TABLES
FOR
DETERMINING THE LATITUDE
BY THE SIMULTANEOUS
ALTITUDES OF
TWO STARS.

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FOR FACILITATING THE DETERMINATION OF THE

LATITUDE AT SEA

BY

THE SIMULTANEOUS ALTITUDES OF TWO STARS.

BY

CHARLES F. A. SHADWELL, Esq. F.R.A.S.

COMMANDER, ROYAL NAVY;

AUTHOR OF "TABLES FOR FACILITATING THE APPROXIMATE PREDICTION OF OCCULTATIONS AND ECLIPSES FOR ANY PARTICULAR PLACE."

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PREFACE.

In offering to the Naval Profession a new collection of Rules and Tables for facilitating the Determination of the Latitude at Sea, by the observed Altitudes of Two Stars, the Author feels that some explanation is necessary of the reasons which have induced him to substitute the title "Simultaneous Altitudes" in lieu of the old established one "Double Altitudes," which seamen have hitherto been accustomed to.

Objections have been raised to the use of the term "Double Altitudes," when applied to the case of the observations of two different bodies taken at the same time, since it is argued that the term *double*, meaning *twice the same*, can only with propriety be used with reference to the observations of the same body made at different times. The phrase "Combined Altitudes" has been suggested as a substitute.

Admitting, to their fullest extent, the force of these criticisms, the Author has been induced to adopt the term "Simultaneous Altitudes," which not only expresses the idea of combination, but also indicates the essential feature of this mode of obtaining the Latitude, viz. the union of the observations at the same instant.

To these reasons may be added the consideration that, as the process of calculation for the Latitude by the "Simultaneous Altitudes" of two Stars possesses few points in common with those developed in the corresponding reduction of "Double Altitudes" of the Sun, the selection of a distinct title, appropriate to the particular character of the observation, is better than allowing a comparatively new kind of computation to remain associated with an old name, which, to the minds of many, only recalls recollections of laborious and oftentimes unsatisfactory calculations.

Advantage has been taken of the present opportunity to introduce to the notice of navigators a new method of computation, which, from its brevity and conciseness, possesses many points of advantage over the old method, by the direct processes of spherical trigonometry; but since, as an eminent astronomical writer has observed, "the substitution of a new method a little more simple and somewhat less long forms no good reason for getting rid of the old one, and for disturbing the technical memory of those who work by fixed rules," the Author has re-computed the Tables for the solution of the problem by the old method also.

All the examples given in illustration of the rules are founded on actual observations made at sea, the dates of the years having been in some cases altered in order to afford an exercise in the use of the Tables.

The Author trusts, in conclusion, that an increasing appreciation of the advantages of stellar observation may procure for these pages a favourable reception. The records of recent disasters at sea tend to shew, if proof were still wanting, the advantages which might arise from careful observations of the stars; but since confidence in the hour of uncertainty cannot be improvised for the occasion, and can only be the result of habitual practice, it is much to be desired that young officers should lose no opportunity of acquiring proficiency in this very essential, but too often neglected, branch of Nautical Astronomy.

Royal Naval College, Portsmouth,
March 1849.

LATITUDE

BY

SIMULTANEOUS ALTITUDES.

ON THE DETERMINATION OF THE LATITUDE AT SEA, BY THE ALTITUDES OF TWO STARS OBSERVED SIMULTANEOUSLY.

Introductory Observations.

THE necessity of providing the seaman with a substitute for the meridional observation at noon for the determination of his latitude, in case of failure from cloudy weather, or other causes, seems to have engaged the attention of mathematicians and astronomers from an early period in the history of modern navigation.

The method of determining the latitude by two altitudes of the sun and the time elapsed between them was first proposed by Mr. Robert Hues, in his treatise on the globes published in 1594, and for many years the ingenuity of mathematicians was directed towards the perfection of the "Double Altitude" problem, with a view to reducing it to such a form as would be suited to the practical wants of seamen.

Among those who have taken part in these discussions we may mention Douwes, Maskeleyne, Lax, Ivory, Brinkley, &c.* the results of whose labours, in the shape of tables of

* Douwes was the author of an ingenious approximate method of solving this problem, which he published in 1754, in the "Actes de l'Académie de Haarlem." The rules given in Moore's, Mackay's, and Norie's treatises on navigation are founded on his investigations, and the special tables for facilitating the computation, given in those works, and which are also to be found in the "Requisite Tables" published by

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"Half-elapsed Time," "Middle Time," "Log Rising," &c., although now somewhat out of date, are probably familiar, at least in name, to many of our readers.

The celebrated astronomer, Dr. Brinkley, appears to have been the first who considered this problem with reference to its applicability to the case of the fixed stars, and in the year 1825 he published, in the Appendix to the "Nautical Almanac," "A convenient practical Rule for computing the Latitude at Sea from the Altitudes of two Fixed Stars observed at the same time." The solution of the problem was facilitated by a table of certain arcs for given pairs of stars, depending on their right ascensions and declinations, and, therefore, constants for those stars.

In the Appendix to the "Nautical Almanac" for 1829, Mr. Lax published a similar table of auxiliary quantities for several pairs of stars, with the necessary rules for the computation of the latitude by their means.

Lastly, the author of these pages, in the year 1836, published a set of tables for the same purpose, in which the arcs were to be applied to a direct method of solution, according to the principles of Dr. Inman's rule.

Many years' experience of the convenience and utility of this method of determining the latitude at sea, obtained in different parts of the world, and under a variety of circumstances, has convinced the author that, although this kind of observation has hitherto found favour among seamen in but a partial degree, it is only because its merits are too little known, and that its peculiar advantages fully justify its being again pressed upon the notice of naval officers and others who are interested in the application of astronomy to the purposes of navigation. Among the advantages attending this method of obtaining the latitude by the simultaneous observation of two fixed stars we would especially enumerate,—

Maskeleyne, were first computed by Admiral Campbell. Lax also gave similar tables in his work.

An ingenious method of solving the problem, without reference to the latitude by account, was given by Mr. Ivory in the "Philosophical Magazine" for August 1821, and subsequently improved by Mr. Riddle (see the same work for September 1822). Mr. Riddle's improvement on Ivory's solution is the basis of the rule given in Riddle's, Norie's, and Galbraith's works.

First, that the method being independent of the necessity for using a chronometer for noting the times of the observation,* both the observation itself, and also the subsequent computations, are much simplified.

Secondly, the time when it is necessary to take the altitudes of the stars not being limited to any precise moment, as in the case of their meridional culminations, the observer is frequently enabled so to arrange his operations as to ensure the concurrence of circumstances favourable to the accuracy of the result.

Thirdly, the great precision with which the observations can be made at morning and evening twilights, at which time the stars are clearly visible, and the sea horizon is plainly defined. The facility and accuracy with which the altitudes of the stars can be taken at morning and evening twilights was quoted by Brinkley as a special recommendation of this method, and few seamen are perhaps fully aware of the elegance and precision of twilight observations.

These advantages are independent of those arising generally from the greater brevity and conciseness of the calculations, as compared with those required in the ordinary form of "Double Altitude" computation; a recommendation which, although of a lower order than those mentioned above, is by no means altogether to be disregarded.

Meridian altitudes of the fixed stars are no doubt to be preferred to any other less direct method of deducing the latitude from stellar observations, whenever opportunities for taking them occur; but the limitation of the times of taking them to precise moments, when perhaps clouds may accidentally intervene and prevent the observation, coupled with the remarkable fact, that the conspicuous bright stars suited for use at sea are very unequally distributed in the heavens, being crowded together in some parts and separated by a wide interval of right ascension in others, very much circumscribes the practical application of this method. Under these circumstances, the method of simultaneous observation of two stars

* Except when the altitudes of both stars are taken by the same observer, when the time is required to be noted, for the purpose of reducing the altitudes to what they would have been if observed at the same instant (see Examples, pp. 13 and 23).

steps in as a valuable auxiliary to the seaman, giving him a choice of convenient objects, and the power of repeating the observation, when desirable, by independent couplets, whereby the errors of the results are neutralised (more especially if one pair of stars be observed north and the other south of the zenith). It is, moreover, always available whenever the distinctness of the sea horizon permits observations to be taken at all.

On the importance of being able to have recourse to an easy and expeditious mode of determining the ship's position in latitude, whether before the setting in of darkness or after the termination of the night, when navigating in a dangerous sea, expecting to make the land, or when approaching a port, it is needless further to expatiate. Every seaman must at once admit its value, and allow that any method of obtaining that knowledge which is readily applicable to the wants of practice, must be of the highest utility.

Impressed with these convictions, we consider that no apology is necessary for again offering to the naval profession a set of convenient rules and tables for attaining this end, adapted to the present epoch, and illustrated with examples exhibiting their practical application.

We propose therefore to give rules by two methods for reducing the observations, and thence determining the latitude.

The first method is the same as that formerly pursued by the Author in his pamphlet in 1836, adapted to a direct method of computation by the solution of two cases of spherical trigonometry, following out the method given by Dr. Inman.

Those who are in the habit of using Dr. Inman's "Navigation," and acquainted with his rules, will find this first method well suited to their previous habits, as they have but little new to learn, and nothing to unlearn.

Table I. will give them by inspection Arcs 1 and 2, and the remainder of the computation is analogous to what they have been already accustomed to in the computation of double altitudes of the sun.

Those who have not previously wedded themselves to any particular mode of calculation, will find the second method well worthy of their attention.

Founded on a very elegant investigation, the leading idea of which is derived from Mr. Ivory's solution, the formulæ of computation afford some peculiar facilities for calculation, which we think are likely to recommend them to the favourable consideration of intelligent navigators.

The auxiliary quantities to be used in connexion with this method will be found in Table II.

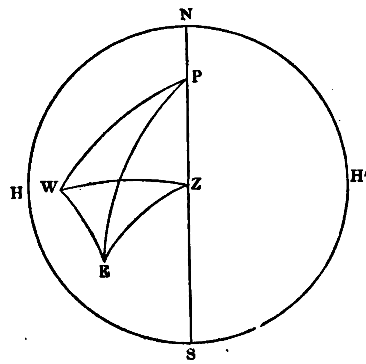
The demonstration of the mathematical principles on which the Rules and Tables are founded is given in a separate form at the close of this work, so as not to interfere with the purely practical part of the book, or to encumber the text with lengthy explanations.

With these preliminary observations we shall now at once proceed to the practical discussion of the problems.

On the Determination of the Latitude at Sea by the Altitudes of two Stars observed simultaneously.

Let N H S H′ (*Fig.* 1) be the rational horizon, N S the meridian, Z the zenith, and P the pole of the heavens.

Fig. 1.

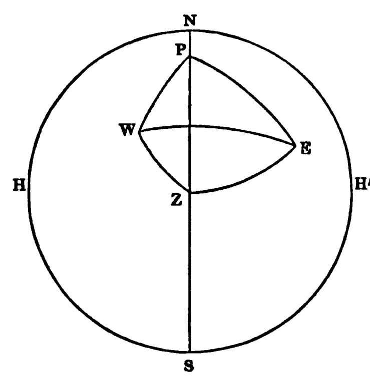


W and E the places of two fixed stars.

Then, supposing the altitudes of the stars W and E to be observed simultaneously, we know Z W and Z E their zenith distances by the observation; also P W and P E their polar distances, as well as W P E the polar angle, or difference of right ascensions, are known from the "Nautical Almanac."

Hence, in the triangle W P E, knowing the angle W P E and the two sides P W and P E, we can compute, by the direct processes of spherical trigonometry, the side W E (the distance arc between the two stars), or Arc 1, and also the angles P W E and P E W, or Arc 2, which quantities being constant, or nearly so, may be previously computed for any given pairs of stars, and arranged for use in a table such as Table I.

Fig. 2.



Again, in the triangle Z W E, knowing the three sides, we can find the angles Z W E and Z E W, or Arc 3.

Then the difference or sum of the angles, P W E and Z W E, or of P E W and Z E W (using always the angles at that star which is the farthest from the meridian), will give the angle P W Z, or P E Z, or Arc 4. The difference or sum of Arcs 2 and 3 being taken according to the position of the stars with reference to the pole and zenith (see *Figs.* 1 and 2).* Lastly, in the triangle P W Z, knowing two sides, P W and Z W, and the included angle P W Z; or in the triangle P E Z, knowing P E and Z E, and the angle P E Z, we can find the remaining side, P Z, the co-latitude.

Such is the principle of the solution on which this method of computing the latitude depends; the management of the observation and the subsequent calculations must be guided by the following rules.

On the Selection of Stars for the Determination of the Latitude by "Simultaneous Altitudes," and the Limits within which the Observations should be taken.

In selecting a couplet of stars for simultaneous observation, it will not be proper to take those which at their meridian passage pass very near the zenith, or which pass the meridian the one north and the other south of the zenith.

The stars ought to pass the meridian either *both* between the zenith and the elevated pole, or *both* outside the zenith and the elevated pole.†

The altitudes of the two stars should neither of them be less than 8° or 10°, on account of the errors of observation arising from the uncertainty of refraction at low altitudes.

* *Fig.* 1 represents the case in which the stars are both on the same side of the meridian, W being the star farthest from the meridian, and the stars passing the meridian outside the zenith and the pole. *Fig.* 2 exhibits a case when the star E is farthest from the meridian, and where the stars pass the meridian between the pole and the zenith. In the former case Arc 4 is the difference of Arcs 2 and 3, in the latter their sum.

† For instance, in the case of example No. 1, p. 11, the latitude by account being 50° N., *Arcturus* and *Altair* (No. 17) were a proper couplet to select for observation, because the declinations of the two stars being respectively about 20° N. and 8° N., neither of them at their meridian transit would pass very near the zenith.

But in that latitude *α Cassiopeiæ* and *Capella* would not be suitable for observation, since these stars, having respectively their declinations 56° N. and 46° N., would at their meridian passages be very near the zenith, the one to the northward and the other to the southward.

Again, in example No. 2, p. 12, *α Eridani* and *α Argûs* (No. 4) were properly chosen for observation, because the declinations of these stars being respectively 58° S. and 53° S., they would pass the meridian between the zenith and the south pole, and

When the stars are *both* east or *both* west of the meridian, the difference of bearing of the two bodies, reckoned from the same point, north or south, should exceed the less bearing.

When the stars are on different sides of the meridian—that is, the one on the east and the other on the west—what this difference of bearing wants of 180° or 16 points should exceed the less bearing, and in *both* cases the *greater* the excess the better.*

The seaman may in most cases determine with sufficient accuracy by the eye when the two bearings fall within the above limits. If the bearings are estimated by the compass a rough allowance should be made for the variation, so as to get the true bearings.

Taking the Observations.

If there are two observers, the altitudes of both stars are to be taken at the same instant; if the observations are all taken

would not at their transits approach near the zenith, the latitude of the observer being 24° S.

But suppose that on that occasion *Antares* and *Fomalhaut* (No. 19), whose declinations are 26° S. and 30° S. respectively, had been proposed for observation, this would have been an objectionable pair of stars to choose, since both of them at their meridian culmination would pass very near the zenith.

A little consideration of the relative positions of the stars with respect to the zenith, at the time of their culminations, and attention to the limits of bearing mentioned in the text, will soon enable the observer to perceive what couplets of stars should be selected for observation, in any given position of geographical latitude, and what should be rejected.

* For instance, suppose the bodies, being both on the same side of the meridian, the one bears S. by W., and the other W. S. W., this would be a good case, because the difference of bearing, five points, exceeds the less bearing, one point.

Again, suppose one star to bear N. W. by W., and the other N. N. E., this is also a good case, because the supplement of the difference of bearings, nine points, exceeds the less bearing, two points.

But if one star should bear S. E. and the other E. by S., this would be an objectionable case, because the difference of bearing, three points, is less than the least bearing, four points.

And again, the bodies being on different sides of the meridian conceive the one to bear E. S. E. and the other S. W. by W.; this also would not be an advantageous condition for observation, since the supplement of the difference of bearings or five points does not exceed the less bearing, which is also five points.

These are the limits of observation laid down by Dr. Inman, and investigated in the Appendix to his "Navigation," p. 23.

by one observer, which is far preferable, the corresponding times must be noted as follows:—

Let the observer take the altitude of the star which is farthest from the meridian* (and whose motion in altitude will therefore generally be the quickest), noting the time by a good pocket watch (with a second hand), then the altitude of the other star, and lastly that of the first one again, noting the times.

Then the change of altitude of the first star being proportional to the time elapsed in the interval, its altitude, when the second star was taken, may be found by direct proportion, in the same manner as the altitudes of the sun and moon are reduced to the time corresponding to the distance in a lunar observation, when both the altitudes and the distance are taken by one observer, a process with which most seamen are familiar.

This reduction can be conveniently effected by the aid of a table of proportional logarithms, and the change of altitude thus found is to be *added* to the altitude of the first star when it is *increasing*, and *subtracted* when *decreasing*, and it will give the altitude corresponding to the time when the other star was observed. (See Example 3, p. 13, and Example 3, p. 23.)

The altitudes thus found are to be corrected for index error, dip, and refraction, and to be subtracted from 90°, to give their true zenith distances.

Computation of the Latitude.

Method the First.

Take from Table I., for the particular couplet of stars which have been observed, Arc 1, and also the Arc 2, and

* Sometimes, when the state of distinctness of the sea horizon under the one star happens to be decidedly more favourable for accuracy of observation than under the other, it may be advisable to take the altitude of that star the first which is so situated, without reference to its position relative to the meridian, and afterwards reduce its altitude, as explained above, to the time of observation of the other star; but if no such reason exists for a preference, it will be better to observe the altitudes of the stars in the manner pointed out in the text.

the polar distance* belonging to the star whose bearing from the meridian is the greatest. If the stars should be on opposite sides of the meridian, and their bearings should happen to be equal, either body may be assumed as that of greater bearing, and the computation be made on that supposition accordingly. Correct these quantities when necessary for annual variation: note down also the sign of application for Arc 4, according to the position of the stars with respect to the zenith.

Then, under Arc 1 put the zenith distance at the greater bearing, and take their difference, under which place the zenith distance at the less bearing, take their sum and difference, half sum and half difference.

Then, to the *log cosecants* of the two first terms in this form add the *log sines* of the two last. The sum, rejecting the tens from the index, will be the *log haversine* of Arc 3.

Take the sum or difference of Arcs 2 and 3, according as the sign of application given in the table is + or —, and call it Arc 4.

Lastly, to the *log haversine* of Arc 4, add the *log sines* of the polar and zenith distances at the greater bearing, rejecting the tens in the index of the sum. With the result as a *log haversine* take out the arc in degrees. To the *natural versine* of this arc, add the *natural versine* of the difference between the polar and zenith distances at greater bearing. The sum will be the *natural versine* of the colatitude, which take from the tables; if greater than 90° subtract 90° from it, and if less than 90° subtract it from 90°; the result will be the latitude, of the same, or a contrary name to the polar distances of the stars observed, according as the arc found from the table of *versines* is less or greater than 90°.

* The polar distances of the stars are in all cases to be the complements of their declinations (like as they are given in Table I.), and they are *not* to be reckoned from the elevated pole in cases where the latitude of the observer and the declinations of the stars of different names.

This arrangement is necessary in order to avoid a distinction of cases in the precepts of the rule.

Example, No. I.

On July 5th, 1850, at $9^h\ 30^m$ P.M., the following observations were made for the determination of the latitude:—

Obs. Alt. *Arcturus*.. 50° 34′ 0″ True Bearing .. S.W. by W.
Obs. Alt. *Altair* .. 30 55 0 True Bearing .. S.E. by E. ½ E.
Index error —1′ 30″. Height of the eye above the sea, 18 feet.

Correcting the altitudes, we have—

	°	′	″
Zen. Dist. of *Altair* at G.B.	59	12	20
Zen. Dist. of *Arcturus* at L.B.........	39	32	30

Also, from Table I. (No. 17),

Arc 1	81	12	2
Arc 2 (*Altair*)	70	57	26
Pol. Dist. at G.B.	81	31	26 N.

Hence,

Diff. Pol. and Zen. Dist. at G.B.	22	19	6

Sign of application for Arc 4 —, the stars being south of the zenith.

To find Arc 3.

° ′ ″		
81 12 2	Cosec...	0·005143
59 12 20	Cosec...	0·066008
21 59 42		
39 32 30		
61 32 12		
17 32 48		
30 46 6	Sine....	9·708882
8 46 24	Sine....	9·183425
	Haversine	8·963458
	Arc 3 ..	35° 18′ 0″

	° ′ ″
Arc 3	35 18 0
Arc 2	70 57 26
Arc 4	35 39 26

Hav. Arc 4	8·971953
Sine P.D. at G.B.	9·995231
Sine Z.D. at G.B.	9·933992
Haversine	8·901176
	32° 47′ 0″

Nat. Vers. (32° 47′ 0″)	0159276
Nat. Vers. {Diff. P. and Z. D.	0074901
Nat. Vers. {At G. B.........	11
Nat. Vers.	0234188
	40° 1′ 14″
Latitude	49 58 46 N.

Example, No. II.

On June 2nd, 1848, in lat. by account 23° 50′ S., the following altitudes were observed at the same time: required the true latitude.*

True Alt. α *Eridani* ..	52° 3′ 0″	Bearing .. S. by W.
True Alt. α *Argûs*	43 12 15	Bearing .. S.E.

Here, correcting the quantities from Table I. (No. 4), for two years' annual variation we get,

	° ′ ″
Arc 1	39 24 46
Arc 2 (α *Argûs*)	52 34 38
Polar Dist. at G. B.................	37 23 8 S.

Also,

Zen. Dist. at G.B.	46 47 45
Diff. Polar and Zenith Dist. at G. B...	9 24 37
Zen. Dist. at L.B.	37 57 0

Sign of application for Arc 4 +, the stars being south of the zenith.

° ′ ″			° ′ ″
39 24 46	0·197295	Arc 3........	55 52 30
46 47 45	0·137321	Arc 2........	52 34 38
7 22 59	9·585877	Arc 4........	108 27 8
37 57 0	9·420933		
45 19 59	9·341426		9·818406
30 34 1			9·783334
22 39 59			9·862679
15 17 0			9·464419
			65° 20′ 15″

	0582661
	66
	0013428
	28
	0596183
	66° 10′ 51″
Latitude ..	23 49 9 S.

* This example is taken from the Royal Naval College Examination Paper. Questions set, Oct. 9th, 1848.

Example, No. III.

On September 22nd, 1855, at $5^h\ 15^m$ A.M., the following observations for the determination of the true latitude were made by one observer :—

Times by Watch.	Obs. Alt. *Procyon* bearing N.E.	Obs. Alt. *Aldebaran* bearing N. by W. ½ W.
h m s	° ′ ″	° ′ ″
5 9 19	36 53 40	
5 10 0		34 50 10
5 12 38	37 18 0	

Index error + 3′ 50″. Height of the eye above the sea, 13 feet.

Here we have for the reduction of the altitude of *Procyon*,

$3^m\ 19^s : 0^m\ 41^s :: 24'\ 20'' :$ Change of Alt. in $0^m\ 41^s$

and using proportional logarithms,

Prop. Log.	$0^m\ 41^s$	2·42064
Prop. Log.	24′ 20″	0·86907
		3·28971
Prop. Log.	$3^m\ 19^s$	1·73457
Prop. Log.		1·55514 .... 5′ 0″

Hence 5′ 0″ is the required reduction to be added to the first altitude of *Procyon*, 36° 53′ 40″, because its altitude is increasing.

Hence we obtain 36° 58′ 40″ as the altitude of *Procyon* corresponding to the time by watch ($5^h\ 10^m\ 0^s$), when the altitude of *Aldebaran* was observed.

Correcting the altitudes, we obtain,

	° ′ ″
Zen. Dist. at G. B.	53 2 17
Zen. Dist. at L. B.	55 10 53

And from Table I. (No. 5), we have, correcting the quantities for six years' annual variation,

	° ′ ″
Arc. 1	46 18 36
Arc 2 (*Procyon*)	72 52 12
Pol. Dist. at G. B.	84 24 34 N.

Hence,

Diff. Pol. and Zen. Dist. at G. B.	31 22 17

Sign of application for Arc 4 +, the stars being north of the zenith.

To find Arc 3.

° ′ ″				
46 18 36	0·140821			
53 2 17	0·097437			
6 43 41	9·711261			° ′ ″
55 10 53	9·613124			
61 54 34	9·562643	Arc 3	......	74 22 15
48 27 12		Arc 2	......	72 52 12
30 57 17		Arc 4	......	147 14 27
24 13 36				

9·963996
9·997928
9·902563
9·864487
117° 38′ 30″
1463812
129
0146146
42
1610129
127° 35′ 56″
Latitude .. 37 35 56 S.

Computers who are not in the habit of using a table of *log haversines* may, instead of following the preceding rule, proceed as follows:—

Take from Table I. Arc 1, and the other quantities, as directed in the preceding rule.

Then, under Arc 1 place the zenith distance at greater bearing and take their difference, under which put the zenith distance at the less bearing, take their sum and difference, half sum, and half difference.

Then, to the *log cosecants* of the two first terms in this form (rejecting the tens from the indices) add the *log sines* of the two last; half the sum will be the log sine of *half* Arc 3.

Under this place *half* Arc 2; their sum or difference, according to the sign of application given in Table I., will be *half* Arc 4.

Lastly, to the *constant logarithm* 6·301030 add twice the *log sine* of *half* Arc 4 and the *log sines* of the polar and zenith distances at the greater bearing. Reject the tens from the index of the sum, and take out the *natural number* of the resulting logarithm, to which add the *natural versine* of the difference of the polar and zenith distances at the greater bearing. The sum will be the *natural versine* of the true colatitude, or of the supplement of the true colatitude, which take from the tables.

If less than 90° subtract it from 90°, if greater reject 90° from it. The result will be the true latitude, of the same or a contrary name to the polar distances of the stars observed, according as the arc found from the table of *versines* is less or greater than 90°.

Observation.—Sometimes, in high latitudes, observations may be made of stars when they are below the pole of the heavens; that is to say, the stars being circumpolar, or nearly so, at the place of observation, they may be in such a part of their diurnal course that their altitudes are less than the altitude of the pole (or the latitude of the observer): in this case the sign of application for Arc 4 is to be reversed.*

The following example will illustrate both this peculiarity and also the above modification of the preceding rule.

Example, No. IV.

On November 5th, 1852, at 8^h 15^m P.M., the true altitudes of *Canopus* and *α Crucis*, observed under the south pole, were as follows:—

True Alt. *Canopus*....	21° 46′ 30″	Bearing S.E. ½ S.
True Alt. *α Crucis* ..	14 16 50	Bearing S. by W.

Required the true latitude.

* In making an observation of a circumpolar couplet of stars, it will be advisable, in order to avoid any possibility of an error in the sign of application for Arc 4, only to observe the stars when the position in altitude of *both* of them, with respect to the pole, is either decidedly *above* or decidedly *below* it. Particular attention should likewise be paid to the limits of bearing mentioned in pp. 7 and 8.

Also, by drawing a figure to represent the circumstances of the case, after the manner of *Figs.* 1 and 2, p. 6, the observer will easily see how the combination of Arcs 2 and 3 is to be effected to produce Arc 4.

Here we have, from Table I. (No. 11), correcting the quantities for three years' annual variation,

	°	′	″
Arc 1	45	4	32
Arc 2 (*Canopus*)	41	3	39
Pol. Dist. at G. B...........	37	22	59 S.

Also,

Zen. Dist. at G. B...........	68	13	30
Diff. Pol. and Zen. Dist. G. B.	30	50	31
Zen. Dist. at L. B...........	75	43	10

Sign of application for Arc 4, from the table, +, the stars being south of the zenith; but this sign is to be reversed, since the stars are under the pole.

To find Arc 3.

° ′ ″			
45 4 32	0·149947		
68 13 30	0·032149		
23 8 58	9·880613		
75 43 10	9·646218		
98 52 8	19·708927		° ′ ″
52 34 12	9·854463	½ Arc 3 ..	45 39 45
49 26 4		½ Arc 2 ..	20 31 49
26 17 6		½ Arc 4 ..	25 7 56
	6·301030		
	9·628109		
	9·628109		
	9·783292		
	9·967851		
	5·308391		203419
			141338
			76
			344833
			49° 4′ 4″
		Latitude	40 55 56 S.

On the Determination of the Latitude at Sea by the Altitudes of two Stars observed Simultaneously.

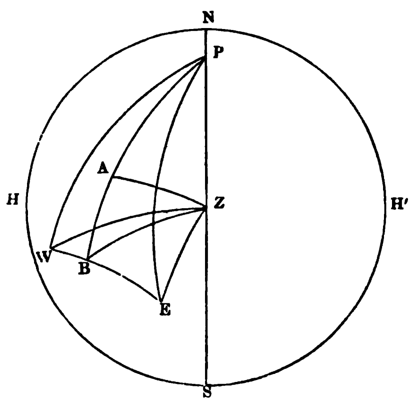
Method the Second.

We shall now proceed to explain the general principles on which the determination of the latitude by "Simultaneous Altitudes" by the second method depends.

Let N H S H′ be the rational horizon, N S the meridian, P the pole of the heavens, and Z the zenith of the spectator.

Let W and E be two fixed stars, W the *western* and E the *eastern* star, Then, supposing their altitudes to be observed simultaneously, we know Z W and Z E from the observation; also P W and P E their polar distances, as well as W P E their polar angle, or difference of right ascension, are known from the "Nautical Almanac."

Fig. 3.



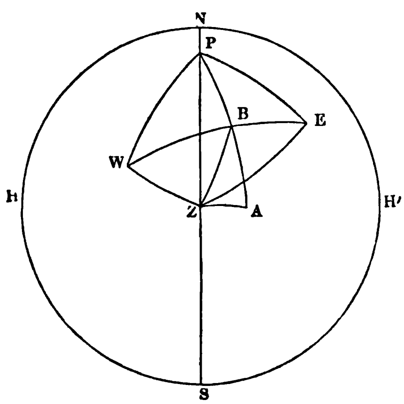
Let W E be the arc of a great circle joining the places of the two bodies, and conceive P B to be an arc of a great circle drawn from the pole, perpendicular to W E, and cutting it in B.

Also let Z A be the arc of another great circle drawn from the zenith perpendicular to the arc P B, *Fig.* 3, or to P B produced, *Fig.* 4, and cutting it in A. Also let Z B be the zenith distance of of the point B.

Now, by the relations subsisting between the spherical triangles formed by these arcs, it is easy, as will be shewn hereafter, to obtain expressions for computing P B, B A, and Z A.

The difference or sum of P B, and B A, according to the position of the stars with reference to the pole and zenith, as shewn in *Figs.* 3 and 4,* will give P A.

Fig. 4.



Then in the right-angled triangle P A Z, two sides P A and Z A being known, the third side P Z, the co-latitude, can be determined.

Such are the leading features of this method of determining the latitude; with reference to its practical application the seaman must attend to the following rules.

On the Selection of Stars for the Determination of the Latitude by "Simultaneous Altitudes," the Limits within which the Observations should be taken, and on making the Observations.

All that we have stated on this head as a preliminary to the first method of computation† applies equally to this method also. We would only remark in addition, that although in making the computation (as will be seen further on) there is no distinction to be attended to between the star of greater or less bearing from the meridian, as in the former case, yet, that in selecting stars for observation, attention to the limits of bearing under which they may be advantageously observed is equally important in this as in the former method.

* *Fig.* 3 represents the case in which the bodies W and E pass the meridian outside the pole and zenith; *Fig.* 4 that in which they pass the meridian between the pole and zenith. In the former case P A is the difference between P B and B A; in the latter their sum.

In the rule for the computation P B is called Arc 1, B A Arc 2, and P A Arc 3.

† See pp. 7 and 8.

It is for this reason that in stating the data of the subsequent examples we have been careful to record the circumstances of bearing from the meridian under which the altitudes of the stars were taken, in order that the attention of observers might be especially directed to the necessity of strictly attending to them.

Computation of the Latitude.

Method the Second.

Correct the observed altitudes for index error, dip, and refraction, and thus determine the true altitudes. Mark the true altitude of the *western* star (W) and that of the *eastern* star (E).*

Take from Table II., for the particular couplet of stars which have been observed, Arc 1, the natural numbers A and B, and the logarithms C and D.

Correct these quantities, when necessary, for annual variation. If the number B has an asterisk (*) prefixed, mark it down; and note also the sign of application for Arc 3 (according to the position of the stars observed with respect to the zenith).

Mark down headings (1), (2), and (3).

Under (1) place the *log sine* of the true altitude of the *eastern* star, increasing the index by 2; under (2), *log.* C; under (3), *log.* D, and under (1), (2), and (3), the log sine of the true altitude of the *western* star.

Take the difference between the logarithms under (1), and find the *natural number* of the resulting logarithm, which call N.

Always take the *difference* between N and A; and take the *sum* of N and B, except in those instances where B is marked with an asterisk (*) in the table: in which case their difference is to be taken instead.

* In Table II. the *western* star of each couplet is marked (W), and the *eastern* (E). Care must be taken to avoid mistakes in noting this, as the correctness of the subsequent computations depends on it.

Place the *logarithm* of the *difference* between N and A under (2), and the *logarithm* of the *sum* or *difference* of N and B under (3).

Add together the logarithms under (2), rejecting the ten from the index of the sum. Look out the resulting logarithm as a *log sine*, and place the corresponding *log secant* under (3).

Add together the logarithms under (3), rejecting the tens from the index of the sum; look out the resulting logarithm as a *log cosine*, and take out the corresponding arc, which call Arc 2.

Take the *sum* or *difference* of Arcs 1 and 2, according as the sign of application for Arc 3 is + or −. Call the result Arc 3.

Lastly, from the *log cosine* of Arc 3 subtract the *log secant* previously used under (3). The resulting logarithm will be the *log sine* of the true latitude, which take from the table and mark *north* or *south* with the same name as the latitude by account.

Observation.—In case this method of computation were employed for the reduction of observations made very near the equator, where the latitude by account might be doubtful, it might be uncertain whether the latitude resulting from the computation should be marked *north* or *south;* nor would it be possible to give any general precept for deciding this point which would not mar the simplicity of the preceding rule, and probably be liable to misapprehension.

Under such circumstances, which would doubtless be of very rare occurrence in actual practice at sea, it would be advisable to have recourse to the mode of computation by method the first, which is free from this ambiguity.

We shall now proceed to give some examples.

Example, No. I.

On November 20th, 1844, the following altitudes of stars were observed at the same time, in latitude by account 36° 46′ N.

Obs. Alt. *Sirius* bearing S.W.	Obs. Alt. *Spica* bearing E.S.E.
27° 50′	12° 56′

The index correction was +0′ 55″, and height of the eye 8 feet. Required the true latitude.*

Correcting the altitudes we have,

	° ′ ″
True Alt. *Sirius* (W)	27 46 15
Spica (E)	12 50 0

And from Table II. (No. 12) correcting the quantities for five years' annual variation,

	Arc 1	69° 15′ 21″	
A	63·23	Log. C	7·906834
B	144·35	Log. D	7·778486

Sign of application for Arc 3 +, the stars being south of the zenith.

(1)		(2)		(3)	
Log. Sine Alt. (E)	11·346579	Log. C. ..	7·906834	Log. D.	7·778486
—— (W)	9·668326	—— ..	9·668326	——	9·668326
	1·678253	Log. 15·56	1·192010	Log. 192·02..	2·283346
		Log. Sine	8·767170	Cor. Log. Sec.	10·000745
				Log. Cos. ..	9·730903

N.	47·67	N.	47·67
A.	63·23	B.	144·35
Diff.	15·56	Sum	192·02

	° ′ ″
Arc 2	57 26 30
Arc 1	69 15 21
Arc 3	126 41 51

Log. Cos. Arc 3..	9·776386
Log. Sec. used under (3)	10·000745
Log. Sine	9·775641

Latitude .... 36° 37′ 30″ N.

* This example is taken from the Royal Naval College Examination Paper. Questions set Oct. 7th, 1844.

Example, No. II.

On September 7th, 1857, at 4^h 30^m A.M., in latitude by account 41° 10′ N., the following simultaneous observations were made for the determination of the true latitude:—

True Alt. *Aldebaran*	(W) 63° 51′ 40″	Bearing S. by E ¼ E.
True Alt. *Pollux*	(E) 43 25 45	Bearing E ½ S.

Here we have from Table II. (No. 6), correcting the quantities for eight years' annual variation,

		Arc 1	61° 11′ 50″
A	170·09	Log. C	7·913550
B*	20·31	Log. D	8·061254

The number B being marked with an asterisk (*), the difference between N and B is to be taken in the computation, instead of their sum as usual.

Also, sign of application for Arc 3 —, the stars being south of the zenith.

	(1)	(2)	(3)
	11·837246	7·913550	8·061254
	9·953150	9·953150	9·953150
	1·884096	1·970858	1·750277
		9·837558	0·139198
N. 76·58	N. 76·58		9·903879
A. 170·09	B. 20·31		
93·51	56·27		

Arc 2	36° 43′ 45″
Arc 1	61 11 50
Arc 3	24 28 5

9·959138
0·139198
9·819940

Latitude .... 41° 20′ 45″ N.

Example, No. III.

On April 21st, 1850, in latitude by account 9° 20′ S., at 9^h P.M., the following observations were made by one observer, for the determination of the true latitude:—

Times by Watch. h m s	Obs. Alt. *Regulus* bearing N.N.W. ° ′ ″	Obs. Alt. *Arcturus* bearing N.E. ½ E. ° ′ ″
9 0 59		35 23 0
9 2 57	62 53 30	
9 5 4		36 12 0

Index error + 3′ 10″. Height of the eye, 17 feet.

Hence for the reduction of the altitude of *Arcturus* we have,

$$4^m\ 5^s : 1^m\ 58^s :: 49'\ 0'' : \text{Change in } 1^m\ 58^s$$

and using proportional logarithms,

Prop. Log.	.. $1^m\ 58^s$	1·96154
——	.. 0° 49′ 0″	0·56508
		2·52662
——	.. $4^m\ 5^s$	1·64426
——		0·88236 .... 23′ 36″ = the

required reduction to be added to the first altitude of *Arcturus,* because the altitude is increasing; whence we obtain 35° 46′ 36″ as its altitude, corresponding to the time by watch ($9^h\ 2^m\ 57^s$), when the other star was observed.

Correcting the altitudes, we have,

	° ′ ″
True Alt. *Regulus* (W)	62 52 6
Arcturus (E)	35 44 20

And from Table II. (No. 14), we have—

		Arc 1	69° 45′ 49″
A	155·34	Log. C	7·866733
B	20·69	Log. D	7·951306

Also, sign of application for Arc 3 +, the stars being north of the zenith.

	(1)	(2)	(3)
	11·766467	7·866733	7·951306
	9·949364	9·949364	9·949364
	1·817103	1·952841	1·936111
		9·768938	0·091905
N. 65·63	N. 65·63		9·928686
A. 155·34	B. 20·69		
89·71	86·32		

	°	′	″
Arc 2	31	56	30
Arc 1	69	45	49
Arc 3	101	42	19

9·307193
0·091905
9·215288

Latitude .... $9^{\circ}\ 27'\ 0''$ S.

We shall conclude these examples by giving one in which the altitudes of the stars were observed under the pole; a peculiarity which renders necessary the reversal of the sign of application for Arc 3 given in the table.*

Taking for this purpose the same example as before given at p. 15, we shall have, moreover, an opportunity of comparing the details of the calculation by the two methods.

Example, No. IV.

On November 5th, 1852, at $8^{h}\ 15^{m}$ P.M., in latitude by account $40^{\circ}\ 50'$ S., the following altitudes, observed simultaneously under the south pole, were taken for the determination of the latitude:—

True Alt. *Canopus* (W)	$21^{\circ}\ 46'\ 30''$	Bearing S. E. ½ S.
True Alt. *α Crucis* (E)	14 16 50	Bearing S. by W.

* See the observation and note, p. 15.

Hence we have from Table II. (No. 11), correcting the quantities for three years' annual variation,

		Arc 1	23° 30′ 13″
A	111·40	Log. C	8·087703
B	52·28	Log. D	7·848226

Also, the sign of application for Arc 3 from the Table is + (the stars being south of the zenith); but since they are also under the pole, this sign is to be reversed, and will become —.

	(1)	(2)	(3)
	11·392075	8·087703	7·848226
	9·569330	9·569330	9·569330
	1·822745	1·652343	2·074707
		9·309376	0·009217
N. 66·49	N. 66·49		9·501480
A. 111·40	B. 52·28		
44·91	118·77		

Arc 2	71° 30′ 0″
Arc 1	23 30 13
Arc 3	47 59 47

9·825546
0·009217
9·816329

Latitude .... 40° 55′ 45″ S.

which result agrees with that obtained by the first method of computation. (*See* p. 16.)

EXPLANATION OF THE TABLES.

TABLE I.

This table contains, for certain pairs of stars situated in relative positions, which are suitable for the determination of the latitude by "Simultaneous Altitudes," the auxiliary quantities required for the reduction of the observations, according to the first method of computation given in the preceding pages.

These quantities are, the polar distances of the stars, the arc measuring the distance between them (or Arc 1), and the angles at each star between its polar distance and the distance arc (or Arc 2).

These several quantities are given for the mean positions of the stars at the epoch, January 1st, 1850.

The annual variations* by which these quantities are to be reduced to their corresponding values for any future year are given in the adjoining columns with their proper signs, + signifying that the corrections are to be added to, and — that they are to be subtracted from, the original quantities.†

In correcting the arcs and polar distances for the reduction of observations made at sea, it will in general be sufficient

* These variations have been obtained by computing the quantities for the mean places on the 1st of January, 1860, and taking the tenths of the differences of their values in 1850 and 1860 for their annual variations.

The mean places for 1860 have been obtained from the standard catalogue for 1840 in the "Nautical Almanac" for 1848, by means of the formulæ of reduction given in the preface to the second edition of the "Nautical Almanac" for 1834, p. xiv.

† If the quantities from the table are required for the reduction of any observations made prior to 1850, the corrections for annual variation must be applied with their signs reversed. (*See* Examples, pp. 14 and 21.)

to correct them for the nearest whole year, and it will not be necessary to take into account the fractional portion of the year elapsed since the 1st of January, except where the variations are large, when the computer must exercise his own judgment on the point.

The limits of accuracy of these auxiliary quantities will always be far within what can reasonably be expected in the other parts of the calculation, which depend on altitudes observed with the sea horizon; they are, therefore, always sufficiently accurate for the determination of the latitude at sea; but since these quantities are computed for, or may be adjusted to, the mean places of the stars only (which may never correspond with their apparent places), they can only be expected, if used for the determination of the latitude on shore by the artificial horizon, to give good approximate results, and should, therefore, never be resorted to when other more accurate means are at hand.

The columns headed "Sign of Application for Arc 4" shew the manner in which the combination of Arcs 2 and 3 is to be effected to obtain Arc 4, which, if the observations have been made in accordance with the principles explained in pages 7 and 8, will depend on the position of the stars north or south of the zenith, and will rarely be ambiguous.

TABLE II.

This table contains, for the same pairs of stars as are given in Table I., the auxiliary quantities required in the second method of computation, which has been just explained.

These quantities are an arc, which we designate Arc 1; two natural numbers, A and B; and two constant logarithms, C and D, all computed for the mean places of the stars on the 1st of January, 1850.

Two other columns also contain the sign of application for Arc 3, which, as in the last table, depends on the position of the stars with respect to the zenith.

The observations just made on Table I., as to the deduction of the annual variations, their application to the correction of the original quantities, and the caution as to the use of the table for the reduction of observations made on shore, apply equally to this table also, and nothing remains to add, but a remark as to the meaning of the asterisks (*) prefixed in some instances to the number B.

When an asterisk (*) is prefixed to the number B, it indicates that in the computation of the latitude by means of these quantities the difference between B and the number N, which comes out in the process of the calculation, is to be taken instead of their sum, as directed by the general rule. The reason for this departure from the usual precept is explained in the demonstration of the formulæ.

EPOCH. TABLE I.

No.	Stars' Names.	Mag.	Polar Distances.	Annual Variation.	Arc 1 (*Distance Arc*).	Annual Variation.
1	α Andromedæ (*Alpheratz*) and α Tauri (*Aldebaran*)	2 1	61° 44′ 16″ N. 73 47 49 N.	−19″·9 − 7·7	62° 9′ 12″	+0″·1
2	α Cassiopeiæ (*Schedar*) and α Aurigæ (*Capella*)	Var. 1	34 17 10 N. 44 9 40 N.	−19·8 − 4·3	42 30 18	+0·3
3	β Ceti and β Orionis (*Rigel*)	2 1	71 11 20 S. 81 37 14 S.	+19·8 + 4·6	66 23 15	−0·2
4	α Eridani (*Achernar*) and α Argûs (*Canopus*)	1 1	32 0 0 S. 37 23 4 S.	+18·4 − 1·8	39 24 46	−0·1
5	α Tauri (*Aldebaran*) and α Canis Minoris (*Procyon*)	1 1	73 47 49 N. 84 23 41 N.	− 7·7 + 8·8	46 18 38	−0·4
6	α Tauri (*Aldebaran*) and β Geminorum (*Pollux*)	1 1·2	73 47 49 N. 61 36 59 N.	− 7·7 + 8·2	45 2 17	−0·6
7	α Aurigæ (*Capella*) and α Ursæ Majoris (*Dubhe*)	1 2	44 9 40 N. 27 26 26 N.	− 4·3 +19·3	49 16 55	+0·2
8	β Orionis (*Rigel*) and α Hydræ (*Alphard*)	1 2	81 37 14 S. 81 59 20 S.	+ 4·6 −15·3	62 30 12	....

TABLE I. JAN. 1ST, 1850.

Arc 2 (Angle at *Western* Star).	Annual Variation.	Arc 2 (Angle at *Eastern* Star).	Annual Variation.	Sign of Application for Arc 4.		No.
				Stars North of the Zenith.	Stars South of the Zenith.	
(*Alpheratz*) 85° 44′ 20″	+ 0″·2	(*Aldebaran*) 66° 9′ 41″	−19″·1	+	−	1
(*Schedar*) 73 28 24	+ 5·4	(*Capella*) 50 49 18	−28·4	+	−	2
(*β Ceti*) 88 54 19	− 3·3	(*Rigel*) 73 3 47	+19·8	−	+	3
(*Achernar*) 65 31 35	−14·8	(*Canopus*) 52 35 43	+32·8	−	+	4
(*Aldebaran*) 97 52 11	+20·7	(*Procyon*) 72 54 9	−19·5	+	−	5
(*Aldebaran*) 65 49 53	+19·0	(*Pollux*) 95 14 30	−20·2	+	−	6
(*Capella*) 37 23 36	+28·1	(*Dubhe*) 66 38 54	−11·6	+	−	7
(*Rigel*) 85 17 18	−19·7	(*Alphard*) 84 41 2	+13·0	−	+	8

EPOCH.

TABLE I.

No.	Stars' Names.	Mag.	Polar Distances.	Annual Variation.	Arc 1 (*Distance Arc*).	Annual Variation.
9	α Orionis (*Betelguese*) and	Var.	82° 37′ 33″ N.	− 1″·1	62° 27′ 20″	−0″·4
	α Leonis (*Regulus*)	1·2	77 18 7 N.	+17·4		
10	α Argûs (*Canopus*) and	1	37 23 4 S.	− 1·8	35 27 7	+0·1
	η Argûs	2	31 6 12 S.	−18·7		
11	α Argûs (*Canopus*) and	1	37 23 4 S.	− 1·8	45 4 32	−0·1
	α Crucis	1	27 44 1 S.	−19·9		
12	α Canis Majoris (*Sirius*) and	1	73 29 7 S.	− 4·6	96 10 21	+0·2
	α Virginis (*Spica*)	1	79 37 23 S.	−18·9		
13	β Geminorum (*Pollux*) and	1·2	61 36 59 N.	+ 8·2	87 26 2	+0·6
	α Boötis (*Arcturus*)	1	70 2 4 N.	+18·9		
14	α Leonis (*Regulus*) and	1·2	77 18 7 N.	+17·4	59 44 16	−0·8
	α Boötis (*Arcturus*)	1	70 2 4 N.	+18·9		
15	α Virginis (*Spica*) and	1	79 37 23 S.	−18·9	45 54 15	....
	α Scorpii (*Antares*)	1·2	63 54 21 S.	− 8·5		
16	α Boötis (*Arcturus*) and	1	70 2 4 N.	+18 9	59 1 38	+2·2
	α Lyræ (*Vega*)	1	51 21 11 N.	− 3·1		

TABLE I. JAN. 1ST. 1850.

Arc 2 (Angle at *Western* Star).	Annual Variation.	Arc 2 (Angle at (*Eastern* Star).	Annual Variation.	Sign of Application for Arc 4.		No.
				Stars North of the Zenith.	Stars South of the Zenith.	
(*Betelguese*) 79° 29′ 8″	+20″·2	(*Regulus*) 88° 13′ 53″	−10″·3	+	−	9
(*Canopus*) 53 36 7	−32·7	(*η Argûs*) 71 5 36	+13·3	−	+	10
(*Canopus*) 41 5 17	−32·7	(*α Crucis*) 59 2 6	− 3·3	−	+	11
(*Sirius*) 77 13 50	−21·0	(*Spica*) 71 54 44	− 8·1	−	+	12
(*Pollux*) 68 38 10	+22·7	(*Arcturus*) 60 39 34	+11·5	+	−	13
(*Regulus*) 74 6 41	+12·5	(*Arcturus*) 86 37 45	+10·4	+	−	14
(*Spica*) 63 33 49	+ 6·7	(*Antares*) 101 15 39	−20·3	−	+	15
(*Arcturus*) 56 9 29	−12·6	(*Vega*) 88 17 1	+26·5	+	−	16

D

EPOCH.

TABLE I.

No.	Stars' Names.	Mag.	Polar Distances.	Annual Variation.	Arc I (*Distance Arc*).	Annual Variation.
17	α Boötis (*Arcturus*) and α Aquilæ (*Altair*)	1 1·2	70° 2′ 4″ N. 81 31 26 N.	+18″·9 − 9·1	81° 12′ 2″	+1″·7
18	α Centauri and α Pavónis	1 2	29 47 22 S. 32 47 25 S.	−15·1 +11·0	41 34 57	+3·3
19	α Scorpii (*Antares*) and α Piscis Aust. (*Fomalhaut*)	1·2 1·2	63 54 21 S. 59 35 4 S.	− 8·5 +19·0	82 50 54	+0·2
20	α Lyræ (*Vega*) and α Andromedæ (*Alpheratz*)	1 2	51 21 11 N. 61 44 16 N.	− 3·1 −19·9	67 6 0	....
21	α Aquilæ (*Altair*) and α Pegasi (*Markab*)	1·2 2	81 31 26 N. 75 36 3 N.	− 9·1 −19·3	47 46 46	−0·6
22	α Pavonis and α Eridani (*Achernar*)	2 1	32 47 25 S. 32 0 0 S.	+11·0 +18·4	40 6 50	....
23	α Cygni (*Deneb*) and α Cassiopeiæ (*Schedar*)	2·1 Var.	45 15 12 N. 34 17 10 N.	−12·6 −19·8	37 59 16	....
24	α Pegasi (*Markab*) and α Tauri (*Aldebaran*)	2 1	75 36 3 N. 73 47 49 N.	−19·3 − 7·7	79 0 25	....

TABLE I. JAN. 1ST. 1850.

Arc 2 (Angle at *Western* Star).	Annual Variation.	Arc 2 (Angle at *Eastern* Star).	Annual Variation.	Sign of Application for Arc 4. Stars North of the Zenith.	Sign of Application for Arc 4. Stars South of the Zenith.	No.
(*Arcturus*) ° ′ ″ 84 7 10	″ −12·6	(*Altair*) ° ′ ″ 70 57 26	″ +20·2	+	−	17
(*α Centauri*) 54 29 42	+19·9	(*α Pavonis*) 48 18 50	−32·9	−	+	18
(*Antares*) 59 33 22	+19·9	(*Fomalhaut*) 63 52 19	− 7·2	−	+	19
(*Vega*) 71 18 52	−25·1	(*Alpheratz*) 57 8 16	− 0·4	+	−	20
(*Altair*) 78 12 44	−17·7	(*Markab*) 91 34 30	+ 5·1	+	−	21
(*α Pavonis*) 53 59 28	+30·9	(*Achernar*) 55 45 42	+14·8	−	+	22
(*Deneb*) 51 37 14	−21·7	(*Schedar*) 81 14 39	− 4·9	+	−	23
(*Markab*) 75 54 2	− 5·4	(*Aldebaran*) 78 1 57	−19·3	+	−	24

EPOCH. **TABLE II.** JAN. 1ST. 1850.

No.	Stars' Names.	Mag.	Arc 1.	Annual Variation.	Numbers A and B, and Logarithms C and D.	Annual Variation.	Sign of Application for Arc 3. Stars N. of Zenith.	Stars S. of Zenith.
1	α Androm. (W) (*Alpheratz*) and α Tauri (E) (*Aldebaran*)	2 1	61° 26′ 41″	−19″·7	A 58·93 B 593·04 C 8·049340 D 7·189878	− 0·003 + 0·156 +0·000002 −0·000104	+	−
2	α Cassiop. (W) (*Schedar*) and α Aurigæ (E) (*Capella*)	Var. 1	32 41 14	−17·6	A 86·83 B 274·65 C 8·162257 D 7·449933	− 0·004 + 0·103 +0·000004 −0·000124	+	−
3	β Ceti (W) and β Orionis (E) (*Rigel*)	2 1	71 9 30	+19·5	A 45·19 B 1593·76 C 8·037292 D 6·786114	+ 0·006 − 1·872 −0·000001 +0·000499	−	+
4	α Eridani (W) (*Achernar*) and α Argûs (E) (*Canopus*)	1 1	28 50 10	+12·5	A 93·69 B 168·00 C 8·183213 D 7·596288	+ 0·006 − 0·087 −0·000011 +0·000145	−	+
5	α Tauri (W) (*Aldebaran*) and α Can. Min. (W) (*Procyon*)	1 1	72 2 0	−15·8	A 35·00 B* 222·53 C 8·097267 D 7·770465	− 0·020 − 0·089 −0·000046 +0·000208	+	−
6	α Tauri (W) (*Aldebaran*) and β Geminor. (E) (*Pollux*)	1 1·2	61 10 34	+ 9·8	A 170·36 B* 20·44 C 7·912742 D 8·061638	− 0·034 − 0·017 +0·000101 −0·000048	+	−
7	α Aurigæ (W) (*Capella*) and α Ursæ Maj. (E) (*Dubhe*)	1 2	25 1 45	+15·1	A 123·71 B 33·00 C 8·018951 D 7·906315	− 0·009 + 0·014 +0·000023 −0·000040	+	−
8	β Orionis (W) (*Rigel*) and α Hydræ (E) (*Alphard*)	1 2	80 23 42	− 5·7	A 95·63 B 112·90 C 7·993258 D 7·739622	+ 0·065 − 0·206 −0·000135 +0·000431	−	+

No.	Stars' Names.	Mag.	Arc 1.	Annual Variation.	Numbers A and B, and Logarithms C and D.	Annual Variation.	Sign of Application for Arc 3. Stars N. of Zenith.	Stars S. of Zenith.
9	α Orionis (W) (*Betelguese*) and α Leonis (E) (*Regulus*)	Var. 1·2	77° 10′ 55″	+15″·5	A 171·26 B 16·64 C 7·814543 D 7·963772	− 0·071 + 0·036 +0·000165 −0·000084	+	−
10	α Argûs (W) (*Canopus*) and η Argûs (E)	1 2	29 15 21	−15·8	A 107·76 B 46·46 C 8·195959 D 7·852467	+ 0·005 − 0·025 −0·000015 +0·000070	−	+
11	α Argûs (W) (*Canopus*) and α Crucis (E)	1 1	23 31 5	−17·4	A 111·39 B 52·32 C 8·087742 D 7·848106	+ 0·005 − 0·015 −0·000013 +0·000040	−	+
12	α Can. Maj. (W) (*Sirius*) and α Virginis (E) (*Spica*)	1 1	69 14 1	−16·2	A 63·36 B 144·11 C 7·906554 D 7·778996	+ 0·027 − 0·048 −0·000057 +0·000102	−	+
13	β Gemin. (W) (*Pollux*) and α Boötis (E) (*Arcturus*)	1·2 1	55 1 8	+18·7	A 71·83 B 143·70 C 7·919072 D 7·747858	− 0·013 + 0·028 +0·000025 −0·000055	+	−
14	α Leonis (W) (*Regulus*) and α Boötis (E) (*Arcturus*)	1·2 1	69 45 49	+20·4	A 155·34 B 20·69 C 7·866733 D 7·951306	+ 0·019 − 0·013 −0·000046 +0·000032	+	−
15	α Virginis (W) (*Spica*) and α Scorpii (E) (*Antares*)	1 1·2	61 44 14	− 0·3	A 244·19 B* 40·05 C 7·723994 D 8·109827	− 0·101 − 0·017 +0·000216 −0·000037	−	+
16	α Boötis (W) (*Arcturus*) and α Lyræ (E) (*Vega*)	1 1	51 19 15	− 2·0	A 182·90 B 4·47 C 7·804290 D 7·989819	+ 0·050 − 0·020 −0·000118 +0·000046	+	−

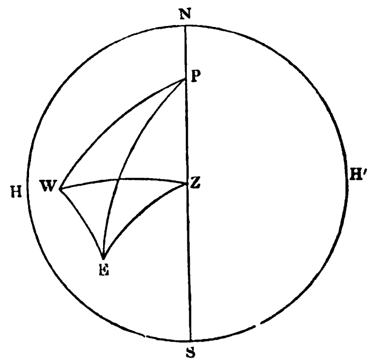
No.	Stars' Names.	Mag.	Arc 1.	Annual Variation.	Numbers A and B, and Logarithms C and D.	Annual Variation.	Sign of Application for Arc 3. Stars N. of Zenith.	Stars S. of Zenith.
17	α Boötis (W) (*Arcturus*) and α Aquilæ (E) (*Altair*)	1 1·2	69° 13′ 13″	+14″·7	A 43·17 B 335·12 C 7·988526 D 7·438797	+ 0·024 − 0·306 −0·000029 +0·000353	+	−
18	α Centauri (W) and α Pavonis (E)	1 2	23 51 22	− 5·4	A 96·86 B 124·83 C 8·155265 D 7·677019	− 0·007 + 0·066 +0·000005 −0·000127	−	+
19	α Scorpii (W) (*Antares*) and α Pis. Aust. (E) (*Fomalhaut*)	1·2 1·2	50 44 13	+ 9·4	A 115·10 B 83·46 C 7·845376 D 7·860134	− 0·028 + 0·026 +0·000060 −0·000057	−	+
20	α Lyræ (W) (*Vega*) and α Androm. (E) (*Alpheratz*)	1 2	47 43 5	−12·0	A 75·84 B 191·01 C 8·003297 D 7·606063	+ 0·010 − 0·076 −0·000017 +0·000126	+	−
21	α Aquilæ (W) (*Altair*) and α Pegasi (E) (*Markab*)	1·2 2	75 31 0	−19·9	A 168·71 B* 13·18 C 7·900817 D 8·037799	+ 0·011 + 0·006 −0·000030 +0·000017	+	−
22	α Pavonis (W) and α Eridani (E) (*Achernar*)	2 1	25 58 55	+19·2	A 100·88 B 93·64 C 8·161796 D 7·740138	− 0·002 + 0·015 +0·000005 −0·000034	−	+
23	α Cygni (W) (*Deneb*) and α Cassiop. (W) (*Schedar*)	2·1 Var.	33 49 55	−19·9	A 117·37 B 19·45 C 8·138904 D 7·935744	...... − 0·001 −0·000001 +0·000003	+	−
24	α Pegasi (W) (*Markab*) and α Tauri (E) (*Aldebaran*)	2 1	69 57 7	−17·2	A 112·21 B 84·39 C 7·868633 D 7·845822	− 0·026 + 0·029 +0·000059 −0·000065	+	−

Explanation of the Formulæ from which the preceding Rules and Tables are deduced.

Method the First and Table I.

Let N H S H′ (*Fig.* 1) be the horizon, N S the meridian, Z the zenith, and P the pole of the heavens.

Fig. 1.



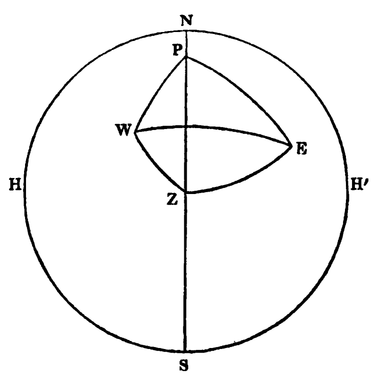
Let W and E be two fixed stars whose altitudes have been observed at the same instant.

Then Z W and Z E, their zenith distances, are known by observation, while P W and P E, their polar distances, as well as W P E, the polar angle, or difference of right ascension, are known from the "Nautical Almanac."

Hence, in the triangle W P E, knowing the angle W P E and the two sides P W and P E, the third side W E (the distance arc between the two stars), and also the angles P W E and P E W can be computed.

Again, in the triangle W Z E, knowing the three sides, we can find the angles Z W E and Z E W.

Fig. 2.



Then the difference or sum of the angles P W E and Z W E, or of P E W and Z E W (using always in practiee the angles at that star which is the farthest from the meridian), will give the angle P W Z or P E Z.

Their difference or sum being taken according to the position of the stars with reference to the pole and zenith, as shewn in *Figs.* 1 and 2.

Lastly, in the triangle P W Z, knowing two sides P W and Z W, and the included angle P W Z; or in the triangle P E Z, knowing P E and Z E, and the angle P E Z, the remaining side P Z, the colatitude of the observer, can be determined.

Now,

Let the Polar Distances P W and P E	$= p$ and p'
the Zenith Distances Z W and Z E	$= z$ and z'
the Distance Arc W E	$= \delta$
the Colatitude P Z	$= \lambda$
the Polar Angle W P E	$= \alpha$
the Angles P W E and P E W	$= \beta$ and β'
the Angles Z W E and Z E W	$= \gamma$ and γ'
the Angles P W Z and P E Z	$= \iota$ and ι'

Then, in the triangle W P E, for the determination of the distance arc δ, we have the formula

$$\left.\begin{aligned} \text{vers}\ \delta^{*} &= \text{vers}\,(p-p') + \text{N} \\ \text{where} \qquad \text{N} &= 2 \sin p \sin p' \sin^2 \frac{\alpha}{2} \end{aligned}\right\} \quad (1)$$

Again, to find the angles at the base of the triangle, we have the well-known formulæ

$$\left.\begin{aligned} \tan \tfrac{1}{2}(\beta+\beta') &= \frac{\cos \frac{1}{2}(p'-p)}{\cos \frac{1}{2}(p'+p)} \cdot \cot \frac{\alpha}{2} \\ \tan \tfrac{1}{2}(\beta-\beta') &= \frac{\sin \frac{1}{2}(p'-p)}{\sin \frac{1}{2}(p'+p)} \cdot \cot \frac{\alpha}{2} \\ \text{whence} \qquad \beta &= \tfrac{1}{2}(\beta+\beta') + \tfrac{1}{2}(\beta-\beta') \\ \beta' &= \tfrac{1}{2}(\beta+\beta') - \tfrac{1}{2}(\beta-\beta') \end{aligned}\right\} \quad (2)$$

These last formulæ are too well known to require any further explanation, as they will be found, in the form analogous to that in which they are given above, in all treatises on spherical trigonometry.†

From these formulæ (1) and (2), the arcs given in Table I. have been computed for certain couplets of stars lying in suitable positions for simultaneous observation.

* See note, p. 42.

† See Snowball's "Spherical Trigonometry," p. 17, and Jeans's "Trigonometry," Part ii. p. 65.

Let us now suppose that the remainder of the calculation is to be made on the presumption that the star W is the farthest from the meridian; then in the triangle W Z E we have

$$\text{haversine } \gamma^* = \frac{\sin \frac{1}{2}(z' + \overline{\delta - z}) \sin \frac{1}{2}(z' - \overline{\delta - z})}{\sin \delta \sin z} \qquad (3)$$

or

$$\sin \frac{\gamma}{2} = \sqrt{\frac{\sin \frac{1}{2}(z' + \overline{\delta - z}) \sin \frac{1}{2}(z' - \overline{\delta - z})}{\sin \delta \sin z}} \qquad (4)$$

The angle γ being known from either of the above formulæ, and β being found from Table I., we obtain

$$\iota = \beta \mp \gamma;$$

Where the upper sign refers to the case where the stars are in

* By the fundamental formula of spherical trigonometry,

$$\cos \gamma = \frac{\cos z' - \cos \delta \cos z}{\sin \delta \sin z}$$

$$\therefore \quad 1 - \cos \gamma = 1 - \frac{\cos z' - \cos \delta \cos z}{\sin \delta \sin z}$$

$$= \frac{\cos (\delta - z) - \cos z'}{\sin \delta \sin z}$$

or

$$\text{versine } \gamma = \frac{2 \sin \frac{1}{2}(z' + \overline{\delta - z}) \sin \frac{1}{2}(z' - \overline{\delta - z})}{\sin \delta \sin z}$$

$$\therefore \quad \left.\begin{matrix}\frac{1}{2}(\text{versine } \gamma) \\ \text{or} \\ \text{haversine } \gamma\end{matrix}\right\} = \frac{\sin \frac{1}{2}(z' + \overline{\delta - z}) \sin \frac{1}{2}(z' - \overline{\delta - z})}{\sin \delta \sin z}$$

or again

$$2 \sin^2 \frac{\gamma}{2} = \frac{2 \sin \frac{1}{2}(z' + \overline{\delta - z}) \sin \frac{1}{2}(z' - \overline{\delta - z})}{\sin \delta \sin z}$$

and

$$\sin \frac{\gamma}{2} = \sqrt{\frac{\sin \frac{1}{2}(z' + \overline{\delta - z}) \sin \frac{1}{2}(z' - \overline{\delta - z})}{\sin \delta \sin z}}$$

Tables of log haversines are given in Inman's collection of "Nautical Tables;" and the same table, under a different name, is given in Raper's "Practice of Navigation," as "Log Sine Square."

The equation is $$1 - \cos x = 2 \sin^2 \frac{x}{2}$$

$$\therefore \quad \frac{1 - \cos x}{2} \text{ or } \tfrac{1}{2} \text{ versine } x = \sin^2 \frac{x}{2}$$

Mr. Raper derives his name from the right-hand side of the equation, and Dr. Inman from the left, contracting ½ Versine into *Haversine.*

the relative positions as to the pole and zenith shewn in *Fig.* 1, and the lower one to that represented in *Fig.* 2.

Lastly, in the triangle P W Z, to find the colatitude λ, we have

$$\left.\begin{array}{ll} & \text{vers } \lambda^* = \text{vers } (p-z) + \text{vers } \theta \\ \text{where} & \text{haversine } \theta = \text{hav } \iota \sin p \sin z \end{array}\right\} (5)$$

$$\left.\begin{array}{ll} \text{or} & \text{vers } \lambda = \text{vers } (p-z) + \text{N} \\ \text{where} & \text{N} = 2 \sin p \sin z \sin^2 \frac{\iota}{2} \end{array}\right\} (6)$$

In a similar manner γ' ι' and λ might be found, in case the star E were the farthest from the meridian.

Equations (3) and (5) give the general rule for the computation of the latitude stated in page 10 and equations (4) and (6); the modification of it given in page 14.

In these rules Arc 3 is the angle γ, and Arc 4 the angle ι, or $(\beta \mp \gamma)$.

* In the triangle P W Z we have, by the fundamental formula,

$$\cos \iota = \frac{\cos \lambda - \cos p \cos z}{\sin p \sin z}$$

$$\therefore 1 - \cos \iota = 1 - \frac{\cos \lambda - \cos p \cos z}{\sin p \sin z}$$

$$= \frac{\cos (p-z) - \cos \lambda}{\sin p \sin z}$$

$$\frac{1-\cos \iota}{2} \text{ or haversine } \iota = \frac{\cos (p-z) - \cos \lambda}{2 \sin p \sin z}$$

Hence $\cos \lambda = \cos (p-z) - 2 \sin p \sin z \text{ hav } \iota$

$$1 - \cos \lambda = 1 - \cos (p-z) + 2 \sin p \sin z \text{ hav } \iota$$

and $\text{vers } \lambda = \text{vers } (p-z) + \text{vers } \theta$

If $\text{hav } \theta = \sin p \sin z \text{ hav } \iota$

or again, $1 - \cos \iota \text{ or } 2 \sin^2 \frac{\iota}{2} = \frac{\cos (p-z) - \cos \lambda}{\sin p \sin z}$

$$\therefore \cos \lambda = \cos (p-z) - 2 \sin p \sin z \sin^2 \frac{\iota}{2}$$

$$1 - \cos \lambda = 1 - \cos (p-z) + 2 \sin p \sin z \sin^2 \frac{\iota}{2}$$

and $\text{vers } \lambda = \text{vers } (p-z) + \text{N}$

Where $\text{N} = 2 \sin p \sin z \sin^2 \frac{\iota}{2}$

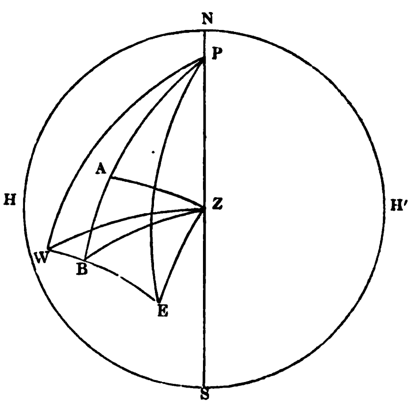
By proceeding in a similar manner in the triangle W P E, it may be shewn that

$$\left.\begin{array}{ll} & \text{vers } \delta = \text{vers } (p-p') + \text{N} \\ \text{where} & \text{N} = 2 \sin p \sin p' \sin^2 \frac{\alpha}{2} \end{array}\right\} \text{See Eq. (1), p. 40.}$$

Explanation of the Formulæ of Method the Second and Table II.

In the annexed figures, representing, as before, the relative positions, as to the pole and zenith, of two stars observed simultaneously, conceive P B, the arc of a great circle, to be drawn from the pole, perpendicular to the distance arc W E, and cutting it in B.

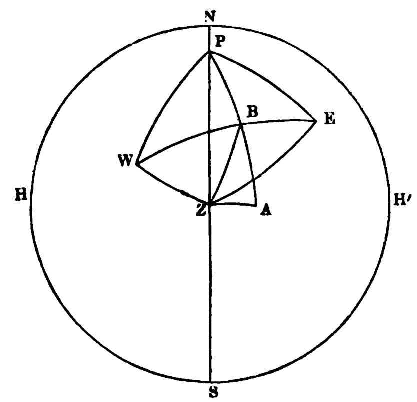
Fig. 3.



Let Z A also be the arc of another great circle drawn from the zenith, perpendicular to the Arc P B, *Fig.* 3, or to P B produced, *Fig.* 4, and cutting it in A. Also let Z B be the zenith distance of the point B.

Retaining in other respects the same notation as before, let also the altitude of the *western* star $=$W, and that of the *eastern* star $=$E.

Fig. 4.



Then, in the triangle W P E, the base δ and the base angles β and β', are known by formulæ (1) and (2).

Also in the right-angled triangles P B W and P B E,

$$\tan BW = \cos \beta \tan p \qquad (7)$$
$$\tan BE = \cos \beta' \tan p' \qquad (8)$$

whence B W and B E are known,

and check .... $BW + BE = WE$ or ∂

Again,

$$\text{check} \ldots \left\{ \begin{array}{l} \sin PB = \sin \beta \sin p \\ \sin PB = \sin \beta' \sin p' \end{array} \right\} (9)$$

From both of which equations P B becomes known.

Then, in the triangle Z B W,

$$\cos ZBW = \frac{\cos ZW - \cos ZB \cos BW}{\sin ZB \sin BW}$$

and in the triangle Z B E,

$$\text{or} \quad \left.\begin{array}{r} \cos ZBE \\ -\cos ZBW \end{array}\right\} = \frac{\cos ZE - \cos ZB \cos BE}{\sin ZB \sin BE}$$

whence,

$$\cos ZW = \cos ZB \cos BW + \sin ZB \sin BW \cos ZBW$$
$$\cos ZE = \cos ZB \cos BE - \sin ZB \sin BE \cos ZBW$$

Instead of cos Z B W substitute its equal sin Z B A.

Also let $\cos BE = A \cos BW$

and $\sin BE = B \sin BW$

That is, let A and B be determined by the equations

$$A = \frac{\cos BE}{\cos BW}$$

$$B = \frac{\sin BE}{\sin BW}$$

remembering, also, that in the right-angled triangle Z A B

$$\sin ZB \sin ZBA = \sin ZA$$

and making these changes in the above equations, we obtain

$$\cos ZW = \cos ZB \cos BW + \sin BW \sin ZA$$
$$\cos ZE = A \cos ZB \cos BW - B \sin BW \sin ZA$$

Taking the sum and difference of these two equations, and putting W and E, the true altitudes of the stars, in place of their zenith distances, and we have—

$$\sin W + \sin E = (1 + A) \cos ZB \cos BW + (1 - B) \sin BW \sin ZA$$
$$\sin W - \sin E = (1 - A) \cos ZB \cos BW + (1 + B) \sin BW \sin ZA$$

from which we get

$$\cos ZB = \frac{\sin W + \sin E}{(1 + A) \cos BW} - \frac{(1 - B) \sin BW \sin ZA}{(1 + A) \cos BW}$$

and $$\cos ZB = \frac{\sin W - \sin E}{(1-A)\cos BW} - \frac{(1+B)\sin BW \sin ZA}{(1-A)\cos BW}$$

Equating these values of cos Z B, and reducing

$$\left\{\frac{(1+B)}{(1-A)} - \frac{(1-B)}{(1+A)}\right\} \sin BW \sin ZA = \frac{\sin W - \sin E}{(1-A)} - \frac{\sin W + \sin E}{(1+A)}$$

$$\therefore (A+B)\sin BW \sin ZA = A \sin W - \sin E$$

and $$\sin ZA = \frac{A \sin W - \sin E}{(A+B)\sin BW} \quad (10)^*$$

whence Z A may be found.

Again, to find B A.

Equating the values of cos Z B W from the triangles Z B W and Z B E, and expressing as above the functions of the arc B E in terms of the arc B W, we have—

$$\frac{\cos ZW - \cos ZB \cos BW}{\sin ZB \sin BW} = -\frac{\cos ZE - A \cos ZB \cos BW}{B \sin ZB \sin BW}$$

Reducing this equation,

$$A \cos ZB \cos BW + B \cos ZB \cos BW = B \cos ZW + \cos ZE$$

$$\therefore \cos ZB = \frac{B \sin W + \sin E}{(A+B)\cos BW}$$

But in the right-angled triangle Z A B,

$$\cos ZB = \cos BA \cos ZA$$

$$\therefore \cos BA = \frac{B \sin W + \sin E}{(A+B)\cos BW} \cdot \text{séc. } ZA \quad (11)^*$$

* It will be observed that equations (10) and (11) are perfectly general, and include the particular case where the polar distances of the two bodies are equal, as in the case of a "double altitude" of the sun, when the declination is supposed to remain invariable during the period of the observation, which is the assumption in Mr. Ivory's solution.

For if the triangle W P E become isosceles, the perpendicular P B bisects the base; consequently B W = B E; $\therefore$ A = 1 and B = 1.

Hence equation (10) becomes,

$$\sin ZA = \frac{\sin W - \sin E}{2 \sin BW}$$

or, $$\sin ZA = \frac{\cos \frac{1}{2}(W+E) \sin \frac{1}{2}(W-E)}{\sin BW} \quad (15)$$

Also, equation (11) becomes,

$$\cos BA = \frac{\sin W + \sin E}{2 \cos BW} \cdot \text{sec. } ZA$$

or, $$\cos BA = \frac{\sin \frac{1}{2}(W+E) \cos \frac{1}{2}(W-E)}{\cos BW} \cdot \text{sec. } ZA \quad (16)$$

which equations are the same as those given in Mr. Riddle's modification of Ivory's formulæ. (See "Philosophical Magazine" for 1822, p. 167, and Riddle's "Navigation," p. 292.)

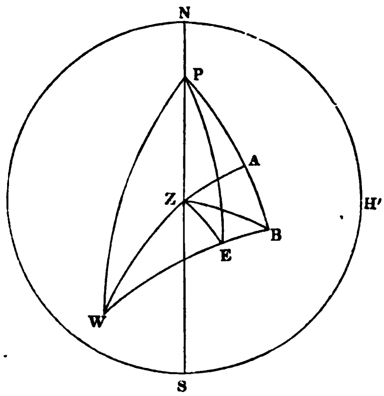
Then $$PB \mp BA = PA$$

The upper sign referring to the relative positions of the stars, as to the pole and zenith, represented in Fig. 3, and the lower to that shewn in Fig. 4.

Lastly, in the right-angled triangle P A Z,

$$\cos PZ \text{ or } \sin \text{Lat} = \cos PA \cos ZA \qquad (12)$$

Fig. 5.



Sometimes the perpendicular, from the pole P B, will fall without the triangle W P E on W E produced, as in Fig. 5, instead of within the triangle W P E, as in Figs. 3 and 4.

In this case it will be found, proceeding in a manner precisely similar to that already pursued, that

$$\sin ZA = \frac{A \sin W - \sin E}{(A-B) \sin BW} \qquad (13)$$

and

$$\cos BA = \frac{B \sin W - \sin E}{(B-A) \cos BW} \cdot \sec. ZA \qquad (14)$$

In order to reduce equations (10) and (13), (11) and (14), to a form more convenient for calculation, let N be determined by the equation,

$$N = \frac{\sin E}{\sin W}$$

$$\therefore \sin E = N \sin W$$

Hence, by substitution, equations (10) and (13) become

$$\sin ZA = \frac{(A-N) \sin W}{(A \pm B) \sin BW}$$

$$= C\,(A-N) \sin W$$

if

$$C = \frac{1}{(A \pm B) \sin BW}$$

and again, equations (11) and (14) become—

$$\cos BA = \frac{(B \pm N) \sin W}{(A \pm B) \cos BW} \cdot \sec. ZA$$

$$= D\,(B \pm N) \sin W. \sec. ZA$$

if $$D = \frac{1}{(A \pm B) \cos BW}$$

In order to verify the correctness of the computation of the Logarithms of C and D,

Since $$C = \frac{1}{(A \pm B) \sin BW}$$

$$\therefore \quad C = \frac{1}{\left(\frac{\cos BE}{\cos BW} \pm \frac{\sin BE}{\sin BW}\right) \sin BW}$$

$$= \frac{1}{\frac{\cos BE \sin BW \pm \sin BE \cos BW}{\cos BW}}$$

$$= \frac{\cos BW}{\sin (BW \pm BE)}$$

$$= \frac{\cos BW}{\sin \delta}$$

and therefore

$$\sin \delta = \frac{\cos BW}{C}$$

In a similar manner it will appear that

$$\sin \delta = \frac{\sin BW}{D}$$

These independent determinations of the value of the distance arc will afford a convenient check on the accuracy of the computations.

In actual practice, in order to avoid the use of quantities in the computations, which are wholly fractional, it is more convenient to make

$$A = \frac{100 \cos BE}{\cos BW}, \quad B = \frac{100 \sin BE}{\sin BW}, \quad N = \frac{100 \sin E}{\sin W},$$

and this has been done in the construction of the rule and table.

Recapitulating the formulæ for the sake of clearness, we have as follows:—

(1.) *Formulæ of Construction of Table II.*

The distance arc δ, and the angles β and β' in the triangle W P E, having been computed by equations (1) and (2),—

$$\tan BW = \cos \beta \tan p \qquad \tan BE = \cos \beta' \tan p'$$

check .... $BW \pm BE = WE$ or δ

check .... $\left\{\begin{array}{l}\sin PB = \sin \beta \sin p \\ \sin PB = \sin \beta' \sin p'\end{array}\right\}$

$$A = \frac{100 \cos BE}{\cos BW} \qquad B = \frac{100 \sin BE}{\sin BW}$$

$$C = \frac{1}{(A \pm B) \sin BW} \qquad D = \frac{1}{(A \pm B) \cos BW}$$

check .... $\left\{\begin{array}{l}\sin \delta = \dfrac{\cos BW}{100\, C} \\ \sin \delta = \dfrac{\sin BW}{100\, D}\end{array}\right.$

In the table the arc P B is denominated Arc 1.

(2.) *Formulæ of Computation of Method the Second.*

$$N = \frac{100 \sin E}{\sin W}$$

$$\sin ZA = C\,(A - N) \sin W$$

$$\cos BA = D\,(B \pm N) \sin W \sec. ZA$$

$$PA = PB \mp BA$$

The difference, or sum, being taken according to the position of the stars relative to the pole and zenith, as shewn in Figs. 3 and 4.

Lastly, $\cos PZ$ or $\sin Lat = \cos PA \cos ZA$.

In the rule the arc B A is called Arc 2, and P A, Arc 3; also, in the arrangement of Table II. the number B is marked with an asterisk (*) in those cases in which the perpendicular P B falling without the triangle W P E, on W E produced, as in Fig. 5, the difference of B and N is to be taken instead of their sum.

THE END.

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