.* |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | bush. | | shill. | | bush. |
| Ex. 5. | If | 8 | : | 9.5 | :: | 35 × 36 = 1260 |
| | | .9030900 | : | .9777236 | :: | 3.1003705 |
| | | | | | | .9777236 |
| | | | | | | 4.0780941 |
| | | | | | | .9030900* |
| | | | | | | 3.1750041 = 1496.25 |
| | | | | | | = 74*l.* 16*s.* 3*d.* |

## INVOLUTION BY LOGARITHMS.

Rule. *Multiply the logarithm of the root by the index of the power;* thus, to square any number, multiply its logarithm by 2; and to cube a number, multiply its logarithm by 3, and so on.

Ex. 1. What is the square of 25; and the 5th power of 2?

25 = log. 1.3979400
25 2
625 = log. 2.7958800

$2^5$. = .301030 × 5 = 1.505150 = 32.

Ex. 2. What is the third power of $\frac{7}{8}$? See p. 142.

I bring $\frac{7}{8}$ to a decimal = .875, the logarithm of this is —1.9420008; this multiplied by 3 gives — 1.826024, which answers to the decimal .67. And it will be found that $\frac{7}{8} \times \frac{7}{8} \times \frac{7}{8} = \frac{343}{512} = .67$. This ad-

---

NOTE.

* By the table, I find that the number answering to this logarithm is more than 1490: by the rules already given, I put down

$\left.\begin{matrix}.1731863\\.1760913\end{matrix}\right\}$ 29050 first diff., $\left.\begin{matrix}.1750041\\.1731863\end{matrix}\right\}$ 18178 second diff., therefore As 29050 : 1 :: 18178 : 625, which is to be annexed to 149 = 1499.25, because the index is 3.

mits of other proofs: thus the log. 7 = .845098
——— 8 = .903090

therefore $\frac{7}{8} = -\,1.942008$, this multiplied by 8, gives $-\,1.826024$, as above.* Or $\left.\frac{7}{8}\right|^{3} = \frac{.845098}{.903090} \times 3 = \frac{2.535294}{2.709270}$, and, subtracting the denominator from the numerator, we have $-\,1.826024 = .67$.

## EVOLUTION BY LOGARITHMS.

To extract the root of any number.

RULE. *Divide the logarithm of the number by the proposed index, and the number answering to the quotient is the required root.*

Ex. 1. What is the square root of 225?

Log. of 225 = 2.3521825, which ÷ by 2, gives 1.1760912, which is the log. of 15.

Ex. 2. What is the square root of 6561?

$$\text{Logarithm of } 6561 = \left\{\begin{array}{r} 3.8169038 \\ 656 \end{array}\right.$$
$$\begin{array}{r} 2)\underline{3.8169694} \\ 1.9084847 \end{array}$$

1.9084847, which answers to 81; of course the square root of 6561 is found to be 81.

Ex. 3. What is the 5th root of 1024? See p. 143.

$$\text{Logarithm of } 1024 = \left\{\begin{array}{r} 3.0086002 \\ 16948 \end{array}\right.$$
$$\begin{array}{r} 5)\underline{3.0102950\dagger} \\ .6020590 \end{array}$$

.6020590, which answers very nearly to 4, the fifth root of 1024.

Ex. 4. What is the square root of .25? See p. 144.

The log. of .25 = — 1.39794, which ÷ 2, gives .69897 = .5 Answer.

Ex. 5. What is the square root of 144?

Ex. 6. What is the cube root of 1728?

---

NOTES.

* The learner may in this way work all the examples in p. 142.

† It may not occur at first sight to the reader, how — 1.942008 × 3 should give — 1.826024; but if he divide the expression into two sums, which he may, as — 1 and .942008, and multiply each by 3, he gets — 3 and 2.826024. The *plus* 2 in the latter expression will destroy — 2 of the former; the remainder will of course be — 1.826024.

# INTEREST.

Interest, *is the sum of money paid, or allowed for the loan or use of some other sum, lent for a certain time, according to a fixed rate.*

*The sum lent, and on which the interest is reckoned, is called the* Principal.

*The sum per Cent. agreed on as interest, is called the* Rate.

*The principal and interest added together, is called the* Amount.

*Interest is distinguished into* Simple *and* Compound.

Simple Interest, *is that which is reckoned on the principal only, at a certain rate for a year, and at a proportionately greater or less sum, for a greater or less term:* thus, if 5*l.* is the rate of interest of 100*l.* for one year, 10*l.* is the interest for two years, 20*l.* for four years, and so on.

Rule (1). *Multiply the principal by the rate, and divide the product by* 100, *and the quotient is the interest for one year:*

Thus the interest of 250*l.* at 5 per cent. is $\frac{250 \times 5}{100} = 12l.\ 10s.$

(2). *Multiply the interest for one year by the number of years, and the product is the interest for the same:*

Thus the interest of 250*l.* for 7 years is 12*l.* 10*s.* × 7 = 87*l.* 10*s.*

(3). *If parts of a year be given, they must be worked for by the aliquot parts of a year, as in Practice, or by the Rule of Three Direct.*

Ex. 1. What is the interest of 853*l.* 10*s.* for 4 years and 8 months, at 5 per cent. per annum?*

---

NOTE.

* By law, more than 5 per cent. cannot be received as interest of money in this country; though at various periods of our history different rates of interest have been allowed, as will be evident from the following table:

| | | | |
|---|---|---|---|
| £. s. | | | £. s. d. |
| 853 10 | 6 | $\frac{1}{2}$ | 42 13 6 = interest for one year. |
| 5 | 2 | $\frac{1}{3}$. | 4 |
| £. 42.67 10 | | | 170 14 0 |
| 20 | | | 21 6 9 |
| shill. 13.50 | | | 7 2 3 |
| 12 | | | £. 199 3 0 |
| pence 6.00 | | | |

Here I multiply the interest for one year by 4, for the number of years, and take parts for the 8 months, by saying 6 months is the half, and 2 months the third of that, as in Practice.

Answer, - 199*l.* 3*s.* 0*d.*

To find the *amount*, I must add the principal to the interest. In this example, the amount is equal to 853*l.* 10*s.* + 199*l.* 3*s.* = 1052*l.* 13*s.*

Ex. 2. What is the amount of 142*l.* 10*s.* for four years and 52 days, at 4½ per cent?*

| | |
|---|---|
| £. 142 10 | £. s. d. |
| 4½ | 6 8 3 = interest for one year. |
| 570 0 | 4 |
| 71 5 | 25 13 0 = interest for four years. |
| 41 5 | |
| 20 | |
| .25 | |
| 12 | |
| e 3.00 | |

To find the interest for the 52 days, I say,

---

NOTE.

| | | £. | s. | d. | | |
|---|---|---|---|---|---|---|
| In 1255 | — | 50 | 0 | 0 | per cent. per annum was given as interest. | |
| 1270 to 1307 | — | 45 | 0 | 0 | —— | —— |
| 1422 to 1470 | — | 15 | 0 | 0 | —— | —— |
| 1545 it was restricted to | | 10 | 0 | 0 | —— | —— |
| 1625 reduced to | — | 8 | 0 | 0 | —— | —— |
| 1645 to 1660 | — | 6 | 0 | 0 | —— | —— |
| 1660 to 1690 | — | 7 | 6 | 6 | —— | —— |
| 1690 to 1697 | — | 7 | 10 | 0 | —— | —— |
| 1697 to 1706 | — | 6 | 0 | 0 | —— | —— |
| 1714 to the present time | | 5 | 0 | 0 | —— | —— |

In many parts of the world a much higher rate of interest is given, and also in the colonies belonging to this country. In India, for instance, 12 per cent. is the legal interest for money: and in the English settlements in New South Wales, the rate of interest is fixed at 8 per cent.

* In the courts of law, interest is always computed in years, quarters, and days; but in computing the interest on the public

| days. | £. | s. | d. | | days. |
|---|---|---|---|---|---|
| If 365 : | 6 | 8 | 3 | :: | 52 |

20

128

12

1539

52

3078

7695

365)80028(219¼ (12)

18 3¼ = interest for 52 days.

| £. | s. | d. | |
|---|---|---|---|
| 25 | 13 | 0 | |
| 0 | 18 | 3¼ | |
| 26 | 11 | 3¼ | = interest |
| 142 | 10 | 0 | = principal |
| 169 | 1 | 3¼ | = amount. |

Ex. 3. What is the interest of 461*l.* at 4 per cent. for 5 years?

Ex. 4. What is the interest of 230*l.* 15*s.* for 6½ years, at 5 per cent. per annum?

Ex. 5. What is the amount of 225*l.* for 7 years, at 3½ per cent. per annum.

Ex. 6. How much shall I have to receive at the end of 5 years for 350*l.* supposing 4½ per cent. be allowed as interest?

In most computations relating to simple interest, the work is shortened, if the interest of 1*l.* for a given term is known, as the interest of any other sum for the same term will then be found by only multiplying by the given sum.

The interest of 1*l.* for a year must be in the same proportion as the interest of 100*l.* to its principal; therefore, at 5 per cent., we say, as 100*l.* : 5*l.* : : 1*l.* : .05*l.* Hence the interest of 1*l.* for one year

| £. | £. |
|---|---|
| At 3 per cent. is | ,03 |
| 3½ | ,035 |
| 4 | ,04 |
| 4½ | ,045 |
| 5 | ,05 |

Ex. 7. What sum will one penny amount to in 1808 years, at 5 per cent.?*

---

NOTE.

bonds of the South-Sea and East-India companies, the time generally taken is in calendar months and days; and on Exchequer-bills, in quarters of a year and days.

* Here the sum is $\frac{1}{240}l.$ = .004166; this multiplied by 1808, and the product multiplied by .05, gives something more than 7*s.* 6¼*d.* See COMPOUND INTEREST, where the difference between Simple and Compound Interest will be put in a most striking point of view by this same question.

The Interest of One Pound for any number of Years.

| Years. | 3 per Cent. | $3\frac{1}{2}$ per Cent. | 4 per Cent. | $4\frac{1}{2}$ per Cent. | 5 per Cent. |
|---|---|---|---|---|---|
| 10 | ,3 | ,35 | ,4 | ,45 | ,5 |
| 20 | ,6 | ,7 | ,8 | ,9 | 1,0 |
| 30 | ,9 | 1,05 | 1,2 | 1,35 | 1,5 |
| 40 | 1,2 | 1,4 | 1,6 | 1,8 | 2,0 |
| 50 | 1,5 | 1,75 | 2,0 | 2,25 | 2,5 |
| 60 | 1,8 | 2,1 | 2,4 | 2,7 | 3,0 |
| 70 | 2,1 | 2,45 | 2,8 | 3,15 | 3,5 |
| 80 | 2,4 | 2,8 | 3,2 | 3,6 | 4,0 |
| 90 | 2,7 | 3,15 | 3,6 | 4,05 | 4,5 |
| 100 | 3,0 | 3,5 | 4,0 | 4,5 | 5,0 |

The 365th part of the yearly interest is always considered as the proper interest for a day, and its multiples as the interest for any number of days: thus, at 5 per cent., the legal rate, the interest for a day is $\frac{.05}{365} = .0001369$; and the interest for 12 days, at the same rate, is .0001369 × 12 = 0016428. Hence, by means of the following table, all calculations at 5 per cent. Simple Interest are easily performed, for any number of days.

| days | Amount. | days | Amount. | days | Amount. | days | Amount. |
|---|---|---|---|---|---|---|---|
| 1 | ,0001369 | 26 | ,0035616 | 51 | ,0069863 | 76 | ,0104109 |
| 2 | ,0002739 | 27 | ,0036986 | 52 | ,0071232 | 77 | ,0105479 |
| 3 | ,0004109 | 28 | ,0038356 | 53 | ,0072602 | 78 | ,0106849 |
| 4 | ,0005479 | 29 | ,0039726 | 54 | ,0073972 | 79 | ,0108219 |
| 5 | ,0006849 | 30 | ,0041095 | 55 | ,0075342 | 80 | ,0109589 |
| 6 | ,0008219 | 31 | ,0042465 | 56 | ,0076712 | 81 | ,0110958 |
| 7 | ,0009589 | 32 | ,0043835 | 57 | ,0078082 | 82 | ,0112328 |
| 8 | ,0010958 | 33 | ,0045205 | 58 | ,0079452 | 83 | ,0113698 |
| 9 | ,0012328 | 34 | ,0046575 | 59 | ,0080821 | 84 | ,0115068 |
| 10 | ,0013698 | 35 | ,0047945 | 60 | ,0082191 | 85 | ,0116438 |
| 11 | ,0015068 | 36 | ,0049315 | 61 | ,0083561 | 86 | ,0117808 |
| 12 | ,0016438 | 37 | ,0050684 | 62 | ,0084931 | 87 | ,0119178 |
| 13 | ,0017808 | 38 | ,0052054 | 63 | ,0086301 | 88 | ,0120547 |
| 14 | ,0019178 | 39 | .0053424 | 64 | ,0087671 | 89 | ,0121917 |
| 15 | ,0020547 | 40 | ,0054794 | 65 | ,0089041 | 90 | ,0123287 |
| 16 | ,0021917 | 41 | ,0056164 | 66 | ,0090411 | 91 | ,0124657 |
| 17 | ,0023287 | 42 | ,0057534 | 67 | ,0091780 | 92 | ,0126027 |
| 18 | ,0024657 | 43 | ,0058904 | 68 | ,0093150 | 93 | ,0127397 |
| 19 | ,0026027 | 44 | ,0060274 | 69 | ,0094520 | 94 | ,0128767 |
| 20 | ,0027397 | 45 | ,0061643 | 70 | ,0095890 | 95 | ,0130137 |
| 21 | ,0028767 | 46 | ,0063013 | 71 | ,0097260 | 96 | ,0131506 |
| 22 | ,0030137 | 47 | ,0064383 | 72 | ,0098630 | 97 | ,0132876 |
| 23 | ,0031506 | 48 | ,0065753 | 73 | ,0100000 | 98 | ,0134246 |
| 24 | ,0032876 | 49 | ,0067123 | 74 | ,0101369 | 99 | ,0135616 |
| 25 | ,0034246 | 50 | ,0068493 | 75 | ,0102739 | 100 | ,0136986 |

Rule. *Multiply the figures corresponding with the number of days by the sum:*

Thus, if the interest of 75*l.* for 61 days be required: I find opposite to 61, the number .0083561, which multiplied by 75, gives .626705 of a pound, which reduced, is equal to 12*s.* 6¼*d.*[*]

Ex. 1. What is the interest of 155*l.* for 49 days; for 76 days, for 184[†] days, and for 198 days?

Ex. 2. How much do I lose by suffering 375*l.* to lie at my bankers 99 days, instead of laying it out in Exchequer bills or India Bonds?[‡]

* * *

# COMMISSION AND BROKERAGE.[§]

Commission *is an allowance of a certain sum per cent. to a correspondent or agent, for buying vnd selling goods for his employer, or to a banker for drawing bills and managing accounts.*

Brokerage, though of a different name, is of the same nature as Commission.

Ex. 1. A salesman at Smithfield, in the course of a year, sells for his correspondents 1120 loads of hay, at the

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NOTES.

[*] Though it is the most convenient in common practice to make use of tables for finding the interest for days, yet the same may be found by the following

Rule. *Multiply the given sum by the number of days, and divide by* 7300. Ex. What is the interest of 712*l.* for 56 days?

$$\frac{712 \times 56}{7300} = 5l.\ 9s.\ 2\tfrac{3}{4}d.$$

[†] The interest for any greater number of days than are contained in the table, is easily found by means of it: thus, to find the interest of 100*l.* for 165 days; by the table, the interest for 100 days is .0136986 × 100 = 1.36986, and for 65 days it is .0089041 × 100 = 89041; these sums added together give 2.26027 = 2*l.* 5*s.* 2¼*d.* for the interest required.

[‡] See the next section but one, p. 168, &c.

[§] This and the following rules of Insurance, Buying and Selling of Stocks, are all worked in a similar manner to the rule of Simple Interest. If, therefore, the pupil is ready in the examples already given, he will find no difficulty in what follows.

average price of 5*l.* 10*s.* per load; and 620 loads of straw, at 55*s.* per load: I wish to know the commission money, at 2¼ per cent?

1120
5½
———
5600
560
———
£.6160 = what the hay sold for.
1705
———
£.7865
2¼
———
15730
196625
———
176.9625
20
———
19.25
12
———
3.00

620
2½ ½
———
1240
310
155
———
1705 = what the straw sold for.

Answer, 176*l.* 19*s.* 3*d.*

Ex. 2. A Manchester manufacturer allows his agent in London 4¼ per cent. for goods sold by him; in the course of the year 1807 he sold to the amount of 15,400*l.*, what was his commission for that year, and how much was the agent's clear gains, supposing his losses on the year's account, by bad debts, amounted to 225*l.* 10*s.* 6*d.*?

Ex. 3. A Liverpool merchant sells goods in a year, for his American correspondents to the amount of 144,454*l.* 10*s.*, on which he reckons his clear gains at the rate of ⅜ per cent., what is his income on this one concern?

Ex. 4. What is the commission of 1206*l.* 10*s.* 6*d.* at 3¾ per cent.?

Ex. 5. A bookseller in London allows his agent in America 5 per cent. commission; what does he pay him for the remittance of 8540*l.* 15*s.* 9*d.*?

Ex. 6. What is the brokerage of 1210*l.*, at ¼ per cent.?

Ex. 7. What is the claim of a broker, at 3⅜ per cent. on 1550*l.* 10*s.* 10*d.*?

Ex. 8. What is the commission on 1000 guineas, at ⅝ per cent.?

Ex. 9. What have I to pay my broker for the sale of goods to the amount of 9950*l.* 9*s.*, at 1¼ per cent.?

Ex. 10. What will the commission of a country banker amount to on 12314*l.* 8*s.* 9*d.*, at ⅛ per cent.?

Ex. 11. What is the brokerage of 1526*l.* 13*s.* 6*d.*, at 1½ per cent.?

# BUYING AND SELLING STOCKS.

STOCK *is a general name for the capitals of our trading companies. It also signifies any sum of money which has been lent to Government, on condition of receiving a certain interest till the money is repaid.*

*The price of stocks, or rates per cent, are the several sums for which* £.100 *of those respective stocks sell at any given time.*

Thus, on the 2d of March, 1808, I bought 500*l.* Consols, at the rate of 64*l.* per cent.; of course the purchase cost 320*l.* But I paid the broker $\frac{1}{8}$th, or 2*s.* 6*d.** per cent. for the purchasing, that is, 12*s.* 6*d.*, so that my purchase cost me 320*l.* 12*s.* 6*d.*; for which I shall receive interest 15*l.* per annum, so long as I keep the same.†

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NOTES.

* The brokerage is 2*s.* 6*d.* (or $\frac{1}{8}$) per cent. on the capital purchased: on TERMINABLE Annuities it is 2*s.* 6*d.* per cent. on the sum laid out. See §. 6. of this note.

† I shall in this note give the price of stocks for one day, and an explanation, so as to render the information, on this head, contained in the papers, intelligible to the youngest reader.

PRICE OF STOCKS.—FEB. 20.

| | |
|---|---|
| Bank Stock 226 | Omnium 1$\frac{1}{4}$ |
| India Stock —— | India Bonds 2*s.* dis. |
| 3 per Cent. Red. 62$\frac{7}{8}$ 63$\frac{1}{8}$ 63 | Imp. Ann. 8 1-16 |
| 3 per Cent. Cons. 62$\frac{3}{8}\frac{5}{8}\frac{1}{2}$ | Ex. Bills 1*s.* dis. 1*s.* pre. |
| 4 per Cent. Cons. 80$\frac{7}{8}$ 81$\frac{1}{8}$ | 3 per Cent. Imp. 62$\frac{3}{3}\frac{1}{2}$ |
| 5 per Cent. Navy 95$\frac{7}{8}$ 96$\frac{1}{8}$ 96 | Lottery Tickets 18*l.* |
| Bank Long Ann. 17$\frac{7}{8}$ 18 | Cons. for Feb. 25. 62$\frac{1}{2}$ |

1. Bank Stock 226; that is, 226*l.* must be be given on that day to purchase 100*l.* of that Stock, the annual interest of this is about 10 or 11 per cent.

2. India Stock——; none of this stock was sold on the day

3. 3 per Cent. Red. 62$\frac{7}{8}$, 63$\frac{1}{8}$, 63. The price of this stock fluctuated in the course of the day; it began at 62$\frac{7}{8}$, or 62*l.* 17*s.* 6*d.*; it rose to 63$\frac{1}{8}$, or 63*l.* 2*s.* 6*d.*; and when the market, as it is called, closed, the value of 100*l.* in the 3 per Cent. Reduced was 63*l.* exactly.

4. 3 per Cent. Cons. 62$\frac{3}{8}$, $\frac{5}{8}$, $\frac{1}{2}$. This stock fluctuated as the last, viz. from 62*l.* 7*s.* 6*d.* to 62*l.* 12*s.* 6*d.*, and then back to 62*l.* 10*s.* The reason of this stock being of less value than the 3 per Cent. Reduced is, that more interest was due upon the former than on the latter; that is, half a year's interest is due at Lady Day on the Reduced, but the half year's interest on the Consols is not due till Midsummer.

Ex. 1. What will 500*l.* 3 per cent. Consols cost, at $61\frac{1}{2}$ per cent.?

| | | | | | |
|---|---|---|---|---|---|
| 500*l.* at 61*l.* 10*s.* | = | £307 | 10 | 0 |
| Brokerage | - | 0 | 12 | 6 |
| | | 308 | 2 | 6 |

Ex. 2. What will 125*l.* 4 per cent Annuities cost, at $69\frac{1}{4}$ per cent.?

Ex. 3. What will 1128*l.* 6*s.* 8*d.* Reduced 3 per cents. cost, at $61\frac{3}{8}$ per cent.?

Ex. 4. A person sells 1000*l.* 3 per cent. Consols, at $70\frac{1}{4}$ per cent., and purchases 635*l.* 17*s.* 6*d.* Navy 5 per cents. at $94\frac{3}{4}$ per cent., what additional interest does he get?

---

NOTE.

5. 4 per cent. Cons., 5 per cent. Navy, and 3 per cent. Imp., will be understood from what has been said.

6. Bank-Long Ann. $17\frac{7}{8}$ to 18. This refers to certain annuities granted for a term of years; the market price of which on this day was from $17\frac{7}{8}$ to 18 years, that is, if I wish to purchase 50*l.* per ann. of these annuities, I must at the lowest price pay $50l. \times 17\frac{7}{8}$, or 893*l.* 15*s.*, and at the highest 50 × 18, or 900*l.*: and for this 893*l.* 15*s.*, or 900*l*, I should be entitled to 50*l.* per annum for about 52 years; the time when these annuities *terminate*. Hence these are called *terminable* annuities.—Imp. Ann. 8 1-16, or $8\frac{1}{16}$, is of the same kind, but worth only $8\frac{1}{16}$ years purchase, because they terminate so much sooner; that 50*l.* per annum in these might be purchased for 403*l.* 2*s.* 6*d.*

7. Omnium, $1\frac{1}{4}$ pre. This is a word that refers to the several sorts of stocks in which a new loan is made: for instance, if government borrow 20 millions, and give to each lender, for every 100*l.* so purchased, 100*l.* 3 per cent. Consols, 50*l.* in the Reduced, and the rest in Long Annuities; then this stock, the moment it is subscribed, is saleable, and while the different articles are sold together, it is styled *omnium*; and $1\frac{1}{4}$ premium means, that a person, to purchase 100*l.* of this loan, must pay $1\frac{1}{4}$, or 1*l.* 5*s.* *more* than the original lender; had it been $1\frac{1}{4}$ discount, then the purchase would have been 1*l.* 5*s.* *less* than the original cost, or 98*l.* 15*s.*

8. India-Bonds, 2*s.* dis.: this phrase shews, that the bonds of 100*l.* given by the East India Company are at 2*s.* each discount; that is, to purchase 9 of these I must pay 899*l.* 2*s.* instead of 900*l.*

9. Ex. Bills, 1*s.* dis., 1*s.* pre., shews that Exchequer bills of 100*l.* each, fluctuated in value from 1*s.* discount to 1*s.* premium; at one part of the day 10 of them would have been purchased for 10 shillings less than 1000*l.*, and at the close of the market 10 shillings more than 1000*l.* must have been given for them. It may be observed, that India-Bonds and Exchequer-bills are convenient stocks to lay money out in, because they may be sold at any time, and the rise and fall are seldom more than a few shillings per cent.

10. Lottery Tickets, 18*l.*, shews the price of Lottery Tickets for the time being.

11. Consols for Feb. 25, $62\frac{1}{2}$, shews that some persons had bought stock in anticipation, and agreed to give for it on the day mentioned at the rate of 62*l.* 10*s.* per cent.

Ex. 5. What will 50*l.* per annum Long Annuities cost, at $17\frac{1}{4}$ years purchase?

Ex. 6. What shall I receive for 450*l.* Bank Stock, at $213\frac{1}{4}$ per cent.?

Ex. 7. How much 3 per cent. Consols must I purchase, to produce an income of 120*l.* per annum?

Ex. 8. How much reduced 3 per cents. can I purchase for 500*l.*, the price being $60\frac{5}{8}$ per cent.?

Ex. 9. What will 2197*l.* 13*s.* 4*d.* 4 per cents. cost, at $78\frac{3}{4}$ per cent.?

Ex. 10. What is the difference on 1200*l.* 3 per cent. Consols bought at $59\frac{7}{8}$, and sold at $61\frac{3}{8}$?

Ex. 11. A person has 730*l.* to invest in the 3 per cent. Conosls, which are at $60\frac{1}{8}$; what sum must he give an order to his broker for, so that including the commission it may cost exactly the sum he has to lay out?

Ex. 12. How much a year in the Long Annuities can I purchase for 1000*l.*, the price being $17\frac{3}{16}$ years purchase?

## A general Method for finding the Value of any Quantity of Stock sold or purchased, which is much used.

RULE. *Multiply the price of the stock by the quantity, observing, instead of the fractional part of the price, if any, to affix the respective figures agreeing thereto, in the following scale, with a point on the left side; then, for the* Perpetual Annuities, *as* 3, 4, *or* 5 *per cents.*, *&c.*, *point off two more figures from the product than were affixed for the decimal part of the price, and the figures on the left side of the point will be pounds; the remaining figures, being multiplied by* 20, *and the same number again pointed off, will be the shillings, if any; and the remaining figures being multiplied by* 12, *and pointed off, as before, will be the pence, if any; which pounds, shillings, and pence, make up the whole of the purchase-money.*

*For the* Terminable Annuities, *as Long, Short, Imperial, &c., mark off so many figures only, in the products, were affixed for the fractional part of the price.*

SCALE.

| | | | |
|---|---|---|---|
| $\frac{1}{16}$ | ,0625 | $\frac{9}{16}$ | ,5625 |
| $\frac{1}{8}$ | ,125 | $\frac{5}{8}$ | ,625 |
| $\frac{3}{16}$ | ,1875 | $\frac{11}{16}$ | ,6875 |
| $\frac{1}{4}$ | ,25 | $\frac{3}{4}$ | ,75 |
| $\frac{5}{16}$ | ,3125 | $\frac{13}{16}$ | ,8125 |
| $\frac{3}{8}$ | ,375 | $\frac{7}{8}$ | ,875 |
| $\frac{7}{16}$ | ,4375 | $\frac{15}{16}$ | ,9375 |
| $\frac{1}{2}$ | ,5 | | |

### EXAMPLES.

What is the purchase of £816, three per cent, Consols, at $72\frac{5}{8}$ per cent.?

Look in the scale, and $\frac{5}{8}$ is found equal ,625; therefore

$$\begin{array}{r}
72{,}625 \times 816. \\
816 \\
\hline
435750 \\
72625\phantom{0} \\
581000\phantom{00} \\
\hline
592{,}62000 \\
20 \\
\hline
12{,}40 \\
12 \\
\hline
4{,}8
\end{array}$$

Answer, £592 12*s.* 4*d.*

What is the value of £58, Long Annuity, at $21\frac{3}{16}$ years purchase? $\frac{3}{16}$ = ,1875; therefore,

$$\begin{array}{r}
21{,}1875 \\
58 \\
\hline
1695000 \\
1059375\phantom{0} \\
\hline
1228{,}8750 \\
20 \\
\hline
17{,}5000 \\
12 \\
\hline
6{,}0000
\end{array}$$

Answer, £1228 17*s.* 6*d.*

What is the purchase of £2470, five per cent. of 1797, at $104\frac{1}{2}$ per cent.?

$$\begin{array}{r}
2470^{*} \\
104{,}5 \\
\hline
12350 \\
9880\phantom{0} \\
2470\phantom{000} \\
\hline
2581{,}150 \\
20 \\
\hline
3{,}000
\end{array}$$

Answer, £2581 3*s.* 0*d.*

---

NOTE.

* It is immaterial whether the price or the quantity of stock is made the multiplier.

What is the value of £26, Short Annuity, at $5\frac{15}{16}$ years purchase?

5,9375
26

356250
118750

154,3750
20

7,5000
12

6,0000 Answer, £154 7*s.* 6*d.*

# INSURANCE.

**Insurance** is a security given in consideration of a premium of so much per cent. paid by the proprietors of goods, &c., to the insurers, for which they engage to answer for the damage of houses, ships, goods, &c., by fires, dangers of the sea, and other accidents.

To find what premium must be given for an insurance of property, at any rate per cent.

Rule. *Multiply the value of the property to be insured by the rate, and the product divided by* 100 *is the premium.*

Ex. 1. I insure my house and goods for £1700, for which I pay a premium of $\frac{1}{10}$th per cent. to the Phœnix Office, and $\frac{1}{8}$th per cent. is paid to government for duty, what do I pay annually?

1700
$\frac{1}{10}$*

1.70
20

14.00

| | £. | *s.* | *d.* | |
|---|---|---|---|---|
| Ans. | 1 | 14 | 0 | Insurance. |
| | 2 | 2 | 6 | Duty. |
| Ans. | 3 | 16 | 6 | = sum annually paid. |

1700.0
$\frac{1}{8}$

2.125
20

2.500
12

6.000

Ex. 2. How much must be paid for the insurance of hazardous property, value 5400*l.*, at the rate of $1\frac{1}{8}$ per cent., and duty $\frac{1}{8}$ per cent.?

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NOTE.

* To multiply by a fraction, whose numerator is unity, or one, is, as we have seen, to divide the sum to be multiplied by the denominator of the fraction.

# SEA INSURANCES.

The premium is a per centage on the sum insured, and is usually taken in guineas, but sometimes in pounds. All Sea Insurances pay a duty of 5*s.* per cent. for foreign voyages, and 2*s.* 6*d.* per cent. for coasting voyages to and from any part of the united kingdom.

Ex. 1. A merchant in London has consigned to him from Jamaica 100 hogsheads of sugar, valued at 20*l.* per hogshead, which he insures for the voyage at 6 guineas per cent.

| | |
|---|---|
| £2000 at 6*l.* 6*s.* is | £126 |
| Duty, - | 5 |
| | 131 |

Ex. 2. What will the insurance of 1600*l.* come to, from Embden to London, at 4½ guineas per cent.?

Ex. 3. What will the insurance come to of 500 casks of butter, valued at 2800*l.*, from Waterford to London, at 2¾ guineas per cent.?

Ex. 4. What will the insurance of 700*l.*, from London to Baltimore, come to, at 2¼*l.* per cent.?

Ex. 5. What will the insurance come to of 10,000*l.*, from Rio Janeiro to the Cape of Good Hope, and from thence to Calcutta, at 4 guineas per cent.?

Most persons who have occasion to make insurances of this kind employ a broker, who receives a shilling for each guinea or pound of the premium for his commission.

Ex. 6. What does an underwriter receive for insuring 700*l.* from Hamburg to London, at 2¼ guineas per cent.?

| | | | |
|---|---|---|---|
| £.700 at £.2 7*s.* 3*d.* per cent. is | £.16 | 10 | 9 |
| Duty - | 1 | 15 | 0 |
| | 18 | 5 | 9 |
| Brokerage, 1*s.* per guinea | 0 | 15 | 9 |
| | 17 | 10 | 0 |

Ex. 7. What does an underwriter receive for insuring 500*l.* from Newcastle to Southampton, at 2*l.* per cent.?

In time of war, insurances are generally done at a much higher premium, with a condition to return a certain part of it, if the ship sails with convoy, and arrives at her destined port.

Ex. 8. What is to be paid for insurance of 2000*l.* from Jamaica to London, at 12 guineas per cent., with an agreement to return five guineas per cent. if the ship depart with convoy for the voyage and arrive. The ship having had convoy for the voyage and arrived?

| | | | |
|---|---|---|---|
| £2000 at £ 12 12*s.* per cent is | £ 252 | 0 | 0 |
| Duty - | 5 | 0 | 0 |
| | 257 | 0 | 0 |
| Return for Convoy - | 105 | 0 | 0 |
| | 152 | 0 | 0 |

Ex. 9. What is to be paid for insurance of 1200*l.* from Stockholm to Plymouth, at 6 guineas per cent., to return 2 guineas per cent. if the ship departs from the Sound with convoy for the Downs, and 1 guinea per cent. more, if with convoy from thence for the voyage and arrive? The ship having had convoy for the whole voyage and arrived.

Ex. 10. What is an underwriter to receive from a broker for insuring 1000*l.* from Liverpool to Dantzic, at 10 guineas per cent., to return two pounds per cent. if the ship depart from the place of rendezvous with convoy for the voyage and arrive? The ship having had such convoy and arrived.

Ex. 11. What is an underwriter to receive from a broker for insuring 300*l.* from London to Buenos Ayres, at 10½ guineas per cent., to return 4*l.* per cent. if the ship depart with convoy for the voyage and arrive, or 2½ pounds per cent. if with convoy for St. Helena, and arrive? The vessel having had convoy to St. Helena, and being arrived.

In case it appears that the value of the goods actually shipped is less than the sum ordered to be insured, a return of premium is made on the short interest, deducting 10*s.* per cent.

A general average does not affect the stipulated returns for sailing with convoy; but in case of a particular average, the returns for convoy are not allowed on such part of the sum insured as is claimed for the average.

Ex. 12. £1500 is insured on sugar, valued at 30*l.* per hogshead, from Grenada to London, at 12 guineas per cent., to return 6 guineas per cent. if the ship sails with convoy for the voyage and arrives. The ship had convoy for the voyage and arrived; but it appeared that only 45 hogsheads had been shipped; the insured is therefore entitled to a return of premium for short interest on 150*l.*, and likewise claims a general average, amounting to 1*l.* 7*s.* 5*d.* per cent. What balance has he to pay on this insurance?

| | | |
|---|---|---|
| £1500 at £12 12*s.* per cent. is | | £189 0 0 |
| Duty - | | 3 15 0 |
| Return for Short Interest | 18 18 0 | 192 15 0 |
| Deduct ½ per cent. | 0 15 0 | |
| | 18 3 0 | |
| Return for Convoy on 1350*l.* | 85 1 0 | |
| General average on ditto | 18 10 1½ | |
| | | 121 14 1½ |
| | | £71 5 10½ |

Ex. 13. £900 is insured from Hull to Tonningen, at 5½ guineas per cent., to return 2 guineas per cent. if the ship sails with convoy and arrives. The ship had convoy for the voyage and arrived; but having met with bad weather, a general average is adjusted, amounting to 17*l.* 6*s.* 6*d.*, and a particular average at 29*l.* 5*s.* What is the sum to be paid or received?

# DISCOUNT.

DISCOUNT is an allowance made for advancing money on securities before they are due. Thus I receive a note of £60, which is payable at the end of two months, but having immediate occasion for the money, I must pay any person who will give me cash, as much as the legal interest at 5 per cent. on £60 for two months.

I. *In business, it is usual to calculate after the rate of one penny per pound per month.**

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NOTE.

* This is, at the rate of 5 per cent. per annum, the legal interest of the country; for, if 100 pounds yield 100 shillings in a year, one pound will yield one shilling, or 12 pence in a year, or one penny per month.

The Rule of Discount is, in fact, the same thing as what we had under that of Simple Interest; but the rule given in the text is that which is in general use, and admitted by custom, and in practice, though it is at a rate somewhat higher than 5 per cent.

Perfect accuracy would require us to find the *present worth* of the bill or bills to be discounted. Thus the present worth of 100*l.* due one year hence, at 5 per cent., is not 95*l.*, but 95*l.* 4*s.* 9¼*d.* nearly; and therefore he who allows 5*l.* for the discount, allows 4*s.* 9¼*d.* too much: it is, however, on the *true principles* that Smart's tables are calculated, and those are chiefly in use by persons in the habit of discounting bills. To find the *present worth* of any sum payable at any time, tables are given which are calculated thus:

As the amount of 100*l.* for the given rate and time,
Is to the interest of 100*l.* for that time;
So is the given sum
To the discount required: and
The discount subtracted from the sum is the *present worth.*

Upon this principle: To find the discount of 100*l.* for one year, at 5 per cent. interest, we say, as 105 : 5 : : 100 : 4*l.* 15*s.* 2¼*d.*

To find the discount of 60*l.* for 3 months, at 5 per cent., we say, as 101*l.* 5*s.* : 1*l.* 5*s.* : : 60*l.* : 14*s.* 9¼*d.*; but, by the common rule in the text, it would be reckoned 15*s.*, or 2¾*d.* too much.

When the sum is large, and the time long, the difference between these modes of calculation is an object deserving of attention.

Ex. Suppose I have a bill of 1000*l.* payable two years hence, then, by the rule in the text, I must pay 100*l.* for discounting the same; but, by this last rule, I say, as

100*l.* : 10*l.* : : 1000*l.*
10

110)10000(90*l.* 18*s.* 2*d.*, which is the true discount, which makes a difference of 9*l.* 1*s.* 10*d.*

In this case the discount will be twice 60 pence, that is, 120 pence, or 10 shillings.

Ex. 1. I have just received two bills of £.70 each, the one is payable at two months, and the other at four, how much must I pay for discounting them?

| | £. | | d. | £. | s. | d. |
|---|---|---|---|---|---|---|
| Discount of | 70 | for 2 months | = 70 × 2 = | 0 | 11 | 8 |
| | 70 | — 4 —— | = 70 × 4 = | 1 | 3 | 4 |
| Answer | | | - | 1 | 15 | 0 |

Ex. 2. I have in my possession the following bills, which I wish to get discounted, what shall I have to pay the person who will give me cash for them?

| | | | | |
|---|---|---|---|---|
| A bill of | 540*l.* 0*s.* | due | 3 | months hence. |
| A do. | 29*l.* 0*s.* | — | 6 | ditto. |
| A do. | 355*l.* 0*s.* | — | 5 | ditto. |
| A do. | 85*l.* 10*s.* | — | 6 | ditto. |

II. To find the discount of a sum of money for any number of days.

**Rule.** *Multiply the number of pounds by the number of days, and divide by 365, the answer is in shillings, because the interest of one pound is one shilling for a year.*

Ex. 1. What is the discount of £.1000 for 25 days?

$$\frac{1000 \times 25}{365} = 3l.\ 8s.\ 6d. \text{ nearly.}$$

Ex. 2. What is the discount of a bill of £.87 10*s.* that has 28 days to run?

$$\frac{87l.\ 10s. \times 28}{365} = \frac{87.5 \times 28}{365} = 6s.\ 8\tfrac{1}{2}d.\tfrac{70}{365}.$$

Ex. 3. How much must I pay for discounting the following bills?

100*l.* at 75 days; 245*l.* at 42 days; 987*l.* at 68 days; 92*l.* at 140 days.

Ex. 4. On the first of May I want to discount a bill of 150*l.* due the 10th of July; how much must I pay for the same?

Ex. 5. What is the discount of 330*l.* for 95 days?

Ex. 6. How much ought I to pay for discounting four bills of 75*l.* each, at 2 and 4 months, six weeks, and 75 days?

III. To find the time at which several bills payable at different times, may be paid at once, or exchanged for one bill, without loss either to the holder or receiver.

RULE. *Multiply each payment by the time at which it becomes due, and divide the sum of the products by the sum of the payments, and the quotient will be the medium time required.*

Ex. 1. I have to receive £967 in notes, as follow; viz. £135 in 3 months, £473 in 5 months, £167 in 6 months, £69 in 9 months, and £123 in 15 months; but as it is more convenient to have the whole in one note, for what time must it be given?

$135 \times 3 = 405$
$473 \times 5 = 2365$
$167 \times 6 = 1002$
$69 \times 9 = 621$
$123 \times 15 = 1845$

967 6238, and $\frac{6238}{967} = 6\frac{436}{967}$, or 6 months and 13 days.

Ex. 2. I have to pay £.250 12*s*. at three payments, viz. £.50 10*s*. 6*d*. at 2 months, £.90 9*s*. 9*d*. at 4 months, and the remainder at six; what length of time must a single note be, to pay the whole at once?*

| £. | *s.* | *d.* | | | | £. | *s.* | *d.* |
|---|---|---|---|---|---|---|---|---|
| 50 | 10 | 6 | (due at 2 months) | × 2 | = | 101 | 1 | 0 |
| 90 | 9 | 9 | ( —— 4 —— ) | × 4 | = | 361 | 19 | 0 |
| 109 | 11 | 9 | ( —— 6 —— ) | × 6 | = | 657 | 10 | 6 |
| 250 | 12 | 0 | | | | 1120 | 10 | 6 |

250 12 0
20
5012
12
60144

1120 10 6
20
22410
12 mon. days.
60144)268926( 4 14 nearly.
240576
28350
30
60144)850500(14

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NOTE.

* If the times of payment, or debts, are of different denominations, as days, weeks, or months, and pounds, shillings, or pence, they must be reduced to the same denomination before the several multiplications take place.

Ex. 3. At how long date must I have one bill of £.906 4*s*. 9*d*. for the following notes; viz. £426 5*s*. 3*d*. payable at 55 days, £229 12*s*. 2*d*. payable at 99 days, and the remainder at 135 days?

| £. | *s.* | *d.* | | | | £. | *s.* | *d.* |
|---|---|---|---|---|---|---|---|---|
| 426 | 5 | 3 | × | 55 | = | 23444 | 8 | 9 |
| 229 | 12 | 2 | × | 99 | = | 22731 | 4 | 6 |
| 250 | 7 | 4 | × | 135 | = | 33799 | 10 | 0 |
| 906 | 4 | 9 | | | | 79975 | 3 | 3 |
| 20 | | | | | | 20 | | |
| 18124 | | | | | | 1599503 | | |
| 4 | | | | | | 4 | | |
| 72499* | | | | | | 72499)6398013(88 days. | | |

Answer - 88 days.

Ex. 4. On the 5th of April, I have in my possession one bill for 185*l*. 10*s*. due August 10th; one for 236*l*. due Sept. 1; one for 95*l*. 7*s*. 6*d*. due Sept. 8th; another 723*l*. 3*s*. 9*d*. due Oct. 1; one for 83*l*. due Oct. 14; and one for 1122*l*. 7*s*. 5*d*. due Nov. 12: at what date ought I to take one bill for the same?

Ex. 5. I go on the 13th of December to a country banker with the following bills, viz. one for 68*l*. 9*s*. 4*d*. due Feb. 1, one for 171*l*. 4*s*. 2*d*. due Feb. 17, one for 238*l*. due Jan. 1, one for 333*l*, 7s. 10*d*. due Jan. 28th, and one for 210*l*. 2s. 4*d*. at two months after date; and he gives me 350*l*. in cash, and his bill on London at two months for 600*l*., for which bill he charges me an eighth per cent. commission, and 7*s*. 6*d*. for the stamp: how much will remain due to me or to the banker?

Ex. 6. A traveller in the wine trade has received in local notes, 13 of 5 guineas each, 3 of 5*l*. each, 17 of 2*l*. each, and 28 of one guinea each; also a 40*l*. bill due in 23 days, a 73*l*. bill due in six weeks, another for 217*l*. 8*s*. 8*d*. due in 47 days, and another for 105*l*. due in 56 days. At what date ought a country banker to give him a bill on London for the same, supposing him to charge for drawing such bill an eighth per cent commission, and 7*s*. 6*d*. for the stamp?

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NOTE.

* In both cases the money is brought into threepences, which is somewhat easier than to bring it into pence.

# PROFIT AND LOSS,

*Is a rule that discovers what is gained or lost on the prime cost in the purchase and sale of goods, and it teaches persons how to fix the price of their goods so as to gain so much per cent.*

Questions in this rule are performed by the Rule of Three Direct, upon this principle, that quantities, or sums of money, which gain or lose at the same rate, are to one another as their gains or losses.

Ex. 1. A tallow chandler has this day purchased mottled soap, at 102*s.* 6*d.* per cwt., at how much per lb. must he retail it out to gain 10 per cent. profit ?

$$\begin{array}{lllll}
\text{£.} & & \text{£.} & & s.\ \ d. \\
100 & : & 110 & :: & 102\ \ 6 \div 112 \\
\ \ 20 & & 102\ \ \ 6 & & \\
\hline
2000 & & 220 & & \\
 & & 1100 & & \\
 & & \ \ \ 55 & & \\
\hline
\multicolumn{3}{r}{2.000)11.275} & & \\
\end{array}$$

$$\text{£. } 5.6375 \text{ and } \frac{5.6375}{112} = 1s\,\tfrac{75}{112} = 1s.\ 0\tfrac{3}{4}d. \text{ nearly.}$$

Ex. 2. How much per cent. is gained at the rate of 2*d.* in a shilling?

Ex. 3. If 3*s.* is gained, in selling at a guinea, at what rate per cent. is that ?

Ex. 4. Three pounds of tobacco are bought at 5*s.* 9*d.* and sold for 7*s.* 6*d.*, what is the gain upon the sale of what cost 100*l.* ?

Ex. 5. Bought cheese at 3*l.* 3*s.* per cwt., and sold it again at 10½*d.* per lb. : what is the gain per cent., supposing the loss in weight to be 4 lb. per cwt.

Ex. 6. Bought silk stockings at 12*s.* 9*d.* per pair : what must they be sold for to gain 20 per cent. profit ?

Ex. 7. If 375 yards of cloth be sold for 290*l.*, and there be 20 per cent. profit, what did it cost per yard ?

Ex. 8. Sold 1 cwt. of hops, at 6*l.* 15*s.*, at the rate of 25 per cent. profit : what would have been the gain per cent. if I had sold them for 8 guineas per cwt. ?

Ex. 9. If 90 ells of cambric cost 120*l.*, for how much must I sell it per yard to gain 18 per cent. ?

Ex. 10. A plumber sold 5 fother of lead for 102*l.* 2*s.* 6*d.* (the fother being 19½ cwt.), and gained after the rate of 12*l.* 10*s.* per cent. : what did it cost him per cwt. ?

Ex. 11. Bought 218 yards of cloth, at the rate of 8*s.* 6*d.* per yard, and sold it for 10*s.* 4*d.* per yard : what was the gain of the whole ?

Ex. 12. Paid 69*l.* for one ton of steel, which is retailed at 8*d.* per lb., what is the profit or loss by the sale of 12 tons ?

# PARTNERSHIP

Is a general rule, by which merchants, &c., trading in company with a joint stock, are enabled to ascertain each person's particular share of the gain or loss, in proportion to his share in the stock.*

This rule divides itself into two parts, viz. 1. Partnership without regard to time; and 2. Partnership with time.

## I. PARTNERSHIP WITHOUT TIME.

RULE. "*As the whole stock is to the whole gain or loss, so is each man's share in the stock to his share of the gain or loss.*"

Ex. 1. Two merchants embark in business, the one puts in as capital £.5550, and the other £.3420, and they gain in the first year £.1260, what is each man's gain?

£. 5550
3420

8970 = joint stock.

8970*l.* : 1260*l.* : : 5550*l.* : 779*l.* 12*s.* nearly; of course the profits of the other are 1260*l.* — 779*l.* 12*s.* = 480*l.* 8*s.*

Ex. 2. Three persons trade together: A puts in 100*l.*; B 150*l.*; C 200*l.*; and they gain 900l: what is each man's gain?

Ex. 3. A, B, and C, enter into partnership; A puts in 3640*l*., B, 4820*l.*, and C 5000*l.*, and they gained 8670*l.*: what is each man's share in proportion to his stock?

Ex. 4. Four merchants, B, C, D, and E, make a stock; B put in 2270*l.*, C 3490*l.*, D 1150*l.* and E 4390*l.*; in trading they gained 4280*l.*: I demand each merchant's share of the gain?

Ex. 5. Three persons, D, E, and F, join in company; D's stock was 3750*l.*, E's 2800*l.*, and F's 2500*l.*, and at the end of 12 months they gained 3420*l.*; what is each man's particular share of the gain?

## II. PARTNERSHIP WITH TIME.

RULE. *As the sum of the product of each man's money and time is to the whole gain or loss, so is each man's product to the share of the gain and loss.*

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NOTE.

* This rule is of great use in various concerns; by it a bankrupt's estate may be accurately divided among his creditors. Legacies are also adjusted by it, when there is not money enough left to answer all the demands of the legatees.

Ex. 1. Two persons lay out 1500*l.* in trade, in the proportion of 3 to 2: that is, A put in 900*l.*, and B 600*l.*; A leaves his money in the concern 9 months, and B does not want his for 12 months: what profits belong to each, supposing they gain 250*l.*?

$$\begin{array}{rrrrr} £.\ 900 \times 9 & = & 8100 & & \\ 600 \times 12 & = & 7200 & & \\ \hline & & 15300 & & \\ 15300 & : & 250 & :: & 8100 \\ & & & & 250 \\ \hline \end{array}$$

$$153{,}00)20250{,}00(132l.\ 7s.$$

| | | | | |
|---|---|---|---|---|
| Answer, | A's share of profit | £. 132 | 7 | 0 |
| | B's - - - - - - | 117 | 13 | 0 |
| | | £. 250 | 0 | 0 |

Ex. 2. A puts into a concern 2080*l.* for 2 months, B 970*l.* for 5 months, and C 400*l.* for 15 months; they gain among them 650*l.*; what must each receive for his share of profit?

Ex. 3. Three merchants join in company for 18 months: D put in 500*l.*, and at 5 months end took out 200*l.*; at 10 months end put in 500*l.*, and at the end of 14 months takes out 130*l.* E puts in 400*l.*, and at the end of 3 months 270*l.* more; at 9 months he takes out 140*l.*, but puts in 100*l.* at the end of 12 months, and withdraws 99*l.*, at the end of 15 months. F put in 900*l.*, and at 6 months took out 200*l.*; at the end of 11 months put in 600*l.*, but takes out that and 100*l.* more at the end of 13 months. They gained 200*l.* I desire to know each man's share of the gain?

* * *

# ALLIGATION

Teaches to mix things of different values, so as to ascertain the price of the mixture. There are two cases in this rule.

1. To find the mean value of a mixture composed of several quantities of different values.

RULE. *Multiply each quantity by its respective value, and divide the sum of the products by the sum of the quantities.*

Ex. 1. A tea-dealer mixes 3½ cwt. of tea, at 9*s.* per lb., with 2 cwt. at 7*s*., and $4\frac{1}{4}$ cwt. at 5*s*. 6*d*., at how much per lb. can he sell the whole mixture?

$$\left.\begin{array}{llll} 3\frac{1}{2} & \times\ 112 & = & 392 \\ 2 & \times\ 112 & = & 224 \\ 4\frac{1}{4} & \times\ 112 & = & \underline{476} \\ & & & 1092 \end{array}\right\} \text{and} \left\{\begin{array}{llll} 392 & \times\ 9 & = & 3528 \\ 224 & \times\ 7 & = & 1568 \\ 476 & \times\ 5.6 & = & \underline{2618} \\ & & & )7714(7s.\ 0\frac{3}{4}d.\frac{84}{1092} \\ & & & \underline{7644} \\ & & & ..70 \end{array}\right.$$

Answer - 7*s*. $0\frac{3}{4}$*d*.

Ex. 2. What is a lb. of sugar worth which is compounded of 3 cwt. at 46*s*.; 2 cwt. at 59*s*.; $1\frac{1}{2}$ cwt. at 84*s*.; and 56 lb. at 60*s*.?

Ex. 3. What is the average earnings of workmen, 48 of whom earn 2*l*. each per week; 80 earn 45*s*. each; 120 in inferior business will get only 25*s*. each?

Ex. 4. A tobacconist mixes 80 lb. of tobacco at 20*d*. per lb.; 150 lb. at 2*s* 3*d*. per lb.; and 40 lb. at 3*s*. 10*d*. per lb.; what will be the value of the mixture per oz.?

## II. To find how much of different things of different values, must be taken, in order to make a mixture of a certain mean value.

**Rule** (1). *Set down the names of the things to be mixed, together with their prices; then, finding the difference between each of these, and the proposed price of the mixture; place these differences in an alternate order, and they will shew the proportion of the ingredients.*

Ex. 1. Orange wine, at 9*s*. per gallon, is to be mixed with raisin wine at 6*s*. per gallon; what will be the proportions, so as to sell the mixture at 7*s*, per gallon?

| | | | Proposed price. | |
|---|---|---|---|---|
| Orange, | - | 9*s*. | 7*s*. | 1 |
| Raisin | - | 6*s*. | | 2 |

A mixture therefore of these wines in the proportion of 1 orange to two raisin, will be the answer.

Ex. 2. A spirit at 16 shillings, and another at 12 shillings per gallon, are to be mixed with low wines at 6 and 5 shillings, in order to produce a mixture worth 9 shillings per gallon; what must the quantities of each be?

| | | | | |
|---|---|---|---|---|
| Spirit, | - | 16 | 9 | 3 |
| Ditto, | - | 12 | | 4 |
| Wine, | - | 6 | | 7 |
| Ditto, | - | 5 | | 3 |

The answer is, 3 gallons at 16*s*., 4 at 12*s*.; 7 at 6*s*.; and 3 at 5*s*.; will make a mixture that may be sold for 9 shillings per gallon: for

$3 \times 16 = 48$
$4 \times 12 = 48$
$7 \times 6 = 42$
$3 \times 5 = 15$

$17 \qquad 153$ and $\frac{153}{17} = 9s.$ Proof.

Ex. 3. A tea-dealer would mix four sorts of tea together, viz. at 4*s.*, 4*s.* 6*d.*, 5*s.* 6*d.*, 6*s.*, and 7*s.* per lb.; in order that he may sell the whole mixture at 5*s.* 6*d.* per lb., what proportion of each will he use?

Ex. 4. How much snuff, at 4*s.*, 3*s.* 6*d.*, 2*s.* 3*d.*, and 2*s.* per lb., will compose a mixture worth 2*s.* 6*d.* per lb.?

III. When the prices of all the things to be mixed are given, likewise where the quantity of one, and the mean rate are also given, to find the several quantities of the others.

Rule. (1) *Take the difference between each price and the mean rate as before.* (2) *As the difference of that thing, whose quantity is given, is to the rest of the differences severally; so is the quantity given to the several quantities required.*

Ex. 1. A rectifier of compounds has 200 gallons of spirit that he can sell for 12*s.* 6*d.* per gallon, but he means to mix it with three other kinds of spirit at 13*s.* 4*d.*, at 15*s.*, and 18*s.* 4*d.* per gallon, in order that he may sell the whole at 14*s.* 2*d.* per gallon; how much must he use of each?

I reduce the several prices to pence, which stand as follow:

| | | | |
|---|---|---|---|
| 170 | 150 | 50 | |
| | 160 | 10 | 50 : 10 : : 200 : 40 |
| | 180 | 10 | 50 : 10 : : 200 : 40 |
| | 220 | 20 | 50 : 20 : : 200 : 80 |

The answer is; to 200 gallons, at 12*s.* 6*d.*, must be added 40 at 13*s.* 4*d.*, 40 at 15*s.*, and 80 at 18*s.* 4*d.*; the truth of which is proved thus;

| | | *s.* | *d.* | | £. | *s.* | *d.* |
|---|---|---|---|---|---|---|---|
| 200 | at | 12 | 6 | = | 125 | 0 | 0 |
| 40 | at | 13 | 4 | = | 26 | 13 | 4 |
| 40 | at | 15 | 0 | = | 30 | 0 | 0 |
| 80 | at | 18 | 4 | = | 73 | 6 | 8 |
| 360 | | | | | 255 | 0 | 0 |

and $\frac{255}{360} = 14s.\ 2d.$ Proof.

Ex. 2. A grocer has 100 lb. of tea worth 4*s.* per lb., which he means to mix with others at 12*s.* 3*d.*, 10*s.*, and 6*s.* per lb.; in order to sell the whole at 8*s.*, how much of each must be used?

IV. When the price of each thing is given, also the quantity and the mean rate, to find how much of each sort will make that quantity.

Rule. (1) *Take the difference between each price and the mean rate as before:* then (2) *As the sum of the differences is to each particular difference, so is the quantity given to the quantity required.*

Ex. 1. A wine merchant means to mix 860 gallons of wine to sell for 8*s.* a gallon, out of other wines that he already sells for 12*s.*, 9*s.*, 6*s.*, and 5*s.* per gallon, how much must he take of each?

| | | | |
|---|---|---|---|
| 8 | 12 | 3 | 10 : 3 : : 860 : 258 |
| | 9 | 2 | 10 : 2 : : 860 : 172 |
| | 6 | 1 | 10 : 1 : : 860 : 86 |
| | 5 | 4 | 10 : 4 : : 860 : 344 |
| Sum of differences = | | 10 | 860 |

The answer is 258 gallons at 12*s.*; 172 at 9*s.*; 86 at 6*s.*; and 344 at 5*s.* per gallon, may be mixed and sold at 8*s* per gallon.

Ex. 2. A goldsmith has four sorts of gold, viz. of 24, 10, 18, and 15 carats fine, wishes 126 oz. of the fineness of 17 carats, how much will he want of each sort?

Ex. 3. A drug grinder has bark worth 16*s.* per lb., some at 10*s.* and some at 4*s.*; but he is desirous of making up two parcels, viz. one containing a cwt. at 9*s.*, and the other 84 lb. at 12*s.*; what proportions of each must be used?

* * *

# POSITION.

Position, or as it is sometimes called, the Rule of False, is a rule, that by means of any supposed numbers, others that are true, and that answer to the terms of the question, are found. There are two kinds of Position, viz. Single and Double.

Single Position is performed, by using a supposed number, and working with it as the true one, till the real number is found.

Rule. *Take any number and perform the work with it,* *as if it were the right number: then say, As the result of this work is to the position, so is the result in the question to the number required.*

Ex. 1. A person counting some guineas, being asked how many he had, replied: "If you had as many, and as many more, and half as many, and one quarter as many, you would have 264." How many had the person who was counting his gold?

By way of supposition, I take 80 as the number; then, by the terms of the question, it will be

| | | | |
|---|---|---|---|
| | 80 | | 96 |
| As many more, | 80 | 220 : 264 : : 80 | 96 |
| Half as many, | 40 | 80 | 48 |
| $\frac{1}{4}$th as many, | 20 | | 24 |
| | | 220)21120(96 Answ. | |
| | 220 | | 264 Proof. |

Ex. 2. A person, after spending $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{6}$th of his money, finds he had 500*l.* left, what was his original property?

I take a number divisible by 2, 4, and 6, for the supposition, viz. 60.

| | | | |
|---|---|---|---|
| Suppose | 60 | 60 — 55 = 5, therefore | Proof. |
| | | As 5 : 60 : : 500 | $\frac{1}{2}$ = 3000 |
| $\frac{1}{2}$ | 30 | 60 | $\frac{1}{4}$ = 1500 |
| $\frac{1}{4}$ | 15 | | $\frac{1}{6}$ = 1000 |
| $\frac{1}{6}$ | 10 | 5)30.000 | |
| | | | 5500 |
| | 55 | Answer, 6.000 £.* | 500 rem. |

Ex. 3. Three persons bought goods at Manchester which cost 600*l.* The first person was to have a third part more than the second, and the third a fourth part more than the first; what was each man's share?

Ex. 4. In a leaky vessel there were three pumps of different capacities; the first would empty the hold of the ship in 20 minutes, the second would require double that time, and the third would not perform the business in less than an hour; how long would all three together take in doing it?

---

NOTE.

* Any other number, as 12 for instance, would have answered the same purpose: then it would have been 12 — 11 = 1, and
1 : 12 : : 500 : 6000*l.*

# DOUBLE POSITION.

Questions in this rule are resolved by making suppositions of two numbers, which *may* both prove false; in that case the errors are made to correct each other.

Rule. (1) *Place each error against its respective position, and multiply them cross ways.* (2) *If the errors are alike, that is, both greater or both less than the given number, take their difference for a divisor, and the difference of their products for a dividend. But if unlike, take their sum for a divisor, and the sum of their products for a dividend, the quotient will be the answer.*

Ex. 1. Three persons have obtained the 20,000*l.* prize in the lottery, and it is to be so divided, that the second is to have 600*l.* more than the first, and the third 800*l.* more than the second, what is each person's share?

| | | | |
|---|---|---|---|
| Suppose the first had | 5000 | Suppose the first had | 5600 |
| Then the second had | 5600 | The second had | 6200 |
| and the third had | 6400 | The third had | 7000. |
| | 17000 too little by 3000 | | 18800 too little by 1200. |

$$\left.\begin{array}{ccc}3000 & & 5000\\ & \times & \\ 1200 & & 5600\end{array}\right\}\text{ that is, }\left\{\begin{array}{l}3000 \times 5600 = 16800000\\ 1200 \times 5000 = 600000\end{array}\right.$$

Diff. of Products, - 10800000 = dividend.

3000 — 1200 = 1800 (diff. of errors) for a divisor.

$$\text{Therefore, } \frac{10.800.000}{1800} = 6000\ \text{£.}$$

6000
6600
7400

£. 20.000 Proof.

Ex. 2. A gentleman, at Christmas, wished to give several poor families 5 shillings each, but he found he had 16*s.* 8*d.* too little; he then gave them 3*s.* 6*d.* each, and found he had 4*s.* 4*d.* left, how many families were there?

Ex. 3. A person purchased a house and land, together with a carriage and horses, for 1500*l.*; he paid 4 times the price of the carriage and horses for the land, and 5 times the price of the land for the house, what was the value of each separately?

# COMPOUND INTEREST AND ANNUITIES.

**Compound Interest**, or interest upon interest, is that which is paid not only for the use of the money lent, but also for the use of the interest as it becomes due.*

There are three methods of working Problems in this Rule, viz. by Common Arithmetic; by Decimals; and by Logarithms: I shall give examples under each.

## I. By Common Arithmetic.

**Rule 1.** *Find the amount of the given principal for the time of the first payment by simple interest.* 2. *Consider this amount as the principal for the second payment, the amount of which is to be calculated as before, and so on through all the payments to the last, still reckoning the last amount as the principal for the next payment.*

Ex. 1. What is the amount of 550*l.* for three years, at 5 per cent. compound interest?

| | | | | |
|---|---|---|---|---|
| | 20)550 | 0 | 0 | given principal. |
| | 27 | 10 | 0 | first year's interest. |
| | 20)577 | 10 | 0 | second year's principal. |
| | 28 | 17 | 6 | second year's interest. |
| | 20)606 | 7 | 6 | third year's principal. |
| | 30 | 6 | 4½ | third year's interest. |
| Answer - | 636 | 13 | 10½ | |

Ex. 2. What is the amount of 400*l.* for four year's, at 5 per cent. compound interest?

Ex. 3. What is the compound interest of 600*l.* for five years, at 5 per cent. compound interest?†

---

NOTES.

* It is not lawful to lend money at compound interest: but in granting or purchasing annuities, leases, or reversions, compound interest for money is allowed.

† Here, when the *amount* is found, the principal must be taken from it, and the remainder is the compound interest. Thus, in the first example, the compound interest is 636*l.* 13*s.* 10½*d.* — 550*l.*, or 86*l.* 13*s.* 10½*d.* In short periods compound interest differs but little from simple interest; in this case, for instance, the simple interest

## II. By Decimals.

RULE 1. *Find the amount of* 1*l. for a year, at the given rate per cent.* 2. *Involve* the amount thus found, to such a power as is denoted by the number of years.* 3. *Multiply this power by the principal, or given sum, and the product will be the amount required.* 4. *Subtract the principal from the amount, and the remainder will be the interest.*

Ex. 1. What is the compound interest of 550*l.* for 3 years, at 5 per cent. per annum?

1.05 = amount of 1*l.* for a year, at 5 per cent.;
Then $1.05 \times 1.05 \times 1.05 = 1.157625$, and
$1.157625 \times 550 = 636.69375$ = amount,
$636.69375 - 550 = 86.69375$ = 86*l.* 13*s.* 10½*d.*

Ex. 2. What is the amount of 400*l.* for 4 years, at 5 per cent. per annum?

Ex. 3. What is the compound interest of 620*l.* for 5 years, at 5 per cent.?

## III. By Logarithms.

RULE. *Multiply the logarithm of the amount of* 1*l. for a year, by the number of years, and to the product add the logarithm of the principal, and the answer is the amount required.*

Ex. 1. What is the amount of 550*l.* for 3 years, at 5 per cent. per ann. compound interest?

0.0211893 = log. of 1.05
3
0.0635679 = log. of $1.05 \times 3$
2.7403627 = log. of 550
2.8039306† = 636*l.* 13*s.* 10½*d.*, as before, = amount.

---

NOTES.

would be 82*l.* 10*s.* When, however, the interest is suffered to accumulate for many ages, the difference between simple and compound interest is almost beyond belief. To give an example, we have seen, p. 164, that a penny put out to simple interest at the birth of Christ would, at the end of the year 1800, only amount to about 7*s.* 6*d.*; whereas the same sum, for the same period, at Compound Interest, would have amounted to a sum greater than could be contained in six hundred millions of globes, each equal to the earth in magnitude, and all of solid gold.

* See p. 142.

† The nearest logarithm to 2.8039306 is 2.8034571, which answers to 636; the difference between the logarithms is 4735, which I

Ex. 2. What is the amount of 400*l.* for four years, at 5 per cent. per annum compound interest?*

Ex. 3. What is the compound interest of 609*l.* for 5 years, at 5 per cent. per annum?

Ex. 4. What is the amount of 845*l.* for 14 years, at 5 per cent. compound interest?

We shall now proceed to consider this subject more generally by the help of tables, the *construction* of which will be shewn in the note below.†

---

NOTE.

multiply by 20 to bring into shillings, and divide by the common difference found at the margin of the table; thus,

4735
20
6823)94700(18
88699
6001
12
6823)72012(10
68230
3782
4
6823)15128($\frac{1}{2}$
13646
1482

* We have purposely given the same examples in all three methods, in order that the pupil may compare the difference in working.

† Although we have intentionally abstained from the use of symbols or letters, yet it seems necessary to give some account of the tables by which Compound Interest, Life Annuities, Reversions, &c. are calculated. This will be perfectly intelligible to those who have made themselves masters of what is gone before; and in a work on Algebra, preparing for the press, will be given a demonstration of the rules of common arithmetic:

Let $r =$ the amount of 1*l.* for one year,
$n =$ the number of years,
$p =$ principal,
$a =$ amount, or principal and interest of the given sum for the time required.

By proportion we say, as 1*l.* principal is to the amount of 1*l.* for one year, so is the amount of 1*l.* for one year, to the amount for two years, and so on: thus

$1 : r :: r : r^2 =$ amount or 1*l.* in 2 years.
$1 : r :: r^2 : r^3 =$ amount of 1*l* in 3 years.
$1 : r :: r^3 : r^4 =$ amount of 1*l.* in 4 years.
$1 : r :: r^4 : r^5 =$ amount of 1*l.* in 5 years.

### NOTES.

Hence is seen the reason of the foregoing rules; for, by involving the amount of 1*l.* for a year, to such a power as is denoted by the number of years, we get the amount; that is, the principal and interest of 1*l.* for the given number of years; that is $r^n$, or $r$ raised to the power whose exponent is the number of years, will be the amount of 1*l.* in those years; and as

$1 : r^n :: p : a =$ the amount of any given principal in the same time.

We have seen in Proportion, or the Rule of Three, that when three of the terms are given, we readily find the fourth; hence the following Theorems.

Theo. I. The principal, time, and rate of interest are given, to find the amount?

Since $1 : r^n :: p : a$, therefore $p \times r^n = a =$ amount.

Ex. Suppose the amount of 520*l.* for 15 years be required, at 5 per cent. compound interest; then

$$500 \times \overline{1.05)}^{15} = a.$$

Theo. II. The amount, time, and rate are given, to find the principal? then

$$\frac{a}{r^n} = p = \text{principal.}$$

Ex. What is the principal that will amount to 636*l.* 13*s.* 10½*d.* in 3 years, at 5 per cent. compound interest?

$$\frac{636l.\ 13s.\ 10\tfrac{1}{2}d.}{\overline{1.05)}^3} = p.$$

Theo. III. The principal, amount, and time, are given, to find the rate?

$$\sqrt[n]{\frac{a}{p}} = r = \text{amount of } 1l. \text{ for one year, and } r - 1 = \text{rate.}$$

Ex. At what rate per cent. will 550*l.* amount to 635*l.* 13*s.* 10½*d.* in 3 years?

$$\sqrt[3]{\frac{636l.\ 13s.\ 10\tfrac{1}{2}d.}{550}} = r.$$

Theo. IV. The principal, amount, and rate, are given, to find the time?

$$\frac{a}{p} = r^n \left\{ \begin{array}{l} \text{Therefore } \frac{a}{p} \text{ being divided by } r \text{ till nothing remains,} \\ \text{the number of divisions will } = n. \end{array} \right.$$

The foregoing theorems are more easily expressed by logarithms;

Thus, I. $\text{Log.}\ p + n \times \log.\ r. = \log.\ a.$

II. $\text{Log.}\ a - n \times \log.\ r. = \log.\ p.$

III. $\frac{\text{Log.}\ a - \log.\ p}{n} = \log.\ r.$

IV. $\frac{\text{Log.}\ a - \log.\ p}{\text{Log.}\ r} = n.$

## TABLE I.

Shewing the Sum to which 1*l*. Principal will increase at 5 per cent. Compound Interest, in any number of years not exceeding a hundred.

| Yrs. | Amount. | Yrs. | Amount. | Yrs. | Amount. | Yrs. | Amount. |
|---|---|---|---|---|---|---|---|
| 1 | 1.05 | 26 | 3.555672 | 51 | 12.040769 | 76 | 40.774320 |
| 2 | 1.1025 | 27 | 3.733456 | 52 | 12.642808 | 77 | 42.813036 |
| 3 | 1.157625 | 28 | 3.920129 | 53 | 13.274948 | 78 | 44.953688 |
| 4 | 1.215506 | 29 | 4.116135 | 54 | 13.938696 | 79 | 47.201372 |
| 5 | 1.276281 | 30 | 4.321942 | 55 | 14.635630 | 80 | 49.561441 |
| 6 | 1.340095 | 31 | 4.538039 | 56 | 15.367412 | 81 | 52.039513 |
| 7 | 1.407100 | 32 | 4.764941 | 57 | 16.135783 | 82 | 54.641488 |
| 8 | 1.477455 | 33 | 5.003188 | 58 | 16.942572 | 83 | 57.373563 |
| 9 | 1.551328 | 34 | 5.253347 | 59 | 17.789700 | 84 | 60.242241 |
| 10 | 1.628894 | 35 | 5.516015 | 60 | 18.679185 | 85 | 63.254353 |
| 11 | 1.710339 | 36 | 5.791816 | 61 | 19.613145 | 86 | 66.417071 |
| 12 | 1.795856 | 37 | 6.081406 | 62 | 20.593802 | 87 | 69.737924 |
| 13 | 1.885649 | 38 | 6.385477 | 63 | 21.623492 | 88 | 73.224820 |
| 14 | 1.979931 | 39 | 6.704751 | 64 | 22.704667 | 89 | 76.886061 |
| 15 | 2.078928 | 40 | 7.039988 | 65 | 23.839900 | 90 | 80.730365 |
| 16 | 2.182874 | 41 | 7.391988 | 66 | 25.031895 | 91 | 84.766883 |
| 17 | 2.292018 | 42 | 7.761587 | 67 | 26.283490 | 92 | 89.005227 |
| 18 | 2.406619 | 43 | 8.149666 | 68 | 27.597664 | 93 | 93.455488 |
| 19 | 2.526950 | 44 | 8.557150 | 69 | 28.977548 | 94 | 98.128263 |
| 20 | 2.623297 | 45 | 8.985007 | 70 | 30.426425 | 95 | 103.034676 |
| 21 | 2.785962 | 46 | 9.434258 | 71 | 31.947746 | 96 | 108.186410 |
| 22 | 2.925260 | 47 | 9.905971 | 72 | 33.545134 | 97 | 113.595730 |
| 23 | 3.071523 | 48 | 10.401269 | 73 | 35.222390 | 98 | 119.275517 |
| 24 | 3.225099 | 49 | 10.921333 | 74 | 36.983510 | 99 | 125.239293 |
| 25 | 3.386354 | 50 | 11.467399 | 75 | 38.832685 | 100 | 131.501257 |

### I. To find by means of the table what any sum will amount to in a given number of years.

RULE. *Multiply the number in the table, opposite to the term of years, by the sum, and the product will be the answer.*

Ex. 1. To what sum will 500*l*. amount in 44 years, at 5 per cent. compound interest ?*

---

NOTES.

On these principles all tables of Compound Interest are calculated ; of which we shall give those that are the most useful and applicable to other subjects, of which we shall have occasion to treat.

* The legal interest of the country being 5 per cent., we have restricted our tables and reasoning to this. The pupil will, however, readily apply this and any questions, either to higher or lower rates, of which tables are to be found in divers books. See PRICE's Annuities.

Opposite to 44 in the table I find 8.557150, this I multiply by 500, and the answer is 4278*l.* 11*s.* 6*d.*

Ex. 2. What will 350*l.* amount to in 25 years, at 5 per cent. compound interest?

Ex. 3. A prudent young man marries at the age of 22; the fortune which he has with his wife is 2500*l.*, half of which he readily gives into the hands of trustees to be accumulated at 5 per cent. compound interest; what will it amount to, supposing he lives 32 years, which he may reasonably expect?*

Ex. 4. The year 1808, is that in which the late Mr. Pitt calculated there would be four millions surplus to be applied to the payment of the national debt; I demand to how much this single four millions will accumulate in half a century, at 5 per cent. compound interest?†

[*See other questions on this subject after the next table.*]

## II. To find the number of years in which a given sum will increase to another given sum, in consequence of being improved at Compound Interest.

RULE. *Divide the latter sum by the former, and the sum in the table which is nearest to the quotient will shew the term required.*

---

NOTES.

* We shall, page 201, give a table on this subject, and the reasons upon which it is founded; *as a general rule*, and which will answer in all ages except the very young, and the very old, we may give this as the mode of estimating the expectation of life: "*Subtract your own age from 86, and divide by 2: thus the young man of 22, taking that number from 86, and dividing the remainder by 2, finds his expectation of life* 32 *years.*" This is not perfectly accurate; by the table it will be found that his expectation is 32.39, that is 32 years, four months, and a fortnight. Take another example; I am now 45; by the rule just given my expectation of life is $\frac{86-45}{2} = \frac{41}{2} = 20\frac{1}{2}$; and by the table to be given hereafter (p. 201) it will be found that a person of 45 may expect 20.52; in this case the difference is but a week.

† It will be seen by the table, that *one* pound put out to compound interest, will be about $94\frac{1}{2}$ years in accumulating to 100*l.* principal. It is frequently said, that money doubles itself at 5 per cent. compound interest, in 14 years, this is by no means accurate, as is evident from the table; if money doubled itself in 14 years, then 1*l.* in 98 years would produce 128*l.*, whereas in that time it only produces 119*l.* 5*s.* 6*d.* But an *annuity* of 1*l.* suffered to accumulate at 5 per cent. compound interest, will produce 100*l.* in somewhat less than 37 years. "See Table in the opposite page."

Ex. 1. In what time will 200*l.* increase to 1500*l.*, if improved at 5 per cent. compound interest?

$\frac{1500}{200} = 7.5$. The nearest number in the table to 7.5 is 7.391988, opposite to which is 41, the number of years. Of course 200*l.* in a little more than forty-one years would, by being accumulated at compound interest, at 5 per cent., amount to 1500*l.*

Ex. 2. In what time will 100*l.* increase to 500*l.* at the same rate of interest?

Ex. 3. In what time will 860*l.* increase to 0,000*l.*?

Ex. 4. In how long would five millions be in paying the national debt, which in January, 1806, was upwards of 580 millions?

Ex. 5. Admiral Rainier has left, this month, May, 1808, 25,000*l.* towards paying off the national debt, when will it have accumulated to a million at 5 per cent. compound interest?

## TABLE II.

Shewing the sum to which 1*l.* *per annum* will increase at 5 per cent. Compound Interest, in any number of years not exceeding a hundred.

| Yrs. | Amount. | Yrs. | Amount. | Yrs. | Amount. | Yrs. | Amount. |
|---|---|---|---|---|---|---|---|
| 1 | 1,0000 | 26 | 51,1135 | 51 | 220,8154 | 76 | 795,4864 |
| 2 | 2,0500 | 27 | 54,6691 | 52 | 232,8562 | 77 | 836,2607 |
| 3 | 3,1525 | 28 | 58,4026 | 53 | 245,4990 | 78 | 879,0738 |
| 4 | 4,3101 | 29 | 62,3227 | 54 | 258,7739 | 79 | 924,0274 |
| 5 | 5,5256 | 30 | 66,4388 | 55 | 272,7126 | 80 | 971,2288 |
| 6 | 6,8019 | 31 | 70,7608 | 56 | 287,3482 | 81 | 1020,7903 |
| 7 | 8,1420 | 32 | 75,2988 | 57 | 302,7157 | 82 | 1072,8298 |
| 8 | 9,5491 | 33 | 80,0638 | 58 | 318,8514 | 83 | 1127,4713 |
| 9 | 11,0266 | 34 | 85,0670 | 59 | 335,7940 | 84 | 1184,8448 |
| 10 | 12,5779 | 35 | 90,3203 | 60 | 353,5837 | 85 | 1245,0871 |
| 11 | 14,2068 | 36 | 95,8363 | 61 | 372,2629 | 86 | 1308,3414 |
| 12 | 15,9171 | 37 | 101.6281 | 62 | 391,8760 | 87 | 1374,7586 |
| 13 | 17,7130 | 38 | 107,7095 | 63 | 412,4698 | 88 | 1444,4964 |
| 14 | 19,5986 | 39 | 114,0950 | 64 | 434,0933 | 89 | 1517,7212 |
| 15 | 21,5786 | 40 | 120,7998 | 65 | 456,7980 | 90 | 1594,6073 |
| 16 | 23,6575 | 41 | 127,8398 | 66 | 480,6379 | 91 | 1675,3377 |
| 17 | 25,8404 | 42 | 135,2317 | 67 | 505,6698 | 92 | 1760,1045 |
| 18 | 28,1328 | 43 | 142,9933 | 68 | 531,9533 | 93 | 1849,1098 |
| 19 | 30,5390 | 44 | 151,1430 | 69 | 559,5510 | 94 | 1942,5653 |
| 20 | 33,0659 | 45 | 159,7002 | 70 | 588,5285 | 95 | 2040,6935 |
| 21 | 35,7192 | 46 | 168,6852 | 71 | 618,9549 | 96 | 2143,7282 |
| 22 | 38,5052 | 47 | 178,1194 | 72 | 650,9027 | 97 | 2251,9146 |
| 23 | 41,4305 | 48 | 188,0254 | 73 | 684,4478 | 98 | 2365,6103 |
| 24 | 44,5020 | 49 | 198,4267 | 74 | 719,6702 | 99 | 2484,7859 |
| 25 | 47,7271 | 50 | 209,3480 | 75 | 756,6537 | 100 | 2610,0252 |

### 1. To find in what time a given annuity will amount to a given sum at compound interest.

RULE. *Divide the given sum by the given annuity, and the number in the table nearest to the quotient will be the answer.*

**Ex. 1.** A person owes 1000*l.* and resolves to appropriate 20*l.* per annum, to be accumulated at 5 per cent. per ann. compound interest, in how many years will the debt be paid?

$\frac{1000}{20} = 50$. The nearest number in the table to 50 is 51.1135, and the number answering to this is 26, so that in less than 26 years a debt of 1000*l.* would be extinguished by laying by, and accumulating at compound interest, annually 20*l.* per annum. If the rate of interest had been 6 per cent. 24 years would have paid the debt, but at 4 per cent. it would have taken between 28 and 29 years.

Ex. 2. How long will 75 guineas a year be in accumulating to 2000*l.*, at the same rate?

Ex. 3. In what time will an annuity of 25*l.* amount to 3575*l.*, at the same rate?

Ex. 4. How long will the national debt, left at the time of Mr. Pitt's death, viz. 581 millions, be in paying off, supposing five millions annually be appropriated for that purpose, and the rate of compound interest 5 per cent.?

Ex. 5. The national debt was, at Midsummer 1807, 756* millions of pounds, out of which the commissioners had redeemed 117 millions and a half, how long would the remainder take in paying off, if eight millions be applied annually, at the rate of 5 per cent. compound interest for the purpose?†

---

NOTES.

* This includes a debt of nearly 65 millions, the debt of Ireland; the whole is of course the debt of the United Kingdom.

† The fund applied to reducing the national debt was in (1808) 8 millions, which, as has been shewn in this question, will, in 33 years, reduce a debt of 639 millions to nothing. It cannot however be expected that 5 per cent. can be obtained through the whole progress, still the operation of this sum, at 4 per cent. even, would be almost omnipotent. In the year 1806, the amount, to which 8 millions per annum would accumulate, at different periods during the present century, if improved at 4 and 5 per cent. compound interest, as stated in round numbers as follows—

Ex. 6. During the quarter between Lady-day and Midsummer 1809, upwards of 2,800,000*l.* was appropriated towards paying off the national debt: supposing as much set apart every quarter, or 11,200,000*l.* annually, in how long time will such an annuity pay off 1,000,000,000*l.*, a sum to which the debt must accumulate in a few years?

## II. To find how much a given annuity will amount to in a given term, at 5 per cent. compound interest.

RULE. *Multiply the given annuity by the number in the table standing opposite to the given term of years.*

Ex. 1. I can lay by 50*l.* per annum with its interest; that is, I can appropriate 50*l.* a year to be accumulated at 5 per cent. compound interest, how much shall I have saved if I live 21 years?

Opposite to 21 years I find 35.719, which multiplied by 50, gives 1785.962550. Answer, 1785*l.* 19*s.* 2*d.*

Ex. 2. How much will an annuity of 35*l.* amount to in 83 years?

Ex. 3. To what sum will an annuity of 100 guineas amount in 19 years, at 5 per cent. compound interest?

Ex. 4. To what sum will 60*l.* per annum amount to in 25 years?

III. *The* PRESENT VALUE *of an annuity is that sum which, if improved at compound interest, would be sufficient to pay the annuity.* For this the following table is adapted.

---

NOTES.

| | | 4 per cent. Millions. | 5 per cent. Millions. |
|---|---|---|---|
| In the year | 1810, or in 4 years | 33 | 34 |
| | 1820 | 146 | 156 |
| | 1830 | 312 | 356 |
| | 1840 | 558 | 680 |
| | 1850 | 923 | 1209 |
| | 1860 | 1463 | 2070 |
| | 1900 | 7782 | 15540 |

The reader will see at once, in what manner the sums standing under 5 per cent. are obtained, from the table in p. 193. The calculation was made in 1806, of course, in 1810, the sum will have been FOUR years accumulating; multiply therefore, 4,3101, which is opposite 4 in the table, by 8 millions, and we get for the answer 34.403, or almost 34 millions and a half; and so of the others.

## TABLE III.

Shewing the present Value of an Annuity of 1*l.* for any number of Years not exceeding 100, at 5 per cent. per annum, Compound Interest.

| Yrs. | Value. | Yrs. | Value. | Yrs. | Value. | Yrs. | Value. |
|---|---|---|---|---|---|---|---|
| 1 | ,952381 | 26 | 14,375185 | 51 | 18,338977 | 76 | 19,509495 |
| 2 | 1,859410 | 27 | 14,643034 | 52 | 18,418073 | 77 | 19,532853 |
| 3 | 2,723248 | 28 | 14,898127 | 53 | 18,493403 | 78 | 19,555098 |
| 4 | 3,545950 | 29 | 15,141074 | 54 | 18,565146 | 79 | 19,576284 |
| 5 | 4,329477 | 30 | 15,372451 | 55 | 18,633472 | 80 | 19,596460 |
| 6 | 5,075692 | 31 | 15,592810 | 56 | 18,698545 | 81 | 19,615677 |
| 7 | 5,786373 | 32 | 15,802677 | 57 | 18,760519 | 82 | 19,633978 |
| 8 | 6,463213 | 33 | 16,002549 | 58 | 18,819542 | 83 | 19,651407 |
| 9 | 7,107822 | 34 | 16,192904 | 59 | 18,875754 | 84 | 19,668007 |
| 10 | 7,721735 | 35 | 16,374194 | 60 | 18,929290 | 85 | 19,683816 |
| 11 | 8,306414 | 36 | 16,546852 | 61 | 18,980276 | 86 | 19,698873 |
| 12 | 8,863252 | 37 | 16,711287 | 62 | 19,028834 | 87 | 19,713212 |
| 13 | 9,393573 | 38 | 16,867893 | 63 | 19.075080 | 88 | 19,726869 |
| 14 | 9,898641 | 39 | 17,017041 | 64 | 19,119124 | 89 | 19,739875 |
| 15 | 10,379658 | 40 | 17,159086 | 65 | 19,161070 | 90 | 19,752262 |
| 16 | 10,837770 | 41 | 17,294368 | 66 | 19,201019 | 91 | 19,764059 |
| 17 | 11,274066 | 42 | 17,423208 | 67 | 19,239066 | 92 | 19,775294 |
| 18 | 11,689587 | 43 | 17,545912 | 68 | 19,275301 | 93 | 19,785994 |
| 19 | 12,085321 | 44 | 17,662773 | 69 | 19,309810 | 94 | 19,796185 |
| 20 | 12,462210 | 45 | 17,774070 | 70 | 19,342677 | 95 | 19,805891 |
| 21 | 12,821153 | 46 | 17,880066 | 71 | 19,373978 | 96 | 19,815134 |
| 22 | 13,163003 | 47 | 17,981016 | 72 | 19,403788 | 97 | 19,823937 |
| 23 | 13,488574 | 48 | 18,077158 | 73 | 19,432179 | 98 | 19,832321 |
| 24 | 13,798643 | 49 | 18,168722 | 74 | 19,459218 | 99 | 19,840306 |
| 25 | 14,093945 | 50 | 18,255925 | 75 | 19,484970 | 100 | 19,847910 |

### To find the present value of an annuity for a term of years.

RULE. *Multiply the number in the table opposite to the given term of years, by the sum, and the product is the answer.*

Ex. 1. What is the present value of an annuity of 126*l.* for 21 years?

In the table opposite 21 is 12.821153; this multiplied by 126, gives 1615.465278 = 1615*l.* 9*s.* 3*d.*

Ex. 2. What is the present value of an annuity of 75l for 12 years, at 5 per cent.?

Ex. 3. What present sum is equivalent to a nett rent of 45*l.* per annum for 84 years, allowing interest of money at 5 per cent.?*

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NOTE.

* As purchasers of leases generally expect to make more than

# CHANCES.*

*Question I.*—Suppose a counter, having a black and a white face, be thrown up, to see which will be uppermost, after the counter has fallen to the ground, and if the white face appear uppermost, a person is to have 5 shillings, what is the chance, or probability, that he will be entitled to the five shillings?

*Solution.* Since either the black or the white face must be uppermost, there is an equal chance for the appearance of either face, of course the chance, or the probability, may be expressed by $\frac{1}{2}$, or a bystander ought to give him 2*s.* 6*d.* for his chance of getting the five shillings.

*Question II.*—Suppose there are three counters put into a bag, one red, another white, and a third black; out of which, if a person blindfolded take the red he is to have 5 shillings, I demand the value of the chance, or what is the probability of his drawing the red counter?

*Solution.* He has evidently one chance out of three, and therefore the probability may be valued at $\frac{1}{3}$, and another person inclining to purchase his chance, ought to give for it the $\frac{1}{3}$d of 5 shillings, or 1*s.* 8*d.*

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NOTES.

5 per cent. of their money, we shall treat more at large on this subject further on. We may, however, observe in this place, that freehold estates are usually valued at so many years purchase, that is, so many years rent:

| | |
|---|---|
| If 30 years purchase be given for a freehold estate, it is equivalent to putting the money out at a little more than - | $3\frac{1}{4}$ per cent. |
| 25 - - - - - | 4 |
| 20 - - - - - | 5 |
| $16\frac{2}{3}$ - - - - - | 6 |

* It is meant only to give so much of the doctrine of chances as shall enable the pupil to understand upon what ground the doctrine of Annuities, &c. depends. To illustrate this part of the subject, recourse will be had to some familiar instances, which may seem, at first sight, to lead to gaming; but it is believed, that the facts adduced must, if properly considered, deter young persons from this pernicious and destructive vice, which is too much encouraged by the almost perpetual drawing of state lotteries.

In the former case, the chances for the event's happening and failing are equal, and each being equal to $\frac{1}{2}$, the certainty is reckoned as 1, or unity.

In this last case, there is one chance for the event's happening, and two for its failing; in other words, the chance for its happening is $\frac{1}{3}$d, and for its failing are $\frac{2}{3}$ds: here, again, the chances for the happening and failing are equal to unity, because $\frac{1}{3} + \frac{2}{3} = \frac{3}{3} = 1$.

*Question III.*—Suppose there are five counters, two white and three black, out of which, when mixed, a person blindfolded is to draw one of the white, and in that case is to be entitled to 5*s.*, what is his chance for so doing, and what is his expectation worth?

*Solution.* It is plain here are five chances in the whole, of which there are two only out of five for taking a white counter, and the other three for taking a black one; therefore the probability of winning may be expressed by the fraction $\frac{2}{5}$, and of missing $\frac{3}{5}$, and he might sell his expectation of the five shillings for $\frac{2}{5}$ths of that sum, that is, for two shillings.

Ex. 1. At the conclusion of the last state lottery, when there were only five tickets left in the wheel, there were two prizes of 50*l.* each, and three blanks, what was the value of one of those tickets?

Ex. 2. What is the value of one ticket, when only five are left in one wheel, and in the other there is one prize of 100*l.* and four blanks?

Ex. 3. What chance has the holder of a single lottery ticket of a prize, when there are three blanks to a prize?*

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* In general, it is held out as a lure to the thoughtless multitude, that there are only two blanks to a prize, or not two, or not three blanks to a prize; yet, how few buy tickets with the hope of gaining the small prizes, of which the number of prizes is almost wholly made up: All hope for the 20,000 or 30,000*l.*; but to point out the folly of such expectations, we shall quote a fact or two deduced from this subject by one of the ablest mathematicians of the age.

1. Supposing a lottery to consist of 25,000 tickets, of which 20 are prizes of 1000*l.* and upwards; a person to have an equal chance of only one of those prizes must purchase 870 tickets: these, at 18*l.* each, (and tickets are seldom so low as this) would cost 15,660*l.*

2. Supposing there are three prizes of 20,000*l.*, and three of 10,000*l.*, and a person out of 25,000 tickets has purchased 3000, in hopes of gaining one of each of these capital prizes, still, though he has laid out, at 18*l.* a ticket, 54,000*l.*, the chances against such an expectation will be as 12 to 1. See article Chances, Rees's Cyclopedia, said to have been written by William Morgan, Esq.

*Question IV.*—What is the probability of throwing an ace with a single die, in one trial?

*Solution.* There are six faces to a die, of which one only is the ace, therefore the probability of throwing an ace with a single die in one trial is expressed by $\frac{1}{6}$; and the probability of not throwing an ace is $\frac{5}{6}$: here, as before, the chances for not throwing the ace, and that for throwing, are together equal to unity.

*Question V.*—What is the probability of throwing an ace in four throws?

*Solution.* We must consider the probability of failing in the four throws. The probability of missing the first time will be $\frac{5}{6}$; so it is the second, third, and fourth times; therefore the probability of missing in all four throws will be $\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{625}{1296}$; which, subtracted from unity or 1, gives $\frac{1296-625}{1296} = \frac{671}{1296}$, which is the probability of throwing it once or oftener in four turns; therefore the odds of throwing an ace in four times, is as 671 to 625, or rather more than an even chance.

The probability in three throws will be $1 - \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = 1 - \frac{125}{216} = \frac{216-125}{216} = \frac{91}{216}$. Here the odds is against throwing the ace in three throws, as 91 is less than 125.

*Question VI.*—In two heaps of cards, one containing the 18 diamonds, the other the 13 spades, placed promiscuously, what is the probability that, taking one card at a venture, out of each heap, I shall take the two aces?

*Solution.* The probability of taking the ace out of the first heap is $\frac{1}{13}$; the probability of taking the ace out of the second heap is also $\frac{1}{13}$, therefore the probability of taking out both aces is $\frac{1}{13} \times \frac{1}{13}$, or $\frac{1}{169}$, which, subtracted from 1, gives $\frac{168}{169}$, of course the chances against me are as 168 to 1: in other words, I may *expect* to do this once in 169 attempts.

On similar principles the *expectation of life* is found. It is known by accurate observation, that of 46 persons aged 40 years, one will die every year, till they are all dead in 46 years; therefore half 46, or 23 years, will be the expectation of life of a person 40 years of age. That is, the number of years enjoyed by them all, will be just the same as if every one of them had lived 23 years, and then died. The same reasoning applies to all other ages, which leads us to a more particular consideration of the subject.

# EXPECTATION OF LIFE.

From the Bills of Mortality in different places, tables have been constructed which shew how many persons, upon an average, out of a certain number born, are left at the end of each year to the extremity of life. From such tables, which, as we have seen, are founded on the doctrine of Chances, the probability of the continuance of a life, of any proposed age is known.

## TABLE I.

Shewing the Probabilities of the Duration of Human Life, deduced from tne Register of Mortality at Northampton.

| Age. | Persons living. | Decrem. of Life. | Age. | Persons living. | Decrem. of Life. | Age. | Persons living. | Decrem. of Life. |
|---|---|---|---|---|---|---|---|---|
| 0 | 11650 | 3000 | 33 | 4160 | 75 | 66 | 1552 | 80 |
| 1 | 8650 | 1367 | 34 | 4085 | 75 | 67 | 1472 | 80 |
| 2 | 7283 | 502 | 35 | 4010 | 75 | 68 | 1392 | 80 |
| 3 | 6781 | 335 | 36 | 3935 | 75 | 69 | 1312 | 80 |
| 4 | 6446 | 197 | 37 | 3860 | 75 | 70 | 1232 | 80 |
| 5 | 6249 | 184 | 38 | 3785 | 75 | 71 | 1152 | 80 |
| 6 | 6065 | 140 | 39 | 3710 | 75 | 72 | 1072 | 80 |
| 7 | 5925 | 110 | 40 | 3635 | 76 | 73 | 992 | 80 |
| 8 | 5815 | 80 | 41 | 3559 | 77 | 74 | 912 | 80 |
| 9 | 5735 | 60 | 42 | 3482 | 78 | 75 | 832 | 80 |
| 10 | 5675 | 52 | 43 | 3404 | 78 | 76 | 752 | 77 |
| 11 | 5623 | 50 | 44 | 3326 | 78 | 77 | 675 | 73 |
| 12 | 5573 | 50 | 45 | 3248 | 78 | 78 | 602 | 68 |
| 13 | 5523 | 50 | 46 | 3170 | 78 | 79 | 534 | 65 |
| 14 | 5473 | 50 | 47 | 3092 | 78 | 80 | 469 | 63 |
| 15 | 5423 | 50 | 48 | 3014 | 78 | 81 | 406 | 60 |
| 16 | 5373 | 53 | 49 | 2936 | 79 | 82 | 346 | 57 |
| 17 | 5320 | 58 | 50 | 2857 | 81 | 83 | 289 | 55 |
| 18 | 5262 | 63 | 51 | 2776 | 82 | 84 | 234 | 48 |
| 19 | 5199 | 67 | 52 | 2694 | 82 | 85 | 186 | 41 |
| 20 | 5132 | 72 | 53 | 2612 | 82 | 86 | 145 | 34 |
| 21 | 5060 | 75 | 54 | 2530 | 82 | 87 | 111 | 28 |
| 22 | 4985 | 75 | 55 | 2448 | 82 | 88 | 83 | 21 |
| 23 | 4910 | 75 | 56 | 2366 | 82 | 89 | 62 | 16 |
| 24 | 4835 | 75 | 57 | 2284 | 82 | 90 | 46 | 12 |
| 25 | 4760 | 75 | 58 | 2202 | 82 | 91 | 34 | 10 |
| 26 | 4685 | 75 | 59 | 2120 | 82 | 92 | 24 | 8 |
| 27 | 4610 | 75 | 60 | 2038 | 82 | 93 | 16 | 7 |
| 28 | 4535 | 75 | 61 | 1956 | 82 | 94 | 9 | 5 |
| 29 | 4460 | 75 | 62 | 1874 | 81 | 95 | 4 | 3 |
| 30 | 4385 | 75 | 63 | 1793 | 81 | 96 | 1 | 1 |
| 31 | 4310 | 75 | 64 | 1712 | 80 | | | |
| 32 | 4235 | 75 | 65 | 1632 | 80 | | | |

*Note.* 1. Here it must be observed that, of 11650 infants born, 3000 will die in the first year. Of the 8650 who live to be one year old,

**CASE I.** To find, by this Table, the expectation of any single life.

**RULE.** *Divide the sum of all the living in the table, at the age whose expectation is required, and at all greater ages, by the sum of all that die annually at that age, and above it, or, which is the same thing, by the number in the table of the living at that age, and half unity, or .5 subtracted from the quotient will be the expectation required.*

Ex. 1. What is the expectation of a life at 60?

The sum of the living at the age of 60 and upwards, by the table, is 27947,* which divided by 2038, the number of living at that age, gives 13.71, from which subtract .5, and the expectation of a life at 60 is equal to 13.21, or 13 years, 11 weeks nearly.†

Ex. 2. What is the expectation of a life 70 years of age, one of 80, and one of 90?

**CASE II.** To find the probability that a given life shall continue any number of years, or attain a given age.

**RULE.** *Make the number in the table, opposite to the proposed age, the numerator of the fraction, and for the denominator take the number opposite the present age.*

Ex. What is the probability that I, who am 45, shall live to 60?

The number against 60 = 2038 } Therefore the chances in my favour are 20 : 12 nearly,
The number against 45 = 3248 } or as - 5 : 3.

For, since the probability of living is equal to $\frac{2038}{3248}$, the chance of dying during that period is $1 - \frac{2038}{3248} = \frac{3248 - 2038}{3248} = \frac{1210}{3248}$. The

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NOTES.

1367 will die in the course of the second year. Therefore, of the 11650 new born infants, the chance of living to the end of the year, is to that of dying within that period, as 8650 : 300, or almost 3 to 1.

Again, the chance which an infant, just born, has of living two years, is as the number of living at the end of two years, is to the number that have died in that time, or as 7283 to (3000 + 1367) 4367, or nearly 2 to 1.

* This number is found by adding all the numbers up from 2038 to 1 inclusive.

† Lest the youth should mistake the meaning of the phrase "*Expectation of Life*," let him be warned that no more is meant by it than that a set of lives, as 100, aged 60, will, one with another, enjoy 13 years 11 weeks each of existence, some of them enjoying a duration as much longer as others fall short of it.

denominators being the same, the chance of life is to the probability of dying as 2038 to 1210, or as 20 to 12, or as 5 to 3 nearly.

Ex. 2. What is the probability that a person aged 21, shall attain to 54?

Ex. 3. What is the probability that a person aged 15 should live till 70?

Ex. 4. What chance has a person aged 70 of living 10 years longer?

## From the foregoing table is formed

### TABLE II.

Shewing the Expectation of Human Life at every Age, according to the Probabilities found by Table I.

| Age. | Expectation. | Age. | Expectation. | Age. | Expectation. | Age. | Expectation. |
|---|---|---|---|---|---|---|---|
| 0 | 25,18 | 25 | 30,85 | 50 | 17,99 | 75 | 6,54 |
| 1 | 32,74 | 26 | 30,33 | 51 | 17,50 | 76 | 6,18 |
| 2 | 37,79 | 27 | 29,82 | 52 | 17,02 | 77 | 5,83 |
| 3 | 39,55 | 28 | 29,30 | 53 | 16,54 | 78 | 5,48 |
| 4 | 40,58 | 29 | 28,79 | 54 | 16,06 | 79 | 5,11 |
| 5 | 40,84 | 30 | 28,27 | 55 | 15,58 | 80 | 4,75 |
| 6 | 42,07 | 31 | 27,76 | 56 | 15,10 | 81 | 4,41 |
| 7 | 41,03 | 32 | 27,24 | 57 | 14,63 | 82 | 4,09 |
| 8 | 40,79 | 33 | 26,72 | 58 | 14,15 | 83 | 3,80 |
| 9 | 40,36 | 34 | 26,20 | 59 | 13,68 | 84 | 3,58 |
| 10 | 39,78 | 35 | 25,68 | 60 | 13,21 | 85 | 3,37 |
| 11 | 39,14 | 36 | 25,16 | 61 | 12,75 | 86 | 3,19 |
| 12 | 38,49 | 37 | 24,64 | 62 | 12,28 | 87 | 3,01 |
| 13 | 37,83 | 38 | 24,12 | 63 | 11,81 | 88 | 2,86 |
| 14 | 37,17 | 39 | 23,60 | 64 | 11,35 | 89 | 2,66 |
| 15 | 36,51 | 40 | 23,08 | 65 | 10,88 | 90 | 2,11 |
| 16 | 35,85 | 41 | 22,56 | 66 | 10,42 | 91 | 2,09 |
| 17 | 35,20 | 42 | 22,04 | 67 | 9,96 | 92 | 1,75 |
| 18 | 34,58 | 43 | 21,54 | 68 | 9,50 | 93 | 1,37 |
| 19 | 33,99 | 44 | 21,03 | 69 | 9,05 | 94 | 1,05 |
| 20 | 33.43 | 45 | 20,52 | 70 | 8,60 | 95 | 0,75 |
| 21 | 32,90 | 46 | 20,02 | 71 | 8,17 | 96 | 0,50 |
| 22 | 32,39 | 47 | 19,51 | 72 | 7,74 | | |
| 23 | 31,88 | 48 | 19,00 | 73 | 7,33 | | |
| 24 | 31,36 | 49 | 18,49 | 74 | 6,92 | | |

## To find the expectation of any given life.

Rule. *Seek in the table the given age, and opposite to it is the expectation.*

Thus, the chance of life to an infant just born is 25.18, or rather more than 25 years; to a person of 45 years of age 20.52, as we have found before, see p. 192; and to a person of 69, just 9 years.

Upon these tables is founded the doctrine of

# LIFE ANNUITIES.

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Life Annuities are annual payments to continue during any life or lives. These are generally purchased or sold for a present sum of money.

"The present value of a life annuity" *is the sum that would be sufficient (allowing for the chance of life failing, which has been considered in the preceding pages) to pay the annuity without loss.*

If money bore no interest, the value of an annuity of 1*l.* would be equal to the expectation of life. Thus, Table II. p. 202, the value of an annuity for a life of 20 years of age, if money bore no interest, would be equal to nearly 33 years and a half purchase; that is, 33*l.* 10*s.* in hand for each life, would be sufficient to pay to any number of such lives 1*l.* per annum.

If money is capable of being improved by being put out to interest, the sum just mentioned would be more than the value, because it would be more than sufficient to pay the annuity; and it will be as much more than sufficient as the interest is greater. As an example,

If money can be improved at 5 per cent. compound interest, the half of 33*l.* 10*s.*, or 16*l.* 15*s.*, will, as we have seen, p. 192, in little more than 14 years, produce the 33*l.* 10*s.* required.

It must not however be supposed, that 16*l.* 15*s.* is the true value of an annuity of 1*l.* during a life of 20. The value of an annuity certain for a term equal to the expectation, always exceeds the true value, because, in a number of life annuities, many of the payments would not be to be made till a much more remote period than the term equal to the expectation.

Upon this principle the following table is computed, from which it appears that the present value of an annuity of 1*l.* on a life of 20 years of age, is equal to 14*l.* and a small fraction only; that is, 14*l.* in hand for each life, improved at compound interest, will be sufficient to pay to any number of such lives 1*l.* per annum.

## TABLE I.

Shewing the Value of an Annuity of 1*l.* on a Single Life, at every Age, according to the probabilities of the Duration of Human Life at Northampton, reckoning interest at 5 per cent.

| Age. | Value. | Age. | Value. | Age. | Value. | Age. | Value. |
|---|---|---|---|---|---|---|---|
| Birth. | 8.863 | 25 | 13.567 | 50 | 10.269 | 75 | 4.744 |
| 1 year | 11.563 | 26 | 13.473 | 51 | 10.097 | 76 | 4.511 |
| 2 | 13.420 | 27 | 13.377 | 52 | 9.925 | 77 | 4.277 |
| 3 | 14.135 | 28 | 13 278 | 53 | 9.748 | 78 | 4.035 |
| 4 | 14.613 | 29 | 13.177 | 54 | 9.567 | 79 | 3.776 |
| 5 | 14.827 | 30 | 13 072 | 55 | 9.382 | 80 | 3.515 |
| 6 | 15.041 | 31 | 12.965 | 56 | 9.193 | 81 | 3.263 |
| 7 | 15.166 | 32 | 12.854 | 57 | 8.999 | 82 | 3.020 |
| 8 | 15.226 | 33 | 12.740 | 58 | 8.801 | 83 | 2.797 |
| 9 | 15.210 | 34 | 12.623 | 59 | 8.599 | 84 | 2.627 |
| 10 | 15.139 | 35 | 12.502 | 60 | 8.392 | 85 | 2.471 |
| 11 | 15.043 | 36 | 12.377 | 61 | 8.181 | 86 | 2.328 |
| 12 | 14.937 | 37 | 12.249 | 62 | 7.966 | 87 | 2.193 |
| 13 | 14.826 | 38 | 12.116 | 63 | 7.742 | 88 | 2.080 |
| 14 | 14.710 | 39 | 11.979 | 64 | 7.514 | 89 | 1.924 |
| 15 | 14.588 | 40 | 11.837 | 65 | 7.276 | 90 | 1.723 |
| 16 | 14.460 | 41 | 11.695 | 66 | 7.034 | 91 | 1.447 |
| 17 | 14.334 | 42 | 11.551 | 67 | 6.787 | 92 | 1.153 |
| 18 | 14.217 | 43 | 11.407 | 68 | 6.536 | 93 | 0.816 |
| 19 | 14.108 | 44 | 11.258 | 69 | 6.281 | 94 | 0.524 |
| 20 | 14.007 | 45 | 11.105 | 70 | 6.023 | 95 | 0 238 |
| 21 | 13.917 | 46 | 10.947 | 71 | 5.764 | 96 | 0.000 |
| 22 | 13.833 | 47 | 10.784 | 72 | 5.504 | | |
| 23 | 13.746 | 48 | 10.616 | 73 | 5.245 | | |
| 24 | 13.658 | 49 | 10.443 | 74 | 4.990 | | |

### To find the value of an annuity for a person of any given age.

RULE. *Multiply the number in the table against the given age, by the sum, and the product is the answer.*

Ex. 1. What should a person, aged 45, give to purchase an annuity of 60*l.* per annum during life, interest being reckoned 5 per cent?

The value in the table against 45 years is 11.105, and this multiplied by 60 gives the answer, 666*l.* 6*s.*

Ex. 2. A person aged 69 years would purchase an annuity of 200*l.* for life, what must he pay for it in ready money at the same rate of interest?

Ex. 3. A merchant marries a lady aged 28, whose fortune for life is 300*l.* per annum, being desirous of converting the same into money, what ought he to have for it, allowing interest 5 per cent.?

Ex. 4. What is the value of an annuity of 200*l.* during the life of a person aged 25 years?

Ex. 5. What is the value of 50*l.* per annum, payable during the life of a person aged 41 years?

Ex. 6. What is the value of a clear annuity of 75*l.* during the life of an old man aged 76?

Ex. 7. What is the value of a landed estate during the life of a person aged 38, producing nett 30*l.* 9*s.* per annum?

Ex. 8. What is the life interest of a person aged 53, in 1250*l.* 3 per cent. Consols worth?

Ex. 9. A gentleman aged 60, who receives an annuity of 150*l.* per annum, for life, out of a freehold estate, wishes to exchange his life for that of his wife, aged 32: what ought to be required of him for so doing?

Ex. 10. A person having an annuity of 100*l.* during a life of 37 years, agrees to exchange it for an equivalent annuity during a life of 45; what annuity should be granted him?

Ex. 11. What annuity will 100*l.* purchase during the life of a person aged 28?

Ex. 12. A parish means to raise a sum of money for building a workhouse, by life annuities; at what ages should they grant 7, 8, and 9 per cent.?*

Ex. 13. What is the difference in value between an annuity of 40*l.* during a life of 36, and an annuity certain for 20 years?†

Ex. 14. A person aged 27 is possessed of 60*l.* per annum in the government long annuities, which have 51 years to run, and which he is willing to relinquish for an annuity during his life; what should the equivalent annuity be?

Ex. 15. What annuity should be granted to a person aged 57 during his life, for 2,000*l.* five per cent. stock, which is now at $99\frac{5}{8}$?

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NOTES.

* Questions of this sort are answered by dividing 100*l.* by the rates per cent., and opposite to the numbers in the table that are nearest the quotient, are the required ages: thus, to find at what age a life annuity of 9 per cent. should be granted, $\frac{100}{9} = 11.111$, the nearest number in the table is 11.105, by the side of which is 45, hence, to ages of 45, an annuity of 9 per cent. may be granted.

† See Tables, p. 204 and 196.

## TABLE II.

Shewing the Value of an Annuity during the joint continuance of Two Lives, according to the probabilities of Life at Northampton, reckoning interest at 5 per cent.

| Ages. | Value. | Ages. | Value. | Ages. | Value. | Ages. | Value. |
|---|---|---|---|---|---|---|---|
| 5-5 | 11.984 | 15-35 | 10.655 | 30-30 | 10.255 | 45-70 | 5.195 |
| 5-10 | 12.315 | 15-40 | 10.205 | 30-35 | 9.954 | 45-75 | 4.206 |
| 5-15 | 11.954 | 15-45 | 9.690 | 30-40 | 9.576 | 45-80 | 3.197 |
| 5-20 | 11.561 | 15-50 | 9.076 | 30-45 | 9.135 | 50-50 | 7.522 |
| 5-25 | 11.281 | 15-55 | 8.403 | 30-50 | 8.596 | 50-55 | 7.098 |
| 5-30 | 10.959 | 15-60 | 7.622 | 30-55 | 7.999 | 50-60 | 6.568 |
| 5-35 | 10.572 | 15-65 | 6.705 | 30-60 | 7.292 | 50-65 | 5.897 |
| 5-40 | 10.102 | 15-70 | 5.631 | 30-65 | 6.447 | 50-70 | 5.054 |
| 5 45 | 9.571 | 15-75 | 4.495 | 30-70 | 5.442 | 50-75 | 4.112 |
| 5-50 | 8.941 | 15-80 | 3.372 | 30-75 | 4.365 | 50-80 | 3.140 |
| 5-55 | 8.256 | 20-20 | 11.232 | 30-80 | 3.290 | 55-55 | 6.735 |
| 5-60 | 7.466 | 20-25 | 10.989 | 35-35 | 9.680 | 55-60 | 6.272 |
| 5-65 | 6.546 | 20-30 | 10.707 | 35-40 | 9.331 | 55-65 | 5.671 |
| 5-70 | 5.472 | 20-35 | 10.363 | 35-45 | 8.921 | 55-70 | 4.893 |
| 5-75 | 4.362 | 20-40 | 9.937 | 35-50 | 8.415 | 55-75 | 4.006 |
| 5-80 | 3.238 | 20-45 | 9.448 | 35-55 | 7.849 | 55-80 | 3.076 |
| 10-10 | 12.665 | 20-50 | 8.861 | 35-60 | 7.174 | 60-60 | 5.888 |
| 10-15 | 12.302 | 20-55 | 8.216 | 35-65 | 6.360 | 60-65 | 5.372 |
| 10-20 | 11.906 | 20-60 | 7.463 | 35-70 | 5.382 | 60-70 | 4.680 |
| 10-25 | 11.627 | 20-65 | 6.576 | 35-75 | 4.327 | 60-75 | 3.866 |
| 10-30 | 11.304 | 20-70 | 5.532 | 35-80 | 3.268 | 60-80 | 2.992 |
| 10-35 | 10.916 | 20-75 | 4.424 | 40-40 | 9.016 | 65 65 | 4.960 |
| 10-40 | 10.442 | 20-80 | 3.325 | 40-45 | 8.643 | 65-70 | 4.378 |
| 10-45 | 9.900 | 25-25 | 10.764 | 40-50 | 8.171 | 65-75 | 3.665 |
| 10-50 | 9.260 | 25-30 | 10.499 | 40-55 | 7.654 | 65-80 | 2.873 |
| 10-55 | 8.560 | 25-35 | 10.175 | 40-60 | 7.015 | 70-70 | 3.930 |
| 10-60 | 7.750 | 25-40 | 9.771 | 40-65 | 6.240 | 70-75 | 3.347 |
| 10-65 | 6.803 | 25-45 | 9.301 | 40-70 | 5.298 | 70-80 | 2.675 |
| 10-70 | 5.700 | 25-50 | 8.739 | 40-75 | 4.272 | 75-75 | 2.917 |
| 10-75 | 4.522 | 25-55 | 8.116 | 40-80 | 3.236 | 75-80 | 2.381 |
| 10-80 | 3.395 | 25-60 | 7.383 | 45-45 | 8.312 | 80-80 | 2.018 |
| 15-15 | 11,960 | 25-65 | 6.515 | 45-50 | 7.891 | 85-85 | 1.256 |
| 15-20 | 11.585 | 25-70 | 5.489 | 45-55 | 7.411 | 90-90 | 0.909 |
| 15-25 | 11.324 | 25-75 | 4.396 | 45-60 | 6.822 | | |
| 15-30 | 11.021 | 25-80 | 3.308 | 45-65 | 6.094 | | |

### Case 1. To find the value of an annuity on the longest of two single lives.

Rule. *From the sum of the values of the single lives subtract the value of their joint continuance, and the remainder will give the value of the longest of the lives.*

Ex. 1. What is the value of the longest of two lives aged 10 and 15?

Table I. { The value of a life at - - - - - - 10 = 15.139
— — — — - - - - - - - 15 = 14.588

29.727

Table II. The value of the joint continuance of two lives of - - - - - - - - 10 and 15 = 12.302

Value of the longest of the two lives - - - 17.425

Therefore an annuity of 100*l.* a year upon the longest of two lives, one 10 and the other 15, would be worth nearly 17 years and a half purchase, or more accurately, 1742*l.* 10*s.*

Ex. 2. What is the value of an annuity on the longest of two lives whose ages are thirty and forty.

## CASE II. To find the value of an annuity on three joint lives.

RULE. *Take the value of the two elder, and find the age of a single life equal to that; then find the value of the joint lives of this now found, and the youngest.*

Ex. 1. Let the three lives be 20, 30, and 40.

The value of the joint continuance of the two eldest; viz of 30 and 40 (by Table II.) is equal to 9.576, which answers to a single life (by Table I.) of 54. Now, the value of the joint lives of 20 and 54 by Table II., or the ages which come nearest, viz. 20 and 55, is 8.216 * for the value sought: hence an annuity of 40*l.* on three joint lives would be worth about 328*l.* 12*s.*

Ex. 2. To find the value of 3 joint lives of the ages 15, 30, and 45.

Ex. 3. What is the value of an annuity of 150*l.* on the joint-continuance of three lives of the ages 50, 60, and 70?

## CASE III. To find the value of the *longest* of any three lives.

RULE. *From the sum of the values of all the single lives, subtract the sum of the values of all the joint lives, combined two and two. To the remainder add the value of the three joint lives, and the sum will be the value of the longest of the three lives.*

Ex. 1. What is the value of the longest of three lives, whose ages are 20, 30, and 40?

Table I. { Value of a life of - 20 = 14.007
— — — — - 30 = 13.072
— — — — 40 = 11.837

38.916

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NOTE.

* The numbers 9.576 and 8.216, are not quite accurate, because the limits of this book do not admit of a table giving the combinations of all ages.

Value of two joint lives of 20 and 30 = 10.707
— — — — — — 20 and 40 = 9.937
— — — — — — 30 and 40 = 9.576

30.220

38.916
30.220

8.696 + 8.216 (the value of the joint lives found in Ex. 1. Case II.) = 16.912 = the value of the longest of the three lives.

Ex. 2. What is the value of the longest of three lives, whose ages are 15, 30, and 45 ?

Ex. 3. What is the value of an annuity on the longest of three lives, whose ages are 50, 60, and 70 ?

### EXAMPLES FOR PRACTICE.

Ex. 1. What is the present value of an annuity of 50*l.*, on the joint lives of two persons, each 30 years of age ?

Ex. 2. What is the present value of an annuity of 65*l.*, during the joint lives and the life of the survivor, of a man aged 45, and his wife aged 35 ?

Ex. 3. What is the value of a lease producing 27*l.* 13*s.* per annum, on the longest of two lives aged 60 and 45 ?

Ex. 4. What is the value of an annuity of 40*l.* on two joint lives of 70 and 5 years ?

Ex. 5. What is the value of an annuity of 50*l.* on the longest of two lives of 70 and 5 years ?

## CASE IV. To find the value of an annuity on a given life for any number of years.

RULE. *Find the value of a life as many years older than the given life as are equal to the term for which the annuity is proposed. Multiply this value by 1l. payable at the end of this term, and also by the probability that the life will continue so long. Subtract the product from the present value of the given life, and the remainder multiplied by the annuity will be the answer.*

Ex. 1. What is the value of an annuity of 50*l.* per ann. for 14 years, on a life of 35 ? 35 + 14 = 49.

The value of a life of 49 (14 years older than the given life, by Table I.) - - - - - - - - = 10.443

The value of 1*l.* payable at the end of 14 years (Table, p. 209) - - - - - - - - - - = .505068

The probability that a life of 35 will continue 14 years (Table, p. 200, and the 2d Case in p. 201.) $= \frac{2936}{4010}$

$10.443 \times .505068 \times \left(\frac{2936}{4010}\right) .7322 = 3.861$, which, subtracted from 12.502, the value of a life of 35, Table I. gives 8.641 ; and 8.641 × 50 = 432*l.* 1*s.*

Ex. 2. What is the value of an annuity of 80*l.* per annum for 20 years, provided a person aged 45 live so long ?

## TABLE,

Shewing the present Value of 1*l.* to be received at the end of any number of years, not exceeding 100; discounting at 5 per Cent. Compound Interest.

| Yrs. | Value. | Yrs. | Value. | Yrs. | Value. | Yrs. | Value. |
|---|---|---|---|---|---|---|---|
| 1 | .952381 | 26 | .281241 | 51 | .083051 | 76 | .024525 |
| 2 | .907029 | 27 | .267848 | 52 | .079096 | 77 | .023357 |
| 3 | .863838 | 28 | .255094 | 53 | .075330 | 78 | .022245 |
| 4 | .822702 | 29 | .242946 | 54 | .075743 | 79 | .021186 |
| 5 | .783526 | 30 | .231377 | 55 | .068326 | 80 | .020177 |
| 6 | .746215 | 31 | .220359 | 56 | .065073 | 81 | .019216 |
| 7 | .710681 | 32 | .209866 | 57 | .061974 | 82 | .018301 |
| 8 | .676839 | 33 | .199873 | 58 | .059023 | 83 | .017430 |
| 9 | .644609 | 34 | .190355 | 59 | .056212 | 84 | .016600 |
| 10 | .613913 | 35 | .181290 | 60 | .053536 | 85 | .015809 |
| 11 | .584679 | 36 | .172657 | 61 | .050986 | 86 | .015056 |
| 12 | .556837 | 37 | .164436 | 62 | .048558 | 87 | .014339 |
| 13 | .530321 | 38 | .156605 | 63 | .046246 | 88 | .013657 |
| 14 | .505068 | 39 | .149148 | 64 | .044044 | 89 | .013006 |
| 15 | .481017 | 40 | .142046 | 65 | .041946 | 90 | .012387 |
| 16 | .458112 | 41 | .135282 | 66 | .039949 | 91 | .011797 |
| 17 | .436297 | 42 | .128840 | 67 | .038047 | 92 | .011235 |
| 18 | .415521 | 43 | .122704 | 68 | .036235 | 93 | .010700 |
| 19 | .395734 | 44 | .116861 | 69 | .034509 | 94 | .010191 |
| 20 | .376889 | 45 | .111297 | 70 | .032866 | 95 | .009705 |
| 21 | .358942 | 46 | .105997 | 71 | .031301 | 96 | .009243 |
| 22 | .341850 | 47 | .100949 | 72 | .029811 | 97 | .008803 |
| 23 | .325571 | 48 | .096142 | 73 | .028391 | 98 | .008384 |
| 24 | .310068 | 49 | .091564 | 74 | .027039 | 99 | .007985 |
| 25 | .295303 | 50 | .087204 | 75 | .025753 | 100 | .007604 |

In order to find the present worth of any sum which is to be received at the end of a certain number of years—*Multiply the number in the table opposite to the term of years, by the sum, and the product will be the answer.*

Ex. 1. What is the present value of 750*l.*, to be received at the expiration of 9 years?

The number in the table even with 9 years is .644609, which is to be multiplied by 750.

.644609
750
3223045
4512263
483.45675
20
9.1350
12
1.620
4
2.48

Answer, - 483*l.* 9*s.* 1½*d.*

Ex. 2. What is the present value of 574*l.* 10*s.* 6*d.*, to be received 15 years hence?

**CASE V.** To find the value of a given sum payable at the decease of a person, whenever that shall happen. That is, to find the value of an assurance of any given sum on the whole duration of life.

RULE. *Subtract the value of the life from the perpetuity.*[*] *Multiply the remainder by the product of the given sum into the rate, and this last product divided by* 100*l. increased by its interest for a year, will give an answer in a single present payment. This payment divided by the value of the life, will give the answer in annual payments during the continuance of life.*

Ex. 1. What ought I, who am now 45, to pay, to assure on my life £1000; that is, what ought I to pay annually, to insure to my children at my decease £1000, allowing money at 5 per cent.?

The value of a life of 45, by Table, p. 204, is 11.105, and the perpetuity is $\frac{100}{5} = 20$. Therefore, by the rule;

$20 - 11.105 = 8.895$, which, multiplied by 5000, gives 44475; this, divided by 105, or $\frac{44475}{105} =$ 423*l.* 11*s.* 5*d.*, equal the answer in a single present payment. Therefore $\frac{423l.\ 11s.\ 5d.}{11.105} =$ 38*l.* 1*s.* 10*d.* nearly, in annual payments continued during life.[†]

Ex. 2. Let the life be 30: the sum £100, and the rate 5 per cent.?

The value of a life of 30 is, by Table, p. 204, equal to 13.072, and the perpetuity 20. Therefore, $20 - 13.072 = 6.928$, which, multiplied by 500, gives 3464, which, divided by 105, or $\frac{3464}{105} =$ 33*l.* nearly, being the sum to be paid in a single payment; and

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NOTES.

[*] See Note, p. 215, for an explanation of the word *Perpetuity.*

[†] Something more than this will be demanded at the most respectable offices, as the Royal Exchange, and Equitable Insurance Offices, because, in all their calculations, they do not suppose that 5 per cent. can at all times be made of money. The difference between 4 and 5 per cent. will be seen in the next question.

$\frac{33}{13.072} = 2l.\ 10s.\ 6d.$ nearly, in annual payments continued during life.

If the interest of money be supposed 4 per cent., then the value of a life of 30 is equal 14.68,* and the perpetuity is equal $\frac{100}{4} = 25$. Therefore $25 - 14.68 = 10.32$. This multiplied by $400l. = 4128$. And $\frac{4128}{104} = 9l.\ 14s.$ nearly; and $\frac{39l.\ 14s.}{14.68} = 2l.\ 14s.$

Hence it appears, that when the values are required in a single payment, the difference in the rate per cent. is considerable, though but trifling when made in annual payments during life. In this question, if money be improved at 5 per cent., the value of the single payment would be 33l.; but at 4 per cent. it would be 39l. 14s., which is one fifth more in the latter case than in the former: but, when the value is paid in annual sums during life; at 5 per cent., each payment is 2l. 10s. 6d., and at 4 per cent. it is 2l. 14s., making a difference of 3s. 6d. per annum, being an increase of less than one-fourteenth.

If the first of the annual payments is to be made immediately, then the single payment is to be divided by the value of the life, with unity added to it, so that at 5 per cent. it will be $\frac{33}{14.072} = 2l.$ 6s. 11d. nearly; and at 4 per cent. it will be $\frac{39l.\ 14s.}{15.68} = 2l.\ 9s.\ 4\frac{1}{2}d.$

Ex. 3. Let the life be 25, the sum 1000l., and the rate 5 per cent.

Ex. 4. Let the life be 60, the sum 1000l., and the rate 5 per cent.

## CASE VI. To determine the value of an annuity certain on a given life for any number of years.

RULE. *Find the value of a life as many years older than the given life as are equal to the term for which the annuity is proposed. Multiply this value by 1l. payable at the end of this term, and also by the probability that this life will continue so long. Subtract the product from the present value of the given life, and the remainder multiplied by the annuity will be the answer.*

Ex. 1. Let the annuity be 50l., the age of the given life 30 years, and the term proposed 15 years; interest 5 per cent.

The value of a life of 45, or 15 years older than the given life, by Table, p. 204, = 11.105. The value of 1l. payable at the end of 15 years is, by table, p. 209, = .481; and the probability that the life of

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NOTE.

* This is taken from a table not in this book. See Price's Reversionary Payments, and Morgan's Doctrine of Annuities, &c.

30 will exist so long, is by Table, p. 200 $= \frac{3248}{4385} = .74$ nearly. Therefore $11.105 \times .481 \times .74 = 3.953$. And the present value of the given life, by Table, p. 204, $= 13.072$; therefore $13.072 - 3.958 = 9.119$, and this multiplied by $50 = 455$*l.* 19*s.*

Had the interest been only 4 per cent. the value would have been about 490*l.*; that is, in the one case 455*l.* 19*s.*, and in the other 490*l.*, by a person who would insure an annuity of 50*l.* per ann. for 15 years certain, which depends on the contingency of the life of a person aged 30.

Ex. 2. Let the annuity be 40*l.*, the age of the given life 40, and the term proposed 20 years.

**CASE VII.** To find the value of a given sum payable at the decease of a person, should that happen within a given term. In other words: What ought a person to give for having his life assured to him for a certain term?

**RULE.** *From the value of an annuity certain for the given term, subtract the value of the life for the same term, and reserve the remainder. Multiply the value of 1l. due at the end of the given term, by the perpetuity,* and also by the probability that the given life shall* FAIL *in the given term. The product is to be added to the reserved remainder, and the sum multiplied by the given sum: this last product divided by the perpetuity increased by unity, gives the value in one present payment.*

Ex. 1. A merchant at Liverpool, aged 30, expects to realize a considerable property in the next 15 years; but as he may die before he can accomplish his views, he is willing to insure on his life, during that period, the sum of 5000*l.*, what must he pay for the same.

The value of an annuity certain for 15 years, by Table, p. 196, is equal to 10.379; and by example, p. 211, the value of an annuity certain for 15 years on a life of $30 = 9.119$; therefore $10.379 - 9.112 = 1.26 =$ reserved remainder.

The value of 1*l.* to be received at the end of 15 years, by Table, p. 209, $= 481$; and the probability that a life of 30 shall fail in 15 years, is $\frac{1142}{4385} = .26$ :† and the perpetuity is $\frac{100}{5} = 20$. Therefore,

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NOTES.

* For the meaning of the word perpetuity, see note to p. 215.

† The probability of a life's failing, is always equal to the probability of its continuing, subtracted from unity. Thus the probability

$.481 \times .26 \times 20 = 2.5$, and this added to the reserved remainder $1.26 = 3.76$, which multiplied by 5000, the given sum, and divided by 21 (the perpetuity increased by unity) is equal 895l. 5*s.* nearly, the value required in a single payment. That is, a person of 30 must give 895l. 5*s.* to secure to his heirs 5000l. supposing he dies within 15 years. Or he must pay annually during the 15 years, if he live so long, 985l. 5*s.* divided by 9.119, or 98l. 3*s.* 4*d.** for the same security.

If money can be improved at 4 per cent. only, then the sum to be paid at once will be 929l. 4*s*, 2*d.*, and the annual payments will be 101l. nearly.

Ex. 2. If I live 7 years, I shall receive 2000l.; what must I give to insure my life for that period, being now 46 years of age?

## CASE VIII. To explain, by examples, the mode of granting annuities by the British Government established in the year 1808.

[The following examples are deduced from the tables printed and circulated by Government, and which may be had, gratis, at the Office, Bank Buildings, Royal Exchange, London.]

Ex. 1. By the tables it appears, that for every 100l. stock in the 3 per cent. consolidated annuities, will be given annually for life, to a person of 46 years, 5l. 11*s.*† If, therefore, a person of that age transfer 1000l. stock, he will receive an annuity for life of 55*l.* 10*s.* But he will receive interest 30l. and keep his capital; and to insure 660l. at the Equitable, or Royal Exchange Offices, he must pay rather more than 4 per cent.; that is, he must pay between 26 and 27l. annually, during life, to insure to his heirs at his death the 660l., which he transfers to Government: he will of course be a loser, by the transfer, of between one and two pounds per annum,

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NOTES.

of a life of 30 continuing 15 years, is by table, p. 200, $= \frac{3248}{4385} = .74$, and the probability of its failing $= 1 - \frac{3248}{4385} = \frac{4485 - 3248}{4385} = \frac{1137}{4385} = .26$. See Chances, p. 198 and 199.

* The payments are supposed to be made at the end of every year. But in all assurances, the first premium is paid immediately, and the remaining ones at the beginning of every year after; hence the proper divisor will be the value of the life for one year less than the given term added to unity, or, in this case, the value of a life for 14 years. And generally: the divisor for determining the annual payments must be increased by unity, whenever it is proposed that the first payment should be made immediately. See p. 211.

† Supposing stocks to be at 66, which they are at present.

It is therefore obvious, that no one, when stocks are at 66, can join in the plan held out by Government, who is not willing to give up his capital.

Ex. 2. When stocks are at 60, he will receive for 1000*l.* stock, 52*l.* 10*s.*; and to insure 600*l.* must pay more than 24l. to insure his life, and will of course be a loser of 1l. 10*s.* per annum.

Ex. 3. When stocks are at 80, as they may be, he will receive for the 1000l. stock 62l.; but to insure 800l., he must pay annually rather more than 32l.; in this case there will be his interest left, and he will be neither gainer nor loser.

These examples will suffice for the whole.

# REVERSIONS.

REVERSIONS, or Reversionary Annuities, are those which do not commence till after a certain number of years, or till the decease of a person, or some other future event has happened.

CASE I. To find the present value of an annuity for a term of years, which is not to commence till the expiration of a certain period.

RULE. *Subtract from the value of an annuity for the whole period, the value of an annuity to the time when the reversionary annuity is to commence.*

Ex. 1. What is the present value, at 5 per cent, compound interest, of 80*l.* per annum for 24 years, commenoing at the end of 8 years? 24 + 8 = 32.

The present value of an annuity (Table, p. 196,) for 32 years, is 15.802677, and the value of one for 8 years is 6.463213, therefore

15.802677
6.463213

9.339464 × 80 = 747.15712 = 747*l.* 3*s.* 1½*d.*

Ex. 2. What is the present value of an annuity of 55*l.* for 15 years, to commence at the end of 15 years?

Ex. 3. What is the present value of an annuity for 49 years, to commence at the end of 47 years?

CASE II. To find the value of an annuity certain for a given term, after the extinction of any life or lives.

RULE. *Subtract the value of the life or lives from the per-*

*petuity,*[*] *and reserve the remainder. Then say, as the perpetuity is to the present value of the annuity certain, so is the reserved remainder, to the number of years purchase required.*

Ex. 1. What is the value of an annuity certain for 14 years, to commence at the death of a person aged 35, allowing 5 per cent.?

The value of a life of 35 (Table, p. 204) = 12.502; this subtracted from 20, the perpetuity, leaves 7.498 = reserved remainder. Then, as 20 : 9.898[†] :: 7.498 : 3.7107 = number of years purchase.

Ex. 2. A and his heirs are entitled to an annuity certain for 25 years, to commence at the death of a cousin aged 45 years; what can A sell his interest in this annuity for?

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NOTES.

[*] PERPETUITY, is the number of years purchase to be given for an annuity which is to continue for ever; and it is found by dividing 100*l.* by the rate of interest; thus, allowing 5 per cent., the perpetuity is 20 years, or $\frac{100}{5} = 20$; and at the rates most usually adopted, the perpetuity is as follows:

At 3 per cent. $\frac{100}{3} = 33.33$, &c.
$3\frac{1}{2}$ ditto $\frac{100}{3.5} = 28.57$, &c.
4 ditto $\frac{100}{4} = 25$.
$4\frac{1}{2}$ ditto $\frac{100}{4.5} = 22.22$, &c.
5 ditto $\frac{100}{5} = 20$
6 ditto $\frac{100}{6} = 16.66$, &c.
7 ditto $\frac{100}{7} = 14.28$, &c.
8 ditto $\frac{100}{8} = 12.5$

These are the number of years purchase to be given for a perpetual annuity, on the supposition that it is receivable yearly: but, as annuities are more commonly received half-yearly, and the interest of money likewise paid half-yearly; in this case the perpetuity will be somewhat greater or less than the above, as the periods at which the annuity is payable are more or less frequent than those at which the rate of interest is here supposed payable.

[†] The value of an annuity certain for 14 years. Table, p. 196.

**Case III.** To find the value of an annuity for a term certain; and also for what may happen to remain of a given life after the expiration of this term.

Rule. *Find the value of a life as many years older than the given life, as are equal to the term for which the annuity certain is proposed. Multiply this value by* 1*l. payable at the end of the given term, and also by the probability that the given life will continue so long. Add the product to the value of the annuity certain for the given term, and the sum will be the answer.*

Ex. 1. What is the value of an annuity of 60*l.* for 14 years, and also for the remainder of a life now aged 35, after the expiration of that term? $35 + 14 = 49$.

The value of a life aged 49 (Table 1. p. 204) - - $= 10.443$
The value of 1*l.* payable at the end of 14 years (Table, p. 209.) - - - - - - $= .505068$
The probability that the life will exist so long, (Table, p. 200 and 201) - - - - $= \frac{2936}{4010}$

Therefore, $10,443 \times .505068 \times \frac{2936}{4010} = 3.861$; this added to 9.898, the value of an annuity certain for 14 years, (see Table, p. 195) $=$ 13.759, the number of years purchase; and $13.759 \times 60 = 825$*l.* 10*s.* 9½*d.*

Ex. 2. What is the value of an annuity of 75*l.* for 10 years, and also the remainder of a life now aged 24, after the expiration of that term?

**Case IV.** To find what annuity can be purchased for a given sum, during the joint lives of two persons of given ages, and also during the life of the survivor, on condition that the annuity shall be reduced one-half at the extinction of the joint lives.

Rule. *Divide twice the given sum by the sum of the value of the two single lives, and the quotient will give the annuity to be paid during the joint lives, one-half of which is therefore the annuity to be paid during the remainder of the surviving life.*

Ex. 1. A man and his wife, aged 35 and 27, are desirous of sinking 2000*l.*, in order to receive an annuity during their joint lives, and also another annuity of half the value during the remainder of the serviving life: what annuities ought to be granted them?

$$\left.\begin{array}{l}\text{The value of a life of } 27\\ \text{The - - - - - - - - - } 35\end{array}\right\} \text{Table 1, p. 204.} \left\{\begin{array}{l} = 13.377\\ = 12.502\end{array}\right.$$

$$\overline{25.879}$$

Therefore, $\frac{4000 \text{ (twice the sum)}}{25.879} = 154l.\ 11s.\ 3d.$ = annuity during their joint lives: and $77l.\ 5s.\ 7\frac{1}{2}d.$ annuity during the life of the survivor.

Ex. 2. A single man, aged 60, possessed of 1500*l.* is desirous of purchasing with it an annuity for himself and his sister, aged 40, during their joint lives, with one of half the value, during the remainder of the life of the survivor, at the death of either; what will be the value of the annuities?

Ex. 3. A man, possessed of 1000*l.*, which he will sink in the same way, and for the same purposes, during the joint lives of himself and father; the age of the one is 55, of the other 80: what annuities can be given for it?

## V. To find the value of the expectation of a perpetual annuity, provided one person of a given age survives another of a given age.

### (1). IF THE EXPECTANT BE THE ELDER.

RULE. *Find the value of an annuity on two equal joint lives, whose common age is equal to the age of the oldest of the two proposed lives; subtract this value from the perpetuity, and take half the remainder: then say,*

*As the expectation of the duration of life of the younger,*
*Is to that of the elder;*
*So is the half remainder to a fourth proportional:*

*which will be the number of years purchase, if the expectant is the older.*

### (2). IF THE EXPECTANT BE THE YOUNGER.

*Add the value found, as above, to that of the joint lives, and let the sum be subtracted from the perpetuity, and the remainder is the answer.*

Ex. 1. What is the value of B's expectation, (aged 30), of an estate of 50*l.* per annum, provided he survive A, aged 20?

Value of two joint lives, aged 30, (Table II. p. 206) = 10.255, the difference between which and 20, (the perpetuity), is 9.745, the half of which is 4.872: therefore,

$$\text{As } 33.43 \left\{\begin{array}{c}\text{The expectation of A.}\\ \text{Table, p. 202.}\end{array}\right\} : 28.27 \left\{\begin{array}{c}\text{Expectation}\\ \text{of B.}\end{array}\right\} :: 4.872$$

$: 4.119 = 205l.\ 19s.$

Ex. 2. What is the value as above, when B is 20, and A 30?

Then, to 4.119, just found, add
10.707, value of the joint lives (Table II. p. 206.)
14.826; this subtracted from 20, the perpetuity, and the remainder, 5.174 × 50 = 258*l.* 14*s.* is the true answer.

## EXAMPLES FOR PRACTICE.

Ex. 1. What is the additional value of an annuity certain for the term of 28 years, if extended to the longest of 3 lives that may survive that term, each now 30 years of age?

Ex. 2. What is the additional value of an annuity certain for the term of 40 years, for the longest of 3 lives that may survive that term, each now 32 years of age?

Ex. 3. What is the difference in the value of an annuity of 20*l.* certain for 30 years, and an annuity of the same amount on the longest of two lives, aged 25 and 40?

Ex. 4. What is the value of an estate of 150*l.* per annum, held on the longest of two lives, aged 40 and 50, subject to the payment of an annuity of 14*l.* to a life of 62, and another annuity of 18*l.* to a life of 65?

Ex. 5. What is the present worth of 2000*l.* to be received at the decease of a person aged 65?

Ex. 6. What is the present value of 36*l.* a-year, being the third part of a farm in Essex, after the death of a person aged 54 years?

Ex. 7. What is the present value of a reversionary annuity of 259*l.* 3*s.* 8*d.* during the life of a person aged 24, in case he survives his brother, aged 34?

Ex. 8. What should be the consideration to be paid at the death of a person aged 80, for 1000*l.* now advanced to a person aged 25, in case the latter survives the former?

Ex. 9. What is the reversion worth of 2000*l.* 3 per cent. consols, if a life of 37 survives one of 53?

Ex. 10. What is the present worth of the reversion of 3075*l.* 3 per cent. consols, if a life of 37 survives two lives, one aged 58 years, and the other 30?

Ex. 11. What is the value of the reversion of 91*l.* per annum for ever, after the death of a person aged 53?

Ex. 12. A person aged 52, is entitled to 800*l.* at the death of another aged 76, provided the former survives the latter: what is its present worth?

Ex. 13. What is the present value of an annuity on the longest of two lives, now aged 25 and 30, the annuity not to commence till 14 years hence?

Ex. 14. What is the value of an annuity of 135*l.*, secured by, and payable out of, the dividends on 3 per cent. Reduced Bank Annuities, to which the purchaser will become entitled on the decease of a lady in her 72d year, and to enjoy the same during the life of a healthy gentleman, now in his 40th year, if he is the surviver?

# LEASES.

A Lease is a conveyance of any lands and tenements, made, in consideration of rent, or of a present sum of money, for life, or for a term of years.

The purchaser of a Lease may be considered as the purchaser of an annuity equal to the rack-rent of the estate; its value must therefore be calculated on the same principles as that of an annuity.

The sum paid down for the grant of a lease is so much, as being put out to interest will enable the landlord to repay himself the rack-rent of the estate, or the yearly value of his interest therein.

The value of the lease depends on the length of the term, and the rate of interest which the landlord can make of his money.

The value of leases at 5 per cent. compound interest may be found from Table, p. 196.

Thus, the value of a lease for 14 years, of a farm worth 150*l.* per ann., is, by that table 9.898641 × 150 = 1484*l.* 15*s.* 11*d.*

Ex. 1. What ought to be given for a lease of 26 years of an estate of 18*l.* per annum clear annual rent, * in order that a purchaser may make 5 per cent. of his money?

Ex. 2. A friend has just purchased the lease of a house for 54 years, for which he gave 550*l.*, and he is to pay a ground-rent of 1*l.* per annum: how much ought the house to let for, allowing 5 per cent. interest only?

Leases are generally calculated at a higher rate of interest; we shall therefore insert the following

## TABLE, †

Shewing the Number of Years Purchase that ought to be given for a Lease, for any Number of Years not exceeding 100, at 6, 7, and 8 per cent. interest.

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NOTES.

* That is, the next surplus rent, after deducting the reserved rent, if any, and all taxes and other annual charges.

† See Gregory's Dictionary of Arts and Sciences: also Baily on Leases.

| Yrs. | 6 per C. | 7 per C. | 8 per C. | Yrs. | 6 per C. | 7 per C. | 8 per C. |
|---|---|---|---|---|---|---|---|
| 1 | .9433 | .9345 | .9259 | 51 | 15.8130 | 13.8324 | 12.2532 |
| 2 | 1.8333 | 1.8080 | 1.7832 | 52 | 15.8613 | 13.8621 | 12.2715 |
| 3 | 2.6730 | 2.6243 | 2.5770 | 53 | 15.9009 | 13.8898 | 12.2884 |
| 4 | 3.4651 | 3.3872 | 3.3121 | 54 | 15.9499 | 13.9157 | 12.3041 |
| 5 | 4.2123 | 4.1001 | 3.9927 | 55 | 15.9905 | 13.9399 | 12.3186 |
| 6 | 4.9173 | 4.7665 | 4.6228 | 56 | 16.0288 | 13.9265 | 12.3320 |
| 7 | 5.5823 | 5.3892 | 5.2063 | 57 | 16.0649 | 13.9837 | 12.3444 |
| 8 | 6.2097 | 5.9712 | 5.7466 | 58 | 16.0989 | 14.0034 | 12.3560 |
| 9 | 6.8016 | 6.5152 | 6.2468 | 59 | 16.1311 | 14.0219 | 12.3669 |
| 10 | 7.3600 | 7.0235 | 6.7100 | 60 | 16.1614 | 14.0391 | 12.3765 |
| 11 | 7.8868 | 7.4986 | 7.1389 | 61 | 16.1900 | 14.0553 | 12.3856 |
| 12 | 8.3838 | 7.9426 | 7.5360 | 62 | 16.2170 | 14.0703 | 12.3941 |
| 13 | 8.8526 | 8.3576 | 7.9037 | 63 | 16.2424 | 14.0844 | 12.4020 |
| 14 | 9.2949 | 8.7454 | 8.2442 | 64 | 16.2664 | 14.0976 | 12.4092 |
| 15 | 9.7122 | 9.1079 | 8.5594 | 65 | 16.2891 | 14.1099 | 12.4159 |
| 16 | 10.1058 | 9.4466 | 8.8513 | 66 | 16.3104 | 14.1214 | 12.4222 |
| 17 | 10.4772 | 9.7632 | 9.1216 | 67 | 16.3306 | 14.1321 | 12.4279 |
| 18 | 10.8276 | 10.0590 | 9.3718 | 68 | 16.3496 | 14.1422 | 12.4333 |
| 19 | 11.1581 | 10.3355 | 9.6035 | 69 | 16.3676 | 14.1516 | 12.4382 |
| 20 | 11.4699 | 10.5940 | 9.8181 | 70 | 16.3845 | 14.1603 | 12.4428 |
| 21 | 11.7[illegible]40 | 10.8355 | 10.0168 | 71 | 16.4005 | 14.1685 | 12.4470 |
| 22 | 12.0415 | 11.0612 | 10.2007 | 72 | 16.4155 | 14.1762 | 12.4509 |
| 23 | 12.3033 | 11.2721 | 10.3710 | 73 | 16.4297 | 14.1834 | 12.4546 |
| 24 | 12.5503 | 11.4693 | 10.5287 | 74 | 16.4431 | 14.1901 | 12.4579 |
| 25 | 12.7833 | 11.6535 | 10.6747 | 75 | 16.4558 | 14.1963 | 12.4610 |
| 26 | 13.0031 | 11.8257 | 10.8099 | 76 | 16.4677 | 14.2022 | 12.4639 |
| 27 | 13.2105 | 11.9867 | 10.9351 | 77 | 16.4790 | 14.2076 | 12.4666 |
| 28 | 13.4061 | 12.1371 | 11.0510 | 78 | 16.4896 | 14.2127 | 12.4691 |
| 29 | 13.5907 | 12.2776 | 11.1584 | 79 | 16.4996 | 14.2175 | 12.4713 |
| 30 | 13.7648 | 12.4090 | 11.2577 | 80 | 16.5091 | 14.2220 | 12.4735 |
| 31 | 13.9290 | 12.5318 | 11.3497 | 81 | 16.5180 | 14.2261 | 12.4754 |
| 32 | 14.0840 | 12.6465 | 11.4349 | 82 | 16.5264 | 14.2300 | 12.4772 |
| 33 | 14.2302 | 12.7537 | 11.5138 | 83 | 16.5343 | 14.2337 | 12.4789 |
| 34 | 14.3681 | 12.8540 | 11.5869 | 84 | 16.5418 | 14.2371 | 12.4805 |
| 35 | 14.4982 | 12.9476 | 11.6545 | 85 | 16.5489 | 14.2402 | 12.4819 |
| 36 | 14.6209 | 13.0352 | 11.7171 | 86 | 16.5556 | 14.2432 | 12.4833 |
| 37 | 14.7367 | 13.1170 | 11.7751 | 87 | 16.6618 | 14.2460 | 12.4845 |
| 38 | 14.8460 | 13.1934 | 11.8288 | 88 | 16.5678 | 14.2486 | 12.4856 |
| 39 | 14.9490 | 13.2649 | 11.8785 | 89 | 16.5734 | 14.2510 | 12.4867 |
| 40 | 15.0462 | 13.3317 | 11.9246 | 90 | 16.5787 | 14.2533 | 12.4877 |
| 41 | 15.1380 | 13.3941 | 11.9672 | 91 | 16.5836 | 14.2554 | 12.4886 |
| 42 | 15.2245 | 13.4524 | 12.0066 | 92 | 16.5883 | 14.2574 | 12.4894 |
| 43 | 15.3061 | 13.5069 | 12.0432 | 93 | 16.5928 | 14.2592 | 12.4902 |
| 44 | 15.3831 | 13.5579 | 12.0770 | 94 | 16.5969 | 14.2610 | 12.4909 |
| 45 | 15.4558 | 13.6055 | 12.1084 | 95 | 16.6009 | 14.2626 | 12.4916 |
| 46 | 15.5243 | 13.6500 | 12.1374 | 96 | 16.6046 | 14.2641 | 12.4922 |
| 47 | 15.5890 | 13.6916 | 12.1642 | 97 | 16.6081 | 14.2655 | 12.4928 |
| 48 | 15.6500 | 13.7304 | 12.1891 | 98 | 16.6114 | 14.2668 | 12.4933 |
| 49 | 15.7075 | 13.7667 | 12.2121 | 99 | 16.6145 | 14.2680 | 12.4938 |
| 50 | 15.7618 | 13.8007 | 12.2334 | 100 | 16.6175 | 14.2692 | 12.4943 |

## CASE I. To find the sum that ought to be given for a lease.

RULE. *Look in the table against the number of years for which the lease is to continue, and on the line even with it, under the given rate of interest, is the number of years purchase that ought to be given for the same.*

Ex. 1. What sum ought to be given for the lease of an estate of 17 years, of the clear annual rent of 75*l.* allowing the purchaser to make 7 per cent. interest of his money?

Answer, 9.7632 × 75 = 732.24 = 732*l.* 4*s.* 9½*d.**

Ex. 2. What must be given for a lease of 21 years, at the clear annual rent of 50 guineas, allowing 8 per cent. for money?

Ex. 3. What is the worth of a lease of 83 years of an estate of 78*l.* per annum, interest being 6 per cent.?

Ex. 4. What sum ought to be given for a lease of 69 years, of a farm of 150*l.* per annum, the purchaser being allowed 6 per cent. for his money?

Ex. 5. What sum ought to be given for the lease of 46 years, of an estate estimated at 200*l.*, but which is charged with the payment of a reserved rent of 70*l.* 15*s.*, besides taxes and incidental expenses to the amount of 49*l.* 12*s.* annually; allowing the purchaser 6 per cent. interest for his money?

Ex. 6. What sum ought to be given for the ground-rent of a house of 15*l.* per annum, for 18 years, allowing the purchaser 8 per cent.?

## CASE II. To find the annual rent corresponding to any given sum paid for a lease.

RULE. *Divide the sum† paid for the lease by the number of years purchase that are found against the given term, and under the rate of interest intended to be made of the purchase money, the quotient will be the annual rent required.*

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NOTES.

* This sum of 732*l.* 4*s.* 9½*d.*, put out to compound interest at the rate of 7 per cent., will produce a clear income of 75*l.* per annum for 17 years; consequently, if it be agreed that 7 per cent. is the proper interest, then the landlord has a just equivalent for his grant.

† The purchaser ought to include in this sum, the money paid down for the lease, and every expense that may be incurred previously to entering upon it.

Ex. 1. I am asked 1500*l.* for a 40 years' lease, to what annual rent is that equivalent, allowing 6 per cent. for money?

$$\text{Answer, - } \frac{1500}{15.046} = 99l.\ 13s.\ 11d.$$

Ex. 2. If I sell the lease of my house, which has 81 years to run, for 800 guineas, at what rent will the purchaser stand, who will have a ground rent of 5*l.* 5*s.* per annum to pay likewise, allowing 7 per cent.?

## CASE III. To find the number of years purchase given for a lease that cost a certain sum of money.

RULE. *Divide the sum paid for the lease by the clear annual rent of the estate for which it is given, and the quotient will be the number of years purchase required.*

Ex. 1. The lease of a house, at the clear annual rent of 116*l.* was sold for 1630*l.*, what number of years purchase was given for it?

$$\frac{1630}{116} = 14 \text{ yrs. } 0 \text{ mo. } 2 \text{ weeks, } 4 \text{ days.}$$

Ex. 2. How many years purchase did the lease of a house sell for which cost 800*l.*, and the rent was 60 guineas?

For the Renewal of Leases, see p. 224.

* * *

# FREEHOLDS.

## CASE I. To find the gross sum which ought to be paid for a freehold estate.*

RULE. (1) "*Multiply the number of years purchase by the annual rent.*" Or, (2) "*Multiply the annual rent by* 100, *and divide the product by the rate of interest which it is proposed to make of money; the quotient will be the sum required.*"

---

NOTE.

* We have already shewn, (Note, p. 215,) the number of years purchase that ought to be given for the perpetuity of a freehold, according to the several rates of interest which the purchaser may make of his money.

Ex. What ought I to give for a freehold, the rent of which is 75*l.* per annum, supposing I mean to make 4 per cent. of my money?

By the 1st Rule, the answer is $25 \times 75 = 1875l.$

By the 2d. — — — — $\frac{75 \times 100}{4} = 1875l.$

If I had wanted 5 per cent. for my money, the answer would have been - - 1st. $20 \times 75 = 1500l.$

2d. $\frac{75 \times 100}{5} = 1500l.$

But if I were contented with 3 per cent., then I might afford to give for it 2500*l.* very nearly, for

1st. $33.333 \text{ \&c.} \times 75 = 2499l.\ 19s.\ 11d.$

2d. $\frac{75 \times 100}{3} = 2499l.\ 19s.\ 11d.$

CASE II. To find the clear annual rent which a freehold ought to produce, so as to allow the purchaser a given rate of interest for his money?

RULE. *Multiply the sum paid for the same, by the given rate per cent., and divide by 100, the quotient will be the annual rent required.*

Ex. A person has given 3000 guineas for a freehold estate, and wishes to let it so as to have 4½ per cent. for his money, what must be the annual rent?

Answer, - $\frac{3150 \times 4\frac{1}{2}}{100} = 141l.\ 15s.$

CASE III. To find the value of a freehold, to be entered upon after a certain term.

RULE. *Subtract the value of that certain term, from the value of the perpetuity, and the difference will be the true value.*

Ex. 1. What sum should be given for the reversion of a freehold after 14 years, allowing interest 6 per cent., and the clear annual rent 85*l.*

Value of a lease of 14 years, Table, p. 220, = 9.295; which, subtracted from 16.667, the perpetuity, leaves 7.372; and this multiplied by 85*l.* gives the value = 626*l.* 12*s.* 4¾*d.*

Ex. 2. What ought I to give for the reversion of a freehold worth 120*l.* per annum; but a lease of which is sold for 5 years to come supposing interest 5 per cent.

# RENEWAL OF LEASES.

CASE I. To ascertain what fine should be given for the renewal of any number of years lapsed in a lease originally granted for 21 years. *See Bailey on Leases.*

RULE. This is done by means of the following

## TABLE,

For Renewing any Number of Years lapsed in a lease for Twenty-one Years.

| Years | 3 per Ct. | 4 per Ct. | 5 per Ct. | 6 per Ct. | 8 per Ct. | £.11.564 per Cent. |
|---|---|---|---|---|---|---|
| 1 | .538 | .439 | .359 | .294 | .199 | .100 |
| 2 | 1.091 | .895 | .736 | .606 | .413 | .213 |
| 3 | 1.661 | 1.370 | 1.132 | .936 | .645 | .338 |
| 4 | 2.249 | 1.863 | 1.547 | 1.287 | .895 | .477 |
| 5 | 2.854 | 2.377 | 1.983 | 1.658 | 1.165 | .633 |
| 6 | 3.477 | 2.911 | 2.441 | 2.052 | 1.457 | .806 |
| 7 | 4.119 | 3.466 | 2.922 | 2.469 | 1.773 | 1.000 |
| 8 | 4.780 | 4.043 | 3.428 | 2.911 | 2.113 | 1.216 |
| 9 | 5.461 | 4.644 | 3.958 | 3.380 | 2.481 | 1.457 |
| 10 | 6.162 | 5.269 | 4.515 | 3.877 | 2.878 | 1.726 |
| 11 | 6.885 | 5.918 | 5.099 | 4.404 | 3.307 | 2.026 |
| 12 | 7.629 | 6.594 | 5.713 | 4.962 | 3.770 | 2.361 |
| 13 | 8.395 | 7.296 | 6.358 | 5.554 | 4.270 | 2.734 |
| 14 | 9.185 | 8.027 | 7.035 | 6.182 | 4.810 | 3.151 |
| 15 | 9.998 | 8.787 | 7.745 | 6.847 | 5.394 | 3.616 |
| 16 | 10.835 | 9.577 | 8.492 | 7.552 | 6.024 | 4.135 |
| 17 | 11.698 | 10.399 | 9.275 | 8.299 | 6.705 | 4.713 |
| 18 | 12.586 | 11.254 | 10.098 | 9.091 | 7.440 | 5.359 |
| 19 | 13.502 | 12.143 | 10.962 | 9.931 | 8.234 | 6.079 |
| 20 | 14.444 | 13.068 | 11.869 | 10.821 | 9.091 | 6.882 |
| total. | 15.415 | 14.029 | 12.821 | 11.764 | 10.017 | 7.779 |

Ex. 1. What ought to be given as a fine for the renewal of 15 years lapsed, or expired in a lease for 21 years, allowing the tenant 5 per cent. interest, and estimating the clear and improved rent at 60 guineas per annum.

Against 15 in the table, and under 5 per cent., is 7.745, and this multiplied by 63*l.* gives 487.935 = 487*l.* 18*s.* 8¼*d.*[*]

If the interest agreed on had been 6 or 8 per cent., the answers would have been

6.847 × 63 = 431*l.* 7*s.* 2*d.*
Or, 5.394 × 63 = 339*l.* 16*s.* 5*d.*

Ex. 2. What ought to be given to a landlord for adding seven years to a lease, of which fourteen years are unexpired, allowing the tenant 6 per cent. interest for his money, and the improved [†] rent to be 60*l.* per annum?

### CASE II. To ascertain the value of the fine which ought to be paid for renewing a given number of years in any lease.

RULE. *The value for renewing an additional term, or for adding any number of years to the unexpired part of an old lease, is equal to the difference between the value of the lease for the whole term, and the value of the unexpired part.*

Ex. 1. What ought to be given for the addition of seven years to a lease, of which 13 are unexpired; allowing 6 per cent. for money?

The whole term for which the new lease is to be granted is 20 years; therefore, Table p. 220, under 6 per cent., and
against 20 is 11.469, and
against 13 is 8.852; therefore this last subtracted from the former, will leave 2.617 for the number of years' purchase which ought to be given for the renewal.

Ex. 2. What should be given for the completing a 60 years' lease, of which a tenant has an unexpired term of 15 years, allowing him 7 per cent. for his money?

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NOTES.

[*] This is the sum which, put out to interest at 5 per cent. would, after the next six years, the remainder of the lease, produce a clear annual income of 63*l.* for 15 years; and therefore is the true sum that ought to be given for the advance of these 15 yearly rents of 63*l.* each, and which the landlord would not otherwise receive till the end of the seventh and 14 following years.

[†] It often happens, that when a tenant applies for the renewal of the years lapsed in a lease, the estate has incseased in value since it has been in his possession; and in such cases the landlord usually demands a fine in proportion to what he conceives the rent ought to be from its increased value. This is called the *improved rent.*

Ex. 3. I have a house for a lease of 48 years, but I wish to extend the lease to 97 years: how much must I pay for it, supposing the house worth 50*l.* per annum, and the interest 8 per cent?

It will be seen, by working Ex. 2, of Case 1, by this rule, that the answer will be precisely the same by both methods: for the whole term for which the new lease is granted is 21 years; the value of a lease for this term is, by Table, p. 220, 11.764, and the value of the 14 years' lease yet to come is 9.295; this, subtracted from the other, gives 2.469, as before, which, multiplied by 60, and the answer is 148*l.* 2*s.* 9½*d.*

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The value of leases or estates for single or joint lives, or for the longest of two or three lives, is found by the same rules that have been given, p. 204—8, for finding the value of annuities for the same terms.

When estates are held on two or three lives, and one of the lives nominated in the lease becomes extinct, the tenant is often desirous of replacing such life, or of putting in a new one, in order that the estate may continue to be held on the same number of lives in being, and thereby his interest in the same may be prolonged. In such cases it is customary, if the estate has improved in value since the original grant of the lease, for the landlord to demand a fine proportionate to such improved value; and to the age of the person intended to be added to those already in possession.

The tenant will, as it is his interest, add one of the best lives he can find, that is, a life which has the greatest expectation of living, according to the best tables of mortality; and such a life will be about eight or ten years: at any rate few persons will be disposed to put in a life above the age of twenty.

The following table will comprehend the cases that most frequently occur at the rate of 5 and 6 per cent.

## TABLE.

For Renewing, with One Life, the Lease of an Estate held on Three Lives.

| Life put in. | Age of lives in possession. | 5 per Ct. | 6 per Ct, |
|---|---|---|---|
| 10 | 30—30 | 1.741 | 1.305 |
| | 30—40 | 2.035 | 1.521 |
| | 30—50 | 2.431 | 1.832 |
| | 30—60 | 2.838 | 2.160 |
| | 30—70 | 3.277 | 2.535 |
| | 30—75 | 3.462 | 2.571 |
| | 40—40 | 2.397 | 1.792 |
| | 40—50 | 2.916 | 2.204 |
| | 40—60 | 3.451 | 2.637 |
| | 40—70 | 3.914 | 3.032 |
| | 40—70 | 4.264 | 3.273 |
| | 50—50 | 3.563 | 2.723 |
| | 50—60 | 4.206 | 3.242 |
| | 50—70 | 4.873 | 3.819 |
| | 50—75 | 5.174 | 4.062 |
| | 60—60 | 5.023 | 3.911 |
| | 60—70 | 6.161 | 4.917 |
| | 60—75 | 6.452 | 5.142 |
| | 70—70 | 7.556 | 6.124 |
| 15 | 30—30 | 1.572 | 1 191 |
| | 30—40 | 1.857 | 1.407 |
| | 30—50 | 2.227 | 1.699 |
| | 30—60 | 2.600 | 1.996 |
| | 30—70 | 3.052 | 2.381 |
| | 30—75 | 3.127 | 2.408 |
| | 40—40 | 2.224 | 1.687 |
| | 40—50 | 2.701 | 2.067 |
| | 40—60 | 3.205 | 2.474 |
| | 40—70 | 3.641 | 2.839 |
| | 40—75 | 3.943 | 3.076 |
| | 50—50 | 3.289 | 2.536 |
| | 50—60 | 3.910 | 3.039 |
| | 50—70 | 4.546 | 3.579 |
| | 50—75 | 4.816 | 3.819 |
| | 60—60 | 4.692 | 3.678 |
| | 60—70 | 5.780 | 4.627 |
| | 60—75 | 6.034 | 4.849 |
| | 70—70 | 7.125 | 5.805 |
| 20 | 30—30 | 1.404 | 1.079 |
| | 30—40 | 1.673 | 1.284 |
| | 30—50 | 2.019 | 1.557 |
| | 30—60 | 2.363 | 1.831 |
| | 30—70 | 2.813 | 2.218 |
| | 30—75 | 2.845 | 2.241 |
| | 40—40 | 2.027 | 1.558 |
| | 40—50 | 2.467 | 1.908 |
| | 40—60 | 2.943 | 2.293 |
| | 40—70 | 3.358 | 2.641 |
| | 40—75 | 3.615 | 2 873 |
| | 50—50 | 3 010 | 2.341 |
| | 50—60 | 9.607 | 2.828 |
| | 50—70 | 4.208 | 3.337 |
| | 50—75 | 4.474 | 3.576 |
| | 60—60 | 4,347 | 3.433 |
| | 60—70 | 5.386 | 4.338 |
| | 60—75 | 5.636 | 4.558 |
| | 70—70 | 6.695 | 5.489 |

RULE. *The years' purchase in the table, multiplied by the improved annual value of the estate, beyond the rent payable under the lease, gives the fine to be paid for putting in the new life.*

Ex. What must be given to put in a life of 10 years, when the ages of those in possession are 40 and 50, allowing 6 per cent. for money?

Answer, 2.204, or not quite 2¼ years' purchase.

If the life to be added be 15 years, the answer would be 2.067, or very little more than 2 years' purchase. And,

If the life to be added be 20 years, the answer would be 1.908, or less than 2 years' purchase.

# PERMUTATIONS AND COMBINATIONS.

The Permutation of quantities is the changing or varying the order of things.

The Combination of quantities is the shewing how often a less number of things can be taken out of a greater, and combined together, without considering their places, or the order in which they stand.

Case I. To find the number of changes that can be made of any given number of things, all different from each other.

Rule. *Multiply all the terms one into another, and the last product will be the number of changes required.*

Ex. 1. How many changes can be rung on 12 bells?

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 = 479,001,600.$$

Ex. 2. How many days can eight persons be placed in a different position at a dinner table?

Case II. Any number of different things being given, to find how many changes may be made out of them, by taking a given number of quantities at a time.

Rule. *Multiply the number of things given by itself less* 1, *and that product by the same number less* 2, *diminishing each succeeding multiplier by an unit, till there are as many products, except one, as there are things taken at a time the last product will be the answer.*

Ex. How many changes can be rung with 4 bells out of 12?

$$12 \times \overline{12-1} \times \overline{12-2} \times \overline{12-3} = 12 \times 11 \times 10 \times 9 = 11880.$$

Ex. 2. How many changes can be rung with 5 bells out of 10?

Ex. 3. What number of words, containing each 6 letters, can be formed out of the 24 letters in the alphabet supposing any 6 to form a word?

Case III. To find the combinations of a less number of things out of a greater, all different.

Rule. *Take the series* 1, 2, 3, 4, *&c. up to the less number of things, and multiply them continually together for a divisor: then take a series of as many terms, decreasing,* *each by an unit, from the greater number of things, and multiply them continually together for a dividend. Divide the latter product by the former, and the quotient will be the answer.*

Ex. 1. How many combinations can be made of 10 things out of 100?

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$$
(the number to be taken at a time) $= 3,628,800$.

$$100 \times 99 \times 98 \times 97 \times 96 \times 95 \times 94 \times 93 \times 92 \times 91$$
(the same number of terms taken from 100)
$= 62,815,650,955,529,472,000$.

$$\text{and } \frac{62815650955529472000}{3628800} = 17310309456440.^*$$

Ex. 2. How many combinations can be made of 3 letters out of the 24 letters in the alphabet?

Ex. 3. A club of 21 persons agreed to meet weekly, five at a time, so long as they could, without the same five persons meeting together, how long would the club exist?

CASE IV. To find the compositions of any number, in sets of equal numbers, the things or persons themselves being different.

RULE. *Multiply the number of things in every set continually together, and the product is the answer.*

Ex. 1. There are three parties of cricketters, in each eleven men, in how many ways can 11 of them be chosen, one out of each?

Answer, $11 \times 11 \times 11 = 1331$.

Ex. 2. In how many ways can the four suits of cards be taken, four at a time?

Ex. 3. There are four parties of whist-players; in one there are 6, in the second 5, in the third 4, and in the fourth 3 persons, how often can the set differ with these persons?

---

NOTE.

* Operations of this sort are shortened by the following mode:

$$\frac{100 \times 99 \times 98 \times 97 \times 96 \times 95 \times 94 \times 93 \times 92 \times 91}{10 \times 9 \times 2 \times 6 \times 8 \times 5 \times 1 \times 3 \times 4 \times 7}$$

$= 10 \times 11 \times 49 \times 12 \times 19 \times 31 \times 23 \times 13 \times 94 \times \frac{97}{6} =$ as above; and dividing the 12 by 6, we place 2 among the numerators, and get rid of all the denominators.

# EXCHANGE.

By Exchange is meant the bartering, or exchanging, the money of one place for that of another, by means of an instrument in writing, called *a bill of exchange.*

Exchanges are carried on by merchants and bankers all over Europe, and are transacted on the Royal Exchange of London, the Royal Exchange of Dublin, the Exchange of Amsterdam, and those of the principal cities of the continent.

When an exchange is mentioned between two places, one place gives a determined price, to receive an undetermined one.

The determined price is called *certain ;* thus,

London gives a pound sterling, which is a certain price, to receive from Paris a number of francs, more or less, to be paid or received there. Again, London gives 100*l.*, which is a certain price, to Dublin and other parts of Ireland, for an uncertain number of pounds, shillings, and pence Irish, to be paid or received there, viz. from 105*l.* to 115*l.* Irish, as the exchange may be.

The undetermined price is called *uncertain*, because it is always subject to variation; for instance,

London pays an uncertain price to Spain, as a number of pence sterling, to receive a dollar which is certain in exchange.

The *real money* of a state signifies one piece or more, of any kind of metal coined, and made current by public authority, as guineas, shillings, &c. of England.

The *imaginary money* is chiefly used in keeping accounts, as pounds sterling, for which there is no coin to answer.

The *par of exchange* is the quantity of the money, whether real or imaginary, of one country, which is equal in value to a certain quantity of the money of another; thus,

£. 100 sterling is equal in value to 108*l.* 6*s.* 8*d.* Irish: and 100*l.* sterling is worth 140*l.* of the currency in the West Indies, and equal to 166*l.* 13*s.* 4*d.* currency of the United States.*

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NOTE.

* In the following note is subjoined the par of exchange between London and some of the principal commercial and other places in Europe.

The *course of exchange* is the value agreed upon by merchants and others, and is continually fluctuating above or below the par of exchange, according as the demand for bills is greater or less.*

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NOTES.

| | Par in Sterling. | | £. | s. | d. |
|---|---|---|---|---|---|
| Rome | 1 crown | = | 0 | 6 | 1⅔ |
| Naples | 1 ducat | = | 0 | 3 | 4½ |
| Florence | 1 crown | = | 0 | 5 | 4⅝ |
| Sicily | 1 crown | = | 0 | 5 | 0 |
| Vienna | 1 rix-dollar | = | 0 | 4 | 8 |
| Franckfort | 1 florin | = | 0 | 3 | 0 |
| Bremen | 1 rix-dollar | = | 0 | 3 | 6 |
| Berlin | 1 ditto | = | 0 | 4 | 0 |
| Embden | 1 ditto | = | 0 | 3 | 6 |
| Dantzic | 13½ florins | = | 1 | 0 | 0 |
| Stockholm | $34\frac{4}{7}$ dollars | = | 1 | 0 | 0 |
| Petersburgh, and other parts of Russia | 1 ruble | = | 0 | 4 | 5 |
| Turkey | 1 asper | = | 0 | 4 | 6 |

In addition to the above, we may observe, that in Switzerland, at Nuremberg, Leipsic, Dresden, many parts of Poland, Denmark, and Norway; at Riga, Revel, &c. the rix-dollar is, at the par of exchange, 4*s.* 6*d.*; and as it is worth more or less than this in exchange with other places, the course of exchange is said to be for or against these places.

* The demand for bills depends upon what is called *the balance of trade*, which is *for* or *against* a country, according as more or less goods are *exported* or *imported* by that country, in comparison of some other. Thus, if London ships to Hamburgh goods to the amount of 500,000*l.*, and Hamburgh in the same time sends to London goods only to the amount of 400,000*l.*, the balance of trade is said to be in favour of London; and as Hamburgh can discharge only to the amount of 400,000*l.* by bills of exchange in the way of trade, that is, to the amount of the value of goods sent to London, there is a balance against her of 100,000*l.*, which she must pay by bills of exchange procured elsewhere, and for these she must pay a premium. If, in this case, Hamburgh pays 1*l.* per cent. for bills, she will, to liquidate the debt, have to pay 1000*l.* premium; in this way the balance of trade affects the fluctuation of exchanges. An unfavourable state of exchange furnishes a motive for exportation. The merchant can, in such case, afford to sell his commodities as much cheaper, as the premium which he is obliged to pay for a bill of exchange amounts. Hence the course of exchange always tends to an equilibrium; and it can never exceed the expense of sending gold or silver bullion to the place upon which the bill is drawn,

*Agio* denotes the difference in Amsterdam and other places, between current money, and the exchange or bank-money, the latter being finer than the former.

*Usance* is a certain space of time allowed by one country to another for the payment of bills of exchange.* Bills are either payable at sight, or at a certain number of days after sight: at *usance*, *double usance*, or *half usance*. At *one*, *two*, &c. usance, means at *one*, *two*, &c. months' *date*. Half usance is 15 days, be the month what it may.

*Days of grace* are a certain number of days allowed for the payment of bills of exchange, after the expiration of the term specified in such bills, and are variable in different countries. In England three days are allowed.†

RULES for finding what quantity of the money of one country will be equal to a given quantity of the money of another, according to a given course of exchange.

CASE I. When the course of exchange is given, how much money of one country answers to a certain sum of another, as of Great Britain?

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NOTES.

since this is the money of the commercial world, and will every where be accepted in payment.

The principal exchanges of Europe are governed by those of London, Amsterdam, and Venice; and the exchanges from foreign countries are to be only had from the merchants and bankers residing abroad.

When England remits an *uncertain* price to any other kingdom, as Spain or Portugal, the *lower* the price of exchange, the more is it to the advantage of England, as giving 35 pence to Spain for a dollar, instead of 36 or 38 pence; or, giving 60 pence to Portugal for a milreis, instead of 65 pence. See page 230.

When England remits the certain price, the higher the exchange is the better, as for instance, giving to France 1*l.* for 26 francs, is better than for 24 or 25; or 100*l.* sterling, for 112*l.* Irish, is better than for 108*l.* 6*s.* 8*d.*, which is the par of Exchange. See p. 235.

* This space of time varies according to the custom of countries, and frequently in proportion to the distance of the places from each other.

† On the third day they must be paid; that is, a bill drawn at two months, on the 14th of July, must be paid on the 17th of September. Bills due on Sunday must be paid on Saturday; for those *at sight* no days of grace are allowed.

RULE. *As the given course of exchange, is to one pound sterling, so is the given sum in foreign money, to its corresponding value in sterling money.*

Ex. 1. How much sterling money can I have for 2035 Flemish shillings, when the course of exchage is 37 shillings for 1*l.*?

Here I say, As 37 : 1 :: 2035 : 55 = pounds sterling.

Ex. 2. How much sterling money can I get for 4086 florins, 4 stivers, 6 penings banco, supposing 1*l.* is worth 38 schillings and 2 grotes?*

| schil gr. | : | £. | :: | florins st. p. | |
|---|---|---|---|---|---|
| 38 2 | : | 1 | : : | 4086 4 6 | |
| 12 | | | | 40 | |
| 458 | | | | 163440 | grotes |
| | | | | 8 | grotes = 4 stivers |
| | | | | ¾ | of a grote = 6 penings |

458)163448¾(356*l.* 17*s.* 6*d.* Answer.

Ex. 3. What sterling money will 293*l.* 10*s.* 6*d.* Irish fetch, when the exchange is 114*l.* Irish for 100*l.* sterling?

114*l.* : 100*l.* : : 293*l.* 10*s.* 6*d.* : 257*l.* 9*s.* 6½*d.*

Ex. 4. Dublin remits to London 826*l.* 13*s.*, what must be received there, exchange being 110*l.* per cent.?

Ex. 5. Jamaica remits to London 287*l.* 0*s.* 10½*d.* currency, what must be received for it, exchange being 135*l.* per cent.?

CASE II. Given the course of exchange, to bring any quantity of sterling money into the money of another country.

RULE. *As 1l. sterling is to the course of exchange, so is the given sum, in sterling money, to its corresponding value in foreign money.*

Ex. 1. How much Flemish money will 233*l.* 6*s.* 8*d.*

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NOTE.

| | | | | | *s.* | *d.* |
|---|---|---|---|---|---|---|
| * 8 penings | make | 1 grote, or penny | | = | 0 | 0.54 |
| 2 grotes | —— | 1 stiver | - - - | = | 0 | 1.09 |
| 12 grotes | —— | 1 schilling | - - - | = | 0 | 6.56 |
| 20 schillings | —— | 1 pound Flemish | - | = | 10 | 11.13 |
| 40 grotes | —— | 1 guilder, or florin | | = | 1 | 9.8 |

sterling be worth, when the exchange is 34*s.* per 1*l.* sterling?

1*l.* : 34*s.* : : 233*l.* 6*s.* 8*d.* : 393*l.* 13*s.* 4*d.* Answer.

Ex. 2. How much Flemish money must be given for 628*l.* 10*s.* sterling, when the exchange is 33*s.* 8*d.* per £. sterling.

CASE III. To reduce the currency of any state into bank or exchange money.

RULE. *As* 100, *with the agio added to it, is to* 100, *so is any given sum current to its value in bank money.*

Ex. 1. How much bank money can a merchant in Amsterdam have for 5550 guilders, when the agio is 4½ per cent.?

104½ : 100 : : 5550 : 5311 $\frac{5}{104.5}$ Answer.

Ex. 2. How many florins bank will 3000 currency purchase, agio being 6¼ per cent.?

CASE IV. To reduce bank money into currency.

RULE. *As* 100 *is to* 100, *with the agio added to it, so is the bank money to the currency.*

Ex. 1. How much currency can I have in Venice for 1500 ducats bank, when the agio is 15 per cent.?

100 : 115 : : 1500 : 1725.

Ex. 2. How much currency can I have for 5000 bank florins; agio being 8 per cent.?

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## IRELAND.

ACCOUNTS are kept in Ireland as in England, viz. in pounds, shillings, and pence.

The par of exchange in Ireland is 108*l.* 6*s.* 8*d.*; that is, 108*l.* 6*s.* 8*d.* Irish, is equal in value to 100*l.* sterling; or 1*s.* 1*d.* Irish, is equal to one shilling English.

The course of exchange varies from 105*l.* to 115*l.* according to the balance of trade. See p. 230.

Ex. 1. London remits to Dublin 300*l.* sterling, what must be received for it, exchange being 106*l.* Irish per cent., and also when it is 112 per cent.?

Here I say, As 100 : 106 : : 300 : 318*l.* Answer.
and 100 : 112 : : 300 : 336*l.* Answer.

Here it is evident, that when England remits the *certain* price to another country, the higher the exchange, the greater advantage is derived by England; for when the exchange is 106, she will receive 318*l.* for her 300*l.*, and when it is 112*l.*, she will receive 336*l.* for the same sum. See p. 230.

Ex. 2. Dublin remits to London 700*l.* Irish, what is it equal to when the exchange is 108*l.*, and also when it is 110*l.*?

Here Dublin remits the *certain*, and London gives the uncertain price, and I say, As 108 : 100 : : 700 : 648*l.* 2*s.* 11½*d.* Answer.
110 : 100 : : 700 : 636*l.* 7*s.* 3*d.* Answer.

Here Dublin is gainer when the exchange is low, because, in that case, 700*l.* purchases 648*l.* 2*s.* 11½*d.*, and in the other it purchases only 636*l.* 7*s.* 3*d.*

Ex. 3. London remits to Dublin 545*l.* 10*s.* sterling, for how much Irish must London be credited, exchange being 110½?

Ex. 4. Dublin remits to London 900*l.* 15*s.*, how much sterling must be received, exchange being 112*l.*?

Ex. 5. I purchase sundry books in Dublin, for which I give as follows;

| | | | | |
|---|---|---|---|---|
| for the first - - | £. 0 | 9 | 6 | Irish, |
| second - - - | 0 | 18 | 0 | |
| third - - - | 0 | 5 | 6 | |
| fourth - - - | 1 | 5 | 0 | |

what are they worth in English money?

## AMERICA AND THE WEST INDIES.

Accounts are kept in these places as in England, in pounds, shillings, and pence.

In America the par of exchange is 166*l.* 13*s.* 4*d.*; in the West Indies it is 140*l.*

Ex. 1. London remits to Barbadoes 945*l.* 17*s.* sterling, how much currency will this amount to, when the exchange is 140 currency?

Ex. 2. Sir Francis Baring writes word, that he has received for me a remittance of one quarter's dividend on 4000 dollars, at 5½ per cent. interest, and the exchange is 164 per cent., what has he to pay me?

In this case the regular interest is 55 dollars, which, at 4*s.* 6*d.* each, when exchange is at par, or at 166*l.* 13*s.* 4*d.*, would be 12*l.* 17*s.* 6*d.*, but the exchange is 164; therefore I say,

As 164 : 166*l.* 13*s.* 4*d.* : : 12*l.* 7*s.* 6*d.* : 12*l.* 11*s.* 6*d.* Answer.
Or by decimals,
164 : 166.66 &c. : : 55 : 55.899 dollars = 12*l.* 11*s.* 6*d.**

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NOTE.

* Exchange being lower than the par, I am a gainer of 4*s.*; for a person who takes a bill of exchange is always benefitted by a *low* course of exchange.

The following is a TABLE of the COURSE OF EXCHANGE, taken, with slight variations, from the Monthly Magazine for the 1st of May, 1808.

## COURSE OF EXCHANGE.

| | | April 5. | | April 12. | |
|---|---|---|---|---|---|
| Hamburgh | gives | 34.5 | —— | 34.6 | for 1*l.* |
| Altona - - | gives | 34.7 | —— | 34.7 | for do. |
| Amsterdam | gives | 35.5.2U. | —— | 35.4.2U. | do. |
| Ditto, sight | gives | 34.9 | —— | 34.8 | for do. |
| Paris, 1. d. | gives | 23.13 | —— | 1. d. 24.0 | for do. |
| Leghorn | receives | $49\frac{3}{4}$ pence | —— | $49\frac{3}{4}$ | for 1 pezza of 8 rials |
| Naples - - | rec. | 42 ditto | —— | 42 | for 1 ducat |
| Genoa - - | rec. | 45 ditto | —— | $45\frac{1}{2}$ | for 1 pezza |
| Lisbon - - | rec. | 60 ditto | —— | 60 | for 1 milrea |
| Oporto - - | rec. | 65 ditto | —— | 65 | |
| Madrid - - | rec. | $38\frac{3}{4}$ do. Eff. | —— | — | for 1 dollar |
| Palermo - - | rec. | 92 per oz. | —— | 92 per oz. | |
| Dublin - - | rec. | $110\frac{1}{4}$*l.* | —— | 110 | for 100 |
| Agio of Bank of Holland - - | | $6\frac{1}{2}$ per cent. | - | $6\frac{3}{4}$ per cent. | |

This table, in addition to what is gone before, will afford an opportunity of explaining every thing that a man of business will wish to be acquainted with.

On the 5th of April, the exchange between Hamburgh and London was at the rate of 34 schillings, 5 grotes, for a pound sterling; that is, if a merchant in London sell a bill on Hamburgh for 500*l.*, he would be paid for it $34.5 \times 500 = 17208$ schil. 4 gro.; but on the 12th, such a bill would have fetched $34.6 \times 500 = 17250$ shillings. Here, the higher the exchange the greater the advantage to England; for the merchant, in this instance, gains 41 schil. 8 gro. by the rise in the exchange.

For Altona, the course of exchange is the same on both days, viz. the £. is worth 37 schil. 7 gro.: and for Amsterdam, the course of exchange falling, the merchant in London would be a loser, who put off his market from the 5th to the 12th.

In this case 35.5. 2U. means, that a pound sterling is worth, on the 5th, 35 schil. 5 gro., allowing it to be payable at two months' date: but if it is payable at sight, it is then worth only 34 schil. 9 gr. This difference, which on a bill of 100*l.* is equal to 34 schil. 4 gr., is instead of the interest of money for the interval.

The course of exchange rose between London and Paris from the 5th to the 12th of April. On the first of these days 1*l.* was at 1 d., that is, at one day's sight, worth 23.13, or 23 francs, and 13 cents.; but on the 12th its value was 24 francs.

Leghorn receives $49\frac{3}{4}$ pence for 1 pezza of 8 rials, that is, a bill of exchange of 5000 pezza would be worth 4*s.* $1\frac{3}{4}$*d.* multiplied by

5000, or 1036*l.* 9*s.* 2*d.* A Naples ducat was worth 3*s.* 6*d.*: a Genoa pezza 3*s.* 9*d.*: a milrea of Lisbon 5 shillings, and one of Oporto 5*s.* 5*d.*

Madrid receives 38¾*d.* *Eff.* for 1 piastre of 8 rials,* that is, a Spanish piastre of exchange was worth 3*s.* 2¾*d.*

A species of paper money, denominated *vales rials*, is circulated in Spain, the value of which independently of interest on them, is this: —Vales rials for 600 dollars are worth 9035 rials, 10 maravedis of *vellon*,† that is, (as 34 maravedies is equal to one rial) 1 dollar payable in this sort of paper is worth 15 rials, 2 maravedies. The paper is transferable by indorsement; and, by law, should be received in payment according to the nominal value; but as it experiences depreciation, it is necessary in drawing on Spain for effective money, to insert the words "*payable in effective*" in the body of the bill, which might otherwise be payable in vales rials: hence the word *Eff.* in the table, which is an abridgment of "*in effective.*"

Palermo 92 pence per oz. In Sicily exchanges are made per onza by the ounce of silver, for which, on the day referred to Palermo received 92 pence, or 7*s.* 8*d.*‡

Dublin 110¼ for 100*l.*, that is, at the date of the table there would have been given on the exchange of London a bill on Dublin for 110*l.* 5*s.* for 100*l.* pound sterling. See page 234.

By the agio of the Bank of Holland is meant, as we have seen, p. 232, the difference between cash and bank money, which, by the table, is on the 5th of April, 6½, or 6*l.* 10*s.* per cent.; that is, 106*l.* 10*s.* currency must be given for 100*l.* bank, and so in proportion.

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NOTES.

* In some parts of Spain they reckon by silver money, which is of two kinds, viz. old and new plate, the former is the most valuable: thus the piastre of exchange consists of 8 rials old plate, or of 10 rials new plate, the rial being at the par of exchange worth little more than 5¼*d.*

† The copper money of Spain is called vellon.

In Madrid, and the principal places of Spain, accounts are kept in piastres (called also *dollars*) rials, and maravedies; and sometimes in ducats.

TABLE.

| | | | | | s. | d. |
|---|---|---|---|---|---|---|
| 34 | maravedies | make | 1 rial | = | 0 | 5⅜ |
| 8 | rials | make | 1 piastre | = | 3 | 7 |
| 375 | maravedies | make | 1 ducat | = | 4 | 11⅜ |

Hence the piastre at par is 3*s.* 7*d.*, and the ducat at par 4*s.* 11¼*d.*; but the course of exchange of the piastre varies from 35 to 45 pence.

‡ The Sicilian ounce is 600 grains, and the monies are regulated by the following Table:

| | | |
|---|---|---|
| 10 grains | make | 1 carlin, |
| 2 carlins | make | 1 tarin, |
| 30 tarins | (600 gr.) | 1 ounce. |

A crown (scudo) is equal 240 grs., therefore 5 crowns = 2 ounces.

### *Exchange between London and other Places in this Country.*

The several cities, towns, &c. in Great Britain, exchange with London for a small premium in favour of London, as from ½ to 1, or 1½ per cent. The premium is more or less, according to the greater or less distance, and according to the demand for bills.

Ex. York draws on London for 560*l.* 10*s.*, exchange being ¾ per cent.; how much money must be paid at York for the bill?

| | | | | |
|---|---|---|---|---|
| $\frac{1}{2}$ | $\frac{1}{200}$ | 560 | 10 | 0 |
| 4 | $\frac{1}{2}$ | 2 | 16 | $0\frac{1}{2}$ |
| | | 1 | 8 | $0\frac{1}{4}$ |
| | £. | 564 | 14 | $0\frac{3}{4}$ |

To avoid paying the premium, which, in some cases, would not be just, it is the usual practice to take the bill payable a certain number of days after date. On this principle, interest being 5 per cent. 78 days are equivalent to 1*l.* per cent. because $\frac{365}{5} = 73$.

Ex. A friend at Exeter has received for me 68 guineas, in which he is no ways interested, and having no means of sending the money but by a bill of exchange, he agrees with his banker to draw it 30 days after date, rather than pay the premium of ½ per cent., is my friend, or the banker, the gainer, allowing 5 per cent.?

#### EXAMPLES FOR PRACTICE.

Ex. 1. How much currency will 6630 guilders, bank-money, be worth in Holland, agio being 8¼ per cent.?

Ex. 2. What is the agio of 3310 guilders, at 6¼ per cent.?*

Ex. 3. What is the agio of 5000 dollars, at 4⅜ per cent., and how much bank money will the 5000 currency purchase?

Ex. 4. A London merchant draws on Amsterdam for 1564*l.* sterling; how many pounds Flemish, and how many guilders will that amount to, exchange being 34 schil. 8 gro. per £. sterling.† See Table, p. 233.

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NOTES.

* If the agio only be required, say, as 100 : agio per cent. : : so is the given sum to the agio required: here, as 100 : 6¼ : : 3310 : to the required sum.

† The money in Holland is sometimes reckoned in guilders and stivers, as well as in schillings and grotes. To reduce Flemish pounds and schillings into guilders and stivers, multiply by 6; and if there be any pence multiply them by 8 for penings: or divide

Ex. 5. How much sterling money will pay a Portuguese bill of exchange of 1654∽372 millreas; that is, of 1654 millreas and 372 reas, exchange being 65½ pence sterling per millrea? *

Ex. 6. How many Portuguese reas will 750*l.* sterling amount to, exchange being 64⅝ per millrea?

Ex. 7. A Spanish merchant imports from Seville goods to the value of 1081 piastres, 6 rials: how much sterling money will this amount to, exchange being, on the day of payment, 41½ pence per piastre? See Table, p. 236-7?

Ex. 8. I want to purchase goods at Cadiz, and for this purpose pay into a Spanish house 1000*l.*: how much value, in piastres, may I expect, exchange being 3*s.* 6½*d.* per piastre?

## ARBITRATION OF EXCHANGES.

The course of exchange, between nation and nation, naturally rises or falls, as we have seen, according as the circumstances and balance of trade may happen to vary. To draw upon, and to remit money to foreign places, in this fluctuating state of exchange, in the way that will turn out most profitable, is the design of arbitration.

Arbitration of Exchange, then, is a method of finding such a rate of exchange between any two places, as shall

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NOTES.

the Flemish pence by 40, and the quotient will be guilders; and half the remainder, if there be any, will be stivers: thus, to bring 338*l.* 17*s.* 4*g.*, or 81328 Flemish pence into guilders:

£. *s.* *gr.*
338 17 4
6
—————
2033 4 0 = 2033 4 guild. stiv.

4,0)8132,8
—————
2033 8 = 2038 4 guild. stiv.

* In Portugal accounts are kept in reas, and millreas, the latter being equal to 1000 of the former; and they are distinguished from each other by some such mark as that in the question.

The millrea, in exchange with this country, is at par 67½ sterling, or 5*s.* 7½*d.*, and the course usually runs from 5*s.* 3*d.* to 5*s.* 8*d.*

TABLE — Par in sterling.

| | | | | *s.* | *d.* | *f.* |
|---|---|---|---|---|---|---|
| | | 1 rea | = | 0 | 0 | 0.27 |
| 400 reas | make | 1 crusade | = | 2 | 3 | 0 |
| 1000 reas | make | 1 millrea | = | 5 | 7 | ½ |

The reas being the thousandth parts of the millreas, are annexed to the integer, and the work proceeds as in decimals.

be in proportion with the rates assigned between each of them and a third place.

By comparing the par of exchange thus found, with the present course of exchange, a person is enabled to find which way to draw bills, or remit the same to most advantage.

[Questions in this rule are performed by the Rule of Three.]

Arbitration of exchange, is either *simple* or *compound*.

In *simple* arbitration, the rates of exchange from one place to two others are given, by which is found the correspondent price between the said two places, called the arbitrated price.

An example or two will make the subject clear.

Ex. 1. If exchange between London and Amsterdam be 34 schil. 9 grotes per £. sterling, and if exchange between London and Genoa be 45 pence per pezza (see Table, p. 236,) what is the par of arbitration between Amsterdam and Genoa?

Here 1*l.* = 240 pence: therefore, as

$$240d. : 34s.\ 9gr. :: 45d. : 78\tfrac{45}{240}gr.$$

Answer, 78 Flemish grotes, or pence per pezza Genoa.

Ex. 2. If exchange from London to Amsterdam be 33*s.* 9*d.* per £. and if exchange from London to Paris be 32*d.* per crown, what must be the rate of exchange from Amsterdam to Paris?

Ex. 3. If exchange from Paris to London be 32*d.* per crown, and if exchange from Paris to Amsterdam be 54*d.* Flemish per crown, what must be the rate of exchange between London and Amsterdam, in order to be on a par with the other two?

Ex. 4. Amsterdam exchanges on London, at 35 schil. 5 gro. per £. sterling; and the exchange between London and Lisbon is 60 pence per milrea, what is the exchange between Amsterdam and Lisbon?

The course of exchange being given, and the par of arbitration found, we obtain a method of drawing and remitting to advantage.

Ex. 5. If exchange from London to Paris be 32 pence sterling per crown, and to Amsterdam 405 Flemish per £., and if I learn that the course of exchange between Paris and Amsterdam is fallen to 52 pence Flemish per crown;

what may be gained per cent., by drawing on Paris and remitting to Amsterdam?

By Ex. 2, the par of arbitration between Paris and Amsterdam is 54*d.* Flemish per crown: then

*d.* cr. £. cr.
32 : 1 :: 100 : 750 drawn at Paris.
cr. *d.* Fl. cr. *d.* Fl.
1 : 52 :: 750 : 39000 credit at Amsterdam.
*d.* Fl. £. *d.* Fl. £. *s.* *d.*
405 : 1 :: 39000 : 96 5 11 to be remitted;
therefore 100*l.* — 96*l.* 5*s.* 11*d.* = 3*l.* 14*s.* 1*d.* = gain per cent.

If the course of exchange between Paris and Amsterdam be at 56 Flemish per crown, instead of 52; and if I would gain by the negotiation, I must draw on Amsterdam and remit to Paris: thus

£. *d.* Fl. £. *d.* Fl.
1 : 405 :: 100 : 40500 drawn at Amsterdam.
*d.* Fl. cr. *d.* Fl. cr.
56 : 1 :: 40500 : 723 credit at Paris.
cr. *d.* cr. £. *s.*
1 : 32 :: 723 : 96 8
therefore 100*l.* — 96*l.* 8*s.* = 3*l.* 12*s.* gain per cent.

In these cases, credit at one foreign place pays the debt at the other.

We might carry the subject of Exchanges to almost any length; but we have said enough to render the theory and practice easy; and from what the pupil has seen he will be able to apply the foregoing principles and rules to the practice of any merchant's counting-house in which he may be situated. We shall, however, give an example in Compound Arbitration.

## COMPOUND ARBITRATION.

In Compound Arbitration, the rate of exchange between three or more places is given, to find how much a remittance passing through them all will amount to at the last place: or to find the arbitrated price, or par of arbitration, between the first and last place.

Examples of this kind may be worked by several successive statings in the Rule of Three, or according to the following Rules.

(1) *Distinguish the given rates, or prices, into antecedents and consequents, placing the antecedents in one*

*column, and the consequents in another, with the sign of equality between them.*

(2) *The first antecedent, and the last consequent to which an antecedent is required, must be of the same kind.*

(3) *The second antecedent must be of the same kind with the first consequent, and the third antecedent of the same kind with the second consequent, &c.*

(4) *Multiply the antecedents together for a divisor, and the consequents together for a dividend, and the quotient will be the answer required*

Ex. If a merchant in London remit 500*l.* sterling to Spain by way of Holland, at 35 shillings Flemish per pound sterling, thence to France at 58 pence per crown, thence to Venice at 10 crowns for 6 ducats, and thence to Spain at 360 mervadies per ducat; how many piastres of 272 mervadies will the 500*l.* amount to in Spain?

1*l.* = 35*s.* or 420*d.* Fl.
58*d.* = 1 crown
10 cr. = 6 ducats
1 duc. = 360 mervadies
272 mer. = 1 piastre
How many piastres = 500*l.*?

Omitting the units, we have by the rule, $\frac{420 \times 6 \times 360 \times 500}{58 \times 10 \times 272}$, and this fraction, reduced to its lowest terms, gives $\frac{21 \times 3 \times 45 \times 500}{29 \times 17} = \frac{1417500}{493} = 2875\frac{1}{4}$ piastres, which is the answer.

By the Rule of three we should have said,

| | | | | | | |
|---|---|---|---|---|---|---|
| 1*l.* | . | 420*d.* | : : | 500*l.* | : | 210000*d.* |
| 58*d.* | : | 1 cr. | : : | 210000*d.* | : | 3620 cr.* |
| 10 cr. | : | 6 duc. | : : | 3620 cr. | : | 2172 duc. |
| 1 duc. | : | 360 mer. | : : | 2172 duc. | : | 781920 mer. |
| 272 mer. | : | 1 pias. | : : | 781920 mer. | : | $2875\frac{1}{2}$ piastres. |

If the course of direct exchange to Spain were $42\frac{1}{2}$ pence sterling, then the 500*l.* remitted would only amount to $2823\frac{1}{2}$ piastres, of course $2875\frac{1}{2} - 2823\frac{1}{2}$, gives 52, which is the number of piastres gained by the negotiation.

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NOTE.

* The fractions are omitted, and on that account the answer by this method will not be quite accurate.

# DUODECIMALS.

DUODECIMALS, or *Cross Multiplication*, is made use of by artificers in measuring their several works, and is performed by means of the following table:

| | | |
|---|---|---|
| $12''''$ fourths | - | make 1 third. |
| $12'''$ thirds | - - | 1 second. |
| $12''$ seconds | - - | 1 inch. |
| $12'$ inches | - - | 1 foot. |

Glaziers, Masons, and others, measure by the square foot.—Painters, Paviors, Plasterers, &c., by the square yard.—Slating, tiling, flooring, &c., by the square of 100 feet.—Brickwork is measured by the rod of 16½ feet, the square of which is 272¼. See p. 42.

RULE. (1) *Arrange the terms of the multiplier under the same denominations of the multiplicand.* (2) *Multiply each term in the multiplicand, beginning at the lowest,* by the feet in the multiplier, and write the result of each under its respective term, observing to carry one for every twelve.* (3) *Multiply, in the same manner, by the inches, and set the result of each term one place removed to the right-hand of those in the multiplicand.*† (4) *Multiply then by the seconds, setting the result of each term two places removed to the right hand of those in the multiplicand.*

Multiply 9 ft. 4 in. 8 sec. by 5 ft. 8 in. 6 sec.

| ft. | in. | sec. | | |
|---|---|---|---|---|
| 9 | 4 | 8 | | |
| 5 | 8 | 6 | | |
| 46 | 11 | 4 | | |
| 6 | 3 | 1 | $4'''$ | |
| | 4 | 8 | 4 | $0''''$ |
| 53 | 7 | 1 | 8 | 0 |

I multiply by 5, saying 5 times 8 are 40, 4 and carry 3; 5 times 4 are 20 and 3 are 23, 11 and carry 1; 5 times nine are 45 and 1 are 46. For the second line I say, 8 times 8 are 64, 4 and carry 5, but the 4 over are thirds; and so of the rest.

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NOTES.

* Hence the origin of the term *cross* multiplication, the operation being crossways, compared with multiplication in the common way. It is called Duodecimals, because the feet, inches, &c. are divided into *twelve* parts, whereas in decimals the unit is divided into *tenths*.

| | | |
|---|---|---|
| † Feet | multiplied into feet | give feet. |
| Feet | multiplied into inches | give inches. |
| Feet | multiplied into seconds | give seconds. |
| Inches | multiplied into inches | give seconds. |
| Inches | multiplied into seconds | give thirds. |
| Seconds | multiplied into seconds | give fourths. |

Ex. 1. How much must I pay for a slab of marble 7 ft. 4 in. long, and 2 ft. 1 in. 6 sec. broad, at the rate of 7*s*. per square foot?

Ex. 2. What will be the expense of glass for a window that measures, in the clear, 10 ft. 6½ in. in height, and 4 ft. 9 in. in width, at 1*s*. 9*d*. per foot?

Ex. 3. How much will a room cost in painting, at 9½*d*. per yard; the sides are 18 ft. 10 in. by 10 ft. 3 in., and the two ends are 16 ft. 6 in. by 10 ft. 3 in.?

Ex. 4. What shall I have to pay for statuary marble about my fireplace, at 14*s*. per foot; the hearth measures 6 ft. 4 in. by 2 ft. 3 in., the three fronts are each 4 ft. 2 in. by 8 in., and the mantle-piece slab is 6 ft. by 9 in.?

Ex. 5. What will the paving of a court-yard come to, at 1*s*. 2*d*. per foot, the yard being 74 feet long, and 56 ft. 8 in. wide?

Ex. 6. How much shall I have to pay for slating a house, consisting of two sloping sides, each measuring 24 ft. 5 in. by 15 ft. 9 in. at the rate of 41*s*. per square of 100 feet?

Ex. 7. What will the tiling of 10 houses come to, the roof of each house consisting of two sides, each 18 feet by 14, and the price of tiling at 28*s*. per square?

Ex. 8. How many square rods are there in a brick wall 44 ft. 6 in. long, and 7 ft. 4 in. high, and 2½ bricks thick?*

Ex. 9. If an oblong garden be 254 ft. 6 in. long, and 184 ft. 8 in. wide, what will a wall cost 10 ft. 6 in. high, and 2½ bricks thick, at 15*l*. 15*s*. per square rod?

Ex. 10. How much shall I have to pay for the plate-glass of four windows; each window consists of 16 panes, and each pane measures 20½ inches by 15¾ inches at 9*s*. 6*d*. per foot?

Ex. 11. How many solid feet of fir are there in a piece of timber 35 ft. 4 in. long, and 13⅜ inches by 14⅝ inches?†

Ex. 12. How many solid feet of oak are there in a piece 14 feet 3½ inches long, and 2 feet 10¼ by 2 feet 2 inches?

Ex. 13. How many solid feet of fir are there in 46 joists, each 14 feet 3¼ inches long, 7¼ inches deep, by 3½ inches broad?

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NOTES.

* Bricklayers value their work at the rate of a brick and a half, or three half bricks thick; and if the wall be more or less than this, it must be reduced to that thickness by the following rule:—" Multiply the measure found by the number of half bricks, and divide by three:" thus, if the wall be 2½ bricks thick, I multiply by 5, and divide the product by 3.

Ex. If the wall be 50 feet long, and 9 high, and 2 bricks thick, it will be $50 \times 9 \times \frac{4}{3} = 600$ feet; and $\frac{600}{272\frac{1}{4}} = 2\frac{1}{4}$ squ. rods nearly.

† Carpenters' rules are divided into *eighths*, so that in these cases the *eighths* must be reduced to twelfths, or the whole must be worked by decimals. In this and the following questions, the length, breadth, and thickness, must be multiplied into one another.

# GEOGRAPHICAL CALCULATIONS.

3 feet - - - make 1 yard,
1760 yards - - - —— 1 mile,
69¼ miles, or 60 geographical miles —— 1 degree.*

The degree is usually reckoned in round numbers, at 69½ miles; but if accuracy be attended to, the number in the table is too large: the real length of a degree is 365184 English feet, or 69 miles 288 yards: this has been ascertained by actual measurement, so that the circumference of the earth is equal to 69 miles, 288 yards × 360 (because in every circle there are 360 degrees) = 25000 miles nearly.

Geographers reckon on the globe two kinds of degrees, viz. degrees of latitude, and degrees of longitude.* The degrees of latitude, which are measured, from north to south, on the meridian, are all of one length, as above. But the degrees of longitude, or the circles which pass round the earth in each parallel of latitude, continually diminish in proceeding from the equator towards the Poles, but at the equator they are of the same length as those of latitude. The following is a Table of the length of the degrees of longitude, carried to three places of decimals, in every 5 degrees of latitude.

TABLE.

| Lat. | Eng. miles. | Lat. | Eng. miles. | Lat. | Eng. miles. | Lat. | Eng. m. |
|---|---|---|---|---|---|---|---|
| 0 | 69.200 | 25 | 62.716 | 50 | 44.481 | 75 | 17.910 |
| 5 | 68.936 | 30 | 59.929 | 55 | 39.691 | 80 | 12.016 |
| 10 | 68.149 | 35 | 56.685 | 60 | 34.600 | 85 | 6.031 |
| 15 | 66.842 | 40 | 53.010 | 65 | 29.245 | 90 | 0.000 |
| 20 | 65.026 | 45 | 48.931 | 70 | 23.667 | | |

Here it is evident, that at latitude 40°, the degree is little more than 53 miles in length; at 70° it is only 23½ miles; and at the pole, or 90°, it comes to nothing, it being supposed to be a point.

## I. To find the distance, in miles, between any two places, having the same degree of latitude.

Rule. *Having found the distance between the places, in degrees, multiply the number so found by the number in the table opposite the given degree of latitude.*

Ex. How many miles distant is Madrid, in Spain, from Bursa, in Natolia; the latitude of both is 40° N., but the long. of Madrid is about 3° W., and that of Bursa 29° E.?

---

NOTE.

* Longitude expresses the distance of meridians, or circles, which are supposed to pass over the head from north to south; and latitude expresses the distance of a place north and south from the equator.

The difference in longitude is 3° + 29° = 32°, this multiplied by 53.01, the number of miles in a degree at the given latitudes, gives 1696 for the miles between Madrid and Bursa.

### II. To find the distance between any two places, having the same degree of longitude.

Rule. *Multiply the number of degrees between the places by 69.2, and the answer is in miles.*

Ex. How far is London from Mount Atlas in Africa, the former is $51\frac{1}{2}^{\circ}$ N. L., the latter $31\frac{1}{2}^{\circ}$ N. L.?

Here the distance is 20°, and 20 × 69.2 = 1384 miles.

TIME is measured by the revolution of the earth about its axis: every revolution is completed in 24 hours; and as there are 360° in the great circle of the earth, so $\frac{360^{\circ}}{24} = 15^{\circ}$ = 1 hour of time.—Hence this Table:

| | | |
|---|---|---|
| 15° of motion answers to | 60′ in time, | or 1 hour. |
| 1° - - - | 4′ - - - | |

### I. To convert time into motion.

Rule. *Multiply the hours by 15, and divide the minutes by 4, and the answer is in degrees, &c.*

Thus 4 h. 20 min. in time, answer to 65° in motion.

### II. To convert motion into time.

Rule. *Divide the given number of degrees of motion by 15, and the answer is in time.*

Thus 65° of motion answer to 4 h. 20 min. in time:

15)65(4
60
5
60
15)300(20
300

Ex. 1. What o'clock is it at Athens, which is 23° 57′ east longitude of London, now it is 12 at the metropolis?

Athens being east of London, the clocks there will be before the clocks here.

15)23° 57′(1 hour.
15
8
60
15)537(36 min. nearly.

Answer. When it is 12 o'clock at London, it will be 36 min. past 1 at Athens.

Ex. 2. What o'clock is it at Philadelphia in America, now it is 12 at London?

Philadelphia is 75° 8′ west longitude of London, of course the clocks there are behind those here.

15)75° 8'(5 ho. 0 min. 32 sec.
75
———
8
60
———
15)480(32

In this case the answer is.
12 h.—5 h. 0 m. 32 s. = 6 h. 59 m. 28 s.
or very nearly 7 in the morning.

In many maps the longitude is reckoned from Ferro, one of the Canary Islands, which is 17° 45' west of London.

III. To reduce the longitude of Ferro to that of London.

Rule 1. *If the place be* EAST *of London, subtract from it* 17° 30', *and the remainder is the longitude east of London.*

Thus, from Ferro, Constantinople is 46° 44'; to reduce this to the longitude reckoned from the meridian of London, we say,

46° 44' — 17° 45' = 28° 59'.

2. *If the place be* WEST *from Ferro, add to the given longitude* 17° 45'.

Thus, Boston is 52° 48' west of Ferro, but it is west of London 52° 48' + 17° 45' = 70° 33'.

3. *If the place lies between Ferro and London, its longitude will be obtained by subtracting its longitude east of Ferro from* 17° 45'.

Thus, Lisbon is 8° 40' east of Ferro, and it is west of London 17° 45' — 8° 40' = 9° 5'.

By a reverse method may be reduced the longitude from London to that of Ferro.

The earth being globular, it is a useful problem to ascertain the extent of the visible horizon: or

IV. To find the distance to which a person can see at any given height of the eye.

Rule. *Multiply the square-root of the height of the eye, in feet, by* 1.2247, *and the product is the distance in miles to which we can see from that height.* See p. 143.

Ex. 1. How far can a sailor see, standing at the top-mast of a ship, 144 feet high?

The square-root of 144 is 12; therefore 1.2247 × 12 = 14.7 miles. Thus, in this situation, a sailor might, on a very clear day, descry land at the distance of 5 leagues, nearly; and he might see the top-mast of another ship at a still greater distance.

Ex. 2. To what distance could a person see from the top of St. Paul's, which is 340 feet high?

$\sqrt{340} \times 1.2247 = 18.44 \times 1.2247 = 22.58$ miles, or something more than $22\frac{1}{2}$ miles.

# A TABLE
## OF THE
# LOGARITHMS OF NUMBERS,
## FROM 1 TO 1000.

*(See also page* 155.*)*

| No. | Logarithms. | No. | Logarithms. | No. | Logarithms. |
|---|---|---|---|---|---|
| 1 | 0.0000000 | 34 | 1.5314789 | 67 | 1.8260748 |
| 2 | 0.3010300 | 35 | 1.5440680 | 68 | 1.8325089 |
| 3 | 0.4771213 | 36 | 1.5563025 | 69 | 1.8388491 |
| 4 | 0.6020600 | 37 | 1.5682017 | 70 | 1.8450980 |
| 5 | 0.6989700 | 38 | 1.5797863 | 71 | 1.8512583 |
| 6 | 0.7781513 | 39 | 1.5910646 | 72 | 1.8573325 |
| 7 | 0.8450980 | 40 | 1.6020600 | 73 | 1.8683229 |
| 8 | 0.9030900 | 41 | 1.6127839 | 74 | 1.8692317 |
| 9 | 0.9542425 | 42 | 1.6232493 | 75 | 1.8750613 |
| 10 | 1.0000000 | 43 | 1.6334685 | 76 | 1.8808136 |
| 11 | 1.0413927 | 44 | 1.6434527 | 77 | 1.8864907 |
| 12 | 1.0791812 | 45 | 1.6532125 | 78 | 1.8920946 |
| 13 | 1.1139434 | 46 | 1.6627578 | 79 | 1.8976271 |
| 14 | 1.1461280 | 47 | 1.6720979 | 80 | 1.9030900 |
| 15 | 1.1760913 | 48 | 1.6812412 | 81 | 1.9084850 |
| 16 | 1.2041200 | 49 | 1.6901961 | 82 | 1.9138139 |
| 17 | 1.2304489 | 50 | 1.6989700 | 83 | 1.9190781 |
| 18 | 1.2552725 | 51 | 1.7075702 | 84 | 1.9242793 |
| 19 | 1.2787536 | 52 | 1 7160033 | 85 | 1.9294189 |
| 20 | 1.3010300 | 53 | 1.7242759 | 86 | 1.9344985 |
| 21 | 1.3222193 | 54 | 1.7323938 | 87 | 1.9395193 |
| 22 | 1.3424227 | 55 | 1.7403627 | 88 | 1.9444827 |
| 23 | 1.3617278 | 56 | 1.7481880 | 89 | 1.9493900 |
| 24 | 1.3802112 | 57 | 1.7558749 | 90 | 1.9542425 |
| 25 | 1.3979400 | 58 | 1.7634280 | 91 | 1.9590414 |
| 26 | 1.4149733 | 59 | 1.7708520 | 92 | 1.9637878 |
| 27 | 1.4313638 | 60 | 1.7781513 | 93 | 1.9684829 |
| 28 | 1.4471580 | 61 | 1.7853298 | 94 | 1.9731279 |
| 29 | 1.4623980 | 62 | 1.7923917 | 95 | 1.9777236 |
| 30 | 1.4771213 | 63 | 1.7993405 | 96 | 1.9822712 |
| 31 | 1.4913617 | 64 | 1.8061800 | 97 | 1.9867717 |
| 32 | 1.5051500 | 65 | 1.8129134 | 98 | 1.9912261 |
| 33 | 1.5185139 | 66 | 1.8195439 | 99 | 1.9956352 |
|  |  |  |  | 100 | 2.0000000 |

| No. | Logarithms. | Diff. | No. | Logarithms. | Diff. |
|---|---|---|---|---|---|
| 101 | 2.0043214 | | 146 | 2.1643529 | |
| 102 | 2.0086002 | 4278 | 147 | 2.1673173 | 2964 |
| 103 | 2.0128372 | 4237 | 148 | 2.1702617 | 2944 |
| 104 | 2.0170333 | 4196 | 149 | 2.1731863 | 2924 |
| 105 | 2.0211893 | 4156 | 150 | 2.1760913 | 2905 |
| 106 | 2.0253059 | 4116 | 151 | 2.1789769 | 2885 |
| 107 | 2.0293838 | 4077 | 152 | 2.1818436 | 2866 |
| 108 | 2.0334238 | 4039 | 153 | 2.1846914 | 2847 |
| 109 | 2.0374265 | 4002 | 154 | 2.1875207 | 2829 |
| 110 | 2.0413927 | 3966 | 155 | 2.1903317 | 2811 |
| 111 | 2.0453230 | 3930 | 156 | 2.1931246 | 2792 |
| 112 | 2.0492180 | 3895 | 157 | 2.1958997 | 2775 |
| 113 | 2.0530784 | 3880 | 158 | 2.1986571 | 2757 |
| 114 | 2.0569049 | 3826 | 159 | 2.2013971 | 2740 |
| 115 | 2.0606978 | 3793 | 160 | 2.2041200 | 2722 |
| 116 | 2.0644580 | 3760 | 161 | 2.2068259 | 2705 |
| 117 | 2.0681859 | 3727 | 162 | 2.2095150 | 2689 |
| 118 | 2.0718820 | 3692 | 163 | 2.2121876 | 2672 |
| 119 | 2.0755470 | 3665 | 164 | 2.2148438 | 2656 |
| 120 | 2.0791812 | 3634 | 165 | 2.2174839 | 2640 |
| 121 | 2.0827854 | 3604 | 166 | 2.2201081 | 2624 |
| 122 | 2.0863598 | 3574 | 167 | 2.2227165 | 2608 |
| 123 | 2.0899051 | 3545 | 168 | 2.2253093 | 2592 |
| 124 | 2.0934217 | 3516 | 169 | 2.2278867 | 2577 |
| 125 | 2.0969100 | 3488 | 170 | 2.2304489 | 2562 |
| 126 | 2.1003705 | 3460 | 171 | 2.2329961 | 2548 |
| 127 | 2.1038037 | 3433 | 172 | 2.2355284 | 2532 |
| 128 | 2.1072100 | 3406 | 173 | 2.2380461 | 2517 |
| 129 | 2.1105897 | 3379 | 174 | 2.2405492 | 2503 |
| 130 | 2.1139434 | 3353 | 175 | 2.2430380 | 2488 |
| 131 | 2.1172713 | 3328 | 176 | 2.2455127 | 2474 |
| 132 | 2.1205739 | 3302 | 177 | 2.2479733 | 2460 |
| 133 | 2.1238516 | 3277 | 178 | 2.2504200 | 2446 |
| 134 | 2.1271048 | 3253 | 179 | 2.2528530 | 2433 |
| 135 | 2.1303338 | 3229 | 180 | 2.2552725 | 2419 |
| 136 | 2.1335389 | 3205 | 181 | 2.2576786 | 2406 |
| 137 | 2.1367206 | 3181 | 182 | 2.2600714 | 2392 |
| 138 | 2.1398791 | 3158 | 183 | 2.2624511 | 2379 |
| 139 | 2.1430148 | 3135 | 184 | 2.2648178 | 2366 |
| 140 | 2.1461280 | 3113 | 185 | 2.2671717 | 2353 |
| 141 | 2.1492191 | 3091 | 186 | 2.2695129 | 2341 |
| 142 | 2.1522883 | 3069 | 187 | 2.2718416 | 2328 |
| 143 | 2.1553360 | 3047 | 188 | 2.2741578 | 2316 |
| 144 | 2.1583625 | 3026 | 189 | 2.2764618 | 2304 |
| 145 | 2.1613680 | 3005 | 190 | 2.2787536 | 2291 |
| | | 2984 | | | 2279 |

| No. | Logarithms. | Diff. | No. | Logarithms. | Diff. |
|---|---|---|---|---|---|
| 191 | 2.2810334 | 2267 | 236 | 2.3729120 | 1836 |
| 192 | 2.2833012 | 2256 | 237 | 2.3747483 | 1828 |
| 193 | 2.2855573 | 2244 | 238 | 2.3765770 | 1820 |
| 194 | 2.2878017 | 2232 | 239 | 2.3783979 | 1813 |
| 195 | 2.2900346 | 2221 | 240 | 2.3802112 | 1805 |
| 196 | 2.2922561 | 2210 | 241 | 2.3820170 | 1798 |
| 197 | 2.2944662 | 2199 | 242 | 2.3838154 | 1790 |
| 198 | 2.2966652 | 2187 | 243 | 2.3856063 | 1783 |
| 199 | 2.2988531 | 2176 | 244 | 2.3873898 | 1776 |
| 200 | 2.3010300 | 2166 | 245 | 2.3891661 | 1769 |
| 201 | 2.3031961 | 2155 | 246 | 2.3909351 | 1761 |
| 202 | 2.3053514 | 2144 | 247 | 2.3926969 | 1754 |
| 203 | 2.3074960 | 2134 | 248 | 2.3944517 | 1747 |
| 204 | 2.3096302 | 2123 | 249 | 2.3961993 | 1740 |
| 205 | 2.3117539 | 2113 | 250 | 2.3979400 | 1733 |
| 206 | 2.3138672 | 2103 | 251 | 2.3996737 | 1726 |
| 207 | 2.3159703 | 2093 | 252 | 2.4014005 | 1720 |
| 208 | 2.3180633 | 2083 | 253 | 2.4031205 | 1713 |
| 209 | 2.3201463 | 2073 | 254 | 2.4048337 | 1706 |
| 210 | 2.3222193 | 2063 | 255 | 2.4065402 | 1699 |
| 211 | 2.3242825 | 2053 | 256 | 2.4082400 | 1693 |
| 212 | 2.3263359 | 2043 | 257 | 2.4099331 | 1686 |
| 213 | 2.3283796 | 2034 | 258 | 2.4116197 | 1680 |
| 214 | 2.3304138 | 2024 | 259 | 2.4132998 | 1673 |
| 215 | 2.3324385 | 2015 | 260 | 2.4149733 | 1667 |
| 216 | 2.3344538 | 2005 | 261 | 2.4166405 | 1660 |
| 217 | 2.3364597 | 1996 | 262 | 2.4183013 | 1654 |
| 218 | 2.3384565 | 1987 | 263 | 2.4199557 | 1648 |
| 219 | 2.3404441 | 1978 | 264 | 2.4216039 | 1642 |
| 220 | 2.3424227 | 1969 | 265 | 2.4232459 | 1635 |
| 221 | 2.3443923 | 1960 | 266 | 2.4248816 | 1629 |
| 222 | 2.3463530 | 1951 | 267 | 2.4265113 | 1623 |
| 223 | 2.3483049 | 1943 | 268 | 2.4281348 | 1617 |
| 224 | 2.3502480 | 1934 | 269 | 2.4297523 | 1611 |
| 225 | 2.3521825 | 1925 | 270 | 2.4313638 | 1605 |
| 226 | 2.3541084 | 1917 | 271 | 2.4329693 | 1599 |
| 227 | 2.3560259 | 1908 | 272 | 2.4345689 | 1593 |
| 228 | 2.3579348 | 1900 | 273 | 2.4361626 | 1588 |
| 229 | 2.3598355 | 1892 | 274 | 2.4377506 | 1582 |
| 230 | 2.3617278 | 1884 | 275 | 2.4393327 | 1576 |
| 231 | 2.3636120 | 1876 | 276 | 2.4409091 | 1570 |
| 232 | 2.3654880 | 1867 | 277 | 2.4424798 | 1565 |
| 233 | 2.3673559 | 1860 | 278 | 2.4440448 | 1559 |
| 234 | 2.3692159 | 1852 | 279 | 2.4456042 | 1553 |
| 235 | 2.3710679 | 1844 | 280 | 2.4471580 | 1548 |

| No. | Logarithms. | Diff. | No. | Logarithms. | Diff. |
|---|---|---|---|---|---|
| 281 | 2.4487063 | 1542 | 326 | 2.5132176 | 1330 |
| 282 | 2 4502491 | 1537 | 327 | 2.5145478 | 1326 |
| 283 | 2.4517864 | 1531 | 328 | 2.5158738 | 1322 |
| 284 | 2.4533183 | 1526 | 329 | 2.5171959 | 1318 |
| 285 | 2.4548449 | 1521 | 330 | 2.5185139 | 1314 |
| 286 | 2.4563660 | 1515 | 331 | 2.5198280 | 1310 |
| 287 | 2.4578819 | 1510 | 332 | 2.5211381 | 1306 |
| 288 | 2.4593925 | 1505 | 333 | 2.5224442 | 1302 |
| 289 | 2.4608978 | 1500 | 334 | 2.5237465 | 1298 |
| 290 | 2.4623980 | 1495 | 335 | 2.5250448 | 1294 |
| 291 | 2.4638930 | 1489 | 336 | 2.5263393 | 1290 |
| 292 | 2.4653829 | 1484 | 337 | 2.5276299 | 1286 |
| 293 | 2.4668676 | 1479 | 338 | 2.5289167 | 1283 |
| 294 | 2.4683473 | 1474 | 339 | 2.5301997 | 1279 |
| 295 | 2.4698220 | 1469 | 340 | 2.5314789 | 1275 |
| 296 | 2.4712917 | 1464 | 341 | 2.5327544 | 1271 |
| 297 | 2.4727564 | 1459 | 342 | 2.5340261 | 1268 |
| 298 | 2.4742163 | 1454 | 343 | 2.5352941 | 1264 |
| 299 | 2.4756712 | 1450 | 344 | 2.5365584 | 1260 |
| 300 | 2.4771213 | 1445 | 345 | 2.5378191 | 1257 |
| 301 | 2.4785665 | 1440 | 346 | 2.5390761 | 1253 |
| 302 | 2.4800069 | 1435 | 347 | 2.5403295 | 1249 |
| 303 | 2.4814426 | 1431 | 348 | 2.5415792 | 1246 |
| 304 | 2.4828736 | 1426 | 349 | 2.5428254 | 1242 |
| 305 | 2.4842998 | 1421 | 350 | 2.5440680 | 1239 |
| 306 | 2.4857214 | 1417 | 351 | 2.5453071 | 1235 |
| 307 | 2.4871384 | 1412 | 352 | 2.5465427 | 1232 |
| 308 | 2.4885507 | 1407 | 353 | 2.5477747 | 1228 |
| 309 | 2.4899585 | 1403 | 354 | 2.5490033 | 1225 |
| 310 | 2:4913617 | 1398 | 355 | 2,5502283 | 1221 |
| 311 | 2.4927604 | 1394 | 356 | 2.5514500 | 1218 |
| 312 | 2.4941546 | 1389 | 357 | 2.5526682 | 1214 |
| 313 | 2.4955443 | 1385 | 358 | 2.5538830 | 1211 |
| 314 | 2.4969296 | 1381 | 359 | 2.5550944 | 1208 |
| 315 | 2.4983106 | 1376 | 360 | 2.5563025 | 1204 |
| 316 | 2.4996871 | 1372 | 361 | 2.5575072 | 1201 |
| 317 | 2.5010593 | 1367 | 362 | 2.5587086 | 1198 |
| 318 | 2,5024271 | 1363 | 363 | 2.5599066 | 1194 |
| 319 | 2.5037907 | 1359 | 364 | 2.5611014 | 1191 |
| 320 | 2.5051500 | 1355 | 365 | 2.5622929 | 1188 |
| 321 | 2.5065050 | 1350 | 366 | 2.5634811 | 1185 |
| 322 | 2.5078559 | 1346 | 367 | 2.5646661 | 1181 |
| 323 | 2.5092025 | 1342 | 368 | 2.5658478 | 1178 |
| 324 | 2.5105450 | 1338 | 369 | 2.5670264 | 1175 |
| 325 | 2.5118834 | 1334 | 370 | 2.5682017 | 1172 |

| No. | Logarithms. | Diff. | No. | Logarithms. | Diff. |
|---|---|---|---|---|---|
| 371 | 2.5693739 | 1169 | 416 | 2.6190933 | 1042 |
| 372 | 2.5705429 | 1165 | 417 | 2.6201861 | 1040 |
| 373 | 2.5717088 | 1162 | 418 | 2.6211763 | 1037 |
| 374 | 2.5728716 | 1159 | 419 | 2.6222140 | 1035 |
| 375 | 2.5740313 | 1156 | 420 | 2.6232493 | 1032 |
| 376 | 2.5751878 | 1153 | 421 | 2.6242821 | 1030 |
| 377 | 2.5763414 | 1150 | 422 | 2.6253124 | 1028 |
| 378 | 2.5774918 | 1147 | 423 | 2.6263404 | 1025 |
| 379 | 2.5786392 | 1144 | 424 | 2.6273659 | 1023 |
| 380 | 2.5797836 | 1141 | 425 | 2.6283889 | 1020 |
| 381 | 2.5809250 | 1138 | 426 | 2.6294096 | 1018 |
| 382 | 2.5820634 | 1135 | 427 | 2.6304279 | 1015 |
| 383 | 2.5831988 | 1132 | 428 | 2.6314438 | 1013 |
| 384 | 2.5843312 | 1129 | 420 | 2.6324573 | 1011 |
| 385 | 2.5854607 | 1126 | 430 | 2.6334685 | 1006 |
| 386 | 2.5865873 | 1123 | 431 | 2.6344773 | 1006 |
| 387 | 2.5877110 | 1120 | 432 | 2.6354837 | 1004 |
| 388 | 2.5888317 | 1117 | 433 | 2.6364879 | 1001 |
| 389 | 2.5899496 | 1115 | 434 | 2.6374897 | 999 |
| 390 | 2.5910646 | 1112 | 435 | 2.6384893 | 997 |
| 391 | 2.5921768 | 1109 | 436 | 2.6394865 | 994 |
| 392 | 2.5932861 | 1106 | 437 | 2.6404814 | 992 |
| 393 | 2.5943926 | 1103 | 438 | 2.6414741 | 990 |
| 394 | 2.5954962 | 1100 | 439 | 2.6424645 | 988 |
| 395 | 2.5965971 | 1098 | 440 | 2.6434527 | 985 |
| 396 | 2.5976952 | 1095 | 441 | 2.6444386 | 982 |
| 397 | 2.5987905 | 1092 | 442 | 2.6454223 | 981 |
| 398 | 2.5998831 | 1089 | 443 | 2.6464037 | 979 |
| 399 | 2.6009729 | 1087 | 444 | 2.6473830 | 977 |
| 400 | 2.6020600 | 1084 | 445 | 2.6483600 | 974 |
| 401 | 2.6031444 | 1081 | 446 | 2.6493349 | 972 |
| 402 | 2.6042261 | 1078 | 447 | 2.6503075 | 970 |
| 403 | 2.6053050 | 1076 | 448 | 2.6512780 | 968 |
| 404 | 2.6063814 | 1073 | 449 | 2.6522463 | 966 |
| 405 | 2.6074550 | 1071 | 450 | 2.6532125 | 964 |
| 406 | 2.6085260 | 1068 | 451 | 2.6541765 | 961 |
| 407 | 2.6095944 | 1065 | 452 | 2.6551384 | 959 |
| 408 | 2.6106602 | 1063 | 453 | 2.6560982 | 957 |
| 409 | 2.6117233 | 1060 | 454 | 2.6570559 | 955 |
| 410 | 2.6127839 | 1057 | 455 | 2.6580114 | 953 |
| 411 | 2,6138418 | 1055 | 456 | 2.6589648 | 951 |
| 412 | 2.6148972 | 1052 | 457 | 2.6599162 | 949 |
| 413 | 2.6159501 | 1050 | 458 | 2.6608655 | 947 |
| 414 | 2.6170003 | 1047 | 459 | 2.6618127 | 945 |
| 415 | 2.6180481 | 1045 | 460 | 2.6627578 | 943 |

| No. | Logarithms. | Diff. | No. | Logarithms. | Diff. |
|---|---|---|---|---|---|
| 461 | 2.6637009 | 941 | 506 | 2.7041505 | 857 |
| 462 | 2.6646420 | 939 | 507 | 2.7050080 | 855 |
| 463 | 2.6655810 | 937 | 508 | 2.7058637 | 854 |
| 464 | 2.6665180 | 935 | 509 | 2.7067178 | 852 |
| 465 | 2.6674530 | 932 | 510 | 2.7075702 | 850 |
| 466 | 2.6683859 | 931 | 511 | 2.7084209 | 849 |
| 467 | 2.6693169 | 929 | 512 | 2.7092700 | 847 |
| 468 | 2.6702459 | 926 | 513 | 2.7101174 | 845 |
| 469 | 2.6711728 | 925 | 514 | 2.7109631 | 844 |
| 470 | 2.6720979 | 923 | 515 | 2.7118072 | 842 |
| 471 | 2.6730209 | 921 | 516 | 2.7126497 | 840 |
| 472 | 2.6739420 | 919 | 517 | 2.7134905 | 839 |
| 473 | 2.6748611 | 917 | 518 | 2.7143298 | 837 |
| 474 | 2.6757783 | 915 | 519 | 2.7151674 | 835 |
| 475 | 2.6766936 | 913 | 520 | 2.7160033 | 834 |
| 476 | 2.6776069 | 911 | 521 | 2.7168377 | 832 |
| 477 | 2.6785184 | 909 | 522 | 2.7176705 | 831 |
| 478 | 2.6794279 | 907 | 523 | 2.7185017 | 829 |
| 479 | 2.6803355 | 905 | 524 | 2.7193313 | 828 |
| 480 | 2.6812412 | 903 | 525 | 2.7201593 | 826 |
| 481 | 2.6821451 | 901 | 526 | 2.7209857 | 824 |
| 482 | 2.6830470 | 900 | 527 | 2.7218106 | 823 |
| 483 | 2.6839471 | 898 | 528 | 2.7226339 | 821 |
| 484 | 2.6848454 | 896 | 529 | 2.7234557 | 820 |
| 485 | 2.6857417 | 894 | 530 | 2.7242759 | 818 |
| 486 | 2.6866363 | 892 | 531 | 2.7250945 | 817 |
| 487 | 2.6875290 | 890 | 532 | 2.7259116 | 815 |
| 488 | 2.6884198 | 889 | 533 | 2.7267272 | 814 |
| 489 | 2.6893089 | 887 | 534 | 2.7275413 | 812 |
| 490 | 2.6901961 | 885 | 535 | 2.7283538 | 811 |
| 491 | 2.6910815 | 883 | 536 | 2.7291648 | 809 |
| 492 | 2.6919651 | 881 | 537 | 2.7299743 | 808 |
| 493 | 2.6928469 | 880 | 538 | 2.7307823 | 806 |
| 494 | 2.6937269 | 878 | 539 | 2.7315888 | 805 |
| 495 | 2.6946052 | 876 | 540 | 2.7323938 | 804 |
| 496 | 2.6954817 | 874 | 541 | 2.7331973 | 802 |
| 497 | 2.6963564 | 872 | 542 | 2.7339993 | 800 |
| 498 | 2.6972293 | 871 | 543 | 2.7347998 | 799 |
| 499 | 2.6981005 | 869 | 544 | 2.7355989 | 797 |
| 500 | 2.6989700 | 867 | 545 | 2.7363965 | 796 |
| 501 | 2 6998377 | 866 | 546 | 2.7371926 | 794 |
| 502 | 2.7007037 | 864 | 547 | 2.7379873 | 793 |
| 503 | 2.7015680 | 862 | 548 | 2.7387806 | 791 |
| 504 | 2.7024305 | 860 | 549 | 2.7395723 | 790 |
| 505 | 2.7032914 | 859 | 550 | 2.7403627 | 788 |

| No. | Logarithms. | Diff. | No. | Logarithms. | Diff. |
|---|---|---|---|---|---|
| 551 | 2.7411516 | 787 | 596 | 2.7752463 | 728 |
| 552 | 2.7419391 | 786 | 597 | 2.7759743 | 727 |
| 553 | 2.7427251 | 784 | 598 | 2.7767012 | 725 |
| 554 | 2.7435098 | 783 | 599 | 2.7774268 | 724 |
| 555 | 2.7442930 | 781 | 600 | 2.7781513 | 723 |
| 556 | 2.7450748 | 780 | 601 | 2.7788745 | 722 |
| 557 | 2.7458552 | 779 | 602 | 2.7795965 | 720 |
| 558 | 2.7466342 | 777 | 603 | 2.7803173 | 719 |
| 559 | 2.7474118 | 776 | 604 | 2.7810369 | 718 |
| 560 | 2.7481880 | 774 | 605 | 2.7817554 | 717 |
| 561 | 2.7489629 | 773 | 606 | 2.7824726 | 716 |
| 562 | 2.7497363 | 772 | 607 | 2.7831887 | 714 |
| 563 | 2.7505084 | 770 | 608 | 2.7839036 | 713 |
| 564 | 2.7512791 | 769 | 609 | 2.7846173 | 712 |
| 565 | 2.7520484 | 768 | 610 | 2.7853298 | 711 |
| 566 | 2.7528164 | 766 | 611 | 2.7860412 | 710 |
| 567 | 2.7535831 | 765 | 612 | 2.7867514 | 709 |
| 568 | 2.7543483 | 763 | 613 | 2.7874605 | 707 |
| 569 | 2.7551123 | 762 | 614 | 2.7881684 | 706 |
| 570 | 2.7558749 | 761 | 615 | 2.7888751 | 705 |
| 571 | 2.7566361 | 760 | 616 | 2.7895807 | 704 |
| 572 | 2.7573960 | 758 | 617 | 2.7902852 | 703 |
| 573 | 2.7581546 | 757 | 618 | 2.7909885 | 702 |
| 574 | 2.7589119 | 755 | 619 | 2.7916906 | 701 |
| 575 | 2.7596678 | 754 | 620 | 2.7923917 | 699 |
| 576 | 2.7604225 | 753 | 621 | 2.7930916 | 698 |
| 577 | 2.7611758 | 752 | 622 | 2.7937904 | 697 |
| 578 | 2.7619278 | 750 | 623 | 2.7944880 | 696 |
| 579 | 2.7626786 | 749 | 624 | 2.7951846 | 695 |
| 580 | 2.7634280 | 748 | 625 | 2.7958800 | 694 |
| 581 | 2.7641761 | 746 | 626 | 2.7965743 | 693 |
| 582 | 2.7649230 | 745 | 627 | 2.7972675 | 692 |
| 583 | 2.7656686 | 744 | 628 | 2.7979596 | 691 |
| 584 | 2.7664128 | 743 | 629 | 2.7986506 | 689 |
| 585 | 2.7671559 | 741 | 630 | 2.7993405 | 688 |
| 586 | 2.7678976 | 740 | 631 | 2.8000294 | 687 |
| 587 | 2.7686381 | 739 | 632 | 2.8007171 | 686 |
| 588 | 2.7693773 | 738 | 633 | 2.8014037 | 685 |
| 589 | 2.7701153 | 736 | 634 | 2.8020893 | 684 |
| 590 | 2.7708520 | 735 | 635 | 2.8027737 | 683 |
| 591 | 2.7715875 | 734 | 636 | 2.8034571 | 682 |
| 592 | 2.7723217 | 733 | 637 | 2.8041394 | 681 |
| 593 | 2.7730547 | 731 | 638 | 2.8048207 | 680 |
| 594 | 2.7737864 | 730 | 639 | 2.8055009 | 679 |
| 595 | 2.7745170 | 729 | 640 | 2.8091800 | 678 |

| No. | Logarithms. | Diff. | No. | Logarithms. | Diff. |
|---|---|---|---|---|---|
| 641 | 2.8068580 | | 686 | 2.8363241 | |
| 642 | 2.8075350 | 677 | 687 | 2.8369567 | 632 |
| 643 | 2.8082110 | 676 | 688 | 2.8375884 | 631 |
| 644 | 2.8088859 | 674 | 689 | 2.8382192 | 630 |
| 645 | 2.8095597 | 673 | 690 | 2.8388491 | 629 |
| 646 | 2.8102325 | 672 | 691 | 2.8394780 | 628 |
| 647 | 2.8109043 | 671 | 692 | 2.8401061 | 628 |
| 648 | 2.8115750 | 670 | 693 | 2.8407332 | 627 |
| 649 | 2.8122447 | 669 | 694 | 2.8413595 | 626 |
| 650 | 2.8129134 | 668 | 695 | 2.8419848 | 625 |
| 651 | 2.8135810 | 667 | 696 | 2.8426092 | 624 |
| 652 | 2.8142476 | 666 | 697 | 2.8432328 | 623 |
| 653 | 2.8149132 | 665 | 698 | 2.8438554 | 622 |
| 654 | 2.8155777 | 664 | 699 | 2.8444772 | 621 |
| 655 | 2.8162413 | 663 | 700 | 2.8450980 | 620 |
| 656 | 2.8169038 | 662 | 701 | 2.8457180 | 620 |
| 657 | 2.8175654 | 661 | 702 | 2.8463371 | 619 |
| 658 | 2.8182259 | 660 | 703 | 2.8469553 | 618 |
| 659 | 2.8188854 | 659 | 704 | 2.8475727 | 617 |
| 660 | 2.8195439 | 658 | 705 | 2.8481891 | 616 |
| 661 | 2.8202015 | 657 | 706 | 2.8488047 | 615 |
| 662 | 2.8208580 | 656 | 707 | 2.8494194 | 614 |
| 663 | 2.8215135 | 655 | 708 | 2.8500333 | 613 |
| 664 | 2.8221681 | 654 | 709 | 2.8506462 | 612 |
| 665 | 2.8228216 | 653 | 710 | 2.8512583 | 612 |
| 666 | 2.8234742 | 652 | 711 | 2.8518696 | 611 |
| 667 | 2.8241258 | 651 | 712 | 2.8524800 | 610 |
| 668 | 2.8247765 | 650 | 713 | 2.8530895 | 609 |
| 669 | 2.8254261 | 649 | 714 | 2.8536982 | 608 |
| 670 | 2.8260748 | 648 | 715 | 2.8543060 | 607 |
| 671 | 2.8267225 | 647 | 716 | 2.8549130 | 607 |
| 672 | 2.8273693 | 646 | 717 | 2.8555192 | 606 |
| 673 | 2.8280151 | 645 | 718 | 2.8561244 | 605 |
| 674 | 2.8286599 | 644 | 719 | 2.8567289 | 604 |
| 675 | 2.8293038 | 643 | 720 | 2.8573325 | 603 |
| 676 | 2.8299467 | 642 | 721 | 2.8579353 | 602 |
| 677 | 2.8305887 | 642 | 722 | 2.8585372 | 601 |
| 678 | 2.8312297 | 641 | 723 | 2.8591383 | 601 |
| 679 | 2.8318698 | 640 | 724 | 2.8597386 | 600 |
| 680 | 2.8325089 | 639 | 725 | 2.8603380 | 599 |
| 681 | 2.8331471 | 638 | 726 | 2.8609366 | 598 |
| 682 | 2.8337844 | 637 | 727 | 2.8615344 | 597 |
| 683 | 2.8344207 | 636 | 728 | 2.8621314 | 597 |
| 684 | 2.8350561 | 635 | 729 | 2.8627275 | 596 |
| 685 | 2.8356906 | 634 | 730 | 2.8633229 | 595 |
| | | 633 | | | 594 |

| No. | Logarithms. | Diff. | No. | Logarithms. | Diff. |
|---|---|---|---|---|---|
| 731 | 2.8639174 | | 776 | 2.8898617 | |
| 732 | 2.8645111 | 593 | 777 | 2.8904210 | 559 |
| 733 | 2.8651040 | 592 | 778 | 2.8909796 | 558 |
| 734 | 2.8656961 | 592 | 779 | 2.8915375 | 557 |
| 735 | 2.8662873 | 591 | 780 | 2.8920946 | 557 |
| 736 | 2.8686778 | 590 | 781 | 2.8926510 | 556 |
| 737 | 2.8674675 | 589 | 782 | 2.8932068 | 555 |
| 738 | 2.8680564 | 588 | 783 | 2.8937618 | 555 |
| 739 | 2.8686444 | 588 | 784 | 2.8943161 | 554 |
| 740 | 2.8692317 | 587 | 785 | 2.8948697 | 553 |
| 741 | 2.8698182 | 586 | 786 | 2.8954225 | 552 |
| 742 | 2.8704039 | 585 | 787 | 2.8959747 | 552 |
| 743 | 2.8709888 | 584 | 788 | 2.8965262 | 551 |
| 744 | 2.8715729 | 584 | 789 | 2.8970770 | 550 |
| 745 | 2.8721563 | 583 | 790 | 2.8976271 | 550 |
| 746 | 2.8727388 | 582 | 791 | 2.8981765 | 549 |
| 747 | 2.8733206 | 581 | 792 | 2.8987252 | 548 |
| 748 | 2.8739016 | 581 | 793 | 2.8992732 | 548 |
| 749 | 2.8744818 | 580 | 794 | 2.8998205 | 547 |
| 730 | 2.8750613 | 579 | 795 | 2.9003671 | 546 |
| 751 | 2.8756399 | 578 | 796 | 2.9009131 | 546 |
| 752 | 2.8762178 | 577 | 797 | 2.9014583 | 545 |
| 753 | 2.8767950 | 577 | 798 | 2.9020029 | 544 |
| 754 | 2.8773713 | 576 | 799 | 2 9025468 | 543 |
| 755 | 2.8779469 | 575 | 800 | 2.9030900 | 543 |
| 736 | 2.8785218 | 574 | 801 | 2.9036325 | 542 |
| 757 | 2.8790959 | 574 | 802 | 2.9041744 | 541 |
| 758 | 2.8796692 | 573 | 803 | 2.9047155 | 541 |
| 759 | 2.8802418 | 572 | 804 | 2.9052560 | 540 |
| 760 | 2.8808136 | 571 | 805 | 2.9057960 | 540 |
| 761 | 2.8813847 | 571 | 806 | 2.9063350 | 539 |
| 762 | 2.8819550 | 570 | 807 | 2.9068735 | 538 |
| 763 | 2.8825245 | 559 | 808 | 2.9074114 | 537 |
| 764 | 2.8830934 | 568 | 809 | 2.9079485 | 537 |
| 765 | 2.8836614 | 568 | 810 | 2.9084850 | 536 |
| 766 | 2.8842288 | 567 | 811 | 2.9090209 | 535 |
| 767 | 2.8847954 | 566 | 812 | 2.9095560 | 535 |
| 768 | 2.8853612 | 565 | 813 | 2.9100905 | 534 |
| 769 | 2.8859263 | 565 | 814 | 2.9106244 | 533 |
| 770 | 2.8864907 | 564 | 815 | 2.9111576 | 533 |
| 771 | 2.8870544 | 563 | 816 | 2.9116902 | 532 |
| 772 | 2.8876173 | 562 | 817 | 2.9122220 | 531 |
| 773 | 2.8881795 | 562 | 818 | 2.9127533 | 531 |
| 774 | 2.8887410 | 561 | 819 | 2.9132839 | 530 |
| 775 | 2.8893017 | 560 | 820 | 2.9138139 | 530 |
| | | 560 | | | 529 |

| No. | Logarithms. | Diff. | No. | Logarithms. | Diff. |
|---|---|---|---|---|---|
| 821 | 2.9143432 | 528 | 866 | 2.9375179 | 501 |
| 822 | 2.9148718 | 528 | 867 | 2.9380191 | 500 |
| 813 | 2.9153998 | 527 | 868 | 2.9385197 | 500 |
| 824 | 2.9159272 | 526 | 869 | 2.9390198 | 499 |
| 825 | 2.9164539 | 526 | 870 | 2.9395193 | 498 |
| 826 | 2.9169800 | 525 | 871 | 2.9400182 | 498 |
| 827 | 2.9175055 | 524 | 872 | 2.9405165 | 497 |
| 828 | 2.9180303 | 524 | 873 | 2.9410142 | 497 |
| 829 | 2.9185945 | 523 | 874 | 2.9415114 | 496 |
| 830 | 2.9190781 | 522 | 875 | 2.9420081 | 496 |
| 831 | 2.9196010 | 522 | 876 | 2.9425041 | 495 |
| 832 | 2.9201233 | 521 | 877 | 2.9429996 | 494 |
| 833 | 2.9206450 | 521 | 878 | 2.9434945 | 494 |
| 834 | 2.9211661 | 520 | 879 | 2.9439889 | 493 |
| 835 | 2.9216865 | 519 | 880 | 2.9448427 | 493 |
| 836 | 2.9222063 | 519 | 881 | 2.9449759 | 492 |
| 837 | 2.9227255 | 518 | 882 | 2.9454686 | 492 |
| 838 | 2.9232440 | 518 | 883 | 2.9459607 | 491 |
| 839 | 2.9237620 | 517 | 884 | 2.9464523 | 491 |
| 840 | 2.9242793 | 516 | 885 | 2.9469433 | 490 |
| 841 | 2.9247960 | 516 | 886 | 2.9474337 | 489 |
| 842 | 2.9253121 | 515 | 887 | 2.9479236 | 489 |
| 843 | 2.9258276 | 514 | 888 | 2.9484130 | 488 |
| 844 | 2.9263424 | 514 | 889 | 2.9489018 | 488 |
| 845 | 2.9268567 | 513 | 890 | 2.9493900 | 487 |
| 846 | 2.9273704 | 513 | 891 | 2.9498777 | 487 |
| 847 | 2.9278834 | 512 | 892 | 2.9503649 | 486 |
| 848 | 2.9283959 | 511 | 393 | 2.9508515 | 486 |
| 849 | 2.9289077 | 511 | 894 | 2.9513375 | 485 |
| 850 | 2.9294189 | 510 | 895 | 2.9518230 | 485 |
| 851 | 2.9299296 | 510 | 896 | 2.9523080 | 484 |
| 852 | 2.9304396 | 509 | 897 | 2.9527924 | 483 |
| 853 | 2.9309490 | 508 | 898 | 2.9432763 | 483 |
| 854 | 2.9314579 | 508 | 899 | 2.9537597 | 482 |
| 855 | 2.9319661 | 507 | 900 | 2.9542425 | 482 |
| 856 | 2.9324738 | 507 | 901 | 2.9547248 | 481 |
| 857 | 2.9329808 | 506 | 902 | 2.9552065 | 481 |
| 858 | 2.9334873 | 505 | 903 | 2.9556878 | 480 |
| 859 | 2.9339932 | 505 | 904 | 2.9561684 | 480 |
| 860 | 2.9344985 | 504 | 905 | 2.9566486 | 479 |
| 861 | 2.9350032 | 504 | 906 | 2.9571282 | 479 |
| 862 | 2.9355073 | 503 | 907 | 2.9576073 | 478 |
| 863 | 2.9360108 | 502 | 908 | 2.9580858 | 478 |
| 864 | 2.9365137 | 502 | 909 | 2.9585639 | 477 |
| 865 | 2.9370161 | 501 | 910 | 2.9590414 | 477 |

| No. | Logarithms. | Diff. | No. | Logarithms. | Diff. |
|---|---|---|---|---|---|
| 911 | 2.9595184 | 476 | 956 | 2.9804579 | 454 |
| 912 | 2.9599948 | 476 | 957 | 2.9809119 | 453 |
| 913 | 2.9604708 | 475 | 958 | 2.9813655 | 453 |
| 914 | 2.9609462 | 474 | 959 | 2.9818186 | 452 |
| 915 | 2.9614211 | 474 | 960 | 2.9822712 | 452 |
| 916 | 2.9618955 | 473 | 961 | 2.9827234 | 451 |
| 917 | 2.9623693 | 473 | 962 | 2.9831751 | 451 |
| 918 | 2.9628427 | 472 | 963 | 2.9836263 | 450 |
| 919 | 2.9633155 | 472 | 964 | 2.9840770 | 450 |
| 920 | 2.9637878 | 471 | 965 | 2.9845273 | 449 |
| 921 | 2.9642596 | 471 | 966 | 2.9849771 | 449 |
| 922 | 2.9647309 | 470 | 967 | 2.9854265 | 448 |
| 923 | 2.9652017 | 470 | 968 | 2.9858754 | 448 |
| 924 | 2.9656720 | 469 | 969 | 2.9862238 | 447 |
| 925 | 2.9661417 | 469 | 970 | 2.9867717 | 447 |
| 926 | 2.9666110 | 468 | 971 | 2.9872192 | 447 |
| 927 | 2.9670797 | 468 | 972 | 2.9876663 | 446 |
| 928 | 2.9675480 | 467 | 973 | 2.9881128 | 446 |
| 929 | 2.9680157 | 467 | 974 | 2.9885590 | 445 |
| 930 | 2.9684829 | 466 | 975 | 2.9890046 | 445 |
| 931 | 2.9689497 | 466 | 976 | 2.9894498 | 444 |
| 932 | 2.9694159 | 465 | 977 | 2.9898946 | 444 |
| 933 | 2.9698816 | 465 | 978 | 2.9903389 | 443 |
| 934 | 2.9703469 | 464 | 979 | 2.9907827 | 443 |
| 935 | 2.9708116 | 464 | 980 | 2.9912261 | 443 |
| 936 | 2.9712758 | 463 | 981 | 2.9916690 | 442 |
| 937 | 2.9717396 | 463 | 982 | 2.9921115 | 442 |
| 938 | 2.9722028 | 462 | 983 | 2.9925535 | 441 |
| 939 | 2.9726656 | 462 | 984 | 2.9929951 | 441 |
| 940 | 2.9731279 | 461 | 985 | 2.9934362 | 440 |
| 941 | 2.9735896 | 461 | 986 | 2.9938769 | 440 |
| 942 | 2.9740509 | 460 | 987 | 2.9943172 | 439 |
| 943 | 2.9745117 | 460 | 988 | 2.9947569 | 439 |
| 944 | 2.9749720 | 459 | 989 | 2.9951963 | 438 |
| 945 | 2.9754318 | 459 | 990 | 2.9956352 | 438 |
| 946 | 2.9758911 | 458 | 991 | 2.9960737 | 438 |
| 947 | 2.9763500 | 458 | 992 | 2.9965117 | 437 |
| 948 | 2.9768083 | 457 | 993 | 2.9969492 | 437 |
| 949 | 2.9772662 | 457 | 994 | 2.9973864 | 436 |
| 950 | 2.9777236 | 456 | 995 | 2.9978231 | 436 |
| 951 | 2.9781805 | 456 | 996 | 2.9982593 | 435 |
| 952 | 2.9786369 | 455 | 997 | 2.9986952 | 435 |
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| 954 | 2.9795484 | 454 | 999 | 2.9995655 | 434 |
| 955 | 2.9800034 | 454 | 1000 | 3.0000000 |  |

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